ABSTRACT

Title of Dissertation: QUANTIFYING DYNAMIC PITCH ADJUSTMENT DECISION STRUCTURES IN STRING QUARTET PERFORMANCE

Nicholas John Tavani III, Doctor of Musical Arts, 2021

Dissertation directed by: Professor David Salness, Strings Department

What does it mean to have a unique group sound? Is such a thing quantifiable? If so, are there noticeable differences between groups, and any correlations to the time each group spends together? It is important to note a caveat right off the bat: music is generally understood to be created by and listened to by humans, and thus any attempts at quantifiable answers to the above questions will be, at best, orthogonal to its main purpose. It is also clear from anecdotes and interviews with professional musicians that qualitatively distinguishable characteristics of group sound and interpretation absolutely do exist and are noticeable to the listener. Paul Katz, cellist of the Cleveland Quartet, describes the multiple layers of such a group identity: “When one spends that many hours per day and years together, there is a meshing of taste, an unspoken unification of musical values, an intuitive understanding of each other's timings and shapings, and even a merging of how one produces sounds, makes a bow change, or varies vibrato, that is deeper than words or

conscious decision making."[1] This dissertation concerns itself with the general question of whether or not it is possible to detect and define, in a quantifiable sense, the patterns and elements of a unique group sound identity, specifically in the intonation domain. Original research was carried out, consisting of recording four string quartets with high-quality equipment under controlled conditions, to begin to answer this question.

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[1] Janof, Tim. 10. "Conversation with Paul Katz." *cello.org.* 10 05. Accessed 08 25, 2020. http://cello.org/Newsletter/Articles/katz/katz.htm.

QUANTIFYING DYNAMIC PITCH ADJUSTMENT DECISION STRUCTURES IN STRING QUARTET PERFORMANCE

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
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Advisory Committee:
Professor David Salness, Chair
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Dedication

To my Dad, my Mom, and my wife Alexandra, who never gave up on me.

Acknowledgements

A huge acknowledgement and heartfelt thank you is due to my Professor, David Salness. Without your invaluable insights about string quartet intonation and unfailing personal and professional support of me through the years, I would never have completed this thesis.

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Chapter 1: Introduction: Group Sound Identity

“Spending Years Together” – Professional String Quartets as the Forge of Group Identity

In 1808, Prince Razumovsky commissioned Ignaz Schuppanzigh to form ‘the finest string quartet in Europe.’[2] The resultant group was provided with a salary to rehearse string quartets together regularly, specifically the quartets of Beethoven, whose middle period quartets demanded far more virtuosity and endurance from all four players than music by previous composers. Usually considered the first professional string quartet, the Schuppanzigh quartet started a trend of professional foursomes playing string quartets together full time, a trend which has continued up until the present day. This produced a phenomenon somewhat unprecedented in Western music history: the idea that the same four people would commit to rehearsing music together on a consistent basis with the purpose of presenting a performance unified in interpretational intent. Newspaper reviews, correspondence, and interviews from the early nineteenth century onward speak about quartets as having a unified group identity based in sound and interpretation; a whole greater than the sum of its parts. [3] In addition, the concept of group identity in the form of

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[2] Gingerich, John M. 2010. "Ignaz Schuppanzigh and Beethoven's Late Quartets." *The Musical Quarterly* 450-513.

[3] Ibid.

transactive memory systems has been confirmed to exist in small collaborative groups of multiple types.[4]

This brings us to the initial generative questions of this dissertation: What does it mean to have a unique group sound? Is such a thing quantifiable? If so, are there noticeable differences between groups, and any correlations to the time each group spends together? It is important to note a caveat right off the bat: music is generally understood to be created by and listened to humans, and thus any attempts at quantifiable answers to the above questions will be, at best, orthogonal to its main purpose. It is also clear from anecdotes and interviews with professional musicians that qualitatively distinguishable characteristics of group sound and interpretation absolutely do exist and are noticeable to the listener. Paul Katz, cellist of the Cleveland Quartet, describes the multiple layers of such a group identity: “When one spends that many hours per day and years together, there is a meshing of taste, an unspoken unification of musical values, an intuitive understanding of each other's timings and shapings, and even a merging of how one produces sounds, makes a bow change, or varies vibrato, that is deeper than words or conscious decision making.”[5] This dissertation concerns itself with the general question of whether or not it is possible to detect and define, in a quantifiable sense, the patterns and elements of a unique group sound identity, specifically in the intonation domain. Original research

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[4] Moreland, Richard L. and Larissa Myaskovsky. 2000. "Exploring the Performance Benefits of Group Training: Transactive Memory or Improved Communication?" *Organizational Behavior and Human Decision Processes* 117-133.

[5] Janof, Tim. 10. "Conversation with Paul Katz." *cello.org.* 10 05. Accessed 08 25, 2020. http://cello.org/Newsletter/Articles/katz/katz.htm.

was carried out, consisting of recording four string quartets with high-quality equipment under controlled conditions, to begin to answer this question.

Training for Good String Quartet Intonation

The problem of achieving good intonation in string playing is one which typically occupies a significant fraction of a string player's formative training. Such problems are only multiplied when four players join together in a string quartet, even four players of high quality. Professional string quartets spend years perfecting their group intonation, a goal which is achieved not only by rote memorization of pitch locations, but extremely quick dynamic adjustment of pitch based on auditory and physical feedback.[6] Audiation, or hearing a desired pitch (or set of pitches) in one's head before playing, also plays an essential role. In this way the quartet members might even be said to adjust their intonation "before it happens." Such fine-grained adjustments may be seen in slow-motion videos of violinists like those taken of Jascha Heifetz.[7] Because of the numerous uncertainties and personal decisions inherent to the biological mechanism of playing a string instrument, perfect playing via the mechanics of finger action is unrealistic. Therefore, a fine string quartet member must always be listening to both his own intonation, intonation of other group members, and group intonation, and adjusting his pitch and sound quality to optimize the group sound. Though this process is never finished, it is generally

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[6] Blum, David. 1986. *The Art of String Quartet Playing.* Ithaca: Cornell University Press.

[7] 1951. *Of Men and Music.* Film, directed by Alexander Hammid and Irving Reis. Performed by Jascha Heifetz.

accepted that experienced string quartets are more efficient at bringing intonation to a high level quickly in rehearsal and achieve higher standards of intonation in performance than do inexperienced or “pick-up” Quartets.[8] It is therefore reasonable to assume that much of this rehearsal time is spent building up both a group consensus on ideal intonation, and a mental model of ideal conditional adjustment parameters to achieve this intonation, a model which can be both explicitly discussed and implicitly learned in rehearsal. In fact, this type of specific intonation tendency model is mentioned in “The Art of Quartet Playing,” the seminal series of interviews with the Guarneri Quartet, one of the premiere quartets of the 20th century.[9] The existence of such a model for established string quartets in the time adjustment domain (adjusting note onsets to one another to maintain synchronicity) has already been experimentally established by Timmers, Endo, and Wing.[10] The hypothesis of this dissertation is that, in the pitch domain, a “group identity” might consist of both general principles of intonation and many complex conditional decision trees which, once internalized, greatly speed up the process of playing in tune as a group. This experiment will attempt to detect the existence of such a model and quantify it as far as possible.

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[8] Janof, Tim. 10. "Conversation with Paul Katz." *cello.org.* 10 05. Accessed 08 25, 2020. http://cello.org/Newsletter/Articles/katz/katz.htm.

[9] Blum, David, *The Art*

[10] Wing, Alan M., Satoshi Endo, Adrian Bradbury, and Dick Vorberg. 2014. "Optimal feedback correction in string quartet synchronization." *Journal of the Royal Society Interface.*

Chapter 2: Questions To Be Answered By This Experiment

While a large body of research and writing on the topic of the string quartet in general is extant, there exists little to no quantitative research on the precise analysis and mechanisms of string quartet pitch adjustment patterns. Recent work by Papiotis, Marchini, Perez-Carrillo, and Maestre explores several possible approaches to analyzing interdependence in several dimensions of performance (including intonation) between string quartet members and uses an experimental approach somewhat similar to this experiment.[11] When the term "interdependence" is used, in the Papoitis article and in this document, it means the amount of influence one voice has on another. This influence can be measured in multiple domains, but we concern ourselves with the dynamics of pitch – e.g., how players adjust their pitch in response to other players. While Papiotis et al come to several useful conclusions about which analytical techniques are appropriate, their approach results in a single interdependence value for the entire quartet over an entire playing excerpt, essentially an average of all possible interdependence values between members and across time. This dissertation research attempts to go further in detail about the precise structure and strength of adjustment between players on an inter-note and intra-note scale. We investigate and attempt to quantitatively and qualitatively define the dynamic decision-making process by which a string quartet adjusts individual pitches, in a real

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[11] Papiotis, Panagiotis, Marco Marchini, and Esteban Maestre Gómez. 2012. "Computational analysis of solo versus ensemble performance in string quartets: intonation and dynamics." *Proceedings of the 12th International Conference on Music Perception and Cognition and the 8th Triennial Conference of the European Society for the Cognitive Sciences of Music; 2012 July 23-28; Thessaloniki, Greece.* 7.

time situation, to achieve a group pitch arrangement which is considered optimal; that is, the dynamic process of playing in tune. This process naturally breaks down into three levels of analysis: Vertical Intonation, Horizontal Intonation at Inter-Note Level, and Horizontal Intonation at Intra-Note Level. These levels will be further explained and precisely defined in subsequent sections, but for now will serve as useful initial categories for the primary specific questions asked by this experiment:

1. Vertical Intonation

Is there a single "optimal intonation" system which remains consistent when measured across variables including years of musical training, years of chamber music experience, and, most importantly, years of time working with the same members of a string quartet? If not, does each group tested maintain an optimal intonation system unique to the group? Is general vertical intonation more consistent for more experienced Quartets?

2. Horizontal Intonation, Inter-note Level

Is pitch information at a note level (i.e., taking the average pitch of a note as a single entity) predictive for future note values in certain intonation problem classes? In other words, do string quartet players play notes in an intonation system which is based on and adjusts to prior notes performed, either their own or another player?

3. Horizontal Intonation, Intra-Note Level

Within the boundaries of a single note, is there a consistent pattern of directional influence which emerges in the analysis of pitch contours? If so, does the group adjust to a single “leader,” does each group member adjust equally, or is there some other decisional model? Are these decisional models more defined and consistent for more experienced Quartets across different instances of the same intonation problem class?

Chapter 3: Methodology

Materials

1. 4 clip on condenser microphones

   These microphones were designed for use on string instruments, and consisted of a clip, a flexible gooseneck, and a small condenser microphone capsule with a supercardioid sound field (which means they were designed to reject off-axis sound from other quartet instruments and only pick up sound from the instrument to which they were attached). They were clipped to the strings of the instruments behind the bridge, and each microphone capsule was aimed at the sounding point (where the bow meets the strings) approximately two inches above the instrument.

2. Audio receiver/amplifier (Behringer UMC 404HD USB audio interface)

   This is a low-noise interface which amplified and transferred the sound signal to the computer.

3. Computer with Mixcraft multi-track audio-recording software

   This software allowed all four signals to be recorded with identical timestamps to allow for analysis.

4. Pitch detection software – Praat and Tony

   This software is discussed in detail later in the dissertation, but consists of a visual interface which allows the user to interact with the individual sound files and extract pitch contours (series of time-pitch pairs) based on the Praat and p-yin algorithms.

5. Matlab and Excel software, for data analysis and display

Test Subjects

Test subjects were taken from three different types of string quartets chosen to give the largest possible variety of chamber music experience. The three volunteer quartets were:

a. Two professional string quartets which have played together for >5 years, both self-estimating rehearsal time at 4-5 hours a day, 5-6 days a week
b. A graduate resident quartet at a large university, consisting of experienced chamber musicians with comparable training to string quartet members, but who had played together for >1 year
c. Students still in conservatory with no sustained (<1 year) string quartet experience

Students were volunteers drawn from these institutions or personal acquaintance of the author. During recording and analysis, data was anonymized. Volunteers were not told beforehand about the nature of the experiment so as not to introduce conscious bias into their pitch adjustment strategies. Specific instructions were given verbally and on the score pages.

Procedure

Subjects were recorded on separate days in a room with sound-dampening measures employed. The live volunteers had clip-on condenser microphones attached to their instruments. They were asked to sit in standard quartet formation and to play the music on their stands without vibrato. Each quartet began with a sequence of chords designed to create a baseline for vertical intonation preferences. They then sight-read a number of Bach chorales which presented each group with several different intonational challenges: the first time slowly, without vibrato, the second time quickly, without vibrato, and the third time quickly, with vibrato. The choice was made to let each quartet decide for themselves the exact tempo to play for “slow” and “fast,” rather than dictate metronome markings – the reasoning being that the comparisons between groups would be more equal if each group performed at tempos comfortable for themselves. The Bach chorales played are given here with their BWV numbers, R numbers (according to the popular Reimenschneider collection of 371 chorale harmonizations), and keys.

BWV 256, R31, in A minor
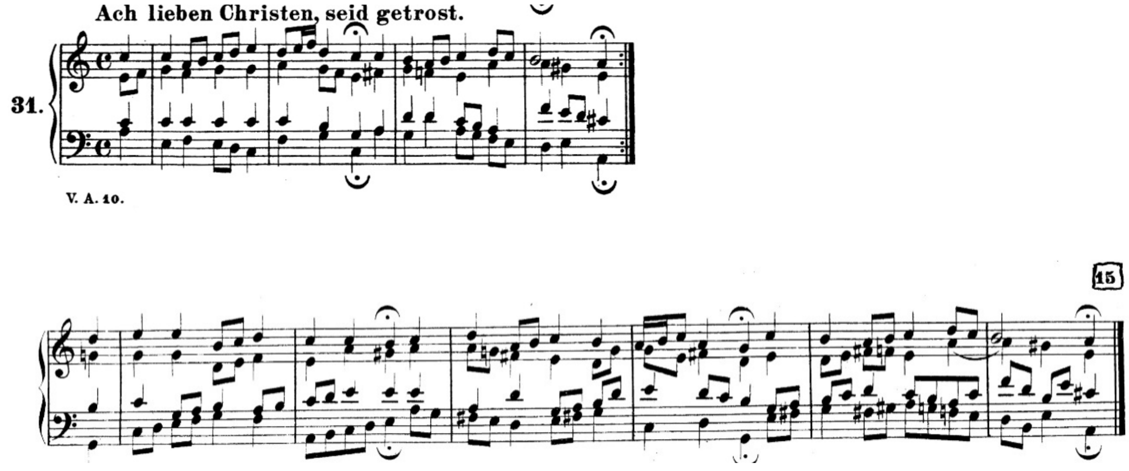
Figure 1 - BWV 256, R31, in A minor

BWV 281, R6, in F Major
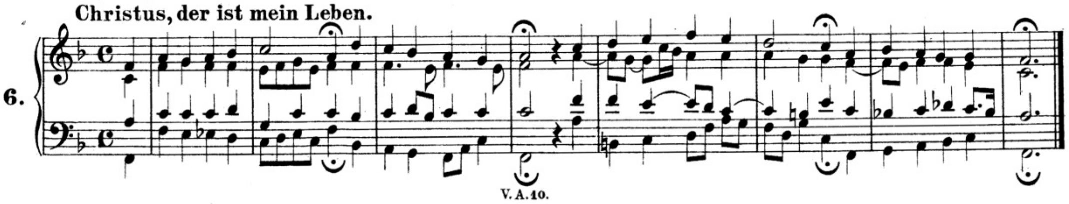
Figure 2 - BWV 281, R6, in F Major

