THE EFFECTS OF AN INTEGRATED LEARNING SEQUENCE ON THE ACQUISITION AND RETENTION OF MATHEMATICS AND SCIENCE BEHAVIORS IN GRADE FIVE

by
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APPROVAL SHEET

Title of Thesis: The Effects of an Integrated Learning Sequence on the Acquisition and Retention of Mathematics and Science Behaviors in Grade Five

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ABSTRACT

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William Lee Gray, Doctor of Philosophy, 1970

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For many years, educators have used the relationship between mathematics and science in the teaching of both subjects. Science examples have been introduced into mathematics programs and often with the intention of facilitating the acquisition of mathematics behaviors. In other cases, mathematics behaviors assumed necessary for the acquisition of certain quantitative science behaviors are taught prior to the presentation of the quantitative science behaviors. There is some support for the notion that teaching the mathematics behaviors assumed necessary for the science behaviors facilitates the acquisition of the science behaviors.

In this experiment, a comparison is made of the effectiveness of two learning sequences in facilitating the acquisition and retention of certain mathematics and science behaviors. In one learning sequence, the related mathematics

and science behaviors are integrated; in the other sequence, they are not. It was hypothesized that the integrated sequence facilitates the acquisition and retention of the mathematics and science behaviors more than the non-integrated sequence.

Three quantitative science behaviors were chosen as the final objectives of the learning sequence. By means of a task analysis procedure, twenty-two objectives were identified as prerequisite for the three terminal objectives. The twenty-five behaviors were then structured in a hierarchy. The three terminal objectives were placed at the top of the hierarchy. The subordinate behaviors were arranged below the terminal objectives in an order suggested by the analysis.

This hierarchy was used as a guide in the construction of the two learning sequences. Each of the twelve lesson sequences was designed to promote the acquisition of the behaviors included in the hierarchy.

A test was constructed which consisted of assessment items designed to test acquisition of each of the mathematics and science behaviors in the hierarchy. This test was administered on two occasions; once, on the day following completion of the learning sequence and, again, nine weeks later.

Nine hundred students in thirty fifth-grade classes in the Baltimore County Public Schools completed all facets of the experiment. The classes were randomly assigned to one of the two sequences.

An analysis of variance procedure was used on the class means to test the acquisition and retention of the mathematics and science behaviors.

The following results were noted:

1. The coefficient of stability for the criterion measure was 0.79.

2. The coefficient of internal consistency was 0.81.

3. The integrated sequence produced a significantly higher overall performance than the non-integrated sequence in acquisition of the mathematical behaviors although there were no significant differences in the effects of the sequence on the rate of forgetting.

4. The two treatments had no differential effects on the overall performance or the rate of forgetting with regard to the science behaviors.

It was concluded that the integrated learning sequence was generally superior to the non-integrated sequence in facilitating acquisition of the mathematical behaviors for the population defined in this study. It could not be established that the two sequences had differential effects on the rate of forgetting of the mathematics or science behaviors.

The results and conclusions suggest that further consideration should be given to the use of integrated learning sequences as an instruction strategy.

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Success in any undertaking of this magnitude can only be achieved with the assistance and cooperation of many people.

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William L. Gray

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CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

Since science was in its infancy, an interdependence between mathematics and science has been exhibited. The demands of science have motivated research in mathematics that has reached increasingly high levels of sophistication. Conversely, many scientific advances have been made because the mathematics necessary to prove certain conjectures theoretically possible has been available for use.

Educators as well as scientists and mathematicians have been aware of the relationship between the two disciplines and have attempted to use this relationship in the teaching of both mathematics and science. Evidence shows that the precedent for teaching certain topics in mathematics through the use of various scientific applications was established over two centuries ago.

In a study of texts used in the United States from 1706 to 1900, Neitz found that a number of the early texts used applied problems and submitted certain tables which dealt with scientific subjects. A procedure found in a number of the texts involved development of a mathematical rule by the analysis and solution of a number of practical problems illustrating the rule. The types of applications used were restricted. Problems related to topics such as weights and measures, astronomy, and navigation.[1]

One of the early twentieth century advocates of change in mathematics education was John Perry. Near the turn of the century, as an influential member of the British Association for the Advancement of Science, he led a movement which promoted the practical or applied approach to mathematics education.

Five basic principles guided the Perry Movement. They included:

1. It is necessary to disregard unnatural barriers between different subjects and parts of subjects.
2. For the great mass of students, utility is of greater importance than philosophical speculation.
3. Mathematics is the language of science and must be taught as such.
4. Much that is laboriously proved true can be assumed and time saved for interesting and original work.

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[1]John A. Neitz, "Evolution of Old Secondary School Arithmetic Books," The Mathematics Teacher, LX (April, 1967), 387-93.

5. Physics and mathematics must be taught together.

Although it is impossible to accurately estimate the influence of Perry on later efforts in mathematics education, a number of his beliefs have recurred during the twentieth century. Whether their reappearance has been due to reference to Perry's work or to original thought by later workers is uncertain.[2]

Relating Mathematics and Physics

The close relationship between mathematics and physics has provided a foundation for a number of studies. It is admitted by several authors that a non-mathematical treatment of physics is possible, but such a treatment is superficial. Most of the literature is concerned with courses in physics which are mathematically oriented.

In the opinion of Robert Carpenter, the most important aim of any physics course, regardless of its rigor, should be to introduce the student to the processes of exact, quantitative thinking with careful observation and analysis of what is observed. For this reason, minimum mathematical competencies were considered necessary for the course. They were identified as two years of algebra and one of geometry.

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[2] Gordon Mock, "The Perry Movement," <u>The Mathematics Teacher</u>, LVI (March, 1963), 130-33.

Carpenter also suggested that the physics course and a trigonometry course be taken concurrently. He charged the mathematics instructor with the responsibility of teaching the prerequisite mathematics, but the physics teacher with the responsibility of applying the mathematics. No mention was made of the mathematics teacher using applied problems as a teaching technique.[3]

Richard A. Parque conducted a three-year study of mathematical deficiencies of senior physics students. By administration of a measuring instrument which tested all of the areas of mathematics contained in the scope of the course, deficiencies were determined. On each of the fifteen areas of mathematics considered by Parque, any student who answered less than sixty per cent of the questions correctly was considered to be deficient in the area.

On the pretest, the mean score of all students on six of the areas was deficient. These areas included work with significant digits, calculation of percentage error, proportions, fractional equations, formulas, and trigonometric functions.

After the deficiencies were identified, modifications

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[3]Robert E. Carpenter, "How Much Mathematics Should Be Required and Used in High School Physics?" School Science and Mathematics, LXII (May, 1962), 374-78.

were made in the course to provide remedial instruction in those areas where improvement was needed. It was not specified whether the remedial teaching used applied problems or not.

Upon completion of the course, a posttest was administered and it was found that the students made significant gains in ten of the fifteen areas. The greatest change was shown in the six areas which had previously shown deficiencies. A follow-up study was suggested to determine the reasons for the deficiencies.[4]

Other studies have been done and articles written about the contributions of mathematics to courses in physical science and the use of physics problems in the teaching of mathematics.

Wick was critical of the type of mathematics teaching which he characterized as a series of sterile manipulations. His opinion was based on a study of a number of texts as well as teaching out of five standard texts.

In order to illustrate a teaching approach which utilized physical situations in the teaching of mathematics

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[4] Richard A. Parque, "An Experimental Study to Investigate the Mathematical Needs of Students in Traditional Physics Courses," School Science and Mathematics, LXI (September, 1961), 405-8.

and to provide a closer coordination between mathematics and science, Wick devised a unit two weeks in length which he called "Physical Mathematics." In this unit, several principles of plane geometry which students had learned during the year were applied in actual situations. Each problem was attacked from a physical point of view.

Unfortunately, no performance results were available from this unit. In a questionnaire, students said that they had enjoyed the unit more than any other that they had had during the school year and the instructor stated that he could not help but feel that the students had learned more under the real conditions than they could have been taught under more traditional conditions. Both of these observations are, of course, subjective. The apparent enthusiasm for this format does suggest that further work in this area is worthy of consideration.[5]

At the college level, Hannon analyzed the mathematics needed for a physical science course. He made the recommendation that the required mathematics be taught during the first three weeks of the course. He did not recommend teaching these mathematical ideas as they were needed in the course.

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[5] John W. Wick, "Physical Mathematics," School Science and Mathematics, LXIII (November, 1963), 619-22.

From the study, Hannon concluded that about sixty per cent of the exercises encountered by the students in the physical science course required computation or the making of generalizations involving mathematics. Additionally, he stated that a large number of mathematical skills are not needed, with most of the skills available from a course in high school algebra if additional emphasis was placed on direct and indirect variation, functions and their graphs, interpretation of data, and ability to generalize. Finally, and most interesting, he made the statement that the understanding of physical principles requires mathematical maturity which can only be gained by presenting a mathematical principle in a variety of situations. No empirical evidence was offered to support these conclusions.

Based on these conclusions Hannon recommended that students be required to show a proficiency in the necessary mathematical skills, be provided with a course in basic mathematics if they fail to meet the standards, and given a booklet in which the mathematical principles are carefully developed with specific examples applying to the physical sciences.[6]

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[6]Herbert Hannon, "An Analysis of the Mathematical Concepts Necessary for the College Physical Science Course," Science Education, Vol. XLIII, No. 1 (February, 1959), pp. 51-55.

Relating Mathematics to Sciences Other Than Physics

Overmire identified four ways in which mathematics could be stressed in high school biology. They included quantitative measurement, presentation of information, estimation, and validation of information. He also presented a number of activities that were designed to give students practice in each of the areas.

In each of these activities the students were expected to use certain mathematical principles in the solution of some scientific investigation. Some of the mathematical principles were ones with which the students were already familiar while others were new to the students and were developed as needed in the biological activity.[7]

Within the past decade, extensive curriculum development has taken place in both mathematics and science. Some of the curriculum projects have been national in scope while others were confined to the local level. Cain and Lee noted that little or no effort had been made to coordinate content and sequence among the different programs in mathematics and science. For this reason, they examined the mathematical content of recent secondary science programs. An analysis

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[7]Thomas C. Overmire, "Mathematics in High School Biology," School Science and Mathematics, LXI (October, 1961), 540-46.

was also made of the scientific applications that have been included in the traditional and recently devised secondary mathematics programs. Specifically, the problem under consideration was whether the newly developed science and mathematics curricula improved the correlation between the mathematical content and sequence of secondary science courses and the content and sequence of secondary mathematics courses.

The materials representative of the new science programs were from the areas of biology, physics, and chemistry. They included materials from the Biological Sciences Curriculum Study, the Chemical Bond Approach Committee, and the Physical Science Study Committee.

The materials representative of the new mathematics programs were from several sources. Included were an outline of the School Mathematics Study Group materials, the report of the Commission on Mathematics of the College Entrance Examination Board, and an outline of materials produced by the University of Illinois Committee on School Mathematics.

Cain and Lee analyzed the programs and used the data from this analysis to compute "coordination ratios." The "coordination ratio" was defined as the ratio of the number of positive correlation symbols in any mathematics-science program combination to the total number of symbols present in that combination. The positive correlation symbols, "b" and "s," referred to situations in which a mathematical concept was taught in the mathematics in some year prior to when it is used in the science. The negative correlation symbols, "a," "x," and "q," refer respectively to situations in which the mathematics concept is taught in the mathematics program after it is used in the science course, is not used in a science course after it is taught in the mathematics course, or if it is not taught in a mathematics course when it is used in a science course.

From the analysis of the program and the "coordination ratios," Cain and Lee made the following observations:

1. The large number of positive correlation symbols between selected mathematics topics and the BSCS, when compared with the number associated with the traditional biology program indicated that many more mathematical behaviors were needed in the new biology program.
2. The number of mathematical behaviors identified as necessary for the CBA and the PSSC was approximately the same as the number identified for the "traditional" chemistry and physics courses.
3. The large number of "x" correlation symbols identified by the analysis indicate that many

of the mathematical concepts taught in both traditional and modern courses which were apparently useful in science courses are not used in the science courses.

4. There are mathematical concepts and processes used in science programs which were not taught in the mathematics programs.

5. Much of the content that has been added to the modern mathematics programs is not directly applicable to the science programs.

6. There is an increase in the coordination between the new programs in mathematics and science as compared to the traditional courses in the two subject areas. The increase, however, was small.

It was not the intention of the authors to suggest that the mathematics or science courses should be built to meet the needs of the other. The suggestion is advanced, however, that improved coordination between the subjects should be included as an objective if neither of the subjects is weakened individually.[8]

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[8]Ralph W. Cain and Eugene C. Lee, "An Analysis of the Relationship Between Science and Mathematics at the Secondary School Level," _School Science and Mathematics_, LXV (December, 1965), 705-13.

If the results of the study by Cain and Lee are indicative of the present relationship between mathematics and science as now taught in our secondary schools, it becomes questionable whether any conscious effort has been made to effect a coordination between the two subjects.

In 1912, Joseph Collins criticized the lack of significant change in the teaching of mathematics and science with regard to relating the subjects. An advocate of the Perry Movement, Collins wrote that there had been a great deal of talk but very little action.[9]

In the succeeding half century changes have occurred within the individual disciplines rather than between them. There has been a continuing interest in teaching the two subjects by some related methods.

Types of Programs Used for Relating Mathematics and Science

The types of programs most often considered in relating mathematics and science are the integrated and correlated programs. Each of the two types of programs possess specific advantages and disadvantages.

An integrated curriculum is, by its nature, a very

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[9] Joseph V. Collins, "The Perry Idea in the Mathematical Curriculum," School Science and Mathematics, XII (April, 1912), 296.

ambitious undertaking. In some cases the program is not restricted to integrating only mathematics and science, but may include other subjects as well. Even with a restriction to mathematics and science, the integrated curriculum is very difficult to implement, particularly at the secondary school level. This is due to the difficulty of finding teachers who have sufficient background to teach elements of both subjects.

In spite of the difficulties of implementing an integrated curriculum, the potential advantages have attracted curriculum workers. One of the primary arguments advanced for integration is that this type of teaching prepares students to adjust themselves to life situations. Charlesworth stressed this consideration, stating that, at the time of his writing, students were leaving high school and college poorly equipped to meet life situations in a rapidly changing world. Charlesworth wrote his article in 1935. Since that time the rate of change that was of so much concern has increased many-fold and the level of complexity of the life of the average citizen has also increased. It is not unreasonable to assume that these changes would have heightened Charlesworth's enthusiasm for the integrated curriculum.[10]

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[10] H. W. Charlesworth, "Mathematics in the Integrated Curriculum," _School Science and Mathematics_, XXXV (June, 1935), 622-26.

Josephs and Brown defined other advantages for the integration of mathematics and science in the same year. They expected an improvement in teaching efficiency due to the more sympathetic attitude of all of the involved teachers toward problems in both fields. They also hypothesized that integration would bring about a saving in the pupils' time and efforts and a broader outlook by the students of both fields.[11]

A number of writers at this time seemed to look upon the integrated curriculum as a panacea for many of education's ills. They were lavish in their praise of this approach although no objective proof was offered to show that the integrated approach was better than the traditional approach.

In his writing Charlesworth had had no doubt that integrated courses would become the rule rather than the exception in years to come. At this time, however, very few successful integrated courses have been reported. Of those reported, a lack of objective evidence is still a problem.

One study which made an attempt to report objective findings was done by Frank H. Gorman. Although the study

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[11]Roswell C. Josephs and F. Martin Brown, "Experiment in Junior High Mathematics and the Sciences," Progressive Education, VII (February, 1930), 16-18.

has technical weaknesses in pupil selection and the definition of the teacher variable, an appraisal was made of its limitations and an interpretation made of the results based on the limitations.

The purpose of Gorman's study was to compare the effectiveness of an integrated plan of teaching seventh and eighth grade science and mathematics with the traditional plan of teaching them separately.

In grade eight, forty-four pupils were divided into two matched groups. The experimental group, which was taught by the author, received the integrated program. Control groups were taught by another instructor who presented the "same" science and mathematics content independently of each other.

Gorman found no significant differences between the levels of achievement of the two groups. He did make two other observations which are of interest. The results seemed to indicate that five periods a week, each of one hour duration, were possibly sufficient for normal progress in seventh and eighth grade science and mathematics under conditions similar to those present in the study. This opinion was based on the fact that the experimental group had averaged approximately a year's growth in mathematics and had completed all of the science units. Although the statement was made with a number of qualifications, it is interesting to consider that the author is stating that five periods may suffice to teach essentially the same material that is now taught in ten periods. Secondly, the author stated that it appeared possible to integrate the topics usually included in seventh and eighth grade science with the topics included in mathematics offered during the same two years. He based this conjecture upon his experience in integrating these programs during the course of the experiment. The preceding observations are in accord with the types of advantages which have been predicted for integrated curricula. Further experimentation was indicated by the results.[12]

Discussion of Studies and Articles Relating Mathematics and Science

The preceding studies and articles are representative of the work that has been attempted or considered in relating mathematics and science. Based on these studies, the following observations were made:

1. Most of the programs attempted were local in nature and of very modest proportion.

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[12] Frank H. Gorman, "An Experiment in Integrating Seventh and Eighth Grade Science and Mathematics," Science Education, XXVII (December, 1943), 130-34.

2. Difficulty was exhibited in evaluating the programs devised.
3. Most of the studies attempted were at the secondary school level.
4. Some of the studies examined ways to provide the mathematics necessary to complete certain topics in science while others used science exercises to aid in learning mathematics.
5. Only the integrated programs sought to improve the learning process in mathematics and science by associating each with the other.

The lack of extensive programs designed to teach mathematics and science as related subjects may be due to a number of factors. One of the primary difficulties is the lack of teachers equipped to teach elements of both subjects if an integrated approach is used. A second problem may arise from a reluctance of the teachers of both disciplines to cooperate fully with each other, thus preventing the close cooperation necessary to successful correlation. A third factor is the reluctance of curriculum workers or administrators to implement new or extensive programs without objective evidence which proves that the new programs will facilitate learning more than a traditional program.

The problem of finding teachers with the background necessary to teach elements of both subjects in an integrated program is particularly acute at the secondary level. This is due to the increased complexity of the topics considered. In addition, the interdepartmental cooperation necessary to implement a correlated program may be lacking at the secondary level due to clearly defined departmental structure. It is, therefore, sensible to consider the elementary school as a place where methods of relating mathematics and science can be explored. The advantages include the predominance of self-contained classrooms in which the same teacher teaches both mathematics and science and the relative simplicity of the mathematics and science taught at this level. Departmental considerations are eliminated by the self-contained classrooms and the topics considered are at a level at which it is reasonable to assume that the individual teacher has the background necessary to teach both subjects.

Evaluation of Learning Sequences

Regardless of the level for which a mathematics-science sequence is designed, its effectiveness must be compared with programs already in use and with other experimental programs. The objectivity of the measurement of the relative effectiveness of the programs is of primary importance.

In evaluating the results of certain studies already mentioned, terms such as "seemed to do better," "seemed to enjoy it more," and "understood it better" were used to indicate that a given program, usually the one in question, was better than the other. Such statements are ambiguous in their meaning and cannot be accepted without objective evidence.

When evaluating a program, a reasonable approach is to consider the objectives of the program and to compare how well these objectives have been met by the given program as compared to other programs with similar objectives. Much of the difficulty associated with the acquisition of objective evidence about the relative effectiveness of various programs can be attributed to poorly defined objectives. Gagné has stated that one cannot draw valid conclusions about differing methods of instruction unless there is an experimental way of controlling content. Since Gagné defines content as descriptions of the expected capabilities of students in specified domains of human activity, the need for well-defined objectives becomes obvious.[13]

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[13] Robert M. Gagné, "Curriculum Research and the Promotion of Learning," AERA Monograph Series on Curriculum Evaluation (Chicago: Rand-McNally and Company, 1967), p. 36.

If the objectives are defined in vague terms such as "the student has a knowledge of," "the student understands," or "the student appreciates," then determining whether these objectives have been met is extremely difficult. Knowing, understanding, and appreciating are global words used to describe many behaviors. In this sense they are ambiguous. This problem would be eliminated if each objective is defined in terms of an observable behavior. An evaluation can then be made of the various programs under consideration by testing the effectiveness with which these behaviors have been acquired.

Construction of learning hierarchies. Using a behavioral approach necessitates the identification of a substantial number of objectives which must be ordered in a logical manner. Gagné has suggested that the behaviors be structured in a hierarchy in which the objectives are defined as behaviors and placed at the top of the hierarchy. Tasks that the learner must be able to do prior to completing the final task are identified. They are placed below the final task in the hierarchy. Tasks which are judged to be approximately equal in complexity are placed at the same level of the hierarchy. The lowest level of the hierarchy contains only those behaviors that the learner is assumed to have acquired prior to the construction of the hierarchy.

Construction of a hierarchy of supporting behaviors necessary for completion of a final task is done by a task analysis procedure. The procedure is begun by asking the question, "What would an individual have to be able to do in order to achieve this behavior, assuming he were given only instructions?" This question will suggest one or more subordinate behaviors that must be acquired. By asking the question successively, the various levels of behavior can be identified until a level is reached where it can safely be assumed that the learner has acquired the behaviors prior to the instructional sequence. As the analysis procedure proceeds, behaviors are defined that are increasingly simple and increasingly general in nature in the sense that they are potentially able to support greater numbers of higher level behaviors or "learning sets."

Gagné and Paradise used the task analysis procedure described above to construct a hierarchy based on the final task "solving linear algebraic equations." They defined a hierarchy of twenty-two learning sets or tasks and three additional basic tasks which were placed at the lowest level of the hierarchy. One of the hypotheses of this study was that the ability to perform a final task defined in the study was dependent upon the successful completion of the relevant subordinate learning sets or behaviors in the hierarchy. Theoretically, each of the subordinate tasks has the function of mediating positive transfer to higher level learning sets throughout the hierarchy.

Validation of learning hierarchies. The prediction of high positive transfer from a given learning set to one at a higher level was tested by noting the pattern of pass and fail which was obtained between lower and higher adjacent sets throughout the hierarchy. If a given task has two or more learning sets subordinate to it, then the theory requires that all of the subordinate learning sets be passed before completion of the given task is possible.

The four possible empirical relationships for passing and failing relevant higher-lower learning set combinations and their implications are:

1. (Higher +, Lower +). This indicates successful accomplishment of a relevant subordinate task and positive transfer to an adjacent higher learning set.

2. (Higher -, Lower -). This indicates failure to perform subordinate tasks and failure to perform an adjacent higher task.

3. (Higher +, Lower -). This indicates successful completion of a higher level task after failure on the relevant subordinate tasks.

4. (Higher -, Lower +). This indicates inability to perform a higher level task after completion of all relevant subordinate tasks.

These four relationships will be expressed symbolically as (+,+), (-,-), (+,-), (-,+). Of the four relationships, only (+,-), which indicates success in a given task after failure in subordinate relevant behaviors, contradicts the theory. The ratio of pass-fail patterns which support the theory is obtained by dividing the number of instances consistent with the hypothesis of positive transfer, [(+,+), (-,-)] by the total testable instances, that is, [(+,+), (-,-), (+,-)].

The theoretical prediction for each ratio computed by the preceding formula for each pass-fail relationship is 1.00.

This theory was tested as part of an experiment conducted with four seventh grade mathematics classes. A total of 118 students completed the eleven-day program which consisted of one day of pretesting, eight days of programmed learning and two additional days of testing at the end. On the last day, questions on the test referred to each of the tasks listed in the hierarchy.

The experimenter conducted each of the sessions although the classroom teacher remained in the room during the period and gave students work to do on another topic after they had completed each day's work.

On the basis of the test administered, the ratio of pass-fail patterns was calculated and no ratio was found to be less than 0.91. Since the theoretical prediction was 1.00 and the values were well above the level of chance and close to 1.00, this result was considered evidence of positive transfer.

