

ABSTRACT

Title of Dissertation: ESSAYS IN CROSS-COUNTRY
 CONSUMPTION RISK SHARING

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This dissertation concerns cross-country consumption risk sharing in a long-run perspective. Financial integration, empirically measured by cross-country holdings of assets and liabilities, has increased dramatically in the past two decades. But what can explain the lack of cross-country risk sharing documented in the literature? Chapters 2 and 3 of this dissertation address this question.

In Chapter 2, we set up a model to illustrate the mechanical difference between a bond economy and an insurance economy. We show that a bond economy can intertemporally smooth consumption in face of transitory output shocks, but not for permanent output shocks; an insurance economy is essential for risk sharing on permanent shocks. We therefore show that when both transitory and permanent output shocks exist, transitory shocks only create “noise” if the focus of interest is on identifying risk sharing in the long run.

In Chapter 3, we specify an empirical nonstationary panel regression model to test long-run consumption risk sharing across a sample of OECD and emerging market countries. This is in contrast to tests in the literature which are mainly

about risks at business cycle frequency. We argue that these existing tests neglected the permanent elements of risks that are of interest and that their model specifications were not rich enough to accommodate heterogeneous short-run dynamics. Since our methodology focuses on identifying cointegrating relationships while allowing for arbitrary short-run dynamics, we can obtain a consistent estimate of long-run risk sharing while disregarding any short-run nuisance factors.

Our results show that, for the period of 1950-2008, the level of long-run risk sharing in OECD countries is similar to that in emerging market countries. However, during the financial integration episode of the past two decades, long-run risk sharing in OECD countries increased more than in emerging market countries. Furthermore, we investigate the relationship between various measures of financial integration and cross-country risk sharing, but only find weak evidence of such linkages.

ESSAYS IN CROSS-COUNTRY CONSUMPTION RISK SHARING

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Dedication

To my wife, Ying Zhang, who has been with me on this from the beginning. Without her understanding and care for the family, it would have been much harder to complete. Half of this dissertation belongs to her.

To my mother and father, who live far away from me, but care enough to give me passion and courage every single moment. This is really for them.

To my daughter, Sonia Qiao, who was born shortly after I began this research. She brought new energy, new happiness, and new challenges. This has all brought new momentum for me to stay steady on this difficult but fascinating process.

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Chapter 1: Introduction

Risk sharing or state contingent insurance is a vintage topic that can be traced back to Arrow (1964), Debreu (1959) and recently Shiller (1993). Obstfeld and Rogoff (1995) showed that, assuming a framework of a complete market with two countries, two periods, and a constant relative risk aversion (CRRA) utility function, consumer utility maximization leads to a “perfectly pooled equilibrium” (Lucas (1982)). This benchmark model implies that full risk sharing leads to perfect cross-country consumption correlation. However, despite the long history and theoretical soundness, Backus, Kehoe and Kydland (1992) documented an important “consumption correlation regularity”, i.e., cross-country consumption correlation is no higher than cross-country output correlation, a contradiction to the model’s prediction. Following Backus, Kehoe and Kydland (1992), explanations of the regularity hinge on the idea of relaxing the consumption utility function to allow, for example, non-addictive non-tradable goods (Backus and Smith, 1993; Tesar, 1993), the inseparability of goods and leisure (Devereux, Gregory and Smith, 1992), and taste shocks (Stockman and Tesar, 1995). The problem is that these models still predict a strong consumption correlation, but empirical tests nevertheless indicate otherwise.¹

Parallel with the development in theoretical models, much of the empirical literature uses panel regression of idiosyncratic consumption growth on idiosyncratic output growth for testing risk sharing (we call this type of regression “conventional panel regression”). Since the benchmark model predicts consumption to be perfectly correlated across countries, consumption should be orthogonal, or inde-

¹Some models transmit strong consumption correlation into other aspects, which leads to irrational testing results on those other aspects.

pendent, to output apart from the common components of world consumption and output.

It is not surprising that the tests rejected the null hypothesis of orthogonality from the benchmark model's prediction, considering that many factors can limit the level of risk sharing in the real world (Mendoza, 1991; Backus, Kehoe and Kydland, 1992 on market frictions and restrictions on market institutions; and Obstfeld and Rogoff, 1995 on moral hazard and sovereign risks). Since the null hypothesis is always rejected, people take a more practical approach to interpret the estimated slope coefficient as a measure of risk sharing. However, this leads to indecisive findings on risk sharing. For example, Canova and Ravn (1996) concluded that risk sharing is almost complete in a short cycle, but not in medium and long cycles. This contradicts the claim of Artis and Hoffmann (2006) that there is more risk sharing in the long run rather than in the short run.

Furthermore, we expect an increase in risk sharing following the recent increase in global financial integration since a country is better off when financial integration can trade away some of its idiosyncratic risk through international diversification. What is puzzling is that much of the literature did not find increases in risk sharing (Bai and Zhang, 2006; Moser, Pointer and Scharler, 2004).²

These empirical findings imply that we need to be cautious on interpreting test results. In order to explain this, we illustrate in Chapter 2 the mechanical difference between intertemporal smoothing and risk sharing and show how the estimate of risk sharing can be biased due to contamination by intertemporal smoothing and other factors.

²Artis and Hoffmann (2006) and Artis and Hoffmann (2007), among a few papers, found risk sharing increased in the recent financial integration period. Labhard and Sawichi (2006), based on a factor analysis approach, even found a slight decrease in risk sharing between UK regions and between UK and other OECD countries. For survey papers, please refer to Kose, Prasad and Terrones (2007) and Corcoran (2008).

Specifically, we set up a model to show that a bond market can intertemporally smooth consumption in face of transitory output shocks, but not for output shocks that have permanent effects. An insurance market is essential for risk sharing on permanent shocks. The types of shocks do not matter in the case of complete insurance because all the risks caused by shocks are fully shared across state of natures and there is no intertemporal smoothing. However, if an insurance market is not complete, the consumption dynamics, driven by the motivation of intertemporal smoothing, is subject to these types of shocks. For example, intertemporal smoothing through a bond market should drive consumption moves more dramatically than output if output is a unit root process, but relatively smooth if output is stationary. Taking this into a panel with heterogeneous cross-country output processes, the estimated slope coefficient in a conventional panel regression captures risk sharing effects, an unknown term, plus a bias caused by correlation between output and the error term. However, if the focus of interest is on identifying risk sharing in the long run, we show that transitory shocks only create “noise” in identifying long-run risk sharing through a nonstationary panel regression. More generally, other nuisance factors, such as taste shocks or short-run dynamics caused by market frictions, also become innocuous.

Chapter 3 tackles the issue noted above by estimating an empirical nonstationary panel regression model. Let us discuss the limitations of the conventional panel regression. First, conventional panel tests do not consider the permanent elements of risks that are of interest. This is because these tests work with differenced data in order to achieve stationarity of the data and therefore avoid the spurious regression problem. However, by doing so, they disregard the permanent element of risks immediately. If the welfare gain of sharing the risks stemming from permanent shocks is larger than that of sharing transitory risks, it is especially important to analyze

the sharing of permanent risks. Secondly, as a result of using specifications that are not rich enough to accommodate the true data generating process (DGP), conventional panel tests omit important factors such as heterogeneous short-run dynamics in output and consumption processes. In many applications, a conventional panel regression is suitable because the panel contains data with a large N dimension and a limited T dimension. The limited/finite T dimension constrains a conventional panel's ability to deal with dynamics, especially heterogeneous dynamics, even if theories indicate the importance of dealing with dynamics. However, in our application, it is important to take the heterogeneous dynamics into consideration simply because we work with countries which are different in so many dimensions.

In light of these limitations, we estimate an empirical nonstationary panel regression model that tests long-run consumption risk sharing. Our methodology focuses on identifying a long-run cointegrating relationship between consumption and output that is induced by risk sharing while allowing for arbitrary short-run dynamics. This implies that we can obtain a consistent estimate of long-run risk-sharing while disregarding any short-run nuisance factors.

This is because a nonstationary panel regression essentially uses time-series properties which take care of dynamics that are unknown. We therefore can be blind about many aspects of the DGP. Specifically, we allow flexibility in both the length and the magnitude of dynamics across countries. This allows us to circumvent many issues that require strong assumptions in the conventional panel. Some may argue that it is nice to apply time-series arguments to macroeconomic tests, but we face data limitations. One of the nice features of nonstationary panels is that it uses data in cross-sectional dimensions to compensate for relatively short data in temporal dimensions in order to achieve reliable estimating and testing results.

We also address an important issue in the empirical work on risk sharing: the

cross country variation in the steady state of risk sharing. On a practical level, different countries will reasonably choose the level of cross-country holdings of assets and liabilities to the extent that costs equal benefits. Given that costs and benefits may differ across countries and across different contingencies, the level of risk sharing should be different. While the nonstationary panel specification allows heterogeneous slope coefficients, the slope coefficient is forced to be common across countries in a conventional panel specification. As a byproduct of allowing the heterogeneity in risk sharing, we can study cross-country risk sharing distribution and link this distribution pattern to static financial integration indicators.

The empirical results of Chapter 3 show that, for the period of 1950-2008, the level of long-run risk sharing in OECD countries is similar to that in emerging market countries. However, during the financial integration episode of the past two decades, long-run risk sharing in OECD countries increased more than emerging market countries. Furthermore, we investigate the relationship between various measures of financial integration and cross-country risk sharing, but only find weak evidences on such linkages.

In sum, this dissertation contributes to the literature on illustrating long-run cross-country consumption risk sharing and to the literature on empirical tests of it. The structure of the rest of this dissertation is as follows. Chapters 2 and 3 are both self-contained essays. Chapter 2 provides a theoretical illustration on cross-country consumption risk sharing. Chapter 3 tests long-run consumption risk sharing across a sample of OECD and emerging market countries. Chapter 4 summarizes the main findings in both essays and discusses future research directions.

Chapter 2: Theoretical Illustrations on Cross-country Consumption Risk Sharing

2.1 Introduction

We assume a world of N small endowment economies with infinite periods. Each economy is endowed with the same single tradable good in each period. Endowments are stochastic, with both stochastic permanent shocks and transitory shocks possible.

The main purpose of this chapter is to show how endowment risks/shocks can be “traded” under different market structures. To that end, and to keep our model as simple as possible, we assume endowment is perishable. For perishable goods, the only way to “save” them is through financial markets.

We assume three market structures: autarky market/economy, risk-free bond market (or bond economy) and Arrow-Debreu state-contingent securities market (or insurance economy). In an autarky economy, people should consume their current endowment if the value of their current endowment perishes to zero in the next period, so risk sharing is trivially zero. For this reason, in the following sections, we focus on just two scenarios: the bond economy and the insurance economy.

In section 2.2, we set up our bond and insurance economy models and solve the optimal consumption paths respectively. We explain that the insurance economy, compared to the bond economy, can achieve better consumption risk sharing since people make sufficient ex-ante asset trading to protect against future contingencies affecting their economic well-being. In section 2.3, we give a model for partial risk sharing and provide an estimation solution on long-run risk sharing. We conclude

in section 2.5.

2.2 Models

Since each country in the world is small, we assume the world interest rate, r . We also assume that people have the same time preference. In a setting with stochastic endowment, people cannot perfectly foresee random outputs, so consumption decisions have to be based on expectations on future outputs. We therefore assume rational expectation on realizations of random outputs in the future.

2.2.1 Bond Economy

We first discuss the question of utility maximization in the bond economy.

Expected utility:

Under the setting of one good and an infinite time horizon, people maximize the discounted expected value of their lifetime utility,

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} E_t [u(C_s)] \quad (1)$$

where C_s is consumption at period s ; $u(\cdot)$ is a period utility function; β is the subjective discounting factor on time preference; and $E_t[\cdot]$ is the conditional expectation operator at period t . U_t is therefore an expected value. To be specific, it is the present expected values of discounted future consumption utilities.

Since expectations need not be correct, people can be surprised by shocks. In a bond economy, people's consumption is therefore contingent on shocks.

Budget constraint:

Since a bond is the only internationally traded asset in this economy, the intertemporal budget constraint is:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s \quad (2)$$

where Y_t is output endowment at period s ; and B_t is the bond holding at the beginning of period t . We restrict Y_t from growing faster than at rate r , i.e., $E|Y_t| < \frac{c}{(1+r)^t} \forall t$, where c is a constant. Under this restriction, equation (2) holds because we assume the transversality condition,³

$$\lim_{s \rightarrow \infty} \left(\frac{1}{1+r} \right)^s B_s = 0 \quad (3)$$

Recall that we assume output is perishable, so people can only save or borrow to smooth consumption by changing their bond holdings.

2.2.2 Insurance Economy

In a bond economy, we limit our discussion of maximizing consumption utility through trading a risk-free bond. However, in today's international financial markets, there are an increasingly wide range of financial instruments besides a risk-free bond. An example is stock. A different feature of stock, compared to a risk-free bond, is that its returns are state-contingent, which means returns vary according to realized states of nature. Will trading state-contingent assets change an economy's reaction to shocks and therefore better mitigate the effects of shocks?

³For more details, please refer to Obstfeld and Rogoff (1995).

To answer the question above, we assume a complete market for insuring output risks, as developed by Arrow (1964) and Debreu (1959). For simplicity, we assume in this section Y_t only has a finite number of possible realizations.

We now introduce the complete Arrow-Debreu market, along with some new notations. We start with a simpler setting of a world of two endowment economies, A and B, that exist for two periods 1 and 2. We assume two outcomes (states of nature), o_1 and o_2 , possible in period 2, which differ only in their associated outputs of A and B. Formally, for the world economy, we have $\Omega = \{o_1, o_2\}$, the sample space of period 2. The complete information σ -algebra field contains $2^2 = 4$ events, namely $e_1 = \{o_1\}$: outcome 1 happens, $e_2 = \{o_2\}$: outcome 2 happens, $e_3 = \{\}$: neither 1 nor 2 happens, and $e_4 = \{o_1, o_2\}$: either 1 or 2 happens. Suppose that people can buy or sell securities with the following payoff structure in period 1: the owner (seller) of the security receives (pays) 1 unit of output in period 2 if o_i occurs on period 2, but receives (pays) nothing if o_j occurs, where $i, j = 1, 2$ and $i \neq j$. We use $p(o_i)$ to denote the period 1 price of such a security, quoted in terms of a sure period 2 output; We denote by $\pi(o_i)$ the associated probability for o_i . Please note that $p(e_3) = 0$ and $\pi(e_3) = 0$; $p(e_4) = p(o_1) + p(o_2)$ and $\pi(e_4) = \pi(o_1) + \pi(o_2) = 1$.

We can generalize this formation into a world of N economies that exists infinitely, with finite N_s outcomes $\{o_1, o_2, \dots, o_{n_s}, \dots, o_{N_s}\}$ possible in period s . The output level of period s now not only depends on outcomes in period s , but also on the history of the economy up to period s . We use h_t to denote the history of the world economy in period t . In this multiperiod setting, h_t is a state of nature that represents current and past outcomes. Thus h_t is a vector valued element in the sample space of period t , denoted as $h_t \in H_t(h_t)$. If outcome $o_{n_{t+1}}$ occurs on period $t + 1$, then $h_{t+1} = (o_{n_{t+1}}, h_t)$ is an element in the sample space of period $t + 1$,

denoted as $h_{t+1} \in H_{t+1}(h_t)$. There are N_{t+1} possible outcomes and so N_{t+1} possible states of nature in period $t + 1$ (from the point of view of period t , given h_t has happened). Progressively, for period $t + 2$, $h_{t+2} = (o_{n_{t+2}}, o_{n_{t+1}}, h_t) = (o_{n_{t+2}}, h_{t+1})$, there are $N_{t+1} \times N_{t+2}$ states of nature in period $t + 2$ sample space $H_{t+2}(h_t)$; and for any period $s > t + 2$, $h_s = (o_{n_s}, h_{s-1})$ there are $N_{t+1} \times N_{t+2} \times \cdots \times N_s$ states of nature in period s sample space $H_s(h_t)$. We can think of a sample space $H_s(h_t)$ as all possible history of the world economy from period t through period s that has $N_{t+1} \times N_{t+2} \times \cdots \times N_s$ elements. The corresponding complete information σ – algebra field contains $2^{N_{t+1} \times N_{t+2} \times \cdots \times N_s}$ events.

Suppose that people can buy or sell securities with the following payoff structure in period t : the owner (seller) of the security receives (pays) 1 unit of output in periods $s > t$ if h_s occurs in period s , but receives (pays) nothing if h'_s occurs, where $h'_s \in H_s(h_t)$ and $h'_s \neq h_s$. We use $p(h_s|h_t)$ to denote the period t price of such a security, quoted in terms of a sure period s output. We denote by $\pi(h_s|h_t)$ the associated probability for h_s to occur. We call such a security the Arrow-Debreu security for h_s . We call a market an Arrow-Debreu market if it constitutes such securities for every $h_s \in H_s(h_t)$, and we call an economy with Arrow-Debreu market a (complete) insurance economy. Please note, after defining the security price and probability for each state of nature, we can yield the “composite” security price and probability for all events. For example, for an event $\{h_s, h'_s\}$, the associated probability equals $\pi(h_s|h_t) + \pi(h'_s|h_t)$ and the price of the “composite” security equals $p(h_s|h_t) + p(h'_s|h_t)$.

An insurance economy has stronger assumptions than the case of a bond economy. We now assume people have complete information on prices of all Arrow-Debreu securities instead of just one price, r , the interest rate of a risk-free bond.

Expected utility:

In an insurance economy, people maximize expected utility as follows:

$$U_t = u(C_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} \left\{ \sum_{h_s \in H_s(h_t)} \pi(h_s | h_t) u[C(h_s)] \right\} \quad (4)$$

where $C(h_s)$ is the uncertain consumption of future period s . Please note that $u[C(h_s)]$, $s \geq t+1$, does not depend on the realized state of nature h_t , i.e., $u[C(h_s)]$ is stable across states of nature. This equation is no different from equation (1). However, it shows explicitly how the expectation in equation (1) is computed because this helps to illustrate that ex-ante contingency consumption arrangements in an insurance economy can achieve stable consumption across states of nature.⁴

Budget constraint:

We can express the Arrow-Debreu budget constraint as

$$\begin{aligned} & C_t + \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[\sum_{h_s \in H_s(h_t)} p(h_s | h_t) C(h_s) \right] \\ &= Y_t + \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[\sum_{h_s \in H_s(h_t)} p(h_s | h_t) Y(h_s) \right] \end{aligned} \quad (5)$$

where $Y(h_s)$ is the uncertain output of future period s . The LHS of equation (5) gives the present value of the country's uncertain consumption stream; the RHS gives the present value of the country's uncertain output.

⁴Arrow Debreu securities do not exist in the real world, but the same results of the insurance economy model can be achieved in a mutual fund economy model. For details on this, please refer to Obstfeld and Rogoff (1995).

2.2.3 Two Types of Shocks

Unit root process

In this paper, we assume output Y_t is $I(1)$ in the sense that⁵:

$$Y_t - Y_{t-1} = u_t \tag{6}$$

where the process $\{u_t\}$ satisfies

$$A(L)u_t = B(L)\varepsilon_t$$

with

$$A(L) = 1 - a_1L - a_2L^2 - \dots - a_pL^p$$

$$B(L) = 1 + b_1L + b_2L^2 + \dots + bL^q$$

where $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables with $E\varepsilon_t = 0$, $E\varepsilon_t^2 = \sigma_\varepsilon^2$, and $E\varepsilon_t^4 < \infty$.

By definition, the process $\{u_t\}$ is an $ARMA(p, q)$ process if $\{u_t\}$ is stationary. If we further assume $A(z)$ and $B(z)$ have no common roots, and $A(z) \neq 0$ and $B(z) \neq 0$ for all $|z| \leq 1$, i.e., all the roots of $A(z)$ and $B(z)$ lie outside the unit

⁵Generally, an output process can be modeled as a unit root with drift. We assume no drift. This is because, in terms of risk sharing, we are sharing risks corresponding to stochastic components of output, or in other words, corresponding to the forecasting variance from a certain time point of view. The drift term is independent of the forecasting error and therefore can be safely dropped. Technically, if $Y_t - Y_{t-1} = \delta + u_t$ is the true DGP, we can always subtract δt from Y_t to make the mean of Y_t constant. However, the variance of Y_t is still time dependent and goes to infinity over time. Note that the statement of infinite variance is loose. Strictly speaking, if the difference of an $I(1)$ process is a causal and invertible $ARMA$ process, as we define immediately below, the variance goes to infinity.

circle, then $\{u_t\}$ is causal and invertible and can be expressed as⁶

$$u_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \quad (7)$$

where

$$\begin{aligned} \psi(L) &= \frac{B(L)}{A(L)} = \frac{1 + b_1L + b_2L^2 + \dots + b_qL^q}{1 - a_1L - a_2L^2 - \dots - a_pL^p} \\ &= \psi_0 + \psi_1L + \psi_2L^2 + \psi_3L^3 + \dots = \sum_{j=0}^{\infty} \psi_j L^j \end{aligned}$$

We assume

$$\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty \quad (8)$$

Permanent and transitory shocks

According to Proposition 17.2 of Hamilton (1994), process (6) can be rewritten as

$$Y_t = u_1 + u_2 + \dots + u_t + Y_0 = \psi(1) \cdot (\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t) + \eta_t - \eta_0 + Y_0 \quad (9)$$

where $\psi(1) = \sum_{j=0}^{\infty} \psi_j$, $\eta_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j}$, $a_j = -(\psi_{j+1} + \psi_{j+2} + \psi_{j+3} + \dots)$, and $\sum_{j=0}^{\infty} |a_j| < \infty$.

This says that for any nonstationary process that satisfies equations (6)-(8), it can be decomposed into a random walk process, $\psi(1) \cdot (\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t)$, the initial condition, $Y_0 - \eta_0$, and a weakly dependent stationary process, η_t . This

⁶For more discussion on causal and invertible ARMA process, please refer to Brockwell and Davis (1991) and Prucha (2004).

decomposition was first observed by Beveridge and Nelson (1981) and therefore called the Beveridge and Nelson decomposition.

Since η_t is a stationary process, only the first term, $\psi(1) \cdot (\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t)$, matters in the long run. This is because when $t \rightarrow \infty$ the other terms converge to zero asymptotically at the rate of $1/\sqrt{T}$.

In order to facilitate the illustration of the difference of intertemporal smoothing on permanent shocks and transitory shocks, we rewrite equation (6) into the form driven by two additive shocks:

$$Y_t = P_t + T_t \tag{10}$$

where

$$P_t = P_{t-1} + \theta_t \tag{11}$$

$$T_t = \sum_{j=0}^{\infty} b_j \zeta_{t-j} \tag{12}$$

where $\{\theta_t\}$ is a sequence of i.i.d. random variables with $E\theta_t = 0$, $E\theta_t^2 = \sigma_\theta^2$, and $E\theta_t^4 < \infty$ ⁷; $\{\zeta_t\}$ is similarly defined. $\{\theta_t\}$ and $\{\zeta_t\}$ are uncorrelated for all leads and lags.⁸ Note that, given the assumption that u_t is causal and invertible, T_t is also a causal and invertible ARMA process.

