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Mechanical Systems with Partial Damping: Two Examples

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ABSTRACT. We discuss the problem of constructing steady state motions of mechanical systems with partial damping. A planar three bar linkage with viscous damping at one of the joints is considered as an example. We show that for a generic set of system parameters all steady state motions are confined to relative equilibria. We also consider the example of two rigid bodies with on-board rotors coupled via a ball-in-socket joint with viscous friction and show that in the steady state, the system is at a relative equilibrium.

KEYWORD. Mechanical Systems, Damping, Relative Equilibria.

1 Introduction

Mechanical systems typically possess some amount of damping. Thus the steady state behavior of systems can be described by restricting the dynamics to a certain maximal invariant set in the state space. An example of such behavior is that of laminar flow of fluids [5]. Even though the overall system is infinite dimensional, in the steady state, the flow is captured by a finite dimensional model and the restriction of the system to a finite dimensional invariant set known as the *inertial manifold* is well-understood in many cases [5].

Here we consider two essentially equivalent methods to compute the steady state behavior of a Lagrangian system with partial dissipation. In the first method, which is

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valid for real analytic systems, dissipation is modeled as a vector field along which the time derivative of the hamiltonian is nonpositive. This then yields the criterion that along any steady state motion all higher order time derivatives of the hamiltonian with respect to the dissipative field should be zero. This in turn gives rise to equations which describe an invariant subset of the state space of the system which contains all steady state motions. This method will be demonstrated by an example of coupled rigid-body system.

Another method to compute steady state motions is the following. Suppose that the dissipation is due to viscous friction at a subset of the joints. Now the configuration space of the system can be thought of as a fiber bundle over the space of joint variables and any motion of the body which yields a relative motion at joints with viscous friction corresponds to negative external work done and hence by the Lagrange-d'Alembert Principle, the total energy of the system will decrease. Thus in the steady state there is no relative motion at joints with viscous friction and vice versa. We will illustrate this with the example of a three-bar linkage mechanism with one of the two joints subject to viscous friction. An interesting aspect of this example is that it displays resonance behavior in the sense that for a nongeneric set of system parameters there can be non-relative equilibrium steady state motions.

2 Hamiltonian Systems with Added Dissipation

A mechanical system with damping (in reduced or unreduced phase space) can be described abstractly as a triple (P, X_H, X^D), where P is a Poisson manifold with a Poisson structure $\{\cdot,\cdot\}$, X_H is a hamiltonian vector field on P with hamiltonian H, and X^D is a vector field on P which describes dissipative terms.

DEFINITION 2.1.

A vector field Y on P is called a *dissipative field* with respect to the hamiltonian H

if,

(1) $Y[H](p) \leq 0, \quad \forall\, p \in P$,

(2) For $p \in P$,

$$Y[H](p) = 0, \quad \text{if and only if} \quad Y(p) = 0.$$

∎

In what follows, we will assume that X^D is a dissipative field.

The fact that in the case of mechanical systems one is led to consider a triple as above can be inferred from the following.

Let Q be the configuration space of a mechanical system with the associated Lagrangian $L : TQ \to \mathbb{R}$. Let $H_L : TQ \to \mathbb{R}$ be the associated energy (see [1] [6] for details). TQ inherits a symplectic structure from the canonical symplectic structure of T^*Q via L in a natural way. Let X_{H_L} denote the hamiltonian vector field on TQ associated to H_L. A dissipative force can be modeled as a horizontal one-form ω on TQ by considering the virtual work on infinitesimal displacements, i.e. special vector fields. Due to the symplectic structure, there is a one-to-one correspondence between horizontal 1-forms and vertical vector fields on TQ. Let X^D be the vertical vector field on TQ which corresponds to ω. As a generalization of the notion of Rayleigh's dissipation, (c.f. e.g. [2],) we assume that the dissipative forces satisfy the following properties,

(1) $\omega(x, 0)(v,\ w) = 0$, for all $x \in Q$, $v,\ w \in T_xQ$.

(2) $\omega(x, \dot{x})(\dot{x}, \ddot{x}) \leq 0$, for all $(x, \dot{x}) \in TQ$, and furthermore, if $\omega(x, \dot{x})(\dot{x}, 0) = 0$ at some $(x, \dot{x})$, then $\omega(x, \dot{x}) = 0$.