In addition to the strong support for positive transfer, a second observation was made by the authors; testing the acquisition of behaviors by an individual by his ability to complete specific tasks in the hierarchy makes it possible to assess in a relatively exact fashion what the individual has learned or has not learned from a learning program. This ability is of great value in helping to diagnose student difficulties.[14]

In a subsequent study conducted by Gagné, Mayor, Garstens, and Paradise, the theory that positive transfer takes place between subordinate relevant tasks in a hierarchy and a given task was again examined. The subject matter in this study was based on addition of integers as taught in Book Seven of the University of Maryland Mathematics Project texts. Additionally, the study was designed to test the effect of variance in repetition of the learning tasks and variance in the integration guidance given. The

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[14] Robert M. Gagné and Noel E. Paradise, "Abilities and Learning Sets in Knowledge Acquisition," Psychological Monographs, LXXVII (1961), 23 pp.

integration guidance was the aid given to a student in the process of attempting a given task after all subordinate tasks had been completed.

A learning program of 126 frames was developed from a chapter outline in the text and a hierarchy of twelve learning sets was derived from the same source. The learning program was then revised in four specific ways to provide high and low amounts of repetition and guidance. The four resulting forms were: Low Repetition, Low Guidance; Low Repetition, High Guidance; High Repetition, Low Guidance; and High Repetition, High Guidance.

Three posttests were given to test learning set achievement, performance, and transfer.

The subjects included 136 seventh grade pupils in four classes. Grouping in the classes was heterogeneous. Each class was divided into a high and low ability group and within each of the half classes, students were assigned randomly to one of the four treatments.

Each subject received a booklet coded to the particular experimental condition to which he was assigned and was always given the same booklet. Students used these booklets over a four-day period. On the next three days the tests were administered.

As in the previous study, ratios of positive transfer were computed and, since no ratio was less than 0.97 compared to the theoretical prediction of 1.00, it was concluded that positive transfer had taken place.

It was found that the experimental variables introduced had no significant effects when tested singly. It was found, however, that the students taking the high repetition, high guidance program did significantly better than those students who took the low repetition, low guidance program.[15]

In a third study conducted by Gagné in conjunction with the staff of the University of Maryland Mathematics Project, further support was found for the theory that positive transfer occurs between subordinate related tasks and a given task.

A learning sequence dealing with certain topics in non-metric geometry was developed which was based on a learning hierarchy consisting of the final tasks and nineteen subordinate items arranged in six levels of complexity.

As in the previous study, the authors wished to show that the attainment of any given learning set in the hierarchy was dependent upon positive transfer from subordinate related behaviors. The two variants in this study were the examples

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[15] Robert M. Gagné, John R. Mayor, Helen Garstens, and Noel E. Paradise, "Factors in Acquiring Knowledge of a Mathematical Task," <u>Psychological Monographs</u>, LXXV (1962), 21 pp.

used to provide repetition of each subordinate task and the amount of time elapsing between the attainment of one task and the next.

Five experimental learning sequences were constructed using the hierarchy as a model. Program E1 had a minimal variety in its problem example; Program E2, an intermediate variety of problem examples; and Program E3 had a maximal variety. Program E0 contained no problem examples except for an initial one required in the completion of each subordinate task. Program EA was the same as E0, but had sets of arithmetic problems inserted so that the program would take the same amount of time.

A total of 116 students in four sixth grades participated in the study. They were randomly assigned to one of the five groups. Four of the programs lasted for eleven days. The fifth, E0, was started three days after the other programs since it ran for a total of eight days.

Two performance tests were administered on the two days following the learning sequence. The test given on the first day consisted of a variety of examples which represented performances associated with the final tasks. The second test tested the acquisition of behaviors subordinate to the final tasks.

An examination of the results showed that the variants had no significant effect on learning. As in the previous studies, strong evidence of positive transfer from relevant subordinate learning sets to higher level sets was exhibited. The range in ratios of positive transfer was from 0.95 to 1.00.

Although the variants of task example variety and elapsed time between task learning revealed no significant differences, it was thought worthwhile to extend the original study by measuring the retention of knowledge in these same students after a time lapse in excess of two months.[16]

Nine weeks after the completion of the preceding study, retention tests were administered to eighty of the students who participated in the original study. Other students were eliminated due to absences and other factors. Two days were used to administer the tests. On the first day, the students were given an achievement test identical to that given immediately following the learning sequence. Measurement was made not only of retention of the final task, but also of the subordinate learning sets. Retention of the subordinate behaviors was tested on the second day.

The retention of ability to do the final task was

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[16] Robert M. Gagné, et al., "Some Factors in Learning Non-Metric Geometry" (unpublished University of Maryland Mathematics Project, 1963), 11 pp.

extremely high for all treatments other than E1. The authors implied from this that the type of material included in the learning program, when learned by means of a carefully constructed instructional program, is highly resistant to forgetting.

Loss of ability to perform the subordinate learning sets, however, was also in evidence. The results left the authors to conclude that, although the subordinate learning sets are of great importance to initial achievement of the final task, their importance decreases after the final task is achieved. In general, the pattern of forgetting subordinate learning sets was found to be irregular when compared with the patterns exhibited immediately after learning had taken place.

Although the forgetting of the subordinate task was not considered important by the authors in relation to the acquisition of the final task, it was stated that these subordinate behaviors would have to be relearned prior to using them as subordinate tasks for a new task. If this is not done, then, as evidenced by the prior studies, learning cannot occur.[17]

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[17] Robert M. Gagné and Otto C. Bassler, "Study of Retention of Some Topics of Elementary Non-Metric Geometry," Journal of Educational Psychology, LIV (June, 1963), 121-31.

Summary of observations on use of learning hierarchies. On the basis of the investigations conducted by Gagné and his associates, the following observations can be made:

1. Strong evidence supports the notion that achievement of each task in a hierarchy is dependent upon prior mastery of relevant subordinate learning sets.
2. A skillfully devised hierarchy is quite helpful in the construction of a logical learning sequence.
3. Evaluation of learning can be efficiently completed through the use of objectives which are stated in behavioral terms.
4. The use of behavioral hierarchies in identifying learning difficulties can be an effective technique in the learning process.
5. Final tasks which are learned by means of a learning sequence based on a hierarchy are highly resistant to forgetting.

The advantages of using behavioral hierarchies and behavioral objectives have received increasing attention from curriculum workers. The most ambitious project to date which has made a total commitment to the behavioral position is that of the American Association for the Advancement of Science Commission on Science Education. The Commission's science project, Science--A Process Approach, has been undertaken with the focus on the creation of materials which assist the learner in the acquisition of a particular collection of behaviors rather than a particular body of content. The instruction in the primary grades emphasizes the development of skills which are basic to further learning. Skills included are Observing, Recognizing and Using Number Relations, Measuring, Recognizing and Using Space/Time Relations, Classifying, Communicating, Inferring, and Predicting.

These skills are refined and reinforced by the exercises designed for the intermediate grades. The exercises are more complex, more comprehensive, and more nearly like the activities engaged in by scientists. Included in the activities are formulating hypotheses, making operational definitions, controlling variables, experimenting, formulating models, and interpreting data. Based on a hierarchy of behaviors, the instructional materials are arranged as an orderly progression of learning experiences, with all objectives clearly specified. Appraisal activities accompany each of the learning experiences. One of the strongest features of the program is the emphasis on appraisal of the students' efficiency in acquiring the behaviors that are stated as objectives. By these means, constant evaluation can be made of the learning sequences.[18]

Use of action words in defining objectives behaviorally. Walbesser has stated that the existence of each curriculum project is dependent upon being able to demonstrate that it accomplishes something. It is not important whether the accomplishment is content assimilation or performance acquisition, but rather the recognition and acceptance of the principle that every curriculum project has the honest and inescapable obligation to supply objective evidence of accomplishment.[19]

In order to evaluate a learning program as efficiently as possible, it is necessary to eliminate sources of ambiguity. It is obvious that goals such as "showing appreciation," or "increasing understanding" are unacceptable from this standpoint. Even if the objectives are defined behaviorally, their meaning may still be unclear unless precise definitions are made of the verbs used in stating the objectives. It is also advantageous to limit the number

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[18] Commission on Science Education, Science--A Process Approach (third experimental edition; Washington: American Association for the Advancement of Science, 1965), Parts 1-7.

[19] Henry H. Walbesser, "Science Curriculum Evaluation: Observations on a Position," The Science Teacher, XXXIII (February, 1966), 34-39.

of classifications of behavior which must be examined. As the number of classifications of behavior that are used increases, the chances of ambiguity correspondingly increase since additional definitions must be made.

Some examples of basic verbs or "action words," which are as unambiguous as possible and which may be used to describe most of the behaviors encountered in any mathematics or science learning sequence are:

1. _Identify_, _recognize_, and _distinguish_ which mean point to, choose, or pick out.
2. _Name_ or _state_ which implies that a verbal statement is needed.
3. _Describe_ which implies a verbal statement.
4. _Order_, or _place in sequence_.
5. _Construct_, _print_, or _draw_.
6. _Demonstrate_.

Construction of assessment items. Behavioral objectives stated in terms of the action verbs can, therefore, make contributions not only in the construction of learning sequences, but also in the efficient evaluation of these sequences. In order to carry out this evaluation, performance assessments, or competency measures as they are called by the AAAS Commission on Science Education, must meet the following criteria:

1. At least one task should be constructed to evaluate each behavior in the hierarchy.
2. Tasks should be designed to elicit behaviors of the sort described in the objectives.
3. The description accompanying each task should tell the instructor what to do.
4. Acceptable performance should be clearly described so that a correct judgment can be made concerning the presence or absence of a given behavior.[20]

The preceding review has dealt with two major areas of concentration. One part of the review considered studies in which mathematics and science are related in one of a variety of ways in a learning sequence. The other area concerned the use of behavioral objectives and behavioral hierarchies in the construction and evaluation of learning sequences.

The Kolb Study. One study has been completed which encompasses elements of both of the areas of concentration previously mentioned. This study by Kolb was conducted at the fifth grade level. The purpose of the investigation was to determine experimentally if an instructional sequence in mathematics based upon a hierarchy of mathematical tasks

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[20] Walbesser, *op. cit.*, p. 37.

assumed necessary for quantitative science behaviors would facilitate the acquisition of the quantitative science behaviors more than an instructional sequence in mathematics not directly related to the science behaviors.

Using the exercises from <u>Science--A Process Approach</u> as a pattern, two science exercises were devised. Among the objectives stated were three quantitative tasks.

By means of the task analysis procedure previously described, a hierarchy was constructed which included three final tasks and twenty-six relevant subordinate mathematical tasks. This hierarchy was used as a guide for developing the instructional sequence in mathematics which was related to the science exercises.

Two tests were constructed. Measure I was designed to test the acquisition of each of the twenty-six items in the mathematics hierarchy and Measure II assessed the acquisition of the behaviors stated as objectives in the two science exercises. These two tests were used both as pretests and posttests.

The time allotment consisted of a two-day testing period prior to the learning sequence, a fourteen-day learning sequence in mathematics, one day to administer Measure I, a six-day instruction period in science followed by the administration of Measure II.

Subjects for the experiment were 275 pupils enrolled in eight fifth grade classes. Students within each class were randomly assigned to one of two treatments. During the learning sequence, those students assigned to treatment A were taught the mathematics that occurred in their books at the time. The students in treatment B, however, were taught the learning sequence that was based on the mathematics hierarchy. The final tasks included in the hierarchy were considered necessary for the quantitative science behaviors. Each teacher taught both of the treatments within her class by meeting with each group on alternate days.

On the basis of the tests administered, Kolb found no significant differences existed between the two groups on the pretests. Significant differences in favor of group B at the 0.01 level of confidence were found on the posttests.

Kolb concluded that the instructional sequence in mathematics related to the science exercises facilitated the acquisition of the quantitative science behaviors for the experimental materials and population used in the study.

As in previous studies involving learning hierarchies, a ratio of positive transfer between relevant lower level tasks and a higher level task was computed using the previously mentioned formula. For this study, it was arbitrarily decided that these ratios should attain or exceed a value of 0.90 in order to conclude that the hierarchy was valid. Seven of the ratios failed to attain a value of 0.90 with one ratio as low as 0.38. The validity of the hierarchy was rejected.

In spite of the problem involving validation of the hierarchy, it was found that those students who received mathematics instruction based on the hierarchy did significantly better on the science posttest. Kolb, therefore, took the position that the task analysis procedure used to define the hierarchy should not be classified as inadequate. Instead, he pointed out that a behavioral hierarchy is derived by an a priori procedure. This procedure requires the experimenter to be guided by knowledge of the tasks under consideration and past experience with hierarchy construction. A problem can occur, however, if the experimenter fails to effectively identify and properly order all of the relevant subordinate tasks due to inexperience. This could produce a hierarchy in which certain higher level performances would be moderately dependent or independent of the supposedly relevant lower level performances. The invalid hierarchy could, therefore, be the result of ineffective utilization of the task analysis procedure.[21]

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[21] John R. Kolb, "The Contributions of an Instructional Sequence in Mathematics Related to Quantitative Science

One of the primary considerations of any study which deals with behaviors structured in a hierarchy must be the validation of that hierarchy. Of the studies examined, only one aspect of the hierarchy validation has been considered, namely, the ratio of positive transfer between lower relevant tasks and a higher level task. This ratio has been previously described by Gagné as being the number of instances which are consistent with positive transfer [(+,+), (-,-)] divided by the total testable instances [(+,+), (-,-), (+,-)].[22]

Walbesser, however, suggests a more rigorous examination of hierarchy validation than that previously attempted. The greater rigor extends not only to the derivation of the ratio, which determines the level of positive transfer from lower relevant learning sets to higher level behaviors, but also examines two other considerations not previously treated in hierarchy validation.

In previous studies, the numerator of the ratio which was a measure of positive transfer consisted of the sum of those two relationships in the learning hierarchy which are

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Exercises in Grade Five" (unpublished Doctoral dissertation, University of Maryland, 1967), 160 pp.

[22] Gagné and Paradise, loc. cit.

consistent with the theory, namely (+,+), (-,-). For this reason, the ratio is called the ratio of consistency.

The Walbesser model for hierarchy validation. Walbesser suggests that the numerator of the ratio measuring consistency contain only the total instances which support positive transfer, that is, (+,+). While it can be said that (-,-) is consistent with the theory, it is certainly not an indication of positive transfer. The inadmissability of the instances of (-,-) has the effect of lowering the ratio achieved. If the ratio agreed upon as acceptable is 0.90, then the number of supportive instances must be greater to attain this level than if the ratio could include instances that were merely consistent with the theory.

This increase in the level of rigor applied to the examination of the consistency of the hierarchy is but the first step. Consistency is considered a necessary, but not sufficient, condition for a valid hierarchy. Consideration must be given to two other factors not treated previously before validation is completed. One of these factors refers to the adequacy of the hierarchy and the other refers to the completeness of the hierarchy.

The adequacy of a hierarchy requires an examination of how often the learner has achieved a higher level learning behavior after relevant subordinate behaviors have been attained. If the instruction is adequate, then a learner who has attained the subordinate behaviors will be able to progress to the higher behavior. The adequacy ratio is defined as the quotient of the number of (+,+) divided by [(+,+), (-,+)]. The level of acceptability for the ratio is 0.90.

High consistency and adequacy ratios are necessary, but not sufficient conditions for claiming validity for the hypothesis of a learning hierarchy. This is true because high consistency and adequacy ratios can be attained which involve only a small number of the actual subjects tested. This occurs when most of the subjects do not acquire the terminal behavior and some of the subordinate behaviors. These cases will fall into the (-,-) category. The author views large numbers of cases in the (-,-) category as evidence of incomplete instruction. The completeness ratio is defined as the quotient of (+,+) divided by [(+,+), (-,-)]. The level of acceptability is 0.90.[23]

The preceding considerations impose on the builder of the hierarchy specific responsibilities. One responsibility involves the accessibility of a given step in the

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[23] Henry H. Walbesser, Science--A Process Approach, An Evaluation Model and Its Application: Second Report (Washington: American Association for the Advancement of Science, 1968), 238 pp.

hierarchy after relevant subordinate steps have been obtained. If many students fail to reach a given higher level upon completion of a lower level, it may be attributed to a weakness in instruction or to the need of an intermediate step between the higher and lower level learning sets. The second responsibility requires that a minimum of instances occur where both the subordinate and higher level behaviors are not attained since this denotes incomplete instruction.

Summary of the Literature

Examination of the literature reveals evidence of a continuing interest in the construction of learning sequences which relate mathematics and science. Some of the studies explored ways in which the learning of mathematics can be aided by the use of scientific applications. Others have investigated the possibility that the learning of scientific principles can be aided by relating appropriate mathematics to the science. In most of the studies, one of the subjects was usually the beneficiary of the other with the second in a subordinate role. The integrated programs, however, tend to equate, at least in theory, the emphasis placed on each of the subjects.

It is also evident from the studies related to learning hierarchies that relevant subordinate learning sets do mediate positive transfer to higher level behaviors. The hierarchy has also been an effective tool in building learning sequences and in evaluating the effectiveness of these sequences.

A Research Proposal

One justification for implementing a new curriculum or learning sequence is whether or not it does a more effective job than that which it is replacing. The evidence of the effectiveness of a program should be obtained by the use of acceptable research procedures which implies the objectivity of the results. Objective evidence of the relative effectiveness of most of the programs reviewed has not been offered.

As each of the articles and studies promoting the relating of mathematics and science is reviewed, one is forced to ask whether the program under consideration is more effective in aiding the learning process than a second study which relates mathematics and science in a different way or a third which teaches by the traditional unrelated method. Unless objective evidence is produced, the question cannot be answered.

It should not be concluded that the only program proposals which receive attention are those which offer objective evidence of their effectiveness. Creative thinkers should be continually encouraged to present proposals which offer alternatives to the conventional.

The research on behavioral hierarchies has also provided a means of making objective comparisons between the relative effectiveness of various programs.

One promising organization for building learning sequences which relate mathematics and science is the integrated approach. As mentioned previously, a limitation of the integrated approach is the academic preparation of the teachers responsible for its presentation. As the level of complexity of the subject matter increases, the number of teachers with sufficient background to teach the sequence decreases. Even with this limitation, the integrated approach may be a more effective method of teaching mathematics and science as related subjects than the other methods considered.

Kolb's study presented evidence that an instructional sequence in mathematics based upon a hierarchy of mathematical tasks assumed necessary for certain quantitative science behaviors would facilitate the acquisition of the quantitative science behaviors more than an instructional sequence in mathematics not directly related to the science behaviors. As one of his suggestions for further research, Kolb suggested the comparison of a learning sequence of the type that he used in his study with a learning sequence in which the mathematics and science are fused or correlated.[24] The following proposal is such an extension of Kolb's study.

The purpose of the proposed study is to compare the effectiveness of two learning sequences in mathematics and science. The first learning sequence is an integrated mathematics and science sequence based on an integrated hierarchy of mathematics and science behaviors. The second learning sequence, designed to promote acquisition and retention of the same behaviors, presents the mathematical behaviors prior to the scientific behaviors with no scientific applications offered during the learning of the mathematical behaviors.

The study will investigate two problems:

1. Will an integrated sequence in mathematics and science facilitate the acquisition and retention of the mathematical behaviors more than a non-integrated sequence?
2. Will an integrated sequence in mathematics and science facilitate the acquisition and retention of the scientific behaviors more than a non-integrated sequence?

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[24] Kolb, _op. cit._, p. 81.

The preceding problems will be investigated by testing the following research hypotheses:

1. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the student in mathematics.

2. Increasing the integration of science and mathematics during instruction decreases the rate at which the mathematical behaviors are forgotten.

3. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the students in science.

4. Increasing the integration of science and mathematics during instruction decreases the rate at which the science behaviors are forgotten.

Summary of the Chapters

Chapter I provided an introduction to the problem and a review of the literature. The design of the experiment and treatment of the data will be discussed in Chapter II. Included will be a description of the population, sample, research hypotheses, measures, and treatments. In Chapter III the findings will be recorded. This will include hierarchy validation and testing of the hypotheses. Chapter IV will include the conclusions and suggestions for subsequent work.

CHAPTER II

THE EXPERIMENT

The purpose of this experiment is to determine whether an integrated mathematics-science learning sequence will facilitate the acquisition and retention of the mathematics and science behaviors involved better than a learning sequence involving the same behaviors in which the mathematics and science are taught separately.

Experimental Subjects

Thirty fifth grade classes from ten elementary schools in Baltimore County, Maryland participated in the experiment. Baltimore County is a county which surrounds Baltimore City on three sides and which encompasses urban as well as suburban areas. The county school system at the time of the experiment had approximately 100 elementary schools and a total student population of about 105,000 students.

One thousand twenty-three students began the experiment. Of these, 123 students were not included in the final data due to absences during the experiment. A total of 900 students in the thirty classes completed the experiment.

Although this sample of classes was not a random sample, the classes represent all levels of ability within the county. The schools involved were located in communities representing various economic levels and geographic areas. The following table identifies the schools that participated by number and indicates the number of classes that each school had in the experiment.

TABLE I

DISTRIBUTION OF CLASSES PARTICIPATING IN THE EXPERIMENT

School	Number of Classes
I	8
II	4
III	3
IV	2
V	2
VI	3
VII	3
VIII	1
IX	2
X	2

With three exceptions, the number of classes participating in the experiment from each of the schools was the total number of fifth grade classes in the school.

Participation on the part of the teachers was voluntary and several teachers did not wish to participate. Three of the teachers who did not participate were new teachers. They had been with their classes less than one month when the learning sequence started. They did not wish to attempt to do the small group activities that were part of the learning sequence. Another teacher felt that the time that would have been spent on these activities was more urgently needed for work in other areas.

All of the classes were taught in self-contained classrooms and each of the classroom teachers taught all of the mathematics and science to her class.

None of the classes had been participating in any experimental mathematics or science program prior to the investigation. In addition, none of the topics covered in the experimental materials had been taught to any of the classes previously.

Prior to the presentation of the experimental sequences, the classes were randomly assigned to one of two treatment groups which were designated Treatment A and Treatment B. A table showing the number of students who completed the learning sequence in each of the experimental classes is found in Appendix A.

Experimental Materials

The basis for the science exercises contained in the experiment was the unit entitled Human Reaction Time which is included in the experimental science materials, Science--A Process Approach, Part VI.[1]

The following final tasks were chosen for the learning sequence:

1. Demonstrate an experiment in which a stimulus is followed by a response with the reaction time measured by indirect means.

2. Construct explanations for reaction time patterns pictured on a graph.

3. Construct graphs involving data accumulated from experimentation.

Mathematical behaviors considered necessary for the acquisition of the quantitative science behaviors were identified. The scientific tasks associated with the acquisition of the final tasks provided scientific applications of the mathematical behaviors.

The Integrated Hierarchy

An integrated hierarchy of mathematics and science behaviors was constructed which included those behaviors which were assumed necessary for completion of the final

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[1] Commission on Science Education, op. cit., Part 6, pp. 167-75.

tasks. The behaviors subordinate to the final tasks were identified by successively asking the question, "What would a student have to be able to do in order to achieve this task?" This procedure was continued until tasks were identified which were assumed to be simple enough to be completed by all of the students involved. This analysis yielded a total of twenty-five tasks which were assumed necessary for, but subordinate to, the acquisition of the final task.

The Pilot Study

Since the validity of the hierarchy is a critical factor in the evaluation of the learning sequences under consideration, an examination of the consistency, adequacy, and completeness of the hierarchy had to be made. Of necessity, the hierarchy was constructed by an a priori analysis procedure. This presents an immediate problem. If the data for the hierarchy validation procedure are gathered concurrently with the data for comparing the two learning sequences under examination and the hierarchy is proved invalid, then the validity of the data comparing the two learning sequences is suspect. In order to eliminate this source of difficulty, a pilot study was conducted.

On the basis of the initial hierarchy constructed, a series of twelve integrated lessons was devised. One fifth grade teacher was asked to teach these twelve lessons to her class. The teacher was also asked to keep a diary during the course of the unit in which comments were made on the length of time that each lesson took, sources of confusion or ambiguity in the lesson plans, deficiencies in the materials of instruction provided, and other general remarks on improvements that could be made in the lessons.

A measuring instrument was also constructed in which at least one item was devised to test acquisition of each of the behaviors found in the hierarchy. The content validity of the items was examined by having several volunteers unfamiliar with the assessment items and the hierarchy components match the items in the testing instrument to the behaviors in the hierarchy. Each of the volunteers had had previous experience in writing behavioral objectives and assessment items. A table showing the results of the matching is found in Appendix B.

