ABSTRACT

Title of dissertation: PRESCHOOLERS' EARLY MATH EXPERIENCES IN VARYING CONTEXTS: PARENT AND CHILD MATH TALK DURING PLAYFUL AND DIDACTIC ACTIVITIES

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The home numeracy environment, particularly parent math talk, are predictive of children's early math development, yet it is not clear what contexts produce high-quality parent-child exchanges about math. Both formal math learning activities and informal activities where math is embedded in the task have been linked to children's math knowledge; however, there is a need for experimental studies investigating the contextual factors that contribute to how parents and children engage in math talk during joint activities. The current study investigated parent and child talk about fractions and numbers during didactic and playful math activities as well as an unguided play context.

Seventy-two dyads of parents and preschoolers were assigned to one of three conditions (Didactic Instruction, Guided Play, Unguided Play) to participate in an activity intended to promote understanding of fractions. The conditions varied in the extent to which the activity was structured, as well as the instructions and materials provided. The quantity and quality of parent and child math talk were analyzed; children's fraction

knowledge was assessed before and after the activity. Parents also completed a survey reporting enjoyment of the task and whether they believed it could promote math learning.

Dyads in the more structured didactic and playful math contexts engaged in greater proportions of, and more diverse, math talk than dyads in the unguided play context. Dyads in the didactic math context also used a greater proportion of, and more diverse, math talk than dyads in the playful math context. Despite the differences found in math talk, no change in children's fraction knowledge was found after participating in the parent-child interaction. Interestingly, parents in the playful math activity context rated the interaction as being as enjoyable as did the parents in the unguided play activity; however, parents in both structured math contexts (playful and didactic) were equally likely to indicate that their respective activities would promote math learning. These findings support the importance of providing guidance to parents for engaging their children in high-quality math talk and highlight the need for further research investigating qualitative differences in parent-child interactions in didactic and playful contexts.

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by

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Chapter 1: Introduction

Statement of the Problem

The current initiatives in U.S. schools to promote science, technology, engineering, and math (STEM; President's Council of Advisors on Science and Technology, 2010) are based in the need for a competitive workforce in the global marketplace. However, a recent report from the U.S. Department of Education (2009) indicates that more than a quarter of U.S. 8th graders fall below basic levels of math achievement. The mathematical knowledge and skills that children possess at the time that they enter school have a lasting impact on subsequent academic performance. For example, Duncan et al. (2007) found that kindergarteners' understanding of math concepts predicted math achievement in later elementary school. Given that there are already gaps in children's math knowledge by the time that they enter preschool (Dowker, 2008; Starkey & Klein, 2008), it is clear that experiences in the home environment contribute to early math development.

There is mounting evidence of a relation between parent talk and engagement in math activities and children's early math knowledge. Studies examining parent reports of the "home numeracy environment" (e.g., frequency of engaging in formal number activities, games involving numbers, or everyday activities requiring numbers such as cooking or telling time) have found links between the quality of the home environment and children's concurrent math abilities (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Skwarchuk, 2009) as well as children's later math abilities (e.g., Anders et al., 2012; Manolitsis, Georgiou, & Tziraki, 2013; Niklas & Schneider, 2013). However, it is still unclear under what circumstances parents most effectively contribute to ensuring that children enter school equipped with the foundational math knowledge that will help them succeed academically. Consequently, while it is known that parents matter in children's early math development, it is still unclear what exactly should be advised to parents regarding how they can promote children's mathematical thinking.

An important step is to identify which contexts promote math development in the home environment. For example, the academic context of formal math activities, in which children's learning is the primary focus (e.g., memorizing math facts, counting) may prompt parents to provide more math-specific guidance that is oriented towards children's learning. On the other hand, informal math activities, in which children's learning is embedded in a playful or everyday activity (e.g., board games, cooking), are likely to occur on a regular basis, and consequently there may be more opportunities for parents to teach math through informal activities. Based on the current literature, it is unclear at this time whether engagement in formal or informal math activities are the best predictors of children's early math knowledge (LeFevre et al., 2009; Ramani, Rowe, Eason, & Leech, 2015). Some studies of parent-reported math activities have not differentiated between formal and informal activity contexts, while those that have parsed apart formal and informal activities have yielded inconsistent findings.

A limitation of parent-reported frequency of home math activities is there is no way to know *how* parents are engaging their children during the activity. For example, it is known that board games offer opportunities for children to learn about numbers (Ramani & Siegler, 2008; Siegler & Ramani, 2008), but it cannot be assumed that all parents engage their children in meaningful number talk while playing board games. While the observational studies that have been conducted provide more insight into the quality of parent-child math talk during joint activities, they often only include one activity so that is unclear how much the context of the activity is driving how the parents engage their children. Few studies have examined contextual effects on math activities. Thus, to move towards a fuller understanding of how parents contribute to children's early math development, there is a need for research that systematically manipulates the context of the activity while controlling the aspects of the activity related to math skills.

Study Rationale

Studies have examined the processes that occur during math-related activities to identify what types of parent-child exchanges are most effective in successfully promoting children's math development. One aspect that appears to have a cumulative effect is the amount of number talk that parents provide to their children (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010), especially more advanced number talk (Gunderson & Levine, 2011). Furthermore, parent talk about specific math concepts is predictive of children's abilities in the same areas of math (Gunderson & Levine, 2011; Pruden, Levine, & Huttenlocher, 2011). Importantly, there is substantial variation in the amount of number talk that children are exposed to in the home (Levine et al., 2010; Saxe, Guberman, & Gearhart, 1987). This is likely related to variations in the quality of the home math environment; children who engage in fewer math-related activities will have fewer opportunities to have math-related exchanges with their parents.

An issue that needs to be addressed is how and why different types of activities in the home math environment contribute to math development. Specifically, the home environment often includes both formal math activities that are directly intended to promote math skills, such as counting or practicing math facts, as well as informal math activities where children may be exposed to math, but it is not the primary goal of the activity (Manolitsis et al., 2013). These informal activities may be further subdivided into categories related to practical everyday applications of math such as measuring ingredients or using money and play activities such as board games or card games that involve number. LeFevre et al. (2009) found that both formal and informal activities reported by parents predicted the math fluency of 5- to 7-year-olds, although it should be noted that Ramani et al. (2015) found that, compared to informal activities, the frequency of formal activities reported by parents was a stronger predictor of math ability in low-income preschoolers. Consequently, there is a need to further explore the conditions under which formal and informal math activities contribute to children's learning in the home environment. Understanding the mechanisms through which parent-child joint activities contribute to children's math development may offer insight into the matter.

From a Vygotskian, or sociocultural, perspective, math-related experiences in the home environment provide an opportunity for children to engage in activities with the guidance of a more experienced partner, often a parent (Vygotsky, 1934/1986). The benefit of working with an advanced partner is that it allows children to participate in activities that they would not be able to accomplish alone. Parents are often able to provide an appropriate level of assistance to children, enabling them to actively participate in the task; as children become more masterful, parents frequently adjust their level of support (Wertsch, 2008; Wood & Middleton, 1975). Using this theoretical framework, the significance of the home math environment is that it allows children to participate in math activities with the support of a parent, where parents gradually transmit the concepts and strategies to the children.

As noted, previous studies on math activities in the home environment have distinguished between formal and informal activities, emphasizing whether children's learning is assumed to be the primary goal of the activity. However, what may be more important to consider is how the activity is structured, regardless of what the goal is. In particular, the structure of the activity may influence how both parents and children engage in the task. For example, the same formal activity intended to promote children's learning could be structured so that it is either adult-driven or child-driven, which may influence how both partners participate. There is a growing body of work examining the roles of teachers and learners in different contexts, such as whether an activity is structured by the teacher or the learner. Guided play, a type of discovery-based learning, has been found to be a more effective approach than either unstructured activities or didactic instruction (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013). The key characteristics of guided play are that it allows the child to take on an active role in their exploration and learning, takes place during an engaging activity, and is structured in a way to guide children towards focusing on the elements of the activity that will promote learning. In other words, it is adult-initiated, but child-directed (Weisberg, Hirsh-Pasek, & Golinkoff, 2013).

Whether or not an activity lends itself to guided play is at least partially contingent on the role that the parent assumes in the activity. Several factors may play a role in whether parents take a guided play approach during an activity. With regard to activities that can promote math learning, some tasks automatically may be more likely to draw parents' (and consequently children's) attention to the mathematical features of the activity because achieving the goal inherently requires talk about math concepts, such as board games or cooking (e.g., Vandermaas-Peeler, Boomgarden, Finn, & Pittard, 2012; Vandermaas-Peeler, Ferretti, & Loving, 2012). In other instances, the relevance to math may be more salient through the materials used, such as items required to play grocery store (e.g., a scale or pretend money; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009) or a storybook that involves numbers (Mix, Sandhofer, Moore, & Russell, 2012).

Studies comparing parent talk and guidance during math activities across varying contexts are quite limited and with the exception of a study of spatial talk (Ferrara, Hirsh-Pasek, Newcombe, Golinkoff, & Lam, 2011) none have specifically examined the construct of guided play. Only a few studies have examined contextual effects on math activities (e.g., Sun & Rao, 2012) and only one study has compared two different activities designed to tap into the same math concepts. Bjorklund, Hubertz, & Reubens (2004) found that parents provided more guidance while working on formal arithmetic problems with their 5-year-old children, compared to during playing a board game that also required the use of similar arithmetic strategies. One potential explanation for the greater amount of parental guidance that Bjorklund et al. (2004) observed in the formal arithmetic problems context is that the clear emphasis on math prompted parents to focus more on teaching their children. In the board game context the opportunity to teach – and how to do so – may have been less obvious to parents. These findings suggest that it is important for parents to be aware of the teaching opportunities in a task in order to structure the interaction in a way that is child-driven and qualifies as guided play.

Other studies have essentially assisted parents in taking a guided play approach (although they do not refer to it as such) by offering suggestions for how to engage children in number talk during informal or playful activities. When parents receive such support, they engaged in greater frequencies of math talk (Vandermaas-Peeler, Boomgarden et al., 2012; Vandermaas-Peeler, Ferretti et al., 2012). This notion of guided play enhancing the quality of parent talk is further exemplified by Ferrara et al.'s (2011) study in which parents in a guided play condition provided more spatial talk to their children than parents in a free play condition; in turn, children in the guided play condition also engaged in more spatial talk.

While these studies pertaining to guided play and parent math talk have not examined differences in children's learning from guided play versus unguided play or formal instruction, additional research suggests that guided play might promote more learning. Fisher and colleagues (2013) found that children who engaged in a guided play activity about geometric concepts with an experimenter later performed better on a shape sorting task than children who worked with the experimenter in a didactic instruction condition where they were taught the same concepts without being able to interact with the materials, or children in an unguided play condition where they were allowed to play with the materials without any guidance or instruction. Fisher et al. suggested that guided play not only directs children's attention to the characteristics of the task relevant to learning the concept but also allows them to explore and experiment with the materials to promote a deeper understanding. In contrast, children in the didactic instruction condition failed to learn the meaning or importance of the concepts behind the activity and children in the unguided play condition failed to attend to the key concepts when not assisted in doing so.

The next step in this line of research is to use the construct of guided play to inform a study where the activity context is manipulated in the extent that it encourages exploration and whether it is parent- or child-driven. Having distinct contexts in which the activity takes place allows for examination of how the context influences the quantity and quality of parent math talk, as well as how variations in parent math talk contribute to children's learning of math concepts. Finally, in order to move towards an understanding of the mechanisms through which parent talk and guidance contributes to children's math learning, child talk and engagement during the joint activity should also be assessed.

Based on sociocultural theory, children learn through joint activities where the more experienced partner assists them in accomplishing something beyond what they are capable of achieving on their own, and gradually children take on more responsibility and apply their newly acquired knowledge to the task while receiving support from the partner. This study aimed to examine what contexts are most optimal for promoting parent support and child engagement in adherence to this sociocultural model.

Study Design

The current study took an experimental approach to examine the conditions under which parents are best able to provide math talk that promotes children's learning of math concepts. Parents and their preschool-age children were randomly assigned to one of three conditions (Unguided Play, Guided Play, and Didactic Instruction) for engaging in an activity to promote math knowledge. These conditions parallel ones that have been used in a previous study examining how to teach math concepts (i.e., shapes) to preschoolers (Fisher et al., 2013); however, they have not been used to examine parent guidance during a math-related activity with preschoolers. As noted, parent-child math activities have frequently been categorized as formal or informal with an emphasis on the goal of the activity but have not paid attention to how parents structure the activity.

Consequently conceptualizing the activities based on the structure (i.e., Unguided Play, Guided Play, and Didactic Instruction) may help to provide a clearer picture of how parents contribute to children's math learning in different activity contexts.

The parent-child dyads were observed engaging in one of the three activity contexts, designed to focus on concepts related to fractions, specifically the concepts of partitioning whole objects into parts and the equal distribution of parts. These concepts were selected to reduce the influence of children's pre-existing math knowledge while increasing the chance to observe growth from pretest to posttest; it is a relatively novel area for preschoolers, but often introduced shortly after in kindergarten and first grade (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Furthermore, Siegler and colleagues (2010) recommended that concepts related to equal sharing and proportional reasoning should be introduced in kindergarten to lay the foundation for later fraction education. Thus the concepts were likely to be relatively unfamiliar yet within the range of what preschoolers may be able to accomplish with assistance. Previous studies have found that 5-year-olds' performance on similar assessments improved with brief sessions of training (Sophian, Garyantes, & Chang, 1997) and guided experience (Singer-Freeman & Goswami, 2001).

Parent and child math talk during the activity was transcribed and analyzed, and children's learning was assessed using a pretest-posttest design. The data were then used to examine how the context of a math activity influenced the frequency and quality of the math talk that parents provide and whether there were differences in the effectiveness of various contexts for teaching math concepts to young children.

Research Aims and Hypotheses

Broadly, the goal of this study was to examine how differences in the context of the joint activity would elicit differences in the quantity and quality of parent and child math talk, ultimately influencing how much children learned from the activity. It was expected that children participating in the guided play activity would learn the most because the activity context would prompt parents to not only engage in more math talk but to do so in a collaborative manner, which would actively engage children in the task in a meaningful way. Specific hypotheses are described next.

Hypothesis 1) The activity context will influence the quantity and quality of parents' and children's talk, such that:

1a: Parents in the Didactic Instruction and Guided Play conditions will engage in more math talk than parents in the Unguided Play condition. Manipulating the context in which the same materials are used, i.e., embedding the activity in a formal (Didactic), semi-formal (Guided Play), or informal (Unguided Play) context, allowed for examination of whether the context affected how the parent and child approached the task. It was hypothesized that the activity context would influence the frequency and quality of parents' math talk, since some contexts may elicit more talk about math than others. In particular, the Didactic Instruction and Guided Play conditions in which the math content of the task was explicitly stated would elicit higher frequencies of parent math talk than the Unguided Play condition. Similarly, the quality, or diversity, of parents' math talk (defined as the number of *different* math words) would be greater in the Didactic Instruction and Guided Play conditions than in the Unguided Play condition.

1b: Children in the Didactic Instruction and Guided Play conditions will engage in more math talk than children in the Unguided Play condition, and children in the Guided Play condition will engage in more math talk than children in the Didactic Instruction condition. In regards to the child behavior, since previous studies have shown a correlation between parent math talk and child math talk (Levine et al., 2010; Ramani et al., 2015), it was expected that the conditions eliciting more parent math talk (Didactic Instruction and Guided Play) would also elicit more child math talk than the Unguided Play condition. Furthermore, it was expected that compared to children in the Didactic Instruction condition, children in the Guided Play condition would engage in more math talk. Sarama and Clements (2008) have proposed that children are especially interested in math when it is clearly embedded in a meaningful task. Similarly, Vandermaas-Peeler, Boomgarden, et al. (2012) observed that children were most engaged in answering their parents' math questions when they were salient to completing a cooking activity, rather than a tangential question. Given the interactive, exploratory nature of the Guided Play condition, it was expected to be the most engaging for children and consequently would elicit the most math talk, both in terms of quantity and diversity.

1c: Parents in the Didactic Instruction condition will engage in a greater amount of talk than their children, thereby driving the interaction more than their children, whereas in the Unguided Play condition, children will drive the interaction and engage in a greater amount of talk than their parents; there will be a balance between parent and child talk in the Guided Play condition. Sociocultural theory proposes that children learn through collaborative exchanges with a more experienced partner, where both partners are active participants (Rogoff, 1998; Wertsch, 2008). It was expected that the Guided Play condition, in which the parent guides the direction of the activity in a way that promotes children's exploration, would be the most likely to equally engage both partners, which would be reflected in a relatively equal share of the conversational load.

Hypothesis 2) Children's learning of fraction concepts will vary as a function of activity context, such that children in the Guided Play condition will demonstrate the greatest improvement and children in the Didactic Instruction condition will demonstrate more improvement than children in the Unguided Play condition. Children's fraction knowledge was assessed before and immediately after the parent-child interaction. Currently there is mixed evidence regarding the relation between different math activities in the home and children's math knowledge: while parent reports of formal and informal activities predict children's math knowledge (e.g., LeFevre et al., 2009; Ramani et al., 2015), studies observing parent-child interactions during informal math activities have failed to find an effect of parent number talk during the activity on children's learning (e.g., Vandermaas-Peeler, Boomgarden et al., 2012). By using an activity that focuses on a very specific math concept (i.e., fractions, as related to partitioning whole objects into parts) and assessing children's understanding of this and related concepts, it may be more plausible to observe growth in a short time period.

Based on previous findings, it was expected that children in the Guided Play condition would demonstrate the most improvement in their fraction knowledge, compared to children in the Didactic Instruction and Unguided Play conditions. Alfieri et al.'s (2011) meta-analysis found that "enhanced discovery-based learning" where learners' exploration was guided was more effective than direct instruction; in contrast, unassisted discovery was not found to be beneficial to learning. Similarly, Fisher et al. (2013) found that preschoolers showed the most improvement in shape knowledge when taught through guided play compared to those taught through didactic instruction or unguided play; children taught through didactic instruction showed slightly more improvement than those in unguided play. It was expected that these previous findings will be upheld by the current study in which parents play the role of teacher.

Hypothesis 3) Differences in children's posttest fraction knowledge will be explained by differences in the parent and child fraction talk during the joint activity, such that:

3a: Greater improvement in children's fraction knowledge will be positively associated with higher frequencies of parents' fraction talk. Previous studies have found that parent talk about math contributes to children's math abilities (Levine et al., 2010), especially when the talk pertains to the same domain of math knowledge as is being assessed (Gunderson & Levine, 2011; Pruden et al., 2011). Thus it was expected that frequencies of parent math talk during an activity related to fraction concepts would contribute to improvement in children's fraction knowledge.

3b: The relation between parent fraction talk and children's learning will be mediated by children's fraction talk during the joint activity. In order to explain why parent fraction talk contributed to children's learning, child fraction talk was also examined. From the sociocultural perspective it is argued that learning occurs through the transmission of language from the advanced partner to the learner. Thus children whose parents engaged them in fraction talk should have more opportunities to use the language themselves, which in turn should result in a stronger understanding.

Contribution to the Field

This study builds upon existing correlational and observational studies by taking an experimental approach to refine the current understanding of how context and parent input contributes to early math learning, and aligns with the tenets of sociocultural theory that learning occurs when embedded in activities that are culturally meaningful and encourage children's active participation. Examining the input that parents provide in three distinct activity contexts allows for drawing conclusions regarding the types of activities that are likely to engage young children in mathematical thinking and discussion. As there is support for the importance of the home numeracy environment (e.g., Anders et al., 2012; Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Manolitsis et al., 2013; Niklas & Schneider, 2013), findings such as those from this study help to clarify whether we should be encouraging parents to spend more time engaging in formal, school-like activities geared towards math or providing them with guidance on how to incorporate more math talk into everyday activities or play.

Additionally, this study offers insight into how parents talk with their young children about fractions. While previous math talk studies have focused on parent talk about whole numbers or spatial concepts, there has not been any research on parent talk about fractions despite the fact that they have been proposed as a particularly critical component of mathematical understanding (Siegler, Fazio, Bailey, & Zhou, 2013). Thus, this study lays the foundation for researching children's early experiences with fractions in the home environment as part of developing foundational math knowledge that may contribute to later math achievement in school.

Chapter 2: Review of the Literature

Overview

This review examines the existing literature on parental contributions to children's numerical development in early childhood. First, the sociocultural theoretical perspectives of how parents contribute to children's early cognitive development are discussed. Next, several examples of how cultural contexts contribute to development of mathematical abilities during early childhood to demonstrate specifically how the sociocultural framework may be used to explain development in the realms of numerical understanding are provided. After reviewing the theoretical perspectives of why parents are an important contributor to children's learning at an early age, the current empirical research findings on parents' contributions to children's mathematical development during preschool are presented.

It is important to note that early mathematical experiences are not limited to number; young children are also developing spatial reasoning, and early experiences and parental input related to spatial reasoning are predictive of children's performance on spatial tasks (Levine, Ratliff, Huttenlocher, & Cannon, 2012; Pruden et al., 2011). While children are acquiring additional math skills in the early home environment, it has been recommended by the Common Core State Standards Initiative that "more learning time in Kindergarten should be devoted to number than to other topics" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Consequently, for the purpose of this paper, the focus is on research related to numeracy-related concepts, such as counting, magnitude, and arithmetic, as these are the skills that are likely to be emphasized upon children's entry into school.

Parental Contributions to Early Learning from a Sociocultural Perspective

One of the central tenets of Vygotsky's sociocultural theory is that human development and behavior cannot be understood without understanding the cultural and historical contexts of an individual's environment (Kozulin & Falik, 1995). In fact, it is the cultural and historical context of the environment that determines *how* individuals develop and, ultimately, how they think. Gauvain, Beebe, and Zhao (2011) describe three ways in which culture influences cognition: social processes in which children's learning is supported; the everyday activities in which these social processes occur; and the socially-transmitted, cultural tools (both symbolic and material) that support thinking (p. 122). All of these aspects occur within a child's Zone of Proximal Development (ZPD), which is described as the range between what an individual can accomplish independently and what they can accomplish with assistance (Gauvain, 2001a; Rogoff, 1998; Vygotsky, 1934/1986; Wertsch, 2008).

Social processes. Within children's ZPDs, experienced partners provide support allowing children to participate in ways that they could not on their own, ultimately advancing children's learning and ability to accomplish the activity more independently (Rogoff, 1998; Wertsch, 2008). These experiences occur in numerous contexts, thus the advanced partner can be a teacher or a more experienced peer; however, given that the focus of this paper is on parent-child interactions, the processes will be described here as occurring during joint activities with parents, although much of the same would apply with another advanced partner.

During joint activities, parents provide varying forms of support, adapting the task in a way that enables children's participation by reducing the amount of responsibility placed on them (Wertsch, 2008). This process is referred to as scaffolding, and includes behaviors such as regulating attention and monitoring progress, simplifying the task, providing feedback and motivation, and modeling how to accomplish the activity (Rogoff, 1998). As children demonstrate mastery or show a need for greater assistance, parents will often adjust the type of support they provide accordingly (Wood & Middleton, 1975). Over time the process reflects a transition from "other-regulation" to self-regulation (Wertsch, 2008).

Modern sociocultural theorists have sought to incorporate the social processes that occur within a broad range of cultural contexts, arguing that during social exchanges there are additional processes beyond explicit instruction that contribute to children's learning. For example, Rogoff (1998) has proposed the concept of guided participation, described as "a perspective for examining people's opportunities to learn through diverse processes of participation in the valued activities of their various communities" (p. 700). This perspective allows for conceptualizing learning not only occurring through direct instruction, but also "side-by-side joint participation" and observation (Rogoff, 1995). Similarly, the concept of "legitimate peripheral participation" by Lave and Wenger (1991) accounts for situations in which children learn through observing more experienced members of the community engage in activities. These theories pertaining to more observational learning highlight an important concept of sociocultural theory: Social processes are dynamic, and both partners share in the exchange. The child is not a passive participant, but is responsible for making meaning of the context, whether it is through direct instruction or peripheral observation (Rogoff, 1998; Wertsch, 2008).

Everyday activities. An essential element in the sociocultural perspective, as addressed in the discussion of guided participation is that the activities in which children participate are those that are valued by the community. These activities will vary greatly, reflecting the culturally-valued skills and knowledge considered to be important for children to acquire as members of that particular society (Gauvain et al., 2011; Rogoff, 1998). Leont'ev's activity theory expands upon Vygotsky and emphasizes participation in activities as the focus in understanding human development (Rogoff, 1998). Leont'ev suggested that both the types of activities available and how individuals participate in them depend on the economic structure of a culture, and as the economic structure of a culture changes, so will the activities (Goncu & Gauvain, 2012). Furthermore, as different activities have different requirements, each offers unique contributions to children's development (Goncu & Gauvain, 2012).

Tudge (2008) provided an example of how much children's everyday experiences vary across cultures in his extensive observational study of 3-year-olds in seven diverse countries, providing detailed descriptions of the types of activities available to children, the settings in which they occurred, and the individuals involved. While all children in all the countries were involved in play activities frequently and to a similar extent, play varied in terms of the materials used (e.g., toys designed for children's play versus using everyday household items or objects from nature) and the types of play (e.g., pretend play versus playing with academic objects). Although countries varied in the amount of play experience children had with academic-related toys, limited play with academic objects did not necessarily mean that children had limited academic experiences. Tudge found that mothers in a Korean city often provided their children with academic materials and toys at home, whereas parents in a Kenyan city often chose to place their children in academically-oriented formal child-care. Thus, both cultures share the goal of preparing their children for school; however, since the requirements for academic play may differ from participating in formal lessons, the children may acquire different skills depending on the setting.

Cultural tools. As noted, different activities have varying requirements, resulting in unique influences on development. In part, different activities require participants to engage and collaborate in different ways, influencing the types of social processes that take place. The other aspect is that different everyday activities will require different material and symbolic tools, which may be specific to the activity and/or the culture. Exposure to (and experience with) the tools of a culture "helps to organize the human mind in culturally specific ways" (p. 128; Gauvain, 2001b). Similar to how children are active participants in their social interactions with others, they are also active participants in learning to use cultural tools. Children are initially exposed to tools in a particular context, and that may contribute to when they select to use them in the future. However, uses of the tools are not predetermined, nor do tools determine human thinking; instead they contribute to thinking in a way that is consistent with the culture (Gauvain, 2001b). For example, children will sometimes use tokens to represent quantities while solving arithmetic problems; this would only be useful in a culture where precise quantities and units are meaningful.

Having broadly described how the sociocultural perspective explains parental contributions to children's early learning, how this theory specifically applies to children's early mathematical learning will now be discussed. In particular, examples from development of numeracy are used to illustrate how participation in everyday activities and experience with cultural tools contribute to the understanding of concepts related to counting, magnitude, and arithmetic.