BWV 278, R371, in E minor
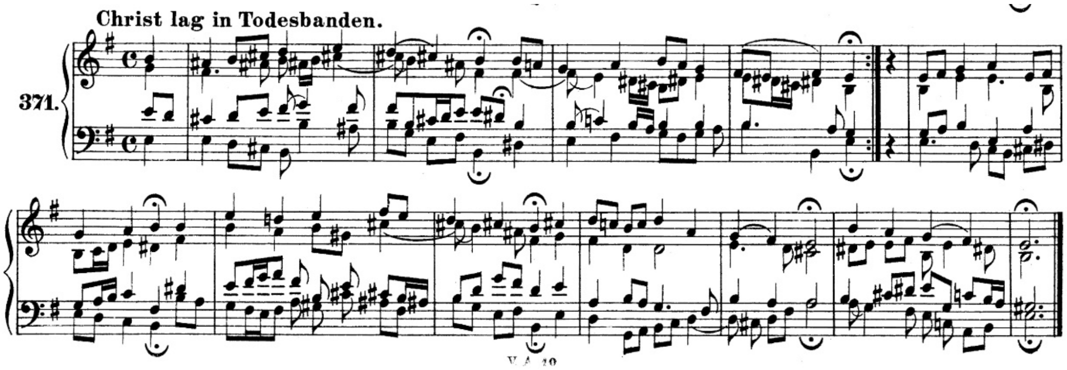
Figure 3 - BWV 278, R371, in E minor

BWV 267a, R5, in G Major
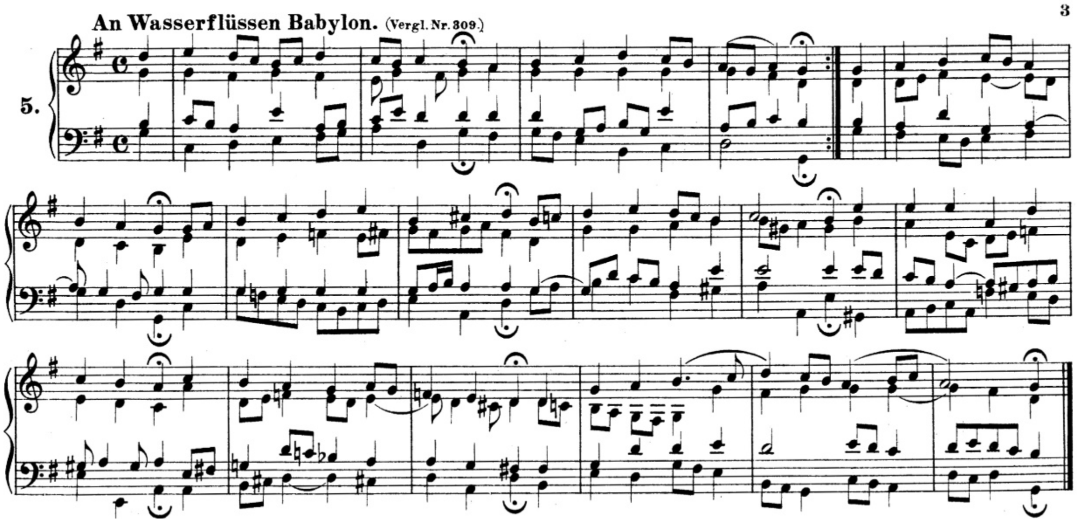
Figure 4 - BWV 267a, R5, in G Major

BWV 267b, R309, in Ab Major
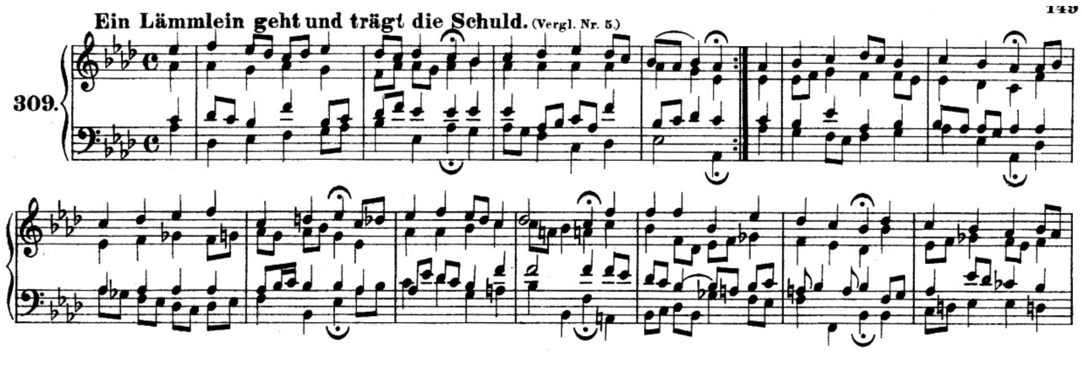
Figure 5 - BWV 267b, R309, in Ab Major

BWV 86, R4, in E Major

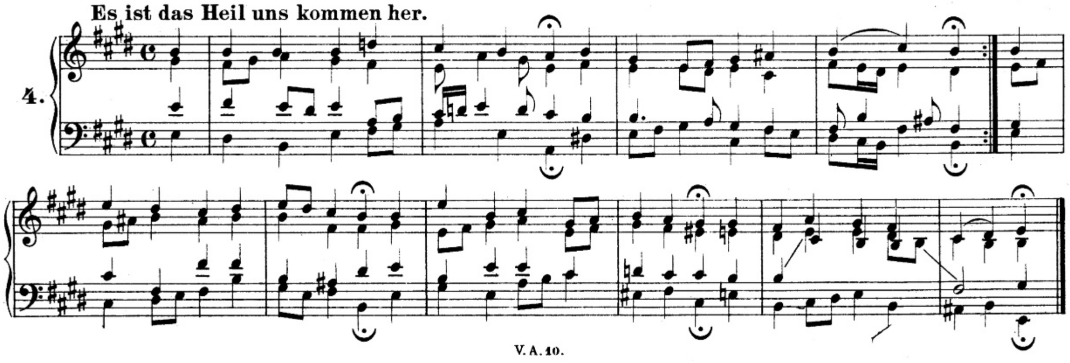

Figure 6 - BWV 86, R4, in E Major

In addition, each group was asked to play a homogeneous chorale excerpt from a piece they had rehearsed and performed together. In the end, only the reference chords and selected slow non-vibrato chorale performances were used for analysis.

Data was recorded to 4 mono wav files, at a 48 kHz sample rate and 32 bit depth for high resolution. As a unified time code is essential to the success of this experiment, the four channels were all routed through a single multi-channel recorder with a unified start time, and contained a sharp noise at the beginning for additional synchronization redundancy. A control for the experiment was established by having each musician play the individual lines of selected chorales without their colleagues present.

After the experiment, the musicians were asked to fill out a short informational survey asking about their number of years of training, chamber music experience. Data was processed in Praat and Tony software as a time series, and exported into Matlab and Excel for further analysis. Settings for the Praat pitch detection algorithm were customized according to the frequency range for each part. Even with the scripting and automated pitch-windowing capabilities of both programs, this was an extremely time-consuming process, as each individual voice required manual inspection and correction of the pitch track. This was a result of the pitch detection software occasionally categorizing the pitch in the wrong octave or picking a different overtone of the fundamental (usually the fifth). In those cases, the software was manually coded to search for the pitch value in a range which corresponded with ~50 cents above or below note in the score. The software still detected the actual pitch contours of the note within those defined boundaries. Pitch values were exported as a table in the form of [t, P_1, P_2, P_3, P_4], with pitch in Hz.

Chapter 4: Data Analysis

Data Processing and Framing

Once the fundamental pitch data has been imported as a time series, the entire wealth of academic research on time series analysis becomes available. However, proper framing of the data is imperative before any analytical methods can be applied. An initial transformation of the data was performed. The initial pitch values, measured in Hz (vibrations per second) were converted to a cents basis via the following equation:

$$P_c = 1200(\log_2(\frac{P_{Hz}}{65.41}))$$

Here, P_c is pitch in cents, measured as an interval against 65.41 Hz (a C2, which is the open C string on the cello, the lowest note in the string quartet) and P_{Hz} is pitch in Hz, estimated by the Praat/Tony algorithms. This allows the logarithmic nature of pitch measured in Hz (meaning that every octave, the Hz value doubles) to be converted to a linear measure, cents. One cent = 1/100 of a half-step, or semi-tone.

A. Defining Beginnings and Endings of Notes

How does one determine where a note begins or ends? Which parts of the note should we measure to determine the pitch of that note? Even to a human listener, these questions can have ambiguous and fuzzy answers. For a quantitative analysis, we need to have consistent mathematical criteria for determining which part of the note we will measure. Auditory and visual inspection of the recorded performances reveal that each note can be divided up into four sections: an ‘unvoiced’ unpitched initial articulation, an initial transient period of rapid adjustment, a period of stability, and an ending transient which is hypothesized to be due to physical finger and bow motion in preparation for the following note or silence.

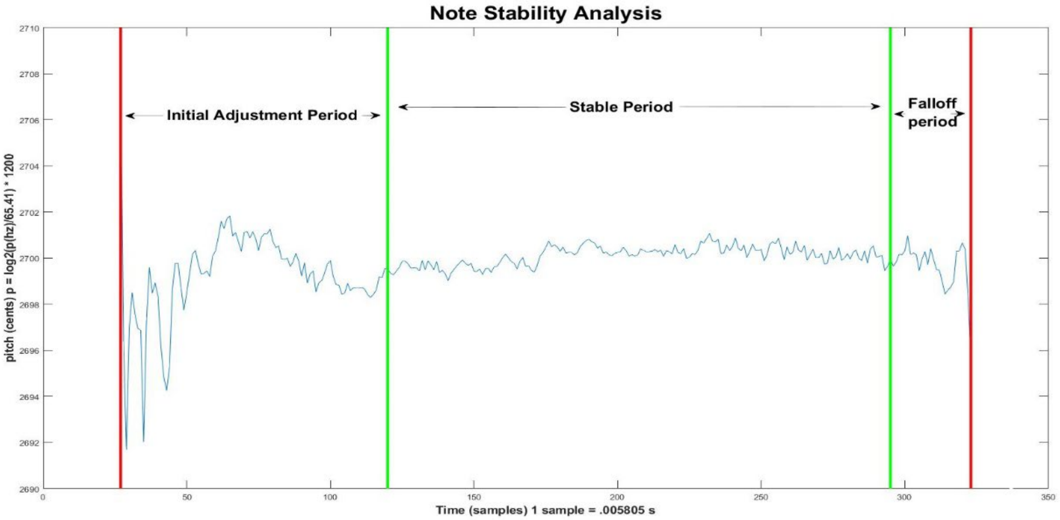

Figure 7 – Note Stability Analysis
The stable period of each note was estimated by taking the standard deviation of a .05 s sliding window, looped through each note's time series from both beginning and end. The beginning and end of the stable period were marked at the outer boundary of the time where 5 consecutive windows registered a standard deviation of < 5 cents.

The initial beginning boundary of each note was determined by either the first continuous (>0.1s) period of pitched sound detected by Praat or Tony, or the first discontinuous (>75 cents over a period of <2 samples) jump in pitch. Endings were determined by unpitched sound after a note, silence, or the aforementioned discontinuous jump in pitch (usually indicating the change of a note under a slur).

B. Stability Analysis

This experiment uses two primary methods of pitch analysis which require different sections of the note to be excluded from the data. For both, the ending transient must be excluded, as it is not assumed to contain intentional adjustment dynamics but is the result of physical left-hand preparation for the following note, or pitch falloff due to bow speed change or release (similar to the well-documented pitch falloff in struck/plucked string instruments like the piano and guitar.) For inter-note analysis, which takes notes to be single auditory entities with possible predictive capabilities, only the stable period is included, with both the initial and ending transients excluded. A single note pitch value is then taken as the average (mean) of this stable period. While visual inspection has been used in past studies, and the use of the mean of the stable period time series drastically reduces the standard error of the mean of the note pitch value (because of the high value of N, number of samples per note; typically more than 100), this algorithmic approach was used for consistencies' sake and to eliminate researcher bias in visual judgement calls. The stable period of each note was estimated by taking the standard deviation of a .05 s sliding window, looped through each note's time series from both beginning and end. The beginning and end of the stable period were marked at the outer boundary of the time where 5 consecutive windows registered a standard deviation of < 5 cents. Visual and auditory inspection confirmed the usefulness of this approach. Once the stable period boundaries were determined, a slope analysis (simple linear regression) was performed, and any notes which exhibited an absolute slope of >10 cents were excluded. This reflects the relatively obvious auditory fact that notes which are constantly rising or falling cannot be said to truly have a single pitch value. After windowing, the mean of the remaining pitch contour was taken to obtain the note pitch value. Since the analysis required all simultaneous pitch structures to be considered separately, there was also the special case of one or more notes being sustained while other notes changed. In this case, boundaries for the stable period of the stationary note were determined by the latest (for beginning) and earliest (for end) stable period of the notes which moved.

For intra-note analysis, which analyzes the dynamics of a single note's pitch track using non-linear methods, a slightly more involved approach was needed. Some initial transient outlier values produced wild initial swings in the pitch tracks of certain notes which did not match an auditory analysis; therefore, any values 100 cents greater than the mean of the stable period were excluded and replaced by linearly interpolated values using Matlab. Other than these outliers, the initial pitched transients were considered to contain real intentional adjustment and so included in the dataset for each note; the ending transients were excluded.

Three Levels of Analysis

When considering any complex dynamical system, there is a tension between analyzing it from the top down or the bottom up; or, equivalently, between qualitative judgements (which audience members and performers make easily and constantly whenever one listens to a musical performance) and quantitative mechanistic analysis. The analysis of even one performance dimension (pitch) is a complex task, and it is not expected that a universal analytic solution will be found, or is even possible to be found, given that we are dealing with the fuzzy and porous boundary between the technical and aesthetic. That being said, through various statistics, this thesis has attempted to capture the mathematical "shadows" cast by what is sometimes referred to as the "fifth member" - the unique quartet identity forged from the melding of musical and technical idiosyncrasies in the forge of rehearsal. In *Gödel Escher Bach,* Douglas Hofstader refers to an ‘epiphenomenon’ – an observable large-scale eventuality which exhibits properties different in quality and greater in quantity than the mere linear sum of its disparate small parts.[12] This thesis attempts to approach the “identity” of pitch adjustment dynamics through the lens of epiphenomena and so analysis was performed on three previously mentioned “meta-levels” of epiphenomena, each of which give some insight into the true nature of the quartet adjustment paradigm.

Although every quartet’s rehearsal process is different, interviews with multiple leading quartets describe intonation rehearsal processes which are fairly consistent in their techniques — recurring motifs include playing slowly, playing in pairs, and choosing players to be “ground truth.”[13] Methods of listening and adjustment include picking a player to listen to, future audiation (imagining what the next note or chord will sound like before it happens), and adjusting by “feel” (allowing the vibration of the string and sympathetic vibrations of the other instruments to influence pitch via micromovements). Combining these top-down insights with analysis of note and interval patterns results in three main level of analysis, in descending order of scale:

1. Vertical Intonation - Overall interval statistics

2. Horizontal Intonation - Inter-note adjustment paradigms

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[12] Hofstadter, Douglas. 1979. *Gödel, Escher, Bach: an Eternal Golden Braid.* Basic Books.

[13] Janof, Tim. *Interview with Paul Katz.*

3. Horizontal Intonation - Intra-note influence pairings via nonlinear coupling coefficient L

Chapter 5: Results

Level 1: Vertical Intonation – Overall Interval Statistics

This meta-level of analysis considers only “vertical” note structures at the inter-note level – that is, it takes each note to be a single perceptual unit (approximated by the mean of the stable period of the pitch track, as described above) and analyzes the interval pairings which result from “simultaneous” (occurring at the same rhythmic point in the score) rhythmic structures in the score. Thus, Level 1 analyses largely ignore the horizontal, dynamic (time based) aspect, and look only at vertical intonation in the aggregate.

A. Reference Chord Tunings

First, the basic triadic and seventh chords tuning recordings were analyzed to find the average interval values that each group considers normative. Quartets were explicitly instructed to hold chords, one at a time, and adjust until all members felt the chord was acceptably in tune.

