ABSTRACT

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In order to improve productivity and reduce costs, manufacturing firms use product families to provide variety while maintaining economies of scale. In a competitive marketplace, designing a successful product family requires considering both customer preferences and the actions of other firms. This dissertation will conduct fundamental research on how to design products and product families in the presence of competition. We consider both single product and product family design problems. We use game theory to construct a model that includes the competition's product design decisions. We use separation, a problem decomposition approach, to replace complex optimization problems with simpler problems and find good solutions more efficiently. We study the well-known universal electric motor problem to demonstrate our approaches. This dissertation introduces the separation approach, optimizes product design with competition, models product family design under competition as a two-player zero-sum game, and models product family design with design and price competition as a two-player mixed-motive game. This dissertation formulates novel

product design optimization problems and provides a new approach to solve these problems.

SEPARATING PRODUCT FAMILY DESIGN OPTIMIZATION PROBLEMS

By

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Dissertation submitted to the Faculty of the Graduate School of the
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Dedication

To my family and all those who made it possible

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Chapter 1: Introduction

1.1 Product Families

A product family is a set of products that are derived from a platform. They share some common components which reduces the manufacturing cost of the products. Product families create greater variety for costumers and lead to higher profit for the manufacturers. Many manufacturers find product families more profitable due to lower manufacturing costs and higher sales in the market. However designing product families may lead the designs to be constrained to the platform and sacrifices their performance if the optimal platform is not selected. For the mentioned reason designing product families can be a large, complex optimization problem with highly nonlinear objective functions which may not lead to optimum solution using current methods. In this research we separate these problems into simpler subproblems to solve these large, complex problems easier with less computational effort.

1.2 Product Family Examples

Many of the products in the market share common components; Ford Motor Company produces the family of Taurus vehicles which share the same frame and body but different engines and some additional features in each product. A family of cordless drills that derive from the same platform based on their battery voltage. Other examples are aircrafts that share many common components which derive based on the number of passengers or freight capacity. Other examples can be ships, locomotives and space shuttles. Intel delivers a family of central processing units to the market.

1.3 Objectives

The research, motivated by the study of design processes, advances in design optimization of single and product families, and design for market systems considering competition, makes contributions to these areas by answering the following research questions. This dissertation uses two-player zero-sum games and two-player mixed motive games to model product family design optimization problems. The results have the potential to improve the practice of design and introduce new horizons to competitive product family design.

1.3.1 Product Family Design

This research increases our understanding of *how a firm can design a product family that can compete with another product family of another manufacturer.* We will also show *how a firm can maximize its product family sales by maximizing the products desirability regardless of what the other firm does.*

1.3.2 Design for Market Systems

This research uses game theory to consider *how should a firm design and price a product in the presence of competition?* This research studies single product and product family design. In particular, we investigate *how a firm should design and price a new product when the other firm's product design and price are not known and the competitor is simultaneously designing its product.*

1.3.3 Design Optimization

In many design problems there is a big issue of *how can we solve the problem using a simpler, faster and more efficient way with less computational effort?* This research gives a very good approach to replace large, complex optimization problems with simpler ones and enable the designer to solve the problem more efficiently. The research yields systematic and rigorous approaches to separation, including the analysis of exact and approximate separations. The concept of separation and the analysis of exact and approximate separations provide a clear theoretical understanding of when a separation can yield a solution that is optimal for the original problem. We also can understand *how separation can help firms to find dominating strategies in order to compete with other firms in the market.*

1.3.4 Product Design Processes

This research helps designers to understand *how optimization approaches such as separation can model a product design process* and *how separation in early stages of design process can help firms to make design decisions.* Also using separation approach we can show *which customer requirements in a product can lead to higher profit.*

1.4 Overview of Dissertation

In this dissertation we introduce a new method to solve large, complex optimization problems. We try to use our approach to solve other market issues such as design in a competitive market and pricing. We first focus on proposing our approach and how this approach helps designers to make engineering decisions in the conceptual design stage.

In Chapter 1 we introduce product families, discuss their importance, and mention some examples. In Chapter 2 we review some previous work in this area. In Chapter 3, we introduce the separation approach. Chapter 4 applies separation to the design of a single product (without considering competition). Chapter 5 applies separation to the design of a product family in the presence of competition, where the objective is to maximize sales. We use a zero-sum game to model this problem. In Chapter 6, we design a single product while competition exists and different strategies lead to different profits gained by competitors. We then expand this example to the family of the products in Chapter 7 and show that designing a family of products has different profitability based on manufacturer's strategy. Chapter 8 summarizes the research and presents some ideas for future work in this area.

Chapter 2: Literature Review

2.1 Product Family

A product family is a group of related products that is derived from a product platform to satisfy a variety of market niches. Many manufacturing firms develop product platforms and design families of products based on these platforms to provide variety while maintaining economies of scale to improve manufacturing productivity. In product family design, sharing components and production processes across a platform of products enables companies to develop differentiated products efficiently by reducing design costs, to increase the flexibility and responsiveness of their manufacturing processes, and to take market share away from competitors that develop only one product at a time. Other benefits include reduced development and production costs and an improved ability to upgrade products [1]. Using a product platform enables designers and manufacturers to upgrade the whole family of products at once by changing design values instead of changing each product individually.

Designing product families requires aligning product, process and supply chain decisions [2]. Studies of product architecture, product variety, product positioning and product line design have increased our understanding of product family design [3]. One aspect that has not been studied yet is product family design in a competitive market, where the competition's product family design decisions are not known when developing the product family. Shiau and Michalek [4] proposed an approach to new product design entering the market while existing products are sold by competitors.

Recently, Luo [5] studied the integration of engineering and marketing criteria in product family optimization and proposed an approach that iterates between the problem of setting design variable configurations to maximize profitability and the problem of adjusting wholesale and retail prices for the new product family and incumbent products.

Ramdas and Sawhney [6] studied the problem of adding new products to an existing product family. Their work considered both the revenue implications and the life-cycle cost implications of component sharing.

D'Souza and Simpson [7] use a multiobjective genetic algorithm to optimize the performance of the products in the resulting family. They illustrate their approach on the design of a family of aircraft. The objective function incorporates multiple attributes related to the cost and performance of the aircraft.

Farrell and Simpson [8] consider the problem of finding the optimal product platform for a family of products. They optimize product size (which relates to product cost) and illustrate their approach on a family of valve yokes.

Heese and Swaminathan [9] consider a problem in which the manufacturer must set the price and quality levels of two products that are offered to two different market segments. Their work does not consider the design variables or the actions of competitors.

Jiao and Zhang [10] consider the impact that a product family has on operational costs and use a genetic algorithm to solve a mixed-integer combinatorial optimization problem. Their work considers different attribute levels but does not consider design variables or the actions of competitors.

Recently, Kumar et al. [11] proposed the Market-Driven Product Family Design (MPFD) methodology, which integrates market considerations with product family design issues to design the most profitable product family. The approach first creates a market segmentation grid, creates a demand model, builds models for product performance, and combines these models for product family optimization, including product positioning decisions.

Li and Azarm [12] present a product family design approach that first generates design alternatives and then solves a product line design evaluation and selection problem. Their work uses a genetic algorithm to solve the problem. The objective function is to maximize the net present value of the total profits. The problem includes a demand estimation model but does not consider the potential actions of competitors. That is, the competition is known and fixed.

2.2 Design for Market Systems

Organizations that develop products and systems want to create the most desirable products in order to maximize their profit. Increasingly, customer needs play a critical role in designer's decisions. The desirability of a product can be defined as meeting customer expectations in a product. Designers are trying to satisfy more customer requirements in a product in order to make their design more desirable.

The decision-based design (DBD) framework [13] is an approach that explicitly addresses the challenge of creating the most profitable design. It starts with the assumption that engineering design is a decision-making process. The framework shows that possible design alternatives should be evaluated based on how they affect the value of the product. A typical bottom-line measurement of value is profit. The framework also indicates that there are uncontrollable variables that affect the value of the product but notes that price is a controllable variable. The framework thus shows that the design problem is to optimize the value of the profit (the expected utility of the profit) by selecting values for all of the design variables and the price. Optimization methods such as analytical target cascading [14] and collaborative optimization [15] have been proposed for solving this complex problem. The influence of retailer decisions on sales has been studied as well [16].

The study of product design has considered two types of market competition: price competition (which has the potential to affect sales in the short-term because prices can be changed quickly) and design competition (which requires more time to affect sales due to the lead time of developing a new product). Previous work has addressed the problems of designing a single product when the competitor's price decision is unknown [17], product positioning and pricing when a dominant retailer must be considered [18], and optimizing a product design while considering the likelihood of retailer acceptance [16]. The spatial pricing problem has been addressed with two-stage approaches that first choose locations in which to sell the product and then select prices for each location [19-25].

Impact of the subjective characteristics on consumer's product preference has been studied using hierarchical Bayesian structural equation model has been proposed to incorporate such impact into the selection of optimal product design [23].

Williams et al. [24] looked at bundling across different product categories, but this is outside the scope of the product families considered in this dissertation.

Although previous work [25] proposed approaches such as Selection-Integrated Optimization (SIO) to optimize product family design with respect to the selection of platform and non-platform design variables, no previous work used game theory to consider market competition in the design of products and product families. Some of the recent work mentioned in Section 2.1 has, like this dissertation, considered both marketing and engineering models in the optimization of product families.

2.3 Optimization

Organizations that develop products and systems want to create the most valuable design that is feasible. The measurement of value, which depends upon the type of organization, may be profitability, life-cycle cost, or system effectiveness, for example. The value of the product or system that is being designed depends upon the decisions that the design engineer (or development team) makes.

The observation that engineering design requires making decisions has motivated a great deal of research, including work on decision analysis, decision theory, concept generation, modeling customer demand, multi-attribute decision-making, enterprise models, product development processes, and decentralized decision-making [26]. Design organizations can be viewed as a set of loosely-coupled decision-makers [27] that generate and share information in order to generate designs [28, 29]. The ultimate goal is to improve the quality of these decisions and increase the value of product development processes [30].

A variety of decision-making processes have been identified [31]. The two that are most relevant to engineering design are the incremental decision process model and optimization. The incremental decision process model [32] presents a structure in which a major decision is implemented as a series of small decisions. This detailed model involves iterating between the following types of activities: recognition, diagnosis, search, screen, design, judgment, analysis, bargaining, and authorization. Designers will easily recognize the similarities between this process and their own activities.

Design optimization is an important engineering design activity and a difficult mathematical problem. In general, design optimization determines values for design variables such that an objective function is optimized while performance and other constraints are satisfied [33, 34, 35]. Formal design optimization is a useful decision-making process when two conditions hold: (1) there exists enough technical knowledge to formulate a mathematical model that can express the value of a design as a mathematical function of the design variables and (2) there is a consensus on the appropriate objective function [31]. The attributes used to describe a design optimization model can be grouped into four areas: scope, variable set, objective function, and model structure [36].

The difficulty of solving large scale optimization problems and multidisciplinary optimization (MDO) problems has motivated various decomposition approaches. In general, these decomposition approaches require multiple iterations to converge to a feasible, optimal solution for a given design optimization model. Model coordination and goal coordination are two common methods for the decomposition of large scale design optimization problems [37, 38]. MDO problems have been the focus of decomposition approaches such as the bi-level integrated system synthesis (BLISS) approach [39], analytical target cascading [40, 41], collaborative optimization [42], and coupled subspace optimization (CSSO) [43, 44]. Yoshimura *et al*. [45] decompose a multi-objective optimization problem into a hierarchy of problems that have two objectives.

The decision-based design (DBD) framework [13] is an approach that explicitly addresses the challenge of creating the most profitable design. It starts with the assumption that engineering design is a decision-making process. The framework shows that possible design alternatives should be evaluated based on how they affect the value of the product. As mentioned above, a typical bottom-line measurement of value is profit. The framework also indicates that there are uncontrollable variables that affect the value of the product but notes that price is a controllable variable. The framework thus shows that the design problem is to optimize the value of the profit (the expected utility of the profit) by selecting values for all of the design variables and the price. The comprehensive nature of the DBD framework has inspired researchers to develop new design optimization models (called enterprise models) that add variables from the marketing and manufacturing domains to models with conceptual design variables and to adapt existing decomposition techniques to solve them [16, 45, 46]. These more extensive design optimization problems reflect the natural desire to handle large, complex problems in an integrated way [47].

This dissertation introduces an approach that replaces a design optimization problem with a set of subproblems to form a decision-based design process. In particular, this section analyzes a version of the DBD framework, identifies conditions under which the separation is exact (the result is optimal), presents sufficient conditions for establishing bounds on the quality of a non-optimal solution, and applies the concept to a specific engineering design problem.

2.4 Game Theory

Game theory is the theory of independent and interdependent decision making. It is concerned with decision making in organizations where the outcome depends on the decisions of two or more autonomous players, one of which may be nature itself, and where no single decision maker has full control [48]. There are many types of games that represent interactions of players, but in this research we consider only two types of games.

The first type is the *zero-sum game* in which the sum of the payoffs is constant. In this type of game, as one player's payoff increases, the other player's payoff decreases by an equal amount. Matching pennies is a well-known example of a two player *zero-sum* game. In this game, each of the players flips a coin. If the coins came up matching (both heads or both tails) then one player (the matcher) wins, so the other players (the mismatcher) pays $1 to the matcher. Thus, the matcher gains $1, and the mismatcher loses $1. If the coins don't match (one head and one tail), then the mismatcher wins, and the matcher pays $1 to the mismatcher. Thus, the mismatcher gains $1, and the matcher loses $1. The gain of one player exactly equals the loss of the other. Some strategies may be dominated by others; in this case a player should not choose the dominated strategies because they cannot be optimal. In general, it is optimal to choose a strategy that has the best worst-case outcome [48].

The second type of game that we consider in this research is the *mixed-motive game,* in which the sum of the payoffs can change from strategy to strategy. Again, some strategies may be dominated and should not be chosen. In general, the optimal solution is to find a Nash Equilibrium point from which none of the players want to deviate [48]. An example of a *mixed-motive* game is shown in the following table in which the sum of the outcomes of each player changes from one strategy to another. Each ordered pair is the payoff for Player 1 and the payoff for Player 2. For example, if Player 1 chooses A and Player 2 chooses C, then Player 1's payoff is 5, and Player 2's payoff is 4.

Table 1: Example of a Mixed-Motive Game.

Strategy	Player 2: C	Player 2: D
Player 1: A	5,4	3,2
Player 1: B	4,1	1,6

In the above game Player 1's best outcome is 5, and Player 2's best outcome is 6. For Player 1, choosing A dominates choosing B because the payoffs for A are greater, no matter what Player 2 does. A Nash Equilibrium is the strategy combination (A, C), which has payoff (5,4). Although it is not the best outcome for Player 2, he has no incentive to change his strategy to D, which would reduce his payoff.

Game theory has been used previously to represent decision-making in design processes, with the players in the game representing different designers (within the same organization) who control different sets of design variables and may have different objectives [48-57]. For example, to design a pressure vessel, Lewis and

Mistree [52] formulated the problem as a two player game in which the first player, who wishes to maximize the volume, controls the diameter and length and the second player, who wishes to minimize the weight, controls the thickness. Chen and Simon [56] used game theory to model various team interactions in concurrent parametric design.

Absent from the work cited here is the problem of designing (and pricing) a product when the competition is simultaneously designing (and pricing) its product. In Michalek *et al*. [58], maximizing profit with respect to the design variables of each producer is considered separately while treating each competitor decisions as constant. The optimization approach iterates among all producers until all producers converge to their final design. Choi *et al.* [59] modeled price decision models in which firms choose their optimal price independently until no firm wants to alter its product strategy and proposed numerical methods to product positioning decision models. Shiau and Michalek [17] proposed an approach to new product design entering the market while existing products are sold by competitors. They assumed that all competitors have their price decisions with known product attributes and costs. In a later study [60], they considered the competition of firms in consumer choice as well as channel structures.

2.4 Summary

This chapter has reviewed the relevant literature on the design of product families and the emerging area of design for market systems, which explicitly considers pricing and market competition during the design of a product. This chapter has also reviewed works on design optimization, the concepts of game theory, and their application in engineering design research.

Despite the great interest in designing product families and design for market systems for single products, absent from the work cited here is the problem of designing a product (or product family) when the competition is simultaneously designing its product (or product family). Game theory is a useful approach to model competition in the market, but it has not been applied to these types of problems.

Chapter 3: Separating Product Design Optimization

This chapter presents a new approach for solving large, complex optimization problems. This general approach will be used to find solutions to the product design and product family design problems considered in this dissertation. This chapter provides an overview of the approach.

3.1 Definition of Separation

In this section we describe an approach that replaces a design optimization problem with a set of subproblems, solves each subproblem once, and produces a feasible solution without iterative cycles. We call this approach separation. The ideal separation produces an optimal solution to the original problem. However, not all separations do.

