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On the Variance Reduction Property of Buffered Leaky Bucket

by L. Kuang

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On the Variance Reduction Property of Buffered Leaky Bucket

Lei Kuang

Department of Electrical Engineering

and

Systems Research Center

University of Maryland at College Park

College Park, MD 20742

Abstract

In this paper, we formalize the intuition that the Leaky Bucket (LB) smooths traffic. By sample path comparisons we show that the inter-departure times of the traffic through a LB are more even than the inter-arrival times of the original traffic in terms of majorization. We also prove that in steady state, the inter-departure time is smaller than the inter-arrival time in the sense of convex ordering. This leads to the conclusion that the coefficient of variation of the inter-departure time is smaller than that of the inter-arrival time.

Key words: leaky bucket, majorization, coefficient of variation.

1 Introduction

Due to the difficulties in reacting to congestion in future Broadband Integrated Services Digital Networks (BISDN), preventive flow control algorithms have received considerable attention in recent years [1]. In particular, one promising preventive flow control algorithm is the so-called Leaky Bucket (LB) [10]. Although there are many variations of LBs, here we are interested in the one analyzed in [3, 4, 8]. A LB is composed of an input buffer, a token pool, and a mechanism for generating tokens at a constant rate. If the capacity of the token pool is reached while a new token is generated, the newly generated token is discarded. Cells (short, fixed length packets) from the source must first obtain tokens from the token pool before entering the network. Cells which have obtained tokens enter the network instantaneously, whereas cells which cannot get a token upon arrival have to wait in the input buffer. If in addition the input buffer is fully occupied, the arriving cell will be either dropped immediately (policed) [2] or marked with a violation tag and passed to the network with the lowest priority [4, 5]. Whenever congestion is detected inside the network, cells with violation tags will be dropped first.

It is widely agreed that such a LB has a smoothing effect on the traffic. However, very few attempts have been made to analyze this property. In [4], the authors show by numerical examples that for a source modeled as a two state Markov modulated Poisson process (MMPP), the LB reduces the peakedness of the traffic. For a Poisson source, expressions of the second moments of the interdeparture times from the LB in steady state were derived in [8], and numerical examples indicate that the coefficient of variation of the traffic is reduced by the LB.

In this paper, we formalize this intuitive notion in some precise sense. In

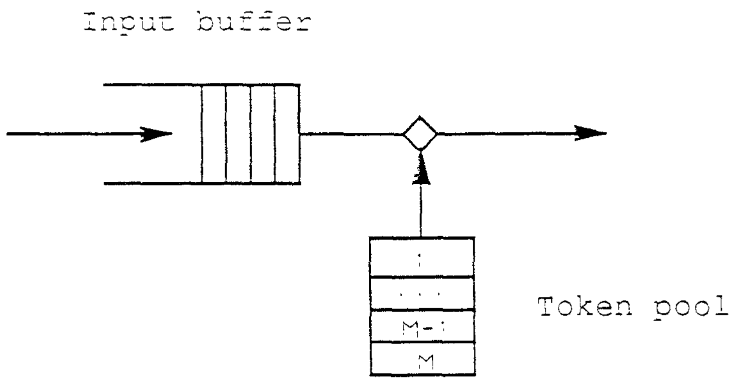

Figure 1: A buffered leaky bucket.

section 2, we describe the model in detail, and present the main results in section 3. After briefly reviewing the necessary material of majorization theory in section 4, we provide a proof of the results in section 5.

2 The Model

If we assume the input buffer to have infinite capacity, a LB (Fig. 1) is characterized by two parameters M and D, where M is the size of the token pool, and D is the token generation period which is assumed constant. Tokens are generated at times $\{kD, k = 0, 1, \cdots\}$. Cells are tagged upon arrival in the order of their arrival; and we assume that the first cell arriving at time $t = 0$ finds an empty buffer. We also assume that the input buffer operates as a FCFS (first come first served) queue. These assumptions are made only for notational convenience and do not affect the results obtained in this paper.

The evolution of the LB can be easily described by two sequences of $\mathbb{R}_+$-valued random variables (rvs) $\alpha \triangleq \{\alpha_n, n = 1, 2, \cdots\}$ and $\delta \triangleq \{\delta_n, n = 1, 2, \cdots\}$ with $\alpha_1 = \delta_1 = 0$. We interpret α_1 and δ_1 as the first arrival and departure epochs, respectively, and for $n = 1, 2, \cdots$, we interpret by α_{n+1} and δ_{n+1} as the inter-arrival and inter-departure times between the n^{th} and the $(n+1)^{st}$ cells, respectively. The inter-arrival times may be zero to represent batch arrivals.

