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On Routing Two-Terminal Nets in the Presence of Obstacles

by

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Abstract

We consider the problem of routing k two-terminal nets in the presence of obstacles in two models: the standard two-layer model and the knock-knee model. Determining routability is known to be NP-complete for arbitrary k. Our main results are polynomial time algorithms to determine whether the given nets are routable in either model for any fixed k. We introduce a technique that reduces the general problem into finding edge-disjoint paths in a graph whose size is propotional to the size of the obstacles. Two optimization criteria are considered: the total length of the wires and the number of vias used.

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1 Introduction

A general wire routing problem consists of interconnecting the terminals of each of a given set of k nets within a given grid, while avoiding a set of rectilinear obstacles representing previously placed modules. The locations of the terminals are assumed to be on the boundaries of the obstacles. Wires can run on a grid in any of several layers with the restriction that wires in a given layer have to be separated by a certain minimum distance. A wire may change layers by using a *contact cut* or *via*. The *standard two-layer* model, where horizontal wires run on one layer and vertical wires run on the other layer, and the *knock-knee* model will be considered in this paper. Our goal is to provide a *detailed* routing of the k nets, whenever such a routing is possible. Two optimization criteria are used. The first is to minimize the *total length* of the wires, and the second is to minimize the number of vias used in the wiring.

This problem has been studied extensively in the literature (e.g. [CO], [CR], [H1], [H2], [HS], [LE], [O], [OT], [WW]). A typical solution consists of a heuristic to order the nets and a method to route each net separately while avoiding the initial obstacles and the wires of the previously routed nets. It seems that no good heuristics exist to perform the ordering satisfactorily. Another alternative is the *rip up and reroute* strategy, where a designer could interactively change the ordering of the nets whenever "blockages" are observed ([DS],[OT]). As for routing a single net while avoiding a set of obstacles, two basic strategies are well-known. The *maze* algorithms initially introduced by Lee [LE] are used extensively. Any such algorithm guarantees an optimal solution, if it exists. However, their time and space requirements are extremely large. The *line search* algorithms introduced by Hightower ([H1]) use much less memory but do not necessarily produce a wiring even if one exists. In ([CO],[CR]), the line search technique was combined with some algorithms from computational geometry to obtain a fast algorithm that always guarantees a solution with minimum length or minimum number of bends, whenever such a solution exists. The running time of this algorithm depends only on the size of the obstacles , unlike that of a maze router whose running time will typically depend on the size of the *overall grid* and the size of the obstacles.

In this paper, we consider the problem of routing the given k nets simultaneously. We restict ourselves to two-terminal nets since otherwise the problem is NP-complete even for a single net. If k is not fixed, the problem is known to be NP-complete ([KV]). Therefore it is highly unlikely that an efficient algorithm exists that guarantees a solution whenever one exists. Our main result provides an algorithm whose running time is polynomial in the size of the constraints and that guarantees a solution whenever one exists for any fixed k. Our main technique is to reduce the general problem to finding edge-disjoint paths in a graph of size $O(k^2n^2)$, where n is the input length of the obstacles. A recent result of [RS] shows how to find such paths efficiently for each fixed k. We also establish stronger results for the case when $k = 2$.

The rest of the paper is organized as follows. The basic definitions and the two wiring models are introduced in the next section, while section 3 contains the solution for the knock-knee and the one-layer models. The proofs for the standard two-layer model are presented in section 4.

2 Definitions

An instance of the k-routing problem consists of a grid specified by its boundaries, a set of rectilinear obstacles specified by line segments that determine their boundaries, and a set of k pairs (a_i, b_i), $1 \leq i \leq k$, representing the terminals of k nets. We assume that any pair of obstacles are separated by at least a unit distance for otherwise they could be combined into a single rectilinear obstacle. The grid without the union of the regions determined by the rectilinear obstacles is called *the routing region.* The *input length* of an instance is determined by the set of line segments specifying the boundaries of the grid and the obstacles, and the pairs $\{(a_i, b_i), 1 \leq i \leq k\}$. Notice that the input length is independent of the size of the given grid. We now introduce the notion of *escape lines.*

A horizontal line segment u *covers* a horizontal boundary segment v if, and only if, they have a nonempty intersection on the X-axis. A similar

