SRC TR 89-19

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† The work of this author was supported in part by the National Science Foundation grants MIP-84-51510 and CDR-88-03012 (Engineering Research Centers Program), and in part by a grant from AT&T.

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Abstract

We show that the adaptive diagnosis approach can be applied to probabilistically diagnosable systems. Three adaptive diagnosis algorithms are developed under both the symmetric and asymmetric test invalidation assumptions. A test selection strategy based on a probabilistic measure of test results is derived and used in each algorithm. We show that the adaptive algorithms are efficient in identifying all faulty units in a system.

I. Introduction

Consider a system denoted by $S(U,T)$ or simply S, where $U = \{u_1, u_2, \ldots, u_n\}$ is a set of n units and T is a connection assignment of test links which assigns each unit to test a subset of the other units. Thus the system S can be represented by a digraph $G = (U,T)$, where U is the set of vertices of G and $T = \{(u_i, u_j)|u_i, u_j \in U$, and u_i is assigned to test $u_j\}$ is the set of directed edges of G. The result of each test is either "pass" (fault-free) or "fail" (faulty) and can be expressed as a binary weight $a(u_i, u_j)$ of edge (u_i, u_j) in T. In particular, $a(u_i, u_j) = 0$ if u_i judges u_j to be fault-free, and $a(u_i, u_j) = 1$ otherwise. The set of all test results of S is called the *syndrome* of S.

Preparata, Metze, and Chien [16] introduced the above graph theoretical model, which is now known as the *PMC model*, to deal with system-level faults in a multiprocessor system. In their model, it is assumed that the test result is reliable only if the testing unit is fault-free; otherwise it is unreliable regardless of the condition of the other unit involved in the test. This is called the *symmetric test invalidation* (abbreviated as STI) assumption. They defined a system S to be *t-diagnosable* if given a syndrome of S, all faulty units in S can correctly be identified, provided the number of faulty units does not exceed the given bound, t. A considerable amount of research has been done on the characterization, diagnosis and diagnosability of t-diagnosable systems under the PMC model [1,5,8,11,18].

A variation of the PMC model, called the *BGM model*, was later introduced by Barsi, Grandoni, and Maestrini [2]. In this model, besides the test invalidation assumption of the PMC model, it is further stipulated that a faulty unit is never evaluated to be fault-free. This is called the *asymmetric test invalidation* (abbreviated as ATI) assumption. A characterization theorem, diagnosis and diagnosability algorithms for t-diagnosable systems under the BGM model have been developed [2,13,15].

In all of the work mentioned above, it is implicitly assumed that all system units have the same probability of failure. In other words, all faults of the system are treated equally in the diagnosis process. On the other hand, Maheshwari and Hakimi [12] studied a probabilistic nature of the processor fault and proposed a probabilistic weighted model. In particular, they considered the *a priori* probability of failure associated with each unit in the system. They defined a system S to be ***probabilistically*** τ***-diagnosable***, or p-τ-*diagnosable* in short, if given a syndrome of S, the set of faulty units whose total *a priori* probability of failure is at least τ, can correctly be identified. Characterization theorems for this model under both the STI and ATI assumptions have been obtained [6,7,12]. An $O(n^3)$ diagnosis algorithm using network flow and vertex-cover set techniques has been developed [4]. Furthermore, the diagnosability problem for p-τ-diagnosable systems has been shown to be NP-complete [19].

In the conventional diagnosis approach discussed so far, one first chooses a set of diagnostic tests, then seeks the results of these tests, and finally proceeds to use the test results to identify the faulty units. Due to the high complexity of the diagnosis algorithms and the requirement for a complete syndrome, the on-line implementation of the diagnosis process becomes very difficult. To overcome this problem, Nakajima [14] suggested a so-called *adaptive diagnosis* approach. In this approach, tests are chosen and performed based on the results of the previous tests until one can identify a fault-free unit. This fault-free unit may then be used as a tester to identify all faulty units. Efficient diagnosis algorithms using this adaptive approach were later developed under both the PMC and BGM models [9]. Additional research work aiming at the reduction of the numbers of tests and testing rounds required for the diagnosis of a system in the PMC model has also been reported [3,10,17]. However, the application of the adaptive diagnosis approach to the more general p-τ-diagnosable systems has yet to be seen.

