ABSTRACT

Title of dissertation: GENERALIZED CONFIRMATORY FACTOR MIXTURE MODELS: A TOOL FOR ASSESSING FACTORIAL INVARIANCE ACROSS UNSPECIFIED POPULATIONS

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Mixture modeling is an increasingly popular analysis in applied research settings. Confirmatory factor mixture modeling can be used to test for the presence of multiple populations that differ on one or more parameters of a factor model in a sample lacking *a priori* information about population membership. There have, however, been considerable difficulties regarding convergence and parameter recovery in confirmatory factor mixture models. The present study uses a Monte Carlo simulation design to expand upon a previous study by Lubke, Muthén, & Larsen (2002) which investigated the effects on convergence and bias of introducing intercept heterogeneity across latent classes, a break from the standard approach of intercept invariance in confirmatory factor modeling when the mean structure is modeled.

Using convergence rates and percent bias as outcome measures, eight design characteristics of confirmatory factor mixture models were manipulated to investigate their effects on model performance: *N*; mixing proportion; number of indicators; factor saturation; number of heterogeneous intercepts, location of intercept heterogeneity, magnitude of intercept heterogeneity, and the difference between the latent means ($\Delta\kappa$) of the two modeled latent classes. A small portion of the present study examined another break from standard practice by having models with noninvariant factor loadings.

Higher rates of convergence and lower bias in the parameter estimates were found for models with intercept and/or factor loading noninvariance than for models that were completely invariant. All manipulated model conditions affected convergence and bias, often in the form of interaction effects, with the most influential facets after the presence of heterogeneity being *N* and $\Delta\kappa$, both having a direct relation with convergence rates and an inverse relation with bias magnitude. The findings of the present study can be used to some extent to inform design decisions by applied researchers, but breadth of conditions was prioritized over depth, so the results are better suited to guiding future methodological research into confirmatory factor mixture models. Such research might consider the effects of larger *N*s in models with complete invariance of intercepts and factor loadings, smaller values of $\Delta\kappa$ in the presence of noninvariance, and additional levels of loading heterogeneity within latent classes.

GENERAL CONFIRMATORY FACTOR MIXTURE MODELS: A TOOL FOR ASSESSING FACTORIAL INVARIANCE ACROSS UNSPECIFIED POPULATIONS

by

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Introduction

Overview of mixture models

Mixture modeling is becoming an increasingly useful tool in applied research settings. At the most basic end of the continuum, such methods might be used to determine whether a single univariate data set arose from one population or from a mixture of multiple populations differing in their univariate distributions (e.g., mean and/or variance). More advanced applications of mixture modeling are used to assess potential mixtures of populations that have different multivariate distributions (e.g., mean vectors and/or covariance matrices). Mixture analyses can even be conducted for samples in which mixtures are hypothesized to exist as the result of sampling from multiple populations differing in latent variable distributions. In all cases, the question of mixtures may be regarded as a question about parameter invariance throughout the data.

Before proceeding further into details about mixture analyses, a definition of the term *mixture analysis* should be developed. In a manner of speaking, any sample that is made up of observations from two or more populations can be thought of as a mixed sample. In ANOVA, for example, the available information about population membership is used to estimate a mean for each population represented in the sample for the purpose of statistically testing the invariance of population means. Advanced multisample latent variable analyses are commonly used in construct validation studies to test the invariance of factor structure across known populations of interest and in test validation settings in which test items themselves are assessed for differential item functioning across multiple populations.

When population membership is not known (or not made available) *a priori*, or when it is not even known whether a mixture of populations exists in a sample, similar statistical questions can be addressed, but the analyses are more complicated. It is in such situations that a mixture analysis is called upon. A mixture analysis is therefore an analysis that estimates parameters for a given number of populations hypothesized to have contributed to a single sample, without the availability of a classification variable or other such *a priori* information about population membership with which to sort the data.

Latent profile analysis (see Gibson, 1959), for example, utilizes patterns in continuous variables to infer the existence of multiple populations in a suspected data mixture and is thus a variation of traditional cluster analysis. Latent class analysis (see Dayton, 1999; McCutcheon, 1987) seeks to identify whether response patterns within categorical data are consistent with the presence of multiple populations (latent classes), each giving rise to a distinct response set in the data. Data-model fit indices (e.g., χ^2, AIC, BIC) allow for model comparison/selection and parameter invariance assessment, and the membership of individual cases in each latent class may be assessed probabilistically.

Models in item response theory (IRT) posit that individual differences along continuous latent variables are responsible for patterns in categorical item responses. The latent variables are typically used to represent cognitive factors (e.g., ability, attitude, etc.), and the measured variables are the observable manifestations of those latent variables. An example of mixture modeling applied in an IRT framework is the work by

Mislevy and Verhelst (1990), who expounded a general method for the probability of examinees' response vectors $\mathbf{x}_i$ that accommodated the possibility of J latent solution strategy classes (each occurring with probability φ_j) with differing Rasch model item parameters $\boldsymbol{\alpha}$:

$$\Pr(\mathbf{x}_i \mid \boldsymbol{\alpha}, \boldsymbol{\varphi}, \boldsymbol{\eta}) = \sum_{j=1}^{J} \varphi_j \int \Pr(\mathbf{x}_i \mid \boldsymbol{\theta}_{ij}, \boldsymbol{\phi}_{ij} = 1, \boldsymbol{\alpha}) \mathrm{g}_j(\boldsymbol{\theta}_{ij} \mid \boldsymbol{\eta}_j) d\boldsymbol{\theta}_{ij} \qquad (1)$$

where $\boldsymbol{\phi}$ indicates solution strategy, $\boldsymbol{\varphi}$ contains strategies' probabilities of usage, and $\boldsymbol{\eta}$ contains parameters specific to subjects using each strategy. By using examinee responses to create one class of apparent guessers and applying a Rasch model to a group of people who seemed to have made a legitimate attempt at responding correctly to the items, the authors employed a mixture model and improved the fit of the model, relative to applying a single-population Rasch model to the data.

Confirmatory factor mixture models

With continuous measured variables, confirmatory factor analysis (CFA) methods allow for the assessment of models positing underlying continuous latent factors. For the single-population (i.e., unmixed) CFA model, the i^{th} person's vector of values, $\mathbf{x}_i$, on the p manifest variables of the m factors, is the function

$$\mathbf{x}_i = \hat{\boldsymbol{\tau}} + \hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\xi}}_i + \hat{\boldsymbol{\delta}}_i \ , \qquad (2)$$

where $\hat{\boldsymbol{\tau}}$ is a p x 1 vector of variable intercept terms, values on the theoretical latent variable hypothesized to cause the manifest variables are contained in the m x 1 vector $\hat{\boldsymbol{\xi}}_i$, the unstandardized slope of the theoretical regression of $\mathbf{x}$ on ξ (i.e., the factor loadings) are contained in the p x m matrix $\hat{\boldsymbol{\Lambda}}$, and $\hat{\boldsymbol{\delta}}_i$ is a p x 1 vector of residuals for the i^{th} individual. For this general CFA model, the first moment implied by the model is

$$\hat{\boldsymbol{\mu}} = \mathrm{E}[\mathbf{x}_i] = \hat{\boldsymbol{\tau}} + \hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\kappa}} , \tag{3}$$

where $\hat{\boldsymbol{\kappa}}$ is the *m* x 1 vector of factor means ($\hat{\boldsymbol{\kappa}}$ is a scalar if there is only one factor). The second moment implied by the model is

$$\mathrm{E}[(\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})'] = \hat{\boldsymbol{\Sigma}} = \hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Lambda}}' + \hat{\boldsymbol{\Theta}} , \tag{4}$$

where $\hat{\boldsymbol{\Phi}}$ is the *m* x *m* factor variance-covariance matrix and $\hat{\boldsymbol{\Theta}}$ is the *p* x *p* variance-covariance matrix of residuals ($\hat{\boldsymbol{\delta}}$).

Assuming multivariate normality (specifically, *p*-variate normality), parameters in $\boldsymbol{\tau}$, $\boldsymbol{\kappa}$, $\boldsymbol{\Lambda}$, $\boldsymbol{\Phi}$, and $\boldsymbol{\Theta}$ in the single-population model are estimated in the full sample by maximizing the likelihood function

$$\prod_{i=1}^{N} (2\pi)^{-p/2} \,|\, \hat{\boldsymbol{\Sigma}} \,|^{-1/2} \exp\Big[(-.5)(\mathbf{x}_i - \hat{\boldsymbol{\mu}})' \hat{\boldsymbol{\Sigma}}^{-1} (\mathbf{x}_i - \hat{\boldsymbol{\mu}})\Big], \tag{5}$$

which is the product across observations of each observation's manifest variable values ($\mathbf{x}_i$) entered into the p-variate normal distribution with model-implied mean $\hat{\boldsymbol{\mu}}$ (Equation 3) and model-implied variance $\hat{\boldsymbol{\Sigma}}$ (Equation 4). This maximization is equivalently accomplished using the maximum likelihood fit function *F*, where

$$\hat{\mathrm{F}} = [\ln|\,\hat{\boldsymbol{\Sigma}}\,| + \mathrm{tr}(\mathbf{S}\hat{\boldsymbol{\Sigma}}^{-1}) - \ln|\,\mathbf{S}\,| - p] + (\mathbf{m} - \hat{\boldsymbol{\mu}})'\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{m} - \hat{\boldsymbol{\mu}}) , \tag{6}$$

expressed using summary statistics in the vector **m** of observed means and matrix **S** of observed variances and covariances (Bollen, 1989). For models across *J* populations for which population membership is known *a priori*, parameters in all *J* subsamples' respective matrices are estimated by maximizing the likelihood function,

$$\prod_{j=1}^{J}\prod_{i=1}^{n_j} (2\pi)^{-p/2} \,|\, \hat{\boldsymbol{\Sigma}}_j \,|^{-1/2} \exp\Big[(-.5)(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j)' \hat{\boldsymbol{\Sigma}}_j^{-1} (\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j)\Big], \tag{7}$$

or equivalently via the multisample maximum likelihood fit function, *G* (Equation 8)

$$\hat{G} = \sum_{j=1}^{J} \left({n_j}/{N} \right) \left\{ \left[\ln|\hat{\boldsymbol{\Sigma}}_j| + \mathrm{tr}(\mathbf{S}_j \hat{\boldsymbol{\Sigma}}_j^{-1}) - \ln|\mathbf{S}_j| - p \right] + \left[(\mathbf{m}_j - \hat{\boldsymbol{\mu}}_j)' \hat{\boldsymbol{\Sigma}}_j^{-1} (\mathbf{m}_j - \hat{\boldsymbol{\mu}}_j) \right] \right\}.$$

If one believes a mixture exists at the latent variable level, that is, that patterns in the measured variables reflect a mixture of multiple subpopulations differing in latent mean, latent variance, and/or latent-to-measured variable relations, then techniques combining mixture modeling with continuous latent variable methods become necessary. Such a situation arose, for example, almost four decades ago when French (1965) learned from participants that different solution strategies might have been used in achievement test responses he had factor analyzed as coming from a single population. Using follow-up questions about the solution strategies participants had employed, he divided participants into groups and found support for the hypothesis that different factor structures were operating for the different solution strategies. In this manner, French first established potential subpopulations and then tested for model and parameter invariance.

When multiple populations are believed to underlie the data but cannot be distinguished in the data *a priori*, then a generalized confirmatory factor mixture model (GCFMM) can be applied. Because we do not know which cases came from which populations (or even if there are multiple populations), we must evaluate each case in the context of each of the *J* hypothesized populations. For each of these hypothesized populations, there is a set of model parameters (i.e., $\hat{\boldsymbol{\tau}}_j, \hat{\boldsymbol{\kappa}}_j, \hat{\boldsymbol{\Lambda}}_j, \hat{\boldsymbol{\Phi}}_j, \hat{\boldsymbol{\Theta}}_j$) to be estimated, along with *J – 1* mixing proportions. Supposing there are two populations believed to underlie the data, all of these quantities are estimated simultaneously by maximizing the product across all observations of

$$L_i = \varphi L_{i1} + (1 - \varphi) L_{i2} \,, \tag{9}$$

where the likelihoods in Equation 9 are

$$L_{i1} = f(\mathbf{x}_i \,|\, \hat{\boldsymbol{\tau}}_1, \hat{\boldsymbol{\kappa}}_1, \hat{\boldsymbol{\Lambda}}_1, \hat{\boldsymbol{\Phi}}_1, \hat{\boldsymbol{\Theta}}_1) \tag{10}$$

and

$$L_{i2} = f(\mathbf{x}_i \,|\, \hat{\boldsymbol{\tau}}_2, \hat{\boldsymbol{\kappa}}_2, \hat{\boldsymbol{\Lambda}}_2, \hat{\boldsymbol{\Phi}}_2, \hat{\boldsymbol{\Theta}}_2) \,. \tag{11}$$

The probability of all observations, assuming independence, becomes

$$\prod_{i=1}^{N} \left[\sum_{j=1}^{2} \varphi_j \, f(\mathbf{x}_i \,|\, \hat{\boldsymbol{\tau}}_j, \hat{\boldsymbol{\kappa}}_j, \hat{\boldsymbol{\Lambda}}_j, \hat{\boldsymbol{\Phi}}_j, \hat{\boldsymbol{\Theta}}_j) \right], \tag{12}$$

or

$$\prod_{i=1}^{N} \left\{ \sum_{j=1}^{2} \varphi_j (2\pi)^{-p/2} \left| \left(\hat{\boldsymbol{\Sigma}}_j \right) \right|^{-1/2} \exp\left[-.5 (\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j)' \left(\hat{\boldsymbol{\Sigma}}_j \right)^{-1} (\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j) \right] \right\}, \tag{13}$$

where

$$\hat{\boldsymbol{\mu}}_j = \hat{\boldsymbol{\tau}}_j + \hat{\boldsymbol{\Lambda}}_j \hat{\boldsymbol{\kappa}}_j \tag{14}$$

and

$$\hat{\boldsymbol{\Sigma}}_j = \hat{\boldsymbol{\Lambda}}_j \hat{\boldsymbol{\Phi}}_j \hat{\boldsymbol{\Lambda}}_j' + \hat{\boldsymbol{\Theta}}_j \,. \tag{15}$$

Note that in addition to

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factor loadings are by convention constrained to be invariant across populations in a mixture model when the mean structure is analyzed.

Various restrictions on the CFA mixture model yield different types of mixture tests. Restricting the corresponding factor means, factor variances, and factor covariances to be equal across populations tests a mixture of indicator covariance patterns among the populations. In this manner, the mixture analysis is essentially a multipopulation CFA but with unknown populations. To test for a mixture at the latent variable level, the factor loadings are fixed to be equal across populations while the factor means, factor variances, or both are freely estimated (along with the manifest variable error variances). Note that with only the factor means freely estimated across populations, the mixture analysis is basically a structured means model but with unknown population membership for the observations (see e.g., Hancock, 2004).

<u>General structural equation mixture models</u>

Equation 13 is a CFA-specific version of the following general formula for a *J*-population latent variable mixture model:

$$\prod_{i=1}^{N}\left\{\sum_{j=1}^{J}\varphi_j(2\pi)^{-p/2}\left|\left(\hat{\boldsymbol{\Sigma}}_j\right)\right|^{-1/2}\exp\left[(-.5)(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_j)'\left(\hat{\boldsymbol{\Sigma}}_j\right)^{-1}(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_j)\right]\right\}. \qquad (16)$$

For confirmatory factor analysis, $\hat{\boldsymbol{\Sigma}}_j$ is replaced per Equation 15, and $\hat{\boldsymbol{\mu}}_j$ is replaced per Equation 14 in order to give Equation 13. For measured-variable path analysis (MVPA) and latent-variable path analysis (LVPA), the substitutions for the model-implied mean vector are different, and for the model-implied variance-covariance matrix, the substitutions are different and quite a bit more complicated.

Measured-variable path analysis mixture models. For MVPA mixture models, the means of the *t* exogenous variables (i.e., variables modeled to cause other variables in the model without themselves modeled to be caused by any variables) are modeled to be the intercepts. The *w* endogenous variables (i.e., variables that are modeled to be caused by one or more variables) are modeled as a function of the exogenous variables and potentially as a function of the other endogenous variables,

$$\mathbf{y}_i = \hat{\boldsymbol{\tau}}_j + \hat{\boldsymbol{\Gamma}}_j \mathbf{x}_i + \hat{\mathbf{B}}_j \mathbf{y}_i + \hat{\boldsymbol{\varepsilon}}_{ij}, \qquad (17)$$

where $\hat{\mathbf{B}}_j$ is a *w* x *w* matrix of the effects of the endogenous variables on each other, $\hat{\boldsymbol{\Gamma}}_j$ is a *t* x *w* matrix of the effects of the exogenous variables on the endogenous variables in the model, and $\hat{\boldsymbol{\varepsilon}}_{ij}$ is the model-implied *w* x 1 vector of error variances for the endogenous variables. The model-implied mean vector for the endogenous variables is therefore

$$\text{E[y]} = \hat{\boldsymbol{\mu}}_{yj} = (\mathbf{I} - \hat{\mathbf{B}}_j)^{-1} \hat{\boldsymbol{\tau}}_j + (\mathbf{I} - \hat{\mathbf{B}}_j)^{-1} \hat{\boldsymbol{\Gamma}}_j \hat{\boldsymbol{\mu}}_{xj}, \qquad (18)$$

where **I** is the identity matrix. The data vector in the general mixture equation, although labeled here with "x", would contain values for the y-variables and for the x-variables. With the y-values appearing first in the column vector and the x-values below them, the model-implied mean vector would first have the values computed per Equation 16 followed by the sample means of the x-variables.

For MVPA mixture models, the *p* x *p* model-implied variance-covariance matrix, $\hat{\boldsymbol{\Sigma}}_j$, where $p = t + w$, and where

$$\hat{\boldsymbol{\Sigma}}_j = \text{E}[(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j)(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_j)'], \qquad (19)$$

can be divided into four submatrices, each of which can be computed separately from the other three. The upper left (UL) submatrix is the w x w model-implied variance-covariance matrix for just the endogenous variables, and is computed as

$$\mathrm{UL} \equiv (\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}(\hat{\mathbf{\Gamma}}_j\hat{\mathbf{\Phi}}_j\hat{\mathbf{\Gamma}}_j{}' + \hat{\mathbf{\Psi}}_j)(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}{}' + \hat{\mathbf{\Theta}}_j, \tag{20}$$

where $\hat{\mathbf{\Psi}}_j$, is the w x w variance-covariance matrix of the errors, $\hat{\boldsymbol{\varepsilon}}_{ij}$ (Jöreskog & Sörbom, 1988). The upper right (UR) submatrix is a w x t matrix of covariances between the endogenous variables and the exogenous variables, the equation for which is

$$\mathrm{UR} \equiv (\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}\hat{\mathbf{\Gamma}}_j\hat{\mathbf{\Phi}}_j. \tag{21}$$

The lower left (LL) submatrix is simply the transpose of the upper right submatrix or

$$\mathrm{LL} \equiv \hat{\mathbf{\Phi}}_j\hat{\mathbf{\Gamma}}_j{}'(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}{}'. \tag{22}$$

The matrix $\hat{\mathbf{\Phi}}_j$ is the t x t variance-covariance matrix of the exogenous variables, making it equal to its transpose and also making it the only quantity in the lower right (LR) submatrix of the overall variance-covariance matrix

$$\mathrm{LR} \equiv \hat{\mathbf{\Phi}}_j. \tag{23}$$

Each of these submatrices is arranged as described in one p x p matrix to form the model-implied variance-covariance matrix to be used in the general equation for MVPA mixture models. Recall that as the parameters in the model-implied mean vectors and the model-implied variance-covariance matrix are being estimated for each of the J populations, the mixing proportions are also being estimated in the iterative process of maximizing the likelihood function of the data.

<u>Latent-variable path analysis mixture models.</u> In LVPA mixture analysis, the exogenous factors have multiple manifest indicators while being modeled to cause one or

more endogenous factors, which themselves have multiple manifest indicators. Exogenous factors may covary amongst themselves, endogenous factors may cause other endogenous factors, and the disturbances of the endogenous factors may covary. If a mixture model is to be estimated, then Equation 16 can again be called upon, with the appropriate substitutions for $\hat{\boldsymbol{\mu}}_j$ and $\hat{\boldsymbol{\Sigma}}_j$ in order to estimate the parameters for each of the J populations and to estimate the mixing proportions.

The model-implied data vector for the manifest indicators of the exogenous factors is computed as per Equation 2, while the model-implied means of the exogenous factors' indicators are computed as per Equation 3. The model-implied data vector for the manifest indicators of the endogenous factors is computed as

$$\mathbf{y}_i = \hat{\boldsymbol{\tau}}_{yj} + \hat{\boldsymbol{\Lambda}}_{yj}\hat{\boldsymbol{\eta}}_j + \hat{\boldsymbol{\varepsilon}}_{ij} \ , \tag{24}$$

where $\hat{\boldsymbol{\eta}}_j$ is the w x 1 model-implied vector of values on the endogenous latent variables, computed by

$$\hat{\boldsymbol{\eta}}_{ij} = \hat{\boldsymbol{\alpha}}_j + \hat{\boldsymbol{\Gamma}}_j\hat{\boldsymbol{\xi}}_i + \hat{\mathbf{B}}_j\hat{\boldsymbol{\eta}}_{ij} + \hat{\boldsymbol{\zeta}}_{ij} \ , \tag{25}$$

where $\hat{\boldsymbol{\alpha}}_j$ is the w x 1 vector of intercepts for the endogenous factors and $\hat{\boldsymbol{\zeta}}_{ij}$ is the w x 1 model-implied vector of disturbances (errors) for the endogenous latent variables. The model-implied mean vector for the y-variables is

$$\hat{\boldsymbol{\mu}}_{yj} = \hat{\boldsymbol{\tau}}_{yj} + \hat{\boldsymbol{\Lambda}}_j\hat{\boldsymbol{\kappa}}_{\eta j} \ , \tag{26}$$

where $\hat{\boldsymbol{\kappa}}_{\eta j}$ is the model-implied mean vector of the endogenous latent variables, computed as

$$\hat{\boldsymbol{\kappa}}_{\eta j} = (\mathbf{I} - \hat{\mathbf{B}}_j)^{-1}\hat{\boldsymbol{\alpha}}_j + (\mathbf{I} - \hat{\mathbf{B}}_j)^{-1}\hat{\boldsymbol{\Gamma}}_j\hat{\boldsymbol{\kappa}}_{\xi j} \ . \tag{27}$$

The model-implied variance-covariance matrix is similar in form to that of MVPA mixture modeling in that the *p* x *p* matrix can be considered in four distinct submatrices: variance-covariance matrix for the endogenous variables (UL); the covariances between the endogenous variables and the exogenous variables (UR); the transpose of that matrix (LL); and the variance-covariance matrix of the exogenous variables (LR). The UL submatrix,

$$\mathrm{UL} \equiv \hat{\mathbf{\Lambda}}_{yj}(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}(\hat{\mathbf{\Gamma}}_j\hat{\mathbf{\Phi}}_j\hat{\mathbf{\Gamma}}_j{}'+\hat{\mathbf{\Psi}}_j)(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}{}'\hat{\mathbf{\Lambda}}_{yj}{}'+\hat{\mathbf{\Theta}}_{\varepsilon j}\,, \qquad (28)$$

incorporates multiple non-unity factor loadings for the endogenous factor by premultiplying the main term of the equation by $\hat{\mathbf{\Lambda}}_{yj}$ and postmultiplying by its transpose (Jöreskog & Sörbom, 1988). For the upper right submatrix, endogenous and exogenous variables are crossed, so instead of using $\hat{\mathbf{\Lambda}}_{yj}$ and its transpose, we use $\hat{\mathbf{\Lambda}}_{yj}$ with the transpose of the loadings of the exogenous factor indicators,

$$\mathrm{UR} \equiv \hat{\mathbf{\Lambda}}_{yj}(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}\hat{\mathbf{\Gamma}}_j\hat{\mathbf{\Phi}}_j\hat{\mathbf{\Lambda}}_{xj}{}'\,, \qquad (29)$$

the transpose of which gives the lower left submatrix,

$$\mathrm{LL} \equiv \hat{\mathbf{\Lambda}}_{xj}\hat{\mathbf{\Phi}}_j\hat{\mathbf{\Gamma}}_j{}'(\mathbf{I}-\hat{\mathbf{B}}_j)^{-1}{}'\hat{\mathbf{\Lambda}}_{yj}{}'\,. \qquad (30)$$

The lower right submatrix, the model-implied variance-covariance matrix for the exogenous variables, is identical to $\hat{\mathbf{\Sigma}}_j$ in CFA,

$$\mathrm{LR} \equiv \hat{\mathbf{\Lambda}}_{xj}\hat{\mathbf{\Phi}}_j\hat{\mathbf{\Lambda}}_{xj}{}'+\hat{\mathbf{\Theta}}_{\delta j}\,. \qquad (31)$$

<u>Implementation issues in continuous latent variable mixture modeling</u>

To date, the critical issues of model identification and parameter estimation have not been explored extensively for continuous latent variable mixture analyses. Primary attention has been given only to a highly restricted form of GCFMM, that of the generalized growth mixture model (GGMM; Muthén, 2001). Traditional latent growth curve models evaluate longitudinal change in a measured variable in terms of specific growth components. Linear models, for example, typically express the amount of the variable at each time point as a function of a latent initial amount and a latent growth rate, where observations are likely to differ in their amounts of these latent factors. With regard to the less restricted GCFMM, however, very little methodological or applied work has been done to date. Such models appear to present unusually difficult problems regarding model identification, solution convergence, and parameter accuracy.