It can be seen easily that these conditions imply that if $X^D[H_L](x, \dot{x}) = 0$ at some $(x, \dot{x}) \in TQ$, then $X^D(x, \dot{x}) = 0$ as well. Each triple (TQ, X_{H_L}, X^D) defines a mechanical system under consideration.

Now let us consider our general mechanical system (P, X_H, X^D). We assume that H is bounded below and all solutions of the system

$$\dot{x} = X_H + X^D, \tag{2.1}$$

are bounded. Let $\mathcal{N} = \{ x \in P \mid X^D[H](x) = 0 \}$. Let $\mathcal{M}$ be the maximal invariant set of the system with respect to the flow of $X_H + X^D$, contained in $\mathcal{N}$. By LaSalle's invariance principle all solutions of the system converge to $\mathcal{M}$ in the steady state. Thus we are interested in the structure of $\mathcal{M}$. Let L_Y denote Lie differentiation with respect to the vector field Y.

LEMMA 2.2.

Suppose that the system is real analytic. Then

$$\mathcal{M} = \{ x \in P \mid L^k_{X_H} L_{X^D} H(x) = 0, \quad k = 0, 1, 2, \cdots \}. \tag{2.2}$$

Proof.

Let $x \in P$ be such that $L^k_{X_H} L_{X^D} H(x) = 0$, $k = 0, 1, 2, \cdots$. Then it follows that $X^D[H] = 0$ along the X_H orbit through p, which is denoted by γ^p. Now by the definition of a dissipative field, $X^D = 0$ on γ^p. Thus γ^p is the $X_H + X^D$-orbit through p and it lies in $\mathcal{N}$. Therefore $p \in \mathcal{M}$. Conversely, if $p \in \mathcal{M}$, then $X^D = 0$ along the X_H and $X_H + X^D$ orbit through p. Therefore $L^k_{X_H} L_{X^D} H(x) = 0$, $k = 0, 1, 2, \cdots$.

∎

In terms of the Poisson structure, we can further write

$$\begin{aligned} \mathcal{M} = \{ p \mid X^D[H](p) = 0, \ \{H, \ X^D[H]\}(p) = 0, \cdots \\ \{H, \{..., \{H, \ X^D[H]\}...\}\}(p) = 0, \cdots \}. \end{aligned} \tag{2.3}$$

This construction will be described via an example of two rigid bodies coupled by a ball-in-socket joint with viscous friction.

Another observation that helps us to compute the set $\mathcal{M}$ is the following. Suppose that damping occurs only at a certain subset of joints. Now let $\phi : Q \to S$ be the projection map from the configuration space to the space of joints with damping. Let us assume that the dissipation on S is due to Rayleigh damping. Now it follows that any infinitesimal displacement X on TQ such that $T\phi(X) \neq 0$ will cause energy loss. Therefore steady state motions should be such that the corresponding joint variables are kept constant. This observation sometimes help us to unravel the otherwise intractable set of equations describing the maximal invariant set. We will illustrate this via an example of three-bar linkage with one dissipative joint.

3 Examples

First we consider the example of two rigid bodies with on-board rotors coupled via a ball-in-socket joint. Figure 3.1 depicts the system configuration. Let B_1, B_2 denote the attitudes of body 1, body 2, respectively, viewed as elements in the rotation group $SO(3)$. One set of rotors, called *driven rotors*, on body 1 are set to spin at constant rates relative to body 1. The other set of rotors, called *damping rotors*, on body 2 provides damping torques according to the following law,

$$T^D = -\alpha\dot{\Phi}, \tag{3.1}$$

where α is a positive-definite matrix and Φ is a vector consisting of relative angles of each damping rotors relative to body 2. Let Ω_1, Ω_2 be the angular velocities of body 1, and 2 relative to their corresponding frame. Let $B = B_1^T B_2$ denote the coordinate transformation from body 2 to body 1, or the *shape* variable. We assume that there is a damping torque exerted at the spherical joint in the form of

$$T^J = -\sigma(\Omega_2 - B^T\Omega_1). \tag{3.2}$$

where σ is also a positive-definite matrix.

