ABSTRACT

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The current study aimed to determine the best method for estimating latent variable interactions as a function of the size of the interaction effect, sample size, the loadings of the indicators, the size of the relation between the first-order latent variables, and normality. Data were simulated from known population parameters, and data were analyzed using nine latent variable methods of testing for interaction effects. Evaluation criteria used for comparing the methods included proportion of relative bias, the standard deviation of parameter estimates, the mean standard error estimate, a relative ratio of the mean standard error estimate to the standard deviation of parameter estimates, the percent of converged solutions, Type I error rates, and empirical power. It was found that when data were normally distributed and the sample size was 250 or more, the constrained approach results in the least biased estimates of the interaction effect, had the most accurate standard error estimates, high convergence rates, and adequate type I error rates and power. However, when sample sizes were small and the loadings were of adequate size, the latent variable scores approach may be preferable to the constrained approach. When data were severely non-normal, all of the methods were biased, had inaccurate standard error estimates, low power, and high Type I error rates. Thus, when data were non-normal, relative comparisons were made regarding the approaches rather than absolute comparisons. In relative terms, the marginal-maximum likelihood approach performed the least poorly of the methods for estimating the interaction effect, but requires sample sizes of 500 or greater. However, when data were non-normal, the latent moderated structure analysis resulted in the least biased estimates of the first-order effects and had bias similar to that of the marginal-maximum likelihood approach. Recommendations are made for researchers who wish to test for latent variable interaction effects.

A COMPARISON OF METHODS FOR TESTING FOR INTERACTION EFFECTS IN STRUCTURAL EQUATION MODELING

By

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Dedication

Dedicated to my wonderful parents (Pamela Weiss and Dennis Weiss), my grandmother (Eleanor Hauck), my brother (Brandt Weiss), and my cousin (Brittany Davis), who have supported me in every decision I have made no matter how extravagant, poor, or eccentric it may have seemed to them at the time. Also dedicated to Mike Slackenerny who provided me with some much needed retrospective humor throughout my graduate studies, and taught me that if he could make it through graduate school then so could I.

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Chapter 1: Introduction

Interactions between continuous variables are frequently hypothesized in the social science literature. An interaction occurs when the relation between a predictor and a criterion variable changes across varying levels of a third variable (often referred to as a moderator variable). For example, the relation between ability and achievement is generally positive, however, it may change depending on the amount of effort students put forth. Specifically, one possible example could be as effort increases, the relation among ability and achievement becomes more positive. Figure 1 shows a graphical depiction of one possible relation between these three variables.

The relation between a predictor and a criterion variable can sometimes vary as a function of either a moderator or a mediator. These two types of third variables are used to specify different types of relations. A moderator variable is introduced when the causal relation between two variables is thought to change as a function of a third variable. A mediator variable is introduced when a predictor is thought to influence a criterion variable through a third variable (i.e., the predictor indirectly affects the criterion variable via a mediator). Moderators affect the direction and strength of the relation between a predictor and a criterion, while mediators account for the relation between the predictor and criterion (Baron & Kenny, 1986). The current study focuses only on the use of moderator variables, which is considered to be synonymous with an interaction in this context.

Methods for estimating interactions are frequently encountered in lower-level statistics courses in undergraduate and graduate studies. These methods include analysis of variance (ANOVA) and multiple regression techniques. When both the predictor and moderator variables are categorical, ANOVA is an acceptable choice of statistical analysis. When either the predictor or the moderator variable is continuous, then multiple regression analysis can be used to estimate the main effects and the interaction effect (Aiken & West, 1991; Cohen, Cohen, Aiken, & West, 1983). The regression equation for estimating interaction effects can be written as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + e, \tag{1}$$

where Y represents the criterion variable, β_0 represents the y-intercept, β_1 and β_2 represent main effects, X_1 represents a predictor variable, X_2 represents a moderator variable, $X_1 X_2$ represents the interaction between the predictor and the moderator variable, β_3 represents the interaction effect, and e represents the residual.

When using ANOVA and multiple regression to examine interaction effects, one assumption that is often overlooked is the assumption that the variables are assumed to be measured without error. Contrarily, most social science researchers hypothesize interactions between latent variables, which contain measurement error. Therefore, when interactions are hypothesized between latent variables, latent variable analyses (such as structural equation modeling (SEM) are more appropriate than the traditional measured variable analyses.

The LISREL specification for the structural portion of a model in which two exogenous variables interact with one another can be written as

$$\eta = \alpha + \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_1\xi_2 + \zeta \,, \tag{2}$$

where η is an endogenous latent variable, ξ_1 and ξ_2 are first-order exogenous latent variables, $\xi_1\xi_2$ is a latent interaction term, α represents an intercept term, γ_1, γ_2, and γ_3 are the direct path estimates, and ζ represents the latent residual.

Limitations of Previous Work

Numerous approaches of testing for latent variable interactions within a structural equation modeling framework exist. These methods typically fall into one of three categories: product indicator methods, ordinary-least-squares regression (OLSR) based methods, or a “new generation” of methods. Each of these categories of methods has advantages and limitations (discussed in detail in Chapter 2).

A number of researchers have conducted simulation studies to compare some of these methods. Three problems exist with previous studies. First, they have focused on comparing a small subset of the latent variable methods available for testing for interaction effects rather than examining the wide array of approaches that have been proposed. Second, there has been a lack of consistency in the specification of these methods across studies. Therefore, even when researchers have compared multiple approaches within a single study, there are discrepancies among the method specifications across studies. Third, these studies do not compare methods across all the three distinct categories of methods that have been described. Some recent studies have compared some product-indicator methods with one or two “new generation” methods. However, the OLSR-based methods have only been compared to methods in the other two categories in theory or based on a single sample and have not been compared in simulation studies.

The Current Study

The current study aimed to determine the best method for estimating latent variable interactions as a function of the size of the interaction effect, the size of the relation between the first-order latent variables, sample size, the loadings of the indicators, and normality. Data were simulated from known population parameters and datasets were tested across nine latent variable methods of testing for interaction effects. Based on the results, recommendations are made for researchers in deciding which method should be used in applied studies.

Chapter 2: Literature Review

The Evolution of the Product-Indicator Models

The Kenny-Judd model. Kenny and Judd (1984) were the first to propose a fully-latent approach for estimating interactions between continuous latent variables. In their model (referred to as the Kenny-Judd model) the main effects and interaction effects on a measured variable y can be conveyed as

$$y = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_1\xi_2 + \zeta \,, \qquad (3)$$

in which all measured variables (*y*- and *x*-indicators) are mean centered, and where γ_1, γ_2, and γ_3 are the regression coefficients, ξ_1 and ξ_2 are exogenous latent variables, $\xi_1\xi_2$ is the latent interaction term between ξ_1 and ξ_2, and ζ is the residual. Because the measured variables were mean centered, the intercept term was thought to be equal to zero and consequently omitted from the equation.

In their model, Kenny and Judd (1984) used two indicators for the latent variables ξ_1 and ξ_2. In order to compare the Kenny-Judd model to other models that will be presented, the current paper will use three indicators for each of the exogenous latent variables. The measurement portion of the six observed variables for ξ_1 and ξ_2 can be shown as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{x1} & 0 \\ \lambda_{x2} & 0 \\ \lambda_{x3} & 0 \\ 0 & \lambda_{x4} \\ 0 & \lambda_{x5} \\ 0 & \lambda_{x6} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}, \tag{4}$$

in which all measured variables are mean centered. In the Kenny-Judd model, the indicators for the interaction term, $\xi_1\xi_2$, were created by using all possible products of the measured variables for ξ_1 and ξ_2. In the case of our three indicator model this would yield nine possible indicators for the interaction term. The measurement portion for these nine indicators for the interaction term can be shown as

$$\begin{bmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{bmatrix} = \begin{bmatrix} x_1x_4 \\ x_1x_5 \\ x_1x_6 \\ x_2x_4 \\ x_2x_5 \\ x_2x_6 \\ x_3x_4 \\ x_3x_5 \\ x_3x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{x7} \\ \lambda_{x8} \\ \lambda_{x9} \\ \lambda_{x10} \\ \lambda_{x11} \\ \lambda_{x12} \\ \lambda_{x13} \\ \lambda_{x14} \\ \lambda_{x15} \end{bmatrix} \begin{bmatrix} \xi_1\xi_2 \end{bmatrix} + \begin{bmatrix} \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \\ \delta_{12} \\ \delta_{13} \\ \delta_{14} \\ \delta_{15} \end{bmatrix}, \tag{5}$$

It is not necessary, however, to use all possible products of the measured variables to create indicators for the interaction. In order for the model to be identified, only one product variable is necessary (Jöreskog &Yang, 1996). Several methods for creating indicators for the latent interaction variable have been suggested. A discussion regarding methods of creating indicators will follow later. For comparison purposes, the current paper will use a matched pairs approach in which one measured variable indicator for ξ_1 will be paired with another measured variable indicator for ξ_2 (Marsh, Wen, & Hau, 2004; Marsh, Wen, & Hau, 2006). Using the matched pairs approach with our three indicator model we would end up with three indicators for the interaction term, such as

$$\begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1 x_4 \\ x_2 x_5 \\ x_3 x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{x7} \\ \lambda_{x8} \\ \lambda_{x9} \end{bmatrix} \begin{bmatrix} \xi_1 \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix}, \tag{6}$$

If we put the equations for all of the indicators of the latent exogenous variables together from Equations 4 and 6, this corresponds to

$$\mathbf{x} = \mathbf{\Lambda_x} \quad \boldsymbol{\xi} + \boldsymbol{\delta} \ . \tag{7}$$

When first introducing the fully-latent approach to estimating latent variable interactions, Kenny and Judd limited their model to effects on a measured variable *y*. It is more frequently the case, however, that researchers wish to test for interaction effects on a latent endogenous variable, η. Hayduk (1987) was the first to expand the Kenny-Judd model to using a latent endogenous variable, η. The LISREL specification for the structural portion of this model can be written as

$$\eta = \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_1 \xi_2 + \zeta \ , \tag{8}$$

For the latent endogenous variable model, the measurement portion of the model would be expanded to include

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \lambda_{y1} \\ \lambda_{y2} \\ \lambda_{y3} \end{bmatrix} [\eta] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}, \tag{9}$$

Constraints. The Kenny-Judd approach imposed several types of constraints upon the model. First, the loadings of the indicators on the interaction term were constrained to be equal to the product of the loadings for the two indicators that created the interaction indicator. That is,

$$\begin{aligned} \lambda_{x7} &= \lambda_{x1}\lambda_{x4}, \\ \lambda_{x8} &= \lambda_{x2}\lambda_{x5}, \\ \lambda_{x9} &= \lambda_{x3}\lambda_{x6}, \end{aligned} \tag{10}$$

This constraint was imposed because the loadings of the product terms are functions of the first-order indicators that created them. For example, algebraically from Equation 4 we can write x_2 and x_5 as

$$x_2 = \lambda_{x2}\xi_1 + \delta_2, \tag{11}$$

$$x_5 = \lambda_{x5}\xi_2 + \delta_5, \tag{12}$$

Using Equations 11 and 12, the product term, $x_2\, x_5$, can be written as

$$x_2 x_5 = \left(\lambda_{x2}\xi_1 + \delta_2\right)\left(\lambda_{x5}\xi_2 + \delta_5\right), \tag{13}$$

then we can rewrite Equation 13 to be

$$x_2 x_5 = \lambda_{x2}\lambda_{x5}\xi_1\xi_2 + \delta_8, \tag{14}$$

where

$$\delta_8 = \lambda_{x2}\xi_1\delta_5 + \lambda_{x5}\xi_2\delta_2 + \delta_2\delta_5, \tag{15}$$

Equation 14 shows that the loading of $x_2\ x_5$ on the latent interaction term $\xi_1\xi_2$ is equal to $\lambda_{x2}\lambda_{x5}$, which is why this type of constraint is reasonable.

Second, assuming that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, and ζ are in mean-deviation form, multivariate normal, and uncorrelated (except ξ_1 and ξ_2 are allowed to relate to one another), then the variance of the interaction latent variable $\xi_1\xi_2$ can also be constrained. Constraining the variance of the interaction we get

$$\phi_{33} = \mathrm{var}\left(\xi_1\xi_2\right) = \mathrm{var}\left(\xi_1\right)\mathrm{var}\left(\xi_2\right) + \mathrm{cov}^2\left(\xi_1, \xi_2\right) \tag{16}$$

Thus, based on Equation 16, the variance of the latent variable interaction was set equal to the product of the variances of ξ_1 and ξ_2 plus the squared covariance between ξ_1 and ξ_2, and can be written as

$$\phi_{33} = \phi_{11}\phi_{22} + \phi_{21}^2 \tag{17}$$

Based on the normality assumption, the covariance between the interaction term and each of the first-order terms were set to zero (i.e., $\phi_{31} = \phi_{32} = 0$). This type of constraint will be referred to as the normality constraint for the present study.

Under these same assumptions that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, and ζ are in mean-deviation form, multivariate normal, and uncorrelated (except ξ_1 and ξ_2 are allowed to covary), the errors of the each of the indicators for the interaction latent variable were constrained based on Equation 15 such that

$$\theta_{\delta 8} = \mathrm{var}\left(\delta_8\right) = \mathrm{var}\left(\lambda_{x2}\xi_1\delta_5 + \lambda_{x5}\xi_2\delta_2 + \delta_2\delta_5\right), \tag{18}$$

Therefore, the errors of each of the indicators for the interaction latent variable were equal to

$$\begin{aligned} \theta_{\delta 7} &= \lambda_1^2\phi_{11}\theta_{\delta 4} + \lambda_4^2\phi_{22}\theta_{\delta 1} + \theta_{\delta 1}\theta_{\delta 4}, \\ \theta_{\delta 8} &= \lambda_2^2\phi_{11}\theta_{\delta 5} + \lambda_5^2\phi_{22}\theta_{\delta 2} + \theta_{\delta 2}\theta_{\delta 5}, \\ \theta_{\delta 9} &= \lambda_3^2\phi_{11}\theta_{\delta 6} + \lambda_6^2\phi_{22}\theta_{\delta 3} + \theta_{\delta 3}\theta_{\delta 6}, \end{aligned} \tag{19}$$

Mean structure. In order to simplify the derivation of the latent product variances and covariances, Kenny and Judd (1984) choose to mean center the observed variables in their model. However, Jöreskog and Yang (1996) argued that even if the observed variables were mean centered, their products would not necessarily be mean centered. This has two consequences on the specification of latent interaction model: 1) the latent interaction variable, $\xi_1\xi_2$, will also not be mean centered, and thus mean structure is necessary, and 2) the intercept, α, will not necessarily be zero.

The former implies that mean structure must always be used when specifying the latent interaction model. Under the assumption that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, and ζ are in mean-deviation form, multivariate normal, and uncorrelated (except ξ_1 and ξ_2 are allowed to covary), Jöreskog and Yang (1996) noted that the mean of the interaction term would be equal to the covariance between ξ_1 and ξ_2 and thus a fourth constraint is imposed upon the model such that

$$\kappa_3 = \phi_{21}, \tag{20}$$

where κ_3 represents the mean of the endogenous latent variable η, and ϕ_{21} represents the covariance between the first-order latent exogenous latent variables. Consequently, the Kenny-Judd model (without mean structure) is only appropriate when the covariance between ξ_1 and ξ_2 is approximately zero. Figure 2 contains a graphical depiction corresponding to the model described so far.

The intercept. The second implication of the non-centered product indicators is that the intercept term, α, will not necessarily equal zero. Jöreskog and Yang (1996) pointed out that although it is tempting to set α equal to zero, there is no way of knowing what its value actually is. The value of α impacts the values of γ_3 and ζ. Consequently, it should not necessarily be omitted from the structural model. Therefore, Equation 2, rather than Equation 8, should represent the structural portion of the model.

Mean centering. When interactions are tested for in a multiple regression context, the interactions are created by taking the product of the predictor and the moderator variables. Adding the interaction into the multiple regression equation could introduce a high amount of multicollinearity into the equation. Multicollinearity could potentially lead to computational difficulties estimating the regression coefficients (Aiken & West, 1991; Cohen, Cohen, West, & Aiken, 2003). Mean centering predictor variables before conducting an analysis circumvents problems that may be caused by multicollinearity between the interaction term and the first-order predictors. Mean centering involves a linear transformation that changes the means of the variables to be equal to zero, but does not change the standard deviation of the variables. Mean centering does not change the relation between the predictor variables, nor does it change the regression coefficient for the interaction. Thus, when interactions are tested using multiple regression, predictors are usually mean centered before conducting the statistical analysis.

When Kenny and Judd (1984) proposed their method for testing for interaction effects using structural equation modeling, the concept of mean centering was carried over from the measured variable world to the latent world. Jöreskog and Yang (1996), however, suggested that mean centering was not necessary in the latent world. This algrabraically changes the measurement portion of the model to include means for the observed latent variables. Thus the measurement portion for the observed indicators of the first-order latent variables of the Jöreskog and Yang (1996) model that corresponds to Equation 2 is

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \tau_{y1} \\ \tau_{y2} \\ \tau_{y3} \end{bmatrix} + \begin{bmatrix} \lambda_{y1} \\ \lambda_{y2} \\ \lambda_{y3} \end{bmatrix} \eta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \tau_{x1} \\ \tau_{x2} \\ \tau_{x3} \\ \tau_{x4} \\ \tau_{x5} \\ \tau_{x6} \end{bmatrix} + \begin{bmatrix} \lambda_{x1} & 0 \\ \lambda_{x2} & 0 \\ \lambda_{x3} & 0 \\ 0 & \lambda_{x4} \\ 0 & \lambda_{x5} \\ 0 & \lambda_{x6} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}, \tag{21}$$

where τ_{y1} through τ_{y3} represent the means of the observed *y* variables, and τ_{x1} through τ_{x6} represent the means of the observed *x* variables. Equation 21 can be used to derive the product variable x_1x_2 such that

$$x_1x_4 = \left(\tau_{x1} + \lambda_{x1}\xi_1 + \delta_1\right)\left(\tau_{x4} + \lambda_{x4}\xi_2 + \delta_4\right), \tag{22}$$

Then Equation 22 can be rewritten to be

$$x_1x_4 = \tau_{x1}\tau_{x4} + \tau_{x4}\lambda_{x1}\xi_1 + \tau_{x1}\lambda_{x4}\xi_2 + \lambda_{x7}\xi_1\xi_2 + \delta_7, \tag{23}$$

where

$$\delta_7 = \tau_{x1}\delta_4 + \tau_{x4}\delta_1 + \lambda_{x1}\xi_1\delta_4 + \lambda_{x4}\xi_2\delta_1 + \delta_1\delta_4, \tag{24}$$

and

$$\lambda_{x7} = \lambda_{x1}\lambda_{x4}, \tag{25}$$

Using Equation 23, the measurement portion for the 3 matched-pairs indicators for the interaction can be written as

$$\begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1x_4 \\ x_2x_5 \\ x_3x_6 \end{bmatrix} = \begin{bmatrix} \tau_{x1}\tau_{x4} \\ \tau_{x2}\tau_{x5} \\ \tau_{x3}\tau_{x6} \end{bmatrix} + \begin{bmatrix} \tau_{x3}\lambda_1 & \tau_{x1}\lambda_4 & \lambda_{x7} \\ \tau_{x5}\lambda_2 & \tau_{x2}\lambda_5 & \lambda_{x8} \\ \tau_{x6}\lambda_3 & \tau_{x2}\lambda_6 & \lambda_{x9} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1\xi_2 \end{bmatrix} + \begin{bmatrix} \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix}, \tag{26}$$

Because using non-centered indicators changes the measurement portion of the model, the third constraint placed on the errors of each of the indicators for the interaction latent variable also changes to include the τ's. Under the assumption that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, and ζ are in mean-deviation form, multivariate normal, and uncorrelated (except ξ_1 and ξ_2 are allowed to covary), the errors are constrained based on Equation 22 to be

$$\begin{aligned} \theta_{\delta 7} &= \tau_1^2\theta_{\delta 4} + \tau_4^2\theta_{\delta 1} + \lambda_{x1}^2\phi_{11}\theta_{\delta 4} + \lambda_{x4}^2\phi_{22}\theta_{\delta 1} + \theta_{\delta 1}\theta_{\delta 4}, \\ \theta_{\delta 8} &= \tau_2^2\theta_{\delta 5} + \tau_5^2\theta_{\delta 2} + \lambda_{x2}^2\phi_{11}\theta_{\delta 5} + \lambda_{x5}^2\phi_{22}\theta_{\delta 2} + \theta_{\delta 2}\theta_{\delta 5}, \\ \theta_{\delta 9} &= \tau_3^2\theta_{\delta 6} + \tau_6^2\theta_{\delta 3} + \lambda_{x3}^2\phi_{11}\theta_{\delta 6} + \lambda_{x6}^2\phi_{22}\theta_{\delta 3} + \theta_{\delta 3}\theta_{\delta 6}, \end{aligned} \tag{27}$$

A summary of all four types of constraints with and without mean centering is shown in Appendix A

Correlated errors. One issue that is not explicitly addressed in the majority of previous studies that investigate interaction effects involves allowing the errors of observed variables to correlate. When product-indicator models are used, systematic variance is introduced into the data. Because the product-indicators are functions of the indicators of the observed variables that were used to create them, it is reasonable to assume they are related. Therefore, it makes sense to allow the errors of the product-indicators to correlate with the errors for the observed variables that created them. In order to model the shared error variance between the product-indicator variables and the observed variables used to create them, the Θ_δ matrix should be

$$\Theta_\delta = \begin{bmatrix} \theta_{\delta 1} & & & & & & & & \\ 0 & \theta_{\delta 2} & & & & & & & \\ 0 & 0 & \theta_{\delta 3} & & & & & & \\ 0 & 0 & 0 & \theta_{\delta 4} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{\delta 5} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{\delta 6} & & & \\ \theta_{\delta 71} & 0 & 0 & \theta_{\delta 74} & 0 & 0 & \theta_{\delta 7} & & \\ 0 & \theta_{\delta 82} & 0 & 0 & \theta_{\delta 85} & 0 & 0 & \theta_{\delta 8} & \\ 0 & 0 & \theta_{\delta 93} & 0 & 0 & \theta_{\delta 96} & 0 & 0 & \theta_{\delta 9} \end{bmatrix} \quad (28)$$

The constrained approach. Algina and Moulder (2001) extended the Jöreskog and Yang model by mean-centering the independently observed variables for exogenous ξ_1 and ξ_2. Algina and Moulder (2001) referred to this model as the “constrained” model, and found that it was more likely to converge, was less biased, and had better Type I error control than the Jöreskog and Yang (1996) uncentered model. Mean centering the observed indicators of ξ_1 and ξ_2 simplifies the measurement portion of the model because the means of the observed variables (i.e., the τ_x's) do not need to be specified in the model. This simplifies both the measurement equations for observed indicators of the exogenous latent variables and the correlations of the measurement errors for the exogenous variables. The LISREL specification for the measurement portion of the mean-centered constrained model can be written as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \lambda_{y1} \\ \lambda_{y2} \\ \lambda_{y3} \end{bmatrix} \eta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_1 x_4 \\ x_2 x_5 \\ x_3 x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{x1} & 0 & 0 \\ \lambda_{x2} & 0 & 0 \\ \lambda_{x3} & 0 & 0 \\ 0 & \lambda_{x4} & 0 \\ 0 & \lambda_{x5} & 0 \\ 0 & \lambda_{x6} & 0 \\ 0 & 0 & \lambda_{x7} \\ 0 & 0 & \lambda_{x8} \\ 0 & 0 & \lambda_{x9} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1 \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix},$$

$$\mathrm{K} = \begin{bmatrix} 0 \\ 0 \\ \kappa_3 \end{bmatrix}, \qquad \Phi = \begin{bmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix},$$

$$\Theta_\varepsilon = diag(\theta_{\varepsilon 1}, \theta_{\varepsilon 3}, \theta_{\varepsilon 3})$$

$$\Theta_\delta = \begin{bmatrix} \theta_{\delta 1} & & & & & & & & \\ 0 & \theta_{\delta 2} & & & & & & & \\ 0 & 0 & \theta_{\delta 3} & & & & & & \\ 0 & 0 & 0 & \theta_{\delta 4} & & & & & \\ 0 & 0 & 0 & 0 & \theta_{\delta 5} & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_{\delta 6} & & & \\ \theta_{\delta 71} & 0 & 0 & \theta_{\delta 74} & 0 & 0 & \theta_{\delta 7} & & \\ 0 & \theta_{\delta 82} & 0 & 0 & \theta_{\delta 85} & 0 & 0 & \theta_{\delta 8} & \\ 0 & 0 & \theta_{\delta 93} & 0 & 0 & \theta_{\delta 96} & 0 & 0 & \theta_{\delta 9} \end{bmatrix}$$

The Algina and Moulder constrained model imposed all four types of constraints discussed previously.

The partially constrained approach. The constraints specified in the constrained model are based on the assumption that ξ_1 and ξ_2 are normally distributed. Wall and Amemiya (2001) pointed out that when this assumption is not met then the covariance between ξ_1 and $\xi_1\xi_2$, and the covariance between ξ_2 and $\xi_1\xi_2$ are not necessarily zero (i.e., $\phi_{31} \neq 0$ and $\phi_{32} \neq 0$), and the constraint on the variance of $\xi_1\xi_2$ does not necessarily hold true (i.e., $\phi_{33} = \phi_{11}\ \phi_{22} + \phi_{21}^2$). Based on this premise, Wall and Amemiya (2001) proposed a generalized appended product indicator (GAPI) approach in which the second constraint was relaxed. This model is also referred to as the partially-constrained approach. In addition to relaxing the second constraint, the partially constrained approach relaxes the normality constraint by allowing $\xi_1\xi_2$ to covary with ξ_1 and ξ_2.

The unconstrained approach. Marsh et al. (2004) introduced an unconstrained model in which all constraints were released. Similarly to the partially-constrained model, this model allows $\xi_1\xi_2$ to covary with ξ_1 and ξ_2, and does not require the stringent assumption that ξ_1 and ξ_2 are normally distributed. Marsh et al. (2006) noted that this unconstrained model was much easier for researchers to implement than the constrained model because it does not necessitate the specification of nonlinear constraints.

Methods for creating product indicators. In their model, Algina and Moulder used a similar approach to Kenny and Judd (1984) for forming indicators for the interaction term, by using all possible products of indicators for ξ_1 and ξ_2 to form the indicators for the interaction term. In our three-indicator model this would yield nine indicators for the latent variable interaction (see Figure 3). In their model, Jöreskog and Yang (1996) used a single product to form an indicator for the interaction term (see Figure 4). In another study, Yang (1998) used a matched-pairs approach in which each indicator of ξ_1 was paired with another indicator of ξ_2. In the matched-pairs approach, each first-order indicator was used in only one product-indicator of the latent variable interaction. In our model with the three indicators for each of ξ_1 and ξ_2, this would yield only three indicators for the latent interaction term, $\xi_1\xi_2$.

Marsh, Wen, and Hau (2004) conducted a simulation study to compare these three methods and found that the matched-pairs method yielded the most precise parameter estimates. Based on this finding, two recommendations were made. First, researchers should use all information that is available (i.e., all observed variables that are indicators of ξ_1 and ξ_2 should be used to form the interaction indicators). Second, information should not be reused (i.e., once an observed variable has been used to form an indicator of the interaction term, that indicator should not be used to form a second indicator of the same interaction term). This second recommendation was made to avoid inducing correlations between the error variances of the indicators for ξ_1 and ξ_2, and $\xi_1\xi_2$.

The residual-centered approach. Little et al. (2006) proposed a residual-centered approach in conjunction with Marsh et al.'s (2004) unconstrained approach. The unconstrained residual-centered approach is a two-step process. Once all product indicators have been created, the first step is to regress all product indicators for the interaction onto all first-order indicators (i.e., not just the indicators for the variables used to create the product). The second step is to use the residuals from the first step as indicators for the interaction effect. This results in indicators for the interaction that are completely uncorrelated with all indicators for the main effects.

In their approach, Little, Bovaird, and Widaman (2006) used all possible products to form the indicators for the interaction. Additionally, they allowed the uniqueness of interaction indicators and their related first-order indicators to covary (i.e., correlated errors). Furthermore, they did not use mean-structure within their model.

Marsh et al. (2004) claimed that if mean-centering was used without specifying a mean-structure, then biased estimates could result. Initially, Marsh, Wen, Hau, Little, Bovaird, and Widaman (2007) thought that because Little et al. (2006) did not use mean-structure in their proposed model, biased parameter estimates could be problematic with the residual-centering approach utilized by Little et al. (2006). Marsh et al. (2007) conducted a study that compared two different models: an unconstrained mean-centered approach, and an unconstrained residual-centered approach. Their paper was a "constructive collaboration" effort to synthesize the unconstrained mean-centered approach with the residual-centered approach. They showed that assuming that ξ_1, ξ_2, ζ, and all errors for measured variables (i.e., δ's) have zero means and are uncorrelated (except that ξ_1 and ξ_2 are allowed to correlate), then the residuals used as indicators for the latent variable interaction in the residual-centered approach will also have zero means by definition of residual scores (Marsh et al., 2007). Therefore, the latent variable interaction will also have a mean of zero. Consequently, because ξ_1, ξ_2, and $\xi_1\xi_2$ all have zero means, η will also have a mean of zero, and thus a mean structure is not needed with the residual-centered approach.

Traditional Methods of Estimating Interaction Effects

Latent variable scores. Another method of testing for interactions between latent variables involves a two-step process that uses latent variable scores in a least squares regression analysis. Latent variable scores represent estimates of individuals' scores on an underlying latent factor. In the first step, latent variable scores for ξ_1, ξ_2, and η are computed. In the second step, the interaction term is created by multiplying the latent variables scores on ξ_1 with the latent variable scores on ξ_2. The latent variable scores are then used in a multiple regression analysis. Procedures for testing for interactions using multiple regression analyses can be found in Cohen, Cohen, West, and Aiken (2003).

One limitation of latent variable scores is that there is no unique solution for the latent variable scores. That is, there is more than one set of solutions that satisfy all necessary conditions for computing latent variable scores. This is known as the factor score indeterminacy problem (Loehlin, 2004). This problem occurs because there are more latent variables and errors of measurement being estimated than there are observed variables (Bollen, 1989). For example, consider a one-factor confirmatory factor model with three observed variables. In this model, we have four unknowns (i.e., one factor score and three error variables) but we only have three measurement model equations (i.e., one for each observed variable). Consequently, there are many possible solutions for the given factor score.

Another limitation of latent variables scores is that because latent variable scores only represent estimates of individuals' scores on an underlying latent factor, they contain measurement error (Bollen, 1989). If the advantage of using latent variable models to estimate interactions is that they remove measurement error, then why would one want to use an analysis that contains measurement error? Although estimating latent variable scores does not completely eliminate measurement error, it does reduce measurement error. Consequently, this approach for estimating interactions between latent variables is still advantageous over using observed scores within multiple regression analyses.

Because there is no one unique solution for estimating latent variable scores, there are several methods available for estimating latent variable scores. First, the least square regression method can be used to estimate an individual's score on a latent factor. This method is readily available in SPSS as the default option for creating factor scores, and it is frequently used within regression analyses. Additionally, Bartlett (as cited in Lastovicka & Thamodaran, 1991) described a method that also used a least squares procedure to estimate latent variable scores which minimizes the sum of squared values. The Bartlett method is also available as an option in SPSS. Anderson and Rubin (1956) extended upon Bartlett's method by forcing factor scores to be orthogonal. The Anderson and Rubin (1956) method is also beneficial in that the sample covariance matrix is exactly equal to the estimated factor covariance matrix (Yang, 1998). The Anderson and Rubin (1956) method of computing latent variable scores is available in SPSS, and is the method used in PRELIS to compute latent variable scores.

Several studies have been conducted to compare the various methods of estimating latent variable scores (e.g., Gorsuch, 1974; Lastovicka & Thamodaran, 1991). Lastovicka and Thamodaran (1991) conducted a parameter-recovery simulation study comparing the least squares regression method, Bartlett's method, Anderson and Rubin's method, and another method proposed by Thurstone (as cited in Lastovicka & Thamodaran, 1991). Additionally, Lastovicka and Thamodaran (1991) used an *ad hoc* procedure using a factor score extension proposed by Dwyer (1937), as well as the commonly utilized method of simply adding up person's responses on all variables (assuming they are coded in the same direction and on a common scale).

Similar results were found among the six estimation methods. The Dwyer extension method resulted in the closest recovery of multiple *R*, and had the lowest standard error of measurement associated with the regression beta weights. The Anderson and Rubin (1956) method resulted in the most accurate recovery of the beta weights, and had a comparable standard error of measurement associated with the regression beta weights to that of the Dwyer extension method. The method of simply summing the scores together resulted in the least accurate recovery of the beta weights, and had the highest standard error of measurement associated with the regression beta weights. The other three methods were somewhat comparable to each other.

Two-stage least squares (2SLS). Another method of testing for interactions between latent variables that involves ordinary least squares regression is the two-stage least squares (2SLS) method. Similarly to Equations 2 and 3, The LISREL specification for the structural portion of a model in which two exogenous variables interact with one another can be written as

$$y = \alpha + \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_1\xi_2 + \zeta \text{,} \tag{29}$$

where y is a measured variable, ξ_1 and ξ_2 are first-order exogenous latent variables, $\xi_1\xi_2$ is an interaction term, α represents an intercept term, γ_1, γ_2, and γ_3 are the direct path estimates, and ζ represents the residual.

For example purposes we will use three indicators for each of the first-order exogenous latent variables. Figure 5 depicts a graphical display of this model. The measurement equations for each of the observed variables are

$$x_1 = \xi_1 + \delta_1, \tag{30}$$
$$x_2 = \tau_{x2} + \lambda_{x2}\xi_1 + \delta_2, \tag{31}$$
$$x_3 = \tau_{x3} + \lambda_{x3}\xi_1 + \delta_3, \tag{32}$$
$$x_4 = \xi_2 + \delta_4, \tag{33}$$
$$x_5 = \tau_{x5} + \lambda_{x5}\xi_2 + \delta_5, \tag{34}$$
$$x_6 = \tau_{x6} + \lambda_{x6}\xi_2 + \delta_6, \tag{35}$$

where τ_{x2}, τ_{x3}, τ_{x5}, and τ_{x6} are intercept terms for Equations 31, 32, 34, and 35, respectively; the δ_i terms have means of zero, and are uncorrelated with ξ_1 and ξ_2 and each other. Equations 30 and 33 do not have τ_{xi} values because they were used to set the scale of their respective latent variables. Equations 22 and 25 can be reordered to solve for the latent variables such that

$$\xi_1 = x_1 - \delta_1, \tag{36}$$
$$\xi_2 = x_4 - \delta_4, \tag{37}$$

Now Equations 36 and 37 can be substituted into Equation 29 such that

$$y_1 = \alpha + \gamma_1 (x_1 - \delta_1) + \gamma_2 (x_4 - \delta_4) + \gamma_3 (x_1 - \delta_1)(x_4 - \delta_4) + \zeta, \tag{38}$$

and can be rewritten as

$$y_1 = \alpha + \gamma_1 (x_1) + \gamma_2 (x_4) + \gamma_3 (x_1)(x_4) + u_1, \tag{39}$$

where u_1 is a linear composite disturbance equal to

$$u_1 = -\gamma_1 \delta_1 - \gamma_2 \delta_4 - \gamma_3 (x_1 \delta_4 + x_4 \delta_1 - \delta_1 \delta_4) + \zeta, \tag{40}$$

Similarly to Equation 29, Equation 39 takes the form of a regression equation. Equation 39, however, involves only observed variables. Ordinary least squares regression is inappropriate for Equation 39 because x_1, x_4, and x_1x_4 will be correlated with u_i unless they are measured perfectly with no measurement error (Bollen & Paxton, 1998). This means that the ordinary least squares regression will lead to biased estimates of γ_1, γ_2, and

γ_3 (Bollen & Paxton, 1998; Jöreskog, Sörbom, du Toit, & du Toit, 2000). This bias can be either positive or negative, small or large (Jöreskog et al., 2000).

To overcome this problem, Bollen and Paxton (1998) introduced a two-step process called two-stage least squares. The first step of this method involves regressing each of the right-hand x variables in Equation 39 (i.e., in this case, x_1, x_4, and x_1x_4) onto a set of instrumental variables. Instrumental variables are observed variables that are correlated with predictors but are uncorrelated with the error in the regression equation (Bollen, 1996; Bollen & Paxton, 1998). In this particular example the instrumental values would be x_2, x_3, x_5, x_6, x_2x_5, and x_3x_6. Of note, any observed variable that was used to set the scale for a latent variable, or is a product of a variable that was used to give a latent variable scale, cannot be used as an instrumental variable because it will be correlated with u_1 and thus violates an ordinary least squares regression assumption. Consequently, in the current example x_1, x_4, and x_1x_4 cannot be used as instrumental variables in this first step.

The predicted values from each of these regressions are saved (i.e., x_1, x_4, and $x_1 x_4$). These predicted values are linear combinations of the instrumental variables, and thus are uncorrelated with the disturbance, u_1. In the second step each of the predicted values replaces its respected observed values in Equation 39. Then ordinary least squares regression can be used to estimate Equation 21.

The two-stage least squares method to estimating interaction effects has many advantages. First, it is easy to understand and is available in many statistical software programs, including SPSS and LISREL. Furthermore, it does not make any distributional assumptions about the latent exogenous variables. This in turn makes the two-stage least squares method an attractive option when observed variables are non-normally distributed.

There are also several disadvantages to the two-stage least squares method. First, the selection of observed variables used as scale indicators for the latent variables may lead to different results (Marsh et al., 2004). Second, the dependent variable, *y*, in Equation 19 is an observed variable. In practice, however, researchers may wish to use multiple indicators of the dependent variable. There is no way to use multiple indicators of the dependent variable with two-stage least squares regression. If one wishes to use multiple indicators of the dependent variable, step two would require a separate regression analysis to be run for each indicator of the dependent variable. This makes the two-stage least squares approach only a partially latent approach.

Modern Methods for Estimating Interaction Effects

One potential problem with the product-indicator models (e.g., constrained, partially constrained, unconstrained, and residual-centering approaches discussed above) is the requirement that researchers create product indicators to be used as indicators for the latent variable interaction. These product indicators can be viewed as artificially measured variables because they are not unique observed variables, instead they are created by the researcher and are thus ad hoc.

Violations of the normality assumption. Another potential problem with the product-indicator models involves the distributional assumptions imposed by the models. First, the constrained approach is based on the assumption that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, and ζ are multivariate normal, and uncorrelated (except ξ_1 and ξ_2 are allowed to relate to one another). When data are non-normal, then the constraints imposed upon the variance of the interaction term (constraint #2) and it's covariance with the first-order terms (normality constraint) do not hold, and thus the constrained approach is not appropriate.

Second, even when the indicators of the first-order latent variables, ξ_1 and ξ_2, are normally distributed (and thus ξ_1 and ξ_2 are also assumed to be normally distributed), the interaction is known to be non-normally distributed (Jöreskog & Yang, 1996). The product-indicator models use maximum-likelihood estimation which is based on the assumption that all indicators in the model are multivariately normally distributed. Because the indicators for the interaction are known to be non-normally distributed, this assumption is violated when maximum-likelihood is used.