In a recent unpublished investigation, Lubke, Muthén, and Larsen (2002) conducted a Monte Carlo study in an attempt to investigate these problems and to develop potential remedies. These authors simulated data for eight measured variables all loading on a single factor, where the data were a mixture of three latent classes (φ_1 = .4, φ_2 = .3, and φ_3 = .3). The factor loadings were constrained to be equal across classes as were the error variances. The researchers varied the number of classes in the latent mixture model fit to the data, the number of intercepts that were free to vary across classes (zero, two, four, or eight out of eight), and the percentage of observations out of N = 5000 for which the true group membership was included in the analysis. Their results demonstrated that relative to a model of complete invariance, the presence of at least two noninvariant item intercepts yields solutions that have much better parameter recovery and greater efficacy at placing observations into their correct classes. They also found that using prior knowledge of group membership improved the accuracy of parameter estimates.

Manipulation of model conditions yielded useful results in the above study, but several potentially influential design characteristics were held constant, so the results are somewhat limited. Simulation studies have found that convergence rate and parameter recovery in single-population CFA models are affected by sample size, the number of indicators, and the magnitude of the loadings (Gagné & Hancock, 2002; Marsh, Hau, Balla, & Grayson, 1998). These conditions may also affect the performance of CFA mixture models, as could the mean structure ($\boldsymbol{\kappa}$ and/or $\boldsymbol{\tau}$) and the mixing proportions (φ).

As there are several design characteristics of confirmatory factor mixture models that influence convergence and parameter estimate accuracy, a thorough, systematic treatment of all of them in one study would be unwieldy. The present study is therefore meant to expand the Lubke et al. (2002) study by exploring more design characteristics than they did but varying each by incorporating only two or three levels. The cursory manipulation of design characteristics in the present study limits its capacity to assist applied researchers in making design decisions, as does the fact that most of the design characteristics manipulated herein are not actually under the control of an applied researcher. Such assistance, however, is but a secondary purpose of the present study. The present study primarily seeks to inform the direction of future research that might explore fewer design characteristics but explore them in greater depth.

Chapter 2

Method

Primary design: Partial invariance of intercepts

Before describing the conditions that were manipulated in the present study, the design characteristics that were held constant will be presented. As was the case in the Lubke et al. (2002) study, only single-factor models were used. The factor variance in each data-generating population was 1, but for model estimation, the factor variance was neither fixed to 1 nor constrained equal across classes. The number of latent classes was also not varied, but holding it at two in the present study offered some variability relative to the three latent classes modeled by Lubke et al.

The sample size of 5000 used in the Lubke et al. (2002) study is reasonable for simulating large-scale assessments, but there is strong potential for the application of GCFMM to situations involving smaller sample sizes. The present study used simulated data for whole samples of 200, 500, and 1000 observations to investigate such situations. The number of manifest variables (p) in the cells of the design was varied at four and eight for the lone factor, but within each cell, the number of indicators of the factor was constant for the two latent classes. Among the design characteristics manipulated in the present study, N and p were the only characteristics which, in an applied setting, are under the control of the researcher to a functional extent.

The magnitudes of the factor loadings (λ) were combinations of .8 and .4, which differed within and across latent classes, depending on the cell. Three within-cell loading combinations (lc) were used: 1) 100% $\lambda = .4$ ($lc = 4$); 2) 50% $\lambda = .8$, 50% $\lambda = .4$ ($lc = 6$);

and 3) 100% $\lambda = .8$ ($lc = 8$). When both .8 and .4 were used in the same factor, corresponding loadings across classes were equal (e.g., with eight indicators, $lc = 6$ had λ_{11} through $\lambda_{41} = .8$ in both classes and λ_{51} through $\lambda_{81} = .4$ in both classes). The error variances were .36 for indicators that had factor loadings of .8, and the error variances were .84 for indicators with $\lambda = .4$. For factor identification purposes, the loading of the second indicator was fixed to its true value in all cells. (Note: This does not artificially improve parameter recovery; fixing the loading to any other value would necessitate rescaling all parameter values in order to examine bias which would then yield the same values of proportional bias.)

Three conditions of intercept noninvariance were investigated: completely invariant intercepts; one noninvariant intercept; and two noninvariant intercepts. Lubke et al. (2002) found that relative to a model with completely invariant intercepts, parameter recovery improved appreciably when there were two noninvariant intercepts across the populations, but having more than two generally yielded slight or no improvement beyond that found with two. By examining the effects of having only one intercept free to vary, the present study made an effort to clarify whether the gain in parameter accuracy is a function of increasing the number of free intercepts or simply a function of having any free intercepts. When the intercepts were homogeneous across classes, the values were arbitrarily chosen to be {2 0 4 5} and {6 0 7 2 1 4 8 3} for the cells with four and eight indicators, respectively.

For the heterogeneous intercept conditions, two additional variables were manipulated: magnitude of the intercept difference across classes (standardized difference of 1.0 and 1.5) and for $lc = 6$, the indicators for which the intercepts differed across classes. With one heterogeneous intercept and all loadings equal, τ_1 was higher in the second latent class. For lc = 6, the p^{th} indicator had a different loading than half of the other indicators; this combination was therefore run once with τ_1 higher in the second class and a second time with only τ_p higher in the second latent class. When two intercepts differed across classes, both τ_1 and τ_p were higher in the second latent class, and two combinations of magnitude of heterogeneity were used: 1.0/1.0 and 1.5/1.5.

The standardized difference between the latent means ($\Delta\kappa$) was manipulated, and it had two levels: 2.0 and 2.5. These two levels of latent mean difference, when multiplied through the factor loadings, contributed a range of additional standardized differences in the observed means from .8 (λ = .4, $\Delta\kappa$ = 2.0) up to 2.0 (λ = .8, $\Delta\kappa$ = 2.5). The mixing proportion (φ) was varied in the present study to be .50 or .70. The class membership of each observation was known, but that information was not incorporated into the analyses, so the analyses were conducted as though the presence and potential nature of a mixture was not known, except for the accurate "hypothesis" that two populations underlie the data. The eight manipulated model conditions (*N*, *p*, *lc*, number of heterogeneous intercepts, $\Delta\tau$, location of intercept heterogeneity, $\Delta\kappa$, and φ) were crossed to the extent possible, which resulted in a total of 408 cells.

<u>Secondary design: Partial invariance of factor loadings</u>

An additional 12 cells were incorporated into the design to investigate partial invariance of factor loadings. It is standard practice to constrain loadings to be equal across classes when the mean structure is modeled (Bollen, 1989), but the effects of having partial invariance in the factor loadings has not been explored in mixture models. To begin to address this issue, a fourth loading combination was included in the design:

one class with a factor with 100% $\lambda = .8$ and the other class with a factor with 75% $\lambda = .8$ and 25% $\lambda = .4$. This combination created asymmetry in the factor structure between the classes, so .50 was the only value of φ used for these cells. When indicators have heterogeneous loadings across classes, the issue of intercept invariance is not meaningful, so a set of intercepts was chosen such that τ_1 and τ_p differed across classes by 1.5 standard deviations. The remaining design characteristics (N, p, and $\Delta\kappa$) had all of and only their levels described for the primary study.

Description of outcome measures

Convergence. With an upper limit of 20,000 replications, enough replications were attempted for each cell to obtain 500 properly converged replications. A replication was considered properly converged if it both converged to a solution according to the program's default convergence criterion and had parameter estimates that were within the range of possible values (e.g., no negative variances). Convergence was measured by the number of replications needed to acquire 500 properly converged replications (C), with failure to achieve 500 after 20,000 replications described as $C > 20{,}000$. For cells with $C > 20{,}000$ but at least 200 proper solutions, the number of proper solutions was specifically reported instead of C, and bias was computed but was not included in detailed accounts of bias behavior. A stop criterion was used such that after every 2000 replications, if the percentage of properly converged solutions was statistically significantly less than 1% ($p < .025$), then the simulation was ended for that cell. Such cells were designated to have $C > 20{,}000$, and it was inferred that they would not have reached the 200/20000 designated for bias computation.

Bias. Averaging across the proper solutions within a cell, the accuracy of the parameter estimates in each cell was assessed by computing the percent bias,

$$\text{percent bias} = 100(\text{average estimate} - \text{parameter}) / \text{parameter}. \qquad (32)$$

Bias in the loading, the error variance, and the intercept for the first and p^{th} indicators of the factor in each latent class was evaluated, as was bias in the variance of the factor in each class, the difference between the means of the two factors, and the mixing proportion. Positive values for percent bias occurred for estimates that were above the true value by the percent magnitude listed, whereas negative values for percent bias indicate that the average estimate was the percent magnitude below the true value.

Computer software and programs

Two statistical software packages were used for the simulations, SAS (v8.1) and Mplus (v2.02; Muthén & Muthén, 1999), in a four-stage process. Stage 1 was the generation of the mixed sample, which was done in SAS. Data were drawn from each of two multivariate normally distributed populations in accordance with the mixing proportion for a particular cell. Intercepts and applicable contributions to the scores of the factor mean were then added to the values. The cases were then combined into a single sample and exported out of SAS to Mplus for stage 2, which was the mixture analysis itself. The models in stage 2 were always the correct model in terms of factor structure, with all manifest variables loading onto a single factor, and in terms of intercept heterogeneity, with the number of intercepts free to differ across classes in the computer program being the same as the number of noninvariant intercepts across the data-generating populations.

In stage 3, SAS was used to obtain the quantities of interest from the Mplus output. Stages 1-3 were repeated until any one of the aforementioned conditions for simulation termination was met. In the final stage, SAS was used to compute the averages and variances across the successful replications in each cell and then to export that information, along with the convergence information, into text files. Appendix A contains an example of a SAS program used in this study, the supporting batch file for the SAS code, and an example of Mplus code used for conducting the mixture analyses.

Chapter 3

Results

Data regarding convergence, bias, and standard errors are provided for the 72 cells with homogeneous intercepts, the 12 cells with heterogeneous intercepts and heterogeneous loadings, and then the 336 cells with only heterogeneous intercepts. Convergence data are discussed all in one section, but the bias information is separated into three sections. Convergence data have their own tables, while percent bias data tables include or are followed by tabulated standard errors of the corresponding parameters.

For the presentation of bias, no formal cutoffs are used to label bias as high, low, or anything in between. In general, estimates that contained 10% bias or more in either direction were considered definitely biased, and biases less than 3% in magnitude were regarded as being quite small. The 10% "cutoff" is applied casually, but the 3% level has an important implication: The patterns described herein of the changes in percent bias as a function of the design characteristics are *not* assumed to hold once the magnitude of the bias drops below 3%. Some of the trends held below 3%, but quite a few yielded to erratic or indiscernible patterns.

For exhibition purposes, standard errors are presented for each of the parameter estimates. For the cells with heterogeneous intercepts but homogeneous factor loadings, the standard errors for a given parameter estimate are provided in a separate table immediately following the bias table for that parameter estimate. The other cells have the standard errors presented in the same table as the bias data. Bias in the standard errors could not be computed, because true standard errors could not be obtained, so the listed standard errors are simply the estimated standard errors, not percent biases. Except for the mixing proportion, the standard errors are the average standard error across the successful replications for a given parameter in a given cell. For the mixing proportion, a standard error was not available for each replication, so an empirical standard error was directly computed as the variance of the mixing proportion across the successful replications in a given cell.

Convergence

Convergence data for the 72 cells in which all intercepts were homogeneous across classes can be found in Table 1. Although *C* is the primary quantity of interest for convergence, most of the cells with homogeneous intercepts did not have a value of *C*, for failing to achieve 500 successful replications in the maximum allotted 20,000 attempts. Such cells instead have the attained number of proper solutions tabulated. A distinction is made between cells that had a convergence rate of at least 1% after 20,000 attempts and those that were stopped from reaching 20,000 attempts for having a convergence rate significantly ($p < .025$) below 1% at a rate checkpoint. Table 1 therefore contains numbers in three different type settings. Standard typeface is used for values of *C*. Italics are used for the number of successful replications in cells that were stopped before 20,000 attempts, with all but one of those cells ($p = 4$, $\varphi = .7$, $\Delta\kappa = 2.5$, and lc = 6 with $N = 200$) being stopped after 2000 attempts (the one exception ran 6000 attempts). For cells that did run the full 20,000 but failed to reach 500 successful replications, the number of successful replications is underlined. For the 12 cells that had heterogeneous factor loadings, Table 2 provides convergence information, using the same key for the typefaces as Table 1. Table 3 contains values of *C* for the 336 cells that had at least one heterogeneous intercept but homogeneous factor loadings.

For the homogeneous intercept cells, convergence rates were very low, with 50 cells being stopped after 2000 replications (33 of which had 0 successes at that point). Of the 6 cells that were not stopped before reaching 20,000 but failed to reach 500 successes, the highest convergence rate was 1.94%. Fifteen cells did attain 500 replications that had a proper solution, including all 12 cells that had four manifest variables loading at .8. In these cells, *C* ranged from a high of 12,746 (convergence rate = 3.92%) to a low of 794 (convergence rate = 62.97%). *C* had an inverse relation with *N* and with $\Delta\kappa$, and a direct relation with φ.

For the 12 cells in which factor loadings were heterogeneous across classes and two intercepts varied across classes, convergence rates were strongly related to *p*. All six cells that had eight manifest variables had perfect convergence. With $p = 4$ and $\Delta\kappa = 2.5$, all cells reached 500 successes, with $C_{N=200} = 2972$, $C_{N=500} = 1945$, and $C_{N=1000} = 1522$. With $p = 4$ and $\Delta\kappa = 2.0$, two of the cells ran the full 20,000 replications without reaching 500 successes, and one had 500 successes but had $C = 18764$. The successful cell was, curiously, the cell with $N = 200$.

All 336 cells that had homogeneous factor loadings but at least one heterogeneous intercept had 500 successful replications. The highest value of *C* was 8489 (convergence rate = 5.89%), and only 13 other cells had $C > 2000$. The lowest value of *C* was 500 (i.e., perfect convergence), which occurred in 100 cells. Sample size and *C* were inversely related but for a few exceptions at $lc = 6$ as *C* approached 500. *C* was alsoinversely related to $\Delta\kappa$ (with a couple of scattered exceptions), $\Delta\tau$, and the number of

Table 1: Convergence Data for Cells with Homogeneous Intercepts

p	φ	Δκ	lc	*N* = 200	*N* = 500	*N* = 1000
4	.5	2	4	*3*	*5*	*6*
4	.5	2.5	4	*3*	*3*	*7*
4	.7	2	4	*4*	*5*	*11*
4	.7	2.5	4	*3*	*6*	*3*
8	.5	2	4	*0*	*0*	*0*
8	.5	2.5	4	*0*	*0*	*0*
8	.7	2	4	*0*	*0*	*0*
8	.7	2.5	4	*0*	*0*	*0*
4	.5	2	6	*9*	216	388
4	.5	2.5	6	223	19748	9700
4	.7	2	6	*10*	229	351
4	.7	2.5	6	*45*	368	11851
8	.5	2	6	*0*	*0*	*0*
8	.5	2.5	6	*0*	*0*	*0*
8	.7	2	6	*0*	*0*	*0*
8	.7	2.5	6	*0*	*0*	*0*
4	.5	2	8	8513	5698	4397
4	.5	2.5	8	3228	1338	794
4	.7	2	8	12746	9741	6989
4	.7	2.5	8	4692	1961	1169
8	.5	2	8	*0*	*0*	*0*
8	.5	2.5	8	*2*	*0*	*4*
8	.7	2	8	*0*	*0*	*0*
8	.7	2.5	8	*0*	*1*	*0*

Simulation ended due to convergence rate significantly below 1% ($p < .025$)

Cell reached 20,000 replications attempted

Table 2: Convergence Data for Cells with Heterogeneous Factor Loadings

p	φ	Δκ	*N* = 200	*N* = 500	*N* = 1000
4	.5	2	18764	373	374
4	.5	2.5	2972	1945	1522
8	.5	2	500	500	500
8	.5	2.5	500	500	500

Cell reached 20,000 replications attempted

Table 3: Values of C for Cells with Heterogeneous Intercepts

				$N = 200$						$N = 500$						$N = 1000$					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	3325	1839	995	635	--	--	1897	1014	650	533	--	--	1419	943	557	503	--	--
4	.5	2.5	4	2566	1231	797	583	--	--	1217	671	548	505	--	--	742	578	515	502	--	--
4	.7	2	4	2579	1365	1083	726	--	--	1229	700	700	577	--	--	887	581	576	551	--	--
4	.7	2.5	4	1797	1003	770	614	--	--	816	579	565	527	--	--	605	529	520	518	--	--
8	.5	2	4	8177	7108	526	505	--	--	3976	3075	504	500	--	--	2538	1413	500	500	--	--
8	.5	2.5	4	3774	2964	516	501	--	--	1587	1173	501	500	--	--	861	608	501	500	--	--
8	.7	2	4	8489	8108	524	504	--	--	5073	3609	511	503	--	--	3350	1678	504	500	--	--
8	.7	2.5	4	4085	3699	521	504	--	--	1936	1316	501	501	--	--	899	656	502	501	--	--
4	.5	2	6	798	570	539	509	552	514	592	510	501	500	527	500	535	502	500	500	507	500
4	.5	2.5	6	574	519	517	507	517	503	505	500	500	500	501	500	500	500	502	500	500	500
4	.7	2	6	692	681	536	513	580	526	527	585	503	505	547	501	501	546	504	504	530	500
4	.7	2.5	6	579	604	517	512	526	526	529	602	505	504	507	501	513	600	504	511	502	500
8	.5	2	6	1220	968	500	500	528	512	728	575	500	500	517	501	557	505	500	500	504	500
8	.5	2.5	6	662	630	501	500	509	501	512	503	500	500	500	500	501	500	500	500	500	500
8	.7	2	6	1350	1015	503	500	537	519	698	552	500	500	524	501	538	501	500	500	514	500
8	.7	2.5	6	705	619	500	500	507	501	508	500	500	500	501	500	501	501	500	500	500	500
4	.5	2	8	1385	760	556	509	--	--	708	545	505	500	--	--	532	504	500	500	--	--
4	.5	2.5	8	1716	798	537	504	--	--	745	517	501	500	--	--	529	500	500	500	--	--
4	.7	2	8	846	582	508	500	--	--	551	505	500	500	--	--	507	500	500	500	--	--
4	.7	2.5	8	855	570	502	500	--	--	531	502	502	500	--	--	500	500	500	500	--	--
8	.5	2	8	581	552	501	500	--	--	523	508	500	500	--	--	504	500	500	500	--	--
8	.5	2.5	8	508	502	501	500	--	--	500	500	500	500	--	--	500	500	500	500	--	--
8	.7	2	8	588	560	501	500	--	--	525	505	500	500	--	--	509	502	500	500	--	--
8	.7	2.5	8	513	506	500	500	--	--	501	501	500	500	--	--	500	501	500	500	--	--

Note: "$\tau_{1,p}$" means "τ_1 and τ_p"

heterogeneous intercepts. Values of C were lower when τ_p was the lone heterogeneous intercept than when τ_1 was the lone heterogeneous intercept. The effect on C of φ was inconsistent.

Finally, there was a complex interaction involving p and the location of intercept heterogeneity. When τ_1 varied across classes, cells with $p = 4$ had lower values of C than corresponding cells with $p = 8$, but when τ_p was heterogeneous (with or without τ_1 being heterogeneous), lower values of C were found in cells with eight manifest variables than in the corresponding cells with four manifest variables. This interaction was further complicated by an interaction with lc: The advantage of $p = 4$ when τ_1 was noninvariant was clearly less at $lc = 6$ than at $lc = 4$, and at $lc = 8$, values of C were consistently lower in cells with $p = 8$ than in cells with $p = 4$, regardless of the location of intercept heterogeneity.

<u>Bias in cells with homogeneous intercepts</u>

For the cells with homogeneous intercepts, Tables 4-8 contain data about the bias in the estimates of the model parameters and of the mixing proportion, but data are presented for only the pairings of p and lc for which there was at least one value of C: $p = 4$, $lc = 6$ and $p = 4$, $lc = 8$. Patterns regarding bias, however, will be discussed only for the 12 cells that paired $p = 4$ with the largest loading combination, because the bias estimates for the other pairing are based on averages across a differing number of replications (ranging from 9 to 500). This unfortunately limited the number of potentially influential design characteristics to three (N; $\Delta\kappa$; and φ), but a few patterns were evident.