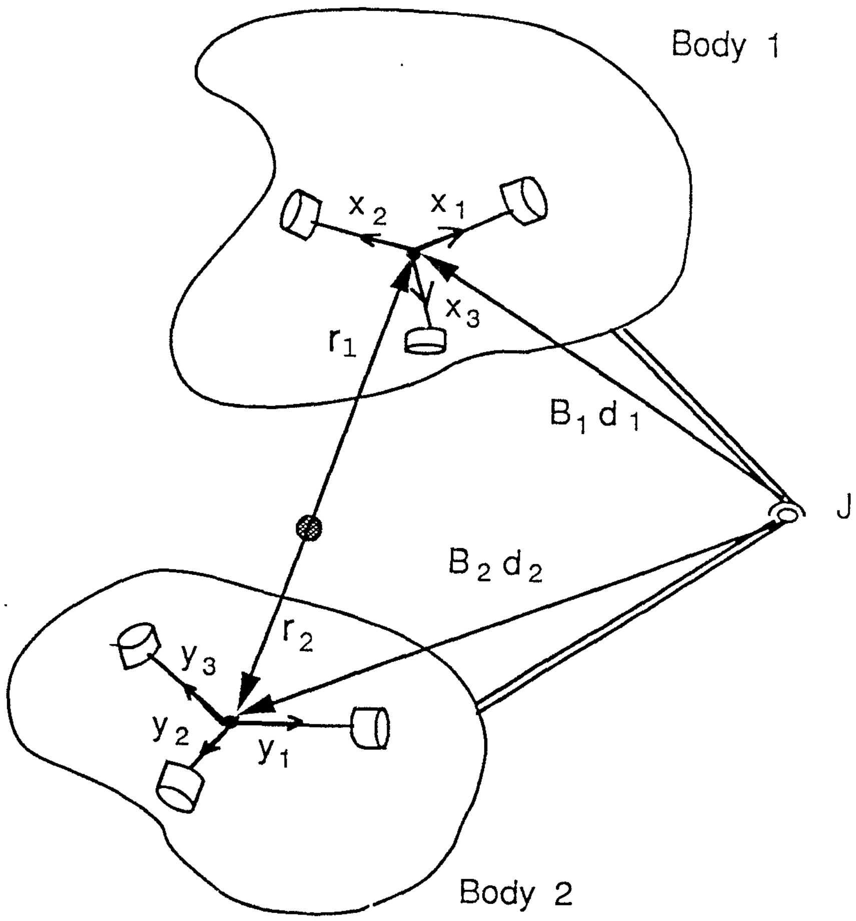

Figure 3.1. Two Rigid Bodies with Rotors

The system under investigation can be set into the framework illustrated in Section 2 with the following entities. First, the configuration space can be modeled as

$$Q = SO(3) \times (S^1)^3 \times SO(3) \times (S^1)^3,$$

where the first $(S^1)^3$ represents the driven rotors and the second one represents the damping rotors. Since each of the driven rotors spins at a constant rate, the system without damping is a gyroscopic system, cf. [6], [7]. Thus it can be shown that the energy functional H_L can be written as

$$
\begin{aligned}
H_L \; &= \frac{1}{2} < \Omega_1,\ \mathbf{J}_1\Omega_1 > + \frac{1}{2} < \Omega_2,\ \mathbf{J}_2\Omega_2 > + \epsilon < \Omega_1,\ d_1 \times B(d_2 \times \Omega_2) > \\
&\quad + \frac{1}{2} < \dot{\Phi}, \mathbf{I}^D \dot{\Phi} > + < \Omega_2,\ \mathbf{I}^D \dot{\Phi} >,
\end{aligned}
\tag{3.3}
$$

where $\mathbf{J}_1$, $\mathbf{J}_2$ are the moments of inertia of body 1 and body 2 with locked rotors respectively, the constant $\epsilon = (m_1 m_2)/(m_1 + m_2)$ is the reduced mass, and the diagonal matrix $\mathbf{I}^D$ consists of three moments of inertia of damping rotors with respect to their corresponding spinning axis. In (3.3) and what follows, the symbol $< \cdot, \cdot >$ denotes the Euclidean inner product. The exterior force ω can be constructed from the damping torques T^D, T^J in (3.1), (3.2). By evaluating the one-form ω on the vector field X_{H_L}, we obtain,

$$
\omega(X_{H_L}) \; = \; - < \dot{\Phi},\ \alpha\dot{\Phi} > \; - \; < \Omega_2 - B^T\Omega_1,\ \sigma(\Omega_2 - B^T\Omega_1) > \; \leq \; 0.
$$

Accordingly, by the definition in Section 2, ω is a dissipative force. The maximum invariant set $\mathcal{M}$ can be then found to be the same as the set of relative equilibria. From the discussion in Section 2, it follows that all steady state motions should be at relative equilibria of the undamped system. See [7] for detailed discussions.

