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by

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Abstract

Suppose two instruments I_0 and I_1 measure the same quantity with the same resolution, where it is known I_0 is more reliable. The second, I_1 , is assumed a distortion of I_0 in some sense. A method is outlined whereby: 1. The information from both I_0, I_1 is combined to increase the reliability of I_0 . 2. The distortion is quantified. An example is given in terms of ship borne precipitation radar and a space borne radiometer both measuring rain rate.

1 Introduction

Suppose two instruments I_0 and I_1 measure the same quantity with the same resolution, where it is known I_0 is more reliable. The second, I_1 , is assumed a distortion of I_0 in some sense. A method is outlined whereby: 1. The information from both I_0, I_1 is combined to increase the reliability of I_0 . 2. The distortion is quantified.

The method is general, but to keep things simple, throughout the paper the quantity of interest is positive rain rate measured over a space-time box by two instruments.

A particular example is combining a space borne precipitation radar (PR), I_0 , and a space borne radiometer, I_1 , both measuring rain rate in mm/hr over a given space-time box, both operating at the same resolution. Another example is I_0 representing a network of rain gauges—ground truth—and I_1 a PR.

Referring to the PR/Radiometer example, in principle both instruments after appropriate adjustment give the same information. In practice however they differ, but in what way and by how much? The essential idea of the paper is that the radiometer is a distorted radar, and the question is how to recover the radar component from the radiometer and combine it with the radar data to produce more reliable statistics. In other words, a radiometer is a “radar in disguise” and the goal is to recover the masked radar information buried in the radiometer and combine it with the “true” radar data for estimation purposes.

The approach is based on a semiparametric model used in statistical case-control studies where the parametric part quantifies the distortion, and the non-parametric part refers to the probability distribution of the quantity of interest as measured by the more reliable instrument.

The first part of the paper—sections 2,3,4—deals with statistical considerations, the second—section 5—with a real PR/Radiometer data example. The connection with the Tropical Rainfall Measuring Mission is described in section 5-a.

2 A Semiparametric Model

For clarity consider the PR/Radiometer case. So, given a random rain rate observation x , it could be a radar measurement or a radiometer measurement. Let y be the corresponding indicator: $y = 0$ means x is a radar measurement, and $y = 1$ indicates x is a radiometer observation, where $P(y = 1) = \pi$. A reasonable

assumption is to model—quite generally—the conditional binary probability by the logistic form,

$$P(y = 1|x) = \frac{\exp(\alpha^* + \beta h(x))}{1 + \exp(\alpha^* + \beta h(x))} \equiv \psi(x) \quad (1)$$

for some function $h(x)$, possibly vector valued with coefficient β of the same dimension. Suppose a radar measurement x has a probability density function (pdf) $g(x)$, and that $g_1(x)$ is the pdf if x were a radiometer observation. Then, upon appealing to Bayes rule, $g(x)$ and $g_1(x)$ are related by the equation (see Kay and Little (1987)),

$$g_1(x) = \exp(\alpha + \beta h(x))g(x) \quad (2)$$

where $\alpha = \log(1 - \pi)/\pi + \alpha^*$.

An attractive feature of model (2) is that both densities $g(x)$ and $g_1(x)$ are modeled nonparametrically except for an "exponential tilt" used to relate one density to the other. Therefore, inference based on model (2) should be more robust than inference based on a full parametric model.

Needless to say, we do not lay claim to full generality, but surprisingly enough, quite a few familiar distributions give rise to (1) and (2) including the normal, lognormal, exponential, binomial, and Poisson distributions. The normal and lognormal cases are discussed in some detail below.

Evidently, by our formulation, the part common to both radar and radiometer manifests itself by the pdf $g(x)$, and the distortion factor is expressed by the exponential weight function $\exp(\alpha + \beta h(x))$. Clearly, since $g(x)$ is a pdf, the distortion is really embodied by β , and when $\beta = 0$ the two instruments behave alike.