In order to obtain the results, we need to impose some conditions on both the input and output traffic and on the LB. To this end, we introduce the notion of convex stability. For any $\mathbb{R}_+$-valued sequence of rvs $\boldsymbol{\zeta} \triangleq \{\zeta_n, n = 1, 2, \cdots\}$, we say that $\boldsymbol{\zeta}$ is *convexly stable* if there exists an integrable $\mathbb{R}_+$-valued rv ζ (i.e., $\mathbb{E}[\zeta] < \infty$) such that for any convex function $\phi : \mathbb{R} \to \mathbb{R}$,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \phi(\zeta_i) = \mathbb{E}[\phi(\zeta)] \qquad a.s. \tag{1}$$

We call ζ the asymptotic version of $\boldsymbol{\zeta}$. Note that the convex stability of $\boldsymbol{\zeta}$ implies

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \zeta_i = \mathbb{E}[\zeta] \qquad a.s. \tag{2}$$

In particular if the inter-arrival time sequence $\boldsymbol{\alpha}$ is convexly stable, then the (long-run) cell arrival rate λ is well defined by

$$\lambda = \left(\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \alpha_i \right)^{-1}.$$

We shall say that the LB is stable if

$$\lambda D < 1. \tag{3}$$

In this paper we mainly consider the case where (3) is satisfied, although the transient results hold without this condition.

With these definitions we are ready to present our main results.

3 Main Results

Our main result is Theorem 3.1 which asymptotically orders the inter-arrival times and the inter-departure times of the LB in the sense of convex ordering. We say that an $\mathbb{R}$-valued rv X is smaller than another $\mathbb{R}$-valued rv Y in the sense of *convex ordering*, and write $X \leq_{cx} Y$, if

$$\mathbb{E}[\phi(X)] \leq \mathbb{E}[\phi(Y)] \tag{4}$$

for all convex functions $\phi : \mathbb{R} \to \mathbb{R}$ whenever the expectations exist in (4) [9].

Theorem 3.1 *Assume $\boldsymbol{\alpha}$ to be convexly stable and (3) to hold, then if $\boldsymbol{\delta}$ is also convexly stable, we have*

$$\delta \leq_{cx} \alpha, \tag{5}$$

where α and δ are the asymptotic versions of $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$.

A result in the vein of (5) can be developed to cover the case where (3) does not hold. Before proving Theorem 3.1 and its extension, we first look at its implications and give examples where the convex stability condition holds. The proof of Theorem 3.1 will be postponed until section 5.

The following corollary is a well-known consequence of the definition of convex ordering.

Corollary 3.2 *Under the assumptions of Theorem 3.1, we have*

$$\mathbb{E}[\alpha] = \mathbb{E}[\delta], \tag{6}$$

and

$$\mathbb{E}[\delta^k] \leq \mathbb{E}[\alpha^k], \qquad k = 2, 3, \cdots. \tag{7}$$

A frequently used measure for variability of rvs is the *coefficient of variation.* The coefficient of variation $c^2(\xi)$ of an $\mathbb{R}_+$-valued rv ξ is defined by

$$c^2(\xi) \triangleq \frac{\mathrm{Var}(\xi)}{(\mathbb{E}[\xi])^2}.$$

It immediately follows from Corollary 3.2 that

Corollary 3.3 *Under the assumptions of Theorem 3.1,*

$$c^2(\delta) \leq c^2(\alpha). \tag{8}$$

Now we have formally shown that the LB does indeed smooth the traffic in terms of the coefficient of variation.

The convex stability condition (1) is not too restrictive; it holds for many arrival processes of interest as we now indicate.

Example 1 – Renewal processes. By the strong law of large numbers, the convex stability condition is satisfied for renewal processes which have finite first moment. This example covers the situation discussed in [8].

Example 2 – Stationary processes. The convex stability condition is justified for stationary ergodic processes with finite first moment by the Strong Ergodic Theorem [6, Theorem 5.4, p. 483].

As an example, we can show that the MMPP is convexly stable if the underlying Markov chain is stationary.

The convex stability condition holds for many non-stationary processes as well. For example, any deterministic and periodic arrival processes with finite interarrival times are convexly stable, but they are obviously non-stationary except in the trivial cases where the period is one.