The second example is a planar three bar linkage mechanism with one dissipative joint. Here we consider three planar objects coupled to each other via pin joints, see Figure 3.2. The center of mass of the second body is assumed to lie on the straight line segment from joint 1 to joint 2. Joint 1 is assumed to be dissipative and joint 2 is frictionless. Thus in the steady state, there is no relative motion at joint 1. We exploit this fact to compute the maximal invariant set. A direct attempt at this computation leads to an intractable problem.

Our approach is the following. Since there is no relative motion between links 1 and 2, we lump them into one body and consider the dynamics of a two-body system. However since the internal torque at joint 1 is equal to 0, the system should satisfy an extra constraint. This will show that links 1 and 2 should be aligned in a straight line. Now the constraint equation simplifies even further and will lead to a set of polynomial equations

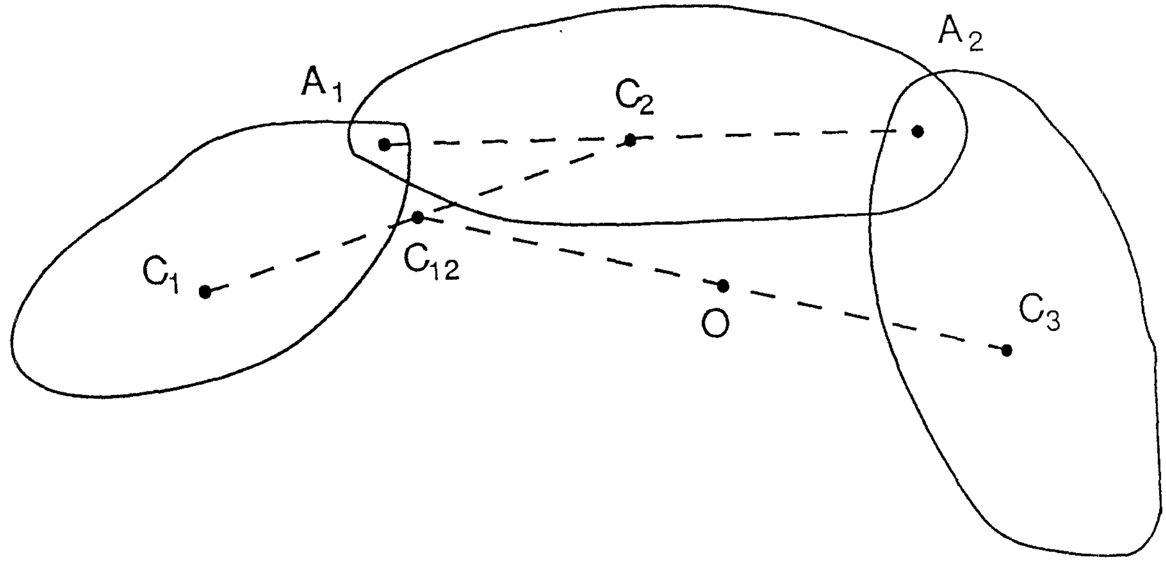

Figure 3.2. A Planar Three Bar Linkage

which should be satisfied in order for non-relative equilibrium steady state motions to exist. Let

O: center of mass of the system,

C_{12}: center of mass of links 1 and 2,

$\overline{A_2C_2}$, $\overline{A_1A_2}$ etc. will have obvious meanings and $|\overline{A_2C_2}|$, $|\overline{A_1A_2}|$ etc. will denote their Euclidean norms, respectively,

$$m_{12} = m_1 + m_2,$$

$$m = m_{12} + m_3,$$

ω_i = angular velocities of link i,

I_i = moment of inertia of link i about its center of mass.