After data were collected from the pilot study, the ratios of consistency, adequacy, and completeness were computed using the formulas suggested by Walbesser. The minimum acceptable ratio for any measure of consistency, adequacy, or completeness was established at 0.90.[2]

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[2]Walbesser, Science--A Process Approach, p. 234.

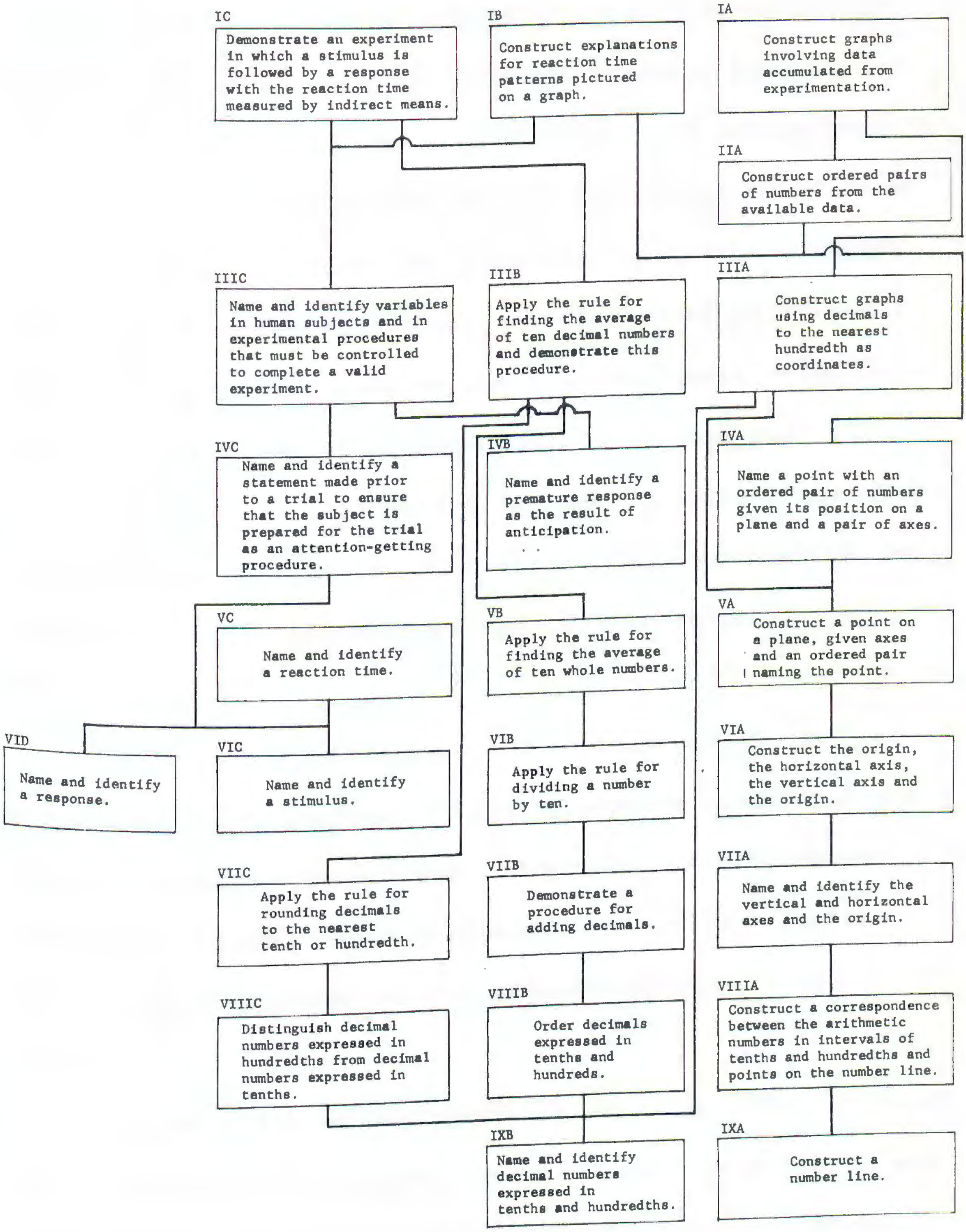

FIGURE 1

THE INTEGRATED HIERARCHY

The ratios computed revealed several areas in the hierarchy where evidence of positive transfer between a given task and those behaviors assumed to be subordinate to it could not be established. In some cases the ratios obtained were well below the minimum agreed upon. Based upon these data, the hierarchy was revised to eliminate relationships established on the a priori basis which showed little evidence of dependence when examined.

The twelve lesson plans for the integrated learning sequence were revised on the basis of the revisions in the hierarchy and the suggestions made by the teacher of the pilot program.

The pilot program served two purposes. First, it made possible the revision of the hierarchy upon which the learning sequences were based. Secondly, it offered an opportunity to identify and correct difficulties in the learning materials and general procedures used in the lessons.

On the basis of the hierarchy obtained from the pilot study, two series of lessons were devised. Each lesson was designed to last between fifty and sixty minutes. Each learning sequence contained twelve lessons and was designated Treatment A or B. Copies of the lesson plans are located in Appendices C and D.

Treatment A involved lessons in which the mathematical behaviors were taught as they were needed for acquisition of the quantitative science behaviors. Scientific applications were used in learning the mathematical behaviors.

All of the mathematical behaviors needed for the quantitative science behaviors included in the integrated hierarchy are taught in the first five lessons of Treatment B. As they are taught, however, no applications to the quantitative science behaviors are made.

Lessons nine through twelve of both Treatments A and B are identical. The design of the investigation planned that differences in learning efficiency between the two treatments could be ascribed to the differing methods of presentation encountered in lessons one through eight in each treatment provided other confounding variables were held constant or randomized.

Some revision was made of the measuring instrument based on the changes made in the hierarchy. At least one assessment item was still provided to test acquisition of each of the tasks in the hierarchy. The same instrument was administered on two different occasions. The only change made was on the cover page. On the first administration it was entitled Measure I; on the second, Measure II. The first administration occurred on the day following the learning sequence. The second administration occurred nine weeks after the first administration.

The teachers involved in teaching the learning sequences were responsible for the administration of the measures. Specific directions were provided for standardizing the administration of the tests. A copy of the instrument and the directions is found in Appendix E.

The Procedure

Before the learning sequence began, the experimenter visited each of the participating schools to meet with the teachers involved for the purposes of orientation. At these meetings each teacher was given all of the materials necessary to teach the twelve lessons and the set of lesson plans designed for the treatment to which the teacher's class was assigned. The teachers were told that two types of learning sequences were being compared, but no mention was made of one treatment being favored over another.

The individual schools or teachers were required to furnish no materials. They were furnished sufficient copies of each of the practice pages written for the lessons in their treatment. Other apparatus necessary for the student experimentation was also provided.

Since eight days were required to visit the various schools, the orientation sessions were started two weeks prior to the day when instruction was scheduled to begin. All teachers were asked to begin the sequence on the same day and to give each of the achievement measures on the same day. This procedure made it possible to eliminate time between administration of the tests as a factor affecting achievement. It was impossible, however, to control the time of day when each lesson was scheduled in each class. By randomly assigning classes to one of the treatments, the bias resulting from classes being given at different times was controlled.

Before the learning sequence ended, the achievement measures were distributed to all schools so that they could be administered on the same day. All copies of Measure I were returned to the experimenter immediately following its administration.

The second achievement measure was distributed to the schools no more than three days prior to the date selected for its administration. They were also returned as soon as they were administered.

Since a number of the assessment items on the measures required special interpretation, the experimenter established specific criteria for identifying acceptable responses for each item. In order to lessen the possibility of inconsistency in the grading of the achievement measures, the experimenter graded all of the measures personally.

Consideration of the Data

Researchers in education may consider several alternatives when deciding on the method which will be employed to compare the effectiveness of the learning sequences.

One of the alternatives involves an analysis of the data obtained from a posttest which is administered immediately after completion of the learning sequences. Usually accompanied by a pretest, this approach gives the researcher an opportunity to measure the learning gains which have taken place as a result of the learning sequences.

While the results obtained from the preceding design are of interest, they offer no clues to the effect of the learning sequence on the retention over time of the behaviors acquired. It can be argued that the number of behaviors retained over a period of time is of greater interest than the number acquired initially. For this reason, a design must be devised which can test the effects of the learning sequence on retention as well as the general effect of the treatments.

To test retention over time, a design is often used which employs an immediate posttest following the learning sequence and one or more additional posttests administered at selected intervals thereafter. Usually, only one additional posttest is administered due to various difficulties encountered in performing further administrations. When the learning task is not present in the behavioral repertoire of the subjects, it is not necessary to administer a pretest.

Types of errors in measurement. The effect of the treatments in facilitating acquisition and retention of the behaviors associated with the learning sequences are made on the basis of the class means achieved on the criterion measures. Not all of the differences in the mean scores, however, can be attributed to the treatment effects. The differences are, in part, due to errors or extraneous factors. The errors are of three types as identified by Lindquist.[3] The errors are associated with subjects, treatment groups, and replications.

"Type S" errors are associated with the subjects. If subjects are randomly assigned to a treatment, that part of the observed treatment effect which is due solely to the assignment of subjects to treatment groups is a "Type S" error.

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[3] Everet F. Lindquist, Design and Analysis of Experiments in Psychology and Education (Boston: Houghton-Mifflin Company, 1953), pp. 8-11.

Even in the absence of "Type S" errors or differences in treatment effects, any single replication of the experiment may exhibit substantial differences between the criterion means. These differences may result from factors which affect a given treatment group, but have no effect on other groups. These "Type G" errors, as they are called by Lindquist, include such factors as difference in teaching ability, the time of day when instruction is given, or a recalcitrant student in the class.

Any variations in treatment effects from replication to replication which are due neither to Type S errors nor Type G errors, but are, instead, genuinely characteristic of the individual replications or sub-populations are called "Type R" errors.

In order to control these errors, a randomization procedure is used. By randomly assigning the classes involved to a given treatment, any systematic bias favoring a given treatment due to a "Type S" error is eliminated. The "Type G" errors are eliminated by randomizing or controlling factors such as differences in the method of administering the tests, ability of the teacher, and time of day when the class is given. A careful examination of all procedures associated with the experiment should be made so that as many sources of "Type G" errors are considered as possible. According to Lindquist, factors that arise during an experiment which cannot be recognized are usually accidental and without bias. The "Type R" errors, characteristic of individual replications, can be controlled by a random selection of replications.

Measuring retention effects. Care must be taken in choosing the proper method of treating the data gathered from the posttests. Wodtke discussed several considerations that should be made in testing the effects of different instructional treatments on retention.[4]

Wodtke proposed that the retention effects of the treatments to be assessed by an examination of the "rate of forgetting." This rate was indicated by curves which could be constructed on the basis of the means computed from the posttests administered. By limiting this discussion to the consideration of a situation involving only two posttests, the curves constructed would be represented as simple line graphs. The graphs of three hypothetical situations that could arise under the preceding restrictions are shown in Figure 2.

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[4]Kenneth H. Wodtke, "On the Assessment of Retention Effects in Educational Experiments," The Journal of Experimental Education, XXXV (Summer, 1967), 28-36.

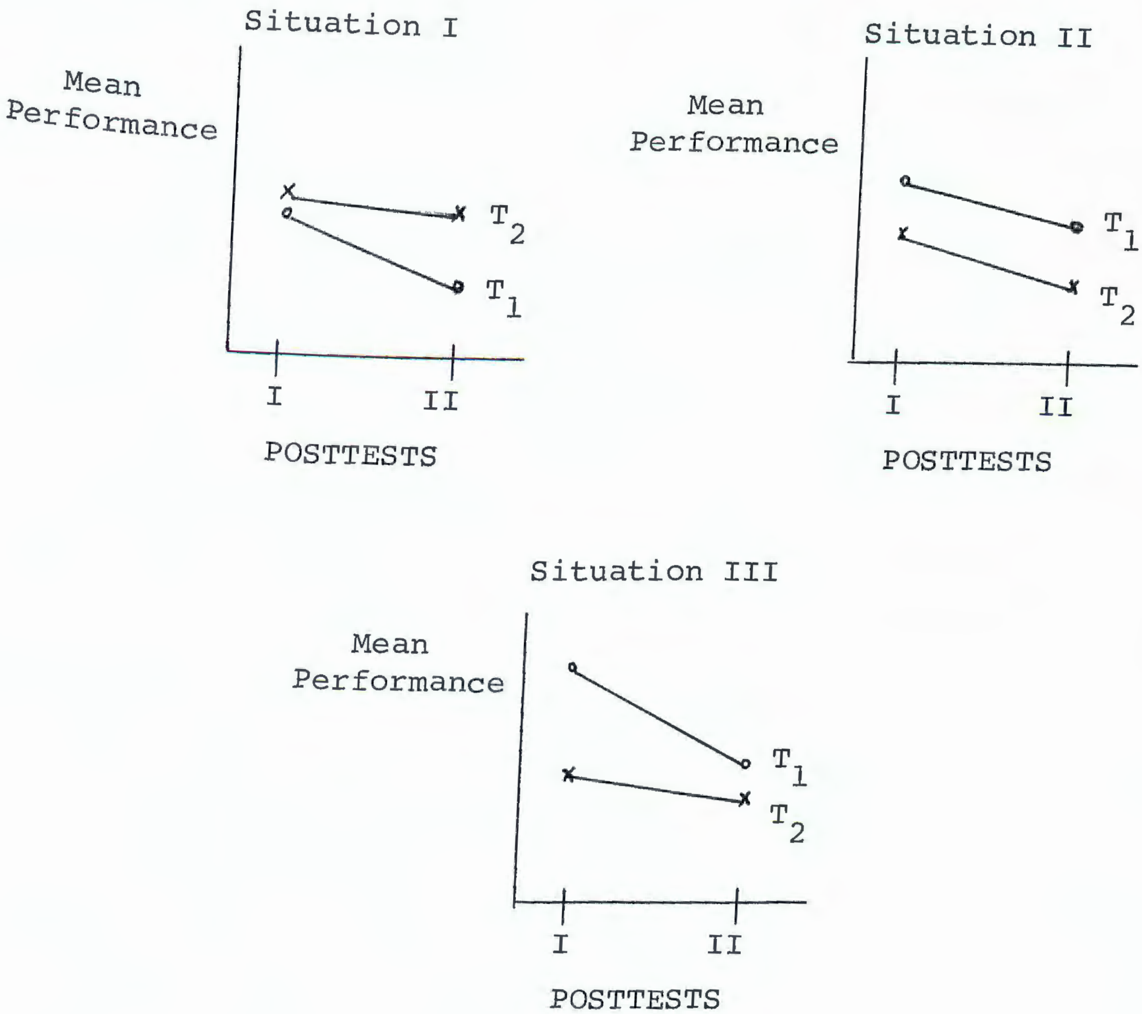

FIGURE 2

A GRAPHIC COMPARISON OF DIFFERENT RATES OF FORGETTING

The first graph illustrates a situation in which no significant difference is detected between the two treatments as measured by the means of the immediate posttest. On the delayed posttest, however, the mean of Treatment 2 is significantly higher than the mean of Treatment 1. In this case, it is possible to conclude that Treatment 2 is more effective than Treatment 1 in facilitating retention over time.

The effect of the treatments on retention is more difficult to assess in Situation II. This is caused by the fact that the mean score indicated for Treatment 1 is significantly higher than the mean score of Treatment 2. If a judgment of the retention effect were made solely on the basis of the scores on the delayed posttest, it would be concluded that Treatment 1 is more effective than Treatment 2 in facilitating retention over time. An examination of the curves, however, reveals that, although Treatment 1 facilitates more initial learning than Treatment 2, the effects of the treatments on retention are the same. This is shown by the parallel curves which are indicative of the rate of forgetting. Therefore, it cannot be said that treatment one is superior to treatment two in facilitating retention independent of its effect on learning.

Situation III illustrates a significant difference between Treatments 1 and 2 on the immediate posttest. Treatment 1 is significantly higher in facilitating learning than Treatment 2. On the delayed posttest, no significant difference is found in the means associated with the two treatments. If the effect of the treatments on retention were based on the means of the delayed posttest, no significant differences would be found. The existence of a significant difference between the means on the immediate posttest and an absence of a significant difference on the means of the delayed posttest indicates that differential retention effects exist. The results of Situation III suggest that Treatment 2 results in relatively less forgetting than Treatment 1 although it also results in less initial learning.

Two factor analysis of variance. One of the methods suggested by Wodtke to be used in analyzing retention data is a repeated measures analysis.[5] This analysis explores two aspects of the treatment effects. It considers the overall effects of the experimental treatments and also tests differences in the slopes of the forgetting curves obtained from successive posttest measures.

The purpose of the analysis is to test the following research hypotheses:

1. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the student in mathematics.

2. Increasing the integration of science and mathematics during instruction decreases the rate at which the mathematical behaviors are forgotten.

3. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the student in science.

4. Increasing the integration of science and

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[5] Wodtke, op. cit., p. 30.

> mathematics during instruction decreases the rate at which the science behaviors are forgotten.

The analysis of variance procedure used to test the statistical hypotheses considered is based upon the procedure described by Winer for a two-factor experiment with repeated measures.[6] Factor I consists of the two treatment comparisons. Factor II, the repeated measures factor, would consist of two measures of retention over time. A statistically significant overall main effect would indicate that one instructional treatment was generally superior to the other and a test of the difference in the slopes would be made to determine differences in rate of forgetting.

The layout for the study is shown in Figure 3.

The datum entered into a particular slot (see X) would be the mean score for the children who received Treatment A from Teacher 2 on Test 1.

The linear model upon which the analysis is based has the following form:

$$X_{jkm} = \mu + \alpha_k + \beta_j + \theta_{k(m)} + \gamma_{jk} + \beta\theta_{jk(m)} + \varepsilon_{jkm}$$

where:

X_{jkm} is the score found in the j^{th} test, in the k^{th} treatment group with the m^{th} teacher.

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[6] B. J. Winer, Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, Inc., 1962), pp. 56-62.

		1st Test	2nd Test
	T_1		
Treatment A	T_2	X	
	.		
	.		
	.		
	T_{15}		
	T_{16}		
Treatment B	T_{17}		
	.		
	.		
	.		
	T_{30}		

FIGURE 3

LAYOUT OF THE STUDY

μ is the population mean.

α_k is the effect of being in the k^{th} treatment group. This effect is estimated by:

$$\hat{\alpha}_A = \bar{X}_A - \bar{X}_T \quad \text{and} \quad \hat{\alpha}_B = \bar{X}_B - \bar{X}_T$$

β_j is the effect of the test. It is estimated by subtracting each test mean from the grand mean.

$\Theta_{k(m)}$ is the effect of being in the m^{th} teacher's class within treatment k. This effect is estimated by subtracting each teacher mean from its respective treatment mean.

γ_{jk} is the interaction of treatments and tests. It is estimated by finding the mean score of all scores in a specific treatment group and on a particular test. This mean minus the grand mean estimates the cell effect. The alpha effect for this cell and the beta effect is also subtracted out and the resulting number is the estimated interaction effect.

$\beta\theta_{jk(m)}$ is the interaction of teachers and tests within a particular treatment. The interaction effect can be estimated for each datum in the analysis by the following formula:

$$\hat{\beta\theta} = jk(m) = \bar{X}_{jkm} - \bar{X}_m - \bar{X}_{j \text{ for treat. } k} + \bar{X}_k$$

ε_{jkm} is the error term

The partitioning of the total variation is shown in Table II.

A test of the null hypothesis that $\alpha_k = 0$ is accomplished by comparing the mean square for treatments with the mean square for error between. If the assumptions underlying the analysis of variance are satisfied, the ratio of MS_{treat} to $MS_{error\ between}$ is distributed as F with (k-1,m-k) degrees of freedom.

A test of the null hypothesis that $\gamma_{jk} = 0$ for all jk is accomplished by comparing the mean square for interaction, treatments by tests, with the mean square for error within. As before, if the assumptions are satisfied, the ratio is distributed as F with [(j-1)(k-1), (m-k)(j-1)] degrees of freedom.

TABLE II

PARTITIONING OF THE TOTAL SUM OF SQUARES IN A TWO-FACTOR EXPERIMENT

Source of Variation	Degrees of Freedom	Mean Square
Between Teachers	m-1	
Treatments	k-1	MS_{treat}
Teachers within Treatments	m-k	
Within Teachers	m(j-1)	
Tests	j-1	MS_{tests}
Interaction: (Treat. X Tests)	(j-1)(k-1)	$MS_{treat\ X\ test}$
Interaction: (Test X Teacher within Treat.)	(m-k)(j-1)	
Total	N-1	

If the ratios computed to test the statistical hypothesis, $\alpha_k = 0$, exceeds the critical values of the F distribution for (k-1,m-k) degrees of freedom, then the null hypothesis will be rejected. It would then follow that $\mu_A \neq \mu_B$ in the population since $\alpha_k \neq 0$.

If the ratios computed to test the statistical hypothesis $\gamma_{jk} = 0$ exceeds the critical values of the F distribution for $[(j-1)(k-1), (m-k)(j-1)]$ degrees of freedom, then the null hypothesis is rejected. This is evidence of interaction between treatments and tests in the population indicating a difference in the rate of forgetting.

Assumptions Underlying the Two-Factor Analysis of Variance

1. Subjects are randomly assigned to treatments and teachers.

2. The variances within each cell are homogeneous.

3. The variances of the interactions between tests and subjects within each treatment group are homogeneous.

4. The data are at least of interval measure.

5. Scores are normally distributed.

From a practical standpoint, it is difficult to satisfy the first condition since experimenters do not have that much control over the assignment of students in schools. It is possible, however, to randomize the available subjects with reference to cells. As Lindquist mentions, this forces the definition of the population to which inference may be drawn to fit the sample of available subjects.

Assumption two is tested by finding the variances of the teacher means about each treatment mean and testing the homogeneity of these variances with an F-test.

The homogeneity of variances of interaction between tests and subjects is tested by finding the variance of the interaction of tests and teachers within treatments for each treatment and then comparing these variances for homogeneity by means of an F-test.

Since the data are expressed as ratios, assumption four is satisfied.

Assumption five is not of serious concern. Lindquist reviewed a study by Norton in which it was shown that the F-test is, in general, insensitive to non-normality in distribution. Unless the departure from normality is extreme, such that it can be detected by inspection, the effect on the validity of the F-ratio is not appreciable.

Use of the Analysis of Variance Procedure

The analysis of variance procedure described will be applied twice. The first application will test the effects of the treatments on the acquisition and retention of the mathematical behaviors associated with the learning sequences. The second application will consider the acquisition and retention of the science behaviors associated with the same sequences.

Summary of Chapter II

The purpose of this chapter was to discuss the various considerations made with respect to the experimental design. A description was given of the population and experimental materials used in the study. The use of the pilot study as a means of refining the hierarchy and experimental materials was also discussed. The analysis of variance procedure used to test the hypotheses was also described.

CHAPTER III

ANALYSIS OF THE DATA

This chapter reports the analysis of the data accumulated during the course of this investigation. Included are the results of the analyses used to validate the hierarchy, test the statistical hypotheses, verify the assumptions of the analysis of variance, and test the reliability of the instrument.

Validity of the Measuring Instruments

As stated by Anastasi, "Content Validity involves essentially the systematic examination of the test content to determine whether it covers a representative sample of the behavior domain to be measured."[1] The assessment items included in the instrument used for this investigation were designed to test acquisition of each of the behaviors contained in the hierarchy.

The following procedure was employed as a means of determining whether the test items sampled the behavior

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[1] Anne Anastasi, Psychological Testing (New York: The Macmillan Company, 1961), pp. 135-36.

may be due to uncontrolled test conditions such as distractions of various types, or by the condition of the subject himself. The coefficient of stability, a measure of temporal stability is a correlation coefficient obtained by examining scores achieved by the same subjects on two administrations of a test.

Item specificity as a source of error variance refers to the extent that scores on a test depend upon factors specific to the particular selection of items.

The homogeneity of a test refers to the consistency of performance on all items within a test. Although homogeneous tests are preferred because their scores permit fairly unambiguous interpretation, a single homogeneous test is not an adequate predictor of a highly heterogeneous criteria.

