ABSTRACT

Title of dissertation: DESIGN AND ANALYSIS OF
VEHICLE SHARING PROGRAMS:
A SYSTEMS APPROACH

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Transit, touted as a solution to urban mobility problems, cannot match the addictive flexibility of the automobile. 86% of all trips in the U.S. are in personal vehicles. A more recent approach to reduce automobile dependence is through the use of Vehicle Sharing Programs (VSPs). A VSP involves a fleet of vehicles located strategically at stations across the transportation network. In its most flexible form, users are free to check out vehicles at any station and return the vehicle at a station close to their destination. Vehicle fleets are comprised of bicycles, cars or electric vehicles. Such systems offer innovative solutions to the larger mobility problem and can have positive impacts on the transportation system as a whole by reducing urban congestion. This dissertation employs a network modeling framework to quantitatively facilitate design and operate VSPs. At the strategic level, the problem of determining the optimal VSP configuration is studied. A bilevel optimization model and associated solution methods are developed and implemented for a large-scale case study in Washington D.C. The model explicitly considers the

intermodalism, and views the VSP as a 'last-mile' connection of an existing transit network. At the operational level, by transferring control of vehicles to the user for improved system flexibility, exceptional logistical challenges are placed on operators who must ensure adequate vehicle stock (and parking slots) at each station to service all demand. Since demand in the short-term can be asymmetric (flow from one station to another is seldom equal to flow in the opposing direction), service providers need to redistribute vehicles to correct this imbalance. A chance-constrained program is developed that generates least-cost redistribution plans such that most demand in the near future is met. Since the program has a non-convex feasible region, two methods for its solution are developed. The model is applied to a real-world car-sharing system in Singapore where the value of accounting for inherent stochasticities is demonstrated. The framework is used to characterize the efficiency of *Vélib'*, a large-scale bicycle sharing system in Paris, France.

DESIGN AND ANALYSIS OF VEHICLE SHARING PROGRAMS: A SYSTEMS APPROACH

by

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Chapter 1: The Thesis

Vehicle sharing programs (VSPs) have been gaining ground around the world for providing an environment-friendly, socially responsible and economical mode of transport. These programs involve a fleet of vehicles positioned strategically at stations across the transportation network. In its most flexible form, users are free to lease a vehicle to complete a trip and drop the vehicle at a station close to their destination. The shared vehicle fleet can be comprised of cars, electric vehicles or bicycles. Such systems offer innovative solutions to the larger mobility problem and can have positive impacts on the transportation system as a whole by affecting modal choice. They do so in multiple ways. For short trips, these systems can be construed as an alternate mode of transport. Users enrolled in sharing programs have been shown to undertake fewer trips [46]. When viewed in conjunction with transit networks, a VSP can serve to increase transit use.

Compared to the automobile, transit services in the United States (US) do not enjoy high levels of patronage, in part due to low accessibility, and lack of flexibility and convenience. The US has also been behind the trend in transit adoption when compared with other industrialized countries. VSPs have the potential to improve the accessibility of a public transit system by offering a competitive 'last mile' connection, the lack of which dissuades potential transit riders. The effective catchment area of a transit line is increased by providing a vital leg of an intermodal route. By transferring control of vehicles to the user, the system becomes vastly more flexible, offering more choices with regard to departure time, destinations and transit routes.

These projected improvements can serve to attract new transit ridership.

Vehicle sharing arose from social experiments in sustainable transportation and now finds willing private participants (even a company like Exxon Mobil has a pilot project in Baltimore City called AltCar [25]). Public agencies positioned to leverage this increased interest from the private sector profit, because this form of transportation provides net social benefit. The predominant revenue source for car and electric vehicle programs are from customer usage fees, while advertising revenue supports bicycle sharing programs. As opposed to central, structured, resource-intensive solutions that are typically employed to alleviate congestion, proponents of vehicle sharing schemes claim it offers a distributed, unstructured, sustainable, and economical solution. In essence, VSPs provide a market opportunity for private entities that aid in achieving public goals of affecting a modal shift from road to transit. To maximize transit mode share, a public-private partnership can be forged that shares the same objective of increased ridership. Several design decisions are critical for the success of such systems (e.g. pricing, station locations, fleet composition and size). Determining the optimal system configuration is vital to their success.

This dissertation formulates models and develops associated solution algorithms that quantitatively facilitate design, assessment, and operation of vehicle sharing systems based on probabilistic information on demand, modal preferences, and the existing transit system configuration. A network modeling framework is employed as a basis to support the formulation and analysis.

The principle components of the research include the following. The key de-

mand and supply processes, and their interaction, are characterized. The VSP design is approached from the perspective of intermodalism, whereby the existing transit network is viewed as the backbone with VSPs serving as a 'last-mile' connection. Fleet management strategies for system operators are devised that account for the stochastic nature of demand. The tradeoffs between level-of-service and operating costs are examined within the context of a specific strategy. Critical design parameters are evaluated for real-world systems under a wide range of scenarios to assess performance.

1.1 Motivation and Background

Transit, touted as a solution to urban mobility problems, cannot match the addictive flexibility of the automobile. 86.5% of all trips in the U.S. are in personal vehicles [55]. Public transit does not compare favorably when attributes that influence mode choice (travel times, costs, accessibility, convenience and flexibility) are considered and accounts for just 1.5% of all trips and 4.7% of all commute trips. Additionally, the majority of federal investment in transportation infrastructure is used for capital improvements to existing highways. Despite the increasing disutility of automobiles (worsening congestion, price of gasoline and the supply of infrastructure that has been outpaced by demand), transit continues to be underappreciated as a viable alternative [52].

Transit is efficient and offers low negative socio-environmental external costs (defined as the collective adverse impact of transport not borne by the user, e.g. pol-

lution) [20]. Planners have strived to improve transit patronage through measures that increase transit utility by alleviating its shortcomings. To increase accessibility, transit is integrated with other modes, such as cars (in the form of park-and-ride), bicycles [54], and other slow mode connections [42, 47]. The emphasis on the interface of transit to other modes has been demonstrated to increase catchment areas of transit lines and provide viable intermodal solutions.

A more recent approach to reduce automobile dependence is through the use of VSPs. For a user, a shared vehicle reduces cost of ownership minimally impacting flexibility. Vehicles, viewed as a resource, spend most of their time idle and depreciating in value. More efficient use of this resource implies lower costs. Shared vehicle fleets offer a mechanism for exploiting this down-time. In addition to cost benefits to individual users, there are system-wide benefits from a decrease in motor vehicles in the system (recent European study estimates one shared vehicle leads to a reduction of between four and 10 privately owned cars; estimates for North America range from six to 23 cars [45]). These programs typically have pricing structures that charge based on usage, which has been shown to reduce travel amongst participants [46]. In essence, a smaller fleet of vehicles offers a comparable level of service to users as vehicle ownership, and provides system-wide benefits.

Users around the world have found VSPs to be profitable. As of 2007, car sharing programs exist in 600 cities around the world [45], and bicycle sharing programs in 102 cities [12]. By membership, North America accounts for 35% of car sharing programs worldwide. Vélib', the bicycle sharing program in Paris, has 20,600 bicycles spread over 1,450 stations across the city with an average of 120,000

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trips daily [21].

VSPs build on para-transit concepts of on-demand 'peer-to-peer' transport with flexible routes and no schedules. The principle difference is that VSPs transfer control of vehicles to users. Para-transit aims, as articulated in [30], to be an intermediate solution between the flexibility of automobiles and the rigidness of transit. A unimodal view of paratransit is espoused by Kirby [30] as shown in Figure 1.1, where the appeal of different modes is limited to specific distance windows. When the true intermodal nature of transport is considered, these limitations are not insurmountable. These limitations depend solely on the ease with which users can interface between different modes.
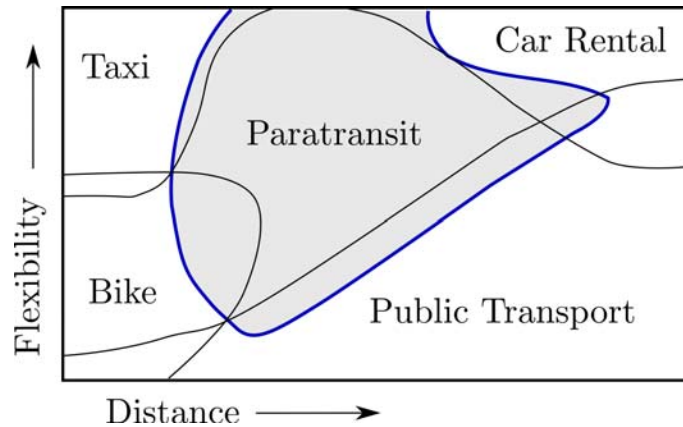
Figure 1.1: A Unimodal View of Urban Transport, adapted from [30]

The weak enthusiam for transit and other non-automobile based modes is partly a result of an auto-centric culture. The true cost of automotive travel is not borne by drivers. Several cost components can be shown to be effectively subsidized. One example is parking, where business owners are forced to offer subsidized (or free) parking to lure customers, recuperating the costs through higher commodity prices

[57]. These factors make the transport market an uneven playing field. With fewer resources, transit operators are expected to match the level-of-service of automobiles. VSPs offer a mechanism for private players to participate in aiding public goals of modal shift. As on-demand feeder services, VSPs can help realize the economies of scale that transit offers.

Earlier generations of VSPs, mainly bicycles, were plagued by theft and vandalism. VSP operators now have substantial Information Technology (IT) infrastructure for various functions, including tracking of vehicles for theft prevention, smart cards for member access, vehicle availability across the network, charging consoles for electric vehicles, payment systems, and online traveler information services. This data-rich environment provides a real-time awareness of the system that can be leveraged for better fleet management. These technologies, taken together, have proven to be a great enabler. System managers have real-time information on the location of the fleet, and users can be made more accountable (though the Vélib' in Paris reports losing 3,000 bicycles annually, or around 15% of its fleet). This reliance on real-time information lends the system to data-intensive operations research methods to provide decision support. Thus, technology is a key driver of current innovations.

The literature on VSPs is dominated by qualitative demand-side studies that predominately aim at market potential and feasibility aspects of the system. These studies are discussed in relevant chapters. The supply side of the equation relating to design and operations has by contrast received very limited attention. The tools developed herein address this gap in the literature. From a theoretical perspective,

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the relevance of this research is in analyzing innovative transport solutions that fall outside the traditional realms. From a societal perspective, as urban communities grapple with finding ways to provide efficient, sustainable mobility to their populace, the developed tools provide a quantitative framework for the analysis of one strategy, namely the shared-vehicle program.

1.2   Research Objectives

Driven by the gap in literature, unique challenges associated with VSPs, the need for sustainable urban mobility solutions, and the increased interest in VSPs across the world, this research has the following objectives.

Address vital strategic network design needs of VSP operators. The research seeks to determine the optimal VSP configuration. Two distinct perspectives underline the model development. From the perspective of users seeking to optimize travel itineraries, the sharing program must offer value-added service, either as a mode or by making transit more attractive by serving as a 'last-mile' connection. From the perspective of VSP service providers who benefit from increased usage, resources are directed to areas with a critical mass of users that warrant the investment.

Develop operational strategies for VSPs. From an operational standpoint, the operator reliquishes control of vehicles to users (who decide where to check out vehicles, how long to use them, and where they are returned). This makes the system highly uncertain and places exceptional logistical challenges on operators. This research seeks to answer the following key questions. (1) Given the uncertainty

of user demand, what operational strategies ensure adequate levels-of-service? (2) What fleet management strategies can minimize cost to operators?

Develop a network-based conceptual framework for VSP strategic design and operations. Mathematical models for the identified problems are formulated that account for inherent uncertainty, multiple stakeholder objectives, and equilibrium conditions, and explicitly model intermodalism. The models yield (1) the optimal VSP configuration (locations of stations, vehicle inventory, and station sizes), and (2) operational fleet management plans that aid in maintaining desired level-of-service at minimal cost to operators.

Develop exact and heuristic solution approaches for identified problems. The developed bilevel and chance-constrained programming models are computationally challenging to solve. Several exact algorithmic approaches and a meta-heuristic scheme are developed for their solution.

Demonstrate applicability of developed models and solution algorithms to real-world instances. The developed methodologies are implemented for several real-world case studies and on synthetic networks to demonstrate the value of a network optimization based approach to analyze VSPs. The strategic network design model is implemented for a large-scale model in Washington D.C. and several synthetic networks. The operational model is implemented for a car-sharing system in Singapore, and a bike-sharing system in Paris.

1.3 Specific Problems Addressed

1.3.1 Strategic Level: Equilibrium Network Design

Taking the perspective of a VSP service provider working with a public transit agency to increase transit mode share, the primary aim of this work is to determine optimal configuration of VSP resources (in terms of where to locate stations, the size of stations, and the vehicle fleet) to increase transit share. To quantify the effects of VSPs, a network representation of the system and models of underlying processes that govern mobility within the system are conceptualized. The principle components of this framework, models of the demand-side process, and supply of transport (links, modes and interfaces) are developed. Since response of potential users to innovative transport solutions is inherently uncertain, simple quantative measures are used to depict preferences of system users.

A bilevel, mixed-integer, equilibrium network design model is developed to determine the optimal VSP system configuration. The model uses a leader-follower framework to consider the differing objectives. Within this framework, the VSP operator (leader) provides the system configuration that users (followers) utilize to improve their travel utilities. In the overall network modeling framework, the presence of a sharing program alters the supply conditions and, therefore, demand for the service is revised. This supply-demand interaction iteratively equilibrates to a VSP design that supports the estimated demand based on the offered level-of-service. The optimal VSP configuration, therefore, represents a supply-demand equilibrium, where the VSP configuration (supply) supports its utilization (demand). The de-

sign of a VSP pertains to location and size of stations and estimation of resource requirements to achieve target level-of-service.

Bilevel programs, in general are computationally challenging to solve due to a non-convex feasible region. An exact technique is proposed that uses the Karush-Kuhn Tucker conditions of the lower-level to transform the problem to a generalized linear complementarity problem. Using additional transformations, the program is converted to a mixed-integer program that is more readily solvable by existing optimization suites.

1.3.2   Operational Level: Fleet Management

Operational fleet management strategies under demand uncertainty are developed. Specifically, a mixed-integer, chance-constrained, stochastic program for anticipative fleet redistribution in VSPs is formulated that generates least-cost fleet plans such that most near-term demand is met. By transferring control of vehicles to the user, exceptional logistical challenges are placed on operators who must ensure adequate vehicle stock (and parking slots) at each station to service all demand. Since demand in the short-term can be asymmetric (flow from one station to another is seldom equal to flow in the opposing direction), service providers need to redistribute vehicles to correct this imbalance. The developed model accounts for demand uncertainty and generates partial redistribution plans when resources are insufficient.

Chance-constrained programs are difficult to solve due to a non-convex feasible

region. Two exact solution techniques are developed that exploit the discrete-valued nature of demand using the theory of p-efficient points. A equal failure apportionment bound is also presented.

1.3.3 Case Studies

The strategic and operational models are implemented for several synthetic networks and three real-world case studies. The strategic network design model is employed to aid the District of Columbia (D.C.) Department of Transportation in expanding their bicycle sharing program. The operational model is employed on a carsharing system in Singapore. A framework used for the operational model is used to characterize *Vélib'*, a large-scale bicycle sharing system in Paris. These real-world case studies demonstrate the applicability of developed models and methods to real-world instances.

1.4 Contributions

This thesis makes the following contributions to the literature on sustainable transportation solutions for urban mobility problems.

At the *strategic level*, a network modeling framework for the discrete equilibrium network design problem is formulated as a mixed-integer bilevel program. This framework explicitly considers the modal interface of VSPs with other modes. Bilevel programs are intractable and their solution presents a significant computational hurdle. An exact solution approach is developed that exploits the convex-

ity of the lower level. The Karush-Kuhn Tucker conditions are used to transform the bilevel program into an mathematical program with equilibrium constraints (MPEC). Using an additional transformation, the MPEC is transformed to a mixed-integer linear program that can be readily solved through the application of off-the-shelf optimization suites such as CPLEX. This exact solution approach is tested on several synthetic networks. A metaheuristic approach using concepts of genetic algorithm is developed for large networks and is applied to aid the D.C. Department of Transportation in designing their bicycle sharing program expansion.

At the *operational level*, a chance-constrained stochastic program to generate fleet management strategies is developed. When resources are insufficient, the program recovers partial redistribution plans that utilize the available resources in the most efficient manner. Two solution algorithms are developed for exact solution of the problem. The first method solves multiple MIPs for special realizations of the random vector called p-efficient points (PEP). This method relies on the enumeration of PEPs where a novel divide-and-conquer PEP generation is presented, a contribution that transcends the current application. A second cone generation method employs concepts from the column generation approach in linear programming to probabilistically constrained programs. The method uses a slave optimization model to determine promising PEPs to consider in the master problem.

Three large-scale, real-world case studies are implemented for the proposed models to demonstrate the efficacy and applicability of the conducted research. The strategic network design model is implemented for a new bike-sharing system in Washington, D.C. The model outputs a near-optimal VSP configuration, quanti-

fying the flows between stations and the improvement in transit accessibility for various neighborhoods within the District. The forecast inter-station flows and effects on transit accessibility are measures that have not been previously studied in the context of VSPs and they are studied herein.

The operational model is implemented for a car-sharing system in Singapore, and the framework used to analyze a bicycle sharing system in Paris. A simulation framework is developed for the analysis of various redistribution plans. The benefit of accounting for inherent uncertainty ('value of stochastic solution') is demonstrated by a comparative analysis with static methods.

1.5   Dissertation Outline

Chapter 2 describes the strategic equilibrium discrete network design problem. Chapter 3 presents the fleet management models and the case study for the Singapore carsharing system. Chapter 4 documents the design of a bicycle sharing system for Washington, D.C. Chapter 5 presents the application of the fleet management model to characterize a large-scale bicycle sharing system in Paris. The research conclusions are summarized in Chapter 6.

Chapter 2: System Design

2.1 Introduction

Since established transport solutions are unsustainable, changes in the way transport is supplied and consumed are imminent. Planners have long embraced the idea that transit will solve urban mobility woes. However, data on users travel choices shows transit to be a losing proposition [55]. Transit simply cannot match the flexibility of the automobile. In recent years, shared-vehicle systems have garnered the interest of urban communities as an integral component in the basket of mobility solutions for the future.

The focus of this chapter is on the design of flexible shared-vehicle systems that allow users to check out vehicles (bicycles, electric vehicles, or cars) close to their origins and drop them off at VSP stations near their destinations. The system is envisioned to be utilized in two ways. For short trips, the shared-vehicles serve as an individual modal alternative. For longer trips, they serve as a vital leg of an intermodal route. In the latter case, VSPs increase travel utility by improving flexibility, offering greater departure time choice, and increasing transit accessibility. The existing transit network is viewed as the backbone of a hierarchical network, with the shared-vehicle system serving as a feeder system (see Figure 2.1). When viewed in conjunction with transit networks, shared-vehicle systems offer a competitive 'last-mile' connection, the lack of which dissuades potential riders.

For the purposes of this chapter, the term 'VSP operator' is designated as
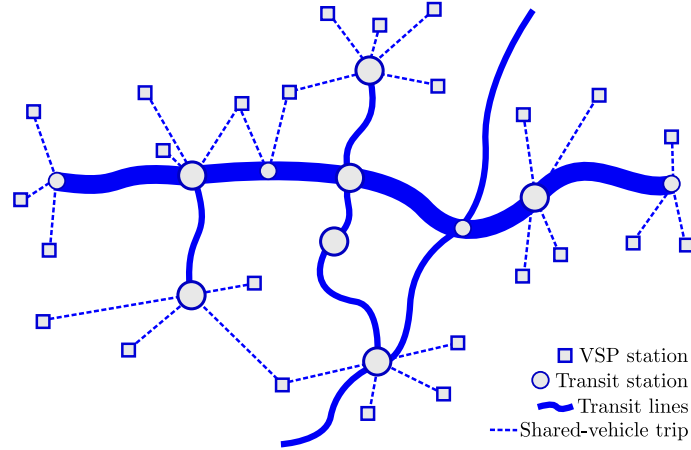
Figure 2.1: Hierarchical transit-VSP network

the entity responsible for designing the VSP system. In practice, the organizational structure behind the design and operation of VSPs varies considerably for different cities. The primary stakeholders involved with the design tasks can include public transportation agencies, public transit authorites, non-profit organizations, private for-profit companies, and advertising companies. Local transportation agencies typically initiate the process by developing operating standards. These standards can pertain to location of VSP stations, shared-vehicle inventories, integration of payment systems, and desired level-of-service. The agency may then work with external service providers (non-profit entities, commercial providers, advertising agencies) to build out the system and operate it. These external service providers are paid through revenue-sharing agreements with the city or through other means, such as advertising rights at VSP stations and vehicles. Another business model involves private companies solely determining the configuration of the system, choosing to provide services where they may be most utilized. Given the myriad forms in which the organizational roles can be structured, the VSP operator can therefore be either

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a public agency, or a private participator.

VSP operators want to decide (1) where to locate stations, (2) how many slots to locate, and (3) how many vehicles to place at each station. In this chapter, a bilevel programming model is developed that optimally determines a VSP configuration. The model recognizes the operator's lack of control over the utilization of the system, since the usage of the VSP is driven by myopic decisions on the part of patrons who seek to maximize their travel utility. The framework also incorporates the intermodal nature of transport and links the shared-vehicle system to existing transit. At the upper level, the VSP operator determines the optimal configuration of the system (supply). At the lower level, users respond to the VSP configuration and optimize their personal itineraries (demand). The VSP operator in turn adjusts the VSP configuration to maximize ridership. At equilibrium, the optimal configuration is one that supports travellers who derive utility from using the shared-vehicle to complete trips. Since bilevel programs are computationally challenging to solve, two solution approaches, one exact and the other heuristic, are developed. The exact technique exploits the convex nature of the lower level problem to derive a large mixed-integer program (MIP) that can be solved using existing MIP solvers. The second meta-heuristic based approach that is developed is suitable for the solution of large instances, as would arise in the real-world. It is presented in Chapter 4.

The literature on design of shared-vehicle systems is very limited. Many studies focus on the qualitative characteristics that aim at determining the feasibility of such systems [23, 53] and the market potential based on surveys and demographic information [28, 46]. Awasthi et al. [3, 4] present a multicriteria decision mak-

ing tool based on the analytic hierarchy process (AHP). Their framework relies on experts ranking stations based on various criterion, including landuse, population density, vehicle ownership, transit access, and presence of target groups. Another pertinent aspect of the literature is the integration of transit with other modes for improving mobility. Several works have studied the role of 'feeder' modes to transit lines [42, 50, 48, 47, 54]. The main focus of these studies have been in integrating bicycles and other slow modes with transit to provide better accessibility. From a conceptual standpoint, the developed model is strongly aligned with the access network design problem, which arises in other sectors, such as telecommunications [5, 11], computer networks [37], and capacity expansion problems in various industries. These previous studies are tailored to problems where the same entity allocates the resources and then determines how it is utilized. In the case of VSPs, however, the operator who designs the network has little control over the fleet of vehicles. Users independently decide where to check out vehicles, the duration of the trip, and where to return them. The control of the resource effectively rests in the hands of users. This important distinction precludes the use of previously proposed models and warrants the development of problem specific tools.

It appears that no previous work has studied the design of shared-vehicle systems from a quantitative, network-based approach. The main contribution of this work is developing a quantitative framework for designing a shared-vehicle system that works in conjunction with existing transit services and infrastructure. While the high level of flexibility of VSPs is a boon for users, operators often rely on qualitative information to decide on the best utilization of their shared-vehicle fleet. The

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work presented herein closes this gap in the literature by providing a quantitative framework for designing shared-vehicle systems.

2.2 Formulation

Given (a) an existing transit network configuration, (b) a set of candidate sites for sharing stations, (c) resource and other site and equity constraints for the VSP operator, (d) behavioral assumptions of transit riders, and (e) static demand, we seek an optimal VSP system configuration that maximizes ridership on the shared-vehicle system.

The VSP system configuration is defined by where VSP stations are located, the capacity (or slots) at each station and the number of vehicles at the station. The role of VSPs within the overall context of urban transportation is explicitly considered. While for shorter trips VSPs can be construed as a modal alternative, longer intermodal trips can involve the use of shared-vehicle segments for access to transit-based services. Since users act independently of the VSP operator, a leader-follower framework is developed to model the differing objectives. The resulting optimal VSP configuration respresents a supply-demand equilibrium, where the VSP configuration (supply) supports its utilization (demand). The framework relies on a network representation of demand and supply processes which are described next.

2.2.1 Supply Processes

To explicitly consider the interaction of VSPs with transit, the network representation of supply stems from the unique characteristics of transit systems. Transit networks possess characteristics that differ from highway networks mainly due to the manner in which users navigate through the system. A transit network must account for walk access, waiting at transit stops, and intermodal paths, and additionally model various services that allow users to reach their destinations. The introduction of a new shared-vehicle system to access the transit network adds an additional means of access. Users moving from origins to destinations could potentially use the VSP to complete their trips should the VSP provide greater travel utility than using transit. Alternatively, users could couple the VSP with transit to generate an intermodal route that maximizes travel utility. This coupling is achieved through the provision of efficient modal interfaces between VSP and transit systems. In practice, a quick transfer from VSP to transit would imply physical proximity of shared stations to transit stations, unified payment systems for VSP and transit for quick transfers, and adequate VSP station capacity near transit stations to ensure level-of-service. The resulting network has a hierarchical structure as shown in Figure 2.2.

The network is represented by a graph $G(V, A)$, where V is a set of nodes and A is a set of arcs connecting nodes of the network. The set A includes a subset of non-frequency based arcs denoted by $\underline{A}$. These include all sharing links, transfer links, and walk links. For frequency-based links that represent transit services, each
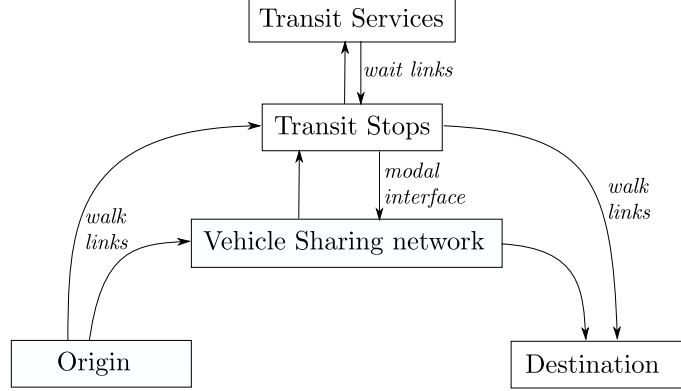
Figure 2.2: Network configuration

link has an associated frequency parameter f_{ij}, $(i,j) \in A \setminus \underline{A}$. Each arc $(i,j) \in A$ is characterized by a pair (c_{ij}, g_{ij}), where c_{ij} is a nonnegative travel time (or cost) and g_{ij} is the distribution function for the waiting time for transit service [51]. For arcs where no waiting is involved, $g_{ij} = 0$.

A VSP operator aims to configure a shared-vehicle system around existing transit lines. The graph $G(V, A)$ includes all candidate sites that a VSP operator considers. The sharing sub-network is denoted as $G_s(V_s, A_s)$, where V_s is the set of candidate sharing sites and $V_s \subseteq V$. Since users of shared vehicles can potentially return vehicles at any station, G_s is a strongly connected graph. The shared-vehicle edge set A_s can be construed in two ways, either as having a single arc for each pair of sharing stations, or as a set of arcs that link each sharing station to the transport network. Similar to each arc in G, each arc in the sharing network G_s has an associated travel cost (or time).

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2.2.2 Demand Processes

The core of any system designed to provide and improve mobility is its users. Users are assumed to behave in a myopic manner to maximize their personal travel utilities. Demand for VSP services is determined by value addition of shared-vehicles to existing routes. Should the VSP configuration provide users with improved trip times (or lower costs), the VSP would attract trips. From a modeling perspective, the demand aspect is incorporated by explicitly considering this utility-maximizing user behavior. However, two considerations limit the sophistication with which user behavior can be incorporated. First, VSPs are a novel and innovative mobility solution. The reaction of users to innovation is inherently uncertain and difficult to ascertain. To this extent, the user reaction is based solely on trip attributes of travel time and cost, while the alternative specific utility (or disutility) is omitted. In the context of mode choice models, alternative specific utilities quantify the innate preferences of users for specific modal alternatives. Secondly, with the inclusion of transit networks, a model of how users navigate through transit is needed. Over the past few decades, several works have focused on transit assignment problems that deal with this issue. More recent developments such as those that apply optimal strategies [51], hyperpaths [35, 24, 59, 33], and dynamic intermodal paths [60, 32], incorporate the probabilistic aspect by which users choose transit services. These models aim to increase the realism of transit-based trips, where waiting users may adapt paths during travel to take advantage of more frequent services.

In this chapter, the *optimal strategy* concept of Spiess and Florian [51] is

extended to include the VSP component. An optimal strategy is defined by a set of arcs denoted by $\bar{A}$. A strategy can include any number of simple paths and defines elements of the user's route/service choice until the destination is reached. Users waiting at a transit stop with multiple service lines choose the service that is more frequent, thereby reducing waiting time. This approach is used in state-of-the-practice transit assignment software, such as EMME/2. The problem of finding an optimal strategy can be formulated as a linear program. In this work, the optimal strategy is developed that considers VSP access links and capacity constraints on the VSP portion of the strategy.

2.2.3 Model Development

With a network representation of supply and the characterization of demand through optimal strategies, the problem faced by the VSP operator is formulated. The VSP operator has at its disposal shared-vehicle resources that need to be allocated to various parts of the existing network. A 'good' allocation leads to better utilization of VSP resources and presumably more revenue for the operator. However, VSP utilization is governed by users who will employ shared-vehicles in their trips only if there is increase in travel utility.