One potential solution to this violation of multivariate normality has been to use weighted-least squares estimation (instead of maximum-likelihood estimation) with the product-indicator models. Weighted-least squares estimation is asymptotically distribution-free and therefore provides asymptotically correct standard errors for parameter estimates. Previous simulation studies have found that weighted-least squares estimation leads to biased parameter estimates when sample sizes are small and underestimates standard errors (Jöreskog & Yang, 1996; Schermelleh-Engel, Klein, & Moosbrugger, 1998). Many SEM software packages are able to provide users with robust standard error estimates that are corrected for non-normality. However, the parameter estimates obtained by the weighted-least squares estimation are still biased. Studies comparing weighted-least squares estimation to maximum-likelihood estimation have found that maximum-likelihood estimation leads to less biased results with small sample sizes than weighted-least squares estimation and is somewhat robust to non-normality at large sample sizes (Jöreskog & Yang, 1996; Schermelleh-Engel, Klein, & Moosbrugger, 1998).

Latent moderated structural equations. Another potential solution for the violation of multivariate normality when estimating the interaction effect is the latent moderated structural equations (LMS) method proposed by Klein and Moosbrugger (2000). The LMS approach is advantageous in that it does not require the creation of indicators for the interaction and recognizes the non-normal distribution of the interaction. The LMS method utilizes a mixture of multivariate normal distributions that are implied by the interaction model. The Expectation-Maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977) is used to compute maximum-likelihood parameter estimates.

The general structural equation for an interaction model using the LMS approach can be written as

$$\eta = \alpha + \mathbf{\Gamma}\xi + \xi'\mathbf{\Omega}\xi + \zeta \,, \qquad (41)$$

where η is a endogenous latent variable, α is an intercept term, $\mathbf{\Gamma}$ is a (1 x k) vector of coefficients, $\boldsymbol{\xi}$ is a (k x 1) vector of latent exogenous variables, $\mathbf{\Omega}$ is an upper triangular (k x k) matrix, and ζ is a disturbance term. In Equation 39, the structural parameters have been separated into two matrices, one containing the linear effects (i.e., $\mathbf{\Gamma}$) and one containing the non-linear effects (i.e., $\mathbf{\Omega}$).

In the case of the model with a single interaction, the **Ω** matrix is an upper triangular matrix, and is specified as

$$\Omega = \begin{bmatrix} 0 & \gamma_3 \\ 0 & 0 \end{bmatrix}, \qquad (42)$$

where γ_3 represents the interaction effect and is located in the upper triangular. Zeros are located on the diagonal because there are no quadratic effects in the current structural equation model. If one wanted to simultaneously estimate quadratic effects with the interaction effect, then the parameters on the diagonal could be freed.

Applying the general structural equation shown in Equation 39 to the case in which two exogenous latent variables interact and affect a single endogenous latent variable, the structural equation model can be written as

$$\eta = \alpha + \begin{pmatrix} \gamma_1 & \gamma_2 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \xi_1 & \xi_2 \end{pmatrix} \begin{bmatrix} 0 & \gamma_3 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \zeta , \qquad (43)$$

where η is an endogenous latent variable, α is an intercept term, ξ_1 and ξ_2 are first-order exogenous latent variables, γ_1, γ_1, and γ_3 are direct path estimates, and ζ is a disturbance term.

The measurement portion of this model can be written as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_{y2} \\ \lambda_{y3} \end{bmatrix} \eta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{x2} & 0 \\ \lambda_{x3} & 0 \\ 0 & 1 \\ 0 & \lambda_{x5} \\ 0 & \lambda_{x6} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}, \tag{44}$$

A graphical depiction of this model is shown in Figure 6.

The LMS method is based on the assumption that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3 are multivariately normally distributed. Additionally, it is assumed that δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3 have expected values of zero and are uncorrelated with ξ_1 and ξ_2. Finally, ζ has an expected value of zero and is assumed to be uncorrelated with ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3. In contrast to the product-indicator methods (i.e., the constrained, partially constrained, unconstrained, and residual-centered unconstrained methods), η is not assumed to be normal.

The elementary interaction model case (in which two exogenous latent variables interact and affect a single endogenous latent variable, and there are three indicators for each of ξ_1, ξ_2, and η) has a nine-dimensional indicator vector $(\mathbf{x}, \mathbf{y}) = (x_1, \ldots, x_6, y_1, \ldots, y_3)$ and can be represented as a finite mixture of multivariate normal distributions. The indicator **x** is assumed to be normally distributed, whereas indicator **y** is not assumed to be normally distributed because the product term $\xi_1\xi_2$ is in the structural equation. Thus linear and non-linear effects are separated and decomposed into independent random **z** variables using the Cholesky decomposition of the covariance matrix $\mathbf{\Phi}$. $\mathbf{z}$ is made up of vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ which represent the nonlinear and linear effects, respectively. From this, a continuous mixture of normal densities with $\mathbf{z_1}$ as the mixing vector can be derived. Then the partitioned mean vector and covariance matrix can be obtained. If an interaction exists, and thus γ_3 differs from zero, then the integral of the mixture cannot be solved analytically. In this case it is approximated by Hermite-Gaussian quadrature formulas of numerical integration, which are used to calculate mixture probabilities and mixture components (Klein & Moosbrugger, 2000).

The LMS method can be implemented using the software program Mplus (Muthén & Muthén, 1998-2005). One limitation of the LMS approach is that it is based on the assumption that indicators of first-order effects are normally distributed (Klein & Moosbrugger, 2000).

Quasi-maximum likelihood (QML). Klein and Muthén (2007) developed a Quasi-Maximum Likelihood (QML) approach to handle more complex models with multiple interaction and quadratic effects that could not be handled by the LMS approach. Like the LMS approach, the QML approach does not require researchers to create product-indicators of the latent variable interaction, no distributional assumptions of the interaction effect are made, and indicators of first-order effects are assumed to be normally distributed (Klein & Muthén, 2007). However, while the LMS approach utilizes a mixture of multivariate normal distributions, the QML approach utilizes a product of normally distributed and conditionally normally distributed distributions.

For a model in which two exogenous latent variables interact and affect a single endogenous latent variable, and there are three indicators for each of ξ_1, ξ_2, and η, the structural equation and the measurement portion of the model is the same as that used for LMS and is shown in Equations 41 and 42, respectively. The QML method is based on the same assumptions as the LMS method. That is, it is assumed that ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3 are multivariately normally distributed; δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3 have expected values of zero and are uncorrelated with ξ_1 and ξ_2; ζ has an expected value of zero and is assumed to be uncorrelated with ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, δ_5, δ_6, ε_1, ε_2, and ε_3; and η is not assumed to be normal.

In the QML method the nine-dimensional indicator vector $(\mathbf{x}', \mathbf{y}')'$ is transformed so that only the variable used to set the scale for η (in this case this would be y_1 with a loading set to 1.0) is non-normally distributed. Then the conditional mean and variance of the non-normal y_1 are derived. The conditional mean and variance are then used to develop the QML estimation procedure. Thus the non-normal density function $f(x, y)$ of indicator vector $(\mathbf{x}', \mathbf{y}')'$ is approximated by the non-normal density $f^*(x, y)$, which is a product of a normal and conditionally normally distributed densities. QML maximizes the quasi-log-likelihood function which is the log likelihood function based on the maximization of the non-normal density $f^*(x, y)$ (Klein & Muthén, 2007).

The QML approach is not available in any commercial software programs. However, a stand-alone unpublished software program, QML, is available by request (Klein, unpublished). Currently, the program is a time-limited prototype version, in which the numbers of indicators, latent exogenous variables, latent endogenous variables, and sample size is limited.

Two-stage method of moments (2SMM). Another approach in which no distributional assumption of the interaction effect is made, was proposed by Wall and

Amemiya (2000, 2003). In their two-stage method of moments (2SMM) approach a general polynomial structural equation is represented as

$$\eta = \boldsymbol{\alpha}' h(\boldsymbol{\xi}) + \zeta, \qquad (45)$$

in which η represents an endogenous latent variable, $\boldsymbol{\alpha}$ represents a (r x 1) vector of unknown coefficients, $\boldsymbol{\xi}$ represents a (k x 1) vector of latent variables, $h(\boldsymbol{\xi})$ represents a (r x 1) vector with each component being a pure mixed power of elements of $\boldsymbol{\xi}$, and ζ represents a disturbance term (Marsh et al, 2004). Equation 45 becomes Equation 2 when

$$\boldsymbol{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}',$$

$$h(\boldsymbol{\xi}) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1\xi_2 \end{bmatrix}' \qquad (46)$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}'$$

The 2SMM involves a two-stage process in which in the first stage the parameters of the measurement model are estimated using linear factor analysis. In the second stage, the conditional moments of the products of latent variables are calculated, and the method-of-moments procedure is used with the conditional moments to estimate the structural equation parameters.

In the first stage, confirmatory factor analysis is used to estimate the loadings of the measurement model in Equation 44 and the variances and covariances of the errors of the indicators. These are used to calculate latent variable scores for each individual using Bartlett's method. Then the variances of the estimation error of the latent variable scores are estimated for ξ_1, ξ_2, and η. Finally, the higher-order moments of **e** = (e_1, e_2, e_3)' are estimated. For a model in which two exogenous latent variables interact and affect a single endogenous latent variable the higher moments needed are

$$
\begin{aligned}
&E(e_1 e_2 e_3) = \mu_1^3 \\
&E(e_2^2 e_3) = \mu_2^3 \\
&E(e_2 e_3^2) = \mu_3^3 \\
&E(e_2^2 e_3^2) = \mu_1^4
\end{aligned}
\qquad (47)
$$

In the second stage the factor scores and errors obtained in the first stage are used to fit the structural model. To do this, Equation 2 needs to be rewritten as an errors-in-variables model

$$
\begin{aligned}
&\eta_i = \alpha + \gamma_1 \xi_1 + \gamma_2 \xi_{2i} + \gamma_3 \xi_{1i} \xi_{2i} + \zeta_i, \\
&\eta_i = \eta_i + e_{1i}, \\
&\xi_{1i} = \xi_{1i} + e_{2i}, \\
&\xi_{2i} = \xi_{2i} + e_{3i},
\end{aligned}
\qquad (48)
$$

Using the errors-in-variables model values, **M** and **m** must be found such that

$$E\left(\mathbf{M} \mid \mathbf{X_1} \ldots \mathbf{X_n}\right) = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(\mathbf{X_i'}\mathbf{X_i}\right),$$
$$E\left(\mathbf{m} \mid \mathbf{X_1} \ldots \mathbf{X_n}, \eta_i \ldots \eta_n\right) = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(\mathbf{X_i'}\eta_i\right), \quad (49)$$

where

$$\mathbf{X}_i = \left(1, \xi_{1i}, \xi_{2i}, \xi_{1i}\xi_{2i}\right), \quad (50)$$

The equation $\mathbf{M\Gamma} = \mathbf{m}$ can then be used to estimate $\mathbf{\Gamma}$ without bias. The final part of the second stage is to obtain the 2SMM estimator $\mathbf{\Gamma}' = \left(\alpha, \gamma_1, \gamma_2, \gamma_3\right)$ using

$$\mathbf{\Gamma} = \mathbf{M}^{-1}\mathbf{m}, \quad (51)$$

The 2SMM approach can also be used with more complex models that involve multiple interaction and polynomial effects. Similarly to the LMS and QML approaches, the 2SMM method is beneficial in that no assumption regarding the distribution of the interaction effect is made. Unlike the LMS and QML approaches, the 2SMM does not make distributional assumptions about ξ_1 and ξ_2. The 2SMM method is not currently available in any commercial software programs, although a version of the method is outlined by Wall and Amemiya (2003).

Marginal maximum likelihood (MML). Another approach which makes no distributional assumption regarding the latent variable interaction is the marginal maximum likelihood (MML) approach. While MML is not a new method, it was only recently introduced as an approach for testing for latent variable interaction effects by Cudeck, Harring, and du Toit (2009). The MML approach uses Gaussian-Hermite quadrature to approximate a multidimensional integral and compute the marginal distribution of the measurement model then uses the result to obtain maximum-likelihood estimates.

The general structural equation shown in Equation 2 can be rewritten as a function with one nonlinear and one linear latent variable (Jöreskog, 1998). For example, the regression of η on ξ_1 for a given ξ_2 can be written as

$$\eta = \left(\alpha + \gamma_2 \xi_2\right) + \left(\gamma_1 + \gamma_3 \xi_2\right)\xi_1 + \zeta \,, \tag{52}$$

In Equation 52, ξ_1 is the linear latent variable, and ξ_2 is the nonlinear latent variable. The measurement model in which three indicators represent each latent variable is similar to Equation 44 and can be rewritten as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{y1} & 0 & 0 \\ \lambda_{y2} & 0 & 0 \\ \lambda_{y3} & 0 & 0 \\ 0 & \lambda_{x1} & 0 \\ 0 & \lambda_{x2} & 0 \\ 0 & \lambda_{x3} & 0 \\ 0 & 0 & \lambda_{x4} \\ 0 & 0 & \lambda_{x5} \\ 0 & 0 & \lambda_{x6} \end{bmatrix} \begin{bmatrix} \eta \\ \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}, \tag{53}$$

$$\mathbf{z} \quad = \quad \mathbf{\Lambda_x} \quad \boldsymbol{\xi} \; + \; \boldsymbol{\delta}$$

Because ξ_2 is nonlinear, one cannot simply use algebra to integrate over ξ_2. The MML approach uses Guassian-Hermite quadrature in which an integral over a function of the type $u(t) = f(t)\exp(-t^2)$ is approximated by a sum

$$\int_t f(t)\exp(-t^2)df \approx \sum_{k=1}^{Q} w_k f(x_k), \qquad (54)$$

where w_k and x_k are the weights and nodes of the Hermite polynomial of degree Q (Cudeck et al., 2009; Harring et al., under review). The log-likelihood function with a sample of N observations, $\mathbf{y}_1, \ldots, \mathbf{y}_N$, can be written as

$$\ln L\left(\theta \mid \mathbf{y}_1, \ldots, \mathbf{y}_N\right) = \sum_{i=1}^{N} \ln f(\mathbf{y}_i), \qquad (55)$$

To estimate the structural model, the MML approach maximizes the log-likelihood function, shown in Equation 55, using any of several optimization techniques. The MML approach can be implemented in SAS using PROC NLMIXED. SAS uses the dual quasi-Newton algorithm as the default optimization technique for maximizing the log-likelihood function.

One limitation of the MML approach is that it becomes increasing slow to converge as the number of latent variables increases. Specifically, Harring et al. (under review) suggested that when the number of latent variables is greater than three or four the MML approach may be very slow to converge. In the current study the structural model shown in Equation 2 has one nonlinear term (i.e., a single interaction effect) and three latent variables. Therefore, using the MML approach to estimate Equation 2 seems feasible. However, the MML approach may be inappropriate for structural models containing multiple nonlinear effects.

Advantages and Limitations of Three Categories of Methods

Numerous approaches of testing for latent variable interactions within an SEM framework exist. These methods typically fall into one of three categories: product indicator methods, ordinary-least-squares regression (OLSR) based methods, or a "new generation" of methods.

The product-indicator methods (i.e., the constrained, partially-constrained, unconstrained, and residual-centered approaches) use products of observed variables as indicators of the latent variable interaction. They are advantageous in that they are fully-latent approaches, and can be implemented in some SEM software programs (e.g., LISREL). These product-indicator approaches are limited in that they can be computationally intensive to specify. This limitation particularly applies to the specification of the constraints on the errors of product indicators for the interaction term (constraint #3 in Appendix A). While the specification of these constraints is feasible when the number of indicators is small (e.g., 2 or 3 indicators per latent variable), researchers are still prone to make mistakes specifying them (Schumacker, 2002). The complication of specifying these constraints becomes more complex and infeasible as the number of observed indicators per latent variable increases.

Additionally, these product-indicator models are limited in that they necessitate the use of special SEM software programs that allow for the use of constraints, such as LISREL. Frequently utilized SEM software programs such as AMOS and EQS cannot be used with the constrained and partially constrained models because they do not allow for researchers to specify non-linear constraints. Furthermore, these models require researchers to alter their measurement model to fit their structural model by creating artificial observed variables (i.e., the product-indicators) to represent the latent variable interaction. Finally, these product-indicator models are typically used with maximum-likelihood estimation which is based on the assumption that the latent variables are multivariately normally distributed. The interaction effect, however, is known to be non-normally distributed (Jöreskog & Yang, 1996). Therefore, this assumption is violated when maximum-likelihood is used. Although, robust standard error estimates can be obtained, the parameter estimates will still be biased.

The OLSR-based methods (i.e., latent variable scores with moderated multiple regression, and the two-stage least squares approach) are often viewed as inferior because they are not considered to be fully latent approaches (i.e., they do not completely remove measurement error from the model). However, they are beneficial in that they are easy to understand, easy to implement, and readily available for practitioners in commonly utilized statistical software packages (e.g., SPSS, SAS, STATA, and LISREL). Furthermore, these methods are beneficial because least squares regression is not based on the assumption of multivariate normality as maximum-likelihood estimation is, thus the non-normality of the latent variable interaction can be incorporated into the model without violating any assumption (Schermelleh-Engel, Klein, & Moosbrugger, 1998).

More recently, a “new generation” of methods for estimating interactions between latent variables has evolved (e.g., latent-moderated structural equations, quasi-maximum likelihood, the two-stage method of moments, and marginal-maximum likelihood). These methods are beneficial in that they provide alternative approaches to estimating interaction effects that do not require the creation of product indicators. Consequently, researchers do not have to alter their measurement model to fit their structural model. This also makes these methods somewhat easier to specify in comparison to some of the product-indicator methods because they do not necessitate the specification of non-linear constraints. Additionally, these newer methods are not based on the assumption that the interaction effect is multivariately normally distributed (Schermelleh-Engel, Klein, & Moosbrugger, 1998).

One of the major disadvantages of these newer methods is that most of these methods are not currently available in commercial software programs (with the exception of the latent moderated structural equations (LMS) procedure which is available in Mplus), making it infeasible for researchers to use in practice. SAS PROC NLMIXED can be used as one-way to estimate the MML approach. Some of these methods are also limited in that they are based on the assumption that indicators of first-order effects are normally distributed (Klein & Moosbrugger, 2000).

Comparing the Methods

The current paper presented nine different proposed approaches for estimating latent variable interaction effects in structural equation modeling. With the numerous approaches available for testing for latent variable interactions, how is a researcher supposed to decide which method to use? A number of simulation studies have been conducted that compare some of the methods discussed (e.g., Algina & Moulder, 2001; Jaccard & Wan, 1995; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Little et al., 2006; Marsh et al., 2004; Moulder & Algina, 2002; Wall & Amemiya, 2003). Results from these studies have been mixed. A list of the simulation studies that have been conducted to compare the models discussed within the current paper is shown in Table 2.

Schermelleh-Engel et al. (1998) found that the 2SLS approach was relatively unbiased for standard error estimates, but it had low power to detect interaction effects and higher standard error of measurement than the constrained approach (with the normality constraint released) and the LMS approach. Similarly, Moulder and Algina (2002) also found that the 2SLS approach had low power to detect interaction effects and high standard error estimates. However, Moulder and Algina (2002) found that the 2SLS procedure resulted in biased estimates of the interaction effect in comparison to the constrained approach with- and without mean-centering.

Marsh et al. (2004) found similar results in terms of bias, and standard error estimates for the constrained, partially constrained, unconstrained, and QML approaches. They found that the constrained approach was slightly less biased than the partially constrained and unconstrained approaches. The QML approach had higher power to detect an interaction effect, however, it also had higher Type I error rates.

Klein and Muthén (2007) found that the QML approach was less biased in terms of parameter estimates and standard error estimates than the constrained, partially constrained, unconstrained, and LMS approaches. They also found that the QML approach had higher statistical power.

Only one study was found that compared the 2SMM approach to other types of approaches for testing for interaction effects. Wall and Amemiya (2003) found that the 2SMM and the partially constrained approaches resulted in similar bias to each other. They found that both the 2SMM and the partially constrained approaches were less biased than the 2SLS and the constrained approaches.

After a review of the literature, only one study was found that compared the latent variable scores approach using the Anderson and Rubin (1956) method to the constrained approach (without mean centering) of testing for interactions among latent variables (Schumacker, 2002). Schumacker (2002) generated data for a single sample and compared these two methods. Results indicated that the same estimates for γ_1 and γ_2 were found across the two methods. The estimate of γ_3 was slightly, but not notably, closer to the value in the population-generating model. Of interest, was the smaller standard error of measurement values associated with the estimation of γ_3 using the latent variable scores approach in comparison to the constrained approach. Based on these outcomes, Schumacker (2002) suggested that future research should be conducted to examine the differences of the standard errors associated with the parameter estimates of the interaction effect.

Little et al. (2006) conducted a simulation study to compare their unconstrained residual-centered approach with Marsh et al.'s (2004) unconstrained mean-centered approach. They found similar results using both methods. However, they did not use mean structure with the unconstrained mean-centering approach. Marsh et al. (2007) showed that because residuals are mean-centered, mean structure is not necessary when residual centering is used. However, Jöreskog and Yang (1996), showed that even when indicators for first-order terms are mean-centered, their products will not necessarily be mean-centered. Therefore, mean structure is always necessary with the product-indicator models. Thus, results based on the Little et al. (2006) study cannot be interpreted because they compared the residual-centered model to an unconstrained model which is known to be incorrect. Therefore, it is unknown how the unconstrained residual-centering approach compares to the unconstrained mean-centering approach with mean structure.

Limitations of Previous Studies

Although many studies have been conducted to compare the various approaches for testing for interaction effects, there has been a lack of consistency across these studies in recommending which method results in the least biased parameter estimates and has the most accurate standard error estimates. Therefore, it is still unclear as to which method should be recommended for researchers to use in applied studies. Additionally, eight problems exist with previous studies.

First, these studies do not compare methods across the three distinct categories of methods. Some recent studies have compared some product-indicator methods with one or two "new generation" methods. Klein and Moosburgger (2000) conducted the only study that compared methods across the three distinct categories. They compared the constrained, 2SLS, and LMS methods in a simulation study, but they only used one condition. Consequently, it is unknown how robust these methods are across factors such as effect size, sample size, size of loadings, and violations of normality. Table 2 shows the simulation studies that have been conducted to compare methods of testing for interaction effects and the methods that have been compared in each of those studies. Second, previous studies have focused on only comparing two to four methods rather than examining the wide array of approaches that have been proposed.

Third, there has been a lack of consistency in the specification of these methods across studies (e.g., mean-structure, centering, correlated errors for product indicators, methods of forming interaction indicators, and whether or not an intercept term in specified in the structural equation). Therefore, even when researchers have compared multiple approaches within a single study, there are discrepancies between the model specifications across studies.

Fourth, in regards to interaction effects, the latent variable scores approach has never been compared to any of the other methods using simulation studies. Simulation studies have compared the latent variable scores approach to several other methods for quadratic effects and have found that the latent variables scores approach may be promising (Weiss & Hancock, 2009; Harring, Weiss, & Hsu, under review). The only study that has compared the latent variable scores approach to other approaches for testing for interaction effects used a single sample (Schumacker, 2002).

Fifth, there was only one simulation study that compared the residual-centered unconstrained approach to the mean-centered unconstrained approach (Little et al., 2006). However, this study incorrectly specified the mean-centered unconstrained approach because mean structure was not used. Therefore, it is unknown how the unconstrained residual-centered approach compares to the unconstrained mean-centered approach with mean structure.

Sixth, only one study has been conducted to compare the 2SMM method to product-indicator methods for testing for interaction effects (Wall & Amemiya, 2003). They found that the 2SMM and the partially constrained approaches were less biased than the 2SLS and the Kenny-Judd model. However, the Kenny-Judd model is known to be incorrect because it does not use mean structure (Jöreskog & Yang, 1996). Additionally, they used the all-possible-products method to create indicators of the interaction term, which is known to be an inferior method compared to the matched-pairs approach (Marsh et al., 2004). Furthermore, the only feature that was manipulated in the Wall and Amemiya (2003) was sample size, meaning they only compared these four methods across three conditions. Thus, it is unknown how the 2SMM method compares to other methods when other conditions are manipulated such as: effect size, loading size, correlation between ξ_1 and ξ_2, and normality.

Seventh, the marginal maximum likelihood method has not been compared to other methods in simulation studies in the context of interaction effects. One study was conducted to compare the marginal maximum likelihood method to several other methods for testing for quadratic effects (Harring, Weiss, & Hsu, under review). Results from this study showed that when data for the first-order factor was normally distributed, the marginal maximum likelihood approach was less biased and had more accurate standard error estimates then the unconstrained, latent variable scores, and LMS approaches. It is unknown whether this will be true for interaction effects.

Finally, previous studies have only investigated the impact of mild non-normality on estimating interaction effects (Marsh et al., 2004; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; and Wall & Amemiya, 2001). Specifically, for the non-normal conditions within these studies, data were generated for ξ_1 and ξ_2 from distributions with skew ranging from -2.0 to 1.5, and kurtosis ranging from -1.5 to 6.0. Kline (2005) suggested that extreme skew is defined by skew values greater than an absolute value of 3.0, and extreme kurtosis is defined by absolute kurtosis values ranging from 8.0 to over 20.0. He further suggested that kurtosis values greater than the absolute value of 20.0 may indicate serious problems with non-normality. Based on Kline's rule-of-thumb values for skew and kurtosis, the skew and kurtosis values for previous studies have been considered to be mild.

The current study aimed to explore similarities and differences among the methods of testing for interaction effects discussed: the constrained, partially-constrained, unconstrained, residual-centered unconstrained, latent variable scores using moderated multiple regression, two-stage least squares, latent moderated structural equations, quasi-maximum likelihood, two-stage method of moments, and the marginal-maximum likelihood approaches.

Chapter 3: Methods

The current study aimed to explore similarities and differences among the following methods of testing for interaction effects in structural equation modeling: the constrained approach, the partially-constrained approach, the unconstrained approach, the residual-centering approach, the latent variable scores with moderated multiple regression, two-stage least squares, latent moderated structural analysis, two-stage method of moments, and marginal-maximum likelihood approaches.

The goal of the current study was to compare all current methods of testing for interaction effects, including the quasi maximum likelihood approach (QML). The QML approach is not available in any commercial software programs. However, a stand-alone unpublished software program, QML, is available by request (Klein, unpublished). Currently, the program is a time-limited prototype version, in which the numbers of indicators, latent exogenous variables, latent endogenous variables, and sample size is limited. Additionally, the version of the QML software program that was provided to us by Klein is limited in that in can only be used with single samples, and therefore cannot be used in simulation studies. Sometimes when a software program can only be used with single datasets, DOS can be used to run the statistical software (Gagné & Furlow, 2009). In this manner, a DOS batch file can be used to automate the process of running analyses for multiple datasets. For the current study, DOS was used to call the QML prototype version, however, the attempt was unsuccessful. DOS was able to open the QML program, but would not open or run the input file for QML. This same procedure was used to successfully run the analyses using LISREL for the constrained, partially constrained, unconstrained, residual-centered, and latent variable scores approaches, as well as to successfully run the analyses using Mplus for the LMS approach. Thus, it was concluded that the problem existed with the currently available version of QML. Because it would be nearly impossible to analyze the 54,000 datasets used in the current study with QML one-at-a-time, QML was not used for the current study.

Simulation Design

The different methods of estimating latent variable interaction effects were compared using Monte Carlo simulation. Monte Carlo simulation empirically generates random samples from known populations (Mooney, 1997). By generating many random samples of data, one can monitor the behavior of a fit statistic across varying data conditions (e.g., differing numbers of manifest and latent variables, effects sizes, and sample sizes). All variables were simulated to come from a population in which

$$\eta = 0.4\xi_1 + 0.4\xi_2 + \gamma_3\xi_1\xi_2 + \zeta \,, \tag{56}$$

where ξ_1 and ξ_2 are standard normal variables. Thus, while the γ_1 and γ_2 paths were set equal to 0.4 based on values used by Marsh et al. (2004), the γ_3 path and ζ varied depending on the magnitude of the interaction effect. As stated by Marsh et al. (2004), varying the values of γ_1 and γ_2 will not affect the estimation of the interaction effect. This is because the latent interaction effect is uncorrelated with the latent first-order effects (Algina & Moulder, 2001; Jaccard & Wan, 1995; Jöreskog Yang, 1996; Kenny & Judd, 1984; Marsh et al., 2004; Schermelleh-Engel et al., 1998; Wall & Amemiya, 2001) Thus these values were not altered in the current study.

The effect size represents the additional variance that the interaction effect explains in η above and beyond that which can be explained by the first-order effects, and is equal to the value expressed by (Marsh et al., 2004)

$$R^2_{\gamma_3} = \frac{\gamma_3^2(\phi_{11}\phi_{22} + \phi_{12}^2)}{\sigma_\eta^2} \quad (57)$$

Jaccard and Wan (1995) did a review of the social science literature and found that interaction effect sizes typically accounted for 5% and 10% of the variance in the dependent variable. Several other studies found that interaction effects accounted for 3% to 8% of the variance in the dependent variable in multiple regression analyses (Champoux & Peters, 1987; Chaplin, 1991). Table 3 shows the previous studies that have been conducted to compare methods of testing for latent variable interaction effects. The $R^2_{\gamma_3}$ values used by Jaccard and Wan (1995) are similar to the values that have been used in other studies investigating interaction effects (Little et al., 2006; Klein & Muthén, 2007; Marsh et al., 2004; Moulder & Algina, 2002). These values are also typical of what have been used in previous studies investigating quadratic effects (Harring, Weiss, Hsu, under review; Weiss & Hancock, 2009). The current study investigated similar effect sizes for interaction effects in which the proportion of variance in η accounted for by the interaction effect was set equal to .0 (to investigate Type I error rates), .05, and .10 (to investigate power).

Three different sample sizes were used in the current study (small, medium, and large) which corresponded to n = 100, 250, and 500, respectively. These sample sizes were chosen for several reasons. First, past simulation studies investigating interactions between latent variables have used similar (but not exactly the same) sample sizes (Jaccard & Wan, 1995; Klein & Muthén, 2007; Marsh et al., 2004; Moulder & Algina, 2002; Schermelleh-Engel et al., 1998). Second, Hu and Bentler (1998; 1999) found that RMSEA and SRMR fit indices tend to over-reject true population models with small samples and recommended using samples greater than 250. Based on this finding researchers may aim to have complete data for at least 250 cases. Third, Wall and Amemiya (2003) evaluated methods using sample sizes as large as 1000. However, little difference was found in the bias of parameter estimates between sample sizes of 500 and 1000. Fourth, Little et al. (2006) used a large sample size of 1500 in their study and found that their residual-centered approach performed similarly to the unconstrained approach (without mean structure).

Previous simulation studies using smaller sample sizes with quadratic effects have found that the residual-centered approach did not perform well with small sample sizes (Weiss & Hancock, 2009). This has not been evaluated with interaction effects yet. Because the OLS regression approaches are not based on iterative processes with convergence criterions, they can be used with small samples. Thus, a sample size of 100 was used in the current study in order to investigate whether the OLS regression methods were less biased at small sample sizes than the product-indicator and newer generation methods.

The loadings relating each indicator to its latent variable were also manipulated. Based on past research the loadings were set to either 0.5 (constant across all indicators) or 0.8 (constant across all indicators) within the population-generating model. The loading of 0.5 was selected to investigate the impact that measurement error had on estimating the structural parameters. Although measurement error is taken into account by the models, parameter estimates will be more accurate when indicators are more psychometrically sound (Kline, 2005). Thus using low loadings allows researchers to evaluate the methods under reasonably difficult conditions (Klein & Muthén, 2007). The loading of 0.8 was selected to represent adequate loading size and is comparable to what has been used in previous studies (Jaccard & Wan, 1995; Klein & Muthén, 2007; Little et al., 2006; Marsh et al., 2004; Moulder & Algina, 2002; Schermelleh-Engel et al., 1998).

The correlation between the two first-order latent variables ξ_1 and ξ_2 was also varied. Jaccard and Wan (1995) conducted a review of social science literature and found correlation values of .20 and .40 were typically observed. Previous studies used similar values in the .20 to .40 range (see Table 3; Jaccard & Wan, 1995; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Little et al, 2006; Marsh et al., 2004; Moulder & Algina, 2002; Schermelleh-Engel et al., 1998).

In addition to the values .20 and .40, the current study manipulated the correlation between the two first-order latent variables ξ_1 and ξ_2 to be .60. When first-order latent variables are strongly related, the standard errors associated with the gamma estimates will become very large (Cohen et al., 2003). Thus, for the current study a larger value for ϕ_{12} was selected to investigate the robustness of the standard errors when ϕ_{12} was high. Therefore, the current study manipulated ϕ_{12} to be either .20, .40, or .60 in the population-generating model (given that the latent variables have unit variance and ϕ_{11} = ϕ_{22} = 1).

Finally, the distributions of ξ_1 and ξ_2 were manipulated to be either normal or severely non-normal. The distributions at the indicator level were not manipulated. This decision was based on the premise that if latent variables are non-normally distributed, then indicators formed from them would also be non-normally distributed, and is consistent with previous studies (Klein & Moosbrugger, 2000; Klein & Muthén, 2007). The distributions of the errors were also not manipulated. The normality of the errors does not effect the structural relations between the latent variables. Because the focus of the current study was on the estimation of the structural interaction effect, it was not necessary to manipulate the distribution of the errors.

When ξ_1 and ξ_2 are non-normal, the second constraint on the variance of $\xi_1\xi_2$ and the normality assumption (i.e., ϕ_{31} = ϕ_{32} = 0) do not hold true. Consequently, the constrained approach should result in systematically biased parameter estimates. Furthermore, because $\xi_1\xi_2$ is known to be non-normally distributed, product-indicator methods (i.e., the constrained, partially constrained, unconstrained, and residual-centered unconstrained methods) that use maximum likelihood estimation may result in biased parameter estimates. The LMS approach allows for the non-normal distribution of $\xi_1\xi_2$. However, the LMS approach is still based on the assumption that ξ_1 and ξ_2 are normally distributed. Consequently, when ξ_1 and ξ_2 are non-normal the LMS approach may lead to biased parameter estimates. Unlike the LMS approach, the 2SMM approach does not require the assumption that ξ_1 and ξ_2 are normally distributed. Therefore, it may be less biased than the other approaches when ξ_1 and ξ_2 are severely non-normally distributed.

While a small number of simulation studies have investigated the impact of mild non-normality on estimating interaction effects (Marsh et al., 2004; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Wall & Amemiya, 2001), none of these studies have investigated the impact of severe non-normality on estimating interaction effects. Most of these studies generated data using either a uniform distribution, chi-square (df=6) distribution, or a chi-square (df=9) distribution to simulate non-normal data for ξ_1 and ξ_2 (Marsh et al., 2004; and Wall & Amemiya, 2001). Using distributions such as these generates data with skew ranging from 0 to 1.15 and kurtosis ranging from -1.5 to 2.0. Two studies used slightly more extreme values with skew values as large as -2 and kurtosis as large as 6 (Klein & Moosbrugger, 2000; Klein & Muthén, 2007). Kline (2005) suggested that extreme skew is defined by values greater than an absolute value of 3.0, and extreme kurtosis is defined by absolute values ranging from 8.0 to over 20.0. He further suggested that kurtosis values greater than the absolute value of 20.0 may indicate serious problems with non-normality. Based on Kline's rule-of-thumb values for skew and kurtosis, the skew and kurtosis values for previous studies have not been considered to be extreme. Consequently, the current study manipulated the distribution of ξ_1 and ξ_2 to be either normal, or severely non-normal with skew of 3.0 and kurtosis of 22.0. For the severely non-normal condition, data were generated using Fleishman's (1978) polynomial transformation procedure with Vale and Maurelli's (1983) intermediate correlation procedure.

In summary, the following features were manipulated in a fully-crossed factorial design: a) the magnitude of the interaction as represented by the amount of variance that the interaction explains in η above and beyond the first-order latent variables ($R^2_{\gamma_3}$ = .00,

.05, or .10); b) sample size (n = 100, 250, or 500); c) factor loadings on first-order latent variables (0.5 or 0.8); d) the correlation between the two first-order latent variables (ϕ_{12} = .2, .4, or .6); and e) the normality of the first-order latent variables (normal or skew=3, kurtosis=22). Nine methods were used to analyze each of the datasets as a within-design method. Based on the manipulations of these five features, a 3 x 3 x 2 x 3 x 2 x (9) factorial design was utilized. This resulted in 108 conditions across 9 different methods of estimating latent variable interaction effects. A summary of the manipulated features is shown in Table 4.

The variances of ξ_1 and ξ_2 were set equal to 1 (i.e., $\phi_{11} = \phi_{22} = 1$). The variance of the interaction term varied depending on the correlation between the two first-order latent variables ξ_1 and ξ_2 (i.e., ϕ_{12}). Specifically, the variance of the interaction term was set equal to $\phi_{33} = \phi_{11}\,\phi_{22} + \phi_{21}^2$. The variance of η was set equal to 1 (i.e., $\psi = 1$).

Three indicators were used for each of the latent variables where y_1, y_2, and y_3 were indicators of η, x_1, x_2, and x_3 were indicators of ξ_1, and x_4, x_5, and x_6 were indicators of ξ_2. All errors were normally distributed and variances of y_1, y_2, y_3; x_1, x_2, x_3, x_4, x_5, and x_6 were equal to 1.0. Errors were chosen to give unit variance to the indicators and thus were chosen based on the size of the loadings.

The squared multiple correlation is dependent upon γ_3 (which changed depending on $R^2_{\gamma_3}$) and ϕ_{12}, and therefore it was not directly manipulated for the current study. The squared multiple correlation is equal to the value expressed by (Marsh et al., 2004)

$$R^2 = \frac{\gamma_1^2\phi_{11}^2 + \gamma_2^2\phi_{22}^2 + 2\gamma_1\gamma_2\phi_{12} + \gamma_3^2\left(\phi_{11}\phi_{22} + \phi_{12}^2\right)}{\sigma_\eta^2} \qquad (58)$$

Corresponding to the three values of $R^2_{\gamma_3}$ and the three values of ϕ_{12}, the resulting R^2 values were .384, .448, .512; .434, .498, .562; and .484, .548, and .612, respectively. These values are consistent with what has been used in other studies (see Table 3; Jaccard & Wan, 1995; Klein & Muthén, 2007; Marsh et al., 2004).

Although data were simulated to come from a population in which Equation 56 is true, and thus $\alpha = 0$ in the population, α is not necessarily zero when the model is estimated. Thus, the structural model that was estimated using each of these methods included α and was equal to Equation 2. Based on recommendations by Marsh et al. (2004), the "matched-pairs" method was used in the current study to create three product indicators of the interaction latent variable, $\xi_1\xi_2$, for the product-indicator methods.

Data were generated in SAS 9.00. To check the data simulation process to ensure results were plausible, several approaches were taken. First, several datasets of sample size 100,000 were simulated. Parameter estimates based on the large sample sizes of 100,000 were equal to the population generating parameters. Second, to ensure that the non-normal data generated using Fleishman's (1978) polynomial transformation with Vale and Maurelli's (1983) intermediate correlation procedure for the several datasets of 100,000 were also generated. The means, standard deviations, skew, and kurtosis values of these datasets were equal to the means, standard deviations, skew and kurtosis values from the population generating model. Additionally, the correlation coefficients between the latent variables were also equal to the correlation coefficients from the population generating model.