For the factor loadings, there was relatively little bias, with the magnitude of the bias exceeding 3% in only two cells, both of which were for the estimate of the p^{th}

Table 4: Percent Bias and Standard Errors of λ_{11} and λ_{p1} in Cells with Homogeneous Intercepts

	p	φ	Δκ	lc	Both classes		
					N = 200	*N* = 500	*N* = 1000
λ_{11}	4	.5	2	6	-1.36	-0.02	0.12
	4	.5	2.5	6	-0.05	0.17	-0.02
	4	.7	2	6	2.36	0.45	-0.12
	4	.7	2.5	6	-0.38	0.87	0.03
	4	.5	2	8	0.24	0.29	0.22
	4	.5	2.5	8	0.41	0.03	0.04
	4	.7	2	8	0.10	0.16	0.11
	4	.7	2.5	8	0.07	0.11	0.09
SE λ_{11}	4	.5	2	6	.0784	.0455	.0398
	4	.5	2.5	6	.0472	.0344	.0256
	4	.7	2	6	.0917	.0414	.0331
	4	.7	2.5	6	.0490	.0354	.0253
	4	.5	2	8	.0523	.0296	.0221
	4	.5	2.5	8	.0402	.0248	.0177
	4	.7	2	8	.0531	.0388	.0248
	4	.7	2.5	8	.0427	.0265	.0185
λ_{p1}	4	.5	2	6	3.97	1.09	0.45
	4	.5	2.5	6	1.37	1.02	0.51
	4	.7	2	6	8.75	0.63	0.61
	4	.7	2.5	6	0.21	1.33	-0.10
	4	.5	2	8	1.01	0.36	0.23
	4	.5	2.5	8	0.71	0.47	-0.01
	4	.7	2	8	1.01	0.22	0.26
	4	.7	2.5	8	0.65	0.33	0.17
SE λ_{p1}	4	5	2	6	.0546	.0348	.0372
	4	5	5	6	.0467	.0295	.0210
	4	7	2	6	.0638	.0355	.0251
	4	7	5	6	.0508	.0307	.0217
	4	5	2	8	.0471	.0310	.0213
	4	5	5	8	.0392	.0250	.0176
	4	7	2	8	.0493	.0439	.0235
	4	7	5	8	.0437	.0261	.0186

Table 5: Percent Bias and Standard Errors of δ_{11} and δ_{p1} in Cells with Homogeneous Intercepts

					Class 1			Class 2		
	p	φ	Δκ	lc	N = 200	N = 500	N = 1000	N = 200	N = 500	N = 1000
δ_{11}	4	.5	2	6	-37.04	-54.71	-45.89	11.85	0.61	-0.10
	4	.5	2.5	6	-52.05	-28.04	-18.17	-2.36	-1.05	0.18
	4	.7	2	6	-30.86	-62.15	-38.72	-10.44	0.01	0.80
	4	.7	2.5	6	-35.75	-36.26	-17.47	4.47	-1.53	0.07
	4	.5	2	8	-18.05	-8.79	-17.56	1.23	-0.20	0.24
	4	.5	2.5	8	-6.47	-5.75	-0.62	0.43	-0.48	-0.54
	4	.7	2	8	-24.16	-15.06	-13.97	-1.06	-0.02	-0.39
	4	.7	2.5	8	-13.12	-4.34	-0.05	1.08	0.63	-0.68
SE δ_{11}	4	.5	2	6	.1986	.1355	.1803	.1310	.0644	.0597
	4	.5	2.5	6	.1187	.1550	.0901	.0913	.0655	.0494
	4	.7	2	6	.4719	.1107	.1529	.0899	.0618	.0484
	4	.7	2.5	6	.1230	.0876	.0879	.0927	.0728	.0559
	4	.5	2	8	.2070	.2304	.1672	.0633	.0370	.0251
	4	.5	2.5	8	.1748	.1017	.0548	.0633	.0433	.0330
	4	.7	2	8	.1870	.2321	.2154	.0550	.0512	.0312
	4	.7	2.5	8	.1852	.1010	.0612	.0720	.0547	.0410
δ_{p1}	4	.5	2	6	3.29	21.34	36.72	6.30	-0.05	-0.40
	4	.5	2.5	6	11.40	12.05	3.37	3.12	0.15	-0.13
	4	.7	2	6	35.27	29.73	28.91	0.67	0.88	0.04
	4	.7	2.5	6	14.12	3.76	4.36	2.24	0.60	0.38
	4	.5	2	8	-16.16	-14.72	-10.07	-0.28	0.39	0.53
	4	.5	2.5	8	-6.09	-3.77	-1.41	-1.18	-0.29	-0.21
	4	.7	2	8	-23.23	-15.86	-11.00	0.62	0.81	-0.07
	4	.7	2.5	8	-7.04	-5.92	-2.33	-0.08	0.24	0.65
SE δ_{p1}	4	5	2	6	.3348	.5429	.6170	.0961	.0592	.0504
	4	5	5	6	.4465	.3886	.2381	.1022	.0766	.0555
	4	7	2	6	1.2147	.6319	.5063	.1589	.0686	.0487
	4	7	5	6	.6180	.3048	.2247	.0982	.0826	.0679
	4	5	2	8	.2185	.2093	.2026	.0562	.0371	.0253
	4	5	5	8	.1790	.0928	.0541	.0619	.0436	.0329
	4	7	2	8	.1932	.2363	.2230	.0577	.0440	.0333
	4	7	5	8	.2021	.1018	.0537	.0777	.0538	.0415

Table 6: Percent Bias and Standard Errors of τ_1 and τ_p in Cells with Homogeneous Intercepts

	p	φ	Δκ	lc	Both classes N = 200	N = 500	N = 1000
τ_1	4	.5	2	6	-59.28	-76.07	-69.46
	4	.5	2.5	6	-51.69	-35.72	-19.43
	4	.7	2	6	-69.40	-67.65	-55.01
	4	.7	2.5	6	-67.57	-40.15	-23.61
	4	.5	2	8	-64.51	-57.45	-50.39
	4	.5	2.5	8	-33.42	-16.77	-6.68
	4	.7	2	8	-70.30	-58.68	-43.22
	4	.7	2.5	8	-37.71	-17.09	-8.42
SE τ_1	4	.5	2	6	.4002	.2119	.2236
	4	.5	2.5	6	.2030	.2050	.1851
	4	.7	2	6	.2065	.1692	.1818
	4	.7	2.5	6	.2172	.1497	.1348
	4	.5	2	8	.2840	.3104	.2795
	4	.5	2.5	8	.2702	.2239	.1588
	4	.7	2	8	.2255	.2635	.2502
	4	.7	2.5	8	.2636	.1580	.1055
τ_p	4	.5	2	6	-13.92	-15.50	-14.01
	4	.5	2.5	6	-10.61	-7.37	-4.02
	4	.7	2	6	-15.39	-13.64	-11.14
	4	.7	2.5	6	-14.16	-8.08	-4.73
	4	.5	2	8	-26.24	-22.96	-20.16
	4	.5	2.5	8	-13.52	-6.87	-2.65
	4	.7	2	8	-28.35	-23.48	-17.30
	4	.7	2.5	8	-15.22	-6.84	-3.39
SE τ_p	4	5	2	6	.2532	.1545	.1339
	4	5	5	6	.1654	.1378	.1059
	4	7	2	6	.1763	.1277	.1089
	4	7	5	6	.1922	.1022	.0805
	4	5	2	8	.2887	.3105	.2775
	4	5	5	8	.2733	.2243	.1586
	4	7	2	8	.2181	.2645	.2334
	4	7	5	8	.2690	.1588	.1051

Table 7: Percent Bias and Standard Errors of Φ_{11} and $\Delta\kappa$ in Cells with Homogeneous Intercepts

					Class 1			Class 2		
	p	φ	Δκ	lc	N = 200	N = 500	N = 1000	N = 200	N = 500	N = 1000
Φ_{11}	4	.5	2	6	-92.93	-88.52	-86.99	106.01	85.45	85.10
	4	.5	2.5	6	-80.89	-64.84	-40.98	121.42	97.38	55.18
	4	.7	2	6	-93.83	-87.37	-85.54	52.40	75.20	79.73
	4	.7	2.5	6	-91.62	-67.23	-48.84	109.15	93.02	77.26
	4	.5	2	8	-75.81	-74.36	-72.49	71.12	74.31	76.43
	4	.5	2.5	8	-51.26	-30.95	-14.59	77.42	47.37	19.47
	4	.7	2	8	-83.34	-79.29	-76.52	71.51	76.28	79.31
	4	.7	2.5	8	-64.53	-40.17	-24.88	103.08	70.30	48.67
SE Φ_{11}	4	.5	2	6	.4688	.1477	.1627	.2491	.1959	.1520
	4	.5	2.5	6	.1527	.2104	.2128	.3171	.2831	.2285
	4	.7	2	6	.3082	.1440	.1668	.3163	.1866	.1420
	4	.7	2.5	6	.1138	.1702	.1913	.3095	.2832	.2399
	4	.5	2	8	.2234	.2572	.2401	.2626	.2172	.1565
	4	.5	2.5	8	.3083	.2542	.2006	.3667	.2860	.2084
	4	.7	2	8	.1588	.2303	.2023	.2678	.2024	.1526
	4	.7	2.5	8	.2545	.2387	.1701	.3908	.3138	.2497
Δκ	4	.5	2	6	--	--	--	32.47	50.56	42.11
	4	.5	2.5	6	--	--	--	12.13	0.78	-1.32
	4	.7	2	6	--	--	--	21.06	21.09	5.20
	4	.7	2.5	6	--	--	--	6.70	-7.59	-11.92
	4	.5	2	8	--	--	--	41.10	32.03	22.90
	4	.5	2.5	8	--	--	--	8.34	1.54	0.48
	4	.7	2	8	--	--	--	26.29	12.06	-6.13
	4	.7	2.5	8	--	--	--	-9.24	-13.44	-10.96
SE Δκ	4	5	2	6	--	--	--	.3654	.2846	.2637
	4	5	5	6	--	--	--	.2751	.2202	.1662
	4	7	2	6	--	--	--	.2818	.2354	.2365
	4	7	5	6	--	--	--	.3048	.2421	.2321
	4	5	2	8	--	--	--	.3509	.3576	.3217
	4	5	5	8	--	--	--	.3005	.2035	.1119
	4	7	2	8	--	--	--	.2920	.3408	.3052
	4	7	5	8	--	--	--	.3774	.2663	.2060

Table 8: Percent Bias and Empirical Standard Errors of φ in Cells with Homogeneous Intercepts

	p	φ	Δκ	lc	Class 1 N = 200	Class 1 N = 500	Class 1 N = 1000		Class 2 N = 200	Class 2 N = 500	Class 2 N = 1000
φ	4	.5	2	6	-90.38	-91.38	-90.36		90.38	91.38	90.36
	4	.5	2.5	6	-80.25	-64.51	-38.28		80.25	64.51	38.28
	4	.7	2	6	-91.02	-89.37	-88.27		212.4	208.5	206.0
	4	.7	2.5	6	-86.60	-63.66	-47.04		202.1	148.5	109.8
	4	.5	2	8	-81.82	-81.37	-81.32		81.82	81.37	81.32
	4	.5	2.5	8	-51.48	-31.37	-13.12		51.48	31.37	13.12
	4	.7	2	8	-87.83	-85.83	-82.12		204.9	200.3	191.6
	4	.7	2.5	8	-63.41	-39.58	-24.78		147.9	92.35	57.82
Empirical SE φ	4	.5	2	6	.0319	.0773	.0830		.0319	.0773	.0830
	4	.5	2.5	6	.1348	.2023	.2076		.1348	.2023	.2076
	4	.7	2	6	.0677	.1272	.1106		.0677	.1272	.1106
	4	.7	2.5	6	.1623	.2551	.2734		.1623	.2551	.2734
	4	.5	2	8	.1227	.1276	.1227		.1227	.1276	.1227
	4	.5	2.5	8	.1999	.2011	.1525		.1999	.2011	.1525
	4	.7	2	8	.1253	.1490	.1417		.1253	.1490	.1417
	4	.7	2.5	8	.2318	.2540	.2299		.2318	.2540	.2299

loading when the p^{th} loading was .4. Bias in the error variances was larger in magnitude than in the factor loadings, but only in class 1. In class 2, there was no bias greater than 3% in magnitude for the estimates of δ, whereas in class 1, there were six cells that had negative bias in excess of 13% in magnitude. The effect of N on bias was generally inverse, but a notable exception occurred when $\varphi = .5$ and $\Delta\kappa = 2.0$, where negative bias in δ_{11} from $N = 500$ to $N = 1000$ increased in magnitude from 8.79% to 17.56%. The effect of $\Delta\kappa$ on bias was consistently and strongly inverse. Bias tended to be larger in magnitude in cells with $\varphi = .7$ than in cells with $\varphi = .5$.

Bias in the estimates of the intercepts was consistently negative, and for τ_1, bias was generally copious, with five magnitudes in excess of 50%; by contrast, the largest magnitude of bias for τ_p was 28.35%. Sample size and $\Delta\kappa$ separately had strong inverse effects on the magnitude of the bias, and they had an interaction effect: The effect of N was stronger when $\Delta\kappa = 2.5$ than when $\Delta\kappa = 2.0$. The same effect of φ seen with the bias in values of δ occurred for the intercept bias, with the magnitudes being larger for cells with $\varphi = .7$ than with $\varphi = .5$.

The estimates of Φ_{11} were substantially biased in both classes, with large negative biases in class 1, large positive biases in class 2, and no biases below 14% in magnitude. Several cells had bias that exceeded 70% in magnitude, with one cell having +103.08% bias in the class 2 estimate of the factor variance. There did not seem to be an effect of N in cells with $\Delta\kappa = 2.0$, but when $\Delta\kappa = 2.5$, bias magnitude clearly decreased as N increased. The mixing proportion again had a consistently direct relation with bias magnitude.

For the estimation of $\Delta\kappa$, the bias magnitudes and the effects of design characteristics on them was largely a function of φ. In cells with $\varphi = .5$, bias was consistently positive, bias was clearly larger in the cells with $\Delta\kappa = 2.0$, and bias decreased as N increased. In cells with $\varphi = .7$, bias magnitude and changes in it were difficult to describe; the reader is simply referred to the relevant portion of Table 7. Bias in the estimation of φ generally was even larger in magnitude than the bias for the factor variance. Sample size interacted separately with $\Delta\kappa$ and with φ, having an inverse relation to bias magnitude overall, but with the effect being stronger at the larger values of $\Delta\kappa$ and φ.

Bias in cells with heterogeneous factor loadings

Table 9 and Table 10 contain the percent bias and the standard errors for the parameter estimates in the 12 cells that had heterogeneous intercepts and heterogeneous factor loadings. Among all of the parameter estimates, there were only a few bias magnitudes in excess of 3% and only three that exceeded 6% (max = +11.24%). With the biases generally so low, it was difficult to locate a pattern in the change in the bias that could not be described as trivial.

Bias in cells with heterogeneous intercepts

The tables for the percent bias of the parameter estimates for the 336 cells in which factor loadings were homogeneous while at least one heterogeneous intercept varied across classes are not specifically referenced in this chapter, but for exhibition purposes, they are presented in Appendix B. The results are summarized below, with the order of presentation for the parameters being: λ_{11}; λ_{p1}; δ_{11}; δ_{p1}; τ_1; τ_p; Φ_{11}; $\Delta\kappa$; and φ. Within each parameter, a general description of bias magnitudes is provided,

Table 9: Percent Bias and Standard Errors of Parameter Estimates in Cells with Heterogeneous Factor Loadings

				Class 1			Class 2	
	p	φ	Δκ	N = 200	N = 500	N = 1000	N = 200	N = 500	N = 1000
λ_{11}	4	.5	2	4.52	1.09	0.06	0.59	-1.24	-0.36
	4	.5	2.5	2.97	0.62	-0.04	-0.71	-0.02	0.30
	8	.5	2	0.98	0.40	0.31	1.36	-0.48	-0.06
	8	.5	2.5	1.51	0.32	0.20	-0.10	0.29	-0.18
SE λ_{11}	4	.5	2	.1215	.0749	.0522	.1242	.0691	.0483
	4	.5	2.5	.1185	.0713	.0496	.1156	.0664	.0463
	8	.5	2	.1091	.0653	.0450	.0885	.0533	.0379
	8	.5	2.5	.1072	.0631	.0445	.0849	.0517	.0365
λ_{p1}	4	.5	2	3.41	1.59	0.73	-7.00	-5.09	-1.63
	4	.5	2.5	1.43	0.38	-0.18	-4.28	-1.51	-1.28
	8	.5	2	0.29	0.37	0.21	0.29	-1.50	-0.53
	8	.5	2.5	0.78	0.48	0.37	-0.11	0.02	-0.13
SE λ_{p1}	4	.5	2	.1170	.0723	.0506	.1433	.0803	.0553
	4	.5	2.5	.1153	.0696	.0483	.1325	.0746	.0523
	8	.5	2	.1039	.0624	.0435	.1174	.0698	.0494
	8	.5	2.5	.1028	.0614	.0432	.1107	.0681	.0477
δ_{11}	4	.5	2	1.01	0.40	0.53	-6.11	-1.17	0.37
	4	.5	2.5	-3.03	-0.86	-0.51	-2.81	-0.91	-0.23
	8	.5	2	-2.79	-0.15	-0.19	-1.57	-0.76	0.10
	8	.5	2.5	-3.16	-0.46	0.21	-3.67	-1.77	-1.24
SE δ_{11}	4	.5	2	.0801	.0498	.0351	.0976	.0585	.0406
	4	.5	2.5	.0754	.0471	.0330	.0912	.0539	.0384
	8	.5	2	.0698	.0421	.0300	.0720	.0438	.0312
	8	.5	2.5	.0639	.0406	.0288	.0663	.0417	.0292
δ_{p1}	4	.5	2	-0.85	-0.49	-0.93	-2.21	0.21	-0.17
	4	.5	2.5	-2.37	-0.81	-0.62	-4.58	-1.16	-0.67
	8	.5	2	-2.50	-1.40	-1.27	-2.52	-1.08	-0.45
	8	.5	2.5	-2.89	-1.41	0.13	-3.14	-0.93	-0.78
SE δ_{p1}	4	.5	2	.0740	.0470	.0332	.1387	.0815	.0571
	4	.5	2.5	.0728	.0455	.0321	.1270	.0785	.0560
	8	.5	2	.0632	.0394	.0276	.1253	.0786	.0558
	8	.5	2.5	.0601	.0382	.0275	.1191	.0771	.0546

Table 10: Percent Bias and Standard Errors of Parameter Estimates in Cells with Heterogeneous Factor Loadings

	p	φ	Δκ	Class 1 N = 200	Class 1 N = 500	Class 1 N = 1000	Class 2 N = 200	Class 2 N = 500	Class 2 N = 1000
τ_1	4	.5	2	3.47	0.46	0.11	0.41	0.77	0.01
	4	.5	2.5	2.97	0.59	0.11	1.94	0.29	-0.04
	8	.5	2	0.09	0.05	0.07	-0.12	0.17	0.09
	8	.5	2.5	0.08	0.09	0.01	0.13	-0.02	0.04
SE τ_1	4	.5	2	.1762	.0963	.0649	.3345	.1864	.1294
	4	.5	2.5	.1534	.0848	.0584	.3627	.2055	.1424
	8	.5	2	.1435	.0813	.0561	.2440	.1411	.0984
	8	.5	2.5	.1308	.0767	.0532	.2681	.1574	.1106
τ_p	4	.5	2	0.64	0.06	0.00	1.32	0.45	-0.08
	4	.5	2.5	0.83	0.19	0.07	1.13	0.33	0.18
	8	.5	2	0.13	0.05	0.09	0.21	0.33	0.18
	8	.5	2.5	0.08	0.16	0.03	0.10	0.08	0.01
SE τ_p	4	.5	2	.1579	.0877	.0598	.3524	.1960	.1339
	4	.5	2.5	.1385	.0787	.0543	.3763	.2131	.1493
	8	.5	2	.1289	.0748	.0518	.2884	.1687	.1185
	8	.5	2.5	.1193	.0717	.0500	.3176	.1922	.1346
Φ_{11}	4	.5	2	11.24	0.48	-1.05	5.45	-0.85	-3.03
	4	.5	2.5	5.68	-0.32	0.12	-0.41	-0.61	-0.45
	8	.5	2	0.81	0.15	0.21	-1.43	-0.08	-0.36
	8	.5	2.5	-1.27	0.52	-0.31	-0.96	-0.25	-0.60
SE Φ_{11}	4	.5	2	.2805	.1630	.1124	.2699	.1546	.0918
	4	.5	2.5	.2681	.1580	.1107	.2161	.1317	.0925
	8	.5	2	.2484	.1495	.1045	.1874	.1184	.0833
	8	.5	2.5	.2373	.1477	.1029	.1853	.1158	.0809
Δκ	4	.5	2	--	--	--	-1.88	-2.37	-1.69
	4	.5	2.5	--	--	--	-1.95	-0.54	-0.32
	8	.5	2	--	--	--	-1.05	0.13	-0.24
	8	.5	2.5	--	--	--	-0.08	-0.21	0.04
SE Δκ	4	.5	2	--	--	--	.2129	.1213	.0841
	4	.5	2.5	--	--	--	.1954	.1161	.0817
	8	.5	2	--	--	--	.1869	.1139	.0799
	8	.5	2.5	--	--	--	.1804	.1125	.0790
φ	4	.5	2	4.50	0.52	-0.22	-4.50	-0.52	0.22
	4	.5	2.5	2.16	0.15	0.07	-2.16	-0.15	-0.07
	8	.5	2	0.12	0.25	0.21	-0.12	-0.25	-0.21
	8	.5	2.5	0.14	0.15	0.06	-0.14	-0.15	-0.06
SE φ	4	.5	2	.0813	.0315	.0136	.0813	.0315	.0136
	4	.5	2.5	.0533	.0145	.0105	.0533	.0145	.0105
	8	.5	2	.0272	.0147	.0098	.0272	.0147	.0098
	8	.5	2.5	.0209	.0120	.0079	.0209	.0120	.0079

followed by a more detailed account of changes in bias as a function of interactions among the design characteristics.

$\underline{\lambda_{11}}$. The percent bias for the first factor loading was generally positive, with no negative bias exceeding 1%. Bias was largest (above +50% in some cells) when the factor had eight indicators all loading at .4 with $\Delta\tau_1 = 1$. The bias in λ_{11} was smallest (less than 1%) when only the p^{th} manifest variable's intercept differed across classes. The effect of having only the p^{th} intercept differ across classes was so strong that there was no sample size effect on the bias of λ_{11} in those cells. For the other cells, bias decreased as *N* increased, with three notable interactions occurring. The effect of sample size was stronger when $\Delta\kappa = 2.5$ than when $\Delta\kappa = 2.0$. It was also stronger when there were four manifest variables rather than eight and when $\Delta\tau = 1.5$ instead of 1.

The number of manifest variables in the model was involved in a complicated interaction with *lc* and the location of intercept heterogeneity. When all $\lambda = .8$, bias was higher with four indicators than with eight. For the other two loading combinations, bias was smaller at $p = 4$ when only τ_1 differed across classes, but bias was smaller at $p = 8$ when both τ_1 and τ_p differed. Regarding other design characteristics, the higher value of $\Delta\kappa$ generally yielded smaller biases in λ_{11}, with the benefit of having the larger $\Delta\kappa$ increasing as *N* increased (as per the interaction described in the previous paragraph). There was an interaction between φ and *p* such that the cells with $\varphi = .7$ had less bias than the corresponding cells with $\varphi = .5$ when $p = 4$, but the bias was greater in the cells with $\varphi = .7$ when $p = 8$.

There was also an interaction between $\Delta\tau$, *p*, number of heterogeneous intercepts, *lc*, and *N*. With two heterogeneous intercepts, the biases in cells with $\Delta\tau = 1$ were consistently higher than in the corresponding cells with $\Delta\tau = 1.5$. This was also the case with only τ_1 being heterogeneous while $p = 4$. At $p = 8$, there was actually a direct relation between the magnitude of the bias and $\Delta\tau$, but only for $lc = 4$ and $lc = 6$ at $N = 200$, and only with $lc = 4$ at $N = 500$.

λ_{p1}. Patterns in the bias of the p^{th} factor loading were difficult to detect, because the bias was relatively low in most cells. Like the bias of λ_{11}, the bias of λ_{p1} was smallest when its manifest variable's intercept was held equal across classes, exceeding 3% in only one of the cells in which τ_p was equal across classes while τ_1 was heterogeneous. Where appreciable bias was present, it was positive bias, with the largest biases occurring in the cluster of cells under $N = 200$ that had two heterogeneous intercepts with four manifest variables, all loading at .4. The only conditions that clearly affected the bias were N and $\Delta\kappa$. As N increased, bias decreased, and cells with $\Delta\kappa = 2.5$ generally had less bias than the corresponding cells with $\Delta\kappa = 2.0$. The interaction between N and $\Delta\kappa$ described for the bias of λ_{11} were less consistent for the bias of λ_{p1}, and there was little, if any, evidence of the other aforementioned interactions.

δ_{11}. The magnitude of the biases for the error variance of the first indicator substantially varied across classes. In class 1, with eight manifest variables, all biases that exceeded 2% in magnitude were negative. The largest biases occurred in cells that had τ_1 varying across classes and all $\lambda = .4$, with a few negative biases exceeding 60% in magnitude and several more in excess of 40%. With four manifest variables, however, the only biases that surpassed 10% in magnitude were positive and occurred only in cells that had all $\lambda = .8$, τ_1 varying across classes, and $\varphi = .5$. In cells with two heterogeneous intercepts, no bias exceeded 10% in magnitude, and in cells that had only τ_p differ across classes, no |bias| was larger than 3%.