The model uses a leader-follower paradigm to accomodate differing objectives and the VSP operator's lack of control over system resources. VSP operators take on the role of *leaders* supplying a VSP configuration (VSP stations and vehicles). Users, or the *followers*, respond to the VSP configuration by adjusting travel itineraries.

The leader can then adjust the configuration to maximize their objective, which can result in new flow patterns of users. At an optimal solution, equilibrium is reached between the objectives of the leader and followers. For the VSP operator, the equilibrium solution represents a configuration that is designed for flows that users will generate following their myopic objectives.

From the set of candidate sites, the VSP operator must decide where to build new VSP stations. Additionally, the operator must determine the capacity of each built station and the base inventory of vehicles at each station that is available at the start of any time period. The VSP operator's decision variables are x_i, a binary variable on whether or not to build at site $i, i \in V_s$; y_i, an integer variable on the capacity at node i; and z_i, the number of vehicles located at i. The location and capacities of shared-vehicles are determined by equilibrium flows, which in turn are determined by where the shared-vehicles are located. A shared-system configuration is characterized by the tuple (x, y, z).

The upper-level problem faced by the VSP operator is to determine the optimal VSP configuration (x, y, z) such that the shared-vehicle flows are maximized. The VSP operator is subject to site limitations for each candidate site, budget constraints that limit the number of stations that can be built, and additional political or equity constraints. The total available budget is denoted by C. The cost components include station setup costs c_s, additional cost for an additional parking slot c_p, and unit cost of a vehicle c_v.

The flows of shared-vehicles are determined by the lower-level problem by users who have different objectives from the upper-level VSP operator. Users traveling

between various origin-destination (OD) pairs minimize their travel and waiting time. OD pairs are denoted by the set K (indexed by k). A link from node i to j has an associated flow for each OD pair denoted by v_{ijk}. Additionally, each node of the transit network has an associated quantity w_{ik}, representing the total waiting time experienced by users in OD pair k at node i. The flows (v) and waiting times (w) are continuous decision variables of the lower-level model. The notation used is summarized next.

	Sets
V	Set of nodes, includes candidate sharing stations
V_s	Set of candidate sharing stations
A	Set of all arcs
A_s	Set of all sharing arcs
$\underline{A}$	Set of all non-frequency arcs (e.g. walk access)
K	Set of OD pairs

	Indices
i, j	Index to set of nodes V or V_s
k	Index into set of OD pairs K
(i, j)	Index to any set of arcs

	VSP operator inputs
c_s	Fixed costs associated with opening one station
c_p	Incremental cost of providing a vehicle slot

c_v	Cost of one vehicle
C	Total budget
y^{ub}	Maximum allowable capacity of sharing station due to site limitations

VSP operator decision variables

x_i	Binary variable indicating if station is located at node i
y_i	Capacity of station at i in terms of number of slots
z_i	Number of vehicles located at node i

Lower level parameters

c_{ij}	Cost (or time) incurred in traversing arc (i, j)
g_{ik}	Demand from node i to destination node associated with OD pair k
f_{ij}	Frequency parameter associated with transit service link (i, j)
a	Checkout-returns replacement ratio. $a = 1$ implies avaiable capacity must meet both checkouts and returns. $a > 1$ implies returns may exceed capacity in anticipation of future checkouts.

Lower level decision variables

v_{ijk}	Flow on link (i, j) for OD pair k
w_{ik}	Total waiting at node i from users on OD pair k

Other

M	A large constant

$\alpha, \beta, \gamma,..$ Dual variables for lower level problem

$u_.^l$ Binary decision variable used to model complementarity constraints

With these preliminaries, the bilevel network design problem (BLNDP) can be formulated as follows.

Upper level:

$$\max_{x,y,z} \sum_k \sum_{(i,j) \in A_s} v_{ijk} \tag{2.1}$$

subject to

$$\sum_{i \in V_s} c_s x_i + c_p y_i + c_v z_i \quad \leq \quad C \tag{2.2}$$

$$Mx_i \quad \geq \quad y_i \qquad i \in V_s \tag{2.3}$$

$$z_i \quad \leq \quad y_i \qquad i \in V_s \tag{2.4}$$

$$y_i \quad \leq \quad y^{ub} \qquad i \in V_s \tag{2.5}$$

$$x_i \quad \in \quad \{0, 1\} \tag{2.6}$$

$$y_i, z_i \quad \in \quad \mathbb{Z}_+^n \tag{2.7}$$

lower-level:

$$\min_{u,v} \sum_k \left(\sum_{(i,j) \in A} c_{ij} v_{ijk} + \sum_{i \in V} w_{ik} \right) \tag{2.8}$$

subject to

$$\sum_{j,(i,j)\in A} v_{ijk} - \sum_{j,(j,i)\in A} v_{jik} \;=\; g_{ik} \qquad i \in V,\, k \in K \qquad (\alpha_{ik}) \qquad\qquad (2.9)$$

$$v_{ijk} \;\leq\; f_{ij}w_{ik} \qquad (i,j) \in A \setminus \underline{A},\, k \in K \qquad (\beta_{ijk})\ (2.10)$$

$$Mx_i \;\geq\; \sum_k v_{ijk} \qquad (i,j) \in A_s \qquad (\gamma_{ij}) \qquad\qquad (2.11)$$

$$Mx_j \;\geq\; \sum_k v_{ijk} \qquad (i,j) \in A_s \qquad (\delta_{ij}) \qquad\qquad (2.12)$$

$$\sum_k \sum_{j,(i,j)\in A_s} v_{ijk} \;\leq\; z_i \qquad i \in V_s \qquad (\zeta_i) \qquad\qquad (2.13)$$

$$\sum_k \sum_{j,(j,i)\in A_s} v_{jik} \;\leq\; a(y_i - z_i) \qquad i \in V_s \qquad (\eta_i) \qquad\qquad (2.14)$$

$$w_{ik} \;\geq\; 0 \qquad i \in V,\, k \in K \qquad (\lambda_{ik}) \qquad\qquad (2.15)$$

$$v_{ijk} \;\geq\; 0 \qquad (i,j) \in A,\, k \in K \qquad (\mu_{ijk}) \qquad\qquad (2.16)$$

The VSP operator seeks to maximize flow through the sharing network (2.1). The flow, however, is not determined by the operator. The operator provides a network configuration (x, y, z) subject to budget constraint (2.2). Constraints (2.3) state that slots can be available only if a station exists at the site. Constraints (2.4) restrict the number of vehicles assigned to a particular site to be less than the site capacity. The site limitations are stated in constraints (2.5). Constraints (2.6) restrict the location variables x_i to be binary and constraints (2.7) restrict the decision variables y_i and z_i to be integer.

In the lower-level problem, users react to a VSP configuration (x, y, z) by minimizing their travel costs and waiting times (2.8). The set of flow conservation relations for each node are given by constraints (2.9) . Constraints (2.10) relate the travel time and waiting time for frequency based links. The detailed derivation of

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this set of constraints from the optimal strategies concept is presented in Spiess and Florian [51]. Constraints (2.11) and (2.12) restrict flow on sharing arcs that have stations to support the travel. Constraints (2.13) and (2.14) are capacity constraints of the sharing stations. Flow and waiting times must be non-negative as enforced by constraints (2.16) and (2.15). Note that the upper-level formulation has an objective involving lower-level variables. In addition, the dual variables for each of the lower-level constraints are indicated in parenthesis.

The equilibrium network design problem is a bilevel, mixed-integer program. Programs of this class typically have non-convex feasible regions rendering them intractable for most cases. For cases when the lower-level portion of the formulation is convex, transformation techniques have been presented in the literature that convert bilevel programs to mathematical programs with equilibrium constraints (MPEC). The optimal strategies formulation possesses the required convexity property motivating its use in the presented formulation. The lower-level convexity is exploited for an exact solution approach presented next.

2.3   Solution Algorithm

The presented bilevel program is transformed to a large mixed integer program (MIP) that can be solved using off-the-shelf MIP solvers. The transformation outlined in this section exploits the convexity of the lower-level problem.

2.3.1 An Exact Solution Approach

The basic solution concept follows methods proposed in the literature [2, 14]. For a given design tuple (x, y, z), the lower level problem is a program with linear objective and linear constraints. This is equivalent to the optimal strategy program of Spiess and Florian [51] with the additional constraints to model VSP flows and capacities. Since it is a convex program and satisfies linear constraint qualifications (CQ), the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient. The lower level optimization problem can be replaced by the KKT conditions to yield a mathematical program with equilibrium constraints (MPEC). The only nonlinearities are due to the presence of complementarity constraints, which can be transformed to a set of linear disjunctive constraints using auxilary binary variables. The resulting program is a large MIP that can be solved using existing MIP solvers.

2.3.1.1 KKT Conditions for Lower Level

Before deriving the KKT conditions for the lower level program defined by Equations (2.8)–(2.16), the existence of a solution needs to be demonstrated.

Proposition 2.3.1 *If the subgraph $G'(V', A')$, where $V' = V \setminus V_s$ and $A' = \{(i, j) \mid i, j \in V', (i, j) \in A\}$, is strongly connected, the lower level problem defined by Equations (2.8)–(2.16) is feasible for all design vectors (x, y, z).*

Proof If the subgraph $G'(V', A')$ is strongly connected, there exists a feasible transit path from every origin to every destination. Since there are no capacity constraints

on transit links, the optimal strategy for each OD pair is never infeasible, as it can involve at least one feasible transit path. This is independent of the design vector (x, y, z). A set of path flows uniquely determines arc flows (Theorem 3.5 Ahuja et al. [1]), therefore, the lower level program is always feasible. ∎

Using the strongly connected property, the KKT conditions are derived for the lower level program defined by objective (2.8) and constraint set (2.9)–(2.16). For brevity, the conditions are merely stated, since the derivation from the Lagrangian function is straighforward (but tedious).

The stationarity conditions are

$$1 - \lambda_{ik} - A = 0 \qquad i \in V,\ k \in K \tag{2.17}$$

and

$$c_{ij} + \alpha_{ik} - \alpha_{jk} - \mu_{ijk} + B + C = 0 \qquad (i, j) \in A,\ k \in K, \tag{2.18}$$

where

$$A = \begin{cases} \sum_{k \in K} \beta_{ijk} f_{ij} & \text{if} (i, j) \in A \setminus \underline{A} \\ 0 & \text{o.w.} \end{cases} \qquad B = \begin{cases} \beta_{ijk} & \text{if} (i, j) \in A \setminus \underline{A} \\ 0 & \text{o.w.} \end{cases}$$

$$C = \begin{cases} \gamma_{ij} + \delta_{ij} + \zeta_i + \eta_j & \text{if} (i, j) \in A_s \\ 0 & \text{o.w.} \end{cases}$$

The primal feasibility constraints are (2.9)–(2.16). Dual feasibility implies the following.

$$\beta_{ijk} \ \geq \ 0 \qquad (i,j) \in A \setminus \underline{A}, \ k \in K \qquad\qquad (2.19)$$

$$\gamma_{ij}, \ \delta_{ij} \ \geq \ 0 \qquad (i,j) \in A_s \qquad\qquad\qquad (2.20)$$

$$\zeta_i, \ \eta_i \ \geq \ 0 \qquad i \in V_s \qquad\qquad\qquad (2.21)$$

$$\lambda_{ik} \ \geq \ 0 \qquad i \in V, \ k \in K \qquad\qquad\qquad (2.22)$$

$$\mu_{ijk} \ \geq \ 0 \qquad (i,j) \in A, \ k \in K \qquad\qquad (2.23)$$

The set of complementarity conditions are as follows.

$$\beta_{ijk} \left(v_{ijk} - f_{ij} w_{ik} \right) \ = \ 0 \qquad (i,j) \in A \setminus \underline{A}, \ k \in K \ (u^1_{ijk}) \ (2.24)$$

$$\gamma_{ij} \left(\sum_k v_{ijk} - M x_i \right) \ = \ 0 \qquad (i,j) \in A_s \qquad (u^2_{ij}) \qquad (2.25)$$

$$\delta_{ij} \left(\sum_k v_{ijk} - M x_j \right) \ = \ 0 \qquad (i,j) \in A_s \qquad (u^3_{ij}) \qquad (2.26)$$

$$\zeta_i \left(\sum_k \sum_{j,(i,j) \in A_s} v_{ijk} - z_i \right) \ = \ 0 \qquad i \in I_s \qquad (u^4_i) \qquad (2.27)$$

$$\eta_i \left(\sum_k \sum_{j,(i,j) \in A_s} v_{ijk} - a(y_i - z_i) \right) \ = \ 0 \qquad i \in I_s \qquad (u^5_i) \qquad (2.28)$$

$$\lambda_{ik} w_{ik} \ = \ 0 \qquad i \in V, \ k \in K \qquad (u^6_{ik}) \qquad (2.29)$$

$$\mu_{ijk} v_{ijk} \ = \ 0 \qquad (i,j) \in A, \ k \in K \qquad (u^7_{ijk}) (2.30)$$

The MPEC is defined by objective (2.1), upper-level constraints (2.2)–(2.7), KKT primal feasibility constraints (2.9)–(2.16), KKT stationarity conditions (2.18) and (2.17), dual feasibility constraints (2.19)–(2.23), and complementarity constraints (2.24)–(2.30). All functions involved are linear, with decision variables that

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are both integer and continuous. The only problematic nonlinear constraint set is the complementarity constraint set (2.24)–(2.30).

2.3.1.2   Dealing with Complementarity

The complementarity constraints can be made linear by using a binary variable. In general, a complementarity constraint of the form $z(q + Qz) = 0$ can be modeled as two linear constraints using a binary variable, u, as $z \leq Mu$ and $q + Qz \leq M(1 - u)$, where M is a large constant that provably exists [2]. Using this transformation, all complementarity constraints can be reduced to linear constraints as follows.

$$\beta_{ijk} \leq Mu_{ijk}^1 \qquad (i, j) \in A \setminus \underline{A}, \ k \in K \qquad (2.31)$$

$$v_{ijk} - f_{ij}w_{ik} \leq M(1 - u_{ijk}^1) \qquad (i, j) \in A \setminus \underline{A}, \ k \in K \qquad (2.32)$$

$$\gamma_{ij} \leq Mu_{ij}^2 \qquad (i, j) \in A_s \qquad (2.33)$$

$$\sum_k v_{ijk} - Mx_i \leq M(1 - u_{ijk}^2) \qquad (i, j) \in A_s \qquad (2.34)$$

$$\delta_{ij} \leq Mu_{ijk}^3 \qquad (i, j) \in A_s \qquad (2.35)$$

$$\sum_k v_{ijk} - Mx_j \leq M(1 - u_{ijk}^3) \qquad (i, j) \in A_s \qquad (2.36)$$

$$\zeta_i \leq M u^4_{ijk} \qquad i \in V_s \tag{2.37}$$

$$\sum_k \sum_{j,(i,j)\in A_s} v_{ijk} - z_i \leq M(1 - u^4_{ijk}) \qquad i \in V_s \tag{2.38}$$

$$\eta_i \leq M u^5_i \qquad i \in V_s \tag{2.39}$$

$$\sum_k \sum_{j,(i,j)\in A_s} v_{ijk} - a(y_i - z_i) \leq M(1 - u^5_i) \qquad i \in V_s \tag{2.40}$$

$$\lambda_{ik} \leq M u^6_i \qquad i \in V, \, k \in K \tag{2.41}$$

$$w_{ik} \leq M(1 - u^6_i) \qquad i \in V, \, k \in K \tag{2.42}$$

$$\mu_{ijk} \leq M u^7_i \qquad (i,j) \in A, \, k \in K \tag{2.43}$$

$$v_{ijk} \leq M(1 - u^7_i) \qquad (i,j) \in A, \, k \in K \tag{2.44}$$

The BiLevel Network Design Problem (BLNDP) can, therefore, be expressed as a mixed integer program (MIP). The BLNDP is defined by objective (2.1) upper-level constraints (2.2)–(2.7), KKT primal feasibility constraints (2.9)–(2.16), KKT stationarity conditions (2.18) and (2.17), dual feasibility constraints (2.19)–(2.23), and transformed complementarity constraints (2.31)–(2.44). This large MIP can be solved exactly by using existing MIP solvers, such as CPLEX.

The large constant M must be chosen *apriori* and is critical from a computational standpoint. A very large value for M introduces numerical stability issues. An M that is too small risks cutting off the optimal solution. In the implementation, the M was computed as a factor of maximum possible flow times a factor of 2.5.

2.4 Experimental Results

The proposed transformation scheme is implemented in Java and solved using the CPLEX 11.2 solver. The model is tested on five randomly generated transit networks for varying problem parameters. The random networks are built by generating random coordinates for nodes within the unit kilometer interval. Nodes can represent VSP candidate sites, transit stops, transit service, or origin/destination nodes. Links representing transit service, walk access, VSP acess, modal transfers, and origin/destination connectors are generated by randomly connecting pertinent nodes. Each node is assigned a degree greater than two, to ensure that the network is strongly connected. The links are classified into three categories. Walk links are assumed to be traversed at a speed of 4.5 km/h. Shared-vehicle links are traversed at a speed of 12 km/h. Transit service links run at 30 km/h. Users are assumed to minimize travel time in going from origin to destination. Figure 2.3 depicts the five tested networks.

For each instance, the four problem parameters (budget, cost of stations, cost of vehicles, and cost of slots) are varied to test sensitivity of the model. The solution of one run is used as a starting solution of the next to provide a warm start to the CPLEX solver. The solver finds good solutions (MIP gap $< 5\%$) relatively early in the branch-and-bound tree for most instances. However proving optimality for some instances takes considerable time. Therefore, each run is limited to a run time of one hour and the resulting MIP gap is reported.

The main outputs of the models are the VSP design configuration, the shared-

(a) 24 Nodes   (b) 35 Nodes   (c) 46 Nodes

(d) 50 Nodes   (e) 64 Nodes
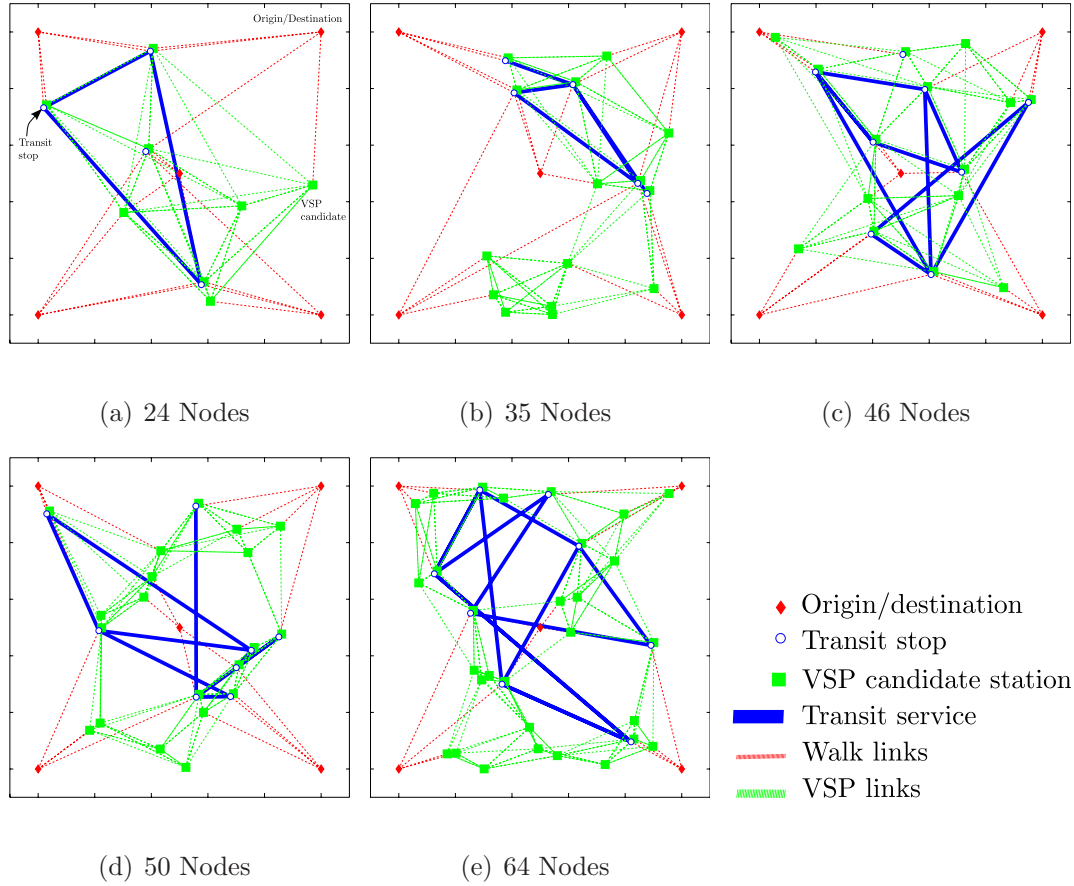
Figure 2.3: Random transit networks tested

flows that result for each budget scenario, and the effect of various cost components on the optimal design. The results of runs are summarized in Tables (2.1)–(2.5). The tables show the input cost vector (c_s, c_p, c_v), along with total budget C for various runs. The summary shows the shared-flow captured increases as a result of larger budgets. The variation of various cost components shows an interesting trend. When station setup costs are high and parking slots costs are comparatively low (as is the case for bicycle sharing systems), more stations are built than compared to the alternate scenario, where the setup costs are low, but costs of parking slots are high (as is the case for car sharing systems). Thus, the higher cost of parking slots

presents agglomeration effects for the system where available resources are pooled
at fewer stations.

c_s	c_p	c_v	C	$\sum x_i$	$\sum y_i$	$\sum z_i$	Shared flow	MIP Gap (%)	Run time (sec)
5	1	2	0	0	0	0	0	-	5.58
			50	2	18	11	10.5	3.343%	35.63
			100	3	39	23	23	2.174%	175.3
			150	4	59	35	35	1.504%	3537.45
			200	6	77	46	46	2.975%	3576.95
			300	8	118	71	70.5	0.426%	13.63
1	4	2	0	0	0	0	0	-	4.5
			50	2	9	5	5	10%	152.17
			100	2	19	11	11	2.02%	623.98
			150	3	28	17	16.5	2.831%	73.47
			200	4	38	22	22	3.238%	3591.67
			300	4	57	34	34	0.452%	320.27

Table 2.1: Results for Instance24 (24 nodes × 129 links)

c_s	c_p	c_v	C	$\sum x_i$	$\sum y_i$	$\sum z_i$	Shared flow	MIP Gap (%)	Run time (sec)
5	1	2	0	0	0	0	0	-	1.52
			50	2	18	11	10.5	4.762%	16.2
			100	3	39	23	23	0.791%	158.59
			150	4	60	35	35	1.504%	3577.91
			200	6	77	46	46	1.902%	3582.02
			300	8	118	71	70.5	0.784%	3592.83
1	4	2	0	0	0	0	0	-	1.52
			50	2	9	5	5	10%	60.97
			100	2	19	11	11	3.567%	3.36
			150	3	28	17	16.5	3.03%	1221.72
			200	4	38	22	22	3.322%	3578.01
			300	5	56	34	33	3.567%	3596.55

Table 2.2: Results for Instance35 (35 nodes × 200 links)

The flow patterns and VSP configuration are graphically summarized in Figure
2.4 for the 46 node instance when the cost of stations $c_s = 5$ and cost of parking
slots $c_p = 1$. The results from other instances follow similar characteristics and are

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c_s	c_p	c_v	C	$\sum x_i$	$\sum y_i$	$\sum z_i$	Shared flow	MIP Gap (%)	Run time (sec)
5	1	2	0	0	0	0	0	-	0.53
			50	2	18	11	10.5	4.762%	81.13
			100	3	39	23	23	2.174%	203.83
			150	4	60	35	35	1.504%	3563.33
			200	6	77	46	46	2.975%	3603.69
			300	8	118	71	70.5	0.784%	3609.06
1	4	2	0	0	0	0	0	-	0.52
			50	3	9	5	5	10.286%	3599.44
			100	2	19	11	11	3.567%	3.27
			150	4	28	17	16.5	3.03%	1152.38
			200	4	38	22	22	3.322%	3612.47
			300	8	56	34	33	3.567%	3610.06

Table 2.3: Results for Instance46 (46 nodes × 221 links)

omitted. The VSP stations chosen for construction are highlighted along with the associated number of slots and vehicles in parenthesis. The VSP configuration can be seen to grow in terms of number of stations when a larger budget is presumed. However, comparing the budget scenarios in Figures 2.4(b) and 2.4(c), though the budget increases by 50 units, the number of stations chosen for construction is reduced. However the capacities at the two stations is augmented so as to maximize shared-flow. When the budget is 300 units, the budgetary constraint is no longer binding, leading to configurations where stations are chosen for construction, but without capacity. The shared-flow corresponding to this budget scenario represents the maximum shared-flow that can be captured.

2.5   Conclusions

The design of a shared-vehicle system in conjunction with a transit network is formulated as a bilevel program. The problem is transformed to a more readily

(a) Budget = 0   (b) Budget = 50   (c) Budget = 100

(d) Budget = 200   (e) Budget = 300
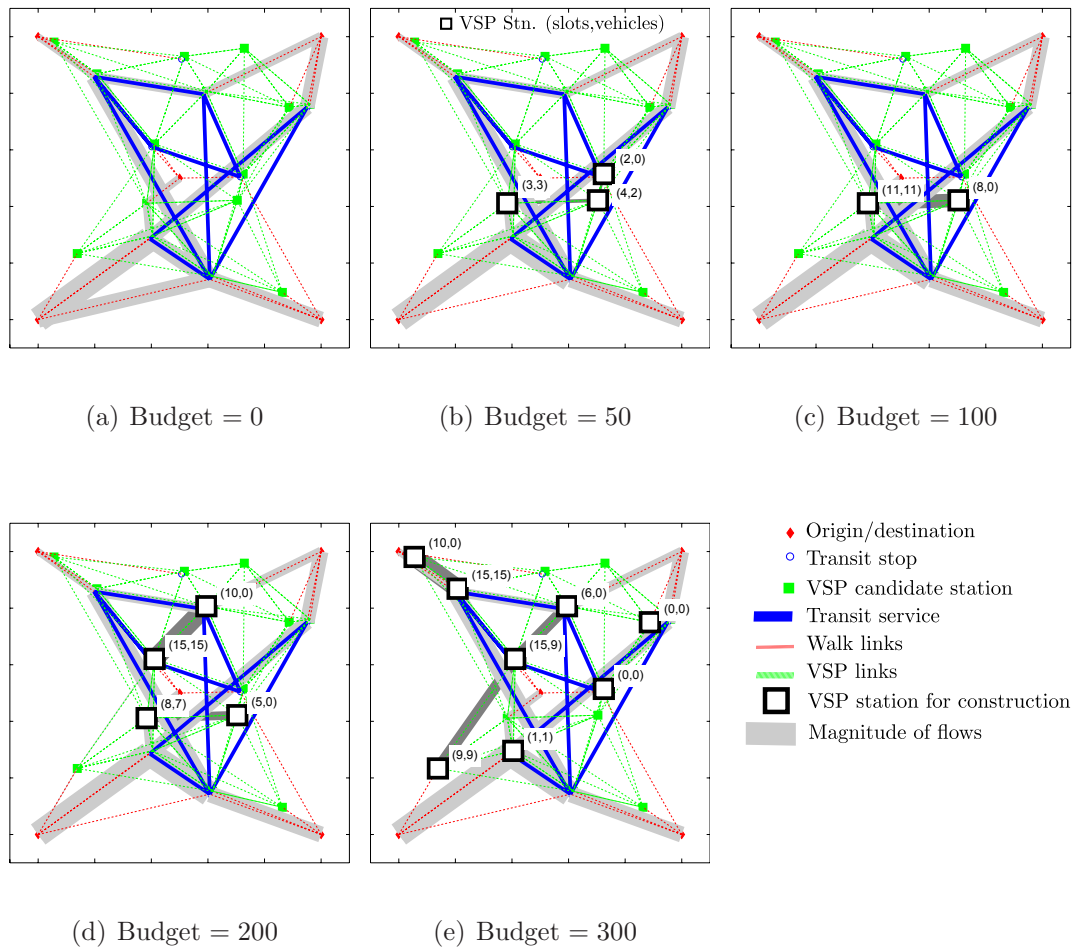
Figure 2.4: Flow patterns and VSP configuration for 46 node instance

c_s	c_p	c_v	C	$\sum x_i$	$\sum y_i$	$\sum z_i$	Shared flow	MIP Gap (%)	Run time (sec)
5	1	2	0	0	0	0	0	-	2.91
			50	2	18	11	10.5	4.762%	345.5
			100	3	39	23	23	2.174%	249.02
			150	5	57	34	34	4.489%	3608.41
			200	6	77	46	46	2.975%	3610.16
			300	9	117	69	69	2.975%	3610.97
1	4	2	0	0	0	0	0	-	2.89
			50	4	6	4	1	465.385%	3613.75
			100	2	19	11	11	3.533%	30.56
			150	3	28	17	16.5	3.473%	3610.61
			200	7	37	22	22	3.567%	3612.42
			300	4	57	34	34	0.521%	211.02

Table 2.4: Results for Instance50 (50 nodes $\times$ 263 links)

solvable mixed integer program. Tests on random networks illustrate the applicability of the model for determining optimal VSP configurations and yield insights into optimal system configurations. These include agglomeration effects for various cost inputs. When station setup costs are low and parking slot costs are high (as is the case in car sharing systems), fewer stations are developed and resources are aggregated. The proposed model incorporates the transit assignment routine of Spiess and Florian, though any transit assignment routine can be explored as alternatives in the lower level of the formulation. The proposed model can be extended along several dimensions. Though the fixed-demand assumption is common in practice for strategic design problems, a framework where variable demand is considered as a function of VSP level-of-service can yield additional insights. Congestion effects on transit networks can be incorporated despite the relatively low volumes of transit flows in the US. An extension of this work could consider joint optimization of prices and design (see [13] for example) and include price sensitivity of users in route

c_s	c_p	c_v	C	$\sum x_i$	$\sum y_i$	$\sum z_i$	Shared flow	MIP Gap (%)	Run time (sec)
5	1	2	0	0	0	0	0	-	1144.78
			50	2	18	11	10.5	4.762%	1719.56
			100	3	39	23	23	2.174%	786.23
			150	4	59	35	35	1.504%	3609.42
			200	6	78	46	46	2.975%	3605.58
			300	8	118	71	70.5	0.784%	3602.3
1	4	2	0	0	0	0	0	-	1149.23
			50	3	9	5	5	13.146%	3613.61
			100	2	19	11	11	3.567%	11.47
			150	4	28	17	16.5	3.475%	3606.94
			200	4	38	22	22	3.497%	3604.56
			300	7	56	34	33	3.497%	3604.25

Table 2.5: Results for Instance64 (64 nodes × 365 links)

choice.

