ABSTRACT

Title of Dissertation: THE DYNAMICS OF VARIABILITY IN INTRODUCTORY PHYSICS STUDENTS' THINKING: EXAMPLES FROM KINEMATICS

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Physics education research has long emphasized the need for physics instruction to address students' existing intuitions about the physical world as an integral part of learning physics. Researchers, however, have not reached a consensus-view concerning the nature of this intuitive knowledge or the specific role that it does (or might) play in physics learning. While many early characterizations of student misconceptions cast students' intuitive thinking as largely static, unitary in structure, and counter-productive for the purpose of learning correct physics, much of contemporary research supports a conceptualization of intuitive thought as dynamic, manifold in structure, and generative in the development of expertise. This dissertation contributes to ongoing inquiry into the nature of students' intuitive thought and its role in learning physics through the pursuit of dynamic systems characterizations of student reasoning, with a particular focus on how students settle into and shift among multiple patterns of reasoning about motion.

In one thread of this research, simple experimental designs are used to demonstrate how individual students can be predictably biased toward and away from different ways of thinking about the same physical situation when specific parameters of questions posed to students are varied. I qualitatively model students' thinking in terms of the activations and interactions among fine-grained intuitive knowledge and static features of the context. In a second thread of this research, case studies of more dynamic shifts in students' conceptual reasoning are developed from videos of student discussions during collaborative classroom activities. These show multiple local stabilities of students' thinking as well, with evidence of group-level dynamics shifting on the time scale of minutes.

This work contributes to existing research paradigms that aim to characterize student thinking in physics education in two important ways: (1) through the use of methods that allow for forms of empirical accountability that connect descriptive models of student thinking to experimental data, and (2) through the theoretical development of explanatory mechanisms that account for patterns in students' reasoning at multiple levels of analysis.

THE DYNAMICS OF VARIABILITY IN INTRODUCTORY PHYSICS STUDENTS' THINKING: EXAMPLES FROM KINEMATICS

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2009

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Acknowledgements

This research discussed in this document has been funded in part by the National Science Foundation under Grants No. DUE 05-24987, No. DUE 03-41289, No. DUE 03-41333, and No. REC 0440113. Any opinions, conclusions, or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

Foremost, I would like to thank my advisors, Rachel Scherr and David Hammer, for not only providing me with the tremendous opportunity to join the Physics Education Research Group at the University of Maryland, but for providing me with the *time and space* to both pursue and struggle with my own intellectual pursuits. Their patience, restraint, and compassion throughout all of this have been invaluable.

I would like to also thank the following people at Maryland and beyond who have all made substantial contributions to my intellectual development and emotional well-being: Andy Elby, Steve Kanim, Luanna Gomez, Joe Redish, Ayush Gupta, Luke Conlin, Renee Michelle Goertzen, Matty Lau, and Rosemary Russ.

Special thank goes to my entire family: Bethany, Biddy & Jan, Dave & Andrea, Goo & Kelly, Kapil, and Wizzy. Their support and encouragement is always there.

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Chapter 1: Introduction and Dissertation Overview

Historical and Current Directions In Physics Education

Over its history, the physics education community has consistently oriented its basic research and instructional reforms to be increasingly attentive to the substance of student thinking. In this chapter, I would like to (1) provide a brief overview of this broad orientation in physics education in order to situate the research presented in this document and (2) introduce and provide an overview for the structure of this dissertation as an inquiry into the dynamics of physics students' thinking.

The Substance of Student Thinking

Research in physics education has long emphasized the need for physics instruction to address students' existing intuitions about the physical world as an integral part of physics learning. It has become somewhat of a cliché to state that students are not 'blank slates' (with respect to knowledge about how the world works) ready to receive unadulterated physics knowledge from lectures and textbooks. Students, instead, arrive to the physics classroom already with particular ways of perceiving, thinking, and talking about the physical world, based partially on their own personal experiences with that world, but also through their participation in a culture that communicates these experiences. The physics education research community has consistently oriented its basic research, curricular design, and instructional reforms to be more attentive to the substance of student thinking about physical phenomena. Researchers, however, have not reached any consensus-view concerning the nature of students' intuitive knowledge or the role that it does *or should* play in students' learning of physics.

Basic Research on Student Thinking

In research, many early characterizations of student misconceptions in physics cast students' intuitive thinking as largely static, unitary in structure, and counter-productive for the purpose of learning correct physics. Students were understood to arrive to the classroom with robust and incorrect ideas about the physical world already intact (e.g., Clement, 1982). These flawed ideas were described as obstacles that not only survive well-intentioned physics instruction, but actually distort the learning of correct physics concepts (e.g., Driver, 1981; McCloskey, 1983; Carey, 1986). Progress was characterized as the process, sometimes revolutionary change, by which students abandon their naïve and incorrect ideas about the physical world and adopt more normative understandings (e.g., Posner et al., 1982). Such characterizations have been criticized along a variety of dimensions. One major criticism has been its focus on students' ideas as deficits for learning- a framework that fails to identify any generative basis for productive student learning (Smith, diSessa, Roschelle, 1993).

In contrast, some of contemporary research has argued for an alternative conceptualization of students' intuitive thought as dynamic, manifold in structure, and generative in the development of expertise (e.g., Hammer, Elby, Scherr, & Redish, 2004). Students are characterized as arriving to the classroom with vast repertoires of physical intuition and discursive tools that generate a variety of different ways of thinking and talking about the physical world (Strike and Posner, 1992). Rather than being committed to some set of ideas, students generate rather local understandings of physical phenomena in ways that may depend greatly on the immediate context (e.g., diSessa, 1993). Progress toward expertise has been described as a process of refining students' existing repertoires and gradually forging new stabilities; rather than as a process of replacing misconceptions about physics with more normative understandings. In this way, physical intuition is viewed as an essential substrate for novices' learning of physics that plays an important, although different, role even in experts' understanding of physics.

Both of these perspectives on the nature of students' intuitive knowledge and reasoning have grown alongside many instructional innovations that have aimed to establish learning environments in which students' own ideas become the starting place for physics instruction, if not also a destination to be visited again and again. Different ways of incorporating students' ideas into curriculum and instruction have emerged.

Curricular Orientations to Student Thinking

There are many different kinds of instructional reform that have taken place along side basic research on student thinking, ranging from modifications of physics laboratories (e.g., Thornton, 1987), to the use of structured curriculum worksheets in small group settings (e.g, McDermott, Shaffer, and the Physics Education Group and the University of Washington, 1998), to the introduction of technologies that increase student engagement in lectures (Mazur & Summer, 1999; Crouch, Watkins, Fagen, Mazur, 2007). Many of these reforms incorporate students' ideas into instruction by having students interact with each other or with various educational technologies. Here I describe two different orientations toward incorporating student thinking into curriculum.

Some curricula have been described as utilizing an approach called *elicit-confront-resolve* (e.g., McDermott, 1991). Students are first asked to use their intuitive ideas to reason about a physical situation. These physical situations are often ones known ahead of time that students will likely think about incorrectly and arrive at a wrong answer. After students spend time applying their own intuitive thinking, students are usually confronted with another situation or argument that illustrates an inconsistency with their reasoning. Students are then helped to resolve the inconsistency through application of correct physics concepts. Part of the intention of this kind of curriculum is for students to spend time articulating their tacit misconceptions about the world so that they can be confronted and resolved.

Another kind of curriculum has been described as utilizing an approach known as *refining intuitions* (e.g., Elby, 2001). Students are often asked to apply their ideas to reason about several situations. For some of these situations, it is expected that students will likely answer correctly based on their intuitive thinking. For the other, it is expected that students' intuitive thinking will lead to incorrect answers. Students are helped to see connections (and to seek connections) among these different situations and to figure out when and why their intuitions do and don't apply. The intention is for students to refine their intuitive ideas about the world and to begin looking for ways to make connections with their own ideas and correct physics concepts.

While there are some philosophical and pragmatic differences between these two approaches, there are perhaps more similarities than differences. Both involve getting students to use their own ideas in making sense of physical situations, to think about their own ideas critically, and to make progress in learning correct physics concepts through that process of reflection of one's thinking. In this sense, these curricula view learning physics not only as a matter of learning about physics concepts that apply to the world, but also learning about one's own mind and the ways that one's mind can and does think about that world.

In this historical context of our community's basic research and instructional reform becoming increasingly attentive to the substance of students' thinking, it is prudent to ask by what methods physics education researchers have gathered data on students' thinking in physics and the impact of reformed physics instruction. To a large extent, such data has been collected using written assessments that are administered to students in pencil-and-paper forms before, during, and after instruction.

The Paradigm of Pre-Post Testing

Some branches of physics education research began as pragmatic endeavors to develop instructional innovations (such as those described above) that help students to develop a deeper understanding of physics concepts beyond the levels typically achieved through traditional instruction. Many of these instructional innovations were motivated by ongoing research documenting surprising gaps in students' understanding of even the most basic physics concepts (e.g., see review by Van

Heuvelen, 1991). While students were leaving physics classroom quite able to solve traditional "end-of-the-chapter" physics problems, these same students were demonstrating significant difficulties applying even basic physics concepts to reason qualitatively about simple physical situations (Mazur, 1997). Physics education researchers have often found and still find themselves in the position of having to motivate and advocate for reform through the careful documentation of such deficits, often using methods involving written assessments to measure what students know or have learned.

Written Assessments in Physics Education

As a means for identifying and documenting such difficulties, physics education researchers have primarily used written assessments that ask students to provide answers to physics questions. Large numbers of students may be given paper-and-pencil assessments before, during, or after instruction. These assessments range from brief free-response questions (e.g, Shaffer & McDermott, 2005) to rather lengthy diagnostic multiple-choice tests (e.g., Thornton & Sokoloff, 1998; Hestenes, Wells, Swackhamer, 1992; Beichner, 1994) Researchers typically analyze large numbers of student responses and begin a process of characterizing the patterns of responses they observe. Hypotheses about the meaning of student responses are offered. These hypotheses are often explored further in one-on-one interviews with students. This process is repeated with refinements made both to the assessments administered and the hypothesis developed.

Based on this iterative research to identify student difficulties, an informed-design process may begin in order to develop some type of instructional intervention (e.g., McDermott & Shaffer, 1992). Similar pre-post testing is often used to measure the efficacy of various interventions or instructional innovation (e.g., Hake, 1998; Crouch & Mazur, 2001). These results may even be compared to some non-intervention or control group. This process ultimately leads to an even broader cycle of revising assessments, hypotheses, and instructional interventions, until some desired level of stability in research interpretations and efficacy of instruction is achieved.

Throughout this entire research and design process, pre-posting testing to measure what students know plays a crucial role. It is used to initially identify prevalent student difficulties, to inform and refine researcher's interpretations of those difficulties, and to measure the efficacy of instruction to help students overcome those difficulties. Given the widespread and diverse use of written assessments and pre-post testing, it is increasingly important to understand how it is that researchers conceptualize the process and products of administering and analyzing these assessments.

To a large extent, researchers use written assessments as tools for measuring—measuring what it is students know or don't know, measuring how much students have learned over some period of time, measuring the effectiveness of one curriculum over another. The word *measurement*, however, implies some assumptions regarding the nature of the interactions between the questions we as researchers have developed and the subjects that we as researchers wish to learn something about with those instruments.

Reflections on Measurement

It's an important question to ask, "What does it mean to for a community to conceptualize a complex interaction as a measurement?" While there is a wide and varied literature in the field of psychometrics concerning the use of instruments and analytical methods for measuring and quantifying attributes of persons, I want to briefly reflect upon a simple case of measurement in physical science.

Consider a thermometer as measuring instrument. A thermometer is a specialized physical system that, under certain conditions, can be understood to measure something about another physical system. Such measurement is possible because these two physical systems interact with each other in particular ways. Stepping back, it is clear that a thermometer can't directly "read" the temperature of another system. It might better be said that the thermometer undergoes changes that indicate the temperature of itself as it *changes in interaction* with that system. The changes that the thermometer undergoes itself are indicated through dynamic but reliable changes (e.g., expansion of a liquid). Under certain conditions, scientists conceptualize the changes that take place in the thermometer as reflecting small changes around a relatively stable state of the other system that they are interested in knowing more about. In a historical context, the thermometer only gradually came into existence as an *instrument for measuring* as various empirical and theoretical techniques in thermodynamics developed that brought insight into the nature of these interactions.

We can imagine stepping back from thinking about the thermometer as simply measuring something and begin asking questions such as, " How does this work?" or, “What’s going on here?” Even knowing nothing about thermodynamics, a person, so-inclined, could find a rather rich phenomenology of thermometer-based interactions in the world to explore and try to explain. For example, you could stick a thermometer in a cup of hot water, and you’d find that the thermometer reading changes quickly at first and then changes slower and slower, until seemingly stabilizing at some temperature. You could then begin to think about what might be happening with the water and the thermometer that would explain this. You could also try putting the same thermometer in a warm cup of water and then a hot cup of water, and you’d find that the thermometer takes more time to stabilize in its changes when in the hotter water than when in the warm water. You could continue in this way, exploring the rich phenomenology of thermometers by *mucking about* and speculating on plausible explanations.

Along the way, you might begin to hypothesize about the nature and kinds of properties of the systems you are exploring, or about the existence of structures and processes that are hidden from direct observation. These structures, properties, and processes become the ontological basis for constructing coherent narratives to explain the rich phenomenology you’ve explored. Over a time, a thermometer becomes conceptualized as a complex system itself—one that you can describe in terms of its own properties and structures. It is through a community’s understanding of these real-time interactions occurring across dynamic systems that the thermometer becomes a tool for possibly measuring something in the world.

Turning back to the use of written assessments as tools for measuring what our students are thinking or what they are learning, the physics education research community remains only in its infancy in the pursuit of theoretical and empirical frameworks that are likely to bring insight into the nature of these complex dynamics. We still struggle with the question of what it even means when student respond to physics questions or what dynamics take place between students and the settings in which we pose questions?

In many other fields related to education and psychology, researchers also use questions to measure something about the knowledge or abilities of the persons or populations they study. Historically, arguments that collections of questions (e.g., standardized assessments) can be used to measure something about individuals or populations have concerned the high-degree of correlation that often exists among scores determined from these instruments, and from other measures of reliability and validity that can be ascertained from such data. Doing well on one kind of test strongly predicts doing well on another kind of test. Evidence for stability and reliability in interaction is certainly one important aspect of measurement. Looking *only* to stability, however, ignores the importance of understanding the *mechanisms* by which stability either does or doesn't arise. We depend on reliable instruments like thermometers for measurements, but reliability alone doesn't make an instrument a tool for measurement.

Physics education researchers have certainly strived to construct questions for their research that produce reliable and stable results when posed to students (at least under certain conditions). It is considered part of the craft of physics education to be able to design just the right questions to do the job. The pursuit of such stability may have been motivated by the need of the physics education to appear "scientific" to the broader physics community they are embedded. The tacit argument being that physics education researchers must be able to demonstrate repeatable and reliable empirical results in the same way the natural sciences have often been able to in their investigations of the physical phenomena.

This tacit framing of science as the pursuit of empirical regularity, however, ignores the aspect of science that is the pursuit of understanding mechanism. In other words, we don't just care that a thermometer can produce reliable results, which can then be used show correlations in the world. We care about the details in *how* that thermometer works and what that tells us about the *dynamics* of that world. We shouldn't just care that the questions we pose to students can be used to demonstrate reliable patterns of responses (often wrong ones, since after all we are advocating for reform), we should care about the details of *how* students assemble responses to questions as real-time activity and what that tells us about the *dynamics* of their thinking. It is these very questions of how and by what dynamics that this dissertation aims to explore.

Concerning phenomena of student thinking in response to physics questions, I want to suggest that there is still much productive exploration to be done—that progress can be made through thoughtful exploration of the phenomena of student thinking itself and through constrained speculation concerning mechanisms by which patterns arise. Doing so will involve stepping back from frameworks that conceptualize questions as measurement of what students know or are able to do.

Instead, we'll need to build from frameworks for thinking about interacting systems—how students interact with the contexts in which we ask questions.

Dissertation Overview

This dissertation as a whole represents an inquiry into the dynamic of students' thinking, attempting to describe interactions that take place among students' intuitive knowledge about motion and the local circumstances of their thinking. In some cases those local circumstances will merely be questions on a page, and in other cases those circumstances will be rich discussions among peers. In order to pursue this agenda of characterizing how students settle into patterns of thinking in these contexts, the research in this dissertation relies upon both emerging theoretical frameworks that allow researchers to describe the dynamics of student thinking as real-time activity and emerging methodological tools that provide researchers with access to richer data concerning student thinking.

In Chapter 2, I begin with a review of historical and contemporary research in cognitive development and science education. I describe how different frameworks locate and granulate cognition throughout space and time in different ways. This discussions serves to motivate the general cognitive framework that forms this basis for describing student thinking through out this document.

In Chapter 3, I review research from psychology and science education concerning students' thinking about motion along with the models researchers use to describe this thinking. In this chapter, I also describe a toy cognitive model of students' intuitive thinking about kinematical relation. I describe properties of this model and use it to describe students' thinking about motion in several examples.

This simple model forms the basis for describing much of student thinking in this document and for motivating much of empirical work in Chapters 4 and 5.

In Chapter 4, I describe an experimental design motivated from the model described in the previous chapter. The purpose of this experiment is to detect variability in students' thinking by altering the presentation of physics questions about identical situations. Two different questions are employed. One question is about a ball rolling off the edge of a table. The second question is about a ball being tossed vertically. The experiment involves of the administration of two different versions of written surveys (that include both of these questions) to large numbers of students. Variation in the distributions of student answers and explanations are analyzed and compared with predictions of our toy cognitive models and other models as well.

In Chapter 5, I describe several case studies of students' thinking as an inquiry into the dynamics by which groups of students settle into and out of different patterns of thinking about tickertape representations of motion. These case studies involve the analysis of video taken from actual classrooms, where student are discussing motion in small groups. These case studies illustrate multiple patterns of thinking that individual students and groups shift among. From this analysis, arguments are developed concerning role of plausible mechanisms that contribute to these patterns of thinking exhibiting local stability.

In Chapter 6, I take a broader perspective on student thinking that was described in Chapter 5. I describe a particular case of one group of students, focusing on how these students change the context around themselves in ways that are consequential to their own thinking. By taking a more distributed view of the unit of cognition, some insight is gained into the role of context in stabilizing patterns of thinking.

In Chapter 7, I reflect upon the contributions of this document to research in physics education, particularly to the nature and role of intuitive thinking, and suggest future directions for research.

Chapter 2: Review of Literature

Science has been described as the study of systems and their underlying mechanisms (Machamer, Darden, and Carver, 2000). Whether it concerns the study of physical, biological, political, or *cognitive systems*, researchers face the issue of defining both the boundaries and constituents of those systems. Necessarily, choices concerning the location of spatial and temporal boundaries influence the kinds of structures and processes that are chosen to describe the system. In turn, choices about what structures and processes to use in describing the system help to define spatial and temporal boundaries.

A major aim of this dissertation is to pursue the development of cognitive accounts of student thinking from a systems perspective—focusing on how constituent elements of cognition assemble in space and time to generate patterns of behavior and thinking. It is the goal of this chapter to (1) layout the relevant spatio-temporal boundaries and structures and processes that will be used to account for phenomena of student thinking and behavior and (2) draw connections between the specific pursuit of systems account of student thinking in this document and a broad literature from science education and cognitive development. In doing so, I defend the choice in this dissertation to borrow from those frameworks with a focus on students' thinking and behavior as the result of cognitive assemblies occurring in *real time* – borrowing both from frameworks that conceptualize assemblies taking place among elements of individuals' knowledge and structures that are social and material in nature.

Issues of Boundary and Constituency

Choosing boundaries in space means drawing lines between the structures and processes that are internal to systems and the structures and processes that act as external influences upon them. Based on the particular system and the lines one draws, it may be that some of the internal and external dynamics decouple from each other. We can think such a system as having an integrity that is rather independent of its environment. For example, a computer may be thought of as the same basic computational device independent of who is using it or what external devices it is connected to. We may even speak of the "intrinsic" dynamics of such a system – how it behaves when not under any particular external influence. Alternatively, it remains possible that for some systems the internal dynamics depends so strongly on the particulars of the external environment, that the issue of how the system is embedded takes center stage. In such case we may even think of the boundaries of our system as being "fuzzy" or extended. In contrast to a computer, we may think of the Internet as a system that has rather fluid boundaries and a non-stationary internal dynamics.

In a similar way, the boundaries of systems *in time* define when to begin and cut off any account of the system dynamics. In some cases, the dynamics of the distant past have little impact on the current state of affairs. It is sufficient to know the current state of the system. Other systems, however exhibit strong hysteresis that couple the details of the past to the present in surprising ways. It may not only matter what state the system is in, but how the system got there. In physics, the force exerted by an ideal spring depends only upon its current displacement from equilibrium. However, unlike the ideal spring, the force exerted by real rubber bands does not only depend upon its current position. It depends upon how the rubber band was stretched in the recent past and strongly upon which direction it came.

Spatial and temporal boundaries not only defines edges, they also constrain the scale-size for structures and the processes that describe system dynamics. The study of systems requires the ability of researchers to move across many levels. For example, the study of evolution entails the study of systems from the level of molecules to the level ecosystems. These different levels involve processes reaching far into the past (occurring slowly over time) and processes occurring over nanosecond intervals. Similarly, the study of human behavior also involves issues of scale, ranging from the very small and fast (e.g., neurological processes) to the very large and slow (e.g, sociological processes).

In studying *cognition*, researchers have often disagreed about the appropriate system(s) to be studied. Some researchers have focused on how learning and development takes place over fairly broad time scales – hours, days, weeks, months, and years. Other researchers have focused their attention on how behavior and thinking take place in relatively short periods of time - milliseconds, seconds, and minutes. Across the range of these time scales, some researchers have described cognition in terms of structures residing in individuals' minds – knowledge, attitudes, and beliefs – and the processes that create, modify and coordinate them. Others have focused on structures residing in the world – social institutions, patterns of discourse, cultural tools, and communities of practice – and the processes that mediate human participation in them. These different perspectives in cognitive and educational research can be characterized in a variety of ways. I focus here on describing how these various accounts of *cognition* locate and granulate cognition as assemblies taking place throughout space and time in qualitatively different ways.

To begin, I describe an example of research concerning how baseball players catch fly balls.

Catching Fly Balls: Cognition in Time and Space

The question of where to focus cognitive accounts in space and time reflects very basic questions about cognitive performance: Who (or what) is doing the work of accomplishing the cognitive task? When and where is the work of accomplishing the cognitive task taking place? In this section, I explore a range of possible focal points in the context of the cognitive task of catching fly balls.

Researchers investigating human perception and action have directed significant efforts toward trying to answer the question, "How do baseball players know where to go in order to catch a fly ball?" (Chapman, 1968; McBeath, Shaffer, and Kaiser, 1995; Oudejans, Michaels, Bakker, and Dolne, 1996). It is tempting to think that baseball players accomplish this task because their mind predicts the ball's future location by means of performing some approximate, intuitive calculation equivalent to solving some set of differential equations that describe the ball's motion. After all, evolution seems to have equipped our brains with the machinery to track the motion of objects quite effectively. These tools could be harnessed for the task of catching fly balls. After performing this 'calculation', one could head out as quickly as possible toward the predicted location, along the way refining and updating the prediction based on continued information.

As it turns out, the task of getting one's body to arrive at the right place at the right time is not necessarily the same as predicting ahead of time where the ball will be. The problem of getting one's body to the right place at the right time may be solved by following a simple rule: move so as to maintain a constant viewing angle with the ball. This simple rule specifies a local move that, over the course of many moves (done correctly), results in the baseball player ending up at the right place at the right time. Oddly enough, people aren't aware that this is the *kind* of solution they use to accomplish the task.[1] People just have an intuitive sense that they know where the ball is headed and follow their instincts. The solution doesn't require an all-purpose cognitive machinery capable of calculating trajectories. In fact, the solution is much more unique to the problem at hand – that of arriving at the right place at the right time. The uniqueness of the solution means that it doesn't easily generalize to other situations involving 'predicting' where balls will land. The rule is of little use, for example, in trying to predict from the stands whether, at the crack of a bat, a baseball is going to be a homerun or a pop fly. That problem requires different strategies and rules.

Knowing which rule gets the job done doesn't entirely answer the question, "How do baseball players catch fly balls?" either. What has been learned, however, is that the question, "How do baseball players *predict* where fly ball's will land?" is potentially misleading. It turns out that, if we are concerned with how individuals go about catching fly balls, a better question might be, "How do baseball players *locally decide* what's the best way to move?" Transforming the task of early prediction to

[1] Baseball players actually move as to keep the tangent of viewing angle constant (Chapman, 1968).

one of local decision-making doesn't necessarily reduce its complexity. How a baseball player decides the *best* next move likely involves a complex task of processing ongoing visual information and coordinating that information with ongoing plans for motor movements. Even if researchers come to understand what goes into the making of each of those local decisions, they still don't necessarily understand how the baseball player *learned over time* to make those best moves. Nor is it always obvious why a simple rule should work at all. Someone who only knew and followed the rule would be surprised each time the baseball landed at his feet or knocked him over the head. Baseball players, on the other hand, don't seem to be aware the rule they are following, but are not surprised to find themselves in a position to catch the ball. Despite the simplicity of the rule, there are complex dynamics occurring at many different levels of organization and across many different time scales. It is these structures and processes interacting at multiple levels of analysis that is the essence of what is meant system dynamics.

When people are asked the question, "How do baseball players catch fly balls?" most assume (often without being aware of it) that the task must be solved by some means of calculating and predicting trajectories. Framing the task as *calculation* locates the complexity of the task (in its entirety) within capacities of the individual mind. The cognitive system in this case is confined to the abilities of the individual. Alternatively, framing the task as *deciding best moves* distributes the complexity over a set of local actions that work partially by exploiting key features of the situation at hand (like the fact that the ground is flat and that the ball must eventually come down). The cognitive system in this case is extended across actions that occur throughout time and space. From this perspective, the individual person acting in the context of a flat earth and gravity pulling down together can be analyzed as a single cognitive system. It is these situated actions in concert that do the work of "calculating" the final position of the ball. The baseball player functions as just one piece of that system (albeit a very central piece) acting in concert with the rest of the system.

As this baseball example illustrates, one issue in interpreting human behavior is the location of complex cognitive performances throughout space and time. When observing complex behavior, it is certainly tempting to locate the entirety of the capacity for performance within the individual's mind. A person's physical body, the tools they use, and their immediate settings (including the people around them) are not conceived as playing an active role in the dynamics of an individual's complex behavior. Rather they are conceived as the stage on which complex behavior and cognition take place. In cognitive and education research, this difference is one of the central arguments between researchers espousing knowledge-based accounts of cognition (with a focus on how individuals knowledge comes to play in those situations) and situated or distributed accounts of cognition (who tend to focus on inextricable relationships between human activity and setting). In a similar way, there is a tendency to think of complex behavior as resulting from mental activity that has already taken place— *un fait accompli.* A person's ongoing actions don't contribute to the complexity of behavior. They simply result from complex mental activity that have taken place prior or arise from structures that already existed in completed forms.

There are many different spatiotemporal focal points to choose among in developing accounts of cognitive performance. For catching fly balls, there is existence of structures in brain whose purpose is to the track the movement of objects in space. There is the learning that takes place in the individual's lifetime. There are the real-time macroscopic actions that unfold as baseball players catch fly balls and the real-time sensory-motor processing within the mind. Such accounts focus on cognitive assemblies as taking place across different temporal and spatial scales.

Cognitive Assemblies in Time

In this section, I compare and contrast various perspectives from cognitive development and science education. These perspectives differ in how they focus their explanations of cognition in time. On one end of the spectrum, there are researchers who focus on explaining cognitive performances in terms of cognitive dynamics taking place in the past and occurring across long periods of time. On the other end, there are researchers who account for cognitive performances in terms of processes and actions occurring in real time. I discuss these contrasting perspective in the context of research in two fields: child development and science education. In child development, the division between research focusing on assemblies of the past and research focusing on real-time assemblies is evident in the different descriptions that are typical of stage-based constructivist approaches (following the work of Piaget) and those typical of dynamic systems approaches (Thelen and Smith, 1993; van Geert, 1994). In science education, a similar division is apparent in the different accounts of student thinking put forth by 'knowledge-frameworks' perspectives

(Driver, 1981; Carey, 1986; Posner et al, 1982) and the accounts put forth by 'knowledge-in-pieces' perspectives (Smith, diSessa, and Roschelle, 1992; diSessa, 1993; Hammer, 1996).

Cognitive Assemblies in the Past

On the far end of locating accounts of cognition in the achievements and assemblies of the past, there is the nativist focus on describing how particular cognitive performances are possible because evolutionary mechanisms have equipped the mind with particular modules or tools for getting the job done. Humans learn languages effectively because of the existence of a universal grammar (Chomsky, 1965). Children represent and keep track of objects because of the existence of core knowledge components (Spelke, 2004; Spelke and Kinzler, 2007). These accounts locate the capacity for certain cognitive performance largely in the dynamics of the far evolutionary past, which have assembled fixed structures in the mind designed for performing specific tasks. The thrust of much of this work is to characterize the nature of these fixed systems in order to better understand how they constrain thinking and learning in the domains they influence. Carey and Spelke (1996) argue that, while such core knowledge components may be used to build new knowledge structures (via mappings across various components), the core knowledge itself does not change over a person's lifetime.

Structural Achievements and Acquisitions in Cognitive Development

Moving closer to a focus on cognitive dynamics in the present, there are researchers who focus on accomplishments and assemblies taking place during an individual's life. These researchers are also largely concerned with characterizing the nature of the existing structures (although not ones that are fixed over a lifetime).

For example, Piaget (1954) accounted for the fact that most children under the age of seven months do not reach for objects that go out of view (and older children do) in terms of the development of the concept of object permanence. This error has been studied, quite famously, in an experiment known as the A-not-B task. In the experiment, infants are habituated to reach toward one location (the A location), where a toy is hidden over several trials. The error is made when the toy is now hidden at another location (the B location), and the child reaches to the original location (to A) and not to the location where the toy was actually hidden (to B). According to staged-based developmental account, older children reach for objects in locations where they were last seen because they have formed the *concept* that objects exist independently of whether they are seen. Younger children, not having developed this concept, are observed to go looking for an object in the last place they were able to find it (rather than where they saw it disappear from view). These children don't have robust expectations that objects persist when out of view. From the research perspective, the capacity for current performance is accounted for in terms whether or not the individual has, in the past, assembled a particular cognitive structure.

Similarly, Smith, Carey, and Wiser (1985) describe the capacity for children to differentiate the concepts of weight (heavy) and density (heavy for size) in terms of the achievement of sufficiently differentiated conceptual structures in the mind. Children who have not differentiated the concepts sufficiently tend to confuse the two in comparison and seriation tasks. For example, a child might say that small object seems heavier than a bigger object that actually weighs more. Children who have differentiated the two concepts don't make these mistakes, or at least make them much less often. While their experimental work led to results that disagreed with Piaget's contention that children of this age have not differentiated concepts of volume (size) and weight (heft), the fundamental characterization of the child's development is identical. Current patterns in performance are accounted for in terms of the past achievement of differentiation in the mind. This differentiation is a fixed attribute of the developing mind.

While nativist perspectives have focused on understanding how primitive structures from the evolutionary past constrain thinking and learning, the focus on structural achievements in cognitive development focuses on how existing structures of the mind (built in the individuals' past) determine or influence current thinking and behavior. Both of the accounts above focus on how past assemblies, resulting in new structures, change and influence cognitive performance in the present, without careful attention to the dynamics by which these behaviors take place in the present. The acquisition of object permanence and the achievement of differentiated structures are explanations that signal landmarks in the developing child's behavior in the world, but do not explain how such behaviors unfold in the here and now.

Knowledge Frameworks and Misconceptions in Conceptual Change

In science education, it was common in the late 1970s and early 1980s to describe students' conceptual difficulties in terms of students' large-scale and robust alternative knowledge frameworks or misconceptions (Driver; 1981; Carey, 1985;

Clement, 1983, Halloun and Hestenes, 1985). These kinds of accounts also located much of cognition in the individuals' past assembly of structures. Researchers directed efforts toward characterizing the nature of students' alternative conceptual structures surrounding science content in order to better understand their naïve thinking. In physics alone, researchers documented faulty conceptions such as the belief that 'motion implies force' (Clement, 1982; Champagne et al, 1981), that 'unsupported objects fall straight down' (Whitaker, 1983), and that 'objects move via an internal impetus' (McCloskey, 1983). From the knowledge framework perspective, students are believed to have assembled beliefs, theories, or conceptions based on their experiences in world. These knowledge frameworks are understood to characterize the students' knowledge during some period of learning and development, which are the cause of resilient misconceptions about the way the world works.

The knowledge-framework movement in conceptual change was arguably motivated by both Kuhn's (1962) philosophical work concerning the nature of scientific change and trends in cognitive science toward domain-specificity (Fodor, 1983; Carey and Spelke, 1994). Kuhn described scientific change in the history of science as a revolutionary process by which new paradigms for doing science overthrow prior ones. Some researchers in science education adopted a similar view of conceptual change in *individuals*. Students undergo conceptual change as the result of a 'change of state' from holding one conception to another (Posner, Strike, Hewson, and Gertzog, 1982). Students' new understandings may be wholly incompatible with their olds ways of understanding. Like Piaget's account of cognitive performances, this revolutionary model of conceptual change locates the cause of students' cognitive performances in the products of past accomplishments. Students arrive to the science classroom with previously-built knowledge frameworks. Students' behavior changes with the acquisition of new knowledge frameworks that succeed the previous ones. Domain-specificity, on the other hand, gave theoretical support to viewing students' ideas about relatively independent science domains as being relatively independent knowledge structures. Researchers could study students' ideas about force and motion independently of their ideas related to heat and temperature.

An example of a conceptual knowledge framework that students have been characterized as having prior to instruction in physics is that "motion implies force". From the correct Newtonian perspective, a force causes a change in motion. However, many students, both before and after instruction reason as if a force is needed to maintain an objects motion. If asked to identify forces for moving objects, they will often indicate a force in the direction of the objects' motion. Clement stated about this specific difficulty that, "When students with these alternative knowledge structures produce incorrect answers…the cause is the stability of the student's alternative knowledge structures," (Clement, 1983, p. 338). This kind of characterization directly maps the property of student answers to properties of the mind. Students continue to provide incorrect answers because they have stably wrong ways of thinking.

A more thorough discussion of research on student thinking about motion is discussed in Chapter 3.

Characteristics of Accounts Focusing on Past Assemblies

I describe here a family of attributes that loosely characterizes many of the perspectives in cognitive development and science education that have focused on development over broad time scales and on explaining current behavior in terms of the dynamics from the past. No single account encompasses all of these characteristics. However, these attributes are common to many accounts.

One characteristic is a methodological metaphor of probing. A focus on explaining present cognition as the result of past accomplishments and acquisitions has led many science education researchers to understand their methods of investigation via the metaphor of "probing" (McCloskey, 1983; Halloun and Hestenes, 1985; White and Gunstone, 1992). Students already have ideas and ways of thinking that exist. With the right questions and techniques, these structures can be exposed. The metaphor of 'probing' fits with the idea that the cognitive assemblies exist in the mind, but that they may be hidden. Therefore, there is a need for tools to "probe beneath the skin." These tools range from carefully crafted questions and surveys or well-designed clinical interviews.

A second characteristic is that the knowledge "state" or developmental stage of an individual refers to a state that characterize the individual over an extended period of time or development. Vosniandou and Ioannides have expressed that cognitive science informs science education by "provid[ing] rich descriptions of the knowledge states of students at different ages and phases in the acquisition of expertise," (Vosniadou & Ioannides, p. 1214, 1998). The acquisition of object permanence characterizes changes to children's patterns in reaching performances broadly over a period of time. Having differentiated knowledge about weight and density (Smith, Carey, Wiser, 1985) is a description that characterizes the child broadly over some time period. Similarly, novice students are understood to reason broadly via “motion implies force” (Clement, 1982), until they replace that knowledge with an appropriate understanding of Newton’s Laws.

A third characteristic, closely related to this second, is the use of singular attributions to characterize this knowledge state. Children either possess the concept of object permanence or not. Children have either differentiated weight from density or they have not. Similarly, Posner et al described students’ conceptual change as a series of changes from holding *one* conception to another to yet another. There is a single description that captures their behavior and thinking during that period. Children and students are typically not characterized by having a multiplicity of possible conceptual understandings within a single domain. They are simply in a stage or have a misconception.

A fourth characteristic is a tight phenomenological correspondence. In other words, characteristic patterns in the phenomenology of behavior and thinking are directly mapped to properties of the knowledge that result in that behavior. For instance, children who act as if objects persist have the concept of object permanence. Children who fail to differentiate weight from density have knowledge that is undifferentiated with respect to weight and density. Students who give answers that there is a force in the direction of motion do so because they have a knowledge framework that “motion implies force”. There is a close correspondence principle applied both to the domain attributed to the phenomenology (e.g., students’ answers to question about force arise from their knowledge about force) and the patterns found in the phenomenology (e.g., not differentiating in tasks corresponds to undifferentiated knowledge). This tight correspondence principle is quite analogous to pre-mechanical-revolution science, in which essences were attributed to objects to explain behavior. For instance, attributing levity to air to explain why it rises. Students' behavior is being similarly described in terms of closely corresponding properties.

A focus on explaining cognition in terms of past dynamics is often associated with a focus on the development of new cognitive abilities or acquisition of new patterns of behaviors that did not seem to exist during an earlier period of time but appear broadly so at a later time. Understanding the conditions, stages, or time frames in which new patterns of behavior emerge is useful for documenting change across broad time scales. However, it often establishes little about how those behaviors happen or fail to happen in real time. For instance, Piaget was not concerned with the question of why, when adults lose their keys, they often go looking for them in places where they habitually find them (and not necessarily where they lost them), despite having the concept of object permanence. Smith, Carey and Wiser were not concerned with why college students often confuse the concepts of weight and density when reasoning about buoyancy (Loverude, Kautz, and Heron, 2003) despite having developed the cognitive distinction at an earlier age. Similarly, science education researchers, working within the paradigm of the expert-notice shift, were not so concerned with when and why experts occasionally make similar mistakes as novices. The failure of people to perform in a given moment (like losing your keys), despite the existence of the proper structures, has often been ignored by making a distinction between a person's competence (the ability to perform a task because the structure exists) and performance (the ability to do so in a specific situation, where other demands may limit or mask the perceived competence). For these researchers, the concern isn't the dynamics of how these behaviors happen in the moments they occur. Rather the concern is whether these behaviors happen at all and what properties of individuals can account for that behavior.

These characteristics- a methodological metaphor of probing, the state of the individual characterizing behavior over broad times scale, the use of singular attributions, tight-phenomenological correspondence, and competence-performance distinction- each arise from thinking about cognition as being constrained or determined by what the individual has achieved in the past. Often, much less attention is paid to the details of what is happening in the moment of behavior to explain the phenomena.

Recall that the end goal of this chapter is to be able to articulate and defend the choice to pursue accounts of student thinking in terms of real-time cognitive activity. In this section, I have so far only reviewed research and research perspectives that I have characterized as accounting for cognition in terms of previous unitary constructions. In the following section, I move on to discuss perspectives in cognitive development and conceptual change that locate cognition much more in the present. To a large extent, this research will concern much of the same phenomenology in human development and science education that has already been described. Differences will largely concern how these researchers conceptualize this phenomenology (as real-time activity) and how such reconceptualizations motivate researchers to pursue novel investigations of these phenomena.

Cognitive Assemblies in Real Time

On the other end of locating cognitive explanations in time are accounts that focus on how cognition takes place in real-time. In this section I describe how dynamic systems approaches in cognitive development have emphasized cognition as taking place in the real-time actions and, separately, how knowledge-in-pieces have emphasized cognition as taking place in real-time activations of elemental structures.

Dynamic Systems Approaches to Cognitive Development

Dynamics systems approaches to cognitive development have focused more on understanding how cognitive tasks are accomplished in real-time rather than on just characterizing cognitive structures built in the past that reflect cognitive performance (Thelen and Smith, 1993; Van Geert, 1994; Lewis, 2000). While the traditional view from Piaget is that children's developing performances reflect the formation of structures (i.e., reaching performances reflect the formation of object permanence), researchers coming from a dynamics systems perspective have taken the stance that 'knowing' things like object permanence may actually be emergent from the cognitive task itself and how that task is coupled to past stabilities of behavior. (Smith, Thelen, Titzer, and Mclin, 1999). This is similar to the shift in perspective described with catching baseballs. The baseball player doesn't necessarily know ahead of time where the ball is going to land, but knowing, in a sense, emerges out of the ongoing activity. Such ongoing activity is understood to arise from how stabilities of behavior (forged in the past) come together in the moment, not just from the existence of such stabilities. In this way, issues of development and change *over time* are understood to be strongly coupled the *here-and-now*.

Researchers espousing dynamic systems approaches have exposed a rich phenomenology of both child and adult reaching performances to support their view. For example, children's strong tendencies to reach for the A location in the A-not-B task are diminished when infants are trained (habituated) and then tested in slightly different contexts. For example, by training infants with weights on their arms and testing them without the weights, researcher's lessened children's tendency to reach for the habitual location lessens (Thelen, E., Schoner, G., Scheier, C. & Smith, L. B, 2001). Similar effects are observed when infants are trained and tested in different postures, so that the error disappears when training them while sitting and then testing while being held more upright (Smith, L. B., Thelen, E., Titzer, R., & McLin, D, 1999). On the opposite end of development, older children who have to remember where an object is buried in a sand box show tendencies similar to infants. Their errors in remembering where objects are buried during the testing tends to move in the direction where the objects were hidden during the training, and this tendency increases the longer they have to wait before being allowed to find it.

Such results emphasize that acting in ways consistent with the knowledge of object permanence is closely tied to the particulars of the immediate environment. Changing the context changes the pattern of behavior. Infants can be made to act in ways that suggest they know object permanence when they shouldn't based on developmental stages. Older children can be made to make reaching errors as well. Such evidence would suggest that acting in ways consistent with knowledge of object permanence is more fluid than would captured by an "all-or-none" account in which object permanence is a concept that is acquired as a single unit.

Such results and perspectives gradually lead to the development of a model of children's preservative reaching called the Dynamic Field Model (Thelen, Schoner, Scheier, and Smith, 2001). In this model, features of the immediate visual field combine with both memories of recent visual field and memories of recent reaching trials to create time-evolving attractor states for motor planning to particular locations. This account models the cognition of reaching performances as softly-assembling motor plans – real time planning that is affected by recent histories and current perceptual attention in particular ways. Note, too, that such an account required a shift in the ontology researchers used to describe the phenomena. While Piaget worked within a framework of thinking about the structures children had or didn't have in the mind, the frameworks described here involved thinking about the formation of behavioral stabilities among sensory-motor systems and memory.

Complex Knowledge Systems Approaches in Science Education

In science education, there are also a variety of approaches that have shifted the focus of research on students' conceptual thinking away from static characterizations and towards descriptions of cognitive dynamics taking place in real time. In this section I focus on a family of complex knowledge systems (CKS) approaches which share a focus on multiplicity and contextuality in knowledge systems (Strike and

Posner, 1992; Smith, diSessa, and Roschelle, 1993; diSessa, 1993; Hammer, Elby, Redish, and Scherr, 2004).

Complex knowledge systems approaches in science education focus on modeling student thinking as emergent from the activation and coordination of fine-grained cognitive elements. Early conceptual change approaches focused on describing student thinking in terms of students having singular cognitive frameworks or conceptions. CKS approaches, in contrast, are committed to a cognitive ontology in which numerous and diverse knowledge elements make up what has been called a *conceptual ecology* (Strike & Posner, 1992). Those elements within the conceptual ecology both constitute and generate student thinking. Researchers working in this framework have reinterpreted many traditional student misconceptions as misapplication of otherwise useful knowledge, rather than as counter-productive conceptions (Smith, diSessa, Roschelle, 1993; Wittmann, 2002). They have also emphasized how both novice and expert knowledge are comprised of many, many knowledge elements of different kinds, which provide for multiplicity of thought (Smith, diSessa, Roschelle, 1993; diSessa and Sherin, 1998; Hammer 1998).

Examples of diverse cognitive elements in the conceptual ecology include intuitive knowledge pieces for thinking about physical mechanism (diSessa, 1993), epistemological resources for thinking about knowledge and knowing (Hammer and Elby, 2002), and knowledge related to sense-making with mathematical symbols (Sherin, 2001). Others still have emphasized the need for accounts which include motivation, goals, and attitudes as part the conceptual ecology (Strike and Posner, 1992). Each of these accounts describe elements within the conceptual ecology at a much finer-grain size than, for example, accounts in terms of students having alternative knowledge frameworks. Instead of students being committed to a "motion implies force" framework, diSessa (1993) argues that students employ a variety of different meanings for force in different contexts- including *force as a mover*, *force as a deflector,* and *force as a spinner*. Instead of being in stage of epistemological development (e.g., Hofer & Pintrich, 1997), Hammer and Ebly argue that students have a variety of ways of thinking about knowledge depending on the circumstances— such as *knowledge as free creation* or *knowledge as propagated stuff.* Whether it's for thinking about concepts like force or for thinking about the nature of knowledge, these researchers argue that individuals are better characterized in terms of diverse pool of resources they use (and how and when they use them).

Many researchers working within CKS approaches have conceptualized the conceptual ecology of mind much like a network– many cognitive elements that are connected to each other. Elements of knowledge are understood to be activated or not, possibly being cued by the features in the environment. Through the connections that exist among these elements, the activation of one element can lead to the activation of others. Network-like drawings of students reasoning patterns have even been proposed to represent various kinds of conceptual reasoning and change (Wittmann, 2006; Sabella & Redish, 2007).

It is important to note that such networks are conceptualized either as networks of current activity or as weighted networks upon which there is dynamic activity.

They are not conceived as static networks of related knowledge, such as knowledge networks describing what differentiated or undifferentiated knowledge look like. In this way, complex knowledge systems approaches have emphasized descriptions of student thinking in terms of the knowledge elements that are currently *active* rather than the knowledge that *exists*. Researchers focus on what knowledge is playing a role right now. Because knowledge elements may or may not be active at any time, one goal of CKS approaches has been to document how specific contexts influence whether particular knowledge elements become and possibly remain active. Redish (2004) writes, "The principle [of context dependences] tells us to pay attention to context dependence, but doesn't tell us how or when," (p. 17). Similarly, Strike and Posner (1992) argue that researchers should be more interested in *how* misconceptions are generated (from elements in a conceptual ecology) rather than just characterizing the nature of misconceptions. Thus, the focus of research has often become more about the dynamics of existing knowledge (e.g., how and when they arise) than just the structure of knowledge.

These assumptions about the diversity of knowledge elements and context-dependent activations allow CKS descriptions to flexibly account for variability in student thinking (Hammer, 2004). Students may think one way one moment or context, and in different way in a different moment or context, similar to the way children's reaching performances may vary with subtle shifts in context. Microgenetic approaches in cognitive development research have also underscored the importance of variability in learning and development (Siegler, 1994; Church &

Goldin-Meadow, 1986). Knowledge-in-pieces perspectives play a similar role in the characterization of student thinking in science education.

Characteristics of Accounts focusing on Real-time Assembly

Previously, I described how cognitive accounts with a focus on past assemblies share many of the following characteristics: a methodological metaphor of probing, a description of knowledge states as characteristic of individuals over broad time scales, the state of an individual described in terms of to singular abilities or conceptions, and a tight correspondence between cognitive phenomena and cognitive structure.

Cognitive accounts with a focus on real-time assemblies can be loosely characterized by different properties. In addition to probing, perturbing serves as an additional metaphor for conceptualizing methods of gathering and interpreting data on student thinking. Researchers have conceptualized individuals as systems, capable of responding in a variety of ways depending both upon the local circumstances as well as the current state of the system. Acting upon that system *induces* patterns of dynamic responses by perturbing them. While questions (and the contexts in which they are presented) may sometimes simply uncover existing cognitive structures, they may just as often contribute to the generation soft-assemblies that may be quite fragile and fleeting. Seemingly fixed stabilities in how children and adults reach for objects, for example, can disappear or reappear depending on the how they are 'perturbed'.

Viewing individuals as a system coincides with a shift in the meaning of a cognitive state as well. The state of an individual as a system is conceptualized as a temporally local description of current activity rather than as global tendencies. For example, in the dynamic field model, the individual is characterized at any moment by a pattern of activations in the motor planning field – activations which change in time. In various knowledge-in-pieces models, the individual is characterized at any moment by a pattern of active knowledge elements. Cognitive states change over the time scales of second and minutes – not just over hours, days, or years. Because of variability at this short time scale, there is a necessary rejection of singular attributions (at least at this scale), which necessitates the need for finer-grained descriptions of cognitive elements. Individuals may be better characterized by the multiplicity and variability that characterizes their cognition.

Lastly, dynamic and complex knowledge systems approaches have largely attempted to refrain from cognitive accounts that *demand* a tight-phenomenological correspondence. Instead, researchers have tried to focus on how cognitive phenomena emerge from local actions and mechanisms (Minsky, 1986; diSessa, 1993; Hammer, Elby, Scherr, Redish, 2004). Two ways in which this has been implemented is through a loosening of domain-specificity (focusing on generative knowledge that apply across many domains) and through attention to mechanism (trying to detail how patterns arise from underlying structures and processes). For example, in child development, Maouene, Hidaka and Smith, (in press) have argued that children's difficulties with reaching performances arise partially from mechanisms – related to how information is bound embodied action– that actually facilitate language learning. In science education, diSessa's (1993) prototypical knowledge element, Ohm's p-prim, represents the idea that more cause produces more effects. This idea applies across many domains both within and outside of science. Ohm's p-prim then may be understood to be a domain-general element (or a collection of domain-specific elements) of cognition that comes to play a role in student thinking in ways that is domain-specific and, more importantly, context-specific.

Brief Summary

In this section, I have discussed how various cognitive frameworks locate and granulate cognition as assemblies taking place in and across time. A major theme throughout this essay has been evaluating the degree to which researcher's interpret thinking and behavior as largely resulting from structures of the mind (and body) that have already assembled *versus* structures that may largely be assembling in real time as behavior unfolds.

I have certainly posed those accounts that focus on cognition as having occurred in the past and those focus on cognition taking place the present as being in opposition to one another. In truth, the distinction is perhaps more subtle, having more to do with the emphasis that these perspectives place on cognition rather than in having a principled stance. A focus on the past means thinking about using the dynamics of the present to tell you about some general state of the individual that has been forged in the past. Researchers with this focus must still present arguments that these present dynamics actually measure something generally about the state of the child or student. Similarly, a focus on the present means using the dynamics of the present to tell you about the local state of cognition right now. These researchers still need to present arguments for how the real-time actions assemble from *something* that existed prior to that moment.

Thus a central concern is not whether we can only talk about the past or talk about the present in accounting for behavior. We must certainly talk about both. The issue really concerns what should we attribute to having already been assembled in the past and what should we attribute to being assembled in the present. What stabilities are already present and held together by their own integrity- either as pieces of knowledge for thinking or as sensory-motor patterns for acting? And what emerging stabilities are merely being held together only locally by the current activity in a particular setting? And importantly, how would we know the difference?

The reader may also be left wondering why, in a dissertation concerning college students' intuitive thinking about physics, there would be so much space given to a review of research on children's developing reaching performances. One reason for including this is to reflect upon how dynamic systems perspectives in child development offered new ways of conceptualizing the same phenomena that Piaget had explored decades early. While Piaget came to conceptualize children's reaching performances as *a measurement* of development, Smith and her collaborators saw their reaching performances as dynamic events in themselves. Instead of using the A-not-B error as a measurement that could be used to explore development over time, they viewed the A-not-B error as a dynamic instantiation of development in real-time. This new characterization of the phenomena didn't just offer *yet another way* of describing the same phenomena in different language. It motivated novel approaches for further exploring that phenomena and drew attention to previously ignored (or under appreciated) aspects of those phenomena, which ultimately led to improved insights into the nature of development and learning.

Complex knowledge systems approaches offer some of the very same possibilities for reconceptualizing well-known phenomena from science education by focusing on student thinking as involving a rich real-time dynamic. An example of this is diSessa's (1993) redescription of students' misconceptions about motion and force in terms of local instantiations of p-prims, which had been previously characterized from the perspective of students espousing alternative knowledge frameworks. These approaches are likely to be generative for researchers to begin characterizing variability and context-dependence in students' thinking using novel methods as well (e.g., Thaden-koch, Dufresen, and Mestre, 2006). However, there is still the need for researchers to carefully document such phenomena in both unexplored and previously explored areas of student thinking.

Finally, recall that the goal of this chapter is to get to a place where we have some footholds upon what kinds of cognitive elements will be generative in describing students' thinking in real-time. Up until this point, I have only considered perspectives that describe structures of cognition as belonging to the individual. Toward this goal, we have certainly made progress in describing some aspects of the cognitive elements of individual that are conducive for describing thinking as it happens in real time. In the following section, we shift gears as a little bit as I review research perspectives concerning the boundaries of cognition in space. Such accounts focusing on describing cognition as taking place within boundaries beyond the individual.

Cognitive Assemblies in Space

In this section I contrast various perspectives concerning the location of cognition in space. On one end of the spectrum, many researchers have located cognition as assemblies taking place solely within individuals' minds. The perspectives discussed above, accounting for cognition either in terms of knowledge frameworks or knowledge-in-pieces, share an ontology of cognitive objects existing in the mind. On the other end, researchers have argued that cognition is better located in much broader systems distributed throughout space in the interactions of humans and various technological and cultural tools.

Cognitive Assemblies in the Mind

So far, we have mostly considered accounts of cognitive performance that are understood to arise from assemblies occurring within the mind of individuals. Some of these assemblies are understood to exist as structures entirely built in the past, which largely determine action and thought. Other assemblies are understood to happen, in part, during moments of acting, thinking, and behaving. In both cases, the elements being assembled have been understood to be elements that belong to the individual.

For all of these perspectives, however, the world still plays an important role. Piaget's accounts of cognitive development describe the individual as building structures in the mind through interactions with the world (Piaget, 1970).

Knowledge-framework accounts in science education have emphasized that students' frameworks are built based largely on their past experiences in the world. Dynamic systems perspectives emphasize how the immediate context of acting in the real world influences behavior in different ways depending on the particulars of those contexts. Knowledge-in-pieces accounts have focused on how particular local contexts affect which elements of knowledge are cued. None of these perspectives deny the importance of the world in describing and accounting for cognitive development or cognitive performance.

These perspectives do, however, maintain a rather distinct spatial boundary separating the internal cognitive system from its external surroundings. For some accounts this boundary is the individual's mind and in other accounts it is the individual's body. The individual possesses structures (knowledge frameworks, p-prims, sensory-motor stabilities, concepts), which assemble either in the past (and now are existing structures) or assemble in the present due to real-time contextual influences.

