ABSTRACT

Title of Dissertation: DESIGN AND OPERATIONS ON THE SUPPLY SIDE OF ONLINE MARKETPLACES

Wenchang Zhang
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Dissertation Directed by: Wedad Elmaghraby, Professor
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Online platforms like eBay, Upwork, Airbnb, and Uber have transformed their markets, and many more are about to emerge. The rise of platforms has become one of the predominant economic and social developments of our time. Moreover, it has created many opportunities and challenges for both practitioners and researchers. My dissertation focuses on the design and operations on the supply side of online marketplaces. In particular, I study supply-side levers (e.g., listing policy and information provision policy) in different marketplace context (e.g., auction marketplace and service platform), with the consideration of strategic behavior of market participants and various friction involved in transactions (e.g., participation cost, information asymmetry, and supply adjustment friction).

The first essay investigates how a one-sided liquidation auction marketplace maximizes its revenue by managing the supply-side market thickness under an exogenous supply inflow. The second essay examines the operational impacts of service platforms' information disclosure regarding service providers' qualities and revealing

their mechanisms. The last essay studies whether two-sided marketplaces benefit or suffer from sellers' quantity competition under unanticipated demand shocks. We further show that marketplaces can maneuver the competition in favorable directions by manipulating the supply adjustment friction. Overall, the findings from the three essays show that marketplaces' operational levers on the supply side have significant effects on the strategies of all participants, which impacts the marketplaces' operational performance. The dissertation offers both theoretical insights on the mechanisms of the studied supply-side levers and practical implications on how these levers should be designed and implemented.

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by

Wenchang Zhang

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Advisory Committee:
Dr. Wedad Elmaghraby, Chair/Advisor
Dr. Kostas Bimpikis, Co-Advisor
Dr. Margrét Bjarnadóttir
Dr. Tunay Tunca
Dr. Michel Wedel
Dr. Leonid Koralov, Dean's Representative

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Chapter 1: Introduction and Overview

The rise of online marketplaces has become one of the predominant economic and social developments of our time. Some marketplaces have become the most prominent players in various markets, such as merchandise, service, transportation, and travel accommodation, once dominated by non-platform companies. Meanwhile, numerous opportunities and challenges regarding the design and operations in online marketplaces are emerging. My research focuses on the supply-side management and design of marketplaces. Since most marketplaces cannot control sellers' behavior directly, they need to rely on other market design levers, such as listing policy and information provision policy, to influence the supply. Moreover, different market structures have different design options available. In each essay of the dissertation, I study the supply-side design of a specific market structure.

In my first essay, we study the revenue impact of listing policy design in online B2B liquidation auction platforms, where salvaging inventory arrives exogenously. We show that platforms can employ listing policies to schedule the ending time of auctions to adjust market thickness (i.e., daily supply), which can further improve revenues. In particular, the high market thickness can incentivize demand, meanwhile, exacerbate cannibalization between auctions. Our structural model enables platforms to identify the optimal market thickness that balances the two competing effects by modeling buyers' behavior and estimating the model primitives. In this study, we establish the critical role of market thickness in B2B auction markets and demonstrate how platforms can use practical operational levers (e.g., listing policies and recommendation systems) to optimize the market thickness.

In my second essay, we build a dynamic model to investigate the effect of information disclosure policies on peer-to-peer service platforms, where the quality of service providers is ex-ante unknown. Due to the information asymmetry, such a platform suffers an under-experimentation issue that the employment rate of new providers is low. The issue then leads to a slow discovery of talented providers and a small proportion of them on the platform. After the platform learns the quality of a provider, it can decide whether or not to disclose it to customers using informational levers. To our surprise, delaying the disclosure of the quality information of some high-quality providers can boost platform revenues. We also identify two effects of the informational delay: experimentation effect and scarcity effect. Our work contributes to the market design literature by identifying the operational value of information disclosure policy in peer-to-peer platforms with quality uncertainty.

In my third essay, we study the effect of sellers' supply adjustment friction on two-sided marketplaces' reactions to unexpected demand shocks using an empirically-validated analytical model. In the model, sellers are heterogeneous in terms of their quality, and they engage in a quantity competition under a given demand. When the demand structure changes, sellers strategically adjust their supply to maximize their profit, incurring a cost for deviating from the original supply level. We find that sellers' strategic responses can either benefit or hurt the marketplace, and adjustment friction is an effective factor in influencing sellers' strategic decisions. By varying the adjustment friction, the marketplace can amplify positive effects under favorable demand shocks and reduce negative effect from unfavorable ones.

To conclude, the findings from the three essays highlight that marketplaces' design levers on the supply side, including listing policy and information provision policy, can have significant operational effects on the marketplace. Moreover, they demonstrate that possible online friction, such as participation cost, information obfuscation, and supply adjustment cost, plays a critical role in affecting the behaviors of all market participants. Through innovative design, online marketplaces can benefit substantially from the friction.

Chapter 2: Managing Market Thickness in Online B2B Markets

Abstract. We explore marketplace design in the context of a B2B platform specializing in liquidation auctions. Even when the platform's aggregate levels of supply and demand remain fixed, we establish that the platform's ability to use its design levers to manage the availability of supply over time yields significant value. We study two such levers, each using the platform's availability of supply as a means to incentivize participation from buyers who decide strategically when/how often to participate. First, the platform's *listing policy* sets the ending times of incoming auctions (hence, the frequency of market clearing). Exploiting a natural experiment, we illustrate that consolidating auctions' ending times to certain weekdays increases the platform's revenues by 7.3% mainly by inducing a higher level of bidder participation. The second lever is a *recommendation system* that can be used to reveal information about real-time market thickness to potential bidders. The optimization of these levers highlights a novel trade-off. Namely, when the platform consolidates auctions' ending times, more bidders may participate in the marketplace (demand-side competition); but ultimately auctions for substitutable goods cannibalize one another (supply-side competition). To optimize these design decisions, we estimate a structural model that endogenizes bidders' dynamic behavior,

i.e., their decisions on whether/how often to participate in the marketplace and how much to bid. We find that appropriately designing a recommendation system yields an additional revenue increase (on top of the benefits obtained by optimizing the platform's listing policy) by reducing supply-side cannibalization and altering the composition of participating bidders.

Keywords: Online markets; Market clearing; Market thickness, Matching supply with demand; Natural experiment; Structural estimation.

2.1 Introduction

The emergence of Internet-enabled platforms, such as Airbnb and Lyft, has highlighted that online marketplaces greatly reduce frictions that previously prevented buyers and sellers from connecting, thereby increasing the volume of trade in a number of markets. Typically, such platforms neither own nor directly control the goods involved in each transaction, but act as intermediaries. Thus, their success relies heavily on the design features of their respective marketplaces, e.g., the ways in which they organize and present information to the buyers and the timing with which they match and clear (portions of) the market.

The opportunity for online intermediaries to create value has manifested itself not only in the cases exemplified by Airbnb and Lyft, but has also reshaped retail operations, particularly with regard to the handling and resale of liquidation inventory. The present paper explores marketplace design in the context of an e-commerce platform specializing in liquidation inventory including merchandise that either remained unsold in its primary market (e.g., due to low demand levels) or was returned by customers. The secondary market within which the platform operates is of great economic significance: roughly 20% of inventory goes unsold in the fast fashion industry Fe05, and whereas brick-and-mortar retailers encounter a 9% return rate on products – for online retailers, the return rate is a staggering 30%.[1] Overall, it is estimated that in 2012, the size of this excess/return product market was $424 billion or 2.9% of the entire US GDP. However, given the uncertainty surrounding the volume, quality, and composition of their excess and returned merchandise, retailers have come to expect mere cents-on-the-dollar recovery rates from traditional channels. Thus, they typically offload this inventory to business buyers further down the retail food chain, such as discount stores, or donate it to qualifying recipients for tax purposes.

Online business-to-business (B2B) auction platforms connect an increasing number of retailers to deeper pools of potential business buyers, both domestic and foreign. Given the diversity of potential bidders, which range from large wholesale liquidators to small mom-and-pop stores, online auctions crucially facilitate price discovery and constitute one of the major sale mechanisms in secondary markets. In 2016, Liquidity Service Inc., one of the fastest-growing online B2B auction platforms, sold merchandise worth more than $600 million in aggregate retail value.[2] Today, many US chain retailers, including Best Buy, Walmart, Home Depot, Amazon,

[1]http://www.sdcexec.com/article/12037309/statistics-reveal-8-to-9-percent-of-goods-purchased-at-stores-get-returned-and-25-to-30-percent-of-e- retail-orders-are-sent-back

[2]http://investors.liquidityservices.com/phoenix.zhtml?c=195189&p=irol-reportsannual

Target, and Costco, utilize online B2B auction platforms to liquidate their products.

The amount of inventory sold in these online platforms is highly variable, owing to the uncertain and dynamic nature of when products are returned and when excess inventory is pulled from shelves and made available for resale. In turn, this results in a varying number of auctions being open on the platform at any point in time. The uncertainty in supply coupled with the uncertain valuation of potential buyers, who are downstream resellers with access to different resale channels, implies that liquidation platforms face a familiar operational challenge: how to tailor their design so as to profitably match supply with demand.

One relatively under-explored lever that an auction platform can employ to attain this match is its *listing policy* or, more specifically, the timing of auctions' closing dates. By aligning or, conversely, spreading out the closing dates of auctions, the platform can induce different levels of market thickness. In turn, the level of market thickness has first-order revenue implications for the platform as it determines participation in the marketplace. Specifically, buyers who face uncertainty with regard to actual supply levels on a given day may choose to participate only when they expect adequate availability, implying that even a *fixed* level of aggregate supply should be coordinated and allocated with this uncertainty in mind.

To this end, using a proprietary dataset collected from a leading online B2B platform, we investigate the role and efficacy of the platform's choice of listing policy in inducing different levels of market thickness and coordinating the behavior of market participants so as to influence market outcomes (i.e., the auctions' final prices). Notably, although, as we show, the listing policy alters neither the platform's underlying supply of arriving liquidation inventory nor its pool of potential bidders, its role in incentivizing bidder participation in auctions can be of first-order importance. Exploiting a natural experiment, we find that the platform's listing policy significantly impacts its revenues: implementing a listing policy to concentrate ("batch") auctions' ending times to certain days of the week increases sellers' revenues by 7.3%. Further evidence supports the hypothesis that the existence of market frictions, specifically *participation costs* associated with the bidders visiting the platform and determining their bidding strategies, drives this result.

Our finding underscores the economic significance of inducing the optimal thickness in the marketplace. In doing so, the platform faces the following trade-off. On the one hand, increasing supply availability on a given day (by having more auctions ending on that day) can profitably incentivize demand-side participation. On the other hand, having auctions end on the same day can induce them to cannibalize one another. Put differently, daily marketplace demand curves are ultimately downward-sloping in the quantity supplied on that day.

Prescriptively, we study two relatively simple market design levers available to the platform to profitably calibrate its market thickness: the listing policy and (targeted) recommendations that aim to provide real-time information about the state of the platform to (a subset of) bidders. However, optimizing these levers involves complex demand-side behavior. For this purpose, we develop a structural model of bidders' behavior, i.e., whether and when they choose to participate in the marketplace and in which auctions and how much they select to bid. These decisions are based on bidders' equilibrium beliefs about the supply-side availability of auctions on any given day and the demand-side competition from other bidders. The model also incorporates the dynamics of the platform's bidder pool and the heterogeneity of the bidders' participation costs, valuations, and potential demand. Out-of-sample revenue forecasts obtained from our estimated structural model align closely with the treatment effect as obtained from the natural experiment.

By simulating the model, we demonstrate how to design the platform's listing policy so as to induce the revenue-maximizing market thickness for any underlying level of expected supply. Our approach accounts for uncertainty in the actual realization of supply and bidders' equilibrium participation and bidding decisions. Additionally, in an effort to further reduce marketplace uncertainty and frictions, we also consider a recommendation system that notifies potential bidders of the supply conditions on the auction site. This is motivated by the fact that, although potential bidders may form accurate beliefs about the *expected* number of auctions on the platform on any given day, they typically do not know their exact number prior to visiting the platform itself. The recommendation system is built to inform a (randomly) sampled set of recipients about the realized supply on a day without them having to visit the platform, when the supply is higher than a given threshold. We find that appropriately designing such a recommendation system improves the platform's revenue by an additional 1.6% on the days when the platform sends out recommendations. This gain is achieved by reducing cannibalization during days with high supply and shifting the composition of participating bidders toward those with higher valuations.

2.1.1 Related Literature

Online marketplaces face a number of design challenges when seeking to match supply with demand so as to maximize revenue. A recent stream of papers explores how different aspects of marketplace design may be used to shape the incentives of market participants. For two-sided service platforms, [1] and [2] deliver novel pricing prescriptions based on how users respond to higher service levels. On the other hand, [3] consider pricing for spatially dispersed demand in a ride-sharing network. In an online auction setting, [4] introduce the notion of a fluid mean-field equilibrium and illustrate its practical appeal in setting reserve prices. In addition, [5] show empirically that the platform's revenues increase by 3% when it boosts bids in a customized fashion based on bidders' past behaviors. Our work contributes to this literature by empirically demonstrating the impact of market thickness on bidders' participation and bidding decisions on a platform specializing in B2B auctions.[3] In addition, our focus is mainly on the use of non-price levers that affect the availability of supply-side inventory and, in turn, influence demand.[4]

Recent literature also connects participants' transaction costs and information frictions to outcomes in online markets. [13] studies the role of the search engine in reducing transaction costs and improving matches on Airbnb, while [14] explore the disclosure of product information within online marketplaces. [15] study the revenue

[3] Relatedly, [6] and [7] explore strategic behavior in B2B spot markets while [8] focus on the revenue impact of the lot size in the context of a sequence of online auctions.

[4] In recent work, [9], [10], [11], and [12] explore (non-price) interventions to improve efficiency in the context of matching platforms.

and welfare implications of costs associated with customers monitoring a retailer's online channel for changes in the price and availability of inventory in which they are interested. [16] suggests that introducing a signaling feature that allows workers to indicate availability could increase surplus by as much as 6% in an online labor market. Closer in spirit to our research questions, [17] use data from TaskRabbit, a marketplace for domestic tasks, to empirically demonstrate that the growth of online peer-to-peer markets is largely affected by the thickness they induce. [18] find that higher market thickness actually leads to lower matching efficiency in an online peer-to-peer holiday rental platform.[5] We contribute to this line of work by illustrating how design levers such as the platform's listing policy may lead to a sizable increase in the platform's revenues.

Relatedly, prior work explores both the effects of inventory availability on strategic demand and the related benefits of reducing buyers' uncertainty about availability. [20], [21], and [22] consider settings where prospective buyers incur a search or opportunity cost when visiting physical stores with the intent to purchase a product, if it is available. The seller decides on inventory levels, potentially across stores, while customers form beliefs about the resulting availability. Moreover, [21], [23], and [24] examine how the seller may benefit from reducing consumers' uncertainty about its inventory availability. Empirically, [25] study the impact of sharing inventory information on consumer behavior through credible "buy online, pick up in store" offers, concluding that brick-and-mortar stores drew increased traffic by resolving availability risks. In contrast to these settings, the online platform

[5] Relatedly, [19] illustrates the potential for welfare losses in large markets.

that we focus on can neither set prices nor control the arrival of inventory to its marketplace. Nonetheless, we highlight that listing policies and state-contingent recommendations (communicated to a subset of bidders) can successfully complement the platform's efforts in boosting the bidders' participation rates.

Finally, our paper falls within the growing body of literature employing structural estimation methods to study auction markets (e.g., [26], [27], and [28]) and to address questions of operational interest (e.g., [29], [30], and [31]).

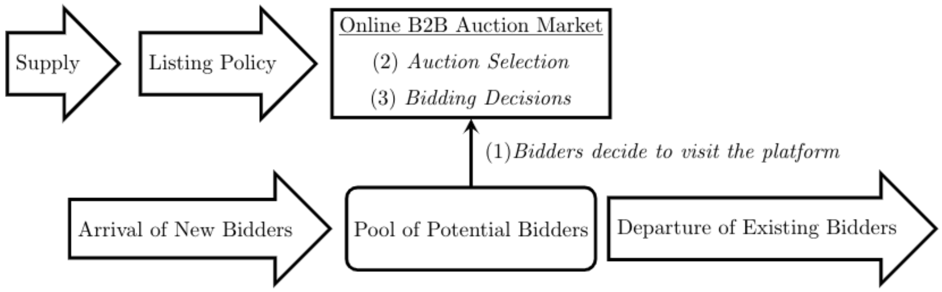
2.2 Data and Background

Our dataset was obtained from a leading online platform managing private B2B auction markets for the liquidation inventory of more than thirty US big-box retailers, such as Costco, Walmart, Sears, and Home Depot (henceforth referred to as *sellers*). In contrast to traditional two-sided online markets, where a multitude of sellers enter and exit freely, each seller on our platform is associated with a private online auction market through which only its own merchandise (excess or returned inventory) can be sold. Under a long-term contract, the platform supports each seller's online auction presence in exchange for a fixed commission taken out of the generated revenues. Sourced from customer returns, trade-ins, and unsold items, the supply of liquidation inventory reaching the platform can be highly stochastic and is beyond the platform's control; rather, it is primarily driven by a seller's reverse logistics. In other words, the supply of these B2B auction markets can be considered exogenous to the platform's design.

Merchandise is sold through an ascending English auction with proxy-bidding (similar to the format used on eBay). Each auction lasts for 1 to 4 days and offers a bundle of similar products for sale in an all-or-nothing auction. While electronics account for most of the platform's annual revenues, which are in the hundreds of millions of U.S. dollars, a broad range of product categories are auctioned on the site, including household appliances, furniture, and apparel. A typical auction lot contains a box of goods from the same product category and in roughly the same condition (e.g., unused, or used and in good condition). Bidders may access the auction's *manifest*, which provides a brief description of the items included in the box. In addition, bidders are able to observe the current second-highest bid (the *standing bid*) and the time remaining in the auction. In contrast to the standing bid, the highest bid currently placed in an auction cannot be observed by bidders.

The demand side of the market consists of downstream resellers specializing in liquidation inventory (henceforth referred to as *bidders*). Reflecting bidders' individual downstream resale channels, both bidders' valuations and levels of demand (i.e., the number of auctions they intend to win) are substantially heterogeneous (see [32]). Bidders need to go through a separate registration process for each such private market in order to be able to view the market's available inventory and submit bids. Finally, the pool of active bidders features the continuous arrival of new registrants and the continuous exit of some existing bidders, both of which are driven by the demand of downstream secondary markets worldwide. Figure 2.1 summarizes the market dynamics and bidders' major decisions within the platform.

Figure 2.1: Market Dynamics and Bidders' Decisions on a Given Day



2.2.1 Markets for iPhones

In what follows, we empirically examine whether the choice of listing policy has any impact on the platform's revenues. To this end, we use data on the iPhone auctions from the platform's two major mobile phone sellers. We restrict attention to iPhone auctions for two reasons. First, to identify the listing policy's revenue impact, we must carefully control for the characteristics of the particular products sold in each auction; iPhones are well-defined products with retail values that are derived in a straightforward way from their observable specifications, such as their model, carrier, condition, and time since the model's release date. This is contrary to other merchandise sold on the platform, such as furniture and household appliances, whose retail values depend on a large set of somewhat subjective features. Second, the revenue generated by iPhone sales alone accounts for approximately 73% of the revenue of the two major cell phone markets we study. We call these two major markets Market A and Market B.

Covering February 2013 to October 2015 (our *observation period*), our dataset tracks the entire bidding history (i.e., every bid's time of submission and dollar amount) in each of the 679 auctions in Market A and each of the 497 auctions in Market B. Moreover, we are able to track bidders' behavior across all auctions in the observation period.

Table 2.1 summarizes the auctions and bidding activity in the two markets. On average, 211 new bidders register per month in Market A, and 261 do so in Market B. Auction lot sizes vary substantially, both within markets (the corresponding coefficients of variation are 0.54 for Market A and 0.53 for Market B) and across markets (the average number of iPhones per auction is 150 in Market A versus 62 in Market B). Participation per auction, whether tallied in bidders or bids, tends to be higher in Market B's auctions. Nonetheless, Market A exhibits slightly higher average per-device revenues ($116.10) than Market B ($106.60).

2.2.2 Bidders

More than 2,200 bidders placed at least one bid in either Market A or Market B during the observation period. In line with prior work on B2B markets Ba04, Pi16 the markets' bidder pools consist of experienced resellers – certified and registered in the market – that are heterogeneous in both their demand profiles and valuations. Over 30% of bidders in both markets exhibit demand for multiple auctions. More specifically, these *multi-unit* (MU) bidders either submit winning bids for two or more concurrent auctions or submit a bid for a new auction shortly after winning an auction. The remaining bidders, whom we call *unit-demand* (UD) bidders, exhibit demand for winning only a single auction lot within our observation period. Though

Table 2.1: Summary Statistics of Auctions in Markets A and B (means with standard errors in parentheses)

	Market A	Market B
Auction duration (in days)	2.60 (1.54)	2.90 (1.09)
Auction lot size (devices)	150.3 (81.0)	61.5 (32.7)
Avg. monthly registrations	211.0 (44.6)	260.7 (63.2)
Number of bidders per auction	4.96 (2.09)	8.20 (2.62)
Number of bids per auction	19.6 (12.1)	27.0 (15.9)
Avg. number of auctions per auction ending day	2.07	1.16
Total number of auctions	679	497
Avg. final per-device price ($)	116.10 (49.97)	106.57 (51.47)

multi-unit bidders are fewer in number, their behavior affects the operations of the platform substantially, as they win over 80% of the auctions in both markets during our observation period.

In addition to their demand characteristics, bidders can also be classified by their registration status in the two markets. We refer to those who are registered in both markets as *cross-market* bidders, and those who are registered in only Market A or Market B during our study period as *Market A* or *Market B* bidders, respectively. Cross-market bidders can observe and participate in auctions from both markets. On the other hand, Market A and Market B bidders can observe auctions only from the single market in which they have registered. In the data, cross-market bidders account for 20% of the bidder population.

In behavior similar to "sniping" in B2C auctions Ba03, we find that bidders predominantly submit bids on an auction's last day. Auctions typically close within the time window from 6pm to 8pm, and most bids are placed during that time. As we report in Table 2.2, the median last bid per bidder per auction comes quite late in both markets (after 99.0% of the auction's total duration in Market A and 80.3% of the auction's total duration in Market B). Moreover, the median winning bid in both markets arrives when 99.7% of the auction's duration has elapsed (similar to 98.3% in Ba03). Because an auction's final price materializes toward its end, we focus on each auction's ending day – as opposed, for example, to the entire time it is open – when studying the impact of the platform's design on revenues.

Table 2.2: Summary Statistics about Bidder Types and Bidding Activity

	Market A	Market B
Percentage of MU bidders (%)	36.4	33.2
Percentage of auctions won by MU bidders (%)	91.9	73.8
Avg. number of auctions a MU bidder participates in	19.65	12.47
Avg. number of auctions won by a MU bidder ever	4.19	1.76
Avg. number of auctions a UD bidder participates in	2.45	1.69
Avg. number of auctions won by a UD bidder ever	0.29	0.12
Median of normalized time of first bid per bidder-auction (%)	97.2	60.0
Median of normalized time of last bid per bidder-auction (%)	99.0	80.3
Median of normalized time of winning bid per auction (%)	99.7	99.7

Table 2.3: Empirical Distribution of Inventory Arrivals (i.e., Auctions' First Day Listed) by Day of Week

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Market A	16.7%	23.5%	20.4%	17.8%	16.3%	5.2%	0.0%
Market B	16.7%	21.4%	20.0%	17.3%	21.8%	3.0%	0.0%

2.2.3 Listing Policies

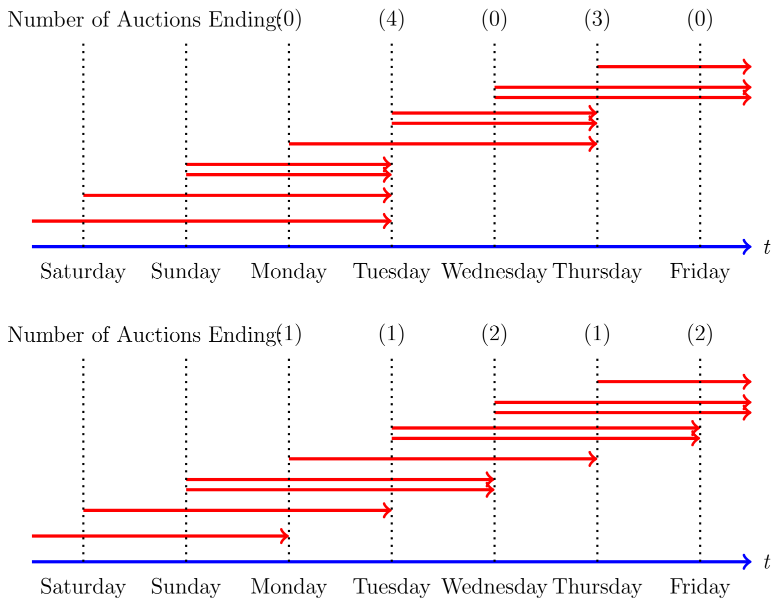
In both Markets A and B, as soon as the seller makes an inventory lot available to the platform (the inventory's *arrival time*), a corresponding auction commences and is listed on the platform. When it lists the auction, the platform must decide and display the auction's ending time. The policy governing how auctions' ending times are determined is the platform's *listing policy*.

Uniform listing. Market B's auctions typically close 3 business days after the time they are listed on the platform (which is the time when the corresponding inventory is made available by the seller). Owing to the fact that the arrival times of inventory are approximately uniformly distributed across the weekdays in both markets (Table 2.3), the number of auctions expected to close is approximately constant across the weekdays. We refer to this listing policy, which sets auction ending times to enforce a fixed duration, as *uniform* listing. Market B uses uniform listing throughout our observation period, while Market A uses it from February 2013 to February 2014.

Batch listing. By contrast, in the second half of our observation period starting in November 2014 (see Section 2.3 for more details), Market A's auctions closed only on Tuesdays and Thursdays. Consequently, the duration for which an auction remains open on the platform varies depending on the arrival time of its associated inventory lot. We use the term *batch* to refer to the listing policy that closes auctions only on Tuesdays and Thursdays.

Effects of different listing policies. Even when receiving the same stream of inventory arrivals, different listing policies result in several salient divergences in outcomes. Figure 2.2 illustrates using the uniform and batch listing policies as examples. First, as shown in Figure 2.2, the number of days on which auctions close is different: two days of the week under batch listing, compared to five days under uniform listing. We call such days the platform's "auction-clearing days." Second, it naturally follows that the average number of auctions cleared on one of the platform's auction-clearing days will depend on the platform's choice of listing policy. Given that bidders predominantly bid in an auction on its last day (Table 2.2), different listing policies result in different levels of supply-side availability of auctions for the bidders, and they induce different levels of thickness in the market. As an example, Figure 2.2 illustrates that compared to uniform listing, batch listing maintains a higher level of supply-side availability on days when auctions are scheduled to close.

Figure 2.2: Batch (above) and Uniform (below) Listing Examples (red arrows represent active auction listings)



2.3 Listing Policies, Market Thickness, and Platform's Revenues

Prior work has studied both empirically and theoretically how to design an (online) marketplace so as to induce a better match between supply and demand. Yet, the primary focus has mostly been on providing incentives for additional supply to join the marketplace (e.g., through surge pricing in ride-hailing platforms) or on smoothing out supply to match exogenous demand. By contrast, in our setting the aggregate levels of supply (incoming inventory) and potential demand (bidder pool) do not seem to be affected by the design decisions we consider. Instead, our analysis illustrates the potential merits of appropriately managing the *effective* availability of supply when the demand side endogenously determines *when* to actively participate in the marketplace.

By exploiting a natural experiment using Market B as a control, we find a substantial benefit attached to Market A's change in listing policy. We estimate that implementing batch instead of uniform listing boosts Market A's revenues by 7.3%, amounting to roughly \$3.8M annually. Our findings suggest that bidders face participation costs that cause them to strategically consider when to participate in the marketplace based on the supply-side availability they expect (i.e., number of active auctions). In other words, much like the platform offering service-level guarantees, assuring adequate market thickness attracts participation. More broadly, the fact that coordinating a fixed supply process can profitably incentivize marketplace participation, points to a novel operational trade-off. On the one hand, thickening the market increases demand-side participation, but, on the other hand, it induces supply-side cannibalization as substitutable auctions compete against one another. In the parlance of classic supply and demand theory, the downward-sloping demand curve dictates that market-clearing prices ultimately fall as the quantity supplied by the market grows.

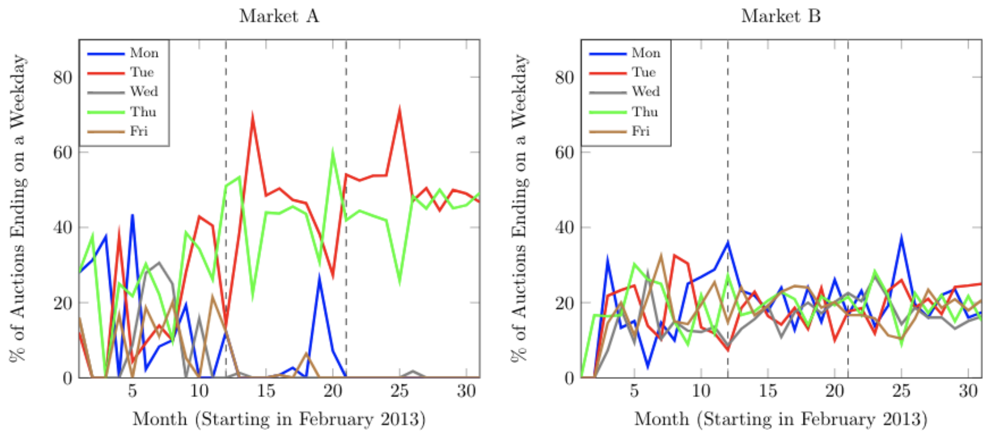
As with any field data, our natural experiment possesses considerable richness; that said, we would like to point out one potential limitation. While our data permits us to carefully account for individual auctions' characteristics and for depreciation in the auctioned phone models (the latter is shared in the form of trends by the two markets), our study is not a perfect natural experiment: a minority of bidders are cross-registrants who bid in both Markets A and B. However, we show that their presence *dilutes* the treatment effect of batch listing as compared to the case of completely independent markets; this is because these cross-registered bidders act as arbitrageurs who bring the treated and control prices closer together (Appendix 2.8.3). Besides, we discuss how cross-registrants' participation is consistent with the hypothesis of participation costs in Section 2.3.4 and Appendix 2.8.5.

2.3.1 Natural Experiment by Change in Listing Policy

Before February 2014, both markets enacted uniform listing and cleared an approximately equal number of auctions on each weekday (see Section 2.2.3). In February 2014, Market A had a change in personnel; management reports that this change was unrelated to the performance of the platform or its marketplaces. Nonetheless, the change triggered Market A to alter its listing policy twice. First, in February 2014, it constrained its auctions to close only on 3 days of the week (Mondays, Tuesdays, and Thursdays). Then, in November 2014, it adopted the batch policy described in Section 2.2.3 to close its auctions only on Tuesdays and Thursdays. Meanwhile, Market B's listing policy remained unaltered throughout the observation period.

Because we are interested in the effect of supply-side availability (and induced market thickness) on participation and revenues, we study a natural experiment: batching the listing policy serves as the relevant treatment of interest. We observe two clearly defined periods: the *pre-treatment* period (February 2013 to February 2014), during which both markets practiced uniform listing, and the *post-treatment* period (November 2014 to October 2015), during which Market A alone batch-listed its auctions. Figure 2.3 depicts the monthly percentages of auctions closing

Figure 2.3: Percentage of Auctions Ending on Each Weekday Aggregated by Month in Market A (*Left*) and Market B (*Right*)



on each of the 5 weekdays, aggregated monthly for each of the Markets A and B: we observe the changes over the pre-treatment and post-treatment periods, as well as the intervening time period.

In addition to the market's listing policy, an auction's per-device revenue is affected by the attributes of the iPhones in the auction, market-level characteristics such as the supply process and the registration rate of new bidders, temporal effects that encompass new product releases, depreciation, and price fluctuations in the overall iPhone market. Observing both markets' pre-treatment period allows us to account for unobserved differences between the two markets. In the data, we note that the differences in these market-level features between the two markets remain roughly constant over time.[6] In Table 2.4, we summarize the numbers of auctions and the newly registered bidders per week for both markets across the pre-

[6]Our average treatment effect analysis explicitly controls for auction-level characteristics such as auction size and product types, as detailed in Section 2.3.3.

Table 2.4: Weekly Supply and Demand Profiles in Markets A and B within Pre- and Post-treatment Periods

	No. of weekly auctions		No. of weekly registrations	
	Market B	Market A	Market B	Market A
Pre-treatment Period	2.49	2.93	53	48
Post-treatment Period	5.11	6.34	60	51
Increase Rate (%)	105%	116%	13%	6%

and post-treatment periods.[7] Market A's aggregate supply grew at a slightly faster rate than that of Market B while its pool of potential bidders (demand) grew at a somewhat slower rate than that of Market B.[8] Using the average treatment effect (ATE) methodology reviewed in [33], we estimate the effect of the platform's listing policy on auctions' final prices while controlling for product attributes, temporal effects, and static market-level differences. Within the observation window, we do not observe changes in any of the market design levers except for the listing policy.

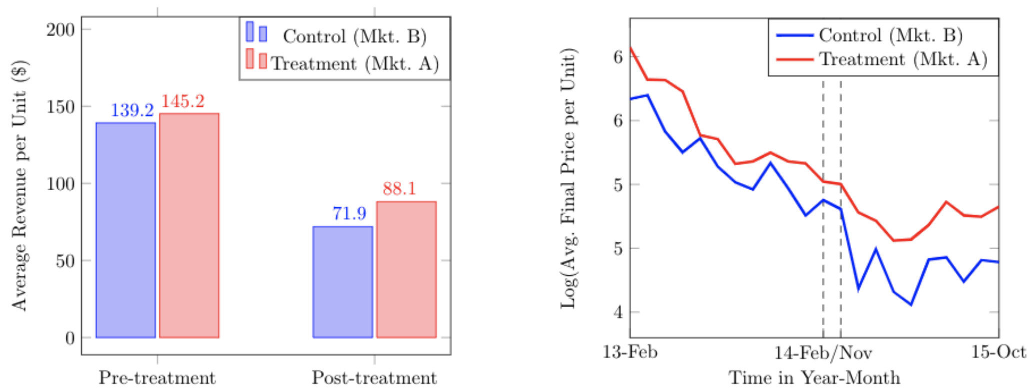
2.3.2 Descriptive Comparison and Difference-in-Differences Analysis

As a primer to the analysis, Figure 2.4 compares the two markets' average, per-device revenues in the pre-treatment and post-treatment periods, respectively.

[7]The trends of the aggregate supply and demand over time are further displayed in Appendix 2.11.1.

[8]According to classic supply and demand theory, we would then expect an increase in Market B's revenues relative to those of Market A. However, we observe the opposite.

Figure 2.4: Averages (a) and Trends (b) of Per-device Prices in Markets A and B across Pre- and Post-treatment Periods



(a) Avg. selling prices across periods (b) Trends of final price per unit in log scale

While iPhone prices fall over time in both markets (as the phone models depreciate), the left plot of Figure 2.4 highlights an emerging gap in the markets' per-device revenues during the post-treatment period relative to the pre-treatment period, consistent with the batch listing policy affording Market A a post-treatment boost in per-device revenue. The right plot of Figure 2.4 shows that the pre-trends in both markets are relatively parallel. Notably, the gap between the price trends increases during the post-treatment period.

A difference-in-differences estimator transparently compares the post-treatment revenue difference between the two markets against any that existed in the pre-treatment period. As shown in Table 2.5, the markets' revenue difference increases by 10.1% post-treatment, which we might attribute to the effect of Market A's new listing policy. Because it remains conceivable that the markets' inventory compositions could have changed across the periods, we carry out an average treatment

Table 2.5: Difference-in-Differences Estimate of Policy Switch on Final Price per Unit

	Dependent Variable
	Log (Final Price per Unit)
Treatment: Batch Policy	10.1%***
	(0.027)

*p<0.1; **p<0.05; ***p<0.01
Confidence intervals constructed via bootstrap.

effect analysis that controls for granular differences in the attributes of individual auction lots and for weekly time trends affecting the phones' (hence lots') valuations.

2.3.3 Average Treatment Effect

First, our analysis controls explicitly for observables, such as phone model, carrier, and auction lot size. Additionally, we include a weekly time fixed effect to account for temporal price fluctuations in the broader iPhone market, leveraging that these effects are simultaneously present in both markets. Finally, we exploit the dataset's pre-treatment period, during which both markets used the uniform listing policy, to measure and account for the effect of unobservable differences between the two markets, including the information they provide about the quality of their products.[9]

For this analysis, we assign an auction lot j at time t to the treatment group if it is posted to Market A (indicator variable A_{jt}) and to the control group if it is instead posted to Market B. Based on its observed attributes, each auction lot exhibits a propensity for being assigned to the treatment group, known as its propensity score. As developed in the related literature (e.g., [34–36]), an efficient estimate of the average treatment effect (ATE), τ, that accounts for such attributes can be obtained as the difference between post-treatment and pre-treatment outcomes appropriately weighted using their associated propensity scores. We estimate separate ATEs for each of the pre-treatment and post-treatment periods (τ_{Pre} and τ_{Post}, respectively) and are ultimately interested in the difference between the two. While both τ_{Pre} and τ_{Post} include the net revenue effects of the markets' unobserved differences (e.g., product quality categories, reputations, and bidder compositions), only τ_{Post} captures the additional revenue effect of batch listings under Market A's revised listing policy. Accordingly, the corresponding estimators, $\hat{\tau}_{\text{Pre}}$ and $\hat{\tau}_{\text{Post}}$, are each obtained by Expression (2.8) in Appendix 2.8.1. As a technical aside, relative to employing propensity score methods, mixed-methods approach combines propensity score weighting and a regression model to improve the precision of the resulting estimate Im04. By "double robustness" Wo07, Gr12, the estimator is consistent if either the parametric propensity score model or the outcome regression model is

[9] More than 90% of Market A iPhone auctions are classified into quality "A/B," and more than 99% of Market B iPhone auctions are classified into quality "Used/Functional." In our sample, we focus exclusively on auctions with these two quality types.

correctly specified. In this study, we adopt an estimator from a class of augmented inverse probability weighting (AIPW) estimators introduced in [37] (see Appendix 2.8.1).

2.3.4 Revenue and Participation Effects of Batch Listing

As reported in Table 2.6, we find the per-device revenue effect of the batch listing policy, $\hat{\tau}_{\text{Post}} - \hat{\tau}_{\text{Pre}}$, to be positive and statistically significant. We find that the batch listing policy, in comparison with the uniform listing policy, yields a 7.3% average increase in Market A's per-device revenue (Table 2.6).[10] This estimate translates to more than $3.8M in additional revenues annually for Market A.[11]

Setting a market's listing policy may appear to be an innocuous choice born of convenience, chance, or custom: a priori it is not clear why the platform would expect anything beyond a marginal impact on its revenues, given that its listing policy does not have an impact on either its exogenous supply of auctions or its demand pool of certified bidders. We examine whether higher supply-side availability of inventory (auctions) on the platform actually results in more bidders participating per auction. First, we estimate the effect of the policy switch on the number of observed bidders per auction by applying a similar average treatment effect anal-

[10] We conduct robustness checks to support the validity of our estimates (see Appendix 2.8.2), including examining the distributional overlap of covariates for the treatment and control groups.In Appendix 2.8.2, we carry out robustness checks on the only covariate that exhibits possibly insufficient overlap (i.e., auction lot size).

[11] Since all posted auctions result in sales in our data, the revenue increase is a direct outcome of an increase in the auctions' average final price.

ysis to the one in the section. All explanatory variables remain the same as those in the ATE analysis of the final price. As shown in the last column of Table 2.6, under the batch policy, Market A attracts 0.49 more bidders per auction, on average. Second, we exploit that cross-market bidders can choose when to participate in untreated Market B based on availability in both markets. We find that the cross-market bidders' participation rate in *Market B* on Tuesdays and Thursdays is substantially higher post-treatment (i.e., 84%) than pre-treatment (i.e., 63%).[12] The findings suggest that it is costly for bidders to participate in the auctions; thus, they strategically decide when to visit the platform (see Appendix 2.8.5). Lastly, a difference-in-differences analysis confirms that the switch of the listing policy has little impact on the bidders' bidding amounts (see Appendix 2.11.4), which implies that the listing policy's revenue gain is not driven by changes in how bidders choose to bid. In summary, our findings support the following mechanism to explain the revenue increase: an increase in market thickness boosts bidder participation rates both on aggregate and per auction. In turn, this additional traffic results in higher revenues for the platform. Underlying this mechanism is the fact that it is costly for bidders to visit the platform on a given day, monitor the set of available auctions, and choose whether and how much to bid in each auction.

Despite the positive revenue effect of thickening the market, there exist potential pitfalls. First, batching too many auctions together may cannibalize and

[12] The post-treatment increase in the number of unique participants per week in Market A (167.1%) likewise exceeds the corresponding figure associated with Market B (88.1%). For more details, refer to Appendix 2.8.4.

Table 2.6: Estimated average Treatment Effect of Policy Switch on Final Price per Device and No. of Bidders per Auction

	Dependent Variables	
	Log (Final Price per Device)	No. of Observed Bidders per Auction
Treatment: Batch Policy	7.3%***	0.49**
	(0.009)	(0.22)

*p<0.1; **p<0.05; ***p<0.01

Confidence intervals constructed via bootstrap.

decrease per-device revenues by increasing the number of immediately available substitutes. Appendix 2.11.3 provides empirical evidence of this cannibalization effect. Second, some bidders may be disincentivized from participating on the platform due to the influx of competing bidders. In what follows, we develop a structural model to characterize the optimal market thickness.

2.4 Structural Model

We present a dynamic, structural model endogenizing the bidders' decisions on whether and when to visit the platform, which auction(s) to participate in, and how much to bid. Our discrete-time model captures the behavior of a dynamic pool of potential bidders who are heterogeneous in their valuations and demand profiles while facing an exogenously stochastic supply of liquidation inventory lots arriving to the platform to be auctioned. We assume that there are S_t auctions on Day t each of which is for a single unit whose valuation is appropriately normalized. Given that auction lots differ from one another, we assume that a bidder's valuation for a given auction has an idiosyncratic term. Motivated by our assertion that cross-market bidders only dampen the price impact resulting from batching auctions' ending times (Appendix 2.8.3), we consider only a single market. To capture heterogeneity in the bidders' demand, we define the following two bidder types: (*Bidder Types)* Bidders are risk-neutral and have private values. Each bidder belongs to one of the following two types:

(i) Unit demand (UD): A UD bidder is interested in winning only one auction throughout her *lifetime* on the platform. Each UD bidder is permanently endowed with a private valuation for winning an auction lot that is drawn independently from distribution F^{UD}.

(ii) Multi-unit (MU): A MU bidder is interested in winning multiple auctions on the platform. In particular, we assume that a MU bidder is interested in winning up to K auctions in a single *day*, where $K > 1$ is exogenously specified, regardless of her prior win history.[13] Each MU bidder's endowed private valuation for an auction lot is drawn independently from distribution F^{MU}.

In contrast to UD bidders, who operate on relatively low volumes, MU bidders represent repeat buyers that interact regularly with the platform. To ensure that the model is tractable, we assume that the bidders' private valuations for each type

[13]This assumption on K is critical to making our model and estimation tractable.

follow the Weibull distribution, which fits the data reasonably well. As described in Section 2.2.2, most bidding activity takes place toward the ending time of an auction, and we assume that bidders submit bids only to auctions that are closing on the day the bid is placed. Because more than 90% of the auctions close within the narrow time window between 6 and 8 PM, we do not focus on the timing of visits within the day and assume that if a bidder decides to visit the platform on a given day, the timing of her visit within the day is exogenous. On each day t, the following sequence of events transpires:

(1) *Supply.* First, S_t new auctions are listed on the platform, with S_t following the count distribution P_{Supply}. Each posted auction possesses characteristics affecting its idiosyncratic fit with a bidder's resale channels. Therefore, Bidder ℓ's valuation for an auction lot j is the sum of Bidder ℓ's endowed product valuation, x_ℓ, and an idiosyncratic term, $\zeta_{\ell j}$. Each $\zeta_{\ell j}$ is independently drawn from mean-zero normal distributions F_ζ^{MU} and F_ζ^{UD}, with standard deviations ν^{MU} for MU bidders and ν^{UD} for UD bidders, respectively.

(2) *Platform Participation.* New bidder arrivals join the existing pool of potential bidders, under an exogenous arrival process. If t is an auction-ending day as defined in Sections 2.2.3 and 2.2.2, all potential bidders in the pool must decide simultaneously whether to visit the platform to bid in ending auctions. Notably, while each bidder ℓ knows her own valuation x_ℓ for a standard unit throughout, a bidder does not know either the current state of the platform (i.e., how many auctions are ending that day and/or the current standing

bids in those auctions) or the idiosyncratic component of her lot valuation $\zeta_{\ell j}$ for any listed auction j. Instead, her decision on whether to visit the platform is based on her expected payoff from visiting, which in turn depends on her ex ante beliefs about the likely state of the platform. Formally, upon visiting the platform, the observable state of the market on Day t is $\omega_{\ell t} \triangleq (n_t, \mathbf{s}_{\ell t})$, where n_t is the market thickness (number of auctions ending) on Day t, with support $\mathcal{N} = \{0, 1, ..., \bar{N}\}$, and $\mathbf{s}_t$ denotes the vector of standing bids of the auctions that are ending on Day t. As discussed in Section 2.4.1, the bidder's ex ante beliefs about $\omega_{\ell t}$ anticipate the platform's equilibrium steady-state distribution. Against her expected payoff, bidder ℓ weighs her daily participation cost, $c_{\ell t}$, on Day t, which captures the cost in time and effort at participating on the platform. A bidder's participation cost is drawn independently each day from an exponential distribution with rate μ^{MU} for MU bidders and μ^{UD} for UD bidders. Thus, across bidders and days, whether or not a bidder visits the platform depends both on her endowed valuation x_ℓ and on the day's realized participation cost $c_{\ell t}$.

(3) *Auction Selection and Bidding.* Upon visiting the platform, a bidder observes the realized state $\omega_{\ell t}$ and the idiosyncratic term of her valuation $\zeta_{\ell j}$ for each available auction j. First, she decides which auction(s) to participate in based on her private valuations and her beliefs about the currently highest rival bids given $\omega_{\ell t}$, and she then determines how much to bid in each auction so as to maximize her expected payoff.

(4) *Departure.* Given the nature of their demand, UD bidders depart with certainty upon winning an auction. In addition, we let α^{MU} and α^{UD} denote the daily retention probabilities of those MU and UD bidders who do not win an auction, respectively. In particular, at the end of Day t, each bidder departs the bidder pool with probability $(1 - \alpha^{TY})$, where $TY \in \{MU, UD\}$.

2.4.1 Bidder Strategies and Equilibrium

The size of the market (i.e., Market A averages about 160 active bidders daily) makes it impractical for an individual bidder to fully track her competitors and the history of their actions. Instead, we assume that bidders respond to their steady-state beliefs about their rivals, which are not meaningfully affected by their own actions Ba16. This assumption approximates well a setting that involves a large group of anonymous bidders, with similarities to other assumptions that have been employed in related settings, such as the notions of oblivious equilibrium (e.g., [38]), stationary competitive equilibrium (e.g., [39]), and mean field equilibrium (e.g., [40,41]). Prior to defining the steady-state equilibrium, we first introduce a bidder's optimal actions on a given day, given her beliefs about the underlying market state Ψ and the highest rival bid G.

Platform Participation. A bidder determines whether or not to participate in the platform on a given day so as to maximize her expected payoff. In particular, for MU bidders, the corresponding maximization problem can be reduced to that of a single period, since their payoffs on any given day are independent of their actions on other days. She receives a payoff of zero if she does not visit. Specifically, a MU bidder's platform participation decision is

$$\sigma_{VST}^{MU}(x_\ell, c_{\ell t}) = \begin{cases} 1, & \text{i.e., visit,} \quad \text{if } r^{MU}(x_\ell; G, \Psi) - c_{\ell t} \geq 0 \\ \\ 0, & \text{i.e., wait,} \quad \text{otherwise,} \end{cases} \tag{2.1}$$

where $r^{MU}(x_\ell; G, \Psi)$ denotes the expected payoff for a MU bidder who visits the platform on Day t. For UD bidders, since they have demand for only one auction throughout their lifetime, their actions on a given day affect their future payoffs. Thus, a UD bidder's platform visit decision is

$$\sigma_{VST}^{UD}(x_\ell, c_{\ell t}) = \begin{cases} 1, & \text{i.e., visit,} \quad \text{if } v^{UD}(x_\ell; G, \Psi) - c_{\ell t} \geq \alpha^{UD} v_f(x_\ell; G, \Psi) \\ \\ 0, & \text{i.e., wait,} \quad \text{otherwise,} \end{cases} \tag{2.2}$$

where $v^{UD}(x_\ell; G, \Psi)$ denotes the aggregated payoff for a UD bidder who visits the platform on Day t, and $v_f(x_\ell; G, \Psi)$ denotes her continuation value if she does not exit the platform after Day t.[14]

Auction Selection and Bidding. After a bidder (of type $TY \in \{MU, UD\}$) participates in the platform, she selects which auctions to bid in, denoted by σ_{SLT}^{TY}, and how much to bid in the selected auctions, denoted by σ_{BID}^{TY}. Naturally, an auction's standing and highest rival bids are not independent. Thus, the bidder updates her beliefs over the highest rival bids after observing the corresponding standing bids and the realized market thickness. Under the updated beliefs, σ_{SLT}^{TY} and σ_{BID}^{TY} jointly maximize her expected payoff on the platform on a given day. For

[14]The characterizations of $r^{MU}(x_\ell; G, \psi), r^{UD}(x_\ell; G, \Psi), v^{UD}(x_\ell; G, \Psi)$, and $v_f(x_\ell; G, \Psi)$ are detailed in Appendix 2.9.1.

MU bidders, the corresponding maximization problem is

$$(\sigma_{SLT}^{MU}, \sigma_{BID}^{MU}) = \underset{\sigma_{SLT}, \sigma_{BID}}{\arg\max} \sum_{j \in \sigma_{SLT}} \int_{s_j}^{\sigma_{BID,j}} (x_\ell + \zeta_{\ell j} - p_j) g_j(p_j|\omega_{\ell t}) dp_j, \tag{2.3}$$

where p_j is the highest rival bid in Auction j, and $g_j(p_j|\omega_{\ell t})$ denotes its conditional PDF. The term $\int_{s_j}^{\sigma_{BID,j}^{MU}} (x_\ell + \zeta_{\ell j} - p_j) g_j(p_j|\omega_{\ell t}) dp_j$ stands for the expected payoff from bidding in Auction j. If the bidder chooses Auction j, her optimal bidding strategy is to bid up to her valuation (i.e., $\sigma_{BID,j}^{MU}(x_\ell; \zeta_{\ell j}, \omega_{\ell t}) = x_\ell + \zeta_{\ell j}$). Then, her optimal auction selection decision $\sigma_{SLT}(x_\ell; \zeta_{\ell 1}, ..., \zeta_{\ell \bar{N}}, \omega_{\ell t})$ is to choose up to K auctions with the highest expected payoffs. For UD bidders, the corresponding maximization problem is

$$\begin{aligned}(\sigma_{SLT}^{UD}, \sigma_{BID}^{UD}) = \underset{\sigma_{SLT}, \sigma_{BID}}{\arg\max} \sum_{j \in \sigma_{SLT}} \int_{s_j}^{\sigma_{BID,j}} & (x_\ell + \zeta_{\ell j} - p_j) g_j(p_j|\omega_{\ell t}) dp_j \\ & + \alpha^{UD} \big(1 - G_j(\sigma_{BID,j}|\omega_{\ell t})\big) v_f(x_\ell; G, \Psi),\end{aligned} \tag{2.4}$$

where $G_j(p_j|\omega_{\ell t})$ denotes the conditional CDF of the highest rival bid in Auction j. The first term in Expression (2.4) is equal to the instantaneous payoff if the bidder wins Auction j, where we establish that $g_j(p_j|\omega_{\ell t})$ is independent of the vector of optimal bids (Appendix 2.9.2). The second term is equal to the bidder's (expected) payoff if she does not win the current auction (which occurs with probability $1 - G_j(b_{\ell j}|\omega_{\ell t})$). Then, we can show that her optimal bid in Auction j is

$$\sigma_{BID,j}^{UD}(x_\ell; \zeta_{\ell j}, \omega_{\ell t}) = x_\ell + \zeta_{\ell j} - \alpha^{UD} v_f(x_\ell; G, \Psi), \tag{2.5}$$

and her optimal auction selection $\sigma_{SLT}^{UD}(x_\ell; \zeta_{\ell \cdot}, \omega_{\ell t})$ is to choose the auction with the highest payoff.[15]

[15]As expected, UD bidders shade their bids by their continuation values. As they are forward-

Note that the bidders' decision whether to visit the platform on any given day largely depends on their beliefs about the steady state of the market (i.e., Ψ) on that day. On the other hand, once a bidder is already on the platform and has observed the number of auctions closing on that day, her decisions regarding which auction(s) to participate in and how much to bid are mainly driven by her beliefs about the highest rival bids (i.e., G). Thus, anticipated market thickness has a more direct impact on a bidder's decision whether to visit the platform compared to which bidding strategy to use. Formally, the notion of steady-state equilibrium we employ is defined as follows. [Equilibrium] A steady-state equilibrium is a tuple $(\{\sigma^{MU}, \sigma^{UD}\}, \{G, \Psi\})$ such that:

- (*Optimality*) For bidder ℓ with type $TY \in \{MU, UD\}$, her best response comprises three decisions on day t; that is, it takes the form $\sigma^{TY}(x_\ell; c_{\ell t}, \zeta_{\ell \cdot}, \omega_{\ell t}) = [\sigma^{TY}_{VST}, \sigma^{TY}_{SLT}, \sigma^{TY}_{BID}]$, where $\sigma^{TY}_{VST}, \sigma^{TY}_{SLT}$, and σ^{TY}_{BID} are defined by Expressions (2.1), (2.2), (2.3), and (2.4), respectively, given the steady-state distributions for the market state Ψ and the highest rival bids G.

- (*Consistency*) Steady-state distributions $\{G, \Psi\}$ are induced by bidders following strategy $\sigma^{TY}(x_\ell; c_{\ell t}, \zeta_{\ell \cdot}, \omega_{\ell t})$, $TY \in \{MU, UD\}$.

In equilibrium, both Ψ and G are functions of the market design (e.g., listing policy). A market design lever impacts the platform's revenues by influencing bidders' beliefs about the market state and the competition level. With the notion

looking and realize that their purchases today come at the expense of winning an auction in the future, they implicitly discount their willingness to pay for a present auction.

of how bidders behave in equilibrium, we then establish that an equilibrium exists under a (mild) technical assumption. Assume that $\nu^{MU} = \nu^{UD} = 0$ and $\bar{N} \leq K$. Then, an equilibrium exists. The proof of Proposition 2.4.1 is in Appendix 2.9.3. As stated in the proposition, equilibrium existence is shown under the assumption that bidders have the same valuation for all auctions and the daily demand of MU bidders is sufficiently large. For the general case where the assumptions of the proposition are relaxed (i.e., $\nu^{MU} \geq 0$, $\nu^{UD} \geq 0$, and $\bar{N} > K$), we provide an algorithm that efficiently converges to the equilibrium (see Appendix 2.11.5), which, in turn, generates a broad set of counterfactuals (see Section 2.6). It is also worthwhile to highlight that given Ψ and G, all of the bidders' decisions can be expressed analytically. This is crucial in enabling us to structurally estimate the model in the following section.

2.5 Structural Estimation

In this section, we outline our structural estimation approach and present our estimates. More specifically, we estimate our structural model on Market A's data exclusively from February 2013 to February 2014 (i.e., the pre-treatment period), throughout which the uniform listing policy was used. By doing so, we are able to: *(i)* validate our structural model by deriving out-of-sample projections for Market A's pre-treatment-to-post-treatment revenue improvement, which we compare against our ATE estimate from Section 2.3.3; and *(ii)* reserve the post-treatment period's data for use in our counterfactual market design analysis.

As is common in the auction literature Ba16, we first normalize the observed bids to adjust for heterogeneity in the products' features and for time fixed effects. As empirically supported in Appendix 2.10.1, bidders freely substitute between the iPhone models available on the platform (i.e., iPhone 4, iPhone 4s, and iPhone 5) after adjusting for differences in their valuations. We likewise treat the iPhone models as substitutes after appropriately normalizing the bids. Post-normalization, bidders' valuations are treated as drawn from a common Weibull distribution throughout the observation period.[16]

The structural estimation follows two steps: we conduct (1) a nonparametric estimation of the platform's steady-state distribution of auctions and bids, followed by (2) a maximum simulated likelihood (MSL) estimation of our modeling primitives for bidders' valuation distributions, participation costs, and retention rates. The pre-treatment period lasts for a year, so it is a fair assumption that Market A's pre-treatment period data are generated from a steady-state equilibrium. The corresponding equilibrium distributions Ψ and G are directly estimated in step (1). Notably, Ψ and G are equilibrium outcomes rather than exogenously specified model primitives. Using the iterative algorithm detailed in Appendix 2.11.5, they are recomputed numerically for each counterfactual scenario corresponding to a given market design (e.g., listing policy) in Section 2.6.[17]

[16] We shift the normalized bids to ensure that they fall within the support of the Weibull distribution.

[17] If multiple equilibria exist, two-step estimation remains consistent under the assumption that a single equilibrium is played. The "correct" equilibrium play is recovered directly from data in step one, and the model primitives are identified by bidders' strategic best responses to such

2.5.1 Estimating the Platform's Steady State

The steady-state belief about the market consists of the distribution of the market thickness as well as the distribution of the corresponding standing bids. In particular, it takes the form of $\Psi(\omega_{\ell t}) = P_{MKT}(n_t)\psi(\mathbf{s}_{\ell t}|n)$, where P_{MKT} is estimated by the empirical distribution of auctions ending on a given day. On the other hand, the PDF of standing bids $\psi(\mathbf{s}_{\ell t}|n)$ conditional on n is estimated using kernel density estimator to minimize misspecification bias.

Furthermore, a bidder forms beliefs about the highest rival bids given a market state $\omega_{\ell t}$. Recall that $g_j(p_j|\omega_{\ell t})$ denotes the PDF of the highest rival bid in the j^{th} auction given the state $\omega_{\ell t}$. Again, we use a kernel density estimator for $g_j(y|\omega_{\ell t})$ to mitigate misspecification bias. The kernel density estimators for $\Psi(\omega_{\ell t})$ and $g_j(p_j|\omega_{\ell t})$ are detailed in Appendix 2.10.2. Lastly, we let the upper bound K on the MU bidders' demand be equal to 14 auctions (i.e., the maximum number of auctions in which a MU bidder submitted a bid on a single day in our observation period).[18]

2.5.2 Estimating the Bidders' Primitives

Before describing in detail how we estimate them, we reiterate that the model primitives for a bidder of type $TY \in \{MU, UD\}$ include: (i) her endowed valuation of the product F^{TY} that is drawn from the Weibull distribution with scale parameter λ^{TY} and shape parameter γ^{TY}; (ii) her daily participation cost that is exponentially

equilibrium play.

[18]During our observation period, no bidder submits bids to more than 14 auctions on any single day.

distributed with rate μ^{TY}; and (iii) her daily retention rate α^{TY} that represents her probability of remaining in the bidder pool for another period. As we argue in Appendix 2.10.4, our dataset exhibits sufficient variation to identify the above primitives.