The program used to test for interaction effects varied for each type of approach. Simulated covariance matrices and mean vectors were analyzed using LISREL 8.8 (Jöreskog & Sörbom, 2001) for the constrained, partially constrained, unconstrained, and residual-centered approaches. Mplus version 4.2 was used for the latent moderated structural equations approach. SAS 9.00 was used to analyze data for the two-stage least squares, the two-step method of moments, and the marginal-maximum likelihood approaches.

As discussed earlier, there are many approaches for estimating factor scores. In previous simulation studies comparing latent variable scores approaches for estimating first-order effects, the Anderson and Rubin (1956) method resulted in the most accurate recovery of the first-order effects. Additionally, the Anderson and Rubin (1956) approach is frequently utilized and is available in LISREL and SPSS. Comparing the numerous latent variable scores approaches was not a research question addressed in the current study. For these reasons the latent variable scores were estimated in the current study using the Anderson-Rubin (1956) method within PRELIS and LISREL. While comparing the various latent variable scores methods was not part of the current study, it is an interesting question and could be investigated in future studies.

After predicting latent variable scores for ξ_1 and ξ_2 from a corresponding confirmatory factor analysis (CFA) model, the interaction term was created by multiplying the latent variable scores on ξ_1 and ξ_2 together for each case. These derived values were used to test for interaction effects between two continuous predictors, following methods described by Cohen, Cohen, West, and Aiken (2003) and were computed in SAS.

Justification for Number of Replications

Data were simulated using data simulated in SAS. Each of the 108 conditions was replicated 500 times. This decision was based on the number of replications used in previous studies, and factors that are known to influence the number of necessary replications for Monte Carlo simulations. Powell and Schafer (2001) conducted a meta-analysis of 219 simulation studies in structural equation modeling that investigated the robustness of the likelihood ratio chi-square. They reported that the number of replications used in these studies ranged from 20 to 1,000, with the median number of replications being 200. Similarly, Bandalos (2006) stated that 500 replications was large for structural equation modeling Monte Carlo simulation studies, and this number of replications would provide stable standard error estimates even when data were generated to come from a non-normal distribution. Table 3 contains the number of replications used in previous studies investigating interaction effects. These values range from 150 to 1000.

The necessary number of replications depends upon many factors, including: the desire to obtain stable parameter estimates (i.e., reduce sampling variability), the purpose of the study, the a priori type I error rate (α), a priori power (1 - β), and the size of the effect one wishes to test for (Bandalos, 2006; Robey & Barcikowski, 1988; Serlin, 2000). When conditions are unstable making estimation difficult, then a large number of replications may be necessary (Bandalos, 2006). Based on previous studies investigating interaction effects and non-linear effects in structural equation modeling, estimation has not been problematic and convergence rates have been high (i.e., 99% to 100%; Weiss &

Hancock, 2009; Harring, Weiss & Hsu, under review; Marsh et al., 2004). Thus, it was not anticipated that a large number of replications were necessary for the current study.

Bandalos (2006) stated that if the purpose of the Monte Carlo simulation is to compare the parameter estimates of models, then a large number of replications are not necessary. However, if the primary focus of a study is to obtain an empirical sampling distribution for use in hypothesis testing, than a larger number of replications may be necessary (Bandalos, 2006). The primary purpose of the current study was to compare parameter estimates between the nine methods. Therefore, a large number of replications was not necessary.

In studies that use too small a number of replications, then power may be too small to detect true differences. On the other hand, if the number of replications is too large, then the study may have excessive power. Some studies have suggested methods of conducting rudimentary power analyses to determine an adequate number of replications (Bradley, 1978; Robey & Barcikowski, 1988; Serlin, 2000). These studies use a priori type I error rates (α), a priori power (1 - β), and the size of the effect one wishes to test for to help determine the number of replications that should be used for Monte Carlo simulations. Generally, for power of .7 to .8, with an a priori α of .05, and an effect size that is classified as being intermediate or liberal (as classified by Bradley, 1978) the number of replications ranges from 400 to 2000. Serlin (2000) stated that as α increases, more liberal effect sizes can be used to determine an adequate number of replications for a study. Based on this statement, in conjunction with the number of replications used in previous studies conducted in this area of structural equation modeling, and the purpose of present study, a more liberal number of replications were used for the current study.

Chapter 4: Results

Criteria for Evaluating Models

Proportion relative bias. The bias of the parameter estimates was the primary measure that was used to compare the nine different methods of testing for interaction effects. Bias is defined by the average difference between the parameter estimates and the population-generating value. For the current study bias was reported in a proportion relative bias form, meaning that the bias values were divided by the population-generating parameter value. In the conditions in which the population-generating parameter value γ_3 was equal to zero the bias cannot be divided by zero, therefore the proportion relative bias was equal to the difference between the average parameter estimates and the population-generating. While the bias of the γ_3 parameter was the primary interest, bias was also examined for the first-order effects γ_1 and γ_2. Tables 4 through 39 contain the proportion of relative bias for all 9 methods across the 108 conditions. The proportion of relative bias for each condition will be discussed in more detail later.

Precision. Previous studies have examined the precision of the parameter estimates for γ_1 , γ_2, and γ_3 using some combination of three measures: the observed standard deviation, the estimated standard error, and the relative ratio of the estimated standard error divided by the observed standard deviation. The observed standard deviation represents the standard deviation of the parameter estimates across the 500 replications within a given cell. This is informative because it shows the true variability of parameter estimates in the sampling distribution.

The estimated standard error represents the mean of the standard errors estimated for each solution. Simply averaging the standard error estimates together yields a biased estimate of precision even if the variances were unbiased to begin with. This is because the standard error estimates that statistical software programs provide are equal to the square root of the variances, and taking the square root of a value is a non-linear transformation. To provide an unbiased estimate of the standard error one must square the standard errors for each solution, take the average of the those variances, and then take the square root

$$SE(\theta) = \sqrt{\frac{\sum \sigma_{\overline{X}}^{2}}{i}} \tag{59}$$

where $\sigma_{\overline{X}}$ represents the estimated standard error for a given replication, and *i* represents the number of replications that converged within a given cell (e.g., in the current study *i* equaled 500 if the solutions for all replications converged).

The estimated standard error and the observed standard deviation were used to compute a relative ratio calculated as

$$\text{Relative Ratio} = \frac{SE(\theta)}{SD(\theta)} - 1 \tag{60}$$

where $SE(\theta)$ represents the estimated standard error, and $SD(\theta)$ represents the observed standard deviation of the parameter estimates. This relative ratio indicates how the average standard error estimates compare to the average empirical variance estimates. Values close to zero are desirable because they indicate that the standard error values that are computed based on the model, are representative of what is in the population. Values less than zero indicate that the model underestimates the standard error estimates, while values greater than zero indicate that the model overestimates the standard error estimates.

The relative ratio was the primary measure of precision that was of interest for the current study. The observed standard deviations and the estimated standard errors were only used as a secondary interest to the relative ratios. The observed standard deviations, the estimated standard errors, and the relative ratios for each condition are reported in Tables 4 through 39 and will be discussed in more detail later.

Convergence. The percent of times a model converged were also kept track for each of the 9 estimated methods across all 108 conditions. The convergence rates are shown in Tables 40 through 43. Because the latent variable scores (LVS) and the two-stage method of moments (2SLS) approaches use ordinary least squares regression, convergence is not an issue with either. Similarly, there were no convergence problems with the latent moderated structural equations (LMS), two-stage method of moments (2SMM), or marginal maximum likelihood (MML) approaches. For the product-indicator models, $R^2_{\gamma 3}$ and ϕ_{12} did not impact convergence rates. Sample size, normality, and loading size, however, did impact convergence rates for these methods. Of these methods, the constrained approach resulted in convergence the most frequently. When data were non-normally distributed, loadings were low, and sample size was 100, the constrained approach was unable to converge for approximately 7 to 10% of the datasets.

When sample size increased to 250, or the size of the loadings increased, the constrained approach reached convergence for almost all datasets.

When the loadings were low, the partially constrained, unconstrained, and residual-centered approaches had some issues with convergence, particularly when the sample size was 100. Convergence was not an issue for these approaches when the sample size increased or when the size of the loadings increased. Of the four product-indicator methods, the constrained and partially-constrained approaches had the fewest issues with convergence.

Type I error rates and empirical power. As previously stated, the proportion of variance in η accounted for by the interaction effect was set equal to .00 (to investigate Type I error rates), .05, and .10 (to investigate power). Type I error rates and power were compared across the 9 approaches and 108 conditions. To compute Type I error rates and power, the null hypothesis H_0: $\gamma_3 = 0$ was used. The Type I error rate was represented by the proportion of converged solutions that had a statistically significant interaction effect in the simulated data when H_0 was true. Power was represented by the proportion of converged solutions that have a statistically significant interaction effect in the simulated data when H_0 was false (Marsh et al., 2004).

Type I error rates. The Type I error rate is represented by the proportion of converged solutions that have a significant interaction effect in the simulated data when the population interaction effect is zero. Type I error rates were computed using an α level of .05, and are shown in Tables 44 and 45.

When the loadings were equal to 0.50, the unconstrained and residual-centering approaches had had high Type I error rates, with between 9% to 25% of the models as having significant γ_3 paths. When the sample size was large (i.e., N=500) and data were normally distributed, the type I error rate for the unconstrained approach improved a little. In general, these rates increased as ϕ_{12} increased.

When data were normally distributed and loadings were equal to 0.50, the partially constrained, LMS, and 2SMM had low Type I error rates, and rejected about 0% to 3% of the models. When loadings were low and the sample size was 100, the LVS and 2SLS methods had Type I error rates closest to the desired α level. When the size of the loadings and the sample increased, all methods (except the MML approach) had Type I error rates close to the desired α level, provided that data were normally distributed. In these conditions the MML approach had very high Type I error rates, rejecting 11% to 30% of true models.

When data were non-normally distributed, the partially constrained and unconstrained approaches had better Type I error rates. When loadings were high, the LVS and 2SLS approaches also had Type I error rates close to the desired α level. In general, the partially constrained approach resulted in lower Type I error rates than the unconstrained approach when data were non-normally distributed. The constrained, residual-centered, LMS, 2SMM, and MML approaches had high Type I error rates when data were non-normally distributed.

Empirical power. Empirical power is represented by the proportion of converged solutions that have a significant interaction effect in the simulated data when the population interaction effect is not equal to zero. Empirical power rates were computed using an α level of .05, and are shown in Tables 46 through 49.

In general, all of the approaches had very low power when loadings were low. When loadings were low and data were normally distributed, the LVS approach had the highest power to detect true interaction effects. Power for the LVS approach increased as loadings increased, sample size increased, and as ϕ_{12} decreased. When data were normally distributed, and loadings were low, the LMS and MML approaches had power levels nearing that of the LVS approach provided that the sample size was 500. When loadings were .80, and data were normally distributed all of the approaches had acceptable levels of power except for the 2SMM approach.

In general all of the approaches had very low power when data were non-normally distributed. The MML approach had the only acceptable level of power, provided that the sample size was 500. In general, power for the MML approach increased as $R^2_{\gamma 3}$ increased, loadings increased, sample size increased, and ϕ_{12} increased.

Results by Type of Method

Overall, the features that impacted the type of method that most accurately detected the interaction effect were the effect size (represented by the proportion of unique variance that the interaction effect explained in η, i.e., $R^2_{\gamma 3}$), and the size of the loadings. The accuracy of parameter estimates changed depending on sample size and the value of the correlation between the two first-order latent variables (i.e., ϕ_{12}). However, in almost all conditions the sample size did not impact the type of method that most accurately detected the interaction effect, and the value of ϕ_{12} never impacted the type of method that was the least biased in detecting the interaction effect.

Constrained approach. The constrained approach was the least biased method when data were normally distributed, loadings were low, and $R^2_{\gamma 3}$ was .05. It was one of the least biased methods when data were normally distributed and loadings were high. The relative ratio was approximately zero when data were normally distributed, the correlation between the two first-order latent variables was .4 or .6, and sample size was at least 250.

The constrained approach is based on the second constraint imposed upon the variance of the interaction term and the assumption that ξ_1 and ξ_2 are normally distributed. Therefore, theoretically the constrained approach should be biased when data is non-normal. As expected, the constrained model resulted in biased parameter estimates of γ_3 in almost all conditions when data were non-normally distributed. When data were non-normal, $R^2_{\gamma 3}$ was .00, and loadings were .80, the constrained method resulted in unbiased estimates of γ_3 and performed similarly to all of the other approaches except for MML. Surprisingly, the constrained approach resulted in the only unbiased estimates of γ_3 when data were non-normal, loadings were low, $R^2_{\gamma 3}$ was .05, and ϕ_{12} was .40 or .60.

The constrained approach resulted in unbiased estimates of first-order effects in nearly all normal and non-normal conditions. Estimates of γ_1 and γ_2 were a little biased when the sample size was 100. The relative ratios for first-order effects were close to zero when data were normal and when the sample size was 250 or greater. When data were non-normal the relative ratios were large.

The constrained approach had high convergence rates across all conditions, even when the sample size was 100. When data were normally distributed, the Type I error rates associated with the constrained approach were approximately equal to the desired alpha level. When data were non-normally distributed, Type I error rates were a little high (around .10 to .16 when ϕ_{12} was equal to .20), and increased as ϕ_{12} increased. The empirical power was low when data were non-normal, loadings were low, or when the sample size was low. In the normal conditions power tended to decrease as ϕ_{12} increased, while in the non-normal conditions power increased as ϕ_{12} increased.

Partially constrained approach. Similarly to the other product-indicator methods and the ordinary-least-squares methods, the partially constrained approach resulted in low bias when the data were normal and the loadings were high. The relative ratio was very large when the sample size was low or when the loadings were low, indicating that in these conditions the partially constrained approach overestimated the standard error associated with the interaction by as much as 745%. Even in the conditions in which the relative ratio was low, other methods resulted in similar bias and more accurate standard error estimates.

The partially constrained approach relaxes both the second constraint on the variance of the interaction term, and the assumption that ξ_1 and ξ_2 are normally distributed. Therefore, theoretically, the partially constrained approach should be unbiased in the non-normal conditions. In comparison to the constrained approach, the partially constrained approach resulted in less biased parameter estimates in the non-normal conditions, when $R^2_{\gamma 3}$ was .00. When $R^2_{\gamma 3}$ was .00, the bias for the partially constrained approach was a little high when the sample size was 100 and when the loadings were .50, however, this bias decreased when the sample size was 250 or greater, or when the loadings increased to. 80. Even though bias was small for the partially constrained approach when $R^2_{\gamma 3}$ was .00, the 2SLS and LVS approaches estimated γ_3 more accurately than the partially constrained approach did. Surprisingly, the partially constrained approach resulted in more biased parameter estimates than the constrained approach when $R^2_{\gamma 3}$ was .05 or .10. When data were non-normally distributed, and $R^2_{\gamma 3}$ was .05 or .10, all approaches, including the partially constrained approach, resulted in biased parameter estimates (underestimating γ_3 by 20% or more).

When the sample size was 100 and the loadings were low, the partially constrained method had convergence rates between 77% to 84%. Convergence was not a problem once the sample size was 250 or more, or the loadings were .80. The partially constrained approach was one of the few methods that had stable Type I error rates, even when data were non-normal. For some of the other methods (e.g., constrained, residual-centered, LVS, LMS, 2SMM, and MML), the Type I error rates increased as ϕ_{12} increased. For the partially constrained approach, however, the Type I error rates remained relatively stable across ϕ_{12} levels. The partially constrained approach resulted in low empirical power, particularly when the sample size was low, the loadings were low, or when data were non-normally distributed.

Unconstrained approach. Similarly to the partially constrained approach, the unconstrained approach was unbiased when data were normally distributed and loadings were high. There were no conditions in which the unconstrained approach was the only unbiased method, meaning that in all conditions in which the unconstrained approach was unbiased, there were several other methods that were also unbiased. In most of the conditions in which bias was low, the relative ratio was high in comparison with that resulting from other methods.

Similarly to the partially constrained approach, the unconstrained approach should theoretically lead to more accurate parameter estimates than the constrained approach when data is non-normally distributed. This is because the second constraint on the variance of the interaction term, and the assumption that ξ_1 and ξ_2 are normally distributed are both relaxed. When data were non-normal and $R^2_{\gamma 3}$ was .00, the unconstrained approach resulted in more accurate estimates of γ_3 than the partially constrained approach and the constrained approach, provided that ϕ_{12} was .20 or .40. When ϕ_{12} was .60, the partially constrained approach resulted into less biased estimates of γ_3 than the unconstrained approach. Even though bias was small when $R^2_{\gamma 3}$ was .00, the 2SLS and LVS approaches resulted in more accurate estimates of γ_3. When $R^2_{\gamma 3}$ was .05 or .10, the unconstrained approach was biased and was more biased than the partially constrained approach, underestimating γ_3 by as much as 30%.

When the sample size was 100 and loadings were low, the unconstrained approach did not convergence for between 20% to 30% of the datasets. Convergence was not a problem once the sample size was increased or the loadings were increased. Type I error rates were high when the sample size was 100 or when loadings were low. When data were non-normally distributed, the Type I error rates were approximately equal to the alpha level. The unconstrained approach resulted in low power when the sample size was 100, loadings were low, or the data were non-normally distributed.

Residual-centered unconstrained approach. Similarly to the other product-indicator methods and the ordinary-least-squares methods, when the data were normally distributed and the loadings were high (i.e., λ_i = .80), the residual-centered approach resulted in minimal bias for both first-order effects and interaction effects. The residual-centered approach resulted in biased estimates of γ_3 when data were non-normal. In most conditions the residual-centered method resulted in high relative ratios compared to the other methods, particularly when the sample size was low or the loadings were low.

When the sample size was 100 and the loadings were low (i.e., $\lambda_i = .50$), the residual-centered approach had low convergence rates (as low as 73%). Type I error rates were high, and power was low, when the loadings were low, or when the data were non-normal.

Latent variable scores approach (LVS). When data were normally distributed , $R^2_{\gamma 3}$ was zero, and the loadings were low, the LVS approach was the least biased approach for estimating γ_3 and had the lowest relative ratio. The LVS approach was one of the least biased approaches when data were normally distributed and loadings were .80, or when data were non-normally distributed, and $R^2_{\gamma 3}$ was zero. When estimating γ_3, the LVS approach had a relative ratio close to zero for all conditions, and the lowest relative ratio when data were non-normal.

For estimating first-order effects, the LVS approach was one of the least biased approaches in all of the conditions, across $R^2_{\gamma 3}$, sample size, loading sizes, ϕ_{12}, and normality. However, relative ratios associated with first-order effects were high and negative in all conditions, indicating that the LVS approach underestimated standard error estimates for first-order effects in all conditions. These standard error estimates became more accurate as loadings increased. Sample size, ϕ_{12}, $R^2_{\gamma 3}$, and normality did not seem to impact the standard error estimates for first-order effects.

Because the latent variable scores approach uses ordinary least square regression, convergence was not an issue. The Type I error rates were close to the desired α level when data were normally distributed and when data were non-normal with sample size 100. Type I error rates were a little high when data were non-normally distributed and the sample size was 250 or 500, but it was not as high as some of the other approaches (e.g., constrained, residual-centered, LMS, and MML). When data were non-normally distributed, the partially-constrained, unconstrained, and 2SLS methods had slightly better Type I error rates than the LVS approach. Power was low when the loadings were low or when data were non-normal. Even though the power was low when loadings were low, the power was still higher than it was for other methods.

Two-stage least squares approach (2SLS). When data were normally distributed and $R^2_{\gamma 3}$ was either .00 or .05, the 2SLS method was one of the least biased methods for estimating the interaction effect γ_3. In the normally distributed conditions in which the 2SLS approach was not biased, the constrained approach or the LVS approach also led to unbiased estimates of γ_3. When data were non-normally distributed and $R^2_{\gamma 3}$ was zero, the 2SLS approach was the least biased method for estimating γ_3 along with the partially constrained and the unconstrained approaches. In these conditions, the 2SLS approach was less biased than the partially constrained and unconstrained approaches when the sample size was 100. The relative ratio associated with γ_3 was small when the sample size was 250 or less in both the normal and non-normal conditions.

For estimating first-order effects, the 2SLS approach was one of the least biased methods. Although most approaches (with the exception of the 2SMM and MML approaches) resulted in unbiased estimates of first-order effects, the 2SLS along with the

LVS method resulted in the most accurate estimates of γ_1 and γ_2. Additionally, the relative ratios associated with γ_1 and γ_2 were close to zero in most conditions, indicating that the standard error estimates resulting from the 2SLS approach were fairly accurate.

Because the 2SLS approach uses ordinary least square regression, convergence was not an issue. Type I error rates were slightly high when loadings were low (Type I error rates were about .06 to .07). Even though at times the Type I error rates were a little high, when data were non-normally distributed the Type I error rates were consistent across sample sizes and ϕ_{12} levels, and were often closer to the desired alpha level then the other approaches. Empirical power was low when the loadings were low, or when the sample size was 100 or 250, or when data were non-normal.

Latent moderated structural equations (LMS). The latent moderated structural equations (LMS) approach resulted in somewhat biased parameter estimates of γ_3 in all normal conditions. It was the least biased method for estimating γ_3 when data were non-normal, $R^2_{\gamma 3}$ was .10, and the loadings were .50. Bias decreased as ϕ_{12} decreased and as sample size increased. In these conditions (i.e., non-normal, $R^2_{\gamma 3}$ = .10, and loadings of .5) the LVS and the 2SLS approaches were more biased, but had more accurate standard error estimates than the LMS approach as evidenced by their relative ratio values being closer to zero. In these conditions, the relative ratio was high and positive, indicating that although unbiased, the LMS approach overestimated standard errors, particularly at small sample sizes. The standard error estimates were fairly accurate when the sample size was 500. In comparison to the constrained, partially constrained, unconstrained, residual-centered, and 2SMM approaches, the LMS approach had relative ratio values closer to zero. When the sample size was 100, the LMS approach resulted in very high and inaccurate standard error estimates for γ_3 (up to 57,600%). This overestimation decreased as ϕ_{12} increased. Therefore, with small sample sizes (i.e., n=100), the standard error estimates for γ_3 resulting from the LMS approach cannot be trusted.

In most conditions the LMS approach lead to unbiased results when estimating the first-order effects, particularly as $R^2_{\gamma 3}$ increased, as loadings increased, and when the sample size was greater than 100. When the data were normally distributed, the Type I error rates were close to the desired alpha level. The Type I error rates tended to become large when data were non-normal and the sample size was 250 or 500 (Type I error rates up to .558). The empirical power was low when the loadings were low (i.e., λ_i = .50), but increased as the sample size increased. The empirical power was acceptable when loadings were high, particularly when the sample size was 250 or more. In all of the non-normal conditions, the empirical power for the LMS approach was low. However, aside from the MML approach, the LMS approach had a higher empirical power rate than the other methods in the non-normal conditions. Surprisingly, in the non-normal conditions, the power for LMS became larger as the correlation between the first-order latent variables (i.e., ϕ_{12}) increased.

Two-stage method of moments (2SMM). Overall, the 2SMM approach resulted in the most biased parameter estimates for first-order and interaction effects in all conditions. Additionally, in most conditions the standard error estimates were drastically inaccurate, particularly when the loadings were low or when the sample size was low. The Type I error rates were close to the desired alpha level when data were normally distributed or when the loadings were low. However, the method had very low power to detect interaction effects when they were present.

Marginal maximum likelihood (MML). The MML approach resulted in parameter estimates that were biased for both first-order effects and interaction effects in all of the normal conditions and most of the non-normal conditions. Although, the MML approach was biased, it was the least biased method for estimating γ_3 , when the data were non-normal, the loadings were .80, and an interaction effect was present (i.e., $R^2_{\gamma 3} \neq$ .00). In these conditions, the MML approach underestimated the value of γ_3 by between 5% to 17%. However, while it was the least biased approach for estimating γ_3 when data were non-normal, loadings were 0.80, and $R^2_{\gamma 3}$ was .00, it resulted in a much higher relative ratios, inaccurately estimating the standard error by between 40% to 1263%.

For first-order effects the MML approach was unbiased when data were normally distributed and when the sample size was 250 or more. Surprisingly, the MML approach resulted in high relative ratios associated with first-order effects when loadings when loadings were 0.80, but was one of the most accurate methods of estimating standard error for first-order effects when loadings were 0.50. The Type I error rates were high (i.e., between .11 and .81) in the non-normal conditions, especially in comparison to the other approaches. Although Type I error rates were high, the MML approach had the highest power to detect interaction effects when data were non-normal, provided that the sample size was 500.

Chapter 5: Discussion

The current study aimed to determine the best method for estimating latent variable interactions. Data were simulated from known population parameters and varied as a function of the size of the interaction effect, sample size, the loadings of the indicators, the size of the relation between the first-order latent variables, and normality in a fully-crossed design. All datasets were analyzed using nine latent variable methods of testing for interaction effects: the constrained, the partially-constrained, the unconstrained, the residual-centering, the latent variable scores with moderated multiple regression, two-stage least squares, latent moderated structural analysis, two-stage method of moments, and marginal maximum likelihood approaches.

Non-Normality

In the current study, when data were non-normally distributed and an interaction effect was present in the population-generating model (present (i.e., $R^2_{\gamma 3} \neq .00$), all methods led to biased estimates of the interaction effect in almost all of the conditions, incorrectly estimating the interaction effect by as much as 10% to 40% in most cases. While previous studies investigating the impact that non-normality had on parameter estimates found that bias tended to increase when data were non-normal, they found that the bias was still minimal for some approaches (Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Marsh et al., 2004; Wall & Amemiya, 2003). This supports the importance of the current study in investigating the impact that severe non-normality has on estimating interaction effects. Consequently, rather than discussing which approaches were unbiased, the question becomes which of the methods resulted in the least biased estimates of the interaction effect.

The constraints specified in the constrained model are based on the assumption that ξ_1 and ξ_2 are normally distributed. Wall and Amemiya (2001) pointed out that when this assumption is not met then the second constraint imposed upon the variance of the interaction term does not hold true. The partially constrained, unconstrained, and the residual-centered unconstrained approaches are not based on the assumption that ξ_1 and ξ_2 are normally distributed, and thus relax the second constraint. Therefore, when ξ_1 and ξ_2 are non-normally distributed the partially constrained, unconstrained, and residual-centered unconstrained approaches should theoretically be less biased than the constrained approach.

In the current study, the constrained approach resulted in biased estimates of the interaction effect in almost all of the non-normal conditions. This finding is not surprising due since the constrained approach is based on the assumption that the first-order latent variables are normally distributed, and is similar to findings in previous studies (Marsh et al., 2004). The residual-centered approach resulted in biased estimates of the interaction effects in all non-normal conditions. This finding is unique to the current study because the residual-centered approach had not previously been evaluated under non-normal conditions.

Surprisingly, the constrained approach was the only method that resulted in unbiased estimates of γ_3 when data were non-normal, loadings were low, $R^2_{\gamma 3}$ was .05, and ϕ_{12} was .40 or .60. In all other non-normal conditions, the partially constrained and unconstrained approaches were less biased than the constrained approach. When data were non-normal and an interaction effect was present, the partially constrained approach resulted in less biased parameter estimates of the interaction effect than the unconstrained and constrained approaches. This finding contradicts Marsh et al.'s (2004) finding in which under non-normal conditions the unconstrained approach was found to result in slightly less biased estimates of the interaction effect than the partially constrained approach. These contradictory findings could be due to the severity of the non-normality in the current study.

Even when ξ_1 and ξ_2 are normally distributed, the interaction is known to be non-normally distributed (Jöreskog & Yang, 1996). The product-indicator models use maximum-likelihood estimation which is based on the assumption that all indicators in the model are multivariately normally distributed. Because the indicators for the interaction are known to be non-normally distributed, this assumption is violated when maximum-likelihood is used. Although standard SEM software packages are able to provide users with robust standard error estimates, the parameter estimates obtained from maximum-likelihood estimation are still expected to be biased. The LMS and MML approaches do not make any distributional assumptions regarding the interaction effect, and thus should theoretically lead to more accurate parameter estimates than the product-indicator approaches in non-normal conditions.

Although all methods were poor at detecting a true interaction effect, the MML approach led to the least biased estimates of the interaction effect when the loadings were adequate. In these conditions the MML had the highest power to detect interaction effects. However, the MML approach had high Type I error rates and very large relative ratios indicating that standard errors were inaccurately estimated by as much as 1263%.

This finding is important because the MML approach has not been compared to other methods in any previous simulation studies.

The LMS approach led to the least biased estimates of the interaction effect when data were non-normal, an interaction effect was present (i.e., $R^2_{\gamma 3} \neq .00$), and the loadings were low. Parameter estimates for the LMS approach became less biased as the size of the interaction effect increased and as the size of the correlation between the first-order latent variables decreased. This contradicts previous findings in which the LMS approach was found to result in unbiased estimates of the interaction effect across all sizes of the interaction effect (Klein & Moosbrugger, 2000; and Klein & Muthén, 2007). These contradictory findings could be due to the severity of the non-normality used in the current study. Also, previous studies set the size of the relation between the first-order latent variables to be .235, and in the current study the LMS approach led to less biased results at this level then when this value was increased to be .4 and .6. In the current study, even when bias was low, the LMS approach led to large standard error estimates, particularly with small sample sizes.

The constrained, LMS, and MML approaches are based on the assumption that ξ_1 and ξ_2 are normally distributed. The constrained, partially-constrained, unconstrained, and residual-centered approaches are based on the assumption that the interaction effects is normally distributed. Therefore, when all exogenous latent variables (i.e., ξ_1, ξ_2, and $\xi_1\xi_2$) are non-normally distributed, the constrained, partially-constrained, unconstrained, residual-centered, LMS, and MML approaches violate at least one distributional assumption necessary for their use. The 2SMM, LVS, and 2SLS approaches make no distribution assumptions regarding the exogenous latent variables and the latent interaction. Therefore, when the latent variables are non-normally distributed, theoretically the 2SMM, LVS, and 2SLS approaches should result in more accurate parameter estimates than the constrained, partially-constrained, unconstrained, residual-centered, LMS, and MML approaches. Furthermore, because the 2SMM approach is considered to be fully-latent while the LVS and 2SLS approaches are partially-latent, one would expect that the 2SMM would provide more accurate parameter estimates than the LVS and 2SLS approaches.

When data were non-normally distributed and an interaction effect was not present, the 2SLS approach, partially constrained, and unconstrained approaches resulted in the least biased estimates of the non-existent interaction effect. When the sample size was 100 in these conditions, the 2SLS resulted in the only unbiased estimates of the non-existent interaction effect. When data were non-normal, the 2SMM and the LVS approaches led to biased estimates of the interaction effect in all of the conditions. The 2SMM approach had very inaccurate standard error estimates associated with the interaction effect. The LVS approach, however, had the most accurate standard error estimates associated with the interaction effect out of all of the methods. Thus, for non-normal data, the LVS approach gave more precise (more accurate standard error estimates) estimates of the wrong value (larger bias of the interaction effect).

When estimating first-order effects, the LVS and 2SLS approaches led to the least biased estimates of the interaction effect in most of the non-normal conditions. The LVS approach tended to underestimate standard errors associated with first-order effects, however, the standard error estimates based on the 2SLS approach tended to be fairly accurate.

These findings are important because previous studies have not examined the 2SMM and LVS approaches under non-normal conditions. Only one previous study investigated the impact of non-normality on the 2SLS approach (Klein & Moosbrugger, 2000). For the single condition reported in their study, the 2SLS was found to result in biased estimates of first-order effects and the interaction effect when data were non-normally distributed. Thus regarding the 2SLS approach, findings for the current study support and build on findings reported by Klein and Moosbrugger (2000).

These findings suggest that even though the 2SMM and LVS approaches make no distributional assumptions regarding the latent variables, these methods tended to lead to inaccurate parameter estimates and standard error estimates. The 2SLS approach accurately estimated first-order effects, the interaction effect, and standard errors in the conditions in which no interaction effect was present (i.e., $R^2_{\gamma 3}$ was .00). However, when an interaction effect was present (i.e., $R^2_{\gamma 3} \neq .00$), the constrained, LMS, or MML approaches resulted in less biased estimates of the interaction effect than the 2SLS approach.

Recommendations

Selecting which method one should use to test for interaction effects depends largely upon whether or not data are normally distributed. When data is normally distributed the constrained model is recommended for use. In the conditions considered in the current study, the constrained approach led to the least biased estimates of the interaction effect, and accurate standard error estimates, particularly when the sample size was 250 or greater and when the correlation between the first-order effects was .4 or greater. Additionally, the constrained approach accurately estimated first-order effects provided that the sample size was 250 or greater. High convergence rates were associated with all normal conditions using the constrained approach. Type I error rates were close to the desired alpha level, particularly when the sample size was 250 or greater. When loadings were low and the sample size was 100, the constrained approach had low power to detect true interaction effects. If the loadings were low, a sample size of at least 500 was necessary to have acceptable power. If the loadings were adequate then a sample size of 250 led to acceptable levels of power. Based on these findings, the constrained approach is recommended for use when data is normally distributed. A sample size of 250 or more is recommended for use with the constrained model, although it performs fairly well with sample sizes of 100 too, provided that the loadings are adequate.

When data is normally distributed and loadings are adequate, the LVS approach is acceptable for use as well. In these conditions, the LVS approach resulted in unbiased estimates of the interaction effect and accurate estimates of the standard errors associated with the interaction effect. The LVS approach resulted in the least biased estimates of first-order effects in all of the normal conditions. The relative ratios associated with the first-order effects were low and negative indicating that the LVS approach underestimated standard errors associated with first-order effects. When loadings were adequate, the LVS approach resulted in acceptable Type I error rates and power, particularly when the sample size was 250 or greater. Out of all the approaches, the LVS and constrained approaches had the highest power to detect true interaction effects at small sample sizes. The LVS is also beneficial in that there are no convergence problems because it is based on ordinary least squares regression. When data is normally distributed, the LVS approach may be preferable to the constrained approach because it is easier to understand, easier to implement because it does not necessitate the use of nonlinear constraints, and readily available for practitioners in commonly utilized statistical software packages (e.g., SPSS, SAS, STATA, and LISREL).

When data were non-normally distributed, it was more difficult to decide which method should be used. When an interaction effect was present, all of the methods resulted in biased parameter estimates, inaccurate standard error estimates, poor Type I error rates, and low power. Therefore, recommendations for the preferred method to use when data are non-normally distributed are based on relative comparisons rather than absolute comparisons. That is, the following recommendations are based on which method performed the least poorly.

When data is non-normal and loadings are of adequate size, then the MML approach has potential. It provided the least biased estimates of the interaction effect, underestimating the interaction by 5% to 17% in these conditions. Additionally, the MML resulted in the highest power to detect true interaction effects in comparison to the other approaches. This high power, however, was accompanied by large Type I error rates. Furthermore, the standard error estimates associated with the interaction effect were vastly underestimated by between 10% to 100%. The interaction standard error estimates based on the MML approach tended to be the most accurate when the sample size was 250. While the bias for interaction effect decreased as the loadings increased, the standard errors associated with the interaction estimates increased as the loadings increased. This finding was surprising because one would expect the standard error estimates to become more accurate as indicators became more reliable. The MML approach also led to biased estimates of first-order effects and high relative ratios associated with first-order effects. This is an indication that the MML approach gave imprecise estimates (large relative ratios) of the wrong value (large bias) for first-order effects.

The MML approach became slow to converge as the number of latent variables increases. In the current study, the MML approach took up to 20 minutes for a single dataset to converge. Therefore, using the MML approach with models with multiple nonlinear effects or very large sample sizes may be impractical because models with more latent variables or more people will inevitably take a longer period of time to converge.

When data is non-normal the LMS approach also has potential, particularly when the loadings are low. In these conditions, the LMS approach lead to the least biased estimates of the interaction effect, incorrectly estimating the interaction effect by between 5% to 20%. The bias tended to decrease as sample size increased, the size of the interaction effect increased, and the size of the relation between the first-order latent variables decreased. The standard error estimates associated with the interaction effect were very high, particularly when the sample size was small. The LMS approach resulted in unbiased estimates of first-order effects, and the standard error estimates of first-order effects were accurate provided the sample size was 500. All methods had low power to detect true interaction effects when data were non-normally distributed. The LMS approach had the second highest power, following only the MML approach. The power for the LMS approach increased as the relation between the first-order latent variables increased and the sample size was 500. Based on these findings, the LMS approach may be appropriate for use when data is non-normally distributed and loadings are low, but it is recommended that a sample size of at least 500 be used.

Although the MML approach had the least biased estimates of the interaction effect when the loadings were of adequate size, the LMS approach had the second least biased estimates of the interaction effect while having more accurate standard error estimates than the MML approach. In these conditions, the sample size did not impact the amount of bias resulting from the LMS approach, but bias did tend to decrease slightly as the relation between the first-order latent variables decreased. Additionally, when the loadings were of adequate size the LMS approach resulted in unbiased estimates of first-order effects, whereas the MML generally resulted in biased estimates of first-order effects. Therefore, if a researcher wishes to obtain parameter estimates of both first-order effects and interaction effects, the LMS approach may be preferable to the MML approach when data is non-normally distributed.

A summary of these four recommended methods (e.g., the constrained, LVS, MML, and LMS approaches) is shown in Table 51. Figure 7 shows a flowchart that can be used by applied researchers to aide them in deciding which approach they should use to test for interaction effects. Of note, the recommendations for the normal conditions are absolute and relative, while the recommendations for the non-normal conditions are only relative. In the non-normal conditions all methods resulted in biased results, inaccurate standard error estimates, large type I error rates, and low power. Thus, recommendations for the non-normal conditions could only be based on comparing the performance of the methods with each other, rather than ideal outcomes.

The partially constrained, unconstrained, and residual-centered approaches resulted in high relative ratios in most conditions. Even in conditions in which the relative ratio was low, other methods resulted in similar bias and more accurate standard error estimates. Additionally, these two approaches had the lowest convergence rates when the sample size was 100 and the loadings were low. Based on this information, the partially constrained, unconstrained, and residual-centered approaches are not recommended for use to test for interaction effects.

When data were non-normally distributed, the 2SLS approach accurately estimated first-order effects, the interaction effect, and standard errors in the conditions in which no interaction effect was present in the population-generating model (i.e., $R^2_{\gamma 3}$ was .00), and was the least biased approach in these conditions when the sample size was 100. When data were normally distributed and an interaction effect was present in the population-generating model (i.e., $R^2_{\gamma 3} \neq .00$), the constrained, LMS, or MML approaches resulted in less biased estimates of the interaction effect than the 2SLS approach. The relative ratios for first-order effects and interaction effects based on the 2SLS approach were small in all conditions, particularly when the sample size was 250 or less. Based on these findings the 2SLS approach is not the preferable method to use to test for interaction effects. While in many conditions parameter estimates were unbiased, and standard error estimates were fairly accurate, the 2SLS was unsuccessful at accurately detecting true interaction effects when they actually existed (i.e., $R^2_{\gamma 3}$ was .05 or .10).

The 2SMM approach resulted in the most biased parameter estimates for first-order effects and interaction effects. Additionally, the standard errors were drastically inaccurate in most conditions, and power was low. Therefore, the 2SMM approach is not recommended for use when testing for interaction effects.

Limitations

Number of indicators. Previous simulation studies have used two or three indicators to represent each latent variable. The current study used three indicators for each latent variable. No studies have been conducted to investigate the impact that varying the number of indicators per latent variable has on the bias and precision of parameter estimates. This is likely due to researchers being more concerned with the quality of the indicators (i.e., the size of the loadings) rather than the quantity of the indicators.

