ABSTRACT

Title of Dissertation: TWO ESSAYS ON SPATIAL ECONOMETRICS

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The first part of the dissertation is a Monte-Carlo study of the small sample properties of various estimators of the parameters of single equation model with a spatially lagged dependent variable and a spatially lagged disturbance term. We focus on the small sample behavior of the maximum likelihood estimator (MLE) and spatial instrumental variable (IV) estimators. These IV estimators are feasible spatial two-stage least squares (FGS2SLS) and series estimators which were suggested by Kelejian and Prucha (1998, 2001), the best GS2SLS estimator which was suggested by Lee (2000).

The findings indicate that the finite sample properties of the IV estimators are almost identical. Furthermore, the advantage of the ML estimator over the spatial IV estimators is very limited or nonexistent in most of the cases considered. These results have important implications in terms of efficiency and computational feasibility of these estimators.

The second part analyses the importance of alternative channels of contagion during the Asian, Russian and Brazilian crisis episodes. We consider four contagion channels relating to the extent of trade, financial links through common lenders (bank lending channel), similarity in risk, and neighborhood effects.

In order to assess the significance of each we apply a spatial modeling technique to weekly stock market returns of a cross-section of countries. The paper improves upon previous contagion studies with similar methodology in two aspects. First, the parameters of the model are estimated by a consistent procedure. This clearly leads to more reliable inferences. Second, we use a data set involving a larger sample of countries. This should alleviate some of the potential sample selection biases inherent in previous studies.

The results indicate that (a) the bank lending channel was important in all three crisis episodes, (b) the trade channel was relevant in the Russian and Brazilian crisis episodes, but not in the Asian crisis, (c) there is some evidence of the risk similarity channel during the Asian crises, but not in the Russian and Brazilian crises, (d) neighborhood effects were important in all three crisis episodes. Furthermore, there is an evidence of negative trade spillovers from Japan during the Asian crisis.

TWO ESSAYS ON SPATIAL ECONOMETRICS

by

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Chapter 1

Introduction

The recent years witnessed a growing interest in econometric models that account for spatial interactions. They have been applied to police expenditure (Kelejian and Robinson (1992)), spatial price competition (Pinske, Slade and Brett (2001)), spending by jurisdiction (Case, Hines and Rosen (1993)), housing prices (Bell and Bockstael (2000)). Theoretical estimation of these models has been developed by Kelejian and Prucha (1998, 1999, 2001), Lee (1999a), Ord (1975), Pinske and Slade (1998). The theoretical results obtained by these authors relate to large samples. The purpose of the first part of the thesis is to study small sample properties of the suggested estimators.

The second part of my dissertation relates to the empirical application of spatial models to study the issues of contagion in international financial markets. In particular, the focus of the study is on the channels of contagion. Understanding the channels of contagion is important for economic policymaking and crisis prevention.

The organization of the dissertation is as follows. Chapter 2 considers the small sample properties of various estimators of the parameters of single equation model with a spatially lagged dependent variable and a spatially lagged disturbance term. Chapter 3 explores the importance of alternative channels of contagion during several crisis episodes. Chapter 4 is the appendix to chapter 2. Chapter 5 is the appendix to chapter 3.

Chapter 2

Finite Sample Properties of Estimators of Spatial Autoregressive Models With Autoregressive Disturbances: Further Results

2.1 Introduction

In cross-sectional and panel data studies units under consideration often interact with each other in such a way that spatial correlation or spatial spill-overs result. This correlation or spill-overs could relate directly to the dependent and independent variables involved, as well as to the error terms. Neglecting the presence of such spatial interactions could lead to inefficient or even inconsistent estimators of the model parameters. Very often such problems are exacerbated by the short time dimension of the data. In such cases traditional methods of estimation (OLS, SUR, fixed and random effects estimators) are not able to account for these effects. For this reason spatial modeling techniques have been developed. Using these techniques relationships involving various forms of spatial correlations and spatial spill-overs can often be estimated with just a single cross section of data. In contrast, as an example, an SUR approach would typically require T cross sections of data, and the corresponding large sample properties would be based on the assumption that $T \to \infty$.

Spatial correlations can be found in a wide variety of empirical models. As one example, the activities of one bank may have an effect on the stability of its partners and, subsequently, on the performance of other banks (Allen and Gale (2000)). As another example, the spread of balance of payments crises across countries, the phenomena called contagion, is directly related to the interactions between countries through different channels such as exposure to a common lender, trade links etc. (Calvo and Reinhart (1997), Kaminsky and Reinhart (2000)). As still another example, decision of state and local governments on the level of public expenditure, taxes and tariffs, to a large extent depends on the decisions of neighboring jurisdictions (Besley and Case (1995), Brueckner (1998), Case, Hines and Rose (1993), Shroder (1995), Stigler (1957)).

Spatial models typically cope with issues relating to spatial spill-overs in three ways, namely, by modeling such spill-overs involving the dependent variables, the predetermined variables, and, finally, the disturbance term. In order to be able to estimate such models in terms of a single cross-section, a great deal of structure is imposed on the relationships involved.[1]

In the early literature the prevalent technique of estimation of spatial models was maximum likelihood (ML). However, later studies showed that this method is computationally imprecise in large samples. As a result, a new computationally simpler method, namely the generalized spatial two stage least squares method, was developed by Kelejian and Prucha (1998). In still later studies a more asymp-

[1] For a general review of spatial models see Anselin (2001), Cliff and Ord (1973, 1981), and Cressie (1993)

totically efficient version of this method was suggested by Lee (1999a), and also by Kelejian and Prucha (2001). The suggestion in Lee (1999a) as well as in Kelejian and Prucha (2001) relate to the instruments used in the procedure.

To date, results relating to the small sample properties of these variations of the original generalized spatial two stage least squares procedure are not available in the literature. The purpose of this thesis, therefore, is to fill this gap via Monte-Carlo techniques. In doing this we will follow the strategy of Das, Kelejian and Prucha (2003) who investigated the small sample properties of the generalized spatial two stage least squares.

This paper is organized as follows. Section 2.2 provides a review of spatial models. Section 2.3 presents a general specification of a single linear equation model with spatially lagged dependent variable and the error term. Section 2.4 describes the estimation techniques which include the maximum likelihood procedure and the generalized spatial two stage least squares procedure proposed by Kelejian and Prucha (1998). Section 2.5 introduces modifications of the generalized spatial two stage least squares method suggested by Lee (1999a), and by Kelejian and Prucha (2001) Section 2.6 describes the iterated versions of the considered estimators. Section 2.7 introduces the design of Monte Carlo experiments. Section 2.8 discusses the results. And, finally, Section 2.9 concludes.

2.2 Review of Spatial Models

2.2.1 Weighting matrix

The key component of spatial models, which captures interactions among units, is a square weighting matrix whose dimension is equal to the sample size . The elements of the weighting matrix are designed to select units that are related to each other in a meaningful way. Such units are considered to be *neighbors*. More specifically, the i,j-th element of the weighting matrix (w_{ij}) describes the extent to which the i-th unit is related to the j-th unit. This relation could reflect the presence of spill-overs, externalities, similarity of markets etc. In other words, it captures the effect of some characteristics of unit j on unit i.

Another important feature of the weighting matrix is that its diagonal elements are equal to zero. This is essentially a normalization of the model; it can also be interpreted as indicating that a unit cannot be a neighbor of itself.

As an example consider the (simple) model:

$$y = b_0 + b_1 x + b_2 W x + \varepsilon, \tag{2.1}$$

where y is an $n \times 1$ vector of observations on the dependent variable, x is an $n \times 1$ vector of observations on an exogenous variable, $W = (w_{ij})$ is an $n \times n$ weighting matrix with zero diagonal elements, and ε is a disturbance vector whose elements are i.i.d. $(0, \sigma^2)$. This model can also be written in the scalar notation:

$$y_i = b_0 + b_1 x_i + b_2 \sum_{j=1}^{N} w_{ij} x_j + \varepsilon_i, \qquad i = 1, ..., N \tag{2.2}$$

$$\frac{\partial E(y_i)}{\partial x_j} = b_2 w_{ij} \tag{2.3}$$

The model in (2.2) suggests that the value of the dependent variable corresponding to the $i-$th unit is related to the values of the independent variables corresponding to the i-th as well as to other units. For example from (2.3) it follows that w_{ij} reflects the effect of x_j on the mean y_i. Note that without spatial interaction between i-th and j-th units, the derivative in (2.3) would be zero for $i \neq j$.

The choice of the elements of the weighting matrix (w_{ij}) is specific to the context of the empirical model. For instance, in many regional studies geographic neighbors are the logical choice of neighboring units. As one example one could take $w_{ij} = 0$ if regions i and j do not have a common border, and otherwise $w_{ij} \neq 0$, otherwise (Kelejian and Robinson (1997), Shroder (1995)). In some cross-sectoral studies the extent of this interaction, w_{ij}, is often measured by a sector's input shares of goods and services produced by the other sector (Conley and Dupor (2003)).

In many studies the weighting matrix is row normalized in the sense that the sum of the elements in every row is unity: $\sum_{j=1}^{N} w_{ij} = 1$, $i = 1, .., N$. Clearly, in terms of the model above, if W is row normalized, y_i relates, among other things, to a weighted average of the exogenous variable corresponding to the neighboring units. As a technical point we note that all the eigenvalues of a row normalized weighting matrix are less than or equal to unity in absolute value. The importance of this will become clear in Section 2.3.

2.2.2 Cliff-Ord models

The classical form of spatial autocorrelation was put forth by Cliff and Ord (1973, 1981) and had the following representation

$$y_n = \lambda W_n y_n + \varepsilon_n \tag{2.4}$$

where y_n is an $n \times 1$ vector of the dependent variable, ε_n is an $n \times 1$ vector of disturbances whose elements are i.i.d.$(0, \sigma^2)$, $W_n = (w_{ij,n})$ is an observed and exogenous $n \times n$ nonstochastic weighting matrix with zero diagonal elements, and λ is a scalar parameter which is typically assumed to be less than unity in absolute value.

This model is a variant of the model that was originally suggested by Whittle (1954) in the context of stationary processes on a plane. Cliff and Ord (1973, 1981) were the first to introduce this model in the regression analysis framework. It was discussed further by Besag (1974), Hordijk (1974), Hordijk and Paelinck (1976), Ripley (1981), and Ord (1975). More recent contributions include Anselin (1988), Kelejian and Prucha (1999), Cressie (1993), Pinske and Slade (1998), Case (1991), McMillen (1992).

In an analogy to the time-series analysis Anselin (1988) refers to (2.4) as a spatial autoregressive process (SAR). The term $W_n y_n$ is often called a spatial lag of y_n. Typical assumptions are that the row and column sums of both W_n and $(I_n - \lambda W_n)^{-1}$ are uniformly bounded in absolute value for all $|\lambda| < 1$ and $n \geq 1$.

An important feature of this model is that the elements of the dependent variable y_n are allowed to depend on the sample size n (it is reflected by the subscript n). Consequently, y_n forms a triangular array whose presence influences the theoretical treatment of the model. Kelejian and Prucha (1999) were among the first to discuss this characteristic of the model. They pointed out that the elements of the weighting matrix W_n typically change with the sample size n. One reason for this is that the weighting matrix is row normalized. This, in turn, implies that the elements of y_n also depend on the sample size. They also note that even if the elements of W_n and ε_n do not change with n, the dependent variable would still represent a triangular array. This becomes evident after observing that $y_n = (I_n - \lambda W_n)^{-1}\varepsilon_n$, and the elements of $(I_n - \lambda W_n)^{-1}$ generally depend on the sample size n, even if the elements of W_n do not depend on it.

The spatial model in its original form (2.4) has not been widely used in the applied regression work except as a specification of spatial correlation involving the disturbance term in a regression model. Consider now such a regression model:

$$
\begin{aligned}
y_n &= X_n\beta + u_n, && && (2.5)\\
u_n &= \rho W_n u_n + \varepsilon_n, && |\rho| < 1
\end{aligned}
$$

where y_n is an $n \times 1$ vector of observations on the dependent variable, ε_n is an $n \times 1$ vector of innovations with i.i.d. $(0, \sigma^2)$ elements, u_n is an $n \times 1$ vector of disturbances, X_n is an $n \times k$ nonstochastic matrix of observations with uniformly bounded elements, $rank(X_n) = k$, β is a $k \times 1$ parameter vector, and ρ is a scalar parameter.

The specification in (2.5) implies that $u_n = (I_n - \rho W_n)^{-1}\varepsilon_n$, so that $E\left(u_n u_n'\right) = \sigma^2(I_n - \rho W_n)^{-1}(I_n - \rho W_n')^{-1}$. Thus, the disturbance term u_n is generally heteroscedastic and spatially correlated since the off-diagonal elements of its variance-covariance matrix are not equal to zero unless $\rho = 0$.

The existence of spatial correlation in the disturbance term is often attributed to the presence of omitted variables that are spatially correlated. For instance, in the case of property prices it is very difficult, if not impossible, to account for all the factors that have an impact on the values of properties in neighboring locations. In this context, Bell and Bockstael (2000) employed this specification to empirically model the price of residential sales in Maryland.[2] They compared the results obtained from two estimation procedures, namely, the maximum likelihood and the GM procedure of Kelejian and Prucha (1999) and found that they were not substantially different.

[2]Other studies that used spatial correlation for modeling property prices are Dubin (1988), and Pace, Barry and Sirmans (1998).

Another important feature of this model is that the regressor matrix X_n may also include spatial lags of exogenous variables. That is, let $H_n = (H_{1n}, H_{2n})$ be a matrix of exogenous variables, then, X_n may contain both H_n and, say, $W_n H_{1n}$, i.e. $X_n = (H_n, W_n H_{1n})$. One of the studies that utilizes this structure of exogenous variables as well as the spatial correlation in the error term is Kelejian and Robinson (1997). It assessed the effect of public capital of neighboring states on the private sector productivity of a given state. In their results the authors underscored the importance of accounting for spatial effects.

Consider yet a more general extension of the Cliff-Ord model. This model involves spatial lags in both the dependent variable and the error term, and referred by Anselin and Florax (1995) as a spatial autoregressive model with autoregressive disturbances of order (1,1), for short SARAR (1,1). Consistent with the previous notation it is given by

$$\begin{aligned} y_n &= X_n\beta + \lambda W_n y_n + u_n, & |\lambda| < 1 \\ u_n &= \rho W_n u_n + \varepsilon_n, & |\rho| < 1 \end{aligned} \tag{2.6}$$

where the assumptions are the same as above.

Since this model is discussed in more details in the next section at this point we present only a brief overview. As noted above the spatial spill-overs enter (2.6) not only via the disturbances, but also directly by the dependent variable. This feature has been widely used in the empirical implementation of the models dealing with strategic interactions of economic agents. For instance, Case, Hines and Rosen (1993) considered this specification to estimate an importance of various factors relating to spending of jurisdictions. Brueckner (1998) estimated the policy reaction functions relating to the growth control for Californian cities. Case (1991) used a discrete choice version of this model in order to explain farmers' decision to adopt a sickle as a rice harvesting tool in rural Java, a province of Indonesia. Pinske, Slade, and Brett (2001), and Kapoor (2003) applied this empirical framework to estimate the nature and extent of spatial price competition in the US gasoline market.

In many of these studies the spatial modelling technique has been used not only to obtain quantitative inferences concerning the explanatory variables, but also to test for a validity of a relevant theory involving spatial interactions.

Besides the mentioned papers there are a number of other studies considering spatial interactions within a wide variety of applications. Shroder (1995) empirically assessed interactions of neighboring states in the game of competitive reduction of public assistance to the poor[3]. The author used a simultaneous equation framework involving spatial lags in the exogenous and endogenous variables in his model. Kelejian and Robinson (1992) considered spatial models in the context of per capita county police expenditure. Another study by Besley and Case (1995) considered a spatial model relating to tax setting interactions of states. Conley and Dupor (2003) investigated productivity co-movements across sectors of the US economy by incorporating spatial interactions measured by economic distance into the covariance matrix approach. Similar technique was exploited by Conley and Topa (2002) in order to explore spatial patterns of unemployment characterized by socio-economic distance and social structure.

All in all, the spatial approach has been proved to be very useful in many empirical application.

[3] see Brown and Oates (1986), Gramilich (1987), and Stigler (1957)

2.3 General specifications of the Single Linear Equation Model

Consider the cross-sectional autoregressive spatial model with autoregressive disturbances:

$$
\begin{aligned}
y_n &= X_n\beta + \lambda W_n y_n + u_n, \qquad & |\lambda| < 1 \\
u_n &= \rho W_n u_n + \varepsilon_n, \qquad & |\rho| < 1
\end{aligned}
\tag{2.7}
$$

where y_n is $n \times 1$ vector of observations on the dependent variable, X_n is the $n \times k$ matrix of observations on k exogenous variables, W_n is an $n \times n$ spatial weighting matrix of known constants, β is the $k \times 1$ vector of regression parameters, u_n is $n \times 1$ vector of disturbances, and ε_n is an $n \times 1$ vector of innovations.

Assumption 1. All diagonal elements of the weighting matrix W_n are zero.

Assumption 2. $(I_n - aW_n)$ is nonsingular for all $|a|< 1$

Assumption 3. The row and column sums of the matrices $W_n, (I_n - \lambda W_n)^{-1}$ and $(I_n - \rho W_n)^{-1}$ are uniformly bounded in absolute value.

Assumption 4. The regressor matrix X_n has full column rank (for n large enough). Furthermore, the elements of X_n are uniformly bounded in absolute value.

Assumption 5. The innovations $\{\varepsilon_{i,n} : 1 \leq i \leq n, n \geq 1\}$ are distributed identically . Further, the innovation $\{\varepsilon_{i,n} : 1 \leq i \leq n\}$ are for each n distributed (jointly) independently with $E(\varepsilon_{i,n}) = 0, E(\varepsilon_{i,n}^2) = \sigma_\varepsilon^2$, where $0 < \sigma_\varepsilon^2 < b$, where $b<\infty$. Additionally the innovations are assumed to possess finite fourth moments.

At the later stage we are going to use the 2SLS method, which exploits a set of instruments. Let H_n denote the $n \times p$ matrix of instruments, and let $Z_n = (X_n, W_n y_n)$ denote the matrix of regressors in (2.7). We are going to specify different sets of instruments in the next sections. At this point we formulate the assumptions that they satisfy.

Assumption 6. The instrument matrices H_n have full column rank $p \geq k+1$ (for n large enough.)

Assumption 7. The instrument H_n satisfy furthermore the following:

$$Q_{HH} = \lim n^{-1} H_n' H_n$$

where Q_{HH} is finite and nonsingular;

$$Q_{HZ} = p \lim n^{-1} H_n' Z_n$$

and

$$Q_{HWZ} = p \lim n^{-1} H_n' W_n Z_n$$

where Q_{HZ} and Q_{HWZ} are finite and have full column rank; furthermore,

$$Q_{HZ} - \rho Q_{HWZ} = p \lim n^{-1} H_n' (I_n - \rho W_n) Z_n$$

has full column rank where $|\rho| < 1$;

$$\Phi = \lim n^{-1} H_n' (I_n - \rho W_n)^{-1} (I_n - \rho W_n')^{-1} H_n$$

is finite and nonsingular where $|\rho| < 1$.

The following assumption ensures that the autoregressive parameter ρ is "identifiably unique" (see Kelejian and Prucha, 1999).

Assumption 8. The smallest eigenvalue of $\Gamma'\Gamma$, say $\lambda_{\min}(\Gamma'\Gamma)$, is bounded away from zero; that is $\lambda_{\min}(\Gamma'\Gamma) \geq \lambda_* > 0$, where

$$\Gamma_n = \frac{1}{n}\begin{pmatrix} 2E(u_n'\overline{u}_n) & -E(\overline{u}_n'\overline{u}_n) & 1 \\ 2E(\overline{\overline{u}}_n'\overline{u}_n) & -E(\overline{\overline{u}}_n'\overline{\overline{u}}_n) & tr(W_n'W_n) \\ E(u_n'\overline{\overline{u}}_n + \overline{u}_n'\overline{u}_n) & -E(\overline{u}_n'\overline{\overline{u}}_n) & 0 \end{pmatrix}$$

and $\overline{u}_n = W_n u_n$ and $\overline{\overline{u}}_n = W_n \overline{u}_n = W_n^2 u_n$.

2.3.1 Model Implications

Given Assumption 2 the reduced form of the model is

$$\begin{aligned} y_n &= (I_n - \lambda W_n)^{-1} X_n \beta + (I_n - \lambda W_n)^{-1} u_n, \\ u_n &= (I_n - \rho W_n)^{-1} \varepsilon_n. \end{aligned} \tag{2.8}$$

Therefore by Assumptions 2 and 5, $E(u_n) = 0$ and so

$$E(y_n) = (I - \lambda W_n)^{-1} X_n \beta \tag{2.9}$$

The result in (2.9) clearly reveals the force of the spatial interactions involving the dependent variable. To see this, suppose $k = 1$ so that X_n is a vector. Then, denoting the $i - th$ elements of y_n and X_n as $y_{i,n}$ and $x_{i,n}$, the result in (2.9) implies that

$$\frac{\partial E(y_{i,n})}{\partial x_{j,n}} = (I - \lambda W_n)^{-1}_{ij,n} \beta$$

where $(I - \lambda W_n)^{-1}_{ij,n}$ is the i, j-th element of $(I - \lambda W_n)^{-1}$. Thus, unless $\lambda = 0$ the effect on the mean of $y_{i,n}$ of a change in the exogenous variable corresponding to j-th unit, has two components. One might be thought of as the direct effect, which in this case is the corresponding coefficient , β. The other is an indirect effect, namely, $(I - \lambda W_n)^{-1}_{ij,n}$, which depends , in general, on all the weights in the weighting matrix. Note, in the absence of the spatial lag $\lambda W_n y_n$ in (2.7)

$$\frac{\partial E\left(y_{i,n}\right)}{\partial x_{j,n}} = 0, \qquad i \neq j$$

It follows from the Assumptions 3 and 5 that

$$\begin{aligned} E(u_n u_n') &= \sigma_\varepsilon^2 (I_n - \rho W_n)^{-1} (I_n - \rho W_n')^{-1} \\ &= \sigma_\varepsilon^2 \Omega_u \end{aligned}$$

where $\Omega_u = (I - \rho W_n)^{-1}(I - \rho W_n')^{-1}$.

Therefore, in general, the elements of u_n will be both spatially correlated and heteroscedastic. It also follows from(2.8) that

$$\begin{aligned} E[(W_n y_n)\, u_n'] &= W_n (I_n - \lambda W_n)^{-1} \sigma_\varepsilon^2 \Omega_u \\ &= \sigma_\varepsilon^2 W_n (I_n - \lambda W_n)^{-1} (I_n - \rho W_n)^{-1} (I_n - \rho W_n')^{-1} \\ &\neq 0 \end{aligned}$$

Therefore, in general, the parameters of (2.7) cannot be consistently estimated by least squares.

2.4 Estimation Procedures

2.4.1 Maximum Likelihood

Assuming $\varepsilon_n \sim N(0, \sigma_\varepsilon^2 I_n)$ so that $u_n \sim N(0, \sigma_\varepsilon^2 \Omega_n)$, the model in (2.7) can be estimated by the maximum likelihood method. The log-likelihood function is[4]

$$
\begin{aligned}
\ln(L) \quad = \quad & -(n/2)\ln(2\pi) - (n/2)\ln\sigma_\varepsilon^2 + \ln|I_n - \rho W_n| + \ln|I_n - \lambda W_n| && (2.10)\\
& -(1/2\sigma_\varepsilon^2)(y_n - \lambda W_n y_n - X_n\beta)' (I_n - \rho W_n)' \times \\
& (I_n - \rho W_n)(y_n - \lambda W_n y_n - X_n\beta)' && (2.11)
\end{aligned}
$$

Two points related to this method are in order. First, there are no formal results on the consistency and asymptotic normality of the ML estimator for this model[5]. However, many researchers suggest that there are "appropriate regularity" conditions such that the ML estimator is consistent, asymptotically normal, and efficient. Furthermore, Monte Carlo studies strongly support this conjecture, at least as it relates to consistency and efficiency.[6]

Second, in large samples the ML procedure will be difficult, if not impossible, to empirically implement. The reason for this is that ML procedure requires the evaluation of the determinants of two $n \times n$ matrices, namely $|I_n - \rho W_n|$ and $|I_n - \lambda W_n|$, for each trial value of ρ and λ in the maximization of $\ln(L)$. Ord (1975) has suggested a simplification in which the determinant of these $n \times n$ matrices can be evaluated in terms of the characteristic roots of W_n which need

[4]For detailed discussion of the ML procedure for spatial models see Cliff and Ord (1981), Ord (1975), Anselin (1988).

[5]An exception is Lee (1999b) who demonstrated these properties of the ML estimator under somewhat restrictive conditions on the parameter space

[6]see Kelejian and Prucha (1999), and Das, Kelejian and Prucha (2003)

only be computed once. However, Kelejian and Prucha (1999) found that it is "very challenging" to accurately determine the roots of nonsymmetric matrices of size 400×400, or larger.

Because of this difficulty with the ML procedure, Kelejian and Prucha (1998, 1999) suggested an alternative procedure which involves three steps which are computationally simple even in large samples.

2.4.2 Feasible Generalized Spatial Two Stage Least Squares

Step 1: Two Stage Least Squares Estimation

In the first step of the procedure suggested by Kelejian and Prucha (1998) consistent estimators of λ and β are obtained by the two-stage least squares (2SLS) technique. Results given in Amemiya(1985) suggest that the conditional mean is an ideal instrument for an endogenous regressor, in (2.7) namely $W_n y_n$. In the context of our model the conditional expectation of $W_n y_n$ can be obtained by premultiplying (2.8) by W_n and then taking expectations

$$E[W_n y_n] = W_n E[y_n] = W_n (I_n - \lambda W_n)^{-1} X_n \beta \tag{2.12}$$

where we have used (2.9) .

Assuming that all of the eigenvalues of λW_n are less than unity in absolute value, the conditional expectation in (2.12) can be written in the form of the infinite sum:

$$E[W_n y_n] = W_n \sum_{i=0}^{\infty} \lambda^i W_n^i X_n \beta \tag{2.13}$$

From (2.13) $E[W_n y_n]$ is seen to be a linear combination of $(W_n X_n, W_n^2 X_n, ..)$. On the basis of this observation Kelejian and Prucha (1999) proposed the instrument set $H_n^r = (X_n, W_nX_n, .., W_n^rX_n)$ [7]. For $r = 1$ the instrument set is $H_n^1 = (X_n, W_nX_n)$. We will henceforth refer to H_n^1 as the minimum set of instruments, as suggested in Kelejian and Prucha (1998). Typically, $r \leq 2$.

Now we are ready to introduce the 2SLS estimator. For simplicity of notation let us rewrite (2.7) as

$$\begin{aligned} y_n &= Z_n\delta + u_n, \\ u_n &= \rho W_n u_n + \varepsilon_n, \end{aligned} \tag{2.14}$$

where $Z_n = (X_n, W_ny_n)$ and $\delta' = (\beta', \lambda)$. Then, the 2SLS estimator of δ is given by

$$\widehat{\delta}_n^{2SLS} = (\widehat{Z}_n' Z_n)^{-1}\widehat{Z}_n' y_n, \tag{2.15}$$

where $\widehat{Z}_n = P_{H_n^r}Z_n = (X_n, \widehat{W_ny_n})$, where $\widehat{W_ny_n} = P_{H_n^r}W_ny_n$, and

$P_{H_n^r} = H_n^r(H_n^{r\prime}H_n^r)^{-1}H_n^{r\prime}$. Under the assumed conditions Kelejian and Prucha (1998) show that $\widehat{\delta}_n^{2SLS}$ is consistent.

Step 2: General Moments Estimator of ρ

Step 2 of the Kelejian and Prucha procedure for the estimation of (2.7) involves a generalized moments (GM) estimator of ρ. This GM estimator is described below, and was introduced by Kelejian and Prucha (1999). At this point we give background results which aid in its comprehension.

[7] Without loss of generality it is assumed that the columns of the indicated instrument set are linearly independent. Otherwise one would have to include in H_n^r only linearly independent columns of $(X_n, W_nX_n, .., W_n^rX_n)$.

A Preliminary: Notation Consider (2.7) and let $\overline{u}_n = W_n u_n$, $\overline{\overline{u}}_n = W_n W_n u_n$ and $\overline{\varepsilon}_n = W_n \varepsilon_n$. Then from (2.7) we have

$$\begin{aligned} u_n &= \rho \overline{u}_n + \varepsilon_n && (2.16) \\ \overline{u}_n &= \rho \overline{\overline{u}}_n + \overline{\varepsilon}_n && (2.17) \end{aligned}$$

Now note that

$$\begin{aligned} \frac{E\varepsilon_n' \varepsilon_n}{n} &= \sigma_\varepsilon^2 \\ \frac{E\overline{\varepsilon}_n' \overline{\varepsilon}_n}{n} &= \frac{E\varepsilon_n' W_n' W_n \varepsilon_n}{n} = \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n' W_n) \\ \frac{E\overline{\varepsilon}_n' \varepsilon_n}{n} &= \frac{E\varepsilon_n' W_n' \varepsilon_n}{n} = \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n') = 0 \end{aligned} \tag{2.18}$$

Given assumptions it can be shown that

$$\begin{aligned} &(a) \quad \frac{\varepsilon_n' \varepsilon_n}{n} \xrightarrow{P} \sigma_\varepsilon^2 \\ &(b) \quad \frac{\overline{\varepsilon}_n' \overline{\varepsilon}_n}{n} - \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n' W_n) \xrightarrow{P} 0 \\ &(c) \quad \frac{\overline{\varepsilon}_n' \varepsilon_n}{n} \xrightarrow{P} 0. \end{aligned} \tag{2.19}$$

The results in (2.19) imply that

$$\begin{aligned} \frac{\varepsilon_n' \varepsilon_n}{n} &= \sigma_\varepsilon^2 + \psi_1 \\ \frac{\overline{\varepsilon}_n' \overline{\varepsilon}_n}{n} &= \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n' W_n) + \psi_2 \\ \frac{\overline{\varepsilon}_n' \varepsilon_n}{n} &= \psi_3 \end{aligned} \tag{2.20}$$

where $\psi_1 \xrightarrow{P} 0$, $\psi_2 \xrightarrow{P} 0$, and $\psi_2 \xrightarrow{P} 0$.

GM estimation The GM procedure produces estimators of ρ and σ_ε^2 on the basis of the residuals from the first step 2SLS estimator of δ. At this point we specify these estimators as if we observe u_n. We will then generalize to the case in which these disturbances are estimated.

