ABSTRACT

Title of Dissertation: ESSAY ON CONTRACT STRUCTURE IN PRINCIPAL-AGENT PROBLEMS WITH BEHAVIORAL BIASES

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Designing employment contracts in a principal-agent relationship is a key problem in the modern firm theory. This dissertation investigates this problem from three different angles, (1) the employment contracts in the labor market with procrastinating workers; (2) the behavior of members and reciprocal leaders in group competitions, where leaders can reward members discretionarily; (3) optimal employment contracts when tasks are endogenously designed.

For the chapter about the employment contracts as a commitment device, I build an adverse selection model in a labor market of one firm against many workers, where the workers, if self-employed, procrastinate due to their own quasi-hyperbolic discounting. In the equilibrium, the model shows that workers with the least procrastination are self-employed and workers with the most procrastination are part-time employees in a separating equilibrium where the workers' hiring contracts differ by their quasi-hyperbolic discounting. In between, there exist specific ranges of quasi-hyperbolic discounting factors, in each of which the workers sign the

same contract in a pooling equilibrium. This model leads to a position hierarchy within the firm as well as separation of paid-employment and self-employment in the labor market.

For the chapter about the behavior of reciprocal leaders and members in group competitions, I model the model equilibrium when the leaders are reciprocal and show the existence of the pure strategy equilibrium. A laboratory group all-pay auctions was run to test for the model predictions.

For the chapter about the optimal employment contracts with endogenous tasks, I examine the optimal job design in a principal-agent setting where tasks are designed by the principal. The principal wants to incentivize agents to exert a given desired amount of effort on an effort space; to minimize the cost of reaching this goal, the principal can freely assign disjoint parts of the effort space to agents as their jobs and partition each job into tasks. I provide a characterization of the optimal number of agents to be hired and optimal partition of each agent's job into tasks.

Essay on Contract Structure in Principal-Agent Problems with Behavioral Models

by

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Introduction

In both lab and field experiments, evidence shows that workers procrastinate, and many procrastinating workers intentionally employ commitment devices for self-control ([1], [2], [3], [4] etc.). Additionally, evidence highlights the importance of workers' self-control in the development of workplace organizations (see [4] for a discussion). For example, evidence implies that firms switched to stricter working disciplines during the industrial revolution because the workers lacked self-control [5]. Clark [5] further argues that "Whatever the workers themselves thought, they effectively hired the capitalists to discipline and coerce them." Based on the evidence, I build a model where sophisticated and procrastinating workers can choose to sign hiring contracts as self-commitment devices for self-control. I also investigate the model's implication on labor market structure, firm structure, and public policy.

The model suggests the importance of workers' self-control in their choices for self-employment. This is supported by anecdotal stories and online columns emphasizing self-control for business owners and independent researchers. For example, an article in Forbes.com [1] argues that it is one key sign for a person to fit in self-employed jobs if she is "a disciplined self-starter". In another article giving advice of time-management to self-employed workers, the author states in the first paragraph that time-management/self-control is more important for self-employed workers than for employees. [2] Specifically, while self-employed workers face the challenges of time-management due to their comparatively unstructured work, employees in a firm typically have a more structured working environment with explicit deadlines and tasks, which is helpful in solving employees' self-control problems. Hence, the trade-off between self-employment and paid-employment for the procrastinating workers can lead to a certain structure of the labor market, where paid-employment is more favorable to more procrastinating workers.

I will briefly illustrate my model in this following example. Imagine a programmer who can either be an employee or a self-employed worker, for example, to develop an App at home. If self-employed, the programmer decides how many hours to spend on coding and how many hours on some costly non-pecuniary activity, for example, exercising in the gym. Assume that both coding and exercising are costly for the programmer but will return positive payoffs in the future. Although the programmer would plan to do lots of coding and exercising beforehand, she knows that she would not do so when the time comes due to her procrastination. Instead, she can sign a hiring contract for coding with a software firm. Suppose that with

[1] "13 Signs You Are Meant To Be Self-Employed," Forbes, https://www.forbes.com/sites/glassheel/2012/04/12/13-signs-you-are-meant-to-be-self-employed/#5205730f275e

[2] "How to Manage Your Time if You Are a Self-Employed Worker," Geston, https://facilethings.com/blog/en/self-employed-time-management.html

perfect monitoring, the firm is able to make its employees fulfill the contracted coding effort. So the worker will code harder by contracting a high amount of coding for self-control while the firm can pay a higher wage to the worker and still be profitable because of the worker's higher productivity. If the expected utility when signing the hiring contract is higher than being self-employed, the worker will sign the contract, as the contract works as a self-commitment device which controls for the worker's procrastination.

In the above example, both the employee and the employer benefit from the hiring contract: the employee earns a higher utility, while the employer is able to extract a certain "commitment rent" by offering this self-commitment device to the employee. One property of this example is that the contracted pecuniary effort will crowd out the non-pecuniary effort. As a result, if the payoff from non-pecuniary effort is large, the worker will never accept a contract with too much pecuniary effort. This reflects a desire for work-life balance of the worker, that a worker will not accept a job which will make her too busy to deal with her own business using non-pecuniary effort.

As in the programmer's example, the firm can offer hiring contracts to workers to control for their procrastination in this model. I use workers' quasi-hyperbolic discounting factors to represent their procrastination (see, for example, [6]; [7]). As in the example, I assume that workers can exert two different efforts: the "pecuniary" effort returns payoff also valuable to the firm and can thus be contracted, and the "non-pecuniary" effort returns payoff only valuable to the worker and thus cannot be contracted. I assume a step payoff function for the non-pecuniary effort, which is just the commonly accepted "fixed cost" production function, and a linear payoff function for pecuniary effort, which can be justified by the existence of the labor market for the pecuniary effort.

Assume that the firm/employer is a monopsonist in offering hiring contracts. [3] As the worker's quasi-hyperbolic discounting factor is private information, the firm offers a contract menu to the workers. Each worker can choose a contract from the menu to sign, or remain self-employed. This introduces an adverse-selection problem where a worker's quasi-hyperbolic discounting factor is the worker's type. The equilibrium of this model is a hybrid one: the workers with the highest quasi-hyperbolic discounting factors and thus the least procrastination will be self-employed; the workers with the lowest quasi-hyperbolic discounting factors and the most procrastination will be in the separating equilibrium with working hours and payments differing on their types. A "position hierarchy" arises for the medium type workers. Each position is a range of workers' quasi-hyperbolic discounting factors where workers sign the same hiring contract with same working hours and pay. [4] I refer to this job in the pooling equilibrium as a "firm position" because in reality, full-time workers in a given firm with a given job are typically paid similar wages and work similar hours every day, in contrast to part-time workers who have more heterogeneity in hours and wages. Hence, I regard workers in the separating equilibrium as the model counterparts to real-life part-time workers, while workers in the pooling equilibrium are the model counterparts to real-life full-time workers. In this model,

[3] See the discussion for a perfectly competitive market in Section 6.

[4] Note that there may exist ranges of quasi-hyperbolic discounting for separating equilibrium between two positions for pooling equilibrium.

given the worker's type, being a full-time worker in the pooling equilibrium yields an extra information rent to the worker, compared to being a part-time worker in the separating equilibrium. This gives a new explanation for the 'part-time penalty' [8], the fact that part-time workers earn less than full-time workers after controlling for measurable characteristics of workers and jobs. With the finding that the most procrastinating workers tend to be part-time, this result also implies a disadvantage for the workers with low cognitive abilities, for example, lack of self-control, pointed out by [9]. The model thus provides a further argument for supporting programs that aim to improve the non-cognitive abilities of children from poor backgrounds to prepare them for the future labor market.

In addition, in the equilibrium of the model, the higher utility of self-employed workers does not come from the higher income but from a lower exerted pecuniary effort. The model thus implements Hurst and Pugsley [10]'s suggestion for non-monetary benefit, as a reason for workers to enter into self-employment despite the lower average income of self-employed workers [11]. Finally, I investigate the implications of the model for policies such as ceilings of working hours and wage. With a uniform distribution of workers' quasi-hyperbolic discounting and other plausible assumptions about the model parameters, the model predicts that a properly designed ceiling of working hours can increase both the workers' surplus and social welfare. The effect of wage ceilings, however, is ambiguous in this model.

Although time-inconsistency is usually considered as an unobservable characteristic of workers, there is indirect evidence implying its relation with workers' labor market performance. Through both experiments and brain activation images, research in neuroscience shows a negative correlation between time-inconsistent behavior and people's personality trait of conscientiousness. Furthermore, the personality trait of conscientiousness has been shown to have a positive correlation to subjects' choices of entrepreneurship and is perceived by human resource departments as an even more important fact for employees' career success than the employees' cognitive ability. [12]

In following Section 1.1, I will discuss the relevant literature. In Section 1.2, I will set up the model and spell out the key assumptions. Section 1.3 defines each player's problem. I will then discuss the properties of the equilibrium and solve the model in Section 1.4. Implications of the analysis for public policies will be given in/ Section 1.5, followed by discussions in Section 1.6. The final section in this chapter concludes.

1.1 Literature Review

1.1.1 Procrastination

In behavioral economics, procrastination of an agent is often modeled as quasi-hyperbolic discounting (see [13] and [7]), with the utility function

$$U(c_0, ..., c_T) = u(c_0) + \beta \sum_{t=1}^{T} \delta^t u(c_t).$$

where $\beta \in [0, 1]$ is the quasi-hyperbolic discounting factor of the agent and $(c_0, ..., c_T)$ is the agent's consumption from period 0 to period T. When $\beta < 1$, the relative importance of the current utility compared to the future utility is higher than that in the comparison of the same pairs of utility assessed in the previous period. As a result, the agent may change her choice to more enjoyment and less effort when the decisions become current, contradicting her previous plans such as working hard, saving a lot, and eating healthy. In this sense, the agent is "time-inconsistent." $\delta \in [0, 1]$ is the traditional intertemporal discounting factor, which I will normalize to one for all the players, to focus the model on the role of workers' quasi-hyperbolic discounting β in players' decisions.

Researchers show that sophisticated people anticipating their procrastination will intentionally apply self-commitment devices to limit their flexibility of deviation from the "plan". For instance, field experiments ([2]; [3]; [4] etc.) show that procrastinating workers in workplaces would like to use costly commitment devices, and the likelihood of the application of commitment devices is positively correlated with the workers' time-inconsistency. Consistent with the idea that workers sign hiring contracts to self-control, evidence implies the role of self-control in the development of workplace organizations (see discussion in [4]). For example, it is shown that during the industrial revolution, workers under flexible working hours and piece-rate pay had unsteady attendance and hours, spent a lot of time socializing at work, and concentrated effort in the latter half of the week leading up to paydays. [5] Clark [5] argues that this low productivity of workers made firms gradually switch to the strict factory discipline, that is, strict working hours, and workers accepted this arrangement due to the awareness of their lack of self-control. This evidence supports the key idea of this model, that the firm offers self-commitment devices for workers.

1.1.2 Labor Market

This chapter separates itself from other works about the market equilibrium of time-inconsistent agents by investigating sophisticated workers' behavior in a labor market setting where workers face trade-offs between contractable/pecuniary and uncontractable/non-pecuniary efforts. Previous research often focuses on the exploit of naïve players in the market ([6]; [14]; [15] etc.) and on the markets where time-inconsistent agents face a trade-off between commitment versus flexibility [16]. In contrast, in this model, there is no future uncertainty and, therefore, no need for flexibility. With the labor market setting of this model, sophisticated workers face the trade-off between contractable and uncontractable actions when choosing the hiring contracts as commitment devices. The equilibrium highlights a market structure with workers' contracted effort differing according to their types, and workers in the equilibrium will never be worse-off with the hiring contracts, if not benefit from them.

The model predicts that given a level of procrastination, employees in a separating equilibrium earn a lower wage than employees in a pooling equilibrium. Note that, in the model, types of workers who fully separate in the equilibrium correspond to real-world part-time employees who are paid based on working hours with a more diversified income. And the pooling employees correspond to full-time workers in reality, who have similar working hours and close incomes in a given firm position. So this result can explain the empirical observation of wage penalty for part-time workers, thus explaining the puzzling lower income of part-time workers compared to full-time workers after controlling for the measurable job and worker characteristics. [8]

The model also contributes to the literature of self-employment where a stable lower income with higher variance is observed for self-employed workers than for employees ([17]; [11] etc.). To explain this observation, Hurst and Pugsley [10] suggest non-monetary benefits such as "more flexibility" and "being one's own boss" as an important reason for self-employment. Consistent with Hurst et al [10]'s finding, in the equilibrium of this model, the higher utility of the self-employed workers comes from their lower exerted pecuniary effort instead of a higher income. The model thus implies a theoretical implication of Hurst and Pugsley's "non-monetary" benefit of self-employment, which explains why people enter into self-employment despite the average lower income of self-employment.

1.1.3 Contract Structure

As a hierarchy arises within the firm based on the workers' quasi-hyperbolic discounting in the equilibrium, this chapter is related to the literature of firm hierarchy. The traditional theories justify the firm's hierarchy as a structure to solve the moral-hazard problem in the principal-agent setting, that is, managers are hired to monitor the workers/agents, in order to keep them from shirking. [5] This chapter offers a new explanation for a firm's hierarchy as a consequence of the firm's optimal contract menu, which maximizes its profit in a labor market with workers who have different tendencies of procrastination.

[5] See [18] for a literature review of studies about firm hierarchy.

This model also relates to the literature about low-powered incentives, which is a motive scheme where the workers get the promotion or are fired discretely by their performance instead of claiming part of their production as in the continuous piece-rate pay [19]. The traditional model justifies the popularity of low-powered incentives in workplaces by cost of possible opportunism ([19]), imperfect observability of tasks (see, for example, [20], [21]) or worker heterogeneity [22]. With the mere idea that workers use hiring contracts as commitment devices, this model offers a simple alternative explanation: as a commitment device for procrastinating workers, the hiring contract must impose a discrete punishment, for example, firing the worker, once an established goal of effort exerting is not accomplished.

1.2 Model Setting

A worker/agent can decide to either work by herself in self-employment or sign a contract with an employer/principal as an employee at $t = 0$. There are two different types of effort $e_1, e_2 \in R^+$ that the worker can exert at $t = 1$. Suppose that the payoff of effort e_1 is valuable for both the worker and the firm, and can thus be contracted. The payoff of e_2 is only valuable for the worker and cannot be contracted. I will thus refer to e_1 later as "pecuniary effort" and e_2 "non-pecuniary" effort. Examples of e_2 include the worker's effort to accompany one's family, exercise to keep healthy, the worker's effort to develop a good habit and so on, which is valuable for the worker herself but has no value to other people, for example, the employer.

Assume that the exerted non-pecuniary effort returns a given payoff $G > 0$ at $t = 2$ once the amount of the exerted non-pecuniary effort is higher than a fixed threshold E. This formulation assumes a fixed cost type of production technology and the payoff function of the non-pecuniary effort e_2 is thus a step function $g(e_2)$, that is,

$$g(e_2) = \begin{cases} G, \text{ if } e_2 \geq E, \\ \\ 0, \text{ otherwise.} \end{cases}$$

On the other hand, as e_1 is not only valuable to the worker herself but also to other people, we can imagine that there is a market where pecuniary effort e_1 can be traded competitively. So the payoff function of e_1 equals the market price of e_1 times the amount of e_1 exerted by the worker, that is, e_1 returns Γe_1 at $t = 2$ where $\Gamma > 0$ is the market price of pecuniary effort e_1. The worker pays an immediate cost $\frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \sigma e_1 e_2$ for exerting e_1 and e_2 with a substitution effect $\sigma \in (0, 1)$ between the two efforts, which is a simple form of the effort-substitution cost function introduced by [20].

Suppose that there is no technology for either the worker or the firm to change the timing of payoff realization. (i.e., there is no bank to borrow from and the firm has zero budget, so the worker cannot be paid before Γe_1 is realized.) The timeline of the game is shown in Figure 1.1.

As shown in Figure 1.1, a worker can decides whether to sign a hiring contract to become an employee or keep self-employed at $t = 0$. After a hiring contract $C = (e_1, w) \in R^+ \times R^+$ is signed at $t = 0$, the employee must exert the contracted pecuniary effort e_1 at $t = 1$. The employee can still decide freely the amount of

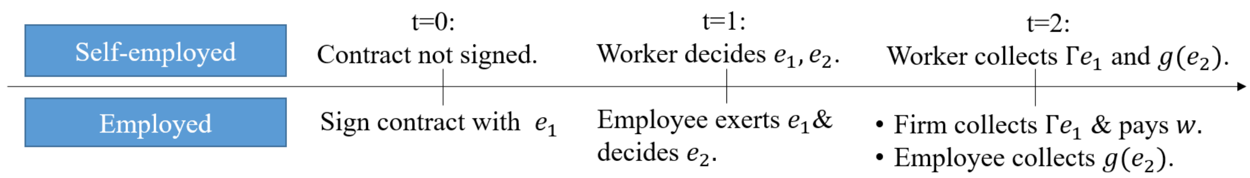

Figure 1.1: Timeline of the interaction between the firm and hired workers

the non-pecuniary effort e_2 to exert at $t = 1$. At $t = 2$, the non-pecuniary effort e_2 generates $g(e_2)$ to the employee. Meanwhile, the employer collects the payoff Γe_1 generated by e_1 and pays the employee a lump-sum wage $w \geq 0$ as in the hiring contract $C = (e_1, w)$. If the worker does not sign the contract at $t = 0$ and remains self-employed, she chooses both the pecuniary effort e_1 and non-pecuniary effort e_2 at $t = 1$ and collects the payoffs Γe_1 and $g(e_2)$ at $t = 2$. As a tie-breaking rule, assume that the worker will choose paid-employment at $t = 0$ if indifferent between paid-employment and self-employment. Also, assume that an employee will choose the contract with higher contracted e_1 among contracts yielding the same utility to her.

Suppose that the worker's quasi-hyperbolic discounting factor β is drawn in a probability distribution function (PDF) $f(\beta) > 0$ with $[\beta \in \underline{\beta}, 1]$ with a lower support $\underline{\beta} \in (0, 1)$. For simplicity, assume that the intertemporal discounting factor of both the employer and workers equals to one. Also, assuming that the hyperbolic discounting factor β is a private information of a worker, the employer offers a menu of contracts, $M \subset R^+ \times R^+$, to workers to maximize its profit. Later I will denote a worker's quasi-hyperbolic discounting factor β as this worker's "type" in this adverse selection problem. All the proofs for later results can be found in Appendix A.

1.2.1 Assumptions

All results of the model rely on the following two assumptions.

Assumption 1: $\Gamma > \sigma E$.

Assumption 1 guarantees that the productivity of the pecuniary effort e_1 is high enough, so the socially optimal e_1 is positive when the worker exerts the minimum e_2 to generate the positive non-pecuniary payoff, that is, the socially optimal e_1 satisfies $e_1 > 0$ when $e_2 = E$.

Assumption 2: $G > G^* = \max\left\{\frac{\Gamma^2+E^2}{2}, \frac{E\Gamma}{\sigma}\right\}$.

There are two components in Assumption 2, $G > \frac{\Gamma^2+E^2}{2}$ and $G > \frac{E\Gamma}{\sigma}$. The first component guarantees that the non-pecuniary payoff from exerting the non-pecuniary effort $e_2 = E$ is large enough so $e_2 = E$ is socially optimal. The second component $G > \frac{E\Gamma}{\sigma}$ makes sure that a self-employed worker will exert $e_2 = E$, given that she exerts a strictly positive pecuniary effort e_1.

1.3 Player's Problems

1.3.1 Self-Employed Worker's Problem

The self-employed worker's utility at $t = 0$ is

$$V^0(\beta) = \beta\left[\Gamma e_1 + g(e_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - \sigma e_1 e_2\right]$$

where e_1, e_2 is determined by the worker solving her problem at $t = 1$ as follows,

$$V^1(\beta) = max_{e_1,e_2}\beta\Gamma e_1 + \beta g(e_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - \sigma e_1 e_2$$

$$\text{s.t.} \quad e_1, e_2 \geq 0.$$

Taking derivative of $V^1(\beta)$ with respect to e_1 yields $e_1 = \beta\Gamma - \sigma e_2$ if $e_1 = \beta\Gamma - \sigma e_2 \geq 0$, otherwise $e_1 = 0$. For e_2, there are two potential solutions: $e_2 = 0$ or $e_2 = E$.

If $e_2 = 0$,

$$V^1(\beta \mid e_2 = 0) = \frac{1}{2}\beta^2\Gamma^2.$$

If $e_2 = E$,

$$V^1(\beta \mid e_2 = E) = \begin{cases} \beta\Gamma(\beta\Gamma - \sigma E) + \beta G - \frac{1}{2}(\beta\Gamma - \sigma E)^2 - \frac{1}{2}E^2 - \sigma E(\beta\Gamma - \sigma E), \text{if } \beta \geq \frac{\sigma E}{\Gamma}; \\ \\ \beta G - \frac{1}{2}E^2, \text{if } \beta < \frac{\sigma E}{\Gamma} \end{cases}$$

$$= \begin{cases} \frac{1}{2}\beta^2\Gamma^2 - \beta\Gamma\sigma E - \frac{1-\sigma^2}{2}E^2 + \beta G, \text{if } \beta \geq \frac{\sigma E}{\Gamma}; \\ \\ \beta G - \frac{1}{2}E^2, \text{if } \beta < \frac{\sigma E}{\Gamma}. \end{cases}$$

The graph of $V^1(\beta \mid e_2 = 0)$ and $V^1(\beta \mid e_2 = E)$ for the different choices of e_2 is shown in Figure 1.2.

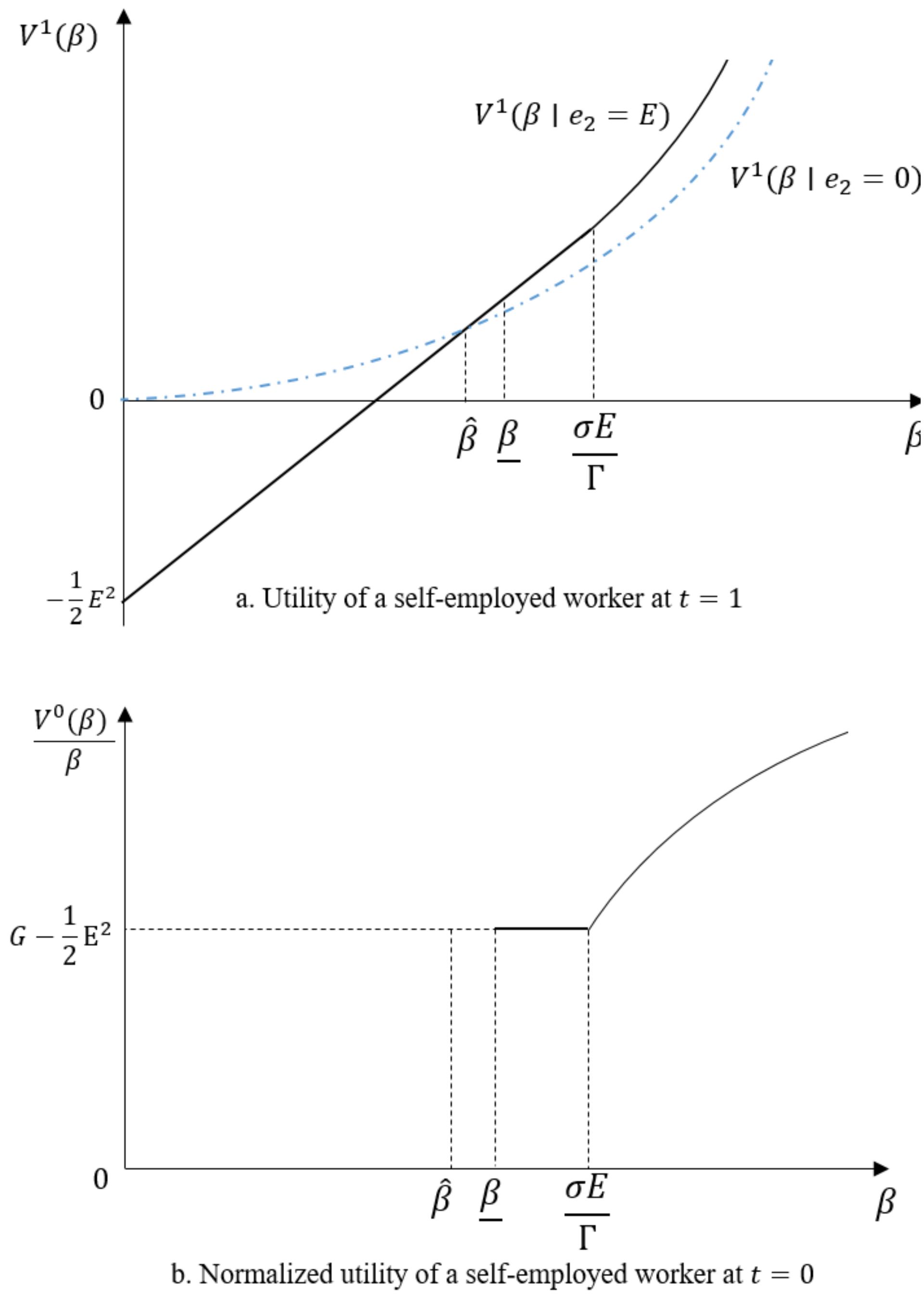

Figure 1.2: Utility of self-employed workers

As in Figure 1.2a, the solid line $V^1(\beta \mid e_2 = E)$ is composed by firstly a linear part and then a quadratic part with a threshold at $\beta = \frac{\sigma E}{\Gamma}$: when $\beta < \frac{\sigma E}{\Gamma}$,

$V^1(\beta \mid e_2 = E) = \beta G - \frac{1}{2}E^2$ and when $\beta \geq \frac{\sigma E}{\Gamma}$, $V^1(\beta \mid e_2 = E) = \frac{1}{2}\beta^2\Gamma^2 - \beta\Gamma\sigma E - \frac{1-\sigma^2}{2}E^2 + \beta G$. The quadratic part of $V^1(\beta \mid e_2 = E)$ is greater than $V^1(\beta \mid e_2 = 0)$, which is the dotted line in Figure 1.2a, once $\beta \geq \frac{(1-\sigma^2)E^2}{2(G-\sigma ET)}$ holds. [6] Note that this condition always holds when $\beta \geq \frac{\sigma E}{\Gamma}$ because $\frac{\sigma E}{\Gamma} > \frac{(1-\sigma^2)E^2}{2(G-\sigma ET)}$ by Assumption 2. As a result, $e_2 = E$ will be chosen by all self-employed workers with $\beta \geq \frac{\sigma E}{\Gamma}$. On the other hand, the linear part of $V^1(\beta \mid e_2 = E)$ is greater than $V^1(\beta \mid e_2 = 0)$ if β is greater than a certain threshold $\hat{\beta}$. [7] Recall that H is the set of all hired employees. The above arguments are formalized in Lemma 1.

Lemma 1. *There exists a threshold $\hat{\beta} \in (0, 1)$ such that all the self-employed workers with $\beta > \hat{\beta}$ will exert the efficient non-pecuniary effort which produces strictly positive non-pecuniary payoff, that is, $e_2(\beta) = E$, $\forall \beta \in [\hat{\beta}, 1] \setminus H$.*

By Lemma 1, self-employed workers will choose $e_2 = E$, which is efficient by Assumption 2, if the worker's type β is higher than a threshold $\hat{\beta}$. Intuitively, if a worker has a very low quasi-hyperbolic discounting β such that $\beta \leq \hat{\beta}$, the worker's behavior, that is, the trade-off between self-employment and paid-employment, will be un-robust as the worker does not care much about future payoffs. As the behavior of those workers with extremely low types is not the focus of this chapter and will bring unnecessary complexity if taken into consideration, I make Assumption 3 below to eliminate this concern.

[6] This result is derived by solving $\frac{1}{2}\beta^2\Gamma^2 - \beta\Gamma\sigma E - \frac{1-\sigma^2}{2}E^2 + \beta G \geq \frac{1}{2}\beta^2\Gamma^2$.