Separation is a type of problem decomposition. Separation is similar (but not identical) to decomposition-based design optimization. Both replace a large design optimization problem with a set of subproblems. In a typical decomposition approach, a second-level problem must be solved to coordinate the subproblem solutions in an iterative manner. (See Figure 1.)

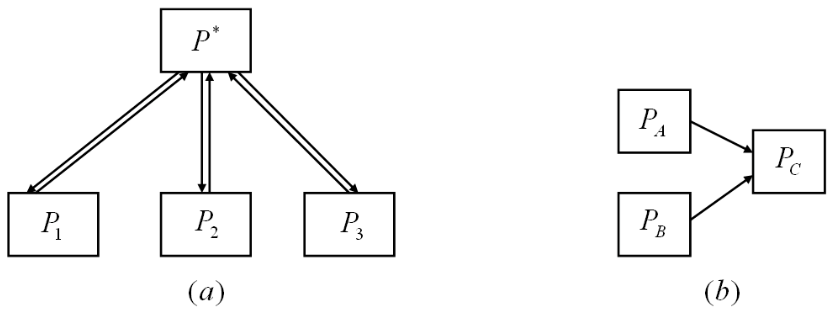

Figure 1. (a) A typical decomposition scheme has multiple first-level subproblems (P1, P2, P3) that receive inputs from a second-level problem (P*), which also coordinates their solutions. (b) Separation yields a set of subproblems. Solving one provides the input to the next.

Separation, on the other hand, does not require subsequent coordination. It is a decentralized and sequential approach related to the concept that is called "factorization" in Pahl and Beitz [64]. A large problem is divided into subproblems. The solution to one subproblem will provide the inputs to one or more subsequent subproblems. However, there is no higher-level problem to coordinate the solution. Note that the separation does not have to be a simple sequence of subproblems; it may have subproblems that are solved in parallel at places. A given separation specifies a partial order in which the subproblems are solved. A different order of subproblems would be a different separation and would lead to a different solution. (Examples of this phenomenon can be found in work on sequential design decision-making [15].)

The subproblems' objective functions are surrogates for the original problem's objective function. These surrogates come from substituting simpler performance measures that are correlated with the original one, eliminating components that are not relevant to that subproblem, or from removing variables that will be determined in another subproblem.

Although it is a unique approach, separation shares some concepts and characteristics with other optimization techniques. The similarities reflect the shared strategy of dividing a large problem into smaller parts, a common approach in decision-making and optimization.

As mentioned above, separation replaces a large optimization problem with a set of smaller ones, like other decomposition approaches do. A key distinctive feature of separation is that, unlike the multiple-discipline-feasible (MDF) and individual-discipline-feasible (IDF) techniques [65-67] or concurrent subspace optimization [43, 44], separation does not iterate until the solution converges. Moreover, the subproblems in a separation do not have to correspond strictly to different disciplines.

The decentralized design that characterizes separation is also discussed by Chanron and Lewis [68], who applied concepts from game theory to study the convergence of various iterative approaches. By contrast, separation does not include iteration, as mentioned above. Moreover, separation allows one to allocate the design variables to different designers and to dictate their objective functions, instead of taking those as given, as the game theory approach does.

Like dynamic programming, separation may solve a set of subproblems and use the solution of one problem to solve another. Typically dynamic programming recursively solves a set of subproblems (corresponding to a set of possible states) starting with a trivial subproblem [68]. By contrast, separation does not contain this special recursive structure; therefore, solving a subproblem considers only the decisions that have been made.

Goal programming [70] prioritizes a set of criteria and finds a solution that meets as many high-priority goals as possible. This approach to multicriteria decision-making uses a single optimization problem that includes all the criteria. Separation, on the other hand, replaces a problem that has one (possibly complex) objective with subproblems that have different objectives. Some separations may resemble goal programming formulations. In general, however, the ordering of subproblems in a separation does not necessarily reflect the importance of their objectives.

Also, despite the similar name, separation is not the same as separable programming, a branch of mathematical programming that concerns nonlinear optimization problems in which the objective function and the constraints are sums of single-variable functions [73]. Separable programming approaches use a linear program to approximate the original problem and employ a type of simplex algorithm to find a solution. By contrast, separation replaces the original problem with a set of subproblems.

3.1.1 Definition of Exact and Approximate Separation

An exact separation is a separation such that finding an optimal solution for each subproblem leads to an optimal solution to the original design optimization problem. If optimal solutions to the subproblems cannot be found, the result may not be an optimal solution to the original design optimization problem.

An approximate separation is a separation such that finding an optimal solution for each subproblem does not lead to an optimal solution to the original design optimization problem.

3.1.2 Separation Example

We demonstrate our approach with the following simple problem and then solve several problems throughout the dissertation.

Suppose that we have the following simple objective function, which we seek to minimize:

$$f(x) = \frac{x_1}{x_2} + \exp(x_1) \tag{1}$$

Table 2 shows lower and upper bounds for the variables.

Table 2: Bounds on Design Variables.

Variable	Lower Bound	Upper Bound
x_1	1	10
x_2	1	10

We can separate the problem into different subproblems. In the first separation, instead of minimizing $f(x)$ we first minimize the surrogate function of $\frac{1}{x_2}$ to get a value for x_2 and then optimize $f(x)$ to get a value for x_1. Results are shown in the following table.

In the second separation, instead of minimizing $f(x)$, we first minimize $\exp(x_1)$ and then optimize $f(x)$.

Third separation includes minimizing $\frac{1}{x_2}$ and $\exp(x_1)$ at the same time to get values of x_1 and x_2 in order to minimize $f(x)$.

Table 3: Optimum Design Values in Different Separations.

	x_1	x_2	$f(x)$	Deviation from A2 (%)
AAO	1	10	2.8183	-
S1	1	10	2.8183	0
S2	1	10	2.8183	0
S3	1	10	2.8183	0

In this example all of the separations lead to an exact solution for the original problem. This is not always the case, however.

3.2 Separating Design Optimization Problems

Separating a design optimization problem is a modeling task that requires understanding the relationships between the design variables, constraints, and objective function. Forming a separation includes identifying the design variables, constraints, and objectives for the subproblems. Although optimization techniques exist for solving the subproblems, there are no automated methods for forming a separation.

Certain natural approaches can be identified. If the problem has a hierarchical structure, the separation can exploit that. Candidate subproblems include those that optimize intermediate values and functions of design variables that are (as a set) independent of other design variables. A separation can first set targets for intermediate values and then set values for design variables to meet these targets.

Alternatively, a separation can set design variables first, using a surrogate objective function that is correlated to the ultimate objective function.

Defining surrogate objectives and appropriate constraints may require additional analysis combined with knowledge (based on experience) about which issues are the most important ones and which solutions are usually poor ones. Subproblems that correspond to different engineering disciplines or engineering tasks (as mentioned by [43]) may be useful. However, it is important to note that the subproblems do not necessarily have to correspond to different engineering disciplines. For highly coupled systems, the use of global sensitivity equations [75] may help identify appropriate subproblems and surrogate objectives. One could find a separation by applying techniques developed for decomposition approaches that rearrange the constraint-parameter incidence matrix formed by the design variables and the constraints [75-76] or the adjacency matrix of the analysis functions and the design variables [77]. Other relevant approaches include using the information gathered during Quality Function Deployment to identify the key design variables [80] and using the value of information to identify simplifications [81].

Engineering design optimization problems for a single product or a family of products with or without competition can be separated using our approach, as the results in this dissertation show. In particular, it is possible to separate problems in which profit is a function of customer attributes (product performance), which are in turn functions of the design variables. The separations reflect the structure of the coupling between the variables, attributes, and objective function. The separations in Chapter 3 illustrate possible separations for these types of problems. In order to separate a product design profit maximization problem; we use intermediate variables to obtain design variables and price to find a feasible solution.

Design optimization problems with competition can be formulated as two-player games. Although each player's decisions affect the other player as well, the interactions are limited to a few key variables, so formulating subproblems for each player creates an effective separation. The analysis of dominance properties also leads to effective separations, as shown in Chapter 6.

Problems without these intermediate variables and functions (and without multiple players) are among the problems that cannot be separated effectively. That is, if all of the variables in the problem are tightly coupled, any attempted separation is unlikely to generate a feasible, near-optimal solution.

3.3 Handling Uncertainty

As we know there are two sources of uncertainty in the design process: (1) aleatory uncertainty and (2) epistemic uncertainty. Many methods such as Reliability Based Design Optimization (RBDO) have been created to handle uncertainty in the design problems. One common approach in most of these methods is to choose a confidence level, and, based on that, we force the search process to explore only those solutions in the design space that are feasible with respect to that confidence level. That is, the confidence level or reliability factor can be added as an additional constraint to the problem, and every solution must satisfy this constraint.

Separations of these types of problems must use appropriate subproblems that also include the confidence level (or reliability factor) in their constraints. If these constraints introduce additional coupling to the problem, then it will be more difficult to find effective separations.

3.4 Summary

This chapter introduced our approach, that we call separation, to solve large, complex optimization problems. We then discussed some approaches to form a separation and identifying subproblems. The chapter also discussed how to handle uncertainty and design for reliability.

Chapter 4: Separating Design Optimization Problems into Decision-Based Design Processes

4.1 Separating the DBD Framework

We now consider a modified version of the DBD framework [13]. (This version ignores any uncertainties, and the demand affects the manufacturer's total lifecycle cost.) First, we will define the following notation:

m = system configuration.

M = the set of all possible configurations.

x = vector of design variables.

$X(m)$ is the set of designs that are feasible for a given configuration m.

p = selling price per unit.

a = vector of product attributes.

D = total demand over the product lifecycle (units).

C = lifecycle cost to manufacturer.

Π = total profit over the product lifecycle ($).

The following functions are given:

$a(x)$ relates the attributes to the design variables.

$D = q(a, p)$ relates the demand to the attributes and the price.

$C(x, D)$ relates the lifecycle cost to the design variables and the demand.

$u(\Pi)$ = utility of profit. We assume that u is monotonically increasing.

Problem P is to choose m, x, and p (the variables) to maximize the utility of the profit:

$$\begin{aligned} \max \quad & u(\Pi) \\ \text{s.t.} \quad & \Pi = Dp - C(x, D) \\ & D = q(a(x), p) \\ & m \in M \\ & x \in X(m) \\ & p \geq 0 \end{aligned} \tag{1}$$

We will separate P into two subproblems, P1 and P2. We will use a graph-like figure to represent a separation. This decision network figure has nodes that correspond to subproblems. An arc from a subproblem node indicates the variables whose values are determined by that subproblem. An arc leading into a node indicates the variables whose values are required by that subproblem. The decision networks corresponding to the original formulation and the separation are shown in Figure 2.

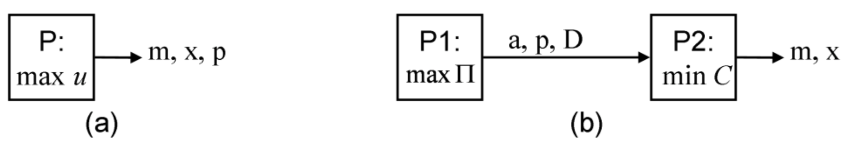

Figure 2. (a) The decision network for the integrated design optimization model. (b) The decision network for the separation.

The variables in P1, the first subproblem in our separation, are *a*, the vector of attributes, and the price *p*. Formulating this subproblem requires defining *A*, the set of all feasible attribute combinations. (A vector of attribute values, sometimes called "targets," is feasible if and only if there is some feasible combination of design variable values that can achieve all of those attributes simultaneously.) It also requires defining $\hat{c}(a, D)$, the approximate life cycle cost if the demand is *D* and the product attributes are *a*. Then, we can let the approximate profitability $\hat{\Pi}$ be the surrogate objective function:

$$\hat{\Pi}(a,p) = q(a,p)p - \hat{c}(a,q(a,p)) \tag{2}$$

Solving P1 provides a solution with values *a** and *p** and also yields $D^* = q(a^*, p^*)$.

$$\begin{aligned} \max \quad & \hat{\Pi}(a,p) \\ \text{s.t.} \quad & a \in A \\ & p \geq 0 \end{aligned} \tag{3}$$

The variables in P2 are *m* and *x*. Solving P2 yields the optimal values $m^* \in M$ and $x^* \in X(m^*)$:

$$\begin{aligned} \min \quad & C(x, D^*) \\ \text{s.t.} \quad & a(x) = a^* \\ & m \in M \\ & x \in X(m) \end{aligned} \tag{4}$$

The quality of this separation is determined by the set *A* and the approximation $\hat{c}(a,D)$. Let *A*(*m*) be the set of attribute combinations that are feasible for a given configuration *m* in *M*:

$$A(m) = \{a(x) : x \in X(m)\} \tag{5}$$

If $A = \bigcup_{m \in M} A(m)$ and $\hat{c}(a,D) = \min_{m \in M, x \in X(m)} \{C(x,D) : a(x) = a\}$, then this is an exact separation. To show this, we need to show that *m**, *x**, and *p** are an optimal solution to Problem P. (The proof that this separation is exact is similar to the analysis of a Stackelberg leader-follower game.)

Suppose not. Then there exists $m' \in M$ and $x' \in X(m')$ and $p' \geq 0$ such that $a' = a(x')$, $D' = q(a', p')$, and $u(D'p' - C(x', D')) > u(D^* p^* - C(x^*, D^*))$.

Because u is monotonically increasing, $D' p' - C(x', D') > D^* p^* - C(x^*, D^*)$. Because m^* and x^* are an optimal solution for P2, we know that $C(x^*, D^*) = \hat{c}(a^*, D^*)$.

Because $a' = a(x')$ and $\hat{c}(a', D') = \min_{m \in M, x \in X(m)} \{C(x, D') : a(x) = a'\}$, we know that $\hat{c}(a', D') \leq C(x', D')$. Therefore,

$$\begin{aligned}
\hat{\Pi}(a', p') &\geq D' p' - C(x', D') \\
&> D^* p^* - C(x^*, D^*) \\
&= D^* p^* - \hat{c}(a^*, D^*) \\
&= \hat{\Pi}(a^*, p^*)
\end{aligned} \tag{6}$$

This contradicts the optimality (from P1) of a^*, p^*. Therefore, m^*, x^*, and p^* are an optimal solution to Problem P. QED.

Having identified sufficient conditions for an exact separation, we now consider an approximate separation. Suppose that the cost function $\hat{c}(a, D)$ is not exact, but we have the following error bound:

$$\left| \hat{c}(a, D) - \min_{m \in M, x \in X(m)} \{C(x, D) : a(x) = a\} \right| < \varepsilon \tag{7}$$

Then we can show that the profitability of m^*, x^*, and p^* must be within 2ε of the optimal profitability as follows. First, let $m' \in M$ and $x' \in X(m')$ and $p' \geq 0$ be an optimal solution to P. Let $a' = a(x')$ and $D' = q(a', p')$. Because m^* and x^* are an optimal solution for P2, we know that $C(x^*, D^*) = \min_{m \in M, x \in X(m)} \{C(x, D^*) : a(x) = a^*\}$.

From this equality, Equation (7), and some rearranging, we have the following:

$$\begin{aligned}
&\hat{c}(a^*, D^*) - C(x^*, D^*) > -\varepsilon \\
&D^* p^* - C(x^*, D^*) > D^* p^* - \hat{c}(a^*, D^*) - \varepsilon
\end{aligned} \tag{8}$$

We also know that $C(x',D') \geq \min_{m\in M, x\in X(m)} \{C(x,D') : a(x) = a'\}$. From Equation (7) we know that $\min_{m\in M, x\in X(m)} \{C(x,D') : a(x) = a'\} > \hat{c}(a',D') - \varepsilon$. Combining these and rearranging terms lead to the following:

$$\begin{aligned} C(x',D') &> \hat{c}(a',D') - \varepsilon \\ D'p' - \hat{c}(a',D') &> D'p' - C(x',D') - \varepsilon \end{aligned} \tag{9}$$

Because *a** and *p** are an optimal solution to P1, we know the following:

$$D^*p^* - \hat{c}(a^*,D^*) \geq D'p' - \hat{c}(a',D') \tag{10}$$

Combining Equations (8), (9), and (10) yields the desired result, which shows that the profitability of *m**, *x**, and *p** is close to the optimal profitability:

$$D^*p^* - C(x^*,D^*) > D'p' - C(x',D') - 2\varepsilon \tag{11}$$

The analysis shows that the quality of this separation depends upon the marketing group's ability to identify feasible attribute combinations and to estimate costs. If marketing selects an infeasible attribute combination, then it will be impossible to design a satisfactory product. If the cost estimates are inaccurate, then the resulting product will be suboptimal.