In this paper, we show for the first time that the adaptive diagnosis approach can be applied to probabilistically diagnosable systems. We develop three adaptive diagnosis algorithms for identifying a fault-free unit under both the STI and ATI assumptions. In the next section, we review the p-τ-diagnosable system model and obtain characterization theorems from previously known such theorems. We derive a test selection strategy based on a probabilistic measure of test results in Section III. In Section IV, we present the adaptive diagnosis algorithms based on the theorems given in Section II. The complexity analysis of each algorithm is also provided. Section V concludes the paper.

II. Preliminaries

Let $S(U,T)$ be a system, where $U = \{u_1, u_2, \ldots, u_n\}$ and $T = \{(u_i, u_j) | u_i, u_j \in U$, and u_i is assigned to test $u_j\}$.

Definition 1: A subset $F \subset U$ is called an *allowable fault set* of S if given a syndrome of S, (1) $u_i \in U - F$ and $a(u_i, u_j) = 0$ imply $u_j \in U - F$ and (2) $u_i \in U - F$ and $a(u_i, u_j) = 1$ imply $u_j \in F$. □

Thus, F is an allowable fault set if the assumptions that the units contained in F are faulty and that the units contained in $U - F$ are fault-free are consistent with the given syndrome.

Let $p(u_i)$ be the *a priori* probability of failure of unit u_i. Assuming that all $p(u_i)$'s are independent, the *a priori* probability of the set of faulty units $F \subseteq U$ in S is given as

$$P(F) = \prod_{u_i \in F} p(u_i) \prod_{u_i \in U-F} (1 - p(u_i)) \ .$$

Definition 2: A system S is called *probabilistically τ-diagnosable* or simply *p-τ-diagnosable* if for any syndrome of S, there exists a unique allowable fault set $F \subseteq U$ such that $P(F) \geq \tau$, assuming that the *a priori* probability of occurrence of the set of faulty units in S is at least τ. □

Let $F \subseteq U$ be a set of faulty units such that

$$P(F) = \prod_{u_i \in F} p(u_i) \prod_{u_i \in U-F} (1 - p(u_i)) \geq \tau.$$

Then

$$\log\,(P(F)) = \sum_{u_i \in F} \log\,(p(u_i)) + \sum_{u_i \in U-F} \log\,(1 - p(u_i)) \geq \log\,(\tau)\ .$$

It follows that

$$\sum_{u_i \in F} \log\,\left(\frac{1 - p(u_i)}{p(u_i)}\right) \leq -\log\,(\tau) + \sum_{u_i \in U} \log\,(1 - p(u_i)) \triangleq K(\tau)\ .$$

If with each unit $u_i \in U$ a weight $w(u_i) = \log\,[(1 - p(u_i))/p(u_i)]$ is associated, then the problem of identifying the set of faulty units in a p-τ-diagnosable system S is equivalent to that of determining the allowable fault set F for which

$$W(F) = \sum_{u_i \in F} w(u_i) \leq K(\tau)\ .$$

The class of conventional t-diagnosable systems then becomes a special case of p-τ-diagnosable systems in which the weight of each unit is identical. Note that in the above model, it is assumed that $p(u_i) < 1/2$ and hence $w(u_i) > 0$ for every $u_i \in U$.

Under the STI assumption, the test result $a(u_i, u_j)$ is reliable only if u_i is a fault-free unit. On the other hand, under the ATI assumption, unit u_j is fault-free if $a(u_i, u_j) = 0$ regardless of the condition of unit u_i. The following theorems give necessary and sufficient conditions for a system to be p-τ-diagnosable under both assumptions. They are derived directly from those given by Fujiwara and Kinoshita [6,7]. Let $S(U, T)$ be a system with each unit $u_i \in U$ having weight $w(u_i)$, where its corresponding digraph $G = (U, T)$ is assumed to be fully connected.