The three equations below are obtained from (2.20) by setting $\varepsilon_n = u_n - \rho\overline{u}_n$ and $\overline{\varepsilon}_n = \overline{u}_n - \rho\overline{\overline{u}}_n$:

$$\begin{aligned} \frac{(u_n - \rho\overline{u}_n)'(u_n - \rho\overline{u}_n)}{n} - \sigma_\varepsilon^2 &= \psi_1 \\ \frac{\left(\overline{u}_n - \rho\overline{\overline{u}}_n\right)'\left(\overline{u}_n - \rho\overline{\overline{u}}_n\right)}{n} - \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n' W_n) &= \psi_2 \\ \frac{\left(\overline{u}_n - \rho\overline{\overline{u}}_n\right)'(u_n - \rho\overline{u}_n)}{n} &= \psi_3 \end{aligned} \tag{2.21}$$

If u_n, and therefore, $\overline{u}_n$ and $\overline{\overline{u}}_n$, were observed the GM estimator of ρ and σ_ε^2 would be obtained from the following minimization problem

$$\min_{\rho,\sigma_\varepsilon^2} \left(\psi_1^2 + \psi_2^2 + \psi_3^2\right)$$

Since u_n is not observed the above described procedure is not feasible. Let $\widehat{u}_n = y_n - Z_n\widehat{\delta}_n^{2SLS}$ where $\widehat{\delta}_n^{2SLS}$ is defined in (2.15); also let $\widehat{\overline{u}}_n = W_n\widehat{u}_n$ and $\widehat{\overline{\overline{u}}}_n = W_n W_n \widehat{u}_n$. Then a three equation system which is analogous to (2.21) but is based on $\widehat{u}_n$, $\widehat{\overline{u}}_n$, and $\widehat{\overline{\overline{u}}}_n$ is

$$\begin{aligned} \frac{\left(\widehat{u}_n - \rho\widehat{\overline{u}}_n\right)'\left(\widehat{u}_n - \rho\widehat{\overline{u}}_n\right)}{n} - \sigma_\varepsilon^2 &= \widehat{\psi}_1 \\ \frac{\left(\widehat{\overline{u}}_n - \rho\widehat{\overline{\overline{u}}}_n\right)'\left(\widehat{\overline{u}}_n - \rho\widehat{\overline{\overline{u}}}_n\right)}{n} - \frac{1}{n}\sigma_\varepsilon^2 Tr(W_n' W_n) &= \widehat{\psi}_2 \\ \frac{\left(\widehat{\overline{u}}_n - \rho\widehat{\overline{\overline{u}}}_n\right)'(\widehat{u}_n - \rho\overline{u}_n)}{n} &= \widehat{\psi}_3 \end{aligned} \tag{2.22}$$

where $\widehat{\psi}_1, \widehat{\psi}_2$, and $\widehat{\psi}_3$ are corresponding residuals. The GM estimator of ρ and σ_ε^2, say $\widehat{\rho}_n^{2SLS}$ and $\widehat{\sigma}_{\varepsilon,n}^2$, defined by Kelejian and Prucha (1999) are obtained from minimization of [8]

$$\min_{\rho,\sigma_\varepsilon^2}\left(\widehat{\psi}_1^2+\widehat{\psi}_2^2+\widehat{\psi}_3^2\right)$$

Kelejian and Prucha (1998) showed that $\underset{n\to\infty}{p\lim}\widehat{\rho}_n^{2SLS}=\rho$. However, the large sample distribution of $\widehat{\rho}_n^{2SLS}$ was not determined, and so tests of hypotheses concerning ρ cannot be based on their procedure.

Step 3: Two Stage Least Squares Estimation of the Transformed Model

In Step 1 we obtained a consistent estimator of δ. However, that estimator did not take into account the spatial correlation of the disturbances which results in a loss of efficiency. In Step 3 an estimator of δ is proposed which accounts for the spatial correlation of the error term u_n.

Consider a spatial Cochrane-Orcutt transformation of (2.7):

$$y_{n*}(\rho)=Z_{n*}(\rho)\delta+\varepsilon_n, \tag{2.23}$$

where

$$\begin{aligned} y_{n*}(\rho) &= y_n-\rho W_n y_n, \\ Z_{n*}(\rho) &= (I_n-\rho W_n)Z_n \\ &= (I_n-\rho W_n)[X_n, W_n y_n]. \end{aligned}$$

Note that the elements of the disturbance vector ε_n in (2.23) are i.i.d. $(0,\sigma_\varepsilon^2)$.

[8]As clarified later, the superscript of an estimator of ρ indicates the estimator of δ which was used to obtain the residuals used in the GM procedure.

This equation cannot be consistently estimated by least squares since, in general, $EZ_{n*}(\rho)\varepsilon_n' \neq 0$. Specifically

$$\begin{aligned} E\left([(I_n - \rho W_n)W_n y_n]\,\varepsilon_n'\right) &= (I_n - \rho W_n)W_n E\left(y_n \varepsilon_n'\right) \\ &= \sigma_\varepsilon^2 (I_n - \rho W_n)W_n(I_n - \rho W_n)^{-1} \\ &\neq 0 \end{aligned}$$

Therefore Kelejian and Prucha (1998) considered an instrumental variable technique.

As pointed out in Step 1, the results in Amemiya (1985) suggest that the ideal instrument for the endogenous regressor $W_n y_n$ is its conditional expectation. Based on the result in (2.13)

$$\begin{aligned} E\left[(I_n - \rho W_n)W_n y_n\right] &= (I_n - \rho W_n)W_n(I_n - \lambda W_n)^{-1}X_n\beta \qquad (2.24) \\ &= (I_n - \rho W_n)W_n \sum_{i=0}^{\infty} \lambda^i W_n^i X_n \beta \end{aligned}$$

Thus, the optimal instrument for $Z_{n*}(\rho)$ in (2.23) would be .

$$\begin{aligned} Z_{n*}^{opt}(\rho) &= (I_n - \rho W_n)[X_n, E\left(W_n y_n\right)] \qquad (2.25) \\ &= (I_n - \rho W_n)[X_n, W_n(I_n - \lambda W_n)^{-1}X_n\beta] \end{aligned}$$

From (2.24) and (2.25) it is easily seen that the optimal instruments are a linear combination of $(X_n, W_n X_n, W_n^2 X_n, ..)$. Following the logic of Step 1 we can approximate the optimal instruments in terms of the instrument set $H_n^r = (X_n, W_n X_n, .., W_n^r X_n)$, where linearly dependent columns are omitted.

Now on the basis of the instrument set H_n^r we are able to define the estimator $\widehat{\delta}_n^{GS2SLS}$ which is termed by Kelejian and Prucha (1998) as the Generalized Spatial

Two-Stage Least Squares (GS2SLS) estimator:

$$\widehat{\delta}_n^{GS2SLS} = (\widehat{Z}_{n*}(\rho)' Z_{n*}(\rho))^{-1} \widehat{Z}_{n*}(\rho)' y_{n*}(\rho) \tag{2.26}$$

where $\widehat{Z}_{n*}(\rho) = P_{H_n^r} Z_{n*}(\rho)$ and $P_{H_n^r} = H_n^r (H_n^{r\prime} H_n^r)^{-1} H_n^{r\prime}$.

In practice, ρ is usually not known. Therefore, a logical step would be to replace it with some consistent estimator of ρ, $\widehat{\rho}_n$. Theoretically it could be any consistent estimator of ρ since Kelejian and Prucha(1998) show that ρ is a nuisance parameter concerning the estimation of δ. The resulting estimator of δ is called the feasible GS2SLS (FG2SLS) estimator and is given by

$$\widehat{\delta}_n^{FGS2SLS} = \left[\widehat{Z}_{n*}(\widehat{\rho}_n)' Z_{n*}(\widehat{\rho}_n) \right]^{-1} \widehat{Z}_{n*}(\widehat{\rho}_n)' y_{n*}(\widehat{\rho}_n), \tag{2.27}$$

where $\widehat{Z}_{n*}(\widehat{\rho}_n) = P_{H_n} Z_{n*}(\widehat{\rho}_n)$ and $P_{H_n} = H_n^r (H_n^{r\prime} H_n^r)^{-1} H_n^{r\prime}$.

Kelejian and Prucha (1998) prove the consistency of this estimator and derived its asymptotic distribution for the case in which $\widehat{\rho}_n$ is any consistent estimator of ρ. They suggest the use of the GM estimator $\widehat{\rho}_n^{2SLS}$ obtained in Step 2.

2.5 The Best Instrumental Variable Estimators

The choice of the instrument set in the estimation procedure described above is based upon the approximation of the optimal instrument given in (2.25). To improve the efficiency of the estimator of δ, at least, in large samples, one would consider a better approximation to the conditional mean of $W_n y_n$.

Two possible procedures involving $\widehat{\lambda}_n^{2SLS}$ and $\widehat{\rho}_n^{2SLS}$ have been considered in the literature. One was proposed by Lee (1999a) and the other by Kelejian and Prucha (2001). In this section these procedures are described.

2.5.1 Lee's Approximation to the Optimal Instruments

In his recent paper Lee (1999a) suggested the following instrumental variable estimator:

$$\widehat{\delta}_n^{Lee} = \left[(\bar{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS})' Z_{n*}(\widehat{\rho}_n^{2SLS})\right]^{-1} \bar{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS})' y_{n*}(\widehat{\rho}_n^{2SLS})$$

where $\overline{Z}_{n*}(\widehat{\rho}_n, \widetilde{\delta}_n)$ is the following approximation of the optimal instrument:

$$\bar{Z}_{n*}(\widehat{\rho}_n, \widehat{\delta}_n^{2SLS}) = (I_n - \widehat{\rho}_n^{2SLS} W_n)\left[X_n, W_n(I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1} X_n \widehat{\beta}_n^{2SLS}\right] \quad (2.28)$$

This estimator requires calculation of $(I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1}$, which, as noted before, might be computationally challenging. Lee (1999a) introduced a numerically simple algorithm for the calculation of $(I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1}$, involving the Choleski decomposition of the matrix $(I_n - \widehat{\lambda}_n^{2SLS} W_n)$.

An advantage of this estimator is that it is consistent and asymptotically efficient, see Lee (1999a). However, despite the fact that Lee's estimator is feasible for the models with large sample sizes it still requires a great deal of computation as well as specific programming of the algorithm that calculates the inverse.

To avoid these difficulties Kelejian and Prucha (2001) proposed a computationally simpler estimator which possesses the same asymptotic properties as Lee's estimator.

2.5.2 Kelejian and Prucha's Approximation to the Optimal Instruments

The instruments suggested by Kelejian and Prucha (2001) are based on the approximation of the polynomial expansion of $(I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1}$:

$$\begin{aligned} (I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1} &= \sum_{i=0}^{\infty} \left(\widehat{\lambda}_n^{2SLS}\right)^i W_n^i \\ &\dot{\approx} \sum_{i=0}^{r_n} \left(\widehat{\lambda}_n^{2SLS}\right)^i W_n^i, \end{aligned} \tag{2.29}$$

where r_n is a natural number such that $r_n \to \infty$ as $n \to \infty$, and $r_n = O(n^\alpha)$, where $\alpha < 0.5$. In practice r_n could be taken as the nearest integer to n^α. For further reference let $r_n(\alpha) = Int(n^\alpha)$, where $Int(\cdot)$ is a function whose outcome is a nearest integer to the argument of the function which is a real number. The approximation of the ideal instrument in Kelejian and Prucha (2001) is given by:

$$\widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha) = (I_n - \widehat{\rho}_n^{2SLS} W_n) \left[X_n, \sum_{i=0}^{r_n(\alpha)} \left(\widehat{\lambda}^{2SLS}\right)^i W_n^{i+1} X_n \widehat{\beta}_n^{2SLS} \right] \tag{2.30}$$

where $\widehat{\lambda}_n^{2SLS}$ and $\widehat{\beta}_n^{2SLS}$ are the estimators obtained in the first step.

The resulting instrumental variable estimator is called a *series* estimator and defined as follows:

$$\widehat{\delta}_{\alpha,n}^{Series} = \left[\widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha)' Z_{n*}(\widehat{\rho}_n^{2SLS}) \right]^{-1} \widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha)' y_{n*}(\widehat{\rho}_n^{2SLS})$$

Kelejian and Prucha (2001) showed that this estimator is also consistent and asymptotically efficient.

An important property of the procedure implementing the series estimator is that its computational count is only $O(N^2)$ in contrast to the Lee's computational count of $O(N^3)$.

2.5.3 Iterated Estimators

The third step of FGS2SLS involves spatial Cochrane Orcutt transformation of (2.7) based on the GM estimator of ρ. It is reasonable to believe that in finite samples the accuracy of the estimator of ρ affects the precision of the FGS2SLS of δ. At the same time, one may expect that the GM procedure using more precisely calculated residuals produces a better estimate of ρ. Therefore, the GM estimator of ρ based on the FGS2SLS residuals is likely to be more accurate than the one based on the 2SLS residuals.

This leads to an evident extension of the FGS2SLS procedure. The extended procedure would use the residuals of FGS2SLS in order to reestimate ρ, and, then, based on the new estimator of ρ repeat step 3, i.e. transform the model and estimate it by 2SLS again. Similar iteration can be conducted with respect to the Lee's and Kelejian and Prucha's modifications. In these modifications the estimator of δ, $\widehat{\delta}_n^{2SLS}$, in (2.28) and (2.30) should be replaced by $\widehat{\delta}_n^{Lee}$ and $\widehat{\delta}_n^{Series}$, respectively. It is important to note that asymptotically this iteration does not produce gains in efficiency; however, in small samples efficiency may be improved. These efficiency issues are the purpose of this Monte-Carlo study.

As a notational convention let the superscript of an estimator of ρ indicate which estimator of δ was used in order to calculate residuals for the GM procedure. For instance, $\widehat{\rho}_n^{2SLS}$ is a GM estimator of ρ based on $u_n^{2SLS} = y_n - Z_n\widehat{\delta}_n^{2SLS}$. Similarly, $\widehat{\rho}_n^{FGS2SLS}$, $\widehat{\rho}_{\alpha,n}^{Series}$, and $\widehat{\rho}_n^{Lee}$ are the GM estimators of ρ based on the residuals $u_n^{FGS2SLS} = y_n - Z_n\widehat{\delta}_n^{FGS2SLS}$, $u_{\alpha,n}^{Series} = y_n - Z_n\widehat{\delta}_{\alpha,n}^{Series}$, $u_n^{Lee} = y_n - Z_n\widehat{\delta}_n^{Lee}$, respectively. Then, the iterated estimators are given by the following formulas:

- Iterated FGS2SLS (IF) based on $\widehat{\rho}_n^{FGS2SLS}$

$$\widehat{\delta}_n^{IF} = \left[\widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS})' Z_{n*}(\widehat{\rho}_n^{FGS2SLS})\right]^{-1} \widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS})' y_{n*}(\widehat{\rho}_n^{FGS2SLS})$$

where $\widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS}) = P_{H_n} Z_{n*}(\widehat{\rho}_n^{FGS2SLS})$ and $P_{H_n} = H_n^r (H_n^{r\prime} H_n^r)^{-1} H_n^{r\prime}$.

- Iterated Series estimator (IS) based on $\widehat{\rho}_{\alpha,n}^{Series}$, $\widehat{\delta}_{\alpha,n}^{Series} = (\widehat{\lambda}_{\alpha,n}^{Series}, \widehat{\beta}_{\alpha,n}^{Series\prime})'$

$$\widehat{\delta}_{\alpha,n}^{IS} = \left[\widetilde{Z}_{n*}(\widehat{\rho}_{\alpha,n}^{Series}, \widehat{\delta}_{\alpha,n}^{Series}, \alpha)' Z_{n*}(\widehat{\rho}_n^{Series})\right]^{-1} \widetilde{Z}_{n*}(\widehat{\rho}_{\alpha,n}^{Series}, \widehat{\delta}_{\alpha,n}^{Series}, \alpha)' y_{n*}(\widehat{\rho}_{\alpha,n}^{Series})$$

where

$$\widetilde{Z}_{n*}(\widehat{\rho}_{\alpha,n}^{Series}, \widehat{\delta}_{\alpha,n}^{Series}, \alpha) = (I_n - \widehat{\rho}_{\alpha,n}^{Series} W_n) \left[X_n, \sum_{i=0}^{r_n(\alpha)} \left(\widehat{\lambda}_{\alpha,n}^{Series}\right)^i W_n^{i+1} X_n \widehat{\beta}_{\alpha,n}^{Series}\right]$$

- Iterated Lee estimator (IL) $\widehat{\rho}_n^{Lee}$, $\widehat{\delta}_n^{Lee} = (\widetilde{\lambda}^{Lee}, \widetilde{\beta}_n^{Lee\prime})'$

$$\widehat{\delta}_n^{IL} = \left[\bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee})' Z_{n*}(\widehat{\rho}_n^{Lee})\right]^{-1} \bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee})' y_{n*}(\widehat{\rho}_n^{Lee})$$

where

$$\bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee}) = (I_n - \widehat{\rho}_n^{Lee} W_n) \left[X_n, W_n (I_n - \widehat{\lambda}_n^{Lee} W_n)^{-1} X_n \widehat{\beta}_n^{Lee}\right]$$

2.6 Monte Carlo Results of Previous Studies

There are only a few Monte Carlo studies in the literature that are related to the estimators of the spatial model. Most of the existing Monte Carlo studies are confined to the issues of testing for spatial correlation which is beyond the scope of this paper. To the best of my knowledge there are only three papers that consider the small sample properties of estimators of spatial models via Monte Carlo experiments.

Kelejian and Prucha (1999) introduced the GM estimator described in section 2.4.2 and applied it to the model involving a spatial lag in the error term (but not in the dependent variable). In addition to their theoretical contribution, they carried out a comprehensive Monte Carlo study related to the small sample properties of the GM estimator and its performance relative to the ML estimator. They considered various specifications of the weighting matrices along with various distributions of the vector of innovations. Their conclusion was that the GM estimator "is virtually as efficient as" the (quasi) ML estimator.

Other results of this study involved estimation of root mean square error response functions relating to the estimation of the autoregressive parameter ρ. These functions help to relate the magnitude of the root mean square error (RMSE) to the parameters of Monte Carlo experiments. The response functions provided a good fit to the data which was reflected by high values of R^2 statistics. The estimation results of the response functions established that RMSEs of the considered estimators are generally higher for weighting matrices with lower degree of sparseness. It was also found that their relationship to the parameter ρ is nonlinear. Namely, the RMSEs were at a maximum for values of ρ between -0.25 and 0.0 (depending on the sparseness of the weighting matrix) and declined as ρ approached the "critical" points ± 1. These patterns were expected by the authors and were given an appropriate explanation.

Further Monte Carlo results are given in a paper by Das, Kelejian and Prucha

(2003) who investigated the small sample properties of the ML and FGS2SLS estimators of (2.7). They also considered other estimators, namely, OLS, 2SLS, GS2SLS, and iterated FGS2SLS. The Monte Carlo experiments in their paper were conducted with respect to several sample sizes and involved weighting matrices varying in their degree of sparseness. They found that although the ML estimator is somewhat more efficient than FGS2SLS, its advantage was, on average, just 7% for the spatial autoregressive parameters λ and ρ relating to the spatial lags of the dependent variable and disturbance term. They also found that the ML and FGS2SLS estimators were virtually equally efficient in the estimation of the parameters relating to the exogenous variables, namely $\beta = (\beta_1, \beta_2)$. Therefore, their suggestion was that in finite samples the difference in efficiency of these two estimators is very small even under the most favorable condition for the ML procedure which involves normally distributed vectors of disturbances.

The paper also emphasizes that the actual difference between RMSEs of the ML and FGS2SLS estimators is not uniform over the parameter space, but instead depends crucially on the true values of λ and ρ. In particular, the RMSEs of the ML estimator of the autoregressive parameter λ are smaller than those of FGS2SLS when λ is negative and ρ is large and positive, the situation is reversed when both λ and ρ are positive and large. It was also confirmed in the study that the OLS and 2SLS estimators of λ and β would generally perform worse than the others due to the inconsistency of the former and inefficiency of the latter.

Another important result of the paper emerges from the comparison of FGS2SLS and GS2SLS (based on the true value of ρ). According to Das, Kelejian and Prucha (2003) RMSE differences between these two estimators are generally small. It indicates that the use of the GM estimator of ρ instead of the true value of ρ results in a "slight" loss in finite sample efficiency.

The findings of these two papers can be summarized in a statement that although the ML estimator is on average somewhat more efficient than the FGS2SLS and GM estimators its advantage is very small. This is very important in light of the major advantage of the FGS2SLS and GM procedures over the maximum likelihood in terms of computational complexity.

Another study by Rey and Boarnet (1998) considered a two equation linear spatial model containing spatially lagged dependent variables as well as systems endogenous variables. The authors explored the small sample efficiency of two-stage least squares estimators of the model parameters. Their estimation procedure in their two equation model could be implemented in terms of the instruments (X_1, X_2), where X_i is a matrix of exogenous variables in the i-th equation, $i = 1, 2$. However, Rey and Boarnet (1998) found that estimation efficiency was improved if their procedure was carried out in terms of the instrument set $(X_1, X_2, W_n X_1, W_n X_2)$, where W_n is their weighting matrix. Interestingly, efficiency was not generally improved when they carried out their procedure in terms of the instrument set $(X_1, X_2, W_n X_1, W_n X_2, W_n^2 X_1, W_n^2 X_2)$.

Further findings of this paper indicate that in finite samples it is preferable to instrument the spatial lag of the dependent variable rather than the dependent variable itself with subsequent multiplication by a weighting matrix. This is consistent with theory and can be illustrated within a single equation framework. Consider a model

$$y_n = X_n \beta + \lambda W_n y_n + \varepsilon_n, \qquad |\lambda| < 1 \tag{2.31}$$

where y_n is $n \times 1$ vector of observations on the dependent variable, X_n is the $n \times k$ matrix of observations on k exogenous variables, W_n is an $n \times n$ spatial weighting matrix of known constants, β is the $k \times 1$ vector of regression parameters, and ε_n is an $n \times 1$ vector of disturbances with elements distributed i.i.d.$(0, \sigma^2)$. Let us rewrite (2.31)

$$y_n = Z_n \gamma + \varepsilon_n,$$

where $Z_n = (X_n, W_n y_n)$ and $\gamma' = (\beta', \lambda)$. Let H_n be a matrix of instruments which contains X_n.[9] Consider $\widehat{Z}_n = P_{H_n} Z_n = (X_n, \widehat{W_n y_n})$, where $\widehat{W_n y_n} = P_{H_n} W_n y_n$, and $\widehat{\widehat{Z}}_n = (X_n, W_n P_{H_n} y_n) = (X_n, W_n \widehat{y}_n)$, where $\widehat{y}_n = P_{H_n} y_n$ and $P_{H_n} = H_n (H_n H_n)^{-1} H_n$. Essentially $\widehat{Z}_n$ and $\widehat{\widehat{Z}}_n$ are matrices which were used by Rey and Boarnet (1998) in their estimation. It is not difficult to show that in the 2SLS framework the use of the instrument matrix $\widehat{\widehat{Z}}_n$ leads to an inconsistent estimator. On the other hand, the use of the instrument matrix $\widehat{Z}_n$ results in a consistent estimation procedure. To be explicit, let

$$\begin{aligned}
\widehat{\gamma}_n &= (\widehat{Z}_n' \widehat{Z}_n)^{-1} \widehat{Z}_n' y_n, \\
\widehat{\widehat{\gamma}}_n &= (\widehat{\widehat{Z}}_n' \widehat{\widehat{Z}}_n)^{-1} \widehat{\widehat{Z}}_n' y_n.
\end{aligned}$$

Then, under further "standard" assumptions

$$\begin{aligned}
p \lim_{n \to \infty} \widehat{\gamma}_n &= \gamma \\
p \lim_{n \to \infty} \widehat{\widehat{\gamma}}_n &\neq \gamma.
\end{aligned}$$

This is consistent with the finding of Rey and Boarnet (1998).[10]

[9] E.g., $H_n = (X_n, W_n X_n)$

[10] Note that if $\widehat{\widehat{\gamma}}_n$ is defined as $(\widehat{\widehat{Z}}_n' Z_n)^{-1} \widehat{\widehat{Z}}_n' y_n$ it would be a consistent estimator of γ.

2.7 Design of The Monte Carlo Study

The Monte Carlo model is

$$
\begin{aligned}
y_n &= \lambda W_n y_n + X_n \beta + u_n, \qquad |\lambda| < 1 \\
u_n &= \rho W_n u_n + \varepsilon_n, \qquad |\rho| < 1
\end{aligned}
\tag{2.32}
$$

where y_n is $n \times 1$ vector of observations on the dependent variable, $X_n = [x_{1n}, x_{2n}]$ is an $n \times 2$ non-stochastic matrix containing two $n \times 1$ vectors of observations on the exogenous explanatory variables x_{1n} and x_{2n}, W_n is an $n \times n$ spatial weighting matrix of known constants, $\beta = [\beta_1, \beta_2]'$ is the 2×1 vector of regression parameters, u_n is $n \times 1$ vector of disturbances, and ε_n is an $n \times 1$ stochastic vector of innovations whose elements are distributed i.i.d. $N(0, \sigma^2)$. Essentially, (2.32) is a version of (2.7) formulated in terms of two explanatory variables.

In a more compact notation (2.32) can be written as

$$
\begin{aligned}
y_n &= Z_n \delta + u_n, \\
u_n &= \rho W_n u_n + \varepsilon_n,
\end{aligned}
\tag{2.33}
$$

where $Z_n = (W_n y_n, X_n)$ and $\delta' = (\lambda, \beta')$.

The Monte Carlo experiments in this study are designed in a way that makes their results comparable to the previous studies, and, in particular, to the results of Das, Kelejian and Prucha (2003). We extend that study in two aspects. First, we consider more experiments involving "extreme" values of the spatial autoregressive parameters λ and ρ, namely the values of 0.9 and -0.9, and, second, there are more estimators under investigation in this study. The first extension allows a wider assessment of the small sample properties of the estimators over the parameter space; the second relates to the theoretical development of efficient estimators in the spatial econometrics literature. Below is a detailed description of the experimental design.

We consider two sample sizes, namely 100 and 400. For each sample size we consider a weighting matrix W_n which is often referred as "3 ahead and 3 behind". This name is given due to the fact that this matrix relates each element of y_n and u_n to the three elements immediately after and before it. More specifically, i-th row has six non-zero elements only in positions $i+1$, $i+2$, $i+3$, and $i-1$, $i-2$, $i-3$, for $i = 4, ..., n-3$. This weighting matrix is circular in a sense that the non-zero elements in the first row are in the positions 2, 3, 4, and n, $n-1$, $n-2$; in the last row the non-zero elements are in positions 1, 2, 3, and $n-1$, $n-2$, $n-3$. The positioning of the non-zero elements in rows 2, 3, $n-1$, and $n-2$ are determined analogously. Furthermore, all non-zero elements of the weighting matrix are equal and the rows sum to unity. Thus, each non-zero element in this weighting matrix is 1/6. This weighting matrix is such that $(I - aW_n)^{-1}$ exists for all $|a| < 1$. Therefore, the reduced form of (2.32)

$$y_n = (I_n - \lambda W_n)^{-1} X_n \beta + (I_n - \lambda W_n)^{-1} (I_n - \rho W_n)^{-1} \varepsilon_n$$

is well defined

We consider seven values considered for λ and seven values for ρ. They are -0.9, -0.8, -0.4, 0.0, 0.4, 0.8, 0.9. We also consider three values of σ^2, namely 0.25, 0.5, 1.0. These values of σ^2 are related to the values of λ and n, and, thus, are woven into the experimental design in the same fashion as in Das, Kelejian, and Prucha (2003)[11]. Table 2.1 describes these values of σ^2.

The combinations of λ, n and σ^2 are such that the average squared correlation

[11] In Das, Kelejian and Prucha (2003) the values of σ^2 were related to the values of λ and n in a way that facilitated estimation of the root mean squared error response functions.

Table 2.1: Design value of the variance of innovations

$n = 100$		$n = 400$	
λ	σ^2	λ	σ^2
-0.9	0.5	-0.9	0.5
-0.8	0.25	-0.8	0.5
-0.4	0.25	-0.8	0.5
-0.4	1.0	-0.4	1.0
0.0	0.5	0.0	0.25
0.4	0.25	0.4	0.5
0.8	1.0	0.8	1.0
0.9	0.5	0.9	0.5

coefficient between y_n and $E[y_n] = (I - \lambda W_n)^{-1} X_n \beta$ over all the experiments corresponding to a given value of λ and n is between 0.60 and 0.90.

The values of the matrix of exogenous variables $X_n = [x_{1n}, x_{2n}]$ in (2.32) are based on the data given in Kelejian and Robinson (1992) on income per capita and on the percent of rental housing in 1980 in 760 counties in the US mid-western states. The 760 observations on the income and rental variables were normalized to have zero mean and unit variance. For experiments in which the sample size is 100 the first 100 observations on these variables were used; the first 400 observations were used in experiments in which the sample size is 400. The same vectors of exogenous variables were used in all the experiments of a given sample size n. Finally, the elements of the parameter vector $\beta = (\beta_1, \beta_2)'$ are taken to be equal to one, i.e. $\beta_1 = \beta_2 = 1$.

All in all, we consider seven values of λ, seven values of ρ, three values of σ^2

which are woven into the experimental design, and two values of n. That leads us to the total of 98 experiments. Each Monte Carlo experiment consists of 5000 trials which generate 5000 different vectors of innovations. The elements of each vector of innovations are distributed i.i.d. $N(0, \sigma^2)$. The same set of 5000 vectors of innovations is used in all experiments that correspond to the same sample size n. Furthermore, the vector of innovations for experiments in which the sample size 100 is taken to be the first 100 elements of the corresponding vector of innovations for the sample size 400.

The efficiency measure of the estimators for each experiments is based on the empirical distribution over the 5000 Monte Carlo trials. For each trial the coefficient are estimated, and the empirical distribution is defined with respect to these 5000 trials. Following Kelejian and Prucha (1998), our efficiency measure is a variation on the root mean squared error and is taken as

$$RMSE^* = \left[bias^2 + [IQ/1.35]^2\right]^{1/2} \tag{2.34}$$

where $bias$ is an absolute difference between the median of the empirical distribution and the true parameter value, and IQ is an interquantile range. That is $IQ = c_1 - c_2$ where c_1 is the 0.75 quantile and c_2 is the 0.25 quantile. Note that if the distribution is normal the median is equal to the mean and $IQ/1.35$ is approximately equal to the standard deviation. An important feature of the measure in (2.34) is that it is based on 0.25, 0.5, 0.75 quantiles which always exist. The standard measure of root mean square error is based on the first and second moments which, as pointed out in Kelejian and Prucha (1999) among others, may not always exist, and, therefore, that measure would not be well defined. For simplicity of presentation we refer to our measure of efficiency as RMSE.