[7] By $\hat{\beta}G - \frac{1}{2}E^2 = \frac{1}{2}\hat{\beta}^2\Gamma^2$, it can be solved that $\hat{\beta} = \frac{E^2}{G+\sqrt{G^2-E^2\Gamma^2}}$. $\hat{\beta} \in (0, 1)$ by Assumptions 1-2. In addition, when G increases, this $\hat{\beta}$ converges to zero. So the condition $\underline{\beta} > \hat{\beta}$ is easy to satisfy with large enough G, which is partially guaranteed by Assumption 2.

Assumption 3: $\underline{\beta} > \hat{\beta}$.

With Assumption 3, the efficient $e_2 = E$ will be exerted by all self-employed workers, as their types β are always higher than $\underline{\beta}$ and thus higher than $\hat{\beta}$. So with Assumption 3, I can substitute the self-employed workers' choices of effort, $e_1 = \begin{cases} \beta\Gamma - \sigma E, \text{ if } \beta\Gamma - \sigma e_2 \geq 0, \\ 0, \text{ otherwise} \end{cases}$ and $e_2 = E$ into $V^0(\beta)$ for the *normalized* utility of self-employed workers,

$$\frac{V^0(\beta)}{\beta} = \begin{cases} (\beta - \frac{1}{2}\beta^2)\Gamma^2 - \frac{1-\sigma^2}{2}E^2 - \sigma\Gamma E + G, \text{if } \beta \geq \frac{\sigma E}{\Gamma}; \\ G - \frac{1}{2}E^2, \text{if } \underline{\beta} \leq \beta < \frac{\sigma E}{\Gamma}. \end{cases}$$

as shown in Figure 1.2b.

Note that in the normalized utility of self-employed workers, I normalize the workers' utility and eliminate the worker's quasi-hyperbolic discounting. By this normalization, I have different workers' normalized utility at $t = 0$ on the same scale of the employer's profit and social welfare, eliminating the effect from their quasi-hyperbolic discounting factors β. This normalized utility can be better illustrated in graphs to show the relation of a signed contract with social welfare and employer's profit, as can be seen later.

1.3.2 Employee's Problem

The employee's problem at $t = 0$ is

$$C(\beta) = \arg\max_{C \in M} w - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2(\beta, e_1) - \sigma e_1 e_2(\beta, e_1) + g(e_2)$$

$$s.t. \quad \frac{U^0(\beta)}{\beta} \geq \frac{V^0(\beta)}{\beta} \tag{IR}$$

where M is the contract menu chosen by the employer. In the above employee's problem, e_2 is determined by the employee at $t = 1$, given the hiring contract (e_1, w) she signed at $t = 0$. Note that if there are multiple solutions of the above maximization, the worker will choose the contract with the highest pecuniary effort by the tie-breaking rule. Denote $H = \left\{\beta \in [\underline{\beta}, 1] \mid \frac{U^0(\beta)}{\beta} \geq \frac{V^0(\beta)}{\beta}\right\}$ as the set of hired employees for following analysis.

1.3.3 Employer's Problem

The employer chooses an optimal menu of contracts $M^* \subset R^+ \times R^+$,

$$M^* \in \arg\max_{M} \int_H [\Gamma e_1(\beta) - w(e_1(\beta))] f(\beta) d\beta = \arg\max_{M \subset \{C\}} \int_H \pi(\beta) f(\beta) d\beta$$

$s.t.$ $\quad C(\beta) = \{e_1(\beta), w(\beta)\}$ solves the problem of the employee with type $\beta, \forall \beta \in H$,

where $H = \left\{\beta \in [\underline{\beta}, 1] \mid \frac{U^0(\beta)}{\beta} \geq \frac{V^0(\beta)}{\beta}\right\}$ is the set of employees hired and $\pi(\beta) = \Gamma e_1(\beta) - w(\beta)$ is an employer's profit from the employee with type β who signs a contract $C(\beta) = (e_1(\beta), w(\beta))$.

1.4 Optimal Contract Menu

1.4.1 Employee's Behavior at t=1

The Revelation Principle implies that the contracting game can be represented by a direct mechanism where agent's strategies are reports of types and that, in equilibrium, reports are truthful. The problem of an employee with $\beta \in H$ at $t = 1$ if signing a contract $C(\beta') = (e_1(\beta'), w(\beta'))$ can thus be written as

$$U^1(\beta, \beta') = \max_{e_2} \beta w(\beta') - \frac{1}{2}e_1^2(\beta') - \frac{1}{2}e_2^2 - \sigma e_1(\beta')e_2 + \beta g(e_2).$$

$$s.t. \quad e_2 \geq 0.$$

There are two potential optimal solutions of e_2, $e_2 = 0$ or $e_2 = E$ which return utilities as follows.

If $e_2 = 0$,

$$U^1(e_2 = 0 \mid \beta, e_1(\beta')) = \beta w(\beta') - \frac{1}{2}e_1^2(\beta'), \tag{1.1}$$

otherwise if $e_2 = E$,

$$U^1(e_2 = E \mid \beta, e_1(\beta')) = \beta w(\beta') - \frac{1}{2}e_1^2(\beta') + \beta G - \frac{1}{2}E^2 - \sigma E e_1(\beta'). \tag{1.2}$$

(1.1) $\leq$ (1.2) if and only if

$$\beta \geq \beta_0(\beta') = \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}.$$

where the threshold $\beta_0(\beta') = \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$ is a function of contracted $e_1(\beta')$ in the signed contract $C(\beta')$. With the decisions for $e_2 = 0$ and $e_2 = E$ respectively at $t = 1$, the employee's utility with type β who signs a contract $C(\beta')$ at $t = 1$ is shown in Figure 1.3a.

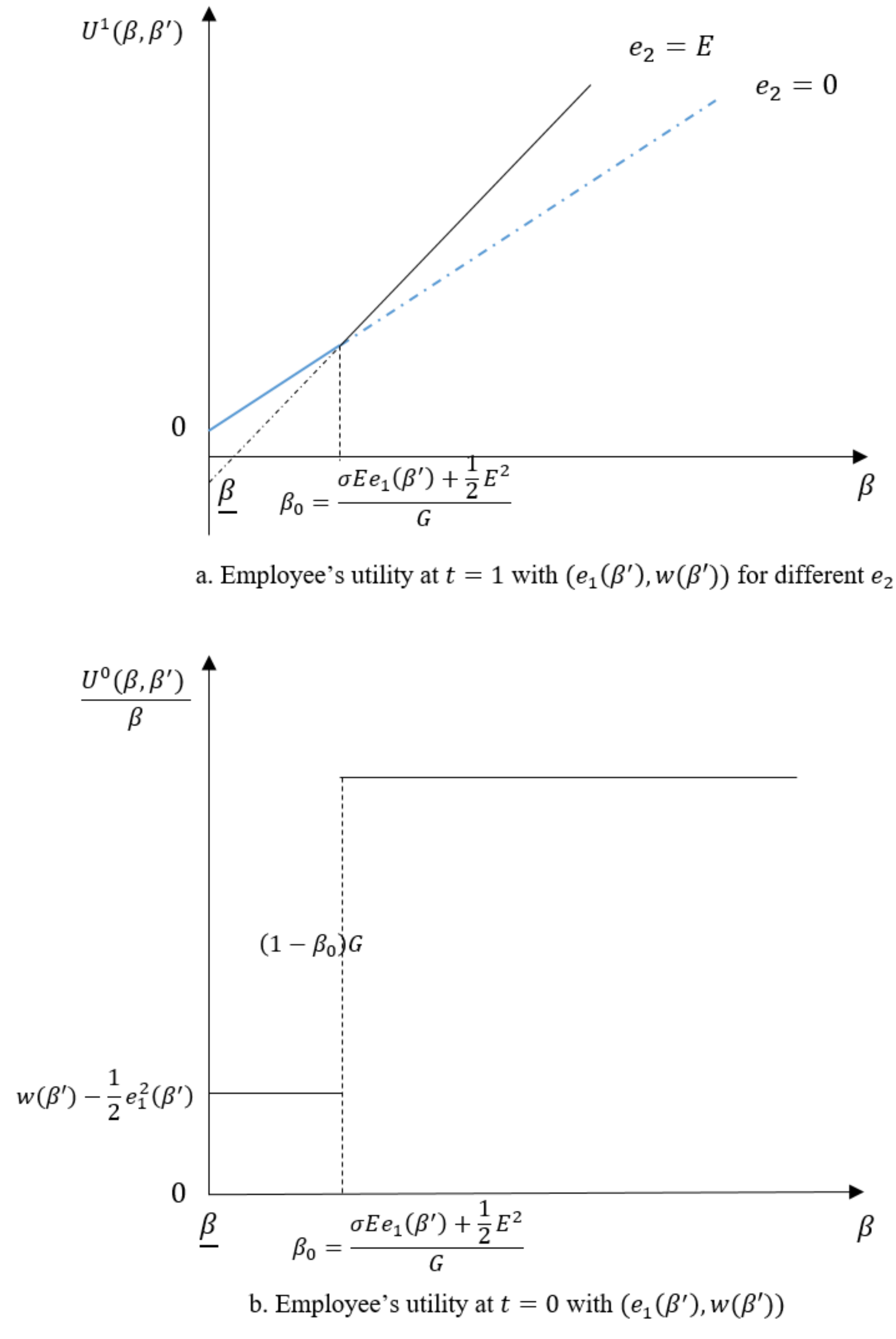

Figure 1.3: Utility of an employee with contract $(e_1(\beta'), w(\beta'))$.

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In Figure 1.3a, given contracted $e_1(\beta')$, at $t = 1$ the employee will choose e_2

which yields the higher utility at that moment, which is

$$e_2 = \begin{cases} 0, \text{ if } \beta < \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G} \\ E, \text{ if } \beta \geq \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G} \end{cases} . \tag{1.3}$$

Substituting the employee's choice at $t = 1$ into her normalized utility at $t = 0$ yields

$$\frac{U^0(\beta, \beta')}{\beta} = \begin{cases} w(\beta') - \frac{1}{2}e_1^2(\beta'), \text{ if } \beta < \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G} \\ w(\beta') - \frac{1}{2}e_1^2(\beta') - \frac{1}{2}E^2 - \sigma E e_1(\beta') + G, \text{ if } \beta \geq \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}, \end{cases} \tag{1.4}$$

as shown in Figure 1.3b. Note that in Figure 1.3b at $\beta = \beta_0(\beta') = \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$, there is a jump on the function of $\frac{U^0(\beta,\beta')}{\beta}$. This can be observed by subtracting the first part of equation (1.4) from its second part,

$$[w(\beta') - \frac{1}{2}e_1^2(\beta') - \frac{1}{2}E^2 - \sigma E e_1(\beta') + G] - [w(\beta') - \frac{1}{2}e_1^2(\beta')]$$

$$= G - \frac{1}{2}E^2 - \sigma E e_1(\beta')$$

$$= G[1 - \beta_0(\beta')] > 0.$$

As in Figure 1.3b, given $C(\beta')$ contracted, the employee's normalized utility $\frac{U^0(\beta,\beta')}{\beta}$ is neither continuous nor differentiable with respect to β. So the traditional method to solve the envelope theorem for adverse selection problems by Milgrom and Segal [23] is not applicable. More importantly, the Spence-Mirrlees condition is not satisfied for this discontinuous utility function. The failure of Spence-Mirrlees condition implies that contracts satisfying the local IC conditions for workers and that the workers will not mimic *any neighboring* types, does not imply the global

IC conditions are satisfied [24], that the workers may mimic further-way types. In this chapter, I employ a different approach by firstly identifying the properties of the optimal contract menu and then looking for the optimal choice variables of the employer, as can be seen later. Additionally, for the pooling equilibrium with limited number of contracts, the discontinuous $\frac{U^0(\beta,\beta')}{\beta}$ function on β results in the discontinuous $\frac{U^0(\beta)}{\beta}$ in the equilibrium, which is one characteristic of the equilibrium.

Given $C(\beta')$ signed by the worker at $t = 0$, Figure 1.4 compares $\frac{U^0(\beta,\beta')}{\beta}$ with the employee's normalized utility from her outside option of self-employment, $\frac{V^0(\beta)}{\beta}$.

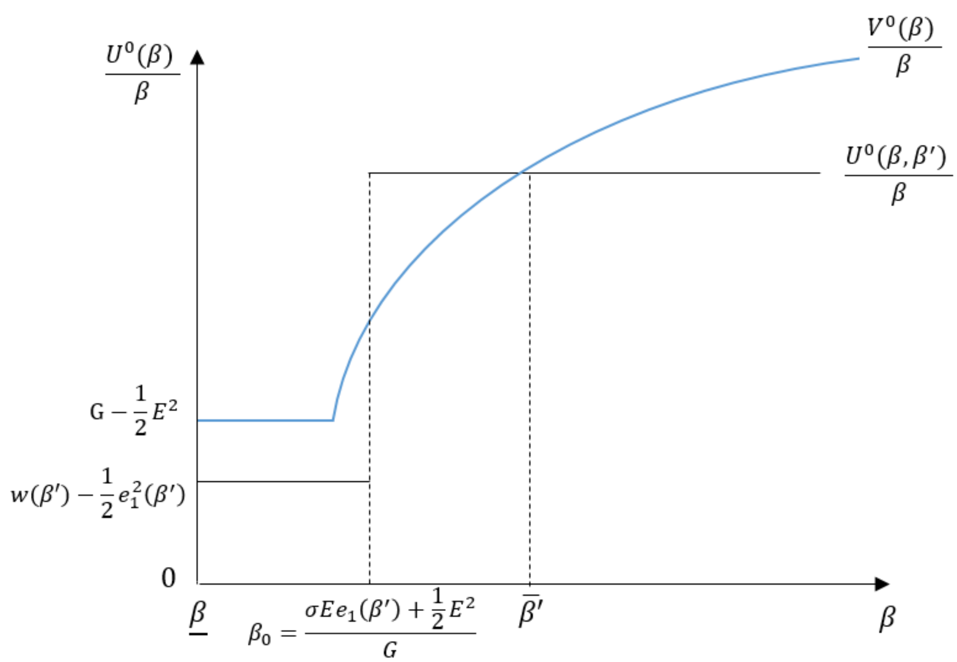


Figure 1.4: The worker's utility as an employee or self-employed worker.

Assumption 2 guarantees the jump of $\frac{U^0(\beta,\beta')}{\beta}$ at $\beta = \beta_0$ to be large enough. As shown in Figure 1.4, it can be proved that $\frac{U^0(\beta,\beta')}{\beta} \leq \frac{V^0(\beta)}{\beta}$ holds for any worker with $\beta < \beta_0(\beta')$, given that the $C(\beta')$ is a profitable contract for the employer. This is formalized in Lemma 2 below.

Lemma 2. *A worker with* $\beta < \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$ *will prefer self-employment to any hiring contract* $(e_1(\beta'), w(\beta'))$ *profitable to the employer.*

With Lemma 2, any employee must have $\beta \geq \frac{\sigma E e_1(\beta) + \frac{1}{2}E^2}{G}$ in the equilibrium. This inequity can be further written as $e_1(\beta) \leq \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$, that is, the pecuniary effort that a worker with type β can accept in a hiring contract in the equilibrium must be smaller or equal to $\frac{\beta G - \frac{1}{2}E^2}{\sigma E}$. This imposes a constraint of a worker's "maximal contractable effort," which is linear on the worker's type β. Also with Lemma 2, as all the employees are constrainted by their "maximal contractable effort," $e_2 = E$ for all employees by (1.3). Combining with Lemma 1, I have Corollary 1 for the amount of non-pecuniary effort exerted by any worker, no matter whether they are employed or self-employed.

Corollary 1. *All workers, no matter employed or self-employed, will exert the effective non-pecuniary effort which produces the positive non-pecuniary payoff, that is,* $e_2 = E$, $\forall \beta \in [\underline{\beta}, 1]$.

In next Section 5, I will apply the effort choices of the self-employed workers and hired employees to derive the properties of an optimal contract menu.

1.4.2 Socially Optimal Pecuniary Effort

In this section, I will solve the socially optimal contract for any worker with $\beta \in [\underline{\beta}, 1]$. Here, after a given contract signed, a worker will still choose her optimal non-pecuniary effort e_2 freely, but the goal of designing the contract is to maximize the produced social welfare instead of the employer's profit. This result will be helpful in the later discussion about the efficiency of the contract menu in the equilibrium.

The social welfare produced by a worker with type β, if self-employed, is

$$SW(e_1, e_2) = \Gamma e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - \sigma e_1 e_2 + g(e_2).$$

Note that the worker's type β is not present in this function of $SW(e_1, e_2)$.

By Lemma 2, the contract can serve as a commitment device for this worker, with the constraint for her "maximal contractable effort" satisfied, that is, $e_1 \leq \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$. So with the constraint of "maximal contractable effort," I have $e_2 = E$ by (1.3), which can be substituted into the above social welfare function, yielding

$$SW(e_1) = (\Gamma - \sigma E)e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 + G.$$

Note that this $SW(e_1)$ achieves its maximum when $e_1 = \Gamma - \sigma E$, disregarding the worker's constraint of "maximal contractable effort." However, as this first-best pecuniary effort may exceed the worker's "maximal contractable effort", the socially optimal e_1^* that can be achieved through an employment contract is

$$e_1^* = \begin{cases} \Gamma - \sigma E, \text{ if } \frac{\beta G - \frac{1}{2}E^2}{\sigma E} \geq \Gamma - \sigma E, \\ \frac{\beta G - \frac{1}{2}E^2}{\sigma E}, \text{ if } \frac{\beta G - \frac{1}{2}E^2}{\sigma E} < \Gamma - \sigma E. \end{cases} \quad (1.5)$$

Denote $\hat{\hat{\beta}}$ for the threshold of the worker's type to choose between the two possible formula for the socially optimal e_1^* above, that is, $\frac{\hat{\hat{\beta}}G-\frac{1}{2}E^2}{\sigma E} = \Gamma - \sigma E$. Hence under the socially optimal contract, e_1^* for a worker with type $\beta < \hat{\hat{\beta}}$ equals her maximal contractable effort $\frac{\beta G-\frac{1}{2}E^2}{\sigma E}$. Otherwise if $\beta \geq \hat{\hat{\beta}}$, the worker will exert e^* equal to the first best pecuniary effort $\Gamma - \sigma E$.

1.5 Characterization of a Hybrid Equilibrium

In this section, I will solve for the properties of an optimal contract menu in the model equilibrium.

Proposition 1. *There exists an optimal contract menu for the employer.*

Proposition 1 directly follows the result of [25]. Based on Proposition 1, a property of the equilibrium is given in Lemma 3 to further investigate the properties of the optimal contract menu.

Lemma 3. *In the equilibrium, if* $\exists \beta' \in H$ *such that* $\beta' > \frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}$*, then* $e_1(\beta) = e_1(\beta')$*,* $\forall \beta \in [\frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}, \beta')$*.*

Intuitively, if there is a hired employee with type $\beta' \in H$ who sign a contract with pecuniary effort strictly less than her "maximal contractable effort," that is, $\beta' > \frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}$, workers with slightly lower types will sign the same contract, which is the only way to satisfy all of those workers' IC constraints. Lemma 3 formalizes this argument. Lemma 3 implies ranges of quasi-hyperbolic discounting factors of employees for a pooling equilibrium, where the lower bound of each range must have the constraint of "maximal contractable effort" for the employee bind, that is, the lower bound of each range for the pooling equilibrium, $\underline{\beta}_j$, satisfies $\underline{\beta}_j = \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$. Theorem 1 follows Lemma 3, which highlights employees' behavior in the hybrid equilibrium with pooling and separating parts.

Theorem 1. *In the optimal contract menu, there exists a set of disjoint collections of employees,* $h = \cup_j h_j \subset H$ where $\bar{\beta}^j \leq \underline{\beta}^{j+1}$ and $h_j = \begin{cases} [\underline{\beta}_j, \bar{\beta}_j], & \text{if } \sup h_j = \sup H, \\ [\underline{\beta}_j, \bar{\beta}_j), & \text{otherwise} \end{cases}$ *a.e. s.t.:*

(1). all employees with $\beta \in h_j$ *sign the same contract* (e_1^j, w^j) *with* $e_1^j = \frac{\underline{\beta}_j G - \frac{1}{2}E^2}{\sigma E}$ *and* $\frac{U^0(\bar{\beta}_j)}{\beta_j} = \frac{V^0(\bar{\beta}_j)}{\beta_j}$; [9]

(2). employees with $\beta \in H \setminus h$ *sign separating contracts with* $e_1(\beta) = \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$ *and* $\frac{U^0(\beta)}{\beta} = \frac{V^0(\beta)}{\beta}$. [10]

The sets of quasi-hyperbolic discounting for pooling equilibrium in Theorem 1, $h = \cup_j h_j$, are directly from Lemma 3. Each $h_j \subset h$ is a connected range of quasi-hyperbolic discounting factors with workers signing the same contract, with the constraint of "maximal contractable effort" binding at its lower bound as stated in Lemma 3. Note that a $h_j \subset h$ is closed on its upper bound if it is for the employees with the highest types, otherwise, $h_j \subset h$ is open on its upper bound. The remaining employees sign contracts with pecuniary effort and lump-sum wage differing on their

[9]The lump-sum wage for this contract can thus be solved as $w^j = [\bar{\beta}_j - \frac{1}{2}(\bar{\beta}_j)^2]\Gamma^2 + \frac{\sigma^2}{2}E^2 - \sigma\Gamma(E - e_1^j) + \frac{1}{2}(e_1^j)^2$.

[10]The lump-sum wage for these contracts can thus be solved as $w(\beta) = (\beta - \frac{1}{2}\beta^2)\Gamma^2 + \frac{\sigma^2}{2}E^2 - \sigma\Gamma[E - e_1(\beta)] + \frac{1}{2}e_1^2(\beta)$.

types $\beta \in H \setminus h$ in this separating equilibrium. Each worker in $H \setminus h$ will exert her "maximal contractable effort" and receive a wage which makes her indifferent between self-employment and paid-employment with this separating contract. Note that in reality, workers in a certain "firm position" work similar hours and earn similar incomes. Hence, one may regard each of the ranges $h_j \subset H$ for a pooling equilibrium as the model counterpart of workers in a "firm position" in reality. In contrast, the part-time workers in reality are paid based on their working hours and have more diversified income. The part-time workers in reality thus correspond to the employees in the separating equilibrium with $\beta \in H \setminus h$ in this model. Note that it is possible that $h = \emptyset$ in the model equilibrium, in which case all the employees are part-time employees in the separating equilibrium.

Based on Theorem 1, from an employee with type β who signs the contract $C = (e_1, w)$, [11] the employer earns profit

$$\pi(\beta) = \Gamma e_1 - w$$

$$= (\Gamma - \sigma E)e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 + G - [w - \sigma E e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 + G]$$

$$= SW(e_1) - \frac{U^0(\beta)}{\beta}.$$

Together with Theorem 1, I can write the employer's profit from an employee with $\beta \in H \setminus h$ as

$$\pi(\beta) = SW\left(\frac{\beta G - \frac{1}{2}E^2}{\sigma E}\right) - \frac{V^0(\beta)}{\beta}$$

$$= \Psi(\beta) - \Phi(\beta)$$

[11] This requires $\beta \geq \frac{\sigma E e_1 + \frac{1}{2}E^2}{G}$ by Lemma 2.

where $\Psi(\beta) = (\Gamma - \sigma E)\frac{\beta G - \frac{1}{2}E^2}{\sigma E} - \frac{1}{2}\left(\frac{\beta G - \frac{1}{2}E^2}{\sigma E}\right)^2$ and

$$\Phi(\beta) = \begin{cases} (\Gamma - \sigma E)(\beta\Gamma - \sigma E) - \frac{1}{2}(\beta\Gamma - \sigma E)^2, \text{ if } \beta \geq \frac{\sigma E}{\Gamma} \\ \\ 0, \text{ otherwise.} \end{cases}$$

Intuitively, $\Psi(\beta)$ is the social welfare produced by the employee with type β who exerts her maximal contractable effort and Φ is the employee's utility from her outside option, after the same term eliminated. Note that $\Psi(\beta)$ and $\Phi(\beta)$ are both quadratic on *beta* and have the same maximal value $\frac{(\Gamma - \sigma E)^2}{2}$ but $\Psi(\beta)$ takes this maximal value at $\beta = \hat{\hat{\beta}} = \frac{\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2}{G} < 1$ and $\Phi(\beta)$ takes this maximal value at the point when $\beta = 1$.

Also, with Theorem 1, I can uniquely identify the contract (e_1^j, w^j) for each $\beta \in h_j$ by the lower and upper bounds of the pooling equilibrium range h_j, $\underline{\beta}_j$ and $\bar{\beta}_j$. So the employer's profit from the employee with $\beta \in h_j \subset h$ is

$$\pi(\beta) = SW\left(\frac{\beta_j G - \frac{1}{2}E^2}{\sigma E}\right) - \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j} = \Psi(\underline{\beta}_j) - \Phi(\bar{\beta}_j).$$

The employer's problem is thus

$$\max_{H,h}\left\{\int_{H\setminus h}\left[\Psi(\beta) - \Phi(\beta)\right]f(\beta)d\beta + \sum_{h_j\subset h}\int_{\underline{\beta}_j}^{\bar{\beta}_j}\left[\Psi(\underline{\beta}_j) - \Phi(\bar{\beta}_j)\right]f(\beta)d\beta\right\},$$
$$s.t. \quad \underline{\beta}_j < \bar{\beta}_j. \tag{1.6}$$

Denote $\tilde{\beta} > 0$ for the crossing point of $\Psi(\beta)$ and $\Phi(\beta)$, that is, $\Psi(\tilde{\beta}) = \Phi(\tilde{\beta})$. Note that $\tilde{\beta} \in (\frac{\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2}{G}, 1)$ because $\Psi(\beta) - \Phi(\beta)$ is a quadratic function on β and

$$\Phi\left(\frac{\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2}{G}\right)$$
$$< \Psi\left(\frac{\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2}{G}\right) = \frac{(\Gamma - \sigma E)^2}{2}$$

and $\Phi(1) = \frac{(\Gamma-\sigma E)^2}{2} > \Psi(1)$. Because the employer earns a profit $\Psi(\beta) - \Phi(\beta)$ from each part-time workers, it is at least profitable for the employer to hire all the workers with types smaller than $\tilde{\beta}$, that is, $\beta \in [\underline{\beta}, \tilde{\beta}]$. In addition, it is easy to prove that the set of hired employees H must be a connected set, with its upper bound greater or equal to $\tilde{\beta}$. This argument is formalized in Proposition 2.

Proposition 2. *In the optimal contract menu, a worker will be hired if and only if her type is lower than a threshold* $\bar{\beta} \in [\tilde{\beta}, 1]$*, that is,* $H = [\underline{\beta}, \bar{\beta}]$ *s.t.* $\bar{\beta} \geq \tilde{\beta}$.

Proposition 2 states that there is a threshold, $\bar{\beta}$, for workers to be self-employed, and all the workers with smaller types will be hired.

With $\Psi(\beta)$ and $\Phi(\beta)$ defined, the employer's profit (1.6) from hiring contracts can be illustrated in Figure 1.5.