One would expect that there is a relation, however, between the quality and quantity of indicators. For example, the current study used a low-indicator condition with loadings being equal to .50. Using low loadings allows researchers to evaluate the methods under reasonably difficult conditions (Klein & Muthén, 2007). As seen with the current study, low indicators led to more biased parameter estimates across all methods in almost all conditions. Kline (2005) suggested that having an insufficient number of indicators per latent variable could lead to specification errors in the model. While three indicators per latent variable is enough for identification purposes, more indicators may be necessary to avoid problems such as non-convergence and inaccurate parameter estimation (Kline, 2005). In the current study, the low-indicator loadings condition often led to biased parameter estimates, however, parameter estimates may have been less biased if more indicators had been used per latent variable.

While using a greater number of indicators for each latent variable may be feasible for some methods, it is more difficult for the constrained and partially constrained approaches. This is because the third type of constraint, which is placed on the errors of the product indicators for the interaction term, is computational intensive to specify. While the specification of these constraints is feasible when the number of indicators is small (e.g., 2 or 3 indicators per latent variable), it becomes exponentially difficult and infeasible when the number of indicators per latent variable increases.

Non-convergence. There is disagreement between methodological researchers about whether or not to remove replications that do not converge. For the current study replications for that did not converge were removed from the analysis. This decision was made because it is most consistent with what would occur with real datasets, and is consistent with past research using Monte Carlo simulation. However, as is frequently the case when a dataset is analyzed using multiple different methods, convergence may be reached for a single dataset with one method, but not for another method. This means that the methods would not be comparing the same datasets for any cells in which convergence was not reached for all replications.

When loadings were adequate or when the sample size was 500, nearly all cases converged for all methods (i.e., approximately 95% to 100% of the replications converged). However, in the few conditions when loadings were low and sample size was small, the partially constrained, unconstrained, and residual-centering approaches converged for approximately 75% to 85% of the replications. Because the replications that did not converge were removed from the analyses, it is difficult to make comparisons across the methods for the conditions in which the loadings were low and the sample size was small. When convergence is not reached, some researchers add additional replications until they get the same number of converged replications for all conditions. In the current study, however, convergence was only problematic for the partially constrained, unconstrained, and residual-centered approaches. Because these four methods also had higher bias, more inaccurate standard error estimates, higher type I error rates, and lower power, it is reasonable to assume that adding additional cases would not improve the results obtained from these methods.

Limitations of software. Analyses were conducted for the current study in SAS, LISREL, and Mplus. Because all methods could not be analyzed using the same software program, there may be differences in the outcomes for the current study due to the type of software used. For example, the different software programs use different numbers of maximum iterations and different starting values. For the current study the default settings within a given software program were used. The default settings were used because it was thought to be most representative of what applied researchers may do. Applied researchers may choose to change the default settings, but this decision would be specific to their particular dataset.

The default number of maximum iterations with Mplus is 1000, while the default for LISREL is set equal to three times the number of free parameters. Therefore, for the product-indicator methods conducting using LISREL not only does the number of maximum iterations change depending on the software program, but it also changes depending on the method used as well, since each method has a different number of free parameters. This means that maximum number of iterations would have been 87, 96, 114, and 114 for the constrained, partially constrained, unconstrained, and residual-centered approaches, respectively.

The default starting values also differ across software programs and could potentially impact results. The default starting values within the respective program were used for all methods with the exception of the MML approach. One limitation of the MML approach is that it becomes increasing slow to converge as the number of latent variables increases. Specifically, Harring et al. (under review) suggested that when the number of latent variables is greater than three or four the MML approach may be very slow to converge. In the current study the structural model had one nonlinear term (i.e., a single interaction effect) and three latent variables. Pilot analyses revealed that the MML approach took up to 20 minutes to converge to a solution for a single dataset. The length of time it took to reach convergence was positively related to the sample size. That is, as sample size increased, the amount of time to reach convergence also increased. In an attempt to reduce the amount of time it took the computer to analyze the 54,000 datasets using the MML approach, starting values were provided for the program for the errors, loadings, covariance between the first-order exogenous latent variables, the variances of the exogenous latent variables, and the structural paths. The starting values were set equal to the values in the population-generating model.

Normality. One of the goals of the current study was to investigate the impact that severe non-normality had on estimating interaction effects. In the current study, when data were normally distributed, the constrained and LVS approaches were favorable. When data were severely non-normal, all methods were poor, but the LMS and MML approaches resulted in the most favorable outcomes. Because only two normality conditions were used in the current study (i.e., normal vs. severely non-normal), it is unknown how non-normal distributions need to be for parameter estimates and their precision to change.

Four previous studies were conducted that compared normal conditions to non-normal conditions (Marsh et al., 2004; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; and Wall & Amemiya, 2001). These previous studies investigated the impact that mild non-normality had on estimating interaction effects. In these studies skew for the non-normal conditions ranged from -2.0 to 1.5, and kurtosis ranged from -1.2 to 6.0. These studies investigated the effects of mild normality deviations in a small number of conditions (see Table 3 for the conditions that were investigated previously). For the current study skew was set at 3, and kurtosis was set at 22 in the population-generating model. More research should be conducted to investigate the impact of mild non-normality across a wider variety of conditions than those used in previous studies to determine how non-normal data needs to be for parameter estimates to become excessively biased.

Methods not examined. The goal of the current study was to compare the current methods for testing for interaction effects in structural equation modeling. Unfortunately, the current study did not include the QML approach. While the QML approach is not available in any commercial software programs, it is available by request as a stand-alone unpublished software program (Klein, unpublished). Unfortunately, the version of the QML software program that was made available by Klein was limited in that it could only be used with single datasets, and therefore could not be used in the current simulation study.

The QML approach does not make distributional assumptions about the interaction effect, and thus is theoretically expected to perform better than the product-indicator methods when data is non-normally distributed. In the current study, all methods resulted in biased estimates of the interaction effect when data were non-normally distributed. However, the estimates of the interaction effect based on the LMS approach were less biased than those resulting from the other approaches in many of the conditions when data were non-normally distributed. One study found that when data were non-normal, the QML approach resulted in more biased estimates of the interaction effect than the partially constrained approach, however, it had smaller standard error estimates, smaller standard deviations of parameter estimates across the replications, and higher power than the constrained, partially constrained, and unconstrained approaches (Marsh et al., 2004). The higher power of the QML approach was accompanied by higher Type I error rates (Marsh et al., 2004).

Another study found that when data were non-normally distributed the LMS approach was slightly less biased than the QML approach (Klein & Muthén, 2007). The QML approach, however, resulted in more accurate estimates of standard errors than the LMS approach did when data were non-normal (Klein & Muthén, 2007). Findings from previous studies suggest that the QML approach may be appropriate when data is non-normally distributed. However, more research needs to be conducted to evaluate this.

The current study compared nine methods of testing for interaction effects in structural equation modeling using an elementary interaction model (i.e., an interaction model that included a single interaction term and no covariates). Previous studies investigating the elementary interaction model had many limitations within studies and many inconsistencies across studies (discussed earlier). Thus, based on findings from previous studies, the question of "Which method is best," had not been thoroughly answered. In practice, however, researchers may hypothesize questions that include multiple interaction effects, nonlinear terms, and covariates. The type of method that most accurately detects a true interaction effect may change when the structural model becomes more complex. The QML approach was developed to handle more complex models with multiple interaction and quadratic effects that could not be handled by the LMS approach. Klein and Muthén (2007) found that the QML approach provided slightly better estimates of interaction effects than the LMS approach when three, two-way interactions were included in the structural model. The 2SMM was also developed to be able to handle more complex models. Although the 2SMM performed poorly in comparison with the other approaches in the current study, it may have potential when testing more complex models.

When conducting research it is inevitable that other researchers are also attempting to answer similar research questions as your own. In July, a study by Mooijaart and Bentler (2010) was published which introduced a new method to test for interaction effects in structural equation modeling. Their method is a "minor extension" of standard structural equation modeling. In addition to using means and covariances, their model fits a selection of third order moments. The third order moments are chosen to reflect the non-normality (specifically the amount of skewness) of the indicators of the latent variable interaction. They conducted a small, preliminary, simulation study investigating a single condition with sample size 400. They compared their method to the LMS method and found that both methods resulted in unbiased estimates of the interaction effect. They found that the LMS approach resulted in slightly smaller standard error estimates. However, their method was advantageous in that it provided a model goodness-of-fit chi-square test statistic, and Lagrange Multiplier tests which were able to detect the presence of an interaction effect, and are not currently available with some of the newer methods of testing for interaction effects (i.e., LMS, MML, and QML). The Mooijaart and Bentler (2010) method is currently available in an experimental version of EQS and will be made available in EQS 7.0. Because their paper was just published, the Mooijaart and Bentler (2010) method was not included in the current study.

Table 1

Summary of Indicator Models for Testing for Interaction Effects

	Mean Structure	Centering	Method of Forming Interaction Indicators	Constraints	Other Information
Kenny-Judd	No	Mean	All possible products	#1, #2, #3, normality	
Jöreskog & Yang	Yes	None	Single product	#1, #2, #3, #4, normality	
Algina-Moulder (Constrained)	Yes	Mean	All possible products	#1, #2, #3, #4 normality	
Wall & Amemiya (Partially Constrained)	Yes		All possible products & Single product	#1, #2, #3	
Marsh et al. (2006) (Unconstrained)	Yes	Mean	Matched Pairs (no overlapping)	None	
Little et al. (Residual Centered)	No	Residual	All possible products	Normality	Correlated errors

Table 2

Summary of Methods Used in Previous Simulation Studies

	Cons	PC	UC	RC	LVS	2SLS	LMS	QML	2SMM	MML
Jaccard & Wan (1995)	X[a]									
Schermelleh-Engel et al. (1998)	X					X	X			
Klein & Moosbrugger (2000) (study 1)	X					X	X			
Klein & Moosbrugger (2000) (study 2)						X	X			
Wall & Amemiya (2001)		X								
Moulder & Algina (2002)	X					X				
Wall & Amemiya (2003)[b]		X				X			X	
Marsh et al. (2004) (study 1)	X	X	X							
Marsh et al. (2004) (study 2)	X	X	X							
Marsh et al. (2004) (study 3)	X	X	X							
Marsh et al. (2004) (study 4)	X	X	X					X		
Little et al. (2006)			X[c]	X			X			
Klein & Muthén (2007) (study 1)							X	X		
Klein & Muthén (2007) (study 2)							X	X		
Klein & Muthén (2007) (study 4)	X	X	X					X		

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.
[a] They did not use mean-structure with the constrained approach. [b] They also examined the Kenny-Judd model without the constraint on Kappa (constraint #4).
[c] They did not use mean-structure with the unconstrained approach

Table 3

Summary of Population-Generating Model Features Used in Previous Simulation Studies

	# of Methods	# of Reps	# of conditions	$R_{\gamma 3}^2$	Multiple R^2	N	Loadings	ϕ_{12}	Normality
Jaccard & Wan (1995)	1	150	48	0, .05, & .10	.30 or .50	175 & 200	.949 & .837	.2 and .4	Normal
Schermelleh-Engel et al. (1998)	3	500	9	small, medium, large	N/A	200, 400, 800	.6 or .7 (within)	.235	N/A
Klein & Moosbrugger (2000; study 1)	3	200	1	zero, small, medium, large	N/A	400	.6 or .7 (within)	.235	Normal
Klein & Moosbrugger (2000; study 2)	2	500	4	zero, small, medium, large	N/A	400	.6 or .7 (within)	.235	Ksi1: skew=-2.0, kurt=6.0; Ksi2: Skew=1.5, kurt=5.0
Wall & Amemiya (2001)	1	1000	9	N/A	N/A	200, 500, 1000	.3, .4, .5, .7,.8 (within)	N/A	Normal Uniform (skew=0, kurt=-1.2) χ^2 (df=9, skew=0.94, kurt=1.33)
Moulder & Algina (2002)	2	200	144	0, .05, & .10	.2 or .5	175 or 400	.71, .84, or .95	.2 or .4	Normal
Wall & Amemiya (2003)	3	1000	3	N/A	N/A	200, 500, 1000	.3, .4, .5, .7,.8 (within)	.5	Normal
Marsh et al. (2004) (study 1)	3	250	9	.047	.46	100, 200, 500	.7	.3	Normal
Marsh et al. (2004) (study 2)	3	250	9	.047	.46	100, 200, 500	.5, .7 or .9 (within)	.3	Normal
Marsh et al. (2004) (study 3)	3	250	12	0 to 0.101	.384 to .552	100, 200, 500	.7 &.9	.2 & .4	Normal
Marsh et al. (2004) (study 4)	4	250	36	0 & .047	.384 to .46	100, 200, 500	N/A	.3 & .7	Normal, Uniform (skew=0.0, kurt=-1.2), χ^2 (df=6, skew=1.15, kurt=2.0)
Little et al. (2006)	3	1000	1	.047	N/A	1500	.7	.3	Normal
Klein & Muthén (2007) (study 1)	2	500	1	.33	N/A	400	.837 &.728	.235	Normal

Klein & Muthén (2007) (study 2)	2	500	1	.33	N/A	400	.837 & .728	.235	ksi1 skew=-2.0 kurt=6.0	ksi2 Skew=1.5 kurt=5.0
Klein & Muthén (2007) (study 4)	4	250	12	0 & .047	.384 to .46	200	N/A	.3 & .7	Normal, Uniform (skew=0.0, kurt=-1.2), χ^2 (df=6, skew=1.15, kurt=2.0)	

Table 4

Summary of Manipulated Features

	1	2	3
$R_{\gamma 3}^{2}$	.00	.05	.10
N	100	250	500
loadings	0.50	0.80	
ϕ_{21}	.20	.40	.60
distributions of ξ_1 and ξ_2	Normal	skew =3, kurtosis = 22	

Table 5

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	-0.001	0.736	0.545	0.350	-0.022	7.184	0.654	9.979	0.002	1.343	0.691	0.943
			PC	0.002	40.165	2.034	18.746	-0.010	1954.366	2.508	778.105	0.005	11.287	1.202	8.386
			UC	-0.005	12.056	1.062	10.354	-0.027	10.771	1.502	6.170	-0.019	559.646	7.285	75.824
			RC	0.003	0.783	0.474	0.653	-0.013	3.805	0.637	4.970	-0.002	18.657	0.815	21.879
			LVS	0.040	0.123	0.396	-0.688	0.018	0.126	0.488	-0.743	0.026	0.141	0.509	-0.723
			2SLS	-0.079	0.415	0.456	-0.089	-0.067	0.429	0.465	-0.078	-0.073	0.420	0.446	-0.057
			LMS	0.061	17.015	0.453	36.585	0.021	0.802	0.497	0.615	0.060	125.235	0.549	227.294
			2SMM	0.350	535.825	3.359	158.497	0.288	1109.998	2.712	408.316	-2.324	408.677	50.258	7.132
			MML	2.425	1.145	52.916	-0.978	0.049	0.599	0.573	0.045	0.057	1.076	0.691	0.558
		250	Cons	-0.027	0.200	0.220	-0.090	-0.031	0.218	0.219	-0.006	-0.018	0.237	0.229	0.039
			PC	-0.017	2.011	0.339	4.928	-0.014	2.768	0.373	6.430	-0.003	69.143	0.421	163.104
			UC	-0.012	4.617	0.416	10.102	-0.023	0.889	0.297	1.995	-0.015	5.396	0.624	7.646
			RC	-0.022	0.246	0.213	0.155	-0.035	0.234	0.213	0.101	-0.030	0.217	0.223	-0.025
			LVS	0.020	0.061	0.214	-0.717	0.018	0.063	0.215	-0.706	0.017	0.065	0.226	-0.712
			2SLS	-0.015	0.270	0.283	-0.046	-0.021	0.278	0.280	-0.011	0.013	0.314	0.316	-0.008
			LMS	0.023	0.221	0.218	0.013	0.021	0.252	0.215	0.171	0.023	0.270	0.232	0.162
			2SMM	0.121	2.896	0.528	4.488	0.033	5.533	0.393	13.062	-0.059	0.595	0.264	1.251
			MML	0.020	0.211	0.225	-0.063	0.028	0.246	0.241	0.023	0.017	0.251	0.244	0.027
		500	Cons	-0.021	0.128	0.126	0.013	-0.025	0.131	0.132	-0.013	-0.012	0.140	0.143	-0.018
			PC	-0.013	2.399	0.159	14.078	-0.018	0.338	0.177	0.908	-0.005	1.789	0.204	7.790
			UC	-0.011	1.606	0.161	8.951	-0.015	0.430	0.173	1.484	-0.001	1.665	0.211	6.881
			RC	-0.023	0.125	0.131	-0.043	-0.022	0.131	0.130	0.007	-0.013	0.146	0.141	0.033
			LVS	0.018	0.041	0.128	-0.677	0.003	0.041	0.130	-0.683	0.014	0.042	0.144	-0.706
			2SLS	-0.003	0.182	0.187	-0.027	-0.003	0.189	0.214	-0.117	0.003	0.207	0.221	-0.063
			LMS	0.021	0.137	0.130	0.051	0.006	0.139	0.131	0.056	0.016	0.149	0.145	0.028
			2SMM	0.074	0.152	0.143	0.063	0.042	0.152	0.156	-0.022	-0.071	0.569	0.177	2.205
			MML	0.019	0.134	0.135	-0.006	0.008	0.137	0.137	0.001	0.013	0.145	0.146	-0.010

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 6

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	-0.021	0.127	0.140	-0.093	-0.016	0.130	0.134	-0.030	-0.017	0.136	0.146	-0.068
			PC	-0.020	0.130	0.139	-0.065	-0.012	0.134	0.138	-0.029	-0.017	0.142	0.152	-0.060
			UC	-0.018	0.128	0.136	-0.058	-0.009	0.133	0.136	-0.021	-0.014	0.142	0.150	-0.058
			RC	-0.016	0.125	0.131	-0.047	-0.011	0.129	0.134	-0.032	-0.013	0.135	0.141	-0.041
			LVS	0.008	0.092	0.131	-0.303	0.019	0.094	0.134	-0.301	0.010	0.097	0.141	-0.311
			2SLS	0.002	0.151	0.161	-0.062	0.013	0.157	0.158	-0.007	-0.003	0.167	0.178	-0.061
			LMS	0.010	0.130	0.134	-0.030	0.018	0.135	0.133	0.015	0.010	0.140	0.142	-0.018
			2SMM	0.062	0.141	0.148	-0.048	0.017	0.136	0.141	-0.034	-0.076	0.124	0.128	-0.038
			MML	0.042	0.130	0.184	-0.293	0.053	0.136	0.197	-0.312	0.044	0.144	0.207	-0.307
		250	Cons	-0.033	0.079	0.082	-0.037	-0.028	0.080	0.082	-0.026	-0.019	0.082	0.081	0.018
			PC	-0.032	0.080	0.081	-0.019	-0.028	0.081	0.083	-0.023	-0.018	0.083	0.081	0.027
			UC	-0.032	0.080	0.082	-0.027	-0.027	0.081	0.082	-0.017	-0.018	0.083	0.081	0.029
			RC	-0.032	0.079	0.081	-0.027	-0.027	0.080	0.082	-0.021	-0.018	0.082	0.081	0.016
			LVS	-0.003	0.058	0.082	-0.292	-0.004	0.058	0.081	-0.285	-0.001	0.059	0.082	-0.278
			2SLS	-0.012	0.094	0.102	-0.074	-0.001	0.095	0.097	-0.014	0.003	0.099	0.101	-0.017
			LMS	-0.005	0.080	0.081	-0.020	-0.002	0.080	0.082	-0.015	-0.002	0.083	0.081	0.031
			2SMM	0.059	0.086	0.086	0.000	0.014	0.083	0.082	0.009	-0.074	0.077	0.079	-0.032
			MML	0.026	0.089	0.117	-0.235	0.024	0.090	0.122	-0.267	0.012	0.089	0.115	-0.230
		500	Cons	-0.023	0.055	0.057	-0.034	-0.021	0.056	0.056	-0.005	-0.020	0.058	0.059	-0.013
			PC	-0.022	0.056	0.057	-0.025	-0.021	0.056	0.057	-0.001	-0.019	0.058	0.059	-0.014
			UC	-0.022	0.056	0.057	-0.019	-0.021	0.057	0.056	0.010	-0.021	0.059	0.058	0.006
			RC	-0.022	0.056	0.057	-0.021	-0.021	0.056	0.055	0.018	-0.019	0.058	0.058	0.003
			LVS	0.007	0.040	0.057	-0.294	0.005	0.041	0.056	-0.269	0.000	0.042	0.058	-0.285
			2SLS	0.006	0.066	0.069	-0.042	0.004	0.067	0.069	-0.027	0.002	0.070	0.075	-0.074
			LMS	0.008	0.056	0.057	-0.022	0.005	0.056	0.055	0.018	0.000	0.059	0.058	0.010
			2SMM	0.061	0.060	0.066	-0.085	0.016	0.058	0.058	0.007	-0.074	0.054	0.054	0.003
			MML	0.041	0.063	0.079	-0.212	0.036	0.063	0.086	-0.263	0.021	0.064	0.080	-0.198

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 7

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	-0.033	15.511	1.012	14.328	-0.019	3.711	0.832	3.460	0.007	4.997	0.967	4.168
			PC	-0.015	14.851	1.266	10.735	0.005	41.903	1.259	32.282	0.019	24.101	4.664	4.168
			UC	-0.025	7.675	0.821	8.346	-0.014	5.767	0.968	4.959	0.021	24.088	3.526	5.832
			RC	-0.041	1.815	0.569	2.191	-0.019	0.911	0.469	0.942	0.004	8.981	0.883	9.167
			LVS	0.014	0.126	0.542	-0.767	-0.137	4.204	3.713	0.132	0.061	0.134	0.509	-0.737
			2SLS	-0.054	0.421	0.415	0.014	-0.035	0.422	0.492	-0.142	-0.019	0.428	0.539	-0.206
			LMS	0.037	2.155	0.452	3.767	0.033	1.845	0.523	2.526	0.058	1.245	0.538	1.316
			2SMM	0.568	9378.216	10.209	917.656	0.338	1652.907	2.909	567.176	0.153	52.071	1.514	33.384
			MML	0.086	0.563	0.685	-0.178	3.709	0.597	81.661	-0.993	0.076	0.700	0.539	0.299
		250	Cons	-0.031	2.396	0.803	1.985	-0.018	0.212	0.206	0.029	-0.022	0.235	0.216	0.090
			PC	-0.006	2.115	0.357	4.931	0.006	3.620	0.511	6.085	0.003	6.218	0.548	10.354
			UC	-0.019	6.217	0.426	13.600	-0.005	1.660	0.332	3.996	0.013	14.496	0.774	17.740
			RC	-0.037	0.375	0.210	0.783	-0.017	0.272	0.203	0.340	-0.019	0.282	0.209	0.349
			LVS	0.013	0.062	0.207	-0.702	0.026	0.064	0.202	-0.684	-0.005	0.061	0.206	-0.706
			2SLS	-0.021	0.265	0.294	-0.099	0.010	0.282	0.301	-0.064	-0.013	0.291	0.297	-0.022
			LMS	0.021	0.335	0.217	0.545	0.034	0.231	0.206	0.120	0.001	0.241	0.206	0.171
			2SMM	0.005	317.188	1.558	202.638	0.012	1.304	0.351	2.719	-0.036	0.609	0.294	1.073
			MML	0.020	0.217	0.235	-0.077	0.034	0.216	0.220	-0.017	-0.003	0.221	0.216	0.025
		500	Cons	-0.030	0.126	0.139	-0.088	-0.020	0.132	0.143	-0.077	-0.009	0.141	0.142	-0.006
			PC	-0.017	0.232	0.173	0.341	-0.020	0.249	0.186	0.338	0.008	1.293	0.224	4.761
			UC	-0.016	9.441	0.319	28.606	-0.020	1.487	0.231	5.443	-0.002	0.442	0.182	1.434
			RC	-0.030	0.139	0.136	0.025	-0.018	0.158	0.140	0.129	-0.006	0.145	0.146	-0.005
			LVS	0.005	0.041	0.137	-0.699	0.011	0.041	0.140	-0.704	0.016	0.042	0.144	-0.706
			2SLS	-0.014	0.192	0.206	-0.070	0.009	0.193	0.219	-0.118	0.001	0.199	0.220	-0.097
			LMS	0.008	0.132	0.135	-0.016	0.014	0.138	0.139	-0.003	0.021	0.150	0.145	0.036
			2SMM	0.053	0.161	0.153	0.054	0.016	0.161	0.160	0.010	-0.074	0.151	0.140	0.074
			MML	0.006	0.130	0.139	-0.060	0.008	0.133	0.134	-0.007	0.015	0.145	0.141	0.025

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 8

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.024	0.127	0.135	-0.060	-0.044	0.127	0.133	-0.047	-0.011	0.135	0.146	-0.076
			PC	-0.021	0.131	0.137	-0.043	-0.041	0.131	0.136	-0.035	-0.008	0.140	0.151	-0.072
			UC	-0.020	0.130	0.136	-0.046	-0.038	0.129	0.135	-0.044	-0.005	0.139	0.152	-0.082
			RC	-0.021	0.126	0.141	-0.104	-0.043	0.126	0.135	-0.064	-0.010	0.134	0.146	-0.084
			LVS	0.013	0.091	0.135	-0.326	-0.021	0.092	0.132	-0.299	0.012	0.096	0.144	-0.336
			2SLS	0.009	0.153	0.168	-0.091	-0.019	0.154	0.165	-0.065	0.003	0.164	0.177	-0.072
			LMS	0.016	0.133	0.136	-0.020	-0.016	0.133	0.132	0.008	0.016	0.139	0.143	-0.023
			2SMM	0.053	0.146	0.137	0.070	0.005	0.140	0.139	0.004	-0.088	0.131	0.132	-0.005
			MML	0.041	0.132	0.192	-0.312	-0.002	0.130	0.182	-0.285	0.045	0.137	0.202	-0.321
		250	Cons	-0.029	0.077	0.084	-0.075	-0.022	0.078	0.088	-0.117	-0.013	0.081	0.084	-0.029
			PC	-0.030	0.079	0.084	-0.071	-0.021	0.079	0.088	-0.103	-0.014	0.082	0.084	-0.021
			UC	-0.028	0.079	0.084	-0.062	-0.020	0.079	0.088	-0.100	-0.012	0.082	0.083	-0.010
			RC	-0.028	0.078	0.084	-0.075	-0.022	0.078	0.091	-0.137	-0.012	0.081	0.085	-0.037
			LVS	0.000	0.056	0.081	-0.306	0.001	0.056	0.089	-0.366	0.001	0.058	0.083	-0.304
			2SLS	-0.002	0.093	0.104	-0.103	0.005	0.094	0.106	-0.109	0.000	0.098	0.102	-0.035
			LMS	0.003	0.080	0.082	-0.026	0.003	0.080	0.088	-0.090	0.003	0.083	0.082	0.005
			2SMM	0.035	0.089	0.087	0.015	0.000	0.086	0.089	-0.033	-0.072	0.080	0.077	0.045
			MML	0.031	0.087	0.123	-0.290	0.025	0.087	0.119	-0.275	0.028	0.091	0.114	-0.203
		500	Cons	-0.031	0.054	0.057	-0.036	-0.021	0.055	0.056	-0.016	-0.016	0.057	0.060	-0.055
			PC	-0.032	0.055	0.058	-0.059	-0.022	0.056	0.056	-0.010	-0.015	0.057	0.060	-0.051
			UC	-0.031	0.055	0.057	-0.026	-0.022	0.056	0.056	-0.006	-0.014	0.057	0.060	-0.049
			RC	-0.030	0.055	0.058	-0.053	-0.020	0.056	0.057	-0.023	-0.014	0.057	0.060	-0.051
			LVS	-0.004	0.040	0.055	-0.283	0.001	0.040	0.055	-0.274	-0.002	0.040	0.059	-0.318
			2SLS	-0.004	0.065	0.072	-0.101	0.003	0.066	0.068	-0.025	0.000	0.069	0.074	-0.073
			LMS	-0.001	0.056	0.056	0.000	0.004	0.057	0.055	0.027	0.000	0.058	0.059	-0.029
			2SMM	0.037	0.062	0.065	-0.036	-0.002	0.061	0.060	0.017	-0.075	0.056	0.060	-0.056
			MML	0.028	0.061	0.080	-0.232	0.028	0.063	0.078	-0.191	0.018	0.063	0.084	-0.246

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 9

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	-0.014	13.417	0.843	14.907	0.003	1.061	0.593	0.790	0.015	7.656	1.513	4.059
			PC	-0.004	8.587	1.126	6.625	0.005	14.197	1.389	9.222	0.023	19.091	1.645	10.604
			UC	0.001	29.050	1.405	19.679	0.013	12.081	1.271	8.503	0.011	10.218	0.981	9.414
			RC	-0.012	1.499	0.555	1.699	0.004	19.211	1.222	14.719	0.008	1.095	0.514	1.129
			LVS	0.091	0.315	0.890	-0.646	-0.151	0.995	4.769	-0.791	0.077	0.127	0.644	-0.803
			2SLS	-0.103	0.432	0.469	-0.078	-0.064	0.435	0.518	-0.160	-0.031	0.439	0.473	-0.073
			LMS	0.056	1.508	0.448	2.364	0.106	5.343	0.561	8.516	0.046	2.565	0.496	4.167
			2SMM	0.032	133.568	1.721	76.604	0.144	188.634	3.099	59.861	-0.216	1408.184	5.986	234.260
			MML	0.035	0.683	0.603	0.134	-0.811	0.685	21.092	-0.968	0.076	0.813	0.662	0.228
		250	Cons	-0.024	0.199	0.208	-0.042	-0.029	0.208	0.213	-0.023	-0.016	0.502	0.268	0.875
			PC	-0.016	5.590	0.460	11.151	-0.020	16.404	0.619	25.507	-0.003	58.294	1.019	56.196
			UC	-0.004	22.029	0.707	30.156	-0.023	7.675	0.697	10.008	0.007	2.797	0.481	4.809
			RC	-0.021	0.337	0.204	0.654	-0.029	0.292	0.206	0.418	-0.014	0.485	0.248	0.958
			LVS	0.012	0.060	0.203	-0.704	0.002	0.061	0.203	-0.697	0.006	0.062	0.221	-0.719
			2SLS	-0.037	0.268	0.305	-0.119	-0.012	0.273	0.310	-0.121	-0.007	0.309	0.345	-0.104
			LMS	0.018	0.218	0.197	0.102	0.010	0.222	0.201	0.109	0.016	0.266	0.221	0.204
			2SMM	0.076	9.042	0.484	17.675	0.020	33.719	0.707	46.681	-0.065	3.389	0.439	6.727
			MML	0.017	0.211	0.212	-0.007	0.009	0.207	0.206	0.008	0.014	0.248	0.247	0.001
		500	Cons	-0.025	0.126	0.132	-0.042	-0.026	0.131	0.141	-0.074	-0.011	0.139	0.141	-0.013
			PC	-0.011	0.472	0.199	1.374	-0.018	3.855	0.374	9.321	-0.001	5.472	0.374	13.646
			UC	-0.016	0.343	0.179	0.912	-0.020	0.317	0.195	0.625	0.000	2.450	0.266	8.217
			RC	-0.025	0.145	0.131	0.106	-0.024	0.152	0.142	0.071	-0.010	0.160	0.138	0.159
			LVS	0.009	0.041	0.129	-0.682	0.009	0.042	0.140	-0.703	0.000	0.041	0.137	-0.701
			2SLS	0.002	0.180	0.203	-0.113	-0.013	0.192	0.248	-0.228	-0.016	0.203	0.235	-0.139
			LMS	0.017	0.132	0.130	0.013	0.014	0.136	0.137	-0.009	0.007	0.145	0.138	0.049
			2SMM	0.042	0.172	0.168	0.021	-0.004	0.161	0.160	0.006	-0.074	0.172	0.156	0.104
			MML	0.015	0.129	0.132	-0.022	0.013	0.134	0.137	-0.021	-0.001	0.140	0.138	0.013

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 10

Parameter Estimates for γ_1 for the Normally Distributed, $R_{\gamma3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	-0.030	0.124	0.135	-0.088	-0.029	0.124	0.143	-0.135	-0.013	0.129	0.140	-0.080
			PC	-0.028	0.129	0.139	-0.069	-0.025	0.130	0.151	-0.141	-0.009	0.134	0.146	-0.078
			UC	-0.024	0.128	0.137	-0.064	-0.025	0.129	0.148	-0.132	-0.008	0.133	0.144	-0.078
			RC	-0.028	0.125	0.138	-0.100	-0.025	0.124	0.146	-0.146	-0.008	0.128	0.143	-0.104
			LVS	-0.001	0.090	0.128	-0.298	-0.005	0.089	0.135	-0.339	0.000	0.090	0.138	-0.347
			2SLS	0.004	0.151	0.161	-0.062	-0.001	0.152	0.174	-0.122	-0.002	0.159	0.177	-0.098
			LMS	0.009	0.131	0.132	-0.003	0.002	0.130	0.140	-0.071	0.005	0.133	0.138	-0.036
			2SMM	0.017	0.147	0.156	-0.059	0.005	0.147	0.165	-0.109	-0.063	0.136	0.136	-0.002
			MML	0.034	0.129	0.190	-0.318	0.032	0.131	0.191	-0.315	0.017	0.136	0.190	-0.285
		250	Cons	-0.029	0.076	0.080	-0.052	-0.028	0.077	0.077	-0.003	-0.021	0.080	0.083	-0.037
			PC	-0.029	0.078	0.081	-0.037	-0.027	0.078	0.079	-0.004	-0.020	0.082	0.086	-0.046
			UC	-0.028	0.078	0.080	-0.032	-0.026	0.078	0.078	0.008	-0.018	0.082	0.085	-0.038
			RC	-0.028	0.077	0.088	-0.126	-0.026	0.078	0.079	-0.022	-0.020	0.081	0.085	-0.046
			LVS	-0.002	0.055	0.080	-0.308	-0.008	0.055	0.075	-0.256	-0.009	0.057	0.081	-0.305
			2SLS	0.002	0.092	0.102	-0.098	-0.004	0.094	0.108	-0.135	-0.011	0.098	0.106	-0.074
			LMS	0.004	0.079	0.080	-0.013	-0.003	0.080	0.075	0.068	-0.006	0.082	0.082	-0.001
			2SMM	0.009	0.092	0.091	0.018	-0.010	0.089	0.094	-0.051	-0.077	0.083	0.080	0.038
			MML	0.024	0.086	0.111	-0.223	0.009	0.085	0.105	-0.191	0.011	0.089	0.115	-0.225
		500	Cons	-0.035	0.053	0.055	-0.040	-0.025	0.054	0.060	-0.095	-0.014	0.056	0.055	0.009
			PC	-0.034	0.054	0.056	-0.044	-0.024	0.055	0.060	-0.093	-0.014	0.056	0.055	0.016
			UC	-0.032	0.054	0.055	-0.018	-0.024	0.055	0.060	-0.085	-0.013	0.057	0.056	0.015
			RC	-0.035	0.054	0.057	-0.057	-0.024	0.055	0.062	-0.123	-0.015	0.056	0.057	-0.014
			LVS	-0.011	0.039	0.054	-0.280	-0.005	0.039	0.059	-0.337	-0.004	0.039	0.054	-0.278
			2SLS	-0.008	0.064	0.071	-0.099	-0.002	0.065	0.076	-0.147	0.002	0.068	0.071	-0.048
			LMS	-0.004	0.055	0.055	0.006	0.000	0.056	0.059	-0.052	0.000	0.057	0.055	0.040
			2SMM	0.019	0.065	0.067	-0.029	-0.015	0.063	0.063	-0.004	-0.073	0.058	0.058	0.003
			MML	0.014	0.060	0.076	-0.212	0.020	0.061	0.081	-0.245	0.021	0.062	0.076	-0.184

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 11

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	0.029	0.504	0.435	0.157	0.042	7.386	0.685	9.783	0.023	5.244	1.202	3.362
			PC	0.144	9.793	0.754	11.980	0.098	2927.346	3.546	824.587	0.156	16.768	1.530	9.958
			UC	0.196	21.661	1.471	13.723	0.125	5.252	0.670	6.834	0.135	416.898	25.857	15.124
			RC	0.031	0.606	0.425	0.425	0.021	3.303	0.503	5.565	0.036	31.176	1.163	25.796
			LVS	0.045	0.104	0.389	-0.734	0.029	0.120	0.493	-0.756	0.033	0.159	0.599	-0.735
			2SLS	-0.057	0.400	0.433	-0.077	-0.050	0.426	0.446	-0.046	-0.083	0.489	0.521	-0.062
			LMS	0.065	1.833	0.446	3.113	0.052	0.985	0.487	1.023	0.032	97.694	0.670	144.837
			2SMM	-0.584	703.363	4.503	155.209	-0.115	851.684	3.893	217.789	-0.097	423.631	3.674	114.299
			MML	12.764	0.390	284.663	-0.999	0.046	0.540	0.511	0.057	0.057	2.964	0.841	2.526
		250	Cons	-0.003	0.177	0.181	-0.020	-0.017	0.213	0.210	0.013	-0.057	0.313	0.288	0.084
			PC	0.042	4.908	0.375	12.089	0.011	1.463	0.315	3.647	-0.001	23.799	0.573	40.500
			UC	0.025	4.190	0.345	11.149	0.019	1.069	0.320	2.335	0.016	4.484	0.694	5.463
			RC	-0.008	0.207	0.181	0.144	-0.007	0.239	0.197	0.214	-0.045	0.273	0.266	0.027
			LVS	0.013	0.056	0.178	-0.686	0.023	0.063	0.199	-0.685	0.022	0.073	0.269	-0.727
			2SLS	0.001	0.246	0.276	-0.111	-0.019	0.279	0.279	0.001	-0.051	0.357	0.382	-0.063
			LMS	0.016	0.196	0.181	0.084	0.028	0.235	0.205	0.146	0.024	0.317	0.276	0.147
			2SMM	-0.291	2.490	0.391	5.376	-0.206	8.858	0.672	12.177	-0.169	0.416	0.262	0.584
			MML	0.019	0.185	0.180	0.029	0.027	0.226	0.221	0.022	0.034	0.303	0.288	0.051
		500	Cons	-0.012	0.115	0.117	-0.020	-0.019	0.130	0.127	0.028	-0.062	0.167	0.173	-0.033
			PC	-0.004	2.138	0.145	13.742	-0.008	0.356	0.167	1.132	-0.042	1.839	0.274	5.704
			UC	-0.008	0.520	0.141	2.694	-0.006	0.429	0.165	1.592	-0.048	0.606	0.238	1.545
			RC	-0.006	0.113	0.118	-0.047	-0.016	0.130	0.126	0.026	-0.060	0.168	0.174	-0.034
			LVS	-0.004	0.038	0.118	-0.674	0.008	0.041	0.126	-0.674	0.009	0.048	0.173	-0.719
			2SLS	-0.007	0.167	0.180	-0.075	-0.009	0.190	0.207	-0.080	-0.011	0.247	0.289	-0.146
			LMS	-0.001	0.122	0.118	0.035	0.010	0.137	0.128	0.070	0.011	0.178	0.173	0.031
			2SMM	-0.251	0.171	0.147	0.169	-0.240	0.149	0.157	-0.047	-0.180	0.450	0.161	1.804
			MML	0.001	0.119	0.118	0.007	0.011	0.134	0.129	0.040	0.018	0.172	0.172	-0.005