As this thesis deals with Western harmonic structures, and tritones, seconds, and major sevenths are unstable (there are many multiple possible ratios in integer-ratio derived systems - see Appendix; Just and Pythagorean Intonation), the intervals played were octaves, thirds (major and minor), fourths, fifths, sixths (major and minor), and minor sevenths. Every group had perfect just octaves, fourths, and fifths assigned to their reference interval profile; thirds, sixths, and sevenths were unique to each quartet. All intervals were reduced to their modulo 1200 equivalent: i.e., any interval which was greater than an octave was reduced to the remainder of a division by 1200. For example, a minor third and a minor tenth were considered the same interval, a perfect fourth and a perfect eleventh the same, and so on.

It should be noted that interval dyad pairings were extracted from all possible interval pairings among the four voices. For the purposes of this thesis, each interval dyad class is treated as a singular fundamental unit in analysis, regardless of voicing or context. It is certainly possible that the voicing and placement of an interval has an influence on its tuning; however, those variables were not considered in this analysis. Every non-perfect interval has multiple possible ratios, even within a single intonation system, but these multiple values are probably most likely to be measured in the minor third and its inverse, the major sixth. This is because the minor third occupies a unique spot in the space of traditionally "consonant" intervals: the m3 as a naturally occurring vibration of a fixed string is far up in the overtone series (the nineteenth harmonic) and therefore hard to hear naturally, the m3 as a notated interval occurs often in Western harmonic structures, and the m3 as an interval class can take on widely different multiple integer ratios. Some of the most common are the Just Intonation 6:5 m3 (315 cents), the Pythagorean 32:27 m3 (294 cents, or two octaves minus three fifths), or the septimal 7:6 m3 (267 cents, or the distance between the fifth and the septimal seventh in a 4:5:6:7 dominant seventh chord). In the extreme of a quartet playing distinguishably different interval values for the same class depending on the interval's position in the chord or scale degree relative to the tonic, this would mean that that specific interval value in the reference interval lookup table would be essentially a linear blend of the multiple values, proportional to their occurrence in the reference chords, and thus the introduced error in the models based on the reference table would also be linear. It also means that stacking these averaged intervals to produce composite multi-pitch average "composite chords" is not meaningful in this context; again, the ground assumption of the analysis is that interval dyad is the fundamental unit, and the interval averages are taken as "ground truth" for the subsequent analysis.

Quartets are labeled as follows:

Quartet A: This group had 8+ years as a professional quartet at the time of the experiment. They convened for 4-6 hours of rehearsal 5-6 days a week, and had training as individuals and as a group at several top 5 US music conservatories.

Quartet B: This group had 5+ years as professional quartet, 4-6 hours of rehearsal 5-6 days a week, and trained as individuals and as a group at several top 5 US music conservatories.

Quartet C: This group had 1.5 years as a semi-professional graduate quartet in residence at a major state university and convened for 3 hours of rehearsal 4 days a week. Their individual training was at a graduate level, and varied between conservatory and university music schools.

Quartet D: This group had ~1 month as student quartet and convened for 2 hours of rehearsal 3 days a week. They all trained as individuals at a top 5 US music conservatory.

Differences were seen among quartets for the mean values of major and minor thirds and sixths:

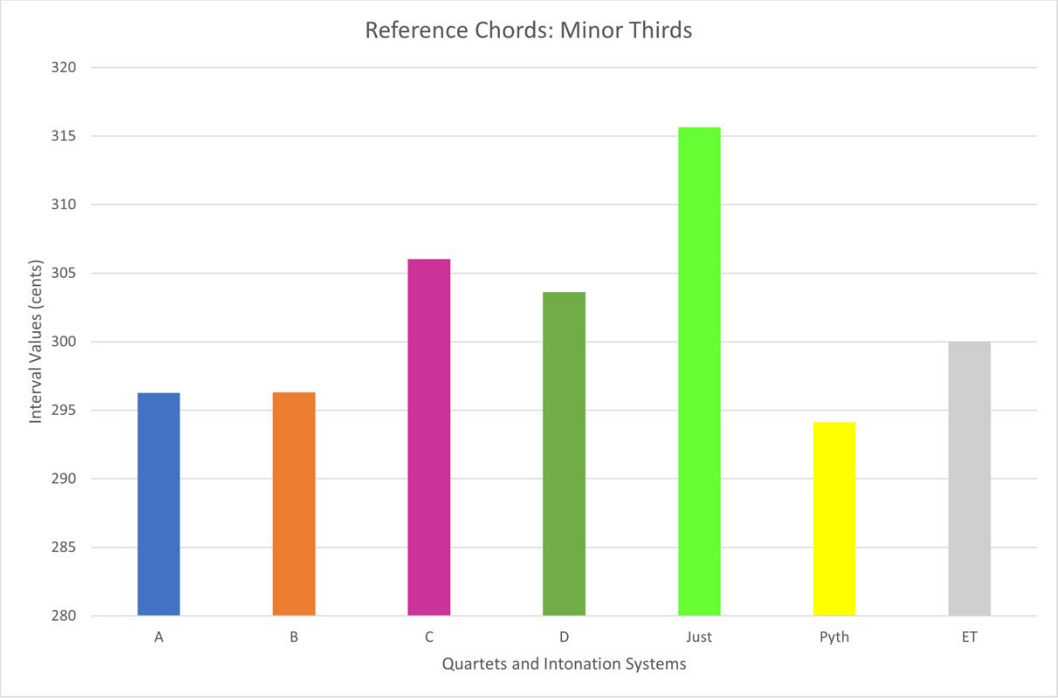

Figure 8 – Reference Chords: Minor Thirds
The letter labels represent the mean of each group's value of a minor third, measured as the average of every minor third equivalent which occurred in the reference chord tunings. "Just," "Pyth," and "ET" refer to the minor third widths as determined by 5 limit Just Intonation, 3 limit JI (Pythagorean), and Equal Temperament systems. This graph shows that Quartets A and B, the two professional groups, had values closer to the small Pythagorean minor third, while Quartets C and D were somewhat in between the neutral ET third and the wide JI third.

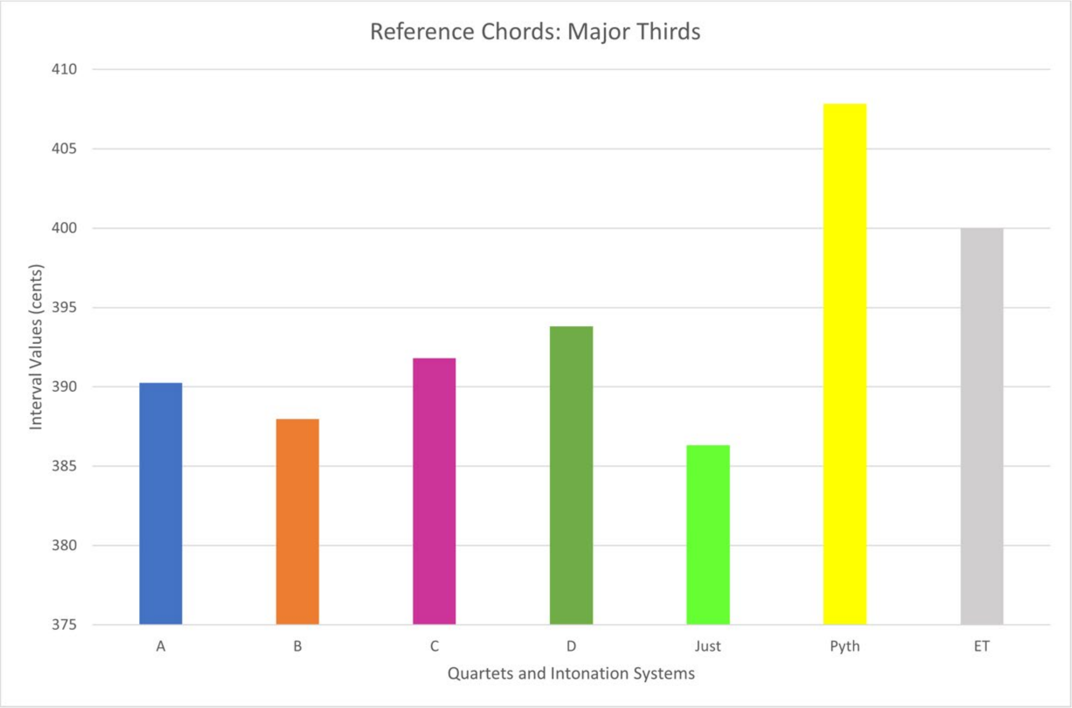

Figure 9 – Reference Chords: Major Thirds

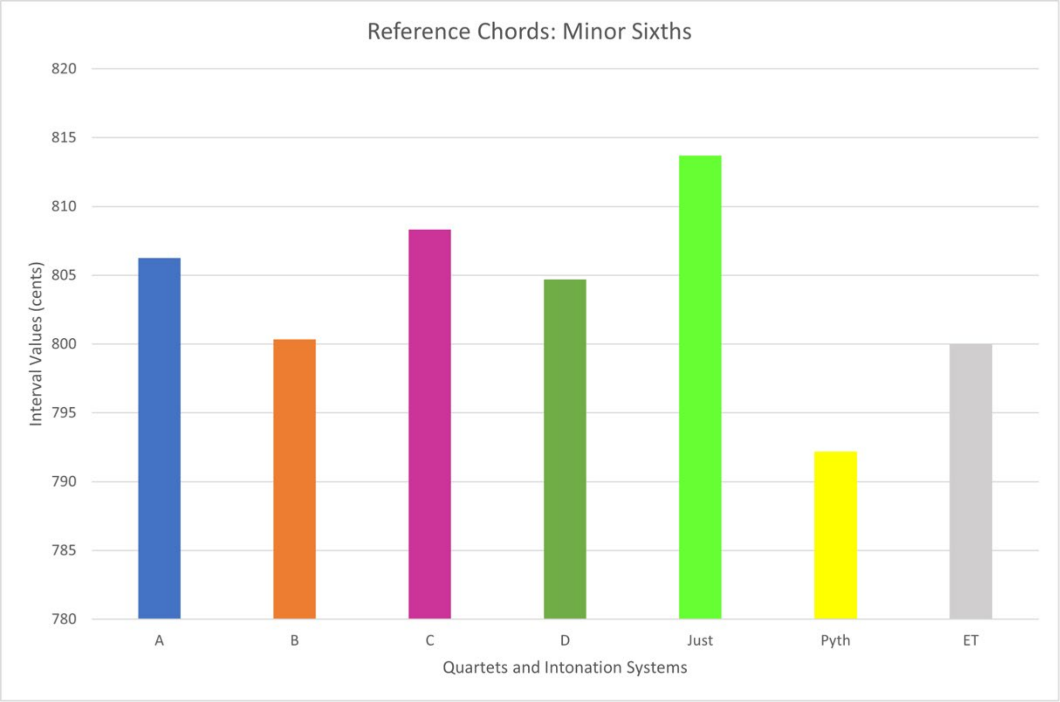

Figure 10 – Reference Chords: Minor Sixths

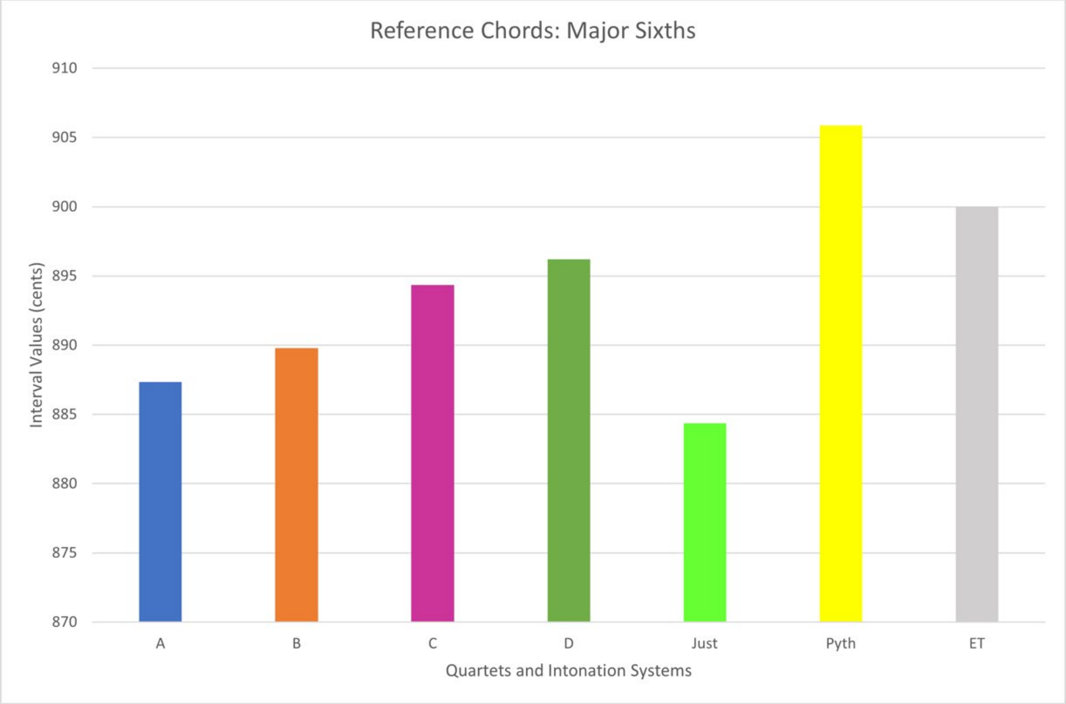

Figure 11 – Reference Chords: Major Sixths

In general, for major thirds and sixths, the more experienced Quartets A and B tended to be closer to the larger intervals of Just Intonation (JI). C (the 1 month pickup group) and D (the semi-pro group) were roughly between JI and Equal Temperament (ET), with C slightly lower for both. For minor thirds, Quartets A and B were quite close to the small Pythagorean m3, and C and D were somewhere in between the medium minor third of ET and the large minor third of Just. This was somewhat surprising, and perhaps the first indication of a unique group intonation identity that transcended a single standard intonation system, as both Quartets A and B had indicated a preference for a small Just major third. That Quartets C and D had m3 values close to ET (ie, between extremes) is perhaps also an indication of a wider spread of values due to the inherently greater variation of the minor third mentioned above. For minor sixths, A, C, and D were between ET and Just, and B was almost exactly at the ET value of 800 cents.

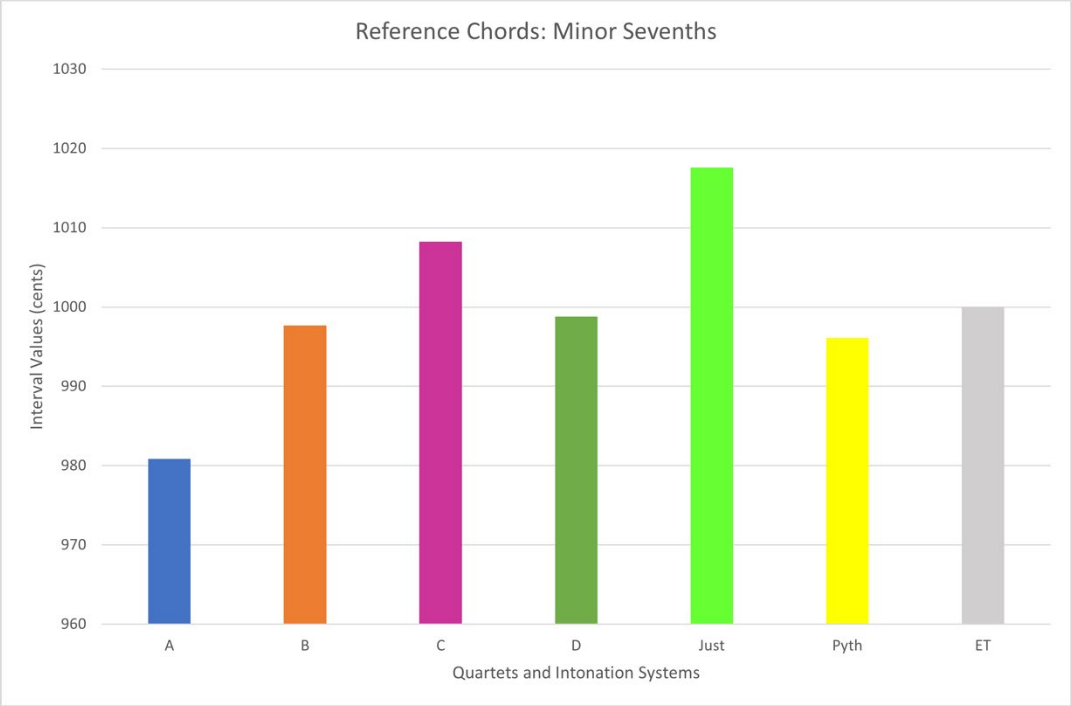

Figure 12 – Reference Chords: Minor Sevenths

Minor sevenths provided the widest variation – A was lower than any system at near 980 cents, B and D were close to the Pyth / ET neutral m7, and C was between ET and JI. It should be noted that there is a another “harmonic” or “septimal” minor 7th with a relatively simple integer ratio – 7:4, or ~969 cents, to which A was closest. From this data, an “interval reference table” for each group was constructed.