Because of the importance of the world in cognition, some researchers have emphasized a much more reciprocal relationship between the structures of the individual minds and the structures in the world than these accounts offer. Greeno (1989), for example, emphasizes that individuals' *abilities* to act or think always take place in environments that provide *affordances* for those actions and thoughts to occur. Abilities, knowledge, or patterns of activation don't exist in a vacuum. Similarly, the world can't provide the context for abilities independent of the individuals who don't possess them. Thinking of abilities and their contexts in this reciprocal way is a bridge to discussing frameworks in which cognition is understood as not just happening *in* the world. Rather, the world is viewed as an irreducible part of cognition assembling. In the following section, I review perspectives in cognition that use the world as their focal point in describing cognition.

Cognitive Assemblies in the World

In this section I consider accounts that examine cognition as assemblies of cognitive elements taking place beyond the minds of individuals. Along the way, I reflect on how these spatially distributed accounts compare and contrast with accounts that focus more on the individual as a cognitive system. At the end of this section, we will be in position to draw from various cognitive frameworks in order to establish some footholds upon an ontology for analyzing student thinking as cognition assembling in space and time.

There are certainly many research perspectives that locate human learning and developmental processes outside of the human mind and body. Socio-cultural perspectives, for example, which take a spatially expansive view of learning and development, encompass many different research traditions from sociology, anthropology, and educational research. Such perspectives broadly view learning and development of any kind as taking place within structures that are social in nature. Lemke (2001a) describes these socio-cultural research as involving community structures that span a broad range— at the smallest scales there are structures of interpersonal social interaction and tool use and at the larger scales there are structures that are institutional and cultural.

Many of these perspectives have their roots in the work of Vygotsky (1924/1978). Vygotsky wrote largely in response to more traditional accounts of development and learning that focus on how individuals learn from being immersed in a platonic world (such as Piaget's account of how children learn about object permanence through experience with such objects). Instead, Vygotsky emphasized that children learn primarily through social participation in human activities that are immersed in language and culture. The concept of 'zone of proximal development' was introduced by Vygotsky to emphasize that individuals can often participate in activities of knowing and doing as part of a social group before they are able to demonstrate such competencies by themselves, arguing that is it only after and through this social participation that one learns to accomplish those tasks more privately. The zone of proximal development can be conceptualized as the difference between what an individual can do alone and what they can do with the help of others. Vygotsky argued the zone of proximal development is a better predictor of future learning than measurements of individual abilities, since it not only takes into account what the individual can do now, but what the individual is likely to be under the right circumstances.

At the broader scale of cognition taking place in the world, there are accounts that describe individual learning as involving shifting patterns of participation within communities of practice rather than as the acquisition of skills or knowledge (Lave and Wenger, 1991). Individuals' changes in the kinds of activities and roles they play (and are allowed to play) in a community are the basis for describing learning. These accounts place the individual within a complex system itself that spans time and space. For example, in ethnographic work describing differences in the trajectories of physics and business students through their college majors, Nespor (1994) describes students as interacting with physical and social structures that ultimately bring them into different places within the communities they are trying to enter. Other researchers have similarly conceptualized learning as a form of 'cognitive apprenticeship' in which the culture, practice, and norms (as well as skill development) constitute relevant structures of learning (Collins, Brown, and Newman, 1989; Schoenfeld, 1985). These accounts focus on how individuals' behavior are influenced by and also act upon broader social institutions in which they are embedded, rather than on understanding how the knowledge that an individual possesses influences their behavior.

For our purposes, I'd like to focus attention on just a slice of these distributed accounts of cognition—those that focus on describing how thinking and knowing take place among people and setting rather than on those that focus on describing how individual's roles in communities change. Two such perspectives are commonly referred to as *distributed cognition* and *situated cognition*.

Distributed Accounts of Socio-technological Systems

Distributed accounts of cognition (Hutchins, 1995a and 1995b) have focused on describing how socio-technological systems, such as naval ships or cockpits, function cognitively due to mutual interactions among persons and artifacts. These distributed views of cognition take observable structures and processes of the world as constituent elements of cognition (instead of largely non-observable processes of the mind). These externals structures are conceptualized as not just influencing the cognition of individuals by providing a context in which person think and behave. These external structures participate in the assembly of cognition because they *are* active elements of that assembly. This perspective thus involves conceptualizing the individual as just a piece of a broader unit of cognition.

Hutchins (1995a), for example, describes how the cockpit of an airplane- as an extended cognitive system- performs the cognitive task of remembering and regulating its speed. The pilot, co-pilot, the various manuals, indicators, and gauges are each viewed as cognitive structures that are capable of storing, representing, and transforming information. The functioning of the cognitive system is described by how information is propagated through the system - creating redundancy and coordinating information - in order to accomplish the task of regulating its speed. Information about the planes speed (and other relevant information like its weight and fuel) is remembered via processes of interaction among these cognitive elements. Both pilot and co-pilot commit speech acts and maintain memories of events. They look and point to entries in table. They turn pages in manuals. They manipulate indicators on gauges that help to reorganize that information. All of these interactions lead to changes of representations within the system that are coordinated at multiple levels. Such changes are propagated throughout the system in a non-linear fashion and serve to maintain a degree of coherence among these various representations of information.

In this kind of analysis, it is the entire cockpit that is conceptualized as remembering the speed. Of course, the pilots must play a key role in regulating this flow of information. The pilot and co-pilot have been trained to perform tasks with their instrument panel and controls, whose purpose is to optimally maintain coherence among these various cognitive media. That is, it is their intent to maintain coherence—one that is hopefully adequate for flying and landing safely. Despite its emphasis on broader systems than the individual, distributed accounts of cognition have not typically ignored the relevance of human knowledge and abilities in their accounts. Rather, they have just emphasized that individuals are just a piece of these socio-technological systems that also can be understood as engaging in the cognitive behavior. Individuals interact with structures in the world, and those structures interact with individuals. It is through these mutual interactions that many cognitive behaviors occur.

Hutchins (1995b) writes that, "Human activity as an integral part of larger systems may bring us a different sense of the nature of individual cognition. Any attempt to explain cognitive properties of such a larger system without reference to its most active part would be deficient. Similarly, though, any attempt to explain the cognitive properties of the integrals parts without reference to the properties othe larger system would also be incomplete," (pp 267-268). Here Hutchins articulates the need to look at the cognitive systems at multiple levels. Individuals (as often the most integral parts of cognition) perform tasks along with other individuals and technological artifacts (as larger cognitive systems). It is through interacting together that these parts comprise a entire cognitive system which can be analyzed at multiple levels.

Situated Accounts of Participant Activity in Setting

The notion that human activity is central to understanding cognition has also been advocated by researchers espousing situated accounts of cognition (Lave, 1988; Brown, Collins, Duguid, 1989; Greeno, 1989), as well as activity-theoretical perspectives (Engstrom, 1987).

One way that activity has been emphasized as central is in the context of mathematical knowing. Lave (1988) describes numerous examples in which people display specific arithmetic understandings in the context of certain everyday activities (shopping, bowling, cooking) but fail to show those same understanding in more formal environments (solving paper-and-pencil problems). Other researchers have similarly described rich mathematical knowing in the context of everyday human activity, such as Scribner's (1984) account of how dairy workers display complex mathematical understanding of fractions in their sorting and counting of milk products. The rich phenomenology of activity-dependent arithmetic that Lave and others uncovered led many researchers to reject the notion of knowledge as a mental construct at all, instead arguing that cognition is best understood as human activities situated within particular settings.

From the situated perspective, human activity is not just the context in which cognition takes place, but rather the 'stuff' of cognition. Similar to the way in which dynamic systems approaches view knowledge as emergent in embodied action, the situated perspective views knowledge as emergent in human (and therefore social) activity – activity that is tightly bound to particular contexts. Relevant to the notion of context in the situated perspective is the distinction between what Lave referred to as *arenas* and *settings*. An arena is understood to consist of the physically-enduring elements of a given situation. A setting is understood as an individual's personally-edited version of that situation (the features that influence and are a part of ongoing activity). It is this person-in-setting, which are irreducibly linked, that comprises the relevant unit to analyze when trying to understanding human behavior, thinking, learning, and development.

Like distributed cognition, physical (and hence observable) actions like speech, gesticulation, and manipulation played prominent roles in situated accounts of cognition. Lave (1988) describes mathematical knowing as activities closely bound to settings in which they take place. In what has become a classic example, Lave describes one dieter's improvisational solution to finding that three-fourths of two-thirds of a cup is ½ of a cup. The dieter creates uses a 2/3rd measuring cup to create a circular mound of flour on a counter. The dieter then cuts the circle into quarters. By taking one of the quarters away, the dieter constructs three-fourths of 2/3rd. In this example the flour, the cups, the knives, and the individual can all be viewed as elements in a cognitive system performing the task of constructing ¾ of 2/3rd of a cup. While the individual alone may not have the knowledge that ¾ of 2/3rd is ½, the extended system of person plus artifacts may be understood as a system that does. The person-in-setting is a system of arithmetic knowing - one is difficult to reduce to merely the arithmetic knowledge that the individual possesses alone.

Many other perspectives also share similar orientations toward looking to human activity, especially to observable actions that take place in settings for that activity, as the basis for describing thinking, knowing, learning, and developing. Givry and

Roth (2006) argue for descriptions of students' conceptions, not as external mental constructs, but in terms of collective patterns of talk, gesticulations, and relevant situational structures. Goodwin (2000) similarly argues that rich description of action, language, human interactions with each other and material artifacts are needed in order to account for unfolding human events. Researchers from these perspectives argue that to reduce these rich complexes of semiotic behavior to pieces of knowledge in the head is to lose the essential nature of the phenomena itself.

Brief Summary

Distributed and situated accounts of cognition emphasize that individuals are always embedded within particular settings for human activity. These accounts tells us to not only pay close attention to the nature of these settings, in how they affect human behavior, but to pay attention to dynamics by which the settings themselves play a role in cognitive behavior. In essence, Hutchins was arguing that, even though humans play a major role in remembering the speed of the plane, it makes more sense to attribute to ability and stability of remembering speed to the entire cockpit itself than to any individual. We might be able describe some of the things that the individual does in the process of this distributed remembering, but the individual alone cannot be attributed with the task. Similarly, Lave was arguing, that even though humans may engage in all kinds of arithmetic activities, that didn't mean that we should attribute "arithmetic" to something that an individual has. Rather arithmetic is something that should be attributed to the individual plus the setting we see this activity take place.

Fundamentally, the issue at hand concerns making choices about where we choose to attribute cognitive ability, or as some researchers (Cobb, 1994) have asked, "Where is the mind?" For situated and distributed accounts, the answer has most often been to attribute cognitive ability to systems broader than the individual. While for many of the accounts described in previous sections, the answer has most often been to attribute cognitive abilities to individuals. Because of these differences, it is certainly easy to characterize individual cognitive approaches and distributed cognitive approaches as being at fundamental odds with one another. Sfard (1998) has argued that both vantage points are necessary but not simultaneously possible in research, and Lemke (2001b) has argued that researchers studying issues of learning need to tackle the difficult task of zooming in and out across these scales of space and time. The problem facing researchers, thus, may not be whether we should *only* attribute cognitive ability to individuals or *only* to individuals and setting. Rather, the issue facing researchers is deciding where to properly attribute cognition ability based on the analysis of data. At times, it may make sense to attribute stabilities of cognition to individuals. At other times, it may make sense to attribute the same or similar stabilities to interactions taking place among individuals and settings.

Concerns about *where* to locate cognition should be reminiscent of the concerns previously described concerning *when* to attribute cognitive stabilities. Is the behavior we observe attributable to some stability forged in the past, or to some stability that is more local to this moment? Is the cognitive behavior we observe attribute to the individual or to the individual in conjunction with a broader setting? We as researchers are thus always faced with decisions about when and where to attribute cognitive stabilities— to attribute the pieces of cognitive behavior to their rightful places in time and space.

This discussion ultimately leads us into the last section of this chapter, where I hope to provide some footholds on the cognitive ontology that will be used as the basis for describing cognitive basis in this dissertation. While I hope to pin down certain aspects of this ontology in place, many of the details of the cognitive ontology will emerge as the result of the research described herein.

Foothold Ideas for Cognitive Ontology

Throughout this dissertation, there is a focus on accounting for student thinking in physics as cognition unfolding in real time. There is a progression in this document from a focus on cognition as taking place within the minds of individuals (Chapters 3 and 4) towards a greater focus on cognition as taking place among individuals and their interactions with structures in the environment (Chapter 5). In this section, I first detail the cognitive ontology of individuals' minds as a unit of analysis, and then discuss more loosely the cognitive ontology for describing broader systems as the unit of analysis.

Cognitive Attributions of the Individual Mind

This dissertation focuses on building characterizations of student thinking in moments of time that do not always directly addressing issues of how students learning across time. Thus the cognitive ontology used throughout this dissertation will need to serve the function of describing patterns of thinking and behavior that span modest expanses of time ranging from seconds to minutes, but not hours, days, or years. In Chapters 3 and 4, I focus primarily on how individual students settle into patterns of thinking in just the brief moments of attending to and responding to specific physical situations or questions. Chapters 5 and 6 focus on more on how patterns of student thinking and behavior cohere across slightly broader times in an instructional period. Our basic question here is, "What kinds of cognitive structure do we attribute to the individual mind?"

Structures and Processes

The phrase "cognition assembling" that has been used in this chapter implies that we are aiming to explain cognitive behavior as arising from real-time processes that bring together cognitive elements. Such real-time assembly may, of course, result in the coming together of existing elements of cognition that has never happened before. These novel assemblies may persist or may be fleeting across the scales of time we observe them. They may also exhibit certain kinds of stabilities and instabilities. What we as researchers must then struggle with is the distinction between the *elements* that come together – pieces that characterize more enduring structures of the system - and the emerging *assemblies* that characterize the real-time state of that system.

From the individual perspective, this dissertation draws heavily from complex knowledge systems perspectives as a cognitive ontology for individual minds. I take the explicit stance that students do *have* already a variety of diverse knowledge elements that may or may not be activated in particular contexts. These existing elements of cognition are the kinds of structures described by Minsky (1986) and diSessa (1993) – simpler elements of cognition that act together to generate the quite rich and complex patterns of behavior and thinking we find in the world. It is these variable activation patterns of fine-grained knowledge elements that constitute the emergent assemblies of students' thinking in the moment, while the elements themselves represent structural stabilities that are likely already been forged in the past.

Since this project primarily involves the study of students' thinking about phenomena of motion, many of the elements described here concern students' physical intuitions about motion. In terms of both the substance and grain-size for describing individual thinking, many of the cognitive elements in this document share much in common with diSessa's account of phenomenological primitives. In terms of substance, p-prims are understood to comprise students' intuitive sense of mechanism. These are intricately related to one's sense of motion. In terms of grain size, p-prims are described at a sufficiently fine grain-size to be useful for modeling the generation of novel patterns of thought. That is, having a diversity of smaller-grained cognitive elements allows one to describe thinking that changes at the small time scales observed throughout this research.

The most central process of any elemental model of individual thinking is activation. In other words, cognitive elements may be on or off. Although the distinction may not always be clear, two separate processes have often been used to describe how a cognitive element of individual mind becomes active. Cueing is often used to describe the process by which a cognitive element is activated by some external feature in the world. For example, seeing a piece of paper may lead to the activation of the cognitive element corresponding to one's sense of "white".

Spreading activation is the process by which a cognitive element becomes active because another "nearby" cognitive element has been activated through association. For example, an element or elements corresponding to "white" may activate other elements such as those corresponding to "milk" or "sugar". In descriptions of students' thinking throughout this research, activation of cognitive elements and especially the cueing of those elements will play a significant role.

Also throughout this document, I have attempted to describe cognitive elements of the individual mind that illustrate an "unproblematic genesis" (diSessa, 1993). That is, the cognitive elements that I attribute to college students in their thinking about motion are the kinds of elements that we can reasonably believe they forged as stabilities in the past. This unproblematic genesis of these cognitive elements may be documented through developmental research showing the development of such ideas. But these attributions should also resonate with our own personal feelings about the reasonableness of these them.

This research does not serve as an inquiry into how these cognitive elements came to be. Nor does this research serve as an inquiry into how new cognitive elements come to be. Rather this research concerns the dynamics of student thinking that arise from assumptions about a reasonable starting place for cognitive attributions of the mind. Given that students have some set of intuitions to draw on when thinking about motion, we are interested in knowing about the dynamics that take place among them.

Methods of Identification

In analyzing students' thinking, it will be necessary to be able to identify individual cognitive elements (that are cued or activated) as well as to identify soft assemblies of multiple cognitive elements arising together. In this section, I broadly describe the methods used to identify such cognitive elements of the individual mind.

In this dissertation, data on student thinking largely comes from two sources: students' written answers and explanations to questions (in an survey-based experimental design) and students' verbal statements and gestures (in video-based case studies). On one hand, the identification of cognitive elements from data concerning student thinking is determined from the specific properties that are attributed to cognitive elements that have been proposed to exist. For example, in the toy cognitive model of students' thinking about kinematical relation (described in Chapter 3), I describe specific properties of all the cognitive elements that comprise this initial toy cognitive model. One such cognitive element, an intuition described as *more speed implies less time,* is characterized in terms of a property concerning the linguistic use of the word 'faster' (that can either mean greater speed or happening in less time). Such linguistic markers may be used to help identify the cueing of such intuitions from verbal data. Throughout this document, I attempt to illustrate particular instantiations of these cognitive elements in students' thinking in specific examples in order to provide some familiarity for the reader, and to draw connections between various instantiations.

On the other hand, there are times when patterns within and among students' statements cannot be described in terms of cognitive elements that have already been proposed- for which we don't know what properties to look for. When this occurs and, common features of students' unaccounted for statements may be used as the basis for suggesting new candidate cognitive elements. Properties of these cognitive elements are described as well with care made to connect these properties back to instantiations in the data.

Identifying soft assemblies of students' thinking (those that consist of multiple intuitions) involves a similar analysis of student statements. Identifying such assemblies is largely the concern of methods employed in the video-based case studies of student thinking (chapter 5) and not of the survey-based experimental design (chapter 4). By examining students' thinking at broader times scales in video data, assemblies of intuitions are identified by the overlapping occurrence and reoccurrence of evidence for single cognitive elements.

Cognitive Attributions in the World

The cognitive ontology used to describe students' thinking will also need to span a modest range of space in the world. In chapters 3 and 4, where we only consider students' responses to questions on the single page with physics questions, I constrain the cognitive analysis to the individual student and the worksheet in front of them. Given the relatively fixed nature of questions printed on that worksheet, the dynamic involvement of that page is surely to be limited. In this case, it may be reasonable to describe cognition as resulting from the cueing of individual cognitive elements in the mind and the dynamics by which they are cued by this rather fixed context.

In chapters 5 and 6, however, where I explore the dynamics of student thinking in classrooms, the degree to which external structures play an active role in cognition becomes an essential task of the research itself. What are the kinds of structures and processes to look for in a broader view of the cognitive systems? How will we identify them?

Structures and Processes in the World

In Hutchins' account of the cockpit remembering its speed, physical artifacts such as speed gauges and tables of numbers are understood to make up structures of the cognitive entire system. These structures are cognitive in the sense that they undergo changes as they interact with other structures—changes that are consequential to the dynamics of the entire cockpit remembering its speed. Of course, not everything in the cockpit changes in the same way nor or all changes consequential to the dynamics of the cognitive systems behavior. Cognitive structures in the world are determined by the nature of their participation in activity, not by there mere existence or assumed function.

Lave made a similar distinction between arenas and settings in describing situated cognition. The setting only concerns those aspects of the arena that influence and take part in cognition activity. For example, in the case of the dieter constructing $3/4^{th}$ of $2/3^{rd}$, the measuring cup and the flour certainly constituted relevant structures of that setting, but maybe not the tile floor she was standing on.

In identifying elements of context that are relevant to human interactions and talk in institutional settings, Schegloff (1992) has employed a construct referred to as *procedural consequentiality.* Relevant structures of context must be those to which participants orient and that are demonstrably consequential to their activity. In this dissertation, I attempt to determine the degree to which structures in the world constitute active cognitive structure through a similar process – trying to identify what it is that students orienting toward and in what way does this orientation have consequences for their behavior and thinking. These structures could be physical artifacts in the world or the outward behaviors of other individual engaged in social activity. For example, in the case studies developed in chapter 6, I argue that the location and arrangement of strips of paper (as artifacts) and students' collective patterns of behavior (as social structure) are both external cognitive elements that students orient to and are consequential to their thinking. These contexts change as students manipulate artifacts around them and shift their posture and gaze in various ways.

Chapter Summary

In this chapter, I have compared and contrasted a variety of research perspectives concerning the nature of thinking, learning, and development in building the case for the particular orientation toward students' thinking in physics as real-time time activity.

Along the way, I have drawn from a variety of disciplines, including cognitive development, science education, and anthropology. I have characterized different perspectives as focusing on issues of human thinking and behavior as being the result of structures and processes that are located and granulated in time and space in different ways. Some of these perspectives focus on describing cognition as arising from entire structures that were forged in the past and determine or constrain thinking and behavior. Other perspectives focus on describing cognition as arising from processes taking place in the present among stabilities that are likely to have been forged in the past. Some of these perspectives focused on structures and processes taking place within individuals, while others focused on structures and processes taking place among individuals and broader aspects of settings for social activity.

At the end of this chapter, I have described some foothold ideas to be used in exploring phenomena of students' thinking and for building cognitive accounts that attempt to explain that phenomena. These footholds draw heavily from both complex knowledge systems perspectives in terms of their focus on multiplicity and contextuality in students' intuitive thinking and from situated/distributed accounts for their focus on the inclusiveness and interdependence of setting.

The goal throughout this dissertation will be to characterize students' thinking as involving a real-time dynamics using these footholds. Beginning with very basic assumptions of fine-grained multiplicity in students' intuitive thinking about motion (described in Chapter 3), this dissertation begins by investigating issues of contextuality in students' intuitive thinking about motion (the subject of Chapters 4 and 5), and builds toward characterizations that take in account the inclusion of settings in that dynamic (the subject of Chapter 6).

Chapter 3: Models of Student Thinking about Motion

Chapter Introduction

In this chapter, I focus on building an account of student thinking along a very narrow slice of space and time by exploring phenomena of student thinking about motion. The goal of this chapter is to motivate, describe, and work out implications for a simple model of student thinking about motion that can be used to make specific predictions about student performance in responding to written questions. This model will also serve as a beginning place for examining patterns of student thinking in the classroom as well.

I begin this chapter with a review of prior research on student thinking about motion. Some of this research on student thinking about motion has emphasized the broadest and most common patterns in student responses (McCloskey, Caramazza, and Green, 1980; Whitaker, 1983; Eckstein & Kozhevnikov, 1997). Other researchers have emphasized variability and lack of systematicity in student thinking about motion (diSessa, 1993; Cooke and Breedin, 1994), or have focused on identifying conditions leading students to depart from the most common incorrect interpretations (Kaiser, Jonides, and Alexander, 1986; Kaiser, Profitt, Whelan, and Hecht, 1992). In response to empirical evidence suggesting both global patterns and local variations, some researchers have remained committed to the notion that students' knowledge about motion is largely fragmented and unsystematic (diSessa, 1993; diSessa, Gillespie, and Esterly, 2004). Other researchers have remained committed to a more coherent and common framework for describing how people make judgments about motion phenomena (Kozhevnikov and Hegarty, 2001).

For this study, a simple knowledge-in-pieces model is developed and deployed to account for a range of student thinking concerning motion phenomena that can be used to make empirical predictions. The specific model is described in some detail, including applications to several examples of student thinking. While most knowledge-in-pieces analyses of student thinking posit accounts that aim to describe the dynamics of student thinking (diSessa, 1993; Wagner, 2006; Scherr, 2007; Parnafes, 2007); a major aim here is to pursue an account of student thinking about motion capable of pinning down testable predictions about patterns in student performance (e.g., Elby, 2000).

Research on Student Thinking about Motion

Student thinking about motion, and specifically to the notion of trajectory, has received much attention in psychology and science education during the last three decades. It is a well-documented result of this research that, despite having a wealth of experience with moving objects in the real world, children and adults tend to make a variety of common errors when asked to predict, describe, or select correct features of moving objects' trajectories. One way researchers have attempted to understand student thinking about motion is to look for systematic tendencies and global patterns in their responses to questions. I first discuss how researchers have characterized these global patterns largely in terms of stable knowledge structures that students have for thinking about motion. These attempts are consistent with alternative knowledge frameworks approaches to conceptual change. I then discuss evidence for variability in students' thinking about motion that challenges the utility of these interpretations.

Evidence for and Accounts of Coherence

An examination of the phenomenology of student thinking about motion shows that there are definite trends and tendencies in people's responses to questions about motion. Many of these incorrect patterns of student responses about motion are common and persist even after instruction (Trowbridge & McDermott, 1980; Halloun & Hestenes, 1985; Shaffer & McDermott, 2005). These global patterns have been interpreted by some researchers to represent stabilities and robustness in students' conceptual knowledge frameworks for thinking about motion. I discuss these patterns and interpretations below.

Existence of Naïve Theories of Motion

McCloskey, Caramazza, and Green (1980) first documented the tendency for novice students to predict that an object moving in a circular fashion maintains a certain degree of circular motion even after the constraints (e.g., external forces) causing the circular motion to occur are no longer present. As a part of the study, students were asked in interviews to predict the motion of a ball as it moved along a horizontal table, after it entered and then exited a circular tube that was horizontally placed on the table. The correct answer is that the ball, upon leaving the tube, moves along a straight path on top of the table. However, many students predict that the ball continues to curve to some degree even after leaving the tube. McCloskey (1983) attributed this error to students having a naïve theory of motion similar to medieval impetus theories. According to this naive theory, physical objects are set in motion by imparting to them an impetus (an internal force), which gives them the power to move. This impetus then either dissipates naturally, causing the object to come to rest, or the impetus lessens as the result of some external influence. The tendency for students to choose trajectories that sustain a degree of circular motion can be understood in terms of such an impetus theory: students believe that the ball (while moving in the tube) gains a *circular impetus* that fades away only gradually after leaving the tube.

The naïve theory characterization of student thinking described by McCloskey additionally accounts for some students' responses concerning the trajectories of objects moving under the influence of gravity. For example, many students indicate a belief that a horizontally projected object under the influence of gravity will continue to move purely horizontally for some time before beginning to descend either gradually or perhaps even abruptly in some cases. In this example, we can also understand these responses in terms of an impetus belief – namely that the object maintains its horizontal impetus for some time before gravity takes over. Although slight variations exist across students' ideas concerning how gravity influences an objects' impetus, McCloskey suggests that the general notion of impetus can be understood to underlie many of these responses. In a similar fashion, student responses to questions about the motion of a pendulum bob after cutting the string seems to follow the same general pattern (Caramazza, McCloskey, and Green, 1981). Many students state that the object continues to move in a way consistent with its previous motion for some time even after the constraints are removed. All of these responses to questions about motion after constraints have been removed can be attributed to students having an impetus conception of motion.

McCloskey described the naïve impetus belief as a well-articulated, strongly-held theory that many students have about motion. The intuitive belief guides predictions across a wide variety of situations and also serves as an interpretive framework through which physics instruction may be misinterpreted. As a research account of student thinking, it accounts for student responses to many different physical situations, which may be viewed a particular strength of the characterization. In effect, this characterization unifies students' seemingly isolated responses to different questions into a single, more-coherent framework; for there is no reason to assume *a priori* that student responses to questions involving circular tubes, pendulum bobs, and vertical descents would all share a similar response pattern.

This account of student thinking is consistent with the alternative knowledge framework perspective, locating cognition largely in the past and exclusively in the mind. Students are understood to arrive with these naïve theories prior to instruction based on their experience in the world. The systematicity of student responses is understood in terms of a systematicity of a single cognitive structure within student minds.

Development of Naïve Theories of Motion

The idea that students have formed naive theories about motion based on their experience in the real world suggests that one might be able to track the development of a naive theory through time. Eckstein and Kozhevnikov (1997) describe the development of students' ideas about motion from elementary school through secondary school prior to any formal physics instruction. In their study students in different age groups are categorized as belonging to particular stages of development (in their understanding of motion) based on their responses to four questions regarding two different physical situations. The authors suggest that the progression of development begins in a *proto-Aristotilean stage*, moving to an *impetus stage*, and finally progressing to a *Newtonian stage.* The proto-Aristotilean stage is marked by a belief that objects fall straight down immediately after losing their supports - an erroneous belief that has been discussed by other researchers as well (e.g., Whitaker, 1983). The impetus stage is marked by answers consistent with those described by McCloskey above. Finally, the Newtonian stage is marked by answers that an expert would give. A significant result of the study was that at least some proportion of students by high school seemed to naturally progress to the Newtonian Stage without formal instruction, at least as evidenced by the small number of questions used in the study.

There are several aspects of this study that are worth noting. First is that the study focused on characterizing the development of students' conceptual frameworks over long time scales from data concerning brief snapshots of student thinking. In order to account for the broad developmental trends, the authors assigned predetermined, fixed stage-categorizations to individual students. These categories were based on prior researchers' characterizations of common conceptions. This choice reflects a commitment to the idea that students can be appropriately categorized as having one and only one belief about motion. Students are understood to be in a stage at the time they are responding to these questions.

The possibility that an individual student might have multiple ways of thinking about motion (see, for example, Taber, 2000) is ruled out by presumption. The second aspect of this study that is worth nothing is that the categorization of the students' thinking is based solely upon their responses to two different physical situations. The small range around which student ideas are explored also reflects a commitment to the idea that students hold singular beliefs, and is consistent with the metaphor of "probing" student existing ideas. If one believes that students can only have one belief, not many questions (if properly designed) are needed to determine which of a small set of beliefs a student holds. While the authors, themselves, explicitly claim to be agnostic concerning why the development of motion ideas in children mirrors stages in the historical development of theories of motion, their methods of data collection and their analysis reflect a commitment to knowledge as consisting of stable, unitary constructs in the head. This analysis excludes the possibility of finding more fragmented or variable student responses.

Evidence for Variability and Context-dependence

The research described above highlights some of the patterns in students' incorrect interpretations of motion phenomena that have been described in terms of students possessing fairly systematic knowledge frameworks. Following the work of McCloskey and his collaborators, other researchers pursued projects of research to test claims about the level of systematicity and robustness in students' conceptual frameworks for thinking about motion. These researchers exposed limits to claims about this systematicity, partially motivating new interpretations of student thinking about motion. I discuss some of this empirical evidence and interpretations of student thinking given from a knowledge-in-pieces perspective.

Phenomenology of Variability

Kaiser, Jonides, and Alexander (1986) documented that incorrect impetus beliefs (or rather students' responses consistent with an impetus view) tended to disappear when they were asked to reason about familiar or everyday situations. For example, they found that the number of incorrect predictions concerning the shape of the trajectory of water exiting a curved water hose was significantly less than the number of incorrect predictions concerning a ball exiting a curved tube. This result suggested to the authors that students often rely on specific experiential knowledge in familiar situations and defaulted to heuristics similar to naïve impetus beliefs in unfamiliar situations. Kaiser, Proffitt, Whelan, and Hecht (1992) additionally documented that students were much less likely to select impetus trajectories when watching animated motions than when selecting trajectories from static sketches. Evidently, students are more likely to reject unphysical trajectories when they can watch them taking place in real time. These two studies suggest that students do not always rely on a single conceptual framework or naïve theory when interpreting or predicting trajectories of motion. There are situations in which students rely on other knowledge (specific experiences) or attend to different information (dynamic visual cues) to make sense of what kinds of motion are reasonable. More recent studies have documented ways in which students' real-time attention to dynamic visual displays are strongly context-dependent and influential upon interpretations of the realism of motion (Thaden-Koch, Dufresne, Mestre, 2006)

While the work of Kaiser and collaborators focused on identifying contexts in which students don't rely on their naïve impetus theories (and are actually reliably correct), other researchers sought to demonstrate a lack of systematicity in order to argue that students do not posssess a naïve theory at all. Cooke and Breedin (1994) examined the variation of individual student thinking across different contextual features in problems posed. They found that the individuals in their study showed little or no correlation across different physical situations (e.g., pendulums, projectiles, rockets) as to whether their responses were indicative of impetus beliefs or Newtonian beliefs. This suggested that responses appearing to be manifestations of naïve impetus beliefs were not as strongly-held or coherent as was suggested by McCloskey. Additionally they found that even minute changes to the manner in which problems are presented are capable of affecting the distribution of student responses. For instance, their study found that the number of students indicating a circular impetus belief in the circular-tube problem significantly went down (although it was not a huge effect) when the shape of the tube where the ball enters the semi-circular tube is extended out parallel to the correct exiting path. Based upon their data, Cooke and Breedin suggest that students are likely to be constructing their responses on-the-fly based on particular contextual features rather than from a previously held, singular belief about motion. Cooke and Breedin suggest the need for a model to account for how student thinking about motion happens on the fly, but do not provide one themselves.

Other researchers have also highlighted the context-dependence of student thinking about motion. Bowden et al (1992), in a phenomenographic study of students' thinking about motion and frames of reference, emphasize the uniqueness and variability of the ways that students think about motion problems. In their study, involving the analysis of clinical interviews with students solving a variety of motion problems, they provide a multitude of different categorizations describing students' understanding and experience of different problems. In particular, their categorizations for students' understanding turn out to be quite specific to particular problems, and they suggest that their data demonstrates that "different problems reflecting the same concepts [are] treated differently by some students," (p. 266).

These studies each emphasize that student thinking about motion involves a variety of different knowledge or knowledge types that comes into play in a context-sensitive way. Student thinking about motion varies based on the familiarity of the contexts, the modality in which contexts are presented, minor variations to visual displays, and also whether or not problems are qualitative or quantitative in nature.

An Ontology for Variability and Systematicity

The studies described above point to evidence of variability and context-dependence in student thinking about motion. While this variability highlights the degree to which the alternative knowledge framework perspective fails to account for much of the phenomenology of student thinking of motion, none of the studies above work toward building a new accounts of student thinking to explain their data. diSessa's account of phenomenological primitives (or p-prims), however, attempts to provide a viable basis for describing the variability in students' thinking about motion as well as mechanisms that generate coherence.

diSessa (1993) also explicitly rejected McCloskey's naïve theory interpretation of students' thinking about motion. Based upon the analysis of extended case studies of students' thinking about force and motion, diSessa argued that students' thinking consists of the activation of many different pieces of knowledge for thinking about physical mechanism that are weakly organized and highly sensitive to context. He called these pieces of knowledge, phenomenological primitives, or p-prims.

diSessa argued that the responses which McCloskey characterizes in terms of naïve impetus belief consists of a dozen or so loosely connected pieces of knowledge that become activated across a fairly narrow range of tasks, denying the global coherence that McCloskey attributed to the naïve theory. In this ontology of mind, p-prims are understood to be at a much finer grain-size than naïve theories of knowledge frameworks, and are, instead, small bits of knowledge about physical mechanism that are "lightly abstracted" from experiences. As a collection, p-prims are understood to be numerous, possibly in the hundreds or thousands. It is this diversity and lack of organization that provide a basis for describing the variability of student thinking observed. diSessa describes some p-prims for thinking about motion phenomena, such more as *more distance implies more time*, that are elements in the toy cognitive model that is described below.

Many researchers working in the knowledge-in-pieces framework have made it a point to state that, despite having emphasized variability and context-dependence, systematicities in student thinking should not be ruled out by the framework (e.g., diSessa, 1993).. The question for many researchers has become how locally coherent patterns of thinking are generated rather than just how they are represented by structures of the mind. For example, Hammer, Elby, Scherr, and Redish (2004) have proposed three different classes of cognitive mechanism that can contribute to local conceptual coherences in student thinking. There are mechanisms relating to the structure of knowledge, those relating the effect of context, and those relating to effect of meta-cognition and epistemology. I delay a further discussion of accounting for local stabilities in student reasoning until chapter 5, and instead move to describe a simple model of students' thinking about motion rooted in a knowledge-in-pieces.

A Simple Model for Student Thinking about Motion

The phenomenological landscape of student thinking about motion includes broad, common patterns as well as variable and context-sensitive responses. One way we have seen researchers account for the broad and common patterns is to assert a broad systematicity to students' conceptual frameworks for thinking about motion. Limitations upon such systematicity, however, have been documented. Students don't always respond as if they held an impetus theory. In face of these limitations, it is certainly possible to maintain a perspective that students do rely on a single conceptual framework, for which there are many exceptions due to a lack of sufficient coherence (e.g, Kozhevnikov & Hegarty, 2001). Alternatively, one can account for broad and common patterns by looking for systematicity, not in students' knowledge, but in the relationships between students' possibly unsystematic knowledge and the range of contexts in which the broad pattern is observed – to look for contextual mechanisms that generate patterns in data. diSessa (1993) seems to be in support of this view in the suggestion that impetus-like responses are generated only in a limited number of contexts where a particular subset of students p-prims are cued (and not because students have a stable impetus conception).

In this section, the central aim is to build a model of student thinking from a knowledge in pieces perspective that can be used to pin down claims about 1) what specific knowledge elements students rely on in the real time of responding to motion questions and 2) how those specific knowledge elements generate patterns of variability in student thinking that is sensitive to contextual cues.

It should be emphasized that initial claims from researchers that students held a singular conceptual framework for thinking about motion motivated research aimed at testing the viability of this assertion. Similarly, claims that students are relying on a variety of fragmented, finer-grained knowledge elements should also lead to generative research that aims test the viability of that assertion as well.

While there are a growing number of "knowledge in pieces" accounts of student thinking in science and mathematics education research - including student thinking about oscillations (Parnafes, 2007), relativity (Scherr, 2007), statistics (Wagner, 2006), waves (Wittman, 2002), biological explanations (Southerland, Abrams, & Cummins, 2001), coordinate systems (Sayre & Wittmann, 2008) - very few accounts have been used for the explicit purpose of the predicting patterns of student thinking in novel situations. Rather these studies have largely focused on developing models that describe student thinking for the particular cases from which those models were developed.

Noted exceptions to this trend are accounts of student thinking reported by Elby (2000) and Smith & Wittmann (2008). Both of these studies account for patterns in student responses in terms of the activation of specific knowledge elements and then use these accounts to make predictions about empirical data. In the following section I discuss a simple model of student thinking with the goal of pinning down specific empirically-testable claims. This model is not intended to be novel. It draws heavily from similar accounts of thinking from research in child development and science education.

Cognitive Elements and their Properties

In this section I describe the cognitive elements that comprise a toy model of students' thinking about motion. This model will be used to make predictions about patterns of student thinking in an experiment designed to variably cue different knowledge that students have for thinking about motion. The particular cognitive elements proposed have connections to both research on the development of motion ideas in young children (Piaget, 1971; Acredolo, Adams, Schmid, 1984) as well as to diSessa's (1993) p-prims. These connections are discussed throughout.

The three cognitive elements in the model concern intuitions for making sense of the interrelations among distance, speed, and time. The three intuitions are described as reflecting the following ideas:

- *more distance implies more time*
- *more speed implies less time*
- *more speed implies more distance*

Positing these intuitions as "elements" in a cognitive model means is to propose that these intuitions exist as fine-grained stabilities of individuals' mind. While these intuitions are understood to have been forged in the past of the individual, the activation of these cognitive elements (and hence participation in thinking) is still dependent upon the context of student reasoning and the activation of other cognitive elements.

In positing these as elements in a cognitive model to describe the thinking of many individual students, we are additionally assuming that individuals in the population (college students enrolled in physics classrooms) share these intuitions as common structures. While their may exist idiosyncrasies in individuals' structures of knowledge, the toy model captures the commonalities of such intuitions across the population in a way that allows us to both describe aspects of individual students' thinking and tendencies in population. The model, as a toy model, certainly overly simplifies certain aspects of student thinking, but will nonetheless prove useful for describing and predicting patterns in student thinking.

More Distance Implies More Time

One intuition for thinking about motion and kinematical relations is the idea that going farther distances takes more time (or that traveling for more time means going for more distance). This intuitive expectation is one that arises often when people think about races or travel. People expect that it will take longer to run a marathon than it will to run a mile. People know that it takes longer to travel across the country than it does to travel across the state. This inference is one of the first speed-distance-time relationships that young children develop. Piaget (1971) showed how this expectation leads young children to misinterpret occluded motions that start and finish at the same time (but occur over different distances) as taking different amounts of time. Children interpret that the longer distance takes more time, despite being able to see when and where the motions begin and end (although they are not allowed to see the objects speed since the motion is occluded).

This intuition is also closely tied to language that people use to talk about distance and time. For example, the words ‘long’ and ‘short’ are almost exclusively used to describe amounts of distance and time. They are not used to describe other amounts like force, mass, or current. The exclusivity of the word longer for distance and time allows for a certain degree of ambiguity. When someone says, “I took the long way home”, that person could be explicitly referring to the distance they traveled as the quantity that is long. Alternatively, they may be just referring to the amount of time it took. More often than not, neither the speaker nor the listener need be explicitly attending to one quantity or the other. The relationship between distance and time in the real world context of motion allows for a productive ambiguity – one that may simultaneously refer to both of these quantities or to their relationship. The ambiguity of words like “longer” is what permits us to make sense of the intuition that *more distance implies more time* as a heuristic that says that “longer means longer”.

It is important to note that the intuitive knowledge element *more distance implies more time* is by itself neither correct nor incorrect. It works much more like a heuristic or a rule-of-thumb – a rule that applies in many situations but not in every situation. Many people who travel by car are familiar with circumstances in which taking the highway (which may be a farther distance) takes less time than taking country roads that are shorter in distance (but at a slower speed). A direct flight across the country may take less time to travel than a shorter trip that involves one or more layovers. In these cases, the intuition *more distance implies more time* fails to produce a reliable inference, just as it did for the children in Piaget's experiments. The intuition can be used in productive or unproductive ways depending on the context. diSessa (1993) suggests that the difference between children and adults in their reasoning about motion phenomena is not differences in the fundamental structures of knowledge, but differences in their reliability of use and cueing priorities. In this interpretation, the children in Piaget's study do not need to abandon (or replace) the useful knowledge that more distance implies more time, but need to more reliably coordinate among multiple ways of attending to physical situations and multiple ways of inferring duration.

From a logical point of view, the rule that *more distance implies more time* is true only for situations in which the comparison being made involves the same speeds - the very information that was occluded from children's view in Piaget's study. However, it need not be the case that the intuition itself is activated in someone's mind encodes for (or activate) knowledge concerning when it is and when it isn't appropriate. Many times, the expectation that longer distances take more time is activated without our explicit attention to whether or not the speeds are the same. In fact there are many situations in which the intuition produces a reliable inference even when the speeds being compared are not the same (e.g., consider a vertically tossed ball thrown to different heights).

In the model of students thinking about motion used in this chapter, a knowledge element corresponding to the idea that going longer takes more time is included. Separate to this knowledge element are other knowledge elements that correspond to when such an idea is appropriate to use (e.g., similar speeds). It is assumed that these elements may or may not be cued together. In another section following, I describe various ways in which these two different knowledge elements may interact to produce different patterns of thinking.

More Speed Implies Less Time

Another intuitive element that students have for thinking about motion is the expectation that *going faster implies taking less time* (or that less time implies having gone faster). People expect that the fastest runner in a race is the one who finishes in the least amount of time. People know that if they leave for work later than usual, they can still get there on time by driving at a greater speed (assuming you don't get pulled over).

Generally, consistent use of this intuition develops later in children than the distance-time relation. It is suspected that the inverse relationship of this intuition makes it more difficult for children. In fact, a common mistake that children make is to make inferences consistent with the idea that speed and time are directly related. While Piaget (1971) and diSessa (1993) describe this improper inference as children tacitly chaining together inferences that more speed leads to more distance, which in turn leads to more time; one study, in particular, challenges this kind of explanation by demonstrating that children assume direct relationships between time and unrelated quantities (Levin, 1979). Levin showed that children are equally biased toward answering that brighter bulbs are on for more time than dimmer bulbs than they are biased into thinking that faster motions persist for longer time (e.g., thinking spinning objects spin for more time when they spin faster. The lack of difference between children's reasoning across these two situations suggested to Levin that children are not explicitly aware of what magnitudes they are attending to (and also should be attending to). Instead, their answers about the time correspond to a more general notion that "more means more" rather than to any explicit ideas about motion.

The intuition that *going faster takes less time* is also an intuition that is reflected in language surrounding rapidity and duration. While the words like "fast" and "quick" often are used to mean a greater rapidity, they are also used to refer to smaller durations (and also just to mean early in time). Consider the example, "You've gotten home quick [sic]. You must have been going fast." In this example it should be clear to most readers that the word "quick" is most likely being used to refer to duration (or temporal ordering) and the word "fast' is being used to refer to rapidity. Although it seems like the speaker is being redundant, they aren't necessarily. In other circumstances however it can become ambiguous (intentionally or not) as to whether such words are meant to indicate an increase in rapidity or a decrease in duration.

Like the intuition that *more distance implies more time*, the intuition that *more speed implies less time* is not necessarily, by itself, correct or incorrect. It functions much like a rule of thumb. Often times the expectation that going faster takes less time applies, and in other circumstances it doesn't. It certainly applies in circumstances where we are comparing travel along equal distances. Here again, there is a need to differentiate the logical implications of the knowledge (in the abstract) and how such knowledge is used by persons. When people form expectations that going faster will take less time they need not be attending to (either explicitly or tacitly) knowledge about the conditions in which that knowledge is appropriate.

In the toy cognitive model being developed here to describe student thinking about motion, we are going to assume that college students have knowledge corresponding to the intuition that *going faster takes less time.* A knowledge element corresponding to thinking that speed and time are directly related (more speed implies more time), is not included. However, it is not ruled out that students may explicitly chain together knowledge elements that are included in the model to produce such an answer. Similarly, a knowledge element corresponding to the conditions in which this intuition is appropriate (e.g., similar distances) is assumed to exist but not necessarily active.

More Speed implies More Distance

A third intuitive element that students have for thinking about motion is the idea that going faster implies covering more distance (or that *more distance implies more* speed). This intuition may underlie the expectation that if you throw a baseball with more speed, it goes farther than one thrown with less speed. This expectation develops in a similar pace to children's expectation that more distance implies more time, and it is generally assumed that this expectation is more easily grasped because of the direct relationships it represents. In Piaget's example of children reasoning about occluded motions with different speeds along different distances, many children recognize that the longer distance implies a faster moving object.

This particular intuition does not have a strong linguistic signature, as do the other two relationships, where words like longer and faster seem to have ambiguous meanings. Historically, researchers have interpreted children's language about motion in ways that suggest some ambiguous meanings among effort and speed. For example, Piaget (1971), throughout most of his work, interpreted children's use of the word "harder" to mean "faster". For instance, if a child said, "I ran hard", this was interpreted as meaning the same as, "I ran fast".

As with the other intuitions discussed, this knowledge is neither correct nor incorrect and its use does not require explicit attention to conditions in which it might be useful. This intuition is included in our model and is combined with other intuitions to form patterns of thinking described in the next section.

Combining and Coordinating Intuitions

Above, I have described three intuitions about physical mechanisms that relate to an individual's thinking about a motion – of course it is just a narrow slice of their intuitive thinking about motion. These three intuitions for thinking about the interrelations among distance, speed and time can be combined, however, in various other ways to make sense of physical situations involving motion: These intuitions can be strung together to make a chained argument. Competing intuitions, for example can be understood to balance out leaving no net effect. Or one intuition can "win out". These intuitions can also be used via feedback loops with knowledge about when to use those intuitions.

One simple way that these intuitions can be combined is to simply apply one intuition to make one inference and then apply another intuition to the first inference drawn. An example of this would be to first apply the intuition that going faster implies going farther and then apply the intuition that going farther implies taking more time. This chaining together of these two intuitions produces a new idea that did not exist before in our model– more speed implies more time. This kind of chaining together of two intuitions emphasizes the fact that these intuitions can be cued without additionally cueing knowledge about when this knowledge is appropriate to use. The first intuition that going faster implies going farther is usually appropriate only when times are the same. The second intuition that going farther implies taking more time is usually only appropriate when the speeds one is comparing are the same. Students who are not attending to ideas about when these intuitions are appropriate to apply may be more likely to chain together such intuitions (inappropriately) than those are specifically monitoring their thinking. In this model, I am using chaining in the sense of spreading activation, where one idea leads to the activation of another, explicitly referring to the activation of two separate knowledge elements. I am not proposing that in the model there is a new intuitive element that more speed implies more time. Rather I am proposing that in certain contexts, the real-time assembly of thinking from more stable intuitions may lead students to such a conclusions.

A second way that students' intuitions can be combined to arrive at a new conclusion is for students to reconcile competing intuitions via a compensation argument. Students may think that going farther *and* going faster implies taking the same time, because *going farther implies taking more time* and *going faster implies taking less time*. These two effects balance out or cancel. In such a case, a student would need to have reason to believe that both the speed and distance increase (or decrease) together. Such information may be provided more directly from the situation at hand, or a student could come to this conclusion via other intuitions (for example, by attending to the idea that *going faster implies covering more distance*). This pattern of reasoning is common in everyday reasoning about travel, in which people make consider which will take less time- slower but shorter back roads or faster but longer highways. We expect college students to not only have these individual intuitions, but to have forged some stabilities in constructing such compensation arguments as well.

Students also have other ways of "dealing" with multiple intuitions. A simple way of dealing with competing intuitions is to choose one over the other (either explicitly or implicitly). A student thinking about the effects of having more speed may cue up both intuitions that more speed implies going farther and more speed implies taking less time. When asked about the time, a student may well decide that the times are the same because the student has decided that the speed affects the distance (and subsequently not the time).

A third dynamic that can occur among these intuitions for thinking about motion concerns feedback between students' intuitions and the knowledge they possess governing when these intuitions are appropriate to use. In many of the examples, I have stressed how the use of these intuitions doesn't imply that the knowledge regarding its appropriate use has also been cued. Here, we consider a case, where students' knowledge about when to use and when not to use the knowledge affects their thinking.

The simplest scenario involves a student who cues up an intuition like *more distance implies more time.* When this knowledge is cued, it may activate the knowledge that this specific inference is only reliable only when comparing the same (or similar) speeds. Perhaps the student checks to make sure if indeed it is the case that the speeds are the same. The student may then either accept or reject the inference *more distance implies more time.* Another possibility is that the knowledge that this intuition is only reasonable when the speeds are the same, may lead the student to infer that the speeds are the same (even for cases when they would have other reasons to believe that the speeds are not the same). Both of these examples are cases in which there is feedback between students' intuitive knowledge and their knowledge about that knowledge. In some cases students' knowledge about the conditions in which knowledge is appropriate can help them to accept or reject the inferences drawn from that knowledge (or to go look for more information). In other cases, students may be led to impose conditions upon the situation that may not be true or may contradict other intuitions like are likely to have.

Above I have described three intuitions for thinking about kinematical relations among speed, distance, and time. Based on research in cognitive development (Piaget, 1978; Levin, 19790) demonstrating the stabilization of these ideas in children by adolescence (and even just our own sense of college students' ideas concerning motion), these intuitions will be assumed to be existing cognitive structures that our students can bring to reason about motion phenomena. In addition, we assume that students we are studying have knowledge for thinking about when these intuitions are going to be applicable or not, although like the intuitions, this knowledge may not be activated as well. Lastly, I have described a few ways that these intuitions and students knowledge about these intuitions can come together to generate different patterns of reasoning. In this next section, I illustrate patterns of student reasoning in the context of oscillator motion and draw connections between students' reasoning using the cognitive elements described above.

Applying the Model: Student Thinking about Oscillators

Before describing how the our toy cognitive model described above may be used to make predictions about the variability of student performance, I want to briefly describe how the model can account for student responses to a physical situation commonly encountered during instruction in mechanics. One fact that students typically encounter at some point during an introductory mechanics course is that, for ideal simple harmonic oscillator, the period of oscillation is independent of amplitude. For many students this is not immediately obvious (nor should it be). Based on just our simple model of student intuitions about motion we account for a range of student responses including for how the period of oscillation for a mass attached to a spring changes with amplitude.

Having the intuition *more distance implies more time*, students may think that it should take more time for a greater amplitude because the mass is covering a greater distance in each cycle (or portion of a cycle). In this case, students would most likely be attending to the distance covered by the object and not necessarily how the speed or the forces exerted on the mass change as the result of having a greater amplitude. Another possibility is that students will think that the greater amplitude should take less time. Students attending to the fact that the forces exerted by the spring on the mass are greater (either on average or at maximum) may think that this would cause the mass to react more and therefore travel with a greater speed and do so in less time. Such a student would be relying on the intuition *more speed implies less time*, but also intuitions for thinking about the effects of springs. diSessa (1993) describes p-prims like *springiness* to describe people's intuitive sense of mechanism like springs – intuitions like the more you stretch or compress a string the hard it resists. A student coming to the conclusion that a greater amplitude implies less time may not be closely attending to the fact that the distance covered by the mass is also increasing. They may be solely focused on how the mass reacts more when stretched more. From these two cases, we can see how students might bring different intuitions to think about the same situation to arrive at different conclusions due partially to what aspects of the situation they are attending.

These intuitions can also be put to good use in making sense of the correct answer as well. While students may not be able to derive from first principles that the period for simple harmonic oscillation is amplitude independent, students do have the intuitions necessary to come to the conclusion that this is a reasonable possibility. One way of achieving this reconciliation is through the compensation argument between the effects of distance and of speed. Students can make sense that the time is the same because these two effects (of increased distance and increased speed) balance or cancel out.

We have described elsewhere (Frank, Kanim, & Gomez, 2008) student responses to a similar problem asking students (enrolled in a algebra-based introductory physics course at Arizona State University) to rank the amount of time it would take a mass attached to a spring to return to equilibrium after being pulled away from equilibrium to three different points. The question was administered in a written survey during the laboratory section of the course, one week after completing a lab in which students explicitly measured the mass dependence, spring-constant dependence, and amplitude dependence of simple harmonic oscillators. Consistent with our model, student responses split among the different possibilities and many student explanations were consistent with the specific intuitions included in our model.

Here is an example of a student explaining why the mass pulled to the farthest distance takes the most time: “I just know the farther you are away, the longer it takes to get there.” This quote is representative of the intuition *more distance implies more time* in several ways. The explanation is stated in a manner that suggests that student believes it to be rather obvious. He chooses to write “I just know” to preface his statement. Additionally the use of “you” also hints that this idea is everyday idea, applicable in a variety of situations.

An example of a student explaining why the greatest distance takes the least time is, “The one at the greatest distance has the greatest velocity, so it will take the least time.” In this quote one can see the student is relying on the intuition that *more speed implies less time.* The student also states that the one at a greater distance has a greater velocity, although it is difficult for us to know exactly why. Other students statements suggest that many students arrive at this conclusion based on their primitive ideas about springs. Here a student writes, “Farther the distance, the greater the force, and the faster it snaps backs”.