Sociocultural Theory in a Mathematical Context

A growing body of research on infants suggests that humans are born with some innate understanding of number. Habituation studies have demonstrated that infants are capable of discriminating between (small) sets of objects, can make judgments regarding ordinality, or comparing set sizes, and may have some implicit understanding of simple arithmetic (see Geary, 2006 for review). Importantly, this early number sense appears to contribute to children's later math abilities. The acuity of infants' approximate number system (ANS), for example, is predictive of children's performance at 3.5 years of age on both standardized assessments of symbolic math knowledge as well as nonsymbolic numeracy (Starr, Libertus, & Brannon, 2013). Additionally, while the ANS is present during infancy, there is evidence that experience can improve the acuity of the system; the precision of ANS increases over development (Halberda & Feigenson, 2008) and training programs have yielded improvements in adults' ANS performance (Park & Brannon, 2013).

While children's implicit awareness of numerosity clearly contributes to their early math development, it is also important to consider how this innate ability interacts with, or is mediated by, cultural tools and concepts, specifically number words and symbols, that advance children's mathematical thinking (Benigno & Ellis, 2008; Geary, 2006; Sarama & Clements, 2008). Subitizing, or automatically recognizing the number of items in a set, may be thought of as the process of mapping specific number words to small quantities that can be implicitly recognized (Sarama & Clements, 2008). While 2-year-olds may be capable of recognizing and distinguishing sets of 2, 3, or 4 objects, it is not until the age of 3 years that children begin to appropriately label small sets with the correct number word (Geary, 2006).

Given that the innate mechanisms only allow for precise representation of up to three or four items, cultural tools that allow for representation of larger sets of items become especially important (Brysbaert, 2005). Following infancy, a substantial portion of children's early math development involves developing a strong understanding of number. During this time, children acquire a set of abilities related to counting, magnitude, and arithmetic, using the symbols and strategies of their particular cultural context. Examples of how sociocultural perspectives explain children's acquisition of these skills will be used to demonstrate how the particular experiences that children have within a specific cultural context will influence their mathematical thought and reasoning.

Counting. Gelman and Gallistel (1978) proposed that children are proficient counters when they understand five counting principles: 1) the one-to-one principle, each number word corresponds with one item to be counted; 2) the stable order principle, number words must always be assigned in the same sequence; 3) the cardinality principle, the last number word when counting a set of objects represents the total number of items in the set; 4) the order irrelevance principle, objects may be counted in any order; and 5) the abstraction principle, these principles apply to any set of objects to be counted. It can take several years from the time that children first attempt to count items to the time when they can successfully coordinate the counting principles (Mix et al., 2012). During this time, children develop each of these concepts through experiences and social exchanges in their cultural setting. For example, Mix (2009) demonstrates how a child's use of number words and early counting emerged between 12 and 38 months of age. It was noted that spontaneous use of number in everyday settings occurred well before the child was able to coordinate the counting principles on standardized math tasks.

For children learning base-10 number systems, number names from one to ten are learned through rote memorization (Ginsburg & Ertle, 2008) and most children learn this sequence by the time they are 4 years old (Geary, 2006). Beyond ten, number systems vary in regularity of number words, and the regularity of the number words appears to be linked to how quickly children learn the system (Benigno & Ellis, 2008; Geary, 2006). Frequently used as an example of a regular system, in Chinese, all words beyond ten are composed of the words one through ten: the word for 12 translates to "ten two" and the word for 25 translates to "two ten five" (Benigno & Ellis, 2008; Geary, 2006). In contrast, the English words for 11 and 12 are still arbitrary, and the pattern for 13-19 (adding "teen" to the end) is not consistent with the pattern for 20 and beyond. Furthermore, children learning the English number system must also learn the word for each decade, such as "twenty" (Ginsburg & Ertle, 2008). Not surprisingly, Chinese-speaking children tend to develop number skills earlier than English-speaking children (Benigno & Ellis, 2008).

As previously mentioned, beyond mastering the counting words, young children must understand additional counting principles. Children often have developed the principles needed for accurate counting (one-one correspondence, stable order, and cardinality) by the time they are 5 years old, although they may still believe that some standard, but not necessary, counting practices (e.g., beginning counting at the end point of a set) are essential (Geary, 2006). For example, social interactions can promote children's acquisition of the counting principles. As noted by Geary (2006), believing that items must be counted in a certain order reflects that children's concepts of counting come from observing others engage in standard counting procedures, such as counting left to right. By this time at 5 years old, children are also able to successfully perform cardinality tasks, demonstrating an understanding that the last number word represents the total quantity (Geary, 2006; Mix et al. 2012). Mix et al. (2012) demonstrated that children's acquisition of the cardinality concept is facilitated through social interactions, where partners label cardinality and then count the set. Gunderson and Levine (2011) also found that children who were exposed to more parent number talk about cardinality demonstrated better understanding of cardinality. Thus, children's understanding of counting develops through exposure to the strategies that more advanced partners use.

Magnitude. Infants and toddlers demonstrate an implicit understanding of magnitude (Geary, 2006), but in order to make more precise, consistent judgments of magnitude, children must be able to coordinate counting and cardinality with an understanding of ordinal relations (Sarama & Clements, 2008). That is, they must know that number words that occur later in the sequence represent larger quantities. Understanding ordinality requires knowledge of the counting sequence (Geary, 2006) and is further supported by the development of a mental number line (Dehaene, Izard, Spelke, & Pica, 2008; Sarama & Clements, 2008; Siegler & Booth, 2004). Siegler and Ramani (2009) proposed that the mental number line may "[serve] as a retrieval structure that improves encoding, storage, and retrieval of numerical information by organizing the information around the numbers' magnitudes" (p. 555).

Development of a mental number line may be influenced by the types of activities available for a child. In general, the accuracy of children's mental number lines for the number range 0-100 increases once they have been exposed to formal math learning in school (Siegler & Booth, 2004). However, some preschoolers possess an accurate mental number line of the numbers from 0-10 prior to entering formal schooling, although this varies with children's exposure to number lines (Opfer, Thompson, & Furlong, 2010; Siegler & Ramani, 2008). Children who have had experience with board games – where numbers are often arranged linearly – often demonstrate a better understanding of numerical magnitudes (Ramani & Siegler, 2008). Board games that present numbers in a line serve as a physical representation of a number line that children may use to help develop a more accurate mental representation, and experience playing linear board games has been associated with better numerical knowledge (Siegler & Ramani, 2009).

Similarly, Moss and Case (1999) found that fourth-grade children were already familiar with the "number ribbon" or progress bar that is typically shown on a computer screen while a program is loading or a file is transferring (e,g., when half of the bar is shaded, there is corresponding text below stating "File transfer: 50% complete). Moss and Case found that they could promote students' math knowledge by using children's pre-existing experience to teach first about percentages and then relating fraction terminology to the already familiar percentage terms. In addition, they used qualitative terms to assist in children's abilities to compare percentages (e.g., 100% means "everything", 99% means "almost everything", 50% means "exactly half").

Another case where language appears to influence children's mathematical thinking is the wording of fraction terms. In East Asian languages, such as Chinese and

Korean, the word for "one fourth" translates to "of four parts, one" (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999). This emphasis on the whole-part relation may explain the fact that Korean children tend to outperform U.S. children on fraction assessments (Paik & Mix, 2003). Furthermore, teaching U.S. children to use terminology that parallels the East Asian fraction terms resulted in significant improvements in their performance, actually beyond that of the Korean children (Paik & Mix, 2003).

These examples demonstrate how the language and the types of activities available to children within a particular culture may influence the types of cognitive tools that are available for children to utilize in conceptualizing magnitudes of both whole numbers and fractions.

Arithmetic. As with magnitude, infants and young children appear to have an implicit knowledge of arithmetic, or the effects of adding or removing objects (Bisanz, Sherman, Rasmussen, & Ho, 2005; Sarama & Clements, 2008). Gradually, as children coordinate their understanding of number concepts, magnitude, and counting, they are able to solve simple addition and subtraction problems more precisely (Geary, 2006). Studies examining children's arithmetic abilities illustrate cultural variations in strategies and the significance of context. Cultural tools intended to aid mathematical thinking can contribute to how individuals think about math even when the tools are not physically available. For example, in some countries children are trained to perform arithmetic tasks on an abacus, an instrument with beads used to represent numerical values. Extensive practice ultimately results in abacus users who are capable of constructing and manipulating a mental image of an abacus to solve arithmetic problems with great speed and accuracy (Pesenti, 2005).

A classic example of sociocultural influences on arithmetic is a study of the arithmetic strategies used by 9- to 15-year-old children in Brazil who worked as street vendors selling food. Carraher, Carraher, and Schliemann (1985) found that these children were quite proficient in solving math problems that were presented in the context of selling goods (e.g., "If one lemon costs $5, how much would 12 lemons cost?"); however they made numerous calculation errors when solving the same problems if they were presented in a formal math problem (e.g., 12 x 5). While the children had developed efficient strategies for mentally calculating informal math problems, they were not able to transfer these strategies to a decontextualized problem. This exemplifies how skills are learned in culturally valued contexts, and that the practices used are influenced by the tools available. In this instance, children were often required to solve problems mentally since paper and pencils were unavailable; the strategies developed allowed them to meet the demands of the situation.

Findings from Bjorklund and Rosenblum's 2002 study also demonstrate how context influences the use of arithmetic strategies. Six- and seven-year-old children were asked to solve similar arithmetic problems in an academic context and during a board game requiring children to move game pieces around a board based on the throw of two dice. Children used more sophisticated strategies in an academic context rather than the board game context. Bjorklund and Rosenblum suggested that the different goals in the two contexts may influence what strategies are selected. For example, during the board game, the children often would solve the problems by counting from one. In the academic context, children were more likely to rely on fact retrieval or counting from the larger addend. The latter strategies were more efficient, but also had a greater risk of error. In the game context, concerns about winning may have led to emphasizing accuracy over speed. Thus, the context in which children are exposed to math concepts may influence what strategies are most valued.

Overall, the theoretical and empirical literature suggest that the particular experiences that children have in their culture - the language and number system, the tools and strategies used to solve mathematical problems, and the contexts in which children are exposed to number - influence how children conceptualize number. While these experiences reflect two facets of sociocultural theory (i.e., cultural tools and participation in everyday activities), they do not take into account social processes, the final aspect identified by Gauvain et al. (2011). Social processes may be considered the bridge between everyday activities and cultural tools; these are the processes that explain how cultural tools are transmitted within everyday activities. The next step is to review the extant research on the social processes to further understand how parents transmit numerical concepts and how the context of the activity influences the interactions.

Parental Contributions to Numeracy Development: Number Talk

A literature search was conducted to find empirical articles involving number talk between parents and children of preschool age or younger. For the purpose of this review, the focus is limited to studies including observational data involving number talk or activities related to numeracy because these studies provided the most detailed information about the nature of number talk during parent-child interactions. Therefore, studies primarily based on parent reports of number-related activities were excluded, although several observational studies also included a parent report component. It is important to note that the majority of studies on parent-child number talk have included samples that are predominantly American, middle-class families. The implications of this will be discussed below.

Parent number talk during math-related activities. Studies examining math-related content in parental talk often have involved observations of parents and children during a wide range of everyday activities or play, in a variety of settings. One of the earliest studies (Durkin, Shire, Riem, Crowther, & Rutter, 1986), took place in a laboratory room and focused on spontaneous talk that mothers provided to their 9- to 36-month-old children using everyday objects such as lights, electrical outlets, and a box of tissues. In contrast, Skwarchuk (2009) allowed parents and preschoolers to select one of three toys to play with, such as Playdoh and cookie cutters or interconnecting blocks and plastic animals. All families then played a game in which the goal was to remove blocks from a wall until a figure on the wall fell off. In another lab-based study, Pan, Gauvain, Liu, and Cheng (2006) analyzed maternal talk during a structured task in a laboratory setting. Mothers of American and Chinese children were asked to help their 5- and 7-year-old children solve a series of problems in which they needed to distribute different amounts of pretend food to a family of troll dolls, based on a fixed proportion of how much each troll would eat (e.g., a 1:2:4 ratio between the baby, mother, and father troll). This structured task required proportional reasoning; however, the use of toys as manipulatives placed the activity somewhere between an academic and play context.

Studies taking place within the participants' homes also have varied in the types of activities observed. Levine and colleagues (2010) used naturalistic observation, following parents and their 14- to 30-month-old children during 90 minutes of their regular day. Parents were not given any instructions to engage in specific activities; toy play, book reading, and times during meals or snacks were frequently observed. Vandermaas-Peeler, Nelson, and Bumpass (2007) examined number talk during a 15-minute free play session that took place in the families' homes. Four-year-olds and their mothers were given a collection of play materials with a post office theme (e.g., mailbox, stationary, stamps, cash register, play money) and were instructed to play in whatever way they chose. Other at-home studies have provided more limited and/or structured activities. For instance, Vandermaas-Peeler and colleagues have observed 4-year-old children and parents in their homes playing a board game with numeracy and literacy elements (Vandermaas-Peeler, Ferretti, et al., 2012), as well as during a cooking activity where materials and instructions were provided (Vandermaas-Peeler, Boomgarden, et al., 2012).

Thus, the contexts in which parent number talk has been studied vary greatly in terms of setting, activities, and the level of structure. Despite variation in contexts, many studies have assessed parents' talk, and in some instances children's talk, with similar coding systems. Number talk has often been coded by specific types of number-related concepts, such as number identification, magnitude, counting, and arithmetic (e.g., Levine et al., 2010; Vandermaas-Peeler, Ferretti, et al., 2012). Some studies have coded number talk in greater detail. For instance, Gunderson and Levine (2011) distinguished between talk about small (1-3 objects) and large (4-10 objects) sets. In another example, Vandermaas-Peeler et al. (2007) coded interactions based on whether numbers were discussed as part of daily use and applications (i.e., concepts related to buying and selling with money) or mathematical concepts (i.e., concepts about numbers more generally, such as quantity, comparison, and counting).

Frequency and variation in parent number talk. Consistently, the findings from studies have revealed much variation in the amount of number-related talk that parents provide while interacting with young children. For example, Vandermaas-Peeler and colleagues (2007) found that during 15 minutes of free play, parents engaged their 4-year-olds in number talk more than once per minute, on average; however, during the entire 15 minutes, the total of numeracy interactions ranged from 6 to 41 per dyad. In observing middle-class parents reading a storybook to their 4-year-old children, Anderson, Anderson, and Shapiro (2004) found that the interactions of just 4 dyads (less than 20% of the sample) accounted for about half of the math-related discourse observed in the study. The impact of this level of variability in number talk is perhaps best explained by Levine and colleagues (2010), in discussing the findings of their longitudinal study of parent number input during everyday interactions with their children between 14 and 30 months of age:

> Some parents produced as few as four number words in more than 7.5 hr of interaction, whereas others produced as many as 257. This variation would amount to a range of approximately 28 to 1,799 number word tokens over a week. (p. 1316)

This extrapolation demonstrates that what may appear to be small differences in number talk during a brief observation could reflect substantial differences in children's cumulative early experiences with number.

It is important to note that, at least in some instances, despite the variability in parents' number-related talk, the amount that they provide is still fairly low on average. For example, when parents were instructed to read and play with their 4-year-old children as they normally would, individual types of number talk were too infrequent to analyze individually and were instead analyzed as a composite of total number talk (Vandermaas-Peeler et al., 2009). Gunderson and Levine (2011) also noted that parents' relative infrequency of number talk with their 14- to 30-month-old children required them to aggregate different kinds of number talk for analyses.

An additional finding that appears to be consistent across studies is that some types of number talk are more common than others. In studies of 4-year-old children, parents were more likely to count and identify numbers compared to the frequency with which they discussed arithmetic, comparing magnitudes, or ordinal relations between numbers. This variation in frequencies of different types of number talk were observed both when dyads played a board game (Vandermaas-Peeler, Ferretti, et al., 2012) and when dyads worked together on a cooking activity (Vandermaas-Peeler, Boomgarden, et al., 2012). This is consistent with an earlier study which found that during everyday activities and play with their 30-month-old children, parents were most likely to use numbers in labeling cardinal values and counting, although identifying numerals was less common (Levine et al., 2010).

Parents' adjustments of number talk to children's needs. According to the sociocultural perspective, a possible explanation for the varying frequencies of different types of number talk is that parents may elect to focus on specific number concepts based on their children's current abilities and knowledge. There is evidence from several studies to support this explanation. For example, Saxe et al. (1987) observed mothers assisting 2- and 4-year-old children during a counting activity and coded mothers' adjustments, or simplifications of the task. Mothers were more likely to simplify the task for younger children or children who performed more poorly on the task when unassisted, or when the task involved larger sets of objects.

More recently, Mix et al. (2012) assessed the types of number-related input that parents provided while reading both a number-oriented book and a non-numerical book with their 3-year-old children. Parents were more likely to label large (rather than small) sets of objects, perhaps reflecting an ability to adapt their input to their children's current ability level – children at this age are often able to independently identify smaller sets of numbers (Mix et al., 2012). Interestingly, parents also made more non-numerical elaborations when talking about large sets of numbers, suggested by the authors to possibly reflect an avoidance of talking about numbers that parents believe to be beyond their children's current understanding.

Findings from several longitudinal studies further demonstrate how parents may adapt their number talk based on their children's development. Vandermaas-Peeler, Ferretti, et al. (2012) assessed parents' number-related guidance during three sessions playing a number board game over the course of two weeks. In addition to coding the types of number skills parents used during the board game, parental feedback to children's responses was also coded. Parents reduced their overall guidance (i.e., asking questions and providing explanations) from the first to the third session of playing the game, reflecting an adjustment of their guidance as their children became more competent; this was especially true for prompting children to identify numbers. In the study by Durkin and colleagues (1986), mother-child number talk was observed longitudinally from 9 to 36 months. Transcripts of the interactions were used to calculate frequencies of number words in both the mothers' and the children's speech. Mothers' use of number words was fairly consistent from 9 months to 36 months; however, there were changes in the frequency of specific number words. Mothers' use of *one* and *two* decreased over time, but use of *three* and *four* increased over time.

Although the findings from these studies suggest that parents will adjust their number talk based on what they view as appropriate for their children, there is also evidence to suggest that parents do not always provide the best types of input for promoting their children's learning. Although an experimental study demonstrated that 3-1/2-year-olds are most likely to learn the cardinal word principle when exposed to a specific type of input (i.e., labeling the quantity of a set, then immediately counting the set), during book reading, only a small proportion of parents' number talk was presented in the label-then-count approach that the researchers had found to be optimal for teaching the cardinal word principle (Mix et al., 2012). Similarly, when Pan and colleagues (2006) observed 5- and 7-year-old American and Chinese children and their mothers during a series of tasks focused on proportional reasoning, less than 40% of the mothers used concept-focused instruction pertaining to mathematical relations, despite the proportional nature of the task. In other words, even when parents are prompted to provide math assistance, the quality of the assistance varies and the input provided by the parents may not tap into the learning potential from the task.

Furthermore, Durkin and colleagues (1986) noted that while parents may often try to incorporate number talk into interactions with children, there are instances where parents' spontaneous teaching may be confusing for children. For example, there may be times that a parent prompts with, "one" intending for the child to repeat, "one", whereas in other instance a parent might say, "one" as prompting the child to continue the number sequence with “two” (Durkin et al., 1986). Thus, the types of math talk that parents provide, as well as the manner in which it is presented, may not be optimal for children to learn mathematical concepts. On the other hand, some may argue that what seems like less-than-ideal guidance may actually be beneficial. Durkin et al. (1986) suggested that the ambiguity and conflict during social interactions related to number may be one pathway to promoting understanding of numerical principles.

In summary, existing research has found that there is great variability in the frequency and types of number talk that parents provide. In some instances, parents may adjust to a child’s existing number skills. However, other studies suggest that parents do not always include the types of number talk that are most conducive to the progression of children’s numeracy.

Contextual influences on parent number talk. While spontaneous number talk does occur in some instances, context can influence the frequency and types of number talk that parents provide. A number of the studies discussed show that parents may engage in number talk without any prompting during everyday activities and play (Anderson et al., 2004; Levine et al., 2010; Vandermaas-Peeler et al., 2007). For example, when mothers and children between the ages of 9 and 36 months were placed in a laboratory observation room containing no toys or objects intended to promote number talk, Durkin and colleagues (1986) found that children were exposed to numbers in both incidental contexts (e.g., numbers used as part of a conversation) and in more “contrived” teaching contexts where it appeared as intending to demonstrate number use to children (e.g., prompting children to count or recite numbers).

Adding numerical elements to the materials or activity appears to promote number talk. For instance, Mix and colleagues (2012) found that parents were more likely to use number talk with their 3-year-olds when reading a number-oriented book rather than a non-numerical book. Vandermaas-Peeler and colleagues (2009) observed that parents and 4-year-olds were more likely to initiate number talk during play (i.e., toys related to playing grocery store such as pretend food and money) than during storybook reading. Some activities inherently require mathematics, and therefore are likely to prompt talk related to numbers and math concepts. For example, playing board games often requires number identification and counting, and is likely to elicit parents' number talk (e.g., Benigno & Ellis, 2004; Bjorklund et al., 2004; Vandermaas-Peeler, Ferretti, et al., 2012). Similarly, parents have been observed to use number talk about quantity and measurements on a regular basis during a cooking activity, even when not explicitly instructed to do so (Vandermaas-Peeler, Boomgarden, et al., 2012).

Beyond the materials themselves eliciting number talk, the instructions that parents receive may also influence the amount of number talk they provide to children; however, simply encouraging parents to focus their talk on mathematics does not necessarily help. Skwarchuk (2009) observed parents while playing with their 4- and 5-year-old children; parents were instructed to focus on mathematical aspects of the activity. The most common type of math talk during the play session was number and operations (compared to geometry, measurement, algebra, and probability); more than a third of the parents were coded as including number-related content in their play. However, a quarter of the parents were coded as not including any type of math content, despite the explicit instructions to do so.

More detailed guidance on *how* to talk about numbers may increase the amount of number talk that parents use. In order to examine how parents incorporate math talk into everyday activities, Vandermaas-Peeler, Boomgarden, et al. (2012) observed parents with their 4-year-old children while following a cooking recipe with six steps. Parents were randomly assigned to one of two groups. Parents in the numeracy group were given suggested number-related activities to incorporate into each step of the recipe; parents in the comparison group were given index cards that simply provided each step of the recipe. Parent-child dialogue was coded for math-related content, such as counting, number identification, and calculation. Dyads in the numeracy group had significantly more numeracy exchanges than the comparison group. Furthermore, although the instructions given to the parents in the numeracy group did not include suggestions related to addition and subtraction, parents in this group engaged their children in more exchanges about addition and subtraction. Therefore, parents who were prompted to talk about numbers did so not only in the ways suggested, but also went beyond those suggested concepts.

Similar findings were observed when parents were given guidance for talking to their children during a board game. Vandermaas-Peeler, Ferretti, et al. (2012) randomly assigned pairs of parents and 4-year-old children to one of two conditions, and asked them to play a board game three times over the course of two weeks. The first session took place in the families' homes and dyads were videotaped; the additional two sessions were audio-taped. Half of the parents were randomly assigned to the numeracy awareness condition and provided with a list of suggested ways to incorporate numeracy into playing the board game, such as reading numbers, counting, and comparing quantities; the other half of the parents in the control condition did not receive these suggestions. Overall, parents who were given suggestions for how to talk about numbers engaged in more number talk with their children during board game play. They prompted their children to do more counting, calculation, and comparisons, as well as modeling more counting themselves.

These findings show that parents are not always aware (or at least, do not take advantage) of opportunities to incorporate math talk into everyday activities and play; however, with minimal prompting, such as providing examples for how to talk about numbers during a board game, parents can engage in a wide range of number-related conversations. Furthermore, these interactions may provide children with more opportunities to be exposed to and practice more advanced number concepts.

When considering the context of the activities, it is important to acknowledge that almost all of the studies reviewed included samples from the United States, with many of them being limited to middle- to upper-class families. As discussed earlier, the sociocultural framework requires that the broader cultural context of participants be considered. In other samples, it is likely that the activities in which parents and children engage together, as well as how parents and children each participate in the activities, will vary based on what is valued in a particular society. Pan and colleagues (2006) found that frequency of maternal involvement in mathematics was predictive of children's performance on a joint proportional reasoning task for American, but not Chinese dyads, even though mothers in both groups reported similar frequencies of engaging in math activities with their 5-year-olds. There were differences, however, in the types of activities that mothers reported, with Chinese mothers reporting more frequent talk about calculation. Consequently, cultural variations in parent-child math interactions, as well as variations in the relation between parent math talk and child outcomes, must be considered.

Children's learning from parent number talk. Beyond what is known about how parents talk with young children about numbers, is there evidence to support that parent input is actually related to children's numeracy development? Many of the studies discussed have found that parent number talk relates to child number talk, as well as children's number knowledge. For instance, in a longitudinal study, Levine et al. (2010) found that cumulative parent number talk between 14 and 30 months of age predicted children's performance on an assessment of cardinality at 46 months of age. When parsing apart different types of number talk, it became evident that only specific types of number talk related to children's understanding of cardinality; parent talk about large sets of present objects predicted cardinal-number knowledge (Gunderson & Levine, 2011).

Findings from short-term studies are less clear. Although Pan et al. (2006) found that children whose mothers provided concept-focused instruction performed better on the proportional reasoning task, regression analyses revealed that concept-focused instruction only predicted children's performance for the Chinese dyads. As mentioned earlier, the best predictor for the American children was the frequency of maternal involvement in at-home math activities, as reported by the mothers. Pan et al. suggested Chinese children's stronger independent math performance as a possible explanation; going into the interaction with better math skills may have been better equipped to understand their mothers' instruction, while the American children appeared to benefit the most from instruction when they had a shared history of these types of interactions.

Other studies have found that while parent number talk related to child number talk during the interaction, it does not relate to improved number knowledge. Vandermaas-Peeler, Boomgarden, et al. (2012) found that children whose parents were prompted to use number talk during a cooking activity did produce more correct math answers during the activity but did not perform better than the comparison children on the post-test math assessment. Similarly, Vandermaas-Peeler, Ferretti, et al., (2012) did not observe growth in children's number knowledge (as measured by their number of correct responses and errors during the interaction) over a two-week period in which they played a number board game at home with their parents, even if their parents were prompted to use more number talk. There was, however, anecdotal evidence from parents' reports that children engaged in more number-related activities in other contexts (e.g., at the grocery store, during imaginative play) after playing the board game. It is unclear whether there was a measurable change or if parents may have had an increased awareness of numeracy opportunities. Thus, although there may not have been an immediate effect from engaging in number activities, doing so may have prompted parents and children to engage in more number talk in other contexts. This could ultimately have a long-term benefit since, as demonstrated by Levine et al. (2010), it appears to be the cumulative amount of parent number talk over time that contributes to children's number knowledge.

Parental Contributions to Numeracy Development: Parent Guidance

So far, the literature reviewed has focused on the content of parent input, or the frequency and types of number talk that parents provide during early childhood. An additional aspect to consider is the manner in which the input is delivered to children.

Parents may simply provide information to children, such as labeling numbers or modeling counting, or they may place more responsibility on the child, such as asking questions about numbers or prompting children to count, and then providing feedback based on the children's performance.