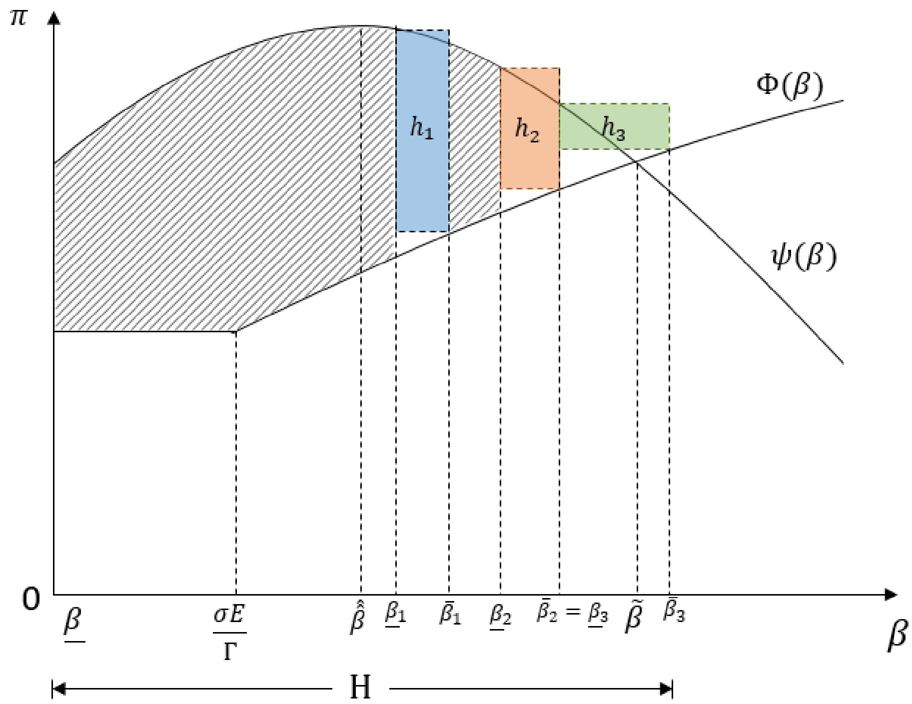


Figure 1.5: Profit of the employer given set of positioned employees h and part-time employees $H \setminus h$

In Figure 1.5, the set of workers' quasi-hyperbolic discounting for pooling equilibrium is $h = h_1 \cup h_2 \cup h_3 \subset H$. From an employee with $\beta \in h_j \subset H$, $j \in \{1,2,3\}$, the employer earns a profit equal to $\Psi(\underline{\beta}_j) - \Phi(\bar{\beta}_j)$. So the profits that the employer earns from employees in a certain firm position $h_j \subset h$, $j \in \{1,2,3\}$ are shown in the three rectangle areas in Figure 1.5. From an employee with $\beta \in H \setminus h$, the employer extracts a profit $\Psi(\beta) - \Phi(\beta)$ so the employer's profit is the shadow area between $\Psi(\beta)$ and $\Phi(\beta)$. The trade-off for the employer between hiring part-time employees versus positioned employees can also be seen in Figure 1.5: compared with positioned/pooling employees in h_j, the employer pays the part-time/separating employees a lower wage which just makes her indifferent between the paid-employment and self-employment. On the other hand, if the workers have high enough types, that is, if $\beta \geq \underline{\beta}_j > \hat{\hat{\beta}}$, the social welfare produced by her as a part-time employee who exerts her own "maximal contractable effort" is lower than that from her being a positioned employee who exerts the "maximal contractable effort" of the lowest type worker in the same position. So there is a trade-off for the employer between hiring full-time employees who exert effort more efficiently but require higher wage versus hiring part-time employees who exert effort less efficiently but require lower wage. In this sense, the full-time employment is more profitable for the employer to impose on the high-type workers, as the benefit from more efficient effort exerting is higher for high-type workers. Based on this idea, Proposition 3 gives a sufficient condition, describing the way in which the employees are positioned in an optimal contract menu.

Proposition 3. *In the optimal contract menu, an employee with type $\beta \in H$ will be in a range for the pooling equilibrium, $h_j \subset h$ a.e., if $\Psi'(\beta) + \Phi'(\beta) < 0$. That is, $\forall \beta \in H$ such that $\Psi'(\beta) + \Phi'(\beta) < 0$, $\beta \in h_j \subset h$ a.e..*

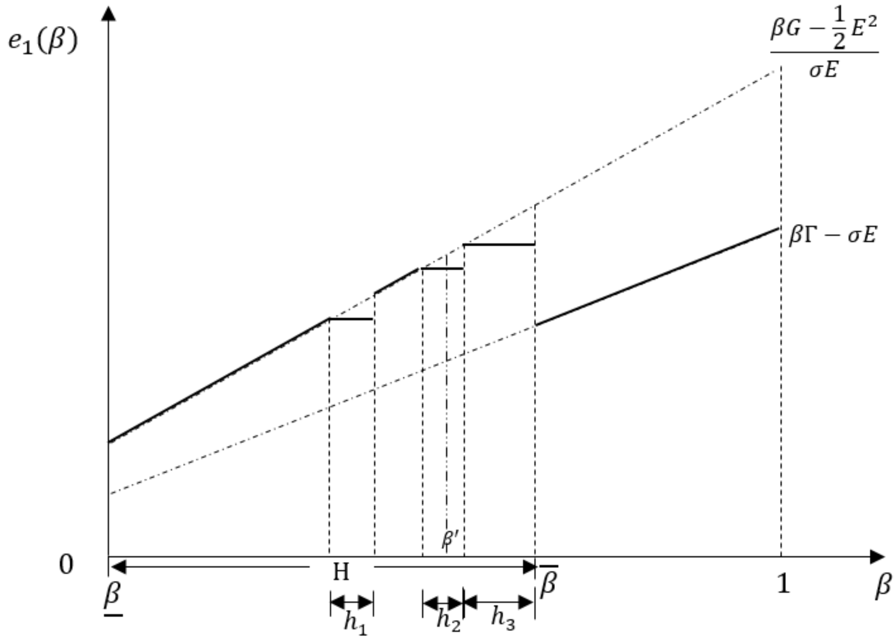
Proposition 3 gives a sufficient condition for an employee with type $\beta \in H$ in the range of a pooling equilibrium, that is, if $\beta \in H$ and $\Psi'(\beta) + \Phi'(\beta) < 0$, then almost everywhere $\beta \subset h$. Note the opposite direction does not hold; in other words, there may exist $\beta \in h$ such that $\Psi'(\beta) + \Phi'(\beta) \geq 0$. Based on Proposition 2, Corollary 2 gives a sufficient condition for the existence of positioned employment/pooling equilibrium.

Corollary 2. *In an optimal contract menu, $h \neq \emptyset$ if $H \neq \emptyset$ and $\Psi'(\tilde{\beta}) + \Phi'(\tilde{\beta}) < 0$.*

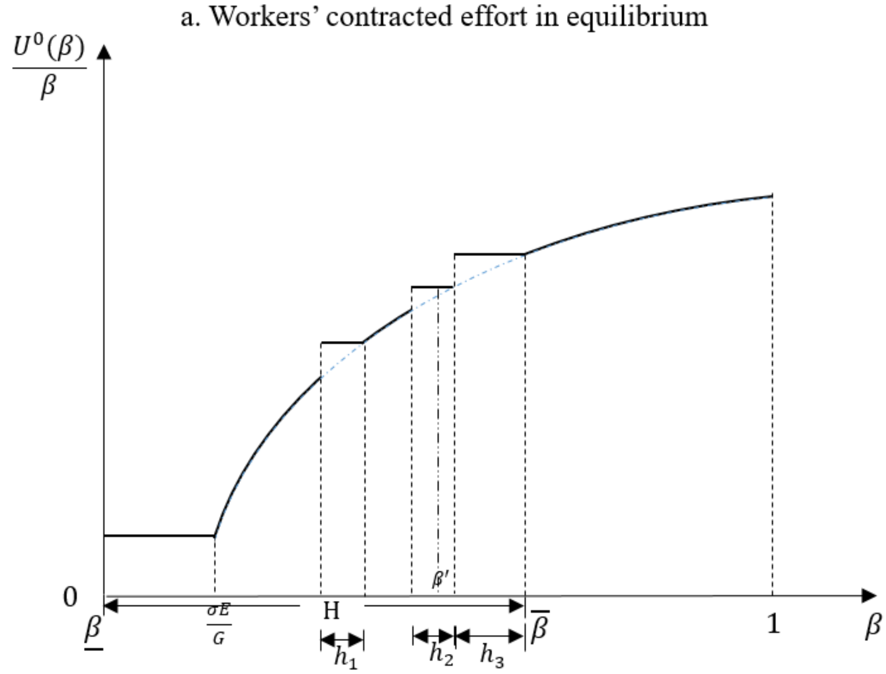
By Corollary 2, the existence of h can be guaranteed for given ranges of model parameters, given $H \neq \emptyset$. Suppose $H \neq \emptyset$. If $\tilde{\beta} < \frac{\sigma E}{\Gamma}$, $\Psi'(\tilde{\beta}) + \Phi'(\tilde{\beta}) < 0$ always hold because $\tilde{\beta} > \hat{\hat{\beta}}$. By Corollary 2, $h \neq \emptyset$ if $\tilde{\beta} < \frac{\sigma E}{\Gamma}$. Otherwise if $\tilde{\beta} \geq \frac{\sigma E}{\Gamma}$, the condition for Corollary 2, $\Psi'(\tilde{\beta}) + \Phi'(\tilde{\beta}) < 0$, is satisfied once G is large enough. [12] So from the above two cases, given $H \neq \emptyset$, there is $h \neq \emptyset$ if G is large enough, which is partly guaranteed by Assumption 2. The existence of the pooling equilibrium can thus be guaranteed by the size of non-pecuniary payoff G.

With Theorem 1 and Propositions 1 to 3, $e_1(\beta)$ and the normalized utility of all workers in an optimal contract menu is shown in Figure 1.6. Recall that $e_2(\beta) = E, \forall \beta \in [\underline{\beta}, 1]$ by Corollary 1.

[12] It can be easily solved that $\Psi'(\tilde{\beta}) + \Phi'(\tilde{\beta}) = \Gamma^2(1 - \tilde{\beta}) - \frac{\tilde{\beta}}{\sigma^2 E^2} G^2 + \frac{G}{\sigma E}(\frac{E}{2\sigma} + \Gamma - \sigma E)$ which is negative with G large enough.



a. Workers' contracted effort in equilibrium



b. Workers' normalized utility in equilibrium

Figure 1.6: Separation of different jobs in the optimal contract menu

In Figure 1.6, all the workers with $\beta \in [\underline{\beta}, \bar{\beta})$ are hired employees where there are three positions $h_1 \cup h_2 \cup h_3 = h$ for those in the pooling equilibrium. At the lower bound of each position where $\beta = \underline{\beta}_j$, $\forall j = \{1, 2, 3\}$, $e_1(\beta)$ is continuous as shown in Figure 1.6a while $\frac{U^0(\beta)}{\beta}$ is discontinuous as shown in Figure 1.6b. This discontinuity of $e_1(\beta)$ is because at $\beta = \underline{\beta}_j$, the constraint of "maximal contractable effort" prevents the employee with types slightly lower than $\underline{\beta}_j$ from mimicking higher type employee who earns a higher wage. The discontinuity in utilities can thus sustain.

On the other hand, at the upper bound $\beta = \bar{\beta}_j$, $e_1(\beta)$ is discontinuous as shown in Figure 1.6a while $\frac{U^0(\beta)}{\beta}$ is continuous as shown in in Figure 1.6b. The continuous $\frac{U^0(\beta)}{\beta}$ is because the employees at the different sides of $\beta = \bar{\beta}_j$ can mimic each other freely with their constraint of "maximal contractable effort" slack, $\frac{U^0(\beta)}{\beta}$ must be continuous. Meanwhile, because $\frac{U^0(\beta)}{\beta}$ is continuous at this point but self-employed workers with types slightly higher than $\sup H$ work significantly less than employees, reflected in the discontinuous $e_1(\beta)$, those self-employed workers must earn a significantly lower monetary payoff than the employees with slightly lower types. This observation could explain the observation that people enter into self-employment despite the lower income of self-employed workers than employees. This explanation is consistent with Hurst and Pugsley [10]'s suggestion for the non-monetary benefit of self-employment, that is, "more flexibility" or "being one's own boss," which is consistent with the lower pecuniary effort exerted by self-employed workers in the model equilibrium.

Finally, as there is a $\beta' \in h_2 \subset H$ such that $\Psi'(\beta') + \Phi(\beta') < 0$ and $\beta' > \frac{\sigma E}{G}$ in Figure 1.6, by Proposition 3 all the employees with $\beta > \beta'$ will be positioned employees in the pooling equilibrium. As a result, there should be no gap between the positions h_2 and h_3, that is, $\bar{\beta}_2 = \underline{\beta}_3$.

1.6 Implications

In this section, I derive properties of the hybrid equilibrium for the model which spells out the firm hierarchy and labor market structure with part-time workers and positions of full-time workers, independent of the distribution of quasi-hyperbolic discounting factor β of workers and based on general assumptions of the model parameters. I will make more assumptions to simplify the solution further, with which I can discuss some policy implication of the model.

1.6.1 First Order Condition

Given e_1^1 in the contract for h_1, $\underline{\beta}_1$ is uniquely determined by $\underline{\beta}_1 = \frac{\sigma E e_1^1 + \frac{1}{2}E^2}{G}$. So for given e_1^1, $\underline{\beta}_1 > \underline{\beta}$ and $\underline{\beta}_1 \leq \underline{\beta}_1$ are both possible. With the properties of the hybrid equilibrium given above, there are two possible forms for the employer's problem according to those different locations of $\underline{\beta}_1$.

If $\underline{\beta}_1 > \underline{\beta}$, the most procrastinating workers with lowest types sign separating contracts. The principal's objective is to maximize profit

$$\Pi(h, H) = \int_{\underline{\beta}}^{\underline{\beta}_1} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta)$$

$$+ \sum_{h_i, h_{i+1} \in h} \left\{ \int_{\underline{\beta}_i}^{\bar{\beta}_i} [\Psi(\underline{\beta}_i) - \Phi(\bar{\beta}_i)] f(\beta) d\beta + \int_{\bar{\beta}_i}^{\underline{\beta}_{i+1}} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta) \right\} + S$$

$$s.t. \quad \underline{\beta}_{i+1} \geq \bar{\beta}_i, \forall h_i, h_{i+1} \subset h. \tag{1.7}$$

If $\underline{\beta}_1 \leq \underline{\beta}$, the most procrastinating workers with lowest types are in the pooling equilibrium h_1 and the first term in (1.6.1) for the profit from part-time employees in $[\underline{\beta}, \underline{\beta}_1)$ is gone. The principal's objective is to maximize the profit

$$\Pi(h,H) = \sum_{h_i,h_{i+1}\in h} \left\{ \int_{\underline{\beta}_i}^{\bar{\beta}_i} [\Psi(\underline{\beta}_i) - \Phi(\bar{\beta}_i)] f(\beta) d\beta + \int_{\bar{\beta}_i}^{\underline{\beta}_{i+1}} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta) \right\} + S$$

$$s.t. \quad \underline{\beta}_{i+1} \geq \bar{\beta}_i, \forall h_i, h_{i+1} \subset h. \tag{1.8}$$

Note that in the above two profit functions, $S = \int_{\underline{\beta}_J}^{\sup H} \pi(\beta) f(\beta) d\beta$ if the number of h_j is finite and equals to J, otherwise $S = \int_{\sup h}^{\sup H} \pi(\beta) f(\beta) d\beta$.

For the above two profit functions, the FOCs for $\underline{\beta}_j$ and $\bar{\beta}_j$ are the same. The FOC for $\bar{\beta}_1$ are the same too. However, the FOCs for $\underline{\beta}_1$ are different, that is,

$$\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1) + \Psi'(\underline{\beta}_1)(\bar{\beta}_1 - \underline{\beta}_1) = 0$$

if $\underline{\beta}_1 > \underline{\beta}$ and

$$\Psi'(\underline{\beta}_1) = 0$$

if $\underline{\beta}_1 \leq \underline{\beta}$. The optimal solution for of the model can thus only be identified after comparing potential solutions of the two sets of FOCs if no further assumption is made. This will make the solution difficult to tract. Because of this, I will make Assumption 4 to further simplify the model.

Assumption 4. *$\underline{\beta} < \hat{\hat{\beta}}$ and $G > \hat{G}$.* [13]

Assumption 4 makes sure that the maximal contractable effort of the lowest type worker, that is, $\beta = \underline{\beta}$, is smaller than the socially optimal pecuniary effort

[13] $G > \hat{G} = \frac{[\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2]\Gamma}{\sqrt{2\sigma E(\Gamma - \sigma E)}}$ which makes sure that $(\hat{\beta}, \hat{\hat{\beta}}) \neq \emptyset$ for $\underline{\beta}$ be able to satisfy both Assumption 3 and $\underline{\beta} > \hat{\beta}$.

$\Gamma - \sigma E$. With Assumption 4, it is optimal for the employer to assign the lowest type employee with $\beta = \underline{\beta}$ a contract with her "maximal contractable effort". The lowest type worker at $\beta = \underline{\beta}$ is thus in the separating equilibrium. This argument is formalized in Proposition 4.

Proposition 4. *In the optimal contract menu, with Assumptions 1-4, the most procrastinating workers with type $\beta = \underline{\beta}$ sign the separating hiring contracts, that is, $\underline{\beta}_1 > \underline{\beta}$.*

By Proposition 4, the FOC of the employer's problem for $\underline{\beta}_1$ is unique so that I can track the solution of optimal contract menu using the unique set of FOCs on the employer's choice variables. Recall that I denote the socially optimal pecuniary effort that can be achieved in hiring contracts as e_1^*, which satisfies $e_1^* \leq \Gamma - \sigma E$. Corollary 3 directly follows the FOC of $\underline{\beta}_1$.

Corollary 3. *With Assumptions 1-4, all the positioned employees exert pecuniary effort more than socially optimal level, that is, $e_1(\beta) > \Gamma - \sigma E \geq e_1^*$, $\forall \beta \in h$.*

By Corollary 3, all the positioned workers overwork. Note that by paying a lower wage, the employer tends to earn a higher profit from part-time employees than from positioned ones. As a result, the employer has the motive to require an effort higher than socially optimal for positioned employees as in Corollary 3, in order to enlarge the range of part-time employees. From Corollary 3, $\underline{\beta}_1 > \hat{\hat{\beta}}$ which is directly from $e_1^1 = \frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E} > \Gamma - \sigma E$.

1.6.2 Uniform Distribution

The solution for the optimal contract menu, as one can easily see, depends on the distribution of β. In order to further discuss the implications of the model, I identify the equilibrium by assuming a simple distribution of β, that is, a uniform distribution. In addition, to avoid the complexity from the kink on $\Phi(\beta)$ for the FOCs of the employer at $\beta = \frac{\sigma E}{\Gamma}$, I make following Assumption 5.

Assumption 5. $\frac{\sigma E}{\Gamma} \leq \hat{\hat{\beta}}$.

With $\hat{\hat{\beta}} = \frac{\sigma E(\Gamma - \sigma E) + \frac{1}{2}E^2}{G} < 1$, Assumption 5 requires the size of pecuniary payoff, Γ, to be big enough. From Corollary 3, $\underline{\beta}_1 > \hat{\hat{\beta}} \geq \frac{\sigma E}{\Gamma}$, so Assumption 5 makes sure that $\underline{\beta}_1$ is at the concave rather than linear part of $\Phi(\beta)$. Based on Assumption 5 and uniform distribution of β, I have Proposition 5.

Proposition 5. *In the optimal contract menu, with Assumptions 1-5, if the distribution of β is uniform, an employee with $\beta \in H$ must sign the same hiring contract if and only if her type β is higher than a threshold $\underline{\beta}_1 \leq \tilde{\beta}$, that is, $h = h_1 = [\underline{\beta}_1, \bar{\beta})$ where $\bar{\beta} \in [\tilde{\beta}, 1]$.*

By Proposition 5, if the distribution of β is uniform and Assumptions 1-5 hold, there is only one position, $h_1 = h$, for employees with comparatively higher types. With Assumptions 1-5 and uniform distribution of β, in the equilibrium workers are partitioned into three sets by their types: lowest-type workers with β such that $\underline{\beta} \leq \beta < \underline{\beta}_1$ are part-time employees in the separating equilibrium, medium-type workers with β such that $\underline{\beta}_1 \leq \beta < \bar{\beta}_1$ are positioned employees in the pooling equilibrium and the highest-type workers with $\beta > \bar{\beta}_1$ are self-employed. By theorem 1, only the positioned employees with medium types benefit from the hiring contract.

With Proposition 5, there are only two choice variables for the employer to decide, the lower bound of types for full-time workers $\underline{\beta}_1$ which determines the working hours of the positioned workers by $e_1^1 = \frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E}$ and the upper bound of types for full-time workers $\bar{\beta}_1$ which determines the positioned employees' wage w^1 by $\frac{U^0(\bar{\beta}_1)}{\bar{\beta}_1} = \frac{V^0(\bar{\beta}_1)}{\bar{\beta}_1}$.

With Proposition 5, the principal's profit is

$$\int_0^{\underline{\beta}_1} \left[\Psi(\beta) - \Phi(\beta)\right] d\beta + \int_{\underline{\beta}_1}^{\bar{\beta}_1} \left[\Psi(\underline{\beta}_1) - \Phi(\bar{\beta}_1)\right] d\beta$$

The policy implications of the model are derived based the above profit function of the employer.

1.6.3 Policy Implications

In this section, I will discuss the marginal effect of ceilings on working hours and wage in the equilibrium. Specifically, I will calculate the marginal effect of decreasing the ceiling which was binding at the equilibrium level without the ceiling. By doing this, I can show the direction of the marginal impact from those regulations on the social welfare and workers' surplus and the potential benefit/loss of imposing those regulations.

Working Hours Regulation

By Corollary 3, all workers in $h \subset H$ work more than the socially optimal level. One natural question is that whether the government can improve the social welfare by imposing a regulation for the maximum allowed working hours, $\hat{e_1}$, in hiring contracts. With Assumptions 1-5, the model gives an affirmative answer to this question. Even more, the model predicts the workers' surplus also rises from a properly designed working hours ceiling.

As $\underline{\beta}_1 = \frac{\sigma E \hat{e}_1 + \frac{1}{2}E^2}{G}$, with a binding ceiling of working hours $\hat{e}_1$, the threshold of workers' types for positioned jobs $\underline{\beta}_1$ decreases and workers with lower types can benefit from the positioned job. On the other hand, the working hour regulation will affect the threshold of self-employment $\bar{\beta}_1$ through its impact on $\underline{\beta}_1$. Applying the implicit function theorem on the FOC of the employer's profit for $\bar{\beta}_1$ yields

$$\frac{d\bar{\beta}_1}{d\underline{\beta}_1} = \frac{\Psi'(\underline{\beta}_1) + \Phi'(\bar{\beta}_1)}{2\Phi'(\bar{\beta}_1) + \Phi''(\bar{\beta}_1)(\bar{\beta}_1 - \underline{\beta}_1)}$$

which can be proved to be strictly negative, when the wage hours ceiling $\hat{e}_1$ just binds at the maximal equilibrium working hours for employees, e_1^1, without the ceiling. Intuitively, the lower working hours makes it cheaper for the employer to hire high-type workers, so more high-type workers will be hired by the employer once the working hours ceiling is imposed. A slightly lower binding ceiling on working hours will thus decrease $\underline{\beta}_1$ and increase $\bar{\beta}_1$. This is good news as the total employment and the full-time employment both increase marginally. The more full-time employment means that more workers can enjoy the surplus from full-time employment and their surplus increases. Also, the pecuniary effort exerted by the employed workers increases and gets closer to the socially optimal level, and social surplus produced by each employee increases. Together with the higher total employment, the total social welfare must increase. As the marginal effect of wage hours ceiling at the equilibrium maximal wage of employees is beneficial for both the social welfare and workers, a properly designed working hours ceiling will increase both the workers' surplus and social welfare.

Wage ceiling

A wage ceiling is a restriction on the highest wage that an employer can pay to the employees. As the wage ceiling will affect the employer's choice of contract menus, it will affect the workers' choice and further the workers' surplus and social welfare. However, the model cannot give a clear implication for the change in the social welfare when a wage ceiling is imposed. The reason is given as follows.

Note that with the uniform distribution of β, w_1^1 for employees in h_1 is the highest wage that can be earned by employees by Proposition 5. When a wage ceiling $\hat{w}$ lower than the equilibrium w_1^1 is imposed, I have

$$\hat{w} = (\bar{\beta}_1 - \frac{1}{2}\bar{\beta}_1^2)\Gamma^2 + \frac{\sigma^2 E^2}{2} + \frac{1}{2}\left(\frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E}\right)^2 + \sigma E \frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E} - \sigma \Gamma E, \quad (1.9)$$

by $\frac{U^0(\bar{\beta}_1)}{\bar{\beta}_1} = \hat{w} - \frac{1}{2}(e_1^1)^2 - \frac{1}{2}E^2 - \sigma E e_1^1 + G = \frac{V^0(\bar{\beta}_1)}{\bar{\beta}_1}$.

With the FOC of the employer's problem with respect to $\underline{\beta}_1$, I can apply the rule for implicit function theorem on (1.9) for $\underline{\beta}_1$ with respect to $\hat{w}$. However, the sign of the resulting $\frac{d\underline{\beta}_1}{d\hat{w}} > 0$ is ambiguous. Also, as the $\underline{\beta}_1$ and $\hat{w}$ will both affect

$\bar{\beta}_1$, with ambiguous change of $\underline{\beta}_1$, the change in $\bar{\beta}_1$ with the binding working hours ceiling imposed is ambiguous too. It is thus unclear how the equilibrium changes with a wage ceiling imposed.

1.7 Discussion

1.7.1 Self-Employment

The model is related to the literature of self-employment where a lower income with higher variance is observed for self-employed workers than for employees ([17]; [11] etc.). This observation cannot be well explained by the existing theoretical literature about self-employment, including the investment and agent model [26] where the self-employment and paid-employment jobs have different earning profiles; matching and learning [27] that the workers have different unknown skills for different sectors that need to be figured out after entering into that sector; and "overconfidence" [28] that entrepreneurs tend to overestimate their success.

As referred to above, this chapter builds a theoretical model consistent with [10] who suggest the importance of non-monetary benefits such as "more flexibility" and "being one's own boss" in choosing self-employment: the higher equilibrium utility of the self-employed workers in this model results from the lower pecuniary effort, that is, the non-monetary benefit, instead of a higher payoff earned. The lower income of self-employment can thus be explained.

In addition, the model can explain the low income of self-employment and its high variance through a second channel. Note that although this model predicts that only the least procrastinating workers will be self-employed, the implicit definition of self-employment in this chapter is different from that in other works. In this chapter, self-employment is the status of the workers without a pecuniary effort committed in the hiring contract, which is closer to the law definition of "self-employment," where self-employment and paid-employment are identified by whether there is a contract "for" services or contract "of" services [11]. However, as the self-employed status is often "self-assessed" in other researches, many workers classified as part-time employees in this chapter could be classified as self-employed workers in those other studies, probably in the form of "freelancers" or independent contractors. [14] As the part-time and self-employed workers are those with extreme types who earn very different incomes in this model, the model predicts that the data with those two types of workers pooling may have lower average income and a higher income variance compared with the income of remaining workers.

This argument is also consistent with the findings of Levine and Rubinstein [17], where the authors suggest using incorporated business owners and unincorporated business owners to identify two different types of self-employed workers. Levine and Rubinstein [17] show that, compared with employees, the incorporated business owners have higher income while the unincorporated business owners have lower income. This diversification of "self-employed" workers is consistent with the diversification between the workers defined as self-employed workers and part-time employees in this model.

[14] See [11] for the discussion of the definition of self-employment in the literature.

1.7.2 Competitive Labor Market

The previous results of the model are all based on a monopsonist firm offering the hiring contracts as commitment devices, where the firm can exploit all the surplus from the employees in the separating equilibrium. In the labor market with competitive firms, the equilibrium will be different.

Consider the firms that compete perfectly to offer the commitment devices. In this case, firms will earn zero profit through competition while employees get all the surplus by receiving $w(e_1) = \Gamma e_1$. The socially optimal pecuniary effort that can be achieved in a hiring contract will be contracted, for all workers in $[\underline{\beta}, 1]$. Among the employees, those with $\beta \in [\underline{\beta}, \hat{\hat{\beta}})$ will be in the separating equilibrium signing the contracts with $e_1(\beta) = \frac{\beta\Gamma - \frac{1}{2}E^2}{\sigma E}$, which is their maximal contractable effort but still smaller than the socially optimal level. Those employees with $\beta \in [\hat{\hat{\beta}}, 1]$ will sign the same contract with socially optimal $e_1 = \Gamma - \sigma E$ in a pooling equilibrium. The social optimum as discussed in Section 1.4.2 can thus be achieved by perfect competition.