Suppose now $\mathbf{x} = (x_1, \dots, x_{n_0})$ is a radar random sample, and $\mathbf{z} = (z_1, \dots, z_{n_1})$ is a radiometer random sample. *By combining the radar and radiometer we mean the estimation of $g(x)$ from the combined radar/radiometer rain rate data $\mathbf{t} = (x_1, \dots, x_{n_0}, z_1, \dots, z_{n_1})$.*

Let $n = n_0 + n_1$. Our goal is to estimate the parameters α, β and the function $g(x)$ from the combined data

$$\mathbf{t} = (\mathbf{x}, \mathbf{z}) \equiv (t_1, \dots, t_n)$$

a. Special Case: Normal Distribution

To demonstrate (1) and (2) are not vacuous, consider two normal distributions with different means but equal variances, and let y be a binary random variable with $P(y = 1) = \pi$. Suppose that given $y = 0$, $x \sim N(\mu_0, \sigma^2)$, and given $y = 1$, $x \sim N(\mu_1, \sigma^2)$. Then, as in (2), the ratio of conditional densities is exponential:

$$\frac{g_1(x)}{g(x)} = \frac{f(x|y=1)}{f(x|y=0)} = \exp(\alpha + \beta x)$$

with

$$\alpha = \frac{\mu_0^2 - \mu_1^2}{2\sigma^2}, \quad \beta = \frac{\mu_1 - \mu_0}{\sigma^2}, \quad h(x) = x$$

This implies the logistic regression model (1) because upon defining α^* by the equation $\alpha = \log \frac{1-\pi}{\pi} + \alpha^*$, Bayes rule gives

$$\frac{P(y=1|x)}{P(y=0|x)} = \exp(\alpha^* + \beta x)$$

and since $P(y=0|x) = 1 - P(y=1|x)$,

$$P(y=1|x) = \frac{\exp(\alpha^* + \beta h(x))}{1 + \exp(\alpha^* + \beta h(x))} = \frac{\exp(\alpha^* + \beta x)}{1 + \exp(\alpha^* + \beta x)}$$

Conditioning normal by a binary variable leads to logistic regression!

When in addition to different means also $\sigma_0^2 \neq \sigma_1^2$, the same argument holds with vectors $\beta, h(x)$,

$$g_1(x) = \exp(\alpha + \beta_1 x + \beta_2 x^2)g(x)$$

where $h(x) = (x, x^2)$ and

$$\alpha = \log \frac{\sigma_0}{\sigma_1} + \frac{\mu_0^2}{2\sigma_0^2} - \frac{\mu_1^2}{2\sigma_1^2}, \quad \beta = (\beta_1, \beta_2) = \left(\frac{\mu_1}{\sigma_1^2} - \frac{\mu_0}{\sigma_0^2}, \frac{1}{2\sigma_0^2} - \frac{1}{2\sigma_1^2} \right)$$

b. Special Case: Lognormal Distribution

In the lognormal case α, β are the same as in the corresponding normal case but x is replaced by $\log x$. So, when $\sigma_0^2 = \sigma_1^2 = \sigma^2$ then $h(x) = \log x$, and when $\sigma_0^2 \neq \sigma_1^2$ then $h(x) = (\log x, \log^2 x)$.

3 Maximum Likelihood Estimation

The pdf $g(x)$ is estimated through the cumulative distribution function (cdf) $G(x)$. So, let $G(x)$ be the cdf of x . A maximum likelihood estimator of $G(x)$ can be obtained by maximizing the likelihood over the class of step cdf's with jumps at the observed values t_1, \dots, t_n . Accordingly, if $p_i = dG(t_i)$, $i = 1, \dots, n$, the likelihood becomes,

$$\mathcal{L}(\alpha, \beta, G) = \prod_{i=1}^n p_i \prod_{j=1}^{n_1} \exp(\alpha + \beta h(z_j))$$

and to find the maximum likelihood estimators $\hat{p}_i, \hat{\alpha}, \hat{\beta}$, the likelihood is maximized with respect to the p_i 's, α and β , subject to the constraints

$$\sum_{i=1}^n p_i = 1, \quad \sum_{i=1}^n p_i [\exp(\alpha + \beta h(t_i)) - 1] = 0$$