From a sociocultural perspective, it is important to consider *how* parents engage in number talk since some forms of parent guidance may be more conducive to children's internalization of the concepts. There is a large body of literature on parents' guidance during a variety of activities such as solving puzzles and planning delivery routes (see Gauvain, 2001a for review) in which the quality of the parent guidance has been emphasized. For example, autonomy support is generally associated with positive outcomes while higher levels of parental control during a task are believed to interfere with children's learning (Bernier, Carlson, & Whipple, 2010; Moorman & Pomerantz, 2008). Similarly, indirect guidance, such as questions or hints, simultaneously challenges and encourages children to take an active role, allowing them more opportunities to practice and internalize the skills and to gain a deeper understanding (Goncu & Rogoff, 1998; Grolnick, Gurland, DeCourcey, & Jacob, 2002; Moorman & Pomerantz, 2008).

Parent guidance during math activities. Compared to the literature on parent support during problem solving, fewer studies have looked at parent guidance during math-related activities with young children. Consequently, there is limited information on the nature of the support and assistance parents provide to their children during numeracy-related exchanges. In the Vandermaas-Peeler, Boomgarden, et al. (2012) cooking activity study, all parents, regardless of condition, were twice as likely to ask a question rather than provide a hint to their children. It should be noted, however, that the numeracy group parents also asked twice as many questions as the parents in the comparison group. Similarly, in the board game study, Vandermaas-Peeler, Ferretti, et al. (2012) found that parents provided numeracy exchanges in the form of questions much more often than they provided explanations about numbers. Parents who were given instructions for engaging in number talk were also more likely to model counting, provide positive feedback, and prompt after children made errors (but were not more likely to provide the correct answer). These findings suggest that parents use number talk in a way that actively engages and shares responsibility with their children.

Contextual influences on parent guidance. Since the previous research has only examined parent guidance during one activity, it is unclear to what extent the context influenced the types of guidance that parents provided. Bjorklund et al. (2004) examined the influence of parenting behaviors on children's use of arithmetic strategies while playing a board game, compared to while working through math problems together. Dyads of 5-year-olds and their parents took part in three weekly sessions where they played an adapted version of Chutes and Ladders and then worked through simple addition problems similar to those that might be encountered during the board game (i.e., addends ranged from 1 to 6). For both contexts, children's strategies were coded (e.g., single-item counting, fact retrieval), as were parent behaviors (e.g., prompt, provide answer, model, instruct).

When analyzing the parents' behaviors, it was found that parents offered more support for children in the math context than in the game context. During the early trials, the parents of children with greater math ability were more likely to use cognitive directives, such as modeling, instructing, or reframing the problem. Children who spontaneously demonstrated more sophisticated strategies received less assistance from their parents, further supporting that parents adapt their behavior to their child's needs. Over the three sessions of the study, children's strategies did not increase in sophistication; however, the authors speculated that if, continued over a longer period of time, children with supportive parents who offered prompts and directives would develop more advanced strategies.

Beyond types of activities, the setting in which parents interact with children may also influence what type of guidance they provide to children. In everyday situations, parent-child exchanges may take place when other adults or children are also involved. As Benigno and Ellis (2004) note, promoting children's learning is not always the central purpose for activities in which parents and children participate together. In the context of a game with multiple people involved, parents may be less concerned with promoting a child's cognitive skills and more concerned with keeping all participants engaged. To examine this, Benigno and Ellis compared parents' guidance when playing a board game alone with a preschooler and when an older sibling was also playing the game, hypothesizing that a larger group would interfere with a parent's ability to provide assistance specifically tailored to the preschooler's needs.

Dyads consisting of a parent and a preschooler and triads consisting of a parent, preschooler, and school-age sibling were observed while playing a board game. The game was designed so that each turn involved several opportunities for counting (counting items on a card, moving game piece, collecting a given number of items). Experimenters coded child's counting strategies and errors, the forms of aid that parents and older siblings provided to preschoolers (e.g., joint counting, counting for child), and teaching behaviors that parents demonstrated during their own turns (e.g., modeling counting, asking child to count).

Benigno and Ellis (2004) found that, overall, parents were responsive to when children needed assistance, demonstrated by the fact that they were more likely to help after a child made a counting error or when a child had drawn a card with a larger number. Parents in the dyadic condition used their own turns as an opportunity to teach counting skills more frequently and invited preschoolers to count during their turns. In the triads, parents and siblings were more likely to count for the younger child and to provide support when it was unneeded. As a result, preschoolers working alone with a parent had a significantly greater number of opportunities to count. Benigno and Ellis (2004) concluded that the addition of a second child may diminish the extent to which parents are able to attend to the specific needs of either child since they must divide their attention between both children.

Summary

Overall, the extant body of literature on parent number input and development of children's numerical understanding offers a window into the types of numeracy experiences that may be available to young children in the home environment, as well as the potential contributions of these experiences to children's number knowledge. Children benefit from more frequent input related to number; however, there is a substantial range in the frequency that parents and young children engage in number talk (Gunderson & Levine, 2011; Levine et al., 2010). A strength of the studies reviewed is that many of them parsed apart types of number talk, distinguishing among parent and child statements involving specific concepts related to numbers (e.g., cardinality, magnitude). Thus, we can see how certain types of number talk may influence certain aspects of number knowledge. This could be advantageous in situations where children have weaknesses in specific areas of number knowledge that need to be targeted.

The literature also offers some insight into the conditions in which parents may be more likely to provide number talk. While parents may talk about numbers in everyday contexts (Durkin et al., 1986; Levine et al., 2010), there appear to be some circumstances in which parents are more likely to engage their children in number talk, such as when numbers are salient to the task or when parents are provided with prompts to talk about numbers. Being able to identify the contexts in which parents are inclined to talk about numbers as well as identifying ways in which activities can be structured to enhance number talk is an important step in offering guidance to parents about how to incorporate math into the home environment.

Addressing the Gaps in the Literature

The current literature demonstrates that parents have the potential to be an important contributor to their children's early mathematical development. The findings that not all parents engage in number talk to the extent or in the manner that would be most beneficial to children indicates, however, that there is room for improvement. It is important for there to be further examination of the circumstances that are most conducive to frequent, *high quality* parent-child interactions about numbers. In particular, the influence of task context needs to be addressed.

As exemplified by the vast range of activities included in the studies on parent-child numeracy interactions, the contexts in which children are exposed to numbers can vary substantially. Studies that have examined correlations between children's number knowledge and number-related activities in the home as reported by parents often include both formal activities where the primary goal is to promote math skills (e.g., counting, practicing math facts) and informal activities where math is embedded in an everyday or playful task (e.g., measuring ingredients, counting money, playing board games; LeFevre et al., 2009; Ramani et al., 2015). It is important to distinguish between formal and informal activities, as they may influence the frequency and types of math talk that parents provide. For example, activities such as cooking inherently require mathematical applications in order to complete the task, consequently requiring some level of talk about math concepts. In contrast, reading a picture book and playing grocery store may not require parents to engage their children in rich number talk, even if they provide opportunities to do so.

Furthermore, in formal contexts where children's learning is seen as the central goal of the activity, parents may offer different forms of guidance (Renshaw & Gardner, 1990). Studies of formal and informal tasks have found that context influences parental guidance during problem-solving and planning activities (e.g., Isman & Tzuriel, 2008; Kermani & Brenner, 2000). For example, Isman and Tzuriel (2008) found that mothers were more likely to relate the immediate activity to a broader context during a free play session compared to during a structured problem-solving task.

It is interesting to note that the majority of observational studies on parent-child number interactions have focused on informal, everyday activities, and very few have included a formal math activity (e.g., Sun & Rao, 2012). In fact, only one study to date, Bjorklund et al. (2004), has systematically compared parent guidance during a formal activity (a worksheet of simple arithmetic problems) and during an informal activity tapping into the same math skills (a board game). The lack of observational studies examining parent-child interactions during formal activities is surprising given survey findings indicating that parents report spending time with preschoolers while engaged in activities directly aimed at promoting numerical understanding (LeFevre et al., 2009; Ramani et al., 2015). There is a need for studies comparing parent-child interactions during playful activities and formal instruction that tap into the same math concepts. Furthermore, there is a need to conceptualize the activities on a spectrum from completely child-driven (Free Play) to completely adult-driven (Didactic Instruction), while also recognizing the middle ground of activities that are adult-guided, but child-driven (Guided Play). Conceptualizing activities in this way allows for the goal of the activity as well as the roles of the child and parent during the activity to be considered.

The current study approached these issues by examining parent-child interactions and children's learning during three conditions all using similar materials aimed at promoting understanding of math concepts related to fractions. The conditions varied in regard to whether there was an emphasis on the activity as a straightforward teaching opportunity, a teaching opportunity embedded in play, or a playful activity with no mention of it being a teaching opportunity. The amount and diversity of parent and child math talk were assessed as a means of examining to what extent the structure of the activity influenced how much parents and children attended to the math concepts embedded in the task. Activities that were structured to prompt parent and child math talk were expected to increase the amount of discussion about math concepts.

Additionally, to test whether the contexts influenced the roles of the child and parent, the extent to which both parents and children engaged in the activity was examined by comparing the portions of the interactions that were driven by the parent versus the child. A well-balanced exchange in which parents and children shared the conversational load was expected to reflect a collaboration that was adult-guided, but child-driven. In other words, the parent's role was to keep the child on task, draw attention to the math learning opportunities, and offer feedback and assistance as needed, but to do so in a way that was responsive to the child and encouraged the child's active involvement and exploration.

Chapter 3: Methods

Participants

Parents and their 4- and 5-year-old children who had not yet entered kindergarten were recruited to participate through the University of Maryland's Infant and Child Studies database, which includes families who have previously expressed interest in being contacted to participate in research studies. There were two criteria used to select which families in the database to contact: 1) the child was at least 4 years old, but had not turned 5 before the current school year (thus presumably had not yet entered kindergarten), and 2) it was reported that English was spoken in the home at least 70% of the time. Families were reached either by phone or email, depending on their indicated preferred means of contact.

Seventy-seven dyads participated in the study; however, 5 dyads were not included in the analyses. In two instances, the families arrived for the study late enough that there was only enough time to do the interaction; the fraction tests and the general math assessment (TEMA) were not administered in these cases. In one case, the video equipment failed and the interaction was not recorded. In one case, it became evident during the pretest that the child had severe limitations in his ability to comprehend and respond to the fraction test, and had some developmental delays. Finally, in one case, a toddler-age sibling ended up present for the interaction, and this seemed likely to have influenced the exchanges between the parent and child.

Of the 72 dyads whose interactions were analyzed, there were 42 girls (58.3%) and 30 boys (41.7%), whose ages ranged from 48 to 68 months ($M = 57.64$, $SD = 5.18$). Based on parent report, 60% of the children were White, 29% were of mixed race, 7%

were Black, and 4% were Asian. The parents in the study included 60 mothers (83.3%) and 12 fathers (16.7%). The race/ethnicity of the parents consisted of 65% White, 13% Black, 11% Asian, and 11% mixed race. Three parents (4%) did not provide information about their education; of the remaining parents, the majority of the parents had either a graduate degree (47%) or a 4-year college degree (39%), with 8% of parents reporting some college and 1% reporting a high school diploma.

Design

The study employed an experimental design testing children's fraction knowledge at pretest and posttest with dyads participating in a joint activity between the two assessment points. Dyads were assigned to one of three experimental conditions for the joint activity: Didactic Instruction (DI), Guided Play (GP), or Unguided Play (UP), which are described below. In order to ensure balanced numbers of boys and girls in each group, there was a stratified assignment of gender into the conditions.

Of the 72 dyads included in the analyses, 24 dyads were assigned to the DI condition, including 14 girls and 10 boys (*M* age = 57.33 months, *SD* = 6.06); 25 dyads were assigned to the GP condition, including 15 girls and 10 boys; (*M* age = 57.20, *SD* = 5.12); and 23 dyads were assigned to the UG condition, including 13 girls, 10 boys (*M* age = 58.43, *SD* = 4.33). Table 1 summarizes the demographic information by condition groups, including the parent-child gender pairs for each condition. While the gender of the children was stratified when assigning conditions, it was not possible to also systematically assign dyads based on parents' gender.

Procedures

Data collection for each family took place during one 60- to 75-minute visit. Visits took place in the observation room of a university research laboratory, equipped with a microphone and video cameras to record the dyadic interactions and containing a table and chairs designed for small children, as well as a full-sized arm chair for the parents. For the majority of the visits, only the examiner was present; in some cases an undergraduate research assistant was also present to observe, run the video equipment, or babysit a sibling. The author was the examiner for most of the studies; however, another trained graduate student served as the examiner and collected data on several occasions when the author was not available.

During the visit, the examiner first went through the consent process with the parent. Then the examiner administered the fraction knowledge pretest and the general math abilities assessment to the child. While the examiner worked individually with the child, the parent completed a background information questionnaire and a survey. In most cases, the parent sat in the room while the examiner worked with the child; however, in several circumstances parents were in a nearby room with their other children and only came into the observation room for the interaction part of the study.

Following the individual testing, the child was offered the chance to take a short break and have a snack. Next, parents were asked to engage their children in an activity for 15 minutes; the specific instructions varied across conditions, as later described. Parents and children were seated on a rug during the interaction; after giving out the materials and reading the instructions, the examiner went into an adjacent observation booth, leaving the dyads alone to work on the activity. Generally the examiner discontinued the activity after 15 minutes; however this varied to some extent. If a child requested to stop before 15 minutes, the activity was stopped early; if dyads in the Didactic Instruction or Guided Play conditions were still going through the worksheet or storybook, they were allowed to finish. Mean times for each condition are presented in Table 2. The joint activity was videotaped for subsequent transcription and analyses.

After the activity, children's learning was assessed through the posttest version of the fraction assessment. While the child completed the posttest with the examiner, the parent was asked to complete a brief questionnaire about the joint activity. As an incentive for participating in the study, families were given a small game or book.

Measures

Background information questionnaire. Parents were asked to complete a questionnaire providing information about their and their child's race and ethnicity, their own education and occupation, as well as the education and occupation of any other parents in the household (see Appendix A).

Speech and language questionnaire. Parents were also asked to complete a survey regarding their children's speech and language. The Speech and Language Assessment Scale (SLAS; Hadley & Rice, 1993; Phenx Toolkit, 2014) is a 19-item parent-report questionnaire assessing young children's speech and language skills (i.e., assertiveness, responsiveness, semantics, syntax, articulation, and talkativeness; see Appendix B for items). This questionnaire was not used as a measure in the study; instead it was used to detract parents' attention from the math-oriented assessments being administered to the children.

General assessment of children's math knowledge. To assess children's general math ability, the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003) was administered. The TEMA-3 is designed to assess the math performance of children between 3 and 8 years old. There are 72 items; however, the starting point is determined by the child's age. Testing is discontinued after a child has 5 consecutive errors. The TEMA is reported to have strong internal reliability (coefficient alphas ranging from .92 to .96). Age-based standard scores were used in analyses.

Only 52 children completed the TEMA; for 10 children the TEMA was discontinued due to the child losing attention or requesting to stop before it was finished. In another 10 instances, the families arrived late and the time constraint resulted in the TEMA either not being administered or discontinued early.

Fraction concepts knowledge assessment. In order to assess the effects of the parent-child joint activity, children's understanding of fractions was assessed both before and after the activity. The items included resembled those on worksheets that are typically used when first introducing the concepts of fractions in school, and are reflective of the emergent understanding of fractions that children possess between the ages of 4 and 7, specifically the concepts of equal sharing and proportional reasoning (Siegler et al., 2010; Siegler et al., 2013 for review). There were three parts to the assessment, described below.

Part 1: Understanding relative size of shared portions. Previous studies have found that children as young as 5 years old have an informal understanding of the concept that when an item (or multiple items) are partitioned into equal shares, the size of each portion is contingent on the number of parts (Sophian et al., 1997). However, 5-

year-olds find it more challenging to apply this logic to continuous items divided into parts rather than discrete objects divided into subgroups (Wing & Beal, 2004). To assess participants' understanding of this concept, they were presented with a series of items in which they were asked to judge the outcomes of partitioning continuous objects into different numbers of parts. Children were introduced to one of two characters who shares food with his friends; items involved the character dividing the same item among different numbers of friends after which the child was asked to judge which scenario would yield the larger (or smaller) share of the item (e.g., "Will Spot get more of the cookie if he shares the cookie with one friend or with two friends?"). Visual representations of the question were presented to the child on a computer screen (see Appendices C and D for example items and script).

Each version of the test included 12 trials: 6 trials asked for the "most amount" of a desirable food (e.g., "Spot loves sandwiches, so when he shares a sandwich with friends, he wants as much of the sandwich as he can get. Will Spot get *more* of the sandwich if he shares it with one friend or two friends?") and 6 trials asked for the "least amount" of an undesirable food (e.g., "Spot does not like watermelon, so when he shares watermelon with friends, he wants as little of the watermelon as possible. Will Spot get *less* of the watermelon if he shares it with one friend or two friends?"). The desirable and undesirable foods of one character were the opposite of the other character; all children were exposed to one character during the pretest and the other character during the posttest; the order of the characters were counter-balanced across participants. Additionally, within each version, it was counter-balanced as to whether children were asked "most amount" or "least amount" items first. Scores were computed separately for the “more” and “less” items, each ranging from 0 to 6, based on the total of correct responses. There was not a significant difference between test conditions, $F(1, 70) = 3.69$, $p = .06$, $\eta^2 = .05$; thus, there did not appear to be an order effect based on receiving the “most” or “least” amount questions first.

Part 2: Partitioning objects into equal shares. To further assess children’s understanding of partitioning objects into equal parts, they were given a task based on those typically used in first-grade classrooms. Children were asked to use a pencil to divide objects into equal parts (5 items into halves, 5 items into fourths). Some items were images of foods closely resembling geometric shapes (e.g., circular watermelon, rectangular granola bar) and other items were simply geometric shapes (see Appendix E for example worksheets). The same 10 items were used in both the pretest and the posttest but were reversed so that the items that the child divided into halves during the pretest were divided into fourths during the posttest; the items to be divided into halves and fourths during the pretest and posttest were counter-balanced across participants.

Each item was scored on a 3-point scale: 0 = *Not divided into correct number of parts,* 1 = *Divided into correct number of parts, but in a way that does not allow for equal sizes*, 2 = *Divided into the correct number of parts with notable disparities in size, but in a way that indicates an understanding of how to divide the objects,* 3 = *Divided into the correct number of pieces, which are relatively the same size*. The distinctions in this rating scale allowed for sensitivity in distinguishing between a child who did not understand the need to divide items symmetrically and a child whose visual-motor development might limit his ability to draw even lines, despite understanding how to divide something equally. To establish reliability, tests from 20 participants (28% of the sample) were scored separately by two independent coders (the researcher and an undergraduate research assistant blind to the specific hypotheses of the study). Given the ordinal nature of the rating scale, Gamma statistics were used to assess the reliability (Goodman & Kruskal, 1972; Liebetrau, 1983). For all 20 participants, the coders' agreement yielded a Gamma statistic over .80 (mean Gamma = .97). Disagreements were resolved by the researcher. Separate scores were calculated for the Half and Fourth items, each ranging from 0 to 15. There was not a significant difference between test conditions, $F(3, 68) = 1.13$, $p = .34$, $\eta^2 = .05$; thus, there did not appear to be a difference in the level of difficulty in the two versions of the test.

Part 3: Proportional reasoning of analogy problems. To assess children's understanding of proportional equivalence, they were presented with a multiple choice task, based on the proportional analogy task used by Singer-Freeman and Goswami (2001). Children were be shown a shape divided into equal portions, with a fraction of the portions shaded; children were asked to identify which of four other shapes had an analogous portion shaded in. Two practice items were provided (i.e., 4/4 circle : 4/4 circle; 1/2 square : 1/2 circle) to ensure that the children understood the task. There were 2 items for each of four levels of difficulty (see Appendix F for example items): same shape, same number of parts (e.g., 2/3 square : 2/3 square); different shape, same number of parts (e.g., 2/4 circle : 2/4 square); same shape, different number of parts (e.g., 2/8 circle : 1/4 circle); and different shape, different number of parts (e.g., 2/6 circle : 1/3 square). All items were presented on a computer screen. There were two versions with different items of equivalent difficulty; one version was administered during the pretest and the other was administered during the posttest. The order of versions presented in the pretest and posttest were counter-balanced across participants. Children received one point for each correct item, resulting in a total score ranging from 0 to 8. There was not a significant difference between test conditions, $F(3, 68) = 1.86$, $p = .14$, $\eta^2 = .08$; thus, the two versions of the test did not appear to differ in level of difficulty.

Post-activity questionnaire. Following the joint activity, parents were asked to respond to a brief set of questions about the task. Specifically, parents were asked about how enjoyable they thought the activity was for their children and for themselves, how intellectually challenging the activity was for their children, how much they felt they knew how to interact during the activity, and how much they believe engaging in such activities could promote different skills (i.e., language ability, creativity, understanding of math concepts; see Appendix G for specific items). For each item, parents responded based on a Likert-type scale of 1 (*Strongly Disagree*) to 7 (*Strongly Agree*).

Joint Activity Materials and Conditions

All dyads were given a set of wooden toy foods (e.g., a loaf of bread, an apple; see Appendix H for a complete inventory and pictures). The foods were partitioned into equal pieces, which could be attached together by Velcro; this allowed for exploration of the concepts of part and whole, as well as fractions (e.g., halves, thirds, fourths). The variations in the materials and instructions provided in each condition are described next.

Unguided Play condition. Dyads assigned to the Unguided Play condition were given a box with the foods listed above, as well as a wooden knife, a cutting board, and four plates. Parents were given the following directions: *"We have some toys for you and your child to play with today. You may play with whichever of the toys you would like to* *play with, and we would like the two of you to play as you typically would play at home. You will have 15 minutes to play with the toys. Have fun!"*

Guided Play condition. Dyads assigned to the Guided Play condition were given the same set of materials as in the Unguided Play condition, but also received further instructions and a book. Parents were given the following instructions: *"We have some toys for you and your child to play with today. These are toys that have been found to be helpful in promoting children's learning about concepts related to fractions. Here is a storybook that you can follow along with as you play. You will have 15 minutes to play; if you finish the storybook early, you may continue to play however you like for the rest of the time. Have fun!"*

Dyads received a book that told a story of four friends having a picnic. During the story there were questions intended to engage the child and parent in exploring the wooden foods in order to answer the questions (see Appendix I for the pages of the book). Three questions were straightforward in prompting children to find the foods that could be fairly shared by two, three, or four friends. Three additional questions encouraged the child to explore the concepts of parts and whole in greater depth such as whether the friends would get more food if they divided it into two or four pieces or what would happen if two friends tried to share a food cut into five pieces.

Didactic Instruction condition. In the Didactic Instruction condition, the dyads received the wooden foods; however, in this condition, the pieces were glued together so that the partitions were visible. In other words, dyads were able to see the number of pieces the food was divided into, but they were not able to explore and manipulate the items. As in the Guided Play condition, parents were told that the activity promoted understanding of fraction concepts, but the words “play” and “toys” were replaced with “use” and “materials”, respectively, in order to downplay the playful nature of the task. The parents were given the following instructions: *“We have some materials for you and your child to work with today. These are objects that have been found to be helpful in promoting children’s learning about concepts related to fractions. Here is a worksheet that you follow along with while you use the materials. You will have 15 minutes for this activity; if you finish the worksheet early, you may continue to use the materials however you like for the rest of the time. Good luck!”* The dyads were given a worksheet (see Appendix J) which posed decontextualized versions of the questions included in the Guided Play book.

Transcription and Measures of Overall Parent and Child Talk

The videotaped parent-child interactions from all of the conditions were transcribed by either the researcher or a trained undergraduate research assistant using the CHAT conventions of the Child Language Data Exchange System (CHILDES; MacWhinney, 2000). Transcripts were verified by another reliable transcriber prior to being analyzed.

Parent and child talk was transcribed into units of utterances, or a sequence of words before or after a pause or a change in conversational turn (Bakeman & Gottman, 1997). The total number of parent and child tokens, or the total number of words produced during the interaction, was used as a measure of quantity of talk. The total number of *different* words produced during the interaction, or word types, was used as a measure of the diversity of parent and child talk. To clarify, while the measure of word tokens counts every time a word is spoken, word types only counts each unique word only once during the interaction. For example, if a child used the word "half" 3 times during an interaction, it would count as 3 word tokens and 1 word type. Including both measures is helpful in capturing differences between a talkative parent who only exposes their child to a limited vocabulary and a parent who may talk less, but provides much richer talk with a wider vocabulary. This is important as previous studies have indicated that the quantity and quality of parent talk are both uniquely related to children's language development (Pan, Rowe, Singer, & Snow, 2005; Rowe, 2012).

In order to examine the extent to which the parent and child were engaged in a balanced collaboration during the task, the mean lengths of parent and child speaking turns (MLT) were computed as the number of tokens per turn. Then, a ratio of child MLT to mother MLT was calculated as the measure of the balance between how much the parent and child were speaking. As explained by MacWhinney (2000), this ratio can be interpreted as how much the parent and child shared the "conversational load", with values close to 1 indicating that the child was speaking about as much as the parent.

A notation was used to indicate when parents were reading text from the storybook in the Guided Play condition or from the worksheet in the Didactic Instruction condition. This allowed for distinguishing between the math talk that was directly from the stimuli provided to parents and the math talk that parents originally produced themselves. Because the math talk from the text was part of the cumulative math talk that children were exposed to during the interactions, initial analyses included both text and non-text utterances in the variables related to parent talk (both overall and math-specific talk). Separate variables excluding the text talk were also created for use in follow-up analyses to examine the extent to which variations in parent talk were driven by parent math talk above and beyond what was provided in the texts.

Measures of Parent and Child Math Talk

Quantity of math talk. The total amount of fraction-related word tokens in three different categories were computed: Formal Fraction Tokens (e.g., *half*, *thirds*), Informal Fraction Tokens (e.g., *two pieces*, *three parts*), and Quantitative Tokens (e.g., *whole*, *equal*; see Table 3 for full lists of the words in each category). Both formal and informal fraction terms were included because while the worksheet in the Didactic Instruction condition included formal fraction words, the storybook in the Guided Play condition did not. Thus it was possible that parents in the Guided Play condition would talk about fraction concepts without using words such as *half* or *third*, and searching for terms such as *two pieces* allowed for detecting this talk. Quantitative words were also included to capture words beyond specific quantities that parents and children might use to talk about fractions, similar to how Levine et al. (2010) coded words such as *count* and *number* when studying parents' elicitation of child number talk.

In order to examine dyads' overall number talk (i.e., not fractions), the total amount of tokens for the words *two* through *twenty* were counted and combined into a category of Number Tokens. Since "one" is often used as a term to refer to an individual object rather than a quantity (e.g., "I want this one"), it was not included in the Number Tokens list; however, "one" was included in the Informal Fraction Tokens list when followed by the word "piece" or "part". A Total Math Tokens variable was calculated as a total of all Formal Fraction, Informal Fraction, Quantitative, and Number Tokens.