The reliability coefficient which measures the item specificity and homogeneity is termed the coefficient of internal consistency. The computation used to produce this coefficient is the Kuder-Richardson Formula 20. The formula is:

$$r = \frac{k}{k-1} \left[1 - \frac{\Sigma pq}{\sigma 2} \right]$$

where r represents the coefficient of internal consistency, k represents the number of test items, p represents the proportion of correct responses to one item, q = 1-p and σ^2

represents the variance of the test scores.

The coefficients of stability and internal consistency computed for this investigation were based on the scores of 180 students from six of the classes that participated. Of the six classes, three were randomly chosen from each of the treatments. The product-moment correlation computed resulted in a coefficient of stability of 0.79.

The scores of the 180 subjects on Measure I were used to compute the coefficient of internal consistency. The scores of these subjects produced a variance of 36.52 and a sum of 7.21 for the proportion of correct responses multiplied by the proportions of the incorrect responses. When the Kuder-Richardson Formula 20 was applied using these values, the coefficient of internal consistency for Measure I was found to be 0.81.

Validation of the Hierarchy

The integrated mathematics-science hierarchy constructed for this investigation was used as the source of the objectives of the lessons developed. The hierarchy also provided a sequence in which the objectives should be considered when constructing the lessons. Finally, the hierarchy provided a basis for construction of the measuring instrument, each assessment item designed to test acquisition of a given behavior in the hierarchy.

Since the hierarchy was, of necessity, constructed by an a priori analysis procedure, it was necessary to test the validity of the hierarchy before proceeding with the experiment. This was one of the major purposes of the pilot study.

The validation of any behavioral hierarchy is based on an examination of the pass-fail patterns which occur between a given terminal behavior contained in the hierarchy and its relevant subordinate behaviors.

If a subject acquires a given terminal behavior in the hierarchy and also acquires all relevant subordinate behaviors, then this is represented symbolically as (1,1).

If a subject fails to acquire a given terminal behavior, but does acquire all relevant subordinate behaviors, then this is represented as (0,1).

If a subject acquires the terminal behavior and does not acquire all of the relevant subordinate behaviors, then this is represented symbolically by (1,0).

If a subject fails to acquire the terminal behavior and all of the relevant subordinate behaviors, then this is represented symbolically as (0,0).

The consistency ratio used in the validation of a behavioral hierarchy is a measure of the support for the hypothesis that acquisition of the terminal behavior implies acquisition of the relevant subordinate behaviors. The ratio is computed by use of the formula:

$$r_{\text{consistency}} = \frac{\Sigma(1,1)}{\Sigma(1,1) + (1,0) + (0,0)}$$

The minimum ration of consistency considered acceptable was established as 0.90.

The adequacy ratio used in the validation of a hierarchy is the measure of support for the hypothesis that acquisition of all relevant behaviors will imply success in acquiring the terminal behavior. This ratio is computed by use of the formula:

$$r_{\text{adequacy}} = \frac{\Sigma(1,1)}{\Sigma(1,1) + (0,1)}$$

A value of 0.90 was also considered acceptable for this ratio.

The completeness ratio tests the ability of subjects to acquire both the terminal behaviors and all relevant subordinate behaviors. If the behaviors chosen for the lowest level of the hierarchy are ones that all of the subjects are assumed to have and this assumption is erroneous, then a low completeness ratio would result. This might suggest that more primitive behaviors should be chosen for the lowest level than those selected. This ratio is computed by use of the formula:

$$r_{completeness} = \frac{\Sigma\,(1,1)}{\Sigma\,(1,1) + (0,0)}$$

The acceptable ration for completeness was also set at 0.90.

On the basis of the data accumulated from the pilot program, the preceding formulas were used to construct ratios for each set of behaviors consisting of a terminal task and all relevant subordinate behaviors. The ratios obtained indicated certain parts of the hierarchy where relevance between the terminal behavior and the subordinate behaviors could not be established. On the basis of this information, the hierarchy was revised.

Following the experiment, a class was chosen from among those participating in the experiment and the ratios of consistency, adequacy, and completeness were computed. The results of this examination are shown on the following pages.

Prior to computing the ratios, it was established that each ratio must attain a level of 0.90 or above to be considered acceptable evidence of a relevance between a terminal behavior and subordinate behaviors. On this basis, it was found that four of the twenty-one consistency ratios

TABLE III

VALIDATION OF THE LEARNING HIERARCHY RATIOS OF CONSISTENCY, ADEQUACY, AND COMPLETENESS

Relevant Tasks	Consistency	Adequacy	Completeness
VIIIA from IXA	1.00	0.97	1.00
VIIA from VIIIA	0.96	0.86	1.00
VIA from VIIA	0.87	1.00	0.93
VA from VIA	0.97	0.97	1.00
IVA from VA	0.97	1.00	1.00
VIIIC from IXB	1.00	1.00	1.00
VIIIB from IXB	1.00	1.00	1.00
VIIB from VIIIB	1.00	0.93	1.00
VIB from VIIB	0.90	0.96	1.00
VB from VIB	0.93	0.93	1.00
IIIA from VA and IXB	1.00	0.90	1.00
VIIC from VIIIC	1.00	0.97	1.00
IIIB from VIIC and VB	0.93	0.88	0.96
IIA from IVA	1.00	0.90	1.00
IA from IIIA and IIA	0.88	0.96	1.00
VC from VIC and VID	0.90	0.64	0.90
IVB from VIC and VID	0.93	1.00	0.97
IVC from VIC and VID	0.93	0.89	1.00
IB from IIIC and IVA	0.87	1.00	1.00
IIIC from IVC and IVB	0.89	0.89	1.00
IC from IIIC and IIIB	0.90	1.00	0.96

TABLE IV

VALIDATION OF THE LEARNING HIERARCHY
FREQUENCY OF PASS-FAIL PATTERNS

Relevant Tasks	(1,1)	(0,0)	(1,0)	(0,1)
VIIIA from IXA	29			1
VIIA from VIIIA	25		1	4
VIA from VIIA	26	2	2	
VA from VIA	28		1	1
IVA from VA	29		1	
VIIIC from IXB	30			
VIIIB from IXB	30			
VIIB from VIIIB	28			2
VIB from VIIB	26		3	1
VB from VIB	26		2	2
IIIA from VA and IXB	27			3
VIIC from VIIIC	29			1
IIIB from VIC and VB	25	1	2	2
IIA from IVA	27			3
IA from IIIA and IIA	26		3	1
VC from VIC and VID	18	2		10
IVB from VIC and VID	28	1	1	
IVC from VIC and VID	25		2	3
IIIC from IVC and IVB	24		3	3
IB from IIIC and IVA	26		4	
IC from IIIC and IIIB	27	1	2	

were below this level. Five adequacy ratios were also found to be below the acceptable limit with one ratio as low as 0.64. All completeness ratios attained levels above the level of acceptability.

Analysis of Variance of Means for Mathematics Items on Measures I and II

Measures I and II were administered to determine the overall effect of the treatments on the acquisition of the

TABLE V

ANALYSIS OF VARIANCE OF MEANS FOR MATHEMATICS ITEMS ON MEASURES I AND II

Source of Variation	df	Mean Squares	F	Action Taken
Between Teachers	29	18.14		
Treatments	1	78.89	4.94	Reject H_o
Teachers within Treatments	28	15.97		
Within Teachers	30	2.13		
Tests	1	45.06		
Interaction: (Treat. X Test)	1	1.74	2.84	Accept H_o
Interaction: (Test X Teacher within Treatments)	28	0.61		
Total	59			

mathematics and science behaviors as well as the rate of forgetting associated with each treatment. Means associated with the acquisition of mathematics behaviors and science behaviors were considered in separate analyses.

The analysis of variance associated with the acquisition of the mathematics behaviors tested the following null hypotheses:

1. The overall effect of increased integration of mathematics and science behaviors in a learning sequence, as indicated by the treatment means, is equal to the overall effect of assignment to a non-integrated learning sequence.

$$H_o: \mu_A = \mu_B$$

2. An increase in the integration of mathematics and science behaviors in a learning sequence produces the same rate of forgetting as a non-integrated sequence.

The subgroup means that were used in the analysis of variance are contained in Appendix F.

By comparing the mean square for treatments with the mean square for error between, an F-ratio of 4.94 was attained. This ratio is sufficient to reject the null hypothesis, $\mu_A = \mu_B$, since the critical F-ratio at the 0.05 level with (1,28) degrees of freedom is 4.20. Thus, there is support for the statement that the treatments have differential effects on overall performance.

Since the means for treatments A and B are 20.6 and

18.3, respectively, the research hypothesis that the integrated mathematics-science learning sequence produces a higher overall performance of the student than the non-integrated sequence is supported.

By comparing the mean square for interaction, treatments by test, with the mean square for error within an F-ratio of 2.84 was obtained. Since the ratio was less than the F-critical ratio at the 0.05 level with (1,28) degrees of freedom, it is necessary to accept the null hypothesis, $\gamma_{jk} = 0$. Thus, the research hypothesis that the treatments had differential effects on the rate of forgetting was not supported.

Statistical Tests of the Homogeneity of Variance for the Treatment Groups

Among the assumptions underlying the two-factor analysis of variance procedure used in this experiment are:

1. The variances within each cell are homogeneous.
2. The variances of the interactions between tests and subjects within each treatment group are homogeneous.

The test for the first assumption is accomplished by finding the variances of the teacher means about each treatment mean and testing the homogeneity of these variances by an F-test. Using this procedure, an F-ratio of 1.58 was obtained. Since the F-critical ratio with (14,14) degrees of freedom is 3.70 @ 0.01 level, the assumption of homogeneity of variance is supported.

The test for the second assumption is accomplished by determining the variance of the interaction of tests and teachers within treatments for each treatment and then comparing these variances for homogeneity by means of an F-test. This procedure resulted in an F-ratio of 1.58. Since the F-critical ratio with (14,14) degrees of freedom is 3.70 @ 0.01 level, this assumption of homogeneity of variance is also supported.

Analysis of Variance of Means for Science Items on Measures I and II

The same procedure described for treating the mathematical data was also applied to the data collected on the science items. The table on the following page summarizes this procedure. The subgroup means for science that were used in the analysis of variance are contained in Appendix G.

On the basis of these data, it was concluded that the treatment to which the groups were assigned had no differential effects on overall performance or rate of forgetting.

TABLE VI

ANALYSIS OF VARIANCE OF MEANS FOR SCIENCE ITEMS ON MEASURES I AND II

Source of Variation	df	Mean Squares	F	Action Taken
Between Teachers	29	6.76		
Treatments	1	10.67	1.61	Accept H_o
Teachers within treatments	28	6.62		
Within Teachers	30	0.34		
Tests	1	1.50		
Interaction: (Treat. X Test)	1	0.54	1.86	Accept H_o
Interaction: (Test X Teacher within Treat.)	28	0.29		
Total	59			

Statistical Tests of the Homogeneity of Variance for the Treatment Groups

Using the procedures previously described to test the variance within cells, an F-ratio of 0.96 was obtained. Since the F-critical ratio with (14,14) degrees of freedom is 3.70 @ 0.01 level of confidence, the assumption of homogeneity of variance is supported.

The test for the homogeneity of variance of the interactions of tests and subjects within each treatment group was also conducted using the procedures previously described. This procedure resulted in an F-ratio of 2.96. Since the F-critical ratio with (14,14) degrees of freedom is 3.70 @ 0.01 level, the assumption of homogeneity of variance is supported.

Summary

This chapter included an examination of the validity and reliability of the measuring instrument used in this experiment, and an anlysis of the data collected. Statistical tests were applied to the data and the following results were obtained:

1. In testing the reliability of the measuring instrument, a coefficient of stability of 0.79 and a coefficient of internal consistency of 0.81 were obtained.

2. The consistency, adequacy, and completeness of the hierarchy were examined. Four of the consistency ratios and five of the adequacy ratios were below the acceptable level of 0.90. All of the completeness ratios were either equal to or greater than 0.90.

3. Treatment A produced a significantly higher overall performance than Treatment B in acquisition of the mathematical behaviors although there were no significant differences between treatments in their effect on rate of forgetting.

4. The two treatments had no differential effects on the overall performance or the rate of forgetting with regard to the science behaviors.

CHAPTER IV

CONCLUSIONS AND SUBSEQUENT WORK

In any investigation which involves statistical inference, the findings are ordinarily applied to a larger domain than those cases actually observed. The larger domain is termed the population and the observed domain is termed the sample.

All of the classes used in the experiment were drawn from the population of fifth grade classes in the Baltimore County Public Schools. Since the classes selected did not constitute a random sample of this population, a hypothetical population was defined to which the inference applies. The classes selected for the investigation were considered a random sample of the hypothetical population. Accordingly, the hypothetical population was defined as the set of fifth grade classes in the Baltimore County Public Schools that would use the experimental materials.

The purpose of this study was to determine whether an integrated learning sequence based on a hierarchy of mathematical and scientific behaviors facilitates the acquisition and retention of the behaviors more than a non-integrated learning sequence based on the same hierarchy.

The comparison of the effectiveness of the two learning sequences was based on the results of the posttests given after completion of the learning sequences. One test was given on the day following completion of the learning sequence, and the second, nine weeks thereafter. Comparisons were made on the basis of the overall effect of the treatments on acquisition of the behaviors and on the rate at which the behaviors are forgotten.

In constructing the experimental design, an attempt was made to control all sources of systematic bias through random assignment to treatments. Other sources of variation were identified and controlled. Therefore, the major difference between the two groups was assignment to the integrated or non-integrated learning sequence. Differences in acquisition and retention of the behaviors are, therefore, attributable to assignment to treatment.

Conclusions

The experiment was conducted to test four research hypothesis. The hypotheses were:

1. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the student in mathematics.

2. Increasing the integration of science and mathematics during instruction decreases the rate at which the mathematical behaviors are forgotten.

3. Increasing the integration of science and mathematics behaviors during instruction increases the overall performance of the student in science.

4. Increasing the integration of science and mathematics during instruction decreases the rate at which the science behaviors are forgotten.

The conclusions drawn were restricted to the specified population and are based on the results of the analysis of variance carried out on the class means.

The first hypothesis was supported by the data. The analysis of variance identified a significant difference in the overall performance that was attributed to assignment to treatment. The integrated sequence produced a significantly higher overall performance than the non-integrated sequence.

No evidence can be offered that supports the second hypothesis. The lack of an interaction between treatments and tests indicates that the rates of forgetting of the mathematical behaviors are approximately the same. Any difference between these rates can be attributed to chance.

As indicated by the following graph, the acquisition of a greater number of mathematical behaviors was exhibited on the initial posttest by those classes of students exposed to the integrated sequence. Those students taking the integrated sequence also exhibited a greater number of mathematical behaviors on the delayed posttest. Thus, although the rate of forgetting was approximately the same for each

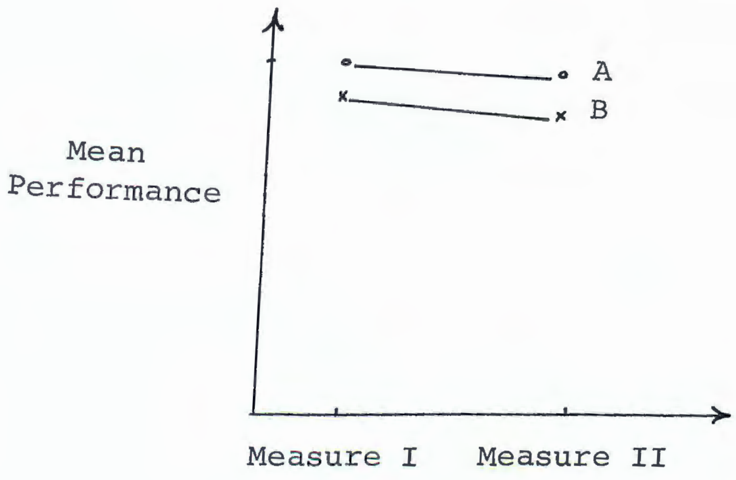


FIGURE 4

GRAPHIC COMPARISON OF RATES OF FORGETTING
OF MATHEMATICS BEHAVIORS

treatment, it is advantageous to use the integrated sequence since a larger number of mathematical behaviors are acquired and retained.

The lack of difference in the rates of forgetting implies that the same number of mathematical behaviors were forgotten regardless of the treatment. If it can be assumed that the number of behaviors forgotten increases as the number of behaviors acquired increases, then it is not unreasonable to assume that at least as many behaviors would be forgotten in the integrated sequence as in the non-integrated sequence.

The third hypothesis was not supported by the data. The overall performance of the students in acquiring science behaviors was not differentially affected by assignment to treatment. Since a number of the assessment items designed to test acquisition of the science behaviors were successfully completed by students assigned to both treatments, it is possible that fourteen science assessment items were too few to produce significant differences in the overall performance. Perhaps several more assessment items designed to test acquisition of the final tasks should have been included in the measure with a greater weight assigned to these tasks for the grading procedure.

The lack of difference in the rate of forgetting predicted by the fourth hypothesis may also be attributed to the need for additional assessment items, particularly those testing acquisition of the terminal behaviors.

Although the preceding conclusions were based on the analysis of variance procedure, other factors which affect the conclusions should also be discussed. These factors include the validity and reliability of the measuring instrument and the validity of the hierarchy.

The measuring instrument. Support for the validity of the measuring instrument was provided by the matching of the assessment items in the instrument with the behaviors in the hierarchy. The matching was done by persons with a background in mathematics, science, and construction of assessment items who had not had prior exposure to the experimental materials. The evidence of validity was, thus, more objective than that which would have been provided by an analysis by the experimenter alone.

The reliability of the instrument was measured by use of the coefficients of stability and internal consistency. Both of these coefficients were at an acceptable level.

Criteria for administering and scoring the instrument were established and the experimenter scored all of the tests personally. In this way, variations in administration and scoring were minimized.

The hierarchy. Validation of the hierarchy was accomplished through an examination of the ratios of consistency, adequacy, and completeness. Although some ratios of consistency were slightly below the level established prior to the examination of the hierarchy, none of the ratios was extremely low. Although these low ratios are not a cause for serious concern, they do indicate the need for further refinement of the hierarchy, or further examination of the assessment items.

In only one case did an adequacy ratio reach a level that could be considered a cause for serious concern. One reason for this low ratio could be an ambiguous assessment item used to test acquisition of the terminal behavior. A second explanation might be less instruction than necessary to acquire the terminal behavior after the subordinate behaviors had been acquired. A third possibility would be the absence of a needed intermediate behavior between the terminal and subordinate behaviors.

All completeness ratios were above the level of acceptability.

Discussion. The implications made on the basis of any educational experiment are limited by the population defined for the experiment. Any implications that are applied to a less restricted population are subject to criticism since there would be no evidence to support them. However, the following possibilities are suggested by the results of the investigation.

Each of the lessons was designed to last for fifty minutes. In the integrated sequence, both the mathematics and science were taught in this amount of time. This was also true of the non-integrated sequence. Usually, when mathematics and science are taught as unrelated subjects, each mathematics or science lesson, itself, will last as long as an integrated lesson. This suggests that the integrated sequence may make more efficient use of time than a sequence in which the mathematics and science are not related.

In the high school, where the behaviors to be acquired in both mathematics and science are complex, an efficient integrated sequence may be difficult to implement. The implementation of such a program would be most restricted by the lack of teachers with the necessary background to teach such a course. The potential benefits of such a program, however, might make the additional difficulties worth consideration.

At the elementary or middle school level, however, the behaviors acquired are less sophisticated and are, consequently, more adaptable to integrated sequences that are within the average teacher's capabilities.

An added advantage of the process used in this investigation is the use of the hierarchy as a guide for constructing the learning sequence. Lessons can be designed to promote acquisition of each of the behaviors included in the hierarchy. Assessment items can be constructed which are specifically designed to test acquisition of the behaviors. In this way, acquisition of each of the behavioral objectives of the learning sequence can be tested when evaluating the effectiveness of the learning sequence.

A validated hierarchy makes it possible to identify related terminal and subordinate behaviors. This aids the teaching process by identifying the behaviors that have not been acquired by the students. This identification makes it possible to isolate sources of learning difficulty.

Implications for Further Research

The process of seeking the solution to one problem in educational research often makes possible the identification of other related problems worthy of consideration. As a result of this investigation, several areas were identified in which further study should prove profitable.

The integrated learning sequence proved effective in facilitating acquisition of the mathematical behaviors with the defined population. Testing similar learning sequences at levels above and below that considered in the investigation should be carried out. The continued success of such sequences in facilitating acquisition of the mathematics behaviors with various populations would suggest that serious thought should be given to designing integrated courses in mathematics and science at the elementary and middle school level. Such a procedure would also encourage the inclusion of more science in the elementary school.

Since the integration or correlation of mathematics and science has often been suggested for the high school, integrated learning sequences could be developed for testing at this level. Since mathematics and science are usually taught as unrelated subjects in the high school today, an investigation could be conducted that involves three treatment groups. The first group would include classes in which mathematics and science are taught as unrelated subjects. The second would be a sequence in which the mathematics needed for acquisition of the science behaviors is taught without science applications being introduced. The third treatment would be an integrated mathematics-science sequence in which the mathematics would be taught as needed through the use of related science problems.

Similar sequences could be developed for testing the effectiveness of such programs as Applied Mathematics, Applied Algebra, or Applied Geometry. Each of these sequences would expose the students to high interest situations in which the acquisition of certain mathematical behaviors was necessary. The situations might include work with automobiles or other mechanical devices for boys and furnishing an apartment and cooking for girls.

The characterization of mathematics as a collection of sterile manipulations does not have to have as much basis in fact as it now enjoys. To dispel this notion, more experimentation should be undertaken in which mathematics is integrated with other subjects. The results of this investigation should provide some encouragement for this approach to the teaching of mathematics.

Summary

This chapter was devoted to a discussion of the conclusions that were reached on the basis of the data obtained from the experiment. The implications of these conclusions were then considered.

The experiment produced evidence that the integrated sequence facilitates acquisition of the mathematical behaviors more than the non-integrated sequence, but had no significant effect on the rate of forgetting.

For the defined population the experimental treatments had no differential effects on the acquisition or retention of the science behaviors.

Based on the success of the integrated sequence in facilitating acquisition of the mathematical behaviors, it is suggested that investigations be undertaken at various grade levels to further explore the utilization of this technique.

It was also suggested that an integrated learning sequence may make more efficient use of time than the usual plan of teaching mathematics and science as unrelated subjects.

The possibility of integrating mathematics with a variety of subjects was also proposed.

APPENDIX A

NUMBER OF PARTICIPATING STUDENTS IN

EACH OF THE CLASSES

TABLE VII

NUMBER OF PARTICIPATING STUDENTS IN EACH OF THE CLASSES

Treatment A		Treatment B	
Class	Number	Class	Number
A_1	37	B_1	37
A_2	25	B_2	37
A_3	31	B_3	27
A_4	33	B_4	27
A_5	29	B_5	24
A_6	20	B_6	26
A_7	29	B_7	30
A_8	39	B_8	36
A_9	30	B_9	35
A_{10}	24	B_{10}	39
A_{11}	43	B_{11}	24
A_{12}	31	B_{12}	17
A_{13}	27	B_{13}	23
A_{14}	29	B_{14}	30
A_{15}	34	B_{15}	27

APPENDIX B

RESULTS OF MATCHING ASSESSMENT ITEMS ON THE MEASURING INSTRUMENTS WITH BEHAVIORS IN THE HIERARCHY

TABLE VIII

RESULTS OF MATCHING ASSESSMENT ITEMS ON MEASURING INSTRUMENT WITH BEHAVIORS IN HIERARCHY

Behavior	Correct Match	Incorrect Match	Per cent of Successful Matches
IXA	6	0	100
IXB_1	6	0	100
IXB_2	6	0	100
IXB_3	5	1	83
IXB_4	5	1	83
$VIIIA_1$	6	0	100
$VIIIA_2$	6	0	100
$VIIIB_1$	6	0	100
$VIIIB_2$	6	0	100
VIIIC	5	1	83
$VIIA_1$	6	0	100
$VIIA_2$	6	0	100
$VIIB_1$	6	0	100
$VIIB_2$	6	0	100
$VIIC_1$	6	0	100
$VIIC_2$	5	1	83
VIA	6	0	100
VIB	6	0	100
VIC	6	0	100
VID_1	5	1	83
VID_2	5	1	83
VA	5	1	83
VB	6	0	100
VC_1	6	0	100

TABLE VIII (continued)

Behavior	Correct Match	Incorrect Match	Per cent of Successful Matches
VC_2	5	1	83
IVA	6	0	100
IVB	5	1	83
IVC_1	5	1	83
IVC_2	5	1	83
IIIA	6	0	100
IIIB	5	1	83
$IIIC_1$	5	1	83
$IIIC_2$	5	1	83
IIA	6	0	100
IA	6	0	100
IB	5	1	83
IC	6	0	100

APPENDIX C

LESSON PLANS FOR TREATMENT A

TREATMENT A

LESSON I

I. Objectives--The student should be able to:

A. Name and identify a response.

B. Name and identify a stimulus.

C. Distinguish a stimulus from a response.

D. Name and identify reaction time.

E. Name and identify a premature response as the result of anticipation.

F. Name and identify decimal numbers expressed in tenths and hundredths.

II. Materials

A. New dollar bill.

B. Paper cut to dollar bill size.

C. Worksheet--Decimals expressed in tenths and hundredths.

III. Vocabulary words:

A. Stimulus

B. Response

C. Anticipation

D. Reaction time

IV. Procedure

A. Tell the students that you are going to show them an old trick. Have a student help. Have the student hold his hand with the palm vertical and his thumb and forefinger approximately one inch apart.