Then following Qin and Lawless (1994), and Qin and Zhang (1996) who assume $h(t) = t$, and with $\rho = n_1/n_0$, it can be shown

$$\hat{p}_i = \frac{1}{n_0} \frac{1}{1 + \rho \exp(\hat{\alpha} + \hat{\beta} h(t_i))} \quad (3)$$

and $\hat{\alpha}, \hat{\beta}$ are the roots of the system of equations,

$$-\sum_{i=1}^n \frac{\rho \exp(\alpha + \beta h(t_i))}{1 + \rho \exp(\alpha + \beta h(t_i))} + n_1 = 0 \quad (4)$$

$$-\sum_{i=1}^n \frac{h(t_i) \rho \exp(\alpha + \beta h(t_i))}{1 + \rho \exp(\alpha + \beta h(t_i))} + \sum_{j=1}^{n_1} h(z_j) = 0 \quad (5)$$

To find the variability of the estimates, it primarily requires the covariance matrix of $\hat{\alpha}, \hat{\beta}$.

For $k = 0, 1, 2$, define,

$$A_k = \sum_{i=1}^n \frac{h^k(t_i) \exp(\alpha + \beta h(t_i))}{1 + \rho \exp(\alpha + \beta h(t_i))} p_i$$

and,

$$\mathbf{A} = \begin{pmatrix} A_0 & A_1 \\ A_1 & A_2 \end{pmatrix}$$

and,

$$\Sigma = \frac{1+\rho}{\rho} \left[\mathbf{A}^{-1} - \begin{pmatrix} 1+\rho & 0 \\ 0 & 0 \end{pmatrix} \right]$$

Then under some regularity conditions and regardless of $h(x)$ the estimators are asymptotically normally distributed,

$$\sqrt{n} \begin{pmatrix} \hat{\alpha} - \alpha_0 \\ \hat{\beta} - \beta_0 \end{pmatrix} \Rightarrow N(\mathbf{0}, \Sigma) \quad (6)$$

where α_0, β_0 are the true parameters.

Thus, for example, $\hat{\beta}$ is approximately normally distributed with variance $1/n$ times the second diagonal element of Σ , a fact from which a confidence interval for β_0 can be constructed.

a. Mean Estimation

The mean of $g(x)$ can be estimated from the combined data using the estimator $\sum_{i=1}^n t_i \hat{p}_i$. Interestingly, equation (5) with $h(t) = t$ implies $\sum_{i=1}^n t_i \hat{p}_i = \bar{x}$. That is, the combined mean estimate is actually the same as the average of x_1, \dots, x_n . However, the “combined” distribution as estimated by the \hat{p}_i is different than the empirical distribution from the x data only. This is illustrated in the data analysis in section 5.

In general (5) gives,

$$\sum_{i=1}^n h(t_i) \hat{p}_i = \frac{1}{n_0} \sum_{i=1}^{n_0} h(x_i)$$

4 Some Simulation Results

a. Example of a Uniform Case

To demonstrate the preceding analysis, consider the case where x is uniformly distributed in $[0, 1]$, so that $g(x) = 1, 0 \leq x \leq 1$, and $g(x) = 0$ otherwise. Assume $\rho = 1$, $\alpha = \beta = 0$, and $h(x) = x$, and observe that with $\alpha = \beta = 0$ the

two instruments are identical. As $n \rightarrow \infty$, the asymptotic covariance matrix Σ can be obtained exactly from

$$A_k = \int_0^1 \frac{t^k \exp(\alpha + \beta t)}{1 + \rho \exp(\alpha + \beta t)} dG(t) = \frac{1}{2(k+1)}, \quad k = 0, 1, 2$$

so that

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/6 \end{pmatrix}$$

and

$$\Sigma = \frac{1+\rho}{\rho} \left[\mathbf{A}^{-1} - \begin{pmatrix} 1+\rho & 0 \\ 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} 12 & -24 \\ -24 & 48 \end{pmatrix}$$