Given a bidder's endowed valuation and her beliefs over the market state and the highest rival bids (which are estimated in Section 2.5.1), her decisions, including whether and when to visit the platform, which auction(s) to participate in, and how much to bid, can be analytically obtained as we described in Section 2.4. This, in principle, allows us to recover all model primitives in an efficient manner by iteratively obtaining the likelihood of a given equilibrium outcome as a function of the set of model primitives using simulation and updating the primitives accordingly.

Bidder ℓ's observed bidding history $\mathbf{X}_\ell$ consists of two parts. The first part is her participation sequence, $\mathbf{B}_\ell = [B_\ell^{t_\ell}, ..., B_\ell^{t_\ell + l_\ell}]$, where $B_\ell^{t_\ell} \in \{0, 1\}$ indicates whether she placed a bid on Day t (here, t_ℓ and $t_\ell + l_\ell$ denote the first and last observed bidding days for the bidder in our sample). The second part comprises her bids on Day t, $\mathbf{b}_{\ell t}$, in the auctions she entered and the standing bids, $\mathbf{S}_{\ell t}$, in the auctions available but not entered within the period from Day t_ℓ to Day $t_\ell + l_\ell$.

Given her endowed valuation x_ℓ and her presence in the bidder pool, we specify the likelihood of each of her observed behaviors. The following formulas apply to both types of bidders; thus we omit the superscripts specifying the bidder type. On Day t, observing a bid by Bidder ℓ implies that she *(i)* visits the market (which occurs with probability $P^V(X_\ell|\theta)$) and *(ii)* places bid $\mathbf{b}_{\ell t}$ (which has likelihood $L_{\ell t}^B(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)$). Thus, the likelihood of her placing bid $\mathbf{b}_{\ell t}$ is

$P^V(X_\ell|\theta)L^B_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)$. On the other hand, if no bid is observed on that day, there are two possibilities: *(i)* she chooses not to visit the platform (which occurs with probability $1 - P^V(x_\ell|\theta)$), or *(ii)* she visits the platform but finds it optimal to not place a bid (which occurs with probability $P^V(x_\ell, \theta)L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)$). Thus, the corresponding likelihood of not observing a bid is $1-P^V(x_\ell|\theta)+P^V(x_\ell|\theta)L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)$.[19] In addition, we specify the likelihood associated with her exit from the bidder pool. Given that the day when she exits from the bidder pool cannot be observed, we assume that she leaves the bidder pool on any day within E days[20] of her last bid (we then take the expectation over the likelihoods of all possible exit days). In summary, the overall likelihood of a bidder's entire bidding history conditional on her endowed valuation x_ℓ is given by $\mathcal{L}_\ell(\mathbf{X}_\ell|x_\ell, \theta) =$

$$\Bigg(\alpha^{l_\ell} \prod_{t=t_\ell}^{t_\ell+l_\ell} \big(\underbrace{P^V(x_\ell|\theta)L^B_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)}_{\text{Placing the bid(s) } \mathbf{b}_{\ell t} \text{ on day } t}\big)^{B^{t_\ell}_\ell} \big(\underbrace{1 - P^V(x_\ell|\theta) + P^V(x_\ell|\theta)L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)}_{\text{Not placing a bid on day } t}\big)^{1-B^{t_\ell}_\ell}\Bigg)$$

$$\cdot \underbrace{\Bigg(1 + \sum_{t'=1}^{E-1} \alpha^{t'} \prod_{t=t_\ell+l_\ell+1}^{t_\ell+l_\ell+t'} \big(1 - P^V(x_\ell|\theta) + P^V(x_\ell|\theta)L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)\big)\Bigg)(1-\alpha)}_{\text{Exiting bidder pool within } E = 14 \text{ days since the last bidding day}}. \tag{2.6}$$

As the bidder's valuation x_ℓ is not observed, we need to consider the unconditional likelihood function, $\mathcal{L}_\ell(\mathbf{X}_\ell|\theta) = \int_{x_\ell} \mathcal{L}_\ell(\mathbf{X}_\ell|x_\ell, \theta)f(x_\ell|\lambda, \gamma)dx_\ell$, where $f(x_\ell|\lambda, \gamma)$ denotes the PDF of the Weibull valuation distribution. As $\mathcal{L}_\ell(\mathbf{X}_\ell|\theta)$ has no closed-form expression, the MLE approach is computationally intractable. To overcome this issue, we construct the simulated likelihood function $\hat{\mathcal{L}}(\mathbf{X}_\ell|\theta)$ by employing Monte

[19] Expressions for $P^V(X_\ell|\theta)$, $L^B_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)$, and $L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)$ are detailed in Appendix 2.10.4.

[20] Our estimates are obtained under $E = 14$ days. We have tested other values (e.g., $E = 21$ days and 28 days) and found that the resulting estimates do not differ significantly.

Carlo integration.[21] The MSL estimate $\hat{\theta}_{MSL}$ is obtained by maximizing the log of the simulated likelihood of all bidders, namely $\hat{\theta}_{MSL} = \arg\max_\theta \sum_\ell log\big(\hat{\mathcal{L}}_\ell(\mathbf{X}_\ell|\theta)\big)$.

2.5.3 Estimation Results

Table 2.7 reports our estimates for MU and UD bidders, including their valuation distribution, average daily participation cost, and daily retention probability. These estimates highlight several differences between MU and UD bidders. On average, MU bidders possess higher endowed valuations than UD bidders, but exhibit substantially lower variability in idiosyncratic valuations within-type across individual bidders. MU type's average valuation is higher by \$20.86 per unit (the average auction lot size in Market A is 150.3 units), which is both statistically and economically significant. By comparison, UD bidders exhibit more variability in their idiosyncratic terms. Despite the non-dominance of valuation components across MU and UD bidders, UD bidders shade their bids (Expression (2.5)), which contributes to MU bidders winning the majority of auctions.

Per daily platform visit, MU bidders incur a substantially higher average participation cost of \$107.52 compared with \$70.64 for UD bidders. On the other hand, MU bidders tend to have a lower *per-auction* participation cost, as they typically participate in multiple auctions within a day. Evidence suggests that the MU bidders' higher cost to visit on a day is associated with having to review, compare, and match downstream channels for multiple auctions: MU bidders, on average, spend

[21] For additional discussion on $\hat{\mathcal{L}}_\ell(\mathbf{X}_\ell|\theta)$, refer to Appendix 2.10.4.

87.8 minutes on the platform per day, compared with 75.9 minutes for UD bidders.[22] Depending on the profit margin of the bidders' downstream resale channels, participation costs in the order of $100 can be substantial, thus affecting their incentives to participate on the platform.

To validate the model, we compare the predicted distributions of the number of bidders per auction and the final price in the post-treatment period with those observed in the dataset. Our model is fairly accurate in predicting both distributions (see Appendix 2.10.5).

2.6 Implications for Platform Design

We explore how our findings from Section 2.5 lead to implications for platform design, with a focus on relating the performance of different listing policies to their induced levels of market thickness.

A platform's listing policy influences revenues by manipulating the induced market thickness. Separate strands in the existing literature contemplate dual, but countervailing, effects from market thickness/availability on the behavior of potential buyers and on revenues. For example, using a dataset of notebook auctions,

[22] As a side remark, note that MU bidders incur higher participation costs than UD bidders. This may be due to the fact that MU bidders have to process more information given that they are bidding and monitoring multiple auctions simultaneously. Similarly, it is plausible that cross-market bidders incur higher participation costs than single-market bidders, as they have to switch between marketplaces and process additional information while cross-bidding. Therefore, the presence of cross-market bidders in the data tends to bias our participation cost estimates upwards.

Table 2.7: Maximum Simulated Likelihood Estimates for the Primitives of the Structural Model

	MU Bidders	UD Bidders
Valuation *per auction* (x)		
Mean	\$18,594	\$15,459
	(\$138)	(\$203)
Standard Deviation	\$876	\$897
	(\$127)	(\$164)
Idiosyncratic Error (SD) (ν)	\$2,978	\$4,313
	(\$72)	(\$244)
Avg. Daily Participation Cost (μ)	\$107.52	\$70.64
	(\$15.38)	(\$10.06)
Retention Rate (α)	0.968	0.843
	(0.001)	(0.014)
Number of Bidders	74	113

Note: Standard deviations of the estimates are in parentheses. Valuations are at normalized scale.

Ch07 suggest that increasing the number of concurrently available and substitutable products reduces bidders' willingness to pay by up to 10.2%. On the other hand, the operations literature suggests that higher product availability may increase the seller's revenue by stimulating demand (e.g., [21]). In a matching context, Ga16 present evidence that thicker markets enhance efficiency by raising matching probabilities. In this subsection, we decompose and analyze the contending effects of supply-side cannibalization and demand-side participation in response to market thickness. Within this framework, we study how market thickness can be adjusted through the platform's listing policy to balance these effects.

In simulating counterfactuals, we evaluate the performance of four listing policies that differ in how auction ending times are distributed throughout the week under six different levels of incoming supply (which together induce a market thickness level). The four listing policies are

(i) Uniform: The number of auctions ending on each weekday is roughly the same.

(ii) Three-Day Batch: Auctions end only on Mondays, Wednesdays, and Fridays.

(iii) Batch: Auctions end only on Tuesdays and Thursdays.

(iv) Single-Day Batch: Auctions end only on Wednesdays.

Given the same supply level, market thickness on auction-ending days increases by switching from policy (i) to policy (iv). We consider six supply levels, denoted by "$\frac{1}{3}$x," "$\frac{1}{2}$x," "1x" (baseline), "2x," "3x," and "5x." In particular, we first derive our "baseline" supply case by fitting Market A's supply data to a gamma distribution to appropriately account for variability observed in the data. To derive the remaining supply levels, we simply scale the average supply level of the baseline case by the corresponding factor while keeping the coefficient of variation the same.

In simulating the demand side of the platform (i.e., bidder behavior) in our counterfactuals, we keep the rate of new bidders joining and leaving the bidder pool the same (except for single-unit bidders departing upon winning), mirroring how the estimated model treats this arrival process as exogenous. Once a bidder joins the pool, she endogenously considers when to visit the platform, in which auctions to bid, and how much to bid in each auction. Thus, the behavior of bidders in the bidder pool is entirely endogenous and based on equilibrium beliefs that are updated to match the counterfactual simulation. Each simulated counterfactual assumes a supply rate at which auction pallets arrive to post on the platform and a listing policy (in Section 2.6.1, we additionally allow for targeted recommendations). While the supply rate is exogenously determined by the retailer's setting and reverse logistics, we vary it to consider how the optimal listing policy and resulting level of market thickness depends on the relative balance of supply and demand levels. The exogenous rate of newly-registered bidders arriving into the bidder pool is estimated non-parametrically from Market A's post-treatment period. Bidders' equilibrium participation and bidding behavior are simulated using an iterative algorithm.[23]

In Table 2.8, we present the ratio of the revenue obtained relative to the

[23] For each listing policy (or recommendation system in Section 2.6.1), we simulate the equilibrium beliefs for the market state and the highest rival bids (i.e., Ψ and G), using the iterative algorithm of Appendix 2.11.5.

revenue under the batch listing policy at that supply level. Therefore, each row reports the relative performance of the four listing policies at the row's associated supply level. In the brackets next to the relative revenues, we display the average market thickness associated with each case.

At the baseline supply level, our counterfactual revenue improvement due to switching Market A from the uniform listing policy to the batch listing policy is roughly 7.0%, offering out-of-sample validation of the 7.3% increase estimated in our ATE analysis of Section 2.3.3 using time-wise separate data.[24] Besides, the optimal policy in the baseline case turns out to be the single-day batch policy, which yields 4.8% more revenue than the batch policy. These relative differences in performance translate into substantial revenue gains. Over the 10 months of the post-treatment period, Market A's revenue from iPhone 4, iPhone 4s, and iPhone 5 auctions amounted to $\$4,608,941$: the 11.3% relative difference between the uniform and the best-performing single-day policy translates to $\$520,810$ of additional revenue for the platform.[25]

The relative performance of the four policies and consequently which listing policy is "optimal" hinges on the platform's underlying supply level. For low levels of supply, "$\frac{1}{3}$x" and "$\frac{1}{2}$x," the single-day batch policy performs best by inducing the thickest market. In this scenario, the participation cost exhibits a dominant

[24] To illustrate the dampening effect of cannibalization, our simulations project an 8.8% increase in revenue if the platform were able to exogenously increase bidder participation to the same level, without adding or batching auctions to attract bidders (i.e., no cannibalization).

[25] Perhaps reflecting our projections, as of January 2017, Market A enforced a single-day listing policy that ends all auctions exclusively on Tuesdays.

Table 2.8: Each Listing Policy's Simulated Revenues as Percentage of Simulated Revenue under Batch Listing

Supply Level	Uniform	Three-Day Batch	Batch	Single-Day Batch
$\frac{1}{3}$x	90.1% [0.6]	96.5% [1.1]	100% [1.6]	**106.1%** [3.2]
$\frac{1}{2}$x	91.3% [0.9]	96.6% [1.5]	100% [2.2]	**105.5**% [4.4]
1x (baseline)	93.5% [2.1]	97.2% [3.4]	100% [5.2]	**104.8%** [10.3]
	($\$-299,581$)	($\$-129,050$)		($\$221,229$)
2x	97.8% [4.2]	**101.0**% [6.9]	100% [10.5]	95.0% [20.6]
3x	109.6% [6.3]	**110.8%** [10.3]	100% [15.7]	88.4% [31.0]
5x	**122.1%** [10.1]	117.8% [16.7]	100% [25.4]	81.9% [50.2]

Note. Values in bold correspond to the row-optimal listing policies with a significance level of 0.001*. Market thickness averages are in brackets.*

effect on the platform's revenues by deterring bidders' visits; thus, the platform finds it optimal to increase the bidders' expected payoff per visit by providing more options each auction-ending day. By contrast, for high levels of supply (e.g., "5x"), the uniform policy and the three-day batch policy perform relatively better than the rest of the policies by maintaining a thinner market. In this case, the cannibalization between auctions becomes the dominant effect; thus, the platform finds it optimal to spread out the auction ending times throughout the week.

The market thickness induced by the platform's listing policy impacts revenues. Intuitively, platform revenues are maximized when the platform's market thickness comes close to the level that effectively balances its trade-off of overcoming bidder participation costs (thus, inducing higher participation rates) against competition as substitutes among auctions on the supply side. Using simulated data from the above counterfactual simulations, Figure 2.5 (the left plot) depicts an auction's average final per-device revenue as a function of the market's (log) average market thickness. Interestingly, low market thickness does not lead to a high average final price, due primarily to the participation costs and the hurdles to entry they present. As shown in the middle plot of Figure 2.5, the expected number of MU bidders per auction drops from 10 to 4 when market thickness decreases from 7 auctions to 1. On the other hand, cannibalization is also evident in the same plot: the number of MU bidders in an auction keeps decreasing when market thickness increases beyond 10 auctions. By comparison, UD bidders are much less sensitive to the shift in market thickness. As displayed in the right plot of Figure 2.5, there is no statistically significant increase in the expected number of UD bidders per auction associated with higher market thickness. Illustrating the trade-off between demand-side participation and supply-side cannibalization as market thickness increases, the average final price is maximized at moderate levels of market thickness.

Given the reduced-formed regressions and the counterfactual simulations, Table 2.9 summarizes the overall effects of the policy switch on bidder behavior, including the rates of platform visit and auction participation, as well as their bidding amount. As bidders' non-bidding website visits are not observed, we employ the counterfactual simulations to estimate the change in the bidders' rate of platform

Figure 2.5: Effect of Market Thickness on Auctions' Final Prices and Bidder Participation (Left plot is a scatter plot of simulated final prices. Center and right plots show the fitted curves for simulated numbers of participants, where grey-curve represents 95%-CI).

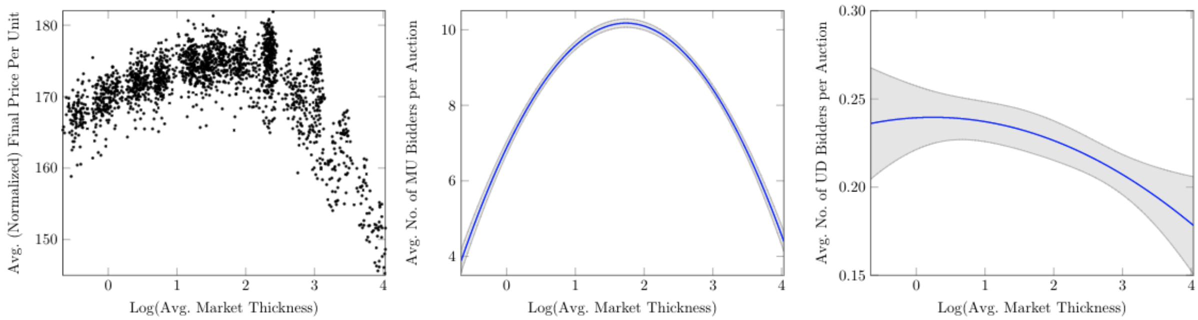


Table 2.9: Revenue Funnel of Participation Effects from Listing Policies

Mechanisms	Change in %	Evidence Type
Platform Visit	11.9%	Counterfactural Simulation
Auction Participation	9.7% (p-value = 0.03)	Observed
Bidding Decision	Not Significant	Observed

visits associated with the policy switch. As evident in Table 2.9, the increase in the platform's revenues is primarily due to the increase in traffic to the platform (which, consequently, results in higher auction participation rates). On the other hand, additional analysis in Appendix 2.11.4 shows that the listing policy has little effect on bidders' bidding decisions.

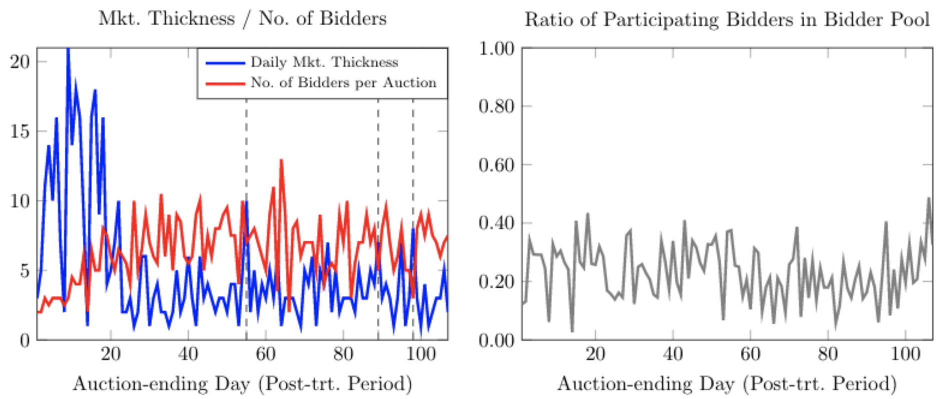
2.6.1 Targeted Recommendations

In this section we discuss the design of recommendation systems for a competitive online market (e.g., online auction markets). Complementing the listing policy, a recommendation system is intended to mitigate cannibalization on days of high market thickness by incentivizing buyers to visit the platform. As is evident in the left plot of Figure 2.6, daily market thickness (blue curve) exhibits large variations even though the listing policy is fixed. On the other hand, bidder participation does not scale up to match the realized supply; thus, the number of bidders per auction (red curve) drops on days where the realized supply is high (e.g., from Day 1 to Day 20 and on Days 55, 89, and 98).[26] As shown in the right plot of Figure 2.6, the ratio of bidders who submit at least one bid to all active bidders in the bidder pool remains consistently below 60%. Therefore, the decline in the number of bidders per auction on these days is not the result of fluctuations in the overall bidder pool. Instead, we argue that the daily supply-demand mismatches are mainly due to the fact that it is costly for bidders to continuously monitor the market and obtain up-to-date information about the realized supply; thus, they decide whether to visit the platform based on their beliefs about the market in steady state. Namely, they cannot account for the inherent variability in the *realized* number of auctions available on the platform.

Given this, the platform can benefit from communicating credible market

[26]The left plot of Figure 2.6 also implies that sellers do not strategize the weekly supply or the time of their listings to match the demand.

Figure 2.6: Variability of Daily Market Thickness and Demand Response in Market A (*market thickness* is defined as the number of auctions ending on a given day, and vertical dashed lines denote the days with supply shocks)



thickness information to bidders as a way to incentivize additional visits. Yet, sharing information about the daily supply to the entire bidder pool may actually backfire. Specifically, our counterfactual simulation of the *full information* policy, which discloses the realized market thickness daily to all bidders, indicates that in the resulting equilibrium the final price drops by 28.3% (on days when the realized supply is low, bidder participation drops significantly and, consequently, the final price per device drops).

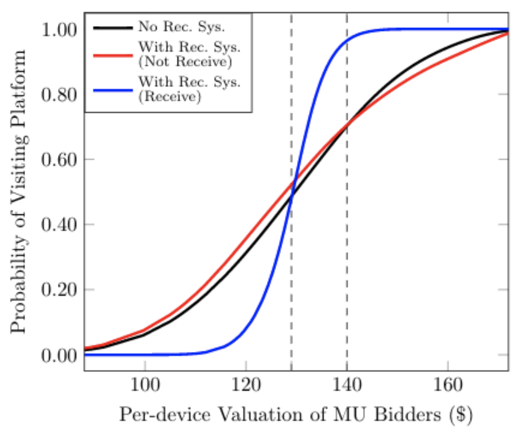
To remedy the adverse effect of the low-supply information revealed from the communication, we consider recommendation systems that send recommendations to disclose the market state only when the market thickness is above threshold κ and only to η fraction of randomly-selected MU bidders. We focus exclusively on MU bidders because they win most auctions (Table 2.2) and UD bidders are much less responsive to market thickness information (right plot in Figure 2.5).

If a bidder receives a recommendation, she will update her belief about the realized supply of auctions on the platform, taking into account both the platform's revealed supply state and how other bidders who received the same recommendation may respond. If she does not receive a recommendation, she will still update her belief by inferring that one of the following two cases must be true: (1) the market thickness on that day is lower than κ; (2) The market thickness exceeds κ, but she was not selected to receive the information. Due to Case (1), the bidder will adjust her belief about market thickness downwards if she does not receive a recommendation. Our counterfactual will elucidate how this affects the participation decisions of various types of bidders.

We optimize over parameters κ and η in Market A, which operates under its optimal single-day batch policy. The optimal system sends recommendations to 10% of the MU bidders when the market thickness on a given day is above 22. Recommendations are sent out roughly on 11.0% of the days in the simulation horizon. Implementing this recommendation system further improves the platform's overall revenue by 0.9%, including a 1.6% increase during the recommendation days.

Figure 2.7 plots MU bidders' equilibrium rates of visiting the platform in the presence and absence of (optimal) recommendations. Bidders' responses to receiving a recommendation (blue curve) vary drastically depending on their valuations. For bidders with lower valuations (below \$131), receiving a recommendation is actually bad news, and substantially decreases their probability of visiting. By contrast, bidders with higher valuations (above \$131) become significantly more likely to visit the platform after receiving a recommendation. Since the supply of auctions is higher on

Figure 2.7: Effects of Recommendations on Bidders' Market Participation (the left vertical dashed line denotes the MU bidder whose participation rate remains unchanged upon receiving a recommendation. The right vertical dashed line denotes the MU bidder whose participation rate remains unchanged upon *not* receiving a recommendation)



days when recommendations are sent out, high-valuation bidders' expected payoff per visit is higher. Besides, their increased platform visits intensify the competition and deters the low-valuation bidders. Thus, the optimal recommendation system increases revenues mainly by altering the composition of participants through self-selection on the market's high-inventory days. Figure 2.7 also reveals how bidders respond to not receiving a recommendation (red curve). First, a high-valuation bidder (above \$140) who did not receive a recommendation in the presence of a recommendation system is slightly less likely to visit the platform than in the absence of a recommendation system (black curve). Such a bidder infers that the supply of auctions may be low, which, in turn, weakens her incentive to participate. On the other hand, for low-valuation bidders (below \$140), not receiving the recommendation can be interpreted as facing less competition on the platform. Thus, the incentives for participation for low-valuation bidders are (slightly) higher.

2.7 Concluding Remarks

In this paper, we empirically illustrate the role of listing policies in inducing the optimal market thickness level and, consequently, in generating higher revenues. In particular, we highlight that optimizing this seemingly innocuous market design lever affects revenues significantly by inducing the appropriate level of market thickness on the platform. We also explore the design of a recommendation system that selectively informs bidders about the market state and establish that it can benefit the B2B auction platform by mitigating cannibalization among substitutable auctions.

Using a proprietary dataset, obtained from a leading online B2B auction platform, we estimate that inducing higher market thickness (by concentrating the auctions' ending times on certain weekdays) leads to a 7.3% increase in the platform's revenues. Additional analysis points to the presence of significant participation costs associated with visiting the platform and bidding in auctions that adversely affects bidders' participation rates. Motivated by the descriptive results, we develop and estimate a structural model, which endogenizes bidders' decision-making including whether and when to visit the platform, which auction(s) to participate in, and how much to bid. Notably, the revenue impact of inducing higher market thickness predicted by counterfactual simulation on the estimated model is consistent with the results from the reduced-form analysis.

Complementary to illustrating the revenue impact of the listing policy, we discuss the design of a recommendation system, which alters the composition of participants through self-selection. Appropriately designing the system yields an additional revenue increase by successively increasing the level of competition between bidders when the daily supply is significantly higher than average.

More broadly, our work highlights that marketplace design can have significantly positive revenue implications for online two-sided platforms by mitigating frictions that impede participation. Given their growing prominence, we believe that exploring the impact of platform design, especially focusing on non-price levers, on market thickness and, consequently, revenues and welfare in the context of other two-sided marketplaces is a very fruitful direction for future research.

2.8 Appendix: Natural Experiment

2.8.1 AIPW Estimator of the Average Treatment Effect

The propensity score e_{jt} of an auction j that gets listed at time t is its probability of being assigned to Market A on the basis of its vector of observable attributes AUC_{jt} and the listing time t at which the auction lot becomes available. To estimate the propensity score, we specify

$$Logit(e_{jt}) = \tilde{\beta}_0 + \tilde{\delta}_t + \tilde{\boldsymbol{\beta}}^T AUC_{jt}, \tag{2.7}$$

where $\tilde{\delta}_t$ and $\tilde{\boldsymbol{\beta}}$ respectively denote the week-t fixed effect and the attribute-coefficient vector. Using the dependent variable A_{jt}, we first estimate model (2.7) by logistic regression to obtain predicted propensity scores $\hat{e}_{jt}$.

The estimator of the ATEs (i.e., τ_{Pre} and τ_{Post}) is characterized as follows:

$$\begin{aligned} \hat{\tau} =& N_{obs}^{-1} \sum_{jt} \Big(\frac{A_{jt} \cdot LFP_{jt}}{\hat{e}_{jt}} - \frac{A_{jt} - \hat{e}_{jt}}{\hat{e}_{jt}} \cdot \widehat{LFP}_{jt,1} \Big) \\ &- N_{obs}^{-1} \sum_{jt} \Big(\frac{1 - A_{jt} \cdot LFP_{jt}}{1 - \hat{e}_{jt}} + \frac{A_{jt} - \hat{e}_{jt}}{1 - \hat{e}_{jt}} \cdot \widehat{LFP}_{jt,0} \Big), \end{aligned} \tag{2.8}$$

where N_{obs} is the period's sample size, corresponding to the pre- and post-treatment periods, respectively, and $\hat{e}_{jt}$ is the estimated propensity score for auction jt obtained by Expression (2.7). We restrict attention to auctions with estimated propensity scores between 0.2 and 0.8 in order to ensure that each unit will not have a weight that is more than 0.025, and that the requisite overlap assumption holds Im04. While the classic ATE estimate is derived as the difference in observed outcomes weighted appropriately using the associated propensity score projections $\hat{e}_{jt}$, Robins'

AIPW estimator adds terms involving the projections $\widehat{LFP}_{jt,1}$ and $\widehat{LFP}_{jt,0}$. In turn, these projections are derived from estimating the following linear models of log-revenue outcomes, as an alternative to the classic ATE:

$$LFP_{jt,A} = \beta_{0,A} + \delta_{t,A} + \boldsymbol{\beta}_A^T AUC_{jt,A} + \epsilon_{jt,A}, \tag{2.9}$$

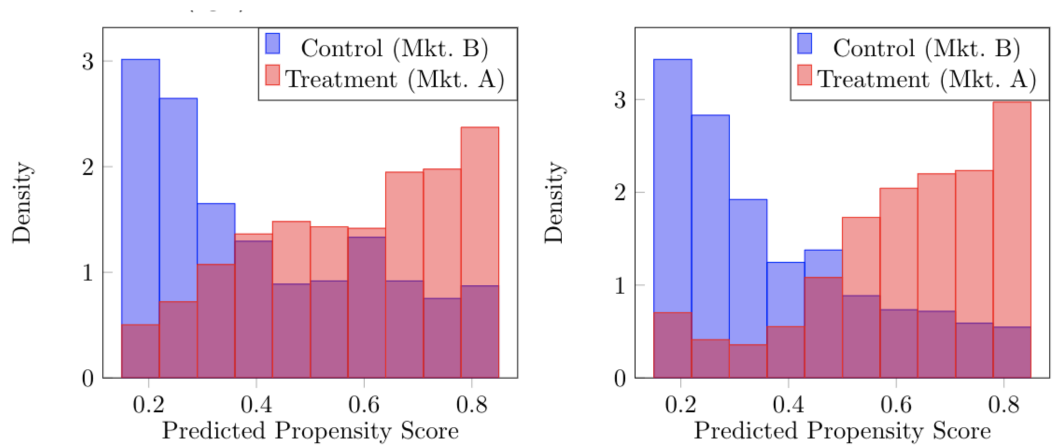
where A is the indicator of Market A, $\beta_{0,A}$ is the intercept, $\delta_{t,A}$ is a fixed effect for week t, and $\boldsymbol{\beta}_A$ denotes the attribute-coefficient vector. Note that Robins' AIPW is a doubly robust estimator; that is, it offers a consistent estimator of the ATE if *either* the classic ATE propensity score model (i.e., Expression (2.7)) or the linear outcome model (i.e, Expression (2.9)) is well specified.

2.8.2 Assessing Overlap

The purpose of this section is to assess the overlap in the covariates of Markets A and B and, subsequently, to argue that our estimation approach is indeed valid. To this end, we restrict attention to observations that have predicted propensity scores between 0.2 and 0.8 and, first, present the corresponding distributions of predicted propensity scores and summary statistics.[27] In our propensity score model, we include variables to capture weekly fixed effects (i.e., the time at which an auction was held), the auction lot size, and fixed effects for product models and carriers. We then calculate the differences between the means of each of the covariates in the treatment and control groups, respectively. A covariate with a difference in means

[27]Given the sample size, the selection of the cutoff points follows the rule that no observation will have a weight that is more than 0.05 Im04

Figure 2.8: Histograms of Propensity Scores in Samples of the Pre-treatment Period (left) and the Post-treatment Period (right)



that is greater than 0.25 standard deviations is considered to be lacking overlap.

During the pre-treatment period, the treatment effect relates to unobserved differences between Markets A and B. As deduced from the left plot of Figure 2.8, there is a sufficiently large overlap between the treatment group (Market A) and the control group (Market B). In the first three columns of Table 2.10, we present the means of the covariates (with standard deviations in parentheses) and the cross-market differences for each covariate. All differences in means lie within 0.14 standard deviations, which indicates that the two groups are well balanced.

During the post-treatment period, the treatment effect relates to both the unobserved differences between Markets A and B and the listing policy switch. In the right plot of Figure 2.8, we provide a histogram with the predicted propensity scores of the treatment group (Market A) and the control group (Market B). Although the overlap is not as large as that in the pre-treatment period, we believe that the level of overlap in the post-treatment period is sufficient. As both distributions spread

Table 2.10: Overlap of Covariates in the Selected Samples of the Pre- and Post-treatment Periods

	Pre-treatment			Post-treatment		
	Ctr. Group	Trt. Group	Diff./S.D.	Ctr. Group	Trt. Group	Diff./S.D.
Auction Lot Size	77.34 (37.58)	80.52 (37.32)	0.08	63.29 (37.65)	92.70 (26.53)	**0.78**
iPh. 4-AT&T	.023 (.152)	.012 (.112)	-0.07	.075 (.264)	.084 (.278)	0.03
iPh. 4-Sprint	.192 (.394)	.194 (.395)	0.01	.104 (.306)	.069 (.254)	-0.11
iPh. 4-Verizon	.291 (.454)	.256 (.436)	-0.07	.037 (.191)	.045 (.207)	0.03
iPh. 4s-AT&T	.175 (.380)	.161 (.368)	-0.03	.162 (.369)	.259 (.438)	0.26
iPh. 4s-Sprint	.103 (.304)	.129 (.336)	0.08	.223 (.416)	.196 (.397)	-0.06
iPh. 4s-Verizon	.133 (.339)	.183 (.386)	0.14	.163 (.369)	.165 (.371)	0.01
iPh. 5-Sprint	.036 (.188)	.031 (.175)	-0.02	.154 (.361)	.105 (.307)	-0.13
iPh. 5-Verizon	.020 (.143)	.014 (.119)	-0.04	.032 (.177)	.039 (.194)	0.04

out within $[0.2, 0.8]$, we have observations from both groups in every propensity score bin. In the last three columns of Table 2.10, we present the covariate means (with standard deviations in parentheses) and the cross-group differences for each covariate. The differences in means for all covariates (with the exception of the auction lot size) are within 0.26 standard deviations. In general, an outcome regression model relies on control observations when predicting the outcome of treated units if they were *not* treated. A large difference in covariate distributions across groups implies that the predictions heavily rely on extrapolation; hence, they are sensitive to the specification of the outcome regression model.

To evaluate how our ATE estimation is affected by the lack of overlap in the auction lot size (in the post-treatment period), we conduct the following robustness checks:

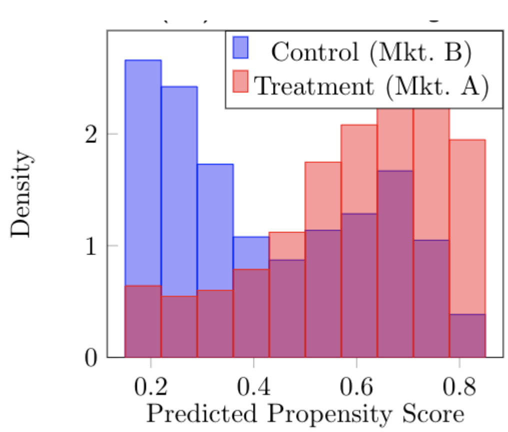
(i) First, we rerun the ATE analysis based on a subsample in which the treatment and control groups have a large overlap and there are sufficient observations to make statistically significant inferences. Although the assumption of overlap in the alternative approach is more credible compared to the original estimation, the estimated effect does not necessarily apply to all observations.

(ii) Second, to test whether the estimation is sensitive to the specification of the outcome regression model, we conduct two additional robustness checks by replacing the auction lot covariate with its log scale value (to make it concave) and by adding a quadratic term for the auction lot (to make it convex).

For the subsample in *(i)*, we select auctions with lot size between 50 and

90. This results in 49 observations for the control group and 58 observations for the treatment group. The subsample accounts for 27% of the original sample. To assess the overlap for the subsample, we present the histogram of predicted propensity scores (left plot of Table 2.11) and the differences in means of the covariates (right table of Table 2.11). Compared with the original sample, the overlap in the subsample is sufficiently higher.

Table 2.11: Histogram of the Predicted Propensity Scores in the Selected Subsample in the Post-treatment Period (left) and a Table Assessing the Overlap of Covariates in the Selected Subsample (right)



	Ctr. Group	Trt. Group	Diff./S.D.
Auction Lot Size	69.11 (11.44)	69.51 (11.30)	0.03
iPh. 4-AT&T	.122 (.327)	.030 (.171)	-0.28
iPh. 4-Sprint	0	0	0
iPh. 4-Verizon	.107 (.309)	.092 (.290)	-0.04
iPh. 4s-AT&T	.109 (.312)	.056 (.230)	-0.17
iPh. 4s-Sprint	.351 (.477)	.400 (.490)	0.10
iPh. 4s-Verizon	.185 (.388)	.129 (.335)	-0.14
iPh. 5-Sprint	.052 (.223)	.093 (.291)	0.18
iPh. 5-Verizon	.048 (.214)	.085 (.280)	0.17

We perform the ATE analysis based on the subsample and then compare the result with that obtained from the original sample. The results can be found in Table 2.12. The estimated ATE of the subsample is significantly positive and on a similar scale to the ATE estimate corresponding to the original sample. Though the subsample estimate may suffer from selection bias, comparison of the two results

Table 2.12: Average Treatment Effect Estimates using the Original Sample and the Selected Subsample

Auction lot size specification	Original Sample	Selected Subsample
Linear	7.3%***	8.1%***
	(0.009)	(0.011)
Note:		*** p<0.01

suggests that the lack of overlap for the auction lot size covariate seems to have a very limited impact on the ATE estimation. In what follows, we conduct another analysis to check whether our ATE estimation is sensitive to the specification of the outcome regression model.

The key issue corresponding to the lack of overlap between treated and controlled units is that the controlled outcome predictions of the treated units with outlying values heavily rely on extrapolation, as few control units are observed in this region when estimating the outcome regression model. In other words, the prediction precision may be sensitive to the specification of the outcome regression model. In our case, the predicted outcomes may be subject to the functional form of auction lot size. In our robustness check *(ii)*, we consider concave (log) and convex (quadratic) specifications regarding the auction lot size in the outcome regression model. As shown in Table 2.13, the ATE estimates remain almost the same under different specifications. In particular, there is a strong linear relationship between the final price of an auction and its size. This is quite intuitive given that the cell

Table 2.13: Robustness Check of Auction Lot Size Specification

Auction lot size specification	Original Sample
Linear	7.3%***
	(0.009)
Concave (Log)	7.2%***
	(0.009)
Convex (Quadratic)	7.3%***
	(0.009)
Note:	*** p<0.01

phones sold in an auction are relatively homogeneous; thus, the price is likely to increase linearly with the lot size. Within a moderate lot size range, economies of scale and demand satiation effects are not significant; thus, the dependence between the final price and the lot size can be well described by a linear relationship. This implies that the estimated ATE is not significantly influenced by outliers, including those with auction lots larger than 90, whose predicted control outcomes heavily rely on extrapolation. In conclusion, our original ATE estimation remains reliable.

2.8.3 Cross-market Bidders

In this section, we establish that the presence of bidders that are cross-registered in both Markets A and B implies that our estimate on the revenue impact of the switch in the listing policy is conservative. In other words, if the markets were truly independent, then switching to a batch listing policy would result in a higher increase in the platform's revenues than our estimated 7.3%. Our approach involves developing a model that features cross-market bidders and computing the platform's revenues as a function of their share in the total population via simulation.

2.8.3.1 Model.

First, we provide an outline of the model.

(a) **Model Setting and Assumptions.**

1. The platform has two markets, A and B. Each market has a single-market bidder pool of fixed size M. Both markets share a cross-market bidder pool of fixed size L. Each bidder has a private valuation x drawn from a common distribution with cdf $F(\cdot)$, which has a positive support. The bidder has demand for K auctions upon entry into the pool.

2. The platform operates over an infinite horizon. All auctions are identical, second-price auctions and last for one day. Under the uniform policy, auctions are listed every day; under the batch policy, auctions are listed every other day.

3. Each day, a bidder first decides whether or not to participate on the platform. If she does, she incurs a participation cost c drawn from the exponential distribution with rate μ. If the bidder chooses to participate on a given day, she selects a number of auctions equal to her unsatisfied demand to participate at random. For each auction j she participates

in, she places a bid equal to $x + \epsilon_j$, where ϵ_j is an idiosyncratic term drawn from $N(0, \sigma_\epsilon^2)$. At the end of the day, she exits the bidder pool with probability $1 - \rho$. As soon as a bidder exits, a new bidder joins so that the pool size remains constant.

(b) **Notation.** We use k and s_m to denote a bidder's demand that is still not satisfied and the number of auctions in Market m that the bidder has chosen to participate in, respectively. Moreover, we introduce the following notation:

1. $G_{mt}(y)$, $g_{mt}(y)$: the CDF and PDF of the highest rival bid in an auction in Market m on Day t.
2. $P_{mt}(w; x, k)$: the probability of a single-market bidder winning w auctions in Market m on Day t.
3. $P_t(w_A, w_B; x, s_A, s_B)$: the probability of a cross-market bidder winning w_A Market A auctions and w_B Market B auctions on Day t.
4. $u_{mt}(x, k)$: a single-market bidder's expected payoff in Market m on Day t.
5. $u_t(x, k)$: a cross-market bidder's expected payoff by visiting the platform on Day t.
6. $v_{mt}(x, k)$: a single-market bidder's aggregate payoff on Day t.
7. $v_t(x, k)$: a cross-market bidder's aggregate payoff on Day t.

(c) **Participation decisions when both markets implement the uniform policy.** Suppose that there are N_m auctions in Market m on each day. Since both markets use the uniform policy, we omit subscript t.

1. Single-market bidders: The expected payoff of a single-market bidder in Market m when she visits the platform is given by

$$u_m(x,k) = \min\{N_m,k\}\int_0^x (x-y)g_m(y)dy + \rho \sum_{w=0}^{\min\{N_m,k\}} P_m\big(w; x, \min\{N_M,k\}\big)v_m(x,k-w). \tag{2.10}$$

   The bidder visits the platform if $u_m(x,k)-c \geq \rho v_m(x,k)$, where $\rho v_m(x,k)$ is the payoff corresponding to waiting. Hence, her aggregate payoff is equal to

$$\begin{aligned} v_m(x,k) &= \mathbb{E}\big[\max\big\{u_m(x,k)-c, \rho v_m(x,k)\big\}\big] \\ &= u_m(x,k) + \frac{1}{\mu}\exp\Big(-\mu\big(u_m(x,k)-\rho v_m(x,k)\big)\Big) - \frac{1}{\mu}. \end{aligned} \tag{2.11}$$

   Lastly, combining (2.10) and (2.11), we can solve for $u_m(x,k)$ and $v_m(x,k)$ numerically for $k = 1, 2, ..., K$, given g_m, P_m and x.

2. Cross-market bidders: The expected payoff of a cross-market bidder when she visits the platform is given by

$$\begin{aligned} u(x,k) =& \frac{1}{|\mathcal{C}(k,N_A,N_B)|}\sum_{(k_A,k_B)\in\mathcal{C}(k,N_A,N_B)}\Bigg(\sum_{m\in\{A,B\}} k_m \int_0^x (x-y)g_m(y)dy \\ &+ \rho\sum_{w_A=0}^{k_A}\sum_{w_B=0}^{k_B} P(w_A,w_B;x,k_A,k_B)v(x,k-w_A-w_B)\Bigg), \end{aligned} \tag{2.12}$$

   where $\mathcal{C}(k,N_A,N_B) = \big\{(k_A,k_B) : k_A+k_B = \min(k, N_A+N_B), k_A = 0,1,...,N_A, k_B = 0,1,...,N_B\big\}$. Then, she will visit the platform if $u(x,k)-c \geq \rho v(x,k)$, where $\rho v(x,k)$ is the payoff corresponding to waiting.

Hence, her aggregate payoff is equal to

$$v(x,k) = \mathbb{E}\big[\max\big(u(x,k) - c, \rho v(x,k)\big)\big]$$
$$= u(x,k) + \frac{1}{\mu}\exp\Big(-\mu\big(u(x,k) - \rho v(x,k)\big)\Big) - \frac{1}{\mu}. \tag{2.13}$$

Lastly, combining Expressions (2.12) and (2.13), we can solve for $u(x,k)$ and $v(x,k)$ numerically, for $k = 1, 2, ..., K$, given g_A, g_B, P, and x.

(d) **Participation decisions when Market A implements the uniform policy and Market B implements the batch policy.** In Market A, there are 0 auctions on odd days, denoted by $t = 1$ below, and $2N_A$ auctions on even days, denoted by $t = 2$ below. In Market B, there are N_B auctions every day.

1. Single-Market-A bidders: For a single-market bidder in Market A, she only visits the platform on even days. Then,

$$u_{A2}(x,k) = \min(2N_A, k)\int_0^x (x-y)g_{A2}(y)dy$$
$$+ \rho^2 \sum_{w=0}^{\min(2N_A,k)} P_{A2}\big(w; x, \min(2N_A,k)\big)v_{A2}(x, k-w). \tag{2.14}$$

She will visit the platform if $u_{A2}(x,k) - c \geq \rho^2 v_{A2}(x,k)$, where $\rho^2 v_{A2}(x,k)$ is the payoff corresponding to waiting. Hence, her aggregate payoff is equal to

$$v_{A2}(x,k) = \mathbb{E}\big[\max\big(u_{A2}(x,k) - c, \rho^2 v_{A2}(x,k)\big)\big]$$
$$= u_{A2}(x,k) + \frac{1}{\mu}\exp\Big(-\mu\big(u_{A2}(x,k) - \rho^2 v_{A2}(x,k)\big)\Big) - \frac{1}{\mu}. \tag{2.15}$$

Lastly, combining (2.14) and (2.15), we can solve for $u_{A2}(x,k)$ and $v_{A2}(x,k)$ numerically for $k = 1, 2, ..., K$, given g_{A2}, P_{A2}, and x.

2. Single-Market-B bidders: For a single-market bidder in Market B, her expected payoff of visiting the platform on day $t \in \{0,1\}$ (let $t' \in \{0,1\}\backslash t$) is

$$\begin{aligned} u_{Bt}(x,k) =& \min(N_B,k)\int_0^x (x-y)g_{Bt}(y)dy \\ &+\rho \sum_{w=0}^{\min(N_B,k)} P_{Bt}\big(w; x, \min(N_B,k)\big)v_{Bt'}(x, k-w). \end{aligned} \tag{2.16}$$

She will visit the platform if $u_{Bt}(x,k) - c \geq \rho v_{Bt'}(x,k)$, where $\rho v_{Bt'}(x,k)$ is the payoff corresponding to waiting. Hence, her aggregate payoff is equal to

$$\begin{aligned} v_{Bt} =& \mathbb{E}\big[\max\big(u_{Bt}(x,k) - c, \rho v_{Bt'}(x,k)\big)\big] \\ =& u_{Bt}(x,k) + \frac{1}{\mu}\Big(-\mu\big(u_{Bt}(x,k) - \rho v_{Bt'}(x,k)\big)\Big) - \frac{1}{\mu}. \end{aligned} \tag{2.17}$$

Lastly, combining Expressions (2.16) and (2.17), we can solve for

$$u_{B1}(x,k), u_{B2}(x,k), v_{B1}(x,k),$$

and $v_{B2}(x,k)$ numerically for $k = 1, 2, ..., K$, given g_{B1}, g_{B2}, P_{B1}, P_{B2}, and x.

3. Cross-market bidders: For a cross-market bidder visiting the platform,

her expected payoffs on odd and even days are given as follows:

$$u_1(x,k) = \min(k,N_B)\int_0^x (x-y)g_{B1}(y)dy + \rho \sum_{w_B=0}^{\min(k,N_B)} P_1\big(0,w_B;x,0,\min(k,N_B)\big)v_2(x,k-w_B), \tag{2.18}$$

$$\text{and } u_2(x,k) = \frac{1}{|\mathcal{C}(k,2N_A,N_B)|}\sum_{(k_A,k_B)\in\mathcal{C}(k,2N_A,N_B)}\Big(\sum_{m\in\{A,B\}} k_m\int_0^x (x-y)g_{m2}(y)dy + \rho\sum_{w_A=0}^{k_A}\sum_{w_B=0}^{k_B} P_2(w_A,w_B;x,k_A,k_B)v_1(x,k-w_A-w_B)\Big), \tag{2.19}$$

where $\mathcal{C}(k,2N_A,N_B) = \big\{(k_A,k_B) : k_A+k_B = \min(k,2N_A+N_B), k_A = 0,1,...,N_A, k_B = 0,1,...,N_B\big\}$. On day $t\in\{0,1\}$ (with $t'\in\{0,1\}\backslash t$), she will visit the platform if $u_t(x,k)-c \geq \rho v_{t'}(x,k)$, where $\rho v_{t'}(x,k)$ is the payoff corresponding to waiting. Hence, her aggregate payoff on Day t is equal to

$$\begin{aligned} v_t(x,k) =& \mathbb{E}\big[\max\big(u_t(x,k)-c, \rho v_{t'}(x,k)\big)\big] \\ =& u_t(x,k) + \frac{1}{\mu}\exp\Big(-\mu\big(u_t(x,k)-\rho v_{t'}(x,k)\big)\Big) - \frac{1}{\mu}. \end{aligned} \tag{2.20}$$

Lastly, combining Expressions (2.18), (2.19), and (2.20), we can solve for $u_1(x,k), u_2(x,k), v_1(x,k)$, and $v_2(x,k)$ numerically for $k = 1,2,...,K$, given $g_{A2}, g_{B1}, g_{B2}, P_1, P_2$ and x.

(e) **Simulation.** We provide a brief summary of how we simulate bids on each day (through Day T):

- Day 0: We initialize $g_{mt}(y)$, $P_{mt}(w; x, k)$, and $P_t(w_A, w_B; x, s_A, s_B)$ for the first $2M$ single-market bidders and L cross-market bidders in the three bidder pools.

- Day t: First, we update the bidder pools by replacing the bidders who exit with new bidders. The new bidders form their beliefs about the state of the market, i.e., $g_{mt}(y), P_{mt}(w; x, k)$, and $P_t(w_A, w_B; x, s_A, s_B)$ based on all bids between Day 1 and Day t. Then, all active bidders make platform-visit decisions and, if they choose to visit the platform, they place bids in a random subset of the auctions. When all bids are placed, auctions are allocated to their winners and we update the latter's demand that has not been yet satisfied.

2.8.3.2 Simulation Results.

(a) **Parameters.** Our simulation scenarios are based on the following parameters:

$$N_A = N_B = 3 \quad 2M + L = 40 \quad F = N(50, 15) \quad F_\epsilon = N(0, 1)$$
$$K = 8 \quad \mu = 0.01 \quad \rho = 0.8 \quad T = 2,000.$$

We chose the parameters above in order to make our simulation scenarios relatively comparable to what we observe in the dataset. We then simulate the model for 1,000 days and analyze the corresponding bids.

(b) **Arbitrage effect of cross-market bidders.** We evaluate the effect of the policy switch under a number of scenarios corresponding to different ratios for the cross-market bidders, ranging from 0% to 80%. Under the parameters

Table 2.14: Average Final Price per Market in Different Simulation Scenarios

Ratio of cross-market bidders	0%	10%	20%	30%	60%	80%
Market A	54.64	55.63	56.81	57.66	59.49	60.67
Market B	49.37	50.61	51.92	53.02	55.67	57.25
Abs. Change	5.28	5.03	4.89	4.64	3.82	3.42
Change in %	10.7%	9.9%	9.4%	8.8%	6.9%	6.0%

noted above, Market A and Market B are identical when they both implement the uniform policy. Thus, we only need to consider the cross-market price difference when Market A is under the batch policy, while Market B is under the uniform policy.

As is evident in Table 2.14, the revenue increase corresponding to the policy switch is decreasing in the proportion of cross-market bidders in the population of bidders. This is due to the fact that cross-market bidders play the role of an arbitrageur and tend to shrink the price gap between the two markets. In particular, when Market A auctions are expected to have higher final prices than those in Market B, a cross-market bidder will certainly choose to bid in Market B, thus intensifying the competition in Market B and resulting in a decrease in the price gap between the two markets. In summary, this analysis implies that our estimate of the revenue impact of the policy switch in Section 2.3.3 is conservative compared to the case where the two markets are completely independent (i.e., when there are no cross-market bidders).

Table 2.15: Average Number of Weekly Participants in Markets A and B (S.D. in the parentheses)

	Market B	Market A
Pre-treatment Period	27.7 (18.3)	17.3 (14.8)
Post-treatment Period	52.1 (18.5)	46.2 (15.4)
Changing Rate (%)	88.1%	167.1%

Therefore, designating our setting as a "natural experiment" is well justified.

2.8.4 Aggregate Participation Increase in Market A

In this subsection, we measure the increase of the overall auction participation in Market A resulted from the policy switch. Explicitly, we count the number of unique bidders participating an auction (not limited to iPhone 4, iPhone 4s, and iPhone 5 auctions) per market per week and aggregate them across markets and across periods. The results are displayed in Table 2.15. As is evident in the table, Market A has a substantial increase of the unique participants per week in the post-treatment period. This indicates that the policy switch from the uniform policy to the batch policy attracts more bidders to Market A.

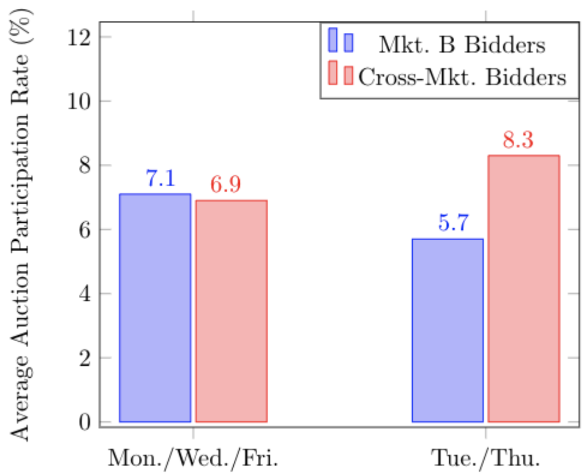
2.8.5 Additional Evidence of Participation Costs

We provide additional evidence for participation costs driving the revenue increase in Market A by examining the platform-visit decisions of cross-market bidders.

To that end, we examine whether cross-market bidders exhibit different participation patterns in Market B across weekdays before and after the adoption of the batch policy in Market A. In particular, we compare bidders' participation rates in Market B on Tuesdays and Thursdays to those on Mondays, Wednesdays, and Fridays by computing the ratio of the total number of participants on Tuesdays and Thursdays over the total number of participants on the remaining weekdays. Assuming that cross-market bidders are equally likely to participate in the market on any given weekday would imply a ratio of 2/3. During the pre-treatment period, we observe the cross-market bidders' participation ratio to be 63% (i.e., close to 2/3) in Market B, consistent with the use of the uniform listing policy in both markets. However, while Market B persisted in employing the uniform listing period throughout the post-treatment period, its participation ratio for cross-registrants increased to 84%, suggesting that these bidders strongly preferred to participate in Market B on the auction-clearing days of the *other* market (i.e., Tuesdays and Thursdays in Market A). Within the same period, an average cross-market bidder bids in a higher fraction of available auctions in Market B on Tuesday/Thursday than on Monday/Wednesday/Friday (as shown in Figure 2.9).[28] This spillover effect in the cross-market bidders' participation rates suggests that the costs involved in visiting the platform, carefully examining the inventory of open auctions, and placing a bid are substantial. Thus, bidders are strategic in their decision to visit the platform and actively participate in auctions. In other words, bidders choose to visit the

[28]We formally analyze this spillover effect in the cross-market bidders' participation rate using a difference-in-differences methodology in Appendix 2.11.2.

Figure 2.9: Average Auction Participation Rates in Market B in the Post-treatment Period



platform only when their expected payoffs from doing so exceed some threshold.

In summary, the above discussion (including Section 2.3.4) suggests the following mechanism to explain the revenue increase: an increase in market thickness boosts bidder participation both on aggregate and per auction. In turn, this additional traffic results in higher revenues for the platform. Underlying this mechanism is the fact that visiting the platform on a given day, monitoring the set of available auctions, and choosing whether and how much to bid entails a significant cost for bidders. In other words, although one expects that the auction platform would eliminate frictions and an auction's ending time would not affect its final price, we demonstrate that this is not the case (at least in the liquidation auctions). Thus, optimizing over listing policies (i.e., auctions' ending times) brings significant benefits to the platform.

2.9 Appendix: Proofs

2.9.1 Characterizations of $r^{MU}(x_\ell; G, \psi), r^{UD}(x_\ell; G, \Psi), v^{UD}(x_\ell; G, \Psi)$, and $v_f(x_\ell; G, \Psi)$

When a MU bidder ℓ participates in the platform and observes market state $\omega_{\ell t}$, her conditional payoff becomes

$$u^{MU}(x_\ell; \zeta_{\ell\cdot}, \omega_{\ell t}, G) = \max_{\sigma_{SLT}, \sigma_{BID}} \sum_{j \in \sigma_{SLT}} \int_{s_j}^{\sigma_{BID,j}} (x_\ell + \zeta_{\ell j} - p_j) g_j(p_j | \omega_{\ell t}) dp_j. \quad (2.21)$$

Therefore, her expected payoff for visiting the platform is

$$r^{MU}(x_\ell; G, \Psi) = \int_\omega \int_{\zeta_{\ell\cdot}} u^{MU}(x_\ell; \zeta_{\ell\cdot}, \omega_{\ell t}, G) d\mathbf{F}_\zeta(\zeta_{\ell\cdot}) d\Psi(\omega_{\ell t}). \quad (2.22)$$

For a UD bidder ℓ, her conditional payoff after visiting the platform is given by

$$\begin{aligned} u_j^{UD}(x_\ell; \zeta_{\ell j}, \omega_{\ell t}, G, \Psi) = \max_{\sigma_{BID,j}} & \int_{s_j}^{\sigma_{BID,j}} (x_\ell + \zeta_{\ell j} - p_j) g_j(p_j | \omega_{\ell t}) dp_j \\ & + \alpha^{UD} \big(1 - G_j(b_{\ell j} | \omega_{\ell t})\big) v_f(x_\ell; G, \Psi). \quad (2.23) \end{aligned}$$

Therefore, her aggregated payoff for participating in the platform is:

$$v^{UD}(x_\ell; G, \Psi) = \int_\omega \int_{\zeta_{\ell j^*}} u_{j^*}^{UD}(x_\ell; \zeta_{\ell j^*}, \omega_{\ell t}, G) dF_\zeta^{UD}(\zeta_{\ell j^*}) d\Psi(\omega_{\ell t}),$$

where $j^* = \sigma_{SLT}^{UD}(x_\ell; \zeta_{\ell\cdot}, \omega_{\ell t})$ denotes the auction in which it is optimal to bid. Lastly, a UD bidder's continuation payoff is characterized by the following Bellman equation:

$$v_f(x_\ell; G, \Psi) = \int_0^\infty \max\Big\{ v^{UD}(x_\ell; G, \Psi) - c, \alpha^{UD} v_f(x_\ell; G, \Psi) \Big\} \mu^{UD} e^{-\mu^{UD} c} dc. \quad (2.24)$$

2.9.2 Independence between $\mathbf{b}_\ell$ and $g_j(p_j|\omega_{\ell t})$

In this subsection, we show that Expressions (2.21) and (2.23) can account for the effect of $\mathbf{b}_\ell$ on the distribution of the highest rival bids.

Recall that the market state is denoted by $\omega_{\ell t} = (n_t, \mathbf{s}_{\ell t})$, and let w_j denote the current highest bid in auction j, which is unobservable. The conditional PDF of w_j is denoted by $h_j(w_j|n_t, \mathbf{s}_{\ell t})$. Let p_j denote the highest rival bid (i.e., the final price) in auction j, and let $k_j(p_j|n_t, \mathbf{s}_{\ell t})$ denote its conditional PDF. The expected payoff of a bidder that places a bid $b_{\ell j}$ in auction j is

$$\int_{s_j}^{b_{\ell j}} \Big(\int_{w_j}^{b_{\ell j}} (x_\ell + \zeta_{\ell j} - p_j) k_j(p_j|n_t, w_j, \mathbf{s}_{\ell t,-j}) dp_j \Big) h_j(w_j|n_t, \mathbf{s}_{\ell t}) dw_j,$$

where s_j denotes the current standing bid in auction j, and $\mathbf{s}_{\ell t,-j}$ is the vector of the standing bids excluding auction j. In turn, this is equal to:

$$\int_{s_j}^{b_{\ell j}} (x_\ell + \zeta_{\ell j} - p_j) \left(\int_{s_j}^{p_j} k_j\big(p_j|n_t, w_j, \mathbf{s}_{\ell t,-j}\big) h_j(w_j|n_t, \mathbf{s}_{\ell t}) dw_j \right) dp_j.$$

Let

$$g_j(p_j|n_t, \mathbf{s}_{\ell t}) = \int_{s_j}^{p_j} k_j\big(p_j|n_t, w_j, \mathbf{s}_{\ell t,-j}\big) h_j(w_j|n_t, \mathbf{s}_{\ell t}) dw_j.$$

We can verify that $g_j(p_j|n_t, \mathbf{s}_{\ell t})$ is a probability density function and that it is independent of $\mathbf{b}_\ell$. ■

2.9.3 Proof of Proposition 2.4.1

To establish the existence of the equilibrium, we first establish that the mapping from beliefs about the highest rival bids to their actual distribution is continuous and compact. Then, we conclude that the mapping has a fixed point, using the Schauder fixed-point theorem.