Chapter 3: Operational Models

3.1 Introduction

This chapter deals with operational strategies of fleet management that VSP operators can pursue to maintain desired levels-of-service. To match automobile flexibility, VSPs transfer control of vehicles to users. This places exceptional logistical challenges on operators who must ensure that demand in the near future is met. For the shared-vehicle user, good service is defined by adequate stock of vehicles at intended station of origin and adequate parking slots at the intended destination station. Since flow from one station to another is seldom equal to flow in the opposing direction, the VSP fleet can become spatially imbalanced. To meet near-future demand, operators must then redistribute vehicles to correct this asymmetry. The focus of this chapter is to provide models that generate efficient, cost-effective operational strategies for fleet management.

The management of VSP fleets differs from previously studied models in related areas. These differences preclude the direct application of prior work and motivate the development of problem specific tools. Firstly, since users determine the trip characteristics, critical system attributes (where vehicles are checked out and returned, and the duration of lease) are beyond the control of operators. Secondly, there is a 'duality' of demand between vehicles and slots. A successful trip needs a vehicle available at the origin station and an available docking slot at the destination. A vehicle checkout reduces vehicle inventory at a station, but increases

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the slot inventory. This duality has significant implications and reduces the range of acceptable inventory levels for vehicles. Thirdly, the inventory is never consumed, but is merely moved. The fleet management strategy involves correcting imbalances at the various stations.

Supply-side analyses of VSPs are predominantly qualitative and the literature dealing specifically with fleet management for VSP's is limited. [7] proposed three strategies to generate redistribution plans. These are based on immediate demand, expected demand and perfect demand information. The time period looks 20 minutes into the future and uses simulation to evaluate the redistribution strategies. No details on how redistribution plans are generated are presented. [8] studied the redistribution problem, but attempt to shift the burden of redistribution on users through two mechanisms of ride splitting and joining. [29] used a mixed-integer program (MIP) to generate redistribution plans and allocate operator staff to redistribution and maintenance activities. Their model uses a time-expanded network, with static, known demand. Unserviced demand is penalized by a penalty cost in the objective function. A simulation model is used to evaluate the redistribution strategy. In these works, the redistribution plans are based on static demand.

In this chapter, the problem of fleet management for shared-vehicle systems is formulated considering demand uncertainty (Section 3.2). The management strategy involves anticipative fleet redistribution that operators initiate to correct short-term demand asymmetry (since flow from one station to another is seldom equal to flow in the opposite direction). When operators have inadequate resources to meet demand, then the model generates partial redistribution plans. The model takes a form of

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a stochastic MIP with joint chance constraints. Stochastic programs of this class have non-convex feasible regions. Two solution methods are presented (Section 3.3). When demand at stations is correlated, the first approach deals with the problem non-convexity using the concept of p-efficient points (PEPs) to transform the problem to a disjunctive MIP that is more readily solvable. A divide-and-conquer algorithm to generate PEPs is designed to reduce the computational burden of existing methods (Section 3.3.1). The algorithm can be applied to any problem with joint chance constraints given that the vector of random variables is discrete. This contribution transcends the current application. A second cone generation solution method (Section 3.3.2), akin to the column generation procedure, is developed when the demand at each station is assumed to be independent. Under this limiting assumption, this method provides quick solutions even for shared systems with large numbers of stations. Additionally, an equal-apportionment bound is derived for the problem that is valid even when demand is correlated (Section 3.4). We compare various redistribution strategies in a real-world application to a car sharing system in Singapore (Section 3.5). Extensive computational experiments and simulation studies show that when the redistribution strategies developed herein are employed, the system operates at a reliability level that would otherwise be possible only with capital improvements to the system.

In the next section, the model formulation is outlined. Section 3.2 describes the model formulation and a solution algorithm is presented in Section 3.3. Section 3.4 describes a failure apportionment bound for the model. A real-world application for a system in Singapore is described in Section 3.5 along with results. The conclusions

are presented in Section 3.6.

3.2   Problem Formulation

Given (a) the system configuration (stations, capacities, fleet size), (b) current inventory levels at each station, (c) relocation costs, and (d) a probabilistic characterization of demand at each station, we wish to find a least-cost fleet redistribution plan such that *most* near future demand scenarios are satisfied.

VSP operators have substantial ITS infrastructure for various functions, including tracking of vehicles for theft prevention, smart cards for member access, vehicle availability across the network, charging consoles for electric vehicles, payment systems, and traveler information services. This data-rich environment provides a real-time awareness of the system that can be leveraged for fleet management. Since individual users decide where and when trips are made, demand at each station is uncertain from a system perspective. The aim of operators is to serve all demand. However, it is typically cost-prohibitive to design the system to satisfy all possible demand realizations and operators can expect demand to outstrip supply in high-demand scenarios. By characterizing demand probabilistically, as can be done using historical information, operators can quantify the existing level-of-service. If the desired level-of-service at a station is not met, then a fleet redistribution action can be initiated to bring the system to an acceptable state.

The VSP system can be defined on a network of n stations. Each station i has capacity, C_i, the maximum number of vehicles it can accommodate. The capacity

represents parking bays for car-sharing, docking slots in bike-sharing systems, or charging stations for electric vehicles. The number of vehicles at station i, termed the station inventory, is denoted by V_i. The cost of relocating vehicles from station i to station j, $i \neq j$, is denoted by a_{ij}. There is a penalty δ to move each vehicle between two stations. The system operator has perfect information on available inventories at each station. The system operator plans for a fixed short-term planning horizon for which demand is known probabilistically. Redistribution tasks are assumed to be completed before the planning period commences. The operator considers redistribution actions periodically throughout the day. At each station i, there are two types of demand processes, one to check out vehicles, ξ_i^1, and the other to return vehicles, ξ_i^2. Both ξ_j^1 and ξ_j^2 are random variables with known probability distributions. The operator seeks a least-cost redistribution plan that would make the system p-reliable during the planning period. That is, the system satisfies all demand at every station $(1, \ldots, n)$, for p proportion of all possible realizations. This can be described by the following joint-chance constraint.

$$
\mathbb{P} \left(\begin{array}{ll}
\text{Available vehicles at stn } 1 \geq \xi_1^1, & \text{Available spaces at stn } 1 \geq \xi_1^2 \\[2mm]
\text{Available vehicles at stn } 2 \geq \xi_2^1, & \text{Available spaces at stn } 2 \geq \xi_2^2 \\
\qquad\qquad \vdots & \qquad\qquad \vdots \\[2mm]
\text{Available vehicles at stn } n \geq \xi_n^1, & \text{Available spaces at stn } n \geq \xi_n^2
\end{array} \right) \geq p.
$$

(3.1)

Equation (3.1) represents a level-of-service constraint for the operator who seeks a p-reliable system. To achieve this, the operator looks at available inventory at each station. If the available resources, both vehicles and free spaces, are adequate

to satisfy p-proportion of all possible demand scenarios, then no further corrective actions are necessary. If the available vehicles are insufficient, then vehicles can be 'borrowed' from adjacent stations. If available spaces are inadequate, then vehicles can be 'lent out' to other stations to free up spaces. Since there are costs involved in these actions, the operator seeks an optimal method to perform this redistribution.

To derive the level-of-service constraint, we note that the available vehicle inventory at each station depends on the current inventory and the number of returns and checkouts during the time period. If the redistribution plan calls for vehicles to be relocated into (or out of) the station, then these vehicles are assumed to be available (or unavailable) at the start of the planning period. This assumption is not restrictive, since redistribution tasks can commence well before the planning period begins. Similarly, the available spaces inventory at each station depends on the current inventory, the number of returns, and the number of vehicles relocated in and out of the station during the planning period. Therefore,

$$
\text{Available vehicles at Stn } i =
\begin{cases}
\text{Vehicle inventory at } i & (V_i) \\
+ \\
\text{Returns at } i & (\xi_i^2) \\
+ \\
\text{Vehicles relocated to } i & (\sum_j y_{ji}) \\
- \\
\text{Vehicles relocated out of } i & (\sum_j y_{ij})
\end{cases}
\tag{3.2}
$$

$$\text{Available spaces at Stn } i = \begin{cases} \text{Spaces inventory at } i & (C_i - V_i) \\ \\ + \\ \\ \text{Checkouts at } i & (\xi_i^1) \\ \\ + \\ \\ \text{Vehicles relocated out of } i & (\sum_j y_{ij}) \\ \\ - \\ \\ \text{Vehicles relocated to } i & (\sum_j y_{ji}). \end{cases} \tag{3.3}$$

It is assumed that redistribution is completed *before* the planning period. Therefore, return (ξ_i^2) and checkout(ξ_i^1) random variables are for the future planning period, while the inventory (V_i) and redistribution variables (y_{ij}) denote operator actions before the planning period commences. The current vehicle inventory, V_i, is known, as is the corresponding spaces inventory, $C_i - V_i$. Let x_{ij} denote a binary decision variable indicating if vehicles are moved from i to j in anticipation of future demand. Let y_{ij} denote an integer decision variable indicating the number of vehicles moved from i to j. In terms of decision variables y_{ij}, the level-of-service constraint (3.1) can be written as

$$\mathbb{P}\begin{pmatrix} V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + \xi_i^2 \geq \xi_i^1, & i = 1, ..n \\ C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + \xi_i^1 \geq \xi_i^2, & i = 1, ..n \end{pmatrix} \geq p. \tag{3.4}$$

Let ξ_i to be the net demand at a station i for the planning period. That is, $\xi_i = \xi_i^1 - \xi_i^2$. The two types of demand (vehicles and spaces) exhibit duality (reduction of one type implies an increase of the other), so the net demand ξ_i encodes both types of demand in one random variable and represents the imbalance

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between demand for vehicles and checkouts. For a particular time period, if the realization of ξ_i is positive, there are more checkouts than returns. Similarly, if ξ_i is negative, there are more returns than checkouts.

The level-of-service constraint (3.4) cannot be met for every demand realization. For example, in scenarios where the demand outstrips available resources, this constraint is infeasible. To recover *partial redistribution plans* that help operators make the best possible use of available resources (though still shy of the desired level-of-service), phantom vehicle and space variables are introduced. For each station, let w_i be the number of phantom vehicles and z_i be the number of phantom spaces. Additionally, let γ be a large penalty cost that forces the use of phantom resources only if necessary. The variables w_i and z_i relax the level-of-service constraint as shown in constraint (3.6). This relaxation allows the model to generate *partial* redistribution plans and maintains feasibility even if resources are inadequate. The optimal fleet redistribution plan can be formulated as a chance constrained model with desired reliability p (CCM-p) as follows.

$$(\text{CCM} - p) \qquad \min \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} x_{ij} + \delta y_{ij}) + \sum_{i=1}^{n} \gamma (w_i + z_i), \qquad (3.5)$$

$$\text{s.t. } \mathbb{P} \left(\begin{array}{ll} V_i + \sum\limits_{j=1}^{n} (y_{ji} - y_{ij}) + w_i \geq \xi_i, & i = 1, \ldots, n \\ C_i - V_i + \sum\limits_{j=1}^{n} (y_{ij} - y_{ji}) + z_i \geq -\xi_i, & i = 1, \ldots, n \end{array} \right) \geq p, \qquad (3.6)$$

$$\sum_{j=1}^{n} y_{ij} \leq V_i \qquad\qquad i = 1, \ldots, n, \qquad\qquad (3.7)$$

$$\sum_{j=1}^{n} y_{ji} \leq C_i - V_i \qquad\qquad i = 1, \ldots, n, \qquad\qquad (3.8)$$

$$y_{ij} \leq M \cdot x_{ij} \qquad\qquad i = 1, \ldots, n, j = 1, \ldots, n, \quad (3.9)$$

$$y_{ij}, w_i, z_j \in \mathbb{Z}_+ \qquad\qquad i = 1, \ldots, n, j = 1, \ldots, n, \quad (3.10)$$

$$x_{ij} \in [0, 1] \qquad\qquad i = 1, \ldots, n, j = 1, \ldots, n. \quad (3.11)$$

The objective (3.5) represents the fixed cost for relocating vehicles, the cost of moving additional vehicles, and the penalty costs for utilizing phantom resources. Fixed cost of redistribution between two stations can be based on distance. The operator seeks to minimize the total cost of redistribution. The probabilistic level-of-service constraint (3.6) states that the redistribution plan must result in inventories that satisfy p proportion of all demand scenarios in the planning horizon. If available resources are insufficient, then this constraint is relaxed using phantom resources. There are capacity constraints (3.7) that limit the number of vehicles relocated out of a station to be no greater than the vehicles available at the start of the planning period. Similarly, there are capacity constraints (3.8) for slots at a station stating that the number of vehicles relocated to a station does not exceed the number of slots available. Constraints (3.9) relates the decision variables. All decision variables are non-negative integer valued (3.10), except x_{ij} which is binary (3.11).

Program CCM-p is a stochastic MIP for determining the optimal redistribution plan to satisfy p proportion of demand. In situations where available resources are inadequate to match anticipated demand, the program yields a partial redistribution

plan. In this case, the phantom resources are utilized ($w_i > 0$ or $z_i > 0$) and the system is no longer p-reliable. The true reliability must be recomputed as shown in Section 3.3.1.2. If phantom resources are not utilized, the joint chance constraint (3.6) states that the probability that demand (in terms of vehicles and slots) exceeds supply is no greater than $1 - p$.

Unless the joint distribution of ξ is log-concave, this stochastic MIP may have a non-convex feasible region, making it computationally challenging to solve. The next section presents two specialized techniques for a solution that deals with this non-convexity. These techniques are applicable when the random vector is only on the right-hand side (RHS) and is discrete as is the case in this application.

3.3 Solving Program CCM-p

CCM-p is a stochastic MIP with joint chance constraints. In general, these programs are difficult to solve, but since the random vectors are discrete and appear only in the RHS of the constraints, a specialized technique involving p-efficient points (PEPs) can be employed. Two solution procedures are presented. In the first method (Section 3.3.1), the main idea is to transform the non-convex feasible space to a disjunctive set of convex spaces. This transformation leads to a family of MIPs, one for each convex set. A single PEP characterizes one such convex set by substituting the chance constraint by linear constraints. A PEP is formally defined shortly, but generally speaking, a PEP is the smallest possible (non-dominated) vector for which the joint chance constraint is valid. For example, if v is a PEP,

then the chance constraint $\mathbb{P}(Ax \geq \xi) \geq p$ can be substituted by a linear constraint $Ax \geq v$, since v represents a realization of ξ that ensures that the chance constraint is met (see Figure 3.1). Solving the family of MIPs yields a set of solutions, the best amongst which is optimal for the original non-convex program.

To generate the family of MIPs, the set of PEPs needs to be enumerated. When the dimension of the random vector is large, enumerating PEPs can be problematic, since the set of PEPs can be very large. Once the set of PEPs is enumerated, the family of MIPs is solved using existing MIP solvers. While this method is not new, our contribution is a PEP enumeration algorithm that aims to address the major bottleneck in the enumeration phase of the algorithm. The proposed divide-and-conquer procedure is more efficient than existing methods [40, 41, 10]. Additionally, we extend the PEP concept to dual-bounded chance constraints.

The second solution method (Section 3.3.2) reduces the computational burden of PEP enumeration, but makes a limiting assumption on the independence of the random vector. The main idea is similar to column generation, where only necessary columns (or PEPs) that improve the objective are generated. The master problem is a convexified linear approximation of the CCM-p. The simplex multipliers from the master problem are used in the subproblem to direct the PEP enumeration phase of the algorithm.

The idea of PEPs was first proposed by Prekopa [38] who also presented ways to deal with the chance constraint if the marginal distribution of the random vector is log-concave [39]. Beraldi et al. [9] documents an application. Prekopa et al. [40] presented a nested algorithm for generating PEPs. The main idea is to

recursively explore the search space while keeping certain dimensions fixed. Beraldi et al. [10] proposed two enumeration schemes, backward and forward, along with hybrid schemes that attempt to avoid complete enumeration of PEPs under some restrictive conditions on the properties of the random vector. Their scheme targets the non-convexity due to integral variables by reducing the number of MIPs to be solved. They also derived conditional bounds that aid in determining if a candidate vector is a PEP. Saxena [43] combined the enumeration scheme with the solution phase to avoid explicit enumeration. They introduced the concept of p-inefficiency to reduce constraints in the resulting program. Dentcheva et al. [18, 19] proposed hybrid methods, called convexification and cone generation methods, with the aim of avoiding explicit PEP enumeration. Though the cone generation method assumes independence of the random vector, this approach is very attractive, since only a limited number of PEPs need to be generated. In this chapter, their approach is adapted to deal with dual-bounded constraints and is presented in Section 3.3.2.

For a joint chance constraint of the form $\mathbb{P}(Ax \geq \xi) \geq p$, where ξ is discrete, the enumeration of PEPs is challenging, since the search space includes all possible realizations of the discrete random vector. The performance of any enumeration scheme depends on (a) the dimensionality of the random vector, (b) the support of the random vector, (c) complexity of evaluating the joint distribution function, and (d) the value of p. Increasing the dimensionality causes a combinatorial explosion in the search space. Increasing the support of the random vector also increases the search space, although not combinatorially. All enumeration schemes must evaluate the joint distribution function repeatedly. Thus, even moderate complexity

in calculating the distribution function negatively impacts performance. Lastly, if the value of p is closer to either 0 or 1, the number of possible PEPs is considerably less than if it is close to 0.5, because there are fewer combinations along different dimensions of the random vector.

3.3.1 Solution based on PEP Enumeration

Since a MIP needs to be solved for each PEP, the solution procedure is designed to reduce the number of MIPs that need to be solved using some problem specific properties. First, as *complete* redistribution plans are preferable to *partial* ones, conditions for which a particular PEP will yield a guaranteed sub-optimal (partial) solution are derived in Section 3.3.1.2. If a complete redistribution plan has been found, these *infeasibility* conditions help in screening PEPs that will yield partial solutions, thereby reducing the number of MIPs to be solved. Second, a zero-cost redistribution plan implies that no imbalance exists, so this forms the absolute lower bound on the problem. Third, since the set of PEPs, denoted by S_p, is large, it may preclude a complete enumeration and alternate termination criteria can be used to settle on an acceptable solution. A partial enumeration provides a solution that is not guaranteed to be optimal. The algorithm for solving CCM-p, given the inventories V_i, $i = 1, \ldots, n$ at each station and the desired reliability level p, is as follows.

Algorithm ENUM-p:

Step 0. **Initalization.** Initialize the objective value of best solution $z^{opt} = \infty$. Set

the PEP counter $k = 1$. Let CR be a boolean flag that is true if a complete redistribution plan exists. Set $CR =$ false.

Step 1. **PEP Enumeration.** Generate the set of all PEPs S_p (see Section 3.3.1.1).

Step 2. For the k-th PEP $(u_k, v_k) \in S_p$ proceed to Step 3.

Step 3. **Feasibility Test.** If $CR =$ true, check all $n + 2$ feasibility conditions (3.14), (3.15), and (3.16) (see section 3.3.1.2) for (u_k, v_k). If feasible or if $CR =$ false, proceed to Step 4; otherwise, set $k = k + 1$ and return to Step 2.

Step 4. **Solve Deterministic Equivalent.** For a PEP (u_k, v_k), solve the program (3.5)–(3.11) by replacing the joint chance constraint (3.6) by the two linear constraints

$$V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \geq v_k \qquad \text{and} \tag{3.12}$$

$$-\left(C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i\right) \leq u_k. \tag{3.13}$$

Step 5. **Solution Check.** If the objective value $z_k < z^{opt}$, then $z^{opt} = z_k$ and save the redistribution plan corresponding to z_k as optimal. If this redistribution plan does not use phantom resources, set $CR =$**true**. If $z_k = 0$, absolute lower bound reached, terminate.

Step 5. If termination criteria are met, stop; else $k = k + 1$ and return to Step 2.

### 3.3.1.1 A Divide-and-Conquer PEP Enumeration Algorithm

As all past developments of PEPs have dealt with random vectors having an upper or lower bound but not both, the concept is extended to the case when the random vector is dual-bounded as is the case in constraint (3.6). Essentially, the procedure is developed for chance constraints of the form $\mathbb{P}(A'x \leq \xi \leq Ax) \geq p$, but also applies to the classical chance constraint $\mathbb{P}(Ax \geq \xi) \geq p$. The principle difference in the treatment of dual-bounded constraints is that PEPs are expressed as vector pairs and the cumulative distribution function is replaced by a function $g(u, v)$ that handles dual bounds. Barring these distinctions, the following development emulates concepts proposed by [41] and others [10, 19].

Definition For $p \in (0, 1)$ a vector pair(u, v), $u \in \mathbb{Z}^n$ and $v \in \mathbb{Z}^n$, is said to be a p-efficient point (PEP) if $g(u, v) \geq p$, where $g(u, v) = \mathbb{P}(u_i \leq \xi_i \leq v_i, \forall i = 1, .., n)$ and there exists no vectors y and z such that $g(y, z) \geq p$, $z \leq v$, and $y \geq u$.

For dual-bounded chance constraints, the definition employs a function $g(u, v)$ instead of the cumulative distribution function to ensure that the lower bound is also met as shown in Figure 3.1.

For this modified dual-bounded case, we first derive a bound that will serve to test if a candidate vector is a PEP similar to the bounds based on the conditional marginal distribution derived by [10].

Proposition 3.3.1 *A vector pair (u, v) is PEP if and only if $l(u) = u$ and $h(v) = v$,*
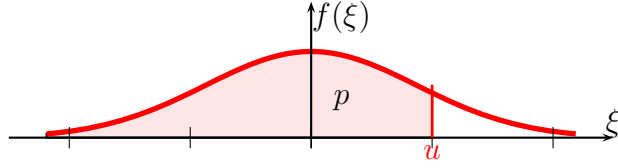
(a) Standard definition when $Ax \geq u \Rightarrow \mathbb{P}(Ax \geq \xi) \geq p$
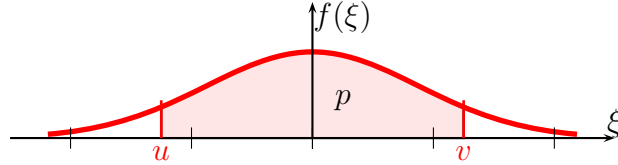
(b) Modified dual-bounded definition when $A'x \leq u$

and $Ax \geq v \Rightarrow \mathbb{P}(A'x \leq \xi \leq Ax) \geq p$

Figure 3.1: Illustration of PEP definitions for a single dimension

where

$$l_i(u) = \arg\max \{k \mid g(u,v) \geq p, u_i = k\}, \; i = 1, .., n \; and$$

$$h_i(v) = \arg\min \{k \mid g(u,v) \geq p, v_i = k\}, \; i = 1, .., n$$

for $g(u,v) = \mathbb{P}(u_i \leq \xi_i \leq v_i, \; \forall i = 1, .., n)$.

Proof Bounds $\Rightarrow$ PEP. The proof is by contradiction and is shown for the lower bound only. Similar arguments can be applied to the upper bound. Assume a pair of vectors (u,v), where $l(u) = u$ and $h(v) = v$. Take a vector pair (y,v) that is PEP, such that $y \leq u$ with $y_k < u_k$ for an arbitrary dimension k. Since $g(u,v)$ monotonically decreases in u, $g(y,v) \geq g(u,v)$ since $y \leq u$. The bounds $l(u) = u$ and $h(v) = v$ imply that $g(u,v) \geq p$. Therefore, (y,v) cannot be PEP, since there exists a larger vector u such that $g(u,v) \geq p$. Now assume another vector pair (y,z) that is a PEP, such that $y \geq u$. For an arbitrary dimension k, where $y_k > u_k$,

construct a new vector w such that $w_i = u_i, \forall i, i \neq k$ and $w_i = y_i$, $i = k$. We know $g(y, z) \geq p$ as (y, z) is a PEP. Since $y \geq u$ and the function $g(w, z)$ monotonically decreases in w, $g(w, z) \geq p$. Therefore, $l_k(u) \leq y_i$, which implies (y, z) is not a PEP, contradicting our assumption.

PEP $\Rightarrow$ Bounds. Follows from the definition, since if (u, v) is PEP, then there exists no larger vector y such that (y, v) is PEP and no smaller vector z such that (u, z) is PEP. ∎

This bound is used to check if a candidate vector is PEP. The concept of the proposed enumeration algorithm is that instead of a linear traversal suggested by previous methods, we exploit the monotonic property of the cumulative distribution function (or $g(u, v)$) to allow us to focus on areas of the search space that contain the p-frontier where $g(u, v) = p$. The function $g(u, v)$ monotonically increases with v and monotonically decreases with u. The search space can be construed as a lattice, since the random vector takes only discrete values. Any arbitrary hyper-rectangle within the search space can be defined by two 'corner' points. The 'low' corner point, consisting of the smallest possible v component and the largest possible u component within the hyper-rectangle and is denoted by (u^s, v^s). The 'high' corner point consists of the largest v and the smallest u components within the hyper-rectangle and is denoted by (u^e, v^e). All lattice points within the hyper-rectangle are candidate PEPs. Due to the monotonic nature of $g(u, v)$, the lowest possible value within the hyper-rectangle is $g(u^s, v^s)$ and the highest possible value it can take is $g(u^e, v^e)$.

Based on the two corner points, three cases may occur as illustrated in Figure 3.2.

Case 1. If $g(u^e, v^e) < p$, then the entire hyper-rectangle can be ignored, since it is guaranteed not to contain a PEP.

Case 2. If $g(u^s, v^s) > p$ then the hyper-rectangle is 'above' the p-frontier. The only possible PEP is the corner point (u^s, v^s). If the corner point is PEP, then it is the sole PEP in the hyper-rectangle, since it would dominate all other candidate solutions. If it is not PEP, then no other PEPs exist within the hyper-rectangle, since they would be dominated by (u^s, v^s).

Case 3. If $g(u^s, v^s) \leq p \leq g(u^e, v^e)$, then the hyper-rectangle may contain one or more PEPs and is marked for further exploration.
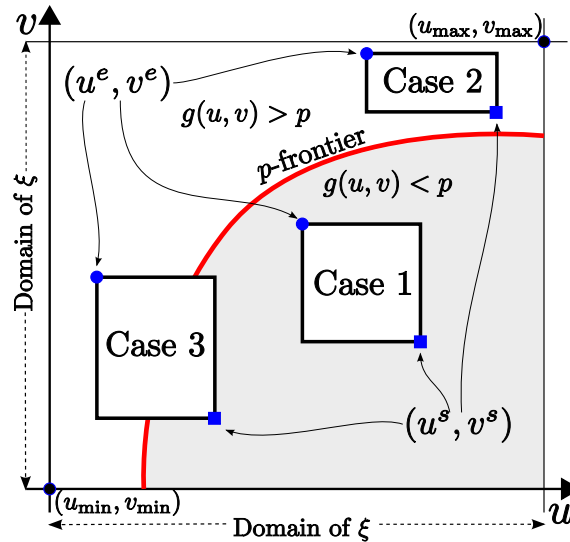

Figure 3.2: Three cases for arbitrary hyper-rectangles in 2-dimensions

Large swaths of the search space can be disregarded quickly using Cases 1 and 2. When the hyper-rectangle is marked for further exploration (Case 3), it can be

partitioned arbitrarily with the same cases applied recursively. With each iteration, the partitions get smaller until they can no longer be divided. The terminal partition is a hyper-rectangle with at most two lattice points along any dimension. For an n-dimensional random vector, enumeration of PEPs in the terminal partition could, in the worst-case, require examination of 2^{2n} candidate vector pairs. The PEPs in this case can be completely enumerated using existing enumeration schemes [40, 41, 10].

This procedure obviates the need for complete enumeration by focusing on areas of the search space that contain the p-frontier where $g(u, v) = p$. Only two evaluations of $g(u, v)$ are needed to determine if a candidate hyper-rectangle contains the p-frontier. Complete enumeration is saved for portions of the search space that are promising. The procedure has a small memory footprint, since the algorithm only keeps track of two vector pairs for each hyper-rectangle. While Prékopa's procedure [40] nests the search along the different dimensions of the random vector, here we nest in the domain of each component of the vector.

The recursive PEP enumeration algorithm is presented for a random vector $\xi = (\xi_1, \xi_2, \ldots, \xi_r)$. Each component of the random vector ξ_i can take values between l_i and u_i.

Step 0. **Initialization**. Define the starting corner vector pairs (u^s, v^s) and (u^e, v^e), where $u_i^s = v_i^e = u_i$ and $v_i^s = u_i^e = l_i$. Initialize the set of PEPs $S_p = \emptyset$ and p the desired probability level.

Step 1. p-**Frontier check**. For two vector pairs (u^s, v^s) (start) and (u^e, v^e) (end) if $g(u^s, v^s) \le p \le g(u^e, v^e)$ then proceed to Step 2, otherwise, terminate.

Step 2. **Partition**. Along an arbitrary dimension $k, k = 1, ..., 2r$, determine a scalar w_k that partitions the hyper-rectangle defined by (u^s, v^s) and (u^e, v^e) into two non-empty, non-overlapping hyper-rectangles. If no partition exists, then go to Step 4.