4.2 Example: Motor Design

A universal electric motor example originally developed by Simpson [80] will be used to demonstrate the concept of separation. Simpson used this example to demonstrate new techniques in product family design. The following example ignores the product family aspect and deals with only a single motor design that should meet given power and torque requirements.

The optimization model for the universal electric motor problem includes nine variables (eight design variables and the price), four customer attributes, twenty-three intermediate engineering attributes, and seven fixed engineering parameters. Table 4 lists the design variables, their lower and upper bounds, and units. The price p is in dollars.

Table 4: Bounds on Design Variables.

Variable	Definition	Lower bound	Upper bound	units
N_c	Turns of wire (armature)	100	1500	turns
N_s	Turns of wire (stator), per pole	1	500	turns
A_{aw}	Cross sectional area of armature wire	0.01	1.0	mm2
A_{sw}	Cross sectional area of stator wire	0.01	1.0	mm2
r_o	Outer radius (stator)	0.01	0.1	m
t_s	Thickness (stator)	0.0005	0.01	m
I	Electric current	0.1	6	A
L	Stack length	0.01	0.2	m

Appendix A describes the engineering parameters and engineering attributes. The derivations of the equations and other background information on universal electric motors can be found in [80, 81]. The four customer attributes are the torque T (in Nm), the power P (in watts), the efficiency η, and the mass M (in kg). They are calculated from the design variables and the engineering attributes as follows:

$$T = K\varphi I$$
$$P = P_{\text{in}} - P_{\text{out}}$$
$$\eta = P / P_{\text{in}} \tag{12}$$
$$M = M_w + M_s + M_a$$

As in Simpson *et al*. [81] we take as given two targets for the power and torque: $P = 300$ W and $T = 0.05$ Nm. There is also a constraint due to the geometry of the motor:

$$r_o > t_s \tag{13}$$

The cost equations were originally derived in Wassenaar and Chen [83]. We simplified the equations slightly. The design cost C_D is assumed to be fixed at \$500,000 while the material cost C_M, labor cost C_L, and capacity cost C_K vary with demand and engineering attributes. (Due to inefficiencies, the capacity cost increases quadratically when the production quantity deviates from the optimal production capacity.)

$$C_D = 500{,}000$$
$$C_M = d\left(M_w C_c + \left(M_s + M_a\right)C_s\right)$$
$$C_L = \frac{3}{7} C_M \tag{14}$$
$$C_K = 50\left(\left(d - 500{,}000\right)/1000\right)^2$$

To predict demand, we used discrete choice analysis (DC) and spline functions that we created to model customer preference. The total demand (d) is the population size (s) multiplied by the probability that a consumer will select a particular design (i.e. estimated market share). We set s = 1,000,000. The following equation shows the common DCA equations developed in [84, 85].

$$\begin{aligned} d &= se^{v}\left[1+e^{v}\right]^{-1} \\ v &= \Psi_1(M)+\Psi_2(\eta)+\Psi_3(P)+\Psi_4(T)+\Psi_5(p) \end{aligned} \tag{15}$$

The attraction value ν is calculated from the following spline functions for the mass, efficiency, power, torque, and price:

$$\begin{aligned} \Psi_1(M) &= 0.5(1-M) \\ \Psi_2(\eta) &= \eta - 0.5 \\ \Psi_3(P) &= -\left(1-\frac{P}{300}\right)^2 \\ \Psi_4(T) &= -\left(1-\frac{T}{0.05}\right)^2 \\ \Psi_5(p) &= \frac{25-4p}{15} \end{aligned} \tag{16}$$

The profit Π of a motor design is a function of the demand (*d*), price (*p*), and the costs discussed above.

$$\Pi = dp - \left(C_D + C_M + C_L + C_K\right) \tag{17}$$

This formulation is related to the notation of Section 4 as follows. The set of configurations has only one element, so the configuration is given. The set $X(m)$ is defined by the upper and lower bounds, the engineering attributes, and the geometry constraint shown in Equation (13). The attributes a are the torque, power, efficiency, and mass. The demand function $q(a, p)$ is determined by the spline functions and the demand functions in Equations (15) and (16). The cost function $C(x, D)$ is determined by the sum of the costs described by Equation (14). Finally, the utility $u(\Pi) = \Pi$.

We conducted numerical tests using different separations of the motor design problem in order to compare their solution quality to the solutions found by solving all-at-once formulations of the problem. (Note that one could consider the all-at-once formulations as "trivial" separations.) The decision networks corresponding to the formulations and separations are shown in Figure 3.

The first formulation (A1) is an all-at-once formulation that determines values for the design variables and price in order to maximize profit. Note that the terms ψ_3 and ψ_4 in the demand model penalize deviations from the power and torque targets.

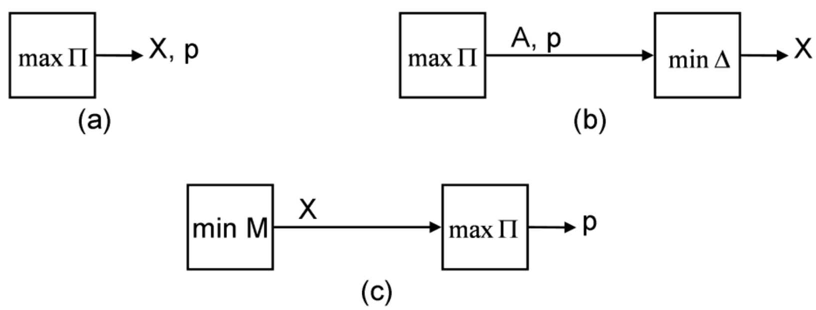

Figure 3. Decision networks (x = the vector of design variables). (a) The all-at-once formulations (a1 and a2) maximize profit. (b) Separation s1 finds the most profitable attribute values and price and then sets the design variables to satisfy them. (c) Separation s2 finds the best design and then sets the price to maximize profit.

The second formulation (A2) is an all-at-once formulation that determines values for the design variables and price in order to maximize profit while enforcing the power and torque requirements (by including them as equality constraints).

Our first separation (S1) has two subproblems, like the one analyzed in Section 4. The first subproblem determines values for the mass, efficiency, and price in order to maximize profit while enforcing the power and torque requirements. This subproblem requires a surrogate cost function that relates the total cost to the customer attributes (power, torque, mass, and efficiency) and price. The first key issue is the material cost, which is a function of the three components' masses, which are not available in this subproblem. Therefore, in the objective function, we replace C_M with $\bar{C}_M = dM\bar{C}$, where $\bar{C}$ is some "average" material cost. Other surrogate cost functions might be possible.

The relationship between mass and efficiency is another important issue. Not all combinations of values for mass and efficiency are feasible; in general, a higher efficiency motor will require more mass. Creating a surrogate constraint for first subproblem in S1 is important to finding a practical solution (one that can be realized) and is critical to employing this particular separation. We will consider two different surrogate constraints in our experimental results.

After solving the first subproblem in S1, we need to determine values for the eight design variables in order to minimize the deviation from the four attribute targets ($M^*, \eta^*, P^* = 300$ and $T^* = 0.05$). We can immediately satisfy the efficiency target by setting the current equal to the value $I^* = P^* / \left(V_t \eta^*\right)$. The second subproblem in S1 then finds values for the other seven design variables in order to minimize a deviation (or loss) function Δ that includes deviations from a target total resistance, the target torque, and the target mass. Achieving these three targets will satisfy all four customer attribute targets.

$$
\begin{aligned}
\Delta_1 &= \left(\frac{\left(R_a + R_s\right) I^{*2}}{P^* / \eta^* - P^* - 2I^*} - 1 \right)^2 \\
\Delta_2 &= \left(\frac{T}{T^*} - 1 \right)^2 \\
\Delta_3 &= \left(\frac{M}{M^*} - 1 \right)^2 \\
\Delta &= \Delta_1 + \Delta_2 + \Delta_3
\end{aligned}
\tag{18}
$$

The second separation (S2) also has two subproblems. The first subproblem in S2 determines values for the eight design variables while satisfying the power and torque requirements. Different versions of this subproblem use different objective functions, including minimizing mass, maximizing efficiency, and minimizing material cost. Given values for the design variables, which set the four customer attributes, the second subproblem in S2 determines the price in order to maximize profit.

4.3 Experimental Results

As mentioned above, the purpose of the numerical experiment was to compare the quality of the solutions that the separations generate to those of the all-at-once formulations and to get some insight into the computational effort. All of the optimization problems were solved using the *fmincon* function in the MATLAB optimization toolbox. Ten initial designs (listed in Appendix B) were found by solving the second subproblem in S1 for ten different randomly-generated combinations of the four customer attributes (power, torque, efficiency, and mass).

For separation S1, we considered four scenarios formed by combining two different sets of surrogate constraints with two values for average material cost. The average material cost $\bar{C}$ was set to \$1.5 per kilogram and \$2 per kilogram. (Note that both values are between the parameters C_c and C_s.) The first set of surrogate constraints (CS1) had the following equations:

$$\begin{aligned} &\eta \leq 0.97 \\ &M \geq 0.15 + 0.05 / (1 - \eta) \end{aligned} \tag{19}$$

The second set of surrogate constraints (CS2) had the following equations:

$$\eta \le 0.97$$
$$M \ge 0.02/(1-\eta) \tag{20}$$

Note that these constraints were added because not all combinations of mass and efficiency can be achieved by feasible values on the design variables. These constraints makes it easier to solve the design subproblem (which must meet these targets).

Tables 5, 6, and 7 show the experimental results for each separation. (Because the subproblems have locally optimal solutions, we solved them with multiple initial points and report the best solution that was found.) The profit of the solution to A1 is slightly higher than the profit of the A2 solution (which is taken as the benchmark), but the A1 solution ($P=$ 315 W and T = 0.0472 Nm) also misses the power and torque targets. The A2 formulation requires many more iterations. The quality of the solution found by separation S1 depends greatly upon the surrogate constraint set. The best solution is found using CS2, which allows mass to become smaller (which is desirable) and thus includes more of the solution space. Of course, it takes more effort to search this larger space. Changing the average material cost does not affect the solution quality as much. Separation S2 shows that, in this case, designs that maximize efficiency (one solution reached nearly 96%) are not as profitable as designs that minimize the material cost or the mass (which are closely related). Note that the high-efficiency solution has a very large mass, which increases costs and reduces profit significantly compared to the low-mass and low-cost designs.

Considering separation S1 in light of the results in Section 4, we note that the constraint sets do not include all of the feasible attribute combinations; indeed, some more profitable combinations are left out. (That is, the set A is incomplete.)

Moreover, using a simpler material cost function ($\bar{C}_M = dM\bar{C}$) introduces an approximation in the surrogate objective function $\hat{\Pi}$. Thus, this separation does not satisfy the conditions for an exact separation. In the worst case (when the stator and armature have a mass of 4.5 kg, the windings have no mass, the efficiency equals 0.96, and the price equals 0 to increase demand), the difference between $\bar{C}_M$ and C_M is over $2,980,000. For the best solution found for formulation A1, when $\bar{C}$ = $1.5 per kilogram, the difference between $\bar{C}_M$ and C_M is only $23,082.

Table 5: Results for each formulation and separation

	Scenario	Function Evaluations (average)	Profit ($)	Deviation from A2 (%)
A1		579	4,000,518	0.29
A2	P = 300, T = 0.05	65037	3,989,027	-
S1	CS1, $\bar{C}$ = 1.5	181	3,317,975	16.82
	CS1, $\bar{C}$ = 2	168	3,580,730	10.24
	CS2, $\bar{C}$ = 1.5	306	3,935,065	1.35
	CS2, $\bar{C}$ = 2	306	3,935,521	1.34
S2	Max Efficiency	312	3,040,692	23.77
	Min Cost	554	3,379,202	15.29
	Min Mass	834	3,379,029	15.29

Table 6: Best design found in each formulation and separation

	N_c	N_s	A_{aw}	A_{sw}
A1	610.6097	285.0453	0.184783	0.184783
A2	655.1343	305.7057	0.180135	0.018042
S1.1	971.5219	56.1535	0.247552	0.094276
S1.2	483.5268	223.3108	1.0000	0.054794
S1.3	391.2846	234.6498	0.235729	0.153379
S1.4	377.6240	177.9033	0.167358	0.172152
S2.1	1280.116	385.2213	0.902201	1.0000
S2.2	374.4015	144.2613	0.042322	0.042731
S2.3	375.9264	143.2410	0.045773	0.039134

	r_o	t_s	I	L	p
A1	0.0100	0.004451	3.184614	0.0100	9.09
A2	0.0100	0.004445	3.054586	0.0100	9.08
S1.1	0.015036	0.0100	3.394331	0.032943	8.42
S1.2	0.018205	0.0100	3.483778	0.0100	8.41
S1.3	0.011727	0.003904	3.056057	0.015274	9.03
S1.4	0.010499	0.001617	3.101400	0.018867	9.01
S2.1	0.010704	0.007590	2.729624	0.0100	8.83
S2.2	0.0100	0.004645	6.0000	0.0100	7.78
S2.3	0.0100	0.004630	6.0000	0.0100	7.78

Table 7: Attributes of best design found in each formulation and separation

	T	P	η	M
A1	0.0472	315	0.8608	0.1026
A2	0.05	300	0.8540	0.1063
S1.1	0.05	300	0.7686	0.3661
S1.2	0.05	300	0.7498	0.3074
S1.3	0.05	300	0.8536	0.1366
S1.4	0.05	300	0.8411	0.1259
S2.1	0.05	300	0.9557	0.5583
S2.2	0.05	300	0.4348	0.0330
S2.3	0.05	300	0.4348	0.0331

4.4 Discussion: Engineering Design Process

The results above show that a design optimization problem can be replaced by a set of subproblems. We now turn to engineering design processes. Separation provides a perspective in which engineering design processes can be considered as heuristics for the problem of finding the most valuable design. From this perspective, separation is a model for a certain class of engineering design processes.

We will use the term *progressive design process* to describe an engineering design process that creates a product or system design through a series of distinct phases. (Thus, this term would not cover prototype-based design processes that iterate through generate-build-test cycles.) The phases generate intermediate results by making decisions about different aspects of the design and generating increasingly detailed information. (The name reflects the similarity to a progressive die, which makes an increasingly complex part through a series of punches.) Pahl and Beitz [64], Asimow [86], Ullman [87], and Ulrich and Eppinger [89] are among those presenting progressive design processes.

Progressive design processes emphasize the movement from one phase to another and the intermediate results that are generated. A progressive design process can be viewed as a heuristic for the value optimization problem discussed at the opening of this section. For instance, if we consider the design process presented by Pahl and Beitz [64], one part of the process is described as optimizing the principle (or concept); another optimizes the layout, form, and material; and another optimizes the production. Moreover, the process is based on a general problem-solving process and ends with a "solution." It seems clear that the entire process is concerned with finding a feasible and valuable system design, even if optimality is not guaranteed.

Previous research has developed models of design processes that focus on the activities that need to be done, as in Gantt charts, the PERT and critical path methods, IDEF, the design structure matrix, Petri nets, and signposting [90]. Such models have been used to estimate the cost and duration of design processes [91-94]. The approach taken in this section provides a way to consider the quality of the design process: how good is the solution that it creates? Answering this question would seem to be a way to extend the principles of decision-based design (including the idea that design should find the most valuable product) from a single decision to a design process.

This dissertation has presented two ways to evaluate the quality of a progressive design process by modeling it as a separation of a design optimization problem. The separation of the DBD framework corresponds to a simple design process in which marketing experts determine the product's price and the attribute values that the product should have; then the engineers have to find the lowest cost design that can meet these targets. Moreover, it indicates mathematically that a progressive design process is a reasonable way to design a product or system, provided that the subproblems are appropriately formulated. It is not necessary to formulate and solve the problem as an integrated whole. The motor design results give additional examples of separations and demonstrate the importance of choosing appropriate surrogate constraints and objective functions.

The proposal to use separations to evaluate design decision-making is in the spirit of research into using game theory concepts to represent design processes, including [82, 68, 48,49, 88]. Some separations correspond exactly to cooperative games, non-cooperative games, and Stackelberg games. However, separations are not limited to these special cases. The analysis of separations studies not only changes in the structure of the separation but also changes to the subproblems' constraints and objectives; these are not taken as given.

This perspective of engineering design is not in conflict with the use of concurrent engineering, in which cross-functional teams consider downstream issues (especially those related to manufacturing) throughout the entire design process. The use of concurrent engineering creates a new separation by modifying the objectives and constraints used to make design decisions and by changing when decisions are made (e.g., some process design activities may be started earlier). However, there is still a separation because the design process is still divided into different subproblems.

Finally, we recognize that creating a separation that corresponds to a real product development process and analyzing its quality are difficult challenges. We are still learning how to do both of these steps, and the results presented here are only the beginning of studying this approach. Progress toward this goal will help us better understand and improve product development processes.