Thus, for example with $n_0 = n_1 = 500$, $n = 1000$, we have approximately: $\text{Var}(\hat{\alpha}) \doteq 12/1000 = 0.012$, $\text{Var}(\hat{\beta}) \doteq 48/1000 = 0.048$, $\text{Cov}(\hat{\alpha}, \hat{\beta}) \doteq -24/1000 = -0.024$, or

$$\text{Var} \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} \doteq \frac{1}{1000} \Sigma = \begin{pmatrix} 0.012 & -0.024 \\ -0.024 & 0.048 \end{pmatrix}$$

A simulation—under the above uniform assumption and $\rho = 1$, $\alpha = \beta = 0$, $h(x) = x$ —was conducted to verify the last result. It generated 100 independent pairs $(\hat{\alpha}, \hat{\beta})$ with $n_0 = n_1 = 500$, $n = n_0 + n_1 = 1000$, giving an estimated (from the sample of 100 pairs $(\hat{\alpha}, \hat{\beta})$) covariance matrix in good agreement with the theoretical one:

$$\frac{1}{1000} \tilde{\Sigma} = \begin{pmatrix} 0.0112 & -0.0223 \\ -0.0223 & 0.0445 \end{pmatrix}$$

Our first pair was $(\hat{\alpha}, \hat{\beta}) = (0.0425, -0.0860)$. Therefore, the corresponding estimated standard errors are $(0.1058, 0.2109)$. The theoretical standard errors are approximately $(0.1095, 0.2191)$.

a. More Simulation Results

Table 1 summarizes further results from various simulations under different conditions where, as above, each simulation generated 100 pairs $(\hat{\alpha}, \hat{\beta})$ under the indicated conditions. The last column is the average from 100 cases together with the corresponding estimated standard errors. Recall that $g(x)$, $g_1(x) = \exp(\alpha + \beta h(x))g(x)$ represent the two instruments.

The first two rows refer to uniform samples, the third to a normal sample, and the last to a lognormal sample. The first 3 rows are with $h(x) = x$; the last with $h(x) = \log x$. For the third row $\rho = 2$, otherwise $\rho = 1$.

5 Combining Real Radar/Radiometer Data

a. Connection With TRMM

Perhaps the most promising method for obtaining rainfall totals on a global scale is through the use of measurements made by satellite receivers. This is because satellites can cover vast areas of the globe—such as over the oceans—where it is impractical to obtain adequate coverage by surface based radars and rain gauges. The Tropical Rainfall Measuring Mission (TRMM) (Simpson et al. (1988)) satellite will be launched in late 1997, and is the first space mission dedicated to measuring rainfall over the tropics and subtropics. The TRMM satellite will carry a microwave radiometer (TRMM Microwave Imager, or TMI) and the first space borne radar (Precipitation Radar, or PR), among other instruments. These types of instruments infer rainfall amounts by converting measured energies to rainfall rates. However, the conversion from reflectivity and microwave temperature to rain rate suffers from several problems including saturation and limited dynamic range. This motivates the idea of combining instruments to improve observation reliability and precision of rainfall estimates. Because it is generally agreed the PR provides more accurate information, the method outlined in the previous sections is applicable to PR/Radiometer combination. Another PR/Radiometer statistical “combined” algorithm for TRMM—based on a Bayesian approach—has been studied in Haddad et al. (1996).