B. Place a new dollar bill between the student's thumb and forefinger so that the picture on the bill is between the thumb and forefinger.

C. Explain to the class that it is very difficult to catch the bill when it is dropped unless one starts to grab for the bill before it is released.

D. Before demonstrating this procedure, explain that usually the person who is trying to catch the dollar bill is told that he can keep the bill if he catches it. Emphasize that this is not the case in the coming demonstration.

E. Drop the bill several times for the student. Tell the students to observe the procedure carefully.

F. Ask what act causes the student to grab for the bill. (The student grabs for the bill when the bill is released by the teacher.)

G. Ask students what happens when the bill is released. (The student grabs for the bill.)

H. The entire procedure therefore involves two separate actions. What are they? (Teacher dropping bill, student grabbing for bill)

I. Explain that in this situation the dropping of the bill is called a stimulus and the student grabbing for the bill is called the response. Write the words stimulus and response on the board for further reference.

J. Give one or two examples of situations involving a stimulus and the response and then let students volunteer similar situations. Have students identify the stimulus and the response in each case.

K. Divide the students into groups of three or four. Using paper cut to dollar bill size, have one drop the paper, another act as the catcher and others in the group act as observers. Positions may be rotated as the activity progresses. Allow about seven minutes for this activity.

L. Following this activity, ask students if any of them were able to catch the bill. If the answer is yes, ask observers if the student in question started his response prior to the release of the bill.

M. Explain that giving a premature response is called an anticipation. Write this word on the board also.

N. Ask students how long they think it takes for the bill to drop through one's fingers. (Various guesses will be made, but students should see at this time that they have no method of measuring the time lapse.)

O. Tell the students that this period of time between the stimulus and the response is called the reaction time.

P. Also tell them that since those periods of time are so short, we must have some method of measuring times less than one second. In order to do this, tell the students that they must have some knowledge of decimals.

Q. Place the following fractions on the board and have the students read them:

$\frac{1}{10}$	$\frac{4}{10}$	$\frac{7}{10}$
$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$
$\frac{3}{10}$	$\frac{6}{10}$	$\frac{9}{10}$

R. Show the students that the fraction $\frac{1}{10}$ is written decimally as 0.1. Explain that this is also read as one tenth. Have students supply the proper decimal notation and names for the remaining fractions.

S. Explain that the decimal point separates whole numbers from fractional numbers and give several examples.

T. Put the fraction $\frac{25}{100}$ on the board and ask students if they can guess how this could be written as a decimal. If they can, give several more examples to reinforce this ability. If they cannot, explain that $\frac{25}{100}$ may be written as 0.25 and then give other examples.

U. Finally, distribute the worksheet, have the students complete it, and then discuss the worksheet with them.

LESSON I

DECIMALS EXPRESSED IN TENTHS AND HUNDREDTHS

Change the following fractions to decimals:

1. $\frac{83}{100}$ 2. $\frac{63}{100}$ 3. $\frac{7}{10}$ 4. $\frac{8}{100}$ 5. $\frac{76}{100}$

Write the following in decimal form:

6. seven tenths -

7. eight hundredths -

8. forty hundredths -

9. ninety-seven hundredths -

10. fifty-three hundredths -

11. sixty-three hundredths -

12. one hundredth -

13. eighty-eight hundredths -

14. three tenths -

15. eleven hundredths -

TREATMENT A

LESSON II

I. Objectives--The student should be able to:

A. Name and identify a centimeter.

B. Name and identify reaction time.

C. Name and identify an attention-getting procedure.

D. Name and identify decimal numbers expressed as tenths and hundredths.

E. Distinguish decimal numbers expressed in hundredths from decimal numbers expressed in tenths.

F. Name and identify variables in human subjects and in experimental procedures that must be controlled to complete a valid experiment.

II. Materials--Meter stick, Reaction time chart

III. Vocabulary

A. Reaction time

B. Centimeter

C. Attention-getting procedure

IV. Procedure

A. Put the following drill on the board or present the situation orally.

Write the sentence containing the stimulus and the sentence containing the response in the following:

A boy is riding a bicycle down the street. He sees a stop sign. He puts the brakes on. The bicycle stops.

B. After students have completed the drill, go over the drill with the students in order to determine their ability to name and identify the stimulus (He sees a stop sign) and the response (He puts the brakes on).

C. Ask students how long they think it would take the boy to brake his bicycle after seeing the stop sign. Again, as in the previous lesson, there will be difficulty in determining how long the action took with only guesses supplying the times.

D. Review the term "reaction time" with students as well as the reading of decimals expressed in hundredths.

E. Give students a copy of the chart provided. Explain to the students that by use of this chart, a fairly accurate measure of reaction time may be made.

F. Show students the meter stick and point out the length of a centimeter, stressing that it is $\frac{1}{100}$ of the total length of the stick.

G. Demonstrate with several pupils how to make use of the chart in measuring reaction time using the following procedure:

> Place a piece of colored tape that the students can see clearly at the 30 cm. mark on the meter stick. When performing the experiment, have the child hold his hand in the same position as it was held when experimenting with the dollar bill. Place the meter stick between his thumb and forefinger with the tape at that point. When the stick is dropped and the child catches it, the scale on the stick is read as the centimeter closest to the top of the child's thumb. When thirty is subtracted from this number, the number of centimeters that it dropped may be determined. By referring to the chart, the time elapsed may be determined.

H. Give a student ten trials and have another student record the reaction time on the board for each trial. With experience, the child should improve, although there is a psychological limit below which he cannot go. Distract the student at least once during the trial and drop the stick when he is unprepared. The stick will then fall further than on previous trials. Ask students if this trial was a good measure of his reaction time. (No, since he was not paying attention.) Ask how the student might insure that the child pays attention while waiting for the stimulus. (Give a warning by saying "ready" or some other attention-getting process.)

I. To prevent children from anticipating the stimulus and grabbing the stick before it is dropped, vary the period between the warning and the dropping so that the child will only respond when the stick is released.

J. Ask students what things they have noticed in the trials that must be done the same by all students to make sure that everyone gets a fair trial. List these on the board. They should include items like the following:
   1. Student must be paying attention.
   2. Meter sticks should be dropped, not pushed downward.
   3. Students must not anticipate.
   4. Students must have equal amounts of practice.

K. Students may think of other considerations which should be listed according to their merits.

V. Summary

A. Ask the following questions:
   1. How can we be sure that a student is ready for a trial?
   2. What is a reaction time?
   3. How can we stop a person from anticipating the release of a stick?
   4. Name the following decimals:

      0.7 0.43 0.56 0.60 0.21 0.6

B. Explain that before further work can be done on the experiments, it will be necessary to learn more about decimals. This will be considered in lesson III.

CHART I

DISTANCE AND TIME TO FALL FROM REST

Centimeters	Seconds
7	.12
8	.13
9-10	.14
11	.15
12-13	.16
14-15	.17
16	.18
17-18	.19
19-20	.20
21-22	.21
23-24	.22
25-27	.23
28-29	.24
30-31	.25
32-34	.26
35-37	.27
38-39	.28
40	.29

TREATMENT A

LESSON III

I. Objectives--The student should be able to:

   A. Distinguish decimal numbers in hundredths from decimal numbers expressed in tenths.

   B. Order decimals expressed in tenths and hundredths.

   C. Demonstrate a procedure for adding decimals.

II. Materials--Two worksheets

III. Procedure

   A. Tell students that before it is possible to do further work with reaction time, it will be necessary to learn more about decimals.

   B. As a drill, put the following decimals on the board. Have the students orally name the decimals.

0.7	0.62	0.59	0.4	0.19
0.17	0.53	0.9	1.7	0.99

   C. After completing the drill, ask students which of the numbers is the largest, the smallest. If they have difficulty, explain to them that any whole number is larger than any decimal fraction less than one and that by annexing a zero to a decimal numeral expressed in tenths, the decimal may be expressed in hundredths.

   D. Working with the students, arrange the numerals in ascending order.

   E. Following this, have students write the following numerals in ascending order.

0.64	1.72	1.5	1.8	0.94
0.58	0.9	0.97	0.89	0.16

F. Go over this ordering with the students to make sure that the students can perform this procedure.

G. Then put the following fractions on the board.

$$\frac{3}{10}$$

$$\frac{2}{10}$$

$$\frac{1}{10}$$

H. Ask the students what the sum of these fractions is and place the sum under the fractions as illustrated below.

$$\frac{3}{10}$$

$$\frac{2}{10}$$

$$\frac{1}{10}$$

$$\frac{6}{10}$$

I. Ask students how each of the above fractions is written as a decimal. Put down these as illustrated below.

$$\frac{3}{10} = .3$$

$$\frac{2}{10} = .2$$

$$\frac{1}{10} = .1$$

$$\frac{6}{10} = .6$$

J. Do a similar procedure using the fractions $\frac{8}{10}$, $\frac{5}{10}$, $\frac{6}{10}$.

K. When you have done, in your judgment, an adequate number of illustrations, ask the students whether they can see a pattern to how decimals should be added. (Decimal points lined up, add as usual.)

L. Using a similar procedure, do several examples involving hundredths.

M. Distribute the worksheet. Have the students complete it and then go over the answers with the students.

N. Follow this by the worksheet on reaction time. Have students complete this and then discuss the answers.

ADDITION OF DECIMALS

ADD THE FOLLOWING DECIMALS:

1)	2)	3)	4)
.2	.7	.26	.93
.6	.8	.71	.63
.4	.6	.51	1.05
.3	1.4	.36	2.13

ARRANGE THE FOLLOWING DECIMALS IN COLUMNS AND THEN ADD:

5) 0.1 + 0.6 + 0.5 + 0.9

6) 0.53 + 0.74 + 0.86 + 0.52

7) .84 + .7 + .5 + 1.3

8) 7.32 + .58 + .72 + .4

LESSON III

TOTAL REACTION TIME

FOUR BOYS TOOK TESTS OF THEIR REACTION TIMES. EACH BOY WAS GIVEN FIVE TRIALS. FIND THE TOTAL AMOUNT OF TIME THAT EACH BOY TOOK ON THE TRIALS.

Jim .42 sec., .40 sec., .29 sec., .36 sec., .33 sec.

Bob .56 sec., .48 sec., .42 sec., 140 sec., .38 sec.

Ike .52 sec., .74 sec., .79 sec., .67 sec., .56 sec.

Art .57 sec., .94 sec., .88 sec., .38 sec., .5 sec.

TREATMENT A

LESSON IV

I. Objectives--The student should be able to:

A. Apply the rule for rounding decimals to the nearest tenth.

B. Apply the rule for rounding decimals to the nearest hundredth.

II. Materials--Worksheet

III. Procedure

A. Put the following decimal numerals on the board prior to the lesson.

0.72	0.6	0.87	0.8	0.91
3.72	4.3	0.5	0.7	0.03

B. Have the students name these numerals. It may be necessary to emphasize the fact that when a decimal numeral contains a whole number as in 3.72, the numeral is read three <u>and</u> seventy-two hundredths.

C. Put the following decimal numerals on the board.

0.534 0.832 0.756 0.842 0.973

D. Ask the students how they think 0.534 would be read as a decimal. If they can read this one successfully, have them read the remaining numbers. If necessary, give the students additional practice on reading these numerals before proceeding.

E. Review the rule for rounding whole numbers to the nearest ten and the nearest hundred. Emphasize the fact that the digit in the column to the right of the column to be rounded determines whether the digit in the column to be rounded stay the same or is raised by one. <u>It is not necessary for the students to formally state this procedure if they can perform the procedure</u>.

F. Use the following method of presenting this topic:

1. Put the following numbers on the board

   57 62 93 79 41

2. Ask the students what each of these numbers would be rounded to the nearest ten. Ask them to explain in their own words how they determined the answer. (Number in the column to the right [units column] determines whether the digit in the tens column stays the same or is raised by one.)

3. Use a similar procedure with the numbers below, rounding to the nearest hundred.

   126 437 596 782 648

G. When the rounding procedure has been reviewed sufficiently, for whole numbers, consider decimals to be rounded to the nearest tenth and the decimals rounded to the nearest hundredth using the same methods.

H. Tell the students that the following decimals represent reaction times in certain experiments. Ask the students to round off the decimals in problem one to tenths and the decimals in problem two to hundredths.

1)	0.63	0.87	0.56	0.94	0.98
2)	0.842	0.927	0.563	0.124	0.072

I. Pass out the worksheet to students and allow them to work on them for the remainder of the period, using this time for giving individual help to students.

J. The worksheet will be finished for homework if time does not remain in the period.

K. The worksheet will be discussed at the beginning of lesson V.

ROUNDING REACTION TIMES

The following decimals represent reaction times determined in various experiments. Round these reaction times to the nearest tenth or hundredth as indicated.

I. Round the following decimals to the nearest tenth. Put your answer in the space provided.

1) 0.72 2) 0.96 3) 0.05 4) 1.44

______ ______ ______ ______

5) 0.67 6) 0.81 7) 0.92 8) 2.99

______ ______ ______ ______

II. Round off the following decimals to the nearest hundredth. Put your answer in the space provided.

1) 0.738 2) 0.972 3) 0.876 4) 5.731

______ ______ ______ ______

5) 0.996 6) 8.898 7) 0.237 8) 0.947

______ ______ ______ ______

TREATMENT A

LESSON V

I. Objectives--The students should be able to:

A. Apply the rule for dividing a number by ten.

B. Apply the rule for finding the average of ten whole numbers to one decimal place.

C. Apply the rule for finding an average of ten decimal numbers.

II. Materials--Worksheet

III. Procedure

A. Discuss the worksheet that was distributed in lesson four. Spend as much time on the discussion as you feel is necessary.

B. Following this discussion, place the following numbers on the board.

20	80	130	210	500

C. Ask the students what the quotient would be if each of the numbers was divided by ten. Put these answers down as shown.

20	80	130	210	500
2	8	13	21	50

D. Ask the student what the only difference is in the number after it has been divided by ten. (The zero is gone.)

E. Ask the student where the decimal point would go in each of these numbers if we wished to put one in. (To the right of the number.) Place the decimal point to the right of each of the numbers as shown below.

20.	80.	130.	210.	500.
2.	8.	13.	21.	50.

F. Explain to the students that a whole number may be written with a zero to the right of the decimal point and have the same value since a number like 21 could be written as 21.0 which would be read as twenty-one and zero tenths which is equal to twenty-one. Annex a zero to each of the quotients on the board as shown below.

20.	80.	130.	210.	500.
2.0	8.0	13.0	21.0	50.0

G. Ask the students what the difference is between the position of the decimal point in the original number and the decimal point in the quotient. (It has been moved one place to the left.)

H. Ask students if they can now tell an easy way to divide a number by ten. (Move the decimal point one place to the left.)

I. Put other multiples of ten on the board and have students show how they would divide the numbers by ten using the rule discovered.

J. When students can do this procedure, put the following numbers on the board.

31.	52.	83.	78.	67.

K. Ask the students how they might apply the same rule to divide the above numbers by ten. The answers should be written as shown below.

31.	52.	83.	78.	67.
3.1	5.2	8.3	7.8	6.7

L. Then have the students apply the rule to divide the following numbers by ten.

3.8	1.72	5.76	7.54	11.53

M. Give further examples if necessary.

N. Following completion of the preceding exercise, put the following numbers on the board.

9, 7, 5, 4, 8, 7, 9, 1, 8, 2

Ask the students how to find the average of these numbers. (Add the numbers and divide by ten.)

Finding averages should not be difficult for the students since they had experience in finding averages in grade four. If they have trouble, however, give them a simple example of finding an average before proceeding.

O. When the process of finding an average has been established, add the numbers in the following way.

$$\begin{array}{r} 9 \\ 7 \\ 5 \\ 4 \\ 8 \\ 7 \\ 9 \\ 1 \\ 8 \\ 2 \\ \hline 60 \end{array}$$

P. Ask students for a quick method of dividing by ten. (Move the decimal point one place to the left.)

Q. Tell students that the following sets of numbers represent reaction times in several experiments. Tell them to find the average reaction time in each case.

1) 9, 5, 6, 7, 8, 6, 4, 9, 2, 3

2) .6, .9, .7, .6, .6, .8, .9, .3, .1, .2

3) .27, .19, .55, .42, .13, .22, .14, .21, .11, .16

R. After doing the preceding problems and discussing them with the students, distribute the worksheet and have the students work on them while individual help is given.

S. The work may be completed for homework and will be discussed in lesson VI.

LESSON V

FINDING AVERAGE REACTION TIME

Find the average reaction time in each of the following cases.

1) .8, .7, .8, .6, .6, .5, .8, .7, .9, .3

2) .2, .6, .5, .4, .8, .3, .7, .6, .8, .1

3) .21, .18, .14, .20, .17, .21, .23, .25, .22, .15

4) .16, .26, .34, .30, .25, .14, .23, .29, .17, .28

TREATMENT A

LESSON VI

I. Objectives--The student should be able to:

Demonstrate an experiment in which a sight stimulus is followed by a response with the reaction time measured by indirect means.

II. Materials

A. Six meter sticks

B. Tally sheets

III. Procedure

A. Put the following statements on the board prior to the lesson.
Statement I--A large rock is laying on a road.
Statement II--A car is coming down the road.
Statement III--The driver sees the rock.
Statement IV--He turns the car to go around the rock.
Statement V--The car continues on its way.

B. Ask students which statement contains the stimulus. (III.)

C. Ask students which statement contains the response. (IV.)

D. Ask students the following question. "Between what two statements would the reaction time come"? (III and IV.)

E. Make sure that students can identify stimulus, response, and reaction time before proceeding.

F. Review with the students the use of the chart and meter stick in measuring reaction time as well as the variables which must be taken into consideration such as the method of dropping the stick, making sure that the subject is paying attention, and the problem of anticipation.

G. Tell the students that all of them will be participating in an experiment. Choose a student with whom you can demonstrate the procedure for the rest of the class.

H. Pass out a worksheet to each of the students. Have a fascimile of the worksheet on the board.

I. Perform the experiment with the student, filling in the chart on the board as the experiment progresses.

J. When the chart has been completed, have the students find the average reaction time by adding the individual reaction times and applying the rule for dividing by ten.

K. Have the students round the average reaction time to hundredths.

L. Discuss with students the general procedure used in the experiment. Make sure that topics such as readiness and anticipation are considered.

M. Divide the class into six groups. Have one student drop the stick, one try to catch it, one record the times for the student catching the stick, and others acting as observers to determine that the procedure is fair.

N. Give the students sufficient time for all of them to have an opportunity to catch the stick and have the results of their efforts recorded.

O. Give students an additional two or three minutes to find the average time that they took.

P. Ask the students to look at their charts and see if they can see any particular pattern to their times. (They may see that they improved their times as the experiment proceeded or that one time is either much greater or smaller than the others. Ask them to give possible reasons for these differences.

Q. Tell students to keep these papers since they will be used again in a later lesson.

REACTION TIME

TRIAL	TIME
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
TOTAL	

AVERAGE TIME - __________

AVERAGE TIME
TO THE NEAREST
HUNDREDTH - __________

TREATMENT A

LESSON VII

I. Objectives--The student should be able to:

A. Construct a number line.

B. Construct a correspondence between the positive rational numbers in intervals of tenths or hundredths and points on the number line.

C. Name and identify the vertical and horizontal axes and the origin.

D. Construct the vertical axis, the horizontal axis, and the origin.

E. Identify a point on a plane given axes and an ordered pair naming the point.

II. Materials

A. Graph paper

B. Rulers

C. Worksheet

III. Vocabulary

A. Vertical axis

B. Horizontal axis

C. Origin

D. Ordered pair

IV. Procedure

A. Discuss the experiment done in lesson VI. Review with the students the differences found in performance between the various trials and the reasons for them such as practice effect, anticipation, attentiveness, and method of dropping the stick.

B. Tell the students that it is often easier to see the difference in performance if they are shown on a graph.

C. Discuss the types of graphs with which the students may be familiar. They may have had some experience with bar graphs. Whether they have had experience with graphs or not, a review of the basic behaviors necessary for graphing will be undertaken.

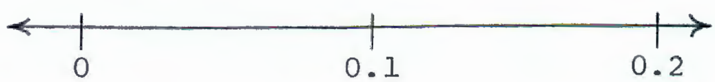
D. Ask a student to come to the board and draw a number line on the board. Have him show how he would place the set of numbers (0, 1, 2, 3, 4, 5) on the line.

E. Have another student expand the number line to ten. Ask students how far the number line could be extended. (Indefinitely.)

F. Put another number line on the board as shown below.

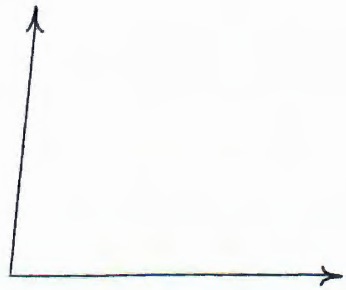
Ask the students how they would place the set of numbers (0.5, 0.9, 1.5, 1.7) on this number line. Assist students in this procedure if necessary. After completing this set, fill in all other units of tenths between 0 and 2.

G. Using a number line set up in the following manner, have students find the set of points (0.01, 0.05, 0.12, 0.16)

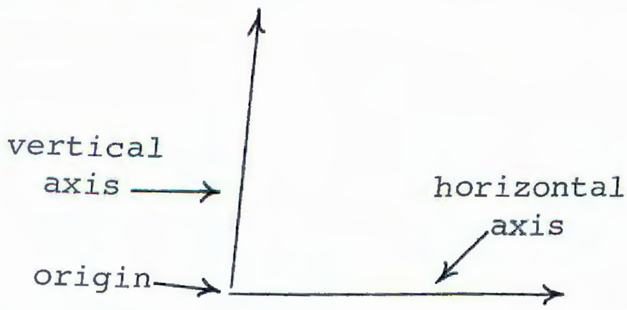


Following this procedure, fill in all units of hundredths between 0 and 0.2.

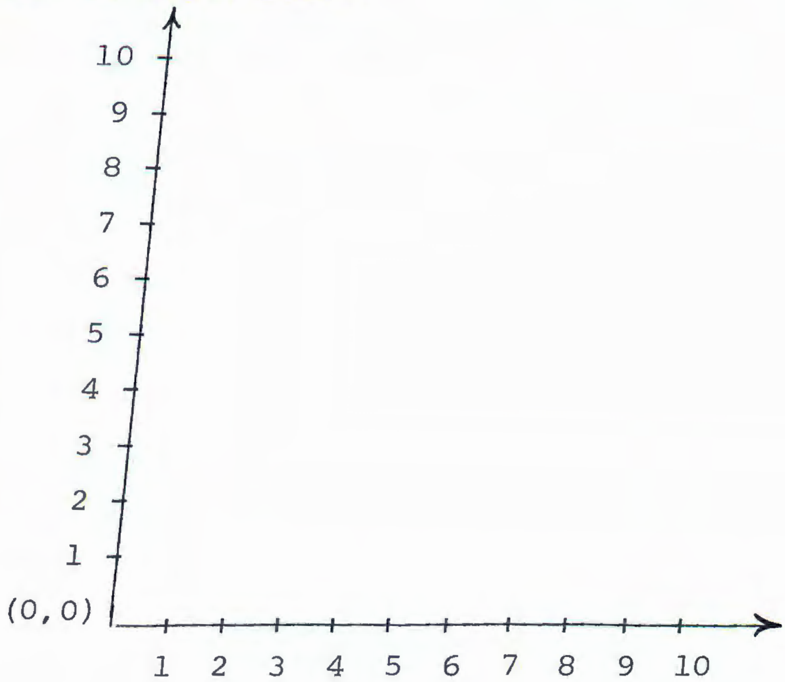
H. Tell students that in order to construct graphs, it is necessary to use two rays placed in the following manner.