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 12

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	-0.018	0.120	0.125	-0.036	-0.036	0.129	0.131	-0.020	-0.065	0.156	0.165	-0.054
			PC	-0.015	0.123	0.128	-0.034	-0.034	0.133	0.136	-0.021	-0.060	0.163	0.169	-0.038
			UC	-0.013	0.122	0.126	-0.031	-0.034	0.131	0.132	-0.006	-0.059	0.162	0.169	-0.044
			RC	-0.014	0.119	0.123	-0.031	-0.035	0.127	0.128	-0.003	-0.063	0.154	0.157	-0.016
			LVS	-0.007	0.088	0.124	-0.291	-0.012	0.093	0.127	-0.263	0.002	0.110	0.159	-0.309
			2SLS	-0.011	0.145	0.153	-0.056	-0.018	0.155	0.168	-0.079	-0.004	0.192	0.201	-0.044
			LMS	-0.005	0.123	0.123	0.002	-0.010	0.132	0.128	0.026	0.006	0.162	0.160	0.008
			2SMM	-0.263	0.143	0.147	-0.029	-0.235	0.139	0.143	-0.028	-0.172	0.129	0.140	-0.084
			MML	0.024	0.117	0.168	-0.307	0.022	0.120	0.182	-0.339	0.044	0.148	0.236	-0.374
		250	Cons	-0.007	0.074	0.074	0.004	-0.023	0.080	0.083	-0.027	-0.061	0.095	0.100	-0.054
			PC	-0.007	0.075	0.073	0.025	-0.022	0.081	0.083	-0.025	-0.060	0.096	0.101	-0.052
			UC	-0.006	0.075	0.073	0.022	-0.020	0.081	0.082	-0.013	-0.059	0.096	0.099	-0.033
			RC	-0.006	0.074	0.073	0.026	-0.021	0.080	0.081	-0.011	-0.059	0.095	0.098	-0.025
			LVS	-0.001	0.054	0.072	-0.250	0.004	0.058	0.080	-0.277	-0.001	0.067	0.098	-0.321
			2SLS	0.003	0.088	0.090	-0.028	0.000	0.096	0.101	-0.048	-0.003	0.115	0.114	0.009
			LMS	-0.001	0.075	0.072	0.039	0.006	0.081	0.082	-0.006	0.002	0.096	0.098	-0.020
			2SMM	-0.248	0.088	0.092	-0.045	-0.234	0.084	0.085	-0.014	-0.182	0.078	0.081	-0.031
			MML	0.032	0.078	0.104	-0.253	0.046	0.084	0.122	-0.307	0.060	0.098	0.137	-0.288
		500	Cons	-0.008	0.052	0.055	-0.054	-0.024	0.056	0.056	-0.004	-0.061	0.066	0.068	-0.026
			PC	-0.008	0.052	0.054	-0.043	-0.024	0.056	0.056	-0.001	-0.060	0.066	0.068	-0.024
			UC	-0.007	0.052	0.054	-0.030	-0.023	0.056	0.056	0.010	-0.058	0.067	0.067	-0.012
			RC	-0.008	0.052	0.054	-0.030	-0.024	0.056	0.055	0.012	-0.060	0.066	0.068	-0.020
			LVS	-0.003	0.038	0.054	-0.294	-0.001	0.040	0.055	-0.271	-0.001	0.047	0.068	-0.311
			2SLS	-0.004	0.061	0.065	-0.050	-0.002	0.066	0.066	-0.001	-0.005	0.080	0.085	-0.058
			LMS	-0.002	0.052	0.054	-0.029	-0.001	0.056	0.055	0.016	-0.002	0.066	0.068	-0.019
			2SMM	-0.258	0.062	0.064	-0.034	-0.232	0.060	0.058	0.024	-0.181	0.055	0.056	-0.010
			MML	0.029	0.055	0.074	-0.249	0.035	0.059	0.077	-0.232	0.054	0.070	0.092	-0.236

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 13

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	0.065	8.362	0.606	12.790	0.031	4.073	0.801	4.082	0.066	4.563	1.152	2.961
			PC	0.130	18.828	1.221	14.419	0.163	27.880	1.114	24.018	0.233	16.433	10.457	0.571
			UC	0.154	9.046	0.840	9.775	0.142	6.075	1.060	4.730	0.210	32.328	2.466	12.111
			RC	0.056	1.164	0.437	1.664	0.020	1.369	0.575	1.383	0.074	14.444	1.311	10.016
			LVS	0.070	0.111	0.388	-0.715	-0.028	0.344	1.802	-0.809	0.165	0.395	2.521	-0.843
			2SLS	-0.023	0.390	0.411	-0.051	-0.079	0.405	0.430	-0.059	-0.096	0.473	0.509	-0.070
			LMS	0.086	2.235	0.433	4.166	0.082	2.626	0.475	4.528	0.131	2.144	1.156	0.854
			2SMM	0.051	6018.238	6.598	911.101	3.533	25036.216	98.458	253.283	-0.404	62.522	1.834	33.088
			MML	0.036	0.393	0.471	-0.166	2.098	0.579	45.609	-0.987	-0.223	1.227	6.723	-0.818
		250	Cons	-0.006	0.190	0.190	0.000	-0.032	0.205	0.211	-0.030	-0.039	0.305	0.305	0.000
			PC	0.024	1.903	0.339	4.612	0.037	3.418	0.474	6.218	0.037	8.216	0.640	11.839
			UC	0.028	4.058	0.332	11.204	-0.005	1.478	0.299	3.947	0.041	20.635	0.768	25.866
			RC	-0.009	0.284	0.188	0.509	-0.030	0.242	0.205	0.183	-0.042	0.356	0.295	0.209
			LVS	0.010	0.056	0.185	-0.696	0.007	0.064	0.211	-0.697	0.037	0.070	0.283	-0.752
			2SLS	-0.018	0.240	0.268	-0.105	-0.031	0.280	0.302	-0.073	-0.043	0.331	0.348	-0.049
			LMS	0.012	0.220	0.190	0.163	0.008	0.233	0.216	0.078	0.034	0.326	0.297	0.097
			2SMM	-0.168	285.359	1.354	209.676	-0.205	1.245	0.292	3.263	-0.186	0.454	0.256	0.770
			MML	0.015	0.185	0.191	-0.032	0.010	0.210	0.214	-0.021	0.042	0.286	0.288	-0.005
		500	Cons	-0.007	0.116	0.117	-0.016	-0.026	0.130	0.138	-0.057	-0.064	0.167	0.155	0.077
			PC	0.002	0.164	0.138	0.190	-0.007	0.310	0.197	0.572	-0.040	1.847	0.307	5.009
			UC	0.007	1.918	0.211	8.087	0.003	2.631	0.309	7.506	-0.037	1.235	0.271	3.556
			RC	-0.007	0.123	0.114	0.079	-0.026	0.155	0.135	0.147	-0.065	0.173	0.163	0.060
			LVS	-0.002	0.039	0.115	-0.666	0.001	0.041	0.135	-0.696	0.003	0.048	0.158	-0.695
			2SLS	-0.008	0.171	0.179	-0.045	-0.016	0.193	0.229	-0.157	-0.006	0.237	0.258	-0.080
			LMS	0.004	0.121	0.114	0.061	0.006	0.137	0.135	0.012	0.006	0.178	0.157	0.133
			2SMM	-0.254	0.149	0.139	0.068	-0.227	0.152	0.149	0.020	-0.180	0.148	0.137	0.076
			MML	0.009	0.119	0.120	-0.011	0.013	0.132	0.134	-0.013	0.015	0.171	0.161	0.065

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 14

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.009	0.117	0.130	-0.098	-0.019	0.127	0.140	-0.088	-0.079	0.150	0.163	-0.081
			PC	-0.006	0.121	0.133	-0.087	-0.017	0.132	0.142	-0.071	-0.073	0.156	0.168	-0.069
			UC	-0.002	0.120	0.131	-0.086	-0.015	0.131	0.142	-0.075	-0.070	0.154	0.164	-0.061
			RC	-0.006	0.116	0.129	-0.098	-0.015	0.128	0.141	-0.094	-0.075	0.149	0.161	-0.076
			LVS	0.000	0.085	0.125	-0.321	0.009	0.092	0.135	-0.317	-0.021	0.106	0.159	-0.332
			2SLS	-0.001	0.141	0.157	-0.101	0.002	0.154	0.161	-0.046	-0.019	0.184	0.203	-0.093
			LMS	0.003	0.121	0.126	-0.041	0.011	0.132	0.138	-0.048	-0.018	0.156	0.160	-0.023
			2SMM	-0.251	0.148	0.151	-0.022	-0.223	0.139	0.144	-0.035	-0.170	0.133	0.145	-0.082
			MML	0.030	0.115	0.169	-0.318	0.035	0.123	0.190	-0.351	0.011	0.139	0.216	-0.358
		250	Cons	-0.008	0.073	0.074	-0.013	-0.025	0.078	0.083	-0.061	-0.058	0.092	0.094	-0.023
			PC	-0.008	0.074	0.075	-0.014	-0.024	0.079	0.083	-0.043	-0.056	0.093	0.093	-0.005
			UC	-0.007	0.074	0.074	-0.004	-0.023	0.080	0.083	-0.037	-0.055	0.093	0.094	-0.009
			RC	-0.008	0.073	0.075	-0.020	-0.024	0.079	0.085	-0.076	-0.056	0.092	0.097	-0.043
			LVS	-0.002	0.053	0.072	-0.261	0.000	0.057	0.081	-0.306	0.000	0.064	0.094	-0.317
			2SLS	0.003	0.087	0.090	-0.028	-0.002	0.095	0.098	-0.033	0.000	0.112	0.114	-0.017
			LMS	0.002	0.075	0.073	0.021	0.003	0.080	0.082	-0.019	0.000	0.094	0.094	0.003
			2SMM	-0.255	0.089	0.088	0.012	-0.231	0.086	0.084	0.018	-0.188	0.080	0.081	-0.020
			MML	0.030	0.077	0.101	-0.239	0.031	0.082	0.112	-0.267	0.043	0.097	0.127	-0.235
		500	Cons	-0.005	0.051	0.054	-0.048	-0.026	0.055	0.060	-0.087	-0.058	0.064	0.068	-0.055
			PC	-0.004	0.052	0.054	-0.041	-0.026	0.056	0.060	-0.075	-0.057	0.065	0.068	-0.048
			UC	-0.004	0.052	0.053	-0.021	-0.025	0.056	0.060	-0.066	-0.057	0.065	0.069	-0.049
			RC	-0.005	0.051	0.055	-0.057	-0.024	0.056	0.063	-0.112	-0.057	0.065	0.068	-0.042
			LVS	0.001	0.037	0.052	-0.289	-0.003	0.040	0.060	-0.333	-0.004	0.045	0.067	-0.324
			2SLS	0.003	0.061	0.064	-0.046	-0.003	0.066	0.074	-0.102	-0.005	0.079	0.087	-0.102
			LMS	0.004	0.052	0.053	-0.003	-0.001	0.056	0.059	-0.050	-0.002	0.066	0.067	-0.026
			2SMM	-0.251	0.062	0.066	-0.051	-0.230	0.060	0.063	-0.043	-0.186	0.056	0.061	-0.086
			MML	0.030	0.055	0.073	-0.254	0.035	0.059	0.083	-0.281	0.046	0.069	0.097	-0.291

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 15

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	0.037	24.253	1.378	16.600	0.009	0.916	0.542	0.691	0.068	9.097	1.160	6.841
			PC	0.192	13.163	1.589	7.283	0.146	25.932	2.070	11.525	0.218	99.153	3.666	26.047
			UC	0.118	15.686	1.012	14.492	0.156	17.224	1.201	13.338	0.254	22.987	1.595	13.413
			RC	0.009	1.454	0.519	1.802	0.033	20.148	1.320	14.263	0.061	1.527	0.692	1.205
			LVS	0.022	0.115	0.431	-0.734	0.048	0.139	0.541	-0.743	0.022	0.145	0.822	-0.823
			2SLS	-0.014	0.397	0.420	-0.054	-0.047	0.427	0.465	-0.082	-0.083	0.473	0.515	-0.081
			LMS	0.059	0.816	0.461	0.772	0.062	0.917	0.565	0.624	0.089	1.634	0.589	1.776
			2SMM	-0.074	362.960	2.934	122.728	-0.272	274.254	3.238	83.686	-0.052	1766.251	5.032	349.983
			MML	0.054	0.549	0.520	0.055	4.026	0.797	88.585	-0.991	0.069	1.170	0.849	0.379
		250	Cons	-0.009	0.180	0.188	-0.043	-0.010	0.202	0.200	0.015	-0.031	0.992	0.416	1.386
			PC	0.059	5.654	0.440	11.855	0.073	15.466	0.716	20.603	0.035	47.663	0.976	47.859
			UC	0.071	10.777	0.428	24.205	0.068	7.347	0.663	10.082	0.034	3.306	0.622	4.312
			RC	-0.008	0.264	0.187	0.407	-0.012	0.331	0.196	0.687	-0.032	0.948	0.396	1.393
			LVS	0.010	0.057	0.187	-0.698	0.027	0.060	0.196	-0.695	0.038	0.071	0.304	-0.766
			2SLS	0.014	0.256	0.298	-0.142	-0.017	0.270	0.304	-0.112	-0.030	0.352	0.363	-0.031
			LMS	0.017	0.197	0.186	0.059	0.029	0.217	0.191	0.135	0.031	0.334	0.281	0.190
			2SMM	-0.275	4.014	0.356	10.271	-0.220	18.010	0.537	32.568	-0.174	1.847	0.333	4.550
			MML	0.021	0.186	0.195	-0.047	0.033	0.202	0.196	0.031	0.047	0.321	0.319	0.008
		500	Cons	-0.005	0.116	0.117	-0.001	-0.020	0.130	0.137	-0.050	-0.057	0.165	0.173	-0.044
			PC	-0.001	0.367	0.174	1.116	0.015	4.482	0.436	9.288	-0.016	8.294	0.523	14.859
			UC	-0.004	0.263	0.152	0.734	-0.009	0.275	0.174	0.577	-0.026	2.108	0.421	4.007
			RC	-0.005	0.132	0.116	0.140	-0.019	0.154	0.138	0.122	-0.058	0.198	0.170	0.161
			LVS	0.005	0.039	0.115	-0.663	0.006	0.041	0.135	-0.694	0.004	0.047	0.170	-0.724
			2SLS	0.003	0.164	0.202	-0.187	0.003	0.195	0.244	-0.200	0.002	0.251	0.271	-0.072
			LMS	0.010	0.121	0.113	0.071	0.012	0.136	0.133	0.025	0.008	0.174	0.172	0.015
			2SMM	-0.270	0.152	0.149	0.019	-0.231	0.152	0.148	0.025	-0.181	0.152	0.137	0.114
			MML	0.011	0.117	0.114	0.026	0.015	0.132	0.136	-0.029	0.015	0.167	0.171	-0.027

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 16

Parameter Estimates for γ_2 for the Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	-0.013	0.116	0.129	-0.096	-0.018	0.124	0.129	-0.039	-0.050	0.150	0.162	-0.073
			PC	-0.010	0.121	0.132	-0.081	-0.017	0.129	0.133	-0.030	-0.048	0.157	0.163	-0.038
			UC	-0.009	0.120	0.131	-0.085	-0.015	0.129	0.134	-0.038	-0.045	0.155	0.162	-0.043
			RC	-0.014	0.117	0.135	-0.137	-0.017	0.124	0.134	-0.073	-0.048	0.148	0.166	-0.108
			LVS	-0.005	0.085	0.126	-0.327	-0.003	0.089	0.125	-0.292	0.010	0.102	0.157	-0.349
			2SLS	-0.004	0.141	0.150	-0.061	0.001	0.151	0.161	-0.064	0.007	0.185	0.190	-0.026
			LMS	0.004	0.122	0.127	-0.038	0.005	0.130	0.129	0.010	0.016	0.155	0.157	-0.009
			2SMM	-0.245	0.145	0.152	-0.045	-0.232	0.142	0.147	-0.035	-0.190	0.131	0.129	0.009
			MML	0.026	0.114	0.173	-0.341	0.023	0.122	0.188	-0.353	0.061	0.146	0.209	-0.301
		250	Cons	-0.014	0.072	0.073	-0.025	-0.020	0.078	0.082	-0.052	-0.052	0.091	0.091	0.004
			PC	-0.013	0.073	0.074	-0.018	-0.019	0.079	0.084	-0.055	-0.050	0.093	0.094	-0.009
			UC	-0.012	0.073	0.074	-0.008	-0.018	0.079	0.083	-0.042	-0.051	0.093	0.092	0.010
			RC	-0.013	0.072	0.076	-0.053	-0.020	0.078	0.083	-0.062	-0.050	0.092	0.090	0.016
			LVS	-0.009	0.052	0.071	-0.267	0.003	0.056	0.079	-0.295	0.001	0.063	0.088	-0.279
			2SLS	-0.003	0.086	0.096	-0.101	0.003	0.094	0.106	-0.114	0.007	0.111	0.112	-0.005
			LMS	-0.003	0.075	0.072	0.040	0.008	0.081	0.080	0.011	0.007	0.094	0.090	0.044
			2SMM	-0.248	0.090	0.091	-0.011	-0.232	0.086	0.092	-0.072	-0.189	0.081	0.081	-0.003
			MML	0.028	0.076	0.097	-0.217	0.039	0.081	0.107	-0.242	0.048	0.096	0.123	-0.220
		500	Cons	-0.006	0.050	0.054	-0.064	-0.022	0.054	0.055	-0.018	-0.051	0.063	0.067	-0.056
			PC	-0.006	0.051	0.054	-0.061	-0.022	0.055	0.056	-0.022	-0.050	0.064	0.067	-0.044
			UC	-0.006	0.051	0.054	-0.053	-0.022	0.055	0.056	-0.009	-0.050	0.064	0.067	-0.047
			RC	-0.004	0.051	0.058	-0.122	-0.022	0.055	0.057	-0.040	-0.049	0.064	0.069	-0.076
			LVS	-0.002	0.036	0.053	-0.310	-0.002	0.039	0.053	-0.270	0.001	0.044	0.067	-0.341
			2SLS	0.003	0.060	0.064	-0.071	0.002	0.065	0.070	-0.072	0.001	0.077	0.081	-0.052
			LMS	0.003	0.052	0.053	-0.018	0.003	0.056	0.054	0.045	0.005	0.065	0.067	-0.029
			2SMM	-0.250	0.063	0.065	-0.030	-0.235	0.061	0.059	0.032	-0.197	0.057	0.056	0.010
			MML	0.036	0.054	0.072	-0.260	0.038	0.058	0.073	-0.212	0.040	0.066	0.090	-0.262

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 17

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	0.052	2.288	1.144	0.999	0.045	0.856	0.824	0.039	0.051	4.484	1.521	1.948
			PC	0.035	100.924	2.938	33.355	0.210	4878.409	6.455	754.792	-0.014	37.816	3.287	10.504
			UC	0.313	98.037	6.077	15.132	-0.031	15.204	1.978	6.685	0.186	601.501	4.013	148.880
			RC	0.015	58.963	4.047	13.568	0.166	14.752	2.294	5.430	0.346	27.700	2.033	12.627
			LVS	-0.001	0.253	0.256	-0.011	0.010	0.269	0.226	0.187	0.002	0.314	0.337	-0.069
			2SLS	-0.008	0.681	0.704	-0.032	-0.039	0.697	0.751	-0.071	0.041	0.689	0.791	-0.129
			LMS	0.038	38.407	0.957	39.144	0.068	1.327	0.653	1.030	-0.016	2.175	1.032	1.108
			2SMM	-0.116	1646.133	9.766	167.558	-0.809	4049.605	12.133	332.776	-1.199	1032.36	21.351	47.351
			MML	8.079	2.023	184.678	-0.989	0.055	0.753	0.751	0.002	0.029	3.762	1.168	2.220
		250	Cons	0.014	0.410	0.467	-0.120	-0.004	0.460	0.496	-0.072	-0.012	0.577	0.637	-0.094
			PC	-0.054	16.562	1.606	9.310	0.021	15.442	1.487	9.388	0.046	299.810	1.803	165.315
			UC	-0.047	50.252	2.945	16.065	0.017	8.433	1.588	4.309	-0.112	49.622	3.757	12.209
			RC	0.047	20.595	1.708	11.057	-0.012	6.809	1.278	4.327	0.197	33.675	2.254	13.941
			LVS	0.003	0.105	0.099	0.059	0.004	0.114	0.109	0.038	0.001	0.127	0.126	0.008
			2SLS	0.013	0.543	0.601	-0.096	-0.007	0.537	0.548	-0.020	0.006	0.600	0.602	-0.004
			LMS	0.009	0.341	0.289	0.179	0.012	0.427	0.351	0.215	0.007	0.465	0.393	0.184
			2SMM	0.061	5.771	0.683	7.445	-0.010	18.367	1.404	12.078	0.003	2.551	0.688	2.706
			MML	0.007	0.303	0.297	0.021	0.018	0.343	0.353	-0.026	0.004	0.384	0.395	-0.027
		500	Cons	-0.001	0.258	0.276	-0.063	-0.013	0.286	0.307	-0.067	0.002	0.343	0.351	-0.021
			PC	-0.008	10.405	0.589	16.673	0.037	2.010	0.672	1.989	0.008	33.195	1.314	24.254
			UC	0.027	7.471	1.274	4.865	0.031	2.119	0.721	1.939	0.013	43.298	1.662	25.045
			RC	0.029	1.129	0.567	0.991	-0.014	2.940	0.871	2.375	0.032	4.803	0.880	4.460
			LVS	0.003	0.068	0.067	0.016	-0.004	0.072	0.072	0.000	-0.002	0.081	0.080	0.011
			2SLS	0.013	0.399	0.427	-0.066	-0.065	0.462	0.524	-0.119	-0.020	0.538	0.575	-0.064
			LMS	0.002	0.199	0.189	0.053	-0.005	0.210	0.199	0.058	-0.004	0.251	0.224	0.121
			2SMM	-0.015	0.742	0.345	1.148	-0.017	0.312	0.258	0.206	-0.014	1.654	0.414	2.996
			MML	-0.004	0.186	0.193	-0.037	-0.003	0.200	0.199	0.009	-0.002	0.231	0.225	0.027

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 18

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	0.005	0.135	0.153	-0.115	-0.004	0.143	0.158	-0.096	-0.001	0.161	0.180	-0.101
			PC	0.004	0.152	0.165	-0.077	-0.002	0.166	0.189	-0.123	-0.001	0.198	0.208	-0.049
			UC	0.005	0.156	0.175	-0.110	0.000	0.167	0.180	-0.069	-0.001	0.247	0.241	0.028
			RC	0.004	0.159	0.178	-0.107	-0.002	0.185	0.206	-0.105	-0.001	0.187	0.197	-0.055
			LVS	0.006	0.100	0.102	-0.017	-0.003	0.106	0.107	-0.012	0.002	0.118	0.120	-0.011
			2SLS	0.005	0.188	0.207	-0.089	0.003	0.201	0.218	-0.079	0.004	0.225	0.215	0.049
			LMS	0.004	0.137	0.143	-0.043	-0.005	0.141	0.148	-0.045	0.006	0.164	0.165	-0.002
			2SMM	0.007	0.136	0.145	-0.065	-0.006	0.132	0.137	-0.037	-0.002	0.121	0.123	-0.014
			MML	0.000	0.117	0.206	-0.431	0.004	0.122	0.204	-0.401	0.003	0.142	0.233	-0.391
		250	Cons	0.004	0.082	0.081	0.011	0.005	0.088	0.091	-0.039	0.000	0.097	0.101	-0.034
			PC	0.003	0.086	0.084	0.025	0.005	0.092	0.095	-0.036	0.000	0.103	0.105	-0.022
			UC	0.003	0.086	0.084	0.023	0.005	0.093	0.096	-0.035	0.000	0.103	0.108	-0.043
			RC	0.003	0.086	0.083	0.031	0.005	0.093	0.096	-0.032	-0.001	0.103	0.108	-0.051
			LVS	0.000	0.061	0.057	0.057	0.005	0.064	0.066	-0.034	0.000	0.070	0.069	0.018
			2SLS	-0.003	0.108	0.110	-0.018	0.009	0.117	0.120	-0.029	0.002	0.130	0.133	-0.025
			LMS	0.002	0.080	0.078	0.033	0.006	0.084	0.087	-0.035	-0.001	0.093	0.093	0.006
			2SMM	0.004	0.079	0.073	0.081	0.002	0.075	0.075	0.004	-0.001	0.070	0.074	-0.061
			MML	0.004	0.079	0.124	-0.359	0.010	0.087	0.127	-0.316	-0.003	0.095	0.147	-0.350
		500	Cons	-0.002	0.057	0.056	0.013	0.002	0.060	0.062	-0.024	-0.002	0.068	0.067	0.018
			PC	-0.003	0.059	0.059	0.005	0.002	0.062	0.063	-0.020	-0.001	0.070	0.069	0.021
			UC	-0.004	0.059	0.059	0.006	0.002	0.062	0.063	-0.017	-0.001	0.071	0.069	0.029
			RC	-0.004	0.059	0.057	0.032	0.002	0.062	0.062	-0.003	-0.001	0.071	0.068	0.035
			LVS	-0.003	0.042	0.041	0.018	0.003	0.044	0.045	-0.025	-0.001	0.048	0.046	0.046
			2SLS	-0.002	0.075	0.075	-0.008	0.004	0.078	0.080	-0.028	-0.005	0.089	0.090	-0.011
			LMS	-0.003	0.055	0.055	0.010	0.004	0.057	0.059	-0.030	-0.002	0.065	0.062	0.047
			2SMM	-0.002	0.055	0.057	-0.040	-0.003	0.053	0.053	-0.012	-0.003	0.049	0.052	-0.066
			MML	-0.001	0.058	0.074	-0.222	0.005	0.060	0.079	-0.242	-0.002	0.069	0.094	-0.261

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 19

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	-0.018	3.694	1.916	0.928	0.036	2.417	1.499	0.613	0.085	2.193	1.564	0.402
			PC	-0.137	25.741	2.276	10.309	-0.177	70.388	2.948	22.878	0.318	35.956	18.139	0.982
			UC	0.165	49.355	4.044	11.204	0.176	55.270	5.153	9.727	0.078	64.020	10.440	5.132
			RC	0.067	34.703	3.223	9.766	-0.011	11.012	1.943	4.668	0.211	9.065	2.172	3.174
			LVS	-0.080	0.247	0.241	0.026	0.202	5.960	5.803	0.027	-0.104	0.498	0.520	-0.041
			2SLS	-0.134	0.679	0.729	-0.068	-0.153	0.647	0.723	-0.105	-0.113	0.739	0.744	-0.008
			LMS	0.299	2.213	0.787	1.812	0.328	4.458	0.986	3.519	0.240	1.454	0.839	0.733
			2SMM	-4.945	47383.795	71.516	661.565	-4.239	124962.4	74.724	1671.32	0.019	67.000	2.439	26.471
			MML	0.244	1.086	0.725	0.497	-1.107	0.931	22.849	-0.959	0.352	2.126	2.528	-0.159
		250	Cons	0.007	4.092	1.971	1.076	-0.003	0.465	0.481	-0.033	-0.009	0.526	0.562	-0.065
			PC	-0.033	5.824	1.033	4.640	0.042	12.378	1.663	6.443	0.013	38.897	2.296	15.945
			UC	0.199	37.733	3.640	9.365	0.063	5.083	1.250	3.067	-0.041	63.001	2.629	22.962
			RC	-0.033	17.149	1.437	10.937	-0.069	9.883	1.450	5.817	-0.149	12.616	1.493	7.451
			LVS	-0.059	0.105	0.129	-0.186	-0.050	0.115	0.140	-0.176	-0.060	0.119	0.133	-0.102
			2SLS	-0.012	0.540	0.594	-0.090	-0.049	0.558	0.587	-0.049	-0.044	0.594	0.638	-0.068
			LMS	0.256	0.414	0.376	0.100	0.252	0.399	0.343	0.162	0.219	0.440	0.381	0.155
			2SMM	0.137	938.854	4.501	207.606	-0.017	5.274	0.799	5.601	-0.053	1.664	0.596	1.791
			MML	0.236	0.328	0.375	-0.126	0.244	0.352	0.347	0.013	0.210	0.377	0.382	-0.012
		500	Cons	0.006	0.274	0.299	-0.084	0.001	0.294	0.279	0.053	-0.019	0.347	0.371	-0.064
			PC	0.014	1.804	0.577	2.126	0.002	1.477	0.668	1.211	-0.025	12.627	1.546	7.169
			UC	0.053	71.079	2.007	34.410	0.020	10.662	1.417	6.526	-0.018	10.431	1.776	4.873
			RC	0.008	16.825	1.827	8.210	-0.015	7.028	1.108	5.340	0.006	3.156	1.102	1.864
			LVS	-0.061	0.069	0.081	-0.145	-0.063	0.072	0.078	-0.078	-0.059	0.080	0.089	-0.100
			2SLS	0.057	0.451	0.493	-0.085	0.010	0.481	0.507	-0.051	0.020	0.497	0.542	-0.083
			LMS	0.232	0.219	0.213	0.027	0.218	0.228	0.209	0.087	0.208	0.267	0.245	0.090
			2SMM	-0.093	0.313	0.262	0.194	-0.085	0.282	0.255	0.104	-0.090	0.280	0.256	0.097
			MML	0.223	0.209	0.214	-0.024	0.209	0.214	0.204	0.049	0.197	0.245	0.242	0.011

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 20

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.001	0.135	0.147	-0.084	-0.001	0.143	0.165	-0.132	-0.020	0.161	0.175	-0.078
			PC	0.006	0.152	0.164	-0.073	0.007	0.163	0.180	-0.098	-0.012	0.190	0.199	-0.044
			UC	0.001	0.166	0.191	-0.128	0.005	0.171	0.198	-0.138	-0.012	0.210	0.209	0.005
			RC	-0.011	0.168	0.179	-0.060	-0.007	0.171	0.195	-0.120	-0.025	0.355	0.226	0.572
			LVS	-0.020	0.098	0.101	-0.032	-0.012	0.104	0.112	-0.069	-0.019	0.116	0.122	-0.050
			2SLS	0.033	0.186	0.194	-0.039	0.048	0.197	0.210	-0.060	0.056	0.234	0.252	-0.072
			LMS	0.050	0.137	0.135	0.012	0.058	0.146	0.149	-0.023	0.048	0.162	0.162	0.004
			2SMM	-0.137	0.141	0.139	0.016	-0.132	0.137	0.142	-0.034	-0.132	0.127	0.133	-0.049
			MML	0.073	0.127	0.191	-0.336	0.080	0.137	0.223	-0.385	0.070	0.143	0.243	-0.410
		250	Cons	0.012	0.083	0.081	0.016	0.004	0.086	0.093	-0.077	-0.021	0.097	0.098	-0.010
			PC	0.015	0.087	0.087	-0.007	0.006	0.090	0.096	-0.065	-0.020	0.103	0.103	-0.002
			UC	0.013	0.089	0.092	-0.027	0.005	0.092	0.102	-0.092	-0.020	0.105	0.108	-0.020
			RC	0.008	0.090	0.090	-0.002	0.001	0.094	0.101	-0.073	-0.024	0.106	0.106	0.007
			LVS	-0.010	0.059	0.058	0.028	-0.011	0.061	0.066	-0.075	-0.023	0.068	0.067	0.006
			2SLS	0.060	0.107	0.113	-0.056	0.050	0.113	0.123	-0.087	0.035	0.129	0.137	-0.056
			LMS	0.061	0.082	0.077	0.068	0.056	0.084	0.088	-0.038	0.039	0.094	0.092	0.030
			2SMM	-0.134	0.080	0.080	-0.002	-0.138	0.077	0.077	-0.007	-0.138	0.072	0.071	0.007
			MML	0.085	0.092	0.126	-0.274	0.073	0.091	0.130	-0.298	0.058	0.104	0.138	-0.247
		500	Cons	0.016	0.057	0.061	-0.064	0.002	0.060	0.065	-0.080	-0.013	0.067	0.068	-0.015
			PC	0.017	0.059	0.062	-0.051	0.002	0.062	0.067	-0.079	-0.012	0.069	0.069	-0.003
			UC	0.017	0.061	0.066	-0.079	0.002	0.063	0.069	-0.091	-0.012	0.070	0.071	-0.010
			RC	0.014	0.062	0.066	-0.068	0.000	0.064	0.068	-0.070	-0.015	0.071	0.070	0.012
			LVS	-0.008	0.041	0.044	-0.063	-0.015	0.043	0.047	-0.095	-0.016	0.047	0.047	-0.014
			2SLS	0.069	0.073	0.081	-0.092	0.052	0.077	0.085	-0.093	0.040	0.087	0.094	-0.077
			LMS	0.062	0.057	0.056	0.006	0.052	0.060	0.062	-0.041	0.047	0.065	0.062	0.034
			2SMM	-0.143	0.055	0.058	-0.057	-0.138	0.053	0.054	-0.024	-0.136	0.049	0.050	-0.030
			MML	0.083	0.064	0.093	-0.312	0.074	0.067	0.096	-0.309	0.066	0.076	0.097	-0.211

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 21

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	0.002	1.784	1.452	0.228	-0.033	1.665	1.309	0.271	0.029	19.033	3.349	4.683
			PC	-0.079	40.287	3.257	11.369	-0.073	53.803	4.282	11.566	0.007	197.539	6.714	28.421
			UC	0.075	264.751	5.274	49.200	-0.160	33.927	3.194	9.623	0.082	64.649	6.173	9.473
			RC	-0.019	9.114	1.802	4.057	-0.104	16.815	2.297	6.321	-0.082	41.289	2.441	15.917
			LVS	-0.077	0.744	0.617	0.206	-0.197	2.219	1.480	0.500	-0.140	0.310	0.310	-0.003
			2SLS	-0.218	0.660	0.710	-0.070	-0.123	0.678	0.807	-0.160	-0.206	0.709	0.807	-0.122
			LMS	0.473	2.903	1.082	1.682	0.408	7.770	0.993	6.824	0.365	1.883	1.038	0.813
			2SMM	0.270	1833.431	12.291	148.174	-0.136	988.438	7.775	126.130	-0.049	6153.9	8.001	768.15
			MML	0.446	2.059	1.312	0.569	9.142	0.905	180.358	-0.995	0.320	1.426	1.273	0.120
		250	Cons	0.017	0.449	0.521	-0.138	-0.005	0.470	0.510	-0.078	0.007	0.623	0.705	-0.117
			PC	0.073	39.220	2.495	14.718	0.145	66.327	2.819	22.528	-0.121	85.335	2.450	33.824
			UC	-0.070	83.417	2.930	27.467	0.132	26.235	3.152	7.324	-0.028	28.073	2.380	10.793
			RC	0.288	14.192	2.389	4.941	0.011	15.434	1.606	8.610	0.017	11.076	1.633	5.783
			LVS	-0.089	0.106	0.135	-0.215	-0.092	0.107	0.127	-0.157	-0.080	0.121	0.149	-0.191
			2SLS	-0.018	0.529	0.634	-0.166	-0.074	0.536	0.643	-0.167	-0.133	0.630	0.747	-0.157
			LMS	0.354	0.439	0.378	0.162	0.329	0.438	0.359	0.221	0.335	0.574	0.445	0.290
			2SMM	-0.093	41.629	1.355	29.724	-0.064	90.446	1.930	45.861	-0.036	8.991	1.073	7.379
			MML	0.338	0.363	0.367	-0.011	0.317	0.366	0.360	0.016	0.350	0.551	0.544	0.011
		500	Cons	0.004	0.287	0.315	-0.089	0.007	0.307	0.317	-0.032	-0.008	0.347	0.380	-0.086
			PC	0.039	2.267	0.772	1.935	0.038	20.320	1.943	9.460	0.024	30.306	1.918	14.799
			UC	-0.010	25.070	1.736	13.438	0.003	2.026	0.901	1.249	0.047	18.026	1.833	8.835
			RC	-0.014	1.789	0.859	1.083	-0.008	3.688	1.141	2.234	0.004	6.995	1.609	3.348
			LVS	-0.091	0.068	0.086	-0.208	-0.085	0.072	0.088	-0.179	-0.084	0.077	0.097	-0.210
			2SLS	0.107	0.415	0.498	-0.167	0.041	0.494	0.636	-0.223	0.071	0.502	0.590	-0.149
			LMS	0.319	0.234	0.219	0.068	0.315	0.247	0.237	0.042	0.294	0.275	0.265	0.037
			2SMM	-0.146	0.301	0.263	0.142	-0.118	0.280	0.237	0.184	-0.110	0.399	0.319	0.253
			MML	0.298	0.218	0.217	0.004	0.304	0.233	0.228	0.021	0.277	0.251	0.254	-0.010

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 22

Parameter Estimates for γ_3 for the Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	0.001	0.136	0.149	-0.092	-0.002	0.141	0.155	-0.088	-0.027	0.159	0.172	-0.076
			PC	0.012	0.154	0.169	-0.088	0.007	0.162	0.174	-0.066	-0.021	0.190	0.194	-0.021
			UC	0.006	0.167	0.185	-0.099	0.003	0.177	0.191	-0.075	-0.023	0.209	0.219	-0.045
			RC	-0.011	0.218	0.211	0.033	-0.013	0.180	0.201	-0.104	-0.037	0.211	0.224	-0.060
			LVS	-0.028	0.097	0.105	-0.081	-0.023	0.100	0.112	-0.113	-0.038	0.110	0.111	-0.009
			2SLS	0.065	0.179	0.198	-0.095	0.073	0.196	0.209	-0.066	0.065	0.227	0.244	-0.071
			LMS	0.071	0.142	0.136	0.039	0.074	0.145	0.146	-0.008	0.056	0.163	0.155	0.053
			2SMM	-0.192	0.139	0.157	-0.118	-0.179	0.136	0.152	-0.102	-0.204	0.124	0.128	-0.026
			MML	0.107	0.136	0.213	-0.360	0.102	0.139	0.209	-0.335	0.087	0.159	0.242	-0.342
		250	Cons	0.010	0.083	0.086	-0.045	-0.004	0.087	0.095	-0.085	-0.014	0.097	0.100	-0.028
			PC	0.014	0.087	0.092	-0.058	0.000	0.092	0.100	-0.078	-0.010	0.104	0.109	-0.047
			UC	0.013	0.091	0.098	-0.070	-0.001	0.096	0.108	-0.108	-0.013	0.109	0.116	-0.064
			RC	0.005	0.094	0.101	-0.074	-0.009	0.098	0.107	-0.091	-0.019	0.113	0.123	-0.087
			LVS	-0.020	0.058	0.064	-0.099	-0.023	0.061	0.069	-0.125	-0.017	0.067	0.071	-0.062
			2SLS	0.073	0.106	0.122	-0.127	0.070	0.113	0.124	-0.093	0.077	0.129	0.146	-0.113
			LMS	0.078	0.083	0.084	-0.009	0.070	0.088	0.090	-0.021	0.077	0.096	0.095	0.020
			2SMM	-0.193	0.084	0.088	-0.051	-0.201	0.079	0.083	-0.053	-0.193	0.073	0.072	0.008
			MML	0.123	0.094	0.134	-0.297	0.102	0.095	0.134	-0.291	0.109	0.107	0.148	-0.274
		500	Cons	0.013	0.058	0.061	-0.058	0.004	0.061	0.067	-0.090	-0.018	0.067	0.070	-0.039
			PC	0.014	0.060	0.064	-0.065	0.006	0.063	0.068	-0.077	-0.016	0.070	0.072	-0.029
			UC	0.013	0.062	0.069	-0.102	0.005	0.065	0.073	-0.105	-0.017	0.072	0.076	-0.055
			RC	0.009	0.063	0.071	-0.103	0.002	0.067	0.073	-0.086	-0.020	0.073	0.078	-0.070
			LVS	-0.020	0.040	0.045	-0.107	-0.017	0.042	0.049	-0.137	-0.025	0.046	0.050	-0.077
			2SLS	0.076	0.072	0.079	-0.080	0.077	0.077	0.087	-0.114	0.062	0.087	0.093	-0.073
			LMS	0.076	0.058	0.057	0.023	0.077	0.061	0.062	-0.010	0.066	0.067	0.066	0.009
			2SMM	-0.203	0.057	0.057	0.009	-0.199	0.054	0.056	-0.043	-0.195	0.050	0.051	-0.017
			MML	0.118	0.067	0.090	-0.258	0.120	0.070	0.094	-0.247	0.104	0.075	0.104	-0.272

