ABSTRACT

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Gross job flows dynamics, defined as the behavior of creation and destruction of jobs at the establishment level, has become a topic of great interest in economics during recent years and researchers have resorted to different empirical methodologies in order to tease out its causes and consequences, as well as its connections to overall economic activity. In this context, my dissertation attempts to contribute to the debate by advancing the usefulness of frequency-domain techniques. I emphasize not only the relevance of the economic questions being examined, but also the unique perspective that frequency-domain techniques can provide. There are three major questions I pursue. The first is why equilibrium search models of labor market frictions have trouble explaining the observed persistence in employment fluctuations. I implement a frequency-domain decomposition of the employment growth rate to isolate the contributions coming from the job creation spectrum, the job destruction spectrum, and the cross-spectrum between the two. Among other results, I show that the failure to generate a negative contemporaneous correlation between job creation and job destruction at business cycle frequencies is behind the

inability of the Mortensen-Pissarides (1994) canonical model to reproduce the empirical spectral shape of the employment growth series. The second question I tackle relates to the direction of causality between aggregate employment fluctuations and gross job reallocation. Recent macroeconomic models suggest an active role for reallocation dynamics over the business cycle, and my results can be interpreted as supporting evidence that such a role indeed exists, but at a low frequency range. The basic idea is to look for different causality relationships at different time-scales by combining wavelet techniques with a standard Granger causality test. Finally, the third question I address investigates the connections between labor productivity growth and the frequency content of the job reallocation series for four-digit level US manufacturing industries. My results indicate that industries with relatively more influential low frequencies display inferior productivity growth. I relate these findings to the literature and propose a simple theoretical model to explain them.

# ESSAYS ON JOB FLOWS DYNAMICS

By

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# Dedication

I dedicate this dissertation to my father Eduardo Marcos Chaves Bastos, my mother Almira Rodrigues, my wife Leatrisse Oba, and my daughter Ana Beatriz Oba Bastos.

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I would have never completed this dissertation in absence of the guidance and support I received throughout my academic journey. Professor John Haltiwanger, my main advisor, provided superb orientation and encouragement. By interacting with him I learned much more than economics. Inadvertently, he taught me the traits of a first-class professional which are valid beyond academic walls. Professor John Shea has contributed enormously to my research through his astonishing ability to extricate "economic juice" from virtually any idea. Professor Mike Pries, a reliable source of advice permanently stationed at Tydings 4th floor, was essential for continuous progress.

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# Chapter 1: Introduction

In his autobiography written on the occasion of the 2003 Nobel Prize in Economics, Clive Granger recounts how his pioneering book "Spectral Analysis in Economic Time Series" (1969) - co-authored with Michio Hatanaka - was produced. According to Granger, when he joined the "Time Series Project" led by Oscar Morgenstern at Princeton, Von Neumann (a close friend of Morgenstern) felt strongly that "economists should be using the Fourier methods with their data". As a result, Granger and Hatanaka pursued that line of research by repeatedly interacting with Princeton statistician John Tukey, whose *modus operandi* was to prescribe a series of computer calculations, provide interpretation for the results, and restart the cycle by asking another batch of work. After a while, a solid body of knowledge emerged and a book was clearly warranted.[1]

The story as told suggests that Granger and Hatanaka's book came to existence partly because Von Neumann had a gut feeling about the potential benefits of spectral analysis in economics. Although I cannot possibly speak to his reasons, it is fair to say that much of the appeal underlying frequency-domain analysis stems from its promise to generate new and rich insights by looking at the data from a different, and yet simple, perspective. Throughout the years, research has reaffirmed that spectral

[1] Granger and Hatanaka told Tukey that they would wait for him to publish his own research first (which included seminal work on the cross-spectrum for a pair of series) before publishing their book. Tukey replied "he was far too busy doing new research to publish, and that they should just go ahead".

techniques are indeed powerful and that they can be used not only as descriptive tools, but also for model analysis.

In this context, Watson (1993) articulates how modern dynamic macroeconomic models can be explicitly tested using frequency-domain concepts. More recently, papers such as Christiano and Vigfusson (1999), Otrok (2001a,b), Otrok, Ravikumar and Whiteman (2002), and Figura (2002, 2001) all relate spectral concepts to particular features of macroeconomic models to be improved, without having the development of formal statistical testing procedures as their main goal. These papers exploit the natural association between some aspects of macroeconomic theory and the frequency-domain.

My dissertation shares this same spirit and it is founded on the idea that models of job flows dynamics have inherent frequency content to be exploited. Each one of the next three chapters is a self-contained essay that pursues a different dimension of this argument. By the end of the dissertation I hope to have instilled in the reader more than a gut feeling (as Von Neumann may once have had) that the frictional nature of the adjustment process underlying job flows dynamics calls for a greater attention to frequency-domain methods.

# Chapter 2: Persistent employment fluctuations and the structure of search models: a frequency-domain perspective

## *2.1 Literature review and motivation*

Employment fluctuations are the result of a complex mix between exogenous shocks and the institutional/decision environment in which firms and workers trade labor services. At the aggregate level, the well-documented positive persistence observed in employment fluctuations is an important sign of how efficiently an economy makes use of its labor input over time. The way we interpret this persistence depends fundamentally on what we believe to be the cause of fluctuations. For example, if exogenous technology shocks shift the demand for labor in a persistent fashion, persistent employment dynamics may be nothing more than healthy Walrasian fluctuations. In contrast, if we believe that persistence results from matching frictions, then policies geared towards improving job finding rates may be socially desirable. So we care about persistence in employment fluctuations because it ultimately reflects deeper structural features of the economy, and the ability of our models to "get persistence right" is crucial for our understanding of labor market dynamics.

As pointed out by Cogley and Nason (1995), early dynamic stochastic general equilibrium models achieved persistent fluctuations without resorting to any internal propagation mechanisms, by simply assuming persistent (exogenous) driving forces. Despite being successful at times, this approach has been criticized for assuming what should be explained. Modern macroeconomic models of employment fluctuations, however, have fully embraced a key non-Walrasian feature of labor markets: time-consuming matching. The main idea behind these models is that escaping the unemployment pool may not be possible even if the agent is willing to accept the current wage and, even though, there are vacancies. A willing-to-work unemployed agent and a willing-to-hire employer need to overcome the problem of finding each other before production can start. Such matching frictions generally take the form of probabilistic hiring, which may be a function of economy-wide variables such as unemployment and vacancies. This friction implies that the stock of employed workers displays sluggish adjustment over time, potentially generating persistent employment dynamics.

A workhorse model in the macroeconomic literature on matching frictions is Mortensen and Pissarides (1994). This model can easily be adapted to study a host of different issues, such as the response of job flows to different labor market policies and institutions. Although the original version of the model is capable of matching important aspects of job flows behavior, recent studies have found important shortcomings in its ability to replicate a broader set of stylized facts.[2] As a result, recent work has attempted to make matching models more reliable tools for policy design. While suggesting modifications and additions to the structural model is an important part of this task, equally important is to pinpoint empirically the source of misalignments present in the baseline framework. This paper does the latter using frequency-domain tools.

[2] These studies include Cole and Rogerson (1999), Shimer (2005, 2003), Pries (2004) and Hall (2005).

The particular shortcoming we focus on here is the lack of persistent fluctuations in the employment growth rate generated by the model *vis a vis* those observed in the data. While the first-order auto-correlation coefficient of the employment growth rate is 0.68 for US manufacturing (calculated at a quarterly frequency for the period 1947 to 1993), the counterpart measure obtained from simulations of the Mortensen and Pissarides model is slightly negative, despite the fact that the simulations use a highly persistent driving force. In order to understand the empirical roots of this result, we propose a frequency-domain decomposition of the employment growth spectrum into the job creation spectrum, the job destruction spectrum and their cospectrum. To help us interpret the decomposition results, we suggest a canonical reduced-form model that illustrates the differences between achieving persistence in levels and in growth rates.

The fact that many models generate weak persistence in employment dynamics has been already pointed out by Hall (1995). More recently, Pries (2004) addressed this problem using a structural model in which match-learning effects induce recurring job losses and sluggish adjustment in employment. In contrast to these papers, we pursue a more descriptive route by comparing model and data spectra for the decomposition of the employment growth rate. We also consider Cole and Rogerson's (1999) reduced-form representation of the Mortensen and Pissarides model to ascertain the limits of re-parameterization in fighting the lack of persistence.

Our results show that temporal decoupling between job creation and job destruction is a quantitatively important element behind the employment growth persistence observed in the data, and that the Mortensen-Pissarides model fails to capture it. Additionally, we show that generating persistence in the employment growth rate is not equivalent to generating decoupling between the creation and destruction margins, and although Cole and Rogerson's parameterization clearly succeeds in the former, it appears to be less successful in the latter. Finally, we perform an exercise applying the Method of Simulated Moments (Gourieroux and Monfort (1996)), in which the moments to be matched are the empirical spectral densities estimated from the data. We find that the deviations between the spectrum implied by Cole and Rogerson's reduced-form model and the data can be further reduced by assuming lower job-finding probabilities. Finally, we relate our empirical findings to a particular structural feature of the Mortensen-Pissarides model: the frictionless determination of meeting rates implied by the zero-profit condition on vacancy posting.

The chapter is organized as follows. Section 2.2 presents an outline of Mortensen and Pissarides (1994), emphasizing how employment dynamics are determined within the model. Section 2.3 describes the dataset used and provides a quick overview of concepts and spectral techniques employed in the paper. Section 2.4 proposes a decomposition of the employment growth rate in the frequency-domain and performs comparisons between the Mortensen-Pissarides model and data spectra. Section 2.5 studies the limits of re-parameterizing the Mortensen-Pissarides model using the reduced-form representation proposed by Cole and Rogerson (1999). Section 2.6 concludes the paper and discusses directions for future research.

## *2.2 Mortensen-Pissarides model and employment dynamics*

### 2.2.1 The environment

The Mortensen-Pissarides (1994) model features an environment populated by risk-neutral entrepreneurs and risk-neutral workers. Existing jobs can either be matched to a worker or vacant, and workers can either be matched to an employee or searching for a job.[3] Job creation takes place when a searching worker meets a vacant job. The meeting process is assumed to be time-consuming, and is modeled according to a matching function which depends on the aggregate levels of unemployment and vacancies. Additionally, vacancies are costly to maintain and any entrepreneur is free to post a vacancy or destroy one already posted. The match surplus generated by labor market frictions is shared via period-by-period wage renegotiations following a Nash bargaining solution.

The productivity of each existing match depends on aggregate and idiosyncratic components. While all new matches are assumed to be created at the maximum idiosyncratic productivity, they experience new idiosyncratic shocks over time and can be terminated at any moment if their corresponding productivity falls below a certain threshold endogenously determined by the model (known as the reservation

[3] The model assumes one worker per firm (entrepreneur), which is generally justified on the grounds of constant returns to scale technology.

productivity). Such terminations contribute to job destruction. A non-degenerate distribution of filled-job productivities is fully consistent with optimizing individual behavior, as matches experience idiosyncratic shocks that are below the upper limit at which they are created but above the threshold level for destruction. Essentially, labor market frictions make it optimal for firms and workers to tolerate filled jobs that are not as productive as brand new jobs.

Mortensen and Pissarides add cyclical dynamics to this setup by allowing the aggregate productivity component of a filled job to follow a three-state Markov process. As a result, the endogenous reservation productivity and matching rates will also fluctuate in response to the current aggregate state. Flows into unemployment will be given by the interaction between the actual distribution of idiosyncratic productivities and the fluctuating reservation productivity values, whereas flows out of unemployment will be given by the interaction between the level of unemployment and job-finding rates implied by the matching function. Once calibrated, the model can be used to generate series of job creation, job destruction and employment. Below we present and briefly discuss basic features of the model affecting the determination of employment dynamics in the presence of aggregate state fluctuations.[4]

[4] The presentation of the main features of the model is intentionally brief and focuses only on what is relevant for our purposes. The interested reader is encouraged to refer to Mortensen and Pissarides (1994).

2.2.2 Basic features

Let $L$ denote the total number of workers in the labor force (which is fixed in the model), $u$ the unemployment rate (unemployed workers as a fraction of the labor force), and $v$ the vacancy rate (total vacancies as a fraction of the labor force). The total number of matches at any point in time is given by $Lm=m(Lv,Lu)$, in which $m(.,.)$ is a matching function assumed to be increasing in both arguments and linearly homogenous. The rate at which vacant jobs are filled is given by $q=m(v,u)/v$, which can also be written as $q(v/u)$ where $q'(v/u)<0$. The job-finding probability for workers is given by $m(v,u)/u$ or, equivalently, $vq(v/u)/u$. Because of the linear homogeneity in the matching function, the vacancy to unemployment ratio $v/u$ (which we will refer to as $\theta$) is sufficient to pin down worker finding and job finding probabilities.

The productivity of a filled job is given by $p+\sigma\varepsilon$, where $p$ is an aggregate shock common to all filled jobs and $\varepsilon$ is an idiosyncratic shock. The aggregate shock follows a three-state Markov process with transition probabilities given by $\pi_{ij}$ (where $i,j=1,2,$ *or* $3$), which are calibrated to display strong persistence.[5] Changes in the idiosyncratic shock follow a Poisson distribution with arrival rate $\lambda$. Once an idiosyncratic shock takes place, a new draw for $\varepsilon$ is taken from a fixed distribution $F(\varepsilon)$. The parameter $\sigma$ controls the amount of dispersion in the job productivity implied by the idiosyncratic shocks.

[5] The transition matrix is constructed to match a first-order auto-correlation coefficient of 0.93.

Let *V(i)* be the asset value of a vacancy in aggregate state *i* (that is, the present discounted value of expected returns associated with a vacant job under a particular aggregate shock) and let $J(\varepsilon,i)$ be the asset value of a filled job with idiosyncratic productivity $\varepsilon$ in aggregate state i. Under perfect capital markets and for any state *i*, *V(i)* must satisfy:

$$V(i) = -c + \frac{1}{1+r}\left[ q(\theta_i)\sum_{j=1}^{3}\pi_{ij}J(\varepsilon_u, j) + (1-q(\theta_i))\sum_{j=1}^{3}\pi_{ij}V(j) \right] \qquad (1)$$

Where *r* is the interest rate, *c* is the vacancy flow cost, $q(\theta_i)$ is the state-contingent job-meeting probability, and $J(\varepsilon_u, i)$ is the asset value of a filled job, evaluated at the upper bound ($\varepsilon_u$) of the distribution of idiosyncratic productivity shocks and at the aggregate state *i*. Because any firm is free to post a new vacancy (free entry) or destroy an existing one in equilibrium, *V(i)* will be instantaneously driven down to zero and equation (1) will determine the value of the endogenous variable $\theta_i$ - which in turn determines job finding and worker finding probabilities. The instantaneous adjustment of $\theta_i$ is an important feature of the model to which we will come back later in the paper.

Because of labor market frictions, matches are valuable objects and they command positive rents in equilibrium. Wages adjust to split the rents between employer and employee according to a Nash bargaining solution. The total match surplus $S(\varepsilon,i)$ equals what both parties get from keeping the employment relationship alive, minus their outside opportunities. If $W(\varepsilon,i)$ is the asset value for an employed worker enjoying idiosyncratic productivity $\varepsilon$ in aggregate state *i*, and $U(i)$ is the asset value for an unemployed worker in aggregate state *i*, the match surplus can be written as:

$$S(\varepsilon,i) = J(\varepsilon,i) - V(i) + W(\varepsilon,i) - U(i) \qquad (2)$$

Job destruction takes place whenever the match surplus falls below zero, which can happen in response to negative shocks in the aggregate or the idiosyncratic components of a job. The state-contingent reservation productivity $\varepsilon_d$ is endogenously determined by the condition $S(\varepsilon_d,i) = 0$.[6] One can solve for $\varepsilon_d$ by using the definitions of $J(\varepsilon,i)$, $V(i)$, $W(\varepsilon,i)$, and $U(i)$ to write (2) as a Bellman Equation whose right-hand side includes the state-contingent $\theta_i$ from (1) and the match surplus $S(x,j)$. The state-contingent reservation productivities can be recovered by iterating on the Bellman Equation to convergence.

So far we have discussed how the state-contingent job-finding probabilities and reservation productivities are determined by the model. Another important endogenous variable is unemployment, which is determined by the intersection in the *v-u space* between the Beveridge curve and the equilibrium vacancy-unemployment ratio *(θ)*. Note that the model is able to pin down employment dynamics because total job creation and total job destruction are fully determined by job-finding

[6] The match surplus is monotonically increasing in the idiosyncratic shock.

probabilities, reservation productivities and unemployment.[7] In particular, total job creation is obtained by multiplying the stock of unemployed workers by the current job-finding probability, while total job destruction is given by the mass of jobs with a non-negative surplus at time *t-1* that no longer exist at time *t*. The evolution of total employment *(N)* is related to total job creation *(C)* and total job destruction *(D)* through a simple identity:

$$N_{t+1} = N_t + C_{t+1} - D_{t+1} \qquad (3)$$

From (3) one can calculate the growth rate of employment and compare its frequency-domain properties to the data.

## *2.3 Data and methodology*

### 2.3.1 Data

We work with quarterly seasonally-adjusted job creation and job destruction rates for US manufacturing from 1947 to 1993 (188 observations), taken from Davis and Haltiwanger (1999). The job creation rate is defined as the sum of employment gains at the plant-level normalized by average employment at all plants in the current and previous period, whereas the job destruction rate is defined as the absolute value of employment losses at the plant-level also normalized by average employment in the

[7] Strictly speaking, both the employment level and the current distribution of filled-jobs across idiosyncratic productivities matter for employment dynamics.

current and previous period.[8] A variable of particular interest in this paper is the net employment growth rate, defined as the job creation rate minus the job destruction rate.

### 2.3.2 Spectral Analysis

The potential benefits of spectral techniques in economics were first outlined by Granger and Hatanaka (1964). Since then, frequency-domain methods have become popular in the profession as an exploratory tool and a device for empirical validation of theories.[9]

Spectral analysis provides an alternative way of looking at time series data. Its appeal as a valid investigative procedure relies on the *spectral representation theorem*, which asserts that any covariance-stationary process can be described as a weighted sum of periodic functions.[10] Once a series can be expressed as a summation of cycles that overlay one another, it is possible for potential regularities in the series to be buried under a cumulative chain of distinct periodic movements.

Spectral techniques are potentially quite useful in studying fluctuations. In particular, they can uncover the contribution of cycles with different frequencies to the behavior of a series (or to the joint behavior of two series). This is accomplished by decomposing the variance of a series (or covariance between two series) according to

[8] This normalization yields what is known as symmetric growth rates.

[9] Examples include Granger (1966, 1969), Nerlove (1964), Engle (1974), Watson (1993), Diebold, Ohanian and Berkowitz (1998), Christiano and Vigfusson (1999), Figura (2001, 2002), Otrok (2001a, 2001b), and Otrok, Ravikumar, and Whiteman (2002).

[10] The reader can find more information about Spectral Analysis in Priestley (1981), Stoica and Moses (1997), Hamilton (1994), or Brockwell and Davis (1996).

their frequencies, which in turn reveals the existence of influential periodic components.[11]

2.3.3 Multivariate spectrum

Let $\{W_t\}_{t=-\infty}^{\infty}$ be a covariance-stationary vector process, where $W_t = [Y_t, X_t]'$ is a (2 x 1) vector and its associated *kth* autocovariance matrix is defined as:

$$\Gamma_k \equiv \begin{bmatrix} \gamma_{YY}^{(k)} & \gamma_{YX}^{(k)} \\ \gamma_{XY}^{(k)} & \gamma_{XX}^{(k)} \end{bmatrix} \equiv \begin{bmatrix} E\left((Y_t - \overline{Y})(Y_{t-k} - \overline{Y})\right) & E\left((Y_t - \overline{Y})(X_{t-k} - \overline{X})\right) \\ E\left((X_t - \overline{X})(Y_{t-k} - \overline{Y})\right) & E\left((X_t - \overline{X})(X_{t-k} - \overline{X})\right) \end{bmatrix} \qquad (4)$$

Assuming that the sequence of autocovariance matrices $\{\Gamma_k\}_{k=-\infty}^{\infty}$ is absolutely summable, the autocovariance-generating function of W can be appropriately transformed to yield the multivariate spectrum, which in this case is a (2 x 2) matrix with complex elements:

$$S_W(\omega) = (2\pi)^{-1} \sum_{k=-\infty}^{\infty} \Gamma_k e^{-i\omega k} \qquad (5)$$

Using De Moivre's theorem and standard mathematical properties of trigonometric functions, the multivariate population spectrum $S_W(\omega)$ can be rewritten as:

[11] A word of caution is necessary. Multivariate spectral techniques alone cannot be used to prove the existence of structural relationships between variables. In particular, such techniques are not able to detect common cycles in two series, but simply whether there are influential cycles of common frequency between them. Spectral analysis is essentially a different way of looking at the variance-covariance matrix of a series.

$$S_W(\omega) = \frac{1}{2\pi}\begin{bmatrix} \sum_{k=-\infty}^{\infty} \gamma_{YY}^{(k)} \cos(\omega k) & \sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \{\cos(\omega k) - i\sin(\omega k)\} \\ \sum_{k=-\infty}^{\infty} \gamma_{XY}^{(k)} \{\cos(\omega k) - i\sin(\omega k)\} & \sum_{k=-\infty}^{\infty} \gamma_{XX}^{(k)} \cos(\omega k) \end{bmatrix} \qquad (6)$$

2.3.4 Univariate spectrum

The diagonal elements of matrix (6) are real-valued functions of $\omega$ representing the (individual) univariate spectra of Y and X, and they can be used to recover any autocovariance $\gamma^{(k)}$. As an illustration, to obtain the *kth* autocovariance of the variable Y, apply the following inversion formula:

$$\gamma_{YY}^{(k)} = \int_{-\pi}^{\pi} \left[ \sum_{k=-\infty}^{\infty} \gamma_{YY}^{(k)} \cos(\omega k) \Big/ 2\pi \right] e^{i\omega k} d\omega = \int_{-\pi}^{\pi} S_Y(\omega) e^{i\omega k} d\omega \qquad (7)$$

Where $S_Y(\omega)$ denotes the univariate spectrum of Y. In particular, the variance of the series can be obtained as a special case of (7):

$$Var(Y) = \gamma_{YY}^{(0)} = \int_{-\pi}^{\pi} \left[ \sum_{k=-\infty}^{\infty} \gamma_{YY}^{(k)} \cos(\omega k) \Big/ 2\pi \right] dw = \int_{-\pi}^{\pi} S_Y(\omega) dw = 2\int_{0}^{\pi} S_Y(\omega) dw \qquad (8)$$

Where the last equality exploits the symmetry of the univariate spectrum. As indicated above, the area below the univariate spectrum, when integrated over its entire period, is equal to the variance of the variable. More importantly, equation (8) can be easily transformed to investigate what portion of the variance of Y is due to cycles that lie within a certain frequency range. This is accomplished by simply setting the integration limits appropriately. Thus, by plotting the spectral function against its angular frequency $\omega$ one can obtain an assessment of how different frequencies contribute to the variance of Y. Frequency regions commanding humps in the spectrum are of particular interest as they suggest important periodic components (or cycles) of a series.

### 2.3.5 Cross spectrum

The off-diagonal elements of matrix (6) are complex-valued functions of $\omega$, and they represent the cross spectra between both series. For interpretation purposes, it is convenient to define, respectively, the cospectrum and the quadrature spectrum:

$$c_{YX}(\omega) = (2\pi)^{-1} \sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \cos(\omega k) \qquad (9)$$

$$q_{YX}(\omega) = -(2\pi)^{-1} \sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \sin(\omega k) \qquad (10)$$

Now the cross spectrum can be written in terms of its two components, which are the cospectrum and the quadrature:

$$S_{YX}(\omega) = c_{YX} + iq_{YX} \qquad (11)$$

The cospectrum between Y and X decomposes the contemporaneous covariance between both series by frequency. As in the case of the univariate spectrum, it is possible to show that integrating (9) over [-π,π] yields the contemporaneous covariance between Y and X.[12] Once again, one would be particularly interested in frequency intervals corresponding to humps of the cospectrum, since they reveal the existence of influential frequencies underlying the contemporaneous correlation between any two series. For our purposes, the cospectrum will help us to address the issue of decoupling between the job creation and destruction margins.