The total amount of parents' math-related tokens was positively correlated with parents' total amount of tokens overall, suggesting that parents who spoke more throughout the interaction also talked about math more (see Table 4). Thus proportions of parent math talk to total parent talk (Math Tokens/All Tokens) were calculated for each category of math tokens; these proportions were used in analyses. While the correlations between children's total amount of word tokens and math-related tokens were more varied across categories (see Table 5), the child math-related tokens were also computed as a proportion of total word tokens for ease of interpretation. Correlations between parent and child talk variables are presented in Table 6.

Quality of math talk. The total amount of fraction-related word types were also computed for the same three categories: Formal Fraction Types (e.g., *half*, *thirds*), Informal Fraction Types (e.g., *two pieces*, *three parts*), and Quantitative Types (e.g., *whole*, *equal*; see Table 3 for full lists of the words in each category). Additionally, the total number of types for the words *two* through *twenty* were also counted and combined into a category of Number Types. Since "one" is often used as a term to refer to an individual object rather than a quantity (e.g., "I want this one"), it was not included in the Number Types list; however, "one" was included in the Informal Fraction Types list when followed by the word "piece" or "part". A Total Math Types variable was calculated as a sum of all Formal Fraction, Informal Fraction, Quantitative, and Number Types. Since the raw number of how many different math-related words parents and children spoke seemed more meaningful, proportion variables were not computed for math-related types as they were done for math-related tokens.

Data Analyses

Preliminary analyses. Prior to testing the study hypotheses, an ANOVA was run comparing the children's general math ability (measured by the TEMA) across the three conditions (Unguided Play, Guided Play, and Didactic Instruction). In the event that there were group differences, general math ability would be used as a covariate in the following analyses; however, given that the dyads will be randomly assigned to the conditions, it was expected that there would not be significant differences across groups in children's math ability. Similar ANOVAs were run to confirm that there were no group differences in children's age or fraction knowledge at the pretest point.

Analyses for Hypothesis 1. To test the hypotheses regarding how the conditions would differ in parent and child talk, a series of ANOVAs and MANOVAs were conducted, described in detail below. All significant ANOVAs and MANOVAs were followed up by Bonferroni post-hoc tests, adjusting for multiple pairwise comparisons.

Hypothesis 1a: Parents in the Didactic Instruction and Guided Play conditions will engage in more math talk than parents in the Unguided Play condition. *To* examine differences in the amount of parent math talk, an ANOVA comparing the total proportion of parent math tokens across the three conditions was run. Next a MANOVA was conducted to compare the proportions of each category of parent math tokens (i.e., Formal Fraction Tokens, Informal Fraction Tokens, Quantitative Tokens, Number Tokens) across the three conditions.

To examine differences in the quality of parent math talk, an ANOVA comparing the total amount of math word types across the three conditions was run. A follow-up MANOVA then was conducted to compare the amount of parents' different word types within each of the math-related categories (i.e., Formal Fraction Types, Informal Fraction Types, Quantitative Types, Number Types) across the three conditions.

Hypothesis 1b: Children in the Didactic Instruction and Guided Play conditions will engage in more math talk than children in the Unguided Play condition, and children in the Guided Play condition will engage in more math talk than children in the Didactic Instruction condition. Parallel to the analyses of parent math talk, children's math talk across conditions was compared by running an ANOVA comparing the total proportion of child math tokens, followed by a MANOVA comparing the proportion of each category of child math tokens across the three conditions; the same ANOVA and MANOVA was also run for child math types to assess the diversity of children's math talk across conditions.

Hypothesis 1c: Parents in the Didactic Instruction condition will engage in a greater amount of talk than their children, thereby driving the interaction more than their children, whereas in the Unguided Play condition, children will drive the interaction and engage in a greater amount of talk than their parents; there will be a balance between parent and child talk in the Guided Play condition. To test the hypothesis that the extent of parent and child involvement would vary as a function of condition, an ANOVA was used to compare the ratio of child mean length of turn (MLT, computed as words per turn) to parent MLT.

Analyses for Hypothesis 2: Children's learning of fraction concepts will vary as a function of activity context, such that children in the Guided Play condition will demonstrate the greatest improvement and children in the Didactic Instruction condition will demonstrate more improvement than children in the Unguided Play

condition. To test the hypothesis that children in the Guided Play condition would demonstrate greater improvement in their fraction knowledge than that of children in the other two conditions, the difference between children's pre- and posttest scores were calculated for each of the three subtests of the Fraction Knowledge Assessment. A MANOVA was then run, including the condition as the between-subjects variable, and the three subtests' change scores as the dependent variables.

Analyses for Hypothesis 3a: Greater improvement in children's fraction knowledge will be positively associated with higher frequencies of parents' fraction talk. To test the hypothesis that parents' fraction talk would account for math talk to children's learning a regression analysis was planned to be conducted predicting children's immediate improvement in fraction knowledge (calculated as the difference between their posttest and pretest fraction knowledge scores). To account for the conditions, dummy-variable coding was used to create two variables that distinguished the three groups (Cohen, Cohen, West, & Aiken, 2003), with the Unguided Play group as the reference group. These two dummy variables were then entered into the regression model before the predictor variable.

Analyses for Hypothesis 3b: The relation between parent fraction talk and children's learning will be mediated by children's fraction talk during the joint activity. To examine whether child math talk during the joint activity mediated the relation between parent math talk and children's improvement of fraction knowledge, a simple mediation analyses would be conducted for improvement at the posttest, using the causal steps strategy for its conceptual simplicity (Baron & Kenny, 1986; Judd & Kenny, 1981). However, given the low power of the causal steps strategy, bootstrapping with bias-corrected confidence estimates to determine significance of the indirect effects would be used (Preacher & Hayes, 2008). Again, dummy variables distinguishing between the three conditions were entered first into the model.

Power analysis. Ferrara et al. (2011) studied parent and child spatial talk during block play, assigning dyads to conditions similar to those in the current study (i.e., Didactic Instruction, Guided Play, Unguided Play). They observed group differences in both parent and child talk with an effect size of $\eta^2 = .26$, or $f = .59$. Given the similar manipulations and analytic approaches, a similar effect size was expected for this study. To detect this effect size with 80% power and an alpha level of .05, a total sample size of 42 would be needed. In order to be able to detect a more conservative effect size, it was determined that data would be collected from 75 participants; in this case there would be 80% power to detect an effect size as small as $\eta^2 = .12$ ($f = .36$) with an alpha level of .05.

Chapter 4: Results

Preliminary Analyses

While dyads were randomly assigned and it was not expected for there to be a need to control for any child or parent characteristics, preliminary analyses were conducted to check for between-group differences. There was no significant between-condition difference in the children's ages, $F(2, 69) = 0.40$, $p = .67$, $\eta^2 = .01$, nor was there a significant difference in the mean TEMA scores, $F(2, 49) = 0.21$, $p = .81$, $\eta^2 = .01$. Consequently, neither child age nor TEMA scores were used as a covariate in the remaining analyses. A MANOVA was conducted to compare the groups' pretest scores on the fraction knowledge test, entering the total scores for Part 1, Part 2, and Part 3 as the dependent variables. There was no significant multivariate effect of condition, $F(6, 136) = 1.63$, $p = .14$, $\eta^2 = .07$, confirming that there were no between-group differences in children's pre-existing fraction knowledge.

Parent and Child Talk during Joint Activity

Total parent and child talk. Table 2 summarizes the overall characteristics of the parent and child talk across groups.

Total parent talk. An MANOVA examined between-condition differences in parents' overall talk as measured by the total number of turns, the mean length of turns (MLT), the total number of types of words, and the total number of tokens. There was a significant multivariate effect, $F(8, 134) = 5.83$, $p < .001$, $\eta^2 = .26$.

Univariate analyses indicated a significant between-condition difference for parent MLT, $F(2, 69) = 15.50$, $p < .001$, $\eta^2 = .31$. Post-hoc tests revealed that parents in the Unguided Play condition ($M = 7.39$, $SD = 1.60$) had a shorter MLT than parents in the

Didactic Instruction (*M* = 11.43, *SD* = 3.96) and Guided Play (*M* = 11.91, *SD* = 3.09) conditions (both $p < .001$), and there was no difference in the MLTs of the latter two conditions. Similarly, there was a significant difference for the total number of parent tokens, $F(2, 69) = 10.22$, $p < .001$, $\eta^2 = .23$. Parents in the Unguided Play condition (*M* = 1046.13, *SD* = 237.34) used fewer tokens than in the Didactic Instruction (*M* = 1379.75, *SD* = 357.64, $p < .01$) and Guided Play (*M* = 1451.16, *SD* = 368.99, $p < .001$) conditions, which did not differ from each other. There were no significant differences in the total number of parent turns, $F(2, 69) = 2.15$, $p = .13$, $\eta^2 = .06$ (Didactic Instruction, *M* = 128.54, *SD* = 35.93; Guided Play, *M* = 125.92, *SD* = 32.09; Unguided Play, *M* = 143.96, *SD* = 28.26), or the total number of types of words that parents used, $F(2, 69) = 2.09$, $p = .13$, $\eta^2 = .06$ (Didactic Instruction, *M* = 258.96, *SD* = 47.46; Guided Play, *M* = 281.16, *SD* = 34.15; Unguided Play, *M* = 264.30, *SD* = 36.49).

Total child talk. Another MANOVA examined between-condition differences in children's overall talk as measured by the same factors used to analyze parents' overall talk (number of turns, MLT, types, and tokens). There was a significant multivariate effect, $F(8, 134) = 3.32$, $p < .01$, $\eta^2 = .17$.

Univariate analyses indicated there were significant differences for the total number of child types, $F(2, 69) = 9.98$, $p < .001$, $\eta^2 = .22$, and child tokens, $F(2, 69) = 5.05$, $p < .01$, $\eta^2 = .13$. Post-hoc tests revealed that children in the Unguided Play condition used more types (*M* = 175.65, *SD* = 40.82) and tokens (*M* = 565.70, *SD* = 192.85) than in the Didactic Instruction condition (types, *M* = 119.50, *SD* = 41.73, $p < .001$; tokens, *M* = 389.29, *SD* = 193.01, $p < .05$) and in the Guided Play condition (types,

$M = 139.24$, $SD = 47.73$; tokens, $M = 415.28$, $SD = 226.36$; both $p < .05$); the latter two groups did not differ from each other.

There was a marginal difference in children's MLTs, $F(8, 134) = 5.79$, $p = .06$, $\eta^2 = .08$. Post-hoc tests indicated that this was driven by children in the Didactic Instruction condition ($M = 3.05$, $SD = 1.36$) having a slightly shorter length of turn compared to children in the Unguided Play condition ($M = 4.00$, $SD = 1.44$, $p = .07$); the MLT of children in the Guided Play condition ($M = 3.28$, $SD = 1.46$) did not differ from that of either condition. There were no significant differences in the total number of child turns, $F(8, 134) = 2.20$, $p = .12$, $\eta^2 = .06$ (Didactic Instruction, $M = 128.38$, $SD = 35.84$; Guided Play, $M = 125.24$, $SD = 32.01$; Unguided Play, $M = 143.70$, $SD = 28.59$).

Thus, the overall amount that parents and children spoke during the interactions varied across conditions, such that parents tended to speak more in the two structured conditions (Didactic Instruction and Guided Play) than they spoke in the unstructured condition (Unguided Play), while the inverse was true for children.

Hypothesis 1a: Parents in the Didactic Instruction and Guided Play conditions will engage in more math talk than parents in the Unguided Play condition.

Quantity. As previously noted, parents' math talk was strongly correlated with their overall talk (see Table 4 for correlations); thus, to examine differences in the amount of parent math talk, an ANOVA comparing the proportion of total parent math talk (sum of all math word tokens/ total word tokens) across the three conditions was conducted (see Table 7 for total math tokens). There was a significant between-conditions difference in the overall amount of math word tokens that parents used, F(2,

69) = 76.78, $p < .001$, $\eta^2 = .69$. As shown in Table 8, post-hoc pairwise comparisons revealed that parents in the Didactic Instruction condition ($M = 0.126$, $SD = 0.039$) used a greater proportion of total math word tokens than parents in the Guided Play ($M = 0.065$, $SD = 0.019$) and Unguided Play ($M = 0.014$ $SD = 0.009$) conditions (both $p < .001$). Parents in the Guided Play condition used a greater proportion of math tokens than parents in the Unguided Play condition ($p < .001$).

Next, a MANOVA was conducted to compare the proportions of each category of parent math word tokens (i.e., Formal Fraction Words, Informal Fraction Words, Quantitative Words, and Number Words) across the three conditions. There was a significant multivariate effect, $F(8, 134) = 21.73$, $p < .001$, $\eta^2 = .57$. Univariate tests and post-hoc comparisons were next examined.

Formal fraction word tokens. Follow-up univariate tests indicated that there was a significant between-condition difference for parents' formal fraction talk, $F(2, 69) = 66.58$, $p < .001$, $\eta^2 = .66$. Parents in the Didactic Instruction condition ($M = 0.027$, $SD = 0.012$) used a greater proportion of formal fraction word tokens than the parents in the Guided Play ($M = 0.004$, $SD = 0.004$) and Unguided Play ($M = 0.003$, $SD = 0.005$) conditions (both $p < .001$); the latter two conditions did not differ from each other.

Informal fraction word tokens. There was also a significant difference for parents' informal fraction talk, $F(2, 69) = 50.32$, $p < .001$, $\eta^2 = .59$. Parents in the Didactic Instruction condition ($M = 0.015$, $SD = 0.006$) used a greater proportion of informal fraction words than the parents in the Guided Play ($M = 0.007$, $SD = 0.003$) and Unguided Play ($M = 0.002$, $SD = 0.004$) conditions (both $p < .001$). Parents in the

Guided Play condition used more informal fraction word tokens than parents in the Unguided Play condition ($p < .001$).

Quantitative word tokens. For parents' use of quantitative word tokens, there was a significant different, $F(2, 69) = 38.70$, $p < .001$, $\eta^2 = .53$. Parents in the Didactic Instruction condition ($M = 0.050$, $SD = 0.018$) used a greater proportion of quantitative word tokens than the parents in the Guided Play ($M = 0.029$, $SD = 0.013$) and Unguided Play ($M = 0.025$, $SD = 0.021$) conditions (both $p < .001$). Parents in the Guided Play condition used more general quantitative word tokens than parents in the Unguided Play condition ($p < .001$).

Number word tokens. Finally, there was a significant difference for parents' number word tokens, $F(2, 69) = 44.09$, $p < .001$, $\eta^2 = .56$. Parents in the Didactic Instruction condition ($M = 0.033$, $SD = 0.016$) used a greater proportion of number word tokens than parents in the Guided Play ($M = 0.026$, $SD = 0.007$; $p < .01$) and Unguided Play ($M = 0.006$, $SD = 0.005$; $p < .001$) conditions. Parents in the Guided Play condition used more number word tokens than parents in the Unguided Play condition ($p < .001$).

Quality. To examine differences in the diversity of parent math talk, an ANOVA comparing the total amount of parents' *different* math word types across the three conditions was conducted. There was a significant between-conditions difference in the overall amount of math word types that parents used, $F(2, 69) = 95.45$, $p < .001$, $\eta^2 = .74$. As shown in Table 9, pairwise comparisons revealed that parents in the Didactic Instruction condition ($M = 18.63$, $SD = 2.20$) used a greater number of different math word types than parents in the Guided Play ($M = 13.12$, $SD = 2.54$) and Unguided Play ($M = 7.69$, $SD = 3.31$) conditions (both $p < .001$). Parents in the Guided Play condition used a greater number of different math word types than parents in the Unguided Play condition ($p < .001$).

Another MANOVA was conducted to compare the diversity of parent math talk, as measured by the number of *different* words uttered within each category (i.e., types of Formal Fraction Word Types, Informal Fraction Word Types, General Quantitative Types, and Number Word Types) across the three conditions. There was a significant multivariate effect, $F(8, 134) = 21.73$, $p < .001$, $\eta^2 = .57$. Univariate tests and post-hoc comparisons were next examined.

Formal fraction word types. Follow-up univariate tests indicated that there was a significant between-condition difference for types of parents' formal fraction words, $F(2, 69) = 71.78$, $p < .001$, $\eta^2 = .68$. Parents in the Didactic Instruction condition ($M = 3.33$, $SD = 0.76$) used more different formal fraction word types than the parents in the Guided Play ($M = 1.28$, $SD = 0.84$) and Unguided Play ($M = 0.74$, $SD = 0.75$) conditions (both $p < .001$). There was a marginal difference where parents in the Guided Play condition used slightly more different formal fraction words than parents in the Unguided Play condition ($p = .06$).

Informal fraction word types. There was also a significant difference for the number of different informal fraction word types used by parents, $F(2, 69) = 71.40$, $p < .001$, $\eta^2 = .67$. Parents in the Didactic Instruction condition ($M = 3.96$, $SD = 0.55$) used a greater number of informal fraction word types than the parents in the Guided Play ($M = 3.28$, $SD = 0.98$, $p < .05$) and Unguided Play ($M = 0.91$, $SD = 1.13$, $p < .001$) conditions. Parents in the Guided Play condition used a greater number of different informal fraction word types than parents in the Unguided Play condition ($p < .001$).

Quantitative word types. There was a significant differences in the number of types of different quantitative words used by parents, $F(2, 69) = 29.33$, $p < .001$, $\eta^2 = .46$. Parents in the Didactic Instruction condition ($M = 6.83$, $SD = 1.05$) used a greater amount of different quantitative word types than parents in the Guided Play ($M = 5.08$, $SD = 1.47$) and Unguided Play ($M = 3.83$, $SD = 1.50$) conditions (both $p < .001$). Parents in the Guided Play condition used more quantitative word types than parents in the Unguided Play condition ($p < .01$).

Number word types. Finally, there was a significant difference for parents' types of number words, $F(2, 69) = 29.33$, $p < .001$, $\eta^2 = .46$. Parents in the Didactic Instruction condition ($M = 4.50$, $SD = 1.25$) used a greater amount of different number word types than parents in the Guided Play ($M = 3.48$, $SD = 0.51$ $p < .01$) and Unguided Play ($M = 2.22$, $SD = 1.17$, $p < .001$) conditions. Parents in the Guided Play condition used more different number word types than parents in the Unguided Play condition ($p < .001$).

Thus, the hypothesis that parents in the structured conditions (Didactic Instruction and Guided Play) would engage in more math talk, specifically related to fractions, was mostly supported. Parents in the Didactic Instruction and Guided Play conditions used more math talk overall, and talk about specific quantities (e.g., numbers and informal fractions) as well as more broad quantitative terms. Parents in the Didactic Instruction condition, but not the Guided Play condition, also used more formal fraction words than the parents in the Unguided Play condition.

Also notably, parents in the Didactic Instruction condition used more math talk overall, as well as within each of the categories of math talk, than parents in the Guided Play condition. Furthermore, there were also condition differences in the diversity of parents' math talk, measured by the number of different math words they used during the activity: the speech of parents in the Didactic Instruction condition included a greater range of math words than that of parents in the Guided Play condition, and parents in both structured conditions included a greater range of math words in their speech compared to parents in the Unguided Play condition.

Hypothesis 1b: Children in the Didactic Instruction and Guided Play conditions will engage in more math talk than children in the Unguided Play condition, and children in the Guided Play condition will engage in more math talk than children in the Didactic Instruction condition.

Quantity. Parallel to the analyses of parent math talk, children's math talk across conditions was compared by conducting an ANOVA comparing the total proportion of child math word tokens across the three conditions (see Table 7 for total math tokens). There was a significant between-conditions difference in the overall amount of math talk that children used, $F(2, 69) = 30.34$, $p < .001$, $\eta^2 = .47$. As shown in Table 8, post-hoc pairwise comparisons revealed that children in the Didactic Instruction condition (M = 0.213, SD = 0.130) used a greater proportion of total math tokens than children in the Guided Play (M = 0.098, SD = 0.048) and Unguided Play (M = 0.031, SD = 0.025) conditions (both $p < .001$). Children in the Guided Play condition used a greater proportion of total math tokens than children in the Unguided Play condition ($p < .05$).

Next, a MANOVA was conducted to compare the proportions of each category of child math word tokens (i.e., Formal Fraction Words, Informal Fraction Words, Quantitative Words, and Number Words) across the three conditions. There was a significant multivariate effect of condition, $F(8, 134) = 7.59$, $p < .001$, $\eta^2 = .31$. Univariate analyses and post-hoc comparisons were next examined.

Formal fraction word tokens. Follow-up univariate tests indicated that there was a significant between-condition difference for children's formal fraction word tokens, $F(2, 69) = 8.88$, $p < .001$, $\eta^2 = .21$. Children in the Didactic Instruction condition (M = 0.017, SD = 0.019) used a greater proportion of formal fraction word tokens compared to the children in the Guided Play (M = 0.003, SD = 0.005) and Unguided Play (M = 0.005, SD = 0.008) conditions (both $p < .01$); the latter two conditions did not differ from each other in the proportion of formal fraction word tokens.

Informal fraction word tokens. There was also a significant difference for children's the proportion informal fraction word tokens, $F(2, 69) = 5.84$, $p < .01$, $\eta^2 = .15$. Children in the Unguided Play condition (M = 0.001, SD = 0.002) used a smaller proportion of informal fraction words than children in the Didactic Instruction (M = 0.006, SD = 0.009; $p < .05$) and Guided Play (M = 0.007, SD = 0.006; $p < .01$) conditions; children in the Didactic Instruction and Guided Play conditions did not differ in their proportions of informal fraction word tokens.

Quantitative word tokens. There was a marginal difference in the proportion of quantitative word tokens used by children, $F(2, 69) = 2.78$, $p = .069$, $\eta^2 = .08$. Children in the Didactic Instruction (M = 0.020, SD = 0.013) and Guided Play (M = 0.020, SD = 0.013) conditions used a slightly greater proportion of quantitative word tokens than the children in the Unguided Play condition (M = 0.013, SD = 0.009); however, these differences were not significant (both $p > .10$). Children in the Didactic Instruction and Guided Play conditions did not differ in their proportions of quantitative word tokens.

Number word tokens. Additionally, there was a significant difference for children's proportion of number word tokens, $F(2, 69) = 24.60, p < .001, \eta^2 = .42$. Children in the Didactic Instruction condition ($M = 0.170, SD = 0.128$) used a greater proportion of talk about numbers than children in the Guided Play ($M = 0.067, SD = 0.043$) and Unguided Play ($M = 0.013, SD = 0.012$) conditions (both $p < .001$), and children in the Guided Play condition used a slightly greater proportion of number tokens than children in the Unguided Condition ($p < .06$).

Quality. To examine differences in the diversity of child math talk, an ANOVA comparing the total amount of children's *different* math word types across the three conditions was conducted. There was a significant between-conditions difference in the overall amount of math talk that children used, $F(2, 69) = 16.13, p < .001, \eta^2 = .32$. As shown in Table 9, post-hoc pairwise comparisons revealed that children in the Didactic Instruction condition ($M = 11.00, SD = 3.89$) used a greater number of different math word types than children in the Guided Play ($M = 7.92, SD = 2.41; p < .01$) and Unguided Play ($M = 5.83, SD = 2.80; p < .001$) conditions. Children in the Guided Play condition used a slightly greater number of different math words than children in the Unguided Play condition ($p < .08$).

A MANOVA was conducted to compare the diversity of child math talk, as measured by the number of different words types uttered (i.e., Formal Fraction Words, Informal Fraction Words, Quantitative Words, and Number Words) across the three conditions. There was a significant multivariate effect, $F(8, 134) = 8.69, p < .001, \eta^2 = .34$. Univariate tests and post-hoc pairwise comparisons were next examined.

Formal fraction word types. Follow-up univariate tests indicated that there was a significant between-condition difference for children's formal fraction word types, $F(2, 69) = 18.74$, $p < .001$, $\eta^2 = .35$. Children in the Didactic Instruction condition ($M = 1.92$, $SD = 1.35$) used more different formal fraction word types than the children in the Guided Play ($M = 0.56$, $SD = 0.51$) and Unguided Play ($M = 0.52$, $SD = 0.59$) conditions (both $p < .001$), and there was no difference in the amount of different formal fraction word types used by children in the latter two conditions.

Informal fraction word types. There was also a significant difference for the amount of different informal fraction word types used by children, $F(2, 69) = 5.97$, $p < .01$, $\eta^2 = .15$. Children in the Unguided Play condition ($M = 0.48$, $SD = 0.73$) used fewer different informal fraction word types than the children in the Didactic Instruction ($M = 1.29$, $SD = 1.27$; $p < .05$) and Guided Play ($M = 1.56$, $SD = 1.26$; $p < .01$) conditions, and children in the latter two groups used similar amounts of informal fraction word types.

Quantitative word types. There were no between-condition differences in the amount of quantitative word types used by children, $F(2, 69) = 0.48$, $p = .70$, $\eta^2 = .01$ (Didactic Instruction, $M = 2.38$, $SD = 1.21$; Guided Play, $M = 2.24$, $SD = 1.13$; Unguided Play, $M = 2.52$, $SD = 1.08$).

Number word types. Finally, there was a significant difference for children's types of number words, $F(2, 69) = 11.24$, $p < .001$, $\eta^2 = .25$. Children in the Didactic Instruction condition ($M = 5.42$, $SD = 3.37$) used a greater amount of different number word types than children in both the Guided Play ($M = 3.56$, $SD = 0.87$; $p < .05$) and Unguided Play ($M = 2.30$, $SD = 1.85$; $p < .001$) conditions. Children in the Guided Play and Unguided Play conditions did not differ in the amount of number word types used.

Thus, the first part of the hypothesis regarding children's math talk was mostly supported: children in the Didactic Instruction and Guided Play conditions used a greater proportions of overall math words, informal fraction words, and number words than children in the Unguided Play condition. However, contrary to the second part of the hypothesis, children in the Guided Play condition did not use more math talk than children in the Didactic Instruction condition. In fact, children in the Didactic Instruction condition used greater proportions of overall math talk, formal fractions, and number words compared to children in the Guided Play condition, as well as a greater amount of different formal fraction and number words.

Hypothesis 1c: Parents in the Didactic Instruction condition will drive the interaction more than their children, whereas in the Unguided Play condition, children will drive the interaction; there will be a balance between parent and child talk in the Guided Play condition. To test the hypothesis that the extent of parent and child involvement would vary as a function of condition, an ANOVA was used to compare the ratio of child to parent mean length of turns (MLT). There was a significant between-condition difference in the ratio of child to parent MLT, $F(2, 69) = 10.54$, $p < .001$, $\eta^2 = .23$. The ratio for the Unguided Play condition ($M = 0.579$, $SD = 0.279$) was significantly higher than the child-to-parent ratios for the Didactic Instruction ($M = 0.310$, $SD = 0.193$) and Guided Play ($M = 0.315$, $SD = 0.211$) conditions (both $p < .001$). The ratios for the latter two conditions did not differ significantly.