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 23

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	-0.026	2.992	1.051	1.848	-0.012	47.100	1.867	24.227	-0.022	17.033	1.954	7.718
			PC	-0.012	10.712	1.194	7.971	-0.009	1671.794	1.621	1030.332	-0.014	31.323	2.089	13.997
			UC	-0.022	10.073	1.065	8.454	0.022	29.339	3.247	8.037	0.000	72.684	2.505	28.019
			RC	0.003	1.511	0.654	1.308	0.003	3.072	0.660	3.655	-0.007	9.158	1.269	6.219
			LVS	0.047	0.205	0.977	-0.790	0.055	0.354	0.995	-0.644	0.074	0.298	1.543	-0.807
			2SLS	-0.119	0.447	0.473	-0.055	-0.078	0.505	0.534	-0.054	-0.080	0.519	0.511	0.015
			LMS	0.060	27.494	0.759	35.239	0.076	854.876	1.015	840.960	0.124	85.949	1.458	57.932
			2SMM	0.172	700.199	1.983	352.049	0.089	1322.828	3.487	378.380	-0.130	100.607	1.279	77.683
			MML	-5.811	1.637	103.936	-0.984	0.067	3.405	1.885	0.806	-0.008	1.753	1.113	0.576
		250	Cons	-0.013	0.198	0.215	-0.081	-0.042	0.247	0.251	-0.017	-0.037	4.502	0.309	13.578
			PC	0.002	2.994	0.474	5.320	-0.020	10.795	0.752	13.348	-0.013	7.944	0.740	9.736
			UC	-0.005	8.835	0.466	17.971	-0.021	0.904	0.382	1.366	-0.013	6.639	0.637	9.427
			RC	0.004	0.229	0.222	0.032	-0.027	0.280	0.268	0.045	-0.021	5.170	0.354	13.619
			LVS	0.014	0.061	0.211	-0.709	0.018	0.068	0.268	-0.745	0.003	0.084	0.461	-0.819
			2SLS	-0.027	0.291	0.301	-0.032	-0.036	0.312	0.347	-0.101	-0.044	0.374	0.395	-0.051
			LMS	-0.012	0.552	0.224	1.465	-0.022	0.319	0.273	0.167	-0.031	0.506	0.336	0.509
			2SMM	0.116	22.065	0.556	38.708	0.037	17.568	0.469	36.444	0.277	2993.5	8.225	362.97
			MML	0.048	0.319	0.296	0.078	0.030	0.387	0.345	0.123	-0.011	0.387	0.375	0.031
		500	Cons	-0.019	0.117	0.126	-0.078	-0.039	0.120	0.137	-0.124	-0.061	0.138	0.160	-0.140
			PC	-0.010	0.574	0.183	2.132	-0.027	1.617	0.247	5.547	-0.040	0.300	0.235	0.275
			UC	-0.004	1.092	0.218	4.004	-0.019	0.856	0.277	2.092	-0.037	5.493	0.243	21.578
			RC	-0.006	0.129	0.131	-0.016	-0.026	0.142	0.155	-0.083	-0.042	0.188	0.189	-0.006
			LVS	0.002	0.040	0.131	-0.691	0.003	0.044	0.155	-0.714	-0.023	0.050	0.189	-0.736
			2SLS	-0.022	0.188	0.211	-0.107	-0.021	0.217	0.237	-0.087	-0.039	0.276	0.326	-0.152
			LMS	-0.026	0.139	0.135	0.025	-0.037	0.158	0.157	0.002	-0.075	0.200	0.193	0.037
			2SMM	0.068	0.179	0.166	0.080	0.011	0.198	0.160	0.238	-0.078	0.341	0.157	1.170
			MML	0.030	0.169	0.174	-0.029	0.010	0.189	0.181	0.046	-0.047	0.213	0.238	-0.107

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 24

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	-0.026	0.139	0.159	-0.128	-0.029	0.148	0.185	-0.200	-0.039	0.169	0.197	-0.141
			PC	-0.020	1.070	0.261	3.105	-0.030	0.210	0.237	-0.113	-0.045	0.235	0.240	-0.021
			UC	-0.016	0.421	0.172	1.450	-0.019	0.224	0.202	0.105	-0.034	0.220	0.231	-0.048
			RC	-0.017	0.131	0.139	-0.054	-0.026	0.148	0.156	-0.055	-0.042	0.184	0.196	-0.063
			LVS	0.000	0.098	0.145	-0.319	0.006	0.110	0.170	-0.355	-0.014	0.125	0.203	-0.385
			2SLS	-0.011	0.187	0.203	-0.082	-0.005	0.210	0.214	-0.020	-0.024	0.242	0.260	-0.068
			LMS	-0.006	0.142	0.146	-0.031	-0.004	0.165	0.174	-0.052	-0.027	0.200	0.207	-0.035
			2SMM	0.043	0.191	0.209	-0.084	-0.017	0.170	0.188	-0.093	-0.104	0.151	0.164	-0.075
			MML	0.083	0.188	0.293	-0.357	0.013	1.854	0.503	2.686	-0.105	0.257	0.473	-0.457
		250	Cons	-0.023	0.077	0.089	-0.132	-0.025	0.079	0.100	-0.214	-0.038	0.091	0.103	-0.119
			PC	-0.026	0.131	0.104	0.256	-0.036	0.095	0.109	-0.129	-0.054	0.113	0.116	-0.025
			UC	-0.020	0.085	0.090	-0.053	-0.030	0.093	0.102	-0.092	-0.049	0.111	0.111	-0.006
			RC	-0.019	0.079	0.081	-0.032	-0.028	0.086	0.097	-0.110	-0.049	0.101	0.105	-0.037
			LVS	-0.002	0.059	0.085	-0.309	-0.004	0.064	0.099	-0.357	-0.020	0.073	0.108	-0.321
			2SLS	-0.006	0.101	0.108	-0.068	-0.003	0.111	0.122	-0.085	-0.012	0.134	0.143	-0.064
			LMS	-0.008	0.082	0.086	-0.052	-0.016	0.090	0.101	-0.109	-0.034	0.105	0.110	-0.043
			2SMM	0.058	0.126	0.141	-0.109	-0.004	0.113	0.114	-0.011	-0.083	0.100	0.106	-0.060
			MML	0.086	0.114	0.189	-0.395	0.015	0.126	0.220	-0.429	-0.280	0.335	3.330	-0.899
		500	Cons	-0.012	0.051	0.056	-0.087	-0.022	0.052	0.060	-0.143	-0.022	0.061	0.072	-0.146
			PC	-0.016	0.056	0.056	-0.004	-0.034	0.062	0.066	-0.053	-0.040	0.075	0.079	-0.050
			UC	-0.014	0.055	0.053	0.040	-0.030	0.062	0.065	-0.044	-0.038	0.075	0.078	-0.036
			RC	-0.014	0.053	0.052	0.026	-0.031	0.058	0.058	-0.006	-0.036	0.069	0.072	-0.036
			LVS	-0.005	0.039	0.052	-0.249	-0.010	0.043	0.061	-0.288	-0.011	0.050	0.074	-0.324
			2SLS	-0.003	0.065	0.065	0.012	-0.006	0.074	0.079	-0.067	-0.007	0.091	0.097	-0.059
			LMS	-0.011	0.055	0.054	0.017	-0.021	0.061	0.062	-0.027	-0.026	0.074	0.076	-0.031
			2SMM	0.054	0.090	0.090	0.007	0.007	0.083	0.088	-0.063	-0.084	0.073	0.078	-0.059
			MML	0.084	0.081	0.129	-0.370	0.009	0.086	0.133	-0.355	-0.121	0.098	0.176	-0.444

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 25

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	-0.005	6.527	1.147	4.692	0.000	2.248	0.631	2.562	-0.027	21.967	1.416	14.513
			PC	0.031	8058.158	2.470	3261.642	-0.014	33.333	2.328	13.316	-0.008	33.306	4.698	6.090
			UC	0.025	7.192	0.869	7.280	0.007	10.315	0.795	11.970	-0.011	160.859	6.605	23.353
			RC	0.008	0.935	0.536	0.744	0.015	1.849	0.672	1.753	-0.005	66.814	2.870	22.280
			LVS	0.020	0.217	0.532	-0.593	0.013	0.148	0.735	-0.798	-0.592	3.096	20.406	-0.848
			2SLS	-0.100	0.427	0.494	-0.136	-0.078	0.460	0.478	-0.038	-0.122	0.575	0.669	-0.140
			LMS	0.062	5.800	0.568	9.214	0.030	13.676	0.817	15.739	0.071	6.464	1.096	4.898
			2SMM	0.044	64.329	1.619	38.740	-0.014	776.938	2.989	258.909	-0.206	2054.903	6.695	305.910
			MML	-0.391	1.474	9.374	-0.843	21.715	1.047	313.068	-0.997	16.061	1.560	305.920	-0.995
		250	Cons	-0.019	0.455	0.230	0.977	-0.033	10.154	0.552	17.392	-0.028	9.045	1.091	7.290
			PC	-0.006	3.808	0.460	7.269	-0.004	2.857	0.490	4.829	0.002	9.192	0.713	11.897
			UC	0.007	9.921	0.402	23.671	-0.001	15.631	0.427	35.625	0.003	3.020	0.749	3.030
			RC	-0.008	0.263	0.220	0.199	-0.018	0.403	0.256	0.570	-0.006	0.755	0.372	1.030
			LVS	0.024	0.068	0.217	-0.688	0.009	0.067	0.250	-0.731	-0.002	0.079	0.378	-0.791
			2SLS	-0.004	0.299	0.328	-0.086	-0.039	0.327	0.342	-0.042	-0.044	0.395	0.423	-0.065
			LMS	0.001	0.283	0.226	0.253	-0.027	0.333	0.245	0.360	-0.066	0.511	0.332	0.537
			2SMM	0.070	2.435	0.360	5.759	0.037	4.163	0.351	10.852	-0.066	8.185	0.460	16.808
			MML	0.069	0.518	0.429	0.208	0.026	0.376	0.368	0.022	-0.059	0.465	0.443	0.051
		500	Cons	-0.014	0.121	0.129	-0.059	-0.042	0.121	0.132	-0.082	-0.052	0.140	0.151	-0.073
			PC	-0.005	2.014	0.256	6.866	-0.019	2.712	0.395	5.864	-0.032	0.575	0.280	1.052
			UC	-0.007	1.141	0.196	4.808	-0.024	26.854	0.560	46.923	-0.029	0.275	0.221	0.245
			RC	-0.007	0.120	0.130	-0.074	-0.029	0.143	0.140	0.020	-0.036	0.191	0.184	0.037
			LVS	0.007	0.040	0.130	-0.691	0.001	0.044	0.140	-0.686	-0.008	0.050	0.184	-0.726
			2SLS	-0.016	0.185	0.204	-0.093	0.005	0.212	0.220	-0.038	-0.009	0.276	0.296	-0.069
			LMS	-0.020	0.140	0.135	0.033	-0.043	0.155	0.144	0.078	-0.057	0.204	0.189	0.082
			2SMM	0.083	0.520	0.220	1.360	0.022	0.170	0.174	-0.023	-0.076	0.160	0.148	0.078
			MML	0.036	0.173	0.177	-0.021	0.011	0.186	0.174	0.069	-0.047	0.207	0.197	0.053

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 26

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.033	0.156	0.174	-0.104	-0.027	0.147	0.173	-0.153	-0.037	0.164	0.187	-0.122
			PC	-0.027	2.267	0.645	2.517	-0.023	2.321	0.254	8.129	-0.040	0.223	0.227	-0.018
			UC	-0.021	30.284	0.193	156.212	-0.015	12.919	0.418	29.923	-0.034	9.091	0.281	31.343
			RC	-0.021	0.134	0.138	-0.029	-0.020	0.146	0.159	-0.084	-0.039	0.178	0.189	-0.058
			LVS	0.000	0.100	0.141	-0.287	0.003	0.108	0.163	-0.337	-0.013	0.123	0.193	-0.362
			2SLS	-0.007	0.191	0.201	-0.048	0.000	0.210	0.212	-0.010	-0.006	0.249	0.260	-0.042
			LMS	-0.005	0.148	0.145	0.020	-0.009	0.164	0.171	-0.039	-0.027	0.194	0.198	-0.019
			2SMM	0.046	0.188	0.207	-0.092	-0.011	0.180	0.196	-0.081	-0.090	0.153	0.171	-0.109
			MML	0.096	0.226	0.327	-0.308	0.038	0.277	0.586	-0.528	0.030	0.537	2.397	-0.776
		250	Cons	-0.019	0.077	0.085	-0.090	-0.020	0.077	0.092	-0.162	-0.029	0.092	0.108	-0.149
			PC	-0.022	0.092	0.090	0.021	-0.029	0.094	0.101	-0.073	-0.044	0.114	0.122	-0.067
			UC	-0.018	0.177	0.098	0.809	-0.025	0.092	0.095	-0.028	-0.038	0.112	0.117	-0.041
			RC	-0.018	0.079	0.080	-0.017	-0.026	0.084	0.088	-0.041	-0.039	0.103	0.108	-0.052
			LVS	0.002	0.058	0.083	-0.294	-0.006	0.063	0.090	-0.303	-0.007	0.073	0.112	-0.348
			2SLS	0.004	0.101	0.110	-0.081	-0.002	0.110	0.113	-0.020	0.000	0.136	0.137	-0.002
			LMS	-0.003	0.082	0.084	-0.022	-0.018	0.089	0.092	-0.037	-0.022	0.107	0.114	-0.063
			2SMM	0.049	0.125	0.132	-0.058	-0.001	0.114	0.123	-0.073	-0.087	0.098	0.108	-0.086
			MML	0.100	0.117	0.193	-0.394	0.029	0.123	0.259	-0.524	-0.100	0.137	0.233	-0.414
		500	Cons	-0.010	0.051	0.057	-0.114	-0.021	0.052	0.062	-0.161	-0.013	0.060	0.068	-0.116
			PC	-0.014	0.056	0.059	-0.038	-0.032	0.063	0.065	-0.028	-0.034	0.073	0.073	-0.005
			UC	-0.011	0.056	0.056	-0.005	-0.028	0.062	0.061	0.013	-0.031	0.073	0.072	0.014
			RC	-0.012	0.053	0.053	0.017	-0.029	0.058	0.058	0.006	-0.031	0.068	0.068	-0.004
			LVS	0.000	0.040	0.054	-0.266	-0.006	0.043	0.060	-0.272	-0.007	0.049	0.070	-0.297
			2SLS	0.001	0.066	0.069	-0.048	0.001	0.074	0.073	0.017	0.003	0.089	0.091	-0.026
			LMS	-0.007	0.055	0.056	-0.008	-0.017	0.064	0.061	0.053	-0.020	0.076	0.075	0.018
			2SMM	0.049	0.091	0.088	0.030	0.017	0.088	0.094	-0.072	-0.080	0.073	0.076	-0.041
			MML	0.088	0.082	0.118	-0.307	0.016	0.088	0.138	-0.363	-0.122	0.100	0.250	-0.600

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 27

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	-0.006	26.353	1.710	14.412	-0.008	10.503	1.371	6.658	-0.017	30.518	1.474	19.708
			PC	0.002	14.366	1.266	10.344	-0.003	63.145	1.965	31.128	-0.023	45.224	2.280	18.835
			UC	0.000	8.869	1.317	5.734	0.006	12.800	1.079	10.868	-0.006	40.122	2.308	16.383
			RC	0.007	2.165	0.757	1.860	0.020	1.293	0.593	1.181	-0.002	21.969	1.892	10.612
			LVS	0.173	0.279	3.711	-0.925	0.194	0.409	2.615	-0.844	-0.009	0.293	1.065	-0.725
			2SLS	-0.095	0.430	0.474	-0.093	-0.079	0.467	0.534	-0.127	-0.119	0.525	0.525	-0.001
			LMS	0.054	258.939	0.676	382.043	0.045	10.205	0.795	11.832	0.007	16.055	1.050	14.292
			2SMM	0.258	485.333	2.970	162.430	69.118	3074646	1536.87	1999.59	-0.299	733.345	5.257	138.508
			MML	-0.542	1.521	13.717	-0.889	0.158	1.916	1.239	0.547	32.970	2.015	512.635	-0.996
		250	Cons	-0.012	0.218	0.227	-0.036	-0.018	0.391	0.308	0.271	-0.024	0.390	0.274	0.424
			PC	0.004	4.812	0.521	8.241	0.016	3.655	0.445	7.207	-0.004	4.096	0.628	5.519
			UC	0.012	4.347	0.467	8.310	0.014	3.225	0.434	6.438	-0.005	6.004	0.717	7.373
			RC	-0.006	0.224	0.228	-0.019	0.000	0.285	0.259	0.101	-0.012	1.322	0.421	2.139
			LVS	0.027	0.061	0.220	-0.722	0.036	0.066	0.253	-0.738	0.079	0.085	0.951	-0.910
			2SLS	-0.011	0.274	0.314	-0.127	-0.052	0.317	0.354	-0.104	-0.050	0.372	0.404	-0.080
			LMS	0.001	0.272	0.227	0.195	-0.006	0.314	0.257	0.222	-0.031	1.382	0.738	0.872
			2SMM	0.066	95.395	0.672	140.971	0.001	33.042	1.167	27.317	-0.059	16.935	0.721	22.493
			MML	0.037	0.285	0.314	-0.091	0.027	0.379	0.375	0.009	-0.025	0.602	0.441	0.366
		500	Cons	-0.019	0.119	0.130	-0.081	-0.028	0.118	0.138	-0.145	-0.043	0.135	0.143	-0.058
			PC	-0.013	6.756	0.339	18.953	-0.007	0.521	0.230	1.259	-0.022	1.811	0.324	4.586
			UC	-0.004	0.548	0.215	1.556	-0.008	0.211	0.169	0.250	-0.017	2.681	0.296	8.043
			RC	-0.014	0.140	0.126	0.108	-0.013	0.140	0.147	-0.047	-0.024	0.192	0.182	0.055
			LVS	0.007	0.041	0.129	-0.684	0.009	0.043	0.148	-0.710	0.007	0.049	0.183	-0.732
			2SLS	-0.009	0.188	0.198	-0.049	0.006	0.230	0.239	-0.037	-0.011	0.264	0.288	-0.083
			LMS	-0.019	0.140	0.130	0.074	-0.031	0.154	0.153	0.005	-0.046	0.208	0.189	0.101
			2SMM	0.079	2.195	0.191	10.476	0.032	2.913	0.207	13.043	-0.061	0.514	0.201	1.559
			MML	0.001	0.151	0.146	0.036	-0.009	0.168	0.176	-0.046	-0.051	0.201	0.195	0.030

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 28

Parameter Estimates for γ_1 for the Non-Normally Distributed, $R_{\gamma3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	-0.036	0.154	0.164	-0.058	-0.030	0.145	0.162	-0.103	-0.034	0.168	0.191	-0.117
			PC	-0.033	0.347	0.187	0.853	-0.032	0.248	0.199	0.245	-0.042	0.490	0.260	0.884
			UC	-0.024	0.290	0.163	0.778	-0.025	0.191	0.183	0.046	-0.035	0.556	0.241	1.308
			RC	-0.023	0.145	0.140	0.031	-0.026	0.150	0.157	-0.045	-0.033	0.188	0.196	-0.043
			LVS	-0.004	0.100	0.141	-0.286	0.000	0.108	0.158	-0.320	-0.007	0.124	0.201	-0.384
			2SLS	-0.005	0.183	0.191	-0.040	-0.005	0.211	0.221	-0.046	-0.017	0.254	0.275	-0.074
			LMS	-0.011	0.149	0.145	0.028	-0.010	0.169	0.161	0.048	-0.022	0.201	0.207	-0.028
			2SMM	0.041	0.202	0.197	0.023	0.006	0.183	0.195	-0.062	-0.094	0.154	0.174	-0.115
			MML	0.056	0.182	0.243	-0.254	0.044	0.186	0.261	-0.288	0.000	0.208	0.307	-0.322
		250	Cons	-0.011	0.076	0.089	-0.148	-0.016	0.077	0.090	-0.140	-0.017	0.091	0.102	-0.109
			PC	-0.014	0.089	0.094	-0.055	-0.025	0.095	0.096	-0.016	-0.032	0.112	0.110	0.012
			UC	-0.010	0.128	0.095	0.344	-0.019	0.092	0.091	0.018	-0.027	0.111	0.110	0.001
			RC	-0.011	0.077	0.084	-0.089	-0.019	0.084	0.085	-0.012	-0.028	0.101	0.103	-0.015
			LVS	0.001	0.057	0.086	-0.332	0.001	0.063	0.087	-0.284	-0.003	0.072	0.107	-0.334
			2SLS	0.002	0.100	0.108	-0.073	0.003	0.111	0.113	-0.016	0.002	0.135	0.139	-0.034
			LMS	-0.004	0.080	0.086	-0.071	-0.012	0.089	0.090	-0.014	-0.016	0.105	0.109	-0.034
			2SMM	0.053	0.124	0.132	-0.054	0.004	0.117	0.118	-0.016	-0.083	0.099	0.104	-0.043
			MML	0.080	0.109	0.145	-0.251	0.055	0.119	0.161	-0.261	-0.002	0.135	0.177	-0.236
		500	Cons	-0.014	0.051	0.060	-0.153	-0.014	0.052	0.061	-0.150	-0.015	0.060	0.069	-0.137
			PC	-0.018	0.057	0.061	-0.065	-0.025	0.063	0.065	-0.032	-0.034	0.073	0.076	-0.035
			UC	-0.015	0.056	0.059	-0.038	-0.023	0.062	0.062	0.000	-0.031	0.073	0.074	-0.014
			RC	-0.015	0.054	0.056	-0.042	-0.024	0.058	0.060	-0.029	-0.031	0.068	0.070	-0.038
			LVS	-0.003	0.040	0.057	-0.304	-0.003	0.043	0.060	-0.280	-0.013	0.049	0.070	-0.310
			2SLS	-0.002	0.066	0.069	-0.045	0.002	0.075	0.076	-0.021	-0.006	0.089	0.096	-0.070
			LMS	-0.010	0.056	0.058	-0.035	-0.014	0.061	0.062	-0.016	-0.027	0.071	0.073	-0.021
			2SMM	0.062	0.093	0.095	-0.017	0.012	0.086	0.087	-0.015	-0.071	0.075	0.077	-0.030
			MML	0.079	0.079	0.108	-0.267	0.057	0.084	0.114	-0.263	-0.021	0.094	0.122	-0.229

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 29

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	0.030	12.608	2.218	4.685	0.025	18.562	1.830	9.144	0.106	267.403	4.917	53.387
			PC	0.177	8.860	2.016	3.395	0.192	342.674	1.165	293.152	0.322	57.346	5.551	9.331
			UC	0.198	15.983	1.723	8.278	0.205	26.696	1.636	15.317	0.337	68.137	2.483	26.437
			RC	0.072	1.771	0.634	1.795	0.044	2.103	0.602	2.496	0.130	6.450	1.159	4.564
			LVS	0.092	0.175	0.675	-0.741	-0.035	0.534	0.872	-0.388	-0.014	0.458	1.405	-0.674
			2SLS	-0.095	0.469	0.495	-0.052	-0.050	0.459	0.479	-0.041	-0.076	0.521	0.544	-0.044
			LMS	0.084	1.585	0.629	1.521	-0.023	399.916	0.635	628.976	-0.042	80.182	1.109	71.311
			2SMM	16.033	1346417	357.3	3767.6	3.501	10691.7	81.0	131.0	0.982	261.504	21.320	11.266
			MML	0.080	2.421	0.855	1.831	1.532	0.926	28.580	-0.968	0.176	2.528	1.673	0.511
		250	Cons	-0.026	0.222	0.228	-0.026	-0.032	0.240	0.252	-0.050	-0.035	3.846	0.283	12.611
			PC	0.033	2.423	0.402	5.029	0.061	19.144	0.952	19.100	0.107	20.566	1.266	15.241
			UC	0.030	6.456	0.417	14.493	0.012	1.276	0.351	2.637	0.082	13.583	1.054	11.883
			RC	-0.012	0.254	0.222	0.147	-0.016	0.258	0.260	-0.005	-0.008	5.024	0.449	10.189
			LVS	0.017	0.062	0.221	-0.720	0.012	0.067	0.257	-0.739	0.042	0.084	0.517	-0.838
			2SLS	-0.024	0.289	0.303	-0.047	-0.023	0.298	0.324	-0.079	-0.009	0.377	0.413	-0.087
			LMS	-0.014	0.273	0.232	0.174	-0.029	0.338	0.273	0.239	-0.031	0.545	0.345	0.581
			2SMM	-0.457	356.728	1.619	219.278	-0.253	46.363	1.220	36.988	-0.738	6680.8	18.40	362.09
			MML	-0.015	0.284	0.277	0.027	-0.008	0.325	0.310	0.046	0.029	0.445	0.382	0.165
		500	Cons	-0.024	0.120	0.134	-0.104	-0.038	0.116	0.130	-0.110	-0.039	0.142	0.161	-0.118
			PC	-0.011	0.511	0.206	1.484	-0.015	0.884	0.214	3.130	-0.014	0.572	0.271	1.111
			UC	-0.009	0.693	0.177	2.917	-0.018	0.196	0.165	0.183	-0.018	0.819	0.233	2.510
			RC	-0.018	0.129	0.131	-0.014	-0.024	0.134	0.139	-0.041	-0.023	0.193	0.192	0.008
			LVS	0.005	0.041	0.133	-0.693	-0.005	0.043	0.141	-0.693	0.007	0.051	0.195	-0.740
			2SLS	-0.005	0.187	0.218	-0.141	-0.015	0.213	0.232	-0.084	-0.001	0.280	0.312	-0.100
			LMS	-0.022	0.142	0.141	0.007	-0.051	0.149	0.144	0.040	-0.043	0.206	0.200	0.030
			2SMM	-0.443	0.248	0.207	0.198	-0.253	0.388	0.216	0.799	-0.047	111.107	0.733	150.492
			MML	-0.019	0.158	0.173	-0.091	-0.024	0.168	0.165	0.022	0.020	0.202	0.211	-0.042

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 30

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	-0.039	0.138	0.155	-0.112	-0.038	0.147	0.161	-0.089	-0.038	0.172	0.186	-0.079
			PC	-0.022	1.133	0.244	3.648	-0.037	0.197	0.193	0.022	-0.042	0.254	0.238	0.065
			UC	-0.016	4.536	0.297	14.285	-0.030	0.177	0.173	0.022	-0.034	0.228	0.232	-0.019
			RC	-0.024	0.136	0.148	-0.084	-0.036	0.149	0.149	-0.001	-0.039	0.187	0.194	-0.033
			LVS	-0.003	0.101	0.148	-0.322	-0.004	0.110	0.154	-0.287	-0.007	0.127	0.196	-0.352
			2SLS	-0.016	0.189	0.197	-0.045	-0.011	0.212	0.205	0.035	0.006	0.249	0.262	-0.050
			LMS	-0.011	0.151	0.153	-0.012	-0.019	0.163	0.161	0.013	-0.021	0.204	0.200	0.020
			2SMM	-0.403	0.215	0.209	0.029	-0.218	0.214	0.225	-0.047	-0.010	0.267	0.252	0.060
			MML	0.012	0.212	0.269	-0.209	0.036	0.163	0.274	-0.406	0.100	0.194	0.297	-0.346
		250	Cons	-0.013	0.077	0.086	-0.105	-0.025	0.079	0.091	-0.134	-0.014	0.091	0.106	-0.144
			PC	-0.015	0.089	0.089	0.008	-0.034	0.095	0.096	-0.016	-0.031	0.113	0.119	-0.050
			UC	-0.011	0.083	0.084	-0.009	-0.029	0.093	0.093	-0.006	-0.026	0.112	0.116	-0.033
			RC	-0.013	0.078	0.079	-0.016	-0.028	0.086	0.087	-0.012	-0.028	0.102	0.108	-0.057
			LVS	0.002	0.058	0.080	-0.269	-0.004	0.064	0.089	-0.286	0.002	0.073	0.109	-0.330
			2SLS	0.003	0.100	0.111	-0.100	-0.001	0.111	0.113	-0.014	0.006	0.135	0.142	-0.049
			LMS	-0.004	0.081	0.081	0.000	-0.015	0.091	0.092	-0.009	-0.012	0.106	0.111	-0.046
			2SMM	-0.425	0.112	0.124	-0.095	-0.240	0.120	0.125	-0.040	-0.030	0.127	0.134	-0.051
			MML	0.000	0.095	0.292	-0.673	0.037	0.106	0.157	-0.324	0.142	0.119	0.191	-0.374
		500	Cons	-0.012	0.051	0.054	-0.046	-0.012	0.052	0.056	-0.075	-0.019	0.061	0.068	-0.103
			PC	-0.017	0.057	0.057	-0.004	-0.023	0.063	0.061	0.035	-0.039	0.074	0.074	0.009
			UC	-0.015	0.056	0.054	0.047	-0.020	0.062	0.058	0.070	-0.036	0.074	0.074	-0.007
			RC	-0.014	0.054	0.051	0.056	-0.020	0.058	0.055	0.065	-0.036	0.069	0.069	0.000
			LVS	0.000	0.040	0.052	-0.234	0.001	0.043	0.056	-0.220	-0.010	0.050	0.072	-0.307
			2SLS	0.000	0.066	0.068	-0.027	0.007	0.074	0.072	0.031	-0.004	0.090	0.096	-0.066
			LMS	-0.006	0.056	0.053	0.050	-0.011	0.061	0.057	0.077	-0.024	0.073	0.074	-0.017
			2SMM	-0.421	0.075	0.074	0.011	-0.245	0.083	0.088	-0.064	-0.026	0.089	0.091	-0.021
			MML	-0.001	0.070	0.114	-0.389	0.061	0.074	0.114	-0.347	0.148	0.086	0.131	-0.342

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 31

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	0.052	7.330	1.468	3.995	0.039	3.478	0.942	2.693	0.062	20.610	1.194	16.261
			PC	0.239	1487.50	4.052	366.112	0.148	25.208	2.671	8.438	0.235	19.431	1.786	9.878
			UC	0.142	5.768	2.423	1.380	0.157	11.979	1.364	7.782	0.238	17.049	3.323	4.131
			RC	0.030	0.567	0.505	0.124	0.043	1.600	0.713	1.245	0.098	11.287	0.914	11.344
			LVS	0.000	0.363	0.674	-0.462	0.046	0.195	0.902	-0.784	0.048	0.201	0.768	-0.739
			2SLS	-0.100	0.444	0.470	-0.056	-0.108	0.479	0.509	-0.058	-0.051	0.533	0.615	-0.133
			LMS	0.048	178.46	0.703	252.949	0.024	49.143	0.756	63.977	-0.001	6.533	0.880	6.423
			2SMM	-1.450	24084	28.7	838.8	-5.992	16536	114.1	144.0	3.765	6483	71.38	89.82
			MML	5.288	3.851	100.839	-0.962	0.298	9.120	3.801	1.399	14.196	2.167	252.672	-0.991
		250	Cons	-0.019	0.225	0.236	-0.048	-0.012	23.929	1.182	19.252	0.003	5.136	0.682	6.531
			PC	0.045	4.070	0.482	7.436	0.017	2.359	0.414	4.696	0.072	17.143	1.162	13.752
			UC	0.038	5.550	0.409	12.557	0.021	15.441	0.474	31.607	0.048	2.922	0.604	3.835
			RC	-0.015	0.244	0.225	0.083	-0.020	0.695	0.285	1.442	0.001	0.710	0.374	0.902
			LVS	0.024	0.064	0.227	-0.716	0.018	0.068	0.276	-0.752	0.030	0.081	0.365	-0.779
			2SLS	-0.026	0.298	0.312	-0.045	-0.020	0.313	0.353	-0.116	-0.058	0.401	0.422	-0.050
			LMS	-0.019	0.970	0.501	0.937	-0.025	0.313	0.245	0.275	-0.015	0.561	0.356	0.578
			2SMM	-0.414	3.724	0.653	4.705	-0.232	7.419	0.611	11.152	-0.026	51.754	1.301	38.793
			MML	0.006	0.367	0.368	-0.002	0.007	0.467	0.426	0.097	0.070	0.465	0.421	0.104
		500	Cons	-0.017	0.119	0.132	-0.102	-0.033	0.122	0.138	-0.117	-0.044	0.142	0.156	-0.092
			PC	0.010	5.456	0.284	18.201	-0.009	1.008	0.290	2.480	-0.007	1.851	0.317	4.843
			UC	-0.001	1.095	0.201	4.460	-0.001	13.161	0.335	38.245	-0.019	0.305	0.229	0.331
			RC	-0.014	0.120	0.130	-0.079	-0.022	0.141	0.141	0.003	-0.025	0.193	0.191	0.013
			LVS	0.008	0.040	0.133	-0.699	0.000	0.043	0.145	-0.701	0.008	0.050	0.193	-0.740
			2SLS	-0.008	0.185	0.203	-0.088	-0.013	0.208	0.231	-0.098	-0.021	0.287	0.295	-0.027
			LMS	-0.020	0.141	0.137	0.033	-0.042	0.156	0.152	0.026	-0.046	0.206	0.197	0.048
			2SMM	-0.464	0.970	0.361	1.684	-0.249	0.327	0.233	0.405	-0.019	0.258	0.242	0.064
			MML	-0.017	0.158	0.162	-0.024	-0.025	0.170	0.167	0.022	0.021	0.206	0.204	0.014

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 32

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.032	0.136	0.152	-0.104	-0.038	0.178	0.171	0.041	-0.031	0.168	0.192	-0.129
			PC	-0.013	0.820	0.237	2.457	-0.038	0.646	0.208	2.112	-0.031	0.228	0.236	-0.034
			UC	-0.013	50.559	0.197	256.288	-0.026	1.532	0.190	7.049	-0.021	4.457	0.242	17.390
			RC	-0.018	0.130	0.137	-0.048	-0.033	0.145	0.155	-0.067	-0.032	0.181	0.188	-0.036
			LVS	-0.005	0.099	0.141	-0.301	-0.009	0.108	0.154	-0.301	0.000	0.124	0.195	-0.360
			2SLS	-0.018	0.188	0.207	-0.094	-0.014	0.225	0.219	0.027	-0.007	0.242	0.255	-0.052
			LMS	-0.012	0.145	0.143	0.014	-0.019	0.173	0.166	0.041	-0.014	0.196	0.198	-0.011
			2SMM	-0.420	0.293	0.239	0.226	-0.214	0.383	0.234	0.637	-0.012	0.204	0.220	-0.075
			MML	0.011	0.150	0.255	-0.412	0.007	1.665	0.402	3.141	0.107	0.199	0.314	-0.364
		250	Cons	-0.019	0.077	0.087	-0.111	-0.023	0.078	0.089	-0.129	-0.023	0.093	0.103	-0.104
			PC	-0.020	0.089	0.091	-0.023	-0.032	0.095	0.100	-0.051	-0.040	0.114	0.116	-0.015
			UC	-0.016	0.091	0.086	0.063	-0.027	0.093	0.095	-0.023	-0.036	0.112	0.113	-0.003
			RC	-0.017	0.078	0.079	-0.013	-0.027	0.085	0.085	0.001	-0.034	0.103	0.102	0.013
			LVS	0.000	0.058	0.080	-0.277	-0.006	0.063	0.088	-0.282	-0.003	0.073	0.106	-0.312
			2SLS	0.000	0.100	0.102	-0.024	-0.005	0.111	0.112	-0.011	0.003	0.136	0.140	-0.022
			LMS	-0.006	0.081	0.082	-0.012	-0.018	0.090	0.091	-0.009	-0.017	0.108	0.109	-0.011
			2SMM	-0.424	0.111	0.117	-0.051	-0.239	0.119	0.130	-0.086	-0.030	0.124	0.137	-0.095
			MML	0.002	0.096	0.153	-0.375	0.041	0.105	0.173	-0.395	0.135	0.122	0.198	-0.384
		500	Cons	-0.015	0.051	0.059	-0.131	-0.008	0.052	0.059	-0.124	-0.021	0.061	0.069	-0.125
			PC	-0.020	0.057	0.061	-0.073	-0.019	0.062	0.064	-0.032	-0.040	0.074	0.076	-0.025
			UC	-0.017	0.056	0.059	-0.046	-0.016	0.061	0.062	-0.008	-0.037	0.074	0.075	-0.017
			RC	-0.017	0.054	0.056	-0.040	-0.017	0.057	0.058	-0.009	-0.036	0.069	0.070	-0.020
			LVS	-0.002	0.040	0.057	-0.304	0.001	0.043	0.059	-0.273	-0.011	0.049	0.072	-0.311
			2SLS	0.002	0.066	0.072	-0.085	0.007	0.073	0.075	-0.027	-0.005	0.090	0.094	-0.045
			LMS	-0.008	0.055	0.058	-0.049	-0.011	0.062	0.060	0.035	-0.025	0.074	0.074	0.009
			2SMM	-0.424	0.075	0.078	-0.037	-0.253	0.085	0.095	-0.103	-0.037	0.088	0.096	-0.082
			MML	0.008	0.070	0.114	-0.379	0.054	0.075	0.112	-0.334	0.146	0.085	0.129	-0.338