Our semiparametric approach provides a way for comparing instruments in general and in particular several of the TRMM instruments. Because the temporal and spatial resolutions usually differ among devices, it can be very difficult to compare rainfall quantities obtained from several measurement sources. The present study will hopefully allow researchers to make more meaningful the comparisons of rainfall quantities obtained from various instruments; for example, between radar and rain gauge, or between radar and radiometer.

b. Description of the Data

This section uses space-time collocated radar- and radiometer- derived positive rain rate samples as input to the model described previously. The region

under consideration is that used for the Third Algorithm Intercomparison Project (AIP-3) (Ebert 1995), which used data collected during the Tropical Ocean Global Atmosphere Coupled Ocean-Atmosphere Response Experiment (TOGA COARE) (Webster and Lukas 1992). TOGA COARE took place in the western equatorial Pacific during the period November 1992 - February 1993, and the area under investigation for AIP-3 was within the TOGA COARE region and bounded by 1 deg N - 4 deg S and 153 deg - 158 deg E.

Microwave radiometer data for AIP-3 was collected by the F10 and F11 Defense Meteorological Satellite Program (DMSP) Special Sensor Microwave/Imager (SSM/I) (Hollinger et al. 1990) satellites. The F10 satellite passed over the AIP-3 region at approximately 1100 and 2300 UTC, and the F11 satellite viewed the region at about 0700 and 1900 UTC. The best spatial resolution for the DMSP SSM/I radiometers are for the 85 GHz channel, having a field-of-view diameter of about 12.5 km. For the current study, microwave brightness temperatures measured by the F10 and F11 for the AIP-3 study are converted to 12.5 km resolution surface rain rates using the algorithm of Kummerow et al. (1996).

Radar data during AIP-3 was obtained from two ship borne Doppler radars that were positioned at approximately 2 deg S, 154 deg E and 2 deg S, 156 deg E. The spatial resolution of the radar data is 2 km and the temporal sampling resolution is 10 min. The radar reflectivities measured by the radars is converted to rain rate using an adaptation of the formulation presented by Tokay and Short (1996).

Because the two satellites passed over the AIP-3 region a total of no more than four times per day, there are only about 400 radar- radiometer coincident scenes available for comparison of the rainfall quantities derived by the radar and radiometer. Furthermore, for many of these coincident scenes, no appreciable rain event took place. The particular event chosen for this section is one in which rainfall was present, namely, that occurring on 22 December 1992 at approximately 2221 UTC, for which data are available from the F11 SSM/I, as well as the two TOGA COARE radars within the AIP-3 domain. The radar-derived rainfall rates are spatially averaged to a 12.5 km resolution to match the SSM/I-derived rain rates. This is done by summing all radar rain rates within 12.5 km of the center of an SSM/I pixel and dividing by the total number of radar rain rates contributing to the sum. The resulting data set consists of the quadruplet (lat, lon, R_{sat}, R_{rain}) where lat and lon are the latitude and longitude of the center of the SSM/I pixel, R_{sat} is the SSM/I-derived rain rate, and R_{rain} is the

radar-derived rain rate. After discarding all quadruplets for which Rsat or Rrain was equal to 0, 81 quadruplets remained for which to use as input to the model.

c. Data Analysis

The data consist of $n_0 = 500$ radar- and $n_1 = 700$ radiometer- derived positive rain rate observations, summarized in histogram form in Figure 1-a and 1-b. Hence $\rho = 1.4, n = 1200$. With $h(x) = x$, the maximum likelihood estimates are

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = \begin{pmatrix} -0.5726 \\ 0.5018 \end{pmatrix}$$

with covariance matrix,

$$\text{Var} \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} \doteq \frac{1}{1200} \hat{\Sigma} = \begin{pmatrix} 0.003319 & -0.003163 \\ -0.003163 & 0.003324 \end{pmatrix}$$

For $\hat{\beta} = 0.5018$ with variance of 0.003324 (standard error 0.057654), the PR and radiometer are quite different.