For further reference let:

$$\begin{aligned} y_n(a) &= y_n - aW_n y_n, \\ Z_{n*}(a) &= Z_n - aW_n Z_n \end{aligned}$$

where a is any finite scalar. Let the independent columns of $H_n = (X_n, W_n X_n, W_n^2 X_n)$ be the set of instruments used in the 2SLS and FGS2SLS procedures, and $P_{H_n} = H_n(H_n' H_n)^{-1} H_n'$ be a projection matrix corresponding to H_n.

There are fourteen estimators of the parameter vector $\delta = (\lambda, \beta_1, \beta_2)'$ of (2.32) considered in this study. The following is a list of these estimators.

- Maximum likelihood estimator based on the maximization of the log-likelihood function (2.10): $\widehat{\delta}_{ML}$.

- Ordinary least squares estimator:

$$\widehat{\delta}_n^{OLS} = (Z_n' Z_n)^{-1} Z_n' y_n$$

- Two-stage least squares estimator:

$$\widehat{\delta}_n^{2SLS} = (\widehat{Z}_n' Z_n)^{-1} \widehat{Z}_n' y_n$$

  where $\widehat{Z}_n = P_{H_n} Z_n = (\widehat{W_n y_n}, X_n)$, and $\widehat{W_n y_n} = P_{H_n} W_n y_n$.

- GS2SLS based on the true value of ρ :

$$\widehat{\delta}_n^{GS2SLS} = (\widehat{Z}_{n*}(\rho)' Z_{n*}(\rho))^{-1} \widehat{Z}_{n*}(\rho)' y_{n*}(\rho)$$

  where $\widehat{Z}_{n*}(\rho) = P_{H_n} Z_{n*}(\rho)$.

- FGS2SLS based on $\widehat{\rho}_n^{2SLS}$

$$\widehat{\delta}_n^{FGS2SLS} = \left[\widehat{Z}_{n*}(\widehat{\rho}_n^{2SLS})' Z_{n*}(\widehat{\rho}_n^{2SLS}) \right]^{-1} \widehat{Z}_{n*}(\widehat{\rho}_n^{2SLS})' y_{n*}(\widehat{\rho}_n^{2SLS}),$$

  where $\widehat{Z}_{n*}(\widehat{\rho}_n^{2SLS}) = P_{H_n} Z_{n*}(\widehat{\rho}_n^{2SLS})$.

- Lee estimator:

$$\widehat{\delta}_n^{Lee} = \left[(\bar{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS})' Z_{n*}(\widehat{\rho}_n^{2SLS})\right]^{-1} \bar{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS})' y_{n*}(\widehat{\rho}_n^{2SLS}),$$

where

$$\bar{Z}_{n*}(\widehat{\rho}_n, \widehat{\delta}_n^{2SLS}) = (I_n - \widehat{\rho}_n^{2SLS} W_n) \left[X_n, W_n(I_n - \widehat{\lambda}_n^{2SLS} W_n)^{-1} X_n \widehat{\beta}_n^{2SLS}\right]$$

- Three series estimators (Series1, Series2, and Series3) corresponding to three values of α which are $\alpha_1 = 0.25$, $\alpha_2 = 0.35$, and $\alpha_3 = 0.45$:

$$\begin{aligned} \widehat{\delta}_{\alpha_i,n}^{Series} &= \left[\widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha_i)' Z_{n*}(\widehat{\rho}_n^{2SLS})\right]^{-1} \times \\ &\quad \widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha_i)' y_{n*}(\widehat{\rho}_n^{2SLS}), \end{aligned}$$

where

$$\begin{aligned} \widetilde{Z}_{n*}(\widehat{\rho}_n^{2SLS}, \widehat{\delta}_n^{2SLS}, \alpha_i) &= (I_n - \widehat{\rho}_n^{2SLS} W_n) \times \\ &\quad \left[X_n, \sum_{j=0}^{r_n(\alpha_i)} \left(\widehat{\lambda}^{2SLS}\right)^j W_n^{j+1} X_n \widehat{\beta}_n^{2SLS}\right], \\ i &= 1, 2, 3 \end{aligned}$$

and $r_n(\alpha_i) = Int(n^{\alpha_i})$ where $Int(\cdot)$ is a function whose outcome is a nearest integer to the argument which is a real number.

- Iterated FGS2SLS (IF):

$$\widehat{\delta}_n^{IF} = \left[\widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS})' Z_{n*}(\widehat{\rho}_n^{FGS2SLS})\right]^{-1} \widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS})' y_{n*}(\widehat{\rho}_n^{FGS2SLS})$$

where $\widehat{Z}_{n*}(\widehat{\rho}_n^{FGS2SLS}) = P_{H_n} Z_{n*}(\widehat{\rho}_n^{FGS2SLS})$.

- Iterated Lee (IL):

$$\widehat{\delta}_n^{IL} = \left[\bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee})' Z_{n*}(\widehat{\rho}_n^{Lee})\right]^{-1} \bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee})' y_{n*}(\widehat{\rho}_n^{Lee})$$

where

$$\bar{Z}_{n*}(\widehat{\rho}_n^{Lee}, \widehat{\delta}_n^{Lee}) = (I_n - \widehat{\rho}_n^{Lee} W_n) \left[X_n, W_n(I_n - \widehat{\lambda}_n^{Lee} W_n)^{-1} X_n \widehat{\beta}_n^{Lee}\right]$$

- Three iterated series estimators (IS1, IS2, IS3) corresponding to the three already defined values of α :

$$\begin{aligned} \widehat{\delta}_{\alpha_i,n}^{IS} &= \left[\widetilde{Z}_{n*}(\widehat{\rho}_{\alpha_i,n}^{Series}, \widehat{\delta}_{\alpha_i,n}^{Series}, \alpha_i)' Z_{n*}(\widehat{\rho}_{\alpha_i,n}^{Series})\right]^{-1} \times \\ & \quad \widetilde{Z}_{n*}(\widehat{\rho}_{\alpha_i,n}^{Series}, \widehat{\delta}_{\alpha_i,n}^{Series}, \alpha_i)' y_{n*}(\widehat{\rho}_{\alpha_i,n}^{Series}) \end{aligned}$$

where

$$\begin{aligned} \widetilde{Z}_{n*}(\widehat{\rho}_{\alpha_i,n}^{Series}, \widehat{\delta}_{\alpha_i,n}^{Series}, \alpha_i) &= (I_n - \widehat{\rho}_{\alpha_i,n}^{Series} W_n) \times \\ & \quad \left[X_n, \sum_{j=0}^{r_n(\alpha_i)} \left(\widehat{\lambda}_{\alpha_i,n}^{Series}\right)^j W_n^{j+1} X_n \widehat{\beta}_{\alpha_i,n}^{Series}\right] \\ i &= 1, 2, 3 \end{aligned}$$

and $r_n(\alpha_i)$ has been defined before. For future reference, the FGS2SLS, Lee, Series estimators, and their iterated versions are referred to as spatial instrumental variable (IV) estimators.

There are six GM estimators of ρ, namely $\widehat{\rho}_n^{2SLS}$, $\widehat{\rho}_n^{FGS2SLS}$, $\widehat{\rho}_n^{Series1}$, $\widehat{\rho}_n^{Series2}$, $\widehat{\rho}_n^{Series3}$, $\widehat{\rho}_n^{Lee}$ which are obtained by applying the GM procedure to the corresponding residuals $u_n^{2SLS} = y_n - Z_n \widehat{\delta}_n^{2SLS}$, $u_n^{FGS2SLS} = y_n - Z_n \widehat{\delta}_n^{FGS2SLS}$, $u_n^{Series1} = y_n - Z_n \widehat{\delta}_n^{Series1}$, $u_n^{Series2} = y_n - Z_n \widehat{\delta}_n^{Series2}$, $u_n^{Series3} = y_n - Z_n \widehat{\delta}_n^{Series3}$, $u_n^{Lee} = y_n - Z_n \widehat{\delta}_n^{Lee}$, We also consider the ML estimator of ρ, $\widehat{\rho}_n^{ML}$, based on the maximization of the log-likelihood function (2.10).

2.8 Results

Tables 2.2-2.9 report the RMSEs of the considered estimators of the parameters $\lambda, \beta_1, \beta_2$, and ρ corresponding to 98 sets of experimental parameter values. These sets of parameter values are based on seven values of λ, seven values of ρ, and two values of n. The values of σ^2 are woven into the 49 combinations of λ and ρ. There are two tables containing RMSEs corresponding to each parameter. The first table relates to the results when $n = 100$; the second table corresponds to the case when $n = 400$.

As a starting point observe that if the experiments involving the values 0.9 and -0.9 of λ and ρ are omitted the sets of parameter values (but not the estimators) of the remaining experiments are identical to those considered by Das, Kelejian and Prucha (2003). Therefore, for these experiments the results reported in their study in Tables 3-10 should be statistically the same as those reported in this study in Tables 2.2-2.9 for the same estimators. In fact, the RMSEs of ML, FGS2SLS, 2SLS, and OLS estimators are virtually the same in both studies. There are some discrepancies stemming from differences in the vectors of innovations used in the Monte Carlo experiments. These discrepancies are within the range of the statistical error[12].

[12]For an additional check we ran our program using vectors of innovations of Das, Kelejian and Prucha (2003). The resulting RMSEs turned out to be identical to those of Das, Kelejian and Prucha (2003).

Table 2.2. Root mean square error of the estimators of λ, *N=100*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.092	0.092	0.095	0.094	0.101	0.098	0.092
-0.9	-0.8	0.50	0.093	0.092	0.095	0.093	0.099	0.095	0.092
-0.9	-0.4	0.50	0.094	0.094	0.095	0.095	0.098	0.094	0.093
-0.9	0	0.50	0.104	0.104	0.107	0.104	0.105	0.102	0.103
-0.9	0.4	0.50	0.123	0.124	0.126	0.124	0.126	0.124	0.130
-0.9	0.8	0.50	0.137	0.153	0.177	0.169	0.171	0.173	0.178
-0.9	0.9	0.50	0.135	0.159	0.223	0.213	0.212	0.213	0.217
-0.8	-0.9	0.25	0.065	0.064	0.066	0.065	0.067	0.066	0.064
-0.8	-0.8	0.25	0.065	0.064	0.066	0.065	0.066	0.065	0.064
-0.8	-0.4	0.25	0.066	0.066	0.067	0.066	0.068	0.067	0.066
-0.8	0.0	0.25	0.074	0.074	0.075	0.074	0.075	0.074	0.074
-0.8	0.4	0.25	0.090	0.088	0.091	0.089	0.090	0.089	0.091
-0.8	0.8	0.25	0.107	0.114	0.126	0.122	0.122	0.121	0.125
-0.8	0.9	0.25	0.108	0.117	0.147	0.143	0.143	0.143	0.145
-0.4	-0.9	0.50	0.078	0.079	0.081	0.081	0.081	0.081	0.081
-0.4	-0.8	0.50	0.079	0.079	0.081	0.081	0.081	0.081	0.081
-0.4	-0.4	0.50	0.083	0.084	0.084	0.084	0.084	0.084	0.084
-0.4	0.0	0.50	0.097	0.096	0.097	0.097	0.096	0.097	0.097
-0.4	0.4	0.50	0.124	0.123	0.126	0.124	0.123	0.124	0.124
-0.4	0.8	0.50	0.164	0.181	0.207	0.199	0.199	0.198	0.199
-0.4	0.9	0.50	0.163	0.196	0.292	0.276	0.276	0.276	0.277
0.0	-0.9	1.00	0.085	0.087	0.089	0.088	0.088	0.088	0.088
0.0	-0.8	1.00	0.087	0.088	0.089	0.089	0.089	0.089	0.089
0.0	-0.4	1.00	0.093	0.093	0.095	0.094	0.094	0.094	0.094
0.0	0.0	1.00	0.112	0.111	0.111	0.111	0.111	0.111	0.111
0.0	0.4	1.00	0.150	0.149	0.150	0.149	0.149	0.149	0.149
0.0	0.8	1.00	0.223	0.254	0.296	0.283	0.283	0.283	0.284
0.0	0.9	1.00	0.218	0.302	0.474	0.445	0.451	0.455	0.459
0.4	-0.9	0.25	0.028	0.029	0.028	0.029	0.029	0.029	0.029
0.4	-0.8	0.25	0.028	0.029	0.029	0.029	0.029	0.029	0.029
0.4	-0.4	0.25	0.031	0.031	0.032	0.032	0.032	0.032	0.032
0.4	0.0	0.25	0.037	0.037	0.038	0.037	0.038	0.038	0.038
0.4	0.4	0.25	0.055	0.053	0.054	0.054	0.055	0.055	0.054
0.4	0.8	0.25	0.109	0.109	0.114	0.109	0.109	0.109	0.109
0.4	0.9	0.25	0.129	0.139	0.165	0.162	0.160	0.161	0.162

Table 2.2 (cont). Root mean square error of the estimators of λ, *N=100*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.111	0.361	0.095	0.094	0.106	0.101	0.096
-0.9	-0.8	0.50	0.107	0.330	0.095	0.094	0.105	0.100	0.095
-0.9	-0.4	0.50	0.098	0.230	0.096	0.095	0.105	0.102	0.096
-0.9	0	0.50	0.104	0.156	0.107	0.106	0.111	0.111	0.106
-0.9	0.4	0.50	0.136	0.134	0.127	0.125	0.129	0.128	0.128
-0.9	0.8	0.50	0.312	0.590	0.166	0.156	0.154	0.155	0.160
-0.9	0.9	0.50	0.533	1.086	0.184	0.169	0.167	0.169	0.172
-0.8	-0.9	0.25	0.078	0.200	0.066	0.066	0.067	0.066	0.065
-0.8	-0.8	0.25	0.075	0.182	0.065	0.065	0.066	0.065	0.065
-0.8	-0.4	0.25	0.068	0.127	0.067	0.066	0.069	0.067	0.066
-0.8	0.0	0.25	0.074	0.093	0.076	0.074	0.076	0.075	0.074
-0.8	0.4	0.25	0.097	0.099	0.091	0.090	0.091	0.090	0.090
-0.8	0.8	0.25	0.235	0.393	0.119	0.115	0.114	0.115	0.116
-0.8	0.9	0.25	0.392	0.796	0.127	0.120	0.120	0.120	0.121
-0.4	-0.9	0.50	0.093	0.267	0.080	0.080	0.080	0.080	0.080
-0.4	-0.8	0.50	0.090	0.242	0.081	0.081	0.081	0.081	0.081
-0.4	-0.4	0.50	0.087	0.162	0.084	0.084	0.084	0.084	0.084
-0.4	0.0	0.50	0.096	0.111	0.098	0.098	0.098	0.098	0.098
-0.4	0.4	0.50	0.132	0.148	0.127	0.126	0.126	0.126	0.126
-0.4	0.8	0.50	0.314	0.608	0.195	0.183	0.183	0.183	0.183
-0.4	0.9	0.50	0.517	0.972	0.242	0.220	0.219	0.223	0.221
0.0	-0.9	1.00	0.099	0.289	0.090	0.088	0.088	0.088	0.088
0.0	-0.8	1.00	0.096	0.258	0.089	0.089	0.089	0.089	0.089
0.0	-0.4	1.00	0.096	0.161	0.096	0.095	0.095	0.095	0.095
0.0	0.0	1.00	0.111	0.110	0.112	0.113	0.113	0.113	0.113
0.0	0.4	1.00	0.156	0.220	0.153	0.153	0.153	0.153	0.153
0.0	0.8	1.00	0.349	0.683	0.282	0.275	0.274	0.276	0.276
0.0	0.9	1.00	0.564	0.879	0.439	0.388	0.385	0.397	0.405
0.4	-0.9	0.25	0.031	0.042	0.029	0.029	0.029	0.029	0.029
0.4	-0.8	0.25	0.030	0.039	0.029	0.029	0.029	0.029	0.029
0.4	-0.4	0.25	0.031	0.033	0.032	0.032	0.032	0.032	0.032
0.4	0.0	0.25	0.037	0.039	0.038	0.038	0.038	0.038	0.038
0.4	0.4	0.25	0.055	0.070	0.055	0.055	0.055	0.055	0.055
0.4	0.8	0.25	0.142	0.247	0.112	0.113	0.113	0.112	0.112
0.4	0.9	0.25	0.228	0.407	0.154	0.147	0.145	0.146	0.148

Table 2.2 (cont). Root mean square error of the estimators of λ, $N=100$

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.014	0.015	0.015	0.015	0.015	0.015	0.015
0.8	-0.8	0.50	0.015	0.015	0.015	0.015	0.015	0.015	0.015
0.8	-0.4	0.50	0.017	0.017	0.017	0.016	0.017	0.017	0.017
0.8	0.0	0.50	0.021	0.021	0.021	0.021	0.021	0.021	0.021
0.8	0.4	0.50	0.031	0.031	0.030	0.031	0.031	0.031	0.031
0.8	0.8	0.50	0.077	0.070	0.069	0.073	0.075	0.074	0.073
0.8	0.9	0.50	0.103	0.108	0.109	0.114	0.125	0.120	0.116
0.9	-0.9	0.50	0.007	0.008	0.008	0.007	0.008	0.008	0.007
0.9	-0.8	0.50	0.008	0.008	0.008	0.008	0.008	0.008	0.008
0.9	-0.4	0.50	0.009	0.009	0.009	0.009	0.009	0.009	0.009
0.9	0	0.50	0.011	0.011	0.011	0.011	0.011	0.011	0.011
0.9	0.4	0.50	0.017	0.017	0.017	0.017	0.017	0.017	0.017
0.9	0.8	0.50	0.045	0.039	0.038	0.041	0.045	0.044	0.042
0.9	0.9	0.50	0.064	0.064	0.062	0.061	0.074	0.071	0.068

Column Average	0.081	0.085	0.096	0.094	0.095	0.095	0.095
Col.Av.w/o |λ|,|ρ|=(	0.081	0.083	0.087	0.086	0.086	0.086	0.086
Col. Av. w/o ρ=0.9	0.072	0.074	0.077	0.076	0.077	0.076	0.076

Table 2.2 (cont). Root mean square error of the estimators of λ, *N=100*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.015	0.016	0.015	0.015	0.015	0.015	0.015
0.8	-0.8	0.50	0.015	0.016	0.015	0.015	0.015	0.015	0.015
0.8	-0.4	0.50	0.017	0.017	0.017	0.016	0.017	0.017	0.017
0.8	0.0	0.50	0.021	0.023	0.021	0.021	0.021	0.021	0.021
0.8	0.4	0.50	0.031	0.042	0.030	0.031	0.031	0.031	0.031
0.8	0.8	0.50	0.073	0.132	0.069	0.081	0.080	0.080	0.080
0.8	0.9	0.50	0.110	0.183	0.106	0.123	0.121	0.119	0.122
0.9	-0.9	0.50	0.008	0.008	0.008	0.007	0.008	0.008	0.007
0.9	-0.8	0.50	0.008	0.008	0.008	0.008	0.008	0.008	0.008
0.9	-0.4	0.50	0.009	0.009	0.009	0.009	0.009	0.009	0.009
0.9	0	0.50	0.011	0.012	0.011	0.011	0.011	0.011	0.011
0.9	0.4	0.50	0.016	0.020	0.017	0.017	0.017	0.017	0.017
0.9	0.8	0.50	0.040	0.067	0.039	0.045	0.049	0.048	0.047
0.9	0.9	0.50	0.059	0.097	0.060	0.070	0.079	0.075	0.074

Column Average	0.127	0.233	0.092	0.090	0.091	0.091	0.091
Col.Av.w/o |λ|,|ρ|=0.9	0.101	0.170	0.086	0.085	0.086	0.086	0.085
Col. Av. w/o ρ=0.9	0.091	0.166	0.076	0.076	0.077	0.076	0.076

Table 2.3. Root mean square error of the estimators of λ, *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.057	0.057	0.057	0.057	0.058	0.057	0.057
-0.9	-0.8	0.50	0.056	0.056	0.056	0.056	0.057	0.056	0.057
-0.9	-0.4	0.50	0.056	0.056	0.056	0.056	0.056	0.056	0.056
-0.9	0	0.50	0.059	0.060	0.060	0.060	0.061	0.060	0.059
-0.9	0.4	0.50	0.068	0.072	0.073	0.073	0.073	0.074	0.070
-0.9	0.8	0.50	0.077	0.092	0.097	0.096	0.096	0.097	0.105
-0.9	0.9	0.50	0.076	0.095	0.111	0.108	0.107	0.109	0.118
-0.8	-0.9	0.50	0.055	0.056	0.056	0.056	0.056	0.055	0.056
-0.8	-0.8	0.50	0.055	0.055	0.055	0.055	0.055	0.055	0.055
-0.8	-0.4	0.50	0.055	0.056	0.056	0.055	0.055	0.055	0.056
-0.8	0.0	0.50	0.059	0.060	0.060	0.060	0.060	0.060	0.059
-0.8	0.4	0.50	0.070	0.073	0.074	0.073	0.074	0.074	0.072
-0.8	0.8	0.50	0.080	0.096	0.101	0.100	0.100	0.101	0.103
-0.8	0.9	0.50	0.080	0.100	0.119	0.115	0.114	0.114	0.118
-0.4	-0.9	1.00	0.066	0.068	0.068	0.069	0.069	0.069	0.069
-0.4	-0.8	1.00	0.067	0.068	0.068	0.069	0.069	0.069	0.069
-0.4	-0.4	1.00	0.069	0.070	0.070	0.069	0.069	0.069	0.069
-0.4	0.0	1.00	0.078	0.078	0.078	0.078	0.078	0.078	0.078
-0.4	0.4	1.00	0.097	0.101	0.102	0.100	0.101	0.100	0.100
-0.4	0.8	1.00	0.116	0.152	0.171	0.163	0.163	0.163	0.163
-0.4	0.9	1.00	0.111	0.166	0.242	0.215	0.215	0.215	0.215
0.0	-0.9	0.25	0.026	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	0.25	0.026	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	0.25	0.028	0.028	0.028	0.028	0.028	0.028	0.028
0.0	0.0	0.25	0.032	0.032	0.032	0.032	0.032	0.032	0.032
0.0	0.4	0.25	0.043	0.043	0.043	0.043	0.043	0.043	0.043
0.0	0.8	0.25	0.072	0.075	0.079	0.077	0.077	0.077	0.077
0.0	0.9	0.25	0.076	0.088	0.102	0.097	0.097	0.097	0.097
0.4	-0.9	0.50	0.023	0.024	0.024	0.024	0.024	0.024	0.024
0.4	-0.8	0.50	0.024	0.024	0.024	0.024	0.024	0.024	0.024
0.4	-0.4	0.50	0.026	0.026	0.026	0.026	0.026	0.026	0.026
0.4	0.0	0.50	0.030	0.030	0.030	0.030	0.030	0.030	0.030
0.4	0.4	0.50	0.043	0.043	0.043	0.043	0.043	0.043	0.043
0.4	0.8	0.50	0.084	0.090	0.093	0.090	0.090	0.090	0.090
0.4	0.9	0.50	0.093	0.118	0.142	0.133	0.131	0.133	0.133

Table 2.3 (cont). Root mean square error of the estimators of λ, *N=400*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.067	0.450	0.057	0.057	0.058	0.058	0.057
-0.9	-0.8	0.50	0.064	0.411	0.056	0.056	0.058	0.057	0.056
-0.9	-0.4	0.50	0.059	0.280	0.057	0.056	0.057	0.056	0.057
-0.9	0	0.50	0.060	0.163	0.060	0.060	0.061	0.060	0.061
-0.9	0.4	0.50	0.078	0.081	0.073	0.073	0.073	0.073	0.074
-0.9	0.8	0.50	0.192	0.693	0.094	0.090	0.091	0.091	0.094
-0.9	0.9	0.50	0.353	1.220	0.099	0.094	0.094	0.094	0.096
-0.8	-0.9	0.25	0.066	0.435	0.056	0.056	0.056	0.056	0.056
-0.8	-0.8	0.25	0.063	0.395	0.055	0.055	0.056	0.055	0.055
-0.8	-0.4	0.25	0.058	0.264	0.056	0.055	0.055	0.055	0.056
-0.8	0.0	0.25	0.060	0.148	0.060	0.060	0.060	0.060	0.060
-0.8	0.4	0.25	0.079	0.089	0.074	0.074	0.074	0.074	0.074
-0.8	0.8	0.25	0.196	0.711	0.098	0.095	0.095	0.095	0.096
-0.8	0.9	0.25	0.358	1.204	0.105	0.099	0.099	0.099	0.100
-0.4	-0.9	0.50	0.080	0.563	0.068	0.069	0.069	0.069	0.069
-0.4	-0.8	0.50	0.077	0.508	0.068	0.068	0.068	0.068	0.068
-0.4	-0.4	0.50	0.072	0.316	0.070	0.069	0.069	0.069	0.069
-0.4	0.0	0.50	0.078	0.136	0.078	0.078	0.078	0.078	0.078
-0.4	0.4	0.50	0.107	0.208	0.102	0.102	0.102	0.102	0.102
-0.4	0.8	0.50	0.268	0.914	0.159	0.151	0.151	0.151	0.151
-0.4	0.9	0.50	0.470	1.206	0.200	0.174	0.173	0.176	0.177
0.0	-0.9	1.00	0.030	0.105	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	1.00	0.029	0.093	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	1.00	0.029	0.053	0.028	0.028	0.028	0.028	0.028
0.0	0.0	1.00	0.032	0.032	0.032	0.032	0.032	0.032	0.032
0.0	0.4	1.00	0.046	0.089	0.043	0.043	0.043	0.043	0.043
0.0	0.8	1.00	0.121	0.418	0.077	0.076	0.076	0.076	0.076
0.0	0.9	1.00	0.220	0.685	0.093	0.088	0.088	0.088	0.088
0.4	-0.9	0.25	0.026	0.075	0.024	0.024	0.024	0.024	0.024
0.4	-0.8	0.25	0.026	0.064	0.024	0.024	0.024	0.024	0.024
0.4	-0.4	0.25	0.026	0.032	0.026	0.026	0.026	0.026	0.026
0.4	0.0	0.25	0.030	0.041	0.030	0.030	0.030	0.030	0.030
0.4	0.4	0.25	0.045	0.123	0.043	0.043	0.043	0.043	0.043
0.4	0.8	0.25	0.117	0.399	0.092	0.091	0.091	0.091	0.091
0.4	0.9	0.25	0.197	0.528	0.130	0.121	0.120	0.120	0.121

Table 2.3 (cont). Root mean square error of the estimators of λ, *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.011	0.012	0.012	0.012	0.012	0.012	0.012
0.8	-0.8	0.50	0.012	0.012	0.012	0.012	0.012	0.012	0.012
0.8	-0.4	0.50	0.013	0.014	0.014	0.013	0.013	0.013	0.013
0.8	0.0	0.50	0.016	0.016	0.016	0.016	0.016	0.016	0.016
0.8	0.4	0.50	0.024	0.024	0.024	0.024	0.024	0.024	0.024
0.8	0.8	0.50	0.059	0.057	0.056	0.056	0.057	0.056	0.056
0.8	0.9	0.50	0.079	0.092	0.095	0.095	0.098	0.095	0.095
0.9	-0.9	0.50	0.004	0.005	0.005	0.004	0.004	0.004	0.004
0.9	-0.8	0.50	0.004	0.005	0.005	0.004	0.004	0.004	0.004
0.9	-0.4	0.50	0.005	0.005	0.005	0.005	0.005	0.005	0.005
0.9	0	0.50	0.006	0.006	0.006	0.006	0.006	0.006	0.006
0.9	0.4	0.50	0.009	0.009	0.009	0.009	0.009	0.009	0.009
0.9	0.8	0.50	0.024	0.023	0.023	0.024	0.024	0.024	0.024
0.9	0.9	0.50	0.041	0.039	0.039	0.040	0.043	0.042	0.041

Column Average	0.050	0.055	0.059	0.057	0.057	0.057	0.058
Col.Av.w/o |λ|,|ρ|=(	0.051	0.054	0.055	0.055	0.055	0.055	0.055
Col. Av. w/o ρ=0.9	0.045	0.047	0.048	0.048	0.048	0.048	0.048

Table 2.3 (cont). Root mean square error of the estimators of λ, *N=400*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.013	0.016	0.012	0.012	0.012	0.012	0.012
0.8	-0.8	0.50	0.013	0.014	0.012	0.012	0.012	0.012	0.012
0.8	-0.4	0.50	0.014	0.016	0.014	0.013	0.013	0.013	0.013
0.8	0.0	0.50	0.016	0.035	0.016	0.016	0.016	0.016	0.016
0.8	0.4	0.50	0.024	0.079	0.024	0.024	0.024	0.024	0.024
0.8	0.8	0.50	0.061	0.184	0.058	0.061	0.062	0.061	0.061
0.8	0.9	0.50	0.099	0.210	0.094	0.098	0.099	0.098	0.098
0.9	-0.9	0.50	0.005	0.005	0.005	0.004	0.004	0.004	0.004
0.9	-0.8	0.50	0.005	0.005	0.005	0.004	0.004	0.004	0.004
0.9	-0.4	0.50	0.005	0.006	0.005	0.005	0.005	0.005	0.005
0.9	0	0.50	0.006	0.010	0.006	0.006	0.006	0.006	0.006
0.9	0.4	0.50	0.009	0.023	0.009	0.009	0.009	0.009	0.009
0.9	0.8	0.50	0.024	0.075	0.023	0.025	0.025	0.025	0.025
0.9	0.9	0.50	0.040	0.101	0.040	0.043	0.046	0.044	0.043

Column Average	0.086	0.284	0.056	0.055	0.055	0.055	0.056
Col.Av.w/o |λ|,|ρ|=0.9	0.068	0.214	0.055	0.054	0.054	0.054	0.054
Col. Av. w/o ρ=0.9	0.059	0.208	0.048	0.047	0.048	0.047	0.048