4 Majorization

Before proceeding to the proof of Theorem 3.1 and its extension, we first review some definitions and results in majorization theory which we shall use. For details on majorization and its applications, the reader is referred to the book by Marshall and Olkin [7].

For any vector $\boldsymbol{x} = (x_1, \ldots, x_n)$ in $\mathbb{R}^n$ we denote by $x_{[i]}$ the i^{th} largest element of $\boldsymbol{x}$, $i = 1, 2, \cdots, n$, i.e.,

$$x_{[1]} \geq x_{[2]} \geq \cdots \geq x_{[n]}. \tag{9}$$

For vectors $\boldsymbol{x} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$ in $\mathbb{R}^n$ such that

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i, \tag{10}$$

we say that $\boldsymbol{x}$ majorizes $\boldsymbol{y}$, and write $\boldsymbol{y} \prec \boldsymbol{x}$, if

$$\sum_{i=k}^{n} y_{[i]} \geq \sum_{i=k}^{n} x_{[i]}, \qquad k = 1, \cdots, n. \tag{11}$$

If condition (10) is dropped from the definition, we say that $\boldsymbol{x}$ weakly supermajorizes $\boldsymbol{y}$, and write $\boldsymbol{y} \prec^w \boldsymbol{x}$, if (11) holds.

Majorization and weak majorization are related by the following lemma [7, p. 11].

Lemma 4.1 *For $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{R}^n$, $\boldsymbol{y} \prec^w \boldsymbol{x}$ iff there exists a vector $\boldsymbol{z}$ in $\mathbb{R}^n$ such that $\boldsymbol{y} \prec \boldsymbol{z}$ and $\boldsymbol{z} \geq \boldsymbol{x}$, where $\boldsymbol{z} \geq \boldsymbol{x}$ means $z_i \geq x_i$, $\quad i = 1, 2, \cdots, n$.*

The following lemma is also called the closure property of majorization under concatenation [7, Proposition 5.A.7, p. 121].

Lemma 4.2 *For vectors $\boldsymbol{x}$, $\boldsymbol{y}$ in $\mathbb{R}^n$, and $\boldsymbol{b}$, $\boldsymbol{c}$ in $\mathbb{R}^m$, define vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in $\mathbb{R}^{n+m}$ by*

$$\boldsymbol{u} = (x_1, \ldots, x_n, b_1, \cdots, b_m) \quad \textit{and} \quad \boldsymbol{v} = (y_1, \ldots, y_n, c_1, \cdots, c_m),$$

then

$$\boldsymbol{y} \prec \boldsymbol{x} \quad \textit{and} \quad \boldsymbol{c} \prec \boldsymbol{b} \quad \textit{imply} \quad \boldsymbol{v} \prec \boldsymbol{u}.$$

The result remains true if $\prec$ is replaced by $\prec^w$.

An important characterization of majorization that we shall need is contained in the following lemma [7, Proposition 4.B.1, p. 108].

Lemma 4.3 *For vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{R}^n$, we have*

$$\boldsymbol{x} \prec \boldsymbol{y} \quad \textit{iff} \quad \sum_{i=1}^{n} \phi(x_i) \leq \sum_{i=1}^{n} \phi(y_i),$$

for all convex functions $\phi : \mathbb{R} \to \mathbb{R}$.

For weak majorization, we have a similar characterization which is an immediate consequence of [7, Theorem 3.A.8, p. 59].

Lemma 4.4 *For vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{R}^n$, we have*

$$\boldsymbol{x} \prec^{w} \boldsymbol{y} \quad \textit{iff} \quad \sum_{i=1}^{n} \phi(x_i) \leq \sum_{i=1}^{n} \phi(y_i),$$

for all decreasing convex functions $\phi : \mathbb{R} \to \mathbb{R}$.

5 Proofs and Discussions

5.1 Proofs

As we shall see shortly, Theorem 3.1 is a consequence of the transient behavior of the traffic to and from the LB, and the convex stability assumption. The following lemma and its corollary describe this behavior. To simplify the notation, we use the following convention. For any sequence $\boldsymbol{\xi} = \{\xi_n, n = 1, 2, \cdots\}$ of $\mathbb{R}$-valued rvs, we define the $\mathbb{R}^{m-n+1}$-valued rvs $\boldsymbol{\xi}_{m,n}$, $m \leq n, n = 1, 2, \cdots$ by

$$\boldsymbol{\xi}_{m,n} \triangleq (\xi_m, \cdots, \xi_n).$$

Let $Z^{(1)}$ be the collection of tags of cells which obtain tokens immediately after their arrival, i.e., all cells whose tags are in $Z^{(1)}$ do not need to wait in the LB. For these cells, we have the following results whose proof will be given later.

