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A Stochastic Modeling for the Characterization of Radom Tool Motion during Machining

by T-W. Hwang and G.M. Zhang

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A Stochastic Modeling for the Characterization of Random Tool Motion during Machining

Tsu-wei Hwang and Guangming Zhang
Department of Mechanical Engineering and Systems Research Center
University of Maryland
College Park, Maryland 20742

Abstract

This paper presents the development of a new stochastic approach to characterize random tool motion during machining. The complexity of cutting mechanism is represented by a random excitation system related to physical properties of the material being machined. A Markov-chain based stochastic approach is developed to model the random tool motion as the response of a machining system under the random excitation. In considering a turning operation, a concept of group distributions is introduced to characterize the global effect on the cutting force due to the variation of a certain material property. A model of segment excitation is used to describe its micro function within an individual revolution. A distribution pattern observed in the material property is represented by a transition model. The simulation of random tool motion during machining resembles the generation of Markov chains. Microstructure analysis and image process are used to collect data, calculate relevant statistics, and estimate the system parameters specified in the developed stochastic model. As illustrated in this paper, the developed stochastic model can be effectively used to simulate the random tool motion and to learn rich information on the performance measures of interest such as machining accuracy and finish quality. The new approach represents a major advance to create a fundamental scientific basis for the realization of a reliable and effective prediction system for information processing in sensor-based manufacturing.

1 Introduction

Computer integrated manufacturing systems have emerged in response to the requirements for great flexibility, productivity, and high quality of the product. As the computer technology advances, the manufacturing industry is now seeking a higher degree of production automation.

Sensor-based manufacturing leads a new direction to the technological development. Instead of relying on intelligent human operators on the shop floor, sensing devices are intended to on-line collect signals related to the machining performance. Through signal processing, interpretors detect the process abnormalities, and built-in controllers take actions to return the process to a normal state. In this regard, mathematical modeling of random tool motion will be a necessity to ensure the sensitivity of monitoring systems to improve the machining precision, efficiency, and product quality. Great efforts have been devoted to the discovery of quantitative descriptions of random tool motion. This statistical approach characterized by the development of a dynamic data system (DDS) was proposed in the 1970s [Pa 83]. Discrete stochastic models, such as AR and ARMA models, are used to identify the transfer function of the physical system which is subject to random tool excitation. However, these models are in general used to describe stationary stochastic signals. For the application to model random tool motion which is a non-stationary time-varying process, these models are not suited and effective. Therefore, there exists a substantial distance towards the general adoption of these methods simply because of the complexity of manufacturing environments.

The hidden Markov model is a powerful tool in describing non-stationary time-varying process [RabJu 86]. It has been successfully applied to the modeling, processing, and recognition of speech signals which is well-known to be highly non-stationary. Due to the similarities between the speech signal and the random tool motion signal, the concept of hidden Markov model may be the best theoretical foundation to initiate the modeling of random tool motion during machining.

In this paper, a new and stochastic approach is formulated for an analytical evaluation of random tool motion during machining. The developed approach consists of three statistical models to capture the dynamic characteristics of the cutting environment. A group distribution model is used in a turning operation to imitate the cutting process during one revolution as an assignment of known stochastic processes. For an assigned process, a normal distribution model is to represent the random excitation system with mean and variance as its two parameters characterizing its average and variation levels. A transition model is to recognize possible patterns or inherent correlations of the tool motion between consecutive revolutions.

The paper is organized into following sections. In section 2, the basic methodology is described. Section 3 outlines the procedure to identify the state transition probability matrix which is the key for modeling the transition pattern of microstructures observed in the workpiece material. A case study through both the experiment and computer simulation is presented in Section 4. It demonstrates the application of using the proposed approach to study the finish quality of machined surfaces caused by random tool motion. Results from the experiment and computer simulation to validate the proposed approach are discussed in Section 5. Section 6 presents conclusions and future directions for this research.

2 Basic Methodology

2.1 Concept of Random Tool Motion

During a machining operation, the cutting tool moves along a path defined by the kinematic motion of the machine tool which is determined by the cutting parameters. In the case of a turning operation, spiral tool paths are most common. In addition to the kinematic motion, the cutting tool vibrates about its moving path in small magnitudes. Close examination suggests that tool vibration is of random nature and should be dealt stochastically.

It has been well known that the built-up edge, tool wear, and unevenly distributed material properties (such as microhardness) are the major sources of observed random tool motion. Among these sources, the random tool motion caused by the built-up edges can be controlled by increasing the cutting speed, a common practice on the shop floor. It is believed that random tool motion becomes severe as tool wear progresses during machining; and at the same time, the non-uniform distribution of hardness in the material being cut beats the cutting tool in a random manner. However, to distinguish the effects of tool wear and distribution of material properties on random tool motion during machining seems extremely difficult although not impossible.

As a strategy to control the random tool vibration, in this research we treat the random tool motion from tool wear as an 'abnormal variation' to the machining process, and the random tool motion from other sources except tool wear as the 'natural variation' to the machining process. This strategy, therefore, suggests that the unevenly distributed material properties become the main source for introducing the 'natural' random tool motion if a high cutting speed is used for diminishing the effects caused by the built-up edge. In the following sections the physical meaning of the unevenly distributed material property, such as microhardness, related to the random tool motion during machining, and the stochastic modeling of this relation is discussed.

2.2 Dynamic Characteristics of the Cutting Environment

From the cutting dynamics point of view, it is beneficial if we treat the cutting process as a sampling process in which the cutting tool meets a series of small samples of the workpiece material (as illustrated in Fig. 1). Each of the small samples (or blocks) in Fig. 1 may possess distinct characteristics in its microstructure. The cutting force generated in this process then varies in a random nature as the microhardness value of each of the samples varies. As a result, the random variation of the cutting force introduces random tool vibration during machining. Accordingly, it is necessary to identify the characteristics of the cutting environment embedded in the microstructures of workpiece material.

As a first attempt to model the cutting dynamics in microscale, microstructural analysis of the workpiece material should be performed. Figure 2a shows the micro-photographs of four representative samples taken from different cross-sections of a rolled AISI 1020 steel bar. The photographs were then scanned and digitized as bitmap files (pictures in Fig. 2 are actually bitmaps). The ferrite structures (bright part or 0's in the bitmap file) and pearlite structures (dark part or 1's in the bitmap file), which are much harder than the ferrite, are clearly shown in the photographs.