Thus, the hypothesis that parents would be more involved than children in the Didactic Instruction condition and that children would be more involved than parents in the Unguided Play condition, and that parents and children would be equally involved in the Guided Play condition, was not supported. In all conditions, parents had a greater number of utterances than their children, and were as engaged in the Guided Play condition as they were in the Didactic Instruction condition.

Change in Children's Fraction Knowledge

The means and standard deviations for children's performance on the fraction knowledge measures before and after the interaction are presented in Table 10. To test Hypothesis 2, that children in the Guided Play condition would demonstrate greater improvement in their fraction knowledge than that of children in the other two conditions, a MANOVA was used to compare the change between pre- and posttest fraction test scores of the children in the three conditions (Didactic Instruction, Guided Play, and Unguided Play). Change variables were computed by subtracting children's pretest score from their posttest score separately for each of the three subtests. There was no significant multivariate effect of condition on children's change in fraction test scores, $F(6, 136) = 0.36$, $p = .90$, $\eta^2 = .02$. Since no improvement in fraction knowledge was observed in any of the conditions, these findings did not support the hypothesis that children in the structured conditions would demonstrate more growth in fraction knowledge compared to children in the Unguided Play condition.

Mechanisms for Fraction Learning during Joint Activities

Given that there was no evidence to support that children's fraction knowledge improved between the pre- and posttests, it was not possible to investigate what factors during the joint activity might account for children's learning or to test the mediation model proposed in Hypothesis 3. It was possible, however, to examine whether parent fraction talk predicted child fraction talk during the joint activity. Since establishing the relation between the predictor variable and the mediating variable is a step in mediation analyses, testing this relation would at least provide insight into whether the mediation model is plausible and should be further tested in future research.

Predicting quantity of child fraction talk. In order to examine whether the quantity of parents' fraction talk predicted the quantity of children's fraction talk, a hierarchical linear regression analysis was used (see Table 11 for summary of results). In the first step, to account for condition differences, two dummy variables were entered, with the Unguided Play condition as the reference group. The total number of parents' formal fraction tokens was entered in a second step. The full model accounted for 35% of the variance in children's formal fraction word tokens, $F(3, 68) = 12.44$, $p < .001$, $R^2 = .35$. After accounting for condition differences, parents' formal fraction tokens accounted for 21% of the variance, $\beta = .69$, $p < .001$, in children's formal fraction tokens.

Predicting quality of child fraction talk. Another hierarchical linear regression analysis was used to examine whether the quality of parents' fraction talk (word types) predicted the quality of children's formal fraction talk. Again, dummy variables were entered in the first step to account for condition differences. The full model accounted for 41% of the variance in children's formal fraction word types, $F(3, 68) = 11.81$, $p < .001$, $R^2 = .41$. After accounting for the conditions, parents' formal fraction types accounted for 6% of the variance, $\beta = .43$, $p < .01$, in children's formal fraction types.

In summary, after accounting for condition differences, the frequency and diversity of parents' talk about formal fractions predicted the frequency and diversity of children's talk about formal fractions during the joint activity. Thus, if growth in children's knowledge about fractions had been observed, and it was possible to test parent fraction talk as a predictor of children's fraction knowledge, it appears that children's fraction talk would be a plausible mediator for the pathway between parent fraction talk and children's learning.

Post-Activity Survey

As a manipulation check, a MANOVA was conducted to compare parents' responses to the post-activity survey across the three conditions (see Table 12). There was a significant multivariate effect, $F(14, 128) = 4.78$, $p < .001$, $\eta^2 = .34$.

Enjoyment. Follow-up univariate tests indicated that there was a significant between-condition difference for parents' perceptions that their children enjoyed the activity, $F(2, 69) = 6.98$, $p < .01$, $\eta^2 = .17$. Parents in the Didactic Instruction condition ($M = 5.38$, $SD = 1.10$) rated their children's enjoyment as lower than the parents in the Guided Play ($M = 6.04$, $SD = 0.90$; $p < .05$) and Unguided Play ($M = 6.35$, $SD = 0.71$; $p < .01$) conditions rated their children's enjoyment; the latter two conditions did not differ from each other. There was also a significant difference for parents' rating of their own enjoyment, $F(2, 69) = 4.72$, $p < .05$, $\eta^2 = .12$. Parents in the Didactic Instruction condition ($M = 6.04$, $SD = 0.86$) rated their own enjoyment slightly lower than the parents in the Guided Play condition ($M = 6.48$, $SD = 0.51$; $p = .07$) and lower than parents in the Unguided Play condition ($M = 6.61$, $SD = 0.58$; $p < .05$); there were no differences between the Guided Play and Unguided Play conditions in how parents rated their own enjoyment.

Challenging. Additionally, there was a significant difference for parents' ratings of how challenging the activity was for their children, $F(2, 69) = 20.23$, $p < .001$, $\eta^2 = .37$. Parents in the Unguided Play condition ($M = 4.57$, $SD = 1.34$) rated the activity as less challenging than parents in the Didactic Instruction ($M = 6.13$, $SD = 0.74$) and Guided Play ($M = 6.20$, $SD = 0.82$) conditions (both $p < .001$); parents in the latter two conditions rated the activity as equally challenging. There were no condition differences in parents' rating regarding if they felt unsure about how to interact with their children during the activity, $F(2, 69) = 3.66$, $p = .12$, $\eta^2 = .06$ (Didactic Instruction, $M = 2.83$, $SD = 1.49$; Guided Play, $M = 2.44$, $SD = 1.19$; Unguided Play, $M = 2.04$, $SD = 1.19$).

Promotes learning. Furthermore, there was also a significant difference for parents' ratings of how much the activity promoted children's math learning, $F(2, 69) = 6.37$, $p < .01$, $\eta^2 = .16$. Parents in the Unguided Play condition ($M = 5.96$, $SD = 0.98$) rated the activity as less promoting of children's math knowledge than parents in the Didactic Instruction ($M = 6.71$, $SD = 0.62$, $p < .01$) and Guided Play ($M = 6.60$, $SD = 0.71$, $p < .05$) conditions; the latter two conditions did not differ in the parent ratings of the activity promoting math knowledge. There were no differences in parents' ratings of the activity promoting children's language and reading skills, $F(2, 69) = 1.14$, $p = .59$, $\eta^2 = .02$ (Didactic Instruction, $M = 5.08$, $SD = 1.67$; Guided Play, $M = 5.48$, $SD = 1.50$; Unguided Play, $M = 5.43$, $SD = 1.16$).

Promotes creativity. Finally, there was also a significant difference for parents' ratings of how much the activity promoted children's creativity, $F(2, 69) = 5.76$, $p < .01$, $\eta^2 = .14$. Parents in the Didactic Instruction condition ($M = 5.58$, $SD = 1.41$) rated the activity as less promoting of creativity than parents in the Guided Play ($M = 6.32$, $SD = 0.69$, $p < .05$) and Unguided Play ($M = 6.48$, $SD = 0.59$, $p < .01$) conditions; the latter two conditions did not differ in the parent ratings of the activity promoting creativity.

These findings suggest that parents viewed the Guided Play activity as being as enjoyable and as likely to promote creativity as the Unguided Play activity; however, parents also saw the Guided Play activity as being as challenging and as likely to promote math knowledge as the Didactic Instruction condition.

Follow-Up Analyses: How Much Does Text Account for Differences in Parent Talk?

In order to determine how much the provided text in the Didactic Instruction and Guided Play conditions (i.e., worksheet and storybook, respectively) contributed to differences in parent talk during the activity, the analyses of parent talk were reexamined. All utterances in which parents were reading directly from the text, were excluded, with the newly created variables referred to from this point forward as “non-text talk”.

Total parent non-text talk. An MANOVA examined between-condition differences in parents’ overall non-text talk as measured by the total number of turns, the mean length of turns (MLT), the total number of types of words, and the total number of tokens. There was a significant multivariate effect, $F(8, 134) = 4.05$, $p < .001$, $\eta^2 = .20$.

Univariate analyses indicated a significant between-condition difference for parent MLT, $F(2, 69) = 8.71$, $p < .001$, $\eta^2 = .20$. Post-hoc pairwise comparisons revealed that parents in the Unguided Play condition ($M = 7.39$, $SD = 1.60$) had a shorter MLT than parents in the Didactic Instruction ($M = 10.89$, $SD = 3.79$; $p < .001$) and Guided Play ($M = 9.79$, $SD = 2.93$; $p < .05$) conditions, and there was no difference in the MLTs of the latter two conditions. Similarly, there was a significant difference for the total number of parent tokens, $F(2, 69) = 3.57$, $p < .05$, $\eta^2 = .09$. Parents in the Unguided Play condition ($M = 1046.13$, $SD = 237.34$) used fewer tokens than in the Didactic Instruction condition ($M = 1311.37$, $SD = 372.87$; $p < .05$); the total number of parents’ non-text tokens in the

Guided Play condition (*M* = 1173.76, *SD* = 384.85) did not differ from either of the other two conditions.

There was a marginal difference in the total number of parent turns, *F*(2, 69) = 2.88, *p* = .06, η^2 = .08. Parents in the Unguided Play condition (*M* = 143.96, *SD* = 28.26) had slightly more turns than parents in the Guided Play condition (*M* = 122.56, *SD* = 31.70); the number of turns for parents in the Didactic Instruction condition (*M* = 127.67, *SD* = 35.64) did not differ from either of the other two conditions. There were no significant differences in the total number of types of words that parents used *F*(2, 69) = 0.67, *p* = .52, η^2 = .02 (Didactic Instruction, *M* = 254.13, *SD* = 48.92; Guided Play, *M* = 250.52, *SD* = 41.20; Unguided Play, *M* = 264.30, *SD* = 36.49).

Quantity of parents' non-text math talk. To examine differences in the amount of parent math talk, an ANOVA comparing the proportion of total parent non-text math word tokens across the three conditions was conducted. There was a significant between-conditions difference in the overall amount of math word tokens that parents used, *F*(2, 69) = 43.48, *p* < .001, η^2 = .56. As shown in Table 8, pairwise comparisons revealed that when excluding text, parents in the Didactic Instruction condition (*M* = 0.123, *SD* = 0.042) used a greater proportion of math word tokens than parents in the Guided Play (*M* = 0.068, *SD* = 0.023) and Unguided Play (*M* = 0.014, *SD* = 0.009) conditions (both *p* < .001). Parents in the Guided Play condition used a greater proportion of math tokens than parents in the Unguided Play condition (*p* < .001).

Next, a MANOVA was conducted to compare the proportions of each category of parent math word tokens (i.e., Formal Fraction Words, Informal Fraction Words, Quantitative Words, and Number Words) across the three conditions. There was a significant multivariate effect, $F(8, 134) = 11.90$, $p < .001$, $\eta^2 = .42$.

Formal fraction word tokens. Follow-up univariate tests indicated that there was a significant between-condition difference for parents' formal fraction word tokens, $F(2,69) = 31.08$, $p < .001$, $\eta^2 = .47$. Parents in the Didactic Instruction condition (*M* = 0.024, *SD* = 0.014) used a greater proportion of formal fraction word tokens than the parents in the Guided Play (*M* = 0.005, *SD* = 0.004) and Unguided Play (*M* = 0.003, *SD* = 0.005) conditions (both $p < .001$); the latter two conditions did not differ from each other.

Informal fraction word tokens. There was also a significant difference for parents' informal fraction word tokens, $F(2, 69) = 36.23$, $p < .001$, $\eta^2 = .51$. Parents in the Didactic Instruction condition (*M* = 0.014, *SD* = 0.007) used a greater proportion of informal fraction word tokens than the parents in the Guided Play (*M* = 0.007, *SD* = 0.004) and Unguided Play (*M* = 0.002, *SD* = 0.004) conditions (both $p < .001$); and parents in the Guided Play condition used more informal fraction word tokens than parents in the Unguided Play condition ($p < .001$).

Quantitative word tokens. There was a significant difference in the proportion of quantitative word tokens used by parents, $F(2, 69) = 23.60$, $p < .001$, $\eta^2 = .41$. Parents in the Didactic Instruction condition (*M* = 0.049, *SD* = 0.020) used a greater proportion of quantitative word tokens than parents in the Guided Play (*M* = 0.033, *SD* = 0.016; $p < .01$) and Unguided Play (*M* = 0.025, *SD* = 0.021; $p < .001$) conditions; parents in the Guided Play condition used more quantitative word tokens than parents in the Unguided Condition ($p < .001$).

Number word tokens. Finally, there was a significant difference for parents' number word tokens, $F(2, 69) = 36.22$, $p < .001$, $\eta^2 = .56$. Parents in the Didactic Instruction condition ($M = 0.035$, $SD = 0.017$) used a greater proportion of number word tokens than parents in the Guided Play ($M = 0.023$, $SD = 0.009$) and Unguided Play ($M = 0.006$, $SD = 0.005$) conditions (both $p < .001$), and parents in the Guided Play condition used more number word tokens than parents in the Unguided Condition ($p < .001$).

Quality of parents' non-text math talk. To examine differences in the diversity of parent math talk, an ANOVA comparing the total amount of parents' different math word types across the three conditions was conducted. There was a significant between-conditions difference in the overall amount of math word types that parents used, $F(2, 69) = 81.83$, $p < .001$, $\eta^2 = .70$. As shown in Table 9, post-hoc pairwise comparisons revealed that parents in the Didactic Instruction condition ($M = 18.13$, $SD = 2.19$) used a greater amount of different math word types than parents in the Guided Play ($M = 12.84$, $SD = 2.79$) and Unguided Play ($M = 7.69$, $SD = 3.31$) conditions (both $p < .001$). Parents in the Guided Play condition used a greater amount of different math word types than parents in the Unguided Play condition ($p < .001$).

Another MANOVA was conducted to compare the diversity of parent math talk, as measured by the number of different words types uttered (i.e., types of Formal Fraction Words, Informal Fraction Words, Quantitative Words, and Number Words) across the three conditions. There was a significant multivariate effect, $F(8, 134) = 15.03$, $p < .001$, $\eta^2 = .47$.

Formal fraction word types. Follow-up univariate tests indicated that there was a significant between-condition difference for parents' formal fraction words, $F(2, 69) =$

50.42, $p < .001$, $\eta^2 = .59$, such that parents in the Didactic Instruction condition (M = 3.17, SD = 1.01) used more different formal fraction word types than the parents in the Guided Play (M = 1.28, SD = 0.84) and Unguided Play (M = 0.74, SD = 0.75) conditions (both $p < .001$), and parents in the Guided Play and Unguided Play conditions did not differ from each other in the amount of formal fraction word types.

Informal fraction word types. There was also a significant difference for the number of different informal fraction word types used by parents, $F(2, 69) = 47.44$, $p < .001$, $\eta^2 = .58$. Parents in the Didactic Instruction condition (M = 3.88, SD = 0.80) used a greater number of informal fraction word types than the parents in the Guided Play (M = 3.04, SD = 1.24; $p < .05$) and Unguided Play (M = 0.91, SD = 1.13; $p < .001$) conditions; and parents in the Guided Play condition used a greater number of different informal fraction word types than parents in the Unguided Play condition ($p < .001$).

Quantitative word types. There was a significant differences in the number of types of different quantitative words used by parents, $F(2, 69) = 23.99$, $p < .001$, $\eta^2 = .41$. Parents in the Didactic Instruction condition (M = 6.58, SD = 1.10) used a greater amount of quantitative word types than parents in the Guided Play (M = 5.08, SD = 1.47) and Unguided Play (M = 3.83, SD = 1.50) conditions (both $p < .001$), and parents in the Guided Play condition used more different quantitative word types than parents in the Unguided Condition ($p < .01$).

Number word types. Finally, there was a significant difference for parents' types of number words, $F(2, 69) = 29.32$, $p < .001$, $\eta^2 = .46$. Parents in the Didactic Instruction condition (M = 4.50, SD = 1.25) used a greater amount of different number word types than parents in the Guided Play (M = 3.44, SD = 0.51; $p < .01$) and Unguided Play (M =

2.22, *SD* = 1.17; $p < .001$) conditions, and parents in the Guided Play condition used more different number word types than parents in the Unguided Condition ($p < .001$).

Ratio of child and parent mean length of turns, excluding text. A new ratio of child to parent MLT was calculated, excluding parent utterances where they were reading text, and a new ANOVA was conducted to compare conditions. There was a significant between-condition difference in the ratio of child to parent MLT, $F(2, 69) = 6.02$, $p < .01$, $\eta^2 = .15$. Post-hoc pairwise comparisons revealed that the ratio for the Unguided Play condition (*M* = 0.58, *SD* = 0.28) was significantly higher than that for the Didactic Instruction condition (*M* = 0.33, *SD* = 0.21; $p < .01$) and marginally higher than the Guided Play condition (*M* = 0.40, *SD* = 0.28; $p = .052$). The ratios for the latter two conditions did not differ significantly.

In summary, excluding parents' utterances where they read text attenuated the effect size for some of the condition differences. There were two notable changes. First, while there were no condition differences between the total number of parents' speaking turns when text utterances were included, there was a significant difference after excluding text utterances. Parents in the Guided Play condition had slightly fewer speaking turns than parents in the Unguided Play condition. Second, the difference between the total number of word tokens for Unguided Play and Guided Play parents was no longer significant when text utterances were eliminated. Overall, however, the differences in the quantity and quality of parent talk remained significant when only examining parents' non-text speech.

Chapter 5: Discussion

The present study sought to extend the current research on how parents contribute to their children's early math development by examining how activity context influences how parents talk to preschoolers about math concepts. An experimental approach where the structure of a fraction-related activity was manipulated was used to test hypotheses regarding the extent to which the structure of the activity would affect parent and child talk about math concepts, as well as how parents and children engaged in the activity together. Furthermore, by evaluating children's understanding of fractions before and after the joint activity, the study aimed to examine in which contexts parents would be most successful in effectively teaching fraction concepts to their children.

In general, the manipulation of the materials and instructions appeared to have an effect on how parents and children engaged in the joint activity. These findings are consistent with previous studies where changes in the instructions, materials, or context influenced the extent to which parents and children talked about mathematical concepts (e.g., Bjorklund et al., 2004; Ferrara et al., 2011; Vandermaas-Peeler, Ferretti et al., 2012). In addition to substantial differences in how parents and children talked about math during the activity, the survey given to parents afterwards yielded responses indicating that parents in each context perceived the activity differently in terms of both enjoyment and learning. These findings regarding parent and child engagement and parents' ratings are discussed further next.

Parent and Child Overall Talk

Parent overall talk. Parents in the didactic and playful math contexts differed from parents in the unguided play context in their overall talk during the activity. The mean length of parents' speaking turns (words per turn) was longer in the two more structured contexts, even when text-reading utterances were excluded. Parents in the didactic and playful math contexts also used more words overall during the interaction compared to parents in the unguided play context, although the diversity of their overall talk did not differ. Thus, parents in the structured contexts did not speak more often nor did they use a more diverse vocabulary than parents in the less structured context, but they did use more words and spoke longer during the activity.

Importantly, the differences found between the contexts remained even after excluding parents' utterances while reading directly from the text. Although reading the text did not completely account for the differences in parents' overall talk, the presence of the printed stimuli in the structured math contexts likely encouraged parents to speak more since they were provided with prompts to discuss with their children. Additionally, with preschool-aged children whose reading abilities were limited, asking the dyads to work from a text immediately placed a greater role on the parents to drive the activity forward. The implications for how printed materials contribute to parents' engagements will be further considered when discussing future directions.

Child overall talk. Similar to the findings for parent overall talk, there were significant differences in children's overall speech across contexts; however, the effects were in the opposite direction. Children in the unguided play context had longer speaking turns compared to children in the didactic context. Additionally, children in the unguided play context also used more diverse language and more words overall than children in the structured math contexts. In the unguided play context, dyads were instructed to play as they would typically play at home. It is likely that in everyday play interactions, children are more likely to take responsibility for making decisions about the goals of their play and what role each play partner takes on (Garvey, 1990; Pellegrini, 2009). Similarly, during unguided play in the present study, many children were observed designating the roles for themselves and their parents, and determining what the goal of the activity was (e.g., having a picnic, playing restaurant). In these instances, children frequently narrated what they were doing (sometimes prompted by parents, but other times initiated by the children themselves), which would account for the greater amount of child talk in the unguided play context.

Proportions of child and parent talk. For the aim of this study, the overall amount of parent and child talk was examined as a way of measuring the extent to which each partner was engaged in the joint activity and shared responsibility for progressing the activity forward. From a sociocultural perspective this balance between partners is important to consider. The process of learning occurs through dynamic, shared exchanges in which both partners are active participants and children gradually take on a greater role (Rogoff, 1998; Wertsch, 2008). Thus, we can only make sense of the parent and child talk when examining how they relate to each other. To do so, the proportions of child and parent speaking turns, measured as a ratio of child and parent mean lengths of turns, were assessed.

The length of children's speaking turns, relative to the length of parents' speaking turns, was shorter for the structured math contexts than for the unguided play context. In all contexts, parents typically had longer speaking turns than children, but in the structured math contexts, there was less of a balance between the amount of time that parents and children were speaking, with parents speaking more than the children. Since parents in the structured contexts rated the activity as being more challenging than parents in the unguided play context, the fact that parents spoke more in relation to the children in the structured contexts could be reflective of their scaffolding, where parents provided more support and guidance to assist their children with the more challenging and unfamiliar task (Rogoff, 1998; Wertsch, 2008; Wood & Middleton, 1975). In contrast, the familiar unguided play scenario likely allowed children to be more autonomous. For example, it was not uncommon for the children to assign themselves an active role (e.g., the chef) and assign their parents a more passive role (e.g., the customer), with some children specifically instructing their parents to not do anything while they "prepared" the food.

It is possible that if parents and children were observed engaging in the structured fraction activities for a longer time, a shift would be observed as children became more familiar with the activity, in which children gradually took on more of an equal, or even greater, speaking role compared to parents' speaking turns. Previous studies have observed a shift in the amount of physical involvement (i.e., handling activity materials) from the beginning to end of a problem-solving task, with parents reducing the amount they handled the materials as their children became more familiar with assembling a puzzle, which was interpreted as giving the children more responsibility for completing the task (Winsler, Diaz, McCarthy, Atencio, & Chabay, 1999). Therefore, for the task in the present study, where answering the questions and completing the task was accomplished through discussion, it seems reasonable that as children's understanding of the fraction-related concepts increased, they would take on a greater speaking role, where the parents asked the questions and children led the discussion in finding the answers.

Parent and Child Math Talk

Quantity and quality of parent math talk.

Parent math talk in structured versus unstructured contexts. The total amount of math-related words that parents used during the interaction differed across contexts, as was predicted by the first hypothesis. When parents were provided with a structured, math-related goal for the activity, they used substantially more math words than when they were asked to play as they would typically play at home. On average, parents in the playful math context used almost four times as many math words as parents in the unguided play context and parents in the didactic context used more than six times as many math words as parents in the unguided play context.

In addition to differences in the quantity of parent math talk, there were also differences in the diversity of parents' math talk across contexts, as measured by the number of different math words spoken at least once during the activity. Like the total quantity of talk, the diversity of parent math words was greater in the structured math activities: on average, parents in the didactic and playful math contexts used about twice as many different math words as parents in the unguided play context. This pattern was consistent for all specific categories of math talk, and was upheld when only examining parents' non-text talk.

It should be noted that the context differences in the amount and diversity of parent math talk were not driven strictly by words related to fractions; parents in the unguided play context on average only used between six and seven number words and around two different number words throughout the activity (compared to 37 words and between 3 and 4 different words in the playful math context, and 46 words and between 4 and 5 different words in the didactic math context). Thus, the higher quantity and diversity of math words in the structured math contexts cannot be attributed to simply being boosted by more talk about fractions, nor was it the case that talk about whole numbers or general quantitative terms was comparable across contexts.

These findings support previous research that parents do provide some number talk without prompting (e.g., Anderson et al., 2004; Durkin et al., 1986; Levine et al., 2010). However, they are more likely to do so when numerical elements are especially clear or relevant (e.g., Mix et al., 2012; Vandermaas-Peeler et al., 2009) or when they are prompted to talk about math, either through a broad suggestion (e.g., Skwarchuk, 2009) or specific guidance on how to do so (Vandermaas-Peeler, Boomgarden, et al., 2012; Vandermaas-Peeler, Ferretti, et al., 2012). Similar findings have been found for spatial talk, as well, with parents engaging in more spatial talk when the goal of an activity was defined (i.e., providing steps to construct a structure out of blocks) rather than undefined (Ferrara et al., 2011).

Parent math talk in didactic versus playful structured contexts. It was not hypothesized that parents in the didactic math context would use greater amounts of math talk compared to parents in the playful context; rather, it was expected that the two contexts, which each included six parallel questions about fractions, would yield similar amounts of parent math talk. Instead parents in the didactic context exceeded parents in the playful math context in the quantity and diversity of all categories of math talk. One possible explanation for this is that although both contexts were explicitly stated as being intended to promote children's understanding of fractions, this goal remained more prominent for the didactic context. Parents in the didactic context had more explicit reminders throughout the activity since the worksheet questions included formal fraction words. The storybook questions in the playful math context, on the other hand, were intended to be more of an everyday context and did not include formal fraction words; thus, there was less of a clear, recurring cue for parents to talk about fractions.

Previous work by Bjorklund and colleagues (2004) yielded similar results where parents engaged in teaching more arithmetic strategies when solving formal arithmetic problems with their children compared to while playing a board game. The lower rate of parent teaching in the board game context could be attributed to the fact that they were instructed to play the game as they typically would. If it had been suggested that the board game could be used to teach arithmetic, parents may have engaged in more teaching, as was found in a later study (Vandermaas-Peeler, Ferretti, et al., 2012). The findings from the present study support the notion that drawing parents' attention to the math relevance of an informal activity increases how much parents talk about math (i.e., parents in the playful math context used more math talk than parents in the unguided play context), but still may not engage in as much math talk as they would in a formal teaching activity.

It is also possible that the absence of extra materials in the didactic math context contributed to the amount of math talk parents used and the prominence of the math-related goal. Despite the introduction as a fraction activity and the math relevant questions in the storybook, dyads in the playful math context were given extra materials (i.e., plates, a knife, and a cutting board) and extra information in the text regarding the characters and plotline of the story. Furthermore, the Velcro-attached food pieces were able to be taken apart. These features are likely to have contributed to why parents in the playful math context rated their own and their children's enjoyment as higher than parents in the didactic context; however, they also may have resulted in the presence of multiple goals during the activity. When comparing the approaches that dyads in the didactic and playful contexts used to the same question (worded differently based on context: which items can be split in half?" or "which items can the two friends share fairly?"), the path to arriving at the answer was often more elaborate for dyads in the playful math context (see Appendix K for sample transcripts). When following the storybook, dyads in the playful math context often chose to assign plates to characters, select foods, cut foods using the knife, and distribute the pieces to the plates. In contrast, beyond identifying the foods, there was little else that dyads in the didactic context could do, and consequently they answered questions more efficiently and directly. The implications of such differences on how these activities may promote children's learning will be discussed later in the document.