In particular, the analysis of Section 4 assumed that there was no uncertainty in order to simplify the exposition. Considering the expected utility of profit or changing other aspects of the original problem formulation would lead to different conditions for exact and approximate separations.

4.5 Summary

In this chapter exact and approximate separations were defined. We showed that some separations give near the optimum solutions, however we found that in some problems we exactly get the same solutions as original problem. We applied our approach to the well-known electric motor problem and showed that in different separations we get different results. For instance we showed that minimizing material cost with respect to certain constraints lead to minimum deviation from the original problem's optimum solution. In the next chapter we discuss design of product families while competition exists in the market. We use separation to replace the problem with a set of subproblem and solve the product family problem in an easier way.

Chapter 5: Optimizing a Product Family under Competition

5.1 Product Family Design: Formulation

This section presents the product family design problem that we consider. There are two manufacturers (Players 1 and 2) whose product families will compete against each other in the marketplace. The marketplace comprises N distinct and independent markets, and each player has one product in each market. Player 1 designs and sells a family of products, numbered from 1 to N. Player 2's products are numbered from $N+1$ to $2N$.

Both players design simultaneously. The payoff for each player is their total sales (for all of their products). The size of each of the N markets is fixed, so any gain in sales by one manufacturer means an equivalent loss in sales by the other. Both manufacturers want to maximize their total sales across all N markets, but the sum of the total sales of the two manufacturers is constant. This problem can be modeled as a zero-sum game. We consider the problem from Player 1's perspective and assume that Player 1 has already decided to create a product family using a product platform (thus he does not design his products independently). Moreover, Player 1 has already identified the platform variables for this family. However, we make no similar assumption about Player 2's products because we assume Player 2 will produce the best design possible for each individual product. In some sense this is the worst-case for Player 1 and is, therefore, a reasonable way for Player 1 to proceed.

For modeling demand, there are various probabilistic choice models based on Discrete Choice Analysis (DCA). In this work, we use the logit model for its simplicity and capability to predict the demand of future product designs. The utility value is based on product characteristics that can be related to engineering design. The logit model is used to determine the market share for each product in each market. The market share of product j depends upon δ_j, the mean utility of product j, which measures the desirability of the product [56]. In this problem, a product's utility depends upon its customer attributes, which do not include price. As in a Cournot duopoly, we assume that the prices are externally fixed quantities [50].

Let S_j be the size of market j. Note that, in each market, there are only two products: products j and $N+j$. For product j designed and sold by Player 1, let q_j be its market share and let Q_j be its sales:

$$q_j = \frac{e^{\delta_j}}{e^{\delta_j} + e^{\delta_{N+j}}} \qquad Q_j = q_j S_j \tag{21}$$

Let Q be the total sales for Player 1's product family, which is the sum of the individual product sales:

$$Q = \sum_{j=1}^{N} Q_j = \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta_{N+j}}} \tag{22}$$

Let $D = (D_1, ..., D_N)$ be Player 1's product family, let $E = (E_1, ..., E_N)$ be Player 2's products, and let F_i be the set of feasible products for Player i. Then, for $j = 1, ..., N$, δ_j depends upon D_j and δ_{j+N} depends upon E_j. Player 1 needs to solve the following optimization problem:

$$\underset{D \in F_1}{Max}\ \underset{E \in F_2}{Min}\ Q \tag{23}$$

5.2 Product Family Design: Solution Approach

To solve this problem, we will first consider Player 2 and find a dominant strategy for Player 2. This strategy yields more sales for Player 2 than any other strategy, no matter what Player 1 does. (Note that a strategy for Player 1 is a choice of D, and a strategy for Player 2 is a choice of E.)

Because Player 2 does not necessarily use a platform, Player 2's dominant strategy is to find, for each market $j = 1, \ldots, N$, the E_j that maximizes δ_{N+j}, which will maximize his sales in that market. Let δ^*_{N+j} be the corresponding maximum product utility.

Then, because Player 2 will follow his dominant strategy, we have the following objective function for Player 1:

$$Q = \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta^*_{N+j}}} \tag{24}$$

Thus, Player 1's optimal strategy is to find D^* that maximizes Q. Thus, we have separated the original problem into $N+1$ subproblems that can be solved in two stages. The first stage solves N subproblems to find Player 2's products, which can be done in parallel. The second stage finds D^* to maximize Q.

5.3 Application

5.3.1 Motor Design

To illustrate this approach to product family design, we will use the universal electric motor family developed by Simpson [80]. The product family has ten products, each designed to produce a different torque. Each motor has eight design variables. Following Simpson [80], six of these eight design variables will be used to define the product platform (as discussed above, the choice of the platform variables has been made already). These variables will have the same values for all ten products. Each product in the family will have unique values for the remaining two design variables. Tables 8 and 9 show the platform design variables and product design variables with their bounds.

Table 8. Bounds on Platform Design Variables.

Variable	Definition	Lower bound	Upper bound	units
N_c	Turns of wire (armature)	100	1500	*turns*
N_s	Turns of wire (stator), per pole	1	500	*turns*
A_{aw}	Cross sectional area of armature wire	0.01	1.0	mm^2
A_{sw}	Cross sectional area of stator wire	0.01	1.0	mm^2
r_o	Outer radius (stator)	0.01	0.1	*m*
t_s	Thickness (stator)	0.0005	0.01	*m*

Table 9. Bounds on Product Design Variables.

Variable	Definition	Lower bound	Upper bound	units
I	Electric current	0.1	6	*A*
L	Stack length	0.01	0.2	*m*

Table 10. Torque Requirement Values.

Market	T_j	S_j
1	0.05	200,000
2	0.10	250,000
3	0.125	200,000
4	0.15	100,000
5	0.20	100,000
6	0.25	50,000
7	0.30	25,000
8	0.35	25,000
9	0.40	25,000
10	0.50	25,000

Because the product family has ten products, there will be ten sets of I and L, which yields twenty design variables. These plus the six platform design variables give a total 26 design variables.

The four customer attributes are the torque T (in Nm), the power P (in watts), the efficiency η, and the mass M (in kg). They are calculated from the design variables and the engineering attributes as follows:

$$\begin{aligned} T &= K\varphi I \\ P &= P_{\text{in}} - P_{\text{out}} \\ \eta &= P / P_{\text{in}} \\ M &= M_w + M_s + M_a \end{aligned} \tag{25}$$

All of the products should have a power of 300 watts. The ten different torque requirements, which define the ten independent markets, are shown in Table 10.

Because the power and torque must meet specific requirements dictated by the marketplace and the prices are fixed, we assume that the mean utility for each product depends only upon its mass and efficiency:

$$\delta_j = \Psi_1(M_j) + \Psi_2(\eta_j) \tag{26}$$

As discussed in the previous section, Player 1 can maximize his sales by solving the following optimization problem:

$$Max \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta^*_{N+j}}} \tag{27}$$

subject to

$$\begin{aligned} T_1 &= 0.05 \\ T_2 &= 0.10 \\ T_3 &= 0.125 \\ T_4 &= 0.15 \\ T_5 &= 0.2 \\ T_6 &= 0.25 \\ T_7 &= 0.3 \\ T_8 &= 0.35 \\ T_9 &= 0.4 \\ T_{10} &= 0.5 \end{aligned}$$

$$P_j = 300 \quad j = 1, \ldots, 10$$

For this application, we can further simplify the problem by eliminating the product design variables as follows. Given values for the platform design variables, it is possible to find values for I and L so that product j satisfies the power constraint P = 300 and the torque requirement T for that product j. (In the following we will drop the subscripts for convenience.)

To do this, we first determine I as a function of the torque requirement:

$$I = \frac{T}{K\varphi} \tag{28}$$

Using the relationships from Appendix A, we can rewrite this and define Z and W as follows:

$$I^2 = \frac{T\Re\pi}{N_C N_S}$$

$$\Re = \frac{1}{L}\left(\frac{\pi(2r_0 + t_s)}{4\mu_{steel}\mu_0 t_s} + \frac{1}{\mu_{steel}\mu_0} + \frac{7\times10^{-4}}{\mu_0(r_0 - t_s - 7\times10^{-4})}\right) = \frac{Z}{L} \tag{29}$$

$$I^2 L = \frac{T\pi Z}{N_C N_S} = W$$

The last of these equations can be rewritten to express L in terms of I:

$$L = \frac{W}{I^2} \tag{30}$$

Now we consider power, which is the difference between input power and output power:

$$P = P_{\text{in}} - P_{\text{out}} \tag{31}$$

Using the relationships for P_{in} and P_{out} from Appendix A, we can derive the following expression:

$$P = 115I - \left(I^2(R_a + R_S) + 2I\right) \tag{32}$$

After including the definitions of R_a and R_S from Appendix A and defining the quantities C_1, C_2, C_3, and C_4 (all of which depend only upon the platform design variables), we have an expression for power in terms of I and L:

$$C_1 = 1.69\times10^{-8} \times 10^6 = 1.69\times10^{-2}$$

$$C_2 = \frac{2N_c}{A_{aw}} + \frac{4N_s}{A_{sw}}$$

$$C_3 = 4(r_0 - t - 7\times10^{-4})\frac{N_C}{A_{aw}} \tag{33}$$

$$C_4 = 8(r_0 - t)\frac{N_S}{A_{sw}}$$

$$P = 113I - C_1 I^2\left(C_2 L + C_3 + C_4\right) \tag{34}$$

Because P = 300, substituting Equation (33) into Equation (34) yields a quadratic function of I :

$$C_1\left(C_3+C_4\right)I^2-113I+C_1C_2W+P=0 \tag{35}$$

Solving this expression will give us two values for I. We use the smaller value because the larger value exceeds the upper bound on I. From this value of I, we can determine L using Equation 30. This procedure can be used to determine the design variables for each of the ten products in the product family. Therefore, it is not necessary to include the product design variables in the search to solve the optimization problem, which can be reformulated as a problem with just the six platform design variables.

Although Player 2 uses the same type of universal motor technology, we don't make any assumptions about Player 2's platform. The best that Player 2 can do is to design independent products, one for each torque requirement, with no platform design variables. Thus, to find Player 2's dominant strategy, we separate the problem into ten parallel problems, one for each of the ten products. The objective of each problem is to maximize the mean utility $\delta_j=\Psi_1(M_j)+\Psi_2(\eta_j)$ subject to the power and torque constraints.

5.3.1.1 Results

First we considered Player 2's problem. For each torque requirement we found values for all eight design variables to maximize the product utility (cf. [61]). Appendix B.2 lists the values of the design variables and customer attributes for Player 2's products. (Again, the power equals 300 watts for all of the designs.)

Then we considered Player 1's product family design problem. In this problem we propose two approaches. In the first approach we optimize Q (Equation 27) using the markets shown in Table 10. The optimization problem was solved ten times using ten different initial points for the platform design variables (listed in Appendix B). All ten runs yielded the same values for the platform design variables (these are shown in Table 11).

From these values, the ten torque requirements, and the power constraint, we determined values for the product design variables using the procedure presented in the previous section. Table 12 shows the values of the product design variables and the customer attributes for the ten products. (Recall that the power equals 300 watts for all of the designs.) We will call this set of products Family A.

Given these solutions and a market size of 1,000,000, we determined the sales of each player (shown in Table 13). As expected, Player 2's sales are slightly higher. However, it is interesting to note that the difference is less than 4,000 units, which shows that, by adopting a product platform, Player 1 does not give up much in terms of sales.

In the second approach we solve a different subproblem to find the product platform. We replace the original objective function (total sales) with the following quantity:

$$X = \sum_{j=1}^{N} e^{\delta_j} \tag{36}$$

Table 11. Player 1's Optimal Product Platform.

Variable	Optimum Value	units

N_c	1170.6427	*turns*
N_s	500	*turns*
A_{aw}	0.2890	mm^2
A_{sw}	0.2892	mm^2
r_o	0.01	*m*
t_s	0.0045	*m*

Table 12. Player 1's Product Designs and Customer Attribute Values for each Torque Value for Family A.

Torque	I	L	M	η
0.05	2.9234	0.0037	0.1661	0.8923
0.10	3.0076	0.0071	0.2118	0.8674
0.125	3.0498	0.0087	0.2327	0.8553
0.15	3.0922	0.0101	0.2524	0.8436
0.2	3.1771	0.0128	0.2886	0.8211
0.25	3.2625	0.0152	0.3208	0.7996
0.3	3.3482	0.0152	0.3495	0.7791
0.35	3.4344	0.0192	0.3751	0.7596
0.4	3.5209	0.0209	0.3979	0.7409
0.5	3.6953	0.0237	0.4360	0.7059

Table 13. Total Sales for Each Player (Player 1 uses Family A).

Player	*Sales*
1	498,698
2	501,302

This optimization problem was solved ten times using ten different initial points for the platform design variables (listed in Appendix B). This generated ten different solutions, but seven of them had the same objective function value. Table 14 shows one of the best solutions found.

Given this product platform, for each torque, we calculated the appropriate values for I and L. Table 15 lists the values of I and L and the customer attributes for each product. (Recall that the power equals 300 watts for all of the designs.) We will call this set of products Family B.

Because Family B is different from Family A, the total sales for Player 1 (across all ten markets) is different (and smaller). Player 2 still uses the product family shown in Table 11. Table 16 shows the total sales for each player.

Table 14 compares the sales in each market for each family. As shown, the products in Family A have larger market shares (compared to those of Family B) in the larger markets (those with a torque requirement of 0.05, 0.10, and 0.125). Although Family B has more sales in some smaller markets, Family A has greater total sales. Interestingly, the difference in total sales is small, indicating that using a simpler objective function to determine the product platform variables is a good heuristic.

Figure 4 shows the mass and efficiency of each product in each family. Although customers want high efficiency, low mass products, there is a tradeoff to achieve this customer requirement. For a given power and torque, high-efficiency motors require more mass, as shown in Figure 4.

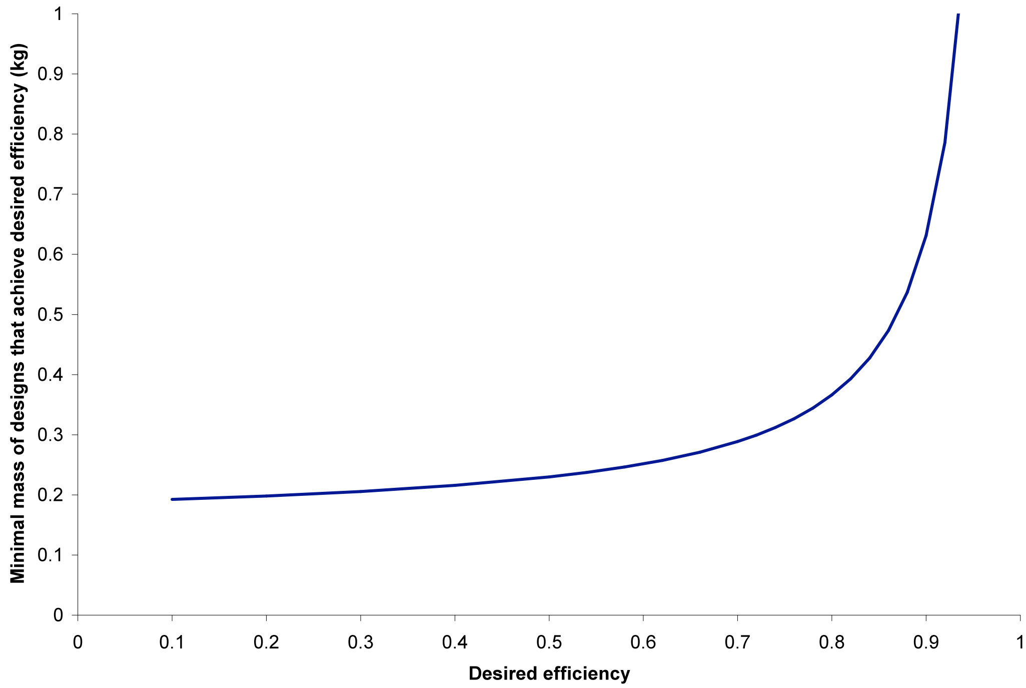

Figure 4. The mass and efficiency tradeoff when power = 300 W and torque = 0.05 Nm.

Player 1's high-torque products have higher mass and slightly lower efficiency than Player 2's high-torque products. The mass and efficiency of the players' medium-torque products are very close. Player 1's low-torque products have slightly lower mass but lower efficiency than Player 2's low-torque products.

Table 14. Player 1's Product Platform (Family B).

N_C	N_S	A_{aw}	A_{sw}	r_0	t_s
970.1875	500.0000	0.2850	0.2854	0.0100	0.0032

Table 15. Player 1's Product Designs and Customer Attribute Values for each Torque Value for Family B.