### 2.3.6 Estimation

A natural estimator of the spectrum is the sample multivariate periodogram:

$$\hat{S}_W(\omega) = (2\pi)^{-1} \sum_{k=-T+1}^{T-1} \hat{\Gamma}_k e^{-i\omega k} \qquad (12)$$

Where T is the sample size and $\hat{\Gamma}_k$ and $\overline{W}$ are:

$$\hat{\Gamma}_k = T^{-1} \sum_{t=k+1}^{T} (W_t - \overline{W})(W_{t-k} - \overline{W})' \qquad (13)$$

[12] In this paper, we will be interested in decomposing the contemporaneous covariance by frequency. So we do not discuss the concept of coherency, which would be important if one wants to assess the importance of cycles of frequency $\boldsymbol{\omega}$ controlling for out-of-phase (non-contemporaneous) dynamics.

$$\overline{W} = T^{-1}\sum_{t=1}^{T} W_t \quad (14)$$

Although it can be shown that the sample periodogram is asymptotically unbiased, its variance is often very large and, most critically, does not decrease with the sample size. Indeed, the number of autocovariances to be estimated increases together with the sample size rendering the sample periodogram an inconsistent estimator of the spectrum. As a result, other approaches are usually employed to achieve consistency. One possibility is to estimate a Vector Auto Regression (VAR) and take advantage of its rational spectra – see Hamilton (1994: chapter 6). This approach is said to be parametric, as it assumes that the process under investigation can be captured by a (preferably low-order) VAR structure. If such an assumption is correct, estimates can be substantially improved given the simplified structure of the auto-covariance matrix. In contrast, if the process cannot be well described by a VAR representation, non-parametric techniques are better. Because they do not rely on restrictive assumptions regarding the data-generating-process, non-parametric methods are generally preferred when one is unsure about the mechanism generating the data.[13]

Under the typical nonparametric strategy, the spectrum at a given frequency ω is estimated by averaging values of the spectrum evaluated in a neighborhood around that frequency. This is sometimes referred to smoothing the spectrum. The researcher must first choose a kernel $k(\omega_{j+m}, \omega_j)$ (also known as the spectral window), which defines a weighting scheme associated with the frequencies being considered. The

[13] Non-parametric methods have their own hurdles as well and we discuss them in the text.

kernel assigns specific weights according to the distance between the frequency $\omega_j$ and its neighbors $\omega_{j+i}, i \le m$. Hence, a nonparametric estimator of the multivariate spectrum can be calculated as:

$$\hat{S}_{NP}(\omega) = \sum_{m=-h}^{h} k(\omega_{j+m}, \omega_j)\hat{S}_W(\omega_{j+m}), \text{ where } \sum_{m=-h}^{h} k(\omega_{j+m}, \omega_j) = 1 \qquad (15)$$

Where $\hat{S}_W(\omega_{j+m})$ denotes the sample periodogram, computed as in (12) – (14). The particular non-parametric approach adopted in this paper is Welch's averaged periodogram method.[14] An important step of the estimation procedure is to choose the shape of the spectral window and the bandwidth parameter *h* (or alternatively, to choose the lag window and its length if the spectrum is estimated by weighting auto-covariances). Although there are several window shapes one can adopt, our estimates are robust across different choices, so we do not discuss issues related to the trade-off between smearing and leakage that underpin the choice of window shape.[15]

In contrast, the choice of the bandwidth parameter *h* merits a more careful discussion, as this parameter matters for our results. A wide spectral window (large *h*) means that spectral estimates at "many" neighbor frequencies are used to estimate the spectrum at each particular ω. This reduces the variance of the estimated spectrum, but at the

[14] See Stoica and Moses (1997) for an in-depth discussion of Welch's method. The process of obtaining a consistent non-parametric estimator of the spectrum can be equivalently described as weighting the auto-covariances in the sample periodogram (12). In this case, the weighting scheme is called the lag window instead of the spectral window. To avoid peaks at zero-frequencies, all variables are linearly detrended prior to estimation.
[15] See Stoica and Moses (1997) for a discussion on this issue.

same time causes loss of resolution (bias) as a result of strong smoothing.[16] A narrow spectral window (small *h*) has the opposite effect, and generates high resolution with large variance. The same trade-off between resolution and variance can be described from the perspective of lag windows. A wide lag window means that long auto-covariances are being used to form an estimate of the spectrum at each particular ω, which delivers high resolution (long auto-covariances can capture richer dynamics) and large variance (long auto-covariances reduce degrees of freedom). A narrow lag window considers only short auto-covariances, which implies smoother estimates and lower variance.[17]

There is no clear-cut best value for the bandwidth parameter *h*, and Hamilton (1994: 167) points out that "one practical guide is to plot an estimate of the spectrum using several different bandwidths and rely on subjective judgment to choose the bandwidth that produces the most plausible estimate". While addressing this problem, we follow the common practice of looking at estimation results across different bandwidth parameters and window types. Overall we believe we have a reliable set of results.

## *2.4 Model and data spectra*

### 2.4.1 Employment decomposition and comparisons among spectral densities

The first-order auto-correlation coefficient of the simulated employment growth series is around -0.2 for the original baseline parameterization of the Mortensen-

[16] The concept of resolution is related to the ability of distinguishing close (but distinct) influential frequencies.
[17] Note that a wide spectral window corresponds to a narrow lag window, and vice-versa.

Pissarides model, compared to a first-order auto-correlation coefficient of 0.68 for US quarterly manufacturing data. This is a dramatic failure of the model. This section resorts to spectral techniques in order to pinpoint the source of the problem and to provide a quantitative assessment of the factors leading up to this counterfactual result. We start by proposing a frequency-domain decomposition of the employment growth rate based on the following identity linking the employment growth rate (NET) to job creation (JC) and job destruction (JD) rates:

$$NET_t = JC_t - JD_t \quad (16)$$

The autocovariance-generating function (ACGF) of NET can be written as a combination of auto-covariances and cross-covariances:

$$G_{NET}(z) = G_{JC}(z) + G_{JD}(z) - G_{JC,JD}(z) - G_{JD,JC}(z) \quad (17)$$

One can translate the time-domain decomposition above into the frequency-domain by applying Fourier transforms. This will provide a decomposition of the employment growth rate spectrum as the sum of job creation and job destruction spectra minus the real part of each cross-spectrum:

$$S_{NET}(\omega) = S_{JC}(\omega) + S_{JD}(\omega) - real\{S_{JC,JD}(\omega)\} - real\{S_{JD,JC}(\omega)\} \quad (18)$$

Where ω denotes frequency and *real {.}* represents the real part of the cross-spectrum at each frequency. The equation above can also be written as:

$$S_{NET}(\omega) = S_{JC}(\omega) + S_{JD}(\omega) - 2 * real\{S_{JC,JD}(\omega)\} \quad (19)$$

This decomposition is applied to both data and simulated series extracted from the Mortensen-Pissarides model. We initially adopt the parameterization of the original paper, except that we use a 10-state aggregate shock representation (instead of a 3-state representation). We do so to avoid the possibility that few aggregate states with large persistence would naturally deliver long spells without any state change and artificially tilt the spectrum to lower frequencies.[18] Table 2.1 presents a summary of parameter changes resulting from using a 10-state version of the model.

The first set of results is reported in Figure 2.1.[19] The first column presents spectra for the data and the second column for the simulated series. The estimated spectra indicate that actual employment growth rate fluctuations are largely the result of (1) influential business cycle frequencies observed in job creation and job destruction and (2) the negative correlation between creation and destruction (decoupling) at business

[18] In order to implement this modification we used the algorithm proposed by Tauchen (1986) to generate a 10-state markov chain for the aggregate productivity shocks. The parameters describing the underlying auto-regression for the shocks were the same as in Mortensen and Pissarides (1994) – zero unconditional mean, first-order auto-correlation coefficient of 0.93 and a standard error of the innovation equal to 0.011. After increasing the dimension of the state space we re-solved the model numerically adjusting the distribution of idiosyncratic productivities accordingly. The model was simulated for $2^{12}$ periods – a power of two is computationally efficient.

[19] The empirical spectral densities were estimated using a Hanning 20 window and spectral densities for the simulated series were estimated with a Hanning 40 window (this is also the case in figures 2.2 and 2.8). Results are robust across alternative window shapes and a wide range of window lengths. Confidence intervals (95 % level) presented in the picture are narrower for the model because we are able to simulate much longer series, which also explains why we can afford to use a wider lag window.

cycle frequencies. Note that this negative correlation adds to the hump in the employment growth spectrum because of the minus sign in the decomposition formula (19).

When one compares the empirical and model spectra, several differences emerge. Most evident is that, in the model, fluctuations in the employment growth rate are not predominantly driven by business cycle frequencies; its spectrum displays a "climbing pattern". The absence of a dominant business cycle frequency is not unexpected given the lack of persistence already revealed in the first-order auto-correlation coefficient. Indeed, positive persistence implies spectral shapes that peak at low frequencies and then decline for higher frequencies. A natural question is how the various components of the simulated employment growth rate contribute to this odd-looking spectrum. With the help of the decomposition formula (19), we can sort out the roles played by job creation, job destruction and their comovement.

The second column of Figure 2.1 indicates that higher-than-business-cycle frequencies seem quite influential for employment behavior in the model. This result should be interpreted with caution. Firstly, it should be noted that the actual employment growth rate is approximately 50% more volatile than its simulated counterpart. Even though high frequencies look very important in the model, they in fact do not generate an unrealistic amount of variation.[20] Hence, influential high

[20] It can be verified that the area beneath the spectrum of the actual employment growth rate is approximately 1.5 times the area beneath the spectrum of simulated series.

frequencies reflect less of a "magnitude" problem and more of a "distribution of power" one, which takes us to the next point.

The *relatively* large role of high frequencies in the model is mostly due to the coupled dynamics between creation and destruction at low frequencies, which through the negative term in the decomposition formula (19) wipes out the influential lower frequencies contained in job creation and destruction. The lack of decoupling is crucial to generating the climbing shape of the employment spectrum and the lack of persistence in employment growth observed in the Mortensen-Pissarides model. To reinforce this point, we simulate the model again assuming less persistence in the aggregate shocks, with results presented in Figure 2.2.[21] While such a change is enough to push the humps of the job creation and job destruction spectra towards more realistic ranges, the coupled dynamics between creation and destruction at low frequencies once more neutralizes these influential frequencies in the employment spectrum.[22]

### 2.4.2 A canonical reduced-form model for levels and growth rates

While the Mortensen-Pissarides model fails to generate persistent employment growth rates, it does generate persistence in the employment level.[23] We develop a

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[21] Using Tauchen's method we generated a 10-state markov chain with an underlying auto-correlation coefficient of .53, simulated the model, and then estimated the relevant spectral densities.

[22] High frequencies also become more influential for job destruction dynamics when shocks are less persistent, since jobs become less resistant to bad idiosyncratic shocks in "non-persistent" good times and because the economy now moves from better to worse states more often (triggering small bursts of separations).

[23] Simulations of the Mortensen-Pissarides model indicate a first-order auto-correlation coefficient of the employment level of around 0.7. The Cogley and Nason (1995) criticism about persistent driving forces applies here. See Figure 2.3 for an illustration.

canonical reduced-form model that sheds some light into the differences between achieving persistence in levels and growth rates. The model motivates why achieving a persistent growth rate is particularly important, and it provides insights as to why the Mortensen and Pissarides model has trouble delivering results consistent with the data.

Consider the following reduced-form model describing the dynamic behavior of a variable $y$ (defined as the log of an economic variable Y):

$$y_t = \rho y_{t-1} + A_t, \qquad (20)$$

$$A_t = \theta A_{t-1} + \varepsilon_t \qquad (21)$$

Where

$$0 \leq \rho < 1, \quad 0 \leq \theta < 1, \quad \varepsilon \sim iid\ (0, \sigma^2) \qquad (22)$$

Equation (20) states that the value assumed by the variable y at time t is the sum of two terms. The term $\rho y_{t-1}$ is a reduced-form representation of frictions in the underlying structural model, whereas the term $A_t$ captures an exogenous shock that is potentially correlated across time. The coefficients $\rho$ and $\theta$ represent the friction intensity and the persistence level of the exogenous shock. As we show, both are necessary to produce persistent dynamics in the *growth rate* of y, even though persistence in levels can be achieved by either $\rho$ or $\theta$ alone.

Equation (21) can be rewritten as an infinite-order moving-average process and substituted back into (20). In this case, an ARMA process with a sufficiently high order for the MA parameter (q) yields a good approximation of (20). So we can approximate the specification above as:

$$y_t = \psi(L)\varepsilon_t \qquad (23)$$

$$\psi(L) = \frac{B(L)}{A(L)} = \frac{(1+\theta L + \theta^2 L^2 + \theta^3 L^3 + \ldots + \theta^q L^q)}{(1-\rho L)} \qquad (24)$$

The spectrum of y in levels is given by:[24]

$$s_y(\omega) = H(\omega)(2\pi)^{-1}\sigma^2 \qquad (25)$$

$$H(\omega) = \left| \frac{(1+\theta e^{-i\omega} + \theta^2 e^{-2i\omega} + \theta^3 e^{-3i\omega} + \ldots + \theta^q e^{-qi\omega})}{(1-\rho e^{-iw})} \right|^2, \quad i = \sqrt{-1} \qquad (26)$$

[24] See Hamilton (1994, Chapter 6) for details.

Where ω denotes frequency and the expression $H(\omega)$ is the transfer function of the ARMA (1,q) model. Equations (25) and (26) describe how the power (variance) of y is distributed across frequencies.

Increasing the persistence of a series will lead to humps at lower frequencies in its spectrum. This happens because shocks die out slowly and their accumulation across time dwarfs the importance of high frequency movements. Increasing ρ and/or θ will cause the spectrum of y in levels to shift to the left at higher peaks, eventually yielding the celebrated Granger's "typical spectral shape". But how does the spectrum of the growth rate of y change as the friction intensity (ρ) and the persistence of aggregate shocks (θ) vary? This question can be answered using the fact that the spectrum of the growth rate of a series can be obtained by applying a first-difference filter to its level:

$$s_{\Delta y}(\omega) = (2 - 2\cos(\omega))s_y(\omega) \qquad (27)$$

Now we can calculate the spectral densities for any pair (ρ, θ) satisfying the stationarity conditions in (22). To compute the percentage of the variance in the growth rate series that is explained by a frequency range $[\overline{\omega}_1, \overline{\omega}_2]$, we integrate over the relevant interval and then normalize by the total area beneath the spectrum:

$$PVE\Big|_{\overline{\omega}_1}^{\overline{\omega}_2} = \int_{\overline{\omega}_1}^{\overline{\omega}_2} s_{\Delta y}(\omega)d\omega \Bigg/ \int_0^{\pi} s_{\Delta y}(\omega)d\omega \qquad (28)$$

To illustrate how persistent dynamics (and more generally different influential frequency ranges) arise from the combination of shocks and frictions, we entertain a simple experiment. First we define three ranges of frequencies over the domain $[0, \pi]$: (i) high-frequency ($\omega \in [0.8,3.14]$), (ii) business-cycle frequency ($\omega \in (.19,0.78)$), and (iii) low frequency ($\omega \in (.19,0)$). For a quarterly series, these ranges correspond approximately to zero-to-two, two-to-eight, and over-eight-year ranges. Next, for each of these frequency ranges, we consider the value of (28) for all possible pairs (ρ, θ). This generates the surfaces presented in the appendix – figures 2.4 to 2.6 – capturing the percentage of the variance of the growth rate of y that can be explained by each of the three frequency ranges, as a function of the friction intensity (ρ) and the persistence of aggregate shocks (θ).

Figures (2.4) through (2.6) clearly show that a combination of positive friction and persistent shocks is necessary to generate persistent dynamics (or, equivalently, influential business cycle frequencies) in the growth rate of y.[25] One without the other is unable to command strong business cycle periodicities or substantial persistence in the Δy series.[26] The pictures indicate that if both elements of the pair (ρ, θ) are pushed too close to one, then influential periodicities start to fall into the low-frequency range, whereas if the elements of (ρ, θ) are not large enough, the high-frequency

[25] The figures where generated with 100 grid-points for ρ and 100 grid-points for θ. The MA parameter (q) adopted was 1000.

[26] An important underlying condition for the validity of this statement is that the internal propagation mechanism is represented by an AR1. If it was the case, like in Pries (20004), that the internal frictions called for an AR2 representation, then even in the absence of persistent exogenous shocks it would be possible to generate influential business cycle frequencies.

range dominates the spectrum. In other words, to generate influential business cycle frequencies in $\Delta y$, one must have a model that features significant internal frictions as well as persistent exogenous shocks.[27]

There is no doubt that the benchmark Mortensen-Pissarides model contains persistent exogenous shocks (since the aggregate component of productivity is assumed to be highly persistent), but the employment growth rate shows no persistence at all. Our canonical model offers guidance for interpreting this result by suggesting that the Mortensen-Pissarides model suffers from insufficient internal frictions. Although "the need for friction" in search models is a conclusion that can be found elsewhere in the literature, we provide here precise links between internal propagation mechanisms, driving forces and growth rates. In the next section we further discuss the lack of internal frictions within the Mortensen-Pissarides model, demonstrating how the "right" combination of persistence in aggregate productivity shocks and magnitudes of job-finding probabilities (capturing internal frictions) play a crucial role for replicating the persistence of employment growth observed in the data.

[27] It is important to mention that our canonical reduced-form model does not map directly into the corresponding employment equation of the Mortensen-Pissarides model. To put Mortensen-Pissarides into a canonical model, we would need to model employment as a Markov switching equation in order to account for time-varying coefficients triggered by fluctuating job-finding probabilities and reservation productivities. While more realistic, this modification would reduce tractability with no impact on the basic message of this subsection. Another point one could raise is that the symmetric growth rate used in the job flows literature has a transfer function that is different from the first-difference filter. We conducted simulation exercises in which we estimated the transfer function of the symmetric growth rate, and concluded that it behaves much like the standard first-difference filter.

## *2.5 Parameters and structure*

### 2.5.1 Can re-parameterization solve the problem?

We have shown that the lack of decoupling between job creation and destruction is the central element generating insufficient persistence of employment growth in the baseline model. Following up on this point, one could ask whether persistent fluctuations in employment growth can be achieved by re-parameterizing the model. We address this issue in the context of Cole and Rogerson (1999). The authors develop a reduced-form representation of the Mortensen-Pissarides model and then search for parameter values that allow the model to match certain stylized facts.[28] The gains from such a "non-structural approach" lie in the ability to search directly over key outcome variables of the model, such as job-finding probabilities and reservation productivities.

The reduced-form characterization proposed by Cole and Rogerson consists of equations for job creation, job destruction and unemployment. The job creation flow *(c)* is determined by:[29]

$$c_t = \alpha_{s_{t-1}}(1-n_{t-1}) \qquad (29)$$

[28] Cole and Rogerson try to replicate the main stylized facts regarding job flows articulated by Davis et al (1999): job creation that is more volatile than employment and less volatile than job destruction, strong positive autocorrelation in the series for job flows and employment, and negative contemporaneous correlation between creation and destruction.

[29] The model is setup in terms of the levels of flows. Before conducting spectral analysis we translate these flows into rates to obtain job creation and job destruction rates.

where $\alpha_s$ is the state-contingent job-finding rate, and *(1-n)* measures the number of unemployed workers. The job destruction flow *(d)* is given by:

$$d_t = (\delta_{s_t} + \phi\delta_0)n_{t-1} \qquad (30)$$

$$\phi = \begin{cases} 1 & if\ s_{t-1} = h\ and\ s_t = l \\ 0 & otherwise \end{cases} \qquad (31)$$

where $\delta_s$ is the state-contingent destruction rate (determined by state-contingent reservation productivities), and $\phi$ is an indicator variable that captures whether the economy has just moved from boom (*h*) to recession *(l)* - in which case there is an extra burst of job destruction.[30] Employment dynamics can be inferred from a combination of creation and destruction flows:

$$n_{t+1} = n_t + c_{t+1} - d_{t+1} \qquad (32)$$

Cole and Rogerson's preferred parameterization is:[31]

$$\{\alpha_l, \alpha_h, \delta_l, \delta_h, \delta_0, \pi\} = \{0.21, 0.30, 0.069, 0.044, 0.01, 0.2\} \qquad (33)$$

[30] In the Mortensen-Pissarides model, the distribution of jobs over idiosyncratic productivities changes depending on the histories of shocks and so the existence of a unique state-contingent $\delta$ is valid as an approximation only. Cole and Rogerson argue that, for sufficiently persistent aggregate shocks, the approximation is good.

[31] The variable $\pi$ denotes the probability of an aggregate state change.

The authors show that, under these values, the proposed reduced-form version of the Mortensen-Pissarides model is capable of replicating a broader set of business cycle facts characterizing US manufacturing than the parameterization presented in the original paper of Mortensen and Pissarides. In particular, the persistence of employment growth is now significantly larger and closer to the data – the first-order autocorrelation coefficient is estimated to be around 0.57 in the simulated data, compared to the empirical auto-correlation of 0.68.

This favorable result was achieved by simply "re-parameterizing" the model; no structural modification affecting the reduced-form representation was implemented.[32] Table 2.2 compares the original parameterization of Cole Rogerson (1999) and the parameterization implied by a 2-state version of the Mortensen-Pissarides model.[33]

Table 2.2

Cole and Rogerson (Original) versus Mortensen and Pissarides (2-state version)

| | $\alpha_l$ | $\alpha_h$ | $\delta_l$ | $\delta_h$ | $\delta_0$ | $\pi$ |
|---|---|---|---|---|---|---|
| Cole and Rogerson (reduced-form) | 0.12 | 0.30 | 0.069 | 0.044 | 0.01 | 0.2 |
| Mortensen and Pissarides (structural) | 0.55 | 0.65 | 0.054 | 0.045 | 0.067 | 0.06 |

[32] Here we are using the term "parameter" in a loose sense. The parameters in the reduced-form representation include true parameters in the structural model (like persistence of aggregate shocks) and endogenous variables in the structural model (job-finding probabilities and reservation productivities).

[33] To produce this last parameterization we applied the strategy described in footnote 17 to produce a 2-state model. The job-finding probabilities endogenously obtained in the structural model are reported in the table. The values for the δ's – which represent fractions of jobs destroyed in different states – are not directly implied by the model's equations; to retrieve them we (1) simulated long series of job creation, job destruction and employment and (2) calculated fractions of jobs destroyed as sample averages during good times, bad times (ignoring the first period after good-to-bad state changes to purge out initial destruction bursts), and the first periods after good-to-bad state changes.

Table 2.2 indicates that to obtain persistence in the employment growth rate, Cole and Rogerson reduce the job-finding rates ($\alpha_l$ and $\alpha_h$) as well as the one-shot burst in destruction following a change from the good to the bad aggregate state ($\delta_0$). This change of parameterization eliminates the spike in job creation right after the economy enters a recession implied by the baseline Mortensen-Pissarides model, both by reducing the initial surge in unemployment and by reducing the share of the newly unemployed who find jobs quickly. This point can also be seen in Figure 2.7, where we compare the two models' responses of job creation and job destruction rates to a permanent switch from the good to the bad state. In Mortensen and Pissarides, job creation shoots up so quickly that employment growth becomes positive in the period immediately following the bad shock, which explains the counterfactual negative auto-correlation of employment growth. In contrast, low job-finding rates together with a smaller initial burst of destruction causes the recovery of employment growth to be much slower in Cole and Rogerson's parameterization.

US data for unemployment spells seem hard to reconcile with the job-finding probabilities imputed by Cole and Rogerson. Indeed, the spells implied by the authors' parameterization are much longer than the ones observed in the data. Nevertheless, Cole and Rogerson argue that their numbers are reasonable if one takes into account transitions from out-of-labor-force directly into employment. If one were to count out-of-labor force workers as unemployed, the implied unemployment spells in the data would be more consistent with those implied by Cole and Rogerson's parameterization. Let's accept Cole and Rogerson's argument for now and use our decomposition formula to check whether the frequency content of their simulated series fits the data.