Theorem 1: S is p-τ-diagnosable under the STI assumption if and only if there exists no 2-partition (V_1, V_2) of U such that $W(V_1) \leq K(\tau)$ and $W(V_2) \leq K(\tau)$. □

Theorem 2: S is p-τ-diagnosable under the ATI assumption if and only if there exists no 3-partition $(V, \{u\}, \{v\})$ of U such that $W(V)+w(u) \leq K(\tau)$ and $W(V)+w(v) \leq K(\tau)$. □

III. Test Selection Strategy

In order to achieve efficient diagnosis using the adaptive approach, the selection of tests among various units in a system is very important in all diagnosis algorithms [3,9,10]. For t-diagnosable systems, the performance of such an algorithm is simply judged by the number of tests used in the diagnosis, since the probability of failure is assumed to be equal for each unit in the system. In this case, when selecting a new unit in the adaptive diagnosis process, only the connection assignment of that unit has to be considered. However, for p-τ-diagnosable systems, the performance of an algorithm should also be judged by the probability of the occurrence of an adaptive test sequence. This in turn requires the consideration of the probability of failure for each unit in the test selection strategy.

Let $S(U, T)$ be a system with each unit $u_i \in U$ having *a priori* probability of failure $p(u_i)$. Let $P_0(u_i, u_j)$ denotes the probability that the test result $a(u_i, u_j) = 0$.

Under the STI assumption, it is assumed that (1) $a(u_i, u_j) = 0$ if both u_i and u_j are fault-free, and (2) $a(u_i, u_j) = 0$ or 1 if u_i is faulty. A natural interpretation of the second case is that $P_0(u_i, u_j) = 1/2$ if u_i is faulty. In this paper, however, we treat this case in a more general setting, by introducing two probabilistic measures. Let α (resp., β) be the probability of the occurrence of $a(u_i, u_j) = 0$ when u_i is faulty and u_j is fault-free (resp., faulty). Note that $0 \leq \alpha,\ \beta \leq 1$. Furthermore, it is reasonable to stipulate that $\alpha + \beta \leq 1$ since if the testing unit u_i is faulty, it is more likely to have $a(u_i, u_j) = 1$ than

$a(u_i, u_j) = 0$. Based on these assumptions, $P_0(u_i, u_j)$ can be expressed as

$$P_0(u_i, u_j) = (1 - p(u_i))(1 - p(u_j)) + \alpha p(u_i)(1 - p(u_j)) + \beta p(u_i)p(u_j) .$$

Under the ATI assumption, (1) $a(u_i, u_j) = 0$ if both u_i and u_j are fault-free, (2) $a(u_i, u_j) = 1$ if both u_i and u_j are faulty, and (3) $a(u_i, u_j) = 0$ or 1 if u_i is faulty and u_j is fault-free. Using the same probabilistic measures α and β introduced above, the second case implies that $\beta = 0$ while the third case can be treated with α in the same way as under the STI assumption. Thus, we have

$$P_0(u_i, u_j) = (1 - p(u_i))(1 - p(u_j)) + \alpha p(u_i)(1 - p(u_j)) .$$

Theorem 3: For any two units $u_i, u_j \in U$, if $p(u_i) \leq p(u_j)$, then $P_0(u_j, u_i) \geq P_0(u_i, u_j)$ under both the STI and ATI assumptions.

Proof: Under the STI assumption, we have

$$P_0(u_j, u_i) = (1 - p(u_j))(1 - p(u_i)) + \alpha p(u_j)(1 - p(u_i)) + \beta p(u_j)p(u_i) \text{ , and}$$

$$P_0(u_i, u_j) = (1 - p(u_i))(1 - p(u_j)) + \alpha p(u_i)(1 - p(u_j)) + \beta p(u_i)p(u_j) .$$

It follows that

$$\begin{aligned} P_0(u_j, u_i) - P_0(u_i, u_j) &= \alpha p(u_j)(1 - p(u_i)) - \alpha p(u_i)(1 - p(u_j)) \\ &= \alpha(p(u_j) - p(u_i)) \geq 0 . \end{aligned}$$