Process $\{T_t\}$ is weakly dependent since the covariance of T_t and T_{t+h} tends to zero as h tends to infinity.⁹ This is a very different property from the process $\{P_t\}$

⁷Process $\{P_t\}$, such defined, is a random walk process. It is a special case of a unit root process. The feature of a random walk process emphasized repeatedly in this paper is its persistence, or in other words, its non-weakly-dependence. We say a random walk is highly persistent since (as you will see in the text shortly) $E_t(P_{t+s}) = P_t$ for all $s > t$.

⁸For a formal proof that process (6) can indeed be rewritten into process (10) - (12), please refer to Chapter 13 of Hamilton (1994).

⁹We know $cov(T_t, T_{t+h}) \neq 0$ even as $h \rightarrow \infty$ if the AR part exists. However, because the speed with which $cov(T_t, T_{t+h})$ tends to zero occurs at a geometric rate, we consider the process

that is highly persistent, since $E(P_{t+h}|P_t) = P_t$ for all $h \geq 1$, i.e., the predicted value of P_{t+h} , conditional on information available at period t , always equals the value of period t regardless of how large h is.

So far, we assume the transitory component follows a weakly stationary process (12). It is a general process where b_j has a complicated coefficient structure on an infinite-order moving average process. The reason to start with such a general specification is because we assume that cross-country output processes are heterogeneous in the sense that $\{T_t\}$ is different across countries. However, when model illustrations are in the context of one certain small open economy, we assume a special case of equation (12) for simplicity,¹⁰

$$T_t = \sum_{s=-\infty}^t \rho^{t-s} \zeta_s \quad (13)$$

where $0 \leq \rho < 1$. $\{T_t\}$ in equation (13) is a stationary $AR(1)$ process, expressed as an infinite MA process.¹¹ If we let $j = t - s$, equation (13) can be rewritten into $T_t = \sum_{j=0}^{\infty} \rho^j \zeta_{t-j}$. This confirms that equation (13) is indeed a special case of equation (12).¹²

2.2.4 Optimal Conditions

Bond economy:

Using the Lagrange Multiplier to maximize utility equation (1) under budget constraint equation (3), or equivalently, plug into equation (1) the current account

$\{T_t\}$ to be “short memory”.

¹⁰We use the simple AR (1) process below to show that intertemporal smoothing cannot share permanent risks and only share transitory risks through borrowing and lending. This conclusion should apply to the general class of an ARMA(p, q) process.

¹¹A little algebra shows that the AR (1) representation is $T_t = \rho T_{t-1} + \zeta_t$.

¹²Please refer to Appendix A for an illustration on the differences of permanent and transitory shocks in terms of forecast.

identity equation (14),

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t \quad (14)$$

where CA_t denotes the current account balance at period t , we get the following first order condition:

$$u'(C_t) = (1+r)\beta E_t \left[u'(C_{t+1}) \right] \quad (15)$$

If we assume $(1+r)\beta = 1$ and a linear quadratic utility function, $u(C_t) = C_t - \frac{\theta_0}{2}(C_t)^2$, equation (15) leads to Hall's (1978) result:

$$E_t C_{t+s} = C_t \quad (16)$$

for all $s > t$.

From equation (16) and budget constraint equation (2), we can derive a reduced form consumption

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t(Y_s) \right] \quad (17)$$

Defining $\widehat{Y}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t(Y_s)$, equation (17) can be rewritten into $C_t = rB_t + \widehat{Y}_t$. This says that consumption is the sum of the return on bond holding, rB_t , and the permanent income, \widehat{Y}_t , where \widehat{Y}_t is the weighted average of life-time income.

From equation (17), we can yield consumption changes,

$$\begin{aligned}
C_{t+1} - C_t &= r(B_{t+1} - B_t) + (\widehat{Y}_{t+1} - \widehat{Y}_t) \\
&= r(Y_t - \widehat{Y}_t) + (\widehat{Y}_{t+1} - \widehat{Y}_t)
\end{aligned} \tag{18}$$

where the last equality holds by substituting $B_{t+1} - B_t$ with the result of plugging $C_t = rB_t + \widehat{Y}_t$ into equation (14). Equation (18) says change in consumption is the sum of change in the return on the current account and change in expected permanent income when new information comes.

Rearranging equation (18), we yield

$$C_{t+1} - C_t = \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-(t+1)} [E_{t+1}(Y_s) - E_t(Y_s)] \tag{19}$$

Change in consumption is now expressed as the weighted average of changes in expectations on output. So, in a bond economy, change in consumption is a function of unexpected shocks.

Now let's look at the difference of consumption intertemporal smoothing under different output processes.

- First, let's check the case when output is stationary, i.e., the case when $Y_t = P_t + T_t$ and $\theta_t = 0$. In this case, $Y_t = T_t$, an $AR(1)$ process as defined by equation (13).¹³

For an $AR(1)$ process, we know $E_t(Y_s) = \rho^{s-t}Y_t$; and we know changes in predictions due to shock, ζ_t : $E_t(Y_t) - E_{t-1}(Y_t) = \zeta_t$, $E_t(Y_{t+1}) - E_{t-1}(Y_{t+1}) = \rho\zeta_t$, $E_t(Y_{t+2}) - E_{t-1}(Y_{t+2}) = \rho^2\zeta_t$, and $\lim_{j \rightarrow \infty} E_t(Y_{t+j}) - E_{t-1}(Y_{t+j}) = \lim_{j \rightarrow \infty} \rho^j \zeta_t = 0$.

¹³We assume $AR(1)$ process with no constant term for simplicity. The results below should hold for the case with constant term by replacing Y_t into $Y_t - c$, where c is the constant term.

Plugging $E_t(Y_s) = \rho^{s-t}Y_t$ into equation (17) yields

$$C_t = rB_t + \frac{r}{1+r-\rho}Y_t \quad (20)$$

Plugging changes in predictions into equation (19) yields

$$C_t - C_{t-1} = \frac{r}{1+r-\rho}\zeta_t \quad (21)$$

Equation (21) gives one of the key results of the bond economy. The marginal utility of consumption in period t , $u'(C_{t-1} + \frac{r}{1+r-\rho}\zeta_t)$, is not too different from marginal utility in period $t-1$, $u'(C_{t-1})$. In other words, marginal utility is not much affected when shock is transitory since the change in consumption is only $\frac{r}{1+r-\rho}\zeta_t$, a small fraction of ζ_t . Comparing equation (21) to equation (16), we see that ex-post consumption is close to the ex-ante consumption plan. This implies that the potential benefit of risk sharing through insurance is small from period $t-1$ to period t given intertemporal smoothing exists.¹⁴

To further investigate the implication of equation (21), let's consider a shock in period t and set all shocks to zero in periods $s > t$. We assume B_0 is the bond holding at the beginning of $t-1$ and, for simplicity, zero output before period $t-1$. From equation (20), we have the evolution of consumption, output, and bond holding summarized in the table below:

s	Y_s	C_s	B_s
t-1	0	rB_0	B_0
t	ζ_t	$rB_0 + \frac{r}{1+r-\rho}\zeta_t$	B_0
t+1	$\rho\zeta_t$	$rB_0 + \frac{r}{1+r-\rho}\zeta_t$	$B_0 + \frac{1-\rho}{1+r-\rho}\zeta_t$

¹⁴Baxter and Crucini (1995) has concluded that intertemporal smoothing can act as a close substitute for risk sharing if output shocks are transitory.

t+2	$\rho^2 \zeta_t$	$rB_0 + \frac{r}{1+r-\rho} \zeta_t$	$B_0 + \frac{1-\rho^2}{1+r-\rho} \zeta_t$
t+3	$\rho^3 \zeta_t$	$rB_0 + \frac{r}{1+r-\rho} \zeta_t$	$B_0 + \frac{1-\rho^3}{1+r-\rho} \zeta_t$
...

The table shows that consumption in period $t-1$ equals interest income on bond holding. On the impact of a positive (negative) output shock, people consume permanent income and run a current account surplus (deficit) through lending (borrowing) a portion of current output (Friedman (1957)). Over time, the current account surplus (deficit) decreases as output decreases (increases) and consumption is maintained at its permanent income level.

Let's define short-run changes of consumption and output under the impact of shock ζ_t as $C_t^{SR} \equiv \frac{\partial C_t}{\partial \zeta_t} \zeta_t$ and $Y_t^{SR} \equiv \frac{\partial Y_t}{\partial \zeta_t} \zeta_t$, respectively, where the superscript SR stands for short run. The relationship between C_t^{SR} and Y_t^{SR} is

$$C_t^{SR} = \frac{r}{1+r-\rho} Y_t^{SR} \quad (22)$$

since $C_t^{SR} = \frac{r}{1+r-\rho} \zeta_t$ and $Y_t^{SR} = \zeta_t$ (from the table above).

Notice that Y_t^{SR} jumps by the level of shock ζ_t while C_t^{SR} changes much less dramatically, a confirmation on the implication of equation (21). We call equation (22) a short-run relationship between consumption and output in the bond economy; and the coefficient in equation (22) a measure of the short-run intertemporal smoothing effect.

Let's now define long-run effects of the shock as $C_t^{LR} \equiv \lim_{s \rightarrow \infty} \frac{\partial C_{t+s}}{\partial \zeta_t} \zeta_t$ and $Y_t^{LR} \equiv \lim_{s \rightarrow \infty} \frac{\partial Y_{t+s}}{\partial \zeta_t} \zeta_t$. The relationship between C_t^{LR} and Y_t^{LR} in this case is not well-defined since $C_t^{LR} = \frac{r}{1+r-\rho} \zeta_t$ and $Y_t^{LR} = \lim_{s \rightarrow \infty} \rho^s \zeta_t = 0$.

- Secondly, let's look at the case when output dynamics follows process (11),

i.e., $Y_t = P_t + T_t$ and $\zeta_t = 0$. Now $Y_t = P_t$, a random walk process defined by equation (11).

In this case, $E_t(Y_{t+s}) = Y_t$. Substitute it into equation (17), we get

$$C_t = rB_t + Y_t \tag{23}$$

Equation (23) says people consume all interest earnings and all current output. There is no changes in bond holding.

Notice that consumption and output move one-to-one, indicating no intertemporal smoothing at all. This is another key result of the bond economy, but it is very different compared to the case of transitory shocks. The marginal utility now jumps up/down driven by the level of shock θ_t since ex-post marginal utility in period t is $u'(C_{t-1} + \theta_t)$, different from marginal utility of period $t-1$, $u'(C_{t-1})$, by the magnitude of the shock θ_t . This implies that although people prefer constant consumption and plan for it ex-ante (equation (16)), it turns out that constant consumption is not achievable ex-post. Intuitively, this is where the benefit of risk sharing through insurance comes.

For a random walk shock θ_t in period t and no further shocks in periods $s > t$, the evolution of consumption, output, and bond holding are as follows:

s	P_s	C_s	B_s
t-1	0	rB_0	B_0
t	θ_t	$rB_0 + \theta_t$	B_0
t+1	θ_t	$rB_0 + \theta_t$	B_0
...

The dynamics in this table confirm that the current account is zero in every

period, meaning a country cannot borrow/lend against a permanent shock without violating its budget constraints. It therefore confirms that the shock's effect cannot be shared through intertemporal smoothing.

In terms of C_t^{SR} and Y_t^{SR} , the short-run relationship is

$$C_t^{SR} = Y_t^{SR} \quad (24)$$

since $C_t^{SR} = \theta_t$ and $Y_t^{SR} = \theta_t$.

The long-run relationship is the same as the short-run relationship

$$C_t^{LR} = Y_t^{LR} \quad (25)$$

since $C_t^{LR} = \theta_t$ and $Y_t^{LR} = \theta_t$. This indicates no intertemporal smoothing in the long run as well.

- In the real world, output is subject to both permanent and transitory shocks.

In this case, $Y_t = P_t + T_t$, $\theta_t \neq 0$ and $\zeta_t \neq 0$.

We can rewrite process Y_t into the following *ARI*(1, 1) representation,

$$Y_{t+1} - Y_t = \varrho(Y_t - Y_{t-1}) + \varepsilon_{t+1} \quad (26)$$

Backward recursiveness results in

$$Y_{t+1} = Y_t + \sum_{s=-\infty}^{t+1} \varrho^{t+1-s} \varepsilon_s \quad (27)$$

where ϱ and ε_t , corresponding to parameter ρ and innovations θ_t and ζ_t , are each some complicated function of ρ , σ_θ , and σ_ζ .¹⁵

¹⁵I did not work out the exact form of ϱ and ε_t since it does not provide any extra insight for

From equation (27), we have the following results: $E_t(Y_t) - E_{t-1}(Y_t) = \varepsilon_t$, $E_t(Y_{t+1}) - E_{t-1}(Y_{t+1}) = (1 + \varrho)\varepsilon_t$, $E_t(Y_{t+2}) - E_{t-1}(Y_{t+2}) = (1 + \varrho + \varrho^2)\varepsilon_t$, $E_t(Y_{t+3}) - E_{t-1}(Y_{t+3}) = (1 - \varrho^{s-t})/(1 - \varrho)\varepsilon_t$, and $\lim_{j \rightarrow \infty} E_t(Y_{t+j}) - E_{t-1}(Y_{t+j}) = 1/(1 - \varrho)\varepsilon_t$.¹⁶

Substituting these expressions into equation (17) and equation (19), we can get,¹⁷

$$C_t = rB_t + Y_t + \frac{\varrho}{(1 + r - \varrho)}(Y_t - Y_{t-1}) \quad (28)$$

$$C_t - C_{t-1} = \frac{1 + r}{1 + r - \varrho}\varepsilon_t \quad (29)$$

Note that consumption function (28) now has an error correction term in it. The evolution of output, consumption, and bond holding can be summarized as:

s	Y_s	C_s	B_s
t-1	0	rB_0	B_0
t	ε_t	$rB_0 + \frac{1+r}{1+r-\varrho}\varepsilon_t$	B_0
t+1	$(1 + \varrho)\varepsilon_t$	$rB_0 + \frac{1+r}{1+r-\varrho}\varepsilon_t$	$B_0 - \frac{\varrho}{1+r-\varrho}\varepsilon_t$
t+2	$(1 + \varrho + \varrho^2)\varepsilon_t$	$rB_0 + \frac{1+r}{1+r-\varrho}\varepsilon_t$	$B_0 - \frac{\varrho(1+\varrho)}{1+r-\varrho}\varepsilon_t$
...
t+s	$(1 + \varrho + \dots + \varrho^s)\varepsilon_t$	$rB_0 + \frac{1+r}{1+r-\varrho}\varepsilon_t$	$B_0 - \frac{\varrho(1+\varrho+\dots+\varrho^{s-1})}{1+r-\varrho}\varepsilon_t$
...
∞	$1/(1 - \varrho)\varepsilon_t$	$rB_0 + \frac{1+r}{1+r-\varrho}\varepsilon_t$	$B_0 - \frac{\varrho}{(1-\varrho)(1+r-\varrho)}\varepsilon_t$

The defined short-run relationship is

the purpose of this paper. If you are interested in more details, please refer to Chapter 4 of Hamilton (1994).

¹⁶Please note that if we let $j = t + 1 - s$, we can rewrite equation (27) into $Y_{t+1} = Y_t + \sum_{j=0}^{\infty} \varrho^j \varepsilon_{t+1-j}$. We can see it is a special case of equation (6).

¹⁷For the details on deriving equation (28) and (29), please refer to Appendix B.

$$C_t^{SR} = \frac{1+r}{1+r-\varrho} Y_t^{SR} \quad (30)$$

since $C_t^{SR} = \frac{1+r}{1+r-\varrho} \varepsilon_t$ and $Y_t^{SR} = \varepsilon_t$. Notice that in period t , consumption moves more dramatically than GDP since consumption moves to its permanent level immediately while output movement is a gradual process.¹⁸

In the long run,

$$C_t^{LR} = \frac{1+r-\varrho-r\varrho}{1+r-\varrho} Y_t^{LR} \quad (31)$$

since $C_t^{LR} = \frac{1+r}{1+r-\varrho} \varepsilon_t$ and $Y_t^{LR} = \frac{1}{1-\varrho} \varepsilon_t$. C_t^{LR} and Y_t^{LR} are similar when r is small and ϱ is less than 1¹⁹. To see this, dividing C_t^{LR} by Y_t^{LR} we have $\frac{C_t^{LR}}{Y_t^{LR}} = \frac{1+r-\varrho-r\varrho}{1+r-\varrho}$. So there is a small blip, $r\varrho$, between consumption and output due to intertemporal smoothing on transitory shocks. However, given r is small and $0 < \varrho < 1$, consumption and output move closely, indicating intertemporal smoothing is small and empirically negligible in the long run.²⁰

To conclude this case: intertemporal smoothing in a nonstationary output process is only important in terms of short-run effects. Since the bond market can not share risks induced by permanent shocks, consumption follows output closely in the long run.

¹⁸This is essentially the Deaton paradox in empirical literature.

¹⁹When ϱ equals 1, the output process is an I(2) process, a case we do not consider in this paper.

²⁰When $\varrho = 0$, output degenerates into a pure random walk process and, therefore, consumption and GDP comove perfectly.

Insurance economy

With output uncertainty in a bond economy, individuals keep expected marginal utility stable over time through borrowing and lending (since a risk-free bond is the only financial instrument) after learning of shocks.

Of course, people would prefer stable consumption across states as well. However, this cannot be achieved in the absence of an insurance market. The essence of state contingent claims (Arrow-Debreu securities) is that it can transfer purchase power over time as well as across states. To see this, we would like to focus our concern on the optimal conditions of an insurance economy.

The first order conditions of maximizing equation (4) under budget constraint equation (5) are

$$\left(\frac{1}{1+r}\right)^{s-t} p(h_s | h_t) u'(C_t) = \pi(h_s | h_t) \beta^{S-t} u'[C(h_s)] \quad (32)$$

We continue to assume $(1+r)\beta = 1$. Furthermore, we assume actuarial fairness, i.e., $p(h_s | h_t)/\pi(h_s | h_t) = p(h'_s | h_t)/\pi(h'_s | h_t)$. The assumption $p(h_s | h_t)/\pi(h_s | h_t) = p(h'_s | h_t)/\pi(h'_s | h_t)$ is a counterpart of $(1+r)\beta = 1$. In a bond economy, if $(1+r)\beta \neq 1$, this will induce a consumption tilting over time. Similarly, if $p(h_s | h_t)/\pi(h_s | h_t) \neq p(h'_s | h_t)/\pi(h'_s | h_t)$, this will induce a consumption tilting across states of nature. Since the purpose of this chapter is to discuss the different mechanisms of risk sharing in a bond economy and an insurance economy, these assumptions help focus on the key point and avoid distractions.²¹

Under the assumptions above, equation (32) leads to

$$u'(C_t) = u'[C(h_s)] = u'[C(h'_s)] \quad (33)$$

²¹The assumption of actuarial fairness implies that we assume lenders are risk neutral, i.e., the marginal utility of consumption is constant for lenders.

where $h'_s \in H_s(h_t)$ is a different history of world economy through date s (different from h_s). This is the Euler equation of the insurance economy, indicating equalized marginal utilities across states and over time.

Note that equation (32) implies the Euler equation in the bond economy,

$$u'(C_t) = E\{u'[C(h_s)]\}$$

since $E\{u'[C(h_s)]\} = \sum_{h_s \in H_s(h_t)} \pi(h_s|h_t) u'[C(h_s)] = u'[C(h_s)] \sum_{h_s \in H_s(h_t)} \pi(h_s|h_t) = u'(C_t)$, where the last equality holds because $\sum_{h_s \in H_s(h_t)} \pi(h_s|h_t) = 1$. This says that consumption smoothing achieved in the bond economy can also be achieved in the insurance economy.

Under a quadratic utility function, equation (33) implies

$$\bar{C} = C_t = C(h_s) = C(h'_s) \quad (34)$$

The equalities remind us of equation (16) in the bond economy, but now it has stronger implications: today's consumption not only equals expected future consumption, but also equals future consumption no matter what happens between today and the future. Consumption now is actually constant across states and dates.

Plug equation (34) into equation (5) and we have

$$\bar{C} = \frac{r}{1+r} \left\{ Y_t + \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[\sum_{h_s \in H_s(h_t)} p(h_s | h_t) Y(h_s) \right] \right\}$$

This equation looks similar to equation (17) in the bond economy, but the implication is again different. The former equation holds both before and after shocks

happen since risks have been insured ex-ante. Equation (17), however, shows that changes in consumption is a function of unexpected shocks, which implies that consumption varies one-for-one with output realization. In other words, consumption is not insured ex-ante.

We can see that in the case of insurance, consumption is independent from its own realized output. This implies that no matter what the effects of idiosyncratic shocks are, permanent or transitory, the induced risks are fully shared.²²

An example on bond and insurance economies

We use the following example in order to see the sharp differences between the bond economy and the insurance economy.

Assume a world consists of N rudimentary endowment economies. In order to keep our example simple, we assume all economies are identical in every aspect until period t . In period t , each economy is subject to i.i.d. mean zero idiosyncratic endowment shocks, and there are no further shocks in periods $s > t$.

We assume there are no aggregate shocks, meaning world output is constant over time, $Y_t^W = \sum_{i=1}^N Y_{it} = N \times \bar{Y}$, where Y_t^W is the total world output in period t ; and \bar{Y} is the pre-shock output level of an economy.

Let's review what happens in a bond economy. In the first case, suppose shocks are purely temporary, meaning the shocks' effects are so transitory that it does not affect outputs in period $t + 1$. We therefore can write the output process of country i as $Y_{it} = \bar{Y} + \epsilon_{it}$ where $E_{t-1}\epsilon_{it} = 0$ for all i .

Suppose that country i 's output shock is positive, say, in period t , $Y_{it} = \bar{Y} + \$2$.

²²In a general equilibrium framework with market clearing conditions, Debreu showed that consumption is a constant portion of world output. Together with the derived result here that consumption is constant, the result implies there is no growth in world output, meaning that there is no aggregate output shocks. This is why in empirical studies, world output is subtracted from individual country's consumption.

Since there is no aggregate shock, the total output in the rest of the world in period t is $(N - 1)\bar{Y} - \$2$. All countries' outputs return to \bar{Y} in periods $s > t$. According to the analysis in Section 2.2.4, country i smooths its consumption by consuming $C_{it} = \bar{Y} + r/(1+r) \times \2 and lending to rest of the world (running a current account surplus) of $1/(1+r) \times \$2$. In period $s > t$, there are no further shocks, and i 's consumption will remain higher since it receives returns from its period t lending.

In the second case, let's assume output shocks are permanent, i.e., $Y_{it} = Y.N_{i,t-1} + \epsilon_{it}$. Please note that $Y_t^W = \sum_{i=1}^N Y_{it} = N \times \bar{Y}$ holds since there is no aggregate shock. In this case, i 's consumption increases by $\$2$ permanently, and consumption in the rest of the world decreases by $\$2$ permanently (the current account will therefore be zero).

The results in this example confirm our conclusion in section 2.2.4: intertemporal smoothing in a bond market can achieve consumption smoothing against transitory shocks, but not against permanent shocks.

In an insurance economy, however, people can trade Arrow-Debreu securities for all possible state of natures, i.e., every realization of the random shock has been hedged ex-ante. In this example, by the time of period t , all the countries have already insured each other against any country idiosyncratic output shocks. If country i 's output is $Y_{it} = \bar{Y} + \$2$ and output of the rest of the world is $(N-1)\bar{Y} - \$2$, country i 's consumption equals $C_{it} = \bar{Y}$; if country i 's output is $Y_{i,t-1} + \$2$ and output of the rest of the world is $(N-1)Y_{i,t-1} - \$2$, country i 's consumption equals $C_{it} = Y_{i,t-1} = \bar{Y}$. With insurance, consumption is independent of the $\$2$ shocks, regardless of whether the shock is permanent or transitory.