A. Variation from Group Interval Reference Chords

Now we move to analyzing the two chorales at the meta-level of overall interval statistics. For convenience, the string quartet voices will always be referred to as follows: Cello: B (Bass), Viola: T (Tenor), Violin 2: A (Alto), Violin 1: S (Soprano). In a string quartet, six non-ordered interval pairings are possible: BT, BA, BS, TA, TS, and AS. For each quartet, each interval pairing of a certain type was subtracted from its corresponding value in the interval reference value lookup table to produce a series of difference values I_d. The absolute value of the difference was then taken. We first examine group mean I_d, the average of all interval differences across all interval types mentioned above:

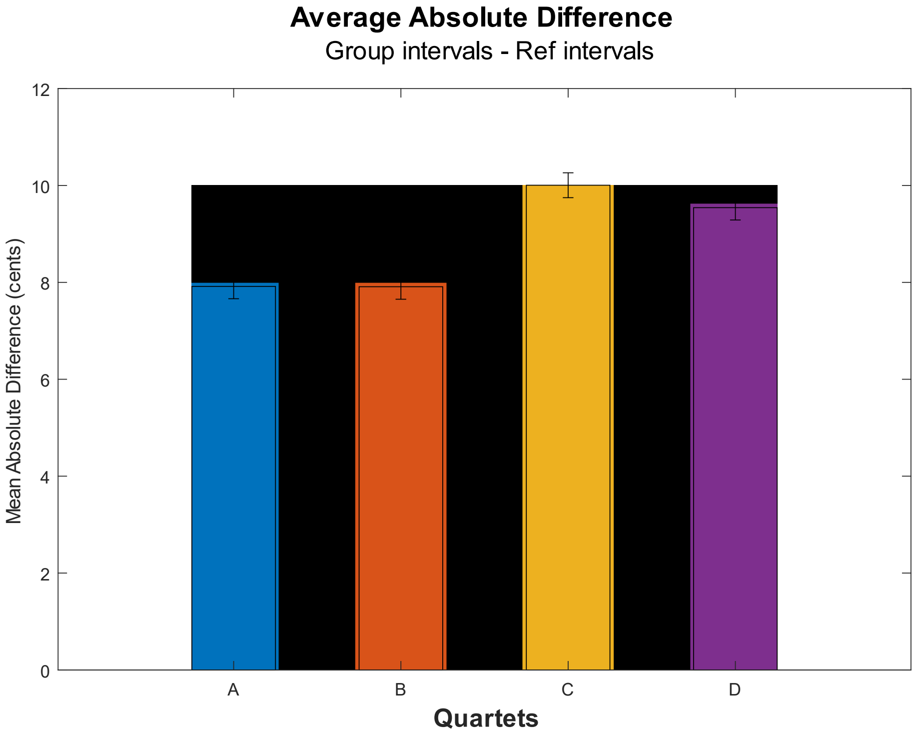

Figure 13 – Average Absolute Difference I_d
This is the average of the difference between every octave, fourth, fifth, third (major and minor), sixth, and minor seventh in the sight-read chorale performances vs reference chord tunings. A lower bar means the quartet tended

to hew more closely to its "ideal" interval tuning as measured in the reference chords. Error bars represent a 95% confidence interval.

Quartet Pairings	**Lower Bound** (95% CI)	**Difference**	**Upper Bound** (95% CI)	**p value**
AB	-0.9247	0.0067	0.9381	1.0000
AC	-3.0168	-2.0881	-1.1595	0.0000
AD	-2.5496	-1.6253	-0.7010	0.0000
BC	-3.0306	-2.0948	-1.1590	0.0000
BD	-2.5634	-1.6320	-0.7006	0.0000
CD	-0.4658	0.4628	1.3915	0.5755

Figure 14 – Multiple Comparison Table
Shows all possible pairings between group means and which were significantly different from one another at the $p < .05$ level based on an ANOVA. Quartets A and B were significantly different from C and D, but AB and CD were not significantly different from one another.

Through the lens of overall average (taking the mean of all interval class differences for all pairs together), the more experienced Quartets A and B hew closer to their "standard" intervals as measured by reference chords than groups C and D. But, as will quickly become a theme in this thesis, the structure of a group's pitch adjustment identity is often obscured by such a simplifying statistic as an overall mean. Looking at interval pairings tells a more subtle story:

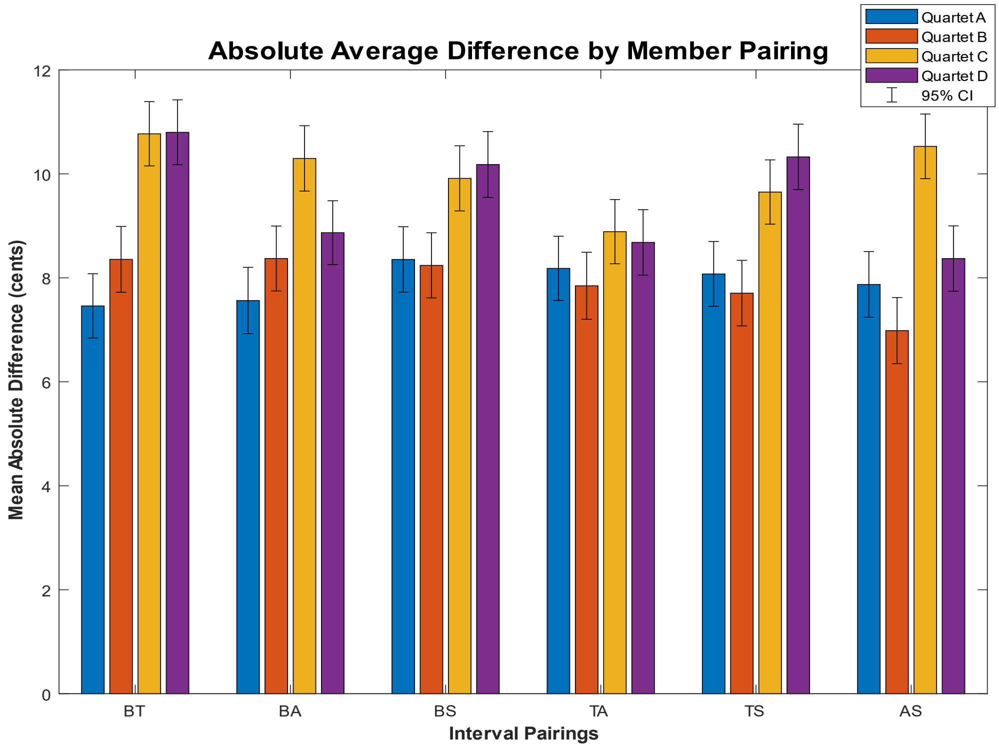

Figure 15 – Average Absolute Difference by Member Pairing

Each cluster of four bars represents one of the six possible member pairings. Quartets are identified by color. A lower bar means the quartet pairing tended to hew more closely to its "ideal" interval tuning as measured in the reference chords.

Here we see that the general trend seen above in I_d still holds, with some nuances. A large contributor of Group C's large absolute difference comes from the BT (Cello/Viola) and AS (Violin 2/Violin 1) pairings. The A and B quartets continue their trend of playing intervals more accurately (to their respective reference interval values) on average than C and D in all 6 pairings, although in the Viola/Violin 2 pairing, no differences between groups are significant at the $p > .05$ level. The strict Quartet AB/CD divide (meaning both Quartets A and B were more accurate than both Quartets C and D) was significantly different in two out of three pairings with Violin 1, suggesting perhaps that Violin 1 is more of an intonational anchor in the professional groups than in the student/semiprofessional groups. It is also interesting to note the group’s consistency of accuracy across their own pairings. The interval errors for professional Groups A and B are all statistically indistinguishable from each other – meaning their errors are spread evenly among pairings – while Groups C and D clearly favor certain pairings to the detriment of others. This could imply a higher level of optimization of the base vertical/horizontal intonation problem – to distribute the tensions inherent in string quartet intonation (see Appendix 1) in an even and aesthetically pleasing way. It is important to note again that these differences are in comparison to each group’s reference chords. The meaning behind a group’s increased discrepancies with their own reference chords is murkier to extract but suggest that the group is simply less consistent with the way they play intervals due to a less well-formed group intonational identity.

Averages - even across all possible pairings - do not tell the entire story: to a listener, consistency of vertical intonation across time could also be influential in the perception of a cohesive group identity. The distribution of interval errors by group shows a clear pattern:

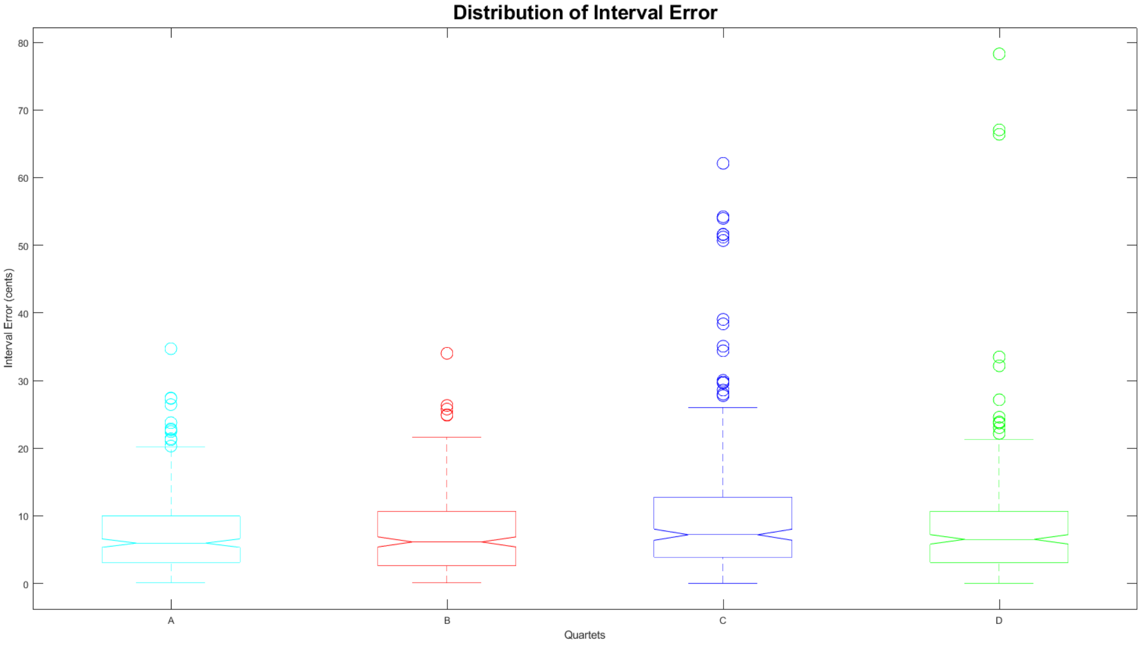

Figure 16 -Distribution of Absolute Interval Error

Line is median, boxes outline middle two quartiles of data for each, and circles represent outliers.

Quartets A and B show much tighter clustering of errors around their means, with no outliers greater than 40 cents, while C and D both have significant outliers and wider clustering. What this means from a qualitative "group sound" perspective, and what is immediately apparent upon listening to the experiment recordings, is that the professional quartets tend to play all consonant intervals more consistently similar, with fewer extreme outliers. It is reasonable to assume (and certainly appears to this researcher) that such outliers are fair to include in the data because they met the stability criteria and contribute greatly to the overall qualitative impression of a group sound/intonation identity.

B. Consistency of interval accuracy across separate chorales

The final statistic we examine in level 1 is consistency of interval accuracy across the two chorales R5 and R309. The reason for picking these specific chorales for this statistic is simple – they are identical in every way except key. R5 is in G Major, while R309 is in Ab Major. Thus, it is possible that accuracy in R5 is partially influenced by fact of shared open-string references. This means simply that when a group plays something in a key whose diatonic scale includes a note which matches one of the five open strings in a quartet (C, G, D, A, or E), those open strings function as an absolute reference point consistent across all players (assuming open strings are in tune with one another). This statistic asks the question: what is the difference $\Delta\ \varepsilon$ between each interval pairing error for each chorale? Or, more simply, how consistent were the quartets in their pattern of interval accuracy, across two causally separated events (two separate performances of two chorales?)

This statistic is calculated by finding the absolute difference between means of each interval pairing difference for each chorale:

$$\Delta\ \varepsilon = \left| \mu_{e1} - \mu_{e2} \right|$$

Where μ_{e1} and μ_{e2} are the respective interval differences of chorales R5 and R309.

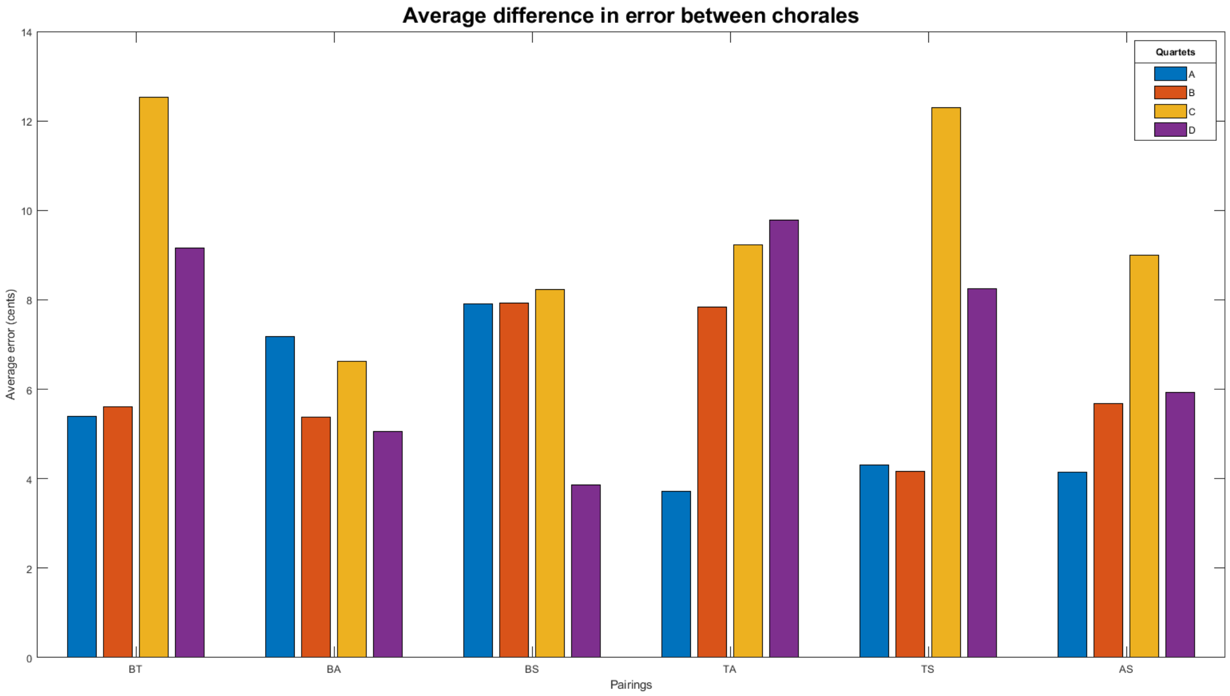

Figure 17 – Average difference in error between chorales

We see a similar pattern here, though with more variability: Quartets A and B exhibit more consistency in interval accuracy in the BT, TA, TS, and AS pairings.