Mapping Γ.

We denote the unconditional CDF of the highest rival bids by $\mathbf{G}$. Its j^{th} component is the following $G_n^j(y_n^j) = \int_\omega G_n^{(j)}(y_n^j|\omega)d\Psi(\omega)$, which is the unconditional distribution of the highest rival bid y_n^j in the auction with the $j^{th}-$lowest standing bid. We then specify the mapping between $\mathbf{G}$ and the resulting unconditional rival bids distribution $\hat{\mathbf{G}}$ as

$$\begin{aligned}\hat{\mathbf{G}}(\cdot) = \Gamma(\cdot;\mathbf{G}) &= P\big(\mathbf{f}_h(\mathbf{x}_{MU}, \mathbf{m}_{MU}, \mathbf{b}, \mathbf{m}_{UD}, o, n) \leq \cdot; \mathbf{G}\big) \\ &= \int \Big(\sum_{o,n,\mathbf{m}_{MU},\mathbf{m}_{UD}} \mathbb{1}\big(\mathbf{f}_h(\mathbf{x}_{MU}, \mathbf{m}_{MU}, \mathbf{b}(\mathbf{x}_{UD};\mathbf{G}), \mathbf{m}_{UD}, o, n) \leq \cdot\big) \\ &\qquad \cdot P(o,n)P^{MU}(\mathbf{m}_{MU};\mathbf{x}_{MU},\mathbf{G})P^{UD}\big(\mathbf{m}_{UD};\mathbf{b}(\mathbf{x}_{UD};\mathbf{G}),\mathbf{G}\big)\Big)P(d\mathbf{x}_{MU}, d\mathbf{x}_{UD}).\end{aligned}$$

The notation we use above can be summarized as follows:

(i) o: the exogenous order of bidders in the pool visiting the platform.

(ii) n: number of auctions (i.e., market thickness).

(iii) $\bar{N}$: upper bound of n.

(iv) $\mathbf{m}_{MU}, \mathbf{m}_{UD}$: sets of MU and UD bidders who visit the platform.

(v) $\mathbf{x}_{MU}, \mathbf{x}_{UD}$: vectors of valuations of MU and UD bidders in the bidder pool.

(vi) $\mathbf{b}(\mathbf{x}_{UD};\mathbf{G})$: bidding function of UD bidders.

(vii) $\mathbf{f}_h(\cdot)$: vector of functions that generate the final price in each auction.

We can ignore all discrete variables (i.e., $\mathbf{m}_{MU}$, $\mathbf{m}_{UD}$, o, and n) as they only take a finite number of values. It is sufficient to establish the continuity and compactness of the following mapping $\Gamma_{sub} : \mathcal{H} \to \mathcal{H}$, where $\mathcal{H}$ is the space of probability CDFs, with fixed $\mathbf{m}_{MU}$,$\mathbf{m}_{UD}$, o, and n. Specifically,

$$\begin{aligned}\tilde{\mathbf{G}}(\cdot) &= \Gamma_{sub}(\cdot; \mathbf{G}) \\ &= \int \Big(\mathbb{1}\big(\mathbf{f}_h(\mathbf{x}_{MU}, \mathbf{m}_{MU}, \mathbf{b}(\mathbf{x}_{UD}; \mathbf{G}), \mathbf{m}_{UD}, o, n) \leq \cdot\big) \\ &\qquad \cdot P(o, n) P^{MU}(\mathbf{m}_{MU}; \mathbf{x}_{MU}, \mathbf{G}) P^{UD}\big(\mathbf{m}_{UD}; \mathbf{b}(\mathbf{x}_{UD}; \mathbf{G}), \mathbf{G}\big)\Big) P(d\mathbf{x}_{MU}, d\mathbf{x}_{UD}).\end{aligned}$$

Note that $\Gamma = \sum \Gamma_{sub}$. In addition, we have the following specifications for the components of $\Gamma_{sub}(\mathbf{G})$:

$$\begin{aligned}&P^{MU}(\mathbf{m}_{MU}; \mathbf{x}_{MU}, \mathbf{G}) \\ &= \prod_{\ell^{MU}} P\big(c_{MU} \leq r^{MU}(x_{\ell^{MU}}; \mathbf{G})\big)^{\mathbb{1}(\ell^{MU} \in \mathbf{m}_{MU})} P\big(c_{MU} > r^{MU}(x_{\ell^{MU}}; \mathbf{G})\big)^{\mathbb{1}(\ell^{MU} \notin \mathbf{m}_{MU})}, \text{ and}\end{aligned}$$

$$\begin{aligned}&P^{UD}(\mathbf{m}_{UD}; \mathbf{b}, \mathbf{G}) \\ &= \prod_{\ell^{UD}} P\big(c_{UD} \leq r^{UD}(b_{\ell^{UD}}; \mathbf{G})\big)^{\mathbb{1}(\ell^{UD} \in \mathbf{m}_{UD})} P\big(c_{UD} > r^{UD}(b_{\ell^{UD}}; \mathbf{G})\big)^{\mathbb{1}(\ell^{UD} \notin \mathbf{m}_{UD})},\end{aligned}$$

where $r^{MU}(\cdot; \mathbf{G})$ and $r^{UD}(\cdot; \mathbf{G})$ denote the expected payoffs per market visit for MU and UD bidders, respectively. Furthermore, Proposition 2.9.4 (specified in Appendix 2.9.4) implies that bidders simply choose to bid in the auction(s) with the lowest standing bid(s).[29] The expected payoffs per market visit for both types of bidders

[29] We provide proofs for the propositions in the remainder of the Appendix.

are:

$$r^{MU}(x_{\ell^{MU}};\mathbf{G}) = \sum_{n=1}^{\bar{N}} p_n \sum_{j=1}^{\min(n,K)} \int_0^{x_{\ell^{MU}}} G_n^j(y)dy, \text{ and } r^{UD}(b_{\ell^{UD}};\mathbf{G}) = \sum_{n=1}^{\bar{N}} p_n \int_0^{b_{\ell^{UD}}} G_n^1(y)dy,$$

where ℓ^{MU} and ℓ^{UD} are indices corresponding to the MU and UD bidders. The probability that there are n auctions is denoted by p_n. The optimal bid $b_{\ell^{UD}}$ placed by UD bidder ℓ^{UD} is:

$$b_{\ell^{UD}} = x_{\ell^{UD}} - \frac{\alpha^{UD}}{1-\alpha^{UD}}\Big(r^{UD}(b_{\ell^{UD}};\mathbf{G}) + \frac{1}{\mu^{UD}} exp\big(-\mu^{UD} r^{UD}(b_{\ell^{UD}};\mathbf{G})\big) - \frac{1}{\mu^{UD}}\Big),$$

where μ^{UD} is the parameter of the participation cost distribution of the UD bidders.

By definition, the vector of functions that generate the final price in each auction can be denoted by $\mathbf{f}_h = (f_h^1, ..., f_h^{\bar{N}})$, and the element f_h^j maps all bidders' auction selections and bidding decisions to the final price in auction j. For ease of exposition, we define f_h^j as a function of the ordered valuations of the participating bidders. In auction j, we use $x_{MU}^{j,(\ell)}, x_{UD}^{j,(\ell)}$ to denote the ℓ^{th}-highest valuation among MU and UD bidders, respectively, and we use $\mathbf{x}_{MU}^j$ and $\mathbf{x}_{UD}^j$ to denote the vectors of valuations of MU and UD bidders who choose auction j. We can write f_h^j as

$$f_h^j(\mathbf{x}_{MU}^j, \mathbf{x}_{UD}^j;\mathbf{G}) = \begin{cases} x_{MU}^{j,(2)}, & if\ x_{MU}^{j,(2)} > b(x_{UD}^{j,(1)}, \mathbf{G}) \\ b(x_{UD}^{j,(2)}, \mathbf{G}), & if\ b(x_{UD}^{j,(2)}, \mathbf{G}) > x_{MU}^{j,(1)} \\ x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}, \mathbf{G}), & otherwise. \end{cases}$$

So far, we have completed the specification of mapping Γ_{sub}. In addition, we note that the steady-state beliefs of the market state (i.e., $\Psi(\omega)$) and the highest rival bids (i.e., $G_j(y|\omega)$) affect bidders' behavior only through $\mathbf{G}$. Thus, to establish consistency, it is sufficient to show that $\mathbf{G}$ is induced by bidders playing their optimal strategies. In what follows, we show the continuity and compactness of Γ_{sub}.

Continuity of Γ_{sub}.

We prove the following three lemmas to establish the continuity of Γ_{sub}. Given $\mathbf{m}_{MU}$, mapping $\Gamma_1(\cdot;\mathbf{G}) \triangleq P^{MU}(\mathbf{m}_{MU};\cdot,\mathbf{G})$ is uniformly bounded and Lipschitz continuous in $\mathbf{G}$.

Proof: The mapping is uniformly bounded since $P_{MU}(\mathbf{m}_{MU};\cdot,\mathbf{G})$ is a probability CDF. To show that Γ_1 is Lipschitz continuous, first note that for all $\mathbf{G},\mathbf{G}' \in \mathcal{H}$ and for all $x_{\ell^{MU}} \in [0,B]$ (recall that B is the upper bound of the valuations), we have:

$$|r^{MU}(x_{\ell^{MU}};\mathbf{G}) - r^{MU}(x_{\ell^{MU}};\mathbf{G}')| \le \sum_{n=1}^{\bar{N}} p_n \sum_{j=1}^{\min(n,K)} \int_0^{x_{\ell^{MU}}} |\mathbf{G}_n^j(y) - \mathbf{G}_n'^j(y)|dy \le \bar{N}^2 B|\mathbf{G}-\mathbf{G}'|_\infty.$$

Therefore, $r^{MU}(\cdot;\mathbf{G})$ is uniformly bounded, and Lipschitz continuous in $\mathbf{G}$.

Second, using the fact that the participation costs follow the exponential distribution, we show that $exp\big(-\mu_{MU}r^{MU}(\cdot;\mathbf{G})\big)$ is uniformly bounded and Lipschitz continuous in $r^{MU}(\cdot;\mathbf{G})$. Note that $0 \le r^{MU}(x_{\ell^{MU}};\mathbf{G}) \le \bar{N}B$. Moreover, for any $u,u' \in [0,\bar{N}B]$, we have

$$|exp(-\mu^{MU}u) - exp(-\mu^{MU}u')| \le \int_{u'}^{u} |\mu exp(-\mu^{MU}t)|dt \le \mu^{MU}|u-u'|,$$

and $exp(-\mu^{MU}u) \in [0,1]$. In turn, this implies that $\exp\big(-\mu^{MU}r^{MU}(\cdot;\mathbf{G})\big)$ is uniformly bounded and Lipschitz continuous in $r^{MU}(\cdot;\mathbf{G})$. From the definition of $P^{MU}(\mathbf{m}_{MU};\cdot,\mathbf{G})$, which is equal to the product between $\exp\big(-\mu^{MU}r^{MU}(x_{\ell^{MU}};\mathbf{G})\big)$ and $1-\exp\big(-\mu^{MU}r^{MU}(x_{\ell^{MU}};\mathbf{G})\big)$ for a finite number of bidders, we conclude that $P^{MU}(\mathbf{m}_{MU};\cdot,\mathbf{G})$ is Lipschitz continuous in $\mathbf{G}$. ∎

Given $\mathbf{m}_{UD}$, mapping $\Gamma_2(\cdot;\mathbf{G}) \triangleq P^{UD}(\mathbf{m}_{UD};\mathbf{b}(\cdot;\mathbf{G}),\mathbf{G})$ is uniformly bounded and Lipschitz continuous in $\mathbf{G}$. *Proof:* The mapping is uniformly bounded as $P^{UD}(\mathbf{m}_{UD}; \mathbf{b}(\cdot; \mathbf{G}), \mathbf{G})$ is a probability CDF. To establish Lipschitz continuity, we decompose Γ_2 into three parts:

(i) Mapping $\Gamma_{2a}(\cdot; \mathbf{G}) \triangleq P^{UD}(\mathbf{m}_{UD}; \cdot, \mathbf{G})$;

(ii) Function $f_{2b}(\mathbf{b}) \triangleq P^{UD}(\mathbf{m}_{UD}; \mathbf{b}, \mathbf{G})$ given $\mathbf{G}$; and

(iii) Mapping $\Gamma_{2c}(\cdot; \mathbf{G}) \triangleq \mathbf{b}(\cdot; \mathbf{G})$.

We show that each part is uniformly bounded and Lipschitz continuous separately. Note that $\mathbf{m}_{UD}$ is fixed. The claim for $\Gamma_{2a}(\mathbf{G})$ follows using the same argument as in Lemma 2.9.3. For $f_{2b}(\mathbf{b})$, it is sufficient to show

$$P\big(c_{UD} \leq r^{UD}(b; \mathbf{G})\big) = 1 - exp(-\mu^{UD} r^{UD}(b; \mathbf{G})),$$

is Lipschitz continuous in b given $\mathbf{G}$. This holds since

$$\Big|\frac{\partial P\big(c_{UD} \leq r^{UD}(b; G)\big)}{\partial b}\Big| = \big|\mu^{UD} exp\big(- \mu^{UD} r^{UD}(b; \mathbf{G})\big) \sum_{n=1}^{\bar{N}} p_n \mathbf{G}_n^1(\mathbf{b})\big| \leq \mu^{UD}.$$

Lastly, mapping $\Gamma_{2c}(\mathbf{G}) : \mathcal{H} \mapsto [0, B]^L$, where recall that L denotes the upper bound of the size of the bidder pool, is uniformly bounded. For a UD bidder with $x_{\ell^{UD}} \in [0, B]$ and for all $\mathbf{G}, \mathbf{G}' \in \mathcal{H}$, according to the optimal bidding decision of UD bidders, we have

$$b = x_{\ell^{UD}} - F(b, \mathbf{G}), \text{ and } b' = x_{\ell^{UD}} - F(b', \mathbf{G}'),$$

where

$$F(b, \mathbf{G}) = \frac{\alpha^{UD}}{1 - \alpha^{UD}} \Big(r^{UD}(b; \mathbf{G}) + \frac{1}{\mu^{UD}} exp\big(- \mu^{UD} r^{UD}(b; \mathbf{G})\big) - \frac{1}{\mu^{UD}} \Big).$$

Note that $b - b' + F(b, \mathbf{G}) - F(b', \mathbf{G}') = 0$, which further implies that

$$b - b' + F(b, \mathbf{G}) - F(b', \mathbf{G}) = F(b', \mathbf{G}') - F(b', \mathbf{G}).$$

The left-hand side of the above equation is equal to $\int_{b'}^{b} 1 + \frac{\partial F}{\partial b}(u, \mathbf{G}) du$. Thus,

$$\frac{\partial F}{\partial b}(u, \mathbf{G}) = \frac{\partial F}{\partial r^{UD}} \frac{\partial r^{UD}}{\partial b}(u, \mathbf{G}) = \frac{\alpha^{UD}}{1 - \alpha^{UD}} \Big(1 - exp\big(-\mu^{UD} r^{UD}(u; \mathbf{G})\big)\Big) \Big(\sum_{n=1}^{\bar{N}} p_n G_n^1(u)\Big) > 0,$$

which implies that $|b - b' + F(b, \mathbf{G}) - F(b', \mathbf{G})| \geq |b - b'|$. On the other hand,

$$|F(b', \mathbf{G}') - F(b', \mathbf{G})| \leq \frac{2\alpha^{UD}}{1 - \alpha^{UD}} |r^{UD}(b'; \mathbf{G}') - r^{UD}(b'; \mathbf{G})| \leq \frac{2\alpha^{UD} \bar{N} B}{1 - \alpha} |\mathbf{G} - \mathbf{G}'|_\infty.$$

Thus,

$$|\mathbf{b} - \mathbf{b}'|_\infty \leq \frac{2\alpha^{UD} \bar{N} B}{1 - \alpha^{UD}} |\mathbf{G} - \mathbf{G}'|_\infty.$$

Finally, we conclude that $\Gamma_{2c}(\mathbf{G})$ is Lipschitz continuous in $\mathbf{G}$, using the triangular inequality. ■

Mapping Γ_3 corresponding to auction j and defined as follows:

$$\Gamma_3(\cdot; \mathbf{G}) \triangleq \int \mathbb{1}\big(f_h^j(\mathbf{x}_{MU}^j, \mathbf{x}_{UD}^j; \mathbf{G}) \leq \cdot\big) P^o(d\mathbf{x}),$$

is uniformly bounded and Lipschitz continuous in $\mathbf{G}$, given $o, n, \mathbf{m}_{MU}, \mathbf{m}_{UD}$. *Proof:* For any $y^j \in [0, B]$, and for any $\mathbf{G}, \mathbf{G}' \in \mathcal{H}$, we have

$$\begin{aligned}
\Gamma_3(y^j; \mathbf{G}) &= \tilde{G}(y^j) \\
&= \int \mathbb{1}\big(b(x_{UD}^{j,(1)}; \mathbf{G}) < x_{MU}^{j,(2)} \leq y^j\big) P^o(d\mathbf{x}) + \int \mathbb{1}\big(x_{MU}^{j,(1)} < b(x_{UD}^{j,(2)}; \mathbf{G}) \leq y^j\big) P^o(d\mathbf{x}) \\
&\quad + \int \mathbb{1}\big(x_{MU}^{j,(2)} \leq b(x_{UD}^{(1)}; \mathbf{G}), x_{MU}^{j,(1)} \geq b(x_{UD}^{j,(1)}; \mathbf{G}), x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}; \mathbf{G}) \leq y^j\big) P^o(d\mathbf{x}).
\end{aligned}$$

Then, $\left|\tilde{G}(y^j) - \tilde{G}'(y^j)\right| \leq I_1 + I_2 + I_3$, for I_1, I_2, and I_3, is defined as:

$$I_1 = \int \left|\mathbb{1}\big(b(x_{UD}^{j,(1)}; \mathbf{G}) < x_{MU}^{j,(2)} \leq y^j\big) - \mathbb{1}\big(b(x_{UD}^{j,(1)}; \mathbf{G}') < x_{MU}^{j,(2)} \leq y^j\big)\right| P^o(d\mathbf{x})$$

$$I_2 = \int \left|\mathbb{1}\big(b(x_{UD}^{j,(1)}; \mathbf{G}) < x_{MU}^{j,(2)} \leq y^j\big) - \mathbb{1}\big(b(x_{UD}^{j,(1)}; \mathbf{G}') < x_{MU}^{j,(2)} \leq y^j\big)\right| P^o(d\mathbf{x})$$

$$I_3 = \int \Big|\mathbb{1}\big(x_{MU}^{j,(2)} \leq b(x_{UD}^{j,(1)}; \mathbf{G}), x_{MU}^{j,(1)} \geq b(x_{UD}^{j,(1)}, \mathbf{G}), x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}; \mathbf{G}) \leq y^j\big)$$
$$- \mathbb{1}\big(x_{MU}^{j,(2)} \leq b(x_{UD}^{j,(1)}; \mathbf{G}'), x_{MU}^{j,(1)} \geq b(x_{UD}^{j,(1)}; \mathbf{G}'), x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}; \mathbf{G}') \leq y^j\big)\Big| P^o(d\mathbf{x}).$$

The claim follows by showing that I_1, I_2, and I_3 are bounded by $|\mathbf{G} - \mathbf{G}'|_\infty$. For brevity, we establish that I_1 is bounded by $|\mathbf{G} - \mathbf{G}'|_\infty$ (the proofs for I_2 and I_3 follow a similar approach). Specifically,

$$\int_{b(x_{UD}^{j,(1)}; \mathbf{G}) \geq x_{MU}^{j,(2)}, b(x_{UD}^{j,(1)}; \mathbf{G}') < x_{MU}^{j,(2)} \leq y^j} \mathbb{1} P^o(\mathbf{x})$$
$$\leq \int_{b(x_{UD}^{j,(1)}; \mathbf{G}') < x_{MU}^{j,(2)} \leq b(x_{UD}^{j,(1)}; \mathbf{G})} \mathbb{1} P^o(d\mathbf{x}) = \int F_{MU}^{j,(2)}\big(b(x_{UD}^{j,(1)}; \mathbf{G})\big) - F_{MU}^{j,(2)}\big(b(x_{UD}^{j,(1)}; \mathbf{G}')\big) P^o(dx_{UD}^{j,(1)})$$
$$\leq |f_{MU}^{j,(2)}|_\infty \int b(x_{UD}^{j,(1)}; \mathbf{G}) - b(x_{UD}^{j,(1)}; \mathbf{G}') P^o(dx_{UD}^{j,(1)}) \leq M_{MU}^{j,(2)} \cdot \frac{2\alpha^{UD} \bar{N} B^2}{1 - \alpha^{UD}} |\mathbf{G} - \mathbf{G}'|_\infty,$$

where $F_{MU}^{j,(2)}(x_{MU}^{j,(2)})$ and $f_{MU}^{j,(2)}(x_{MU}^{j,(2)})$ are the CDF and PDF of the second-highest valuation among MU bidders in auction j, respectively. The last inequality holds from Lemma 2.9.3, and $M_{MU}^{j,(2)} \triangleq \sup_{\mathbf{G}} |f_{MU}^{j,(2)}|_\infty < \infty$, which is independent of $\mathbf{G}$. ■

In summary, combining Lemmas 2.9.3, 2.9.3, and 2.9.3, we conclude that Γ_{sub} is uniformly bounded and Lipschitz continuous in $\mathbf{G}$. Therefore, Γ is a continuous mapping.

Compactness of Γ_{sub}.

Here we show that Γ_{sub} is a compact mapping using the Arzela–Ascoli theorem. To apply the theorem, Γ_{sub} needs to satisfy the following conditions:

1. The image $\Gamma_{sub}(\cdot;\mathcal{H})$ is uniformly bounded.

2. Sequence $\{\Gamma_{sub}(\mathbf{y};\mathbf{G}_n)\}$ is equicontinuous in $\mathbf{y}\in[0,B]^{\bar{N}}$.

The fact that $\Gamma_{sub}(\mathcal{H})$ is uniformly bounded can be derived in a straightforward manner. To establish the equicontinuity of the mapping, we show that for any $\mathbf{G}\in\mathcal{H}$ and for any $\mathbf{y},\mathbf{y}'\in[0,B]^{\bar{N}}$, we have

$$\left|\Gamma_{sub}(\mathbf{y};\mathbf{G})-\Gamma_{sub}(\mathbf{y}';\mathbf{G})\right|\leq K_0\cdot\left|\mathbf{y}-\mathbf{y}'\right|,$$

where K_0 is a constant, which is independent of $\mathbf{G}$. It is sufficient to show that the following inequality holds for the j^{th} auction given y_j, y_j' and fixed $\mathbf{m}_1,\mathbf{m}_2,n,o$:

$$|\Gamma_{sub,j}(y_j;\mathbf{G})-\Gamma_{sub,j}(y_j';\mathbf{G})|\leq J_1+J_2+J_3,$$

where J_1, J_2, and J_3 are defined as follows:

$$\begin{aligned}
J_1 &= \int\left|\mathbb{1}\left(b(x_{UD}^{j,(1)};\mathbf{G})<x_{MU}^{j,(2)}<y_j\right)-\mathbb{1}\left(b(x_{UD}^{j,(1)};\mathbf{G})<x_{MU}^{j,(2)}<y_j'\right)\right|P^o(d\mathbf{x})\\
J_2 &= \int\left|\mathbb{1}(x_{MU}^{j,(1)}<b(x_{UD}^{j,(2)};\mathbf{G})\leq y_j)-\mathbb{1}(x_{MU}^{j,(1)}<b(x_{UD}^{j,(2)};\mathbf{G})\leq y_j')\right|P^o(d\mathbf{x})\\
J_3 &= \int\left|\mathbb{1}\left(x_{MU}^{j,(2)}\leq b(x_{UD}^{j,(1)};\mathbf{G}),x_{MU}^{j,(1)}\leq b(x_{UD}^{j,(1)};\mathbf{G}),x_{MU}^{j,(1)}\wedge b(x_{UD}^{j,(1)};\mathbf{G})\leq y_j\right)\right.\\
&\qquad\left.-\,\mathbb{1}\left(x_{MU}^{j,(2)}\leq b(x_{UD}^{j,(1)};\mathbf{G}),x_{MU}^{j,(1)}\leq b(x_{UD}^{j,(1)};\mathbf{G}),x_{MU}^{j,(1)}\wedge b(x_{UD}^{j,(1)};\mathbf{G})\leq y_j'\right)\right|P^o(d\mathbf{x}).
\end{aligned}$$

In each case, the difference within the absolute value takes value $-1,0$, or 1. For brevity, we only show that J_3 is bounded by $|\mathbf{y}-\mathbf{y}'|$ (Using similar arguments, one can show that J_1 and J_2 are bounded by $|\mathbf{y}-\mathbf{y}'|$ as well). We have

$$
\begin{aligned}
J_3 &= \int \mathbb{1}\Big(x_{MU}^{j,(2)} \le b(x_{UD}^{j,(1)}, \mathbf{G}), x_{MU}^{j,(1)} \le b(x_{UD}^{j,(1)}; \mathbf{G}), y_j < x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}; \mathbf{G}) \le y_j'\Big) P^o(d\mathbf{x}) \\
&\le \int \mathbb{1}\big(y_j < x_{MU}^{j,(1)} \wedge b(x_{UD}^{j,(1)}; \mathbf{G}) \le y_j'\big) P^o(d\mathbf{x}) \\
&= \int \Big(\mathbb{1}(y_j < x_{MU}^{j,(1)} \le y_j') \cdot \mathbb{1}\big(x_{MU}^{j,(1)} \le b(x_{UD}^{j,(1)}; \mathbf{G})\big) + \mathbb{1}\big(y_j < b(x_{UD}^{j,(1)}; \mathbf{G}) \le y_j'\big) \cdot \mathbb{1}\Big(b(x_{UD}^{j,(1)}; \mathbf{G}) \\
&\le \int_{y_j}^{y_j'} P^o(dx_{MU}^{j,(1)}) + \int_{y_j < b(x_{UD}^{j,(1)}; \mathbf{G}) \le y_j'} P^o(dx_{UD}^{j,(1)}). \qquad (2.25)
\end{aligned}
$$

For the first term in Expression (2.25), we have

$$
\int_{y_j}^{y_j'} P^o(dx_{MU}^{j,(1)}) \le \Big|\frac{dF_{MU}^{j,(1)}}{dx_{MU}^{j,(1)}}\Big|_\infty \cdot |y_j' - y_j| \le M_{MU}^{j,(1)} \cdot |y_j' - y_j|,
$$

where $M_{MU}^{j,(1)} \triangleq \sup_{\mathbf{G}} |f_{MU}^{j,(1)}|_\infty < \infty$. For the second term in Expression (2.25), we define vectors $\mathbf{z}, \mathbf{z}'$ as the solutions to the following equations:

$$
z_j = y_j + F(y_j, \mathbf{G}), \text{ and } z_j' = y_j' + F(y_j', \mathbf{G}).
$$

Recall that from Lemma 2.9.3:

$$
F(b, \mathbf{G}) = \frac{\alpha^{UD}}{1-\alpha^{UD}}\Big(r^{UD}(b; \mathbf{G}) + \frac{1}{\mu^{UD}} exp\big(-\mu^{UD} r^{UD}(b; \mathbf{G})\big) - \frac{1}{\mu^{UD}}\Big).
$$

Therefore,

$$
\int_{y_j < b(x_{UD}^{j,(1)}; \mathbf{G}) \le y_j'} P^o(dx_{UD}^{j,(1)}) = \int_{z_j < x_{UD}^{j,(1)} \le z_j'} P^o(dx_{UD}^{j,(1)}) \le M_{UD}^{j,(1)} \cdot |z_j' - z_j|,
$$

where $M_{UD}^{j,(1)} \triangleq \sup_{\mathbf{G}} |f_{UD}^{j,(1)}|_\infty < \infty$. Finally,

$$
|z_j' - z_j| \le |y_j' - y_j| + |F(y_j', \mathbf{G}) - F(y_j, \mathbf{G})| \le \frac{1}{1-\alpha^{UD}} |y_j' - y_j|.
$$

Therefore, we establish that $\{\Gamma_{sub}(\mathbf{y}; \mathbf{G})\}$, $\mathbf{G} \in \mathcal{H}$, is equicontinuous in $\mathbf{y}$. Applying the Arzela–Ascoli theorem, we obtain that Γ_{sub} is a compact mapping. ■

Finally, employing the Schauder fixed-point theorem implies the existence of a fixed point such that $\Gamma(\mathbf{G}) = \mathbf{G}$. This completes the proof of the proposition. ■

2.9.4 UD Bidders' Auction Selection

Assume that $\nu^{MU} = \nu^{UD} = 0$ and $K \geq \bar{N}$; the UD bidders' optimal auction selection decision is to choose the auction with the lowest standing bid.

Proof. The assumptions of the proposition (i.e., $\nu^{MU} = \nu^{UD} = 0$ and $K \geq \bar{N}$) directly imply that MU bidders place a bid in all open auctions (provided that they place at least one bid). Given this observation, the proposition states that it is optimal for a UD bidder to bid in the auction with the lowest standing bid at the time she places her bid, if all other competing UD bidders also bid in the auction with the lowest standing bid at the time they place their bids. In other words, participating in the auction with the lowest standing bid (at the time she determines which auction to participate in) is an equilibrium strategy for a UD bidder.

Prior to proving the proposition, we first state and prove Lemmas 2.9.4, 2.9.4, and 2.9.4 that establish the following: if the *last* agent that places a bid is a UD bidder, it is optimal for her to bid in the auction with the lowest standing bid, assuming that all other UD bidders also place bids in the auctions with the lowest standing bids (at the time they decide which auction to participate in).

Suppose that there are n auctions on the platform, and they are ordered with respect to their standing bids (i.e., auction i's standing bid is no greater than auction j's if $i < j$). It is sufficient to show that the conditional CDFs of the current winning bids satisfy the following relationship:

$$G_1^0(w|\mathbf{s}) \geq G_2^0(w|\mathbf{s}) \geq ... \geq G_n^0(w|\mathbf{s}), \tag{2.26}$$

where $G_j^0(w|\mathbf{s})$ is the conditional CDF of the current winning bid in the auction with the j^{th}-lowest standing bid. In what follows, we focus on the case where the number of UD bidders is known and denoted by N. One can extend the results to the case where N is unknown, by taking the expectation over the number of UD bidders on the platform.

We first start with the simplest case, where there are only two auctions and all bidders are UD bidders. Suppose that there are 2 identical auctions and $N+1$ UD bidders placing bids in an exogenous sequence. It is optimal for the last UD bidder to bid in the auction with the lowest standing bid, assuming that the rest of the UD bidders also place bids in the auctions with the lowest standing bids (at the time they choose which auction to participate in).

Proof. It is sufficient to show that, after the first N UD bidders place their bids, the conditional CDFs of the winning bids given the standing bids (s_1, s_2), with $s_1 \leq s_2$, follow the relationship

$$G_1^0(w|s_1, s_2) \geq G_2^0(w|s_1, s_2),$$

where $G_j^0(w|s_1, s_2)$ is the conditional CDF of the winning bid in auction j.

We use $X_1, X_2, ..., X_N$ to denote the random variables corresponding to the bids of the first N bidders. The corresponding order statistics are denoted by $X_{(1)}, X_{(2)}, ..., X_{(N)}$ (i.e., $X_{(\ell)}$ is the ℓ^{th}-largest bid among these N bids).

One important observation is that after all bids are submitted, the two standing bids (s_1, s_2) and the two (invisible) winning bids, denoted by (W_1, W_2), are comprised of $X_{(1)}, X_{(2)}, X_{(3)}$, and $X_{(4)}$. We can verify this observation by way of contradiction. In particular, suppose that the last four bids are $X_{(1)}, X_{(2)}, X_{(3)}$, and $X_{(\ell)}$ and $X_{(\ell)} < X_{(4)}$. Note that the 4^{th}-highest bid can always be placed (since it is impossible to have both standing bids greater than $X_{(4)}$). Before $X_{(4)}$ is removed from these four remaining bids, it has to first serve as the standing bid of an auction. In this case, $X_{(4)}$ is the lowest standing bid among these two auctions, and it is outbid by a higher incoming bid (which can only be one of $X_{(1)}$, $X_{(2)}$, or $X_{(3)}$). In this case, bidding $X_{(\ell)}$ has no impact on the vector of standing bids, which yields the contradiction.

Given this observation, we note that there are the following three cases to consider:

Cases	(s_1, s_2)	(W_1, W_2)
1	$s_1 = X_{(4)}, s_2 = X_{(3)}$	$W_1 = X_{(1)}, W_2 = X_{(2)}$
2	$s_1 = X_{(4)}, s_2 = X_{(3)}$	$W_1 = X_{(2)}, W_2 = X_{(1)}$
3	$s_1 = X_{(4)}, s_2 = X_{(2)}$	$W_1 = X_{(3)}, W_2 = X_{(1)}$

In Case 1, we have

$$\Delta_1 \triangleq G_1^0(w|s_1, s_2, \text{Case 1}) - G_2^0(w|s_1, s_2, \text{Case 1})$$

$$= P(X_{(1)}|X_{(4)} = s_1, X_{(3)} = s_2) - P(X_{(2)}|X_{(4)} = s_1, X_{(3)} = s_2).$$

Similarly, in Case 2, we have:

$$\Delta_2 \triangleq G_1^0(w|s_1, s_2, \text{Case 2}) - G_2^0(w|s_1, s_2, \text{Case 2})$$

$$= P(X_{(1)}|X_{(4)} = s_1, X_{(3)} = s_2) - P(X_{(2)}|X_{(4)} = s_1, X_{(3)} = s_2)$$

$$= -\Delta_1.$$

We also note that the conditional probabilities of a bidding sequence falling into Case 1 or Case 2 given (s_1, s_2) are identical (i.e., $P(\text{Case } 1|s_1, s_2) = P(\text{Case } 2|s_1, s_2)$). The reason is the following: for any bidding sequence that falls into Case 1, switching the order of bids $X_{(1)}$ and $X_{(2)}$ results in a bidding sequence that falls into Case 2, as (i) the decisions of these two bidders are the same, provided that their bids are larger than the standing bids, and (ii) both bids are invisible (thus, they do not alter the sequence of the following bids). In other words, the total number of bidding sequences that lead to Case 1 is the same as the corresponding number for Case 2. We also note that each bidding sequence has the same chance of occurring (as the order in which bidders place their bids is exogenous); thus, $P(\text{Case } 1|s_1, s_2) = P(\text{Case } 2|s_1, s_2)$ for any s_1, s_2. Finally, in Case 3, we have:

$$\begin{aligned}\Delta_3 &\triangleq G_1^0(w|s_1, s_2, \text{Case } 3) - G_2^0(w|s_1, s_2, \text{Case } 3)\\ &= P(X_{(3)} \leq w|X_{(4)} = s_1, X_{(2)} = s_2) - P(X_{(1)} \leq w|X_{(4)} = s_1, X_{(2)} = s_2).\end{aligned}$$

Note that $\Delta_3 \geq 0$ since $\{X_{(1)} \leq w, X_{(4)} = s_1, X_{(2)} = s_2\} \subseteq \{X_{(3)} \leq w, X_{(4)} = s_1, X_{(2)} = s_2\}$. Summarizing the discussion above, we obtain that

$$\begin{aligned}&G_1^0(w|s_1, s_2) - G_2^0(w|s_1, s_2)\\ &= P(\text{Case } 1|s_1, s_2)\Delta_1 + P(\text{Case } 2|s_1, s_2)\Delta_2 + P(\text{Case } 3|s_1, s_2)\Delta_3 \geq 0.\end{aligned}$$

■

Next, we extend Lemma 2.9.4 to the case where there are n auctions, but still all bidders are UD.

Suppose that there are n identical auctions and $N+1$ UD bidders placing bids in an exogenous sequence. It is optimal for the last UD bidder to bid in the auction with the lowest standing bid, assuming that the rest of the UD bidders also place bids in the auctions with the lowest standing bids (at the time they choose which auction to participate in).

Proof. It is sufficient to show that, after the first N bidders place their bids, the conditional CDFs of the winning bids given the standing bid vector $\mathbf{s}$ follow the relationship specified in Expression (2.26). Equivalently, in the following proof, we show that for any $i < j$, we have

$$G_i^0(w|\mathbf{s}) \geq G_j^0(w|\mathbf{s}).$$

Observe that after the first N bids are submitted, the vector of standing bids $\mathbf{s}$ and the vector of current winning bids $\mathbf{W}$ are comprised of $X_{(\ell)}$, where $1 \leq \ell \leq 2n$. This follows using a similar argument as in the proof of Lemma 2.9.4.

Next, we consider the following three cases separately for auctions i and j.

Cases	(s_i, s_j)	(W_i, W_j)	
1	$s_i = X_{(l)}, s_j = X_{(k)}$	$W_i = X_{(L)}, W_j = X_{(K)}$	Given $K < L < k < l$
2	$s_i = X_{(l)}, s_j = X_{(k)}$	$W_i = X_{(K)}, W_j = X_{(L)}$	i.e., $X_{(l)} < X_{(k)} < X_{(L)} < X_{(K)}$
3	$s_i = X_{(l)}, s_j = X_{(L)}$	$W_i = X_{(k)}, W_j = X_{(K)}$	$\left\{\mathbf{X}_{-(l,k,L,K)} \in (\mathbf{s}_{-i,-j}, \mathbf{W}_{-i,-j})\right\}$

In the table, $\left\{\mathbf{X}_{-(l,k,L,K)} \in (\mathbf{s}_{-i,-j}, \mathbf{W}_{-i,-j})\right\}$ denotes the set of all valid assignments of the remaining $X_{(\ell)}$ ($1 \leq \ell \leq 2n$ and $\ell \neq l, k, L, K$) to the standing bids and the winning bids of all auctions excluding auctions i and j, which are denoted by $\mathbf{s}_{-i,-j}$ and $\mathbf{W}_{-i,-j}$, respectively.

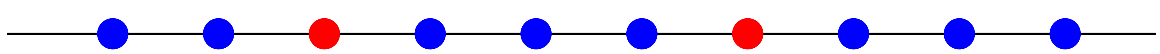
Specifically, for a given assignment in Case 1, denoted by a_1, we have:

$$s_i = X_{(l)}, s_j = X_{(k)}, W_i = X_{(L)}, W_j = X_{(K)} \text{ and } s_m = X_{m'}, W_m = X_{m''},$$

where $m \neq i, j, m' \neq l, k, m'' \neq L, K$, and $m' > m''$. Then, we have:

$$\begin{aligned}\Delta_1(a_1) \triangleq& G_i^0(w|s_i, s_j, a_1) - G_j^0(w|s_i, s_j, a_1)\\ =& P(X_{(L)} \leq w | X_{(l)} = s_i, X_{(k)} = s_j, X_{(m')} = s_m, m \neq i, j, m' \neq l, k)\\ &- P(X_{(K)} \leq w | X_{(l)} = s_i, X_{(k)} = s_j, X_{(m')} = s_m, m \neq i, j, m' \neq l, k).\end{aligned}$$

Based on a given bidding sequence that results in an outcome in assignment a_1, we can develop a new bidding sequence that leads to an outcome in the assignment described in Case 2, denoted by a_2. Specifically, we can switch the order of two bidders who placed bids $X_{(L)}$ and $X_{(K)}$ in the bidding sequence of a_1, to show that the new bidding sequence belongs to a_2 and leads to the same vector of standing bids $\mathbf{s}$ as the original one in a_1. In particular, the following is an illustration of a bidding sequence in a_1:



Each circle stands for a bid, and the two red circles represent bids $X_{(L)}$ and $X_{(K)}$. At the first red circle, a bidder's auction choice is the same regardless of whether her bid is $X_{(L)}$ or $X_{(K)}$. Suppose that the corresponding auction is auction i, given the definition of a_1. After the switch, at the first red circle, $X_{(K)}$ will be placed in auction i. Note that in a_1, the highest losing bid in that auction at the first red circle is no greater than $X_{(l)}$; thus, when $X_{(K)}$ is placed, it does not get revealed as $X_{(K)} > X_{(l)}$. In other words, the subsequence of bidding until the second red circle will not be affected after the switch. Similarly, at the second red circle, the bidder with bid $X_{(L)}$ will choose to bid in auction j (the same decision at this circle in the sequence of a_1). In addition, bid $X_{(L)}$ does not get revealed as the highest losing bid in auction j at the second red circle since it is no greater than $X_{(k)}$, which, in turn, is lower than $X_{(l)}$. In summary, the bidding sequence in a_1 is identical to the new sequence in a_2 except that we switched the bids corresponding to the two red circles. Moreover, these two sequences lead to the same vector of standing bids.

Given a_2 in Case 2, we have:

$$\begin{aligned}\Delta_2(a_2) \triangleq & G_i^0(w|s_i, s_j, a_2) - G_j^0(w|s_i, s_j, a_2) \\ &= P(X_{(K)} \leq w | X_{(l)} = s_i, X_{(k)} = s_j, X_{(m')} = s_m, m \neq i, j, m' \neq l, k) \\ &\quad - P(X_{(L)} \leq w | X_{(l)} = s_i, X_{(k)} = s_j, X_{(m')} = s_m, m \neq i, j, m' \neq l, k) \\ &= -\Delta_1(a_1).\end{aligned}$$

In addition, $P(a_1|\mathbf{s}) = P(a_2|\mathbf{s})$, as the numbers of assignments in Cases 1 and 2 are the same, and their elements can be matched one to one using the above argument.

Given an assignment in Case 3, we have:

$$s_i = X_{(l)}, s_j = X_{(L)}, W_i = X_{(k)}, W_j = X_{(K)} \text{ and } s_m = X_{(m')}, W_m = X(m''),$$

where $m \neq i, j, m' \neq l, L, m'' \neq k, K$, and $m' > m''$. Then, we have:

$$\Delta_3(a_3) \triangleq P(X_{(k)} \leq w | s_i = X_{(l)}, s_j = X_{(L)}, a_3) - P(X_{(K)} \leq w | s_i = X_{(l)}, s_j = X_{(L)}, a_3).$$

Note that $\Delta_3(a_3) \geq 0$, we have

$$\left\{X_{(K)} \leq w, s_i = X_{(l)}, s_j = X_{(L)}, a_3\right\} \subseteq \left\{X_{(k)} \leq w, s_i = X_{(l)}, s_j = X_{(L)}, a_3\right\}.$$

Therefore, by taking the expectation of $\Delta_1(a_1), \Delta_2(a_2)$ and $\Delta_3(a_3)$ over a_1 in Case 1, a_2 in Case 2, and a_3 in Case 3, we have:

$$\begin{aligned}
&G_i^0(w|\mathbf{s}) - G_j^0(w|\mathbf{s}) \\
&= \sum_{a_1 \in \text{Case 1}} P(a_1|\mathbf{s})\Delta_1(a_1) + \sum_{a_2 \in \text{Case 2}} P(a_2|\mathbf{s})\Delta_2(a_2) + \sum_{a_3 \in \text{Case 3}} P(a_3|\mathbf{s})\Delta_3(a_3) \geq 0.
\end{aligned}$$

■

Lastly, we consider the general case, which includes MU bidders. In particular, we establish the following lemma.

Suppose that there are n identical auctions. Furthermore, $N+1$ UD bidders and N' MU bidders place their bids in an exogenous sequence. If the last one is a UD bidder, it is optimal for her to bid in the auction with the lowest standing bid, assuming that the rest of the UD bidders also place bids in the auctions with the lowest standing bids (at the time they choose which auction to participate in).

Proof. It is sufficient to show that, after all MU bidders and the first N UD bidders place their bids, the conditional CDFs of the winning bids given the standing bid vector $\mathbf{s}$ follow the relationship specified in Expression (2.26).

Since a MU bidder participates in all auctions (if she chooses to bid at all), there is at most one MU winner at any given time. Thus, there are two types of outcomes: either all winners are UD bidders (Case 1) or there is a single MU winner (Case 2). In what follows, we show that Expression (2.26) holds in both cases.

In Case 1, where the highest $2n$ bids are placed by UD bidders, the claim follows directly from Lemma 2.9.4.

In Case 2, where there is at least one auction being won by a MU bidder, we denote the winning MU bid by M. Given the vector of standing bids $\mathbf{s}$ after the submissions of all bids, for any two auctions i and j with $i < j$, we aim to show that:

$$G_i^0(w|\mathbf{s}, \text{Case 2}) \geq G_j^0(w|\mathbf{s}, \text{Case 2}).$$

We then use W_i and W_j to denote the winning bids in auctions i and j, and consider the following three cases:

(i) $W_i = M, W_j = M$, where the MU bidder is winning both auctions.

(ii) $W_i = M, W_j = X_{(l)}$, where the MU bidder is winning auction i but not auction j.

(iii) $W_i = X_{(l)}, W_j = X_{(l')}$, where the MU bidder is winning neither auction.

In sub-case *(i)*, we have $G_i^0\big(w|\mathbf{s}, \text{Case 2}(i)\big) = G_j^0\big(w|\mathbf{s}, \text{Case 2}(i)\big)$. In sub-case *(ii)*, we have $X_{(l)} \geq M$; otherwise, the MU bidder is also winning auction j. Thus, $G_i^0\big(w|\mathbf{s}, \text{Case 2}(ii)\big) \geq G_j^0\big(w|\mathbf{s}, \text{Case 2}(ii)\big)$, because $\big\{X_{(l)} \leq w, \mathbf{s}, \text{Case 2}(ii)\big\} \subseteq \big\{M \leq w, \mathbf{s}, \text{Case 2}(ii)\big\}$. In sub-case *(iii)*, we can again apply the argument in the proof of Lemma 2.9.4 to show that $G_i^0\big(w|\mathbf{s}, \text{Case 2}(iii)\big) \geq G_j^0\big(w|\mathbf{s}, \text{Case 2}(iii)\big)$. In summary, we have shown that with the presence of MU bidders, Expression (2.26) holds. ■

Proof of Proposition 3. The optimal bid, given that the bidder has already chosen an auction, can be directly derived using the first-order conditions for the payoff maximization problem. For the auction selection decision, we use induction to show that choosing to bid in the auction with the lowest standing bid is optimal, given that all other UD bidders follow the same strategy.

Step 1. Consider the bidder that places the last bid on a given day (Bidder A). Lemma 2.9.4 directly implies that it is optimal for Bidder A to choose to bid in the auction with the lowest standing bid.

Step 2 (Induction step). We then assume that Auction 1, which has the lowest standing bid, is the optimal auction choice for a bidder given $\mathbf{s}$ and N future competing bidders. Specifically,

$$v_1^N(b, x; \mathbf{s}) \geq v_2^N(b, x; \mathbf{s}) \geq ... \geq v_n^N(b, x; \mathbf{s}),$$

where $v_i^N(b, x; \mathbf{s})$ denotes the expected payoff for the bidder if she places a bid in auction i. The conditional expected payoff $v_i^N(b, x; \mathbf{s})$ can be expressed as

$$v_i^N(b, x; \mathbf{s}) = \int_{s_i}^{b} \Big(\int_{w_i}^{b} (x - p_i)\hat{g}_i^N(p_i|w_i, \mathbf{s}_{-i}) + \alpha^{UD}\big(1 - \hat{G}_i^N(b|w_i, \mathbf{s}_{-i})v_f(x)\big)\Big) g_i^0(w_i|\mathbf{s})dw_i.$$

In auction i, $\hat{g}_i^N(p_i|\mathbf{s})$ and $\hat{G}_i^N(p_i|\mathbf{s})$ respectively stand for the PDF and CDF of the highest rival bid against the *incumbent* winner given standing bids $\mathbf{s}$ and N incoming bidders.

The optimal bid of the next UD bidder (Bidder B) is denoted by b_1.[30] Without loss of generality, we show that for Bidder A, inequality $v_1^{N+1}(b; \mathbf{s}) \geq v_2^{N+1}(b; \mathbf{s})$ holds.

[30]For brevity, we omit the discussion of the case where the next bidder is a MU bidder, since the claim follows in a similar fashion when we classify the information set.

We construct four information sets and establish that the inequality holds in each set.

(i) $b_1 \leq s_1$ (i.e., Bidder B does not submit a competing bid). In this case, it follows from the induction step that it is optimal for Bidder A to place a bid in Auction 1.

(ii) $s_1 < b_1 \leq \min(W_1, s_2)$ (i.e., whether Bidder B submits a bid depends on which auction Bidder A places her bid in). In this case, Bidder B will not bid in any auction if Bidder A bids in Auction 1 because bid b_1 is lower than both the standing bid in Auction 2 and the new standing bid in Auction 1 (i.e., the previously winning bid W_1). If Bidder A chooses to bid in auction 1, she will have only N future competitors and, as a result, the state would be updated from $(s_1, s_2, ..., s_n)$ with $N+1$ incoming bids to $(b_1, s_2, ..., s_n)$ with N incoming bids. Therefore, her conditional expected payoff is

$$\begin{aligned} v_1^{N+1}(b, x; \mathbf{s}) = \int_{b_1}^{b} \Big(\int_{w_1}^{b} (x - p_1) \hat{g}_1^N(p_1 | w_1, \mathbf{s}_{-1}) dp_1 \\ + \alpha^{UD} \big(1 - \hat{G}_1^N(b | w_1, \mathbf{s}_{-1})\big) v_f(x, G, \Psi) \Big) g_1^0(w_1 | b_1, \mathbf{s}_{-1}) dw_1 \\ = v_1^N(b, x; b_1, \mathbf{s}_{-1}). \end{aligned}$$

On the other hand, if Bidder A places a bid in Auction 2, Bidder B will bid in Auction 1. In this case, the standing bid in Auction 1 increases to b_1. Her conditional expected payoff from bidding in Auction 2 is

$$\begin{aligned}v_2^{N+1}(b,x;\mathbf{s}) =& \int_{s_2}^{b} \Big(\int_{w_2}^{b} (x-p_2)\hat{g}_2^N(p_2|b_1,w_2,\mathbf{s}_{-1,-2})dp_2 \\ &+ \alpha^{UD}\big(1-\hat{G}_2^N(b|b_1,w_2,\mathbf{s}_{-1,-2})\big)v_f(x,G,\Psi)\Big) g_2^0(w_2|b_1,\mathbf{s}_{-1})dw_2 \\ =& v_2^N(b,x;b_1,\mathbf{s}_{-1}).\end{aligned}$$

Note that $b_1 \leq s_2 \leq ... \leq s_n$ and, by induction, we conclude that

$$v_1^N(b,x;b_1,\mathbf{s}_{-1}) \geq v_2^N(b,x;b_1,\mathbf{s}_{-1}).$$

(iii) $W_1 < b_1, W_1 < s_2$ (i.e., Bidder B will submit a bid regardless of where Bidder A places her bid). In this case, no matter which auction Bidder A chooses, Bidder B always places a bid in Auction 1. If Bidder A enters Auction 1, Bidder B will also bid in Auction 1 and outbid the new standing bid W_1. Bidder B's bid b_1 therefore serves as the new winning bid in Auction 1. Bidder A's conditional expected payoff can thus be written as

$$\begin{aligned}v_1^{N+1}(b,x;\mathbf{s}) =& \int_{W_1}^{b} \Big(\int_{b_1}^{b} (x-p_1)\hat{g}_1^N(p_1|b_1,\mathbf{s}_{-1})dp_1 \\ &+ \alpha^{UD}\big(1-\hat{G}_1^N(b|b_1,\mathbf{s}_{-1})\big)v_f(x,G,\Psi)\Big) g_1^0(b_1|W_1,\mathbf{s}_{-1})db_1 \\ =& v_1^N(b,x;W_1,\mathbf{s}_{-1}).\end{aligned}$$

Similarly, if Bidder A places a bid in Auction 2, her conditional expected payoff can be written as

$$\begin{aligned}v_2^{N+1}(b,x;\mathbf{s}) =& \int_{s_2}^{b} \Big(\int_{w_2}^{b} (x-p_2)\hat{g}_2^N(p_2|W_1,w_2,\mathbf{s}_{-1,-2})dp_2 \\ &+ \alpha^{UD}\big(1-\hat{G}_2^N(b|W_1,w_2,\mathbf{s}_{-1,-2})\big)v_f(x,G,\Psi)\Big) g_2^0(w_2|W_1,\mathbf{s}_{-1})dw_2 \\ =& v_2^N(b,x;W_1,\mathbf{s}_{-1}).\end{aligned}$$

Note that $W_1 \leq s_2 \leq ... \leq s_n$ and, by induction, we conclude that

$$v_1^N(b, x; W_1, \mathbf{s}_{-1}) \geq v_2^N(b, x; W_1, \mathbf{s}_{-1}).$$

(iv) $s_2 < b_1, s_2 < W_1$ (i.e., Bidder B will also submit a bid regardless of Bidder A's decision). In this case, Bidder B will submit a bid in Auction 2 if Bidder A enters Auction 1, whereas Bidder B will place a bid in Auction 1 if Bidder A places a bid in Auction 2. When Bidder A places a bid in Auction 1 knowing that $W_1 > s_2$, she will update her belief about W_1 accordingly (i.e., the conditional PDF of W_1 becomes $g_1^0(w_1|s_2, \mathbf{s}_{-1})$). Her conditional expected payoff then becomes

$$v_1^{N+1}(b, x; \mathbf{s})$$
$$= \int_{s_2}^{b} \Big(\int_{w_1}^{b} (x - p_1)\hat{g}_1^N(p_1|w_1, \min(b_1, W_2), \mathbf{s}_{-1,-2})dp_1 +$$
$$\alpha^{UD}\big(1 - \hat{G}_1^N(b|w_1, \min(b_1, W_2), \mathbf{s}_{-1,-2})\big)v_f(x, G, \Psi)\Big) g_1^0(w_1|s_2, \min(b_1, W_2), \mathbf{s}_{-1,-2})dw_1$$
$$= v_1^N\big(b, x; s_2, \min(b_1, W_2), \mathbf{s}_{-1,-2}\big),$$

where W_2 is the current winning bid in Auction 2. On the other hand, when Bidder A submits a bid in Auction 2, she expects Bidder B to place a bid in Auction 1. In addition, she only knows that the standing bid is the smaller one between b_1 and W_1 (i.e., $min(b_1, W_1)$). Therefore, her conditional expected payoff of submitting a bid in Auction 2 is

$$v_2^{N+1}(b, x; \mathbf{s}) = \int_{s_2}^{b} \Big($$
$$\alpha^{UD}\big(1 - \hat{G}_2^N(b|\min(b_1, W_1), w_2, \mathbf{s}_{-1,-2})\big)v_f(x, G, \Psi)\Big) g_2^0(w_2|\min(b_1, W_1), s_2, \mathbf{s}_{-1,-2})dw_2$$
$$= v_2^N\big(b, x; \min(b_1, W_1), s_2, \mathbf{s}_{-1,-2}\big).$$

Note that both W_1 and W_2 have the same conditional PDF (i.e., $g_1^0(w_1|s_2, \mathbf{s}) = g_2^0(w_2|s_2, \mathbf{s})$) as Auctions 1 and 2 are indistinguishable given Bidder A's information set. Furthermore, expressions $v_1^N\big(b, x; s_2, \min(b_1, W_2), \mathbf{s}_{-1,-2}\big)$ and $v_2^N\big(b, x; \min(b_1, W_1), s_2, \mathbf{s}_{-1,-2}\big)$ are symmetric in terms of Auctions 1 and 2. Thus,

$$E\big[v_1^N\big(b, x; s_2, \min(b_1, W_2), \mathbf{s}_{-1,-2}\big)\big] = E\big[v_2^N\big(b, x; \min(b_1, W_1), s_2, \mathbf{s}_{-1,-2}\big)\big]$$

and for Bidder A, bidding in Auction 1 is a weakly dominant strategy.

In summary, given N incoming competing bids, it is a weakly dominant strategy for a UD bidder to bid in the auction with the lowest standing bid. This strategy depends on neither the bidders' valuations nor the market state. In the case of an unknown number of incoming bidders, one can derive the same conclusion by taking an expectation over N. ■

2.10 Appendix: Structural Model

2.10.1 Substitutability of iPhone 4, iPhone 4s, and iPhone 5 Models

In this section, we present evidence supporting that bidders view auctions for iPhone 4, iPhone 4s, and iPhone 5 as substitutes. In particular, we test whether bidders significantly prefer one model to the others at both the population and individual bidder levels.

Stated more precisely, we allow that devices substitute for one another after controlling for a price (value) differential. For example, an iPhone 5 may be worth more than an iPhone 4s. Supposing that the normalizing price difference were \$100, bidders would be indifferent in substituting between the two at that differential. That is, if the bidder's valuation for the iPhone 4s is \$200, then she would be willing to purchase instead the iPhone 5 at \$299 but not \$301. In equilibrium, bidders would be willing to substitute freely across the auctions of different phone models, but that the going bids in auctions would tend to differ by fixed, normalizing amounts. In our example, prices would tend to be \$100 higher for iPhone 5 auctions than iPhone 4s auctions.

Accordingly, we examine the substitutability assumption by testing its key empirical prediction that bidders should freely substitute across auctions of the different phone models, in proportion to their availability that day. Thus, we carry out two critical, empirical tests regarding whether the number of bids placed for each model type varies in proportion with its supply.

First, does the market's total number of bids for each type of phone model accrue in proportion to each model's share of the available supply? In other words, suppose that today's marketplace features 50% iPhone 5 and 50% iPhone 4s auctions. If we were to find that a disproportionate 80% of bids are placed for iPhone 5, this would provide evidence refuting the claim that the two models are substitutable to bidders. On the other hand, suppose that 50% of bids are for iPhone 5, followed the next day by 25% of bids when iPhone 5 constitutes 25% of the supply. This pattern supports our hypothesis of substitutability. Thus, our primary statistical test is designed to importantly distinguish these cases, showing that bidders do not exhibit an overriding preference for one model over another as models' supply varies from day to day.

Second, while our primary test provides our most critical piece of evidence, we carry out a second test that provides additional assurance. In particular, even if we observe that the proportion of bids matches the daily, available share of supply for each model, it remains conceivable that some bidders greatly prefer, for example, the iPhone 5 but simply do not bother to bid if they do not see a sufficient supply of iPhone 5 auctions available that day. However, such a bidder would still reveal her strong preference for focusing her bidding in iPhone 5 auctions when tracked across her own visits to the platform. Thus our second, bidder-level statistical test examines whether individual bidders act to flexibly match the available, daily supplies of the phone models when placing bids.

At the population level, given a model pair, i.e., (iPhone) Model 1 and (iPhone) Model 2, we use b_{1t} and b_{2t} to denote the number of bids placed in auctions for Model

1 and Model 2 phones that end on Day t, respectively. Also, we use n_{1t} and n_{2t} to denote the total number of auctions for Model 1 and Model 2 phones that end on Day t, respectively. Then, we propose to test the following null hypothesis: On a given day, the expected ratio of bids placed in Model 1 auctions to bids placed in Model 2 auctions is equal to the ratio of the total number of Model 1 auctions to the total number of Model 2 auctions. Specifically, we focus on the following statistic:

$$\hat{R} = \frac{1}{T} \sum_{t=1}^{T} \left(\frac{b_{1t}}{b_{1t} + b_{2t}} / \frac{n_{1t}}{n_{1t} + n_{2t}} - 1 \right),$$

where T is the number of days when at least one Model 1 auction ends. On a given day, a consistent mismatch between $b_{1t}/(b_{1t} + b_{2t})$ and $n_{1t}/(n_{1t} + n_{2t})$ indicates that bidders prefer one model to the other. For an iPhone model, the statistic $\hat{R}$ manages to capture not only the daily mismatch between its shares of bids and auctions, but also how the trend of bid shares matches the trend of auction shares over time. If $\hat{R}$ differs significantly from zero, we reject Hypothesis 2.10.1 and conclude that one model is preferred to the other at the population level; thus, the models cannot be considered as substitutes.