I. Tell students the names of these lines. Label the horizontal axis, the vertical axis, and the origin. Make sure that the students can identify these before proceeding.



J. Pass out graph paper to the children and have them construct a set of axes. Have them label the graph as shown below.



Explain that the origin is represented by a pair of numbers in parentheses called an ordered pair. (Some students may already be familiar with ordered pair notation.)

K. Explain that these pairs of numbers are called ordered pairs since the first number tells you how many spaces are moved to the right and the second number tells how spaces are moved upward from the horizontal axis. (NOTE: It is not necessary that the children state this procedure if they are able to demonstrate the procedure.)

L. Ask students why they think the origin is represented by the ordered pair (0,0). (Because no spaces are moved to the right of the origin or up from the horizontal axis.)

M. Ask students where the point would be on the plane that is represented by the ordered pair (5,2). Follow this by the ordered pair (2,5). Place these on the graph or have a student do it. Emphasize the importance of order in the procedure.

N. Place the ordered pairs (3,7), (0,1), (5,8), (9,6), and (10,7) on the board and locate these points on the graph with student help.

O. Give out the worksheet, "Plotting Points," and have the students begin work on it. Use this time to give individual help.

P. The worksheet may be completed at home and checked at the beginning of the next lesson.

PLOTTING POINTS

Directions: Construct a set of horizontal and vertical axes on your graph paper. Number from 0 to 10 on the vertical axis and from 0 to 11 on the horizontal axis. Find the points represented by each ordered pair and connect the points by line segments as you find each point.

(3,2)
(3,6)
(7,10)
(9,8)
(9,9)
(10,9)
(10,7)
(11,6)
(11,2)
(8,2)
(8,4)
(6,4)
(6,2)

Finally, draw a line segment between (6,2) and (3,2). WHAT HAVE YOU DRAWN?

TREATMENT A

LESSON VIII

I. Objectives--The students should be able to:

A. Name a point with an ordered pair of numbers given its position on a plane and a pair of axes.

B. Construct a graph using decimals to the nearest hundredths as coordinates.

II. Materials

A. Graph paper

B. Two worksheets

III. Procedure

A. Distribute worksheet I. The students will be expected to construct an ordered pair to represent each of the numbered points on the plane.

B. Construct two or three of the ordered pairs with the help of the students.

C. Have students complete the construction of the ordered pairs. About seven minutes should be sufficient time to complete this exercise. Give individual help while the students are working.

D. Discuss the worksheet after students have completed this exercise.

E. Tell students that in order to graph reaction times it will be necessary to construct graphs in which the coordinates are expressed in hundredths. For this reason, we will practice constructing such graphs.

F. Distribute graph paper to the students. At the same time, distribute worksheet II to the students.

G. Ask students to look carefully at the ordered pairs on the worksheet.

H. Review the meaning of the first and second members of the ordered pairs with the students.

I. Ask the students whether they can tell, just by looking at the ordered pair on the worksheet, the units that should be used as coordinates. (Hundredths.)

J. Tell the students to look only at the first member of each ordered pair. Ask them the highest number represented by any first member. (0.13.)

K. Ask students how knowing the highest number in the set of first members will help them to construct the graph. (The horizontal axis must be numbered from 0 to 0.13.)

L. By a similar procedure, have the students identify 0.21 as the largest member of the set of second members of ordered pairs, requiring the vertical axis to be numbered from 0 to 0.21.

M. Have students construct the axes and place coordinates from 0 to 0.13 on the horizontal axis and from 0 to 0.21 on the vertical axis.

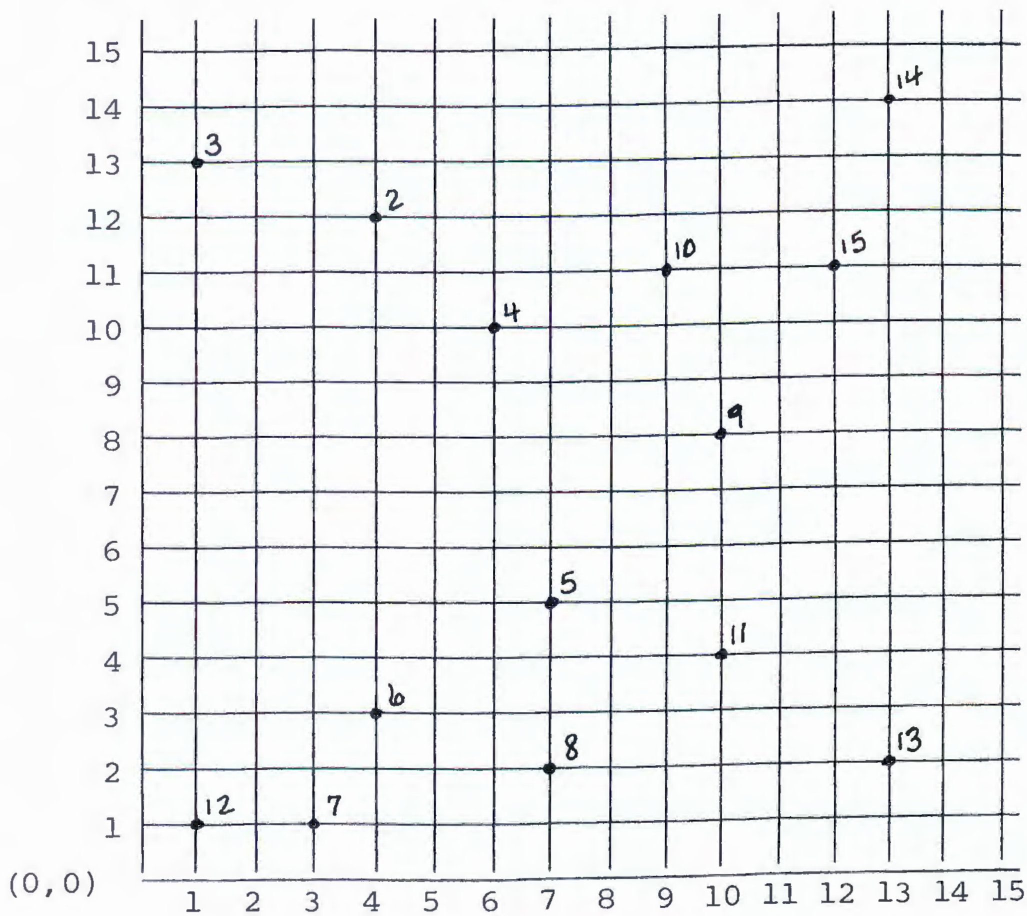
N. Referring to the worksheet, ask a student to come to the board and place the point represented by the ordered pair (0.09,0.07) on a facsimile of the graph constructed by the students. Have the students place the point on their papers. Continue this procedure with successive points from the worksheet, joining them by line segments as you proceed. Construct as many points as you think necessary to reinforce the procedure with the students.

O. Have students complete the graph, using this time to give students individual help.

P. When the students have completed this task, have them compare their effort with the graph supplied. Discuss their graphs and have students offer critical comments on the graph.

CONSTRUCTING ORDERED PAIRS

Construct ordered pairs on the spaces provided below for each of the points shown on the graph.



1) ______________ 6) ______________ 11) ______________

2) ______________ 7) ______________ 12) ______________

3) ______________ 8) ______________ 13) ______________

4) ______________ 9) ______________ 14) ______________

5) ______________ 10) ______________ 15) ______________

GRAPHING ORDERED PAIRS OF DECIMALS

Graph the following points and connect the points by a line segment as you find each point.

(0.09,0.07)

(0.06,0.04)

(0.06,0.07)

(0.08,0.08)

(0.06,0.09)

(0.06,0.10)

(0.03,0.08)

(0.06,0.12)

(0.07,0.13)

(0.06,0.13)

(0.03,0.14)

(0.06,0.15)

(0.08,0.21)

(0.10,0.15)

(0.13,0.14)

(0.10,0.14)

(0.10,0.09)

Finally, draw a line segment between (0.10,0.09) and (0.09,0.07).

TREATMENT A

LESSON IX

I. Objectives

A. Construct ordered pairs of numbers from available data.

B. Construct graphs involving data accumulated from the experiments.

C. Demonstrate an experiment in which a sound stimulus is followed by a response with the reaction time measured by indirect means.

II. Materials

A. Graph paper

B. Meter stick

C. Blindfold

III. Procedure

A. Ask students to place their data sheets from the experiment of lesson VI on their desks.

B. Ask them if they can think of a way to construct ordered pairs from the information on the data sheet. (Let the first member of the ordered pair be a Roman numeral representing a trial and the second member be a decimal numeral expressed in hundredths representing a reaction time.)

C. If the students are unable to think of this method, use the data sheet of a student in the class to construct the first ordered pair as an example and let the students provide the remaining ordered pairs. Then have students construct their own ordered pairs from their own data sheets.

D. When all ordered pairs have been completed, ask the students how these ordered pairs could be used to construct a graph. (Put the Roman numerals from I to X on the horizontal axis and the reaction times on the vertical axis.)

E. Ask the students what units should be used on the vertical axis. (Hundredths.)

F. Ask the students how they can determine how far they must number on the vertical axis. (It depends upon the largest number found in the reaction times.) NOTE: Students may need help in arriving at this conclusion.

G. Have the students construct the coordinate with your guidance as you put a facsimile on the board. NOTE: It would be a good idea to have students place coordinates from 0 to 0.30 on the vertical axis since 0.29 is the limit on the chart used by the students in performing the experiment. If you feel that it would not confuse the students, they could write every other coordinate.

H. After the students have completed the axes, use the ordered pairs from the sample to begin the graphing procedure. Connect the points by line segments as you proceed.

I. When you have done two or three examples, have the students use their own ordered pairs to do their individual graphs. Give individual help during this time.

J. Following completion of these tasks, discuss the graphs with the students in relation to improvement as the experiment progressed, attentiveness, and anticipation.

K. Tell the students that they have now completed an experiment in which the stimulus is a sight stimulus since the student actually sees the stick being dropped.

L. Ask them what senses could provide a stimulus in addition to sight. (Hearing, touching, smelling, and tasting.)

M. Discuss these with the students and ask them which of these we might use in an experiment similar to the one completed. In the discussion, bring out the inadvisability of using smell and taste as stimuli in this type of experiment. Students should bring out touch and hearing as the two senses that might be used.

N. Choose the sound stimulus as the next stimulus on which to base an experiment.

O. Ask students how they might set up an experiment similar to the previous experiment. The major differences between this experiment and the previous experiment would be:
   1. In order for the sight stimulus to be inactivated, the subject must be blindfolded.
   2. Some sound stimulus must be used such as the word, "Now!" or, "Go!"
   3. The person dropping the meter stick must be sure to release the meter stick and give the stimulus as closely as possible to the same time.

P. Put a data chart on the board similar to that given in lesson VI. Select a student to act as a subject.

Q. Give the student a few practice trials after blindfolding him and then give him ten trials having another student record his times on the board. The students should be able to determine these times by consulting their centimeter-time charts.

R. Add the times and compute an average time and then construct the ordered pairs with student assistance.

S. Use these ordered pairs to construct a graph of the results. Have the students give the method of performing this construction.

T. Tell the students that they will conduct the experiment themselves in the next lesson.

TREATMENT A

LESSON X

I. Objectives--The students should be able to:

A. Demonstrate an experiment in which a sound stimulus is followed by a response with the reaction time measured by indirect means.

B. Construct ordered pairs of numbers from available data.

C. Construct graphs involving data accumulated from the experiments.

II. Materials

A. Student data sheets

B. Six meter sticks

C. Student procedure sheets

D. Six blindfolds

E. Graph paper

III. Procedure

A. Distribute the student procedure sheets and discuss them.

B. Divide the students into six groups.

C. Allow the students to perform the experiments.

D. Carry this procedure on to completion, covering all items on the student procedure sheets.

E. Have the students construct the graph and join the points graphed using a colored pencil.

F. Have them also graph points from the experiment completed in lesson VI on the <u>same</u> graph using a pencil of a different color. This is done so that comparisons can be made between the two efforts in lesson XI.

G. If the students do not complete these tasks after a reasonable period of time, the completion should be assigned for homework.

H. Tell the students that these results will be discussed during the next lesson.

STUDENT PROCEDURE SHEET

1. Choose a student to drop the stick.
2. Choose a student to catch the stick.
3. Choose a student to act as a recorder.
4. Other students in the group will watch closely to make sure that the procedure is fair.
5. Change jobs each time a student completes the trials.
6. Give each student four practice trials and ten recorded trials.
7. Observers should help the recorder to compute the times from the centimeter-time sheet.
8. After everyone in the group is finished, return to your seat and compute your average reaction time.
9. Compute the ordered pairs from your results.
10. Construct a graph from the ordered pairs.

REACTION TIME

TRIAL	TIME
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
TOTAL	

AVERAGE TIME - ________________

AVERAGE TIME
TO THE NEAREST
HUNDREDTH - ________________

TREATMENT A

LESSON XI

I. Objectives--The student should be able to:

A. Construct explanations for reaction time patterns pictured on a graph.

B. Demonstrate an experiment in which a touch stimulus is followed by a response with the reaction time measured by indirect means.

II. Materials

A. One meter stick

B. One blindfold

III. Procedure

A. Have several students draw a crude line graph on the board representing their results of the previous experiment. If possible, students could do a more accurate job on the graphs if time was available prior to the lesson.

B. Use these graphs as examples to discuss the results of the experiment. Have the students interpret these graphs in relation to practice effect, attentiveness, dropping procedure, and anticipation.

C. Following this procedure, ask the students if they remember what sense we have not used experimentally that it was decided could be used. (Touch.)

D. Develop, with the students, a format for doing an experiment involving touch.

E. The only new skill that is a cause of difficulty in this experiment is the ability to touch the person and release the stick at the same time.

F. When the procedure has been agreed upon, have one student drop the stick, another catch the stick, and a third record the results on the board as in previous experiments.

G. After recording the data, have the students construct ordered pairs from this data.

H. Finally, have the students tell you the steps in constructing a graph of the data.

I. Complete the graph on the board under student direction and then have the students interpret this graph.

TREATMENT A

LESSON XII

I. Objectives--The student should be able to:

Construct explanations for reaction time patterns pictured on a graph.

II. Materials--Student worksheet

III. Procedure

A. Distribute the worksheets.

B. Allow students to work on the worksheets for as much time as you think necessary. The time required will vary according to the ability of the class.

C. Some individual help may be given during the work period although the students should be encouraged to work with as much independence as possible.

D. Discuss the worksheet with the students stressing the method used for computing the average, trials in which anticipation of the stimulus may have been a factor, the effect of practice on reaction time and the role of attentiveness in any of the trials.

E. Try to have students use the applicable vocabulary as much as possible and have them verbally construct ordered pairs from the graph as they discuss particular points on the graph.

INTERPRETING GRAPHS

1. Did A or B have the fastest reaction time? _____

2. Did A or B have the slowest reaction time? _____

3. What do you think may have caused the reaction time of B in trial VI?

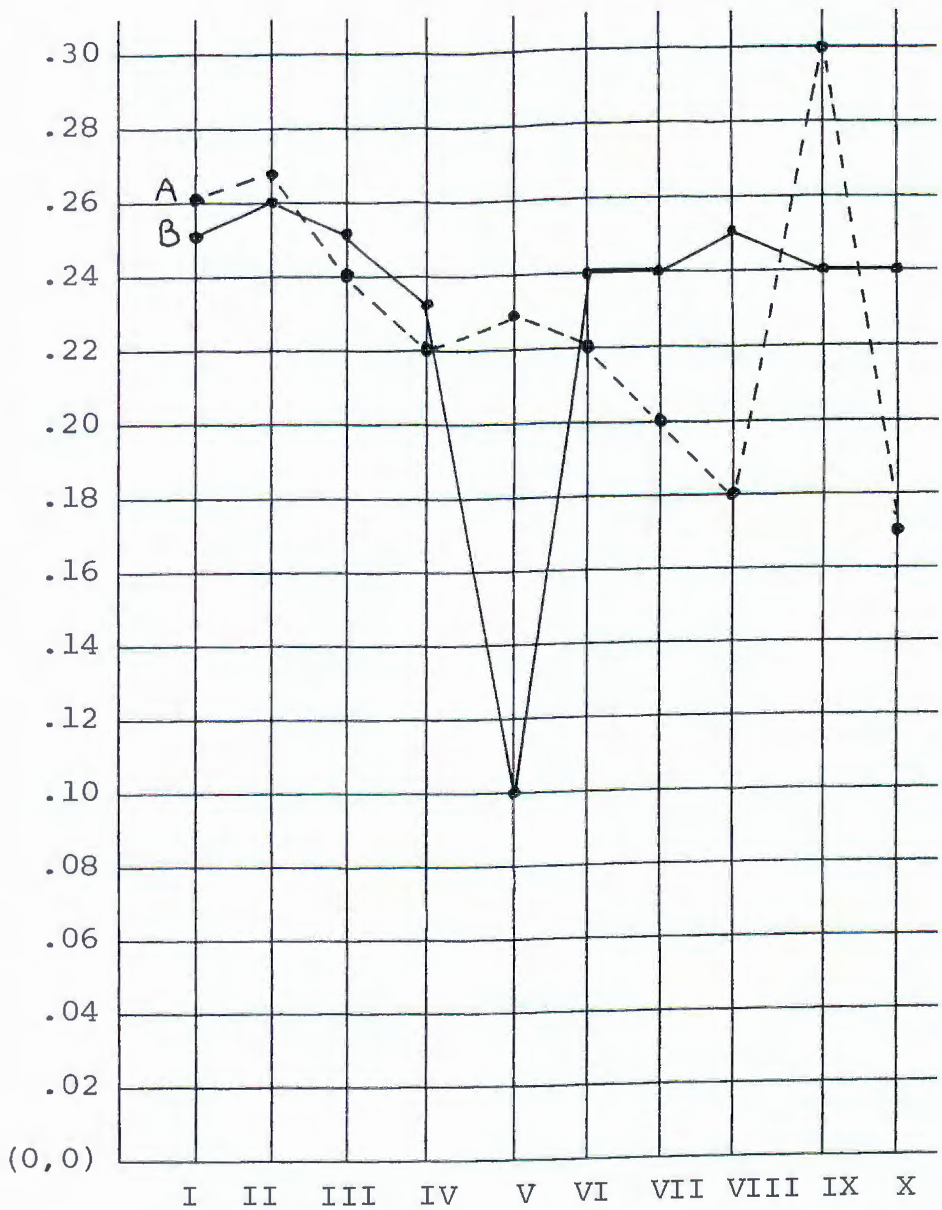
4. What do you think may have caused the reaction time of A in trial IX?

5. Demonstrate how you would find A's average reaction time.

6. Demonstrate how you would find B's average reaction time.

7. Who do you think showed the most improvement as the trials proceeded? Why?

Two students each took ten trials in an experiment involving reaction time. The graph shows the results of the trials. Examine the graph carefully and then answer the questions about the graph.



APPENDIX D

LESSON PLANS FOR TREATMENT B

TREATMENT B

LESSON I

I. Objectives--The student should be able to:

A. Name and identify decimal numbers expressed in tenths and hundredths.

B. Distinguish decimal numbers expressed in hundredths from decimal numbers expressed in tenths.

C. Order decimals expressed in tenths and hundredths.

D. Demonstrate a procedure for adding decimals.

II. Materials--Worksheet

III. Procedure

A. Place the following fractions on the board and have the students read them:

$$\frac{1}{10} \qquad \frac{4}{10} \qquad \frac{7}{10}$$

$$\frac{2}{10} \qquad \frac{5}{10} \qquad \frac{8}{10}$$

$$\frac{3}{10} \qquad \frac{6}{10} \qquad \frac{9}{10}$$

B. Show the students that the fraction $\frac{9}{10}$ is written decimally as 0.9. Explain that this is read "nine tenths." Have students supply the proper decimal notation and names for the remaining fractions.

C. Explain that the decimal point separates whole numbers from fractional numbers and give several examples.

D. Put the fraction $\frac{25}{100}$ on the board and ask students if they can guess how this could be written as a decimal. If they can, give several more examples to reinforce this ability. If they cannot, explain that $\frac{25}{100}$ may be written as 0.25 and then give other examples.

E. As an exercise, have the following decimals on the board. Have the students orally name these decimals.

0.7	0.62	0.59	0.4	0.19
0.16	0.53	0.9	1.7	0.97

F. After completing this exercise, ask students which of the numbers is the largest, the smallest. If they have any difficulty, explain to them that any whole number is larger than any decimal fraction less than one and that by annexing a zero to a decimal expressed in tenths, the decimal may be expressed in hundredths.

G. Working with the students, arrange the preceding decimals in ascending order.

H. Following this, have students write the following numbers in ascending order.

0.64	1.73	1.5	1.8	0.94
0.58	0.9	0.97	0.89	0.15

I. Go over this ordering with the students to make sure that they can perform the ordering procedure.

J. Then, put the following fractions on the board.

$$\frac{3}{10}$$

$$\frac{2}{10}$$

$$\frac{1}{10}$$

K. Ask the students what the sum of these fractions is and place the sum under the fractions as illustrated below.

$$\begin{array}{c} \frac{3}{10} \\[2ex] \frac{2}{10} \\[2ex] \frac{1}{10} \\ \hline \frac{6}{10} \end{array}$$

L. Ask students how each of the above fractions is written as a decimal. Put these down as illustrated below.

$$\begin{array}{ccc} \frac{3}{10} & = & .3 \\[2ex] \frac{2}{10} & = & .2 \\[2ex] \frac{1}{10} & = & .1 \\ \hline \frac{6}{10} & = & .6 \end{array}$$

M. Do a similar procedure using the fractions $\frac{8}{10}$, $\frac{5}{10}$, and $\frac{6}{10}$.

N. When you have done, in your judgment, an adequate number of illustrations, ask the students whether they can see a pattern to how decimals are added. (Decimal points lined up and add as usual.)

O. Using a similar procedure, do several additions involving hundredths.

P. Distribute the worksheet; have the students begin it in class and assign the remaining problems for homework if insufficient time remains to complete the assignment in class.

Q. Discuss the worksheet at the beginning of lesson II.

ADDITION OF DECIMALS

ADD THE FOLLOWING DECIMALS:

1)	2)	3)	4)
.2	.7	.26	.93
.6	.8	.71	.63
.4	.6	.51	1.05
.3	1.4	.36	2.13

ARRANGE THE FOLLOWING DECIMALS IN COLUMNS AND THEN ADD:

5) 0.1 + 0.6 + 0.5 + 0.9

6) 0.53 + 0.74 + 0.86 + 0.52

7) .84 + .7 + .5 + 1.3

8) 7.32 + .58 + .72 + .4

TREATMENT B

LESSON II

I. Objectives--The student should be able to:

   A. Apply the rule for rounding decimals to the nearest tenth.

   B. Apply the rule for rounding decimals to the nearest hundredth.

II. Materials--Worksheet

III. Procedure

   A. Put the following decimal numerals on the board prior to the lesson.

0.72	0.6	0.87	0.8	0.91
3.72	4.3	0.5	0.7	0.03

   B. Discuss worksheet from the preceding day.

   C. Following this, have the students name the numerals. It may be necessary to emphasize the fact that when a decimal numeral contains a whole number as in 3.72, the numeral is read three <u>and</u> seventy-two hundredths.