Although increases in N and in $\Delta\kappa$ did decrease the magnitude of the bias in class 1, and those two design characteristics again demonstrated their aforementioned interaction, the effect of p on bias was clearly stronger. Loading combination also affected bias, but its effect interacted with p. At p = 8, bias consistently decreased in magnitude as the factor loadings increased, while at p = 4, bias generally changed only slightly from lc = 4 to lc = 6, but then increased in the positive direction, sometimes to rather appreciable levels, in cells with lc = .8. The value of the mixing proportion also affected the bias of δ_{11} in that the biases at φ = .7 were more negative than the biases for the corresponding cells with φ = .5.

In class 2, biases of δ_{11} in cells with heterogeneous τ_1 radically changed direction relative to the values found in class 1. The cells that had four indicators on the lone factor had many strongly negative biases, while in cells with eight indicators, all biases that exceeded 10% in magnitude were positive. Cells with two heterogeneous intercepts still had biases that were negative, but the magnitudes of bias were consistently higher in class 2 than in class 1, with several cells having magnitudes above 10% when N = 200. Biases in cells with only heterogeneous τ_p were consistently but only slightly more negative in class 2 than in class 1 when N = 200, but no bias of δ_{11} in these cells exceeded 5% in magnitude.

The inverse relations of bias with N and bias with $\Delta\kappa$ held in class 2 as did their interaction, with N having a stronger effect at the higher value of $\Delta\kappa$ and the proportional differences in the bias between levels of $\Delta\kappa$ increasing with N. The effect of loading combination was peculiar but did not interact with p. From $lc = 4$ to $lc = 6$, bias became more positive in all cells but never enough such that biases switched signs. From $lc = 6$ to $lc = 8$, bias generally became more negative but not enough to put the levels back to their values at $lc = 4$.

Another pattern that emerged only in class 2 was a complex interaction of $\Delta\tau$ with p, number of heterogeneous intercepts, loading combination, and N. When two intercepts differed across classes, the relation between bias and $\Delta\tau$ was inverse. When only one intercept differed, the relation between bias and $\Delta\tau$ at $N = 200$ was inverse at $p = 4$ but direct in cells with $p = 8$. This pattern weakened at $N = 500$, while at $N = 1000$, only $lc = 4$ demonstrated a direct relation between bias and $\Delta\tau$ at $p = 8$.

$\underline{\delta_{p1}}$. The biases in the error variance of the p^{th} indicator were generally negative and relatively low. The largest magnitude for bias in either class was -20.86%, and only 14 other cells had magnitudes above 10%, all occurring with $N = 200$. In class 1, the largest biases in δ_{p1} occurred when τ_p differed across classes, with the largest biases in the cells with only τ_p heterogeneous. In class 2, this pattern held, but the biases in cells that had $p = 4$ and $lc = 4$ had anomolously high magnitudes relative to the other cells. These biases, in fact, clearly exceeded those found in the cells that had only τ_p being heterogeneous (which, by design, had $lc = 6$).

With the bias being so low overall, it was difficult to describe patterns in their change, but a few were notable. In both classes, N was inversely related to bias magnitude. The value of $\Delta\kappa$, however, had an inconsistent effect on bias, even when $|\text{bias}| > 3\%$. In class 2, $\Delta\tau$ was inversely related to bias, but in class 1, this relation was somewhat less consistent. There were faint signs of the previously detailed complex interaction among $\Delta\tau$, *p*, number of heterogeneous intercepts, loading combination, and *N*.

$\underline{\tau_1.}$ The bias of the first intercept was below 1% in magnitude for the cells that had τ_1 equal across classes, so these cells will be ignored for the rest of the summary of the bias of τ_1. In class 1, the direction of bias was a function of *p*, with bias at $p = 4$ being generally positive (i.e., no negative bias in excess of 0.5%) and biases at $p = 8$ being generally negative (i.e., no positive bias in excess of 0.7%). The magnitudes were at their highest (twice exceeding 35%) in the cells in which τ_1 was the only heterogeneous intercept among four manifest variables. The highest magnitudes for the negative biases (three cells exceeding 20%) also occurred when τ_1 was the only heterogeneous intercept, but when $p = 8$.

Sample size and $\Delta\kappa$ clearly demonstrated inverse relations with bias in class 1, and their interaction was also clearly evident. There was an interaction between φ and *p*: When $p = 8$, bias in cells with $\varphi = .7$ was just slightly more negative than in cells with $\varphi = .5$, but when $p = 4$, bias was considerably more negative (though still positive in direction) for $\varphi = .7$ than for the cells with $\varphi = .5$. There were main effects for $\Delta\tau$ and for number of heterogeneous intercepts, with the magnitude of the bias being smaller at the larger levels of these design characteristics. They each also interacted with sample size in such a way that their effects were increasingly apparent as *N* increased. Loading combination interacted with *p* to affect bias such that at $p = 8$, bias consistently decreased in magnitude as the factor loadings increased, while in cells with $p = 4$, bias decreased from $lc = 4$ to $lc = 6$ but then increased from $lc = 6$ to $lc = 8$. These effects were influenced by N in that the decrease was more pronounced as N increased while the ensuing increase from lc = 6 to lc = 8 was more tempered as N increased.

In class 2, the bias in τ_1 was generally negative with the highest magnitudes reaching just above 20%. There were, however, a few positive biases, including three cells in which the magnitude exceeded 10% (where τ_1 varied across classes in cells with four manifest variables all loading at .4). Bias decreased in magnitude as N increased, and bias generally decreased in magnitude as $\Delta\kappa$ increased, with the same N x $\Delta\kappa$ interaction seen previously. Bias was consistently more negative in cells with φ = .7 than in the corresponding cells with φ = .5. The effect on bias of the number of heterogeneous intercepts interacted with p such that the effect was inconsistent in cells with p = 4, but in cells with p = 8, bias was acutely reduced in magnitude when two intercepts varied across classes instead of just τ_1.

$\underline{\tau_p}$. The bias in the estimate of the p^{th} intercept was generally low in both classes, with six cells having negative bias above 10% in magnitude and only one cell having positive bias that reached 10%. The largest bias magnitudes occurred in the cells in which only τ_1 was heterogeneous across classes, which were cells in which τ_p was constrained equal across the two classes. Among the cells with only τ_p or with both τ_1 and τ_p different across classes, only three had |bias| > 5%, making patterns of change in bias difficult to detect. In the cells with τ_1 differing across classes, bias was inversely related to N and generally inversely related to $\Delta\kappa$, with the usual interaction between N and $\Delta\kappa$ of the effect of N being stronger for the larger $\Delta\kappa$ and the benefit of larger $\Delta\kappa$ being stronger as N increased. In the cells with φ = .7, the bias of τ_p was consistently more negative than in the corresponding cells with φ = .5.

$\underline{\Phi_{11}.}$ Bias in the estimate of the factor variance was substantial in many cells, with several magnitudes in excess of 50%, including a few cells in which $N = 1000$. In class 1, the number of manifest variables seemed to determine the direction of bias, except for the cells in which only τ_p varied across classes, which led to negative bias in Φ_{11} regardless of p. Bias in cells with $p = 4$ was generally positive, with no negative bias greater than 4% in magnitude (excepting the τ_p heterogeneous cells). Bias in cells with $p = 8$ was generally negative, with no positive bias greater than 5%. The largest magnitudes appeared in cells with heterogeneous τ_1, with considerably less bias in cells that had τ_p or both τ_1 and τ_p varying across classes.

With the biases in class 1 being so large in magnitude, many patterns in the change in bias were readily apparent. Increasing N decreased bias magnitude except in one combination of conditions: eight manifest variables, all loading at .4, with a 50/50 mixture of the two classes, $\Delta\kappa = 2.0$, and $\Delta\tau_1 = 1$. For this set of conditions, the biases in VF1 were -49.98%, -59.03%, and -58.98% for $N = 200$, 500, and 1000, respectively. The effect on bias of increasing $\Delta\kappa$ was inconsistent in cells with $N = 200$, but at the other levels of N, there was an inverse relation between $\Delta\kappa$ and the magnitude of the bias. Sample size and $\Delta\kappa$ interacted in the same manner as described for the other parameters.

The value of φ interacted with p and the location of intercept heterogeneity to affect bias such that in cells with four manifest variables, bias was smaller in magnitude in cells with $\varphi = .7$, but in cells with eight manifest variables or with τ_p heterogeneous, the cells with $\varphi = .5$ had bias of lower magnitude than their $\varphi = .7$ counterparts. In cells with heterogeneous τ_1 (whether or not τ_p varied across classes), there was an inverse relation between $\Delta\tau$ and bias magnitude except in cells with $N = 200$ and eight manifest variables all loading at .4, where the relation was direct. Loading combination interacted with p to affect bias, with bias decreasing in magnitude as loading combination increased in cells with $p = 8$, while a more complicated pattern occurred in cells with $p = 4$. For those cells, shifting from $lc = 4$ to $lc = 6$ decreased bias, but continuing on to $lc = 8$ tended to increase bias.

In class 2, the cells that had appreciable bias in class 1 tended to have appreciable bias, but with the opposite sign. The negative biases with the largest magnitudes (two greater than 50%) occurred in cells that had only τ_1 varying across classes and four manifest variables all loading at .8. The largest positive biases occurred in cells with τ_1 as the lone heterogeneous intercept and eight manifest variables loading at .4, and these biases were exceptionally high, in excess of 120% in three cells and above 80% in 11 other cells. By contrast, the largest bias, positive or negative, when τ_1 and τ_p were both heterogeneous was -28.48%, with only eight other cells having bias magnitudes above 10%.

As happened in class 1, N was not perfectly inversely related to bias, with three cells showing an increase in bias from $N = 200$ to $N = 500$, all of which were cells in which $\Delta\tau_1 = 1$ with $lc = 4$, $\varphi = .5$, and $\Delta\kappa = 2.0$. The larger value of $\Delta\kappa$ did not always have less bias, surpassing $\Delta\kappa = 2.0$ in eight pairs of cells, all of which were cells with $p = 8$ and with τ_1 as the only heterogeneous intercept. The interaction between N and $\Delta\kappa$ led to the bias in cells with $\Delta\kappa = 2.5$ being of lesser magnitude than the corresponding cells with $\Delta\kappa = 2.0$ at $N = 500$ in all but one case.

Increasing the magnitude of the factor loadings also restored (or further clarified) the advantage of the larger factor mean difference once lc was increased to 8, except in the pair of cells with $p = 8$, $\Delta\tau_1 = 1$, and $\varphi = .7$. This failure of $\Delta\kappa = 2.5$ to regain its advantage was the result of a complex interaction involving φ, p, lc, and the number of heterogeneous intercepts. The magnitude of the bias in Φ_{11} decreased as loading combination increased in cells with $p = 8$; when τ_1 was the only heterogeneous intercept, this effect was relatively weak when $\varphi = .7$, but when two intercepts varied across classes, the effect of loading combination when $p = 8$ was strong in cells with $\varphi = .7$. One final effect found in class 2: $\Delta\tau$ was inversely related to the magnitude of bias, with the effect increasing as N increased.

$\Delta\kappa$. Bias in the standardized difference between the factor means (i.e., bias in the factor mean of class 2) was generally negative, with only one positive bias above 10% (12.21%). Several cells that had only one heterogeneous intercept, but no cell that had two heterogeneous intercepts, had negative biases that exceeded 10% in magnitude. The negative biases with the highest magnitudes were in the mid-30% range and occurred in cells that had eight manifest variables loading at .4 with $\varphi = .7$.

Sample size generally had an inverse relation with bias, but there were a few important exceptions, including two conditions (both with $p = 8$, $\Delta\kappa = 2.0$, $lc = 4$, and $\Delta\tau_1 = 1$) in which bias magnitude steadily increased as N increased. There did not appear to be a consistent main effect for the bias in $\Delta\kappa$ as a function of the value of the parameter itself, but the interaction of N with $\Delta\kappa$ was present. The number of heterogeneous intercepts had a generally inverse relation with the magnitude of the bias, while $\Delta\tau$ had a more definitively inverse relation to bias magnitude.

φ. Although the percent biases in the mixing proportion differed across classes, the summary of the bias will focus on only the bias in class 1. The signs differ across classes, because, with only two classes, bias in the mixing proportion in one class must be compensated in sign by the bias in the mixing proportion of the other class. In fact, for φ = .5, the bias must also be identical in magnitude across classes. When φ = .7, the percent bias varies across classes, but only due to the denominator; raw bias in the mixing proportion of one class must be compensated in both sign and magnitude by the raw bias in the other class. The only reason for addressing both classes would therefore be to make note of the tremendously large biases in φ_2 when φ_2 = .3 in the population.

Bias in φ_1 was generally negative at p = 8 and positive at p = 4, with the exception of negative biases for cells in which only τ_p was heterogeneous regardless of p. The largest positive biases were in the mid-50% range, appearing in cells that had N = 200, φ = .5, $\Delta\kappa$ = 2.0, and $\Delta\tau_1$ = 1. The negative biases with the largest magnitude were in the mid-80% range, and they occurred in cells with $\Delta\kappa$ = 2.0 and $\Delta\tau_1$ = 1 but were not restricted to only the lowest N, with two negative biases at N = 1000 exceeding 70% in magnitude and several others with magnitudes above 20%.

The inverse relations of N with bias magnitude and $\Delta\kappa$ with bias magnitude were clear and consistent, as was their usual interaction effect. Having two heterogeneous intercepts yielded bias lower in magnitude than in corresponding cells with only one heterogeneous intercept, and having τ_p vary across classes resulted in a lower bias magnitude than did having τ_1 be heterogeneous. Regardless of the location of heterogeneity, $\Delta\tau$ had an inverse relation with the magnitude of bias, with the effect increasing as N increased. There was the oft occurring interaction between p and lc such that at p = 8, bias magnitude consistently decreased as loadings increased, while at p = 4, the bias magnitudes were largest with lc = 4 and smallest with lc =6 . There was also an interaction among φ, $\Delta\kappa$, and p. In cells with four manifest variables, the magnitude of the bias in φ was higher for cells with $\varphi = .5$ than for their corresponding cells with $\varphi = .7$, regardless of the value of $\Delta\kappa$. In cells with eight manifest variables, the magnitude of the bias was higher for cells with $\Delta\kappa = 2.0$ than for their corresponding cells with $\Delta\kappa = 2.5$, regardless of the value of φ.

Chapter 4

Discussion

The present study sought to answer two questions. The first question was: In terms of convergence rates and bias under the standard restrictions of homogenous factor loadings and homogeneous intercepts in CFA mixture models, how do CFA mixture models with the standard restrictions relaxed compare? Lubke et al. (2002) found that the presence of two or more heterogeneous intercepts in a CFA mixture model improved the accuracy of the parameter estimates relative to a model with completely invariant intercepts. The present study provides additional detail in answering the heterogeneity question by investigating the effects of having only one heterogeneous intercept and by varying the magnitude of the intercept difference. The second question posed by this study extends the first by asking: What effects do other design characteristics have on the convergence rates of and bias in CFA mixture models? Prior research (e.g., Gagné & Hancock, 2002; Marsh et al., 1998) has demonstrated that sample size, the number of manifest indicators, and factor saturation affect the convergence rates and bias of single-population factor models, so these design characteristics were manipulated in the present study as was the mixing proportion.

Cross-class heterogeneity

The convergence data alone demonstrate such an advantage to models with at least some degree of heterogeneity in the intercepts over completely invariant models that the standard practice of constraining all intercepts to be equal across classes should be reconsidered. Among the cells with complete invariance, 33 out of 72 had a convergence rate of 0%, and none of the convergence rates compared favorably to the convergence rates of corresponding cells with at least one heterogeneous intercept. Whatever theoretical utility there is in forcing homogeneity on a model solution seems slight relative to the obvious futility of having no solution at all.

For the few completely invariant cells for which bias was computed, the bias in the estimation of λ_{11} did tend to be smaller in magnitude than the bias in the estimation of λ_{11} in the corresponding heterogeneous intercept cells. Bias in λ_{p1} tended to be comparable between the two conditions. For all of the other parameter estimates, cells with at least one heterogeneous intercept had clearly smaller bias magnitudes than the homogeneous intercept cells, with a substantial advantage to the heterogeneous cells in estimating both intercepts, $\Delta\kappa$, Φ_{11}, and the mixture proportion.

That the intercept bias was higher in magnitude in the completely invariant models has a more subtle meaning than just the numerical difference in the bias. With intercepts that are invariant in the populations, even a random partitioning of a mixed sample should yield subsamples that have roughly the same intercept as each other, as the full sample, and as any of the data-generating populations. Given that the biases were smaller in magnitude when a full sample (of equal overall *N*) had different intercepts than either of the data-generating populations, it seems that a mixture of intercept-invariant populations somehow yields samples that have more bias in the intercepts than mixture samples from populations with heterogeneous intercepts.

Moving along the heterogeneity continuum to models that have heterogeneous factor loadings had a curious effect on convergence while having an even more beneficial effect on bias than relaxing only the intercept invariance assumption. Convergence for models with eight manifest variables was perfect when two factor loadings differed between classes. Convergence for models that had four manifest variables was substantially worse when two loadings varied across classes than when the loadings (but not the intercepts) were invariant, even with *lc* = 4, a condition with considerably lower factor reliability than the heterogeneous factor loading combination of 100% $\lambda = .8$ in class 1 and 75% $\lambda = .8$ & 25% $\lambda = .4$ in class 2. A possible explanation for this is a confounding of *p* with the percentage of loadings that were heterogeneous across classes. Given the very few heterogeneous loading cells in the present study, however, it is not possible to elaborate further on the potential presence or nature of such a confound beyond that it would have to be an interaction effect (percentage of noninvariant loadings did not diminish the convergence rates of models with $p = 8$).

One final note about heterogeneity should be made regarding the heterogeneity of factor loadings *within* each class. The interaction of *p* and *lc* in cells with noninvariant intercepts was such that with four manifest variables, models with heterogeneous loadings within each class consistently outperformed models that had homogeneous loadings within each class in terms of both higher convergence rates and lower bias magnitudes. At present, no explanation is offered for this effect, except for the possibility of a benefit to there being heterogeneous loadings within classes; additional study of this issue seems warranted.

<u>Other design characteristics</u>

In addition to the presence or absence of cross-class heterogeneity of the intercepts and factor loadings in the models, several other design characteristics were manipulated to examine their effects on convergence and bias. To attempt to summarize these effects efficiently, an effort will be made to rank them in terms of their effects at increasing convergence rates and decreasing the magnitude of bias in the parameter estimates. Ranking the importance of each of the design characteristics is difficult, given the numerous interactions reported in Chapter 3 (some of which are also discussed in this section). Such an undertaking does, however, seem germane in order to inform design decisions of applied mixture modeling researchers.

Methodological studies of CFA when population membership is known for all observations have consistently found that sample size is of paramount importance to model convergence and to the accuracy of the parameter estimates, with recommendations always being that N should be as large as possible. The results of the present study indicate that for mixture CFA, sample size strongly affected model convergence and the bias of the parameter estimates, with larger N leading to better convergence and generally smaller magnitudes of bias. The most important design characteristic, however, was not N. The shift from a completely invariant model to one in which there was any degree of cross-class heterogeneity in the intercepts hugely improved convergence rates, doing so to a clearly greater extent than increasing N for the range of N examined. The presence of heterogeneity also more strongly reduced bias magnitude than did increasing N for the range of N examined.

After N, the next most influential design characteristic was the magnitude of $\Delta\kappa$, which generally had a direct relationship with convergence rate and an inverse relationship with bias magnitude. The ranking of $\Delta\kappa$ as third most important may, however, be unduly low. The smallest value of $\Delta\kappa$ examined was a standardized difference of 2.0, which is a statistically significant difference at the .05-level for a z-test.

Bias in the estimation of this parameter when $\Delta\kappa = 2.0$ was high and positive in the few completely invariant cells that had reasonable convergence rates, suggesting that an already statistically significant mean difference had to be adjusted further upward in order to detect a mixture of populations that differ in their latent means when no other parameters differed across populations. Bias in $\Delta\kappa$ was generally negative, if at all appreciable, when any degree of heterogeneity was introduced for the intercepts, but that seems to allow only the conclusion that the critical value (so to speak) of $\Delta\kappa$ is lower with heterogeneous intercepts than without. The relative importance of the presence of heterogeneity and of N might therefore need to be modified by the phrase "given a factor mean difference of at least 2.0". Smaller and smaller values of $\Delta\kappa$ would eventually render impossible the convergence of a completely invariant model and could have deleterious effects on the convergence of noninvariant models beyond the ability of N to compensate.

Following $\Delta\kappa$ are the effects of the presence of additional heterogeneity in the form of either a second heterogeneous intercept or larger magnitude of the difference between heterogeneous intercepts, both of which yielded higher convergence rates and lower bias magnitudes. For the bias of the factor loadings and, to a lesser extent, for the bias of the error variances, both of these effects were overshadowed by the location of the heterogeneity, with bias seeming to follow wherever the intercept heterogeneity went. For convergence and for the bias of the remaining parameters, however, once there was any heterogeneity, the extent of the heterogeneity was an important characteristic.

It is difficult to speak to the effects of p and of loading combination. The effect of p on bias, although stronger than N in a manner of speaking for certain parameters, was either wild beyond what can be summarized or was involved in the interaction with loading combination. Loading combination had no notable effect on some parameter estimates and had the interaction effect with *p* on other parameter estimates and on convergence. These results are inconsistent with those found in studies of the effects of *p* and *lc* in single-population CFA models (e.g., Gagné & Hancock, 2002; Marsh et al., 1998) in which both *p* and *lc* had a direct (and separate) relation with convergence rate while having an inverse (and separate) relation with bias.

Varying the value of φ did not have a main effect on convergence or bias. It was, however, involved in an interaction with *p* to have a sometimes weak but very consistent effect on convergence and bias. When $p = 4$, convergence rates were lower and bias magnitudes were higher with $\varphi = .5$ than with $\varphi = .7$, but with $p = 8$, convergence rates were lower and bias magnitudes were higher with $\varphi = .7$ than with $\varphi = .5$. For bias in the estimate of $\Delta\kappa$, Φ_{11}, and φ, the φ x *p* interaction effect interacted with the location of intercept heterogeneity such that when τ_p was the lone heterogeneous intercept, convergence rates were lower and bias magnitudes were higher with $\varphi = .7$ than with $\varphi = .5$ regardless of *p*. It is worth reiterating that the interaction effects involving φ were very consistent, indicating that estimated value of φ is important to consider when evaluating the rest of the parameter estimates in a confirmatory factor mixture model.

The only other interaction effect that consistently arose was the interaction between *N* and $\Delta\kappa$. The effects of *N* were stronger at larger levels of $\Delta\kappa$, and the advantage of $\Delta\kappa = 2.5$ over $\Delta\kappa = 2.0$ was greater as *N* increased (or in some cases, first created and then strengthened as *N* increased, because at $N = 200$, there were a number of instances in which $\Delta\kappa = 2.0$ had a slight advantage in terms of higher convergence rate or lower bias magnitude). This interaction effect was quite strong, and as mentioned, was quite consistent.

To summarize the rankings of the import of the design characteristics, the most influential design characteristic was the presence of any degree of noninvariance in the model, with such models having drastically higher convergence rates and generally substantially lower bias magnitudes compared to models with completely invariant intercepts. The next most important characteristic is one that an applied researcher can typically control, and that is sample size: Increasing N yielded higher convergence rates and generally decreased bias magnitudes. The third was the magnitude of the factor mean difference: Models with larger values of $\Delta\kappa$ tended to have higher convergence rates and lower bias magnitudes. To a clearly lesser, but still quite notable, extent than any of the first three characteristics, models with more heterogeneity in the intercepts, either in number of noninvariant intercepts or in the magnitude of the heterogeneity, generally had higher convergence rates and lower bias magnitudes than models with less heterogeneity. Ranking the effects of the number of manifest variables, the magnitude of the factor loadings, the location of intercept heterogeneity, and the mixing proportion is not feasible based on the results of the present study, because the effects of these facets were so entangled with other design characteristics.