Step 3. **Recurse**. If $k \leq r$, then construct $s = (u_1^e, u_2^e, \ldots, w_k, \ldots, u_k^e)$. Go to Step 1 first with the vector pairs $(u^s, v^s), (s, v^e)$ and then again with the vector pairs $(s, v^s), (u^e, v^e)$. If $k > r$, then construct $s = (v_1^e, v_2^e, \ldots, w_k, \ldots, v_r^e)$ and go to Step 1 first with vector pairs $(u^s, v^s), (u^e, s)$ and then with $(u^s, s)(u^e, v^e)$.

Step 4. **Enumerate**. For each candidate vector pair (u, v) in the hyper-rectangle, compute the conditional bounds $l(u)$ and $h(v)$. If $l(u) = u$ and $l(v) = v$, add to set of PEPs $S_p = S_p \cup (u, v)$. Stop.

This procedure terminates with the set S_p required in the solution procedure of CCM-p (Section 3.3.1).

3.3.1.2 Reducing Computational Effort

Conditions that provide a quick test on whether a PEP (u, v) will provide a sub-optimal solution (without solving the MIP) are derived. These conditions are based on the premise that the cost of a partial redistribution plan always exceeds that of a complete redistribution plan, since the phantom resources (the decision variables w_i and z_i) are utilized with a high penalty (γ). As the solution algorithm involves determining redistribution for a series of PEPs, if a complete plan has been found (that is not necessarily optimal), then all successive PEPs that lead to partial

solutions can be safely ignored. A partial plan is used only when resources are outstripped by demand and operators cannot cover p-proportion of demand. These conditions utilize the physical signifance of a PEP pair (u, v). v_i represents the number of vehicles (if positive) needed at station i for the desired level-of-service and u_i, if negative, represents the required number of spaces.

At any station the total number of spaces and vehicles needed cannot exceed the capacity. This condition is termed the *capacity infeasibility* condition and can be expressed as

$$- \min(u_i, 0) + \max(v_i, 0) > c_i \qquad i = 1, \dots, n. \tag{3.14}$$

These capacity constraints are 'local', since they are applied to each station. There are 'global' *supply infeasibility* conditions when the available inventory in the system is insufficient for the operator to meet anticipated demand. These supply infeasibilities can be expressed as

$$\sum_{i=1}^{n} V_i \;<\; \sum_{i=1}^{n} \max(v_i, 0) \qquad \text{and} \tag{3.15}$$

$$\sum_{i=1}^{n} (C_i - V_i) \;<\; -\sum_{i=1}^{n} \min(u_i, 0). \tag{3.16}$$

Eq. (3.15) states that the total inventory of vehicles available across the network is exceeded by total anticipated demand across the entire network. Eq. (3.16) states the same principle for spaces.

When the model suggests partial redistribution, the system operates at reliability levels that are lower than the desired p. The true system reliability in this case can be computed as follows. Let (u^*, v^*) be the PEP for which the (partial)

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redistribution plan is optimal. Let w_i^* and z_i^* be the optimal values of the phantom resources. Then, the true system reliability $\hat{p}$ can be expressed as

$$
\begin{aligned}
\hat{p} &= \mathbb{P}\left(u_i^* + w_i^* \leq \xi_i \leq v_i^* - z_i^*, \ i = 1, \ldots, n\right) \qquad (3.17)\\
&= g(u^* + w^*, v^* - z^*)
\end{aligned}
$$

3.3.2 A Cone Generation Method

When demand across stations is assumed to be independent, the cone generation method proposed by Dentcheva et al. [18, 19] can be employed. The solution algorithm presented here mirrors their procedure, but proposes a new subproblem formulation to deal with the dual-bounded chance constraint. The method can generate redistribution plans quickly and is suitable for large systems where PEP enumeration is prohibitive. The master problem is an approximation of CCM-p and the subproblem generates p-efficient points as needed. The basic idea is similar to column generation, where each PEP can be viewed as a column.

Algorithm CGM-p:

Step 0. **Initialization**. Choose an arbitrary starting PEP pair (u^0, v^0) and set $k = 0$, $J_k = 0$.

Step 1. **Master problem**. Convexify CCM-p (Eqs. (3.5)-(3.11)) relaxing integral constraints and solve the resulting linear program

$$
\min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij} x_{ij} + \delta y_{ij}\right) + \sum_{i=1}^{n} \gamma\left(w_i + z_i\right), \qquad (3.18)
$$

$$V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \geq \sum_{j \in J_k} \mu_j v_i^j \qquad i = 1, \ldots, n, \qquad (3.19)$$

$$C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i \geq - \sum_{j \in J_k} \mu_j u_i^j \qquad i = 1, \ldots, n, \qquad (3.20)$$

$$\sum_{j \in J_k} \mu_j = 1, \qquad (3.21)$$

$$\sum_{j=1}^{n} y_{ij} \leq V_i \qquad\qquad i = 1, \ldots, n, \qquad (3.22)$$

$$\sum_{j=1}^{n} y_{ji} \leq C_i - V_i \qquad\qquad i = 1, \ldots, n, \qquad (3.23)$$

$$y_{ij} \leq M \cdot x_{ij} \qquad\qquad i = 1, \ldots, n, j = 1, \ldots, n, \quad (3.24)$$

$$x_{ij}, y_{ij}, w_i, z_j \geq 0 \qquad\qquad i = 1, \ldots, n, j = 1, \ldots, n, \quad (3.25)$$

$$\mu_j \geq 0 \qquad\qquad j \in J_k \qquad (3.26)$$

Let λ_v^k and λ_u^k be the simplex multipliers of constraints (3.19) and (3.20), respectively.

Step 2. **Upper bound**. Calculate the bound for the subproblem over the set of generated PEPs.

$$\bar{d}(u^k, v^k) = \min_{j \in J_k}(\lambda_v^k)^T \cdot v^j - (\lambda_u^k)^T \cdot u^j \qquad (3.27)$$

Step 3. **Solve subproblem**. Find the PEP pair (u^{k+1}, v^{k+1}) by solving

$$\min \left\{ (\lambda_v^k)^T \cdot v^{k+1} - (\lambda_u^k)^T \cdot u^{k+1} \mid g(u^{k+1}, v^{k+1}) \geq p \right\}, \qquad (3.28)$$

and compute $d(u^k, v^k) = (v^{k+1})^T \cdot \lambda_v^k - (u^{k+1})^T \cdot \lambda_u^k$.

Step 4. **Termination Condition**. If $d(u^k, v^k) = \bar{d}(u^k, v^k)$, then stop; otherwise, set

$J_{k+1} = J_k \cup (k+1)$, $k = k+1$ and goto Step 1.

The subproblem in Step 3 has a nonlinear constraint $g(u, v) \geq p$. When demand at each station is assumed to be independent, this constraint can be written as

$$\ln(g(u^k, v^k)) = \sum_{i=1}^{n} \ln(g_i(u_i^k, v_i^k)) \geq \ln p. \tag{3.29}$$

If each component of the random vector ξ_i takes values between l_i and b_i, the subproblem can formulated as an MIP. Denote y_{ijk} as a binary decision variable that is one if for the i-th dimension of ξ, $u_i = j$ and $v_i = k$ and zero otherwise. For a given set of multipliers $(\lambda_u^k, \lambda_v^k)$ and a desired probability level p, the subproblem can be written as

$$\min \sum_{i=1}^{n} \sum_{j=l_i}^{b_i} \sum_{k=l_i}^{b_i} (\lambda_i^v k - \lambda_i^u j) \, y_{ijk} \tag{3.30}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=l_i}^{b_i} \sum_{k=l_i}^{b_i} \ln(q_{ijk}) y_{ijk} \geq \ln(p) \tag{3.31}$$

$$\sum_{j=l_i}^{b_i} \sum_{k=l_i}^{b_i} y_{ijk} = 1 \qquad\qquad i = 1, \ldots, n \quad \text{(3.32)}$$

$$y_{ijk} \in [0, 1], \tag{3.33}$$

where $q_{ijk} = g_i(j, k) = \mathbb{P}(j \leq \xi_i \leq k)$. The subproblem yields a PEP that can be reconstructed from the decision variables. When $y_{ijk} = 1$, the i-th component of the PEP is given by $u_i = j$ and $v_i = k$. The resulting PEP is then added to the

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master problem as a column and the procedure terminates when the PEP leading to the best solution has been found.

3.4   A Failure Apportionment Bound

If the systemwide reliability level can be translated to a component-level measure, the joint chance constraint can be decoupled to give linear constraints. These constraints provide a bound on the original problem. This transformation requires no assumption of independence across stations. The VSP stations can be viewed as being 'in series', since the unserviced demand at any station implies lower reliability. A system that is p-reliable has an acceptable failure rate of at most $1 - p$. The Boole-Bonferroni inequality [39] implies that the sum of the station failure rates cannot exceed the systemwide failure rate.

$$\sum_{i=1}^{n}(1 - p_i) \leq 1 - p. \tag{3.34}$$

Under an equal apportionment of failure we have $1 - p_i = (1 - p)/n$. Decoupling the joint chance constraint (3.6) results in n joint constraints:

$$\mathbb{P}\left(\begin{array}{l} -\left[C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i\right] \leq \xi_i \\ V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \qquad\qquad \geq \xi_i \end{array}\right) \geq p_i \qquad i = 1, \ldots, n, \tag{3.35}$$

where $p_i = (n - 1 + p)/n$. This set of n joint chance constraints can be further reduced by allowing the acceptable failure rate to be divided among unserviced demand for vehicles and unserviced demand for spaces. This is shown in Figure 3.3.
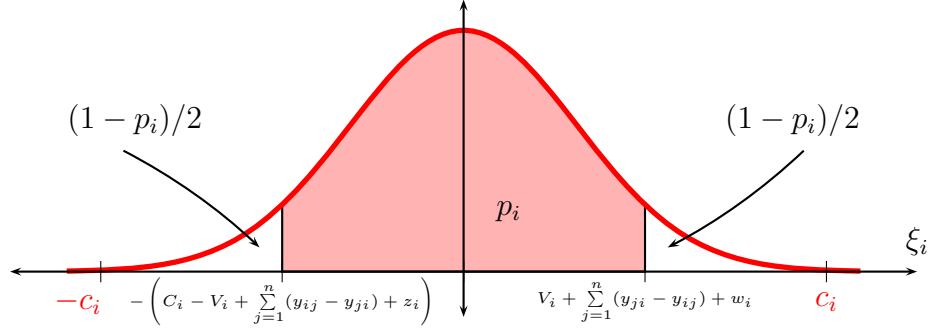
Figure 3.3: Demand scenarios covered by the chance constraint at one station

This results in $2n$ chance constraints:

$$\mathbb{P}\left(V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \geq \xi_i\right) \leq \frac{1-p_i}{2} \qquad i = 1, \ldots, n, \quad (3.36)$$

$$\mathbb{P}\left(-\left[C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i\right] \leq \xi_i\right) \leq \frac{1-p_i}{2} \qquad i = 1, \ldots, n. \quad (3.37)$$

In terms of the inverse marginal distribution, the constraints (3.36) and (3.37) can be derived as

$$V_i + \sum_{j=1}^{n}(y_{ji} - y_{ij}) + w_i \geq F_{\xi_i}^{-1}\left(\frac{1+p_i}{2}\right) \qquad i = 1, \ldots, n, \quad (3.38)$$

$$-\left[C_i - V_i + \sum_{j=1}^{n}(y_{ij} - y_{ji}) + z_i\right] \leq F_{\xi_i}^{-1}\left(\frac{1-p_i}{2}\right) \qquad i = 1, \ldots, n. \quad (3.39)$$

The solution to the MIP defined by the objective (3.5) and constraints (3.7), (3.8), (3.9), (3.10), (3.11), (3.38), (3.39) provides a bound on the optimal solution.

3.5 Application

The Intelligent Community Vehicle System (ICVS) operated by the Honda Motor company in Singapore City, Singapore was a car-sharing system with 14 stations mainly in the downtown region and one at the Changi Airport. The system

was also studied by [29]. The program is no longer operational. Data from the program available from March 2003 through January 2006 documents $45,570$ trips. Across the 14 stations, the system had an assumed capacity of 202 spaces, with 94 vehicles spread around the network. The characteristics of the fleet are not known and are assumed to be homogeneous. The trip characteristics are summarized in Figures 3.4 and 3.5. Figure 3.6 shows the inter and intra station flow patterns. While most trips start and terminate at the same station, a considerable proportion of flows are oneway. Also note that inter-station flows between any two stations are asymmetric and not equal.

The realized demand process is assumed to be the true demand process. This implies that extreme demand scenarios are not represented in the inputs (since they are never observed). The demand-supply interaction is ignored and treated as exogenous and inelastic. Each day is divided into four time periods when redistribution is considered. During each time period at each station, the number of vehicles checked out and the number returned were found to be Poisson distributed. Of all 112 input distributions (one for each station and each period during the day for checkouts and returns), 28 failed the χ^2 test due to the low number of observations. Sample distributions for two stations are shown in Figure 3.7.

For each station j and time period t, the Poisson vehicle checkout rate (λ_{tj}^1) and the Poisson vehicle return rate (λ_{tj}^2) are determined. Since the random variable ξ_i in program CCM-p is the difference of the two, the distribution of the difference is needed. When two random variates are Poisson distributed with means λ_{tj}^1 and λ_{tj}^2, their difference ξ_{tj} is Skellam distributed with pmf
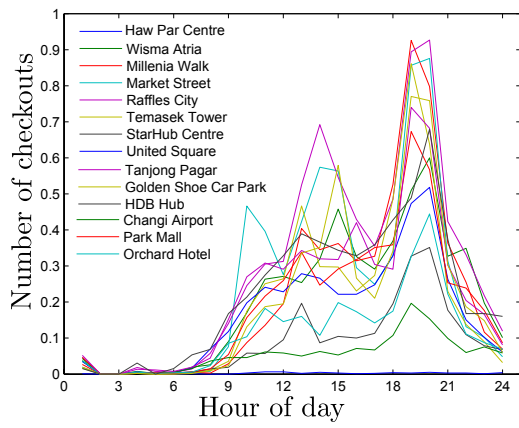
(a) Average Number of Checkouts
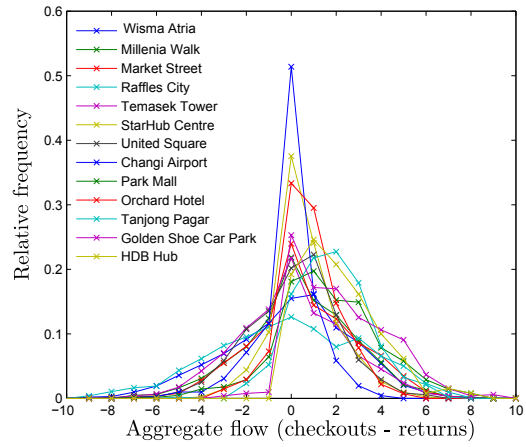
(b) Distribution of Daily Imbalance

Figure 3.4: Characteristics of the IVCS Singapore system
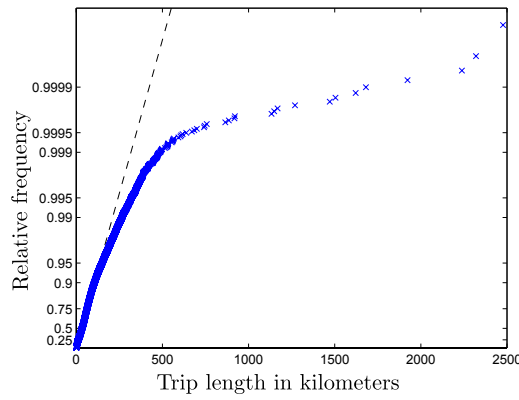
(a) Distribution of Trip Distance
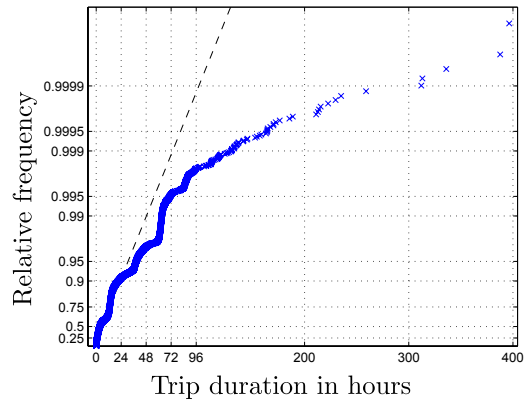
(b) Distribution of Trip Duration

Figure 3.5: Trip characteristics
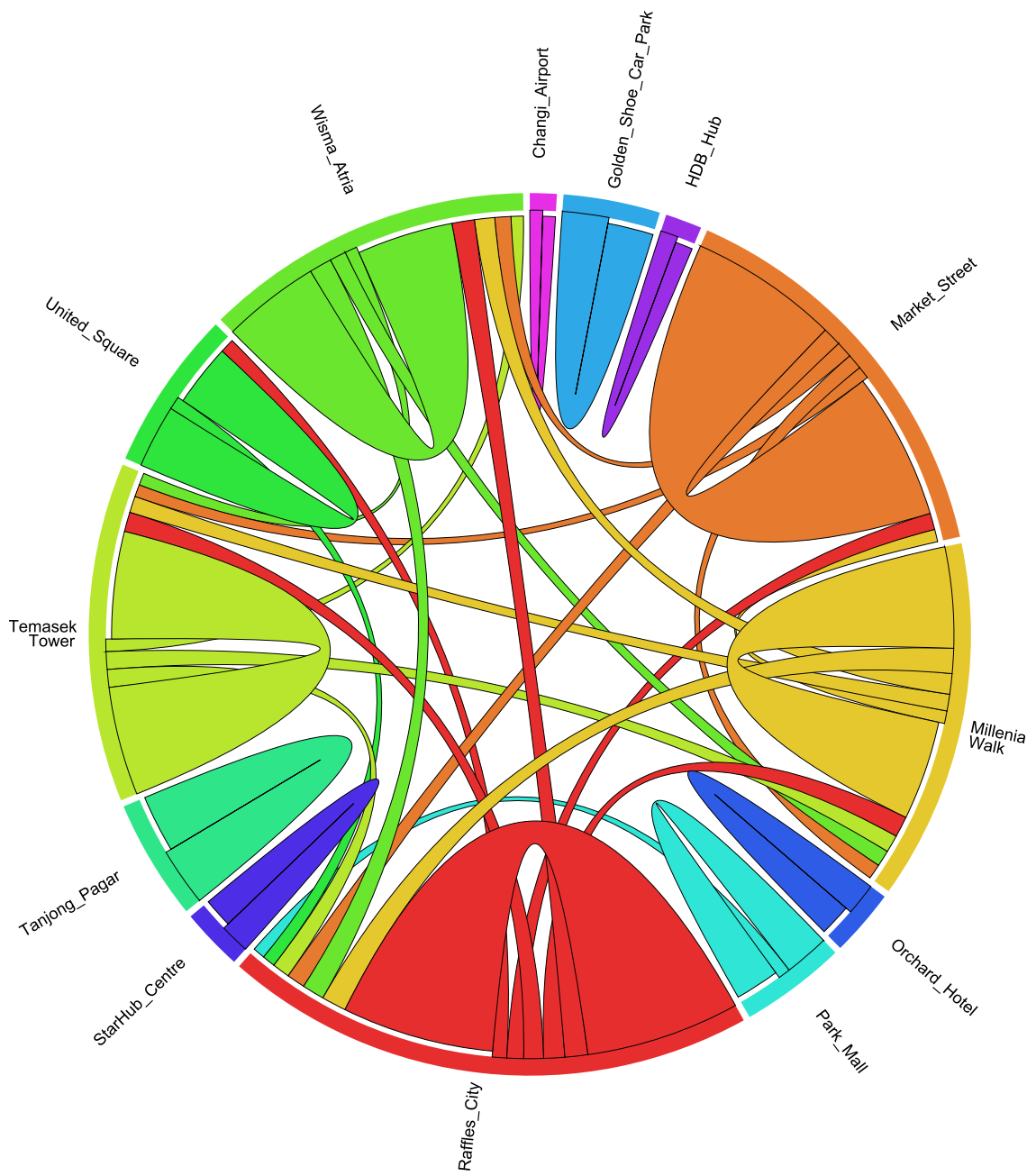
Figure 3.6: Relative inter and intra station flows

$$\mathbb{P}(\xi_{tj} = k) = e^{-(\lambda_{tj}^1 + \lambda_{tj}^2)} \left(\frac{\lambda_{tj}^1}{\lambda_{tj}^2} \right)^{k/2} I_k(2\sqrt{\lambda_{tj}^1 \lambda_{tj}^2}), \tag{3.40}$$

where $I_k(z)$ is the modified Bessel function of the first kind. Since this is a discrete distribution, $F_\xi^{-1}(p)$ exists when we define the inverse function as the infimum and when $0 < p < 1$. Figure 3.8 shows the Skellam distribution for some sample values.

3.5.1 Computational Experiments

Nine strategies for redistribution based on expected value, doing nothing, the enumeration-based solution method, the cone generation method, and the failure apportionment bound are tested. A single day is divided into four planning periods and a redistribution plan is generated for each period using each of the nine strategies. Starting from base inventories in the first period (common for all solution methods), the different redistribution plans are enacted. Consequently, the state of the system at subsequent periods across the different solution methods may not be the same. A demand scenario is randomly generated and the fleet inventories are adjusted to serve as the start point to generate a new redistribution plan. The performance of the system is measured in terms of actualized or true reliability $\hat{p}$, redistribution cost (without the penalties for phantom resources, since this component of the objective functions is what operators will experience) involved in implementing the redistribution plan, and the robustness of the plan over a wide range of demand scenarios.

A do-nothing (DN) approach is when operators relocate only once before the

start of each day, but do not redistribute during the day. An expected value approach (AVG) generates redistribution based on the expected value of demand. The PEP enumeration based solution approach (ENUM-p) is used to generate strategies for three values of p (0.8, 0.9, and 1.0). Since the support of the Skellam distribution is $\mathbb{Z}$, for the last case p is very close to, but not equal to, 1.0. The cone generation method (CGM-p) is run for $p = 0.8, 0.9$. The results for CGM-1.0 and ENUM-1.0 are exactly the same, since only one PEP is considered. Therefore, only results from ENUM-1.0 are reported. The failure apportionment bounds (FAB-p) presented in Section 3.4 are computed for two values of p: 0.8 and 0.9. The proposed procedures (DN, AVG, ENUM-p, CGM-p, FAB-p) were implemented in MATLAB 2009a, java, and CPLEX 11.2. All experiments were run on an Intel Xeon processor running at 3.00 GHz with 16GB of RAM. Since for ENUM-p, the PEP enumeration phase (Step 1) of the algorithm is needed only once, the PEP generation procedure was allowed to run for 24 hours for each stage using $p = 0.8$ and 0.9 (for $p = 1.0$, there is only one PEP). The number of PEPs generated for each period are shown in Table 3.1.

State	Time Period (hrs)	# PEPs $p = 0.8$	# PEPs $p = 0.9$
1	$0 \leq t < 9$	50.10	9.41
2	$9 \leq t < 12$	37.60	22.68
3	$12 \leq t < 18$	64.60	173.80
4	$18 \leq t < 24$	77.20	202.00

Table 3.1: Number of PEPs for each stage (in millions)

Since the set of PEPs is extremely large, the run time for ENUM-p experiments at each time period was restricted to one hour. Consequently, the solutions

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presented next are not guaranteed to be optimal. Should the ENUM procedures run to completion, the results would be exact and match those obtained via the CGM-p method. Since demand at each station is assumed to be independent, the results from CGM-p (where independence is forced) can be directly compared with other strategies.

Results are presented for a 2-day period. Figure 3.9 shows the actualized reliability $\hat{p}$ (averaged over time periods) versus the total relocation costs over the entire day. The relocation costs are computed as the value of the objective function minus the penalties for using the phantom resources. For ENUM-p methods, the family of solutions is depicted for $p = 0.8, 0.9$. CGM-p methods with $p = 0.8$ or 0.9 provide the best actualized reliability at the lowest cost. For the same reliability level p, CGM methods outperform ENUM, since by terminating prematurely all PEPs are not explored for ENUM and, therefore, optimality is not necessarily achieved. FAB-p strategies provide high reliability, but are expensive to implement. The ENUM-1.0 and CGM-1.0 (not reported) methods yield the same results and both perform poorly. Since the resource requirements to satisfy such a high level-of-service are extraordinary, the redistribution plan that the model achieves in this case is always partial, since phantom variables must be utilized. The actualized reliability achieved for this case is very low.

Snapshots of the system at the start of each period illustrate the role of complete and partial redistribution plans. Figure 3.10 shows the actualized reliability and redistribution costs dissaggregated by period for Day 1. After starting at base inventory levels in the first period, the system state at each subsequent period is

different for various strategies as the enacted redistribution plans vary across strate-gies. Hence, the solutions at each snapshot are not commensurate and should be viewed as a time slice through an evolving system. Nonetheless, these figures il-lustrate that when resources are adequate at the start of the day (Figures 3.10(a) and 3.10(b)), most strategies provide complete redistribution plans. When inven-tories are low (Figure 3.10(d)), a complete redistribution plan is only achieved by the CGM-0.8 strategy. Under these circumstances, all other solutions show drops in true reliability and represent partial redistribution plans. When the plans are complete, the FAB-p solutions cost more than the ENUM-p and CGM-p strategies (for a comparable p). When the plans are partial, this need not be the case, since the penalty costs are not included. This is also illustrated in Figure 3.11, where FAB strategies involve the largest number of redistributed vehicles.

To determine the value of considering stochasticity in generating redistribution plans, simulation is employed. The strength of a particular system configuration can be tested over a range of demand realizations. The number of unserviced users, or dropped demand, is measured for each realization. Given a random realization of demand $\bar{\xi}$, the dropped demand for vehicles d_v and spaces d_s can be computed as

$$d_v = \sum_{i=1}^{n} \max(0, \bar{\xi}_i - V_i),$$ (3.41)

$$d_s = -\sum_{i=1}^{n} \min(0, \bar{\xi}_i + C_i - V_i).$$ (3.42)

These quantities are computed for all the strategies and time periods for two days over $100,000$ realizations. Results are summarized in Tables 3.2 and 3.3. The ENUM-0.8, ENUM-0.9, and CGM-p strategies do not drop any demand for a high

proportion of realizations, regardless of resource availability. When resources are adequate, at the start of the day, all strategies do well. During later time periods, the AVG, ENUM-1.0, and DN approaches are more likely to leave demand unserviced. The FAB strategies perform as well as CGM, but these are more expensive to implement. Tables 3.2 and 3.3 also show the worst-case demand realization for each algorithm. The CGM-p methods consistently drop fewer number of demand requests for vehicles and spaces compared to other methods even in the worst-case realization, indicative of robustness of the redistribution strategy.

The solutions yield additional insights on system characteristics. Since the PEPs have a physical interpretation, for the PEP that resulted in the best known solution, the system resource requirements can be computed (see Table 3.4). These numbers directly relate the desired level-of-service with the resources. For example, in Day 1, to achieve a systemwide reliability of 0.9 requires 77 vehicles during the 18:00-24:00 Hours (Hrs) time period for the CGM and ENUM methods. A reliability of 0.8 requires eight fewer vehicles for the same period. These values are contingent on starting inventories, thus, their purpose is illustrative. In a similar vein, the stations which have frequent local infeasibilities can be the target of capacity improvements, because local infeasibilities indicate recurrent imbalance in flows.