Our findings are presented in the first two columns of Table 2.16. For all three model pairs, we cannot reject Hypothesis 2.10.1. This implies that on a daily basis, the number of bids for each model is proportional to the number of auctions for that model. Therefore, each model is not more or less popular than other models at the population level. Thus, we can treat the three models as substitutes.

Furthermore, we test whether the models are substitutable at the individual bidder level. We find that more than 90% of the bidders do not show a strong

Table 2.16: Hypothesis Testing Results regarding Bidders' Preference for iPhone Models

	Population Level Test		Individual Level Test
(Model 1, Model 2)	$\hat{R}$	p-value	% of Bidders *without* a Strong Preference
(iPhone 4, iPhone 4s)	0.055	0.157	94.7%
(iPhone 4, iPhone 5)	0.000	0.987	92.6%
(iPhone 4s, iPhone 5)	-0.039	0.122	96.8%

preference for a specific model. In particular, given Bidder ℓ, we construct the following null hypothesis to test: On a given day, when Bidder ℓ participates in at least one Model 1 or Model 2 auction, the expected ratio of Model 1 auctions to Model 2 auctions that Bidder ℓ participates in is equal to the ratio of the total number of Model 1 auctions to the total number of Model 2 auctions. In other words, we focus on the following statistic for Bidder ℓ:

$$\hat{R}_\ell = \frac{1}{|\mathcal{T}_\ell|} \sum_{t \in \mathcal{T}_\ell} \left(\frac{p_{1t\ell}}{p_{1t\ell} + p_{2t\ell}} / \frac{n_{1t}}{n_{1t} + n_{2t}} - 1 \right),$$

where $\mathcal{T}_\ell$ is the set of days when Bidder ℓ participated in at least one Model 1 or Model 2 auction, and $p_{1t\ell}$ (resp., $p_{2t\ell}$) are the numbers of Model 1 (resp., Model 2) auctions which Bidder ℓ participated in on Day t. If $\hat{R}_\ell$ differs significantly from zero, we reject Hypothesis 2.10.1 and conclude that the bidder has a strong preference for either Model 1 or Model 2. Specifically, we say that the bidder does not exhibit a strong preference for either of the two models if the p-value of the test result is less than 0.01. We report our findings in the last column of Table 2.16. For each model pair, there are more than 90% of bidders without a strong preference. Therefore, the findings provide convincing evidence that supports the idea that auctions of the three models can be viewed as substitutes.

2.10.2 Kernel Density Estimators

In this section, we specify the kernel density estimators for $\Psi(\omega_{\ell t})$ and $g_j(p_j|\omega_{\ell t})$. Assuming the within-day timing of a participating bidder's bids to be exogenous, the steady-state vector $\mathbf{s}_{\ell t}$ of standing bids encountered by bidders per platform visit can be viewed as identically and independently distributed. We use $\mathbf{S}_n = \{\mathbf{s}_{\ell t,n}\}$ and $\mathbf{y}_n = \{y_{\ell t,n}\}$ to denote the observations of the vectors of standing bids[31] and highest rival bids (final prices), respectively, when the market thickness is n auctions. The corresponding sample size is denoted by N_n. The kernel density estimator of the standing bids conditional on market thickness n can be written as $\hat{\psi}(\mathbf{s}_n|n) = \frac{1}{N_n}\sum_{\ell t} K_{h_s}\big(\mathbf{s}_n - \mathbf{s}_{\ell t,n}\big)$, where $K_{h_s}(\mathbf{s})$ is the multivariate Gaussian kernel density with identical bandwidth h_s:

$$K_{h_s}(\mathbf{s}) = \left(\frac{1}{\sqrt{2\pi}h_s}\right)^n \prod_{j=1}^{n} exp\left(-\frac{s_j^2}{2h_s^2}\right).$$

The selection of the optimal bandwidth h_s^* is conducted by minimizing the cross-validation (CV) estimator of the risk function. Finally, the kernel density estimator of the highest rival bid in auction j (i.e., $y^{(j)}$), given market state $\omega = (n, \mathbf{s})$, can be

[31] If a bidder places multiple bids within a day, we assume that the market state is observed upon her last bid of the day, which reflects the most recent market state for her decision.

written as

$$\hat{g}_j(y^{(j)}|\omega) = \frac{\frac{1}{N_n}\sum_{\ell t} K_{h_{y,1}}(y^{(j)} - y^{(j)}_{\ell t,n}) K_{H_y}(\mathbf{s} - \mathbf{s}_{\ell t,n})}{\frac{1}{N_n}\sum_{\ell t} K_{H_y}(\mathbf{s} - \mathbf{s}_{\ell t,n})},$$

where $y^{(j)}_{\ell t,n}$ is the j^{th} element of $y_{\ell t,n}$, and $h_{y,1}$ is the bandwidth associated with the highest rival bid. For tractability, we assume that the bandwidth matrix of standing bids takes the form $H_y = h_{y,2} I_n$. The optimal bandwidths $h^*_{y,1}$ and $h^*_{y,2}$ are determined by minimizing the CV estimator of the risk function, $CV(h_{y,1}, h_{y,2})$, of the conditional density estimation.[32]

2.10.3 Bandwidth Selection in Kernel Density Estimation

In this subsection, we describe the bandwidth selection process we use when we non-parametrically estimate the distributions of the steady state of the market and the highest rival bids, respectively. The risk, or mean integrated squared error $R(f, \hat{f})$, which we define below, is a metric for the distance between a density estimator $\hat{f}$ and the true density f:

$$R(f, \hat{f}) = E_X\Big(\int \big(f(x) - \hat{f}(x|X)\big)^2 dx\Big),$$

where $X \sim f$. As f is unknown, the risk cannot be evaluated. Therefore, we consider the cross-validation (CV) estimator of risk, which is defined as follows:

$$CV(\mathbf{h}) = \int \big(\hat{f}(x)\big)^2 dx - \frac{2}{n}\sum_{i=1}^{n} \hat{f}_{(-i)}(X_i),$$

where $\hat{f}_{(-i)}(X_i)$ is the kernel density estimator obtained by removing the i^{th} observation, X_i. In our case, $\hat{f}$ is the kernel density estimator with bandwidth $\mathbf{h}$, which

[32] We provide additional details on the bandwidth selection procedure for both $\hat{\psi}_n(\mathbf{s}_n)$ and $\hat{g}_j(y|\omega)$ in Appendix 2.10.3.

is the parameter to be optimized (i.e., the bandwidth $\mathbf{h}$ is chosen so that the CV estimator of risk is minimized).

For the kernel density estimator of the standing bids, the CV estimator of risk can be further specified as:

$$CV(h_s) = \frac{1}{N^2|H_s|}\sum_{\ell t}\sum_{\ell' t'} \bar{K}\big(H_s^{-1}(\mathbf{s}_{\ell t}-\mathbf{s}_{\ell' t'})\big) - \frac{2}{N(N-1)|H_s|}\sum_{\ell t}\sum_{\ell' t' \neq \ell t} K\big(H_s^{-1}(\mathbf{s}_{\ell t}-\mathbf{s}_{\ell' t'})\big),$$

where N denotes the total number of observations (here, we use ℓt and $\ell' t'$ as observation indices). Furthermore, the bandwidth matrix is given by $H_s = h_s I_n$ and K, $\bar{K}$ denote the kernel and the corresponding convolution functions, respectively.

Note that when estimating the conditional distribution of highest rival bids, directly optimizing the cross-validation of the risk function may lead to very small bandwidths (due to the fact that multiple vectors of standing bids can be associated with the same vector of highest rival bids). To resolve this issue, we employ bootstrap sampling from the original dataset, drawing only one observation $(y_{\ell t,n}, \mathbf{s}_{\ell t,n})$ per auction (to ensure that we do not duplicate the vectors of highest rival bids within a bootstrap sample). Given a bootstrap subsample b, the subsample CV estimator of risk can be specified as

$$\begin{aligned}
&CV^{(b)}(h_{y,1}, h_{y,2}) = \\
&\frac{1}{N}\sum_{\ell t} \frac{\sum_{\ell' t' \neq \ell t}\sum_{\ell'' t'' \neq \ell t} K_{H_y}(\mathbf{s}^{(b)}_{\ell t} - \mathbf{s}^{(b)}_{\ell' t'}) K_{H_y}(\mathbf{s}^{(b)}_{\ell t} - \mathbf{s}^{(b)}_{\ell'' t''}) K_{\sqrt{2}h_{y,1}}(y^{(b)}_{\ell'' t''} - y^{(b)}_{\ell' t'})}{\big(\sum_{\ell' t' \neq \ell t} K_{H_y}(\mathbf{s}^{(b)}_{\ell t} - \mathbf{s}^{(b)}_{\ell' t'})\big)^2} \\
&- \frac{2}{N}\sum_{\ell t} \frac{\sum_{\ell' t' \neq \ell t} K_{H_y}(\mathbf{s}^{(b)}_{\ell t} - \mathbf{s}^{(b)}_{\ell' t'}) K_{h_{y,1}}(y^{(b)}_{\ell t} - y^{(b)}_{\ell' t'})}{\sum_{\ell' t' \neq \ell t} K_{H_y}(\mathbf{s}^{(b)}_{\ell t} - \mathbf{s}^{(b)}_{\ell' t'})}.
\end{aligned}$$

Lastly, we consider the average of the subsample CV estimators of risk (we draw 50 bootstrap samples) as the objective function to optimize when selecting the bandwidth for the kernel density estimation.

2.10.4 Simulated Maximum Likelihood Estimation

In this subsection, we provide expressions for $P^V(x_\ell|\theta)$, $L^B_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)$, and $L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)$, where $\theta = [\alpha, \mu, \nu, \lambda, \gamma]$, and describe how we derive the simulated likelihood function $\hat{L}_\ell(\mathbf{X}_\ell|\theta)$, using importance sampling. Before presenting the technical details, we preview how each primitive is estimated by exploiting the variations in the data.

The data contain sufficient variations for us to identify all the model primitives. First, in the data, a bidder (whose endowed valuation x_ℓ is given) may choose a subset of auctions on a given day and place bids in the selected auctions. The variations in her auction selections and in her bids can be mainly explained by the idiosyncratic valuation distribution (characterized by ν). In addition to the bid variations of a given bidder, we observe sizable bid variations across bidders (e.g., some bidders consistently submit much higher bids than others in the same auction). The cross-bidder bid variations are characterized by the bidders' endowed valuations (i.e., x_ℓ), whose distribution is characterized by λ^{TY} and γ^{TY}. Second, we observe that each bidder's platform-visit sequence follows a stochastic process with a certain frequency (e.g., some bidders tend to participate on the platform more often than others). In our model, a bidder's platform-visit pattern is determined by the interplay between her expected payoff, which depends on the endowed valuation (i.e., λ^{TY} and γ^{TY}) as well as the idiosyncratic valuation (i.e., ν^{TY}), and the distribution of participation cost (characterized by μ^{TY}). Note that $\lambda^{TY}, \gamma^{TY}$, and ν^{TY} are pinned down by the above bid variations; thus, the variation in bidders' platform visits can be employed to identify μ^{TY}. Lastly, in the data, each bidder exits the platform at some point, while her stay in the bidder pool may vary from a few days to a few months. The variations in the duration of bidders' stays in the bidder pool are then used to estimate the retention rate α. Therefore, our data have enough variations to recover all the modeling primitives.

We use $P^V(x_\ell|\theta)$ to denote the probability that Bidder ℓ visits the platform on Day t:

$$P^V(x_\ell|\theta) = P\big(c_{\ell t} \leq r(x_\ell; \theta, \hat{G}, \hat{\Psi})\big) = 1 - \exp\big(-\mu r(x_\ell; \theta, \hat{G}, \hat{\Psi})\big),$$

where recall that $r(x_\ell; \theta, G, \Psi)$ is the unconditional payoff per platform visit (Section 2.4). Given x_ℓ, θ^{MU}, $\hat{G}$, and $\hat{\Psi}$, a MU bidder's payoff per platform visit r^{MU} can be calculated using Expressions (2.21) and (2.22). For a UD bidder, given x_ℓ, θ^{UD}, $\hat{G}$, and $\hat{\Psi}$, we solve for r^{UD} and the continuation valuation v_f simultaneously from

$$\begin{aligned} & r^{UD}(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) \\ & = \int_\omega \int_{\zeta_{\ell\cdot}} \sum_j^{n(\omega)} \mathbb{1}\big\{\sigma_{SLT}^{UD}(x_\ell; \zeta_{\ell\cdot}, \omega, \theta^{UD}) = j\big\} \\ & \cdot \int_{s_j}^{x_\ell - \alpha v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) + \zeta_{\ell j}} \hat{G}(p_j|\omega) dp_j d\mathbf{F}_\zeta(\zeta_{\ell\cdot}; \nu^{UD}) d\hat{\Psi}(\omega), \end{aligned} \tag{2.27}$$

$$\begin{aligned} & \text{and } v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) = \\ & \frac{1}{1-\alpha^{UD}}\Big(r^{UD}(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) + \frac{1}{\mu^{UD}} \exp\big(-\mu^{UD} r^{UD}(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi})\big) - \frac{1}{\mu^{UD}}\Big), \end{aligned} \tag{2.28}$$

where Expression (2.27) is derived from Expressions (2.5) and (2.23), and Expression (2.28) is derived from Expression (2.24). For computational tractability, we employ Monte Carlo integration to approximate all the integrations over $\zeta_{\ell \cdot}$. Any integration of $\hat{G}$ or $\hat{\Psi}$ can be done quickly due to the selection of the Gaussian kernel.

Conditional on Bidder ℓ visiting the platform on Day t, we let $L^{B}_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta)$ denote the likelihood of her placing bid(s) $\mathbf{b}_{\ell t}$, and we let $L^{NB}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta)$ denote the likelihood of her not placing any bid.

First, assume that ℓ is a MU bidder (who is interested in winning $K = 14$ auctions, which are *all* open auctions, on a given day).[33] Then, when she visits the platform on Day t, she will not bid in auction j if and only if her bid is lower than the current standing bid $s_{\ell j}$ (i.e., $b_{\ell j} = x_\ell + \zeta^{MU}_{\ell j} < s_{\ell j}$). Therefore,

$$L^{NB,MU}_{\ell t}(\mathbf{S}_{\ell t}|x_\ell, \theta^{MU}) = \prod_{j' \in \mathcal{A}^t} \Phi(s_{\ell j'}|x_\ell, \nu^{MU}),$$

where $\Phi(\cdot|x_\ell, \nu^{MU})$ denotes the CDF of the normal distribution with mean x_ℓ and standard deviation ν^{MU}, and $\mathcal{A}^t$ denotes the set of all auctions on Day t. On the other hand, if she places bids $\mathbf{b}_{\ell t} = \{b_{\ell j}\}$, and $\mathcal{J}^t_\ell$ denotes the set of auctions that Bidder ℓ chooses to participate in on Day t, the likelihood of placing these bids is given by $\prod_{j \in \mathcal{J}^t_\ell} \phi(b_{\ell j}|x_\ell, \nu^{MU})$. Therefore,

$$L^{B,MU}_{\ell t}(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t}|x_\ell, \theta^{MU}) = \prod_{j \in \mathcal{J}^t_\ell} \phi(b_{\ell j}|x_\ell, \nu^{MU}) \prod_{j' \in \mathcal{A}^t \setminus \mathcal{J}^t_\ell} \Phi(s_{\ell j'}|x_\ell, \nu^{MU}).$$

Next, assume that Bidder ℓ has unit demand. Then, given that she visits

[33] Recall that K is the observed maximum auction participation on a given day.

the platform on Day t, she will not place a bid if and only if all auctions on the day have standing bids that are higher than her potential bids (i.e., $b_{\ell j} = x_\ell - \alpha^{UD} v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) + \zeta_{\ell j}^{UD} < s_{\ell j}, \forall j \in \mathcal{A}^t$). Therefore,

$$L_{\ell t}^{NB,UD}\big(\mathbf{S}_{\ell t} | x_\ell, \theta^{UD}\big) = \prod_{j' \in \mathcal{A}^t} \Phi\big(s_{\ell j'} | x_\ell - \alpha^{UD} v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}), \nu^{UD}\big).$$

On the other hand, if she places a bid $\mathbf{b}_{\ell t} = b_{\ell j}$ in auction j, this implies that participating in auction j yields the highest payoff among all open auctions on the same day (as was shown in Proposition 2.9.4). In addition, if she chooses auction j (i.e., $\sigma_{SLT}^{UD}(x_\ell; \zeta_{\ell \cdot}, \mathbf{S}_{\ell t}, \theta^{UD}) = j$), the likelihood of placing bid $b_{\ell j} = x_\ell - \alpha^{UD} v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}) + \zeta_{\ell j}^{UD}$ is equal to $\phi\big(b_{\ell j} | x_\ell - \alpha^{UD} v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}), \nu^{UD}\big)$. Thus,

$$L_{\ell t}^{B,UD}\big(\mathbf{b}_{\ell t}, \mathbf{S}_{\ell t} | x_\ell, \theta^{UD}\big) =$$

$$\phi\big(b_{\ell j} | x_\ell - \alpha^{UD} v_f(x_\ell; \theta^{UD}, \hat{G}, \hat{\Psi}), \nu^{UD}\big) \int_{\zeta_{\ell \cdot}} \mathbb{1}\big\{\sigma_{SLT}^{UD}(x_\ell; \zeta_{\ell \cdot}, \mathbf{S}_{\ell t}, \theta^{UD}) = j\big\} d\mathbf{F}_\zeta(\zeta_{\ell \cdot}; \nu^{UD})$$

To obtain the simulated likelihood, $\hat{L}_\ell(\mathbf{X}_\ell | \theta)$, we first rewrite the unconditional likelihood function as

$$\begin{aligned}\mathcal{L}_\ell(\mathbf{X}_\ell | \theta) &= \int_{x_\ell} \mathcal{L}_\ell(\mathbf{X}_\ell | x_\ell, \theta) \frac{f(x_\ell | \lambda, \gamma)}{f_I(x_\ell | \lambda_I, \gamma_I)} f_I(x_\ell | \lambda_I, \gamma_I) dx_\ell \\ &= E_{x_I}\Big(\mathcal{L}_\ell(\mathbf{X}_\ell | x_I, \theta) \frac{f(x_I | \lambda, \gamma)}{f_I(x_I | \lambda_I, \gamma_I)}\Big), \end{aligned} \tag{2.29}$$

where $f_I(x_I | \lambda_I, \gamma_I)$ is PDF of the candidate Weibull distribution with parameters λ_I and γ_I. Then, we draw M samples for the bidder's valuation $x_{m,I}, m = 1, 2, ..., M$ from the candidate Weibull distribution. Finally, we (approximately) compute integral (2.29) using Monte Carlo integration and obtain

$$\hat{\mathcal{L}}_\ell(\mathbf{X}_\ell | \theta) = \frac{1}{M} \sum_{m=1}^{M} \mathcal{L}_\ell(\mathbf{X}_\ell | x_{m,I}, \theta) \frac{f(x_{m,I} | \lambda, \gamma)}{f_I(x_{m,I} | \lambda_I, \gamma_I)}.$$

In our estimation process, we draw $M = 200$ samples and set λ_I, γ_I such that they satisfy the first and second moment conditions of the empirical bid distribution.

2.10.5 Model Validation

To validate the model, we compare the predicted distributions of (i) the number of bidders participating in an auction and (ii) the final price in the post-treatment period, with those observed in the dataset. Given that these distributions concern outcomes in the post-treatment period, they can be viewed as out-of-sample predictions (recall that the model was estimated using pre-treatment data). Our findings provide support for the validity of the model. In particular:

(a) The mean and the standard deviation of the simulated distribution of participating bidders (who placed at least one bid) per auction in the post-treatment period are shown in the first two columns of Table 2.17 (with standard errors in parentheses). As is evident from the table, our counterfactual simulations provide a relatively accurate prediction of the corresponding distribution. In particular, both the mean and the standard deviation of the simulated distribution are not significantly different from those observed in the data.

(b) Similarly, we simulate the mean and the standard deviation of the final price distribution (on a normalized scale) in the post-treatment period. The results are shown in the last two columns of Table 2.17 (with standard errors in parentheses). As one can deduce from the table, the simulations provide an accurate out-of-sample prediction of the average final price in the post-treatment

Table 2.17: Comparison between Observed and Simulated Distributions of the Number of Bidders per Auction and Final Price per Device in the Post-treatment Period

	No. of Bidders per Auction		Final Price per Unit ($)	
	Mean	Standard Deviation	Mean	Standard Deviation
Observed	5.24	2.33	168.75	15.49
Simulated	5.79	2.01	167.97	18.30
	(0.68)	(0.35)	(5.24)	(1.99)

period.

2.11 Appendix: Supporting Material

2.11.1 Effects of Policy Switch on Aggregate Supply and Demand

In this subsection, we first check whether the aggregate supply and the aggregate demand are affected by the policy switch, by plotting their trends during the pre- and post-treatment periods in Markets A and B. We further employ difference-in-differences analyses to test the hypothesis that both the supply and the demand remain parallel over time between two marketplaces.

In Figure 2.10, we plot the weekly number of auctions and weekly number of registrants in Markets A and B, respectively, over time. For both the supply and the demand, the gap between the trends of the two markets does not seem to consistently change across periods.

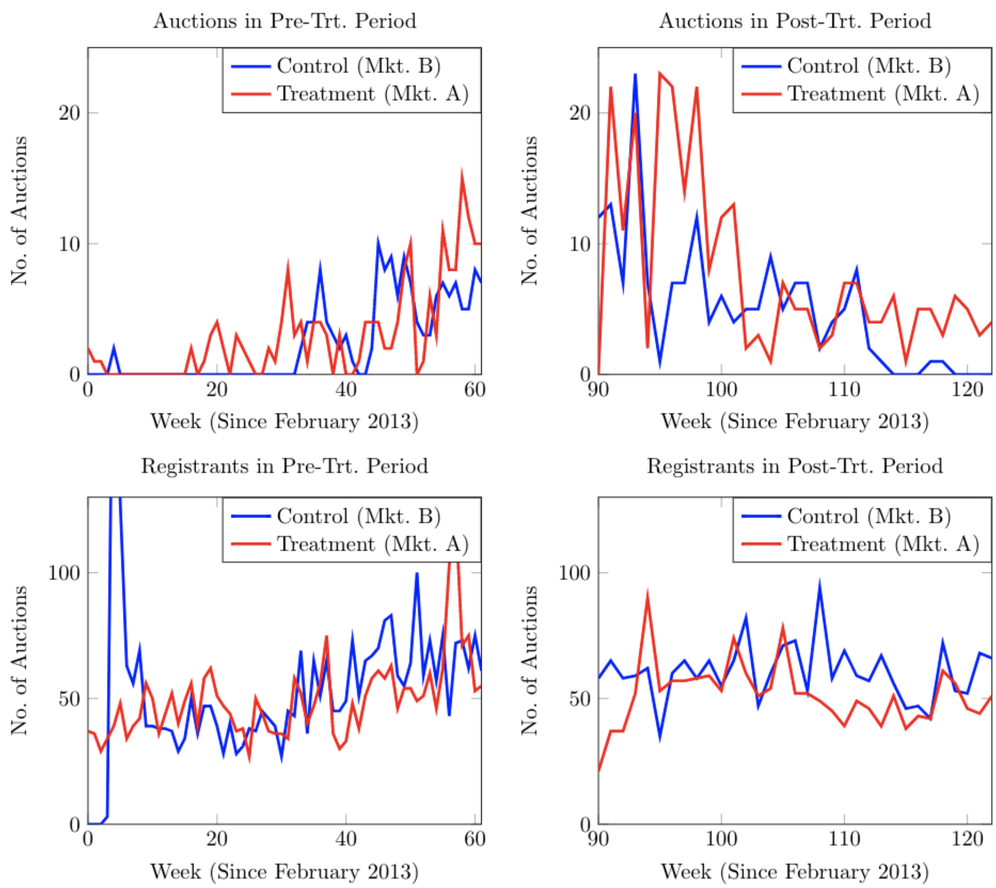
2.11.2 Spillover Effects on Participation Rates

This subsection focuses on auction participation behaviors in Market B and the subset of cross-market bidders (i.e., bidders that are registered in both Markets A and B). The objective is to establish that the change in Market A's market thickness has a strong positive effect on the cross-market bidders' participation rates in Market B. To this end, we specify the following linear model for the bidders' participation rates in Market B:

$$APR_{\ell tw} = \xi_\ell + \eta_w + \beta_1 CMB_\ell + \beta_2 TT_t + \beta_3 CMB_\ell \cdot TT_t + \epsilon_{\ell tw}, \quad (2.30)$$

where the dependent variable $APR_{\ell tw}$ is defined as

Figure 2.10: Trends of Weekly Supply and Weekly Demand



$$APR_{\ell tw} = \frac{\text{No. of Mkt. B auctions Bidder } \ell \text{ bids on Day } t \text{ in Week } w}{\text{No. of Mkt. B auctions ending on Day } t \text{ in Week } w}.$$

In Equation (2.30), ξ_ℓ denotes the fixed effect associated with Bidder ℓ's idiosyncratic participation behavior, and η_w denotes a weekly fixed effect introduced to control for changes over time.

We include fixed effect ξ_ℓ to control for Bidder ℓ's idiosyncratic participation behaviors. We also include fixed effect η_w to control for changes over time (e.g., releases of new products on participation rates) in Week w. Furthermore, CMB_ℓ and TT_t are binary variables denoting whether i is a cross-market bidder and whether the day of the Week t is Tuesday or Thursday, respectively. Thus, coefficient β_1 captures the participation pattern of cross-market bidders in Market B, and β_2 captures the baseline difference in participation rates between [Tuesday, Thursday] and [Monday, Wednesday, Friday]. Finally, the quantity of interest, β_3, is meant to capture the potential spillover effect on the cross-market bidders' participation rates on Tuesdays and Thursdays.

For the results that follow, we focus only on the treatment period and restrict attention to iPhone 4 (thus, we do not need to control for product characteristics). In this sample, cross-market bidders account for 49% of the total Market B bidders. First, we plot in Figure 2.9 the average participation rates corresponding to cross-market bidders and exclusively to Market B bidders on Monday/Wednesday/Friday and Thursday/Thursday, respectively. Although on Monday/Wednesday/Friday, cross-market and Market B bidders have similar participation rates, on Tuesday/Thursday, the participation rate of cross-market bidders is substantially higher than the participation rate of Market B bidders.

Second, we employ difference in differences to estimate coefficient β_3. As we report in Table 2.18, we estimate that the relative increase in the cross-market bidders' participation rates that can be attributed to the change in Market A's market thickness is roughly 39% (2.7% in absolute terms).

Table 2.18: Difference-in-Differences Estimate of Spillover Participation Effect Caused by Batch Listing

	Dependent Variable
	Auction Participation Rate
Spillover Participation Effect $\hat{\beta}_3$	0.027***
	(0.005)
Note:	*p<0.1; **p<0.05; ***p<0.01

The size of the spillover effect provides evidence supporting the presence of participation frictions associated with visiting a market on any given day and actively bidding and monitoring the auctions listed on the platform.

2.11.3 Regression Analysis in Support of Cannibalization among Auctions

In this section, we establish that high market thickness may have a negative impact on the final price of the platform's auctions. To this end, we conduct a regression analysis using data on Samsung Galaxy S3 auctions in Market A. We address the potential issue of demand endogeneity by using an additional dataset tracking the retail price of Galaxy S3 phones that was obtained from Amazon.

Demand Endogeneity.

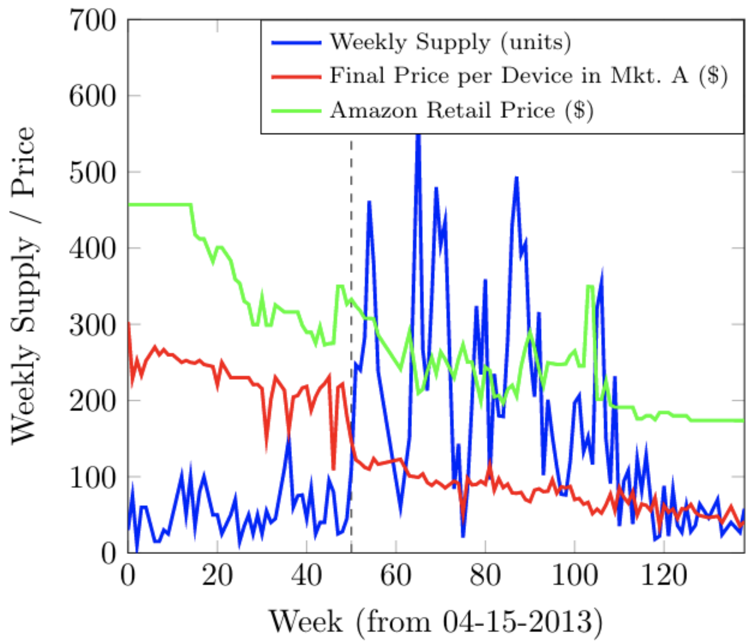
To address the potential issue arising from demand endogeneity, we use a dataset tracking the price of Galaxy S3 phones that was obtained from Amazon. The prices on Amazon, which is a retail market, are largely exogenous to the prices on the platform, which is a secondary market. This allows us to control for market trends in the demand for Galaxy S3 phones; thus, we can attribute price fluctuations we observe on the platform to changes in the induced market thickness.

Data and Variables.

The data we use for this analysis includes (1) detailed Galaxy S3 auction data from Market A, and (2) weekly retail price data from Amazon. Our observation window is from March 15, 2013 to December 7, 2015.[34] In the Galaxy S3 auction

[34] We collect Amazon's retail prices for Samsung Galaxy S3 from CamelCamelCamel.com, a tool tracking prices for products sold on Amazon.

Figure 2.11: Weekly Supply and Prices of Samsung Galaxy S3 (the vertical dashed line highlights the release of Galaxy S5 on 04-11-2014)



dataset, there are 376 observations within the observation window. For each auction, the dataset contains the start and ending times of the auction, its final price, lot size, and carrier information. Market thickness on a given day is measured by two metrics: the number of auctions ending on that day and the number of units in those auctions. The dataset we obtained from Amazon tracks the retail price of a Galaxy S3 phone within our observation window.

In Figure 2.11, we plot the price trends of Galaxy S3 phones in Market A (red curve) and on Amazon (green curve) as well as the weekly supply in Market A (blue curve). The supply shock in the local market (corresponding to the dashed line in the figure) is due to the release of a new Samsung Galaxy model.

Empirical Evidence.

Here, we establish that high market thickness may have a negative impact on the final price of an auction. To this end, first, we compute the correlation between market thickness and the auction's final price without controlling for demand endogeneity. As already mentioned, market thickness on a given day is measured in two ways: (1) by the number of auctions ending on that day and (2) by the number of units in those auctions. We find that the correlation coefficients corresponding to the market thickness metrics are equal to -0.30 and -0.31, respectively, thus establishing the negative impact of market thickness on price.

Next, we address demand endogeneity by explicitly controlling for the market value of Galaxy S3 phones using the Amazon data. In the regression analysis, we select the final price per unit as the dependent variable. We include fixed effects for both the product carrier and new product releases.

The estimation results are displayed in Table 2.19. As is evident in the results for Models 1 and 2 (without including the fixed effect corresponding to a new product release), high market thickness has a significantly negative impact on the final price. Particularly, we note that adding one Galaxy S3 auction to the platform's market thickness results in a decrease in the final price per unit by \$7.63. Similarly, adding 10 units of Galaxy S3 phones decreases the final price per unit by \$1.10.

Furthermore, we control for possible supply endogeneity by considering the exogenous shocks corresponding to new product releases (i.e., the releases of Samsung Galaxy S4, S5, and S6). When a new Samsung Galaxy model is released, we expect

Table 2.19: Regression Estimates in Support of the Cannibalization Effect

	Dependent Variable			
	Final Price per Unit			
	Model 1	Model 2	Model 3	Model 4
No. of units per day	-0.11***		-0.043**	
	(0.02)		(0.017)	
No. of auctions per day		-7.63***		-3.06***
		(1.39)		(1.06)
Auction lot size		-0.15**		-0.08*
		(0.06)		(0.048)
Amazon retail value	0.54***	0.53***	0.226***	0.225***
	(0.02)	(0.02)	(0.027)	(0.027)
FEs of Rel. of S4, S5, S6	No	No	Yes	Yes
FEs of carriers	Yes	Yes	Yes	Yes
Constant	-54.42***	-43.42***	122.44***	128.20***
	(7.39)	(8.02)	(15.56)	(15.61)
Note:		*p<0.1; **p<0.05; ***p<0.01		

a jump in the number of Galaxy S3 phones that become available in the secondary market. As shown in Table 2.19 (Models 3 and 4), the cannibalization effect remains significant even after we account for the release of new products. In particular, our results indicate that adding an auction to the platform's market thickness results in a decrease in the final price per unit by \$3.06, whereas adding 10 Galaxy S3 phones results decreases their unit price by \$0.43.

2.11.4 Listing Policy Has Little Impact on Bids

In this section, we look into how bidders' bidding strategies respond to the listing policy switch in Market A. We focus on a sample of bidders who placed a bid in either only Market A or only Market B in both the pre- and post-treatment periods (i.e., cross-market bidders are excluded). Therefore, in the post-treatment period, Market A bidders are directly subject to the policy switch, while Market B bidders are not. There are 14 Market A bidders, 21 Market B bidders, and 814 total bids observed in the sample. Then, we estimate the following difference-in-differences model to capture any effect on their bidding strategies:

$$log(Bid_i) = \beta_0 + FEProd_i + \beta_1 IsMktA_i + \beta_2 IsPost_i + \beta_3 IsMktA_i \cdot IsPost_i + \epsilon_i,$$

where for each observation i, Bid_i is the variable corresponding to the placed bid (normalized to a single unit), $FEProd_i$ captures fixed effects associated with product features, $IsMktA_i$ is the dummy variable of Market A auctions, and $IsPost_i$ is the dummy variable of the post-treatment period. Thus, the coefficient of interest is β_3, which captures the effect of the policy switch on the size of the bids. As shown

Table 2.20: Difference-in-Differences Estimate of Batch Listing's Effect on Bids

	Dependent Variable
	Log(Bid)
$IsPost$	-0.763^{***}
	(0.118)
$IsMktA$	1.206^{***}
	(0.070)
$IsMktA \cdot IsPost$	-0.173
	(0.146)
FEs of Products	Yes
Constant	3.359^{***}
	(0.765)
Note:	*p<0.1; **p<0.05; ***p<0.01

in Table 2.20, the policy switch does not seem to affect bidders' bidding strategies, given that β_3 is not significantly different from zero. This finding rules out the alternative mechanism that could explain the increase in revenues observed after the listing policy switch, i.e., that the increase is the outcome of bidders changing their bidding strategies.

2.11.5 Algorithm for Estimating a Steady-State Equilibrium

In this subsection, we describe an iterative algorithm that converges to the steady-state equilibrium. Given the stochastic supply, a dynamic bidder pool, and a listing policy, a bidder's decision whether to visit the platform, which auction(s) to participate in, and how to bid are all endogenously determined. Overall, the algorithm starts by initializing the beliefs for the highest rival bid G and the state distribution Ψ, based on which bidders decide whether to visit the platform and how to bid. Once the corresponding bidding data are generated, they are used to re-estimate G and Ψ. The algorithm iterates between the data simulation step and the belief estimation step until they converge (Algorithm 1). The expected payoff $r(x; G, \Psi)$ is calculated numerically for MU and UD bidders separately. We employ kernel density estimation for the conditional distribution of the highest rival bid G and the distribution of standing bids Ψ, which are specified in Section 2.5.1.

Algorithm 1 Estimating a steady-state equilibrium

Assign a convergence threshold ϵ.
Assign a positive value to Δ, such that $\Delta > \epsilon$.
Iteration $i = 0$.
Initialization:
    The highest rival belief $G^{(0)}$.
    Market state distribution $\Psi^{(0)}$.
    $r^{TY}(x_\ell, G^{(0)}, \Psi^{(0)}), \forall \ell$ in the set of all bidders with type $TY \in \{MU, UD\}$.
while $\Delta > \epsilon$ **do**
    Iteration $i \leftarrow i + 1$.
    Simulate bidding history $\Omega^{(i)}$ under beliefs $G^{(i-1)}$ and $\Psi^{(i-1)}$.
    for day $t = 1...T$ **do**
        Update bidder pool on Day t:
            1. Existing bidder exits $w.p.\ 1 - \alpha^{TY}$.
            2. Add new bidders $\sim P^{TY}_{Arr}$.
        Initialize market state ω:
            1. Add new auctions $\sim P_{Supply}$.
            2. Set standing bids.
        for Bidder ℓ in the pool with type $TY \in \{MU, UD\}$ **do**
            Participation cost $c_{\ell t} \sim Exp(\mu^{TY})$.
            if $\sigma^{TY}_{PRT}(x_\ell; c_{\ell t})$ = Visit **then**
                Observe the market state ω and idiosyncratic terms $\zeta_{\ell\cdot}$.
                Select auction(s) according to $\sigma^{TY}_{SLT}(x_\ell; \zeta_{\ell\cdot}, \omega)$.
                Place bid(s) $\sigma^{TY}_{BID}(x_j; \zeta_{\ell\cdot}, \omega)$ in selected auction(s).
                Update standing bids in market state ω.
            end if
        end for
    end for
    Estimate $G^{(i)}$ and $\Psi^{(i)}$ non-parametrically from $\Omega^{(i)}$.
    Compute $r^{TY}(x_\ell, G^{(i)}, \Psi^{(i)}), \forall \ell$ in the set of all bidders with type $TY \in \{MU, UD\}$.
    $\Delta \leftarrow ||G^{(i)} - G^{(i-1)}|| + ||\Psi^{(i)} - \Psi^{(i-1)}||$.
end while

Chapter 3: Information Provision in Service Platforms: Optimizing for Supply

Abstract. This paper explores the interaction between information design and supply-side decisions, including supplier entry/exit and pricing, in peer-to-peer service platforms. We develop a dynamic model of a two-sided platform that allows us to analyze the long-run implications of alternative information-provision policies. Our analysis highlights three mechanisms through which such a policy may increase platform revenues. First, in cases where the platform is not dominant in the market, a downgrading policy increases the volume of transactions on the platform and, surprisingly, may also result in an *increase* in the volume of high-quality providers active in the platform. Second, when the platform is dominant in the market, a downgrading policy helps the platform modulate the composition of suppliers active on the platform, leading to an overall more revenue-efficient set of suppliers. Third, when commission subsidies are used by the platform to incentivize entry of new providers, a downgrading policy helps the platform achieve equivalent new-provider entry while extracting higher revenue per transaction.

Keywords: platform operations, information provision, social learning, product line design, applied game theory

3.1 Introduction

Online peer-to-peer service platforms, such as Upwork, Taskrabbit, and Thumbtack, have been proliferating over the years. They serve the intermediaries to connect service providers with different quality with consumers who have various pending tasks, which have different equality sensitivity. Some work (e.g., article editing) values high-quality services more than others (e.g., plumbing). In other words, such platforms feature substantial heterogeneities on both supply and demand sides of the market. Moreover, like any two-sided marketplaces, participants on both sides are strategic. In addition to participants' entry decisions, consumers choose which provider to transact, and service providers set prices for their service to maximize their utilities, respectively.

Service platforms' revenues primarily come from the commission charged from each transaction, which usually is proportional to the transaction price. Notice that services delivered by high-quality providers charge high prices. Service platforms may benefit from expanding the pool of active, high-quality professionals.

A critical challenge encountered by service platforms is that providers' quality is ex-ante unknown, and it has to be learned through their services delivered on the platform. In other words, the size of high-quality providers population depends on the scale of employment of new providers (i.e., experimentation) on the platform. If a service platform fails to experiment with sufficient new providers in the first place, the discovered high-quality providers will be in short later.

In this paper, we explore the effectiveness of a service platform's information provision policy joint with its commission scheme in maximizing the platform's commission revenue. In particular, we investigate when and how information provision policy complements the commission scheme in incentivizing experimentation on new providers and optimizing provider composition. To address these questions, we develop a discrete-time infinite-horizon model with learning on new providers' quality. The model characterizes the strategic behaviors of all participants and specifies the matching with heterogeneity on both sides of the market.

We focus on a class of information provision policies, which delay the disclosure of high-quality providers to consumers by labeling them as new providers. We henceforth refer to them as informational delay policy. We establish that informational delay policy adds values to a service platform through three distinct mechanisms. First, the informational delay can increase the experimentation on new providers by improving their expected quality, when it is difficult for the platform to do so under a single commission scheme. Second, even though the platform achieves the maximum experimentation on new providers solely using the single-commission scheme, the informational delay can further improve the platform's revenue by optimizing the providers' composition. Third, when the platform implements a differentiated commission scheme, which can incentivize the experimentation on new providers through subsidy, the informational delay can improve the platform's revenue by lowering the subsidy.

3.1.1 Related Literature

Our work ties to the burgeoning literature that addresses the issues resulted from the information friction of the uncertainty of workers' quality in online labor markets. [42] shows that the information friction can lead to market failure. He demonstrates that a labor market could fail by hiring too many mediocre workers and too few novices with potential when the quality of inexperienced workers can only be revealed on the job. Some recent work has established the effectiveness of several online labor marketplaces' information provision mechanisms (e.g., agencies, reviews, and ratings) in mitigating the inefficiencies incurred by the information friction. In a field experiment, [43] observes that a public evaluation significantly increases the chance of inexperienced workers being hired as well as their wages in an online labor marketplace. Based on a proprietary dataset, [44] identify that buyers are willing to pay higher for sellers with a higher reputation (revealed by reviews and ratings). [45] study the role of outsourcing agencies in reducing the information friction by signaling the high quality of affiliated inexperienced workers. They further estimate that the presence of agencies increases the revenue generated per worker by 11%. [46] highlight the research opportunities of platforms' information design to reduce friction. Our work contributes to the literature by studying the design of a straightforward yet effective information provision policy, which exploits the workers' quality uncertainty to maximize the platform's revenues.

Our work is related to the rich literature that focuses on the settings involving experimentation, quality uncertainty, and self-interest agents. In the case where the realized outcomes are observable to all agents (i.e., information transparency), the equilibrium is very likely to be suboptimal regarding social surplus. [47] show that in a market with two-sided learning, sellers' pricing decisions joint with buyers' choices in equilibrium result in excessive experimentation compared to the socially efficient solution. On the other hand, [48] and [49] present that in a multi-armed bandit problem with incentive constraints, the system suffers from the under-experimentation issue. Both papers show that information provision policies can effectively alleviate the issue. In our work of two-sided online service marketplace, we also observe the under-experimentation issue under the full information setting, and information provision policies, especially the one with imprecise information disclosure, can effectively elicit experimentation, which echoes [49].

3.2 Model Description

We consider a dynamic model of a two-sided platform that connects service providers with consumers. The model consists of three types of players, who interact with one another over an infinite discrete-time horizon: (i) the platform, which chooses a commission to be charged per transaction and an information provision policy; (ii) the supply (i.e., a population of service providers), who choose whether to join the platform and, if so, what price to charge for providing service, and (iii) the demand (i.e., consumers), who choose whether to seek service on the platform and, if so, with which service provider.

Consumers.

We assume that in each period there is a short-lived population of consumers with total mass normalized to one who enters the platform seeking service. Consumers are heterogeneous in their willingness to pay for service quality. We use θ_i to denote consumer i's type, and we assume that consumer types are uniformly distributed on the interval $[1, 2]$. A consumer's net utility from transacting with provider j is given by

$$u_i = \theta_i q_j - p_j,$$

where q_j is the provider's service quality and p_j is the provider's service price. Upon entering the platform, each consumer observes the set of available service providers, the price set by each provider, and any information on the provider's service quality provided by the platform (the latter is determined by the platform's information provision policy, described in detail below). Then, each consumer chooses among the available service providers to maximize her expected utility. Apart from the providers available on the platform, consumers also have the option of seeking service outside of the platform; we assume that doing so results in service quality $q_0 \in (0, 1)$ at a price $p_0 \in (0, 1)$. We further assume that $q_0 \geq p_0$, so that the outside option results in non-negative utility for all consumer types.

Service Providers.

We assume that there is a large pool of potential service providers, a fraction $1 - \beta$ of whom cease to exist in each period and are replaced by new potential providers of equal mass. In every period, the potential providers may choose to enter the platform or pursue employment outside of it. Employment outside of the platform yields an expected profit of w_0 per period. Inside the platform, expected profits depend on the platform's commission and information-provision policies, and the resulting equilibrium behavior of the service providers and consumers. We assume that the service quality of each provider j can be high or low, $q_j \in \{q_H, q_L\}$, where we normalize $q_H = 1$ and $q_L = 0$. The probability of a randomly chosen provider being of high quality is denoted by $\gamma := P(q_j = q_H)$. Our analysis will be primarily concerned with environments where the supply of high-quality providers is relatively scarce. Accordingly, we assume that (i) $E[q_j] = \gamma q_h + (1-\gamma) q_l < q_0$ (i.e., although there are high-quality providers in the market, the expected quality of a randomly drawn provider is lower than that of the outside option), and (ii) $\gamma\beta < 1 - \beta$ (i.e., the volume of surviving high-quality providers in each period does not exceed the volume of providers who cease to exist).

We assume that the service quality of a new provider entering the platform is initially unknown, but is perfectly revealed to the platform after the provider engages in a single transaction with a consumer.[1] In every period, each provider, taking into account the platform's commission and information-provision policies, chooses whether to join the platform and, if so, what price to charge for service. Finally, we assume that the price set by a provider is bounded below by b_0, for some $b_0 \in [0, p_0]$ (e.g., this may represent the per-period cost of providing service).

[1] Adding noise to the process of quality revelation does not affect the qualitative nature of our model insights.

Platform.

The platform is long-lived and seeks to maximize its expected per-period profit by choosing a commission rate of τ and an information-provision policy. The commission rate is a percentage fee collected by the platform on any transaction that occurs between a consumer and a provider. The information-provision policy specifies a message or label for each provider that is displayed to the consumers and contains information on the provider's service quality, based on the platform's observations of the provider's past service outcomes. According to the platform's quality-learning process described above, the platform's information about provider j's quality in any given period is summarized by the state $s_j \in \{H, L, U\}$, corresponding to high, low, or unknown quality, respectively. The information-provision policy employed by the platform is expressed as a (possibly stochastic) mapping from the platform's private information about provider j to a "label" which is assigned to the provider and published on the platform, i.e.,

$$g(s_j) = \begin{cases} \mathcal{H} & \text{w. p. } \rho_{s_j}^{\mathcal{H}} \\ \mathcal{L} & \text{w. p. } \rho_{s_j}^{\mathcal{L}} \\ \mathcal{U} & \text{w. p. } \rho_{s_j}^{\mathcal{U}}, \end{cases} \tag{3.1}$$

where $\rho_{s_j}^{\mathcal{H}} + \rho_{s_j}^{\mathcal{L}} + \rho_{s_j}^{\mathcal{U}} = 1$, for all $s_j \in \{H, L, U\}$. Designing an information-provision policy consists of choosing the probability with which each label is assigned to each supplier state. At one extreme, a policy such that $\rho_H^{\mathcal{H}} = \rho_L^{\mathcal{L}} = \rho_U^{\mathcal{U}} = 1$ corresponds to full information disclosure, since the platform's knowledge can be perfectly inferred from the labels it assigns. At the other extreme, any policy with $\rho_{s_j}^{\mathcal{H}}, \rho_{s_j}^{\mathcal{L}}$, and $\rho_{s_j}^{\mathcal{U}}$ chosen independently of s_j corresponds to no information disclosure, since none of the platform's information can be inferred from the labels it assigns to the suppliers. Policies involving intermediate levels of information provision can be constructed by choosing the probabilities $\rho_{s_j}^{\mathcal{H}}, \rho_{s_j}^{\mathcal{L}}, \rho_{s_j}^{\mathcal{U}}$ appropriately "between" the above two extremes.

Concerning the design of information-provision policies, it is straightforward to show that the platform cannot benefit by concealing the quality of a provider who is known to be of low quality. However, as we demonstrate in our analysis, it is far from evident that the same holds for providers known to be of high quality. In the analysis that follows, we will focus on the class of policies satisfying

$$\rho_L^{\mathcal{L}} = \rho_U^{\mathcal{U}} = 1, \text{ and } \rho_H^{\mathcal{U}} = 1 - \rho_H^{\mathcal{H}} =: \alpha \in [0, 1]. \tag{3.2}$$

In words, the platform always assigns label $\mathcal{L}$ to providers of low quality and label $\mathcal{U}$ to providers of unknown quality. However, providers of high quality are assigned label $\mathcal{U}$ with probability $\alpha \in [0, 1]$, and label $\mathcal{H}$ otherwise. At first glance, it may appear counter-intuitive for the platform to conceal the quality of its best providers, given that these providers are its highest earners. However, in Section 3.4, we demonstrate three mechanisms, along with sufficient conditions, through which a policy involving $\alpha > 0$ improves platform profits.

From a practical standpoint, we note that the class of policies described in (3.2) is particularly appealing in that it is operationally equivalent to a policy that delays disclosing information on the quality of a high-quality service provider. To emphasize this connection, we refer to a policy of the form (3.2) as an "information-delay" policy with delay α.

3.3 Equilibrium

Given that the underlying supply and demand processes are time-invariant in our model, our analysis will focus on steady-state equilibria of the supply-demand game, for a fixed platform policy $\{\tau, \alpha\}$. For the existence of a steady-state equilibrium, the supply-demand game must simultaneously satisfy several conditions relating to supply-side participation and pricing, demand-side participation and provider choice, and supply-demand matching. We now describe these conditions in detail.

Consider first the platform participation decisions of individual suppliers. In a steady-state equilibrium, the expected lifetime earnings of a high-quality provider who is assigned label $\mathcal{H}$ in any given period are given by

$$V_{\mathcal{H}}^{H} = \max\left\{\frac{(1-\tau)p_{\mathcal{H}}}{1-\beta}, \frac{w_0}{1-\beta}\right\}. \tag{3.3}$$

That is, such a provider will stay in the platform provided the price he can charge as an $\mathcal{H}$-labeled provider is sufficiently high, or the platform's commission rate is sufficiently low. Otherwise, he will seek employment outside the platform. The expected lifetime earnings of a high-quality provider who is assigned label $\mathcal{U}$ are given by

$$V_{\mathcal{U}}^{H} = \max\left\{\eta(1-\tau)p_{\mathcal{U}} + \beta\left(\alpha V_{\mathcal{U}}^{H} + (1-\alpha)V_{\mathcal{H}}^{H}\right), \frac{w_0}{1-\beta}\right\}, \tag{3.4}$$

where the (endogenous) parameter $\eta \in [0,1]$ here accounts for rationing that may occur if in equilibrium the demand for $\mathcal{U}$-labeled providers is lower than the availability of such providers.[2] Finally, for a supplier of unknown quality (i.e., who has not yet transacted on the platform), the expected lifetime earnings are given by

$$V_{\mathcal{U}}^{U} = \max\left\{(1-\eta)\beta V_{\mathcal{U}}^{U} + \eta\left((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\left(\gamma\left(\alpha V_{\mathcal{U}}^{H} + (1-\alpha)V_{\mathcal{H}}^{H}\right) + (1-\gamma)\left(\frac{w_0}{1-\beta}\right)\right)\right), \frac{w_0}{1-\beta}\right\}. \tag{3.5}$$

For the quantities $V_{\mathcal{H}}^{H}$, $V_{\mathcal{U}}^{H}$, and $V_{\mathcal{U}}^{U}$, they are complicated by the fact that prices are determined endogenously by the demand for each provider type, as well as the competition between providers. Moreover, we note that (i) our model assumes that each provider must receive a minimum payment $b_0 \geq 0$ for providing service, which implies that in any equilibrium with positive i-labeled provider participation, we have $(1-\tau)p_i \geq b_0$, for $i \in \{\mathcal{U}, \mathcal{H}\}$, and (ii) free entry of providers into the platform implies $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$. Assuming that a steady-state equilibrium exists, we use δ_i^j to denote the mass of providers of quality $j \in \{U, H\}$ who are active in the platform

[2] Note that there can never be rationing among providers labeled $\mathcal{H}$ in equilibrium since, in such a case, a provider could increase his earnings by unilaterally lowering his price slightly, which would guarantee being matched to a customer. Note also that if $\eta < 1$ then $(1-\tau)p_{\mathcal{U}} = b_0$, that is, rationing in equilibrium occurs only if the providers' compensation when they provide service is equal to the minimum possible, i.e., b_0.

in any period and who are assigned label $i \in \{\mathcal{U}, \mathcal{H}\}$.[3,4]

Next, we discuss the demand for different provider types (i.e., labels) in a steady-state equilibrium. Recall that each consumer chooses a provider to maximize her expected utility, i.e.,

$$\arg\max_{i \in \{0, \mathcal{U}, \mathcal{H}\}} \; \theta q_i - p_i,$$

where $\{0, \mathcal{U}, \mathcal{H}\}$ represents the set of available options to customers, i.e., transacting with the outside option or with a provider on the platform carrying with label $\mathcal{U}$ or $\mathcal{H}$. According to the information policy (3.2), the expected quality of a provider with label $\mathcal{H}$ is $q_{\mathcal{H}} = q_H = 1$, while the expected quality of a provider with label $\mathcal{U}$ is given by

$$\hat{q}_{\mathcal{U}}(\alpha) = \frac{\delta^U_{\mathcal{U}} q_U + \delta^H_{\mathcal{U}} q_H}{\delta^U_{\mathcal{U}} + \delta^H_{\mathcal{U}}} = \frac{\delta^U_{\mathcal{U}} \gamma + \delta^H_{\mathcal{U}}}{\delta^U_{\mathcal{U}} + \delta^H_{\mathcal{U}}}. \tag{3.7}$$

The latter expression reflects the fact that, as a result of the platform's information-delay policy, label $\mathcal{U}$ may contain providers of high quality in addition to providers of unknown quality, so that $\hat{q}_{\mathcal{U}}(\alpha) \in [q_U, q_H]$. Given expected qualities q_i and equilibrium prices p_i, let ζ_i denote the mass of customers that engage in a transaction with a provider carrying the label i, for $i \in \{\mathcal{U}, \mathcal{H}\}$. The following result describes how the quantities q_i, p_i, and ζ_i are related, and provides the main structure of a

[3] A provider of low quality never remains in the platform: according to (3.2), the platform immediately reveals his quality so that he cannot demand a positive price inside the platform.

[4] Note that these quantities must satisfy the time-invariance conditions

$$\delta^H_{\mathcal{H}} = \beta\big(\delta^H_{\mathcal{H}} + (1-\alpha)(\delta^H_{\mathcal{U}} + \gamma\delta^U_{\mathcal{U}})\big) \text{ and } \delta^H_{\mathcal{U}} = \beta\alpha\big(\delta^H_{\mathcal{U}} + \gamma\delta^U_{\mathcal{U}}\big). \tag{3.6}$$

steady-state equilibrium.

Proposition 3.3.1 *Consider a steady-state equilibrium under a platform policy* $\{\tau, \alpha\}$.

1. *Suppose* $q_{\mathcal{U}}(\alpha) < q_0$. *Then the equilibrium satisfies:*

   - *If* $1 < \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} < \frac{p_{\mathcal{H}}-p_0}{1-q_0} < 2$, *then* $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} - 1$ *and* $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}}-p_0}{1-q_0}$.

   - *If* $\max\left(\frac{p_{\mathcal{H}}-p_0}{1-q_0}, 1\right) < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} < \min\left(\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, 2\right)$, *then* $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} - 1$ *and* $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$.

2. *Suppose* $q_{\mathcal{U}}(\alpha) \geq q_0$. *Then the equilibrium satisfies:*

   - *If* $\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} < 1 < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} < 2$, *then* $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} - 1$ *and* $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$.

   - *If* $1 < \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} < \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$, *then* $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}} - \frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}$ *and* $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{1-q_{\mathcal{U}}}$.

To conclude this section, we establish that a steady-state equilibrium, as described above, indeed exists for any given platform policy.

Proposition 3.3.2 *For any policy* $\{\tau, \alpha\}$, *a steady-state equilibrium exists.*

In summary, a steady-state equilibrium exists for any platform policy $\{\tau, \alpha\}$, and the supply-demand interactions induced by the platform's policy are fully described by the endogenous quantities δ_j^i, ζ_i, and p_i, for $j \in \{U, H\}$ and $i \in \{\mathcal{U}, \mathcal{H}\}$.

3.4 Value Drivers of Information Provision

In this section, we demonstrate the mechanisms through which optimizing the platform's information provision increases the platform's equilibrium profits. The mechanisms through which optimal information provision may benefit the platform depend on the platform's market position, and in particular, the extent to which the platform "covers" the available customer demand.

To illustrate each mechanism, our exposition proceeds in two steps: first, we characterize the equilibrium outcome assuming that platform employs a full-information policy. That is, in any period, the platform discloses all information it processes regarding the quality of each provider active on the platform. Next, we show that employing an appropriately designed information-delay policy of the form given in (3.2) leads to higher profits for the platform, focusing on the intuition underlying each mechanism.

Before proceeding to the first mechanism, we place two assumptions on our model parameters.

Assumption 1 *The following inequalities hold: (a)* $q_0 - p_0 < E[q_j] - b_0$*; (b)* $q_0 < \frac{(2-\beta)\gamma}{2(1-\beta)+\beta\gamma}$.

Assumption 1a ensures that the platform's profit under a full-information policy is positive. In particular, when this assumption is violated, no consumer would ever transact with a provider inside the platform, preferring the outside option instead. Assumption 1b is a technical condition we impose for tractability, which essentially places an upper bound on the quality of the outside option.

3.4.1 Information Obfuscation Leads to Experimentation

The first setting we consider involves markets where it is not profitable for the entire population of customers to engage in transactions within the platform. In other words, the customers' outside option provides sufficiently high utility to a subset of customers. Formally, in this section, we restrict attention to markets that satisfy the following assumption.

Assumption 2 *There exists a consumer type $\bar{\theta} \in (1, 2)$ such that $\bar{\theta} q_0 - p_0 > \bar{\theta} E[q_j]$*

.

The implication of Assumption 2 is that, irrespective of the platform's chosen commission τ, a subset of customers in equilibrium will always prefer to use the outside option. That is, when Assumption 2 holds, the platform's market coverage under full information is only, which is formalized in the following result.

Proposition 3.4.1 *Suppose Assumption 2 holds. Then, when the platform employs a full-information policy (i.e., $\alpha = 0$), the mass of customers that choose the outside option under the optimal commission is strictly positive.*

Proposition 3.4.1 suggests that the platform may increase its revenues by incentivizing a higher volume of transactions. Indeed, Proposition 3.4.2 below establishes that setting α appropriately (and higher than zero) leads to a higher mass of customers engaging with the platform. However, this (direct) effect does not imply that the platform's revenues would also increase. Besides, we show that the overall increase in the volume of transactions has a second (indirect) effect. In essence, customers transact with a higher mass of novice providers and, thus, the mass of providers labeled $\mathcal{H}$ at the resulting steady-state is also higher than the case when the platform follows a full-information provision policy.

Proposition 3.4.2 *Suppose Assumption 2 holds. Then, the optimal information provision policy for the platform features positive delay, i.e.,* $\alpha^* > 0$*. Moreover, the overall volume of transactions and the volume of transactions with providers labeled* $\mathcal{H}$ *is higher than under the case when the platform uses a full-information provision policy.*

The intuition behind Proposition 3.4.2 can be best described as follows: setting $\alpha > 0$ results in bundling a subset of high-quality providers with novices. In turn, the expected quality of transacting with a provider labeled $\mathcal{U}$ is higher than under a full-information provision policy. Customers find transacting within the platform more attractive, which leads to an increase in the overall volume of transactions. Finally, the higher overall volume of transactions has a second (indirect) effect. Although each provider in the platform has a lower number of transactions labeled as $\mathcal{H}$ (given that the platform "delays" labeling a high-quality provider as $\mathcal{H}$), the mass of providers labeled $\mathcal{H}$ at each period is higher than under a full information provision policy. Obfuscating information leads to more experimentation with novice providers and results in a higher mass of providers labeled $\mathcal{H}$ available on the platform.