As shown in Figure 2.8, Cole and Rogerson's parameterization can successfully produce cyclical (or near-cyclical) decoupling of creation and destruction. However, the cospectrum also seems to reveal coupled dynamics at very low frequencies, which in turn washes out influential lower-frequencies in the job creation and job destruction series that would otherwise lead to a counterfactually large role for low frequency shocks to employment growth. The end result is a well-behaved overall spectrum for employment growth, but the "composition" of this spectrum is not entirely satisfactory. Having said that, it must be acknowledged that this shortcoming of the Cole-Rogerson parameterization is much less dramatic than the one observed in the original parameterization of Mortensen and Pissarides.

### 2.5.2 Spectral loss function: a new metric

In order to provide an assessment of Cole and Rogerson's specific choice of parameters from a frequency-domain perspective, we construct a measure of fit designed to capture which combinations of job-finding probability and persistence of aggregate shock can best match the joint behavior of the empirical spectra and cospectrum for job creation and job destruction. This measure is calculated for a limited set of parameter choices and without taking into account the mapping between reservation productivities and job-finding probabilities generated by the structural model.[34] Let $\hat{S}_j^d(\omega)$ denote the spectra of job creation (*j=jc*), job destruction (*j=jd*) or the cospectrum between the two (*j=jc,jd*) estimated from the data; and let $\hat{S}_j^m(\omega;\Omega)$ be the analogous measures estimated from simulated series of Cole and Rogerson's reduced form model. We experiment with different values of two parameters, contained in the vector $\Omega$: the job-finding probability in the low aggregate state ($\alpha_l$) and the probability of aggregate state change ($\pi$).[35] We define the following spectral loss function as our equation (34):

$$\Psi(\Omega)=\sum_{\omega=0}^{\overline{\omega}}\left(\hat{S}_{jc}^m(\omega;\Omega)-\hat{S}_{jc}^d(\omega)\right)^2+\sum_{\omega=0}^{\overline{\omega}}\left(\hat{S}_{jd}^m(\omega;\Omega)-\hat{S}_{jd}^d(\omega)\right)^2+\sum_{\omega=0}^{\overline{\omega}}\left(\hat{S}_{jc,jd}^m(\omega;\Omega)-\hat{S}_{jc,jd}^d(\omega)\right)^2$$

The upper limit $\overline{\omega}$ is set to 0.7854, which means that we are considering periodicities higher than two years. The loss function measures deviations between model and data spectra at each particular frequency up to $\overline{\omega}$ as a function of the parameter space $\Omega$, and it assigns equal weights to all three components of the employment growth rate spectrum. We plot the tri-dimensional surface generated by $\Psi(\Omega)$ as a function of the pair ($\alpha_l,\pi$).[36] Figure 2.9 shows that Cole and Rogerson's proposed parameterization

[34] In particular, the job destruction percentages are held fixed at Cole and Rogerson's values while we vary job-finding probabilities and levels of persistence of the aggregate state. Addressing these points would require one to solve for the endogenous relationship between reservation productivities, job-finding probabilities and persistence. We leave this for future work.

[35] The job-finding probability in the high aggregate state is set to be 50% higher than its low state counterpart. We relax this assumption latter.

[36] The window selection for estimating spectral densities from simulated series of Cole and Rogerson model was Hanning 40. Results were robust across window shapes. Regarding the window size, we found that increasing smoothness of the estimates tilted $\Psi(\Omega)$ in favor of more persistent aggregate shocks.

is in the "valley region" of the function, but that even lower job-finding probabilities or less persistent aggregate shocks could further reduce the value of $\Psi(\Omega)$.

To investigate further, we minimize the loss function $\Psi(\Omega)$ over the pair of job-finding probabilities $(\alpha_l, \alpha_h)$ keeping all other variables at Cole and Rogerson's original parameterization shown in Table 2.2 (including the persistence of the aggregate shock).[37] The results indicate that the optimum job-finding probabilities are 7.41 % in the low aggregate state and 10.46 % in the high aggregate state. These values can be used to generate series of job creation and destruction rates from which we can check conformity between data and model spectra. Figure 2.10 shows that, under these job-finding probabilities, the decoupling pattern is indeed closer to the data. Although a more definitive result on the "optimal parameterization" requires a more formal investigation, our results indicate that the reduced-form model's ability to reproduce the cospectrum shape from the data calls for even lower job-finding probabilities than found by Cole and Rogerson, implying even more unrealistically long unemployment spells. For these reasons we do not feel comfortable concluding that the persistence issue can be fully resolved by re-parameterizing the model, and so we are led to look into the structure of the Mortensen-Pissarides model for answers.

[37] This procedure is nothing more than a Simulated Method of Moments procedure (Gourieroux and Monfort (1996)), in which the "moments" we are trying to match are the empirical spectral densities. We are assuming the identity matrix as our weighing matrix.

### 2.5.3 Are there enough frictions in the Mortensen-Pissarides model?

In section 2.4, we used our canonical reduced-form model to argue that the lack of persistent growth rates even in the presence of highly persistent driving forces is a symptom of insufficient internal frictions. This view is corroborated by the fact that the reduced-form representation could achieve persistent employment growth fluctuations by reducing job-finding probabilities (a natural way to increase the role of internal frictions).

The problem is that unless we push job-finding probabilities down to unrealistically low levels (below the ones proposed by Cole and Rogerson), the Mortensen-Pissarides model misses the decoupling between job creation and destruction present in the data - which is quantitatively very important for employment fluctuations. In other words, it seems that the Mortensen-Pissarides model is in need of additional sources of friction. In Section 2, we pointed out that the Mortensen-Pissarides model pins down three endogenous variables in each state: the job-finding rate[38], the reservation productivity, and the level of unemployment. The first two are jointly determined by so-called *asset equations* (a free-entry condition and an optimal job destruction condition) and capture the rate at which unemployed workers escape unemployment, and the cut-off value for idiosyncratic productivity shocks below which it becomes optimal for a worker-firm pair to dissolve their employment relationship. The third endogenous variable is unemployment, determined *ex post* as a function of job creation and destruction.

[38] This variable is fully determined by the vacancy-unemployment ratio (v/u) given the assumptions on the matching function.

When the economy is hit by aggregate shocks, the current job-finding rate and reservation productivity adjust instantaneously to their new equilibrium levels. This happens because of two optimality conditions: (i) exhaustion of all vacancy rents (or free-entry) and (ii) no excess surplus on the lowest productivity job. Such conditions guarantee that firms will post/close vacancies up to the point where the return is zero, and will destroy any marginally non-optimal job immediately, so that the optimality conditions are always met. More importantly, this is true for every point in time even in the face of fluctuating aggregate states. In a sense, the model is completely frictionless when it comes to the determination of the job finding rate and the reservation productivity; changes in equilibrium values of these variables occur instantaneously. In contrast, unemployment is a sluggish variable and its convergence to new equilibrium levels is not instantaneous. This happens because job creation is probabilistic in nature, and so it takes time to fully absorb any changes in the job-finding probability caused by aggregate shocks.

Our empirical results can be interpreted as suggesting a need for richer dynamics in the determination of job-finding probabilities, which would be a way to introduce further frictions into the model.[39] Indeed, as shown in Figures 2.11 and 2.12, time series of creation and destruction produced by the Mortensen-Pissarides model or

[39] Allowing for such modification does not necessarily entail major changes in the modeling approach. Indeed, Pries (2004) provides such an example in which the introduction of learning effects creates the concept of job-finding probabilities for enduring matches (on top of the traditional period-by-period job finding probability). Alternatively, Fujita (2003) suggests the incorporation of vacancy creation lags in the Mortensen-Pissarides model to overturn the free entry condition which would then preclude job-finding probabilities from being "jump variables".

Cole and Rogerson's reduced-form representation do not mimic the data well, which can be viewed as a reflection of poor job-finding probability dynamics. While we may not have needed spectral techniques to reach this conclusion, such techniques are essential to quantify the importance of decoupling, and they provide a natural metric for assessing future changes to the model. Our results indicate that marked decoupling between creation and destruction is indeed crucial to matching fluctuations observed in the data, and that such decoupling poses a non-trivial challenge to the Mortensen-Pissarides model.

## *2.6 Concluding remarks and future work*

This paper applies frequency-domain tools to pinpoint why the Mortensen-Pissarides workhorse model is unable to generate employment growth persistence. By comparing model and data spectra, we conclude that a marked decoupling between job creation and job destruction at cyclical frequencies is the missing link. We then investigate the potential for model re-parameterization to solve the problem. Adopting a reduced-form representation proposed by Cole and Rogerson (1999), we conclude that replicating the decoupling pattern observed in the data is far from trivial even for re-parameterized versions of the Mortensen-Pissarides framework. In doing so, we propose a new metric for evaluating how well models can reproduce employment fluctuations observed in the data. Such a metric is based on a spectral loss function that measures the extent to which model and data spectra deviate from each other.

There are several directions for future work. A first is to incorporate structural features of the endogenous mapping between job-finding probabilities and the reservation productivities in the minimization of the spectral loss function or, even better, to optimize the loss function over deep parameters of the structural model. Although the latter is more elegant, computation complexities may lead us to pursue the former first. Another future extension is to incorporate richer dynamics for job-finding probabilities into the reduced-form representation proposed by Cole and Rogerson, and then evaluate their contribution to the cospectrum behavior. This would be in line with recent research by Fujita (2003), who allows for temporary violation of the free-entry condition in the Mortensen-Pissarides model by assuming vacancy creation lags.

A third extension would be to model richer dynamics for job-finding probabilities from first principles. Although creation lags in the spirit of Fujita (2003) can preclude the vacancy-unemployment ratio from adjusting instantaneously to aggregate shocks, Fujita does not model the phenomenon from first principles, and a deeper investigation into the dynamics of job finding rates could yield a high payoff.

# Chapter 3: Job reallocation dynamics and aggregate employment fluctuations in the US manufacturing: a Granger causality study across time-scales

## *3.1 Reallocation intensity and macroeconomics*

Plant-level studies for the US manufacturing show that job reallocation (measured as the sum of job creation and job destruction) is largely a within-sector phenomenon. This evidence has been rationalized as an indication that idiosyncratic shocks dominate aggregate ones along the reallocation process. As a result, there has been a renewed interest on models highlighting producer heterogeneity that are capable of accounting for diverse fortunes across businesses even when they belong to the same narrowly defined sectors. In this context, a question that comes to mind is: should macroeconomics as a discipline care about all of this - in other words, do models designed to explain aggregate fluctuations have to incorporate producer heterogeneity?

Macroeconomic literature certainly provides us with particular examples in which modeling underlying heterogeneity is crucial (the study of macroeconomics of incomplete markets, for one). Nonetheless, recent studies that emphasize the interplay between job reallocation dynamics and the business cycle (e.g. Caballero and Engel (1993) and Caballero, Engel and Haltiwanger (1997)) have put forth a much broader case for why modeling heterogeneity is indispensable for understanding aggregate fluctuations. A key issue in this debate is whether time-varying reallocation intensity actively influences aggregate fluctuations. While theoretical research has already identified possible channels through which reallocation intensity can affect cyclical dynamics, empirical studies are still trying to fully determine the connections between the two. This chapter fits into the latter category.

Our work has a similar motivation to Figura (1999), who uses plant-level data to ask whether reallocation is related to the cycle. Through a decomposition based on band-pass filtering techniques, he argues that "permanent reallocation accounts for approximately 30% of cyclical fluctuations in aggregate employment". In doing so, Figura pointed to the existence of so-called "frequency content" in the reallocation series and we exploit this same insight, though the technique we use and the precise question we consider are different. Our paper investigates whether there is Granger causality between reallocation and employment across different frequency bands (or time-scales to be more precise) using aggregate data for US manufacturing. In order to isolate frequencies of interest we resort to wavelet techniques, which present some methodological advantages for carrying out frequency-domain analysis of economic series.[40]

As is well known, a Granger causality test looks for temporal precedence between two variables. In our context, we are interested in examining whether lagged values

[40] See Gençay, Selçuk, and Whitcher (2002) and Schleicher (2002) for an accessible introduction to wavelets in economics. Other useful books are Percival and Walden (2000), Mallat (1998) and Vidakovic (1999). Recent papers using wavelets in Economics are Conway and Frame (2000), Ramsey (2002), Ramsey (1999), and Ramsey and Lampart (1998a, 1998b).

of reallocation have predictive power for aggregate employment (and vice-versa). While this approach does not provide information about “structural causation” between the variables, it can provide meaningful evidence to check the consistency of different theories. More importantly, we examine how (and if) the nature of this relationship changes across different frequencies. As an illustration of why this might be useful, consider for instance the possibility that high-frequency reallocation reflects adjustments of labor input that are mostly reactive to common shocks driving employment, and therefore does not play an important role in determining aggregate dynamics; while, in contrast, low-frequency reallocation captures a more fundamental restructuring behavior in gross job flows which indeed influences future aggregate employment dynamics. If this story was true, we would expect to find that reallocation does not Granger cause employment at high frequencies, but does at low frequencies.

The idea of using wavelets to test for Granger causality across different frequency bands has been suggested by Ramsey and Lampart (1998a,b) who studied the relationship between (i) money and income and (ii) money and expenses. In a sense, it is fair to say that this ides follows the notion of band-spectrum regression – Hannan (1963a,b) and Engle (1974) – which is well-suited to deal with a frequency-dependent relationship between variables. In the next section we provide a brief overview of the empirical approach adopted here, including an intuitive presentation of the multi-scale decomposition achieved via wavelets.

## *3.2 Empirical methodology*

### 3.2.1 Wavelets: an overview

Traditional spectral analysis techniques – see Priestley (1981) – allow us to decompose the power (variance) of a series across different frequencies. While such information may provide useful insights, it leads to what is known as "complete loss of time resolution". In other words, to obtain information about the power distribution of a series, we surrender the ability to locate in time the role of different frequency components. Additionally, to the extent that conventional frequency-domain methods rely on Fourier Transforms and require (second-order) stationarity of the underlying data, the contribution of different frequency components is not supposed to vary over time. Wavelet techniques can handle these problems – that is, they can provide us with frequency information that is localized in time, and they are able to handle non-stationary data.

We use wavelets in this chapter to generate time-series of reallocation and employment that are driven by different frequency components. Mechanically, this task can be accomplished within the realms of conventional frequency-domain methods by resorting to band-pass filters, but if the chosen technique relies (as most do) on Fourier Transforms then the assumption that the frequency components of a series are stable across time is necessary. So wavelets provide a more general framework for time-frequency decompositions of an economic series since it can naturally accommodate more fundamental changes in its dynamics.[41]

To accomplish such flexibility, wavelets rely on bases of finite length (Figure 3.1 display one basis), in sharp contrast to the sine and cosine bases of the Fourier transform which run from minus to plus infinity. A generic basis called a "mother wavelet" – which we denote by $\psi_{u,s}(t)$ - is chosen and can be stretched/compressed to capture cycles of different frequencies, and shifted from the beginning to the end of a series to provide time localization. Stretching/compression is determined by a scale parameter and time position is determined by a location parameter – these correspond to s and u, respectively, in the expression for the "mother wavelet" below. Wavelet coefficients characterizing a particular series x(t) are obtained through the following projection – also known as the continuous wavelet transform (CWT):

$$W(u,s) = \int_{-\infty}^{\infty} x(t)\psi_{u,s}(t)dt \quad (1)$$

The wavelet coefficients W(u,s) will be relatively large if the series x(t) has an important component of scale s at location u. More importantly, these coefficients can be used to reconstruct the series x(t) at different scales by using the inverse continuous wavelet transform. At low scales (small values of s), the corresponding wavelets are very compressed and so they are appropriate for measuring rapidly

[41] On the downside, wavelets are a fairly recent technique and, although they have strong mathematical foundations, the statistical properties are still being developed.

changing movements of the series (high frequencies). The opposite is true for high scales where a stretched wavelet captures slow-moving changes (low frequencies).

Figure 3.1

Symlet12 Wavelet

### 3.2.2 Multi-scale decomposition

Although, the CWT generates coefficients corresponding to all possible scales and positions, there are computational advantages in working with a finite subset of sample points taken from the (u,s) space. Such points are obtained by critical sampling of the two-dimensional space (u,s) and they can be used to generate the discrete wavelet transforms (DWT). Theoretical results have established the sampling procedures to be followed such that accuracy in reconstructing the series is assured when performing an inverse DWT. One of the consequences of such procedures is that our scale decomposition will follow a dyadic (power of two) pattern – that is, for quarterly data, our scales will be 2 quarters, 4 quarters, 8 quarters, 16 quarters, and so on.

We will use the DWT to achieve the following multi-scale decomposition of a time series $x_t$:

$$x_t = D1_t + D2_t + D3_t + D4_t + D5_t + A5_t \qquad (2)$$

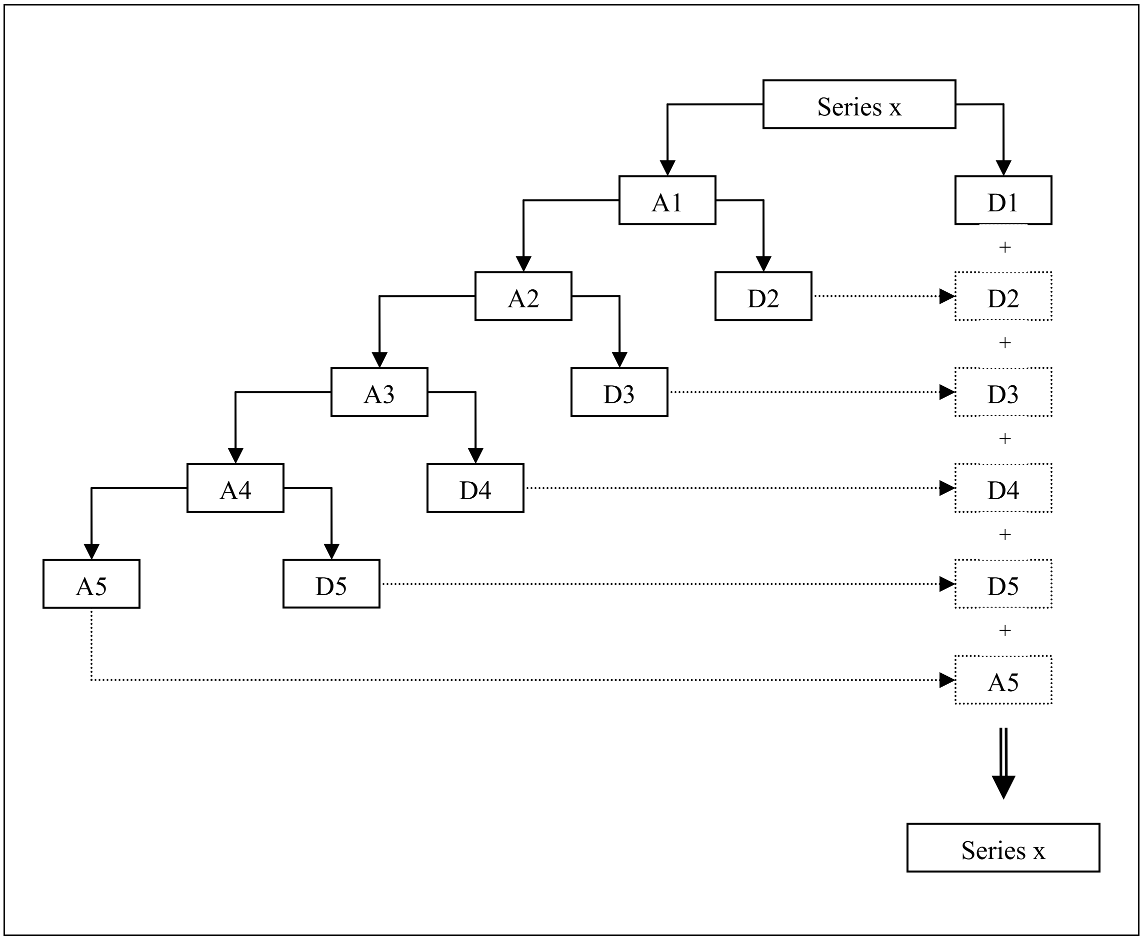
The series $D1_t$ denotes the first-level wavelet detail of series $x_t$, which captures movements associated with scale 1 – for quarterly data, scale 1 goes from $2^1$ to $2^2$ quarters (six months to one year). The second-level wavelet detail $D2_t$ captures changes associated with the range $2^2$ to $2^3$ quarters (one to two years). Wavelet details $D3_t$ (two to four years), $D4_t$ (four to eight years), and $D5_t$ (eight to sixteen years) are defined analogously. The series $A5_t$ denotes the wavelet approximation of level 5, and provides a smooth series describing the low-frequency behavior of $x_t$ – it basically adds up all other remaining wavelet details greater than 5.

The decomposition is implemented through a pyramid algorithm due to Mallat (1989), based on the idea of sequential filtering of the wavelet approximation component – see Figure 3.2. Basically, one starts with the original series $x_t$ and then performs a single-level discrete wavelet transform to write $x_t$ as $D1_t + A1_t$. Next, another single-level discrete wavelet transform is applied to $A1_t$ so that we obtain $A1_t = D2_t + A2_t$. The process goes on until we reach $A5_t$.[42]

[42] Actually, the sequential filtering of wavelet approximations can be repeated up to J times – where J= $\log_2$(Sample Size). We chose to stop at scale 5 which already deals with fairly low frequency components (eight to sixteen years) for an economic series. In order to keep the exposition straightforward we are omitting from the figure 3.2 the downsampling procedure involved in the algorithm. Interested readers should refer to Mallat (1989) or Gençay et all (2002) for a discussion about downsampling as well as for a formal exposition of the concepts.

Figure 3.2

Sequential filtering and the Pyramid Algorithm




When implementing a DWT, it is first necessary to decide which wavelet basis to use. There are many possibilities that vary in terms of shape (Haar, Daubechies, Symlet, etc) and length (longer wavelet bases display a higher number of oscillations). We follow Ramsey and Lampart (1998), who picked a Symlet (12) on the grounds that it "is an intermediate choice in that it has reasonably narrow, compact support, is fairly smooth, is nearly symmetric, and has a moderate degree of flexibility". Our results were robust to the choices of Daubechies (4) and Daubechies (12) as well.[43]

3.2.3 Granger-causality tests across time-scales

Besides wavelets, this chapter also relies upon another empirical methodology: bivariate Granger-causality tests. Let $x_t$ and $y_t$ be the time series of interest and assume they are covariance stationary.[44] To implement the aforementioned test we first estimate a VAR as follows:

$$x_t = \alpha_0 + \sum_{i=1}^{k} \alpha_{x,i} x_{t-i} + \sum_{i=1}^{k} \alpha_{y,i} y_{t-i} + \varepsilon_{x,t} \qquad (3)$$

$$y_t = \beta_0 + \sum_{i=1}^{k} \beta_{x,i} x_{t-i} + \sum_{i=1}^{k} \beta_{y,i} y_{t-i} + \varepsilon_{y,t} \qquad (4)$$

where the disturbances $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ are assumed to be independent white noise processes. Using an F-test we can check whether the following null hypotheses are rejected or not:

$$H_0: \quad \alpha_{y,1} = \alpha_{y,2} = \alpha_{y,3} = \ldots = \alpha_{y,k} = 0 \qquad (5)$$

$$H_0: \quad \beta_{x,1} = \beta_{x,2} = \beta_{x,3} = \ldots = \beta_{x,k} = 0 \qquad (6)$$

[43] We used the method of symmetrization to address the problem of border distortion that naturally arises when one tries to filter a signal of finite length.
[44] If the series are not stationary, one can still perform Granger causality tests using first-differenced series together with an error correction term – see Lutkepohl (1993).

The first null is that y does not Granger-cause x, while the second one asserts that x does not Granger-cause y. If both hypotheses are rejected, then we say to have found evidence of a feedback relationship between x and y (that is, Granger-causality runs in both directions); if only one of the null hypotheses is rejected, we say that x (or y) Granger-causes y (or x); and if neither hypothesis is rejected, the test is inconclusive. As mentioned in the introduction, the concept of Granger-causality captures whether lagged values of one variable have predictive power (in the sense of reducing the mean square error of forecasts) over the other and should not be viewed as an indication of structural causation per se. However, the test can still provide important insights into the behavior of economic variables. Moreover, given the multi-scale decomposition discussed in the previous section, we can apply the Granger-causality test between corresponding wavelet details belonging to different series, exploiting richer implications. We now turn to those issues.

