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by

M. Morari, E. Zafiriou, and B.R. Holt

SYSTEMS RESEARCH CENTER

UNIVERSITY OF MARYLAND

COLLEGE PARK, MARYLAND 20742

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DESIGN OF RESILIENT PROCESSING PLANTS. NEW CHARACTERIZATION OF THE EFFECT OF RHP ZEROS

MANFRED MORARI[†] and EVANGHELOS ZAFIRIOU[‡]
Chemical Engineering, 206-41 California Institute of Technology, Pasadena, CA 91125, U.S.A.

and

BRADLEY R. HOLT
Department of Chemical Engineering, University of Washington, Seattle, WA 98195, U.S.A.

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Abstract—Right Half Plane (RHP) zeros restrict the achievable closed loop performance independent of controller design. A new characterization of all achievable closed loop setpoint/output transfer matrices is provided in terms of "zero-directions". The zero directions also give some insight into what forms of partial decoupling are preferable.

INTRODUCTION

Through extensive studies by a number of researchers during the last few years it has been established rigorously that for both Single-Input–Single-Output (SISO) and Multi-Input–Multi-Output (MIMO) systems RHP zeros limit the achievable closed loop performance *independent* of the control system design. RHP zeros are characteristics of the plant which can be affected only by changes of the plant itself, for example the selection of a different set of manipulated variables. A thorough understanding of the effect of RHP zeros on the achievable closed loop behavior can help the design engineer avoid process options which have inherently bad dynamic performance regardless of how well the control system is designed.

Holt and Morari (1985) (referenced as H&M in the following) have reviewed the different definitions of zeros and have characterized for special cases how MIMO zeros affect the achievable closed loop setpoint/output behavior. The reader is assumed to be familiar with the definitions and results in this paper. To a large extent we will retain the nomenclature introduced there. In this paper we will use the concept of "zero-directions" to characterize the achievable transfer matrices in a convenient and very general manner.

DEFINITIONS AND GENERAL DISCUSSION

The plant is described by the transfer matrix $G(s)$ which satisfies the following assumptions: (1) $G(s)$ is square ($n \times n$); (2) all RHP zeros $z_1, \ldots, z_m$ are of degree one; (3) no RHP poles are located at $z_1, \ldots, z_m$. These assumptions are not restrictive for most practical situations and can be easily relaxed at the expense of a more involved notation. A more general treatment is suggested by the results of Zafiriou and Morari (1985).

Definition 1

Let z_i be a zero of $G(s)$. The vector λ_i ($\lambda_i \neq 0$) satisfying $\lambda_i^T G(z_i) = 0$ is called the *direction* of the zero z_i.

Note that $G(z_i)$ is of rank $n-1$ because the zero was assumed to be of degree one. λ_i is the eigenvector of $G(z_i)^T$ associated with the eigenvalue zero. λ_i is called zero direction because for any system input with frequency z_i the output in the direction of λ_i is identically equal to zero.

Let H_{oi} denote the transfer matrix between output o and input i. In particular, with respect to Fig. 1A we can define the following four relations

$$H_{ur} = C(I+GC)^{-1} \tag{1}$$

$$H_{ud} = -H_{ur} \tag{2}$$

$$H_{yr} = GC(I+GC)^{-1} = GH_{ur} \tag{3}$$

$$H_{yd} = (I+GC)^{-1} = I - H_{yr} = I - GH_{ur}. \tag{4}$$

Note that eqs (2)–(4) can be expressed in terms of the plant G and the IMC-controller H_{ur}. It can be shown (e.g. Callier and Desoer, 1982) that necessary and sufficient conditions for the internal stability of the system in Fig. 1A are

$R1$: H_{ur} stable

$R2$: $(I - GH_{ur})G$ stable.

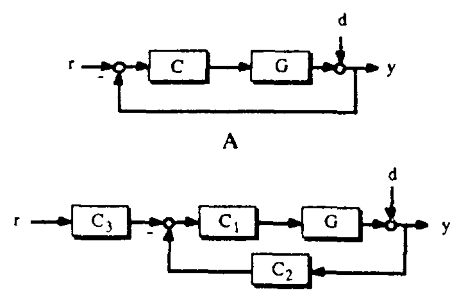

Fig. 1.

[†]Author to whom correspondence should be addressed.
[‡]Current address: Chemical and Nuclear Engineering Department and Systems Research Center, University of Maryland, College Park, MD 20742, U.S.A.

Let us investigate the consequences of the requirements $R1$ and $R2$ on the achievable closed loop performance. Because of $R1$, H_{ur} cannot cancel any RHP zeros of G and they will appear unchanged in H_{yr} (3). Furthermore from eq. (4) and definition 1 we find

$$\lambda_i^T H_{yd}(z_i) = \lambda_i^T(I - GH_{ur}(z_i)) = \lambda_i^T \tag{5}$$

which implies that the magnitude of any disturbance d entering along the zero direction λ_i and passing through to the output y is unaffected by feedback. Thus RHP zeros affect both the achievable H_{yr} and H_{yd}.