   D. Put the following decimals on the board.

0.534	0.832	0.756	0.842	0.973

   Ask the students how they think 0.534 would be read as a decimal. If they can read this one successfully, have them read the remaining numbers. If necessary, give the students additional practice on reading these numerals before proceeding.

   E. Review the rule for rounding whole numbers to the nearest ten and the nearest hundred. Emphasize the fact that the digit in the column to the right of the column to be rounded off determines whether the digit in the column to be rounded stays the same or is raised by one. <u>It is not necessary for the students to formally state the rule if they can perform the procedure</u>.

F. Use the following method of presenting this topic:

1. Put the following numerals on the board:

   57 62 93 79 41

2. Ask students what each of these numbers would be rounded to the nearest ten. Ask them to explain in their own words how they determined the answer. (Number in the column to the right [units column] determines whether the digit in the tens column stays the same or is raised by one.)

3. Use a similar procedure with the numbers below, rounding to the nearest hundred.

   126 437 596 782 648

G. When the rounding procedure has been reviewed sufficiently for whole numbers, consider decimals to be rounded to the nearest tenth and to the nearest hundredth using the same methods.

H. The following two sets of decimals may be used to teach the rounding procedure for tenths and hundredths.

1) 0.63 0.87 0.56 0.94 0.98

2) 0.842 0.927 0.563 0.124 0.072

I. Pass out the worksheet to students and allow them to work on them for the remainder of the period, using this time for giving individual help to the students.

J. The worksheet will be finished for homework if time does not remain in the period.

K. The worksheet will be discussed at the beginning of the next lesson.

ROUNDING DECIMALS

I. Round the following decimals to the nearest tenth. Put your answer in the space provided.

1) 0.72 ______ 2) 0.96 ______ 3) 0.05 ______ 4) 1.44 ______

5) 0.67 ______ 6) 0.81 ______ 7) 0.92 ______ 8) 2.99 ______

II. Round off the following decimals to the nearest hundredth. Put your answer in the space provided.

1) 0.738 ______ 2) 0.972 ______ 3) 0.876 ______ 4) 5.731 ______

5) 0.996 ______ 6) 8.898 ______ 7) 0.237 ______ 8) 0.947 ______

TREATMENT B

LESSON III

I. Objectives--The students should be able to:

A. Apply the rule for dividing a number by ten.

B. Apply the rule for finding the average of ten whole numbers to one decimal place.

C. Apply the rule for finding the average of ten decimal numbers.

II. Materials--Worksheet

III. Procedure

A. Discuss the worksheet that was distributed in lesson two. Spend as much time on the discussion as you feel necessary.

B. Following this discussion, place the following numbers on the board.

20	80	130	210	500

C. Ask the students what the quotient would be if each of the numbers was divided by ten. Put these answers down as shown.

20	80	130	210	500
2	8	13	21	50

D. Ask the student what the only difference is in the number after it has been divided by ten. (The zero is gone.)

E. Ask the students where the decimal point would go in each of these numbers if we wished to put one in. (To the right of the number.) Place the decimal point to the right of each of the numbers shown below.

20.	80.	130.	210.	500.
2.	8.	13.	21.	50.

F. Explain to the students that a whole number may be written with a zero to the right of the decimal point and have the same value since a number like 21 could be written as 21.0 which would be read as twenty-one and zero tenths which is equal to twenty-one. Annex a zero to each of the quotients on the board as shown below.

20.	80.	130.	210.	500.
2.0	8.0	13.0	21.0	50.0

G. Ask the students what the difference is between the position of the decimal point in the original number and the decimal point in the quotient. (It has been moved one place to the left.)

H. Ask students if they can now tell an easy way to divide a number by ten. (Move the decimal point one place to the left.)

I. Put other multiples of ten on the board and have students show how they would divide the numbers by ten using the rule discovered.

J. When students can do this procedure, put the following numbers on the board.

31. 52. 83. 78.

K. Ask the students how they might apply the same rule to divide the above numbers by ten. The answers should be written as shown below.

31.	52.	83.	78.
3.1	5.2	8.3	7.8

L. Then have the students apply the rule to divide the following numbers by ten.

3.8 1.72 5.76 7.54 11.53

M. Give further examples if necessary.

N. Following completion of the preceding exercise, put the following numbers on the board.

9, 7, 5, 4, 8, 7, 9, 1, 8, 2

Ask the students how to find the average of these numbers. (Add the numbers and divide by ten.)

Finding averages should not be difficult for students since they had work in finding averages in grade four. If they have trouble, however, give them a simple example of finding an average before proceeding.

O. When the process of finding an average has been established, add the numbers as shown below.

$$\begin{array}{r} 9 \\ 7 \\ 5 \\ 4 \\ 8 \\ 7 \\ 9 \\ 1 \\ 8 \\ 2 \\ \hline 60 \end{array}$$

P. Ask students for a quick method of dividing the sum by ten. (Move the decimal point one place to the left.)

Q. Find averages of the following sets of numbers.

1) 9, 5, 6, 7, 8, 6, 4, 9, 2, 3

2) .6, .9, .7, .8, .6, .6, .8, .9, .3, .1, .2

3) .27, .18, .56, .42, .13, .22, .14, .21, .11, .16

R. After doing the preceding problems and discussing them with the students, distribute the worksheet and have the students work on them while individual help is given.

S. The work may be completed for homework and will be discussed in the next lesson.

FINDING AVERAGES

Find the averages of the following sets of numbers.

1) .8, .7, .8, .6, .6, .5, .8, .7, .9, .3

2) .2, .6, .5, .4, .8, .3, .7, .6, .8, .1

3) .21, .18, .14, .20, .17, .21, .23, .25, .22, .15

4) .16, .26, .34, .30, .25, .14, .23, .29, .17, .28

TREATMENT B

LESSON IV

I. Objectives--The student should be able to:

A. Construct a correspondence between the positive rational numbers in intervals of tenths and hundredths and points on the number line.

B. Construct a number line.

C. Name and identify the vertical and horizontal axes and the origin.

D. Construct the vertical axis, the horizontal axis, and the origin.

E. Identify a point on a plane given axes and an ordered pair naming the point.

II. Materials

A. Graph paper

B. Ruler

C. Worksheet

III. Vocabulary

A. Vertical axis

B. Horizontal axis

C. Origin

D. Ordered pair

IV. Procedure

A. Discuss the types of graphs with which the students may be familiar. They may have had some experience with bar graphs. Whether they have had experience with graphs or not, a review of the basic behaviors necessary for graphing will be undertaken.

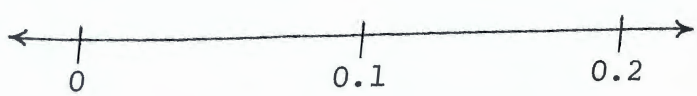
B. Ask a student to come to the board and draw a number line on the board. Have him show how he would place the set of numbers (0, 1, 2, 3, 4, 5) on the line.

C. Have another student expand the number line to ten. Ask students how far the number line could be extended. (Indefinitely.)

D. Put another number line on the board as shown below.

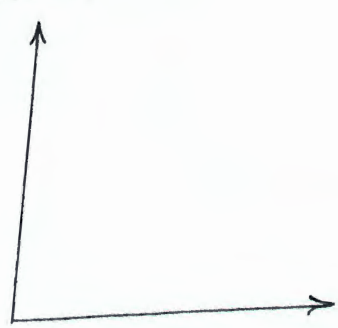
Ask the student how he would place the set of numbers (0.5, 0.9, 1.2, 1.7) on this number line. Assist students in this procedure if necessary. After completing this set, fill in all other units of tenths between 0 and 2.

E. Using a number line constructed in the following manner, have students find the set of points (0.01, 0.05, 0.12, 0.16).

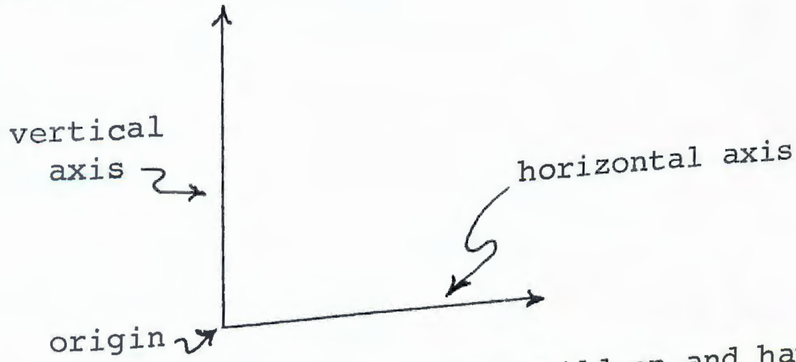


Following this procedure, fill in all units of hundredths between 0 and 0.2.

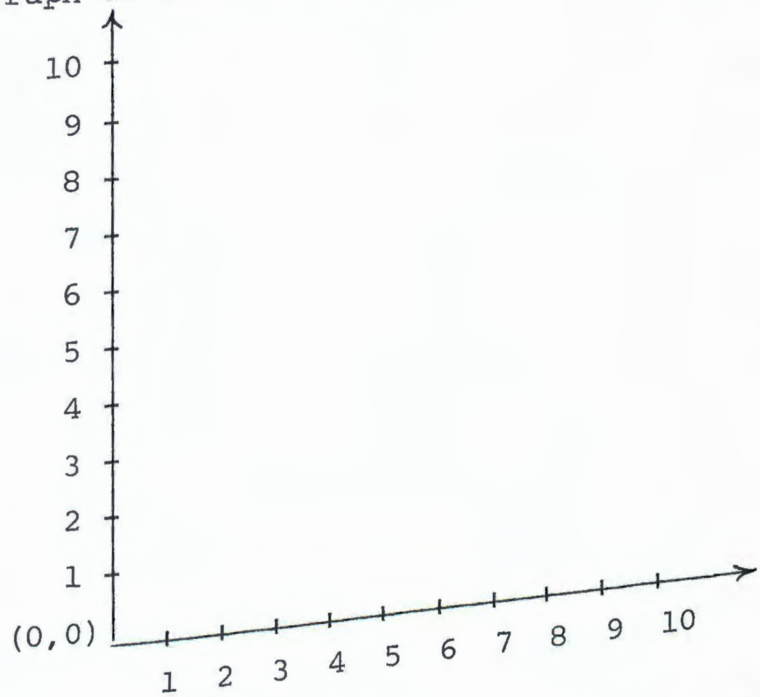
F. Tell students that in order to construct graphs, it is necessary to use two rays placed in the following manner.



G. Tell students the names of these lines. Label the horizontal axis, the vertical axis, and the origin. Make sure that the students can identify these before proceeding.



H. Distribute graph paper to the children and have them construct a set of axes. Have them label the graph as shown on the following drawing.



Explain that the origin is represented by a pair of numbers in parentheses called an ordered pair. (Some students may already be familiar with ordered pair notation.)

I. Explain that these pairs of numbers are called ordered pairs since the first number tells you how many spaces are moved to the right from the origin and the second number tells how many spaces are moved up from the horizontal axis.

J. Ask the students why they think that the origin is represented by the ordered pair (0,0). (Because no spaces are moved to the right of the origin or up from the horizontal axis.)

K. Ask students where the point would be on the plane that is represented by the ordered pair (5,2). Follow this by the ordered pair (2,5). Place these on the graph or have a student do it. Emphasize the importance of order in the procedure.

L. Place the ordered pairs (3,7), (0,1), (5,8), (9,6), and (10,7) on the board and locate these points on the graph with student help.

M. Give out the worksheet, "Plotting Points," and have the students begin work on it. Give individual help at this time.

N. The worksheet may be completed at home and checked at the beginning of the next lesson.

PLOTTING POINTS

Directions: Construct a set of horizontal and vertical axes on your graph paper. Number from 0 to 10 on the vertical axis and from 0 to 11 on the horizontal axis. Find the points represented by each ordered pair and connect the points by line segments as you find each point.

(3,2)
(3,6)
(7,10)
(9,8)
(9,9)
(10,9)
(10,7)
(11,6)
(11,2)
(8,2)
(8,4)
(6,4)
(6,2)

Finally, draw a line segment between (6,2) and (3,2). <u>WHAT</u> <u>HAVE</u> <u>YOU</u> <u>DRAWN</u>?

TREATMENT B

LESSON V

I. Objectives--The students should be able to:

A. Name a point with an ordered pair of numbers given its position on a plane and a pair of axes.

B. Construct a graph using decimals to the nearest hundredth as coordinates.

II. Materials

A. Graph paper

B. Two worksheets

III. Procedure

A. Distribute worksheet I. The students will be expected to construct an ordered pair to represent each of the numbered points on the plane.

B. Construct two or three of the ordered pairs with the help of the students.

C. Have students complete the construction of the ordered pairs. About seven minutes should be sufficient time to complete this exercise. Give individual help while the students are working.

D. Discuss the answers to the worksheet before proceeding.

E. Distribute graph paper to the students. At the same time, distribute Worksheet II to the students.

F. Ask students to look carefully at the ordered pairs on the worksheet.

G. Review the meaning of the first and second members of the ordered pairs with the students.

H. Ask the students whether they can tell, just by looking at the ordered pairs on the worksheet, the units that should be used as coordinates. (Hundredths.)

I. Tell the students to look only at the first member of each ordered pair. Ask them the highest number represented by any first member. (0.13.)

J. Ask students how knowing the highest number in the set of first members will help them to construct the graph. (The horizontal axis must be numbered from 0 to 0.13.)

K. By a similar procedure, have the students identify 0.21 as the largest member of the set of second members of ordered pairs, requiring the vertical axis to be numbered from 0 to 0.21.

L. Have students construct the axes and place coordinates from 0 to 0.13 on the horizontal axis and from 0 to 0.21 on the vertical axis.

M. Referring to the worksheet, ask a student to come to the board and place the point represented by the ordered pair (0.09,0.07) on a facsimile of the graph constructed by the students. Have the student place the point on their papers. Continue this procedure with successive points from the worksheet, joining them by line segments as you proceed. Construct as many points as you think necessary to reinforce the procedure with the students.

N. Have students complete the graph, using this time to give students individual help.

O. When the students have completed this task, have them compare their effort with the graph supplied. Discuss their graphs and have students offer critical comments on the graph.

GRAPHING ORDERED PAIRS OF DECIMALS

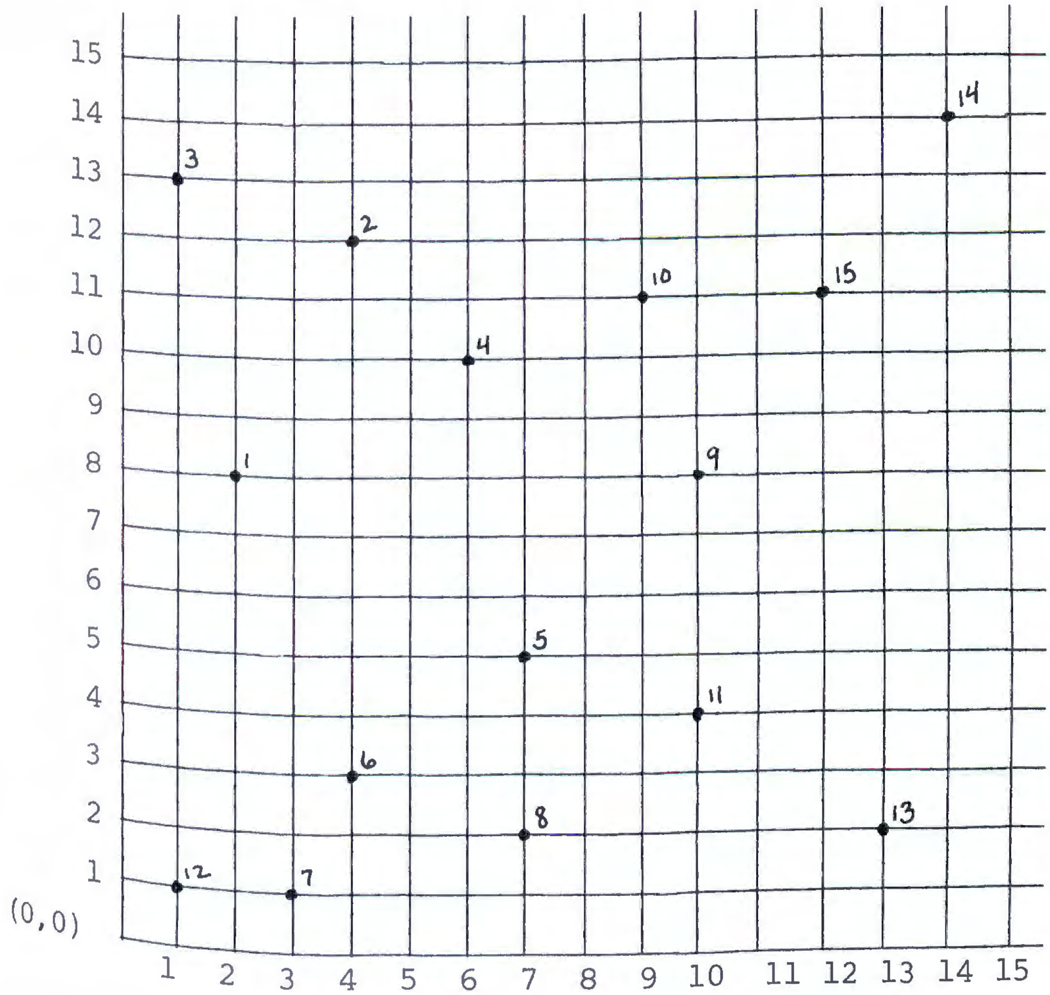
Graph the following points and connect the points by a line segment as you find each point.

(0.09,0.07)
(0.06,0.04)
(0.06,0.07)
(0.08,0.08)
(0.06,0.09)
(0.06,0.10)
(0.03,0.08)
(0.06,0.12)
(0.07,0.13)
(0.06,0.13)
(0.03,0.14)
(0.06,0.15)
(0.08,0.21)
(0.10,0.15)
(0.13,0.14)
(0.10,0.14)
(0.10,0.09)

Finally, draw a line segment between (0.10,0.09) and (0.09,0.07).

CONSTRUCTING ORDERED PAIRS

Construct ordered pairs on the spaces provided below for each of the points shown on the graph.



1) __________ 6) __________ 11) __________

2) __________ 7) __________ 12) __________

3) __________ 8) __________ 13) __________

4) __________ 9) __________ 14) __________

5) __________ 10) __________ 15) __________

TREATMENT B

LESSON VI

I. Objectives--The student should be able to:

  A. Name and identify a response.

  B. Name and identify a stimulus.

  C. Distinguish a stimulus from a response.

  D. Name and identify a premature response as the result of anticipation.

II. Materials

  A. New dollar bill

  B. Paper cut to dollar bill size

III. Vocabulary words

  A. Stimulus

  B. Response

  C. Anticipation

IV. Procedure

  A. Tell the students that you are going to show them an old trick. Have a student help. Have the student hold his hand with the palm vertical and his thumb and forefinger approximately one inch apart.

  B. Place a new dollar bill between the student's thumb and forefinger so that the picture on the bill is between the thumb and forefinger.

  C. Explain to the class that it is very difficult to catch the bill when it is dropped unless one starts to grab for the bill before it is released.

D. Before demonstrating this procedure with the student, explain that usually the person who is trying to catch the dollar bill is told that he can keep the bill if he catches it. Emphasize that this is not the case in the coming demonstration.

E. Drop the bill several times for the student. Tell the students to observe the procedure carefully.

F. Ask what act causes the student to grab for the bill. (The student grabs for the bill when the bill is released by the teacher.)

G. Ask the students what happens when the bill is released. (Student grabs for the bill.)

H. The entire procedure involves, therefore, two separate actions. What are they? (Teacher dropping bill, student grabbing for bill.)

I. Explain that in this situation the dropping of the bill is called a stimulus and the student grabbing for the bill is called a response. Write the words stimulus and response on the board for further reference.

J. Give one or two examples of situations involving a stimulus and the response and then let students volunteer similar situations. Have students identify the stimulus and the response in each case.

K. Divide the students into groups of three or four. Using paper cut to dollar bill size, have one drop the paper, another act as the catcher, and the others in the group act as observers. Positions may be rotated as the activity progresses. Allow about seven minutes for this activity.

L. Following this activity, ask students if any of them were able to catch the bill. If the answer was yes, ask observers if the student in question started his response prior to the release of the bill.

M. Explain that giving a premature response is called an anticipation. Write this word on the board also.

N. Ask students how long they think it takes for the bill to drop through one's fingers. (Various guesses will be made, but students should see at this time that they have no method of measuring the time lapse.)

O. Tell students that in the next lesson, a method of determining short periods of time will be considered.

V. Summary--Using some situations involving stimulus and response, have students identify the stimulus and the response.

TREATMENT B

LESSON VII

I. Objectives--The student should be able to:

A. Name and identify a centimeter.

B. Name and identify reaction time.

C. Name and identify an attention-getting procedure.

D. Name and identify variables in human subjects and in experimental procedures that must be controlled to complete a valid experiment.

II. Materials--meter stick, Reaction time chart

III. Vocabulary

A. Reaction time

B. Centimeter

C. Attention-getting procedure

IV. Procedure

A. Put the following drill on the board.

Write the sentence containing the stimulus and the sentence containing the response in the following:

A boy is riding a bicycle down the street. He sees a stop sign. He puts the brakes on. The bicycle stops.

B. After students have completed the drill, go over the drill with the students in order to determine their ability to name and identify the stimulus. (He sees a stop sign) and the response (He puts the brakes on).

C. Ask students how long they think it would take the boy to brake his bicycle after seeing the stop sign. Again, as in the previous lesson, there will be difficulty in determining how long the action took with only guesses supplying the times. This will lead students to discover that a better method of measuring the time must be found.

D. Tell the students that the term "reaction time" will refer to the interval of time between the stimulus and the response.

E. Give students a copy of the chart provided. Tell them that a fairly accurate measure of reaction time may be made by using this chart.

F. Demonstrate with several pupils how to make use of the chart in measuring reaction time using the following procedure.

> Place a piece of colored tape that the students can see clearly at the 30 cm. mark on the stick. When performing the experiment, have the child hold his hand in the same position as it was held when experimenting with the dollar bill. Place the meter stick between his thumb and forefinger with the tape at that point. When the stick is dropped and the child catches it, the scale on the stick is read as the centimeter closest to the child's thumb. When thirty is subtracted from this number, the number of centimeters that it dropped may be determined. By then referring to the chart, the time elapsed may be determined.

G. Give a student ten trials and have another student record the reaction time on the board for each trial. With experience, the child should improve, although there is a psychological limit below which he cannot go. Distract the student at least once during the trials and drop the stick when he is unprepared. The stick will then fall further than on previous trials. Ask students if this trial was a good measure of his reaction time. (No, since he was not paying attention.) Ask how the student might insure that the child pays attention while waiting for the stimulus. (Give a warning by saying "ready" or some other attention-getting process.)

H. To prevent children from anticipating the stimulus and grabbing the stick before it is dropped, vary the period between the warning and the dropping so that the child will only respond when the stick is released.

I. Ask students what things they have noticed in the trials that must be done the same by all students to make sure that everyone gets a fair trial. List these on the board. They should include items like the following:

1. Student must be paying attention.
2. Meter sticks should be dropped, not pushed downward.
3. Students must not anticipate the stimulus.
4. Students must have equal amounts of practice.

J. Students may think of other considerations that should be listed according to their merits.

V. Summary

Ask the following questions:

1. How can we be sure that a person is ready for a trial?
2. What is a reaction time?
3. How can we stop a person from anticipating the release of the stick?

CHART I

DISTANCE AND TIME TO FALL FROM REST

Centimeters	Seconds
7	.12
8	.13
9-10	.14
11	.15
12-13	.16
14-15	.17
16	.18
17-18	.19
19-20	.20
21-22	.21
23-24	.22
25-27	.23
28-29	.24
30-31	.25
32-34	.26
35-37	.27
38-39	.28
40	.29

TREATMENT B

LESSON VIII

I. Objectives--The student should be able to:

Demonstrate an experiment in which a sight stimulus is followed by a response with the reaction time measured by indirect means.