It can be proven that the microhardness distributed within each cross-section tends to be a normal distribution [ZhKa 90] (as also shown in Fig. 2a). Moreover, depending on the microstructures along the bar, the said distributions for different cross-sections may be statistically different from (or similar to) each other as those shown in the group 1 or group 2 of Fig. 2a. If the microstructures are homogeneously distributed along the steel bar, the difference of microstructures among the cross-sections may not be sensed by the cutting tool when it passes along the bar with a preset feed rate during machining. In this case, a single (normal) distribution will be sufficient for modeling the microhardness distribution in the entire workpiece material.

However, if a sample was taken from the longitudinal direction of the bar, such as the one shown in Fig. 2b, the microstructures will be quite different from those shown in Fig. 2a. It is obvious that the pearlite structures in Fig. 2b tend to be distributed as stripes with certain pattern along the longitudinal direction of the bar which were formed during fabrication by the rolling process. As a result, when the tool passes from one revolution to the next, the cutting force generated will also possess a similar pattern which is then transmitted to the cutting tool in a form of vibration.

Based on the above observation, when the tool is cutting around one revolution of the material, a specific distribution which is applied statistically to estimate the microhardness of each blocks in the cross-section should be assigned. As the tool moves to the next revolution of the material, a distribution which may or may not be the same as the previous one should also be assigned. The assignment of these distributions is discussed in the next section. On the other hand, when the microhardness of each small block around one revolution, which is next to the previous one, is to be assigned based on the current distribution, it has to simultaneously follow the transition pattern as observed in the longitudinal direction of the material. It should be pointed out that the cutting parameter settings (e.g., feed rate, depth of cut, and spindle speed) will have significant effects on the determination of the statistics for the microstructures distributed in the material (both in the cross-sectional and longitudinal directions). These issues will be covered in detail in the following sections.

2.3 A Stochastic Approach to Simulate the Micro Cutting Dynamics

2.3.1 Assignment of Group Distributions

As mentioned earlier, it is perceivable that the microhardness in different cross-sections of the same material has different distributions. For example, if we take ten samples from ten different cross sections of a steel bar, it is very likely that each sample has a different mean (μ_i, $i = 1, 2, ..., 10$) and variance (σ_i^2, $i = 1, 2, ..., 10$) of microhardness distributions. However, these statistics are not truely different from each other (from a statistical point of view). If we perform the significance tests on means, the ten distributions could be statistically 'grouped' into groups under certain criterion. Each group may then be characterized by a new mean and variance based on the statistics of the members in each group.

Figure 3 presents such a case that the distributions of ten samples were divided into three groups with their new means (μ_{g_j}, $j = 1, 2, 3$) and variances ($\sigma_{g_j}^2$, $j = 1, 2, 3$). These three distributions are called *Group Distributions*. Therefore, one can choose any of the three group distributions to re-characterize the original ten samples as well as any other samples from different cross-sections of the same steel bar. Beside the mean and variance, the probability of each group distribution (p_{g_j}), such that any cross section within the material will be characterized by the distribution, could also be estimated[1]. For clarity, we use different background shadings to represent different group distributions as shown in Fig. 3. It is emphasized that these group distributions are determined by the microstructural analysis of the workpiece material which is technically feasible. Therefore, if there is enough information from the microstructural analysis, the group distributions can be determined as a known property of the material. Based on the said group distribution probabilities, one can then assign the group distributions to the consecutive cross-sections (it can be done by using a random number generator) which will be encountered by the cutting tool during machining (as shown in Fig. 3).

[1]For example, in Fig. 3 three out of ten samples are grouped as distribution one; hence, the probability that distribution one will occur in any cross section is 0.3.

2.3.2 Segment Model for Single Revolutions

After the assignment of group distributions to each of the cross sections, it is intuitive that one can then determine the microhardness of each block within a specific revolution (or *segment*) as previously shown in Fig. 1. For example, if the first revolution is assigned by group distribution two, the microhardness of each block in this revolution can be directly determined by the mean and variance of the group distribution two (since it is a normal distribution). Therefore, it is possible to determine the microhardness of each block in any other segments of the material in the same fashion.

As observed earlier, there exists a certain pattern in the microstructures along the longitudinal direction of the steel bar. In other words, there is an underlying correlation between the consecutive segments. This pattern has to be identified before the microhardness in the blocks of consecutive segments can be further assigned. This issue is discussed in the next section.

2.3.3 Transition Model for Pattern Recognitions

Research in the speech recognition process by applying the basic theory of Markov chains [RabJu 86] has provided the insight in this work to model the pattern of microstructures as seen in Fig. 2b. With a good model, we can predict the outcome and learn as much as possible via simulation of the process. How does one apply the Markov chains as a tool to model the transition pattern encountered during a machining process? We need first to understand the basic mechanism of Markov chains. A brief description and definitions of a Markov chain are presented as follows.

Consider a system which may be described at any time as being in one of a set of mutually exclusive states $(S_1, S_2, ..., S_N)$. According to a set of probabilistic rules, the system may, at certain discrete instants of time, undergo *changes of state* (or *state transition*). One could number the particular event of a time instant $(q_1, q_2, ..., q_t)$ at which transitions may occur. A Markov chain or discrete-state Markov process is a random process $\{q_t, t = 1, 2, ...\}$ that takes on a finite or countable number of states (e.g., $S_1, S_2, ..., S_N$) and satisfies the following condition:

$$P(q_t = S_j | q_{t-1} = S_i, q_{t-2} = S_k, ..., q_1 = S_m) = P(q_t = S_j | q_{t-1} = S_i) \tag{1}$$

for all $t > 0$ and all possible states (say, N of them) [Dr 67]. In other words, a Markov chain is a discrete random process in which the next state entered depends only on the current state, but not on the previous states. The transitions between states are governed by a set of state transition probabilities $\{p_{ij}\}$, where p_{ij} is the probability of going directly from state i to state j as i and j vary over all possible states, or

$$p_{ij} = P(q_t = S_j | q_{t-1} = S_i) = P(S_j | S_i),\ \forall\, i, j \in [1, N];\ p_{ij} \text{ independent of } t \tag{2}$$

with the following properties

$$p_{ij} \geq 0 \text{ and } \sum_{j=1}^{N} p_{ij} = 1,\ \forall\, i \in [1, N] \tag{3}$$