In summary, fleet management strategies that explicitly account for demand stochasticity offer greater reliability than plans based on static methods. Redistribution strategies based on the proposed stochastic MIP also weather scenarios in which demand outstrips supply. In simulation studies, the CGM-p, and even the po-

Time (Hrs)	Algorithm	Probability of no dropped vehicle demands	Average number of dropped vehicle demands	Number of dropped vehicle demands for worst-case	Probability of no dropped spaces	Average number of dropped space demands	Number of dropped space demands for worst-case	Partial Redistribution	Relocation Costs	$\hat{p}$
0-9	DN	0.998	0.00	2	1.000	0.00	3	-	0.00	0.599
	AVG	0.998	0.00	2	1.000	0.00	3	-	0.00	0.599
	FAB-0.9	0.998	0.00	3	1.000	0.00	2	No	38817.33	0.994
	FAB-0.8	1.000	0.00	2	1.000	0.00	2	No	36291.29	0.992
	ENUM-1.0	0.998	0.00	2	1.000	0.00	3	Yes	0.00	0.599
	ENUM-0.9	1.000	0.00	3	1.000	0.00	3	No	11590.26	0.984
	ENUM-0.8	1.000	0.00	2	1.000	0.00	3	No	10749.17	0.961
	CGM-0.9	0.999	0.00	2	1.000	0.00	3	No	10250.17	0.954
	CGM-0.8	0.999	0.00	2	0.999	0.00	3	No	10077.12	0.910
9-12	DN	0.487	0.80	8	0.391	1.16	9	-	0.00	0.289
	AVG	0.754	0.34	7	0.423	1.06	9	-	1495.01	0.483
	FAB-0.9	0.988	0.01	4	0.990	0.01	4	No	11335.09	0.988
	FAB-0.8	0.997	0.00	3	0.986	0.02	4	No	5294.02	0.995
	ENUM-1.0	0.487	0.80	8	0.391	1.16	9	Yes	1279.01	0.289
	ENUM-0.9	0.985	0.02	4	0.970	0.04	5	No	3626.02	0.978
	ENUM-0.8	0.998	0.00	3	0.822	0.24	6	No	3959.04	0.984
	CGM-0.9	0.990	0.01	4	0.944	0.07	5	No	5294.04	0.982
	CGM-0.8	0.989	0.01	4	0.817	0.24	6	No	1668.03	0.959
12-18	DN	0.121	2.94	16	0.219	2.42	16	-	0.00	0.044
	AVG	0.323	1.66	14	0.328	1.75	14	-	592.01	0.168
	FAB-0.9	0.773	0.40	11	0.882	0.18	8	Yes	34089.16	0.767
	FAB-0.8	0.854	0.24	10	0.829	0.27	9	Yes	28025.11	0.840
	ENUM-1.0	0.121	2.94	16	0.226	2.38	16	Yes	0.00	0.045
	ENUM-0.9	0.852	0.24	10	0.773	0.37	9	Yes	29310.14	0.843
	ENUM-0.8	0.866	0.21	7	0.835	0.26	9	No	30574.13	0.853
	CGM-0.9	0.954	0.06	7	0.710	0.50	10	Yes	28554.15	0.912
	CGM-0.8	0.934	0.09	7	0.690	0.53	10	No	6043.06	0.876
18-24	DN	0.022	4.86	18	0.024	4.88	20	-	0.00	0.018
	AVG	0.156	2.68	15	0.220	2.34	17	-	4175.05	0.140
	FAB-0.9	0.924	0.11	7	0.477	1.12	13	Yes	25102.27	0.919
	FAB-0.8	0.919	0.12	7	0.480	1.10	13	Yes	22069.24	0.915
	ENUM-1.0	0.022	4.86	18	0.026	4.78	20	Yes	0.00	0.019
	ENUM-0.9	0.912	0.13	7	0.473	1.13	13	Yes	16865.20	0.908
	ENUM-0.8	0.841	0.25	9	0.602	0.75	11	Yes	10984.16	0.839
	CGM-0.9	0.950	0.07	7	0.398	1.39	14	Yes	14639.17	0.945
	CGM-0.8	0.906	0.13	8	0.458	1.14	12	No	13413.16	0.887

Table 3.2: Summary of 100,000 simulation runs for Day 1

Time (Hrs)	Algorithm	Probability of no dropped vehicle demands	Average number of dropped vehicle demands	Number of dropped vehicle demands for worst-case	Probability of no dropped spaces	Average number of dropped space demands	Number of dropped space demands for worst-case	Partial Redistribution	Relocation Costs	$\hat{p}$
0-9	DN	0.998	0.00	3	1.000	0.00	2	-	0.00	0.599
	AVG	0.998	0.00	3	1.000	0.00	2	-	0.00	0.599
	FAB-0.9	0.998	0.00	3	1.000	0.00	1	No	38817.33	0.994
	FAB-0.8	1.000	0.00	1	1.000	0.00	1	No	36291.29	0.992
	ENUM-1.0	0.998	0.00	3	1.000	0.00	2	Yes	0.00	0.599
	ENUM-0.9	1.000	0.00	1	1.000	0.00	2	No	11590.25	0.985
	ENUM-0.8	0.999	0.00	3	1.000	0.00	2	No	10575.22	0.957
	CGM-0.9	0.999	0.00	3	1.000	0.00	2	No	10250.17	0.954
	CGM-0.8	0.999	0.00	3	0.999	0.00	2	No	10077.12	0.910
9-12	DN	0.965	0.04	4	0.332	1.33	11	-	0.00	0.341
	AVG	0.965	0.04	4	0.332	1.33	11	-	0.00	0.341
	FAB-0.9	0.995	0.01	3	0.998	0.00	3	No	4950.03	0.995
	FAB-0.8	0.999	0.00	3	0.989	0.01	4	No	1125.02	0.997
	ENUM-1.0	0.965	0.04	4	0.332	1.33	11	Yes	0.00	0.341
	ENUM-0.9	0.999	0.00	2	0.975	0.03	5	No	0.00	0.987
	ENUM-0.8	0.999	0.00	2	0.979	0.02	4	No	2542.03	0.974
	CGM-0.9	0.991	0.01	4	0.958	0.05	5	No	2501.02	0.967
	CGM-0.8	0.989	0.01	4	0.908	0.12	6	No	1641.03	0.932
12-18	DN	0.494	1.05	11	0.089	3.95	21	-	0.00	0.098
	AVG	0.495	1.05	11	0.115	3.50	20	-	1125.01	0.116
	FAB-0.9	0.896	0.16	7	0.846	0.24	9	Yes	13059.07	0.859
	FAB-0.8	0.947	0.08	7	0.780	0.36	10	Yes	11480.08	0.893
	ENUM-1.0	0.494	1.05	11	0.089	3.95	21	Yes	0.00	0.098
	ENUM-0.9	0.946	0.08	6	0.789	0.35	10	Yes	13418.09	0.910
	ENUM-0.8	0.888	0.17	8	0.841	0.25	10	No	12392.19	0.874
	CGM-0.9	0.963	0.05	6	0.671	0.59	11	Yes	12119.17	0.910
	CGM-0.8	0.955	0.06	6	0.632	0.68	11	No	8908.14	0.887
18-24	DN	0.052	3.76	20	0.005	7.28	27	-	0.00	0.031
	AVG	0.330	1.60	16	0.083	3.80	23	-	4577.07	0.262
	FAB-0.9	0.917	0.12	10	0.485	1.09	15	Yes	26147.22	0.913
	FAB-0.8	0.927	0.11	9	0.400	1.39	17	Yes	20664.19	0.922
	ENUM-1.0	0.052	3.76	20	0.005	7.28	27	Yes	0.00	0.031
	ENUM-0.9	0.914	0.12	10	0.536	0.94	15	Yes	19417.15	0.909
	ENUM-0.8	0.844	0.24	11	0.629	0.69	13	No	19441.17	0.841
	CGM-0.9	0.952	0.07	9	0.395	1.42	17	Yes	18299.14	0.944
	CGM-0.8	0.910	0.13	10	0.501	1.04	16	No	13766.12	0.904

Table 3.3: Summary of 100,000 simulation runs for Day 2

Day 1		Required Vehicles				Required Spaces				Local Infeasibility			
Strategy	p	0-9	9-12	12-18	18-24	0-9	9-12	12-18	18-24	0-9	9-12	12-18	18-24
AVG	-	2.49	8.97	25.73	27.15	14.55	6.72	17.79	10.47	0	0	0	0
FAB	0.8	22	47	76	84	74	51	69	54	0	0	8	6
FAB	0.9	19	41	71	79	68	48	63	48	0	0	5	2
ENUM	1.0	137	149	245	240	161	141	228	207	13	13	13	13
ENUM	0.9	21	45	71	77	67	46	66	54	0	0	5	2
ENUM	0.8	20	41	62	69	61	48	62	51	0	0	0	0
CGM	0.9	17	39	77	77	53	32	51	34	0	0	0	0
CGM	0.8	14	36	63	69	46	26	41	28	0	0	0	0

Day 2		Required Vehicles				Required Spaces				Local Infeasibility			
Strategy	p	0-9	9-12	12-18	18-24	0-9	9-12	12-18	18-24	0-9	9-12	12-18	18-24
AVG	-	2.49	8.97	25.73	27.15	14.55	6.72	17.79	10.47	0	0	0	0
FAB	0.8	22	47	76	84	74	51	69	54	0	0	8	6
FAB	0.9	19	41	71	79	68	48	63	48	0	0	5	2
ENUM	1.0	137	149	245	240	161	141	228	207	13	13	13	13
ENUM	0.9	22	48	71	76	65	46	68	57	0	0	5	3
ENUM	0.8	21	45	63	70	60	44	63	52	0	0	0	0
CGM	0.9	17	39	76	77	53	32	53	34	0	0	0	0
CGM	0.8	14	36	63	69	46	26	41	28	0	0	0	0

Table 3.4: Systemwide resource needs based on various strategies with the number of stations with local capacity infeasibilities

tentially suboptimal ENUM-0.9 and ENUM-0.8 strategies, demonstrate robustness over all sampled demand scenarios.

3.6   Conclusions

Fleet management strategies that explicitly consider demand stochasticity are developed for vehicle sharing systems. In developing these management strategies, no assumptions are made on the specific operational characteristics and demand processes of a particular system. However, system specific attributes can be incorporated with relative ease. For example, if a sharing system allows advance reservations of vehicles, then the demand process splits into a static known component and an uncertain one. By adjusting start inventories at each stage, the static

portion can be guaranteed service.

The main contributions of this work are in formulating the VSP fleet management problem as a stochastic MIP. The approach taken herein overcomes the limitations of prior works that assume static or known demand. The proposed framework quantifies the systemwide level-of-service offered based on a probabilistic characterization of demand. Two solution techniques, one based on enumeration and the other on cone generation, are presented. For the enumeration-based technique, the PEP enumeration algorithm improves on existing tools by using a divide-and-conquer paradigm that is able to quickly eliminate areas of the search space that are guaranteed not to contain PEPs. Our technique has a smaller memory and computational footprint than previously proposed methods. Additionally, the concept of PEP is extended to include dual-bounded chance constraints. For a more restrictive case when demand is assumed to be independent across stations, a second solution technique can be employed which is quick even for large systems. An equal-failure apportionment bound is also derived that is applicable even when demand across stations is correlated. Under these limiting assumptions (independence or equal failure probability), exact solutions can be quickly obtained. In an application of the proposed framework to a system in Singapore, the operational strategies were found to be robust in simulation studies. Additionally, trade-offs between redistribution costs and level-of-service were explored.

Future work along this direction could relax some assumptions, namely immediate fleet relocation, incorporate staff availability to perform redistribution, and tackle heterogeneous fleets. To address large-scale systems, such as the bicycle-

sharing system in Paris, faster heuristics can be developed. The exact solution techniques proposed herein can be used to provide benchmark solutions for use in evaluating these heuristics. One might also study the assignment and routing of relocation teams to carry out fleet redistribution.
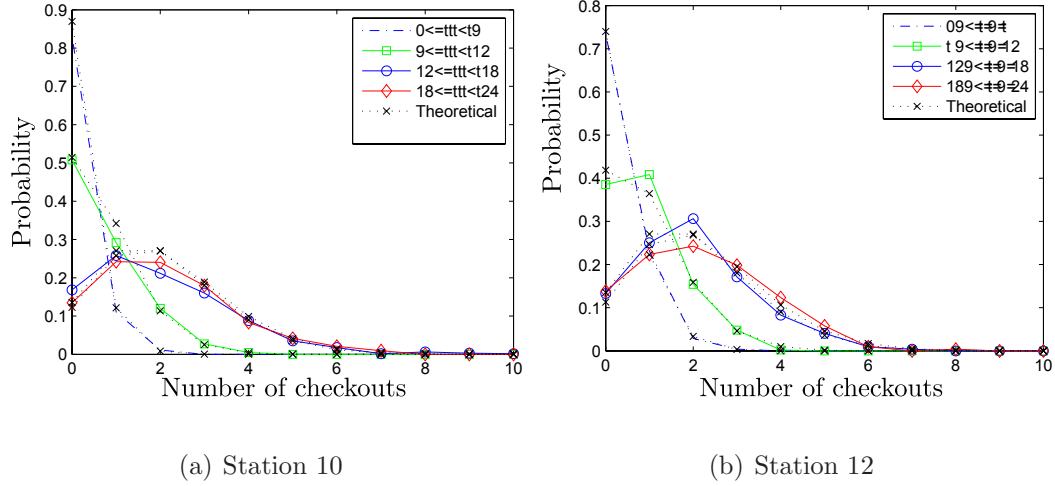
(a) Station 10

(b) Station 12

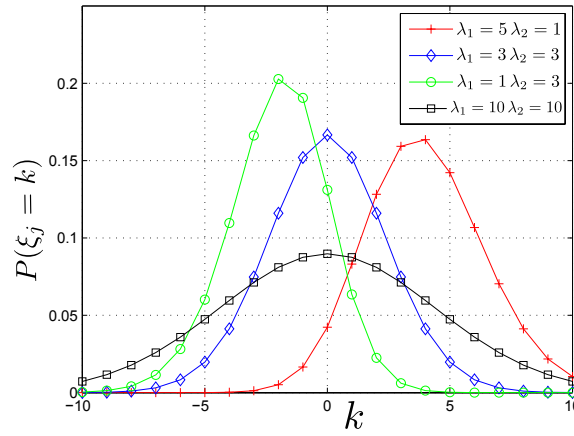Figure 3.7: Actual and theoretical demand distributions for two Stations

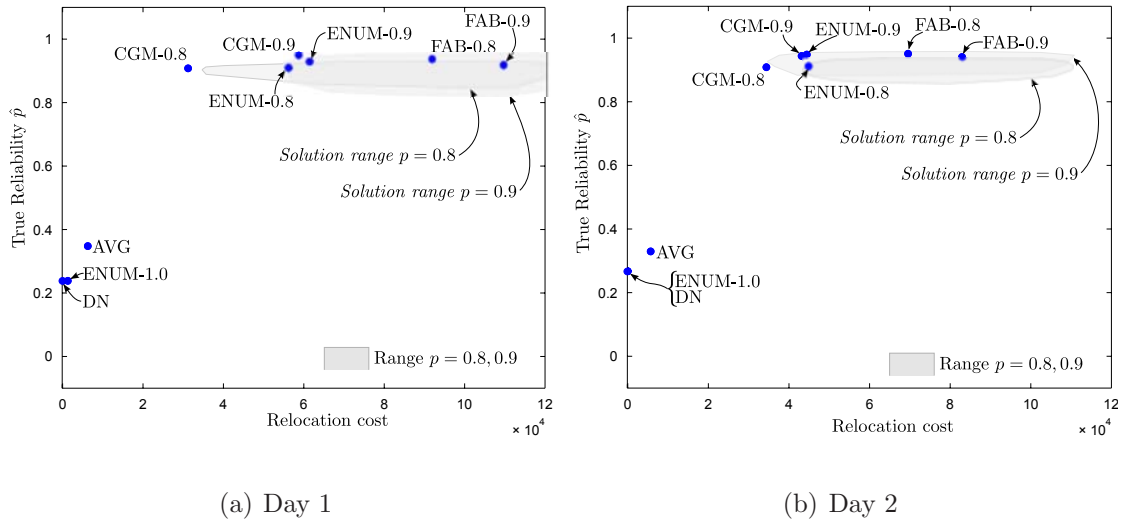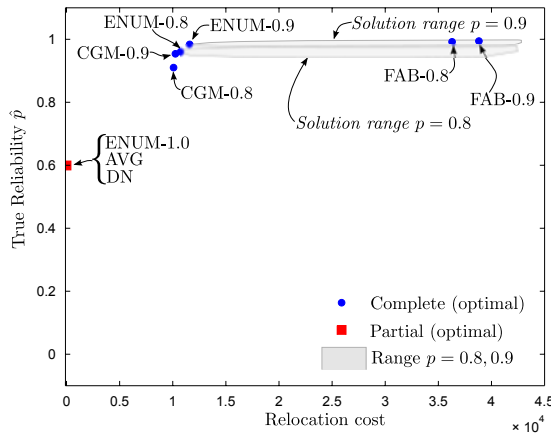Figure 3.8: The Skellam distribution function for sample λ combinations.
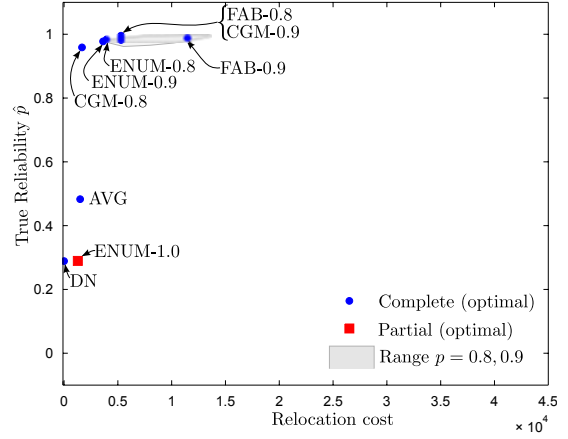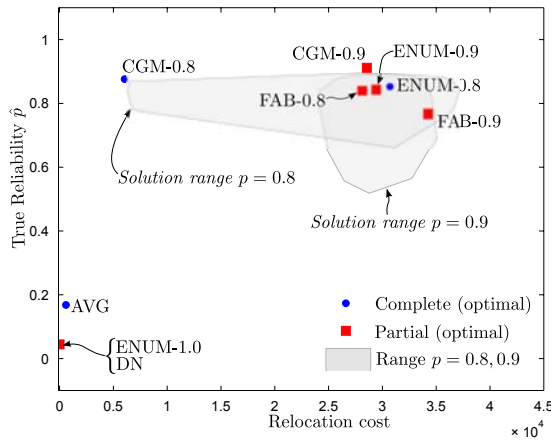

(a) Day 1

(b) Day 2

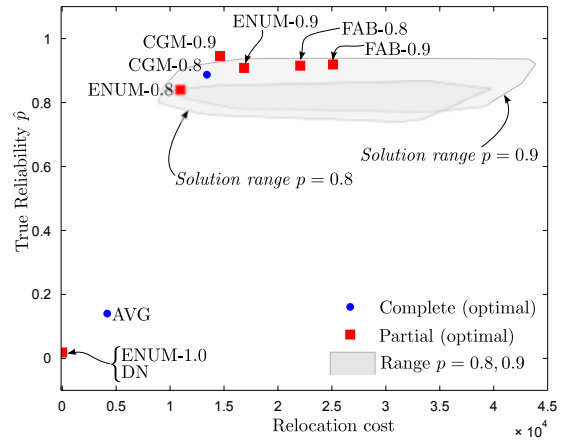Figure 3.9: Average actualized reliabilty vs. relocation costs over entire day

(a) Period 00:00 to 09:00 Hrs

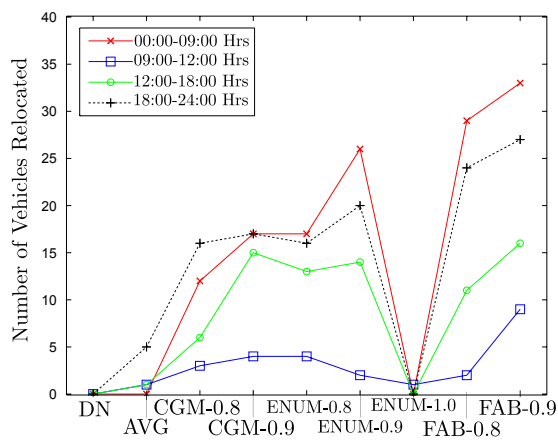(b) Period 09:00 to 12:00 Hrs

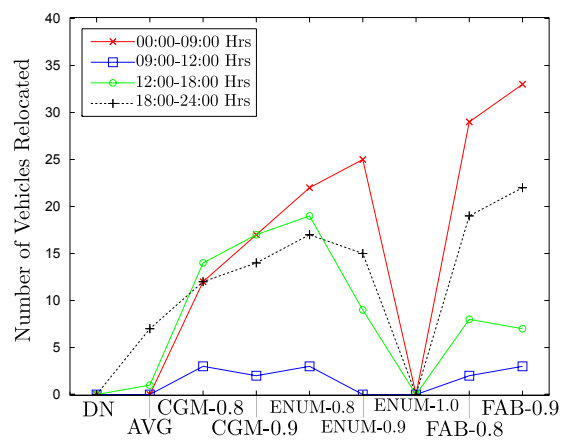(c) Period 12:00 to 18:00 Hrs

(d) Period 18:00 to 24:00 Hrs

Figure 3.10: System snapshots for various time periods (Day 1)

(a) Day 1   (b) Day 2

Figure 3.11: Number of vehicles relocated

Chapter 4: System Design For Washington D.C.

4.1 Introduction

The Washington D.C. metropolitan region ranks second in traffic congestion in the U.S. [44]. To address the growing transportation problems in the region, the District of Columbia Department of Transportation (DDOT) has actively promoted the use of alternate solutions to highway congestion. These strategies include facilitating increased bicycle use, improvement of pedestrian facilities, and the promotion of shared-vehicle systems. These goals are being met through the innovative use of transportation related policy. In the North American context, DDOT has pioneered the use of on-street parking for shared cars to increase the utility of shared vehicles for users. The Washington Metropolitan Area Transit Authority (WMATA) exclusively reserves several parking spots at its transit stations to integrate shared-vehicle and transit use.

In August, 2008, Washington D.C. was the first city in the US to implement a bicycle sharing program named SmartBikeDC. As a public-private partnership, the program started with 10 stations, with capacities ranging from 10 to 15 bicycles in the downtown areas of Washington. Following the success of this initial system, in March 2010, DDOT in collaboration with neighboring Arlington County, announced a new system of 100 stations and 1000 bicycles that will supersede SmartBikeDC. The new system will be operational in the Fall of 2010 and will use the 'Bixi' modular concept already in place in cities like Montreal. The Bixi concept was

developed for Montreal to overcome some design and implementation issues faced by earlier generation of systems. By making solar-powered stations and using wireless communications, the setup costs are reduced since utilities need not be extended to each station. This also allows for quick installation, easy removal of stations over the winter, and capacity enhancements.

In this chapter, based on the framework introduced in Chapter 2, an optimal configuration for the proposed bike sharing system is determined. In discussions with DDOT representatives, several site and equity constraints were established. The new program, termed *Capital BikeShare*, spans Washington D.C. and Arlington County, Virginia. Several program characteristics were determined in negotiations between the city and the service provider. The program is to have 114 stations, with 100 stations in the District of Columbia and 14 stations in Arlington County. Tentative pricing information released indicates an annual membership of $80 that would allow for unlimited number of rides under 30 minutes. Trips lasting more than 30 minutes will be charged a yet undecided fee for every additional half-hour. Day-use memberships with a cost of $5 and monthly memberships for $30 are also planned.

The main contributions of this chapter are in extending the bilevel VSP network design problem presented in Chapter 2 for a real-world case study for Washington D.C. The proposed bicycle sharing program expansion has problem specific characteristics that are incorporated into the VSP design problem. A Genetic Algorithm (GA) based meta-heuristic approach is developed for its solution. In addition to determining the near-optimal VSP configuration, the system usage patterns are

forecasted and flow potential between stations is quantified. Improvement in travel times through the use of shared-vehicles is also estimated.

4.1.1 Setting

As the nation's captial, Washington D.C. is home to several large federal institutions, museums, foreign embassies, and relatively dense residential neighborhoods. Key relevant social and urban characteristics of the city are summarized in Figures 4.1 and 4.2.

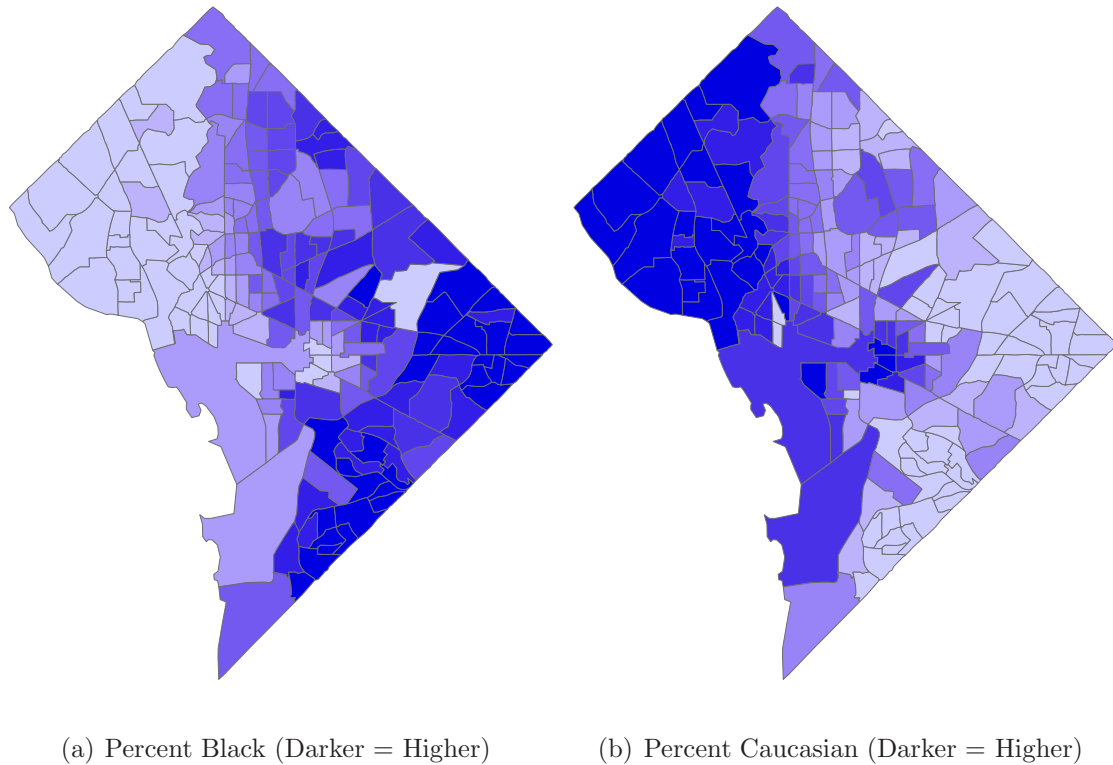(a) Percent Black (Darker = Higher)   (b) Percent Caucasian (Darker = Higher)

Figure 4.1: Social characteristics of Washington D.C.

Transportation related issues for the greater Washington region are handled through the Metropolitan Washington Council of Governments (MWCOG), a regional planning entity that serves the Greater Washington metropolitan area, in-

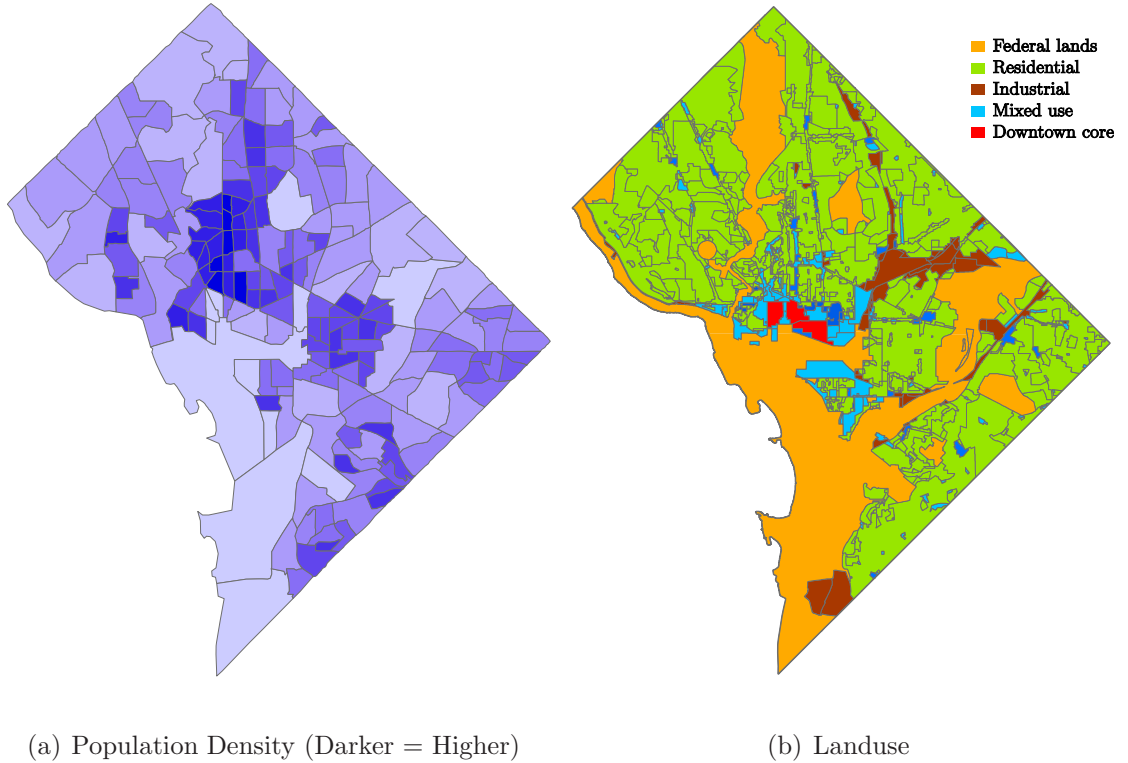(a) Population Density (Darker = Higher)   (b) Landuse

Figure 4.2: Urban characteristics of Washington D.C.

cluding D.C., Maryland, and Virginia. The main transit agency for the region is WMATA that runs a metro system with six lines, 88 stations, and 106 miles of transit lines. In addition, WMATA operates the Metrobus service consisting of 319 routes, serving 12,227 bus stops across the region. Transit plays a vital role in the region, especially in the urban core, where 42% of all work-based trips use transit [58]. As shown in Figure 4.3, access to metro stations in the downtown region of D.C. are predominantly by walk or bicycle, while stations in the outer suburban areas have large parking facilities that attract park-and-ride users from the suburban residential communities [36].
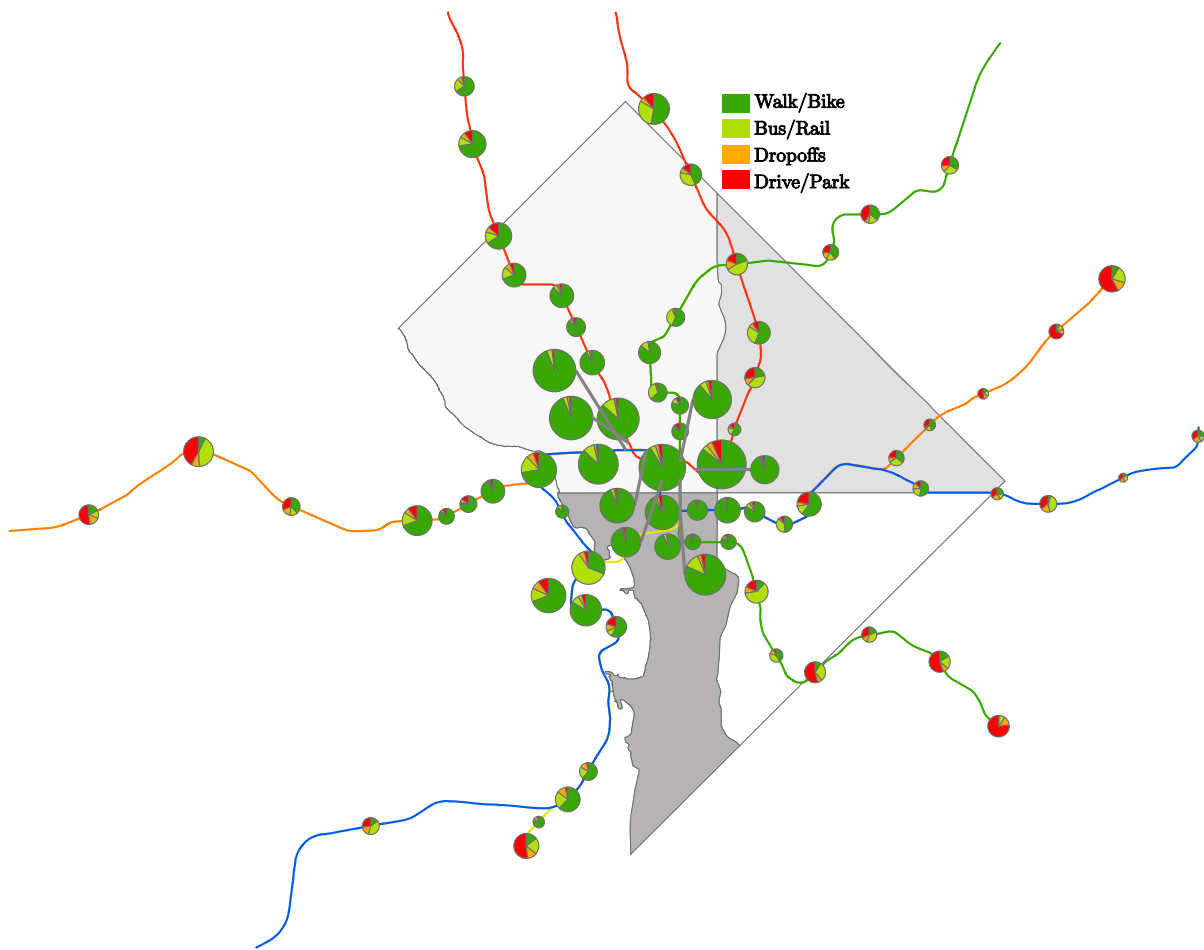
Figure 4.3: Access modes of passengers to metro stations

4.1.2 Model Components

Following the conceptual developments in Chapter 2, the key supply and demand characteristics need to be represented in a network modeling framework. The network is defined by a set of nodes that represent various elements in the transit and VSP system. Nodes can represent transit stops, transit services, origins, destinations, link intersections for walk, bicycle and road networks, and candidate VSP stations. Network links are used to connect the nodes and can represent walk links, shared-vehicle movement, transit services, transit access and egress, waiting links, bicycle links, and road links. The case study network uses data collected from

various sources. The transit information is based on data from WMATA, while the walk and bicycle network information are from the D.C. Office of Planning. DDOT provided information on VSP candidate stations. The data attributes and its limitations are described in greater detail next.