Torque	I	L	M	η
0.05	2.9525	0.0036	0.1745	0.8835
0.10	3.0293	0.0068	0.2142	0.8612
0.125	3.0678	0.0083	0.2325	0.8503
0.15	3.1065	0.0097	0.2499	0.8398
0.2	3.1840	0.0123	0.2821	0.8193
0.25	3.2620	0.0147	0.3110	0.7997
0.3	3.3403	0.0168	0.3371	0.7810
0.35	3.4190	0.0187	0.3606	0.7630
0.4	3.4982	0.0204	0.3817	0.7457
0.5	3.6577	0.0233	0.4177	0.7132

Table 16. Total Sales when Player 1 uses Family B.

Product Family	Sales
Player1	497,684
Player2	502,316

Table 17. Sales in Each Market.

Market	Product Family A	Product Family B
1	99,468	98,818
2	125,000	124,538
3	100,022	99,778
4	49,993	49,929
5	49,915	49,951
6	24,895	24,958
7	12,412	12,462
8	12,376	12,443
9	12,340	12,420
10	12,276	12,378
Total	498,698	497,684

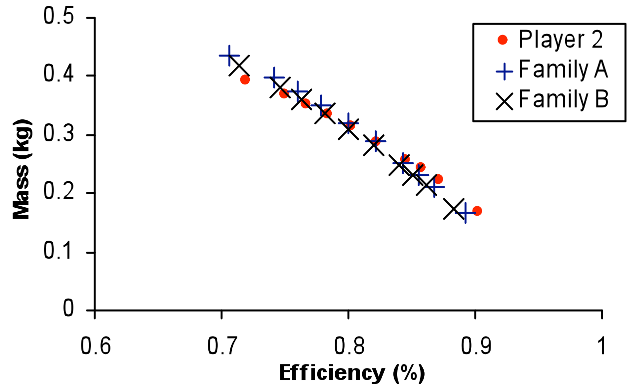

Figure 5. The mass and efficiency of each player's products.

5.3.1.2 Discussion: Product Family Design Process

The analysis of the product family design problem and the results from the motor design example show that a design optimization problem that initially appears to be quite complex can be solved by replacing it with an appropriate set of subproblems, which we call a separation [61]. Unlike decomposition-based design optimization, which requires multiple iterations to converge to a solution, each subproblem in the separation is solved only once. In this section we propose two approaches to solve the problem. In the first approach we try to maximize total sales for our products based on competitor's best design.

In the second approach however instead of maximizing total sales (Q), we maximize a simpler intermediate quantity (the quantity X) that is sufficient to identify a dominant strategy. This leads to a suboptimal solution, but the separation requires less effort because it ignores Player 2. In the motor design example, we can eliminate the product design variables from the problem for Player 1 and solve to find just the platform design variables, which reduces the size of the search space. Finally, for

Player 2, we separate his problem into ten subproblems, one for each of his products. Using this separation, we get an exact optimal solution to the original problem. This is not always the case, unfortunately.

5.4 Summary

In this chapter we used game theory to model market competition to solve product family design problems. We divided the market in different segments and showed that losing profit for one manufacturer causes gaining profit for the other. We replaced the problem of maximizing the total sales with a surrogate function and showed that our separation gives exact results as our original problem. We showed that using game theory can be a good approach to model competition and as we showed in our electric motor example Player 1 could able to compete with Player 2 although adapted a product platform which can be a constraint for optimum product design.

Chapter 6: Optimizing the Profitability of a Product Design under Competition

In this section we consider the problem of designing a product with the objective of maximizing profit when the competition's product is not yet known because the competitor is simultaneously designing its product and setting its price. We formulate the problem as a mixed-motive (i.e., non-zero-sum) two-player game and present an approach that finds the optimal product design. We find an exact separation that first optimizes the product design and then finds the equilibrium point of a mixed-motive two-player game to determine the optimal prices for both firms. We apply the approach to a motor design example and consider the impact of changes to the competitor's cost model. As mentioned before, in a mixed motive game, unlike a zero-sum (or constant sum) game, the gain of one player does not necessarily equal the loss of the other (for a given combination of the players' choices).

6.1 Problem Formulation

This section formulates the problem of optimal product design under competition. There are two firms, whom we call "Player 1" and "Player 2". First, we will define the following notation:

p_i = Price for Player *i*

d_i = Performance of Player *i*'s product; this is constrained to the range $[0, d_{\max}]$.

$h_i(d_i)$ = Unit cost of Player i's product as a function of product performance.

X_i = Desirability of the Player *i*'s product.

Π_i = Profit for Player *i*.

S = Total market size ($S > 0$).

We assume that $h_i(d_i)$ is a strictly increasing function of d_i over $[0, d_{\max}]$ and that $h_i(d_i) > 0$ for all values of d_i.

For modeling demand, there are various probabilistic choice models based on the Discrete Choice Analysis (DCA). In this work, we use the logit model for its simplicity and capability to predict the demand of future product designs. The utility value is based on product characteristics that can be related to engineering design. Like [58], we formulate the utility function in terms of observable characteristics to model the entire market, not individual preferences. Although there exist many possible forms for the utility function, this study, like other studies in this area, assumes that the product's utility is a linear function of its price and key product attributes (cf. [14, 16]). We will define product performance as the total utility associated with the product attributes. (See Section 5 for an example.) Then, the product's utility is the difference between its performance and its price (multiplied by the appropriate parameter) and express its desirability as follows:

$$X_i = e^{u_i} = e^{d_i - \beta_i p_i} \tag{44}$$

We use the logit model to estimate each player's sales. Each firm's profit depends upon the net revenue and sales:

$$\Pi_i = (p_i - h_i(d_i)) \frac{X_i}{X_1 + X_2} S \tag{45}$$

Both firms seek to set d_i and p_i in order to maximize their own profit subject to $0 \le d_i \le d_{\max}$, $p_i \ge 0$, and $p_i > h_i(d_i)$.

We exclude fixed costs from this formulation because we assume that changes to the design cause insignificant changes to the fixed costs. Thus, they are essentially constant and can be ignored in this analysis. Future work will have to analyze the problem with significant fixed costs that vary greatly as a function of the design.

6.2 Solution Approach

In order to analyze this two-player mixed-motive game, we will first find a set of non-dominated strategies for each player. Let R_i be the profit per unit sold for Player *i*. For a value of $R_i > 0$, consider the set of strategies (p_i, d_i) for Player *i* that all have a profit per unit of R_i:

$$R_i = p_i - h_i(d_i) \tag{46}$$

To find the non-dominated strategy in this set, we will focus on Player 1 and then apply the results to Player 2.

Theorem 1. Given $R_1 > 0$ and two strategies (p_a, d_a) and (p_b, d_b) that satisfy $p_a - h_1(d_a) = p_b - h_1(d_b) = R_1$, $\Pi_1(p_a, d_a) > \Pi_1(p_b, d_b) \quad \forall X_2 > 0$ if and only if $d_a - \beta_1 h_1(d_a) > d_b - \beta_1 h_1(d_b)$.

Proof. Let $X_a = e^{d_a - \beta_1 p_a}, X_b = e^{d_b - \beta_1 p_b}$. Then $\Pi_1(p_a, d_a) > \Pi_1(p_b, d_b) \quad \forall X_2 > 0$ if and only if

$$R_1 \frac{X_a S}{X_a + X_2} > R_1 \frac{X_b S}{X_b + X_2} \quad \forall X_2 > 0 \tag{47}$$

Because $S > 0$ and $R_1 > 0$, this is true if and only if

$$\frac{X_a}{X_a + X_2} > \frac{X_b}{X_b + X_2} \quad \forall X_2 > 0 \tag{48}$$

Because X_2 is always positive, this is true if and only if

$$X_a > X_b \tag{49}$$

This is equivalent to the following:

$$d_a - \beta_1 p_a > d_b - \beta_1 p_b \tag{50}$$

$$d_a - \beta_1 h_1(d_a) - \beta_1 R_1 > d_b - \beta_1 h_1(d_b) - \beta_1 R_1 \tag{51}$$

$$d_a - \beta_1 h_1(d_a) > d_b - \beta_1 h_1(d_b) \tag{52}$$

Q.E.D.

This theorem (when applied to both firms) implies that the non-dominated strategies for Player *i* include d_i^* that maximizes $d_i - \beta_i h_i(d_i)$. Let x_i be the vector of design variables for Player *i*'s product such that $d_i = w_i(x_i)$ and $h_i(d_i) = h_i\left(w_i(x_i)\right) = c_i(x_i)$ is the unit cost function. Finding d_i^* is equivalent to finding x_i^* that maximizes $w_i(x_i) - \beta_i c_i(x_i)$.

Thus, Player 1 should find his optimal design, Player 2 should find his optimal design, and then they should determine prices that maximize each one's profit. This is also a mixed-motive two-player game, but it is simpler than the original problem because each player now has only one variable.

Suppose that Player *i* has found d_i^*. Player *i* chooses price p_i, which determines the price and profit:

$$X_i = e^{d_i^* - \beta_i p_i} \tag{53}$$

$$\Pi_i = \left(p_i - h_i\left(d_i^*\right)\right)\frac{X_i S}{X_i + X_{3-i}} \tag{54}$$

In order to calculate the Nash equilibrium we have to get first order derivative with respect to p_i.

$$\frac{\partial X_i}{\partial p_i} = e^{d_i^* - \beta_i p_i} \cdot \left(\frac{\partial d_i^*}{\partial p_i} - \frac{\partial \beta_i p_i}{\partial p_i} \right) = e^{d_i^* - p_i} \left(0 - \beta_i\right) = -\beta_i e^{d_i^* - p_i} = -\beta_i X_i \tag{55}$$

$$\frac{\partial \Pi_i}{\partial p_i} = \left(p_i - h_i\left(d_i^*\right)\right) \cdot \frac{\partial}{\partial p_i}\left(\frac{X_i S}{X_i + X_{3-i}} \right) + \frac{X_i S}{X_i + X_{3-i}} \tag{56}$$

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{-\beta_i \left(p_i - h_i\left(d_i^*\right)\right) X_i X_{3-i} S}{(X_i + X_{3-i})^2} + \frac{X_i S}{X_i + X_{3-i}} \tag{57}$$

To find the Nash equilibrium, we find p_1 and p_2 such that

$$\begin{gathered} \frac{\partial \Pi_1}{\partial p_1} = 0 \\ \frac{\partial \Pi_2}{\partial p_2} = 0 \end{gathered} \tag{58}$$

These conditions (shown in Figure 1) are equivalent to the following:

$$\begin{gathered} \beta_1 \left(p_1 - h_1\left(d_1^*\right)\right) \frac{X_2}{X_1 + X_2} = 1 \\ \beta_2 \left(p_2 - h_2\left(d_2^*\right)\right) \frac{X_1}{X_1 + X_2} = 1 \end{gathered} \tag{59}$$

This system of equations has a unique solution, so there is a unique Nash equilibrium, and this is a non-dominated strategy. To find the solution, let $R_i = p_i - h_i(d_i^*)$ for $i = 1$ and 2, find R_2 such that $\beta_2 R_2 / \left(\beta_2 R_2 - 1\right) - \beta_2 R_2 - \left(d_1^* - \beta_1 h_1(d_1^*)\right) + \left(d_2^* - \beta_2 h_2(d_2^*)\right) = \ln\left(\beta_2 R_2 - 1\right)$, and set $R_1 = \frac{1}{\beta_1\left(\beta_2 R_2 - 1\right)} + \frac{1}{\beta_1}$. Then, the prices p_1 and p_2 can be determined.

6.3 Separation

The solution approach described in Section 6.2 is an exact separation [61]. It replaces the large scale design optimization problem with a set of subproblems, solves each subproblem once, and produces a feasible solution without iterative cycles. Each firm seeks to maximize its profit, which depends upon its product's price and performance and the decisions of the other firm. Finding the set of non-dominated strategies allows each firm to solve a simpler problem first. Namely, each firm must find x_i^* that maximizes $w_i(x_i) - c_i(x_i)$, the difference between performance and cost. After this step, each firm needs to set its price (represented by the surrogate variables R_1 and R_2). The solution to this simpler mixed-motive game gives the optimal price decisions for each firm.

This separation of a complex design optimization problem into subproblems reduces the computational effort required to solve the problem. Moreover, in this case, the separation provides an optimal solution to the original problem.

This analysis depends upon the form of the desirability function. Changing the desirability function will change the set of non-dominated strategies by changing the objective function that is used to find the optimal product design, but an exact separation is still possible.

6.4 Example: Motor Design

To illustrate the approach, we will consider the universal electric motor design optimization problem developed by Simpson [80]. The optimization model includes nine variables (eight design variables and the price), four customer attributes, twenty-three intermediate engineering attributes, and seven fixed engineering parameters (described in Appendix A). Table 18 lists the design variables, their lower and upper bounds, and units. The price p is in dollars.

Table 18: Bounds on Design Variables.

Variable	Definition	Lower bound	Upper bound	units
N_c	Turns of wire (armature)	100	1500	turns
N_s	Turns of wire (stator), per pole	1	500	turns
A_{aw}	Cross sectional area of armature wire	0.01	1.0	mm^2
A_{sw}	Cross sectional area of stator wire	0.01	1.0	mm^2
r_o	Outer radius (stator)	0.01	0.1	m
t_s	Thickness (stator)	0.0005	0.01	m
I	Electric current	0.1	6	A
L	Stack length	0.01	0.2	m

The four customer attributes are the torque T (in Nm), the power P (in watts), the efficiency η, and the mass M (in kg). They are calculated from the design variables and the engineering attributes as follows:

$$\begin{aligned} T &= K\varphi I \\ P &= P_{\text{in}} - P_{\text{out}} \\ \eta &= P / P_{\text{in}} \\ M &= M_w + M_s + M_a \end{aligned} \tag{60}$$

The motor should have a power of 300 watts and torque of 0.05 Nm. The total market size S = 1,000,000. The performance of the product depends upon these four attributes as follows:

$$\begin{aligned}
\Psi_1(M) &= 0.5(1-M) \\
\Psi_2(\eta) &= \eta - 0.5 \\
\Psi_3(P) &= -\left(1-\frac{P}{300}\right)^2 \\
\Psi_4(T) &= -\left(1-\frac{T}{0.05}\right)^2
\end{aligned} \tag{61}$$

Let d_i be the performance of Player i's product, which is the sum of the four attribute's performance:

$$d_i = w_i(x_i) = \Psi_1(M_i) + \Psi_2(\eta_i) + \Psi_3(P_i) + \Psi_4(T_i) \tag{62}$$

The unit cost of a product (which includes labor and other processing costs) is proportional to the unit material cost. Let α_i be the cost coefficient of Player i's product. The unit cost function depends upon the mass and material cost of each component of the motor:

$$c_i(x_i) = \alpha_i \left(M_{wi}C_c + \left(M_{si} + M_{ai}\right)C_s\right) \tag{63}$$

For Player 1, following our previous work [57], we assume that $\alpha_1 = 10/7$. For Player 2, we consider different values of α_2 in order to evaluate the impact of changes to that firm's cost structure. For both players, we assume three different price parameters for player 1 to evaluate our desirability model. We assume price parameter $\beta_1 = 0.5,1,2$ while $\beta_2 = 1$. We realized increasing price parameter will decrease player 1's profit because weight of cost function increases and player 1 decreases its price in order to compete with other player which leads to lower profit. Results are shown in Tables 18-20.

6.5 Experimental Results

First we determined the best design for Player 1. The optimization problem, which maximizes $w_i(x_i) - \beta_i c_i(x_i)$, was solved ten times with ten different initial points. This was repeated for each value of β_1. We kept the solutions (shown in Table 19) that gave the best value of objective function.

Table 19: Best Designs for Player 1 for different price parameters.

β_1	N_c	N_s	A_{aw}	A_{sw}	r_o	t_s	I	L
0.5	623.3301	295.0772	0.1447	0.1447	0.0100	0.0044	3.1721	0.0100
1.0	589.1725	275.1006	0.1092	0.1092	0.0100	0.0044	3.3863	0.0100
2.0	623.2663	242.8017	0.0821	0.0821	0.0100	0.0050	3.7331	0.0100

For Player 2 we solved the optimization problem with three different cost coefficients. (Again, ten different initial points were used to solve each problem.) For each cost coefficient, we kept the best design found. These solutions are shown in Table 20.

Table 20: Best Designs for Player 2 for different cost coefficients.

α_2	N_c	N_s	A_{aw}	A_{sw}	r_o	t_s	I	L
1.5	562.0179	262.4602	0.0942	0.0942	0.0100	0.0044	3.5424	0.0100
2.0	585.5484	273.4140	0.1069	0.1069	0.0100	0.0044	3.4064	0.0100
2.5	540.9784	252.6670	0.0851	0.0851	0.0100	0.0044	3.6733	0.0100

The next step is to solve the mixed-motive two-player game by finding the Nash equilibrium, which required finding R_1 and R_2. We then verified that the solution was a Nash equilibrium by checking that the second derivatives were negative, as shown in Tables 24-26. We did this for each of the three cost coefficients. Tables 21, 22, and 23 show the results for both firms. In the scenarios with the larger unit cost for the second firm's product, the second firm's sales and profit are lower.