And as to be expected, many students give explanations indicative of compensation arguments. One student explains why the times are the same by stating that, “As the distance increases, so does the velocity and acceleration, but so does the distance.” The fact that the student says “but so does the distance” supports the interpretation that the student was thinking about the competing effects of increased speed and increased distance. Of course, not all students gave intuitive explanations in arriving at correct answers. Some students merely stated that period is independent of amplitude. Surprisingly, very few students explicitly mentioned the experimental result from the week before (where they found that the period didn’t change with amplitude) in their explanation for why the times would be the same.

This brief analysis of student responses to questions about simple harmonic oscillators and masses attached to springs provides several insights into the application of the model.

First, it highlights how student responses to this question are possibly being constructed on the fly from finer-grained intuitions and not from pre-assembled conceptual frameworks. It seems rather obvious that, for the topic of simple harmonic oscillation, students would not necessarily have a stable or deeply rooted conceptions about oscillators. The variability of student thinking in the population is perhaps to be expected. Nonetheless, this aspect of student thinking is reflected in the data and in the analysis. It is an experimental question whether similar sensitivity in student responses will be observed in physical contexts where students have more experience.

Second, students use their intuitions partially based on what aspects of the physical situation they are attending. This conclusion is consistent with other research reflecting how students use the word ‘fast’ differently to describe oscillation in different ways depending on what is salient and also depending on what aspects of the situation they are attending to (Parnafes, 2008). This attention-sensitive aspect of how student knowledge is cued is the mechanism we exploit in the experiment (described in the following section) to tip students into relying on different aspects of their intuition in thinking about a different physical situation involving gravity.

Third, student use of their kinematical intuitions may be combined with other knowledge that students have for thinking about the physical situation. In this example, students relied on intuitions they have for thinking about springs – that stretching a spring further causes it to snap back harder (or faster). In the experiment described in the next chapter, students rely on other knowledge and intuitions they have for thinking about effects of gravity.

Finally, students may rely on different knowledge for reasons having to do with a construct known as *epistemological framing* – students’ own sense of what kind of ‘knowledge-activity’ is taking place (Hammer, Elby Scherr, Redish, 2004; Redish, 2004; Hammer and Scherr, 2009). Student who write statements like, “I just know the farther you are away, the longer it takes to get there” seem to be relying on intuitions and experiences to make sense of the new physical situation. The student who wrote, “period is independent of amplitude” is quoting a rule. These two different explanations may not only reflect different content knowledge, but a different sense of the activity involved. These different responses may reflect different senses for what kind of ideas and explanations are relevant for the task at hand – relying on experiences in one case and quoting a rule in the other. We will see in the experiment described in the next chapter, not only differences in the intuitions students rely on, but differences in the ways students explain their ideas and the kinds of knowledge they draw upon.

Theoretical Applications of the Model

In this last section, I describe how this simple model of student thinking can be used as a generative starting place for conducting research on the dynamics of student thinking about motion phenomena.

First, the model begins with an assumption of multiplicity in student thinking. This means that individual students can attend to, respond to, and make sense of physical situations (and physics questions) in a variety of different ways, both across contexts and even within a given context. In particular, the model specifies a very small set of intuitions that, when activated, generate patterns of student thinking about the interrelations among speed, distance, and time. This set of intuitions establishes particular kinds of reasoning patterns that may be expected in a broad class of motion problems – any time students are reasoning about speed, distance, and time. Generally speaking, the model should account for at least some of student reasoning patterns that arise in response to situations involving comparative inferences of duration, distance, or speed. We'd expect the model to be particularly applicable for problems that involve explicit variation of one parameter (either duration, distance, or speed) and ask for how that variation affects the non-varied parameters. Just as we encountered other intuitive ideas arising in students' thinking about the simple harmonic oscillator, we should expect to find other intuitions in different situations as well.

Importantly, we are interested in more than just identifying patterns of reasoning that arise when students are asked to respond to different questions. We'd like to be able to use the model to help make sense of the *dynamics* by which different patterns of reasoning are actualized, and not just describe the patterns we observe.

I have argued that these three intuitions are used by students in thinking about motion in a variety of situations. As diSessa (1993) notes in his description of many motion p-prims (used by children), we should expect that these intuitions are quite reliably used by college students in arriving at correct interpretations of motion phenomena for lots of situations involving motion. For example, if you ask students whether it takes more time or less time to travel a farther distance than a shorter distance (at same speed), they are likely going to answer correctly. Ask college students whether it takes more time or less time to travel a certain distance going faster or slow, they are going to answer correctly. In such situations, students reliably arrive a stable interpretation of the situation. In order to observe any dynamics of student reasoning with these intuitions, it will be necessary to setup situations in which students' thinking is likely to exhibit variability.

One way to tap into variable aspects of student thinking is through the context-dependent nature of the activation of knowledge. Thus, we might expect to be able to construct situations that reliably bias students toward and away from certain intuitions by changing particular aspects of the context that we present to students.

For example, I previously described several ways that students respond to a question about the amount of time taken for a mass on a spring to return to equilibrium. Some students think that the spring that was stretched the most would take the most time because it was the farthest away. Other students think it would take less time because the spring (now stretched more) would bring the mass back with greater speed. Yet, other students reason to the correct answers- that the times could be the same- because the increase in distance and speed compensate for each other. The question, however, explicitly asks the question in the context of emphasizing the fact that the mass was pulled to different distances. Drawing attention to the different *distances*, we might expect this feature of the problem to bias students toward the intuition *more distance implies more time*. The question is, "Is it possible to construct different versions of the question that bias students toward thinking that stretching the spring more would lead to the mass springing back faster (and in less time)?"

Exploratory Investigation

As a way to explore the possibility of biasing students toward and away from different patterns of reasoning, a modified version of the mass on the spring problem was written. Instead of emphasizing the fact that the mass is pulled to different distances, our modified version of the question drew attention to the fact the mass was pulled so that it was being tugged on by the spring to varying amounts. We expected that more students would now answer that the spring takes less time to get back to equilibrium because it would increase the probability of activating intuitions like *springiness* and *more speed implies less time*. In other words, by drawing students' attention to the impact of the spring (rather than the location of the mass), we expected it to preferentially lead to the activation of different intuitions.

This exploratory investigation was not intended to be rigorously implemented, but rather to identify a proof of concept that we might be able to biased toward and away from different intuitive ideas for thinking about the same physical situation. We had administered the original version of the question several semesters prior to the administration of the modified version. The courses were taught by different professors. Some of the lab curriculum had been modified during this time. There were few controls. However, we still believed that a positive signal would suggest that we might be able to bias students toward using different intuitions for thinking about the mass on the spring problem, and thus with other problems as well.

The original question, emphasizing the distance, was administered in total to 114 students. In that version, 30% of the students had answered that farthest pull would take the least time. This is the answer that would be consistent with thinking that the mass moving faster would imply getting back in less time. The modified question, emphasizing the tug of the spring back on the mass, was administered to 63 students. Therefore, by drawing attention to this aspect of the problem, we expected that more than 30% of the students would respond that it takes less time when pulled further out so that spring tugged back harder. The results for the modified question were that 45% of the students answered that the strongest pull would take the least time. These differences are on the border of being statistically significant, $p = 0.51$, using a chi-squared test of independence. While this result is not particularly compelling, because of the largely uncontrolled conditions and the relatively small numbers, it was enough of a signal to motivate the design of a more carefully controlled experiment to 'tip' students into thinking differently about the same situation.

Designing a More Rigorous Study

Several factors need to be considered in the design of an experiment to measure variability in students' thinking about motion.

First, as I explained, the physical situations we ask students to reason about need to be ones that students' thinking might plausible vary. Knowledge-in-pieces frameworks don't rule out the possibility of contexts in which students might exhibit stability in their reasoning. If we ask questions for which students' thinking is too stable, we won't be able to detect any variability.

Second, the manner in which we vary the presentations of the problems need be significant enough to induce change in the ways students think about the physical situation, but still minor enough to be compelling that we are observing students' thinking vary around some common physical situation. In contrast, researchers such as Cooke and Breedin (1994) have already documented variability in the ways that students think about wholly different situations such as rocket ships and pendulums. Our goal is to demonstrate that students can be biased toward different patterns of thinking about the same question for a single physical situation when presented in slightly different ways. The variations that we introduce in the presentation of the problems need not only be viewed as the same from an expert perspective, but the problems need to be plausibly viewed as the same from the students as well.

Third, we would like the problems we choose to demonstrate variability in students' thinking about motion that are relevant to historical and contemporary research on student thinking about motion. For this reasons, we might choose problems that are similar to problems other researchers have asked, so that accounts of the data from multiple perspective can be considered. In doing so, not only might we able to use our model to predict patterns of variability, but we can also compare outcomes to predictions made by other accounts.

In terms of the administration of the experiment, it makes sense to randomly assign students to receive one of two versions in a single classroom at the same time. This way we could have some reasonable assurances that the conditions of the experiment and the populations in both conditions are similar in all cases.

The problems we constructed concern the motions of objects in gravity. In one problem, a ball is rolled off of a table horizontally and allowed to hit the floor. In the second problem, a ball is thrown vertically upward. In both problems, students are asked to reason about the duration of motion. In the horizontal projectile problems, students are asked to reason about the amount of time to hit the ground once the ball leaves the edge of the table. In one version, students are asked to compare times for when the ball is either rolled off with different speeds. In the other version, students are asked to compare times for when the ball lands at different locations on the floor. In the vertical toss question, students are asked to reason about amount of time taken for the ball to reach its maximum height. In one version it is explained that the ball is thrown with two different speeds, and in the other version, it is explained that ball reaches two different heights. These questions are described in more detail in the next chapter.

Based on our toy cognitive model, we expected that we could use these problems to bias students toward and away from different intuitions in our model. Given that these problems all ask students to reason about the amount of time, we expected that would be able to bias them toward and away from the intuitions that *more speed implies less time* and *more distance implies more time*. The problems that emphasize the initial speed are expected to lead to more students answering consistent with the intuition that *more speed implies less time*, and the problems that emphasize distances traveled are expected to lead to more students answering consistent with the intuition that *more distance implies more time*.

In the following chapter, I describe in more detail the design, implementation, predictions, and results of the experiment that was conducted.

Chapter 4: Experimental Measures of Variability

Experimental Design and Model-based Predictions

The model described in the previous chapter highlights how student responses to questions about motion can be understood to arise from the real-time cueing and coordination of various fine-grained intuitions for thinking about kinematical relations. One kind of evidence to support the hypothesis that student responses are being assembled "on-the-fly" is the degree of observable variation in student responses. In this section, I describe an experiment used to carefully examine such variability in student thinking about the motion of objects under gravity.

The experiment described in this section is designed to put to the test our simple model by using the model to pin down specific predictions concerning the variability of student thinking.

One kind of variability researchers can observe is within-context variability of individual students. Given a particular situation, we may find that students flexibly go back and forth among multiple stabilities for thinking about the same situation (e.g., Parnafes, 2007). In the simple harmonic oscillation example described in the last chapter, a student may not be able to make up their mind (consciously or not) as whether or not it takes the same time or more time, because they have a variety of different ways of thinking about the physical situation.

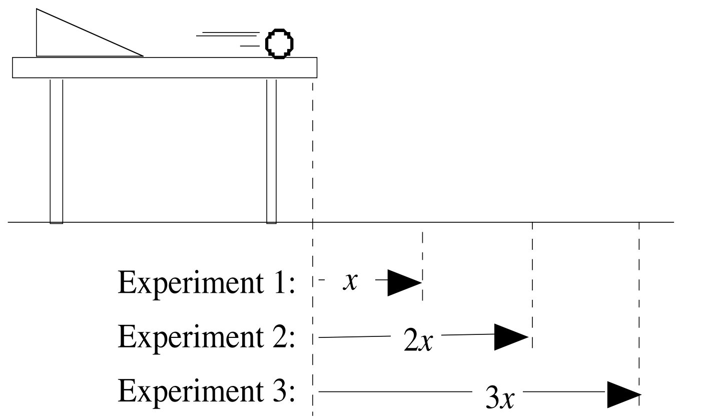
Another kind of variability is across-context variability. By asking students to reason about slightly different situations, researchers can examine how students shift among various stabilities (e.g, Cooke and Breedin, 1994). For the case of simple harmonic oscillators, we may find that students use different intuitions for reasoning about durations depending whether its masses attached to springs or pendulum bobs. Parnafes (2007) analysis suggests that the actual rate of oscillation influences whether students between thinking of *fast* as corresponding to "covering more distance" or to "having more oscillations". Our exploratory investigation also hinted at the possibility of being able to bias students thinking with small changes to the context.

The experiment described below targets several different kind of variability: The primary target is measuring how student responses to motion questions change when the questions about the same physical situation are modified in specific ways that are suggested by the theoretical (toy) model. Second, observations are made for how single students ideas vary across different physical situations, and lastly, for how single students ideas possibly vary within thinking about a single physical situation. In the next section I describe the specific experiment and the questions posed to students.

Experimental Design and Description of Surveys

The experiment that was conducted involves the administration of two different versions of a conceptual survey that both consists of two physics questions. Each of the questions ask students to compare the durations of motion for objects moving under the influence of gravity. The two versions are called the distance-cueing and the speed-cueing surveys. The first question on both surveys, shown in Figure 1 and Figure 2, is referred to as the projectile motion task. The second question on both surveys, shown in Figure 3 and Figure 4, is referred to as the vertical toss task. The distance-cueing survey consists of the two questions shown in Figure 2 and Figure 3.

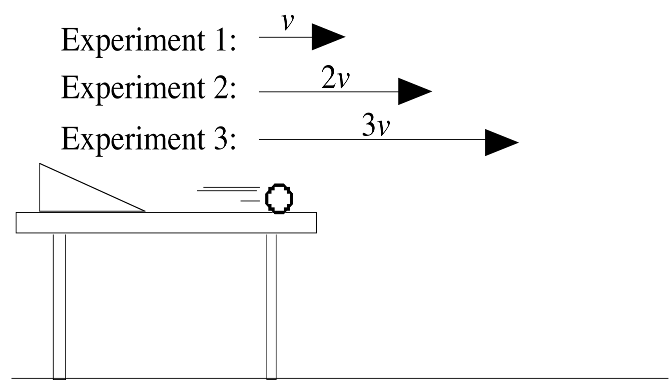
The speed-cueing survey consists of the questions that are shown in Figure 2 and Figure 4.



In 3 separate experiments, a student launches a ball horizontally from a table. The horizontal distance that the ball travels before hitting the ground is x in the first experiment, $2x$ in the second experiment, and $3x$ in the third experiment.

Rank, from greatest to least, the *time* that it takes the ball to travel from the table to the floor for the three experiments. Explain how you determined your ranking.

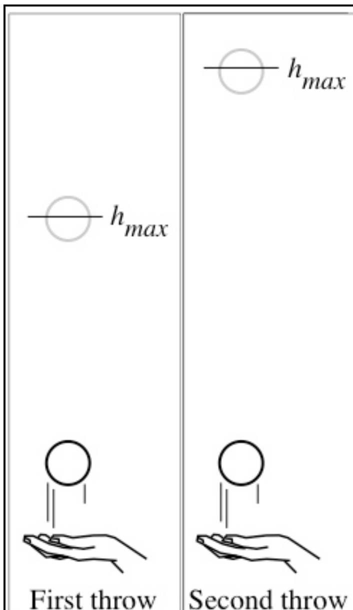
Figure 1: Horizontal Launch Question in Distance-cueing Survey



In 3 separate experiments, a student launches a ball horizontally from a table. The ball leaves the table with a speed v in the first experiment, with a speed $2v$ in the second experiment; and with a speed $3v$ in the third experiment.

Rank, from greatest to least, the *time* that it takes the ball to travel from the table to the floor for the three experiments. Explain how you determined your ranking.

Figure 2: Horizontal Launch Question in Speed-cueing Survey



A student throws a ball straight up in the air and times how long it takes the ball to reach its maximum height. Afterwards, the student throws the same ball such that it goes higher up than before.

Compared to the first throw, will the amount of time taken for the ball to reach its maximum height be *greater, less,* or *the same?*

Figure 3: Vertical Toss Question in Distance-cueing Survey

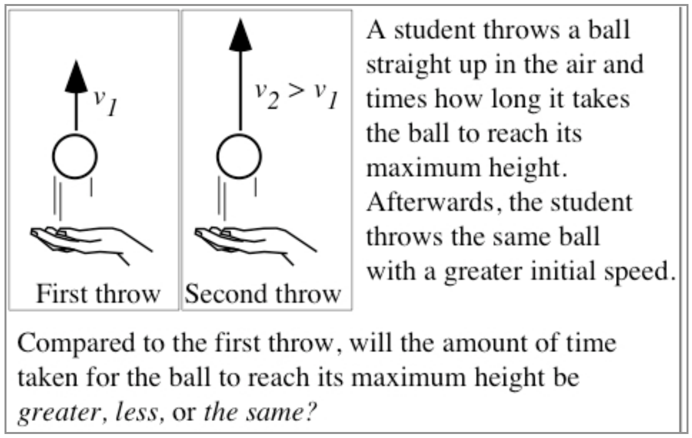


Figure 4: Vertical Toss Question in Speed-cueing Survey

The Horizontal Launch Question

The first question on both surveys asks students to rank the amount of time taken for a ball to reach the ground after rolling off a table. In *the distance-cueing* versions, students are told that the object lands at three different locations (at a distances *x, 2x*, and *3x* from the table's edge). The emphasis on the different distances is represented both in the language used to describe the problem and in the accompanying illustration. In the *speed-cueing* versions, students are told that the object leaves the table with three initial different speeds (*v, 2v,* and *3v*). This emphasis on the different initial speeds is also represented in both the language and illustration.

The correct answer to both versions of the question is that the amounts of time to reach the ground are all the same. In each case, the ball still has to fall the same vertical distance to reach the ground independent of how fast it leaves the table. Because all of the balls begin with a zero-vertical-component of velocity (and the acceleration downward remains the same in each case), the ball takes the same amount of time to fall the same vertical distance. The faster balls simply land farther away because they cover more horizontal distance in the same amount of time (having started with more horizontal speed). The two questions, according to Newton's Laws, are equivalent. Ranking the times based on the distance gives the same answers as ranking the times based on the speeds. As detailed in a later section, these two questions are both qualitatively the same from the perspective of an impetus theory of motion and qualitatively the same from the perspective of students who are *explicitly* asked about it.

The Vertical Toss Question

The second question on both surveys asks students to rank the amount of time for a different physical situation. The question asks students to compare the time for a ball thrown vertically upward to reach its highest point. In the speed-cueing version, students are asked to compare the times for balls that are thrown with different initial speeds. In the distance-cueing versions, students are asked to compare the times for balls that are thrown to different heights. These variations of the questions are, of course, equivalent from a Newtonian perspective, since the faster thrown ball does higher. As with the horizontal launch question, the cues are represented in both the language and representations.

The correct answer is that the ball that is thrown with a greater initial speed (and subsequently goes higher) takes more time. One way of reasoning to this correct answer is to recognize that the acceleration due to gravity is constant. A constant acceleration means that the ball changes its velocity by the same amount in any given period of time. Thus, the ball with the greater amount of speed takes more time to reach a speed of zero (which is at its highest point). It also seems likely that people would simply know the answer to this question from the direct experience.

Model-based Predictions

For both the vertical toss and horizontal launch questions, the model described above arguably supports specific predictions concerning differences in how students should respond to the distance-cueing and speed-cueing versions (see Table 1 below). The distance-cueing versions of the questions plausibly direct students' attention more strongly to the different distances and subsequently create a higher cueing priority for the intuitive knowledge that *more distance implies more time*. Similarly, the speed-cueing versions plausibly direct students' attention more strongly to the differences in initial speeds and subsequently create a higher cueing priority for the intuitive knowledge that *more speed implies less time.*

Based on the simple cognitive model, one prediction is that more students will answer that Experiment 3 takes the longest time on the distance-cueing survey than on the speed-cueing survey. This follows from the fact that, in our model, the likelihood that the intuitive knowledge element *more distance implies more time* activates is increased as students attend to distance features of the physical situation. The distance-cueing version of the question makes the distance the ball travels before hitting the ground salient in two ways – through the written words that describe the problem and the visual depiction of the situation.

Second the model predicts that more students will answer that Experiment 3 takes the least amount of time on the speed-cueing survey than on the distance-cueing survey. The problem makes the speed of the ball of the edge of the table more salient, making it more likely that students will stably attend this aspect of the situation. Ultimately this make it more likely that the intuitive knowledge element ‘more speed implies less time’ will be activated.

Importantly, the model *does not* necessarily predict that more students on the distance-cueing survey should answer that Experiment 3 takes the most time than students answering that Experiment 1 takes the most time. The patterns of attention that the surveys are attempting to establish (attention to the objects’ speed or distance) have the effect of biasing students’ toward and away from the activation of the particular fine-grained intuition. Based on this mechanism, students should more likely to think that *more distance means more time* on the distance-cueing version than when compared to the students responding to the speed-cueing versions; but this doesn’t imply that *more distance means more time* will represent the majority pattern of thought in a given context. One way of understanding this is that the model is not assuming any particular ‘resting’ weights that represent the probability of activation for knowledge elements. The different cues are assumed to change probability of activation, but we don’t know what the probabilities were to begin with. The predictions concern how the distributions of responses changes across the cues, not which answer will be the dominant in a particular version of the question.

The specific predictions for the vertical toss question are that (1) more students will answer that the second toss takes the most time in the distance-cueing version than in the speed-cueing version and that (2) more students will answer that the second toss takes the least time in the speed-cueing version than in the distance cueing versions. Once again, the model makes no specific predictions concerning the relative frequency of answers for a given question. The model predicts how the distributions of responses changes as the cues change.

Table 1: Table of Model-based Predictions for Experiment

	Speed cues	**Distance cues**
Model-based Assumption	Increases the cueing priority of *more speed implies less time.*	Increases the cueing priority of *more distance implies more time*
Empirical Consequence	Different Distribution of Responses Across Different Cues	
Horizontal Projectile Task	Greater frequency of answers T3 < T2 < T1 (compared to frequency with distance cues)	Greater frequency of answers T3 < T2 < T1 (compared to frequency with speed cues)
Vertical Toss Task	Greater frequency of answers that T2 < T1 (compared to frequency with distance cues)	Greater frequency of answers that T2 > T1 (compared to frequency with speed cues)

Secondary to these specific empirical predictions, there are a variety of other response patterns that can be examined in order evaluate the plausibility of the model both in terms of the pieces in the model and the prevalence of particular dynamics. For all questions, students are prompted to give an explanation for their answer. These explanations are coded (as described below) as to the types of explanations they are likely to represent. These explanations may provide further evidence as to the prevalence of other dynamics discussed above, including students stringing together intuitions, students forming compensation arguments, other dynamic feedback among knowledge elements.

Context for Research

The experiment we designed was conducted at the beginning of the second-semester, algebra-based, introductory physics courses at the University of Maryland during the spring semester of the 2006-2007 academic year. The experiment was conducted in two separate lectures taught by two different professors. The students enrolled in this second semester class must have either passed the first-semester physics course or must have placed out of the first-semester course through advanced placement.

The first-semester course covered topics in mechanics. That semester was taught by two different professors (only one of the professors was the same across semesters). Both of those professors covered the relevant material on one- and two-dimensional kinematics during the first semester course that would be needed to correctly answer the survey questions. Based on discussion with these professors and existing syllabi, students had opportunities in lecture, recitation, and homework to learn about kinematical concepts of displacement, velocity, and acceleration, as well as the mathematical concepts of vectors (including vector addition and subtraction). In particular, both of these professors discussed a specific lecture demonstration called the shooter-dropper demonstration. In this demonstration two identical steel balls are released at the same time and allowed to fall the ground (through the same height). One of the balls is released with zero initial velocity, while the other ball is projected horizontally. Students observe (by watching and listening) that the two balls hit the ground at the same time. Both professors spent time in the lecture working out why the balls do, in fact, hit the ball at the same time. This question may be particularly relevant to student responses because of its similarity to the horizontal launch question in the survey (in which balls are launched horizontally with different speeds instead of being dropped and launched).

A majority of students enrolled in the algebra-based physics sequence are life science majors. Because of the structure of their academic programs and because physics is a topic that is covered in the Medical College Admissions Test (MCAT), many students choose to take the physics sequence in their junior or senior year.

The experiment was conducted on either the first or second day of lecture, as attempts were made to accommodate the needs of the professors teaching the course. In the professor's class where the experiment was conducted on the first day of class, the professor introduced himself, discussed the syllabus of the course, and then allocated 15 minutes for the experiment. In the second professor's class, the experiment was conducted during the first 15 minutes of lecture on the second day of class. These two experiments were conducted two days apart in different sections no student could be enrolled in both). However, it remains possible that students who took the experiment first may have talked about the questions to students who would take part in the experiment later. Additionally, it remains possible that a small number of students took the experiment twice if students attended both lectures. Students were explicitly asked not to take part in the experiment twice. Since students were not asked to identify themselves on the survey, we have no way of independently verifying that no students took the survey twice.

In each of the professor's class, every student took a single survey with the two different questions that involved the same bias—either two questions with distance cues or two questions with speed cues. The choice to not mix speed cues and distance cues in a single survey (or to randomly assign different perturbations and orderings) was made based on the assumption that interaction effects among the questions and cues were inevitable (e.g., student thinking about one question impacts their thinking on other). Using the same cues for both questions on the survey certainly doesn't eliminate any interaction effect. However, it does plausibly make for a more consistent biasing effect. Students are biased toward the same intuition for both questions. While it may have been possible to run all possible variations, the ability to detect differences in the distributions would be weakened by having fewer students in each condition. Since the primary goal of this experiment was to examine the biasing effects of the cues (and not the interaction effects across questions), it made the most sense to use the same cues for both questions and maintain greater power in detecting an effect.

The method of administering the surveys was simply to pass out the two versions of the surveys in an alternating fashion to all students who elected to participate. This was done in a large lecture hall, and students were instructed to work on the survey independently. However, in such a large lecture hall it is impossible to guarantee that students did not look at each other's answers. All that can be said is that it appeared as if overt discussions were at a reasonable minimum during the implementation of the survey. Additionally, there was no grade-incentive for students to cheat. As per the Internal Review Board application, students were not required to take the survey. The surveys were not graded or shown to the instructors. Students were not asked to identify themselves in anyway.

The majority of students appeared to have finished the surveys well in advance of the fifteen-minute time frame. The surveys were collected at the end of the fifteen-minute period by having the students pass the surveys toward the ends of the aisles. No effort was made to keep the surveys in any particular order corresponding to time of completion, seating arrangement, or any other factor.

Primary Analysis: Student Responses Across Cues

In this section I describe the results and analysis of the surveys. The primary analysis consisted of coding student answers and statistically comparing distributions in order to test the hypotheses described above concerning shifts in distributions. A secondary analysis involved coding (and recoding) of student explanations in order to make a qualitative comparison of these written artifacts with particular aspects of the model.

Categorization of Student Answers

For the horizontal launch question, student responses were categorized along the following categories: *Exp1>Exp2>Exp3, Exp1<Exp2<Exp3*, *Exp1=Exp2=Exp3*, *No Answer, Other Answer.*

For the majority of cases, students' written responses were easily categorized. In a small number of cases, students explicitly wrote down one answer in the form of an inequality (e.g., "T3 < T2 < T1") and then also wrote down a contradictory answer in English (e.g, "experiment three takes the most time). In such cases, the verbal statement was given precedent over the mathematical inequality. The *no answer* category was given both to students who left that section of the paper blank or who explicitly gave an answer like "I don't know". The *other answer* category was given to a small number of students who gave any other ranking than the ones explicitly shown above (e.g., Exp1> Exp3 >Exp2).

Similarly, in the vertical toss questions student responses were categorized as to the answer they most likely represented. The categories were as follows: *Toss 1 > Toss 2, Toss 1 < Toss 2, Toss 1 = Toss 2, No Answer, and Other Answer*. For this question, the category other answer was given to answers such as "it depends" (a category of student responses discussed in a later section).

Following the categorization of data, students who gave "other rankings" were removed from quantitative analysis, as were students who gave no answer. The fraction of students giving "other rankings" was 3% of the total number of responses on the speed-cueing task and 1% on the distance-cueing task. The removal of these responses was done for two reasons. First, it was done to simplify the analysis to allow the use of chi-squared tests of independence (which are not valid for such small frequencies). Second, this removal of data was done because this subset of the data bears no direct relevance to any of the predictions concerning distributions of answers.

After student answers were categorized for each the problem separately, a simple count of the number of responses in each category was done. This was done for each of the four different problems separately. The count was also done separately for the two different courses in which the experiment was conducted. This count resulted in a distribution of answers for each problem for each course. Next, for each problem, a chi-squared test of independence was run for the pair of distributions from the two different classes. In each of the four cases, the distributions of responses were not statistically different. Based on this fact, the distributions across the two courses were collapsed together for each of the four problems. Therefore, the following discussion of distributions of students answers concern this collapsed data set.

Across-Cue Analysis of Student Answers

In order to test the predictions given by the model, a chi-squared test of independence was run on the distributions of student answers. This test was used to determine if there were any significant differences between the distributions of responses across different cues. Our predictions were based on the claims that the distance cues should increase the cueing priority of the intuition that more distance implies more time and speed cues should increase the cueing priority of the intuition that more speed implies less time. Our specific predictions for this experiment concern differences in the distributions of responses across the two surveys.

On the horizontal launch task, our toy cognitive model predicts more students should answer that Experiment 3 takes the most time on the distance-cueing survey than on the speed-cueing survey. In addition, more students should answer that Experiment 3 takes the least time on the speed-cueing survey than on the distance-cueing survey. For the vertical toss task, this means that more students should answer that the second toss takes the most time (which is also the correct answers). In addition, more students should answer that the second toss takes less time on the speed-cueing version that on the distance-cueing version.

In addition to calculating for statistical significance, confidence intervals of 95% were constructed for the proportions of students answering in relevant categories.

Answers to Horizontal Launch Task

For the horizontal launch task, the number of students answering correctly overall was 62% for the speed-cueing version and 60% for distance-cueing version. These proportions are not statistically different. This suggests that the different cues do not have an observable effect on student performance in terms of correctness. Both tests were equally as difficult. However, the different cues do appear to have an effect on the distribution of answers that are incorrect. Percentages of incorrect answers (indicated with 95% confidence intervals) are shown in Figure 5.

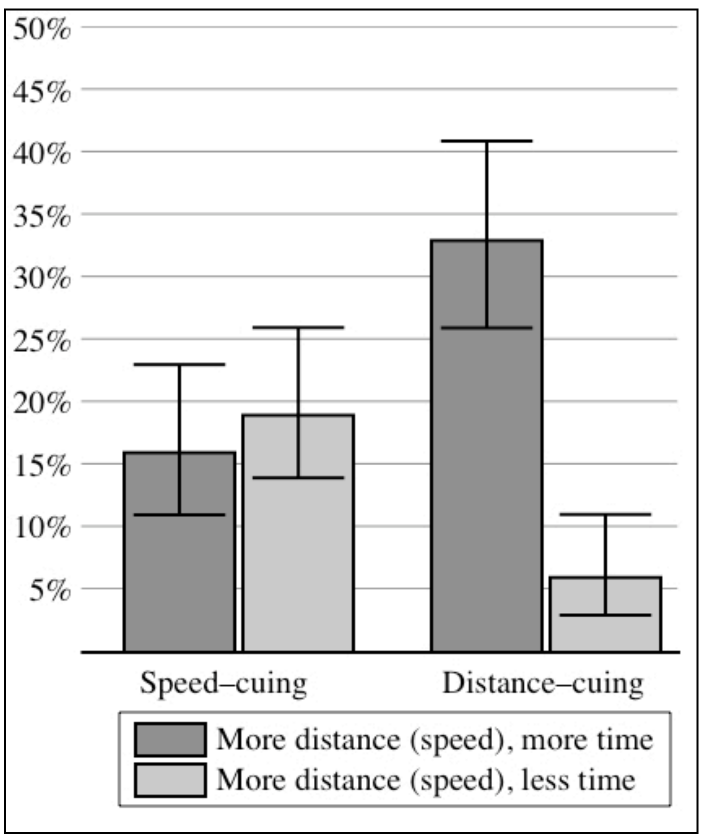


Figure 5: Distributions of Incorrect Answers to the Horizontal Launch (N= 318)

A chi-squared test of independence indicates a significant difference in the distribution of students' responses between the two different cues, $\chi^2(2, N = 318) =$

21.1, $p < 0.001$. The particular pattern of responses support both of the hypotheses concerning how the distribution of student answers should compare across the two versions. There are more student answers consistent with the intuition *more distance implies more time* on the distance-cueing versions (33%) than on the speed-cueing version (17%). Similarly, there are more students giving answers consistent with the intuition *more speed implies less time* on the speed-cueing versions (19%) than on the distance-cueing version (6%). These results are consistent with the predictions of the model. The cues presented in the problems appear to have biased students toward and away from various intuitions they can bring to thinking about the problem.

Answers to Vertical Toss Task

As with the horizontal launch task, the percentages of students answering correctly on the vertical toss task seemed to not be affected by the different cues presented in the problem. For the question, 49% of the students gave correct answers to the speed-cueing version and 48% gave correct to the distance-cueing version, which are not significantly different. Once again, the difference in the distributions was evident in the distributions of incorrect answers, shown below in Figure 6.

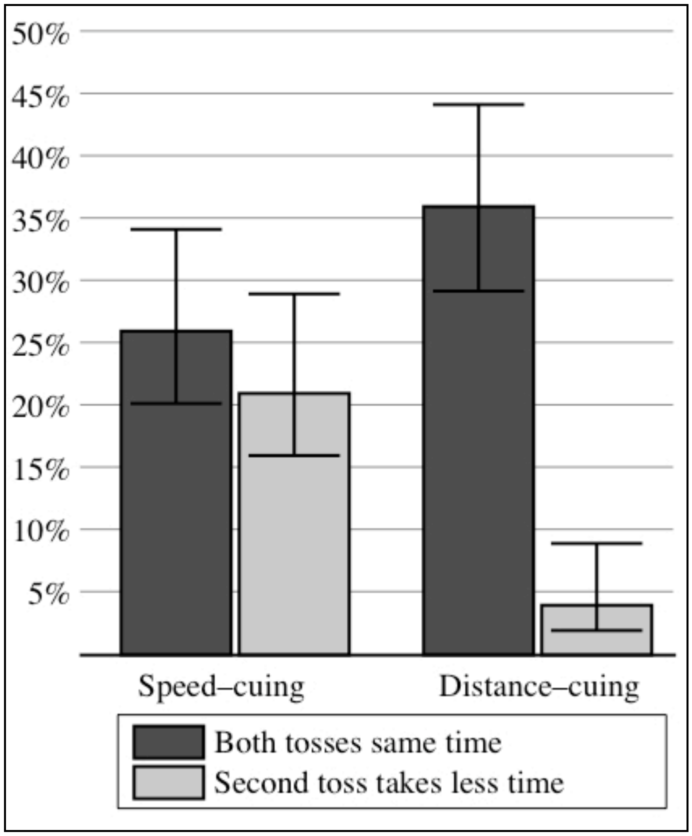


Figure 6: Distribution of Wrong Answers on the Vertical Toss Question (N= 309)

A chi-squared test of independence indicates a significant difference in the distribution of student responses between the two different cues, $\chi^2(1, N = 309) = 19.6$, p <0.001. The distribution of student responses supports one of the two original hypotheses: 21% of students taking the speed-cueing survey answered that the second toss would take less time (the answer consistent with cueing the intuition that *more speed implies more time*) compared to only 4% on the distance-cueing survey. However, the distance-cueing survey did not bias students toward the correct answer; the same percentages of students gave the correct answer independent of the cues that were presented. We expected that the distance-cueing survey would increased the number of correct answers since the increased cueing of *more distance implies more time* should lead more students to the correct answers. It can also be noted that the distance-cueing version led to 36% of students answering that both tosses take the same amount of time to reach the top, while the speed-cueing versions led to 26% of the students giving this answer.

Summary of Primary Analysis

The most basic result demonstrated by this analysis of student answers is that the speed-cueing and distance-cueing surveys generated different distributions of answers. Independent of the details of these differences, this suggests that the student population as a whole responded differently to these questions depending on their presentation. This most basic finding is consistent with our general theoretical position that individual students have a variety of intuitions for making sense of physical phenomena that depend on context. The different presentations of these questions can be understood to have biased students toward and away from different intuitions they may have brought to reason about these physical situations.

More specifically, our particular toy cognitive model was used as the basis for making predictions concerning *how* the distribution of should be different. From the toy cognitive model, we made four hypotheses concerning the relative frequency of answers. For both the horizontal launch and vertical toss problems, speed-cueing versions produced more students giving answer consistent with the intuition that *more speed implies less time* than distance-cueing versions. For the horizontal launch question, but not the vertical toss question, the distance-cueing version produced more students giving answers consistent with the intuition that more distance implies more time than speed-cueing versions. These results are illustrated in Figure 7 below, which shows the difference between the frequency of particular answers on the speed-cuing and distance-cuing versions. These differences are indicated with 95% confidence intervals. The directions of various predictions (based on the toy model) are also indicated with an arrow.

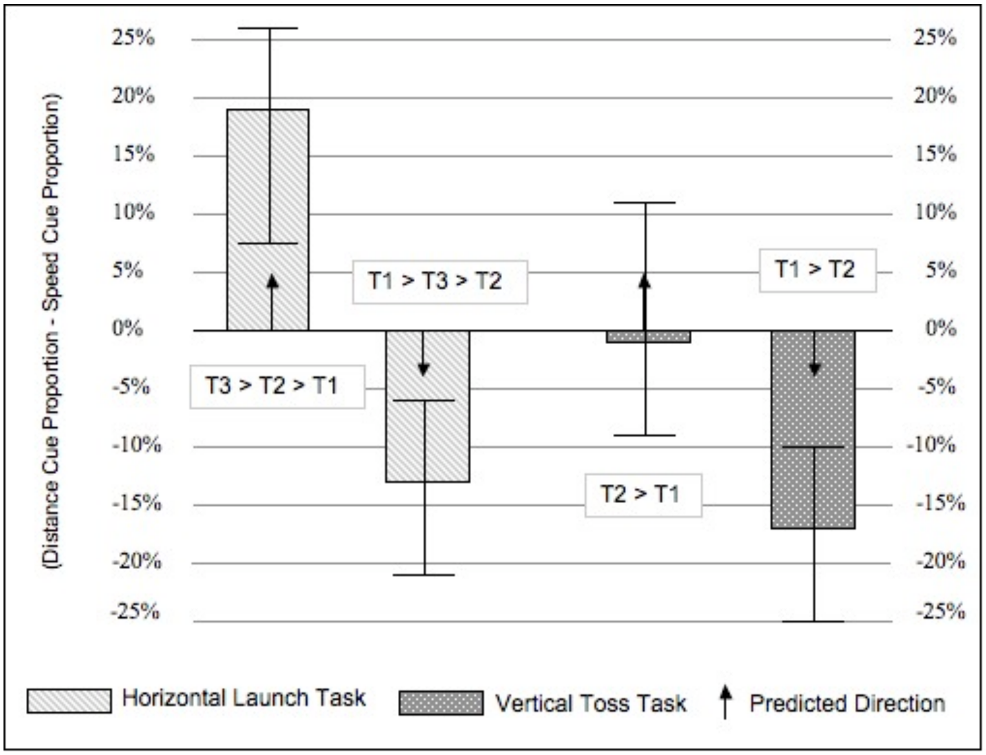


Figure 7: Difference in the Frequency of Student Answers Across Cuing Conditions

These results support our hypothesis that the speed cues presented in problems about duration bias students toward thinking that *more speed implies less time*. For both problems, problems with speed cues had more students giving answers consistent with this intuition. Our hypothesis concerning the effect of distance cues is not as conclusive, since only one of our predictions concerning the biasing effect of distance-cues was supported by our analysis. It could be that the distance-cues still has an overall biasing effect, which was masked by other effects that are yet to be determined; or it may be possible to construct alternative explanations for the differences in distributions that do not include the biasing the activation of intuitions like *more distance implies more time.*

Secondary Analysis: Student Explanations

Students were also asked as a part of completing the surveys to write written explanations for the answer they choose. One reason for including an analysis of students' explanations is to explore the degree to which student explanations do or do not match with intuitions from our model.

Categorization of Written Explanations

Whereas student *answers* were categorized into a fixed categorization scheme, the categorization of student explanations was conducted via an iterative process. Initially student explanations were coded as to whether or not they represented patterns of thinking that are consistent with the patterns of thinking discussed in the model. Initially, student explanations were coded only if they reflected (1) the intuition *more distance implies more time* (either by itself or in conjunction with the intuition *more speed implies more distance*), (2) the intuition *more speed implies less time* (either by itself or in conjunction with the intuition that *more distance implies more speed*), or (3) if they reflected compensation arguments, suggesting both intuitions are activated.

It was not expected that this initial coding scheme would span the data set. Nonetheless it was a useful analytical tool for seeing how patterns of particular explanations (that are based on patterns of thinking in the model) correspond to patterns of answers. For example, our model assumes that students answering that the longer distance took more time were doing so because of the cueing of the closely corresponding intuitive element. It was assumed that students answering that the faster motions took more time did so because they explicitly chained together the intuitions that more speed implies more distance and more distance implies more time. This initial coding of student explanations examines how representative these patterns of thinking based on the model are apparent in students' explicit explanations.

During this initial round of categorizations, several new categories emerged in order to better span the data set. These new categories are discussed in the context of each of the questions and are subsequently discussed in relation to various models of student thinking. One common pattern across both of the questions is explanations that express thinking that more speed takes longer to be overcome (or be stopped) by gravity. Other explanations however were unique to the particular questions, such as students concluding that the answer to the vertical toss question depends on information not stated in the problem. The two tables below provide examples of the initial and emergent categories for students' explanation on both the horizontal launch question and the vertical toss question.

Table 2: Categories for Coding Student Explanations on Horizontal Launch Question

Initial Coding Categories	Variations	Example
More Distance Implies more Time	More Distance, More Time	*If the distance is increasing, the time to get there must be increasing as well.*
	Less Distance, Less Time	*It is obvious that distance x being the smallest distance will take the least time*
	More Speed, More Distance; More Distance, More Time	*I expected that with a greater speed in the x-direction there will be increase of displacement along the x-direction, so it will take longer time.*
	Same Velocity Explicit	*The less distance traveled, w/ same velocity, will result in less time*
More Speed Implies less Time	More Speed, Less Time	*The faster the ball, the lesser the time*
	Less Distance, Less Speed; Less Speed, More Time	*The velocity must be less for 1 than for 2 and 3... I know this because it travels a smaller distance. The smaller the velocity, I assume that it took a longer time.*
	Same Distance Explicit	*The greater v, the smaller the time it would take for the ball to travel a same distance*
Compensation Argument	Proportional Explanation	*In order to reach a distance of 3x and 2x the ball must leave the table at different initial velocities in the x-direction... the velocities are proportional to the distance traveled so the time is the same.*
	Balancing Explanation	*If the ball travels farther in horizontal distances, it'd mean that the initial velocity is higher. Therefore, even though the horiz. Distance for all expt's differ, the different initial velocity balances the amount of time.*
Emergent Coding Categories	**Variations**	**Examples**
Common Features Argument	Gravity is the Same	*Same because gravity acts on the ball in the same amount.*
	Same Vertical Distance	*Because the table height is constant, the time to fall in all three cases will be the same.*
	Same Vertical Speed	*Equal when they hit the ground because their initial vertical velocities are the same.*
Faster Stays Up Longer	Slower, Less Time	*Experiment 1 would take the least amount of time because the horizontal velocity is smaller than the others. Gravity would therefore have a large effect on it.*
	Faster, More Time	*The other two with higher velocity will take longer because they will fall more gradually.*

Table 3: Categories for Coding Student Explanations for Vertical Toss Question

Initial Coding Categories	Variations	Examples
More Distance Implies more Time	More Distance, More Time	*Common sense tells me the faster ball should take longer to reach the max height, because it will travel a farther distance.*
	More Speed, More Distance; More Distance, More Time	*The time will be greater for the second ball thrown because it had a greater initial velocity, so it will travel higher and therefore take more time.*
	Same Velocity Explicit	*The less distance traveled, w/ same velocity, will result in less time*
More Speed Implies less Time	More Speed, Less Time	*The second throw will achieve the height faster b/c its traveling @ a greater speed.*
	Less Distance, Less Speed; Less Speed, More Time	*The time for the ball to reach max height will be less b/c it has a greater velocity so it will reach the distance faster.*
	Same Distance Explicit	*Less because velocity is greater. Since distance to maximum height is the same in both cases and the velocity is increased, time must be decreased.*
Compensation Argument	Proportional Explanation	*Same since the maximum height of ball 2 will be larger than that of ball 1 in proportion to its greater initial velocity.*
	Balancing Explanation	*If the ball travels farther in horizontal distances, it'd mean that the initial velocity is higher. Therefore, even though the horiz. Distance for all expt's differ, the different initial velocity balances the amount of time.*
Emergent Coding Categories	**Variations**	**Examples**
Common Features Argument	Gravity is the Same	*It will be the same. Gravity is the only force acting on the ball.*
More Speed to Overcome	More time to Slow to a Stop	*"Because the second one is released with a greater speed, the negative acceleration (g) will take longer to slow the ball down to zero.*
	More time to Overcome	*With more velocity, the ball can overcome gravity for a longer time.*
It Depends	Depends on Speed	*It depends on the initial speed and the height. It could be the same. It could be less and greater. It all depends on the initial speed.*

A disclaimer concerning the relationships among students' answers, their thinking, and their explanations is worth briefly mentioning before going onto present specific examples.

It would be a mistake to interpret students' written explanations as directly corresponding to students' intuitive sense of what's happening in the physical situation or physics problem. Based on our framework of students' intuitive thinking, students have a variety of intuitive ideas about physical mechanism. However, it need not be the case that all of these intuitive patterns of thinking be connected to students' discursive repertoire for talking about their ideas. The intuitive elements in our toy cognitive model certainly have drawn from examples in which students seem able to talk about their ideas about kinematical relation; but this need not be true, generally, nor in any given situation involving student explanation.

Even if students do have discursive repertoires for talking about the substance of their own intuitive thinking, what students choose to write must certainly be affected by students' sense of what kind of explanation they are supposed to give. Some students may perceive the activity of explaining as being about describing their own thought processes. Other students may perceive the activity as being about citing a law of physics. We will certainly observe differences in students' explanations that have to do with the use of everyday language versus more formal mathematical or physics language.

Explanations on the Horizontal Launch Task

In the final round of coding student explanations for the horizontal launch questions, the inter-rater reliability for two raters resulted in 94% agreement across a sample of 64 randomly selected explanations. Coders were not blind to the condition. All disagreements were in regard to whether a particular explanation was clear enough to be coded as one specific type or whether it should be was unclear, in which case it was coded as “other.” Students providing no written record of their reasoning (less than 7% of total) were removed from further analysis. Therefore, percentages given below are of those students who provided an explanation of any kind, including non-categorical explanations.

Explanations for Experiment 3 takes the Most Time

For the distance-cueing horizontal launch question, 89% of the students answering that Experiment 3 takes the most time wrote explanations that could be coded as reflecting the intuition that *more distance implies more time.* Student explanations for this answer are rather consistent with our model in the sense that we hypothesized that answer is biased by preferentially cueing this intuition. This result also indicates that a large fraction of students are aware of this aspect of their thinking and that they have a vocabulary for describing that thinking that we as researchers can recognize.

Some of these written explanations were expressed as reflecting students’ own everyday thinking. One student writes, “Common sense, you would think #3 takes the longest because it travels the most distance.” In this case, the student refers to his or her own thinking as “common sense,” in expressing this intuition. Other students' responses, however, seem to point to a more general idea of proportionality between time and distance. One student writes, “Time is proportional to distance. As the distance increases from a certain point, the time to reach that distance increases the further away the ball gets from its initial starting point."

Students given the speed-cueing version also provided explanations consistent with this intuition, often reasoning first that greater speed implies going farther. One such student writes, "The faster the ball leaves the table, it means it travels farther. Because it travels farther, it will hit the ground at a later time." In this example, first this student articulates an explanation consistent with the intuition that *greater speed implies more distance*, and then writes that is because of going farther that it hits the ground at a later time. This kind of explanation that involve these two particular intuitions applied in succession, however, only accounts for 55% of all students explanations for why the third experiment takes the most time in the speed-cueing version.

The second most common explanation written by students for why experiment three takes the most time was not a part of the initial coding scheme. These explanations ended up accounting for 42% of the student explanations on the speed-cueing horizontal launch question and 4% on the distance-cueing survey. The following student explanations for why it takes more time in Experiment 3 illustrates this emergent category:

> *"The other two with higher velocity will take longer because they will fall more gradually."*

“It will spend the most time traveling horizontally before it begins to move in a vertical direction.”

“Experiment 1 would take the least amount of time because the horizontal value is smaller than the others. Gravity would therefore have a large effect on it.”

“Faster the ball is going, more time it spends going hori[zontal] in the air"

In all of these explanations, speed is described as if it were something that help to keeps the object up or to help resist the pull of gravity. The explanation seems to be that having more speed allows an object to fall more gradually (or vice versa, that having less speed implies falling downward at a faster rate). This kind of explanation in which students state that more speed implies more time to fall (because gravity takes longer to pull it down) is a category that emerged in the iterative process of coding student explanations. These explanations resemble notions of the naïve impetus theory described by McCloskey. Students seem to be articulating an idea that increased forward motion (or impetus) makes it so the ball can keep it’s forward motion going for more time before the downward motion occurs. The connection to impetus notions is discussed more thoroughly in a section below called Tertiary Analyses: Consistency and Variability

The prevalence of this explanation highlights two important features. First, not all explanations for why Experiment 3 takes the most time are articulated as expectations that increased distance implies greater time. Although it represents the vast majority of explanations for the distance-cueing survey, it only represents about half of the explanations for the speed-cueing. Second, the distribution of explanations given by students for why Experiment 3 takes the most time depends on the version of the question asked. The two different versions of the survey not only have an effect on the distribution of answers. Even for the same answer across the cues, students give a different distribution of explanations. These “impetus-like” explanations were more prevalent for the speed-cueing version than the distance-cueing version.

Explanations for Experiment 3 takes the Least Time

On the speed-cueing version of the horizontal launch question, 93% of students answering that Experiment 3 takes the least time gave explanations that were coded as reflecting the intuition that *more speed implies less time*. As with the other intuitions, some students expressed this in everyday language while others used more formal language. One student writes, “It’s like a pitcher throwing a baseball. If he throws it with a greater velocity, it will reach the catcher faster.” This explanation involves seeing the horizontally rolling ball as analogous to another familiar situation involving horizontally moving objects moving faster and slower. Another student writes, “Velocity is inversely proportional to time. As your velocity increases, the shorter time will be till you hit the floor.”

As is evident from the data on student answers to the horizontal launch question (Figure 5), very few students taking the distance-cueing version gave the answer that Experiment 1 took the least time. Six of the nine students who gave this answer (and provided any explanation) were coded as reflecting the intuition that *more speed implies taking less time*. One example comes from a students writing, “The velocity must be less for 1 than 2 than 3…I know this because it travels a small distance. The smaller the velocity, I assume it took longer." It is evident in this example that the students first explains that less distance implies less speed and then that less speed implied more time. Another student explanations coded as reflecting this intuition, "The speed of the ball would have to be greater to go farther, so it will take less to go the farther distance." The other three students gave explanations that were coded as unclear.

Explanations for All Experiments Take The Same Time

The correct answer to either version of the horizontal launch question is that all the experiments take the same amount of time. Many students gave an explanation of this answer in terms of compensation arguments (see Table 4)– a pattern of reasoning that can be described in terms of our simple model. On the speed-cueing version, 15% of explanations for correct answers were coded as compensation arguments. On the distance-cueing version, 10% of the explanations for correct answers were coded as compensation arguments. One student writes, "The ones with a faster velocity will travel faster, and go farther, so it will take the same amount of time." Here the student draws attention to both changes to speed and distance and concludes that the time is the same.

Other students more explicitly mentioned balancing or canceling in their explanations. One student writes, "If the ball travels farther in horizontal distances, it'd mean that the initial velocity is higher. Therefore, even though the horizontal distance for all experiments differ, the different initial velocity balances the amount of time." The use of the "even though" in this explanation is particular interesting. It may suggest that the student is explaining why the times are, in fact, the same, when it might otherwise seem that the longer distances should take more time. This might well imply that the student initially thought that it would take more time, and then reconsidered. Or it, could just imply that explaining the compensation is closely tied in an epistemological way to the explaining away of the counterfactual. Either interpretation supports modeling the dynamics of the reasoning as involving the intuition that *more distance implies more time* and the intuition that *more speed implies less time*. This is in contrast to other explanations that seems to involve just reasoning about either the relation between speed and time or distance and time alone.

As has been the case in other students' explanations, some students expressed this idea more formally, in this case appealing to proportionality. One student writes, "I feel like the times for the ball to hit the ground are all the same. The ball may be traveling faster and may travel a horizontal distance that is greater, but the time is equal... If d increases and the v increases by a proportional amount, the t remains the same." Similar to the above example involving 'even though', the use of word "may" here implies suggests the student is alluding to an outcome (e.g., the times not being the same) that does not occur because of the proportionality.

The most common type of explanations for a correct answer to the horizontal launch question involves students pointing features of the physical situation or motion that are common to all the three experiments. Typically, students giving these explanations point out some aspect(s) of the vertical motion that is identical, either the vertical distance, initial vertical speed, acceleration, or force. While it is possible to see these explanations as indicating that students are reasoning from normative knowledge based related to Newton's Laws, we can also view many of these explanations as involving refinements of how some of basic intuitions in our cognitive model are applied. Consider, for example, the following explanation: "Because the table height is constant, the time to fall in all three cases is the same." This explanation is similar to explanations that *more distance implies more time*, except here being used to explain why the *same distance implies same time.* Here, however, the student applies this idea to just the vertical aspect of the motion. Another explanation comes from this student who states, "Equal when they hit the ground, because their initial vertical velocities are the same". This explanation may be viewed as involving a refinement of the idea that *more speed implies less time*, here being applied to just the vertical aspect of the motion. The refinement of these ideas may just come from learning that such intuitions can be applied in just certain aspects of the motion, or rather from having a degree of control over how and in what way these intuitions are applied.

Other students seem to reference the acceleration or force due to gravity being the same: "Time is the same with both balls because acceleration due to gravity acting on the balls is 9.8 m/s^2" It seems less obvious how such an explanation may correspond to a refinement of an intuition that is in our model. I revisit these kinds of explanations (and others as well) further after describing students' explanations to the vertical toss question.

Other Student Explanations

Explanations were coded as “other” if they were unclear if they reflected a particular intuition or not or if they were clear but idiosyncratic enough to be quite rare. An example of a student explanation that may reflect a compensation argument, but was not coded as such is this: “The time is the same because the proportion.” Another example is, the time is the same if velocities vary.” Two examples of idiosyncratic explanations are, “They will all be the same time because they will be slow down by air resistance,” and “Less time because velocity is the derivative of displacement.”