Table 2.4. Root mean square error of the estimators of B1, N=100

λ	ρ	σ2	ML	GS2SLS	FGS2SLS	LEE	SER1	SER2	SER3
-0.9	-0.9	0.50	0.060	0.060	0.061	0.061	0.062	0.063	0.061
-0.9	-0.8	0.50	0.061	0.060	0.061	0.061	0.062	0.062	0.061
-0.9	-0.4	0.50	0.061	0.060	0.061	0.060	0.061	0.061	0.061
-0.9	0	0.50	0.062	0.061	0.061	0.061	0.061	0.061	0.061
-0.9	0.4	0.50	0.064	0.064	0.064	0.064	0.064	0.064	0.064
-0.9	0.8	0.50	0.069	0.070	0.074	0.074	0.074	0.074	0.075
-0.9	0.9	0.50	0.068	0.071	0.084	0.086	0.085	0.086	0.087
-0.8	-0.9	0.25	0.043	0.043	0.043	0.043	0.044	0.044	0.043
-0.8	-0.8	0.25	0.043	0.043	0.043	0.043	0.044	0.044	0.043
-0.8	-0.4	0.25	0.043	0.043	0.043	0.043	0.043	0.043	0.043
-0.8	0.0	0.25	0.043	0.043	0.043	0.043	0.043	0.043	0.043
-0.8	0.4	0.25	0.045	0.045	0.045	0.045	0.045	0.045	0.045
-0.8	0.8	0.25	0.049	0.050	0.052	0.052	0.052	0.052	0.052
-0.8	0.9	0.25	0.050	0.050	0.057	0.057	0.057	0.057	0.057
-0.4	-0.9	0.50	0.062	0.062	0.062	0.063	0.063	0.063	0.063
-0.4	-0.8	0.50	0.062	0.062	0.063	0.063	0.063	0.062	0.063
-0.4	-0.4	0.50	0.062	0.061	0.062	0.062	0.062	0.062	0.062
-0.4	0.0	0.50	0.061	0.061	0.061	0.061	0.061	0.061	0.061
-0.4	0.4	0.50	0.062	0.062	0.062	0.062	0.062	0.062	0.062
-0.4	0.8	0.50	0.068	0.068	0.070	0.071	0.070	0.070	0.071
-0.4	0.9	0.50	0.068	0.071	0.079	0.084	0.084	0.084	0.085
0.0	-0.9	1.00	0.090	0.090	0.090	0.090	0.090	0.090	0.090
0.0	-0.8	1.00	0.090	0.089	0.090	0.089	0.089	0.089	0.089
0.0	-0.4	1.00	0.089	0.088	0.089	0.089	0.089	0.089	0.089
0.0	0.0	1.00	0.087	0.087	0.087	0.087	0.087	0.087	0.087
0.0	0.4	1.00	0.087	0.087	0.086	0.086	0.087	0.086	0.086
0.0	0.8	1.00	0.091	0.092	0.093	0.097	0.098	0.098	0.098
0.0	0.9	1.00	0.092	0.096	0.102	0.120	0.128	0.127	0.126
0.4	-0.9	0.25	0.046	0.045	0.046	0.046	0.046	0.046	0.046
0.4	-0.8	0.25	0.046	0.045	0.046	0.046	0.046	0.046	0.046
0.4	-0.4	0.25	0.045	0.045	0.045	0.045	0.045	0.045	0.045
0.4	0.0	0.25	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4	0.4	0.25	0.043	0.043	0.043	0.043	0.043	0.043	0.043
0.4	0.8	0.25	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4	0.9	0.25	0.045	0.046	0.047	0.048	0.048	0.049	0.049

Table 2.4 (cont). Root mean square error of the estimators of $B1$, $N=100$

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.069	0.062	0.061	0.061	0.064	0.064	0.061
-0.9	-0.8	0.50	0.066	0.061	0.061	0.061	0.063	0.063	0.061
-0.9	-0.4	0.50	0.062	0.060	0.060	0.060	0.061	0.061	0.061
-0.9	0	0.50	0.061	0.060	0.061	0.061	0.062	0.063	0.061
-0.9	0.4	0.50	0.067	0.066	0.064	0.064	0.065	0.065	0.064
-0.9	0.8	0.50	0.106	0.089	0.073	0.072	0.072	0.072	0.073
-0.9	0.9	0.50	0.154	0.097	0.078	0.075	0.075	0.075	0.077
-0.8	-0.9	0.25	0.049	0.046	0.044	0.044	0.044	0.044	0.044
-0.8	-0.8	0.25	0.047	0.045	0.044	0.044	0.044	0.044	0.044
-0.8	-0.4	0.25	0.044	0.043	0.043	0.043	0.043	0.043	0.043
-0.8	0.0	0.25	0.043	0.043	0.043	0.043	0.043	0.043	0.043
-0.8	0.4	0.25	0.046	0.046	0.045	0.045	0.045	0.045	0.045
-0.8	0.8	0.25	0.074	0.066	0.051	0.051	0.050	0.050	0.051
-0.8	0.9	0.25	0.115	0.081	0.053	0.052	0.051	0.051	0.052
-0.4	-0.9	0.50	0.071	0.069	0.063	0.063	0.063	0.063	0.063
-0.4	-0.8	0.50	0.069	0.067	0.063	0.063	0.062	0.063	0.063
-0.4	-0.4	0.50	0.063	0.062	0.062	0.062	0.062	0.062	0.062
-0.4	0.0	0.50	0.061	0.061	0.061	0.061	0.061	0.061	0.061
-0.4	0.4	0.50	0.064	0.064	0.062	0.062	0.062	0.062	0.062
-0.4	0.8	0.50	0.091	0.089	0.070	0.070	0.069	0.069	0.070
-0.4	0.9	0.50	0.126	0.102	0.076	0.077	0.076	0.077	0.077
0.0	-0.9	1.00	0.103	0.105	0.091	0.091	0.091	0.091	0.091
0.0	-0.8	1.00	0.100	0.101	0.091	0.090	0.090	0.090	0.090
0.0	-0.4	1.00	0.090	0.091	0.090	0.090	0.090	0.090	0.090
0.0	0.0	1.00	0.087	0.086	0.088	0.087	0.087	0.087	0.087
0.0	0.4	1.00	0.089	0.093	0.087	0.087	0.087	0.087	0.087
0.0	0.8	1.00	0.120	0.136	0.093	0.095	0.094	0.095	0.096
0.0	0.9	1.00	0.134	0.149	0.101	0.111	0.115	0.118	0.120
0.4	-0.9	0.25	0.053	0.053	0.046	0.046	0.046	0.046	0.046
0.4	-0.8	0.25	0.051	0.051	0.046	0.046	0.046	0.046	0.046
0.4	-0.4	0.25	0.046	0.046	0.045	0.045	0.045	0.045	0.045
0.4	0.0	0.25	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4	0.4	0.25	0.044	0.047	0.043	0.043	0.043	0.043	0.043
0.4	0.8	0.25	0.056	0.091	0.044	0.044	0.044	0.044	0.044
0.4	0.9	0.25	0.080	0.135	0.046	0.047	0.047	0.047	0.047

Table 2.4 (cont). Root mean square error of the estimators of *B1* , *N=100*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.065	0.064	0.064	0.065	0.065	0.065	0.065
0.8	-0.8	0.50	0.065	0.064	0.064	0.065	0.064	0.065	0.065
0.8	-0.4	0.50	0.064	0.064	0.065	0.065	0.064	0.065	0.065
0.8	0.0	0.50	0.063	0.063	0.063	0.063	0.063	0.063	0.063
0.8	0.4	0.50	0.061	0.060	0.061	0.061	0.061	0.061	0.061
0.8	0.8	0.50	0.059	0.058	0.059	0.059	0.060	0.059	0.059
0.8	0.9	0.50	0.059	0.058	0.059	0.064	0.067	0.067	0.067
0.9	-0.9	0.50	0.063	0.063	0.064	0.064	0.064	0.064	0.064
0.9	-0.8	0.50	0.064	0.063	0.064	0.064	0.064	0.064	0.064
0.9	-0.4	0.50	0.064	0.063	0.065	0.064	0.064	0.065	0.065
0.9	0	0.50	0.063	0.063	0.063	0.063	0.063	0.063	0.063
0.9	0.4	0.50	0.061	0.061	0.061	0.061	0.061	0.061	0.061
0.9	0.8	0.50	0.058	0.058	0.058	0.059	0.059	0.059	0.059
0.9	0.9	0.50	0.058	0.058	0.058	0.061	0.063	0.063	0.063

	ML	GS2SLS	FGS2SLS	LEE	SER1	SER2	SER3
Column Average	0.061	0.062	0.063	0.064	0.064	0.064	0.064
Col.Av.w/o |λ|,|ρ|=0.9	0.061	0.060	0.061	0.061	0.061	0.061	0.061
Col. Av. w/o ρ=0.9	0.061	0.061	0.062	0.062	0.062	0.062	0.062

Table 2.4 (cont). Root mean square error of the estimators of *B1*, *N=100*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.075	0.075	0.064	0.065	0.065	0.065	0.065
0.8	-0.8	0.50	0.072	0.072	0.065	0.065	0.065	0.065	0.065
0.8	-0.4	0.50	0.066	0.066	0.065	0.065	0.065	0.065	0.065
0.8	0.0	0.50	0.063	0.065	0.063	0.063	0.063	0.063	0.063
0.8	0.4	0.50	0.063	0.072	0.061	0.061	0.061	0.061	0.061
0.8	0.8	0.50	0.081	0.142	0.058	0.059	0.059	0.059	0.059
0.8	0.9	0.50	0.108	0.172	0.059	0.064	0.063	0.063	0.063
0.9	-0.9	0.50	0.075	0.074	0.064	0.063	0.064	0.064	0.064
0.9	-0.8	0.50	0.072	0.072	0.064	0.064	0.064	0.064	0.064
0.9	-0.4	0.50	0.066	0.066	0.065	0.065	0.065	0.064	0.064
0.9	0	0.50	0.063	0.064	0.063	0.063	0.063	0.063	0.063
0.9	0.4	0.50	0.063	0.070	0.061	0.061	0.061	0.061	0.061
0.9	0.8	0.50	0.082	0.139	0.058	0.059	0.059	0.059	0.059
0.9	0.9	0.50	0.106	0.169	0.058	0.061	0.060	0.060	0.061
Column Average			0.076	0.080	0.062	0.063	0.063	0.063	0.063
Col.Av.w/o |λ|,|ρ|=0.9			0.067	0.072	0.061	0.061	0.061	0.061	0.061
Col. Av. w/o ρ=0.9			0.069	0.072	0.062	0.062	0.062	0.062	0.062

Table 2.5. Root mean square error of the estimators of *B1* , *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.034	0.034	0.034	0.034	0.034	0.034	0.035
-0.9	-0.8	0.50	0.034	0.034	0.034	0.034	0.034	0.034	0.035
-0.9	-0.4	0.50	0.035	0.035	0.035	0.035	0.035	0.035	0.035
-0.9	0	0.50	0.037	0.037	0.037	0.037	0.037	0.037	0.037
-0.9	0.4	0.50	0.040	0.040	0.040	0.040	0.039	0.040	0.040
-0.9	0.8	0.50	0.043	0.046	0.046	0.046	0.046	0.047	0.049
-0.9	0.9	0.50	0.044	0.047	0.050	0.050	0.050	0.051	0.053
-0.8	-0.9	0.50	0.035	0.034	0.034	0.035	0.035	0.035	0.034
-0.8	-0.8	0.50	0.035	0.034	0.034	0.034	0.035	0.034	0.034
-0.8	-0.4	0.50	0.035	0.035	0.035	0.035	0.036	0.035	0.035
-0.8	0.0	0.50	0.037	0.037	0.037	0.037	0.037	0.037	0.037
-0.8	0.4	0.50	0.039	0.040	0.040	0.039	0.039	0.039	0.039
-0.8	0.8	0.50	0.043	0.046	0.046	0.046	0.046	0.046	0.047
-0.8	0.9	0.50	0.043	0.047	0.050	0.051	0.051	0.051	0.051
-0.4	-0.9	1.00	0.051	0.051	0.051	0.052	0.052	0.052	0.052
-0.4	-0.8	1.00	0.051	0.051	0.051	0.051	0.051	0.051	0.051
-0.4	-0.4	1.00	0.052	0.051	0.051	0.052	0.052	0.052	0.052
-0.4	0.0	1.00	0.053	0.053	0.053	0.053	0.053	0.053	0.053
-0.4	0.4	1.00	0.055	0.055	0.055	0.055	0.055	0.055	0.055
-0.4	0.8	1.00	0.060	0.063	0.064	0.064	0.064	0.064	0.064
-0.4	0.9	1.00	0.059	0.065	0.074	0.076	0.075	0.075	0.075
0.0	-0.9	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.0	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.4	0.25	0.028	0.028	0.028	0.028	0.028	0.028	0.028
0.0	0.8	0.25	0.030	0.030	0.030	0.030	0.030	0.030	0.030
0.0	0.9	0.25	0.031	0.032	0.033	0.033	0.033	0.033	0.033
0.4	-0.9	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	-0.8	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	-0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.0	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.8	0.50	0.040	0.040	0.041	0.040	0.040	0.040	0.040
0.4	0.9	0.50	0.041	0.043	0.043	0.044	0.044	0.044	0.044

Table 2.5 (cont). Root mean square error of the estimators of $B1$, *N=400*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.041	0.043	0.034	0.034	0.035	0.035	0.035
-0.9	-0.8	0.50	0.040	0.042	0.034	0.034	0.035	0.035	0.035
-0.9	-0.4	0.50	0.037	0.038	0.035	0.035	0.035	0.035	0.035
-0.9	0	0.50	0.037	0.037	0.037	0.037	0.037	0.037	0.037
-0.9	0.4	0.50	0.043	0.043	0.040	0.040	0.040	0.040	0.040
-0.9	0.8	0.50	0.084	0.076	0.046	0.045	0.046	0.046	0.047
-0.9	0.9	0.50	0.150	0.099	0.048	0.047	0.047	0.047	0.047
-0.8	-0.9	0.25	0.042	0.048	0.035	0.035	0.035	0.035	0.035
-0.8	-0.8	0.25	0.040	0.046	0.035	0.035	0.035	0.035	0.035
-0.8	-0.4	0.25	0.037	0.040	0.035	0.035	0.036	0.036	0.035
-0.8	0.0	0.25	0.037	0.037	0.037	0.037	0.037	0.037	0.037
-0.8	0.4	0.25	0.042	0.042	0.040	0.040	0.039	0.039	0.040
-0.8	0.8	0.25	0.083	0.081	0.046	0.045	0.046	0.045	0.046
-0.8	0.9	0.25	0.145	0.111	0.048	0.047	0.047	0.047	0.047
-0.4	-0.9	0.50	0.062	0.098	0.051	0.051	0.051	0.051	0.051
-0.4	-0.8	0.50	0.060	0.091	0.051	0.051	0.051	0.051	0.051
-0.4	-0.4	0.50	0.054	0.068	0.052	0.052	0.052	0.052	0.052
-0.4	0.0	0.50	0.053	0.055	0.053	0.053	0.053	0.053	0.053
-0.4	0.4	0.50	0.058	0.064	0.055	0.055	0.055	0.055	0.055
-0.4	0.8	0.50	0.104	0.149	0.063	0.063	0.063	0.063	0.063
-0.4	0.9	0.50	0.177	0.185	0.068	0.067	0.067	0.068	0.068
0.0	-0.9	1.00	0.032	0.041	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	1.00	0.031	0.038	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	1.00	0.028	0.030	0.027	0.027	0.027	0.027	0.027
0.0	0.0	1.00	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.4	1.00	0.029	0.035	0.028	0.028	0.028	0.028	0.028
0.0	0.8	1.00	0.050	0.115	0.030	0.030	0.030	0.030	0.030
0.0	0.9	1.00	0.084	0.183	0.032	0.032	0.032	0.032	0.032
0.4	-0.9	0.25	0.047	0.057	0.039	0.039	0.039	0.039	0.039
0.4	-0.8	0.25	0.046	0.053	0.039	0.039	0.039	0.039	0.039
0.4	-0.4	0.25	0.041	0.042	0.039	0.039	0.039	0.039	0.039
0.4	0.0	0.25	0.039	0.042	0.039	0.039	0.039	0.039	0.039
0.4	0.4	0.25	0.042	0.070	0.039	0.039	0.039	0.039	0.039
0.4	0.8	0.25	0.067	0.198	0.040	0.041	0.041	0.041	0.041
0.4	0.9	0.25	0.112	0.257	0.043	0.043	0.043	0.043	0.044

Table 2.5 (cont). Root mean square error of the estimators of $B1$, $N=400$

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.054	0.055	0.056	0.055	0.055	0.055	0.055
0.8	-0.8	0.50	0.054	0.056	0.056	0.056	0.056	0.056	0.056
0.8	-0.4	0.50	0.056	0.056	0.057	0.057	0.057	0.057	0.057
0.8	0.0	0.50	0.056	0.056	0.057	0.057	0.057	0.057	0.057
0.8	0.4	0.50	0.056	0.056	0.056	0.056	0.056	0.056	0.056
0.8	0.8	0.50	0.055	0.056	0.055	0.056	0.056	0.055	0.056
0.8	0.9	0.50	0.055	0.057	0.057	0.059	0.061	0.061	0.061
0.9	-0.9	0.50	0.038	0.039	0.039	0.038	0.038	0.038	0.038
0.9	-0.8	0.50	0.038	0.039	0.039	0.039	0.039	0.039	0.039
0.9	-0.4	0.50	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.9	0	0.50	0.040	0.040	0.040	0.040	0.040	0.041	0.041
0.9	0.4	0.50	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.9	0.8	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.9	0.9	0.50	0.039	0.040	0.039	0.040	0.040	0.040	0.040

Column Average	0.042	0.042	0.043	0.043	0.043	0.043	0.043
Col.Av.w/o |λ|,|ρ|=(	0.043	0.043	0.043	0.043	0.043	0.043	0.043
Col. Av. w/o ρ=0.9	0.041	0.042	0.042	0.042	0.042	0.042	0.042

Table 2.5 (cont). Root mean square error of the estimators of $B1$, $N=400$

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.067	0.068	0.056	0.055	0.055	0.055	0.055
0.8	-0.8	0.50	0.065	0.065	0.056	0.055	0.055	0.055	0.055
0.8	-0.4	0.50	0.059	0.060	0.057	0.056	0.057	0.057	0.056
0.8	0.0	0.50	0.056	0.073	0.057	0.057	0.057	0.057	0.057
0.8	0.4	0.50	0.060	0.128	0.056	0.056	0.056	0.056	0.056
0.8	0.8	0.50	0.094	0.272	0.055	0.056	0.056	0.056	0.056
0.8	0.9	0.50	0.160	0.302	0.056	0.059	0.058	0.059	0.060
0.9	-0.9	0.50	0.047	0.047	0.039	0.038	0.038	0.038	0.038
0.9	-0.8	0.50	0.046	0.045	0.039	0.038	0.039	0.038	0.039
0.9	-0.4	0.50	0.042	0.042	0.040	0.040	0.040	0.040	0.040
0.9	0	0.50	0.040	0.047	0.040	0.040	0.040	0.041	0.041
0.9	0.4	0.50	0.043	0.074	0.040	0.040	0.040	0.040	0.040
0.9	0.8	0.50	0.067	0.217	0.039	0.039	0.039	0.039	0.039
0.9	0.9	0.50	0.111	0.280	0.039	0.040	0.039	0.039	0.040

Column Average	0.062	0.090	0.043	0.043	0.043	0.043	0.043
Col.Av.w/o |λ|,|ρ|=0.9	0.052	0.077	0.043	0.043	0.043	0.043	0.043
Col. Av. w/o ρ=0.9	0.050	0.071	0.042	0.042	0.042	0.042	0.042

Table 2.6. Root mean square error of the estimators of *B2* , *N=100*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.076	0.075	0.075	0.075	0.077	0.078	0.075
-0.9	-0.8	0.50	0.076	0.075	0.076	0.075	0.076	0.076	0.076
-0.9	-0.4	0.50	0.076	0.074	0.075	0.075	0.075	0.076	0.075
-0.9	0	0.50	0.076	0.076	0.076	0.076	0.077	0.077	0.076
-0.9	0.4	0.50	0.077	0.078	0.078	0.077	0.079	0.079	0.078
-0.9	0.8	0.50	0.080	0.082	0.086	0.086	0.087	0.087	0.086
-0.9	0.9	0.50	0.080	0.083	0.096	0.098	0.099	0.099	0.099
-0.8	-0.9	0.25	0.054	0.053	0.054	0.054	0.054	0.054	0.054
-0.8	-0.8	0.25	0.054	0.053	0.054	0.054	0.053	0.054	0.054
-0.8	-0.4	0.25	0.054	0.053	0.053	0.053	0.053	0.053	0.053
-0.8	0.0	0.25	0.054	0.053	0.054	0.054	0.054	0.054	0.054
-0.8	0.4	0.25	0.054	0.054	0.054	0.054	0.055	0.055	0.054
-0.8	0.8	0.25	0.057	0.058	0.060	0.060	0.060	0.060	0.060
-0.8	0.9	0.25	0.058	0.058	0.065	0.065	0.065	0.065	0.066
-0.4	-0.9	0.50	0.078	0.078	0.078	0.077	0.077	0.078	0.077
-0.4	-0.8	0.50	0.077	0.078	0.078	0.077	0.077	0.077	0.077
-0.4	-0.4	0.50	0.076	0.076	0.077	0.077	0.077	0.077	0.077
-0.4	0.0	0.50	0.076	0.075	0.076	0.076	0.076	0.076	0.076
-0.4	0.4	0.50	0.075	0.076	0.075	0.075	0.075	0.075	0.075
-0.4	0.8	0.50	0.080	0.081	0.083	0.083	0.083	0.083	0.083
-0.4	0.9	0.50	0.080	0.083	0.094	0.100	0.100	0.100	0.100
0.0	-0.9	1.00	0.112	0.112	0.112	0.112	0.112	0.112	0.112
0.0	-0.8	1.00	0.111	0.113	0.112	0.111	0.111	0.111	0.111
0.0	-0.4	1.00	0.110	0.110	0.110	0.110	0.110	0.110	0.110
0.0	0.0	1.00	0.108	0.108	0.108	0.108	0.108	0.108	0.108
0.0	0.4	1.00	0.106	0.105	0.105	0.106	0.106	0.106	0.106
0.0	0.8	1.00	0.108	0.111	0.112	0.117	0.118	0.118	0.118
0.0	0.9	1.00	0.108	0.115	0.128	0.143	0.152	0.153	0.153
0.4	-0.9	0.25	0.057	0.057	0.057	0.057	0.057	0.057	0.057
0.4	-0.8	0.25	0.057	0.057	0.057	0.057	0.057	0.057	0.057
0.4	-0.4	0.25	0.056	0.056	0.056	0.056	0.056	0.056	0.056
0.4	0.0	0.25	0.054	0.054	0.055	0.055	0.054	0.054	0.055
0.4	0.4	0.25	0.052	0.052	0.052	0.052	0.052	0.052	0.052
0.4	0.8	0.25	0.053	0.053	0.053	0.053	0.053	0.053	0.053
0.4	0.9	0.25	0.054	0.055	0.056	0.058	0.058	0.059	0.059

Table 2.6 (cont). Root mean square error of the estimators of $B2$, $N=100$

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.090	0.083	0.076	0.076	0.079	0.079	0.076
-0.9	-0.8	0.50	0.087	0.081	0.076	0.076	0.078	0.079	0.076
-0.9	-0.4	0.50	0.079	0.076	0.076	0.076	0.076	0.077	0.075
-0.9	0	0.50	0.076	0.074	0.076	0.076	0.077	0.078	0.076
-0.9	0.4	0.50	0.081	0.081	0.078	0.077	0.079	0.079	0.078
-0.9	0.8	0.50	0.137	0.114	0.084	0.083	0.082	0.082	0.084
-0.9	0.9	0.50	0.195	0.136	0.088	0.087	0.086	0.087	0.088
-0.8	-0.9	0.25	0.064	0.061	0.054	0.054	0.054	0.054	0.054
-0.8	-0.8	0.25	0.062	0.059	0.054	0.054	0.054	0.054	0.054
-0.8	-0.4	0.25	0.056	0.054	0.054	0.054	0.054	0.054	0.054
-0.8	0.0	0.25	0.053	0.052	0.054	0.054	0.054	0.054	0.054
-0.8	0.4	0.25	0.057	0.057	0.055	0.054	0.055	0.054	0.054
-0.8	0.8	0.25	0.097	0.085	0.059	0.059	0.058	0.058	0.059
-0.8	0.9	0.25	0.151	0.104	0.060	0.059	0.060	0.059	0.060
-0.4	-0.9	0.50	0.091	0.085	0.078	0.078	0.078	0.078	0.078
-0.4	-0.8	0.50	0.088	0.083	0.078	0.077	0.077	0.077	0.077
-0.4	-0.4	0.50	0.080	0.078	0.077	0.077	0.077	0.077	0.077
-0.4	0.0	0.50	0.075	0.074	0.076	0.076	0.076	0.076	0.076
-0.4	0.4	0.50	0.078	0.078	0.075	0.075	0.075	0.075	0.075
-0.4	0.8	0.50	0.121	0.094	0.083	0.081	0.081	0.081	0.081
-0.4	0.9	0.50	0.161	0.093	0.090	0.089	0.090	0.090	0.090
0.0	-0.9	1.00	0.131	0.126	0.112	0.112	0.112	0.112	0.112
0.0	-0.8	1.00	0.127	0.122	0.112	0.111	0.111	0.111	0.111
0.0	-0.4	1.00	0.115	0.112	0.110	0.111	0.111	0.111	0.111
0.0	0.0	1.00	0.108	0.107	0.108	0.108	0.108	0.108	0.108
0.0	0.4	1.00	0.108	0.108	0.107	0.107	0.107	0.107	0.107
0.0	0.8	1.00	0.147	0.123	0.112	0.113	0.113	0.114	0.113
0.0	0.9	1.00	0.164	0.124	0.127	0.133	0.136	0.143	0.145
0.4	-0.9	0.25	0.067	0.068	0.057	0.058	0.057	0.058	0.058
0.4	-0.8	0.25	0.065	0.065	0.057	0.057	0.057	0.057	0.057
0.4	-0.4	0.25	0.058	0.058	0.056	0.056	0.056	0.056	0.056
0.4	0.0	0.25	0.054	0.054	0.055	0.054	0.054	0.054	0.054
0.4	0.4	0.25	0.054	0.055	0.052	0.052	0.052	0.052	0.052
0.4	0.8	0.25	0.072	0.087	0.053	0.053	0.053	0.053	0.053
0.4	0.9	0.25	0.102	0.117	0.055	0.056	0.056	0.056	0.056

Table 2.6 (cont). Root mean square error of the estimators of $B2$, $N=100$

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.082	0.082	0.082	0.082	0.083	0.083	0.082
0.8	-0.8	0.50	0.082	0.082	0.082	0.083	0.083	0.083	0.082
0.8	-0.4	0.50	0.080	0.080	0.081	0.080	0.080	0.080	0.080
0.8	0.0	0.50	0.078	0.077	0.079	0.078	0.078	0.078	0.078
0.8	0.4	0.50	0.074	0.073	0.074	0.074	0.074	0.075	0.074
0.8	0.8	0.50	0.071	0.071	0.071	0.072	0.072	0.072	0.072
0.8	0.9	0.50	0.071	0.072	0.072	0.077	0.080	0.080	0.080
0.9	-0.9	0.50	0.081	0.082	0.082	0.081	0.082	0.082	0.082
0.9	-0.8	0.50	0.081	0.082	0.082	0.081	0.082	0.082	0.082
0.9	-0.4	0.50	0.080	0.080	0.080	0.081	0.081	0.081	0.080
0.9	0	0.50	0.078	0.077	0.078	0.079	0.078	0.078	0.078
0.9	0.4	0.50	0.074	0.073	0.074	0.074	0.074	0.074	0.074
0.9	0.8	0.50	0.071	0.070	0.070	0.071	0.072	0.072	0.072
0.9	0.9	0.50	0.070	0.070	0.070	0.073	0.075	0.076	0.076
Column Average			0.075	0.076	0.077	0.078	0.078	0.078	0.078
Col.Av.w/o |λ|,|ρ|=(			0.074	0.074	0.075	0.075	0.075	0.075	0.075
Col. Av. w/o ρ=0.9			0.075	0.075	0.076	0.076	0.076	0.076	0.076

Table 2.6 (cont). Root mean square error of the estimators of $B2$, *N=100*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.097	0.097	0.082	0.082	0.083	0.082	0.082
0.8	-0.8	0.50	0.093	0.093	0.082	0.082	0.082	0.082	0.083
0.8	-0.4	0.50	0.082	0.082	0.081	0.081	0.081	0.080	0.081
0.8	0.0	0.50	0.077	0.078	0.078	0.078	0.078	0.078	0.078
0.8	0.4	0.50	0.076	0.083	0.074	0.074	0.074	0.074	0.074
0.8	0.8	0.50	0.099	0.144	0.072	0.072	0.072	0.072	0.072
0.8	0.9	0.50	0.125	0.173	0.071	0.077	0.075	0.077	0.077
0.9	-0.9	0.50	0.097	0.096	0.083	0.081	0.082	0.082	0.081
0.9	-0.8	0.50	0.093	0.093	0.083	0.081	0.082	0.082	0.082
0.9	-0.4	0.50	0.083	0.082	0.081	0.081	0.081	0.080	0.080
0.9	0	0.50	0.077	0.078	0.078	0.079	0.078	0.078	0.078
0.9	0.4	0.50	0.076	0.083	0.074	0.074	0.074	0.074	0.074
0.9	0.8	0.50	0.099	0.147	0.071	0.072	0.072	0.072	0.072
0.9	0.9	0.50	0.123	0.179	0.070	0.073	0.073	0.073	0.073
Column Average			0.095	0.093	0.077	0.077	0.077	0.077	0.077
Col.Av.w/o |λ|,|ρ|=0.9			0.084	0.083	0.075	0.075	0.075	0.075	0.075
Col. Av. w/o ρ=0.9			0.086	0.086	0.076	0.076	0.076	0.076	0.076