Quantity and quality of child math talk.

Child math talk in structured versus unstructured contexts. The hypothesis that children in the structured math contexts would engage in greater amounts of math talk than children in the unguided play context was supported. Children in the didactic context used more than four times the total number of math words compared to children in the unguided play context; children in the playful math context used about twice as many math words compared to children in the unguided play context. While the informal fraction, quantitative, and number words used by children in both the playful and didactic math contexts exceeded those used by children in the unguided play context, only children in the didactic context used more formal fraction words than children in the unguided play context. In other words, children in the playful math and unguided play contexts did not differ in the amount of formal fraction words they used.

Additionally, the diversity of children's math language, measured as the number of different math words spoken at least once during the activity, differed across groups, with children in the structured math contexts using a greater range of math words compared to children in the unguided play context. Children in both the didactic and playful math contexts used a greater range of informal fraction words and number words compared to children in the unguided play context, and children in the didactic context also used a greater range of formal fraction words compared to children in the unguided play context.

The most likely explanation for the differences in the amount and diversity of children's math talk is the amount of parent math talk. Previous studies have found strong positive correlations between parent and child math talk (Levine et al., 2010; Ramani et al., 2015); thus, the same contexts that elicited more parent math talk in turn would be expected to elicit more child math talk. In particular, the structured math contexts included questions for the parent to ask related to fraction concepts, prompting parent-child discussions that would naturally include more math talk. Correlation analyses for the present study support this, with significant positive relations between parent and child overall math word quantity and diversity, and specifically for fraction and number words. These relations were not as strong for informal fraction words and quantitative words. It is possible that informal fraction terms and general quantitative words are more familiar to children and commonly used in everyday interactions; consequently children possess greater facility to use such words without parents' prompting, compared to formal fraction words. This is similar to the findings from Ramani et al. (2015) where the correlations between parent and child number talk were much stronger for talk about more advanced concepts such as arithmetic and ordinal relations compared to the correlations between parent and child talk more simpler concepts such as counting and numeral identification.

Child math talk in didactic versus playful structured contexts. The second part of the hypothesis pertaining to child math talk, however, was not supported. It was predicted that children in the playful math context would use more math talk than children in the didactic context; however, children in the didactic context used about twice as many math words as children in the playful math context, and also used a greater range of different math words during the activity. These differences were not consistent for all of the specific categories of math talk, however. While children in both structured math contexts used comparable amounts of informal fraction words, children in the playful math context used fewer formal fraction and number words compared to children in the didactic context. Additionally, the diversity in the formal and informal fraction words and number words used by children in the playful math context were less than those used by children in the didactic context.

It was expected that children in the playful math context would engage in more math talk because the exploratory nature of the task would promote more parent-child discourse about the questions in the storybook. As noted earlier when discussing parent math talk, the extraneous materials included in the playful math context, intended to replicate everyday informal experiences in a playful context, may have resulted in multiple goals beyond simply talking about fractions (e.g., cutting food, acting out the story). Consequently, both parents and children may have split their attention and conversation between the math-oriented and other goals, resulting in less math talk compared to dyads in the didactic context.

Another factor that may have played a role is the extent to which parents elaborated and deviated from the didactic context. A previous study using similar didactic and playful contexts involved children being taught by an experimenter rather than a parent, allowing for stringent adherence to the didactic protocol (Fisher et al., 2013). In contrast, beyond the introduction, worksheet, and materials, in the present study there were no limits set on how the parents could engage their children in the didactic context. Consequently, many of the parents in the didactic context appeared, overall, to be successful in adapting the activity to be more engaging for their children. For instance, some parents spontaneously reframed the questions to be in a real-life context, so that they closely resembled the questions in the playful math context. This is not entirely surprising given the body of literature supporting that parents are often adept at adapting activities based on their children's needs (e.g., Saxe et al., 1987; Wood & Middleton, 1975).

Parent Ratings of the Joint Activity

Parents were asked to complete a survey following the activity that rated their enjoyment and their perception of their children's enjoyment, as well as the extent to which they believed the activity could promote children's development of reading and math skills and creativity. Parents rated both playful activities (playful math and unguided play contexts) as being equally enjoyable for both themselves and their children, and more enjoyable than the didactic activity. At the same time, parents saw the structured math activities (playful math and didactic) as being equally challenging and likely to promote children's learning of math concepts; the unguided play activity was seen as less challenging and less likely to promote math concepts.

Consequently, it would appear that the added structure and content of the playful math context resulted in it posing more of a challenge for children. This did indeed draw dyads' attention to the math-relevance of the activity more than the unguided play context, which was expected to be more representative of how parents and children would typically play with the toys. Furthermore, providing the storybook context and additional toys resulted in the playful math activity being rated as more enjoyable than the didactic activity. It should be noted, however, that it is not clear whether the playful context actually made engaging in math talk more enjoyable for dyads, or whether having the math talk broken up by enjoyable, playful tasks (e.g., playing with the pretend food) only made the math aspect of the activity more palatable. This is worth distinguishing because it could influence how much children were engaged in the math talk. In other words, were children actually motivated to explore the math questions or were they eager to answer a question so that they could move on to playing with the food? Bjorklund and Rosenblum (2002), for instance, observed that children's tendency to use arithmetic strategies that favored accuracy versus efficiency varied depending on whether there was motivation for ensuring accuracy. More behavioral coding of the parent-child exchanges during the activity may help to shed light on these two possibilities.

While the post-activity survey primarily was intended as a manipulation check to ensure differences between the experimental conditions, the findings regarding parents' perceptions of the activities may help to inform future studies and interventions to promote math in the home environment. These implications will be elaborated to a greater extent later in the discussion.

Improvement in Children's Fraction Knowledge

It was predicted that since children in the didactic and playful math contexts would be engaged in more fraction talk during the joint activity compared to children in the unguided play context, improvement in fraction understanding would only be observed for children in the structured math contexts. Furthermore, since children in the playful math context were expected to engage in more fraction talk than children in the didactic context, it was also predicted that children in the playful math context would show the greatest improvement in fraction understanding. Comparisons of the pre- and posttest fraction assessment scores revealed that there was no change in the scores for children from any of the groups. Consequently, there was no evidence to confirm the hypothesis that contexts would differ in the extent to which children learned from the activity; in turn, the lack of improvement in fraction understanding prohibited the possibility of examining the potential mechanisms through which parents would contribute to their young children's early math learning. In light of this, instead of discussing the findings related to these hypotheses, this section of the discussion will explore the possibilities for why growth was not observed.

The fraction assessment was based on previous measures used with preschool-aged children (Singer-Freeman & Goswami, 2001; Sophian et al., 1997; Wing & Beal, 2004) and there was a wide range of scores on all three parts. Thus, it was not the case that the assessments were too challenging for all of the children, nor was it the case that there was a ceiling effect during the pretest that would limit the possibility of observing growth. Consequently, it seems more likely that the lack of change in children's fraction test scores can be attributed to factors related to the joint activity rather than the method of assessment.

Previous studies observing improvement. The questions asked during the joint activity were designed specifically to map on to the concepts tested in the assessment: partitioning objects into equal shares, understanding the relative size of shared portions, and identifying equivalent proportions (e.g., one half equals two fourths). One reason that these concepts were targeted was that previous studies have indicated that five-year-olds' performance on similar tasks improved after brief training sessions. For example, Sophian and colleagues (1997) used an 8-trial training session during which children were able to observe and compare the outcomes of dividing quantities in two different partitions (e.g., dividing a quantity into 2 partitions versus 3 partitions). This training was effective in improving children's responses on a posttest with similar questions. Because children improved from pretest to posttest on both trained items and new items, it was argued that the training improved children's general understanding of the inverse relation between number of recipients and share size, rather than simply remembering the outcome of specific contrasts. Similarly, Singer-Freeman and Goswami (2001) found that giving children experience comparing food items cut into mixed denominators (i.e., half of a pizza was cut into fourths and the other half was cut into eighths) where they could perceptually match up equivalent proportions later improved their success in proportional reasoning.

If such minimal, brief experiences in previous studies have led to improvement in children's fraction knowledge, why was that not the case in the present study? One possibility is that while the underlying concepts of the joint activity and the fraction assessment were similar, the tasks themselves were too different. In other words, it is possible that children did learn from the joint activity but were not able to transfer their acquired understanding to the fraction assessment. In contrast, the previous studies assessed children's performance after training or extra experience with the same materials. In the present study, if children had been tested afterwards by asking them to answer the same questions, or similar questions in a slightly different version of the task, perhaps more improvement (and notable group differences in improvement) would be observed. It should be noted, however, that the decision to use a different task was influenced by the difficulty in developing a posttest that shared features of both the didactic and playful math contexts.

Using the same task in assessment and training also makes it difficult to determine whether children had grasped a conceptual understanding of fraction concepts, or simply learned the procedures for completing the joint activity. Follow-up studies could clarify this issue by working further to refine an assessment method that is more sensitive to detecting development of children's conceptual understanding. Additionally, future analyses could investigate children's talk about the fraction questions during the joint activity; coding children's responses at the microgenetic level (Flynn & Siegler, 2007) may also reflect growth in their understanding prior to children being able to transfer the knowledge to a different task.

Math learning from parent-child interactions. While the lack of growth in the present study differs from previous studies with experimenter-led interventions (Fisher et al., 2013; Singer-Freeman & Goswami, 2001; Sophian et al., 1997), this is similar to an earlier study that assessed children's learning after parent-led, math-related activities. For example, Vandermaas-Peeler, Boomgarden, and colleagues (2012) observed that when parents were prompted to talk about numeracy during a cooking activity with four-year-olds, the children and parents engaged in more number talk than a comparison group; however, children's general number knowledge on a posttest did not significantly differ from the scores for the comparison group. Again, it is possible that the difference between the cooking activity and the posttest was too substantial for children to be able to transfer what they learned.

Given that some of the most compelling evidence of parent math talk as a predictor of children's math knowledge is from longitudinal studies reflecting the cumulative effects of parent math talk (Gunderson & Levine, 2011; Levine et al., 2010), it is possible that one brief parent-child activity would not be enough to observe changes in children's learning. While experimenter-led trainings such as the ones described earlier (Fisher et al., 2013; Singer-Freeman & Goswami, 2001; Sophian et al., 1997) are direct, targeted, and controlled, the greater variation in parent-child joint activities may mean that parents' contributions to children's learning are more likely to be apparent in the long-term. The implications of this for future research, as well as application, are discussed more later on.

Future Directions

Based on the findings of this study, there are three paths for future research. These issues in particular are emphasized as they target questions arising from the current study and work towards building a more comprehensive understanding of how parents contribute to children's early math learning.

Behavioral coding. A theme throughout this discussion is that several remaining questions that have emerged through the findings of the present study would likely be addressed by further coding of parent and child talk and behaviors at a microgenetic level. The process of transcription and analyzing parent and child math talk successfully showcased the differences in the quantity and diversity of math talk across different activity contexts, particularly between unstructured and structured activities. However, there may be some qualitative differences in the parent-child exchanges that are not captured in this approach; these differences especially may be important for distinguishing between the didactic and playful math contexts.

Coding the mathematical content of parent and child speaking turns could provide insight into how the math words are being used and how both partners in the dyad respond to each other. For example, examining parents' math questions to their children and how the children respond could help highlight to what extent it appears that children are actually comprehending the math content, particularly in the playful math context where there were extra materials and distractions. While the transcripts provide evidence of how long children's speaking turns were, it is not clear whether children's turns using math words were below, at, or above the mean length of a turn. Longer math-related turns could potentially indicate that children are not only sharing in the overall conversational load, but in particular is sharing in the math content of the conversation. In contrast, children who are using math words in brief utterances that are shorter than their typical speaking turns would suggest that they are more engaged in other aspects of the activity. Relatedly, behavioral coding could, as previously mentioned, be used to detect growth in children's understanding of the concepts before they are able to be transferred to a posttest assessment, such as examining whether there is a shift in the balance between the child and parent's conversation as the activity continues. Such coding would be beneficial in helping to further align the observations from this study with sociocultural theory, offering insight into the roles of each partner and the developmental changes that occur over time during the interaction.

Behavioral coding could also examine parents' reactions to child responses, especially when a child responds incorrectly. A parent who perseveres with offering hints or explanations, or encourages the child to try again, is likely to be more focused on the math-related goal of the activity. Understanding how contexts of math activities influence parents' questions, explanations, and responses to children is especially important given that some forms of parent guidance are more likely than others to encourage children's engagement and allow them to gain a deeper understanding (Goncu & Rogoff, 1998; Grolnick et al., 2002; Moorman & Pomerantz, 2008). Contingency analyses, in which math-related exchanges are coded at the dyadic level, could provide further support for context influences how both partners respond to each other: when a parent is most likely to focus on children's understanding, and when children are most likely to follow along with the math-oriented goals.

Further exploration of contextual variables. A strength of the current study is the use of similar materials and activities within different contexts. Through discussing the possible explanations for various findings, however, it is clear that there are multiple variables contributing to creating the different contexts, and therefore it is difficult to identify which specific variable may account for differences between the contexts. In particular, there are several factors that, in the present study, were presumed to make the playful activities more engaging and motivating for children, but also may have served as distractors and caused both parents and children to veer from the math-related goal. Future studies could focus on isolating these variables to determine which ones are most important in creating an effective, engaging math activity for parents and children and to pinpoint if any factors seem to especially impede parent and child talk about math during joint activities.

Printed material differences. Earlier it was noted that the questions in the didactic context contained formal fraction words, while the playful math context did not, which likely accounted for the difference in parent and child use of formal fraction words. However, the questions were also presented in distinct formats: in the didactic context, the questions were delivered on one page with black and white text, while in the playful math context, the questions were in a multi-page storybook that included illustrations. Beyond being more visually appealing, the storybook could also enable the children to be more engaged because they could use the illustrations to help them comprehend the questions. For example, seeing the addition of a third character in the picture would help children anticipate that they now need to find foods that three friends can share. On the other hand, the illustrations may have led to more child talk that was not directly related to the math goal (e.g., "Wait, which one is Molly?" while trying to identify the characters). Future studies in which the printed materials provided are manipulated to more varying degrees could clarify how this impacts children's engagement.

Objects used for learning. The playful and didactic contexts in the present study varied in whether the dyads were able to take apart the food items into pieces or simply view division lines on food items that were glued together. The ability for children to explore and manipulate objects in a way that draws attention to concepts relevant to the learning goals is a central component to the notion of guided play. Fisher et al. (2013) found that children learned less about shapes when observing someone else sorting them, rather than when they were able to actively sort the shapes themselves. A distinction between this previous study and the current study, however, is that the didactic instruction context in the current study still allowed the children to handle the objects and explore them, even though they could not manipulate them. Therefore they were still somewhat actively involved. It is unclear how the parent and child engagement would have been different if parents were instructed to only show the objects to their children. From a practical standpoint, it should be noted that the external validity and value of such a study would be quite limited, as it seems unlikely that parents would typically engage their children in activities where children were not permitted to handle the materials.

It would, however, be valuable to examine differences in parents' approaches to teaching depending on whether the objects were able to be manipulated, and how this influences children's learning. In sociocultural theory, cultural tools support cognition, and social processes with partners typically provide children with the initial information for how to use tools within a particular context. Thus, children's observations of how parents used the food items may contribute to children's future use of these items as a tool in problem solving. Many parents in the didactic context resorted to gestures to try to demonstrate dividing the objects into different numbers of pieces, as did children. A common exchange (in both math contexts) while searching for items that could be divided in half involved a child selecting an item divided into three or five pieces, the parent asking, “How would you divide that in half?”, followed by the child gesturing slicing down the center of the object, even though there was not a pre-existing line there. Sophian and colleagues (1997) noted that it was the opportunity to observe multiple outcomes of dividing something into different numbers of shares that promoted children’s understanding. While the number of outcomes was limited in the playful context since the items were precut, children were still more able to observe the outcomes of dividing compared to in the didactic context where foods were glued together. Further research in which only the ability to manipulate materials is varied while controlling the other contextual factors could provide insight into how the materials available might influence parents’ exploration and children’s learning.

A separate issue related to the materials provided is that even though the didactic context was intended to remove all “everyday” elements from the activity, the objects used were still pretend food. This possibly made it more likely that parents would find a way to reframe the questions to discuss “sharing food” rather than “dividing objects”, as the worksheet questions were worded. It is unclear whether this everyday reframing and real-life application made the activity more engaging and easy to comprehend for children. A future study could further decontextualize the didactic activity by using 3D shapes rather than food items; it would even be possible to use the same food items, but paint them different, solid colors so that they were not recognizable as food.

Presence of extraneous materials. As noted earlier, the inclusion of extra materials in the playful contexts (e.g., plates, cutting board, knife) could have divided both children and parents’ attention or de-emphasize the math goal of the activity by creating a competing goal. While these items were included in the playful math context so that it more closely resembled the unguided play context, future studies should consider the extent to which these materials may promote or detract from children's thinking about mathematical elements of the activity. It has been proposed that cutting the foods into pieces may have helped children explored the outcomes of dividing objects; however, there is no reason to argue that cutting the food with the wooden knife rather than separating them by hand, would add to children's learning. Thus, the use of the knife and corresponding talk about it, such as parents telling children how to use the knife may have interfered with children attending to the math goal.

In contrast, the plates seemed at times to enhance children's understanding of the fraction concepts. There were often times in the playful math context that the plates were used to divide food items between a given number of individuals. This was especially useful for facilitating parent and child discussion about the exploration questions regarding dividing objects cut in fourths and odd numbers of pieces evenly between two people. Even in the unguided play context, there was often talk between parents and children comparing quantities of food on different plates. Therefore, there could be some extra materials that provide support for exploration rather than interference.

Designing studies to observe fraction learning. One proposed explanation for the lack of growth in children's fraction understanding in the current study was that the manipulations could only attempt to influence how parents interacted with their children rather than providing them with a script, as would be used in studies where experimenters followed a protocol for while teaching children. Still, the evidence of the home numeracy environment and especially parent math talk as important predictors in young children's math knowledge (Anders et al., 2012; Blevins-Knabe & Musun-Miller, 1996;

LeFevre et al., 2009; Levine et al., 2010; Manolitsis et al., 2013; Niklas & Schneider, 2013; Skwarchuk, 2009) suggests that finding ways to promote high quality parent-child math exchanges is a valuable avenue to pursue in research. Future studies should be designed with the goal of maximizing the chance of observing improvement in children's fraction understanding.

One aspect to be considered if the extent to which parents are given clear guidance for how to talk with their children during the activities. The current study had built-in fraction questions to ensure that dyads in the math structured contexts talk about fraction concepts. Changing or adding more structure to the activity could take it a step further to increase the likelihood of parents and children talking in a way that promotes children's understanding. Some parents were observed asking their children to explain their responses (e.g., asking why the children thought that the characters would get more watermelon if they split it between two friends rather than four friends), but not all parents did this. Adding follow up questions such as "Why?" or "How do you know?" could boost the in-depth discussions that children and parents have during the activity.

Another aspect of the research design to consider is the time frame in which the dyads participate in the activity. As noted, parent math talk appears to have a cumulative effect on children's math knowledge (Gunderson & Levine, 2011; Levine et al., 2010), and parent interventions stretched across several time points rather than one short interaction may be more likely to yield results. Unfortunately, the one study that has examined parent-child math activities over an extended period of time, where children and parents played a board game 3 times over the course of 2 weeks, did not include a posttest assessment of children's math knowledge (Vandermaas-Peeler, Ferretti et al., 2012). Therefore it is not clear whether such a time frame would be sufficient for observing growth in children's fraction understanding. Evidence from classroom-based interventions with preschoolers, however, may provide some insight the issue. For example, Ramani, Siegler, and Hitti (2012) observed improvement in preschoolers' numeracy after playing a number board game with paraprofessionals during 5 sessions over 3 to 4 weeks. A similar time frame and number of sessions may be reasonable for expecting to observe growth in children's fraction understanding after parent-child activities. Practically, it should be noted, recurring sessions with parents would likely be more feasible if dyads were given the materials to take and use at home. This would also provide additional information about how such activities would work as a part of families' regular routines.

Study Limitations

When interpreting the findings from this study, there are several points worth noting. First, it is important to recognize that the parents in the sample were highly educated, with a large majority having obtained at least a 4-year degree college degree and almost half of the parents having a graduate degree. Socioeconomic and cultural variability beyond the current sample may reveal an even greater range of parent and child math talk. There also may be cultural differences in how parents and children respond to the didactic and playful math contexts, based on what types of experiences are most common in their everyday lives (Goncu & Gauvain, 2012).

Additionally, there are a number of factors beyond the activity context that might have influenced parent and child exchanges during the interactions, and account for the variability of math talk within each context. For example, a parent who was more concerned with ensuring that their child understood the fraction concepts might engage their child in more math talk compared to a parent who is primarily concerned with making sure that they successfully finish the task (Moorman & Pomerantz, 2010). Alternately, variables related to children's cognition or temperament might influence their behavior during the activity and how much math talk occurs. Notably, the dynamic nature of the joint activity means that there could also be interaction effects between parent and child variables. Thus, it is clear that the present study did not take into account all the potential factors that may contribute to the quantity and quality of parent and child math talk. A related point is that the current study only examines a brief interaction between parents and children who have a shared history. For example, the frequency in which they participate in math-related activities at home could have influenced how comfortable they were with doing so during the research study.

Finally, it should be noted that the author was also the primary experimenter who both administered the math and fraction assessments and provided instructions to dyads prior to the joint activity. Consequently, she was not blind to the study hypotheses, nor was she blind to children's performance on the pretest while providing joint activity instructions or administering the posttest. While both the joint activity instructions and assessments were highly scripted and structured, it is possible that expectations and prior knowledge of the hypotheses would lead to differences in the experimenter's affect during either part of the study. Scheduling conflicts and limited resources for this present study made it difficult to have additional individuals (blind to study hypotheses and/or conditions) assist with data collection; however, ideally there would have been separate individuals administering the fraction tests and providing joint activity instructions.

Implications and Application

Since previous work has found that parents' talk about more advanced concepts is especially predictive of children's math knowledge, and children's knowledge in specific areas is related to parents' talk about the same math concepts (Gunderson & Levine, 2011; Pruden et al., 2011), there is value in identifying the best ways to prompt parents to use more complex, diverse talk about math concepts. In the current study, both structured math activities yielded more frequent and diverse talk about math compared to the unguided play context. It is also notable that the didactic context, in which formal fraction words were introduced on the worksheet prompted parents and children to use the terms "third" and "fourth" in addition to "half". While some children in the unguided play and playful math contexts did use the word "half", it was not always correctly used (e.g., "I'm going to cut the banana in half into three pieces."). In contrast, parents and children in the didactic context often discussed the precise definitions of half, third, and fourth, drawing children's attention to dividing objects into different numbers of pieces.

As noted when discussing future directions for research, there are a number of factors that need to be further explored to determine what particular features in different contexts contribute to the quantity and quality of parent and child math talk. However, it is critical to keep in mind that parent-child interactions take place outside the lab in much "messier" contexts. Thus, rather than identifying the optimum contexts for the best math exchanges to occur, the goal of systematically examining different contextual variables should be to identify what matters the most. For example, even if a didactic context yields more parent math and child math talk, it may become apparent that the math talk only contributes to children's learning if it is accompanied by physical representations of concepts that the children are able to explore.

It is interesting that parents viewed the playful math context as being just as challenging and math-relevant as the didactic context, but also viewed it as being as enjoyable as the unguided play context. Particularly for potential future studies and interventions where parents would be asked to engage their children in math activities at home over the course of several weeks, this is important to take into consideration. The extent to which parents and children enjoy the activity may impact the likelihood of them actually following through with the activity, as well as how much children engage in the activity. It is also interesting that although parents rated the playful math context as being as likely to promote math knowledge as the didactic context, parents in the playful math context did not engage in as much math talk. This highlights the apparent need to provide parents with specific guidance for engaging their children in complex math talk. Books and printed material may be especially useful since they create a demand for parents to take an active role and offer starting points for using math talk.

Another benefit to providing some structure and guidance for how parents and children can engage in math activities is that it can create opportunities for children to explore mathematical concepts to a further extent than they would during incidental exposures to math in every day contexts. For example, Sophian et al. (1997) pointed out that while children might experience sharing and observe the quantitative effects of sharing on a regular basis, they would not necessarily explore outcomes of different ways of sharing and develop a deeper understanding of quantitative relations. Since exploratory experiences are less likely to occur spontaneously, encouraging parents to take the time to engage in such activities may be an important element to promoting children's early mathematical understanding.

Conclusion

This study highlights the value of drawing parents' attention to mathematical elements of the activities they engage in with preschoolers. The current study did not provide evidence that engaging in more math talk during play promotes short-term learning for young children. However, when paired with previous studies showing a link between parents' math talk and children's math knowledge (Levine et al., 2010; Ramani et al., 2015), the finding that structured activities elicits more parent and child math talk offers insight into how children's math learning can be increased in the early home environment. In particular, activities where math is the central goal and parents and children can focus on the math elements of the task are likely to produce the most frequent and diverse math talk.

What remains an unanswered question is whether encouraging parents to engage in more didactic, direct teaching activities with their preschoolers is the best option from a practical perspective. Since parents rated their children as enjoying the playful math activity as much as unguided play and more than the didactic activity, it is possible that parents and children would be more receptive to incorporating some elements of guided play (more structure and math content) into their everyday routine rather than devoting time to directly teaching math. If the goal is to encourage long-term, ongoing engagement in the home math environment, culturally-relevant, playful or everyday activities that produce less math talk in one sitting but occur more regularly, might be just as, or more, effective than direct teaching.

Table 1

Summary of Child and Parent Characteristics by Condition

	Didactic Instruction *n* = 24	Guided Play *n* = 25	Unguided Play *n* = 23
Child age (*M, SD*)	57.33 (6.06)	57.20 (5.12)	58.43 (4.33)
Child gender			
Girls	14 (58.3%)	15 (60.0%)	13 (56.5%)
Boys	10 (41.7%)	10 (40.0%)	10 (43.4%)
TEMA score[a]	107.11 (14.04)	110.53 (12.71)	108.35 (19.52)
Parent gender			
Mothers	20 (83.3%)	23 (92.0%)	17 (73.9%)
Fathers	4 (16.7%)	2 (8.0%)	6 (26.1%)
Parent-child dyads			
Mother-boy	8 (33.3%)	10 (40.0%)	6 (26.1%)
Mother-girl	12 (50.0%)	13 (52.0%)	11 (47.8%)
Father-boy	2 (8.3%)	0	4 (17.4%)
Father-girl	2 (8.3%)	2 (8.0%)	2 (8.7%)
Parent education			
Graduate degree	11 (45.8%)	11 (44.0%)	12 (52.2%)
4-year degree	8 (33.3%)	12 (48.0%)	8 (34.8%)
Some college	2 (8.3%)	2 (8.0%)	2 (8.7%)
High school	1 (4.2%)	0	0
No response	2 (8.3%)	0	1 (4.3%)

Note. TEMA = Test of Early Mathematics Achievement, Standard Score.
[a]Mean TEMA standard scores are based on data from 52 children (18 in Didactic Instruction, and 17 each in Guided Play and Unguided Play).