### *3.3 Reallocation and aggregate fluctuations: results*

We work with quarterly seasonally-adjusted job creation and job destruction rates for US manufacturing from 1947 to 1993 (188 observations) as compiled by Davis and Haltiwanger (1999). The job creation rate is defined as the sum of employment gains at the plant-level normalized by average employment at all plants in the current and previous periods, whereas the job destruction rate is defined as the absolute value of employment losses at the plant-level, also normalized by average employment in the current and previous periods. Variables of particular interest for our purposes are the job reallocation rate (the sum of the job creation and job destruction rates) and the net employment growth rate (the job creation rate minus the job destruction rate). We start by applying the multi-scale decomposition described in the previous section to these two variables.

Figure 3.3 shows the decomposition for the employment growth rate. The first-level wavelet detail (D1) is associated with sudden changes in the series and roughly isolates periods of recession (peak to trough) in the US economy by means of clear spikes during such periods. Details 2 through 4 are associated with typical business cycle frequencies fluctuations (1 to 8 years) and they drive much of the volatility of the employment series. Detail 5 and approximation 5 are much less influential, and they indicate that low-frequency components are not important for the dynamics of employment. These conclusions change when we consider the reallocation series. In fact, approximation 5 indicates a trend toward lower levels of reallocation overtime and detail 5 reveals that frequencies between 32 and 64 quarters are important for the series. Additionally, details 2 through 4 provide evidence that business cycle frequencies do command important fluctuations in reallocation; while detail 1 also isolates recession periods.

Figures 3.3 and 3.4 also reveal differences between recession episodes in the 70's and the early 80's in the US economy. The wavelet details 3 and 4 associated with the reallocation series indicate that the recessions of the 70's generated a "deeper" process of adjustment than the recession of the early 80's. Such conclusions are sensible in light of the permanent restructuring caused by oil price shocks in the 70's. At the same time, it is interesting to note that the difference between the two recessions cannot be readily seen when one considers the employment series alone. Such contrast can be construed as another example of the advantages associated with looking at gross flows.

The multi-scale decomposition also allows us to break the reallocation and employment series into frequency-specific components and check for Granger-causality between corresponding wavelet details of different scales. A priori, one could imagine that, at high frequency bands (low scales), reallocation reflects a noisy process in which firms make fast-paced decisions about their labor inputs that do not affect aggregate net dynamics; while, at low frequency bands (high scales), reallocation captures a more fundamental reorganization of production activity which affects the evolution of net flows dynamics. The key idea here is to look for overlaying relationships across frequency-bands that may be buried in the original series.

Before implementing the causality test, we check for non-stationarity in our series by applying Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. The Phillips-Perron test allows for possible heteroskedasticity of disturbances through Newey-West standard errors. Lag selection in the ADF test is achieved by checking the auto-correlation function (ACF) for lack of serial correlation. The tests are specified with a constant and without a deterministic trend, except for the original reallocation series, in which case we include a deterministic trend. The results lead us to reject the null of non-stationarity for all series – see table 3.1 in the appendix for details.

The next step is to determine the number of lags to include in the vector auto-regressions used to test for Granger-Causality. The Schwartz Bayesian information criterion (SBIC) was adopted for this purpose. Given the concern that the SBIC could generate "excessively parsimonious" lag order specifications in some cases, we also ran a Lagrange-multiplier test to make sure that there was no auto-correlation left in the residuals of the estimated VAR's. Table 3.4 reports results from Granger-Causality tests between reallocation and employment performed at different time-scales. Tables 3.2 and 3.3 in the appendix provide more detailed information on the estimation results.

Table 3.4
Granger causality across time-scales

| SERIES | GRANGER-CAUSALITY | SIGNIFICANCE LEVEL |
|---|---|---|
| Original | NET ↔ REL | 5 % |
| Wavelet Detail 1 (D1) | NET → REL | 1 % |
| Wavelet Detail 2 (D2) | NET ↔ REL | 1 % |
| Wavelet Detail 3 (D3) | NET ↔ REL | 5 % |
| Wavelet Detail 4 (D4) | NET ↔ REL | 1 % |
| Wavelet Detail 5 (D5) | REL → NET | 1 % |

The first row of results indicates that the original series of employment and reallocation display a feedback relationship – that is, we reject the null that reallocation does not Granger-cause employment as well as the null that employment does not Granger-cause reallocation. Interestingly, the feedback relationship between the two series does not hold across all different frequency bands. At high frequencies (first wavelet detail) we find that employment Granger-causes reallocation; while at low frequencies (fifth wavelet detail) the reverse is true. Nonetheless, the feedback relationship between reallocation and employment holds at intermediate frequency bands (second to fourth wavelet details), which capture typical business cycle frequencies. Hence, the results isolate "eight-to-sixteen years" as a frequency-band in which reallocation dynamics have predictive power over aggregate dynamics (and not the other way around). These results are consistent with theories emphasizing a prominent role for permanent and long-termed gross job flows, and the evidence uncovered suggests that wavelets were successful in teasing out overlaying relationships contained in the original employment and reallocation series.

### *3.4 Concluding remarks and Extensions*

In this chapter we investigated the causality relationship between aggregate employment fluctuations and job reallocation. While a Granger-causality test indicated a feedback relationship between the two variables, wavelet techniques allowed us to isolate frequency-bands in which such causality goes in one direction only. Moreover, we argued that our findings are consistent with theories emphasizing the role of reallocation dynamics in driving aggregate fluctuations.

A natural extension of this chapter is to combine the wavelet details and approximation obtained with a structural vector auto-regression (SVAR) approach, rather than running Granger-causality tests. By doing so, we would be able to trace movements in the job flows series back to hypothetical structural shocks “more easily” by exploiting identifying restrictions that operate in different frequency-bands.

# Chapter 4: Productivity growth and the frequency content of job reallocation: a cross-section study of US manufacturing industries.

## *4.1 Productivity growth and reallocation*

The relationship between reallocation and productivity growth has been the focus of recent attention in macroeconomics.[45] Many studies have examined the sources of productivity growth by considering decomposition exercises using plant-level data. In contrast, this chapter takes on the issue at a higher level of aggregation and, like Schuh and Triest (1998), capitalizes on the opportunity to combine 4-digit level LRD data on job flows with the NBER productivity database in order to investigate the connection between productivity growth and reallocation. Schuh and Triest's conclusion is that "increased reallocation normally is not correlated with increased trend productivity". The authors also note that their evidence "does not conform well to theories that posit improvements in investment and productivity through creative destruction channels". This work revisits the issue of productivity performance and reallocation from a frequency-domain perspective.

[45] References include Dunne, Roberts, and Samuelson (1989); Baily, Hulten, and Campbell (1992); Dunne, Haltiwanger, and Troske (1996); Baily, Bartelsman, and Haltiwanger (1996); Olley and Pakes (1996); Liu and Tybout (1996); and Foster, Haltiwanger, and Krizan (1998).

We first use disaggregated LRD data to estimate the spectral densities for the total reallocation series of each 4-digit industry in US manufacturing. Then, we manipulate the estimated spectra to generate industry-specific measures of the relative importance of low and high frequencies for the fluctuations of the reallocation series. These measures are then combined with industry-specific labor productivity growth and other control variables, calculated from the NBER productivity database as averages over the same period used to estimate the spectral densities. Then, through OLS regressions, we investigate whether labor productivity growth and our reallocation indices are correlated.

The motivation underlying our empirical approach comes from the idea that reallocation has a distinct frequency content that can be mapped into existing theories. For example, the hypothesis of technological sclerosis put forth by Caballero and Hammour (1996) says that contracting inefficiencies between capital and labor may cause excessively slow restructuring. More importantly, the notion of inefficiently slow restructuring can be cast in the frequency-domain as the reallocation series being driven by sub-optimally low frequencies. Hence, a sensible implication to be tested is whether industries with relatively more influential low frequencies display lower productivity growth.

An additional link between our testing approach and the literature relates to the dichotomy between short- and long-term job flows featured in many macroeconomic models. Models of permanent job flows highlight the role of productivity-enhancing and protracted reallocation that is related to the restructuring of assets.[46] In contrast, models emphasizing short-term job flows feature reallocation mainly as a process of accommodation to temporary shocks. This class of models focuses on changes in utilization of plant-assets (as opposed to restructuring of assets) which are not naturally associated with improvements in productivity. If this dichotomy really holds, then one would expect to see industries with a greater share of their reallocation driven by lower-frequencies to experience greater productivity growth. Existing empirical evidence does not support this view, however. As mentioned before, Schuh and Triest (1998) conclude that "increased reallocation normally is not correlated with increased trend productivity" and our results indicate that, if anything, labor productivity growth is *negatively* correlated with how influential low-frequencies are in the behavior of reallocation. Our results are then consistent with the phenomenon of technological sclerosis. Next we discuss in detail our three proposed measures for assessing the frequency content of the reallocation series. The three measures are designed to capture the same basic idea; working with all three adds to the reliability of our results, and allows for alternative measurement strategies.

### *4.2 Measuring the importance of low frequencies*

#### 4.2.1 Data

We work with seasonally-unadjusted quarterly job flows series from 1973 to 1993 (83 observations across time) at the 4-digit level constructed by Davis et al. (1996). As usual, the job creation rate is defined as the sum of employment gains at the plant-

[46] For instance, in Caballero and Hammour (1996), the restructuring process involves getting rid of outdated technologies and, in Mortensen and Pissarides (1994), restructuring implies the termination of low-productivity job sites.

level normalized by average employment in current and previous periods, whereas the job destruction rate is defined as the absolute value of employment losses at the plant-level also normalized by average employment in current and previous periods. We also use the NBER manufacturing productivity database, which provides annual information on 450 manufacturing industries (4-digit-level) from 1958 to 1996.[47] The wealth of information provided by this database allows us to control for several industry-specific factors during the regression exercises.

### 4.2.2 Reallocation index one

This measure is based on the normalized cumulative spectral distribution associated with the total reallocation series of each industry. The idea is that if the cumulative spectral density sharply increases at low frequencies, then fluctuations in reallocation have a more long-term nature. We call this measure reallocation index one (RI_1); it is calculated in a three-step procedure:

<u>Step one:</u> Estimate the normalized cumulative spectral distribution associated with the total reallocation series of each industry *i*.

$$\Omega_i(\omega) = \left( \int_0^{\omega} \hat{S}_{NP_i}(\omega) dw \right) \left( \int_0^{\pi} \hat{S}_{NP_i}(\omega) dw \right)^{-1}, \quad \omega \leq \pi \qquad (14)$$

The first term on the right-hand-side is the cumulative integral of the spectrum. The more low frequencies are important for a series, the higher the value of its cumulative

[47] See Bartlesman and Gray (1996) for technical documentation.

integral for small $\omega$'s. In contrast, for series in which low frequencies are not influential, one would expect this term to be close to zero for low values of the angular frequency. The second term in parentheses normalizes the cumulative spectrum by the total area beneath the spectrum. This forces $\Omega_i(\omega)$ to take values in [0, 1] and allows for comparisons across industries with different reallocation variances. Figure 4.1 illustrates the case in which reallocation in industry *i* is driven by lower frequencies than in industry *j*.

Step two: Integrate the normalized cumulative spectral densities from the zero frequency to a cut-off value $\overline{\omega}$, chosen to minimize the influence of seasonal effects.

$$M_i(\overline{\omega}) = \int_0^{\overline{\omega}} \Omega_i(\omega) d\omega, \quad \overline{\omega} \leq \pi \qquad (15)$$

We choose $\overline{\omega}$ to be 1.21 (approximately five quarters).[48] Note that, even though we determine a cut-off value for seasonal effects, our approach does not require establishing cut-offs between cyclical and permanent fluctuations (which is certainly much more problematic to do). Indeed, up to a linear trend, we consider the entire mass of frequencies below seasonal fluctuations that command variation in the series, treating cyclical and permanent fluctuations jointly.

[48] The use of windowing schemes in non-parametric estimation may lead to what has been called "leakages through the edges of the window". This problem may cause frequencies in the neighborhood of four quarters to pick up seasonal effects, so we pick 5 quarters as the cut-off. Inspection of the estimated spectra suggests that this cut-off value corresponds to frequencies not substantially affected by leakage.

It is important to stress that the area beneath the cumulative integral up to $\varpi$ conveys different information from the value of $\Omega_i(\varpi)$. In particular, even if two sectors possess identical values of $\Omega_i(\varpi)$, one of them may accumulate low frequencies at a faster rate than the other. Indeed, our index is capable of capturing how fast (in comparison to other sectors) a particular cumulative integral "takes off".[49] One problem is that industries experience different seasonal effects – that is, they have different $\Omega(\varpi)$ - and this will distort the measure $M_i(\varpi)$. In fact, those industries in which seasonal frequencies are very important (in the food sector, for instance) present a cumulative spectral distribution that is relatively flat up to the cut-off value $\omega = \varpi$, and so $M_i(\varpi)$ will be naturally low - as illustrated by industry j in figure 4.2. The third step addresses this issue.

<u>Step three:</u> Control for heterogeneous seasonal frequencies by regressing $M_i(\varpi)$ on a polynomial in $\Omega_i(\varpi)$. The estimated residuals of this regression form RI_1.

$$\underbrace{\begin{bmatrix} RI\_1_1 \\ RI\_1_2 \\ \dots \\ RI\_1_I \end{bmatrix}}_{Ix1} = \underbrace{\begin{bmatrix} M_1(\varpi) \\ M_2(\varpi) \\ \dots \\ M_I(\varpi) \end{bmatrix}}_{Ix1} - \underbrace{\begin{bmatrix} 1 & \Omega_1(\varpi) & (\Omega_1(\varpi))^2 & \dots & (\Omega_1(\varpi))^p \\ 1 & \Omega_2(\varpi) & (\Omega_2(\varpi))^2 & \dots & (\Omega_2(\varpi))^p \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \Omega_I(\varpi) & (\Omega_I(\varpi))^2 & \dots & (\Omega_I(\varpi))^p \end{bmatrix}}_{Ix(p+1)} \underbrace{\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \dots \\ \hat{\beta}_p \end{bmatrix}}_{(p+1)x1} \qquad (16)$$

[49] Measuring the importance of low frequencies by comparing "how soon" is the cumulative spectral density take-off works exactly only if densities that take-off early do not cross others later on. If that is not the case, then the argument will still hold as an approximation as long as the area "re-gained" by integrals taking-off later is not significant. We ran several checks to see whether this was the case and we found that, up to the seasonal frequency, the distortion introduced by "crossing" is minor.

Where $\hat{\beta}_{(p+1)x1}$ is a vector of OLS estimates obtained from a cross-industry regression of $M_i(\overline{\varpi})$ on powers of $\Omega_i(\overline{\varpi})$. Note that, because OLS can be viewed as imposing a moment condition of orthogonality between regressors and the residuals, *RI_1* is uncorrelated with the seasonal effects $\Omega_i(\overline{\varpi})$. The total number of industries (*I*) in the regression is 389.[50]

#### 4.2.3 Reallocation index two

Our second measure *RI_2* is similar to the previous index, but deals with the issue of heterogeneous seasonal effects in a different manner. Instead of trying to purge seasonals through a polynomial regression, we normalize the cumulative spectral distributions (second term on the right-hand-side of equation (14)) by their respective non-seasonal variance rather than the total variance – that is, we integrate over the range: $0 < \omega < \overline{\varpi}$, where $\overline{\varpi} = 1.21$. This will force the normalized cumulative distributions to assume the value one at $\omega = \overline{\varpi}$ for all industries, eliminating the need to correct for differential seasonal effects. The index can be calculated in two steps.

<u>Step one:</u> Estimate the normalized cumulative spectral distribution associated with the total reallocation series of each industry *i* (but now under a new normalization as shown in Figure 4.3):

[50] Some sectors were dropped due to incomplete job flows series.

$$\Omega_i(\omega) = \left( \int_0^{\omega} \hat{S}_{NP_i}(\omega) dw \right) \left( \int_0^{1.21} \hat{S}_{NP_i}(\omega) dw \right)^{-1}, \quad \omega \leq 1.21 \qquad (17)$$

Where the interpretation is analogous to step one of *RI_1*.

Step two: Integrate the normalized cumulative spectral densities from the zero frequency to a cut-off value $\overline{\omega}$:

$$RI\_2_i = \int_0^{\overline{\omega}} \Omega_i(\omega) d\omega \qquad (18)$$

Where $\overline{\omega}$ is again equal to 1.21.

#### 4.2.4 Reallocation index three

In contrast to the first two, the third measure assumes a frequency cut-off (8 years) separating permanent from cyclical fluctuations, and so does not treat the whole mass of frequencies jointly. This measure equals the ratio between the percentage of variance (of the reallocation series) explained by frequencies lower than eight years and the percentage explained by frequencies between eight and two years. This index captures the relative importance of low frequencies for fluctuations in reallocation, but it relies on an arbitrarily chosen frequency threshold. The index is calculated in two steps:

Step one: Estimate the normalized cumulative spectral distribution associated with the total reallocation series of each industry *i*.

$$\Omega_i(\omega) = \left( \int_0^{\omega} \hat{S}_{NP_i}(\omega) dw \right) \left( \int_0^{\pi} \hat{S}_{NP_i}(\omega) dw \right)^{-1}, \quad \omega \leq \pi \qquad (19)$$

This step is identical to the first one used in the calculation of *RI_1*.

Step two: Calculate *RI_3* as the ratio between the percentage of variance explained by low and high frequencies based on a cut-off point of eight years.

$$RI\_3_i = \frac{\Omega_i(\overline{\omega}_a) - \Omega_i(\overline{\omega}_b)}{\Omega_i(\overline{\omega}_c) - \Omega_i(\overline{\omega}_a)} = \frac{\Omega_i(\overline{\omega}_a)}{\Omega_i(\overline{\omega}_c) - \Omega_i(\overline{\omega}_a)} \qquad (20)$$

Where $\overline{\omega}_a$ = 0.2271, $\overline{\omega}_b$ = 0, and $\overline{\omega}_c$ = 0.8327. These numbers correspond to an approximate partition of frequency ranges into (i) frequencies lower than eight years (the numerator) and (ii) frequencies between eight and two years (the denominator). An industry displaying a high *RI_3* is one in which low frequencies are relatively more influential. Figure 4.4 illustrates a situation in which industry *i* will have a higher RI_3 than industry *j*.

## *4.3 Empirical results*

### 4.3.1 Productivity growth and the frequency content of reallocation

As mentioned before, the three measures are similar in spirit. We choose to work with all of them in order to check the robustness of our results and because it is not clear which measurement strategy is the best. It is our belief, though, that the first two measures are superior because they do not require an arbitrarily set threshold. One may argue, for instance, that the threshold for what should be considered "permanent fluctuations" may vary across industries. At the same time, the third measure is a natural one to consider since the practice of assuming cut-off frequencies is common in filtering. We can learn more about how the reallocation indices relate to each other by looking at their correlation matrix. As shown in Table 4.1, our last measure RI_3 is the most dissimilar among the three, even though all the reallocation indices display strong positive correlation among themselves.

Next we investigate the connection between industry-specific measures of low-frequency reallocation and productivity growth. As a starting point, we report simple correlation coefficients between productivity growth and our reallocation indices in Table 4.2. The correlation coefficients are all negative and statistically significant. Next, we include industry-level controls available from the NBER Manufacturing Productivity Database. Table 4.3 reports OLS results from estimating the following specification:

$$Labor\_prod\_growth_{sector\ i} = const + \gamma RI_{sector\ i} + \Lambda\, controls_{sector\ i} + \varepsilon_{sector\ i} \quad (21)$$

Before we address the results of the regressions, it is worthy to discuss the issue of which controls to use. Our indices capturing cross-industry differences in the frequency content of the reallocation series are supposed to proxy for permanent reallocation affected by underlying inefficiencies. Clearly, permanent reallocation and low-frequency changes are very different things and we have no hope to use the latter as an absolute measure of the former even after controls. However, we do believe that it is possible to use our indices as *relative* measures of permanent reallocation as long as we account for differential factors across industries.

For example, it is reasonable to expect that industries will have at least portion of its reallocation motive associated to the schedule of replacement/updating of their capital. To the extent that industries with higher capital intensity follow a slower schedule of replacement/updating than industries with lower capital intensity, this effect will show up in their spectral density and we would like to control for it. Unfortunately, the list of candidate controls may be quite long. Ideally, we should have a theoretical model spelling out the control variables in our context. While later in the chapter we present a simple model that helps to organize some of our empirical results, we leave for future work this task. Hence, in addressing the issue of which controls to use, we start with a minimal specification (table 4.2) and then proceed to add variables that could potentially capture differential factors such as the one listed above. We report simple correlation coefficients (table 4.2) and a more elaborate specification (table 4.3). The coefficient on the reallocation index was robust to more parsimonious specifications.

Table 4.3 indicates that, even after controlling for several industry-specific features, the negative association between productivity growth and our reallocation indices persists.[51] It is interesting to note that the total level of reallocation, though positively related to productivity growth, is not significant in any regression (and this is the case even if we run the regressions without the reallocation indices). This finding is consistent with the conclusions of Schuh and Triest (1998), who find no evidence linking reallocation and productivity. Although their empirical strategy is different from ours, we may say that our results confirm their message that protracted reallocation is not synonymous with superior productivity performance. Indeed, we find that, if anything, industries with relatively more influential low frequencies in the reallocation series tend to display lower productivity growth.

We are not suggesting that low-frequency-driven reallocation processes cause poor productivity growth. A number of factors (such as demand fluctuations, sector-specific technological change, market structure or the regulatory environment) may affect our indices and we make no effort to disentangle this mix. What the results do suggest is the existence of a more complex relationship between reallocation and

[51] The results were derived using a Hanning window with length 20. They were robust across alternative window shapes (Blackman and Bartlett), but different window sizes were found to have some influence on the p-value of $\gamma$, though never reversing its sign. In particular, the relationship seems to be stronger for smoother spectra (generated by a narrow lag window or, equivalently, a wide spectral window) than for high resolution spectra (generated by a wide lag window or, equivalently, a narrow spectral window). Inspection of estimated spectra revealed that window sizes out of the 20 to 30 range would either generate very jagged spectra with larger variance (long lag windows) or very smooth spectra with non-informative wide humps (short lag windows).

productivity than would be suggested by a pure cleansing view that permanent reallocation is necessarily productivity-enhancing. Additionally, our results are consistent with the phenomenon of technological sclerosis as described by Caballero and Hammour (1996), according to whom contracting inefficiencies between capital and labor can yield an excessively slow pace of restructuring. Indeed, if this is the case, one would expect the reallocation series to be driven by sub-optimally low frequencies (or, alternatively, sub-optimally long periodicities) and the negative association between productivity growth and our indices could be explained.

#### 4.3.2 Synchronization: a further look into inefficiencies

So far we argued that, under technological sclerosis, low-frequency components of the reallocation series would reflect the sub-optimally slow pace of restructuring. The reallocation indices we proposed were able to quantify the relative importance of low frequencies and we found that they were negatively correlated with productivity growth, a result that is consistent with technological sclerosis. In this sub-section we further exploit spectral concepts to investigate whether inefficiencies play a role along the reallocation process.

According to Caballero and Hammour (1996), technological sclerosis refers to excessively slow renovation caused by inefficiencies in the contracting environment involving capital and labor. The authors argue that an additional implication of such inefficiencies is that the creation and destruction margins become decoupled and unsynchronized over the cycle as the economy no longer concentrates restructuring at times of recession (when the opportunity cost is low). We use the concept of phase shift, which captures synchronization between any two series, to investigate whether less synchronized creation and destruction is associated with lower productivity growth.