Lemma 5.1 *We have*

$$\boldsymbol{\delta}_{1,n} \prec \boldsymbol{\alpha}_{1,n}, \qquad n \in Z^{(1)}. \tag{12}$$

Combining (12) with Lemma 4.3, we then conclude

Corollary 5.2 *For any convex function $\phi : \mathbb{R} \to \mathbb{R}$, we have*

$$\frac{1}{n}\sum_{i=1}^{n} \phi(\delta_i) \leq \frac{1}{n}\sum_{i=1}^{n} \phi(\alpha_i), \qquad n \in Z^{(1)}. \tag{13}$$

Note that the results obtained so far hold without any conditions. We are now ready to the proof of Theorem 3.1.

Proof of Theorem 3.1: It immediately follows from assumption (3) that $Z^{(1)}$ is countably infinite. Taking limits on both sides of (13) as $n \to \infty$ along the subsequence $Z^{(1)}$, and applying the convex stability assumptions, we get

$$\mathbb{E}[\phi(\delta)] \leq \mathbb{E}[\phi(\alpha)], \tag{14}$$

for any convex function $\phi : \mathbb{R} \to \mathbb{R}$ whenever the expectations exist. Theorem 3.1 then follows from the definition of convex ordering.

■

In order to prove Lemma 5.1, we need to look at the traffic patterns more closely.

For $n = 1, 2, \cdots$, the arrival epoch a_n and the departure epoch d_n of the n^{th} cell can be expressed as

$$a_n \triangleq \sum_{i=1}^{n} \alpha_i \text{ and } d_n \triangleq \sum_{i=1}^{n} \delta_i, \qquad n = 1, 2, \cdots.$$

For $t \geq 0$, we define

$P(t)$: the number of tokens available in the token pool at time t;

$Q(t)$: the number of cells waiting in the input buffer at time t.

By convention, if at time t a cell arrives exactly at a token generation epoch, $P(t)$ and $Q(t)$ remain unchanged. This convention removes the ambiguity when the token pool happens to be full when this situation occurs. Although different conventions lead to different sample path behavior, they will not affect the results obtained in this paper. From the definition of the LB, we see that for all $t \geq 0$ at least one of the quantities $Q(t)$ or $P(t)$ must be zero, i.e., $Q(t)P(t) = 0$; and both quantities $Q(t)$ and $P(t)$ can be uniquely recovered from their difference $S(t)$ defined by

$$S(t) \triangleq Q(t) - P(t), \qquad t \geq 0.$$

If $S(t) \geq 0$, then $Q(t) = S(t)$ and $P(t) = 0$; and if $S(t) < 0$, then $Q(t) = 0$ and $P(t) = -S(t)$. Under the assumptions made in section 2, the initial condition at $t = 0$ is necessarily $S(0) = 0$.

With $l_1^{(1)} \triangleq 1$, we recursively define

$$l_k^{(2)} \triangleq \min\{i > l_k^{(1)} : S(a_i) > 0\}, \qquad k = 1, 2, \cdots,$$

and

$$l_{k+1}^{(1)} \triangleq \min\{i > l_k^{(2)} : S(a_i) \leq 0\}, \qquad k = 1, 2, \cdots.$$

For $n = 1, 2, \cdots$, we say that the n^{th} cell is of the first kind if $l_k^{(1)} \leq n < l_k^{(2)}$, for some $k = 1, 2, \cdots$. A cell which is not of the first kind is called a cell of the second kind. We define the k^{th} *burst cycle* of the LB by