Table 21: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = 0.5, \beta_2 = 1$

Cost Coefficient	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
α_2	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
1.5	4.0754	1.9637	4.3072	2.1182	50.93	49.07	2,075,400	963,700
2.0	4.1147	1.9458	4.3465	2.182	51.39	48.61	2,114,700	945,800
2.5	4.1453	1.9323	4.3771	2.1629	51.75	48.25	2,145,300	932,600

Table 22: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = \beta_2 = 1$

Cost Coefficient	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
α_2	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
1.5	2.0037	1.9963	2.1764	2.1508	50.09	49.94	1,003,700	996,300
2.0	2.0228	1.9777	2.1954	2.214	50.56	49.44	1,022,800	977,700
2.5	2.0376	1.9638	2.2102	2.1944	50.92	49.08	1,037,600	963,800

Table 23: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = 2, \beta_2 = 1$

Cost Coefficient	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
α_2	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
1.5	0.9777	2.0467	1.1055	2.2012	48.86	51.14	477,700	1,046,700
2.0	0.9868	2.0272	1.1146	2.2635	49.33	50.67	486,800	1,027,200
2.5	0.9938	2.0126	1.1216	2.2431	49.69	50.31	493,800	1,012,600

Table 24: Second Derivatives for Nash Equilibrium for Different Cost Coefficients. $\beta_1 = 0.5, \beta_2 = 1$

Cost Coefficient α_2	$\frac{\partial^2 \Pi_1}{\partial p_1^2}$	$\frac{\partial^2 \Pi_2}{\partial p_2^2}$
1.5	-0.5187	-0.4907
2.0	-0.5283	-0.4861
2.5	-0.5357	-0.4825

Table 25: Second Derivatives for Nash Equilibrium for Different Cost Coefficients. $\beta_1 = \beta_2 = 1$

Cost Coefficient α_2	$\frac{\partial^2 \Pi_1}{\partial p_1^2}$	$\frac{\partial^2 \Pi_2}{\partial p_2^2}$
1.5	-0.5009	-0.4991
2.0	-0.5056	-0.4944
2.5	-0.5092	-0.4908

Table 26: Second Derivatives for Nash Equilibrium for Different Cost Coefficients. $\beta_1 = 2, \beta_2 = 1$

Cost Coefficient α_2	$\frac{\partial^2 \Pi_1}{\partial p_1^2}$	$\frac{\partial^2 \Pi_2}{\partial p_2^2}$
1.5	-0.4942	-0.5114
2.0	-0.4966	-0.5067
2.5	-0.4984	-0.5031

6.6 Summary

In this chapter we discussed the problem of designing a single product considering competition while different player's strategies lead to different profit. We separated the problem of profit maximization into a set of subproblems. In the motor example we replaced the problem of maximizing the profit with value of the product which is the difference between performance and cost to reach that performance. Then we find Nash Equilibrium points to set the price in which none of the players want to deviate from that. We showed that cost coefficient affects profit. Higher cost coefficients lead to lower profit. In the next chapter we used mixed motive strategy to design family of products.

Chapter 7: Optimizing the Profitability of a Product Family under Competition

7.1 Problem Description

In this problem Player 1 is trying to design a family of products that will compete with a family of products from Player 2 in which each player is seeking to maximize his profits. We model this problem as a mixed motive game (instead of a zero-sum game) because the sum of the profits gained by the players is not constant. In order to design a product family Player 1 first should design a product platform and then derive product design variables for each individual product. In order to investigate Player 1's optimal strategy, we first consider the worst case scenario that can happen to Player 1. The answer is in that scenario, Player 2 designs the best products that he can without a product platform.

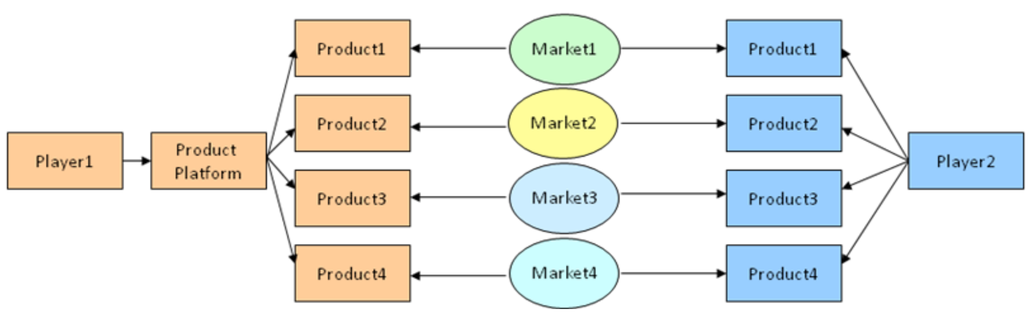

Figure 5. Product family in a competitive market.

We will define the following notation:

p_{i1} = Price for Player 1's Product *i*.

d_{i1} = Performance of Player 1's Product *i*; this is constrained to the range $[0, d_{\max}]$.

h_{i1} = Unit cost of Player 1's Product i as a function of product performance.

X_{i1} = Desirability of the Player 1's Product *i*.

Π_{i1} = Profit for Player 1's Product *i*.

S_i = Product *i*'s market size ($S > 0$).

We assume that $h_{i1}(d_{i1})$ is a strictly increasing function of d_{i1} over $[0, d_{\max}]$ and that $h_{i1}(d_{i1}) > 0$ for all values of d_{i1}.

The profit function can be expressed with equation 64.

$$\Pi_1 = \sum_{i=1}^{n} (p_{i1} - h_{i1}) \frac{X_{i1}}{X_{i1} + X_{i2}} \times S_i \tag{64}$$

7.2 Solution Approach

In order to solve the problem we separate the problem into three sets of subproblems. Figure 6 shows how we separate the all-at-once problem into sets of subproblems. In the first set of subproblems Player 2 maximizes the performance of each of his products by maximizing υ_{i2} $i = 1, \ldots. n$ (n is number of products). The second set has just one subproblem. In this subproblem Player 1 determines his product family design by maximizing the following sum:

$$\max \sum_{i=1}^{n} \upsilon_{i1} \tag{65}$$

For each individual product,

$$\upsilon_{i1} = d_{i1} - \beta_{1i} h(d_{i1}) \quad i = 1, \ldots. n \tag{66}$$

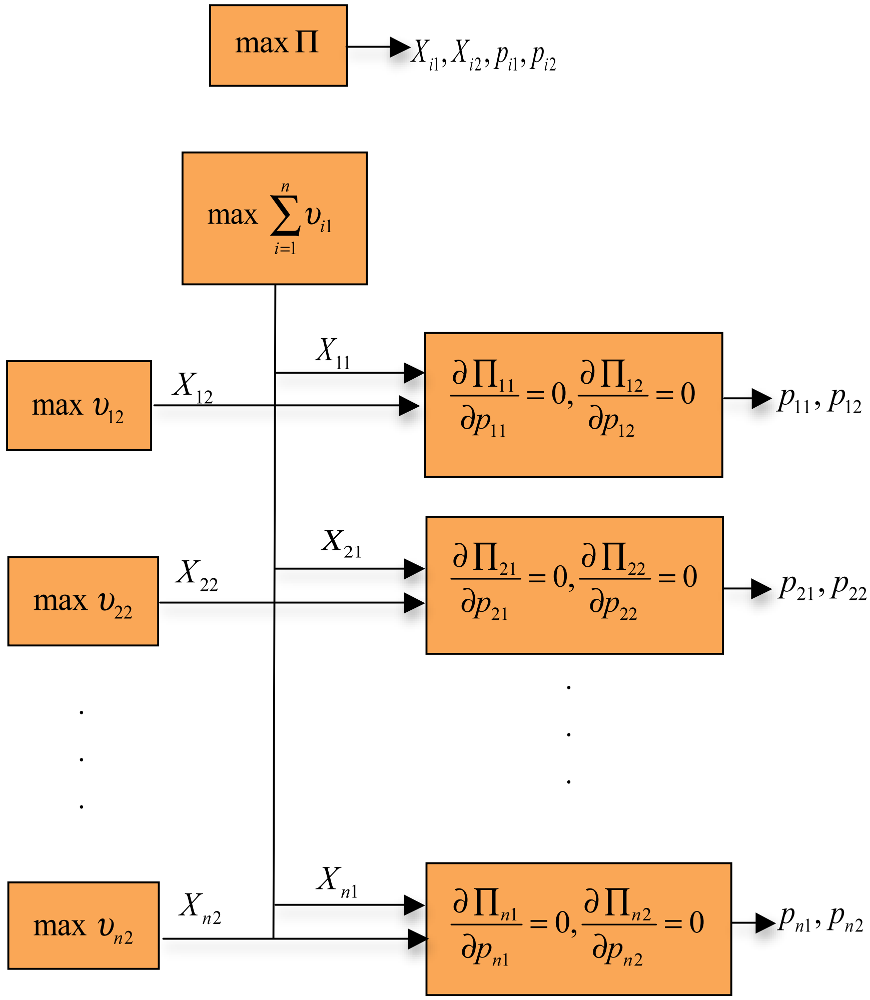

Figure 6. Separating product family design. This figure shows all of the subproblems.

The third set of subproblems includes, for each product market, a mixed-motive game to find the optimal prices for both players' products in that market. We do this for each market independently of the others. Players 1 and 2 have found υ_{ij} $i = 1, \ldots n$ *and* $j = 1$ *and* 2 and d_{ij}^{*} (i = 1, …, n; j = 1, 2). Player j's price for product i is p_{ij}, which determines Player j's profit for this product as follows:

$$X_{ij} = e^{d_{ij}^{*} - \beta_{ij} p_{ij}} \tag{67}$$

$$\Pi_{ij}\left(p_{ij},d_{ij}^{*}\right)=\sum_{i=1}^{n}(p_{ij}-h_{ij})\frac{X_{ij}}{X_{ij}+X_{ij}}\times S_{i}\quad j=1\ \ and\ \ 2 \tag{68}$$

In order to calculate the Nash equilibrium for market i we have to get the first-order derivative with respect to p_{ij}.

$$\frac{\partial X_{ij}}{\partial p_{ij}}=e^{d_{ij}^{*}-\beta_{ij}p_{ij}}\cdot\left(\frac{\partial d_{ij}^{*}}{\partial p_{ij}}-\frac{\partial \beta_{ij}p_{ij}}{\partial p_{ij}}\right)=-\beta_{ij}X_{ij} \tag{69}$$

$$\frac{\partial \Pi_{ij}}{\partial p_{ij}}=\left(p_{ij}-h_{ij}\left(d_{ij}^{*}\right)\right)\cdot\frac{\partial}{\partial p_{ij}}\left(\frac{X_{ij}S}{X_{ij}+X_{3-ij}}\right)+\frac{X_{ij}S}{X_{ij}+X_{3-ij}} \tag{70}$$

$$\frac{\partial \Pi_{ij}}{\partial p_{ij}}=\frac{-\beta_{ij}\left(p_{ij}-h_{ij}\left(d_{ij}^{*}\right)\right)X_{ij}X_{3-ij}S}{(X_{ij}+X_{3-ij})^{2}}+\frac{X_{ij}S}{X_{ij}+X_{3-ij}} \tag{71}$$

To find the Nash equilibrium, we find p_{i1} and p_{i2} such that

$$\begin{gathered}\frac{\partial \Pi_{i1}}{\partial p_{i1}}=0\\ \frac{\partial \Pi_{i2}}{\partial p_{i2}}=0\end{gathered} \tag{72}$$

These conditions are equivalent to the following:

$$\begin{gathered}\beta_{i1}\left(p_{i1}-h_{i1}\left(d_{i1}^{*}\right)\right)\frac{X_{i2}}{X_{i1}+X_{i2}}=1\\ \beta_{i2}\left(p_{i2}-h_{i2}\left(d_{i2}^{*}\right)\right)\frac{X_{i1}}{X_{i1}+X_{i2}}=1\end{gathered} \tag{73}$$

Now, for j = 1 and 2, let $R_{ij}=p_{ij}-c_{ij}(x_{ij}^{*})$; these conditions imply that $\beta_{ij}R_{ij}>1$ and $R_{ij}>0$. They are equivalent to the following system of equations:

$$\begin{gathered}\left(\beta_{i2}R_{i2}-1\right)-1/\left(\beta_{i2}R_{i2}-1\right)+\ln\left(\beta_{i2}R_{i2}-1\right)=\left(d_{i2}^{*}-\beta_{i2}c_{i2}(x_{i2}^{*})\right)-\left(d_{i1}^{*}-\beta_{i1}c_{i1}(x_{i1}^{*})\right)\\ \left(\beta_{i1}R_{i1}-1\right)\left(\beta_{i2}R_{i2}-1\right)=1\end{gathered} \tag{74}$$

Because the left-hand side of the first equation is strictly increasing and the right-hand side is a constant independent of R_{i1} and R_{i2}, there is a unique value R_{i2}^{*} that solves this system of equations. Consequently, there is also a unique value R_{i1}^{*} that can be determined from the second equation. From R_{i1}^* and R_{i2}^*, we can determine the optimal prices p_{i1}^* and p_{i2}^*. Because both R_{i1}^* and R_{i2}^* are positive, each player's profit is positive at this point.

This point is a Nash equilibrium. That is, if Player *j* changes his price to any value not equal to p_{ij}^*, then his profit will decrease. When $p_{ij} = c_{ij}(x_{ij}^*)$, $\Pi_{ij}\left(p_{ij}, x_{ij}^*\right) = 0$. The first derivative $\partial \Pi_{ij} / \partial p_{ij}$ is positive if and only if $\beta_{ij} p_{ij} + \ln\left(\beta_j \left(p_{ij} - c_{ij}\left(x_{ij}^*\right)\right) - 1\right) < d_{ij}^* - \ln X_{3i-ij}$, which is true if and only if $p_{ij} < p_{ij}^*$. As p_{ij} increases, the profit first increases until it reaches its maximum at $p_{ij} = p_{ij}^*$, where the first derivative equals 0. For $p_{ij} > p_{ij}^*$, $\partial \Pi_{ij} / \partial p_{ij}$ is negative. As p_{ij} increases, the profit decreases and approaches 0. In other words, $\Pi_{ij}\left(p_{ij}, x_{ij}^*\right)$ is strongly quasiconcave, and p_{ij}^* is the unique global optimal solution.

The set of strategies $\left\{\left(p_{i1}^*, x_{i1}^*\right), \left(p_{i2}^*, x_{i2}^*\right)\right\}$ is a Nash equilibrium for the original two-player game. Because $\left\{p_{i1}^*, p_{i2}^*\right\}$ is a Nash equilibrium to the two-player pricing game, we know that if either player changes only his price, that will decrease his profits. Recall that $z_{ij}(x_{ij}^*) = z_{ij}^*$. As a result of Theorem 1, if Player *j* changes his design to x_{ij} such that $z_{ij}(x_{ij}) < z_{ij}^*$, then, for any price p_{ij}, his profit decreases because $\Pi_{ij}\left(p_{ij}, x_{ij}\right) < \Pi_{ij}\left(p_{ij} - c_{ij}\left(x_{ij}\right) + c_{ij}\left(x_{ij}^*\right), x_{ij}^*\right) \le \Pi_{ij}\left(p_{ij}^*, x_{ij}^*\right)$.

There may be multiple Nash equilibria if there are multiple designs for Player j that maximize $z_{ij}(x_{ij}) = w_{ij}(x_{ij}) - \beta_j c_{ij}(x_{ij})$. In this case, because they all have the same value of $w_{ij}(x_{ij}) - \beta_j c_{ij}(x_{ij})$, they all lead to same equilibrium prices and profits.

Each design optimization subproblem (maximizing $z_{ij}(x_{ij})$ subject to $x_{ij} \in A_{ij}$) has one player's design variables, and the effort required to solve it depends upon the properties of $z_{ij}(x_{ij})$ and A_{ij}. The pricing subproblem can be solved directly from (17).

7.3 Application: Motor Design

To illustrate this approach to product family design, we will use the universal electric motor family mentioned in Section 4.2. Player 2's Best Products and Customer Attribute Values for each Torque Value are shown in Table B.2. We will consider two different separations for this problem by changing the objective of Player 1's product platform design optimization platform. In the first separation, Player 1 seeks to $\max \sum_{i=1}^{n} \upsilon_{i1}$. In the second separation, Player 1 seeks to minimize the total material cost $c_i(x_i) = \alpha_i \left(M_{wi} C_c + (M_{si} + M_{ai}) C_s \right)$.