1.8 Conclusion

This chapter investigates an adverse selection problem where a monopsonist firm offers hiring contracts as commitment devices to sophisticated workers with different procrastination. Those sophisticated workers, when choosing the hiring contracts as commitment devices, face a trade-off between contractable and uncontractable efforts. I further discuss the model's implication for the firm structure, labor market structure and public policies regarding ceilings of working hours. The model explains the separation of self-employment and paid-employment in the labor market, as well as the position hierarchy within the firm among employees due to their procrastination. The prediction of the model is consistent with the empirical observation in the literature about the wage penalty of part-time workers and the lower income of self-employment. It can also help explain the popularity of low-powered incentives in workplaces. Finally, in a market with firms that compete perfectly to offer the commitment devices, the most efficient hiring contracts will be offered. In the uniform case, the model predicts that a properly designed ceiling on working hours can increase both the social welfare and workers' surplus, while the effect of wage ceiling is ambiguous in this model.

Based on this chapter, further investigations can be made to explore the employment market with procrastinating workers. As mentioned in this chapter, the final structure of equilibrium and firm positions depend on the distribution of workers' quasi-hyperbolic discounting. It would be an interesting question to characterize the equilibrium with different distribution of workers' types. Besides, in this model, as the market structure depends on the model parameters including the productivity of pecuniary/non-pecuniary effort, it can offer a new channel for economy cycles which affects workers' productivity, to affect the full-time and part-time employment. Finally, in this model, all workers are assumed to have the same productivity. Introducing the heterogeneity in workers' productivity can bring an extra dimension into the market, which can explain more practical problems.

Chapter 2: Role of Leaders with Different Genders in Group All-Pay Auctions

Introduction

This chapter studies how behavior in a group all-pay auction is affected when there is a "reward" stage to reward bidders for their winning bids. This game analogizes real competitions such as awarding of monopoly licenses, political campaigns, research/development races, and wars where members make a contribution to their groups, and once the contribution is made, members' contributions cannot be returned, no matter win or lose. Previous research shows that there is usually no pure strategy equilibrium in full-information all-pay auctions without the reward stage (see the literature review in [29]). Note that in real competitions, a discretionary reward is common after group competitions; for example, a promotion or a bonus, occurs according to members' performance in the group. In this chapter, we thus introduce a leader of the group who can allocate the prize among the members as rewards after the competition. We show that with this leader who can reward a pure strategy equilibrium does exist in the group all-pay auction. We also predict the pattern of members and leaders' behavior in the equilibrium and run a pilot experiment to test for those predictions.

In the equilibrium, we show that leaders' reward to a specific member increases on this members' bid and decreases on other members' bid in the same group. Intuitively, given members' bids, leaders with higher reciprocity reward more. Also, when the members are told of their leaders' genders, the model predicts that members should bid more when assigned to leaders with the more reciprocal gender. This prediction contributes to the management literature for comparison between female and male leaders in business. [30]

In this chapter, members and leaders interact in a two-stage group all-pay auction where each group consists of three members and one leader. In the first stage, two groups compete in a group all-pay auction with full information for a given prize. Each member bids from one's initial endowment and the group bid is the sum of its members' bids. The group wins the competition for sure if its group bidding is strictly higher. At a tie, each group wins the competition with 50% probability. The members are told of their leader's gender when they bid. In the second stage, leaders of those groups allocate the prize, if any, among the group. In the model, we assume that leaders are reciprocal and thus reward those members to maximize their reciprocity utility function (see [31] for the relevant literature). Specifically, as those members who bid higher are perceived by the leaders as 'kinder', the leader will prefer to allocate them a higher amount of reward. A pilot lab experiment is run in order to test the predictions of the model and the initial findings are consistent with the theoretical predictions.

In Section 2.1, we introduce the model for reciprocal leaders in the two stage all-pay auction where the leaders can reward members after the auction. Section 2.2 gives the predictions of the model. Section 2.3 presents the experimental procedure and observations of a pilot study. Other theories that can be relevant for explaining subjects' behavior are discussed in Section 2.4. Section 2.5 concludes the chapter.

2.1 Model Setting

In this section, we introduce the setup for the specific parameters of the model. Note that the parameters such as the prize and initial endowments can be generalized easily. Suppose that there are two leaders, $L \in \{1, 2\}$. Each leader L is assigned to a group with three members, $j \in \{1, 2, 3\}$. Members are given 360 points as an initial endowment, from which they can bid for their group. The Members in a group and across groups make simultaneous bids in an all-pay auction setting. Leaders are given 180 points as the initial endowment, but they can not do anything on the first bidding stage. The group with the higher sum of bids by members win a prize of 2400 points. If the group wins, the leader may allocate member j with $r_j \geq 0$ points after observing the member's bid b_j from the winning prize of 2400 points, where $\sum_{j=1}^{3} r_j \leq 2400$. After the experiment, the leader's payoff is $\pi_L = 180 + 2400 - \sum_{j=1}^{3} r_j$, and the member j's payoff is $\pi_j = 360 - b_j + r_j$.

With the standard selfish utility for leaders, leaders in the equilibrium will never reward anything to members even if the group wins. we conjecture that bidders who believe that the leaders are reciprocal will not follow this equilibrium suggested by the standard selfish model.

To explain the behavior of reward by leaders and the members' positive bids, we model the leaders' objectives by incorporating their reciprocity motivation. We can show that such a reciprocity model (see the literature review by [31]) can explain the leaders' rewarding decisions and the correlations between the reward and members' bids.

If the group wins, the optimization problem of a reciprocal leader L on the rewarding stage is, [1]

$$\max_{r_1,r_2,r_3} u_L(\pi_L) + \gamma \sum_{j=1}^{3} b_j k(r_j, b_j), \tag{2.1}$$

s.t.

$$r_j \geq 0, \text{ for any } j \in \{1, 2, 3\}$$

where

$$k(r_j, b_j) = u_M(\pi_j(b_j, r_i)) - \underline{u}_M.$$

$k(r_j, b_j)$ is the leader's reciprocal 'kindness', measured as the difference between a member's utility, $u_M(\pi_j)$, and a certain threshold $\underline{u}_M$. $\underline{u}_M$ is a member's utility when the leader is neutral and does not behave kindly or unkindly to the member. $k(r_j, b_j)$ is multiplied by the member's bid b_j in the leader's objective function, implying that it is more beneficial for the leader to be kind to a member who bid higher. $u_L(\pi_L)$ is the leader's utility from his/her monetary payoff π_L. Both $u_M(\pi)$ and $u_L(\pi)$ are monotone and strictly concave with respect to the received monetary payoff π.

[1] Note that we do not consider the upper bound for the leaders' reward $\sum_{j=1}^{3} r_j \leq 2400$ as we expect that few of the leaders in reality will reach this upper bound and left zero rewards for the leader-self. In the pilot experiment, none of the leaders reached this upper bound.

2.2 Model Predictions

By comparing the first order condition (FOC) of problem 2.1 with respect to the reward r_j for different members j in the same group, we have the following comparison between the net earnings of different members of a group. Proofs for all theoretical results in this chapter can be found in Appendix B.

Proposition 1. Among members of a winning group who bid positive amounts and received positive rewards, a bidder with higher bid gets a higher net payoff after the leader's reward, that is, for members j, k in the same winning group, if $b_j > b_k > 0$ and $r_j, r_k > 0$, then $\pi_j > \pi_k$.

Proposition 1 gives a critical prediction by the reciprocity model: in the same winning group, bids and net payoffs of members, once members received a positive reward, should end up with the same orders. It is a strong result because after the first stage before the leader rewarded the members, higher bidders should have fewer remaining points, and the ranks of bids and members' payoffs should be ordered in opposite ways. According to Proposition 1, the leader's reward should be large enough to reverse the ranks of payoffs among members, making the highest(lowest) bidder end up with the highest (lowest) payoff.

Proposition 1 is about the comparison of members' bids and their payoffs in the same group, while Proposition 2 below addresses the mere effect of a member's bid on his/her payoff, given bids of other members in the same group.

Proposition 2. The reward of a member in a winning group increases with his/her bid, that is, $\frac{\partial r_j(b_1,b_2,b_3,\gamma)}{\partial b_j} > 0$ for $j \in \{1,2,3\}$ if the group wins.

Proposition 2 predicts how a member's reward relates to the member's own bid if the group wins. Note that it is not a corollary of Proposition 1. Proposition 1 addresses the relative orders of net payoffs and bids in a certain group, while Proposition 2 is about the absolute change in members' reward on their bids. Proposition 3 below further addresses how the member's reward relates to other members' bids in the same group.

Proposition 3. Member j's reward decreases with other members' bids in the same group if the group wins, that is, $\frac{\partial r_j(b_1,b_2,b_3,\gamma)}{\partial b_i} < 0$ for members $i, j \in \{1,2,3\}$ and $i \neq j$ in a winning group.

As Proposition 2 states a positive correlation between the leader's reward to a particular member and this member's bid in winning groups, Proposition 3 suggests a negative correlation between the reward to a particular member and the bids of other members in the same group. Combining Propositions 2 and 3, we know that a leader's reward to a specific member increases with this member's bid and decreases with other members' bid. As a result, reciprocal leaders apply a "tournament-type" rewarding scheme in each group, which, as proven in [32], can effectively motivate their members in the environment with common shocks in members' performance.

In other words, reciprocal leaders' strategy adapts to this environment even when they are the last movers of the game.

Proposition 4. Given bids submitted by a winning group, a leader with a higher reciprocity coefficient will reward the members more, that is, $\frac{\partial r_j(b_1,b_2,b_3,\gamma)}{\partial \gamma} > 0$.

Proposition 4 illustrates how the leaders' reward changes with the leaders' levels of reciprocity. Given the bids of members, it is intuitive that a more reciprocal leader rewards more than the less reciprocal one. We can test whether there is a significant difference in the reward by leaders with different genders and identify which gender of leaders is more reciprocal based on Proposition 4.

In this model, members are the first movers of the game. So even if they are reciprocal, the motive of reciprocity does not play any role in the members' utility function as they do not perceive any kindness from either the leaders or other members before they bid. We will thus model members as classical selfish subjects who only care about their own monetary payoffs and identify an equilibrium for members' bids by solving the members' problem on the first stage of the game, given their belief for leaders' reciprocal level. One observable factor which may relate to the agents' reciprocity level is the leader's gender. Previous literature ([33], [34] etc.) suggests a higher reciprocity level of female subjects compared with male subjects. In the pilot experiments, I informed bidders of the gender of their leader to test for the predictions of the model, given heterogeneity in reciprocal types of leaders with different genders.

By doing this, we can further look for evidence in members' bids to verify the belief held by members for leaders' reciprocity level. In the following analysis, denote leader's gender as $G \in \{G_1, G_2\}$. Note that the theory only assumes that there are two reciprocal types, I interpret it as gender as my pilot experiments test for heterogeneity of reciprocity by leaders with different genders. however, results can be applied for types due to some other observable differences between leaders. In the pilot experiment, the bidders know their own leader's gender, but not the gender of the competing team's leader. Hence members need to form a belief to guess the equilibrium bids of the competing team, which depends on the leader's gender in this competing team. Assume that P_i ratio of leaders in the population have gender $G = G_i$ for $i \in \{1, 2\}$, such that $P_1 + P_2 = 1$. Denote the member j's bid when assigned to a leader with gender G_i as b_j^i. The member's belief for a leader with gender G_i to have a reciprocity coefficient γ can be denoted as $f(\gamma|G_i)$.

Definition 1. Members believe that leaders with gender G_1 are more reciprocal than leaders with gender G_2 if $f(\gamma|G_1)$ first order stochastic dominates (FOSD) $f(\gamma|G_2)$.

Given members' belief for the more reciprocal gender of leaders, we can model members' choices of bids in a payoff-maximizing problem and study the properties of the equilibrium bidding behavior of the members.

To be able to solve the bidding behavior of the members, we will study members with utility functions of constant absolute risk aversion (CARA), that is,

$u_M(\pi) = 1 - e^{-R\pi}$. We also need a technical assumption that restricts the risk aversion parameter of the members. Precisely, we need to assume that there exists a cutoff $R' > 0$ such that a member's risk aversion parameter, R, cannot exceed this cutoff. [2] These two assumptions are not needed for Propositions 1-4, and those properties of leaders' reward in Propositions 1-4 hold with any strictly concave utility function of members and leaders.

Proposition 5. If members believe that leaders with gender G_1 are more reciprocal than leaders with gender G_2, there exists an equilibrium where members bid a higher bid with leaders of gender G_1 than G_2, i.e. $b_j^1 = b^1 \geq b_j^2 = b^2$ for any member j.

Proposition 5 guarantees the existence of an equilibrium with $b_j^1 = b^1 \geq b_j^2 = b^2$ for any member j if members believe that leaders with gender G_1 are more reciprocal than those with gender G_2. The result stated in Proposition 5 can be tested by our initial experiments, to see whether there is difference of reciprocity of leaders with different gender, and whether this gender-dependent reciprocity can be predicted by the members.

[2] R' can be defined by $\frac{\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial b_j \partial \gamma}}{\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial \gamma}} = R'$.

2.3 Experiment

We ran some pilot laboratory experiments for a simple test of the above predictions.

We recruited 31 male leaders, 29 female leaders, and 174 members. Every three members were randomly assigned to one group. Each of the leaders was shown four different groups and each group of members was shown to four different leaders. Hence we needed $(29 + 31) * 3 = 180$ members to make up those groups. In the experiment, we reused the bid of six of the recruited members to make up the 60 groups of three members.

The experiment was conducted using experimental "points." In the group all-pay auction, every two groups were paired to compete for a prize of 2400 points. The exchange rate between one RMB (Chinese Yuan) and one experimental point was 1 RMB to 30 experimental points. At the beginning of the game, members were given 360 points for their initial endowment and leaders were given 180 points for their initial endowment. [3] Members were told of their leader's gender, and they could bid integer points from their endowments for their group. Members paid their bid no matter whether their group won or lost. Members only bid once, while leaders rewarded members using the strategy method: after groups were paired and the results of competitions decided, each leader was shown all individual members' bids and the competition results of the four groups assigned. For each winning group

[3] On average, each subject earned average 15 RMB payoff plus 5 RMB show-up fee, which was equivalent to about 3.2 USD. The hourly wage for a student worker in the campus was 15 RMB per hour or about 2.4 USD.

within these four groups, the leader decided how to allocate the 2400 point prize among the leader and three members. The leader could not reward anything to a losing group. In other words, each leader saw the contribution of $3 * 4$ members, and decided the reward if the group won. After leaders made the point reward decisions, the experimenter randomly pick one group (out of four) for each leader, and calculated the leader's and the members' payment based on the leader's decisions made for that group.

The experiments were run using printed instructions and experimental forms. All members and leaders had to complete some questions to test their understanding of the rules. The understanding test for the later session was harder than that for the first session to be sure of the subjects' understanding of the experiment. There was no significant difference in the behavior of subjects who passed the harder understanding test or easier understanding test. In future experiments, a single understanding test should be given to all subjects.

After the understanding test, the competition stage was run where all members wrote down their bids on their experimental forms with the gender of their leaders written on top. The experimenter then collected all the forms, assigned members into groups randomly, calculated the sum of bids in each group, and got the competition results for randomly paired groups. In the following reward stage, the leaders received experimental forms which showed the individual bids of members and competition results in four groups, and made their decisions.

In the experiment, leaders were told of the members' genders in the first smaller session. This extra information was eliminated in the later session, as leaders did not seem to respond to this information. Though there is no significant difference in the subjects' behavior when this information is given or not, the model setting should be consistent in future experiments, i.e. all leaders are not told of their members' genders. In both sessions, the leaders wrote down their reward to each of those members on their experimental forms. [4] Although all the members' bids were shown to leaders, those who did not pass the understanding test questions only received their show-up fee. Their bids would not be used in the analysis for members' bid. Leaders also had to pass the understanding test to get paid in addition to the show-up fee and have their data used in the analysis.

A considerable number of leaders passed the understanding test: 20 out of 29 female leaders and 24 out of 31 male leaders passed the test. As we showed each leader four groups, leaders made reward decisions in groups for $44 \times 4 = 176$ times, coming from 44 groups of members. 90 of those groups won and 86 lost. [5] Those leaders thus made $3 = 270$ decisions of reward for each member in the winning groups, which was later used to analyze the leaders' strategy of reward. Among the 174 members recruited, 122 of them passed all questions in the understanding tests and will be used to investigate whether members bid differently according to the genders of their leaders.

One experimental session was run in Shenzhen University. To get more data, another session was run in Fujian Agriculture and Forestry University. The locations of experiments were changed because one of the experimenters changed locations.

[4] If the group lost in a reward treatment, the leader still saw the bids and competition result, that is, losing, but did not have any prize to share.

[5] The number for winning and losing groups are not equal as some groups are dropped in this data as their leaders did not pass the understanding tests.

Note that both the schools are mid-rank public universities in cities of China and they both admit undergraduate students from all over the country where the students are screened by standard university entrance exams. Due to the similarity of the two schools, there is no reason to believe that the pool of students changed significantly in different sessions. As anticipated, the rewarding behavior of leaders and bids of members were quantitatively the same between those two sessions. For further experiments, subjects should be recruited from the same school to avoid possible confounds. In both sessions, the instructions for all treatments were read aloud. The total time for these experiments was 20 minutes. A show-up fee of 5CNY was paid to each subject in addition to their earnings in the session in which they participated.

2.4 Results

With experimental data collected in the pilot study, we are able to check whether the predictions by the reciprocity model were consistent with our observations. When calculating p-values for the leaders' reward, one problem is that we get multiple and potentially dependent observations from the multiple groups assigned to one leader. To control for the dependency among those observations, I use a cluster regression, using the leaders' ID as the cluster variable. Note that due to the complex form of the possible dependency of observations by the same leader within and among the groups, a cluster regression may not be able to capture all the effect. In future experiments, a leader should only be assigned to one single group, to avoid the interference between observations in different groups rewarded by the same leader.

Observation 1.

Recall that Proposition 1 suggests that the members' net payoff and their bids should be correlated positively in winning groups. In the data, leaders who passed the understanding test questions made 270 decisions of reward to members in 90 winning groups. In 89 out of those 90 groups, leaders rewarded non-zero points to all three members. [6] Among the 89 groups where members received the non-zero reward, 48 groups had members' payoffs in the same order as their bids, that is, 53.9% of the winning groups are consistent with the prediction of Proposition 1. On the other hand, there are only six groups where members bid differently but ended up with the same payoff, [7] and three groups where the payoff of members in the same group had the opposite order as the bids of those members; in other words, the highest (lowest) bidder received the lowest (highest) payoff.

Observation 2.

Proposition 2 predicts how a bidder's reward depends on her own bid for given bids of other members, which is tested in Observation 2.

The top 25 percentile of bids received by leaders was 300 points or more. Leaders rewarded members who bid more than 300 points with 608.91 points on average. The bottom 25 percentile of bids received by leaders was 200 or fewer points. Lead-

[6]The three members in the remaining one group were all allocated with zero rewards.

[7]If we relax the equality condition and allow variance in members' payoffs, i.e. groups where members bid differently but the difference in their final net payoffs is smaller than 30 points, there are still only seven groups that falls into this category.

ers rewarded members who bid less than 200 points with 455.98 points on average. The difference in leaders' reward to the top 25 percentile and bottom 25 percentile bidders is significant with a p-value of 0.00 by both t-test and Kolmogorov-Smirnov test. Also, the Pearson pairwise correlation between the reward for members and the members' own bids is 0.3293 with a p-value of 0.00. Those results above are consistent with the prediction of Proposition 2.

Observation 3.

Proposition 3 predicts that members' reward should decrease on bids by other members in the same group.

Leaders who received at least one bid among the top 25 percentile of bids, as discussed in Observation 2, rewarded another member in the group with 503.97 points on average. Leaders who received at least one bid from the bottom 25 percentile bids rewarded another member in the group with 572.12 points on average. This difference in rewards is significant with p-value 0.00 by both t-test and Kolmogorov-Smirnov test. Also, the Pearson pairwise correlation between the reward for a particular member and the bid of a different member in the same group is -0.14 with p-value 0.00. Those results above are consistent with the prediction of Proposition 3.

Observation 4.

Proposition 4 proposes that the more reciprocal leaders should reward more, given the members' bids.

In the experiment, female leaders rewarded 584.63 points on average while male leaders rewarded 506.87 points on average. The difference in rewards is significant with both t-test and Kolmogorov-Smirnov test with p-value 0.01. However, one may argue that this difference in rewards can be caused by the different levels of bids from members facing leaders with different genders. Hence, we use a clustered linear regression to compare the reward level of female and male leaders. As each leader makes decisions of reward to members in four groups, we use leaders' ID as the cluster variable to control for the correlation between the rewards made by the same leader. The regression result is shown in following Table 2.1.

Variables	(1)	(2)
Male Leader	-77.76*	-79.45*
	(42.28)	(41.12)
Bid		0.772***
		(0.265)
Sum of Others' Bid		-0.441**
		(0.214)
Constant	584.6***	614.4***
	(18.33)	(138.6)
Observations	270	270
R-squared	0.032	0.153

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 2.1: Regression of Reward on leader's Gender and Bids

In both regressions of Table 2.1, the coefficient of the gender dummy (Male=1, Female=0) is negative and significant at 0.10 with a p-value of 0.073 and 0.060 respectively, implying that male leaders on average allocated a lower amount of reward than female leaders. A problem of this above regression is that the leaders are shown four groups at the same time and there may exist interactions between bids by those groups in the leader's reward. We are unable to eliminate this by introducing the group fix effects, as the variable of the leader's gender will be a linear combination of those group fix effects. In the future experiments, each leader should only be assigned to one group and reward members in this assigned group in a decision method, to avoid this issue.

Observation 5.

As female leaders are more reciprocal than male leaders as suggested by Observation 4, Observation 5 shows that members anticipated that female leaders are more reciprocal as follows.

On average, female leaders received 242.16 points of bids while male leaders received 217.33 points of bids on the competition stage. This difference is significant by t-test at a significance level of 0.10 (p-value, 0.083). Also, for both female and male members, they bid qualitatively higher when assigned to female leaders than male leaders although the difference is not significant.

The distribution of members' bid to leaders with different genders can be seen in following Figure 2.2.

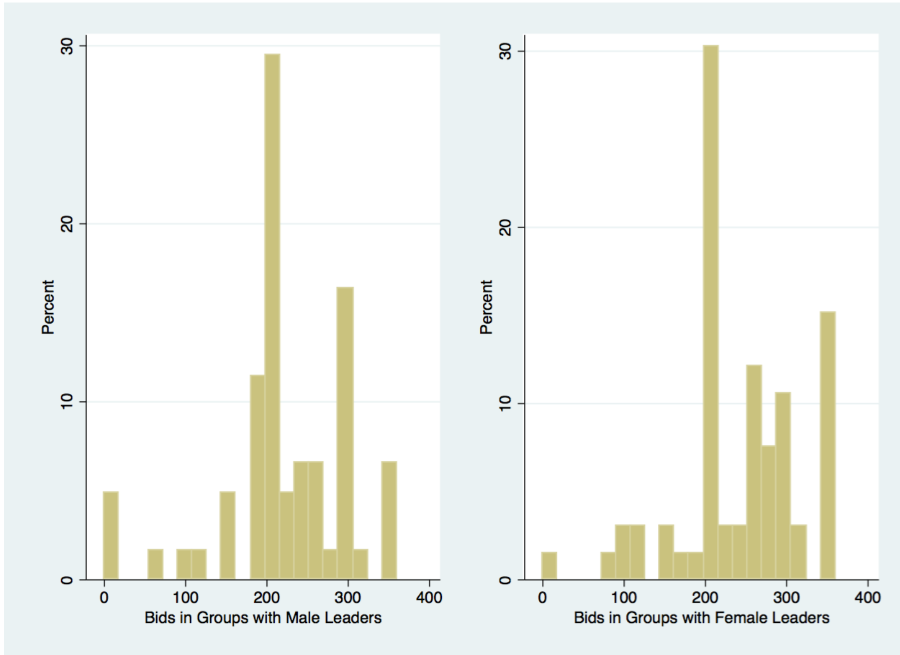


Figure 2.1: Histogram of Bids to Female and Male Leaders

The first and second histograms in Figure 2.2 are the probability distributions of bids to female and male leaders respectively. The two histograms are similar with a peak of bids at 200 points with a probability of 30%. One may notice that there are more members who bid 0 to male leaders than female leaders. Also, there are more members who bid the highest 360 points to female leaders than to male leaders. This difference in the extreme amount of bids led to the final difference in the average amount of bids to male and female leaders.

Observation 6.

Members bid higher to female leaders so that female leaders won often as reported in Observation 5. Meanwhile, female leaders rewarded more than male leaders if they won as reported in Observation 4. So a natural question to ask whether female leaders do end up with higher net payoffs after the experiments, i.e. whether female leaders' more reciprocal strategies pays off with higher monetary payoffs in this experiment. Pooling winning and losing groups together, female leaders earn 540.73 points of net payoffs while male leaders earn 423.38 points of net payoffs after the experiment. [8] The difference is significant by t-test with p-value 0.02 and Kolmogorove-Smirnov test with p-value 0.01. As a result, female leaders get a higher average net payoff than male leaders in this experiment. In other words, female leaders' strategy of higher reward/reciprocity pays off in this environment.

2.5 Discussion

In this chapter, we use the reciprocity model to explain the leaders' positive reward to the winning group members. Note that leaders' reward cannot be explained by the standard selfish utility model where subjects are assumed to only care about their monetary payoffs. As the leaders are the last movers in a one-shot sequential move game, the standard economic theory would predict that leaders never reward members, and they keep all the points that their groups win to themselves. Other than the reciprocity model, the inequity aversion model seems like a natural alternative behavioral model to explain any positive reward by the leaders. However, one may incorporate an inequity aversion model (see [31] for relevant literature) in our setup and show that some of our experimental findings would be inconsistent with the predictions of such a model. For example, the experimental data shows that the

[8] Here, we calculate the leaders' payoffs based on all the four groups they are matched with, i.e., we include the four counterfactual payoffs for each leader in this analysis.

ranks of payoffs are the same as the bids' ordering in more than half of the groups after the leaders' reward, that is, the highest (lowest) bidder ends with the highest (lowest) payoff after rewarding, as discussed in Observation 1. This observation cannot be explained by the inequity aversion model since an inequity-averse leader would prefer a smaller gap of payoffs within the group but never sacrifice his/her payoff to make the ranks of members' payoffs same as the ordering of bids.

2.6 Conclusion

In this chapter, we derive a pure strategy equilibrium for a first-prize all-pay auction with complete information where reciprocal leaders can allocate the prize based on the members' bids in winning groups.

The result predicts that the leaders' reward will make the members in a group end up with the same orders of net payoffs and bids. Also, a member's reward will increase on his own bid and decrease on other members' bids in the same group. The model also predicts that there exists an equilibrium where members bid higher to more reciprocal leaders and more reciprocal leaders also reward more. We run a pilot experiment to test for the predictions, where we observe consistent patterns.

A more complete experiment is needed in the future to investigate the performance of the leaders and members in this environment and check the prediction power of the theory. Moreover, the current theory is built based on the discretionary reward option of the leaders. In reality, there exists other motivating scheme. For example, in some environments, leaders can punish members after the competition.

By checking the leaders' behavior and the model's implications in different environments, we can get a more complete understanding for the group competition, as well as fir the performance of male and female leaders in providing discretionary motives.