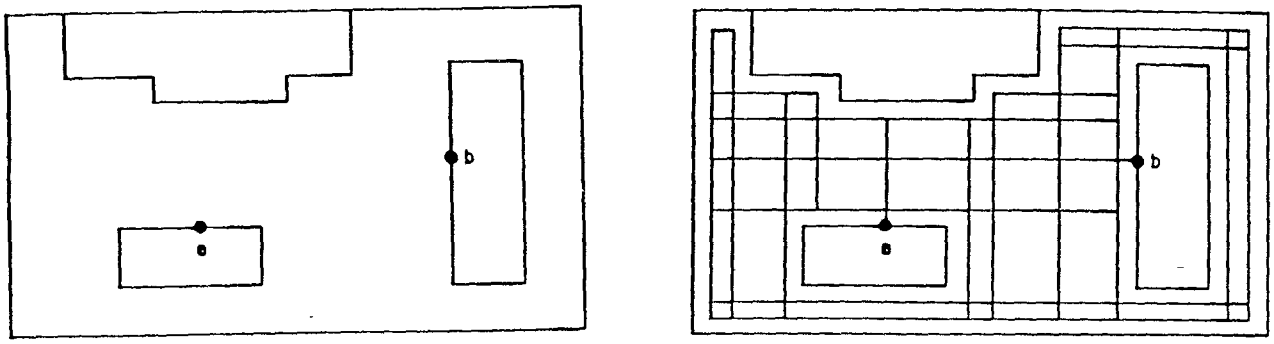

Figure 1: Escape Lines

definition holds for vertical segments. The main technique of this paper relies on generating *maximal* line segments in the routing region that cover, and are within a small distance of, the boundary segments. The typical notion of *escape lines* consists of generating two types of line segments. The *type I* escape lines are maximal line segments that cover boundary segments and are within a unit distance of these boundary segments. The *type II* escape lines are maximal horizontal or vertical segments in the routing region that intersect a terminal (see Figure 1). As we will see later the routing problem could be restricted to finding paths in a graph that consists of escape line segments.

As was mentioned before, two routing models will be considered. The *standard two-layer* model restricts the routing to two layers such that horizontal wires run one layer and vertical wires run on the other layer. On the other hand, the *knock-knee* model allows arbitrary rectilinear wiring as long as no two wire segments overlap. Hence wires could intersect either at a *crossing point* or at a *knock-knee* (See Figure 2). It was shown in [BB] that four layers suffice for any routing in the knock-knee model.

We assume that the reader is familiar with the basic techniques for *channel routing* and in particular with the notion of *constraint graphs*.

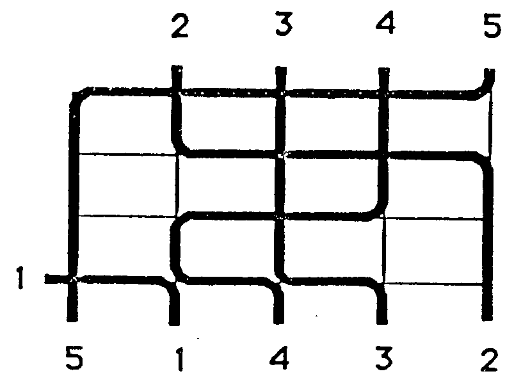

Figure 2: Routing in Knock-knee Model

3 Routing in the Knock-Knee Model

Given an instance of the k-routing problem, we are supposed to determine whether or not the corresponding nets are routable in the knock-knee model. We start by extending the notion of escape lines and then establish the main algorithm of this section.

Let u be the maximal line segment in the routing region which covers a boundary segment and whose distance from that boundary segment is one. For each such segment, generate as many as possible but no more than $\lceil \frac{k}{2} \rceil$ parallel maximal segments on each side u as shown in Figure 3. These segments will be called *type I escape lines.* Notice that for each boundary segment there are at most $2 \lceil \frac{k}{2} \rceil + 1$ type I escape lines. We will later see that in general we cannot eliminate any of them. Let v be a maximal line segment in the routing region with one endpoint being a terminal. As before, generate the maximum number $\leq \lceil \frac{k}{2} \rceil$ of maximal line segments parallel to v (on each side) in the routing region as shown in Figure 3. These segments will be called *type II escape lines.*