By assumption, C_2 lies on the line segment $\overline{A_1A_2}$. Consider a steady state motion. First lump links 1 and 2 together to form link (1,2). For the sake of nontriviality, assume that the system is not at a relative equilibrium and the total angular momentum is nonzero. We will only consider the case when $C_{12} \neq A_2$ here. Now for the planar two-body (links (1,2) and 3) problem, for the ensuing motion let $\omega_3(t)$, $\omega_{1,2}(t)$ denote the angular velocities of links 3 and (1,2) respectively and let $\beta(t)$ be the angle between them. We will first show that C_1, A_1 and A_2 are collinear. The dynamical equations are (see [3] [4]),

$$\begin{pmatrix} J_{1,2} & \gamma\cos\beta \\ \gamma\cos\beta & J_3 \end{pmatrix}\begin{pmatrix} \dot\omega_{1,2} \\ \dot\omega_3 \end{pmatrix} = \gamma\sin\beta\begin{pmatrix} \omega_3^2 \\ -\omega_{1,2}^2 \end{pmatrix} \tag{3.4}$$

where

$$\begin{aligned} \epsilon &= \frac{m_3 m_{1,2}}{m_3 + m_{1,2}}, \\ J_{1,2} &= I_{1,2} + \epsilon|\overline{A_2C_{12}}|^2, \\ J_3 &= I_3 + \epsilon|\overline{A_2C_3}|^2, \\ \gamma &= \epsilon|\overline{A_2C_3}|\,|\overline{A_2C_{12}}|, \end{aligned}$$

and $I_{1,2}$ is the moment of inertia of link (1,2). Hence

$$\begin{aligned} \ddot\beta &= \dot\omega_{1,2} - \dot\omega_3 \\ &= \frac{\gamma\sin\beta}{J_{1,2}J_3 - \gamma^2\cos^2\beta}\left((J_{1,2}+\gamma\cos\beta)\omega_{1,2}^2 + (J_3+\gamma\cos\beta)\omega_3^2\right). \end{aligned} \tag{3.5}$$

Now we show that there exists some $t_0 > 0$ such that $\beta(t_0) = \pi$. For, assume otherwise, i.e. $\beta(t) \in (-\pi,\pi)$ for all $t > 0$. Since we are considering non-relative equilibrium steady motions, we may discard the possibility that $\beta(t)$ converges to a limit as $t \to \infty$. Now let $\bar t > 0$ such that either $\beta(\bar t) > 0$ and $\beta(\bar t)$ is a local maximum or $\beta(\bar t) < 0$ and $\beta(\bar t)$ is a local minimum. Then, $\dot\beta(\bar t) = 0$ and $\ddot\beta(\bar t)\sin\beta(\bar t) \leq 0$. However, from (3.5),

$$\ddot\beta(\bar t)\sin\beta(\bar t) = \frac{\gamma\sin^2\beta(\bar t)}{J_{1,2}J_3 - \gamma^2\cos^2\beta(\bar t)}\left((J_{1,2}+J_3) + 2\gamma\cos\beta(\bar t)\right)\omega_3(\bar t)^2,$$

and its right hand side is positive since $J_{1,2}J_3 > \gamma^2$. This contradiction proves our assertion that there exists some $t_0 > 0$ such that $\beta(t_0) = \pi$. For simplicity let $t_0 = 0$.

Let us now focus on the dynamic force balance of the system at $t = 0$. From (3.4) it follows that $\dot\omega_{1,2}(0) = 0 = \dot\omega_3(0)$.

Let us denote the acceleration vector of point x relative to y at time $t = 0$ by a_{xy}, namely, $a_{xy} \triangleq \ddot{\overline{yx}}$. Let $\omega_i(0) = \Omega_i$. It is easy to see that

$$a_{C_{12},O} = \frac{1}{m}\left(m_3\Omega_3^2\,\overline{A_2C_3} + m_3\Omega_{1,2}^2\,\overline{C_{12}A_2}\right),$$

where $m = m_1 + m_2 + m_3$. Since $\beta(0) = \pi$, it follows that

$$a_{C_{12},O} = \lambda\,\overline{A_2C_3},$$

for some positive constant λ. For an arbitrary point P on $\overline{C_1A_1}$,

$$a_{P,O} = a_{P,A_1} + a_{A_1,O} = \Omega_{1,2}^2\,\overline{PA_1} + a_{A_1,O}.$$