4.1.2.1   VSP Candidate Stations

Washington, D.C. consists of eight wards. Wards are political districts used to elect members to the City Council. The design of any public system faces equity concerns regarding the distribution of facilities across wards. Therefore, in determining the candidate shared-bicycle stations, an adequate number of candidate facilities must be considered for each ward. To generate the set of candidate stations, all major transit stations within the D.C. jurisdiction were considered. Additionally, from the 1,403 signalized intersections in D.C., a randomly selected subset was chosen to serve as candidate VSP sites. Signalized intersections are chosen as candidate sites, since they tend to have greater visibility in an urban setting. Figure 4.4 shows the candidate stations along with ward boundaries. A total of 455 candidate sites for VSP stations are considered. The modular Bixi bicycle sharing technology uses solar-powered stations and wireless communications. Since the proposed VSP stations are self-supporting, the selection of VSP candidate stations is limited only by physical space and not by site accessibility to electricity or communication utilities. Site accessibility to utilities is not considered in determining candidate sites.
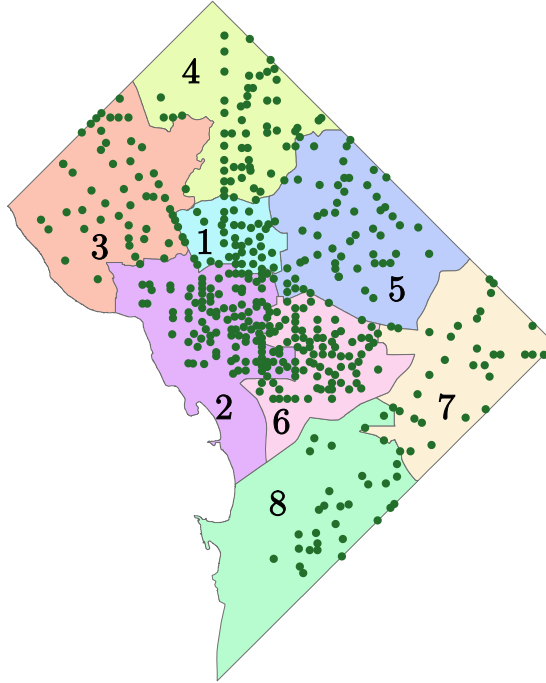
Figure 4.4: Candidate VSP stations with ward boundaries

4.1.2.2 Travel Demand

The VSP design is based on flow patterns in the region. The demand for travel for the analysis is based on the Maryland Statewide Transportation Model (MSTM) [15]. The MSTM has 89 zones within D.C and uses the four-step transportation planning process to determine flows between zone pairs for base year 2000. Starting from landuse patterns, the models use trip generation, distribution, and mode choice. Additionally, MSTM performs a temporal allocation for four different time periods during the day. Inputs from a coarser regional model that has visitor and long-distance travel models are also incorporated.

For the purposes of this study, the total demand (without the mode choice

component) from the MSTM model provides input to determine flow potential of the proposed shared-bicycle network. An ideal representation of the demand process would include the transit-based flows along with utility functions that could be used to estimate the fraction of automobile trips that are induced due to the improved service from the presence of VSPs. However, three reasons preclude such a detailed representation. First, the fraction of auto users that shift to VSPs requires detailed stated preference information that is typically gleaned from surveys. Such an analysis is beyond the resources available for this study and is outside the focus of this research. Second, the response of users to new and innovative programs is highly uncertain and difficult to predict. In this light, the value of stated preference surveys can be limited. Third, transit flows from MSTM are undergoing revisions to address quality issues while this research was being conducted. Should better estimates of demand be available, these can be easily incorporated.

Since the state-wide model includes long distance trips from outside the region, five external zones were defined that serve as proxy zones for all external zones. Each zone outside D.C. is assigned to one of the external zones. Only inter-zonal flows that generate more than 20 trips/hour are considered. This corresponds to 1,000 origin-destination pairs. Each zone is associated with a centroid, where the flows are assumed to originate. Each zone centroid is linked to the walk network to allow flows to access transit services.

4.1.2.3 Transit Network

The transit component of the network is built from the Google Transit Feed Specification (GTFS) data from WMATA. While the data includes stops, service, and schedule information for all WMATA services in the region, computational issues involved in modeling a large transit network preclude the use of the entire network. Instead only the main transit services are considered including the metrorail lines. For these services, travel times and frequencies are computed from the schedule information. Each transit stop within D.C. is associated with a VSP candidate station.

4.1.2.4 Walk and Bicycle Network

Transit access is considered through walk and bicycle modes. Based on available data for metro access, for regions within D.C., a large majority of transit based trips are accessed by these two modes (see Figure 4.3). Transit and bicycle access networks are based on the network data provided by the D.C. Office of Planning. The network information includes information on bicycle lanes, travel times, and metro entry points, thereby providing reasonably accurate estimates of access times. The VSP candidate stations are located on the bicycle network and topologically linked to the bicycle network.

4.2 Formulation

The problem faced by D.C. Department of Transportation (DDOT) is to determine locations of VSP stations that provide an efficient and equitable configuration so as to maximize the flow potential of the proposed system. The number of stations to be located is known *a priori*; however their locations are to be determined. Unlike the network design problem addressed in Chapter 2, the station capacities and base vehicle inventories at each station are not considered for the following reasons. The technology used by DDOT for the proposed system developed by Bixi is modular in design. This implies that vehicle and slot inventories can be easily altered to better match demand. Secondly, many of the program parameters (number of stations, pricing, and technology) have already been determined by policy makers, so the value of this case study is in determining the optimal locations for VSP stations.

Given (a) an existing transit configuration, (b) the bicycle and walk network, (c) a set of candidate bicycle sharing stations, (d) equity and resource constraints for DDOT, (e) behavioral assumption of users, and (f) fixed demand, DDOT seeks an optimal VSP configuration that maximizes VSP flow potential.

Following the notation and development of Chapter 2, the VSP operator decision variable is $x_i, i \in V_s$, which is a binary variable indicating if the bicycle sharing station is located at node i and V_s is the set of candidate sharing stations. Let p represent the number of stations to be located and W be the set of wards in the district (indexed by w). Define $\delta_{iw}, w \in W, i \in V_s$, as an indicator variable that is set to one if the VSP station candidate i is located in ward w. Additionally, denote

$w_{\min}$ as the minimum number of stations at each ward that represents an equitable resource allocation.

The model presented in this chapter and used for the D.C. case study is a variant of the BLNDP (Chapter 2, Section 2.2) and is denoted by BLNDP-DC. At the upper level, VSP operators decide the optimal locations for VSP stations subject to equity constraints and the number of stations. DDOT seeks to maximize the potential shared-flow on the proposed bicycle sharing system. The lower level model mirrors the one presented in Section 2.2. OD pairs are denoted by the set K (indexed by k). A link from node i to j has an associated flow for each OD pair denoted by v_{ijk}. Additionally, each node of the transit network has an associated quantity w_{ik}, representing the total waiting time experienced by users in OD pair k at node i. The flows (v) and waiting times (w) are continuous decision variables of the lower level model.

The network design problem, BLNDP-DC, can be formulated as a bilevel mixed-integer program as follows.

Upper level:

$$\max_{x,y,z} \sum_k \sum_{(i,j)\in A_s} v_{ijk} \tag{4.1}$$

subject to

$$\sum_{i\in V_s} x_i = p \tag{4.2}$$

$$\sum_{i\in V_s} \delta_{iw} x_i \geq w_{\min} \qquad w \in W \tag{4.3}$$

$$x_i \in \{0,1\} \tag{4.4}$$

Lower level:

$$\min_{u,v} \sum_k \left(\sum_{(i,j)\in A} c_{ij}v_{ijk} + \sum_{i\in V} w_{ik} \right) \qquad (4.5)$$

subject to

$$\sum_{j,(i,j)\in A} v_{ijk} - \sum_{j,(j,i)\in A} v_{jik} = g_{ik} \qquad i \in V, \, k \in K \qquad (4.6)$$

$$v_{ijk} \leq f_{ij}w_{ik} \qquad (i,j) \in A \setminus \underline{A}, \, k \in K \qquad (4.7)$$

$$Mx_i \geq \sum_k v_{ijk} \qquad (i,j) \in A_s \qquad (4.8)$$

$$Mx_j \geq \sum_k v_{ijk} \qquad (i,j) \in A_s \qquad (4.9)$$

$$w_{ik} \geq 0 \qquad i \in V, \, k \in K \qquad (4.10)$$

$$v_{ijk} \geq 0 \qquad (i,j) \in A, \, k \in K \qquad (4.11)$$

The BLNDP-DC is defined by the upper level objective for DDOT of max-
imizing VSP flow potential (4.1). Constraint (4.2) limits the number of bicycle
sharing stations chosen for construction to the maximum allowable number of sta-
tions, which is determined *a priori* and is equal to 100. Constraints (4.3) ensure
that each ward in D.C. receives a minimum number of stations for the configuration
to be equitable. The upper level decision variables are restricted to be binary (4.4).

The lower level program is formulated from the perspective of users who seek
to minimize their travel time. Objective 4.5 minimizes the travel time and waiting
time for transit services for all OD pairs. Constraints (4.6) are the flow conservation
constraints. Constraints (4.7) relate waiting time at nodes to frequency of services on
transit links servicing that node. Constraints (4.8) permit flow on links originating
from a candidate VSP station only if a station exists. Similarly, constraints (4.9)
ensure that shared-bicycle flows terminate at a node only if a station exists at the

node. The flow and waiting variables are non-negative and continuous as required by constraints (4.10) and (4.11).

This model differs from the one presented in Section 2.2 on the following aspects. The VSP configuration is simply defined by station locations. Therefore, the upper level problem is solely a location problem. The station capacities and vehicle fleet inventories are not considered in this case study for two reasons. First, accurate estimates of demand were unavailable for this study (for reasons documented in Section 4.1.2.2). Since the vehicle inventory and VSP station sizes serve as capacity constraints in the BLNDP formulation, their inclusion could result in suboptimal configurations in the face of 'true' demand. Therefore, this variant recognizes limitations of the available data. Second, the Bixi technology is highly modular. This implies that stations can easily be reconfigured to match observed demand levels. The value of determining VSP station sizes and vehicle inventories is therefore limited. VSP station locations, on the other hand, are less flexible. Frequent users expecting a bicycle sharing station at a particular location are likely to be surprised if the entire station is relocated. Therefore, changes to VSP stations can only be done judiciously. Instead of total VSP system flows, the BLNDP-DC problem measures flow potential that represents the market potential of the VSP system at a particular configuration. Despite the fewer VSP configuration design variables, the size of the network precludes the direct application of the exact solution method presented in Section 2.3. A metaheuristic scheme for the solution of the problem is presented next.

4.3   Genetic Algorithm Based Solution Method

A GA is proposed for the solution to the BLNDP-DC. The use of a GA is motivated by the computational challenges that bilevel mixed-integer programs present. In addition, the size of a real-world instance such as that involving the proposed bicycle sharing program in D.C. make the exact solution method presented in Section 2.3 impractical. Details of GAs have been comprehensively discussed elsewhere [26, 27] and are omitted here. The basic concept in the context of optimization involves a set of solutions (termed a population). Each solution is represented by a vector (termed a chromosome) that encodes a solution to the problem. A solution vector can be a vector of real numbers, a list of nodes, a path, or a bit string. The representation of the solution is highly problem-dependent and can be determined by the algorithm designer. The algorithm progresses by ranking the chromosomes based on a fitness function. At each generation, chromosomes reproduce through a variety of *operators* to create a new set of chromosomes. Chromosomes with good fitness values are preferred to ones with poor fitness values. With subsequent generations, the chromosomes evolve towards the optimal solution. The exact nature of operators to use, the proportion of the population to use them on, and the manner in which newly create chromosomes are included or disregarded are left to the algorithm designer. Some typical operators include *elitism*, whereby the best solution (or solutions) are retained and directly carried over to the next generation; *mutation*, which involves random changes to a chromosome; and *crossover*, where two chromosomes are combined to generate two or more new solutions. For constrained

programs, such as the upper-level program in BLNDP-DC, several techniques exist to address the feasibility of solutions. The penalty method involves penalizing infeasible solutions to drive them to extinction. Alternatively, the operators can be designed in a manner that no infeasible solution is generated. Finally, the algorithm terminates when there is inadequate improvement in the fitness function.

The main idea behind the GA developed herein is that the two levels of the bilevel program can be solved separately. The mutual dependence of the two levels can be replaced by a scheme where the upper-level variables are generated externally and the resulting flows recorded by solving the lower-level problem. The generation of upper-level location variables can be performed in a genetic algorithmic framework. The design vector is encoded as a genome and uses the lower-level transit assignment based model to evaluate the fitness of the design vector. A design configuration that maximizes the shared-vehicle flow is considered as propogated through the generations to yield an near-optimal configuration. The advantage of such an approach is that the optimization component of the problem is reduced to solving multiple linear programs (as opposed to MIPs), while the GA operators are used to deal with the integrality constraints. Steps in developing a GA for the BLNDP-DC are presented next.

4.3.1   Solution Representation

The chromosome is characterized simply by the location variables x and is represented by a bit string. If the candidate VSP site is to be constructed, the

respective bit has the value 1, and 0 otherwise. The entire chromsome is a bit string that has the length equal to the number of candidate sites. This representation is straightforward and allows for operates to verify feasibility of a particular chromosome easily.

4.3.2 Initialization

The initial population is generated by solving multiple times the binary integer program

$$\max \quad \sum_i \xi_i x_i \tag{4.12}$$

$$s.t. \quad \text{Constraints } (4.2),(4.3),(4.4) \ ,$$

where ξ_i is a randomly generated coefficient in the unit interval. By changing the objective coefficient, the value of each station varies randomly, thereby generating a diverse set of starting solutions. By this method, all generated initial solutions are feasible, since all constraints of the BLNDP-DC are explicitly considered.

4.3.3 Evaluation

To evaluate the fitness of a particular solution, the lower-level portion of the BLNDP-DC, i.e. (4.5)–(4.11), needs to be solved. Note that when the design vector x is fixed, the lower-level problem is a linear program that represents a transit assignment model with side constraints to deal with shared-vehicle flows. The evaluation step requires the solution of a linear program. If a network has m links, n nodes, and k OD pairs, the number of continuous variables in this formulation is

$k(n+m)$. If the network is large with many OD pairs, as is the case with the D.C. network, this may result in millions of variables. For such cases, the problem can be solved independently for each OD pair and the link flows on each link summed across all OD pairs. This dissaggregated approach is employed for this case study. For smaller networks, where the lower-level model is solved jointly for all OD pairs, one additional insight aids in reducing run times. Since the design vector occurs only on the right hand side of the lower-level model, the optimal basis from one function evaluation can saved as a starting basis for the next evaluation after changing the right hand side. For large networks this may be computationally advantageous since new problem instances need not be recreated and few simplex iterations may be needed to find the optimal solution.

4.3.4 Evolution

To evolve the population of solutions from one generation to the next, three operators are utilitized. Methods for handling infeasibilities are presented, along with the description of the operators.

Crossover: The crossover operator works on two solution strings jointly. A solution substring, representing the location configuration for a randomly selected ward, is swapped between the two solutions. Essentially, a spatially proximate subset of the VSP configuration is exchanged between the solutions. Two new solutions are generated whose feasibility is not guaranteed, since the total number of stations in the configuration may not equal the number of required stations p. To rectify po-

tential infeasibilities, two corrective procedures are initiated. For the new solution, if the number of VSP stations exceeds the designated number of stations p, then the required number of VSP stations are randomly dropped in a manner so as not to violate the minimum number of stations in each ward $w_{\min}$. If the number of VSP stations is less than the designated p, then the required number of stations is randomly added to the solution string.

Mutation: The mutation operator is applied to one randomly selected solution in the population. Two VSP candidate sites within a ward, one with and the other without a VSP station, are selected and swapped. This operator effectively relocates one VSP station from one candidate site to another. By selecting two candidate sites within the same ward the total number of VSP stations within the ward remains unchanged. Therefore, if the original solution is feasible, the result of the mutation operator is also feasible.

Elitism: The elite, a small percentage of chromosomes that have the best fitness function value, are directly moved to the subsequent generation.

4.3.5   Termination Criteria

Several criteria are considered jointly to terminate the algorithm. If any one of the criterion are met, the procedure terminates. The maximum number of generations is limited to 100. The number of stall generations, defined as the number of generations that are evaluated without an improvement in objective function, is set to 5. The amount of time without improvement in objective function is set to

$10,000$ seconds.

The proposed GA is implemented in MATLAB with the lower-level transit assignment routines run using CPLEX 11.2 accessed through Java. Results and analysis from the GA are presented next.

4.4   Analysis

The D.C. study area resulted in a network with $10,380$ nodes and $32,402$ links as summarized in Table 4.4. Flows between $1,000$ OD pairs were considered. The chosen OD pairs generate 20 trips/day or more. The metaheuristic solution method described ran to termination in 41 generations. The procedure terminates since there is no improvement in objective function over 5 generations. The values for the best upper-level objective function and its corresponding lower-level objective at each generation are shown in Figure 4.5. Note that at termination, the final solution for the upper-level does not correspond to the best lower-level solution. A better lower-level solution is found at generation 30 (Figure 4.5(b)) though this does not lead to a better VSP configuration for the VSP operator. This demonstrates that the objectives are not complementary and that what is best for the VSP operator is not necessarily the best for individual system users. Additionally, the lower-level objective represents the total travel and waiting time for all OD pairs and is not weighted by demand for travel for that OD pair, which is taking into account only at the upper level.

Next, the VSP configuration chosen by the GA upon termination is discussed.

Nodes		Links	
Type	# Num	Type	# Num
VSP candidates	455	Bicycle network	28,502
Origins/destinations	89	Transit services	226
Transit stops	88	Transit access	454
Transit services	236	Walk network	2,226
Bike network	9514	VSP station connector	910
		VSP-transit modal interface	84
Total	10,382	Total	32,402

Table 4.1: Summary of network component for Washington D.C.
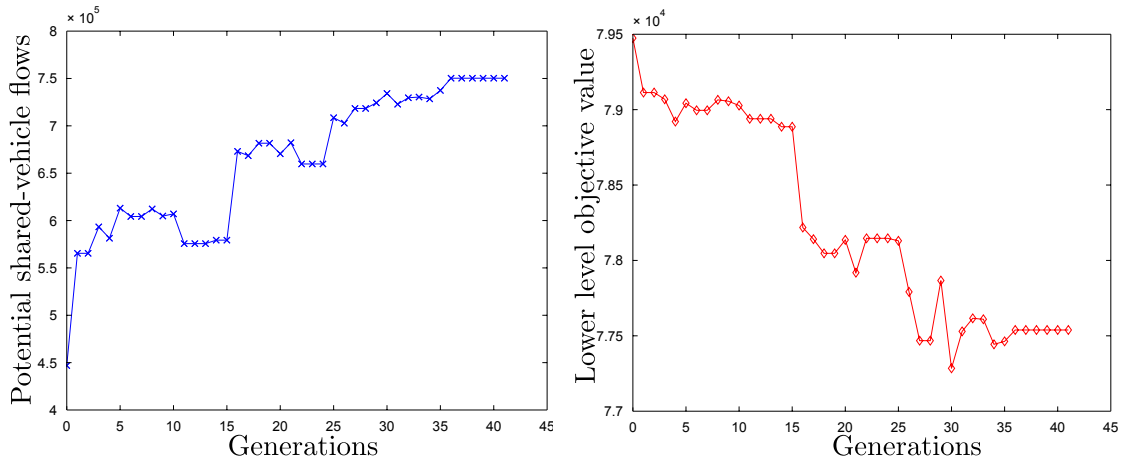
(a) Upper-level objective function   (b) Lower-level objective function

Figure 4.5: Best solution at each generation of GA

The map of recommended VSP stations along with induced flows are presented in Figure 4.6. Of the 100 stations, the near-optimal solution locates 19 stations at metro stations for VSP integration with transit. In comparison, the base flows are shown in Figure 4.7.

The improvements in travel times for users can be dissaggregated by zone to see the effects of introducing the bicycle sharing system. Figure 4.8 shows the average travel time reduction for trips by origin and destination zone with darker zones indicating higher improvements.

In addition to the configuration of the bicycle sharing program, the inter-station flows can be forecasted. The results of the lower-level model include path information. All intermodal paths that use a shared-vehicle component are examined and the origin and destination VSP stations are recorded. These statistics are aggregated across all OD pairs. The result is a large matrix of inter-station flows. To discern flow patterns, the matrix of flows is plotted as a circular diagram using Circos [31] as shown in Figure 4.9. The stations are represented along the circumference of the diagram. The length of each arc represents the flow potential at each station relative to the system. The bands between two stations are wider when the flow potential is greater. Based on Figure 4.9 the flow potential at transit stations and commuter hubs (such as Union station) are largest. This provides a key insight into the configuration of VSP stations and the important of VSP-transit integration. There are considerable asymmetries in flow potential from one station to another.

Table 4.2 shows the number of stations per ward associated with the solution. The number of stations for each ward satisfies the upper-level equity constraints
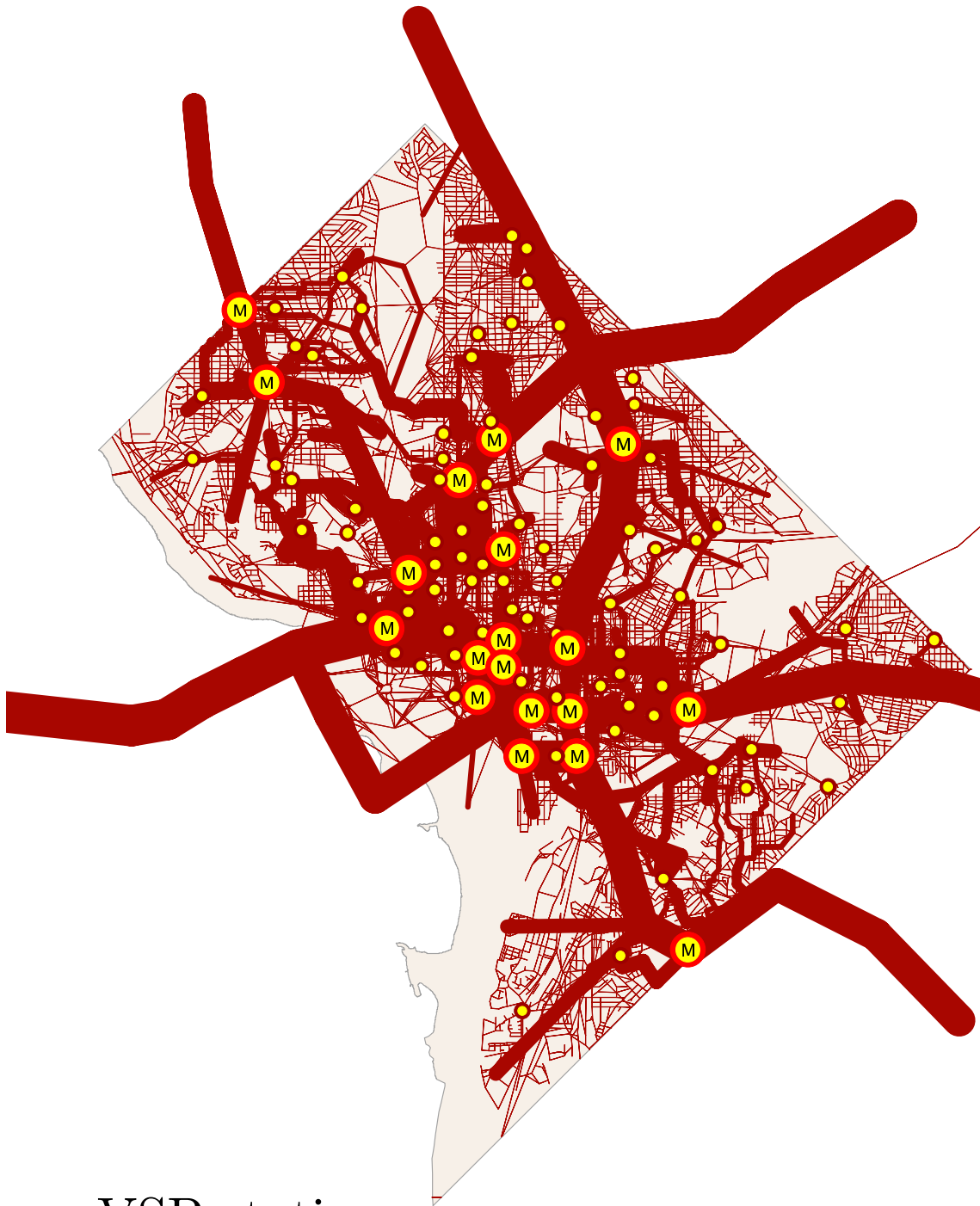
- VSP station
- Ⓜ VSP station at Metro stop

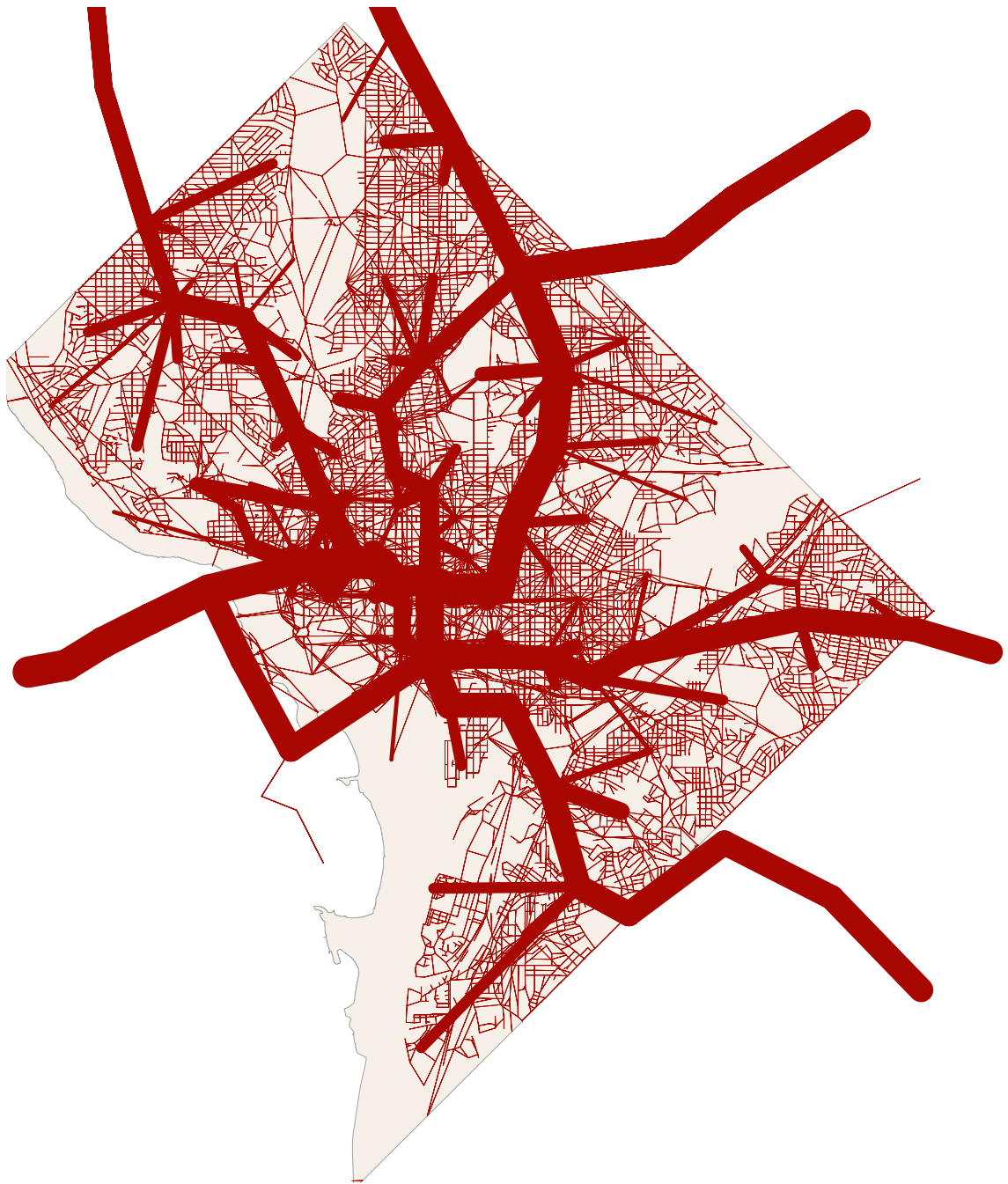Figure 4.6: Transit-based flows with VSP showing recommended VSP configuration

Figure 4.7: Transit-based flows without VSP
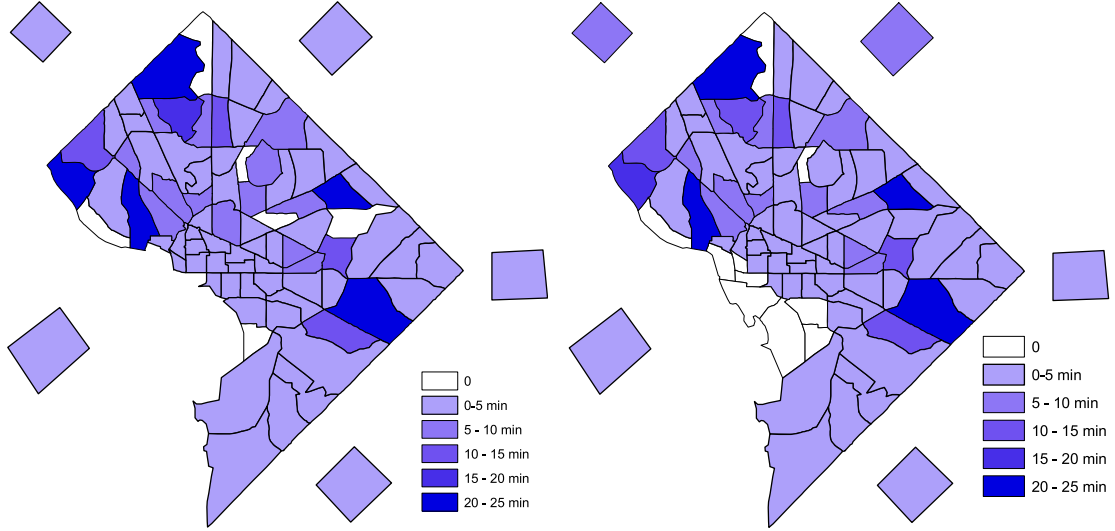
(a) for trips originating from zone

(b) for trips terminating in zone

Figure 4.8: Average travel time reduction in minutes

Ward	# VSP Stns
1	11
2	24
3	12
4	10
5	13
6	18
7	7
8	5

Table 4.2: Number of VSP stations by ward

(4.3). This equity constraint is binding only for Ward 8. Should the equity constraints be relaxed, it is beneficial to the VSP operator to relocation VSP stations from within Ward 8 to other parts of the network. Therefore, the utilization of the stations located in Ward 8 can be expected to be lower than in the rest of the network.