3.4.2 Improving the Composition of Service Providers

The next setting we consider involves markets where it is profitable for the entire population of customers to engage in transactions within the platform. In other words, the customers' outside option is not sufficiently attractive to customers, who find it optimal to seek service from the platform's service providers. In particular, in this section, we impose the following assumption.

Assumption 3 *There exists a consumer type* $\underline{\theta} \in (1, 2)$ *such that* $\underline{\theta} q_0 - p_0 < \underline{\theta} E[q_j]$.

We establish that when Assumption 3 holds, and the platform discloses all the information it has at its disposal, all customers would choose the platform under the optimal commission.

Proposition 3.4.3 *Suppose Assumption 3 holds and* $b_0/w_0 < \rho$ *for a constant* $\rho \in (0, 1)$. *Then, when the platform follows the full-information provision policy and sets the single commission optimally, the entire population of customers engages in transactions within the platform.*

Proposition 3.4.3 states that under Assumption 3, the platform finds it optimal to set its commission so that the entire population of customers transacts within the platform at equilibrium. In this case, the platform cannot increase its revenues by inducing more transactions. However, appropriately designing its information provision policy can still add value: Proposition 3.4.4 below establishes that setting α appropriately (and introducing a delay in revealing high-quality providers) leads to higher revenues for the platform. Although the volume of transactions remains the same, introducing the optimal level of delay, leads to an increase in the average commission per transaction.

Proposition 3.4.4 *Suppose Assumption 3 holds and $b_0/w_0 < \rho$ for a constant $\rho \in (0,1)$. Then, the optimal information provision policy for the platform features positive delay, i.e., $\alpha^* > 0$. Moreover, the volume of transactions with providers labeled $\mathcal{H}$ is lower than under the case when the platform uses a full-information provision policy.*

Intuitively, the mechanism described in Proposition 3.4.4 is analogous to price discrimination by reducing the quality of a portion of a firm's output, i.e., "damaging" a firm's product solely to offer two versions of it and engaging in price discrimination. In particular, by setting $\alpha^* > 0$, the platform essentially downgrades a subset of its high-quality providers. In turn, the price set by the providers labeled $\mathcal{H}$ at equilibrium is higher than in the case that the platform uses a full-information provision policy. Thus, the volume of transactions with providers labeled $\mathcal{H}$ decreases, the revenue per transaction increases, and the platform's revenue benefits.

3.4.3 Experimentation via Commission Subsidies

So far, our analysis has exclusively considered the case when the platform sets the same commission rate for every transaction. The single commission scheme is widely adopted in practice and allows us to transparently describe the mechanisms through which delayed information provision may benefit a platform. In this section, we extend our original setting by considering the case when the platform may set different commission rates for transactions with providers of different labels, i.e., the platform sets $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$, i.e., the commission rates for transactions with providers labeled $\mathcal{U}$ and $\mathcal{H}$, respectively.

Moreover, we restrict attention to settings where it is not profitable for the platform to cover the entire market. In such a case, a natural alternative to induce a higher volume of transactions (and more experimentation with new providers) is to "subsidize" transactions with providers labeled $\mathcal{U}$ by setting $\tau_{\mathcal{U}} \leq \tau_{\mathcal{H}}$. The proposition below establishes that when the platform uses a full-information provision policy setting $\tau_{\mathcal{U}} \leq \tau_{\mathcal{H}}$ is indeed optimal.

Proposition 3.4.5 *When the platform follows the full-information provision policy, it finds it optimal to set the commission rate for transactions with providers labeled $\mathcal{U}$ lower than that for transactions with providers labeled $\mathcal{H}$.*

The intuition behind this proposition is straightforward: the platform benefits from inducing a higher volume of transactions in markets, as doing so also increases the rate at which it reveals high-quality providers. The platform can incentivize an increase in the volume of transactions with new providers by setting the corresponding commission rate lower, i.e., effectively subsidizing engaging in transactions with them.

Finally, Proposition 3.4.6 below establishes that appropriately designing the platform's information-provision policy leads to benefits even in this case where commission rates associated with different provider labels may be different. To state the proposition, we will abuse notation slightly by letting $\tau^*_{\mathcal{U}}(\alpha), \tau^*_{\mathcal{H}}(\alpha)$ denote the optimal commission rate for transactions with providers labeled $\mathcal{U}$, $\mathcal{H}$, respectively, when the platform uses an information delay policy with parameter α.

Proposition 3.4.6 *Then, the optimal information provision policy for the platform features positive delay, i.e.,* $\alpha^* > 0$*. Moreover, the commission rate for transactions with providers labeled* $\mathcal{U}$ *is higher than the case when the platform uses a full-information provision policy, i.e.,* $\tau^*_{\mathcal{U}}(\alpha^*) > \tau^*_{\mathcal{U}}(0)$.

Proposition 3.4.6 suggests that introducing a delay in revealing high-quality providers can complement setting a lower commission rate for providers labeled $\mathcal{U}$ in increasing the overall volume of transactions and subsequently the platform's revenues. The optimal policy uses both design levers, i.e., the information provision policy and different commission rates, and, thus, establish the value of such joint optimization.

3.5 Concluding Remarks

In this paper, we investigate the benefits of information provision policy in online peer-to-peer service platforms. Services providers are strategic in terms of entering the platform and pricing their service. Their service qualities are heterogeneous and learned through transactions. Consumers are strategic in terms of their hiring decisions. Therefore, when designing the information provision policy and commission fee structure, the platform needs to take into account the strategic behaviors of all participants, as well as the learning process of providers' qualities.

We establish that the informational delay can benefit the platform implements either a single commission scheme or a differentiated commission scheme. Under an optimal single commission scheme, the informational delay can further increase the platform's revenue by increasing the experimentation of new providers or optimizing the provider composition and inducing profitable price discrimination. Under an optimal differentiated commission scheme, the informational delay can further improve the platform's revenue by lowering the revenue loss incurred by subsidizing new providers.

3.6 Appendix: Technical propositions and lemmas

Definition 3.6.1 *Given an information provision policy $\{\alpha\}$ and commission structure $\{\tau_{\mathcal{U}}, \tau_{\mathcal{H}}\}$, if $q_{\mathcal{U}} \leq q_0$, then we classify the equilibrium into the following four types as their characterization are different.*

- *E1: There is no rationing among providers with label $\mathcal{U}$ (i.e., $\eta = 1$), and all customers choose the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}} = 1$).*
- *E2: There is rationing among providers with label $\mathcal{U}$ (i.e., $\eta < 1$), and all customers choose the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}} = 1$).*
- *E3: There is rationing among providers with label $\mathcal{U}$ (i.e., $\eta < 1$), and not all customers choose the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}} < 1$).*
- *E4: There is no rationing among providers with label $\mathcal{U}$ (i.e., $\eta = 1$), and there not all customers choose the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}} < 1$).*

Lemma 3.6.1 *The equilibrium quantities of H providers with label $\mathcal{U}$ and label $\mathcal{H}$ can be simplified as:*

$$\delta_{\mathcal{U}}^{H} = \beta\gamma\lambda\delta_{\mathcal{U}}^{U} \text{ and } \delta_{\mathcal{H}}^{H} = \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^{U}, \tag{3.8}$$

where $\lambda = \frac{\alpha}{1-\beta\alpha}$.

Proof. The claim immediately follows from (3.6). □

Lemma 3.6.2 *In equilibrium, the free-entry condition for new providers (i.e.,* $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$*) is equivalent to*

$$F(\lambda,\eta) \triangleq \frac{\beta\gamma}{1-\beta}\cdot\frac{1-(1-\beta)\lambda}{1+\beta\gamma\lambda\eta}\big((1-\tau_{\mathcal{H}})p_{\mathcal{H}} - w_0\big) = \frac{w_0}{\eta} - (1-\tau_{\mathcal{U}})p_{\mathcal{U}}, \tag{3.9}$$

where $\lambda = \frac{\alpha}{1-\beta\alpha}$.

Proof. First, by (3.5), $V_{\mathcal{U}}^{H} \geq \frac{w_0}{1-\beta}$, and $V_{\mathcal{H}}^{H} \geq \frac{w_0}{1-\beta}$, we rewrite $V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}$ as:

$$(1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta p_{\mathcal{U}}^{H}\gamma V_{\mathcal{U}}^{H} + \beta(1-p_{\mathcal{U}}^{H})\gamma V_{\mathcal{H}}^{H} - \beta\gamma\frac{w_0}{1-\beta} = \frac{w_0}{\eta}.$$

Second, we substitute $V_{\mathcal{U}}^{H}$ and $V_{\mathcal{H}}^{H}$ in the above equality using (3.4) and (3.3), respectively, and obtain:

$$(\delta_{\mathcal{U}}^{U} + \eta\delta_{\mathcal{U}}^{H})\Big((1-\tau_{\mathcal{U}})p_{\mathcal{U}} - \frac{w_0}{\eta}\Big) + \delta_{\mathcal{H}}^{H}\Big((1-\tau_{\mathcal{H}})p_{\mathcal{H}} - w_0\Big) = 0.$$

Lastly, by substituting $\delta_{\mathcal{H}}^{U}$ and $\delta_{\mathcal{H}}^{H}$ in the above equality using (3.8) and letting $\lambda = \frac{\alpha}{1-\beta\alpha}$, we obtain (3.9). Therefore, we have shown that the free-entry condition is equivalent to (3.9). □

3.6.1 Single commission scheme

In this section, we introduce technical lemmas and propositions related to the single-commission structure (i.e. $\tau_{\mathcal{U}} = \tau_{\mathcal{H}}$).

Lemma 3.6.3 *Suppose* $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$ *and* $\lambda = \frac{\alpha}{1-\beta\alpha} \geq 0$. *Then, in equilibrium, the quantity and the expected quality of hired providers with label* $\mathcal{U}$ *are* $\delta_{\mathcal{U}} = (1 + \beta\gamma\lambda\eta)\delta_{\mathcal{U}}^{U}$ *and* $q_{\mathcal{U}} = \frac{\gamma+\beta\gamma\lambda\eta}{1+\beta\gamma\lambda\eta}$, *respectively. The remaining quantities have different characterizations across types:*

If an E1 holds,

- *Quantity of hired new providers:* $\delta_{\mathcal{U}}^{U} = \frac{1-\beta}{1-\beta+\beta\gamma}$.
- *Prices of providers with label* $\mathcal{U}$ *and providers with label* $\mathcal{H}$*:*
  - $p_{\mathcal{U}} = \frac{w_0}{1-\tau} - \frac{\beta\gamma}{1-\beta+\beta\gamma}\big(1-(1-\beta)\lambda\big)\big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)(1-\gamma)$.
  - $p_{\mathcal{H}} = p_{\mathcal{U}} + \big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)(1-\gamma)$.
- *Rationing rate:* $\eta = 1$.
- *Platform revenue:* $\pi_{r,1}(\tau,\lambda) = \frac{\tau}{1-\tau} w_0$.
- *The equilibrium satisfies the following conditions:*
  - *Providers with label* $\mathcal{U}$ *are not financially constrained. That is,* $(1-\tau)p_{\mathcal{U}} \geq b_0$.
  - *The customer at* $1+\delta_{\mathcal{U}}$ *prefers providers with label* $\mathcal{U}$ *to the outside option. That is,* $p_{\mathcal{U}} \leq p_0 - \big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$.

If an E2 holds,

- *Quantity of hired new providers:* $\delta_{\mathcal{U}}^{U} = \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)}$.

- *Prices of providers with label* $\mathcal{U}$ *and providers with label* $\mathcal{H}$*:*

  · $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$.

  · $p_{\mathcal{H}} = \frac{b_0}{1-\tau} + \big(\frac{1}{1+\beta\gamma\lambda\eta} + \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)}\big)(1-\gamma)$.

- *Rationing rate:* η *is solved from* (3.9).

- *Platform revenue:* $\pi_{r,2}(\tau, \lambda) = \tau\Big(\frac{b_0}{1-\tau} + \big(\frac{1}{1+\beta\gamma\lambda\eta} + \delta_{\mathcal{U}}^{U}\big)\delta_{\mathcal{H}}^{H}(1-\gamma)\Big)$.

- *The equilibrium satisfies the following conditions:*

  · *Providers with label* $\mathcal{U}$ *get rationed. That is,* $0 < \eta < 1$.

  · *The customer at* $1+\delta_{\mathcal{U}}$ *prefers providers with label* $\mathcal{U}$ *to the outside option. That is,* $1 + \delta_{\mathcal{U}} \leq \frac{p_0 - p_{\mathcal{U}}}{q_0 - q_{\mathcal{U}}}$.

If an E3 holds,

- *Quantity of hired new providers:* $\delta_{\mathcal{U}}^{U} = \frac{p_0 - \frac{b_0}{1-\tau}}{q_0 - \gamma - \beta\gamma\lambda\eta(1-q_0)} - \frac{1}{1+\beta\gamma\lambda\eta}$

- *Prices of providers with label* $\mathcal{U}$ *and providers with label* $\mathcal{H}$*:*

  · $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$.

  · $p_{\mathcal{H}} = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\big(1 - (1-\beta)\lambda\big)\delta_{\mathcal{U}}^{U}\Big)(1-q_0)$.

- *Rationing rate:* η *is solved from* (3.9).

- *Platform revenue:* $\pi_{r,3}(\tau, \lambda) = \frac{\tau}{1-\tau}\big(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta}\big)w_0\delta_{\mathcal{U}}^{U}$.

- *The equilibrium satisfies the following conditions:*

  · *Providers with label* $\mathcal{U}$ *get rationed. That is,* $0 < \eta < 1$.

  · *There are customers choosing the platform. That is,* $\delta_{\mathcal{U}}^{U} > 0$.

  · *Not all customers choose the platform. That is,* $\big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)\big)\delta_{\mathcal{U}}^{U} < 1$.

If an E4 holds,

- *Quantity of hired new providers:* $\delta_{\mathcal{U}}^{U} = \frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - \frac{w_0}{1-\tau}) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - (q_0-\gamma) - (1-q_0)\beta\gamma\lambda}{(\frac{\beta\gamma}{1-\beta})^2(1-q_0) + (q_0-\gamma) - \Big(\frac{1-\beta+2\beta\gamma}{1-\beta}(1-q_0) - (q_0-\gamma)\Big)\beta\gamma\lambda}$.

- *Prices of providers with label* $\mathcal{U}$ *and providers with label* $\mathcal{H}$*:*

  · $p_{\mathcal{U}} = p_0 - (\frac{1}{1+\beta\gamma\lambda} + \delta_{\mathcal{U}}^{U})\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$.

  · $p_{\mathcal{H}} = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^{U}\Big)(1-q_0)$.

- *Rationing rate:* $\eta = 1$.

- *Platform revenue:* $\pi_{r,4}(\tau, \lambda) = \frac{\tau}{1-\tau} \cdot \frac{1-\beta+\beta\gamma}{1-\beta} w_0 \delta_{\mathcal{U}}^{U}$.

- *The equilibrium satisfies the following conditions:*

  · *Providers with label* $\mathcal{U}$ *are not financially constraint. That is,* $(1-\tau)p_{\mathcal{U}} \geq b_0$.

  · *There are customers but not all choosing the platform. That is,* $0 < \delta_{\mathcal{U}}^{U} < \frac{1-\beta}{1-\beta+\beta\gamma}$.

Proof. First of all, the quantity of hired providers with label $\mathcal{U}$ is $\delta_{\mathcal{U}} = \zeta_{\mathcal{U}} = \delta_{\mathcal{U}}^{U} + \eta\delta_{\mathcal{U}}^{H} = (1+\beta\gamma\lambda\eta)\delta_{\mathcal{U}}^{U}$, where the last equality is obtained from (3.8). Then, the expected quality of hired providers with label $\mathcal{U}$ is:

$$q_{\mathcal{U}} = \frac{\delta_{\mathcal{U}}^{U}\gamma + \eta\delta_{\mathcal{U}}^{H}}{\delta_{\mathcal{U}}^{U} + \eta\delta_{\mathcal{U}}^{H}} = \frac{\gamma + \beta\gamma\lambda\eta}{1 + \beta\gamma\lambda\eta}.$$

The second equality follows from (3.8).

Next, we characterize the rest equilibrium quantities by types:

E1: In this case, there is no rationing among providers with label $\mathcal{U}$ (i.e., $\eta = 1$), and all customers hire providers from the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}}^{H} = 1$). From these two equalities and (3.8), we solve for $\delta_{\mathcal{U}}^{U} = \frac{1-\beta}{1-\beta+\beta\gamma}$. We then characterize the prices based on Case 2 of the equilibrium definition. In particular, $p_{\mathcal{H}} = p_{\mathcal{U}} + (1+\zeta_{\mathcal{U}})(1-q_{\mathcal{U}}) = p_{\mathcal{U}} + \big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)(1-\gamma)$, which is a function of $p_{\mathcal{U}}$. Then, we solve for $p_{\mathcal{U}}$ from (3.9). To characterize the platform revenue, we have $(1-\tau)(\delta_{\mathcal{H}}^{H} p_{\mathcal{H}} + \delta_{\mathcal{U}} p_{\mathcal{U}}) = w_0$ from (3.9), from which we obtain $\pi_{r,1} = \frac{\tau}{1-\tau} w_0$.

E2: In this case, providers with label $\mathcal{U}$ are financially constraint, so their price is $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$. Note that all customers choose to hire providers from the platform (i.e., $\delta_{\mathcal{U}} + \delta_{\mathcal{H}}^{H} = 1$). So, we solve for $\delta_{\mathcal{U}}^{U}$ from the last equality and (3.8). Then, we characterize $p_{\mathcal{H}}$ based on Case 2 of the equilibrium definition. In particular, $p_{\mathcal{H}} = p_{\mathcal{U}} + (1+\zeta_{\mathcal{U}})(1-q_{\mathcal{U}}) = \frac{b_0}{1-\tau} + \big(\frac{1}{1+\beta\gamma\lambda\eta} + \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta)}\big)(1-\gamma)$. Given the characterization of $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$, we determine η from (3.9). Lastly, for the platform revenue, we have $\pi_{r,2} = \tau(p_{\mathcal{U}}\delta_{\mathcal{U}} + p_{\mathcal{H}}\delta_{\mathcal{H}}^{H}) = \tau\big(\frac{b_0}{1-\tau} + (\frac{1}{1+\beta\gamma\lambda\eta} + \delta_{\mathcal{U}}^{U})\delta_{\mathcal{H}}^{H}(1-\gamma)\big)$, where the second equality is obtained by substituting $p_{\mathcal{H}}$ using the above characterization.

E3: In this case, providers with label $\mathcal{U}$ are financially constraint, so their price is

$p_{\mathcal{U}} = \frac{b_0}{1-\tau}$. Then, by the Case 1 of the equilibrium definition, we solve for $\delta_{\mathcal{U}}^U$ and $p_{\mathcal{H}}$. Given the characterization of $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$, we determine η using (3.9). Lastly, for the platform revenue, we obtain $\delta_{\mathcal{U}} p_{\mathcal{U}} + \delta_{\mathcal{H}}^H p_{\mathcal{H}} = \frac{w_0}{1-\tau}\big(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta}\big)\delta_{\mathcal{U}}^U$, from which we obtain the characterization of $\pi_{r,3}$.

E4: In this case, there is no rationing (i.e., $\eta = 1$). By Case 1 in the equilibrium definition, we obtain $p_{\mathcal{U}} = p_0 - (1+\delta_{\mathcal{U}})(q_0 - q_{\mathcal{U}}) = p_0 - \big(\frac{1}{1+\beta\gamma\lambda} + \delta_{\mathcal{U}}^U\big)\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$ and $p_{\mathcal{H}} = p_0 + \big(2 - \delta_{\mathcal{H}}^H\big)(1-q_0) = p_0 + \big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U\big)(1-q_0)$. Notice that $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ only depends on $\delta_{\mathcal{U}}^U$, so we can plug them into (3.9) and solve for $\delta_{\mathcal{U}}^U$ in closed from. Lastly, we obtain $\pi_{r,4}$ by letting $\eta = 1$ in $\pi_{r,3}$.

Therefore, we have completed the equilibrium characterization for all four types. □

Lemma 3.6.4 *Suppose $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$ and $\alpha = 0$. Then, each equilibrium type can be characterized as follows.*

In E1,

- *Prices of new providers and $\mathcal{H}$-label providers: $p_{\mathcal{U}} = \frac{w_0}{1-\tau} - \frac{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)}{(1-\beta+\beta\gamma)^2}(1-\gamma)$, $p_{\mathcal{H}} = p_{\mathcal{U}} + (1+\delta_{\mathcal{U}}^U)(1-\gamma)$.*

- *Quantities of new providers and $\mathcal{H}$-label providers: $\delta_{\mathcal{U}}^U = \frac{1-\beta}{1-\beta+\beta\gamma}$, $\delta_{\mathcal{H}}^H = \frac{\beta\gamma}{1-\beta+\beta\gamma}$.*

- *Rationing rate: $\eta = 1$.*

- *The equilibrium satisfies the following conditions:*

  - *The customer at $1+\delta_{\mathcal{U}}^U$ prefers the new provider to the outside option (i.e., $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$).*

· *New providers are not financially constrained (i.e.,* $(1-\tau)p_{\mathcal{U}} \geq b_0$*).*

- *Platform revenue:* $\pi_{r,1}(\tau) = \frac{\tau}{1-\tau} w_0$.

In E2,

- *Prices of new providers and* $\mathcal{H}$*-label providers:* $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$, $p_{\mathcal{H}} = p_{\mathcal{U}} + (1 + \delta_{\mathcal{U}}^{U})(1-\gamma)$

- *Quantities of new providers and* $\mathcal{H}$*-label providers:* $\delta_{\mathcal{U}}^{U} = \frac{1-\beta}{1-\beta+\beta\gamma}$, $\delta_{\mathcal{H}}^{H} = \frac{\beta\gamma}{1-\beta+\beta\gamma}$.

- *Rationing rate:* $\eta = \frac{1-\beta}{\beta\gamma} \frac{w_0}{\frac{1-\beta+\beta\gamma}{\beta\gamma} b_0 - w_0 + \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(1-\tau)(1-\gamma)}$.

- *The equilibrium satisfies the following conditions:*

  · *The customer at* $1 + \delta_{\mathcal{U}}^{U}$ *prefers the new provider to the outside option (i.e.,* $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$*).*

  · *New providers get rationed (i.e.,* $0 < \eta < 1$*).*

- *Platform revenue:* $\pi_{r,2}(\tau) = \tau\big(\frac{b_0}{1-\tau} + \frac{\beta\gamma(2(1-\beta)+\beta\gamma)}{(1-\beta+\beta\gamma)^2}(1-\gamma)\big)$.

In E3,

- *Prices of new providers and* $\mathcal{H}$*-label providers:* $p_{\mathcal{U}} = \frac{b_0}{1-\tau}$, $p_{\mathcal{H}} = p_0 + (2 - \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U})(1-q_0)$.

- *Quantities of new providers and* $\mathcal{H}$*-label providers:* $\delta_{\mathcal{U}}^{U} = \frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} - 1$, $\delta_{\mathcal{H}}^{H} = \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U}$.

- *Rationing rate:* $\eta = \frac{w_0}{\frac{\beta\gamma}{1-\beta}(1-\tau)p_{\mathcal{H}} - \frac{\beta\gamma}{1-\beta} w_0 + b_0}$.

- *The equilibrium satisfies the following conditions:*

    · *Not all customers choose the platform (i.e.,* $\delta_{\mathcal{U}}^U + \delta_{\mathcal{H}}^H < 1$*).*

    · *New providers get rationed (i.e.,* $0 < \eta < 1$*).*

    · *There are customers choosing the platform* $\delta_{\mathcal{U}}^U > 0$.

- *Platform revenue:*

$$\pi_{r,3}(\tau) = \Big(1+\frac{\beta\gamma}{1-\beta}\cdot\frac{1}{\underline{\eta}}\Big)\frac{\tau}{q_0-\gamma}\Big(p_0-(q_0-\gamma)-\frac{b_0}{1-\tau}\Big)\Big(\frac{b_0}{1-\tau}+\frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}\Big)$$

In E4,

- *Prices of new providers and* $\mathcal{H}$*-label providers:* $p_{\mathcal{U}} = \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}\frac{w_0}{1-\tau}+(\frac{1}{\underline{\eta}}-1)p_0-\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0)}{\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma}}$, $p_{\mathcal{H}} = p_0 + (2-\frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U)(1-q_0)$.

- *Quantities of new providers and* $\mathcal{H}$*-label providers:* $\delta_{\mathcal{U}}^U = \frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} - 1$, $\delta_{\mathcal{H}}^H = \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$.

- *Rationing rate:* $\eta = 1$.

- *The equilibrium satisfies the following conditions:*

    · *Not all customers choose the platform (i.e.,* $\delta_{\mathcal{U}}^U + \delta_{\mathcal{H}}^H < 1$*).*

    · *New providers are not financially constrained (i.e.,* $(1-\tau)p_{\mathcal{U}} \geq b_0$*).*

    · *There are customers choosing the platform* $\delta_{\mathcal{U}}^U > 0$.

- *Platform revenue:*

$$\pi_{r,4}(\tau) = \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma(1-\beta)}\frac{w_0}{(q_0-\gamma)(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})}\frac{\tau}{1-\tau}\Big(p_0+\frac{2\beta\gamma}{1-\beta+\beta\gamma}\big(1-\frac{1}{2}\underline{\eta}\big)(1-q_0)-\frac{w_0}{1-\tau}\Big)$$

Proof. We show Lemma 3.6.4 by letting $\alpha = 0$ in Lemma 3.6.3. □

Proposition 3.6.1 specifies the equilibrium type under a given single commission with full information.

Proposition 3.6.1 *Suppose $\tau_{\mathcal{U}} = \tau_{\mathcal{H}} = \tau$ and $\alpha = 0$. Then, an equilibrium exists. Moreover,*

If $p_0 \le q_0 - \gamma$, then there is no provider on the platform.

If $q_0 - \gamma < p_0 \le \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)$ and

1. *If $0 < b_0 \le \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, then*

$$\frac{b_0}{p_0 - (q_0 - \gamma)} < \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0 - \gamma)(\frac{2}{\underline{\eta}} - 1)} \le \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1 - q_0) - (\frac{1}{\underline{\eta}} - 1) p_0}. \tag{3.10}$$

   Moreover,

   (a) *If $0 \le \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0-(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}$, then an E3 holds.*

   (b) *If $1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0-(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0} \le \tau < 1 - \frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}$, then an E4 holds.*

   (c) *Otherwise, there is no provider on the platform.*

2. *If $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 < b_0 \le w_0$, then*

   (a) *If $0 < \tau < 1 - \frac{b_0}{p_0-(q_0-\gamma)}$, then an E3 holds.*

   (b) *Otherwise, there is no provider on the platform.*

If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, *then*

$$\frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0 \leq \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0. \quad (3.11)$$

Moreover,

3. *If* $0 < b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0$, *then*

$$\frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} \leq \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)} \frac{w_0-b_0}{1-\gamma}. \quad (3.12)$$

Moreover,

(a) *If* $0 < \tau < 1 - \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)} \frac{w_0-b_0}{1-\gamma}$, *then an E2 holds.*

(b) *If* $1 - \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)} \frac{w_0-b_0}{1-\gamma} \leq \tau \leq 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}$, *then an E1 holds.*

(c) *If* $1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} < \tau < 1 - \frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}$, *then an E4 holds.*

(d) *Otherwise, there is no provider on the platform.*

4. *If* $\frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, *then*

$$\frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} < \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0} < \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}. \quad (3.13)$$

Moreover,

(a) *If* $0 < \tau \leq 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$, *then an E2 holds.*

(b) *If* $1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} < \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}$, *then an E3 holds.*

(c) *If* $1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0} \leq \tau < 1 - \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}$, *then an E4 holds.*

(d) *Otherwise, there is no provider on the platform.*

5. *If* $\frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 \leq b_0 \leq w_0$, *then*

(a) *If* $0 < \tau \leq 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$, *then an E2 holds.*

(b) $1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} < \tau < 1 - \frac{b_0}{p_0-(q_0-\gamma)}$, *then an E3 holds.*

(c) *Otherwise, there is no provider on the platform.*

If $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, *then*

$$\frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 < \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0. \tag{3.14}$$

Moreover,

6. *If* $0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, *then*

$$\begin{aligned} \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} &< \frac{w_0}{p_0 + \frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} \\ &< \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)}\frac{w_0-b_0}{1-\gamma}. \end{aligned} \tag{3.15}$$

Moreover,

(a) *If* $0 < \tau < 1 - \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)}\frac{w_0-b_0}{1-\gamma}$, *then an E2 holds.*

(b) *If* $1-\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}\le\tau\le 1-\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}$, *then an E1 holds.*

(c) *If* $1-\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}<\tau<1-\frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}$, *then an E4 holds.*

(d) *Otherwise, there is no provider on the platform.*

7. *If* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0<b_0\le\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, *then*

$$\begin{aligned}\frac{b_0}{p_0-q_0+\gamma}\le&\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}\\ \le&\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}\le\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}.\end{aligned} \tag{3.16}$$

Moreover,

(a) *If* $0<\tau<1-\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}$, *then an E2 holds.*

(b) *If* $1-\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}\le\tau\le 1-\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}$, *then an E1 holds.*

(c) *If* $1-\frac{w_0}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}<\tau\le 1-\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}$, *then an E4 holds.*

(d) *If* $1-\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}<\tau<1-\frac{b_0}{p_0-q_0+\gamma}$, *then an E3 holds.*

(e) *Otherwise, there is no provider on the platform.*

8. *If* $\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0<b_0\le w_0$, *and*

(a) *If* $0 < \tau \leq 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$*, then an E2 holds.*

(b) *If* $1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} < \tau < 1 - \frac{b_0}{p_0-q_0+\gamma}$*, then and E3 holds.*

(c) *Otherwise, there is no provider on the platform.*

Proof. By the equilibrium characterization in Lemma 3.6.4, to show the existence of equilibrium of a given type, it suffices to verify the conditions it satisfy. In particular,

- To show that E1 occurs, we need to verify:
    - The customer at $1 + \delta_{\mathcal{U}}^{U}$ prefers the new provider to the outside option (i.e., $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$).
    - New providers are not financially constrained (i.e., $(1-\tau)p_{\mathcal{U}} \geq b_0$).
- To show that E2 occurs, we need to verify:
    - The customer at $1 + \delta_{\mathcal{U}}^{U}$ prefers the new provider to the outside option (i.e., $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$).
    - New providers get rationed (i.e., $0 < \eta < 1$).
- To show that E3 occurs, we need to verify:
    - Not all customers choose the platform (i.e., $\delta_{\mathcal{U}}^{U} + \delta_{\mathcal{H}}^{H} < 1$).
    - New providers get rationed (i.e., $0 < \eta < 1$).
    - There are customers choosing the platform $\delta_{\mathcal{U}}^{U} > 0$.
- To show that E4 occurs, we need to verify:

- Not all customers choose the platform (i.e., $\delta_{\mathcal{U}}^U + \delta_{\mathcal{H}}^H < 1$).
- New providers are not financially constrained (i.e., $(1-\tau)p_{\mathcal{U}} \geq b_0$).
- There are customers choosing the platform $\delta_{\mathcal{U}}^U > 0$.

First, it is straightforward to verify that when $p_0 \leq q_0 - \gamma$, no providers choose to stay on the platform.

Second, we show the cases when $q_0 - \gamma < p_0 \leq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)$. There are two groups of cases corresponding to $0 < b_0 \leq \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$ and $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 < b_0 \leq w_0$, respectively.

1. In the case with low b_0, we first show that the inequalities in (3.10) hold. In particular, it is straightforward to verify that both inequality in (3.10) are equivalent to $b_0 \leq \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$.

   (a) Then, to show an E3 arises in case 1-(a), it suffices to verify conditions $\delta_{\mathcal{U}}^U + \delta_{\mathcal{H}}^H < 1$, $0 < \eta < 1$, and $\delta_{\mathcal{U}}^U > 0$. Notice that $\delta_{\mathcal{U}}^H = 0$ and $\delta_{\mathcal{H}}^H = \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$. The first condition is equivalent to $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$, which is equivalent to $p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < \frac{b_0}{1-\tau}$. The last inequality holds since its left-hand side is non-positive while the right-hand side is positive. Then, by Lemma 3.6.4, we have the closed-form characterization of η. Using the characterization, we show $\eta > 0$, since it is equivalent to

$$\tau < 1 - \frac{w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}, \tag{3.17}$$

which holds because of

$$\tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}. \tag{3.18}$$

Moreover, we show $\eta < 1$, which is equivalent to (3.18). Lastly, we verify the third condition (i.e., $\delta_{\mathcal{U}}^U > 0$). Notice that it is equivalent to

$$\tau < 1 - \frac{b_0}{p_0 - q_0 + \gamma}, \tag{3.19}$$

which holds because of (3.18) and (3.10). Therefore, an E3 occurs under in this case.

(b) Then, to show an E4 arises in case 1-(b), it suffices to verify conditions $\delta_{\mathcal{U}}^U + \delta_{\mathcal{H}}^H < 1$, $(1-\tau)p_{\mathcal{U}} \geq b_0$, and $\delta_{\mathcal{U}}^U > 0$. For the first condition, it is equivalent to show that $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$, which holds because $\tau < 1$, $b_0 > 0$, and $p_0 \leq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)$. By Lemma 3.6.4, we have a closed-form characterization of $p_{\mathcal{U}}$ in E4. By the characterization, we show that the second condition (i.e., $(1-\tau)p_{\mathcal{U}} \geq b_0$) is equivalent to

$$\tau \geq 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}, \tag{3.20}$$

which is the given condition. Besides, the last condition (i.e., $\delta_{\mathcal{U}}^U > 0$) holds as it is equivalent to

$$\tau < 1 - \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0 - \gamma)(\frac{2}{\underline{\eta}} - 1)}, \tag{3.21}$$

which is the given condition. Therefore, an E4 occurs.

(c) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

2. In the case with high b_0, there are two cases to consider.

   (a) In case 2-(a), we show that an E3 arises. Similar to case 1-(a), it suffices to show conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$ hold. Follow the same argument as in case 1-(a), we show these conditions hold for this case.

   (b) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

Third, we show the cases when $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\eta}-1}$. Notice that (3.11) holds as it is equivalent to $p_0 \leq \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\eta}-1}$, which is given. There are three groups of cases corresponding to $0 < b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\eta}-1)} w_0$, $\frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\eta}-1)} w_0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\eta}-1)} w_0$, and $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\eta}-1)} w_0 \leq b_0 \leq w_0$, respectively.

3. In the case of low b_0, we first show that (3.12) holds. In particular, it is equivalent to $0 < b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\eta}-1)} w_0$, which is given.

   (a) Then, to show an E2 arises in case 3-(a), it suffices to verify conditions $0 < \eta < 1$ and $\frac{p_0 - p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$. We notice that $\eta < 1$ holds as it is equivalent to

$$\tau < 1 - \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)} \frac{w_0 - b_0}{1-\gamma}, \tag{3.22}$$

which is given. Moreover, notice that $\eta > 0$ is equivalent to

$$\tau < 1 - \frac{1-\beta+\beta\gamma}{2(1-\beta)+\beta\gamma} \cdot \frac{w_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma} b_0}{1-\gamma}, \tag{3.23}$$

which holds given (3.22) and $\frac{1-\beta+\beta\gamma}{2(1-\beta)+\beta\gamma} \cdot \frac{w_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma} b_0}{1-\gamma} < \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)} \frac{w_0-b_0}{1-\gamma}$. The last inequality holds as it is equivalent to $w_0 > 0$. Lastly, we notice that condition $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ is equivalent to

$$\tau < 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}. \tag{3.24}$$

To show the last inequality holds, it suffices to show $\frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} \leq \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)} \frac{w_0-b_0}{1-\gamma}$, which holds as it is equivalent to $b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0$. Therefore, an E2 occurs.

(b) Then, to show an E1 holds in case 3-(b), it suffices to verify conditions $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ and $(1-\tau)p_{\mathcal{U}} \geq b_0$. The first condition is equivalent to

$$\tau \leq 1 - \frac{w_0}{p_0 + \frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}, \tag{3.25}$$

which is given. The second condition is equivalent to

$$\tau \geq 1 - \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\big(2(1-\beta)+\beta\gamma\big)} \frac{w_0-b_0}{1-\gamma}, \tag{3.26}$$

which is also given. Therefore, an E1 occurs.

(c) Then, to show an E4 holds in case 3-(c), it suffices to verify conditions $\delta_{\mathcal{U}}^{U} + \delta_{\mathcal{H}}^{H} < 1$, $\delta_{\mathcal{U}}^{U} > 0$, and $(1-\tau)p_{\mathcal{U}} \geq b_0$. Notice that the first condition is equivalent to $\delta_{\mathcal{U}} < \frac{1-\beta}{1-\beta+\beta\gamma}$, which is equivalent to

$$\tau > 1 - \frac{w_0}{p_0 + \frac{\big(2(1-\beta)+\beta\gamma\big)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}, \tag{3.27}$$

and it is given. Notice $\delta_{\mathcal{U}}^{U} > 0$ is equivalent to (3.21), which is also given. Lastly, we notice that $(1-\tau)p_{\mathcal{U}} \geq b_0$ holds because $(1-\tau)p_{\mathcal{U}} = \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (1-\tau)\left(\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0) - \left(\frac{1}{\underline{\eta}}-1\right)p_0\right)}{\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma}}$ is increasing in τ (as $\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0)-\left(\frac{1}{\underline{\eta}}-1\right)p_0 > 0$) and $(1-\tau)p_{\mathcal{U}} \geq b_0$ when $\tau = 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}$, which is the threshold of shifting from E1 to E4. The last inequality holds because of the continuity of the equilibrium regarding τ.[5] Therefore, an E4 occurs.

(d) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

4. In the case of moderate b_0, we first show that (3.13) holds. In particular, the first inequality of (3.13) holds because it is equivalent to $b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, which is given. The second inequality of (3.13) holds because it is equivalent to $b_0 > \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which is also given.

(a) Then, to show an E2 holds in case 4-(a), it suffices to verify conditions $0 < \eta < 1$ and $\frac{p_0 - p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$. By the proof of case 3-(a), we notice that $\eta > 0$ is equivalent to (3.23), and $\eta < 1$ is equivalent to (3.22). Using $w_0 > 0$, we can show that (3.23) holds if (3.22) holds. Next, we show that (3.22) holds under the following given condition:

$$\tau \leq 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}. \tag{3.28}$$

In particular, we can show $\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} > \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}$ as it is

[5]The continuity of equilibrium is straightforward to show, and we omit it for brevity.

equivalent to $b_0 > \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0$, which is given. Thus, we have shown $0 < \eta < 1$. Lastly, condition $\frac{p_0 - p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ holds as it is equivalent to (3.28). Therefore, an E2 occurs.

(b) Then, to show an E3 holds in case 4-(b), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$, similar to case 1-(a). Notice that $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to

$$\tau > 1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}. \tag{3.29}$$

which is given. Then, we notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.19). To show (3.19) holds under

$$\tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1) p_0}, \tag{3.30}$$

it suffices to show $\frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}} - 1) p_0} > \frac{b_0}{p_0 - (q_0-\gamma)}$. The last inequality is equivalent to $b_0 < \frac{p_0 - (q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, which is given. Then, in case 1-(a), we have shown $\eta > 0$ is equivalent to (3.17), and $\eta < 1$ is equivalent to (3.18). Since $w_0 > 0$, (3.17) holds as long as (3.18) holds. Besides, (3.18) is given in this case. Therefore, an E3 occurs.

(c) Then, to show an E4 holds in case 4-(c), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $(1-\tau)p_{\mathcal{U}} \geq b_0$, similar to the proof of case 1-(b). Notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.21), which is given, and $(1-\tau)p_{\mathcal{U}} \geq b_0$ is equivalent to (3.20), which is also given. Lastly, we show $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$. In particular, we notice that $\delta_{\mathcal{U}}^U$ is decreasing in τ by the characterization of $\delta_{\mathcal{U}}^U$ in Lemma 3.6.4, and $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ at

$\tau = 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}$, which the threshold of shifting from E3 to E4. Therefore, $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ holds in this case, and an E4 occurs.

(d) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

5. In the case of high b_0, there are two cases to consider.

(a) In case 5-(a), we show an E2 holds by verifying conditions $0 < \eta < 1$ and $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ as in case 3-(a). Notice that $\eta > 0$ is equivalent to (3.23), $\eta < 1$ is equivalent to (3.22), and $\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ is equivalent to (3.24). (3.24) holds as it is given. As in the proof of case 3-(a), (3.23) will hold if (3.22) holds. To show (3.22) given (3.24), it suffices to show $\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0 - b_0}{1-\gamma} < \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$. The last inequality holds as it is equivalent to $b_0 > \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which holds as $b_0 \geq \frac{p_0 - (q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, which is given, and (3.11). Therefore, an E2 occurs.

(b) Then, to show an E3 holds in case 5-(b), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$, similar to case 1-(a). Notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.19), which is given, and $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to (3.29), which is also given. Moreover, $\eta > 0$ is equivalent to (3.17), and $\eta < 1$ is equivalent to (3.18). Besides, (3.17) will hold if (3.18) holds. (3.18) holds in case 5-(b) because $\frac{b_0}{p_0-(q_0-\gamma)} \geq \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}$, and the last inequality is equivalent to $b_0 \geq \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, which is given. Therefore, an E3 occurs.

(c) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

Fourth, we show that when $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, (3.14) holds. In fact, (3.14) is equivalent to $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$. Then, there are three groups corresponding to $0 < b_0 \le \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 < b_0 \le \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, and $\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0 < b_0 \le w_0$, respectively.

6. In the case of low b_0, we first show that (3.15) holds. Given $\underline{\eta} > 0$, it is straightforward to show the first inequality of (3.15) holds. Then, we can show that its second inequality is equivalent to $b_0 < \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$. The last inequality holds given $b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$ and (3.14).

   (a) Then, to show an E2 arises in case 6-(a), it suffices to verify conditions $0 < \eta < 1$ and $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \ge \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$. By the proof of case 3-(a), we notice that $\eta > 0$ is equivalent to (3.23), and $\eta < 1$ is equivalent to (3.22). Using $w_0 > 0$, we can show that (3.23) holds if (3.22) holds. Notice that (3.22) is given in case 6-(a). Moreover, $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \ge \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ is equivalent to (3.28). (3.28) holds in this case as $\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} < \frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma}$. The last inequality is equivalent to $b_0 < \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which holds by (3.14). Therefore, an E2 occurs.

   (b) Then, to show an E1 arises in case 6-(b), it suffices to verify conditions

$\frac{p_0 - p_{\mathcal{U}}}{q_0 - \gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$, which is equivalent to (3.25), and $(1-\tau)p_{\mathcal{U}} \geq b_0$, which is equivalent to (3.26). Notices that (3.25) and (3.26) are given in this case. Therefore, an E1 occurs.

(c) Then, to show an E4 holds in case 6-(c), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $(1-\tau)p_{\mathcal{U}} \geq b_0$, similar to the proof of case 1-(b). Notice that $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to (3.27), which is given, and $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.21), which is also given. Lastly, we show $(1-\tau)p_{\mathcal{U}} \geq b_0$ holds. Notice that it is equivalent to

$$\tau \leq 1 - \frac{(\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}} - 1)p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}. \tag{3.31}$$

To show (3.31) under (3.21), it suffices to show $\frac{(\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}} - 1)p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)} < \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}$. The last inequality is equivalent to $b_0 < \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, which is given. Therefore, an E4 occurs.

(d) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

7. In the case of moderate b_0, we first show that (3.16) holds. The first inequality holds because it is equivalent to $b_0 \geq \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, which is given. Then second and third inequalities are both equivalent to $b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which is also given.

(a) Then, we show an E2 arises in case 7-(a). We omit the proof as it is the same as case 6-(a).

(b) Then, we show an E1 arises in case 7-(b). We omit the proof as it is the same as case 6-(b).

(c) Then, to show an E4 arises in case 7-(c), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $(1-\tau)p_{\mathcal{U}} \geq b_0$, similar to the proof of case 1-(b). Notice that $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to (3.27), which is given. Then, we notice that $(1-\tau)p_{\mathcal{U}} \geq b_0$ is equivalent to (3.31), which is given. Lastly, we notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.21). (3.21) holds under (3.31) because $\frac{w_0}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} < \frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}$, which is equivalent to $b_0 > \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$. Notice that the last inequality is given. Therefore, an E4 occurs.

(d) Then, to show an E3 arises in case 7-(d), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$, similar to case 1-(a). Notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.19), which is given. Then, we notice that $\eta > 0$ is equivalent to

$$\tau > 1 - \frac{(\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0 - w_0}{(\frac{1}{\underline{\eta}} - 1)p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}, \tag{3.32}$$

and $\eta < 1$ is equivalent to

$$\tau > 1 - \frac{(\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}} - 1)p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}. \tag{3.33}$$

Under $w_0 > 0$ and $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, it is straightforward to show that $\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)} < \frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}$, which implies that (3.32) holds as long as (3.33) holds. We further notice that (3.33) is given. Lastly, we notice that $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to (3.29). To show (3.29) holds, it suffices to show that $\frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} \geq \frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)}$. Then, we can show that the last inequality is equivalent to $b_0 \leq \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-}$ which is given. Therefore, an E3 occurs.

(e) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

8. In the case of high b_0, there are three cases to consider.

(a) To show an E2 holds in case 8-(a), it suffices to verify conditions $0 < \eta < 1$ and $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$. Notice that $\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma} \geq \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}$ is equivalent to (3.28), which is given. Then, we notice that $\eta > 0$ is equivalent to (3.23), and $\eta < 1$ is equivalent to (3.22). Using $w_0 > 0$, we can show that (3.23) holds if (3.22) holds. Moreover, (3.22) holds under (3.28) because $\frac{(1-\beta+\beta\gamma)^2}{\beta\gamma\left(2(1-\beta)+\beta\gamma\right)}\frac{w_0-b_0}{1-\gamma} < \frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$ holds. The last inequality is equivalent to $b_0 > \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which is given. Therefore, an E2 occurs.

(b) To show an E3 holds in case 8-(b), it suffices to verify conditions $0 < \delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ and $0 < \eta < 1$, similar to case 1-(a). Notice that $\delta_{\mathcal{U}}^U > 0$ is equivalent to (3.19), which is given, and $\delta_{\mathcal{U}}^U < \frac{1-\beta}{1-\beta+\beta\gamma}$ is equivalent to (3.29), which is also given. Then, we notice that $\eta > 0$ is equivalent to (3.32) and $\eta < 1$ is equivalent to (3.33). Under $w_0 > 0$ and $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, we can show that (3.32) holds as long as (3.33) holds, following the argument in case 7-(d). Then, we show that (3.33) holds under (3.29) because $\frac{(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0-\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)} > \frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$. The last inequality is equivalent to $b_0 > \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}w_0$, which is given. Therefore, an E3 occurs.

(c) For bigger τ, no new providers engage in the platform. We omit the proof for brevity.

Therefore, we have completed the proof of Proposition 3.6.1. □

Proposition 3.6.2 specifies the revenue optimal single commission under a full-information policy. We use $\tau^*(0)$ to denote the optimal single commission under the full information.

Proposition 3.6.2 *We let $\tau_4^* = 1 - \frac{2w_0}{p_0+w_0+\frac{2\beta\gamma}{1-\beta+\beta\gamma}(1-\frac{1}{2}\underline{\eta})(1-q_0)}$. Under the full information, $\tau^*(0)$ can be characterized as follows:*

If $q_0-\gamma < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)$ and

1. *If $0 < b_0 \le \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0$, then*

   (a) *If $b_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\left(1-\frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0)+(1-\frac{1}{2}\underline{\eta})(1-q_0)}\right)w_0$, then*

   $$\tau^*(0) = \arg\max \pi_{r,3}(\tau)\mathbb{1}\left\{0 < \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}\right\}, \tag{3.34}$$

   and an E3 holds.

   (b) *Otherwise, $\tau^*(0) = \tau_4^*$ and an E4 holds.*

2. *If $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)}w_0 < b_0 \le w_0$, then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau)\mathbb{1}\left\{0 < \tau < 1 - \frac{b_0}{p_0 - (q_0 - \gamma)}\right\},$$

and an E3 holds.

If $\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma) < p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, *then*

3. *If* $0 < b_0 \leq \frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0$, *then*

$$\tau^*(0) = \max\left\{\tau_4^*, 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0 - \gamma)(\frac{1}{\underline{\eta}} - 1)}\right\} \tag{3.35}$$

Moreover, if

$$p_0 \geq w_0 + \frac{2(1-\beta)}{1-\beta+\beta\gamma}\Big(\frac{3(1-\beta)+\beta\gamma}{2(1-\beta+\beta\gamma)} - \frac{1-\beta}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}}\Big)(q_0 - \gamma), \tag{3.36}$$

then an E1 holds; Otherwise, an E4 holds.

4. *If* $\frac{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0 < b_0 < \frac{p_0-(q_0-\gamma)}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, *then*

   (a) *If* $b_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\Big(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0)+(1-\frac{1}{2}\underline{\eta})(1-q_0)}\Big) w_0$, *then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau)\mathbb{1}\left\{1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)} \leq \tau < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1 - q_0) - (\frac{1}{\underline{\eta}} - 1)p_0}\right\}, \tag{3.37}$$

   and an E2 or E3 holds.

(b) *Otherwise,* $\tau^*(0) = \tau_4^*$, *and an E4 holds.*

5. *If* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 \le b_0 \le w_0$, *then*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau)\mathbb{1}\left\{1 - \frac{b_0}{p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)} \le \tau < 1 - \frac{b_0}{p_0-(q_0-\gamma)}\right\}, \tag{3.38}$$

and an E2 or E3 holds.

If $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, *then*

6. *If* $0 < b_0 \le \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$, *then* $\tau^*(0)$ *is characterized by* (3.35). *Moreover, if* (3.36) *holds, then an E1 holds. Otherwise, an E4 holds.*

7. *If* $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 < b_0 \le \frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0$, *then*

   (a) *If* $b_0 \le \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\left(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0)+(1-\frac{1}{2}\underline{\eta})(1-q_0)}\right) w_0$, *then* $\tau^*(0)$ *is characterized by* (3.35). *Moreover, if* (3.36) *holds, then an E1 holds. Otherwise, an E4 holds.*

   (b) *Otherwise,*

$$\tau^*(0) = \arg\max \pi_{r,3}(\tau)\mathbb{1}\left\{1 - \frac{(\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma})b_0 - \frac{1-\beta+\beta\gamma}{\beta\gamma}w_0}{(\frac{1}{\underline{\eta}}-1)p_0 - \frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)} < \tau < 1 - \frac{b_0}{p_0-q_0+\gamma}\right\}, \tag{3.39}$$

   and an E3 holds.

8. *If* $\frac{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}{p_0+\frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)} w_0 < b_0 \le w_0$, *then* $\tau^*(0)$ *is characterized by* (3.38), *and an E2 or E3 holds. If* $\tau^*(0) = 1 - \frac{b_0}{p_0-\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0-\gamma)}$, *then an E2 occurs. Otherwise, an E3 occurs.*

Proof. We show this proposition based on three observations.

First of all, by Lemma 3.6.4, the platform revenue increases in τ under E1 and E2. Therefore, the first observation is that $\tau^*(0)$ is not in the interior of the commission intervals in cases 3-(a), 3-(b), 4-(a), 5-(a), 6-(a), 6-(b), 7-(a), 7-(b), and 8-(a) in Proposition 3.6.1. By the continuity of the equilibrium with respect to τ, we only need to search for $\tau^*(0)$ in the boundary and interior of the commission intervals in the remaining cases where $E3$ and $E4$ holds.

The second observation is that, if $\tau^*(0)$ is in the interior of the commission interval where E4 arises, we show that $\tau^*(0) = 1 - \frac{2w_0}{p_0 + w_0 + \frac{2\beta\gamma}{1-\beta+\beta\gamma}(1-\frac{1}{2}\underline{\eta})(1-q_0)} \triangleq \tau_4^*$. By the definition of $\pi_{4,r}(\tau)$ in Lemma 3.6.4, it is equivalent to maximize the following quadratic function:

$$(u-1)\left(\frac{1}{w_0}\Big(p_0 + \frac{2\beta\gamma}{1-\beta+\beta\gamma}(1-\frac{1}{2}\underline{\eta})(1-q_0)\Big) - u\right),$$

where $u = \frac{1}{1-\tau}$. By maximizing the above expression, we obtain $\tau^*(0) = \tau_4^*$.

The third observation is that if E3 and E4 can occur in the same case when changing τ (e.g., case 1 in Proposition 3.6.1), then there does not exist $\hat{\tau}_3$ and $\hat{\tau}_4$ such that $\pi'_{r,3}(\hat{\tau}_3) = 0$, $\pi'_{r,4}(\hat{\tau}_4) = 0$, and $\hat{\tau}_3$ and $\hat{\tau}_4$ follow the same order as the order of E3 and E4 occurring when τ increases in a given case.[6] In other words, we cannot have interior local optimal commission in E3 and E4 at the same time. In particular, by the equilibrium characterization of E3 and E4 in Lemma 3.6.4, we notice that the platform revenue under E3 and E4 can be written as $\pi = \tau h(p_{\mathcal{U}})$,

[6]In cases 1 and 4, we require $\hat{\tau}_3 < \hat{\tau}_4$ as E3 occurs first and in case 7, we require $\hat{\tau}_3 > \hat{\tau}_4$ as E4 occurs first.

where $h(p_{\mathcal{U}})$ is a function of $p_{\mathcal{U}}$. In particular,

$$\begin{aligned} h(p_{\mathcal{U}}) &= \delta_{\mathcal{U}}^{U} p_{\mathcal{U}} + \delta_{\mathcal{H}}^{H} p_{\mathcal{H}} \\ &= \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U}\left(\frac{1-\beta}{\beta\gamma}p_{\mathcal{U}} + p_0 + \left(2 - \frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U}\right)(1-q_0)\right) \\ &= \frac{1+\frac{\beta\gamma}{1-\beta}\frac{1}{\underline{\eta}}}{q_0-\gamma}\Big(p_0 - (q_0-\gamma) - p_{\mathcal{U}}\Big)\left(p_{\mathcal{U}} + \frac{\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0) - \left(\frac{1}{\underline{\eta}}-1\right)p_0}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}\right). \end{aligned}$$

Notice that τ influences $h(p_{\mathcal{U}})$ only through $p_{\mathcal{U}}$. Therefore, $\pi'(\tau) = 0$ is equivalent to $\tau p'_{\mathcal{U}} = -\frac{h(p_{\mathcal{U}})}{h'(p_{\mathcal{U}})}$, where $p'_{\mathcal{U}}$ is the derivative of $p'_{\mathcal{U}}$ with respect to τ. Notice that

$$\frac{h'(p_{\mathcal{U}})}{h(p_{\mathcal{U}})} = -\frac{1}{p_0 - (q_0-\gamma) - p_{\mathcal{U}}} + \frac{1}{p_{\mathcal{U}} + \frac{\left(2+\frac{\beta\gamma}{1-\beta}\right)(1-q_0) - \left(\frac{1}{\underline{\eta}}-1\right)p_0}{\frac{1-\beta}{\beta\gamma}+\frac{1}{\underline{\eta}}}},$$

which is decreasing in $p_{\mathcal{U}}$. Then, we show the observation by considering the following two possible types of cases:

1. In cases 1 and 4, where E3 occurs before E4 as we increase τ, we show that such $\hat{\tau}_3$ and $\hat{\tau}_4$ do not exist simultaneously. We show this by the way of contradiction. Suppose such $\hat{\tau}_3$ and $\hat{\tau}_4$ exist. Thus, we have $\hat{\tau}_3 < \hat{\tau}_4$, $\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) = -\frac{h\left(p_{\mathcal{U}}(\hat{\tau}_3)\right)}{h'\left(p_{\mathcal{U}}(\hat{\tau}_3)\right)}$, and $\hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4) = -\frac{h\left(p_{\mathcal{U}}(\hat{\tau}_4)\right)}{h'\left(p_{\mathcal{U}}(\hat{\tau}_4)\right)}$. Moreover, we show that $p'_{\mathcal{U}}(\hat{\tau}_3) < p'_{\mathcal{U}}(\hat{\tau}_4)$, which can be rewritten as (by Lemma 3.6.4) $\frac{b_0}{(1-\hat{\tau}_3)^2} < \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} \cdot \frac{w_0}{(1-\hat{\tau}_4)^2}$. To show the last inequality holds, it suffices to show that

$$b_0 \le \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0. \tag{3.40}$$

We also notice that $b_0 < \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$ in cases 1 and 4. Then, to show (3.40), it suffices to show $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 < \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0$. The last inequality is equivalent to $p_0 < \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, which holds in cases 1 and

4. Therefore, we have shown $p'_{\mathcal{U}}(\hat{\tau}_3) < p'_{\mathcal{U}}(\hat{\tau}_4)$. Given the last inequality, we obtain $\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) < \hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4)$ and $-\frac{h'(p_{\mathcal{U}}(\hat{\tau}_3))}{h(p_{\mathcal{U}}(\hat{\tau}_3))} > -\frac{h'(p_{\mathcal{U}}(\hat{\tau}_4))}{h(p_{\mathcal{U}}(\hat{\tau}_4))}$, which results in a contradiction. Therefore, such $\hat{\tau}_3$ and $\hat{\tau}_4$ do not exist simultaneously.

2. In case 7, where E4 occurs before E3 as we increase τ, we show that such $\hat{\tau}_3$ and $\hat{\tau}_4$ do not exist simultaneously. We show this by the way of contradiction. Suppose such $\hat{\tau}_3$ and $\hat{\tau}_4$ exist. Thus, we have $\hat{\tau}_3 > \hat{\tau}_4$, $\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) = -\frac{h\left(p_{\mathcal{U}}(\hat{\tau}_3)\right)}{h'\left(p_{\mathcal{U}}(\hat{\tau}_3)\right)}$, and $\hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4) = -\frac{h\left(p_{\mathcal{U}}(\hat{\tau}_4)\right)}{h'\left(p_{\mathcal{U}}(\hat{\tau}_4)\right)}$. Moreover, we show that $p'_{\mathcal{U}}(\hat{\tau}_3) > p'_{\mathcal{U}}(\hat{\tau}_4)$, which can be rewritten as (by Lemma 3.6.4) $\frac{b_0}{(1-\hat{\tau}_3)^2} > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} \cdot \frac{w_0}{(1-\hat{\tau}_4)^2}$. To show the last inequality holds, it suffices to show that

$$b_0 \geq \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0. \tag{3.41}$$

We also notice that $b_0 > \frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0$ in case 7. Then, to show (3.41), it suffices to show $\frac{p_0-(q_0-\gamma)}{p_0+\frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)(\frac{2}{\underline{\eta}}-1)} w_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\frac{1}{\underline{\eta}}} w_0$. The last inequality is equivalent to $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta} \cdot \frac{1-q_0}{\frac{1}{\underline{\eta}}-1}$, which holds in case 7. Therefore, we have shown $p'_{\mathcal{U}}(\hat{\tau}_3) < p'_{\mathcal{U}}(\hat{\tau}_4)$. Given the last inequality, we obtain $\hat{\tau}_3 p'_{\mathcal{U}}(\hat{\tau}_3) > \hat{\tau}_4 p'_{\mathcal{U}}(\hat{\tau}_4)$ and $-\frac{h'(p_{\mathcal{U}}(\hat{\tau}_3))}{h(p_{\mathcal{U}}(\hat{\tau}_3))} < -\frac{h'(p_{\mathcal{U}}(\hat{\tau}_4))}{h(p_{\mathcal{U}}(\hat{\tau}_4))}$, which results in a contradiction. Therefore, such $\hat{\tau}_3$ and $\hat{\tau}_4$ do not exist simultaneously.