The *escape graph* $G = (V, E)$ is defined as follows. V is the set of all the intersection points of all the escape lines plus all the terminals. E is the set of all *escape segments* joining two vertices in V. Notice that if n is the size of the input representing the boundary segments of the obstacles, then the sizes of V and E are bounded by $O(k^2n^2)$. Our goal is to prove that a routing exists if, and only if, there exist k edge-disjoint paths in G between the pairs of vertices corresponding to the terminals of the given nets. Before we can establish the main result of this section, we have to

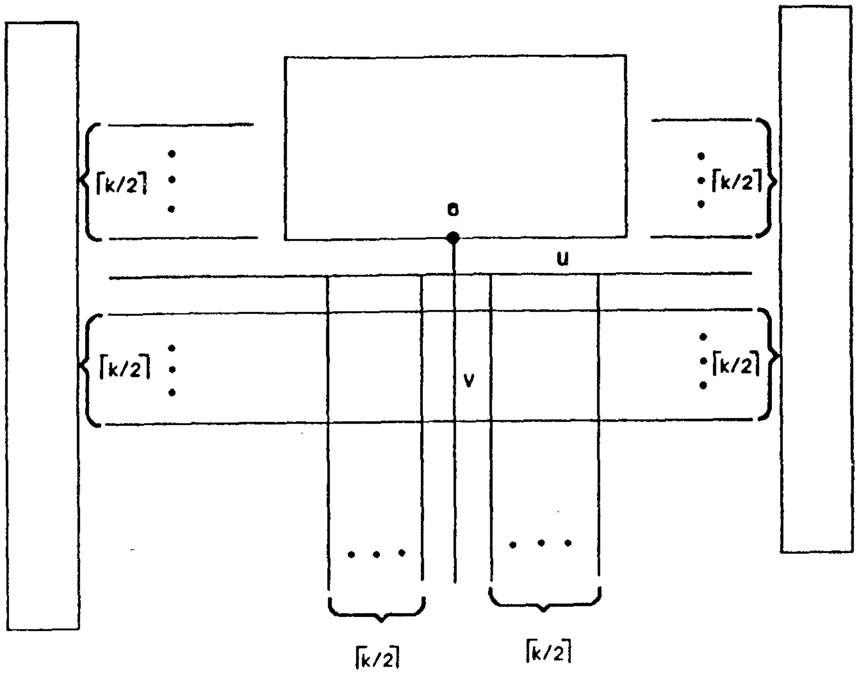

Figure 3: Escape Lines in the Knock-Knee Model

introduce a characterization ([FR]) for the existence of edge-disjoint paths in a rectilinear grid.

Let R be a rectilinear grid with n columns and m rows. Suppose we are given a set of pairs (x_i, y_i) representing the terminals of a set of nets, where x_i and y_i lie on the boundary of R. A necessary and sufficient condition for the routability of these nets in the knock-knee was given in ([FR]). A *vertical cut* at column $c < n$ is the region of R between the columns c and $c + 1$. The *congestion* of c is the number of terminal pairs separated by the vertical cut at c. A column is *saturated* if its congestion is equal to m. Similar definitions hold for horizontal cuts. Suppose there are t saturated horizontal cuts, say $r_1, r_2, \ldots, r_t$, and let c be any column. The saturated rows induce $t+1$ regions to the left of c, say $T_1, T_2, \ldots, T_{t+1}$ (see Figure 4). Join each terminal by an imaginary edge. Call the resulting graph the *extended graph.* A set T is *odd* if the number of edges leaving T is odd. The number of odd sets T_i is called the *parity congestion* of c. The *revised congestion* of c is the sum of the parity congestion and the congestion of c. The *revised column criterion* states that the revised congestion of any column is at most m. We can similarily define the *revised row criterion.*

Theorem1 [FR]: A routing within a rectangle exists if and only if the revised row and column criteria hold.

We are ready to state the main result of this section.