Table 2.7. Root mean square error of the estimators of *B2* , *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.037
-0.9	-0.8	0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.037
-0.9	-0.4	0.50	0.037	0.037	0.037	0.037	0.037	0.037	0.037
-0.9	0	0.50	0.039	0.039	0.039	0.039	0.040	0.040	0.040
-0.9	0.4	0.50	0.041	0.042	0.042	0.042	0.042	0.042	0.042
-0.9	0.8	0.50	0.043	0.046	0.046	0.047	0.046	0.046	0.049
-0.9	0.9	0.50	0.043	0.046	0.049	0.049	0.049	0.049	0.052
-0.8	-0.9	0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.036
-0.8	-0.8	0.50	0.036	0.036	0.037	0.036	0.036	0.037	0.037
-0.8	-0.4	0.50	0.037	0.037	0.037	0.037	0.037	0.038	0.037
-0.8	0.0	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
-0.8	0.4	0.50	0.041	0.041	0.041	0.041	0.042	0.041	0.041
-0.8	0.8	0.50	0.043	0.046	0.046	0.046	0.046	0.046	0.047
-0.8	0.9	0.50	0.043	0.047	0.049	0.050	0.050	0.049	0.050
-0.4	-0.9	1.00	0.052	0.052	0.052	0.052	0.052	0.052	0.052
-0.4	-0.8	1.00	0.052	0.052	0.052	0.052	0.052	0.052	0.052
-0.4	-0.4	1.00	0.053	0.053	0.053	0.053	0.053	0.053	0.053
-0.4	0.0	1.00	0.055	0.055	0.055	0.055	0.055	0.055	0.055
-0.4	0.4	1.00	0.057	0.057	0.057	0.057	0.057	0.057	0.057
-0.4	0.8	1.00	0.059	0.064	0.064	0.064	0.064	0.064	0.064
-0.4	0.9	1.00	0.058	0.065	0.073	0.074	0.074	0.074	0.074
0.0	-0.9	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.0	0.25	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.4	0.25	0.028	0.028	0.028	0.028	0.028	0.028	0.028
0.0	0.8	0.25	0.030	0.030	0.030	0.030	0.030	0.030	0.030
0.0	0.9	0.25	0.031	0.032	0.033	0.033	0.033	0.033	0.033
0.4	-0.9	0.50	0.038	0.039	0.039	0.039	0.039	0.039	0.039
0.4	-0.8	0.50	0.038	0.038	0.039	0.039	0.039	0.039	0.039
0.4	-0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.0	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.8	0.50	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.4	0.9	0.50	0.040	0.043	0.044	0.045	0.044	0.045	0.044

Table 2.7 (cont). Root mean square error of the estimators of $B2$, *N=400*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
-0.9	-0.9	0.50	0.043	0.055	0.036	0.036	0.036	0.036	0.037
-0.9	-0.8	0.50	0.042	0.053	0.036	0.036	0.036	0.036	0.037
-0.9	-0.4	0.50	0.039	0.045	0.037	0.037	0.037	0.037	0.037
-0.9	0	0.50	0.039	0.041	0.039	0.040	0.040	0.040	0.040
-0.9	0.4	0.50	0.044	0.043	0.041	0.041	0.042	0.042	0.042
-0.9	0.8	0.50	0.084	0.084	0.046	0.045	0.045	0.045	0.046
-0.9	0.9	0.50	0.143	0.122	0.047	0.046	0.046	0.046	0.047
-0.8	-0.9	0.25	0.043	0.051	0.036	0.036	0.036	0.036	0.036
-0.8	-0.8	0.25	0.042	0.049	0.036	0.036	0.036	0.036	0.036
-0.8	-0.4	0.25	0.039	0.043	0.037	0.038	0.038	0.037	0.037
-0.8	0.0	0.25	0.039	0.040	0.039	0.039	0.039	0.039	0.039
-0.8	0.4	0.25	0.043	0.043	0.041	0.041	0.042	0.041	0.042
-0.8	0.8	0.25	0.083	0.077	0.046	0.045	0.045	0.045	0.046
-0.8	0.9	0.25	0.140	0.106	0.047	0.046	0.047	0.047	0.047
-0.4	-0.9	0.50	0.063	0.057	0.052	0.052	0.052	0.052	0.052
-0.4	-0.8	0.50	0.060	0.056	0.052	0.052	0.052	0.052	0.052
-0.4	-0.4	0.50	0.056	0.055	0.053	0.053	0.053	0.053	0.053
-0.4	0.0	0.50	0.055	0.055	0.055	0.055	0.055	0.055	0.055
-0.4	0.4	0.50	0.060	0.058	0.057	0.057	0.057	0.057	0.057
-0.4	0.8	0.50	0.107	0.066	0.064	0.063	0.063	0.063	0.063
-0.4	0.9	0.50	0.157	0.066	0.069	0.068	0.067	0.068	0.068
0.0	-0.9	1.00	0.032	0.032	0.027	0.027	0.027	0.027	0.027
0.0	-0.8	1.00	0.031	0.031	0.027	0.027	0.027	0.027	0.027
0.0	-0.4	1.00	0.028	0.028	0.027	0.027	0.027	0.027	0.027
0.0	0.0	1.00	0.027	0.027	0.027	0.027	0.027	0.027	0.027
0.0	0.4	1.00	0.029	0.029	0.028	0.028	0.028	0.028	0.028
0.0	0.8	1.00	0.051	0.042	0.030	0.030	0.030	0.030	0.030
0.0	0.9	1.00	0.086	0.048	0.032	0.032	0.032	0.032	0.032
0.4	-0.9	0.25	0.046	0.047	0.039	0.039	0.039	0.039	0.039
0.4	-0.8	0.25	0.045	0.045	0.039	0.039	0.039	0.039	0.039
0.4	-0.4	0.25	0.041	0.041	0.039	0.039	0.039	0.039	0.039
0.4	0.0	0.25	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.4	0.4	0.25	0.041	0.044	0.039	0.039	0.039	0.039	0.039
0.4	0.8	0.25	0.068	0.076	0.040	0.041	0.041	0.041	0.041
0.4	0.9	0.25	0.106	0.091	0.043	0.043	0.042	0.043	0.043

Table 2.7 (cont). Root mean square error of the estimators of *B2*, *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.054	0.055	0.056	0.055	0.055	0.055	0.055
0.8	-0.8	0.50	0.055	0.055	0.056	0.055	0.055	0.055	0.055
0.8	-0.4	0.50	0.055	0.056	0.055	0.055	0.055	0.055	0.055
0.8	0.0	0.50	0.056	0.056	0.056	0.056	0.056	0.056	0.056
0.8	0.4	0.50	0.055	0.055	0.055	0.055	0.055	0.055	0.055
0.8	0.8	0.50	0.054	0.054	0.053	0.054	0.054	0.054	0.054
0.8	0.9	0.50	0.054	0.055	0.054	0.058	0.060	0.060	0.059
0.9	-0.9	0.50	0.038	0.039	0.039	0.038	0.038	0.038	0.038
0.9	-0.8	0.50	0.038	0.039	0.039	0.038	0.039	0.038	0.038
0.9	-0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.9	0	0.50	0.040	0.040	0.040	0.040	0.039	0.039	0.039
0.9	0.4	0.50	0.039	0.039	0.039	0.039	0.039	0.039	0.039
0.9	0.8	0.50	0.038	0.038	0.038	0.038	0.038	0.038	0.038
0.9	0.9	0.50	0.038	0.038	0.038	0.039	0.039	0.039	0.039

Column Average	0.042	0.043	0.043	0.043	0.043	0.043	0.043
Col.Av.w/o |λ|,|ρ|=(	0.043	0.044	0.044	0.044	0.044	0.044	0.044
Col. Av. w/o ρ=0.9	0.042	0.042	0.042	0.042	0.042	0.042	0.042

Table 2.7 (cont). Root mean square error of the estimators of $B2$, *N=400*

λ	ρ	σ2	**TSLS**	**OLS**	**IF**	**ILEE**	**ISER1**	**ISER2**	**ISER3**
0.8	-0.9	0.50	0.066	0.066	0.055	0.055	0.055	0.055	0.055
0.8	-0.8	0.50	0.064	0.064	0.056	0.055	0.055	0.055	0.055
0.8	-0.4	0.50	0.058	0.058	0.055	0.055	0.055	0.055	0.055
0.8	0.0	0.50	0.056	0.059	0.056	0.056	0.056	0.056	0.056
0.8	0.4	0.50	0.057	0.074	0.055	0.055	0.055	0.055	0.055
0.8	0.8	0.50	0.089	0.127	0.054	0.055	0.054	0.055	0.054
0.8	0.9	0.50	0.122	0.140	0.054	0.058	0.057	0.058	0.058
0.9	-0.9	0.50	0.047	0.046	0.039	0.038	0.038	0.038	0.038
0.9	-0.8	0.50	0.045	0.045	0.039	0.038	0.039	0.038	0.038
0.9	-0.4	0.50	0.041	0.041	0.039	0.039	0.039	0.039	0.039
0.9	0	0.50	0.040	0.041	0.040	0.040	0.039	0.039	0.039
0.9	0.4	0.50	0.041	0.048	0.039	0.039	0.039	0.039	0.039
0.9	0.8	0.50	0.065	0.104	0.038	0.038	0.038	0.038	0.038
0.9	0.9	0.50	0.098	0.131	0.038	0.039	0.038	0.039	0.039
Column Average			0.061	0.060	0.043	0.043	0.043	0.043	0.043
Col.Av.w/o |λ|,|ρ|=0.9			0.052	0.053	0.044	0.044	0.044	0.044	0.044
Col. Av. w/o ρ=0.9			0.051	0.053	0.042	0.042	0.042	0.042	0.042

Table 2.8. Root mean square error of the estimators of ρ, *N=100*

λ	ρ	σ2	**ML**	**TSLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.227	0.247	0.254	0.254	0.263	0.266	0.254
-0.9	-0.8	0.50	0.229	0.241	0.254	0.254	0.260	0.263	0.253
-0.9	-0.4	0.50	0.230	0.225	0.240	0.238	0.240	0.242	0.238
-0.9	0	0.50	0.202	0.197	0.206	0.205	0.207	0.207	0.206
-0.9	0.4	0.50	0.151	0.157	0.153	0.152	0.153	0.153	0.154
-0.9	0.8	0.50	0.068	0.122	0.082	0.080	0.082	0.082	0.081
-0.9	0.9	0.50	0.039	0.144	0.062	0.061	0.061	0.062	0.060
-0.8	-0.9	0.25	0.215	0.233	0.242	0.240	0.242	0.242	0.241
-0.8	-0.8	0.25	0.219	0.231	0.241	0.241	0.241	0.243	0.241
-0.8	-0.4	0.25	0.218	0.215	0.230	0.228	0.228	0.229	0.228
-0.8	0.0	0.25	0.193	0.189	0.198	0.197	0.197	0.196	0.197
-0.8	0.4	0.25	0.144	0.149	0.147	0.147	0.145	0.146	0.147
-0.8	0.8	0.25	0.064	0.101	0.074	0.073	0.073	0.073	0.074
-0.8	0.9	0.25	0.038	0.104	0.050	0.050	0.050	0.050	0.049
-0.4	-0.9	0.50	0.220	0.241	0.250	0.250	0.250	0.250	0.250
-0.4	-0.8	0.50	0.223	0.239	0.250	0.250	0.250	0.250	0.250
-0.4	-0.4	0.50	0.228	0.226	0.237	0.237	0.237	0.237	0.237
-0.4	0.0	0.50	0.203	0.197	0.204	0.204	0.204	0.204	0.204
-0.4	0.4	0.50	0.155	0.162	0.157	0.158	0.158	0.158	0.158
-0.4	0.8	0.50	0.074	0.129	0.093	0.090	0.090	0.090	0.090
-0.4	0.9	0.50	0.044	0.161	0.080	0.074	0.073	0.073	0.074
0.0	-0.9	1.00	0.226	0.251	0.260	0.260	0.260	0.260	0.260
0.0	-0.8	1.00	0.232	0.249	0.261	0.260	0.260	0.260	0.260
0.0	-0.4	1.00	0.236	0.237	0.249	0.248	0.248	0.248	0.248
0.0	0.0	1.00	0.217	0.213	0.221	0.219	0.219	0.219	0.219
0.0	0.4	1.00	0.175	0.185	0.180	0.174	0.174	0.174	0.174
0.0	0.8	1.00	0.098	0.182	0.141	0.132	0.132	0.132	0.131
0.0	0.9	1.00	0.056	0.268	0.186	0.136	0.121	0.122	0.125
0.4	-0.9	0.25	0.207	0.226	0.238	0.238	0.237	0.238	0.238
0.4	-0.8	0.25	0.211	0.225	0.239	0.239	0.239	0.239	0.239
0.4	-0.4	0.25	0.213	0.216	0.228	0.229	0.229	0.229	0.229
0.4	0.0	0.25	0.190	0.191	0.197	0.197	0.197	0.197	0.197
0.4	0.4	0.25	0.147	0.148	0.149	0.150	0.150	0.150	0.150
0.4	0.8	0.25	0.082	0.103	0.086	0.085	0.085	0.085	0.085
0.4	0.9	0.25	0.052	0.110	0.072	0.071	0.070	0.071	0.071

Table 2.8. (cont.) Root mean square error of the estimators of ρ, *N=100*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.207	0.227	0.240	0.241	0.240	0.241	0.241
0.8	-0.8	0.50	0.211	0.228	0.240	0.242	0.240	0.240	0.241
0.8	-0.4	0.50	0.215	0.217	0.230	0.230	0.230	0.230	0.230
0.8	0.0	0.50	0.192	0.194	0.199	0.200	0.198	0.199	0.200
0.8	0.4	0.50	0.150	0.151	0.152	0.152	0.153	0.153	0.153
0.8	0.8	0.50	0.099	0.112	0.096	0.097	0.100	0.099	0.098
0.8	0.9	0.50	0.078	0.130	0.112	0.101	0.106	0.104	0.103
0.9	-0.9	0.50	0.206	0.226	0.238	0.241	0.238	0.238	0.240
0.9	-0.8	0.50	0.211	0.226	0.238	0.241	0.238	0.239	0.240
0.9	-0.4	0.50	0.212	0.215	0.227	0.228	0.226	0.227	0.227
0.9	0	0.50	0.191	0.191	0.196	0.196	0.196	0.197	0.196
0.9	0.4	0.50	0.146	0.147	0.146	0.148	0.147	0.147	0.148
0.9	0.8	0.50	0.094	0.101	0.088	0.091	0.093	0.091	0.091
0.9	0.9	0.50	0.074	0.109	0.096	0.090	0.099	0.096	0.094

Column Average	0.163	0.187	0.182	0.180	0.180	0.180	0.180
Col.Av.w/o |λ|,|ρ|=0.9	0.176	0.188	0.188	0.187	0.187	0.187	0.187
Col. Av. w/o ρ=0.9	0.182	0.194	0.197	0.196	0.197	0.197	0.196

Table 2.9. Root mean square error of the estimators of ρ , *N=400*

λ	ρ	σ2	**ML**	**TSLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	0.114	0.123	0.124	0.124	0.124	0.124	0.125
-0.9	-0.8	0.50	0.115	0.123	0.124	0.124	0.124	0.124	0.125
-0.9	-0.4	0.50	0.114	0.115	0.118	0.117	0.117	0.117	0.118
-0.9	0	0.50	0.101	0.101	0.103	0.102	0.103	0.103	0.102
-0.9	0.4	0.50	0.075	0.080	0.077	0.077	0.078	0.078	0.076
-0.9	0.8	0.50	0.034	0.064	0.041	0.040	0.040	0.040	0.043
-0.9	0.9	0.50	0.020	0.075	0.027	0.026	0.026	0.026	0.029
-0.8	-0.9	0.50	0.113	0.123	0.123	0.124	0.124	0.124	0.124
-0.8	-0.8	0.50	0.115	0.123	0.123	0.123	0.123	0.123	0.124
-0.8	-0.4	0.50	0.114	0.115	0.117	0.117	0.116	0.117	0.117
-0.8	0.0	0.50	0.101	0.101	0.103	0.103	0.103	0.103	0.103
-0.8	0.4	0.50	0.076	0.080	0.077	0.077	0.078	0.078	0.077
-0.8	0.8	0.50	0.035	0.065	0.042	0.041	0.041	0.041	0.042
-0.8	0.9	0.50	0.020	0.076	0.028	0.028	0.027	0.027	0.028
-0.4	-0.9	1.00	0.119	0.133	0.134	0.133	0.133	0.133	0.133
-0.4	-0.8	1.00	0.122	0.132	0.133	0.133	0.132	0.133	0.133
-0.4	-0.4	1.00	0.122	0.124	0.128	0.127	0.127	0.127	0.127
-0.4	0.0	1.00	0.111	0.112	0.113	0.112	0.112	0.112	0.112
-0.4	0.4	1.00	0.086	0.094	0.090	0.089	0.089	0.089	0.089
-0.4	0.8	1.00	0.041	0.093	0.060	0.058	0.058	0.058	0.058
-0.4	0.9	1.00	0.022	0.125	0.053	0.046	0.046	0.046	0.046
0.0	-0.9	0.25	0.103	0.113	0.115	0.115	0.115	0.115	0.115
0.0	-0.8	0.25	0.104	0.113	0.116	0.115	0.115	0.115	0.115
0.0	-0.4	0.25	0.104	0.108	0.110	0.110	0.110	0.110	0.110
0.0	0.0	0.25	0.093	0.095	0.096	0.096	0.096	0.096	0.096
0.0	0.4	0.25	0.073	0.073	0.073	0.073	0.073	0.073	0.073
0.0	0.8	0.25	0.036	0.054	0.041	0.040	0.040	0.040	0.040
0.0	0.9	0.25	0.022	0.060	0.030	0.029	0.029	0.029	0.029
0.4	-0.9	0.50	0.104	0.116	0.118	0.117	0.117	0.117	0.117
0.4	-0.8	0.50	0.106	0.116	0.118	0.117	0.117	0.117	0.117
0.4	-0.4	0.50	0.106	0.111	0.112	0.112	0.112	0.112	0.112
0.4	0.0	0.50	0.097	0.098	0.099	0.099	0.099	0.099	0.099
0.4	0.4	0.50	0.078	0.079	0.079	0.079	0.079	0.079	0.079
0.4	0.8	0.50	0.047	0.068	0.055	0.053	0.053	0.053	0.053
0.4	0.9	0.50	0.029	0.081	0.053	0.047	0.046	0.047	0.047

Table 2.9. (cont.) Root mean square error of the estimators of ρ, *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	0.102	0.118	0.120	0.118	0.119	0.118	0.118
0.8	-0.8	0.50	0.105	0.117	0.119	0.118	0.119	0.118	0.118
0.8	-0.4	0.50	0.107	0.111	0.114	0.113	0.113	0.113	0.113
0.8	0.0	0.50	0.099	0.101	0.102	0.102	0.102	0.102	0.102
0.8	0.4	0.50	0.082	0.084	0.083	0.083	0.083	0.083	0.084
0.8	0.8	0.50	0.066	0.078	0.069	0.065	0.067	0.065	0.065
0.8	0.9	0.50	0.053	0.104	0.093	0.074	0.075	0.074	0.073
0.9	-0.9	0.50	0.099	0.111	0.113	0.113	0.113	0.113	0.113
0.9	-0.8	0.50	0.101	0.111	0.113	0.113	0.113	0.113	0.113
0.9	-0.4	0.50	0.101	0.107	0.108	0.108	0.108	0.108	0.108
0.9	0	0.50	0.090	0.093	0.094	0.093	0.093	0.093	0.093
0.9	0.4	0.50	0.071	0.073	0.072	0.072	0.072	0.072	0.072
0.9	0.8	0.50	0.049	0.052	0.048	0.048	0.049	0.048	0.048
0.9	0.9	0.50	0.044	0.059	0.052	0.047	0.051	0.049	0.048
Column Average			0.082	0.098	0.091	0.090	0.090	0.090	0.090
Col.Av.w/o |λ|,|ρ|=0.9			0.089	0.098	0.095	0.094	0.094	0.094	0.094
Col. Av. w/o ρ=0.9			0.091	0.100	0.098	0.097	0.098	0.098	0.098

Table 2.10. Bias of the estimators of ρ , *N=100*

λ	ρ	σ2	**ML**	**TSLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
-0.9	-0.9	0.50	-0.022	0.022	-0.043	-0.050	-0.028	-0.029	-0.051
-0.9	-0.8	0.50	-0.021	0.013	-0.043	-0.048	-0.028	-0.026	-0.049
-0.9	-0.4	0.50	-0.023	-0.012	-0.037	-0.038	-0.025	-0.024	-0.040
-0.9	0	0.50	-0.019	-0.024	-0.025	-0.025	-0.017	-0.017	-0.025
-0.9	0.4	0.50	-0.011	-0.031	-0.016	-0.013	-0.012	-0.012	-0.009
-0.9	0.8	0.50	-0.005	-0.045	-0.015	-0.006	-0.007	-0.008	0.002
-0.9	0.9	0.50	-0.004	-0.081	-0.019	-0.005	-0.003	-0.003	0.001
-0.8	-0.9	0.50	-0.024	0.016	-0.045	-0.051	-0.046	-0.047	-0.052
-0.8	-0.8	0.50	-0.024	0.008	-0.045	-0.050	-0.044	-0.045	-0.051
-0.8	-0.4	0.50	-0.022	-0.014	-0.038	-0.040	-0.034	-0.035	-0.040
-0.8	0.0	0.50	-0.019	-0.024	-0.026	-0.025	-0.020	-0.022	-0.025
-0.8	0.4	0.50	-0.010	-0.029	-0.015	-0.011	-0.009	-0.009	-0.011
-0.8	0.8	0.50	-0.004	-0.032	-0.009	-0.003	-0.003	-0.003	0.000
-0.8	0.9	0.50	-0.003	-0.049	-0.009	-0.002	-0.001	-0.001	0.000
-0.4	-0.9	1.00	-0.025	0.006	-0.050	-0.054	-0.054	-0.054	-0.054
-0.4	-0.8	1.00	-0.025	0.000	-0.049	-0.053	-0.053	-0.053	-0.053
-0.4	-0.4	1.00	-0.025	-0.021	-0.042	-0.043	-0.043	-0.043	-0.043
-0.4	0.0	1.00	-0.022	-0.029	-0.031	-0.028	-0.027	-0.027	-0.027
-0.4	0.4	1.00	-0.015	-0.035	-0.022	-0.016	-0.015	-0.015	-0.015
-0.4	0.8	1.00	-0.007	-0.049	-0.024	-0.010	-0.009	-0.008	-0.007
-0.4	0.9	1.00	-0.005	-0.096	-0.033	-0.010	-0.007	-0.004	-0.003
0.0	-0.9	0.25	-0.025	-0.003	-0.055	-0.057	-0.057	-0.057	-0.057
0.0	-0.8	0.25	-0.026	-0.011	-0.055	-0.055	-0.055	-0.055	-0.055
0.0	-0.4	0.25	-0.026	-0.030	-0.049	-0.046	-0.046	-0.046	-0.046
0.0	0.0	0.25	-0.023	-0.039	-0.041	-0.032	-0.032	-0.032	-0.032
0.0	0.4	0.25	-0.020	-0.047	-0.035	-0.021	-0.020	-0.020	-0.020
0.0	0.8	0.25	-0.013	-0.095	-0.066	-0.017	-0.024	-0.016	-0.019
0.0	0.9	0.25	-0.009	-0.189	-0.111	-0.030	-0.023	-0.011	-0.014
0.4	-0.9	0.50	-0.027	-0.015	-0.056	-0.058	-0.058	-0.058	-0.058
0.4	-0.8	0.50	-0.027	-0.020	-0.055	-0.057	-0.057	-0.058	-0.058
0.4	-0.4	0.50	-0.029	-0.031	-0.050	-0.050	-0.049	-0.049	-0.050
0.4	0.0	0.50	-0.025	-0.037	-0.038	-0.036	-0.036	-0.036	-0.036
0.4	0.4	0.50	-0.021	-0.033	-0.024	-0.021	-0.020	-0.020	-0.020
0.4	0.8	0.50	-0.010	-0.032	-0.023	-0.010	-0.009	-0.009	-0.009
0.4	0.9	0.50	-0.007	-0.057	-0.031	-0.011	-0.009	-0.007	-0.007

Table 2.10. (cont.) Bias of the estimators of ρ , *N=100*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	-0.031	-0.017	-0.058	-0.065	-0.062	-0.063	-0.065
0.8	-0.8	0.50	-0.030	-0.022	-0.056	-0.063	-0.060	-0.061	-0.063
0.8	-0.4	0.50	-0.032	-0.035	-0.050	-0.052	-0.050	-0.051	-0.052
0.8	0.0	0.50	-0.027	-0.039	-0.040	-0.038	-0.037	-0.038	-0.038
0.8	0.4	0.50	-0.020	-0.036	-0.028	-0.021	-0.021	-0.021	-0.021
0.8	0.8	0.50	-0.012	-0.041	-0.038	-0.009	-0.018	-0.007	-0.011
0.8	0.9	0.50	-0.011	-0.083	-0.075	-0.017	-0.027	-0.005	-0.018
0.9	-0.9	0.50	-0.033	-0.015	-0.055	-0.067	-0.060	-0.062	-0.064
0.9	-0.8	0.50	-0.033	-0.019	-0.054	-0.066	-0.060	-0.061	-0.063
0.9	-0.4	0.50	-0.034	-0.032	-0.049	-0.056	-0.050	-0.051	-0.054
0.9	0	0.50	-0.028	-0.037	-0.038	-0.039	-0.036	-0.037	-0.039
0.9	0.4	0.50	-0.021	-0.033	-0.026	-0.023	-0.021	-0.022	-0.022
0.9	0.8	0.50	-0.007	-0.035	-0.029	-0.006	-0.016	-0.005	-0.008
0.9	0.9	0.50	-0.007	-0.067	-0.062	-0.019	-0.029	-0.006	-0.018

Column Average	-0.019	-0.034	-0.040	-0.033	-0.031	-0.030	-0.032
Col.Av.w/o |λ|,|ρ|=0.9	-0.021	-0.031	-0.038	-0.032	-0.032	-0.031	-0.032
Col. Av. w/o ρ=0.9	-0.022	-0.025	-0.039	-0.036	-0.034	-0.034	-0.036

Table 2.11. Bias of the estimators of ρ , *N=400*

λ	ρ	σ2	ML	TSLS	FGS2SLS	LEE	SER1	SER2	SER3
-0.9	-0.9	0.50	-0.007	0.008	-0.013	-0.012	-0.014	-0.014	-0.010
-0.9	-0.8	0.50	-0.007	0.005	-0.012	-0.012	-0.014	-0.014	-0.010
-0.9	-0.4	0.50	-0.007	-0.003	-0.011	-0.010	-0.011	-0.011	-0.009
-0.9	0	0.50	-0.006	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
-0.9	0.4	0.50	-0.004	-0.011	-0.007	-0.006	-0.006	-0.005	-0.008
-0.9	0.8	0.50	-0.002	-0.018	-0.005	-0.005	-0.004	-0.003	-0.009
-0.9	0.9	0.50	-0.001	-0.032	-0.005	-0.005	-0.004	-0.004	-0.007
-0.8	-0.9	0.50	-0.006	0.007	-0.013	-0.012	-0.014	-0.014	-0.012
-0.8	-0.8	0.50	-0.007	0.004	-0.013	-0.012	-0.014	-0.013	-0.011
-0.8	-0.4	0.50	-0.007	-0.003	-0.011	-0.010	-0.011	-0.010	-0.010
-0.8	0.0	0.50	-0.006	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
-0.8	0.4	0.50	-0.004	-0.011	-0.007	-0.006	-0.006	-0.006	-0.006
-0.8	0.8	0.50	-0.002	-0.018	-0.006	-0.006	-0.005	-0.005	-0.008
-0.8	0.9	0.50	-0.001	-0.033	-0.006	-0.005	-0.005	-0.005	-0.007
-0.4	-0.9	1.00	-0.008	0.006	-0.015	-0.013	-0.013	-0.013	-0.013
-0.4	-0.8	1.00	-0.007	0.003	-0.014	-0.012	-0.012	-0.012	-0.012
-0.4	-0.4	1.00	-0.007	-0.005	-0.013	-0.011	-0.011	-0.011	-0.011
-0.4	0.0	1.00	-0.007	-0.012	-0.012	-0.010	-0.010	-0.010	-0.010
-0.4	0.4	1.00	-0.006	-0.016	-0.012	-0.010	-0.010	-0.010	-0.010
-0.4	0.8	1.00	-0.002	-0.033	-0.019	-0.014	-0.014	-0.014	-0.014
-0.4	0.9	1.00	-0.002	-0.072	-0.023	-0.012	-0.012	-0.012	-0.012
0.0	-0.9	0.25	-0.007	0.002	-0.016	-0.015	-0.015	-0.015	-0.015
0.0	-0.8	0.25	-0.007	-0.001	-0.015	-0.015	-0.015	-0.015	-0.015
0.0	-0.4	0.25	-0.008	-0.007	-0.013	-0.012	-0.012	-0.012	-0.012
0.0	0.0	0.25	-0.008	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009
0.0	0.4	0.25	-0.007	-0.010	-0.007	-0.006	-0.006	-0.006	-0.006
0.0	0.8	0.25	-0.003	-0.014	-0.007	-0.006	-0.006	-0.006	-0.006
0.0	0.9	0.25	-0.002	-0.024	-0.008	-0.006	-0.006	-0.006	-0.006
0.4	-0.9	0.50	-0.008	-0.002	-0.018	-0.016	-0.016	-0.016	-0.016
0.4	-0.8	0.50	-0.008	-0.004	-0.017	-0.015	-0.015	-0.015	-0.015
0.4	-0.4	0.50	-0.008	-0.009	-0.015	-0.013	-0.014	-0.013	-0.013
0.4	0.0	0.50	-0.007	-0.012	-0.012	-0.010	-0.010	-0.010	-0.010
0.4	0.4	0.50	-0.007	-0.013	-0.010	-0.008	-0.008	-0.008	-0.008
0.4	0.8	0.50	-0.005	-0.020	-0.018	-0.012	-0.012	-0.012	-0.012
0.4	0.9	0.50	-0.002	-0.042	-0.024	-0.013	-0.013	-0.013	-0.013

Table 2.11. (cont.) Bias of the estimators of ρ , *N=400*

λ	ρ	σ2	**ML**	**GS2SLS**	**FGS2SLS**	**LEE**	**SER1**	**SER2**	**SER3**
0.8	-0.9	0.50	-0.007	-0.004	-0.018	-0.017	-0.017	-0.017	-0.017
0.8	-0.8	0.50	-0.007	-0.006	-0.018	-0.017	-0.016	-0.017	-0.017
0.8	-0.4	0.50	-0.008	-0.012	-0.016	-0.013	-0.014	-0.014	-0.013
0.8	0.0	0.50	-0.007	-0.013	-0.013	-0.010	-0.010	-0.010	-0.010
0.8	0.4	0.50	-0.005	-0.015	-0.011	-0.007	-0.006	-0.007	-0.007
0.8	0.8	0.50	-0.004	-0.030	-0.030	-0.013	-0.014	-0.013	-0.013
0.8	0.9	0.50	-0.005	-0.067	-0.062	-0.024	-0.017	-0.018	-0.019
0.9	-0.9	0.50	-0.007	-0.003	-0.016	-0.016	-0.016	-0.016	-0.016
0.9	-0.8	0.50	-0.008	-0.005	-0.016	-0.015	-0.015	-0.016	-0.016
0.9	-0.4	0.50	-0.007	-0.009	-0.014	-0.013	-0.013	-0.013	-0.013
0.9	0	0.50	-0.006	-0.011	-0.011	-0.009	-0.009	-0.009	-0.009
0.9	0.4	0.50	-0.005	-0.010	-0.008	-0.006	-0.006	-0.005	-0.006
0.9	0.8	0.50	-0.002	-0.015	-0.011	-0.004	-0.006	-0.005	-0.004
0.9	0.9	0.50	-0.003	-0.032	-0.028	-0.012	-0.012	-0.011	-0.011

Column Average	-0.006	-0.013	-0.014	-0.011	-0.011	-0.011	-0.011
Col.Av.w/o |λ|,|ρ|=0.9	-0.006	-0.011	-0.013	-0.011	-0.011	-0.011	-0.011
Col. Av. w/o ρ=0.9	-0.006	-0.008	-0.013	-0.011	-0.011	-0.011	-0.011

It follows from the tables relating to the parameters λ, β_1, and β_2 that typically RMSEs of the OLS estimator are the largest while those of the ML estimator are the lowest. This relates to the theoretical notion of inconsistency of the OLS estimator and consistency and efficiency of the ML estimator (assuming standard ML theory applies for the considered model). The relatively low RMSE values of the ML estimator are also likely to be due to the normality assumption on the disturbances. The RMSEs of the 2SLS estimator are typically lower than those of the OLS, but typically larger than the other estimators under consideration. This result is due to the fact that the 2SLS estimator is consistent but inefficient because, in contrast to the other estimators (except the OLS), it does not take into account the spatial structure of the error term.