Given the above observations, we can characterize $\tau^*(0)$ and the equilibrium type under $\tau^*(0)$ in each case:

1. In case 1, it is straightforward to show that if $\tau_4^* < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma} w_0 - (\frac{1}{\underline{\eta}} + \frac{1-\beta}{\beta\gamma}) b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0) - (\frac{1}{\underline{\eta}}-1) p_0}$ is equivalent to

$$b_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}} \Big(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0) - (\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0) + (1-\frac{1}{2}\underline{\eta})(1-q_0)}\Big) w_0. \tag{3.42}$$

Based on the third observation, if (3.42) holds, then $\tau^*(0)$ is determined by (3.34) and an E3 occurs under $\tau^*(0)$. Otherwise, $\tau^*(0) = \tau_4^*$ and an E4 occurs under $\tau^*(0)$.

2. The characterization of case 2 is straightforward so we omit the proof for brevity.

3. We establish (3.35) as the characterization of $\tau^*(0)$ in case 3 based on the first and the second observations. Then, it is straightforward to show that $\tau_4^* \leq 1 - \frac{w_0}{p_0 + \frac{\left(2(1-\beta)+\beta\gamma\right)(1-\beta)}{(1-\beta+\beta\gamma)^2}(q_0-\gamma)(\frac{1}{\underline{\eta}}-1)}$ is equivalent to (3.36). Therefore, if (3.36) holds, the platform revenue is decreasing in τ in the commission interval associated with E4 (i.e., case 3-(c) in Proposition 3.6.1), and an E1 holds under $\tau^*(0)$. Otherwise, an E4 holds.

4. In case 4, the proof is similar to case 1. If (3.42) holds, then $\tau_4^* < 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}$. Therefore, $\tau^*(0)$ is obtained by (3.37), and an E2 or an E3 occurs under $\tau^*(0)$. Otherwise, we have $\tau^*(0) = \tau_4^*$ and an E4 occurs.

5. We can show case 5 based on the first observation. We omit the proof for brevity.

6. We can show case 6 based on the first and second observation. Since the proof is similar to case 3, we omit the proof for brevity.

7. In case 7, based on the third observation, we only need to search for $\tau^*(0)$ when an E1 or an E4 occurs if $\tau_4^* \leq 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0 - (\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}$, which is equivalent

to $b_0 \leq \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\left(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0)+(1-\frac{1}{2}\underline{\eta})(1-q_0)}\right)w_0$. In this case, $\tau^*(0)$ is given by (3.35), and we can determine whether an E1 or an E4 occurs under $\tau^*(0)$ based on (3.36) (similar to the proof of case 3). On the other hand, we can show $\tau^*(0)$ is characterized by (3.39) if $\tau_4^* > 1 - \frac{\frac{1-\beta+\beta\gamma}{\beta\gamma}w_0-(\frac{1}{\underline{\eta}}+\frac{1-\beta}{\beta\gamma})b_0}{\frac{2(1-\beta)+\beta\gamma}{1-\beta}(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}$, which is equivalent to $b_0 > \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma/\underline{\eta}}\left(1 - \frac{(2+\frac{\beta\gamma}{1-\beta})(1-q_0)-(\frac{1}{\underline{\eta}}-1)p_0}{\frac{1-\beta+\beta\gamma}{2\beta\gamma}(p_0+w_0)+(1-\frac{1}{2}\underline{\eta})(1-q_0)}\right)w_0$. By all three observations, we can show that in this case $\tau^*(0)$ is characterized by (3.39) and an E3 occurs under $\tau^*(0)$.

8. We can show case 8 based on the first observation. We omit the proof for brevity.

Therefore, we have completed the characterization of $\tau^*(0)$ as well as the corresponding equilibrium type in each case. □

Lemma 3.6.5 *Suppose an E1 holds under $(\tau^*(0), 0)$ and $\underline{\eta} < 1$. Then, $\lambda^* > 0$. Besides, the optimal delay policy results in lower $\delta_{\mathcal{H}}^H$ compared the full-information policy with $\tau^*(0)$.*

Proof. In this proof, we first show that $\lambda^* > 0$ by finding a policy with $\lambda > 0$, which results in an E1 with higher platform revenue than that under $(\tau^*(0), 0)$. We then show that for any policy with $\lambda > 0$, the corresponding quantity of providers with label $\mathcal{H}$ is less than $\frac{\beta\gamma}{1-\beta+\beta\gamma}$, which is the quantity of providers with label $\mathcal{H}$ under $\lambda = 0$ and E1.

First, we notice that the platform revenue in E1 is $\frac{\tau}{1-\tau}w_0$ by Lemma 3.6.3. Therefore, within E1, the conditional optimal τ given λ is the maximum τ such that

E1 holds. By Lemma 3.6.3, to show an E1 holds, we need to check two conditions: $p_{\mathcal{U}} \leq p_0 - \big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$ and $(1-\tau)p_{\mathcal{U}} \geq b_0$. Next, we show that at the optimal commission given λ, denoted by $\tau^*(\lambda)$, the first condition is binding while the second condition is not binding. In particular, it is straightforward to verify that the left-handed side of both conditions (i.e., $p_{\mathcal{U}}$ and $(1-\tau_{\mathcal{U}})p_{\mathcal{U}}$) are increasing in τ. Therefore, given λ, $\tau^*(\lambda)$ is solved from $p_{\mathcal{U}} = p_0 - \big(\frac{1}{1+\beta\gamma\lambda} + \frac{1-\beta}{1-\beta+\beta\gamma}\big)\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$, which is equivalent to

$$\frac{w_0}{1-\tau} = p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\Big(\frac{1}{\underline{\eta}} - 1\Big)\Big(1 + \frac{1-\beta}{1-\beta+\beta\gamma}(1+\beta\gamma\lambda)\Big),$$

from which we solve for $\tau^*(\lambda) = 1 - \frac{w_0}{p_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\Big(\frac{1}{\underline{\eta}}-1\Big)\Big(1+\frac{1-\beta}{1-\beta+\beta\gamma}(1+\beta\gamma\lambda)\Big)}$. Besides, we notice that $\tau^*(\lambda)$ is increasing in λ. In other words, if both policies $(\tau^*(\lambda), \lambda)$, where $\lambda > 0$, and $(\tau^*(0), 0)$ result in an E1, then $\tau^*(\lambda) > \tau^*(0)$. Hence, the first policy with delay leads to a higher platform revenue than the second policy, which is the full-information policy with the optimal commission. To show such $(\tau^*(\lambda), \lambda)$ policy exists, it suffices to show that condition $(1-\tau)p_{\mathcal{U}} \geq b_0$ holds under the given policy. It is straightforward to verify that the left-handed side of the condition is increasing in λ. Therefore, if policy $(\tau^*(0), 0)$ satisfies the condition, then policy $(\tau^*(\lambda), \lambda)$ will satisfy the condition. In sum, we have establish that $\lambda^* > 0$ under the given conditions.

Second, we show that any policy with a positive delay (i.e., $\lambda > 0$) results in fewer providers with label $\mathcal{H}$ compared a full-information, which results in an E1. We let $\delta_{\mathcal{H}}^{H+}$ and $\delta_{\mathcal{H}}^{H0}$ denote the quantity of providers with label $\mathcal{H}$ under the given delay policy and the given full-information policy, respectively. By Lemma 3.6.3, we have $\delta_{\mathcal{H}}^{H0} = \frac{\beta\gamma}{1-\beta+\beta\gamma}$. Then, we show $\delta_{\mathcal{H}}^{H+} < \delta_{\mathcal{H}}^{H0}$ holds, when E1, E2, E3, and E4 occurs under the given delay policy ($\lambda > 0$):

1. If an E1 holds under the given delay policy, then $\delta_{\mathcal{H}}^{H+} = \frac{\beta\gamma}{1-\beta+\beta\gamma}\big(1-(1-\beta)\lambda\big)$ (by Lemmas 3.6.1 and 3.6.3). Therefore, $\delta_{\mathcal{H}}^{H+} < \delta_{\mathcal{H}}^{H0}$.

2. If an E2 holds under the given delay policy, then by Lemma 3.6.3 the quantity of hired new providers is $\frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta}-\beta\gamma\lambda(1-\eta)} < \frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta}-\beta\gamma\lambda}$. The last inequality holds because $\eta > 0$. Then, by the characterization of $\delta_{\mathcal{H}}^{H}$ in Lemma 3.6.1, we obtain $\delta_{\mathcal{H}}^{H+} < \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta}-\beta\gamma\lambda}$. Then, it is straightforward to show that the right-handed side of the last inequality is decreasing in λ, which hence is maximized at $\lambda = 0$. Therefore, $\delta_{\mathcal{H}}^{H+} < \frac{\beta\gamma}{1-\beta+\beta\gamma} = \delta_{\mathcal{H}}^{H0}$.

3. If an E3 holds under the given delay policy, then by Lemma 3.6.3 the quantity of hired new providers is less than $\frac{1}{\frac{1-\beta+\beta\gamma}{1-\beta}-\beta\gamma\lambda(1-\eta)}$. By following the same proof as in case E2, we can show $\delta_{\mathcal{H}}^{H+} < \delta_{\mathcal{H}}^{H0}$.

4. If an E4 holds under the given delay policy, then by Lemma 3.6.3, the total quantity of hired providers is less than 1. That is, $(1+\beta\gamma\lambda)\delta_{\mathcal{U}}^{U} + \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^{U} < 1$, which is equivalent to $\delta_{\mathcal{U}}^{U} < \frac{1-\beta}{1-\beta+\beta\gamma}$. Then, by the characterization of $\delta_{\mathcal{H}}^{H}$ in Lemma 3.6.1, we obtain $\delta_{\mathcal{H}}^{H+} < \delta_{\mathcal{H}}^{H0}$.

Therefore, we have shown that a full-information policy, which results in an E1, have more providers with label $\mathcal{H}$ than an policy with a positive delay. $\square$

3.6.2 Differentiated commissions

In this sections, we introduce technical lemmas and proposition for the case of differentiated commissions.

Lemma 3.6.6 *The revenue maximization problem over informational delay and general commission rates can be formulated as*

$$\max_{\tau_{\mathcal{U}},\tau_{\mathcal{H}},\lambda\geq 0} \tau_{\mathcal{U}} p_{\mathcal{U}}(\delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H) + \tau_{\mathcal{H}} p_{\mathcal{H}} \delta_{\mathcal{H}}^H$$

$$s.t.$$

$$\textit{(Steady state condition) } (3.6),\ \delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H + \delta_{\mathcal{H}}^H \leq 1$$

$$\textit{(Expected quality of unrevealed providers) } q_{\mathcal{U}} = \frac{\delta_{\mathcal{U}}^U \gamma + \eta\delta_{\mathcal{U}}^H}{\delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H},$$

$$\textit{(Price of newcomers) } p_{\mathcal{U}} = \begin{cases} p_0 - (1 + \delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H)(q_0 - q_{\mathcal{U}}), & q_{\mathcal{U}} \leq q_0 \\ p_0 + (2 - \delta_{\mathcal{U}}^U - \eta\delta_{\mathcal{U}}^H - \delta_{\mathcal{H}}^H)(q_{\mathcal{U}} - q_0), & q_{\mathcal{U}} > q_0 \end{cases}$$

$$\textit{(Price of } \mathcal{H}\textit{-label providers) } p_{\mathcal{H}} = \begin{cases} p_0 + (2 - \delta_{\mathcal{H}}^H)(1 - q_0), & q_{\mathcal{U}} \leq q_0 \\ p_{\mathcal{U}} + (2 - \delta_{\mathcal{H}}^H)(1 - q_{\mathcal{U}}), & q_{\mathcal{U}} > q_0 \end{cases}$$

$$\textit{(Free-entry condition) } (3.9)$$

$$\textit{(Stay of } \mathcal{H}\textit{-label providers) } (1 - \tau_{\mathcal{H}}) p_{\mathcal{H}} \geq w_0$$

$$\textit{(Financial constraint) } (1 - \tau_{\mathcal{U}}) p_{\mathcal{U}} \geq b_0$$

Proof. In this proof, we only focus on the characterization the price functions of unrevealed and $\mathcal{H}$-label providers as other components are straightforward to verify. We consider the four matching structures in the equilibrium definition in Section 3.3, separately. First of all, we notice that $\zeta_{\mathcal{U}} = \delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H$ and $\zeta_{\mathcal{H}} = \delta_{\mathcal{H}}^H$, which are the market clearing conditions.

1. In case 1 ($q_{\mathcal{U}} < q_0$), we have $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0} - 1$ and $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_0}{1 - q_0}$. Then by the market-clearing conditions, we obtain $p_{\mathcal{U}} = p_0 - (1 + \delta_{\mathcal{U}}^U + \eta \delta_{\mathcal{U}}^H)(q_0 - q_{\mathcal{U}})$ and $p_{\mathcal{H}} = p_0 + (2 - \delta_{\mathcal{H}}^H)(q_H - q_0)$.

2. In case 2 ($q_{\mathcal{U}} \leq q_0$), we have $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}} - 1$ and $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}}$. Then by the market clearing conditions, we obtain $p_{\mathcal{H}} = p_{\mathcal{U}} + (2 - \delta_{\mathcal{H}}^H)(q_H - q_{\mathcal{U}})$. Besides, customer with sensitivity $\zeta_{\mathcal{U}} + 1$ (weakly) prefer the providers with label $\mathcal{U}$ to the outside option (i.e., $(\zeta_{\mathcal{U}} + 1)q_{\mathcal{U}} - p_{\mathcal{U}} \geq (\zeta_{\mathcal{U}} + 1)q_0 - p_0$ or, equivalently, $p_{\mathcal{U}} \leq p_0 - (\zeta_{\mathcal{U}} + 1)(q_0 - q_{\mathcal{U}})$). Next, we show that $p_{\mathcal{U}} < p_0 - (\zeta_{\mathcal{U}} + 1)(q_0 - q_{\mathcal{U}})$ is suboptimal. If the inequality holds, the platform can increase the revenue by raising the commissions. In particular, by (3.9), increasing $\tau_{\mathcal{H}}$ or $\tau_{\mathcal{U}}$ or both can increase $p_{\mathcal{U}}$ given all other equilibrium outcomes fixed. In other words, if $p_{\mathcal{U}} < p_0 - (\zeta_{\mathcal{U}} + 1)(q_0 - q_{\mathcal{U}})$ holds, the platform can always increase its revenue by slightly increasing either commission. Therefore, if the optimal policy leads to the matching structure specified in case 2, equality $p_{\mathcal{U}} = p_0 - (\zeta_{\mathcal{U}} + 1)(q_0 - q_{\mathcal{U}})$ should hold. Therefore, we obtain $p_{\mathcal{U}} = p_0 - (1 + \delta_{\mathcal{U}}^U + \eta \delta_{\mathcal{U}}^H)(q_0 - q_{\mathcal{U}})$ and $p_{\mathcal{H}} = p_0 + (2 - \delta_{\mathcal{H}}^H)(1 - q_0)$.

3. In this case ($q_{\mathcal{U}} \geq q_0$), we have the same matching structure as case 2. Thus, we have $p_{\mathcal{H}} = p_{\mathcal{U}} + (2 - \delta_{\mathcal{H}}^H)(q_H - q_{\mathcal{U}})$. Besides, customer with sensitivity 1 (weakly) prefers unrevealed providers over the outside option (i.e., $q_{\mathcal{U}} - p_{\mathcal{U}} \geq q_0 - p_0$ or, equivalently, $p_{\mathcal{U}} \leq p_0 + q_{\mathcal{U}} - q_0$). Follow the same argument in case 2, if the optimal policy leads to the matching structure specified by case

3, we have $p_{\mathcal{U}} = p_0 + q_{\mathcal{U}} - q_0$. Therefore, $p_{\mathcal{H}} = p_{\mathcal{U}} + (2 - \delta_{\mathcal{H}}^H)(q_H - q_{\mathcal{U}})$, and $p_{\mathcal{U}} = p_0 + (2 - \delta_{\mathcal{U}}^U - \eta\delta_{\mathcal{U}}^H - \delta_{\mathcal{H}}^H)(q_{\mathcal{U}} - q_0)$, where $\delta_{\mathcal{U}}^U + \eta\delta_{\mathcal{U}}^H + \delta_{\mathcal{H}}^H = 1$.

4. In this case ($q_{\mathcal{U}} > q_0$), we have $\zeta_{\mathcal{U}} = \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}} - \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}$ and $\zeta_{\mathcal{H}} = 2 - \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{1 - q_{\mathcal{U}}}$. By the market clearing conditions, we obtain $p_{\mathcal{H}} = p_{\mathcal{U}} + (2 - \delta_{\mathcal{H}}^H)(q_H - q_{\mathcal{U}})$ and $p_{\mathcal{U}} = p_0 + (2 - \delta_{\mathcal{U}}^U - \eta\delta_{\mathcal{U}}^H - \delta_{\mathcal{H}}^H)(q_{\mathcal{U}} - q_0)$.

Therefore, we have formulated the platform's revenue maximization problem using information provision policy and differentiated commissions. □

Lemma 3.6.7 *Suppose $\gamma < \frac{1-\beta}{\beta}$ holds. Then, the optimal information-commission policy results in no rationing (i.e., $\eta = 1$) in equilibrium. Moreover, under the full information (i.e., $\lambda = 0$), the optimal commission scheme results in no rationing in equilibrium.*

Proof. It suffices to show that given a policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$, which leads to $\eta < 1$ in equilibrium, we can always find another policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$, which leads to a strictly higher platform revenue.

First, by (3.9), we can re-write the platform revenue as:

$$\pi_r = \delta_{\mathcal{U}}^U \left(\frac{\beta\gamma}{1-\beta} \big(1 - (1-\beta)\lambda\big) p_{\mathcal{H}} + (1 + \beta\gamma\lambda\eta) p_{\mathcal{U}} - \left(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta} \right) w_0 \right). \quad (3.43)$$

Note that (3.43) is independent of the commissions. Then, we consider the following two cases:

- $q_{\mathcal{U}} \leq q_0$. In this case, by the characterization of $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ from Lemma 3.6.6, π_r can be further written as:

$$\pi_r = \delta_{\mathcal{U}}^U \Bigg(\Big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda(1-\eta) \Big) p_0 - \big(q_0 - \gamma - \beta\gamma\lambda\eta(1-q_0) \big) + \frac{2\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)(1-q_0) - \Big(\frac{\beta\gamma}{1-\beta} + \frac{1}{\eta} \Big) w_0 - \Big(\frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big) \Big)^2 (1-q_0)\delta_{\mathcal{U}}^U - (1+\beta\gamma\lambda\eta)\big(q_0-\gamma-\beta\gamma\lambda\eta(1-q_0)\big)\delta_{\mathcal{U}}^U \Bigg). \tag{3.44}$$

Given λ, η, and $\delta_{\mathcal{U}}^U$, it is evident that π_r is fully determined (i.e., it is independent of $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$). Noticeably, we can show that $\frac{\partial \pi_r}{\partial \eta} > 0$. To see this, $\frac{\partial \pi_r}{\partial \eta}$ has the same sign as:

$$\beta\gamma\lambda p_0 + \beta\gamma\lambda(1-q_0) + 2(\beta\gamma\lambda)^2(1-q_0)\eta\delta_{\mathcal{U}}^U + \beta\gamma\lambda(1+\gamma-2q_0)\delta_{\mathcal{U}}^U + \frac{w_0}{\eta^2} > 0 \tag{3.45}$$

The last inequality holds as $1+\gamma-2q_0 > 0$ (note that $\underline{\eta} < 1$ and $\gamma < \frac{1-\beta}{\beta}$).

Then, given $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$, which results in $\delta_{\mathcal{U}}^U$, η and $p_{\mathcal{U}}$, we let $\tilde{\lambda} = \lambda$, and we choose $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that the following holds:

$$\frac{\beta\gamma}{1-\beta}\frac{1-(1-\beta)\lambda}{1+\beta\gamma\lambda}\big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where $\tilde{p}_{\mathcal{H}} = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U\Big)(1-q_0)$, and $\tilde{p}_{\mathcal{U}} = p_0 - \delta_{\mathcal{U}}^U\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big)$. Therefore, under policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$, the resulting $\tilde{\delta}_{\mathcal{U}}^U = \delta_{\mathcal{U}}^U$, yet, $\tilde{\eta} = 1$. Besides, it is straightforward to verify that $(1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} \geq w_0$ and $w_0 \geq (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}} \geq b_0$. Lastly, by (3.45), we show that policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$ strictly outperforms policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$ in revenue.

- $q_{\mathcal{U}} > q_0$. In this case, we first show that given a policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$ with $\delta_{\mathcal{U}}^U$ and η in equilibrium, we can find an alternative policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$ such that its equilibrium outcomes satisfy $\tilde{\delta}_{\mathcal{U}}^U = \delta_{\mathcal{U}}^U$, $\tilde{\eta} = 1$, and $\tilde{\lambda} = \lambda\eta$. Note that if we can find such policy, it will have the same expected quality of unrevealed providers (i.e., $\tilde{q}_{\mathcal{U}} = q_{\mathcal{U}}$). In particular, we choose $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that:

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-(1-\beta)\tilde{\lambda}}{1+\beta\gamma\tilde{\lambda}}\big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where $\tilde{p}_{\mathcal{U}} = p_0 + \big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U\big)(q_{\mathcal{U}} - q_0)$ and $\tilde{p}_{\mathcal{H}} = \tilde{p}_{\mathcal{U}} + \big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\tilde{\lambda}\big)\delta_{\mathcal{U}}^U\big)(1-q_{\mathcal{U}})$. It is straightforward to show that we can solve for $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ from the above two equations.

Next, we show that the alternative policy leads to a higher platform revenue. We let $u \triangleq \lambda\eta = \tilde{\lambda}\tilde{\eta} = \tilde{\lambda}$, which is the same under both policies. Then, given u, we can write down the platform revenue as (by (3.43) and Lemma 3.6.6):

$$\pi_r(\lambda) =$$
$$\delta_{\mathcal{U}}^U\Bigg(\Big(\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda + \beta\gamma u\Big)\Big(p_0 + \big(2 - (\frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda + \beta\gamma u)\delta_{\mathcal{U}}^U\big)(q_{\mathcal{U}} - q_0)\Big)$$
$$+ \frac{2\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)(1-q_{\mathcal{U}}) - \Big(\frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\Big)^2(1-q_{\mathcal{U}})\delta_{\mathcal{U}}^U - \big(\frac{\beta\gamma}{1-\beta} + \frac{\lambda}{u}\big)w_0\Bigg). \tag{3.46}$$

To simplify the notation, let $A(\lambda) \triangleq \frac{1-\beta+\beta\gamma}{1-\beta} - \beta\gamma\lambda + \beta\gamma u$, $B(\lambda) \triangleq \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)$, $a \triangleq q_{\mathcal{U}} - q_0 = \frac{\beta\gamma u(1-q_0)-(q_0-\gamma)}{1+\beta\gamma u} \geq 0$, and $b \triangleq 1 - q_{\mathcal{U}} = \frac{1-\gamma}{1+\beta\gamma u} \geq 0$. Then, the revenue function becomes:

$$\pi_r(\lambda) = \delta_{\mathcal{U}}^U \Bigg(A(\lambda)\Big(p_0 + \big(2 - A(\lambda)\delta_{\mathcal{U}}^U\big)a\Big) + 2B(\lambda)b - B^2(\lambda)b\delta_{\mathcal{U}}^U - \Big(\frac{\beta\gamma}{1-\beta} + \frac{\lambda}{u}\Big)w_0 \Bigg).$$

Notice that u and $\delta_{\mathcal{U}}^U$ are fixed while we change the policy following the above rules, it suffices to show that $\pi_r(\lambda)$ is decreasing in λ given u and $\delta_{\mathcal{U}}^U$. Besides, it is straightforward to verify that $\frac{\partial \pi_r}{\partial \lambda}$ has the same sign as

$$-p_0 - 2a\big(1 - A(\lambda)\delta_{\mathcal{U}}^U\big) - 2b\big(1 - B(\lambda)\delta_{\mathcal{U}}^U\big) - \frac{w_0}{\beta\gamma u} < 0.$$

The above inequality holds as $A(\lambda)\delta_{\mathcal{U}}^U = \delta_{\mathcal{U}} + \delta_{\mathcal{H}}^H \leq 1$ and $B(\lambda)\delta_{\mathcal{U}}^U = \delta_{\mathcal{H}}^H < 1$. In other words, we have shown that $\frac{\partial \pi_r}{\partial \lambda} < 0$ and the proposed policy without rationing results in a higher platform revenue than the original policy with rationing. Therefore, we conclude that in the case $q_{\mathcal{U}} \geq q_0$, the policy with $\eta < 1$ is suboptimal.

In sum, any general commission scheme with $\eta < 1$ is suboptimal. Besides, the above argument applies in the set of full-information policies. □

In Lemma 3.6.8, we characterize the quantity of hired new providers resulted from the optimal general commission scheme given an informational delay.

Lemma 3.6.8 *Suppose $\gamma < \frac{1-\beta}{\beta}$ holds. Then, given $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$, the number of hired new providers under the optimal commission scheme is*

$$\delta_{\mathcal{U}}^{U*}(\lambda) = \min\left(\frac{1}{2} \cdot \frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - \beta\gamma\lambda(1-q_0)}{q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0) - \beta\gamma\lambda\big(1 + \gamma - 2q_0 + \frac{2\beta\gamma}{1-\beta}(1-q_0)\big)}, \frac{1-\beta}{1-\beta+\beta\gamma} \right). \tag{3.47}$$

Besides, $\delta_{\mathcal{U}}^{U*}(\lambda)$ is increasing in $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$.

Proof. By Lemma 3.6.7 (under $\gamma < \frac{1-\beta}{\beta}$), we only need to consider the equilibrium with $\eta = 1$. For $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$, we have $q_{\mathcal{U}} \leq q_0$. From (3.44), we note that π_r can be rewritten as:

$$\begin{aligned}
\pi_r(\delta_{\mathcal{U}}^U, \lambda) =& \delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - \big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big) + \frac{2\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)(1-q_0) \\
& - \Big((1+\beta\gamma\lambda)\big(q_0 - \gamma - \beta\gamma\lambda(1-q_0)\big) + \big(\frac{\beta\gamma}{1-\beta} - \beta\gamma\lambda\big)^2(1-q_0) \Big)\delta_{\mathcal{U}}^U \Bigg) \\
=& \delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0) - \beta\gamma\lambda(1-q_0) \\
& - \Big(q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0) - \beta\gamma\lambda\big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\big)\Big)\delta_{\mathcal{U}}^U \Bigg),
\end{aligned} \tag{3.48}$$

which is maximized by $\delta_{\mathcal{U}}^{U*}(\lambda)$ characterized in Lemma 3.6.8.

Given $\delta_{\mathcal{U}}^{U*}(\lambda)$, we can find corresponding $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$ such that the equilibrium with $\delta_{\mathcal{U}}^{U*}(\lambda)$ holds. In particular, we can verify that $\tau_{\mathcal{U}}^* = 1 - \frac{w_0}{p_{\mathcal{U}}^*}$ and $\tau_{\mathcal{H}}^* = 1 - \frac{w_0}{p_{\mathcal{H}}^*}$, where $p_{\mathcal{U}}^* = p_0 - \big(1 + (1+\beta\gamma\lambda)\delta_{\mathcal{U}}^{U*}(\lambda)\big)(q_0 - q_{\mathcal{U}})$ and $p_{\mathcal{H}}^* = p_0 + \Big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^{U*}(\lambda)\Big)(1-q_0)$ satisfy the free-entry condition, stay of $\mathcal{H}$-label providers, and the financial constraint.

Next, to show that $\delta_{\mathcal{U}}^{U*}(\lambda)$ is increasing in λ, it suffices to show that it holds when $\delta_{\mathcal{U}}^{U*}(\lambda) < \frac{1-\beta}{1-\beta+\beta\gamma}$. In this case, we note that $\frac{d\delta_{\mathcal{U}}^{U*}}{d\lambda}(\lambda)$ has the same sign as:

$$
\begin{aligned}
&\Big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\Big)\Big(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)\Big)\\
&-(1-q_0)\Big(q_0-\gamma+\big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0)\Big)\\
>&\big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\big)\\
&\cdot\Big(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)+\frac{\beta\gamma}{1-\beta}(1-q_0)\Big)-(1-q_0)\Big(q_0-\gamma+\big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0)\Big)\\
=&\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)\big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\big)\\
&+\frac{1-\beta+\beta\gamma}{1-\beta}(1-q_0)\big(\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)\big)\\
>&\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)\big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\big)\\
>&0.
\end{aligned}
$$

The third and second to last inequalities hold because $\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)>0$ (Assumption 1). □

Proposition 3.6.3 characterizes the optimal full-information policy with differentiated commissions.

Proposition 3.6.3 *Suppose $\gamma < \frac{1-\beta}{\beta}$ holds. Then, the optimal general commissions under the full-information policy, $\tau_{\mathcal{U}}^*(0)$ and $\tau_{\mathcal{H}}^*(0)$, can be characterized as:*

$$\tau_{\mathcal{U}}^*(0)=1-\frac{w_0}{p_{\mathcal{U}}^*(0)}\ \textit{and}\ \tau_{\mathcal{H}}^*(0)=1-\frac{w_0}{p_{\mathcal{H}}^*(0)}, \tag{3.49}$$

where

$$\delta_{\mathcal{U}}^{U*}(0)=\min\left(\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)}{2\Big(q_0-\gamma+\big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0)\Big)},\frac{1-\beta}{1-\beta+\beta\gamma}\right), \tag{3.50}$$

$$p_{\mathcal{U}}^*(0)=p_0-\big(1+\delta_{\mathcal{U}}^{U*}(0)\big)(q_0-\gamma),\ \textit{and}\ p_{\mathcal{H}}^*(0)=p_0+\Big(2-\frac{\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^{U*}(0)\Big)(1-q_0).$$

Besides, $\tau_{\mathcal{U}}^*(0) < \tau_{\mathcal{H}}^*(0)$.[7]

Proof. First, by Lemma 3.6.8 (under $\gamma < \frac{1-\beta}{\beta}$) and $\lambda = 0$, it is straightforward to verify that $\delta_{\mathcal{U}}^{U*}(0)$ is characterized by (3.50).

Next, we characterize the optimal commissions given $\lambda = 0$ (i.e., $\tau_{\mathcal{U}}^*(0)$ and $\tau_{\mathcal{H}}^*(0)$). The optimal commissions may not be unique as they can result in the same equilibrium. Among the equivalent commission pairs, we choose the commission pair with the largest $\tau_{\mathcal{H}}$. In particular, given an equilibrium with $p_{\mathcal{U}}$ and $p_{\mathcal{H}}$ under λ, the following conditions hold at the optimal commissions:

$$(1 - \tau_{\mathcal{U}})p_{\mathcal{U}} = w_0 \text{ and } (1 - \tau_{\mathcal{H}})p_{\mathcal{H}} = w_0. \tag{3.51}$$

Notice that $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{H}}$ determined by the above conditions satisfy the free-entry condition ((3.9)). On the other hand, by the condition that providers with label $\mathcal{H}$ stay on the platform in Lemma 3.6.6 (i.e., $(1-\tau_{\mathcal{H}})p_{\mathcal{H}} \geq w_0$), any higher $\tau_{\mathcal{H}}$ becomes infeasible.

Lastly, by (3.51), we obtain (3.49). Moreover, given $\lambda = 0$ and $\eta = 1$, $p_{\mathcal{U}}^*(0)$ and $p_{\mathcal{H}}^*(0)$ in (3.49) are characterized by Lemma 3.6.6. □

In Lemma 3.6.9, we provide a sufficient condition under which the platform serves all customers under the full information with the optimal differentiated commissions.

---

[7]The optimal commissions are not unique and we choose the pair with the largest $\tau_{\mathcal{H}}$. In the following analysis, if there are multiple optimal commission pairs, we present the pair with the largest $\tau_{\mathcal{H}}$.

Lemma 3.6.9 *Suppose* $\gamma < \frac{1-\beta}{\beta}$ *and* $\underline{\eta} < \frac{1}{2}$*. Then,* $\delta_{\mathcal{U}}^{U*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$*, where* $\delta_{\mathcal{U}}^{U*}(0)$ *is characterized by* (3.50)*.*

Proof. By (3.50), to show $\delta_{\mathcal{U}}^{U*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$, it is equivalent to show

$$\frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1 - q_0)}{2\big(q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1 - q_0)\big)} \geq \frac{1-\beta}{1-\beta+\beta\gamma}.$$

It is straightforward to verify that the above inequality is equivalent to

$$\frac{1-\beta+\beta\gamma}{1-\beta}\frac{p_0 - w_0}{q_0 - \gamma} + \frac{2(1-\beta)}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}} \geq 1 + \frac{2(1-\beta)}{1-\beta+\beta\gamma}. \tag{3.52}$$

Notice that $p_0 \geq w_0$. Therefore, to show (3.52) holds, it suffices to show $\frac{2(1-\beta)}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}} \geq 1 + \frac{2(1-\beta)}{1-\beta+\beta\gamma}$, which is equivalent to $\underline{\eta} \leq \frac{2(1-\beta)}{3(1-\beta)+\beta\gamma}$. Then, the last inequality holds given $\gamma < \frac{1-\beta}{\beta}$ and $\underline{\eta} < \frac{1}{2}$. □

In Lemma 3.6.10, we establish that all customers choose the platform under the optimal differentiated commissions given $\lambda = \frac{q_0 - q_N}{\beta\gamma(1-q_0)}$.

Lemma 3.6.10 *Suppose* $\gamma < \frac{1-\beta}{\beta}$ *holds. Then, we have* $\delta_{\mathcal{U}}^{U*}\big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big) = \frac{1-\beta}{1-\beta+\beta\gamma}$*, where* $\delta_{\mathcal{U}}^{U*}(\lambda)$ *is characterized in Lemma 3.6.8.*

Proof. Given the characterization of $\delta_{\mathcal{U}}^{U*}(\lambda)$ in Lemma 3.6.8 (which holds under $\gamma < \frac{1-\beta}{\beta}$), it is equivalent to show:

$$\frac{1}{2} \cdot \frac{\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1 - q_0) - (q_0 - \gamma)}{q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1 - q_0) - \frac{q_0-\gamma}{1-q_0}\big(1 + \gamma - 2q_0 + \frac{2\beta\gamma}{1-\beta}(1 - q_0)\big)} \geq \frac{1-\beta}{1-\beta+\beta\gamma}.$$

The above inequality can be rewritten as:

$$
\begin{aligned}
&\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)+\frac{2\beta\gamma}{1-\beta}(1-q_0)-2(q_0-\gamma)\\
\geq&\frac{2(1-\beta)}{1-\beta+\beta\gamma}\left(q_0-\gamma+\Big(\frac{\beta\gamma}{1-\beta}\Big)^2(1-q_0)-\frac{q_0-\gamma}{1-q_0}\Big(\big(1+\frac{2\beta\gamma}{1-\beta}\big)(1-q_0)-(q_0-\gamma)\Big)\right)\\
=&\frac{2(1-\beta)}{1-\beta+\beta\gamma}\left(\Big(\frac{\beta\gamma}{1-\beta}\Big)^2(1-q_0)-\frac{2\beta\gamma}{1-\beta}(q_0-\gamma)+\frac{(q_0-\gamma)^2}{1-q_0}\right)\\
=&\frac{2(1-\beta)}{1-\beta+\beta\gamma}\cdot\frac{1}{1-q_0}\Big(\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)\Big)^2.
\end{aligned}
$$

Note that $p_0-w_0>0$, to make the above inequality holds, it suffices to show that

$$
2>\frac{2(1-\beta)}{1-\beta+\beta\gamma}\cdot\frac{1}{1-q_0}\Big(\frac{\beta\gamma}{1-\beta}(1-q_0)-(q_0-\gamma)\Big)=\frac{2\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta}),
$$

The above inequality holds as $\beta\gamma<1-\beta$ ($\gamma<\frac{1-\beta}{\beta}$) and $\underline{\eta}>0$ (Assumption 1). □

In Lemma 3.6.11, we specify the shape of the platform revenue regarding the informational delay λ under the optimal differentiated commissions.

Lemma 3.6.11 *Suppose* $\gamma<\frac{1-\beta}{\beta}$ *holds and* $\delta_{\mathcal{U}}^{U*}(\lambda)$ *is characterized by* (3.47) *in Lemma 3.6.8. Then, for* $\lambda\in[0,\frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$*, If* $\delta_{\mathcal{U}}^{U*}(\lambda)<\frac{1-\beta}{1-\beta+\beta\gamma}$*, the platform revenue under the optimal commission rates is convex in* λ*. Otherwise, the platform revenue under the optimal commission rates is linearly increasing in* λ*.*

Proof. First, we show that when $\delta_{\mathcal{U}}^{U*}(\lambda)<\frac{1-\beta}{1-\beta+\beta\gamma}$ the platform revenue is convex in

λ. To simplify the notation, we introduce

$$A \triangleq \frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0), \tag{3.53}$$

$$B \triangleq \beta\gamma(1-q_0), \tag{3.54}$$

$$C \triangleq q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0), \tag{3.55}$$

$$\text{and } D \triangleq \beta\gamma\big(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\big). \tag{3.56}$$

Then, by (3.48), the platform revenue under the optimal commission rates given λ (i.e., at $\delta_{\mathcal{U}}^{U*}(\lambda)$) is $\pi_r^*(\lambda) = \frac{1}{4}\cdot\frac{(A-B\lambda)^2}{C-D\lambda} = \frac{\left(A-\frac{BC}{D}\right)^2}{4(C-D\lambda)} + \frac{B}{2D}\big(A-\frac{BC}{D}\big) + \frac{B^2}{4D^2}(C-D\lambda)$. Therefore, $\pi_r^*(\lambda)$ is convex in λ as $C - D\lambda > 0$.

Next, by (3.48), it is straightforward to verify that when $\delta_{\mathcal{U}}^{U*}(\lambda) = \frac{1-\beta}{1-\beta+\beta\gamma}$ the platform's revenue under the optimal commission rates is linearly increasing in λ. Besides, the coefficient of λ has the same sign as:

$$\begin{aligned}&-(1-q_0) + \frac{1-\beta+2\beta\gamma}{1-\beta+\beta\gamma}(1-q_0) - \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\\ &= \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-q_0) - \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\\ &> 0.\end{aligned}$$

The last inequality holds as $\frac{1-\beta}{\beta\gamma}\cdot\frac{q_0-\gamma}{1-q_0} < 1$ (Assumption 1). $\square$

In Lemma 3.6.12, we characterize the optimal informational delay and a sufficient and necessary condition for the optimal informational delay to be positive within $[0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$.

Lemma 3.6.12 *Suppose* $\gamma < \frac{1-\beta}{\beta}$ *holds and* $\lambda \in [0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$*. Then, if* $\delta_{\mathcal{U}}^{U*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$*, then* $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$*. Otherwise,* $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ *if* (3.57) *holds, and* $\lambda^* = 0$ *if* (3.57) *does not hold.*

Proof. Note that in the interval of $\big[0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big]$, we have $q_{\mathcal{U}} \le q_0$. Besides, by Lemma 3.6.7 (under $\gamma < \frac{1-\beta}{\beta}$), we only need to consider the policies with $\eta = 1$.

First, we show that the optimal informational delay is $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, if all customers choose the platform under the full information and the optimal differentiated commissions (i.e., $\delta_{\mathcal{U}}^{U*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$). The result holds by Lemma 3.6.11 (under $\gamma < \frac{1-\beta}{\beta}$), where we show that the platform revenue is linearly increasing in λ.

Next, we consider the case where not all customers choose the platform under the full information and the optimal differentiated commissions (i.e., $\delta_{\mathcal{U}}^{U*}(0) < \frac{1-\beta}{1-\beta+\beta\gamma}$).

On the one hand, at $\lambda = 0$ we plug $\delta_{\mathcal{U}}^{U*}(0)$ into π_r ((3.48)), and we have:

$$\pi_r(0) = \frac{1}{4} \cdot \frac{\Big(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) - (q_0 - \gamma) + \frac{2\beta\gamma}{1-\beta}(1-q_0)\Big)^2}{q_0 - \gamma + \big(\frac{\beta\gamma}{1-\beta}\big)^2(1-q_0)} = \frac{1}{4} \cdot \frac{A^2}{C},$$

where A and C are defined by (3.53) and (3.55). On the other hand, at $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ we have $\delta_{\mathcal{U}}^{U*}\big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big) = \frac{1-\beta}{1-\beta+\beta\gamma} = \bar{x}$ by Lemma 3.6.10 (under $\gamma < \frac{1-\beta}{\beta}$). Besides, we can characterize the platform revenue as (by (3.48))

$$\begin{aligned}
\pi_r\Big(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\Big) &= \bar{x}\Big(A - B\frac{q_0-\gamma}{\beta\gamma(1-q_0)} - \Big(C - D\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\Big)\bar{x}\Big) \\
&= \bar{x}\Big(A - C\bar{x} + (D\bar{x} - B)\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\Big),
\end{aligned}$$

where B and D are defined by (3.54) and (3.56).

It is straightforward to verify that $\bar{x}\big(A - C\bar{x} + (D\bar{x} - B)\frac{q_0-\gamma}{\beta\gamma(1-q_0)}\big) \ge \frac{1}{4} \cdot \frac{A^2}{C}$ is equivalent to $\frac{q_0-\gamma}{\beta\gamma(1-q_0)} \ge \frac{(A-2\bar{x}C)^2}{4C\bar{x}(\bar{x}D-B)}$, which is (3.57). Notice that π_r is convex in λ (Lemma 3.6.11), we conclude that the optimal informational delay within [0,

$\frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$ is $\lambda \in \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. Similarly, if (3.57) does not hold, then $\frac{q_0-\gamma}{\beta\gamma(1-q_0)} < \frac{(A-2\bar{x}C)^2}{4C\bar{x}(\bar{x}D-B)}$, which is equivalent to $\pi_r\big(\frac{q_0-q_N}{\beta\gamma(1-q_0)}\big) < \pi_r(0)$. Notice that π_r is convex in λ (Lemma 3.6.11), we conclude that the optimal informational delay within $[0, \frac{q_0-q_N}{\beta\gamma(1-q_0)}]$ is zero.

□

In Lemma 3.6.13, we characterize the optimal informational delay when $\lambda \geq \frac{q_0-q_N}{\beta\gamma(1-q_0)}$.

Lemma 3.6.13 *Suppose $\gamma < \frac{1-\beta}{\beta}$ holds. Then, the revenue optimal policy within $\big[\frac{q_0-\gamma}{\beta\gamma(1-q_0)}, \frac{1}{1-\beta}\big)$ has delay $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$.*

Proof. By Lemma 3.6.7 (under $\gamma < \frac{1-\beta}{\beta}$), we only need to search for the optimal policy among those with $\eta = 1$ in equilibrium.

First, we show that given a policy $(\tau_{\mathcal{U}}, \tau_{\mathcal{H}}, \lambda)$ with $\delta^U_{\mathcal{U}}$, $\eta = 1$, and $\lambda > \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ in equilibrium, we can find an alternative policy $(\tilde{\tau}_{\mathcal{U}}, \tilde{\tau}_{\mathcal{H}}, \tilde{\lambda})$ with $\tilde{\delta}^U_{\mathcal{U}} = \delta^U_{\mathcal{U}}$, $\tilde{\eta} = 1$, and $\frac{q_0-\gamma}{\beta\gamma(1-q_0)} \leq \tilde{\lambda} < \lambda$. In particular, we select $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ such that

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-(1-\beta)\tilde{\lambda}}{1+\beta\gamma\tilde{\lambda}}\big((1-\tilde{\tau}_{\mathcal{H}})\tilde{p}_{\mathcal{H}} - w_0\big) = w_0 - (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}}, \text{ and } (1-\tilde{\tau}_{\mathcal{U}})\tilde{p}_{\mathcal{U}} = (1-\tau_{\mathcal{U}})p_{\mathcal{U}},$$

where $\tilde{p}_{\mathcal{U}} = p_0 + \big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta^U_{\mathcal{U}}\big)(\tilde{q}_{\mathcal{U}} - q_0)$ and $\tilde{p}_{\mathcal{H}} = \tilde{p}_{\mathcal{U}} + \big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\tilde{\lambda}\big)\delta^U_{\mathcal{U}}\big)(1-\tilde{q}_{\mathcal{U}})$. It is straightforward to show that we can solve for $\tilde{\tau}_{\mathcal{U}}$ and $\tilde{\tau}_{\mathcal{H}}$ from the above two equations.

Then, we show the alternative policy results in a higher platform revenue compared to the original policy. It suffices to show that given $\eta = 1$ and $\delta^U_{\mathcal{U}} \in [0, \frac{1-\beta}{1-\beta+\beta\gamma}]$, we have $\frac{\partial \pi_r}{\partial \lambda}(\delta^U_{\mathcal{U}}, \lambda) < 0$, where $\pi_r(\delta^U_{\mathcal{U}}, \lambda)$ is the platform revenue. By

(3.46) and $\eta = 1$, we have:

$$\pi_r(\delta_{\mathcal{U}}^U, \lambda) = \delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0 - w_0) + \frac{1-\beta+\beta\gamma}{1-\beta}\big(2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U\big)\frac{\beta\gamma\lambda(1-q_0)-(q_0-\gamma)}{1+\beta\gamma\lambda} + \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\Big(2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U\Big)\frac{1-\gamma}{1+\beta\gamma\lambda}\Bigg).$$

To simplify the notation, we let $\theta_{\mathcal{U}0} \triangleq 2 - \frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$ and $\theta_{\mathcal{HU}} \triangleq 2 - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U$. Then, $\frac{\partial \pi_r}{\partial \lambda}(\delta_{\mathcal{U}}^U, \lambda)$ can be expressed as

$$\begin{aligned}
&\frac{\partial \pi_r}{\partial \lambda}(\delta_{\mathcal{U}}^U, \lambda)\\
=&\delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}\frac{\beta\gamma(1-\gamma)}{(1+\beta\gamma\lambda)^2}\theta_{\mathcal{U}0} - \frac{\beta\gamma(1-\gamma)}{1+\beta\gamma\lambda}\theta_{\mathcal{HU}}\\
&+ \frac{\beta\gamma(1-\gamma)}{1+\beta\gamma\lambda}(2-\theta_{\mathcal{HU}}) - \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\frac{\beta\gamma(1-\gamma)}{(1+\beta\gamma\lambda)^2}\theta_{\mathcal{HU}}\Bigg)\\
=&\delta_{\mathcal{U}}^U \Bigg(\frac{1-\beta+\beta\gamma}{1-\beta}\frac{\beta\gamma(1-\gamma)}{(1+\beta\gamma\lambda)^2}\theta_{\mathcal{U}0} - \frac{1-\beta+\beta\gamma}{1-\beta}\frac{\beta\gamma(1-\gamma)}{(1+\beta\gamma\lambda)^2}\theta_{\mathcal{HU}} + \frac{\beta\gamma(1-\gamma)}{1+\beta\gamma\lambda}(2-\theta_{\mathcal{HU}})\Bigg)\\
=&\delta_{\mathcal{U}}^U \Big(-\frac{1-\beta+\beta\gamma}{1-\beta}\cdot\frac{\beta\gamma(1-\gamma)}{(1+\beta\gamma\lambda)^2}(\theta_{\mathcal{HU}}-\theta_{\mathcal{U}0}) + \frac{\beta\gamma(1-\gamma)}{1+\beta\gamma\lambda}(2-\theta_{\mathcal{HU}})\Big)
\end{aligned}$$

Thus, $\frac{\partial \pi_r}{\partial \lambda}(\delta_{\mathcal{U}}^U, \lambda)$ has the same sign as:

$$-\frac{1-\beta+\beta\gamma}{1-\beta}\cdot\frac{\theta_{\mathcal{HU}}-\theta_{\mathcal{U}0}}{1+\beta\gamma\lambda} + 2 - \theta_{\mathcal{HU}} = -\frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U + \frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U < 0.$$

The last inequality holds as $\frac{1-\beta+\beta\gamma}{1-\beta}\delta_{\mathcal{U}}^U$ is the quantity of total hired providers and $\frac{\beta\gamma}{1-\beta}\big(1-(1-\beta)\lambda\big)\delta_{\mathcal{U}}^U$ is quantity of providers with label $\mathcal{H}$. Notice that the proposed policy has lower delay compared with the original policy (i.e., $\tilde{\lambda} < \lambda$), so the proposed policy results in higher platform revenue.

Therefore, for $\lambda \in [\frac{q_0-\gamma}{\beta\gamma(1-q_0)}, \frac{1}{1-\beta})$, the platform revenue is decreasing in the informational delay, and the optimal policy has informational delay $\frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. □

Proposition 3.6.4 characterize the optimal information-commission policy under the differentiated commission scheme.

Proposition 3.6.4 *Suppose* $\gamma < \frac{1-\beta}{\beta}$ *holds. Then, if* $\delta_{\mathcal{U}}^{U*}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$, *then* $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$. *Otherwise,* $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ *if*

$$\frac{q_0-\gamma}{\beta\gamma(1-q_0)} \geq \frac{\left(\left(\frac{1-\beta+\beta\gamma}{1-\beta}(p_0-w_0)-(q_0-\gamma)+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)-2\left(\frac{1-\beta}{1-\beta+\beta\gamma}\right)\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)\right)\right)^2}{4\left(q_0-\gamma+\left(\frac{\beta\gamma}{1-\beta}\right)^2(1-q_0)\right)\left(\frac{1-\beta}{1-\beta+\beta\gamma}\right)\left(\frac{1-\beta}{1-\beta+\beta\gamma}\left(\beta\gamma\left(1+\gamma-2q_0+\frac{2\beta\gamma}{1-\beta}(1-q_0)\right)\right)-\beta\gamma(1-q_0)\right)}, \tag{3.57}$$

and $\lambda^* = 0$ *if* (3.57) *does not hold.*

Moreover, if $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, *then*

$$\tau_{\mathcal{U}}^*(\lambda^*) = 1 - \frac{w_0}{p_{\mathcal{U}}^*(\lambda^*)} \text{ and } \tau_{\mathcal{H}}^*(\lambda) = 1 - \frac{w_0}{p_{\mathcal{H}}^*(\lambda^*)}, \tag{3.58}$$

where $p_{\mathcal{U}}^*(\lambda^*) = p_0$ *and* $p_{\mathcal{H}}^*(\lambda^*) = p_0 + \left(\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\right)(1-q_0)$. *If* $\lambda^* = 0$, *then the optimal policy is given by Proposition 3.6.3.*[8]

Proof. We characterize the global optimal informational delay by characterizing the optimal delay within two complement intervals $[0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$ and $(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}, \frac{1}{1-\beta})$. We have $q_{\mathcal{U}} \leq q_0$ in the first interval and $q_{\mathcal{U}} > q_0$ in the second interval.

First, we search for the optimal delay within $[0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$. In particular, the optimal informational delay is characterized by Lemma 3.6.12. Moreover, Lemma 3.6.12 provides a sufficient and necessary condition for the optimal informational delay to be optimal.

[8]The optimal commissions are not unique and we choose the pair with the largest $\tau_{\mathcal{H}}$. In the following analysis, if there are multiple optimal commission pairs, we present the pair with the largest $\tau_{\mathcal{H}}$.

Second, we search for the optimal delay within $\left(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}, \frac{1}{1-\beta}\right)$. In Lemma 3.6.13, we establish that $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ is revenue optimal.

By the continuity of the equilibrium regarding λ at $\lambda = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ as well as Lemmas 3.6.12 and 3.6.13, we conclude that the revenue optimal delay found within $[0, \frac{q_0-\gamma}{\beta\gamma(1-q_0)}]$ dominates that found within $\left(\frac{q_0-\gamma}{\beta\gamma(1-q_0)}, \frac{1}{1-\beta}\right)$. Therefore, the characterizations of the optimal informational delay as well as the sufficient and necessary condition for the optimal informational delay to be positive in Lemma 3.6.12 apply to the entire interval (i.e., $\lambda \in (0, \frac{1}{1-\beta})$).

Lastly, we characterize the optimal commissions when $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ (i.e., $\tau_{\mathcal{U}}^*(\lambda^*)$ and $\tau_{\mathcal{H}}^*(\lambda^*)$). By (3.51), we obtain (3.58), where $p_{\mathcal{U}}^*(\lambda^*)$ and $p_{\mathcal{H}}^*(\lambda^*)$ are determined by Lemma 3.6.6.

In sum, we have proved Proposition 3.6.4. $\square$

In Corollary 3.6.1, we compare the optimal commissions when $\lambda^* > 0$ with those under the full information policy (i.e., $\lambda = 0$).

Corollary 3.6.1 *Suppose $\gamma < \frac{1-\beta}{\beta}$ and $\underline{\eta} < \frac{1}{2}$ hold. Then, we have $\tau_{\mathcal{U}}^*(0) < \tau_{\mathcal{U}}^*(\lambda^*)$ and $\tau_{\mathcal{H}}^*(0) < \tau_{\mathcal{H}}^*(\lambda^*)$, where $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$ and $\left(\tau_{\mathcal{U}}^*(\lambda), \tau_{\mathcal{H}}^*(\lambda)\right)$ are the optimal commission rates for providers with label $\mathcal{U}$ and label $\mathcal{H}$, respectively, given λ.*

Proof. First, by Lemma 3.6.9 and Proposition 3.6.4, we know $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$.

Second, we compare $\tau_{\mathcal{U}}$. At $\lambda = 0$, we notice that $\tau_{\mathcal{U}}^*(0) = 1 - \frac{w_0}{p_0-(1+\delta_{\mathcal{U}}^{U*}(0))(q_0-\gamma)}$ by (3.49), where $\delta_{\mathcal{U}}^{U*}(0)$ is characterized by (3.50). Then, at $\lambda = \lambda^*$, we have $\tau_{\mathcal{U}}^*(\lambda^*) = 1 - \frac{w_0}{p_0}$ (as $p_{\mathcal{U}} = p_0$ at $\lambda^* = \frac{q_0-\gamma}{\beta\gamma(1-q_0)}$) by Proposition 3.6.4. Therefore, we have $\tau_{\mathcal{U}}^*(0) < \tau_{\mathcal{U}}^*(\lambda^*)$.

Third, we compare $\tau_{\mathcal{H}}$. At $\lambda = 0$, we notice that $\tau^*_{\mathcal{H}}(0) = 1 - \frac{w_0}{p^*_{\mathcal{H}}(0)}$, where $p^*_{\mathcal{H}}(0) = p_0 + \left(2 - \frac{\beta\gamma}{1-\beta}\delta^{U*}_{\mathcal{U}}(0)\right)(1-q_0)$, by (3.49). Moreover, we can simplify $p^*_{\mathcal{H}}(0)$ as $p^*_{\mathcal{H}}(0) = p_0 + \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(1-q_0)$ as $\delta^{U*}_{\mathcal{U}}(0) = \frac{1-\beta}{1-\beta+\beta\gamma}$ under $\underline{\eta} < \frac{1}{2}$ (Lemma 3.6.9). At $\lambda = \lambda^*$, we obtain $\tau^*_{\mathcal{H}}(\lambda^*) = 1 - \frac{w_0}{p^*_{\mathcal{H}}(\lambda^*)}$, where $p^*_{\mathcal{H}}(\lambda^*) = p_0 + \big(\frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\big)(1-q_0)$, by (3.58). Then, to show $\tau^*_{\mathcal{H}}(0) < \tau^*_{\mathcal{H}}(\lambda^*)$, it is equivalent to show $p^*_{\mathcal{H}}(0) < p^*_{\mathcal{H}}(\lambda^*)$. Given $\underline{\eta} > 0$ and the above characterizations of $p^*_{\mathcal{H}}(0)$ and $p^*_{\mathcal{H}}(\lambda^*)$, it is straightforward to show the last inequality holds. □

3.7 Appendix: Proofs for Section 3.3

Proof of Proposition 3.3.2

We take three steps to establish the existence of an equilibrium. First, we construct an auxiliary normal form game with finitely many players and convex and compact strategy spaces. Second, we establish the existence of the auxiliary game by [50]. Third, we show that the equilibrium of the auxiliary game corresponds to an equilibrium defined in Section 3.3.

In the first step, we construct the auxiliary game. In this auxiliary game, we assume there are 15 agents. In what follows, we characterize each agent's strategic space and payoff function, denoted by u_i. Given agent i, we further establish that u_i is the upper semi-continuous (u.s.c.) in the actions of all agents and quasi-concave (q.c.) in agent i's action, and $\max u_i$ is lower semi-continuous (l.s.c.) in the actions of all agents other than agent i. For the notation, we let $\mathbf{a}$ denote the action vector of all agents and let $\mathbf{a}_{-i}$ denote the action vectors of all agents except agent i. We then let $\{M_i\}$ a set of large constants such that $M_j \gg M_i \gg 0$ if $i < j$.

- Agent 1: We denote the agent's action by $\tilde{\delta}_{\mathcal{U}}^{U} \in [0, M_1]$ and the payoff function by

$$u_1(\tilde{\delta}_{\mathcal{U}}^{U}, \mathbf{a}_{-1}) = -\eta \tilde{\delta}_{\mathcal{U}}^{U} \Big| V_{\mathcal{U}}^{U} - \frac{w_0}{1-\beta} \Big|,$$

  where η is the action of agent 13 with action space $[0, M_1]$, and $V_{\mathcal{U}}^{U}$ is the action of agent 6 with action space $[0, M_4]$. Therefore, u_1 is u.s.c. in $\mathbf{a}$ and q.c. in $\tilde{\delta}_{\mathcal{U}}^{U}$. Besides, $\max_{\tilde{\delta}_{\mathcal{U}}^{U}} u_1 = 0$, which is l.s.c in $\mathbf{a}_{-1}$.

- Agent 2: We denote the agent's action by $\delta_{\mathcal{U}}^{H} \in [0, M_2]$ and the payoff function by

$$u_2(\delta_{\mathcal{U}}^{H}, \mathbf{a}_{-2}) = -\Big| \delta_{\mathcal{U}}^{H} - \frac{\beta \gamma \rho_{H}^{\mathcal{U}}}{1 - \beta \rho_{H}^{\mathcal{U}} s_{\mathcal{U}}} \eta \tilde{\delta}_{\mathcal{U}}^{U} \Big|,$$

  where $s_{\mathcal{U}}$ is the action of agent 4 with action space $[0, 1]$. Therefore, u_2 is u.s.c. in $\mathbf{a}$ and q.c. in $\delta_{\mathcal{U}}^{H}$. Besides, $\max_{\delta_{\mathcal{U}}^{H}} u_2 = 0$, which is l.s.c. in $\mathbf{a}_{-2}$.

- Agent 3: We denote the agent's action by $\delta_{\mathcal{H}}^{H} \in [0, M_3]$ and the payoff function by

$$u_3(\delta_{\mathcal{H}}^{H}, \mathbf{a}_{-3}) = -\Big| \delta_{\mathcal{H}}^{H} - \frac{\beta(1 - \rho_{H}^{\mathcal{U}})}{1 - \beta s_{\mathcal{H}}} \big(s_{\mathcal{U}} \delta_{\mathcal{U}}^{H} + \gamma \eta \tilde{\delta}_{\mathcal{U}}^{U} \big) \Big|.$$

  Therefore, u_3 is u.s.c. in $\mathbf{a}$ and q.c. in $\delta_{\mathcal{H}}^{H}$. Besides, $\max_{\delta_{\mathcal{H}}^{H}} u_3 = 0$, which is l.s.c. in $\mathbf{a}_{-3}$.

- Agent 4: We denote the agent's action by $s_{\mathcal{U}} \in [0, 1]$ and the payoff function by

$$u_4(s_{\mathcal{U}}, \mathbf{a}_{-4}) = -s_{\mathcal{U}} \max\Big(\frac{w_0}{1-\beta} - V_{\mathcal{U}}^{H}, 0 \Big) - (1 - s_{\mathcal{U}}) \max\Big(V_{\mathcal{U}}^{H} - \frac{w_0}{1-\beta}, 0 \Big),$$

where $V_{\mathcal{U}}^H$ is the action of agent 7 with action space $[0, M_3]$. Therefore, u_4 is u.s.c. in $\mathbf{a}$ and q.c. in $s_{\mathcal{U}}$. Besides, $\max_{s_{\mathcal{U}}} u_4 = 0$, which is l.s.c. in $\mathbf{a}_{-4}$.