Chapter 3: A Principal-Agent Problem with an Endogenous Task Design

3.1 Introduction

Designing tasks, assigning them to employees for their jobs, and determining how to monitor and pay the employees are central issues in modern firm theory. An employer often evaluates and pays her employees by different measures in workplaces. Some jobs are paid and promoted by the completeness of many individual tasks. For example, secretaries are being monitored and promoted based on the completion of tasks including scheduling appointments and answering phone calls; students are being graded based on tasks including midterm exam, final exam, and homework. Meanwhile, some jobs are being measured and paid as a whole. For example, some writers are being paid on a royalty based on the sales of their books; plumbers are being paid on the completeness of plumbing. In observing these examples, one may naturally ask what properties of the jobs determine those different payments, for example, should an employer divide a particular job into different tasks or keep it as a whole in a single task? Should employees be paid for each task or the completion of all the tasks? When can the employer choose the number of jobs to create and hence the number of employees to hire, how is this number being determined, and how should the employer partition jobs into tasks on which the payment can be conditioned?

The current research characterizes the optimal task and job design in a principal-agent setting, while previous research about job design mainly focuses on how to allocate exogenous tasks to employees without discussing the underlying reasons for the formation of tasks. In particular, most of the literature assumes exogenous job structure with symmetric tasks. However, in several applications, the principal (employer) often has some flexibility to decide the measurements on which the agents (employees) will get paid on. For example, the employer can determine whether to impose multiple tasks on different parts of the job and measure the employee's performance in each of them, or to check the completion of the whole job. Without the endogenous task formation embedded in the model, the previous principal-agent models cannot justify the assumption for symmetric tasks and can thus be regarded as incomplete. The current chapter characterizes situations where a certain structure of tasks is optimal for the principal and hence provides a foundation for some of the existing models.

To solve the principal's problem, I build a model for a principal-agent problem where the principal can assign parts of the effort space as agents' jobs and partition each part/job into tasks. The total effort on the effort space is T and the principal's goal is to incentivize agents to exert an exogenous D amount of effort, where $D \leq T$, at the lowest cost. I also assume that the probability for each task to succeed or fail is a function of the task's incompleteness, which will be defined later. The hazard rate for the task return function to return a failure is assumed to be concave. Under those assumptions, if the hazard rate for the task return function is an increasing function of the task's incompleteness, it is optimal to hire one agent, design the whole effort space as a single task, and assign this to the agent as his job. Otherwise, if the hazard rate for the task return function is a decreasing function, the principal should just assign any hired agent the amount of effort that the principal wants this agent to exert. In other words, in the equilibrium, there will be no incompleteness left in each task after the agent exerts effort. Also in this case, there exist multiple equilibria where it is optimal for the principal to hire any number of agents and partition each of their jobs into any number of tasks where each of the tasks contains enough amount of effort that can be exerted.

In Section 3.2, I introduce the related literature to the job design problem. I then describe the model setting in Section 3.3. The principal's problem in Task Design Stage and Job Design Stage is solved sequentially in Section 3.4, following by the conclusion in Section 3.5.

3.2 Literature Review

In most literature on the employment contracts, tasks are assumed to be exogenous. Holmstrom and Milgrom[35] paper is built on this setting, where they assume that exerting specific effort increases the probability of success more than other types of effort. This specific effort is thus more observable as the principal only sees the successes and failures of tasks. Holmstrom and Milgrom[35] concludes that a fixed wage should be paid to avoid the situation that the more observable effort crowds out the less observable one. As the function of observed outcome with respect to effort is exogenous in [35], it does not embed the endogenous task design in the model.

Dewatripont and Tirole [36] studies an advocate model, assuming a direct conflict between two tasks (for example, prosecution and defense for a suspect on the court). In their model, higher effort in one task will decrease the probability of success in the other task so that an agent will not exert full effort if assigned both tasks. Based on this setting, Dewatripont and Tirole [36] argues that it is optimal for the principal to assign contradicting tasks to different agents as advocates. In their model, this direct conflict between tasks, which affects tasks' outcomes, is exogenously imposed. As there is no formation of tasks, it does not address the endogenous task design problem either.

Laux [37] discusses the optimal incentive scheme when there are multiple identical and independent tasks. In this model, as the agents are paid on the number of successful tasks, it is optimal to assign tasks to one single agent as the agent can be punished harshly for shirking by eliminating his payoff from all the tasks. Bond and Gomes [38] imposes a budget constraint on Laux [37]'s model which limits the maximum bonus that can be paid to the agent. As Laux [37] and Bond and Gomes [38] focus on the optimal reward scheme with exogenous and identical tasks, their models keep silent on the formation of tasks.

Abreu et al. [39] builds a model where players continuously make decisions and receive accumulated information with a lag. It finds that shortening the lag of receiving information does not have a monotonic effect on the probability of agents' cooperation, and there exists an optimal lag. Their model is dynamic, with effort exerted continuously, and information received with delays. On the contrary, the current model is a static one with outcomes from all tasks received by the principal at the same time after the agent exerts effort. As a result, Abreu et al. [39]'s model describes the frequency of monitoring instead of the formation of tasks compared with the current model.

Another strand of the relevant literature is the information system literature (see, for example, [40], [41]). In this literature, the principal-agent model consists of two stages: the choice of an information system and the design of an optimal incentive contract. Although this information system is endogenous, it is an abstract mapping from agents' action space to the space of outcome received by the principal. It does not interact with the design of the contract due to the complexity of the problem. As a result, it is not the same as the endogenous task design problem here.

3.3 Model Setting

This chapter has a simple principal-agent setting, where the principal aims to minimize the cost to motivate agents to exert a given desired amount of effort on an effort space. As in the standard moral-hazard model, the effort is not directly observable. The principal can see whether a task succeeds or fails after the effort is exerted, and reward each agent based on the outcomes of his tasks. Different from traditional principal-agent models, in addition to the reward scheme, the principal in this chapter also designs 'jobs' and 'tasks'; she decides how much of the effort space to assign to each agent as his job and how to partition each agent's job into multiple parts where each part corresponds to a task. Thus, there are two stages for the principal to design jobs. (1) Job Design Stage: the principal decides the optimal number of agents to hire, and how much of the effort space to be allocated to each agent as his job. (2) Task Design Stage: the principal partitions the part of the effort space which is assigned to each agent in Job Design Stage, and the elements of this partition are the tasks for that agent. The reward scheme for each agent is allowed to depend on the agents tasks designed on this Task Design Stage.

A structure of jobs and tasks on the two stages is illustrated in Figure 3.1.

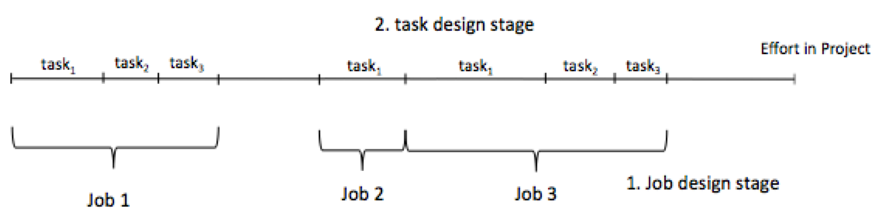


Figure 3.1: Task and job structures

In Figure 3.1, I think of the effort space as an interval of real numbers with length T. The principal assigns parts of this interval to agents as their jobs and partitions each job to tasks. In this particular example, note that the principal assigns three disjoint parts of the effort space to three agents as their jobs: Job 1, Job 2 and Job 3, that is, these agents are hired. The three jobs can include different amounts of effort that can be exerted, as shown in the different lengths of those jobs on the effort space in Figure 3.1. The principal can further partition each job into tasks. A job can include only one task as in Job 2 or can consist of multiple tasks as in Job 1 and Job 3. Each task may include different amounts of effort, represented by the tasks' different length on the effort space in Figure 3.1. The principal will supply an employment contract to each agent with a reward scheme, specifying the payment to the agents based on the outcomes of his tasks. That is, agent 1 will be rewarded based on the outcomes of the three tasks in Job 1, agent 2 will be rewarded based on the outcome of the single task in Job 2 and so on. Given the principal's goal to motivate agents to exert D, the reward schemes should be designed to incentivize agents to exert this D amount of effort, and a properly designed structures for jobs and tasks can decrease the cost for the principal. This optimal structures of jobs and tasks will be investigated in this chapter.

The principal can hire as many agents as she wants, and can assign any disjoint part of the effort space to any agent, as agents are all symmetric. After jobs are designed, the principal and agents interact in a traditional way. There is a three-period game between the principal and agents. The timeline of the game is shown in Figure 3.2.

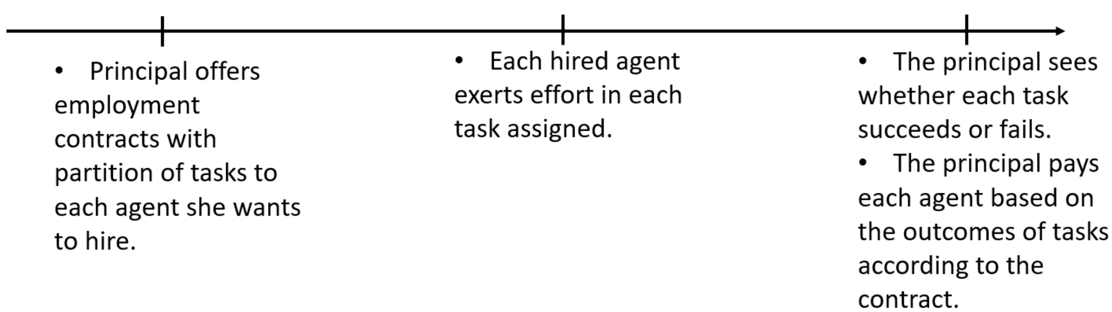


Figure 3.2: Timeline of the interaction between the principal and agents

As shown in Figure 3.2, in the first period, the principal offers employment contracts to a certain number of agents on the market, who accept or reject the jobs offered. In the second period, an agent that accepts an employment contract decides how much effort to exert in each task. In period three, the principal sees whether each task succeeds or fails, and pays the agents according to their employment contracts.

Next, I will give an example to illustrate the characteristics of this model and how it corresponds to a problem in reality, and then the formal model settings and notations will be introduced.

3.3.1 Example

Suppose that a power company wants to hire employees to maintain a transmission line of electric power with length T. The company wants to incentivize hired employees to check at least D length of the transmission line where $D \leq T$, but cannot directly see whether the effort is exerted. Instead, it can assign each employee a part of the transmission line and partition each part further into separate districts. Once there are one or more short occurring in one district, there will be a outrage in the district which can be detected by the power company. Hence, the employee who is in charge of the district can be punished according to the observed outrage, and thus motivated to exert effort. The power company wants to find the optimal number of employees to hire, how much transmission line to assign to each of them, and how to partition those parts of the transmission line into districts to minimize its cost. The company can achieve this goal through following two stages.

On the Job Design Stage, the power company decides the number of employees to hire, z. For any hired employee j, s.t. $j \in \{1, 2, ..., z\}$, the power company assigns him a part of the power line with length T^j as his job, s.t. $\sum_{j=1}^{z} T^j \leq T$. With the employment contract, agent j decides to check D^j length out of the power line in his charge, s.t. $D^j \leq T^j$. As the power company wants the total length of power line checked by employees to be at least D, it should design the employment contracts such that $\sum_{j=1}^{z} D^j \geq D$.

On the Task Design Stage, the power company partitions the job of employee j, which is a power line with length T^j, into y^j districts, with length $\{t_1^j, t_2^j, ..., t_{y^j}^j\}$ respectively, such that $\sum_{i=1}^{y^j} t_i^j = T^j$. With the employment contract and the partitions of districts, the employee's decision of checking D^j length of power line is consist of the $\{d_1^j, d_2^j, ..., d_{y^j}^j\}$ length of power line to check in each of his districts, s.t. $d_i^j \leq t_i^j$ for any district i and $\sum_{i=1}^{y^j} d_i^j = D^j$. Suppose that a short will only occur on the district i's range that has not been checked in a Poisson distribution. The event rate for the Poisson distribution is λ, which represents the average number of shorts occurred per unit of length unchecked. The probability of a outrage in the district i is thus 1 minus the probability of no short occurring, that is, $1 - e^{-\lambda(t_i^j - d_i^j)}$. As can be seen later, this probability of outrage occurring satisfies all assumptions for the task return function in the current model. This power company's problem can thus be solved under the current framework, given the employees' outside option for not being hired by the power company is low enough.

3.3.2 General Setting

A principal wants to hire agents to exert effort on an effort space with a total amount of effort T. Her goal is to motivate agents to exert an exogenous D amount of effort with the lowest cost, s.t. $D \leq T$. There are an infinite number of agents on the market that the principal can hire. Each agent can accept the employment contract offered to him, or reject it and accept an outside option with a utility 0. Both principal and agents are risk-neutral.

3.3.3 Job Design Stage

The principal assigns a part of the effort space with T^j effort to agent j as his job. So $\mathbf{T} = \{T^1, T^2, ..., T^z\}$ represents the amount of effort in all hired agents' jobs s.t. $\sum_{j=1}^{z} T^j \leq T$. Recall that in the power line example, T^j is the length of the power line that the company assigns to employee j, and the total length of the power line is T. Note that the T^j effort in agent j's job will be further partitioned into tasks on the Task Design Stage, which will be discussed later.

The principal designs an employment contract for agent j, which incentivizes agent j to exert a total amount of D^j effort, s.t. $D^j \leq T^j$. So $\mathbf{D} = \{D^1, D^2, ..., D^z\}$ represents the amount of effort that the z hired agents are incentivized to exert. Recall that the principal's goal is to motivate agents to exert at least D effort, so there must be $\sum_{i=1}^{z} D^j \geq D$ in the equilibrium. Assume that agents are homogeneous and pay the cost of effort $\Psi(d)$ when exerting d amount of effort. I assume that the cost function is quadratic on the amount of effort exerted, that is, $\Psi(d) = \frac{1}{2}\gamma d^2$, which is a general setting in the job design literature yielding an increasing marginal cost of effort. $\gamma > 0$ measures how fast the marginal cost of effort increases with respect to the total amount of effort exerted.

3.3.4 Task Design Stage

Consider an agent denoted by j who is assigned a job with the amount of effort T^j on the Job Design Stage. The problem for the principal on this Task Design Stage is to partition the job into tasks to minimize the cost to incentivize agent j to exert the D^j effort, which has been determined in the Job Design Stage. Corresponding to the power company example, the company as a principal can partition each range of power line maintained by an employee into districts, on which the employee's performance will determine the employee's payoff.

Let t_i^j denote task i for agent j in his job. Agent j's job structure can be represented as $\mathbf{t}^j = \{t_1^j, t_2^j, ..., t_{y^j}^j\}$, s.t. $\sum_{j=1}^{y^j} t_i^j = T^j$. Hence, the job structures for all hired agents 1 to z can be denoted as $\mathbf{t} = \{\mathbf{t}^1, \mathbf{t}^2, ..., \mathbf{t}^z\}$. Also, assume that the number of tasks, y^j, for any agent j is limited by a capacity constraint M, which represents the maximal number of tasks that can be assigned to an agent, that is, $y^j \leq M$ for any $j \in \{1, 2, ..., z\}$. In the power company example, this M is the maximum number of districts that can be assigned to an employee.

With the employment contract signed, agent j decides for how much effort d_i^j to exert in each of his tasks $i \in \{1, ..., y^j\}$, s.t. $d_i^j \leq t_i^j$. Note that in this Task Design Stage, the principal will design the employment contract such that $\sum_{i=1}^{y^j} d_i^j = D^j$

where D^j is determined on the Job Design Stage. I can write the effort exerted by agent j in his tasks as $\mathbf{d}^j = (d_1^j, d_2^j, ..., d_{y^j}^j)$. The effort exerted by all z hired agents in their tasks can be denoted as $\mathbf{d} = \{\mathbf{d^1}, \mathbf{d^2}, ..., \mathbf{d^z}\}$. Next, I will introduce the task return function, with which a certain outcome is returned from each task.

3.3.5 Signaling Process

Task i for agent j returns a binary outcome of either failure or success to the principal after the effort is exerted, that is, $s_i^j \in \{0, 1\}$ where 0 represents failure. Let $\mathbf{s^j} = \{s_1^j, s_2^j, ..., s_{y^j}^j\}$ denote the outcomes of the y^j tasks assigned to agent j. The probability for a task to return a certain outcome will be determined by the task's incompleteness as defined below.

Definition 1. (Task's incompleteness) A task's incompleteness is defined as the total amount of effort in this task minus the amount of effort exerted by the agent in this task, that is, for task i of agent j, its incompleteness is $w_i^j = t_i^j - d_i^j$.

Recall that in the power company example, this w_i^j is the district i's range that has not been checked by agent j. Assume that the probability of a task to return a failure outcome is an increasing function on its incompleteness. Hence, the more effort agent j exerts on the assigned task i and less incomplete task i is, task i will be less likely to fail. I denote $F(s_i^j|w_i^j)$ as the probability for an outcome s_i^j to be returned by task i assigned to agent j, given the incompleteness w_i^j for the task. Moreover, assuming that the outcomes from different tasks are independently distributed and their distributions are identical, the probability to receive a set of outcomes $\mathbf{s^j}$ from agent j's y^j tasks is $\Pi_{i\in\{1,\ldots,y^j\}}F(s_i^j|w_i^j)$.

Assumption 1. The more incomplete a task is, the more likely it fails, that is, $\frac{dF(0|w)}{dw} > 0$.

With Assumption 1, I have $\frac{dF(1|w)}{dw} = \frac{d[1-F(0|w)]}{dw} < 0$. Denote $F_d(s|w) = \frac{\partial F(s|w)}{\partial d}$ where $w = t - d$ and $F_w(s|w) = \frac{\partial F(s|w)}{\partial w}$. Hence $F_d(s|w) = -F_w(s|w)$. Once Assumption 1 is satisfied, the probability function of tasks to fail satisfies Monotone Likelihood Ratio Property (MLRP) of d_i^j because

$$\frac{F_d(1|w_i^j)}{F(1|w_i^j)} = \frac{-F_w(1|w_i^j)}{F(1|w_i^j)} > 0 > \frac{F_d(0|w_i^j)}{F(0|w_i^j)} = \frac{-F_w(0|w_i^j)}{F(0|w_i^j)}.$$

With MLRP, exerting extra effort in a certain task will increase the percentage change in the successful rate more than that in the failure rate, which makes it possible for the principal to incentivize the agent to exert effort by rewarding successes.

As the reward scheme from the principal can only depend on the received outcomes, it is a function from the outcome space to a positive real number for the payment, that is, $R^j : \{0,1\}^{y^j} \rightarrow R^+$. An employment contract can thus be denoted as the combination of the agent's job structure and the corresponding reward scheme: $\{\mathbf{t}^j, R^j\}$. Further, I denote the hazard rate for the task return function to return a failure as $H(w) = \frac{F_w(0|w))}{F(1|w)}$. Based on those notations, next I will impose some structure on the model which will be useful in the characterization of the optimal task design in a job.

Assumption 2. The hazard rate for the task return function to return a failure is monotonic and concave with respect to the task's incompleteness w, that is, $H'(w) \geq 0$, $\forall w \in [0,T]$ or $H'(w) \leq 0$, $\forall w \in [0,T]$; and $H''(w) \leq 0$.

Assumption 2 gives properties for a well-behaved hazard rate of the task return function in later analysis. The set of probability functions with monotonic hazard rate is rich as discussed in [42]. [1]

In the power company example, the probability for a outrage on a particular district of the transmission line, with w_i^j of length unchecked, is a Poisson Probability distribution with $F(0|w_i^j) = 1 - e^{-\lambda w_i^j}$. As such a Poisson distribution has a hazard rate $H(w_i^j) = \lambda$, it satisfies Assumptions 1 and 2.

Assumption 3. The hazard rate of the task return function is bounded from above, i.e. $H(x) \leq \tilde{H}$. [2]

Assumption 3 makes sure that the principal pays a large enough wage to incentivate the agents to satisfy the agents' incentive-compatibility (IC) condition, so that the agents' individual rationality (IR) condition will not bind at this solution. Otherwise, if the hazard rate of task return functions is greater than the given threshold $\tilde{H}$, a small wage is enough to make sure that the agents will not shirk, as shirking will rapidly increase the probability for the task to fail. As this little wage may not be enough for the agent to choose the hiring contract over his outside option, the principal's optimal wage scheme will be determined by the agents' individual rationality (IR) condition. As this chapter focuses on the optimal job and task structures to better incentivize agents, I rule out this case by limiting the size of the hazard rate of the task return function in Assumption 3. [3]

[1] The common families of probability distribution satisfying both Assumptions 1 and 2 include the Gamma Distribution when $\alpha > 1$, the Weibull distribution when $1 < p < 2$, and some truncated normal distributions.

[2] $\tilde{H} = \frac{2}{D}$.

[3] The job structure determined by the agents' IR condition is also not an interesting one. In the model, since the agents' cost of effort is quadratic, the total effort cost to exert a certain D

3.4 Solution

Next, I will first solve the principal's problem on the Task Design Stage to partition tasks for a given job with a total amount of effort of T^j and the desired effort of D^j for the agent to exert. Then I will solve the principal's problem on the Job Design Stage by finding the optimal number of agents to hire (i.e., jobs to create) and optimal T^j and D^j for each of them.

3.4.1 Task Design Stage

On the Task Design Stage, the principal's problem is to find the optimal employment contract $\{\mathbf{t}^j, R^j\}$, given T^j and D^j for the job. The optimal T^j and D^j for agent j will be solved in the next session for the Job Design Stage. The principal's problem on the Task Design Stage can be written as

$$\{\mathbf{t}^{j*}, R^{j*}\} = \arg \min_{\{\mathbf{t}^j, R^j\}} \sum_{\mathbf{s}^j} \left[R^j(\mathbf{s}^j) \Pi_{i \in \{1,\ldots,y^j\}} F(s_i^j | w_i^j) \right] \tag{3.1}$$

s.t.

$$\mathbf{d^{j*}} = \arg \max_{\mathbf{d}^j} \sum_{\mathbf{s}^j} \left[R^j(\mathbf{s}^j) \Pi_{i \in \{1,\ldots,y^j\}} F(s_i^j | w_i^j) \right] - \Psi(\sum_{i=1}^{y^j} d_i^j) \tag{IC}$$

$$\sum_{i=1}^{y^j} d_i^{j*} \geq D^j \tag{3.2}$$

$$\sum_{i=1}^{y^j} t_i^{j*} = T^j \tag{3.3}$$

$$y^j \leq M \tag{3.4}$$

effort converges to zero as agents share the effort equally and the number of agents converge to infinity, i.e. $\lim_{Z \to \infty} \sum_{z=1}^{Z} \frac{1}{2} \gamma \left(\frac{D}{Z} \right)^2$. So agents' IR condition can always be satisfied by hiring a large enough number of agents to share the effort desired as each agent's cost of effort converges to zero.

$$\sum_{\mathbf{s}^j} \left[R^j(\mathbf{s}^j)\Pi_{i\in\{1,\ldots,y^j\}}F(s_i^j|w_i^j)\right] - \Psi(\sum_{i=1}^{y^j} d_i^{*j}) \geq 0. \tag{IR}$$

Proposition 1. Given the amount of effort T^j in the job, the desired effort to exert, D^j, and the capacity constraint M, the optimal reward scheme for agent j is

$$R^j(\mathbf{s}^j) = \begin{cases} \frac{\Psi'(D^j)}{H\left(\frac{T^j-D^j}{y^j}\right)\Pi_{i\in\{1,\ldots,y^j\}}F\left(1|\frac{T^j-D^j}{y^{j*}}\right)}, \text{ if } \mathbf{s}^j = \{1,1,...,1\} \\ 0, \text{ otherwise.} \end{cases}$$

Also, it is optimal for the principal to partition the job into y^j tasks with $\sum_{i=1}^{y^j} t_i^j = T^j$ and $t_i^j \geq \frac{T^j-D^j}{y^j}$ for any task i of agent j.

Proof. Replacing the agent's (IC) constraint with its FOC condition [4] , the above principal's problem can be written as (3.1), subject to (3.2), (3.4), (IR), and the (IC') constraint below

$$\sum_{\mathbf{s}^j} R^j(\mathbf{s}^j)\left[\Pi_{i\in\{1,\ldots,y^j\}}F(s_i^j|w_i^j)\right]\frac{F_d(s_i^j|t_i^j - d_i^{j*})}{F(s_i^j|t_i^j - d_i^{j*})} = \Psi'(\sum_{i=1}^{y^j} d_i^{j*}), \text{ for all i,} \tag{IC'}$$

which is the FOC for the original (IC) condition. In the optimal contract problem, the constraint $\sum_{i=1}^{y^j} d_i^{j*} \geq D^j$ must bind in the solution to minimize the principal's cost so I can substitute

$$\sum_{i=1}^{y^j} d_i^{j*} = D^j \tag{3.5}$$

into the right hand side of the (IC') constraint. Summing all the (IC') conditions for $i = 1, ..., y^j$, I get a relaxed (IC") condition,

$$\sum_{\mathbf{s}^j}\left\{R^j(\mathbf{s}^j)\left[\Pi_{i\in\{1,\ldots,y^j\}}F(s_i^j|w_i^j)\right]\sum_{i=1}^{y^j}\frac{F_d(s_i^j|t_i^j - d_i^{j*})}{F(s_i^j|t_i^j - d_i^{j*})}\right\} = y^j\Psi'(D^j). \tag{IC"}$$

[4]When $\Psi''(x) = \gamma$ is big enough, the agent's problem in the IC condition is concave and the IC constraint can be replaced with the FOC condition.

I first solve the principal's problem

$$\{\mathbf{t}^{j*}, R^{j*}, \mathbf{d}^{j*}\} = \arg \min_{\{\mathbf{t}^j, R^j, \mathbf{d}^j\}} \sum_{\mathbf{s}^j} \left[R^j(\mathbf{s}^j) \Pi_{i\in\{1,\ldots,y^j\}} F(s_i^j | w_i^j) \right] \tag{3.6}$$

with respect to (3.5), (3.4), and the relaxed condition (IC"), ignoring the (IR) condition. Later, I can show that this solution satisfies the (IC') and (IR) conditions.

Note that the left hand side of the (IC") condition equals to the principal's objective function with each term multiplied by a weight $\sum_{i=1}^{y^j} \frac{F_d(s_i^j | t_i^j - d_i^{j*})}{F(s_i^j | t_i^j - d_i^{j*})}$. Given y^j, $\mathbf{t}^{j*}$ and $\mathbf{d}^{j*}$, which can minimize this objective function while keeping the (IC") condition held, the optimal reward scheme R^{j*} and thus $R^{j*}(\mathbf{s}^j)\Pi_{i\in\{1,\ldots,y^j\}} F(s_i^j | w_i^j)$ should be positive only for the state s^* which has the largest weight of $\sum_{i=1}^{y^j} \frac{F_d(s_i^j | t_i^j - d_i^{j*})}{F(s_i^j | t_i^j - d_i^{j*})}$, and $R^j(s) = 0$ for any other state $s^j \neq s^{j*}$.

By Assumption 1, $\frac{F_d(s_i^j = 0 | t_i^j - d_i^{j*})}{F(s_i^j = 0 | t_i^j - d_i^{j*})} < 0$ for $s_i^j = 0$, and $\frac{F_d(s_i^j = 1 | t_i^j - d_i^{j*})}{F(s_i^j = 1 | t_i^j - d_i^{j*})} \geq 0$ for $s_i^j = 1$. So given y^{j*}, $\mathbf{t}^{j*}$ and $\mathbf{d}^{j*}$, the weight $\sum_{i=1}^{y^j} \frac{F_d(s_i^{j*} | t_i^j - d_i^{j*})}{F(s_i^{j*} | t_i^j - d_i^{j*})}$ is the largest when $\mathbf{s^{j*}} = \{1, 1, ..., 1\}$, compared with any other $\mathbf{s}^j \neq \mathbf{s^{j*}}$.