2.3 Intermediate Case

The illustration in section 2.2 considers consumption risk sharing in a bond economy and an insurance economy separately. Such models are clearly extreme. In the real world, it is well known that the financial market is incomplete, i.e., we are facing an intermediate case where insurance is between 0 and 100 percent.

We now model consumption risk sharing under general output process (6) in an incomplete market with partial insurance. We assume that the $1 - \lambda$ portion of the risks is insurable. So, equation (6) can be rewritten as

$$Y_t - Y_{t-1} = (1 - \lambda)u_t + \lambda u_t$$

We define $(1 - \lambda)u_t = u_t^I$ and $\lambda u_t = u_t^U$, where the superscripts I and U index the insurable and uninsurable respectively. Similarly, we define $(1 - \lambda)Y_t = Y_t^I$ and $\lambda Y_t = Y_t^U$ such that $Y_t = Y_t^I + Y_t^U$. The evolutions of the processes of Y_t^I and Y_t^U are therefore

$$Y_t^I - Y_{t-1}^I = u_t^I$$

$$Y_t^U - Y_{t-1}^U = u_t^U$$

where each process has similar statistical properties as process (6).

This conceptual partition is in line with some of the recent work on exploring the implications of incomplete markets where there do not exist sufficient contracts that would allow people to fully allocate their consumption and resources across states and over time. The incompleteness can be thought of as both exogenous and endogenous. When it is endogenous, it may come from the inability to write

contracts against certain risks or to enforce contracts even if contracts are writable. When it is exogenous, it could be that some people are prevented from insurance markets or some goods are non-tradable in nature.²³

This section will follow in two parts. In the first part, we will show how risk sharing can be identified in the long run in a nonstationary regression framework for an individual country. In order to facilitate this purpose, we again take the special process (26) of the general output process (6). In the second part, we will show long-run risk sharing can be consistently identified even in a nonstationary panel of countries with heterogeneous output processes; that is, the output process in different countries takes a certain form from the general output process (6).

2.3.1 Identifying Risk Sharing in Long Run

Short-run and long-run relationships

For insurable risks, results from the insurance economy should apply, i.e., consumption is constant and therefore independent of output; likewise, for uninsurable risks, results from the bond economy apply. For example, when the output process is $ARI(1, 1)$ (equation (26)), combining results from the bond and insurance economies, consumption is

$$\begin{aligned}
 C_t &= C_t^I + C_t^U \\
 &= (1 - \lambda)\bar{C} + (rB_t + Y_t^U + \frac{\varrho}{1 + r - \varrho}\Delta Y_t^U) \\
 &= (1 - \lambda)\bar{C} + rB_t + \lambda(Y_t + \frac{\varrho}{(1 + r - \varrho)}\Delta Y_t)
 \end{aligned}$$

²³Although the cause of market incompleteness can be endogenous, we do not tackle the issue of endogenous λ in estimation. Rather, we take a practical stance by interpreting λ as a measure of de facto level of risk sharing.

where C_t^I and C_t^U are defined correspondingly to Y_t^I and Y_t^U ; $B_t = Y_{t-1}^U + (1+r)B_{t-1} - C_{t-1}^U$ which only involves Y_t^U and C_t^U since bond holding in an insurance market is zero; and $\Delta Y_t^U = Y_t^U - Y_{t-1}^U$ and $\Delta Y_t = Y_t - Y_{t-1}$.

Likewise, we can also solve consumption in special cases where output processes follow an $AR(1)$ (equation (13)) and a random walk (equation (11)). We summarize results in the table below:

	Y_t	C_t	$C_t - C_{t-1}$
Case1	$Y_t = \rho Y_{t-1} + \zeta_t$	$(1 - \lambda)\bar{C} + rB_t + \frac{\lambda r}{1+r-\rho} Y_t$	$\frac{\lambda r}{1+r-\rho} \zeta_t$
Case2	$Y_t = Y_{t-1} + \theta_t$	$(1 - \lambda)\bar{C} + rB_t + \lambda Y_t$	$\lambda \theta_t$
Case3	$\Delta Y_t = \varrho \Delta Y_{t-1} + \varepsilon_t$	$(1 - \lambda)\bar{C} + rB_t + \lambda(Y_t + \frac{\varrho}{(1+r-\varrho)} \Delta Y_t)$	$\frac{\lambda(1+r)}{1+r-\varrho} \varepsilon_t$

where $C_t - C_{t-1}$ equals the results from the bond economy multiplied by λ because $C_t^I = (1 - \lambda)\bar{C}$ is constant.

Recalling C_t^{SR} and C_t^{LR} defined in bond economy, we can write the counterparts in the incomplete economy as $C_t^{\bar{SR}} = \lambda C_t^{SR}$ and $C_t^{\bar{LR}} = \lambda C_t^{LR}$. These two relationships hold again because $C^I = (1 - \lambda)\bar{C}$ is constant. The defined short-run and long-run outputs stay the same. The table below summarizes the short-run and long-run risk sharing relationships:

	Y_t	Short run	Long run
Case1	$Y_t = \rho Y_{t-1} + \zeta_t$	$C_t^{\bar{SR}} = \frac{\lambda r}{1+r-\rho} Y_t^{SR}$...
Case2	$Y_t = Y_{t-1} + \theta_t$	$C_t^{\bar{SR}} = \lambda Y_t^{SR}$	$C_t^{\bar{LR}} = \lambda Y_t^{LR}$
Case3	$\Delta Y_t = \varrho \Delta Y_{t-1} + \varepsilon_t$	$C_t^{\bar{SR}} = \frac{\lambda(1+r)}{1+r-\varrho} Y_t^{SR}$	$C_t^{\bar{LR}} = \lambda(1 - \frac{r\varrho}{1+r-\varrho}) Y_t^{LR}$

The short-run relationships depend on serial correlation parameters of the output process, besides λ . This is because the effect of intertemporal smoothing in the bond market is important in the short run.

The long-run relationships, however, when they are well-defined, only depend on λ , besides a small blip (in Case3).

Estimating long-run relationships

We first derive the testing equation on long-run risk sharing and show the derived testing equation can consistently estimate the true risk sharing relationship. We will then go back to see that the testing equation based on consumption and output growth cannot consistently estimate risk sharing.

In the case of $ARI(1,1)$ output: $\Delta Y_t = \rho \Delta Y_{t-1} + \varepsilon_t$, we have derived the consumption process,

$$C_t = (1 - \lambda)\bar{C} + rB_t + \lambda\left(Y_t + \frac{\rho}{(1 + r - \rho)}\Delta Y_t\right) \quad (35)$$

and we can derive the bond holding process,

$$B_t = B_0 - \frac{\lambda\rho}{1 + r - \rho}(Y_{t-1} - Y_{-1}) \quad (36)$$

where equation (36) holds if we recursively plug C_{t-1}^U to C_1^U into $B_t = Y_{t-1}^U + (1 + r)B_{t-1} - C_{t-1}^U$ and use $Y^U = \lambda Y$.

By plugging equation (36) into equation (35), we have

$$\begin{aligned} C_t &= (1 - \lambda)\bar{C} + r\left[B_0 - \frac{\lambda\rho}{1 + r - \rho}(Y_{t-1} - Y_{-1})\right] + \lambda\left(Y_t + \frac{\rho}{1 + r - \rho}\Delta Y_t\right) \\ &= \left[(1 - \lambda)\bar{C} + rB_0 + r\frac{\lambda\rho}{1 + r - \rho}Y_{-1}\right] - r\frac{\lambda\rho}{1 + r - \rho}Y_{t-1} + \lambda\left(Y_t + \frac{\rho}{1 + r - \rho}\Delta Y_t\right) \\ &= \alpha - r\frac{\lambda\rho}{1 + r - \rho}\left(Y_t - \sum_{s=-\infty}^t \rho^{t-s}\varepsilon_s\right) + \lambda\left(Y_t + \frac{\rho}{1 + r - \rho}\sum_{s=-\infty}^t \rho^{t-s}\varepsilon_s\right) \end{aligned}$$

where the last equality holds because we let the constant term $(1 - \lambda)\bar{C} + rB_0 + r\frac{\lambda\varrho}{1+r-\varrho}Y_{-1} = \alpha$, and we substitute Y_{t-1} and ΔY_t using $Y_t = Y_{t-1} + \sum_{s=-\infty}^t \varrho^{t-s}\varepsilon_s$ (equation (27)), which, as we have shown is $\Delta Y_t = \varrho\Delta Y_{t-1} + \varepsilon_t$ expressed in another form.

Recollecting terms,

$$C_t = \alpha + \lambda\left(1 - \frac{r\varrho}{1+r-\varrho}\right)Y_t + \frac{(1+r)\lambda\varrho}{1+r-\varrho} \sum_{s=-\infty}^t \varrho^{t-s}\varepsilon_s$$

This is a testable equation that can be written into the following fashion:

$$C_t = \alpha + \beta^{LR}Y_t + u_t \tag{37}$$

where $\beta^{LR} = \lambda\left(1 - \frac{r\varrho}{1+r-\varrho}\right)$; u_t has properties of a causal and invertible *ARMA* process with $u_t = \sum_{s=-\infty}^t \varphi^{t-s}\varepsilon_s$ where φ is a function of ϱ , λ and r ; and $C_t \sim I(1)$; $Y_t \sim I(1)$.

We know that the OLS estimator of β^{LR} , $\hat{\beta}^{LR}$, is a super-consistent estimate of β^{LR} when equation (37) satisfies the above properties. $\hat{\beta}^{LR}$ is super-consistent because its asymptotic properties are driven by the series that is $I(1)$. For this reason, u_t is asymptotically irrelevant even though it has complicated dynamics since it is $I(0)$.²⁴ In Appendix C.1, we showed that $plim_{T \rightarrow \infty} E(\hat{\beta}^{LR}) = \lambda\left(1 - \frac{r\varrho}{1+r-\varrho}\right)$.

²⁴We derived equation (37) under the assumptions of a quadratic utility function and no aggregate shocks. If the utility function is in the form of constant relative risk aversion, equation (37) holds in log terms (Hall (1978)) since now consumption can only be approximated as a random walk process. If aggregate shocks exist, then we need to first subtract them out since aggregate shocks cannot be shared. So, in a general specification, the testing equation will be $c_t - c_t^w = \alpha + \beta^{LR}(y_t - y_t^w) + u_t$, where lower case letters c and y denote log consumption and output. c^w and y^w are the world average log consumption and output. We subtract them from log consumption and output in order to get the idiosyncratic consumption and output.

The term $\lambda \frac{r\varrho}{1+r-\varrho}$ is close to zero if the interest rate r is small and $0 \leq \varrho < 1$. So, even though λ and the estimated β^{LR} are not identical, the difference is small empirically given a small interest rate is the case in the real world.

Key message: we can consistently estimate long-run risk sharing by exploring the nonstationary relationship between consumption and output when the output process is $I(1)$.

Please note that when $\varrho = 0$, output process (26) degenerates into random walk process (11): $Y_t = Y_{t-1} + \varepsilon_t$ where $\varepsilon_t = \theta_t$. This is Case2 in the tables above. Equation (37) can still consistently estimate the true risk sharing relationship (which is λ) since now, $\beta^{LR} = \lambda$ and $u_t = 0$ which can be thought of as an i.i.d. white noise $(0, 0)$ process.

Estimating short-run relationships using differenced series

When $\Delta Y_t = \varrho \Delta Y_{t-1} + \varepsilon_t$, we have derived,

$$C_t - C_{t-1} = \frac{\lambda(1+r)}{1+r-\varrho} \varepsilon_t \quad (38)$$

Substitute ε_t of equation (38),

$$\begin{aligned} C_t - C_{t-1} &= \frac{\lambda(1+r)}{1+r-\varrho} (\Delta Y_t - \varrho \Delta Y_{t-1}) \\ &= \frac{\lambda(1+r)}{1+r-\varrho} (Y_t - Y_{t-1}) - \frac{\lambda(1+r)}{1+r-\varrho} \varrho \Delta Y_{t-1} \\ &= \frac{\lambda(1+r)}{1+r-\varrho} (Y_t - Y_{t-1}) - \frac{\lambda(1+r)}{1+r-\varrho} \varrho \sum_{s=-\infty}^{t-1} \varrho^{t-1-s} \varepsilon_s \end{aligned}$$

where the last equality holds because we substitute ΔY_{t-1} using equation (27).

We can rewrite $C_t - C_{t-1}$ into

$$C_t - C_{t-1} = \beta^{SR}(Y_t - Y_{t-1}) + \mu_t \quad (39)$$

where $\beta^{SR} = \frac{\lambda(1+r)}{1+r-\varrho}$; μ_t has properties of a causal and invertible *ARMA* process with $\mu_t = \sum_{s=-\infty}^{t-1} \psi^{t-1-s} \varepsilon_s$ where ψ is some function of ϱ , λ and r ; and $C_t - C_{t-1} \sim I(0)$; $Y_t - Y_{t-1} \sim I(0)$.

We know that the OLS estimator of β^{SR} , $\beta^{\hat{SR}}$, is not a consistent estimate of β^{SR} when the properties of equation (39) is as defined above. Comparing testing equations (37) and (39), the reason that β^{LR} can be consistently estimated while β^{SR} cannot is because we explore the nonstationary relationship in equation (37), which is robust with respect to the dynamics in the error term. However, $\beta^{\hat{SR}}$ is contaminated by the dynamics in the error term of equation (39).²⁵ Please refer to Appendix C.2 for the probability limit of $\beta^{\hat{SR}}$.

If using differenced series to test risk sharing in the special case of an *AR*(1) output process, we cannot achieve a consistent estimate for a similar reason. We can briefly illustrate this.

When output follows an *AR*(1) process as $Y_t = \rho Y_{t-1} + \zeta_t$, we have derived,

$$C_t - C_{t-1} = \frac{\lambda r}{1+r-\rho} \zeta_t \quad (40)$$

Plug equation $Y_t = \rho Y_{t-1} + \zeta_t$ into equation (40)

²⁵This regression can correctly identify short-run effects in special cases of complete insurance or random walk output. It is simple to check the case of complete insurance where the true relationship is full risk sharing, i.e., $\lambda = 0$. When $\lambda = 0$, $\beta^{SR} = 0$ and $\mu_t = 0$ and, therefore, $\beta^{\hat{SR}}$ is a consistent estimate of the true relationship in equation (39). In the case of a random walk output, $\varrho = 0$, which makes $\beta^{SR} = \lambda$ and $\mu_t = 0$. Again, $\beta^{\hat{SR}}$ is a consistent estimate of the true relationship in equation (39).

$$\begin{aligned}
C_t - C_{t-1} &= \frac{\lambda r}{1+r-\rho}(Y_t - \rho Y_{t-1}) \\
&= \frac{\lambda r}{1+r-\rho}(Y_t - Y_{t-1}) + \frac{\lambda r}{1+r-\rho}(1-\rho)Y_{t-1} \\
&= \frac{\lambda r}{1+r-\rho}(Y_t - Y_{t-1}) + \frac{\lambda r}{1+r-\rho}(1-\rho) \sum_{s=-\infty}^{t-1} \rho^{t-1-s} \zeta_s
\end{aligned}$$

where the last equality holds because we substitute Y_{t-1} using equation $Y_{t-1} = \sum_{s=-\infty}^{t-1} \rho^{t-1-s} \zeta_s$, which, as we have shown, is the same output process as $Y_t = \rho Y_t + \zeta_t$.

We can rewrite the last equality of $C_t - C_{t-1}$ into the following fashion,

$$C_t - C_{t-1} = \beta^{SR}(Y_t - Y_{t-1}) + \mu_t$$

where $\beta^{SR} = \frac{\lambda r}{1+r-\rho}$; μ_t has properties of a causal and invertible *ARMA* process with $\mu_t = \sum_{s=-\infty}^{t-1} \phi^{t-1-s} \zeta_s$ where ϕ is some function of ρ , λ and r ; and $C_t - C_{t-1} \sim I(0)$; $Y_t - Y_{t-1} \sim I(0)$.

We know that the OLS estimator of β^{SR} , $\hat{\beta}^{SR}$, is not a consistent estimate of β^{SR} when the properties of the testing equation are as defined above.

We have finished the discussion on short-run and long-run relationships and the estimations on them. We would like to summarize the main results into the following lemma:

Lemma 1

If output is an $I(1)$ process with $AR(1)$ dynamics, defined as equation (26), the short-run relationship between consumption and output cannot be consistently estimated through equation (39), except in

special cases. However, the level of risk sharing, measured by λ , can be approximated by the estimate on the slope coefficient in equation (37). Mathematically, $\lambda \approx \beta^{\hat{L}R} = \lambda(1 - \frac{r\varrho}{1+r-\varrho})$.

Lemma 1 is drawn from the special output process (26), but it can be generalized for an $I(1)$ output process with causal and invertible $ARMA(p, q)$ dynamics. This is because no matter what the $ARMA$ process is, as long as it is weakly dependent, intertemporal smoothing is small in the long run as effects of $ARMA$ shocks die out. We formalize this generalization into the following lemma:

Lemma 2

If output is an $I(1)$ process with causal and invertible $ARMA(p, q)$ dynamics, defined as equation (6), the short-run relationship between consumption and output cannot be consistently estimated through equation (39), except in special cases. However, the level of risk sharing, measured by λ , can be approximated by the estimate on the slope coefficient in equation (37). Mathematically, $\lambda \approx \beta^{\hat{L}R} = \lambda(1 - o(1))$, where $o(1)$ denotes some small number which is significantly smaller than 1.

2.3.2 Identification under General Output Process (6)

So far, in order to achieve analytical results, we have chosen the special output process (26). However, An output process does not have to be in this special form. That is, the transitory components of outputs can be $AR(1)$ in one country, $AR(2)$ in another, and more generally, some $ARMA$ process in a third country. If we test risk sharing using data from different countries, results from a panel short-run regression will be noisy in the sense that each individual country estimate is biased

and the biases are driven by different dynamics in different countries.

If we know the output dynamics of each single country or we can estimate the output dynamics of each single country, we can make corrections and adjustments on the estimate of the slope coefficient of a short-run regression to achieve a consistent estimate of risk sharing and intertemporal smoothing effects. However, the problem is that we do not know what the true output process is, and it is difficult to provide a reliable estimate on output dynamics, especially in the case of country level studies where we normally only have a couple of decades of time-series data.

Applying the results in Lemma 2, however, a long-run regression of a panel of countries will still be able to consistently identify the true level of risk sharing, apart from a small blip. So, our analysis is useful for designing empirical panel tests on long-run risk sharing when people are faced with both types of shocks.

2.3.3 Other Short-run Distortions

This section is an extension on section 2.3.1. The distortions in this section are important empirical questions, but they can be handled similarly as transitory output shocks. This is because the distortions only have short-run effects and die out over time without influencing the long run relationships.

Everything else being equal, the relationship between consumption and output can be affected by taste shocks. In other words, since taste shocks influence consumption given a particular output process, the estimated β^{SR} moves away from the risk sharing relationship that we intend to identify. When such shocks are large, unknown, and erratic, it posts challenges for an empirical study since it is hard to control them by using observable data. For example, Stockman and Tesar (1995) have shown that people may choose a non-smooth consumption path due to changes in preferences. This implies the high frequency relationship between

consumption and output can be undetermined for a given degree of risk sharing. However, realizing that taste shocks are transitory shocks, this is not an issue for the estimation of long-run risk sharing.

Specifically, in terms of long-run relationships, $Y_t^{LR} = 1/(1 - \rho)\varepsilon_t$ because the output process is the same as before; C_t^{LR} should still equal $(\frac{1+r}{1+r-\rho}\varepsilon_t)$ because $E_t(C_{t+s}) = C_t$ when $s \rightarrow \infty$ as the effects of taste shocks die out. Thus, the long-run relationship is the same as in section 2.3.1, implying that taste shocks are irrelevant in the long run. Please note that a similar argument applies to market frictions, and other types of short-run distortions, as long as their effects are transitory.

2.4 Conclusion

The literature on testing risk sharing has found limited risk sharing across countries. This is in contrast to well documented facts of financial integration in the past two decades, measured by cross-country holdings of assets and liabilities (Lane, 2001; lane and Milesi-Ferretti, 2003). This paper provides a potential explanation on the findings of low risk sharing. We illustrate that a bond economy can intertemporally smooth consumption in face of transitory output shocks, but not for permanent output shocks. An insurance economy is essential for risk sharing on permanent shocks. This mechanical difference requires a careful study of implications in consumption risk sharing given a certain output process. We have therefore shown that, when both transitory and permanent shocks exist, an estimate of risk sharing in a short-run panel regression is inconsistent and contaminated by intertemporal smoothing and other short-run distortions. However, we can achieve a consistent estimate of risk sharing through a long-run panel regression.

Chapter 3: Empirical Tests on Cross-country Consumption Risk Sharing

3.1 Introduction

The complete market benchmark model on consumption risk sharing across countries predicts that a country's consumption equals a constant portion of current world output that depends on the country's initial share of world wealth (Obstfeld and Rogoff, 1995). This implies that a country's consumption is independent of, or orthogonal to, GDP, apart from the global components of its consumption and GDP. Much of the empirical literature has used panel regressions of country specific consumption growth on output growth in testing this orthogonal implication (I call this type of regression a "conventional panel regression").²⁶

What is puzzling is the indecisiveness of the findings in using conventional panel regressions. It is not surprising that the test and estimate results found limited risk sharing considering many factors can limit the level of risk sharing in the real world (Mendoza, 1991; Backus, Kehoe and Kydland, 1992 on market frictions and restrictions on market institutions; and Obstfeld and Rogoff, 1995 on moral hazard and sovereign risks). It is indecisiveness that makes people doubt if risk sharing indeed exists in practice. For example, Canova and Ravn (1996) concluded that risk sharing is almost complete in a short cycle, but not in medium and long cycles. This contradicts the claim of Artis and Hoffmann (2006) that there is more risk sharing in the long run than in the short run. Moreover, despite the theoretical

²⁶Kollmann (1995), using nonstationary time-series techniques to test risk sharing, found rejection of the null hypothesis of full risk sharing in all country pairs. However, the method he used, besides the problem of potential low power and high size distortion in a time series context, can only do a test of full risk sharing or not, but cannot test the degree of risk sharing.

prediction that globalization should reinforce risk sharing through easier access to more diversified contingency contracts, much of the literature nevertheless did not find increases in risk sharing following the recent increase in global financial integration (Bai and Zhang, 2006; Moser, Pointer and Scharler, 2004).²⁷ Labhard and Sawichi (2006), based on a factor analysis approach, even find a slight decrease in risk sharing between UK regions and between UK and other OECD countries. For survey papers, please refer to Kose, Prasad and Terrones (2007) and Corcoran (2008).