- Agent 5: We denote the agent's action by $s_{\mathcal{H}} \in [0, 1]$ and the payoff function by

$$u_5(s_{\mathcal{H}}, \mathbf{a}_{-5}) = -s_{\mathcal{H}} \max\Big(\frac{w_0}{1-\beta} - V_{\mathcal{H}}^H, 0\Big) - (1 - s_{\mathcal{H}}) \max\Big(V_{\mathcal{H}}^H - \frac{w_0}{1-\beta}, 0\Big),$$

where $V_{\mathcal{H}}^H$ is the action of agent 9 with action space $[0, M_2]$. Therefore, u_5 is u.s.c. in $\mathbf{a}$ and q.c. in $s_{\mathcal{H}}$. Besides, $\max_{s_{\mathcal{H}}} u_5 = 0$, which is l.s.c. in $\mathbf{a}_{-5}$.

- Agent 6: We denote the agent's action by $V_{\mathcal{U}}^U \in [0, M_4]$ and the payoff function by

$$\begin{aligned} &u_6(V_{\mathcal{U}}^U, \mathbf{a}_{-6}) = \\ &-\Bigg|V_{\mathcal{U}}^U - \frac{\eta}{1-(1-\eta)\beta}\Big((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\gamma\rho_{\mathcal{U}}^H s_{\mathcal{H}} V_{\mathcal{U}}^H + \beta\gamma(1-\rho_{\mathcal{U}}^H) s_{\mathcal{H}} V_{\mathcal{H}}^H \\ &+ \beta\big(\gamma\rho_{\mathcal{U}}^H(1-s_{\mathcal{U}}) + \gamma(1-\rho_{\mathcal{U}}^H)(1-s_{\mathcal{H}}) + 1 - \gamma\big)\frac{w_0}{1-\beta}\Big)\Bigg|, \end{aligned}$$

where $p_{\mathcal{U}}$ is the action of agent 14 with action space $[\frac{b_0}{1-\tau_{\mathcal{U}}}, M_1]$. Therefore, u_6 is u.s.c. in $\mathbf{a}$ and q.c. in $V_{\mathcal{U}}^U$. Besides, $\max_{V_{\mathcal{U}}^U} u_6 = 0$, which is l.s.c. in $\mathbf{a}_{-6}$.

- Agent 7: We denote the agent's action by $V_{\mathcal{U}}^H \in [0, M_3]$ and the payoff function by

$$u_7(V^H_{\mathcal{U}}, \mathbf{a}_{-7}) =$$
$$-\Bigg|V^H_{\mathcal{U}}$$
$$-\frac{1}{1-\beta\rho^H_{\mathcal{U}}s_{\mathcal{U}}}\Big(\eta(1-\tau)p_{\mathcal{U}} + \beta(1-\rho^H_{\mathcal{U}})s_{\mathcal{H}}V^H_{\mathcal{H}} + \beta\big(\rho^H_{\mathcal{U}}(1-s_{\mathcal{U}}) + (1-\rho^H_{\mathcal{U}})(1-s_{\mathcal{H}})\big)\frac{w_0}{1-\beta}\Big)\Bigg|.$$

Therefore, u_7 is u.s.c. in $\mathbf{a}$ and q.c. in $V^H_{\mathcal{U}}$. Besides, $\max_{V^H_{\mathcal{U}}} u_7 = 0$, which is l.s.c. in $\mathbf{a}_{-7}$.

- Agent 8: We denote the agent's action by $V^H_{\mathcal{H}} \in [0, M_2]$ and the payoff function by

$$u_8(V^H_{\mathcal{H}}, \mathbf{a}_{-8}) = -\Big|V^H_{\mathcal{H}} - \frac{(1-\tau_{\mathcal{H}})p_{\mathcal{H}}}{1-\beta}\Big|,$$

where $p_{\mathcal{H}}$ is the action of agent 12 with action space $[\frac{b_0}{1-\tau_{\mathcal{H}}}, M_1]$. Therefore, u_8 is u.s.c. in $\mathbf{a}$ and q.c. in $V^H_{\mathcal{H}}$. Besides, $\max_{V^H_{\mathcal{H}}} u_9 = 0$, which is l.s.c. in $\mathbf{a}_{-8}$.

- Agent 9: We denote the agent's action by $\zeta_{\mathcal{H}} \in [0,1]$ and denote the payoff function by

$$u_9(\zeta_H, \mathbf{a}_{-9}) = -\left|\zeta_{\mathcal{H}} - \left(2 - \max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\}\right)\right|,$$

where $q_H = 1$. Therefore, u_9 is u.s.c. in $\mathbf{a}$ and q.c. in $\zeta_{\mathcal{H}}$. Besides,

$$\max_{\zeta_{\mathcal{H}}} u_9 = \begin{cases} 0, & \text{if } \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 1\right\} \le 2, \\ -\left|2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 1\right\}\right|, & \text{otherwise.} \end{cases}$$

Therefore, $\max_{\zeta_{\mathcal{H}}} u_9$ is l.s.c. in $\mathbf{a}_9$.

- Agent 10: We denote the agent's action by $\tilde{\zeta}_{\mathcal{U}} \in [0,1]$ and the payoff function by

$u_{10}(\tilde{\zeta}_{\mathcal{U}}, \mathbf{a}_{-10}) =$

$$-\left|\tilde{\zeta}_{\mathcal{U}} - D_{\mathcal{U}}(\mathbf{a}_{-10})\right|\Big(\mathbb{1}_{\{p_{\mathcal{U}}<p_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{p_{\mathcal{U}}>p_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{q_{\mathcal{U}}<q_0\}}(\mathbf{a}_{-10}) + \mathbb{1}_{\{q_{\mathcal{U}}>q_0\}}(\mathbf{a}_{-10})\Big),$$

where $D_{\mathcal{U}}(\mathbf{a}_{-10}) =$

$$\begin{cases} \max\left\{\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1, & \text{if } q_{\mathcal{U}} < q_0, \\ \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\}, 1\right\} & \\ -\min\left\{\max\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\}, 1\right\}\right\}, & \text{if } q_{\mathcal{U}} > q_0, \\ \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1, & \text{if } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} < p_0, \\ 0, & \text{if } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} \geq p_0. \end{cases}$$

In the characterization of $D_{\mathcal{U}}(\mathbf{a}_{-10})$, $q_H = 1$ and $q_{\mathcal{U}}$ is the action of agent 15 with action space $[\gamma, 1]$. Next, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} \neq q_0\} \cup \{\mathbf{a}_{-10}|p_{\mathcal{U}} \neq p_0\}$. By the definition, it follows that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\} \cup \{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$.

Then, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point within $\{\mathbf{a}_{-10}|q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} < p_0\}$. Given $\underline{p} < p_0$, it suffices to show that

$$\lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \underline{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^-, \underline{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \underline{p}), \quad (3.59)$$

$$\text{and} \quad \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \underline{p}^-)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \lim_{(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) \to (q_0^+, \underline{p}^+)} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = D_{\mathcal{U}}(q_0, \underline{p}). \quad (3.60)$$

The first equalities in (3.59) and (3.60) hold because $D_{\mathcal{U}}$'s is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\}$ and $\{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$, respectively. To show the second equality in (3.59) hold, we notice that when $q_{\mathcal{U}}^i \to q_0^-$ and $p_{\mathcal{U}}^i \to \underline{p}^+$, we have

$$D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) = \max\left\{\min\left\{\frac{p_{\mathcal{U}}^i - p_0}{q_{\mathcal{U}}^i - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}^i}{q_H - q_{\mathcal{U}}^i}, 2\right\}, 1\right\} - 1$$

converges to

$$D_{\mathcal{U}}(q_0, \underline{p}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - \underline{p}}{q_H - q_0}, 2\right\}, 1\right\} - 1,$$

as $\frac{p_{\mathcal{U}}^i - p_0}{q_{\mathcal{U}}^i - q_0} \to +\infty$ and $\frac{p_{\mathcal{H}} - p_{\mathcal{U}}^i}{q_H - q_{\mathcal{U}}^i} \to \frac{p_{\mathcal{H}} - \underline{p}}{q_H - q_0}$. Similarly, to show the second equality in (3.60), we notice that when $q_{\mathcal{U}}^i \to q_0^+$ and $p_{\mathcal{U}}^i \to \underline{p}^+$, we have

$$\begin{aligned} D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) =& \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}^i}{q_H - q_{\mathcal{U}}^i}, 2\right\}, 1\right\} \\ &- \min\left\{\max\left\{\frac{p_{\mathcal{U}}^i - p_0}{q_{\mathcal{U}}^i - q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}^i}{q_H - q_{\mathcal{U}}^i}, 2\right\}, 1\right\}\right\} \end{aligned}$$

converges to

$$D_{\mathcal{U}}(q_0, \underline{p}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - \underline{p}}{q_H - q_0}, 2\right\}, 1\right\} - 1,$$

as $\frac{p_{\mathcal{U}}^i - p_0}{q_{\mathcal{U}}^i - q_0} \to -\infty$ and $\frac{p_{\mathcal{H}} - p_{\mathcal{U}}^i}{q_H - q_{\mathcal{U}}^i} \to \frac{p_{\mathcal{H}} - \underline{p}}{q_H - q_0}$. Therefore, $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any points with $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = \underline{p} < p_0$.

Next, we show that $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any point within $\{\mathbf{a}_{-10} | q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} > p_0\}$. Given $\bar{p} > p_0$, it suffices to show that

$$\lim_{(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) \to (q_0^-, \bar{p}^-)} D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) = \lim_{(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) \to (q_0^-, \bar{p}^+)} D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) = D_{\mathcal{U}}(q_0, \bar{p}), \quad (3.61)$$

$$\text{and} \lim_{(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) \to (q_0^+, \bar{p}^-)} D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) = \lim_{(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) \to (q_0^+, \bar{p}^+)} D_{\mathcal{U}}(q_{\mathcal{U}}^i, p_{\mathcal{U}}^i) = D_{\mathcal{U}}(q_0, \bar{p}). \quad (3.62)$$

The first equalities in (3.61) and (3.62) hold because $D_{\mathcal{U}}$'s is continuous in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} < q_0\}$ and $\{\mathbf{a}_{-10}|q_{\mathcal{U}} > q_0\}$, respectively. To show the second equality in (3.61) hold, we notice that when $q^i_{\mathcal{U}} \to q_0^-$ and $p^i_{\mathcal{U}} \to \bar{p}^+$, we have

$$D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_H - q^i_{\mathcal{U}}}, 2\right\}, 1\right\} - 1$$

converges to $D_{\mathcal{U}}(q_0, \bar{p}) = 0$, as $\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0} \to -\infty$ and $\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_H - q^i_{\mathcal{U}}} \to \frac{p_{\mathcal{H}} - \bar{p}}{q_H - q_0}$. Similarly, to show the second equality in (3.62), we notice that when $q^i_{\mathcal{U}} \to q_0^+$ and $p^i_{\mathcal{U}} \to \bar{p}^+$, we have

$$\begin{aligned} D_{\mathcal{U}}(q^i_{\mathcal{U}}, p^i_{\mathcal{U}}) =& \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_H - q^i_{\mathcal{U}}}, 2\right\}, 1\right\} \\ & - \min\left\{\max\left\{\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0}, 1\right\}, \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_H - q^i_{\mathcal{U}}}, 2\right\}, 1\right\}\right\} \end{aligned}$$

converges to $D_{\mathcal{U}}(q_0, \bar{p}) = 0$, as $\frac{p^i_{\mathcal{U}} - p_0}{q^i_{\mathcal{U}} - q_0} \to +\infty$ and $\frac{p_{\mathcal{H}} - p^i_{\mathcal{U}}}{q_H - q^i_{\mathcal{U}}} \to \frac{p_{\mathcal{H}} - \bar{p}}{q_H - q_0}$. Therefore, $D_{\mathcal{U}}(\mathbf{a}_{-10})$ is continuous at any points with $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = \bar{p} > p_0$. In sum, we have established the continuity of $D_{\mathcal{U}}(\mathbf{a}_{-10})$ in $\{\mathbf{a}_{-10}|q_{\mathcal{U}} \neq q_0\} \cup \{\mathbf{a}_{-10}|p_{\mathcal{U}} \neq p_0\}$.

Then, based on continuity of $D_{\mathcal{U}}(\mathbf{a}_{-10})$, we rewrite the $u_10(\tilde{\zeta}_{\mathcal{U}}, \mathbf{a}_{-10})$ as follows:

$$u_{10}(\tilde{\zeta}_{\mathcal{U}}, \mathbf{a}_{-10}) = \begin{cases} 0, & \text{if } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} = p_0 \\ -\left|\tilde{\zeta}_{\mathcal{U}} - D_{\mathcal{U}}(\mathbf{a}_{-10})\right|, & \text{if } q_{\mathcal{U}} > q_0 \text{ and } p_{\mathcal{U}} = p_0 \\ & \text{or } q_{\mathcal{U}} < q_0 \text{ and } p_{\mathcal{U}} = p_0 \\ & \text{or } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} > p_0 \\ & \text{or } q_{\mathcal{U}} = q_0 \text{ and } p_{\mathcal{U}} < p_0 \\ -2\left|\tilde{\zeta}_{\mathcal{U}} - D_{\mathcal{U}}(\mathbf{a}_{-10})\right|, & \text{if } q_{\mathcal{U}} > q_0 \text{ and } p_{\mathcal{U}} > p_0 \\ & \text{or } q_{\mathcal{U}} < q_0 \text{ and } p_{\mathcal{U}} > q_0 \\ & \text{or } q_{\mathcal{U}} > q_0 \text{ and } p_{\mathcal{U}} < q_0 \\ & \text{or } q_{\mathcal{U}} < q_0 \text{ and } p_{\mathcal{U}} < q_0 \end{cases}$$

Based on the above characterization, it immediately follows that u_{10} is u.s.c. in $\mathbf{a}$ and q.c. in $\tilde{\zeta}_{\mathcal{U}}$. Besides, $\max_{\tilde{\zeta}_{\mathcal{U}}} u_{10} = 0$, which is l.s.c.

In addition, we observe that when $q_{\mathcal{U}} \neq q_0$ or $p_{\mathcal{U}} \neq p_0$, we have

$$\zeta^*_{\mathcal{H}} + D_{\mathcal{U}}(\mathbf{a}_{-10}) \leq 1 \tag{3.63}$$

where

$$\zeta^*_{\mathcal{H}} = \arg\max_{\zeta_{\mathcal{H}}} u_9 = \max\left\{2 - \max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\}, 0\right\}.$$

In particular, we show (3.63) holds in four possible cases (1) $q_{\mathcal{U}} < q_0$, (2) $q_{\mathcal{U}} > q_0$, (3) $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} < p_0$, and (4) $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} > p_0$, separately.

(1) In this case, we notice that

$$D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{\min\left\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1\right\} \leq 1.$$

Therefore,

$$\zeta^*_{\mathcal{H}} + D_{\mathcal{U}}(\mathbf{a}_{-10}) =$$

$$\left\{ D_{\mathcal{U}}(\mathbf{a}_{-10}), 1 + \min\left\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\} - \max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\} \right\}.$$

Then, to show the above expression is no greater than 1, it is equivalent to show

$$\min\left\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\} \le \max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\}. \quad (3.64)$$

It is straightforward to verify that under $q_{\mathcal{U}} < q_0$, either

$$\frac{p_{\mathcal{H}} - p_0}{q_H - q_0} \le \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}} \le \frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0} \text{ or } \frac{p_{\mathcal{H}} - p_0}{q_H - q_0} \le \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}} \le \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}$$

holds. In the first case, (3.64) is equivalent to $\min\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\} \le \max\left\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 1\right\}$, which holds. In the second case, (3.64) is equivalent to $\min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, 2\right\} \le \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, 1\right\}$, which holds.

(2) In this case, we notice that

$$D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{ \min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1 \right\} - \min\left\{ \max\left\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, 1\right\}, \max\left\{ \min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1 \right\} \right\} \le 1.$$

Then, to show (3.63) holds, it is equivalent to show

$$\max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\} - \max\left\{ \min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1 \right\} + \min\left\{ \max\left\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, 1\right\}, \max\left\{ \min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1 \right\} \right\} \ge 1.$$

The above inequality holds as the difference between the first two terms is non-negative, and the third term is no less than 1.

(3) In this case, we notice that

$$D_{\mathcal{U}}(\mathbf{a}_{-10}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1 \leq 1.$$

Then by the characterization of $\zeta_{\mathcal{H}}^*$, to show (3.63) holds, it is equivalent to show

$$\max\left\{\frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1\right\} - \max\left\{\min\left\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\right\}, 1\right\} \geq 0,$$

which has been established in case (2).

(4) Notice that $D_{\mathcal{U}}(\mathbf{a}_{-10}) = 0$ in this case. Since $\zeta_{\mathcal{H}}^* \leq 1$, (3.63) follows.

Therefore, we have shown (3.63) when $q_{\mathcal{U}} \neq q_0$ or $p_{\mathcal{U}} \neq p_0$.

- Agent 11: We denote the agent's action by $\zeta_{\mathcal{U}} \in [0, 1]$ and the payoff function by

$$u_{11}(\zeta_{\mathcal{U}}, \mathbf{a}_{-11}) = -\Big|\zeta_{\mathcal{U}} - \min(\tilde{\zeta}_{\mathcal{U}}, 1 - \zeta_{\mathcal{H}})\Big|.$$

Therefore, u_{11} is u.s.c. in $\mathbf{a}$ and q.c. in $\zeta_{\mathcal{U}}$. Besides, $\max_{\zeta_{\mathcal{U}}} u_{11} = 0$, which is l.s.c. in $\mathbf{a}_{-11}$.

- Agent 12: We denote the agent's action by $p_{\mathcal{H}} \in [\frac{b_0}{1-\tau_H}, M_1]$ and the payoff function by

$$u_{12}(p_{\mathcal{H}}, \mathbf{a}_{-12}) = \Big(p_{\mathcal{H}} - \frac{b_0}{1 - \tau_{\mathcal{H}}}\Big)(\zeta_{\mathcal{H}} - s_{\mathcal{H}}\delta_{\mathcal{H}}^H).$$

Therefore, u_{12} is u.s.c. in $\mathbf{a}$ and q.c. in $p_{\mathcal{H}}$. Besides, $\max_{p_{\mathcal{H}}} u_{12}(p_{\mathcal{H}}, \mathbf{a}_{-12}) = \max\Big\{\Big(M_1 - \frac{b_0}{1-\tau_{\mathcal{H}}}\Big)(\zeta_{\mathcal{H}} - s_{\mathcal{H}}\delta_{\mathcal{H}}^H), 0\Big\}$, which is l.s.c. in $\mathbf{a}_{-12}$.

- Agent 13: We denote the agent's action by $\eta \in [0, M_1]$ and the payoff function by

$$u_{13}(\eta, \mathbf{a}_{-13}) = -\Big|\zeta_{\mathcal{U}} - \eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}})\Big|.$$

Therefore, u_{13} is u.s.c. in $\mathbf{a}$ and q.c. in η. Besides, $\max_\eta u_{13}(\eta, \mathbf{a}_{-13}) = \min\Big\{ -\zeta_{\mathcal{U}} + M_1(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}), 0\Big\}$, which is l.s.c. in $\mathbf{a}_{-13}$.

- Agent 14: We denote the agent's action by $p_{\mathcal{U}} \in [\frac{b_0}{1-\tau_{\mathcal{U}}}, M_1]$ and the payoff function by

$$u_{14}(p_{\mathcal{U}}, \mathbf{a}_{-14}) = \Big(p_{\mathcal{U}} - \frac{b_0}{1-\tau_{\mathcal{U}}}\Big)(\eta - 1).$$

Therefore, u_{14} is u.s.c in $\mathbf{a}$ and q.c. in $p_{\mathcal{U}}$. Besides, $\max_{p_{\mathcal{U}}} u_{14}(p_{\mathcal{U}}, \mathbf{a}_{-14}) = \max\Big\{\Big(M_1 - \frac{b_0}{1-\tau_{\mathcal{U}}}\Big)(\eta - 1), 0\Big\}$, which is l.s.c. in $\mathbf{a}_{-14}$.

- Agent 15: We denote the agent's action by $q_{\mathcal{U}} \in [\gamma, 1]$ and the payoff function by

$$u_{15}(q_{\mathcal{U}}, \mathbf{a}_{-15}) = -\Big|q_{\mathcal{U}} - \frac{\eta\gamma(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}{\eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}) + s_{\mathcal{H}}\delta^H_{\mathcal{H}}}\Big|\eta\tilde{\delta}^U_{\mathcal{U}}.$$

Therefore, u_{15} is u.s.c. in $\mathbf{a}$ and q.c. in $q_{\mathcal{U}}$. Besides, $\max_{q_{\mathcal{U}}} u_{15} = 0$, which is l.s.c. in $\mathbf{a}_{-15}$.

Therefore, we have specified the auxiliary game.

In the second step, we show that the auxiliary game has a pure strategy Nash equilibrium. It follows by the corollary of Theorem 2 in [50]. Then, we observe the following in any equilibrium:

- By the equilibrium actions of agents 2 and 3, we have:

$$\delta_{\mathcal{U}}^{H} = \frac{\beta\gamma\rho_{H}^{\mathcal{U}}}{1-\beta\rho_{H}^{\mathcal{U}}s_{\mathcal{U}}}\eta\tilde{\delta}_{\mathcal{U}}^{U} \text{ and } \delta_{\mathcal{H}}^{H} = \frac{\beta(1-\rho_{H}^{\mathcal{U}})}{1-\beta s_{\mathcal{H}}}\big(s_{\mathcal{U}}\delta_{\mathcal{U}}^{H} + \gamma\eta\tilde{\delta}_{\mathcal{U}}^{U}\big). \tag{3.65}$$

- By the equilibrium actions of agents 6, 7, and 8, we have:

$$\begin{aligned} V_{\mathcal{U}}^{U} =& \frac{\eta}{1-(1-\eta)\beta}\Big((1-\tau_{\mathcal{U}})p_{\mathcal{U}} + \beta\gamma\rho_{\mathcal{U}}^{H}s_{\mathcal{H}}V_{\mathcal{U}}^{H} + \beta\gamma(1-\rho_{\mathcal{U}}^{H})s_{\mathcal{H}}V_{\mathcal{H}}^{H} \\ & + \beta\big(\gamma\rho_{\mathcal{U}}^{H}(1-s_{\mathcal{U}}) + \gamma(1-\rho_{\mathcal{U}}^{H})(1-s_{\mathcal{H}}) + 1-\gamma\big)\frac{w_0}{1-\beta}\Big), \quad (3.66) \\ V_{\mathcal{U}}^{H} =& \frac{1}{1-\beta\rho_{\mathcal{U}}^{H}s_{\mathcal{U}}}\Big(\eta(1-\tau)p_{\mathcal{U}} + \beta(1-\rho_{\mathcal{U}}^{H})s_{\mathcal{H}}V_{\mathcal{H}}^{H} + \beta\big(\rho_{\mathcal{U}}^{H}(1-s_{\mathcal{U}}) \\ & + (1-\rho_{\mathcal{U}}^{H})(1-s_{\mathcal{H}})\big)\frac{w_0}{1-\beta}\Big), \quad (3.67) \\ \text{and } V_{\mathcal{H}}^{H} =& \frac{(1-\tau_{\mathcal{H}})p_{\mathcal{H}}}{1-\beta}. \quad (3.68) \end{aligned}$$

Furthermore, if $\eta\tilde{\delta}_{\mathcal{U}}^{U} > 0$ occurs in the equilibrium, we have the following observations :

- By the equilibrium action of agent 1, we have:

$$V_{\mathcal{U}}^{U} = \frac{w_0}{1-\beta}. \tag{3.69}$$

- By the equilibrium action of agent 15, we have:

$$q_{\mathcal{U}} = \frac{\eta\gamma(\tilde{\delta}_{\mathcal{U}}^{U} + s_{\mathcal{U}}\delta_{\mathcal{U}}^{H}) + s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}}{\eta(\tilde{\delta}_{\mathcal{U}}^{U} + s_{\mathcal{U}}\delta_{\mathcal{U}}^{H}) + s_{\mathcal{H}}\delta_{\mathcal{H}}^{H}}. \tag{3.70}$$

- By the equilibrium action of 14, we have

$$0 < \eta \leq 1. \tag{3.71}$$

We show (3.71) by the way of contradiction. Suppose $\eta > 1$, then $p_{\mathcal{U}} = M_1 \to +\infty$ by the equilibrium action of agent 14. As a result, $\tilde{\zeta}_{\mathcal{U}} = 0$ by the equilibrium action of agent 10, and hence $\zeta_{\mathcal{U}} = 0$ by the equilibrium action of agent 11. Further, we have $\eta = 0$ by the equilibrium action of agent 13 (note that $\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}} \geq \tilde{\delta}^U_{\mathcal{U}} > 0$), which yields a contradiction.

In addition, by the equilibrium action of agent 13, we have:

$$\zeta_{\mathcal{U}} = \eta(\tilde{\delta}^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}}). \tag{3.72}$$

- By the payoff function of agents 12, we have:

$$\zeta_{\mathcal{H}} = s_{\mathcal{H}}\delta^H_{\mathcal{H}}. \tag{3.73}$$

We first show $\zeta_{\mathcal{H}} \leq s_{\mathcal{H}}\delta^H_{\mathcal{H}}$ by the way of contradiction. Suppose $\zeta_{\mathcal{H}} > s_{\mathcal{H}}\delta^H_{\mathcal{H}}$. Then, agent 12's equilibrium action is $p_{\mathcal{H}} = M_1 \to +\infty$. Then, we have $\frac{p_{\mathcal{H}}-p_0}{q_H-q_0} \to +\infty$ and $\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}} \to +\infty$. By the equilibrium action of agent 9, we have $\zeta_{\mathcal{H}} = 0$, which yields a contradiction. Then, we show $\zeta_{\mathcal{H}} \geq s_{\mathcal{H}}\delta^H_{\mathcal{H}}$ by the way of contradiction. Suppose $\zeta_{\mathcal{H}} < s_{\mathcal{H}}\delta^H_{\mathcal{H}}$. Then, agent 12's action is $p_{\mathcal{H}} = \frac{b_0}{1-\tau_{\mathcal{H}}} < \frac{w_0}{1-\tau_{\mathcal{H}}}$, which then results in $V^H_{\mathcal{H}} < \frac{w_0}{1-\beta}$. By the equilibrium action of agent 5, we have $s_{\mathcal{H}} = 0$, which yields a contradiction.

In the third step, we show the equilibrium of the auxiliary game coincides with the equilibrium we defined in Section 3.3. It is straightforward to verify that (i) the free-entry condition holds because of (3.69). (ii) Providers' lifetime earnings, (3.5), (3.4), and (3.3), coincide with (3.66), (3.67), and (3.68), respectively. (iii) Providers'; retention decisions, $s_{\mathcal{U}}$ and $s_{\mathcal{H}}$, coincide with the equilibrium actions of agents 4 and 5. (iv) The mass of providers, (3.6), coincide with (3.65). (v) Condition

$$\zeta_{\mathcal{U}} \leq \delta^U_{\mathcal{U}} + s_{\mathcal{U}}\delta^H_{\mathcal{U}} \text{ and } \zeta_{\mathcal{H}} \leq s_{\mathcal{H}}\delta^H_{\mathcal{H}}$$

holds because of (3.72) and (3.73). (vi) Minimum payment constraints hold because of the action spaces agents 12 and 14.

Next, we verify that $\zeta_{\mathcal{U}}$ and $\zeta_{\mathcal{H}}$ are equal to the mass of customers who choose providers with label $\mathcal{U}$ and label $\mathcal{H}$ as their best choice. For the notation, we use $\Theta_{\mathcal{U}}$ and $\Theta_{\mathcal{H}}$ denote the set of customers within $[1,2]$, whose best choice are providers with label $\mathcal{U}$ and label $\mathcal{U}$, respectively. We use $\mu(\cdot)$ to denote the measure a given customer set.

We first verify that $\zeta_{\mathcal{H}} = \mu(\Theta_{\mathcal{H}})$, where $\zeta_{\mathcal{H}}$ is characterized by the equilibrium action of agent 9. Notice that given $\theta \in \Theta_{\mathcal{H}}$, we have $\theta q_H - p_{\mathcal{H}} \geq \theta q_0 - p_0$ and $\theta q_H - p_{\mathcal{H}} \geq \theta q_{\mathcal{U}} - p_{\mathcal{U}}$, from which we obtain $\theta \geq \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}\right\}$. Therefore,

$$\Theta_{\mathcal{H}} = \left[\max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}\right\}, +\infty\right) \cap [1,2],$$

and $\mu(\Theta_{\mathcal{H}}) = \max\left\{2 - \max\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 1\right\}, 0\right\} = \zeta_{\mathcal{H}}$.

Then, we verify $\zeta_{\mathcal{U}} = \mu(\Theta_{\mathcal{U}})$, where $\zeta_{\mathcal{U}}$ is characterized by the equilibrium action of agent 11. We verify the condition in the following 5 possible cases:

1. Suppose $q_{\mathcal{U}} < q_0$ occurs in equilibrium. Notice that for any $\theta \in \Theta_{\mathcal{U}}$, we have $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_0 - p_0$ and $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_H - p_{\mathcal{H}}$, which results in $\theta \leq \min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}\right\}$. Therefore,

$$\Theta_{\mathcal{U}} = \left(-\infty, \min\left\{\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}\right\}\right] \cap [1,2],$$

   and $\mu(\Theta_{\mathcal{U}}) = \max\left\{\min\left\{\frac{p_{\mathcal{H}}-p_0}{q_H-q_0}, \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\right\}, 1\right\} - 1 = \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}}$, where the last equality holds because of (3.63) and the equilibrium action of agent 11.

2. Suppose $q_{\mathcal{U}} > q_0$ occurs in equilibrium. Notice that for any $\theta \in \Theta_{\mathcal{U}}$, we have

$\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_0 - p_0$ and $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_H - p_{\mathcal{H}}$, which results in $\frac{p_{\mathcal{U}}-p_0}{q_{\mathcal{U}}-q_0} \leq \theta \leq \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}$. Therefore,

$$\Theta_{\mathcal{U}} = \left[\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}\right] \cap [1, 2],$$

and

$$\mu(\Theta_{\mathcal{U}}) =$$

$$\max\Big\{ \min\Big\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\Big\}, 1\Big\} - \min\Big\{ \max\Big\{\frac{p_{\mathcal{U}} - p_0}{q_{\mathcal{U}} - q_0}, 1\}, \max\Big\{ \min\Big\{\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 2\Big\}, 1\Big\}\Big\}$$

$$= \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}},$$

where the last equality holds because of (3.63) and the equilibrium action of agent 11.

3. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} < p_0$ occur in equilibrium. Notice that for any $\theta \in \Theta_{\mathcal{U}}$, we have $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_0 - p_0$ and $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_H - p_{\mathcal{H}}$, which results in $\theta \leq \frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}$. Therefore,

$$\Theta_{\mathcal{U}} = \Big(- \infty, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}\Big] \cap [1, 2],$$

and $\mu(\Theta_{\mathcal{U}}) = \max\Big\{ \min\Big\{\frac{p_{\mathcal{H}}-p_{\mathcal{U}}}{q_H-q_{\mathcal{U}}}, 2\Big\}, 1\Big\} - 1 = \tilde{\zeta}_{\mathcal{H}} = \zeta_{\mathcal{U}}$, where the last equality holds because of (3.63) and the equilibrium action of agent 11.

4. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} > p_0$ occur in equilibrium. Notice that for any $\theta \in \Theta_{\mathcal{U}}$, we have $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_0 - p_0$ and $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_H - p_{\mathcal{H}}$. However, the first inequality cannot hold for any θ. Therefore, $\Theta_{\mathcal{U}} = \emptyset$, and $\mu(\Theta_{\mathcal{U}}) = 0 = \tilde{\zeta}_{\mathcal{U}} = \zeta_{\mathcal{U}}$, where the last equality holds because of (3.63) and the equilibrium action of agent 11.

5. Suppose $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = p_0$ occur in equilibrium. Notice that for any $\theta \in \Theta_{\mathcal{U}}$, we have $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_0 - p_0$ and $\theta q_{\mathcal{U}} - p_{\mathcal{U}} \geq \theta q_H - p_{\mathcal{H}}$, which results in $\theta \leq \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}$. Besides, since providers $\mathcal{U}$ and customers' outside option are identical, customers are indifferent in choosing between them, and any

$$\Theta_{\mathcal{U}} \subseteq \Big(-\infty, \frac{p_{\mathcal{H}} - p_0}{q_H - q_0} \Big] \cap [1, 2]$$

is valid. Therefore, $\mu(\Theta_{\mathcal{U}}) \leq \max\Big\{ \min\Big\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 2 \Big\}, 1 \Big\} - 1$. On the other hand, $\zeta_{\mathcal{U}} \leq 1 - \zeta_{\mathcal{H}}$, since $\tilde{\zeta}_{\mathcal{U}}$ can take any value within $[0, 1]$. To establish $\mu(\Theta_{\mathcal{U}}) = \zeta_{\mathcal{U}}$ for given $\Theta_{\mathcal{U}}$ or given $\zeta_{\mathcal{U}}$, it is equivalent to show that $\max\Big\{ \min\Big\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 2 \Big\}, 1 \Big\} - 1 = 1 - \zeta_{\mathcal{H}}$. By the characterization of $\zeta_{\mathcal{H}}$ from the equilibrium action of agent 9, it is equivalent to show

$$\begin{aligned}
2 - \max\left\{ \min\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 2 \right\}, 1 \right\} &= \max\left\{ 2 - \max\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, \frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}}, 1 \right\}, 0 \right\} \\
&= \max\left\{ 2 - \max\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 1 \right\}, 0 \right\} \\
&= 2 + \max\left\{ -\max\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 1 \right\}, -2 \right\} \\
&= 2 - \min\left\{ \max\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 1 \right\}, 2 \right\}.
\end{aligned}$$

The second equality holds as $\frac{p_{\mathcal{H}} - p_{\mathcal{U}}}{q_H - q_{\mathcal{U}}} = \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}$ in this case. Then, it is straightforward to verify that

$$\max\left\{ \min\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 2 \right\}, 1 \right\} = \min\left\{ \max\left\{ \frac{p_{\mathcal{H}} - p_0}{q_H - q_0}, 1 \right\}, 2 \right\}.$$

Therefore, we show that given $\zeta_{\mathcal{U}}$, there exists $\Theta_{\mathcal{U}}$ such that $\mu(\Theta_{\mathcal{U}}) = \zeta_{\mathcal{U}}$.

In sum, we have established the existence of the equilibrium defined in Section 3.3. □

3.8 Appendix: Proofs for Section 3.4

3.8.1 Proof of Proposition 3.4.1

If $\frac{p_0}{q_0-\gamma} < 1 + \frac{1-\beta}{1-\beta+\beta\gamma}$, then cases 1 and 2 of Proposition 3.6.2 occur. So, E3 or E4 arises under the optimal full-information policy. Besides, by Section 3.6.4, there are customers choosing the outside options in E3 and E4. □

3.8.2 Proof of Proposition 3.4.2

We take two steps to show the proposition. In the first step, we show that the optimal delay is $\lambda^\star = \frac{q_0-\gamma}{\beta\gamma(1-q_0)} > 0$ and the optimal commission is

$$\tau^\star = 1 - \frac{w_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\left(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\right)}$$

under the given conditions. In the second step, we show that under the given condition, $\delta_{\mathcal{H}}^{H}$ is higher under the optimal delay policy than that under the full-information policy.

In the first step, we first show that E1 holds under $\lambda^\star$ and $\tau^\star$. It is straightforward to verify that under $\lambda^\star$ and $\tau^\star$, we have $q_{\mathcal{U}} = q_0$ and $p_{\mathcal{U}} = p_0$. By Lemma 3.6.3, it suffices to verify that at $\lambda^\star$, the customer at $1+\delta_{\mathcal{U}}$ prefers $\mathcal{U}$-label providers to the outside option and $\mathcal{U}$-label providers are not financially constraint. That is, $p_{\mathcal{U}} \le p_0$ and $(1-\tau)p_{\mathcal{U}} \ge b_0$. Besides, we need that the free-entry condition (i.e., (3.9)) holds. For $p_{\mathcal{U}} \le p_0$, it holds because $p_{\mathcal{U}} = p_0$. Then, for $(1-\tau)p_{\mathcal{U}} \ge b_0$, it is equivalent to show:

$$\tau^\star \geq 1 - \frac{w_0 - b_0}{\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)}.$$

The above inequality is equivalent to:

$$\frac{w_0 - b_0}{\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} \geq \frac{w_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)}.$$

To show the above inequality hold under $b_0 < p_0 - (q_0 - \gamma)$ (Assumption 1), it suffices to show that

$$\frac{w_0 - \big(p_0 - (q_0 - \gamma)\big)}{\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} \geq \frac{w_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)}.$$

Notice that the above inequality is equivalent to:

$$\frac{p_0}{q_0 - \gamma} \leq 1 + \frac{p_0}{p_0 + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)} \cdot \frac{w_0}{q_0 - \gamma}.$$

Since $p_0 > q_0 - \gamma$, to show the above inequality, it suffices to show the following holds (we substitute p_0 with $q_0 - \gamma$):

$$\frac{p_0}{q_0 - \gamma} \leq 1 + \frac{w_0}{q_0 - \gamma + \frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)},$$

which satisfies Assumption 2. Lastly, for the free-entry condition to hold, it is equivalent to show the following equality holds (notice that $\eta = 1$ and $\delta_{\mathcal{U}}^{U} = \frac{1-\beta}{1-\beta+\beta\gamma}$ under E1):

$$\frac{\beta\gamma}{1-\beta} \cdot \frac{1-\underline{\eta}}{1 + \frac{q_0-\gamma}{1-q_0}}\big((1-\tau)p_{\mathcal{H}} - w_0\big) = w_0 - (1-\tau)p_{\mathcal{U}},$$

where

$$(1-\tau)p_{\mathcal{H}} = w_0 + (1-\tau)\Big(\frac{1-\beta}{1-\beta+\beta\gamma} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\Big)\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(q_0-\gamma)\Big), \text{ and}$$

$$(1-\tau)p_{\mathcal{U}} = w_0 - (1-\tau)\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1 - q_0 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big).$$

Then, the free-entry condition becomes:

$$
\begin{aligned}
&(1-\tau)\frac{\beta\gamma}{1-\beta}\cdot\frac{1-\underline{\eta}}{1+\frac{q_0-\gamma}{1-q_0}}\Big(\frac{1-\beta}{1-\beta+\beta\gamma}+\frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\Big)\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big)\\
=&(1-\tau)\frac{\beta\gamma}{1-\beta+\beta\gamma}(1-\underline{\eta})\Big(1-q_0+\frac{1-\beta}{1-\beta+\beta\gamma}(1-\gamma)\Big).
\end{aligned}
$$

The above equality holds because it is straightforward to verify that the following equality always holds:

$$
\frac{\beta\gamma}{1-\beta}\frac{1}{1+\frac{q_0-\gamma}{1-q_0}}\Big(\frac{1-\beta}{1-\beta+\beta\gamma}+\frac{\beta\gamma}{1-\beta+\beta\gamma}\underline{\eta}\Big)=\frac{\beta\gamma}{1-\beta+\beta\gamma}.
$$

Next, we show that the equilibrium under $\lambda^\star$ and $\tau^\star$ is the same as the equilibrium under the optimal information-commission policy with differentiated commissions. In particular, by Proposition 3.6.4 and Lemma 3.6.9, we notice that when $\underline{\eta}<1/2$ the optimal information-commission policy with differentiated commissions has delay $\frac{q_0-\gamma}{\beta\gamma(1-q_0)}$, which is the same as $\lambda^\star$. By Lemma 3.6.6, it is straightforward to verify that under the optimal information-commission policy with differentiated commissions, the λ, η, $\delta^U_{\mathcal{U}}$, $p_{\mathcal{U}}$, and $p_{\mathcal{H}}$ are the same as those under $\lambda^\star$ and $\tau^\star$. Therefore, they have the same revenue by (3.43). In other words, $\lambda^\star$ and $\tau^\star$ is the optimal information-commission policy with single commission.

In the second step, we show that under $\lambda^\star$ and $\tau^\star$, the $\delta^H_{\mathcal{H}}$ is higher than that under the full-information policy. In particular, under $\lambda^\star$ and $\tau^\star$, the quantity of providers with label $\mathcal{H}$ is $\delta^{H\star}_{\mathcal{H}}=(1-\underline{\eta})\frac{\beta\gamma}{1-\beta+\beta\gamma}$. On the other hand, under the full-information policy, the quantity of providers with label $\mathcal{H}$ is $\delta^{H0}_{\mathcal{H}}=\frac{\beta\gamma}{1-\beta}\big(\frac{p_0-p_{\mathcal{U}}}{q_0-\gamma}-1\big)$ (under E3 or E4). To show $\delta^{H\star}_{\mathcal{H}}>\delta^{H0}_{\mathcal{H}}$, it suffices to show:

$$
\delta^{H\star}_{\mathcal{H}}>\frac{\beta\gamma}{1-\beta}\Big(\frac{p_0}{q_0-\gamma}-1\Big)>\delta^{H0}_{\mathcal{H}}.
$$

In fact, the first inequality holds because it is equivalent to $\frac{p_0}{q_0-\gamma} < 1 + \frac{1-\beta}{1-\beta+\beta\gamma}(1-\underline{\eta})$ (Assumption 2). Besides, the second inequality holds as $p_{\mathcal{U}} \geq 0$.

Therefore, we have shown that $\lambda^* > 0$ under the given conditions. Besides, under the optimal information-commission policy with single commission, more providers with label $\mathcal{H}$ are hired compared with the full-information policy. □

3.8.3 Proof of Proposition 3.4.3

First, we let $\underline{\theta} = 1 + \frac{1-\beta}{1-\beta+\beta\gamma} \in (1, 2)$. Then, Assumption 3 is equivalent to $p_0 > \frac{2(1-\beta)+\beta\gamma}{1-\beta+\beta\gamma}(q_0 - \gamma)$. Therefore, to characterize $\tau^*(0)$ and determine the equilibrium type, we only need to consider cases 3, 4, 5, 6, 7, and 8 in Proposition 3.6.2. Second, under condition $b_0/w_0 < \rho$, for some $\rho \in (0, 1)$, we only need to focus on cases 3 and 6 in Proposition 3.6.2, as other cases do not hold when b_0 is sufficiently small. Lastly, we show that under $\underline{\eta} < 1/2$, (3.36) holds. In particular, the right-handed side of (3.36) is less than w_0 because $\frac{3(1-\beta)+\beta\gamma}{2(1-\beta+\beta\gamma)} - \frac{1-\beta}{1-\beta+\beta\gamma}\frac{1}{\underline{\eta}}$ is negative under $\underline{\eta} < 1/2$ and $\gamma < \frac{1-\beta}{\beta}$. Then, by $p_0 > w_0$, we show (3.36) holds. Therefore, by Proposition 3.6.2, an E1 occurs under $\tau^*(0)$. By the definition of E1 (Definition 3.6.1), the platform serve all customers.

3.8.4 Proof of Proposition 3.4.4

The proposition holds because of Proposition 3.4.3 and Lemma 3.6.5.

3.8.5 Proof of Proposition 3.4.5

The proposition holds because of Proposition 3.6.3. □

3.8.6 Proof of Proposition 3.4.6

The proposition holds because of Corollary 3.6.1. □

Chapter 4: Demand Shocks and Supply Adjustment Friction in Two-sided Marketplaces

Abstract. We explore the effect of sellers' supply adjustment friction on two-sided marketplaces' reactions to unexpected demand shocks using an empirically-validated analytical model. In the model, sellers, which are heterogeneous in terms of their quality, engage in a quantity competition under a given demand. When the demand structure changes, sellers strategically adjust their supply to maximize their profit, incurring a cost for deviating from the original supply level. We find that sellers' strategic responses can either benefit or hurt the marketplace, and adjustment friction is an effective factor in influencing sellers' strategic decisions. By varying the adjustment friction, the marketplace can amplify positive effects under favorable demand shocks and reduce negative effect from unfavorable ones. In particular, a marketplace that maximizes the total revenue (social welfare) benefits from increasing (decreasing) the friction if the demand expands or the quality sensitivity level increases, and it benefits from decreasing (increasing) the friction if the demand shrinks or the quality sensitivity level decreases. We further validate our model empirically by testing its predictions regarding demand impacts on sellers based on data collected from a low-friction marketplace and empirical findings in a high-friction

marketplace documented in the literature.

Keywords: Two-sided marketplaces, matching supply and demand, quantity competition, difference-in-differences.

4.1 Introduction

The prosperity of online two-sided marketplaces, such as eBay, Airbnb, and Upwork, has highlighted the online platform as a successful business model, which promptly matches supply with demand at a broad scale and can grow virally. Some of them have become the most prominent players in many traditional markets, including retailing, and short-term rentals, and labor markets, once dominated by offline companies. Nevertheless, the operations of online marketplaces encounter new challenges that are little concerned by the traditional business models, such as retailing and manufacturing.

One primary challenge is rooted in the decentralized nature of online marketplaces. Marketplaces' value creation solely depends on matching supply and demand; however, marketplaces control neither of the components. On the demand side, buyers or consumers decide whether or not to join a given marketplace and which sellers to transact with. On the supply side, sellers or service providers decide how many products or services to list on the marketplace. Moreover, the transaction prices of many marketplaces (e.g., eBay and Airbnb) result from buyers' and sellers' decisions, and marketplaces lack direct levers to manipulate them.

Many marketplaces feature heterogeneity on both sides of the market. On the one hand, sellers differ not only horizontally by what products or services they offer but also vertically by the quality of their offerings. On the other hand, buyers make trade-offs between quality and price, and they differ in terms of their valuation for quality. Intuitively, between a set of products, buyers are willing to pay more for high-quality products compared with low-quality ones. We henceforth use *quality sensitivity* to characterize the trade-offs that buyers make between quality and price. In a proprietary data set collected from an online B2B marketplace, we further observe that buyers exhibit significantly different quantity sensitivities.

Moreover, among marketplaces specializing in different product types (e.g., merchandise, rental properties, and services), their sellers experience different levels of friction when they adjust their supply. On eBay, sellers are relatively flexible in adjusting their listed quantities. Relying on multiple sourcing channels, eBay sellers can quickly scale up or shrink their listings. By contrast, it is difficult for Airbnb hosts to modify the number of their listed properties. We show that the adjustment friction is a catalyst to market expansion in some cases, yet, an obstacle in others.

Like any business, marketplaces operate in a dynamic environment, which consists of unexpected demand shocks. Some demand shocks seem beneficial to marketplaces. For example, when brand or bulk sellers enter a marketplace, they bring their own buyers, who may later transact with other sellers in the market. Some demand shocks seem detrimental—for example, demand shrinks after the entry of competing marketplaces, natural disasters, or economic contraction. However, sellers' strategic reactions to demand shocks increase the uncertainties of the actual impacts on the marketplace. Therefore, it remains a challenge for marketplaces' managers to identify the impacts of unanticipated demand changes, and more importantly, magnify gains under favorable shocks and reduce loss under unfavorable ones.

Our first goal is to characterize how demand changes in size and quality sensitivity affect a marketplace's total revenue and social welfare. The second goal is to compare these effects between marketplaces with different adjustment friction. To achieve these goals, we develop an analytical model which specifies sellers' supply decision under adjustment friction and buyers choices. Moreover, it considers heterogeneity on both sides of the market and parameterizes demand's size and quality sensitivity. To validate the model, we derive a series of hypotheses based on the model and test them employing a proprietary data set collected from an online B2B liquidation marketplace.

First, we show that when the demand's sensitivity level increases, the marketplace's total revenue, and social welfare can improve even though the overall demand size shrinks. In other words, a demand contraction is not necessarily an unfavorable situation depending on the change of its sensitivity distribution. Second, we show that under the same demand shock, marketplaces with different adjustment friction are affected differently. After a favorable (unfavorable) demand change, such as demand expands (shrinks) and demand's sensitivity level increases (decreases), the total revenue increase (decrease) more when a marketplace has higher adjustment friction, while social welfare increase (decrease) more when a marketplace has less adjustment friction. Based on these observations, a revenue-maximization marketplace should increase (decrease) sellers' adjustment friction when the demand becomes favorable (unfavorable). A social-welfare-maximization marketplace should decrease (increase) sellers' adjustment friction when the demand becomes favorable (unfavorable).

4.1.1 Related literature

Our paper connects to the literature that examines market design levers under various demand structures. In a ride-sharing platform, [51] examine the effectiveness of spatial pricing under various demand networks. They show that the platform achieves the maximum profit when the demand pattern is "balanced". In the same platform, [52] characterize the optimal pricing under a demand shock.

Then, our paper is akin to the literature that investigates how demand-side friction impact buyers' behavior and the marketplace's performance. Using a data set collected from a holiday property-rental platform, [53] characterize the negative impact of customer's search friction, and how the friction is affected by the market thickness. In an online B2B marketplace, [54] design the listing policy with the consideration of buyers' participation cost. This work is one of the few papers investigating the impact of supply-side friction.

4.2 Theoretical Framework

We employ a two-period quantity competition in a vertically differentiated duopoly to analyze how sellers of a marketplace react to unanticipated demand changes. The demand varies across the two periods. The model characterizes sellers' adjustment friction, which sellers incur when deviating their supply in period 2 (after the demand change) from the supply level in period 1 (before the demand change). It also specifies demand changes in both size and sensitivity to seller quality. Using the model, we characterize demand effects on the total revenue and the social welfare of a marketplace. Furthermore, we establish that these effects vary substantially across marketplaces with different friction levels. In this section, we describe the model setup and the equilibrium concept.

4.2.1 Model setup

We assume there are two periods, denoted by $t \in \{1, 2\}$, and there are two sellers, denoted by seller L and seller H, in a marketplace. Sellers list the same generic product but differ regarding their quality. Seller H has a high-quality, θ_H, given that its product descriptions are accurate and credible. In comparison, seller L has a low-quality, θ_L, given that its product descriptions are ambiguous (i.e., $0 < \theta_L < \theta_H$). We then assume that both sellers have the same marginal cost, and we normalize it to zero.

At a given period, seller i's profit, where $i \in \{L, H\}$, is characterized as follows:

$$V_{i,t} = \begin{cases} p_{i,1}Q_{i,1}, & \text{if } t = 1 \\ p_{i,2}Q_{i,2} - c(Q_{i,2} - Q_{i,1})^2, & \text{if } t = 2 \end{cases} \tag{4.1}$$

where $Q_{i,t}$ denotes its listed quantity, and $p_{i,t}$ denotes the price of its product. Notice that at $t = 1$, sellers are not forward-looking as they do not expect the demand change in the next period. After the unanticipated demand change at $t = 2$, seller i adjusts its listed quantity $Q_{i,2}$ to maximize its profit. At this period, term $c(Q_i - Q_{i,0})^2$ denotes the cost incurred by making the supply adjustment, where $Q_{i,1}$ is the supply level at period 1.

At period t, the demand consists of a continuum of buyers with mass $\mu_t > 0$ who are heterogeneous regarding their quality sensitivities. We let x denote a buyer's quality sensitivity, and x is drawn from a cumulative distribution function (CDF), denoted by $F_t(x)$, with support on $[0, 1]$. Each buyer's demand is infinitesimally small and hence denoted by dx. Their utility derived from transacting with a seller with quality θ or taking the outside option is:

$$U_t(x, i) = \begin{cases} x\theta_i - p_{i,t}, & \text{if } i \in \{L, H\}\text{, i.e., choosing seller with quality } \theta_i, \\ 0, & \text{if } i = 0\text{, i.e., choosing the outside option.} \end{cases} \tag{4.2}$$

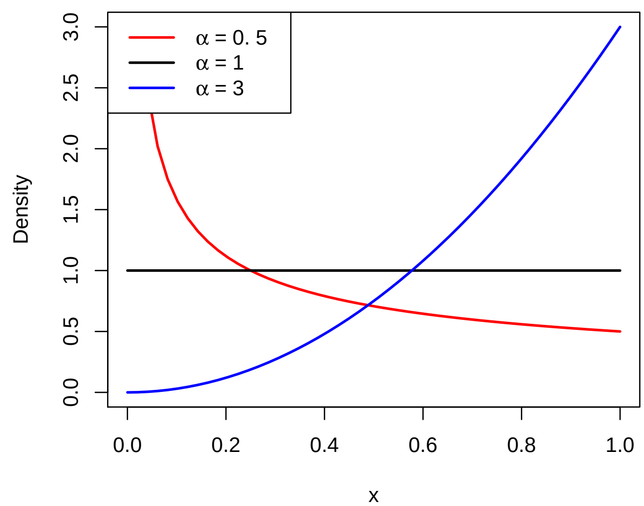
If the buyer transacts with a seller, $x\theta_i$ is the buyer's willingness-to-pay, and $p_{i,t}$ is the transaction price.

Both the size and sensitivity of the demand may change. First, we change μ to capture the change of demand size after the site launch. If the marketplace's demand expands (shrinks), we assume that μ increases (decreases) in period 2. Second, we alter F to capture the changes in demand sensitivity. If the demand sensitivity increases (decreases), we make F skew to the left (right) in period 2. To obtain crystal implications of demand sensitivity effects, we make the following assumption:

Assumption 4 *The sensitivity distribution F changes within the following class of distributions:*

$$\mathcal{F} = \{\mathcal{F}_\alpha(\S) | \textit{CDF satisfies: } \mathcal{F}_\alpha(\S) = \S^\alpha, \textit{where } \S \in [/, \infty] \textit{ and } \alpha > /\},$$

Figure 4.1: Probability density functions of three examples in $\mathcal{F}$.



where we call α the sensitivity parameter.

We make this assumption for two reasons: (1) The class includes a broad spectrum of distributions on [0, 1]. Its elements cover both left-skewed (i.e., $\alpha > 1$) and right-skewed (i.e., $\alpha < 1$) distributions (as demonstrated in Figure 4.1). In addition, the uniform [0, 1], which is commonly assumed in the literature on competition with vertical differentiation, is in the class (i.e., $\alpha = 1$). (2) All distributions in $\mathcal{F}$ are ranked based on their sensitivity. In particular, if $F_{\alpha_1}, F_{\alpha_2} \in \mathcal{F}$ and $\alpha_1 < \alpha_2$, then F_{α_2} first-order stochastically dominates F_{α_1}, indicating that given $x_0 \geq 0$, the percentage of buyers with sensitivity $x > x_0$ in F_{α_2} is higher than that of buyers with $x > x_0$ in F_{α_1}. That is, the sensitivity parameter α fully characterizes the rank of the distributions in $\mathcal{F}$.

4.2.2 Equilibrium

For the notation associated with seller $i \in \{L, H\}$ at period $t \in \{1, 2\}$, we let $Q_{i,t}$ denote its quantity, $p_{i,t}$ denote the price of its products, and $D_{i,t}$ denote the mass of customers, who transact with it. Using the notation, we formally introduce the notion of equilibrium:

Definition 4.2.1 *An equilibrium under a demand change from $\{\mu_1, \alpha_1\}$ to $\{\mu_2, \alpha_2\}$ consists of $\{Q_{i,t}, p_{i,t}, D_{i,t}\}$, where $i \in \{L, H\}$ and $t \in \{1, 2\}$, such that at given period t:*

- *Sellers set their quantity $Q_{i,t}$ to maximize their $V_{i,t}$.*
- *Customer x makes choice to maximize their $U_t(x, i)$.*
- *The demand and supply match (i.e., $D_{L,t} = Q_{L,t}$ and $D_{H,t} = Q_{H,t}$).*

First of all, the above defined equilibrium exists.

Proposition 4.2.1 *An equilibrium exists under any $\{\mu_1, \alpha_1\}$, $\{\mu_2, \alpha_2\}$, and c.*

Then, we characterize the equilibrium prices as follows:

Lemma 4.2.1 *Suppose Assumption 4 holds. Then, the equilibrium prices can be characterized as:*

$$p_{L,t} = \theta_L\Big(1 - \frac{Q_{L,t}}{\mu_t} - \frac{Q_{H,t}}{\mu_t}\Big)^{\frac{1}{\alpha_t}} \text{ and } p_{H,t} = (\theta_H - \theta_L)\Big(1 - \frac{Q_{H,t}}{\mu_t}\Big)^{\frac{1}{\alpha_t}} + \theta_L\Big(1 - \frac{Q_{L,t}}{\mu_t} - \frac{Q_{H,t}}{\mu_t}\Big)^{\frac{1}{\alpha_t}}.$$

where $t \in \{1, 2\}$.

Besides, we characterize buyers' equilibrium choice as follows:

Lemma 4.2.2 *In equilibrium, buyers with quality sensitivity* $x \in [1 - Q_{H,t}, 1]$ *transact with the high-quality seller, buyers with quality sensitivity* $x \in [1 - Q_{L,t} - Q_{H,t}, 1 - Q_{H,t})$ *transact with the low-quality seller, and the remaining buyers choose the outside option.*

Lastly, in two stylized cases, where $c = 0$ and $c = +\infty$, we provide the closed-form characterizations of sellers' quantity under any demand structure.

Lemma 4.2.3 *Suppose Assumption 4 holds. Then, the equilibrium quantities at* $t = 1$ *are:*

$$Q_{L,1} = Q_L(\mu_1, \alpha_1) \text{ and } Q_{H,1} = Q_H(\mu_1, \alpha_1),$$

where $Q_L(\mu, \alpha) = \frac{\alpha\mu}{1+\alpha+\alpha^2 A(\alpha)}$, $Q_H(\mu, \alpha) = \frac{\alpha^2 A(\alpha)\mu}{1+\alpha+\alpha^2 A(\alpha)}$, *and* $A(\alpha) = \frac{\theta_L \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}} + (\theta_H - \theta_L)\left(1+\frac{1}{\alpha}\right)^{\frac{1}{\alpha}}}{\theta_L \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}-1} + (\theta_H - \theta_L)\left(1+\frac{1}{\alpha}\right)^{\frac{1}{\alpha}-1}}$. *For equilibrium quantities at* $t = 2$,

- *If* $c = 0$, *then* $Q_{L,2} = Q_L(\mu_2, \alpha_2)$ *and* $Q_{H,2} = Q_H(\mu_2, \alpha_2)$.

- *If* $c = +\infty$, *then* $Q_{L,2} = Q_{L,1}$ *and* $Q_{H,2} = Q_{H,1}$.

4.3 Impacts of demand and adjustment friction

For the growth of many online marketplaces, they focus on maximizing either the total revenue or social welfare. In this section, we first characterize various demand impacts on the total revenue and the social welfare of a given marketplace. We show that the marketplace can benefit from an increase in its demand sensitivity even though its demand size shrinks simultaneously. Moreover, we compare these demand impacts across different adjustment friction. To our surprise, marketplaces with high adjustment friction benefit from some demand changes more than those with low friction.

Our analyses focus on two stylized types of marketplaces depending on how difficult a seller can adjust its supply level instantly in response to unexpected demand variation. The first type features zero adjustment friction (i.e., $c = 0$), where sellers can immediately adjust their quantity after a demand change. Merchandise marketplaces, such as eBay and Amazon, normally belong to this type, where sellers vary their number of listings to react to demand dynamics. The second type features infinite friction (i.e., $c = +\infty$), where it is hugely costly for sellers to change their supply within a short period. Service platforms, such as Upwork and TaskRabbit, fall into this category, where the number of tasks listed per user is inelastic to demand dynamics Cu20.