For the Level 1 meta-analyses of mean interval error, it appears that the more experienced quartets A and B exhibit a lower interval error across almost all pairings, and also exhibit a more consistent interval error profile between different chorales.

Level 2: Inter-note prediction and adjustment matrix

Here we leave the realm of vertical intonation and consider the role of time and predictive audiation in quartet pitch adjustment profiles, though still at the inter-note level. Some hypothetical thought about possible mechanisms is here warranted (thought this thesis is primarily inductive in nature, some deduction about what is possible from an aural and physical perspective is necessary, as it will inform the construction of a predictive model).

Mausch, Frieler, and Dixon performed an experiment analyzing pitch drift and pitch memory in solo singers and created a time-based algorithm that could predict future notes.[14] They found that a linear model which relied on partial memory of the *implied root pitch* immediately prior to each note, combined with a partial sensitivity to immediately previous "error" (deviation from implied root pitch), could predict future notes for the singer with statistical significance. Essentially, Mausch et al are stating that a singer (or musician) uses past pitch information to measure future pitches. An aside on implied root pitch: this means essentially that rather than considering each note as an unrelated absolute entity from which to measure an interval, musicians tend to hear pitches (in a tonal music) as part of a system relative to a "root" pitch – the tonic of whatever key they are playing in. Root pitch is here calculated in the same way as Mausch. Given a note G_i of pitch G in cents where i is the index value of the note's interval above the root value R and I_i is the cents value of the reference interval:

$$R = G_i - I_i$$

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[14] Mauch, Matthias, Klaus Frieler, and Simon Dixon. 2014. "Intonation in unaccompanied singing: Accuracy, drift, and a model of reference pitch memory." *The Journal of the Acoustical Society of America* 401-411.

In other words, one begins with an initial table of cents values for each note and voice:

B	**T**	**A**	**S**	**B pitch**	**T pitch**	**A pitch**	**S pitch**
Ab3	C4	Ab4	Eb5	2014.343	2398.245	3224.204	3921.926
Db3	Db4	Ab4	F5	1307.24	2519.147	3221.962	4106.739
Db3	C4	Ab4	F5	1304.544	2404.492	3226.092	4103.476
Eb3	Bb3	G4	Eb5	1505.401	2216.231	3104.74	3922.528
Eb3	Bb3	G4	Db5	1510.264	2214.154	3108.445	3711.265
F3	F4	Ab4	C5	1732.818	2921.874	3221.066	3612.903
F3	F4	Ab4	Db5	1720.661	2917.398	3222.105	3704.215
G3	Bb3	G4	Eb5	1916.596	2230.178	3105.394	3914.985
Ab3	C4	G4	Eb5	2012.842	2395.825	3107.75	3914.941
Bb3	Db4	F4	Db5	2214.545	2515.206	2924	3716.852
Bb3	F4	Ab4	C5	2208.324	2914.245	3215.427	3612.562
Eb3	Eb4	Ab4	Db5	1518.676	2714.38	3214.047	3716.943
Eb3	Eb4	G4	Db5	1515.9	2717.373	3104.42	3715.428
Ab3	Eb4	Ab4	C5	2023.401	2712.356	3213.415	3604.582
G3	Eb4	Bb4	Bb4	1911.808	2719.624	3417.449	3414.518
Ab3	Eb4	Ab4	C5	2002.97	2718.545	3223.21	3612.524
G3	Eb4	Ab4	C5	1922.804	2721.662	3224.503	3610.608
F3	Ab3	Ab4	Db5	1712.747	2018.122	3227.285	3712.463
F3	Bb3	Ab4	Db5	1710.775	2209.774	3223.543	3712.938
C3	C4	Ab4	Eb5	1211.915	2407.767	3217.934	3913.806
C3	Ab3	Ab4	Eb5	1207.739	2021.3	3216.474	3915.292
Db3	F4	Ab4	Db5	1297.731	2903.858	3221.516	3713.618
Db3	F4	Ab4	C5	1319.915	2908.084	3218.978	3608.113
Eb3	Bb3	Ab4	Bb4	1522.222	2219.711	3218.547	3408.709
Eb3	C4	Ab4	Ab4	1515.263	2400.621	3216.64	3223.602
Eb3	Db4	G4	Bb4	1515.894	2511.672	3100.8	3414.176
Ab2	C4	Eb4	Ab4	822.1598	2400.429	2713.652	3218.62

Figure 18 – Table of note names absolute cents values as intervals above C2

These values are converted to a table of normalized root pitch variations for each voice and note:

B	**T**	**A**	**S**	**B root**	**T root**	**A root**	**S root**
Ab3	C4	Ab4	Eb5	14.34316	8.002623	24.20444	19.38074
Db3	Db4	Ab4	F5	10.49827	22.40491	21.96171	19.39056
Db3	C4	Ab4	F5	7.801993	14.24977	26.09155	16.1269
Eb3	Bb3	G4	Eb5	2.855115	16.23052	4.739731	19.98259
Eb3	Bb3	G4	Db5	7.718549	14.15406	8.445	14.52287
F3	F4	Ab4	C5	45.4691	34.52496	21.06583	22.66085
F3	F4	Ab4	Db5	33.31232	30.04959	22.10458	7.47303
G3	Bb3	G4	Eb5	16.5956	30.17761	5.394274	12.4391
Ab3	C4	G4	Eb5	12.84246	5.582826	7.75011	12.39482
Bb3	Db4	F4	Db5	14.54457	18.46418	36.65165	20.10994
Bb3	F4	Ab4	C5	8.324324	26.89623	15.427	22.31989
Eb3	Eb4	Ab4	Db5	16.13012	11.83388	14.04656	20.20131
Eb3	Eb4	G4	Db5	13.35433	14.82763	4.420369	18.68601
Ab3	Eb4	Ab4	C5	23.40131	9.810515	13.41522	14.34038
G3	Eb4	Bb4	Bb4	11.80752	17.07866	17.44931	14.5177
Ab3	Eb4	Ab4	C5	2.970167	15.99894	23.20994	22.28198
G3	Eb4	Ab4	C5	22.80444	19.11581	24.50269	20.36626
F3	Ab3	Ab4	Db5	25.39829	18.12177	27.2848	15.72098
F3	Bb3	Ab4	Db5	23.42608	9.773875	23.54334	16.19599
C3	C4	Ab4	Eb5	21.6727	17.52509	17.93433	11.26062
C3	Ab3	Ab4	Eb5	17.4967	21.29965	16.47365	12.74674
Db3	F4	Ab4	Db5	0.989147	16.50958	21.51642	16.87638
Db3	F4	Ab4	C5	23.17346	20.73498	18.97817	17.871
Eb3	Bb3	Ab4	Bb4	19.67604	19.71135	18.54697	8.708946
Eb3	C4	Ab4	Ab4	12.71696	10.37879	16.64039	23.60192
Eb3	Db4	G4	Bb4	13.34803	14.92965	0.800442	14.17646
Ab2	C4	Eb4	Ab4	22.15976	10.18737	11.10611	18.61962

Figure 19 – Table of note names and Implied Root Pitch values

The simplest way to analyze horizontal intra-note adjustment tendencies is to analyze two-note segments when one voice has a common tone between the two – i.e., the note does not change:

Figure 20 -Chorale R309

Blue circled note sequences are examples of horizontal common tones.

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This is one of the clearest examples of the tension between horizontal vs vertical intonation. John Dalley of the Guarneri Quartet weighs in on this very problem when asked the following by David Blum:

“Since you’re often playing a middle voice, John, are there times when you have to adjust a repeated note in different directions?”

Dalley: “Absolutely. Look, for example, at the following passage from the slow movement of Beethoven’s Opus 132:

I have six successive middle C’s. The other players should be aware of this and, in principle, adjust to me, as these are relatively ironclad notes. But at the moment of performance I'll have to be ready, if necessary, to adjust to any fluctuations going on around me."[15]

The extreme cases are as follows: a quartet dedicated to pure horizontal intonation in all voices will show no movement of any root pitch; a quartet dedicated to prioritizing horizontal intonation in common tones will show movement only in other voices; a quartet dedicated to minimizing root movement in a particular voice will show no movement in that voice. Of course, it is to be expected that the actual data will lie somewhere in between these extremes; but examining the proportion of movement in common tone vs non-common-tone voices will give an idea of a group's tendency.

Once a list is compiled of all two note sequences where only one voice has a common tone, we can first analyze the mean absolute change in common tone and non-common tone voice pitches, or ΔG:

$$\Delta G_{common} = G_{in} - G_{i(n-1)}$$

$$\Delta G_{noncommon} = G_{in} - G_{j(n-1)}$$

Where G is the root pitch in cents of note, n is the note number ($n = 1$ is the first note of the chorale, and the notes are numbered sequentially), and i and j are the note indexes (measured as semitones above the root pitch).

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[15] Blum, David. *The Art*
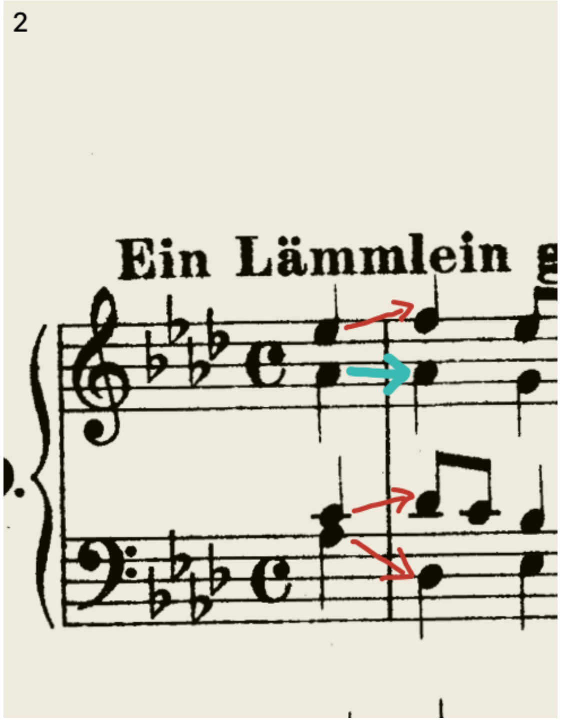
Figure 21 - A common tone movement (blue arrow).
Non-common tone movement in red.

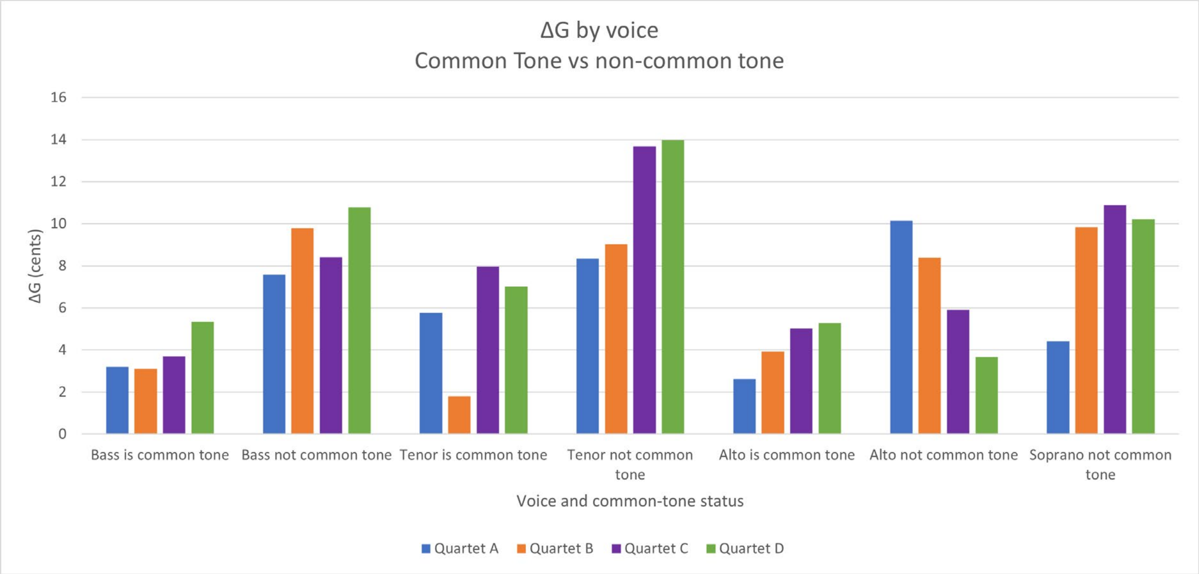

Figure 22 – ΔG by voice.
This is the difference in root pitch between one note and the next, for each voice in each quartet.
(Note that there were no soprano common tones in R309 and R5)

For all voices with common tone movement in Chorales 5 and 309 (soprano voice had no common tones), Quartets A and B exhibited less overall pitch movement between notes than Quartets C and D, suggesting that the more experienced groups were more committed to a common – tone model than the less experienced. (Because of the smaller sample size, these statistics are more exploratory rather than definitive in nature). For not-common-tone movement, the picture is more complex and varies by voice – though almost all voices show more tendency to move their root pitch when they are not the common tone (more on that in the next section), second violin (alto) in Quartets C and D shows an almost equal tendency to adjust regardless of common tone status, while Quartets A and B show almost double the average motion, suggesting that the adjustment profile of Quartets A and B is possibly tied more to common tone status, and adjustment profile of Quartets C and D is, at least for the alto voice, tied more directly to voice. In other words, the professional quartets showed an intonation adjustment profile that was more *global* and *principle-based* – common tones in any voice were prioritized – while quartets C and D showed more of a series of individual adjustment profiles unique to each voice.

Level 3: Intra-note adjustment and influence diagrams

So far, this analysis has only analyzed chorale interval statistics (Level 1) and inter-note pitch dynamics (Level 2). But quartet players acknowledge that "in performance there is a great deal of fluctuation of intonation."[16] Such fluctuation can occur intentionally or as a result of error, and in either case necessitates intra-note pitch adjustment – i.e., a player making small, very fast adjustments to pitch based on

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[16] Blum, David. *The Art*

the pitch movement of other players within the body of a single note. This kind of conscious, fast-twitch adjustment based on listening (and in some cases, physical sensation of the finger angle and string vibration) is mentioned by Heifetz, Flesch, and many other well-known performers and pedagogues as being an essential skill for a professional musician.[17] Rather than time-averaged values as in the prior two meta-analysis levels, this requires the use of the pitch contour time series themselves, which give a fine-grained approximation of perceived pitch over a single note, to make inferences about causality.

Many statistics for measuring the linkage between two or more time series exist. Papiotis et al tested four different measures of quartet pitch interdependence, and found significant results for directional causal linking for only one measure – the "nonlinear coupling coefficient" ***L***.[18] They postulated that simpler measures such as Pearson correlation and Granger causality failed because of the inherent non-linearity of the pitch contour data, a property which is clear upon visual inspection of a note:

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[17] Flesch, Carl. 2000. *The Art of Violin Playing.* Carl Fischer Music.

[18] Papiotis, Panagiotis, Marco Marchini, and Esteban Maestre Gómez. 2012. "Computational analysis of solo versus ensemble performance in string quartets: intonation and dynamics." *Proceedings of the 12th International Conference on Music Perception and Cognition and the 8th Triennial Conference of the European Society for the Cognitive Sciences of Music; 2012 July 23-28; Thessaloniki, Greece.* 7.