Under the ATI assumption, we have

$$P_0(u_j, u_i) = (1 - p(u_j))(1 - p(u_i)) + \alpha p(u_j)(1 - p(u_i)) \text{ , and}$$

$$P_0(u_i, u_j) = (1 - p(u_i))(1 - p(u_j)) + \alpha p(u_i)(1 - p(u_j)) .$$

Hence,

$$P_0(u_j, u_i) - P_0(u_i, u_j) = \alpha(p(u_j) - p(u_i)) \geq 0 . \square$$

The main idea of the adaptive diagnosis approach is to identify a fault-free unit as quickly as possible. Under the ATI assumption, once a "0" test result is obtained, the unit tested is fault-free. Under the STI assumption, the accumulation of "0" test results as well as that of "1" test results will lead to the identification of a fault-free unit. Theorem 3 implies that in order to produce a "0" test result probabilistically quicker, a unit with a higher probability of failure should test a unit with a lower probability. Without loss of generality, we can assume, in the remainder of the paper, that the units in U are sorted in nondecreasing order of their *a priori* probabilities of failure; that is, $p(u_1) \leq p(u_2) \leq \cdots \leq p(u_n)$, or equivalently, $w(u_1) \geq w(u_2) \geq \ldots \geq w(u_n)$.

Theorem 4: For any three units $u_i, u_j, u_k \in U$, if $p(u_i) \leq p(u_j) \leq p(u_k)$, then $P_0(u_j, u_i) \geq P_0(u_k, u_i) \geq P_0(u_k, u_j)$ under both the STI and ATI assumptions.

Proof. Under the STI assumption, we have

$$P_0(u_j, u_i) = (1 - p(u_j))(1 - p(u_i)) + \alpha p(u_j)(1 - p(u_i)) + \beta p(u_j)p(u_i) \ ,$$

$$P_0(u_k, u_i) = (1 - p(u_k))(1 - p(u_i)) + \alpha p(u_k)(1 - p(u_i)) + \beta p(u_k)p(u_i) \ , \text{ and}$$

$$P_0(u_k, u_j) = (1 - p(u_k))(1 - p(u_j)) + \alpha p(u_k)(1 - p(u_j)) + \beta p(u_k)p(u_j) \ .$$

Therefore, we derive

$$\begin{aligned}
P_0(u_j, u_i) - P_0(u_k, u_i) &= (p(u_k) - p(u_j))(1 - p(u_i)) + \alpha(p(u_j) - p(u_k))(1 - p(u_i)) \\
&\quad + \beta(p(u_j) - p(u_k))p(u_i) \\
&= (p(u_k) - p(u_j))[1 - p(u_i) - \alpha(1 - p(u_i)) - \beta p(u_i)] \\
&= (p(u_k) - p(u_j))[1 - \alpha - (1 - \alpha + \beta)p(u_i)] \\
&\geq (p(u_k) - p(u_j))[1 - \alpha - (1 - \alpha + \beta) \cdot \frac{1}{2}] \quad \left(\text{since } p(u_i) < \frac{1}{2}\right) \\
&= (p(u_k) - p(u_j)) \left(\frac{1}{2}\right) (1 - \alpha - \beta) \geq 0 \quad (\text{since } \alpha + \beta \leq 1) \ , \text{ and}
\end{aligned}$$

$$
\begin{aligned}
P_0(u_k,u_i) - P_0(u_k,u_j) &= (1-p(u_k))(p(u_j)-p(u_i)) + \alpha p(u_k)(p(u_j)-p(u_i)) \\
&\quad + \beta p(u_k)(p(u_i)-p(u_j)) \, . \\
&= (p(u_j)-p(u_i))(1-p(u_k)+\alpha p(u_k)-\beta p(u_k)) \\
&= (p(u_j)-p(u_i))[1-(1-\alpha+\beta)p(u_k)] \geq 0 \\
&\quad (\text{since } p(u_k) < \frac{1}{2} \text{ and } 1+\beta-\alpha \leq 2) \, .
\end{aligned}
$$