It is often convenient to display these transition probabilities as members of a square matrix, i.e.,

$$[p] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{bmatrix} \quad (4)$$

As a simple example, a two-state ($N = 2$) Markov process with a 2×2 transition probability matrix:

$$[p] = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

is demonstrated here. Suppose that a coin tossing experiment is performed, and a sequence of observations are given; e.g., an observed sequence of the outcome would be $Q = \{S_1, S_1, S_2, S_2, S_1\}$, where S_1 stands for heads and S_2 stands for tails. It is desired to determine the probability of Q if the model is given (namely, it follows Eqs. 2 and 3). This probability can be expressed as

$$\begin{aligned} P(Q|Model) &= P(\{S_1, S_1, S_2, S_2, S_1\}|Model) \\ &= P(S_1)P(S_1|S_1)P(S_2|S_1)P(S_2|S_2)P(S_1|S_2) \\ &= \epsilon_1 p_{11} p_{12} p_{22} p_{21} \end{aligned}$$

where the notation

$$\epsilon_i = P(q_1 = S_i), \ \forall\, i \in [1, N] \quad (5)$$

stands for the initial state probabilities (it is usually set to 1) [RabJu 86, Ro 83].

It is also important to find out what the probability would be if a known state is fixed in that state for exactly d observations [RabJu 86]. This probability can be evaluated as the probability of the observation sequence $Q = \{q_1 = S_i, q_2 = S_i, \ldots, q_{d_i} = S_i, q_{d_i+1} = S_{j,j\neq i}\}$, given the model, which is

$$P(Q|Model, q_1 = S_i) = (p_{ii})^{d_i - 1}(1 - p_{ii}) = f(d_i) \quad (6)$$

The quantity $f(d_i)$ is the discrete probability density function of duration d in state i. Therefore, the expected number of observations (duration) in a state conditioned on starting in that state is

$$E(d_i) = \sum_{d_i=1}^{\infty} d_i f(d_i) = \sum_{d_i=1}^{\infty} d_i (p_{ii})^{d_i - 1}(1 - p_{ii}) = \frac{1}{1 - p_{ii}}, \ \forall\, i \in [1, N]. \quad (7)$$

As long as the state transition probabilities $\{p_{ij}, \ \forall\, i, j \in [1, N]\}$ are defined, the pattern existing in the microstructures along the longitudinal direction of the steel bar can be recovered. Suppose that a two state Markov process (assuming that state one possesses the softest microhardness distribution) is given as the transition model, and a sample block of the first revolution has been identified being under state one. Then the state of a neighboring sample block in the second revolution which is just next to the first one can be determined based on the transition probabilities of p_{1j}, $j = 1, 2$. For example, if the transition probability from state one to itself, p_{11}, is 0.8, and the transition probability from state one to state two, p_{12}, is 0.2 (recall Eq. 3), the next state will most likely be still under state one. This work can be easily done by using a random number generator.

From the proposed stochastic approach, the relation between the microstructures of workpiece material and the random tool motion could be studied through computer simulation. This study could then serve as a tool for us to learn more about the random nature of a machining process. Before doing so, it is imperative to develop a method for identifying the parameters of the transition model, i.e., the states and transition probability matrix. Such a method is presented in the next section.

3 Identification of the Transition Model Parameters

The states and state transition probabilities are important to model a Markov process. This section will present a method about how to identify them from an observed image of microstructures of the workpiece material similar to the one shown in Fig. 2b.

3.1 Determination of State Boundaries and States

Suppose that the cutting parameter settings, such as feed and cutting speed in a turning process are given. The image in Fig. 2b can be divided into subdivisions along both the longitudinal (feed) and vertical (cutting) directions based on the settings. For example, the microstructures shown in Fig. 4a is divided into six subdivisions along the longitudinal direction and two subdivisions along the vertical direction (based on the cutting parameter settings). Since the image is stored as a bitmap file (0's and 1's), one can calculate the ratio of the number of black pixels (hard spots) to the total number of pixels in each subdivision. We call this a *state ratio* and is defined as

$$r_{kl} = \frac{\textit{Number of Black Pixels within the } (k,l)\textit{th Subdivision}}{\textit{Total Number of Pixels within the } (k,l)\textit{th Subdivision}}, \quad k \leq m, \ l \leq n. \tag{8}$$

where m is the number of subdivisions in the cutting speed direction, and n is the number of subdivisions in the feed direction. The purpose of evaluating these ratios is to determine the *state* of each subdivision associated with the hardness variation within the given image area. If the number of states are determined in advance (the number of states could be two, three, four, or even more), the *state* of each subdivision can be determined based on the *state ratios*.

To fix the idea, a map of *state ratios*, calculated in each subdivision based on Eq. 8, is shown in Fig. 4b. The mean ($\overline{r}$) and standard deviation (σ_r) of these ratios can also be evaluated. It is true that if the number of observations (i.e., the total number of subdivisions in the image) are very large, the distribution of *state ratios* will tend to be a normal distribution.

From the above distribution, the *state ratios* are then divided into regions based on the number of states to be considered. The boundaries of each region can be determined by using the 3σ criteria; i.e., a) divide the ratios between $[\overline{r} - 3\sigma_r, \overline{r} + 3\sigma_r]$ by N (the number of states) and call this number dr ($= 6\sigma_r/N$); b) the boundaries will be $(-\infty, \overline{r} - 3\sigma_r + dr]$, $(\overline{r} - 3\sigma_r + dr, \overline{r} - 3\sigma_r + 2dr]$,..., and $(\overline{r} + 3\sigma_r - dr, +\infty)$.

Suppose that the number of states of the hardness variation is two, the hardness of each subdivision can be either at state I (soft) or state II (hard). Therefore, any ratio in the map which is smaller than the mean value ($\overline{r}$) can be assigned at state I, and those larger than the mean value can be assigned at state II. In other words, the mean value of the ratios is the *state boundary* for a two-state model. The upper-right corners of each subdivision in Fig. 4b shows the states after they are assigned. This specific assignment of states is called a *state map*. The state map of a three-state model is also shown in Fig. 4b (lower-left corners) for comparison.

It is obvious that if the cutting parameters (feed or speed) are changed, the *state ratios* (and hence the state map) will also be changed. For example, if the feed rate is reduced by one half, the same image will have more subdivisions (as those divided by dashed lines in Fig. 4a). Thus, the pixels in each subdivision will be different, and the *state ratios* as well as the state maps will also be different.