Summary of Explanations for Horizontal Launch Task

The table below gives a breakdown of student explanations on the horizontal launch question by showing the frequency of different explanations given for a particular answer for a particular cueing survey. It shows that the most common type of explanations for the correct answer that the time is the same is a common feature argument. Compensation arguments represented a small fraction of explanations given. For the answer that Experiment 3 takes the most time, the most common explanations were consistent with the intuition that *more distance implies more time*. However, it was much more common (89%) in the distance-cueing version that the speed-cueing version (55%). Another common explanations on the speed-cueing version for why Experiment 3 takes the most time is that the faster balls stay up in the air for longer. The most common explanation for why Experiment 3 takes the least time is it takes less time because the ball is moving faster.

Overall, the breakdown of student explanations is largely consistent with our toy cognitive model assumption about the intuitions playing a role in students thinking. Many students giving answers consistent with the intuition that *more distance implies more time* also gave explanations consistent with the intuition. Many students giving answers with the intuition that *more speed implies less time* also gave explanations consistent with the intuition. Other explanations were provided as well that did not directly correspond to intuitions in the model. While some of the common features arguments can be expressed as applications of intuitions in our model (applied to aspects of the vertical motion), student explanations for why a faster ball stays up in the air for longer are not reconcilable with out model.

Table 4: Breakdown of Student Explanations on Horizontal Launch Task

All Experiments Same Time	Distance-cueing (N =93)	Speed-cueing (N = 94)
Compensation Argument	15%	10%
Common Feature Argument	80%	88%
Other	5%	2%
Experiment 3 Takes Most Time	Distance-cueing (N = 52)	Speed-cueing (N = 24)
More Distance Implies More Time	89%	55%
Faster Stays Up Longer	4%	42%
Other	7%	3%
Experiment 3 Takes Least Time	Distance-cueing (N = 9)	Speed-cueing (N = 27)
More Speed Implies Less Time	67%	93%
Other	33%	7%

Explanations on the Vertical Toss Task

As with the horizontal launch question, student explanations to the vertical toss task were coded. In the final round of coding for the vertical toss task, inter-rater reliability was 89% for a sample of 64 student explanations. As with the horizontal launch task, disagreements concerned whether there was sufficient evidence to code the explanation as a particular kind of explanation or to code it as unclear. Students who provided no written explanation were removed from this analysis.

Explanations for Second Toss Takes Less Time

As was seen with the horizontal launch question, students both gave answers and explanations consistent with the intuition that *more speed implies less time* (see

Table 5 below). Thirty-three of the thirty-four students giving the answer that the second throw takes less time wrote explanations consistent with the intuition that *more speed implies less time.*

In one example, a student writes, “If you do anything at a faster speed, it will take less time to reach that destination.” For many of these explanations, explicit consideration for how the maximum height changes with increased speed were vague at best. In the above case, the students’ use the word ‘destination’ could imply thinking about both balls reach the same maximum height.

Other students, however, explicitly stated that the maximum height would be the same. One such student wrote, “Since distance to maximum height is the same in both cases and the velocity is increased, the time must be decreased.” In these cases, students apply the intuition that more speed implies less time with what would be an appropriate caveat – more speed does imply less time when distance are equal. However, these students fail to consider other intuitions that would suggest to them this is not the case (e.g., more speed implies more distance). It could be argued that these students have simply misinterpreted the question as actually implying the distances are the same (in referring to maximum height). We would still be, nonetheless, able to support from the evidence our conclusion that these students are attending to the intuition that *more speed implies less time* and failing to attend to (or follow up on) intuitive ideas that would suggest its implausible that they attain the same maximum height. Similar phenomena of reasoning, involving inappropriately holding quantities constant, have been documented by other researchers studying student reasoning in different contexts, including electric circuits (Cohen, Eylon, Ganiel, 1982) and thermodynamics (Rozier and Viennot, 1991).

However, it is certainly not the case that all students cueing into the idea that more speed implies less time were explicit about the heights being the same (or vague). To the contrary, some students explicitly mention the heights being different as part of their explanation. Here a student explicitly states that the maximum heights are different, and that the second ball takes less time because it is faster: “It will be less because with a greater initial speed the ball will be able to travel a greater distance faster which means it will reach its maximum height in a shorter time.” In this example we see reasoning suggestive of the intuition that *more speed implies more distance* (“with greater initial speed the ball will be able to travel a greater distance”) and for the intuition that *more speed implies less time* (“faster which means it will reach its maximum height in a shorter time”). Here is another example of student explaining that the faster ball goes higher and in less time: “The time will be less to reach max height when there is a greater initial speed. The second throw started from nothing, but since the second throw had more starting speed it can go higher faster”

Observations of these two kinds of explanations – both involving the intuition that *more speed implies less time* but with different ideas about how the distance compares – are consistent with the description of our model of students’ thinking described in the previous chapter. First, these different explanations reflect the fact that the intuition that *more speed implies less time* may or may not be cued along with resources for thinking about when that intuition applies. Students applying the intuition *more speed implies less time* while also stating that the distances are the same may have first cued into the kinematical intuition itself, which then activated resources for thinking about its applicability. This idea about its applicability may have then led them to impose this condition on the situation (even know it's not true). In the other explanation, students who are explicit about the fact that the distances are different, seem unlikely to be thinking about how the intuition applies only when they are the same. The fact that we find these two different reasoning patterns supports our view of these finer-gained knowledge elements as quasi-independent cognitive elements that can activate together in various ways.

Explanations for Second Toss Takes Most Time

Students answering that the second throw takes more time are correct, in fact, in their answer. For both the speed-cueing or distance-cueing versions, about half of the students answered correctly, independent of the version. However, the distribution of explanations varied across the two different surveys.

One kind of explanation given by students for both surveys for this answer is that *more distance implies more time*. On the distance-cueing 70% of students explanations were coded as indicating more distance implies more time. On the speed-cueing survey, however, only 30% were coded as indicating the idea that more distance implies less time. For student explanation on the distance-cueing survey, many students made simple statements such as, "The time is greater because the distance is greater." Another student described this as, "I think the second ball will take a greater amount of time as it will travel the same path as the first ball plus some more." Here the student emphasizes that the second ball travels the *same path* as the first and then some additional, explaining why the time is greater.

For the speed-cueing survey, many of the students' explaining why more height implies more time, also explained that the ball reaches a greater height because of its greater speed. Here a student reasons, "The harder the student throws the ball, the higher the maximum height will be, thus taking the ball longer to get reach it." Another explains, "The higher velocity of the second ball will drive it higher making it take a longer time." Here we see explanations consistent with the intuitions that *more speed implies more distance* and *more distance implies more time* in reaching this conclusion. It is worth noting that this reasoning leads to an incorrect answer for the horizontal launch question, but it leads to the right answer for the vertical toss question.

The second kind of explanation given by students involves thinking about how long it takes gravity to slow down ball's throw with different amounts of speed. Here are several examples to illustrate it:

> *"The acceleration due to gravity has to slow something down that's going faster, so it will take longer."*

> *"A little more time, since it is thrown with greater upward speed in the y direction, g will have to work harder to v back to 0 at the maximum."*

"It will take a longer time for gravity to counter act the initial velocity of the second throw"

"The time will be greater because at the max height for each throw, v = 0, but the second throw has to undergo a greater change in speed working against the same forces."

"Greater because gravity is forcing the ball downward, in both cases, but takes a longer period of time when the initial velocity is greater."

In all of these examples, the students seem to reason that speed of the ball has to be stopped or slowed down to zero at the maximum height and that it should take more time for gravity to slow it down (by pulling downward on it) when the ball has more speed to begin with. These explanations, like similar ones from the horizontal launch, do not seem to correspond to any intuitive element or combination of intuitive elements the initial model described. There are likely many different ways of describing this reasoning in terms of different decompositions of elemental structures of knowledge. I'd like to refrain, for the moment, from speculating about a cognitive decomposition behind this written explanation, and instead simply refer to this phenomenological pattern as the explanation *gravity takes more time to overcome more speed*, and to briefly note it's similarity to students' explanations on the horizontal launch question about gravity having more effect on the ball's with less speed (pulling them down in less time).

While the number of students answering correctly were the same independent of the version of the question posed, the distribution of explanations were not identical across both version of the survey. On the speed-cueing survey, this kind of explanation accounts for 70% of the codes for the students answering that the second ball takes more time. For the distance cueing survey, it only accounts for 20% of the explanations coded. This difference in distribution of explanations across the two versions of the explanation could be explained in a variety of ways. It could be that student thinking across the two surveys is quite similar (characterized by the activation of the same cognitive elements), with differences being only in how students think they are supposed to explain their answer. It may be that students taking the distance-cueing problem think they are supposed to explain their answer based on the distance. These students might then only give explanations consistent with the idea that more *distance implies more time*, even if other intuitive explanations were available to them. Similarly students taking the speed-cueing version, may only give explanations more directly reasoning from the difference in speed, to explain that *gravity takes more time to overcome more speed.*

Explanations for Second Toss Takes Same Time

Student explanations for the answer that the two balls reach the top at same time fall into two categories, one corresponding to a compensation argument and the other corresponding to a common feature argument.

For students answering that the times are the same, 70% of student explanations were coded as compensation arguments across both versions of the surveys. These were not evenly distributed across the two versions of the surveys, however. A total for the distancing-cueing survey, 83% of the explanations were coded as compensation arguments. However, only 55% of the explanations on the speed-cueing survey were coded as compensation arguments.

Here is an example of a student giving a compensation argument: "The same amount of time … since the 2nd is moving at a greater velocity, it will reach a higher height which will take longer, but is faster so the time is equal." Once again we see a similar pattern in the way students articulate this explanation suggesting that both the intuitions that *more distance implies more time* and *more speed implies less time* are activated. The student even seems to first allude to an actual motion that goes high and takes longer ("it will reach a higher max height which will take longer"), before explaining that the time is, in fact, the same because it has more speed ("but is faster so the time is equal"). Another student argues, "It would take the same amount of time…The first throw is moving at a slower rate so even though the height is lower, it will take longer to get there." Here, too, the student seems to allude to a motion that is slower and takes longer, but doesn't because of a difference in height. Across both of these examples, we see students using words like 'but' and 'even though' suggesting that students are not only explaining why it's the same, but alluding to why it's not a different answer that might be expected.

The other pattern of explanation that was given by students for why the times are the same is that gravity is the same. For the distance-cueing survey, these explanation only accounted for 5% of explanations for the same time answer. For the speeding cueing survey, it accounted for 30% of the explanations. We noted this pattern of reasoning as an explanation given in the horizontal launch question as well. It is worth pointing out that this pattern of explanation was associated with correct answers in the horizontal launch question. Here it is associated with an incorrect answer.

Summary of Explanations to Vertical Toss Task

Table 5, shown below, gives the distribution of student explanations on the vertical task broken down by answer and cues. The table highlights that two different kinds of explanations were prevalent for students answering that the second toss take more time. One kind of explanation was consistent with our model (more distance implies more time), and another explanations were not (more speed to overcome). The former explanation was more common on the distance-cueing version (68%) and less common on the speed-cueing version (31%). The table also highlights that two different kinds of explanations were prevalent for students answering that the second toss takes the same time. Students gave explanations consistent with compensation arguments, while other students wrote explanations that the time is same because gravity acts the same on the ball. The distribution for both of these answers shows that student explanations were qualitatively different across the two different cues even for the same answer.

The table also highlights how student explanations for why the second toss take less time were consistent with the intuition that *more speed implies less time*. This was independent of the cues presented.

Like the explanations on the horizontal launch task, explanations for the vertical toss task show some consistency with our toy cognitive model. Students gave explanations consistent with the intuitions we assumed were contributing to students giving particular answers, but other explanations not consistent with our model were also prevalent.

Table 5: Breakdown of Student Explanations on Vertical Toss Task

Second Toss Takes More Time	Distance-cueing (N=70)	Speed-cueing (N=71)
More Distance Implies More Time	68%	31%
More Speed to Stop/Overcome	26%	69%
Other	6%	0%
Second Toss Takes Same Time	Distance-cueing (N=64)	Speed-cueing (N=40)
Compensation	**83%**	55%
Gravity is the Same	6%	35%
Other	1%	10%
Second Toss Takes Less Time	Distance-cueing (N=7)	Speed-cueing (N=31)
More speed implies less time	86%	**87%**
Other	14%	13%

Summary of Secondary Analysis

The analysis of students' written explanations helps to establish some confidence in the inclusion of particular intuitive elements comprising our model of student thinking. There is evidence for students' giving explanations consistent with many of the intuitions we assumed were playing a role in students' thinking, including more *distance implies more time*, *more speed implies less time*, as well as patterns of thinking involving multiple intuitions such as compensation arguments and chaining together intuitions like *more speed implies more distance* and *more distance implies more time.* We also see evidence of dynamics involving the students' intuitions have and the knowledge they have about when its appropriate to apply those intuitions, such as when students reason that the two balls thrown at different initial speeds reaches the same maximum height. These explanations that are consistent with the model not only give confidence that these are some of intuitions playing a role in the dynamic, but they give indications that students are consciously aware of some of these intuitions and have vocabularies for describing those aspects of their thinking.

For certain scenarios, there is actually high degree of correspondence between students' answers and the codes given to their explanations. For example, in the distance-cueing horizontal launch question, 89% of student explanations for why Experiment 3 takes the most time were coded as reflecting the intuition that *more distance implies more time*. However, student explanations for this answer on the speed-cueing split between codes for *more distance implies more time* and *gravity takes longer to overcome more speed*. This pattern of cue-dependent distributions of explanation is also reflected in the vertical toss question, where 68% of the students answering correctly in the distance-cueing version gave explanations coded as *more distance implies more time* (but only 31% on the speed-cueing version). Across both questions and both cues, answers consistent with the intuition that *more speed implies less time* corresponded to a high degree to explanations that were coded as such.

Other explanations given by students do suggest that other intuitions also play a role in the dynamics of students' settling on an answer for these questions. This is to be expected. In our description of student thinking about the mass on the spring in the previous chapter, we found that students used intuitions for thinking about s*pringiness* in thinking about the amount of time to reach equilibrium again. In students' explanations for these questions, there is evidence for students relying on intuitions and knowledge they have for thinking about gravity. We see students explaining that gravity should take more time to overcome or stop an object that has more speed. In the vertical toss question, this leads students to the right answer, but it leads students to wrong answer in the horizontal launch question.

We also see students pointing out that gravity acts the same on all objects in explaining why the answer that the times are the same on the horizontal launch question. While this many not reflect an intuitive idea about gravity (more like a school-learned fact), this piece of knowledge seems to have an impact of students' answers to the question. In some respects, this seems like a reasonable explanation for the why all the experiments in the horizontal launch question take the same time. It is true that gravity has the same effect on all the balls independent of how fast they are moving, but the fact the end result is same amount of time to fall has just as much to do with the other aspects of the vertical motion being identical as well. To this effect, the explanation "gravity is the same" arises in students incorrect answers in the vertical toss question for why the two times would be the same. In this case, it's certainly not the case that all aspects of the vertical motion are identical.

Understanding the nature and dynamics of these explanations (and the intuitive thinking that may underlie them) may be difficult to address from this experiment alone. At this point, I'd like to refrain from speculating on any cognitive basis for these explanations, although some will be offered later in this chapter. For the moment, I'd like to shift toward an analysis of student explanations across and within tasks that give insight into the consistency and variability of student thinking.

Tertiary Analyses: Consistency and Variability

In this section I consider patterns of student answers across the two different questions as well as within single questions. Recall that each student received the two different motion questions- the horizontal launch question and vertical toss question- in a single survey. There were no mixed-cueing surveys, involving both speed and distance cues, so no student received a speed-cueing horizontal launch question with a distance-cueing vertical toss question. Patterns that are discussed in this section are (1) students answering correctly to both questions across the survey and (2) students answering in a manner consistent with a naïve impetus theory and (3) students showing evidence of multiple patterns of thinking in a single question.

Across-Task Analysis of Student Answers

Newtonian Consistency

In total, for all of students taking either survey (N= 316), only 37% of the students were able to provide a correct answer to both the vertical toss and horizontal launch questions. This relatively poor outcome (for students having, not just taken, but passed a course in introductory mechanics) highlights much of the concerns of the physics education research community that students often leave our introductory physics classroom unable to apply basic concepts to make sense of simple physical situations. The distribution of correctness, shown in Table 6, was slightly different between the two versions of the question.

Table 6: Breakdown of Correct Answers for Each Survey

	Speed-cueing	Distance-cueing
Both Correct	40%	34%
One Correct	25%	41%
Neither Correct	25%	25%

For the speed-cueing surveys, 40% gave correct answers to both questions, 35% gave only one correct answers, and 25% of gave no correct answers. On the distance-cueing survey, students did slightly worse. Only 34% gave correct answers to both questions, 41% correct on only one, and 25% correct one neither.

Recall that for the horizontal launch question, 60% of students gave the correct answer independent of whether it from the speed-cueing or the distance-cueing survey. For the vertical toss question, 50% of students gave the correct answer independent of the survey as well. While the difficulty of each question came out independent of the cues presented, the distance-cueing survey was slightly more difficult over all, with fewer students answering both correctly.

From the viewpoint of trying to understand the sources of students' intuitive thinking in response to these questions, it is rather intriguing that students did better on the horizontal launch question than they did on the vertical toss question. Questions like the horizontal projectile motion question are commonly known to be counter-intuitive. For whatever the reasons, many students don't expect the balls to reach the ground at the same time. It is because of this unexpected result that demonstrations like the shooter-dropper have become widely used as a part of instruction. The vertical toss question, on the other hand, would seem to be a more familiar example from students' experience and would seem to have a more obvious answer. Even if students haven't ever carefully timed such tosses, there would seem to be a wide range of experiences people would have to draw on to answer this question.

Based on just our own intuitions about the difficulty of these questions, we might have expected students to do quite well on the vertical toss task, and quite poorly on the horizontal launch task. Anecdotally, many colleagues initially expressed doubt that any variability would be evident in students' thinking about the vertical toss task, because nearly all the students would know the right answer independent of how it is asked. The results, however, show that students did better on the horizontal launch task than they did on the vertical toss task. Students answered 60% correctly on the horizontal launch and only 50% on the vertical toss task. The fact that even 60% of the students answered correctly to the horizontal launch task may be attributable to the fact that students observed and discussed the shooter-dropper demonstration. Having observed and discussed, it is possible that students could have simply memorized the correct answer or have come to understand this situation quite well. It is quite puzzling, however, that only 50% of the students would answer the vertical toss question correctly, when it seems that students could simply think about their own experiences of tossing objects in the air. What might be contributing to students doing so poorly given that they would seem to have so many productive experiences to draw on?

As one source of speculation about why students might have done so poorly, consider the following student quotes taken from explanations given from students who did produce correct answers for the vertical toss question:

> *"The amount of time would be greater [when thrown higher] because when you think of just the experience of throwing a ball up and down, the*

ball reaches its peak a lot quicker at less height than when you throw it higher"

"The time is greater in the second throw. I just imagined tossing a ball 5 inches in the air and tossing a ball a mile into the air. It does not make any sense that it would take the same amount of time"

Both these statements involve students alluding to actual experiences of throwing balls. The first student writing, "When you think of just experiences of throwing a ball up and down" and the second student writing, "I just imagined tossing a ball." Both the students correctly conclude that it should take more time for the second toss. In this first example, the student concludes that the first ball reaches its peak a lot quicker (in less time) when the ball is thrown to a lesser height. In the second example, we see the student simply stating that it doesn't make sense that it would take the same time. For these students, thinking about actual experiences of throwing balls vertically leads students to the correct answer and steers them away from wrong ones. They productively draw on there own experiences in making sense of the answer to the question.

We can contrast these written explanations with students who arrive at incorrect answer that the time is the same. Recall that 35% of the students who answered that the time is the same on the speed-cueing version of the vertical toss task wrote that the time is same because gravity is the same. It's difficult to imagine that these students are tapping into their own intuitive experiences with throwing balls in this scenario. Instead, it seems that students are cueing into more formal school knowledge that gravity is a constant. Many of these explanations explicitly mentioned that gravity is “9.8” or “9.8 m/s^2”, which is certainly more formal school knowledge than intuitive sense of the effect of gravity. In these cases, relying on facts learned from school may be steering some students toward an incorrect answer, and more importantly, away from productive intuitions they have for making sense of the physical situation.

This is certainly not to say that students’ intuitive sense for physical situations is going to be generally more productive than their more formal school knowledge in trying to sense of physical situations or for arriving at correct conclusions. For example, I described earlier a students who relies on their own experiences of pitching a baseball to incorrectly conclude that the fastest ball rolling off the edge of the table hits the ground first. It is certainly interesting, however, that more school-ish knowledge such as “gravity is 9.8” might be leading students away from aspects of their intuitive understandings that are arguably productive. The issues of where students look for consistency in their thinking is certainly an interesting question that this result brings up. Do they look for consistency with other physical experiences they have (such as throwing a ball really high and really low or such as pitching a baseball) or do they look for consistency with facts they have learned (such gravity acts the same on all bodies)? The simple toy model of students’ thinking only considered a few fine-grained intuitions students might use to make sense of situations involving motion, and did not take in account the impact of students’ more formal school knowledge or consider the different ways in which students might draw on different experiences in looking for consistency.

Naïve Theory Consistency

In the previous section, I analyzed patterns of students' answers across the two surveys in terms of correctness. Another way we can look at patterns of students' answers to these questions is see the extent to which students gave answers consistent with a naïve impetus theory of motion.

For the vertical toss task, McCloskey's description of the theory should lead students to the right answers. Students should answer that it takes more time to reach the top in the second throw because it will take gravity more time to change the motion that was initially imparted to the ball when there is more impetus, allowing it travel higher up into the air. For the horizontal launch task, McCloskey's description of the theory should lead students to a wrong answer. Students should answer that the third experiment takes the most time for the same reasons as the vertical toss. The greater impetus of the ball resists the pull of gravity for more time allowing it carry further out. Therefore, by looking at the percentages of students giving both answers consistent with impetus theory, we can get a gauge on the degree to which students' thinking reflects a consistent application of the naïve theory.

On the speed-cueing version of the survey, 13% of the total number of students gave answers consistent with naïve impetus theory. On the distance-cueing survey, only 7% of the students gave answers consistent with the naïve impetus theory. In total, independent of the survey students were given, only 10% of the students gave answers consistent with the naïve impetus theory. For some reference point, consider that there are roughly nine possible combinations of answers students could give—three different rankings for the horizontal launch task and three different comparisons for the vertical toss task, ignoring only the most rare answers. If all students were randomly guessing among these possibilities, we'd expect 11% of the students to give answers consistent with the naïve impetus theory. It seems unlikely that even the observed 10% of students are thinking about these problems in terms of an underlying consistent theory like the impetus theory. Instead, the data points to there being a systematicity between how the problem is presented and the particular finer-grained intuitions students apply in making sense of the question. In other words, we are observing not a systematicity in the structure of students' knowledge, but rather a systematicity in the contextual dependence of students' knowledge.

Within-Task Analysis of Student Explanations

The experiment described in this chapter demonstrates that the population of students *as a whole* can be biased toward and away from particular patterns of responses in predictable ways. From the observed variation in the aggregate patterns of responses, we infer the existence of individual students in the population who would answer the speed-cueing and distance-cueing questions differently. In this section, I explore some evidence of variability in students' written explanations. Instead of making observations of how students' thinking changes when cues are varied, we are able to look for evidence of dynamic variability in students' thinking when reasoning about the same question. Below, I present evidence for variability in individual students' thinking through erasures and consideration of multiple answers.

Student Erasures and Scratched out Answers

By the far the most common way of getting access to variability in students' thinking about a single question comes from when students write down an initial answer (or possibly also explanation), and then they change their mind and write down a new answer and explanation. Often students attempt to erase or scratch out their previous answer. These erasures and partially scratched out explanations provide a trace of the dynamics of individual students' thinking.

In the following example, a student initially gives a correct answer to the vertical toss question, and then changes his answer to an incorrect one:

> *"~~Greater because the ball is traveling farther.~~ It's the same because the ball is traveling faster even though it is traveling farther."*

Here it seems that the student initially cues into the idea that *more distance implies more time.* This answer is crossed out, and the student writes an incorrect answer that seems to be based on a compensation argument. The student argues that the time is the same because it's traveling faster and farther. We have been modeling the compensation argument as involving both of the intuitions that *more distance implies more time* and *more speed implies less time*. In this explanation, then, we see evidence for a dynamic in students' thinking beginning with the initial cueing of *more distance implies more time*. This intuition is initially cued, and then something is likely to have activated the idea that there must be more speed as well (perhaps *more distance implies more speed*). As these other intuitions are activated, a network of ideas forms (including the initially cued intuition *more distance implies more time* and other intuitions), which generates the new pattern of thinking about compensation.

In a different example, a student initially explains that the time is the same because gravity and mass are the same. The student crosses out this answer, and then writes an explanation that a less hard throw results in a lesser maximum height, which takes less time. Here we can see evidence that the student finally cues into the idea that *more distance implies more time*:

> *~~"The same time because the student uses the same ball, the mass of the object is the same for both throws. The gravity acts on the ball the same way in both cases~~. Greater than the first throw, because there is not a lot of force pushing the 1st ball up, so it's maximum height will be lower. Therefore, the 1st ball will take a shorter time to reach a lower maximum height."*

In another example, a student changes his mind about the horizontal launch task. He first gives an explanation consistent with the intuition that *more speed implies less time* (reasoning first that in order to go a farther distance it must have had a greater velocity). He crosses out this answer and gives a common feature explanation for why the time is the same for each experiment:

"~~In order to push the ball a further distance, a larger velocity is required, and therefore take less time.~~ All of the balls will hit the ground at the same time. The force of acceleration down is same in all 3."

Here a students does the opposite, changing his answers from the time being the same to the time being less for the faster motion:

"~~They are all the same time because~~... The faster it is moving the further it goes and the less time it takes to drop."

It is difficult to engage in a rigorous analysis of the variability of individual students' thinking using survey data alone. For starters, we only get direct evidence of such variability when students' provide a written record of the change. We would never know if a student changed their mind before writing anything and then wrote down only their explanation for their later thinking. Even when erasures are present, it isn't always possible to decipher what the student originally wrote, especially when the students erase their answer, and then write their new explanation on top of the erasure.

However, finding evidence that even some students change their mind, highlights the multiplicity inherent in students' thinking about physical situations. Students changed from correct answers to incorrect answers. Students also changed incorrect to correct answers. I stated before that the observed population variability in student response distributions implied that there must be some individual students who would answer differently to different answered questions. These examples of variability in single students' thinking support this conclusion.

The analysis of student erasures also provides some additional evidence to support our primary claims that distance cues increase the cuing priority of the intuition more distance implies more time and that speed cues increase the cuing priority of the intuition more speed implies less time. On the horizontal launch task, we analyzed the nature of changed answers by noting students first answers and final answer when possible. For this analysis, we coded students' answers (e.g., "Experiment 3 > Experiment 2 > Experiment 1") as well as explanations as evidence (e.g., "faster ball takes less time"). For the distance-cuing task, 11 erasures indicated a change from the answer T3 > T2 > T1 (answer consistent with the cuing of more distance implies more time to T3 = T2 = T1 (the correct answer), and zero answers indicating a change from T3 < T2 < T1 (answer consistent with the cuing of more speed implies less time). For the speed-cuing task, 7 erasures indicated a change from T3 < T2 < T1 to the correct answers, and zero change answers from T3 > T2 > T1 to the correct answers. This trend indicates an increased cuing priority for the predicted intuitions even for students who eventually settle on the correct answer. In this sense, the number of students who are biased toward thinking that more distance implies more time (with distance cues) and more speed implies less time (with speed cues) is higher than is indicated by students' final answers alone. Some students seem to initially cue into the intuitions that are being biased, but that pattern of thinking is not reliably sustained as they settle upon different patterns of thinking.

Multiple Answers from Students

A second source of variability in students' thinking comes from students who explicitly consider multiple answers in their written explanation. One student gives an answer based on his "common sense" and another answer based on "an educated guess" for the horizontal launch question:

> *"Using common sense the shorter the distance traveled the shorter the time, however as distance increases so does v proportionally, so therefore I would venture an educated guess that they are equal."*

This written explanation is interesting for a variety of reasons. First, it illustrates the high cueing priority of the intuition that more distance implies more time for the distance-cueing version of the horizontal launch question. His initial reaction to the situation is that shorter distances take less time. However, the student then presents a different idea as well, constructing pieces of a compensation argument. He suggests that as the distance increases so too does the velocity, implying that the times might be the same. The second reason this example is interesting is because the student is both aware of his own thinking and has ways of thinking about the kinds of thinking he has. He refers to his initial thinking as "using his common sense" and his later thinking as "venturing an educated guess".

Other students also considered multiple answers. This students explains that she could see the time being the same or the time being greater for the second toss:

"I think in some instances it could be the same, however it is possible for the second throw to take longer. My answer is greater."

The student finally decides that their answer is greater time for the second toss. Unfortunately, the student doesn't provide much explanation for either answer.

Other students giving multiple answers wrote rather bizarre explanations. In this case, the student seems to give two different answers, one for how much time the balls spend going horizontally and another for how time the balls will spend going vertically in the horizontal launch task:

"The faster the launch speed, the more time the ball will spend going horizontally, but they will all spend equal time going vertical."

The bizarreness of this explanation stems from the fact that the student seems to be implying that there are two completely independent movements, so that the ball can hit the ground at one time horizontally and another time vertically. One possible explanation for this students' response is that the student recalls that the correct answer is that they all hit the ground at the same time. The student knows that this is correct (and may even know that this has to do with the vertical motion), but yet they still retain the intuitive sense that the faster ball will remain in the air for longer. Instead of reconciling these competing ideas, the student maintains aspects of both reasoning in this strange manner.

Similar to erasures, explanations involving the consideration of multiple answers illustrate the multiplicity inherent in students' thinking about physical situations. This data supports the idea that students don't have a single way of thinking about the world, but a variety of intuitions (and experiences and school knowledge) to draw upon. For these students we see explicitly the occurrence of this multiplicity in their written explanations. In the first case I described above, the reconciling of multiple ways of thinking seemed to be done in a rather sophisticated way. The student was aware that they had two different competing ideas – one that more distance implied more time and the other that the distance and speed balanced out. The student also had ways of thinking about those different kinds of knowing – one as common sense and one as an educated guess. He then decided upon one of those answers and not the other. In this last example, the student also seemed to have multiple ways of thinking about the situation as well. This student managed to include both aspects of their thinking (same time to fall, but more time to go outward) in a way that seems largely non-physical. A single ball can't hit the ground at two different times.

Summary of Tertiary Analysis

The purpose of this tertiary analysis was to further explore the degree to which consistency and variability is evident in students' thinking. By looking at students answered to both problems on the survey, we found that only 37% of the students answered both question correctly and only 10% of students gave answers that would appear to be consistent with espousing a naïve impetus theory. These results demonstrate that fairly low numbers of students were able to consistently apply correct physics concepts (based on Newton's Laws) in thinking about these two physical situations involving motion in gravity. It also demonstrates that, while students have difficulties applying correct physics concepts, their difficulties do not seem to be the result of a coherent knowledge framework or singular misconception.

In terms of variability, we also found evidence of multiplicity in individual students' thinking as evidenced by erasures and students explicitly considering multiple answers. Students changed their minds, sometimes in ways that we would identify as shifts among intuitions in our model. Other students seem to contend with multiple competing intuitions and deal with them in different ways.

Refinements and Reflections upon the Model

The results of the experiment described in this chapter highlight specific fine-grained sensitivities in students' thinking about motion phenomena.

Through much of the analysis presented above, variability and multiplicity in student thinking seems to be a major theme. Distributions of student answers varied when features of the presentation of problems were changed. Distributions of students' explanations also show variation across different versions of the question even for same answers. Even in looking for patterns of consistency across the question, there were few students providing answers that were either consistent with a Newtonian understanding or a Naïve theory understanding. Lastly, there were even cases of individual students showing variability among the particular intuitions in our model within single problems.

The toy cognitive model that was proposed to characterize student thinking prior to the experiment consisted of just three intuitions for thinking about the interrelationships among speed, distance, and time. The experiment was designed in order to bias students toward and away from two (among many possible stable states). Distance cues were hypothesized to bias students toward thinking that more distance would imply more time and away from thinking that more speed would imply less time. Speed cues were hypothesized to bias students toward thinking that more speed would less time and away from thinking that more distance implies less time. This simple model along with these assumptions was able to anticipate some of the observed variation in distributions of answers.

The model, however, was certainly not intended to capture every aspect of student thinking. Hence, I have referred to this cognitive model as 'toy' model. The model reduces the complexity of student thinking to the cueing and activation of just a small set of intuitions. The model was able to capture many aspects of students' explanations and predict how variations to the presentation of question would affect the distribution of answers. However, the analysis above points to many other aspects that student reasoning that are playing a role in the dynamic of settling upon answers. In this section, I would like reflect upon limitations of this toy cognitive model and suggest possible refinements.

I first discuss refinements having to do with intuitions students bring to reasoning about the effects of gravity, speculating on the cognitive basis for student explanations indicating that gravity has less effect upon (or takes more time to effect) objects that have more speed. Second, I offer some speculations concerning the dynamics of how and where students look for consistency in their intuitive reasoning.

Student Reasoning about Speed and Gravity

In both the vertical toss and the horizontal launch questions, patterns of student explanations emerged that could not be easily accounted for in terms of our toy cognitive model. In the vertical toss question, students explained that it would take more time for the ball thrown with more speed to reach the top because gravity would take more time to slow it down. Similarly, in the horizontal launch question, students explained that it would take more time for the balls rolling with more speed to hit the ground because they would fall more gradually. Both of these explanations seem to concern the role of speed in keeping an object from falling down. These explanations also seem rather consistent with McCloskey's observations of student reasoning about motion, which he characterized as denoting a naïve impetus theory. In this section, I'd like to consider some plausible candidate structures that may account for this reasoning.

Plausible Cognitive Structures

Many of the students providing explanations for why having more speed implies taking more time for the vertical toss or the horizontal launch articulate gravity as being an *agent* that acts on the ball. Students are thinking about how gravity exerts influence on the ball in ways that changes its motion. This is in contrast to students merely having the intuitive sense that *going faster implies taking less time* or *going farther implies taking more time*, which concern intuitive sense of kinematical relation. These intuitive senses for kinematical relations may arise with or without students explicitly thinking about any role of gravity as effecting the duration of motion. Thinking about a farther distance simply creates the intuitive sense of taking more time. Thinking about going faster simply creates a sense of taking less time.

Students' written explanations for why more speed takes more time seem to indicate that students are conceptualizing gravity as an *agent,* which has to "work" to pull downward. The pulling downward can have different effects.

In the case of the vertical toss, many students seem to conceptualize gravity as *overcoming* the speed of the ball until it is stopped at its peak. An example of this can be seen in the following student explanations:

> *"The time should be greater, since it will require more time for the force of gravity to overcome the initial velocity"*

> *"Less, a greater initial velocity will allow the ball to overcome gravity more easily."*

In the horizontal launch, many students seem to conceptualize gravity as *deflecting* the path of the ball. An example of this can be seen in the following student explanation:

> *"It will be greater because gravity has to do more work to change the direction of the ball."*

In both situations, a ball moving with more speed means that gravity has to *work harder* (or take more time working the same amount of effort) to either *overcome* the ball (so it stops at the maximum height) or to *deflect* the ball downward until it reaches the floor. Thus, gravity acts as an agent that can either deflect or slowdown by pulling downward on objects.

Such explanation can be viewed as arising from a variety different p-prims that diSessa (1993) used in describing students' thinking about motion. Students conceptualizing gravity as an agent seems to indicate the use of *Ohm's p-prim* in making sense of this cause and effect. Students also seem to be conceptualizing the final event of the motion (either reaching the peak or hitting the floor) as a result of *overcoming* and/or *deflecting.*

As with many of the finer-grained intuitions (or p-prims) that describe students' reasoning, it is difficult and likely unproductive to think about these intuitions as being fundamentally correct or incorrect. Rather, the important thing to pay attention to is when they arise and how they are applied. Thinking about gravity as an *agent* that pulls downward and either *overcomes* and/or *deflects* is certainly not incorrect. In fact, it seems like a strong intuitive basis for qualitatively making sense of the equation of motion $\Delta v_y = -g\Delta t$, and for understanding why projectiles follow parabolic paths. These intuitions do seem to lead students to the correct answer on the vertical toss task (more time for second toss), but to the wrong answer on the horizontal launch task (more time for Experiment 3). It is interesting to note that while in the horizontal launch task, it doesn't take more time for gravity to accelerate the ball through a fixed vertical distance; it would take more time for the ball's deflection angle to change (having more horizontal speed to begin with). So the reasoning that gravity takes more time to affect the faster ball in the horizontal launch also has some truth to it, it just depends on what result one is looking for.

The other category of explanation that emerged when analyzing students' writing from the surveys that involved mentioning gravity was explanations like "time is the same because gravity is the same". I described earlier that it seems unlikely that the knowledge that gravity acts the same (or is "9.8" as many students write) is unlikely to reflect some intuitive ideas about gravity, but rather school learned facts. However, we can still speculate as to why this knowledge might lead students to think that gravity being the same implies the time being the same. Recall that this category of explanation had some correlation with students' correct answer that the horizontal launch experiments all take the same time and with students' incorrect answer that the vertical tosses take the same time.

It seems that this kind of explanation could stem from the activation of a cognitive element such as *same conditions leads to same results.* Students know that gravity acts the same, because they have learned this in school. Student may even recall the counter-intuitive observation from the shooter dropper demonstration (which was performed in class, showing that dropped and horizontally launched ball hit at the same time). In coming to the wrong answer for the vertical toss questions, students may be merely over generalizing. This over generalization may even be exacerbated by the structure of the survey, which asks students the horizontal launch question before the vertical toss question.

These two patterns of explanation involving gravity give us insight into how we might expand our toy cognitive model to account for broader aspects of students' thinking about the vertical toss and horizontal launch question. By speculating about some underlying structure, we may also gain insight into the design of future experiments that may be used to both uncover other interesting dynamics in the real-time thinking of students' as they respond to these questions and provide us with some evidence to support the inclusion of particular elements that are merely speculative at this point. At the end of this chapter, I suggest some additional experiments that could be implemented.

Cognitive Assemblies of Multiple Elements

One way that the design and analysis of this experiment has oversimplified students' thinking is to overestimate the degree to which a given answer corresponds to the use of a single intuition (or cognitive element from our model). I have attempted to be careful to refer to students' answers as merely being consistent with one intuition or another, and not to equate an answer with an intuition. I have also attempted to be careful to refer to students' written explanations as resulting from dynamics that include the intuitions that arise in their thinking, but are certainly not equivalent to them. This is to say the cognitive model describes underlying structures that give rise to observable phenomena, but they are not equivalent.

The analyses of student answers and explanations in this chapter shows that students are likely arriving at some of the same answers to the questions that were posed through different patterns of reasoning. Students showed evidence of thinking that it takes more time because a ball covers a greater distance when thrown faster. Other students showed evidenced that they think it takes more time because gravity has to work harder to deflect or overcome a greater speed. Other answers also seem to involve the coordinating of multiple intuitions. Some of these were able to take into account such as compensation arguments, and other are not accounted for in terms of the model.

In this section, I would like to briefly offer some further speculation on how students' might be coordinating among multiple intuitions to arrive at answers, such that students' thinking (and thus responses to these questions) arise from assemblies of cognitive elements.

First, consider the answer on the vertical toss task that the time taken to reach the top is the same, independent of how high it is thrown. Two explanations that have accompanied this answer are that the effects of greater speed and greater distance compensate and that gravity is the same. Here a single student explains why they think the time is the same, incorporating elements of both types of explanation:

> *"The amount of time to reach maximum height would probably be the same - because gravity still acts on the ball the same way. The second throw may result in the ball traveling a greater distance in terms of height due to a greater initial speed, but therefore its max height is higher than that of the distance from throw one, therefore time must compensate"*

Here we might model this students' thinking as involving the activation of two different ideas that both support the same conclusion. One idea involves applying the heuristic *same conditions implies same results* along with the school knowledge that gravity is a constant. The other ideas relates to a plausible account of how the times could end up being the same— *more speed implies more distance* and the effect of more distance and more speed compensate. While this student explicitly provides explanations from two of the categories in our coding scheme, it remains possible that other students' thinking reflecting similar assemblies of many different intuitions (even when they only give one explanation). It's also impossible from this data to know if one of these patterns of thinking arose first and then another, or whether they co-emerged.

Similar assemblies of multiple intuitions may comprise students' thinking for other answers as well. For example, in students' answers for the why Experiment 3 takes the most time in the horizontal launch question, we saw a split between explanations that more distance takes more time and more speed keeps the ball up for more time. While explanations were coded as categorically distinct, it seems likely that students' real-time intuitive thinking consists of assemblies of many cognitive elements— *more distance implies more time* and *more speed implies more distance* for thinking about kinematical relation and other intuitions such as *Ohm's p-prim, overcoming, deflecting* for thinking about the role of gravity. This assembly of cognitive elements may provide a local stability of this pattern of reasoning beyond the initial cueing of the readily available intuitions (such as just *more distance implies more time*).

Suggestions for Further Experimentation

The method used in this experiment to measure variability in students' thinking about motion involved varying features in the presentation of physics questions and then randomly assigning these altered questions to students in a population. The choice to vary the two questions in these particular ways was (1) based on a description of a toy cognitive model of students' thinking about motion with particular properties and (2) was based on trying to strike a balance between having enough variation to actually detect any biasing of students' thinking and having subtle enough variations that it plausibly represent students' thinking around a single situation. These experiments were also carried out with a specific population using a particular method of administration.

Having carried out this specific experiment and observed patterns of variation consistent with aspects of our model, I'd like to suggest a few experiments that might bring further insight to the dynamics of students' intuitive thinking about these physical situations.

Further Exploration of Conceptual Dynamics

First, the analysis of students' explanations led to some speculations for refinements to the model that might be able to account for other patterns observed in the data. In the previous section I described some plausible intuitions that might play a role in students' thinking about gravity as an *agent* acting on the ball to overcome it or deflect it (similar in many respects to diSessa's account of students' thinking about the vertical toss). In our experiment, some students explicitly attended to this sense of agency in their explanations, where as other students' explanations seemed to only concern kinematical relations with out any explicit regard for the role of gravity. These refinements suggest that it may be possible to introduce variations of these questions that cue students toward and away from these intuitions for thinking about gravity's role as *agent* for change as well. Some exploratory work may be required to gain some insight into the best way to do this, as was the case with the tipping between strictly kinematical intuitions.

Second, the manner in which we varied the problems was intended to bias students toward or away from the intuition *more distance implies more time* or *more speed implies less time*. We did this by only drawing attention the distance in one version (with both language and visual depictions) and drawing attention to the speed in the other version (with both language and visual depictions). Arguably, this is a not a modest variation. Since we now know that we are able to detect significant variations in students' thinking with this dramatic of a change in the presentation of questions, we might now look for more subtle ways to change the problems and also find sensitivities in students' thinking. A relevant place to begin looking concerns introducing variations that appeal to the specific properties of the intuitions as I have described them.

For example, we might imagine constructing variations of these problem that only draw attention to differences in distance – one that attempts to do this through linguistic cues only (explicitly using polysemous words such as shorter and longer) and another that does this through visual cues only (under exaggerating or over exaggerating the differences in distances traveled). We might also try to manipulate cues to activate *agency* in thinking about gravity. These more subtle variations may give us more insight into the specific features that contribute to the activation of these intuitions. Are these intuitions more strongly bound to linguistic cues or to spatial cues? Are the magnitude of the effects proportional the spatial cues we present?

There are certainly other variations one could imagine exploring in order to get fine-grained details concerning both the structure of this intuitive knowledge and the dynamics by which they are cued. While some of these may not be particularly illuminating for physics education researchers oriented toward improving instruction, they may be of interest to those with orientation toward cognitive psychology.

Dynamics in Diverse Settings and Populations

Another avenue for experimentation on the conceptual dynamics of students' thinking about these situations is to conduct the experiments via different methods of assessment and/or with different populations. In some recent exploratory work involving asking these questions as a part of physics instruction, we asked the two versions of the vertical toss tasks to an entire class using "clickers". "Clickers" are mobile electronic devices that allow students to anonymously answers questions that are posed by instructors in lecture halls. The distance-cueing and speed-cueing versions of the vertical toss question were asked in sequence to students in a calculus-based introductory physics course. Even with these questions asked in sequence (not randomly assigned), differences in the distributions of student responses were observed. These differences were consistent with our model: 21% of students answered that the second toss takes less time in the speed-cueing version and only 2% answered that the second toss take less in the distance-cueing versions; and 78% answering more time on the distance-cueing version and 61% for the speed-cueing. This suggests a certain degree of robustness in our initial results, even when asked in different populations using different methods.

This method to 'tip' students into different ways of thinking about the same situation may also be used across different populations. While across-age studies, such as those carried out by Eckstein (1997), demonstrate changes in the most common patterns of students' thinking across development, the experimental methods used in our study may be used to demonstrate changes in the variability of students' thinking over time. It may be that students steadily and gradually become less and less susceptible to such biasing as they progress in learning physics. It's also possible that students may begin with more stability in their naïve thinking (before any instruction in physics) and only develop such sensitivities in transition to learning more normative physics concepts.

Exploring Other Dynamics

Another pattern observed in students' explanations concerned the kind of knowledge students seemed to using in explaining their reasoning. Some students seemed to be tapping into their everyday experiences in explaining their answers (e.g., a pitcher throwing a baseball), while other students seemed to be explaining more formal ideas (e.g., the acceleration due to gravity is 9.8 m/s^2). These differences certainly reflect different elements of knowledge, but the dynamics by which they are cued may have less to do with patterns of conceptual attention (such as attending to distance or speed) than with other factors such as students' tacit sense of what they are supposed to be doing while answering these questions. Do students see themselves as explaining everyday experiences they are already familiar with or do student see themselves as trying to remember the right physics fact? These issues concern what Hammer, Elby, Scherr, and Redish have described as *epistemological framing.* Colleagues working at the University of Maryland have presented some initial findings (Goertzen, Hutchison, Hammer, 2007) that suggest students can be biased into different epistemological framings while answering questions on worksheets that do impact the answers they give. Given students' relatively poor performance on the seemingly obvious question of vertical toss task, this question seems ripe for exploring epistemological dynamics by which students answer this question differently due how they variably frame the task of completing the survey.

Chapter Summary

In this chapter I have described an experiment to explore dynamics of student thinking about motion phenomena. The experiment and analysis was motivated by an account of a toy cognitive of model of students' thinking about motion that was originally described in Chapter 3. The toy model assumed that students' have a variety of different intuitions for making sense of motion phenomena, of which the model of only specified three for making predictions about the kinds of variability to expect in the experiment. The results of this experiment demonstrate specific contextual-dependencies in students' thinking based on how distributions of students' answers vary when questions are presented in different ways. Variability in student thinking was demonstrated in a variety of ways – in the answers they gave across different versions, in the written explanations they gave across different versions, and even in written explanations for a single question. This analysis supported certain aspects of the toy cognitive model that was originally proposed, as well as suggested the need for refinements of the model to take into account other cognitive structures and dynamics.

This research is certainly not the first to document variability in students' thinking about motion in empirical data (Kaiser, Jones, & Alexander, 1986; Cooke & Breedin, 1994). Nor is it the first to employ fine-grained knowledge descriptions of students' thinking about motion (diSessa, 1993; Thaden-Koch et al, 2004; Paranafes, 2007). One contribution of this research, however, is in documenting a very specific way in which students' reasoning about motion phenomena depends on context and in connecting this variability to a cognitive model. The model we used was even used to predict patterns of variability—not just describe pattern after the fact. Pressing upon the specificity of the cognitive frameworks we employ to predict phenomena rather than merely describe them is certainly generative for research in physics education— not for the purposes of proving that one's framework is the *right one*, but for holding ourselves accountable to a certain level of specificity and empirical rigor. Continuing to explore terrains of fine-grained sensitivities in student's thinking is likely to prove vital in improving our understanding of the dynamics by which student settle into local patterns of thinking and forge new stabilities as they learn across longer time scales.

The research described so far has concerned variability in student thinking in response to what may be called *micro-contextual* variations. In other words, students' thinking was understood to change in response to how words and figures were differently presented on worksheets. These variations were carefully manipulated by researchers in rather static contexts. This is certainly a narrow slice of what many researchers understand as the context of student thinking. Context is certainly construed as being a more expansive and dynamic construct than what has been captured so far in this document (Lave, 1988; Lemke, 2001; Finkelstein, 2005). In the subsequent chapters, I begin to explore these more expansive and dynamic constructs of context and their participation in the dynamics of student thinking through case studies of students' thinking in actual classrooms.

Chapter 5: Student Thinking in the Classroom

Chapter Introduction

In the previous chapter, I examined variability in student thinking that stemmed from the preferential cueing of different fine-grained intuitions that students could bring to reason about physical situations involving motion. The experiment conducted involved biasing students toward and away from just two (of many) different ways of making sense of those physical situations. This variability concerned *individual students* and the *individual cognitive elements* that comprise students' thinking in just the brief moments of privately answering questions on sheet of paper. At this scale of space and time, it was possible to predict certain aspects of this variability (as evidenced in the distributions of student responses) by using just a toy cognitive model of students' intuitive knowledge and some additional assumptions about the effects of parameters we could manipulate. The variability that was observed stands in contrast to many traditional accounts of students' knowledge about motion phenomena that characterized student as committed to more coherent frameworks.

In this chapter, I analyze variability in student reasoning via a very different approach, by exploring student thinking about motion in actual classroom practice employing video analysis techniques. While in our experimental design, we as researchers were able to closely control the conditions students were asked to respond to questions, the classroom is a much more dynamic place. Even in a fixed curriculum, the idiosyncrasies in the trajectories of student thinking and behavior, especially in groups, are plentiful. While we certainly lose control over some of these aspects, we gain, instead, richness in the data of student thinking and behavior that could not possibly be captured through data collected on survey instruments. This richness will be invaluable for characterizing the moment-by-moment dynamics by which students settle into and shift among multiple ways of thinking and behaving.

Due to the very different nature of this data, the goal of this chapter is to explore variability in student reasoning at multiple scales of space and time, breaking down this analysis of student thinking in three ways. At the smallest scale of space and time, I examine evidence of how individual students bring different fine-grained intuitions to bear when reasoning about motion in short periods of time. This analysis is similar to our previous analysis in that it concerns students' immediate responses to questions posed. At a slightly broader scale, I examine collective patterns of student reasoning which appear to involve multiple intuitions cohering together over slightly longer times scales. In particular, I describe two distinct patterns of reasoning that both appear to exhibit various degrees of stability under different conditions. Similar to the way students could be biased toward and away from thinking in the experimental design, we see evidence that students dynamically shift between these two broader coherences of thinking.

Lastly, at an even broader scale of space, I offer an analysis of how these patterns of reasoning emerge from dynamics taking place among individuals and between individuals and material artifacts. In this account, structures that assemble in space play an integral role in the account of the cognitive dynamics.

By the end of this chapter, we'll have in place a better understanding of the variability and coherences of thinking evident at these multiple scales as well as some evidence for mechanisms that account for why these patterns of thinking cohere together and exhibit the stabilities they do. In order to get there, I begin by reflecting upon the diversity of ways in which researchers have accounted for why patterns of thinking hold together and exhibit stability.

Accounting for Variability and Stability

We will be building our account of student thinking in this chapter beginning from very local moments of reasoning and expanding to somewhat broader patterns of reasoning that cohere over time. In this section, I describe various approaches to accounting for issues of consistency, stability, and coherence in students' reasoning. There are certainly many different approaches not considered here. The approaches described here fall into two categories: explanations in terms of the structure of knowledge and explanations in terms of the contexts that support them.

Explanations in terms of the Structure of Knowledge

One way that researchers have tried to describe consistency or systemacity in student reasoning involves attributing various stabilities to the structures of knowledge that students possess or to processes occurring among knowledge.

For instance, a variety of researchers worked to explain common and robust patterns in student thinking in terms of the knowledge frameworks, misconceptions, or naïve theories that students forged in the past due to their experiences in the world (Driver, 1981; Clement, 1982; McCloskey, 1983). I discussed many of these examples in Chapter 2 and 3, along with many of the criticisms that such accounts have faced, including the difficulties and failures of these descriptions to account for the variability of students' thinking.

Other researchers have also attempted to describe persistent stabilities in students' thinking by alluding to the stability of some structure of the mind, even if these structures are not necessarily conceptual in nature.

For instance, researchers have described some of students' persistent difficulties in physics in terms of having inappropriate ontological classifications (Chi, 1992; Chi & Slotta, 1993; Chi, Slotta, & De Leeuw, 1994; and Slotta, Chi, & Koram, 1995). For example, such accounts explain students' persistent difficulties with thinking about concepts like force or current as stemming from students having tacitly categorized these concepts as belonging to a *matter* category instead of a *process* category. In other words, they about force and current as if they were both "stuff". As a result, even if students' thinking about these concepts isn't fixed completely, it is highly constrained by the category to which it has cognitively been assigned. These accounts have been criticized as well for being overly static accounts of student reasoning about such concepts, failing to recognize the ontological flexibility of both experts and novices in thinking about such concepts (Gupta, Hammer & Redish, 2009).

Other accounts of the stability of students' reasoning have been explained in terms of stabilities in learner's *epistemological presuppositions* (Vosniadou, 1994). In one such account, childrens' stabilities in reasoning about force are explained, not in terms of a misclassification of force, but as arising from an epistemological stability in childrens' thinking that motion is a phenomena that needs to be explained (where as in a Newtonian Framework, only a change in motion needs to be explained). These accounts assume that the individual has some fixed epistemological supposition about the concept that (like Chi's ontological categories) highly constrain the individual thinking. Some of the stability that Ioannides and Vosniadou (2002) attributed to developing ideas about force have been challenged two quasi-replication studies (diSessa, Gillespie, & Esterly, 2004; and Gokhan & Clark, 2009).

From a complex knowledge systems or knowledge-in-pieces framework, researchers also aim to explain stabilities in individuals thinking in terms of the structure and relations among elements in conceptual ecology. One such account is diSessa and Sherin's (1998) coordination class, which is intended to be a model for a concept that allows an individual to "see" the same information across a variety of contexts. The coordination class consists of both knowledge pertaining to how to get information from the world (readout strategies) and knowledge pertaining to what that information means (causal net). The complex relations among these different kinds of elements allow the individual to see the same thing in different contexts. Wagner (2006) stresses that coordination class accounts differs from many other accounts since it described how the structure of knowledge manages to deal with context rather than ignore it.