7.3.1 Results

First we considered Player 2's problem. For each torque requirement we found values for all eight design variables to maximize the product utility (cf. [61]). Appendix B.2 lists the values of the design variables and customer attributes for Player 2's products. (Again, the power equals 300 watts for all of the designs.)

Then we considered Player 1's product family design problem. In this problem we propose two approaches. In the first approach we try to maximize the value of the product family shown in equation (58). Then we find R_2 and R_1 by finding Nash Equilibrium points in order to find prices for each product in the family. The optimization problem was solved ten times using ten different initial points for the platform design variables (listed in Table B). We found based values for the product platform based on different values of price coefficients (β_1 = 0.5, 1, 2). Table 27 shows the best product platforms, Tables 27, 28 and 29 show the product family values, and Figures 7, 8, and 9 graph the mass and efficiency of the product families for β_1 = 0.5, 1 and 2. Tables 30, 31, 32, 33, 34, and 35 show the prices, product-specific profits, and total profits for each player for the different values of the price coefficients. We see that, as the price coefficient β_1 increases, Player 1's prices must decrease, and the firm's total profits decrease.

Table 27: Best Product Platform Designs for Player 1 for different price coefficients.

β_1	N_c	N_s	A_{aw}	A_{sw}	r_o	t_s
0.5	291.6347	320.4783	0.1921	0.1344	0.0166	0.0047
1.0	194.3221	335.0022	0.1262	0.1924	0.0167	0.0033
2.0	970.7074	128.5253	0.1791	0.1889	0.0250	0.0087

Table 28: Best Product Family Designs for Player 1 for β_1=0.5.

Torque	I	L	M	η
0.05	3.3158	0.0085	0.1353	0.7867
0.10	3.5681	0.0147	0.1907	0.7311
0.125	3.6973	0.0171	0.2122	0.7056
0.15	3.8288	0.0191	0.2304	0.6813
0.2	4.0989	0.0222	0.2584	0.6364
0.25	4.3795	0.0243	0.2773	0.5957
0.3	4.6720	0.0257	0.2892	0.5584
0.35	4.9779	0.0264	0.2956	0.5241
0.4	5.2993	0.0266	0.2976	0.4923
0.5	6.0000	0.0259	0.2916	0.4348

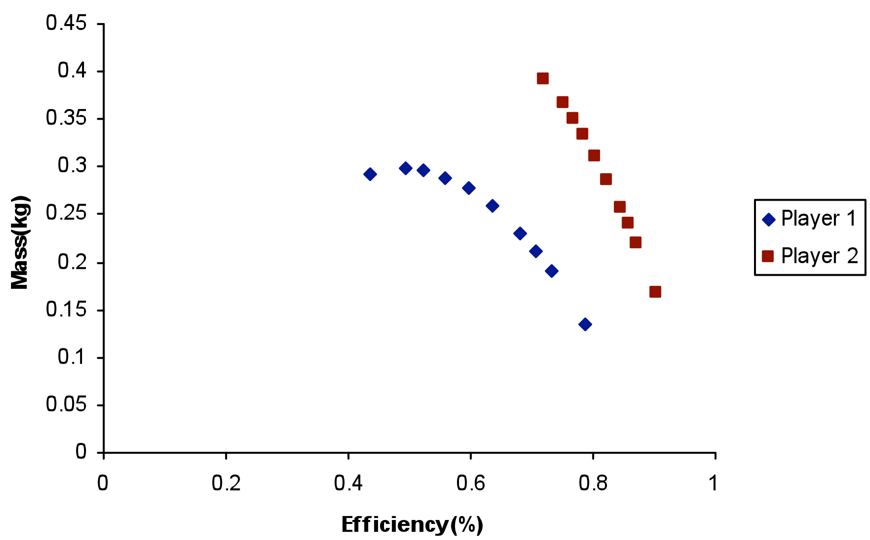

Figure 7. The mass and efficiency of each player's products for β_1=0.5.

Table 29: Best Product Family Designs for Player 1 for β_1=1.

Torque	I	L	M	η
0.05	3.2671	0.0116	0.1807	0.7985
0.10	3.5227	0.0200	0.2578	0.7405
0.125	3.6533	0.0232	0.2877	0.7141
0.15	3.7858	0.0260	0.3129	0.6891
0.2	4.0570	0.0301	0.3514	0.6430
0.25	4.3372	0.0330	0.3774	0.6015
0.3	4.6274	0.0348	0.3939	0.5638
0.35	4.9286	0.0357	0.4030	0.5293
0.4	5.2422	0.0361	0.4064	0.4976
0.5	5.9139	0.0355	0.4005	0.4411

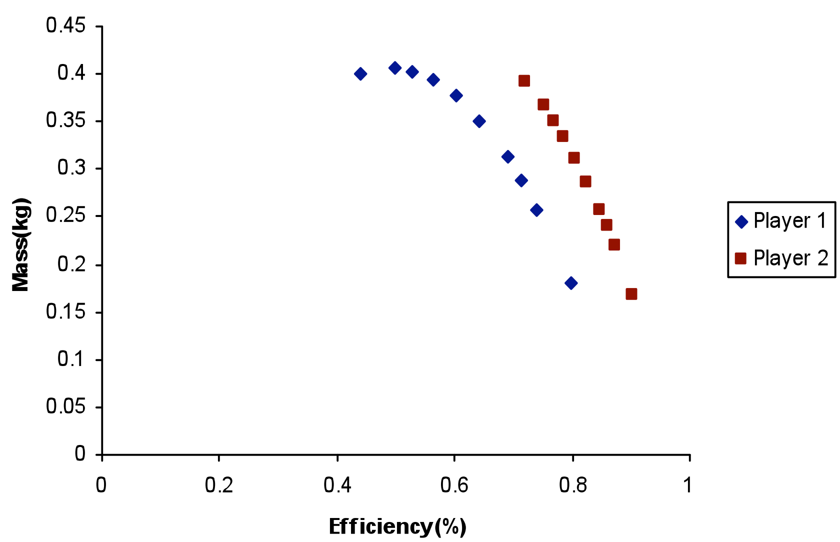

Figure 8. The mass and efficiency of each player's products for β_1 = 1.

Table 30: Best Product Family Designs for Player 1 for β_1=2.

Torque	I	L	M	η
0.05	3.5804	0.0040	0.2015	0.7286
0.10	3.7766	0.0072	0.2617	0.6908
0.125	3.8784	0.0085	0.2868	0.6726
0.15	3.9829	0.0097	0.3089	0.6550
0.2	4.2009	0.0116	0.3453	0.6210
0.25	4.4331	0.0130	0.3721	0.5885
0.3	4.6826	0.0140	0.3907	0.5571
0.35	4.9538	0.0146	0.4019	0.5266
0.4	5.2537	0.0148	0.4064	0.4965
0.5	5.9978	0.0142	0.3949	0.4349

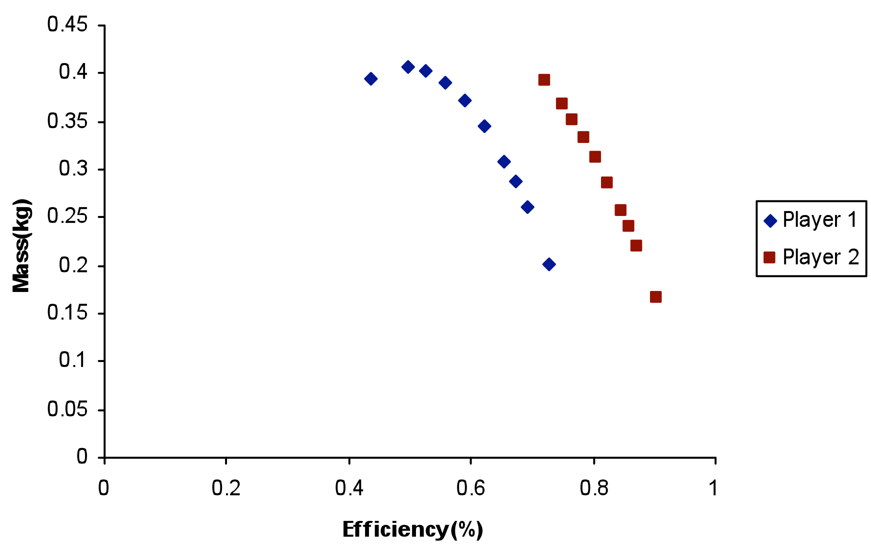

Figure 9. The mass and efficiency of each player's products for β_1 = 2.

Table 31: Prices and Profit for Both Firms for Different Price Coefficients. $\beta_1 = 0.5, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	4.0517	1.9748	4.37	2.31	50.64	49.36	2,051,700	974,800
0.10	4.0791	1.962	4.50	2.41	50.97	49.03	2,079,100	962,000
0.125	5.7628	1.5315	6.22	2.03	65.29	34.71	3,762,800	531,500
0.15	4.0971	1.9537	4.59	2.49	51.19	48.81	2,097,100	953,700
0.2	4.1082	1.9487	4.65	2.54	51.32	48.68	2,108,200	948,700
0.25	4.117	1.9448	4.69	2.59	51.42	48.58	2,117,000	944,800
0.3	4.1325	1.9379	4.73	2.63	51.60	48.40	2,132,500	937,900
0.35	4.1496	1.9304	4.76	2.66	51.80	48.20	2,149,600	930,400
0.4	4.1614	1.9253	4.77	2.69	51.94	48.06	2,161,400	925,300
0.5	4.197	1.9103	4.80	2.72	52.35	47.65	2,197,000	910,300

Table 32: Total Profit for Each Player. $\beta_1 = 0.5, \beta_2 = 1$

Player	Profit ($)
1	22,856,400
2	9,019,400

Table 33: Prices and Profit for Both Firms for Different Price Coefficients. $\beta_1 = 1, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	1.9368	2.0675	2.36	2.40	48.37	51.63	936,800	1,067,500
0.10	1.917	2.0905	2.48	2.54	47.84	52.16	917,000	1,090,500
0.125	2.661	1.602	3.28	2.10	62.42	37.58	1,661,000	602,000
0.15	1.9022	2.1085	2.56	2.64	47.43	52.57	902,200	1,108,500
0.2	1.8921	2.121	2.62	2.72	47.15	52.85	892,100	1,130,100
0.25	1.8849	2.1301	2.66	2.78	46.95	53.05	884,900	1,130,100
0.3	1.8853	2.1296	2.69	2.82	46.96	53.04	885,300	1,129,600
0.35	1.8894	2.1244	2.71	2.86	47.07	52.93	889,400	1,124,400
0.4	1.8924	2.1206	2.72	2.88	47.16	52.84	892,400	1,120,600
0.5	1.9084	2.1008	2.73	2.91	47.60	52.40	908,400	1,100,800

Table 34: Total Profit for Each Player. $\beta_1 = 1, \beta_2 = 1$

Player	Profit ($)
1	9,769,500
2	10,604,100

Table 35: Prices and Profit for Both Firms for Different Price Coefficients. $\beta_1 = 2, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	0.8723	2.3431	1.39	2.68	42.68	57.32	372,300	1,343,100
0.10	0.861	2.3851	1.48	2.83	41.93	58.07	361,000	1,385,100
0.125	1.1559	1.7624	1.81	2.26	56.74	43.26	655,900	762,400
0.15	0.8512	2.4235	1.55	2.96	41.26	58.74	351,200	1,423,500
0.2	0.8433	2.4563	1.60	3.05	40.71	59.29	343,300	1,456,300
0.25	0.8375	2.4813	1.64	3.13	40.30	59.70	337,500	1,481,300
0.3	0.8348	2.4936	1.66	3.19	40.10	59.90	334,800	1,493,600
0.35	0.834	2.4968	1.68	3.23	40.05	59.95	334,000	1,496,800
0.4	0.8347	2.4938	1.69	3.25	40.10	59.90	334,700	1,493,800
0.5	0.8432	2.4567	1.68	3.27	40.71	59.29	343,200	1,456,700

In the second separation, Player 1 seeks to minimize the material cost. The price coefficient does not affect the product platform design. The best product platform design variables are shown in Table 37. Table 38 shows the corresponding product designs, and Figure 10 shows the products' mass and efficiency. Note that the product mass is much lower than those of the product families found using the first separation. Tables 39, 40, 41, 42, 43, and 44 show the prices, product-specific profits, and total profit for each player. Again, as Player 1's price coefficient increases, the firm must decrease their prices, which reduces their profits.

To compare the separations, we compare the profits of the product families for different values of the price coefficient. Tables 45, 46, 47, 48, 49 and 50 show the change in profits for each product and for total profits. The values reported are the profits found using the first separation minus the profits found using the second separation. Thus, a positive difference indicates that the first separation found a more profitable product family. Table 51 summarizes these results.

Table 36: Total Profit for Each Player. $\beta_1 = 2, \beta_2 = 1$

Player	Profit ($)
1	3,767,900
2	13,792,600

Table 37: Best Product Platform Designs for Player 1 using the second separation.

N_c	N_s	A_{aw}	A_{sw}	r_o	t_s
971.5294	56.2648	0.1494	0.1546	0.0100	0.0067

Table 38: Best Product Family Designs for Player 1.

Torque	I	L	M	η
0.05	4.2227	0.0354	0.2022	0.6178
0.10	5.7624	0.0380	0.2160	0.4527
0.125	6.5562	0.0367	0.2091	0.3979
0.15	7.3674	0.0348	0.1995	0.3541
0.2	9.0467	0.0308	0.1782	0.2884
0.25	10.8117	0.0270	0.1579	0.2413
0.3	12.6772	0.0235	0.1398	0.2058
0.35	14.6623	0.0205	0.1239	0.1779
0.4	16.7938	0.0179	0.1099	0.1553
0.5	21.6681	0.0134	0.0864	0.1204

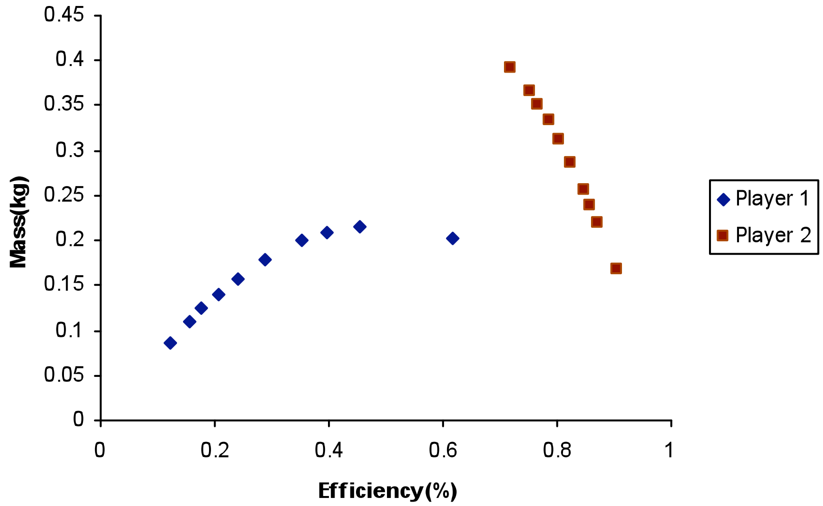

Figure 10. The mass and efficiency of each player's products.