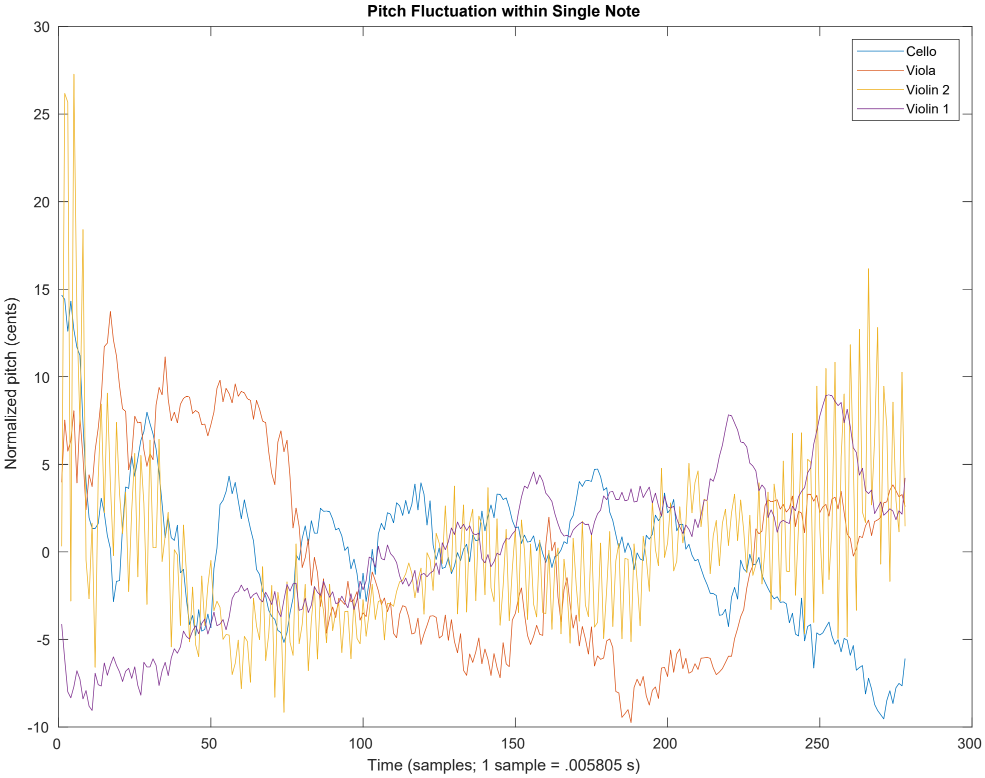

Figure 23 - Pitch over time; single note window.
X axis is pitch sample windows: each window = .005805 s. Y axis is pitch deviation in cents about the normalized pitch (normalized to mean pitch across the window for each voice in this case).

L is a measure which uses the concept of phase space embedding to draw causal links between all possible non-similar directional pairings. Phase space is a concept from physics – a multi-dimensional space in which a set of variables, expressed as a vector, capture complete information about all aspects of a dynamical system (any system that changes over time).[19] Embedding is a technique which recreates these dynamics by approximating them with vectors composed of rolling

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[19] Chicharro, Daniel, and Ralph G. Andrzejak. 2009. "Reliable detection of directional couplings using rank statistics." *Physical Review E* 026217.

windows of lagged (past) values of each timeseries. ***L*** then measures the Euclidean distance between these lagged embedded phase space vectors and the current non lagged vector, and presents (in the case of the 4-member string quartet) a 4x4 matrix ***L*** of directional influence strengths based on the mean rankings of those lags according to minimum distance:

	B Influencer	**T Influencer**	**A Influencer**	**S Influencer**
→ B	0	0.291	0.1745	0.5077
→ T	0.4208	0	0.2475	0.5052
→ A	0.1399	0.3869	0	0.2007
→ S	0.6488	0.5357	0.2361	0

Figure 24 - Matrix of L influence couplings for a single note.
Each column represents an influencing voice, and the rows represent voices influenced by the column voices' pitch contour. L does not detect self-influence, so the diagonal will always be 0. L varies from 0 to 1, with 0 implying no causal linkage and 1 implying complete synchrony.

L is a dimensionless measurement which varies from 0 to 1, with 1 implying total causal linkage and 0 implying no causal linkage. In a later paper, Leguia et al demonstrated accurate reconstruction of the topology of interconnected networks of semi-chaotic coupled Lorenz attractors[20]. They also produced reconstructions of causally linked EEG nodes in the brains of epilepsy patients in the midst of seizures – which matched clinical evidence for physically and causally linked brain areas. Their conclusion was that the nonlinear coupling coefficient L can be used to accurately reproduce both the topology (existence of directional causal coupling) and the relative strengths of such couplings, both of which are of interest in our string quartet

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[20] Leguia, Marc G., Cristina G. B. Martinez, Irene Malvestio, Adria Tauste. 2019. "Inferring directed networks using a rank-based connectivity measure." *Physical Review E, 2019 - APS* 10.

analysis. Furthermore, L was found to have optimal accuracy in situations with heterogenous dynamics (time series not moving in near synchrony), medium coupling strength, and the addition of some noise. All of these qualities accurately describe the dynamics of string quartet pitch adjustment at small timescales: heterogenous (usually, pitch contours, or the dynamics of how a pitch moves in time, are not initially moving in the same direction at the same time), moderately coupled (it is unlikely that one voice would move in absolute perfect synchrony with another), and somewhat noisy (some noise, distributed normally about a windowed mean, is present in all the pitch data – possible reasons include windowing effects from the pitch algorithm and random electronic fluctuation in the microphone/pre-amp circuitry). It should be noted that, despite the noisiness of some of the pitch data, smoothing was considered and discarded for both the reasons mentioned above (some noise aids the L algorithm in the network topology reconstruction)[21] and because the smoothing constraints were rather arbitrary and affected different voices asymmetrically.

Significant pre-processing of the data was required due to the inherent sensitivity of the algorithm to small changes. The pitch contours were windowed by note as before. Because each pitch measurement was to be taken on all four pitch contours simultaneously, the maximum starting time and minimum ending time across all four voices was taken as the window start/end time to ensure continuous signal across the window. Extreme pitch artifacts (> 100 cents) or momentary gaps were removed and replaced by linearly interpolated values. Preprocessing and the L algorithmic analysis were both implemented in Matlab, using code designed by the

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[21] Leguia, Marc G. et al. “Inferring directed networks" *Physical Review E*

author (for preprocessing) and a custom-made Matlab function (for construction of the L matrix) provided by the authors Legiua et al on their website.

Legiua's algorithm requires four main inputs: $m, tau, krec, W$, where m is the embedding dimension (how many data points the algorithm uses as dimensions in phase space to plot dynamics), tau is the time delay for the causative time series, $krec$ is the number of nearest neighbors used in the reconstruction, and W is the Theiler correction – how many temporally nearest neighbors to exclude from the reconstruction. As in Papiotis et al, I found that the variables $m, krec, W$ had little effect on the resultant L matrix and so used similar values for those variables to their paper. tau was set at 0.1 seconds to match the hypothesized minimum auditory – physical reaction time discussed in papers by Landry and Pain.[22] [23]

In the influence note presented above from Quartet B, the Bass has strong influence over Tenor and Soprano, Soprano has strong influence over Bass and Tenor, and Tenor has strong influence over Soprano. Alto seems to be independent in both being influenced and influencing others' pitch movements. From the L matrix above one can construct a directed influence graph:

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[22] Pain, Matthew TG, and Angela Hibbs. 2007. "Sprint starts and the minimum auditory reaction time." *Journal of Sports Sciences* 79-86.

[23] Landry, Simon P., and François Champoux. "Musicians react faster and are better multisensory integrators." *Brain and cognition* 111 (2017): 156-162.

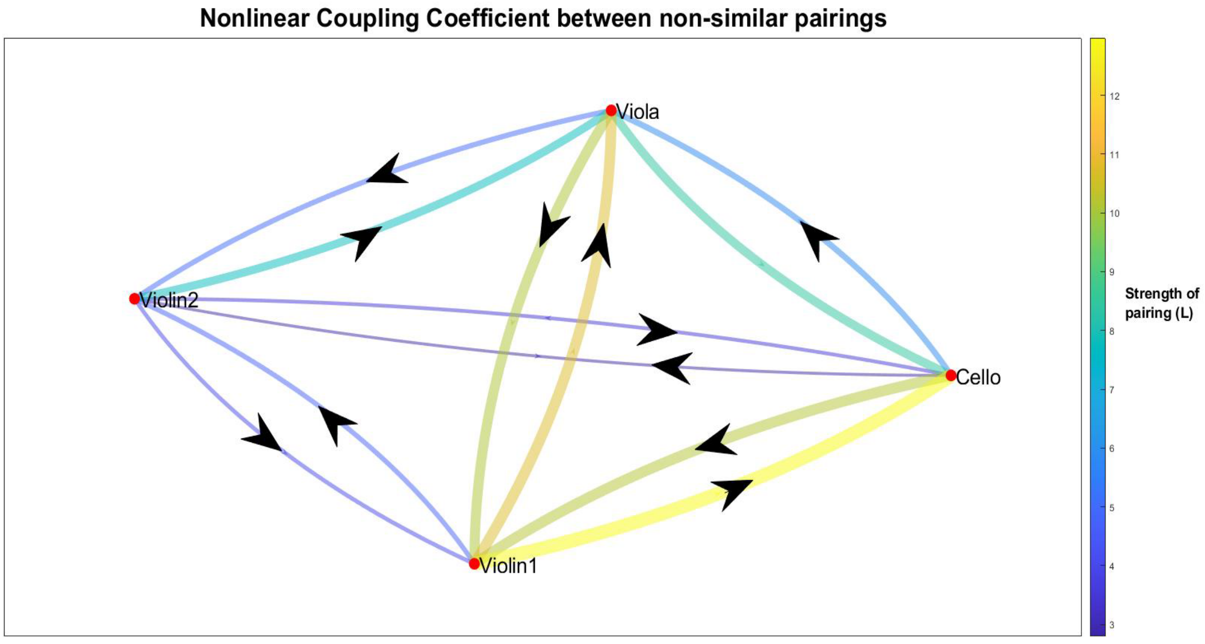

Figure 25 – Nonlinear Coupling Coefficient between non-similar pairings
Color gradient shows strength of coupling relative to other pairings in the quartet; width of line shows absolute strength of L (measured in dimensionless units from 0 to 1 and expanded linearly from 0 to 15 for clarity).

The number of variables which could be analyzed against this influence data is staggering; this thesis does not aim to create a general analysis covering all possible dimensions of intra-note quartet adjustment. We therefore will analyze interdependence data from only longer sustained notes – i.e. cadence notes and reference chord tunings.

A. Reference Chords and Cadences

In a way, reference chords provide an almost idealized laboratory for studying intonation tendencies; each chord is causally separate (not part of linear intonation and separated by silences in between) and held for a long period of time. Cadences are similar, although not completely causally separated. Therefore we will use reference chords and cadences for our nonlinear causality analysis. They have the added benefit of being longer than ~100 milliseconds, the minimum reaction time threshold established above.

Here we analyze $\boldsymbol{\overline{L}}$**,** the mean of all influence matrices taken across individual elements. (Taking the mean across multiple instances of a directed network is the same method that Leguia et al use when inferring networks in multiple numerically generated instances of the same underlying Lorenz attractor.)[24]

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[24] Leguia, Marc G. et al. "Inferring directed networks" *Physical Review E*

Nonlinear Coupling Coefficient ***L***

Between Non-Identical Pairings

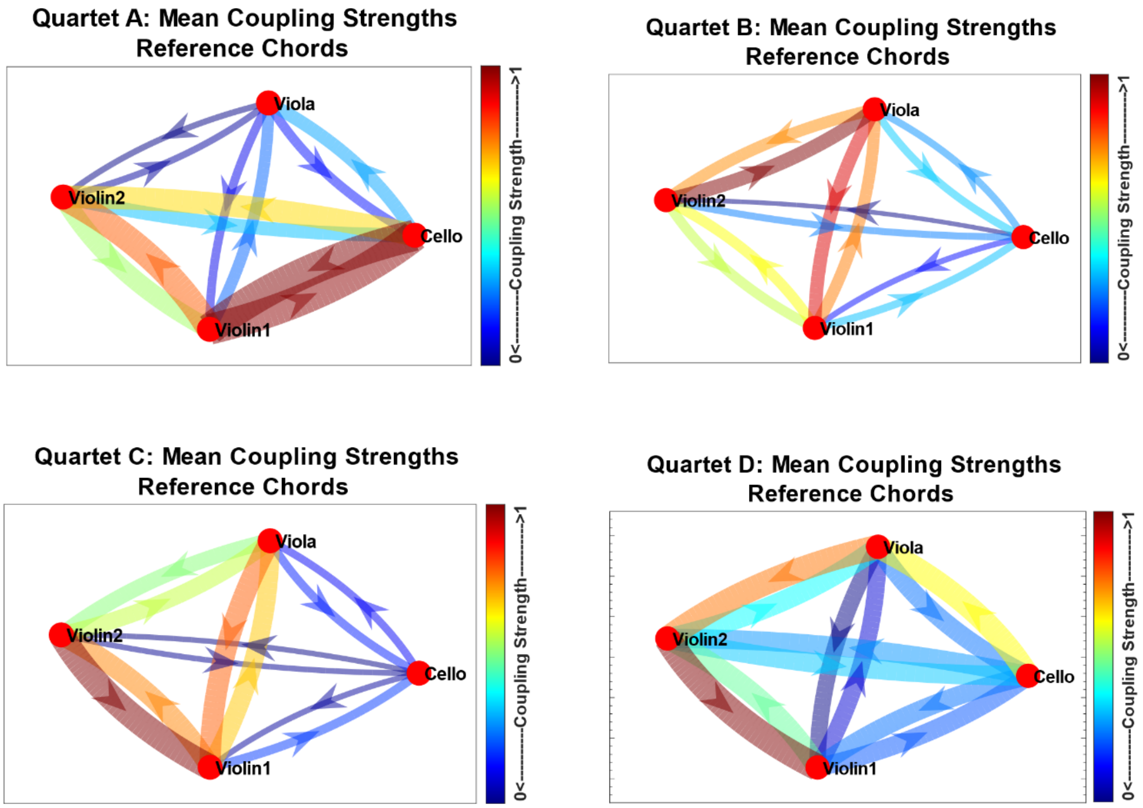

Figure 26 - Mean Coupling Strengths of each quartet, during reference chord tuning and cadence notes. Color gradient shows strength of coupling relative to other pairings in the quartet; width of arrow shows absolute strength of L (measured in dimensionless units from 0 to 1 and expanded linearly for visual clarity). In an ANOVA, Quartets A and C showed statistically asymmetric ($p < .01$) coupling strengths between different voice pairings, while Quartets B and D did not ($p = 0.6994$).

Quartet A

	B Influencer	T Influencer	A Influencer	S Influencer
→ B	0	0.3686	0.4827	0.5663
→ T	0.3137	0	0.2514	0.2992
→ A	0.3782	0.2643	0	0.4463
→ S	0.5636	0.3467	0.5146	0

Quartet B

	B Influencer	T Influencer	A Influencer	S Influencer
→ B	0	0.2776	0.2224	0.2509
→ T	0.2884	0	0.3551	0.3795
→ A	0.2737	0.3921	0	0.3292
→ S	0.2862	0.3575	0.338	0

Quartet C

	B Influencer	T Influencer	A Influencer	S Influencer
→ B	0	0.283	0.2364	0.2469
→ T	0.2954	0	0.401	0.4682
→ A	0.2422	0.4167	0	0.5139
→ S	0.3049	0.4441	0.4591	0

Quartet D

	B Influencer	T Influencer	A Influencer	S Influencer
→ B	0	0.4911	0.4511	0.4468
→ T	0.4467	0	0.506	0.418
→ A	0.4568	0.4632	0	0.5307
→ S	0.4433	0.4281	0.4745	0

Figure 27 – Coupling Matrices
Each matrix shows the directional coupling strengths for the mean values of L taken across all reference chords and cadences.

For the first time, differences emerge which are not along the line dividing professionals from semi-student. Quartets A and C demonstrated significantly different coupling strengths between players; that is, there were certain players which tended to be more influential relative to others for specific pairings. An ANOVA analysis on the mean coupling strengths between all pairings confirmed this (Quartet A: $F = 4.4372$, $p < .0009$), Quartet C: $F = 8.3607$ $p < 0.0001$). In particular, Quartet

A had strong couplings in the B→S (Cello influencing V1) and S→B pairings, A→ S and S→A pairings, and A→B pairings. It is interesting to note that these couplings are mostly symmetric, implying that if a player was influencing another player, they were also more likely to be listening to that player and being influenced by them in turn. Quartet C has a similar pattern of symmetry, though the viola here tends to be more influential than the cello. Quartets B and D, on the other hand, both exhibit significantly equal coupling strengths across all voices, and Quartet D has significantly higher coupling strength as an average taken across all voices than Quartets A or B. Does this mean that Quartet D, the least experienced quartet, is listening more to each other than the professional quartets? Not necessarily.