$$B_k \triangleq [a_{l_k^{(1)}}, a_{l_{k+1}^{(1)}}), \qquad k = 1, 2, \cdots.$$

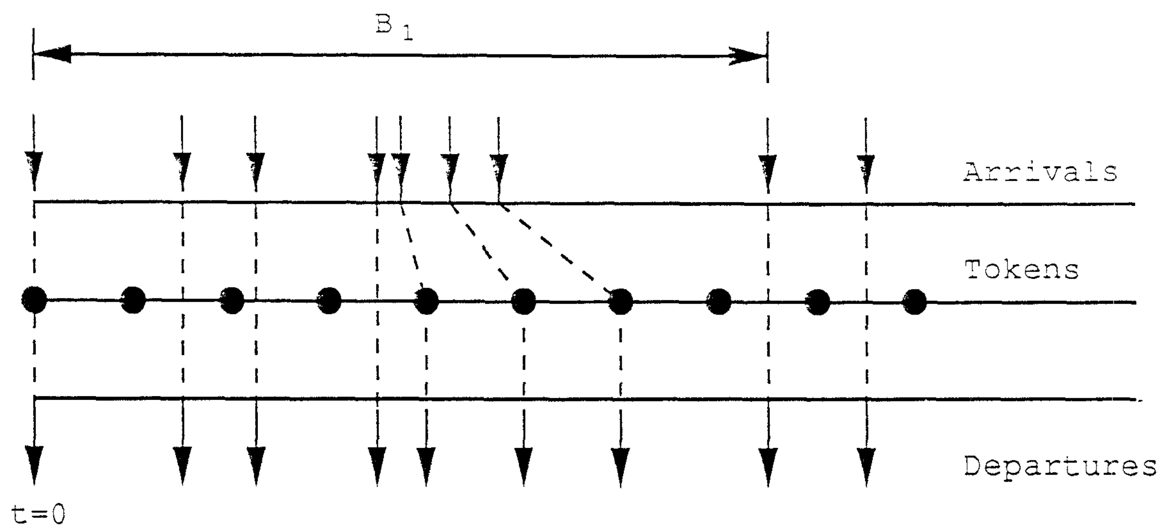

Figure 2: Input/output traffic and the burst cycle

From these definitions we see that $l_k^{(1)}$ is the tag of the first cell (of the first kind) in the k^{th} burst cycle, and $l_k^{(2)}$ is the tag of the first cell of the second kind in the k^{th} burst cycle. We see also that the set $Z^{(1)}$ is just the collection of all the tags of cells of the first kind, i.e.,

$$Z^{(1)} = \{n = 1, 2, \cdots : l_k^{(1)} \leq n < l_k^{(2)} \text{ for some } k = 1, 2, \cdots\}.$$

A typical realization of the burst cycles is shown in Fig. 2.

It is easily seen that cells of the first and second kinds are separated by each other into groups. The first burst cycle begins with a group of cells of the first kind followed by a group of cells of the second kind, and this pattern repeats itself forming successive burst cycles. Cells of the first kind do not need to wait in the input buffer as they pass through the LB without any delay; cells of the second kind, however, have to wait for tokens to be generated in order to leave the LB.

Since all burst cycles have an identical structure, we can invoke the closure property of majorization under concatenation to conclude that Lemma 5.1 need only be proved for the first burst cycle, that is for $1 \leq n < l_2^{(1)}$. From Fig. 2 we see that Lemma 5.1 is trivially true for $1 \leq n < l_1^{(2)}$. By the closure property of majorization under concatenation again, we conclude that we only need to prove the following lemma.

Lemma 5.3 *We have*

$$\boldsymbol{\delta}_{l_1^{(2)}, l_2^{(1)}} \prec \boldsymbol{\alpha}_{l_1^{(2)}, l_2^{(1)}}. \tag{15}$$

Proof: We observe from Fig. 2 that

(i) $\alpha_{l_1^{(2)}} < \delta_{l_1^{(2)}} < D \leq \delta_{l_2^{(1)}} < \alpha_{l_2^{(1)}}$;

(ii) $\delta_i = D$, $l_1^{(2)} < i < l_2^{(1)}$.

Since the $l_2^{(1)}$-th cell is of the first kind, $d_{l_2^{(1)}} = a_{l_2^{(1)}}$ and

$$\sum_{i=l_1^{(2)}}^{l_2^{(1)}} \delta_i = \sum_{i=l_1^{(2)}}^{l_2^{(1)}} \alpha_i.$$

Furthermore observe that the departure times of cells of the second kind are delayed, so that

$$a_i < d_i, \qquad l_1^{(2)} \leq i < l_2^{(1)}.$$

Thus,

$$\sum_{i=l_1^{(2)}}^{k} \alpha_i \leq \sum_{i=l_1^{(2)}}^{k} \delta_i, \qquad l_1^{(2)} \leq k < l_2^{(1)}.$$

Now Lemma 5.3 follows from the next lemma which mimics the structure of the inter-arrival times and the inter-departure times of cells of the second kind in one burst cycle.