Figure 4.9: Flow potential between designed stations
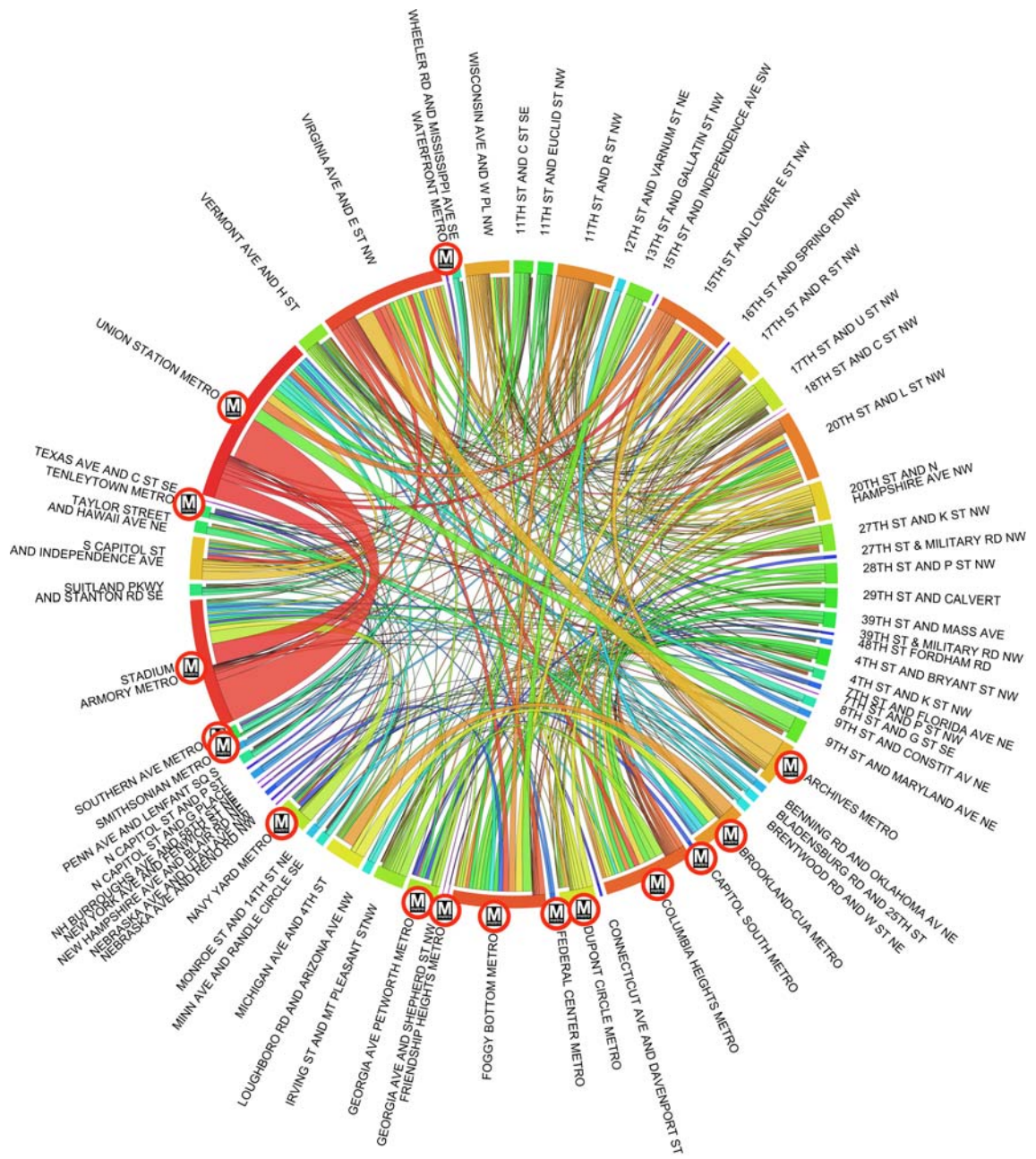
4.5 Conclusions

A bilevel mixed-integer program for the design of a bicycle sharing system in Washington D.C. is developed. A GA, designed for the solution of large real-world problem instances, is proposed. The metaheuristic solution recommends a VSP system configuration that provides several insights. 19% of all stations are located at metro stations indicating that optimal VSP configurations integrate with existing transit facilities to maximize flows on the shared-vehicle segments. The improvement in travel times as the result of VSP introduction is quantified. From the case study analysis, the average travel time for a trip can be reduced by 3.55 minutes solely due to the use of the shared-bicycle program. In addition to the effects on the environment of using a green mode, such information can potentially be used for economic analysis to justify (or oppose) investment in shared-vehicle systems. The analysis results also provide forecasts of flows between the proposed stations. Since the flows are asymmetric, operational policies can be developed for newly designed systems that can employ such information to maintain a desired level-of-service.

Several extensions of this research warrant further investigation. Since the genetic algorithm-based solution method separates the two levels of the bilevel program, the lower-level linear program can be sustituted by computationally more efficient transit assignment procedures, such as one that employs a hyperpath formulation (eg. [24, 35]). A new hyperpath procedure that accounts for the probabilistic availability of vehicles at VSP stations can also be developed. Large flow asymme-

tries between proposed stations imply larger operational costs involved in relocating bicycles. An alternate formulation could include the future operational costs into the design process so that asymmetries are avoided. The proposed framework can be placed in the context of the larger transportation problem by the inclusion of various modal alternatives, mode choice functions, and congestion effects.

Chapter 5: *Vélib'* Case Study

5.1 Setting

The draft strategic plan of the United States (US) Department of Transportation (DOT) seeks to diversify the transportation landscape in the US [56] through new livable community initiatives. With coordinated policies and investment strategies across federal entities in housing and the environment, the strategic plan refocuses the goal of providing safe, clean, and sustainable transport. Amongst several initiatives that aim to reduce automobile dependence, the draft strategic plan also places greater credence on pedestrian and bicycle modes. These federal level initiatives mirror the efforts of some local DOT's who grapple with increasing congestion and mobility problems. Though transit has long been preferred by transportation planners as a solution to congestion woes, data on traveler choices in the US shows it to be a losing proposition with just 1.5% of all trips and 4.7% of all commute trips [55]. A more recent approach to reduce automobile dependence in urban areas is through the use of shared-vehicle programs.

Shared-vehicle programs involve a network of strategically located stations that host a fleet of vehicles (bicycles, cars, or electric vehicles). In its most flexible form, users making a trip can check out a vehicle close to their origin and return it close to their destination. A shared-vehicle system can be construed as an individual mode (for short trips), or as a vital segment of an intermodal route (for longer trips). In the latter case, it serves as a vital 'last-mile' connection, the lack of which dissuades

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potential riders. In this role, shared-vehicle systems increase transit accessibility. They are strongly aligned with integrated transit systems explored in the past that also aim to increase the catchment area of transit ([54, 42, 47]).

The emphasis on livability at the federal level, growing urban mobility problems at the local level, and the potential benefits of shared-vehicles in alleviating some of the problems have lead to an increased interest in these systems for the U.S. The widespread adoption of such innovations in the urban transport sector is hindered to some extent by the uncertainty surrounding the response of the traveling public to such systems. Though bicycle sharing systems exist in over 100 cities [12], in the U.S. context, local transportation agencies and policymakers have limited precedent to aid in developing such systems for their communities. Several papers have studied the shared-vehicle system through the prism of marketing aspects [16], policy considerations [49], environmental concerns [28], technology challenges [54], growth trends [45], political issues [17], and user response [46]. The focus of this paper is on system configuration and operational aspects. Using trip information from the pioneering *Vélib'* bicycle sharing system in Paris, France, key system design aspects are highlighted. The system accounts for as many as 120,000 trips daily [21] and is considered very successful.

For users, shared-bicycles offer increased travel utility through flexibility and cost. They are free to choose their departure time, routes, and destinations. Compared to other modes, these systems are attractively priced. For the *Vélib'* system, a nominal annual membership fee ($37.50 as of July, 2010) entitles a member to and unlimited number of free trips that last 30 minutes or less for a year. To ensure

circulation, trips lasting longer are charged for every additional half hour accrued. *Vélib'* also offers daily or weekly passes. Additionally, transit fare passes (*Passe Navigo*) also work on the *Vélib'*,though there are no preferential fares for transit customers using *Vélib'*.

Increased flexibility of shared-bicycles places an exceptional logistical challenge on *Vélib'* operators who need to ensure future short-term demand for vehicles and parking slots are met. Since flow from one station to another is seldom equal to flow in the opposing direction, the VSP fleet can become spatially imbalanced over time. To meet near-future demand, operators must then redistribute vehicles between stations to correct this asymmetry. Since future demand is not known exactly and is highly variable (as shown in later sections), the challenges faced by operators is amplified. There is limited literature on fleet management for shared vehicles. A variety of approaches have been studied including approaches that employ simulation [7], mixed-integer programming [29], and multi-stage stochastic programming with recourse [22]. The mixed-integer chance-constrained program presented in Chapter 3 generates redistribution plans to meet a target reliability level. Should resources in the system be insufficient to meet the desired levels of reliability, the model generates partial redistribution plans that utilize existing resources at lower levels-of-service. While generating redistribution plans is not the focus of this chapter, the chance constrained program provides a probabilistic characterization of the system. Such a characterization quantifies key system performance measures that elucidates the efficiency of stations in dealing with uncertain demand. These measures allow for a quantitative, reliability-based analysis that is performed on the *Vélib'* system.

In this chapter key attributes of the *Vélib'* bicycle sharing program pertaining to system configuration and utilization are discussed. The public transit-*Vélib'* connection is explored. Flow patterns based on observed data and the extent of flow asymmetries between stations are presented. The demand for bicycles and parking slots is characterized probabilistically. Using the chance-constrained fleet management model from prior work [34] described above, several reliability-based performance measures are quantified to provide insights into the workings of a successful bicycle sharing system.

5.2   System Characteristics

As one of the largest bicycle sharing programs in the world, *Vélib'* has a fleet of 20,000 bicycles spread across 1,450 stations. Figure 5.1 shows the map of all stations in the Paris region. The operator of the system, JCDeaux, has provided trip data for a four month period from 1st March, 2009 through 9th July, 2009. Due to the proprietary nature of the dataset, the scale of some descriptive statistics are not presented though patterns suggested by the data are highlighted. The system logged 10,392,808 trips during this period, at an average of 79,945 trips per day. The temporal distribution of trips over the course of a day is plotted in Figure 5.2, showing that usage on a weekday follows familiar travel patterns with two peaks. The morning rush hour is shorter in duration than the evening one. The weekend use is void of commuter peaks and shows a different pattern.

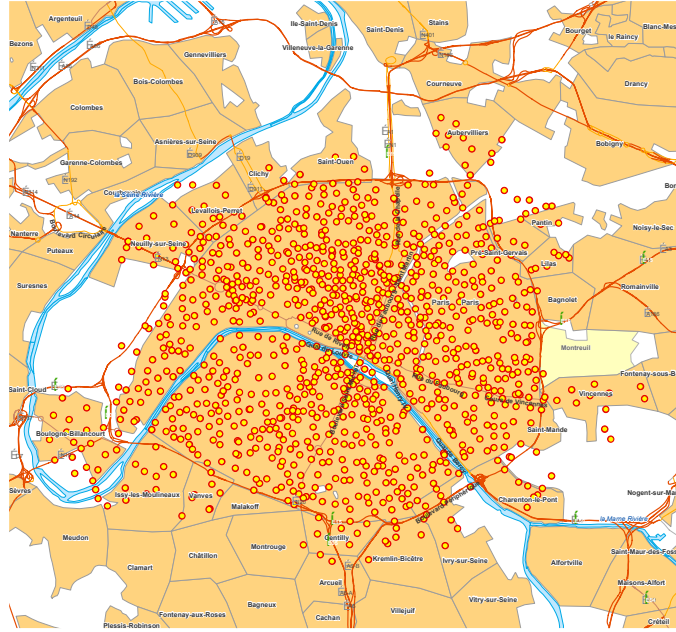The system is designed for a quick turnaround of bicycles and the pricing

Figure 5.1: Spatial distribution of *Vélib'* stations

structure discourages long term rentals. Figure 5.3(a) shows that majority of trips are concluded within the alloted free time of 30 minutes. A subset of stations, called *Vélib'+* are located in areas of low bicycle accessibility (high altitude or in the outskirts of Paris). The allotted maximum free travel time to these stations is increased to 45 minutes to allow for the additional travel time. Figure 5.3(a) shows 98% of all trips are completed within 45 minutes. Therefore, the overage charge is incurred by a marginal proportion of users.

5.2.1 Transit-*Vélib'* Interaction

The Paris Métro is one of the busiest transit systems in the world. The system consists of 16 underground lines and 300 stations. With a high density of stations within the urban core of Paris, it offers good transit accessibility and high levels-
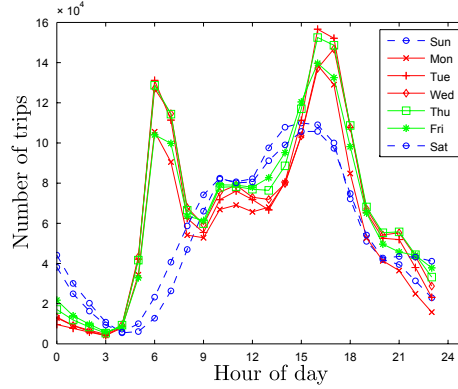
Figure 5.2: Temporal distribution of trips

(a) Travel time

(b) Travel distance
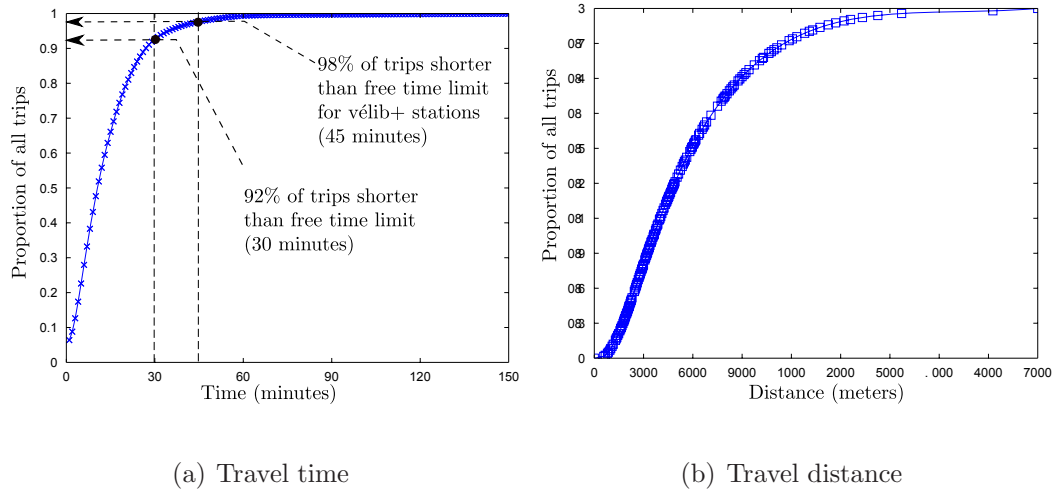
Figure 5.3: Cumulative distribution of travel characteristics

of-service. For transit users, the *Vélib'* offers a potential segment of transit-based intermodal trips. The configuration of the metro system and *Vélib'* are juxtaposed to evaluated the effectiveness of modal transfers between the two systems. Accessibility is used as a proxy for ease with which users can transfer between the systems. The nearest transit stop to each *Vélib'* station is determined. Figure 5.4 shows the distribution of distances of *Vélib'* stations and their corresponding closest transit stops. With the majority of the *Vélib'* stations being well within the pedestrian catchment area for transit (the zone defined by the maximum distance a transit

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Figure 5.4: Distribution of distances from *Vélib'* stations to closest transit stops

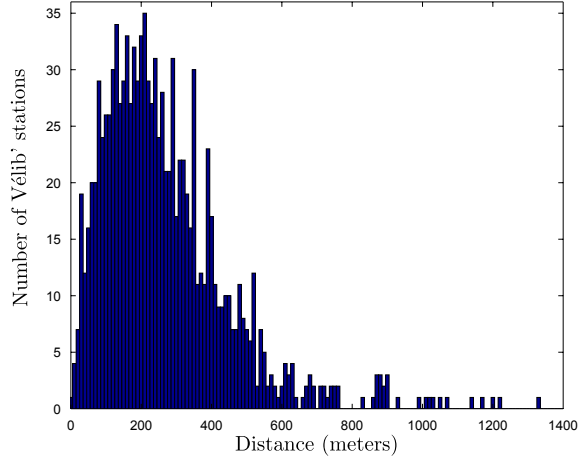user would walk to reach transit services) considered to be around 400 meters, *Vélib'* appears to be deeply coupled with the Paris Métro. By evaluating the utilization of *Vélib'* stations for the study period, the effectiveness of placing bicycle sharing stations close to transit stops hypothesized herein can be confirmed.

From the trip data, the stations are ranked based on utilization. The top 20 ranked stations are reported in Table 5.1 for incoming flows and Table 5.2 for outgoing flows. It can be seen that highly ranked *Vélib'* stations typically have spatially proximate transit stops and services. The exception to the rule being the *Vélib'* station 'Francs Bourgeois' located in the Marais district that houses several hotels.

Further evidence of the importance of the close coupling of transit and *Vélib'* is shown in Figure 5.5. Stations are represented as segments along the circumference with the size of the segment proportional to flows (both incoming and outgoing) handled by a particular station. Therefore, a large segment implies a *Vélib'* station with

Rank	Vélib' station	Closest metro stop (line)	Distance (m)
1	BEAUBOURG RAMBUTEAU	Rambuteau (m11)	139.87
2	FAUBOURG DU TEMPLE PLACE DE LA REPUBLIQUE	Republique (m3-5-8-9-11)	118.59
3	SAINT PAUL PAVÉE	Saint-Paul (m1)	79.00
4	QUAI DE LA LOIRE	Jaures (m2-5-7bis)	61.06
5	HOTEL DE VILLE	Hotel de Ville (m1-11)	84.83
6	LES HALLES SAINT EUSTACHE	Chatelet - Les Halles (rA-B-D) /m(1-4-7-11-14)	173.93
7	TRAVERSIERE	Ledru-Rollin (m8)	97.69
8	BASTILLE RICHARD LENOIR	Bastille (m1-5-8)	59.35
9	FAIDHERBE CHALIGNY	Faidherbe - Chaligny (m8)	132.59
10	TURENNE BRETAGNE	Filles du Calvaire (m8)	67.32
11	FRANCS BOURGEOIS	Saint-Paul (m1)	443.80
12	BOULEVARD VOLTAIRE	Voltaire (m9)	131.04
13	TOLBIAC NATIONALE	Olympiades (m14)	55.92
14	BASTILLE	Bastille (m1-5-8)	81.18
15	LOBAU	Hotel de Ville (m1-11)	83.84
16	RIVOLI SAINT DENIS	Chatelet (m1-4-7-11-14)	145.02
17	GARE DE LYON VAN GOGH	Gare de Lyon (m1-14)(rA-D)	44.68
18	CROZATIER	Ledru-Rollin (m8)	113.67
19	JACQUES BONSERGENT	Jacques Bonsergent (m5)	138.03
20	ODEON QUATRE VENTS	Odeon (m4-10)	26.13

Table 5.1: Top 20 Vélib' stations with highest inflows with distance to closest transit stop

high utilization. Figure 5.5 shows all inter-station flows that average 10 trips/day or more. The major Vélib' stations, as depicted by the size of the segment on the circumference, can be seen to have close correspondence to transit stops (denoted by 'M'). Though not presented here, the data also reveals the existence of secondary Vélib' stations that serve as a buffer to major ones when they are full.

5.2.2 Flow Asymmetry at Stations

By completing trips from one station to another, users shift bicycle inventory from one portion of the network to another. For operators, this presents a logisti-
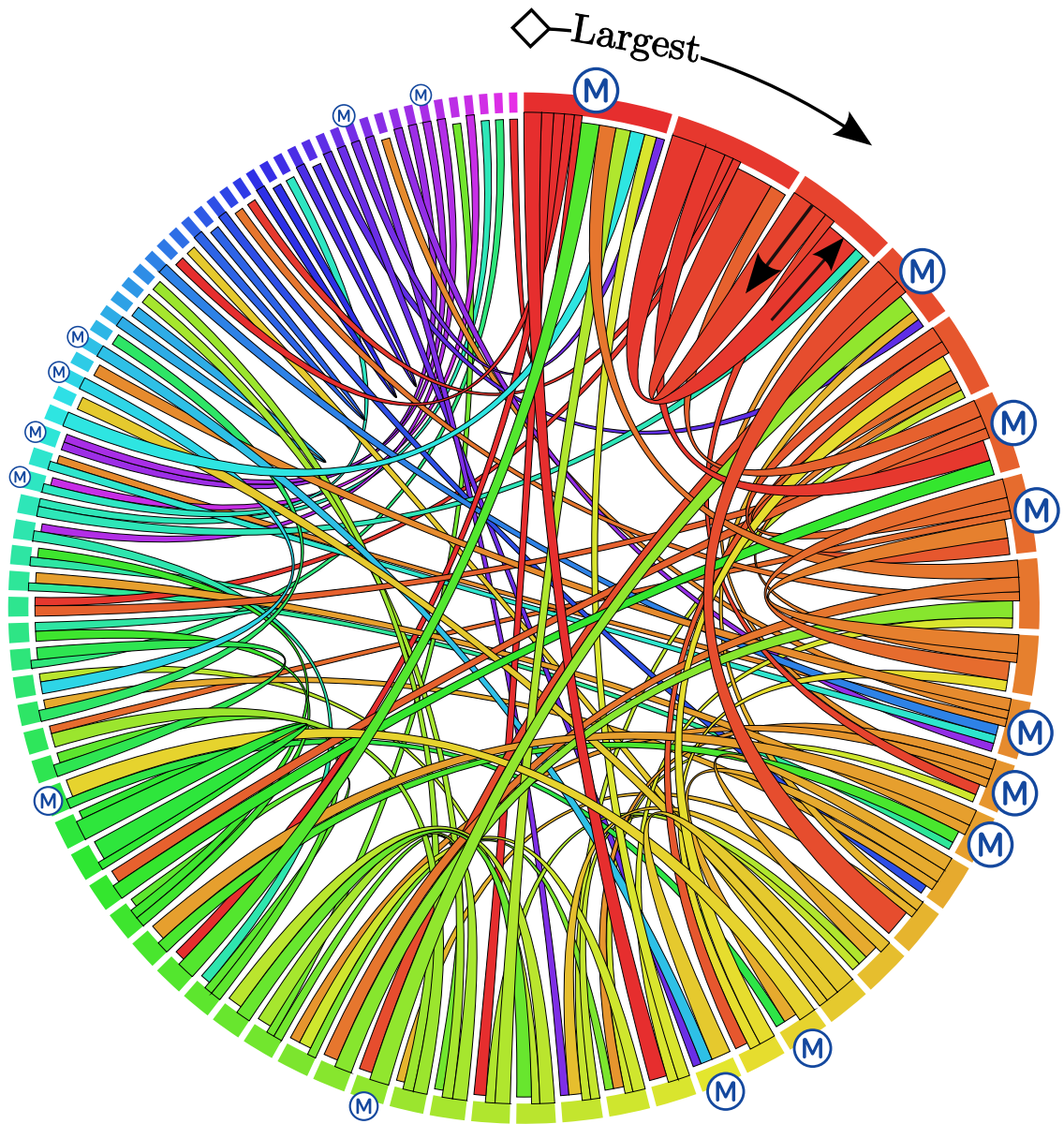
Figure 5.5: Average inter-station flows exceeding 10 trips/day highlighting *Vélib'* stations within 100 meters of Métro station

| Rank	*Vélib'* station	Closest metro stop (line)	Distance (m)
1	BEAUBOURG RAMBUTEAU	Rambuteau (m11)	139.87
2	FAUBOURG DU TEMPLE PLACE DE LA REPUBLIQUE	Republique (m3-5-8-9-11)	118.59
3	SAINT PAUL PAVEE	Saint-Paul (m1)	79.00
4	QUAI DE LA LOIRE	Jaures (m2-5-7bis)	61.06
5	HOTEL DE VILLE	Hotel de Ville (m1-11)	84.83
6	HALLES SAINT EUSTACHE	Chatelet - Les Halles (rA-B-D) /m(1-4-7-11-14)	173.93
7	TRAVERSIERE	Ledru-Rollin (m8)	97.68
8	BASTILLE RICHARD LENOIR	Bastille (m1-5-8)	59.35
9	FAIDHERBE CHALIGNY	Faidherbe - Chaligny (m8)	132.59
10	TURENNE BRETAGNE	Filles du Calvaire (m8)	67.32
11	FRANCS BOURGEOIS	Saint-Paul (m1)	443.80
12	BOULEVARD VOLTAIRE	Voltaire (m9)	131.04
13	TOLBIAC NATIONALE	Olympiades (m14)	55.91
14	BASTILLE	Bastille (m1-5-8)	81.18
15	GARE DE LYON VAN GOGH	Gare de Lyon (m1-14)(rA-D)	44.68
16	RIVOLI SAINT DENIS	Chatelet (m1-4-7-11-14)	145.02
17	LOBAU	Hotel de Ville (m1-11)	83.83
18	CROZATIER	Ledru-Rollin (m8)	113.67
19	JACQUES BONSERGENT	Jacques Bonsergent (m5)	138.03
20	ODEON QUATRE VENTS	Odeon (m4-10)	26.13

Table 5.2: Top 20 *Vélib'* stations with highest outflows with distance to closest transit stop

cal problem, since they must initiate corrective actions to redistribute bicycles to stations to ensure that short-term future demands are met. It could potentially be cost-prohibitive to satisfy all demand needs, thus operators need to maintain adequate inventories such that levels-of-service are maintained. For policymakers and transportation agencies developing new systems, the extent of operational cost and associated personnel is difficult to forecast. The purpose of this section is to highlight the extent of flow asymmetries in the *Vélib'* system. *Vélib'* operates several teams for redistribution and maintenance activities, including a team on barges that move up and down the river Siene.