Here one must consider what exactly ***L*** is measuring. Leguia et all use ***L*** to measure the *dynamic* independence between separate time series; that is, the tendency of one time series (in this case, the pitch contour of an instrument, created intentionally by a human musician) to influence the values of another time series in the future. [25] This means, possibly, that the more the players tend to adjust their pitch (in some synchrony), the higher the L coupling value will be. One possible explanation is that Quartet D is adjusting more often because they are taking longer to settle to an accepted chordal intonation, and their pitch fluctuations are larger due to overcorrection. When one examines the average interquartile range of pitch in cents of each voice between B and D, the difference becomes apparent:

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[25] Leguia, Marc G. et al. “Inferring directed networks" *Physical Review E*

Mean Interquartile Range (cents)

	B	T	A	S
Quartet B	1.7452	2.2405	1.9643	1.9696
Quartet D	2.9051	5.4644	4.2376	2.9107

Figure 28 – Mean Interquartile Range (cents)
This is the size of the range between which the middle 50% of data points (pitch) fell, over all notes measured. It gives an idea of the general amount of pitch fluctuation going on during the adjustment period in each voice.

Quartet D had significantly ($p > .01$) more pitch fluctuation about the mean pitch for each note and each voice. The average time spent per chord bears this out as well (recall that quartets were asked to tune the chords until they all were comfortable; this was signaled usually by a period of pitch stability followed by a nonverbal visual signal). Quartet B took 2.0706 seconds on average to find an appropriate tuning; Quartet D took 4.7067. Although this is inductive reasoning, it seems reasonable to assume that the higher interdependence in Quartet D is due to these factors. Critical listening to the experiment recordings bears this out as well.

Chapter 6: Conclusions – A Subtle Multivariate Dance

This conclusion will discuss each of three meta-levels separately, then move on to some practical application for string quartet rehearsal and what further research might be needed.

Level 1 – Vertical Intonation

A positive correlation between years of study as a group and both the existence and consistency of a group intonation model (measured by mean interval error) was found. That is, the two professional groups tended to play most intervals closer in value to their "preferred" reference values across multiple instances, and their reference intervals were clustered tighter around their mean. This is perhaps the least surprising and most easily understood result. In addition, the professional groups' consistency with their group intonation profile was more independent of key (as noted in Results, when playing the same chorales in two different keys, the error values of groups C and D fluctuated significantly more than groups A and B). As intonation is inherently subjective, part of a string quartet's typical rehearsal process often involves explicit discussion about their intonation preferences, both in the general sense ("I like major thirds low") and in specific applications. One somewhat interesting result was both groups' preferences for minor thirds which were significantly narrower than the comparatively large minor third of 5-limit Just Intonation (which can be thought of as where to put the minor third in a minor triad so that the 'upper' major third is a typical JI narrow major third). In the minor seventh interval, the professional group Quartet A also exhibited a preference for a much narrower minor seventh than JI – closest, as mentioned previously, to the colorful 7:4 ‘septimal’ minor seventh, found explicitly in the microtonal works of composers like Ben Johnston.[26] This was contrary to my initial assumptions that most quartets would converge on 5 – limit JI after sufficient rehearsal. There is no question that this was intentional coloring – the clustering around such intervals is tight, as noted in Figure 9. Regardless, this portion of the analysis showed that the professional quartets showed a clear, consistent group sound identity in the form of playing intervals consistently close to their “preferred” intonation profiles, which sometimes differed significantly from any single mathematically derived intonation system.

Level 2 – Horizontal Intonation: Inter-note adjustment

In general, for common-tone harmonic movement, the more experienced Quartets A and B exhibited a stronger preference for keeping common tones at the same absolute pitch value and moving the root pitches of other voices around them to achieve accurate vertical intonation. This shows in both the relative root pitch movement between common-tone and non-common tone voices (common-tone root pitch movement was always lower, implying that the common tones tended to stay the same) and in the fact that Quartets A and B showed less common-tone movement in each voice than Quartets C and D. Thus, the two professional quartets seemed to be guided more by a principle (common tones are sacrosanct) regardless of individual player or context, and the non-professional quartets’ adjustment profiles seemed to be much more individually variable.

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[26] Johnston, Ben. 1990. *String Quartet No. 4.* Baltimore, Maryland: Smith Publications

Level 3 – Horizontal Intonation, intra-note level

Here, we see a more complex picture. Quartets C and D actually showed more coupling strength on average than Quartets A and B. Also, for the first time, the professional quartets were split in terms of the way they adjust – Quartets A and C showed statistically significant differences between directional coupling strengths, while B and D did not. In plain terms, this means Quartets A and C had certain members (Cello and First Violin for A, First Violin and Second Violin for C) which exhibited more intonational influence than the others. But as mentioned before, context and other variables must be taken in to account. One possible explanation for the higher average values of C and D is their higher interquartile range of fluctuation, i.e., these groups were making more adjustments in the body of a note because their pre-note audiation via inter-note adjustment profiles was not as well defined (as mentioned above).

Practical Applications

The primary nature of this research was academic rather than practical, but this does not preclude the drawing of practical conclusions from the data. Some results will not be surprising to any experienced chamber musician; that a group who rehearses together is more likely to play certain intervals the same way is not surprising. However, Levels 2 and 3 show that perhaps more emphasis should be placed on pre-note audiation than dynamic adjustment in the moment, especially in a string quartet context. While the scope of this analysis is limited, and more research needs to be done on more complex intonation problems, the professional quartets had smaller fluctuations, shorter “settling times” (initial pitch detection to stable period), and a generally more well-defined horizontal intonation profile than the non-professional quartets. Rehearsal techniques to improve this skill could include placing silent gaps before difficult chords or modulations, playing slowly in pairs, practicing with the score in front of each player, and singing parts together.

Epiphenomena: The Qualitative and the Quantitative

At the beginning of this document, I discussed the tension between the qualitative and quantitative in music, which is related to the tension between small-scale mechanism and epiphenomenon. When one considers the sheer number of these mechanisms that must work in concert to allow us to experience hearing air pressure oscillations as music, the mind boggles. Even more incredible is the fact that we can control our muscles in direct response to these stimuli to produce music of our own. But this is not even the most miraculous part about music making. Consider what is actually taking place when four people perform in synchrony with one another: each player, ears alert, ears mind processing four sounds with rich multilayered overtone spectra, responding to each minute change in pitch, dynamic, articulation, phrasing, and a hundred other micro-variables at time scales of tenths of a second or quicker by incredibly subtle movements of the fingers, arms, and body. Any human-created mechanism with an analogous complexity would be surely doomed to fail. That all quartet players spend years practicing to achieve mastery, and then spend years more rehearsing together to achieve some kind of harmonious group sound, is indeed a testimony to the difficulty of the endeavor. The most miraculous part of it all to me is that somehow, despite the messiness and flawed humanness of the entire operation (or perhaps because of it), real communication on multiple levels – levels of time, pitch, even aesthetic meaning — does occur. The painstaking and sometimes mundane work of developing a group identity transforms a collection of pressure waves in a world governed by physics into music in a world governed by Beauty. If this experiment and study has taught me anything, it is that what our ears, minds, and fingers can do in synchrony together is a subtle, almost infinitely variable dance, more wonderful and far deeper than any set of numbers or graph can capture. The attempt to quantify this dance does not reduce the mystery of such a thing – on the contrary, glimpsing the mathematical 'shadows' of group identity has only given me a deeper appreciation for the beauty that results when four committed players come together to form a sound greater than the sum of its parts.

Appendix – Necessary Scientific and Biological Background

1. Intonation – Individual and Quartet

Though performing the traditional Western Classical repertoire does involve the reproduction of fixed sets of pitches, success in musical performance is in no way represented by mere mechanically accurate reproduction of those pitches; rather, the goal is a musically satisfying and emotional moving performance. Therefore, though pitch accuracy and precision is an important part of achieving such a goal, the inherently subjective nature of these success criteria and inherent ambiguities and multiple conflicting systems of intonation mean that it is effectively impossible to define a single, universal intonation system for musical performance. Moreover, there is a well-established dichotomy at play in string ensemble performances between melodic intonation, which focuses on producing the most pleasing "horizontal" series of notes over time, and harmonic intonation, which focuses on producing a pleasing vertical combination of notes which occur simultaneously. That being said, there do exist several mathematical systems which have roots in the historical development of music. Though they were developed for and applied initially to instruments of fixed pitch reproduction capabilities, and string players have a continuous spectrum of pitch available to them and are not bound to a single system as keyboard instruments are, they can serve as useful frameworks for discussing intonation. They are briefly defined below:

A. “5 limit” Just Intonation (JI)

Based on the harmonic series of the fundamental, which is obtained by multiplying F_0 by increasing integer ratios. The frequency ratios used in calculation of intervals are expressed: $\frac{F_{higher}}{F_{lower}}$. For intervals of the fifth (3/2), fourth (4/3), major third (5/4), and minor third (6/5), which correspond to the 3rd, 4th, 5th, and 6th harmonics of the overtone series, and indeed for the entire diatonic scale based on the fundamental, this system results in a pure-sounding and resonant interval structures. Intervals between scalar pitches higher in the overtone series derived from the just system begin to deviate wildly, and triads and chords formed from such pitches are increasingly out of tune with one another. As such, pure JI is impossible for keyboard and other fixed-pitch instruments to adhere to. That being said, a pure JI definition of a triad and seventh chord is what most string players will gravitate toward when playing long sustained intervals. The “5 limit” term comes from the fact that it uses intervals constructed from integer ratios only up to the number 5.

B. Pythagorean Intonation (PI)

A variant of JI based on the interval of the perfect fifth at a ratio of 3/2, which is extrapolated through the whole circle of fifths to obtain all chromatic pitches. This results in the octave at the enharmonic being slightly different than the fundamental, a difference called the syntonic comma. This is also sometimes referred to as “3 limit” Just Intonation, as it uses intervals constructed from integer ratios only up to the number 3.

C. Equal Temperament (ET)

Based on dividing a perfect octave into 12 steps divided logarithmically equally so that the frequency of each semitone is 21/12 higher than the last, and each semitone is divided in a like manner into 100 logarithmically spaced steps called cents - in essence, spreading the syntonic comma out evenly over each semitone. This results in fifths which are 2 cents narrower than a perfect fifth, and major thirds which are ~14 cents wider than a JI third.

Experiments seem to show that, when playing scales without accompaniment, violinists tend to play pitches equally close to a Pythagorean and equal tempered intonation system.[27] When practicing with drones, or in ensembles, generally accepted practice is to adhere to a context-based just intonation – i.e., play notes which are as much as possible in resonance with the tonic note of whatever chord one is playing or whatever drone note one is tuning to. Horizontal or “melodic” intonation is often encouraged in solo playing, with the aim of coloring certain intervals to emphasize their melodic direction or dissonant qualities and produce an emotional effect. Common practice is to raise leading tones and slightly exaggerate major and minor thirds by respectively widening and narrowing them. The open fifth tuning of the string quartet produces several inherent problems with either system, due to the fact that notes played solo on an instrument generally sound most resonant when they are played in consonance with an overtone of an open string. One example of this is the

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[27] Loosen, Franz. "Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations." *The Journal of the Acoustical Society of America*, 1993: 525-539

problem of playing in keys which have open strings as thirds in tonic or dominant chords. In C Major, all three notes of the tonic chord are found as open strings on the instruments (C and G on the cello and viola, and G and E on the violins). If lower strings play in tune with their open C and G and violinists play in tune with their open E, an unacceptably wide major third results. If playing a piece in such a key, quartets will often tune their open strings somewhat narrower ("tight tuning") to reduce this quality, with the tradeoff of some sympathetic resonance being lost due to the imperfect quality of the fifths. Other quartets merely use fingers to make a consistent adjustment (violinists will play low, or lower strings high). Several of the proposed exercises will attempt to exploit these types of intonational paradoxes, as they almost always require some sort of group consensus to be made through explicit discussion in order to achieve consistency in intonation.

2. Pitch

This experiment will attempt to quantify and describe the adjustment processes that occur as a direct result of the way musicians hear pitch. As such, a successful pitch-detection algorithm will attempt to process sound in a way that returns pitch values as close as possible to those heard by humans listening to the same sound. This necessitates an understanding of both the current research on human pitch perception and the spectrum of available mathematical tools to use to extract a pitch value from a recorded signal. Even the definition of “pitch” is variable depending on whether one takes it from a psychoacoustic or a signal-processing perspective: ANSI describes psychoacoustic pitch in the following way: “pitch is that auditory attribute of sound according to which sounds can be ordered on a scale from low to high." In signal processing and pitch-detection algorithms, where pitch must be quantified, it is simply equated with frequency. These two definitions coincide exactly for pitches produced by pure sine waves; however, a string instrument produces for every pitch a power vs frequency spectrum that consists of a fundamental frequency (F_0) plus a number of overtones at integer multiples of F_0. In such cases, the pitch is generally equated with F_0, and this often corresponds with perceived pitch. There are some cases, however, in which the fundamental is weakly represented or nonexistent, yet the perceived pitch is that of the missing fundamental. Such a phenomenon occurs, in fact, on notes lower than about a D on the violin – resonance modes for the fundamentals of those notes are weak, yet we perceive the pitches to be those of the fundamentals. Inharmonicity and multiple pitches additionally complicate the issue of pitch detection. Since any inherent inharmonicity of strings is cancelled by the phase-locking behavior of a bowed string, the players will be playing without vibrato, and quartet instruments will be individually miked, such issues should not be a consideration with this experiment; however, as the topics of pitch perception and algorithmic pitch detection are far from straightforward, an overview of them will be beneficial.

A. Sound detection and pitch perception in the human ear

The mechanics of human hearing are well understood. Sound enters the ear as pressure waves in the air formed by vibration. They hit the eardrum and are processed by the brain as sound. The basic processing chain of a sound from ear to brain is as follows: Outer ear --> Middle ear ---> Inner ear (cochlea) ---> Auditory nerve--->Midbrain--->Cortex.