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 33

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma 3}^{2}$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^{2}$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	0.052	8.745	0.743	10.773	0.111	19.273	1.494	11.898	0.163	411.534	5.410	75.063
			PC	0.122	17.425	0.918	17.983	0.233	2576.483	3.313	776.715	0.378	24.470	2.031	11.047
			UC	0.142	7.606	1.575	3.829	0.166	6.876	1.184	4.808	0.287	63.302	3.022	19.946
			RC	0.038	1.282	0.511	1.508	0.062	3.358	0.756	3.440	0.191	13.189	1.325	8.951
			LVS	-0.183	1.342	3.891	-0.655	-0.212	0.901	5.821	-0.845	0.567	1.490	13.488	-0.890
			2SLS	-0.042	0.442	0.455	-0.028	-0.069	0.479	0.488	-0.017	-0.076	0.519	0.508	0.022
			LMS	0.030	113.767	0.621	182.168	0.075	18.765	0.882	20.283	0.030	15.028	0.897	15.760
			2SMM	47.564	1547732	988.8	1564.2	-0.387	6374.7	8.712	730.7	11.506	39005	173.7	223.6
			MML	-0.053	1.077	1.388	-0.224	0.076	2.575	1.300	0.981	-0.052	1.546	1.349	0.146
		250	Cons	-0.028	0.194	0.204	-0.050	-0.042	0.226	0.211	0.075	-0.034	0.380	0.276	0.375
			PC	0.058	9.909	0.644	14.380	0.024	4.841	0.506	8.566	0.072	7.469	0.745	9.022
			UC	0.064	5.998	0.595	9.084	0.037	8.802	0.445	18.763	0.077	9.806	0.669	13.649
			RC	-0.016	0.209	0.195	0.071	-0.032	0.237	0.229	0.034	-0.001	0.911	0.359	1.536
			LVS	0.011	0.063	0.209	-0.699	-0.010	0.066	0.227	-0.712	-0.079	0.086	1.003	-0.914
			2SLS	-0.020	0.280	0.320	-0.124	-0.043	0.327	0.366	-0.107	-0.036	0.379	0.392	-0.032
			LMS	-0.014	0.274	0.224	0.224	-0.052	0.284	0.230	0.233	-0.059	1.036	0.517	1.005
			2SMM	-0.447	138.183	1.506	90.778	-0.193	53.885	1.907	27.255	-0.057	60.576	2.311	25.217
			MML	-0.030	0.244	0.259	-0.056	-0.062	0.263	0.253	0.038	-0.042	0.505	0.378	0.336
		500	Cons	-0.022	0.118	0.127	-0.068	-0.031	0.117	0.138	-0.151	-0.039	0.131	0.142	-0.080
			PC	0.027	5.212	0.364	13.326	-0.010	0.541	0.200	1.704	0.017	6.442	0.527	11.235
			UC	0.011	0.814	0.217	2.745	-0.008	0.209	0.162	0.286	0.020	8.610	0.545	14.791
			RC	-0.013	0.123	0.130	-0.057	-0.018	0.135	0.143	-0.054	-0.018	0.184	0.178	0.034
			LVS	-0.001	0.040	0.132	-0.697	-0.001	0.042	0.145	-0.709	0.003	0.048	0.178	-0.729
			2SLS	-0.013	0.194	0.192	0.010	-0.002	0.229	0.221	0.039	-0.003	0.265	0.299	-0.113
			LMS	-0.032	0.140	0.132	0.061	-0.048	0.150	0.151	-0.007	-0.053	0.197	0.181	0.087
			2SMM	-0.445	5.844	0.320	17.285	-0.264	8.560	0.461	17.552	-0.045	0.995	0.359	1.776
			MML	-0.036	0.142	0.143	-0.010	-0.049	0.153	0.162	-0.052	-0.028	0.186	0.177	0.047

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 34

Parameter Estimates for γ_2 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	-0.025	0.145	0.165	-0.119	-0.031	0.145	0.172	-0.156	-0.035	0.172	0.189	-0.091
			PC	-0.014	1.242	0.246	4.040	-0.032	0.208	0.201	0.035	-0.034	0.552	0.251	1.202
			UC	0.000	4.400	0.389	10.320	-0.019	0.244	0.207	0.178	-0.022	0.574	0.238	1.407
			RC	-0.017	0.144	0.142	0.012	-0.020	0.146	0.150	-0.024	-0.034	0.195	0.188	0.037
			LVS	0.011	0.101	0.149	-0.323	0.001	0.107	0.157	-0.321	-0.002	0.125	0.193	-0.356
			2SLS	-0.004	0.187	0.204	-0.088	-0.003	0.209	0.213	-0.020	-0.006	0.241	0.264	-0.087
			LMS	0.004	0.151	0.157	-0.036	-0.012	0.165	0.164	0.010	-0.017	0.211	0.200	0.055
			2SMM	-0.416	0.241	0.188	0.283	-0.243	0.256	0.207	0.238	-0.010	0.218	0.240	-0.095
			MML	-0.019	0.144	0.235	-0.386	-0.034	0.159	0.253	-0.371	-0.022	0.186	0.265	-0.298
		250	Cons	-0.019	0.077	0.086	-0.112	-0.021	0.077	0.087	-0.118	-0.024	0.091	0.099	-0.081
			PC	-0.019	0.090	0.093	-0.031	-0.029	0.094	0.094	-0.009	-0.041	0.112	0.110	0.020
			UC	-0.016	0.118	0.092	0.293	-0.025	0.092	0.092	-0.005	-0.035	0.111	0.107	0.037
			RC	-0.017	0.078	0.079	-0.011	-0.026	0.084	0.087	-0.028	-0.036	0.102	0.100	0.016
			LVS	-0.001	0.058	0.081	-0.283	-0.005	0.063	0.088	-0.286	-0.008	0.072	0.105	-0.314
			2SLS	0.001	0.103	0.109	-0.056	0.002	0.110	0.115	-0.045	0.004	0.136	0.136	-0.005
			LMS	-0.006	0.080	0.082	-0.026	-0.015	0.089	0.089	-0.005	-0.023	0.105	0.105	0.002
			2SMM	-0.419	0.108	0.116	-0.069	-0.245	0.118	0.125	-0.053	-0.031	0.124	0.130	-0.050
			MML	-0.034	0.094	0.134	-0.299	-0.037	0.103	0.143	-0.280	0.001	0.121	0.172	-0.299
		500	Cons	-0.008	0.051	0.055	-0.084	-0.012	0.052	0.062	-0.170	-0.013	0.060	0.071	-0.155
			PC	-0.013	0.057	0.057	-0.014	-0.024	0.062	0.066	-0.058	-0.034	0.073	0.078	-0.063
			UC	-0.010	0.056	0.055	0.026	-0.021	0.062	0.063	-0.015	-0.031	0.073	0.078	-0.061
			RC	-0.009	0.054	0.052	0.028	-0.021	0.058	0.058	-0.005	-0.031	0.068	0.071	-0.041
			LVS	0.003	0.040	0.053	-0.251	-0.002	0.043	0.061	-0.293	-0.007	0.049	0.073	-0.334
			2SLS	0.002	0.066	0.065	0.011	0.003	0.074	0.078	-0.048	-0.001	0.089	0.097	-0.083
			LMS	-0.004	0.055	0.054	0.029	-0.013	0.061	0.063	-0.034	-0.021	0.072	0.076	-0.053
			2SMM	-0.423	0.076	0.076	-0.003	-0.251	0.085	0.088	-0.030	-0.037	0.090	0.099	-0.088
			MML	-0.029	0.068	0.097	-0.298	-0.023	0.073	0.098	-0.255	0.011	0.085	0.115	-0.263

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 35

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.50	100	Cons	0.304	9.426	3.074	2.066	0.273	295.201	5.798	49.910	0.394	557.269	11.414	47.823
			PC	0.132	47.138	3.586	12.147	0.106	1987.21	2.753	720.737	0.079	24.478	2.268	9.795
			UC	0.055	80.135	4.305	17.615	0.094	40.111	4.860	7.253	0.106	41.774	2.700	14.471
			RC	0.064	19.655	2.392	7.217	0.125	9.335	2.354	2.966	0.306	28.790	1.718	15.758
			LVS	0.050	0.622	0.645	-0.035	0.122	0.939	1.594	-0.411	0.094	1.335	0.919	0.452
			2SLS	0.043	0.626	0.711	-0.119	0.027	0.612	0.641	-0.046	0.117	0.575	0.605	-0.049
			LMS	0.367	84.212	2.261	36.240	0.603	43.428	2.671	15.257	0.735	55.538	3.590	14.469
			2SMM	120.162	11782138	1873.5	6287.7	2.885	19860	76.8	257.5	1.459	239.083	27.524	7.686
			MML	11.651	158.952	179.045	-0.112	7.749	5.777	110.977	-0.948	0.727	3.932	2.188	0.797
		250	Cons	0.133	0.658	0.674	-0.023	0.200	0.518	0.570	-0.093	0.257	0.612	0.782	-0.217
			PC	0.007	13.720	1.303	9.530	0.063	29.346	1.549	17.940	-0.025	16.987	1.334	11.734
			UC	-0.056	42.363	1.912	21.152	0.016	7.469	1.562	3.783	0.086	15.715	1.377	10.416
			RC	0.157	10.832	1.551	5.984	0.107	2.551	0.863	1.956	0.237	28.836	3.421	7.428
			LVS	0.049	0.130	0.139	-0.067	0.062	0.105	0.110	-0.042	0.053	0.102	0.091	0.121
			2SLS	0.032	0.566	0.588	-0.037	0.050	0.442	0.520	-0.149	0.050	0.387	0.474	-0.183
			LMS	0.327	0.717	0.584	0.228	0.407	0.521	0.407	0.279	0.396	0.505	0.436	0.160
			2SMM	0.412	1602.0	8.389	189.97	0.073	19.577	0.871	21.484	0.256	2213.877	6.131	360.080
			MML	0.576	0.625	0.745	-0.161	0.650	0.511	0.522	-0.021	0.631	0.477	0.500	-0.046
		500	Cons	0.129	0.333	0.360	-0.076	0.189	0.268	0.302	-0.112	0.224	0.201	0.256	-0.215
			PC	0.019	4.629	0.846	4.468	-0.022	5.497	0.654	7.401	-0.011	0.663	0.264	1.509
			UC	-0.012	60.881	4.591	12.260	-0.012	7.317	1.949	2.753	-0.010	21.353	0.391	53.600
			RC	0.090	6.063	0.742	7.174	0.118	0.640	0.379	0.686	0.162	1.504	0.333	3.517
			LVS	0.038	0.075	0.078	-0.034	0.050	0.061	0.066	-0.075	0.056	0.048	0.059	-0.186
			2SLS	0.009	0.411	0.463	-0.111	0.031	0.324	0.357	-0.094	0.031	0.249	0.275	-0.094
			LMS	0.320	0.326	0.280	0.166	0.361	0.239	0.223	0.072	0.343	0.193	0.207	-0.067
			2SMM	0.052	0.350	0.239	0.463	0.074	0.451	0.219	1.054	0.089	147.365	0.948	154.460
			MML	0.554	0.346	0.368	-0.058	0.612	0.296	0.326	-0.093	0.623	0.261	0.307	-0.149

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 36

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .00, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.00	.80	100	Cons	0.028	0.206	0.261	-0.211	0.038	0.189	0.205	-0.077	0.060	0.152	0.162	-0.065
			PC	0.019	17.457	1.887	8.251	-0.020	0.486	0.335	0.450	-0.003	0.346	0.241	0.435
			UC	-0.016	8.602	0.646	12.316	-0.012	0.688	0.434	0.586	-0.004	0.235	0.227	0.035
			RC	0.028	1.815	0.754	1.407	0.027	0.322	0.256	0.258	0.052	0.184	0.189	-0.026
			LVS	0.022	0.139	0.139	0.003	0.013	0.127	0.122	0.038	0.025	0.102	0.101	0.008
			2SLS	0.004	0.351	0.308	0.143	0.004	0.308	0.283	0.087	-0.001	0.232	0.258	-0.102
			LMS	0.044	0.246	0.204	0.206	0.049	0.223	0.170	0.313	0.063	0.136	0.142	-0.044
			2SMM	0.008	0.253	0.233	0.086	0.041	0.191	0.210	-0.091	0.071	0.348	0.261	0.330
			MML	-0.714	0.198	18.947	-0.990	0.128	0.180	0.259	-0.302	0.148	0.140	0.239	-0.414
		250	Cons	0.030	0.103	0.114	-0.093	0.049	0.082	0.084	-0.034	0.049	0.060	0.063	-0.056
			PC	-0.007	0.379	0.208	0.823	0.006	0.094	0.089	0.058	-0.003	0.067	0.064	0.053
			UC	0.002	0.138	0.136	0.012	0.005	0.097	0.095	0.015	-0.004	0.065	0.063	0.020
			RC	0.030	0.127	0.132	-0.040	0.047	0.087	0.091	-0.049	0.048	0.058	0.060	-0.029
			LVS	0.015	0.071	0.072	-0.013	0.022	0.058	0.058	-0.008	0.017	0.045	0.046	-0.037
			2SLS	0.008	0.167	0.208	-0.198	0.009	0.113	0.118	-0.037	-0.002	0.079	0.079	0.002
			LMS	0.040	0.097	0.103	-0.058	0.057	0.081	0.081	-0.009	0.047	0.056	0.063	-0.119
			2SMM	0.022	0.095	0.113	-0.159	0.034	0.092	0.109	-0.156	0.041	0.082	0.108	-0.242
			MML	0.150	0.099	0.222	-0.555	0.150	0.083	0.144	-0.422	0.128	0.071	0.207	-0.659
		500	Cons	0.030	0.064	0.068	-0.056	0.042	0.048	0.051	-0.067	0.049	0.036	0.042	-0.131
			PC	0.002	0.073	0.071	0.025	-0.001	0.048	0.049	-0.022	0.002	0.037	0.041	-0.096
			UC	0.001	0.076	0.075	0.014	-0.002	0.048	0.049	-0.014	0.001	0.037	0.041	-0.091
			RC	0.028	0.074	0.077	-0.036	0.041	0.046	0.050	-0.078	0.051	0.034	0.040	-0.135
			LVS	0.014	0.045	0.045	-0.012	0.016	0.034	0.036	-0.047	0.017	0.027	0.030	-0.093
			2SLS	0.004	0.092	0.092	-0.003	0.002	0.060	0.063	-0.054	0.004	0.046	0.050	-0.090
			LMS	0.040	0.061	0.064	-0.050	0.047	0.045	0.053	-0.160	0.044	0.357	0.041	7.599
			2SMM	0.013	0.056	0.064	-0.130	0.027	0.059	0.072	-0.183	0.032	0.053	0.069	-0.224
			MML	0.135	0.066	0.107	-0.388	0.129	0.056	0.112	-0.501	0.132	0.049	0.094	-0.476

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 37

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .05, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.50	100	Cons	0.216	50.720	3.067	15.537	-0.046	4.696	1.986	1.364	0.500	7.873	3.964	0.986
			PC	-0.070	14899	11.131	1337.6	-0.128	65.853	5.515	10.940	-0.192	46.014	10.386	3.430
			UC	-0.203	30.777	3.036	9.136	-0.175	43.448	2.971	13.623	-0.165	20.793	2.547	7.165
			RC	0.029	31.713	3.107	9.207	-0.141	6.154	1.324	3.649	0.113	10.411	1.490	5.985
			LVS	-0.312	2.044	2.154	-0.051	-0.174	0.391	0.475	-0.177	-0.361	4.914	3.813	0.289
			2SLS	-0.260	0.619	0.638	-0.031	-0.142	0.587	0.653	-0.101	-0.103	0.577	0.634	-0.091
			LMS	-0.042	168.794	2.555	65.074	0.190	26.277	1.052	23.980	0.285	4.597	1.928	1.384
			2SMM	11.448	12125	358.2	32.8	44.657	11195.372	952.401	10.755	8.873	9580.9	226.3	41.3
			MML	-1.601	3.867	156.836	-0.975	15.615	24.798	292.396	-0.915	3.730	3.521	80.031	-0.956
		250	Cons	-0.120	0.780	0.722	0.082	-0.007	0.695	0.731	-0.049	0.020	0.502	0.498	0.008
			PC	-0.239	34.653	2.774	11.494	-0.195	12.503	1.365	8.160	-0.250	24.493	1.124	20.783
			UC	-0.282	81.521	3.029	25.916	-0.266	62.220	1.133	53.900	-0.263	6.490	0.810	7.010
			RC	-0.200	10.218	1.157	7.832	-0.046	28.077	2.010	12.969	-0.056	1.229	0.479	1.567
			LVS	-0.185	0.139	0.139	-0.003	-0.157	0.107	0.121	-0.115	-0.144	0.086	0.096	-0.102
			2SLS	-0.223	0.583	0.634	-0.081	-0.171	0.432	0.454	-0.048	-0.109	0.394	0.399	-0.012
			LMS	0.083	1.295	0.663	0.954	0.156	0.589	0.478	0.234	0.163	0.479	0.356	0.345
			2SMM	-0.136	9.730	1.269	6.669	-0.117	7.222	0.787	8.173	-0.075	47.159	1.049	43.964
			MML	0.382	0.890	0.902	-0.013	0.420	0.553	0.623	-0.113	0.448	0.502	0.572	-0.122
		500	Cons	-0.129	0.362	0.448	-0.190	-0.016	0.284	0.325	-0.124	0.034	0.205	0.246	-0.169
			PC	-0.221	15.814	1.121	13.113	-0.181	7.115	0.848	7.393	-0.202	2.906	0.371	6.833
			UC	-0.234	22.990	2.045	10.242	-0.223	133.369	2.689	48.589	-0.197	0.389	0.249	0.564
			RC	-0.187	1.751	0.644	1.719	-0.106	0.685	0.406	0.689	-0.030	0.178	0.198	-0.100
			LVS	-0.182	0.077	0.081	-0.051	-0.153	0.064	0.072	-0.111	-0.140	0.049	0.053	-0.080
			2SLS	-0.247	0.437	0.488	-0.104	-0.193	0.324	0.353	-0.081	-0.151	0.246	0.253	-0.028
			LMS	0.115	0.400	0.312	0.282	0.181	0.251	0.249	0.009	0.152	0.239	0.215	0.112
			2SMM	-0.176	1.042	0.394	1.642	-0.142	0.446	0.277	0.614	-0.130	0.191	0.189	0.009
			MML	0.352	0.357	0.432	-0.173	0.418	0.302	0.321	-0.061	0.419	0.263	0.312	-0.157

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 38

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2 = .05$, $\lambda = 0.80$ Conditions

				$\phi_{12} = .20$				$\phi_{12} = .40$				$\phi_{12} = .60$			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.05	.80	100	Cons	-0.194	0.226	0.310	-0.269	-0.162	0.197	0.229	-0.142	-0.143	0.136	0.157	-0.132
			PC	-0.217	8.891	1.150	6.730	-0.227	8.528	0.703	11.132	-0.214	0.209	0.203	0.031
			UC	-0.218	224.099	0.791	282.378	-0.206	41.236	1.250	31.988	-0.206	50.904	1.015	49.165
			RC	-0.204	0.570	0.413	0.381	-0.169	1.995	0.528	2.776	-0.151	0.176	0.174	0.015
			LVS	-0.200	0.143	0.160	-0.107	-0.186	0.122	0.117	0.042	-0.174	0.096	0.099	-0.028
			2SLS	-0.224	0.382	0.382	-0.001	-0.189	0.322	0.344	-0.064	-0.185	0.220	0.222	-0.010
			LMS	-0.179	0.225	0.240	-0.062	-0.144	0.200	0.178	0.121	-0.139	0.127	0.137	-0.075
			2SMM	-0.159	0.499	0.344	0.452	-0.168	0.631	0.202	2.126	-0.135	0.141	0.197	-0.283
			MML	0.158	71.124	5.217	12.633	-0.050	1.431	0.384	2.724	-0.088	0.162	0.265	-0.388
		250	Cons	-0.190	0.103	0.115	-0.102	-0.163	0.076	0.081	-0.067	-0.139	0.064	0.064	-0.004
			PC	-0.215	0.257	0.172	0.494	-0.210	0.087	0.089	-0.022	-0.190	0.069	0.064	0.079
			UC	-0.231	1.221	0.318	2.834	-0.211	0.088	0.088	0.000	-0.191	0.068	0.065	0.053
			RC	-0.191	0.297	0.211	0.408	-0.167	0.081	0.081	-0.002	-0.141	0.065	0.071	-0.088
			LVS	-0.203	0.073	0.070	0.038	-0.189	0.054	0.057	-0.043	-0.172	0.047	0.045	0.042
			2SLS	-0.216	0.171	0.204	-0.164	-0.202	0.106	0.105	0.005	-0.189	0.085	0.080	0.066
			LMS	-0.179	0.099	0.097	0.018	-0.154	0.071	0.079	-0.106	-0.139	0.059	0.064	-0.071
			2SMM	-0.199	0.090	0.110	-0.183	-0.168	0.085	0.107	-0.208	-0.156	0.076	0.105	-0.281
			MML	-0.075	0.103	0.193	-0.468	-0.076	0.082	0.396	-0.791	-0.056	0.077	0.143	-0.460
		500	Cons	-0.190	0.063	0.068	-0.079	-0.167	0.048	0.047	0.038	-0.145	0.035	0.038	-0.091
			PC	-0.218	0.071	0.075	-0.053	-0.210	0.050	0.047	0.067	-0.192	0.035	0.035	0.024
			UC	-0.218	0.074	0.081	-0.097	-0.210	0.051	0.049	0.041	-0.192	0.035	0.034	0.027
			RC	-0.192	0.073	0.079	-0.071	-0.167	0.048	0.048	0.012	-0.143	0.033	0.036	-0.100
			LVS	-0.206	0.045	0.046	-0.027	-0.192	0.035	0.032	0.073	-0.177	0.026	0.027	-0.022
			2SLS	-0.217	0.090	0.095	-0.051	-0.208	0.062	0.057	0.096	-0.190	0.044	0.044	-0.005
			LMS	-0.179	0.062	0.065	-0.053	-0.162	0.838	0.046	17.387	-0.149	0.035	0.064	-0.454
			2SMM	-0.208	0.055	0.066	-0.176	-0.177	0.058	0.073	-0.202	-0.160	0.053	0.066	-0.196
			MML	-0.070	0.069	0.135	-0.488	-0.064	0.056	0.103	-0.455	-0.065	0.049	0.086	-0.432

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 39

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.50 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.50	100	Cons	0.428	7.837	2.982	1.629	0.222	27.148	8.679	2.128	0.240	1637.924	20.812	77.700
			PC	-0.288	13.120	2.307	4.686	-0.249	3957.9	6.392	618.222	-0.268	30.584	2.681	10.408
			UC	-0.262	28.496	3.665	6.776	-0.275	112.030	6.735	15.634	0.010	68.231	3.043	21.421
			RC	-0.103	6.209	1.278	3.859	0.046	6.267	1.425	3.396	0.054	14.668	1.786	7.214
			LVS	-0.359	1.467	1.446	0.015	-0.503	4.801	5.242	-0.084	-0.135	1.258	1.308	-0.038
			2SLS	-0.378	0.640	0.676	-0.054	-0.239	0.626	0.662	-0.055	-0.182	0.531	0.561	-0.054
			LMS	0.084	965.6	1.7	576.2	0.028	27.579	1.685	15.364	0.176	3.041	1.502	1.024
			2SMM	36.211	986235	583.3	1689.7	-82.324	4759550	1831.7	2597.4	-4.963	109070	104.7	1040.8
			MML	0.778	2.024	9.303	-0.782	0.425	4.348	3.538	0.229	38.679	1.803	557.758	-0.997
		250	Cons	-0.201	0.615	0.684	-0.101	-0.096	0.564	0.661	-0.146	-0.038	0.445	0.535	-0.168
			PC	-0.275	33.062	2.112	14.651	-0.322	17.023	1.377	11.366	-0.300	14.856	1.112	12.357
			UC	-0.399	19.988	1.811	10.035	-0.309	13.065	1.378	8.482	-0.303	13.045	1.081	11.063
			RC	-0.222	14.395	1.722	7.358	-0.173	2.594	0.668	2.882	-0.095	3.020	0.620	3.871
			LVS	-0.274	0.129	0.138	-0.062	-0.234	0.108	0.113	-0.052	-0.223	0.086	0.098	-0.120
			2SLS	-0.338	0.521	0.576	-0.094	-0.239	0.456	0.501	-0.090	-0.226	0.405	0.453	-0.106
			LMS	0.023	0.784	0.557	0.409	0.120	0.551	0.433	0.273	0.098	0.637	0.440	0.448
			2SMM	-0.301	326.387	3.587	89.984	-0.190	35.183	1.869	17.828	-0.193	52.454	1.893	26.704
			MML	0.172	0.585	0.701	-0.165	0.274	0.487	0.531	-0.084	0.280	0.424	0.491	-0.136
		500	Cons	-0.190	0.339	0.394	-0.138	-0.106	0.267	0.280	-0.047	-0.044	0.191	0.239	-0.200
			PC	-0.306	49.483	2.121	22.328	-0.331	4.745	0.500	8.490	-0.310	20.743	1.468	13.125
			UC	-0.350	15.582	1.829	7.518	-0.309	0.823	0.369	1.227	-0.271	11.460	0.687	15.672
			RC	-0.223	5.231	0.573	8.130	-0.154	0.907	0.358	1.530	-0.103	0.219	0.192	0.143
			LVS	-0.272	0.075	0.079	-0.053	-0.246	0.060	0.066	-0.083	-0.223	0.047	0.055	-0.147
			2SLS	-0.307	0.432	0.433	-0.004	-0.300	0.413	0.386	0.071	-0.244	0.243	0.288	-0.156
			LMS	0.027	0.356	0.331	0.076	0.059	0.270	0.218	0.238	0.062	0.194	0.191	0.016
			2SMM	-0.269	5.321	0.325	15.354	-0.209	3.881	0.363	9.692	-0.194	1.371	0.331	3.137
			MML	0.195	0.320	0.406	-0.211	0.246	0.257	0.286	-0.102	0.287	0.225	0.263	-0.144

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 40

Parameter Estimates for γ_3 for the Non-Normally Distributed, $R_{\gamma 3}^2$ = .10, λ = 0.80 Conditions

				ϕ_{12} = .20				ϕ_{12} = .40				ϕ_{12} = .60			
$R_{\gamma 3}^2$	λ	N	Method	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio	% Bias	SE	SD	Rel. Ratio
.10	.80	100	Cons	-0.285	0.218	0.274	-0.204	-0.242	0.181	0.215	-0.155	-0.210	0.146	0.191	-0.236
			PC	-0.314	5.370	0.682	6.874	-0.289	0.665	0.362	0.839	-0.269	0.723	0.344	1.102
			UC	-0.319	11.907	1.850	5.437	-0.306	0.862	0.519	0.662	-0.280	0.495	0.265	0.867
			RC	-0.265	4.579	0.804	4.695	-0.247	1.241	0.409	2.037	-0.221	0.333	0.319	0.043
			LVS	-0.301	0.143	0.145	-0.016	-0.270	0.118	0.118	0.003	-0.249	0.098	0.100	-0.018
			2SLS	-0.288	0.365	0.416	-0.123	-0.286	0.291	0.305	-0.043	-0.257	0.229	0.279	-0.178
			LMS	-0.276	0.223	0.219	0.014	-0.227	0.188	0.179	0.053	-0.209	0.135	0.146	-0.075
			2SMM	-0.258	0.594	0.353	0.682	-0.232	0.550	0.243	1.264	-0.208	0.146	0.189	-0.228
			MML	-0.177	0.171	0.306	-0.441	-0.134	0.164	0.253	-0.355	-0.115	0.136	0.227	-0.399
		250	Cons	-0.284	0.100	0.111	-0.094	-0.251	0.078	0.077	0.006	-0.221	0.061	0.062	-0.021
			PC	-0.312	0.205	0.165	0.238	-0.297	0.088	0.083	0.061	-0.274	0.070	0.064	0.101
			UC	-0.312	1.231	0.460	1.679	-0.298	0.088	0.080	0.113	-0.275	0.087	0.069	0.262
			RC	-0.283	0.145	0.157	-0.078	-0.255	0.081	0.077	0.049	-0.221	0.061	0.063	-0.029
			LVS	-0.297	0.070	0.072	-0.023	-0.278	0.055	0.054	0.020	-0.254	0.045	0.043	0.048
			2SLS	-0.308	0.182	0.190	-0.043	-0.296	0.109	0.114	-0.048	-0.274	0.084	0.081	0.030
			LMS	-0.271	0.097	0.099	-0.026	-0.241	0.074	0.079	-0.066	-0.223	0.055	0.059	-0.061
			2SMM	-0.298	0.085	0.101	-0.159	-0.259	0.085	0.115	-0.265	-0.234	0.075	0.090	-0.174
			MML	-0.169	0.102	0.164	-0.378	-0.131	0.085	0.155	-0.451	-0.113	0.083	0.142	-0.413
		500	Cons	-0.279	0.063	0.067	-0.071	-0.254	0.048	0.051	-0.052	-0.225	0.034	0.039	-0.118
			PC	-0.309	0.069	0.072	-0.039	-0.297	0.049	0.048	0.027	-0.273	0.034	0.036	-0.051
			UC	-0.309	0.072	0.079	-0.093	-0.297	0.049	0.047	0.048	-0.274	0.034	0.036	-0.052
			RC	-0.281	0.068	0.073	-0.068	-0.254	0.047	0.048	-0.010	-0.224	0.032	0.037	-0.149
			LVS	-0.295	0.044	0.044	0.007	-0.279	0.034	0.034	0.007	-0.257	0.026	0.028	-0.072
			2SLS	-0.303	0.087	0.089	-0.022	-0.296	0.062	0.063	-0.015	-0.271	0.043	0.043	-0.011
			LMS	-0.266	0.060	0.062	-0.032	-0.249	0.046	0.048	-0.045	-0.231	0.349	0.039	7.930
			2SMM	-0.298	0.057	0.064	-0.114	-0.266	0.060	0.072	-0.171	-0.245	0.052	0.066	-0.206
			MML	-0.150	0.070	0.119	-0.409	-0.128	0.062	0.123	-0.500	-0.097	0.055	0.109	-0.493

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 41

Convergence for Normally Distributed Data and λ=.5

		ϕ_{12} = .20			ϕ_{12} = .40			ϕ_{12} = .60		
N	Method	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10
100	Cons	97.0	96.8	96.4	98.0	96.0	95.8	93.8	94.6	97.0
	PC	84.4	79.2	80.2	80.4	80.2	80.2	80.2	78.8	76.8
	UC	76.2	75.0	73.0	70.0	72.8	76.2	72.6	72.6	71.2
	RC	77.2	78.4	78.2	81.4	81.0	79.8	75.8	72.2	80.0
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
250	Cons	99.6	100.0	98.8	99.8	100.0	98.8	99.8	99.6	99.6
	PC	92.8	90.0	90.6	92.0	90.8	93.4	91.4	89.8	90.0
	UC	83.4	82.0	86.0	81.8	81.8	81.2	80.4	80.8	82.2
	RC	87.8	87.6	89.6	86.4	86.2	88.4	83.8	85.6	87.6
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
500	Cons	99.2	99.6	100.0	99.8	100.0	99.8	99.2	100.0	100.0
	PC	97.8	96.0	96.6	97.0	97.0	96.8	97.6	96.8	92.8
	UC	95.2	92.6	93.0	92.6	93.4	90.6	89.2	90.8	88.4
	RC	92.8	95.4	96.4	92.4	92.0	94.6	92.4	92.4	93.4
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 42

Convergence for Normally Distributed Data and λ=.8

N	Method	ϕ_{12} = .20			ϕ_{12} = .40			ϕ_{12} = .60		
		$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10
100	Cons	99.4	100.0	99.4	99.6	100.0	99.8	99.4	100.0	100.0
	PC	100.0	99.8	99.8	98.8	100.0	100.0	98.4	99.8	100.0
	UC	99.2	99.6	98.6	99.8	99.0	99.6	100.0	100.0	99.4
	RC	99.8	99.8	100.0	100.0	99.6	100.0	99.6	99.8	99.8
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
250	Cons	99.8	99.8	99.4	99.2	99.2	100.0	99.8	99.6	100.0
	PC	98.8	99.8	100.0	100.0	99.6	100.0	100.0	99.0	100.0
	UC	99.8	100.0	99.4	100.0	100.0	99.2	99.8	100.0	99.6
	RC	100.0	100.0	99.8	100.0	100.0	99.6	99.2	100.0	100.0
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
500	Cons	99.8	99.4	100.0	99.8	99.4	100.0	99.8	99.2	100.0
	PC	100.0	98.4	100.0	100.0	100.0	100.0	100.0	100.0	99.0
	UC	99.0	100.0	99.8	97.8	100.0	99.8	99.0	98.8	100.0
	RC	99.8	99.8	99.6	99.8	99.8	99.6	99.8	100.0	100.0
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 43

Convergence for Non-Normally Distributed Data and λ=.5

		ϕ_{12} = .20			ϕ_{12} = .40			ϕ_{12} = .60		
N	Method	$R_{\gamma 3}^2$=.00	$R_{\gamma 3}^2$=.05	$R_{\gamma 3}^2$=.10	$R_{\gamma 3}^2$=.00	$R_{\gamma 3}^2$=.05	$R_{\gamma 3}^2$=.10	$R_{\gamma 3}^2$=.00	$R_{\gamma 3}^2$=.05	$R_{\gamma 3}^2$=.10
100	Cons	90.8	92.0	89.6	90.4	93.0	90.4	91.6	91.4	93.0
	PC	70.4	72.6	70.8	76.6	78.0	77.8	82.4	81.6	83.2
	UC	71.8	69.6	70.2	73.4	72.4	74.0	79.0	78.2	80.0
	RC	73.6	76.0	74.2	78.4	80.6	77.0	75.4	80.0	82.6
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
250	Cons	100.0	100.0	100.0	100.0	99.8	99.6	100.0	99.4	99.6
	PC	89.4	89.0	89.2	94.2	94.4	93.8	96.6	94.4	94.6
	UC	83.0	86.0	86.4	88.6	94.4	90.4	95.4	95.0	93.2
	RC	86.4	87.6	89.8	91.4	91.2	91.0	93.6	92.2	94.8
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
500	Cons	100.0	100.0	100.0	100.0	100.0	99.8	99.8	100.0	100.0
	PC	96.6	94.4	94.8	97.2	99.4	97.8	99.4	99.6	99.2
	UC	90.6	89.6	90.0	96.4	97.0	96.0	98.0	99.6	99.4
	RC	93.4	90.8	91.2	97.0	96.4	95.6	98.6	98.2	99.0
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 44

Convergence for Non-Normally Distributed Data and λ=.8

		ϕ_{12} = .20			ϕ_{12} = .40			ϕ_{12} = .60		
N	Method	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10	$R_{\gamma3}^2$=.00	$R_{\gamma3}^2$=.05	$R_{\gamma3}^2$=.10
100	Cons	99.8	100.0	100.0	99.8	99.8	99.8	100.0	99.6	99.6
	PC	99.0	97.2	97.4	99.4	98.2	98.8	99.6	99.8	98.8
	UC	95.4	95.6	93.4	97.0	97.0	96.4	98.6	99.8	98.8
	RC	93.4	94.8	94.8	95.2	97.0	96.8	98.8	99.0	99.2
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
250	Cons	99.8	100.0	100.0	100.0	100.0	99.8	99.8	100.0	100.0
	PC	99.8	99.2	98.6	99.8	100.0	98.2	99.8	99.8	99.6
	UC	98.8	99.4	96.8	99.2	100.0	100.0	99.4	99.6	100.0
	RC	98.4	99.4	99.2	99.6	99.8	99.8	99.8	99.8	99.8
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									
500	Cons	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	PC	99.8	99.8	100.0	99.0	100.0	100.0	99.6	99.8	100.0
	UC	99.4	99.2	99.8	100.0	99.2	100.0	100.0	99.2	100.0
	RC	99.6	99.8	100.0	100.0	99.8	100.0	100.0	99.0	99.0
	LVS									
	2SLS									
	LMS									
	2SMM									
	MML									

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 45

Type I error rates for $R_{\gamma 3}^2$=.00 and λ=.50

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.00	.50	100	Cons	0.025	0.033	0.038	0.073	0.073	0.138
			PC	0.000	0.005	0.005	0.011	0.010	0.015
			UC	0.129	0.123	0.118	0.095	0.063	0.020
			RC	0.161	0.133	0.179	0.201	0.161	0.164
			LVS	0.066	0.068	0.044	0.070	0.046	0.082
			2SLS	0.056	0.070	0.060	0.076	0.070	0.082
			LMS	0.018	0.018	0.024	0.026	0.022	0.074
			2SMM	0.000	0.000	0.000	0.008	0.010	0.034
			MML	0.050	0.046	0.032	0.084	0.116	0.118
		250	Cons	0.050	0.048	0.038	0.108	0.198	0.308
			PC	0.006	0.004	0.000	0.007	0.019	0.019
			UC	0.091	0.066	0.107	0.082	0.041	0.023
			RC	0.180	0.218	0.255	0.236	0.206	0.235
			LVS	0.038	0.052	0.054	0.100	0.106	0.106
			2SLS	0.072	0.064	0.068	0.056	0.092	0.072
			LMS	0.024	0.018	0.028	0.096	0.138	0.204
			2SMM	0.006	0.010	0.010	0.034	0.036	0.050
			MML	0.024	0.024	0.040	0.206	0.308	0.400
		500	Cons	0.052	0.044	0.042	0.166	0.382	0.579
			PC	0.012	0.012	0.012	0.017	0.033	0.054
			UC	0.065	0.082	0.083	0.091	0.048	0.063
			RC	0.144	0.195	0.247	0.203	0.285	0.465
			LVS	0.026	0.044	0.046	0.098	0.176	0.262
			2SLS	0.076	0.062	0.062	0.072	0.082	0.078
			LMS	0.026	0.036	0.028	0.298	0.458	0.558
			2SMM	0.036	0.014	0.010	0.052	0.102	0.104
			MML	0.032	0.032	0.028	0.496	0.676	0.812

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 46

Type I error rates for $R_{\gamma 3}^2$=.00 and λ=.80

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.00	.80	100	Cons	0.078	0.066	0.074	0.106	0.104	0.094
			PC	0.080	0.063	0.059	0.048	0.044	0.034
			UC	0.073	0.064	0.056	0.059	0.047	0.037
			RC	0.070	0.058	0.060	0.077	0.086	0.075
			LVS	0.068	0.042	0.046	0.060	0.058	0.052
			2SLS	0.060	0.056	0.036	0.026	0.044	0.054
			LMS	0.074	0.070	0.074	0.092	0.124	0.124
			2SMM	0.082	0.070	0.088	0.140	0.182	0.242
			MML	0.304	0.238	0.276	0.404	0.414	0.394
		250	Cons	0.056	0.065	0.052	0.120	0.144	0.140
			PC	0.051	0.054	0.054	0.064	0.054	0.058
			UC	0.046	0.058	0.050	0.073	0.052	0.060
			RC	0.042	0.068	0.044	0.104	0.137	0.122
			LVS	0.046	0.052	0.038	0.078	0.072	0.076
			2SLS	0.058	0.070	0.062	0.066	0.050	0.044
			LMS	0.048	0.056	0.052	0.142	0.198	0.202
			2SMM	0.040	0.044	0.070	0.158	0.186	0.236
			MML	0.230	0.186	0.200	0.488	0.556	0.526
		500	Cons	0.050	0.056	0.030	0.106	0.166	0.232
			PC	0.048	0.052	0.032	0.036	0.053	0.060
			UC	0.053	0.049	0.022	0.040	0.062	0.066
			RC	0.046	0.042	0.026	0.102	0.164	0.238
			LVS	0.060	0.040	0.022	0.062	0.094	0.124
			2SLS	0.042	0.072	0.046	0.046	0.068	0.080
			LMS	0.068	0.060	0.030	0.160	0.250	0.296
			2SMM	0.066	0.060	0.070	0.124	0.174	0.264
			MML	0.110	0.130	0.164	0.580	0.640	0.714