Under the ATI assumption, we have

$$P_0(u_j,u_i) = (1-p(u_j))(1-p(u_i)) + \alpha p(u_j)(1-p(u_i))$$

$$P_0(u_k,u_i) = (1-p(u_k))(1-p(u_i)) + \alpha p(u_k)(1-p(u_i)) \, , \text{ and}$$

$$P_0(u_k,u_j) = (1-p(u_k))(1-p(u_j)) + \alpha p(u_k)(1-p(u_j)) \, .$$

Therefore, we obtain

$$
\begin{aligned}
P_0(u_j,u_i) - P_0(u_k,u_i) &= (1-p(u_i))(p(u_k)-p(u_j)) + \alpha(1-p(u_i))(p(u_j)-p(u_k)) \\
&= (1-\alpha)(1-p(u_i))(p(u_k)-p(u_j)) \geq 0 \, , \text{ and} \\
P_0(u_k,u_i) - P_0(u_k,u_j) &= (1-p(u_k))(p(u_j)-p(u_i)) + \alpha p(u_k)(p(u_j)-p(u_i)) \\
&= (1-(1-\alpha)p(u_k))(p(u_j)-p(u_i)) \geq 0 \, . \, \Box
\end{aligned}
$$

The above theorems suggest the following test selection strategy. First, use tests of the form (u_{i+1}, u_i) rather than those of the form (u_i, u_{i+1}). Then, perform the test (u_2, u_1) rather than the test (u_3, u_2), and proceed to carry out the tests $(u_3, u_1), (u_4, u_1), \ldots,$ until unit u_1 is hopefully identified to be fault-free; otherwise repeat the process starting with the test (u_3, u_2) and so forth. Based on this strategy, we develop adaptive diagnosis algorithms in the next section.

IV. Adaptive Diagnosis Algorithms

Let $S(U,T)$ be a system with each unit $u_i \in U$ having weight $w(u_i)$. Recall the assumption that the units in U are sorted in nondecreasing order of their *a priori* probabilities of failure, or equivalently in nonincreasing order of their weights; that is, $w(u_1) \geq w(u_2) \geq \ldots \geq w(u_n)$. Before we proceed to present adaptive diagnosis algorithms, we introduce optimal base sets for p-τ-diagnosable systems under the STI and ATI assumptions. They play a key role in the algorithms. Let $G = (U,T)$ be the digraph representing system S. We say that a subset $V \subseteq U$ of S is p-τ-diagnosable if the subsystem of S defined by the induced subgraph $(V, T(V))$ of G is p-τ-diagnosable, where $T(V) = \{(u,v) \in T | u, v \in V\}$. Namely, the subsystem consists of the units in V and the test links of T which connect units in V.

Definition 3: Let $S(U,T)$ be a p-τ-diagnosable system under the STI (resp., ATI) assumption. A smallest cardinality subset $B \subseteq U$ of S that is p-τ-diagnosable under the STI (resp., ATI) assumption, is called an ***optimal base set*** of S under the STI (resp., ATI) assumption. □

Given a p-τ-diagnosable system $S(U,T)$, one can construct an optimal base set B_s under the STI assumption in the following manner: if $\sum_{i=1}^{n} w(u_i) \leq 2K(\tau)$, set $B_s = U$; otherwise, find index b such that $\sum_{i=1}^{b-1} w(u_i) \leq 2K(\tau) < \sum_{i=1}^{b} w(u_i)$, and set $B_s = \{u_1, u_2, \ldots, u_b\}$. Similarly, an optimal base set B_a under the ATI assumption can be obtained as follows: if $\sum_{i=2}^{n} w(u_i) \leq K(\tau)$, set $B_a = U$; otherwise, find index b such that $\sum_{i=3}^{b-1} w(u_i) \leq K(\tau) - w(u_1) < \sum_{i=3}^{b} w(u_i)$, and set $B_a = \{u_1, u_2, \ldots, u_b\}$.

We now show an adaptive diagnosis algorithm which identifies a fault-free unit under the STI assumption. Let $B_s = \{u_1, u_2, \ldots, u_b\}$ be an optimal base set of S.