This implies that given y^{j*}, $\mathbf{t}^{j*}$ and $\mathbf{d}^{j*}$, the principal should pay a positive reward only when a series of outcomes $\mathbf{s^{j*}} = \{1, 1, 1, ...\}$ is received for the relaxed principal's problem (3.7). In other words, the principal only rewards the agent when all his tasks are successful. Substituting (3.5) and this R^{j*} into the (IC") condition yields

$$R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}} F(1|w_i^j) \sum_{i=1}^{y^j} \frac{F_d(1|t_i^{j*} - d_i^{j*})}{F(1|t_i^{j*} - d_i^{j*})} = y\Psi'(D^j). \tag{IC*}$$

Given (IC^*), to minimize $R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}} F(1|w_i^j)$ for the principal's objective function, the optimal $\mathbf{t}^{j*}$ and $\mathbf{d}^{j*}$ should maximize $\sum_{i=1}^{y^j} \frac{F_d(1|t_i^{j*} - d_i^{j*})}{F(1|t_i^{j*} - d_i^{j*})}$. As $\frac{F_d(1|t_i^{j*} - d_i^{j*})}{F(1|t_i^{j*} - d_i^{j*})}$ is a concave function with respect to $t_i^{j*} - d_i^{j*}$ by Assumption 2, it is optimal to have all tasks of agent j with equal size of task incompleteness, that is, $t_i^{j*} - d_i^{j*} = \frac{T^j - D^j}{y^j}$ if $T^j - D^j > 0$. Note that when the task incompleteness is symmetric for all tasks in the optimal partition, the (IC') condition is satisfied and can be written as

$$R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}}F(1|w_i^j) = \frac{\Psi'(D^j)}{H(\frac{T^j - D^j}{y^j})}. \tag{3.7}$$

Now I will prove that such a set of symmetric task incompleteness can be achieved by a task structure where $\sum_{i=1}^{y^j} t_i^j = T^j$ and $t_i^j \geq \frac{T^j - D^j}{y^j}$. This can be seen by checking the agent's FOC,

$$R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}}F(1|w_i^j)H(w_i^j) = \Psi'(D^j). \tag{3.8}$$

for any task i with task incompleteness $w_i^j = t_i^j - d_i^j$. Since $R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}}F(1|w_i^j)$ is the same for all task i and $H(w_i^j)$ is concave and monotonic on w_i^j, the agent j will choose d_i^j and d_k^j for any task i and k such that $w_i^j = w_k^j = \frac{T^j - D^j}{y^j}$ once $t_i^j, t_k^j \geq \frac{T^j - D^j}{y^j}$. The symmetric task incompletness can thus be achieved with a task structure where $\sum_{i=1}^{y^j} t_i^j = T^j$ and $t_i^j \geq \frac{T^j - D^j}{y^j}$ for any task i of the agent.

Note that the left-hand side in the agent's FOC (3.7) is the principal's cost. (3.7) thus gives the minimal cost for the relaxed principal's problem, with a optimal task structure where $\sum_{i=1}^{y^j} t_i^j = T^j$ and $t_i^j \geq \frac{T^j - D^j}{y^j}$ is optimal, and the optimal reward scheme is R^{j*} which rewards the agent only when all tasks are successful. The principal's choice of the optimal number of tasks y^{j*} will be discussed in following Lemma 1.

Now this solution can be shown to satisfy the (IR) condition as well. In the relaxed principal's problem (3.7), agent j earns an expected payoff,

$$\begin{aligned} R^{j*}(\mathbf{s}^{j*})\Pi_{i\in\{1,\ldots,y^j\}}F(1|w_i^j) - \Psi(D^j) &= \frac{\Psi'(D^j)}{H(\frac{T^j-D^j}{y^{j*}})} - \Psi(D^j). \\ &= \frac{\gamma D^j}{H(\frac{T^j-D^j}{y^{j*}})} - \frac{1}{2}\gamma(D^j)^2 \\ &= \gamma D^j\left[\frac{1}{H(\frac{T^j-D^j}{y^{j*}})} - \frac{1}{2}D^j\right] \\ &\geq \gamma D^j\left[\frac{1}{2}D - \frac{1}{2}D^j\right] \\ &\geq 0 \end{aligned}$$

where the last second inequity is by Assumption 3 which guarantees $H(\frac{T^j-D^j}{y^{j*}}) \leq \frac{2}{D}$. Q.E.D.

Proposition 1 gives the properties of an optimal task structure for a certain job, given the total amount of effort T^j and the amount of effort that the principal wants the agent to exert D^j. The optimal number of tasks in each job, however, depends on whether the hazard rate is increasing or decreasing with respect to the task's incompleteness, as discussed in following Lemma 1.

Lemma 1.

(1). When $D^j < T^j$,

- if the hazard rate of the task return function is an increasing function of the task's incompleteness, it is optimal to have one task in each job, that is, $y^{j*} = 1$ if $H'(w) \geq 0$;
- if the hazard rate of the task return function is an decreasing function of the task's incompleteness, it is optimal to have the maximal possible number of tasks in each job, that is, $y^{j*} = M$ if $H'(w) < 0$.

(2). When $T^j = D^j$, it is optimal for the principal to partition agent j's job into any number of tasks once the total number of tasks is smaller than its upper bound M.

Proof. From (3.7), the expected cost for the principal to motivate an agent to exert effort D^j out of T^j effort in his job is

$$R^j(\mathbf{s}^j)\Pi_{i\in\{1,\ldots,y^j\}}F\left(1|\frac{T^j - D^j}{y^{j*}}\right) = \frac{\Psi'(D^j)}{H\left(\frac{T^j - D^j}{y^j}\right)}.$$

it can be easily seen that when $T^j - D^j > 0$, to minimize the principal's cost, it is optimal to have the maximal possible number of tasks for the agent, that is, $y^{j*} = M$, if $H'(w) < 0$. Otherwise if $H'(w) \geq 0$ and $T^j - D^j > 0$, it is optimal to have the minimal possible number of tasks, that is, $y^{j*} = 1$ if $H'(w) \geq 0$. If $T^j - D^j = 0$, the choice of y^j does not affect the principal's cost so it is optimal to partition the job into any number of tasks, once this number is smaller than its upper bound M. Q.E.D.

Lemma 1 is obvious following the optimal task structure given in Proposition 1. By Lemma 1, the optimal number of tasks depends on whether the hazard rate of the task return function is increasing or decreasing. For the former case, it is optimal to have one single task in each job. Hence it can be seen that such a job is measured and paid only on its final outcome, as in the earlier example of book royalty where the agent is given one task as her job (i.e., finishing the book) and rewarded only when the outcome of that task is successful. Otherwise, if the hazard rate of the task return function is decreasing, it can be seen that the job is measured and paid by the completion of multiple tasks, as in the example of grading for a student where there are multiple tasks counted such as submitting a project, writing an exam, attending classes, and the student is graded based on the completion of all the tasks.

3.4.2 Job Design Stage

With the optimal employment contract solved in the last section for the Task Design Stage, the principal's remaining problem is to choose how to assign the jobs, that is, the optimal number of agents to hire, the optimal amount of effort T^j in any agent j's job, and how much effort D^j to incentivize agent j to exert in the contract. Given D desired amount of effort, the principal's problem at the Job Design Stage can be written as

$$\{\mathbf{T}^*, \mathbf{D}^*, z^*\} = \min_{\mathbf{T},\mathbf{D},z} \sum_{j=1}^{z} \frac{\Psi'(D^j)}{H(\frac{T^j - D^j}{y^*})} \tag{3.9}$$

s.t.

$$\sum_{j=1}^{z} D^{j*} \geq D$$

$$\sum_{j=1}^{z} T^{j*} \leq 1$$

where y^* is the optimal number of tasks for each job determined by Lemma 1.

Proposition 2. (1). When the hazard rate of the task return function decreases with the task incompleteness, there exist multiple optimal employment contracts where the principal can hire any number of agents. The principal will require each task to be performed completely, that is, $T^j = D^j$ for all $j \in \{1, 2, ..., z^*\}$.

(2). When the hazard rate of the task return function increases with the task incompleteness, the principal will hire one agent and assign the whole effort space and incentivize this agent to exert the desired amount of effort, that is, $z^* = 1$, $T^1 = 1$ and $D^1 = D$.

Proof. Note that the constraint $\sum_{j=1}^{z} D^{j^*} \geq D$ must bind in the solution for this cost minimization problem. Substituting $\Psi'(D^j) = \gamma D^j$ into the objective function (3.9) yields,

$$\{\mathbf{T}^*, \mathbf{D}^*, z^*\} = \min_{\mathbf{T},\mathbf{D},z} \gamma \sum_{j=1}^{z} \frac{D^j}{H(\frac{T^j - D^j}{y^{j*}})}$$

s.t.

$$\sum_{j=1}^{z} D^{j*} = D$$

$$\sum_{j=1}^{z} T^{j*} \leq 1.$$

Also replacing $\mathbf{T}$ with $\mathbf{T} - \mathbf{D}$ as a choice variable, I have

$$\{\mathbf{T}^* - \mathbf{D}^*, \mathbf{D}^*, z^*\} = \min_{\mathbf{T}-\mathbf{D},\mathbf{D},z} \gamma \sum_{j=1}^{z} \frac{D^j}{H(\frac{T^j - D^j}{y^{j*}})} \tag{3.10}$$

s.t.

$$\sum_{j=1}^{z^*} D^{j*} = D$$

$$\sum_{j=1}^{z^*} (T^{j*} - D^{j*}) \leq T - D.$$

This problem (3.10) can be solved by checking following two cases.

CASE 1. $H'(w) < 0$.

When $H'(w) < 0$, as $T^j - D^j \geq 0$, it is optimal to have $T^j - D^j = 0$ for any $j \in \{1, 2, ..., z\}$ to minimize each term $\gamma \frac{D^j}{H\left(\frac{T^j - D^j}{y^*}\right)}$ in the principal's objective function in (3.10). At this time, the cost function of the principal in (3.10) becomes

$$\gamma \sum_{j=1}^{z} \frac{D^j}{H(0)} = \gamma \frac{D}{H(0)} \tag{3.11}$$

which is independent of the total number of agents hired, z, and how much $T^j = D^j$ effort assigned to each hired agent j.

CASE 2. $H'(w) \geq 0$.

Prove that it is optimal to hire only one agent in CASE 2 by contradiction. Suppose that there exists $z > 1$ agents in the optimal job assignment. Without loss of generality, assume that $\frac{T^1-D^1}{y^*} \geq \frac{T^2-D^2}{y^*} \geq ... \geq \frac{T^z-D^z}{y^*}$ in (3.10). As $H'(w) \geq 0$, $H(\frac{T^{1*}-D^{1*}}{y^*}) \geq H(\frac{T^{2*}-D^{2*}}{y^*}) \geq ... \geq H(\frac{T^{z*}-D^{z*}}{y^*})$. At this time, the cost for the principal is

$$\frac{D^1}{H(\frac{T^1-D^1}{y^*})} + \frac{D^2}{H(\frac{T^2-D^2}{y^*})} + ... + \frac{D^z}{H(\frac{T^z-D^z}{y^*})} \leq \frac{D}{H(\frac{T^1-D^1}{y^*})} < \frac{D}{H(\frac{T-D}{y^*})}.$$

So hiring z agents, s.t. $z > 1$, is dominated by hiring only one agent and assign him all the effort space, which yields a cost $\frac{D}{H(T-D)}$ for the principal. Q.E.D.

Proposition 2 states how the principal should assign parts of the effort space to agents as their jobs, given the optimal employment contract solved in Proposition 1 and Lemma 1.

Proposition 2 depends on the quadratic cost function of effort. If the degree of the polynomial for agents' cost function does not equal to two, (3.11) no longer holds for the principal's cost. In other words, the number of agents hired does affect the principal's cost to incentivize D effort exerted. In this case, the principal may tend to hire more agents if the degree is higher than two or fewer agents if the degree is lower than two.

Combining Proposition 2 with Proposition 1 and Lemma 1, we have the optimal job design for different jobs. If the hazard rate of the task return function is increasing for one job, the principal should hire only one agent by Proposition 2. Also, with Proposition 1 and Lemma 1, only the final outcome of the job will be monitored. This is consistent with the book royalty example, where the writer is paid based on finishing of the writing and a book is usually written by a single writer. If the hazard rate of the task return function decreases, the principal can split the job among multiple agents and monitor each agent on several tasks. This is consistent with the example of the secretary jobs. In reality, secretary jobs are paid based on the completion of multiple tasks including answering phones and scheduling meetings. Also, depending on the amount of work, multiple secretaries could be hired, as predicted by Proposition 2.

3.5 Conclusion

In this chapter, I investigate a two-stage job design problem where the principal assigns parts of the project to agents as their jobs in the Job Design Stage and partitions each job into tasks in the Task Design Stage. The model justifies the optimality of symmetric partitions of tasks under certain conditions, and supplies foundations for some of the previous literature. The model also offers condition for the stochastic task return functions and explains when it is optimal to partition a job into several tasks or monitor it as a whole.

Based on those results, the job design problem can be further discussed, implementing other possible situations. First, the effort in the current model is homogeneous. When there exist heterogeneous efforts, it is interesting to discuss the job design further, that is, whether those different efforts should be partitioned into the same or different tasks, and how this partition interacts with the job assignment to different agents. Second, the current model is based on the assumption that the hazard rate of tasks to fail is concave with respect to the tasks' incompleteness. It would be interesting to discuss the same problem when this assumption is relaxed, for example, the hazard rate of the task return function could be convex. With a convex hazard rate, one may guess that an asymmetric task structure will be preferred by the principal, i.e., some tasks will be larger than others even when all effort is homogeneous. This result can shed a light on the existence of some asymmetric task structure in reality. Finally, some other constraints can be added to the model, for example, a budget constraint for the principal, or a binding outside option for agents. With those situations discussed, the current model can explain more real-life observations.

Appendix A: Proofs in Chapter 1

A.1 Lemma 1

Proof. Note that if $\beta \geq \frac{\sigma E}{\Gamma}$,

$$V^1(e_2 = E|\beta) = \frac{1}{2}\beta^2\Gamma^2 - \beta\Gamma\sigma E - \frac{1-\sigma^2}{2}E^2 + \beta G \geq V^1(e_2 = 0|\beta) = \frac{1}{2}\beta^2\Gamma^2$$

if

$$\beta \geq \frac{(1-\sigma^2)E^2}{2(G-\sigma E\Gamma)}$$

which always holds as $\frac{\sigma E}{\Gamma} > \frac{(1-\sigma^2)E^2}{2(G-\sigma E\Gamma)}$ by

$$\left[\frac{1-\sigma^2}{2\sigma} + \sigma\right] E\Gamma = \frac{1}{2}\left(\frac{1}{\sigma} + \sigma\right) E\Gamma < \frac{E\Gamma}{\sigma} < G$$

with Assumption 2. So I have

$$e_2(\beta) = E, \forall \beta \geq \frac{\sigma E}{\Gamma} > \frac{(1-\sigma^2)E^2}{2(G-\sigma E\Gamma)} \tag{A.1}$$

for self-employed workers with $\beta \geq \frac{\sigma E}{\Gamma}$. Also, when $\underline{\beta} \leq \beta < \frac{\sigma E}{\Gamma}$, $\beta G - \frac{1}{2}E^2 > \frac{1}{2}\beta^2\Gamma^2$ always holds as $\beta \geq \underline{\beta} > \hat{\beta}$. So $e_2(\beta) = E$, for all self-employed workers with $\beta < \frac{\sigma E}{\Gamma}$. Q.E.D.

A.2 Lemma 2

Proof. $\forall \beta < \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$,

$$\frac{U^0(\beta, \beta')}{\beta} = w(\beta') - \frac{1}{2}e_1(\beta')^2 \leq \Gamma e_1(\beta') - \frac{1}{2}e_1(\beta')^2 \leq \frac{\Gamma^2}{2}$$

where the first inequality is because the employer's profit from the contract $C(\beta')$, $\Gamma e_1(\beta') - w(\beta')$ must be non-negative and thus $\Gamma e_1(\beta') \geq w(\beta')$. Recall

$$\frac{V^0(\beta)}{\beta} = \begin{cases} (\beta - \frac{1}{2}\beta^2)\Gamma^2 - \frac{1-\sigma^2}{2}E^2 - \sigma\Gamma E + G \geq G - \frac{1}{2}E^2, \text{if } \beta \geq \frac{\sigma E}{\Gamma}; \\ G - \frac{1}{2}E^2, \text{if } \beta \leq \frac{\sigma E}{\Gamma}, \end{cases}$$

so when $\beta < \frac{\sigma E e_1(\beta) + \frac{1}{2}E^2}{G}$, $\frac{V^0(\beta)}{\beta} \geq G - \frac{1}{2}E^2 \geq \frac{\Gamma^2}{2} \geq \frac{U^0(\beta,\beta')}{\beta}$ where the second inequality is by Assumption 2. Q.E.D.

Corollary 1

Proof. From Lemma 2, $\beta \geq \frac{\sigma E e_1(\beta) + \frac{1}{2}E^2}{G}$, $\forall \beta \in H$. So by (1.3), $e_2(\beta) = E$, $\forall \beta \in H$.

Also, $\forall \beta \in [\underline{\beta}, 1] \setminus H$, $e_2(\beta) = E$ by Lemma 1. Q.E.D.

A.3 Preparation for Proposition 2 and Theorem 1

Lemma 3

Proof. If $\exists \beta' \in H$ s.t. $\beta' > \frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}$, by (1.4),

$$\frac{U^0(\beta)}{\beta} \geq \frac{U^0(\beta, \beta')}{\beta} = w(\beta') - \frac{1}{2}e_1^2(\beta') - \frac{1}{2}E^2 - \sigma E e_1(\beta') + G = \frac{U^0(\beta')}{\beta'}$$

$\forall \beta \in [\frac{\sigma E e_1(\beta') + \frac{1}{2}E^2}{G}, \beta')$.

On the other hand,

$$\frac{U^0(\beta')}{\beta'} \geq \frac{U^0(\beta',\beta)}{\beta'} \geq \frac{U^0(\beta)}{\beta}$$

where the second inequality is by (1.4) when $\beta' \geq \beta$. Combining the above two inequalities yields

$$\frac{U^0(\beta')}{\beta'} = \frac{U^0(\beta)}{\beta} = \frac{U^0(\beta',\beta)}{\beta'} = \frac{U^0(\beta,\beta')}{\beta}$$

, $\forall \beta \in [\frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}, \beta')$ with the optimal contract M. In other words, the workers in $[\frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}, \beta']$ must be indifferent between $C(\beta')$ and their own contracts. Recalling the assumption that the workers will choose the contract with higher contracted effort if indifferent between different contracts, all the employees in $[\frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}, \beta']$ must sign the same contract, i.e. $e_1(\beta) = e_1(\beta')$, $\forall \beta \in [\frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}, \beta')$. Q.E.D.

The steps to prove Proposition 2 and Theorem 1 are as following. By firstly showing the equilibrium $e_1(\beta)$ for the employees in h and $H \setminus h$ in Claim 1, I can prove the form of the optimal normalized utility, which can identify $w(\beta)$ for the employees in h and $H \setminus h$ in Claims 2-3. All the employees' contracts are determined except for the employees at the upper bound of each position $\bar{\beta}_j$. With Claims 1 to 3, proposition 2 can be proved by showing that all the workers in $[\underline{\beta}, \tilde{\beta}]$ will be hired. And based on Proposition 2, the employee's decision at $\beta = \bar{\beta}_j$ can be determined, which adds to Claims 1 to 3 and reaches Theorem 1.

CLAIM 1. There exists $h = \cup h_j \subset H$ s.t. $\min h_j = \underline{\beta}_j$ and $\sup h_j = \bar{\beta}_j$ where $e_1(\beta) = \frac{\underline{\beta}_j G - \frac{1}{2}E^2}{\sigma E}$, $\forall \beta \in h_j$.

PROOF: With Lemma 2, all employees must have $\beta \geq \frac{\sigma E e_1(\beta)+\frac{1}{2}E^2}{G}$. In the optimal contract menu, if $\forall \beta \in H$, $\beta = \frac{\sigma E e_1(\beta)+\frac{1}{2}E^2}{G}$, I have $h = \emptyset$ and the result holds trivially.

If $\exists \beta_1' \in H$ s.t. $\beta_1' > \frac{\sigma E e_1(\beta_1')+\frac{1}{2}E^2}{G}$, I have $e_1(\beta) = e_1(\beta_1'), \forall \beta \in \left[\frac{\sigma E e_1(\beta_1')+\frac{1}{2}E^2}{G}, \beta_1'\right]$ by Lemma 3. I can thus define h_1' s.t. $h_1' = \{\beta \in H \mid e_1(\beta) = e_1(\beta_1')\}$. h_1' is thus closed at its lower bound $\underline{\beta}_1 = \frac{\sigma E e_1(\beta_1')+\frac{1}{2}E^2}{G}$ by Lemma 3 while it is not sure whether h_1' is closed or open at its upper bound $\bar{\beta}_1 = \sup\{\beta \mid e_1(\beta) = e_1(\beta_1')\}$. Note that by the definition of $\bar{\beta}_j$, we have either $\bar{\beta}_1 \geq \beta_1' > \frac{\sigma E e_1(\beta_1')+\frac{1}{2}E^2}{G} = \frac{\sigma E e_1(\bar{\beta}_1)+\frac{1}{2}E^2}{G}$ if h_1' is closed at $\bar{\beta}_1$ or there exists small enough $\epsilon > 0$ s.t. $\bar{\beta}_1 - \epsilon \geq \beta_1' > \frac{\sigma E e_1(\beta_1')+\frac{1}{2}E^2}{G} = \frac{\sigma E e_1(\bar{\beta}_1-\epsilon)+\frac{1}{2}E^2}{G}$ if h_1' is open at $\bar{\beta}_1$. In either case, by Lemma 3, h_1' is a connected set, i.e. $e_1(\beta) = e_1(\beta_1')$, $\forall \beta \in h_1'$.

From h_1', I can further identify a list of h_j', $j \geq 2$ as following. If $\exists \beta_j' \in H \setminus \left(\cup_{i=1}^{j-1} h_i'\right)$ s.t. $\beta_j' > \frac{\sigma E e_1(\beta_j')+\frac{1}{2}E^2}{G}$, denote $h_j' = \{\beta \in H \mid e_1(\beta) = e_1(\beta_j')\}$ with its lower bound $\underline{\beta}_j = \frac{\sigma E e_1(\beta_j')+\frac{1}{2}E^2}{G}$ by Lemma 3 and its upper bound $\bar{\beta}_j = \sup\{\beta \mid e_1(\beta) = e_1(\beta_j')\}$. Similarly, h_j' is closed at $\underline{\beta}_j$ while it is not sure by now whether it is closed or open at $\bar{\beta}_j$. Also as in the h_1' case, $e_1(\beta) = \frac{\underline{\beta}_j G - \frac{1}{2}E^2}{\sigma E}$, $\forall \beta \in h_j'$. With the above process, I can include all $\beta \in H$ s.t. $\beta > \frac{\sigma E e_1(\beta)+\frac{1}{2}E^2}{G}$ in the list of sets $\{h_1', h_2', ...\}$ where $h_i' \cap h_j' = \emptyset, \forall i \neq j$ and all employees in h_j' sign the same contract.[1]

Define $h = \{h_1, h_2, ...\} \subset H$ as a permutation of $\{h_1', h_2', ...\}$ s.t. $\bar{\beta}_j \leq \underline{\beta}_{j+1}$. By Lemma 1 if $\beta \in H$, $\beta \geq \frac{\sigma E e_1(\beta)+\frac{1}{2}E^2}{G}$. In other words, $e_1(\beta) \leq \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$, $\forall \beta \in H$. As all the $\beta \in H$ with $e_1(\beta) < \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$ are contained in h in the above process,

[1] This argument requires that those h_j' are countable, which holds by the countable chain condition, given by [43].

$\beta = \frac{\sigma E e_1(\beta)+\frac{1}{2}E^2}{G}$ or $e_1(\beta) = \frac{\beta G-\frac{1}{2}E^2}{\sigma E}$, $\forall \beta \in H \setminus h$. Q.E.D.

Note that it is not sure whether h_j's are closed at their upper bounds, i.e. $\beta = \bar{\beta}_j$ by Claim 1, which will be clarified after Proposition 2. With Claim 1, if

$$\frac{U^0(\beta)}{\beta} = \frac{V^0(\beta)}{\beta}$$

, $\forall \beta \in H \setminus h$ and (*)

$$\frac{U^0(\beta)}{\beta} = \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j},$$

$\forall \beta \in h_j \subset h$, IR condition binds for all the contracts for $\{h_1, h_2, ...\}$ and $H \setminus h$. So if I can prove that all the IC conditions also hold with (*), (*) must be the normalized utility of employees in the optimal contract menu. To reach this goal, I will start from the following Claim 2 to prove that with (*), an employee in $H \setminus h$ will never mimic another employee in $H \setminus h$.

CLAIM 2. In the optimal contract menu, if $\forall \beta \in H \setminus h$, $\frac{U^0(\beta)}{\beta} = \frac{V^0(\beta)}{\beta}$, an employee with $\beta \in H \setminus h$ will never mimic another employee in $H \setminus h$.

PROOF. We can see that there is no β term in $\frac{U^0(\beta,\beta')}{\beta}$ by (1.4) if $\beta \geq \frac{\sigma E e_1(\beta')+\frac{1}{2}E^2}{G}$. So by Claim 1, if $\beta_1 > \beta_2 = \frac{\sigma E e_1(\beta_2)+\frac{1}{2}E^2}{G}$,

$$\frac{U^0(\beta_1,\beta_2)}{\beta_1} = \frac{U^0(\beta_2)}{\beta_2} = \frac{V^0(\beta_2)}{\beta_2} \leq \frac{V^0(\beta_1)}{\beta_1} = \frac{U^0(\beta_1)}{\beta}_1.$$

and the employee with $\beta_1 > \beta_2$ will never mimic the employee with β_2 who exert a lower contracted effort by Claim 1, for $\beta_1, \beta_2 \in H \setminus h$.

If $\beta_1 < \beta_2 = \frac{\sigma E e_1(\beta_2)+\frac{1}{2}E^2}{G}$,

$$\frac{U^0(\beta_1,\beta_2)}{\beta_1} < \frac{V^0(\beta_1)}{\beta_1} = \frac{U^0(\beta_1)}{\beta_1}$$

by Lemma 2 and the employee with $\beta_1 < \beta_2$ will never mimic the employee with β_2, for $\beta_1, \beta_2 \in H \setminus h$. Q.E.D.

Based on Claim 2, Claim 3 continues to justify that the IC conditions with (*) hold for all employees and (*) is the optimal assignment for employees' normalized utility given h and H.

CLAIM 3. In the optimal contract,

$$\frac{U^0(\beta)}{\beta} = \frac{V^0(\beta)}{\beta}$$

, $\forall \beta \in H \setminus h$;

$$\frac{U^0(\beta)}{\beta} = \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j},$$

$\forall \beta \in h_j \subset h$.

PROOF. We already know that a part-time employee in $H \setminus h$ will never mimic another employee in $H \setminus h$ by Claim 2. To prove the IC condition holds for any part-time employee in $H \setminus h$, with Lemma 2, I just need to prove that the part-time employee with $\beta \in H \setminus h$ will never mimic any positioned employees with $\beta' \in h_j \subset h$,

$$\frac{U^0(\beta,\beta')}{\beta'} \leq \frac{U^0(\beta)}{\beta}$$

, s.t. $\beta' < \bar{\beta}_j \leq \beta$. This inequity holds because with (*),

$$\frac{U^0(\beta,\beta')}{\beta} = \frac{U^0(\beta')}{\beta'}$$

$$= \frac{U^0(\bar{\beta}_j)}{\bar{\beta}_j} = \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j} \leq \frac{V^0(\beta)}{\beta} = \frac{U^0(\beta)}{\beta}.$$

So the IC conditions hold for all $\beta \in H \setminus h$. Note that I do not need to consider the case for the part-time employee mimicking a positioned employee with a higher type because $e_1(\beta') = \frac{\bar{\beta}_j G - \frac{1}{2}E^2}{\sigma E} > \frac{\beta G - \frac{1}{2}E^2}{\sigma E}$ by Claim 1 and thus $\frac{U^0(\beta,\beta')}{\beta'} < \frac{V^0(\beta)}{\beta} = \frac{U^0(\beta)}{\beta}$ by Lemma 1, $\forall \beta' \in h_j$ s.t. $\beta' \geq \bar{\beta}_j > \beta$.