3.2 Estimation of Transition Probability Matrix, $[p]$

3.2.1 Two-State Markov Process

How can one evaluate the transition probabilities or the transition probability matrix $[p]$ from the observed state map? It is clear from Eq. 7 that as long as the expected duration of a state i is estimated, the probability for transition from state i to itself can be estimated by

$$\hat{p}_{ii} = 1 - \frac{1}{\hat{E}(d_i)}. \tag{9}$$

The evaluation of $\hat{E}(d_i)$ can be done by counting the number of occurrences of state i[1] and the duration of each occurrence of state i observed from the state map; i.e.,

$$\hat{E}(d_i) = \frac{\sum_{j=1}^{K_i} d_{i_j}}{K_i} \tag{10}$$

where d_{i_j} is the j^{th} duration of occurrence at state i and K_i is the number of occurrences of state i. For example, there are three occurrences of state I in the map of Fig. 4b (i.e., $K_1 = 3$). The duration of each occurrence can also be counted; i.e.,

$$d_{i_1} = Length(I, I, I, I) = 4,\ d_{i_2} = Length(I) = 1,\ d_{i_3} = Length(I, I) = 2$$

where $i = 1$ (at state I). Therefore, the estimated expectation of duration at state I from the observation is

$$\hat{E}(d_1) = \frac{d_{1_1} + d_{1_2} + d_{1_3}}{Number\, of\, occurrences\, of\, state\, I} = \frac{4+1+2}{3} = \frac{7}{3}$$

[1]It should be noted that an occurrence of state i is defined as the continuous observation of staying at state i without changing to any other states.

The meaning of the above number (7/3) is that the expected duration of staying in state I is about 2.3 (feeds) in the feed direction. The probability for transition from state I to itself is then calculated from Eq. 9:

$$\hat{p}_{11} = 1 - \frac{1}{\hat{E}(d_1)} = \frac{4}{7}$$

Similarly, there are four occurrences of state II in the map of Fig. 4 ($K_2 = 4$). The estimated expectation of duration at state II from the observation is

$$\hat{E}(d_2) = \frac{d_{2_1} + d_{2_2} + d_{2_3} + d_{2_4}}{Number\,of\,occurrences\,of\,state\,II} = \frac{1+1+2+1}{4} = \frac{5}{4};$$

therefore,

$$\hat{p}_{22} = 1 - \frac{1}{5/4} = \frac{1}{5}.$$

It is obvious that from Eq. 3 the transition probabilities of $\hat{p}_{12}$ and $\hat{p}_{21}$ can also be estimated. Therefore, the estimated transition probability matrix of this two-state Markov process based on the image given in Fig. 4a is

$$[\hat{p}] = \begin{bmatrix} 4/7 & 3/7 \\ 4/5 & 1/5 \end{bmatrix}.$$

3.2.2 N-State Markov Process (N > 2)

If the number of states is not two but three, four, or even more, how does one estimate the transition probability matrix? Based on the state map of a three-state Markov process (Fig. 4b), and follow the calculations similar to that of the two-state model, the estimated expectation of duration at state I, II, and III are

$$\hat{E}(d_1) = \frac{1+2}{2} = 1.5,\ \hat{E}(d_2) = \frac{3+1+2}{3} = 2.0,\ \text{and}\ \hat{E}(d_3) = \frac{1+2}{2} = 1.5,$$

respectively. Therefore, from Eq. 9, we can estimate the following transition probabilities:

$$\hat{p}_{11} = 1 - \frac{1}{\hat{E}(d_1)} = \frac{1}{3},\ \hat{p}_{22} = 1 - \frac{1}{\hat{E}(d_2)} = \frac{1}{2},\ \text{and}\ \hat{p}_{33} = 1 - \frac{1}{\hat{E}(d_3)} = \frac{1}{3}.$$

Since the transition probabilities of p_{ii}'s have been estimated, the problem now is how to estimate other transition probabilities, $p_{ij,j\neq i}$, in the $[p]$ matrix.

Suppose that each state can only change to its neighboring states or to itself (e.g., state i can only change to state $i+1$, $i-1$, or i), the transition probability matrix will have a form as follows

$$[p] = \begin{bmatrix} p_{11} & 1-p_{11} & 0 & 0 & \cdots & 0 \\ b_1 & p_{22} & 1-p_{22}-b_1 & 0 & \cdots & 0 \\ 0 & b_2 & p_{33} & 1-p_{33}-b_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & b_{N-2} & p_{N-1\,N-1} & 1-p_{N-1\,N-1}-b_{N-2} \\ 0 & \cdots & \cdots & 0 & 1-p_{NN} & p_{NN} \end{bmatrix} \tag{11}$$

where b_i, $i = 1, 2, ..., N-2$, are the probabilities for transition from state $i+1$ to state i. This type of transition pattern is sometimes referred as the *birth and death* Markov process [Ch 67]. It is obvious that the estimation of above $[p]$ is much easier than the estimation of Eq. 4 since only the b_i's are to be determined in the above $[p]$ matrix. Actually, based on the observed microstructures of the workpiece material, this type of transition seems the most favorable for our purpose.

To determine the b_i's, a stationary distribution of the (N) states, $\underline{\pi} = [\pi_1\, \pi_2 \ldots \pi_N]$ where π_i is the 'long-term fraction' of state i observed in the state map (e.g., Fig. 4b), should be first defined, and it should satisfy the matrix equation [Ch 67]

$$\underline{\pi} = \underline{\pi}[p]. \tag{12}$$

The π_i's can be estimated by

$$\hat{\pi}_i = \frac{\sum_{j=1}^{K_i} d_{i_j}}{K_{obs}} \tag{13}$$

where d_{i_j} is the j^{th} duration of occurrence at state i, K_i is the number of occurrences of state i, and K_{obs} is the total number of observations in the state map (i.e., the number of subdivisions in the cutting speed direction, m, times the number of subdivisions in the feed direction, n). For example, from the state map of Fig. 4b, the estimated π_i's are

$$\hat{\pi}_1 = \frac{d_{1_1} + d_{1_2}}{\textit{Total Number of Observations in the State Map}} = \frac{1+2}{2 \times 6} = \frac{1}{4},$$

similarly,

$$\hat{\pi}_2 = \frac{3+1+2}{2 \times 6} = \frac{1}{2}, \text{ and } \hat{\pi}_3 = \frac{1+2}{2 \times 6} = \frac{1}{4}.$$

From Eq. 12, one can manipulate the matrix equation such that b_i's can be evaluated based on the linear regression method. The derivation of b_i's is explained in the Appendix. After solving the b_1 (=1/4) in this three-state model, the transition probability matrix is

$$[\hat{p}] = \begin{bmatrix} 1/3 & 2/3 & 0.0 \\ 1/4 & 1/2 & 1/4 \\ 0.0 & 2/3 & 1/3 \end{bmatrix}.$$

4 Case Study

A turning operation of machining an AISI 1020 steel bar with diameter of 72 mm and length of 300 mm was studied to verify the developed stochastic approach for modeling the random tool vibration during machining. The study consists of both the experimental work and computer simulation. They are discussed in this section.