Algorithm SYM:

$i \leftarrow 1;\ F \leftarrow \phi;\ k \leftarrow K(\tau)$

while $i \leq b$ **do**

  if $W(F) + w(u_i) > K(\tau)$ **then return** $*/u_i$ is fault-free $/*$

  else $V_0 \leftarrow \{u_i\};\ V_1 \leftarrow \phi;\ j \leftarrow i + 1$

    while $j \leq b$ **do**

      Perform test (u_j, u_i)

      if $a(u_j, u_i) = 0$ **then** $V_0 \leftarrow V_0 \cup \{u_j\}$

        if $W(V_0) > k$ **then return** $*/u_i$ is fault-free $/*$

      else $V_1 \leftarrow V_1 \cup \{u_j\}$

        if $W(V_1) > k$ **then** $F \leftarrow F \cup \{u_i\};\ k \leftarrow k - w(u_i);\ i \leftarrow i + 1;$ **break**

$*/u_i$ is faulty $/*$

        if $W(F) + w(u_i) > K(\tau)$ **then return** $*/u_i$ is fault-free $/*$

      $j \leftarrow j + 1$

    endwhile

  $i \leftarrow i + 1$

endwhile $\square$

Let $\lceil x \rceil$ denote the smallest integer that is greater than or equal to x.

Theorem 5: Let $S(U, T)$ be a system which satisfies the condition of Theorem 1, and let B_s be an optimal base set of S with $|B_s| = b$. Under the STI assumption, Algorithm SYM always identifies a fault-free unit in S using at most $bx - x(x+1)/2$ tests, where $x = \lceil (b-1)/2 \rceil$.

Proof: We first note that the set V_1 contains those units that are faulty if unit u_i is assumed to be fault-free. Therefore, if $W(V_1) > k$, V_1 cannot be a set of faulty units, and hence the assumption that u_i is fault-free is wrong. Thus, u_i must be faulty. This implies that F always contains faulty units which have correctly been identified so far. Therefore, if the termination condition that $W(F) + w(u_i) > K(\tau)$ is met, u_i must be fault-free. On the other hand, the set V_0 contains those units that are faulty if unit u_i is assumed to be faulty. Using the same reasoning as above, if the algorithm terminates when $W(V_0) > k$, u_i is fault-free. Finally, it is not difficult to see that the algorithm terminates on the third line only when $F = \phi$ and $w(u_1) > K(\tau)$. Apparently u_1 cannot belong to any allowable fault set of cardinality at most $K(\tau)$, and hence u_1 is fault-free.

We have shown that the algorithm correctly identifies a fault-free unit if it terminates. We now show that it does terminate using at most $bx - x(x+1)/2$ tests. Recall that the units in the optimal base set B_s are sorted in nonincreasing order of their weights, that is, $w(u_1) \geq w(u_2) \geq \ldots \geq w(u_b)$. Let $V_x = \{u_i | 1 \leq i \leq x\}$ and $V_y = U - V_x - \{u_{x+1}\}$, where $x = \lceil (b-1)/2 \rceil$. Clearly $W(V_x) \geq W(V_y)$ and hence $W(V_x) + w(u_{x+1}) > K(\tau)$ since $\sum_{i=1}^{b} w(u_i) > 2K(\tau)$ due to the property of B_s. Therefore, in the worst case in which no unit in V_x is identified as fault-free in the diagnosis process, u_{x+1} will be diagnosed as fault-free. Clearly, the total number of tests used is at most $(b-1)+(b-2)+\ldots+(b-x) = bx - x(x+1)/2$. □

We now present two adaptive diagnosis algorithms which identify a fault-free unit under the ATI assumption. The first algorithm is applicable to systems which satisfy the condition of Theorem 1. Note that if a system is p-τ-diagnosable under the STI assumption, it is so under the ATI assumption. Let $B_s = \{u_1, u_2, \ldots, u_b\}$ be an optimal base set of S.