■

Lemma 5.4 *If vectors $\boldsymbol{x} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$ in $\mathbb{R}_+^n$ satisfy*

(a) $\quad x_1 \leq y_1 \leq D$,

(b) $\quad D \le y_n \le x_n$,

(c) $\quad y_2 = y_3 = \cdots = y_{n-1} = D$,

(d) $\quad \sum_{i=1}^{k} x_i \le \sum_{i=1}^{k} y_i, \qquad k = 1, 2, \cdots, n-1$,

(e) $\quad \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$,

then

$$\boldsymbol{y} \prec \boldsymbol{x}.$$

Proof. From the definition of majorization, by taking into account (e), we only need to show that

$$\sum_{i=k}^{n} y_{[i]} \ge \sum_{i=k}^{n} x_{[i]}, \qquad k = 2, \cdots, n.$$

This is trivial since for $k = 2, \cdots, n$, we have

$$\sum_{i=k}^{n} y_{[i]} = \sum_{i=1}^{n-k+1} y_i \ge \sum_{i=1}^{n-k+1} x_i \ge \sum_{i=k}^{n} x_{[i]}.$$

The first equality follows from (a)–(c), the second inequality follows from (d), and the last inequality follows from (9).

■

5.2 Extensions

In the case where (3) fails, we may no longer be able to take limits along the subsequence $Z^{(1)}$ as we did in the proof of Theorem 3.1, because in this case $Z^{(1)}$ may contain only finitely many elements. So without the assumption (3) Theorem 3.1 may not hold. A weaker result, however, can be obtained by taking advantage of the notion of weak majorization. We first extend Lemma 5.1 to an arbitrary n.

Lemma 5.5 *We have*

$$\boldsymbol{\delta}_{1,n} \prec^{w} \boldsymbol{\alpha}_{1,n}, \qquad n = 1, 2, \cdots. \tag{16}$$

Proof: By a similar argument as in the proof of Lemma 5.1, we only need to prove that

$$\boldsymbol{\delta}_{l_1^{(2)},n} \prec^{w} \boldsymbol{\alpha}_{l_1^{(2)},n}, \qquad l_1^{(2)} \leq n < l_2^{(1)}. \tag{17}$$

Define $\boldsymbol{\zeta}$ by

$$\zeta_i = \begin{cases} \alpha_i, & i \neq n; \\ \alpha_n + d_n - a_n, & i = n. \end{cases} \tag{18}$$

We can interprate $\boldsymbol{\zeta}$ as a modified input sequence which is the same as $\boldsymbol{\alpha}$ except that the n^{th} arrival is forced to delay a certain amount of time so that the second burst cycle begins earlier at the n^{th} arrival epoch. So by Lemma 5.1, we get

$$\boldsymbol{\delta}_{l_1^{(2)},n} \prec \boldsymbol{\zeta}_{l_1^{(2)},n}.$$

It is clear that

$$\boldsymbol{\zeta}_{l_1^{(2)},n} \geq \boldsymbol{\alpha}_{l_1^{(2)},n},$$

and (17) now follows from Lemma 4.1.

∎

From Lemmas 4.4 and 5.5, we conclude to the following result.

Corollary 5.6 *For any decreasing and convex function $\phi : \mathbb{R} \rightarrow \mathbb{R}$, we have*

$$\frac{1}{n}\sum_{i=1}^{n} \phi(\delta_i) \leq \frac{1}{n}\sum_{i=1}^{n} \phi(\alpha_i), \quad n = 1, 2, \cdots. \tag{19}$$

Prompted by (19) we define the decreasing convex ordering by requiring (4) to hold only for those mappings $\phi : \mathbb{R} \rightarrow \mathbb{R}$ which are decreasing and convex, in which case we write $\leq_{dcx}$. Now upon using (19), we can prove a theorem similar to Theorem 3.1.

Theorem 5.7 *Assume α to be convexly stable, then if δ is also convexly stable, we have*

$$\delta \leq_{dcx} \alpha. \tag{20}$$

We remark here that $\delta \leq_{dcx} \alpha$ does not imply corollary 3.3.

The results obtained so far were derived under the assumption that the input buffer is infinite. If the input buffer is finite as is usually the case in practice, we may consider the arrival process in two stages: Cells arriving at the LB are granted entrance into the LB depending upon whether or not the input buffer is full; cells who are not admitted into the input buffer (those who meet a full buffer) will never come back (and are thus rejected). Those who enter the LB see the input buffer as if it were of infinite size. Therefore, for LB with finite input buffer we have the following result.

Corollary 5.8 *If the traffic that actually enters the LB satisfies the conditions in Theorem 3.1, then the output traffic is less variable than the actual input traffic in the sense of Corollary 3.3.*

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