$R2$ is implied by $R1$ if G is stable. If G is unstable, the RHP poles of G have to be canceled by RHP zeros of $(I - GH_{ur})$ which imposes further restrictions on the choice of H_{ur} and thus indirectly on H_{yr} and H_{yd}. Thus RHP poles also impose restrictions on the achievable H_{yr} and H_{yd}.

Now consider the general feedback system shown in Fig. 1B. For the disturbance behavior H_{yd} it is irrelevant if the controller is implemented as one block C as in Fig. 1A or as two blocks C_1 and C_2 as in Fig. 1B. Thus H_{yd} is restricted both by the RHP zeros and the RHP poles of the system as discussed previously. The situation is different for H_{yr} as we will explain next.

Let us assume now that a stabilizing controller C for satisfactory disturbance response has been found. Let us split C into two blocks C_1 and C_2 such that C_1 is minimum phase and C_2 is stable. Then it is easy to see that the only RHP zeros of the stabilized system $GC_1(I + GC_1C_2)^{-1}$ are those of the plant G. Thus C_3 can be designed without regard for the RHP poles of G and H_{yr} is restricted by the RHP zeros of G *only*.

In summary, the achievable disturbance response of a system is restricted by the presence of the plant RHP zeros and poles regardless of how complicated a controller is used. If the Two-Degree-of-Freedom controller shown in Fig. 1B is employed the achievable setpoint response is restricted by the plant RHP zeros only. For a more rigorous discussion the reader is referred to Vidyasagar (1985).

CHARACTERIZATION OF ACHIEVABLE H_{yr}

The foregoing discussion implies the following theorem.

Theorem 1

H_{yr} is achievable by a set of controllers C_1, C_2 and C_3 such that the closed loop system is internally stable if and only if there exists a stable Q such that $H_{yr} = GQ$.

Proof. The necessity follows directly from $R1$. For a stable plant G, $R1$ is also sufficient for internal stability. Indeed a controller yielding H_{yr} is given by $C_1 = Q(I - GQ)^{-1}$, $C_2 = C_3 = I$. As discussed in the previous section an unstable plant can be stabilized by C_1 and C_2 without adding RHP zeros. C_3 can then be designed for the stabilized plant.

A direct test for the existence of a stable Q is provided by the following theorem.

Theorem 2

There exists a stable Q and thus a set of stabilizing controllers C_1, C_2 and C_3 such that the setpoint/output transfer matrix is H_{yr} equal to a desired transfer matrix if and only if

$$\lambda_i^T H_{yr}(z_i) = 0 \tag{6}$$

for all RHP zeros z_i of the plant $G(s)$ where λ_i is the direction of the zero $z_i(\lambda_i^T G(z_i) = 0)$.

Proof. See Appendix.

Before we discuss the implications of theorem 2 and put it in relation to the results obtained by H&M, we will illustrate it through an example.

Example 1. Consider the plant G_6 from H&M

$$G_6 = \frac{1}{s+1}\begin{bmatrix} 1 & 1 \\ 1+2s & 2 \end{bmatrix} \tag{7}$$

which has a z..o at $s = z = 1/2$. The zero direction $\lambda = (2, -1)$ satisfies

$$\lambda^T G_6(z) = \lambda^T \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = 0. \tag{8}$$

We will use condition (6) to construct decoupled and one-way decoupled H_{yr}'s for which stable Q's and therefore stabilizing C's exist. Trivially for the decoupled plant

$$H_{yr} = \begin{bmatrix} \dfrac{-2s+1}{2s+1} & 0 \\ 0 & \dfrac{-2s+1}{2s+1} \end{bmatrix}. \tag{9}$$

$H_{yr}(z) = 0$ and therefore (6) is satisfied. Let us postulate

$$H_{yr}^{LT} = \begin{bmatrix} 1 & 0 \\ x_1 & \dfrac{-2s+1}{2s+1} \end{bmatrix} \tag{10}$$

and

$$H_{yr}^{UT} = \begin{bmatrix} \dfrac{-2s+1}{2s+1} & x_2 \\ 0 & 1 \end{bmatrix} \tag{11}$$

where x_1 and x_2 are to be determined. We find from (6)

$$2 - x_1(z) = 0 \quad \text{or} \quad x_1(z) = 2 \tag{12}$$

$$2x_2(z) - 1 = 0 \quad \text{or} \quad x_2(z) = \tfrac{1}{2}. \tag{13}$$

If we postulate x_1 and x_2 to be of the form $\beta s/(2s+1)$ (with β a constant to be determined) so that there are no steady state interactions, then we find from (12)

$$x_1(z) = \frac{\beta}{4} = 2$$

and

$$x_1(s) = \frac{8s}{2s+1}. \tag{14}$$

Similarly from (13)

$$x_2(s) = \frac{2s}{2s+1}. \tag{15}$$

As illustrated by the example, theorem 2 has the following implications:

(1) λ_i is a constant possibly complex vector. Equation (6) requires that each column of H_{yr} evaluated at the plant zero z_i has to be orthogonal to λ_i. For a particular element of the input vector r the elements of the output vector y cannot be selected independently but have to satisfy the linear "interpolation conditions" [eq. (6)]. The presence of RHP plant zeros requires some relationship between the elements of a column of H_{yr} but the columns themselves can be selected independently of each other.