Algorithm ASYM1:

$i \leftarrow 2;\ F \leftarrow \phi$

while $i \leq b$ **do**

  if $W(F) + w(u_1) > K(\tau)$ **then return** $*/u_1$ is fault-free $/*$

  else Perform test (u_{i+1}, u_i)

    if $a(u_{i+1}, u_i) = 0$ **then return** $*/u_i$ is fault-free $/*$

    else $F \leftarrow F \cup \{u_{i+1}\}$

  $i \leftarrow i + 2$

endwhile

print "u_1 is fault-free"$\square$

Theorem 6: Let $S(U,T)$ be a system which satisfies the condition of Theorem 1, and let B_s be an optimal base set of S with $|B_s| = b$. Under the ATI assumption, Algorithm ASYM1 always identifies a fault-free unit in S using at most $\lceil (b-1)/2 \rceil$ tests.

Proof: If $a(u_{i+1}, u_i) = 0$ occurs for some i, then clearly u_i is a fault-free unit under the ATI assumption. Suppose that the results of all tests performed are "1". Let $V_1 = \{u_i \in B_s | i \text{ is odd and } i \geq 3\}$ and $V_2 = \{u_i \in B_s | i \text{ is even }\}$. Since $w(u_{2j-1}) \geq w(u_{2j})$ for $j = 1, 2, \ldots,\ \lceil (b-1)/2 \rceil$, $w(u_1) + W(V_1) \geq W(V_2)$. Due to the property of the optimal base set B_s under the STI assumption and Theorem 1, $w(u_1) + W(V_1) > K(\tau)$. Since $F = V_1$, $w(u_1) + W(F) > K(\tau)$. Thus, the algorithm terminates and u_1 is a fault-free unit. Clearly, the total number of tests used is at most $\lceil (b-1)/2 \rceil$. $\square$

The last algorithm we present is applicable to systems which satisfy the condition of Theorem 2. Let $B_a = \{u_1, u_2, \ldots,\ u_b\}$ be an optimal base set of S.

Algorithm ASYM2:

$i \leftarrow 2;\ j \leftarrow 2;\ F \leftarrow \phi;$

while $i \leq b$ **do**

  If $w(u_1) + W(F) > K(\tau)$ **then return** $*/u_1$ is fault-free $/*$

  else Perform test (u_i, u_1)

    if $a(u_i, u_1) = 0$ **then return** $*/u_1$ is fault-free $/*$

    else $j \leftarrow i + 1$

    while $j \leq b$ **do**

      Perform test (u_j, u_i)

      if $a(u_j, u_i) = 0$ **then return** $*/u_i$ is fault-free $/*$

      $j \leftarrow j + 1$

    endwhile

  $F \leftarrow F \cup \{u_i\}$

  $i \leftarrow i + 1$

endwhile □

Theorem 7: Let $S(U,T)$ be a system which satisfies the condition of Theorem 2, and let B_a be an optimal base set of S with $|B_a| = b$. Under the ATI assumption, Algorithm ASYM2 always identifies a fault-free unit using at most $(b-1)b/2 - 1$ tests.

Proof: Let $V = \{u_i | 3 \leq i \leq b\}$. Due to the property of the optimal base set B_a under the ATI assumption, $w(u_1) + W(V) > K(\tau)$. During the first cycle, if the results of all tests that involve unit u_2 are "1", u_2 is faulty; otherwise, all units in $\{u_1\} \cup V$ would be faulty and their total weight would exceed $K(\tau)$. Using a similar reasoning, during the i-th cycle, if the results of all tests that involve unit u_{i+1} are "1", u_{i+1} is faulty. If no fault-free unit is found in $V - \{u_b\}$, $F = \{u_2\} \cup V - \{u_b\}$. Since $w(u_2) \geq w(u_b), W(F) \geq W(V)$ and hence $w(u_1) + W(F) > K(\tau)$. Thus, the algorithm terminates and unit u_1 is fault-free.

Therefore, the algorithm always identifies a fault-free unit. Clearly, the total number of tests used is at most $(b-1)+(b-2)+\ldots+2=(b-1)b/2-1$. $\square$

V. Conclusions

We have presented efficient adaptive diagnosis algorithms for probabilistically diagnosable systems under both the symmetric and asymmetric test invalidation assumptions. Using a probabilistic measure of test results and the property of an optimal base set, we have derived a test selection strategy. Thus, each algorithm terminates with a higher probability of using a minimum number of tests. We have given the worst case complexity analysis of each algorithm for identifying a fault-free unit. Such an analysis for the identification of a faulty unit can easily be done. These analyses are useful when executing the algorithms in either without repair or with repair processes. The adaptive approach has been shown to provide simple and efficient fault diagnosis even for the more general system-level model.