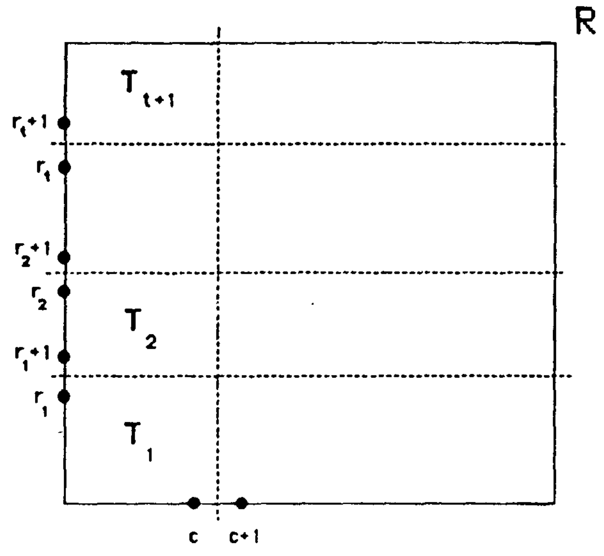

Figure 4: Routing Within a Rectangle

Theorem2: Given an instance of the k-routing problem, a solution exists if and only if there exist k edge disjoint paths connecting the corresponding terminals in the escape graph.

Proof: Suppose a solution to an instance of the k-routing problem exists and let $p_j(a_j, b_j)$ be the path determined by the wiring of the jth net. Without loss of generality, we can assume that p_j consists of rectilinear segments $\{r_{j1}, r_{j2}, \ldots, r_{jt_j}\}$, where for any two successive segments one is horizontal and the other is vertical. We will show how to transform this wiring into paths in the escape graph G.

Suppose a path p_j has some segments outside G. Let r_{ji} be the minimally indexed routing segment of $p_j(a_j, b_j)$ that is not an escape segment. Without loss of generality, assume that r_{ji} is a horizontal segment. Let R be the maximal routing region containing r_{ji} and $\lceil \frac{k}{2} \rceil + 1$ horizontal escape segments above r_{ji} and $\lceil \frac{k}{2} \rceil + 1$ below r_{ji} (See Figure 5). Since r_{ji} is not escape segment, such a region always exists. A path p_k may enter R at some point x_k and leave R at x'_k. We want to alter the routing within R so that all the horizontal segments used are escape segments. Moreover, the columns used are the entry and the exit columns of the given wiring in R and possibly one additional column which is used by a previous wire. It is easy to see that the routing in R is a channel routing problem (because of the maximality of R) with $2\lceil \frac{k}{2} \rceil + 2$ tracks (the escape segments) such that each net has two terminals. It is clear that in this case there are no

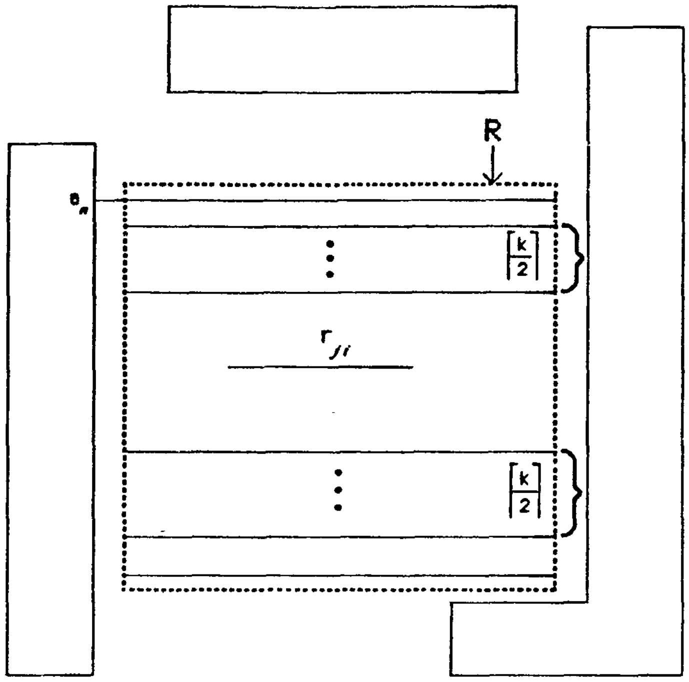

Figure 5: The Rectangle R in the Proof of Theorem2

saturated columns. The congestions of all the rows are identical. If all the rows are saturated, then a routing exists if and only if every induced net is a trivial net. Assume that none of the rows is saturated. Then using Theorem1 the problem always has a solution with $2\lceil \frac{k}{2} \rceil + 2$ tracks and with one additional column. Choose this column to be an escape segment if possible. Notice that r_{ji} is now an escape segment and none of the escape segments used by any path has been replaced by a non-escape segment. Therefore the proof follows.

Using a result from ([RS]) we have the following.

Theorem3: Given an instance of the k-routing problem, we can determine a routing whenever it exists in time polynomial in the input length for all fixed k.