The next observation relates to the comparison of the FGS2SLS and Lee estimators. The theory indicates that asymptotically the Lee estimator is more efficient than the FGS2SLS. Our results show that in finite samples, namely of sizes 100 and 400, the difference between RMSEs of these estimators averages to just 2% for the parameter λ. The RMSEs of the Lee and FGS2SLS estimators of β_1 and β_2 are, on average, virtually the same. Generally, our results suggest that in finite samples the Lee estimator is somewhat more efficient than the FGS2SLS, however, its efficiency gains are practically negligible. This finding is important in light of the computational and programming simplicity of FGS2SLS estimator relative to the Lee estimator.[13]

The results also indicate that the difference between the RMSEs of these

[13] As shown later in this section the Lee and series estimators are very close in terms of the efficiency in finite samples. Therefore, the results on comparison of the FGS2SLS and the Lee estimators apply to the comparison of the FGS2SLS and the series estimators.

estimators is not uniform over the parameter space. Although the RMSEs of the Lee estimators of λ, β_1 and β_2 are generally the same or somewhat lower than those of the FGS2SLS estimators, the FGS2SLS estimator generally dominates the Lee estimator when λ and ρ have high and positive values. This observation is probably due to the fact that the Lee estimator involves the inversion of the matrix $(I_n - \widehat{\lambda}_n^{2SLS} W_n)$ which under certain circumstances may be close to being singular. In particular, when the true value of λ is large, the estimated value of λ could be close to 1 which is a singular point, or even exceed it. This could cause problems for the Lee estimator. On the other hand, the FGS2SLS estimator only relates to the 2SLS estimator indirectly through the disturbance estimates and, therefore, does not possess this vulnerability.

Comparing the Lee and Series estimators one can readily see that their performance is virtually the same. On average, the difference between the RMSEs of the Lee and the three series estimators do not exceed 1% for the parameter λ, and 0.5% for the parameters β_1 and β_2. The performance of these estimators is similar not only in terms of averages but also over the whole parameter space. More specifically, the difference between the RMSEs of the Lee and Series3 (based on $\alpha = 0.45$) estimators typically does not exceed 5% in any of the experimental sets of parameter values when the sample size is 100 and 3% when the sample size is 400. The fact that the differences between RMSEs of the two estimators become smaller as the sample size increases is consistent with the equivalence of asymptotic distributions of the Lee and series estimators. These findings imply that one could use the computationally simpler series estimator without much loss of efficiency.

Another result emerges from the comparison of the series estimators based on different values of α, namely Series1, Series2, and Series3 estimators. Interestingly, there seems to be no indication that a series estimator with a higher value of α, which corresponds to a better approximation of the optimal instrument, dominates a series estimator with a lower value of α. More specifically, the Series3 estimator does not dominate the Series2 or Series1 estimators. Furthermore, the Series2 estimator does not outperform the Series1 estimator. Therefore, one may conjecture that in moderate to reasonably large samples a series estimator based on $\alpha = 0.25$ provides a sufficient approximation of the optimal instrument.

It is also interesting to compare the RMSEs of the ML estimator to the FGS2SLS, Lee, and series estimators, which are spatial instrumental variable (IV) estimators. Consider first the set of the experiments that do not contain $\rho = 0.9$. Over these experiments the gain in efficiency of the ML estimator relative to the spatial IV estimators averages to just 6-7% for the parameter λ. For the parameters β_1 and β_2 the ML and the spatial IV estimators are roughly the same in terms of the averages over this set of experiments. Therefore, we can say that if the value of ρ is not close to 1, the loss of efficiency of the spatial IV estimators relative to the ML estimator is generally small or nonexistent.[14]

If all the experiments are considered the difference between RMSE averages of the ML and the spatial IV estimators rises up to 16-18% for λ and between 2-4% for β_1 and β_2. The reason for such disparity is that for certain combinations of the parameters λ and ρ, namely those involving negative λ and high and positive ρ, the RMSEs of spatial IV estimators are considerably higher than those of the

[14]For the purpose of the comparison to Das, Kelejian and Prucha (2003) we also report the averages over the experiments not involving values 0.9 and -0.9 of λ and ρ. These averages are almost identical to the averages over the experiments not involving $\rho = 0.9$.

ML. This may be due to the fact that such combinations of the parameter values are associated with high RMSEs of the 2SLS estimator whose residuals are used in the FGS2SLS, Series, and Lee procedures for estimation of ρ. Therefore, it is reasonable to believe that iterating on the spatial IV estimators would improve their performance (see the discussion in Section 2.5.3). In fact, for the parameter λ the average difference between RMSEs of the ML and the iterated FGS2SLS estimators goes down to 14%, and between the ML and the iterated Lee and series estimators to 11-12%. For the parameters β_1 and β_2 these differences go down to just 2-3%. These results suggest that the advantage of the ML over the spatial IV estimators is still relatively small given that the experiments are conducted under the most favorable conditions for the ML procedure involving normally distributed vectors of disturbances. This finding is important in light of the computational simplicity of the spatial IV estimators considered in this study relative to the ML estimator which is often not feasible because of severe computational problems.

As a general observation we note that iterating on the spatial IV procedures typically does not reduce the efficiency of the estimators, but it substantially improves that efficiency in cases involving negative λ and high and positive ρ. Therefore, in practice, it would be advisable to use the iterated version of the FGS2SLS, Lee, and Series estimators.

The average difference between RMSEs of GS2SLS and FGS2SLS estimators of λ is 13% for $n = 100$ and 8% for $n = 400$. This difference decreases to 5% and 2%, respectively, if only the experiments which do not involve $\rho = 0.9$ are considered. Furthermore, the RMSEs of the iterated FGS2SLS of λ are, on average, higher than those of the GS2SLS by 8% for the sample size 100 and by

3% for the sample size 400. For the parameters β_1 and β_2 the average RMSEs of FGS2SLS and iterated FGS2SLS estimators are, on average, higher than those of the ML estimator by at most 2% if the sample size is 100, and by 1% if the sample size is 400. These results suggest that the loss of finite sample efficiency due to the use of GM estimator of ρ is small in moderate to large samples.

Tables 2.8-2.9 relate to the estimators of ρ. Generally the ML estimator of this parameter is better than the others, while the performance of the GM estimators based on the residuals of the FGS2SLS, Lee, and series procedures are very similar throughout the parameter space. The efficiency of the GM estimator of ρ based on the 2SLS residuals is similar to the other GM estimators if the experiments not involving $\rho = 0.9$ are considered. Over these experiments the GM estimators are on average roughly 8% worse than the ML estimator. If all the experiments are considered the average difference with the ML estimator is roughly 16% for the 2SLS estimator, and 10% for the others. The bias of the estimators of ρ is reported in Tables 2.10-2.11. If one compares it to the root mean squared error of these estimators it is readily seen that the bias is very small relative to RMSE, and, therefore, its contribution to RMSE is minimal and becomes smaller as the sample size increases. In most cases, however, the value of the bias is negative, and in absolute terms it is typically the smallest in case of the ML estimator.

We also note that the values of RMSEs of almost all the considered estimators in the tables corresponding to λ, β_1 and β_2 generally decrease as the sample size increases. An exception to this is the OLS estimator which is not consistent and whose RMSEs, as a result, often increase with the sample size. These findings are in accordance with the asymptotic properties of these estimators.

Finally we note that the relative performance of the estimators in sample size

100 are roughly the same as in sample size 400. As an illustration, over all the experiments considered the ratio of the RMSE of the ML estimator of λ relative to that of the FGS2SLS estimator is, roughly 0.85 for the samples of size 100 as well as for the samples of size 400. The corresponding ratio for the ML and FGS2SLS estimators of β_1 and β_2 are 0.975 in samples of size 100 and in samples of size 400.

2.9 Conclusion

This paper conducted a Monte Carlo study aimed to explore the finite sample properties relating to the ML, FGS2SLS, Lee and the series estimators of a linear spatial model with lagged dependent variable and autocorrelated disturbances.

The findings indicate that, on average, the advantage of the ML estimator over the spatial IV estimators is limited. It seems most beneficial to use the ML estimator when λ is negative and ρ is large and positive. In the other cases the difference between the ML estimator and the spatial IV estimators is small or nonexistent. This is important since the ML estimator is computationally impossible to implement in large samples, while for the spatial IV estimators this consideration is not an issue. We also considered iterated versions of the spatial IV estimators and found them to be rarely less efficient than the corresponding non-iterated IV estimators, and considerably more efficient in the cases when λ is negative and ρ is large and positive. Therefore, in practice it is advisable to use iterated versions of the FGS2SLS, Lee, and the series estimators.

Of the spatial IV estimators the Lee estimator while asymptotically efficient in the class of IV estimators is computationally burdensome relative to the other spatial IV estimators. We found that the computationally and programmingly simpler Kelejian-Prucha series estimator, which is asymptotically equivalent to the Lee estimator, has virtually the same finite sample properties as the Lee estimator. We also found that the efficiency of the series estimator does not seem to relate for values of α considered, namely 0.25, 0.35, and 0.45. This is somewhat contrary to the intuition because large values of α lead to a better approximation of the optimal instrument.

Furthermore, the results indicate that the loss of efficiency of the FGS2SLS estimator relative to the Lee estimator is minimal. Thus, the model can be estimated at even smaller computational cost, and practically without loss of efficiency.

We have also explored the finite sample efficiency of the ML and GM estimators relating to the parameter ρ. Again, the ML estimator is usually superior to the others when λ is negative and ρ is large and positive. Furthermore, in these cases the GM estimators based on the residuals of FGS2SLS, Lee, or series procedures are more efficient than the GM estimator based on the 2SLS residuals. For all other values of λ and ρ the considered GM estimators of ρ are virtually the same, and the their finite sample properties are similar to those of the ML estimator.

For future research it would be interesting to design rules determining an optimal number of instruments for the FGS2SLS procedure and an optimal expansion for the series estimator. To the best of our knowledge the existing literature does provide an answer to this issue. One of the ways it can be addressed is by conducting a Monte Carlo study.

Chapter 3

Estimating Contagion: A Spatial Approach

3.1 Introduction

The devastating consequences of the financial crises of the last decade left economists and policymakers wondering how a crisis starting in one country could travel within and beyond its original neighborhood to other countries, leaving behind inflated exchange rates, ballooned interest rates and economic stagnation. This phenomena, which was a common feature of the major recent crises, is referred by economists as "contagion".

Understanding channels of contagion is of great importance for prescribing economic policies when dealing with a crisis or preventing it from spreading to other economies. For instance, if trade is the reason for contagion, a country would be advised to diversify its export base and/or trading partners. However, policy implications change when contagion is due to other factors such as financial linkages among countries, imperfections in the world capital markets, herd behavior etc. In these cases, one can make an argument for intervention by international financial organizations, or for a change in regulations of capital markets in the major financial centers.

While theoretical research put forth a number of models highlighting different channels that could explain the existence of contagion, the empirical literature has not reached a firm consensus regarding the propagation mechanisms of contagion.[1]

This paper tries to identify channels of contagion by looking at the patterns of co-movement of weekly stock market returns during the three recent crisis episodes in Asia, Russia and Brazil. We distinguish between four channels of contagion highlighted in the literature: bilateral trade, financial links through major banking centers (bank lending channel), similarity in risk, and neighborhood effects.

Our empirical analysis is based on a spatial model in which the dependent variable relates to stock market returns. The model contains four spatial lags in the dependent variable. Each spatial lag reflects a channel of contagion transmission which describes how the stock market returns of the countries involved are interrelated. We test for the significance of each channel of transmission.

Some previous papers used a similar methodology to assess contagion effects. The closest study to this one was conducted by Hernandez and Valdes (2001) who also considered weekly stock market returns and used weights to compare the relative importance of different contagion channels. However, there are considerable differences in the empirical and methodological frameworks in that study and ours. First, we use a consistent estimation procedure for this type of model, which is based on the generalized spatial two-stage least squares procedure suggested by Kelejian and Prucha (1998). Second, we consider a larger sample of

[1] See discussion in Dornbusch and Claessen (2000), Kaminsky and Reinhart (2000), Caramazza et. al. (2000), Van Rijckeghem and Weder (2001) among others.

countries which helps to alleviate possible sample selection biases which may be contained in earlier studies. Third, we use more comprehensive accounting for a *common shock* (or common factor), an issue largely ignored in empirical studies on contagion. And, finally, in our methodology the importance of each channel of contagion can be determined when they are considered simultaneously.

Section 3.2 provides a brief review of the literature, while Section 3.3 highlights some of the methodological issues. The empirical model is given in Section 3.4 and 3.5. Section 3.6 describes the data, and results are discussed in Section 3.7. Conclusions are given in Section 3.8.

3.2 Review of Related Literature

3.2.1 Definition of contagion

To date there is no consensus on the definition of contagion. Researchers use different definitions depending upon the objective of their study, see, e.g. the discussion in Masson (1998), Kaminsky and Reinhart (2000), and Forbes and Rigobon (2001). In this paper we use a definition based on what Calvo and Reinhart (1996) call a "fundamental-based contagion", namely, a contagion is a transmission of a crisis from one country to another through real and financial interdependence between them. In the context of this paper this transmission is manifested through the co-movement of stock market returns, which we try to explain by the existence of various types of links among countries.

3.2.2 Trade links

In the contagion literature the notion of real interdependence is often related to trade links. Countries may be connected through trade in two ways: first, by trading directly with each other, and, second, by competing with each other for exports to a third country. Clearly, both connections may contribute to the transmission of a crisis.

In the first case, a fall in aggregate demand in a crisis country would adversely affect imports from its trading partners. As a result, it would create pressures on exchange rates in economies that have a large share of exports going to the crisis country. These pressures could eventually result in a sizable devaluation of domestic currency and a full scale financial crisis. This argument was formalized by Gerlach and Smets (1994) with respect to EMU crisis in 1992.

In the second case, a currency devaluation in one country would reduce competitiveness of countries that export their goods to the same markets. The resulting competitive disadvantage would create incentives for competitors to devalue their currency, see Corsetti et al.(2000) for a formal treatment.

There are a number of empirical studies that find the trade channel to be a significant explanation for contagion. Eichengreen et al. (1996) analyzed contagion using data on 20 industrial economies from 1959 to 1993. They found that the probability of a crisis in a country increases in the presence of a crisis elsewhere, and that this increase is better explained by trade links among countries than by macroeconomic similarities. Glick and Rose (1999) used a much larger sample of countries and found that trade competition in third markets had high power in explaining contagion across countries in five major crisis episodes between 1971 and 1997. Forbes (2001) used disaggregated trade data which lead to a more accurate measure for export competition. She finds that both bilateral trade and competition in third markets are robust and significant determinants of co-movements in stock market returns in times of crisis.

3.2.3 Financial links

Similar to trade links, financial links can be of two types – direct and through a third party. The first type relates to direct financial interdependence which results from, among other things, cross border investments among countries. In the presence of these links, a crisis in, say, Thailand, could cause financial difficulties for, say, Malaysian companies that invest in Thailand. If the aggregate financial exposure was high, the consequences for the Malaysian economy may be quite severe.

The second type of financial link which received much more attention is called a "common lender" link. It arises from the fact that countries borrow money from large financial institutions that are concentrated in financial centers such as Europe, Japan and the US. Therefore, these countries are interconnected by the financial system. In this sense, Europe, Japan and the US can be viewed as three big creditors and, hence, are usually referred to as common lenders. Exposure to a common lender may propagate a crisis in the following way. A country experiencing a crisis would generate losses to a common lender, and, if those losses are large enough they may adversely affect its liquidity. Thus, the common lender may be forced to sell the securities of other countries, driving down their prices. In some studies, contagion during the Asian crisis was attributed to the fact that Japanese banks, that were already experiencing difficulties, suffered losses followed by the Thai devaluation, and, as a result, had to liquidate their portfolios in other countries in the region, leading to a crisis propagation. The study by Kaminsky and Reinhart (2000) was among the first that stressed the role of this channel of contagion.

It has also been shown that a presence of informational asymmetries may amplify the common lender effect. In Calvo (1999) the amplification mechanism works through the existence of large highly leveraged informed investors and uninformed investors. The informed investors experiencing a margin call have to sell their assets to restore liquidity. The uninformed investors, in turn, receive a mixed signal about the quality of the assets, and, as a result of signal extraction, follow informed investors creating fall in prices not warranted by fundamentals.

In a study by Kodres and Pritsker (2001) it was shown that the presence of informational asymmetries among investors can generate contagion across countries that do not even share macroeconomic risks. In their modeling the transmission mechanism involves a rebalancing of investors' portfolios.

There are other explanations for financial contagion which are based on imperfections and institutional arrangements in capital markets. Calvo and Mendoza (2000) built a model where the presence of fixed informational cost concerning a country's fundamentals, and increasing diversification opportunities in the global capital markets give rise to the role of rumors and escalate herd behavior.

Other observers attribute contagion to shifts in investor sentiment, such as increased risk-aversion. In this scenario a crisis serves as a "wake-up call" for investors, making them reassess risks involved in other countries. As a result, economies with similar fundamentals may suffer (see Goldstein (1998)).

Empirical studies of contagion which allow for financial links usually find them significant, and sometimes "overshadowing" the trade channel. Kaminsky and Reinhart (2000) found that the probability of a crisis increases significantly when a country shares a common lender with a crisis country, although, the authors caution that generally it is difficult to distinguish between the trade and a common lender channel since both of them have a regional pattern.

Van Rijckeghem and Weder (2001) employ different measures of financial links and find strong evidence of financial spill-overs through a common creditor. Hernandez and Valdes (2001) analyze co-movement of bond spreads and stock market returns across countries during the Asian, Russian and Brazilian crises. They find that financial links are important in explaining bond spreads, but these links seemed to effect stock prices only during the Russian crisis.

Still other studies found evidence of investment practices that create contagion. For instance, Kaminsky Lyons and Schmukler (1999) showed a presence of a momentum strategy in the behavior of mutual funds. This strategy creates co-movements in asset prices not warranted by fundamentals.[2]

3.3 Some Methodological Issues

Essentially, one can distinguish three approaches in the contagion literature relating to channels of transmission.

The approach used by Kaminsky and Reinhart (2000) originates from the

[2]There are a number of papers which address the issues of contagion by looking at correlation in asset prices (see Calvo and Reinhart (1996), Valdes (1997), Rigobon (1999), Forbes and Rigobon (2001), Bordo and Murshid (2000)), and changes in volatility (see Edwards (1998), Park and Song (1998)). These studies focus on establishing existence of "excess" co-movements in asset prices or geographical direction of contagion. However, they do not focus on the issues of propagation mechanisms of contagion, and , therefore, are beyond the scope of this study.

methodology introduced by Kaminsky, Lizondo and Reinhart (2000) in their study on currency crises. The channels of contagion are identified by comparing the unconditional probability of a crisis to the probability conditional on a country being in the same region as crisis country, or in the same trade or financial cluster. Typically in this framework various statistics are considered in order to verify the robustness of inferences .

Another approach was introduced by Glick and Rose (1999) and widely used in subsequent studies. First, it requires identification of a "ground-zero country" – a country where a crisis started. Second, a crisis variable is regressed on a set of fundamentals and variables reflecting trade or financial links to the "ground-zero country".

The advantage of the latter approach is that it is relatively simple and it allows testing not only for the presence of contagion, but also to distinguish between different channels of transmission. One of the drawbacks of this methodology is that it does not take into account a so called "cascade effect" (following the terminology of Glick and Rose (1999)). The "cascade effect" implies that a country may experience a crisis not only because of its direct links to the "ground-zero" country, but also due to spill-overs from countries that were already affected by the original crisis. For example, if Brazil was affected by the crisis in Russia, Argentina may also experience a crisis not because of its links to Russia, but because of its links to Brazil. Missing these links in an empirical model would lead to inconsistent estimates.

This point was recognized in a series of papers on contagion, see e.g. De Gregorio and Valdes (2001), and Hernandez and Valdes (2001). In their empirical models a contagion indicator of one country depends on a weighted average of the indicators of other countries. They construct different types of weights based on macroeconomic similarities, trade, financial links through major banking centers, and neighborhood effects, and check which set of weights better fits the data. For instance, Hernandez and Valdes (2001) model weekly stock market returns using the following equation:

$$y_{it} = c + \alpha \sum_{j=1}^{n} w_{ij} y_{jt} + x_t B + u_{it} \tag{3.1}$$

where y_{it} is a measure of weekly stock market return in country i in time period t, w_{ij} are the weights linking country i to country j, x_t is an observable exogenous common shock, u_{it} is a heteroscedastic disturbance term, and α and B are parameters. The model is normalized by setting $w_{ii} = 0$ for all i. The presence of contagion would be reflected by a non-zero value of α. Due to the interactions of the values of the dependent variable, this is a simultaneous equation model.

While recognizing the simultaneity problem the authors estimate their model by ordinary least squares. They argue that a bias of their estimator is proportional to the true value of parameters and is not present when the true value of α is zero (under the null of the absence of contagion).

Their approach, while informative about the presence of contagion channels, has certain disadvantages. First, models such as (3.1) are not consistently estimated by least squares procedure. Since the extent of inconsistencies are not known the parameter describing contagion, namely α, cannot be accurately estimated. Second, it becomes tricky to test for different channels of contagion simultaneously since in the presence of two sets of weights in one equation the biases of both coefficients are interrelated and OLS estimation gives little information about relative relevance of each coefficient.

These disadvantages would be resolved if a consistent estimation procedure were applied to an expanded version of the model which contains more than one channel of transmission[3]. This is a strategy that we are going to pursue in the paper.

The empirical setup in this study builds on the work of Hernandez and Valdez (2001) who apply this empirical methodology to measure contagion through bond spreads and stock market returns. We consider a larger sample of countries which helps to alleviate some of the sample selection biases. Furthermore, we utilize a consistent estimation procedure which is a special case of generalized spatial two-stage least squares proposed by Kelejian and Prucha (1998).

3.4 Specification

In our empirical framework we try to explain the behavior of weekly stock market returns of a cross-section of countries during the Asian, Russian, and Brazilian crises.

The presence of contagion is captured by the fact that the stock market return y_{it} of country i in time period t is determined not only by exogenous variables but also by a weighted average of returns of other countries in time period t. We allow for several kinds of weights in the equation simultaneously. Each set of weights corresponds to a specific channel of contagion. Our model is:

[3]Lee (2000) shows that under certain restrictive conditions the OLS estimator is consistent and efficient. However Lee's conditions are not satisfied by the typical contagion model.

$$
\begin{aligned}
y_{it} &= c_t + \alpha_1 \sum_{j=1, i\neq j}^{n} w_{ij}^{Trade} y_{jt} + \alpha_2 \sum_{j=1, i\neq j}^{n} w_{ij}^{Fin} y_{jt} \\
&\quad + \alpha_3 \sum_{j=1, i\neq j}^{n} w_{ij}^{\mathrm{Si}\,milarity} y_{jt} + \alpha_4 \sum_{j=1, i\neq j}^{n} w_{ij}^{Neighbor} y_{jt} \\
&\quad + x_i B_t + z_{it}\gamma + u_{it}, \\
t &= 1, .., T, \qquad n = 1, .., N
\end{aligned} \tag{3.2}
$$

where y_{it} is a measure of weekly stock market returns of country i in period t, denominated in US\$, $w_{ij}^{Trade}, w_{ij}^{Fin}, w_{ij}^{\mathrm{Si}\,milarity}$, $w_{ij}^{Neighbor}$ are the weights reflecting how country i is connected to country j through, respectively, bilateral trade, financial links, similarity in risk, and the geographical neighborhood; x_i and z_{it} are variables capturing common shocks and u_{it} is a heteroscedastic disturbance term which we assume to be independently distributed over i and t. The parameters of the model are $\alpha_1, \alpha_2, \alpha_3$, α_4, B_t, and γ.

Following the methodology of Hernandez and Valdes (2001), for each crisis episode we take a three-month window starting from the month a crisis starts – July in the Asian crisis, August in the Russian crisis, and January in the Brazilian crisis. This procedure creates a time dimension equal to 12 in each case, i. e., 12 panels for each of the three crises. The cross-sectional dimension consists of 50 countries for the Asian crisis, and 54 for the Russian and Brazilian crises[4].

[4]The full list of countries includes Argentina, Australia, Brazil, Bulgaria, Chile, China, Colombia, Croatia, Cyprus, Czech Republic, Ecuador, Egypt, Estonia, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Israel, Jordan, Kenya, Korea, Kuwait, Latvia, Lebanon, Lithuania, Malaysia, Malta, Mauritius, Mexico, Morocco, New Zealand, Nigeria, Pakistan, Peru, Philippines, Poland, Portugal, Romania, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, Sri Lanka, Taiwan, Thailand, Tunisia, Turkey, Ukraine, Venezuela,

3.4.1 Weighting Matrices

Trade

Following the previous literature w_{ij}^{Trade} is taken to be the ratio of exports from country i to country j to the total exports of country i. The matrix is subsequently row-normalized, i.e. each row is scaled such that it sums up to one. Using evident notation

$$w_{ij}^{Trade} = \frac{Export_{i,j}}{Export_i}, \qquad Export_i = \sum_{j=1}^{N} Export_{i,j}$$

where N is the total number of countries involved.

Financial Links

It is more difficult to find an intuitive measure of the financial interdependence through a common lender. The previous literature has proposed several ways to account for it.

The first measure of financial links was originally based on a formula for trade competition in third markets and was proposed by Glick and Rose (1999). Van Rijckeghem and Weder (2001) used their formula to measure financial interdependence between two countries through a common lender (sometimes referred as a bank lender). They called it a competition for funds indicator - the extent to which country i competes with country j for funding from the same bank lenders. Following previous studies we consider three bank lenders corresponding to the three major financial center: Europe, Japan and the US.

Zimbabwe. For the Asian crisis we had to exclude Bulgaria, Tunisia, Saudi Arabia, Ukraine due to unavailability of data.

There are two measures for competition for funds in the literature: absolute and relative. The absolute measure relative to common lender C is:

$$w_{ij,Abs}^{C} = \frac{b_{j,C} + b_{i,C}}{b_j + b_i}\left(1 - \frac{|b_{j,C} - b_{i,C}|}{b_{j,C} + b_{i,C}}\right) \tag{3.3}$$

where $b_{i,C}$ is the debt of country i to a common lender C, and b_i is the total foreign debt of country i ($b_i = \sum_C b_{i,C}$), $C = \{Europe, Japan, US\}$.

The first term of (3.3) reflects the importance of a common lender C to country i and j. The second term captures similarity in the borrowing patterns between country i and j. If they owe the same amount to the common lender the second term takes its highest value of one; if the difference between their debt is large it is close to zero.

Interestingly, after some simple manipulations we can write (3.3) as

$$w_{ij,Abs}^{C} = 2 * \frac{\min\{b_{j,C}, b_{i,C}\}}{b_j + b_i}$$

In this representation it has a different interpretation. Let us assume that $b_{j,C} = \min\{b_{j,C}, b_{i,C}\}$. Then, the formula suggests that the financial link between countries i and j is determined by the debt of country j to lender C, $b_{j,C}$, relative to the total debt of countries i and j. Put differently, the value of $b_{i,C}$ does not play any role as long as $b_{i,C} > b_{j,C}$.

In order to construct an aggregate financial link between two countries we form the three matrices $W_{Abs}^{C} = \left(w_{ij,Abs}^{C}\right)$ corresponding to each of the three financial centers $C = \{Europe, Japan, US\}$ and after row normalizing each of them we sum them up:

$$w_{ij,Abs}^{Fin} = \frac{1}{3}\left(\frac{w_{ij,Abs}^{Eur}}{\sum_j w_{ij,Abs}^{Eur}} + \frac{w_{ij,Abs}^{Jap}}{\sum_j w_{ij,Abs}^{Jap}} + \frac{w_{ij,Abs}^{US}}{\sum_j w_{ij,Abs}^{US}}\right). \tag{3.4}$$

The relative competition for funds indicator is similar to the absolute one with the only difference being that it uses shares of funds obtained from the same creditor instead of absolute values. It is calculated as follows:

$$w_{ij,\mathrm{Re}\,l}^{C} = \frac{b_{j,C} + b_{i,C}}{b_j + b_i} \left(1 - \frac{|b_{j,C}/b_j - b_{i,C}/b_i|}{b_{j,C}/b_j + b_{i,C}/b_i}\right) \tag{3.5}$$

Similar to the absolute competition for funds weighting matrix the elements of the financial weighting matrix $W_{\mathrm{Re}\,l}^{Fin} = (w_{ij,\mathrm{Re}\,l}^{Fin})$ based on (3.5) are given by

$$w_{ij,\mathrm{Re}\,l}^{Fin} = \frac{1}{3}\left(\frac{w_{ij,\mathrm{Re}\,l}^{Eur}}{\sum_j w_{ij,\mathrm{Re}\,l}^{Eur}} + \frac{w_{ij,\mathrm{Re}\,l}^{Jap}}{\sum_j w_{ij,\mathrm{Re}\,l}^{Jap}} + \frac{w_{ij,\mathrm{Re}\,l}^{US}}{\sum_j w_{ij,\mathrm{Re}\,l}^{US}}\right). \tag{3.6}$$

It is worth noting that, before row-normalization, the matrices constructed on the basis of (3.3) and (3.5) are symmetric. This implies that country i is affected by country j through this channel in the same way that country j is affected by country i. Clearly, this situation does not seem to be plausible when one imagines countries such as China and Bangladesh.