In a related way, knowledge-in-pieces accounts have described stabilities in student reasoning is in terms of reliability feedbacks among elements of knowledge. DiSessa (1993) describes *reliability priorities* as a property of knowledge in two ways: phenomenologically as the likelihood that some structure of knowledge remains active once it has already been turned on, and structurally in terms of the number of network pathways that lead away from that element and then back to it. In this sense, an element may contribute to its own stability (once activated) by activating other elements of knowledge that feedback into the activation of the initial element. The more pathways lead back to that element, the more reliable that element is.

Knowledge-in-pieces accounts have also been used described local stabilities of student reasoning in terms of dynamic epistemological stances that students take on. Hammer, Elby, Scherr, and Redish (2004) describe a physics student whose entire approach to thinking in a physics classroom changes by tapping into different epistemological resources (Hammer & Elby, 2002), resources he readily employed in other contexts. Scherr and Hammer (2009) describe how students' working in groups work within different epistemological framings that influence the substance of their reasoning about physical phenomena. These accounts do not assumed fixed epistemological beliefs that provide stability, but stabilities arising from context-dependent epistemological framings.

All of these explanations concern the properties and relations of cognitive elements that individuals possess. In this sense, patterns of reasoning are held together by the nature of the cognitive elements – whether they are conceptual, ontological, or epistemological in nature. In some of these examples the reasoning is held together because the knowledge is belief-like and robust. In other examples the reasoning is held together by being constrained by other related knowledge, and in other examples the reasoning is held together through mutual interactions with other elements of knowledge that make the entire reasoning self-sustaining.

Explanations in terms of the Contexts that Support them

Other ways of accounting for patterns of student thinking that hold together concern the ways in which contexts support that thinking, what Hammer, Elby, Scherr, Redish (2004) refer to as contextual mechanisms. For example, many of us have had the experience of being able to make sense an idea in a lecture, only to walk away and not be able to create that understanding, even moments later. In this sense, the context of listening and engaging with the speaker provides a context for stabilizing one's thinking. Once that context is removed, the pattern of thinking can't always be held together in the same way.

At the simplest level, we have already explored in this dissertation how contexts can initially cue a pattern of thinking. This was the case with students thinking that throwing a ball with more speed would reach its peak in less time. We don't think that students respond this way because they have some fixed stability of their knowledge concerning vertically tossed balls. Rather the local stability of this reasoning arises from how features of the context support the activation (and possibly the sustained activation) of a finer-grained stability of the mind. We don't expect, however, that all patterns of thinking that are initially cued by features of context will exhibit stability for very long. For example, in the vertical toss question, we saw examples of students initially reaching one conclusion (based on intuitions with high cueing priorities) only to settle upon a different way of thinking about the problem moments later.

Contexts can also support broader stabilities of thinking and behavior by providing settings for distributed activity. In Chapter 2, I reviewed accounts of thinking and learning from distributed and situated perspectives. In this review, I described evidence for a rich phenomenology of activity-dependent arithmetic. For example, Lave (1988) described individuals that could perform identical mathematical operations in the context of bowling or grocery shopping that they could not perform in written assessments. In these cases, activities such as keeping track of bowling scores or deciding what's the best bargain are contexts that support mathematical practice in ways that is not supported by other contexts (like answering a word problem).

Individuals, however, don't just find themselves in contexts randomly nor are these contexts static. Individuals change the contexts of their activity by altering these environments and the circumstances of that activity. Martin and Schwartz (2005), for example, describe several ways in which individuals interact with their environment as part of cognitive activity. They describe *off-loading* as activity in which individuals who hold fairly stable ideas manipulate aspects of environments with which they are familiar. These individuals do so in ways that make performing some cognitive task easier. In physics, we may think of drawing a diagram on a sheet of paper or blackboard as a way of off-loading some of the cognitive demand for keeping of spatial organization. The authors also describe *repurposing* as activity in which individuals hold fairly stable ideas as well, but in next environments they are less familiar. In these cases, individual may improvise in their use of the environment to support cognitive activity. For example, In Hutchins' (1995) description of the cockpit, he describes how pilots repurposed the use of indicators on speed gauges in novel ways that facilitated their ability to anticipate change in the state of the system.

These two ways of changing the environment, however, concern how individuals use their environments as the result of the stability of the mind (that facilitate activity of expert physicists and pilots), not how contexts themselves work to establish a cognitive stability that may not have an internalized stability. In the context of novices thinking about and learning physics, we don't necessarily expect our students to have stable ideas about physical phenomena or stable routines for changing their environments in support of those ideas. To this end, Martin and Schwartz also argue that there are also some kinds of activities in which both the *ideas individual have* and *the environments in which the activity takes place* exhibit flexibility that allow for different stabilities to arise. They call such activities- those in which internal and external elements of cognition exhibit variability- instances of *physically-distributed learning* in order to differentiate it from more *inductive activities* (where unstable ideas forge stabilities in more stable environments). Because of the dynamic nature of the students' intuitive thinking and the dynamic nature of students' actual classroom activity, it is this dynamic interplay among changing settings for activity and intuitive thinking that will be crucial for understanding of how patterns of students' thinking hold together (or cohere) across moments of time.

Brief summary

In this chapter, the goal is to describe patterns of students' thinking and the dynamics by which these patterns of thinking hold together— what some have referred to as *local coherences* (Hammer, Elby, Scherr, Redish, 2004) and others as *conceptual stabilities* (Leander and Brown, 1999).

The analysis will begin as an examination of students' intuitive thinking in brief moments of time. This analysis serves as a means for identifying and illustrating aspects of the finer-grained intuitive elements students bring to reason.

The analysis shifts to more expansive moments of time, describing the structures of intuitive thought that comprise broader patterns of thinking that seem to persist and exhibit some stability. Engaging in this analysis will involve attending to two different kinds of mechanisms that may contribute to the stability of students' thinking. One kind of mechanism contributing to stability of reasoning involves the interrelations among internal cognitive elements that individual have (such as the intuitions students bring to thinking about motion). The other kind of mechanism concerns the ways in which external elements in the world provide contextual stabilities.

Context and Setting for Research

The cases that form the basis of our analysis of student thinking in this chapter come from videos of student discussion in tutorial. Tutorials are now a common, research-based curricular module whose goal is to help students to develop stronger conceptual understandings of physics concepts. Tutorials, like *Tutorials in Introductory Physics* (McDermott, Shaffer, and the Physics Education Group at the University of Washington, 1998) and *Maryland Tutorials in Physics Sense-making* (Elby et al.), are carefully crafted worksheets that students work on collaboratively in small groups. They usually target specific conceptual difficulties that have often been documented in the literature. They typically consume about one hour of instruction and were designed to be a replacement for more traditional recitation sections at the college level, where students traditionally have mostly passively listened to teaching assistants explain how to solve homework problems. In tutorials, teaching assistants typically function as facilitators of student discussion rather than as lecturers. Students work together through questions on the worksheets, and TAs circulate to help students make progress.

Instructional Setting

Tutorials as Instructional Setting

The following examples of student thinking about motion in tutorial are drawn from undergraduate courses at the University of Maryland. Students enrolled in the algebra-based introductory physics courses at the University of Maryland (PHYS 121) spend three hours in lecture with a professor. The lecture portion is usually independently taught by several different professors in a given semester. Regardless of which professor students are assigned to, students spend an additional two hours in laboratory and additional one hour in a recitation section with a graduate teaching assistant (TA). In the fall semesters, the recitation sections for this course are structured around the use of *Maryland Tutorials in Physics Sense-making*, a research-based curriculum modeled on Tutorials in Introductory Physics. In these tutorials, students work together through a guided-inquiry worksheet. These worksheets emphasize conceptual topics in physics that students often struggle with.

Graduate assistants in tutorial function as facilitators for student discussions as the students work through these worksheets.

Maryland Tutorials in Physics Sense-making are geared toward the needs of population that typically takes the algebra-based course. The tutorials explicitly address conceptual content for topics ranging from kinematics, dynamics, energy, and waves. The tutorials specifically attempt to build on students' intuitions, with the intent of helping them to think about their own intuitive thinking. Rather than emphasizing how their intuitions are inadequate, the tutorials try to foster a sense that everyday intuitions are useful but often in need of refining. For example, in a tutorial about Newton's 2nd law, students are led to refine an intuition that a *force is needed for motion* rather than to reject it outright. They are guided to see that this intuition can be refined slightly to produce an idea more consistent with Newton's 2nd law that a *force is needed to initiate (or change) motion*. This attention to students' thinking and the need for refinement is made explicit in the curricular materials themselves.

Additionally, the tutorials often address epistemological issues with the learning of physics. The tutorials at times ask students to think about and to discuss issues relating to their own learning of physics. For example, in one tutorial about drawing force diagrams, students are asked a question about why it's useful to draw force diagrams (if at all). In another tutorial, students are asked to reflect upon the usefulness of continuing to think about a question whose answer was known after completing the first page. Similar sorts of questions are distributed throughout the collection of tutorials in hopes of helping students to develop productive attitudes and habits of mind about their own thinking and learning in physics.

The Meaning of Speed Tutorial

The tutorial that is the context for this research is one designed to help students make sense of some basic kinematical concepts with which they are likely to have some familiarity. “The Meaning of Speed” (or “Speed”) tutorial is the first tutorial of the semester for students enrolled in the algebra-based introductory physics course. The tutorial focuses on the use of tickertape representations of motion to help students develop and articulate an understanding of the concepts of constant, instantaneous, and average velocity and to apply them to make sense of various physical situations involving motion. The first page of the tutorial is shown below (the entire tutorial can be found in the appendix).

Prior to the tutorial, a long strip of tickertape was generated by means of attaching the tickertape to uniformly accelerating cart. As the accelerated cart moves, it pulls the tickertape through a tapping device that leaves dots on the strip every $1/40^{th}$ of a second. This long strip of paper is then cut into smaller sections that each contains only six dots. This means that the strips are cut to different lengths that represent different speeds. Cutting it so that each contains six dots makes it so that each section represents the same interval of time (approximately 5/40th of a second). Because the strips represent such a very short period of time, each strip appears to represent constant velocity, despite the fact that it was generated from a cart that was accelerating (changing speed). In the context of the accelerating motion represented by the entire strip, each small segment can be understood to represent the instantaneous velocity of the cart (since it’s over a very small interval of time).

At the beginning of the tutorial, each student is given a single segment of tickertape (which appears to represent uniform motion). Since students are working in groups of four, the group has four segments in total. In the tutorial, students are told that the segments of paper represent the motion of a cart. They are told that the strips of paper were generated by being attached to a cart which pulled the strips through a tapping device and that the tapping device leaves dots on the strip at a constant rate. However, students are not told that the segments of paper are, in fact, parts of a larger piece that representing changing speed. Students discover this later in the tutorial.

I. Recording motion with a tapper

Physics is, to a large extent, the study of the motion of objects. There are any number of ways to record the motion of a cart. One simple method is to attach a ribbon of paper so that it drags behind the cart and through a tapping device; the tapper leaves dots as it taps the ribbon at a constant rate. Your TA has one of these devices for you to examine.

To begin this tutorial, each person in your group will need a ruler and at least one segment of paper ribbon from the staff. All the paper segments were generated using the same tapping device. Please don't write on or fold the paper segments – other classes need to use the same ones.

A. Compare your paper segment with those of your partners. What kind of motion does each represent?

1. How does the time taken to generate one of the short segments compare to the time to generate one of the long ones? How can you tell?

2. Arrange the paper segments in order by speed. How do you know how to arrange them?

B. Suppose the tapper that made the dots strikes the ribbon every 1/40th of a second.

1. How far did the object that generated your paper segment move in: 1/40th of a second? 2/40th of a second? 3/40th of a second?

2. Predict how far the object would move in:

   i. 1 second

   ii. 1/80th of a second

Why are these *predictions,* rather than just calculations? That is, what assumption(s) do you use to make them?

3. Determine the speed of the object that generated each of your paper segments (in cm/s). Write the speed on a small sticky note and attach it to the paper segment.

I briefly describe the questions that are presented to students in the beginning of this tutorial (see Appendix for entire tutorial). I also discuss the answers that students are expected to be able to construct.

At the top of the first page is a paragraph explaining how the strips were generated. In part A, the tutorial asks the students to compare the strips of paper and to answer a few questions. The students are first asked, "What kind of motion does each represent?" This question is intended to orient students to the fact that each strip *appears* to represent constant speed or uniform motion, because the dots are equally spaced out.

The next question asks, "How does the time taken to generate one of the short segments compare to the time taken to generate one of the longs ones." This questions is meant to direct students attention to the fact that each of the strips represents the same amount of time because 1) there are the same amount of dots on each strips and 2) the tapping device makes those dots at a constant rate. Either of these explanations might be viewed as adequate answers to the question asking students, "How do you know?"

Assuming that students recognize that the strips of paper each represent constant speed for the same interval of time, students are then asked to "Arrange the paper segments by speed." Students are expected to be able to come to the conclusion that the longer strips represent greater speed. Because each of the strips represents the same amount of time, the strip that was made when the cart was traveling with a greater speed should be longer. Students are also again asked how they know.

In the next part of the tutorial, the students are asked some more quantitative questions after being given a specific rate of dotting (one dot every 1/40th of second). They are asked to calculate various quantities related to their own strip of paper. Students are asked how far the object (that generated their strip) moves in 1/40th of a second, 2/40th of a second, and 3/40th of a second. Students are expected to do this either by measuring the three relevant distances (or by scaling the first measurement appropriately). Students are then asked to predict how far the object would move in 1 second and in 1/80th of a second. They are expected to be able to this by appropriately scaling their measurement for the 1/40th of a second (by multiplying by 40 and dividing by 2).

The next question asks the students to explain why the last two calculations were predictions rather than just calculations, and also to explain what assumptions went into making these predictions. Students are expected to realize that these are predictions (not just calculations) because there is no physical record (i.e., dot mark) indicating where the object was located at those specific times. It is an assumption that the object was traveling at constant velocity for all times during the first second. This assumption allows students to infer how much distance was traveled for both times located between dots (1/80th of a second) and for times beyond the strip of paper (1 second).

The last question on this page of the tutorial asks students to determine a number for the speed represented by the strips of paper. Students may realize that they have already calculated the number when they answered how far the object goes in one second. Alternatively, many students use the formula v = x / t in order to calculate the speed represented by their strips.

Collection and Selection of Data

For this and other research, our research group has set up two video cameras in the classroom where tutorial are run in order to document student discussions at two different tables. Each video camera is set up so as to capture a single group of students working during an entire tutorials, and often for tutorial sessions throughout an entire semester. Each video camera is mounted on a tripod that is located on the other side of the room. The video camera is far enough away from the students that it is not invasive to the their work. A built-in microphone is covered by a small cage at the center of the table, which captures the student conversations. The cage takes up a small amount of space on the tutorial table. The cameras are typically turned on at the beginning of the tutorial and run the entire time with no person required to man them.

Since the video cameras record student groups from across the room, certain actions are sometimes difficult to observe. For example, it is impossible to tell from the video what students write in their tutorial worksheets, unless students read out loud what they are writing or have written. Also, because of the positioning of the camera, one pair of students in a group is always in front another pair of students. At times, the bodies of the two students in the front block the view of the students who are behind them. Such obstructions are minimized by angling the cameras downward on student groups from above, but they still occur. Physical space restrictions in the room means that it is not always possible or practical to position the camera in a way that allows for ideal capture of video. Additionally, at times students or teaching assistants may locate themselves somewhere that obstructs the view of groups being videotaped. This often happens when teaching assistants are interacting with the students at a table being videotaped.

Students who participated in this tutorial for the purpose of this research were videotaped either during the fall semester of 2006 or during the fall semester of 2007. In total, forty-two different student groups were recorded as they worked through "The Meaning of Speed" tutorial. Of these forty-two original tapes, thirty-one were of sufficient quality to be included in the sample. Reasons for excluding tapes from the sample include tapes with off-centered image (that excluded one or more students from the picture), tapes with poor sound quality, and tapes with no students present at the table. Several tapes were also excluded because teaching assistants did not implement the tutorial as directed. For example, instead of having students work through the tutorial together in small groups, one TA gave a twenty-minute lecture on kinematics. From this sampling, student conversations were transcribed.

Analysis of Fine-grained Intuitions

In this section, I discuss patterns of student thinking in the "Speed" tutorial in terms of how students use various intuitions from our toy cognitive model in order to make sense of the questions that are presented to them. In essence, the analysis described here closely ties to our analysis of students' explanations in the previous chapter, concerning the substance of student reasoning in response to survey questions. Attention is given to how particular patterns of student thinking coincide with particular patterns of attention (as evidenced by gestures, gaze, and also verbal statements). In this section, descriptions of student thinking about the tutorial are broadly organized by the particular tutorial questions.

Arguments are built that (1) students make sense of the motion phenomena and representations in this tutorial using many of the same intuitions that describe students thinking in response to the kinematical surveys and (2) that the cueing of these intuitions are influenced by the similar attention-driven dynamics described earlier.

Intuitive Thinking about Time Ranking

One of the first tasks students are asked to complete in the Speed Tutorial is to order the strips of paper by the amount of time taken to generate them. The exact phrasing of the prompt is, “How does the time taken to generate one of the short segments compare to the time to generate one of the long ones? How can you tell?” The correct answer to this question again is that all of the strips represent (approximately) the same amount of time, because each strip contains the same number of dots that were made at the same rate of dotting.

One particularly relevant feature of this question is that the students are asked to make the time comparison between *shorter* strips and *longer* ones. While the question could have referred to the fact that the students have different strips without pointing out any specific characteristics that make them different, the particular phrasing of this question draws attention to differences in distance.

In many respects this makes this question similar to the distance-cueing horizontal launch question. The correct answer is that the time is the same, distance and speed co-vary, and the question draws attention to differences in distance. For both questions, compensation arguments may lead students to a correct answer in these questions. However, attention toward other aspects of the situation is essential to understanding the mechanisms responsible for why the times are the same. In the horizontal launch question, reasoning that the time is the same involves reasoning about aspects of the vertical motion. For the tickertape representations of motion, recognizing that the time is the same involves attending to the fact that the tapping device taps at the same rate for each strip and reasoning that, therefore, the same amount of dots implies the same amount time. Based on the similarities of these questions, we should expect to find at least some similar reasoning patterns.

More Distance Implies More Time

Based on the toy model of student thinking, it is not particularly surprising that a common response to this question is that the shorter distances take less time and that the longer distances take more time. Drawing attention to the distance features of the strips, according to our toy cognitive model, increase the probability cueing the intuition that *more distance implies less time* (when compared student responses to a similar question that does not specifically draw attention to this feature). More specifically, the question uses the particular words 'short' and 'long' in pointing out this difference. The activation of this intuitive idea in response to these cues occurs despite the fact that the tutorial has just explained how the strips were made, which would suggest that the times are the same.

Below, I present several examples of student responses to the time ranking question that are highly indicative that these students are cued into the intuition that *more distance implies more time*. I use these examples to illustrate several features of these responses that are characteristic of student responses using this intuition:

> *"Isn't the time to make a short one, shorter, than to make a long one?"*
>
> *"I was thinking it would be shorter on the shorter segments like"*
>
> *"The longer it is apart, the longer it takes"*
>
> *"It takes longer for the longer ones and shorter for the shorter ones"*

In each of these examples above, students use the words 'short' or 'long' twice in close proximity to each other. In one place, the word is seemingly used to refer to distance – as in "shorter segments" or "the longer it is". In the other place, the word is seemingly used to refer to duration – as "would be shorter" and "longer it takes". This particular use of language by the students is consistent with the substance of the intuition (that *more distance implies less time*) and consistent with the property of this intuition (that it is closely connected to this particular linguistic structure). By using the same word (e.g., 'longer' or 'shorter') to describe both the distance and the time, in this particular grammatical construct (e.g., 'longer for longer' and 'shorter for shorter'), the language is suggestive of the very kinematical relation that is trying to be expressed.

Another property of student responses relates to the manner in which students state this intuitive idea. In students' written explanations on the written survey (previous chapter), we observed variation in the manner in which students expressed the intuitive idea that more distance implies more time. Sometimes students expressed them as formal proportions, while other students stated them as being more related to their common sense. Consider the following examples of students talking to other students about their answer to the time-ranking question:

> *"How does the time taken to generate one of ...[mumbled reading of the question that trails off]... Obviously it takes less time to generate the more closely spaced dots."*

> *"How does the time taken to generate one of the shorter segments compare to the [several second of silence while reading rest of question quietly]... it's shorter!"*

The two examples above illustrate two features that are common to the manner in which many students respond when using this intuition. First is that students often state their answer to the question immediately after reading the prompt. Students do not seem to deliberate over the question, nor do spend any time contemplating their answer after stating it. In this sense, it is a 'knee-jerk' response. Students read the question, and immediately respond with their intuitive sense that the shorter strips take less time.

The second property is related to the sense in which students state the answer as if it were obvious to them. In the first example, the student explicitly prefaces their statement with the word 'obviously'. In the second example, the student uses a particular rising and falling pitch when saying, "It's shorter". The student also draws out the *i* sound in "its" and the trailing *r* sound in "shorter" The sing-songy nature of his response seems to suggest that the student views the answer as obvious as well. Although more formal mathematical sounding were observed in the surveys, no instances of students expressing formal mathematical relations were observed during this portion of the tutorial. This may have to do with the fact that students are expressing their ideas to their peers, rather than as writing an explanation for an authority (which may well be how some students perceive the audience for their explanation in the survey experiments).

These two features of student responses – the short duration of time between reading and responding and the linguistic (and paralinguistic) cues suggesting students view their response as obvious – are likely related. In fact, students may subconsciously use the duration of time to reach an answer as an indicator that this kind of knowing is 'obvious'. In this sense, we not only observe the rapid convergence of their thinking here, but we observe a bit of students' intuitive knowledge about their own knowing. In other words, many students seem to know that this kind of knowing is the 'obvious' kind.

In the following exchange between two students, we can observe all three of the above-mentioned features of student responses:

Student1: *How does the time taken to generate one of the short segments compare to time to generate one of the long ones?*

Student2: *Shorter? [Laughing]*

Student1: *Yeah isn't it pretty much the shorter ones are shorter.*

The example has the properties of a rapid response. Student 2 immediately responds to Student1's reading of the question with "Shorter". Student2's statement also is said with a tone suggesting he thinks this is obvious. While the Student2's tone of voice initially seems to indicate a question, the laughter afterwards seems to better indicate that he is actually scoffing at something. Here I am suggesting that the student is actually laughing at how mundane the question is for having such an obvious answer (rather than him questioning his own answer). In this sense, not only is the question obvious, it's too obvious. Independent of what the student's actual reason for the questioning tone and laugh may be, Student1 seems to join in with this interpretation of the question and answer as being somewhat obvious, by saying, "Yeah, isn't it pretty much". The explicit use of the words 'pretty much' together with the negation ("isn't it") suggest that the answer is obvious to Student2 as well. Finally, this example illustrates the linguistic properties of student responses in their multiple uses of the word shorter (e.g., "shorter ones are shorter")

Of course, not all student responses using this particular intuition share all of these properties – rapidity, obviousness, and polysemy. Many student responses seem to just clearly indicate the substance of the intuition, connecting distance and time as directly related. For example, one student merely states, "I guess it would take less time to generate that because it has less space to go through". Another student states, "It's shorter, so it means there's less time between the two." While many of the other examples seem to emphasize the linguistic structure of students' statements, these examples have a more visual aspect to them. The first example alludes to an amount of 'space to go through', implying that there is some movement through that space. In the second example, the student mentions “between the two”, possibly alluding to a beginning and end. Observations of students using this intuition while also referring to visual depiction of space and motion is not surprising either. In our attempts to tip students toward this intuition in the survey experiments, we included visual depictions (visually indicating the length that the ball travels off the edge of the table or up in the air) that emphasized differences in distances as well as language. In fact, it is arguable that the most obvious visual characteristic of the strips is that they have different lengths.

One last feature of student responses is worth mentioning before going on to other intuitive responses to the time ranking question. In chapter 3, I discussed how the intuition *more distance implies more time* may or may not be activated with other ideas about when such intuitive knowledge is applicable. For instance we observed students imposing that a ball thrown with different speeds reaches the same height in concluding that the faster one takes less time. In all the above examples, however, students seem to rapidly respond that shorter strips take less time, for which there is little subsequent discussion about the answer. The intuition that *more distance implies more time* can always be correctly applied when comparing motions with the same (or similar enough) speeds. Therefore, the intuition fails to apply correctly in these cases, because students are comparing the amount of time for strips of paper that represent motions with different amounts of speed. In the above example, and throughout the entire corpus of data, there were few examples of students discussing the fact that this idea (that more distance implies more time) only applies when comparing similar speeds.

In contrast, a student says, “If it was based solely on the length, the longer ones would be longer. But I mean you can’t tell how fast the thing was going.” In this example, the student seems to be aware that one would need to be able to know something about the speed (how fast the thing was going) to conclude that the longer ones take more time. In this case, the student cues into the kinematical intuition that *more distance implies more time*, and even uses the word “longer” twice (as is common). However, the student not only recognizes the intuition they are thinking about, but also the idea that it only applies when you know something about the speed. It doesn’t seem that the student has (yet) recognized that you actually can tell how fast the strips were going based on their length, but they do realize the speed is something they will need to know. In doing so, the student seems to be going beyond the initial knee-jerk response that *more distance implies more time*. The student reflects upon what they are thinking and if and when on this intuition would imply.

What we do not know from this example is why this particular individual activates knowledge about the applicability of the kinematical intuition, while many of the other students do not. It is unlikely that the other students simply do not have the same resources that this student demonstrates for thinking about one’s assumptions or the need to be careful—both may be considered productive stances to take at times in the doing and learning of science. Perhaps a more plausible account is related to students’ sense of obviousness of the question and answer. What I mean by this is that students who have an intuitive sense that the answer is obvious would probably be less likely to reflect upon their own reasoning and go look for reasons to doubt (or confirm) their intuitive sense. If one thinks that the answer is obvious (or trivial), there is little need to further reflect upon the thinking that generated one's answer.

All the examples discussed above are consistent with students cueing into the intuition that *more distance implies more time*, an intuitive knowledge element that is a part of the toy cognitive model. Several properties of students' responses are noted: the use of polysemous language, the rapidity of student responses to the question, and the obviousness with which student present the idea. Other intuitions from our toy cognitive model also arise in students thinking about motion as well.

More Speed Implies Less time

Responses to the time-ranking question consistent with the intuition that *more speed implies less time* were not nearly as common as students responding with the intuition that *more distance implies more time*. Here I discuss a few examples of students using this intuition and point out some of the features that characterize student responses in this tutorial.

The language of the time ranking question, which asks students to make the ranking for long and short strips, draws attention to the different distance of the strips. There is nothing about the strips themselves that is apparently related to speed (e.g., there is no dynamic visual depiction of any motion), nor does the language of the time-ranking question directly draw students' attention to the fact the strips (might) differ in speed. Given there is nothing directly pointing to the speed of the strips, we should expect to find patterns of thinking that include thinking that *more speed implies less time* to involve students chaining together several intuitions. For example, when reasoning about the ball rolling off the edge of the table to different locations on the floor below, students reasoned that in order to get farther the ball must have been going faster. Going faster, students would reason, it must have taken less time. Given the similarity of the tutorial question to the survey question (asking about time rankings when drawing attention to differences in distance), we should expect to find similar patterns of reasoning about the tickertapes as with horizontal launch question.

The specific ordering of questions in the tutorial first asks students to rank the strips by time and then to rank them by speed. Possibly because of this ordering, it turns out that most student groups have not even begun discussing the speed of the strips when they begin answering the time-ranking question. It turns out that students discussing the idea that *more speed implies less time* are groups either returning to the time-ranking question (after having moved onto later parts of the tutorial that involve thinking about the speed) or are groups that spontaneously began discussing the speed ordering of the strips before reading the time question at all. It makes sense that students who are already thinking about the strips in terms of the speed they represent, would be more likely to coordinate their thinking around the idea that *more speed implies less time*.

One particular clear example of a student using this intuition when thinking about the time ordering of the strips comes from a group of students that has already agreed that the longer strips represent greater speed. When they begin discussing the time ranking question, one student in the group states:

"It takes more time to generate this one [pointing to the shortest one], because it's moving through it slower."

The fact that the student points to the shortest strip is consistent with the groups' earlier agreement that the shorter strips represent slower speed, which was based on the intuition that *more distance implies more speed*. His explanation that the time is less because of its slower speed indicates he is thinking about the intuition that *more speed implies les time*. In this sense, the reasoning stems from chaining together the ideas that *more distance implies more speed* with *more speed implies less time*. Despite having attended to the distance earlier to make inferences about the speed, the student doesn't now cue into the idea the distance would influence the time (e.g., *more distance implies more time*). As described in the model and evidenced in examples from student explanations in the surveys, the cueing of these intuitions need not arise from a logical chain of events or accompany the activation of knowledge concerning the applicability of that knowledge.

A few moments later, when restating his idea to a teaching assistant, the same student asks a question also indicating this intuition:

"Would it be this [pointing to a longer strip] goes through faster, because you can tell it's been moved through faster?"

Here the student switches from pointing to the shorter one as slower, to the longer was as faster, but the idea is the same. These two statements are consistent with each other, coordinating around the same intuition that speed and time are inversely related. In restating his idea this way, we see the use of polysemous language with the word faster. The first use of the word faster in his statement seems to indicate he is referring to time (evidenced by the fact that the question they are trying to answer is about the time), while the second use seems to indicate he is referring to speed (as evidenced by using the words 'moved through'). The statement also emphasizes the structure of the reasoning that we are modeling as involving the activation of two intuitive knowledge elements – first inferring that greater distances imply greater speed, and then that greater speed implies less time. In the second part of his statement, the student states you can *tell* it's been moved through faster (meaning more speed). His use of the 'tell' seems to indicate it's something he thinks is readily observable from the strips themselves. He is reading this information from the strip. While in the first part of the sentence, he is asking the TA if he is correct in inferring that it takes less time (because of this greater speed). In this case, the inference about the speed is given, and he is questioning the validity of the inference.

In a separate example below, two students express the idea that more speed implies less time together in a brief exchange with another student. In this group, the students are returning to the question of the time ranking after having already measured the length of their strips and assigned numerical quantities to the speed of their strips. In doing so, they have also spent a lot of time thinking about the strips as representing speed, and have come to (correctly) agree that the longer strips represent greater speed.

Student1: *So it takes less time to generate...*

Student2: *The bigger segments.*

Student1: *Cause if this is moving 80 cm/s, it's gonna [sic] be generated faster than the 24 cm/s.*

In this example, the two students state in tandem that it's less time for the bigger strips. From the students' first statement (" So it takes less time to generate the bigger segments") it is clear the two students are not cueing into the intuition that *more distance implies more time*. They are making the opposite conclusion. Instead, Student1 explains that it's less time because the strip moving at 80 cm/s is made 'faster' than the one traveling at 24 m/s. The ambiguous use of the word faster here is highly characteristic of the intuition that *more speed implies less time*. We could interpret this statement as saying 80 cm/s is generated with more speed than 24 cm/s ,which then explains why it's less time. Or we could interpret this statement as saying that something moving at 80cm/s is going to be generated in less time than a strip moving at 24 cm/s. Either way the polysemous use of the word faster does not interfere with out ability to understand the idea being expressed – *more speed implies less time*

Intuitive Thinking about Speed Ranking

In this section I discuss students' intuitive thinking about the speed-ranking task. I describe one pattern of student thinking using one of the intuitive knowledge elements from our toy model, the intuition that *more speed implies more distance*.

More Speed implies more Distance

Many students correctly arrive at the speed ranking for the strips by cueing into the intuition that *more speed implies more distance.* There are many examples of students cueing into this idea. Here I would like to point to some relevant aspects concerning students' explanations consistent with this intuition: variation in the distance students attend to, use of the word pulling, and the use of grammatical structure. Here are several examples of student statements:

"The faster it goes through, the longer the distance."

"You pulled it faster that's why the length is greater."

"Is it fair to say that a longer piece of paper was being pulled faster?"

"If it goes faster, then it would generate a longer segment."

In the above examples, all of the students seem to be the thinking about how a greater speed leads to an entire strip of paper being a greater distance. Students' attention here is to the entire length of the strip, and not to the spacing of the dots. However, in the examples below, students seem to be thinking about a how a greater speed implies a greater distance between dots.

“The dots are going, so as it’s pulling through, if it’s pulled faster, it’ll be like, more, more distance.”

“The closer dots are together are slower, because if it’s moving fast, then the machine will hit at larger intervals.”

“If it’s going slower, more dots are going to be closer together.”

“If you pull something through, and you pull it through slowly, the dots are going to be closer together.”

This difference, between reasoning that more speed implies a greater total length and that more speed implies a greater distance between dots, illustrates how the intuition can be applied with attention to different aspects of the physical situation. Either way it leads students the correct answer for the speed ranking, because either way both the entire strips and the space between two dots represent the same amount of time.

The second feature that I’d like to point out is the prevalence of students’ using the world “pulling” when describing their reasoning that a greater speed leads to a greater distance. Students do not only describe the motion of the strip as being faster or slower (e.g., “it’s moving faster”), students describe the action of pulling as being faster or slower (e.g., “it’s pulled faster”).

The last feature that I'd like to point out has to do with the grammatical structure of the statements made by students as they make statements consistent with this intuition. Recall that with the intuition *more distance implies more time*, we saw that many students used the grammatical structures like "shorter for the shorter ones" or "shorter on the shorter ones" and they often expressed this idea as if it were obvious. In the above cases in which students express the intuition that *more speed implies more distance*, students tended use a different kind of grammatical structure. Many students used *if-then* constructions to explain their reasoning, as well as using words like "so", "because" to form more complex sentences. While obviousness seemed to characterize students' use of the intuition that *more distance implies more time*, these intuitions seem to be characterized by the use of hypotheticals. Students aren't just reading off what seems obvious to them. Instead, the language they use suggests that they view themselves as making inferences from what they think has taken place.

Other Students Responses for Speed Ranking

Students also gave incorrect answers to the speed-ranking question. Most often students stated that the shortest strips are fastest and the longest strips are slowest. The basis for students' thinking this is not immediately obvious from any brief statements made by students. For that reason, I defer describing the details of these patterns of thinking for now. An account of this pattern of thinking will emerge in our analysis of more extended student discussions and will involve a new intuition reflecting the idea that *bunched up implies fast.* This intuition is not an element in our original toy model of student thinking about motion. Just as the case with the simple harmonic oscillator and the questions about projectile motion, intuitions specific to the context at hand arise in conjunction with the intuitions described in our toy model.

Brief Summary

The above analysis of students' statements shows that many of the same intuitions from our original toy model of students' thinking are used by students in thinking about questions from the Speed Tutorial, including *more distance implies more time*, *more speed implies less time*, and *more speed implies more distance*. These patterns of thinking are not surprising due to the fact that the structure of the questions in the tutorial are quite similar to structure of the questions we constructed in the surveys.

Many students respond to the time-ranking question with the immediate response that the longer strips take more time. I discussed examples that illustrate how the intuition *more distance implies more time* is readily available to students (and rapidly cued), how this intuition is closely tied to use of the word "shorter" or "longer", and how this intuition is often expressed as an obvious statement. There is some variation concerning what students are paying attention to when making this inference. Some students seem to be paying attention to the entire length of the strips when using this intuition, while other students seem to be attending to the distance between dots. It is certainly arguable that the most readily available cue from the strips themselves is that they have varying lengths. Even the phrasing of the question itself directs students' attention to the length of the strips by using the words "shorter" and "longer". Therefore, it is not surprising that this intuition is commonly cued.

Other students, after having spent time thinking about the strips as representing speed, conclude that the faster strips take less time. The cueing of intuition that *more speed implies less time* after having thought about the strips as representing speed makes sense. Since the strips themselves do not readily give information about speed, students first arrive at ways of thinking about the strips as representations of speed (e.g., *more speed implies more distance*) and then cue into the intuition that *more speed implies less time*.

In an even smaller number of cases not discussed, there is evidence that students arrive at the correct answer that the amount of times taken to generate all the strips are the same via compensation arguments. We also see students concluding that longer segments represent more speed, seemingly tapping into the intuition that *more speed implies more distance*.

These examples demonstrate the multiple possibilities for intuitive thinking that can arise from a relatively small number of intuitions.

Up until this point, I have mostly illustrated the multiplicity in students' thinking from brief snapshots of students' statements in response to isolated questions from the tutorial, only describing patterns that seem to reflect the activation of single cognitive element in the model. In the following section, I would like to demonstrate that these intuitions do not arise randomly. Instead, certain intuitions tend to arise together to form patterns of reasoning that persist on the scale of minutes.

Analysis of Local Stabilities in Student Thinking

In this section, I would like to describe two different patterns of thinking that arise as students think about the tickertape representations of motion in the Speed

Tutorial. Then I would like to construct toy models of these patterns of thinking in terms of two different assemblies of fine-grained intuitions.

The first pattern of student thinking that I described in the previous section involves students' arriving at the incorrect answers that the shorter strips are faster and take less time than the longer strips. The second pattern of students' thinking involves students' arriving at the correct answers that the shorter strips are slower and were made in the same amount of time as the longer strips. Similar to the manner in which we observed variability in individual students' thinking about projectile motion (as evidenced by erasures on the written surveys), case studies of students' thinking about the tickertape demonstrate shifts in students' thinking on the scale of minutes. The goals of this section are to (1) point out defining characteristics of these two patterns of reasoning, (2) describe plausible cognitive models (in terms of assemblies of intuitions) that account for these patterns of reasoning, and (3) suggest some plausible mechanisms that contribute to the stability of these patterns of reasoning. These goals are accomplished through the development of several case studies involving these patterns of reasoning.

I begin this analysis with a presentation of three cases that illustrate the first of two patterns of thinking- one that is common to many individuals and groups of students in the Speed Tutorial. Through a discussion of these cases, I describe key features of this pattern of thinking and present evidence that this pattern of thinking exhibits some stability under certain conditions. A main goal will be to build toward a characterization of this pattern of students' thinking in terms of a real-time assembly of cognitive elements (corresponding to various fine-grained intuitions), and offer a plausible account of its stability in terms of the properties of those cognitive elements and features of the students' immediate context. Along the way, I consider another plausible account of students' thinking and evaluate its merit against these cases.

Each of the three case studies is broken into two parts. The first part involves a single, incorrect pattern of student thinking that is shared in all the cases. In the second part of each case, a different pattern of thinking is described that is representative of students' correct thinking, which will involve their attention to the physical mechanisms that generated the strips.

Here is a brief description of the three cases to be discussed:

Case Study One (CS1) involves an individual student, "Nora"[2], who quickly comes to the incorrect conclusion that the shorter segments of tickertape are faster and take less time. I first discuss features of Nora's thinking and compare and contrast this with features of students' thinking in the other two cases. When we revisit this case later, Nora's groupmates disagree with her and Nora struggles to make sense of particular aspects of their reasoning.

In Case Study Two (CS2), a group of students are led by one student, Mark, into reaching these same conclusions as Nora that the shorter strips are faster and take less time. When we revisit this case later, Mark suddenly realizes that one of their

[2] This name and all subsequent names are gender-indicative pseudonyms.

answers about the speed ranking is wrong and another student realizes that the time ranking is wrong as well.

In Case Study Three (CS3), a different group of students collectively reach these same conclusions about the tickertape strips as well. When we revisit this case, one student realizes that they have also answered the speed ranking question wrong.

Case Study 1, Part 1: Nora’s Initial Understanding

This first case study (CS1) begins by examining the initial statements made by a student named Nora, shortly after the tutorial has begun and the teaching assistants have finished introducing themselves. During this introduction, the TAs say a little about the Speed Tutorial itself, including a description of how the strips of paper were generated. Nora and the other students begin by quietly reading from their worksheets. A few moments later, Nora looks up and reads aloud the time-ranking question from the tutorial worksheet. The transcript presented below picks up after Nora finishes reading the question:

Nora: It depends on how fast it was going, doesn't it? I mean how far apart the dots are.

[No one follows up on Nora’s statement. About a minute passes as the group tries to figure out if two of the strips are identical or just similar.]

Nora: Well, I was thinking it would be shorter on the shorter segments like.

Student 4: Yeah, But by how many times, I guess...

[Student 4 begins counting the number of dots on one of strips. The other students become distracted by a loud student who has entered the class late. The students watch for a while and then reorient back to their worksheets.]

Nora: Well I mean there's six dots.

Student 4: Oh, there's six on both of them.

Nora: Except these are just closer together.

Student 4: Right.

Nora: So I guess this is faster *[points to shorter strip]*.

In this brief scene, Nora has seemingly come to some rather quick conclusions about the first two questions of the tutorial.

From the very beginning, Nora says, "It depends on how fast it was going, doesn't it? I mean how far apart the dots are." By "it depends", it is likely that Nora means that *the time depends*, since she has just read the question asking them to compare the times. She thinks that the time depends, "on how fast it was going". By "it" here, Nora could mean something specific— how fast the strip was moving, how fast the cart was moving, or possibly even how fast the tapping device was tapping. It's also possible she's not referring to any specific motion at all. She's merely thinking that the time depends on how fast *something* was happening. Immediately after saying this, she adds that she means how far apart the dots are. It is not clear whether if by using the word "means" she is correcting what she thinks the time depends on (the spacing of the dots rather than the speed), or whether she is clarifying what is she means by fast (that by fast she means the spacing of dots). Despite some ambiguity, Nora seems to be saying she thinks that the time might depend on how fast *something was going* and/or how far apart the dots are spaced.

No one in the group follows up on Nora's statement about what the time depends on. Instead, the students become engaged in a brief discussion about how the strips of paper compare. It turns out that two of strips given to the group are nearly identical, and the group tries to decide if the two strips are exactly the same or if they are just close. They examine closely where the dots are located on each of strips, realizing that some of the marks on one of the strips are just accidental pencil marks. One student mentions that it probably doesn't matter whether or not they are exactly the same or not, and they reorient back to their worksheets.

A few moments later, Nora looks up and says, "I was thinking it would be shorter on the shorter segments." Here, too, it seems that Nora is offering an answer to the time-ranking question, implying that the time is less for the shorter segments. One of the other students responds, "Yeah, but by how many times?" This student begins counting the dots on one of strips. It's not clear if the student finishes counting the number of dots or not, but the group soon becomes distracted by a particularly loud student who has arrived late to the classroom. They watch this student for a short while and then look back down at their worksheets. After about ten seconds of silence, Nora addresses the group again, saying, "Well, there's six dots, except these are just closer together." Here Nora makes a comparison of the spacing of the dots between one of the longer strips and another strip that is shorter.

She points out correctly that they each have six dots, and identifies the shorter strip as just having dots that are "just closer together".

One of the other students remarks with a little bit of surprise, "Oh, there's six on both of them". Nora nods and then points to the shorter segments, saying, "So I guess this was faster." It's not entirely clear what Nora means by "faster" here either. Earlier she had used the phrase, "how fast it was going". In that statement, it was difficult to know for sure what "it" was referring to. However, she did use of the word "going", which seemed to imply some kind of movement. Here she just states that "this is faster", simply identifying the strip as 'faster'.

As already stated, it turns out that Nora's answers to these questions are not unique. Her thinking about the strips is common to many individuals and groups in the Speed Tutorial. At this point, I'd like to present two other cases of a student thinking that are similar to Nora's, before revisiting all the cases together.

Case Study 2, Part 1: A Group's Initial Understanding

The transcript shown below begins before a different group of students has even started trying to answer any of the questions on the tutorial. One student, Mark, has gathered all the strips in front of him on the table. Another student Kara, seated next to him, is leaning toward Mark and trying to look at the strips. There are two other students in the group, Ryan and Mona, who both speak only once during this first scene.

Mark: Yours is a little bit slower than mine *[referring to a long strip of paper he took from Kara]*

Kara: I don't even know where the dots are on that one. Where are they? I don't even…

Mark: They're right along here *[pointing along strip]*

Kara: Is that one, two, three?

Mark: Yes. Umm 1, 2, 3, 4, 5, 6. 1, 2, 3, 4, 5, 6. 1, 2, 3, 4, 5, 6. 1, 2 ,3, 4, 5, 6. *[pointing and counting along each of the four strips]*

Ryan: Just put them in order by length.

Mark: *[Arranges them in order by length]* This is fastest to slowest *[pointing from shorter to longest],* and they are all six ticks, whatever that means

Kara: They're all six?

Mark: Yes.

Mark: *[reading*] Time taken to generate one of the short segments compared to the… it's <u>shorter</u>!

Kara: Well *[pause while looking at the strips]* …yeah

Mona: Are we saying that it's faster (**Kara:** faster) to do a little one?

Kara: It's faster cause the clicks are closer together, even though it's still six.

Mark: <u>Faster</u>! *[pointing to a shorter strip]* Hey check that out. I already did number two, and I wasn't even thinking about it.

[Students are quiet for about fifteen seconds while writing]

Mark: Arrange your paper segments in order. How do you know how to arrange them. *[reading]* Cause.

Kara: How do you arrange them by speed?

Mark: The distance between dots indicates how fast

At the very beginning of this scene, Mark refers to one of the longer strips as being 'slower' than a strip that is shorter. It is difficult to know for sure what might be leading Mark to identify the longer strip as slower, since there is little elaboration on his thinking. He simply identifies this strip with the word 'slower'.

Kara comments that she can't tell where the dots are on one of the strips. This leads Mark, and to a lesser extent Kara, to examine closely the location of the dots on the strips. Mark counts the number of dots on the strips, counting out loud from one to six on each as he points to the dots. Mark then arranges the strips in order by their length after being prompted by another student Ryan. After doing so, Mark again refers to the shortest strip as fastest and the longest one as slowest. He does this by pointing and moving his finger along the arrangement he has constructed. He then adds "And they're all six ticks, whatever that means." Kara asks, "They're all six?" and Mark confirms this is so.

Mark and then Kara both look down at their tutorial worksheets. After a few seconds, Mark begins reading the time-ranking question from the tutorial worksheet. He trails off a little bit while reading the question and then blurts out, "It's shorter!" He says this with in a very particular way, drawing out the *i* sound in the word "it's" and the final *r* sound in the word "shorter". The tone of voice combined with the timing makes his statement sound similar to a sports radio announcer giving a play-by-play. Kara looks up and says, "Well" with a bit of hesitation in voice. She leans over to look at the strips that are still in front of Mark. She looks at the strips for a few seconds and then leans away while saying, “Yeah’.

Mona, speaking for the first time, asks, “Are we saying it’s faster to do a little one?” Mona’s question to confirm what they are saying about the strips is consistent with Mark’s prior identification of the shorter strips as faster than the longer ones. While Mark used the words, “This is fastest,” Mona here uses the phrase “faster to do”. This particular construction may indicate that Mona is using the word “faster” to mean “takes less time” rather than “going with greater speed”.

Kara affirms this is what they are saying, adding that it is faster because the clicks are closer together, “even though it’s still six.” This is the first time we get an explicit reason for why they have identified the shorter strips as taking less time – the proximity of the dots. However, it seems likely the closeness of the dots has played a role in their thinking about the fastness of the strips all along, since we know that both Mark and Kara were attending closely to the location of dots on the strips around the time of their statements. They also knew that each of the strips varied in length and had the same amount of dots.

Mark, who had been looking down at this tutorial worksheet, suddenly looks up and blurts out “Faster!” as he points to one of the shorter strips. He says this with the same emphasis and tone of voice he had given to the word “shorter” when had given his answer to the time-ranking question. Mark seems to be giving an answer the speed-ranking question, commenting that he has already done the second question, ‘without even thinking about it”. Mark then looks back down at this worksheet, reading aloud in a mumbled voice the speed-ranking question, including the part that asks how you know how to arrange them. Mark blurts out, “Cause”. He says this word with two syllables (“cuh-uzz”), employing the same emphasis and tone of voice given to both of the words “shorter” and “faster”. Mark doesn’t finish his statement. Instead he looks down at his worksheet and continues to write in his worksheet. Kara asks how they know to arrange them and Marks says, “The distance between dots indicate how fast.”

There are various similarities between Nora’s thinking about the tickertape strips and the conversation between Mark and his groupmates described here. Before describing these similarities in detail, I’d like to present one more example of students’ thinking also similar to Nora’s thinking.

Case Study 3, Part 1: Another Group’s Initial Understanding

The transcript below begins as yet another group of students begin the Speed Tutorial. This group, like the other groups, has just finished listening to the TAs introduce themselves and say a little about the tutorial itself. After the TAs are done talking, the group immediately calls over one of the TAs and asks him to explain again how the strips were made. The TA says to them, “So if you read the first paragraph, it says it's connected to the motion of a cart. So the cart was moving and something else was tapping on it. Something else was putting those dots on the cart as it was moving. And so that's how these strips were generated.” The TA walks away and the students begin examining the strips:

Rita: Yours is faster *[pointing to shorter strip of Dani],* and mine is slower.

Fran: Ours look the same *[referring a similarly sized strip of Judi]*

Dani: Mine is faster than yours *[referring to longer strips of Fran and Judi]*

Rita: And mine is the fastest

Judi: And ours look about the same *[referring to strip of Judi]*

Dani: So yours *[points to the shortest strip of Ria]* was faster right?

Rita: Hmm, yeah. Mine was the fastest, so it takes less time.

Dani: So it takes less time, right? You can tell because the dots are closer together. These *[points to dots along longer strip]* are like more spread out.

Rita: They all have the same number of dots. So they are close together *[pointing to dots on a shorter strip]*. But same number of dots.

Judi: So the short segment was moving faster through the thing, we think that right?

Dani: Mm-hmm

Rita: Those two are the same *[points to strips of Judi and Fran]*

Judi: They look like same.

Fran: This one's a little shorter.

Dani: A little shorter.

Judi: Yeah

Dani: Basically they are in size order.

Fran: The speed is increasing with the shorter one?

The scene above begins with the students making comparisons of the strips they each have. Collectively the students make several statements and gestures indicating that Rita's strip (the shortest one) is the "fastest", and that Dani's strip (longer than Rita's) is "faster" than Fran's and Judi's strips which are "about the same" (both longer still). They do this by pointing to each other's strips and saying that one strip is faster than another. It's difficult to know exactly what the students are referring to when they used the word "faster". It's possible that they don't have any one specific meaning. They, like the other groups, are simply identifying the shorter strips as faster and the longer strips as slower.

Rita then comments that her strip (the shortest) is the fastest, adding, "so it takes less time," and Dani adds that you can tell it's less time because the dots are closer together. Dani and Rita here both give different reasons for the why the time is less for the shorter strip. Rita's comment is that it's less time because it's faster. Dani comment is it's less time because the dots are closer together. It may be that Dani is giving what she believes to be an additional explanation for why the time is less. Or it's possible that Dani views the spacing of the dots as indicating the fastness, which explains why it's less time. Either way, Dani appears to be agreeing with Rita about the answer to the question.

Rita then points out that all of the strips have the same number of dots, and then points to a shorter one and says, "They are closer together, but same number of dots."

Judi then asks a question to confirm if they are saying the shorter segment was moving faster through the thing. Here it seems clearer that Judi is using the word faster to refer to some motion. However, the word “faster” still could refer to more speed or less time in reference to that motion. She mentions it being faster to move “through the thing”. By “thing”, it seems likely that she is referring to the tapping device. Dani affirms this is what they are saying.

The students then continue to compare the two strips that are very similar in appearance. Although they had originally said that the two strips were about the same, they now seem to decide that one of them is a little shorter. At the end of this scene, Dani comments, “Basically they are in size order”, and Frannie asks a question to make sure about their answer regarding the speeds.

Characterizing Students’ Initial Thinking

Above I have described three brief cases of student thinking about the tickertapes that all share some common features. In this section, I’d like to point out the details of these similarities and offer two plausible accounts of students’ thinking.

Similarities Across the Case Studies

All three groups seem aware that the strips have varying lengths, and that they each have the same number of dots that are variably spaced. In CS1, Nora comments that they each have six dots and that one has dots that are closer together. In CS2, Mark arranges the strips in order by their length and also comments that they each have six dots. Kara also notes that the six dots are close together on one of them. In CS3, Rita comments twice that they have the same number of dots (and that on one they are just closer together), and Dani notes that the strips are in size order.

All three groups identify the longer strips as “slower” and the shorter strips as “faster”. In CS1, Nora refers to a shorter strip as being faster. In CS2, Mark makes this comparison several times throughout the scene, as do Kara and Mona. The proximity of the dots on the strips seems to play a role in their indications of the strips as being faster and/or shorter. In CS2, Mark and Kara explicitly say this is what indicates that strips are either faster or slower. In CS1, Nora doesn’t explicitly say so, but she makes this comparison immediately after commenting that the six dots are closer together on the shorter one. In CS3, DFJR repeat these claims several times while comparing the strips they each have, although they don’t give any reasons for it.

All three groups express thinking that the shorter strips take less time. Two of groups use the word “shorter” to describe the time as being less. In CS1, Nora says that she thinks it’s “shorter for the shorter segments”. In CS2, Mark blurts out, “It’s shorter,” after reading the time ranking questions. In CS3, Dani explains that it’s less time (for a shorter one) because the dots are closer together.

In a broad sense, the students seem to be mostly examining the physical features of the strips in order to characterize the strips and also to answer tutorial questions. They notice and compare the locations, numbers, and spacings of dots, as well as the overall length of the strips. They even refer to these features in reference to the conclusions they are making about how the time and the speed of strips compare. In CS1, Nora mentions that the dots are closer together in stating that it’s faster. In CS2, Kara mentions that it’s faster because the clicks are closer, even commenting "even though it's still six." Mark also says that the distance between dots indicates how fast. In CS3, Dani says you can tell it's less time because the dots are closer together. In all of these cases, students are making claims about the strips based on their physical features.

Largely absent from students' statements is either talk about how the tickertape strips were made or talk about the motion of the cart (which the strips are intended to represent). Although the students use words like "faster" and "slower", very few of the students' statement seem to be about explicitly describing any translational motion of the strips of paper (or the cart) or about describing the up and down motion of the tapping device onto the strips. If we look at students' statements involving the word "faster" and "slower", we see that the statements are mostly identifying statements. In CS1, Nora says, "This is faster." In CS2, Mark says, "Yours is a little bit slower," and "This is fastest to slowest." Kara also says, "It's faster cause," and then talks about features of the strips. In CS3, the students make similar statements when comparing their strips: "Yours is faster and mine is slower," and "Mine is the fastest." In all of these cases, the students use the words faster and slower as *adjectives* to describe or compare strips. In this sense, it is not explicit in their statements as to what the students are using the word 'faster' to refer. The students may privately have a specific meaning for the word 'faster' that refers to a particular sense of movement. In this case, they may they just be using the identifying statements as shorthand. When saying "This is faster", they might really be thinking, "This was moving faster". Alternatively, it's possible that the students do not have a clear and unambiguous referent for the word "faster" separate from the strip itself being “faster”. In this case, we’d might expect that students if explicitly asked what they meant by faster, then that they would have to do some work to think about what they meant.