Using notation from equations (9) and (10) in chapter 1, the phase shift between two series X and Y can be calculated by using the cospectrum $c_{xy}$ and quadrature spectrum $q_{yx}$ as follows:

$$Phase\_Shift(\omega) = \left[ \tan^{-1}\left( \frac{q_{yx}(\omega)}{c_{xy}(\omega)} \right) \right] \Big/ \omega \quad (22)$$

Whenever the phase shift is different than zero, then the series Y and X are not fully synchronized at that particular frequency – that is, one series is either leading or lagging the other. More importantly, if Y and X are job creation and job destruction rates, then we can derive a measure of synchronization between the two margins from their estimated multivariate spectrum and investigate its association with productivity growth along the same lines of the previous section. We consider two measures based on the phase shift concept:

$$Unsync\_1 = \left( \sum_{\omega=0}^{\bar{\omega}_1} \left| Phase\_Shift(\omega) \right| \right) \Big/ I_1 \qquad (23)$$

$$Unsync\_2 = \left( \sum_{\omega=0}^{\overline{\omega}_2} \left| Phase\_Shift(\omega) \right| \right) \Big/ I_2 \qquad (24)$$

Where $I_1$ and $I_2$ denote, respectively, the number of frequencies in the sequence $\{0,...,\overline{\omega}_1\}$ and $\{0,...,\overline{\omega}_2\}$. In words, equations (23) and (24) represent the average of absolute values taken by the phase shift across a set of frequency points belonging to intervals $[0,\overline{\omega}_1]$ and $[0,\overline{\omega}_2]$. The two intervals are chosen to include only frequencies lower than 8 years (*unsync_1*) and frequencies lower than seasonals (*unsync_2*). These indices quantify the lack of synchronization between creation and destruction rates. The next step is to run an OLS regression similar to (21), but using the new indices:

$$Labor\_prod\_growth_{\sec tor\ i} = const + \gamma usync_{\sec tor\ i} + \Lambda\, controls_{\sec tor\ i} + \varepsilon_{\sec tor\ i} \qquad (25)$$

Results reported in table 4.4 suggest that industries with less synchronized job creation and job destruction also display lower productivity growth.[52] These findings provide support for the argument of Caballero and Hammour that unsynchronized job flows dynamics are a symptom of inefficient restructuring, which ultimately hinders productivity growth.

[52] As in the previous section, results were obtained using a Hanning window with size 20. They were robust to alternative window shapes (Blackman and Bartlett), but varying the window sizes affected the p-values of $\gamma$. Even though there was no sign reversal of the estimated coefficient, our results in this section are less robust to alternative window sizes than the ones we obtained for equation (21).

## *4.4 Insights from a simple theoretical model*

In the previous section we presented empirical evidence relating the frequency content of the reallocation series for four-digit-level industries to their corresponding productivity growth performance. One of our findings was that industries in which reallocation is (relatively) "more driven" by low frequencies tend to display lower productivity growth. Along the way, we made reference to a story consistent with such a relationship: in a model with vintage effects, productivity growth is positively affected by how fast reallocation occurs. Although this statement may seem plausible, there are several dimensions of the argument that need to be further scrutinized.

This section proposes a simple theoretical model that allows us to examine key points of the "vintage story" just mentioned. We do not intend to propose a full-blown model and, as will become clear, there are several extensions to be added before we can claim success in achieving a satisfactory mapping between theory and the empirical evidence presented in this chapter. Although we leave this task for future work, the model presented here articulates in clear form some features that are important for generating the results seen in the data.

### 4.4.1 The model

Even though our empirical evidence relies on aggregate sectoral data, the unit of analysis in the model is the firm. The firm is subjected to vintage effects, such that profits are a decreasing function of the firm's age. The vintage effects are associated with the match (organizational vintage effects) and not with capital, which is not present in the model. There is one worker per firm, and the only decision the firm needs to make during the current period is whether it wants to (1) continue producing and allow the match to age one more period or (2) to destroy the match, which implies paying a fixed cost, relinquishing any production during that period, and resetting vintage effects for future production.

We assume a partial equilibrium set-up where prices are exogenous. Each active firm faces the following dynamic programming problem:

$$V_{ac}(a,z,s) = Max\{\pi(a)zs + \beta EV_{ac}(a+1,z',s'), V_{in}(s)\} \qquad (26)$$

$$V_{in}(s) = -f + \beta EV_{ac}(1,z',s') \qquad (27)$$

Where the subscripts *ac* and i*n* appearing in the value functions denote, respectively, active and inactive firm.  The variable $a$ indicates age of the firm, while $z$ and $s$ are idiosyncratic and aggregate shocks. The former type of shock is i.i.d. and the latter Markovian with positive persistence. The function $\pi(a)$ is the profit-vintage schedule, which we assume to be a decreasing function of age. In each period, an active firm decides whether to remain active and become one period older, or to pay the fixed cost $f$ to terminate the job and start a brand-new one in the following period.

Note the assumption that if a firm destroys its current match then it will necessarily restart production in the next period under a brand new vintage (age 1). We can easily relax this assumption by allowing an inactive firm to restart production with some positive probability next period. Such modification (under the assumption that the restarting probability is an i.i.d. idiosyncratic shock) would not change our qualitative results. Nonetheless, incorporating a truly endogenous entry decision process would require more structure and we leave it for future work. Because the model features i.i.d. idiosyncratic shocks and Markovian aggregate shocks, there is not a unique age threshold for destruction, but rather a collection of scrapping ages that vary with the current exogenous states.

### 4.4.2 Dynamics

We numerically solve the model by value function iteration. Once we obtain the decision rules, it is possible to simulate a panel of firms by drawing one sequence of aggregate shocks and a sequence of idiosyncratic shocks for each firms. Then, using the simulated panel of firms we can obtain job reallocation rates and productivity growth for the industry (both at the firm-level and averages for the industry). We can then vary the fixed cost of job destruction to see how barriers to vintage updating affect productivity growth and the frequency content of reallocation.

Everything else constant, a higher fixed cost will increase the lifespan of firms since it becomes more costly to destroy a job. As a result, the contribution of each firm to the industry reallocation process (measured as an individual occurrence of job destruction immediately followed by a job creation) is more spaced across time. The fact that firms are subject to idiosyncratic and aggregate shocks makes this argument less exact (a very bad shock can cut short the operation of a young firm), but it still holds on average when we consider a large number of firms. An important issue is how this micro pattern translates into aggregate reallocation. Will higher fixed costs induce a more slow-moving industry reallocation series – that is, a reallocation series in which lower frequencies are more influential? This is essentially an aggregation question. If firms are sufficiently synchronized in terms of their destruction decisions, then the industry reallocation series will reproduce the positive relationship between temporally spaced reallocation and fixed costs that holds at the firm-level. However, iid idiosyncratic shocks work against synchronization and so we need a counteracting force. This role is played by the aggregate shocks in the model, which once in a while promote a burst of destruction and induce some synchronization.[53]

Another important aspect highlighted by the model is the distinction between productivity levels and productivity growth, where our empirical evidence relates the latter to properties of the reallocation series. For the sake of the argument, hold idiosyncratic and aggregate shocks constant and first consider continuing firms. While, by definition, vintage effects contribute negatively to the productivity level of a continuously active firm over time, the same cannot be said for productivity growth. The behavior of a firm's productivity growth over its lifetime depends on the shape of

[53] Another way to induce synchronization is by assuming that the entire profit schedule is vintage specific. This way some firms would lock in a profit vintage schedule for long periods and innovations to the schedule itself would generate a stronger motive for updating.

the profit-vintage schedule – that is, on how fast profits fall with age. A linearly decreasing profit-vintage schedule delivers a constant negative productivity growth rate for an individual continuing firm across time, while a concave decreasing profit-vintage schedule delivers an increasingly negative growth rate. Under a linear profit-vintage schedule, industry productivity growth of continuing firms is independent of the underlying age distribution of firms, since the slope of the profit-vintage is constant, but under a concave profit-vintage this is no longer true.[54] In this case, an increase in the number of old firms will tend to reduce aggregate productivity growth among continuing firms since more of them will be sitting on a region with a stronger negative slope – see figure 4.5.

Hence, to the extent that a higher fixed cost of destruction causes firms to stay active for longer, it will also lead to lower productivity growth for continuing firms under a concave profit-vintage schedule. Once we re-introduce idiosyncratic and aggregate shocks into the, the profit-vintage schedule becomes stochastic and the argument still holds on average for the industry under reasonable parameterizations - that is, for parameters values that do not imply a complete dominance of shocks over vintage effects.

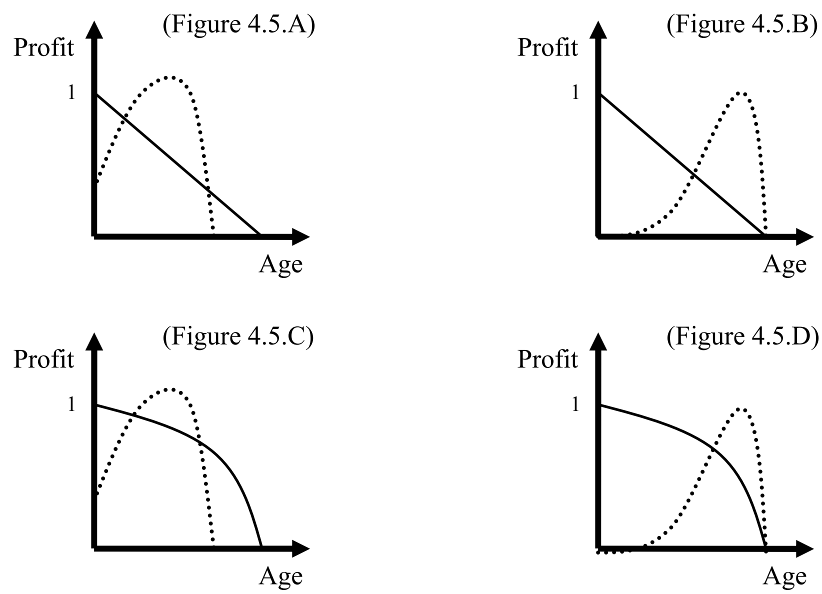
More importantly, we have articulated the conditions under which an increase in the barriers to vintage updating (fixed cost of destruction) can reduce productivity growth and tilt the spectrum of the reallocation series to lower frequencies, consistent with

[54] Because firms have equal size, employment-weighted industry productivity growth of continuing firms can be calculated as simple averages of the underlying individual growth rates.

the empirical results shown in the previous section.[55] Two features were crucial to the argument: a concave profit-vintage schedule and some synchronization of destruction decisions.

Figure 4.5

Profit-vintage schedules




Straight line: profit-vintage schedule – dotted line: underlying distribution of firms

Now consider the task of accounting for productivity growth coming from the entry margin, temporarily putting aside the occurrence of any shocks. A brand-new firm will yield the maximum productivity level, but its productivity growth is not well defined since that particular firm was inactive in the previous period. We can obtain

[55] Appendix III illustrates by means of an example that this relationship between the spectral shape and infrequent updating holds even for an individual firm (in which case adjustment is a binary event for our model). Such an example is helpful in motivating the argument, but later we will be concerned with the spectral densities of the entire industry, in which case adjustment is not a binary event anymore (that is, one may have multiple firms both adjusting or not adjusting at any time).

the contribution of entry to aggregate productivity growth by comparing the average productivity level of all firms that exited last period with the productivity level of brand new firms in the current period.[56] Here, the presence of older firms actually increases productivity growth coming from entry. This is because the difference in average productivity levels between exiting firms and new firms is higher when exiting firms are older.[57] This fact poses a potential complication since it counteracts the link between higher fixed costs and lower productivity growth established for continuing firms. We come back to this issue in the next subsection.

#### 4.4.3 Simulation results

As mentioned before, we want to construct a simulated panel of firms and investigate whether the simple model proposed here is able to generate the relationship between the reallocation index and productivity growth we documented in the data. The idea is to simulate panels for different fixed costs parameterizations and assess whether industries with stronger barriers to updating can generate (simultaneously) lower productivity growth and higher reallocation indices. In order to perform such an exercise, we parameterize the model as in table 4.5.

The first row of table 4.5 indicates that the profit-vintage schedule corresponds to the first quadrant of a unit-radius circle – which implies it is concave. The discount factor

[56] In order to generate a productivity growth figure for the entire industry we average the contribution coming from continuing firms and from the entry/exit process, using the current share of brand new firms as weights. The reader should also note the existence of richer productivity growth decomposition in the literature – see Foster, Haltiwanger and Krizan (1998).

[57] As a result productivity growth from entry is not independent of age distribution even under a linear profit-vintage schedule.

is .9 and the aggregate shock (s) is modeled as a two-state Markov chain with a non-symmetric transition matrix designed to capture strong recessionary periods that are not very persistent.[58] The idiosyncratic shock follows a uniform distribution, and we consider two possible levels for the fixed cost.

Table 4.5
Model Parameterization

| | |
|---|---|
| $\pi(a)$ | $.01 * \sqrt{10^4 - (a-1)^2}$ |
| $\beta$ | .9 |
| $s$ | $\bar{s} = \begin{bmatrix} s_b = 1.0 \\ s_r = 0.5 \end{bmatrix}, \quad \Pi = \begin{bmatrix} p_{bb} = 0.95 & p_{br} = 0.05 \\ p_{rb} = 0.4 & p_{rr} = 0.6 \end{bmatrix}$ |
| $z$ | Uniform distribution with support [.9, 1.1] |
| $f$ | First case: 3 – second case: 1 |

Figure 4.6 presents the value functions and scrapping age schedules characterizing the firms' decisions at different points of the state space. As expected, the value function for an active firm is decreasing with age, and job destruction occurs when the value of an active firm is less than or equal to the value of an inactive firm. Destruction also depends on the aggregate and idiosyncratic shocks, so to visualize the value functions we graph them conditioned on particular realizations of the shocks. The results show that, for any given age, good realizations of the exogenous shocks delay destruction. Additionally, because idiosyncratic shocks are not serially correlated, there is a unique scrapping age schedule for each realization of that shock (conditioned on the aggregate state). As indicated in Figure 4.6, the cut-off age for destruction is increasing in the magnitude of the idiosyncratic shock for a given aggregate state. The

[58] We further discuss the role of aggregate shocks below.

figure also confirms that a higher fixed cost causes delayed updating. Finally, aggregate shocks have a larger impact on scrapping ages when the fixed cost is low. This happens because a near-death firm facing high fixed costs to updating is sitting on a region of the profit-vintage schedule in which productivity is already relatively low regardless of the aggregate state – recall that the aggregate shock is multiplying the profit-vintage schedule. In contrast, a near-death firm facing low fixed costs is sitting on a more favorable region of the profit-vintage schedule and changes in the aggregate shock have a greater marginal effect on the payoffs to production.

Next, a panel of firms for each parameterization of the fixed cost is generated using the decision rules implied by the value functions illustrated in Figure 4.6. The panel is then used to produce industry-level time-series for reallocation and productivity growth, which in turn generate scalar values for the reallocation index and average productivity growth.

The reallocation index adopted is the RI_2 measure discussed in subsection 4.2.3, and the average productivity growth for the industry has two components: an average for continuing businesses and a component coming from the entry/exit process. The former is obtained by (1) calculating the change in productivity levels for each continuing firm (firm-level growth rates), (2) averaging the firm-level growth rates cross-sectionally in order to generate a time-series for continuing-firm productivity growth at the industry level, and (3) averaging this series over time. The contribution coming from the entry/exit process is obtained by (1) calculating the average difference between the average productivity of firms that exit in t-1 and firms entering in period t, and (2) averaging this figure over time. Finally, by using the share of entrants (firms with age 1) in any period we can produce a weighted average for productivity growth at the industry level and then generate its average over time.

Table 4.6
Reallocation index and productivity growth I (simulated data)

| Fixed Cost | Reallocation Index | Average productivity growth (%) | | |
|---|---|---|---|---|
| | | Continuing Businesses *(mean share)** | Entry/exit *(mean share)** | Weighted Average |
| f = 1 | 42.4451 | 2.1893 *(0.9834)* | 12.4210 *(0.0186)* | 3.1060 |
| f = 3 | 43.7521 | 1.9058 *(0.9879)* | 34.9166 *(0.0121)* | 3.0786 |

**Will not deliver figures in weighted average column. See footnote below.*

Table 4.6 presents the results of this experiment and indicates that the model is able to generate responses of the reallocation index and productivity growth that are consistent with the empirical evidence presented earlier.[59] Note that the change in the fixed cost *f* from 1 to 3 can be construed as an increase in the barrier for updating that causes a more slow-moving reallocation process (higher reallocation index) and reduced productivity growth (which arises from the combination of an "older" age distribution and a concave vintage-profit schedule). Notice that the impact of fixed costs on productivity is stronger for continuing businesses than for the overall

[59] Note the positive values of average productivity growth for the continuing businesses. Vintage effects off course call for negative productivity growth, but aggregate and idiosyncratic shocks override this tendency in the model. Also note that the weighted average column is not readily obtained from the information contained in the previous columns. The share figures are averages over the period, while the weighted average was produced using period-specific shares.

economy. This is because the offsetting contribution coming from entry/exit increases with fixed costs as discussed above.

Alternatively, we can measure average industry productivity growth by (1) averaging firm productivity level cross-sectionally in any given period, (2) generating a series of industry productivity growth, and (3) taking the average of this series. This corresponds to a different weighting (based on output rather than employment), which means that the productivity growth figures will be different from the ones presented in table 4.6. Nonetheless, we obtain the same result as before according to which a lower fixed cost is associated to a lower reallocation index and higher average productivity growth for the industry. This is shown in table 4.7 below.

Table 4.7
Reallocation index and productivity growth II (simulated data)

| Fixed Cost | Reallocation Index | Average Productivity Growth (%) |
|---|---|---|
| f = 1 | 42.4451 | 2.3744 |
| f = 3 | 43.7521 | 2.3462 |

Our objective in this section was to build a simple framework capable of producing qualitative results consitent with the empirical evidence presented before. Although we leave for future work the task of achieving a realistic parameterization of the model such that quantitative implications can be further exploited, we investigated the robustness of the findings in table 4.6 across different parameterizations of the model. We experimented with a range of different values for the fixed cost, the support of the idiosyncratic distribution and the aggregate shock (including its realization values and the transition matrix). Throughout the exercises it was clear that the particular choice of the profit-vintage schedule implies that strong vintage effects only kick in after many periods. As a result we need to specify a baseline aggregate state (boom) that is sufficiently persistent so that vintage effects have time to operate. At the same time, we need a negative aggregate shock to be "sufficiently" bad in order to trigger enough synchronization of reallocation decisions. Additionally, idiosyncratic shocks have to be limited in magnitude (relative to the aggregate shock) so as not to override the synchronization effects. With these constraints in mind, results regarding the responses of the reallocation index and the productivity growth were fairly robust.

## *4.5 Concluding remarks and extensions*

This chapter used frequency-domain techniques to document an empirical relationship between productivity growth and job reallocation across four-digit US manufacturing. I found that industries in which low-frequencies are relatively influential in the behavior of reallocation have lower productivity growth. We then suggested a theoretical model that can qualitatively accommodate this fact.

The model articulates key features that could generate the relationship documented in the data. However, more work is needed before we can fully exploit its quantitative implications. A central issue to be addressed regards the parameterization of the profit-vintage schedule. This is an empirical question and, as suggested in the last section, it can affect the parameterization of the entire model. Incidentally, in the light of firm-learning models, one might even suggest that the profit-vintage schedule should feature increasing regions at earlier ages (which would help to dampen the contribution coming from the entry/exit margin in our model). Another important improvement to be pursued is to make the entry decision endogenous, which will require adding a general equilibrium flavor to the model (either through search externalities affecting equilibrium job-finding probabilities as in Mortensen-Pissarides (1994) or through endogenous output price determination that responds to fluctuations in the industry supply).

# Chapter 5: Concluding remarks

The study of job flows dynamics has become remarkably popular in the last decade, assuming a prominent spot in the macroeconomic research agenda. The underlying reasons for such popularity are the non-trivial firm-level heterogeneity that impacts labor allocation decisions, and the incomplete (often times misleading) picture provided by aggregate labor market dynamics. As a result, the last ten years or so have witnessed an increasing worldwide effort devoted to the compilation and development of rich datasets allowing for the measurement of gross job flows at the establishment or firm level. In parallel, theories featuring simultaneous creation and destruction of jobs as well as its relationship to the business cycle and productivity have emerged.

In this context, this dissertation illustrates the high payoff associated with the application of frequency-domain techniques to study job flows dynamics. We believe that the high payoff stems not only from the relatively small number of studies pursuing such methods, but because many of the relevant questions have an inherent frequency-domain flavor or, at least, can be significantly refined through spectral methods. We hope to have demonstrated such claims throughout the chapters and, also importantly, to have convinced the reader about the potential benefits of combining frequency-domain techniques with the study of job flows dynamics.

# Appendices

## Appendix I: Tables

Table 2.1
Mortensen and Pissarides Model (10- vs 3-state Aggregate Shocks)

Aggregate Shock

Discretized grid

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| -0.0917 | -0.0713 | -0.0509 | -0.0306 | -0.0102 | 0.0102 | 0.0306 | 0.0509 | 0.0713 | 0.0917 |

*(10 state)*

| | | |
|---|---|---|
| -0.0530 | 0.0000 | 0.0530 |

*(3 state)*

Transition Matrix

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.6435 | 0.3433 | 0.0132 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0868 | 0.6018 | 0.3019 | 0.0095 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0010 | 0.1071 | 0.6229 | 0.2622 | 0.0068 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0015 | 0.1315 | 0.6374 | 0.2249 | 0.0048 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0022 | 0.1593 | 0.6448 | 0.1904 | 0.0033 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0033 | 0.1904 | 0.6448 | 0.1593 | 0.0022 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0048 | 0.2249 | 0.6374 | 0.1315 | 0.0015 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0068 | 0.2622 | 0.6229 | 0.1071 | 0.0010 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0095 | 0.3019 | 0.6018 | 0.0868 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0132 | 0.3433 | 0.6435 |

*(10 state)*

| | | |
|---|---|---|
| 0.9330 | 0.0670 | 0.0000 |
| 0.0170 | 0.9660 | 0.0170 |
| 0.0000 | 0.0670 | 0.9330 |

*(3 state)*

Outcome Variables

Reservation Productivities

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.6680 | 0.5440 | 0.4360 | 0.3400 | 0.2440 | 0.1480 | 0.0600 | -0.0320 | -0.1200 | -0.2240 |

*(10 state)*

| | | |
|---|---|---|
| 0.4840 | 0.2440 | -0.0160 |

*(3 state)*

Job-Finding Probabilities

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.3048 | 0.3737 | 0.4443 | 0.5094 | 0.5695 | 0.6255 | 0.6781 | 0.7278 | 0.7749 | 0.8165 |

*(10 state)*

| | | |
|---|---|---|
| 0.4374 | 0.5983 | 0.7325 |

*(3 state)*

Table 3.1
Unit root tests

| Series | Augmented Dickey-Fuller (p-value) | Phillips-Perron (p-value) |
|---|---|---|
| **NET** | 0.0065 | 0.0000 |
| **REL** | 0.0467 | 0.0163 |
| **NET (Detail 1)** | 0.0001 | 0.0000 |
| **REL (Detail 1)** | 0.0000 | 0.0000 |
| **NET (Detail 2)** | 0.0000 | 0.0000 |
| **REL (Detail 2)** | 0.0000 | 0.0000 |
| **NET (Detail 3)** | 0.0000 | 0.0297 |
| **REL (Detail 3)** | 0.0000 | 0.0134 |
| **NET (Detail 4)** | 0.0000 | 0.0225 |
| **REL (Detail 4)** | 0.0000 | 0.0056 |
| **NET (Detail 5)** | 0.0000 | 0.0103 |
| **REL (Detail 5)** | 0.0004 | 0.0577 |

Results on lag coefficients not reported. Linear trend included for reallocation series. Number of Newey-West lags: 14. H0: Series is non-stationary