To overcome this problem we propose a second measure which is a variation of the one given in Caramazza et al. (2000).

Motivation for this measure comes from the simple logic that contagion between two countries sharing a common lender may occur if two conditions are fulfilled. First, the exposure of the common lender to a crisis country should be large enough to bring about significant losses that would affect the liquidity of that common lender. Second, the debt of an affected country to the common lender must also be large so that the country would be vulnerable to the common lender's actions.

In light of this discussion an intuitive measure of the link between country i and j through a common lender C, which we call an asymmetric measure, can be captured by the following formula:

$$w_{ij,Asym}^{C} = \frac{b_{i,C}}{GDP_i} * \frac{b_{j,C}}{b_C} \tag{3.7}$$

where b_C is the total portfolio of the common lender $b_C = \sum_i b_{i,C}$. The first component reflects the importance of common lender C to country i, while the second component measures exposure of the common lender to country j. The product of these two terms reflects the potential impact of country j on country i through the common lender channel.

A matrix based on (3.7) will not be symmetric and, therefore, would embrace the mentioned asymmetries in the links between countries. In constructing an aggregate financial link we do not row normalize this matrix. However, we scale the matrices corresponding to each common lender by the sum of all their elements divided by the number of cross-sectional units, i. e. the number of countries. The resulting expression is given by

$$w_{ij,Asym}^{Fin} = \frac{1}{3}\left(\frac{w_{ij,Asym}^{Eur}}{\frac{1}{N}\sum_i \sum_j w_{ij,Asym}^{Eur}} + \frac{w_{ij,Asym}^{Jap}}{\frac{1}{N}\sum_i \sum_j w_{ij,Asym}^{Jap}} + \frac{w_{ij,Asym}^{US}}{\frac{1}{N}\sum_i \sum_j w_{ij,Asym}^{US}}\right). \tag{3.8}$$

As becomes clear from the appendix this scaling ensures that the resulting weighting matrix $W_{Asym}^{Fin} = \left(w_{ij,Asym}^{Fin}\right)$ is absolutely summable.

It is also important to note that the financial weighting matrices are based on the banking statistics which does not include mutual funds, hedge funds, and other institutional investors. Therefore, they reflect the linkage through the banking sectors of the major economies, and these linkages are further often referred to as bank lending channel.

Similarity in Risk

For calculating similarity in risk we use a distance measure of credit ratings between two countries. One can view that as a proxy for macroeconomic similarity between two countries, the aspect that received a lot of attention in the literature on contagion.[5] The rationale for the importance of this measure is the "wake-up call" theory of contagion. This theory suggests that the occurrence of a crisis in one country makes investors look at other countries with similar macroeconomic conditions or falling in the same risk category, and update their risk assessment of these countries. This channel is based on the presence of incomplete information, which creates cross-country informational externalities.

We use the data on credit rating of countries compiled by Institutional Investor Magazine. Risk distance is calculated using the formula suggested by De Gregorio and Valdes (2001):

$$d_{ij} = \exp\{-|x_i - x_j|\}$$

where x_i is a credit rating of country i. The variable x_i is standardized to have mean zero and standard deviation 1. The risk similarity matrix based on this measure of distance is a row normalized matrix whose elements are given by

$$w_{ij}^{\mathrm{Si}\,mil} = \frac{d_{ij}}{\sum_j d_{ij}}$$

Neighborhood

There are several reasons for inclusion of a neighborhood effect into the empirical model. First, it reflects direct financial links among countries. Their regional

[5] See Golstein (1998), Eichengreen et. al. (1996), Rigobon (1998), De Gregorio and Valdes (2001).

pattern may originate from bilateral trade which also tend to be regional. Second, it may capture many of the non-linearities and residual terms of the trade links since the trade matrix may not be of very precise functional form and the neighborhood matrix may correct for it. Third, there might be other economic and non-economic regional links that connect countries and may contribute to the existence of spill-overs. In other words, many things that are regional and not accounted for by the other three matrices may be reflected by neighborhood effects.

The "neighborhood" matrix is constructed by assigning a weight of one if two countries belong to the same region, and a weight of zero otherwise. All the countries are divided into four regions: Europe, South and South-East Asia, Latin America, Middle East and North Africa, and South African region. The list of the countries divided by region is given in Table 3.1.

Table 3.1. Neighborhood clusters

Europe	**Latin America**	**South and South East Asia**	**Middle East and North Africa**	**Africa**
Bulgaria	Argentina	Australia	Egypt	Kenya
Croatia	Brazil	China	Israel	Mauritius
Cyprus	Chile	Hong Kong	Jordan	Nigeria
Czech	Colombia	India	Kuwait	South Africa
Estonia	Ecuador	Indonesia	Lebanon	Zimbabwe
Greece	Mexico	Korea	Morocco	
Hungary	Peru	Malaysia	Saudi Arabia	
Iceland	Venezuela	New Zealand	Tunisia	
Latvia		Pakistan		
Lithuania		Philippines		
Malta		Singapore		
Poland		Sri Lanka		
Portugal		Taiwan		
Romania		Thailand		
Russia				
Slovakia				
Slovenia				
Turkey				
Ukraine				

Common Shock Variables

There are three groups of variables in the model that capture the effect of common shocks: time effects represented by c_t in the model, so called "quasi-fixed effects" represented by the vectors x_i, and z_{it}. The first group, time effects c_t, controls for common shocks in time period t that affect all countries equally.

There are a number of reasons to believe that some common shocks have a different effect on different groups of countries. For instance, it is documented that an increase in the US interest rates has a more dramatic effect on the Latin American region than any other. Therefore, the effect of a shock originating in, say, US should be properly weighted by the extent of country's ties to the US economy. In our framework these ties can be either trade or financial. Furthermore, we assume that common shocks originate in the three major world economies - Europe, Japan and the US - and their impact on other countries is proportional to the trade and financial ties to these economies.

The set of vector x_i accounts for shocks propagated through the trade linkages, and z_{it} through financial linkages.

The vector x_i is specified as follows:

$$x_i = \left[\frac{Export_{i,Eur}}{GDP_i}, \frac{Export_{i,Jap}}{GDP_i}, \frac{Export_{i,US}}{GDP_i} \right]$$

Note that the coefficient corresponding to this vector, namely, B_t, is time-variant. It reflects the spill-overs in time period t, coming from Europe, Japan and the US through a trade channel. In a way, it can be interpreted as a time-effect proportionate to trade. This is the reason why we sometimes refer to the set of variables x_i as "quasi-fixed effects". Among other things it would capture trade competition among countries in the third market, given that the large share of their exports goes to the three major economies.

The financial common shock is more difficult to account for. At first glance, one may argue that we can construct the same variables as for trade with debt instead of export ratios. However, the problem with this approach is that among other things a time-variant coefficient before these variables (analog of B_t in the trade case) would also absorb the financial shocks transmitted across countries, in our sample through Europe, Japan and the US (a common lender channel). As a result, it would be impossible to identify whether a country is influenced by troubles of the major economies, or it is suffering from a shock transmitted through a common lender channel. Since the common lender channel is of great interest to us we cannot use this approach.

To overcome this problem we construct the vector z_{it} which interacts debt ratios with stock market returns in three financial centers:

$$z_{it} = \left[\frac{b_{i,Eur}}{GDP_i} y_{t,Eur}, \frac{b_{i,Jap}}{GDP_i} y_{t,Jap}, \frac{b_{i,US}}{GDP_i} y_{t,US}\right]$$

where $b_{i,C}$ is the debt of country i to financial center C, and $y_{t,C}$ is the weekly stock market return in country C at time period t, where, again, $C = \{Europe, Japan, US\}$.

Clearly, the stock market components of z_{it}, $y_{t,Eur}$, $y_{t,Jap}$, and $y_{t,US}$ are likely to be endogenous, i.e. they might be affected by the stock markets in, for instance, Asia or Eastern Europe during crises. Therefore, it is necessary to construct instruments for these variables.

In order to find proper instruments, we obtain sub-indices of the total stock market index which are inherently domestic: non-cyclical consumer services, real estate and utilities. Second, we regress the stock market return variables $y_{t,Eur}, y_{t,Jap}, y_{t,US}$ on the corresponding changes in the sub-indices and oil price. And finally, the fitted values from these regression are substituted into the formula for z_{it}. The resulting instrument becomes

$$\hat{z}_{it} = \left[\frac{b_{i,Eur}}{GDP_i} \hat{y}_{Eur,t}, \frac{b_{i,Jap}}{GDP_i} \hat{y}_{Jap,t}, \frac{b_{i,US}}{GDP_i} \hat{y}_{US,t} \right].$$

So far we have specified the model and explained all the variables and the coefficients involved. Next we are going to discuss the estimation of the model.

3.5 Matrix Notation And Estimation

In order to describe the estimation procedure it will be useful to write the model in the matrix notation. First, at time period t :

$$\begin{aligned} y_{it} &= c_t + \alpha_1 W^{Trade} y_{.t} + \alpha_2 W^{Fin} y_{.t} + \alpha_3 W^{\mathrm{Si}milarity} y_{.t} \\ &\quad + \alpha_4 W^{Neighbor} y_{.t} + X B_t + z_{.t}\gamma + u_{.t}, \qquad t = 1, .., T, \end{aligned} \tag{3.9}$$

where $y_{.t} = (y_{1t}, .., y_{nt})'$, $X = (x_1', .., x_n')'$, $u_{.t} = (u_{1t}, .., u_{nt})'$, $z_{.t} = (z_{1t}', .., z_{nt}')'$, B_t and γ are 3×1 vectors of parameters, and $c_t, \alpha_1, \alpha_2, \alpha_3$, and α_4 are scalar parameters of the model. Essentially (3.9) is a panel data models. We can stack the data in the usual way and obtain:

$$\begin{aligned} Y &= \mathbf{E}C + \alpha_1 \mathbf{W}^{Trade} Y + \alpha_2 \mathbf{W}^{Fin} Y + \alpha_3 \mathbf{W}^{\mathrm{Si}milarity} Y \\ &\quad + \alpha_4 \mathbf{W}^{Neighbor} Y + \mathbf{X} B + Z\gamma + u \end{aligned} \tag{3.10}$$

where $Y = (y_{.1}', .., y_{.T}')'$ is a stacked dependent variable, $\mathbf{E} = (I_T \otimes e_N)$ is a matrix of dummy variables, which relate to the time period involved, $\mathbf{W}^{Trade} = (I_T \otimes W^{Trade})$, $\mathbf{W}^{Fin} = (I_T \otimes W^{Fin})$, $\mathbf{W}^{\mathrm{Si}milarity} = (I_T \otimes W^{\mathrm{Si}milarity})$, $\mathbf{W}^{Neighbor} = (I_T \otimes W^{Neighbor})$ are block diagonal weighting matrices, $\mathbf{X} = (I_T \otimes X)$ is a matrix of quasi-fixed effects, $Z = (z_{.1}', .., z_{.T}')'$ is matrix of financial common shock variables, $u = (u_{.1}', .., u_{.T}')'$ is a stacked heteroscedastic disturbance term, and $B = (B_1', ..., B_T')'$, $C = (c_1, ..., c_T)'$ are parameters of the model.

The last expression resembles a typical spatial model of the type considered in Cliff and Ord (1973)[6]. There are four endogenous variables on the RHS besides Z. They are $\mathbf{W}^{Trade}Y, \mathbf{W}^{Fin}Y, \mathbf{W}^{\mathrm{Si}\,milarity}Y, \mathbf{W}^{Neighbor}Y$. In the literature on spatial model these variables are called spatial lags of the dependent variable, and they are obviously correlated with the error term and therefore need to be instrumented[7]. The list of instruments for this model is inspired by the work of Kelejian and Prucha (1998) and given by

$$\begin{aligned} H &= [\mathbf{X}, \hat{Z}, \mathbf{W}^{Trade}\mathbf{X}, \mathbf{W}^{Fin}\mathbf{X}, \mathbf{W}^{\mathrm{Si}\,milarity}\mathbf{X}, \mathbf{W}^{Neighbor}\mathbf{X}, \mathbf{W}^{Trade}\hat{Z}, \\ &\quad \mathbf{W}^{Fin}\hat{Z}, \mathbf{W}^{\mathrm{Si}\,milarity}\hat{Z}, \mathbf{W}^{Neighbor}\hat{Z}], \end{aligned}$$

where $\hat{Z}$ is defined the same way as Z with hats on corresponding variables.[8]

One of the important conditions for consistency given by Kelejian and Prucha (1998) is that the weighting matrices possess the property of absolute summability. In Appendix we give a definition of this concept and show that all our weighting matrices satisfy this property. For detailed discussion and proofs of consistency and asymptotic normality of this estimator see Kelejian and Prucha (1998).

[6]The only difference with the typical Cliff-Ord model is that it contains several weighting matrices and the regressor Z is endogenous. However, it does not complicate the analysis, and it stays essentially the same.

[7]For discussion of spatial models and their estimation see Anselin (1988), Cliif and Ord (1973, 1981), Cressie (1993), Kelejian and Prucha (1998, 1999), Ord (1975).

[8]For detailed motivation of this set of instruments see Kelejian and Prucha (1998).

3.6 Data

All the data on stock market indices and exchange rates were extracted from Datastream. The trade data and GDP figures were taken from Direction of Trade statistics compiled by IMF and World Development Indicators, respectively. The source of the data on the financial matrix deserves special attention. We use Bank of International Settlements (BIS) data on consolidated claims of banks of 18 developed countries on other individual countries in order to calculate a proxy for a total debt of an individual country to Europe, US and Japan.

There are several points that need to be mentioned about these data. First, it does not cover certain financial institutions such as hedge funds, institutional investors, dedicated mutual funds. Nonetheless, it seems to be a valid (and the only available) proxy for financial involvement of the industrial countries in the rest of the world given the substantial role of banks during crises (see discussion in Van Rijckeghem Weder (2001)). Second, the data covers only on-balance sheet positions ignoring off-balance positions that can be used to hedge risk. As pointed out by Van Rijckeghem Weder (2001) it could play a significant role only when crisis is widely anticipated such as Brazilian crisis, and is of less importance in the Russian and Asian crises. Moreover, they argue that it does not seem to be feasible to account for the off-balance positions.

Another imprecision in the BIS data which has not been as widely recognized before may arise from the fact that a lot of investment in emerging markets went through offshore financial centers and zones, see Wincoop and Yi (2000) for discussion. This fact would result in a bias in debt figures that may work both ways. For instance, some funds invested by European banks to Thailand through, say, Cayman Islands would not be reflected in BIS statistics as claims of European banks on Thailand. Hence debt of Thailand to Europe would be underestimated. From this angle, the BIS statistics of European, Japanese and American claims on some of the countries would be underestimated.

On the other hand, some countries in our sample themselves have offshore zones on their territories[9]. That means that part of their debt claimed to be owed to, say, Europe may turn out to be money of European banks actually invested in neighboring countries. It implies that for those countries the reported debt is an overestimation of the actual amount. Given a large concentration of funds in the offshore centers the latter bias is likely to be more substantial. We proceed keeping these limitations in mind.

3.7 Empirical Results

We start our empirical analysis by replicating some of the results of Hernandez and Valdes (2001) henceforth referred as HV. Next, we illustrate the differences in the results arising from more comprehensive accounting for common shocks, using a larger sample of countries, and utilizing a consistent estimation procedure. Finally, we use this procedure to estimate the full model that includes all contagion links at the same time.

First, we estimate the regression of HV containing a single weighting matrix, namely, based on bilateral trade. A common shock in this regression is captured by the US stock market return. In scalar notation the estimated equation is given

[9]By classification of Erico and Musalem (1999) there are fourteen countries in our sample falling under this category. They are Australia, Cyprus, Hong Kong, Hungary, Israel, Kuwait, Lebanon, Malaysia, Malta, Mauritius, Philippines, Russia, Singapore, Thailand.

by:

$$y_{it} = c + \alpha \sum_{j=1, i \neq j}^{n} w_{ij}^{Trade} y_{jt} + x_t B + u_{it}$$

where y_{it} is a weekly stock market return of country i in time period t, w_{ij} are the weights based on bilateral trade, x_t is a weekly return on the US stock market in time period t, and u_{it} is a heteroscedastic disturbance term, c, α and B are scalar parameters.

We estimate this equation using the same estimation technique as HV, which is OLS, the same time windows for each crisis episode which are three months starting from the month of a crisis, and the same sample of seventeen countries. The estimated values of the parameter of interest, namely, α are reported in the first column of Table 3.2. They are very close to those of HV, except for the Russian crisis in which our estimate is 0.79 while HV's is 0.45. The discrepancies might be due to differences in stock market indices used in the data as well as somewhat different methodology in constructing the trade matrices[10].

[10] There are two such differences. First, we use export statistics to construct the trade links while HV use the sum of export and import figures. Forbes (2001) argue that the contagion effects via export links and import links work in different directions. However, she found that the effect of import links is insignificant while that of export links is very siginificant. Therefore, it seems reasonable to use just export statistics for constructing bilateral trade links. Second, in order to constuct export figures, we use a reflection of the import statistics since the import statistics are presumed to be more precise.

Table 3.2. Comparison of Different Estimation Procedures, Different Accounting For Common Shocks and Different Sample Sizes.*

Estimation:	OLS	OLS	2SLS	2SLS
Sample size:	Small Sample	Large Sample	Large Sample	Small Sample
Accounting for common shock:	US stock market	As in the model	As in the model	As in the model
Asian crisis	0.76 (7.33)	0.13 (0.98)	0.42 (2.37)	0.22 (0.80)
Number of obs.	204	600	600	204
Russian crisis	0.79 (7.86)	0.21 (1.83)	0.71 (3.34)	1.12 (1.82)
Number of obs.	204	648	648	204
Brazilian crisis	0.42 (3.20)	0.12 (1.01)	0.80 (3.49)	-0.54 (-1.47)
Number of obs.	204	648	648	204

* t-statistics based on robust standard errors are in parenthesis

Note that HV estimated their model for both stock market returns and for bond spreads as dependent variables. Since bond spreads are available only for a limited number of countries, they consider only 17 countries in both bond spread and stock market returns regressions in order to make the results comparable. However, this approach may have some hidden biases. The data on bond spreads is compiled by JP Morgan, and, apparently, the choice of countries for which the bond spreads are calculated is not random. This fact implies that there is a potential for sample selection bias in the model.

In order to investigate this problem we check whether HV's results would hold if we expand the sample from 17 to 50 countries for the Asian crisis and to 54 for the Russian and Brazilian crises, and improve accounting for common shock. Again the model is estimated by OLS. Astonishingly, none of the coefficients corresponding to the weighting matrix turns out to be significant in any of the crisis episodes. Thus, the results change completely.

So far, we have run two OLS regressions differing in the sample of countries and accounting for common shocks. We have found that all of them produce different and sometimes opposite results. It suggests that even if we use the estimation procedure of previous studies the results change substantially depending on the sample of countries under consideration, and the way of accounting for common shocks. However, these assertions are based on least squares estimation which, as already mentioned, is clearly inconsistent. Therefore, we now introduce the results obtained with a consistent estimation and compare them to what we have found so far.

Columns 3 and 4 of Table 3.2 report the 2SLS estimates of the coefficients based respectively on the small and large samples of countries[11]. Two comments are in order. First, if we compare these two columns we can observe a substantial difference in both the values and the significance of the coefficients. It confirms that the sample selection bias may be quite substantial. Second, the OLS and 2SLS results based on the large sample of countries (columns 2 and 3, respectively) produce strikingly different outcomes. Specifically, results based on 2SLS indicate significance of the trade matrix coefficient in all crisis episodes, while the corresponding least squares estimates are insignificant at the 5% level in all cases. Hence, it may be misleading to base conclusions on the least squares estimates in this type of model. Clearly, a consistent estimation procedure is needed in order to obtain reliable inferences in such models.

We used the above exercise in order to illustrate that the estimation procedure matters in the empirical model *a la* HV. However, in order to estimate the model consistently one would have to include all the weighting matrices corresponding to different channels of contagion into one equation. The rest of this section discusses the results of the main specification given by equation (3.2).

The main specification (3.2) contains all four weighting matrices in one equation. We run three regressions for each crisis episode. They differ only in the specification of the financial matrix included in the equation. The financial matrices were described in Section 3.4.

Table 3.3 contains the estimates of the coefficients corresponding to the weighting matrices $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$. It can be seen that the results are sensitive to the definition of the financial matrix. For example, in case of the Russian crisis the

[11]Unfortunately, it is not feasible to apply the consistent estimation procedure used in the paper to the specification that accounts for common shocks *a la* HV.

channels that are significant in the first column (absolute competition financial matrix) are insignificant in the second and the third columns (relative competition and asymmetric financial matrices) and *visa versa*. This finding is not surprising because the weighting matrices themselves may be correlated due to the regional pattern of trade and financial links. Thus, when one of the matrices is misspecified the others may gain significance by absorbing the effect of misspecification. This situation may be a consequence of potential multicollinearity among these weights discussed in Kaminsky and Reinhart (2000). Keeping this limitation in mind we proceed to the next step which finds the financial weighting matrix that fits our empirical model the best.

Table 3.3. Estimated coefficients corresponding to the weighting matrices*

Financial Matrix:	Absolute Competition	Relative Competition	Asymmetric
Asian Crisis			
Trade	0.24 (1.31)	0.26 (1.33)	0.20 (1.03)
Financial Link	**0.79** **(3.25)**	0.40 (0.62)	0.01 (0.20)
Similarity	*0.88* *(1.77)*	0.68 (1.35)	0.79 (1.54)
Neighborhood	**0.38** **(2.60)**	**0.43** **(2.75)**	**0.49** **(3.22)**
Number of obs.	600		
Russian Crisis			
Trade	**0.46** **(2.18)**	0.07 (0.31)	0.13 (0.51)
Financial Link	**1.06** **(4.54)**	0.14 (0.18)	0.06 (1.06)
Similarity	0.33 (0.83)	**1.07** **(2.70)**	**1.05** **(2.66)**
Neighborhood	*0.30* *(1.65)*	**0.67** **(3.61)**	**0.62** **(3.11)**
Number of obs.	648		
Brazilian Crisis			
Trade	**0.46** **(2.05)**	0.21 (0.92)	0.30 (1.11)
Financial Link	**1.00** **(4.30)**	-0.94 (-0.90)	**0.18** **(2.70)**
Similarity	1.04 (1.57)	*1.28* *(1.95)*	**1.33** **(1.92)**
Neighborhood	**0.47** **(2.74)**	**0.75** **(4.32)**	**0.64** **(3.41)**
Number of obs.	648		

* t-statistics based on robust standard errors are in parenthesis, estimates in bold imply 5% significance, in italic – 10%

In order to find a preferred financial weighting matrix we simultaneously include two different financial weighting matrices into one regression equation and see which one performs better[12]. There are three distinct pairs of the different financial matrices. Hence we run three regressions for each crisis episode covering all the different combinations of the financial matrices. The results are reported in Table 3.4. It is readily seen that the absolute competition financial matrix is always significant when it is present in the equation while the other two are never significant when paired with absolute competition weighting matrix. It clearly indicates that the absolute competition for funds financial matrix dominates the others in terms of capturing financial links among countries.

Table 3.5 reports the results for the preferred specification of (3.2) which includes absolute competition for funds weighting matrix as a financial weighting matrix.

[12] Another way to find a preferred specification is to include all three financial weighting matrices into one equation. However this method may encounter the problem of degrees of freedom in the first stage of 2SLS procedure.

Table 3.4. Comparison of different financial weighting matrices.*

		Coefficient	T-statistics	Tail probability
Asian Crisis	**Absolute competition**	0.76	2.97	0.00
	Relative competition	-0.41	-0.62	0.54
	Absolute competition	0.71	2.91	0.00
	Asymmetric	-0.03	-0.71	0.48
	Relative competition	0.35	0.57	0.57
	Asymmetric	0.01	0.22	0.82
Russian Crisis	**Absolute competition**	1.21	5.62	0.00
	Relative competition	-0.99	-1.28	0.20
	Absolute competition	1.20	5.14	0.00
	Asymmetric	-0.09	-1.28	0.20
	Relative competition	0.19	0.23	0.82
	Asymmetric	0.07	1.24	0.22
Brazilian Crisis	**Absolute competition**	1.24	5.40	0.00
	Relative competition	-1.78	-1.73	0.09
	Absolute competition	1.05	3.87	0.00
	Continuous	0.05	0.63	0.53
	Relative competition	-0.44	-0.44	0.66
	Continuous	0.18	2.60	0.01

* The financial weighting matrices in the double line boxes were included in the empirical specification simultaneously.

Table 3.5. Summary of the results of the preferred specification**.***

		Asian Crisis	*Russian Crisis*	*Brazilian Crisis*
Coefficients before the weighting matrices	Trade	0.24 (1.31)	**0.46** **(2.18)**	**0.46** **(2.05)**
	Financial Link	**0.79** **(3.25)**	**1.06** **(4.54)**	**1.00** **(4.30)**
	Similarity	*0.88* *(1.77)*	0.33 (0.83)	1.04 (1.57)
	Neighborhood	**0.38** **(2.60)**	*0.30* *(1.65)*	**0.47** **(2.74)**
Coefficients corresponding to financial spillovers	Europe	-0.18 (-0.58)	-0.11 (-0.38)	-0.25 (-0.47)
	Japan	-0.03 (-0.05)	0.52 (0.89)	0.86 (1.12)
	US	-0.72 (-0.22)	1.10 (0.65)	1.19 (0.42)
Sum of the coefficients corresponding to trade spillovers of each major economy *$\sum B_{t,M}$ t=1,..,12* *M={Eur, Jap, US}*	Europe	6.3 *Prob. =* 0.82	*66.7* *Prob. =* *0.09*	-18.1 *Prob. =* 0.41
	Japan	**-281.6** ***Prob. =*** **0.01**	34.5 *Prob. =* 0.81	-117.2 *Prob. =* 0.24
	US	36.2 *Prob. =* 0.32	15.5 *Prob. =* 0.80	14.1 *Prob. =* 0.76
	Number of obs.	600	648	648

* t-statistics based on robust standard errors are in parenthesis, estimates in bold imply 5% significance, in italic – 10%

The results on the importance of bilateral trade linkages indicates that their role are crisis specific. They are insignificant in the case of the Asian crisis. This result is consistent with Baig and Goldstein (1998) who found that bilateral trade cannot explain contagion during the Asian crisis. It also follows from the table that during the Russian crisis bilateral trade linkages were important in propagation of a shock. This is somewhat surprising since for this particular crisis episode the bilateral trade channel has not been much emphasized in the cross-sectional contagion studies. The reason for such finding is that our sample contains many countries that have strong trade ties to Russia, namely, those from Eastern Europe and the former Soviet Union. In other studies these countries were often underrepresented in the sample due to data considerations. This result underscores the importance of sample selection issues in cross-sectional contagion studies.

In the case of the Brazilian crisis bilateral trade spill-overs also played a significant role. These results can be attributed to the fact that similar to Russia Brazil is a major country in its region. It is the biggest economy in Latin America and a major trading partner in the MERCOSUR trade agreement while Russia is a major economy among the former Soviet republics. In contrast, the South East Asian countries are not as much integrated among themselves in term of bilateral trade (see Kaminsky and Reinhart (2000) for more detailed discussion).

There is a clear evidence of the importance of the bank lending channel. It is significant in all crisis episodes! (See coefficients corresponding to the financial link in Table 3.5.) We note that in our results the relevance of all the other channels is crisis specific while the bank lending channel is relevant in all the considered crisis episodes. This result confirms the findings of many empirical studies that consider financial links as a propagation mechanism of a crisis.

The risk similarity channel does not seem to be important in any of the crisis episodes. The coefficient before the risk similarity matrix is never significant at 5% level, and significant at 10% level only in the Asian crisis episode. This result goes in line with Eichengreen Rose and Wyplosz (1996) and De Gregorio and Valdes (2001) who did not find similarities among countries to be a significant explanation of contagion.

The neighborhood matrix is significant at 5% level in Asian and Brazilian crisis episodes, and at 10% in the Russian crisis. This result suggests that the co-movements of stock market returns are regional and this regional pattern is not explained by the other links in the regression. The source of neighborhood effect may stem from bilateral financial links, residual trade effects, or other relevant regional connections between countries that are difficult, if not impossible, to account for in an aggregate empirical model.

An important implication of the above results is that the channels of contagion are specific to each crisis. This finding suggests that an empirical model that utilizes pooled data from several crisis episodes may lead to unreliable inferences.

The next block of Table 3.5 shows the results for the financial spill-overs from the major economies through their banking systems. None of the coefficients corresponding to this factor turns out to be significant. This suggests that there were no such financial spill-overs that were a consequence of domestic shock to Japan, Europe or US. This result come at odds with some observers' claims that contagion during the Asian crisis was largely attributed to the sluggish Japanese economy and, in particular, to the troublesome Japanese banking system.

In order to assess the common factors that spread through the trade channel we summed up the coefficients B_t ("quasi-fixed effects") for each crisis episode. This sum represents estimated spill-overs over the chosen time window. Then we tested a null hypothesis that the sum of "quasi-fixed effects" corresponding to each major economy is equal to zero. The last block of Table 3.5 report these sums and the tail probability of the hypothesis that they are equal to zero. It is readily seen that the trade spill-overs from Japan during the Asian are highly significant and negative. The interpretation of this coefficient is that if a country has a 10% share of Japan in its exports, then *ceteris paribus* the stock market index would go down by 28% over the three months of the Asian crisis. However the exact source of these spill-overs cannot be determined. Given the nature of our empirical model it is not possible to say whether the spill-overs were due to a domestic shock to the Japanese economy or to the competition among countries in the Japanese market. Furthermore, it may be the case that the trade statistics would better capture financial links than BIS statistics, and the above result is just a reflection of the financial difficulties of Japan.