Table 39: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = 0.5, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	3.868	2.0707	4.34	2.40	48.29	51.71	1,868,000	1,070,700
0.10	3.859	2.0759	4.37	2.53	48.17	51.83	1,859,000	1,075,900
0.125	5.4161	1.5855	5.91	2.08	63.07	36.93	3,416,100	585,500
0.15	3.897	2.0543	4.36	2.59	48.68	51.32	1,897,000	1,054,300
0.2	3.9403	2.0308	4.36	2.63	49.24	50.76	1,940,300	1,030,800
0.25	3.9823	2.0089	4.35	2.65	49.78	50.22	1,982,300	1,008,900
0.3	4.0319	1.9843	4.36	2.68	50.40	49.60	2,031,900	984,300
0.35	4.0763	1.9632	4.37	2.7	50.94	49.06	2,076,300	963,200
0.4	4.1103	1.9477	4.37	2.71	51.34	48.66	2,110,300	947,700
0.5	4.1842	1.9157	4.39	2.73	52.20	47.80	2,184,200	915,700

Table 40: Total Profit for Each Player. $\beta_1 = 0.5, \beta_2 = 1$

Player	Profit ($)
1	21,365,400
2	9,637,000

Table 41: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = 1, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	1.8625	2.1594	2.34	2.49	46.31	53.69	862,500	1,159,400
0.10	1.8539	2.1711	2.36	2.62	46.06	53.94	853,900	1,171,100
0.125	2.5828	1.6318	3.07	2.13	61.28	38.72	1,582,800	631,800
0.15	1.8769	2.1404	2.34	2.67	46.72	53.28	876,900	1,140,400
0.2	1.9041	2.106	2.32	2.70	47.48	52.52	904,100	1,106,000
0.25	1.9308	2.0744	2.30	2.72	48.21	51.79	930,800	1,074,400
0.3	1.9608	2.0408	2.29	2.73	49.00	51.00	960,800	1,040,800
0.35	1.9878	2.0123	2.28	2.74	49.69	50.31	987,800	1,012,300
0.4	2.0093	1.9908	2.27	2.75	50.23	49.77	1,009,300	990,800
0.5	2.0542	1.9486	2.26	2.76	51.32	48.68	1,054,200	948,600

Table 42: Total Profit for Each Player. $\beta_1 = 1, \beta_2 = 1$

Player	Profit ($)
1	10,023,100
2	10,275,600

Table 43: Prices and Profit for Both Firms for Different Cost Coefficients. $\beta_1 = 2, \beta_2 = 1$

Torque	Nash equilibrium		Price ($)		Market Share (%)		Profit ($)	
T	R_1	R_2	p_1	p_2	Player 1	Player 2	Π_1	Π_2
0.05	0.8686	2.3565	1.34	2.69	42.44	57.56	368,600	1,356,500
0.10	0.8613	2.3839	1.37	2.83	41.95	58.05	361,300	1,383,900
0.125	1.1762	1.7394	1.67	2.24	57.49	42.51	676,200	739,400
0.15	0.8755	2.3316	1.34	2.87	42.89	57.11	375,500	1,331,600
0.2	0.8931	2.2719	1.31	2.87	44.02	55.98	393,100	1,271,900
0.25	0.9107	2.2174	1.28	2.86	45.10	54.90	410,700	1,217,400
0.3	0.9298	2.1632	1.26	2.86	46.23	53.77	429,800	1,163,200
0.35	0.9472	2.118	1.24	2.85	47.21	52.79	447,200	1,118,000
0.4	0.9617	2.0829	1.22	2.84	48.01	51.99	461,700	1,082,900
0.5	0.9912	2.018	1.20	2.83	49.55	50.45	491,200	1,018,000

Table 44: Total Profit for Each Player. $\beta_1 = 2, \beta_2 = 1$

Player	Profit ($)
1	4,415,300
2	11,682,800

Table 45: Profit Comparison between First and Second Approach. $\beta_1 = 0.5, \beta_2 = 1$

Torque	Player 1	Player 2
0.05	0.1871	-0.0975
0.10	0.2235	-0.1155
0.125	0.3529	-0.0549
0.15	0.2031	-0.102
0.2	0.1715	-0.0837
0.25	0.1373	-0.0653
0.3	0.105	-0.0484
0.35	0.0779	-0.0348
0.4	0.0543	-0.0237
0.5	0.0162	-0.0068

Table 46: Total Profit Difference for Each Player. $\beta_1 = 0.5, \beta_2 = 1$

Player	Total Profit Difference ($)
1	1,491,000
2	-617,600

Table 47: Profit Comparison between First and Second Approach. $\beta_1 = 1, \beta_2 = 1$

Torque	Player 1	Player 2
0.05	0.0625	-0.0783
0.10	0.0525	-0.0679
0.125	0.06	-0.0231
0.15	0.0156	-0.0199
0.2	-0.0227	0.0285
0.25	-0.0555	0.0681
0.3	-0.085	0.1011
0.35	-0.1089	0.1255
0.4	-0.1271	0.1428
0.5	-0.1546	0.163

Table 48: Total Profit Difference for Each Player. $\beta_1 = 1, \beta_2 = 1$

Player	Total Profit Difference ($)
1	-253,600
2	328,500

Table 49: Profit Comparison between First and Second Approach. $\beta_1 = 2, \beta_2 = 1$

Torque	Player 1	Player 2
0.05	0.022	-0.0763
0.10	0.0167	-0.0612
0.125	0.0085	-0.0091
0.15	-0.0087	0.0314
0.2	-0.0356	0.1266
0.25	-0.0602	0.209
0.3	-0.0828	0.2778
0.35	-0.1021	0.3309
0.4	-0.1175	0.3697
0.5	-0.1428	0.4173

Table 50: Total Profit Difference for Each Player. $\beta_1 = 2, \beta_2 = 1$

Player	Total Profit Difference ($)
1	-647,400
2	2,109,800

Table 51: Total Profit for Each Player and Change in Profit due to Change in Separation.

β_1, β_2	Player	Separation 1	Separation 2	Difference
0.5,1	1	22,856,400	21,365,400	1,491,000
	2	9,019,400	9,637,000	-617,600
1,1	1	9,769,500	10,023,100	-253,600
	2	10,604,100	10,275,600	328,500
2,1	1	3,767,900	4,415,300	-647,400
	2	13,792,600	11,682,800	2,109,800

7.3.2 Discussion: Product Family Design with Mixed Motive Strategy

The analysis of the product family design problem and the results from the motor design example show that competition between manufacturers is an important issue for designers to consider while designing their products. As shown in Tables 31, 33, and 35, Player 1's market share and total profit decreased as Player 1's price coefficient increased. Player 1's reaction to profit loss is to decrease the price in order to keep up with the competition. As the price coefficient increases, the need to design lighter products (product with less material cost) decreases. Tables 28, 29 and 30 show increase in product mass. As product torque increases throughout the family there is a need to design longer rotor to produce higher current for motor rotation. For this reason increasing torque will increase mass of the product.

As shown in Figures 8, 9, and 10, Player 1 sacrifices mass and efficiency of the products in order to compete with Player 2 and although these customer requirements in Player 1's product are worse than Player 2, it still does not give up the competition. As shown in Tables 31, 33, 35, 39, 41, and 43, as price coefficient increases; efficiency of the product drops due to increasing current throughout the family. In the second separation Player 1 seeks to minimize the material cost in order to maximize profit. In this approach Player 1's market share and total profit decreases and we see profit loss as the price coefficient increases. Figure 11 however shows that the second separation minimizes mass, but at the same time the efficiency of the products drops dramatically. We calculated the profit difference between first and second approach to compare different approaches. Results are shown in Table 24, 26 and 30.

$$\Delta\Pi = \Pi_{T1} - \Pi_{T2} \tag{68}$$

The first separation gives better results for Player 1 in most market segments. Total profit is only higher in $\beta_1 = 0.5, \beta_2 = 1$ scenario in the first separation. The second separation reduces product mass, which is correlated with product cost. This separation shows better total profits when β_1 = 1 and 2 because the price is more important to the customer. In the second separation Player 2 performs better in terms of total profit. As shown in Figure 11, Player 1's products can not compete with the other player with respect to efficiency and Player 1's product mass and efficiencies are much worse than Player 2. This gives us some conclusions that the second separation sacrifices product performance in order to compete with the other manufacturer. While separation 1 performs better when customers care less about price, when the price coefficient increases (that is, customers care more about price) the first separation performs worse than the second separation.

7.4 Summary

In this chapter we studied mixed motive strategy to investigate the effect of different strategies on product family design. We considered a two player game in which one player adapts a platform and still compete with the other player which designs the best for each product in the family. We separated the problem into a set of subproblems and replaced the problem of maximizing the profit with a surrogate function that maximized the value of the whole family to set product platform variables first and then we find Nash Equilibrium point to set the price. The example showed that the relative quality of different separations can depend upon customer preferences (in this case, the relative importance of performance and price).

Chapter 8: Summary and Conclusions

This chapter summarizes the results of this dissertation, discusses the contributions of the work, and presents ideas for future work.

8.1 Summary of Research Results

This dissertation studied product design and product family design optimization problems in the presence of competition. In order to solve these problems, we developed a new approach for solving design optimization problems. We defined "separation" and distinguished it from decomposition-based design optimization, Analytical Target Cascading and all-at-once techniques, which can be computationally extensive and expensive due to iterative nature of the algorithms. A separation replaces a large, complex subproblem with a series of subproblems and solves each subproblem once and sequentially. Separation provides a different way to find solutions to design optimization problems. However, a separation must be carefully designed to provide a valuable solution. Our results showed that the quality of approximate separations depends upon the constraints and objectives used in the subproblems.

This dissertation studied four product design and product family design problems.

We first studied the problem of maximizing profit without competition. Our formulation followed the decision-based design framework for product design. We showed that the problem can be separated to solve it faster and easier. We used the universal electric motor problem to illustrate this approach and showed that focusing on certain customer requirements lead to better results. A separation that optimized the product attributes first and then set the design variables to minimize the deviation from those targets led to a more profitable design than other approximate separations.

In the second problem we considered the problem of maximizing profit with the existence of competition. Our analysis of the problem identified a set of non-dominated strategies. We then replaced the original mixed-motive two-player game (which included the design and price variables for each firm) with an exact separation that has three subproblems. The first two subproblems, which can be solved in parallel, find the optimal design for each firm by maximizing the value of the product. The third sets the optimal prices for both firms in order to maximize their profits.

The third problem that we considered was the problem of maximizing the sales for a family of products in a two-firm market. We formulated the problem as a two-player zero-sum game and found dominant strategies for each player. We constructed an exact separation in which Player 2 designs each of his products and then Player 1 designs a product family that maximizes his sales.

In the fourth problem, the two firms are designing product families and seek to maximize their profit. We constructed an approximate separation that has three sets of subproblems. First, Player 2 designs each of his products. Then, Player 1 designs a product family. Finally, we have to set optimal prices for both players in each market. We compared the performance of two different approximate separations to the motor product family design problem. The separations' relative performance depended upon the customer preferences about product performance and price.

For all of these problems, we developed a separation that can be used as to solve the problem in general and also illustrated the solution approach by using the motor design problem as an example.

8.2 Contributions and Limitations

This dissertation makes two types of contributions. First, it presents a new way to solve large design optimization problems. Separations can solve large, complex problems easier. It is more efficient, needs less computations and iterations are eliminated due to the sequential nature of the process.

From a methodological perspective, we contribute to the literature by proposing and testing a new approach to solve design optimization problems. Because it avoids iteration of decomposition, a separation may reduce the time needed to find a feasible solution, which could be useful when development time is limited and the designer is willing to accept a suboptimal, feasible design. Such a separation is helpful. However, a separation that fails to find a feasible solution must be replaced with a better separation.

The dissertation demonstrates the usefulness of separation by separating product design and product family design problems with competition. We showed that product design can be separated into subproblems of designing and pricing of the product separately in order to maximize sales or profit.

Second, this dissertation contributes to the literature on product design optimization by studying new problems not previously considered. In particular, we considered problems in which a firm must both design and price a new product or product family while the competition is simultaneously doing the same thing. This led to game-theoretic models of product design unlike those used in previous work to model collaborative design. In this work, the players in the game are directly competing against each other.

This dissertation has studied only some of the problems that can occur in the area of design for market systems. It has not addressed problems with different distribution channels like those discussed in Williams et al. [16].

Clearly, the consideration of problems with only two players is limited. The problems studied here do not cover situations with more than two firms competing in the marketplace. In addition, these problems do not cover leader-follower scenarios.

Because the demand models assume that the markets for different products in the product family are completely distinct, they may be invalid in cases where consumers will choose among different product sectors.

8.3 Future Work

In this dissertation, we studied the separation of product family design problems with and without competition. Areas for future work include separating reliability-based design optimization problems, robust optimization problems, and separating multi-objective, multidisciplinary optimization problems. Examples in designing large, complex systems such as ships, aircrafts can be investigated to illustrate these approaches and other future extensions of separation. Design of these large, complex systems can be separated into smaller subproblems throughout the design process. Also we need to address guidelines on how to separate design optimization problems, how to automate this process, and how to get these guidelines.

Appendices

Appendix A. Engineering Parameters and Attributes

Engineering Parameters

Length of air gap l_g = 7.0 ×10-4 m

Terminal voltage V_t = 115 V

Resistivity of copper ρ = 1.69×10-8 $Ohms \bullet m$

Permeability of free space μ_o = 4π ×10-7 H/m

Number of stator poles p_{st} = 2

Cost of copper C_c = 2.2051 $\$/kg$

Cost of steel C_s = 0.882 $\$/kg$

Density of copper δ_c = 8,960 kg/m^3

Density of steel δ_s = 7,861.09 kg/m^3

Engineering Attributes

Magnetizing intensity [Ampere turns/ m] $H = N_c I / (l_c + l_r + 2l_g)$

Mean path length within the stator [m] $l_c = \pi(2r_o + t_s)/2$

Diameter of armature [m] $l_r = 2(r_o - t_s - l_g)$

Input power [W] $P_{in} = V_t I$

Power losses due to copper and brushes [W] $P_{out} = I^2(R_a + R_s) + 2I$

Armature wire length [m] $l_{aw} = (2L + 2l_r)N_c$

Stator wire length [m] $l_{sw} = p_{st}(2L + 4(r_o - t_s))N_s$

Armature wire resistance [Ohm] $R_a = \rho l_{aw} / A_{aw} \times 10^6$

Stator wire resistance [Ohm] $R_s = \rho l_{sw} / A_{sw} \times 10^6$

Mass windings [kg] $M_w = (l_{aw}A_{aw} + l_{sw}A_{sw})\delta_c \times 10^{-6}$

Mass of stator [kg] $M_s = \pi L(r_o^2 - (r_o - t_s)^2)\delta_s$

Mass of armature [kg] $M_a = \pi L(r_o - t_s - l_g)^2 \delta_s$

Motor constant [dimensionless] $K = N_c / \pi$

Magneto magnetic force [A turns] $\Im = N_s I$

Magnetic flux [Wb] $\varphi = \Im / \Re$

Total reluctance [A turns/ Wb] $\Re = \Re_s + \Re_a + 2\Re_g$

Stator reluctance [A turns/ Wb] $\Re_s = l_c /(2\mu_{steel}\mu_o A_s)$

Armature reluctance [A turns/ Wb] $\Re_a = l_r /(\mu_{steel}\mu_o A_a)$

Reluctance of one air gap [A turns/ Wb] $\Re_g = l_g /(\mu_o A_g)$

Cross sectional area of stator [m^2] $A_s = t_s L$

Cross sectional area of armature [m^2] $A_a = l_r L$

Cross sectional area of air gap [m^2] $A_g = l_r L$

Relative permeability of steel [dimensionless]

$$\mu_{steel} = -0.2279H^2 + 52.411H + 3115.8 \quad H \le 220$$
$$\mu_{steel} = 11633.5 - 1486.33\ln(H) \quad 220 < H \le 1000$$
$$\mu_{steel} = 1000 \quad H > 1000$$

Appendix B. Initial Designs

Table B.1: Initial designs for Separations S1 and S2.

N_c	N_s	A_{aw}	A_{sw}	r_o	t_s
622.4461	10.2789	0.1798	0.0855	0.0296	0.0087
971.5237	41.0603	0.1669	0.2469	0.0299	0.0036
622.4460	10.2836	0.2004	0.1590	0.0294	0.0051
971.4546	52.0387	0.2522	0.9947	0.0113	0.0021
373.0639	21.7458	0.2813	1	0.0159	0.0008
971.5356	48.7206	0.2976	0.9760	0.0142	0.0025
383.8721	33.1335	0.2371	1	0.0103	0.0005
971.5287	56.2628	0.2115	0.2162	0.0123	0.0013
483.5892	223.6510	0.0644	1	0.0243	0.0005
970.7074	128.5235	0.2849	0.2776	0.0181	0.0098

Table B.2: Player 2's Best Products and Customer Attribute Values for each Torque Value.

Torque	N_C	N_S	A_{aw}	A_{sw}	r_0	t_s	I	L	M	η
0.05	624.4289	331.9046	0.2936	0.2936	0.0100	0.0040	2.8878	0.0100	0.167	0.9034
0.10	970.0895	441.3717	0.2955	0.2955	0.0100	0.0046	2.9927	0.0100	0.2203	0.8717
0.125	1045.8406	488.0866	0.2951	0.2951	0.0100	0.0044	0.0389	0.0100	0.2399	0.8584
0.15	1142.0968	500	0.2937	0.2937	0.0100	0.0042	3.0842	0.0100	0.2562	0.8458
0.2	1207.8453	500	0.2921	0.2921	0.0100	0.0037	3.1682	0.0109	0.2863	0.8234
0.25	1240.4244	500	0.2909	0.2909	0.0100	0.0034	3.2466	0.0119	0.3118	0.8035
0.3	1273.7696	500	0.2896	0.2897	0.0100	0.0031	3.3230	0.0128	0.3331	0.785
0.35	1306.6778	500	0.2884	0.2884	0.0100	0.0029	3.3984	0.0136	0.3513	0.7676
0.4	1338.0306	500	0.2870	0.2870	0.0100	0.0028	3.4735	0.0142	0.3669	0.751
0.5	1394.6589	500	0.2845	0.2845	0.0100	0.0026	3.6239	0.0153	0.392	0.7198

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