To prove the IC condition holds for full-time employees in positions with $\beta \in h_j \subset h$, by Lemma 2, I just need to prove

$$\frac{U^0(\beta,\beta')}{\beta'} \leq \frac{U^0(\beta)}{\beta}.$$

, $\forall \beta' < \underline{\beta}_j$ s.t. $\beta' \in H$ with (*). I have two cases for whether $\beta' \in h$ to check in order to justify the above inequality.

Case 1. If $\beta' \in H \setminus h$,

$$\frac{U^0(\beta,\beta')}{\beta'} = \frac{U^0(\beta')}{\beta'} = \frac{V^0(\beta')}{\beta'} \leq \frac{V^0(\beta)}{\beta} \leq \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j)} = \frac{U^0(\beta)}{\beta}.$$

Case 2. If $\beta' \in h_i \subset h$ s.t. $i < j$,

$$\frac{U^0(\beta,\beta')}{\beta'} = \frac{U^0(\beta')}{\beta'} = \frac{U^0(\bar{\beta}_i)}{\bar{\beta}_i} = \frac{V^0(\bar{\beta}_i)}{\bar{\beta}_i} \leq \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j} = \frac{U^0(\bar{\beta}_j)}{\bar{\beta}_j} = \frac{U^0(\beta)}{\beta}.$$

So IC conditions satisfy for $\beta \in h$ and the employee with β would never mimic any other employee.

As all IC conditions hold and IR conditions bind for all $\beta \in H$ with (*) as checked above, (*) is the normalized utility for employees in the optimal contract menu. Q.E.D.

A.3.1 Employer's profit from contracts

With Claim 1 and Claim 3, from an employee with type $\beta \in H$ who takes the contract $C = (e_1, w)$, the employer earns a profit

$$\pi(\beta) = \Gamma e_1 - w$$

$$= (\Gamma - \sigma E)e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 + G - [w - \sigma E e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 + G]$$

$$= SW(e_1) - \frac{U^0(\beta)}{\beta}.$$

So I can write the employer's profit from an employee with $\beta \in H \setminus h$ as

$$\pi(\beta) = SW\left(\frac{\beta G - \frac{1}{2}E^2}{\sigma E}\right) - \frac{V^0(\beta)}{\beta}$$

$$= \Psi(\beta) - \Phi(\beta)$$

where $\Psi(\beta) = (\Gamma - \sigma E)\frac{\beta G - \frac{1}{2}E^2}{\sigma E} - \frac{1}{2}\left(\frac{\beta G - \frac{1}{2}E^2}{\sigma E}\right)^2$ and

$$\Phi(\beta) = \begin{cases} (\Gamma - \sigma E)(\beta\Gamma - \sigma E) - \frac{1}{2}(\beta\Gamma - \sigma E)^2, \text{ if } \beta \geq \frac{\sigma E}{\Gamma} \\ 0, \text{ otherwise.} \end{cases}$$

$\Psi(\beta)$ and $\Phi(\beta)$ have the same maximal value $\frac{(\Gamma - \sigma E)^2}{2}$ but $\Psi(\beta)$ takes this maximal value at the point when $\frac{\beta G - \frac{1}{2}E^2}{\sigma E} = \Gamma - \sigma E$, i.e. $\beta = \hat{\hat{\beta}}$, and $\Phi(\beta)$ takes this maximal value at the point when $\beta\Gamma - \sigma E = \Gamma - \sigma E$ or $\beta = 1$.

Also, with Claims 1 and 3, I can uniquely identify the contract (e_1^j, w^j) for each h_j by its lower and upper bounds $\underline{\beta}_j$ and $\bar{\beta}_j$. The employer's profit from the employee with $\beta \in h_j \subset h$ is thus

$$\pi(\beta) = SW\left(\frac{\beta_j G - \frac{1}{2}E^2}{\sigma E}\right) - \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j} = \Psi(\underline{\beta}_j) - \Phi(\bar{\beta}_j).$$

The employer's problem is

$$\max_{H,h}\left\{\int_{H\setminus h}\left[\Psi(\beta)-\Phi(\beta)\right]f(\beta)d\beta+\sum_{h_j\subset h}\int_{\underline{\beta}_j}^{\bar{\beta}_j}\left[\Psi(\underline{\beta}_j)-\Phi(\bar{\beta}_j)\right]f(\beta)d\beta\right\},$$

$$s.t.\quad \underline{\beta}_j<\bar{\beta}_j. \tag{A.2}$$

Denote $\tilde{\beta}>0$ such that $\Psi(\tilde{\beta})=\Phi(\tilde{\beta})$. Note that $\tilde{\beta}\in(\frac{\sigma E(\Gamma-\sigma E)+\frac{1}{2}E^2}{G},1)$ because $\Psi(\beta)-\Phi(\beta)$ is a quadratic function on β and

$$\Phi\left(\frac{\sigma E(\Gamma-\sigma E)+\frac{1}{2}E^2}{G}\right)$$
$$<\Psi\left(\frac{\sigma E(\Gamma-\sigma E)+\frac{1}{2}E^2}{G}\right)=\frac{(\Gamma-\sigma E)^2}{2}$$

and $\Phi(1)=\frac{(\Gamma-\sigma E)^2}{2}>\Psi(1)$. Proposition 2 is based on $\tilde{\beta}$ identified.

A.4 Proposition 2

Proof. If $\underline{\beta}\geq\tilde{\beta}$, either $H=\emptyset$ or $H=h$ because part-time employment will not be profitable for the employer for any part-time employee yields profit $\Psi(\beta)-\Phi(\beta)\leq 0$ for $\beta\geq\underline{\beta}\geq\tilde{\beta}$. In the first case, $\bar{\beta}=\tilde{\beta}$ so $H=[\underline{\beta},\bar{\beta}]=\emptyset$. In the second case, $\forall\beta'\in h_j\subset H=h$, I must have $\underline{\beta}_j<\tilde{\beta}$ otherwise the contract would not be profitable to the employer. As $\bar{\beta}_j\geq\beta'\geq\underline{\beta}\geq\tilde{\beta}$, there is only one h_j where the employee with β' can be. So $j=1$ and $H=h=h_1=[\underline{\beta}_1,\bar{\beta}_1]$. Let $\bar{\beta}=\bar{\beta}_1$ and Proposition 2 holds.

So I just need to consider the case when $\underline{\beta}<\tilde{\beta}$. The result holds directly following Claims 4.

CLAIM 4. $[\underline{\beta},\tilde{\beta}]\subset H$.

PROOF: Prove by contradiction. Suppose $\exists \beta' \in [\underline{\beta}, \tilde{\beta}]$ and $\beta' \notin H$ in an optimal contract menu.

Note that the pecuniary effort exerted under self-employment for the worker with β' must be less than the maximal accepted effort in the contract by Assumption 2, i.e. $\beta'\Gamma - \sigma E < \frac{\beta' G - \frac{1}{2}E^2}{\sigma E}$ as $G > \frac{E\Gamma}{2\sigma}$. Construct a new contract menu M' which is the same as M except including an extra contract (e_1', w'). Let $e_1' = \frac{\beta' G - \frac{1}{2}E^2}{\sigma E}$ and w_1' is determined by $\frac{U^0(\beta')}{\beta'} = \frac{V^0(\beta')}{\beta'}$ so the worker with β' will take the new contract. Note that none of the employees with $\beta > \beta'$ will deviate to this new contract because

$$\frac{U^0(\beta, \beta')}{\beta} = \frac{U^0(\beta')}{\beta'} = \frac{V^0(\beta')}{\beta'} \leq \frac{V^0(\beta)}{\beta} \leq \frac{U^0(\beta)}{\beta}$$

and also all employees with $\beta > \beta'$ must have $e_1(\beta) > e_1(\beta')$ according to Claim 1.

Also, no employees with $\beta < \beta'$ will take this contract by Lemma 2. The employer earns a positive profit from this contract (e_1', w') because it returns a profit

$$\pi(\beta') = \Psi(\beta') - \Phi(\beta') \geq 0$$

as $\beta' \leq \tilde{\beta}$. Q.E.D.

By above Claim 4, I can prove that H is connected as following. If $\exists \beta' > \tilde{\beta}$ s.t. $\beta' \in H$, $\beta' \in h$ because if $\beta' \in H \setminus h$ and $\beta' > \tilde{\beta}$, $\pi(\beta) = \Psi(\beta) - \Phi(\beta) < 0$, the principal will be better off by not providing the contract for this β'. So $\forall \beta' > \tilde{\beta}$ s.t. $\beta' \in H$, $\exists h_J \subset h$ s.t. $\beta' \in h_J = [\underline{\beta}_J, \bar{\beta}_J] \subset h$. In this case, I have $\underline{\beta}_J < \tilde{\beta}$ otherwise if $\underline{\beta}_J \geq \tilde{\beta}$, I have $\pi(\beta) = \Psi(\underline{\beta}_J) - \Phi(\bar{\beta}_J) < \Psi(\underline{\beta}_J) - \Phi(\underline{\beta}_J) \leq 0$, and the employer is better off by not providing the contract for h_J. Notice that there could be only one h_J with $\underline{\beta}_J < \tilde{\beta}$ and $\bar{\beta}_J > \beta' > \tilde{\beta}$, so if $\exists \beta' > \tilde{\beta}$ s.t. $\beta' \in H$, $H = h_J \cup [\underline{\beta}_1, \tilde{\beta}]$ with $\underline{\beta}_J < \tilde{\beta}$ otherwise $H = [\underline{\beta}, \tilde{\beta}]$. $H = [\underline{\beta}, \bar{\beta}]$ is connected in either case, with $\bar{\beta} = \bar{\beta}_1 > \tilde{\beta}$ in the first case and $\bar{\beta} = \tilde{\beta}$ in the second case.

Q.E.D.

A.5 Theorem 1

Proof. By Claim 1 and Claim 3, any employee in $[\underline{\beta}_j, \bar{\beta}_j) \subset h_j \subset h$ must sign the contract (e_1^j, w^j) with $e_1^j = \frac{\underline{\beta}_j G - \frac{1}{2}E^2}{\sigma E}$ and $\frac{U^0(\beta)}{\beta} = \frac{V^0(\bar{\beta}_j)}{\bar{\beta}_j}$ which lead to $w^j = [\bar{\beta}_j - \frac{1}{2}(\bar{\beta}_j)^2]\Gamma^2 + \frac{\sigma^2}{2}E^2 - \sigma\Gamma(E - \bar{\beta}_j) + \frac{1}{2}(\bar{\beta}_j)^2$. To prove Theorem 1, I just need to check the workers' behavior at the upper bound $\bar{\beta}_j$ of $h_j \subset h$ with Claims 1-3 and Proposition 2.

(1). If $\bar{\beta}_j = \sup H$, by the tie breaking rule, workers at $\beta = \bar{\beta}_j = \sup H$ will choose employment so h_j is closed at $\bar{\beta}_j$.

(2). If $\bar{\beta}_j \neq \sup H$ and $\bar{\beta}_j = \underline{\beta}_{j+1}$, the worker at $\beta = \bar{\beta}_j = \underline{\beta}_{j+1}$ will choose the contract in the higher hierarchy, (e_1^{j+1}, w^{j+1}). So h_j is open at $\bar{\beta}_j$.

(3). If $\bar{\beta}_j \neq \sup H$ and $\bar{\beta}_j < \underline{\beta}_{j+1}$, there must exist part-time employment for workers in $(\bar{\beta}_j, \underline{\beta}_{j+1})$. Whether h_j is closed or open at $\bar{\beta}_j$ depends on whether the employer supplies the part-time contract to the worker at $\bar{\beta}_j$. Note that the decision of whether to supply this part-time contract yields the zero marginal profit to the employer so in this case, the contracts menus with h_j closed or open at their upper bounds are all in the set of optimal contract menu. It is safe to say $h_j = [\bar{\beta}_j, \bar{\beta}_j)$ a.e. in this case.

Q.E.D.

A.6 Proposition 3

Proof. Prove by contradiction. Suppose that there exists a set $(a, b) \subset H \setminus h$ and $\Psi'(\beta) + \Phi'(\beta) < 0$, $\forall \beta \in (a, b)$ in the optimal contract menu M.

Consider a new contract menu M' same as M except for a range $[\underline{\beta}', \bar{\beta}') \subset (a, b)$ s.t. $\bar{\beta}' \geq \underline{\beta}'$ with a contract (e_1, w) s.t. $e_1 = \frac{\beta' G - \frac{1}{2}E^2}{\sigma E}$ and w determined by $\frac{U^0(\bar{\beta}')}{\beta'} = w - \frac{1}{2}e_1^2 - \frac{1}{2}E^2 - \sigma E e_1 + G = \frac{V^0(\bar{\beta}')}{\beta'}$. Note when $\underline{\beta} = \bar{\beta}'$, M' is the same as the previous contract menu M, where all employees in (a, b) are part-time employees.

This contract menu M' yields profit for the employer

$$\int_{H \setminus (a,b)} \pi(\beta) f(\beta) d\beta + \int_a^{\underline{\beta}'} [\Psi(\beta) - \Phi(\beta)] f(\beta) d\beta + \int_{\underline{\beta}'}^{\bar{\beta}'} [\Psi(\underline{\beta}') - \Phi(\bar{\beta}')] f(\beta) d\beta + \int_{\bar{\beta}'}^{b} [\Psi(\beta) - \Phi(\beta)] f(\beta) d\beta.$$

$\forall \underline{\beta}' \in (a, b)$, taking derivative of the above employer's profit with respect to $\bar{\beta}_j$ yields

$$[\Psi(\underline{\beta}') - \Psi(\bar{\beta}')] f(\bar{\beta}') - \Phi'(\bar{\beta}') [F(\bar{\beta}') - F(\underline{\beta}')]$$

which equals to zero when $\bar{\beta}' = \underline{\beta}'$.

Also, $\forall \underline{\beta}' \in (a, b)$, taking second order derivative of the above employer's profit with respect to $\bar{\beta}_j$ yields

$$-[\Psi'(\bar{\beta}') + \Phi'(\bar{\beta}')] f(\bar{\beta}') + [\Psi(\underline{\beta}') - \Psi(\bar{\beta}')] f'(\bar{\beta}') - \Phi''(\bar{\beta}') [F(\bar{\beta}') - F(\underline{\beta}')]$$

which collapses to $-[\Psi'(\bar{\beta}') + \Phi'(\bar{\beta}')] f(\bar{\beta}') > 0$ when $\bar{\beta}' = \underline{\beta}'$.

When $\bar{\beta}' = \underline{\beta}' \in (a, b)$, as the derivative of the profit function with respect to $\bar{\beta}'$ is zero and the second derivative with respect to $\bar{\beta}'$ is strictly positive, it must be more profitable to have $\bar{\beta}' > \underline{\beta}'$ rather than $\bar{\beta}' = \underline{\beta}'$. So the original contract menu M is not optimal. Contradiction. Q.E.D.

Corollary 2.

Proof. If $\Psi'(\tilde{\beta}) + \Phi'(\tilde{\beta}) < 0$, by the continuity of $\Psi'(\beta) + \Phi'(\beta)$, there must exist a range $(\beta', \tilde{\beta}) \subset [\underline{\beta}, \tilde{\beta})$ s.t. $\Psi'(\beta) + \Phi'(\beta) < 0, \forall \beta \in (\beta', \tilde{\beta})$. So $(\beta', \tilde{\beta}) \subset h \neq \emptyset$. Q.E.D.

A.7 Proposition 4

Proof. With Theorem 1 and Propositions 1-3, the employer's problem can be written as

$$\int_{H \setminus [\underline{\beta}_1, \bar{\beta}_1)} \pi(\beta) f(\beta) d\beta$$

$$+ \int_{\underline{\beta}_1}^{\bar{\beta}_1} \left[(\Gamma - \sigma E) \left(\frac{\underline{\beta}_1 G - \frac{1}{2} E^2}{\sigma E} \right) - \frac{1}{2} \left(\frac{\underline{\beta}_1 G - \frac{1}{2} E^2}{\sigma E} \right)^2 - \Phi(\bar{\beta}_1) \right] f(\beta) d(\beta)$$

where $\pi(\beta)$ is the profit the employer earns from the worker with type β. Note that $f(\beta) = 0$ if $\beta < \underline{\beta}$. Taking derivative of $\underline{\beta}_1$ yields

$$\pi(\underline{\beta}_1) f(\underline{\beta}_1) - \left[(\Gamma - \sigma E) \left(\frac{\underline{\beta}_1 G - \frac{1}{2} E^2}{\sigma E} \right) - \frac{1}{2} \left(\frac{\underline{\beta}_1 G - \frac{1}{2} E^2}{\sigma E} \right)^2 - \Phi(\bar{\beta}_1) \right] f(\underline{\beta}_1) + \left(\Gamma - \sigma E - \frac{\underline{\beta}_1 G - \cdots}{\sigma E} \right.$$

which is strictly positive with $f(\underline{\beta}_1) = 0$ for $\underline{\beta}_1 < \underline{\beta}$ and $\frac{\underline{\beta}_1 G - \frac{1}{2} E^2}{\sigma E} < \frac{\underline{\beta} G - \frac{1}{2} E^2}{\sigma E} < \Gamma - \sigma E$ by Assumption 3. So $\underline{\beta}_1 \geq \underline{\beta}$. . Q.E.D.

A.8 Corollary 3

Proof. If $\underline{\beta}_1 \geq \underline{\beta}$, the principal's problem can be written as

$$\Pi(h,H) = \int_{\underline{\beta}}^{\underline{\beta}_1} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta)$$

$$+ \sum_{h_i, h_{i+1} \in h} \left\{ \int_{\underline{\beta}_i}^{\bar{\beta}_i} [\Psi(\underline{\beta}_i) - \Phi(\bar{\beta}_i)] f(\beta) d\beta + \int_{\bar{\beta}_i}^{\underline{\beta}_{i+1}} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta) \right\} + S$$

s.t.

$$\underline{\beta}_{i+1} \geq \bar{\beta}_i, \forall h_i, h_{i+1} \subset h. \tag{A.3}$$

Taking the derivative of $\underline{\beta}_1$ on the objective function of employer (A.3) yields

$$[\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1)] f(\underline{\beta}_1) + \Psi'(\underline{\beta}_1)[F(\bar{\beta}_1) - F(\underline{\beta}_1)] \leq 0 \tag{A.4}$$

where the inequality is from the constraint $\underline{\beta}_1 \geq \underline{\beta}$.

Because the first term in (A.4) is positive as $\bar{\beta}_1 > \underline{\beta}_1$, the second term must be negative and thus $\Psi'(\underline{\beta}_1) \leq 0$. As

$$\Psi'(\underline{\beta}_1) = (\Gamma - \sigma E - \frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E}) \frac{G}{\sigma E} \leq 0,$$

$\frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E} \geq \Gamma - \sigma E$. Further, I have $e_1(\beta) = \frac{\underline{\beta}_j G - \frac{1}{2}E^2}{\sigma E} \geq \frac{\underline{\beta}_1 G - \frac{1}{2}E^2}{\sigma E} \geq \Gamma - \sigma E$, $\forall \beta \in h_j \subset h$. Q.E.D.

Corollary 4.

Proof. Firstly if $\underline{\beta} \leq \tilde{\beta}$, the non-empty set $[\underline{\beta}, \tilde{\beta}] \subset H \neq \emptyset$.

For the other direction, it is equivalent to prove if $\underline{\beta} > \tilde{\beta}$, $H = \emptyset$. With $\underline{\beta} > \tilde{\beta}$, the employer will never supply part-time contracts which return profit $\pi(\beta) =$

$\Psi(\beta)-\Phi(\beta)<0$ for any worker with $\beta \geq \underline{\beta} > \tilde{\beta}$. If supplying a positioned contract, the employer earns profit $\Pi(\beta)=\Psi(\underline{\beta}_j)-\Phi(\bar{\beta}_j) < \Psi(\underline{\beta}_j)-\Phi(\underline{\beta}_j) < 0$ for any $\beta \in h_j$ as $\underline{\beta}_j \geq \underline{\beta}_1 > \underline{\beta} > \tilde{\beta}$, by Proposition 4. Q.E.D.

A.9 Proposition 5

Lemma 4. *If the distribution of β is uniform, (1) $\bar{\beta}_j = \underline{\beta}_{j+1}$, $\forall h_j, h_{j+1} \subset h$; (2) $\sup h > \tilde{\beta}$.*

Proof. (1). Taking derivative of $\bar{\beta}_j$ on the principal's objective function if $h_j, h_{j+1} \subset h$ yields

$$[\Psi(\underline{\beta}_j)-\Psi(\bar{\beta}_j)]f(\bar{\beta}_j)-\Phi'(\bar{\beta}_j)[F(\bar{\beta}_j)-F(\underline{\beta}_j)]$$

$$=\Psi(\underline{\beta}_j)-\Psi(\bar{\beta}_j)-\Phi'(\bar{\beta}_j)(\bar{\beta}_j-\underline{\beta}_j) \geq 0. \tag{A.5}$$

Taking derivative of $\underline{\beta}_j$ yields

$$\Phi(\bar{\beta}_1)-\Phi(\underline{\beta}_1)+\Psi'(\underline{\beta}_1)(\bar{\beta}_1-\underline{\beta}_1) \leq 0. \tag{A.6}$$

Adding (A.5) and (A.6) yields

$$[\Psi(\underline{\beta}_j)-\Psi(\bar{\beta}_j)+\Psi'(\underline{\beta}_1)(\bar{\beta}_1-\underline{\beta}_1)]+[\Phi(\bar{\beta}_1)-\Phi(\underline{\beta}_1)-\Phi'(\bar{\beta}_j)(\bar{\beta}_j-\underline{\beta}_j)]$$

which is strictly positive by the concavity of $\Psi(\beta)$ and $\Phi(\beta)$. So the inequality of (A.5) must be strict and the constraint $\underline{\beta}_{j+1} \geq \bar{\beta}_j$ must bind.

(2). Similarly, if $\bar{\beta}_j = \sup h \leq \tilde{\beta}$, taking derivative of $\bar{\beta}_j$ on the principal's objective function yields

$$[\Psi(\underline{\beta}_j)-\Psi(\bar{\beta}_j)]f(\bar{\beta}_j)-\Phi'(\bar{\beta}_j)[F(\bar{\beta}_j)-F(\underline{\beta}_j)]$$

$$= \Psi(\underline{\beta}_j) - \Psi(\bar{\beta}_j) - \Phi'(\bar{\beta}_j)(\bar{\beta}_j - \underline{\beta}_j) \geq 0. \tag{A.7}$$

Taking derivative of $\underline{\beta}_j$ yields

$$\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1) + \Psi'(\underline{\beta}_1)(\bar{\beta}_1 - \underline{\beta}_1) \leq 0. \tag{A.8}$$

Adding the above two inequalities yields

$$[\Psi(\underline{\beta}_j) - \Psi(\bar{\beta}_j) + \Psi'(\underline{\beta}_1)(\bar{\beta}_1 - \underline{\beta}_1)] + [\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1) - \Phi'(\bar{\beta}_j)(\bar{\beta}_j - \underline{\beta}_j)] > 0$$

by concavity of $\Psi(\beta)$ and $\Phi(\beta)$. So the inequality in the FOC w.r.t to $\bar{\beta}_j$ when $\bar{\beta}_j = \sup h \leq \tilde{\beta}$ must be strict,

$$\Psi(\underline{\beta}_j) - \Psi(\bar{\beta}_j) - \Phi'(\bar{\beta}_j)(\bar{\beta}_j - \underline{\beta}_j) > 0. \tag{A.9}$$

I must have $\bar{\beta}_j \geq \tilde{\beta}$ in the optimal contract menu.

Based on $\bar{\beta}_j \geq \tilde{\beta}$, the employer's problem can be written as

$$\Pi(h, H) = \int_{\underline{\beta}}^{\underline{\beta}_1} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta)$$

$$+ \sum_{h_i, h_{i+1} \in h} \left\{ \int_{\underline{\beta}_i}^{\bar{\beta}_i} [\Psi(\underline{\beta}_i) - \Phi(\bar{\beta}_i)] f(\beta) d\beta + \int_{\bar{\beta}_i}^{\underline{\beta}_{i+1}} [\Psi(\beta) - \Phi(\beta)] f(\beta) d(\beta) \right\} + \int_{\underline{\beta}_J}^{\bar{\beta}_J} [\Psi(\underline{\beta}_J) - \Phi(\bar{\beta}_J)] f(\beta)$$

s.t. $\underline{\beta}_J < \tilde{\beta}$ and $\bar{\beta}_J = \sup H \geq \tilde{\beta}$.

Taking derivative of $\bar{\beta}_J$ yields

$$\Psi(\underline{\beta}_J) - \Phi(\bar{\beta}_J) - \Phi'(\bar{\beta}_J)[F(\bar{\beta}_J) - F(\underline{\beta}_J)],$$

which is strictly positive when $\bar{\beta}_J = \tilde{\beta}$ because

$$\Psi(\underline{\beta}_J) - \Phi(\tilde{\beta}) - \Phi'(\bar{\beta}_J)[F(\bar{\beta}_J) - F(\underline{\beta}_J)] = \Psi(\underline{\beta}_J) - \Psi(\tilde{\beta}) - \Phi'(\bar{\beta}_J)[F(\bar{\beta}_J) - F(\underline{\beta}_J)] > 0$$

where the strict inequality is from (A.9) for $h_J = \sup H \leq \tilde{\beta}$. So $\bar{\beta}_J > \tilde{\beta}$ and $\bar{\beta}_J = \bar{\beta}$.

Q.E.D.

Proposition 5

Proof. Prove by contradiction. Suppose there are more than two different h_j's such that $h = \{h_1, h_2, ...\}$ in the optimal contract M. I can denote $h_1 = [\underline{\beta}_1, \underline{\beta}_2)$ and $h_2 = [\underline{\beta}_2, \bar{\beta}_2)$ by Lemma 4. The profit earned by the principal can be written as

$$\Pi_M = \int_{\underline{\beta}}^{\underline{\beta}_1} [\Psi(\beta) - \Phi(\beta)] f(\beta) d\beta + \sum_{k=1}^{M} \int_{\underline{\beta}_j}^{\bar{\beta}_j} \left[\Psi(\underline{\beta}_j) - \Phi(\bar{\beta}_j)\right] f(\beta) d\beta$$

As M is the optimal contract, the FOC of $\underline{\beta}_1$ can be written as

$$-\frac{\Psi'(\underline{\beta}_1)}{f(\underline{\beta}_1)} - \frac{\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1)}{F(\bar{\beta}_1) - F(\underline{\beta}_1)} = 0 \tag{A.10}$$

Now construct a new contract M' where $h'_{j-1} = h_j$ for $j > 2$ and $h'_1 = h_1 \cup h_2 = [\underline{\beta}_1, \bar{\beta}_2)$. I then have the change in the employer's total profit by applying M' as

$$\begin{aligned}
\Pi(M') - \Pi(M) &= \int_{\bar{\beta}_1}^{\bar{\beta}_2} \left[\Psi(\underline{\beta}_1) - \Psi(\bar{\beta}_1)\right] f(\beta) d\beta - \int_{\underline{\beta}_1}^{\bar{\beta}_1} \left[\Phi(\bar{\beta}_2) - \Psi(\bar{\beta}_1)\right] f(\beta) d\beta \\
&= \left[\Psi(\underline{\beta}_1) - \Psi(\bar{\beta}_1)\right] \left[F(\bar{\beta}_2) - F(\bar{\beta}_1)\right] - \left[\Phi(\bar{\beta}_2) - \Phi(\bar{\beta}_1)\right] \left[F(\bar{\beta}_1) - F(\underline{\beta}_1)\right] \\
&= \left[F(\bar{\beta}_2) - F(\bar{\beta}_1)\right] \left[F(\bar{\beta}_1) - F(\underline{\beta}_1)\right] \left[\frac{\Psi(\underline{\beta}_1) - \Psi(\bar{\beta}_1)}{F(\bar{\beta}_1) - F(\underline{\beta}_1)} - \frac{\Phi(\bar{\beta}_2) - \Phi(\bar{\beta}_1)}{F(\bar{\beta}_2) - F(\bar{\beta}_1)}\right] \\
&> \left[F(\bar{\beta}_2) - F(\bar{\beta}_1)\right] \left[F(\bar{\beta}_1) - F(\underline{\beta}_1)\right] \left[\frac{\Psi(\underline{\beta}_1) - \Psi(\bar{\beta}_1)}{F(\bar{\beta}_1) - F(\underline{\beta}_1)} - \frac{\Phi(\bar{\beta}_1) - \Phi(\underline{\beta}_1)}{F(\bar{\beta}_1) - F(\underline{\beta}_1)}\right] \\
&= \left[F(\bar{\beta}_2) - F(\bar{\beta}_1)\right] \left[F(\bar{\beta}_1) - F(\underline{\beta}_1)\right] \left[\frac{\Psi(\underline{\beta}_1) - \Psi(\bar{\beta}_1)}{F(\bar{\beta}_1) - F(\underline{\beta}_1)} + \frac{\Psi'(\underline{\beta}_1)}{f(\underline{\beta}_1)}\right]
\end{aligned}$$

where the last equality is by (A.10), which is strictly positive by the concavity of $\Psi(\beta)$. This is a contradiction to M being the optimal contract. Q.E.D.