References

[1] F. J. Allan, T. Kameda, and S. Toida, "An approach to the diagnosability analysis of a system," *IEEE Trans. Comput.*, vol. C-24, pp. 1040-1042, Oct. 1975.

[2] F. Barsi, F. Grandoni, and P. Maestrini, "A theory of diagnosability of digital systems," *IEEE Trans. Comput.*, vol. C-25, pp. 585-593, June 1976.

[3] M. Blecher, "On a logical problem," *Discrete Math.*, vol. 3, pp. 379-380, 1982.

[4] A. T. Dahbura, "An efficient algorithm for identifying the most likely fault set in a probabilistically diagnosable system," *IEEE Trans. Comput.*, vol. C-36, pp. 354-356, Apr. 1986.

[5] A. T. Dahbura and G. M. Masson, "An $O(n^{2.5})$ fault identification algorithm for diagnosable systems," *IEEE Trans. Comput.*, vol. C-33, pp. 486-492, June 1984.

[6] H. Fujiwara and K. Kinoshita, "Connection assignments for probabilistically diagnosable systems," *IEEE Trans. Comput.*, vol. C-27, pp. 280-283, Mar. 1978.

[7] H. Fujiwara and K. Kinoshita, "Some existence theorems for probabilistically diagnosable systems," *IEEE Trans. Comput.*, vol. C-27, pp. 379-384, Apr. 1978.

[8] S. L. Hakimi and A. T. Amin, "Characterization of connection assignment of diagnosable systems," *IEEE Trans. Comput.*, vol. C-23, pp. 86-88, Jan. 1974.

[9] S. L. Hakimi and K. Nakajima, "On adaptive system diagnosis," *IEEE Trans. Comput.*, vol. C-33, pp. 234-240, Mar. 1984.

[10] S. L. Hakimi and E. F. Schmeichel, "An adaptive algorithm for system level diagnosis," *J. Algorithms*, pp. 526-530, 1984.

[11] T. Kameda, S. Toida, and F. J. Allan, "A diagnosis algorithm for networks," *Informat. Contr.*, vol. 29, pp. 141-148, 1975.

[12] S. N. Maheshwari and S. L. Hakimi, "On models for diagnosable systems and probabilistic fault diagnosis," *IEEE Trans. Comput.*, vol. C-25, pp. 228-236, Mar. 1976.

[13] G. G. L. Meyer, "A diagnosis algorithm for the BGM system level fault model," *IEEE Trans. Comput.*, vol. C-33, pp. 756-758, Aug. 1984.

[14] K. Nakajima, "A new approach to system diagnosis," in *Proc. 19th Annu. Allerton Conf. Commun. Contr. and Comput.*, Sept. 1981, pp. 697-706.

[15] J. Narasimhan and K. Nakajima, "An algorithm for determining the fault diagnosability of a system," *IEEE Trans. Comput.*, vol. C-35, pp. 1004-1008, Nov. 1986.

[16] F. P. Preparata, G. Metze, and R. T. Chien, "On the connection assignment problem of diagnosable systems," *IEEE Trans. Electron. Comput.*, vol. EC-16, pp. 848-854, Dec. 1967.

[17] E. F. Schmeichel, S. L. Hakimi, M. Ohtsuka, and G. Sullivan, "Minimizing the Number

of Test Rounds," in *Dig. 18th Int. Symp. on Fault-Tolerant Computing*, Tokyo, Japan, July 1988, pp. 266-271.

[18] G. Sullivan, "A polynomial time algorithm for fault diagnosability," in *Proc. 25th Annu. Symp. Found. of Comp. Sci.*, Oct. 1984, pp. 148-155.

[19] G. Sullivan, "System-level fault diagnosability in probabilistic and weighted models," in *Dig. 17th Int. Symp. on Fault-Tolerant Computing*, Pittsburgh, PA, June 1987, pp. 190-195.