Since the internal torque at A_1 is equal to zero, it follows from the Newton's equation that,

$$a_{A_1,O} = \mu\,\overline{C_1A_1}, \tag{3.6}$$

for some $\mu \in \mathbb{R}$. Now

$$\begin{aligned} a_{A_1,O} &= a_{A_1,C_{12}} + a_{C_{12},O} \\ &= \Omega_{1,2}^2\,\overline{A_1C_{12}} + \lambda\,\overline{A_2C_3}, \\ &= \Omega_{1,2}^2\,\overline{A_1C_{12}} + \theta\,\overline{C_{12}A_2}. \end{aligned} \tag{3.7}$$

where θ is a positive constant. However, since C_2 lies on $\overline{A_1A_2}$,

$$\overline{A_1C_1} = \eta_1\,\overline{A_1C_{12}} - \frac{m_2}{m_1}\eta_2\,\overline{C_{12}A_2}, \tag{3.8}$$

where η_1, η_2 are positive constants. From (3.6), (3.7) and (3.8) it now follows that $\overline{A_1C_{12}}$ and $\overline{C_{12}A_2}$ are linearly dependent. Therefore A_1, A_2 and C_1 are collinear.

Let β denote the angle between $\overline{A_2C_3}$ and $\overline{A_2C_{12}}$. For $x_1(t)$, $x_2(t) \in \mathbb{R}^2$, let the acceleration vector of $x_1(t)$ relative to $x_2(t)$ be denoted by $\alpha_{x_1,x_2}(t)$. For $\theta \in \mathbb{R}$, let

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

and define the angle of inclinations of $\overline{A_2C_3}$ and $\overline{A_2C_{12}}$ by

$$\begin{aligned}\overline{A_2C_3} &= R(\theta_3)|\overline{A_2C_3}|\,\mathbf{i}, \\ \overline{A_2C_{12}} &= R(\theta_{1,2})|\overline{A_2C_{12}}|\,\mathbf{i},\end{aligned}$$

where $\mathbf{i} = (1\ \ 0\ \ 0)^T$. Now by a straightforward calculation, it can be checked that

$$\alpha_{C_{12},O} = \frac{m_3}{m}(\dot\theta_3^2\,\overline{A_2C_3} - \dot\theta_{1,2}^2\,\overline{A_2C_{12}} - \ddot\theta_3 R(\theta_3)|\overline{A_2C_3}|\,\mathbf{j} + \ddot\theta_{1,2}R(\theta_{1,2})|\overline{A_2C_{12}}|\,\mathbf{j}),$$

where $\mathbf{j} = (0\ \ 1\ \ 0)^T$. Hence

$$\alpha_{C_1,O} = \frac{m_3}{m}\left(\dot\theta_3^2\,\overline{A_2C_3} - \ddot\theta_3 R(\theta_3)|\overline{A_2C_3}|\,\mathbf{j} + \varepsilon\left(\dot\theta_{1,2}^2\,\overline{A_2C_{12}} - \ddot\theta_{1,2}R(\theta_{1,2})|\overline{A_2C_{12}}|\,\mathbf{j}\right)\right),$$

where ε is a real constant. Now since the internal torque at A_1 is equal to zero, by Newton's equation we obtain,

$$0 = m_1\alpha_{C_1,O} \times \overline{A_1C_1} + I_1\ddot\theta_{1,2}\,\mathbf{k},$$

where $\mathbf{k} = \mathbf{i} \times \mathbf{j}$. After simplifying this we obtain,

$$\delta\frac{m_3}{m}\left((\dot\theta_3^2\sin\beta + \ddot\theta_3\cos\beta)\,|\overline{A_2C_3}|\,|\overline{A_2C_{12}}| + \varepsilon\,\ddot\theta_{1,2}|\overline{A_2C_{12}}|^2\right) + I_1\ddot\theta_{1,2} = 0, \quad (3.9)$$

where δ is a constant.