Three statements made by the students (one in each case study) stand out in contrast to the above usages of the word faster. In CS1, Nora first asks, “It depends on how fast it was going, doesn’t it?” in reference to the time-ranking question. In CS2, Mona asks, “Are we saying it’s faster to do a little one?” after Mark answers it’s shorter (time) for a shorter segment. In CS3, Judi asks, “So the shorter segment was moving faster through the thing, we think that right?” after other students give reasons for why the shorter strips take less time. In two of these statements the word “fast” or “faster” is used, not as an adjective, but as an adverb. Nora refers to the *going as fast*, Mona seems to refer to the *doing as faster* (even though it’s not used as an adverb), and Judi refer to the *moving as faster.* Also, in each of these statements, the students are asking questions, not making declarations. Nora suggests an idea about what the time might depend upon in the form of a question. Both Judi and Mona are making sure what the group’s answers are.

A Plausible Account

Here I’d like to briefly describe a plausible account of the pattern of thinking that is illustrated in the above examples. This account is based on students having an explicit sense of mechanism for how the strips were generated.

One way in which the students’ statements about the tickertape strips could make sense is if they are privately thinking that the tapping device taps at a different rate for each of the strips and that each of the strips was pulled through the device with the same speed. Of course this isn't how the strips were actually made, but many of them could have easily misinterpreted the description given by the teaching assistants and the tutorial worksheets. This would also mean that the spacing of dots on the strips simply represent the tapping frequency of the tapping device and not the motion of the cart.

If this were the case, all of the students' statements about the fastness or slowness could be interpreted as referring to the rate at which the tapping device taps, rather than the speed of that the strip was moving through the tapping device. It would make sense that higher (or faster) rate of tapping would cause the dots to be closer together. Since the tape moves at the same speed, the strip of paper would have moved less distance before the next dot comes down (because there's less time). Looking to the case studies, we see that there are statements in each case study referring to shorter strips as the faster. There are even statements explicitly saying that it's faster because the dots are more closely spaced. This would be consistent with students' thinking this is how the strips were made.

It would also make sense that a higher rate of tapping would produce six dots in less time. In each case study, there are statements that the time is less for the shorter strips, There is also evidence that each group is aware that there are six dots on each strip. Knowing this information, and thinking that the tapping is variable, would make these statements seem quite sensible.

In addition, the word 'faster' used across many of the students' statements would apply correctly to the strips whether it was being used to mean "greater frequency" or "occurring in less time". This is so because the shorter strips would be ‘faster’ in the sense of representing a greater rate of tapping and in the sense of representing less time (because six dots are made in less time). Based on this interpretation of their thinking, all of statements made by the students can be viewed as correct, just correct about the wrong situation.

It is certainly easy to sympathize with this interpretation of the students’ thinking about the tickertape strips. It offers a coherent account of what the students could be thinking “behind the scenes”, which leads in a logical way to conclusions consistent with their statements. The account values the students as rational agents—persons who have sound reasons for reaching the conclusions they do. It suggests that Nora’s and the other students’ difficulty is a matter of a simple misunderstanding. The problem is that they think that the tapping device has a variable rate, which isn’t true. They need to realize that the tapping device always taps at the same rate and that it is the speed of the strip’s motion that is different.

We should, however, be somewhat skeptical about this interpretation of the students’ thinking, because of certain presumptions that this interpretation makes. First, we might be skeptical of this account because it describes the students’ thinking about the tickertapes as the result of mental processing that has already taking place. The students have each already formulated an underlying sense of how the tickertape strips work. Their statements merely follow from this understanding that is stable. In this sense, it is an account of her reasoning that is similar to accounts of how baseball players catch fly balls by calculating ahead of time where the ball will land. Cognition happens first in the mind, and then behavior results.

A second reason to be skeptical would be that this account presumes that their reasoning follows a logical sequence. Their inferences about how speed and time compare follow logically from the explicit (although incorrect) understanding they have for the situation. In building an interpretation of their reasoning that can be viewed as logically correct, there is a risk of asserting rationality into the cognitive account *a priori*. A more cautious approach may be to account for how aspects of rationality emerge from cognition.

A third reason to be skeptical of this account is that it seems require that all the individuals who participate in this pattern of thinking have the same misunderstanding about how the tickertape strips were generated— each student in the group having misinterpreted the situation as involving a variable tapping device. From this account, the observation that this pattern of thinking seems to include multiple individuals (at least in the second two cases) is explained by having each individual in the group share the same cognitive state. Students agree because they have the same understanding (or are at least unwilling to socially disagree).

Despite these reasons to be skeptical about it, I'd like to keep this interpretation of their thinking in our back pocket. At this point, we don't have any explicit statements from any of these students concerning how they might be thinking about how the strips were made. No one in any of the group mentions the motion of tapping device or the motion of the cart or strips specifically. What we do know is that both groups seems to have oriented toward certain physical features of the tickertape strips—that there are longer and shorter strips, that they each have six dots, and that those dots are variously spaced. We also know that they have some ideas about how the speed and time compares for the strips- the shorter strips are faster and take less time.

In this next section, I'd like to build an account of the students' thinking that avoids the problematic presumptions in the account described above.

A Finer-grained Account

Here I'd like to describe how this pattern of thinking could emerge from an assembly of cognitive elements that occur in real time as students attend to various aspects of context around them. In contrast to the above account of student reasoning in terms of logical inferences following from an explicit understanding for how the strips were made, here I seek to describe students' thinking about the tickertape strips in terms of an assembly of fine-grained intuitions concerning kinematical relations and features of the tickertape strips. The toy cognitive model I describe here consists of two intuitions from our original model and a new intuitive element, which is motivated from aspects of student thinking in the above cases. The three elements are:

- *More Distance Implies More Time*
- *Bunched Up Implies Fast*
- *More Speed Implies Less Time*

The first of the three intuitions is idea that *more distance implies more time*. Evidence that this intuition plays a role in students' thinking comes from explicit statements made by students immediately after reading the time-ranking question. Both Nora (CS1) and Mark (CS2) make statements that the shorter segments are "shorter." The fact that they both use the word "shorter" to indicate the time being less is consistent with the activation of this particular intuition. Dani (CS3) doesn't use the word shorter, but she does explain that she can tell it's less time because the dots are closer together. Here she is looking at not the total length of the strips, but just the small distance between dots to make sense of why it would take less time. It important to remember that the intuition *more distance implies more time* is understood to arise independently from other knowledge concerning its applicability. In this sense, the intuition *more distance implies more time* is activated *not because* it is a logical inference from some explicit understanding (such as variable tapping) or because students believe that the speeds they are comparing are the same. The physical intuition simply comprises the students' sense that the shorter segments take less time, *because they are shorter*. In these moments, it appears obvious to the students that the shorter strips take less.

The second intuition posited to accounts for students' thinking is one that has not been discussed up to this point. The intuition concerns how students make sense of the proximity of the dots on the tickertape strips. Recall that because each of the strips has six dots, the shorter strips have dots that are more closely spaced (and the longer strips have dots are more spread out). The intuition I'd like to suggest that students' use to make sense of this physical feature of the strips can be described as *bunched up means fast (or spread out means slow).*

I have already discussed some of the various meaning of words like "fast", "slow", and "quick" that demonstrate a degree of polysemy with these words. Faster can mean greater rapidity as in greater speed. Faster can also mean taking place in less time or as occurring sooner in time. Faster can also mean occurring more frequently. Parnafes (2008), for example, describes this particular meaning as one of the meanings of 'fast' that students tap into when making sense of oscillatory motions. Parnafes describes "fast" as a word possibly connected to various specific meanings (e.g., more distance in the same time) and also to a general sense of *More X occurring in the Same Y*.

One way of understanding the meaning of fast as 'occurring frequently' is in terms of events happening in rapid succession in time. For example, we might hear someone at the beach saying, "Those waves are coming in fast." This person would be likely referring to the frequency with which waves are arriving (if not also their propagation speed). However, the word "fast" and other related words may also be used to make sense of spatial frequencies. A mountain range in the distance can be described as "having peaks and valleys that rise and fall *quickly*". Musical notation (which represents time as space on a page) exploits this sense of fastness by drawing notes that are supposed to be played "quickly" as bunched up on page. Lakoff and Johnson (1980) have described space as one of the primary metaphors that we use to talk about time, so it may not be surprising to see flexible use of the word "fast" for both temporal and spatial frequencies. In this account of students' thinking, I am using the intuition *bunched up means fast* to denote the sense in which a visual display with many elements in close proximity is perceived as being rapid (or fast) in comparison to elements that are more spaced out (like in the example of musical notation). For the tickertape strips, the shorter strips with the more closely spaced dots would be perceived as being faster than the more spread out dots which would be slower.

Evidence for this intuition comes from students' identification of the strips as being faster or slower than other strips. In particular, they use the words faster and slower as adjectives, which would be consistent with them describing the features on the strips as "fast" rather than as the strips as representing a motion that was "fast". There are also explicit statements made by students that seem to indicate that when they are referring to the shorter strips as being faster than longer strips that they are attending to the closeness of the dots on those strips. This account in terms of the activation of this intuition differs from students coming to this conclusion from a logical inference of variable tapping, since students need only be attending to physical features of the strips (not to any physical sense of motion). Nor does this account require students engaging in a logical deduction. Students merely attend to spacing of the dots and the intuition *bunched up mean fast* is cued. It is a perceptual-linguistic dynamic not a logical-deductive dynamic.

The third intuition playing a role in students' thinking is one we have discussed before. Evidence for the intuition that *more speed implies less time* is not as readily apparent from explicit statements made by students as with the other two intuitions discussed so far. Rita (CS3) is the only student who explicitly expresses that a faster motion would imply less time when she says, "Mine's the faster, so it take less time." Nora's statement (CS1) that "[the time] depends on how fast it was going" provides some evidence that the intuition *more speed implies less time* is playing a role. She doesn't explicitly state that the time would be less because of a greater speed, but she is clearly thinking that a greater speed would have affect on the time.

It seems likely that this is the intuition she is relying on when making this statements. There is no explicit statement from the second case study, involving Mark or any of the other students. It's possible that this intuition still contributes to students' individual or collective thinking without any verbal declarations as such. We might view Mona's use of the phrase "faster to do" to be a reflection of ideas relating speed to time, and thus possibly the intuition *more speed implies less time*.

In terms of this fine-grained account of students' thinking, the first two intuitions I described above, *more distance implies more time* and *bunched up means fast,* concern how students make sense of more-or-less directly observable features of the strips. The students perceive shorter distances as taking shorter amounts of time. The students perceive the close proximity of dots as being faster. The intuition *more speed implies less time* plays a slightly different role in the sense that the intuitive sense of fastness (from the proximity of dots) and their intuitive sense of duration (from the length of the strips) are both connected to the intuition that *more speed implies less time*. We might think of the first two elements as being cued as a result to particular features of the strips, and the intuition more speed implies less time being subsequently activated due to the activation of the first two.

We can think of these three cognitive elements, each corresponding to a fine-grained intuition about kinematical relation, as comprising an assembly of intuitions that together account for the pattern of student thinking in each of the above cases. In Table 7 shown below, I show quotes from each of the case studies that are consistent with students' using these intuitions.

Table 7: Student Statements Illustrating Intuitions Across Cases Studies

	Case Study One	Case Study Two	Case Study Three
Faster implies less time	*"It depends on how fast it going, doesn't it?"*	--	*"Mine was the fastest, so it takes less time"*
Less distance implies less time	*"I was thinking it would be shorter on the shorter segments"*	*"It's Shorter!"*	*"It takes less time, right? You can tell because the dots are closer together"*
Bunched up means fast	*"Except these are just closer together… So I guess this was faster."*	*"It's faster because the clicks are closer together"*	--
	--	*"The distance between dots indicates how fast"*	--

In the following section, I continue along with each of the cases in order to gather more data and re-evaluate our cognitive accounts of students' initial incorrect thinking about the tickertape strips. In case study one, Nora's groupmates are going to disagree with Nora, and provide different reasoning about the strips. In case study two, Mark is going to realize they have made a mistake. In case study three, the group is also going to realize they have reached some wrong conclusions. Examining these cases will involve trying to understand the new pattern of thinking that emerges, which will also help to bring further insight into the nature of students' incorrect thinking.

Case Study 1, part 2: Nora's Groupmates Disagree

At the end of Case Study One (part 1), Nora had just indicated that she thought that a shorter strip was faster. Immediately after Nora says this, her groupmates make it known that they think it's the opposite- that the longer strips are faster. The transcript below follows immediately from where we left off.

S4: Umm.

S0: <u>That</u> one is pulled slower *[referring to one Nora called faster]*

Nora: Which one? <u>This</u> one *[points to longer one]?*

S0: Uhh.

S4: No, that should be faster, yeah?

S0: Yeah the one with more space is faster

Nora: This one's faster? Why?

S4: Because if it's going at the same rate

S2: It's pulled faster

S4: Right, so the pulling should be taking it a little bit faster. If. If it's going slower, more dots are going to be closer together than if it's faster it will be more spread out.

Nora: Oh, OK, OK, OK, OK, OK. I gotcha

[Students look down at their worksheets for a few seconds]

S4: Wait, the time taken. The time is the same.

Nora: No, it's not. The time. So for this *[points to longer one]* one it's slower, right?

S4: Shouldn't the time be the same to generate the six dots? It's just...for being pulled faster

Nora: No but the space...

S2: That's what I thought. I thought he said it was constant, like the same 40 beats per second.

S4: Yeah. That's what I thought.

S0: Yeah.

Nora: So then why are they different?

Immediately after Nora says, “So I guess, this is faster,” one of the students, S4, says, “Umm.” This is said in a hesitant way that suggests S4 either disagrees with Nora or is unsure about what Nora has just said.

S0 makes this disagreement more explicit by saying that the strip Nora had just called faster is actually “pulled slower”. Nora asks, “Which one?” and then points to one of the shorter ones and asks again. S0 says, “Uhh,” in a similarly hesitant tone. S4 then says, “No, that should be faster, yeah?” S0 agrees with S4 by saying, “Yeah, the one with more space is faster.” Here the students are disagreeing about which strips are the faster ones. Nora had said that the shorter strips are faster. The other students are saying that the longer strips are longer.

Nora seems to understand what they are saying in terms of the answer, even if she doesn’t understand their reasoning. Nora asks the other students why the longer one would be faster, and S4 explains, “Because if it’s going at the same rate”. Here it seems plausible that S4 is referring to the constant rate of the tapping device. She could, however, be referring to the rate of motion on strip is constant. S2 also adds that, “it’s pulled faster.”

The three other students have offered three different statements to explain why the longer strips are faster. One is that the strips with *more space* are faster. Two is that it’s *going at a constant rate*. Three is it’s *pulled* faster. S4 summarizes some of these statements together, saying that it’s the pulling that makes it go faster. S4 seem to be talking about what is causing the motion to be faster – the pulling. Then S4 explains that if it's slower the dots will be closer together, and if it's faster the dots will be farther apart. Here S4 seems to be explaining the consequence of the strip being pulled faster or slower.

In response to these explanations, Nora simply says, "Oh" and then repeats the word, "OK" six times in rapid succession before ending with, "I gotcha." It's hard to know if Nora is genuinely conveying that she has made sense of what they are saying, or if she is just saying this in order to avoid having it known that she doesn't understand. The students seem to take it that Nora does understand what they have said. They all look back down toward their worksheets as if to continue along.

S4 suddenly pops up and says, "Wait, The time taken. The time is the same." It seems at this point that S4 has realized that the question they are supposed to be answering is about the time taken to make the strip (and not the speed which they had been talking about up until this point). She also suggests an answer to the question that the times are all the same.

Nora expresses a disagreement. Nora says, "No it's not. The time. So for this one, it's slower, right?" As she says this, she points to one of the longer segments, suggesting as she did before that the longer strips take more time. In this case, it seems as if Nora is using the word "slower" to mean more time, since it is in the context of her disagreement about the time. However, the word "slower" here is also consistent with her original idea that the shorter strips are "faster." The ambiguity either way is consistent with both of her previous statements.

S4 explains that it should be the same time to generate the six dots. This would seem to be the correct reasoning for why the times are the same. Earlier in this transcript, S4 had explained, “that it’s going at the same rate.” It seems likely that S4 is aware that the tapping device goes at the same rate. In the prior transcript, we also know that S4 is aware that each strip has six dots. It seems likely that S4 is coming to the conclusion that because the tapping device goes at the same rate, it should take the same time for each strip (which each have six dots).

S4 also adds, “It’s just… for being pulled faster.” It’s hard to know exactly what S4 is referring to here since there is a gap in her sentence. It seems like she could be meaning, “It’s just different for being pulled faster,” or possibly even “more spaced out”. Either way, it seems to be that S4 is explaining not only why the times are the same, but why the strips are different. The times are the same because it’s same amount of dots. The strips are different because they were pulled differently.

Nora tries to explains, “No, but the space…” Here Nora seems to be pointing out physical features of the strips, possibly the space between dots. Nora is cut off, however, as S2 makes it known that she also agrees with what S4 is saying, adding, “I thought he said it was constant, like the same 40 beats per second.” Here S2 is makes it more explicit that the students are attending to the fact that the tapping device taps at a constant rate. The other students take turns agreeing.

This scene ends with Nora asking, “So then why are they different?”

The Other Students’ Thinking

The other students in the group seem to have a pretty good understanding of the correct mechanism by which the strips were generated. They explain that the strips were being pulled, which caused them to have different speeds. They also explain that the tapping device taps at the same rate.

They also seem to have a pretty good understanding of the implications of this mechanism. They explain that the dots are either more closely spaced or more spread out depending on whether it was pulled slowly or pulled fast. They also explain that it takes the same amount of time to make six dots.

The students' thinking here is different from Nora's thinking (and the other students' thinking previously discussed) in a variety of ways. Obviously, the students are arriving at different (and also correct) answers to the questions. The longer strips do represent faster strips, and all the strips were generated in the same amount of time. However, this is not the only difference.

In discussing their ideas, the students are explaining why the features of the strips are they way they are because of how the strips were generated. In this sense, they are reasoning about what the strips should look like due to how the strips were made. The students don't seem to be just looking at the strips to making inferences about what that means. Instead, they are reasoning that a faster motion would create strips that have more spaced between dots.

This attention to how the strips were made is reflected in how the students use the word 'faster'. While in the previous pattern of students' thinking, we saw most of the statements involving the word "faster" were used to identify the strips, here the students use the word faster to describe actions upon strips. S0 says, "This is one is pulled slower." S2 says, "It's pulled faster." S4 says, "Right, so the pulling should be taking it a little bit faster."

Nora's Continued Thinking

There is certainly some evidence to suggest that Nora isn't completely making sense of the arguments presented by the other students. Although Nora does say that she now gets what they are saying when they explain why the longer strips are faster, we have reasons to be skeptical.

First, Nora's statement about understanding them does not come across as particularly genuine. She quickly says, "OK" six times, before saying "I gotcha." She could be saying that she understands simply because there are three other students disagreeing with her and all explaining the same thing. She may not want it to appear that she doesn't get understand what seems obvious to them.

Second, Nora doesn't participate in discussing any aspect of the mechanism that the other students are talking about. She doesn't talk about the pulling or the tapping, or what those actions would imply. She doesn't even engage in this kind of conversation either to agree or disagree with the students, only to question what the answer is (which ones are faster and slower).

Third, Nora continues to maintain aspects of her original thinking. Specifically, when S4 explains that the times are the same, Nora disagrees by saying that the time is slower for a longer one. It is particularly compelling that she uses the word 'slower' here to refer to the time as being greater. In part 1, Nora had both identified the longer strips as being slower and as taking more time. The fact that she uses the word 'slower' here as well suggests that she may not only be disagreeing with them about the duration. She may also still be thinking that the longer strips are 'slower' in reference to having less speed as well.

Other aspects of her original thinking continue as well. Although Nora doesn't say a lot in this scene, she makes two statements in reference to physical features of the strips. When the other students repeat their explanation for why the times are the same, Nora simply rebuttals, "No, but the space." The students had been talking about the tapping of the tapping device and the pulling of the strips, but Nora's disagreement isn't in regard to their understanding of the how the strips were made. Instead, Nora simply refers back to physical features of the strips. Even at the end of the transcript, Nora asks, "So then why are they different?" Here, too, Nora seems to be drawing attention to the physical features of the strips. She is looking for an explanation, but is not clear what kind of explanation she is looking for.

In this case study, Nora seems to have one way of thinking about the strips and her groupmates seems to have another. Aspects of Nora's thinking that was evident in part one now seem to persist in her thinking now despite her groupmates attempts to explain their thinking. (In fact, Nora continues to struggle in making sense of the other groups' thinking for several minutes as they continue to explain how the strips were made. Eventually the group decides moves ahead in the tutorial, but there is sparse evidence to suggest that Nora comes to understand their thinking). Similarly, her groupmates remain confident in their understanding despite Nora's protests otherwise. Both groups seem to exhibit a stability in their thinking about the tickertape strips. Nora's thinking seems to be focused on making inferences from the physical features of the strips, while her groupmates' thinking seems to be focused on how the strips were made. We'll return to the issue of accounting for the local stability of these patterns of thinking after revisiting the other two cases.

Case Study 2, part 2: Mark's New Thinking

At the end of the first part involving this group, the students were quietly writing in their tutorial worksheets after having decided that the shorter strips are faster and take less time. The students continue to write quietly in their worksheets for a few minutes. At the start of this scene, Mark pops his head up from his tutorial worksheet and addresses the group.

Mark: Actually… this is the fastest one *[pointing to the longest one]*

Kara: Why?

Mark: Umm. Because it taps at the same rate *[tapping gesture],* and when you pull this through quickly *[pulling gesture]* the dots get farther apart. If you pull through slowly *[pulling gesture]* the dots are closer together *[tapping gesture]*.

Ryan: Oh-oh

Mark: My bad

Kara: You're right.

Mark: When you pull it *[pulling gesture]* through that thing, it's like tapping *[tapping gesture]* at the same rate the whole time, so if you pull through really quick [pulling gesture] it's gonna keep tapping *[tapping gesture]* and the dots are going to be farther apart. If you pull it slowly, the dots are going to be close together.

Ryan: So it all takes the same time to generate the same. *[tapping on the table]*

Mark: Yeah, it's the same

Ryan: It takes the same time to generate each strip, cause they're all six dots.

Kara: Oh, so you mean the strips are going through slower.

Mona: Yeah, cause if you took like *[picks up one of the shorter strips],* and you went like really, really slow *[pulls the strips across the table.]*

Mark: So it takes the same amount of time, right?

Ryan: Right, cause they each have six dots.

Mark: There are a certain amount of dots per second. *[tapping gesture]*

In this scene, Mark looks up from his work sheet to tell everyone that he thinks that they have the speed-ranking wrong. He points to one of the longer strips and says, "Actually…this one is faster." After Mark says this, Kara looks up and asks, "Why?" Mark first explains, "Because it's tapping at the same rate". As he says this, he taps his finger up and down on the table. He then goes on to say, "When you pull this through quickly, the dots get further apart." As he says this he draws his hand horizontally across the table in front of his body. He then explains, "If you pull it slowly, the dots are closer together." As he says this, Mark repeats the same hand motion he did before, just slower this time.

Ryan says, "Oh-oh," indicating that he understands Mark's explanation, and Kara says, "You're right." Mark then goes on to explain his reasoning again. He repeats his explanation again, this time with a little bit more emphasis on the fact that the tapping is the same. He also repeats the tapping and pulling gestures.

When Mark is done with his explanation, Ryan comments, "So it takes the same time to generate the same." As he says this, Ryan taps his finger up and down on the table in the same way Mark had earlier. Mark then agrees with Ryan, and Ryan adds, “It takes the same time to generate each strip, cause they’re all six dots.”

Kara then jumps in, saying, “Oh, so you mean the strips are going through slower.” She says this with a particular emphasis on the word “going”. Mona then picks up a shorter strip and says, “Yeah, cause if you took like, and you went really, really slow.” Mona doesn’t finish her sentence, but she pulls the strip horizontally across the table as Mark had done.

Mark asks a question to confirm that they are saying it takes the same amount of time, and Ryan repeats that it’s, “[be]cause they each have six dots”. Mark adds, “There are a certain amount of dots per second,” while he taps his finger again.

Features of their New Thinking

Mark’s explanation for why the longer strip is faster has many of the same elements that we observed in Nora’s groupmates’ correct thinking as well. Mark is now thinking about how the strips were made and then coming to conclusions about what they should look like based on that. He explains that the tapping device is tapping at the same rate. He even uses his hand to act this tapping out. He explains that the strips of paper can be pulled quickly or slower. He also enacts this out with his hands. These statements and gestures all reflect his attention to actions that actually took place in creating the strips, not just to the physical features of the strips. He uses these actions to explain that a faster pull results in the dots being spaced further out, and that a slower pull results in the dots being spaced closer together.

As with the previous cases involving Nora's groupmates disagreeing (CS1, part 2), paying attention to the actions involved in making the strips is accompanied in changes in how words like "faster" and "slower" are used. Before Mark made statements such as, "This is fastest," and "Yours is a little bit slower", using "faster" and "slower" as adjectives to describe the strips. Mark now uses the words as adverbs to describe the action of pulling. Mark makes statements like, "If you pull through really quickly." He makes three other statements like this as he explains his idea.

Both Kara and Mona also make statements that they are now thinking about these actions as well, which also involve using the word faster as adverbs. Kara says, "The strips are going through slower," and Mona says, "If it went really, really slowly." Kara using "slower" to describe the verb going, and Mona using the word slowly to describe the verb went. It seems that Mark's shift in talking about the fastness and slowness of the strips seems to be taken up by other members of the group as well. His gestures are taken up by others as well. Ryan mimics his tapping gestures, and Mona mimics his pulling gesture.

Ryan doesn't participate in talking about the movement of the strips. Mark's explanation, however, does seem to contribute to Ryan realizing that the strips were all made in the same time. Ryan explains that they were all made at the same time because they all have the same amount of dots, and Mark adds that it's because there are certain number of dots per second. Recall that Mark had earlier commented that he didn't know what the six dots meant. Now, as the group seems to shift along with Mark in his new thinking about the strips in terms of what was physically happening with the strips in terms of pulling and tapping, the meaning of the six dots becomes readily apparent. Thinking about the role of tapping, in particular to it being constant, seems relevant to their conclusion.

Case Study 3, Part 2: Rita Changes her Mind

In the third case study, I have chosen not to present the actual transcript for period of time when Rita, Fran, Judi, and Dani change their thinking and realize they have the speed-ranking wrong. Part of this, has to do with the amount of off-task talk, overlapping speech, and incomplete sentences that make the scene difficult to follow. Here I just describe some features of their thinking that are relevant.

Several minutes after having decided the shorter strips take less time and are faster, the students are measuring the distances on each of their strips in order to calculate the speed. While doing this, Rita realizes they have the speed-ranking question wrong. She explains to the other students that the longer strips must be faster because, “it moves the furthest.” The other students seem to quickly realize this is correct as well. Several comments are made by the students that about their thinking. One student says, “We were not thinking, smartly,” and another students says, “I don’t know what we were thinking. That was stupid.” It certainly seems that these disparaging remarks merely represent a way to dismiss their earlier thinking that was “wrong”. However, it may also be the case that the students really don’t know what they were thinking before, precisely because they had no explicit sense of understanding about how the strips were made. Several minutes after, Rita also realizes that the times must be the same as well, because there are the same amount of dots.

Revisiting our Cognitive Accounts

In this section, I'd like to accomplish two things. Now that we have more evidence concerning students' thinking, I'd like to re-evaluate the two cognitive accounts I presented for students' incorrect thinking about the tickertape strips. Second, I'd to suggest a plausible account for students' correct thinking in terms of some fine-grained intuitions.

Accounts of Students' Initial Thinking

Previously I had described two different accounts of the students' incorrect reasoning about the tickertape strips. One account was in terms of students having an explicit understanding of the mechanism by which the strips were generated that was incorrect. Students were privately thinking that the strips were generated with a variable tapping device. The other account was in terms of the activation of fine-grained intuitions for making inferences from particular features of the strips that didn't require any explicit sense of mechanism.

First, consider Nora's thinking after her groupmates disagree with her. Nora's groupmates make arguments concerning the mechanisms they believe were responsible for making the strips. They explicitly discuss thinking that the strips were pulled at different speeds (faster or slower) at the tapping device tapped at constant rate (40 beats per second). Nora, however, does not make any statements that suggest she disagrees with their description of the mechanism. Neither does she make any statements that she agrees with this description of the mechanism. Instead, Nora maintains her attention to the physical features of the strips. She points out the space between dots, and continues to ask why the strips are different.

In addition Nora uses the word "slower" to describe the time for the longer strip. While slower seems to be used in a manner to mean "more time", this word is also consistent with her statements from before regarding the shorter strips being faster. The ambiguous use of the word faster here may support the earlier suggestion that the intuition *more speed implies less time* was responsible for her saying that the time depends on how fast it was. Although it is not shown in the transcript in the scene above, Nora even later the tutorial can be heard asking, "Wouldn't the time be slower for this one?" Here, too, referring to the time as being "slower", likely meaning more time.

Aspects from the second case study with Mark also support the account of students' initial (and wrong) pattern of thinking in terms of a collection of fine-grained intuitions. When Mark realizes that they have the speed-ranking question wrong, he explains why in terms of the mechanism by which the strips were made. He explicitly refers to the pulling as being variable (faster or slower) and the tapping and being constant (same rate the whole time). However, he doesn't seem to articulate this understanding of how the strips were made as being in contrast to some earlier misperceived sense of how the strips were generated. He merely explains how he is now thinking about they are made *and* what consequence that has for the speed-ranking of the strips. Mona and Kara join in on talking about the pulling of the strips, and Ryan suggests that a consequence of the constant tapping is that all the strips were made in the same time. While this certainly doesn't prove that students were not thinking about variable tapping earlier, we might have expected the students to mention that they had been thinking something else earlier or have couched this new idea as a change in mind about the mechanism. Instead, Mark couches it is a change to the answer to which strip is the fastest.

In the third case study, I have chosen not to present the actual transcript for period of time when Rita, Fran, Judi, and Dani change their thinking and realize they have the speed-ranking wrong. Part of this, has to do with the amount of off-task talk, overlapping speech, and incomplete sentences that make the scene difficult to follow. Here I just describe some features of their thinking that are relevant to the fine-grained account.

In the third case study, the students seemed unable to easily access what their prior thinking had been when they were thinking incorrectly. It may be that the activation of these prior intuitions had required little attention to their own thinking. The shorter segments obviously took less time, and the bunched up strips were obviously faster. These intuitions were cued initially as they were paying attention to just the physical features of the strips, but now that the students shift to thinking about the mechanisms by which they were generated, they no longer have access to their earlier thinking.

In these cases, students' attention to the mechanism by which the strips were generated led to students to productive uses of different intuitions for thinking about the tickertape strips. When students changed their answers, students' explanation did not indicate that they were previously thinking about a different mechanism. Instead, it seems as if they are just now closely paying attention to that mechanism. Nora, on the other hand, persists in prior thinking and persists in her attention to just the physical features of the strips. These aspects provide some evidence against an account of students' incorrect thinking as the result of a simple misunderstanding about how the strips were made. Instead, it supports the finer-grained account in terms of how physical features of the strips contribute to the activation of elements of knowledge corresponding to intuitive sense of motion and visual representation.

Accounting for Students' Correct Thinking

In this section, I'd like to reflect on just a limited aspect of students' correct understanding. In the second parts of each of these case studies, students expressed ideas that seemed to indicate a correct understanding of the mechanism by which the strips were generated and correct inferences about what that mechanisms implies about the time and speed ranking. Students reasoned that the strips were pulled at various speeds leading them have differently spaced dots- faster motions leading to more spread out dots. Students also concluded that the same number of dots implied that the times to make each strips were same because of the constant tapping. Here I'd like to describe a toy cognitive model that captures certain aspects of this reasoning as well. This toy model consists of three intuitions that can plausibly account for students' correct thinking about the pulling of the strips and what that implies about the distance:

- *More Speed Implies More Distance*
- *Continuous Force (causes Steady Motion)*
- *Ohm's P-prim*

One intuition that seems to strongly play a role in students' thinking is the intuition that *more speed implies more distance.* Students expressed thinking that more speed implies more distance between dots on the strips or less speed implied more closely spaced dots in all the examples discussed. I also described other examples of students' using these intuitions earlier. While students' incorrect thinking involved reading out information about time from distances, here students are connecting the distance to the speed of the strips.

The other two intuitions proposed seemed to play a strong role in students' attention to the role of pulling. Likely candidates for this aspect of students' thinking are intuitions (or p-prims) such as *continuous push* and *Ohm's p-prim*. The activation of the p-prim *continuous push* describes students' thinking that some continuous action causes the steady motion of the strip. In this sense it is through the action of some agent pulling on the strip that the strip is caused to move. Often students described that agent as "you". *Ohm's p-prim* also describe students' sense that the pulling with various amount of effort can lead to it going faster or slower. diSessa (1993) describes these two p-prims as activating together with high reliability.

It is certainly interesting that students' using the intuition *more speed implies more distance* would arise with these other two intuitions for the thinking about the dynamics of "pulling". The inclusion of these intuitions seems consistent with the manner in which students expressed thinking that the larger strips represent more speed. Students don't seem to just infer the speed by directly reading it from the distance on the strips. Rather students seem to be attending to the physical mechanism of pulling to reason that the faster strips should be longer (or have more spread out dots). In other words, students are explaining how the action of pulling leads to differences in how strips should look, *not* explaining how the features of strips indicate what the speed is. It is subtle difference, but an important one. In this case, students are reasoning to evidence based on actions and mechanisms they believe to have taken place. In contrast, when students were using intuitions such as *more distance implies more time,* students seemed to be only attending to features of the strips as evidence for make inferences about motions. One way of making sense of this relation is that *Ohm's p-prim* and *continuous force* may be understood to connect to either "more speed" or "more distance". In other words, greater pulling may cause more speed, or greater pulling effort may cause more distance (or to a more ambiguous sense of greater motion). Attention to the action of pulling thus creates a network of ideas that helps to stabilize the interpretation of distance as meaning speed (and not time).

This basic toy model of three intuitions ultimately fails to take into account the role of students' thinking about the tapping, which also characterized students' shift in thinking as well. It seems likely that a variety of different conceptual ideas must come into play here as well, including ideas for making sense of what "constant" means (*e.g.,* same numbers of dots in same amount of time) and for visualizing the actual movement itself up and down. Careful attention to thinking about the pulling as being *different* for each strip and the tapping as being the *same* are also likely to be important conceptual distinctions that help to differentiate and specify meanings of words like fast.

Brief Summary

In these three case studies, I have illustrated how students' thinking in the beginning of the Speed Tutorial falls into two distinct patterns.

One pattern of thinking involves students closely attending to physical features of the strips. As students make sense of these patterns on the strips students cue into various intuitions that lead them to conclude that the shorter strips are faster and take less time.

A second pattern of thinking involves students closely attending to the physical mechanisms that made the strips. As students make sense of these actions, students cue into various intuitions that lead them to conclude that the longer strips are faster and take the same amount of time.

We might characterize one major difference between these two patterns of thinking as concerning what it is that students infer from the distance on the strips. In the first case, students view the distance on the strips as indicating time, cueing into the intuition that *more distance implies more time* (as well as other intuitions). In the second case, students view the distance on the strips as indicating speed, cueing into the intuition that *more speed implies more distance* (as well as other intuitions). Students variably cue into different intuitions much in the same way that we encountered in the experiment described in chapter 4.

In the experimental design, we were largely concerned with describing the dynamics by which particular intuitions were initially cued by features of the context we could manipulate. Here, we see a similar dynamic of different intuitions arising with different patterns of attention. Students paying attention only to static features of the strips cueing into the intuition that *more distance implies more time* (and others). Students paying attention to physical mechanism cueing into the intuition that *more speed implies more distance*.

In the following section, I would like to offer a plausible account of how students' initial thinking holds together and exhibits some stability.

Mechanisms Contributing to Stability

I have argued that, from a diversity of fine-grained intuitions that students might bring to make sense of the tickertape strips, two distinct patterns of thinking arise.

One of those patterns of students' thinking leads students to the incorrect conclusions that the shorter strips are faster and take less time. I described a toy cognitive model to explain how this pattern of thinking might arise from an assembly of intuitions: *bunched up means fast, more distance implies more time,* and *more speed implies less time.* These intuitions arose as students paid attention almost exclusively to certain aspects of the physical features— the location of dots, the number of dots, the closeness (or density) of dots, particular distances between dots, and the overall length of the strips—and to the questions posed at the beginning of the tutorial that asked them to compare the strips and order them by time and speed.

Another pattern of students' thinking lead students to the correct conclusion that the shorter strips are slower are made in the same time. I described a plausible toy cognitive model to explain certain aspects of this pattern of thinking as well. This model was described as a collection of intuitions such as *more speed implies more distance, continuous force,* and *Ohm's p-prim.* These intuitions arose as students paid attention to specific mechanisms by which the strips were made—the pulling of the strips through the tapper and the constant making of dots from the tapper.

In this section, I would like to reflect upon some plausible mechanisms that contribute to these patterns of thinking persisting over time and exhibiting some stability.

Knowledge Structure Mechanisms

In students' incorrect thinking about the tickertapes, I pointed out how students used the words like 'faster' in particular ways when referring to the strips. The vast majority of students' statements used the word "fast', "slow", "slower", "faster" as adjectives to describe the strips, not as adverbs for describing actions. In doing so, it was not often clear to what movement the description 'fast' was being applied (if to any specific sense of movement at all). Most of the time it seemed as if students were using the word "faster" to merely describe physical attributes of the strips (e.g., "it's faster cause the clicks are closer together") or to identify objects (e.g., "That's the fastest". Other times it seemed as if students were using the word "faster" to mean something taking place in less time (e.g., "faster to do a little one"). Students also used the word faster to describe their answer to the speed-ranking question (e.g, "How do you know how to arrange them by speed…Faster!").

This polysemy of words like "faster" suggests that they are connected to multiple intuitions for thinking about kinematical relation. The linguistic overlap may serve to stabilize the activation of many intuitions reflecting these many meanings. Students may variously cue into multiple intuitions all related to the word 'faster'. Each of these activations helps to stabilize the activation of the others. These linguistic relations in the structure of these intuitive knowledge elements serves as one mechanism by which the pattern of thinking is self-sustaining for a period of time.

Students may not even be aware that they themselves (or others) are cueing into different shades of meaning around the word “faster”, especially given the use of ambiguous referents. Consider in case study one (CS1), for example, that Nora says, “the time is slower for this one” while pointing to one of longer strips of paper. Nora may being cueing into both intuitions related to word “slower”— one as meaning *more time* and one as meaning *spread out*. In this way Nora is simultaneously cueing into the intuitions that *more speed implies less time* and *bunched up means fast*. She may not notice that she is cueing into two different intuitions because of this polysemy.

In other cases, the stability of reasoning due to polysemy may be understood as being more distributed across individuals. In case study two (CS2), for example, Mona asks, “Is it faster to do a little one?” Mona seemed to using the word “faster” to mean taking place in less time (in response to Mark’s suggestion that the shorter strips take less time). Kara then gives an explanation for why the strips are faster, “It’s faster because the clicks are closer together.” Mona and Kara may, in fact, be using the word “faster” in relation to different intuitions without noticing.

Students’ correct reasoning that the longer strips are faster also give some insight into how polysemy may provide some stability for students’ incorrect reasoning. Consider that when students arrived at correct answers, students used words like “faster” in qualitatively different ways then they did when arriving at incorrect answers. They used words like “faster” more often as adverbs to describe specific actions (e.g., “if you pull it more slowly”) rather than do describe features or to as adjectives for an ambiguous referent. Even in case study two (CS2), Mark gestures repeatedly to act out these specific meanings of “faster” or “slower”. Other students even join in on and mimic these specific gestures.

This shift to thinking about a particular action may serve to constrain the possible meanings of “faster” for both the speaker and listeners as referring to the magnitude of translational speed. Attention to a particular action may stabilize the activation of certain intuitions that arise with that particular meaning (e.g., *more speed implies more distance*) and suppress other meanings (e.g., *bunched up mean fast*). The use of dynamic gestures themselves may serve to stabilize students’ thinking as well. Many researchers have argued that such embodied understandings have a significant role in thinking and communication (Goldin-Meadow, 2003; or see review by Scherr, 2008).

As one mechanism contributing to stability of students’ thinking, the polysemy of the word ‘faster’ may play a role by binding together multiple intuitions that share a linguistic overlap. The activation of one cognitive element feeds into the activation of another and vice-versa. This mutual reinforcement need only be context dependent, however. From an individual perspective, the linguistic overlap may only serve to reinforce multiple activations of cognitive elements (related to fast) when other cognitive elements are not activated that would locally differentiate among these meanings. For example, the activation of *continuous push* may locally serve to reinforce the intuition *more speed implies more distance,* and by doing so cognitively differentiate among multiple meanings of fast. Among individuals, statements and gestures that use the word "faster" in largely ambiguous ways (e.g., "this is the fastest" while pointing) may allow students cueing into different intuitions to fail to notice these differences. When students use statements or gestures with more specific referents (e.g., "when you pull it slowly like this"), students may be more likely to notice discrepancies. This may suggest the need for students, as a matter of learning physics (and learning how to learn physics), to begin developing habits of mind that are aimed at identifying and locating such ambiguities in their own thinking and discussions.

Contextual Mechanisms

Beyond the structure of knowledge, there are also specific aspects of context of the tutorial that provide a degree of stability for students' incorrect thinking.

I have already mentioned that the time-ranking question uses the words "shorter" and "longer" to describe the different strips. This linguistic cue contributes to the initial activation of the intuition that *more distance implies more time.* However, there are certainly other aspects of the context that support the continued activation of this and other intuitions comprising this patterns of thinking

The strips themselves have a particular structure to them. Importantly, each of the strips has the same number of dots. This means that the long strips are made of parts that are also long. The long strip is made up five 'mini-segments' that are also long. Likewise, the short strips are made up of parts that are short. In this way, whether students are attending to the entire length of the strips (and how they compare across strips) or to just the space between dots (and how they compare across strips), these features both support the continued activation of intuition *more distance implies more time.* The activation of the intuition is resilient to shifts in this kind of attention.

Beyond just attending to the distance of an entire strip or just to the distance between two dots, students also seem to pay attention the overall closeness (or density) of the dots on the strips. I have argued that students' make sense of this feature of the strips using the intuition that *bunched up means fast.* This pattern of attention may also bring students' to the conclusion that the shorter strips take less time because of the intuition that *more speed implies less time*. This also leads students to the conclusion that the shorter strips take less time.

Together, the context of students' thinking provides a degree of stability because the particular assembly of intuitions are resilient to certain shifts in students' attention. Students may attend to words on the page (e.g. "shorter" and "longer"). Students may attend to how the total lengths of the strips compare. Students may attend to how the space between dots compare. Students may attend to the overall proximity of the dots on the strips. Each of these patterns of attention plausibly contributes to sustained activation of cognitive elements in the model. It is in this way that the particular context in the Speed Tutorial can serve to locally stabilize students' thinking that the shorter strips are faster and take less time.

Implications

I have proposed two plausible mechanisms by which students' incorrect thinking about the tickertape strips exhibits some stability. The first mechanism concerns relationships among cognitive elements (as intuitions for making sense of "fast") that lead to their sustained mutual activation through polysemy. The second mechanism concerns how different aspects of the context provide multiple inroads to the activation of the same cognitive elements. Here I want to briefly reflect upon what these two accounts of stability imply about perturbations that might disrupt the local stability of this pattern of thinking. I first discuss implications for polysemy as a stabilizing mechanism and then upon the implication of attentional resiliency as a stabilizing mechanism.

Implications of Polysemy as Stabilizing Students' Thinking

I have argued that students' have multiple intuitions strongly connected to the word "faster" that they use to make sense of tickertape strips. In describing polysemy as a mechanism contributing to some of the stability, I have implied that these multiple intuitions keep each other active through a linguistic overlap. According to this account then, students cue into different intuitions for making sense of "fast" and do so without noticing. This implies, of course, that students are neither closely monitoring certain aspects of the substance of their own thinking (in case of Nora) nor aspects of the substance of other's thinking (in the case of other groups).

So what might students be monitoring? It is arguable that students are more closely monitoring for agreement upon the answers they are supposed to write down then they are for the substance of their own others' ideas.

For example, in case study one (CS1), Nora quickly gives answers to both the time-ranking and speed-ranking questions without much explanation. In the second part, when the students disagree with her, Nora never responds to what the other students are saying about the how the strips were made. Instead, she notices that they have different answers. In fact, she seems to go along with what they are saying about how the strips were made, until it is obvious that she has a different answer for the time-ranking question. As they attempt to explain their reasoning in terms of the mechanisms by which the strips were generated, Nora persists in just attending to physical features of the strips and to fact that her answer is different.

In case study two (CS2), Mark simply blurts out one-word answers to the questions from tutorial. When Mark first blurts out, “Shorter,” as an answer to the time-ranking question, Kara seems to hesitate for a moment. But then she simply says, “Yeah,” and goes back to writing in her worksheet. What seemed like a moment to explore the substance of each other’s ideas turns into a missed opportunity. Later when Mark blurts out, “Faster,” in response to the speed-ranking question, Kara does ask, “How do you know how to arrange them?” This might seem to be a moment where Kara is trying to explore aspects of Mark’s reasoning as well, but Kara is simply reading word-for-word a question from the tutorial.

In case study (CS3), the students begin by just making simple declaration such as, “Yours was faster,” and “Mine’s the fastest” without much explanation for why they think this. As they continue, we also see several examples of the students checking in with each other to make sure they agree upon the answers. One student says, “So it takes less time, right?” Another says, “So the shorter segment was moving faster through the thing, we think that right?” And yet another says, “The speed is increasing with the shorter one?” Each of these statements reflects how students are making sure they have the same answer. It is not clear that they are checking in to make sure they share similar reasoning.

Based on these examples, I am suggesting that part of the stability by which polysemy acts is through the fact that students aren't closely paying attention to meaning in the substance of their own or each others' ideas. The multiple meanings of "faster" go unnoticed in their own minds and across each other's statements because they are paying more close attention to their answers then they are to intuitive ideas that go behind these answers. This suggests that one way of disrupting the stability of this pattern of thinking is through students being more attentive to their own thinking or to the thinking of others. By doing so, students may come to distinguish in their own minds (or in the minds of others) the multiple intuitions concerning "fast" that are being used to make sense of the tickertape strips. Of course, as I have already suggested, part of learning how to learn physics involves developing habits of mind to locate ambiguities in one's thinking. This activity requires attention to the substance of one's thinking (and to the thinking of others' as well).

There is some need here to carefully distinguish this kind of differentiation of ideas in the moment (e.g., students in a moment noticing they are or others are cueing into different intuitions) from other accounts of differentiate of ideas more generally (e.g., student either have or lack the ability to differentiate). I am referring here only to a local differentiation among multiple intuitions concerning the word 'fast', which assumes that students already have in place abilities for attending to these differences in meaning, not to some deficit in which students are generally characterized as being unable to differentiate among related ideas (e.g., Smith, Carey, & Wiser, 1985; Trowbridge & McDermott, 1980). In particular, I am arguing that part of the local dynamics by which students differentiate among these meanings (or not) concerns what students are meta-cognitively monitoring (Flavel1, 1979), or how they are epistemologically framing the learning activity (Scherr & Hammer, 2009). This monitoring or framing might apply at an individual level, such as Nora not closely paying attention to her own thinking. It might also apply at a group-level.

Testable Implications for How Context Stabilizes Students' Thinking

The second mechanism I proposed concerned how features of the context provide a degree of stability for students' incorrect thinking. Whether students attend to particular words on the page or various features on the strips, the same intuitions may be continually activated by sustained attention to those features. This contextual stability could be disrupted in a variety of ways. The most obvious way this happens, which is apparent from the above cases, is that students' shift away from attending to just the physical features of the strips. In all of the examples of students' correct thinking, we find students attending to how the strips were made. The contextual stability is disrupted because the assembly of intuitions is only resilient to certain shifts in attention, not all.

I want to also suggest some empirically testable (at least plausibly testable) perturbations that would disrupt some of the stability of this pattern of thinking based on the particular account of contextual stability. Since it is particular aspects of the tutorial (that were designed by its writers) that contribute to the stability of students' thinking, it should be possible to modify the tutorial in ways that make this pattern of reasoning less prevalent. First, one could reword the time-ranking question not to draw attention to the distance features of the strips. Instead of using the words “shorter” and “longer”, the question could use a more neutral phrasing. This is likely to significantly decrease the initial cueing of intuitions such as *more distance implies less time* that make up for parts of the pattern of thinking.

A second change would be to cut the strips of tickertape so that not all the strips represent the same amount of time (i.e., have the same number of dots). When the strips represent the same amount of time, both the entire length of the strips and the parts of the strips are either longer or shorter. I argued that this particular construction provided a degree of stability for cueing the intuition *more distance implies more time* because both the space between dots and the whole strip both indicate the same information. It would be easy to setup a situation where the strips are cut so that some of the strips representing less speed (with less space between dots) are actually longer in their entire length. This would mean that they would need to represent more time and thus have more dots. Therefore as students shift their attention from the entire length of the strip to the length of its parts, the context no longer provides the same stability because the longer strips are made of shorter parts, and the shorter strips are made of longer parts.

These empirically testable implications should not be misconstrued as suggestions for improvement to the curriculum. These are not normative suggestions about what is best for students in terms of the learning. It *may be* that changing the context in the ways describe would leads to more students settling on the correct answers initially. This may be productive, or it may not. Seen one way, it may be viewed as productive for students to not spend too much time thinking incorrectly about the tickertape strips in order to move on to other important conceptual learning. Seen another way, if learning to locate ambiguities in one's thinking is a desirable result of instruction, then it may be beneficial to provide a context in which succeeding involves careful attention to one's thinking. These suggestions, however, are merely intended to be implications of the particular contextual mechanism described here.

Chapter Summary

In this chapter, I have accomplished two of the three analyses concerning students' thinking in the Speed Tutorial that have been set out.

The first analysis concerned looking at isolated statements made by students from across the entire data set. Examinations of these statements illustrated the use of many of fine-grained intuitions from our original toy cognitive model that was described in Chapter 3. These examples highlighted various general features of these intuitions as well as features relevant to particular instantiations in the Speed Tutorial.

The second analysis concerned looking at student thinking across slightly broader expanses of time. I first described two different patterns of reasoning that commonly arise and the different properties that characterize them. Next, as a part of this analysis, I suggested plausible cognitive accounts to explain these patterns and evaluated these accounts against three cases of student thinking. Last, I suggested two plausible mechanisms by which one of these patterns of reasoning appears to exhibit some stability, and offered a few implications for how the local stabilities may be disrupted.

Here I want to briefly draw some connections between this analysis in case studies and the analysis of student thinking in the survey-base experimental design. Beyond just involving many of the same intuitions from our toy cognitive model, there is another similarity between the two investigations. Both investigations concern modeling the dynamics of two quasi-stable cognitive states. What I mean by this is that students, in both investigations, exhibit variability in their thinking about a single situation. This variability largely concerns how students settle into one of two possible ways of thinking – possibilities that are not stable across all perturbations, but do appear to exhibit enough stability to persist across time scales that we can observe them.

The experiment design described in the previous chapter concerned trying to 'tip' students toward and away from two finer-grained stabilities. Students could either think that more *distance implies more time* or *more speed implies less time* in thinking about the amount of time taken for some event to occur. The experimental data and toy modeling suggests that these two fine-grained stabilities arise, in part, due to how students' attention are drawn to specific features of the context that we could manipulate.

In these case studies, there are also two different quasi-stable cognitive states representing students' thinking. At times and in certain contexts, students think that the shorter strips are faster and take less time. At other times and in different contexts, students think that shorter strips are slower and take the same time. I have argued through these case studies that these patterns of thinking also arise in part due to how patterns of attention are influenced by particular features in the context. Although we could not manipulate students' attention, we see evidence in these cases for different patterns of attention from students' verbal statements and gestures.

It's likely to have occurred to the reader, that along the way, our analysis has shifted away from one concerning only individual students' intuitions in response to specific questions, to an analysis of patterns of thinking that arise among students' as they discuss their ideas with each other. In many respects, the cognitive unit of analysis has subtly shifted beneath our eyes from one of individual to one of collections of individuals. In describing plausible mechanisms by which students' thinking cohere, I described mechanisms that might plausibly occur either within individuals or among individuals. Nora persists in her own thinking, in part, due to how polysemous words help sustain the activation of multiple intuitions in her own thinking and in how she understands her groupmates. Mark, Kara, and Mona contribute to the persistence of their group's thinking by failing to notice subtle differences in their own intuitive thinking due to the use of polysemous words in ambiguous ways. In a similar way, the account of contextual mechanisms shares the same applicability to an individual or among individuals. An individual student may shift their attention from one thing to the next (such as total length or length of the part of a strip), or two different students may maintain different patterns of attention that both contribute to the activation of the same intuitive elements in their own minds.

In third analysis that follows in the next chapter, I would like to more carefully engage in the analysis of students' thinking as embedded in a larger cognitive unit – one that includes multiple individuals and relevant aspects of the context with which they dynamically interact.

Chapter 6: Dynamics Shifts Among Multiple Stabilities

Chapter Introduction

In this chapter, I continue the analysis of student thinking in "The Meaning of Speed" tutorial by presenting a new case study of a single group's thinking. A careful analysis of students' physical behavior is carried out in order to better understand the dynamics by which these students shift between the two patterns of thinking we've already encountered. Like the other groups discussed, these students settle into different patterns of thinking at different times—each exhibiting some local stability and persisting on the scale of minutes. At times, their discussion seems to coordinate around thinking that the shorter strips take less time and are faster. At other times, they coordinate their thinking around the idea that the strips pulled faster are longer. Whereas the previous analysis focused on the nature of those local stabilities, this chapter's analysis brings insight into the dynamics of how students transition between these two local stabilities of thought.

In the case study, the students are characterized as not only responding to the static context presented in the tutorial (like features of questions and features of strips), but students are described as constructing and reconstructing new contexts for thinking through their own actions. Particular attention is paid to how the students construct and reconstruct context and how these constructions are consequential in the dynamic of their thinking. In order to reach this goal, I carry out an analysis of students' non-verbal behavior (in addition to verbal statements) that involves attending closely to student interactions with each and other material artifacts. The details of this analysis rely on a methodological framework for categorizing students' physical behavior in tutorial, which was initially described by Scherr and Hammer (2009). As with their study, this observational tool provides insight into nature of dynamics occurring between students' behavior and reasoning and between individuals and groups.

This chapter begins with brief motivation for why we should pay close attention to students' physical behavior at all. I then describe the method of analysis in some detail. Using this observational tool, I discuss an extended case study of student thinking that involves multiple transitions in behavior and reasoning. At the end of this chapter, I reflect upon some implications of this case study for research on student thinking and on implications for instruction.