The maximum likelihood estimate of $p_i = dG(t_i)$ is,

$$\hat{p}_i = \frac{1}{500} \frac{1}{1 + 1.4 \exp(-0.5726 + 0.5018t_i)}$$

This gives a mean of $\sum_{i=1}^{1200} t_i \hat{p}_i = 0.7914$, which is the same as \bar{x} since $h(x) = x$.

Figure 1-c, the “combined” histogram, was derived by summing over the \hat{p}_i corresponding to each class,

$$\sum_{i=1}^n \hat{p}_i I(t_i \in C_j), \quad j = 1, \dots, 15,$$

where $I(t_i \in C_j)$ denotes the indicator of the event $(t_i \in C_j)$, and C_j denotes the class.

Evidently, as seen from Figure 1-c, the estimated “combined” distribution is a hybrid of the PR and radiometer empirical distributions. To illustrate the difference between the PR empirical distribution from the radar data only and the “combined” distribution from the combined data—both estimate $g(x)$ —the variance of the first is 3.1713, and of the second 0.8257. *The “combined” distribution has a much smaller variance.* It follows that an approximate “combined” 95% confidence interval for the mean of positive rain rate is

$$\bar{x} \pm 1.96 \sqrt{0.8257/500} = (0.7118, 0.8710)$$

From the PR data alone, the 95% interval is wider,

$$\bar{x} \pm 1.96\sqrt{3.1713/500} = (0.6353, 0.9475)$$

6 Remark Concerning Probability Matching

The so called probability matching method (PMM) is a method that approximates a $Z - R$ relationship by finding a relation between the quantiles of the corresponding probability distributions. Treating Z, R as monotonically related random variables, the method actually connects quantiles corresponding to the same “matched” probability. See Krajewski and Smith (1991) for a critical review of the method, and Rosenfeld et al. (1994) for a recent application.

The semiparametric approach can be applied in the estimation of quantiles of Z (radar reflectivity) as well, for example by combining several radars. This problem has not been studied before, and it is mentioned here only as a possibility for further research.

7 Summary

The problem of combining two instruments was cast in terms of a distribution $g(x)$ and its exponential shift $\exp(\alpha + \beta h(x))g(x)$, where $g(x)$ is unspecified. Combining the instruments means the semiparametric estimation of the common factor $g(x)$ through the corresponding cdf $G(x)$ from data—positive rain rate in our case—obtained from *both devices*—data fusion—a novel feature of this paper. The estimation of α, β , again from the combined data, provides a measure of bias or difference between the instruments. Since the “combined” method uses more data—data from both devices—it provides more precise estimates of $g(x)$ and its mean than can be obtained from a single instrument.

The choice of $h(x)$ depends on the problem at hand, but $h(x) = x$ is a reasonable first guess. For normal (lognormal) samples with equal variances $h(x) = x$ ($h(x) = \log x$), and $h(x) = (x, x^2)$ ($h(x) = (\log x, \log^2 x)$) for unequal variances.

The method makes no assumption about $g(x)$, it is not tied to any geometry and/or coregistration, and it can be easily extended to any finite number of instruments. The extension and its mathematical underpinning will be published elsewhere.

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Table 1: Each row corresponds to a simulation that generated 100 pairs $(\hat{\alpha}_i, \hat{\beta}_i)$. The last column gives $\bar{\alpha} = \sum_{i=1}^{100} \hat{\alpha}_i/100$, $\bar{\beta} = \sum_{i=1}^{100} \hat{\beta}_i/100$.