4.1 Experimental Work

To verify the basic methodology used to simulate the micro cutting dynamics, experiments were carried out and the experimental results were compared with the results from computer simulation. The experimental work consists of:

1. Microstructural analysis: This analysis provides the information regarding the microhardness distributions in the workpiece material, as discussed in previous sections. Samples taken from the steel bar were polished and etched for taking microphotographs under an optical microscope or a scanning electron microscope. Pictures of each sample were then scanned and image data files were stored. Statistical analysis was performed on the image files to get the related statistics for further study.

2. Machining tests: The machining of the AISI 1020 steel bar was performed on a lathe with the following settings:

   - Cutting data: feed=0.23 mm/rev, depth of cut=0.5 mm, spindle speed=470 rpm (the built-up edge was insignificant at this speed).
   - Cutting tool: nose radius=0.6 mm, rake angle=0°, lead angle=20°.
   - Cutting fluid: none.
   - Cutting time: less than one minute (no tool wear was observed).

3. Measurement of surface profiles: The surface roughness along the feed direction (axial direction of the steel bar) was measured using a Talysurf surface profilometer. Twenty five surface profiles were taken in parallel with a $0.2mm$ interval inside a rectangular area of $4.8 \times 4.0\,mm^2$ to form a surface topography as the one shown in Fig. 5a. A representative surface profile from the experiment is shown in Fig. 5b. The measured surface roughness indices, such as the roughness average values (R_a), root mean squared values (R_q), peak to valley values (PTV), and the standard deviation of each of them are listed in Table 1.

4.2 Computer Simulation

From the basic methodology described in Section 2.3, computer programs were written to simulate the micro cutting dynamics. The programs were all written using the FORTRAN 77 language and compiled on a Unix based operating environment. They are portable to any system with a FORTRAN 77 compiler (except that a graphics display module is device dependent). The main structure of the programs is based on previous research of the analysis of cutting dynamics which has been presented in the reference [ZhHw 90].

The statistical approach mentioned in Section 2.3 is coded as subroutines and appended to the original programs. The same cutting parameter settings as chosen in the experiment, the digitized images from samples of the material, and the group distributions[1], are read as input data files. The state transition probability matrix is then estimated. The assignment of group distributions, the prediction of segment microhardness values and state map of the first revolution, assignment of the state maps of consecutive revolutions (or segments), and prediction of the microhardness values of all segments are then carried out one after another. As long as the microhardness values of each revolution are available, they are stored as one of the input data files for the evaluation of dynamic random tool motion. We call this set of input data as *Case 1*.

[1] Three group distributions were estimated from the microstructural analysis, they are: $\mu_{g_1} = 110.0$ BHN, $p_{g_1} = 0.2$, $\mu_{g_2} = 143.0$ BHN, $p_{g_2} = 0.5$, $\mu_{g_3} = 165.0$ BHN, $p_{g_3} = 0.3$, and the standard deviations are the same ($\sigma_{g_{j,j=1,2,3}} = 12.0$ BHN).

Table 1: Surface Roughness Indices

Units in μm	Experiment	*Case 1*	*Case 2*	*Case 3*
$\overline{R}_a$	4.20	4.13	4.00	3.65
σ_{R_a}	0.32	0.22	0.19	0.06
$\overline{R}_q$	5.04	5.05	4.88	4.37
σ_{R_q}	0.44	0.28	0.27	0.09
$\overline{PTV}$	20.28	16.97	16.93	15.38
σ_{PTV}	1.70	0.94	0.91	0.50

In order to see the effects of different microstructures on the random tool motion and simulated surface topographies, two other sets of microstructural models are also used (recall Section 2.2). The characteristics of these two input sets are:

- *Case 2:* Instead of assigning group distributions to each segment, only one normal distribution (μ_g = 143.0 BHN, σ_g = 12.0 BHN) is applied as the segment model in this case, and the transition model is still kept to simulate the pattern along the longitudinal direction. This case assumes that the microhardness distribution in one cross section of the material is similar to that of any other cross section.

- *Case 3:* This case assumes that the distribution of microhardness in the entire workpiece material is characterized by a single normal distribution and there is even no transition pattern along the longitudinal direction [ZhKa 90]. For example, we can heat treat a steel bar to spheroidize the pearlite structure into a matrix of soft, machinable ferrite structure [As 84]. Thus, the transition pattern existed along the longitudinal direction is destroyed and microstructures are almost homogeneously distributed as a single distribution in this case.

The cutting force, dynamic response of tool vibration, surface profiles, and surface topography are part of the outputs available from the computer simulation. To check the validity of the proposed models, we compare the surface profiles and surface quality indices with experimental results (since they also represent the outcome of a machining process). A simulated surface topography from *Case 1* is shown in Fig. 5a. From the three cases of simulation, representative surface profiles along the feed direction are also shown in Fig. 5b. Based on the simulated surface profiles, we can then evaluate the surface roughness indices as those listed in Table 1.

5 Discussion of Results

A qualitative comparison is made by first looking at the two surface topographies shown in Fig. 5a. The common characteristics are the waviness of the surface texture along the cutting speed direction which is caused by tool vibration and the arc chain pattern which is caused by the tool motion along the feed (or longitudinal) direction. By comparing the surface profiles shown in Fig. 5b, it is found that profiles from *Case 1* and *2* are closer to the profile from the experiment because the height variations of profiles from *Case 1, 2,* and experiment are more prominent than that from *Case 3*. This can be explained by the transition pattern observed in the steel bar which is considered during simulation in *Case 1* and *2* but not in *Case 3*. The difference between *Case 1* and *2* is not easy to tell from the profiles. However, it is anticipated that the height variation of *Case 1*, where three group distributions are used, is more prominent than that of *Case 2*, where only one distribution exists (refer to the shaded bars of *Case 1* and *2* in Fig. 5b). This qualitative comparison suggests that the validity of proposed models is very promising.