II. Materials

A. Six meter sticks

B. Tally sheets

III. Procedure

A. Put the following statements on the board prior to the lesson.

Statement I--A large rock is laying on a road.
Statement II--A car is coming down the road.
Statement III--The driver sees the rock.
Statement IV--He turns the car to go around the rock.
Statement V--The car continues on its way.

B. Ask students which statement contains the stimulus. (III.)

C. Ask students which statement contains the response. (IV.)

D. Ask students the following question: "Between what two statements would the reaction time come?" (III and IV.)

E. Make sure that students can identify stimulus, response, and reaction time before proceeding.

F. Review with the students the use of the chart and meter stick in measuring reaction time as well as the variables which must be taken into consideration such as the method of dropping the stick, making sure that the subject is paying attention, and the problem of anticipation.

G. Tell the students that all of them will be participating in an experiment. Choose a student with whom you can demonstrate the procedure for the rest of the class.

H. Pass out a worksheet to each of the students. Have a facsimile of the worksheet on the board.

I. Perform the experiment with the student, filling in the chart on the board as the experiment progresses.

J. When the chart has been completed, have the students find the average reaction time by adding the individual reaction times and applying the rule for dividing by ten.

K. Have the students round the average reaction time to hundredths.

L. Discuss with students the general procedure used in the experiment. Make sure that topics such as readiness and anticipation are considered.

M. Divide the class into six groups. Have one student drop the stick, one try to catch it, one record the times for the student catching the stick, and others acting as observers to determine that the procedure is fair.

N. Give the students sufficient time for all of them to have an opportunity to catch the stick and have the results of their efforts recorded.

O. Give students an additional two or three minutes to find the average time that they took.

P. Ask the students to look at their charts and see if they can see any particular pattern to their times. (They may see that they improved their times as the experiment proceeded or that one time is either much greater or smaller than the others. <u>Ask them to give possible reasons for these differences</u>.)

Q. Tell students to keep these papers since they will be used again in a later lesson.

REACTION TIME

TRIAL	TIME
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
TOTAL	

AVERAGE TIME - ______________

AVERAGE TIME
TO THE NEAREST
HUNDREDTH - ______________

TREATMENT B

LESSONS IX THROUGH XII

Lessons IX, X, XI, and XII are exactly the same as the lessons of the same number in Treatment A.

APPENDIX E

THE ACHIEVEMENT MEASURE WITH

ACCOMPANYING TEST GUIDE

TESTING DIRECTIONS

1. Place a sign on the classroom door which says, "TESTING, PLEASE DO NOT DISTURB."
2. Make sure that each student has a pencil and ruler.
3. Distribute one piece of scratch paper to each student.
4. Tell students, "I AM GOING TO DISTRIBUTE THE TEST. PLEASE DO NOT LOOK AT THE TEST OR OPEN THE TEST UNTIL I TELL YOU TO BEGIN."
5. Distribute the tests.
6. Say to the students, "WRITE YOUR NAME AND MY NAME ON THE TEST BOOKLET IN THE SPACES PROVIDED."
7. Say to the students, "YOU WILL BE GIVEN PLENTY OF TIME TO FINISH THE TEST. WORK STEADILY, BUT CAREFULLY. <u>DO NOT HURRY</u>! IF YOU FINISH BEFORE TIME IS UP, YOU MAY READ A LIBRARY BOOK FOR THE REMAINDER OF THE TESTING TIME.
8. Say to the students, " DO <u>NOT</u> DO QUESTIONS ONE AND TWO. FOLLOW THE DIRECTIONS FOR EACH OF THE QUESTIONS CAREFULLY. START WITH QUESTION III. <u>YOU MAY BEGIN</u>!
9. Answer no questions as the test proceeds.
10. At the end of 75 minutes, say, "STOP! PUT YOUR PENCILS DOWN. DO NOT WRITE ANOTHER WORD. CLOSE YOUR TEST BOOKLET AND PUT IT AT THE UPPER LEFT HAND CORNER OF YOUR DESK.
11. Collect the tests and then collect the scratch paper. Throw the scratch paper away.

MEASURE II

STUDENT'S NAME ______________________________

TEACHER'S NAME ______________________________

ANSWER THE FOLLOWING QUESTIONS USING THE SPACE PROVIDED

1. The metric measurement that your teacher is illustrating is called a ________________.

2. What unit in the metric system is larger than a millimeter and smaller than a meter? ________________.

3. Construct a number line and show how you would place the numbers 0 to 5 on the line.

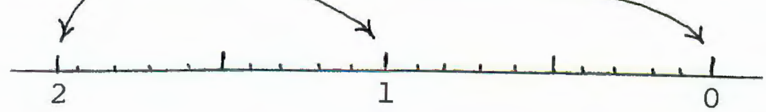
4. The decimal 0.31 is read ____________________________.

5. The decimal 0.6 is read ____________________________.

6. The decimal expression for seven tenths is:
   a. 0.70
   b. 7.0
   c. 0.07
   d. 0.7

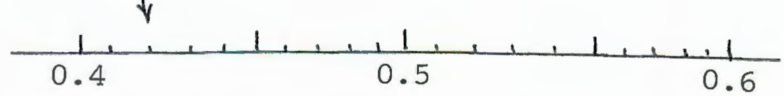
7. The decimal expression for fifty-seven hundredths is:
   a. 5.7
   b. 0.57
   c. 0.057
   d. 5700

8. Given the decimals 0,1,2,0.1,0.2,0.3,0.4,1.3,1.7,1.9 and the number line.



Finish connecting the points on the number line to the decimals.

9. Given the decimals 0.42, 0.53, 0.59, 0.60, and 0.46 and the number line.



Finish connecting the points on the number line to decimals.

10. Order the following decimals from smallest to largest.

1.4 0.4 0.8 0.1 1.3 0.5

smallest ____ ____ ____ ____ ____ ____ largest

11. Order the following decimals from smallest to largest.

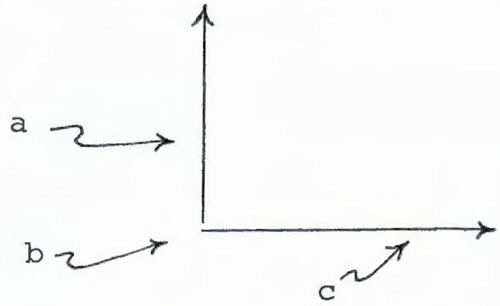
2.30 0.23 0.26 0.16 0.57 0.03

smallest ____ ____ ____ ____ ____ ____ largest

12. Place a circle around the decimals that are expressed in tenths.

0.73 0.7 0.50 0.8 3.7 0.07

13. In the following drawing, the horizontal axis is labeled _____, the vertical axis is labeled _____, and the origin is labeled _____.



14. Given the decimals .24, .56, .30, .72, and .16, show how you would find their sum.

15. Given the decimals .6, .8, .9, .2, .7, and .8, show how you would find their sum.

16. Construct a pair of axes. Show the horizontal axis, the vertical axis, and the origin.

17. In the space provided, round the following numbers to the nearest tenth.

a. .53 b. 2.75 c. .88 d. 1.97

______ ______ ______ ______

- -

Read the following paragraph and then answer the questions. Circle the correct answers.

1. A man is driving a car down a road.
2. He sees another car coming out of a side street.
3. He puts his foot on the brake.
4. The car stops.

18. The stimulus in this situation appears in:

a. Statement 1

b. Statement 2

c. Statement 3

d. Statement 4

19. The response in this situation appears in:

a. Statement 1

b. Statement 2

c. Statement 3

d. Statement 4

20. The man's reaction time is the amount of time between:

    a. Statements 1 and 2
    b. Statements 2 and 3
    c. Statements 3 and 4
    d. Statements 2 and 4

21. Name the stimulus.

22. Name the response.

23. Show, by drawing an arrow, how you would divide the numbers below by ten. Then write the answers in the spaces provided.

    a. 3.85 ________________
    b. 21.4 ________________
    c. 4.7 ________________

24. In the spaces provided, round the following numbers to the nearest hundredth.

    a. .583 b. .943 c. .955 d. .706

    ______ ______ ______ ______

25. Read the following statements. Put an "S" next to the statement containing the stimulus and an "R" next to the statement containing the response.

__________ A boy walks into the kitchen.

__________ He smells something cooking.

__________ He lifts the top from the pot.

__________ He replaces the lid.

26. A boy was doing an experiment that had ten trials. On the fourth trial, his reaction time was much less than on any other trial. This may be due to: (Circle one)

a. being ready for the stimulus

b. practice

c. anticipating the stimulus

27. When a person anticipates a stimulus, his reaction time will probably be: (Circle one)

a. about the same as his average time

b. much less than his average time

c. much more than his average time

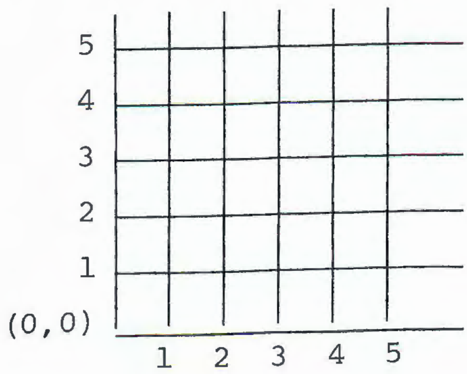
28. An attention-getting procedure is used to: (Circle one)

a. keep a student from responding too fast

b. make sure the student stands straight

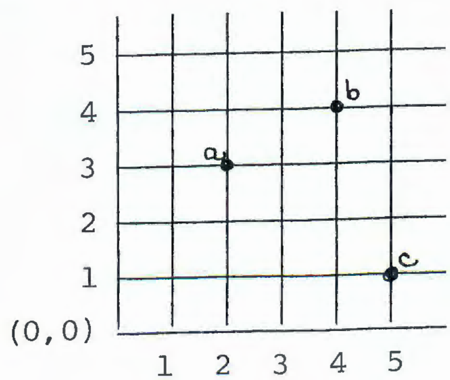
c. make sure that a student is ready for the trial

29. On the graph below, place the points that are represented by the pairs of numbers (2,3), (4,2), and (1,5).



30. Show how you would find the average of the following numbers. 6,4,7,8,5,7,6,9,8,10

31. Name each of the points on the graph using an ordered pair of numbers.



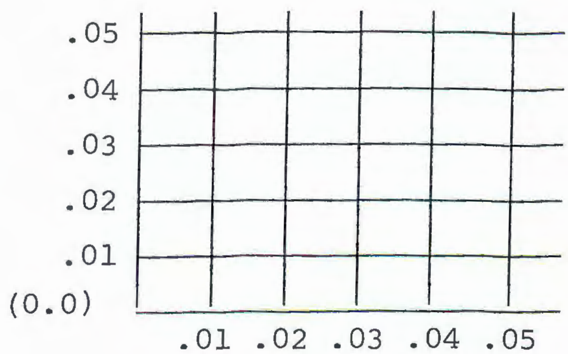
a. ________________

b. ________________

c. ________________

32. Using the graph below, find the points corresponding to each ordered pair and connect the points by line segments.

(.01, .04) , (.02, .03) , (.03, 0) , (.05, .03)



33. Show how you would find the average of the following numbers.

.26, .23, .37, .84, .56, .21, .42, .33, .88, .42

34. An attention-getting procedure might be:

a. saying the word "ready" after the stimulus

b. saying the word "ready" before the stimulus

c. saying the word "ready" after the response

35. Name four things that you would control very carefully to make sure that an experiment is fair.

1.

2.

3.

4.

36. From the following list, circle the items that you would control to make sure that an experiment is fair.

a. temperature of the room

b. order in which the students do the experiment

c. attention of the student

d. anticipation of the stimulus

e. practice

f. anticipation of the response

g. what the students had for breakfast

h. how the stick is dropped

37. On the next page, you will find twelve pictures showing how an experiment on reaction time is done. The pictures are not in order. In the spaces that are provided below, put the numbers of the pictures in the proper order.

____

____

____

____

____

____

____

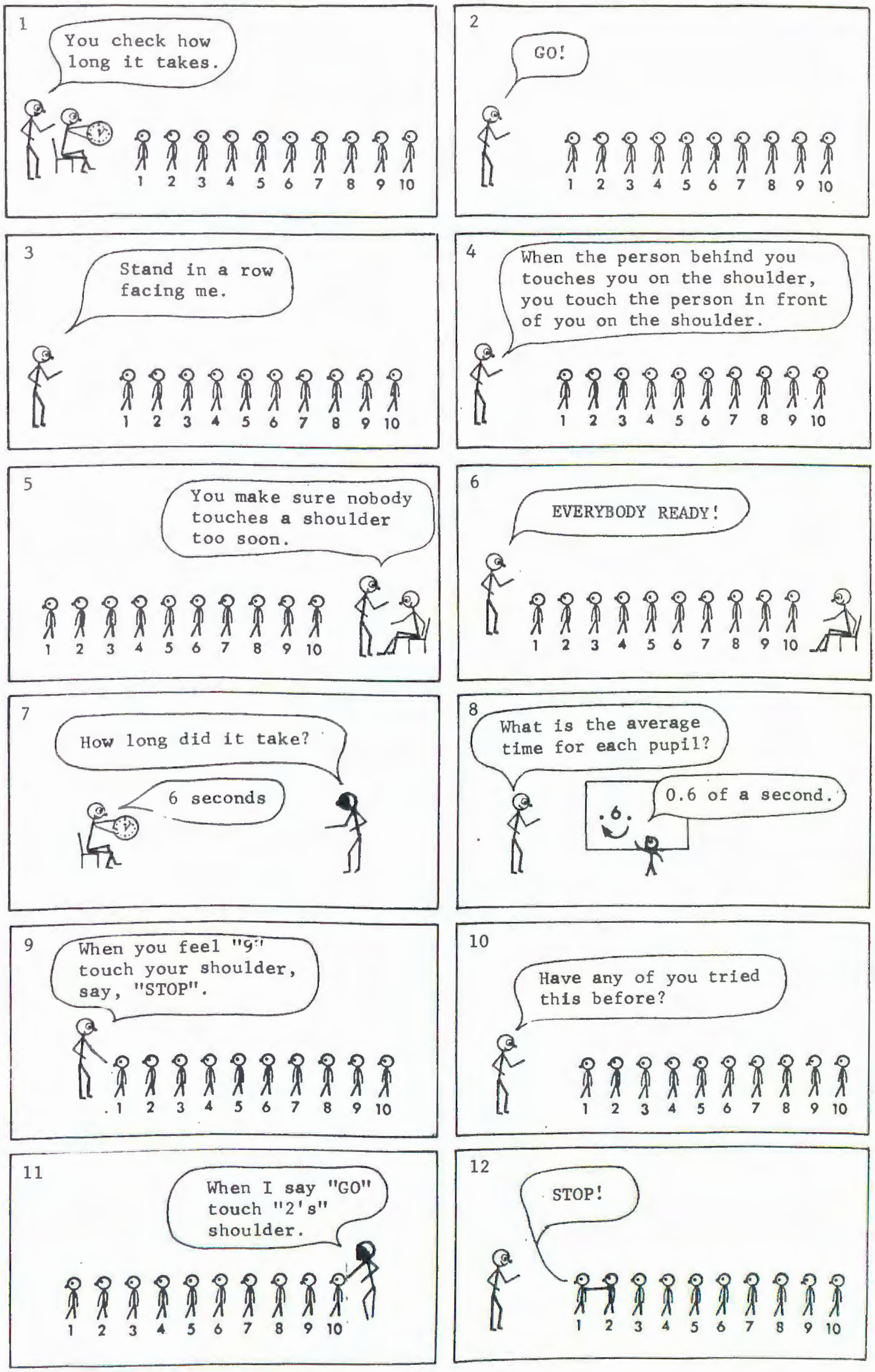
____

____

____

____

____


1
You check how long it takes.
2
GO!
3
Stand in a row facing me.
4
When the person behind you touches you on the shoulder, you touch the person in front of you on the shoulder.
5
You make sure nobody touches a shoulder too soon.
6
EVERYBODY READY!
7
How long did it take?
6 seconds
8
What is the average time for each pupil?
.6.
0.6 of a second.
9
When you feel "9" touch your shoulder, say, "STOP".
10
Have any of you tried this before?
11
When I say "GO" touch "2's" shoulder.
12
STOP!

38. On the chart below are results from an experiment on reaction time. Construct ordered pairs from this information that you could use to construct a graph.

TRIAL	REACTION TIME IN SECONDS
I	.27
II	.26
III	.22
IV	.19
V	.20

- -

<u>ORDERED PAIRS</u>

____

____

____

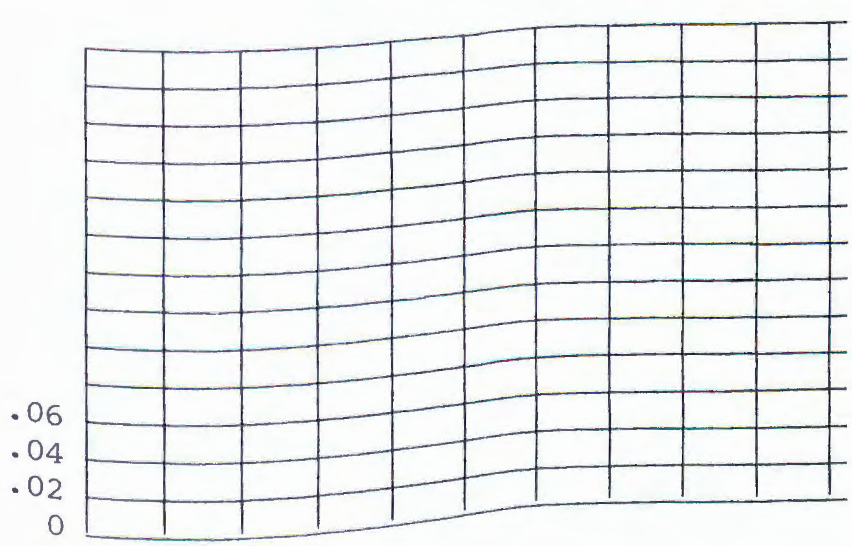
____

____

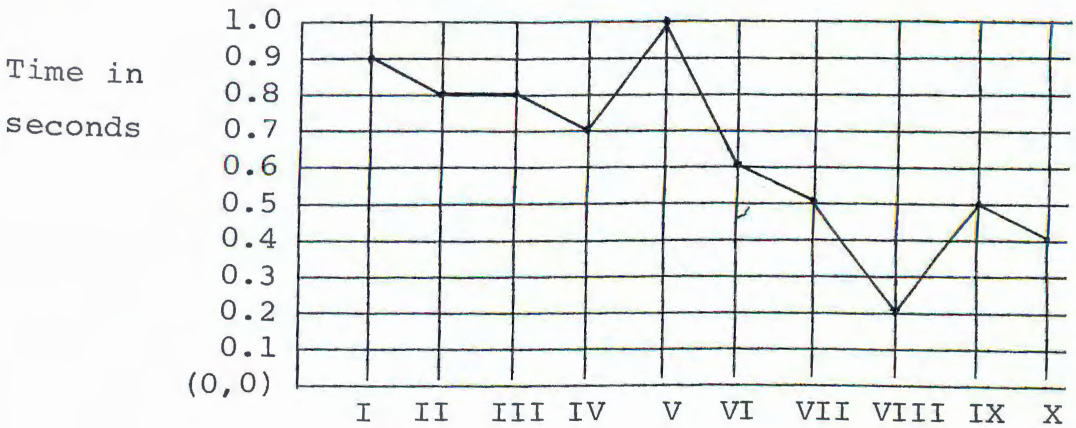
39. In a certain experiment involving reaction time, the average time for ten groups was as follows:

GROUP	REACTION TIME IN SECONDS
I	.18
II	.14
III	.22
IV	.20
V	.09
VI	.16
VII	.21
VIII	.11
IX	.14
X	.10

Graph this information on the graph provided below.



40. Read the following graph representing the results of an experiment and answer the questions below the graph. It involves one boy doing ten trials.



A. What was the boy's lowest reaction time? ________

B. What was the boy's highest reaction time? _______

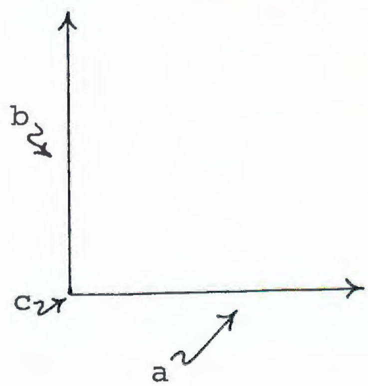
C. What was the average reaction time for the boy on the ten trials?

D. What may have happened on trial V?

E. What may have happened on trial VIII?

F. From the graph, does it look like the boy was improving as he performed the trials? WHY?

41. Look at the following drawing. Write the name of each part in the space provided.



The part labeled a is usually called the ________________.

The part labeled b is usually called the ________________.

The part labeled c is usually called the ________________.

APPENDIX F

THE GROUP MEANS FOR POSTTEST I (MATHEMATICS)

TABLE IX

THE GROUP MEANS FOR POSTTEST I (MATHEMATICS)

Treatment A	Mean	Treatment B	Mean
A_1	22.2	B_1	18.3
A_2	22.9	B_2	23.0
A_3	20.8	B_3	18.3
A_4	23.6	B_4	17.0
A_5	24.5	B_5	21.3
A_6	20.7	B_6	18.5
A_7	22.0	B_7	23.3
A_8	23.6	B_8	20.5
A_9	21.0	B_9	22.8
A_{10}	21.3	B_{10}	16.9
A_{11}	19.6	B_{11}	16.3
A_{12}	17.9	B_{12}	15.6
A_{13}	16.4	B_{13}	18.4
A_{14}	20.8	B_{14}	18.3
A_{15}	22.6	B_{15}	22.1

APPENDIX G

THE GROUP MEANS FOR POSTTEST II (MATHEMATICS)

TABLE X

THE GROUP MEANS FOR POSTTEST II (MATHEMATICS)

Treatment A	Mean	Treatment B	Mean
A_1	21.3	B_1	17.2
A_2	22.7	B_2	22.4
A_3	18.0	B_3	16.1
A_4	22.9	B_4	13.7
A_5	23.3	B_5	20.3
A_6	19.8	B_6	15.7
A_7	19.4	B_7	23.1
A_8	23.4	B_8	18.4
A_9	20.0	B_9	22.3
A_{10}	20.2	B_{10}	13.7
A_{11}	16.9	B_{11}	13.4
A_{12}	16.8	B_{12}	12.2
A_{13}	13.3	B_{13}	14.4
A_{14}	18.9	B_{14}	15.6
A_{15}	22.1	B_{15}	21.0

APPENDIX H

THE GROUP MEANS FOR POSTTEST I (SCIENCE)

TABLE XI

THE GROUP MEANS FOR POSTTEST I (SCIENCE)

Treatment A	Mean	Treatment B	Mean
A_1	9.8	B_1	9.0
A_2	11.6	B_2	12.1
A_3	8.1	B_3	8.5
A_4	11.7	B_4	9.7
A_5	11.5	B_5	9.3
A_6	8.7	B_6	7.8
A_7	11.1	B_7	11.0
A_8	12.5	B_8	9.3
A_9	9.5	B_9	11.1
A_{10}	10.7	B_{10}	7.3
A_{11}	9.3	B_{11}	7.3
A_{12}	8.8	B_{12}	7.6
A_{13}	6.6	B_{13}	7.6
A_{14}	9.1	B_{14}	8.6
A_{15}	8.1	B_{15}	11.1

APPENDIX I

THE GROUP MEANS FOR POSTTEST II (SCIENCE)

TABLE XII

THE GROUP MEANS FOR POSTTEST II (SCIENCE)

Treatment A	Mean	Treatment B	Mean
A_1	9.7	B_1	8.8
A_2	11.5	B_2	12.4
A_3	7.7	B_3	7.8
A_4	12.3	B_4	9.1
A_5	11.8	B_5	10.3
A_6	8.6	B_6	7.2
A_7	11.7	B_7	11.5
A_8	12.8	B_8	9.1
A_9	9.5	B_9	11.8
A_{10}	10.0	B_{10}	6.2
A_{11}	8.7	B_{11}	5.1
A_{12}	8.1	B_{12}	7.2
A_{13}	6.0	B_{13}	6.0
A_{14}	8.0	B_{14}	8.1
A_{15}	8.8	B_{15}	9.1

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