(2) By assumption, z_i is a zero of $G(s)$ of degree one. Assume that H_{yr} is selected to be diagonal, i.e. completely decoupled. Then according to the theorem the degree of z_i in H_{yr} has to be at least equal to the number of nonzero entries in λ_i. Usually this number is larger than one, generally equal to n. Thus, requiring a decoupled response generally leads to the introduction of RHP zeros not originally present in the plant $G(s)$. This is the price to be paid for decoupling (Desoer and Gündes, 1986).

(3) The zero z_i is "pinned" to the outputs corresponding to nonzero entries in λ_i (Bristol, 1980; H&M, p. 67): The zero has to affect at least one of these outputs and it cannot affect any of the outputs corresponding to zero entries in λ_i.

Example 2 (*from H&M*). The system

$$G_4(s) = \frac{1}{s+2}\begin{bmatrix} -s+1 & -s+1 \\ 1 & 2 \end{bmatrix} \tag{16}$$

has a zero at $s = 1$ with the direction $\lambda^T = (1, 0)$. It can only affect the first output of H_{yr}, i.e. it is "pinned" to the first output.

(4) Theorem 14 (from H&M, rephrased): Assume that the kth element λ_{ik} of the zero direction λ_i is nonzero. Then it is possible to obtain perfect control on all outputs $j \neq k$ with the remaining output exhibiting no steady state offset.

This and the next result follow trivially from theorem 2 of this paper.

(5) Theorem 16 (from H&M, rephrased): Assume that $G(s)$ has a single zero z and that the kth element λ_k of the zero direction λ is nonzero. Then H_{yr} can be chosen of the form

$$H_{yr} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\beta_1 s}{s+z} & \frac{\beta_2 s}{s+z} & \cdots & \frac{\beta_{k-1} s}{s+z} & \frac{-s+z}{s+z} & \frac{\beta_{k+1} s}{s+z} & \cdots & \frac{\beta_n s}{s+z} \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{17}$$

where

$$\beta_j = -\frac{2\lambda_j}{\lambda_k} \quad \text{for } j \neq k. \tag{18}$$

The interaction terms will be insignificant if $\lambda_k \gg \lambda_j (\forall j \neq k)$, i.e when the zero is aligned predominantly with output k. If for some j, $\lambda_j \gg \lambda_k$ then the zero is aligned predominantly with output j. It can be pushed to output k only at the cost of generating significant interactions (large β's).

As a demonstration of the alignment effect recall example 1 with the zero aligned with the first output [$\lambda^T = (2, -1)$]. As shown by H&M, pushing the zero to the second output [eq. (10)] leads to an ISE of four while aligning the zero with the first output [eq. (11)] is much more favourable (ISE = 1). Thus, if one way decoupling is contemplated the zero direction should be used as a guideline.

CONCLUSION

The zero directions have been shown to be a convenient tool to judge the feasibility of alternate forms of decouplers. Though generically all kinds of decouplers are almost always feasible, they might not be advisable in terms of performance. If a zero direction is aligned predominantly with one output, the best overall performance for the MIMO system is achieved when this alignment is preserved in the closed loop transfer matrix.

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APPENDIX: PROOF OF THEOREM 2

Assume there exists a stable Q such that $H_{yr} = GQ$. Then $\lambda^T H_{yr}(z_i) = \lambda^T GQ(z_i) = 0$.

Find the partial fraction expansion of G^{-1}.

$$G(s)^{-1} = \frac{1}{s - z_i} R_i + P(s) \tag{A1}$$

where R_i is the matrix of residuals and $P(s)$ is a remainder term with no poles at $s = z_i$. Postmultiply both sides of eq. (A1) by $G(s)$

$$I = \frac{1}{s - z_i} R_i G(s) + P(s)G(s). \tag{A2}$$

Because the LHS of eq. (A2) is the identity the RHS must not have a pole at $s = z_i$. $P(s)G(s)$ does not have a pole at $s = z_i$ and therefore it must be that

$$R_i G(z_i) = 0. \tag{A3}$$

Because z_i is of degree one, $G(z_i)$ is of rank $(n - 1)$. Hence R_i is of rank 1 and as a result of definition 1, the rows of R_i are multiples of λ_i^T. Therefore $\lambda_i^T H_{yr}(z_i) = 0$ implies

$$R_i H_{yr}(z_i) = 0. \tag{A4}$$

Now postmultiply both sides of eq. (A1) by H_{yr}.

$$Q = G^{-1} H_{yr} = \frac{1}{s - z_i} R_i H_{yr}(s) + P(s) H_{yr}(s). \tag{A5}$$

$P(s)H_{yr}(s)$ does not have a pole at $s = z_i$. Hence eq. (A4) implies that Q does not have a pole at $s = z_i$.