Table 2

Summary of Means, Standard Deviations, and Ranges and Condition Comparisons for Parent and Child Overall Talk

	Didactic Instruction *n* = 24		Guided Play *n* = 25		Unguided Play *n* = 23		
Overall talk	*M (SD)*	Min - Max	*M (SD)*	Min – Max	*M (SD)*	Min - Max	Comparisons
Turns							
Par. w/ text	128.54 (35.93)	72 – 204	125.92 (32.09)	74 – 176	143.96 (28.26)	86 – 194	
Par. no text	127.67 (35.64)	71 – 203	122.56 (31.70)	73 – 174	143.96 (28.26)	86 – 194	UP > GP
Child	128.38 (35.84)	71 – 203	125.24 (32.01)	73 – 175	143.70 (28.59)	86 – 194	
MLT							
Par. w/ text	11.43 (3.96)	7.11 – 19.96	11.91 (3.09)	7.10 – 20.65	7.39 (1.60)	4.99 – 10.08	DI, GP > UP
Par. no text	10.89 (3.79)	6.06 – 19.27	9.79 (2.93)	5.61 – 18.50	7.39 (1.60)	4.99 – 10.08	DI, GP > UP
Child	3.05 (1.34)	1.25 – 5.91	3.28 (1.46)	1.60 – 6.83	4.00 (1.44)	1.76 – 7.35	UP > DI
Types							
Par. w/ text	258.96 (47.46)	158 – 383	281.16 (34.15)	219 – 381	264.30 (36.49)	200 – 326	
Par. no text	254.13 (48.92)	144 - 382	250.52 (41.20)	168 - 368	264.30 (36.49)	200 – 326	
Child	119.50 (41.73)	39 – 190	139.24 (47.73)	72 – 257	175.65 (40.82)	98 – 266	UP > DI, GP
Tokens							
Par. w/ text	1379.75 (357.64)	939 – 2136	1451.16 (368.99)	844 – 2379	1046.13 (237.34)	543 – 1550	DI, GP > UP
Par. no text	1311.37 (372.87)	775 – 2127	1173.76 (384.85)	535 – 2132	1046.13 (237.34)	543 – 1550	DI > UP
Child	389.29 (193.01)	122 – 804	415.28 (226.36)	125 – 1154	565.70 (192.85)	213 – 951	UP > DI, GP
Chi-Par MLT ratio							
With text	0.31 (0.19)	0.07 – 0.72	0.32 (0.21)	0.09 – 0.96	0.58 (0.28)	0.22 – 1.35	UP > DI, GP
No text	0.33 (0.21)	0.07 – 0.75	0.40 (0.28)	0.10 – 1.22	0.58 (0.28)	0.22 – 1.35	UP > DI, GP
Duration (seconds)	860.90 (147.06)	541 – 1137	976.52 (136.45)	825 – 1428	939.83 (110.16)	808 – 1401	GP > DI

Note. MLT = Mean Length of Turn (Words per turn). W/ text = includes all parent utterances. No text = excludes parent utterances marked as reading text. UP = Unguided Play. GP = Guided Play. DI = Didactic Instruction.

Table 3

List of Formal Fraction, Informal Fraction, and Quantitative Words

Formal fraction words	Informal fraction words	Quantitative words
Half (Halves)	[One … Five] part(s)	Part(s)
Third(s)	[One … Five] piece(s)	Piece(s)
Fourth(s)	[One … Five] portion(s)	Whole
Quarter(s)		"How many"
Fifth(s)		More
Sixth(s) [a]		Less
Seventh(s)		Fewer
Eighth(s)		Divide
Ninth(s)		Equal
Tenth(s)		Same
		Portions

[a]It was unexpected that terms for sixths or greater would be used since the food items had no more than five pieces.

Table 4

Intercorrelations for Parent Talk Variables

	Overall				Math words									
					Formal fraction		Informal fraction		Quantitative		Number		Total math	
	Turns	MLT	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens
Overall														
Turns	--													
MLT	-.58***	--												
Types	.33**	.27*	--											
Tokens	.26*	.60***	.65***	--										
Form. frac.														
Types	-.18	.38***	-.07	.27*	--									
Tokens	-.06	.30*	-.09	.30**	.77***	--								
Inf. frac.														
Types	-.23*	.55***	.07	.44***	.60***	.53***	--							
Tokens	-.22	.61***	.00	.52***	.62***	.67***	.77**	--						
Quant.														
Types	-.17	.48***	.09	.41***	.60***	.53***	.63***	.64***	--					
Tokens	-.12	.62***	.04	.64***	.58***	.65***	.69***	.88***	.65***	--				
Number														
Types	-.20	.37**	.03	.24*	.62***	.49***	.65***	.47***	.62***	.49***	--			
Tokens	-.13	.59***	.13	.60***	.61***	.59***	.74***	.78***	.62***	.72***	.70***	--		
Total math														
Types	-.23	.53***	.04	.41***	.82***	.68***	.86***	.75***	.86***	.72***	.84***	.79***	--	
Tokens	-.13	.60***	.04	.60***	.72***	.81***	.76***	.92***	.69***	.94***	.61***	.87***	.82***	--

Note. MLT = Mean Length of Turn (Words per turn).
**p* < .05. ***p* < .01. ****p* < .001.

Table 5

Intercorrelations for Child Talk Variables

	Overall talk				Math words									
					Formal fraction		Informal fraction		Quantitative		Number		Total math	
	Turns	MLT	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens
Overall														
Turns	--													
MLT	.03	--												
Types	.45***	.80***	--											
Tokens	.48***	.87***	.90***	--										
Form. frac.														
Types	-.01	-.03	-.10	-.06	--									
Tokens	.10	.12	-.05	.13	.66***	--								
Inf. frac.														
Types	.01	.14	.08	.12	.09	.05	--							
Tokens	.01	.26*	.13	.23	.06	.07	.83***	--						
Quant.														
Types	.10	.56***	.28*	.30*	.09	.25*	.28*	.18	--					
Tokens	-.05	-.37**	.38***	.53***	.09	.31**	.61***	.69***	.59***	--				
Number														
Types	-.12	-.09	-.23	-.17	.18	.22	.16	.19	.05	.11	--			
Tokens	.05	-.17	-.36**	-.16	.41***	.38***	.11	.16	-.03	.04	.58***	--		
Total math														
Types	-.06	.08	-.08	-.01	.47***	.44***	.54***	.46***	.45***	.47***	.81***	.54***	--	
Tokens	.08	.01	-.23	.02	.48***	.56***	.31**	.38***	.15	.35**	.94***	.94***	.67***	--

Note. MLT = Mean Length of Turn (Words per turn).
**p* < .05. ***p* < .01. ****p* < .001.

Table 6

Correlations between Parent and Child Talk Variables

	Child overall talk				Child math words									
					Formal fraction		Informal fraction		Quantitative		Number		Total math	
Parent talk	Turns	MLT	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens	Types	Tokens
Overall														
Turns	**1.00*****	.021	.45***	.47***	-.01	.10	.01	.00	.10	-.04	-.12	.05	-.06	.08
MLT	-.58***	**-.51*****	-.69***	-.69***	.13	-.06	.11	.02	-.21	.15	.06	.15	.09	.07
Types	.32**	-.25*	**.08**	-.08	-.05	-.11	.06	.02	-.03	-.08	-.33**	-.18	-.23	-.19
Tokens	.26*	-.55***	-.38***	**-.38*****	.14	.07	.12	.01	-.03	-.17	.00	.28*	.07	.21
Form. frac.														
Types	-.18	-.14	-.36**	-.20	**.62*****	.47***	.14	.15	.13	.11	.41***	.65***	.55***	.67***
Tokens	-.06	-.29*	-.43***	-.28*	.69***	**.58*****	-.03	-.04	.06	-.05	.27*	.68***	.40***	.66***
Inf. frac.														
Types	-.23*	-.28*	-.47***	-.34**	.36**	.28*	**.26***	.22	.09	.15	.38***	.52***	.48***	.54***
Tokens	-.21	-.41***	-.60***	-.46***	.35**	.23	.10	**.08**	.01	-.08	.35**	.61***	.38***	.55***
Quant.														
Types	-.18	-.46***	-.62***	-.49***	.29*	.18	.12	.07	**-.08**	-.05	.36**	.55***	.37**	.50***
Tokens	-.12	-.48***	-.61***	-.49***	.34**	.25*	.00	-.05	.04	**-.16**	.36**	.63***	.35**	.54***
Number														
Types	-.20	-.27*	-.42***	-.35*	.50***	.28*	.17	.17	.05	.03	**.45*****	.59***	.53***	.57***
Tokens	-.13	-.48***	-.59***	-.47***	.38***	.21	.12	.09	-.03	-.12	.31**	**.67*****	.35**	.59***
Tot. math														
Types	-.23*	-.35**	-.57***	-.42***	.51***	.35**	.20	.18	.07	.06	.47***	.68***	**.56*****	.66***
Tokens	-.13	-.48***	-.63***	-.49***	.49***	.35**	.04	.01	.03	-.13	.36**	.73***	.42***	**.66*****

Note. MLT = Mean Length of Turn (Words per turn).
**p* < .05. ***p* < .01. ****p* < .001.

Table 7

Summary of Means, Standard Deviations, and Ranges for Parent and Child Math Word Tokens by Condition

	Didactic Instruction *n* = 24		Guided Play *n* = 25		Unguided Play *n* = 23	
Tokens	*M (SD)*	Min - Max	*M (SD)*	Min – Max	*M (SD)*	Min - Max
Total math						
Parent w/ text	172.58 (68.84)	89 – 349	97.12 (42.96)	42 – 182	26.39 (20.63)	2 – 71
Parent no text	159.58 (69.92)	67 – 328	82.16 (43.53)	27 – 165	26.39 (20.63)	2 – 71
Child	69.17 (31.85)	12 – 139	35.60 (16.78)	6 – 71	16.00 (12.04)	2 – 50
Formal fraction						
Parent w/ text	36.88 (22.09)	9 – 109	5.32 (5.43)	0 – 25	3.65 (5.64)	0 – 22
Parent no text	32.42 (23.27)	5 – 108	5.32 (5.43)	0 – 25	3.65 (5.64)	0 – 22
Child	5.79 (6.27)	0 – 21	1.20 (1.66)	0 – 6	2.43 (5.21)	0 – 24
Informal fraction						
Parent w/ text	20.33 (9.23)	9 – 39	10.72 (5.91)	3 – 26	1.96 (3.31)	0 – 11
Parent no text	18.67 (9.44)	1 – 36	8.88 (6.04)	1 – 24	1.96 (3.31)	0 – 11
Child	2.50 (3.41)	0 – 14	2.92 (2.84)	0 – 10	0.52 (0.79)	0 – 3
Quantitative						
Parent w/ text	68.83 (31.77)	30 – 157	43.96 (27.38)	14 – 128	14.00 (8.64)	1 – 32
Parent no text	64.42 (32.31)	23 – 150	40.84 (27.40)	11 – 124	14.00 (8.64)	1 – 32
Child	8.04 (7.14)	0 – 30	8.52 (6.67)	0 – 26	6.61 (5.07)	1 – 23
Number						
Parent w/ text	46.54 (21.18)	18 – 91	37.12 (14.39)	19 – 65	6.78 (6.08)	0 – 25
Parent no text	44.08 (20.31)	16 – 87	27.12 (14.85)	11 – 56	6.78 (6.08)	0 – 25
Child	52.83 (28.14)	7 – 111	22.96 (12.73)	2 – 52	6.43 (6.32)	0 – 28

Note. No text = excludes parent utterances marked as reading text.

Table 8

Summary of Means, Standard Deviations, and Ranges and Condition Comparisons for Parent and Child Proportions of Math Tokens

	Didactic Instruction *n* = 24		Guided Play *n* = 25		Unguided Play *n* = 23		
Proportion of tokens	*M (SD)*	Min - Max	*M (SD)*	Min – Max	*M (SD)*	Min - Max	Comparisons
Total math							
Parent w/ text	.126 (.039)	.056 – .193	.065 (.019)	.035 – .103	.014 (.009)	.001 – .040	DI > GP > UP
Parent no text	.123 (.042)	.050 – .190	.068 (.023)	.030 –.110	.014 (.009)	.001 – .040	DI > GP > UP
Child	.213 (.130)	.056 – .534	.098 (.048)	.025 – .201	.031 (.025)	.004 – .094	DI > GP > UP
Form. frac							
Parent w/ text	.027 (.012)	.007 – .051	.004 (.004)	.000 – .016	.003 (.005)	.000 – .018	DI > GP,UP
Parent no text	.024 (.014)	.004 – .051	.005 (.004)	.000 – .019	.003 (.005)	.000 – .018	DI > GP,UP
Child	.017 (.019)	.000 – .069	.003 (.005)	.000 – .019	.005 (.008)	.000 – .026	DI > GP,UP
Inf. frac							
Parent w/ text	.015 (.006)	.005 – .028	.007 (.003)	.003 – .013	.002 (.004)	.000 – .013	DI > GP > UP
Parent no text	.014 (.007)	.001 – .029	.007 (.004)	.001 – .014	.002 (.004)	.000 – .013	DI > GP > UP
Child	.006 (.009)	.000 – .037	.007 (.006)	.000 – .024	.001 (.002)	.000 – .005	DI, GP > UP
Quant.							
Parent w/ text	.050 (.018)	.018 – .097	.029 (.013)	.012 – .071	.025 (.021)	.002 – .088	DI > GP > UP
Parent no text	.049 (.020)	.017 – .102	.033 (.016)	.012 – .082	.025 (.021)	.002 – .088	DI > GP > UP
Child	.020 (.013)	.000 – .051	.020 (.013)	.000 – .050	.013 (.009)	.001 – .033	
Numbers							
Parent w/ text	.033 (.016)	.017 – .074	.026 (.007)	.017 – .044	.006 (.005)	.000 – .018	DI > GP > UP
Parent no text	.035 (.017)	.015 – .078	.023 (.009)	.013 – .045	.006 (.005)	.000 – .018	DI > GP > UP
Child	.170 (.128)	.033 – .523	.067 (.043)	.013 – .166	.013 (.012)	.000 – .048	DI > GP > UP

Note. Proportions = math tokens / all tokens. No text = excludes parent utterances marked as reading text. UP = Unguided Play. GP = Guided Play. DI = Didactic Instruction.

Table 9

Summary of Means, Standard Deviations, and Ranges and Condition Comparisons for Parent and Child Math Word Types

	Didactic Instruction *n* = 24		Guided Play *n* = 25		Unguided Play *n* = 23		
Types	*M (SD)*	Min - Max	*M (SD)*	Min – Max	*M (SD)*	Min - Max	Comparisons
Total math							
Parent w/ text	18.63 (2.20)	15 – 25	13.12 (2.54)	8 – 18	7.69 (3.31)	2 – 14	DI > GP > UP
Parent no text	18.13 (2.19)	14 – 23	12.84 (2.79)	8 – 18	7.69 (3.31)	2 – 14	DI > GP > UP
Child	11.00 (3.89)	6 – 20	7.92 (2.41)	3 – 11	5.83 (2.80)	2 – 15	DI > UP
Formal fraction							
Parent w/ text	3.33 (0.76)	1 – 4	1.28 (0.84)	0 – 4	0.74 (0.75)	0 – 3	DI > GP, UP
Parent no text	3.17 (1.01)	1 – 4	1.28 (0.84)	0 – 4	0.74 (0.75)	0 – 3	DI > GP, UP
Child	1.92 (1.35)	0 – 4	0.56 (0.51)	0 – 1	0.52 (0.59)	0 – 2	DI > GP, UP
Informal fraction							
Parent w/ text	3.96 (0.55)	3 – 5	3.28 (0.98)	2 – 5	0.91 (1.13)	0 – 3	DI > GP > UP
Parent no text	3.88 (0.80)	1 – 5	3.04 (1.24)	1 – 5	0.91 (1.13)	0 – 3	DI > GP > UP
Child	1.29 (1.27)	0 – 4	1.56 (1.26)	0 – 4	0.48 (0.73)	0 – 3	DI > GP > UP
Quantitative							
Parent w/ text	6.83 (1.05)	5 – 9	5.08 (1.47)	2 – 7	3.83 (1.50)	1 – 6	DI > GP > UP
Parent no text	6.58 (1.10)	4 – 8	5.08 (1.47)	2 – 7	3.83 (1.50)	1 – 6	DI > GP > UP
Child	2.38 (1.21)	0 – 5	2.24 (1.13)	0 – 4	2.52 (1.08)	1 – 4	
Number							
Parent w/ text	4.50 (1.25)	3 – 9	3.48 (0.51)	3 – 4	2.22 (1.17)	0 – 4	DI > GP > UP
Parent no text	4.50 (1.25)	3 – 9	3.44 (0.51)	3 – 4	2.22 (1.17)	0 – 4	DI > GP > UP
Child	5.42 (3.37)	3 – 19	3.56 (0.87)	1 – 5	2.30 (1.85)	0 – 9	DI > GP, UP

Note. No text = excludes parent utterances marked as reading text. UP = Unguided Play. GP = Guided Play. DI = Didactic Instruction.

Table 10

Summary of Means and Standard Deviations for Fraction Pretest and Posttest by Condition

	Fraction subtests						
	Part 1			Part 2			Part 3
Condition	More	Less	Total	Halves	Quarters	Total	Total
Didactic Instruction (*n* = 24)							
Pretest	3.83 (2.22)	4.08 (2.04)	7.92 (3.35)	8.88 (5.67)	5.21 (5.23)	14.08 (8.38)	5.46 (1.32)
Posttest	4.38 (2.02)	4.50 (1.98)	8.88 (3.41)	8.88 (5.59)	5.42 (5.47)	14.29 (8.96)	5.88 (0.99)
Change	0.54 (1.59)	0.42 (1.89)	0.96 (2.24)	0.00 (5.17)	0.21 (2.06)	0.21 (5.52)	0.42 (1.41)
Guided Play (*n* = 25)							
Pretest	3.00 (2.10)	3.60 (2.24)	6.60 (2.92)	9.28 (6.07)	5.12 (4.90)	14.40 (9.89)	5.40 (1.08)
Posttest	3.32 (2.25)	3.92 (2.22)	7.48 (3.31)	9.00 (5.97)	5.00 (5.01)	14.00 (9.57)	5.80 (1.32)
Change	0.32 (1.70)	0.32 (1.49)	0.88 (2.57)	-0.28 (3.88)	-0.12 (2.77)	-0.40 (5.15)	0.40 (1.55)
Unguided Play (*n* = 23)							
Pretest	4.17 (2.17)	4.48 (1.90)	8.65 (3.35)	7.74 (6.52)	3.65 (4.67)	11.39 (9.95)	5.96 (1.33)
Posttest	4.70 (1.82)	4.36 (2.19)	8.97 (3.89)	7.76 (6.51)	4.30 (5.23)	12.06 (10.51)	6.00 (1.48)
Change	0.53 (1.81)	-0.12 (2.19)	0.32 (2.66)	0.02 (5.57)	0.65 (2.09)	0.67 (5.51)	0.04 (1.78)

Note. Part 1 = Understanding Relative Size of Shared Portions, possible scores range from 0-6 each for More and Less, 0-12 Total. Part 2 = Partitioning Objects into Equal Shares, possible scores range from 0-15 each for Halves and Quarters, 0-30 Total. Part 3 = Proportional Reasoning of Analogy Problems, possible scores range from 0-8.

Table 11

Hierarchical Multiple Linear Regression Analyses Predicting Children's Fraction Talk

		Child fraction talk			
		Tokens		Types	
Step	Predictor	Δ R^2	β	Δ R^2	β
1. Dummy variables	Guided play	.149**	-.143	.352***	-.065
	Didactic instruction		-.223		.213
2. Parent fraction talk	Tokens	.205***	.692***		
	Types			.060*	.430*
Total R^2		.354***		.412***	

*p < .05. **p < .01. ***p < .001.

Table 12

Summary of Means, Standard Deviation, and Ranges and Condition Comparisons for Post-Activity Parent Survey

	Didactic Instruction *n* = 24		Guided Play *n* = 25		Unguided Play *n* = 23		
Survey item	*M (SD)*	Range	*M (SD)*	Range	*M (SD)*	Range	Comparisons
Child thought activity was fun	5.38 (1.10)	3 – 7	6.04 (0.90)	4 – 7	6.35 (0.71)	5 – 7	GP, UP > DI
Parent enjoyed activity	6.04 (0.86)	5 – 7	6.48 (0.51)	6 – 7	6.61 (0.58)	5 – 7	UP > DI
Activity was challenging for child	6.13 (0.74)	4 – 7	6.20 (0.82)	4 – 7	4.57 (1.34)	2 – 7	DI, GP > UP
Parent not sure how to interact	2.83 (1.49)	1 – 7	2.44 (1.19)	1 – 5	2.04 (1.19)	1 – 5	
Activity promotes…							
Language and reading	5.08 (1.67)	2 – 7	5.48 (1.50)	2 – 7	5.43 (1.16)	4 – 7	
Math	6.71 (0.62)	5 – 7	6.60 (0.71)	4 – 7	5.96 (0.98)	4 – 7	DI, GP > UP
Creativity	5.58 (1.41)	3 – 7	6.32 (0.69)	5 – 7	6.48 (0.59)	5 – 7	GP, UP > DI

Note. UP = Unguided Play. GP = Guided Play. DI = Didactic Instruction.

Appendix A: Background Information Questionnaire

1. When is your child's birthday? (mm/dd/yyyy) _____________

2. What is your child's gender?
   (a) Female
   (b) Male

3. Of the following list of racial and ethnic categories, which do you consider **yourself** to be? (circle all that apply.)

(a) Asian or Pacific Islander	(d) American Indian/ Alaskan tribe
(b) Black	(e) Hispanic
(c) White	(f) Other ______________________

4. Of the following list of racial and ethnic categories, which do you consider **your child** to be? (circle all that apply.)

(a) Asian or Pacific Islander	(d) American Indian/ Alaskan tribe
(b) Black	(e) Hispanic
(c) White	(f) Other ______________________

5. What is your occupation? ____________________________

6. What is your partner's occupation? ____________________________

7. What other family members live with you?

________________________________ __________________________________

________________________________ __________________________________

________________________________ __________________________________

8. What is your highest educational degree?

(a) some high school	(d) college degree
(b) complete high school (or GED)	(e) graduate school or professional degree
(c) some college	

9. What is your partner's highest educational degree?

(a) some high school	(d) college degree
(b) complete high school (or GED)	(e) graduate school or professional degree
(c) some college	

10) What language is primarily spoken at home?
   (a) English
   (b) other: _______________

Appendix B: Speech and Language Assessment Scale

On a scale of 1-7 with 1 being "Very Low" and 7 being "Very High, please rate your child's language and social skills compared to other children his or her own age.

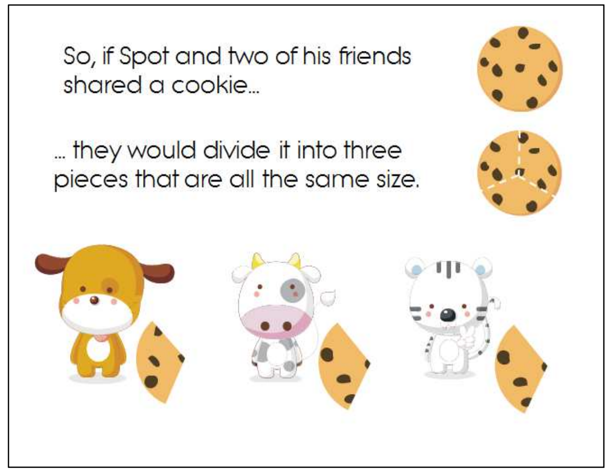
	Very Low			Normal for Age			Very High
1. My child's ability to **ask** questions properly is:	1	2	3	4	5	6	7
2. My child's ability to **answer** questions properly is:	1	2	3	4	5	6	7
3. My child's ability to understand what others say to him/her is:	1	2	3	4	5	6	7
4. My child's ability to say sentences clearly enough to be understood by strangers is:	1	2	3	4	5	6	7
5. The number of words my child knows is:	1	2	3	4	5	6	7
6. My child's ability to use his/her words correctly is:	1	2	3	4	5	6	7
7. My child's ability to get his/her message across when talking to others is:	1	2	3	4	5	6	7
8. My child's ability to understand directions spoken to him/her is:	1	2	3	4	5	6	7
9. My child's ability to follow directions spoken to him/her is:	1	2	3	4	5	6	7
10. My child's ability to use the proper words when talking to others is:	1	2	3	4	5	6	7
11. My child's ability to get what he/she wants by talking is:	1	2	3	4	5	6	7
12. My child's ability to start a conversation, or start talking with other children is:	1	2	3	4	5	6	7
13. My child's ability to keep a conversation going with other children is:	1	2	3	4	5	6	7

	Very Low			Normal for Age			Very High
14. The length of my child's sentences is:	1	2	3	4	5	6	7
15. My child's ability to make "grown up" sentences is:	1	2	3	4	5	6	7
16. My child's ability to correctly say the sounds in individual words is:	1	2	3	4	5	6	7
17. My child's awareness of differences in the way people act, speak, dress, etc. is:	1	2	3	4	5	6	7

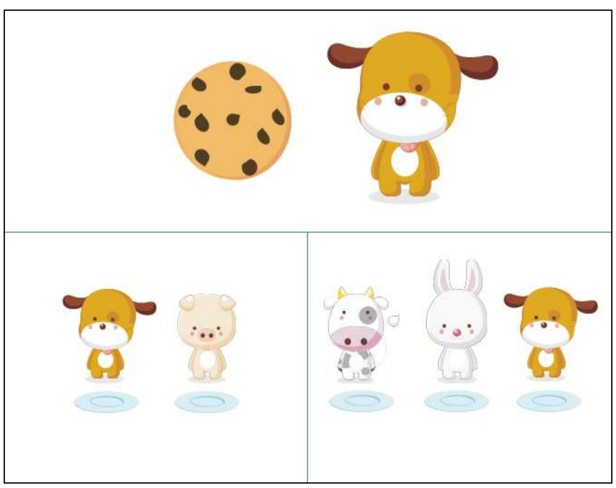
	Too Soft			About Loud Enough			Too Loud
18. My child usually speaks:	1	2	3	4	5	6	7

	Not Often Enough			About Often Enough			Too Often
19. My child usually speaks:	1	2	3	4	5	6	7

Appendix C: Fraction Concepts Assessment
Part 1 Introduction and Example Items



Copies of the two slides used to introduce Part 1 of the Fraction Concept Assessment.



Two items from Part 1 of the Fraction Concept Assessment.