Table 3.2
Bivariate VAR between employment and reallocation at different time-scales

| Lag Order | Coefficient | Original Series | | Detail 1 | | Detail 2 | | Detail 3 | |
|---|---|---|---|---|---|---|---|---|---|
| | | NET | REL | NET | REL | NET | REL | NET | REL |
| Lag 1 | NET | .82$^{1}$ | -.27$^{1}$ | -3.21$^{1}$ | .02$^{N}$ | 1.74$^{1}$ | -.13$^{2}$ | 2.88$^{1}$ | .01$^{N}$ |
| | REL | .30$^{2}$ | .37$^{1}$ | .08$^{N}$ | -3.3$^{1}$ | -.35$^{1}$ | 1.73$^{1}$ | -.41$^{2}$ | 3.01$^{1}$ |
| Lag 2 | NET | -.06$^{N}$ | .06$^{2}$ | -6.87$^{1}$ | .20$^{N}$ | -4.62$^{1}$ | .30$^{1}$ | -3.22$^{1}$ | -.01$^{N}$ |
| | REL | .09$^{N}$ | .26$^{1}$ | .11$^{N}$ | -7.0$^{1}$ | .82$^{1}$ | -4.66$^{1}$ | 1.19$^{2}$ | -3.68$^{1}$ |
| Lag 3 | NET | | | -11.1$^{1}$ | .63$^{2}$ | 5.34$^{1}$ | -.72$^{1}$ | 1.25$^{1}$ | -.11$^{N}$ |
| | REL | | | .07$^{N}$ | -11.1$^{1}$ | -2.2$^{1}$ | 5.53$^{1}$ | -.93$^{N}$ | 1.61$^{1}$ |
| Lag 4 | NET | | | -14.7$^{1}$ | 1.23$^{1}$ | -9.03$^{1}$ | 1.25$^{1}$ | -.91$^{1}$ | .31$^{2}$ |
| | REL | | | .08$^{N}$ | -14.5$^{1}$ | 3.32$^{1}$ | -9.15$^{1}$ | -.09$^{N}$ | -.67$^{N}$ |
| Lag 5 | NET | | | -16.6$^{1}$ | 1.83$^{1}$ | 7.57$^{1}$ | -1.81$^{1}$ | 2.99$^{1}$ | -.27$^{N}$ |
| | REL | | | .24$^{N}$ | -16.0$^{1}$ | -5.56$^{1}$ | 7.97$^{1}$ | -.56$^{N}$ | 2.66$^{1}$ |
| Lag 6 | NET | | | -15.9$^{1}$ | 2.26$^{1}$ | -10.0$^{1}$ | 2.36$^{1}$ | -2.94$^{1}$ | .03$^{N}$ |
| | REL | | | .62$^{N}$ | -15.1$^{1}$ | 6.14$^{1}$ | -10.1$^{1}$ | 1.95$^{2}$ | -3.10$^{1}$ |
| Lag 7 | NET | | | -13.1$^{1}$ | 2.29$^{1}$ | 5.13$^{1}$ | -2.23$^{1}$ | -.36$^{N}$ | -.07$^{N}$ |
| | REL | | | 1.20$^{N}$ | -12.5$^{1}$ | -7.73$^{1}$ | 5.90$^{1}$ | -.28$^{N}$ | -.56$^{N}$ |
| Lag 8 | NET | | | -9.30$^{1}$ | 1.99$^{1}$ | -5.98$^{1}$ | 2.09$^{1}$ | 1.46$^{1}$ | .35$^{N}$ |
| | REL | | | 1.57$^{N}$ | -8.93$^{1}$ | 6.25$^{1}$ | -6.25$^{1}$ | -3.18$^{1}$ | 2.45$^{1}$ |
| Lag 9 | NET | | | -5.53$^{1}$ | 1.44$^{1}$ | -.46$^{N}$ | -.81$^{N}$ | .52$^{N}$ | -.34$^{N}$ |
| | REL | | | 1.50$^{N}$ | -5.36$^{1}$ | -6.05$^{1}$ | .85$^{N}$ | 3.05$^{1}$ | -.03$^{N}$ |
| Lag 10 | NET | | | -2.69$^{1}$ | .86$^{1}$ | -.36$^{N}$ | .04$^{N}$ | -.90$^{2}$ | .01$^{N}$ |
| | REL | | | 1.09$^{N}$ | -2.62$^{1}$ | 2.50$^{N}$ | -1.04$^{N}$ | .18$^{N}$ | -1.65$^{1}$ |
| Lag 11 | NET | | | -.97$^{1}$ | .38$^{1}$ | -4.78$^{1}$ | 1.69$^{2}$ | -1.25$^{1}$ | .14$^{N}$ |
| | REL | | | .58$^{N}$ | -.94$^{1}$ | -1.22$^{N}$ | -2.90$^{2}$ | -1.42$^{N}$ | -.19$^{N}$ |
| Lag 12 | NET | | | -.23$^{1}$ | .10$^{1}$ | 2.66$^{1}$ | -2.10$^{1}$ | 1.77$^{1}$ | -.01$^{N}$ |
| | REL | | | .18$^{N}$ | -.18$^{1}$ | -2.37$^{N}$ | 1.74$^{N}$ | .21$^{N}$ | 1.23$^{1}$ |
| Lag 13 | NET | | | | | -5.06$^{1}$ | 3.06$^{1}$ | -.30$^{N}$ | -.04$^{N}$ |
| | REL | | | | | 2.87$^{N}$ | -3.37$^{1}$ | -.35$^{N}$ | .14$^{N}$ |
| Lag 14 | NET | | | | | 2.29$^{1}$ | -2.53$^{1}$ | -.30$^{N}$ | -.08$^{N}$ |
| | REL | | | | | -4.39$^{1}$ | 1.74$^{3}$ | 2.04$^{1}$ | -1.08$^{1}$ |
| Lag 15 | NET | | | | | -2.93$^{1}$ | 2.51$^{1}$ | -.44$^{N}$ | .15$^{N}$ |
| | REL | | | | | 3.28$^{1}$ | -1.95$^{2}$ | -2.56$^{1}$ | .24$^{N}$ |
| Lag 16 | NET | | | | | .90$^{N}$ | -1.50$^{1}$ | .69$^{1}$ | -.10$^{N}$ |
| | REL | | | | | -2.90$^{1}$ | .78$^{N}$ | 1.43$^{1}$ | .40$^{N}$ |
| Lag 17 | NET | | | | | -.99$^{1}$ | 1.09$^{1}$ | -.28$^{1}$ | .02$^{N}$ |
| | REL | | | | | 1.48$^{N}$ | -.69$^{3}$ | -.34$^{2}$ | -.25$^{1}$ |
| Lag 18 | NET | | | | | .13$^{N}$ | -.39$^{1}$ | | |
| | REL | | | | | -.80$^{1}$ | .19$^{N}$ | | |
| Lag 19 | NET | | | | | -.19$^{1}$ | .18$^{1}$ | | |
| | REL | | | | | .18$^{N}$ | -.12$^{N}$ | | |
| Granger Causality | REL→NET | 0.00 (p-value) | | 0.55 (p-value) | | 0.00 (p-value) | | 0.00 (p-value) | |
| | NET→REL | 0.00 (p-value) | | 0.01 (p-value) | | 0.00 (p-value) | | 0.04 (p-value) | |
| Lagrange Multiplier test (Ho: no auto-corr.) | | 0.715 (p-value) (lag2) | | 0.064 (p-value) (lag12) | | 0.923 (p-value) (lag 19) | | 0.081 (p-value) (lag 17) | |
| Number of Obs | | 186 | | 176 | | 169 | | 171 | |

VARs estimated with constant (omitted from the table). $^{1}$: significant at 1%, $^{2}$: significant at 5%; $^{N}$: not significant. Criterion for lag selection: SBIC.
Linear trend included for original series (omitted from the table).

Table 3.3
Bivariate VAR between employment and reallocation at different time-scales

| Lag Order | Coefficient | Detail 4 | | Detail 5 | |
|---|---|---|---|---|---|
| | | **NET** | **REL** | **NET** | **REL** |
| **Lag 1** | **NET** | $2.15^{1}$ | $-.12^{2}$ | $2.35^{1}$ | $-.10^{N}$ |
| | **REL** | $-.43^{1}$ | $2.46^{1}$ | $-.28^{1}$ | $2.37^{1}$ |
| **Lag 2** | **NET** | $-.85^{1}$ | $.42^{1}$ | $-.99^{1}$ | $.19^{N}$ |
| | **REL** | $1.13^{1}$ | $-1.58^{1}$ | $.68^{1}$ | $-1.05^{1}$ |
| **Lag 3** | **NET** | $-.74^{1}$ | $-.63^{1}$ | $-.94^{1}$ | $.01^{N}$ |
| | **REL** | $.81^{5}$ | $-.26^{N}$ | $-.20^{N}$ | $-.78^{1}$ |
| **Lag 4** | **NET** | $.06^{N}$ | $.53^{1}$ | $.13^{N}$ | $-.15^{N}$ |
| | **REL** | $-.12^{N}$ | $-.13^{N}$ | $-.54^{1}$ | $-.05^{N}$ |
| **Lag 5** | **NET** | $.41^{2}$ | $-.15^{N}$ | $.55^{1}$ | $-.07^{N}$ |
| | **REL** | $.20^{N}$ | $1.13^{1}$ | $.18^{N}$ | $.46^{2}$ |
| **Lag 6** | **NET** | $.09^{N}$ | $-.24^{N}$ | $.08^{N}$ | $.16^{N}$ |
| | **REL** | $.13^{N}$ | $-.83^{1}$ | $.37^{2}$ | $.35^{N}$ |
| **Lag 7** | **NET** | $-.44^{5}$ | $.31^{2}$ | $-.19^{1}$ | $-.04^{N}$ |
| | **REL** | $-.01^{N}$ | $.06^{N}$ | $-.20^{1}$ | $-.31^{1}$ |
| **Lag 8** | **NET** | $.23^{N}$ | $-.05^{N}$ | | |
| | **REL** | $.02^{N}$ | $-.18^{N}$ | | |
| **Lag 9** | **NET** | $.32^{N}$ | $-.20^{N}$ | | |
| | **REL** | $-.13^{N}$ | $.87^{1}$ | | |
| **Lag 10** | **NET** | $-.07^{N}$ | $.23^{N}$ | | |
| | **REL** | $-.40^{N}$ | $-.54^{2}$ | | |
| **Lag 11** | **NET** | $-.28^{N}$ | $-.07^{N}$ | | |
| | **REL** | $.64^{N}$ | $.09^{N}$ | | |
| **Lag 12** | **NET** | $-.10^{N}$ | $.07^{N}$ | | |
| | **REL** | $-.23^{N}$ | $-.17^{N}$ | | |
| **Lag 13** | **NET** | $.01^{N}$ | $-.12^{N}$ | | |
| | **REL** | $.49^{N}$ | $-.43^{2}$ | | |
| **Lag 14** | **NET** | $.13^{N}$ | $-.10^{N}$ | | |
| | **REL** | $-.72^{5}$ | $.49^{2}$ | | |
| **Lag 15** | **NET** | $.19^{N}$ | $.28^{2}$ | | |
| | **REL** | $.18^{N}$ | $.31^{N}$ | | |
| **Lag 16** | **NET** | $-.18^{1}$ | $-.12^{1}$ | | |
| | **REL** | $.08^{N}$ | $-.33^{1}$ | | |
| **Granger Causality** | **REL→NET** | 0.00 (p-value) | | 0.00 (p-value) | |
| | **NET→REL** | 0.00 (p-value) | | 0.36 (p-value) | |
| **Lagrange Multiplier test (Ho: no auto-corr.)** | | 0.802 (p-value) (lag 16) | | 0.597 (p-value) (lag 17) | |
| **Number of Obs** | | 172 | | 181 | |

VAR's estimated with constant (omitted from the table). $^{1}$: significant at 1%, $^{2}$: significant at 5%; $^{N}$: not significant. Criterion for lag selection: SBIC.

Table 4.1
Correlation Matrix for Reallocation Indices

| | ***RI_1*** | ***RI_2*** | ***RI_3*** |
|---|---|---|---|
| ***RI_1*** | 1 | | |
| ***RI_2*** | 0.8541 | 1 | |
| ***RI_3*** | 0.6449 | 0.5935 | 1 |

Table 4.2
Correlation Coefficients between Reallocation Indices and Productivity Growth

| | Correlation with labor productivity growth and (p-value) |
|---|---|
| ***RI_1*** | -0.1688 (0.0008) |
| ***RI_2*** | -0.1093 (0.0311) |
| ***RI_3*** | -0.1204 (0.0139) |

Table 4.3
OLS Results for Productivity Growth and Reallocation Index

$$Labor\_prod\_growth_{\sec tor\ i} = const + \gamma RI_{\sec tor\ i} + \Lambda\, controls_{\sec tor\ i} + \varepsilon_{\sec tor\ i}$$

| | **RI_1** | | **RI_2** | | **RI_3** | |
|---|---|---|---|---|---|---|
| **Labor Productivity Growth** | **Coefficient** | **P-value** | **Coefficient** | **P-value** | **Coefficient** | **P-value** |
| Reallocation Index (RI) | -0.4763 | 0.005 | -0.1253 | 0.066 | -1.2481 | 0.022 |
| Total reallocation rate | 0.0049 | 0.728 | -0.0011 | 0.934 | 0.0042 | 0.763 |
| Capital-Labor ratio | 0.0044 | 0.011 | 0.0045 | 0.009 | 0.0042 | 0.014 |
| Employment | -0.0025 | 0.329 | -0.0028 | 0.27 | -0.0032 | 0.211 |
| Employment growth rate | -13.909 | 0.002 | -13.748 | 0.003 | -14.438 | 0.003 |
| Standard dev emp. growth | 7.4603 | 0.004 | 7.5463 | 0.002 | 7.4308 | 0.002 |
| Inventory to output ratio | 0.5581 | 0.639 | 0.4752 | 0.691 | 0.3972 | 0.739 |
| Energy to output ratio | -3.5056 | 0.19 | -3.5836 | 0.182 | -3.3387 | 0.213 |
| Labor force composition | -1.0819 | 0.193 | -0.9987 | 0.233 | -1.2004 | 0.152 |
| Capital stock composition | 0.3083 | 0.15 | 0.3306 | 0.125 | 0.3049 | 0.156 |
| Investment | 0.0007 | 0.083 | 0.0007 | 0.071 | 0.0007 | 0.064 |
| Investment growth | 0.3256 | 0.623 | 0.2896 | 0.664 | 0.4850 | 0.472 |
| Constant | 1.8156 | 0.012 | 2.7645 | 0.002 | 2.2256 | 0.003 |
| *Number of Observations* | *389* | | *389* | | *389* | |
| *F(11,377) statistic* | *6.17* | | *5.73* | | *5.45* | |
| *R-squared* | *0.16* | | *0.15* | | *0.15* | |

Table 4.4
OLS Results for Productivity Growth and Synchronization Index

$$Labor\_prod\_growth_{sector\ i} = const + \gamma usync_{sector\ i} + \Lambda\, controls_{sector\ i} + \varepsilon_{sector\ i}$$

| | **Usync_1** | | **Usync_2** | |
|---|---|---|---|---|
| **Labor Productivity Growth** | **Coefficient** | **P-value** | **Coefficient** | **P-value** |
| Synchronization index | -0.0599 | 0.077 | -0.19085 | 0.11 |
| Total reallocation rate | 0.0007 | 0.956 | 0.00151 | 0.914 |
| Capital-Labor ratio | 0.0046 | 0.007 | 0.004696 | 0.007 |
| Employment | -0.0024 | 0.34 | -0.00266 | 0.303 |
| Employment growth rate | -12.601 | 0.002 | -12.9664 | 0.001 |
| Standard dev emp. growth | 7.4538 | 0.001 | 7.464958 | 0.001 |
| Inventory to output ratio | 0.5088 | 0.671 | 0.426591 | 0.722 |
| Energy to output ratio | -3.5705 | 0.184 | -3.50674 | 0.193 |
| Labor force composition | -0.8414 | 0.319 | -0.85811 | 0.31 |
| Capital stock composition | 0.3222 | 0.135 | 0.329547 | 0.127 |
| Investment | 0.0006 | 0.14 | 0.000636 | 0.129 |
| Investment growth | 0.1266 | 0.849 | 0.129276 | 0.847 |
| Constant | 2.0550 | 0.005 | 2.150036 | 0.004 |
| *Number of Observations* | *389* | | *389* | |
| *F(11,377) statistic* | *5.71* | | *5.65* | |
| *R-squared* | *0.15* | | *0.15* | |

## Appendix II: Figures

Figure 2.1
Data and Model Spectra I (Mortensen and Pissarides) - Original Calibration
Frequencies lower than 8 years (0 to 0.19), Frequencies between 8 and 2 years (0.19 to 0.78)
Frequencies higher than 2 years (0.78 to π)

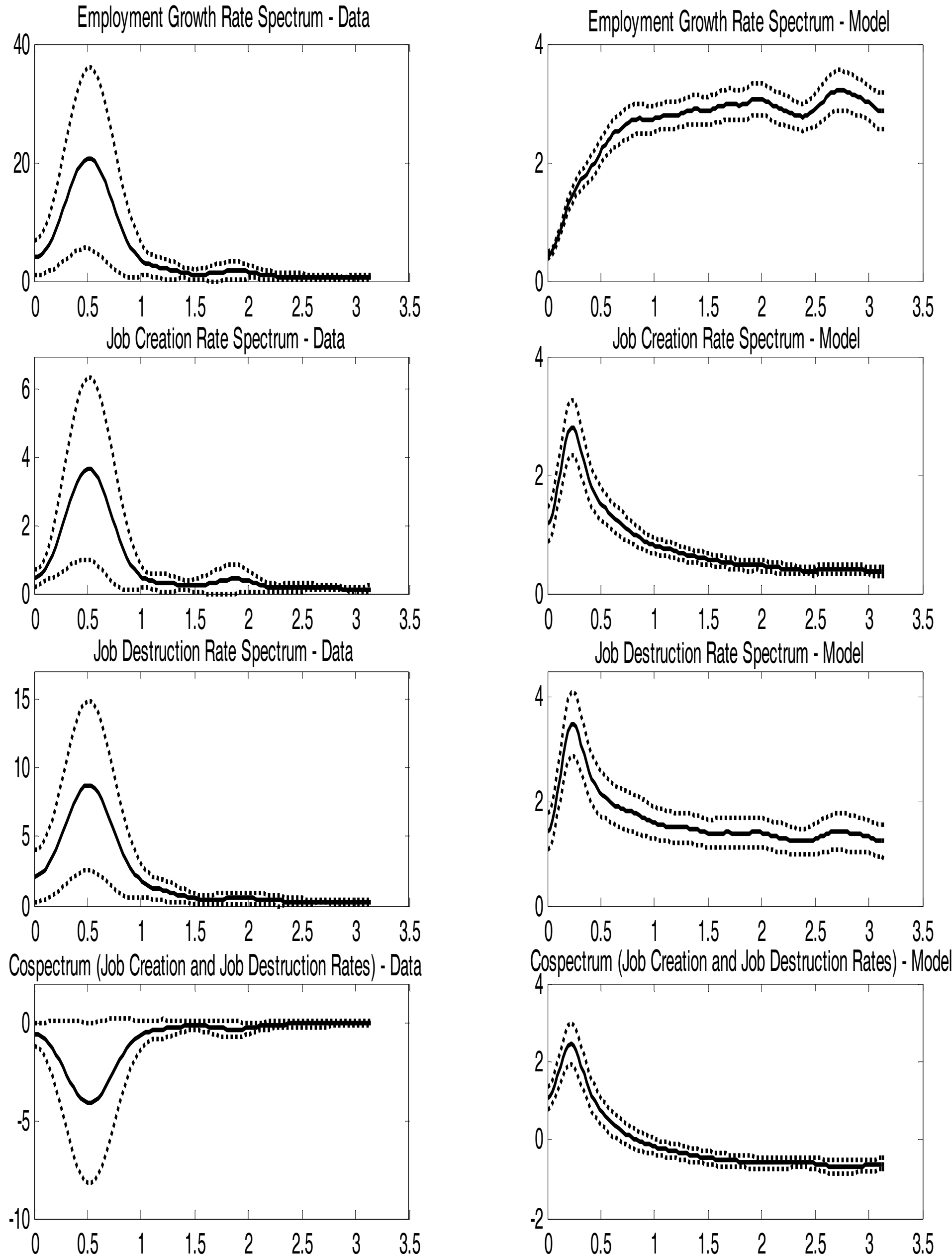


Figure 2.2
Data and Model Spectra II (Mortensen and Pissarides) - Less Persistence ρ=0.53
Frequencies lower than 8 years (0 to 0.19), Frequencies between 8 and 2 years (0.19 to 0.78)
Frequencies higher than 2 years (0.78 to π)

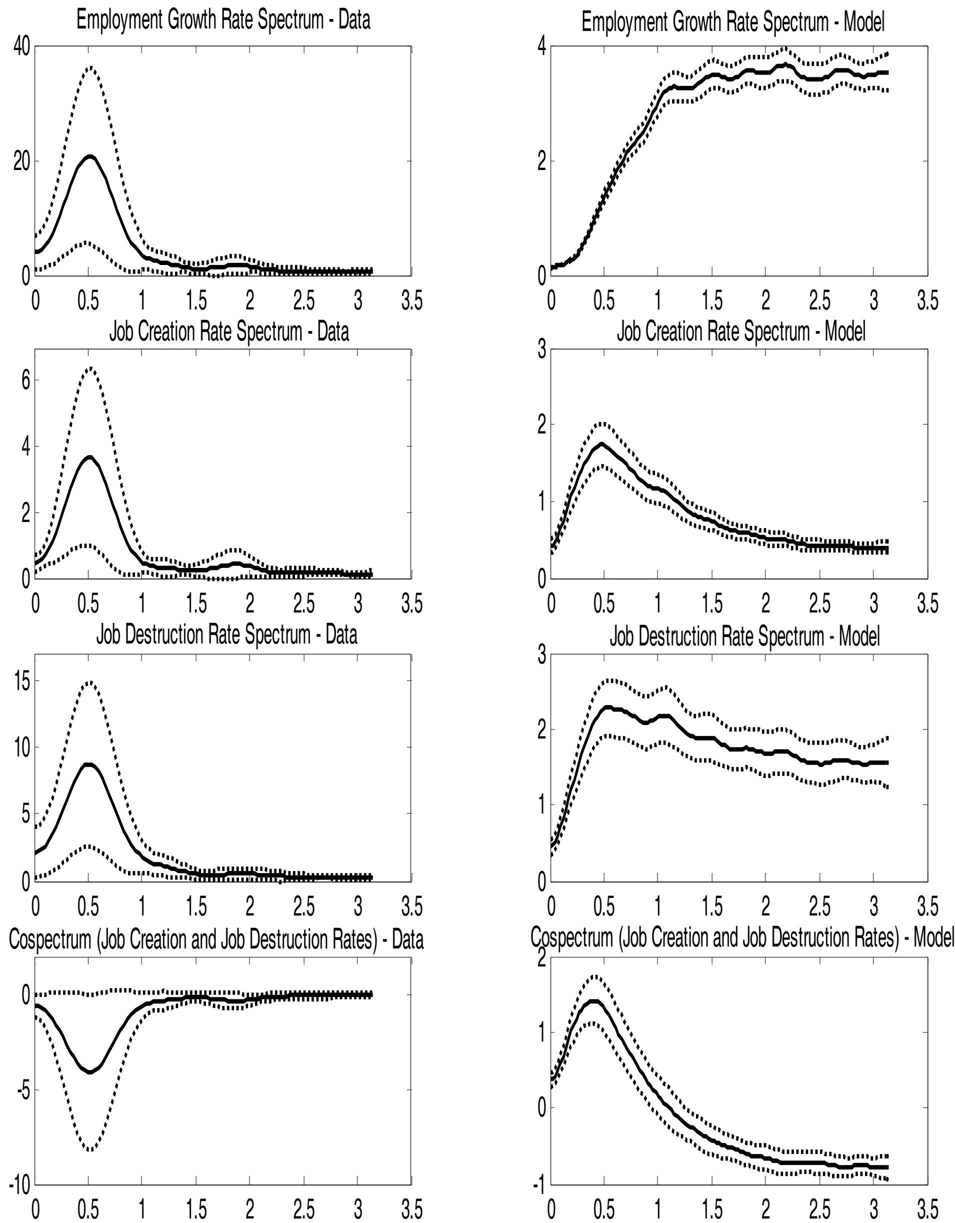
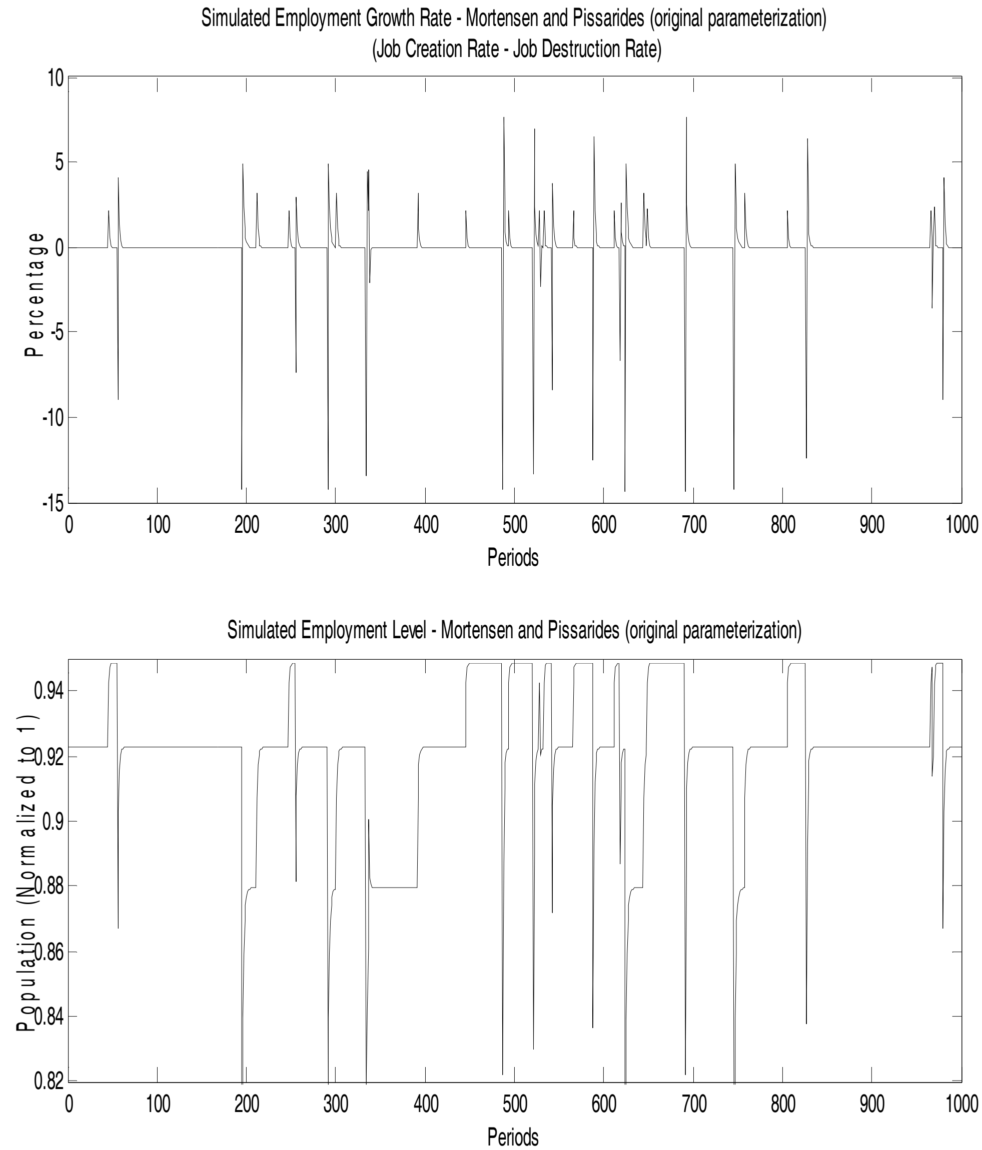