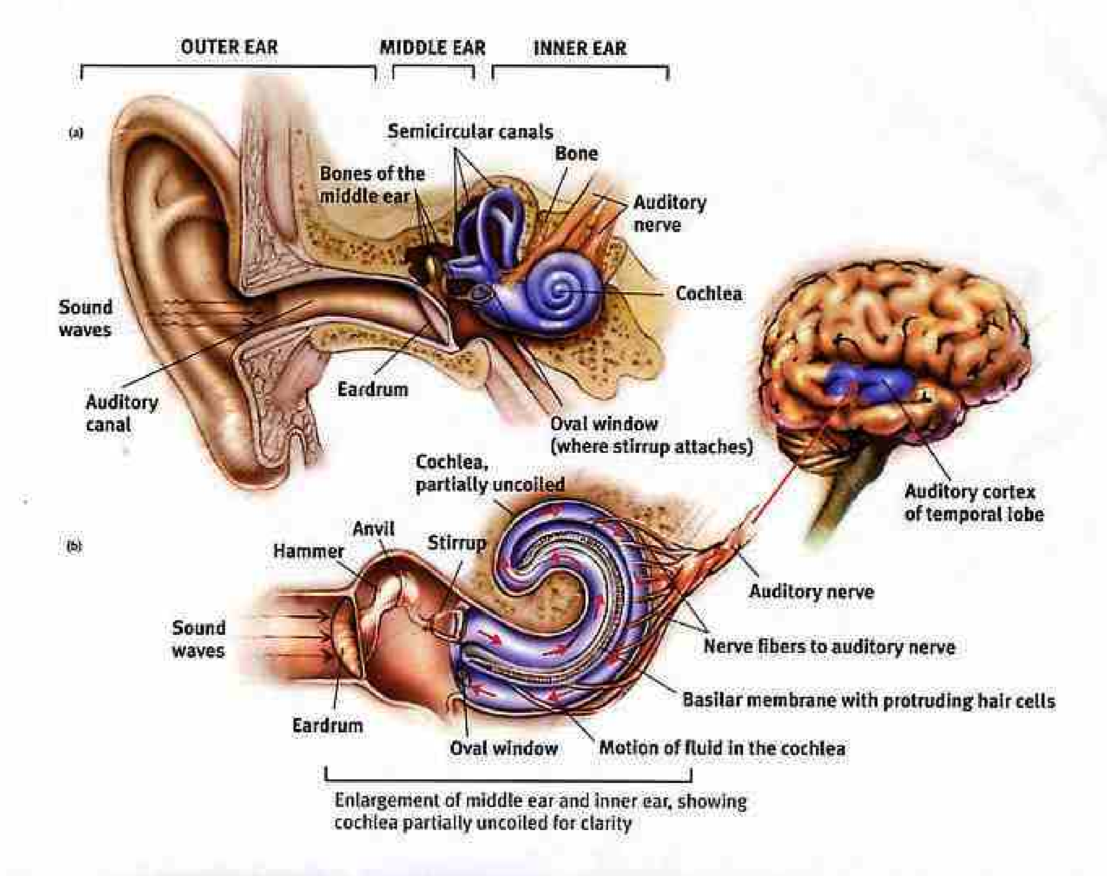

Figure 29 – The Human Ear

The pressure wave is first converted into mechanical vibrational energy at the eardrum at the end of the ear canal, which at the absolute lower threshold of sound is sensitive to vibrations smaller in wavelength than the diameter of a hydrogen atom, giving the ear a dynamic range far exceeding that of sensitive audio equipment. It is worth noting that the ear canal has an inherent natural resonance in roughly the 1000-6000 Hz range - roughly equivalent to the upper third of a piano keyboard. This acts, in essence, as a midrange filter, a fact which some pitch detection algorithms try to emulate in their preprocessing chain. The sound energy is then transferred through vibration of the three middle ear bones to the cochlea. This is where the transfer from acoustic signal to neural signal occurs. The middle ear bones transmit vibration to another membrane covering the oval window at the end of the fluid filled tube called the cochlea. Within this tube, the vibrations are passed down a spiraled cylindrical membrane called the basilar membrane, which has many tiny hair cells which move in sympathy with its vibrations. The movement of these hair cells opens ion channels which triggers the firing of nerve cells attached to each hair.

Thus the audio signal is converted in to an electrochemical signal. A complete mechanism by which the human ear and brain converts this signal into sound and pitch entities, however, has not been definitively established. Two primary concepts for how pitch is assigned in the brain have emerged: frequency-based pattern matching (theory of place), and time-based period analysis (autocorrelation)[28].

i. Theory of Place:

The basilar membrane functions as a physical mechanism which performs the mechanical equivalent to a Fourier transform on the audio signal:

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[28] Boersma, Paul. 1993. "Accurate short-term analysis of the fundamental frequency and the harmonics-to-noise ratio of a sampled sound." *Proceedings of the institute of phonetic sciences* 97-110

$$\hat{a}(\omega) = \int a(t)\, e^{-i\omega t}\, dt$$

This transforms the function $a(t)$ (amplitude as a function of time) into the function $\hat{a}(\omega)$ (amplitude as a function of frequency).

Each section of the membrane has a different resonant frequency range and vibrates with an amplitude corresponding to the amplitude of that particular partial in the complex waveform. There are about 3500 inner hair cells in contact with the basilar membrane - each one is connected to a nerve ending, and when they vibrate, neurotransmitters are released which sends a signal up the nerve. These nerves tend to fire in phase with the frequency of the pitches they are representing. Thus pitch becomes encoded as a function of position on the membrane, and also as a function of the firing phase. There is much evidence which supports this model. In addition to the physical structure and resonance pattern of the basilar membrane, in the brain, individual frequency clusters are assigned to Quartets of neurons. A recent study by Christian Gaser and Gottfried Schlaug showed significant differences between professional musicians, amateur musicians, and non-musicians in several areas of the brain, including increase in the size of "primary motor and somatosensory areas" - i.e., those areas which are related to motor function, specifically finger coordination, and pitch sensitivity and processing. The implications for this are clear - long term practice of musical skillsets results in positive, measurable physical changes to the brain. Whether such skillsets could include the mental quartet intonation adjustment model described above remains to be seen. The "theory of place" was once thought to entirely explain pitch detection in the human ear. However, the Just Noticeable

Difference (JND) between one frequency and another is on the order of 3Hz for pure sine waves and even smaller for complex tones like those which string instruments produce, meaning that humans can functionally distinguish a very large number of tones per octave over the detectable frequency range of 20-20000 Hz. This would require many more pitch-specific locations along the basilar membrane than there are nerves, especially at high frequencies due to the logarithmic nature of pitch position along the basilar membrane. Also, the upper end of the human hearing spectrum is higher than can be resolved by sympathetic resonance of the basilar membrane, which tops out at about 5 khz. This indicates that other mechanisms are probably also in play in fine-grained detection of pitch.

ii. Time-based (Autocorrelation) theory:

Once signals travel up the auditory nerve, they are processed by the midbrain and auditory Cortex. The important fact here is that an incredible amount of specific signal information is preserved through the first several steps of the processing chain - it is only once the signal reaches the auditory cortex that any real compression or lossy filtering activity goes on. The autocorrelation theory postulates that some pitch processing occurs at this level: the brain detects the period of an audio signal by comparing sound-energy events with each other in time, by lagging. The basic idea behind this concept is the autocorrelation function, which for continuous functions is:

$$rx(\tau) \equiv \int x(t)\, x(t + \tau) dt$$

where τ is the lag time between the two signals being compared. This results in maxima where $x(t) = x(t + \tau)$, and if the signal is periodic, the maxima will be at the period T_0 of the signal, with the fundamental frequency $F_{0=}$. The function is evaluated over short segments of time called windows, though the precise mechanism of how the brain does this has not been established.

B. Mathematical pitch detection – theory, algorithms, software

As discussed before, "pitch," as a property of a pure periodic (typically sinusoidal) function, is equal to the frequency in cycles per second at which that function oscillates. As a property of an instrument, specifically a string instrument, its classification and detection is a good deal more complex. Wave theory states that a string with a standing wave has a fundamental frequency which is inversely related to the period of oscillation and many overtones whose frequencies are integer multiples of the fundamental. Helmholtz vibration is produced through the slip-stick action of the bow on the string: tiny hook-like features on the horsehair grab the string and pull it horizontally in the direction of bow motion, producing a sharp corner. When the tension on the string becomes greater than the force of friction holding the string to the bow, the string "slips" rapidly back to a stretched position opposite. At any given time, two corners are traveling rapidly up and down the string. The vibrational energy is transferred via the bridge and soundpost to the front and back plates of the instrument, which vibrate in complex patterns unique to each violin. When a single pitch of an instrument is recorded and a small segment of the signal is analyzed, the Fourier transform of the signal produces a power-to-frequency graph which also has a profile unique to each instrument and is a representation of what players call the "timbre" or "tone color" of the violin. Even a straight-tone pitch, therefore, consists (sometimes) of a fundamental frequency and many overtones.

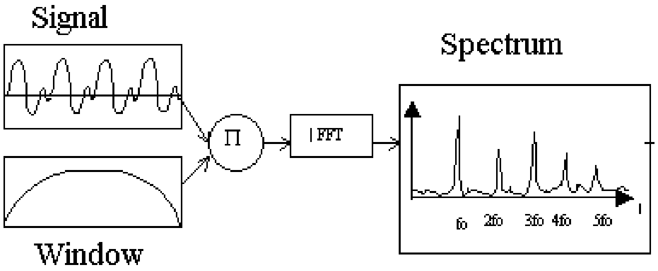

Figure 30 - An illustration of how an amplitude modulated signal is windowed and converted via Fast Fourier Transform to a spectrum of frequencies.

As mentioned above, for many notes of lower frequency in string instruments, the fundamental is not even strongly present. This presents a problem - how to determine F_0 from such a signal?

Like theories of human pitch perception, pitch detection algorithms fall into two main categories which are analogous: frequency domain analysis, which is fundamentally based on a discrete version of the Fourier transform and time domain analysis, which uses the autocorrelation function described earlier as a point of departure. Both divide signals up into segments of time, then multiply by a windowing function which weights the middle of the window over the edges, which can cause errors if the window is not precisely the length of the period of the signal. The limitations of each method as a model for human hearing also apply to their use in algorithmic pitch detection. After research and preliminary experimentation, algorithms which consist of a combination of the two methods and several steps of filtering tend to produce the best results. An excellent freeware software solution which follows this model exists: The primary software to be used in this experiment, called Praat, was developed by Paul Boersma of the University of Amsterdam for phonetic research, and is fundamentally based on the autocorrelation function:

$$rx(\tau) \equiv \int x(t)\, x(t+\tau) dt$$

where τ is the lag.

The algorithm chops the signal into segments, and multiplies each segment by a *windowing function* which minimizes contributions from the edge of the window which can cause errors when the edges of the window are out of phase. The uniqueness of Praat lies in its dividing the autocorrelation of the signal by the autocorrelation of the window itself, eliminating the tendency of the un-normalized autocorrelation to choose the first formant over the fundamental F_0.

The entire process is diagrammed below:[29]

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[29] Boersma, Paul. 1993. "Accurate short-term analysis of the fundamental frequency and the harmonics-to-noise ratio of a sampled sound." *Proceedings of the institute of phonetic sciences* 97-110.

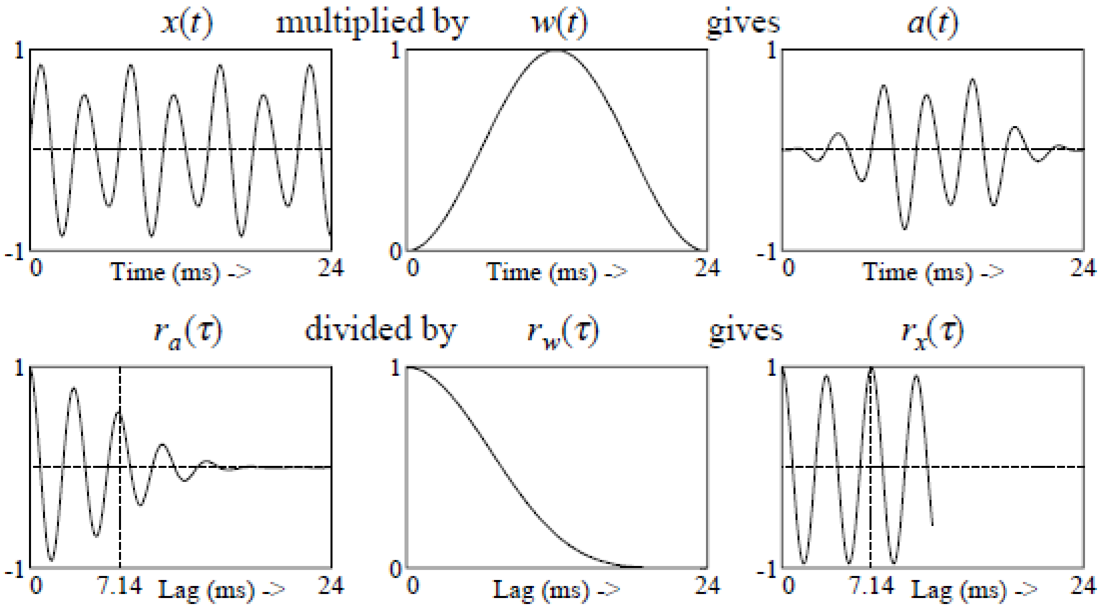

Fig. 1. How to window a sound segment, and how to estimate the autocorrelation of a sound segment from the autocorrelation of its windowed version. The estimated autocorrelation $r_x(\tau)$ is not shown for lags longer than half the window length, because it becomes less reliable there for signals with few periods per window.

Figure 31 - A more detailed example of how Praat windows a signal for autocorrelation pitch analysis.

For intonation research, Praat uses a Gaussian windowing function and the Fast Fourier Transform in the power domain to accurately estimate perceived pitch of an incoming audio signal.

Bibliography

Bach, Johann Sebastian. 1986. *371 Harmonized Chorales and 69 Chorale Melodies with Figured Bass.* Edited by Albert Riemenschneider. G. Schirmer, Inc.

Blum, David. 1986. *The Art of String Quartet Playing.* Ithaca: Cornell University Press.

Chicharro, Daniel, and Ralph G. Andrzejak. 2009. "Reliable detection of directional couplings using rank statistics." *Physical Review E* 026217.

Flesch, Carl. 2000. *The Art of Violin Playing.* Carl Fischer Music.

Gingerich, John M. 2010. "Ignaz Schuppanzigh and Beethoven's Late Quartets." *The Musical Quarterly* 450-513.

1951. *Of Men and Music.* Directed by Alexander Hammid and Irving Reis. Performed by Jascha Heifetz.

Hofstadter, Douglas. 1979. *Gödel, Escher, Bach: an Eternal Golden Braid.* Basic Books.

Jain, Aditya, Ramta Bansal, Avnish Kumar, and K. D. Singh. 2015. " "A comparative study of visual and auditory reaction times on the basis of gender and physical activity levels of medical first year students."." *International Journal of Applied and Basic Medical Research 5, no. 2* 124.

Janof, Tim. 10. *cello.org.* 10 05. Accessed 08 25, 2020. http://cello.org/Newsletter/Articles/katz/katz.htm.

Johnston, Ben. 1990. *String Quartet No. 4.* Baltimore, Maryland: Smith Publications.

Landr, Simon P. and François Champoux. 2017. "Musicians react faster and are better multisensory integrators." *Brain and cognition 111* 156-162.

Leguia, Marc G., Cristina GB Martínez, Irene Malvestio, Adrià Tauste Campo, Rodrigo Rocamora, Zoran Levnajić, and Ralph Gregor Andrzejak. 2019. ""Inferring directed networks using a rank-based connectivity measure [software]."." *Physical Review E.*

Marc G. Leguia, Cristina G. B. Martines, Irene Malvestio, Adria TAuste CAmpo, Rodrigo Rocamora, Zoran Levnajic, and Ralph G. Andrezejak. 2019. "Inferring directed networks using a rank-based connectivity measure." *Physical Review E* 012319.

Marc G. Leguia, Cristina G. B. Martinez, Irene Malvestio, Adria Tauste. 2019. "Inferring directed networks using a rank-based connectivity measure." *Physical Review E, 2019 - APS* 10.

Matthias Mauch, Klaus Frieler, Simon Dixon. 2014. "Intonation in unaccompanied singing: Accuracy, drift, and a model of reference pitch memory." *The Journal of the Acoustical Society of America* 401-411.

Moreland, Richard L. and Larissa Myaskovsky. 2000. "Exploring the Performance Benefits of Group Training: Transactive Memory or Improved Communication?" *Organizational Behavior and Human Decision Processes* 117-133.

Moreland, Richard L., and Larissa Myaskovsky. 2000. "Exploring the performance benefits of group training: Transactive memory or improved communication?" *Organizational behavior and human decision processes* 117-133.

Papiotis, Panagiotis, Marco Marchini, and Esteban Maestre Gómez. 2012. "omputational analysis of solo versus ensemble performance in string quartets: intonation and dynamics." *Proceedings of the 12th International Conference on Music Perception and Cognition and the 8th Triennial Conference of the European Society for the Cognitive Sciences of Music; 2012 July 23-28; Thessaloniki, Greece.* 7.

Wing, Alan M., Satoshi Endo, Adrian Bradbury, and Dick Vorberg. 2014. "Optimal feedback correction in string quartet synchronization." *Journal of the Royal Society Interface.*

Zhang, B. G., Li, W., Shi, Y., Liu, X., & Chen, L. 2017. "Detecting causality from short time-series data based on prediction of topologically equivalent attractors." *BMC systems biology* 128.