We also note that all the other indicators of trade spill-overs are insignificant except for the case of the Russian crisis where the spill-overs from Europe are positive and significant at 10% level.

Finally, the last set of results tests the financial spill-overs from Japan, Europe and the US for endogeneity. In other words, it tests whether the stock markets in these countries were significantly affected by the stock markets from outside in the context of our empirical framework. We conduct a Hausman test for endogeneity of the variables that correspond to financial spill-overs from the major financial centers (Z in (3.2)). The results are reported in Table 3.6. In none of the cases these variables are endogenous. This implies that the major financial centers were not significantly affected by the economic conditions of the countries in the sample. However, that does not mean that the financial centers were not transmitting the shocks from one country to another.

Now (3.2) can be reestimated with Z as exogenous variables. Table 3.7 shows that when (3.2) is reestimated with Z, the conclusions do not change.

Table 3.6. Hausman test for endogeneity of financial spillovers from the major economies**.***

	Asian Crisis	*Russian Crisis*	*Brazilian Crisis*
Chi2(3)	2.74	3.22	5.78
Prob > Chi2	0.43	0.36	0.14
Number of obs.	600	648	648

Table 3.7. Summary of the results of the specification with exogenous financial spillovers**.***

		Asian Crisis	*Russian Crisis*	*Brazilian Crisis*
Coefficients before the weighting matrices	Trade	0.22 (1.21)	0.49 (2.32)	0.51 (2.30)
	Financial Link	**0.81 (3.35)**	**1.10 (4.73)**	**1.04 (4.41)**
	Similarity	*0.88 (1.76)*	0.28 (0.70)	0.95 (1.42)
	Neighborhood	**0.39 (2.70)**	0.27 (1.53)	**0.46 (2.59)**
Coefficients corresponding to financial spillovers	Europe	-0.14 (-0.51)	-0.08 (-0.30)	-0.22 (-0.46)
	Japan	0.09 (0.16)	0.51 (0.87)	0.61 (0.79)
	US	-2.13 (-0.75)	-0.06 (-0.04)	-1.39 (-0.55)
Sum of the coefficients corresponding to trade spillovers of each major economy	Europe	6.1 *Prob.* = 0.83	*69.4 Prob. = 0.08*	-17.5 *Prob.* = 0.43
	Japan	**-277.8** *Prob.* = **0.01**	37.1 *Prob.* = 0.80	-113.7 *Prob.* = 0.26
$\sum B_{t,M}$ *t=1,..,12 M={Eur, Jap, US}*	US	40.3 *Prob.* = 0.27	13.0 *Prob.* = 0.83	18.2 *Prob.* = 0.69
	Number of obs.	600	648	648

* t-statistics based on robust standard errors are in parenthesis, estimates in bold imply 5% significance, in italic – 10%, *Prob.* denotes tail probability of F-test

3.8 Concluding remarks

This paper analyzed channels of contagion by employing a spatial modelling technique to explain co-movements of stock market returns across countries in crisis periods. We considered three recent crisis episodes – the Asian, the Russian and the Brazilian crises.

It was shown that the estimation procedure for spatial models used in the previous literature on contagion leads to inaccurate inferences. Furthermore, it was also shown that improper accounting for common shocks and the presence of sample selection may aggravate potential biases.

This paper corrects for these problems in the following ways. First, it introduces a consistent estimation procedure for this type of model which is a variant of the generalized spatial two stage least squares estimator suggested by Kelejian and Prucha (1998). Second, it proposes a comprehensive way of accounting for common shocks. Third, it considers a larger sample of countries which helps to alleviate sample selection biases.

The results confirm the importance of the bank lending channel which was found to be important in previous studies. It is significant in all crisis episodes. In contrast, the role of the bilateral trade channel varies across crises. It is found to be significant during the Russian and Brazilian crises, but not significant during the Asian crisis. The results on risk similarity channel suggest that it was not present during the Russian and the Brazilian crises, and had only marginal effect during the Asian crisis. Finally, neighborhood effects are found significant in Asian and Brazilian crises and marginally significant in Russian crisis.

It is important to note that some channels of transmission are not present in all crises and are rather crisis specific. Therefore, one should be cautious before obtaining estimates based on a pooled data set of different crises.

We did not find evidence of a common shock spreading through financial ties to the major world economies. Furthermore, the results suggest that the stock market returns of the major economies were not significantly affected by other countries in the considered crisis episodes. There is evidence that during the Asian crisis countries that had higher trade with Japan experienced lower stock market returns. It is not clear whether this result is due to domestic shock to the Japanese economy, or to the competition among countries in the Japanese market. However, the former seems to be more likely since Japan was experiencing recession during that time. On the other hand, one should be careful in "blaming" trade for the transmission of shocks. The reason for this is that bilateral trade links may also reflect financial constraints which can be triggered in crisis times and are best captured by statistics on trade flows[13]. In general, it should also be noted that one should not overemphasize a certain channel based on significance of a corresponding coefficient since a measure of each channel may reflect other channels as well. In this respect the econometric results would be useful for determining vulnerability of certain countries to contagion rather than for discriminating among different theories. In order to distinguish between different theories one would have to construct a general equilibrium model incorporating different links among countries and specify an empirical model on the basis of first order conditions derived from the model. This should be a major challenge for future research on contagion. The empirical strategy employed in this paper will be a useful tool for further research in this area.

[13]For illustration see Paasche (2001). In his model a terms of trade shock is amplified by collateral constraints in the economy.

Chapter 4

Appendix to Chapter 3

4.1 Absolute Summability Of The Weighting Matrices

Definition. Let $W_N = (w_{ij})$ be $N \times N$ matrix. We say that W_N is absolutely summable if $\max_i \sum_{j=1}^{N} |w_{ij,N}| < c_w$ and $\max_j \sum_{i=1}^{N} |w_{ij,N}| < c_w$ for all $N \geq 1$ where c_w is a finite constant.

Observe that in the case of nonnegative elements of matrix W_N we can say that W_N is absolutely summable if the row and column sums of this matrix are uniformly bounded. Given the fact that the weighting matrices in this paper do not contain negative elements we will use this property throughout the appendix.

In order to facilitate the proof of absolute summability it is necessary to make several assumptions related to the economic indicators used in the construction of the weighting matrices. Most of these assumptions are related to the behavior of the economic indicators when the number of countries increases. We note that the increase in the number of countries is purely hypothetical and serves for interpretation of the consistency of the estimators used in our analysis.

Assumption 1. The world GDP does not change as the number of countries increases. Formally:

$$\sum_{i=1}^{N} GDP_{i,N} = GDP, \quad \text{for all } N \geq 1 \tag{4.1}$$

where $GDP_{i,N}$ is a GDP of country i, and GDP is the world GDP. Put differently, this assumption says that the number of countries grows due to division of the existing countries into several regions, rather than due to the addition of new territories.

Assumption 2. As the number of countries increases no country becomes dominant relative to the others and no country becomes infinitely small relative to the others. This statement is formalized by the following inequalities:

$$0 < K_1 \leq N * \frac{GDP_{i,N}}{GDP} \leq K_2 < \infty \;\; \text{for all } i = 1, ..., N, \text{ and all } N \geq 1, \tag{4.2}$$

where K_1 and K_2 are finite positive constants invariant to N.

Other assumptions are formulated as we proceed to the proof of the absolute summability of the weighting matrices.

Trade weighting matrix

The trade weighting matrix is row-normalized, which implies that row-sums are uniformly bounded by construction. Thus, in order to prove absolute summability it is sufficient to show that the column sums are uniformly bounded. For this purpose we make the following assumptions related to the trade variables:

Assumption 3. Export cannot exceed country's GDP:

$$\sum_{i=1}^{N} Exp_{ij,N} = Exp_{i,N} \leq GDP_{i,N} \quad \text{for all } i = 1, ..., N, \text{ and all } N \geq 1. \tag{4.3}$$

where $Exp_{ij,N}$ is an export from country i to country j, $Exp_{i,N}$ and $GDP_{i,N}$ are, respectively, the total export and GDP of country i.

Assumption 4. All countries have a positive export, and

$$0 < C_1 \leq \frac{Exp_{i,N}}{GDP_{i,N}} \leq 1 \quad \text{for all } i = 1, ..., N, \text{ and all } N \geq 1, \tag{4.4}$$

where C_1 is a finite and positive constant invariant to N. The inequalities in (4.4) imply that all the countries are integrated into the world economy by trade.

Using (4.1)-(4.4)we can write the following inequalities related to the j-th column sum of the trade weighting matrix:

$$\begin{aligned}
\sum_{i=1}^{N} w_{ij}^{Trade} &= \sum_{i=1}^{N} \frac{Exp_{ij,N}}{Exp_{i,N}} \\
&= \sum_{i=1}^{N} \frac{Exp_{ij,N}/GDP_{i,N}}{Exp_{i,N}/GDP_{i,N}} \leq \frac{1}{C_1} \sum_{i=1}^{N} \frac{Exp_{ij,N}}{GDP_{i,N}} \\
&= \frac{1}{C_1} \sum_{i=1}^{N} \frac{Exp_{ij,N}/GDP}{GDP_{i,N}/GDP} \leq \frac{N}{K_1 C_1} \frac{Exp_{j,N}}{GDP} \\
&\leq \frac{1}{K_1 C_1} * N \frac{GDP_{j,N}}{GDP} \leq \frac{K_2}{K_1 C_1}
\end{aligned}$$

Thus,

$$\sum_{i=1}^{N} \frac{Exp_{ij,N}}{Exp_{i,N}} \leq const \quad \text{for all } j = 1, ..., N, \text{ and all } N \geq 1.$$

which implies that column sums are uniformly bounded.

Absolute and relative competition for funds financial weighting matrices

The following assumptions are related to the variables involved in the financial weighting matrices. They ensure that the countries are also financially integrated into the world economy.

Assumption 5. All the countries have a positive external debt which satisfies:

$$0 < B_1 \leq \frac{b_{i,N}}{GDP_{i,N}} \leq B_2 \text{ for all } i = 1, ..., N, \text{ and all } N \geq 1, \tag{4.5}$$

where B_1 and B_2 are finite positive constants invariant to N.

Assumption 6. All the countries have a positive debt to the financial centers under consideration which satisfies

$$
\begin{aligned}
0 &< D_1 \leq \frac{b_{i,C,N}}{b_{i,N}} \leq 1 \qquad (4.6)\\
\text{for all } i &= 1, ..., N, \text{ all } N \geq 1, \text{ and } C = \{Europe, Japan, US\}.
\end{aligned}
$$

where D_1 is a finite positive constant invariant to N.

It is readily seen that (4.6) implies that

$$
\begin{aligned}
0 &< D_1 \leq \frac{b_{i,C,N} + b_{j,C,N}}{b_{i,N} + b_{j,N}} \leq 1 \qquad (4.7)\\
\text{for all } i,j &= 1, ..., N, \text{ all } N \geq 1, \text{ and } C = \{Europe, Japan, US\}
\end{aligned}
$$

Utilizing (4.5)-(4.7) we can write the following inequalities related to the j-th column-sum of the absolute competition for funds weighting matrix:

$$
\begin{aligned}
\sum_{i=1}^{N} w_{ij,Abs}^{Fin} &= \frac{1}{3} \sum_{C} \sum_{i=1}^{N} \frac{\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}} \left(1 - \frac{\left|b_{j,C,N}-b_{i,C,N}\right|}{b_{j,C,N}+b_{i,C,N}}\right)}{\sum_{j=1}^{N} \frac{b_{j,C}+b_{i,C}}{b_j+b_i} \left(1 - \frac{\left|b_{j,C}-b_{i,C}\right|}{b_{j,C}+b_{i,C}}\right)} \\
&\leq \frac{1}{3} \sum_{C} \sum_{i=1}^{N} \frac{\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}}}{\sum_{j=1}^{N} \frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}} \left(1 - \frac{\left|b_{j,C,N}-b_{i,C,N}\right|}{b_{j,C,N}+b_{i,C,N}}\right)} \\
&\leq \frac{1}{3} \sum_{C} \frac{1}{D_1} \sum_{i=1}^{N} \frac{1}{\sum_{j=1}^{N} \left(1 - \frac{\left|b_{j,C,N}-b_{i,C,N}\right|}{b_{j,C,N}+b_{i,C,N}}\right)} \\
&\leq \frac{1}{3} \sum_{C} \frac{1}{D_1} \sum_{i=1}^{N} \frac{1}{\sum_{j=1}^{N} \left(1 - \frac{b_{j,C,N}+b_{i,C,N}}{b_{j,C,N}+b_{i,C,N}}\right)} \leq \frac{1}{3} \sum_{C} \frac{1}{D_1(1-D_1)} \\
&= \frac{1}{D_1(1-D_1)}
\end{aligned}
$$

Thus, we showed that the column-sum of the absolute competition for funds weighting matrix does not exceed a sample invariant constant, which implies that it is uniformly bounded. We can perform similar manipulations with the column sum of the relative competition for funds weighting matrix :

$$\sum_{i=1}^{N} w_{ij,\mathrm{Re}l}^{Fin} = \frac{1}{3}\sum_{C}\sum_{i=1}^{N} \frac{\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}}\left(1-\frac{\left|b_{j,C,N}/b_{j,N}-b_{i,C,N}/b_{i,N}\right|}{b_{j,C}/b_{j}+b_{i,C}/b_{i}}\right)}{\sum\limits_{j=1}^{N}\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}}\left(1-\frac{\left|b_{j,C,N}/b_{j,N}-b_{i,C,N}/b_{i,N}\right|}{b_{j,C,N}/b_{j,N}+b_{i,C,N}/b_{i,N}}\right)}$$

$$\leq \frac{1}{3}\sum_{C}\sum_{i=1}^{N} \frac{\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}}}{\sum\limits_{j=1}^{N}\frac{b_{j,C,N}+b_{i,C,N}}{b_{j,N}+b_{i,N}}\left(1-\frac{\left|b_{j,C,N}/b_{j,N}-b_{i,C,N}/b_{i,N}\right|}{b_{j,C,N}/b_{j,N}+b_{i,C,N}/b_{i,N}}\right)}$$

$$\leq \frac{1}{3}\sum_{C}\frac{1}{D_1}\sum_{i=1}^{N} \frac{1}{\sum\limits_{j=1}^{N}\left(1-\frac{\left|b_{j,C,N}/b_{j,N}-b_{i,C,N}/b_{i,N}\right|}{b_{j,C,N}/b_{j,N}+b_{i,C,N}/b_{i,N}}\right)}$$

$$\leq \frac{1}{3}\sum_{C}\frac{1}{D_1}\sum_{i=1}^{N} \frac{1}{\sum\limits_{j=1}^{N}\left(1-\frac{b_{j,C,N}/b_{j,N}+b_{i,C,N}/b_{i,N}}{b_{j,C,N}/b_{j,N}+b_{i,C,N}/b_{i,N}}\right)}$$

$$\leq \frac{1}{3}\sum_{C}\frac{1}{D_1(1-D_1)} = \frac{1}{D_1(1-D_1)}$$

This completes the proof of the absolute summability of the first two financial weighting matrices.

Asymmetric financial weighting matrix

In contrast to the other weighting matrices the asymmetric weighting matrix is not row normalized. Therefore we have to demonstrate that both row and column sums are uniformly bounded.

The following series of inequalities shows that the row-sums of the asymmetric weighting matrix are uniformly bounded. For the i-th row-sum:

$$\sum_{j=1}^{N} w_{ij,Asym}^{Fin} = \frac{1}{3}\sum_{C}\frac{1}{\frac{1}{N}\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}} * \frac{b_{j,C,N}}{b_{C,N}}}\sum_{j=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}} * \frac{b_{j,C,N}}{b_C}$$

$$= \frac{1}{3}\sum_{C}\frac{1}{\frac{1}{N}\sum\limits_{i=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}} * \sum\limits_{j=1}^{N}\frac{b_{j,C,N}}{b_{C,N}}} * \frac{b_{i,C,N}}{GDP_{i,N}}\sum_{j=1}^{N}\frac{b_{j,C,N}}{b_C}$$

$$= \frac{1}{3}\sum_{C}\frac{1}{\frac{1}{N}\sum\limits_{i=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}}}\frac{b_{i,C,N}}{GDP_{i,N}}$$

$$\leq \frac{1}{3}\sum_{C}\frac{N}{NB_1} = \frac{1}{B_1}$$

Similarly, for the j-th column-sum:

$$\sum_{i=1}^{N} w_{ij,Asym}^{Fin} = \frac{1}{3}\sum_{C}\frac{1}{\frac{1}{N}\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}}\frac{b_{j,C,N}}{b_{C,N}}}\sum_{i=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}} * \frac{b_{j,C,N}}{b_C}$$

$$= \frac{1}{3}\sum_{C}\frac{1}{\frac{1}{N}\sum\limits_{i=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}}} * \frac{b_{j,C,N}}{b_C}\sum_{i=1}^{N}\frac{b_{i,C,N}}{GDP_{i,N}}$$

$$\leq \frac{1}{3}\sum_{C}\frac{N}{NB_1}\sum_{i=1}^{N}\frac{b_{i,C,N}}{b_{i,N}}\frac{b_{i,N}}{GDP_{i,N}} \leq \frac{B_2}{B_1}$$

Thus, the row and column sums of the asymmetric financial weighting matrix are uniformly bounded, and, thus, this weighting matrix is absolutely summable.

Similarity matrix

The elements of the similarity matrix represent a measure of distance between countries. Let $d_{ij,N}$ be the risk distance between countries i and j. Note that $d_{ij,N}$ are bounded by construction, i.e. there exists constant D such that $d_{ij,N} < D$ for $i, j = 1, ..., N$, and for all $N \geq 1$.

Assumption 7. There exist $0 < A < \infty$ and $0 < \alpha < 1$ such that for any $1 \leq i \leq N$, and for all $N \geq 1$ there are at least αN countries in the set $B_{i,N}$, where

$$B_{i,N} = \{j : d_{ij,N} > A\}$$

In words, it says that there is proportion α of all countries that does not get closer to any given country in term of risk.

Assumption 7 also implies

$$\sum_{j=1}^{N} d_{ij,N} \geq \sum_{B_{i,N}} d_{ij,N} > \alpha NA$$

Now, the proof of the uniform boundedness of j-th column-sum is evident:

$$\sum_{i=1}^{N} w_{ij}^{\mathrm{Si}\,mil} = \sum_{i=1}^{N} \frac{d_{ij,N}}{\sum\limits_{i=1}^{N} d_{ij,N}} \leq \frac{1}{\alpha NA} \sum_{j=1}^{N} d_{ij,N} \leq \frac{N}{\alpha NA} = \frac{1}{\alpha A}$$

Neighborhood matrix

The neighborhood weighting matrix is a symmetric row-normalized block-diagonal matrix. It can be easily seen that column-sums as well as the row-sums are equal to one. This ensures absolute summability of this matrix.

BIBLIOGRAPHY

[1] Allen, F., and Gale, D., 2000. Financial Contagion. *Journal of Political Economy*, 108, pp. 1-33.

[2] Amemiya, T., 1985. *Advanced Econometrics*, Cambridge, MA, Harvard University Press.

[3] Anselin, L., 1988. *Spatial Econometrics: Methods and Models*. Boston: Kluwer.

[4] Anselin, L., 2001. Spatial Econometrics. In B. Baltagi (ed.), *A Companion to Theoretical Econometrics*. Oxford: Basil Blackwell, 2001, pp. 310-330.

[5] Anselin, L., and Florax RJGM, 1995. Small sample properties of tests for spatial dependence in regression models: Some further results. In: Anselin, L, and Florax, RJGM (eds) *New directions in spatial econometrics*. Springer, Berlin Heidelberg New York.

[6] Baig, T., and Goldfajn, I., 1998. Financial Market Contagion in the Asian Crisis. *International Monetary Fund Working Paper WP/98/155*.

[7] Besag, J., 1974. Spatial interaction and the statistical analysis of lattice systems. Journal of the Royal Statistical Society B 36, pp. 192-225.

[8] Bell, K., and Bockstael, N., 2000. Applying the generalized moments estimation approach to spatial problems involving micro-level data. *The Review of Economics and Statistics,* 82, pp. 72-82

[9] Besley, T., and Case, A., 1995. Incumbent behavior: vote-seeking, tax-setting and yardstick competition. *American Economic Review*, 85, pp. 25-45.

[10] Bordo, M., and Murshid, A. P. 2000. Are Financial Crises Becoming Increasingly More Contagious? What Is The Historical Evidence On Contagion? *NBER Working Paper* 7900.

[11] Brown, C.C., and Oates, W. E., 1986. Assistance to the Poor in a Federal System. *Journal of Public Economics*, 32, pp. 307-330.

[12] Brueckner, J. K., 1998. Testing for strategic interaction among local governments: the case of growth controls. *Journal of Urban Economics*, 44, pp. 438-467.

[13] Calvo, G. A., 1999. Contagion in emerging markets: when the Wall street is a carrier. University of Maryland, mimeo.

[14] Calvo, G. A., and Mendoza, E., 2000. Rational Contagion and the Globalization of Securities Market. *Journal of International Economics* 51, 79-113.

[15] Calvo, S., and Reinhart, C., 1996. Capital Flows to Latin America: Is There Evidence of Contagion Effects? *World Bank Policy Research Working Paper* 55.

[16] Caramazza, F., Ricci, L., and Saldago, R., 2000. Trade and Financial Contagion in Currency Crises. *International Monetary Fund Working Paper WP/00/55.*

[17] Case, A., 1991. Spatial Patterns in Household Demand. *Econometrica*, 59, pp. 953-965.

[18] Case, A., Hines, J.R., and Rosen, H.S. (1993). Budget Spill-overs and Fiscal Policy Interdependence. *Journal of Public Economics*, 52, pp. 285-307.

[19] Cliff A., and Ord J., 1973. *Spatial Autocorrelation.* London: Pion

[20] Cliff A., and Ord J., 1981. *Spatial Processes, Models and Applications.* London: Pion.

[21] Conley, T., 1999. GMM Estimation with Cross Sectional Dependence. *Journal of Econometrics*, 92, pp. 1-45.

[22] Conley, T., and Dupor, W., 2003. Spatial Analysis of Sectoral Complementarity. *Journal of Political Economy*, 111, pp. 311-352.

[23] Conley, T., and Topa, G., 2002. Socio-economic Distance and Spatial Patterns in Unemployment. *Journal of Applied Econometrics*, 17, pp. 303-327.

[24] Corcetti, G., Pesenti, P., Roubini, N., and Tille, C., 2000. Competitive Devaluations: Toward a Welfare-Based Approach. *Journal of International Economics* 51 217-241.

[25] Cressie N.A.C., 1993, *Statistics for Spatial Data.* Wiley Publishing Company, New York.

[26] Das, D., Kelejian, H. H., and Prucha, I. R., 2003. Finite sample properties of spatial autoregressive models with autoregressive disturbances. *Papers in Regional Science*, 82, pp.1-26.

[27] De Gregorio, J. and Valdes, R. O., 2001. Crisis Transmission: Evidence from Debt, Tequila, and Asian Flu Crises. In: Claessens, S., and Forbes, K. (eds) *International Financial Contagion: How It Spreads and How It Can Be Stopped,* Kluwer Academic Publishers.

[28] Dubin, R., 1988. Estimation of Regression Coefficients in the Presence of Spatially Autocorrelated Error Terms. *The Review of Economics and Statistics*, pp.466-474.

[29] Edwards, S., 1998. Interest Rate Volatility, Contagion and Convergence: An Empirical Investigation of the Cases of Argentina, Chile and Mexico. *Journal of Applied Economics*, 1(1) 55-86.

[30] Eichengreen, B., Rose, A. K., and Wyplosz, C., 1996. Contagious Currency crises. *NBER Working Paper* 5681.

[31] Errico, L., and Musalem, A., 1998. Financial Market Contagion in the Asian Crisis. *International Monetary Fund Working Paper WP/98/155.*

[32] Forbes, K., 2001. Are Trade Linkages Important Determinants of Country Vulnerability to Crises? *NBER Working Paper 8194.*

[33] Forbes, K., and Rigobon, R., 2001. Measuring Contagion: Conceptual and Empirical Issues. In: Claessens, S., and Forbes, K. (eds) *International Financial Contagion: How It Spreads and How It Can Be Stopped,* Kluwer Academic Publishers.

[34] Gerlach, S. and Smets, F., 1994. Contagious Speculative Attacks, *CEPR Discussion Paper 1055.*

[35] Glick, R., and Rose, A. K., 1999. Contagion and Trade: Why Are the Currency Crises Regional? *Journal of International Money and Finance* 18(4) 603-18.

[36] Goldstein, M., 1998. *The Asian Financial Crisis.* Washington, DC: Institute for International Economics.

[37] Gramilich, E. M., 1987, Cooperation and Competition in Public Welfare Policies. *Journal of Public Analysis and Management*, 6(3), pp. 417-431.

[38] Hernandez, L. F. and Valdez, R. O., 2001. What Drives Contagion Trade, Neighborhood, or Financial Links? *International Review of Financial Analysis* 10, 203-218.

[39] Hordijk, L., 1974. Spatial Correlation in the Disturbances of a Linear Interregional Model. *Regional and Urban Economics*, 4, pp.117-140.

[40] Hordijk, L. and Paelinck, J., 1976. Some principles and results in spatial econometrics. *Recherches Economiques de Louvain,* 42, pp. 175-197.

[41] Kaminsky, G., Lizondo, S., and Reinhart, C., 2000. The Leading Indicators of Currency Crises. *IMF Staff Papers 45 1-48.*

[42] Kaminsky, G., Lyons, R., Schmukler, S., 1999. Managers, Investors, and Crises: Mutual Fund Strategies in Emerging Markets. *NBER Working Paper 7855.*

[43] Kaminsky, G., and Reinhart, C., 2000. On Crises, Contagion, and Confusion. *Journal of International Economics* 51 145-168.

[44] Kapoor, M., 2003. Panel Data Models with Spatial Correlation: Estimation Theory and an Empirical Investigation of the US Wholesale Gasoline Industry. Ph. D. Thesis.

[45] Kelejian H. H. and Robinson D. P., 1992. Spatial autocorrelation: A new computationally simple test with an application to per capita country police expenditures. *Regional Science and Urban Economics*, 22, pp. 317-333.

[46] Kelejian H. H. and Robinson D. P., 1997. Infrastructure Productivity Estimation and Its Underlying Econometrics Specification: A Sensitive Analysis. *The Journal of REAI*, 76(1), pp. 115-131.

[47] Kelejian, H. H., and Prucha, I. R., 1998. A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances. *Journal of Real Estate Finance and Economics* 17, pp. 99-121.

[48] Kelejian, H. H., and Prucha, I. R., 1999. A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model. *International Economic Review*, 40, pp. 509-533.

[49] Kelejian, H. H., and Prucha, I. R., 2001. Best Series Instrumental Variable Estimator for a Spatial Autoregressive Model With Autoregressive Disturbances. Department of Economics, University of Maryland, College Park, mimeo.

[50] Kodres, L. E., Pritsker, M. 2001. A Rational Expectation Model of Financial Contagion. *FEDS Working Paper 48, The Federal Reserve Board.*

[51] Lee, L-F, 1999a. Best Spatial Two-stage Least Squares Estimators for a Spatial Autoregressive Model With Autoregressive Disturbances. Department of Economics, HKUST, Hong Kong.

[52] Lee, L-F, 1999b. Asymptotic Distributions of Maximum Likelihood Estimators for Spatial Autoregressive Models. Department of Economics, HKUST, Hong Kong.

[53] Lee L-F, 2002. Consistency and Efficiency of Least Squares Estimation for Mixed Regressive, Spatial Autoregressive Models. *Econometric Theory* 18, 252-277.

[54] Masson, P., 1998. Contagion: Monsoonal Effects, Spillovers, and Jumps Between Multiple Equilibria, *International Monetary Fund Working Paper WP/98/142.*

[55] McMillen, D. P., 1992. Probit With Spatial Autocorrelation. *Journal of Regional Science*, 32, pp. 335-348.

[56] Ord, J., 1975. Estimation Methods for Models of Spatial Interaction. *Journal of the American Statistical Association,* 70, pp. 120-126.

[57] Paasche, B., 2001. Credit Constraints and International Financial Crisis. *Journal of Monetary Economics* 48 623-650.

[58] Pace, R. K., Barry, R., and Sirmans C.F., 1998. Spatial Statistics and Real Estate. *Journal of Real Estate Finance and Economics*, 17, pp. 5-13.

[59] Park, Y. C., and Song C. Y., 1999. East Asian Financial Crisis: One Year After. In *IDS bulletin* Vol. 30, No. 1, edited by Stephany-Griffith Jones, Institute of Development Studies, University of Sussex.

[60] Pinkse, J., and Slade, M. E., 1998. Contracting in space: an application of spatial statistics to discrete-choice models. *Journal of Econometrics*, 85, pp. 125-154.

[61] Pinske, J., Slade, M. E., and Brett, C., 2002. Spatial Price Competition: a Semiparametric Approach. *Econometrica*, 70, pp. 1111-1153.

[62] Rey, S.J., and Boarnet, M.G., 1998. A taxonomy of spatial econometric models for simultaneous equation systems. Department of Geography, San Diego State University.

[63] Rigobon, R., 1998. Information Speculative Attacks: Good News is No News. *MIT Working Paper.*

[64] Rigobon, R., 1999. On the Measurement of the International Propagation of Shocks. *NBER Working Paper* 7354.

[65] Ripley, B. D., 1981. *Spatial Statistics*. Wiley, New York.

[66] Shroder, M., 1995. Games the States Don't Play: Welfare Benefits and the Theory of Fiscal Federalism. *The Review of Economics and Statistics*, pp. 183-191.

[67] Stigler, G. J., 1957. The Tenable Range of Functions of Local Government. In Joint Economic Committee, Subcommittee on Fiscal Policy, *Federal Expenditure Policy for Economic Growth and Stability*, Washington, DC, 1957, pp. 213-219.

[68] Valdes, R., 1997. Emerging Markets Contagion: Evidence and Theory. *Mimeo, MIT.*

[69] Van Rijckeghem, C. and Weder B., 2001, Sources of Contagion: Is It Finance or Trade? *Journal of International Economics* 54, 293-308.

[70] Whittle, P., 1954. On stationary processes in the plane. *Biometrica,* 41, pp. 434-449.

[71] Wincoop, E. van, and Yi, K-M, 2000. Asian Crisis Post-Mortem: Where Did the Money Go And Did the United States Benefit? *Federal Reserve Bank of New York Economic Policy Review* 6(3) 51-70.