Figure 2.3
Levels and Growth Rates - Simulated Series



Figures 2.4, 2.5, and 2.6

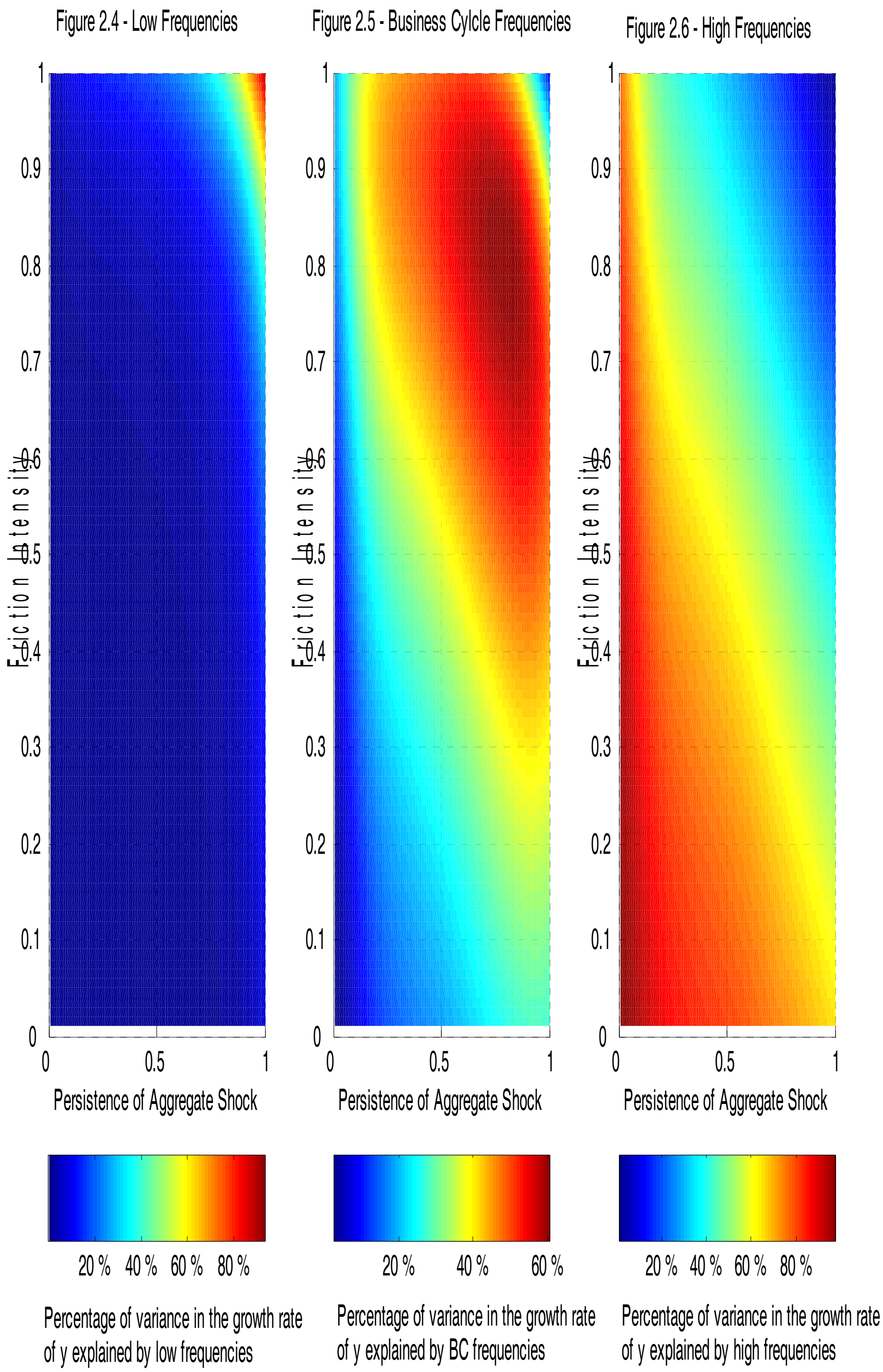


Figure 2.7
Response to Permanent bad shocks in alternative models
Cole and Rogerson (1999) versus Mortensen and Pissarides (1994)

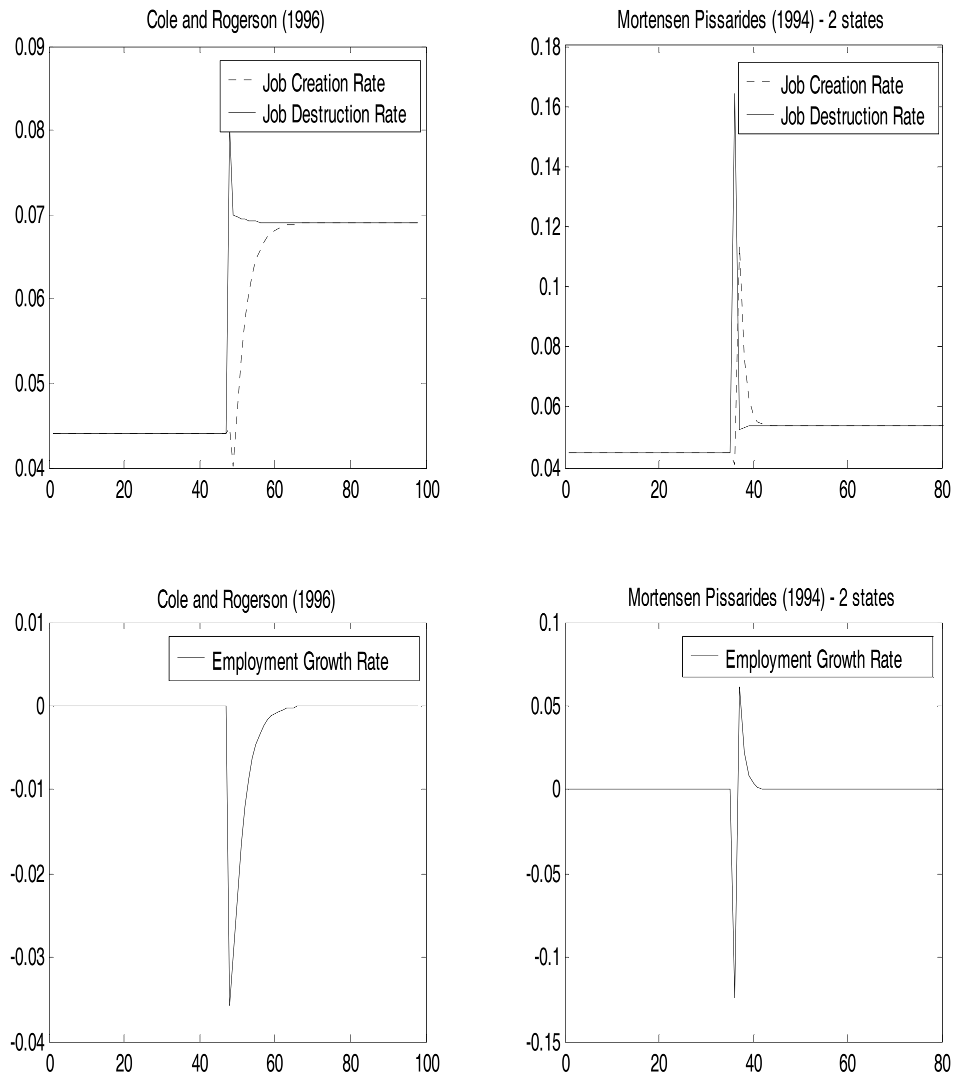


Figure 2.8
Data and Model Spectra III (Cole and Rogerson)
Frequencies lower than 8 years (0 to 0.19), Frequencies between 8 and 2 years (0.19 to 0.78)
Frequencies higher than 2 years (0.78 to π)

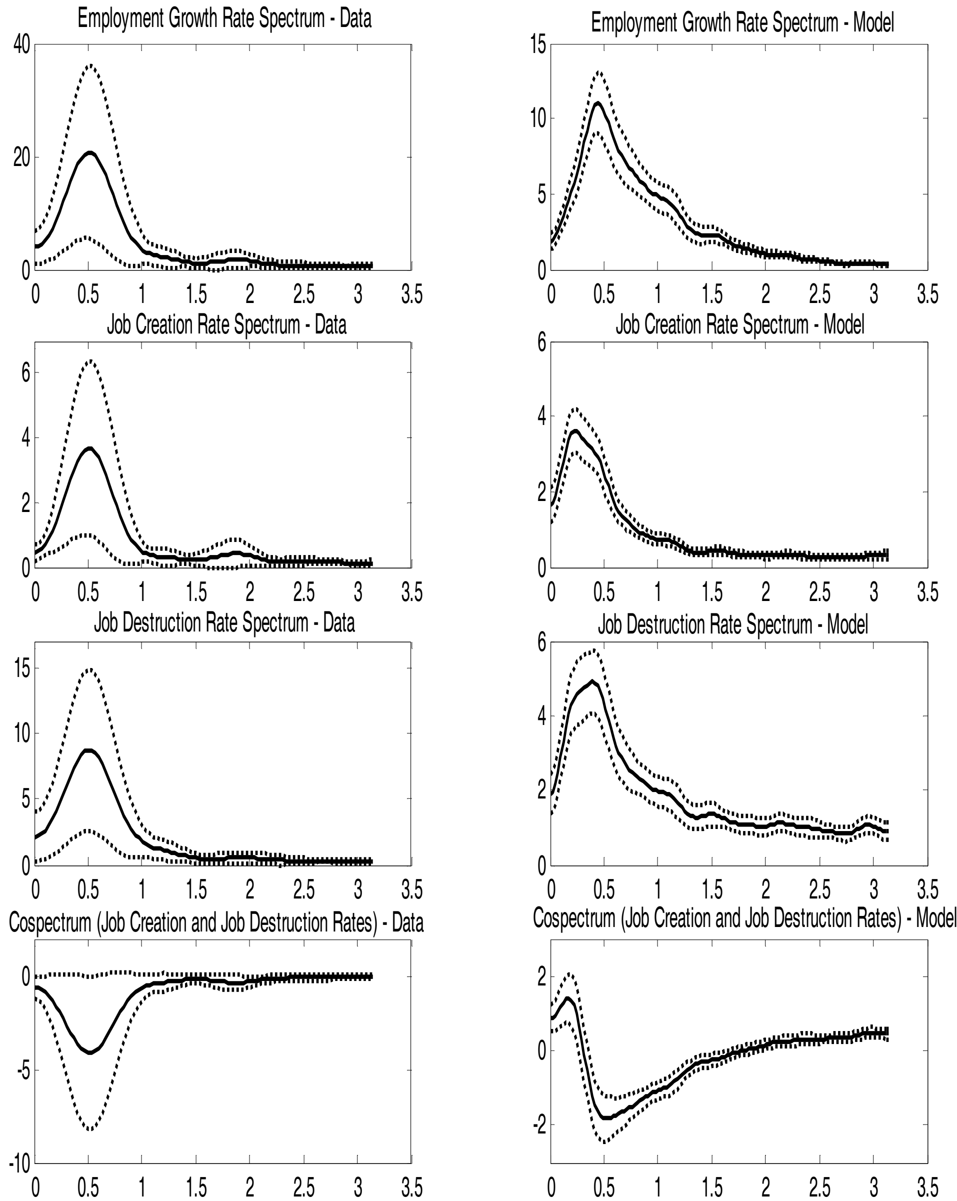


Figure 2.9
Measure of fit $\Psi(\theta)$
Alternative values of job-finding probabilities and persistence of aggregate shocks.

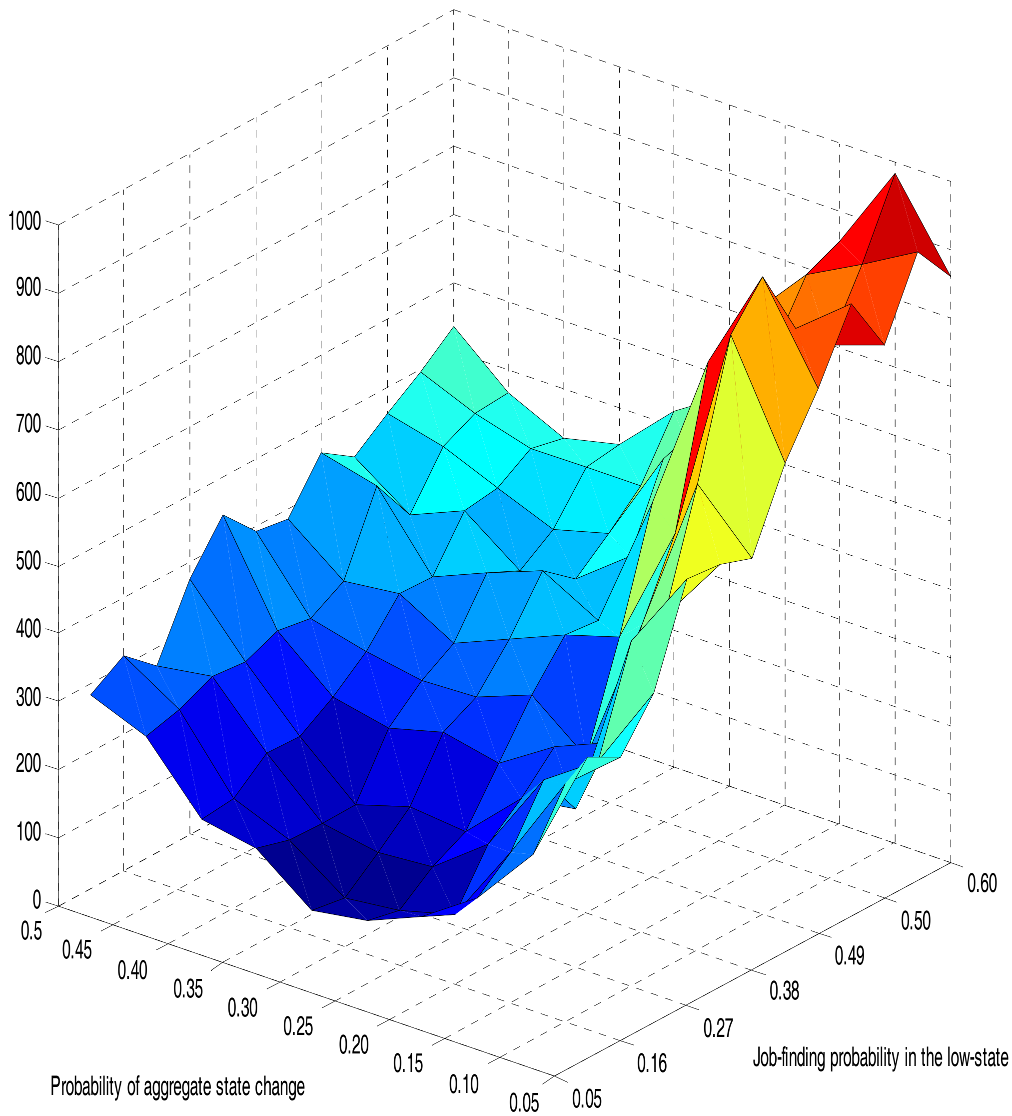


Figure 2.10
Data and Model Spectra IV (optimal job-finding probabilities)
Cole and Rogerson with job-finding probabilities as the minimum of $\Psi(\theta)$
Frequencies lower than 8 years (0 to 0.19), Frequencies between 8 and 2 years (0.19 to 0.78)
Frequencies higher than 2 years (0.78 to π)

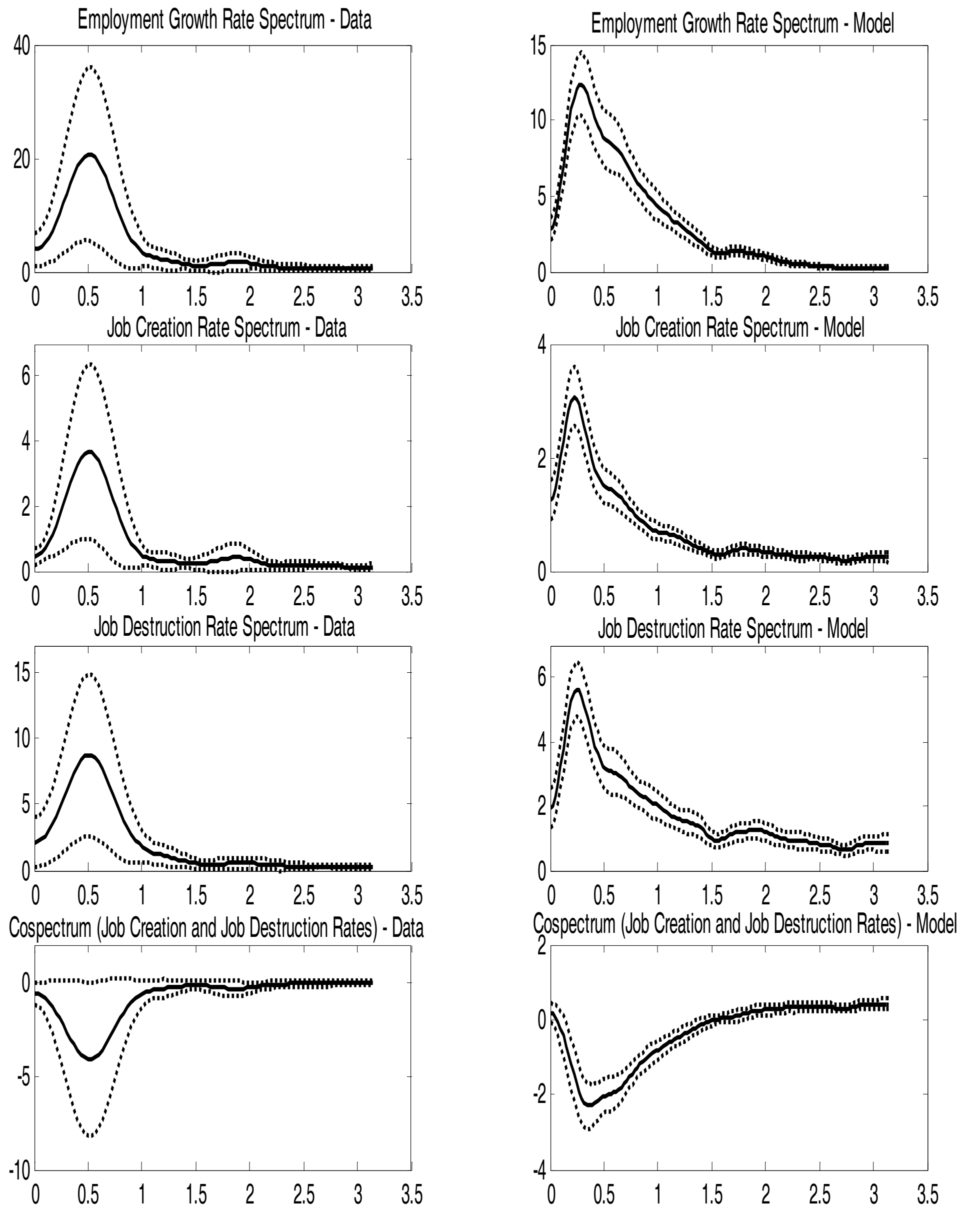


Figure 2.11
Job Creation and Job Destruction Rates I
Data and Simulated Paths for Mortensen and Pissarides

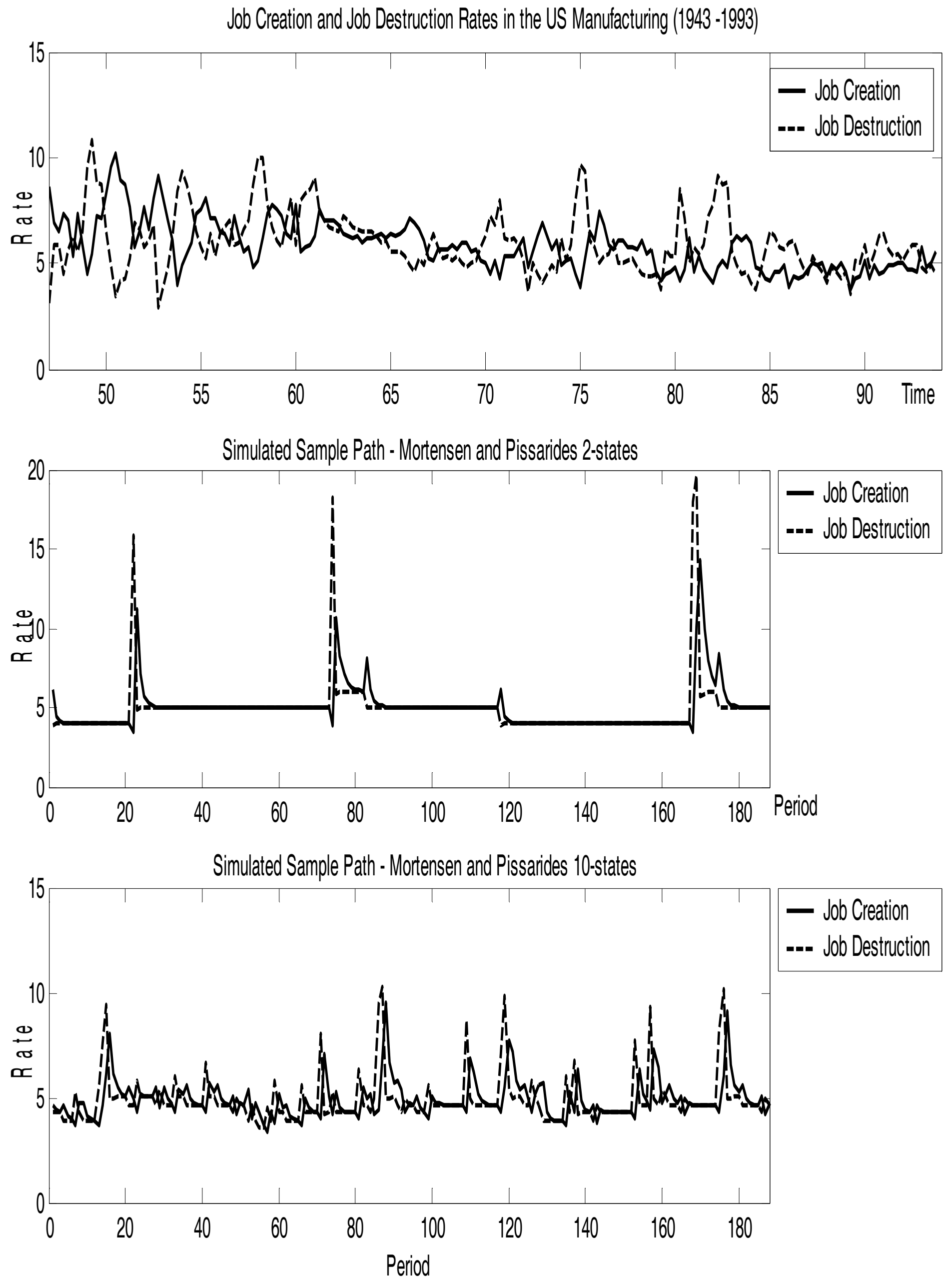


Figure 2.12
Job Creation and Job Destruction Rates II
Data and Simulated Paths for Cole and Rogerson