At a basic level, a conventional panel regression requires stationarity of the data in order to avoid a spurious regression problem and nonstandard distributions for inference. Therefore, in testing risk sharing, researchers routinely first-difference data on consumption and GDP. As a result of differencing, the estimates measure risk sharing on transitory shocks or risks at business cycle frequency. The welfare gain from risk sharing at business cycle frequency has been found small in the literature, for example, Gourinchas and Jeanne (2006), Lucas (1987) and Cole and Obstfeld (1991). The small welfare gain implies the motivation of risk sharing is low and may be dominated by many other motivations. It is therefore not surprising that low risk sharing or no increase of risk sharing has been found in the literature.

If the level of output contains information beyond the information carried through changes in output that is useful for the decision-making on consumption risk sharing, we should include the level of output in our investigation. Specifically, if output is $I(0)$, i.e., it is mean-reversing, the level of output does not give much additional information beyond the differenced output. If output is $I(1)$, differencing would remove the permanent component of output that drives the nonstationarity.

²⁷Artis and Hoffmann (2006) and Artis and Hoffmann (2007), among a few papers, found risk sharing increased in the recent financial integration period.

As discussed below, the welfare gain of risk sharing on permanent shocks should be much higher than that on transitory shocks. We therefore think it is important and interesting to test risk sharing on permanent shocks. In this case, the estimated consumption risk sharing, identified by the cointegrating coefficient in a nonstationary panel regression model, is long-run risk sharing.

Because our methodology focuses on identifying a long-run cointegrating relationship, we can allow for full heterogeneity in short-run dynamics. This implies that we can obtain a consistent estimate of long-run risk sharing while disregarding any short-run nuisance factors. However, in the conventional panel regression model, without further structure assumption on the model, the dynamics are restricted to be homogeneous.²⁸ As a result, they omit important factors such as the heterogeneity in short-run dynamics that are caused by intertemporal smoothing, taste shocks, or market frictions. The recent paper by Artis and Hoffmann (2008) offers a similar insight. They argue that risk sharing has, in fact, increased following the recent financial integration, but both the conventional panel regression and consumption correlation failed to detect this increase due to the change of the output dynamics in the same period.

Athanasoulis and van Wincoop (2001), and a more recent and close cousin of it, Flood, Marion and Matsumoto (2008) are among the recent developments in the literature that have brought us closer to understanding long-run risk sharing. Athanasoulis and van Wincoop (2001) argued “the effect of temporary income shocks on consumption can be buffered through borrowing and lending, but over longer horizons one can expect consumption growth to closely follow the growth rate of income.” They therefore use the techniques developed in Athanasoulis and

²⁸This is essentially because that conventional panel analysis is an extension of cross-sectional analysis where it pools the cross-sectional dimension or averages on the cross-sectional dimension to achieve an estimate. In other words, it relies on cross-sectional asymptotics for inference. Therefore, it cannot allow for country-specific slope coefficients and dynamics.

van Wincoop (2000) to test income risk sharing at different frequencies between U.S. states.

Artis and Hoffmann (2006) is the closest paper in the literature to this paper. They, as we do below, use consumption and GDP levels (instead of growth rates) on testing and estimating risk sharing, which they hope can get rid of the effects of short-run confounding factors. However, their regression is essentially under a conventional panel framework, without taking the nonstationary properties and full heterogeneity in short-term dynamics into account. Moreover, they use OLS and a pooled version dynamic OLS, which do not give an estimate of a cointegrating relationship if the true slope coefficient is heterogeneous.

Our results indicate that, for the period of 1950 to 2008, the level of long-run risk sharing in OECD countries is similar to that in emerging market countries. However, during the financial integration episode of the past two decades, long-run risk sharing in OECD countries has increased much more than in emerging market countries. Furthermore, we investigate the relationship between various measures of financial integration and cross-country risk sharing, but only find weak evidences on such linkages.

This chapter is structured as follows. In section 3.2, we discuss the implications of financial integration on risk sharing and how long-run risk sharing can be estimated in a nonstationary panel. Section 3.3 will illustrate model specifications pertinent to the issues in testing and estimating risk sharing. We will discuss our data and sample selection in section 3.4. Section 3.5 will present our cointegration testing and estimating results. We examine the distribution patterns of risk sharing and link it to some financial integration indicators in section 3.6. Finally, section 3.7 will conclude this chapter.

3.2 Theoretical Motivations

In order to estimate long-run risk sharing, we need to understand how risk sharing happens when countries open up and financially integrate with each other. In fact, financial integration influences a country's consumption, given a certain output dynamic, through two functions: state contingent insurance and intertemporal smoothing. In a financially integrated world, countries facing uncertain output streams buy insurance contracts in an insurance market, such as Arrow-Debreu securities and Shiller portfolios, to share away the idiosyncratic output risks (Arrow, 1964; Debreu, 1959; and recently Shiller, 1993). In practice, such insurance contracts do not exist, so we use cross-country holdings of assets and liabilities as proxies. If an insurance market is not complete, intertemporal smoothing that involves intertemporal reallocation of consumption through borrowing and lending in a risk-free bond market comes into play. If an insurance market is complete, a bond market is redundant (Constantinides and Dufe, 1996).²⁹

We note that intertemporal smoothing may be preferable when a shock can be also insured. This is because costs of insurance contracts are higher than costs of bond contracts due to sovereign risks and moral hazards (Obstfeld and Rogoff, 1995). We are not considering sovereign risk and moral hazard explicitly. However, those types of endogenous imperfections of financial markets can further limit the extent of risk sharing (Becker and Hoffmann, 2006).³⁰ We will discuss this when explaining the empirical results.

These two functions are mechanically different and bear different welfare impli-

²⁹Another risk sharing institution is government transfer. However, since it is relatively small at the country level (Asdrubali, Sorensen and Yosha, 1996), and also because this paper focuses on financial integration, we do not have it explicitly in this paper. However, we should keep in mind that the estimated risk sharing has a small portion of the government transfer effect.

³⁰We call the sovereign risk and moral hazard endogenous incompleteness in order to distinguish them from the exogenous market incompleteness, such as the market for uninsurable non-tradable goods.

cations. Beveridge and Nelson (1981) have illustrated that any time series which exhibits any kind of homogeneous non-stationarity can be decomposed into two additive components, a weakly dependent stationary series and a pure random walk. Specifically, in terms of an output process, transitory shocks only lead to output deviating from its current value temporarily and reversing to its current value in the long run. However, output subject to permanent shocks is not mean-reversing and thus performs as a random walk process. We therefore say transitory shocks, which constitute a stationary $I(0)$ process, are second-order, compared to first-order nonstationary movement caused by the permanent shocks, which is an $I(1)$ process.

Baxter and Crucini (1995) conclude that if an output shock has permanent effects, it can only be shared through an insurance market; a bond market can only share transitory shocks.³¹ Therefore, in the context of risk sharing, the deterministic force on a country's consumption is state contingent insurance. Loosely speaking, the permanent component of output, which is driven by permanent shocks, has an infinite variance over time. People therefore face much larger uncertainty associated with it, compared to the uncertainty associated with the transitory component. Given permanent shocks can only be shared in an insurance market, insurance, compared to intertemporal smoothing, bears a much larger welfare gain (Van Wincoop, 1999; Obstfeld, 1994).³²

We can think of this welfare gain using the following example. Let us imagine

³¹That is, if shocks to GDP are transitory, intertemporal smoothing through borrowing and lending in a bond market can act as a close substitute for risk sharing. However, if shocks to GDP are persistent, intertemporal smoothing is not effective due to the persistent nature of the shocks.

³²The statement that the variance of an $I(1)$ process tends to infinity is not generally true. Strictly speaking, if the difference of an $I(1)$ process is a causal and invertible *ARMA* process, the variance goes to infinity. In the following part of the paper, it is helpful to think that the output process is an $I(1)$ process with a causal and invertible *ARMA* disturbance. For more details on this, please refer to Leeb and Poetscher (1999) and Prucha (2004).

the extreme case of a complete insurance market. There are only two countries in the world, the U.S. and Zimbabwe, which were identical in every aspect 200 years ago. They signed an insurance contract against idiosyncratic future shocks. Let's assume that there was a permanent negative shock driving Zimbabwe's GDP permanently downward and there was a permanent positive shock driving U.S. GDP permanently upward after signing the contract. We assume no further permanent shocks thereafter and reneging of the contract is not possible. We expect today's consumption in the U.S. would be the same as that in Zimbabwe. Clearly, in long run terms, insurance is more important and constitutes most of the welfare gain. It is for this reason a separate investigation of long-run risk sharing is warranted.

Although we are not focusing on risk sharing at business cycle frequency or on transitory shocks, it is fully addressed in the serial correlation properties of nonstationary panel analysis. This is because long-run risk sharing involves I(1) movement of consumption and output while risk sharing on transitory shocks only involves I(0) stationary movements, which is a lesser order of magnitude and therefore can be corrected by using internal instruments.

Specifically, similar to the literature, we use the relationship between idiosyncratic output per capita and idiosyncratic consumption per capita as a measure of the risk sharing effect of financial integration. The difference is that we explore the nonstationarity of this relationship. Suppose $c_{it} - c_t^w$, $t = 1, \dots, T$ has a unit root for each member $i = 1, \dots, N$, and so does $y_{it} - y_i^w$ (where c_{it} is log consumption per capita of country i ; c_t^w is log world average consumption per capita; y_{it} and c_t^w are similarly defined on output; $c_{it} - c_t^w$ and $y_{it} - y_i^w$ are therefore idiosyncratic log consumption per capita and idiosyncratic log output per capita), then $c_{it} - c_t^w$ and $y_{it} - y_i^w$ form a cointegrated panel if some linear combination, $u_{it} = (c_{it} - c_t^w) - \alpha_i - \beta_i(y_{it} - y_i^w)$, is stationary. The slope coefficient β_i is the steady

state cointegrating coefficient which indicates a long-run relationship between two I(1) series that will be maintained forever unless some external shock breaks it. We interpret the estimated β_i as a measure of long-run risk sharing. Since risk sharing on transitory shocks only involves short-run fluctuations towards its steady state equilibrium, it is contained in the error term in such a cointegrated system (Phillips, 1991).

In brief, long-run risk sharing is defined in contrast to the risk sharing on risks at business cycle frequency that dominates the literature, where all of the series are first differenced to render stationarity. The nonstationary panel approach allows us to isolate the long-run steady state relationship from short-run dynamics by wiping out the confounding effect of intertemporal smoothing and other nuisance features.

Another advantage of nonstationary panel analysis is that the group mean Fully Modified OLS (FMOLS) and the group mean Dynamic OLS (DOLS) estimations can address an important issue in empirical work on risk sharing: the cross country variation in the steady state of risk sharing. The intuition on this is straightforward. At the practical level, different countries will reasonably choose the level of cross-country holdings of assets and liabilities to the extent that costs equal benefits. Given that costs and benefits may differ across countries and across different contingencies, the level of risk sharing should be different. While group-mean nonstationary specification allows heterogeneous slope coefficients, the slope coefficient is forced to be common across countries in a conventional panel specification.³³ As a byproduct of allowing heterogeneity in risk sharing, we can study cross-country risk sharing distribution and link this distribution pattern to static financial inte-

³³Without exploring time series asymptotics, it is difficult for the conventional panel model to achieve a reliable estimate on the country specific slope coefficient with enough explanatory power except for the case of Hsiao and Pesaran (2004) where some structures are imposed on their random coefficient model.

gration indicators.

Another reason for doing this long-run analysis is because the short-run analysis in the literature finds no or a limited increase in risk sharing during the recent financial integration period. Lane and Milesi-Ferretti (2003), using carefully collated data, have shown dramatic increases in international capital flows accompanying financial integration. This leaves the puzzle as to whether increased financial integration, as indicated by an increase in capital flows, can, in practice, induce higher risk sharing (Sorensen, Wu, Yosha and Zhu, 2007). Artis and Hoffmann (2008) found that consumption risk sharing has increased during the financial integration period, but the short-run analysis failed to detect it due to the concurrent decline of output volatility in the short run. Therefore, by splitting our data sample into a before and after 1990 period, we test changes in risk sharing associated with financial integration using the nonstationary panel techniques.³⁴

A branch of the short-run analysis takes advantage of the gross national income (GNI) data available from a country's national accounts to estimate state-contingent insurance and intertemporal smoothing separately through an output variance decomposition approach initiated by Asdrubali, Sorensen and Yosha (1996). Using GNI, instead of a consumption series, to estimate state-contingent insurance seems to get rid of the contamination of intertemporal smoothing in the most direct way. In fact, although the contamination is not directly from consumption smoothing in this case, the same arguments apply. The intertemporal consideration can endogenously influence the real level of net factor income recorded in the national accounts, making it different from the potential level of net factor income (Lane, 2001). Therefore, net factor income can be simultaneous with output dynamics, and thus bias estimated insurance in a similar way as the

³⁴This data split is in line with the capital flow patterns found in Lane and Miles-Ferretti (2003) and is consistent with the practice in the literature.

estimate on risk sharing we argued in the paragraphs above. In addition, it is well known that factor income from the BOP accounts is not accounted accurately. This can induce serious measurement problems in a conventional panel regression. Furthermore, capital gains and losses on investment are not captured in GNI, but it will provide some kind of risk sharing. For countries holding large portfolios in equity and FDI, this is especially important since, typically, most returns are in the form of capital gains or losses.

In addition, the nonstationary panel analysis allows some other features that turn out to be particularly convenient in testing and estimating long-run risk sharing. For example, at the macro level, everything depends on everything else, thus it is fair to think that GDP and consumption are interdependent. Just as in time series nonstationary analysis, we do not need to worry about the simultaneity or endogeneity problems in the nonstationary panel analysis simply due to the fact that we are exploring a cointegrating relationship that is an order of magnitude greater than the simultaneous and endogenous problems that plague the conventional panel analysis. For a similar reason, it is also robust to many forms of omitted variables. Meanwhile, in contrast to time series analysis that is well-known to be data-demanding with low power and high size distortion in a finite sample, a nonstationary panel is able to use relatively short time series to infer the long run while maintaining reliable power and size properties (Pedroni, 2000).

3.3 Discussion on Conventional and Nonstationary Panel

Approaches

3.3.1 Conventional Panel

In the literature, many researchers used equation (41) or its variants to measure risk sharing (Appendix D lists studies using conventional panel analysis; for survey papers, refer to Corcoran, 2008 and Kose, Prasad and Terrones, 2007):

$$\Delta c_{it} - \Delta c_t^w = \alpha_i + \beta^{SR}(\Delta y_{it} - \Delta y_t^w) + \varepsilon_{it} \quad (41)$$

where Δc_{it} is the change in log consumption of country i from period $t-1$ to t ; Δc_t^w is the change in world average log consumption from period $t-1$ to t ; Δy_{it} and Δy_t^w are defined in the same way on log outputs. $\Delta y_{it} - \Delta y_t^w$, the relative changes of log output in country i , capture idiosyncratic output risks. β^{SR} is restricted to be the same across countries.

The idea of using equation (41) to test risk sharing comes from the orthogonality condition of the benchmark model: $E(\Delta c_{it} - \Delta c_t^w | X_{it}) = 0$ where X_{it} is a vector of idiosyncratic risk factors of country i , typically output risks. This orthogonality condition implies a testable condition of equation (41), $\beta^{SR} = 0$. However, it is well-known that the real world financial market is incomplete. This led researchers to adopt a pragmatic approach to interpret the estimated β^{SR} as a measure of degree of risk sharing.

ε_{it} is typically assumed to be *i.i.d.*($0, \sigma^2$) white noise. Equation (41) is consequently estimated by using panel pooled OLS or fixed effect techniques. If the maintained assumptions of exogenous regressors (in the case of pooled OLS) or strictly exogenous regressors (in the case of FE) and the rank condition also hold, consistent estimate of β^{SR} can be achieved when $N \rightarrow \infty$ and T is fixed. However,

we argue that, empirically, the estimate of β^{SR} in this model specification is biased for several reasons.

First, if output process has non-trivial short-run dynamics in it, ε_{it} cannot be treated as *i.i.d.*($0, \sigma^2$) process. Actually, ε_{it} and $\Delta y_{it} - \Delta y_t^w$ are correlated and the correlation will not go to zero even asymptotically. Chapter 2 has given a full illustration of this. For the reason of completion and self-containment of this chapter, I have summarized the main results in Appendix E. ³⁵

We can relax the assumption on ε_{it} to allow for heteroskydascity and even homogeneous serial correlations. If ε_{it} is assumed to be serial correlated, it is by construction treating dynamics. However, because the asymptotic properties of estimates depend on $N \rightarrow \infty$ and fixed T in equation (41), the series correlation across i is required to be the same (Arellano and Bond, 1991). A homogeneous dynamics implies that the impulse responses to disturbances are the same across countries in terms of size, shape and convergence speed. In the case of risk sharing, this means the returns of consumption to its long-run equilibrium are the same across countries. This is simply not realistic. For example, it is just not possible that the dynamics of US and Zimbabwe are the same in terms of level, length and even directions. If the latent true dynamic is heterogeneous but is forced to be homogeneous, we will run into trouble in estimating β^{SR} (Smith and Peseran 1995).

Some may argue that we can treat the dynamics in each country up-front by estimating the serial correlation properties in ε_{it} . However, this approach requires very long time series data which are not realistically available, especially at the macro level.

³⁵We assume no aggregate risk in Chapter 2 and in Appendix E. This is why results therein do not have the terms c_t^w and y_t^w . Since aggregate risk is not insurable and cannot be intertemporally smoothed, it is therefore subtracted when applying empirical tests, and only idiosyncratic risk is left as a result.

Furthermore, the slope coefficient β^{SR} is assumed to be homogeneous in equation (41). We turned to believe a heterogeneous coefficient as discussed. If the true slope coefficient is heterogeneous in nature but forced to be homogeneous in regression models, the estimated β^{SR} will be biased. Actually, again, all the arguments of Peseran and Smith (1995) will apply and the OLS estimator, $\hat{\beta}^{SR} \rightarrow 1$ no matter what the true value is.

Second, when taste shocks exist, β^{SR} cannot be interpreted as a measure of risk sharing even if it can be consistently estimated. Taste shocks, and demand side shocks in general, do not get modeled in equation (41), but they influence consumption given a particular output process. As a result, in equation (47) of Appendix E, besides the true risk sharing effect, there will be an extra term in the coefficient, capturing the effect of taste shocks (For more details on this, please refer to Chapter 2).

Taste shock can be thought of as another risk factor besides output risks. That is, the orthogonal condition can be pinned down into $E(\Delta c_{it} - \Delta c_t^w | y_{it}, \tau_{it}) = 0$, where τ_{it} denotes idiosyncratic taste shocks. The effect of taste shock can be isolated from risk sharing if we can find reliable measures of it and thus use them as controls in equation (41). However, taste shocks remain as a black box in the literature and therefore very difficult to find quantifiable measures on it.

Third, market inefficiencies, such as market frictions, can place another layer of latent dynamics into the system. In standard models, in order to facilitate in yielding analytical results, market efficiency is implicitly assumed. For example, in deriving equation (47), we have assumed an efficient bond market. It is debatable if a bond market can be modeled as efficient, but we believe a more general DGP in a cross-section of countries. For example, it is hard to believe that bond markets are well developed in emerging markets and can be modeled in the same way as

that of the U.S. Cavaliere, Fanelli and Gardini (2008) have shown market frictions, which prevent consumption adjusting to its optimal instantaneously but instead gradually, can lead to a lower consumption correlation than that standard models predict. They proceed to attribute the lack of risk sharing documented in previous research to the misspecification of short-term dynamics. In such cases, we have to take into account heterogeneous transitional dynamics caused by different levels of market frictions.

Some literature treats the differencing data at lower frequency in equation (41) as capturing long-run effect of risk sharing (Canova and Ravn (1996)). Again, this is only valid under the strong assumption on dynamics which is that ε_{it} is i.i.d. white noise after differencing at the lower frequency. But for the reason argued above, we tend to believe that we should specify a model that takes as many lags as needed to make sure ε_{it} is white noise, and we believe that this can only be accomplished by using the nonstationary panel that we are turning to shortly.

In general, to summarize the discussion above, under the framework of conventional panel analysis, we have to make restrictive assumptions on how the data are being generated. The problem is, on the one hand, the lack of the unified theoretical model that can completely describe the DGP,³⁶ and on the other hand, the unmeasurability or unavailability of data, for example, the quantifiable measure of taste shocks, hindered the applicability of such empirical specifications. However, it turns out not the case in using the nonstationary panel. In particular, we can be blind on many aspects of the serial correlation properties of the data generating process and still be able to achieve consistent estimates on risk sharing effect.

³⁶Backus, Kehoe and Kydland (1992) had predicted that “five years from now the models that have been developed will differ from this starting point in fundamental ways”, unfortunately, the development has not been fundamental enough until now.

3.3.2 Nonstationary Panel

We know that nonstationarity is typical in a macro panel. The presence of nonstationarity provides us the opportunity to take advantage of its nice properties in analyzing risk sharing.

In this paper, we use the following equation to test risk sharing.

$$c_{it} - c_t^w = \alpha_i + \beta_i^{LR}(y_{it} - y_t^w) + u_{it} \quad (42)$$

$$u_{it} = \Psi_i(L) \cdot \varepsilon_{it} \quad (43)$$

where consumption and output variables are defined the same as those in equation (41). But instead of working on growth, we deal with levels directly. Noticing that if $y_{it} - y_t^w \sim I(1)$, and $u_{it} \sim I(0)$ following some weakly dependent $I(0)$ process, then $c_{it} - c_t^w \sim I(1)$ by construction.³⁷ The subscript i on $\Psi_i(L)$ means the dynamics are allowed to be heterogeneous across countries, and ε_{it} is *i.i.d.* white noise disturbance term.³⁸ Despite simplicity in form, this equation has surprisingly nice features that can take care of the problems discussed above.

The OLS estimate of β_i^{LR} is a consistent estimate. This is because u_{it} is an $I(0)$ weakly dependent stationary process, the impacts of dynamics contained in it is an order of magnitude less than the cointegrating relationship β_i^{LR} that we are estimating. As a result, the convergence of OLS estimate (and FMOLS and DOLS estimate that we will discuss shortly) is determined by the $I(1)$ components.

³⁷Consumption and output being $I(1)$ processes is the necessary condition to explore the cointegration relationship between them. We will test these in the empirical part.

³⁸The regression model of equations (42) and (43) is a generalization of regression model of equation (41). On the one hand, when u_{it} is *i.i.d.*, equations (42) and (43) degenerate to equation (41) by taking first-difference. On the other hand, equations (42) and (43) include the permanent component of consumption and output, and therefore estimate a long-run risk sharing relationship, instead of short-run risk sharing relationship.

Meanwhile, since intertemporal smoothing aims at smoothing out risks at business cycle frequency that are caused by transitory output shocks, it is only important in the short run. Equation (42), however, estimates a long-run relationship, so the effects of intertemporal smoothing are washed out and β_i^{LR} is a proxy on risk sharing through insurance. In light of these, we interpret the OLS estimate of β_i^{LR} a consistent estimate of long-run risk sharing relationship (For more details, please refer to Appendix E and Chapter 2).

So far, we have pushed the data generating features into u_{it} and simply hope it can accommodate them. This is because nonstationary panel analysis applies nonstationary time series properties into the panel. Time series analysis is all about how to take care of dynamics that are unknown when you have enough data on T dimension. Although we do not know the form of $\Psi_i(L)$ in u_{it} , but the estimation procedure (step-down procedure in ADF specification or kernel in nonparametric specification) will give the best estimates on them. This allows us get around many issues that require strong assumptions in the conventional panel.³⁹ Again, we emphasize that as a result of the full heterogeneities in u_{it} , we can achieve consistent estimate on long-run behavior of cross-country risk sharing that are invariant with respect to the finely detailed structure in short-run dynamics. In other words, different as the case of the conventional panel, we are not making assumptions on restricting the DGP, but hoping that the full heterogeneities can be rich enough to include the true data generating mechanism as a special case.