A quantitative comparison is done by comparing the surface roughness indices between the measured and simulated surface profiles. As listed in Table 1, the surface roughness average (R_a), the root mean square of roughness (R_q), and the mean of the peak to valley value (PTV), which are mostly used in practice, are chosen as the surface roughness indices for comparison. It can be seen that the simulation results from *Case 1* are well matched with those from the experiment. For instance, the mean values of R_a, R_q, and PTV from the measured surface are 4.20 μm, 5.04 μm, and 20.28 μm; and those from *Case 1* are 4.13 μm, 5.05 μm, and 16.97 μm, respectively. The tests on means were carried out to see the statistical difference between the mean values of surface indices. It turned out that R_a and R_q are statistically the same (it is 95% confident to say so) between those from *Case 1* and those from the experiment. Similarly, if tests were carried out for *Case 2* and also for *Case 3*, only the R_q from *Case 2* is statistically the same as that from the experiment. This comparison, therefore, provides a much stronger evidence that *Case 1* is the most likely situation happened during the machining of the AISI 1020 steel bar as performed in the experiment. It is, however, very interesting to observe that the PTV values from all simulation cases do not match that from the experiment. This relates to an elastoplastic interaction between the material and cutting tool and is beyond the scope of this paper.

To some extent, the above quantitative comparison has shown that through the proposed stochastic models (*Case 1*) and computer simulation, we can almost reconstruct the finished surface texture obtained from a turning process. On the other hand, the random tool motion during machining can also be characterized by the proposed stochastic approach if the nonhomogeneous material hardness distribution is considered as the most significant source for random excitation.

6 Conclusions

A stochastic approach to characterize the random tool motion during machining has been proposed. It is based on the Markov chain theory to characterize the non-stationary and time-varying signal process. The *group distributions*, *segment model*, and *transition model* are proposed to quantitatively evaluate the random tool motion caused by a random excitation related to microstructures of the workpiece material being cut. A method has also been developed to determine the states and state transition probabilities defined in the transition model. Results from experiments confirm the predictions obtained from computer simulation. It is believed that this research may become an essential part to quantify the random tool vibration during machining, a task which nowadays seems difficult but critical for developing a sensor-based machining system to improve the quality and productivity in the manufacturing industry.

Acknowledgements

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Appendix - Derivation of b_i's

Equation 12 can be rewritten as

$$\underline{b}\Pi = \underline{\pi}\Gamma \tag{14}$$

where $\underline{b} = [b_1, b_2, ..., b_{N-2}]$, Π is a $(N-2) \times N$ matrix defined as

$$\Pi = \begin{bmatrix} \pi_2 & 0 & -\pi_2 & 0 & 0 & \dots & 0 \\ 0 & \pi_3 & 0 & -\pi_3 & 0 & \dots & 0 \\ 0 & 0 & \pi_4 & 0 & -\pi_4 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & \pi_{N-1} & 0 & -\pi_{N-1} \end{bmatrix} \tag{15}$$

and Γ is a $N \times N$ matrix defined as

$$\Gamma = \begin{bmatrix} (1-p_{11}) & -(1-p_{11}) & 0 & \dots & 0 \\ 0 & (1-p_{22}) & -(1-p_{22}) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & (1-p_{N-1\,N-1}) & -(1-p_{N-1\,N-1}) \\ 0 & \dots & 0 & -(1-p_{NN}) & (1-p_{NN}) \end{bmatrix} \quad (16)$$

The matrices Π and Γ can be estimated based on the estimations of π_i's and p_{ii}'s as derived in the paper (Eqs. 13, 10, and 9), respectively. In other words, $\hat{\Pi} = \Pi(\hat{\underline{\pi}})$ and $\hat{\Gamma} = \Gamma(\underline{\hat{p}_{ii}})$. Finally, the $\underline{b}$ can be estimated by minimizing the mean square error of $(\underline{b}\Pi - \underline{\pi}\Gamma)$; i.e., choose $\underline{b}$ such that

$$\| \hat{\underline{b}}\hat{\Pi} - \hat{\underline{\pi}}\hat{\Gamma} \| = \min (\| \underline{b}\hat{\Pi} - \hat{\underline{\pi}}\hat{\Gamma} \|)$$

where $\| \,.\, \|$ stands for the mean-squared norm. Actually, the solution for $\hat{\underline{b}}$ is given by

$$\hat{\underline{b}} = \hat{\underline{\pi}}\hat{\Gamma}\hat{\Pi}^T(\hat{\Pi}\hat{\Pi}^T)^{-1}. \quad (17)$$

Nomenclature

b_i	state transition probability from state $i+1$ to i
d_{i_j}	j^{th} duration of occurrence at state i
m	number of subdivisions in the cutting speed direction
n_s	number of samples in one revolution
p_{ij}	state transition probability from state i to state j
q_t	observed event at time t
$\overline{r}$	mean of the state ratios
K_i	number of occurrences of state i
N	number of states
Q	an observation sequence
R_q	root mean squared value
ϵ_i	initial state probability of state i
μ_i	mean microhardness value in the i^{th} cross-section
σ_{g_j}	j^{th} group standard deviation of microhardness
σ_r	standard deviation of the state ratios
σ_{R_a}	standard deviation of R_a
Π	a matrix consisting of π_i's
d_i	duration of occurrence at state i
$f(d_i)$	discrete probability density function of d_i
n	number of subdivisions in the feed direction
p_{g_j}	probability of j^{th} group distribution for assignment
$[p]$	transition probability matrix
r_{kl}	state ratio within the k,l^{th} subdivision
D	workpiece (feature) diameter
K_{obs}	total number of observations in a state map
PTV	peak to valley value
R_a	roughness average value
S_i	the i^{th} state
μ_{g_j}	j^{th} group mean microhardness value
π_i	long-term fraction of state i observed in a state map
σ_i	standard deviation of microhardness in the i^{th} cross-section
σ_{PTV}	standard deviation of PTV
σ_{R_q}	standard deviation of R_q
Γ	a matrix consisting of $1-p_{ii}$'s