Under Assumption 4, the total revenue, denoted by π, and the social welfare, denoted by σ, at period t can be expressed as:

$$\pi(Q_{L,t}, Q_{H,t}, \alpha_t, \mu_t) = \theta_L(Q_{L,t} + Q_{H,t})\Big(1 - \frac{Q_{L,t}}{\mu_t} - \frac{Q_{H,t}}{\mu_t}\Big)^{\frac{1}{\alpha_t}} + (\theta_H - \theta_L)Q_{H,t}\Big(1 - \frac{Q_{H,t}}{\mu_t}\Big)^{\frac{1}{\alpha_t}}, \tag{4.3}$$

and $$\sigma(Q_{L,t}, Q_{H,t}, \alpha_t, \mu_t) = \mu_t \frac{\alpha_t}{\alpha_t + 1}\left(\theta_H - (\theta_H - \theta_L)\Big(1 - \frac{Q_{H,t}}{\mu_t}\Big)^{1+\frac{1}{\alpha_t}} - \theta_L\Big(1 - \frac{Q_{L,t}}{\mu_t} - \frac{Q_{H,t}}{\mu_t}\Big)^{1+\frac{1}{\alpha_t}}\right) \tag{4.4}$$

Besides, Lemma 4.2.3 further specifies the equilibrium quantities of sellers at period 2 for both friction types. In Sections 4.3.1 and 4.3.2, we analyze the total revenue and the social welfare, separately.

4.3.1 Total revenue

In this section, we focus on the effects of demand changes on the total revenue of a given marketplace, which is characterized by Expression (4.3).

First, Proposition 4.3.1 provides a sufficient and necessary condition regarding α_t and μ_t such that a frictionless marketplace's total revenue increases after the demand change.

Proposition 4.3.1 *Suppose Assumption 4 holds and $c = 0$. Then, the total revenue increases after the demand change if and only if $\mu_2/\mu_1 > \tilde{\pi}(\alpha_1)/\tilde{\pi}(\alpha_2)$, where*

$$\tilde{\pi}(\alpha) = \alpha(1 + \alpha + \alpha^2 A(\alpha))^{-1-\frac{1}{\alpha}} \big(\theta_L(1 + \alpha A(\alpha)) + \alpha A(\alpha)(\theta_H - \theta_L)(1 + \alpha)^{\frac{1}{\alpha}}\big).$$

Besides, $\tilde{\pi}(\alpha)$ is increasing in α.

Proposition 4.3.1 implies that the total revenue is increasing in the sensitivity of the demand. In particular, if the demand sensitivity increases (reduces), the total revenue may increase (decrease), although the demand size shrinks (expands).

Second, we compare the revenue effect of demand changes between marketplaces with different adjustment friction. Notice that if the demand size or sensitivity changes slightly (i.e., $\mu_2 = \mu_1 + \epsilon$ or $\alpha_2 = \alpha_1 + \epsilon$, where ϵ is a infinitesimal amount), the total revenue changes by $\frac{d\pi}{d\mu}\epsilon$ and $\frac{d\pi}{d\alpha}\epsilon$, respectively, where

$$\frac{d\pi}{d\mu} = \frac{\partial\pi}{\partial\mu} + \frac{\partial\pi}{\partial Q_L}\frac{dQ_L}{d\mu} + \frac{\partial\pi}{\partial\mu}\frac{dQ_H}{d\mu} \text{ and } \frac{d\pi}{d\alpha} = \frac{\partial\pi}{\partial\alpha} + \frac{\partial\pi}{\partial Q_L}\frac{dQ_L}{d\alpha} + \frac{\partial\pi}{\partial\alpha}\frac{dQ_H}{d\alpha}.$$

Therefore, $\frac{d\pi}{d\mu}$ and $\frac{d\pi}{d\alpha}$ captures the normalized total revenue effects of demand changes, and they vary across marketplaces with different adjustment friction. In particular, we compare the demand effects between a marketplace with zero adjustment friction (i.e., $c = 0$) and a marketplace with infinite friction (i.e., $c = \infty$). For the notation, we let $\left(\frac{d\pi}{d\mu}\right)_{c=0}$ and $\left(\frac{d\pi}{d\alpha}\right)_{c=0}$ denote the demand effects in the frictionless marketplace, and we let $\left(\frac{d\pi}{d\mu}\right)_{c=\infty}$ and $\left(\frac{d\pi}{d\alpha}\right)_{c=\infty}$ denote the demand effects in the infinite-friction marketplace. Then, we have the following observations regarding the comparison between the two types:

Proposition 4.3.2 *Suppose Assumption 4 holds. Then,*

$$\left(\frac{d\pi}{d\mu}\right)_{c=\infty} > \left(\frac{d\pi}{d\mu}\right)_{c=0} > 0 \textit{ and } \left(\frac{d\pi}{d\alpha}\right)_{c=\infty} > \left(\frac{d\pi}{d\alpha}\right)_{c=0} > 0. \tag{4.5}$$

When the demand enlarges or becomes more sensitive, Proposition 4.3.2 implies that the total revenue increases in both infinite-friction and frictionless cases. Moreover, it suggests that the revenue increase in the infinite-friction case is higher than that of the frictionless case. In other words, it would benefit the total revenue if the platform can deter sellers' strategic reactions to the demand change and make their supply remain stable.

When the demand contracts or becomes less sensitive, Proposition 4.3.2 implies that the total revenue decreases in both cases. Moreover, it suggests that the revenue decrease in the infinite-friction case is higher than that of the frictionless case. In other words, it would benefit the total revenue if the platform can encourage sellers' strategic reactions to the demand change and let them adjust their supply promptly.

4.3.2 Social welfare

In this section, we focus on the effects of demand changes on the social welfare of a given online marketplace, which is characterized by Expression (4.4).

Proposition 4.3.3 provides a sufficient and necessary condition regarding α_t and μ_t such that a frictionless marketplace's social welfare increases after the demand change.

Proposition 4.3.3 *Suppose Assumption 4 holds and $c = 0$. Then, the social welfare increases after the demand change if and only if $\mu_2/\mu_1 > \tilde{\sigma}(\alpha_1)/\tilde{\sigma}(\alpha_2)$, where*

$$\tilde{\sigma}(\alpha) = \frac{\alpha}{1+\alpha}\left(\theta_H - (\theta_H - \theta_L)\Big(\frac{1+\alpha}{1+\alpha+\alpha^2 A(\alpha)}\Big)^{1+\frac{1}{\alpha}} - \theta_L\Big(\frac{1}{1+\alpha+\alpha^2 A(\alpha)}\Big)^{1+\frac{1}{\alpha}}\right).$$

Besides, $\tilde{\sigma}(\alpha)$ is increasing in α.

Proposition 4.3.3 implies that social welfare is increasing in the sensitivity of the demand. In particular, if the demand sensitivity increases (reduces), social welfare may increase (decrease), although the demand size shrinks (expands).

Second, we compare the social welfare effect of demand changes between marketplaces with different adjustment friction. Notice that if the demand size or sensitivity changes slightly (i.e., $\mu_2 = \mu_1 + \epsilon$ or $\alpha_2 = \alpha_1 + \epsilon$, where ϵ is a infinitesimal amount), the social welfare changes by $\frac{d\sigma}{d\mu}\epsilon$ and $\frac{d\sigma}{d\alpha}\epsilon$, respectively, where

$$\frac{d\sigma}{d\mu} = \frac{\partial\sigma}{\partial\mu} + \frac{\partial\sigma}{\partial Q_L}\frac{dQ_L}{d\mu} + \frac{\partial\sigma}{\partial\mu}\frac{dQ_H}{d\mu} \text{ and } \frac{d\sigma}{d\alpha} = \frac{\partial\sigma}{\partial\alpha} + \frac{\partial\sigma}{\partial Q_L}\frac{dQ_L}{d\alpha} + \frac{\partial\sigma}{\partial\alpha}\frac{dQ_H}{d\alpha}.$$

Therefore, $\frac{d\sigma}{d\mu}$ and $\frac{d\sigma}{d\alpha}$ captures the normalized social welfare effects of demand changes, and they vary across marketplaces with different adjustment friction. Similar to the total revenue analysis, we compare the demand effects on social welfare between a marketplace with zero adjustment friction (i.e., $c = 0$) and a marketplace with infinite friction (i.e., $c = \infty$). For the notation, we let $\left(\frac{d\sigma}{d\mu}\right)_{c=0}$ and $\left(\frac{d\sigma}{d\alpha}\right)_{c=0}$ denote the demand effects in the frictionless marketplace, and we let $\left(\frac{d\sigma}{d\mu}\right)_{c=\infty}$ and $\left(\frac{d\sigma}{d\alpha}\right)_{c=\infty}$ denote the demand effects in the infinite-friction marketplace. Then, we have the following observations regarding the comparison between the two types:

Proposition 4.3.4 *Suppose Assumption 4 holds. Then,*

$$\left(\frac{d\sigma}{d\alpha}\right)_{c=0} > \left(\frac{d\sigma}{d\alpha}\right)_{c=\infty} > 0 \textit{ and } \left(\frac{d\sigma}{d\mu}\right)_{c=0} > \left(\frac{d\sigma}{d\mu}\right)_{c=\infty} > 0. \tag{4.6}$$

When the demand enlarges or becomes more sensitive, Proposition 4.3.4 implies that social welfare increases in both infinite-friction and frictionless cases. Moreover, it suggests that the increase in the frictionless case is higher than that of the infinite-friction case. In other words, it would benefit social welfare if the platform can encourage sellers' strategic reactions to the demand change and let them adjust their supply promptly.

When the demand contracts or becomes less sensitive, Proposition 4.3.4 implies that the total revenue decreases in both infinite-friction and frictionless cases. Moreover, it suggests that the decrease in the frictionless case is higher than that of the infinite-friction case. In other words, it would benefit social welfare if the platform can deter sellers' strategic reactions to the demand change and stabilize their supply.

4.4 Model validation by hypothesis testing

Using a proprietary data set collected from an online marketplace, where sellers have considerable flexibility to adjust their supply, we conduct a series of hypothesis testing to validate our theoretical model. The data set records two types of demand changes in the marketplace, and we observe that sellers adjust their supply instantly in response. In particular, we validate our model by testing two groups of hypotheses.

The first set of hypotheses are associated with marketplaces with significant adjustment friction. In this case, we develop hypotheses regarding sellers' average listed quantities and their changing rates of revenue after demand shifts. By empirically rejecting these hypotheses, we verify the alternative case characterized by the model, which corresponds to marketplaces with small friction.

The second set of hypotheses are associated with marketplaces with small adjustment friction. In this case, we develop hypotheses regarding sellers' average listed quantities, their changing rates of quantity and revenue after demand shifts. By confirming these hypotheses using consistent empirical observations, we defend the validity of the model specification.

4.4.1 Hypotheses of marketplaces with large friction

Lemma 4.2.3 states that sellers do not change their quantity to react to any demand changes when the adjustment friction is infinite. Based on the observation, we derive the following hypothesis regarding sellers' responses in marketplaces with large adjustment friction: In a marketplace with large adjustment friction, sellers do not significantly change their listed quantities under any demand change.

Next, we derive hypotheses regarding how sellers' revenues are affected under the large adjustment friction. We let $r_i(\mu_1, \mu_2, \alpha_1, \alpha_2, c) = \frac{p_{i,2}Q_{i,2}}{p_{i,1}Q_{i,1}}$, where $i \in \{L, H\}$, denote the revenue changing rate of seller i after the demand change. Notice that r_i depends on not only the demand structures in both periods but also the adjustment friction c. Then, we have the following observations regarding r_i in a marketplace with infinite friction.

Corollary 4.4.1 *Suppose Assumption 4 holds. Then,*

- *If $\alpha_2 > \alpha_1$ and $\mu_2 > \mu_1$, then $r_H < r_L$.*

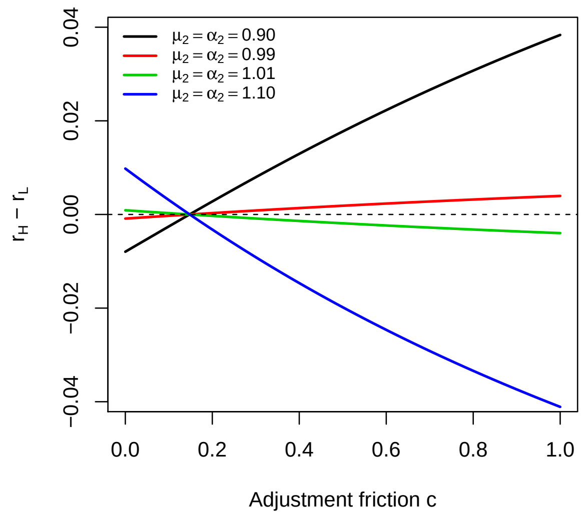
- *If $\alpha_2 < \alpha_1$ and $\mu_2 < \mu_1$, then $r_H > r_L$.*

Corollary 4.4.1 indicates that when it is difficult for sellers to adjust their quantity to react to sudden demand changes, the revenue changing rate of seller H is lower (higher) than that of seller L, when the demand expands (shrinks), and its sensitivity increases more (less).

Moreover, the patterns revealed by Corollary 4.4.1 hold for sufficiently large but finite c. In Figure 4.2, we numerically compute $r_H - r_L$ as we increase c under different demand changes. As evident in the figure, when c is large, it follows that $r_H < r_L$ when the demand expands and becomes more sensitive (i.e., the green and blue curve) and $r_H > r_L$ when the demand shrinks and becomes less sensitive. Therefore, by Corollary 4.4.1 and Figure 4.2, we derive Hypothesis 4.4.1.

In a marketplace with large adjustment friction:

Figure 4.2: Difference between r_H and r_L under different demand changes from $\mu_1 = \alpha_1 = 1$



A. If its demand expands and becomes more sensitive, then the revenue changing rate of high-quality sellers will be lower than that of their low-quality peers. That is, $r_H < r_L$.

B. If its demand shrinks and becomes less sensitive, then the revenue changing rate of high-quality sellers will be higher than that of their low-quality peers. That is, $r_H > r_L$.

In Section 4.5, we describe the empirical setting and the data collected from a low-friction marketplace, which we employ to test and reject Hypotheses 4.4.1 and 4.4.1.

4.4.2 Hypotheses of marketplaces with small friction

Lemma 4.2.3 characterizes sellers' quantity reactions to demand changes when the adjustment friction is zero. Then, we observe the following regarding sellers' average quantity:

Corollary 4.4.2 *Suppose Assumption 4 holds and* $c = 0$*. Then,*

1. *If* $\mu_2 > \mu_1$ *and* $\alpha_2 > \alpha_1$*, then* $\frac{1}{2}(Q_{H,2} + Q_{L,2}) > \frac{1}{2}(Q_{H,1} + Q_{L,1})$*.*

2. *If* $\mu_2 < \mu_1$ *and* $\alpha_2 < \alpha_1$*, then* $\frac{1}{2}(Q_{H,2} + Q_{L,2}) < \frac{1}{2}(Q_{H,1} + Q_{L,1})$*.*

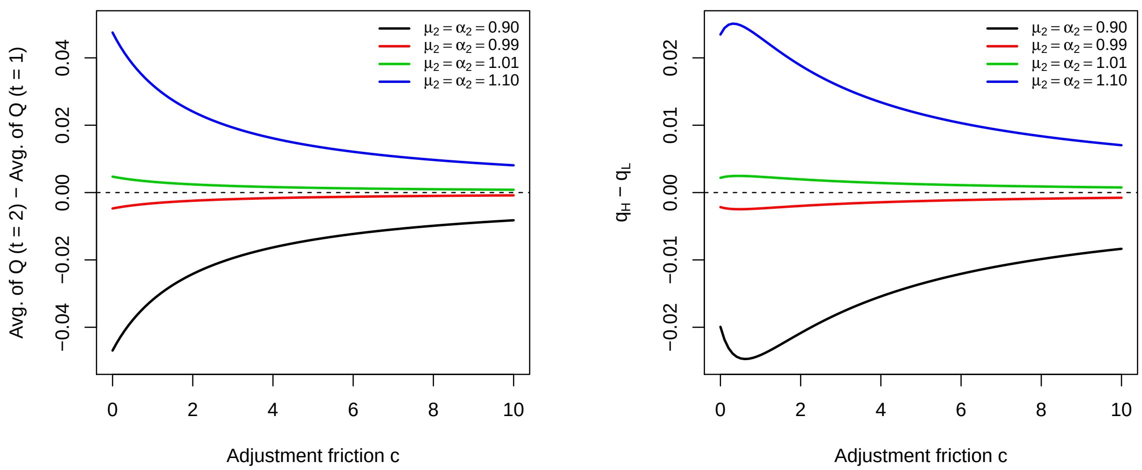
Corollary 4.4.2 implies that when sellers can flexibly adjust their quantity, their average quantity is increasing in the demand size and sensitivity. Moreover, we numerically compute the differences of the average quantity between periods as we increase c and display them in the left plot of Figure 4.3. Notice that the pattern revealed by Corollary 4.4.2 also holds for $c > 0$. Moreover, the difference of average quantity shrinks as we increase c. It implies that in marketplaces with small adjustment friction, we expect significant changes in the average quantity after demand changes. By Corollary 4.4.2 and Figure 4.3, we derive Hypothesis 4.4.2.

In a marketplace with small adjustment friction:

A. If its demand expands and its sensitivity level increases, then the average listed quantity per seller increases significantly.

B. If its demand shrinks and its sensitivity level decreases, then the average listed quantity per seller decreases significantly.

Figure 4.3: Difference between the average quantities (left) and difference between q_H and q_L (right) under different demand changes from $\mu_1 = \alpha_1 = 1$



Next, we derive hypotheses regarding how the quantity of sellers with various qualities are affected differently when the adjustment friction is small. We let $q_i(\mu_1, \mu_2, \alpha_1, \alpha_2, c) = \frac{Q_{i,2}}{Q_{i,1}}$, where $i \in \{L, H\}$, denote the quantity changing rate of seller i after the demand change. Notice that q_i depends on not only the demand structures in both periods but also the adjustment friction c. Then, we have the following observations regarding q_i in a frictionless marketplace.

Corollary 4.4.3 *Suppose Assumption 4 holds, and* $c = 0$*. Then,*

- *If* $\alpha_2 > \alpha_1$*, then* $q_H > q_L$*.*
- *If* $\alpha_2 < \alpha_1$*, then* $q_H < q_L$*.*

Corollary 4.4.3 implies that in a marketplace with zero adjustment friction, the quantity changing rate of the high-quality seller is higher than that of the low-quality seller when the demand becomes more sensitive. On the other hand, the high-quality seller's quantity changing rate lower than that of the low-quality seller when the demand becomes less sensitive. Moreover, for positive c, we numerically compute $q_H - q_L$ under several demand changes in the right plot in Figure 4.3. The plot shows that $q_H > q_L$ when the demand's sensitivity increases and $q_H < q_L$ when the demand's sensitivity decreases, and the patterns are consistent with Corollary 4.4.3. By Corollary 4.4.3 and Figure 4.3, we derive Hypothesis 4.4.2.

In a marketplace with small adjustment friction:

A. If the sensitivity level of its demand increases, then the quantity changing rate of high-quality sellers will be higher than that of their low-quality peers. That is, $q_H > q_L$.

B. If the sensitivity level of its demand decreases, then the quantity changing rate of high-quality sellers will be lower than that of their low-quality peers. That is, $q_H < q_L$.

Lastly, we derive hypotheses regarding how sellers' revenues are affected when the adjustment friction is small. We first observe the following in a frictionless marketplace:

Corollary 4.4.4 *Suppose Assumption 4 holds, ratio θ_H/θ_L is sufficiently large, and $c = 0$. Then,*

- *If $\alpha_2 > \alpha_1$, then $r_H > r_L$.*

- *If $\alpha_2 < \alpha_1$, then $r_H < r_L$.*

Corollary 4.4.4 implies that when sellers can freely adjust their quantity to react to demand changes, the revenue changing rate of seller H is higher (lower) than that of seller L, when the demand's sensitivity level increases (decreases).

Moreover, the patterns revealed by Corollary 4.4.4 hold for sufficiently small but positive c. As evident in Figure 4.2, when c is small, it follows that $r_H > r_L$ when the demand becomes more sensitive (i.e., the green and blue curves) and $r_H < r_L$ when the demand becomes less sensitive (i.e., the black and red curves). Therefore, by Corollary 4.4.4 and Figure 4.2, we derive Hypothesis 4.4.2.

In a marketplace with small adjustment friction:

A. If the sensitivity level of its demand increases, then the revenue changing rate of high-quality unbranded sellers will be higher than that of their low-quality peers. That is, $r_H > r_L$.

B. If the sensitivity level of its demand decreases, then the revenue changing rate of high-quality unbranded sellers will be lower than that of their low-quality peers. That is, $r_H < r_L$.

In Section 4.5, we describe the empirical setting and the data collected from a low-friction marketplace, which we employ to test and verify Hypotheses 4.4.2, 4.4.2, and 4.4.2.

4.5 Data and empirical setting

To test the hypotheses developed in Section 4.4, we collect proprietary data from a leading B2B liquidation platform that hosts multiple online auction marketplaces for retailers to liquidate their big-box salvaging inventory through ascending English auctions.

The platform owns a two-sided marketplace, where all small and midsize retailers list and sell their salvaging inventory. It is open to all buyers and sellers to participate, so we refer it as a *public marketplace*. The public marketplace will be the subject of our empirical analysis, and we will describe it further in Section 4.5.1.

Besides, the platform helps branded retailers, such as Walmart, Costco, and Home Depot, launch their own marketplaces for liquidation, which we refer to as *private sites*. A private site is one-sided, where only one branded retailer can list and sell its merchandise. The platform names each private site after the retailer's brand. After branded retailers launch their private sites, they direct their inventory and buyers to the platform's ecosystem. We exploit the launches of two private sites as exogenous shocks to the public marketplace, which we will specify in Section 4.5.2.

Lastly, we outline the procedures for hypothesis testing. First, we identify the demand impacts on the public marketplace resulted from each launch, and the corresponding methods are specified in Section 4.5.3. Second, we test all hypotheses by identifying sellers' reactions and how their revenues are affected under the identified demand impacts. We describe the methods employed in this step in Section 4.5.4.

4.5.1 Public marketplace

We describe the public marketplace by specifying its participants on the supply and demand sides, as well as their characteristics.

Sellers.

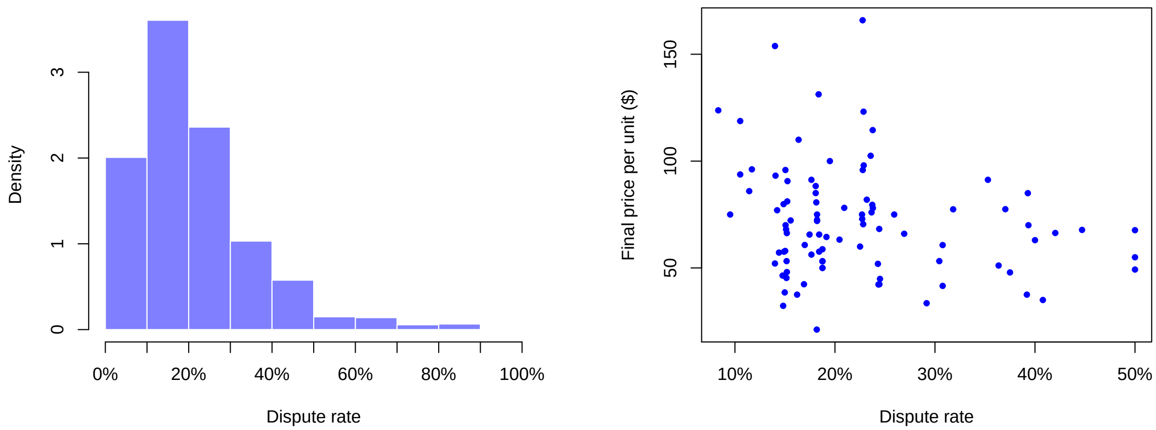
We refer to all the retailers in the public marketplace as *sellers*. Most sellers liquidate a variety of product categories, such as consumer electronics, apparel, and appliances. Sellers create auction listings for their inventory to transact with buyers. A listing contains an auction of multiple products for sale, which typically belong to the same category and have similar conditions. The listing reveals necessary product information, including product names, quantities, and conditions. Listings serve as the only intermediary, where sellers and buyers interact on the platform.

Since auctions determine the prices, the listed quantities become the essential lever for sellers to react to market dynamics. We observe that sellers vary their number of monthly listings substantially in response to the launches of private sites (specified in Section 4.5.2), which implies that sellers in this marketplace incur little friction when adjusting their supply.

Sellers' quality.

Since salvage inventory items come in a variety of different conditions, product information disclosed within listings is instrumental to buyers' evaluation of these products and the decision process. Depending on how accurately each seller describes their products, buyers may later find that the received merchandise is not as it was described in the listing. Hence, buyers bear a risk while transacting with sellers whose listing information has low quality. We then use sellers' dispute rates, added by the platform to each of their listings, to approximate the quality of their

Figure 4.4: Unbranded sellers' dispute rates (left) and buyers' quality-price trade-offs in purchasing salvage iPhone 6 (2016) (right).



listings. If a buyer finds the purchased merchandise not as described, the buyer can file a dispute with the seller. Therefore, sellers' dispute rates capture buyers' feedback on how accurately their listing information describes the products.[1]

Given the central role of listings in the communication between sellers and buyers, buyers discern sellers by their listing qualities. We henceforth use *seller quality* to refer to sellers' qualities of listings and measure the quality of a given seller by its dispute rate. In the left plot of Figure 4.4, we notice that sellers' qualities vary widely in the public marketplace.

[1]In addition to dispute rates, the platform includes other metrics of sellers in their listings such as tenure on the platform, cumulative transactions, and the number of purchases from repeat buyers. Among these metrics, we believe that the dispute rate characterizes listing qualities most accurately.

Buyers and their quality sensitivity

The demand on the platform consists of business buyers, including downstream resellers and refurbishers from across the globe (henceforth referred to as *buyers*). All buyers are for-profit downstream businesses that would prefer to get desired products at the lowest prices. Besides, buyers specialize in different product categories, and some product categories, such as consumer electronics and appliances, have little overlap in terms of the buyers.

In the public marketplace, we observe that buyers make trade-offs between seller quality and price. Among the same type of products, buyers are drawn to listings from high-quality sellers and pay more for their products than those listed by low-quality sellers. This pattern indicates that buyers' willingness-to-pay depends on the seller's quality. We henceforth use *quality sensitivity* to characterize the trade-offs that buyers make between seller quality and price. In the right plot of Figure 4.4, we use salvage iPhone 6 transactions in 2016 as an example to illustrate the quality-price trade-offs that buyers make. The correlation between the final price per unit and the dispute rate is -0.21, which indicates that high (low) seller quality is associated with a high (low) final price per unit.

4.5.2 Launches of private sites

We can view the launches of private sites as exogenous shocks to the public marketplace. Their launching decisions and dates depend on a confidential negotiation process between the branded retailer and the platform, which relates little to the public marketplace. This process may last from weeks to months. Meanwhile, unbranded sellers and all buyers have no clue whether or when a new marketplace will appear.

We build our analysis on two launches of private sites specializing in consumer electronics (henceforth referred to as *CE*).[2] The first launch involves two branded CE retailers who launched their sites on the platform in January and March of 2013, respectively. Since these two launching dates are so close, we consider them as one launch. Both of the private sites have low dispute rates (zero) than the public marketplace, whose dispute rate is 13.3%. Then, we infer that the listing quality in these private sites is higher than the public marketplace. The second launch involves another branded CE retailer who launched its private site in July of 2015. It has particularly a very high dispute rate of 44.4%, which indicates that its listing quality is low.

We exploit the demand shocks resulting from these two launches to test the hypotheses developed in Section 4.4. Both private sites specialize in CE, so their launches primarily affect the public marketplace's supply and demand of the same category. It is worthwhile to mention that once a branded retailer launches its private site, it starts attracting its buyer base to the platform. Since buyers can cross bid in any private sites and the public marketplace, the entry of a private site impacts the public marketplace by either bolstering or weakening its demand, which

[2] Some private sites liquidate merchandise from multiple product categories (e.g., Walmart lists auctions of furniture and apparel), while others specialize in one category (e.g., Macy's only liquidates apparel).

subsequently affects its sellers' behavior.

To participate in the public marketplace or a private site, buyers need to register an account with that marketplace. Using buyers' first registered marketplaces, we can distinguish buyers based on which marketplace brings them to the platform. In this way, we identify the following two groups of buyers for both launches, respectively:

- Original Buyer: a buyer whose first registration is with the public marketplace.
- New Buyer: a buyer whose first registration is with the newly-launched private site.

4.5.3 Identification of private sites' demand impacts

Before testing the hypotheses, which involves sellers' reactions under a given demand shock, we first identify how these two launches affect the public marketplace's demand. Specifically, we characterize the two launches' impacts on the demand size (i.e., μ) and sensitivity to quality (i.e., F) of the public marketplace.

For the effects on μ, we compare the buyer inflow from the newly-launched private site to the public marketplace with the buyer outflow that moves in the reverse direction. In particular, we contrast the New Buyers from the newly-launched private site who transact in the public marketplace (i.e., inflow) with the Original Buyers who transact in the newly-launched private site (i.e., outflow). Then, μ expands (shrinks) if the inflow is more (less) massive than the outflow. We show the results of the t-test comparison in Section 4.6.

For the effects on F, we compare the quality sensitivity between the associated New Buyers, who cross bid in the public marketplace, and the Original Buyers. If the cross-bidding New Buyers are more (less) sensitive to the sellers' quality than are the Original Buyers, then F becomes more (less) sensitive. Moreover, Lemma 4.2.2 and Expression (4.2) imply that buyers with various quality sensitivities tend to choose sellers with different qualities and have different willingness-to-pay, for a given product (i.e., $x\theta$). Therefore, we estimate two regression models regarding buyers' choice of sellers and their willingness-to-pay, respectively.

In addition to sellers' quality, buyers' choice between sellers and their willingness-to-pay for auctions depends on product-related characteristics. For example, some buyers are only interested in purchasing iPhones. These buyers will not participate in auctions of Samsung Galaxy phones regardless of the seller's quality of listings. To tease out quality effects from product-related influences, we focus on buyers' choice among sellers who list very similar products and buyers' bids in response to these listings. After the first launch, we select iPhone 5/5c/5s because they are homogeneous, as well as one of the most frequently listed products. Likewise, we consider iPhone 6/6s after the second launch.

In what follows, we define the variables for each regression. Then, the results of the regression analyses are detailed in Section 4.6.

Choice between sellers.

For the dependent variable, we consider buyers' choice between sellers throughout their lifetime in the public marketplace. The dependent variable of buyer j choosing unbranded seller i is denoted by C_{ij}.

Notice that product-related characteristics are controlled via the sample selection. As a result, a buyer's decision about whether or not to choose a seller depends primarily on the seller's characteristics, including their number of completed transactions ($NumComp_i$), number of repeat buyers ($NumRep_i$), and listing quality, denoted by a dummy variable Qlt_i:

$$Qlt_i = \begin{cases} 1, & \text{if seller } i\text{'s dispute rate is lower than the average.} \\ 0, & \text{otherwise.} \end{cases}$$

Additionally, we include buyer fixed effects, denoted by ν_j, to control for buyer's idiosyncratic preferences, and a dummy variable, $IsNB_j$, to indicate whether the buyer is a New Buyer ($IsNB_j = 1$) or an Original Buyer ($IsNB_j = 0$).[3] The corresponding regression is specified by Expression (4.7).

Willingness-to-pay.

For the dependent variable, we use buyers' bids in a given auction, which is normalized by its retail value. The reason is bidders' willingness-to-pay is unob-

[3]Since high-quality sellers and low-quality unbranded sellers have listings of the selected products simultaneously during more than 83% of the observational weeks, buyers' choice is unlikely to be affected by sellers' availability. Thus, we choose not to include availability-related variables.

served, and their bids serve a valid proxy.[4] Then, we let B_{ij} denote the dependent variable, where i stands for the auction and j stands for the buyer.

Though the major iPhone features are fixed (i.e., iPhone 5 series or iPhone 6 series), a buyer's willingness-to-pay is still subject to other product specifications (e.g., carrier), auction lot size, and the number of concurrent auctions with the same product. Therefore, we explicitly control for specific product features, including model (Mod_i), carrier (Car_i), condition ($Cond_i$), per-unit retail price ($UnitRP_i$), auction lot size (Lot_i), and the number of concurrent cell phone auctions in the public marketplace ($NumAuct_i$). Moreover, unbranded sellers' characteristics, especially their quality of listings, will also impact buyers' bidding decisions. Hence, we include $NumComp_i, NumRep_i$, and Qlt_i (defined in the *Choice between sellers* section) as controls. As in the previous analysis, we use $IsNB_j$ to indicate whether or not buyer j is a New Buyer. Lastly, we add seller fixed effects, buyer fixed effects, and weekly fixed effects, denoted by η_i, ν_j, and μ_i, respectively, to account for unobserved patterns. The corresponding regression is specified by Expression (4.8).

4.5.4 Identification of private sites' supply impacts

After identifying each launch's impacts on demand size and sensitivity, we test hypotheses by estimating how sellers in the public marketplace react to these demand changes using their quantities and how their revenues are affected. In particular, we apply a difference-in-differences (DiD) approach to identify these impacts. We select the monthly CE listings per seller as the *treatment group*, as

[4] If a buyer places multiple bids in an auction, then we select the last bid.

the demand for this category is most affected. Then, we focus on how sellers adjust their monthly CE listings in response to each launch.

Notice that sellers adjust their monthly listings to react to other unobserved platform-level shocks, such as website upgrades and marketing campaigns, in addition to the launches of private sites. To control for these unobserved shocks, we construct a *control group* corresponding to each launch by selecting monthly listings of other product categories in the public marketplace, whose demand is little affected by the New Buyers from the private site. In particular, we select the control category that has the least demand overlap with CE. For the first launch, we choose jewelry and toys as the control category, since the New Buyers participate in only 3% of its listings. For the second launch, we choose appliances as the control category, since the New Buyers bid in only 0.1% of them. As a result, these control groups should barely react to the launches of CE private sites; meanwhile they are subject to platform-level changes that apply to all listings in the public marketplace.

Moreover, we select the pre-launch and post-launch periods such that there are no launches of private sites specializing in the control categories. To further control for seasonality patterns that apply to CE listings (e.g., the release of newer generation products), we select the pre- and post-launch periods such that they cover the same months of a year. For the first site launch, the pre-launch period starts in April 2012 and ends in November 2012, and the post-launch period begins in April 2013 and ends in November 2013.[5] For the second site launch, the pre-launch period

[5]The pre-launch period begins in April 2012 to match the starting month of the post-launch period. The post-launch period ended in November 2013, as another branded CE retailer launched

is from January 2014 to May 2015, and the post-launch period is from January 2015 to May 2016.[6]

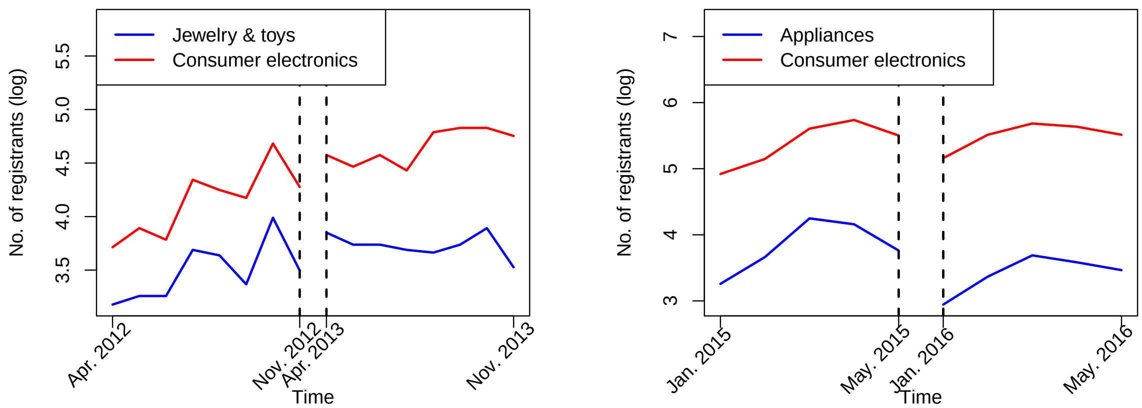
Next, we examine whether pre-trends are parallel in the treatment group and control group concerning buyer registrations and sellers' listings. Figure 4.5 shows the trends of monthly registrants in the public marketplaces across categories. They remain parallel during the pre-launch period in both natural experiments. Then, we plot the trends of sellers' monthly listings across categories in Figure 4.6, where we notice that they are approximately parallel during the pre-launch period. Figures 4.5 and 4.6 imply that the supply and demand of consumer electronics, jewelry and toys, and appliances have similar organic growth rates, and the parallel trend assumption is likely to hold.

Lastly, we specify the variables in the DiD analysis. We denote the dependent variables of interest by Y_{ikt}, where we use i to denote sellers, k to represent product category, and t to signify month. When we estimate the effect on sellers' monthly listings and monthly transacted auctions, Y_{ikt} counts how many auctions are listed and sold. When identifying the impact on sellers' monthly revenues yielded from CE listings, we discretize the outcome value by assigning rounded monthly revenues at

its private site the following month.

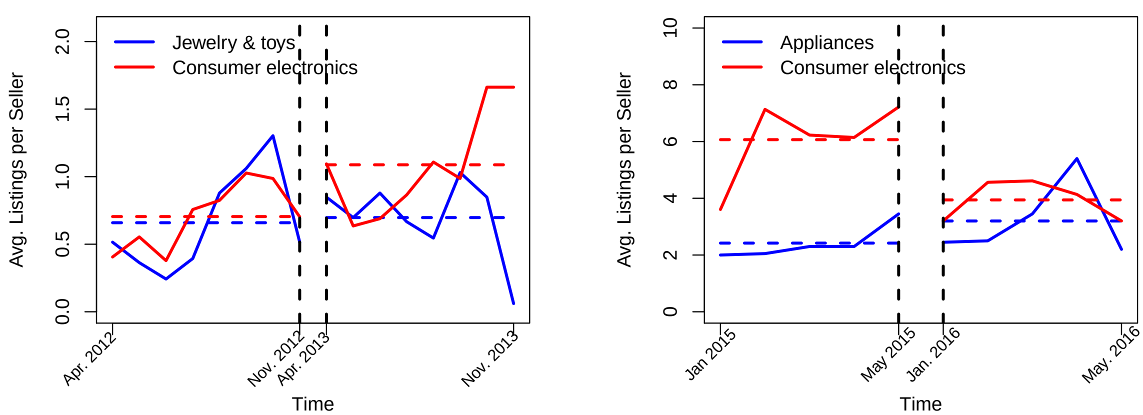
[6]The pre-launch period starts in January 2015, as a branded appliances retailer launched its private site in December 2014. The post-launch period ends in May 2016 because a branded CE retailer closed its private site the following month. We do not consider jewelry and toys as the control category in the analysis because during the post-launch period there was a supply shock of jewelry and toys listings in one of the private sites.

Figure 4.5: Trends of monthly registrants in the public marketplace during the first launch (left) and the second launch (right).



Note. The two vertical dashed lines specify the periods before and after the launches of the private site.

Figure 4.6: Average monthly listings per seller during the first launch (left) and the second launch (right).



Note. The two vertical dashed lines specify the periods before and after the launches of the private site. The horizontal dashed lines specify the average monthly listings per seller per category.

the level of a thousand dollars to Y_{ikt}.[7] Variable Trt_k indicates whether or not the observation is associated with CE (i.e., in the treatment group), and variable $Post_t$ is the dummy variable indicating whether or not month t falls within the post-launch period. Then, we let binary variable Qlt_{it} (defined in the *Choice between sellers* section) denote the quality of seller i. We employ seller fixed effects, η_i, to control for sellers' unobserved time-invariant characteristics. We then consider monthly fixed effects, μ_t, to control for the unobserved temporal trends of the public marketplace. Lastly, we introduce the fixed effects of seller's tenure (in months) on the platform, τ_{it}, to control for any listing patterns associated with sellers' learnings or previous platform experiences. The corresponding regression is specified by Expressions (4.9) and (4.10).

4.6 Demand impacts of private sites' launches

In this section, we characterize the two launches' impacts on the public marketplace's demand. We show that the first launch expands the public marketplace's demand size and increases its sensitivity level. In contrast, the second launch imposes an opposite effect by shrinking the demand size, as well as decreasing its

[7] Given the underlying counting process (i.e., the number of listed auctions), the revenues yielded by multiple auctions cannot be well-characterized by continuous distributions. About 52% of observations have zero revenue due to either no listings or no sales. Additionally, in cases where there is a sale, the revenues rarely take values near zero given the starting price (average of \$374) and high final price per auction (average of \$946). Therefore, considering revenue levels as the dependent variable, despite the loss of granularity, is sensible.

Table 4.1: Average monthly listings purchased by Original Buyers and New Buyers in public marketplace (denoted by Pub. Mkt.) and newly launched private sites (denoted by NLPS).

	Buyer type	Pub. Mkt.	NLPS	Inflow - outflow
After the first launch	Original Buyer	531	115 (outflow)	24^{***}
	New Buyer	140 (inflow)	216	
After the second launch	Original Buyer	626	105 (outflow)	-46^{***}
	New Buyer	59 (inflow)	87	
Note.				***p<0.01

sensitivity level.

First, we establish the two launches' impacts on the size of the marketplace's demand. After a launch, Table 4.1 presents the average number of monthly CE listings in the public marketplace and the newly-launched private site purchased by Original Buyers and New Buyers, respectively. Then, the public marketplace's listings purchased by New Buyers capture the demand inflow to the public marketplace, and the private site's listings purchased by Original Buyers represent the demand outflow from the public marketplace. By comparing the difference between the inflow and outflow with zero, we obtain the demand size impact of each launch. As evident in the last column of Table 4.1, the differences are significant from zero, which implies that the first launch expands the public marketplace's demand size while the second one shrinks it. We further explain the findings using the fact that the first private site brings more buyers relative to its listings to the platform, while the other private site does the reverse.

Second, we establish the two launches' impacts on the sensitivity level of the public marketplace's demand. After a launch, we adopt regression analyses to compare the sensitivity levels between the Original Buyers and the New Buyers based on their choice between sellers and their willingness-to-pay.

For buyers' choice between sellers, Lemma 4.2.2 suggests that buyers, who are sensitive to sellers' quality, tend to choose high-quality sellers more often than insensitive buyers do. Then, we employ a logistic regression to characterize New Buyers' and Original Buyers' preferences between sellers with different qualities, and the model is specified as:

$$
\begin{aligned}
logit(C_{ij}) =& \beta_0 + \gamma_0 IsNB_j + (\beta_1 + \gamma_1 IsNB_j) NumComp_i + (\beta_2 + \gamma_2 IsNB_j) NumRep_i \\
&+ (\beta_3 + \gamma_3 IsNB_j) Qlt_i + \nu_j, \qquad (4.7)
\end{aligned}
$$

where β_3 and $\beta_3 + \gamma_3$ measure how much Original Buyers and New Buyers, respectively, prefer a high-quality seller ($Qlt_i = 1$) to a low-quality seller ($Qlt_i = 0$). The estimates of Expression (4.7) for both site launches are displayed in Table 4.2. The estimate of γ_3 indicates that the New Buyers brought in by the first launch significantly more inclined to choose high-quality sellers than the Original Buyers are, so they are more sensitive to seller qualities than the Original Buyers. Therefore, the sensitivity level of the public marketplace's demand increases after the first launch. By contrast, the New Buyers brought in by the second launch show less preference for high-quality sellers than do the Original Buyers, so they are less sensitive than the Original Buyers. As a result, the sensitivity level of the public marketplace's demand decreases after the second launch.

Table 4.2: Comparison of Original Buyers vs. New Buyers regarding seller choice

	First launch (high-quality)	Second launch (low-quality)
	Dependent variable C_{ij}	
Qlt (β_3)	0.181^{***}	0.572^{***}
	(0.046)	(0.152)
Qlt × IsNB (γ_3)	0.309^{***}	-3.369^{**}
	(0.095)	(1.458)
Buyer Fixed Effects	Y	Y
Observations	24,105	2,527
Log Likelihood	−9,580.414	−736.210
Akaike Inf. Crit.	21,200.830	1,690.421

Note. *p<0.1; **p<0.05; ***p<0.01

For buyers' willingness-to-pay, Expression (4.2) indicates that buyers, who are quality-sensitive, value the high-quality sellers' listings more and, hence, bid higher than quality-insensitive buyers do. Then, we employ linear regression to compare New Buyers' and Original Buyers' bids, which serve as a proxy for their willingness-to-pay, and the model is specified as:

$$
\begin{aligned}
B_{ij} =&\beta_0 + \gamma_0 IsNB_j + (\beta_1 + \gamma_1 IsNB_j)Mod_i + (\beta_2 + \gamma_2 IsNB_j)Car_i \\
&+ (\beta_3 + \gamma_3 IsNB_j)Cond_i + (\beta_4 + \gamma_4 IsNB_j)UnitRP_i + (\beta_5, +\gamma_5 IsNB_j)Lot_i \\
&+ (\beta_6 + \gamma_6 IsNB_j)NumComp_i + (\beta_7 + \gamma_7 IsNB_j)NumRep_i + (\beta_8 + \gamma_8 IsNB_j)Qlt_i \\
&+ (\beta_9 + \gamma_9 IsNB_j)NumAuct_i + \eta_i + \nu_j + \mu_i + \epsilon_{ij},
\end{aligned} \tag{4.8}
$$

where γ_8 measures the difference in bids between New Buyers and Original Buyers in auctions listed by high-quality sellers. We present the regression results of Expression (4.8) for both site launches in Table 4.6, which reveals the same pattern as Table 4.2. Specifically, the estimate of γ_8 indicates that in high-quality listings, the New Buyers from the first (second) private site bid significantly higher (lower) than the Original Buyers do. Therefore, the first private site brings in more quality-sensitive buyers, while the other private site brings more quality-insensitive buyers, to the public marketplace.

	First launch (high-quality)	Second launch (low-quality)
	Dependent variable B_{ij}	
Qlt (β_8)	-0.005 (0.017)	0.209^{***} (0.068)
Qlt × IsNB (γ_8)	0.032^{**} (0.014)	-0.203^{***} (0.058)
Seller Fixed Effects	Y	Y
Buyer Fixed Effects	Y	Y
Weekly Fixed Effects	Y	Y
Observations	1,611	996
R^2	0.893	0.963
Adjusted R^2	0.685	0.856
Residual Std. Error	0.044 (df = 549)	0.045 (df = 259)
F Statistic	4.305^{***} (df = 1061; 549)	9.035^{***} (df = 736; 259)

Note. *p<0.1; **p<0.05; ***p<0.01

In sum, we have identified the impacts of the two launches on the marketplace's demand in the dimensions of size and sensitivity. By Propositions 4.3.2 and 4.3.4, we notice that the first launch leads to a favorable impact on the public marketplace's total revenue and social welfare by expanding its demand and increasing the sensitivity. On the other hand, the second launch results in an unfavorable impact regarding these two metrics by shrinking its demand size and decreasing the sensitivity level.

4.7 Effects of launches on sellers

In this section, we focus on the effects of newly launched CE private sites on sellers in the public marketplace and test all the hypotheses developed in Section 4.4. First, we test Hypotheses 4.4.1 and 4.4.2 by examining the average impacts on sellers' monthly listings. Second, we test Hypotheses 4.4.1, 4.4.2, and 4.4.2 by examining heterogeneous impacts on the monthly listings and monthly revenues of sellers with different qualities.

4.7.1 Average impacts on supply

We reject Hypothesis 4.4.1 by showing that sellers' supply of CE varies significantly after both launches and verify Hypothesis 4.4.2 by showing that their supply increases after the first launch and decreases after the second launch. In particular, we specify the DiD model for these tests as follows:

$$Y_{ikt} = f(\beta_0 + \beta_1 Trt_k + \beta_2 Post_t + \beta_3 Trt_k \times Post_t + \gamma Qlt_{it} + \eta_i + \mu_t + \tau_{it} + \epsilon_{ikt}), \tag{4.9}$$

where f is the negative binomial link function.[8]

We present the regression results associated with both launches in Table 4.3, and the estimate of β_3 captures the average effects of a launch on sellers' listings,

[8] We conduct a Wald test and reject the hypothesis that the data follow a Poisson regression.

transactions, and revenues of CE per month. First, we reject Hypothesis 4.4.1, which states that sellers' listed quantities are irresponsive to any demand shocks when the adjustment friction is large, as the estimates of β_3 are significantly different from zero for both launches (i.e., Models (1), (2), (4), and (5)). Second, we observe that the first launch increases sellers' monthly listings (Model (1)) and monthly transactions (Model (2)) by 114% and 94%, respectively,[9] and the second launch decreases sellers' monthly listings (Model (4)) and monthly transactions (Model (5)) by 64% and 59%, respectively.[10] Given the demand impacts of both launches revealed in Section 4.6, we thus confirm Hypothesis 4.4.2, which characterizes sellers' average reactions to demand shocks when the adjustment friction is small. Therefore, the estimated average supply effects validate our analytical model.

4.7.2 Heterogeneous impacts on supply and revenues

In this section, we first reject Hypothesis 4.4.1 and verify Hypothesis 4.4.2, simultaneously, by showing that the revenue changing rate of high-quality sellers (i.e., r_H) is higher (lower) than that of low-quality sellers (i.e., r_L) after the first (second) launch. Next, we verify Hypothesis 4.4.2 by showing that the changing rate of high-quality sellers' listed quantities (i.e., q_H) is higher (lower) than that of low-quality sellers (i.e., q_L) after the first (second) launch, which is stated by the

[9] By the negative binomial link function, we note that $\exp(1.145) - 1 = 1.14$ and $\exp 1.079 - 1 = 0.94$.

[10] By the negative binomial link function, we note that $1 - \exp(-1.029) = 0.64$ and $1 - \exp(-0.903) = 0.59$.

Table 4.3: Average listing effect of both launches.

	First launch (high-quality)			Second launch (low-quality)		
	Dependent variable					
	Listings	Sold auctions	Revenues	Listings	Sold auctions	Revenues
Model	(1)	(2)	(3)	(4)	(5)	(6)
Trt × Post	1.145***	1.079***	1.142***	−1.029***	−0.903***	−0.694***
	(0.357)	(0.359)	(0.396)	(0.212)	(0.213)	(0.234)
Seller Fixed Effects	Y	Y	Y	Y	Y	Y
Monthly Fixed Effects	Y	Y	Y	Y	Y	Y
Age Fixed Effects	Y	Y	Y	Y	Y	Y
Observations	543	543	543	3,976	3,976	3,976
Log Likelihood	−838	−777	−1,212	−3,526	−3,421	−2,997

Note. **p<0.05; ***p<0.01

hypothesis. Sepcifically, we obtain the DiD model for the tests by interacting the term of sellers' quality (i.e., Qlt) with the cross-term $Trt \times Post$ in Expression (4.9), which is specified as:

$$\begin{aligned} Y_{ikt} = f\big(&\beta_0 + \gamma_0 Qlt_{it} + (\beta_1 + \gamma_1 Qlt_{it})Trt_k + (\beta_2 + \gamma_2 Qlt_{it})Post_t \\ &+ (\beta_3 + \gamma_3 Qlt_{it})Trt_k \times Post_t + \eta_i + \mu_t + \tau_{it} + \epsilon_{ikt}\big), \end{aligned} \tag{4.10}$$

where β_3 and $\beta_3 + \gamma_3$ represent the treatment effect on low-quality sellers and high-quality sellers, respectively.

We present the estimates of Expression (4.10) for the first and second launches in Table 4.4. First, Models (3) and (6) of Table 4.4 estimate that r_H is higher than r_L by 224% after the first launch and r_H is lower than r_L by 58% after the second launch.[11] Therefore, the estimated heterogeneous effects on sellers' revenue contradicts Hypothesis 4.4.1, which specifies the opposite differentiated revenue impacts under large adjustment friction, and confirm Hypothesis 4.4.2, which specifies the same effects under small adjustment friction. Next, Models (1) and (4) estimate that q_H is higher than q_L by 374% after the first launch and q_H is lower than q_L by 56%,[12], which are consistent with Hypothesis 4.4.2's claims. Therefore, the heterogeneous effects revealed by Expression (4.10) further validate our analytical model.

[11] By the negative binomial link function, we note that $\exp(1.446) - 1 = 224\%$ and $1 - \exp(-0.874) = 58\%$.

[12] By the negative binomial link function, we note that $\exp(1.748) - 1 = 374\%$ and $1 - \exp(-0.843) = 56\%$.

Table 4.4: Heterogeneous effects of both launches on sellers with various qualities.

	First launch (high-quality)			Second launch (low-quality)		
	Dependent variable					
	Listings	Sold auctions	Revenues	Listings	Sold auctions	Revenues
Model	(1)	(2)	(3)	(4)	(5)	(6)
Trt × Post	0.378	0.262	0.501	-0.569^{*}	-0.591^{*}	-0.146
	(0.488)	(0.498)	(0.632)	(0.342)	(0.342)	(0.401)
Trt × Post × Qlt	1.748^{**}	1.718^{**}	1.446^{*}	-0.843^{*}	-0.595	-0.874^{*}
	(0.737)	(0.741)	(0.857)	(0.450)	(0.450)	(0.505)
Seller Fixed Effects	Y	Y	Y	Y	Y	Y
Monthly Fixed Effects	Y	Y	Y	Y	Y	Y
Age Fixed Effects	Y	Y	Y	Y	Y	Y
Observations	543	543	543	3,976	3,976	3,976
Log Likelihood	−835	−774	−1,209	−3,523	−3,419	−2,995

Note. *p<0.1; **p<0.05; ***p<0.01

4.8 Additional evidence for model validation

To further validate our analytical model, we collect additional empirical evidence to test the hypotheses developed in Section 4.4. First, we test Hypothesis 4.4.1B by citing an unfavorable demand shock (i.e., Airbnb's entry to compete for demand) on a marketplace with high adjustment friction (i.e., hotel market) documented by [55]. Their findings are consistent with Hypothesis 4.4.1B's claim. Second, we select an alternative category as the control group for both launches and retest all the hypotheses using the DiD method. We obtain the same testing results as revealed in Section 4.7.

4.8.1 Empirical evidence from high adjustment-friction market

[55] empirically investigate the revenue impact of entry of Airbnb on Texas' hotels. The hotel industry is known for its inflexibility in making supply adjustments, so we consider it as a marketplace with large adjustment friction. Besides, when Airbnb enters a state, it substantially weakens the demand for the hotels. Hypothesis 4.4.1B implies that the high-quality hotels suffer less compared with those with lower quality. In Table 5 of [55], they identify the same pattern, where the negative revenue impact exacerbates as the tier of hotel downgrades (i.e., from luxury to budget). Therefore, [55]'s findings enhance our analytical model's validity in characterizing demand shocks on high friction marketplaces.

4.8.2 Alternative control group in DiD method

We show that the empirical investigation in Section 4.7 is robust to the control group selection, by retesting all five hypotheses using another qualified product category as the control group. For both launches, we select listings of a particular category called “mixed lots” in the public marketplace as the control group. Each mixed-lots listing contains a set of unsorted merchandise from any categories, such as consumer electronics, jewelry, toys, and appliances. Sellers list mixed-lots auctions to expedite the liquidation process by reducing the time of sorting items. Mixed lots listings qualify for the control group as they have little overlap with CE listings in terms of buyers. Only 3% mixed-lots listings are participated in by New Buyers from the first CE private site, and only 0.1% mixed-lots auctions are participated in by New Buyers from the second CE private site.

Employing the alternative control group, we rerun the DiD analysis of Section 4.7 entirely. First, the estimated average supply effects of both launches are displayed Table 4.5, which contradict Hypothesis 4.4.1 and verify Hypothesis 4.4.2. Then, the estimated heterogeneous impacts on the listings and revenues of sellers with various qualities are displayed in Table 4.6, which reject Hypothesis 4.4.1 and confirm hypotheses 4.4.2 and 4.4.2. Therefore, our hypothesis testing results in Section 4.7 are robust to the control group selection, which further increases the validity of the analytical model.

Table 4.5: Average listing effect of private site launches (use mixed lots auctions as control group).

	First launch (high-quality)			Second launch (low-quality)		
	Dependent variable					
	Listings	Sold auctions	Revenues	Listings	Sold auctions	Revenues
Model	(1)	(2)	(3)	(4)	(5)	(6)
Trt × Post	0.861^{**}	0.967^{**}	0.947^{**}	-0.412^{**}	-0.426^{***}	-0.731^{***}
	(0.423)	(0.423)	(0.475)	(0.162)	(0.165)	(0.200)
Seller Fixed Effects	Y	Y	Y	Y	Y	Y
Monthly Fixed Effects	Y	Y	Y	Y	Y	Y
Age Fixed Effects	Y	Y	Y	Y	Y	Y
Observations	557	557	557	2,962	2,962	2,962
Log Likelihood	−751	−710	−590	−5,724	−5,560	−4,708

Note. *p<0.1; **p<0.05; ***p<0.01

Table 4.6: Heterogeneous Effects of private sites on unbranded Sellers with Various Reputations

	First launch (high-quality)			Second launch (low-quality)		
	Dependent variable					
	Listings	Sold auctions	Revenues	Listings	Sold auctions	Revenues
Model	(1)	(2)	(3)	(4)	(5)	(6)
Trt × Post	0.386	0.434	0.471	0.179	0.169	−0.091
	(0.503)	(0.509)	(0.556)	(0.251)	(0.259)	(0.308)
Trt × Post × Qlt	$1.398^{\dagger}$	1.484^{*}	1.657^{*}	-0.927^{***}	-0.943^{***}	-1.043^{***}
	(0.864)	(0.853)	(0.977)	(0.316)	(0.323)	(0.396)
Seller Fixed Effects	Y	Y	Y	Y	Y	Y
Monthly Fixed Effects	Y	Y	Y	Y	Y	Y
Age Fixed Effects	Y	Y	Y	Y	Y	Y
Observations	557	557	557	2,962	2,962	2,962
Log Likelihood	−750	−708	−588	−5,717	−5,554	−4,702

Note. †p=0.10; *p<0.1; **p<0.05; ***p<0.01

4.9 Concluding Remarks

In this paper, we build an analytical model to study how a two-sided marketplace reacts to unexpected demand shocks in terms of size and sensitivity. By incorporating the friction of sellers' supply adjustment, we compare various demand impacts on a given marketplace under low and high friction levels. Regardless of whether a given marketplace maximizes the total revenue or social welfare, we establish that manipulating the friction can amplify the positive impact of favorable demand shocks and alleviate the negative impact when the demand shocks are unfavorable.

We further validate our analytical model by a series of hypothesis testing. Employing two demand shocks on an online liquidation marketplace, where sellers' adjustment friction is low, we test the model's hypotheses under both the low-friction and the high-friction scenarios. All test results strengthen the validity of the model. Moreover, we cite a demand shock on a high-friction marketplace empirically investigated by [55] to test our model in the same scenario. As a result, their empirical evidence is consistent with the model's predictions.

4.10 Appendix: Parallel Assumption Checks Using Pseudo Treatment

We apply DiD model (4.9) to the pre-entry period of each entry to check the parallel assumption. We choose the mid-point of the pre-entry period as the pseudo treatment. In other words, if an observation is before (after) the mid-point of the pre-entry period, $Post_t$ is equal to 1 (0). The corresponding estimates are displayed in Table 4.7. We notice that the coefficients of the interaction term in both entries are statistically insignificant, which implies that the trend of each group remains parallel over time.

Table 4.7: Check of parallel assumption using pseudo treatment

	Dependent variable					
	Entry of high-quality retailer			Entry of low-quality retailer		
	Listings	Sold auctions	Revenues	Listings	Sold auctions	Revenues
Model	(1)	(2)	(3)	(4)	(5)	(6)
Trt × Post	−0.352	−0.478	0.657	0.213	0.155	0.478
	(0.338)	(0.341)	(0.464)	(0.278)	(0.279)	(0.318)
Seller Fixed Effects	Y	Y	Y	Y	Y	Y
Monthly Fixed Effects	Y	Y	Y	Y	Y	Y
Age Fixed Effects	Y	Y	Y	Y	Y	Y
Observations	242	242	242	1,658	1,658	1,658
Log Likelihood	−342	−310	−293	−1,473	−1,435	−1,367

Note. *p<0.1; **p<0.05; ***p<0.01

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