The dynamical equations of the two-body system, i.e. links (1,2) and 3, are

$$\begin{pmatrix} J_3 & \gamma\cos\beta \\ \gamma\cos\beta & J_{1,2}\end{pmatrix}\begin{pmatrix}\ddot\theta_3 \\ \ddot\theta_{1,2}\end{pmatrix} = \gamma\sin\beta\begin{pmatrix}-\dot\theta_{1,2}^2 \\ \dot\theta_3^2\end{pmatrix}$$

where J_3, $J_{1,2}$ and γ are defined previously. Hence

$$\begin{pmatrix}\ddot\theta_3 \\ \ddot\theta_{1,2}\end{pmatrix} = \frac{-\gamma\sin\beta}{J_3J_{1,2} - \gamma^2\cos^2\beta}\begin{pmatrix} J_{1,2}\dot\theta_{1,2}^2 + \gamma\cos\beta\dot\theta_3^2 \\ -\gamma\cos\beta\dot\theta_{1,2}^2 - J_3\dot\theta_3^2\end{pmatrix}. \quad (3.10)$$

Substitution of (3.10) in (3.9) yields,

$$a\omega_3^2 + b(\cos\beta)\omega_{1,2}^2 + c(\cos\beta)^2\omega_3^2 = 0, \quad (3.11)$$

for all β in an interval around π, where

$$
\begin{aligned}
a &= \gamma J_3(I_1 + \varepsilon\,|\,\overline{A_2C_{12}}\,|^2) \;+\; \delta\frac{m_3}{m}J_3J_{1,2}|\,\overline{A_2C_3}\,|\,|\,\overline{A_2C_{12}}\,|,\\
b &= \gamma^2(I_1 + \varepsilon\,\delta\frac{m_3}{m}|\,\overline{A_2C_{12}}\,|^2) \;-\; \gamma J_{1,2}\delta\frac{m_3}{m}|\,\overline{A_2C_3}\,|\,|\,\overline{A_2C_{12}}\,|,\\
c &= -\;2\delta\gamma^2\frac{m_3}{m}|\,\overline{A_2C_3}\,|\,|\,\overline{A_2C_{12}}\,|,\\
\omega_3 &= \dot{\theta}_3,\\
\omega_{1,2} &= \dot{\theta}_{1,2}.
\end{aligned}
$$

Clearly a, b, c are constants which are polynomials of system parameters. It now follows that if the system parameters are such that $a = 0 = b = c$, then there are steady state motions which are not at relative equilibria. This is the resonance condition that we referred to in the Introduction.

We will now show that for a generic set of system parameters, all steady state motions coincide with relative equilibria. For fixed (a, b, c) we may consider (3.11) as a polynomial equation in ω_3, $\omega_{1,2}$ and $\cos\beta$. Note that repeated differentiation of (3.11) with respect to time along solutions of (3.10) yield equations which are once again polynomials in ω_3, $\omega_{1,2}$ and $\cos\beta$. ($\sin\beta$ occurs only as a factor and can be omitted for reason of continuity). Now any orbit passes through a point at which $\beta = \pi$. Let us focus on this instant in time. We only need to show that for a generic set of system parameters these equations in ω_3 and $\omega_{1,2}$ are in general position. But since being in general position is generic, we only need to show that there is *one* system for which the equations are in general position.

Now we consider the case,

$$
\begin{gathered}
|\,\overline{A_2C_3}\,| \;=\; 1, \quad |\,\overline{A_2C_2}\,| \;=\; |\,\overline{A_1C_1}\,| \;=\; \frac{1}{2}, \quad |\,\overline{A_2A_1}\,| \;=\; 1, \quad \overline{A_2C_2} \;=\; \overline{A_1C_1},\\
m_1 \;=\; m_2 \;=\; \frac{1}{2}, \quad m_3 \;=\; 1, \quad I_1 \;=\; I_2 \;=\; \frac{1}{4}, \quad I_3 \;=\; \frac{1}{2}.
\end{gathered}
$$

For this system, (3.11) is

$$
9\omega_3^2 \;+\; 6(\cos\beta)\omega_{1,2}^2 \;+\; (\cos\beta)^2\omega_3^2 \;=\; 0. \tag{3.12}
$$

Evaluating (3.12) and its derivative at $\beta = \pi$, we obtain

$$10\omega_3^2 \; - \; 6\omega_{1,2}^2 \; = \; 0, \qquad (3.13)$$

$$-18\omega_3^3 \; + \; 22\omega_3^2\omega_{1,2} \; + \; 46\omega_3\omega_{1,2}^2 \; - \; 18\omega_{1,2}^3 \; = \; 0, \qquad (3.14)$$

Clearly (3.13) and (3.14) have no complex solutions. Thus the system is in general position for this particular set of parameters, and hence for a generic set of parameters. Therefore we conclude that for a generic system all steady state motions are coincident with relative equilibria.

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