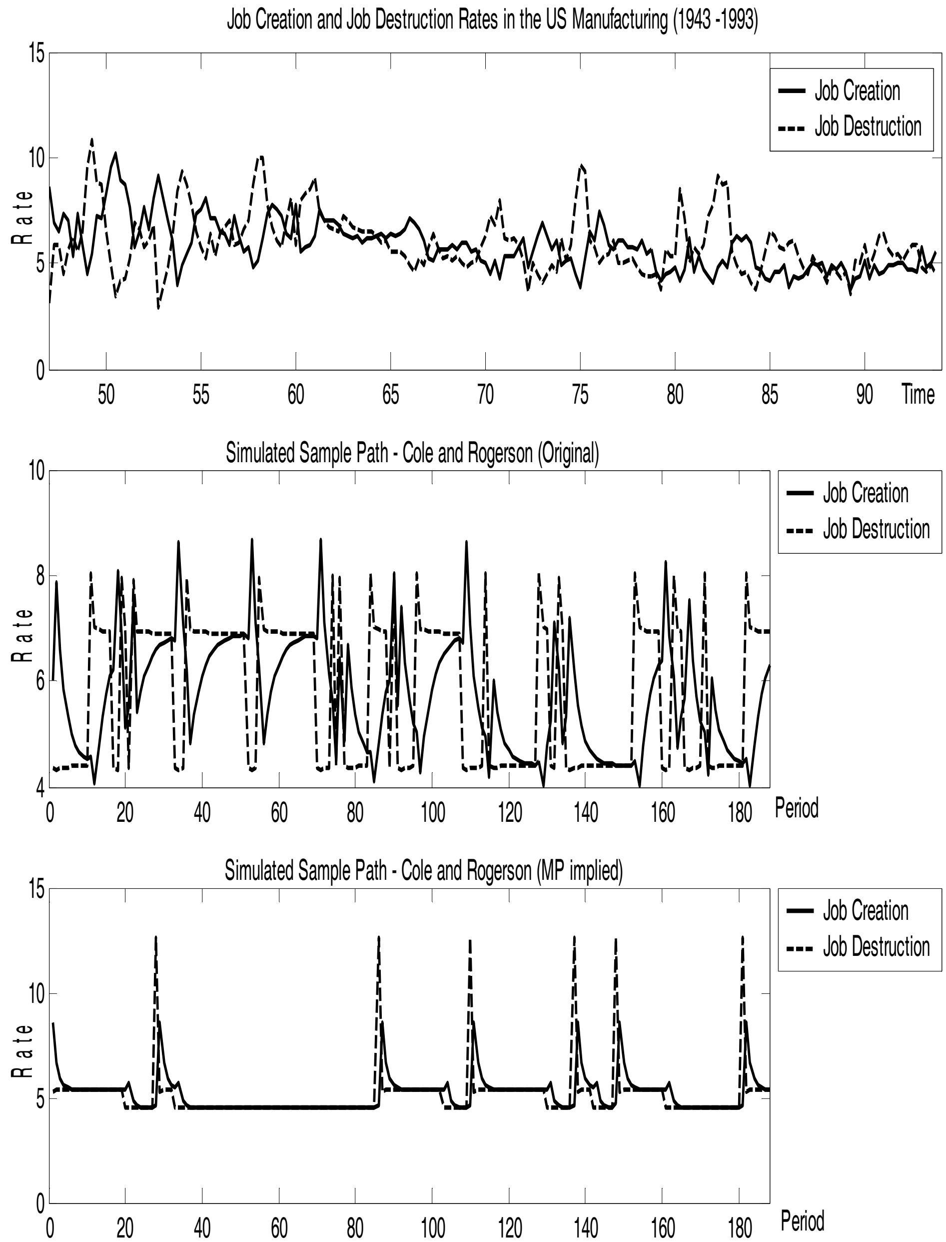


Figure 3.3
Multi-scale Decomposition for Employment

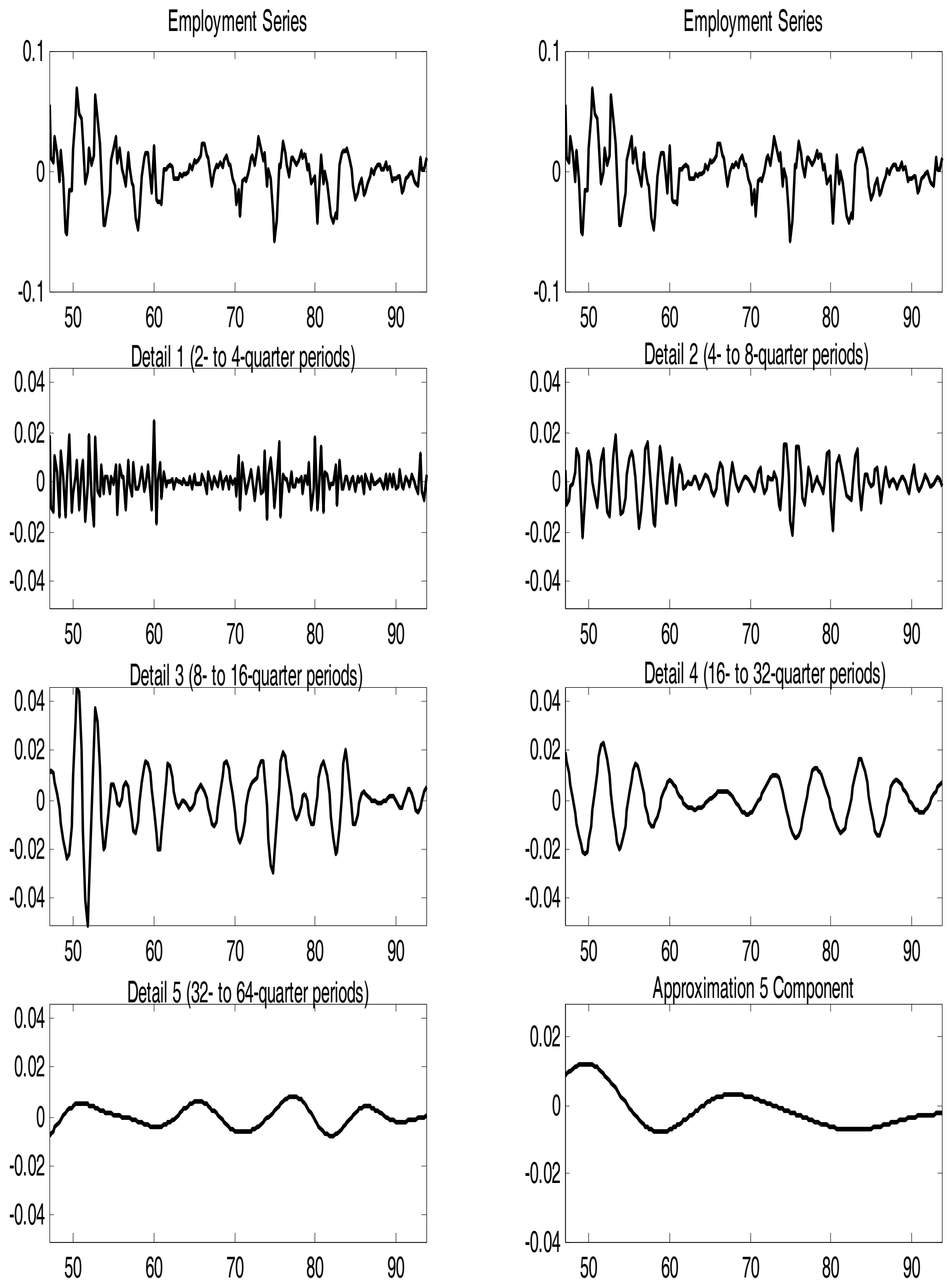


Figure 3.4
Multi-scale Decomposition for Reallocation

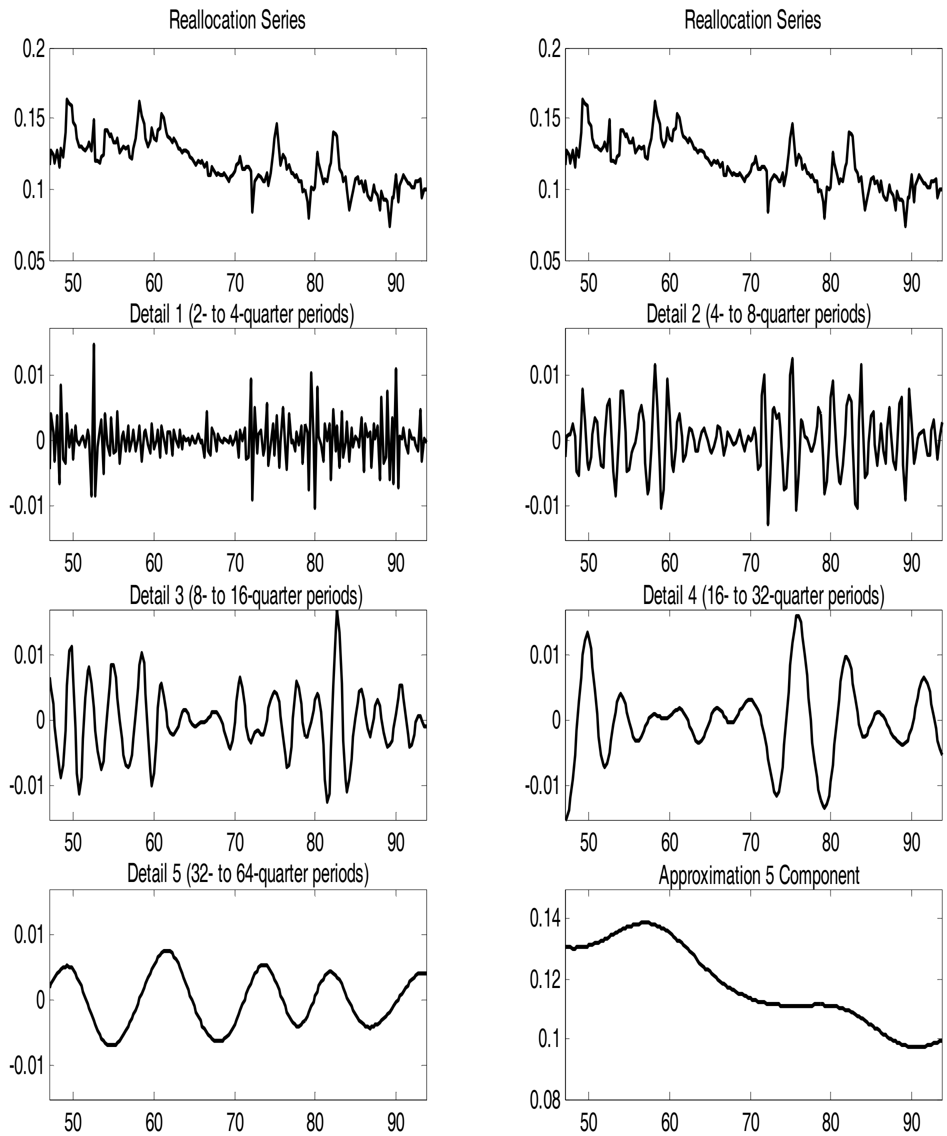


Figure 4.1
Reallocation Index One

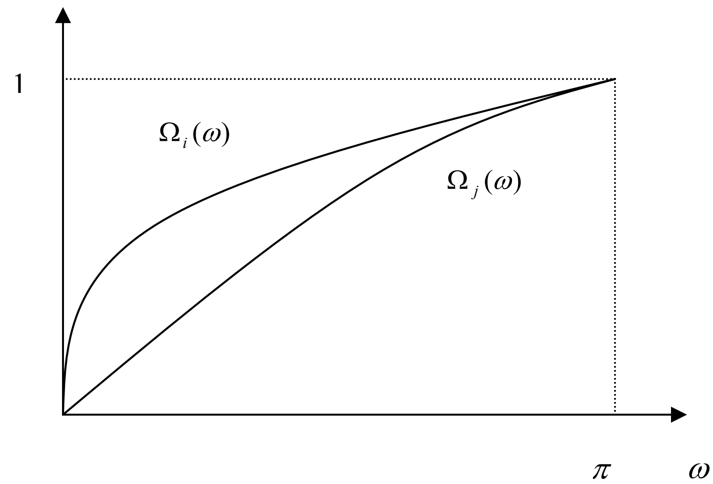



Figure 4.2
Reallocation Index One - Heterogeneous Seasonal Frequencies

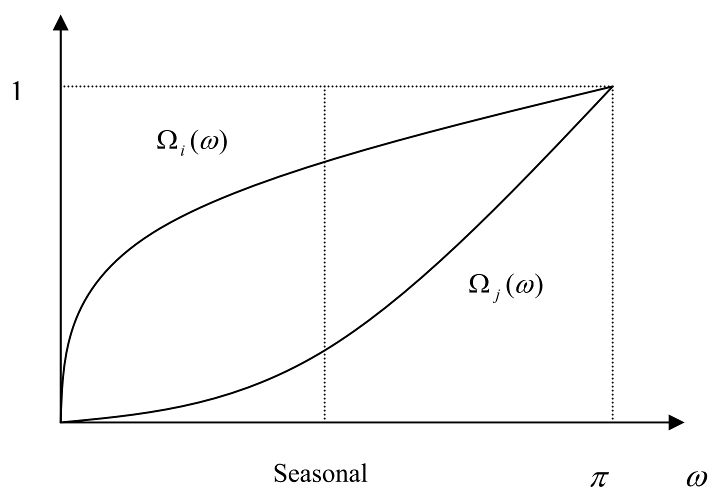


Figure 4.3
Reallocation Index two

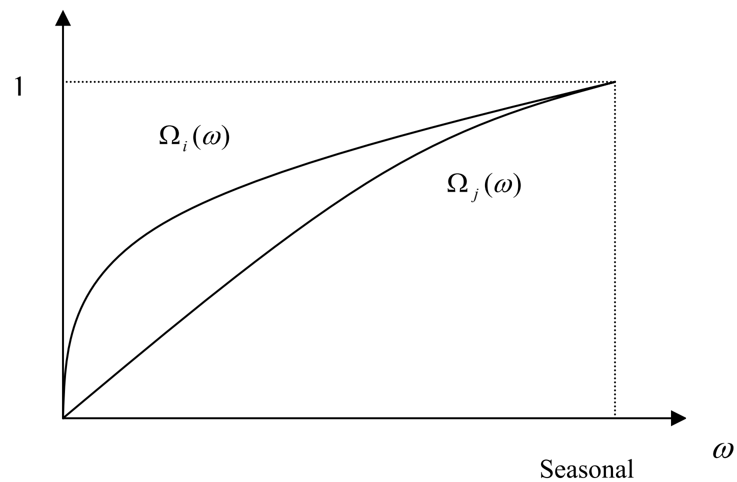



Figure 4.4
Reallocation Index three

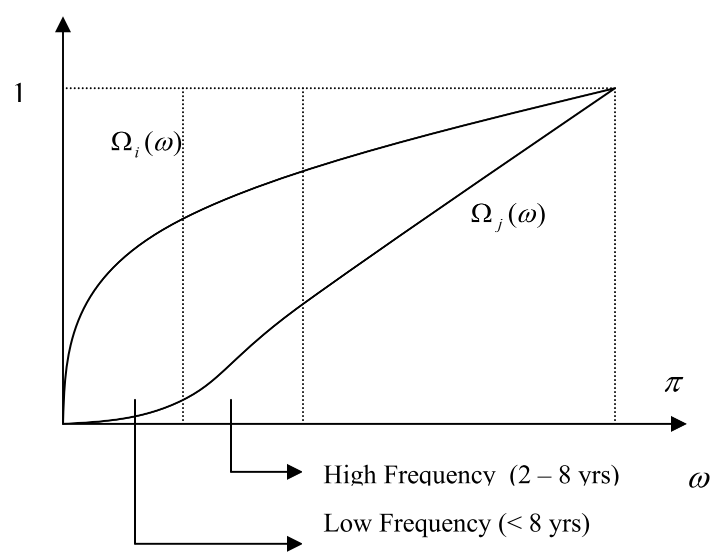
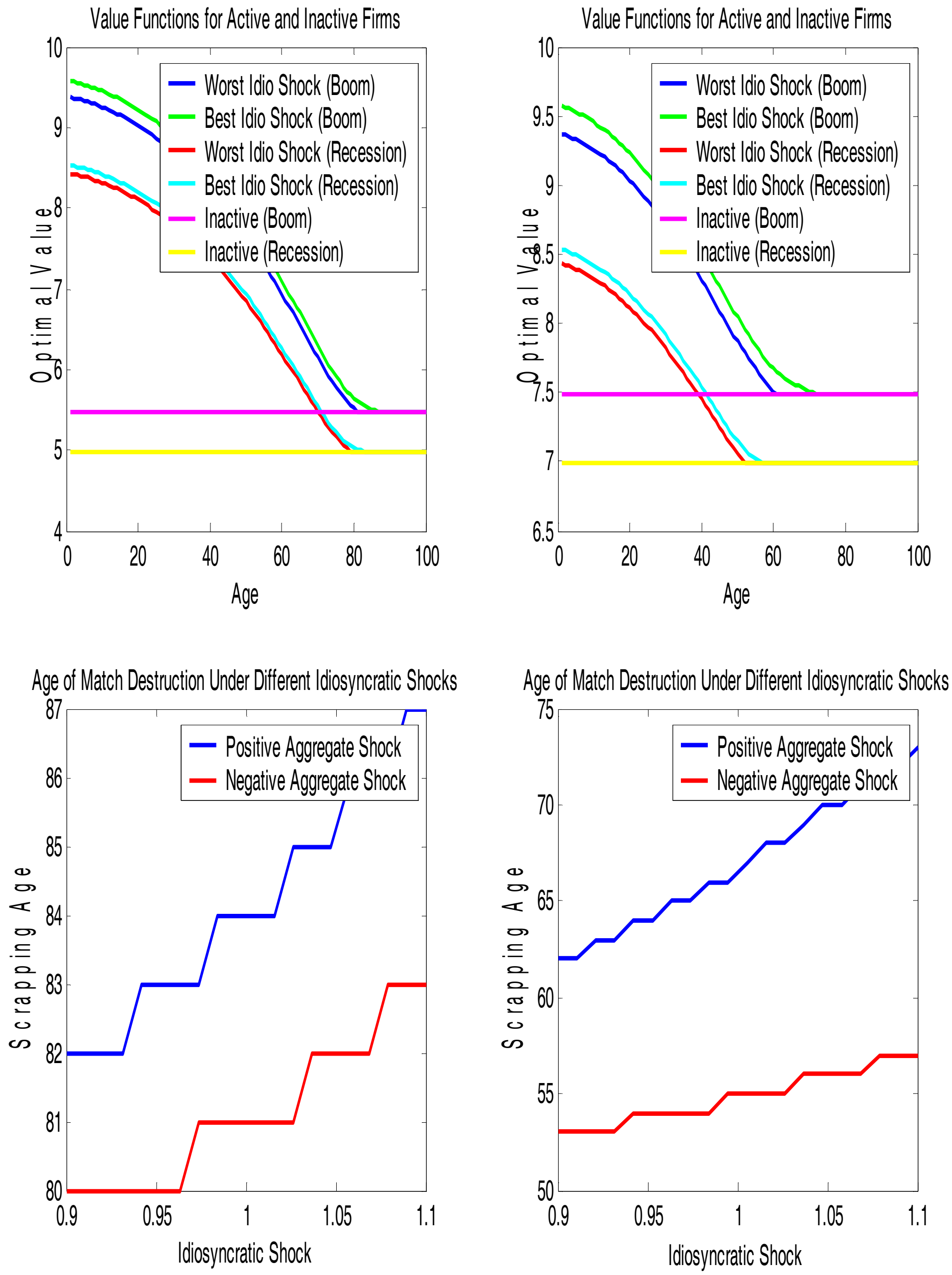


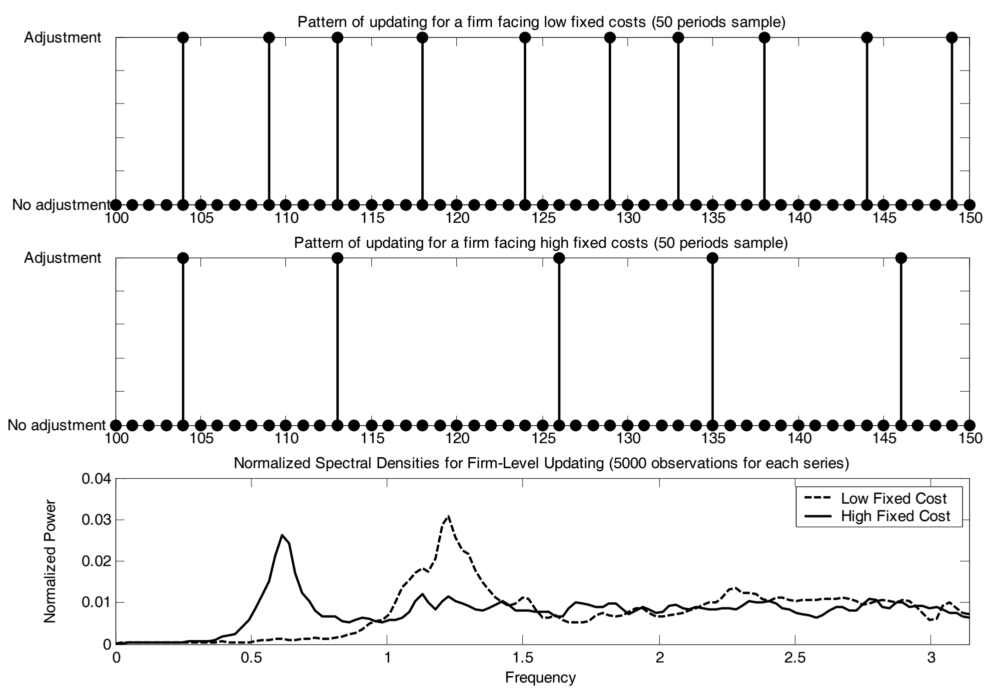
Figure 4.6
Value functions and scrapping ages
First Column: fixed cost *f* =*3* – second column: fixed cost *f*=*1*



## Appendix III: Infrequent Updating and Spectral Shape

Consider two firms that need to decide whether to update or not in each period. Due to different fixed costs, one firm updates every 5 periods on average (low fixed cost firm) while the other updates every 10 periods on average (high fixed cost firm). Even though the decision to update is a binary event, spectral techniques as described in the chapter still reveals that slow updating is associated with influential lower frequency components. The example below illustrates this point. It was constructed by simulating a 5000-period path of updating behavior for each firm and then estimating the corresponding spectral densities.

Figure III.1 – Updating and spectral Shape



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