For example, the reasoning above applies to taste shocks. Taste shocks are not explicitly specified in equation (42), but they are washed out without biasing the estimation on β_i^{LR} since taste shocks are widely regarded as transitory shocks which

³⁹Note that equation (41) estimates risk sharing of transitory shocks. Durlauf and Quah (1999) argued that conventional panel estimated a high frequency relationship by forcing all the low frequency relationships into the fixed effect. In contrast, despite the use of deterministic terms, the slope coefficient in a cointegrating panel picks up a long-run relationship.

are captured by the serial correlation of u_{it} .

A broad class of short-term dynamics of consumption, such as market frictions, can be accommodated in equation (42). In equation (42), the univariants $c_{it} - c_t^w$ and $y_{it} - y_t^w$ both have complicated dynamics and these can lead to more complicated dynamics in u_{it} , but it is OK since the estimation procedure will provide the best "guess" on it.

It is well-known that we face data limitations when applying time series analysis on macroeconomic tests. However, This is not the case for a nonstationary panel. One of the nice features of a nonstationary panel is that it uses the data on cross-sectional dimensions to compensate for the relatively short data on temporal dimensions in order to achieve reliable estimating and testing results (Pedroni, 1997).

An important advantage of nonstationary panel specification is that the equation (42) above allows for heterogeneous slope coefficient, β_i^{LR} , which serves to capture the cross country variations in risk sharing, while in conventional panel approach that involves stationary variables, the slope coefficient, by construction, is forced to be homogeneous, leaving all the heterogeneities into the fixed effect. As we discussed before, the costs and benefits make it hard to believe that the degree of risk sharing in the U.S. and Zimbabwe are the same. This implies that a heterogeneous β_i^{LR} is required.⁴⁰

The reason that β_i^{LR} is allowed to be heterogeneous is because of the way of pooling data in our cointegration test and estimate. There are two ways of pooling the data on cross-sectional dimension and time series dimension based on the commonality explored across sections. One way assumes the commonality across

⁴⁰The variation is also caused by different intertemporal smoothing effect due to heterogeneous output processes across countries. But it is small in the long run and therefore not particularly concerned in empirical tests.

sections comes from a common β^{LR} and produces within estimator on the cointegration relationship. Another way assumes β_i^{LR} is drawn from a common distribution and produces the group mean estimator of cointegration relationship. The panel estimate, therefore, is an estimate on the limit of the average of individual β_i^{LR} . Economically, it measures how much of idiosyncratic consumption risks in the world is shared on average. Pedroni (2000) and Pedroni (2001) emphasize the advantages of using group-mean estimators. Also as a by-product of the group-mean estimator, we can compare the properties of the distribution of individual estimates to group mean values.

So far, we explain the terms of c_t^w and y_t^w in equation (42) as global components of consumption and output. From the theoretical point of view, the risks that are global in nature cannot be shared and thus the subtraction of c_t^w and y_t^w serves to leave only the idiosyncratic component in check. Meanwhile, from the empirical point of view, this subtraction can be interpreted as accounting for certain forms of cross-sectional dependency that may be present in the nonstationary panel. From a pure econometric point of view, the nonstationary panel approach uses the data on cross-sectional dimension to compensate the relatively short data on temporal dimension in order to achieve reliable estimating and testing results. Therefore, we hope time series data are independent across sections and thus the information in individual cross section can add to each other. If data are cross section dependent, that means some information is redundant that reduces power and introduces size distortion. The effectiveness of c_t^w and y_t^w in eliminating cross-sectional dependency depends on the form of true dependency, but it turns out that this simple form performs reasonably well in many cases, for example, in the case that data are in part driven by common global business cycles or by a common stochastic trend.

Up to this point, our discussion takes incomplete market as given, but did not

explain why a market is incomplete. Explaining why a market is not complete is not the purpose of this paper and please refer to Chapter 6 of Obstfeld and Rogoff (1995) for theoretical reasons on endogenous market incompleteness, such as sovereign risk and moral hazard. The point that we want to make is that the estimated slope coefficient in equation (42) reflects those endogenous incompleteness. We point out that it also reflects the impact of exogenous incompleteness, for example, the non-insurability of non-tradable goods and labor incomes. However, we need to be cautious on the interpretation on the non-insurability of non-tradable goods and labor incomes because of a fine point about the assumption on the additivity of the period utility function. Taking the non-tradable goods as an example, if additivity holds, then we can derive a neat equalized marginal rate of substitutions between countries on the tradable goods and therefore we can interpret our estimate on the slope coefficient of equation (42) as proportional to the case of tradable goods since the non-tradable goods are included into the regression. However, if the additivity does not hold, the introduction of the intratemporal elasticity of substitution and its interaction with the intertemporal elasticity of substitution rule out a neat relationship between countries on the tradable goods and therefore, the interpretation can be viewed as a proxy at best.⁴¹ In the end, we can view our risk sharing estimate as a "de facto" measure of risk sharing.

Backus, Kehoe and Kydland (1992)'s simulation results show that, in the case of technology spillover, consumption correlation can be high while output correlation is low even between the autarky economies. Is our measure of risk sharing subject to such spillover bias? We justify this from two aspects. One the one hand, our

⁴¹The simulation results in the literature show that the impact of non-tradable goods is not large enough to generate the as low consumption correlation as it is in the data without assuming extreme intertemporal and intratemporal elasticity of substitution parameters. A similar finding for the case of leisure. This comforts us in not worrying too much on this fine point.

test is a long-run test. If technology spillover is as high as in Backus, Kehoe and Kydland (1992)'s simulation model, we should see GDP convergence, but this is not the case of the data (Pedroni, 2008). On the other hand, we have taken the cross-country dependency of GDP out, and this mutes the impact of technology spillover (or contagions in general) on our estimated coefficient.

3.4 Data and Sample Selection

3.4.1 Dataset

Our data on GDP and consumption are taken from the Penn World Table (PWT) version 6.2, the latest release in September 2006, and World Economic Outlook (WEO) April 2009 Publication. PWT contains a set of annual national accounts economic time series on many countries. It is widely used in the international risk sharing literature and therefore is convenient for our purpose since it has converted the expenditure entries into international dollars so that real quantity cross-country comparisons can be made (for details, please refer to Heston, Summers and Aten, 2006). However, the PWT only has GDP and consumption data up to 2004; in order to achieve the longest possible temporal dimension information, which is, in practice, important for the nonstationary analysis, we therefore extended the data to 2008 by using the national accounts data from WEO.

PWT and WEO covers 188 countries and 176 countries respectively which are literally almost the whole world. However, before rushing to experiment with all the covered countries, we must pay sufficient regard to empirical limitations to this particular sample. The PWT starts from 1950. However, for many developing countries, especially the least developed countries, the data before 1970s are missing and the data quality grades signal that the reliability of the estimates is of concern.

Moreover, the restrictions on capital flows, the high risks associated with those countries, along with the substantial international transfer flows which provides some kinds of de-couple of consumption and GDP through non-financial market mechanism, make it highly debatable if any meaningful risk sharing exists and therefore can be detected in those countries.

Based on those considerations, we picked 45 OECD and emerging market countries for which have a data span available from 1950 to 2008. These 45 countries cover all the 26 OECD countries and all the 22 emerging market countries defined by the FTSE Group and the Economist, except the East European transitional economies and Russia.⁴² Moreover, these 45 countries consist of more than 80 percent of world GDP as of 2008 and thus we believe they are large enough for us to treat them as a proxy for the whole world. We define idiosyncratic GDP per capita and consumption per capita as the country level GDP per capita and consumption per capita minus the world-wide average of GDP per capita and consumption per capita. Therefore, the higher the risk sharing, the less comovement between idiosyncratic GDP per capita and idiosyncratic consumption per capita. From this point on, when we discuss GDP per capita and consumption per capita, we implicitly mean the idiosyncratic ones, which are the demeaned GDP per capita and consumption per capita.

3.4.2 Sample Selection

We have made the decision on the data sample that we are going to explore, but before applying the empirical tests on it, it is worth explaining the strategies used

⁴²The OECD countries include United States, United Kingdom, Austria, Belgium, Denmark, France, Germany, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, Canada, Japan, Finland, Greece, Iceland, Ireland, Portugal, Spain, Turkey, Australia, New Zealand, Mexico, Korea. The emerging market countries include Argentina, Brazil, Chile, China, Colombia, Egypt, Hong Kong, India, Indonesia, Israel, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Saudi Arabia, Singapore, South Africa, South Korea, Thailand, Turkey.

to apply the nonstationary techniques in order to achieve robust and informative results. Basically, any empirical tests are guided by the theoretical models and this is the first strategy. Unfortunately, we are facing real world data limitations. If the data did not show the pattern predicted by the theory, we will not be able to apply tests on that theory. Therefore, another empirical strategy is to investigate what the data tell us and sort out the useful data information in testing theories. In the analysis of this paper, we compromise between the information carried through data and the prediction made by theories, and use both strategies in our tests and hope we can cover basis by doing both.

The panel unit root tests on GDP per capita and consumption per capita, as shown in table 1, signal very strong sign of non-rejection of the null of unit root for the 45 country sample. The tests of cointegration between GDP per capita and consumption per capita, as shown in table 2, indicate significant rejection of the null of unit root on the error term of equation 42, meaning they are cointegrated. These findings are consistent with the predictions of the neoclassical growth model. The neoclassical growth model tells us that a country's GDP per capita should follow some kind of non-mean-reversing process if a country has experienced permanent changes in technologies or in investment rates, and therefore we can model the GDP per capita as a unit root process. Since we find consistency between data and theory, therefore, complying with the first strategy, we test and estimate long-run risk sharing on the whole 45 countries.

The panel test results in table 1 and table 2 are constructed by the test results of the individual country. For example, in table 1, we reported the Im, Pesaran and Shin Augmented Dick-Fuller (IPS ADF) test statistics which are, in its simplest form, an average of the individual ADF test statistics. When taking a closer look at the individual country's unit root test results, reported in table A1, we find,

for some countries, the test statistics reject the null of unit root on the GDP per capita and consumption per capita. This may be due to the high size distortion when coming to time series nonstationary analysis and we should not trust nor pay much attention to it. But, at a practical level, there is nothing restricting the GDP per capita of a country has to follow a unit root process within a certain time period. For example, the technology changes or changes in investment rates may not have been significant enough within the sample period to drive the country to move with unit root characteristics. To include those countries won't break the test based on the whole sample down. This is because although those countries with stationary GDP per capita process are not very informative about the risk sharing relationship that we are interested in, they are an order of magnitude less than the cointegration relationship and therefore irrelevant asymptotically. However, for a finite sample, we realize that it increases the noise-to-signal ratio of the long-run risk sharing analysis. We therefore take out those countries with test results indicating stationary GDP per capita or consumption per capita.

We proceed to conduct cointegration tests after excluding those countries. The panel tests continue showing that consumption and GDP are cointegrated, but individual tests indicate that they are not in many countries (table A2).⁴³ Again, this could be due to the low power for rejection of the null hypothesis on the error terms or due to the high size distortion, but to be on the safe side, we take those countries out. This leaves us with 21 countries, a country sample which contains rich nonstationary information, even for individual countries, and therefore with significantly reduced noise-to-signal ratio. The test results on the 21 country subsample are used as robust checks on the whole sample results.

⁴³In table 2A, only those countries that passed the individual cointegration tests are reported. But the full results are available from the author up on request.

3.5 Interpreting the Risk Sharing Relationship

3.5.1 FMOLS and DOLS

We estimate the slope coefficient, β_i^{LR} , of equation (42) using group mean FMOLS and group mean DOLS techniques and interpret the estimated β_i^{LR} as a measure of “de facto” risk sharing. Depending on the way of pooling the information on time series and cross sectional dimensions of the panel, and depending on the parametric or nonparametric estimation approaches, the econometricians have developed several different versions of estimators on the panel cointegrating coefficient. For the details, please refer to Phillips and Moon (1997), Mark and Sue (1999) and Kao (1997) for the pooled estimators, and Pedroni (2000) and Pedroni (2001) for the group mean estimators.

We pick the group mean estimators, instead of the pooled versions because the pooled versions have a maintained assumption which treats the slope coefficient of the cointegrating relationship as common value. This maintained assumption not only restricts the applicability of the pooled estimators in the context of risk sharing, but also restricts the opportunities for us to investigate cross-country risk sharing distribution. Moreover, the group mean estimators perform better small sample size properties than the pooled estimators in the Monte Carlo simulations shown in Pedroni (2000). In addition, Pedroni (2001) shows that the group mean FMOLS and DOLS both tend to perform well in small samples in terms of size distortion, but since DOLS is a parametric-based test, it does better in terms of power when sample is very short which would be the case of this paper when we apply our test for the period post-1990. Therefore, we do both FMOLS and DOLS in order to cover all bases.⁴⁴

⁴⁴We only report risk sharing estimates using FMOLS since the estimates are similar using DOLS. The DOLS estimates are available up on request.

The FMOLS estimator was first developed by Phillips and Hansen (1990) in the time series context. Pedroni (2000) extended it into panel context and developed the group mean FMOLS estimator, which allows both heterogeneous dynamics and heterogeneous cointegrating vectors. The basic idea of the group mean FMOLS estimator is straightforward and can be interpreted as the cross-country average of the individual country FMOLS estimators, where the individual FMOLS estimator has been corrected for serial correlation and for endogeneity through a long-run covariance matrix. The correction can be achieved because of the fact that the cointegration relationship is an order of magnitude higher than the biases induced by serial correlations and endogeneities and therefore the differentiated regressors can serve as internal instruments to get rid of the biases therein.

In the context of risk sharing, this correction means that the effects of intertemporal smoothing, taste shock and some other serial correlation due to transitional dynamics have been wiped out. Therefore, the estimated slope coefficient in equation (42) represents the long-run steady state relationship between GDP and consumption which survives even with the presence of transitional dynamics which temporarily drives away the economies from the steady state.⁴⁵ For the asymptotic properties of the group mean FMOLS estimator and the steps on how to construct group mean FMOLS in a context of applied econometrics, please refer to Appendix F. Here, we just lay out the group mean FMOLS estimator to see how it is different as the conventional panel estimator and how it allows us to study the distribution of the individual country estimates:

⁴⁵We are not discussing the group mean DOLS estimator since the idea is the same. The difference is the econometric technique to achieve the serial correlation and endogeneity biases. The DOLS uses the parametric adjustment, instead of the nonparametric adjustment used by FMOLS.

$$\widehat{\beta}^{LR\ GFM} = N^{-1} \sum_{i=1}^N \left(\left(\sum_{t=1}^T x_{it}^2 \right)^{-1} \sum_{t=1}^T (y_{it} x_{it}^* - T \widehat{\gamma}_i) \right) \quad (44)$$

where, in order to keep the notation as simple as possible, we use y and x to replace $c - c^w$ and $y - y^w$. $x_{it}^* = x_{it} - \widehat{\Omega}_{21i}/\widehat{\Omega}_{22i} \Delta x_{it}$, indicating the x_{it} has been transformed by an adjusting term which serves as the internal instrument; and $\widehat{\gamma}_i = \widehat{\Gamma}_{21i} + \widehat{\Omega}_{21i}^0 - \widehat{\Omega}_{21i}/\widehat{\Omega}_{22i}(\widehat{\Gamma}_{22i} + \widehat{\Omega}_{22i}^0)$, acting as the long-run covariance matrix.

The point we want to make from equation (44) is that the group-mean FMOLS estimator, $\widehat{\beta}^{LR\ GFM}$, looks very similar to the OLS estimator of conventional panel, except for two features. The OLS estimator achieves the estimate on slope coefficient by minimizing the sum of mean squared errors of x on y . The group mean FMOLS does the same, but on top of a transformation of x and a long-run adjustment. In looking closer to this transformation and adjustment, we can find that this is a specific feature of the nonstationary panel because the transformation and adjustment only survive if the x and y are nonstationary. If, the x and y are $I(0)$ as in the case of conventional data, they are in the same order of magnitude as the transformation and adjustment terms which makes such transformation and adjustment unfeasible.

To summarize, provided x and y are $I(1)$, we can take advantage of the nonstationary panel features to achieve the cointegrating relationship estimate which indicates the level of risk sharing in our context. However, the conventional panel analysis, including the dynamic panel analysis such as Arrellano and Bond GMM, as long as it deals with the $I(0)$ process, is subject to first order biases due to the serial correlations which are hard to correct.

The second feature is that the group mean FMOLS allows us to study the cross-country risk sharing distribution. We have mentioned that we can inter-

pret the group mean FMOLS as the cross-country average of the individual country FMOLS estimator. From equation (44), we can clearly see that $\widehat{\beta}^{LR^{GFM}} = N^{-1} \sum_{i=1}^N (\widehat{\beta}_i^{LR^{FM}})$, where $\widehat{\beta}_i^{LR^{FM}} = (\sum_{t=1}^T x_{it}^s)^{-1} \sum_{t=1}^T (y_{it} x_{it}^* - T \widehat{\gamma}_i)$ is the individual country FMOLS time series estimator.

3.5.2 Conventional Panel Regression Results

We first check the estimates on risk sharing using conventional panel regression techniques, both in difference and in level. The results are reported in Table 2 and Table 3 respectively. Column 1 of each table reports pooled OLS estimates and Column 2 of each table reports fixed effect estimates. The results are similar across the two specifications.

The results are comparable with the findings in the literature. Basically, as shown in the first panel of table 2, for the whole sample period, an estimate of about 32 percent of business cycle frequency risks has been shared. However, this constitutes risk sharing through both insurance and intertemporal smoothing. In the case when risk free bond market can act as a close substitute on insurance market, most of the risk sharing should be carried through intertemporal smoothing because insurance contract is more risky and costly due to the moral hazard or contract enforcement issues, especially at the international level. Therefore, out of the 32 percent, it is fair to reasonably think that only a small portion is through insurance market (for theoretical findings and empirical results on this, please refer to Baxter and Crucini, 1995 and Artis and Hoffmann, 2006).

By comparing the estimates before and after 1990 in the 2nd and 3rd columns of table 2, a conclusion that would have been drawn is that we do not find increasing in risk sharing in the recent financial integration period. This is puzzling and counter-intuitive to the standard model's prediction. Our explanation, in keeping with the

argument of this paper, are two-fold. One is that the low and no increase in risk-sharing through insurance market on business cycle frequency risks is due to the low welfare gains. Another is that the misspecification and restrictive assumptions in the short-run dynamics hinder the capability to achieve an estimate of true β .

Table 3 reports results on estimates of long-run risk sharing by using pooled OLS and FE. The results indicate that less than 9 percent of long-run risks have been shared when estimated by pooled OLS in the whole 1960 to 2008 period, but around 18 percent when estimated by FE. The higher estimates in the FE specification make better sense. Some of the country-idiosyncratic factors, that is, factors beyond idiosyncratic output, cannot be shared through financial market and we should take them into consideration by using fixed effect.

Comparing the estimates before and after 1990 in the 2nd and 3rd columns of table 3, there is still not much increase in risk sharing. The issue is how much we can trust the estimates in table 3 in general. We know that OLS can achieve a consistent estimate on the cointegrating coefficient, but there is a second-order bias associated with it. The second order-bias does not appear even asymptotically. In a finite sample, we suspect that the second-order bias may turn out to be first-order bias, which seriously influences the reliability of these estimates.

3.5.3 Nonstationary Panel Regression Results

We report the long-run risk sharing estimates on the 45 country sample and its subgroups in table 4A. For the panel of 45 countries in the period of 1950-2008, the point estimate shows about 14 percent of long-run risks have been shared. The t-statistics on testing the null hypothesis of full risk sharing is 112.92, which indicates far from complete risk sharing; on the other hand, the t-statistics on testing the null of no risk sharing points to the existence of economically and statistically

significant risk sharing. We also performed estimates by splitting our sample into two periods. In the recent financial integration period, long-run risk sharing among the 45 countries more than doubled that in the pre-1990 period, reaching from 12 percent to 27 percent.

The estimates and test results on sub-country groups offer more insights. The risk sharing of OECD countries are at a similar level as the risk sharing of emerging markets on the whole sample period. However, in the financial integration period, about 34 percent of risks are shared for OECD countries, while only about 23 percent of risks are shared for emerging market countries. More importantly, the benefits of risk sharing are evenly enjoyed within OECD country groups. This is not the case for emerging markets. It seems that most of the benefits of financial integration are enjoyed by the advanced emerging markets.⁴⁶

It looks a bit puzzling that the risk sharing of EU countries is only about 10 percent for the whole sample period, and only about 6 percent for the pre-1990 period. We therefore have done an intra-region risk sharing analysis. The results appear in the memorandum panel of Table 4A. When testing risk sharing among only OECD countries, it shows that risk sharing is higher than risk sharing between OECD countries and the rest of the world for the whole sample period and for the pre-1990 period, but the levels of risk sharing are similar in the post-1990 period. This indicates that the markets between OECD and emerging markets are more isolated before financial integration. A comparison of risk sharing within EU15 countries and the risk sharing between EU15 and the rest of the world, however indicates that EU15 countries used to share risks mostly among themselves, but more risks are shared with the rest of the world in the post-1990 period (risk

⁴⁶One reason is that advanced countries are less debt vulnerable and more FDI-oriented. So it is interesting for future research to test risk sharing across different asset classes. For example, the FDI insurance may perform better than debt insurance since it is not as expected to be paid back as much as debt.

sharing is about 24 percent within EU15 after 1990, but about 36 percent with rest of the world). A similar story applies to other advanced countries. They used to share more risks among themselves, but now share more risks with EU countries and emerging market.

As a robust check, table 4B shows the long-run risk sharing estimates on the 21 country sample. Since we do not have enough countries on the cross-section to do a detailed breakdown on country groups, we only estimate the risk sharing on a sample of 21 countries, a sample of 11 OECD countries and 10 emerging market countries. The results basically show the same picture as the tests on the full sample of 45 countries. We find that the risk sharing estimate on the panel of 21 countries is 14 percent for the whole sample period and increases to 39 percent in the financial integration period. The increase is entirely due to more risk sharing in the OECD countries though.

3.6 Cross-country Risk Sharing Patterns

The group mean FMOLS does not restrict the slope coefficient to be homogeneous, and we can therefore look into the heterogeneous cross-country patterns of risk sharing by looking into the estimates of cointegrating coefficients on individual countries. We know that the estimates are not reliable individually, i.e., each of them is a poor estimate of the true cointegrating relationship due to the high size distortion of our short sample, but each of them is an asymptotically consistent estimate, and so the pooling of the individual estimates should show some consistent pattern. We report in Appendix Tables A3a and A3b the estimates of cointegrating coefficients of individual countries. The difference between Tables A3a and A3b is due to the different strategy we used in data sampling.