Recommendations for applied researchers

Many of the design characteristics manipulated in the present study actually represent different levels of characteristics of nature rather than the characteristics of an applied study that a researcher can control. The degree of intercept heterogeneity, the magnitude of the difference between factor means, and the proportion of the sample that came from each of the hypothesized populations are under nature's control, so knowing convergence and bias patterns as a function of such design characteristics is only helpful in a *post facto* sense. The magnitude of the factor loadings, although somewhat predictable in certain contexts, is typically a feature of the model that a researcher can only reflect on after the data have been analyzed.

Decisions regarding sample size and number of indicators, however, can be made by an applied researcher in the planning phase. The results of the present study lead to the usual recommendation to use the largest N available. Specific guidelines for N are difficult to provide, given that the extent of the effect of N tended to be influenced by variations in nature-controlled design characteristics such as intercept heterogeneity and the magnitude of the factor mean difference, but it can be said with confidence that to improve convergence and to reduce bias in most of the parameter estimates, N should be as large as practically possible.

The results of the present study unfortunately render it difficult to make a straightforward recommendation for the number of manifest variables, because the effect of p on bias and convergence depended heavily on the extent of intercept heterogeneity. Completely invariant models had such difficulty converging that having eight manifest variables to sift through rendered convergence essentially nil. Although clearly improved, convergence was generally very poor when there were only four manifest variables in the completely noninvariant models. In models with one noninvariant intercept, the influence of p was exceedingly complex, but with two noninvariant intercepts, there was a clear advantage of eight manifest variables over four, in terms of both improved convergence rates and smaller bias magnitudes.

For two reasons, the recommendation is made to use a greater number of indicators per factor when theoretically feasible. The first reason is the support for such a recommendation in the single-population CFA research (e.g., Gagné & Hancock, 2002; Marsh et al., 1998). The second reason is the suggestion early in this chapter that researchers move away from the practice of forcing intercepts to be invariant in confirmatory factor models. The advantage of smaller p in completely noninvariant models is essentially meaningless, given that such models had such poor convergence rates. When convergence rates were reasonable, the effect of p was either enigmatic or in favor of larger p.

Directions for future research

As a preliminary investigation into the effects of different design characteristics on convergence and bias in mixture models, the present study manipulated several facets but did so to a very limited extent. Four of the design characteristics manipulated in the present study had only two levels ($\Delta\kappa$, p, $\Delta\tau$, and φ) and none had more than three levels. Due to, and potentially based on, the many interaction effects described in the present study, additional studies are in order that will more extensively investigate a smaller set of design characteristics in order to flesh out their influences on convergence rates and bias.

There were also important design characteristics not manipulated in the present study that deserve some attention. An additional preliminary investigation could be undertaken to examine the same facets manipulated herein but for three or more populations rather than for only two. The number of latent variables in the model could also be expanded; the presence of multiple factors would likely not influence the patterns of bias in the parameters investigated in the current study, but by using models that had only one factor, the present study did not inform the quality of the estimation of factor covariances in mixture CFA or how factor covariances might influence model convergence. Factor variances were freely estimated across classes in the present study, but they were actually equal across the data-generating populations. Given the influence of factor variance on the factor loadings (and thereby on the error variances), the magnitude of the factor variance could be a useful design characteristic to manipulate as could the magnitude of a difference in factor variance across populations (with the present study providing a head start on determining the effects of $\Delta\Phi_{11} = 0$ in the populations).

A subtle but potentially very important confound arose in the present study between number of heterogeneous intercepts in the populations and the number of intercepts allowed to vary in the algorithm estimating the parameters of the mixture models. Cells with two heterogeneous intercepts, for example, demonstrated better convergence rates than cells with one heterogeneous intercept, which in turn, had better convergence rates than cells in which all intercepts were homogeneous. It is possible, that to some extent, convergence rates improved with increasing number of heterogeneous intercepts just by virtue of granting the algorithm the flexibility of not having to force the intercepts to be exactly equal across classes. Some degree of difference (likely a nonsignificant difference) in the values of the intercepts will exist between classes in a sample even if all of the intercepts are equal across the populations. Requiring the estimation algorithm to constrain all of the intercepts to be literally equal across classes could be creating convergence difficulties that might be alleviated to a useful degree if even one intercept is freed. For applied purposes, after freeing up one or more intercepts in the estimation algorithm, a follow-up significance test could be used to determine whether the freely estimated intercepts differ statistically across classes; if all of the tests are statistically nonsignificant, then it can be empirically inferred (rather than forced by convention) that the intercepts are homogeneous in the populations.

The results of the present study also point to the potential utility of expanding mixture modeling research of the cross-class heterogeneity of factor loadings. With only 12 cells incorporated into the pilot study of models with heterogeneous factor loadings, it was not reasonable to draw many meaningful conclusions about the effects of design characteristics on convergence and/or bias in such models. Additional methodological research of mixture models with heterogeneous factor loadings could be conducted by using the design of the primary portion of the present study as a template and making some adjustments. Two such adjustments would, of course, be crossing the heterogeneous loading combination with more of the design characteristics and the inclusion of more levels of factor heterogeneity. A third and very important adjustment would be to control for the potential effect of the percentage of loadings that vary across classes.

Appendix A: Code for Simulation Programs

SAS code

```
options nodate nonumber linesize=90;
proc iml;
goseed1=1000085; goseed2=1000091;
reps=20000;
p=4;
mix={0.5 0.5};
deltakap=2.5;
mixload={0.0 1.0};
load={0.8 0.4};
trnsint1={2 0 4 5};
trnsint2={3 0 4 5};
error={0.36 0.84};
phi={1 1};
n={200,500,1000};
fitstuff=repeat(0,3,12);
classone=repeat(0,3,34);
classtwo=repeat(0,3,34);
keepthis=repeat(0,1500,39);
wk={0,0,0};
do sampsize=1 to 3;
  w=0; needed=0;
  seed1=goseed1+2*(sampsize-1);
  seed2=goseed2+2*(sampsize-1);
  print seed1;
  print seed2;
  holdthis=repeat(0,500,39);
  mormmnts=repeat(0,500,39);
do i=1 to reps;
varcheck=repeat(0,p*2+2,1);
pass=1;
lambda1=repeat(0,p,1);
lambda2=repeat(0,p,1);
thetdel1=repeat(0,p,p);
thetdel2=repeat(0,p,p);
sds1=repeat(0,p,p);
sds2=repeat(0,p,p);
n1=mix[1,1]*n[sampsize,1];
n2=mix[1,2]*n[sampsize,1];
makn1byp=repeat(1,n1,1);
makn2byp=repeat(1,n2,1);
int1=makn1byp*trnsint1;
int2=makn2byp*trnsint2;

if mixload[1,1]>0 then do;
  do jg=1 to (mixload[1,1]*p);
    lambda1[jg,1]=load[1,1];
    lambda2[jg,1]=load[1,1];
    thetdel1[jg,jg]=error[1,1];
    thetdel2[jg,jg]=error[1,1];
  end;
end;
if mixload[1,2]>0 then do;
  do rw=(mixload[1,1]*p+1) to p;
    lambda1[rw,1]=load[1,2];
    lambda2[rw,1]=load[1,2];
    thetdel1[rw,rw]=error[1,2];
    thetdel2[rw,rw]=error[1,2];
  end;
end;

sigma1=lambda1*phi[1,1]*lambda1`+thetdel1;
sigma2=lambda2*phi[1,2]*lambda2`+thetdel2;
sigma3=mix[1,1]*sigma1+mix[1,2]*sigma2;
do h=1 to p;
  sds1[h,h]=root(lambda1[h,1]*lambda1[h,1]*phi[1,1]+thetdel1[h,h]);
  sds2[h,h]=root(lambda2[h,1]*lambda2[h,1]*phi[1,2]+thetdel2[h,h]);
```

```
end;
z1=normal(repeat(seed1,n1,p));
R1=inv(sds1)*sigma1*inv(sds1);
g=root(R1);
D=int1+z1*g*sds1;
z2=normal(repeat(seed2,n2,p));
R2=inv(sds2)*sigma2*inv(sds2);
h=root(R2);
E=int2+(makn2byp*deltakap*lambda2`)+(z2*h*sds2);

file 'C:\mixed.dat';
  do r=1 to n1;
    do c=1 to p;
      put (D[r,c]) +1 @;
        end;
    put;
  end;
  do r=1 to n2;
    do c=1 to p;
      put (E[r,c]) +1 @;
        end;
    put;
  end;
closefile 'C:\mixed.dat';
start system(command);
  call push(" x '",command,"'; resume;");
  pause;
finish;
run system('C:\dissertation\programs\makeitgo');

infile 'C:\dissertation\programs\Mgo_p04.out';
input / / / / / / / / / / / / / / / / / / / / / / / / / / /
      / / / / / / / / / / / / / / / / / / / / / / / / / / /
      / / / / / / / / / / / / / / / / / / / / / / / / / / /
      / / / / / / / / / @33 test $char8.;
if test={"NORMALLY"} then do;
  input / / / / / / / @41 likelih;
  input / / / @49 freeparm / @41 AIC / @41 BIC / @41 BICadj / / @46 entropy;
  input / / / / / / / / / @23 L1c1 @33 se_L1c1 @51 cheknine 7.3;
  if cheknine=999.000 then e1c1=0;
  else if cheknine<999.000 then do;
    input @23 L2c1 @33 se_L2c1 / @23 L3c1 @33 se_L3c1 / @23 L4c1 @33 se_L4c1;
    input / / @23 e1c1 @33 se_e1c1 / / / / @23 e2c1 @33 se_e2c1
          / @23 e3c1 @33 se_e3c1 / @23 e4c1 @33 se_e4c1;
    input / / @23 vf1c1 @33 se_vf1c1 / / / @23 mf1c1 @33 se_mf1c1;
    input / / @23 i1c1 @33 se_i1c1 / @23 i2c1 @33 se_i2c1 / @23 i3c1 @33 se_i3c1
          / @23 i4c1 @33 se_i4c1;
    input / / / / @23 L1c2 @33 se_L1c2 / @23 L2c2 @33 se_L2c2
          / @23 L3c2 @33 se_L3c2 / @23 L4c2 @33 se_L4c2;
    input / / @23 e1c2 @33 se_e1c2 / @23 e2c2 @33 se_e2c2
          / @23 e3c2 @33 se_e3c2 / @23 e4c2 @33 se_e4c2;
    input / / @23 vf1c2 @33 se_vf1c2 / / / @23 mf1c2 @33 se_mf1c2;
    input / / @23 i1c2 @33 se_i1c2 / @23 i2c2 @33 se_i2c2 / @23 i3c2 @33 se_i3c2
          / @23 i4c2 @33 se_i4c2;
    input / / / / / / / / / @37 mixp1 / @37 mixp2;
    varcheck[1,1]=e1c1; varcheck[2,1]=e2c1; varcheck[3,1]=e3c1;
    varcheck[4,1]=e4c1; varcheck[5,1]=vf1c1;
    varcheck[6,1]=e1c2; varcheck[7,1]=e2c2; varcheck[8,1]=e3c2;
    varcheck[9,1]=e4c2; varcheck[10,1]=vf1c2;
  end;
end;
closefile 'C:\dissertation\programs\Mgo_p04.out';
do v=1 to (p*2+2);
  if varcheck[v,1]<=0 then pass=0;
end;
if (test={"NORMALLY"} & pass=1) then do;
  w=w+1;
  needed=needed+1;
  holdthis[w,1]=likelih; holdthis[w,2]=AIC; holdthis[w,3]=BIC;
  holdthis[w,4]=BICadj; holdthis[w,5]=entropy;
  holdthis[w,6]=L1c1; holdthis[w,7]=se_L1c1; holdthis[w,8]=L4c1; holdthis[w,9]=se_L4c1;
```

```
  holdthis[w,10]=e1c1; holdthis[w,11]=se_e1c1; holdthis[w,12]=e4c1;
holdthis[w,13]=se_e4c1;
  holdthis[w,14]=i1c1; holdthis[w,15]=se_i1c1; holdthis[w,16]=i4c1;
holdthis[w,17]=se_i4c1;
  holdthis[w,18]=vf1c1; holdthis[w,19]=se_vf1c1; holdthis[w,20]=mf1c1;
holdthis[w,21]=se_mf1c1;
  holdthis[w,22]=mixp1;
  holdthis[w,23]=L1c2; holdthis[w,24]=se_L1c2; holdthis[w,25]=L4c2;
holdthis[w,26]=se_L4c2;
  holdthis[w,27]=e1c2; holdthis[w,28]=se_e1c2; holdthis[w,29]=e4c2;
holdthis[w,30]=se_e4c2;
  holdthis[w,31]=i1c2; holdthis[w,32]=se_i1c2; holdthis[w,33]=i4c2;
holdthis[w,34]=se_i4c2;
  holdthis[w,35]=vf1c2; holdthis[w,36]=se_vf1c2; holdthis[w,37]=mf1c2;
holdthis[w,38]=se_mf1c2;
  holdthis[w,39]=mixp2;
end;
else needed=needed+1;
tm=mod(i,25);
if tm=0 then do;
  file 'C:\dissertation\tmi';
    put sampsize +1 @;
    put i +1 @;
    put w;
  closefile 'C:\dissertation\tmi';
end;
wk[sampsize,1]=w;
if w=500 then i=reps;
testbail=w/i;
if testbail < .0087 then do;
  if (i=2000 & w<=11) then i=reps;
  else if (i=4000 & w<=28) then i=reps;
  else if (i=6000 & w<=45) then i=reps;
  else if (i=8000 & w<=63) then i=reps;
  else if (i=10000 & w<=81) then i=reps;
  else if (i=12000 & w<=99) then i=reps;
  else if (i=14000 & w<=117) then i=reps;
  else if (i=16000 & w<=136) then i=reps;
  else if (i=18000 & w<=154) then i=reps;
end;
end; /* Replications */

do r=1 to wk[sampsize,1];
  do c=1 to 39;
    keepthis[r+(500*(sampsize-1)),c]=holdthis[r,c];
  end;
end;
do l=1 to 17;
  classone[sampsize,l*2-1]=sum(holdthis[,l+5])/w;
  classtwo[sampsize,l*2-1]=sum(holdthis[,l+22])/w;
  if w < 500 then do;
    do k=1 to w;
         mormmnts[k,l+5]=(holdthis[k,l+5]-classone[sampsize,l*2-1])##2;
         mormmnts[k,l+22]=(holdthis[k,l+22]-classtwo[sampsize,l*2-1])##2;
       end;
    classone[sampsize,l*2]=sum(mormmnts[,l+5])/w;
    classtwo[sampsize,l*2]=sum(mormmnts[,l+22])/w;
  end;
  else do;
    classone[sampsize,l*2]=(sum(holdthis[##,l+5])-((sum(holdthis[,l+5])##2)/500))/500;
    classtwo[sampsize,l*2]=(sum(holdthis[##,l+22])-((sum(holdthis[,l+22])##2)/500))/500;
  end;
end;
do f=1 to 5;
  fitstuff[sampsize,f*2-1]=sum(holdthis[,f])/w;
  if w < 500 then do;
    do g=1 to w;
      mormmnts[g,f]=(holdthis[g,f]-fitstuff[sampsize,f*2-1])##2;
    end;
    fitstuff[sampsize,f*2]=sum(mormmnts[,f])/w;
  end;
```

```
  else fitstuff[sampsize,f*2]=(sum(holdthis[##,f])-((sum(holdthis[,f])##2)/500))/500;
end;
fitstuff[sampsize,11]=w; fitstuff[sampsize,12]=needed;
end; /* Sample sizes */

file 'C:\dissertation\classone.dat';
  do r=1 to 3;
    do c=1 to 34;
      put (classone[r,c]) +1 @;
        end;
    put;
  end;
closefile 'C:\dissertation\classone.dat';
file 'C:\dissertation\classtwo.dat';
  do r=1 to 3;
    do c=1 to 34;
      put (classtwo[r,c]) +1 @;
        end;
    put;
  end;
closefile 'C:\dissertation\classtwo.dat';
file 'C:\dissertation\fitstuff.dat';
  do r=1 to 3;
    do c=1 to 12;
      put (fitstuff[r,c]) +1 @;
        end;
    put;
  end;
closefile 'C:\dissertation\fitstuff.dat';
file 'C:\dissertation\keepthis.dat';
  do samp=1 to 3;
  do r=1 to wk[samp,1];
    do c=1 to 39;
      put (keepthis[r+(500*(samp-1)),c]) +1 @;
        end;
    put;
  end;
  end;
closefile 'C:\dissertation\keepthis.dat';
print wk;
quit;
```

Batch file makeitgo.bat

```
C:\mplus\mplus.exe C:\dissertation\programs\mgo_p04.mpl
copy C:\docume~1\phill\mgo_p04.out C:\dissertation\programs
exit
```

Mplus code

```
title: Mixture CFA61rom SAS generated data, p = 4
data: file=C:\mixed.dat;
variable: names are v1-v4;
          classes=c(2);
analysis: type=mixture;
          miterations=1000;

model:
  %overall%
  f1 by v1*1.5 v2@0.4 v3-v4*1.25;
  f1*;
  %c#2%
  f1*1.2;
  v1-v4*0.9;
  [v1*10.5];
!  [v1*11 v4*10.5];
output: stand;
```

Appendix B: Bias and Standard Errors for Cells with Heterogeneous Intercepts

Included in this Appendix are 29 tables containing biases of parameter estimates and standard errors for every cell of the study that had at least one heterogeneous intercept but homogeneous factor loadings. The order of presentation of the tables is the same as the order of presentation of the parameters in Chapter 3: λ_{11}; λ_{p1}; δ_{11}; δ_{p1}; τ_1; τ_p; Φ_{11}; $\Delta\kappa$; and φ.

Table B1a: Percent Bias of λ_{11}

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	44.33	35.62	49.60	31.54	--	--	35.36	20.73	31.60	10.79	--	--	21.48	7.73	12.95	2.47	--	--
4	.5	2.5	4	40.97	32.47	35.56	24.29	--	--	22.76	12.94	22.23	5.54	--	--	10.82	1.11	5.06	3.70	--	--
4	.7	2	4	30.70	24.59	24.89	23.03	--	--	20.40	6.53	19.46	7.09	--	--	10.77	3.52	5.71	4.33	--	--
4	.7	2.5	4	26.98	21.81	24.21	19.68	--	--	12.26	5.36	13.00	6.15	--	--	4.31	3.60	4.76	2.36	--	--
8	.5	2	4	57.62	77.64	14.46	9.19	--	--	51.88	63.20	4.76	1.60	--	--	44.21	31.83	1.40	1.80	--	--
8	.5	2.5	4	43.51	62.65	9.91	6.30	--	--	33.16	30.40	2.60	2.15	--	--	14.45	7.80	-0.04	0.70	--	--
8	.7	2	4	55.46	82.91	10.19	6.95	--	--	50.84	73.27	2.21	2.02	--	--	47.03	48.27	1.71	0.87	--	--
8	.7	2.5	4	53.13	75.38	11.27	8.05	--	--	42.88	48.07	2.71	2.09	--	--	25.68	23.71	0.21	0.70	--	--
4	.5	2	6	11.82	5.98	5.97	2.64	0.16	0.56	4.91	1.46	2.69	1.07	0.50	0.32	1.59	0.45	0.75	0.65	0.14	0.15
4	.5	2.5	6	7.40	2.50	4.49	1.99	0.52	0.73	2.72	1.23	1.03	0.84	-0.04	-0.14	0.56	0.78	0.56	0.53	-0.05	0.10
4	.7	2	6	5.77	2.86	4.99	1.43	0.26	0.67	2.87	0.64	2.07	0.62	-0.14	-0.01	0.61	0.13	0.78	0.48	0.03	0.02
4	.7	2.5	6	3.92	1.79	3.27	2.49	-0.56	-0.06	1.17	0.80	1.35	-0.11	0.36	0.06	0.69	0.20	0.29	0.56	0.09	0.05
8	.5	2	6	23.93	28.11	2.36	0.89	0.03	0.09	15.17	11.34	0.67	-0.12	0.07	0.22	7.82	2.71	0.02	0.16	-0.01	0.06
8	.5	2.5	6	16.20	16.72	1.16	0.90	-0.24	-0.19	4.52	4.60	0.19	0.01	0.30	0.02	0.62	0.47	-0.03	0.09	-0.10	0.19
8	.7	2	6	28.14	30.78	1.95	0.81	-0.15	0.56	18.41	11.65	0.32	0.06	-0.09	0.05	10.19	4.37	-0.23	0.48	0.06	-0.08
8	.7	2.5	6	22.36	26.30	1.30	0.79	0.57	0.04	9.91	6.20	0.17	-0.26	0.09	-0.04	3.18	0.84	-0.01	0.10	-0.01	-0.01
4	.5	2	8	23.37	19.41	13.02	9.91	--	--	12.86	5.65	4.52	1.59	--	--	6.72	0.81	0.92	-0.08	--	--
4	.5	2.5	8	18.79	15.00	8.52	6.47	--	--	6.61	2.56	0.60	0.80	--	--	0.57	0.27	0.51	-0.01	--	--
4	.7	2	8	9.78	6.03	2.58	0.93	--	--	3.37	1.20	0.69	0.44	--	--	0.46	0.07	-0.28	0.01	--	--
4	.7	2.5	8	6.18	3.81	1.12	0.57	--	--	1.25	0.80	0.09	0.03	--	--	0.04	-0.02	0.11	0.06	--	--
8	.5	2	8	15.10	15.16	1.14	0.41	--	--	9.21	6.18	0.33	-0.12	--	--	4.49	1.99	0.44	-0.01	--	--
8	.5	2.5	8	4.93	2.16	2.47	0.45	--	--	0.20	0.31	-0.55	0.15	--	--	0.28	-0.06	-0.07	-0.07	--	--
8	.7	2	8	24.19	24.30	0.39	0.13	--	--	17.09	12.56	0.35	0.07	--	--	10.39	3.07	0.07	-0.06	--	--
8	.7	2.5	8	13.91	11.31	0.61	-0.07	--	--	5.72	2.86	0.25	0.18	--	--	1.70	0.55	-0.08	0.06	--	--