Appendix B: Proofs in Chapter 2

B.1 Proofs

Taking derivative of the leader's problem (2.1) with respect to r_j yields

$$-u'_L(180 + 2400 - \sum_{i=1}^{3} r_i) + \gamma_i^R b_j u'_M(360 - b_j + r_j) = 0.$$

So the solution of optimal reward for the leader can be written in a system of implicit functions,

$$f(\mathbf{r}, \mathbf{b}, \gamma) = \begin{bmatrix} -u'_L(2580 - r_1 - r_2 - r_3) + \gamma b_1[u'_M(360 - b_1 + r_1)] \\ -u'_L(2580 - r_1 - r_2 - r_3) + \gamma b_2[u'_M(360 - b_2 + r_2)] \\ -u'_L(2580 - r_1 - r_2 - r_3) + \gamma b_3[u'_M(360 - b_3 + r_3)] \end{bmatrix} = \mathbf{0}$$

where $\mathbf{r} = [r_1, r_2, r_3]^T$ and $\mathbf{b} = [b_1, b_2, b_3]^T$.

By implicit function theorem,

$$\frac{D\mathbf{r}}{D\mathbf{b}} = -\left[\frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\mathbf{r}}\right]^{-1} * \frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\mathbf{b}}$$

and

$$\frac{D\mathbf{r}}{D\gamma} = -\left[\frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\mathbf{r}}\right]^{-1} * \frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\gamma}.$$

In order to solve $\frac{D\mathbf{r}}{D\mathbf{b}}$ and $\frac{D\mathbf{r}}{D\gamma}$, we will need $\left[\frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\mathbf{r}}\right]^{-1}$.

For easy notations, denote

$$x_j = \gamma b_j u''_M(\pi_j)$$

and

$$L = u''_L(\pi_L).$$

We have

$$\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}} = \begin{bmatrix} u''_L(\pi_L)+\gamma b_1 u''_M(\pi_1) & u''_L(\pi_L) & u''_L(\pi_L) \\ u''_L(\pi_L) & u''_L(\pi_L)+\gamma b_2 u''_M(\pi_2) & u''_L(\pi_L) \\ u''_L(\pi_L) & u''_L(\pi_L) & u''_L(\pi_L)+\gamma b_3 u''_M(\pi_3). \end{bmatrix}$$

$$= \begin{bmatrix} L+x_1 & L & L \\ L & L+x_2 & L \\ L & L & L+x_3 \end{bmatrix}$$

As solved in Appendix C,

$$\left[\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}}\right]^{-1} = \begin{bmatrix} \frac{1+\frac{L}{x_2}+\frac{L}{x_3}}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} & -\frac{L/x_2}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} & -\frac{L/x_3}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} \\ -\frac{L/x_1}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} & \frac{1+\frac{L}{x_1}+\frac{L}{x_3}}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} & -\frac{L/x_3}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} \\ -\frac{L/x_1}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & -\frac{L/x_2}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & \frac{1+\frac{L}{x_1}+\frac{L}{x_2}}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} \end{bmatrix}.$$

It can also be solved that

$$\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{b}} = \begin{bmatrix} \gamma u'_M(\pi_1)-x_1 & 0 & 0 \\ 0 & \gamma u'_M(\pi_2)-x_2 & 0 \\ 0 & 0 & \gamma u'_M(\pi_3)-x_3 \end{bmatrix}$$

and

$$\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\gamma} = \begin{bmatrix} b_1[u'_M(360-b_1+r_1)] \\ b_2[u'_M(360-b_2+r_2)] \\ b_3[u'_M(360-b_3+r_3)] \end{bmatrix}.$$

Proposition 1.

Proof. The FOC for optimal r_j in the leader's problem is

$$u'_L(\pi_L) = \gamma b_j u'_M(360-b_j+r_j)$$

for all $j \in \{1,2,3\}$. Hence, we must have

$$\gamma b_j u'_M(360-b_j+r_j) = \gamma b_k u'_M(360-b_k+r_k)$$

for member j and member k in the same group s.t. $r_j, r_k > 0$. Without loss of generality, suppose $b_j > b_k$. We thus have $u'_M(360-b_j+r_j) < u'_M(360-b_k+r_k)$. As members' utility function is concave, we must have $360-b_j+r_j > 360-b_k+r_k$ or $\pi_j(b_j,\gamma) > \pi_k(b_k,\gamma)$. Q.E.D.

Proposition 2.

Proof. As $\frac{D\mathbf{r}}{D\mathbf{b}} = -\left(\frac{Df}{D\mathbf{r}}\right)^{-1}\frac{Df}{D\mathbf{b}}$,

$$\frac{dr_j}{db_j} = -\frac{1+\sum_{i\neq j}\frac{L}{x_i}}{L+x_j\left(1+\sum_{i\neq j}\frac{L}{x_i}\right)} * [\gamma u'_M(\pi_j) - x_j]$$

where members i in the same group as member j. Since

$$x_i = \gamma b_i u''_M(\pi_i) < 0$$

and

$$L = u''_L(\pi_L) < 0,$$

we thus have $\frac{dr_j}{db_j} > 0$. Q.E.D.

Proposition 3.

Proof.

$$\frac{dr_j}{db_i} = -\frac{L}{x_i\left[x_j + L\left(1 + \frac{x_j}{x_i} + \frac{x_j}{x_k}\right)\right]} \left({}'_M(\pi_i) - x_i\right) < 0$$

for $i \neq j \neq k$. Q.E.D.

Proposition 4.

Proof. From the FOC conditions $f(\mathbf{r}, \mathbf{b}, r_L^R) = 0$, we have

$$\frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\gamma} = \begin{bmatrix} b_1[u'_M(360 - b_1 + r_1)] \\ b_2[u'_M(360 - b_2 + r_2)] \\ b_3[u'_M(360 - b_3 + r_3)] \end{bmatrix} = \begin{bmatrix} \frac{u'_L(\pi_L)}{\gamma} \\ \frac{u'_L(\pi_L)}{\gamma} \\ \frac{u'_L(\pi_L)}{\gamma} \end{bmatrix}.$$

So

$$\frac{D\mathbf{r}}{D\gamma} = -\left[\frac{Df(\mathbf{r}, \mathbf{b}, \gamma)}{D\mathbf{r}}\right]^{-1} \frac{f(\mathbf{r}, \mathbf{b}, \gamma)}{D\gamma}$$

$$= -\frac{u'_L(\pi_L)}{\gamma} \begin{bmatrix} \frac{1}{x_1 + L\left(1 + \frac{x_1}{x_2} + \frac{x_1}{x_3}\right)} \\ \frac{1}{x_2 + L\left(1 + \frac{x_2}{x_1} + \frac{x_2}{x_3}\right)} \\ \frac{1}{x_3 + L\left(1 + \frac{x_3}{x_2} + \frac{x_3}{x_1}\right)} \end{bmatrix}.$$

So $\frac{dr_j}{d\gamma} > 0$ as $L < 0$ and $x_j < 0$ for $j \in \{1, 2, 3\}$. Q.E.D.

B.2 Proof of Proposition 5

To prove Proposition 5, we need Lemmas 1-4 for preparation. Among these, Lemmas 1-3 can be derived from the properties of the leaders' reward function. The members' problem will not be introduced until Lemma 4.

Lemma 1. A leader with higher reciprocal level γ will increase her/his reward to member j, r_j, faster as the member's bid $b_j > \underline{b}$ increases than those with lower reciprocal levels, for example, $\frac{d^2 r_j(b_1,b_2,b_3,\gamma)}{db_j d\gamma} > 0$.

Proof.

$$\begin{aligned}
\frac{dr_1}{db_1} &= -\frac{1+\frac{L}{x_2}+\frac{L}{x_3}}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)}\left[\gamma u'_M(\pi_1)-x_1\right] \\
&= -\frac{1+\frac{L}{x_2}+\frac{L}{x_3}}{L+x_1\left(1+\frac{L}{x_2}+\frac{L}{x_3}\right)}\left[\gamma u'_M(\pi_1)-x_1\right] \\
&= -\frac{1}{\frac{L}{1+\frac{L}{x_2}+\frac{L}{x_3}}+x_1}\left[\gamma u'_M(\pi_1)-x_1\right] \\
&= -\frac{1}{\frac{L}{\gamma+\frac{L}{b_2 u''_M(\pi_2)}+\frac{L}{b_3 u''_M(\pi_3)}}+b_1 u''_M(\pi_1)}\left[u'_M(\pi_1)-b_1 u''_M(\pi_1)\right]
\end{aligned}$$

which increases on γ. By symmetry of r_1, r_2 and r_3, $\frac{dr_i}{db_i}$ increases on γ for any $i \in \{1,2,3\}$. Q.E.D.

Lemma 2. $\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}$ increases on γ.

Proof.

$$\begin{aligned}
&\frac{\partial}{\partial\gamma}\left[\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\right] \\
&= \frac{\partial}{\partial\gamma}\left[\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial \pi_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\right] \\
&= \frac{\partial u_M^2(\pi_j(b_j,\gamma))}{\partial^2 \pi_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial \gamma}+\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial \pi_j}\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial b_j\partial\gamma}
\end{aligned}$$

To have $\frac{\partial}{\partial\gamma}\left[\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\right] > 0$, we need

$$R < \frac{\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial b_j\partial\gamma}}{\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial \gamma}}.$$

As

$$\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial b_j \partial \gamma} = [u'_M(\pi_j) - b u''_M(\pi_j)] \frac{-\frac{1}{u''_L}}{1 + b_j u''_M(\pi_j)(\frac{\gamma}{u''_L} + \sum_{i\neq j} \frac{1}{b_i u''_M(\pi_i)})},$$

the LHS of above inequity is positive when R converges to zero. So the inequity holds when R is small enough, for example, there exists a $R' > 0$ such that when $R < R'$, $R < \frac{\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial b_j \partial \gamma}}{\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial \gamma}}$. Q.E.D.

Lemma 3. $\frac{d^2 r_1}{d(b_1)^2} < 0$.

Proof.

$$\frac{d^2 \pi_1(b_1,\gamma)}{d(b_1)^2} = \frac{d^2 r_1(b_1,\gamma)}{d(b_1)^2}$$

$$= \frac{d}{db_j}\left\{ \frac{-1}{\frac{1}{\frac{1}{L}+\frac{1}{x_2}+\frac{1}{x_3}} + x_1} [\gamma u'_M(\pi_1) - x_1] \right\}. \qquad (*)$$

The term in the square brackets, $\gamma u'_M(\pi_1) - x_1$, can be written as

$$\gamma u'_M(\pi_1) - x_1$$

$$= \gamma u'_M(\pi_1) - \gamma b_1 u''_M(\pi_1)$$

$$= \gamma R e^{-R\pi_1} + \gamma b_1 R^2 e^{-R\pi_1}.$$

Note that b_1 can change this term through b_1 itself and through π_1. So

$$\frac{d}{db_1}\left[\gamma R e^{-R\pi_1} + \gamma b_1 R^2 e^{-R\pi_1}\right]$$

$$= \gamma R^2 e^{-R\pi_1} - \gamma R^2 e^{-R\pi_1}\frac{d\pi_1}{db_1} - \gamma b_1 R_M^3 e^{-R\pi_1}\frac{d\pi_1}{db_1}$$

$$= \gamma R^2 e^{-R\pi_1}(1 - \frac{d\pi_1}{db_1} - R b_1 \frac{d\pi_1}{db_1})$$

$$= \gamma R^2 e^{-R\pi_1}\left[1 - (\frac{dr_1}{db_1} - 1) - R b_1(\frac{dr_1}{db_1} - 1)\right].$$

Note that when R converges to zero, $\frac{dr_1}{db_1}$ converges to infinity. So when R is small enough as in Assumption 2, $\gamma u'_M(\pi_1) - x_1$ is positive and decreasing on b_1.

For the first term in $(*)$, $\frac{-1}{\frac{1}{\frac{1}{L}+\frac{1}{x_2}+\frac{1}{x_3}}+x_1}$, denote it as $g(b_1, \pi_L, \pi_1, \pi_2, \pi_3) = \frac{-1}{\frac{1}{\frac{1}{L}+\frac{1}{x_2}+\frac{1}{x_3}}+x_1}$ where $L = u''_L(\pi_L)$ and $x_j = \gamma b_j u''_M(\pi_j)$.

$$\frac{dg(b_1, \pi_L, \pi_1, \pi_2, \pi_3)}{db_1}$$

$$= \left[\frac{\partial g}{\partial b_j} + \frac{\partial g}{\partial \pi_L}\frac{d\pi_L}{db_1} + \frac{\partial g}{\partial \pi_2}\frac{d\pi_2}{db_1} + \frac{\partial g}{\partial \pi_3}\frac{d\pi_3}{db_1}\right]$$

Note that

$$g(b_1, \pi_L, \pi_1, \pi_2, \pi_3) = \frac{1}{\frac{1}{\frac{1}{R_L^2 e^{-R_L \pi_L}}+\frac{1}{{}_2 R^2 e^{-R\pi_2}}+\frac{1}{{}_3 R^2 e^{-R\pi_3}}} +_1 R^2 e^{-R\pi_1}}.$$

So $\frac{\partial g}{\partial b_j} < 0$, $\frac{\partial g(b_1,\pi_L,\pi_1,\pi_2,\pi_3)}{\partial \pi_L} > 0$, $\frac{\partial g(b_1,\pi_L,\pi_1,\pi_2,\pi_3)}{\partial \pi_2} > 0$ and $\frac{\partial g(b_1,\pi_L,\pi_1,\pi_2,\pi_3)}{\partial \pi_3} > 0$. Also,

$$\frac{\partial \pi_L}{db_1} = -\frac{\partial r_1}{db_1} - \frac{\partial r_2}{db_1} - \frac{\partial r_3}{db_1} = [{}'_M(\pi_1) -_1 u''_M(\pi_1)]\left[\frac{1}{x_1 + L(1 + \frac{x_1}{x_2} + \frac{x_1}{x_3})}\right] < 0$$

so $\frac{\partial \pi_L}{db_1}\frac{\partial g}{\partial \pi_L} < 0$. Similarly, $\frac{\partial \pi_2}{db_1}\frac{\partial g}{\partial \pi_2} < 0$ and $\frac{\partial \pi_3}{db_1}\frac{\partial g}{\partial \pi_3} < 0$. So $\frac{dg(b_1,\pi_L,\pi_1,\pi_2,\pi_3)}{db_1} = \frac{d^2 r_1}{d(b_1)^2} < 0$.

Q.E.D.

Lemma 4. $\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\frac{1}{u'_M(360-b_j)}$ decreases on b_j.

Proof. $\frac{1}{u'_M(360-b_j)}$ decreases on b_j and is always positive. $\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}$ is positive. So we need to check whether it decreases on b_j to know whether their production $\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\frac{1}{u'_M(360-b_j)}$ decreases on b_j or not.

$$\frac{\partial}{\partial b_j}\left[\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\right]$$

$$= \frac{\partial}{\partial b_j}\left[\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial \pi_j}\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\right]$$

$$= \frac{\partial^2 u_M(\pi_j(b_j,\gamma))}{\partial(\pi_j)^2}\left[\frac{\partial \pi_j(b_j,\gamma)}{\partial b_j}\right]^2 + \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial \pi_j}\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial^2 b_j} < 0$$

as $\frac{\partial^2 \pi_j(b_j,\gamma)}{\partial^2 b_j} = \frac{\partial^2 r_j(b_1,b_2,b_3,\gamma)}{\partial^2 b_j} < 0$ by Lemma 3, with Assumption 5. Q.E.D.

The members' strategy in the pure strategy equilibrium takes different forms in the three cases for $b_j^1 > b_j^2$, $b_j^1 = b^1 < b_j^2 = b^2$ and $b_j^1 = b_j^2$.

Case 1. $b^1 > b^2$.

In this case, if a member j is matched with a leader with gender G_1, his/her problem is

$$\max_{b_j} \frac{1}{2}P_1 u_M(360 - b_j) + (\frac{1}{2}P_1 + 1 - P_1)E[u_M(\pi_j(b_j,\gamma_i))]$$

$$= \frac{1}{2}P_1 u_M(360 - b_j) + (\frac{1}{2}P_1 + 1 - P_1)\int u_M(\pi_j(b_j,\gamma))f_j(\gamma|G_1)d\gamma.$$

Taking derivative of b_j yields FOC,

$$-u_M'(360 - b_j) + (\frac{2}{P_1} - 1)\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_1} f_j(\gamma|G_1)d\gamma = 0,$$

which can be written as

$$\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j}\frac{1}{u_M'(360 - b_j)} f_j(\gamma|G_1)d\gamma = \frac{1}{\frac{2}{P_1} - 1}. \tag{B.1}$$

The member j's utility function with leader's gender G_2 is

$$\max_{b_j}[P_1 + \frac{1}{2}(1 - P_1)]u_M(360 - b_j) + \frac{1}{2}(1 - P_1)E[u_M(\pi_j(b_j,\gamma))]$$

$$= [P_1 + \frac{1}{2}(1 - P_1)]u_M(360 - b_j) + \frac{1}{2}(1 - P_1)\int u_M(360 - b_j + r(b_j,\gamma))f_j(\gamma|G_2)d\gamma.$$

where member j's bid is b_j. Taking derivative of b_j yields FOC,

$$-u'_M(360 - b_j) + \frac{1 - P_1}{1 + P_1} \int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} f_j(\gamma | G_2) d\gamma$$

which can be further written as

$$\int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma | G_2) d\gamma = \frac{1 + P_1}{1 - P_1}. \tag{B.2}$$

Case 2. $b^1 < b^2$.

When $b^1 < b^2$, the FOC condition can be similarly derived as in CASE 1. The FOC conditions are

$$\int \frac{\partial u_M(b_j, \gamma)}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma | G_1) d\gamma = \frac{2 - P_1}{P_1} \tag{B.3}$$

for member j's bid b_j with leader's gender G_1 and

$$\int \frac{\partial u_M(b_j, \gamma)}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma | G_2) d\gamma = \frac{1 - P_1}{1 + P_1} \tag{B.4}$$

for member j with leader's gender G_2.

Case 3. $b^1 = b^2$.

When $b^1 = b^2$, the member j's problem when facing a leader with gender $G \in \{G_1, G_2\}$ is

$$\max_{b_j} \frac{1}{2} u_M(360 - b_j) + \frac{1}{2} E[u_M(\pi_j(b_j, \gamma))]$$
$$= \frac{1}{2} u_M(360 - b_j) + \frac{1}{2} \int u_M(\pi_j(b_j, \gamma)) f_j(\gamma | i) d\gamma_i.$$

Taking derivative of b_j yields FOC,

$$-u'_M(360 - b_j) + \int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_i} f_j(\gamma|i) d\gamma = 0,$$

which can be written as

$$\int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma|i) d\gamma = 1. \tag{B.5}$$

Q.E.D.

Proposition 5.

Proof. We will prove this Proposition 5 by checking two possible cases and show that an equilibrium with $b^1 \geq b^2$ exists in either case.

1. There exists $b_j = b^1 \in [0, 360]$ such that

$$\int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma|G_1) d\gamma = \frac{1}{\frac{2}{P_1} - 1} \tag{B.6}$$

when $b_j = b^1$. So when $b_j = b^1$,

$$\int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma|G_2) d\gamma < \frac{1}{\frac{2}{P_1} - 1} < \frac{1 + P_1}{1 - P_1}$$

where the first inequality is from Lemma 2 and $f_j(\gamma|G_1)$ FOSD $f_j(\gamma|G_2)$. By Lemma 4, $\frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)}$ decreases on b_j, $\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_2) d\gamma$ must decrease on b_j too. If there exists $b_j = b^2$ such that

$$\int \frac{\partial u_M(\pi_j(b_j, \gamma))}{\partial b_j} \frac{1}{u'_M(360 - b_j)} f_j(\gamma|G_2) d\gamma = \frac{1 + P_1}{1 - P_1},$$

we must have $b^2 < b^1$. Otherwise we will have $b_j^2 = 0$. In either case, $b_j^2 = b^2 \leq b_j^1 = b^1$.

CASE 2. $\forall b_j = b^1 \in [0, 360]$,

$$\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_1) d\gamma \neq \frac{1}{\frac{2}{P_1}-1}.$$

Because the left hand side of the above inequality is continuous on b_j, we must have either $\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_1) d\gamma > \frac{1}{\frac{2}{P_1}-1}$, or $\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_1) d\gamma < \frac{1}{\frac{2}{P_1}-1}$, $\forall b_j \in [0, 360]$. In the first case, $b_j = b^1 = 360$ so $b^2 \leq b^1$ must hold. In the second case, we must have

$$\int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_2) d\gamma \leq \int \frac{\partial u_M(\pi_j(b_j,\gamma))}{\partial b_j} \frac{1}{u'_M(360-b_j)} f_j(\gamma|G_1) d\gamma$$

$$< \frac{1}{\frac{2}{P_1}-1} < \frac{1+P_1}{1-P_2}$$

so $b_j = b^2 = 0$ and $b^2 \leq b^1$ holds as well. Q.E.D.

Appendix C: Matrix Inverse in Chapter 2

To solve $\left[\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}}\right]^{-1}$, note that

$$\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}} = \begin{bmatrix} u_L''(\pi_L) + \gamma b_1 u_M''(\pi_1) & u_L''(\pi_L) & u_L''(\pi_L) \\ u_L''(\pi_L) & u_L''(\pi_L) + \gamma b_2 u_M''(\pi_2) & u_L''(\pi_L) \\ u_L''(\pi_L) & u_L''(\pi_L) & u_L''(\pi_L) + \gamma b_3 u_M''(\pi_3) \end{bmatrix}$$

where $\pi_L = 2580 - r_1 - r_2 - r_3$ and $\pi_i = 360 - b_i + r_i$. For easy notations, recall that we denote

$$x_i =_i -u_M''(\pi_i)$$

and

$$L = u_L''(\pi_L)$$

so the above system of equations can be written as

$$\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}} = \begin{bmatrix} x_1 + L & L & L \\ L & x_2 + L & L \\ L & L & x_3 + L \end{bmatrix}.$$

Using linear row reduction to find the inverse matrix, we have

$$\begin{bmatrix} x_1 + L & L & L & 1 & 0 & 0 \\ L & x_2 + L & L & 0 & 1 & 0 \\ L & L & x_3 + L & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 - R_2 \rightarrow \begin{bmatrix} x_1 & -x_2 & 0 & 1 & -1 & 0 \\ L & x_2 + L & L & 0 & 1 & 0 \\ L & L & x_3 + L & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 - R_3 \rightarrow \begin{bmatrix} x_1 & -x_2 & 0 & 1 & -1 & 0 \\ 0 & x_2 & -x_3 & 0 & 1 & -1 \\ L & L & x_3 + L & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 * \frac{1}{x_1} \rightarrow \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & x_2 & -x_3 & 0 & 1 & -1 \\ L & L & x_3 + L & 0 & 0 & 1 \end{bmatrix}$$

$$R_3 - R_1 * L \rightarrow \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & x_2 & -x_3 & 0 & 1 & -1 \\ 0 & L + L\frac{x_2}{x_1} & x_3 + L & -\frac{L}{x_1} & \frac{L}{x_1} & 1 \end{bmatrix}$$

$$R_2 * \frac{1}{x_2} \rightarrow \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & 1 & -\frac{x_3}{x_2} & 0 & \frac{1}{x_2} & -\frac{1}{x_2} \\ 0 & L + L\frac{x_2}{x_1} & x_3 + L & -\frac{L}{x_1} & \frac{L}{x_1} & 1 \end{bmatrix}$$

$$R_3 - R_2 * \left(L + L\frac{x_2}{x_1}\right) \rightarrow \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & 1 & -\frac{x_3}{x_2} & 0 & \frac{1}{x_2} & -\frac{1}{x_2} \\ 0 & 0 & x_3 + L + \frac{x_3}{x_2}\left(L + L\frac{x_2}{x_1}\right) & -\frac{L}{x_1} & \frac{L}{x_1} - \frac{L + L\frac{x_2}{x_1}}{x_2} & 1 + \frac{L + L\frac{x_2}{x_1}}{x_2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & 1 & -\frac{x_3}{x_2} & 0 & \frac{1}{x_2} & -\frac{1}{x_2} \\ 0 & 0 & L + x_3\left(1 + \frac{L}{x_2} + \frac{L}{x_1}\right) & -\frac{L}{x_1} & -\frac{L}{x_2} & 1 + \frac{L}{x_1} + \frac{L}{x_2} \end{bmatrix}$$

$$R_3 * \frac{1}{x_3 + L\left(1 + \frac{x_3}{x_2} + \frac{x_3}{x_1}\right)} \rightarrow \begin{bmatrix} 1 & -\frac{x_2}{x_1} & 0 & \frac{1}{x_1} & -\frac{1}{x_1} & 0 \\ 0 & 1 & -\frac{x_3}{x_2} & 0 & \frac{1}{x_2} & -\frac{1}{x_2} \\ 0 & 0 & 1 & -\frac{L/x_1}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & -\frac{L/x_2}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & \frac{1+\frac{L}{x_1}+\frac{L}{x_2}}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} \end{bmatrix}$$

In this step, we already get the last row of the inverse matrix of $\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}}$. By the symmetry of the matrix and r_1, r_2, r_3, we will directly know the other rows of the inverse matrix, for example,

$$\left[\frac{Df(\mathbf{r},\mathbf{b},\gamma)}{D\mathbf{r}}\right]^{-1} = \begin{bmatrix} \frac{1+\frac{L}{x_2}+\frac{L}{x_3}}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} & -\frac{L/x_2}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} & -\frac{L/x_3}{x_1+L\left(1+\frac{x_1}{x_2}+\frac{x_1}{x_3}\right)} \\ -\frac{L/x_1}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} & \frac{1+\frac{L}{x_1}+\frac{L}{x_3}}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} & -\frac{L/x_3}{x_2+L\left(1+\frac{x_2}{x_1}+\frac{x_2}{x_3}\right)} \\ -\frac{L/x_1}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & -\frac{L/x_2}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} & \frac{1+\frac{L}{x_1}+\frac{L}{x_2}}{x_3+L\left(1+\frac{x_3}{x_2}+\frac{x_3}{x_1}\right)} \end{bmatrix}$$

Note that this matrix is symmetric.

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