Instrument	Sample Sizes		Parameters		Estimated Parameters	
	n_0	n_1	α	β	$\bar{\alpha}$	$\bar{\beta}$
$g(x) \sim U(0, 1)$ $g_1(x) \sim U(0, 1)$	200	200	0	0	0.0026 (0.01774)	-0.0047 (0.03555)
$g(x) \sim U(0, 1)$ $g_1(x) \sim U(0, 1)$	500	500	0	0	-0.0098 (0.0106)	0.0196 (0.0211)
$g(x) \sim N(0, 1)$ $g_1(x) \sim N(3, 1)$	200	400	-4.5	3	-4.5566 (0.06303)	3.0495 (0.04288)
$g(x) \sim LN(0, 1)$ $g_1(x) \sim LN(3, 1)$	200	200	-4.5	3	-4.7072 (0.05894)	3.1335 (0.03721)

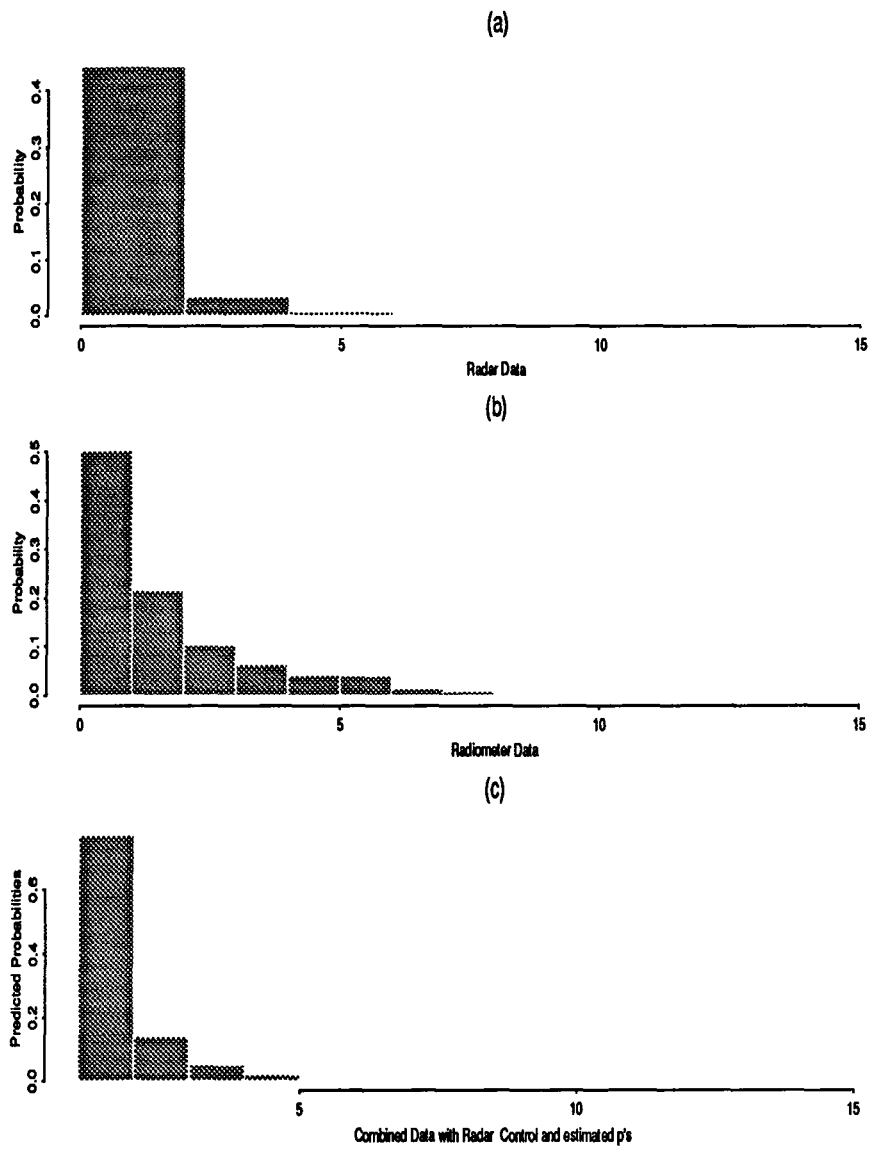


Figure 1: (a) Histogram of the radar data. (b) Histogram of the radiometer data. (c) Histogram from $\hat{p}_i = d\hat{G}(t_i)$, the combined PR/Radiometer estimator.