Appendix D: Fraction Concepts Assessment
Part 1 Script

Version 1A[1]

Example Item:

"When Spot shares things with his friends, everyone gets the same amount.
So, if Spot and his two friends shared a cookie, they would divide it into three pieces that are all the same size. See? Each friend got the same amount of the cookie." (*point to the three characters and three pieces of cookie*)

"Spot loves cookies, so when he shares a cookie with friends, he wants as much of the cookie as he can get. Will he get more of the cookie if he shares the cookie with…

1. "1 friend, or with 2 friends?"
2. "What about if Spot shares the cookie with 3 friends or with 2 friends?"
3. "How about with 1 friend or with 3 friends?"

"Spot also loves sandwiches, so when he shares a sandwich with friends, he wants as much of the sandwich as he can get. Will Spot get more of the sandwich if he shares the sandwich with…

4. "2 friends or with 1 friend?"
5. "What about if Spot shares the sandwich with 2 friends or with 3 friends?"
6. "How about with 3 friends or with 1 friend?"

"Spot does <u>not</u> like pizza, so when he shares a pizza with friends, he wants as <u>little</u> of the pizza as possible. Will Spot get less of the pizza if he shares the pizza with…

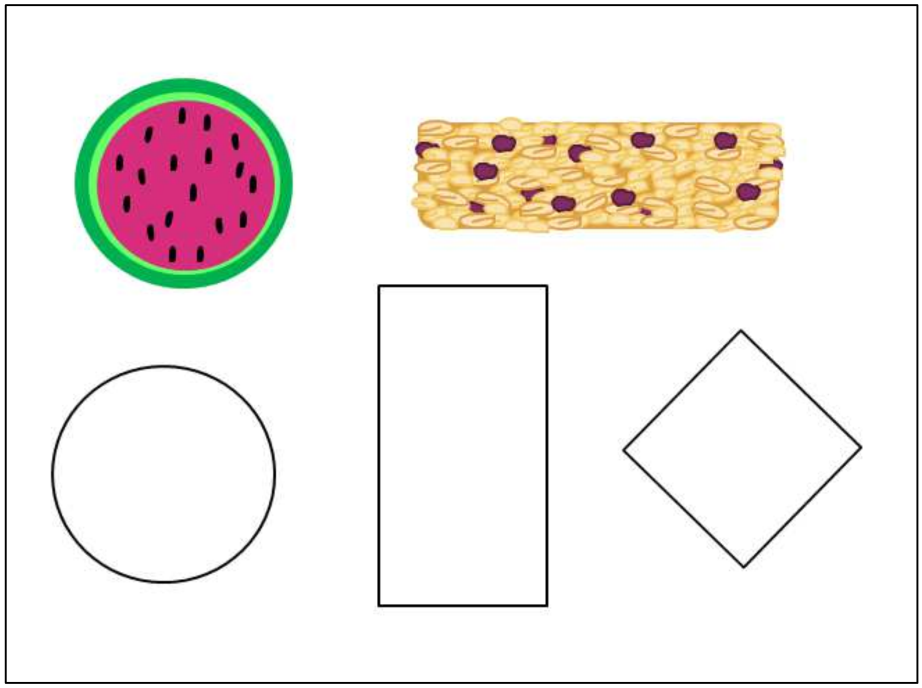
1. "3 friends, or with 2 friends?"
2. "What about if Spot shares the pizza with 1 friend or with 2 friends?"
3. "How about with 1 friend or with 3 friends?"

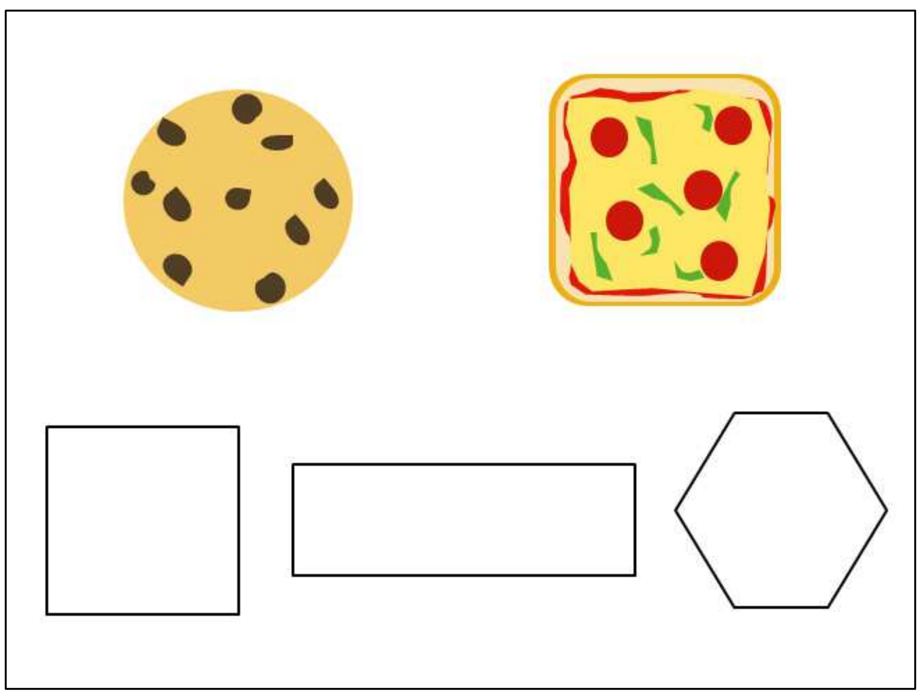
"Spot also does not like watermelon, so when he shares a watermelon with friends, he wants as little of the watermelon as possible. Will Spot get less of the watermelon if he shares the watermelon with…

4. "2 friends or with 3 friends?"
5. "What about if Spot shares the watermelon with 3 friends or with 1 friend?"
6. "How about with 2 friends or with 1 friend?"

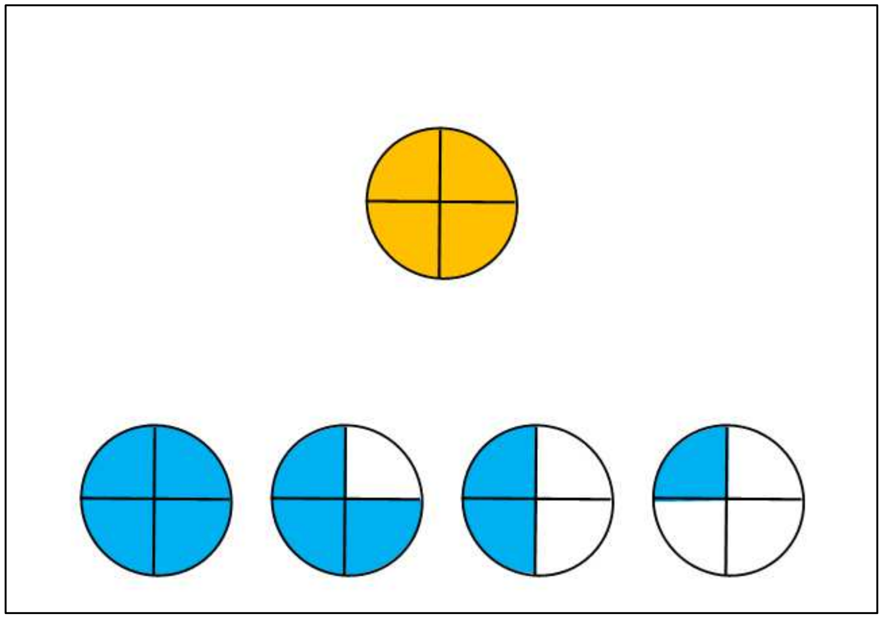
[1]Version 1B used the same items but presents the "less" items first. Versions 2A and 2B used a different character with the inverse preferences of Spot (i.e., loves pizza and watermelon, does not like cookies and sandwiches).

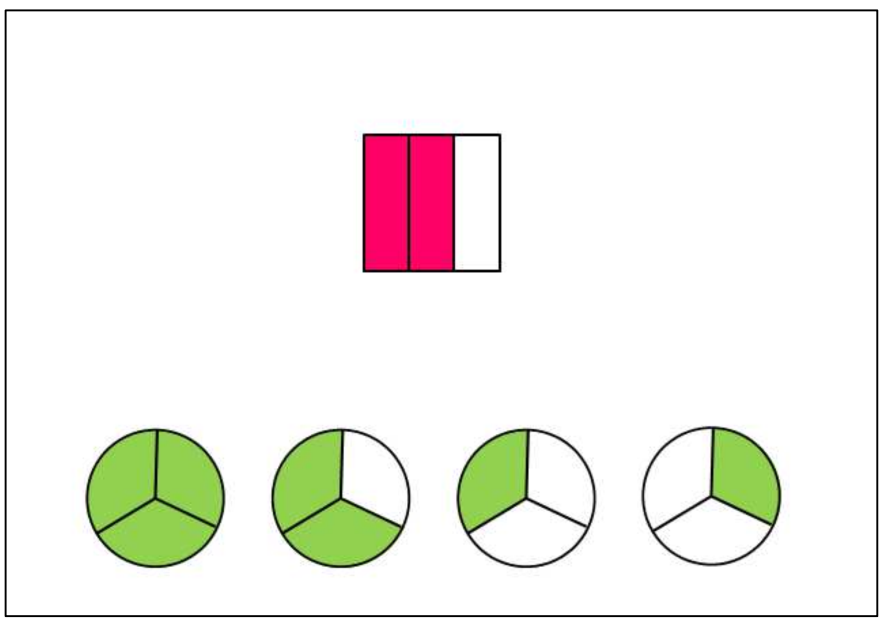
Appendix E: Fraction Concepts Assessment
Part 2 Example Worksheets





Appendix F: Fraction Concept Assessment
Part 3 Example Items





Appendix G: Post-Activity Questionnaire

Based on the activity you just completed with your child, please indicate to what extent you agree or disagree with each of the following statements:

	Strongly Disagree	Disagree	Somewhat Disagree	Not sure/ Neutral	Somewhat Agree	Agree	Strongly Agree
1. My child found the activity to be fun.	1	2	3	4	5	6	7
2. I enjoyed engaging in the activity with my child.	1	2	3	4	5	6	7
3. The activity was intellectually challenging for my child.	1	2	3	4	5	6	7
4. I was not sure how I should interact with my child during the activity.	1	2	3	4	5	6	7
5. I believe this type of activity can promote my child's language and reading abilities.	1	2	3	4	5	6	7
6. I believe this type of activity can promote my child's understanding of math concepts.	1	2	3	4	5	6	7
7. I believe this type of activity can promote my child's creativity.	1	2	3	4	5	6	7

8. Have you or your child encountered any of the materials used during the activity before today?

€ Yes, we have these items in our home.
€ Yes, we have used these items outside our home (e.g., at school, at a friend's house).
€ No, but we have used similar items before.
Please explain:
____
____
€ No, we have never used anything like this before.

9. If you have used these materials, or similar materials, before, please describe how you typically use them:

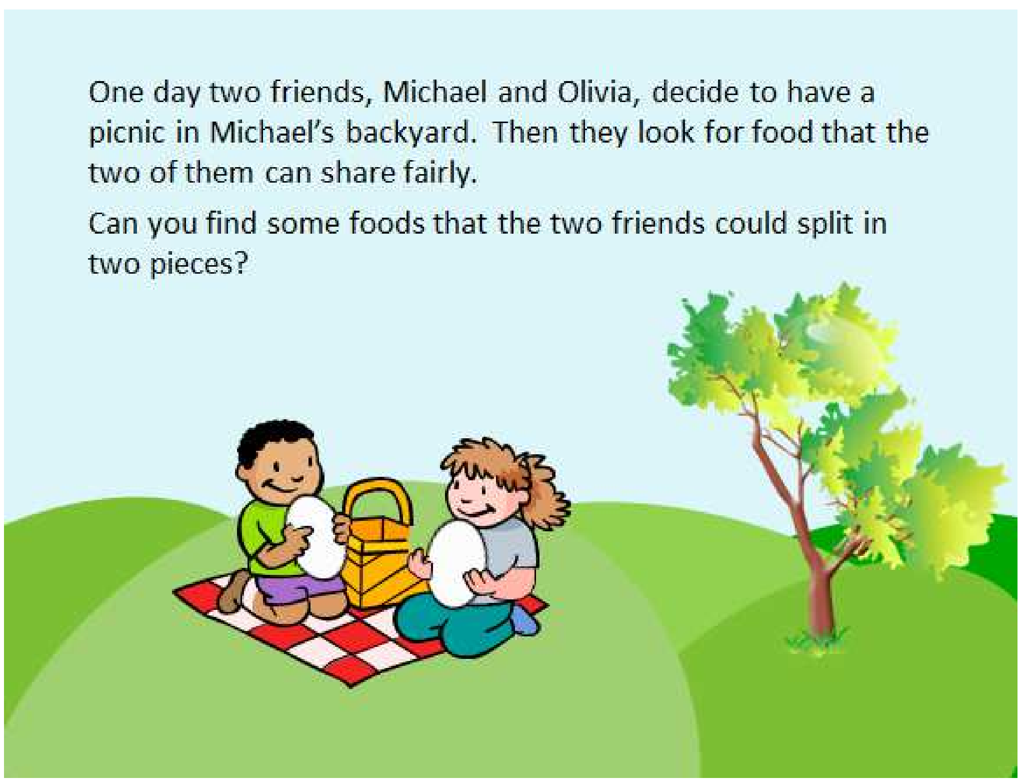
____
____

Appendix H: Toy Foods Included in Joint Activity

Equal halves	Equal thirds	Equal fourths	Extra items
Pear	Banana	Green pepper	Watermelon (4 uneven pieces)
Carrot	Tomato slice[a]	Melon	Bread loaf (5 uneven pieces)
Apple	Sandwich bread	Cucumber	

[a]Not pictured.

Appendix I: Pages from Book for Guided Play Condition



After Michael and Olivia set up their picnic in the backyard, their friend David joins them. Now they need food that the three of them can share.

Can you find some foods that the three friends can split so that they are shared fairly?

Just as Michael, Olivia, and David are getting ready to eat, their friend Molly comes over. *Now* they need to find food that the four of them can share.

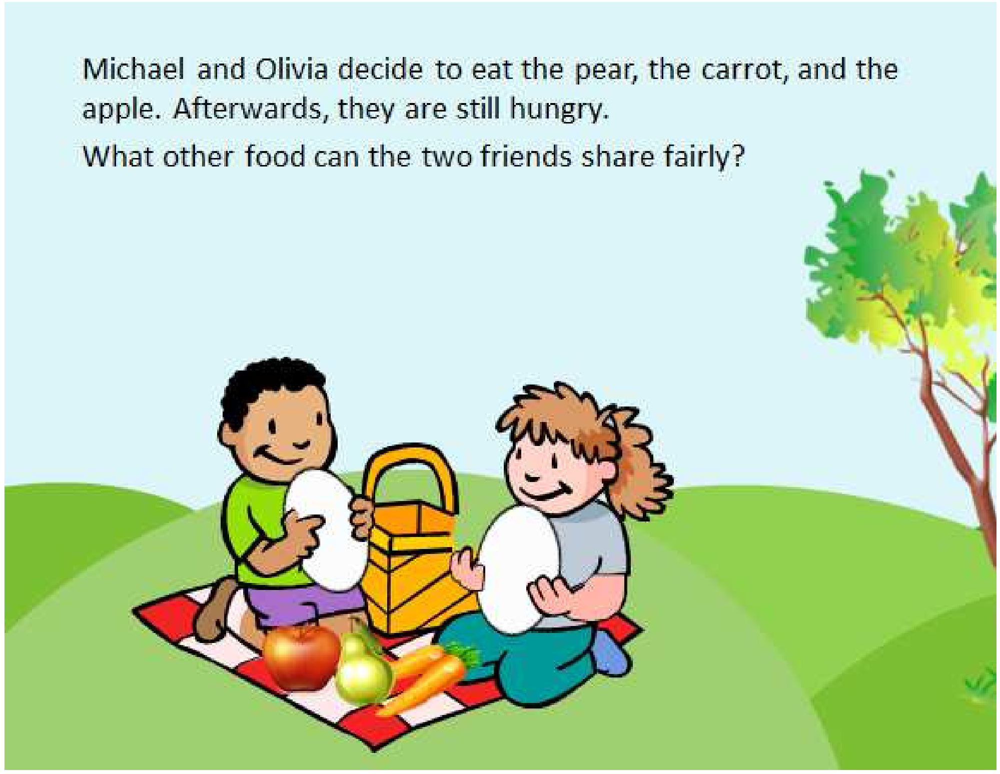
Can you find the foods that the four friends can share fairly?

The friends decide to eat the melon first. They cut it into four pieces that they can share.

Before they start to eat the melon, David and Molly realize it is time to go home.

Michael and Olivia wonder what they should do with the melon.

Will Michael and Olivia each get a bigger piece of melon if they split the melon between the two of them, or if they save some for David and Molly?


Michael and Olivia decide to eat the pear, the carrot, and the apple. Afterwards, they are still hungry.
What other food can the two friends share fairly?


Olivia and Michael find the loaf of bread and wonder if they could share it.
What would happen if Olivia and Michael tried to share the loaf of bread fairly?

Appendix J: Worksheet for Didactic Instruction Condition

Learning about Fractions

Use the materials provided to help you answer the following questions.

1. Which items are divided into halves (two pieces)?
2. Which items are divided into thirds (three pieces)?
3. Which items are divided into fourths (four pieces)?
4. Find the melon. If you divided it in half would each piece be bigger or smaller than if you divided it into fourths?
5. Are there items other than the ones cut in half that could be divided in two parts?
6. Find the loaf of bread. What would happen if you tried to divide the pieces equally in two groups?

Thank you for participating in this activity! Feel free to continue to use the materials however you like for the rest of the time.

Appendix K: Sample Excerpts Transcripts

Didactic Instruction Transcript

*MOT:	+" which items are divided into halves?
*MOT:	or two pieces?
*MOT:	hmm.
*CHI:	two pieces?
%gpx:	holds up orange
*MOT:	two pieces.
*MOT:	alright.
*CHI:	four pieces.
%gpx:	holds up pepper
*MOT:	and that one's four pieces.
*MOT:	so is that one we're looking for?
*CHI:	no.
*MOT:	no.
*MOT:	what else?
*CHI:	did you say three pieces?
*MOT:	+< so this one?
%gpx:	picks up orange
*MOT:	we said halves.
*MOT:	or two pieces.
*CHI:	halves.
*CHI:	<what are> [/] what are halves?
*MOT:	two pieces.
*MOT:	like you said this one.
%gpx:	holds up orange
*MOT:	right?
*CHI:	oh yeah.
*MOT:	one two pieces.
*MOT:	it was a half.
*MOT:	what else?
*MOT:	so is this one a half?
*MOT:	two pieces?
%gpx:	holds up tomato
*CHI:	no.
*MOT:	no?
%gpx:	shakes head
*MOT:	so let's put this one in the no pile.
%act:	puts down tomato
*MOT:	put this one in the yes pile?
%act:	moves orange
*CHI:	0.
%act:	picks up pear

*MOT:	what about that one?
*CHI:	yes pile.
*MOT:	yes pile?
*MOT:	alright.
*MOT:	that one's got two.
%act:	puts down pear
*CHI:	no pile.
%act:	moves watermelon
*MOT:	no pile.
*CHI:	no pile.
%act:	moves pepper
*MOT:	right.
*CHI:	no pile.
%act:	moves cucumber
*MOT:	okay.
*CHI:	yes pile.
%act:	picks up carrot
*MOT:	yes pile.
*CHI:	no pile.
%act:	moves banana
*MOT:	no.
*CHI:	no pile.
%act:	moves bread loaf
*CHI:	no pile.
%act:	moves bread slices
*MOT:	0 [=! gasps].
*MOT:	you did it!
%gpx:	gives CHI high five
*MOT:	alright.
*MOT:	let's mix (th)em back up.
*CHI:	what?
*MOT:	we gotta mix (th)em back up.
*CHI:	mix (th)em back up?
*MOT:	yeah.
*MOT:	mix (th)em back up.
%act:	puts all the food together
*MOT:	alright.
*MOT:	you ready for the next question?
*CHI:	uhhuh.
*MOT:	okay.
*MOT:	+" which items are divided into thirds?
*CHI:	<what> [/] what does thirds mean?
*MOT:	it means three pieces.

*MOT: +" can you find some foods that two friends can split into two pieces?
*MOT: do you want the blue plate or the red plate?
*CHI: um.
*CHI: blue.
%act: puts orange half on a plate
*MOT: so which food can we share fairly?
*MOT: I'll take the red plate.
*CHI: 0.
%act: puts other orange half on plate
*MOT: so this one we can share fairly because +...
*CHI: you can cut it into two pieces.
*MOT: yeah.
*MOT: let's do that one.
*MOT: is there anything else we can share fairly?
*CHI: 0.
%act: picks up pear
*MOT: what's that?
*CHI: uh.
*CHI: <peach> [/] peach.
*MOT: a pear.
*CHI: 0.
%act: puts pear halves on plates
*MOT: okay.
*MOT: so.
*MOT: I get a piece and you get a piece.
*CHI: yeah.
*MOT: what else?
*MOT: I'm hungry for (.) something else.
*CHI: <do you wanna share> [/] do you wanna share some bread with me?
%act: picks up bread slices
*CHI: 0.
%act: puts bread back
*CHI: how about this?
%gpx: holds up pepper
*CHI: these have two pieces.
*MOT: okay.
*CHI: xxx.
*CHI: that way we can both get two pieces of this.
%act: cuts pepper in half
*MOT: can you cut mine in half again?
*MOT: (be)cause they're too big for me to bite into.
*CHI: okay.

*MOT:	mmm!
*MOT:	do you know what this one is?
*CHI:	no.
*MOT:	avocado.
*CHI:	this?
%act:	picks up pepper half
*MOT:	mmhm.
*CHI:	I'll try avocado.
*CHI:	(be)cause I've never tried it before.
*CHI:	0.
%act:	tries to cut pepper half
*MOT:	ooh it's hard to cut into.
*CHI:	uhhuh.
*CHI:	really hard.
*CHI:	I wish we could just break it into little pieces.
*CHI:	so no one has to cut it so hard.
%act:	puts pepper pieces on plates
*CHI:	here.
*CHI:	do you wanna try a carrot?
%gpx:	holds up carrot
*MOT:	sure.
*CHI:	they're good.
*MOT:	oh you should use the cutting board.
%gpx:	points towards cutting board
*CHI:	okay.
*CHI:	I will cut them.
%act:	cuts carrot
*CHI:	ooh!
%act:	puts carrot halves on plates
*MOT:	okay.
*MOT:	anything else that we can share fairly?
*CHI:	this.
%gpx:	picks up tomato
*CHI:	we can both have +...
*CHI:	<that> [/] that only has three.
*CHI:	and if I get two and you get one that's not fair.
%gpx:	shakes head
*CHI:	so let's try this.
%gpx:	picks up melon
*CHI:	<if we> [//] if I cut in the middle then we can both have two.

*MOT:	what do you wanna make?
*MOT:	what looks tasty?
*MOT:	you gonna cut it on the cutting board?
*CHI:	shall we make a salsa?
*MOT:	sure.
*CHI:	come on.
*CHI:	xxx plates.
*MOT:	what do you +/.
*CHI:	you put them on the plates and I cut them.
*MOT:	okay.
*CHI:	what should we put in our salsa?
*CHI:	a banana?
%act:	picks up banana
*MOT:	sure.
*CHI:	how many pieces?
*MOT:	whatever you want.
*MOT:	what do you think?
*MOT:	do you wanna use the cutting board?
%gpx:	points to cutting board
*CHI:	I think we should do one piece.
%act:	moves banana to cutting board
*MOT:	one piece?
*MOT:	like that?
%act:	puts banana piece on plate
*CHI:	mmhm.
*MOT:	okay.
*MOT:	then what?
*CHI:	what's this?
%gpx:	holds up melon
*MOT:	that looks like uh avocado?
*CHI:	0.
%act:	cuts melon in half and puts on plate
*MOT:	two avocados.
*MOT:	okay.
*CHI:	0.
%act:	puts other melon half on second plate
*MOT:	okay.
*CHI:	0.
%act:	cuts another piece of banana
*MOT:	does this need a banana?
%gpx:	points to plate
*CHI:	0.

%act:	puts banana piece on plate
*MOT:	okay.
*MOT:	do you have extra banana?
*CHI:	hmm.
%act:	puts third banana piece on third plate
*CHI:	<what's this> [?]?
*MOT:	now what do you need?
*MOT:	well <that's> [//] what's this?
%gpx:	points to watermelon
*CHI:	watermelon.
%act:	picks up watermelon
*MOT:	watermelon.
*MOT:	bread.
*CHI:	0.
%act:	cuts up watermelon
*MOT:	this looks like a lime.
%act:	picks up pepper
*MOT:	pear.
*MOT:	Jack what do you think?
*MOT:	that's a tomato?
%gpx:	holds up tomato
*CHI:	0.
%act:	cuts more melon
*MOT:	orange.
*CHI:	0.
%act:	arranges food on plates
*MOT:	this is probably a cucumber.
%gpx:	holds up cucumber
*MOT:	right?
*CHI:	0.
%act:	divides melon
*MOT:	so everybody gets a piece of avocado.
*MOT:	okay.
*MOT:	then what?
*CHI:	0.
%act:	cuts up watermelon on cutting board
*CHI:	0.
%act:	puts watermelon on plates
*MOT:	oh this person doesn't get any banana?
%gpx:	taps on plate
*CHI:	0 [=! laughs].
%gpx:	shakes head

Appendix L: IRB Approval Letter

UNIVERSITY OF MARYLAND

INSTITUTIONAL REVIEW BOARD

1204 Marie Mount Hall
College Park, MD 20742-5125
TEL 301.405.4212
FAX 301.314.1475
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DATE: June 13, 2014

TO: Sarah Eason, MS
FROM: University of Maryland College Park (UMCP) IRB

PROJECT TITLE: [608816-1] Parent and Preschooler Interactions during Joint Activities
REFERENCE #:
SUBMISSION TYPE: New Project

ACTION: APPROVED
APPROVAL DATE: June 13, 2014
EXPIRATION DATE: June 12, 2015
REVIEW TYPE: Expedited Review

REVIEW CATEGORY: Expedited review category # *6 & 7*

Thank you for your submission of New Project materials for this project. The University of Maryland College Park (UMCP) IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a project design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

This submission has received Expedited Review based on the applicable federal regulation.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Federal regulations require each participant receive a copy of the signed consent document.

Please note that any revision to previously approved materials must be approved by this committee prior to initiation. Please use the appropriate revision forms for this procedure which are found on the IRBNet Forms and Templates Page.

All UNANTICIPATED PROBLEMS involving risks to subjects or others (UPIRSOs) and SERIOUS and UNEXPECTED adverse events must be reported promptly to this office. Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed.

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to this office.

This project has been determined to be a Minimal Risk project. Based on the risks, this project requires continuing review by this committee on an annual basis. Please use the appropriate forms for this procedure. Your documentation for continuing review must be received with sufficient time for review and continued approval before the expiration date of June 12, 2015.

Please note that all research records must be retained for a minimum of three years after the completion of the project.

If you have any questions, please contact the IRB Office at 301-405-4212 or irb@umd.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within University of Maryland College Park (UMCP) IRB's records.

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