The specific choice of the escape segments will be justified by the following example. Let $k = 8$ and let the obstacles and the nets be as given in Figure 6. We justify the introduction of type II escape lines. The regions above the horizontal line [1,2] and below the horizontal line [4,3] are chosen to be saturated (see discussion preceding Theorem1). Therefore the routability of the given instance will depend on the number of horizontal lines in the region between lines [1,2] and [4,3]. The path congestion at the third column is 8 and hence by Theorem1 we need at least 8 horizontal lines

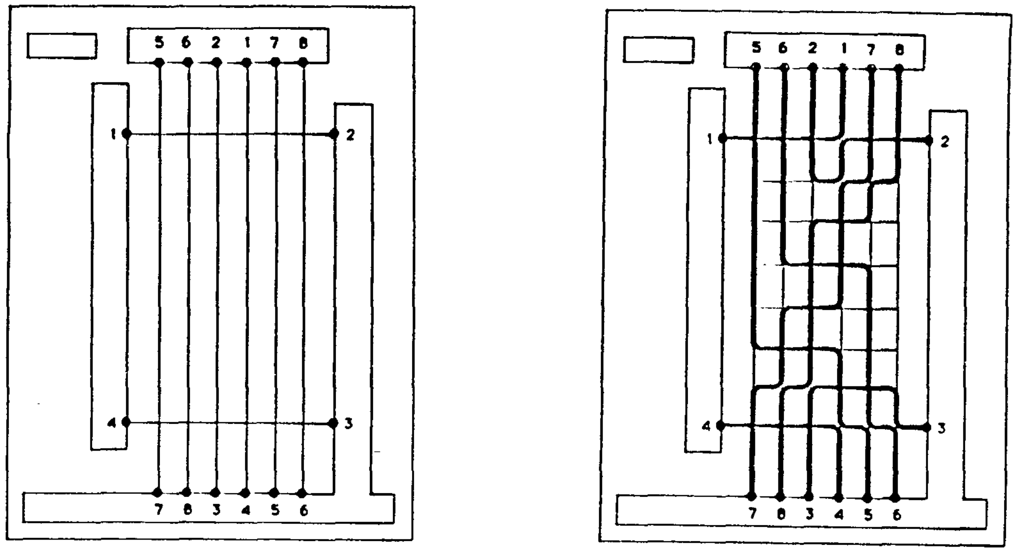

Figure 6: Justification of Escape Lines

in the unsaturated region. Therefore enough lines parallel to an escape II line have to be generated if routability to be tested on the generated lines only.

We end this section with the following theorem concerning one layer routing.

Theorem4: Given an instance of the k-routing problem, a one-layer routing exists if and only if it exists in the escape graph G. Moreover an optimal solution with respect to bend minimization lies in G as well.

The proof of this theorem is similar to that of Theorem2 , except that the routing induced within the rectangle R is a river routing problem.

4 Routing in the Standard Two-Layer Model

The routing problem in the standard two-layer model is considerably more difficult than that in the knock-knee model. For example, determining the minimum number of tracks needed to wire two-terminal nets within a channel is NP-complete ([ST]), while the corresponding problem in the knock-knee model can be solved quite efficiently ([PL]). However the results of the previous section can be extended to the the standard two-layer model.

We first introduce the escape lines associated with the given boundaries and we then show that it is enough to restrict the search to the routing region determined by these escape lines. Finally we show how to map this problem into finding a set of edge-disjoint paths on a graph roughly the same size as the routing region.

The escape lines are generated as before except that $\lceil \frac{3k}{4} \rceil$ parallel lines (whenever possible) are generated rather than the $\lceil \frac{k}{2} \rceil$ needed for the knock-knee model. As before, let $G = (V, E)$ be the corresponding *escape graph* determined by the escape lines. We are ready to prove the first theorem of this section.

Theorem5: An instance of the k-routing problem has a solution if and only if it has a solution within the routing region determined by the escape graph G. Moreover , a solution with the minimum number of bends lies in G as well.