Table B1b: Standard Error of λ_{11}

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.2266	.2186	.3076	.2899	--	--	.1502	.1331	.2051	.1377	--	--	.0907	.0850	.1154	.0813	--	--
4	.5	2.5	4	.2259	.2327	.2922	.2658	--	--	.1471	.1261	.1843	.1190	--	--	.0921	.0765	.0945	.0816	--	--
4	.7	2	4	.2174	.2461	.2586	.2414	--	--	.1728	.1290	.1777	.1247	--	--	.0944	.0830	.0968	.0831	--	--
4	.7	2.5	4	.2172	.2559	.2508	.2619	--	--	.1313	.1117	.1610	.1206	--	--	.0898	.0778	.0918	.0793	--	--
8	.5	2	4	.1376	.1421	.1630	.1383	--	--	.0889	.1003	.0954	.0764	--	--	.0650	.0825	.0619	.0532	--	--
8	.5	2.5	4	.1157	.1383	.1610	.1285	--	--	.0851	.0830	.0864	.0746	--	--	.0624	.0551	.0578	.0509	--	--
8	.7	2	4	.1260	.1633	.1624	.1320	--	--	.0809	.1035	.0945	.0786	--	--	.0731	.0758	.0622	.0535	--	--
8	.7	2.5	4	.1229	.1453	.1451	.1363	--	--	.0880	.0979	.0888	.0754	--	--	.0697	.0659	.0579	.0511	--	--
4	.5	2	6	.1392	.1319	.1500	.1266	.0588	.0580	.0792	.0716	.0873	.0759	.0362	.0347	.0530	.0500	.0576	.0527	.0253	.0245
4	.5	2.5	6	.1295	.1135	.1378	.1194	.0486	.0461	.0753	.0696	.0801	.0736	.0298	.0288	.0504	.0488	.0549	.0518	.0209	.0204
4	.7	2	6	.1295	.1205	.1479	.1227	.0614	.0596	.0800	.0707	.0873	.0743	.0386	.0369	.0528	.0498	.0579	.0525	.0272	.0262
4	.7	2.5	6	.1312	.1126	.1356	.1241	.0521	.0496	.0746	.0688	.0802	.0722	.0322	.0310	.0507	.0479	.0550	.0516	.0227	.0219
8	.5	2	6	.1028	.0875	.0845	.0651	.0454	.0445	.0522	.0476	.0457	.0401	.0282	.0282	.0345	.0309	.0308	.0283	.0199	.0199
8	.5	2.5	6	.0790	.0805	.0714	.0615	.0389	.0388	.0492	.0420	.0420	.0385	.0248	.0247	.0302	.0276	.0293	.0273	.0174	.0175
8	.7	2	6	.0814	.1115	.0789	.0653	.0474	.0465	.0575	.0570	.0456	.0402	.0295	.0294	.0409	.0331	.0307	.0287	.0209	.0208
8	.7	2.5	6	.0847	.0915	.0728	.0627	.0413	.0409	.0538	.0470	.0421	.0388	.0260	.0260	.0339	.0284	.0293	.0273	.0184	.0185
4	.5	2	8	.0861	.0875	.1187	.0758	--	--	.0544	.0498	.0529	.0430	--	--	.0352	.0303	.0333	.0298	--	--
4	.5	2.5	8	.0729	.0734	.0816	.0771	--	--	.0615	.0424	.0454	.0417	--	--	.0309	.0281	.0319	.0289	--	--
4	.7	2	8	.0843	.0749	.0814	.0694	--	--	.0506	.0428	.0482	.0427	--	--	.0329	.0299	.0326	.0298	--	--
4	.7	2.5	8	.0734	.0672	.0754	.0665	--	--	.0443	.0403	.0455	.0412	--	--	.0306	.0284	.0315	.0290	--	--
8	.5	2	8	.0817	.0762	.0788	.0588	--	--	.0476	.0427	.0408	.0367	--	--	.0353	.0283	.0284	.0260	--	--
8	.5	2.5	8	.0725	.0593	.0637	.0565	--	--	.0401	.0363	.0379	.0353	--	--	.0279	.0254	.0267	.0250	--	--
8	.7	2	8	.0848	.0910	.0663	.0591	--	--	.0600	.0545	.0407	.0372	--	--	.0372	.0299	.0283	.0262	--	--
8	.7	2.5	8	.0675	.0675	.0628	.0577	--	--	.0406	.0406	.0385	.0358	--	--	.0259	.0259	.0268	.0252	--	--

Table B2a: Percent Bias of λ_{p1}

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	1.28	1.09	49.67	37.48	--	--	1.36	2.16	28.14	13.90	--	--	0.77	0.89	11.79	2.25	--	--
4	.5	2.5	4	0.11	-0.33	33.31	23.16	--	--	-0.28	0.22	20.51	5.89	--	--	-0.25	0.56	5.71	4.00	--	--
4	.7	2	4	1.74	4.18	24.32	21.56	--	--	0.64	0.15	17.21	9.93	--	--	0.53	0.50	5.04	4.72	--	--
4	.7	2.5	4	1.25	3.37	25.36	24.23	--	--	0.27	0.37	12.28	7.62	--	--	0.11	0.61	4.28	1.79	--	--
8	.5	2	4	3.70	2.04	12.80	6.16	--	--	1.22	1.01	2.69	3.36	--	--	1.00	0.37	2.44	0.11	--	--
8	.5	2.5	4	1.31	1.89	11.60	6.20	--	--	0.02	0.70	0.31	0.16	--	--	-0.09	0.08	0.24	0.93	--	--
8	.7	2	4	2.86	2.46	14.25	8.84	--	--	0.64	1.79	3.63	2.34	--	--	-0.01	0.31	1.72	0.68	--	--
8	.7	2.5	4	1.99	2.13	7.80	5.86	--	--	0.85	0.82	4.80	2.43	--	--	-0.04	0.11	-0.30	0.11	--	--
4	.5	2	6	-0.16	0.28	7.18	2.79	9.01	9.24	0.45	-0.20	3.11	0.22	5.41	2.83	-0.32	-0.06	0.21	0.19	1.14	1.83
4	.5	2.5	6	0.09	-0.06	7.40	0.94	9.22	3.83	0.45	-0.04	0.48	-0.46	1.49	1.08	0.04	0.31	-0.09	0.70	1.04	0.28
4	.7	2	6	-0.41	1.60	3.68	1.47	16.80	13.61	0.72	0.66	2.14	-0.04	10.95	4.84	0.09	-0.19	-0.12	0.75	2.98	0.97
4	.7	2.5	6	-0.39	0.03	3.67	1.36	9.42	6.96	0.43	0.51	1.13	0.57	4.58	1.78	0.15	0.26	-0.02	-0.14	0.84	0.15
8	.5	2	6	-0.36	0.08	0.37	1.64	13.28	9.80	-0.24	0.11	-0.50	-0.48	4.31	-0.01	0.22	0.35	0.55	0.37	1.64	0.08
8	.5	2.5	6	0.29	0.49	1.50	-0.43	3.79	6.28	0.28	0.03	0.19	0.24	1.43	-0.85	-0.21	-0.01	-0.01	-0.01	0.99	1.06
8	.7	2	6	0.52	0.14	1.56	0.77	18.10	16.21	0.31	1.28	-0.33	0.38	10.53	5.73	0.33	0.31	0.04	-0.76	1.96	0.57
8	.7	2.5	6	-0.18	0.53	1.83	1.37	6.47	3.35	-0.03	0.08	0.59	-0.01	0.98	0.67	0.22	0.19	0.56	0.17	0.61	0.01
4	.5	2	8	-0.33	-0.14	13.54	9.70	--	--	0.04	-0.27	5.17	1.83	--	--	-0.15	-0.01	0.86	-0.03	--	--
4	.5	2.5	8	-0.53	-0.15	9.32	5.99	--	--	-0.20	-0.22	0.87	0.59	--	--	-0.04	0.06	0.23	0.44	--	--
4	.7	2	8	-0.59	-0.10	2.14	1.12	--	--	0.18	0.01	0.37	0.64	--	--	-0.01	0.10	-0.08	0.22	--	--
4	.7	2.5	8	-0.11	-0.15	1.79	-0.01	--	--	0.08	-0.09	0.13	-0.01	--	--	0.01	-0.12	-0.03	0.11	--	--
8	.5	2	8	0.42	0.42	1.89	0.38	--	--	0.09	-0.10	0.45	-0.20	--	--	0.07	0.01	0.18	-0.04	--	--
8	.5	2.5	8	0.04	-0.05	2.22	0.45	--	--	0.11	0.04	-0.20	0.38	--	--	0.10	0.10	0.07	0.03	--	--
8	.7	2	8	-0.07	0.13	0.87	-0.28	--	--	0.14	0.27	0.27	0.26	--	--	-0.06	-0.03	0.00	-0.03	--	--
8	.7	2.5	8	-0.06	0.28	0.16	0.14	--	--	-0.15	0.09	0.33	0.19	--	--	0.07	0.17	0.15	0.16	--	--

Table B2b: Standard Error of λ_{p1}

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.0942	.0868	.3009	.3187	--	--	.0572	.0545	.1874	.1496	--	--	.0397	.0376	.1132	.0808	--	--
4	.5	2.5	4	.0784	.0724	.2952	.2663	--	--	.0478	.0449	.1702	.1195	--	--	.0324	.0315	.0963	.0812	--	--
4	.7	2	4	.0975	.0954	.2718	.2536	--	--	.0617	.0568	.1740	.1345	--	--	.0421	.0402	.0975	.0836	--	--
4	.7	2.5	4	.0937	.0819	.2654	.2875	--	--	.0498	.0481	.1405	.1224	--	--	.0349	.0340	.0909	.0784	--	--
8	.5	2	4	.0866	.0766	.1645	.1410	--	--	.0504	.0495	.0932	.0775	--	--	.0353	.0349	.0626	.0530	--	--
8	.5	2.5	4	.0651	.0673	.1580	.1272	--	--	.0414	.0413	.0888	.0744	--	--	.0289	.0290	.0576	.0510	--	--
8	.7	2	4	.0832	.0996	.1642	.1372	--	--	.0672	.0557	.0945	.0785	--	--	.0442	.0356	.0629	.0538	--	--
8	.7	2.5	4	.0738	.0723	.1433	.1328	--	--	.0443	.0446	.0913	.0753	--	--	.0308	.0306	.0570	.0511	--	--
4	.5	2	6	.0520	.0519	.1228	.0985	.1462	.1151	.0329	.0329	.0741	.0586	.0866	.0656	.0232	.0233	.0492	.0412	.0553	.0443
4	.5	2.5	6	.0447	.0445	.1192	.0925	.1223	.0999	.0286	.0286	.0677	.0572	.0749	.0597	.0202	.0203	.0455	.0401	.0508	.0409
4	.7	2	6	.0538	.0545	.1159	.0983	.1143	.1252	.0344	.0346	.0748	.0586	.0802	.0656	.0244	.0245	.0481	.0409	.0553	.0438
4	.7	2.5	6	.0473	.0470	.1072	.0940	.1147	.1020	.0302	.0301	.0662	.0561	.0703	.0607	.0214	.0213	.0453	.0398	.0495	.0412
8	.5	2	6	.0525	.0506	.1119	.0830	.1262	.1126	.0314	.0313	.0602	.0501	.0793	.0608	.0222	.0222	.0413	.0355	.0534	.0414
8	.5	2.5	6	.0434	.0437	.0959	.0781	.1190	.1030	.0276	.0276	.0565	.0492	.0726	.0558	.0195	.0195	.0393	.0348	.0472	.0390
8	.7	2	6	.0518	.0516	.0981	.0820	.1244	.1236	.0328	.0329	.0612	.0499	.0804	.0714	.0233	.0232	.0404	.0353	.0523	.0412
8	.7	2.5	6	.0454	.0463	.0937	.0785	.1081	.0983	.0290	.0290	.0555	.0489	.0699	.0577	.0207	.0206	.0387	.0346	.0460	.0386
4	.5	2	8	.0456	.0456	.1190	.0789	--	--	.0286	.0284	.0514	.0434	--	--	.0201	.0201	.0333	.0298	--	--
4	.5	2.5	8	.0397	.0391	.0832	.0751	--	--	.0255	.0247	.0463	.0415	--	--	.0174	.0176	.0317	.0290	--	--
4	.7	2	8	.0465	.0469	.0816	.0699	--	--	.0300	.0299	.0480	.0426	--	--	.0210	.0211	.0327	.0299	--	--
4	.7	2.5	8	.0416	.0412	.0754	.0656	--	--	.0261	.0262	.0454	.0412	--	--	.0185	.0185	.0315	.0291	--	--
8	.5	2	8	.0455	.0431	.0809	.0594	--	--	.0275	.0273	.0409	.0367	--	--	.0193	.0195	.0284	.0259	--	--
8	.5	2.5	8	.0379	.0376	.0631	.0561	--	--	.0241	.0241	.0381	.0353	--	--	.0170	.0169	.0268	.0249	--	--
8	.7	2	8	.0517	.0457	.0791	.0589	--	--	.0287	.0288	.0407	.0372	--	--	.0203	.0202	.0283	.0262	--	--
8	.7	2.5	8	.0398	.0398	.0620	.0577	--	--	.0254	.0254	.0384	.0357	--	--	.0180	.0180	.0269	.0252	--	--

Table B3a: Percent Bias of δ_{11}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-0.73	7.39	-7.22	-5.64	--	--	1.51	2.73	-3.75	-1.53	--	--	4.18	1.86	-1.98	-0.55	--	--
4	.5	2.5	4	-2.85	-0.26	-5.64	-2.34	--	--	0.05	-0.67	-2.46	-2.56	--	--	0.92	0.00	-1.10	-0.52	--	--
4	.7	2	4	-4.01	-4.68	-8.11	-6.05	--	--	-0.61	-2.78	-3.83	-2.01	--	--	-0.60	-1.84	-1.82	-0.62	--	--
4	.7	2.5	4	-6.02	-5.09	-7.02	-4.77	--	--	-4.03	-1.92	-2.90	-1.87	--	--	-2.27	-0.72	-0.63	-1.09	--	--
8	.5	2	4	-63.56	-60.73	-6.45	-2.83	--	--	-57.67	-48.92	-2.16	-1.58	--	--	-51.82	-24.74	-1.05	-0.87	--	--
8	.5	2.5	4	-52.04	-41.98	-4.23	-2.20	--	--	-34.45	-21.11	-2.49	-1.44	--	--	-17.20	-5.28	-0.71	-0.93	--	--
8	.7	2	4	-66.88	-67.08	-6.36	-2.95	--	--	-62.08	-55.92	-2.11	-0.60	--	--	-52.58	-33.59	-0.29	-0.86	--	--
8	.7	2.5	4	-56.50	-50.28	-5.39	-3.25	--	--	-38.42	-26.38	-1.41	-0.67	--	--	-18.44	-10.59	-0.43	-0.20	--	--
4	.5	2	6	8.58	1.95	-3.33	-4.01	-0.53	-0.19	1.36	-1.10	-3.96	-2.15	2.20	-1.00	0.64	-0.65	-0.28	-1.06	0.46	-0.28
4	.5	2.5	6	2.54	-1.44	-4.26	-5.19	-0.60	-2.87	-1.12	-2.27	-1.96	-0.23	-0.64	-1.02	-1.41	-0.66	-0.80	-0.67	0.26	-0.98
4	.7	2	6	-1.27	-4.20	-4.89	-2.84	0.42	1.60	-2.48	-1.43	-3.40	-2.20	1.31	-0.17	-1.06	-1.31	-1.61	-0.74	-0.49	-0.34
4	.7	2.5	6	-5.77	-2.59	-5.97	-3.54	-0.77	-1.11	-2.99	-0.61	-2.56	-1.29	-1.44	-0.52	-1.98	-0.33	-1.12	-0.53	0.21	-0.12
8	.5	2	6	-41.03	-39.76	-4.13	-0.69	-0.76	0.59	-31.80	-18.11	0.21	-0.07	-0.98	-0.24	-16.93	-5.01	-0.61	-0.41	0.14	-0.25
8	.5	2.5	6	-28.99	-21.28	-2.51	-2.79	-1.33	-0.91	-10.67	-6.51	-0.53	-1.36	-0.44	-0.02	-2.71	-0.48	-1.02	-0.38	0.01	0.74
8	.7	2	6	-52.50	-44.49	-0.95	-1.74	1.44	0.10	-35.94	-18.19	-1.86	-0.69	-0.51	0.08	-20.22	-6.40	-0.84	-0.06	-0.46	-0.89
8	.7	2.5	6	-34.84	-33.14	-3.20	0.11	-1.41	-0.12	-16.31	-8.89	-0.85	-0.49	-0.89	-0.005	-5.69	-1.87	-0.51	0.24	0.31	-0.12
4	.5	2	8	26.50	34.34	-1.19	-2.19	--	--	16.02	11.20	-0.22	-0.13	--	--	8.32	1.67	0.02	-0.11	--	--
4	.5	2.5	8	17.32	19.34	0.15	-1.72	--	--	5.62	3.19	-0.53	-0.79	--	--	-0.66	-1.65	-0.42	-0.75	--	--
4	.7	2	8	7.84	5.19	-2.21	-1.20	--	--	1.98	0.35	-0.69	-1.16	--	--	-0.92	-0.84	-0.41	-0.50	--	--
4	.7	2.5	8	1.79	3.56	-1.76	-0.84	--	--	0.06	0.27	-0.80	-0.82	--	--	-0.76	0.13	-0.74	-0.37	--	--
8	.5	2	8	-21.35	-17.29	-3.30	-2.23	--	--	-15.39	-9.23	-0.84	-0.16	--	--	-8.59	-2.27	-0.66	-0.44	--	--
8	.5	2.5	8	-9.03	-3.11	-0.77	-0.95	--	--	-1.64	-0.75	-0.73	-1.26	--	--	-0.43	-0.32	-0.81	-0.08	--	--
8	.7	2	8	-39.20	-31.95	-1.78	-1.74	--	--	-28.28	-17.01	-1.50	-0.46	--	--	-15.17	-4.07	-0.44	-0.33	--	--
8	.7	2.5	8	-20.92	-13.13	-0.59	-1.25	--	--	-8.56	-3.42	-0.43	0.27	--	--	-2.86	-0.59	-0.43	-0.28	--	--

Table B3b: Standard Error of δ_{11}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.2171	.2621	.2235	.1941	--	--	.1592	.1666	.1375	.1110	--	--	.1097	.1183	.0855	.0754	--	--
4	.5	2.5	4	.2204	.2441	.2035	.1862	--	--	.1560	.1483	.1243	.1048	--	--	.1017	.0986	.0779	.0726	--	--
4	.7	2	4	.1853	.2069	.1789	.1620	--	--	.1422	.1338	.1090	.0916	--	--	.0887	.0924	.0689	.0636	--	--
4	.7	2.5	4	.1976	.2048	.1676	.1554	--	--	.1179	.1161	.0983	.0890	--	--	.0830	.0803	.0651	.0615	--	--
8	.5	2	4	.2641	.2728	.1876	.1626	--	--	.2759	.2301	.1151	.0998	--	--	.2052	.1729	.0763	.0691	--	--
8	.5	2.5	4	.2315	.2550	.1773	.1515	--	--	.2181	.1703	.1009	.0940	--	--	.1262	.1137	.0696	.0651	--	--
8	.7	2	4	.2068	.2238	.1652	.1340	--	--	.2465	.2033	.0906	.0823	--	--	.1709	.1226	.0612	.0562	--	--
8	.7	2.5	4	.2469	.2413	.1363	.1417	--	--	.1798	.1425	.0848	.0779	--	--	.1082	.0831	.0563	.0546	--	--
4	.5	2	6	.1520	.1499	.1324	.1179	.1442	.1103	.0991	.0900	.0780	.0745	.0773	.0648	.0682	.0619	.0547	.0521	.0487	.0450
4	.5	2.5	6	.1372	.1268	.1241	.1121	.1138	.0989	.0829	.0796	.0740	.0718	.0647	.0605	.0566	.0559	.0516	.0503	.0455	.0433
4	.7	2	6	.1277	.1202	.1154	.1085	.1287	.1003	.0817	.0757	.0692	.0643	.0795	.0573	.0557	.0519	.0472	.0454	.0444	.0383
4	.7	2.5	6	.1180	.1083	.1061	.1020	.1050	.0887	.0709	.0686	.0656	.0625	.0590	.0521	.0485	.0474	.0451	.0448	.0387	.0365
8	.5	2	6	.1628	.1465	.0911	.0794	.1019	.0829	.1030	.0735	.0529	.0479	.0527	.0461	.0610	.0446	.0362	.0336	.0348	.0318
8	.5	2.5	6	.1270	.1009	.0776	.0705	.0787	.0711	.0711	.0548	.0475	.0450	.0462	.0434	.0372	.0354	.0330	.0320	.0319	.0308
8	.7	2	6	.1337	.1331	.0721	.0624	.0943	.0837	.0903	.0646	.0434	.0391	.0602	.0408	.0536	.0346	.0291	.0277	.0292	.0261
8	.7	2.5	6	.1159	.1082	.0631	.0607	.0740	.0616	.0618	.0488	.0390	.0374	.0378	.0366	.0341	.0290	.0275	.0266	.0262	.0256
4	.5	2	8	.0900	.1050	.0834	.0738	--	--	.0629	.0616	.0522	.0454	--	--	.0442	.0410	.0341	.0322	--	--
4	.5	2.5	8	.0822	.0970	.0770	.0699	--	--	.0580	.0532	.0453	.0438	--	--	.0373	.0360	.0322	.0312	--	--
4	.7	2	8	.0785	.0765	.0636	.0604	--	--	.0501	.0477	.0396	.0377	--	--	.0349	.0324	.0277	.0267	--	--
4	.7	2.5	8	.0710	.0702	.0606	.0595	--	--	.0441	.0426	.0381	.0368	--	--	.0305	.0297	.0267	.0262	--	--
8	.5	2	8	.1198	.1044	.0704	.0622	--	--	.0713	.0576	.0429	.0394	--	--	.0660	.0376	.0302	.0277	--	--
8	.5	2.5	8	.0790	.0721	.0636	.0606	--	--	.0469	.0437	.0401	.0383	--	--	.0321	.0310	.0279	.0273	--	--
8	.7	2	8	.1367	.1024	.0746	.0516	--	--	.1034	.0585	.0344	.0326	--	--	.0465	.0307	.0244	.0232	--	--
8	.7	2.5	8	.0709	.0709	.0525	.0532	--	--	.0404	.0404	.0333	.0321	--	--	.0257	.0257	.0233	.0227	--	--

Table B3c: Percent Bias of δ_{11}, Class 2

				N = 200						*N* = 500						*N* = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-54.40	-37.35	-19.28	-7.71	--	--	-38.26	-17.78	-5.50	-2.14	--	--	-28.59	-6.89	-1.58	-0.65	--	--
4	.5	2.5	4	-44.60	-27.33	-16.95	-7.39	--	--	-26.60	-7.32	-3.68	-2.12	--	--	-9.68	-1.67	-1.36	-1.20	--	--
4	.7	2	4	-46.67	-26.81	-15.93	-5.80	--	--	-26.28	-7.57	-4.78	-2.58	--	--	-14.95	-0.84	-2.58	-1.25	--	--
4	.7	2.5	4	-39.39	-17.67	-9.97	-3.72	--	--	-13.68	-4.10	-3.08	-2.38	--	--	-4.87	-1.27	-1.39	-0.27	--	--
8	.5	2	4	8.00	25.48	-5.69	-2.85	--	--	9.37	22.88	-1.53	-1.88	--	--	9.59	12.47	-1.11	-0.80	--	--
8	.5	2.5	4	2.43	14.67	-5.42	-2.60	--	--	3.55	7.85	-0.62	-1.54	--	--	2.10	1.88	-0.73	-1.07	--	--
8	.7	2	4	7.74	27.89	-9.23	-4.94	--	--	9.53	26.84	-2.20	-1.29	--	--	11.13	22.87	-0.97	-1.03	--	--
8	.7	2.5	4	3.71	16.83	-5.37	-5.95	--	--	8.42	15.19	-0.84	-1.16	--	--	5.28	9.35	-0.26	-1.19	--	--
4	.5	2	6	-24.52	-7.08	-6.54	-5.27	-2.93	-3.52	-10.48	-3.29	-3.12	-1.36	-0.77	-0.90	-3.82	-1.67	-1.39	-0.58	-1.03	-0.42
4	.5	2.5	6	-12.81	-5.95	-7.90	-3.08	-1.61	-0.92	-3.29	-1.87	-1.50	-1.54	-0.92	-0.48	-1.45	0.21	-0.88	-1.53	-0.50	-0.38
4	.7	2	6	-9.75	-5.11	-6.56	-3.27	-1.77	-4.78	-4.04	-1.17	-3.48	-1.61	1.24	-0.57	-1.61	0.26	-2.16	-2.00	0.04	-0.72
4	.7	2.5	6	-10.76	-0.79	-8.99	-3.48	2.15	-1.93	-1.75	-1.77	-3.23	-1.14	-0.01	-1.36	-1.13	-1.05	-0.04	-0.89	-0.79	-0.33
8	.5	2	6	26.70	50.49	-3.98	-3.51	0.68	-0.63	19.48	22.98	-1.79	-1.02	-0.33	-0.63	10.70	5.78	-1.06	-0.59	-0.14	-0.21
8	.5	2.5	6	11.19	23.75	-2.84	-1.36	-1.90	-1.22	4.78	7.32	-1.61	-1.13	-0.70	-0.34	0.23	0.66	-0.04	0.45	0.08	0.07
8	.7	2	6	32.76	61.87	-4.25	-0.71	-1.92	-0.25	26.06	23.55	-0.77	-1.31	0.32	-1.92	15.57	8.72	-0.70	0.24	0.40	0.07
8	.7	2.5	6	20.86	39.03	-4.81	-2.85	0.07	-0.47	10.84	11.68	-1.29	-1.16	-1.43	0.83	4.76	1.17	-0.87	-0.17	-1.13	0.24
4	.5	2	8	-43.20	-27.23	-10.62	-7.70	--	--	-27.20	-10.60	-3.38	-1.01	--	--	-15.48	-1.90	-1.12	0.02	--	--
4	.5	2.5	8	-37.60	-21.11	-8.58	-4.61	--	--	-13.27	-4.01	-1.22	-0.59	--	--	-1.31	-0.42	-0.84	-1.11	--	--
4	.7	2	8	-23.26	-10.41	-6.07	-0.56	--	--	-9.62	-2.78	-1.55	-1.31	--	--	-1.28	0.04	-1.37	-0.97	--	--
4	.7	2.5	8	-16.52	-6.90	-3.01	-2.03	--	--	-3.28	-2.00	-2.04	-2.16	--	--	-1.23	-0.77	-0.56	-0.65	--	--
8	.5	2	8	15.24	26.80	-3.64	-2.61	--	--	11.46	13.06	-1.40	-1.02	--	--	4.86	2.98	-1.00	-0.06	--	--
8	.5	2.5	8	3.94	1.50	-2.79	-1.83	--	--	-1.38	-0.44	-0.66	-0.58	--	--	-0.38	-0.20	-0.95	-0.44	--	--
8	.7	2	8	33.00	53.79	-3.44	-2.98	--	--	23.00	26.92	-1.77	-0.66	--	--	14.91	7.93	0.07	-0.62	--	--
8	.7	2.5	8	11.23	16.01	-3.80	-2.82	--	--	5.43	4.82	-1.24	-0.01	--	--	2.31	1.43	-0.58	-0.93	--	--