Analyzing Patterns of Student Behavior

Brief Motivation

In the previous chapter, patterns of student thinking and attention were ascertained primarily from students' verbal statements. Verbal statements alone, however, only reflect a narrow slice of the data that is available from video. Students can be seen gesturing, often in qualitatively different ways in conjunction with speech. Students shift their gaze, posture, and the location of their hands. Students also manipulate objects, moving them to different locations and arranging them in different configurations. All of these observable behaviors occur along different channels than speech and each provide access to data on student thinking and attention. These non-verbal behaviors prove relevant to understanding the dynamics of student thinking in our case study. Other researchers have also highlighted the significance and relevance of students' non-verbal behavior for understanding communication and thinking.

Researchers studying the nature and role of gesture, for example, emphasize that gesture and speech are inseparable elements of utterance (Kendon, 2004). The two often co-occur in acts of communication and thinking. Because gestures are not constrained in the same way that speech is (with its sequential and grammatical structure) and are typically performed unconsciously, the analysis of gestures can provide unique insight into a person's thinking (Scherr, 2008). Gestures have been interpreted as reflecting pre-articulate ideas (Church & Goldin-Meadow, 1986) that may precede speech. They have also been interpreted as facilitating the construction of new ideas (Roth & Lawless, 2002; Goldin-Meadow, Nusbaum, Garber, & Church, 2001). Gestures, therefore, represent an important aspect of students' non-verbal behaviors that provide insight into their thinking.

Other forms of student behavior, such as gaze and posture, have also been described as being coupled to the substance of student thinking. In analyzing video of students in similar tutorial settings, Scherr and Hammer (2009) utilize both verbal and non-verbal behavior as evidence for how students frame activities during tutorial. They describe such behaviors as both indicating and communicating to others person's expectations about the nature of current activity. Among individuals, these behaviors may be viewed as promoting certain collective framings by communicating meta-messages about how individuals are framing group activities.

Scherr and Hammer provide evidence for a rich dynamic among students' collective behaviors, how they frame tutorial, and the substance of their thinking.

Brief Introduction to Case Study

In the case study described in this chapter, I aim to describe some of the dynamics by which Paul, Beth, John and Kate (see Figure 8 below) settle into different patterns of thinking and behavior.

Figure 8: Locations of Students

In this case study, I build toward understanding how the contexts that students construct for themselves can be understood to support (and participate in the stability of) different patterns of thinking and attention. Some of these dynamics may be understood as taking place among students' collective physical behavior (such as gesture, gaze orientation, and even posture), students' manipulations of physical artifacts (such arranging and rearranging of the tickertape strips), and students' interactions with their own written artifacts (such as reading and writing). Drawing on method of analysis similar to that described by Hammer and Scherr (2009), I analyze patterns of student behavior with a particular emphasis on (1) how this behavior is influenced by particular material aspects within the setting and (2) how their own behaviors alter that settings in ways that feedback into their behavior.

Methodology for Identifying Patterns of Student Behavior

In this section, I describe the method that was used to identify patterns of student behavior in the Speed Tutorial, beginning with a brief summary of the framework used by Scherr and Hammer to identify broad patterns of student behavior in tutorial more generally.

Underlying Methodological Framework

Scherr and Hammer (2009) analyzed patterns of student behavior in tutorial settings (also at the University of Maryland) by identifying moments of change in clusters of students' behavior by watching video in real time. Their analysis resulted in the identification of four distinct patterns of physical behavior that span nearly all the time students spend in tutorial. These patterns are characterized by the particular behaviors that are present and can be coded easily with a high reliability. For example, they identified one pattern as the *blue behavioral cluster,* in which students' eyes are primarily on their worksheets. This cluster is characterized by the presence of bodies leaning forward, quiet hands, neutral faces, a low tone of voice, and brief glances at peers. Another pattern is the *green behavioral cluster.* This pattern is characterized by the presence of prolific gestures, animated tone of voice, animated faces, upright postures, eye contact, and clear verbal statements.

Scherr and Hammer focused on changes in students' behavior that tend to happen all at once for an entire group of participants. In this sense, the unit of analysis is the entire group, and the extent to which their collective behavior indicates the behavioral cluster. One focus of their work is in understanding the dynamics by which group shift to different behavioral clusters. These shifts often coincide with changes in the substance of student reasoning, changes in the stances that students take toward knowledge, and changes in affect. Scherr and Hammer have interpreted these behavioral clusters as reflecting different epistemological framings that students espouse during tutorial activity. They describe the blue behavioral cluster as promoting and indicating a *completing the worksheet* frame, and a green behavioral cluster as promoting and indicating a *discussion* frame. Importantly, Scherr and Hammer note strong correlations between green behaviors and student discussions about physical mechanisms. As a result of their applying this kind of analysis to aid in development of case studies of student reasoning in tutorial, correlations could be documented among the substance of student reasoning, clusters of behavior, and epistemological framings.

The analysis of student behavior in the Speed Tutorial shares much in common with the method of Hammer and Scherr. Changes in students' gaze, posture, and gestures are used as the basis for establishing broad categories of behavior. Analysis of students' physical behavior serves as a source of data for characterizing the dynamics of transitions in students' tutorial activity. The analysis of student behavior, however, will differ slightly in some important ways. The analysis will concern changing patterns of behavior for individual students, not just entire groups. It will also differ slightly in its overall purpose. Hammer and Scherr were largely concerned with the dynamics taking place among behavior, reasoning, and epistemological framing. Our goal here will be to understand the dynamics taking place among patterns of behavior, patterns of reasoning, and patterns of attention. This shift in focus is not inconsistent with the interpretation of behaviors discussed by Hammer and Scherr. They note that verbal and non-verbal behaviors reflect multiple aspects of human activity. For example, a gesture may simultaneously indicate (and communicate to others) aspects of a persons' thinking that are conceptual, affective, and epistemological in nature. For our case study of student thinking in the Speed Tutorial, patterns of students' physical behavior provide additional source data on the nature of student attention and the dynamics by which transitions of behavior, reasoning, and attention occur.

In the following section, I describe the method of analyzing student behavior with examples drawn from the case study of John, Paul, Beth, and Kate.

Categorizing Patterns of Student Behavior

The method used for analyzing students' physical behavior begins by watching video with the sound turned off. Moments of change in individual students' behavior can be broadly observed in terms of shifts in individual students' posture, gaze, and hand positioning. The exact times of such changes are marked along with a description of the nature of the change. An example helps to make this clearer.

In the snapshot shown below (Figure 9), we can see that John, Paul, and Beth each seem to be oriented toward the center of the table. The three of them each has their gaze directed at the center of the table (indicated with green dot). Beth is leaned further inward and has even placed her hand at the center of the table where they are all looking. Kate, however, is not looking at the center of the table. She is oriented toward her worksheet. She is looking down (and away from the center of the table), her posture is hunched over, and she has a pencil in her hand. Her behavioral orientation toward the worksheet is indicated with a red dot.

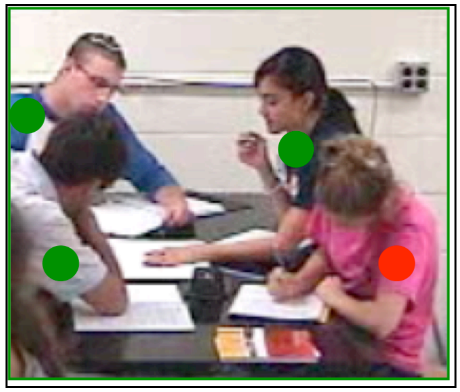


Figure 9: Example #1 of Student Behavior

In the next snapshot (Figure 10), which occurs just a moment later, Beth, John, and Paul are all still oriented toward the center of the table. They are all looking at the center of the table where Beth's hand is located. Paul has even moved his hands closer to the center of the table. Kate is now looking at the center of the table, too. Her posture, which was hunched over before, is now leaned more inward. Both of her hands are also at the center of the table. The entire group's collective behavior is oriented to the center of the table (and the group), which is indicated with a green dot for each student.

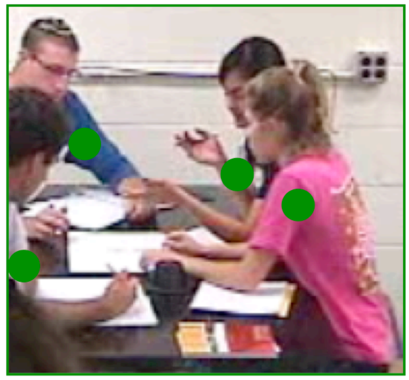


Figure 10: Example #2 of Student Behavior

These two snapshots illustrate what is meant by using students' gaze, posture, and hand positioning to identify moments of *change* in individual students' physical behavior. Kate's behavioral orientation has changed significantly, while the other students have not changed very much.

This process of classifying broad categories of interactive behavior were made across the portions of the video that were identified as being relevant to the students' discussion about the time and speed rankings of the tickertape strips. From these observations, particular patterns of orientation emerged and are described in the following section.

Emerging Patterns of Behavior

Four broad categories of student behaviors were ascertained from the above analysis concerning change in individual students' behavior: orienting toward center of the table (coded as green), orienting toward worksheets (coded as red), orienting toward other individual students (coded as yellow), and orienting away from table (coded as grey). Each of these behaviors could be assigned to each of the four students both before and after marked moments of change. The color codes are additionally used to indicate the *change in orientation* of students in snapshots taken from the video. Example are shown below and in the appendix, where two examples of extended transcripts illustrate some various codes

In the snapshot shown below (Figure 11), three of these categories of orientation can be identified. Paul and John are oriented toward their worksheets (indicated with the red dot). They are both looking down with their posture hunched over. Beth is oriented toward the center of the table (indicated with green dot). Her gaze is directed at the center of the table, her posture is leaned inward, and she has her right hand located at the center of the table. Kate is oriented away from the table (indicated with a grey dot). Her posture is more upright and her gaze is directed away from the table and the other students. No student is looking at another student.

Figure 11: Example #3 of Student Behavior

In another snapshot shown below (Figure 12), Paul and Beth are clearly oriented toward each other (indicated with yellow dots). Their gaze is directed toward each other and their posture is upright. Kate is oriented toward the two of them and her posture is also upright. John is oriented toward his worksheet. His gaze is directed at the worksheet and his hands are located directly over the worksheet. No one is looking at the center of the table, leaning inward, or has their hands located near the center of the table. No one appears to be oriented toward the center of the table or away from the table.

Figure 12: Example #4 of Student Behavior

As I have mentioned, this method for coding individual students' behavior from video is very similar to and was motivated by a methodology for coding clusters of student behavior (in tutorial from video data) that is described by Scherr and Hammer (2009). The two methodologies vary from each other in several important ways. First, their codes were applied to whole groups as indictors of collective activity. The analysis described here applies codes to individual students. Second, their coding was designed to be easily and rapidly implemented by watching videos in real time. Because of this relative ease of use, their analysis could be applied across an entire instructional period for many different groups. The coding described here, however, requires the careful watching of even short segments of video repeatedly. The video for this case study was often analyzed in slow motion or advanced frame by frame to identify moments of behavioral change.

The two methods not only differ in their implementation, but in their purpose as well. The emergent patterns of behavior that were coded for these two separate methodologies do overlap —both have codes for behaviors that can be described as oriented toward worksheets and for behaviors oriented toward other students. However, these codes are used for making different inferences about student activity. The behavioral clusters that were identified by Scherr and Hammer are used as the basis for making inferences about students' *epistemological framings* during tutorial. The patterns of student behavior described here are used as a source of data that indicates (not only what students are paying attention to, but) how they enter into and out different patterns of attention.

At this point, I have only described the patterns of student orientation that emerged from this initial process of identifying moments of change. In the next section, I describe sources of change in students' orientation, and how these sources lead to couplings among students' orientation.

Sources of Change and Couplings among Students' Orientation

Now that we have in place (rather loosely) a methodology for identifying patterns of behavioral orientation, I'd like to shift gears toward an analysis of potential sources of change. What are students actually orienting toward? Why do students change their orientation?

Sources of Change in Students' Orientation

By looking across the many moments of behavioral shifts across the portions of video that were coded, potential sources of change are identified. In this section, I illustrate some specific examples of change in students' patterns of behavioral orientation. Through a discussion of these examples, I suggest several sources of change in students' orientation: Other students' behavior, the location of material objects, and verbal declarations.

One source of change involves students orienting to what other students are doing. Specifically, students notice that other students are oriented in particular ways and orient themselves similarly. In the series of snapshots shown below (Figure 13), Kate changes her orientation in a way that mirrors the other students' orientation.

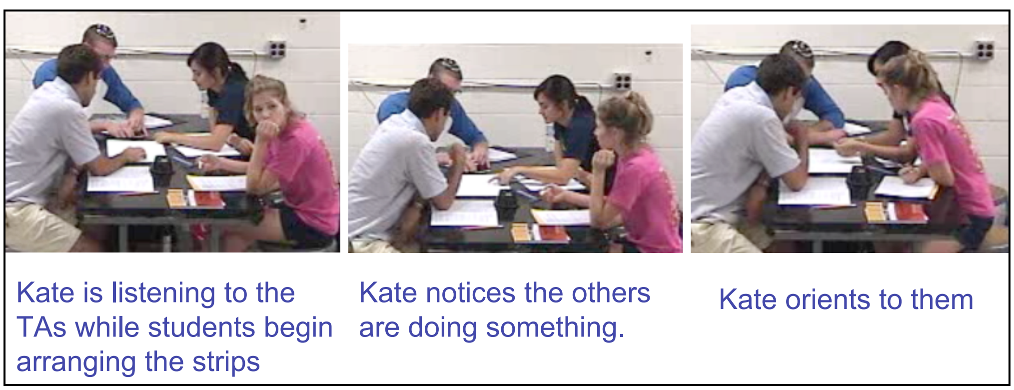


Figure 13: Example of Student Orienting to Other Student Behavior

In the first snapshot above, Kate is oriented away from the table. One of the TAs is still addressing the entire class and Kate is listening. John, Paul, and Beth, however, have already begun the tutorial. Each of them is oriented toward the center of the table. Their gaze is directed toward the center, and each of their hands is located there as well. Before the TA finishes talking, Kate turns around and notices that the other students are doing something. Kate then orients to the center of the table by leaning in closer. This example illustrates how other students' behaviors can serve as source of change.

Another source of change comes from verbal declarations made by students. In the series of snapshots shown below (Figure 14), the students change their orientation after Beth makes a statement. In the first snapshot, the students are all oriented at the center of the table. Then Beth says, "So what do we write about this?" As she says this, she begins to lean away from the center of the table. In the second snapshot, all of students shift their orientation toward their worksheets. In this case, Beth exerts some influence over what the other in the group orient to by verbally directing attention to an activity involving the worksheets.

Figure 14: Example of Students Reorienting after Verbal Statement

Another source of change concerns the location of objects in space. In the series of snapshots shown below (Figure 15), John, Paul, and Beth are oriented toward their worksheets. Kate is slightly oriented toward the center of the table. She is looking at one of the strips located on the table. In the second snapshot, Kate picks up the strip of paper off of the table as she begins speaking. Paul and Beth slightly turn their heads toward the strip in Kate's hand. In the third scene, all of the students are oriented toward Kate and the strip of paper in her hand. John is now looking and sitting up, and Beth has more fully turned her head toward Kate.

Figure 15: Example of Students Orienting to Objects

In this example, Kate's changing of the location of an object becomes a source of change for the other students' physical behavior. Kate was oriented toward the strip of paper at the center of the table and the other students were oriented toward their worksheets. As Kate lifted the paper off the table, the other students shifted their behavior to that object as well. As Kate brought that object close to her, all of the students orient to her.

It should be clear from the following examples that there is not a single source of change in students' orientation, not even across a particular moment of change. In the first example, the Paul, Beth, and John are all oriented toward the center of the table where all of the strips of tickertape are located. These objects are a source of their orientation to the center of the table as well as each other's collective behavior which is oriented to that location. When Kate changes her behavior, she is both orienting to the students' behavior and to the objects at the center of the table.

In the second example, the students are oriented toward the center of the table where the strips are all located there. When Beth says, "So what should we write about this?" she begins to shift her body and gaze toward her worksheet. Beth's verbal statement about writing, Beth's change in behavior toward her worksheet, and the worksheet itself all serve as cues for reorienting the others' behavior.

The third example similarly involves multiple source of change. Kate changes the location of an object, makes a verbal statement while doing so, and changes her orientation. Students change their behavior to be oriented toward each other as each of these changes takes place.

In many respects, none of the general claims made about the sources of change for patterns of student behavior are surprising. Students notice and orient to what other students are doing. Students notice when other student make statements about what they should do next. Students also notice objects around them (especially when they change location) and orient to those locations. However, the details in how these specific sources of change play a role in dynamics of student orientation prove important for understanding the dynamics of student thinking in this setting and, importantly, for the local stability of students' thinking.

Coupling Among Students' Behavior

In the previous section, I described how students change their own behavior in part due to other students' behavior, to locations and relocations of objects, and to students' verbal statements. In this section, I describe how these sources of change lead to couplings among students' individual orientations that generate collective patterns of orientation among students that exhibit stabilities of their own. These collective patterns of orientation include two that were initially described by Hammer and Scherr, and one collective of patterns that arises in part due to students' interaction with the strips of tickertape. The three collective patterns of orientation are discussed here:

- Collective Orientation toward Worksheets
- Collective Orientation toward Tickertape Strips
- Collective Orientation toward Peers

The first collective pattern of behavior is students all being oriented toward their worksheets (see Figure 16). Students' posture is more-or-less hunched over. Students' gaze is directed downward. Students' hands are located in front of them (often writing). This collective behavior is the same as Scherr and Hammer's red behavioral cluster, which indicated to them a completing the worksheet frame. This collective pattern may exhibit a global stability due in part to how students orient to each others' behavior (they notice what each other are writing and continue doing the same) as well as how they orient to their worksheets (objects that are located near to each of the students). It is certainly not irrelevant that these students have probably been immersed in schooling activities their entire lives that involve similar worksheets and worksheet activities. They seem to know both that they are supposed to attend to these worksheets and in what ways they are supposed to interact with it.

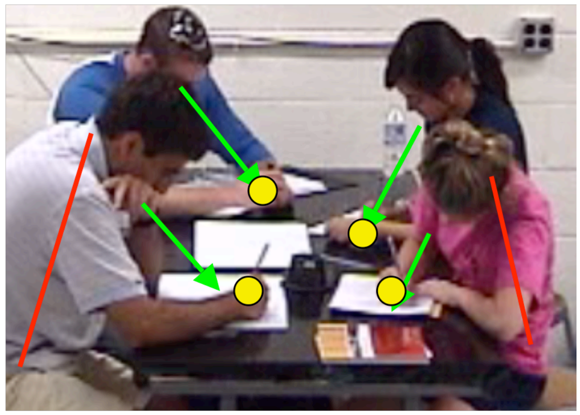


Figure 16: Students Collectively Oriented to Worksheets

A second collective pattern of behavior is students all being oriented toward the center of the table (see Figure 17). Students' postures are mostly leaned inward. Students' gaze is mostly directed inward. Their hands are mostly located at the center of the table. In this case, there are also objects at the center of the table that provide a degree of stability for this collective pattern. It is not irrelevant that physical objects such as the tickertape strips remain where they are placed on the table until moved again. This regularity and stability in the physical world provides a stability for this pattern of behavior. Students must actively change the location of the objects away from the center of the table for those objects to no longer influence their behavior in such a way to be oriented to the center of the table.

Students' hands are also often located at the center of the table. They may be pointing to strips and to locations on the strips. As one student points to a particular location, other students shift their gaze in that direction. They in turn notice things and point to strips and location themselves. The dynamic between pointing and looking that takes place among students may serves to stabilize this pattern for a period of time.

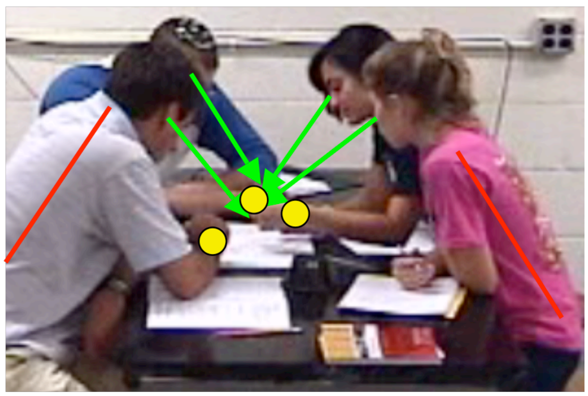


Figure 17: Students Collectively Oriented to the Center of the Table

A third pattern of collective behavior is students being mutually attentive to each other. In Figure 18 (shown below), all of their posture is upright and their gaze is on each other. Kate and John even have their hands in the air away from their worksheets and away from the center of the table. This pattern of behavior is the same as Scherr and Hammer’s blue behavioral cluster, which indicated a discussion frame. This pattern of behavior may be stabilized in particular ways as well. Students notice that other students’ posture is upright and they continue to do so themselves. Students also engage in mutual eye contact. In the scene below, Beth and John are also gesturing their hands around their own bodies, helping to sustain other students’ gaze toward them.

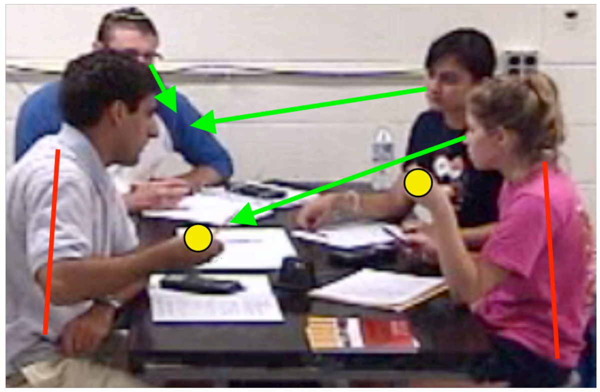


Figure 18: Students Mutually Oriented to Each Other

These three patterns of collective orientation are significant because students' individual orientations are not random, despite exhibiting variability. In fact, there are times in the case study (that will be described) where a single pattern of collective orientation persist for minutes or longer. While individuals may fleetingly move in and out of that collective orientation, the whole groups' collective orientation remains dynamically stable through couplings among students and between students and objects. This is not to say that students are always characterized by a single collective orientation. Rather, students fall into and out of collective patterns. Sometimes they are strongly coupled to each other. Other times they are not. At even broader time scales, students' collective orientations may be seen to oscillate between two dynamic stabilities of collective orientation. For example, during a scene described later, students seem to move only between being collectively oriented toward the worksheets and collectively oriented toward the center of the table. Both of these phenomenological observances of patterns in students' behavior- emergent stabilities and transitions among multiple stabilities-are similar to qualitative dynamics of other complex system such as the Lorenz attractor (Lorenz, 1963).

Putting the Tool to Use for Analyzing Student Thinking

In this section I have described a fairly loose methodology for identifying patterns of student behavior in the Speed Tutorial. The method began by simply observing the video of these particular students as they proceed through the Speed Tutorial with the sound off. Through repeated observations, moments of change in student behavior were marked by attending to changes in students' gaze, posture, and hand locations. From these moments, several broad categories of behavioral orientation emerged. These categories served as a basis for speculating about sources of change to individual students' orientation. Partially arising out of these sources of change, I argued that couplings among individual students' behavior lead to emergent patterns of collective behavior that exhibit stabilities as well. These collective patterns of behavior coincided with two of Hammer and Scherr's behavioral clusters and included one pattern of behavior that is unique to the Speed Tutorial.

For the rest of this chapter, I would like to use the products of this analysis concerning student behavior to bring insight into the dynamics by which John, Paul, Beth, and Kate settle into and shift among multiple patterns of thinking and attention. This application serves to illustrate how the methodology can bring insight into the dynamics taking place in the setting for students' activity and, importantly, also for how those dynamics couple to the dynamics of student thinking. This analysis will involve a shift toward considering the students and the objects around them as being part of an extended cognitive system that exhibits its own dynamics. These dynamics take place at a broader scale of space than those of activation in our toy cognitive models of elements of the mind. These dynamics are observable and concern the physical relations of these objects and students as agents for changing those objects. Similar to way our toy cognitive models represent students' intuitions as cognitive elements that are either activated or not, external structures in the world become focal and consequential to students' activity in different ways. We may think of these structure in the world as cognitive elements that become active and assemble in different ways.

The analysis of John, Paul, Beth, and Kate's behavior thinking throughout this case study is broken into three scenes. The first scene involves students thinking that the shorter strips take less time and are faster. The second scene involves students thinking about the mechanisms by which the strips were made. The third scene involves students explicitly contending with these two different patterns of thinking.

Scene 1: The Dynamics of Students' Initial Thinking

In the first scene, Paul, Beth, John, and Kate settle into an initial pattern of thinking about the strips that we have encountered before—the shorter strips take less time and are faster.

This pattern of thinking emerges as the students construct a particular setting for their activity (by moving and arranging the strips at the center of table), as they behave in particular ways (by orienting their bodies inward toward the strips and downward toward their worksheets), and as they attend to particular features of the tickertape strips (by referring and pointing to physical features of the strips).

Presentation of Data

This initial scene is broken down into five parts. In part one, the students move the strips to the center of the table and Beth arranges the strips in a particular way. In part two, Beth proposes that the shorter strips take less time as an answer to the time-ranking question. In part three, the students point out features of the strips in order to affirm Beth's proposed answer. In part four, Kate wonders about other features of strips that they possibly aren't paying attention to. In part five, the students explicitly answer that speed-ranking question.

Each of the five parts will involve the presentation of the transcript followed by a narrative describing what takes places in that transcript. After each of the five parts are presented and narrated, we'll return to analyze student's thinking and behavior.

Student actions and shifts in orientation are noted in *[italicized brackets]* throughout the presentation of transcripts. Snapshots taken from the video and corresponding behavioral codes can be found in the appendix as well.

Scene 1, Part 1: Moving the Strips to the Center

This first transcript begins as students are just starting the Speed Tutorial. The teaching assistants are still talking as Beth, John, and Paul begin the tutorial.

Beth *[flipping open her worksheet]:* Alright

John *[looking to the center and then down to his worksheet]:* Compare.

Beth *[reading]:* Compare you paper segments with those of your partners… So everybody got one…

[Paul, John, and Beth lean toward the center and each move their strips to the center of the table]

[Kate is looking up listening to one of the TAs who is still introducing himself]

Beth: Did anybody get the same rate? This one has like similar *[pointing to two strips].*

Kate: What, what are their names? *[Kate turns toward the group, looking toward the center]*

Paul: Those are kind of close *[pointing to two of the strips]*

[*Beth leans in farther to examine the strips. Paul and John look lean to the center looking closely at what Beth is doing. Beth is shifting the strips around. Kate looks to her worksheet and then off into the distance again*]

Beth: These two might be the same.

[Kate leans toward the center]

Kate *[turning to her worksheet]:* OK. Wait.

Beth: No. Nobody has the exact same rate

John [*pointing to the shortest strip*]: So I guess that's the fastest.

[Kate quickly turns toward the center. Beth taps her finger over one of the strips several times]

John: The one right there. Yeah

Beth *[turning to her worksheets]:* OK. So what do we write about this?

This scene starts with Beth and John opening up their worksheets reading from the tutorial. They read from the worksheet that they are supposed to compare their strips, and then Paul, Beth and John move all of the strips to the center of table, as is indicated in Figure 19, shown below.

Figure 19: Tickertape Strips Located at Center of Table

John, Paul, and Beth collectively orient to the strips by leaning and reaching inward as they look toward the strips. Kate does not join in on this right away. Instead, Kate is looking off into the distance (see Figure, seemingly listening to one of the TAs who is still talking. Kate finally does notice that the students are collectively doing something and Kate briefly orients her gaze to what they are doing. However she quickly looks away from what they are doing and says, “OK. Wait.” She then orients her gaze back to the worksheet.

Meanwhile Paul, Beth, and John continue comparing the strips at the center of table. They notice that two of the strips are similar (referring to this similarity as the same ‘rate’). They decide that none are exactly the same after checking more closely. Beth continues to manipulate the strips with her hands. She moves them around and places the side-by-side in order by length, as is shown in Figure 20.

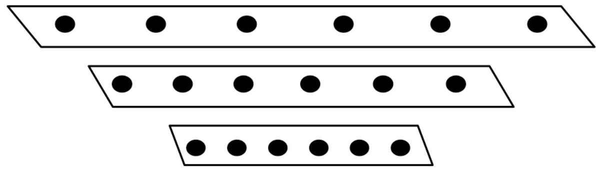


Figure 20: Tickertape Strips in Side-by-Side Arrangement

Afterwards, John points to one of to shortest strips and says, “That’s the fastest.” In doing so, he uses the word “fastest” as an adjective to describe the strip, which is a pattern of use we have observed before. As John finishes saying this, Kate reorients backs to the group (leaning and gazing toward the center). Paul, Beth, and John then shift their attention to their worksheets right after Beth, says, “OK. What do we write about this?” Kate then does the same.

Beth then reads the first question of the tutorial and proposes that the motion is constant and uniform. Kate responds by looking over to Beth and saying, “Yeah, That’s Good!” Beth says back, “Hey! Hey!” in a friendly but slightly sarcastic tone. John adds that it’s (constant and uniform) because they are all evenly spaced out. Beth acknowledges this, and then students continue to write in their worksheets.

Even in this first minute of the tutorial, there are particular patterns that emerge in the way that the students’ orientation toward objects on the table, toward statements made by each other, and toward each other’s posture and gaze. First, in the beginning of this clip, Paul, Beth, and John each orient to the strips that they have moved to the center of the table. It seems particularly relevant the Beth has organized the strips in order by their length the particular way that she has. It emphasizes the differences in length in very clear way. In attending to features of those strips, they refer to similarities and differences in the strips. Beth uses the word “rate” to describe this similarity. John refers to the shortest as the fastest.

Paul, Beth, and John’s behavior throughout this scene is fairly coordinated. These three each enter in and out patterns of mutual orientation toward the strips and collective orientation to their own worksheets with remarkable coordination. They also orient and coordinate around shared attention to objects and statements made by each other. Beth and John coordinate their attention and statements through brief verbal exchanges and gestures, following up on each others’ statements and acknowledging shared meaning. They seem to do some coordinating around a shared meaning of *difference* and *rate* (e.g., they have different rates and one is the fastest).

Throughout, Kate is not orienting to aspects of the situation in the same way. While she does orient toward their activity at times, she opts in and out quite variably. She is the last to join in and the first to opt out. She neither refers to the strips of tickertape in her speech nor does she point to them with her hands or pencil as the other do. Instead Kate seems to be trying to figure out what is going on. First, she wonders what the TAs names are, looking around while the others are comparing the strips. Then, when she notices what the other students are doing, Kate only joins them briefly. She then says, “Wait” as she goes back to read her worksheet. The tone of voice when she says this suggests she is trying to read the worksheet to figure out what is going.

Scene 1, Part 2: More Distance Implies More Time

In this second scene, the students are writing in their worksheets. Beth is about to read the next question of the tutorial.

[The students are all oriented with their gaze and posture toward their own personal space, where their worksheets are located. They all have pencils in their hands and are writing in their worksheets]

Beth: *[reading from her worksheet]* How does the time taken to generate one of …*[mumbled reading of the question that trails off]*. Obviously it takes less time to generate the more closely spaced dots. *[holds two fingers up separated by a small gap]*

[Beth's phone rings and she goes to turn off her phone, turning away from the group. The three other students continue to sit quietly oriented toward their worksheets for about 10 seconds]

[John quickly gazes toward the center of the table (where the strips are located. Kate gazes toward the center of the table, but John looks away back toward his worksheet. Kate glances over at John's worksheet.]

Beth: *[returning to the table, looking at her worksheet]* OK. Sorry. So.

[Paul and John glance up toward Beth. Beth orients slightly toward the center.]

John: *[looking toward the center, as Paul look ups to John and Kate looks down]* So you are saying it takes less time to make the shorter segments? *[quickly looking back toward his own worksheets]*

Beth: Right.

John: Alright.

At the beginning of this scene, the students are all still oriented toward their worksheets and writing their answers from the previous question (that the strips are constant and uniform). As she did with the first question of the tutorial, Beth now reads out loud the second question from the worksheet and proposes an answer. She says, “Obviously it takes less time to generate the more closely spaced dots”.

Immediately after, Beth’s phone rings, and Beth goes to turn it off. In doing she turns away from what everyone else is doing- attending to their worksheets. While waiting for Beth to return, John and Kate stir a little bit, shifting their gaze, but the group remains largely oriented toward their worksheets.

When Beth finally returns, she immediately orients back to her worksheet with the other students. John then looks up and the toward the center of the table and asks, “So you are saying it takes less time to make a shorter segment?” Beth acknowledges this is what she meant and also looks toward the center. In this moment Beth and John seem to be agreeing that shorter strips take less time. In doing so, they orient back toward the strips at center of the table in a mutual way. It is important to note that John does not simply parrot back what Beth said. He uses the phrase “shorter segment”, while Beth said, “more closely spaced dots”, and Beth acknowledges this as what she meant. This suggests that they are not just attending to the use of particular words, but orienting around the particular idea. As with many of the other groups we have observed responding to the time-ranking questions, it seems that the Beth and John are cueing into the intuition that *more distance implies more time.*

Scene 1, Part 3: Attention to the Physical Features

In this scene, Kate is about to read the second part from the time-ranking question.

Kate *[reading from the worksheet]:* How can you tell?

[Beth leans in toward the center, placing her hand over the strips, and taps her pencil on one of them. Kate and John both look toward the center. John quickly looks back at his worksheet and then back at the center]

Beth: You can tell because…

Paul *[glancing toward the center]*: It's a shorter segment

Beth: It's a shorter distance.

John: You've made. You've made more segments in like the same amount of space *[reaches in and indicates a length on one of the strips].* You've made like more little *[spreads his fingers out indicating dots and then waves his hand back and forth over the strips]* things.

Beth: Yeah.

[Kate orients back toward her worksheets]

[Paul reaches toward the center]

Paul: It's like the same amount of dots *[pointing to a strip]* in a shorter piece *[indicating a length on one of the strip by separating two fingers]*

Beth: Yeah. When given the same length of… *[pauses while holding an open palm face down near one of the strips, indicating a length between her thumb and index finger]*

At the beginning of part of this scene, Kate reads off her worksheet, “How can you tell?” All of the students then orient toward the center of the table.

During this time there are many instances of students identifying features of the strips. Paul and Beth state that they can tell because it’s “shorter”. Paul, Beth, and John each articulate in different ways that they are also paying attention to how many dots are segments fit in a certain amount of space. Paul says there is there are more segments in the same space. John says it’s the same dots in a shorter piece. Beth begins to state that something about a fixed length, but then trails off. They seem to coordinating their attention around the idea there are a certain amount of dots “contained within” a certain amount of space.

As they do this, Paul, Beth, and Kate are all strongly oriented toward the center of the table. There are also many instances of pointing to strips and pointing to particular locations on the strips. Beth points to one of the strips with her pencil. John spreads his fingers over some of the dots on a strip (Figure 21). Paul points to a strip and then indicates a length by separating two fingers (Figure 22). Beth indicates a length between her finger and thumb (Figure 23).

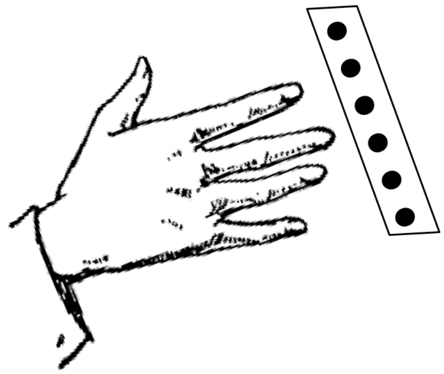


Figure 21: Illustration of John Spreading his Fingers over a Strip

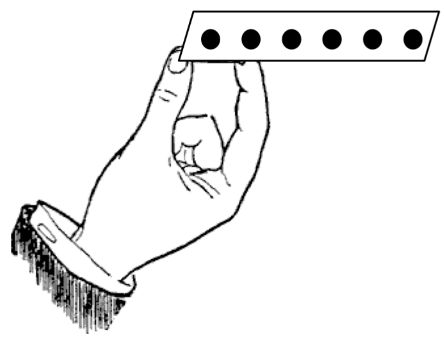


Figure 22: Illustration of Paul Indicating a Length between Two Fingers

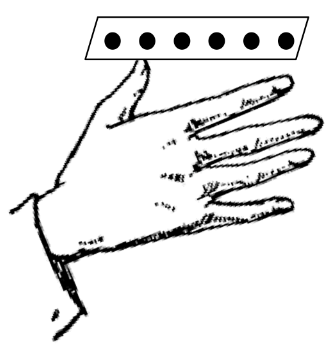


Figure 23: Illustration of Beth Indicating a Length on a Strip

Paul, Beth, and John seem to be collectively engaged in the same activity. They are each oriented to the same objects (the strips and particular features on the strips). They all share common posture and gaze (leaning in and looking toward the center of the table). They also seem coordinating among many of same ideas – that the shorter strips take less time, and that the spacing of dots is different for different strips. Their orientation, statements, and gestures all keep this activity oriented at the center of the table.

Kate does not orient to their activity as strongly. She does orient to the strip as the center at times, but only partially so (she gazes toward the center but does lean in; she looks at the strips but does not point to them). She does not mention any features of the strips or make comparisons as the other do. She is also the first student to orient back toward the worksheets.

Scene 1, Part 4: Shifts and Recognition of Attention

At the end of the previous scene, Kate was oriented toward her worksheet while the other students were oriented toward the center of the table. At the beginning of this scene, Kate abruptly orients to the center of table as she begins talking to the other students.

Kate *[popping up from her worksheet leaning in toward the center]:*

> When we are talking about segments *[pointing to strips with her pencil],* are we like not thinking about how long the total paper is *[bringing her other hand over to the center over the strips]*? Are

we just looking at the marks *[pointing to successive marks with her pencil]?* Are we supposed to be considering – *[pulls her left hand away, keeping her hand with pencil at the center of the table]*.

John *[leaning further toward the center]:* — I'm guessing they like mean from here *[pointing with pencil]* to here *[pointing with pencil]. [points with pencil to the two marks again in succession]*

Kate: Like I wonder why like the papers are all different lengths *[moving her pencil from one strip to another]*

Beth *[bringing her hand closer to the center]:* Cause none of these papers are the exact same si-ize *[speaking quietly]* Except for these two *[pointing to two strips]*

Paul: Right because I think *[moving hand toward the center]* they all have the same amount of dots *[pointing several locations on one of the strips]*.

Beth: Oh-oh

Paul: I think they all have six dots.

Beth: Oh do, they?

Kate *[leaning farther in]:* Is that true? 1, 2, 3, 4, 5, 6 *[pointing to successive dots on a strip]*

John: Yep

Paul: So, it's a shorter amount of time for a shorter piece of paper.

In this scene, Kate pops up from her worksheet and quickly orients herself to what the others are doing and to the strips at the center of the table. She asks a series of question about what the group is doing, including what they are talking about, what they are thinking about, they are looking at, and what they are supposed to be considering. It is important here that she is not pointing out features of the strips in the same the others have (in support of the claim that the shorter strips take les time). Rather she is pointing out features of the strips as a way of asking to what the group is doing and what they are supposed to be doing.

John points to a distance he thinks they (the tutorial writers) mean them to be paying attention to (the distance between dots). Kate then says she wonders why the papers are all different sizes. In doing so she seems to referring to entire length of the strips, not just the distance between dots. Beth seems to pick up on what Kate has said, adding that none of them are the same size.

The answer that Paul gives is that they all have the same amount of dots. When Paul says they all have six dots, Kate asks if this true. Kate then goes to verify this claim by counting the number of dots. She leans far in toward the center of the table and moves her finger along the strips as she counts from one to six. After Kate is done counting to affirm what Paul has said, Paul restates that “It’s a shorter amount of time for a shorter segment.” By doing so, Paul brings up the original idea again that had initially been suggested Beth. The students then all reorient back to their worksheets, seemingly beginning to write their answers to the this question in their worksheets.

Scene1, Part 5: Shorter Papers are Faster

The brief scene occurs a short while later after the students are done writing in their tutorial worksheets. The students read the speed-ranking from the tutorial and quickly settle upon an answer.

Kate *[reading]:* How do you know how to arrange them

[Beth looks back to her worksheet]

Beth *[reading]:* How do you know how to arrange –

Kate *[looking up with Paul]:* Ahh. The shorter the segment the faster the speed

Beth *[quickly looking up and the pointing and nodding to Kate]:* Yeah.

John: Yeah.

[Everyone looks back down at their worksheets again.]

Paul *[looking up slightly, as does Beth]*: Also shorter the paper, it's the same thing.

John*:* Yeah

[Everyone is looks back down to their worksheets again.]

Kate *[looking up]:* Yeah. I don't know. *[no one looks up, Kate touches her hair, eventually Kate looks back down]*

In this last part of scene one, Kate begins reading aloud the speed-ranking question from the tutorial but is interrupted by Beth who takes over. Before Beth finishes reading the question Kate answers," The shorter segments the faster the speed". John and Beth quickly agree.

Kate's statement here is consistent with John's initial statement (from part one) that a shorter strip was faster. Paul then adds, "Also shorter the paper, it's the same thing." Paul seems to be saying here that it's also true that the entire strips of paper is faster, implying that Kate had been pointing out how shorter segments (meaning spaces between dots) means faster. John agrees with that as well that they mean the same things. The students then all orient toward their worksheets as they have done before when settling upon an answer.

At the very end of the tutorial, Kate looks up and says, "Yeah. I don't know." No one else looks up or acknowledges what Kate has said. Kate seems to wait to see if anyone will look up. She touches her hair and then looks back down at her worksheet.

Analysis of Students' Thinking

In the analysis of students' thinking described here I set out to accomplish three tasks: (1) point out relevant features of students' thinking that are representative of our fine-grained account develop in the previous chapter; (2) point out features of students' thinking that are representative of the mechanisms that contribute to the stability of their thinking; and (3) describe how the students' thinking and its stability take place within the dynamics of an extended cognitive system.

Consistency with the Fine-grained Cognitive Account

There are many features of students' thinking here that are consistent with our previous fine-grained account which was developed from other case studies of similar students' thinking.

As in many other cases, these students identify the shorter strips as being faster. John initially does so in part one, when he uses the word "faster" as an adjective to describe a shorter strip. Kate does so again in part five while specifically attending to the speed-ranking question, stating that the shorter segments have faster speed.

These students also state that the shorter strips take less time (in parts two and three). Beth does so initially, and even prefaces her statement with the word "obviously". John, Paul, and Beth each make statements and explanations to support this claim. In giving explanations for why the strips take less time, they only refer to physical features of the strips. Paul and Beth each point to the fact the strips are shorter. Paul, Beth, and John also explain in terms of the density of dots in different ways.

As with the other cases, the students are also closely paying attention to and comparing the physical features of the strips. They are aware that the strips have different lengths (evident in part two and part three). Beth and Paul both make statements about this, and Beth is the one that arranges the strips by length. They are also aware that strips have different densities of dots. John, Paul, Beth each make statements about this (in part three). They are also aware that each of the strips has the same amount of dots. John says so explicitly and Kate counts to make sure (in part four).

As with the other cases, talk about the mechanism by which the strips were made is markedly absent. They don't talk about tapping, or moving, or pulling, or marking or any other actions that would indicate they are explicitly thinking about how the strips were made or what that would imply about physical features of the strips. Instead they talk about features of the strips while using their hands and pencils to point to these physical features.

Consistency with Account of Contextual Mechanisms

There are also features of students' thinking that are consistent with the contextual mechanisms that were described as contributing to the stability of students' thinking.

Beth is the first to state that the shorter strips take less time right after reading the strips. She says, "It's take less time for the more closely spaced dots." Here it is seems that she is attending to the space between dots or possibly to the overall density of dots. When John follows up with her statement, he uses the words "shorter segments" to describe the strip with less time. John could be using the word "segment" to refer to the entire length or to just the space between dots. When Kate asks how they can tell, Paul says it's a "shorter distance" and Beth says it's a "shorter segment".

Then the students shift to talking more specifically about overall density of the dots. John says, "You've made more segments in like the same amount of space." Paul says, "It's like the same amount of dots in a shorter piece." Students shift from thinking about the space between dots to thinking about the overall density of dots on a strip. This shift takes place in context of explaining how they know it's less time for the shorter strips. These differences in attention are subtle, but they are not disruptive to the students' thinking.

In part four, Kate makes a more explicit attempt to perturb students' attention. Kate asks what it is they are paying attention to when talking about the strips.

Specifically she asks if they are just paying attention to the marks or the entire lengths. Kate also states wondering why they all have different lengths, and Beth acknowledges this. It is somewhat interesting that the explanation Paul gives for *why* the strips are different lengths is that they all have the same number of dots. It is interesting because his explanation for why there are differences in the features of the strips (the lengths are different) is to point out features of the strips that are the same (the number of dots is the same). It's possible that Paul is thinking that it makes sense that the longer strips are longer in total because of the fact that they are made of the same amount of "segments" which are also longer. His explanation isn't about what made the strips longer, but rather about identifying features of the strips that seem consistent with some of them being longer and shorter. They are all made of the same number of dots, and the spacing between dots varies according to the length.

After the students shift their attention more to the total length of the strips and the number of dots on each of the strips, Paul repeats the group's initial claim that, "It's a shorter amount of time for a shorter piece of paper."

Kate's explicit call for the group to reflect upon what they are paying attention to and talking about doesn't disrupt their thinking that the shorter strips takes less time. Nor does Kate's mentioning that she is *wondering why* the strips are different lead the group to explain these differences in terms of causes. Rather, they continue to point out features of the strips that seem to be consistent with each other.

The entire episode starting from when Beth initially states that the more closely spaced dots take less time until Paul reaffirms by saying that it's shorter amount of time for the shorter segments can be conceptualized in the following way.

Beth initially cues into the intuition *less distance implies less time*. John asks a question to make sure what she is saying, showing some evidence that he is also cued into this intuition. Over the course a few minutes, the students attend to various features of the strips that serve to sustain the activation of this intuition. Even when Kate makes an explicit move that gets the group to pay attention to additional features they hadn't currently attended to before, these shifts in attention still allow for the sustained activation of the same intuition. Even later, when the students are talking about the speed-ranking question, Paul mentions that it's the "same thing" whether you are looking at the shorter length of just the shorter segments. Across all of these shifts in attention, the students sustain the same pattern of thinking. Although the evidence is not as conclusive from students' verbal statements, it seems likely that the intuitions *faster means bunched up* and *more speed implies less time* also play a role. The students spend a lot of time attending to and talking about the density of the dots. We will also see later that Beth explicitly relating increased speed with taking less time when revisiting their earlier thinking.

Dynamics of the Extended Cognitive System

Here I'd like to describe how some of structures and dynamics in the setting for students' thinking can be understood to support and participate in the stability of students' thinking about the tickertape strips.

Probably the most important change in the setting for students' thinking occurs in part one. John, Paul, and Beth, all move their strips to the center of the table. Their actions locate the strips in a place where they can all sustain a shared orientation. This centralized location makes it easy for them to notice and compare features across the strips. It helps to keep the group oriented to the strips not only because the strips are on the table in a shared space; the group sustains in orienting to the center of the table because everyone is also oriented to the center of the table. In other words, the objects are not only a source for students attending to the center of the table. Other students orienting to the center of the table are also a source of sustaining one's orientation (or reorienting) back to the center of the table. Students leaning in and pointing to locations keeps other students oriented toward the center of the table.

In fact, during parts 1-4 of this scene, where the students are discussing that the shorter strips take less time, there are only two brief moments that the students are *not* either oriented to the strips at the center of the table or oriented to their worksheets. Kate briefly looks up to Beth while saying, "Hey, that's good!" Paul and John briefly look up to Beth when she returns to the table after the phone call disruption. Both of these behavioral shifts consist of just brief glances and do not concern the substance of any ideas. At no times during this whole discussion do any of the students engage in mutual orientation toward each other. Instead students orient to the strip and to the worksheets. Paul, Beth, and John do so with remarkable coordination, collectively moving in out of these collective patterns together, which helps to maintain the visual cuing of intuitions from the physical features of the strips.

The tutorial worksheets also take part in the dynamics of student thinking and behavior. When students are not oriented at the center of the table, they are oriented toward their worksheets (except for Kate who at times is looking off into the distance). Beth, in particular, takes on the role to orient students toward and away from the tutorial worksheets through verbal statements. In part one, Beth, asks, “So what do we write about this?” and everyone orients to the worksheets. Beth also reads questions and proposes answers several times. In part one, she proposes that the strips are constant and uniform. In part two, she also proposes that it’s less time for the more closely spaced dots. After Beth reads and proposes answers to these questions, the students orient (back) to the strips at the center of the table. John and Beth do so as they briefly discuss how rates and fastness compare across the strips. The entire group orients to the center of the table for an extended period of time as they discuss the time-ranking question (with Kate variably moving in and out of that pattern).

This pattern of reading aloud questions and then shifting orientation away from the worksheets to the strips of tickertape (and then back to the worksheets) is part of the dynamic by which students move through the tutorial one question at a time. The students read the prompt to compare the strips, and they move their strips to the center of table. Students then read the question about what kind of motion the strips represent. They orient to the strips at the center of the table, and then go back to write down their answers. The students read the question about the time-ranking.

They orient to the strips at the center of the table again, and then go back to write down their answers. Their activity is arguably more geared at answering questions in the tutorial based on static features of the strips than it is a conversation about describing motion phenomena.

The side-by-side arrangement of the strips that Beth constructs also plays an important role in the dynamics of their thinking. Not only do students notice patterns among the features of the strips, they organize the world around them to reflect the things they are noticing. This particular arrangement, in which the strips are organized by their length, also facilitates the students' sustained attention to distance features of the strips by making differences in distance more readily available than they would otherwise (e.g., if the strips were either located in separate places on the table or if they were arranged more randomly at the center of the table). This physical arrangement of the strips, and the fact that it remains in place until someone actively changes it, contributes to the contextual stability of students' thinking by exhibiting a structural stability in the world.

Brief Summary

In this section I have described how students' thinking in this first scene is consistent with aspects of the fine-grained account described in the previous chapter. One particular intuition that seems to play a significant role in students' thinking here is the intuition that *more distance implies more time*. There is evidence that each of the students in the group shares this intuition when thinking about the strips.

I have also described how aspects pertaining to the stability of their thinking are consistent with the particular contextual mechanisms described in the previous chapter. The contextual stability of students' thinking was described in terms of the resiliency of the particular assembly of intuitions to remain active despite shifts in attention to different physical features of the strips. In the above scene, we students' attention shift to different physical features of the strips all within an extended scene of students cueing into the intuition that *more distance implies more time*. While this account of contextual stability helps to explain why students' thinking is not disrupted when they shift their attention to various physical features of the strips; it doesn't help in explaining why students' attention only varies in this narrow way.

Some insight into the dynamics by which students' attention remains focused on only the physical features of the strips is made by considering a more extended cognitive system. Patterns of student behavior show how relevant structures (such as the location and arrangement of strips, the location of worksheets, and students' own collective orientations) facilitate certain patterns of attention and inhibit others. Some of these structures were put in place by the students themselves as they relocated and manipulated objects. Other structures, such as the worksheets, exert influence through their physical location, but also likely through a history of school culture that sets tacit *norms* for their use (Lave and Wenger, 1991). Students start from the beginning of the worksheet and proceed question-by-question. As they do so, the students orient back and forth between the strips at the center of the table and the worksheets in front of them. They do not look at each other in any significant way. Nor do they discuss aspects of the situation that are not directly relevant to particular tutorial questions or discuss aspects that aren't the outcome of their attention to the strips at the center of table.

Here I have argued that students' thinking that the shorter strips take less time and are faster takes place in a particular setting for students' thinking that they have played a role in constructing. Collective patterns of students' behavior shift between one of two stabilities— mutually oriented to the strips at the center of the table and collectively oriented to their own worksheets. The particular location and arrangement of the tickertape strips are consequential to the dynamics by which students move between these two patterns of attention.

In the following section, I describe the dynamics by which students' thinking changes. As is the case here in describing students' initial thinking, the dynamics of students' new thinking involve changes to external structures that are consequential for patterns of student orientation.

Scene 2: The Dynamics of Students' New Thinking

In the last part of scene one, students were busy writing in their worksheets. After students finish writing down their answers, they proceed to the next part of the tutorial that asks them determine numerical values for the distances traveled by the cart represented by their strip of paper.

Scene two is broken up into two parts. In part one, the students deconstruct the side-by-side arrangement of the strips at the center of the table in order to measure distances on each of their own strips. Part two occurs a little while later as the students are discussing what assumptions they were making in doing these calculations. In this part, Kate begins talking about how the strips were made, which ultimately leads to a shift in students' thinking about the strips in a different way.

Presentation of Data

Scene 2, Part 1: Deconstructing the Side-by-Side Arrangement

In part one, Kate begins by reading a question from the tutorial about how far the object that made their paper traveled in $1/40^{th}$ of a second. This leads to group to reach toward the center of the table and relocate the strips closer to themselves.

Kate *[reading]:* How far did the object that generated your paper move in $1/40^{th}$ of second? *[looks up]* OK, so it's like.

Beth *[looking toward center]:* So should we measure in millimeters?

Kate *[leaning in to grab a strip]:* I guess. Do we each measure our own?

[Beth leans in and grabs a strip. Kate hunches over her strip and starts measuring with a ruler]

Beth: On your paper, yeah. *[leans in and grabs a ruler, and hunches over]*

[John leans in and grab a strip and a ruler, going back to his own work]

Kate: Mine's like a perfect inch measurement. So.

[Paul looks toward the center and grabs a strips, and then looks back down]

In this scene, students' actions are the most relevant. Kate reads aloud a question from the tutorial (as they have repeatedly done before). As the students begin to discuss how they are going to measure distance on the strips, each of the students reaches toward the center of the table. Kate reaches toward the center of the table, grabs a strip, and moves it directly in front of her. One by one, the other students do the same. Beth leans in and grabs her strip moving closer to her. John and Paul do the same. In the process of doing so, they deconstruct the side-by-side arrangement that had remained at the center of the table throughout the entire first scene.

After the transcript ends here, the students proceed to answer each of the questions about how far the objects that generated their strip moved in different amounts of time. The students accomplish this task fairly easily and their work is done fairly quietly. They are mostly busy measuring with their rulers and using calculators to perform some arithmetic. They do check in with each other at times to make sure they are calculating their answers the same way.

Scene 2, Part 2: A New Pattern of Orientation

We pick up with the students' activity as they have just finished up with these calculations and Beth is about to read aloud a question from the tutorial worksheet about their assumptions.

[The students are all oriented toward their worksheets]

Beth *[reading from her worksheet]:* Why are these just predictions rather than just calculations? What assumptions did you use to make these observations?

Paul *[Kate looking over to Paul]:* We assumed that 1/40th of a second.