Proof: The proof is similar to that of Theorem2 . Suppose the given instance of the k-routing problem has a solution and let $p_j(a_j, b_j)$ be the path corresponding to the j-th net. Suppose that one p_j has a segment that is not an escape segment. Let r_{ji} be the minimally indexed routing segment of p_j (say horizontal) that is not an escape segment. As before, let R be the maximal routing rectangle containing r_{ji} and $\lceil \frac{3k}{4} \rceil$ horizontal escape segments above r_{ji} and $\lceil \frac{3k}{4} \rceil$ below r_{ji}. One can check that the routing paths determine a channel routing problem within R. We now show that the routing within R can be done such that all the horizontal segments used are escape segments and the number of bends used is not increased.

We will briefly outline how to route the induced nets by using at most $\lceil \frac{3k}{2} \rceil$ tracks. Let c be the number of nets induced within R. The new routing strategy will use the entry and the exit columns of each net plus one specified extra column (the same for all the nets) such that all the horizontal tracks are escape segments. The columns used will be a subset of those used by the given routing. Start by routing those nets whose terminals lie on one side of R by using one escape line per net. Notice that the number of bends used per net is two and hence the total number of bends used is minimum. For the remaining nets, let's consider the vertical

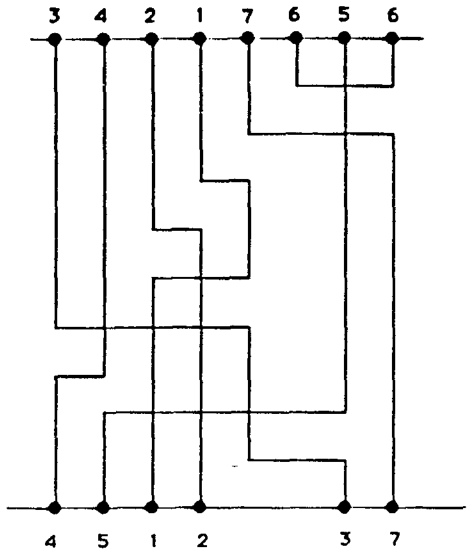

Figure 7: A Cycle in The VC Graph

constraint graph (VC) associated with this channel routing problem. We distinguish between two cases:

(i) The VC graph does not have any cycles. Then it is well-known that c tracks are sufficient and the only columns used are the entry and the exit columns. The wiring of each net has two bends (obviously nets appearing in the same column do not require any bends).

(ii) The VC graph has l cycles. In this case, the given wiring must use at least one extra column in addition to the entry and the exit columns for otherwise the nets are unroutable. Let u be such a column (vertical escape segment, if possible). Each cycle can be broken by using a section of u and an extra track (see Figure 7). If a cycle has q nets, then the number of bends used is $2q + 2$ which is minimum. The remaining nets can be routed as before without any additional columns (one track per net). Since there are at most $\frac{c}{2}$ cyles, at most $\frac{3k}{2}$ will ever be used. Moreover the wiring introduced uses the minimum number of bends possiblem. Therefore the proof of the theorem follows.

We now show how to reduce the above problem to that of finding a set of edge-disjoint paths in a new graph $G' = (V', E')$. G' is constructed from G as follows. For each vertex v_i of V that is an intersection point of two escape lines, create two vertices v_{i1}, v_{i2}. The terminals are also included in V'. The edges E' consist of :

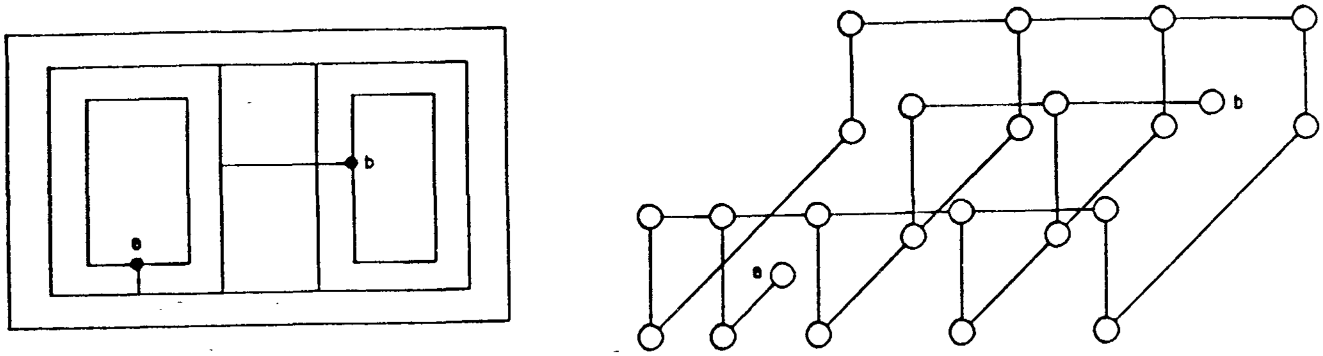

Figure 8: The Escape Graph G and the Corresponding Graph G'