The general mobility patterns of urban residents are reflected in the use of the *Vélib'* system. There is considerable directionality in movement of bicycles in both the spatial and temporal dimensions. As an example, Figure 5.6 highlights the sources and destinations of *Vélib'* traffic for a single *Vélib'* station during two different time periods. The width of connecting segments represents the flow as a proportion of total incoming (or outgoing) flows for that time period. Several observations are worth noting. By comparing Figures 5.6(a) and 5.6(b), inflows and outflows for the same period are from different stations. Figures 5.6(c) and 5.6(d) show flows during the evening rush hour with flows from a greater variety of stations than in the morning. Figure 5.6(d) also shows two heavily utilized routes denoted by the two thick ribbons connect the segments.

5.3 Fleet Management

To match flexibility of other modes, shared-vehicle programs transfer control of bicycles to users. This places exceptional logistical challenges on operators who must ensure demand in the near future is met. For the shared-vehicle users, good service is defined by an adequate stock of vehicles at the intended stations of origin and adequate parking slots at the intended destination stations. Thus users seek two types of resources from the system. As shown in the previous section, flow from one station to another is seldom equal to flow in the opposing direction and the bicycle fleet can become spatially imbalanced. To meet near-future demand, operators must then redistribute vehicles to correct this asymmetry. Redistribution plans can be
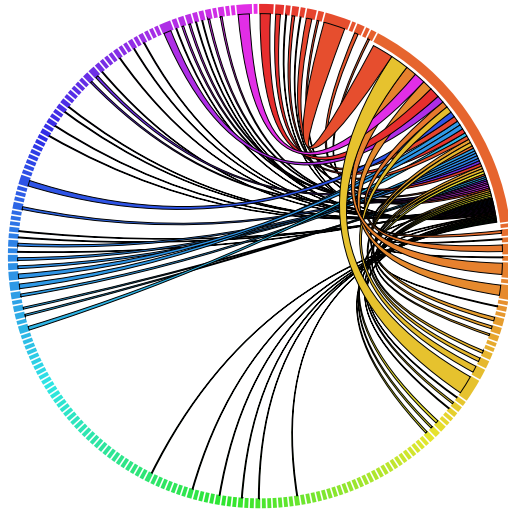
(a) Average 8-9am inflows
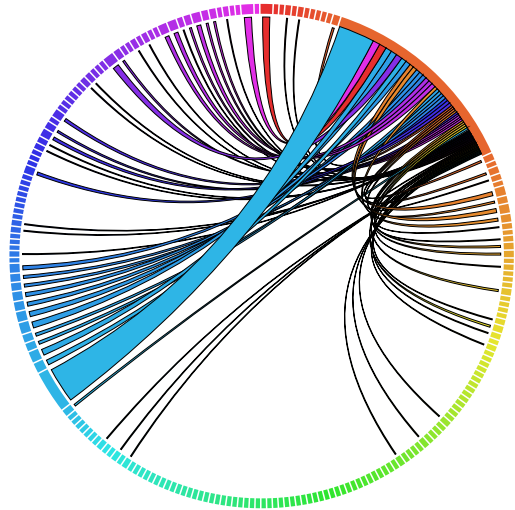
(b) Average 8-9am outflows
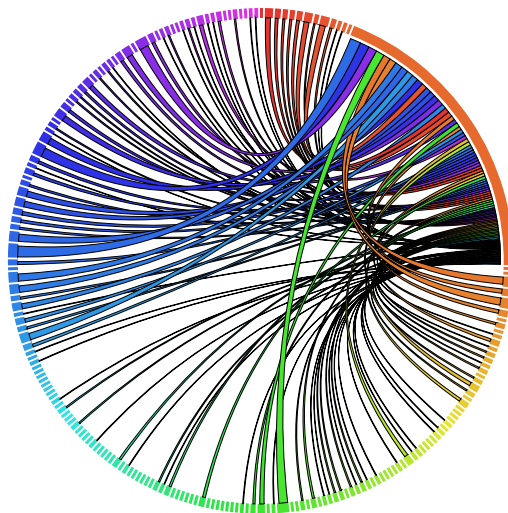
(c) Average 4-5pm inflows

(d) Average 4-5pm outflows

Figure 5.6: Flows to and from Les Halles *Vélib'* station

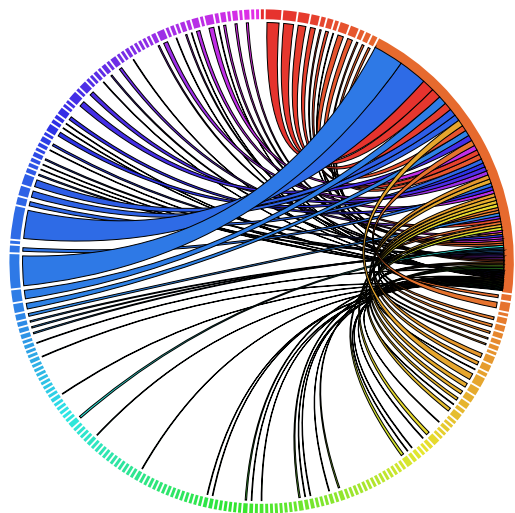based on historical information for services at particular stations. In the event of a 'stock out' when there is an absence of either bicycles or spaces, users need to seek out the nearest alternate station with sufficient resources. This process involves a loss in level-of-service, since travel times are increased and repeated stock-outs harm the perception of the system.

The two types of demand for bicycles and parking slots are related. The increase in one implies a reduction in the other. Users checking out bicycles free up parking spaces for other users to return them. Therefore, at the station-level, the task of the operator is to maintain the vehicle inventory within acceptable limits. Too few bicycles leads to unserviced demand as does having too many bicycles at a station. During the process of redistribution, resources from stations having excess bicycles are transferred to those approaching the 'stock-out' condition. This action balances the fleet across all the stations in the network.

Three aspects limit the extent to which redistribution can be carried out. The task of redistributing vehicles implies operational costs for the operator. Redistribution activities can only be done judiciously to minimize operational costs. Secondly, during high-demand scenarios, the resources in the system could potentially be inadequate to meet future demand. No amount of redistribution can address the shortfall. In these cases, there is a drop in system performance. Therefore, the operator's goal is to meet *most* future demand scenarios.

To determine what constitutes most demand scenarios, the demand for services is probabilistically characterized based on historical information. This allows operators to quantify the state of the system in a reliability measure and allows the

operator to set target levels-of-service that need to be met using redistribution. The development presented herein employs concepts from Chapter 3. Next, the demand processes are described, followed by the system representation and a brief summary of the analytical framework used.

5.3.1 Probabilistic Characterization of Demand

Demand processes at each *Vélib'* station are for bicycles and spaces. It is assumed that at a particular station demand for bicycles is independent of demand for spaces. The operator is interested in knowing the probability distribution of the number of vehicle and parking slot requests for a future time period, termed the planning period. Based on the observed trip patterns, the aggregate resource requests at each station is tabulated. Note that no assumptions are made on trip characteristics determined by users (origins, destinations, and duration). Several distributions were fitted on the observed data on bicycle check outs and returns at each station and the demand process is closest matched by the negative binomial distribution. This distribution is related to the Poisson process, though it exhibits higher dispersion. The Poisson distribution requires the mean and variance to be equal. Negative binomial distribution has a variance that is larger than the mean. This implies that there is high variability in demand for vehicles and spaces.

If a variate x follows the negative binomial distribution, then the probability of x taking a value k is given by

$$P(x = k) = \binom{r + k - 1}{k} p^r (1 - p)^k, \tag{5.1}$$

where r and p are the parameters of the distribution. This is denoted by $x \sim NB(r, p)$.

The two demand processes exhibit duality so the random variable of interest is the difference of the two demand processes. The inherent assumption is that vehicle returns and withdrawals cancel each other out for short planning periods. An 'aggregate' demand random variable is defined to be the difference of demand for vehicles and demand for spaces. This encodes both types of demand in one random variable. When the aggregate demand is positive, then there is greater number of vehicle requests than returns. The redistribution plan could potentially direct bicycles to the station to meet this higher demand. When the aggregate demand is negative, there are more requests for spaces. The operator can free up parking capacity by moving bicycles out of the station to other locations.

If x is a random variable that represents the number of vehicle requests, and y is a random variable that represents the number of return requests, then aggregate demand z is defined as $z = x - y$. z follows the distribution defined by the difference of two negative binomial variates. It can be shown that if $x \sim NB(r_x, p_x)$ and $y \sim NB(r_y, p_y)$, then the distribution of $z = x - y$ can be expressed as

$$P(z = k) = \begin{cases} p_x^{r_x} p_y^{r_y} (1 - p_x)^k \dfrac{(r_x)_k}{k!} F(r_x + k, r_y; k + 1; (1 - p_x)(1 - p_y)) & k \geq 0 \\[3mm] p_x^{r_x} p_y^{r_y} (1 - p_y)^k \dfrac{(r_y)_{|k|}}{|k|!} F(r_x, r_y + |k|; |k| + 1; (1 - p_x)(1 - p_y)) & k < 0 \end{cases} \tag{5.2}$$

where

$$F(\alpha, \beta; \gamma; z) = \sum_0^\infty \frac{(\alpha)_n (\alpha)_n}{(\gamma)_n} \frac{z^n}{n!} \qquad (5.3)$$

is the hypergeometric function [6] and $(a)_n = \Gamma(a + n)/\Gamma(a)$. There is no known closed-form solution for the inverse of this four parameter distribution, which allows for the computation of reliability. Therefore, simulation is employed to calculate the distribution. Figure 5.7 shows the fitted demand distribution for vehicles and spaces along with the simulated four parameter difference distribution for selected stations. In comparison with the Singapore car sharing system presented in Section 3.5, where the aggregate demand distribution was found to be Skellam distributed, the four parameter distribution for *Vélib'* has greater dispersion. This indicates higher demand variability.

5.3.2   Framework

Following the concepts presented in Chapter 3, the *Vélib'* system is depicted as a network of n stations represented as nodes (indexed by i) and all links between all station pairs (a complete graph in graph theory parlance). Each *Vélib'* station i is associated with a 'aggregate demand' random variable, denoted by ξ_i, and a capacity, denoted by c_i. As shown above, the ξ_i random variable can be described by a four parameter distribution. At any given point in time, the system operator is aware of the inventory of bicycles at each station, denoted by v_i. The inventory of parking spaces is correspondingly $c_i - v_i$.

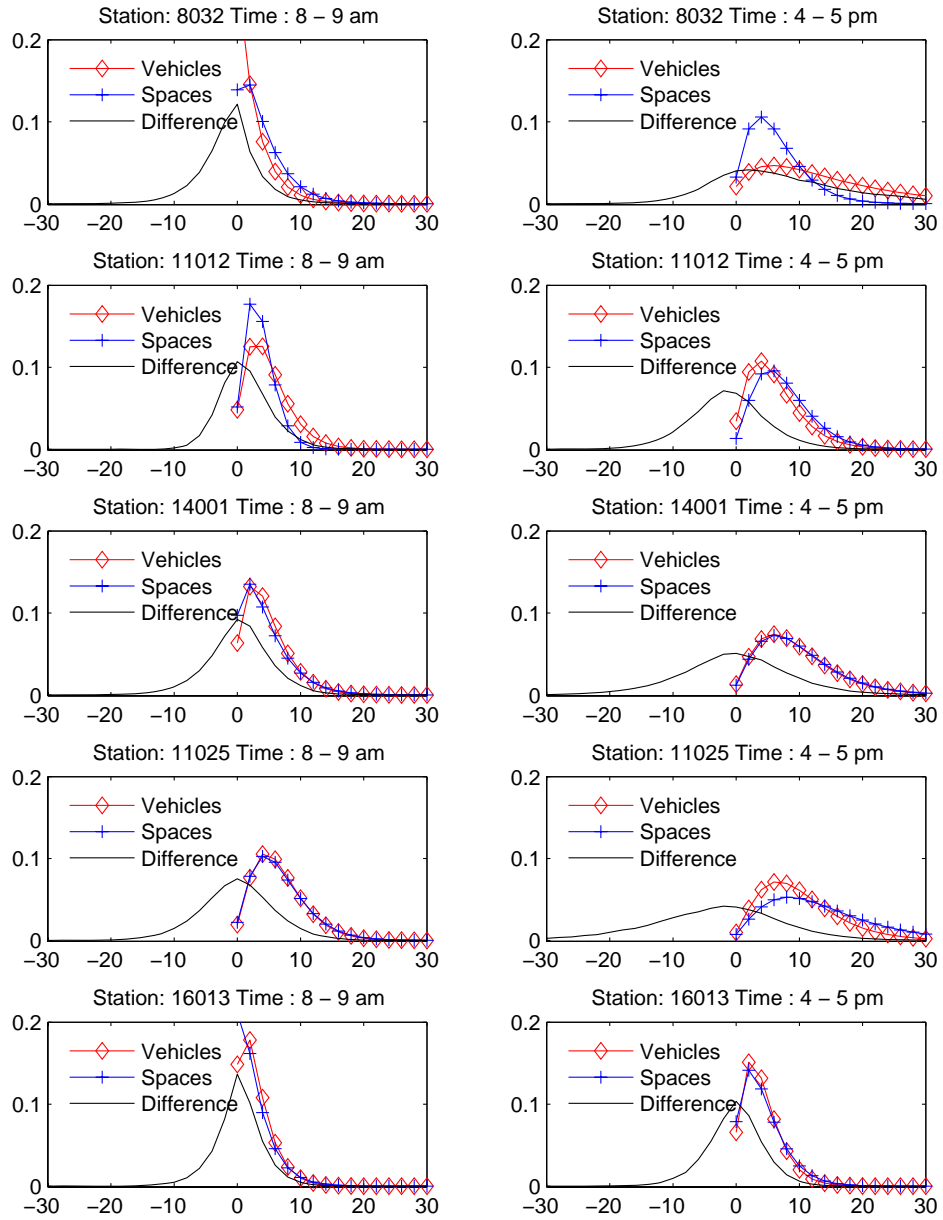The operator considers near-term future demand during the planning period

Figure 5.7: Theoretical demand distribution for selected *Vélib'* stations

(assumed here to be one hour). For this period period, the probability that the current number of bicycles at a station i, termed vehicle inventory, and the number of free parking slots, termed the spaces inventory, can satisfy demand during the planning period can be written as

$$p_i = P(-(c_i - v_i) \leq \xi_i \leq v_i). \tag{5.4}$$

Here, p_i is termed the component-level reliability, since it measures the reliability level at the *Vélib'* station level. Figure 5.8 shows the component reliability for all bicycle inventory levels at selected stations. If demands for bicycles and spaces are assumed to be independent across stations, the systemwide reliability measure can be expressed as

$$p = P(-(c_i - v_i) \leq \xi_i \leq v_i \,, i = 1, \ldots, n) = \prod_{i=1}^{n} p_i. \tag{5.5}$$

The operator can quantitatively set target levels-of-service using the measure p for systemwide reliability, or p_i for component level measures. With a large like *Vélib'*, p can be very small, since it involves the product of component-level measures. In this case, an average component reliability measure can be used to aggregate component-level measures.

To generate redistribution plans for bicycles, these component-level measures can be used to set the desired number of bicycles at all stations. If the operator defines a systemwide reliability level to be met, then the component-level measures can be derived using several techniques outlined in Chapter 3. The most straightforward approach involves the equal apportionment of failure. In this approach,

the acceptable failure rate is divided amongst all stations. Other methods first determine an optimal set of component-level reliabilities, such that the systemwide reliability is met. To generate redistribution plans, an optimization model is used to relocate bicycles from stations with excess bicycles to ones with inadequate stock. Bicycles can also be relocated to free up parking slots should the number of bicycles at a station be too high.

The target level-of-service is constrained by physical capacity. There are demand scenarios that cannot be met with any amount of redistribution. The desired level of service needs to be relaxed and partial redistribution carried out that maximizes the utilization of available resources. *Vélib'* stations with low capacity or high demands that persistently do not meet target reliability levels can be subject to capacity improvements.

5.4 Analysis

The component level reliability for selected stations is plotted in Figure 5.8. At stations with low demand levels such as station 15103, a wide range of bicycle stocks have high reliability. For the *Vélib'* station 1008, during the busy period of 4-5pm, at no level of bicycle inventory is teh component reliability greater than 0.5. This implies that capacity limitations exist at this station. Note that even when stations are empty the component reliability is non-zero since the station still services demands for spaces.

Redistribution plans for the *Vélib'* system are generated for various systemwide

(a) 8-9am   (b) 4-5pm
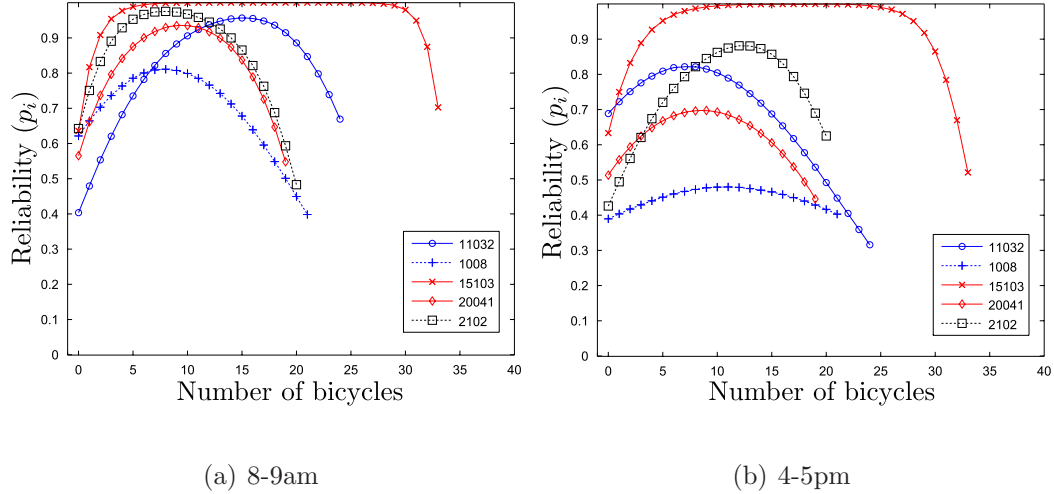
Figure 5.8: Component reliability p_i for selected stations and two planning periods

reliability levels p. Over a period of five days, redistribution is considered for two planning periods, one from 8-9am and the other from 4-5pm. Table 5.3 summarizes the component reliability measures for two cases when redistribution is performed and when it is not. Redistribution is shown to improve the average component reliability in every simulated instance. Additional insights can be gleaned from the model as well by assessing the infeasibility of location stations in handling demand. Stations that experience persistent low levels-of-service can be identified, and the extent of capacity improvements needed can be assessed. Figure 5.9 shows that the majority of stations have sufficient capacity to handle demand uncertainty. However, some key stations require capacity enhancements.

5.5 Conclusions

An empirical analysis of the pioneering *Vélib'* bicycle sharing system in Paris, France is presented. The analysis provides key insights on the functioning of such

Figure 5.9: Stations with persistent shortages
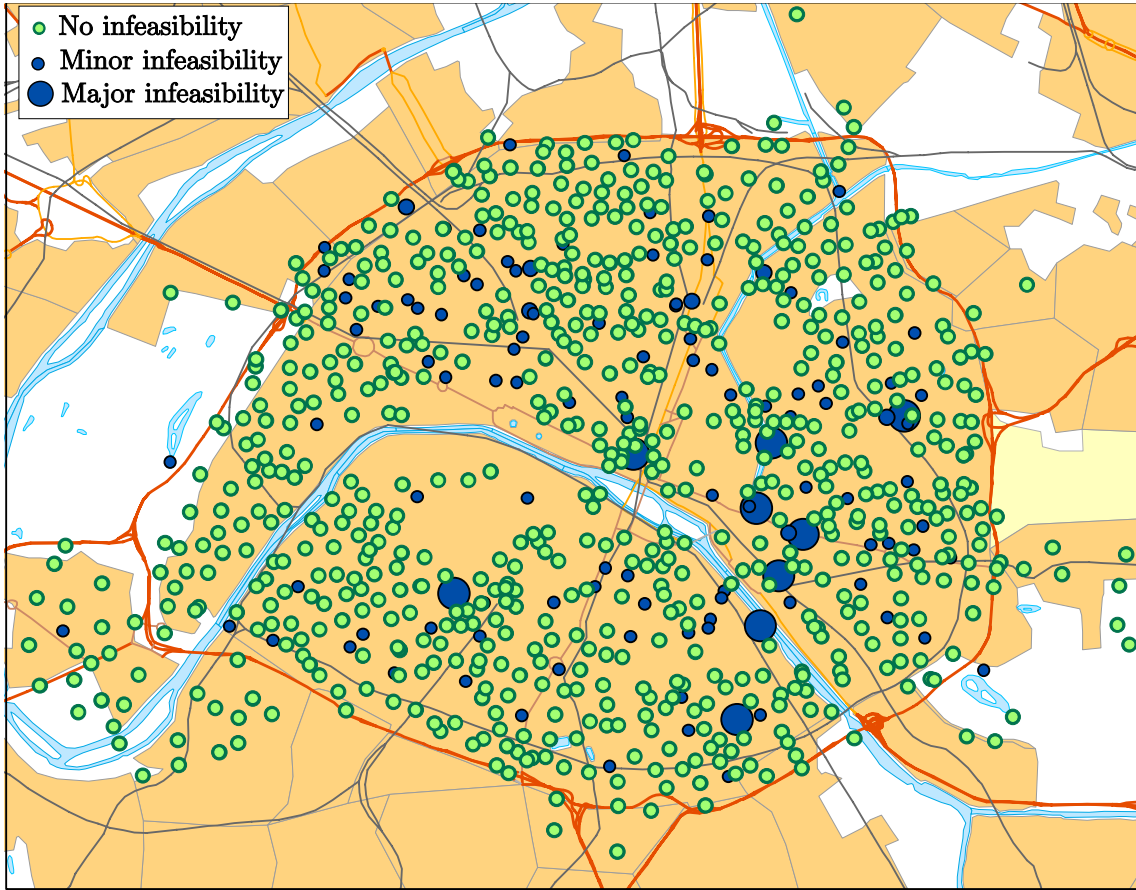
Day	Period	p	Average p_i		Minimum p_i		Improvement in avg(p_i) through redistribution
			No Redistr.	With Redistr.	No Redistr.	With Redistr.	
1	8-9am	0.5	0.78	0.844	0.237	0.254	0.064
		0.7	0.78	0.84	0.237	0.254	0.06
		0.8	0.78	0.836	0.237	0.254	0.056
		0.9	0.78	0.831	0.237	0.254	0.05
	4-5pm	0.5	0.85	0.938	0.054	0.371	0.088
		0.7	0.846	0.932	0.054	0.338	0.086
		0.8	0.844	0.929	0.054	0.338	0.085
		0.9	0.841	0.922	0.054	0.338	0.081
2	8-9am	0.5	0.795	0.838	0.254	0.254	0.043
		0.7	0.791	0.831	0.254	0.254	0.04
		0.8	0.788	0.827	0.254	0.254	0.039
		0.9	0.784	0.82	0.251	0.279	0.037
	4-5pm	0.5	0.847	0.929	0.054	0.366	0.083
		0.7	0.844	0.922	0.054	0.356	0.078
		0.8	0.84	0.917	0.054	0.356	0.077
		0.9	0.834	0.909	0.054	0.356	0.074
3	8-9am	0.5	0.796	0.824	0.269	0.279	0.028
		0.7	0.792	0.819	0.269	0.279	0.026
		0.8	0.789	0.814	0.269	0.279	0.025
		0.9	0.784	0.807	0.269	0.279	0.023
	4-5pm	0.5	0.838	0.908	0.267	0.371	0.07
		0.7	0.835	0.9	0.267	0.356	0.065
		0.8	0.832	0.895	0.267	0.338	0.063
		0.9	0.828	0.886	0.267	0.338	0.058
4	8-9am	0.5	0.794	0.814	0.237	0.279	0.02
		0.7	0.79	0.81	0.237	0.279	0.02
		0.8	0.787	0.806	0.237	0.279	0.019
		0.9	0.781	0.798	0.237	0.279	0.017
	4-5pm	0.5	0.833	0.885	0.054	0.371	0.051
		0.7	0.831	0.878	0.054	0.356	0.047
		0.8	0.829	0.872	0.054	0.356	0.043
		0.9	0.822	0.864	0.054	0.356	0.042
5	8-9am	0.5	0.789	0.799	0.237	0.279	0.01
		0.7	0.785	0.798	0.237	0.279	0.012
		0.8	0.783	0.796	0.237	0.279	0.013
		0.9	0.781	0.795	0.237	0.279	0.014
	4-5pm	0.5	0.824	0.872	0.338	0.371	0.048
		0.7	0.824	0.866	0.338	0.356	0.042
		0.8	0.825	0.863	0.338	0.356	0.038
		0.9	0.824	0.856	0.338	0.356	0.032

Table 5.3: Summray of *Vélib'* simulation results over five days and two time periods

systems and serves to inform policy makers in other urban communities wanting to explore bicycle sharing systems. This paper studies the *Vélib'* system from several aspects, including system characteristics, utilization patterns, the connection between public transit and bicycle sharing systems, and flow imbalances between stations.

The close coupling of transit stops with the bicycle sharing system corresponds with higher utilization of bicycle sharing. This confirms the hypothesis that inter-modal trips with shared-vehicle segments provide value addition for users. Policies that seek to integrate the two systems are, therefore, profitable. Examples of such policies include seamless fare collection, preferential fares for transit users, and prime location of shared-bicycle stations.

From the operator's perspective, the *Vélib'* system is shown to have highly variable demand. By characterizing the demand probabilistically, the target reliability levels at the component or system level can be set and the required fleet and parking slot capacity calculated. Redistribution plans are generated based on these target levels showing related improvement in component-level reliability.

Chapter 6: Conclusions

This dissertation proposes models and associated algorithms for the design and operation of shared-vehicle systems. Using the developed framework, three real-world case studies were conducted to analyze system characteristics and demonstrate the efficacy of proposed models and methods. The analysis of these real-world systems provides vital insights for policy makers and transportation agencies wanting to develop shared-vehicle programs for their communities.

A strategic network design model is developed that takes the form of a bilevel mixed-integer program. The problem is solved using exact and heuristic approaches for synthetic networks and a large real-world case study for Washington, D.C. The output of the model is the optimal design configuration for shared-vehicle systems. For the Washington, D.C. case, 19% of all shared-vehicle stations are located at candidate sites that are at transit stops. Thus, policies that push transit-VSP integration are profitable. Such policies could include seamless fare collection, prime location of shared-vehicles within the confines of transit stops, and preferential fares. The average transit-based trip within Washington, D.C. experiences a reduced travel time of 3.55 minutes solely due to the introduction of the shared-vehicle system.

At the operational level, a fleet management model that generated anticipative fleet redistribution plans is formulated. Two exact solution techniques are also proposed for its solution. The model characterizes demand probabilistically and serves to generate least cost fleet relocation directives that meet most future demand scenarios. Using a car sharing system in Singapore, the efficacy of the formulated

model, and the efficiency of the solution methods is demonstrated. The value of accounting for inherent stochasticities is also quantified. Using the same framework for a large-scale bicycle sharing system in Paris, France, several key insights on the nature of system configuration, utilization patterns, and capacity bottlenecks are identified. The system maintains high levels of service at most locations.

6.1   Benefits to Society

With growing urban congestion problems, rising transportation costs, and a degrading environment, changes in the way transportation resources are supplied and consumed are imminent. This thesis explores one innovation in shared-vehicle systems. As urban communities around the globe are beginning to consider and implement these shared vehicle systems as viable options for reducing congestion, several cities in the US have started, or are planning to start, car sharing and bicycle sharing programs, most notably in Washington, D.C., Boston and Denver. This follows the success of similar programs internationally in cities like Paris, Barcelona and Montreal. While shared-vehicle systems alone will not solve urban congestion woes, they have a place in the basket of mobility solutions for the future. Using optimization tools presented herein, such systems can offer a value added proposition to users through high levels-of-service. As demonstrated in the *Vélib'* system analysis, shared-vehicle systems are closely coupled with transit. This intermodal combination presents an economical and sustainable solution.

6.2   Extensions

The conducted research opens avenues in several promising directions. An economic analysis based on system costs, economic benefit to users and society in terms of reduced travel times and environmental impacts can provide vital input for policy makers. Such an analysis could use principles from network economics to evaluate the scale at which such programs should be implemented. In this respect, the shared-vehicle system is similar to other networked technologies, like communication devices or social networks, where the value of the system increases when there are greater numbers of users and stations. A greater number of users would imply more circulation of vehicle stock. The addition of a single shared-vehicle station increases the value of all other shared-vehicle stations, since the users have more destination choice. Equilibrium effects incorporating rational user behavior also presents a promising direction.

The role of pricing in attracting and retaining a user base can be explored. Given the high cost of automobile ownership, the key question faced by the system operator is what to charge users. Models and methods could include joint system design and pricing of a shared-vehicle system, variable pricing as a demand management tool, or directional pricing to offset operator redistribution costs by having the users perform redistribution.

From the methodological perspective, several extensions can be the focus of future research. At the strategic level, the model assumptions can be relaxed to better represent the existing demand and supply processes. For example, the fixed-demand

assumption can be relaxed to demand that varies as a function of level-of-service. Congestion effects on transit and shared-vehicle links can be incorporated. The use of travel times can be replaced by econometric functions of utility that capture various attributes that users consider in mode and route choice. The strategic model can use the expected value of future operational costs in designing the system. Alternate solution methods for solving large-scale instances can be developed. These techniques could use computationally efficient methods that employ hyperpaths.

The operational level model can be extended to include multi-period relocation costs. Stochastic programs with recourse present an alternate model form for multiple period extension, where current redistribution decisions are the first state variables, and the future redistribution decisions are recourse variables. Redistribution costs at the present period can be reduced if the operator accounts for uncertainty from future periods. A variant of the problem could consider the vehicle routing of redistribution teams through the system. Such work would build on priod advances in the pick-up and drop-off problem in the literature. An online version of the model would trigger redistribution requests by constantly monitoring system state.

As a sustainable solution to urban mobility that uses existing proven technologies, shared-vehicle programs will play a greater role in the years to come.

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