The measures on financial integration are from the updated and extended version

of a dataset constructed by Lane and Milesi-Ferretti (2007). It contains data for the period 1970-2007 and for 178 economies plus the euro area as an aggregate. For each of the countries, it reports total external assets and liabilities and associated breakdowns. We constructed our measure of financial integration by first splitting the data into a pre- and post- 1990 period. We then calculated the average of total assets and liabilities, the average of portfolio equity assets and liabilities, the average of FDI assets and liabilities, and the average of debt assets and liabilities on the split periods for each country of our sample. The panel figure shows the linkage of risk sharing pattern with such calculated financial integration measures.

The first chart in the panel shows that long-run risk sharing is positively correlated with the gross asset and liability to GDP ratio in the pre-1990 period. This is expected from the theoretical model's prediction. The second chart shows a weaker positive relationship for the post-1990 period. As you can see from the x-axis, the gross capital flow, on average, quadrupled compared to the pre-1990 period. If we take out the observation of Ireland as an outlier, then it almost tripled. However, as we have seen in our tables, long-run risk sharing, on average, only doubled during the same episode. This indicates that the pace of increase in long-run risk sharing does not catch up with the pace of increase in financial flows. It is therefore too strong to claim that risk sharing and financial flows are twins separated at birth. Financial integration is the necessary condition for risk sharing, but it is not sufficient, i.e., more liberal financial flows do not necessarily carry out proportionally more risk sharing. As pointed out by Kose, Prasad and Terrones (2007), threshold effects can be a potential explanation.

The middle two charts in the panel show the relationship between long-run risk sharing and the gross FDI and portfolio to GDP ratio. The bottom two charts show the relationship with the debt to GDP ratio. Two features are worth noting.

One is that most of the increase in financial flows in the post-1990 period is driven by the increase of FDI and portfolio. FDI and portfolio as a percent of GDP quadrupled in the post-1990 period compared to the pre-1990 period. But the debt to GDP ratio only doubled if we take out Ireland. The second feature is that they both confirm the relationship of the top two charts, with post-1990 showing a less positive relationship.

3.7 Conclusion

In this chapter, we specify an empirical nonstationary panel regression model that tests long-run risk sharing and allows for richer data generating processes. This is in contrast to the literature on consumption risk sharing, which is mainly about risks at business cycle frequency. Since our methodology focuses on identifying cointegrating relationships while allowing for arbitrary short-run dynamics, we can obtain a consistent estimate of long-run risk sharing while disregarding any short-run nuisance factors. Furthermore, the combination of a focus on the long-run low frequency relationship and the dimensionality of the panel allows us to study the distribution pattern of cross-country risk sharing. We therefore can link the distribution pattern to various measures of financial integration.

Our results show that, for the period of 1950-2008, about 14 percent of long-run risk has been shared in OECD countries and emerging market countries. However, during the financial integration episode of the past two decades, long-run risk sharing in OECD countries increased more than in emerging market countries, with about 34 percent of risks shared in OECD countries and about 23 percent in emerging market countries. These results are robust to our sample selection.

When investigating the relationships between various measures of financial integration and cross-country risk sharing, we find evidence of positive relationships,

i.e., more capital flows are associated with more long-run risk sharing. However, the positive relationships are smaller in the recent financial integration period, indicating that the increase of risk sharing is not proportional to the increase in capital flows.

Chapter 4: Conclusion

This dissertation concerns testing cross-country consumption risk sharing using panel regressions. The existing literature on testing risk sharing has found limited risk sharing across countries. This is in contrast to the prediction of a standard benchmark model and the well documented facts of financial integration in the past two decades. Chapter 2 of this dissertation set up a model that provides a potential explanation on the findings of low risk sharing. We illustrate that a bond economy can intertemporally smooth consumption in face of transitory output shocks, but not for output shocks that have permanent effects. An insurance economy is essential for risk sharing on permanent shocks. This mechanical difference requires a careful study of the implications of risk sharing on consumption given a certain output process. We have therefore shown that, when both transitory and permanent shocks exist, the short-run risk sharing relationship between consumption and output cannot be consistently estimated in a panel regression due to untreated short-run dynamics. However, we can consistently estimate a long-run risk sharing relationship because the distortion caused by short-run dynamics goes to zero asymptotically.

In Chapter 3, we provided empirical tests on long-run risk sharing by estimating a nonstationary panel regression model. Since our methodology focuses on identifying cointegrating relationships while allowing for arbitrary short-run dynamics, we can obtain a consistent estimate of long-run risk sharing while disregarding any short-run nuisance factors.

Our results show that, for the period of 1950-2008, about 14 percent of long-run risk has been shared in OECD countries and emerging market countries. However, during the financial integration episode of the past two decades, long-run risk

sharing in OECD countries increased more than in emerging market countries, with about 34 percent of risks shared in OECD countries and about 23 percent of risks shared in emerging market countries. These results are robust to our sample selection.

When investigating the relationships between various measures of financial integration and cross-country risk sharing, we find evidence of positive relationships, i.e., more capital flows are associated with more long-run risk sharing. However, the positive relationships are smaller in the recent financial integration period, indicating that the increase of risk sharing is not proportional to the increase in capital flows.

Future research will be in two directions. One is to investigate what drives the different level of risk sharing across country groups. Does the level of risk sharing link to certain features of a country, such as income levels and institutional development (leading to differences in asset and liability classes), and, more generally, demographic characteristics? Another direction is to explain why the increase of risk sharing lags behind the increase in capital flows in the recent financial integration period. Sorensen, Wu, Yosha and Zhu (2007) found capital home bias and low risk sharing are twin puzzles separated at birth. To a certain degree, this argument cannot be wrong. But the findings of Chapter 3 indicate there are forces that separate them after their births. It would be interesting to research those forces.

Table 1. Panel Unit Root and Cointegration Test Results (45 countries)

Unit root		GDP	Consumption	
IPS ADF (large sample adjustment values)		3.21***		1.09***
IPS ADF (Bootstrapped)		0.84***		-0.01***
MW (Bootstrapped)		<u>84.73***</u>		<u>89.42***</u>
Cointegration		ADF	PP	Rho
Group mean panel		-2.71***	-4.24***	-3.74***
Pooled Panel		-1.16	-2.67***	-2.06***

Note: Lag truncation: K=4

Table 2: Conventional Panel Regression Results under Different Specifications

	(1)	(2)	(3)	(4)
1960-2008	Pooled OLS	Pooled OLS	FE OLS	FE OLS
GDP growth	0.680 (0.055)***	0.681 (0.059)***	0.669 (0.062)***	0.669 (0.067)***
Constant	0.001 (0.001)*	0.017 (0.007)**	0.001 (0.001)*	0.018 (0.007)***
Observations	2535	2535	2535	2535
R-squared	0.31	0.33	0.29	0.31
Pre 1990				
GDP growth	0.641 (0.070)***	0.642 (0.076)***	0.624 (0.079)***	0.621 (0.087)***
Constant	0.002 (0.001)	0.011 (0.008)	0.002 (0.001)	0.020 (0.007)***
Observations	1680	1680	1680	1680
R-squared	0.27	0.28	0.25	0.26
Post 1990				
GDP growth	0.809 (0.045)***	0.807 (0.045)***	0.803 (0.061)***	0.800 (0.060)***
Constant	0.001 (0.001)	0.003 (0.003)	0.001 (0.001)	0.003 (0.003)
Observations	855	855	855	855
R-squared	0.50	0.51	0.46	0.46
Year dummy	No	Yes	No	Yes
Number of countries	45	45	45	45

Robust standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

Table 3: Level Panel Regression Results under Different Specifications

	(1)	(2)	(3)	(4)
1960-2008	Pooled OLS	Pooled OLS	FE OLS	FE OLS
GDP growth	0.912 (0.006)***	0.912 (0.006)***	0.796 (0.012)***	0.794 (0.013)***
Constant	0.009 (0.004)**	0.022 (0.031)	-0.023 (0.004)***	-0.056 (0.021)***
Observations	2580	2580	2580	2580
R-squared	0.95	0.95	0.78	0.78
Pre 1990				
GDP growth	0.912 (0.007)***	0.912 (0.007)***	0.801 (0.024)***	0.797 (0.026)***
Constant	0.011 (0.005)**	0.022 (0.031)	-0.022 (0.007)***	-0.061 (0.021)***
Observations	1725	1725	1725	1725
R-squared	0.95	0.95	0.65	0.66
Post 1990				
GDP growth	0.911 (0.009)***	0.911 (0.009)***	0.826 (0.024)***	0.824 (0.025)***
Constant	0.006 (0.006)	0.005 (0.032)	-0.015 (0.006)***	-0.023 (0.009)**
Observations	855	855	855	855
R-squared	0.95	0.95	0.71	0.71
Year dummy	No	Yes	No	Yes
Number of countries	45	45	45	45

Robust standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

Table 4A. Country Group Cointegration Coefficient Estimates

Group	Whole sample period			Before 1990			After 1990		
	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)
Full Panel (45)	0.86	112.92	-17.81	0.88	119.56	-13.84	0.73	108.55	-26.56
OECD (26)	0.88	89.07	-11.04	0.90	105.84	-7.43	0.66	77.08	-26.53
EU15	0.91	55.41	-4.59	0.94	80.33	0.54	0.64	56.17	-20.42
Euro area 12	0.88	56.51	-5.79	0.94	82.14	1.11	0.66	55.61	-22.65
Other advanced countries (11)	0.84	72.23	-11.61	0.84	68.92	-12.06	0.68	52.91	-16.94
Emerging market (22)	0.86	86.24	-16.22	0.86	69.86	-14.92	0.77	81.36	-11.92
Advanced emerging markets (8)	0.79	72.22	-12.85	0.79	57.69	-9.87	0.65	21.63	-9.72
Other emerging markets	0.90	52.96	-10.54	0.91	43.53	-11.23	0.84	86.44	-7.53
<i>Memorandum</i>									
Intra region risk sharing									
OECD (26)	0.80	112.99	-22.67	0.86	121.79	-15.2	0.65	69.79	-30.29
EU15	0.84	94.96	-10.02	0.92	97.29	-3.15	0.76	80.65	-24.07
Advanced emerging markets (8)	0.73	48.40	-10.84	0.55	28.57	-9.93	0.71	18.44	-8.78

Note 1: Advanced emerging markets includes all the countries defined by the Economist and Morgan Stanley Capital International (MSCI), which are South Africa, Brazil, Mexico, Israel, Saudi Arabia, Hong Kong, Korea and Singapore, except the two transitional economies: Hungary and Poland.

Note 2: the high coefficients on OECD, esp. on EU 15 and Euro 12 indicate that before financial integration, EU countries did very small risk sharing with rest of the world.

Table 4B. Country Group Cointegration Coefficient Estimates (Countries passed individual tests)

Group	Whole sample period			Before 1990			After 1990		
	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)
Full Panel (21)	0.86	126.14	-19.11	0.83	107.12	-16.63	0.71	94.62	-22.87
OECD (11)	0.90	97.78	-10.62	0.86	89.53	-7.28	0.63	65.36	-22.25
Emerging market (10)	0.80	102.06	-21.69	0.80	72.73	-20.55	0.82	73.55	-10.03

Advanced emerging markets includes all the countries defined by the Economist and Morgan Stanley Capital International (MSCI), which are South Africa, Brazil, Mexico, Israel, Saudi Arabia, Hong Kong, Korea and Singapore, except the two transitional economies: Hungary and Poland.

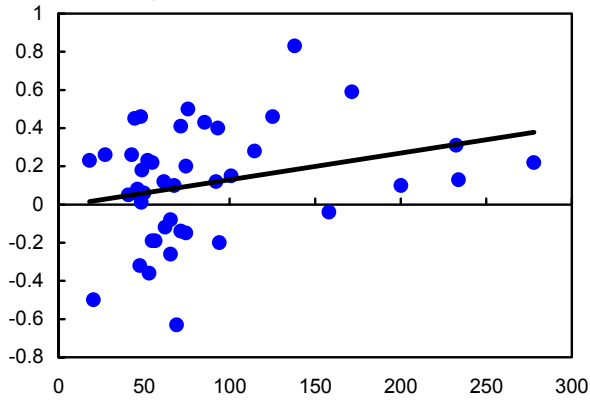
Note: the high coefficients on OECD, esp. on EU 15 and Euro 12 indicate that before financial integration, EU countries did very small risk sharing with rest of the world.

OECD: Austria, Belgium, Luxembourg, Sweden, Switzerland, Canada, Japan, Ireland, Spain, Australia, New Zealand and Korea.

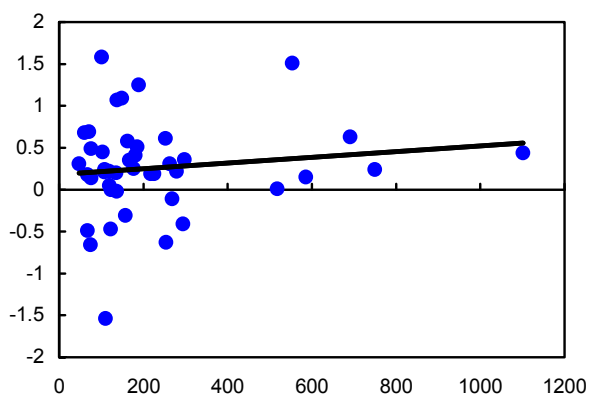
Emerging markets: South Africa, Argentina, Chile, Hong Kong, China, Korea, Malaysia, Pakistan, Singapore, Thailand.

Figure: Cross-country Risk Sharing and Financial Assets

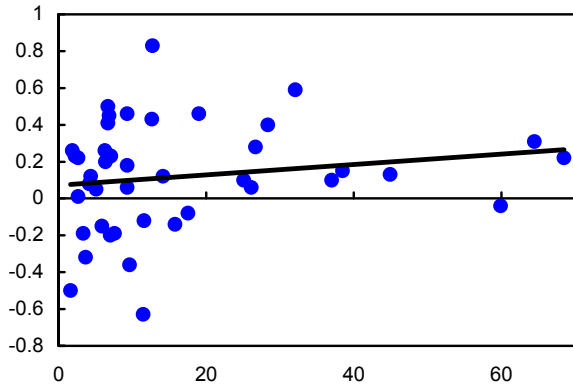
Pre-1990, positive correlation between risk sharing and total capital assets to GDP ratio.



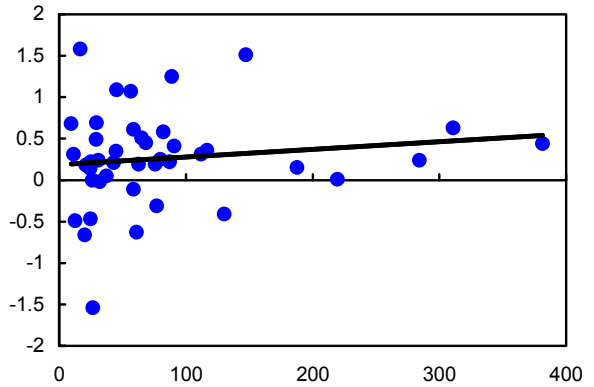
Post-1990, however, we observe a less positive relationship.



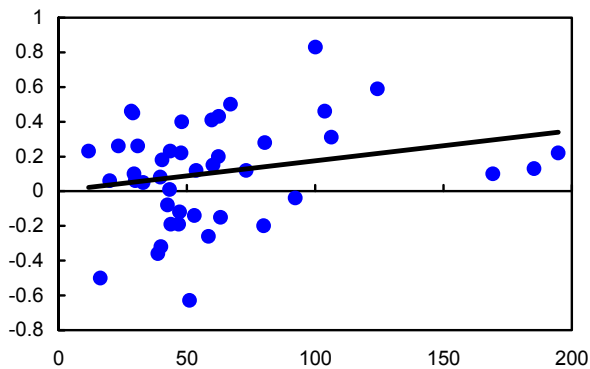
Pre-1990, positive correlation between risk sharing and portfolio + FDI assets to GDP ratio.



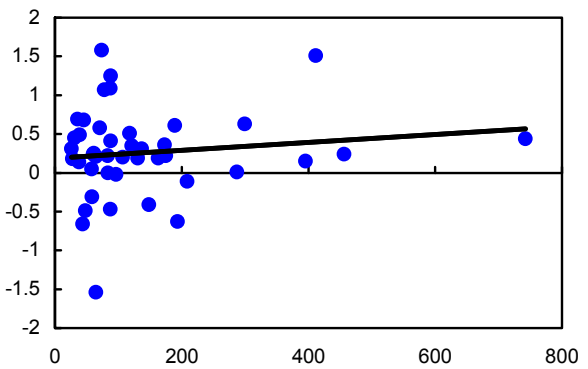
Post-1990, we observe a similar, if not less positive relationship.



Pre-1990, positive correlation between risk sharing and debts to GDP ratio.



Post-1990, however we observe a less positive relationship.



Source: PWT, WEO and EWN II

Table A1. Individual and Panel Unit Root Test Results 1950-2008 (45 countries)

Country	GDP			Consumption		
	ADF	pval	lags	ADF	pval	lags
United States	0.21	0.953	8	-0.98	0.714	8
United Kingdom	-0.94	0.723	8	-0.76	0.777	8
Austria	-1.11	0.676	3	-1.36	0.578	3
Belgium	-1.90	0.304	8	-1.48	0.486	6
Denmark	0.31	0.942	8	0.14	0.937	6
France	-1.23	0.595	8	-1.86	0.328	8
Germany	-0.80	0.737	3	-1.66	0.451	3
Italy	-0.85	0.139	6	-1.42	0.548	3
Luxembourg	-0.92	0.709	5	-1.22	0.628	8
Netherlands	0.18	0.963	6	-1.15	0.666	5
Norway	-1.47	0.505	8	-5.2	0.001	0
Sweden	0.27	0.952	8	0.64	0.98	8
Switzerland	0.25	0.946	7	0.29	0.93	8
Canada	0.41	0.971	8	-0.44	0.855	8
Japan	-1.69	0.426	2	-1.39	0.56	3
Finland	-1.59	0.477	2	-5.25	0.001	0
Greece	-2.38	0.156	8	-1.69	0.417	8
Iceland	-3.75	0.003	0	-1.45	0.519	5
Ireland	-1.87	0.244	8	-2.42	0.112	0
Portugal	-4.35	0.004	0	-1.99	0.251	8
Spain	-1.93	0.300	7	-2.37	0.141	8
Turkey	-4.39	0.001	6	-1.82	0.325	5
Australia	0.59	0.972	8	-0.5	0.835	8
New Zealand	-0.50	0.855	2	-1.33	0.544	8
South Africa	-0.47	0.794	8	-1.54	0.517	1
Argentina	-0.62	0.882	0	-0.81	0.813	0
Brazil	-1.34	0.551	8	-1.64	0.475	2
Chile	-1.93	0.347	5	-2.57	0.099	0
Colombia	-0.63	0.803	8	-0.31	0.907	4
Mexico	-0.27	0.927	2	-2.37	0.15	0
Peru	-0.93	0.762	1	-1.36	0.594	7
Israel	-1.80	0.353	8	-0.23	0.878	7
Saudi Arabia	-1.28	0.562	5	-0.94	0.649	7
Egypt	-0.92	0.740	6	-0.97	0.696	8
Hong Kong	-0.98	0.720	3	-1.69	0.454	1
India	0.38	0.930	8	-2.18	0.202	6
Indonesia	-0.97	0.751	1	-0.88	0.671	8
Korea	-1.00	0.610	5	-0.72	0.688	5
Malaysia	-1.08	0.679	3	-0.74	0.813	1
Pakistan	-1.27	0.575	8	-2.64	0.088	7
Philippines	-1.14	0.673	4	-1.58	0.437	8
Singapore	-1.37	0.563	1	-1.02	0.743	1
Thailand	-0.79	0.694	7	-0.42	0.881	3
Morocco	-3.29	0.025	0	-2.06	0.284	2
China	0.00	0.803	6	0.23	0.898	4
IPS ADF (large sample adjustment values)	3.21	0.999	8	1.09	0.999	8
IPS ADF (Bootstrapped)	0.84	0.800	8	-0.01	0.495	8
MW (Bootstrapped)	84.73	0.637	8	89.42	0.498	8

Note: Lag truncation: K=8

Table A2. Individual and Panel Cointegrated Test Results 1960-2008

Country	ADF	P value	lags	PP	P value
Austria	-3.50	0.012	3	-4.56	0.001
Belgium	-2.06	0.239	3	-2.7	0.07
Luxembourg	-2.86	0.058	1	-4.24	0.002
Sweden	-3.11	0.035	0	-3.15	0.03
Switzerland	-3.24	0.021	0	-3.15	0.027
Canada	-3.18	0.027	1	-2.44	0.139
Japan	-2.84	0.053	1	-3.91	0.006
Ireland	-3.51	0.013	0	-3.71	0.01
Spain	-2.73	0.070	0	-3.04	0.041
Australia	-3.35	0.016	0	-3.57	0.012
New Zealand	-2.79	0.060	0	-2.95	0.04
South Africa	-3.04	0.041	1	-2.6	0.101
Argentina	-3.96	0.003	1	-3.35	0.018
Chile	-2.67	0.091	0	-2.61	0.107
Hong Kong	-5.34	0.002	2	-3.46	0.017
Korea	-3.97	0.004	0	-3.87	0.005
Malaysia	-3.82	0.007	1	-2.93	0.055
Pakistan	-2.81	0.063	3	-2.56	0.11
Singapore	-4.49	0.002	1	-2.84	0.064
Thailand	-3.68	0.010	0	-3.87	0.008
China, P. R.: Ma	-3.22	0.029	2	-3.44	0.013
Group mean panel	-7.25	0.000	4	-7.36	0.0000
Pooled panel	-6.32	0.000	4	-7.02	0.0000

Note: the symbols *, **, *** indicate 10%, 5% and 1% rejection respectively.