[Paul looking to Beth, and vice versa]

[Beth looks down to her worksheet]

Kate: I think we also assumed in the *[Paul looking over to Kate]*

Beth: That's true. *[Glancing up and pointing to Paul]*

Kate *[continuing]:* that these *[picks up a strip of paper with her left hand and everyone immediately looks up and over to her]* were made *[holds the strip horizontally in front of her]* by the speed at which the paper traveled through the tapper *[pulling the strips horizontally in front of her body]* which was different for each paper. Cause like the marks *[pointing to Beth's worksheet with her pencil, where the others also look],* they are telling you the marks were made every 1/40th of a second. *[Beth and Paul both look down]* So like, it's not the marking *[tapping up and down with her pencil in the air]* that's different.

[Paul looks up again and Beth glances up]

We're assuming the rate it was going through *[pulling the strip of paper horizontally again]* is different.

John: Yeah *[pointing to Kate]*

Kate: You know what I mean?

Beth: Yeah

John: *[looking to Kate who looks back]***:** Right *[Beth looking over to Paul],* cause if you move it really fast then *[pulling his hand with pencil in it quickly in front of his body]*,

Kate *[who is looking to Paul]* **:** It's like if each mark is being made a 1/40th of a second *[pointing back to Beth's worksheet or right hand while holding the strip up with her left],*

Beth *[talking over Kate and looking to John]:* That's true! It could depend on how fast the ribbon was pulled *[pulls her hand quickly across her body]*

Kate *[continuing]:* then they are all going to have the same number of marks *[bringing the strip to the center of the table, holding it vertically]*

[Kate withdraws her strips from the center and behind her shoulder and looks down]

John *[nodding while looking to Beth]:* Yeah *[looking down to his worksheet].* So we're yeah

Kate: We're assuming that umm *[holding the strips out in front of her again]*

Beth *[looking up and pointing to the strip Beth is holding]:* That the length is proportional to

[Paul looks up to the strip at the center of the table]

Kate: The speed at which the ribbon was pulled through

Beth: Yeah

[All the students reorient back to their worksheets writing quietly for about 20 seconds]

Beth *[looking up to Paul]:* The speed at which the ribbon was pulled? *[pulling her hand toward her body twice in succession]*

Kate*:* Yeah.

Paul *[looking up to Beth]*: Yeah.

[Students all orient toward their worksheets]

In this scene the students begin reading a question from the tutorial about the assumptions they are making in doing the calculations for how far the objects traveled in different amount of time. After Paul suggests they are assuming 1/40th of a second, Kate goes on a very lengthy description about how they are assuming the strips were made.

Just as she begins this explanation, Kate picks up the strips of the paper that had been on the table and brings up into the air. When she does this everyone orients to her and the strip of paper in her hand. Kate explains that strips were made by going through the tapper, which was different for each strip. As she says this, she pulls the strips of paper across her body. Seemingly building on what Paul said about 1/40th of a second, Kate explains that they (the tutorial writers) are saying that the marks are being made every 1/40th of a second. She then explains that it's not the marking that is different. As she says this she taps her pencil up and down in the air. She explains that it's the pulling that is different. As she explains this she pulls the strips of paper across the front of her body again.

John then says, "Yeah," and Kate asks, "Do you know what I mean?"

John then says, "Right, cause if you pull it really fast then." John doesn't finish his sentence but he pulls his hand across his own body really fast to indicate what he means by the consequence of pulling fast. Beth also joins in on this. She excitedly says, “That’s true! It could depend on how fast the ribbon was pulled.” As she says this she also pulls her hand across her body.

Kate and Beth then state in tandem that they are assuming that the length of the strips are proportional to the speed of the pull.

The students then go write down their answer, and Beth asks one more time to make sure, “The speed at which the ribbon was pulled?” As Beth says this, she pulls her hand toward her body twice, and the students acknowledge that this as correct. As with many other scenes, this one ends with the students orienting back to their worksheets.

Analysis of Students’ Thinking

Consistency with the Fine-grained Cognitive Account

In this scene, there are features of students’ thinking are consistent with our characterization of students’ correct thinking in the previous chapter.

Students, this scene, begin to talk about actions that occurred while the strips were being made. They refer to pulling, traveling, marking, and going all as aspects of how the strips were made. As with other cases, this pattern of thinking is associated with the use of the word ‘faster’ as an adverb to describe actions. John says, “Cause if you move it really fast,” using fast to describe the verb move. Beth says, “It could depend on how fast the ribbon was pulled,” using the word fast to describe the verb pulled.

Like the other groups, they attend to the mechanisms by which the strips were generated (pulling at different speeds with constant tapping) and to the consequences of that mechanism (varying lengths). Beth and Kate explain together that the length of the strips is proportional to the speed at which the ribbon was pulled.

In the previous scene, students were only attending to the physical features of the strips. Here students' attention shifts to thinking about actions that took place in making the strips.

Dynamics of the Extended Cognitive System

In this scene, there are structures and dynamics of the setting that are significantly different than before that are consequential to their thinking.

In part one of this scene, students deconstruct the side-by-side arrangement that was located at the center of the table. This act changes the setting for their activity in two possible ways. First, it makes it so that the strips are no longer arranged in order by length. Before, the difference in lengths (across the strips) had been readily available due to their close proximity and particular ordering. Now the strips are isolated from one another, making their differences in length less readily available. Second, the strips are no longer located in a single place. Before, the objects' location at the center of the table provided a source for drawing students' gaze, hand, and posture to that space. Now with the objects separately located, students are not able to mutually orient to the strips in the same way. They can only orient to one strip at a time, and it is difficult for students to mutually orient to each other's strips.

This change of structure allows Kate to do something different with the strips that the students had not done before. Kate lifts the strips off of the table. The moment she lifts the strip off of the table, all of the students orient their gaze to the strip and toward Kate. Their gaze follows along with the strips as Kate brings it near to herself. In this moment, the students are for the first time mutually oriented toward each other.

Kate then uses the strip of paper and a pencil as props in a re-enactment of mechanism by which the strips were made. She pulls the strips across her body, and she taps with her pencil up and down. The other students watch these gestures and listen to Kate as she explains how the strips were made. Beth and John both even mimic aspects of Kate's re-enactment when they respond to her and also discuss aspects of the mechanisms. John pulls his hand across his body. Beth pulls her hand across her body as well. Just as pointing gestures toward the strips at the center of table (earlier) were a source for sustaining collective orientation, these more dynamic gestures around the students' own bodies serve as a source for sustaining students' collective orientation toward each other.

It is certainly intriguing that students' change to talking about the mechanism by which the strips were made (and progress toward a correct understanding) coincides with the stabilization of a collective pattern of orientation that the students had not entered into before. I am suggesting here that the students' deconstruction of the side-by-side arrangement earlier is relevant and consequential to the actualization of this change— both in the substance of their thinking and in their arriving at a new collective pattern of orientation. Note that I am deliberately refraining from attributing *cause* to this dynamic relation. In the counter-factual *sense* of causation, had the students not deconstructed the side-by-side arrangement, they still may have found other ways to enter into a pattern of mutual attention or enter into a discussion about the mechanisms by which the strips were made. I am only arguing that for the particular trajectory taken to arrive at this state, the deconstruction of that arrangement played a role in setting up the conditions for change. During the time that the strips were at the center of the table, students' collective orientations were highly constrained to be either to the worksheet or to the center of the table. As the side-by-side arrangement was deconstructed, these constraints were relaxed, providing the degrees of freedom to enter into new patterns of collective orientation. As students mutually oriented to each other, they engaged in a kind of conversation they had not before.

It is certainly tempting to speculate about why students' talk about mechanism would coincide with this new collective pattern of mutual orientation. Similar correlations have been observed among this kind of student behavior and this kind of reasoning (Conlin, Gupta, Scherr, & Hammer, 2007), and researchers have offered explanatory accounts for why such correlations might exist (Scherr & Hammer, 2009). I only briefly mention here that Kate's explanation involved creating a micro-world in which the motion that had taken place in the past becomes actualized in the present through the use of props. By doing so, she takes on the role of an actor for a captive audience. Kate shows that she wants to be understood by this captive audience (e.g., by asking, "Do you know what I mean?"), and this captive audience reciprocates by demonstrating that they care to understand (e.g., by looking, listening and repeating back aspects of her ideas).

Reasons to be Skeptical of Stable Change

In the previous section, I argued that a particular change in the state of an extended cognitive system (the relocation of objects) allowed the group of students to change their patterns of thinking and collective behaviors.

If one takes seriously my claims that students' thinking may be rather strongly influenced by the particulars of that extended system, one should be skeptical that the change that has taken place in students' thinking about the tickertape strips is stable in any general way. One should also doubt any claim of attribution of that stability to the students themselves. Instead, one might rightly claim that students' thinking is still, in fact, distributed among the individuals, the materials artifacts in their setting, and in their particular relations to each other. In this section, I describe a few specific reasons why we should maintain a skeptical stance regarding the nature of the change that has taken place.

First, students' thinking that the shorter distances take less time also seemed to exhibit stability, but that stability did not persist indefinitely. It was stable within certain patterns of attention to the physical features of the strips, which I argued were supported by the dynamics of student orientation in an extended cognitive system. I have described how changed aspects of that extended cognitive system were consequential to students' new thinking. The question is, "Do we have any evidence to support the claim that students' new thinking is more stable than their prior thinking?"

I think there is sufficient evidence to say that the answer to this question is no. This evidence mostly comes from what students don't do following their change in thinking. In particular, students do not go back and reflect upon their incorrect answers to the first two questions of the tutorial worksheet. This suggests two things. One is that the students themselves may not be particularly aware of how there thinking now is different from what they are thinking before. That is, they are not carefully paying enough attention to substance of their own thinking to recognize that their earlier thinking conflicts with what they are thinking now. (Note: Kate and the other students do seem to be aware they are thinking something new, but not necessarily different and in conflict with their prior thinking). This suggests that students may be just as likely to slip back into their prior thinking without noticing either. Second, the fact that students haven't gone back and physically changed their answers to those questions leaves artifacts of their prior thinking "lurking" among the distributed cognitive system. While the students, for a brief time, engage in thinking about the mechanisms by which the strips were made, this change in thinking isn't propagated to changes in the tutorial worksheet where students have written their answers. Individually, we see evidence that Kate, John, and Beth each cue into intuitions for thinking about the pulling of the strips and the consequences of that action. For each of them, we see evidence of a shift to a different way of thinking, at least for a short while.

In the next section, I describe the third and final scene comprising this case study of this group's thinking about the tickertapes. I describe some aspects of the dynamics by which the remaining incongruity in the distributed cognitive system becomes recognized by the students and how artifacts of their prior thinking “lurking “ in the distributed cognitive system continue to exert influence.

Scene 3: The Persisting Influence of Context

In the previous section I described how the John, Paul, Beth, and Kate had failed to notice that their new thinking implied that their previous answer about the time-ranking and the speed-ranking were incorrect. In this section, I describe two scenes that arise as consequence of this failure to notice. In the first scene, Beth comes to explicitly notice there is some inconsistency in their thinking, and the students work to reconstruct their correct understanding. In the second part, Beth and Kate go back to change their answers that they now realize must be wrong. As Kate reads her previous written answers to the question, she briefly ‘slips’ back into thinking that the shorter strips take less time.

Presentation of Data

When we left scene two, the students were writing down their answers to the tutorial question about their assumptions. Afterwards, the students go about determining the numerical speeds that are represented by each of their strips. Paul and John do this by realizing that it’s the same number they wrote for their answer to how far the object travels in one second. Beth and Kate do this by using dividing the distance between two dots by $1/40^{th}$ of second. Everyone does this correctly.

Afterwards the students each write the numbers they calculated for the speed of their strips on notepads. These notepads have a sticky back to them, and the students each attach the notepad (with the numerical speed) to the strips of paper. An illustration of this new object is shown in Figure 24 below.

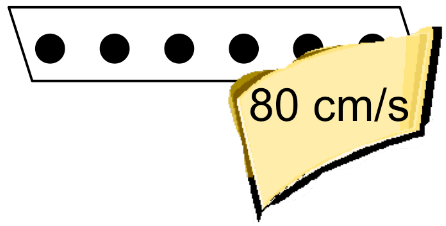


Figure 24: Illustration of Notepad Attached to Strip

Scene 3, Part 1: The Impact of a New Object

This scene begins shortly after the students have finished the first page of the tutorial and the students have put their notepads with the numerical speeds on their strips of tickertape. The students have each turned to the second page of the tutorial. They are all oriented toward their worksheets. Beth is flipping back and forth between the first and second page, seemingly looking back to her answers on the first page. Beth than looks up to Paul and begins speaking.

Beth: *[looking up to Paul]* Wait a second though. Cause mine is 40.4. *[points to her strip]* And then yours is probably 80 point *[points to his strip]*

Paul: 84.

Beth: Oh, I'm sorry. 84. Umm. So, but. *[reaches over the Paul's strip]* Wouldn't yours be going slower than mine, because it took more time to make that same...*[puts finger on his strip, taps, and then slams her palm]* uh

John: Yeah. So the...

Paul: Uh, no. Cause it's not time. *[spreads his finger out and shakes his hand in front of the strip]* We all assume it took 1/40th of a second to make that time, and mine just traveled a farther distance.

Beth: Oh so you are saying the speed of his *[points to John's strip]* ribbon should be less.

Kate: Should be less.

Beth: Than the speed of your ribbon *[Paul's ribbon]*

Paul: No, it should be more *[spreads his finger out again and shakes them],* because we assumed, we're all assuming 1/40th of a second, so it's the exact same time

Kate: Actually it's wrong, it's the larger the segment the faster it was pulled.

John: The larger the segment *[slowly said],* yeah

Beth: No, cause like think about it. *[grabs this strip]* Say if something is hitting at this constant rate *[gestures a constant tapping on the strip],* if it's pulled this slowly *[while gesturing a slow pull of the strip]*

Kate: It's gonna be closer together

Beth: Yeah. But if it's like this *[gestures a very fast pull]*

John: Yeah,

Kate: Right. So the larger the segment, the faster it was

In this scene, Beth looks up to Paul and she begins to talk about the numerical speeds of their strips compare. Beth's strip is shorter and notepad on her strips reads 40.4 cm/s. Paul's strip is longer and the notepad on his strip reads 84 cm/s.

Beth says, "But wouldn't yours be going slower than mine, because it took more time to make the same." Here Beth seems to be saying that since Paul's strip takes more time, his must be going slower. At the same time she seems to be pointing out that this implication contradicts the numbers they have calculated for the speeds of the strips as indicated by the notepads attached to those strips.

Paul says, "No, cause it's not time" and explains that they are assuming $1/40^{th}$ of a second and that his strip just went farther.

Beth and Kate then say together that John's strip should slower than Paul's strip. (Note: John's strip is the longest, so this is wrong). Paul again says no and explains the speed John's strip has the most speed because they assumed the times were the same.

Kate then speaks up and say, "Actually, it's the larger the segment the faster the speed." John slowly repeats back part of her statement, "the larger the segment, yeah"

Then Beth says, "No, cause like think about it." As she says this she picks up a strip, she begins to say, "Say if something is hitting it at this constant rate." She says this as she taps her finger up and down above the strip that it is in her hand. She then says, "If it's pulled slowly," but she doesn't finish her sentence. She does, however, pull the strips slowly from under her finger that is moving up and down.

Kate finishes Beth's statement saying, "It's gonna be closer together." Beth says, "Yeah," in seemingly genuine way. She then says, "But if it's like this." She then pulls the strip quickly this time, but does not finish her statement. John and agrees and the Kate repeats again, "So the larger the segment, the faster it was."

Scene 3, Part 2: Reorienting to Prior Written Artifacts

After Kate makes this statement, the students each turn back to the first page of the tutorial. Beth directs their attention back to the worksheets.

Beth: So, where was that?

Kate: Number two. And one I think? *[reading answer from her worksheet]* Less time is required to generate shorter segments… That's right though, right?

Beth: Uhh.

Kate: Less time is required to generate *[slowly spoken]* it's actually, it shouldn't be time. It's like slower speed

In this scene, Beth and Kate seem to realize that their thinking implies that their previous answers are wrong. Beth asks, "So where was that?" Kate replies that she thinks that it's question two and one (seemingly referring to the time-ranking and speed-ranking questions)

Kate then reads what she was written for the time-ranking question of the tutorial. She reads, "Less time required for the shorter strips." Kate pauses for a brief moment and then says, "That's still right though, right?"

Beth hesitantly says, "Uhh." Kate then slowly rereads what she has written in her worksheet, "Less time is required to generate…" Kate actually doesn't finish reading what she has written this time. She interrupts her own reading by saying, "It's actually, it shouldn't be time. It's like slower speed."

Analysis of Students' Thinking

Variability in Individual Students' Thinking

In this scene, Beth begins by explaining an inconsistency she has noticed. First she points out that Paul has a larger numerical speed (84 cm/s) than hers (40.4 cm/s). Then she explains why Paul's strip should, in fact, be slower. Beth explains that since his took less time, it should be going slower. The numerical results that Paul's number is 84 cm/s is inconsistent with her thinking that Paul's should be slower.

Beth's thinking here is actually consistent with the group's initial incorrect thinking that the shorter strips take less time and are faster. She says that Paul's shorter strip takes less time, which is consistent with the intuition that *more distance implies more time.* She also explains that his should be slower because it took less time, and this is consistent with intuition that *more speed implies less time*. During this time she is pointing to her strip and to Paul's strip. Beth pointing to strips is also consistent with this pattern of thinking. Beth seems to have entered into the pattern of thinking and attention that they had at beginning of the tutorial, which shares many of the features that have characterized this pattern of thinking across many of the case studies.

Paul seems to try to explain that it's not the time that's different, but just the distance. Paul even points to locations on the strips, and spreads his hands out above the strips as he had done previously (scene 1, part 3).

Kate, however, then suddenly changes her mind. She says, "Actually, it's the longer the segment the faster the speed." Here she seems to be cueing into the intuition that *more speed implies more distance.* Her Kate seems to be cueing into ideas from their change thinking.

Beth disagrees and begins to re-enact the making of the strips at the center of the table. She does this by pulling a strip across the table while tapping on it with her finger. As she re-attends back to the mechanism by which the strips were made, and physically acts it out, she seems to realize that what Kate is saying is right. Beth now seems to have entered back into the pattern of thinking and attention they had minutes earlier when Kate originally discussed how they assumed the strips were made.

Kate and Beth realize that this means their previous answers were wrong. However, when she rereads what she had written, "Less time required to generate shorter segments," she seems to think that this is still correct, even if only briefly. Reading the artifacts of her prior thinking reactivates the intuition that *more distance implies more time*. This conclusion again seems reasonable to her.

In this scene, there seems to be a high degree of variability in Beth and Kate's thinking about the tickertape strips. Beth first expresses ideas that are very consistent with their original incorrect thinking, and engages in many of the same patterns of attention while pointing to the strips. Kate seems to at first agree with Beth, but she then changes her mind. Beth, then, goes to act out how the strips were made. In attending to the mechanism by which the strips were made, she again seems to settle again on ideas consistent with their later thinking that the longer strips represent more speed. Finally, in going back to change their answers, Kate is briefly led into thinking that the shorter strips do, in fact, represent less time.

This scene clearly demonstrates that the change in students' thinking that we observed from scene one to scene two was not globally stable. Both Beth and Kate show evidence of shifting between these two stabilities of thinking in just a brief period of time. For Beth, shifting between these two patterns of thinking also coincided with patterns of attention that had accompanied their earlier thinking.

Dynamics of the Extended Cognitive System

This scene illustrates how structure and dynamics in the extended cognitive system continue to exert influence of the dynamics of individual students' thinking. I point out two stand-out features of this dynamic: how the construction of a new object facilitates Beth's noticing of the inconsistency and how Kate's reorienting to written artifacts leads to regeneration of prior ideas.

When Beth explains the apparent inconsistency she has noticed, she points between her strip and Paul's strip that both have their numerical speeds posted on them. Before putting the notepads on their strips of paper, each of the students had only recorded their numerical answers on their worksheets. At that time, for Beth to notice the apparent inconsistency between her thinking that the shorter strips should be faster and their numerical rankings, she would have had to coordinate information that was located in four different locations. Paul's strip was in one location on the table and his numerical ranking was written on his worksheet in a different location. The same is true for Beth's strip and numerical ranking. Her shorter strip of paper was in one location, and her numerical ranking was located in another. By placing the numerical rankings physically onto the strip of tickertape, new objects were created for which Beth only need to coordinate information across two spatial locations—her own strip and Paul' strip.

Just as the deconstruction of the side-by-side arrangement of the strips was consequential to the dynamics by which students' thinking originally changed, the construction of these new object (strips with numbers on them) is consequential to the dynamics of Beth's noticing the apparent inconsistency. The relocation and rearrangement of these objects provided new opportunity for student attention that was not afforded before.

It is also interesting that, for Beth, resolving this inconsistency required recreating some of the context of their prior thinking about the mechanisms by which the strips were made. Beth stops just pointing to her strip and Paul's strip. She picks up the strips as Kate had done, and re-enacts how the strips were made. In this sense, Beth is now coordinating thinking, behavior, and objects from across three separate moments: the thinking and behavior they initially engaged in when thinking about the shorter strips as less time, the thinking and behavior they later engaged in when thinking how the strips were made, as well as the products of their numerical calculations.

The second way that the extended cognitive system exerts influence on students' thinking is more obvious. Kate turns back to read her written ideas from before, and is briefly convinced that they are still true. In the extended cognitive system, these artifacts from students' prior thinking were structures that were always there. However, students were not orienting to them. They had not noticed there were any inconsistencies. In fact, it is arguable that going backwards in a worksheet is not within the norms of worksheet activity. Beth hesitantly flips back and forth between page one and two of the tutorial several times before addressing the group.

As I suggested before, we might think of students' written artifacts as being cognitive element in the distributed system that were not "activated", just as certain intuitions may not be active at a certain times. When Kate orients to her written work, those structures become "activated" and again play a role in the dynamics of their thinking again. Here, cognitive elements in Kate's own mind *(more distance implies more time)* are seemingly activated when she orients to this external cognitive element.

Summary and Implications of Case Study

Summary of Case Study

This extended case study of student thinking in the Speed Tutorial reflects many of the same aspects of the other case studies previously analyzed in chapter 5. Students shift among multiple ways of thinking about the same physical situation. These different ways of thinking coincide with different patterns of attention.

What makes the analysis of this case study different from the others is that more careful attention is paid to how changes in the context are consequential to changes in student thinking. By analyzing patterns of students' physical behaviors, a more specific understanding of the dynamics within that context was possible.

The dynamics of student behaviors themselves appear rich and complex. There are variety of structures in the world that "pull" on students in different ways, including material objects and other various semiotic fields such as speech, gesture, posture, etc. Couplings among students seem to lead to emergent patterns of collective behavior. The method that was outlined here for analyzing patterns in student behavior was certainly cursory. A more rigorous analytical approach may result in a better of understanding of those dynamics. However, a deeper understanding of those dynamics alone was not the goal of this chapter.

The goal of this chapter has been to use the products of this cursory analysis to better understand how some of the dynamics occurring among external structures in the world interact with the substance of students' thinking. This analysis by no means takes into account how all structures interact with the substance of student thinking (e.g., I have merely alluded to some dynamics concerning the role of norms for worksheet activity). Here I summarize a few of the claims made throughout this case study:

In scene one, I argue that the location and arrangement of tickertape strips at the center of the table were consequential to student thinking by providing a strong pull for student orientation. This attractor for student orientation facilitated students' sustained attention to physical features of the strips and to the patterns of thinking that arose with that attention. It also inhibited them from collectively orienting in other ways. There arranging the strips in order by length was an outcome of their noticing these features, but also served to sustain their attention to those features.

I scene two, I argue that the rearrangement of the strips to isolated locations on the table allowed for new patterns of collective orientation. Students settled into a new pattern of mutual orientation and began a discussion that involved attending to how the strips were made. Students' thinking seemed to change as a result, and I suggested reasons to doubt the global stability of this change.

Scene three served to illustrate more specifically how this change in students' thinking was indeed not particularly stable. Two of the students seemed to settle back into their earlier pattern of thinking about the strips. From the perspective id the extended cognitive system, I argued that Beth's noticing of inconsistencies in her own thinking were facilitated by the construction of new objects, and that Kate's brief encounter with her prior written artifacts were consequential to re-cueing of intuitions from prior thinking.

Overall, the case study highlights what situated and distributed perspective have continually advocated—that settings for human activity are interwoven in the dynamic of seemingly private mental affairs. Along the way, I have tried to draw connections between the cognitive elements attributed to dynamic of individual students' thinking (such as various fine-grained intuitions) and the cognitive elements attributed to the dynamic of the extended in cognitive system (such as material artifacts).

Implications for Student Learning and Instruction

In this case study, Paul, Beth, John and Kate initially came to some incorrect conclusions that were common to many tutorial groups. They did so while closely attending to the static physical features of the tickertape strips and while moving through the tutorial one question at a time. I argued that part of the local stability of their initial thinking resulted from how the location and arrangement of material objects influenced patterns of collective behavior. These patterns of behavior, in turn, both reflected and supported the group's sustained attention to static physical features of the strips (which precluded a discussion a physical mechanism) and their rather narrow focus on what answers they are supposed write for particular worksheets questions (which largely precluded attention to the substance of ideas).

There is certainly nothing wrong with students coming to incorrect conclusions as part of learning science. In fact, it is likely to be necessary. Physics education research has highlighted how learning physics involves deliberate reflection upon one's ideas. Through this reflection, one hopes, not to eradicate incorrect ways of thinking, but for students to become increasingly mindful and increasingly in control of their own thinking. While students' incorrect thinking about the tickertape strips certainly involved aspects of their own physical intuition (e.g., *more distance implies more time*), there is little evidence to support a claim that students ever became aware of (and reconciled) the ideas they were originally using to answer questions and make sense of the situation. This lack of awareness and control over their own ideas (as individuals and as a group) is arguably part of the dynamic by which their newly constructed correct reasoning was fleeting and unstable. Beth and Kate, despite showing evidence for engaging in correct thinking while attending to physical mechanism (and evidence for being excited about this new thinking), later cue back into their previous intuitions (such as *more speed implies less time*, and less *distance implies less time*) while focusing back on static features of the strips.

At times, the group did seem to orient to certain forms of coherence in their attention and thinking. Kate wants to make sure they are all attending to same aspects of the strips (and attending to the right aspects). Beth later notices an inconsistency among her own intuitive ideas about the speed ranking and the quantitative measures they calculated. The group, however, never orients toward a more global consistency in their account. Kate and the others never try to make sense of the fact that they had earlier thought that the shorter strips take less time (and why it seemed to make sense to Kate later while reading the answer again). Instead, they merely turn back to change their answers from incorrect to correct. If we want our students to mindful about the substance of their own thinking (and not just it's correctness), students may need additional instructional support that was not provided by the worksheet or by the TAs.

It is certainly arguable that this group (and every other group) possessed the conceptual tools necessary for making sense of the tickertape strips and the motion of objects. The students, on their own, were able to construct correct understandings from their own personal intuitions. What the students were not able to do on their own, or least do well and consistently, was to be mindful of their own thinking and to seek out ambiguities and inconsistencies in their ideas. Instruction that is designed to be more attentive to this aspect of student learning may be important for helping students to develop habits of mind that are productive for learning. In this way, students' own attitudes about what they are supposed to be doing may drive coherence in their own learning in physics, rather than the local stabilities of their thinking being so strongly influenced by the particulars of setting in which they find and construct for themselves. The development of such instruction is, of course, a matter of research itself. Finding the most productive ways to harness students' fleeting moments of coherence-seeking behavior, for the development of more stable habits towards coherence-seeking, is an issue that is not addressed in this research.

Chapter Summary

This chapter initially began with a discussion of a method for analyzing patterns of student orientation from video data in the Speed Tutorial by using gaze, posture, and hand positioning as indictors for change. From this analysis, particular patterns of student orientation were categorized and used as the basis for speculating about sources of change to students' orientation. Student orientation became the basis for defining the relevant features of context in the setting for students' thinking—such as the location and arrangement of tickertape strips, various collective patterns of student orientation, and written artifacts such as worksheets. These structures were construed as being elements in an extended cognitive system, much in the way that Hutchins described structures like tables and speed gauges as cognitive elements in the cockpit. These structures play a role in the dynamics of the extended cognitive system in the MOS tutorial by influencing the behavior of other structures (such as the orientation of students) and by undergoing changes themselves (e.g.. through student manipulation of those structures).

The rest of this chapter concerned the analysis of a single group of students in how their thinking and behavior changes throughout the first page of the Speed Tutorial. This group, like many other groups, exhibited thinking in two substantively different ways about the tickertape strips. Initially, the group discussed how the shorter strips of paper take less time and are faster. Later, the group discussed the mechanisms by which the strips were made. I described how these aspects of students' thinking in this case study were similar to the other cases of student thinking and consistent with the accounts developed in the previous chapter.

The analysis of this case study differed from the analysis in the previous chapter through an examination of how changes in the structure and dynamics of the extended cognitive system were consequential to the substance of students' thinking. For example, I argued that the particular location and arrangement of the tickertape strips, initially constructed by the students at the center of the table, played a role in regulating student orientation. The emergent stabilities in students' collective orientation were consequential to students' attention remaining fixed only to the physical features of the strips and to singular questions from the worksheet.

In describing students' new thinking, I argued that the relocation and rearrangement of the tickertape strips provided possibilities for new collective patterns of orientation. Students thinking about how the strips were made coincided with the students being mutually oriented to each other for the first time. This new pattern of orientation arose as Kate repurposed the use of the strips as props in a re-enactment.

In analyzing the last few scenes of this case study, I argued that certain instabilities in students' thinking about the tickertape strips were represented in the extended cognitive system. I describe how Beth comes to notice a particular inconsistency in her own thinking through the construction of a new object, and how Kate's interaction with her own written artifacts from the past influence her current thinking.

Chapter 7: Dissertation Summary and Future Directions

In chapter one, I began by describing how the physics education research community has oriented much of its basic research and instructional reform toward the substance of student thinking. The research presented in this document contributes to the ongoing inquiry into the nature of students' thinking in physics through an analysis of student thinking about motion as real-time activity. In this final chapter, I discuss specific contributions of this research and reflect upon future directions for inquiry.

Dissertation Summary

In summarizing the work here I find it prudent to be tentative in making of any general claims. Lemke (2001) warns that we should be wary of broad generalizations concerning phenomena of human behavior. He suggests that we only know (and can perhaps only ever know) a few local reasons for why people might do the things they do and only in a very limited number of circumstances. Levrini and diSessa (2008) have similarly stated that we as a community should aim to develop *humble theories* for describing conceptual change.

In many respects this dissertation represents an attempt to start from a very simple description of students' intuitive thinking about motion and to then see where that description leads in exploring a limited set of dynamics in students' thinking.

Specifically, much of the research presented here focused on how context influences students' thinking in ways that introduce moments of variability and stability.

In chapters 3, a toy cognitive model involving just handful of intuitions was originally used as the basis for motivating an experiment to bias students toward and away from different ways of thinking about physics problems involving motion. This experiment demonstrated particular kinds of variability and contextual-dependencies in students' thinking that were consistent with the model (and others that were not consistent with the model).

Complex knowledge systems approaches in science education certainly anticipate being able to find such sensitivities, and research that can carefully document terrains of stability and variability in students' thinking are important toward the continued development of these frameworks. It is moderately compelling that an even an over-simplified model such as the one described in the beginning of this document could make predictions about some sensitivities in students' thinking.

As it turns out, that same toy cognitive model became a generative starting place for describing many of the same patterns of students' thinking about tickertape representations of motion in the classroom. In Chapter 5, case studies were developed demonstrating how students' thinking during one particular segment of a curriculum largely settled into two distinct patterns of reasoning. Similar toy cognitive models were used once again as the basis for speculating about mechanisms that contribute to the local stabilities of these reasoning patterns. These mechanisms were largely individually-focused explanations for the stability of students' thinking—the stability described in terms of relations among knowledge that students have and in terms of patterns of attention that students sustain.

In the final chapter of this dissertation, I described how student behavior influences (and is influenced by) the setting for students' own activity, beginning with a method for analyzing patterns in student orientation. Focusing on dynamic changes taking place in the context of students' thinking provided insight student thinking at a different scale of analysis. I have tried in a very limited way to incorporate aspects from both individually focused and more distributed focused accounts of cognitive behavior. This analysis, however, was only conducted in the context a *single group* of students working in a *particular setting* over a modest expanse of time and space.

Complex knowledge systems analyses and distributed cognition analyses certainly offer accounts of human behavior at different scales of spatial extent, but that certainly does not mean they are incommensurable. It is arguable that accounts of student thinking that attempt to explain stabilities as arising from mechanisms occurring at different scales of space are likely to be generative, if not also difficult. No one expects to be able to explain the evolution of species by focusing on mechanisms occurring at a single grain-size. Much progress has been made by attempting to move across scales of space and time- to explain how mechanisms occurring at different levels act in concert. Thus, I don't find it terribly problematic that this research has concerned a very narrow range of space and time, nor a very specific example. In the history of science, progress has been made through the careful examination of small worlds seemingly unrelated to anything else (e.g., the erratic motion of pollen grains in water).

Future Directions

Conducting research in any field almost always involves narrowing the scope of an investigation so that progress can be made in a limited away along one or several dimensions. Doing so enables researchers to focus in on certain aspects of phenomena under investigation while ignoring others. Here, I offer some brief speculation about future directions for research that are motivated from the limited progress made here.

First, from a complex knowledge systems perspective, there is a need to continue to identify boundaries of stability and instability in students' thinking. This should be done using a variety of different methods, in a variety of different contexts, for a variety of populations, for a variety of topics, and for different kinds of instabilities. I suggested a limited set of fine-grained sensitivities in students' thinking about motion that could be explored as a direct result of the experiment described in Chapter 4. Identifying phenomena that illustrate such boundaries, however, is only a starting place. The development and refinement of models that attempt to explain such phenomena are needed as well.

The question of how we are best to explain phenomena of human behavior has been a major concern since the early beginnings of history and philosophy. In Chapter 2, I discussed just a few of these perspectives in how they attempt to explain phenomena at different scales of space and time. I think looking forward, it will be generative for our field to pursue more investigations of student thinking that connect accounts of human behavior at varying scales of space and time. This kind of research that spans large expanses of space and time, however, is difficult for individual researchers to pursue on their own; and often require the coordination among many researchers.

I think one major impediment to this kind progress in our field is the awkwardly disjointed nature of our community's collective endeavor. There are so many different avenues to explore regarding phenomena of student thinking and learning in physics that is easy for researchers to pursue agendas that do not connect to each other in meaningful ways. Researchers not only approach phenomena from different methodologies and different theoretical perspectives, we are often not even examining much of the same phenomena. I think there is value generated when a community orients to a small number of well-established phenomena. Consider the A-not-B error as a case in point. The A-not-B error in is a fairly robust phenomenon to which researchers in cognitive development and other fields (also using different methodologies and theoretical frameworks) have investigated over decades. During this time, significant theoretical and methodological gains have been made that impact broader communities.

It's not clear at all how the physics education research community should carve out the space of similarity and difference in the phenomena we explore. I am suggesting, however, that careful thought should go into how we as a community collectively orient our research to such phenomena. In particular, I see the convergence of different research upon a common and narrow set of phenomena as one productive route for the community as a whole.

Appendix:

Meaning of Speed Tutorial

I. Recording motion with a tapper

Physics is, to a large extent, the study of the motion of objects. There are any number of ways to record the motion of a cart. One simple method is to attach a ribbon of paper so that it drags behind the cart and through a tapping device; the tapper leaves dots as it taps the ribbon at a constant rate. Your TA has one of these devices for you to examine.
To begin this tutorial, each person in your group will need a ruler and at least one segment of paper ribbon from the staff. All the paper segments were generated using the same tapping device. Please don't write on or fold the paper segments – other classes need to use the same ones.

A. Compare your paper segment with those of your partners. What kind of motion does each represent?

1. How does the time taken to generate one of the short segments compare to the time to generate one of the long ones? How can you tell?

2. Arrange the paper segments in order by speed. How do you know how to arrange them?

B. Suppose the tapper that made the dots strikes the ribbon every 1/40th of a second.

1. How far did the object that generated your paper segment move in: 1/40th of a second?
2/40th of a second? 3/40th of a second?

2. Predict how far the object would move in:

i. 1 second

ii. 1/80th of a second

Why are these *predictions,* rather than just calculations? That is, what assumption(s) do you use to make them?

3. Determine the speed of the object that generated each of your paper segments (in cm/s). Write the speed on a small sticky note and attach it to the paper segment.

II. Interpreting ratios

Physics, admittedly, often involves a lot of calculations. But calculations are no use unless we know what the result of the calculation means. An *interpretation* of a calculated number is a statement that tells you what the number means physically. For example, if 500 g of sand were spread evenly over a 10 cm2 area, and interpretation of the number 50 would be "the number of grams of sand on each square centimeter."

A. Give an interpretation of the speed of the object that generated your paper segment. (If you have trouble, you might try starting your interpretation with "The speed is the number of …".)

☐ *Check your interpretation with an instructor before you proceed.*

B. A model train moving with constant speed travels 60 cm for every 1.5 s that elapses.

1. Is there a name that is commonly given to the quantity represented by the number 40? (40 = 60/1.5) If so, what is the name?

2. Interpret the number 40 for this situation.

3. Interpretations can be useful for calculating, as well as for understanding. Use your interpretation (instead of setting up and solving an equation) to figure out the distance the train moves in 2.5 s.

C. A model train moving with constant speed travels 60 cm for every 1.5 s that elapses.

1. Is there a name that is commonly given to the quantity represented by the number 0.025? (0.025= 1.5/60) If so, what is the name?

2. Interpret the number 0.025 for this situation. (You should be able to do this whether or not you identified a name for this number.)

3. Use your interpretation (not an equation) to figure out the time it takes the train to move 90 cm.

III. Longer ribbons

A. In the space below, sketch a possible paper segment resulting from motion with varying speed. What motion is taking place for the segment that you sketch?

B. Your TA has a long paper ribbon that was generated by the same tapper that generated your paper segment. Bring your paper segment to the TA and follow the instructions for this activity.

C. Suppose your paper segment was in fact part of a longer ribbon that represented varying speed. Look back at your *interpretation* of the speed for your paper segment (part II.A). Would that interpretation be valid for the entire motion that generated your segment? Why or why not?

1. If necessary, propose a new interpretation of the speed represented on your segment so that it applies even when the overall motion varies.

2. What name is given to a speed that is interpreted in this way? (The interpretation is more important than the name, so if you don't happen to know this particular vocabulary word, you can just ask and your TA will tell it to you.)

D. Suppose you selected two widely separated dots on the TA's long ribbon. Imagine measuring the distance between those two dots and dividing that by the time it took the object to move between the dots.

1. What would be an interpretation of that number?

2. What would be the name of that number? (Again, if you don't know, just ask.)

□ *Consult an instructor before you proceed.*

IV. What to take away from this tutorial (at any speed)

In each tutorial, we will alert you to what's important for you to take away from it. We will try to particularly emphasize things that might help in any of your science courses (not only physics). In this tutorial, the two most important "exportable" ideas are *interpretations* and *awareness of assumptions.*

A. Interpretations, as you know by now, are statements that tell you what a calculated number means. Which of the following best represents your attitude about interpretations of calculated numbers? (There's no right or wrong answer to this question; please express your honest opinion and discuss it with your partners. The point is to identify the role of interpretations *for you.*)

i. Most calculated numbers won't have any interpretation. There are just a few, like speed, that have a meaning worth paying attention to.

ii. If a number is worth calculating it's probably because it tells you something about the physical situation, in which you should interpret it to check that it makes sense.

iii. Lots of physics numbers probably have interpretations, but you shouldn't spend your time trying to figure them out if that's not part of the assignment.

B. During this tutorial, you made some assumptions. For example, you probably took it for granted (at least to begin with) that the motion represented on your paper segment represented the overall motion of the object. Which of the following best represents your attitude about assumptions? Again, there's no right or wrong answer to this question; please discuss your opinion frankly with your partners.

i. Ideally, you don't assume anything. Everything should be completely spelled out.

ii. You are always assuming things, often without even realizing it. You can't eliminate assumptions, and it's pointless to try.

iii. You can't eliminate assumptions, but you should try to be aware of what ones you're making, so that you can check later if they're really valid.

iv. You can't eliminate assumptions, and you should try to be aware of what ones you're making because they might help you account for counterintuitive results.

□ *Share your group's opinions with a TA before you go.*

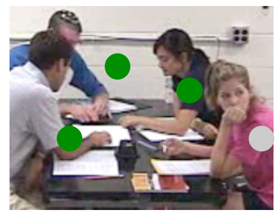
Extended Example #1 of Coding for Student Orientation

[The TAs are finishing up with their introduction. John and Paul are looking at their worksheets. Beth is looking at one of the strips near the center of the table. Kate is looking up.]

Beth *[flipping open her worksheet]:* Alright

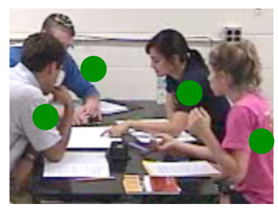
John *[looking to the center and then down to his worksheet]:* Compare.

Beth *[reading]:* Compare you paper segments with those of your partners… So everybody got one…



[Paul, John, and Beth lean toward the center and each move their strips to the center of the table]

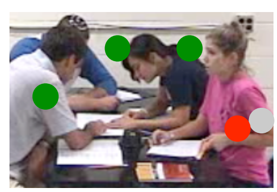
[Kate is looking up listening to one of the TAs who is still introducing himself]



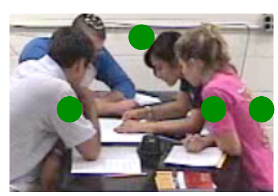
Beth: Did anybody get the same rate? This one has like similar *[pointing to two strips]*.

Kate: What, what are their names? *[Kate turns toward the group, looking toward the center]*

Paul: Those are kind of close *[pointing to two of the strips]*

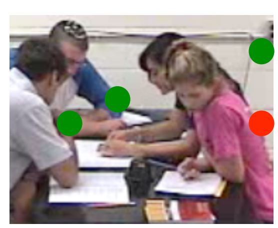


[*Beth leans in farther using a ruler to measure. Paul and John look lean to the center looking closely at what Beth is doing. Beth is shifting the strips around. Kate looks to her worksheet and then off into the distance again*]



Beth: These two might be the same.

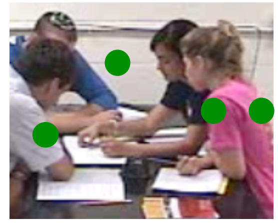
[Kate leans toward the center]



Kate *[turning to her worksheet]:* OK. Wait.

Beth: No. Nobody has the exact same rate

John [*pointing to the shortest strip*]: So I guess that's the fastest.



[Kate quickly turns toward the center. Beth taps her finger over one of the strips several times]

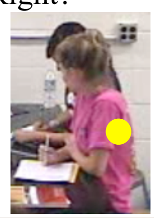
John: The one right there. Yeah

Beth *[turning to her worksheets]:* OK. So what do we write about this?

Kate: I don't…

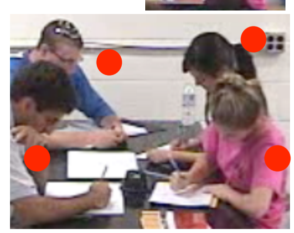
[The student all orient toward their worksheets]

Beth *[reading from the worksheet]*: What kind of motion does each represent? *[then looking up slightly]* Constant? Uniform? Right?

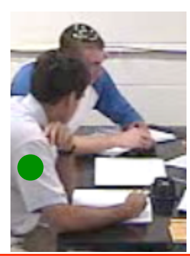


Kate *[looking to Beth]:* Yeah, that's good.

Beth: Hey! Hey!

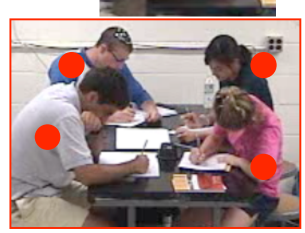


[Everyone looking back down again, writing in their worksheets]



John *[looking toward the center]:* And I guess it's because they are all likely ee-qually spaced out or whatever.

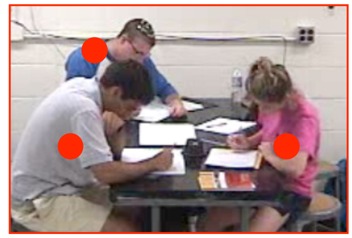
Beth *[nodding]:* Mm-hmm.



[The students are all oriented with their gaze and posture toward their own personal space, where their worksheets are located. They all have pencils in their hands and are writing in their worksheets]

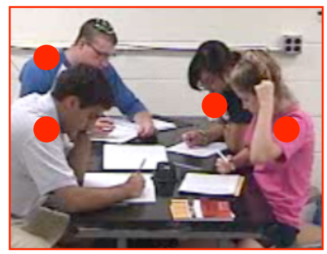
Beth: *[reading from her worksheet]* How does the time taken to generate one of …*[mumbled reading of the question that trails off]*. Obviously it takes less time to generate the more closely spaced dots. *[holds two fingers up separated by a small gap]*

[Beth's phone rings and she goes to turn off her phone, turning away from the group. The three other students continue to sit quietly oriented toward their worksheets for about 10 seconds]

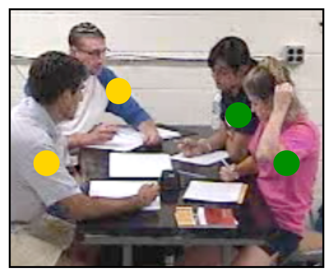


[John quickly gazes toward the center of the table (where the strips are located. Kate gazes toward the center of the table, but John looks away back toward his worksheet. Kate glances over at John's worksheet.]

Beth: *[returning to the table, looking at her worksheet]* OK. Sorry. So.



[Paul and John ***glance up*** *toward Beth. Beth orients slightly toward the center]*

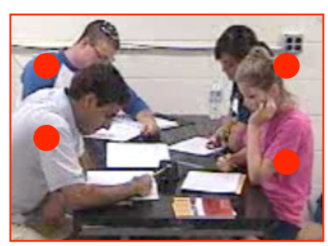


John: *[looking toward the center, as Paul look ups to John and Kate looks down]* So you are saying it takes less time to make the shorter segments? *[quickly looking back toward his own worksheets]*

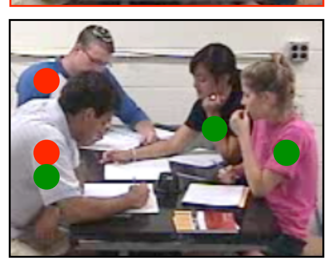
Beth: Right.

John: Alright.

[The students all self-orient toward their worksheets. Beth picks up her pencil.]



Kate *[reading from the worksheet]:* How can you tell?



[Beth leans in toward the center, placing her hand over the strips, and taps her pencil on one of them. Kate and John both look toward the center. John quickly looks back at his worksheet and then back at the center]

Beth: You can tell because…

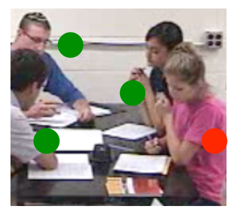
Paul *[glancing toward the center]*: It's a shorter segment

Beth: It's a shorter distance.

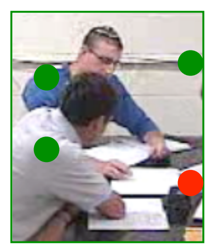
John: You've made. You've made more segments in like the same amount of space *[reaches in and indicates a length on one of the strips]*.

You've made like more little *[spreads his fingers out indicating dots and then waves his hand back and forth over the strips]* things

Beth: Yeah.

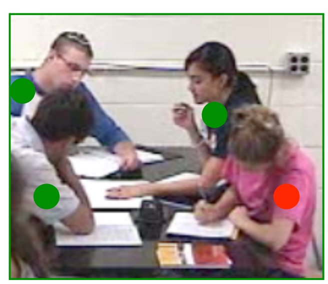


[Kate orients back toward her worksheets]



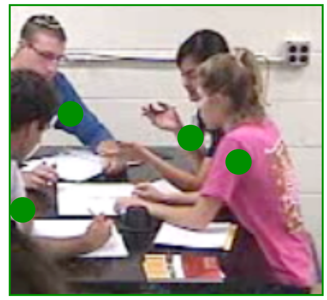
[Paul reaches toward the center]

Paul: It's like the same amount of dots *[pointing to a strip]* in a shorter piece *[indicating a length on one of the strip by separating two fingers]*



Beth: Yeah. When given the same length of… *[pauses while holding an open palm face down near one of the strips, indicating a length between her thumb and index finger]*

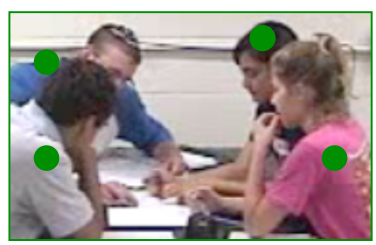
Kate *[popping up from her worksheet leaning in toward the center]:* When we are talking about segments *[pointing to strips with her pencil],* are we like not thinking about how long the total paper is *[bringing her other hand over to the center over the strips]*? Are we just looking at the marks *[pointing to successive marks with her pencil]?* Are we supposed to be considering – *[pulls her left hand away, keeping her hand with pencil at the center of the table]*



John *[leaning further toward the center]:* — I'm guessing they like mean from here *[pointing with pencil]* to here *[pointing with pencil]. [points with pencil to the two marks again in succession]*

Kate: Like I wonder why like the papers are all different lengths *[moving her pencil from one strip to another]*

Beth *[bringing her hand closer to the center]:* Cause none of these papers are the exact same si-ize *[speaking quietly]* Except for these two *[pointing to two strips]*



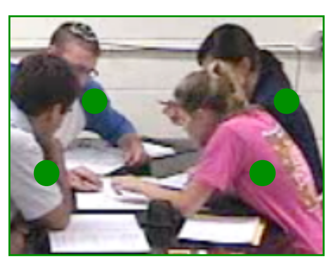
Paul: Right because I think *[moving hand toward the center]* they all have the same amount of dots *[pointing several locations on one of the strips].*

Beth: Oh-oh

Paul: I think they all have six dots.

Beth: Oh do, they?

Kate *[leaning farther in]:* Is that true? 1, 2, 3, 4, 5, 6 *[pointing to successive dots on a strip]*



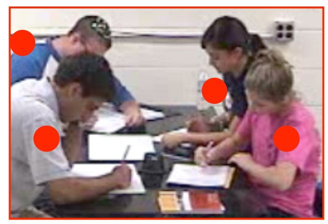
John: Yep

Paul: So, it's a shorter amount of time for a shorter piece of paper

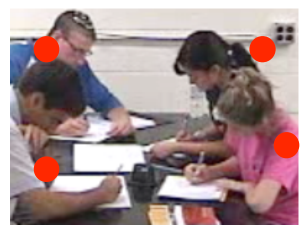
[Kate leans away and looks down]

Beth *[leaning in with her hands]:* So the time it takes to generate six dots *[moving her hand up and down, beating with six dots]* varies for each of them.

[Students all self-orient themselves to their worksheets]



[Students are self-oriented, writing in their worksheets]



Beth *[reading what she is writing]: Each strip is a different length*

Extended Example #2 of Coding for Student Orientation

[The students are all oriented toward their worksheets]

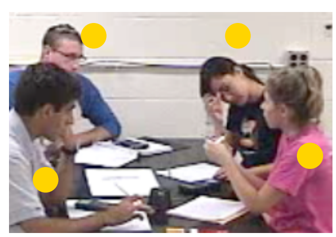
Beth *[reading from her worksheet]:* Why are these just predictions rather than just calculations? What assumptions did you use to make these observations?

Paul *[Kate looking over to Paul]:* We assumed that $1/40^{th}$ of a second. *[Paul looking to Beth, and vice versa]*

[Beth looks down to her worksheet]

Kate: I think we also assumed in the *[Paul looking over to Kate]*

Beth: That's true. *[Glancing up and pointing to Paul]*



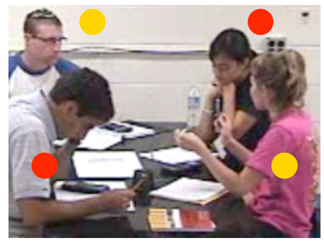
Kate *[continuing]:* that these *[picks up a strip of paper with her left hand and veryone immediately looks up and over to her]* were made *[holds the strip horizontally in front of her]*

by the speed at which the paper traveled through the tapper *[pulling the strips horizontally in front of her body]* which was different for each paper.

Cause like the marks *[pointing to Beth's worksheet with her pencil, where the others also look],* they are telling you the marks were made every 1/40th of a second.

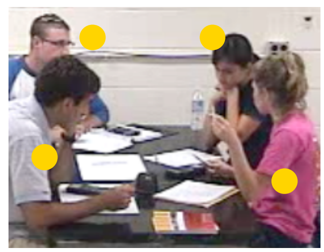
[Beth and Paul both look down]

So like, it's not the marking *[tapping up and down with her pencil in the air]* that's different.



[Paul looks up again and Beth glances up]

We're assuming the rate it was going through *[pulling the strip of paper horizontally again]* is different.



John: Yeah *[pointing to Kate]*

Kate: You know what I mean?

Beth: Yeah

John: *[looking to Kate who looks back]*: Right *[Beth looking over to Paul],* cause if you move it really fast then *[pulling his hand with pencil in it quickly in front of his body]*,

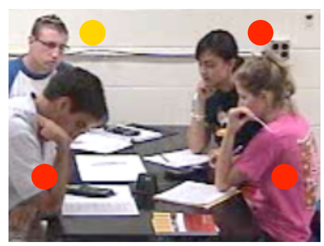
Kate *[who is looking to Paul]* **:** It's like if each mark is being made a 1/40th of a second *[pointing back to Beth's worksheet or right hand while holding the strip up with her left],*

Beth *[talking over Kate and looking to John]:* That's true! It could depend on how fast the ribbon was pulled *[pulls her hand quickly across her body]*

Kate *[continuing]:* then they are all going to have the same number of marks *[bringing the strip to the center of the table, holding it vertically]*

[Kate withdraws her strips from the center and behind her shoulder and looks down]

John *[nodding while looking to Beth]:* Yeah *[looking down to his worksheet]*. So we're yeah



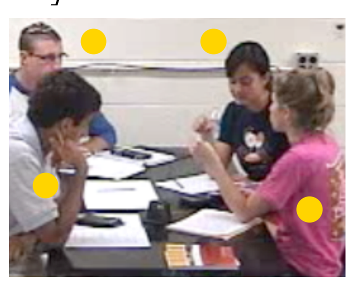
Kate: We're assuming that umm *[holding the strips out in front of her again]*

Beth *[looking up and pointing to the strip Beth is holding]:* That the length is proportional to

[Paul looks up to the strip at the center of the table]

Kate: The speed at which the ribbon was pulled through

Beth: Yeah



[All the students reorient back to their worksheets writing quietly for about 20 seconds]

Beth *[looking up to Paul]:* The speed at which the ribbon was pulled? *[pulling her hand toward her body twice in succession]*

Kate*: Yeah.*

Paul *[looking up to Beth]*: Yeah

[Students all orient toward their worksheets]

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