List of Figures:

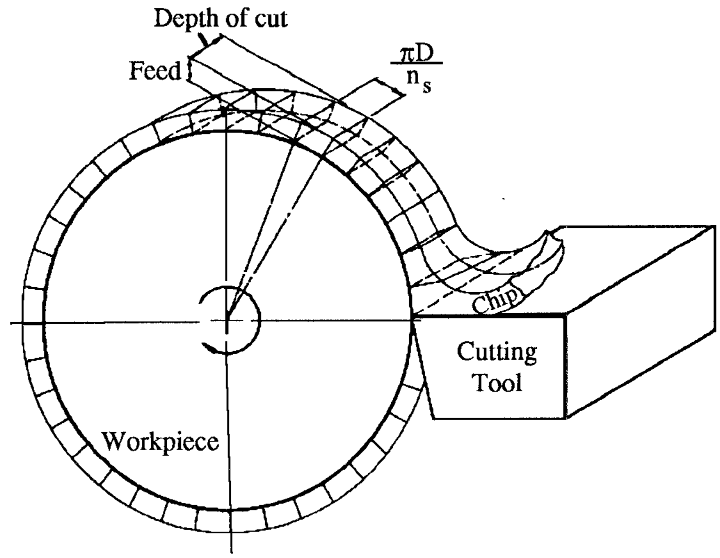
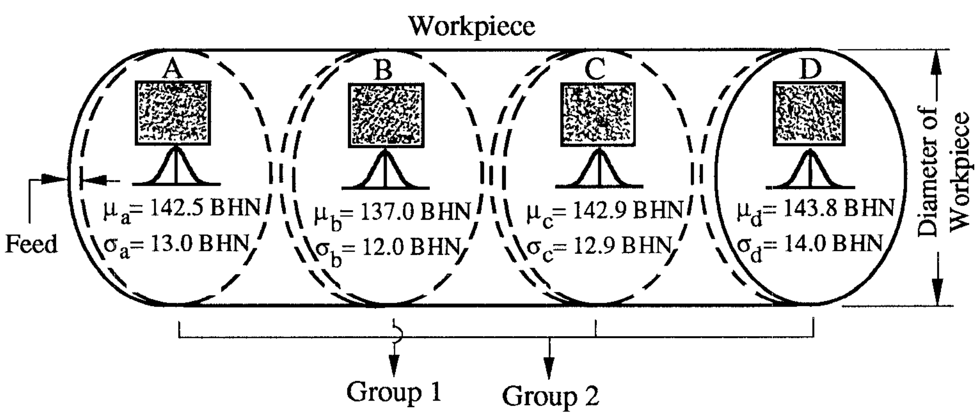
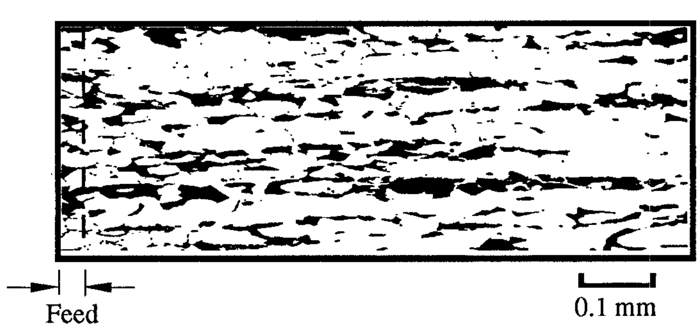


Figure 1: Cutting Process as a Sampling Process



a. Microstructures from Different Cross-Sections



b. Microstructures from Longitudinal Direction

Figure 2: Microstructures from an AISI 1020 Steel Bar

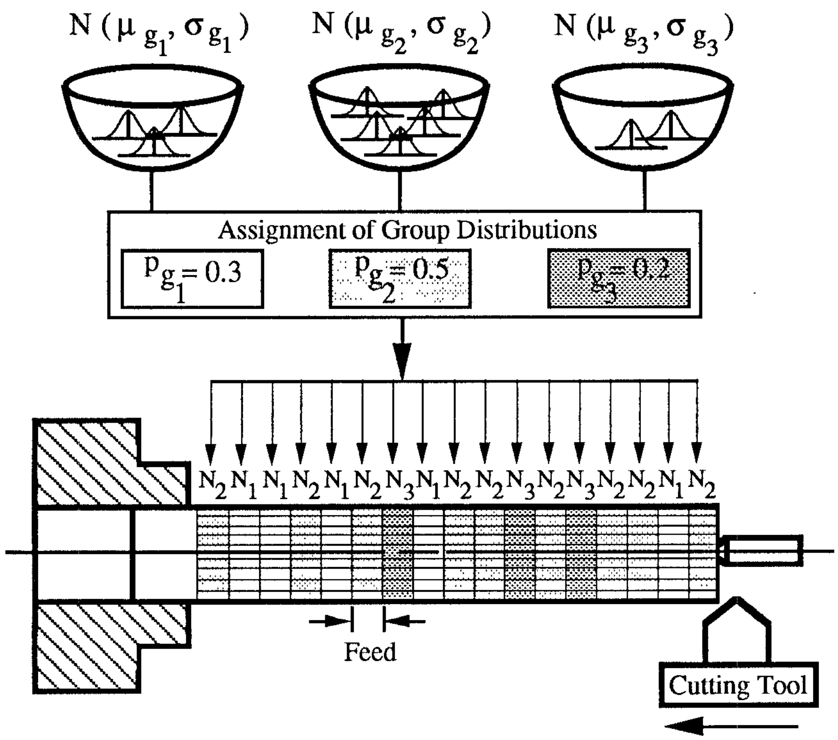
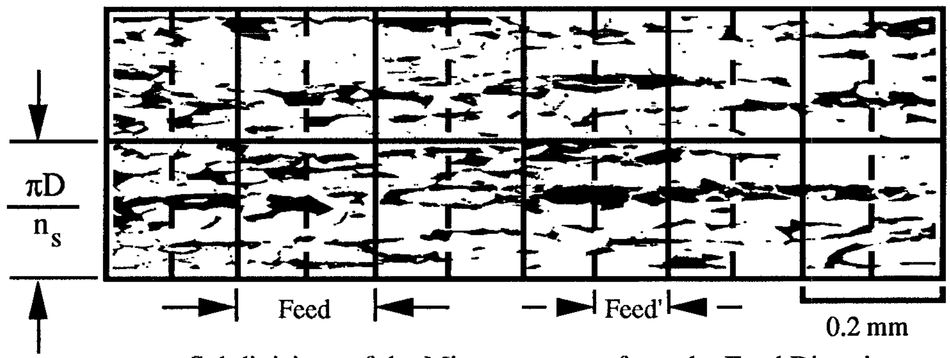
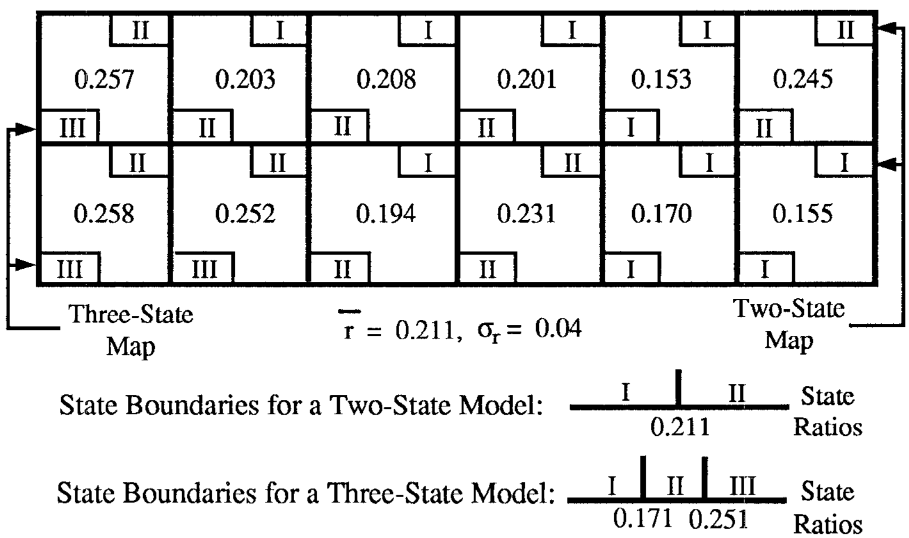