Table B3d: Standard Error of δ_{11}, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.2780	.2751	.2652	.1929	--	--	.2599	.1992	.1452	.1096	--	--	.1778	.1286	.0869	.0757	--	--
4	.5	2.5	4	.3256	.2524	.2360	.1884	--	--	.2017	.1570	.1254	.1036	--	--	.1328	.1034	.0780	.0726	--	--
4	.7	2	4	.2824	.3435	.2628	.2505	--	--	.2802	.2279	.1789	.1445	--	--	.1962	.1660	.1134	.0997	--	--
4	.7	2.5	4	.2950	.3283	.2572	.2354	--	--	.2052	.1962	.1594	.1362	--	--	.1423	.1369	.1040	.0952	--	--
8	.5	2	4	.1696	.1884	.1838	.1691	--	--	.1062	.1314	.1124	.0974	--	--	.0758	.1054	.0759	.0685	--	--
8	.5	2.5	4	.1460	.1858	.1656	.1499	--	--	.1063	.1185	.1028	.0934	--	--	.0827	.0836	.0694	.0652	--	--
8	.7	2	4	.1439	.1919	.2373	.2076	--	--	.1077	.1535	.1619	.1343	--	--	.1094	.1325	.1078	.0939	--	--
8	.7	2.5	4	.1574	.2053	.2143	.2020	--	--	.1323	.1714	.1340	.1248	--	--	.1101	.1258	.0953	.0874	--	--
4	.5	2	6	.1713	.1521	.1503	.1178	.1308	.1268	.1090	.0885	.0802	.0742	.0707	.0651	.0670	.0618	.0551	.0522	.0482	.0450
4	.5	2.5	6	.1385	.1238	.1248	.1159	.1064	.1029	.0831	.0792	.0740	.0713	.0644	.0609	.0575	.0558	.0515	.0503	.0453	.0430
4	.7	2	6	.1993	.1795	.1622	.1508	.1445	.1330	.1298	.1204	.1072	.0932	.0928	.0863	.0922	.0827	.0720	.0648	.0653	.0599
4	.7	2.5	6	.1697	.1646	.1445	.1404	.1360	.1239	.1083	.1030	.0935	.0877	.0864	.0809	.0743	.0712	.0654	.0623	.0601	.0568
8	.5	2	6	.1086	.1064	.0974	.0763	.0827	.0760	.0596	.0615	.0526	.0479	.0519	.0457	.0420	.0405	.0361	.0336	.0342	.0320
8	.5	2.5	6	.0845	.0918	.0777	.0718	.0736	.0700	.0555	.0538	.0472	.0449	.0457	.0434	.0361	.0353	.0334	.0322	.0318	.0307
8	.7	2	6	.1017	.1461	.1150	.1040	.1076	.0999	.0798	.0893	.0752	.0646	.0707	.0624	.0622	.0569	.0499	.0458	.0478	.0436
8	.7	2.5	6	.1064	.1221	.0986	.0946	.0987	.0941	.0760	.0742	.0637	.0599	.0605	.0594	.0505	.0475	.0451	.0425	.0423	.0408
4	.5	2	8	.1400	.1217	.1386	.0823	--	--	.1018	.0708	.0763	.0466	--	--	.0621	.0412	.0346	.0322	--	--
4	.5	2.5	8	.1224	.1140	.1036	.0786	--	--	.0831	.0549	.0475	.0442	--	--	.0384	.0362	.0321	.0312	--	--
4	.7	2	8	.1376	.1247	.1080	.0974	--	--	.0987	.0778	.0671	.0596	--	--	.0619	.0557	.0456	.0420	--	--
4	.7	2.5	8	.1166	.1119	.0961	.0898	--	--	.0726	.0699	.0599	.0568	--	--	.0507	.0481	.0428	.0407	--	--
8	.5	2	8	.0840	.0879	.0899	.0616	--	--	.0529	.0523	.0429	.0389	--	--	.0379	.0352	.0297	.0281	--	--
8	.5	2.5	8	.0717	.0721	.0686	.0602	--	--	.0460	.0441	.0399	.0384	--	--	.0321	.0312	.0282	.0271	--	--
8	.7	2	8	.0970	.1220	.0977	.0812	--	--	.0757	.0776	.0581	.0519	--	--	.0504	.0493	.0410	.0369	--	--
8	.7	2.5	8	.0915	.1027	.0861	.0783	--	--	.0634	.0632	.0529	.0505	--	--	.0445	.0421	.0376	.0356	--	--

Table B4a: Percent Bias of δ_{p1}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-1.53	-2.06	-7.68	-4.87	--	--	-1.12	-0.33	-3.34	-1.44	--	--	-0.58	-0.13	-1.35	-1.17	--	--
4	.5	2.5	4	-2.31	-1.74	-4.70	-2.65	--	--	-0.84	0.23	-2.78	-1.83	--	--	-0.51	-0.26	-0.67	-0.38	--	--
4	.7	2	4	-1.19	-1.17	-6.76	-5.28	--	--	-0.78	0.18	-3.78	-2.91	--	--	0.10	0.44	-1.01	-1.11	--	--
4	.7	2.5	4	-1.70	-1.28	-6.06	-5.23	--	--	-1.07	0.09	-2.06	-2.00	--	--	-0.17	0.46	-1.30	-0.54	--	--
8	.5	2	4	-8.65	-12.98	-4.42	-5.47	--	--	-8.31	-8.48	-2.46	-1.75	--	--	0.31	-0.78	-0.91	-1.31	--	--
8	.5	2.5	4	-6.27	-5.32	-4.01	-2.82	--	--	-3.64	-1.54	-1.76	-0.60	--	--	-1.33	-0.46	-0.80	-0.12	--	--
8	.7	2	4	-2.91	-9.42	-6.88	-4.14	--	--	-6.12	-9.81	-2.54	-1.36	--	--	-1.36	0.77	-0.95	-0.52	--	--
8	.7	2.5	4	-7.30	0.55	-4.51	-2.90	--	--	-4.16	-2.19	-1.99	-1.69	--	--	2.62	0.98	-0.52	-0.30	--	--
4	.5	2	6	-1.80	-2.44	-2.91	-2.11	-15.32	-7.59	-0.28	-0.30	-0.92	-1.30	-5.32	-1.94	-0.02	-0.13	-0.35	-0.23	-0.92	0.24
4	.5	2.5	6	-1.02	-2.54	-2.91	-2.15	-6.83	-5.39	-0.56	-0.29	-0.39	-0.59	-1.10	-0.12	-0.36	-0.41	-0.58	-0.43	-0.88	-0.93
4	.7	2	6	-1.38	0.19	-3.99	-2.76	-20.86	-9.88	-0.19	-0.45	-1.06	-0.94	-10.65	-3.77	-0.03	0.56	-0.49	-0.34	-3.77	-0.90
4	.7	2.5	6	-1.56	-1.14	-1.68	-1.84	-11.86	-8.38	0.20	0.23	-0.81	-0.65	-4.21	-1.88	0.19	-0.01	-0.59	-0.25	-1.20	-0.30
8	.5	2	6	-5.78	-1.63	-3.53	-0.81	-11.69	-5.56	-2.26	-0.60	-0.79	-1.49	-4.15	-1.89	-0.48	-0.76	-0.80	-0.07	-0.72	-0.62
8	.5	2.5	6	-2.26	-1.59	-2.08	-1.57	-5.45	-3.63	-0.56	-0.80	-1.50	-0.37	-1.66	-1.71	-0.86	-0.46	-0.35	-0.25	-0.11	-0.07
8	.7	2	6	-0.01	-1.00	-2.33	-2.36	-14.32	-7.89	-2.31	-0.41	-0.94	-0.77	-7.48	-3.71	-0.68	-0.72	-0.23	-0.54	-2.65	-0.35
8	.7	2.5	6	-4.11	-0.57	-2.07	-1.38	-7.20	-5.81	-0.57	0.45	-0.74	-1.16	-1.80	-1.07	-0.06	-0.15	-0.49	0.06	-0.98	-1.07
4	.5	2	8	-1.27	-1.14	1.73	-0.65	--	--	-0.49	-0.48	-0.17	-0.99	--	--	-0.46	-0.30	-0.39	-0.17	--	--
4	.5	2.5	8	-1.62	-1.70	-2.46	-0.76	--	--	0.20	-0.40	-0.29	-0.72	--	--	-0.43	-0.22	-0.83	-1.13	--	--
4	.7	2	8	-2.04	-0.29	-0.46	-2.28	--	--	-0.42	-0.38	-1.84	-0.97	--	--	0.21	-0.03	-0.46	-0.41	--	--
4	.7	2.5	8	-1.14	-1.48	-1.53	-2.92	--	--	0.06	-0.15	-0.81	-0.52	--	--	0.23	0.32	-0.30	-0.32	--	--
8	.5	2	8	-2.08	0.68	-1.26	-0.68	--	--	-0.47	-0.53	-1.06	-0.41	--	--	-0.08	-0.26	0.15	-0.80	--	--
8	.5	2.5	8	0.09	-1.49	-1.19	-2.38	--	--	-0.41	-0.99	-0.90	-0.98	--	--	-0.11	0.37	-0.72	-0.43	--	--
8	.7	2	8	-0.92	3.34	-2.95	-0.91	--	--	2.12	0.04	-0.39	-0.97	--	--	1.95	0.86	-0.86	-0.18	--	--
8	.7	2.5	8	-2.11	-1.84	-2.55	-2.26	--	--	-0.34	0.19	-0.98	-0.66	--	--	0.12	-0.22	-0.61	-0.38	--	--

Table B4b: Standard Error of δ_{p1}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_{1};1	τ_{1};1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_{p};1	τ_{p};1.5	τ_{1};1	τ_{1};1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_{p};1	τ_{p};1.5	τ_{1};1	τ_{1};1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_{p};1	τ_{p};1.5
4	.5	2	4	.1318	.1411	.2340	.2040	--	--	.0905	.0974	.1318	.1145	--	--	.0674	.0702	.0867	.0752	--	--
4	.5	2.5	4	.1361	.1433	.2076	.1922	--	--	.0994	.0978	.1206	.1045	--	--	.0701	.0684	.0779	.0725	--	--
4	.7	2	4	.1260	.1339	.1815	.1689	--	--	.0857	.0866	.1071	.0943	--	--	.0605	.0598	.0691	.0640	--	--
4	.7	2.5	4	.1435	.1420	.1731	.1606	--	--	.0880	.0813	.0984	.0902	--	--	.0602	.0575	.0653	.0616	--	--
8	.5	2	4	.4507	.3893	.1876	.1658	--	--	.4169	.3185	.1116	.0969	--	--	.3782	.2213	.0754	.0688	--	--
8	.5	2.5	4	.3520	.3302	.1674	.1501	--	--	.2570	.1811	.1023	.0933	--	--	.1454	.0818	.0694	.0650	--	--
8	.7	2	4	.4360	.4065	.1603	.1319	--	--	.4584	.3449	.0925	.0816	--	--	.3608	.1661	.0617	.0570	--	--
8	.7	2.5	4	.4012	.3743	.1392	.1266	--	--	.2507	.1453	.0876	.0770	--	--	.1250	.0743	.0559	.0543	--	--
4	.5	2	6	.1285	.1284	.1474	.1338	.2315	.2139	.0872	.0822	.0904	.0837	.1290	.1214	.0614	.0580	.0638	.0598	.0868	.0844
4	.5	2.5	6	.1314	.1233	.1387	.1285	.1733	.1579	.0821	.0806	.0862	.0825	.1048	.1021	.0585	.0571	.0596	.0580	.0722	.0713
4	.7	2	6	.1148	.1122	.1170	.1128	.1917	.1744	.0735	.0690	.0774	.0701	.1233	.1008	.0505	.0488	.0519	.0492	.0755	.0663
4	.7	2.5	6	.1093	.1072	.1131	.1075	.1665	.1427	.0698	.0673	.0706	.0685	.0907	.0843	.0492	.0480	.0494	.0485	.0589	.0582
8	.5	2	6	.3683	.2832	.1674	.1319	.2158	.1903	.2276	.1310	.0897	.0816	.1208	.1148	.1070	.0645	.0623	.0588	.0850	.0799
8	.5	2.5	6	.2403	.2097	.1344	.1269	.1747	.1654	.1113	.0903	.0829	.0805	.1027	.0977	.0599	.0562	.0587	.0570	.0685	.0683
8	.7	2	6	.3533	.2943	.1196	.1095	.1841	.1876	.1878	.1114	.0746	.0684	.1211	.1078	.0857	.0534	.0508	.0487	.0725	.0645
8	.7	2.5	6	.2276	.1913	.1107	.1059	.1490	.1319	.1034	.0784	.0689	.0665	.0844	.0823	.0543	.0473	.0489	.0476	.0563	.0557
4	.5	2	8	.0584	.0628	.0815	.0737	--	--	.0421	.0430	.0500	.0452	--	--	.0310	.0312	.0338	.0321	--	--
4	.5	2.5	8	.0592	.0631	.0720	.0754	--	--	.0445	.0427	.0457	.0441	--	--	.0314	.0306	.0320	.0311	--	--
4	.7	2	8	.0578	.0575	.0649	.0600	--	--	.0386	.0367	.0398	.0378	--	--	.0271	.0260	.0276	.0267	--	--
4	.7	2.5	8	.0565	.0554	.0611	.0577	--	--	.0370	.0363	.0379	.0370	--	--	.0263	.0258	.0268	.0263	--	--
8	.5	2	8	.1182	.0993	.0687	.0627	--	--	.0683	.0493	.0428	.0392	--	--	.0491	.0333	.0300	.0278	--	--
8	.5	2.5	8	.0732	.0609	.0638	.0593	--	--	.0394	.0372	.0399	.0383	--	--	.0272	.0267	.0282	.0272	--	--
8	.7	2	8	.1502	.1139	.0595	.0518	--	--	.0996	.0575	.0352	.0326	--	--	.0418	.0255	.0243	.0233	--	--
8	.7	2.5	8	.0647	.0647	.0525	.0497	--	--	.0347	.0347	.0331	.0321	--	--	.0225	.0225	.0233	.0227	--	--

Table B4c: Percent Bias of δ_{p1}, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-11.43	-5.61	-19.38	-6.78	--	--	-2.78	-1.39	-7.20	-2.11	--	--	-5.00	-1.15	-1.77	-0.76	--	--
4	.5	2.5	4	-13.60	-3.09	-14.54	-5.72	--	--	-4.26	-0.92	-4.30	-1.62	--	--	-1.38	-0.78	-1.50	-0.97	--	--
4	.7	2	4	-11.70	-5.71	-13.42	-6.27	--	--	-8.26	-0.20	-5.41	-1.88	--	--	-2.16	0.23	-0.42	-0.55	--	--
4	.7	2.5	4	-11.76	-4.51	-11.16	-5.78	--	--	-2.11	-1.20	-2.80	-2.04	--	--	-0.69	-0.22	-2.22	-0.64	--	--
8	.5	2	4	-0.70	-0.51	-7.14	-1.92	--	--	-1.07	-0.58	-2.21	-1.31	--	--	-0.13	-0.11	-0.43	-0.21	--	--
8	.5	2.5	4	0.11	-0.20	-6.68	-2.46	--	--	0.53	-0.63	-0.95	-0.41	--	--	-0.47	0.16	-0.80	-0.89	--	--
8	.7	2	4	-0.57	-1.22	-6.62	-3.88	--	--	0.09	-0.22	-3.19	-2.62	--	--	0.25	-0.30	-2.14	-0.13	--	--
8	.7	2.5	4	-0.06	-0.40	-6.96	-3.66	--	--	0.05	-1.42	-2.27	-2.27	--	--	-0.57	-0.81	-0.26	-1.28	--	--
4	.5	2	6	-2.11	-0.33	-6.71	-3.32	-7.20	-1.46	-0.75	-0.57	-1.84	-0.38	-3.96	-0.92	-0.20	0.04	-0.32	-0.47	-1.71	-0.04
4	.5	2.5	6	-1.51	-1.12	-4.88	-2.08	-5.97	-3.93	-0.25	0.43	-0.20	-0.46	-1.85	-0.49	-0.75	-0.30	-0.53	-0.49	-1.06	-1.04
4	.7	2	6	-3.51	0.62	-5.85	-3.84	-9.67	-2.41	-1.76	-0.10	-0.76	-0.14	-2.57	-0.78	-0.10	-0.93	-0.21	-1.19	-1.70	-1.62
4	.7	2.5	6	-2.09	-0.61	-5.33	-2.70	-5.29	-4.47	-0.44	-0.20	-1.88	-1.34	-2.83	-1.58	-0.02	-0.64	0.13	-0.54	-0.61	-0.37
8	.5	2	6	-0.35	-0.20	-3.96	-2.29	-4.44	-3.44	0.06	0.01	-1.22	-0.81	-1.86	-1.89	-0.76	0.04	-0.05	-0.40	-0.62	-0.35
8	.5	2.5	6	-1.05	-0.97	-2.37	-3.50	-3.71	-2.14	-1.05	0.52	-0.59	-0.29	-1.12	-1.45	-0.13	-0.25	-0.46	-0.73	-0.58	-0.68
8	.7	2	6	-1.20	-1.34	-7.75	-2.76	-4.33	1.29	-0.17	-1.33	-2.06	-1.10	-2.89	-0.57	0.07	-0.90	-0.72	-1.13	-2.10	-1.51
8	.7	2.5	6	-1.15	-0.57	-3.18	-1.18	-6.28	-1.65	-0.80	-1.37	-1.52	-1.26	-1.61	-2.65	0.31	-0.47	-0.36	-0.80	-0.91	-0.55
4	.5	2	8	-3.82	2.55	-13.23	-5.73	--	--	1.42	1.19	-4.04	-1.79	--	--	0.53	0.08	-1.47	-0.68	--	--
4	.5	2.5	8	-4.38	-0.72	-9.50	-2.70	--	--	2.35	-0.07	-2.01	-1.25	--	--	0.06	0.03	-0.10	-0.11	--	--
4	.7	2	8	0.54	-2.79	-6.13	-2.52	--	--	-0.49	-1.42	-2.62	-0.08	--	--	-0.66	-0.56	-0.84	-0.86	--	--
4	.7	2.5	8	0.25	-1.97	-4.95	-5.00	--	--	-0.42	1.55	-0.93	-0.93	--	--	0.74	-0.43	-0.29	-0.81	--	--
8	.5	2	8	-1.96	-1.53	-4.28	-2.06	--	--	-0.61	-0.97	-0.38	-0.64	--	--	-0.45	0.44	-0.71	-0.97	--	--
8	.5	2.5	8	-1.02	-1.47	-3.01	-2.47	--	--	-1.21	-0.02	0.16	-0.51	--	--	0.16	-0.07	-0.90	-0.35	--	--
8	.7	2	8	-1.48	-3.13	-3.67	-2.08	--	--	-0.80	-0.83	-0.67	-1.06	--	--	-0.13	-0.26	-0.39	-0.78	--	--
8	.7	2.5	8	-0.45	-2.20	-4.73	-0.81	--	--	-0.82	-0.83	-1.10	-0.84	--	--	-0.26	-0.43	0.14	-0.45	--	--

Table B4d: Standard Error of δ_{p1}, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.3446	.2511	.2538	.2015	--	--	.2566	.1544	.1440	.1116	--	--	.1654	.0877	.0861	.0752	--	--
4	.5	2.5	4	.3316	.2242	.2330	.1852	--	--	.1804	.1095	.1221	.1040	--	--	.1013	.0698	.0777	.0727	--	--
4	.7	2	4	.3515	.2746	.2793	.2484	--	--	.2436	.1495	.1793	.1492	--	--	.1511	.0976	.1153	.1005	--	--
4	.7	2.5	4	.2855	.2449	.2663	.2388	--	--	.1689	.1330	.1614	.1364	--	--	.1022	.0919	.1042	.0941	--	--
8	.5	2	4	.1137	.1056	.1838	.1700	--	--	.0733	.0744	.1119	.1003	--	--	.0539	.0682	.0772	.0690	--	--
8	.5	2.5	4	.1121	.1199	.1671	.1534	--	--	.0807	.0822	.1008	.0952	--	--	.0614	.0618	.0694	.0651	--	--
8	.7	2	4	.1085	.1171	.2586	.2145	--	--	.0787	.0867	.1553	.1340	--	--	.0673	.0710	.1053	.0940	--	--
8	.7	2.5	4	.1211	.1366	.2129	.2114	--	--	.0916	.0999	.1391	.1234	--	--	.0768	.0787	.0941	.0865	--	--
4	.5	2	6	.2261	.1456	.1680	.1340	.2245	.1957	.1234	.0829	.0924	.0850	.1247	.1209	.0666	.0584	.0635	.0596	.0873	.0840
4	.5	2.5	6	.1536	.1281	.1487	.1279	.1712	.1700	.0855	.0811	.0860	.0819	.1037	.1016	.0591	.0569	.0599	.0579	.0727	.0722
4	.7	2	6	.2172	.1812	.1907	.1819	.2135	.2469	.1217	.1108	.1238	.1120	.1637	.1662	.0830	.0759	.0862	.0780	.1160	.1146
4	.7	2.5	6	.1852	.1636	.1752	.1663	.2076	.2176	.1108	.1057	.1122	.1059	.1371	.1353	.0783	.0743	.0803	.0760	.0981	.0972
8	.5	2	6	.1178	.1169	.1540	.1329	.1883	.1831	.0760	.0761	.0900	.0835	.1233	.1138	.0564	.0565	.0627	.0584	.0836	.0796
8	.5	2.5	6	.1190	.1190	.1408	.1240	.1712	.1668	.0807	.0778	.0841	.0806	.1020	.0985	.0575	.0556	.0588	.0569	.0676	.0683
8	.7	2	6	.1153	.1309	.1906	.1711	.2352	.2412	.0941	.0998	.1243	.1099	.1561	.1827	.0731	.0725	.0843	.0775	.1136	.1067
8	.7	2.5	6	.1358	.1448	.1794	.1630	.2073	.2236	.1016	.0992	.1114	.1035	.1374	.1312	.0750	.0727	.0787	.0742	.0919	.0925
4	.5	2	8	.1679	.1164	.1447	.0848	--	--	.1001	.0583	.0666	.0466	--	--	.0536	.0333	.0351	.0320	--	--
4	.5	2.5	8	.1363	.1042	.1038	.0785	--	--	.0775	.0453	.0468	.0439	--	--	.0331	.0308	.0322	.0313	--	--
4	.7	2	8	.1475	.0974	.1073	.0959	--	--	.0789	.0595	.0653	.0597	--	--	.0455	.0410	.0461	.0421	--	--
4	.7	2.5	8	.1238	.0968	.0944	.0879	--	--	.0614	.0579	.0603	.0568	--	--	.0421	.0399	.0427	.0404	--	--
8	.5	2	8	.0682	.0559	.0692	.0629	--	--	.0377	.0371	.0432	.0392	--	--	.0283	.0275	.0300	.0278	--	--
8	.5	2.5	8	.0595	.0587	.0673	.0602	--	--	.0387	.0374	.0403	.0386	--	--	.0273	.0266	.0281	.0272	--	--
8	.7	2	8	.0772	.0663	.1067	.0817	--	--	.0470	.0480	.0575	.0518	--	--	.0343	.0349	.0403	.0368	--	--
8	.7	2.5	8	.0709	.0714	.0811	.0858	--	--	.0491	.0482	.0536	.0501	--	--	.0360	.0348	.0377	.0359	--	--