Table A3a. Cointegration coefficient estimates

Country	1950-2008			Before 1990			After 1990		
	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)
United States	0.33	1.26	-2.59	0.54	3.27	-2.82	-0.07	-0.58	-9.00
United Kingdom	0.46	1.58	-1.88	0.87	11.63	-1.81	-0.51	-1.54	-4.56
Austria	1.20	53.97	9.17	1.20	59.75	9.98	1.11	25.24	2.44
Belgium	0.88	16.82	-2.19	0.90	18.00	-2.05	0.85	20.98	-3.71
Denmark	1.18	5.44	0.85	0.88	6.22	-0.87	0.78	6.46	-1.86
France	1.03	19.44	0.64	1.14	53.63	6.76	0.64	10.67	-6.03
Germany	0.98	5.17	-0.10	1.63	13.97	5.40	0.81	44.10	-10.03
Italy	1.05	6.35	0.31	1.19	28.09	4.46	0.65	20.89	-11.25
Luxembourg	0.49	3.29	-3.46	0.33	0.98	-2.02	-0.04	-0.20	-5.88
Netherlands	1.13	7.01	0.78	1.04	6.34	0.26	0.99	17.92	-0.14
Norway	0.41	6.89	-9.83	0.57	16.64	-12.44	0.81	7.82	-1.86
Sweden	1.39	11.85	3.31	1.12	8.70	0.95	1.41	19.96	5.77
Switzerland	0.88	24.48	-3.24	0.78	14.55	-4.02	0.76	63.06	-19.87
Canada	1.01	12.29	0.16	0.85	10.19	-1.78	0.59	5.26	-3.73
Japan	0.95	57.09	-2.89	0.95	72.85	-4.06	0.78	35.13	-9.99
Finland	0.85	13.63	-2.43	0.88	25.84	-3.49	0.69	6.45	-2.88
Greece	0.77	5.39	-1.59	0.82	15.53	-3.37	0.80	7.30	-1.86
Iceland	1.20	14.51	2.41	1.19	20.12	3.23	1.63	6.35	2.45
Ireland	0.48	13.11	-14.04	0.41	3.72	-5.47	0.56	26.38	-20.47
Portugal	0.79	15.68	-4.29	0.80	14.85	-3.64	0.39	9.43	-15.03
Spain	0.93	35.88	-2.86	0.94	43.87	-2.98	0.49	3.49	-3.61
Turkey	1.01	4.63	0.03	0.74	6.56	-2.27	1.49	26.34	8.66
Australia	0.83	10.86	-2.29	0.94	14.02	-0.82	0.42	3.53	-4.95
New Zealand	0.96	15.66	-0.65	1.08	14.26	1.07	-0.25	-1.84	-9.08
South Africa	0.75	13.46	-4.46	0.90	9.31	-0.99	0.55	3.30	-2.65
Argentina	0.90	39.29	-4.58	0.92	31.57	-2.70	1.02	24.85	0.57
Brazil	1.05	5.35	0.24	1.36	18.16	4.76	0.31	2.52	-5.56
Chile	0.80	5.04	-1.26	0.72	5.59	-2.18	1.31	36.29	8.58
Colombia	1.40	9.84	2.81	0.74	10.57	-3.79	1.66	33.88	13.42
Mexico	0.82	8.45	-1.81	0.77	6.90	-2.05	0.51	5.00	-4.81
Peru	0.84	6.88	-1.31	0.59	1.69	-1.16	0.76	4.21	-1.32
Israel	0.57	2.64	-1.98	0.54	5.51	-4.63	-0.09	-0.72	-8.45
Saudi Arabia	0.44	2.47	-3.11	0.17	0.66	-3.10	1.47	8.21	2.64
Egypt	0.36	4.64	-8.40	0.50	4.86	-4.89	2.54	19.07	11.56
Hong Kong	1.10	55.80	4.85	1.07	48.47	3.29	1.24	13.24	2.59
India	0.45	2.27	-2.77	1.50	6.01	2.01	0.69	41.58	-18.81
Indonesia	1.67	10.95	4.38	1.32	6.71	1.64	-0.58	-2.01	-5.49
Korea	0.82	83.44	-17.81	0.78	49.24	-14.01	0.86	25.41	-3.99
Malaysia	0.67	18.43	-8.98	0.60	10.19	-6.93	0.75	6.96	-2.35
Pakistan	0.76	4.85	-1.52	0.99	6.39	-0.06	0.32	2.58	-5.45
Philippines	0.72	6.77	-2.59	1.15	6.97	0.93	1.00	9.20	0.01
Singapore	0.73	32.64	-12.25	0.69	24.93	-11.19	0.37	4.23	-7.23
Thailand	0.70	29.55	-12.55	0.55	32.89	-26.88	0.95	18.83	-1.01
Morocco	1.03	12.20	0.37	1.26	20.39	4.24	0.79	12.03	-3.20
China	0.80	40.23	-10.03	0.77	11.43	-3.34	0.82	96.92	-20.75
Panel	0.86	112.92	-17.81	0.88	119.56	-13.84	0.73	108.55	-26.56

Source: PWT and WEO.

We select our countries based on the sample coverage and data justification. Specifically, we select the OECD countries and the emerging market countries which are a total of 51 countries. Then we take out the East European transitional economies Czech. Rep. Hungary, Poland and Slovak Rep. This leaves us with 47 countries in our data sample (25 OECD and 22 emerging markets). we selected countries has data coverage at least from year 1967.

Table A3b. Cointegration coefficient estimates

Country	1950-2008			Before 1990			After 1990		
	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)	Coefficient	t-statistic (b=0)	t-statistic (b=1)
Austria	1.20	53.97	9.17	1.20	59.75	9.98	1.11	25.24	2.44
Belgium	0.88	16.82	-2.19	0.90	18.00	-2.05	0.85	20.98	-3.71
Luxembourg	0.49	3.29	-3.46	0.33	0.98	-2.02	-0.04	-0.20	-5.88
Sweden	1.39	11.85	3.31	1.12	8.70	0.95	1.41	19.96	5.77
Switzerland	0.88	24.48	-3.24	0.78	14.55	-4.02	0.76	63.06	-19.87
Canada	1.01	12.29	0.16	0.85	10.19	-1.78	0.59	5.26	-3.73
Japan	0.95	57.09	-2.89	0.95	72.85	-4.06	0.78	35.13	-9.99
Ireland	0.48	13.11	-14.04	0.41	3.72	-5.47	0.56	26.38	-20.47
Spain	0.93	35.88	-2.86	0.94	43.87	-2.98	0.49	3.49	-3.61
Australia	0.83	10.86	-2.29	0.94	14.02	-0.82	0.42	3.53	-4.95
New Zealand	0.96	15.66	-0.65	1.08	14.26	1.07	-0.25	-1.84	-9.08
South Africa	0.75	13.46	-4.46	0.90	9.31	-0.99	0.55	3.30	-2.65
Argentina	0.90	39.29	-4.58	0.92	31.57	-2.70	1.02	24.85	0.57
Chile	0.80	5.04	-1.26	0.72	5.59	-2.18	1.31	36.29	8.58
Hong Kong	1.10	55.80	4.85	1.07	48.47	3.29	1.24	13.24	2.59
Korea	0.82	83.44	-17.81	0.78	49.24	-14.01	0.86	25.41	-3.99
Malaysia	0.67	18.43	-8.98	0.60	10.19	-6.93	0.75	6.96	-2.35
Pakistan	0.76	4.85	-1.52	0.99	6.39	-0.06	0.32	2.58	-5.45
Singapore	0.73	32.64	-12.25	0.69	24.93	-11.19	0.37	4.23	-7.23
Thailand	0.70	29.55	-12.55	0.55	32.89	-26.88	0.95	18.83	-1.01
China	0.80	40.23	-10.03	0.77	11.43	-3.34	0.82	96.92	-20.75
Panel	0.86	126.14	-19.11	0.83	107.12	-16.63	0.71	94.62	-22.87

Source: PWT and WEO.

We select our countries based on the sample coverage and data justification. Specifically, we select the OECD countries and the emerging market countries which are a total of 51 countries. Then we take out the East European transitional economies Czech. Rep. Hungary, Poland and Slovak Rep. This leaves us with 47 countries in our data sample (25 OECD and 22 emerging markets). we selected countries has data coverage at least from year 1967.

Appendix A: Permanent and Transitory Shocks

This appendix illustrates the difference between shocks that have permanent and transitory effects.

By the Law of Iterated Conditional Expectations, the predicated value of T_s in process (13) is

$$E_t(T_s) = \rho^{s-t}T_t$$

for all $s \geq t$. Since $0 \leq \rho < 1$, this says that the effects of a shock in the $AR(1)$ process die out over time and the long-run effect is zero.

However, for process (11), the predicated value of P_s for all $s \geq t$, conditional on information in period t , is

$$E_t(P_s) = P_t$$

This says that the effect of a shock in a pure random walk process is permanent.

The variance of the forecast error for process (13) is finite even when $s \rightarrow \infty$, since

$$\lim_{s \rightarrow \infty} E(T_{t+s} - T_{t+s|t})^2 = 1/(1 - \rho)\sigma_\zeta^2$$

While the variance of the forecast error for process (11) tends to infinity, since

$$E(P_{t+s} - P_{t+s|t})^2 = s\sigma_\theta^2$$

Appendix B: Deriving Equations

B.1 Deriving Equation (28)

From $Y_{t+1} = Y_t + \sum_{j=0}^{\infty} \varrho^j \varepsilon_{t+1-j}$, we have

$$E_t(Y_{t+1}) = Y_t + (\varrho \varepsilon_t + \varrho^2 \varepsilon_{t-1} + \varrho^3 \varepsilon_{t-2} + \dots)$$

$$\begin{aligned} E_t(Y_{t+2}) &= Y_t + (\varrho \varepsilon_t + \varrho^2 \varepsilon_{t-1} + \varrho^3 \varepsilon_{t-2} + \dots) + (\varrho^2 \varepsilon_t + \varrho^3 \varepsilon_{t-1} + \varrho^4 \varepsilon_{t-2} + \dots) \\ &= Y_t + (\varrho + \varrho^2) \varepsilon_t + (\varrho^2 + \varrho^3) \varepsilon_{t-1} + (\varrho^3 + \varrho^4) \varepsilon_{t-2} + \dots \end{aligned}$$

$$E_t(Y_{t+3}) = Y_t + (\varrho + \varrho^2 + \varrho^3) \varepsilon_t + (\varrho^2 + \varrho^3 + \varrho^4) \varepsilon_{t-1} + (\varrho^3 + \varrho^4 + \varrho^5) \varepsilon_{t-2} + \dots$$

Plug these into equation (17)

$$\begin{aligned}
C_t &= rB_t + \frac{r}{1+r} \{Y_t \\
&\quad + \frac{1}{1+r} (Y_t + \varrho \varepsilon_t + \varrho^2 \varepsilon_{t-1} + \varrho^3 \varepsilon_{t-2} + \dots) \\
&\quad + (\frac{1}{1+r})^2 [Y_t + (\varrho + \varrho^2) \varepsilon_t + (\varrho^2 + \varrho^3) \varepsilon_{t-1} + (\varrho^3 + \varrho^4) \varepsilon_{t-2} + \dots] \\
&\quad + (\frac{1}{1+r})^3 [Y_t + (\varrho + \varrho^2 + \varrho^3) \varepsilon_t + (\varrho^2 + \varrho^3 + \varrho^4) \varepsilon_{t-1} + (\varrho^3 + \varrho^4 + \varrho^5) \varepsilon_{t-2} + \dots] \\
&\quad + \dots \} \\
&= rB_t + \frac{r}{1+r} \{ [1 + \frac{1}{1+r} + (\frac{1}{1+r})^2 + (\frac{1}{1+r})^3 + \dots] Y_t \\
&\quad + \frac{\varrho}{1+r} (1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho+\varrho^2}{(1+r)^2} + \dots) \varepsilon_t \\
&\quad + \frac{\varrho^2}{1+r} (1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho+\varrho^2}{(1+r)^2} + \dots) \varepsilon_{t-1} \\
&\quad + \frac{\varrho^3}{1+r} (1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho+\varrho^2}{(1+r)^2} + \dots) \varepsilon_{t-2} \\
&\quad + \dots \}
\end{aligned}$$

Now, let's define

$$s = 1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho+\varrho^2}{(1+r)^2} + \frac{1+\varrho+\varrho^2+\varrho^3}{(1+r)^3} + \dots$$

\therefore

$$\begin{aligned}
s &= 1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho}{(1+r)^2} + \frac{\varrho^2}{(1+r)^2} + \frac{1+\varrho}{(1+r)^3} + \frac{\varrho^2+\varrho^3}{(1+r)^3} + \dots \\
&= 1 + \frac{1+\varrho}{1+r} (1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots) + \frac{\varrho^2}{(1+r)^2} (1 + \frac{1+\varrho}{1+r} + \frac{1+\varrho+\varrho^2}{(1+r)^2} + \dots) \\
&= 1 + \frac{1+\varrho}{1+r} (1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots) + \frac{\varrho^2}{(1+r)^2} s
\end{aligned}$$

\therefore

$$s = \frac{(1+r)^2}{(1+r-\rho)r}$$

Plug s into consumption function \implies

$$\begin{aligned} C_t &= rB_t + Y_t + \frac{r}{1+r} \left[\frac{(1+r)^2}{(1+r-\rho)r} \left(\frac{\rho}{1+r} \varepsilon_t + \frac{\rho^2}{1+r} \varepsilon_{t-1} + \frac{\rho^3}{1+r} \varepsilon_{t-2} + \dots \right) \right] \\ &= rB_t + Y_t + \frac{r}{(1+r-\rho)r} [\rho \varepsilon_t + \rho^2 \varepsilon_{t-1} + \rho^3 \varepsilon_{t-2} + \dots] \\ &= rB_t + Y_t + \frac{r\rho}{(1+r-\rho)r} \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} \\ &= rB_t + Y_t + \frac{\rho}{1+r-\rho} (Y_t - Y_{t-1}) \end{aligned}$$

B.2 Deriving Equation (29)

This equation is much easier to derive. Substitute $E_t(Y_t) - E_{t-1}(Y_t) = \varepsilon_t$, $E_t(Y_{t+1}) - E_{t-1}(Y_{t+1}) = (1+\rho)\varepsilon_t$, $E_t(Y_{t+2}) - E_{t-1}(Y_{t+2}) = (1+\rho+\rho^2)\varepsilon_t$, $E_t(Y_{t+3}) - E_{t-1}(Y_{t+3}) = (1-\rho^{s-t})/(1-\rho)\varepsilon_t$, and $\lim_{j \rightarrow \infty} E_t(Y_{t+j}) - E_{t-1}(Y_{t+j}) = 1/(1-\rho)\varepsilon_t$ into equation (19),

$$C_t - C_{t-1} = \frac{r}{1+r} \left(\varepsilon_t + \frac{1+\rho}{1+r} \varepsilon_t + \frac{1+\rho+\rho^2}{(1+r)^2} \varepsilon_t + \dots \right)$$

Plug $s = \frac{(1+r)^2}{(1+r-\rho)r}$ into equation above \implies

$$C_t - C_{t-1} = \frac{1+r}{1+r-\rho} \varepsilon_t$$

Done.

Appendix C: Probability Limits of $\beta^{\hat{L}R}$ and $\beta^{\hat{S}R}$

In this appendix, we illustrate the limiting properties of $\beta^{\hat{L}R}$ and $\beta^{\hat{S}R}$ when the output process is $Y_t = Y_{t-1} + u_t$, where $u_t = \sum_{j=0}^{\infty} \varrho^j \varepsilon_{t-j}$

C.1 Long-run Slope Coefficient ⁴⁷

In equation (37),

$$\beta^{\hat{L}R} = \frac{\sum_{t=1}^T C_t Y_t}{\sum_{t=1}^T (Y_t)^2}$$

Plug in results that $C_t = (1-\lambda)\bar{C} + rB_t + \lambda[Y_t + \frac{\varrho}{1+r-\varrho}(Y_t - Y_{t-1})]$ and $Y_t - Y_{t-1} = u_t$

$$\begin{aligned} \beta^{\hat{L}R} &= \frac{\frac{1}{T^2} \sum_{t=1}^T \{(1-\lambda)\bar{C} + rB_t + \lambda[Y_t + \frac{\varrho}{1+r-\varrho}u_t]\} Y_t}{\frac{1}{T^2} \sum_{t=1}^T (Y_t)^2} \\ &= \frac{(1-\lambda)\bar{C} \frac{1}{T^2} \sum_{t=1}^T Y_t + r \frac{1}{T^2} \sum_{t=1}^T B_t Y_t + \lambda \frac{1}{T^2} \sum_{t=1}^T Y_t^2 + \frac{\lambda\varrho}{1+r-\varrho} \frac{1}{T^2} \sum_{t=1}^T u_t Y_t}{\frac{1}{T^2} \sum_{t=1}^T (Y_t)^2} \end{aligned}$$

\therefore According to the Central Limit Theorem (Theorem 17.2 in Chapter 17 of Hamilton (1994)),

$$\frac{1}{T^{3/2}} \sum_{t=1}^T Y_t \xrightarrow{L} \epsilon \cdot \int_0^1 W(r) dr$$

⁴⁷For simplicity, I omitted the constant term α .

$$\frac{1}{T^2} \sum_{t=1}^T (Y_t)^2 \xrightarrow{L} \epsilon^2 \cdot \int_0^1 [W(r)]^2 dr$$

$$\frac{1}{T} \sum_{t=1}^T u_{t-j} Y_t \xrightarrow{L} \begin{cases} \frac{1}{2} \{ \epsilon^2 \cdot [W(r)]^2 - \gamma_0 \} & j = 0 \\ \frac{1}{2} \{ \epsilon^2 \cdot [W(r)]^2 - \gamma_0 \} + \gamma_0 + \gamma_1 + \dots + \gamma_{j-1} & j = 1, 2, 3, \dots \end{cases}$$

where $\epsilon = \frac{\rho}{1-\rho} \sigma_\epsilon$, $\gamma_j = \frac{\rho^j}{1-\rho^2} \sigma_\epsilon^2$, $u_t = Y_t - Y_{t-1} = \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j}$ and where $W(\cdot)$ is the standard Brownian motion.

\therefore

$$\frac{1}{T^2} \sum_{t=1}^T Y_t \xrightarrow{P} 0$$

$$\frac{1}{T^2} \sum_{t=1}^T u_t Y_t \xrightarrow{P} 0$$

where $x_T \xrightarrow{L} y$ means x_T converges to y in distribution; $x_T \xrightarrow{P} y$ means x_T converges to y in probability.

\implies

$$plim_{T \rightarrow \infty} \hat{\beta}^{LR} = \lambda + r \frac{plim_{T \rightarrow \infty} \frac{1}{T^2} \sum_{t=1}^T B_t Y_t}{plim_{T \rightarrow \infty} \frac{1}{T^2} \sum_{t=1}^T (Y_t)^2}$$

So, in order to get $plim_{T \rightarrow \infty} E(\hat{\beta}^{LR})$, it is key to solve $plim_{T \rightarrow \infty} E(\frac{1}{T^2} \sum_{t=1}^T B_t Y_t)$

Using equation (36): $B_t = B_0 - \frac{\lambda \rho}{1+r-\rho} (Y_{t-1} - Y_{-1})$,

$$\frac{1}{T^2} \sum_{t=1}^T B_t Y_t = \frac{1}{T^2} \sum_{t=1}^T [B_0 - \frac{\lambda \varrho}{1+r-\varrho} (Y_{t-1} - Y_{-1})] Y_t$$

\Rightarrow

$$plim_{T \rightarrow \infty} \left(\frac{1}{T^2} \sum_{t=1}^T B_t Y_t \right) = -\frac{\lambda \varrho}{1+r-\varrho} plim_{T \rightarrow \infty} \left\{ \frac{1}{T^2} \sum_{t=1}^T (Y_t)^2 \right\}$$

since

$$plim_{T \rightarrow \infty} \frac{1}{T^2} \sum_{t=1}^T B_0 Y_t = B_0 plim_{T \rightarrow \infty} \frac{1}{T^2} \sum_{t=1}^T Y_t = 0$$

$$plim_{T \rightarrow \infty} \left(\frac{1}{T^2} \sum_{t=1}^T Y_{-1} Y_t \right) = Y_{-1} plim_{T \rightarrow \infty} \left(\frac{1}{T^2} \sum_{t=1}^T Y_t \right) = 0$$

\Rightarrow

$$\begin{aligned} plim_{T \rightarrow \infty} \beta^{\hat{LR}} &= \lambda + r \left\{ -\frac{\lambda \varrho}{1+r-\varrho} \right\} \\ &= \lambda \left(1 - \frac{r \varrho}{1+r-\varrho} \right) \end{aligned}$$

C.2 Short-run Slope Coefficient

In equation (39)

$$\beta^{\hat{SR}} = \frac{\sum_{t=1}^T (C_t - C_{t-1})(Y_t - Y_{t-1})}{\sum_{t=1}^T (Y_t - Y_{t-1})^2}$$

Using equation (38): $C_t - C_{t-1} = \frac{\lambda(1+r)}{(1+r-\varrho)}\varepsilon_t$

$$\begin{aligned}\beta^{\hat{SR}} &= \frac{\sum_{t=1}^T \frac{\lambda(1+r)}{1+r-\varrho} \varepsilon_t u_t}{\sum_{t=1}^T (u_t)^2} \\ &= \frac{\lambda(1+r)}{1+r-\varrho} \frac{\frac{1}{T} \sum_{t=1}^T \varepsilon_t (\varrho u_{t-1} + \varepsilon_t)}{\frac{1}{T} \sum_{t=1}^T (u_t)^2}\end{aligned}$$

where the second equality holds since $Y_t - Y_{t-1} = u_t$ and $Y_t - Y_{t-1} = \varrho(Y_{t-1} - Y_{t-2}) + \varepsilon_t$.

\therefore According to the Central Limit Theorem,

$$\frac{1}{\sqrt{T}} \sum_{t=1}^T \varepsilon_t u_{t-1} \xrightarrow{L} N(0, \sigma_\varepsilon^2 \gamma_0)$$

$$\frac{1}{T} \sum_{t=1}^T \varepsilon_t^2 \xrightarrow{P} \sigma_\varepsilon^2$$

$$\frac{1}{T} \sum_{t=1}^T u_t^2 \xrightarrow{P} \gamma_0$$

where $\gamma_0 \equiv E(u_t^2) = \sigma_\varepsilon^2 \sum_{j=0}^{\infty} \varrho^{2j} = \sigma_\varepsilon^2 [1/(1-\varrho^2)]$.

\therefore

$$\frac{1}{T} \sum_{t=1}^T \varepsilon_t u_{t-1} \xrightarrow{P} 0$$

and therefore

$$\begin{aligned}
plim_{T \rightarrow \infty} \hat{\beta}^{SR} &= \frac{\lambda(1+r)}{(1+r-\varrho)} \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 [1/(1-\varrho^2)]} \\
&= \frac{\lambda(1+r)(1-\varrho^2)}{(1+r-\varrho)}
\end{aligned}$$

So, $\hat{\beta}^{SR}$ is inconsistent even asymptotically since it consists of the true β^{SR} times a bias term $1 - \varrho^2$.

Appendix D: Studies using Conventional Panel Analysis

We list the following studies, but the list is far from exclusive.

Kose et al. 2007

$$\Delta c_{it} - \Delta c_t^w = \alpha_i + \delta_t + (\beta_0 + \beta_1 f_{oit})(\Delta y_{it} - \Delta y_t^w) + \varepsilon_{it}$$

Sorensen et al 2007

$$\Delta c_{it} - \Delta c_t^w = \alpha_i + (\beta_0 + \beta_1 (EHB_{it} - EHB_t^w) + \beta_2 (t - \bar{t}))(\Delta y_{it} - \Delta y_t^w) + \varepsilon_{it}$$

Bai and Zhang 2005

$$\Delta c_t = \alpha_i + \gamma \Delta y_t + \varepsilon_{it}$$

$$\Delta c_{it} = \alpha_i + \eta \Delta c_t^w + \gamma \Delta y_{it} + \varepsilon_{it}$$

Moser et al 2004

$$\Delta c_{it} = \alpha_i + \eta_i \Delta c_t^w + \gamma_i (\Delta y_{it} - \Delta y_t^w) + \varepsilon_{it}$$

Crucini 1999

$$\Delta c_{it} = \eta_i \Delta c_t^w + (1 - \eta_i) \Delta y_{pit} + \varepsilon_{it}$$

Lewis 1996

$$\Delta c_{it}^{T-D} = v_t + \eta_1 \Delta y_{it}^N + \eta_2 \Delta y_{it}^D + \eta_3 \Delta y_{it}^{T-D} + \varepsilon_{it}$$

Obstfeld 1995

$$\Delta c_t = \alpha + \eta \Delta c_t^w + \gamma(\Delta y_t - \Delta i_t - \Delta g_t) + \varepsilon_{it}$$

Appendix E: Technical Illustration on Conventional and Nonstationary Panel

In Chapter 2 of this dissertation, we have illustrated that $\beta^{\hat{S}R}$, the OLS estimate on the slope coefficient of equation (39), cannot consistently capture the true risk sharing effect. However, $\beta^{\hat{L}R}$, the OLS estimate on the slope coefficient of equation (37), can consistently capture the true long-run risk sharing effect. To make this Chapter self-contained, we summarize the main results in Chapter 2 and extend them to the context of equation (41) and (42).