Figure 3: Assignment of Group Distributions to Each Segment

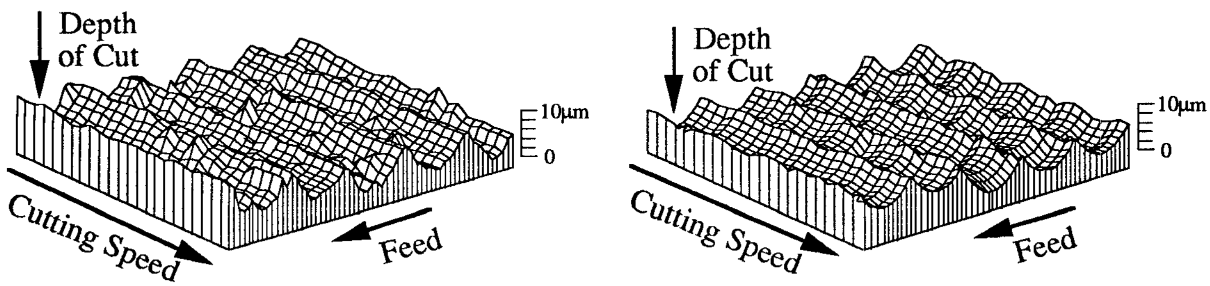


a. Subdivisions of the Microstructures from the Feed Direction



b. State Ratios and State Maps associated with Feed = 0.2 mm

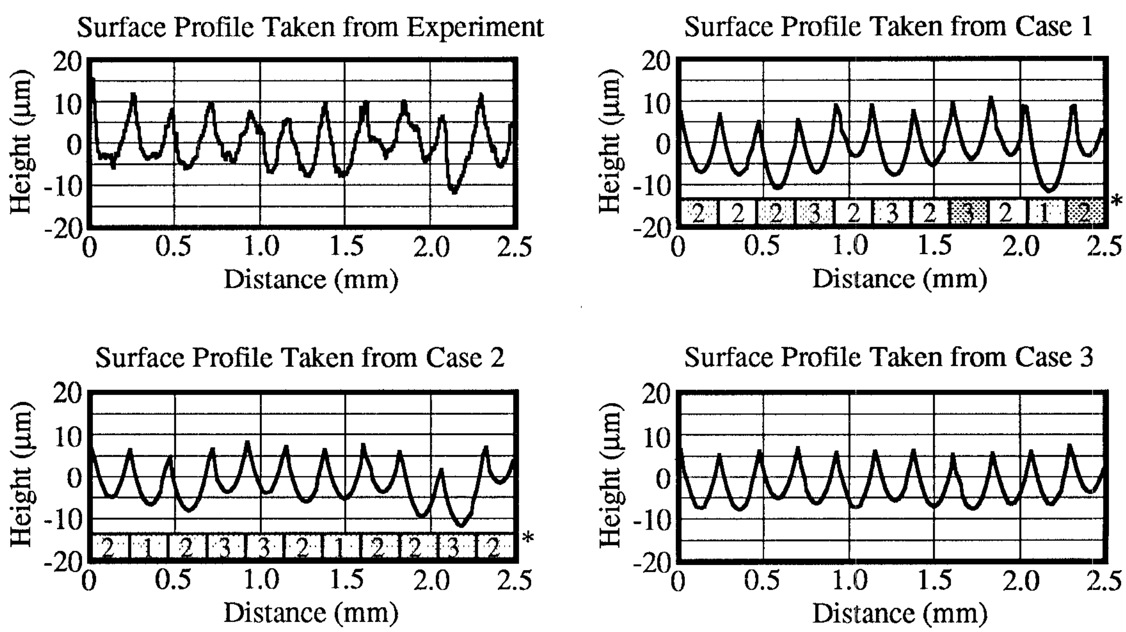
Figure 4: Determination of State Boundaries and State Map
(Two and Three-State Markov Models)



Measured Surface Topography

Simulated Surface Topography (Case 1)

a. Surface Topographies from Experiment and Computer Simulation



* The background shading of each block and the number inside it represent the assigned group distribution (recall Fig. 3) and its state associated with each feed, respectively.

b. Surface Profiles from Experiment and Computer Simulation

Figure 5: Surface Topographies and Profiles from Experiment and Computer Simulation