1. All pairs (v_{i1}, v_{j1}) such that $[v_i, v_j]$ is a *horizontal* escape segment in G.
2. All pairs (v_{i2}, v_{j2}) such that $[v_i, v_j]$ is a *vertical* escape segment in G.
3. All pairs (v_{i1}, v_{i2}) such that $v_i \in V$ is an intersection point of two escape segments.
4. All pairs (x, v_{lm}) where x is a terminal and $(x, v_l) \in E$. $m = 1$ if $[x, v_l]$ is a *horizontal* escape segment, $m = 2$ otherwise.

Notice that $|V'| \leq 2|V|$ and $|E'| \leq 2|E|$.

See Figure 8 for an example.

Theorem6: Let G be the escape graph corresponding to an instance of the k-routing problem in the standard two-layer model and let G' be the graph defined above. Then a solution for the k-routing problem exists in G if and only if there are k edge-disjoint paths in G' connecting the pairs of terminals (a_i, b_i).

Proof: Suppose that G' contains k edge-disjoint paths say $p_i(a_i, b_i)$. We will show how to obtain a legal routing of the k nets in G. For each path p_i collapse any two vertices v_{j1}, v_{j2} corresponding to an intersection point

v_j in G. Let the resulting paths be $q_i(a_i, b_i)$. We claim that these are legal paths in G. No two paths q_i and q_j could share a segment since p_i and p_j are edge (and vertex) disjoint. On the other hand if q_i and q_j intersect at a knock-knee v, then one can easily check that p_i and p_j have to share the edge (v_1, v_2) in G', which contradicts the fact that these paths are edge-disjoint. Hence the q_i's are legal paths in G. The rest of the proof follows similarily.

As before using the result of [RS] the following theorem follows.

Theorem7: Given an instance of the k-routing problem, it is possible to determine whether or not it has a solution in time polynomial in n for all fixed k, where n is the input length of the constraints. Moreover a routing can be found in polynomial time whenever it exists.

For the case of $k = 2$ we can show that a solution with the minimum total wire length can be found in the routing region determined by the escape graph.

Theorem8: Suppose an instance of the 2-routing problem has a solution. Then it has a solution with the minimum total wire length within the routing region determined by the escape graph.

Proof: We use the same strategy as before. We start with any wiring that minimizes the total wire length and we show that the wiring could be done with escape segments without increasing the total wire length. We use the same notation introduced in the proof of theorem . We want to modify the routing within R in such a way that all the horizontal segments used are escape segments, the columns used are a subset of those used in the given wiring and the total wire length is not increased. There are 4 tracks and at most 2 pairs, (x_1, y_1),(x_2, y_2), of terminals in R. Two situations need to be considered:

(i) The terminals (x_i, y_i), $i = 1, 2$ are on one side of R. Route them by using the two upper tracks if the terminals are the top side of R. Use the two lower tracks otherwise. The problem can be easily solved as well if the terminals of one net are on one side while the terminals of the other net are on opposite sides.

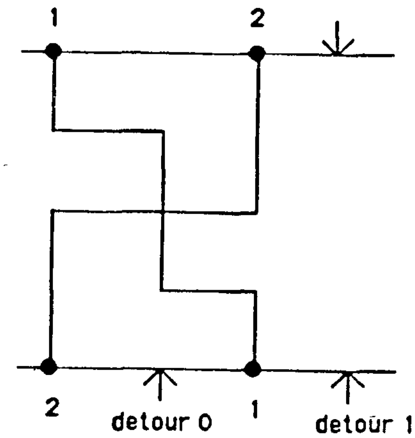

Figure 9: Cycle in the Constraint Graph

(ii) Suppose that the terminals of each net are on opposite sides of R. If the nets are not involved in any cycle, route them according to the order induced. Each net uses one track and the minimum total wire length. If the two nets are involved in a cycle, then an extra column is needed. Either one or more extra columns are used by the given wiring. Use whichever minimizes the length of the "detour" (See Figure 9) and three tracks that are escape segments.

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