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 47

Empirical power estimates for $R_{\gamma 3}^2$=.05 and λ=.50

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.05	.50	100	Cons	0.058	0.046	0.047	0.054	0.088	0.118
			PC	0.010	0.010	0.008	0.017	0.005	0.020
			UC	0.099	0.113	0.061	0.078	0.055	0.049
			RC	0.143	0.151	0.219	0.163	0.144	0.190
			LVS	0.126	0.132	0.092	0.076	0.076	0.104
			2SLS	0.068	0.084	0.078	0.056	0.078	0.078
			LMS	0.050	0.044	0.030	0.030	0.030	0.052
			2SMM	0.000	0.002	0.002	0.010	0.018	0.026
			MML	0.074	0.074	0.046	0.098	0.104	0.124
		250	Cons	0.196	0.130	0.127	0.090	0.182	0.290
			PC	0.058	0.007	0.020	0.007	0.015	0.021
			UC	0.105	0.125	0.104	0.081	0.055	0.025
			RC	0.208	0.174	0.227	0.256	0.213	0.245
			LVS	0.406	0.318	0.258	0.074	0.122	0.132
			2SLS	0.112	0.080	0.074	0.066	0.074	0.070
			LMS	0.300	0.228	0.166	0.078	0.190	0.208
			2SMM	0.010	0.014	0.020	0.038	0.050	0.070
			MML	0.330	0.264	0.174	0.202	0.304	0.400
		500	Cons	0.408	0.316	0.220	0.142	0.360	0.562
			PC	0.204	0.132	0.072	0.017	0.026	0.040
			UC	0.181	0.120	0.110	0.083	0.037	0.046
			RC	0.189	0.187	0.188	0.225	0.259	0.436
			LVS	0.670	0.570	0.414	0.104	0.190	0.232
			2SLS	0.162	0.104	0.112	0.074	0.068	0.080
			LMS	0.632	0.540	0.374	0.278	0.474	0.538
			2SMM	0.048	0.042	0.044	0.040	0.062	0.102
			MML	0.632	0.570	0.370	0.476	0.718	0.794

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 48

Empirical power estimates for $R_{\gamma 3}^2$=.05 and λ=.80

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.05	.80	100	Cons	0.560	0.514	0.360	0.100	0.130	0.106
			PC	0.509	0.482	0.335	0.043	0.053	0.028
			UC	0.504	0.473	0.338	0.075	0.060	0.030
			RC	0.487	0.470	0.325	0.095	0.085	0.107
			LVS	0.554	0.526	0.364	0.062	0.048	0.056
			2SLS	0.332	0.330	0.218	0.060	0.070	0.064
			LMS	0.560	0.536	0.388	0.108	0.108	0.106
			2SMM	0.090	0.118	0.116	0.118	0.174	0.240
			MML	0.648	0.606	0.530	0.392	0.442	0.362
		250	Cons	0.946	0.861	0.697	0.076	0.118	0.130
			PC	0.940	0.853	0.681	0.040	0.050	0.050
			UC	0.932	0.848	0.682	0.050	0.048	0.042
			RC	0.930	0.840	0.672	0.060	0.110	0.134
			LVS	0.946	0.888	0.716	0.036	0.076	0.062
			2SLS	0.762	0.644	0.432	0.062	0.064	0.048
			LMS	0.944	0.890	0.714	0.118	0.180	0.216
			2SMM	0.188	0.156	0.118	0.152	0.194	0.246
			MML	0.862	0.802	0.658	0.480	0.550	0.568
		500	Cons	0.996	0.986	0.950	0.114	0.126	0.212
			PC	0.996	0.986	0.948	0.056	0.042	0.052
			UC	0.994	0.982	0.945	0.054	0.046	0.056
			RC	0.994	0.982	0.942	0.106	0.122	0.206
			LVS	0.998	0.988	0.976	0.078	0.058	0.084
			2SLS	0.964	0.904	0.758	0.060	0.038	0.058
			LMS	0.998	0.990	0.968	0.164	0.224	0.284
			2SMM	0.304	0.272	0.232	0.168	0.208	0.218
			MML	0.982	0.964	0.896	0.600	0.690	0.686

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 49

Empirical power estimates for $R_{\gamma 3}^2$=.10 and λ=.50

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.10	.50	100	Cons	0.083	0.088	0.029	0.067	0.082	0.101
			PC	0.012	0.015	0.005	0.014	0.023	0.002
			UC	0.077	0.110	0.090	0.100	0.054	0.035
			RC	0.128	0.135	0.198	0.181	0.148	0.121
			LVS	0.216	0.198	0.126	0.068	0.084	0.078
			2SLS	0.084	0.088	0.076	0.094	0.078	0.058
			LMS	0.088	0.086	0.034	0.026	0.056	0.056
			2SMM	0.006	0.004	0.004	0.004	0.008	0.018
			MML	0.148	0.134	0.068	0.108	0.110	0.134
		250	Cons	0.374	0.273	0.203	0.096	0.185	0.317
			PC	0.128	0.079	0.027	0.004	0.026	0.023
			UC	0.130	0.116	0.097	0.067	0.066	0.032
			RC	0.176	0.158	0.226	0.218	0.204	0.245
			LVS	0.636	0.500	0.384	0.096	0.122	0.148
			2SLS	0.150	0.116	0.078	0.080	0.078	0.070
			LMS	0.514	0.378	0.278	0.092	0.200	0.262
			2SMM	0.028	0.022	0.020	0.022	0.048	0.088
			MML	0.572	0.416	0.300	0.202	0.338	0.458
		500	Cons	0.642	0.567	0.406	0.168	0.369	0.622
			PC	0.340	0.295	0.121	0.032	0.022	0.036
			UC	0.265	0.214	0.122	0.089	0.044	0.040
			RC	0.253	0.245	0.193	0.252	0.276	0.459
			LVS	0.872	0.834	0.692	0.116	0.158	0.248
			2SLS	0.244	0.226	0.168	0.068	0.096	0.078
			LMS	0.872	0.802	0.652	0.304	0.456	0.596
			2SMM	0.060	0.062	0.046	0.062	0.100	0.130
			MML	0.904	0.838	0.678	0.478	0.700	0.802

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 50

Empirical power estimates for $R_{\gamma 3}^2$=.10 and λ=.80

				Normally Distributed			Non-Normally Distributed		
$R_{\gamma 3}^2$	λ	N	Method	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60	ϕ_{12} = .20	ϕ_{12} = .40	ϕ_{12} = .60
.10	.80	100	Cons	0.835	0.806	0.610	0.114	0.126	0.153
			PC	0.812	0.770	0.546	0.053	0.055	0.059
			UC	0.799	0.743	0.533	0.058	0.071	0.067
			RC	0.776	0.730	0.539	0.082	0.083	0.107
			LVS	0.856	0.790	0.584	0.066	0.058	0.072
			2SLS	0.628	0.554	0.366	0.074	0.046	0.072
			LMS	0.822	0.764	0.588	0.112	0.130	0.140
			2SMM	0.160	0.174	0.110	0.136	0.186	0.236
			MML	0.828	0.796	0.636	0.396	0.402	0.428
		250	Cons	0.998	0.986	0.962	0.078	0.104	0.156
			PC	0.994	0.980	0.960	0.053	0.026	0.050
			UC	0.996	0.978	0.954	0.058	0.024	0.054
			RC	0.996	0.982	0.948	0.075	0.084	0.134
			LVS	0.998	0.982	0.970	0.056	0.048	0.088
			2SLS	0.938	0.880	0.776	0.068	0.056	0.062
			LMS	0.998	0.978	0.962	0.126	0.180	0.230
			2SMM	0.292	0.242	0.182	0.140	0.216	0.216
			MML	0.978	0.958	0.898	0.414	0.532	0.532
		500	Cons	1.000	1.000	0.998	0.106	0.148	0.224
			PC	1.000	1.000	0.998	0.050	0.046	0.052
			UC	1.000	1.000	0.998	0.050	0.058	0.042
			RC	1.000	1.000	1.000	0.104	0.154	0.232
			LVS	1.000	1.000	1.000	0.064	0.080	0.100
			2SLS	1.000	0.992	0.970	0.066	0.060	0.056
			LMS	1.000	1.000	1.000	0.160	0.228	0.322
			2SMM	0.478	0.412	0.334	0.118	0.168	0.198
			MML	1.000	0.996	0.992	0.608	0.702	0.748

Note. Cons=Constrained. PC=Partially constrained. UC=Unconstrained. RC=Residual-centered unconstrained. LVS=Latent variable scores. 2SLS=Two-stage least squares. LMS=latent moderated structural equations. 2SMM=Two-step method of moments. MML=Marginal maximum likelihood.

Table 51

Recommendations for Type of Method Applied Researchers Should Use

	Minimum	First-Order Effects		Interaction Effect		Loadings	
	N	Bias	Relative Ratio	Bias	Relative Ratio	0.50	0.80
Normal (absolute comparisons)							
Constrained	250+	Slight	Good	None	OK if N > 250	Yes	Yes
LVS	100	None	Good	None	Good	No	Yes
Non-Normal (relative comparisons, not absolute)							
MML	500+	Biased	Poor	Most accurate	Poor	Yes	No
LMS	500+	Most accurate	Poor	2nd most accurate	Poor	Yes	Yes

Note. LVS=Latent variable scores. MML=Marginal maximum likelihood. LMS=latent moderated structural equations.

Figure 1: *Relation of Achievement and Ability Moderated by Effort*

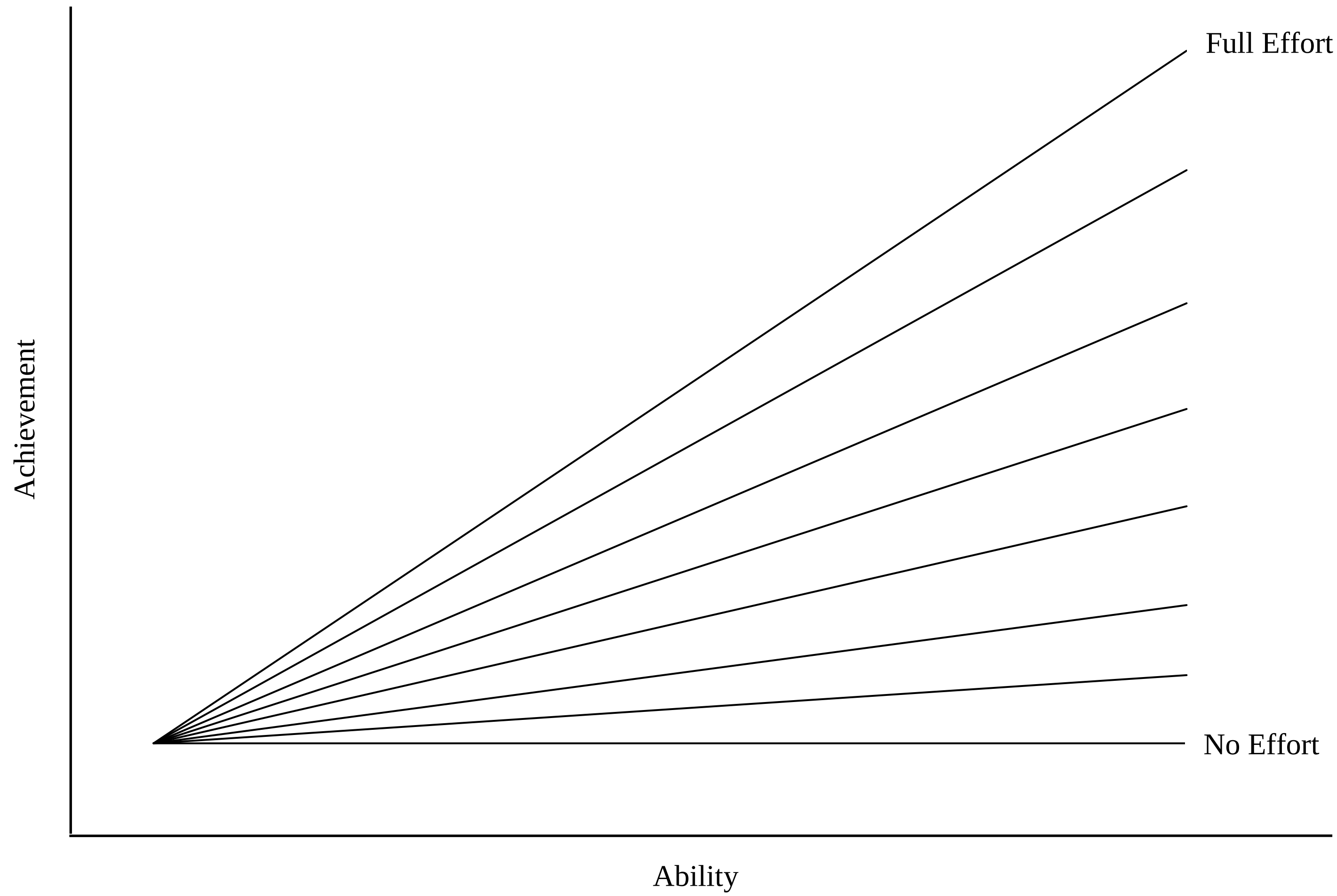


Figure 2: *Matched-Pairs, Product-Indicator Model*

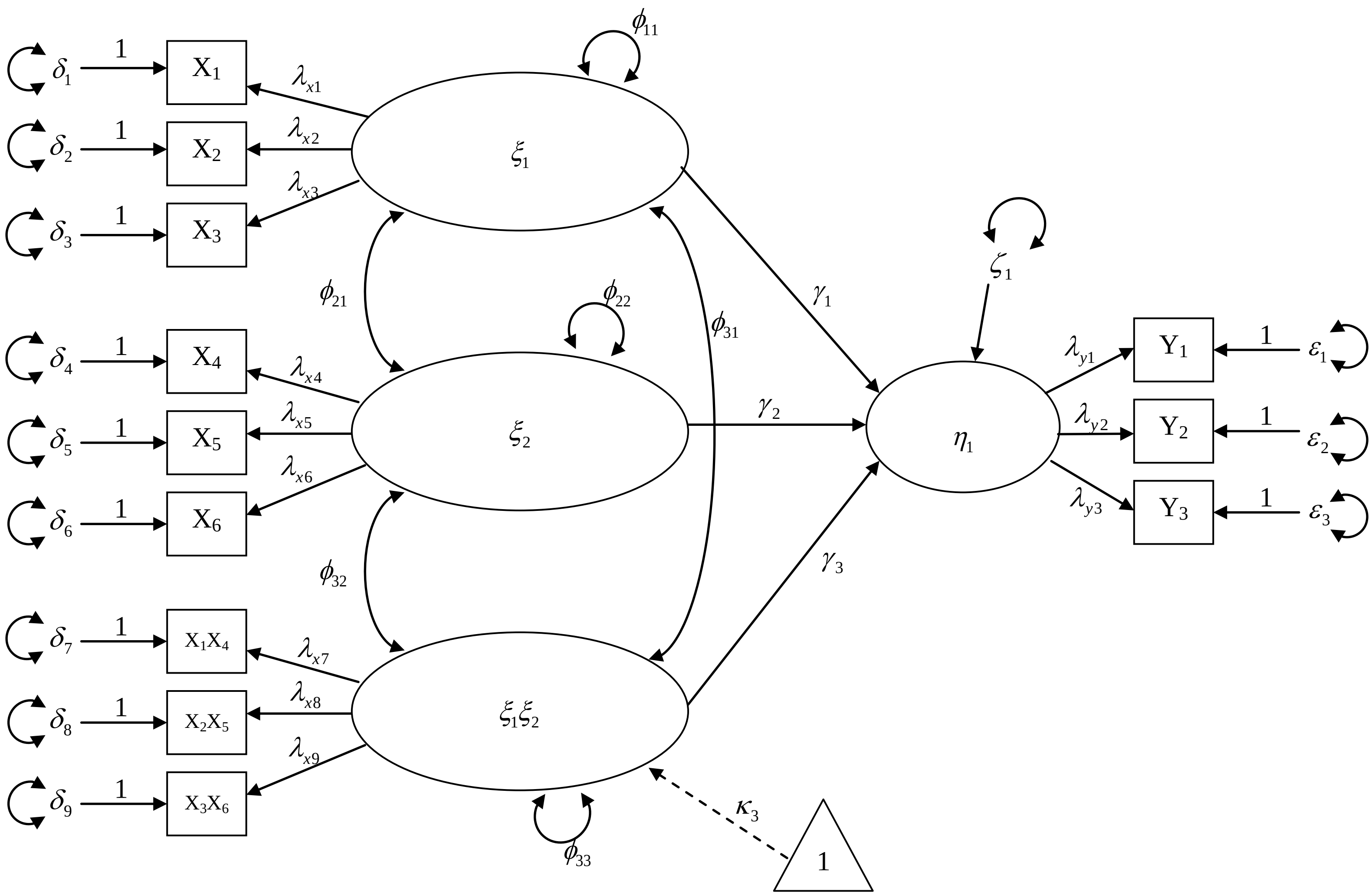


Figure 3: *All Possible Pairs, Product-Indicator Model*

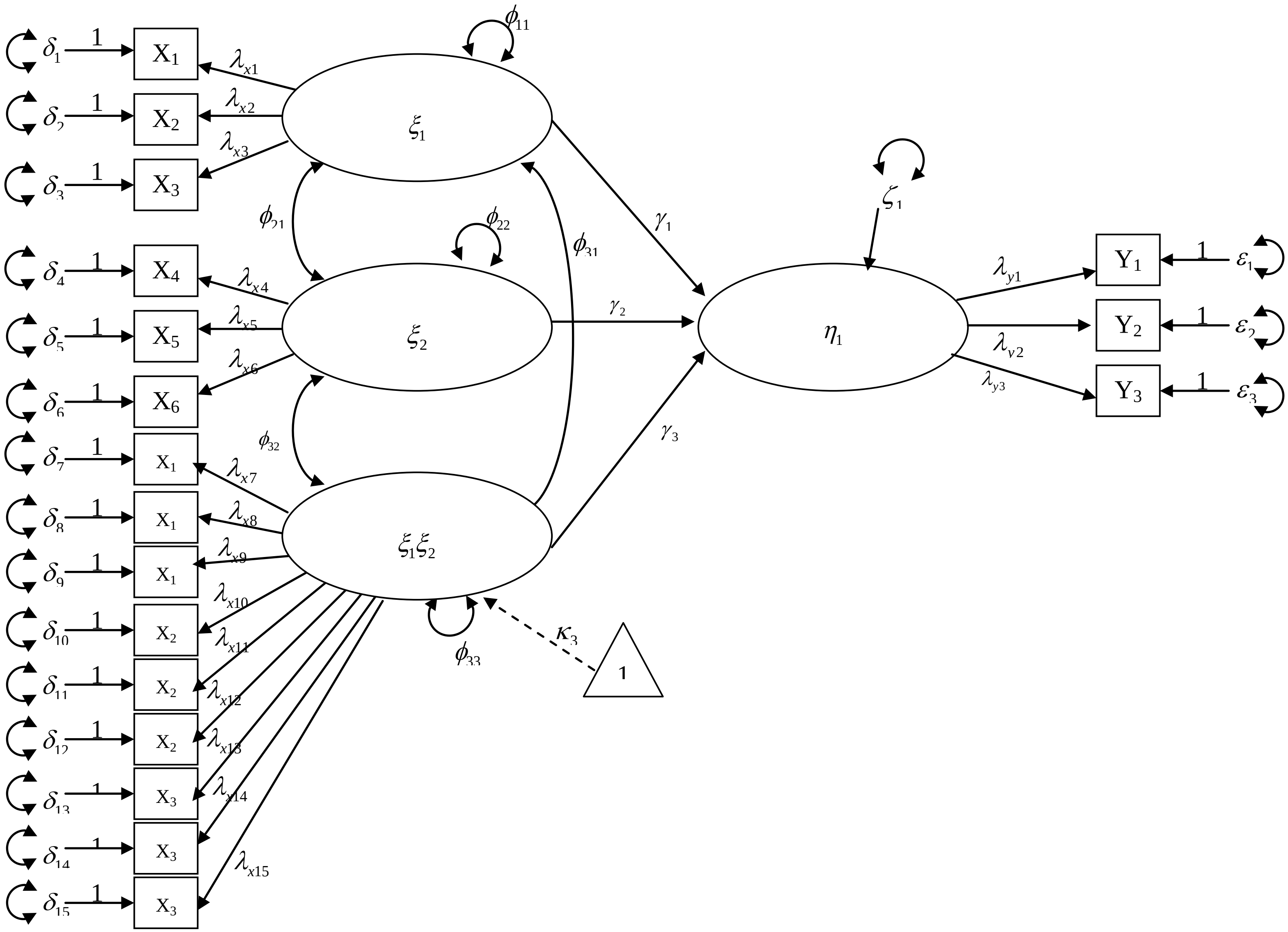


Figure 4: *Single Best Pair, Product-Indicator Model*

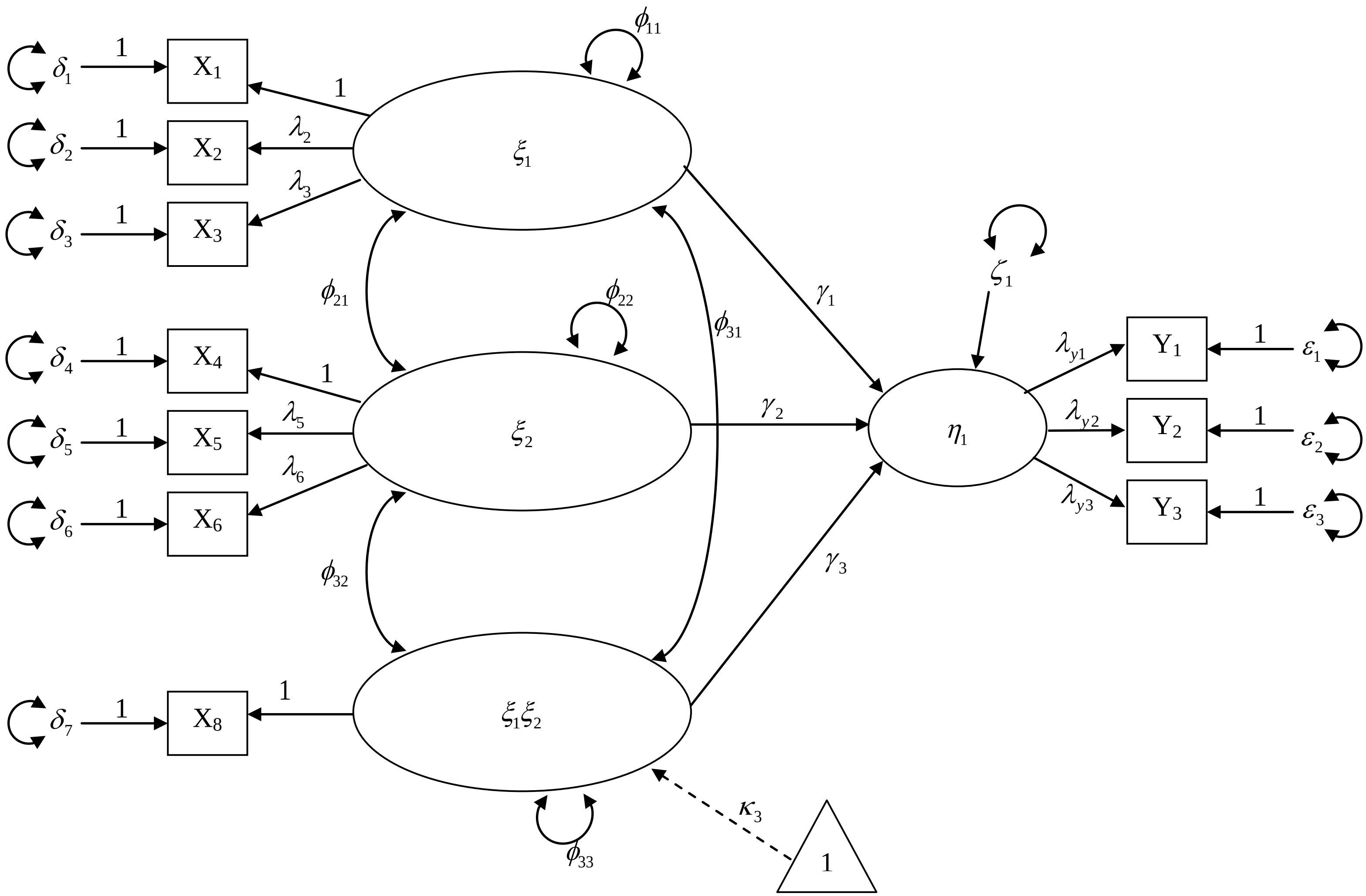


Figure 5: *Two-Stage Least Squares Interaction Model*

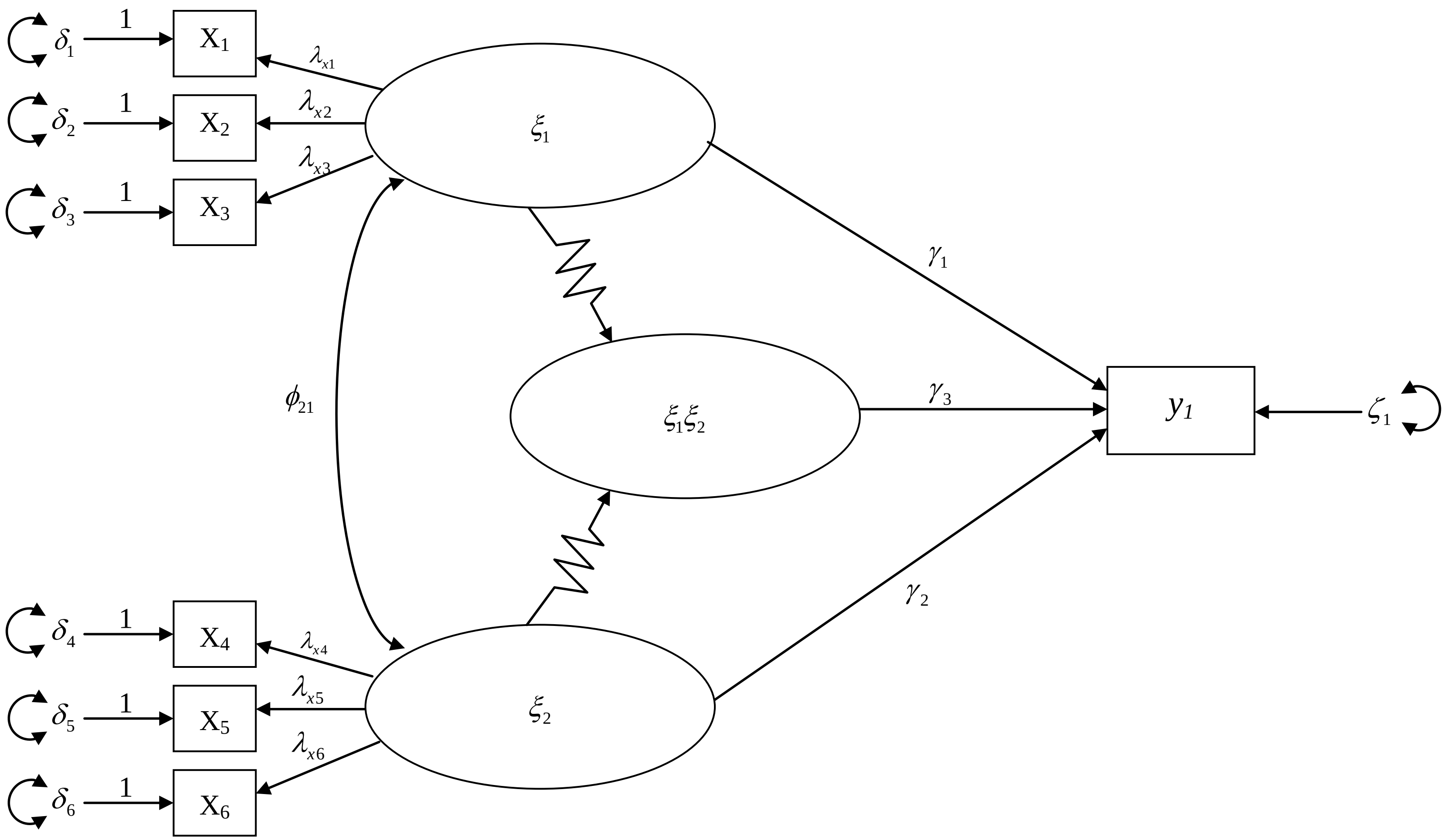


Figure 6: *Interaction Model*

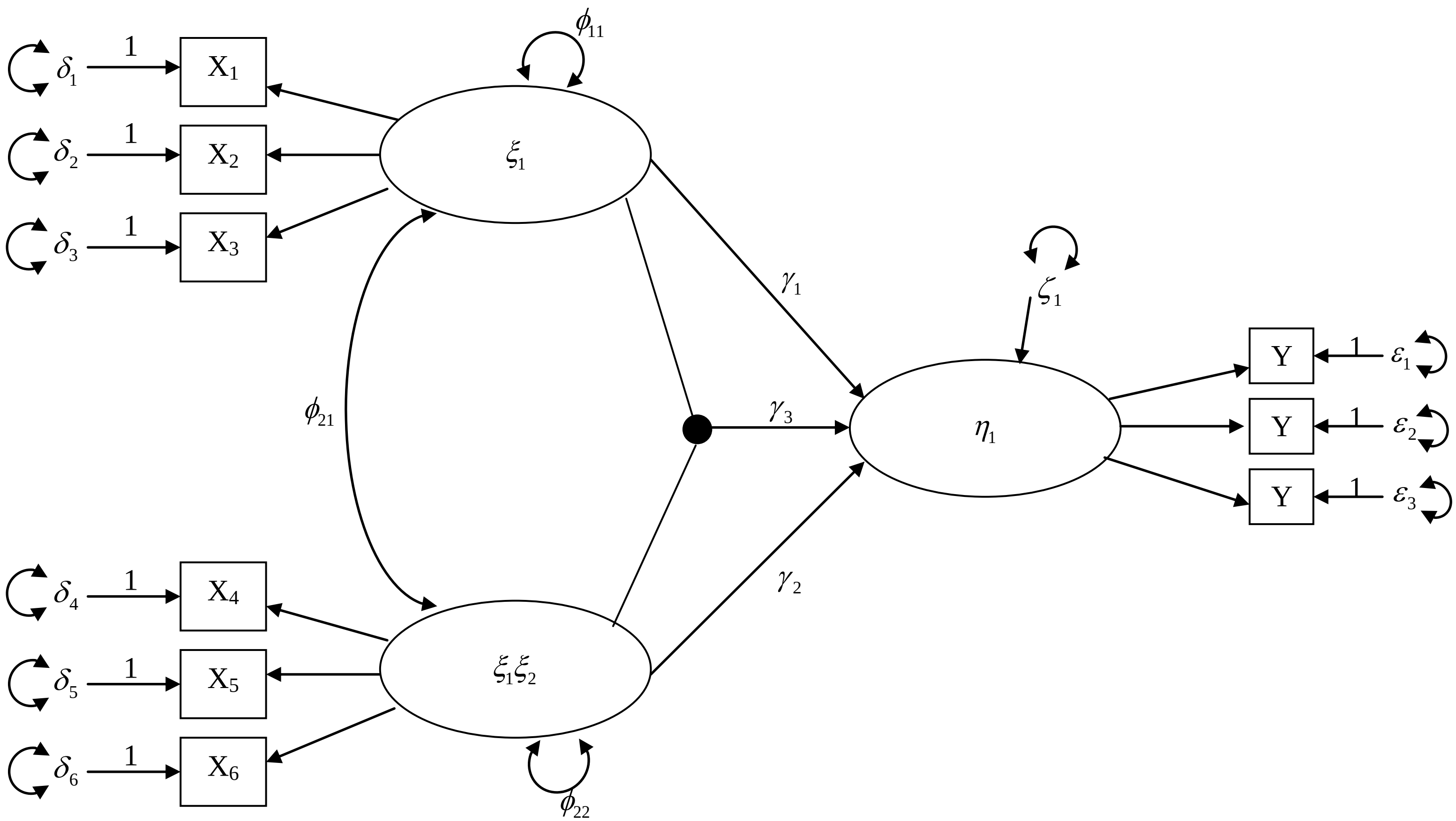
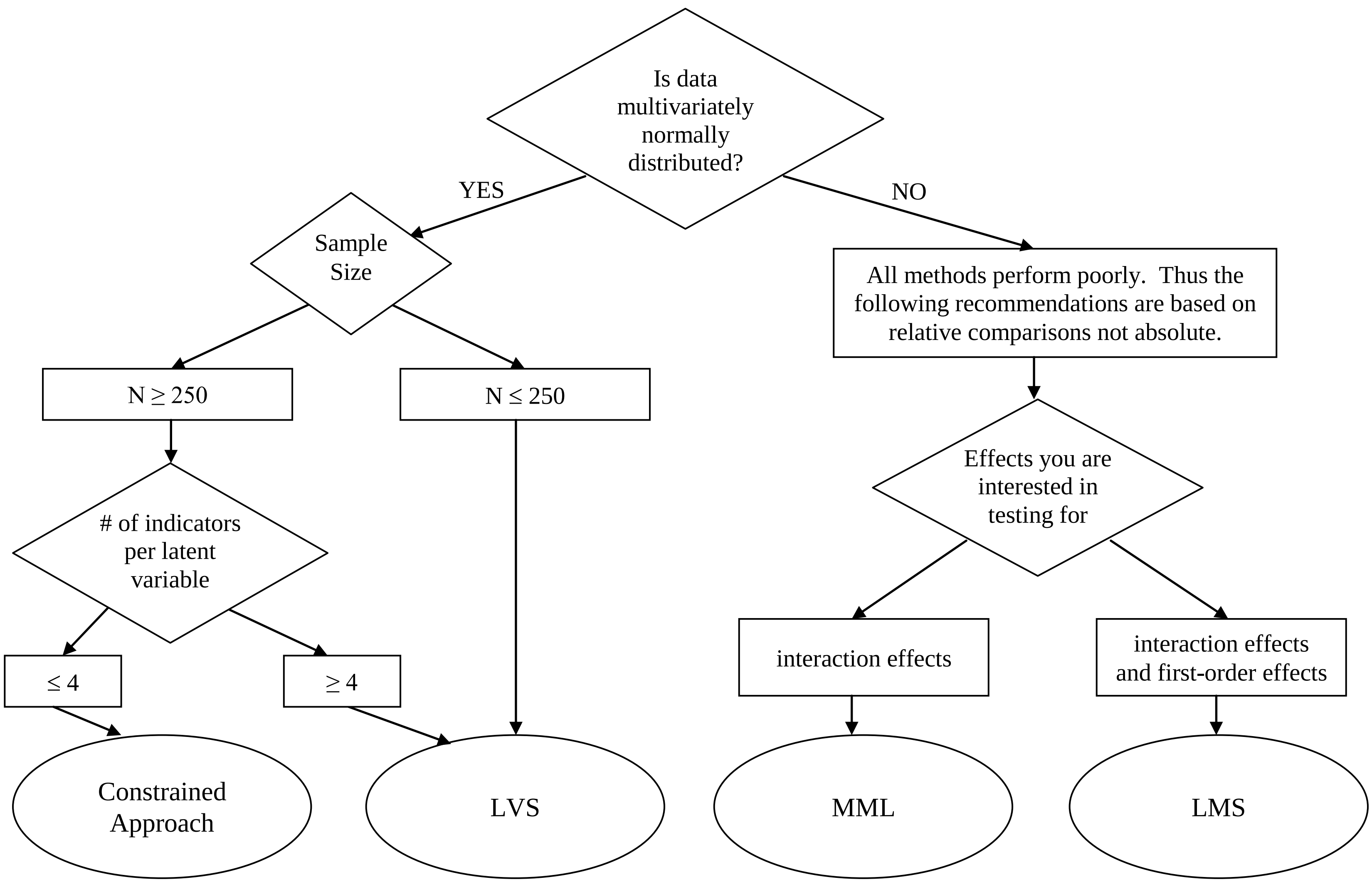


Figure 7: *Recommendations for Type of Procedure*



Appendix A: Summary of Constraints for 3-indicator Interaction Model

1. **Constraint #1 -** the loadings for each of the interaction effects indicators is constrained to equal the product of their associated loadings on ξ_1 and ξ_2

$$\lambda_{x7} = \lambda_{x1}\lambda_{x4},$$
$$\lambda_{x8} = \lambda_{x2}\lambda_{x5},$$
$$\lambda_{x9} = \lambda_{x3}\lambda_{x6},$$

2. **Constraint #2** - the variance of the interaction latent variable is equal to the product of the variances of ξ_1 and ξ_2 plus the squared covariance between ξ_1 and ξ_2 [imposed in conjunction with the normality constraint]

$$\phi_{33} = \phi_{11}\,\phi_{22} + \phi_{21}^{\ 2}$$

3. **Normality Constraint** – The second constraint is based on the assumption that ξ_1 and ξ_2 are normally distributed. If this assumption holds true then the covariance of $\xi_1\xi_2$ and each of the first-order terms (i.e., ξ_1 and ξ_2) is zero, (i.e., $\phi_{31} = 0$ and $\phi_{32} = 0$)]. Thus, the second constraint should also be imposed in conjunction with the normality constraint in which ϕ_{31} and ϕ_{32} are constrained to equal zero.

4. **Constraint #3** – constrains the errors of the each of the indicators for the interaction latent variable

No Mean Centering:

$$\theta_{\delta 7} = \tau_1^2\theta_{\delta 4} + \tau_4^2\theta_{\delta 1} + \lambda_{x1}^2\phi_{11}\theta_{\delta 4} + \lambda_{x4}^2\phi_{22}\theta_{\delta 1} + \theta_{\delta 1}\theta_{\delta 4},$$
$$\theta_{\delta 8} = \tau_2^2\theta_{\delta 5} + \tau_5^2\theta_{\delta 2} + \lambda_{x2}^2\phi_{11}\theta_{\delta 5} + \lambda_{x5}^2\phi_{22}\theta_{\delta 2} + \theta_{\delta 2}\theta_{\delta 5},$$
$$\theta_{\delta 9} = \tau_3^2\theta_{\delta 6} + \tau_6^2\theta_{\delta 3} + \lambda_{x3}^2\phi_{11}\theta_{\delta 6} + \lambda_{x6}^2\phi_{22}\theta_{\delta 3} + \theta_{\delta 3}\theta_{\delta 6},$$

Mean Centering:

$$\theta_{\delta 7} = \lambda_{x1}^2\phi_{11}\theta_{\delta 4} + \lambda_{x4}^2\phi_{22}\theta_{\delta 1} + \theta_{\delta 1}\theta_{\delta 4},$$
$$\theta_{\delta 8} = \lambda_{x2}^2\phi_{11}\theta_{\delta 5} + \lambda_{x5}^2\phi_{22}\theta_{\delta 2} + \theta_{\delta 2}\theta_{\delta 5},$$
$$\theta_{\delta 9} = \lambda_{x3}^2\phi_{11}\theta_{\delta 6} + \lambda_{x6}^2\phi_{22}\theta_{\delta 3} + \theta_{\delta 3}\theta_{\delta 6},$$

5. **Constraint #4** - the mean of the interaction latent variable is constrained to equal the covariance between ξ_1 and ξ_2

$$\kappa_3 = \phi_{21},$$

NOTES

- **Constrained** = constraints #1, 2, 3, 4, normality
- **Partially Constrained** = constraints #1, 3, & 4
- **Unconstrained** = no constraints

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