Short-run and long-run risk sharing relationships

In a rudimentary model of a world of N countries with stochastic endowment outputs of one single tradable good, rational people maximize the discounted expected value of lifetime utility under budget constraints.

For country i , we assume output is $I(1)$ in the sense that

$$Y_t - Y_{t-1} = u_t \tag{45}$$

where the process $\{u_t\}$ satisfies

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{46}$$

where $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables with $E\varepsilon_t = 0$, $E\varepsilon_t^2 = \sigma_\varepsilon^2$, and $E\varepsilon_t^4 < \infty$; $0 \leq \rho < 1$.

We assume there are no aggregate shocks to outputs. We also assume output is

perishable so that the only way to share output risks is through financial markets. Financial markets are incomplete in the sense that only the $1 - \lambda$ portion of output risks can be shared in an insurance market.

Define C_t^{SR} and Y_t^{SR} , the short-run changes of consumption and output under the impact of a output shock, as $C_t^{SR} \equiv \frac{\partial C_t}{\partial \varepsilon_t} \varepsilon_t$ and $Y_t^{SR} \equiv \frac{\partial Y_t}{\partial \varepsilon_t} \varepsilon_t$ respectively. Similarly, define long-run effects of the shock as $C_t^{LR} \equiv \lim_{s \rightarrow \infty} \frac{\partial C_{t+s}}{\partial \varepsilon_t} \varepsilon_t$ and $Y_t^{LR} \equiv \lim_{s \rightarrow \infty} \frac{\partial Y_{t+s}}{\partial \varepsilon_t} \varepsilon_t$ respectively, where superscripts *SR* and *LR* stand for short run and long run.

Assuming a quadratic utility function and using the results from utility maximizations under a bond market and an insurance market, we can derive the defined short-run and long-run risk sharing relationships,

$$C_t^{SR} = \frac{\lambda(1+r)}{1+r-\varrho} Y_t^{SR} \quad (47)$$

$$C_t^{LR} = \lambda \left(1 - \frac{r\varrho}{1+r-\varrho}\right) Y_t^{LR} \quad (48)$$

Testing equations and testing results

Empirically, it is interesting to estimate the short-run and long-run relationships.

In the process of solving utility maximization under a bond market and an insurance market, we yield the following consumption and bond holding processes,

$$C_t = (1 - \lambda)\bar{C} + rB_t + \lambda \left[Y_t + \frac{\varrho}{(1+r-\varrho)} (Y_t - Y_{t-1}) \right] \quad (49)$$

$$C_t - C_{t-1} = \frac{\lambda(1+r)}{1+r-\varrho} \varepsilon_t \quad (50)$$

$$B_t = B_0 - \frac{\lambda \varrho}{1+r-\varrho} (Y_{t-1} - Y_{-1}) \quad (51)$$

where B_t is bond holding at the beginning of period t ; \bar{C} is the constant level of consumption achieved in an insurance market. We use them to derive equations (41) and (42).

1. Deriving equation (41) Plug equation (45)-(46) into equation (50),

$$\begin{aligned} C_t - C_{t-1} &= \frac{\lambda(1+r)}{1+r-\varrho} (\Delta Y_t - \varrho \Delta Y_{t-1}) \\ &= \frac{\lambda(1+r)}{1+r-\varrho} (Y_t - Y_{t-1}) - \frac{\lambda(1+r)}{1+r-\varrho} \varrho \Delta Y_{t-1} \\ &= \frac{\lambda(1+r)}{1+r-\varrho} (Y_t - Y_{t-1}) - \frac{\lambda(1+r)}{1+r-\varrho} \varrho \sum_{s=-\infty}^{t-1} \varrho^{t-1-s} \varepsilon_s \end{aligned}$$

where the last equality holds because we substitute ΔY_{t-1} using $Y_{t+1} = Y_t + \sum_{s=-\infty}^{t+1} \varrho^{t+1-s} \varepsilon_s$ (it is equation (45)-(46) expressed in another form).

We can rewrite $C_t - C_{t-1}$ into

$$C_t - C_{t-1} = \beta^{SR} (Y_t - Y_{t-1}) + \mu_t \quad (52)$$

where $\beta^{SR} = \frac{\lambda(1+r)}{1+r-\varrho}$; μ_t has properties of a causal and invertible *ARMA* process with $\mu_t = \sum_{s=-\infty}^{t-1} \psi^{t-1-s} \varepsilon_s$ where ψ is some function of ϱ , λ and r ; and $C_t - C_{t-1} \sim I(0)$; $Y_t - Y_{t-1} \sim I(0)$.

We know that the OLS estimator of β^{SR} , $\hat{\beta}^{SR}$ is not a consistent estimate of β^{SR} when the properties of equation (52) are as defined above.

So far, we derived equation (52) under the assumptions of a quadratic utility function and no aggregate shocks. If the utility function is in the form of constant

relative risk aversion, equation (52) will hold in log terms (Hall (1978)) since consumption can only be approximated as a random walk process. If aggregate shocks exist, we need to subtract them since aggregate shocks cannot be shared. So, in a general specification, the testing equation will be $\Delta c_t - \Delta c_t^w = \alpha + \beta^{SR}(\Delta y_t - \Delta y_t^w) + \mu_t$, where lower case letters c and y denote log consumption and output. c^w and y^w are the world average log consumption and output. Equation (41) is a natural extension of the last equation in a panel context.

2. Deriving equation (42)

By plugging equation (51) into equation (49), we have

$$\begin{aligned}
C_t &= (1 - \lambda)\bar{C} + r \left[B_0 - \frac{\lambda\varrho}{1 + r - \varrho}(Y_{t-1} - Y_{-1}) \right] + \lambda \left(Y_t + \frac{\varrho}{1 + r - \varrho} \Delta Y_t \right) \\
&= \left[(1 - \lambda)\bar{C} + rB_0 + r \frac{\lambda\varrho}{1 + r - \varrho} Y_{-1} \right] - r \frac{\lambda\varrho}{1 + r - \varrho} Y_{t-1} + \lambda \left(Y_t + \frac{\varrho}{1 + r - \varrho} \Delta Y_t \right) \\
&= \alpha - r \frac{\lambda\varrho}{1 + r - \varrho} \left(Y_t - \sum_{s=-\infty}^t \varrho^{t-s} \varepsilon_s \right) + \lambda \left(Y_t + \frac{\varrho}{1 + r - \varrho} \sum_{s=-\infty}^t \varrho^{t-s} \varepsilon_s \right)
\end{aligned}$$

where the last equality holds because we let the constant term $(1 - \lambda)\bar{C} + rB_0 + r \frac{\lambda\varrho}{1 + r - \varrho} Y_{-1} = \alpha$, and we substitute Y_{t-1} and ΔY_t using $Y_t = Y_{t-1} + \sum_{s=-\infty}^t \varrho^{t-s} \varepsilon_s$.

Recollecting terms,

$$C_t = \alpha + \lambda \left(1 - \frac{r\varrho}{1 + r - \varrho} \right) Y_t + \frac{(1 + r)\lambda\varrho}{1 + r - \varrho} \sum_{s=-\infty}^t \varrho^{t-s} \varepsilon_s$$

This is a testable equation that can be written into the following fashion:

$$C_t = \alpha + \beta^{LR}Y_t + u_t \quad (53)$$

where $\beta^{LR} = \lambda(1 - \frac{r\varrho}{1+r-\varrho})$; u_t has properties of a causal and invertible *ARMA* process with $u_t = \sum_{s=-\infty}^t \varphi^{t-s}\varepsilon_s$ where φ is a function of ϱ , λ and r ; and $C_t \sim I(1)$; $Y_t \sim I(1)$.

We know that the OLS estimator of β^{LR} , $\hat{\beta}^{LR}$, is a super-consistent estimate of β^{LR} when equation (53) satisfies the properties above.

Similarly, in a general specification, the long-run testing equation shall be $c_t - c_t^w = \alpha + \beta^{LR}(y_t - y_t^w) + u_t$. Equation (42) is a natural extension of the last equation in a panel context.

Appendix F: Group-mean FMOLS Estimator: Model Specifications and Estimation Recipes

To simplify the notations used in this appendix, we use y_{1it} to denote $c_{it} - c_t^w$ and y_{2it} to denote $y_{it} - y_t^w$. Equation (42) can be rewritten as

$$y_{1it} = \alpha_i + \beta_i y_{2it} + \varepsilon_{it} \quad t = 1, \dots, T; i = 1, \dots, N \quad (54)$$

where β_i is the slope parameter in which we are interested as defined in the main text; $\{\varepsilon_{it}\}$ is an $I(0)$ stationary weakly dependent disturbance term; and y_{2it} is $I(1)$. Notice that if y_{2it} is $I(1)$ and ε_{it} is $I(0)$, y_{1it} is $I(1)$ by construction.

Equation (54) is our regression model. We assume that the true model can be expressed in the following equation system using the triangular representation:⁴⁸

⁴⁸The structure system below is typical of more general models which can have multiple regressors, multidimensional cointegrationships and with deterministic trends in equation (56)

$$y_{1it} = \alpha_i + \beta_i y_{2it} + \varepsilon_{it} \quad (55)$$

$$y_{2it} = y_{2it-1} + v_{it} \quad t = 1, \dots, T; i = 1, \dots, N \quad (56)$$

where $\mu_{it} = (\varepsilon_{it}, v_{it})'$ are the $I(0)$ stationary weakly dependent disturbance terms.

Since the properties of cointegration tests, cointegrating coefficients estimates, and hypothesis tests in the time series context have been well established, we review some of the propositions in the time series context first. The time series counterparts of equations (55) and (56) are as follows:

$$y_{1t} = \alpha + \beta y_{2t} + \varepsilon_t \quad (57)$$

$$y_{2t} = y_{2t-1} + v_t \quad t = 1, \dots, T \quad (58)$$

We assume that equations (57) and (58) satisfy the assumptions and therefore the results in Proposition 19.2 of Hamilton (1994), which I quote below (note the notation in the proposition is self-contained and should not be confused with the notation outside the proposition):

Proposition 19.2: Let y_{1t} be a scalar and y_{2t} be a $(g \times 1)$ vector. Let $n = g + 1$, and suppose that the $(n \times 1)$ vector $(y_{1t}, y'_{2t})'$ is characterized by exactly one cointegrating relation ($h = 1$) that has a nonzero coefficient on y_{2t} . Let that triangular representation for the system be

$$y_{1t} = \alpha + \gamma' y_{2t} + z_t^* \quad ([19.2.9])$$

(Phillips, 1991). Nevertheless, the discussion remains essentially the same.

$$\Delta y_{2t} = u_{2t} \quad ([19.2.10])$$

Suppose that

$$\begin{bmatrix} z_t^* \\ u_{2t} \end{bmatrix} = \Psi^*(L)\varepsilon_t \quad ([19.2.11])$$

where ε_t is an $(n \times 1)$ *i.i.d.* vector with mean zero, finite fourth moments, and positive variance-covariance matrix $E(\varepsilon_t \varepsilon_t') = PP'$. Suppose further that the sequence of $(n \times n)$ matrices $\{s \cdot \Psi_s^*\}_{s=0}^\infty$ is absolutely summable and that the rows of $\Psi^*(1)$ are linearly independent. Let $\hat{\alpha}_T$ and $\hat{\gamma}_T$ be estimated based on OLS estimation of [19.2.9],

$$\begin{bmatrix} \hat{\alpha}_T \\ \hat{\gamma}_T \end{bmatrix} = \begin{bmatrix} T & \sum y'_{2t} \\ \sum y'_{2t} & \sum y_{2t} y'_{2t} \end{bmatrix} \begin{bmatrix} \sum y_{1t} \\ \sum y_{2t} y_{1t} \end{bmatrix} \quad ([19.2.12])$$

where \sum indicates summation over t from 1 to T . Partition $\Psi^*(1) \cdot P$ as

$$\Psi^*(1) \cdot P = \begin{bmatrix} \lambda_1^* \\ (1 \times n) \\ \Lambda_2^* \\ (g \times n) \end{bmatrix}$$

Then

$$\begin{bmatrix} T^{1/2}(\hat{\alpha}_T - \alpha) \\ T(\hat{\gamma}_T - \gamma) \end{bmatrix} \xrightarrow{L} \begin{bmatrix} 1 & \int [W(r)' dr] \cdot \Lambda_2^{*'} \\ \Lambda_2^* \cdot \int [W(r) dr] & \Lambda_2^* \cdot \int [W(r) \cdot W(r)' dr] \cdot \Lambda_2^{*'} \end{bmatrix}^{-1} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad ([19.2.13])$$

where $W(r)$ is n -dimensional standard Brownian motion, the integral sign denote integration over r from 0 to 1, and

$$\begin{aligned}
h_1 &= \lambda_1^* \cdot W(1) \\
h_2 &= \Lambda_2^* \cdot \left\{ \int_0^1 [W(r) \cdot W(r)' dr] \right\} \cdot \lambda_1^* + \sum_{v=0}^{\infty} u_{2t} z_{t+v}^*
\end{aligned}$$

The holding of [19.2.13] involves the Beveridge and Nelson decomposition on $(y_{1t} \ y'_{2t})'$ and the multivariate functional limiting theorem on $(z_t^* \ u'_{2t})'$. To better understand this OLS estimator, let's consider a simplified case. If we assume y_{2t} is a random walk, z_t^* is white noise and $(z_t^* \ u'_{2t})'$ are Gaussian disturbance processes, the regression model [19.2.9] satisfies the case where the error term is *i.i.d.* Gaussian and is independent of explanatory variables. Under these assumptions, the OLS estimator is normal distributed and the t and F statistics have the exact t and F distributions for inference. If the error term is non-Gaussian, OLS estimator is normal distributed and we can use its associated asymptotic t and F statistics for inference.

What happens if $[z_t^*, u'_{2t}]'$ is autocorrelated and/or z_t^* correlated with Δy_{2t} . The estimated $\hat{\gamma}_T$ by OLS in [19.2.9] is still superconsistent, but now it has a second-order bias. Actually, although Δy_{2t} is mean zero in Proposition 19.2, the superconsistency property survives even in the case $E(\Delta y_{2t}) = \delta_2 \neq 0$. Hansen (1992) has given the generalized result through rotating of variables. This generalization is also applied to the case of FMOLS that we will discuss below. However, the second-order bias, which does not go away asymptotically, may hinder our ability to infer our testing results in finite samples, so the remaining task is how to correct the second order bias created by the serial correlations and endogeneity caused by feedback effects between Δy_{2t} and z_t^* .

Given there are different representations on the equation (57) and (58), it is not

surprising on lack of consensus on the best empirical estimation approach. Phillips and Loretan (1991) has shown the many different representations and the transformations and interchanges among them in the time series context. The asymptotic theory of their paper concluded that the full systems maximum likelihood method (FSML) in the situation where the unit roots are imposed is the optimal approach. Meanwhile, they have also shown that the FMOLS developed by Phillips and Hansen (1990) is optimal as well since FMOLS estimator are asymptotically the same as FSML estimator. Given the limitation of spaces and also for the reason that we will give the recipe for panel FMOLS estimator, please refer to Chapter 19.3 (Hamilton 1994) for the exact formula on the asymptotic distribution of the FMOLS estimator and associated test statistics. But we can intuitively know that, after corrections, the FMOLS estimator becomes well behaved and we can use the standard asymptotic t and F statistics for inference.

Empirically, in the time series context, the inference based on FMOLS estimator suffers from the low power and high size distortion in finite samples. Pedroni (2000) extended Phillips and Hansen (1990) FMOLS approach into panel and developed panel group mean FMOLS estimator of (54).⁴⁹

In the context of double indexed process where both N and $T \rightarrow \infty$, three approaches (sequential limit, diagonal limit and joint limit) are possible, depending on the passage to infinity of the two indexes. Phillips and Moon (2000) has given a generalization on when the sequential limit is equivalent to joint limit.

⁴⁹As stated in Baltagi and Kao (2000), “the focus of panel data econometrics has shifted towards studying the asymptotics of macro panels with large N (number of countries) and large T (length of the time series) rather than the usual asymptotics of micro panel with large N and small T ...the hope of the econometrics of non-stationary panel data is to combine the best of both worlds: the method of dealing with non-stationary data from the time-series and the increased data and power from the cross-section. The addition of the cross-section dimension, under certain assumptions, can act as repeated draws from the same distribution. Thus as the time and cross-section dimension increase panel test statistics and estimators can be derived with converge in distribution to normally distributed random variables.”

Specifically, they first derived the sequential limit of a double index sequence and then verified the joint limit theory applies when $T, N \rightarrow \infty$ and $T/N \rightarrow \infty$. For the macroeconomic series, in most of the cases, we can think them as T is potentially growing while N is relatively constant, so they fit into the scenario where $T, N \rightarrow \infty$ and $T/N \rightarrow \infty$. For this reason, the sequential limit theory is used to develop the asymptotics for the panel group mean FMOLS estimators. This is also consistent with the claim in Baltagi and Kao (2000) that cross section can act as repeated draws from the same distribution. Therefore, we can think the group mean FMOLS estimator below as $T \rightarrow \infty$ being in a sense the true asymptotic feature.

Let's look at the recipe on how to compute the group mean FMOLS estimator and hypothesis test statistics. You will see why the short term dynamics in a cointegrating system can be allowed to be heterogeneous across countries and the regressors can be allowed for complete endogeneity. This is basically in keep with the discussion of Phillips (1991) on why optimal estimation on cointegrating coefficients can be achieved without a finely detailed specification on the short-run dynamics and how the endogeneity bias of the OLS estimation of the time series counterpart of equation (54) can be adjusted. These arguments can be directly applied into panel context.⁵⁰

Step 1: Estimate by OLS the time series cointegration regression for each country and collect estimated residuals $\widehat{\varepsilon}_{it}$.

Step 2: For each country i , using estimated residuals from step 1, form the time series vectors $\xi_{it} = (\widehat{\varepsilon}_{it}, \Delta y_{2it})'$. We can then use these vectors to compute the country specific long-run covariance matrix $\Omega_i = \sum_{j=-\infty}^{\infty} \Psi_{ij}$, where Ψ_{ij} is the j th autocovariance for ξ_i . The matrix Ω_i can be thought of as $\Omega_i = \Sigma_i + \Gamma_i + \Gamma_i'$,

⁵⁰The illustration below on computing step is based on a seminar at the IMF by Peter Pedroni.

where Σ_i is contemporaneous covariance matrix; Γ_i and Γ'_i are the forward and backward spectrum respectively. We can use the Newey-West estimator to estimate Ω_i nonparametrically and get $\widehat{\Omega}_i = \widehat{\Sigma}_i + \widehat{\Gamma}_i + \widehat{\Gamma}'_i$ where $\widehat{\Sigma}_i = 1/T \sum_{t=1}^T \xi_{it}\xi'_{it}$, $\widehat{\Gamma}_i = +1/T \sum_{s=1}^{K_i} [1 - s/(K_i + 1)] \sum_{t=s+1}^T \xi_{it}\xi'_{it-s}$. The bandwidth K_i is typically chosen as a fraction of the sample, such as $K_i = 4(T_i/100)^{2/9}$ (Newey and West (1994)).

Step 3: For each country i , compute the adjustment terms $\widehat{\gamma}_i = \widehat{\Gamma}_{21i} + \widehat{\Sigma}_{21i} - \widehat{\Omega}_{21i}/\widehat{\Omega}_{22i}(\widehat{\Gamma}_{22i} + \widehat{\Sigma}_{22i})$ to correct for country specific serial correlation dynamics; compute $y_{1it}^* = (y_{1it} - \overline{y_{1i}}) - \widehat{\Omega}_{21i}/\widehat{\Omega}_{22i} \Delta y_{2it}$ to correct for country specific endogeneity where the difference in y_{2it} are used as "internal instruments". The terms in $\widehat{\gamma}_i$ and y_{1it}^* are indirectly from the estimates of the long-run covariance matrix Ω_i . To see this, in partition form:

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} \\ \Omega_{21i} & \Omega_{22i} \end{bmatrix}$$

where $\Omega_{11i} = \sigma^2$ is scalar long-run variance of ε_{it} ; $\Omega_{12i} = \Omega_{21i}$ is the scalar long-run covariance between ε_{it} and Δy_{2it} ; Ω_{22i} is the scalar long-run covariance among Δy_{2it} .

Step 4: Compute the country specific FMOLS estimator using the adjustment terms from Step 3:

$$\widehat{\beta}_{FMi}^* = \left[\sum_{t=1}^T (y_{2it} - \overline{y_{2i}})^2 \right]^{-1} \left[\sum_{t=1}^T (y_{2it} - \overline{y_{2i}}) y_{1it}^* - T \widehat{\gamma}_i \right]$$

and the associated t-statistic is:

$$t_{\widehat{\beta}_{FMi}^*} = (\widehat{\beta}_{FMi}^* - \beta_{oi}) \left[\widehat{\Omega}_{11i} \sum_{t=1}^T (y_{2it} - \overline{y_{2i}})^2 \right]^{1/2}$$

⁵¹In the general case when y_{2it} is not a scalar, but a $M \times 1$ vector, then $\Omega_{12i} = \Omega'_{21i}$ is $M \times 1$ vector of long-run covariance between $\widehat{\varepsilon}_{it}$ and Δy_{2it} . The analysis remain essentially the same.

where β_{oi} is the value of the coefficient being tested under the null hypothesis.

Step 5: Compute the group mean FMOLS estimator as

$$\widehat{\beta}_{GFM}^* = N^{-1} \sum_{n=1}^N \widehat{\beta}_{FMi}^*$$

and the associated t-statistic is:

$$t_{\widehat{\beta}_{GFM}^*} = N^{-1/2} \sum_{n=1}^N t_{\widehat{\beta}_{FMi}^*} = N^{1/2} \overline{t_{\widehat{\beta}_{FM}^*}}$$

where $\overline{t_{\widehat{\beta}_{FM}^*}} = N^{-1} \sum_{n=1}^N t_{\widehat{\beta}_{FMi}^*}$ is the group mean.

Step 6: Compare panel statistic from step 5 to critical values of tails of $N(0, 1)$ distribution to reject. Specifically, under $H_0 : \beta_i = \beta_0$ (for all i, or, for most i)

$$t_{\widehat{\beta}_{GFM}^*} \implies N(0, 1)$$

Under $H_A : \beta_i \neq \beta_0$ (for all i, or, for some i)

$$t_{\widehat{\beta}_{GFM}^*} \rightarrow \pm\infty$$

So this is a two-sided test and large absolute values imply rejection of null.

Steps 1 to 6 above provide the recipes for calculating the panel group mean FMOLS estimator and test statistics on it. Please refer to Pedroni (2000) for the theorems of consistency and limiting distribution of the panel group mean FMOLS estimator. Please note that in this appendix, we only work on the FMOLS since the DOLS is just the parametric counterpart of the FMOLS and therefore the same principle applies. Please refer to Pedroni (2001) for the group mean DOLS estimator.

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