Table B5a: Percent Bias of τ_1, Class 1

				N = 200						*N* = 500						*N* = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	25.88	19.44	11.20	2.36	--	--	20.96	9.01	3.82	0.35	--	--	15.56	3.16	0.63	-0.07	--	--
4	.5	2.5	4	25.69	14.01	8.41	1.56	--	--	13.27	2.87	1.74	-0.09	--	--	6.05	0.47	0.13	0.33	--	--
4	.7	2	4	10.14	4.62	1.93	-0.15	--	--	6.91	0.67	-0.10	-0.10	--	--	4.12	-0.25	-0.09	-0.17	--	--
4	.7	2.5	4	9.00	2.69	0.48	-0.35	--	--	2.35	0.05	0.06	0.13	--	--	0.16	0.11	0.03	-0.12	--	--
8	.5	2	4	-20.94	-17.30	0.28	-0.10	--	--	-19.22	-12.73	0.03	0.04	--	--	-14.57	-5.47	-0.04	-0.03	--	--
8	.5	2.5	4	-15.17	-11.18	0.47	0.04	--	--	-8.87	-4.28	-0.14	0.07	--	--	-3.85	-0.86	-0.01	-0.03	--	--
8	.7	2	4	-21.87	-17.53	-0.18	-0.14	--	--	-20.10	-12.40	-0.05	0.02	--	--	-14.29	-5.65	0.04	-0.03	--	--
8	.7	2.5	4	-15.60	-11.56	-0.24	0.07	--	--	-9.39	-4.18	-0.01	0.03	--	--	-3.84	-1.57	-0.004	0.02	--	--
4	.5	2	6	16.04	3.57	5.49	0.58	0.34	-0.72	5.72	0.64	0.54	0.02	-0.32	-0.38	1.45	0.12	0.18	0.06	0.002	-0.10
4	.5	2.5	6	8.05	1.52	2.82	0.10	0.60	0.81	1.61	0.19	0.06	0.08	0.27	0.03	0.26	-0.02	-0.07	0.14	0.02	-0.07
4	.7	2	6	3.63	0.09	0.71	0.16	0.35	-0.18	0.17	-0.13	-0.37	-0.08	-0.70	0.29	-0.10	-0.21	0.07	0.08	-0.05	0.10
4	.7	2.5	6	0.71	-0.32	0.76	0.35	-0.87	-0.52	-0.18	0.14	0.29	-0.08	-0.47	0.04	0.05	-0.19	0.04	-0.01	-0.16	0.10
8	.5	2	6	-10.05	-8.00	0.38	0.09	-0.14	-0.02	-5.78	-2.91	0.02	0.07	-0.16	0.02	-2.64	-0.58	0.02	-0.06	-0.02	-0.06
8	.5	2.5	6	-4.93	-3.70	0.26	0.06	0.26	0.24	-1.52	-0.85	0.12	-0.08	0.13	0.07	-0.24	-0.08	-0.05	-0.05	-0.06	0.04
8	.7	2	6	-9.85	-7.31	0.14	0.09	-0.81	-0.45	-4.64	-2.03	0.01	-0.14	-0.31	-0.14	-2.24	-0.53	-0.001	0.08	-0.04	0.03
8	.7	2.5	6	-5.08	-3.77	0.15	0.02	-0.22	-0.11	-1.70	-0.65	-0.03	0.03	-0.08	-0.06	-0.61	-0.12	-0.02	-0.02	-0.01	0.01
4	.5	2	8	37.61	24.14	15.29	6.80	--	--	22.21	7.61	6.16	0.97	--	--	11.99	0.73	0.97	0.09	--	--
4	.5	2.5	8	37.59	20.71	12.22	5.27	--	--	12.06	3.35	1.72	0.19	--	--	0.68	0.12	0.01	-0.22	--	--
4	.7	2	8	10.26	4.10	1.92	0.31	--	--	3.33	0.46	0.21	0.23	--	--	0.36	-0.01	-0.03	0.02	--	--
4	.7	2.5	8	8.11	3.93	0.71	0.20	--	--	1.06	0.42	0.005	-0.16	--	--	-0.09	0.11	-0.12	0.06	--	--
8	.5	2	8	-4.40	-2.87	0.41	0.17	--	--	-2.93	-1.33	-0.01	-0.12	--	--	-1.40	-0.23	0.08	0.05	--	--
8	.5	2.5	8	-1.58	-0.35	0.63	0.04	--	--	-0.04	-0.03	-0.04	0.01	--	--	0.04	-0.04	-0.02	-0.03	--	--
8	.7	2	8	-5.42	-4.16	0.12	0.09	--	--	-3.78	-1.92	-0.01	0.06	--	--	-2.11	-0.41	0.02	-0.03	--	--
8	.7	2.5	8	-2.57	-1.38	0.09	0.06	--	--	-1.07	-0.41	0.07	-0.08	--	--	-0.30	-0.01	-0.003	0.03	--	--

Table B5b: Standard Error of τ_1, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	$\tau_{1;}1$	$\tau_{1;}1.5$	$\tau_{1,p;}1$	$\tau_{1,p;}1.5$	$\tau_{p;}1$	$\tau_{p;}1.5$	$\tau_{1;}1$	$\tau_{1;}1.5$	$\tau_{1,p;}1$	$\tau_{1,p;}1.5$	$\tau_{p;}1$	$\tau_{p;}1.5$	$\tau_{1;}1$	$\tau_{1;}1.5$	$\tau_{1,p;}1$	$\tau_{1,p;}1.5$	$\tau_{p;}1$	$\tau_{p;}1.5$
4	.5	2	4	.1947	.2567	.2357	.1681	--	--	.1803	.1973	.1562	.0972	--	--	.1624	.1394	.1024	.0641	--	--
4	.5	2.5	4	.2107	.2517	.2282	.1526	--	--	.2023	.1666	.1298	.0850	--	--	.1498	.1082	.0844	.0589	--	--
4	.7	2	4	.1565	.1965	.1580	.1269	--	--	.1498	.1429	.1171	.0764	--	--	.1195	.1025	.0765	.0514	--	--
4	.7	2.5	4	.1813	.1901	.1482	.1144	--	--	.1374	.1161	.0958	.0682	--	--	.1072	.0792	.0638	.0466	--	--
8	.5	2	4	.4206	.3809	.2303	.1645	--	--	.3751	.2802	.1425	.0912	--	--	.3134	.1777	.0908	.0613	--	--
8	.5	2.5	4	.3162	.3316	.1886	.1438	--	--	.2613	.1843	.1107	.0819	--	--	.1493	.1013	.0736	.0558	--	--
8	.7	2	4	.3468	.3464	.1867	.1231	--	--	.3498	.2711	.1063	.0720	--	--	.3077	.1340	.0693	.0483	--	--
8	.7	2.5	4	.3425	.2813	.1490	.1261	--	--	.2320	.1635	.0925	.0653	--	--	.1298	.0812	.0565	.0452	--	--
4	.5	2	6	.2752	.2119	.2353	.1357	.2932	.2021	.1920	.1151	.1216	.0816	.1718	.1115	.1272	.0779	.0801	.0566	.1083	.0750
4	.5	2.5	6	.2444	.1582	.1935	.1261	.2280	.1618	.1346	.0928	.1025	.0761	.1324	.0930	.0888	.0647	.0682	.0531	.0888	.0650
4	.7	2	6	.1935	.1423	.1484	.1129	.1984	.1600	.1383	.0908	.0956	.0647	.1406	.0871	.0930	.0602	.0615	.0453	.0836	.0581
4	.7	2.5	6	.1735	.1205	.1311	.1016	.1664	.1301	.1010	.0745	.0790	.0608	.0997	.0733	.0676	.0503	.0530	.0430	.0654	.0500
8	.5	2	6	.3618	.2703	.2156	.1339	.2543	.1838	.2032	.1263	.1119	.0782	.1652	.1027	.1146	.0722	.0727	.0544	.1056	.0694
8	.5	2.5	6	.2568	.1856	.1677	.1217	.2209	.1720	.1294	.0910	.0935	.0730	.1281	.0888	.0759	.0593	.0635	.0515	.0802	.0612
8	.7	2	6	.2759	.2368	.1520	.1048	.2114	.1612	.1547	.1024	.0877	.0634	.1370	.0959	.0903	.0568	.0562	.0443	.0784	.0542
8	.7	2.5	6	.1978	.1632	.1263	.0985	.1705	.1323	.1028	.0735	.0731	.0597	.0963	.0718	.0605	.0474	.0504	.0423	.0613	.0481
4	.5	2	8	.2209	.2104	.2129	.1487	--	--	.1697	.1243	.1250	.0771	--	--	.1072	.0711	.0681	.0516	--	--
4	.5	2.5	8	.2110	.2036	.1879	.1524	--	--	.1480	.0958	.0933	.0727	--	--	.0775	.0597	.0590	.0496	--	--
4	.7	2	8	.1673	.1351	.1322	.0984	--	--	.1161	.0790	.0768	.0602	--	--	.0760	.0532	.0518	.0420	--	--
4	.7	2.5	8	.1471	.1190	.1158	.0942	--	--	.0885	.0684	.0676	.0576	--	--	.0595	.0472	.0472	.0408	--	--
8	.5	2	8	.3084	.2084	.1708	.1154	--	--	.1678	.1074	.0897	.0706	--	--	.1060	.0680	.0619	.0496	--	--
8	.5	2.5	8	.1940	.1424	.1437	.1108	--	--	.1086	.0833	.0811	.0690	--	--	.0736	.0577	.0565	.0485	--	--
8	.7	2	8	.2903	.1900	.1209	.0944	--	--	.1595	.0991	.0719	.0582	--	--	.0855	.0554	.0492	.0410	--	--
8	.7	2.5	8	.1320	.1320	.1076	.0937	--	--	.0690	.0690	.0659	.0571	--	--	.0467	.0467	.0456	.0401	--	--

Table B5c: Percent Bias of τ_1, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	16.04	5.52	-6.85	-6.81	--	--	9.56	1.10	-8.19	-2.68	--	--	7.62	-0.05	-3.94	-0.59	--	--
4	.5	2.5	4	11.44	0.05	-7.19	-4.54	--	--	4.79	-1.58	-7.45	-1.48	--	--	0.88	-0.06	-1.80	-1.14	--	--
4	.7	2	4	11.01	1.76	-4.41	-5.59	--	--	4.49	0.42	-6.61	-1.62	--	--	3.89	-1.12	-1.57	-1.25	--	--
4	.7	2.5	4	7.17	-2.22	-7.68	-6.96	--	--	0.48	-0.81	-3.22	-1.51	--	--	-0.78	-1.04	-1.73	-0.59	--	--
8	.5	2	4	-16.71	-20.20	-1.36	-0.63	--	--	-15.14	-15.95	-0.48	0.18	--	--	-12.08	-7.70	-0.09	-0.15	--	--
8	.5	2.5	4	-12.96	-15.74	-1.20	-0.61	--	--	-9.34	-7.29	-0.36	-0.10	--	--	-4.20	-1.97	0.17	-0.05	--	--
8	.7	2	4	-17.87	-23.82	-0.84	-0.36	--	--	-15.46	-19.36	-0.15	-0.10	--	--	-13.19	-12.11	-0.09	-0.09	--	--
8	.7	2.5	4	-15.58	-19.05	-1.63	-0.80	--	--	-11.84	-11.34	-0.46	-0.12	--	--	-7.04	-5.65	-0.06	-0.01	--	--
4	.5	2	6	0.63	-1.85	-1.21	-1.04	0.34	-0.72	0.58	-0.35	-1.32	-0.47	-0.32	-0.38	0.01	-0.23	-0.36	-0.32	0.002	-0.10
4	.5	2.5	6	-0.82	-0.55	-1.73	-0.93	0.60	0.81	-1.02	-0.47	-0.82	-0.54	0.27	0.03	-0.26	-0.41	-0.47	-0.30	0.02	-0.07
4	.7	2	6	-1.15	-1.58	-2.32	-0.46	0.35	-0.18	-1.65	-0.51	-1.48	-0.28	-0.70	0.29	-0.45	-0.33	-0.34	-0.10	-0.05	0.10
4	.7	2.5	6	-2.48	-1.98	-1.37	-1.34	-0.87	-0.52	-0.96	-0.50	-0.50	0.10	-0.47	0.04	-0.43	-0.06	-0.31	-0.24	-0.16	0.10
8	.5	2	6	-18.33	-18.03	-0.25	-0.02	-0.14	-0.02	-11.24	-7.43	-0.12	0.01	-0.16	0.02	-5.56	-1.59	0.03	-0.08	-0.02	-0.06
8	.5	2.5	6	-11.45	-10.25	-0.01	-0.01	0.26	0.24	-3.24	-2.52	0.11	-0.04	0.13	0.07	-0.38	-0.16	-0.04	-0.04	-0.06	0.04
8	.7	2	6	-22.10	-21.00	-0.41	-0.13	-0.81	-0.45	-12.97	-6.96	-0.05	-0.13	-0.31	-0.14	-6.64	-2.30	0.09	-0.08	-0.04	0.03
8	.7	2.5	6	-15.79	-15.16	-0.17	-0.05	-0.22	-0.11	-6.17	-3.36	-0.09	0.07	-0.08	-0.06	-1.96	-0.39	0.01	-0.02	-0.01	0.01
4	.5	2	8	1.57	-2.94	-1.14	-4.01	--	--	2.25	-0.69	-0.06	-0.83	--	--	1.35	-0.10	-0.15	0.12	--	--
4	.5	2.5	8	6.31	-2.24	-0.26	-2.42	--	--	0.64	-0.57	0.84	-0.51	--	--	-0.10	-0.16	-0.25	-0.13	--	--
4	.7	2	8	-0.38	-1.72	-0.27	-0.11	--	--	-0.36	-0.73	-0.56	0.04	--	--	-0.10	-0.08	0.22	0.09	--	--
4	.7	2.5	8	1.46	-1.18	-0.17	-0.42	--	--	-0.43	-0.23	-0.08	0.03	--	--	0.07	0.01	-0.15	0.08	--	--
8	.5	2	8	-9.74	-8.35	0.17	0.11	--	--	-6.36	-3.74	-0.001	-0.07	--	--	-3.04	-0.92	-0.03	0.03	--	--
8	.5	2.5	8	-3.48	-1.13	-0.43	-0.10	--	--	-0.08	-0.10	0.10	-0.06	--	--	-0.01	0.02	0.005	0.000	--	--
8	.7	2	8	-16.56	-14.59	0.12	0.02	--	--	-11.06	-6.97	-0.03	-0.03	--	--	-6.65	-1.83	-0.05	-0.03	--	--
8	.7	2.5	8	-8.66	-6.12	-0.11	0.05	--	--	-3.60	-1.73	-0.02	-0.07	--	--	-0.98	-0.19	0.03	0.04	--	--

Table B5d: Standard Error of τ_1, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.6821	.6575	.8410	.6822	--	--	.5747	.3923	.5797	.3316	--	--	.3878	.2530	.3079	.1906	--	--
4	.5	2.5	4	.8123	.6947	.8389	.6505	--	--	.5500	.4002	.5405	.3280	--	--	.3469	.2314	.2833	.2263	--	--
4	.7	2	4	.6931	.6552	.6812	.6090	--	--	.5441	.3738	.4962	.3054	--	--	.3432	.2509	.2663	.2014	--	--
4	.7	2.5	4	.7357	.7088	.7908	.6920	--	--	.4741	.3545	.4568	.3408	--	--	.3147	.2406	.2803	.2226	--	--
8	.5	2	4	.4604	.4787	.4451	.3348	--	--	.3680	.3819	.2633	.1755	--	--	.3069	.2916	.1662	.1227	--	--
8	.5	2.5	4	.4329	.4650	.4820	.3528	--	--	.3553	.3016	.2644	.2020	--	--	.2526	.1960	.1721	.1391	--	--
8	.7	2	4	.3829	.4433	.4663	.3285	--	--	.3433	.4007	.2848	.1948	--	--	.3329	.2967	.1809	.1327	--	--
8	.7	2.5	4	.3830	.4806	.4511	.3724	--	--	.3469	.3717	.2781	.2118	--	--	.2702	.2403	.1807	.1443	--	--
4	.5	2	6	.4836	.3807	.4547	.3186	.2932	.2021	.2852	.1927	.2490	.1866	.1718	.1115	.1822	.1317	.1601	.1291	.1083	.0750
4	.5	2.5	6	.4647	.3376	.4451	.3471	.2280	.1618	.2555	.2044	.2482	.2102	.1324	.0930	.1669	.1428	.1681	.1471	.0888	.0650
4	.7	2	6	.4510	.3466	.4257	.3194	.1984	.1600	.2840	.2015	.2525	.1854	.1406	.0871	.1871	.1366	.1626	.1295	.0836	.0581
4	.7	2.5	6	.4407	.3549	.4304	.3623	.1664	.1301	.2557	.2080	.2484	.2079	.0997	.0733	.1676	.1418	.1680	.1472	.0654	.0500
8	.5	2	6	.5070	.4370	.2981	.1847	.2543	.1838	.3051	.2209	.1529	.1123	.1652	.1027	.1845	.1136	.1012	.0787	.1056	.0694
8	.5	2.5	6	.3829	.3455	.2573	.1950	.2209	.1720	.2316	.1666	.1469	.1208	.1281	.0888	.1170	.0920	.1021	.0852	.0802	.0612
8	.7	2	6	.3872	.4608	.2688	.1946	.2114	.1612	.3028	.2260	.1605	.1161	.1370	.0959	.2157	.1211	.1030	.0815	.0784	.0542
8	.7	2.5	6	.3917	.4021	.2545	.2021	.1705	.1323	.2433	.1822	.1478	.1230	.0963	.0718	.1382	.0961	.1019	.0867	.0613	.0481
4	.5	2	8	.3880	.3083	.3854	.2324	--	--	.2693	.1724	.1826	.1233	--	--	.1706	.0974	.1055	.0829	--	--
4	.5	2.5	8	.3547	.3054	.3230	.2349	--	--	.2209	.1443	.1620	.1331	--	--	.1187	.0934	.1072	.0912	--	--
4	.7	2	8	.3364	.2543	.2665	.2008	--	--	.2146	.1408	.1532	.1209	--	--	.1327	.0942	.1024	.0845	--	--
4	.7	2.5	8	.3042	.2425	.2613	.2144	--	--	.1705	.1365	.1524	.1311	--	--	.1142	.0936	.1058	.0923	--	--
8	.5	2	8	.3831	.3095	.2252	.1579	--	--	.2373	.1660	.1247	.0982	--	--	.1620	.0945	.0845	.0694	--	--
8	.5	2.5	8	.2930	.2120	.2216	.1739	--	--	.1572	.1209	.1266	.1082	--	--	.1054	.0841	.0882	.0760	--	--
8	.7	2	8	.4128	.3670	.2186	.1630	--	--	.2909	.2192	.1276	.1015	--	--	.1844	.1041	.0870	.0710	--	--
8	.7	2.5	8	.3259	.2598	.2160	.1760	--	--	.1874	.1458	.1278	.1102	--	--	.1183	.0862	.0885	.0769	--	--

Table B6a: Percent Bias of τ_p, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	5.08	3.08	4.13	1.00	--	--	3.91	1.27	1.52	0.20	--	--	2.91	0.29	0.17	-0.10	--	--
4	.5	2.5	4	5.52	2.62	3.43	0.72	--	--	2.94	0.53	0.67	-0.01	--	--	1.31	0.02	-0.02	-.0001	--	--
4	.7	2	4	2.19	0.72	0.66	-0.01	--	--	1.36	0.11	-0.16	-0.08	--	--	0.84	-0.10	-0.05	-0.02	--	--
4	.7	2.5	4	2.14	0.60	0.28	-0.03	--	--	0.63	0.11	-0.05	0.01	--	--	0.09	-0.04	-0.03	-0.01	--	--
8	.5	2	4	-11.44	-11.10	0.44	-0.23	--	--	-9.61	-7.93	-0.06	0.30	--	--	-6.82	-3.45	-0.02	-0.08	--	--
8	.5	2.5	4	-7.67	-7.24	0.91	0.11	--	--	-4.54	-2.94	-0.32	0.17	--	--	-2.09	-0.93	0.13	-0.02	--	--
8	.7	2	4	-11.12	-11.37	-0.60	0.005	--	--	-7.58	-7.81	-0.18	-0.05	--	--	-5.06	-3.90	0.05	-0.14	--	--
8	.7	2.5	4	-7.20	-6.61	-0.24	0.35	--	--	-4.32	-3.15	-0.17	-0.10	--	--	-2.34	-1.30	-0.02	0.01	--	--
4	.5	2	6	1.91	0.21	1.21	0.15	-1.90	-0.91	0.69	0.11	0.10	-0.05	-0.68	-0.19	0.22	-0.001	0.07	-0.02	-0.13	-.0004
4	.5	2.5	6	1.00	0.16	0.87	0.19	-0.53	0.18	0.20	0.12	-0.07	-0.14	0.09	0.01	0.02	-0.01	-0.10	0.03	-0.06	-0.09
4	.7	2	6	0.53	-0.08	0.07	0.11	-3.18	-1.35	0.04	0.01	-0.04	0.001	-2.10	-0.38	-0.08	-0.01	-0.06	-0.02	-0.60	-0.10
4	.7	2.5	6	0.08	-0.15	0.19	-0.03	-1.99	-0.82	-0.06	-0.05	-0.05	0.01	-0.63	-0.10	0.04	-0.06	-0.02	0.04	-0.16	-.0002
8	.5	2	6	-9.12	-7.81	0.43	0.01	-1.54	0.04	-5.31	-3.26	-0.10	0.08	-0.69	-0.18	-2.57	-0.60	-0.02	-0.04	-0.28	0.01
8	.5	2.5	6	-5.00	-3.83	0.43	0.20	-0.14	0.27	-1.50	-0.77	0.04	-0.05	-0.04	0.12	-0.07	0.02	0.03	0.06	0.04	0.01
8	.7	2	6	-10.01	-8.01	-0.28	0.20	-3.06	-1.94	-4.99	-2.39	-0.08	-0.16	-1.94	-0.92	-2.24	-0.56	-0.01	0.03	-0.67	0.08
8	.7	2.5	6	-5.59	-4.46	0.11	0.12	-1.52	-0.70	-1.68	-0.75	0.01	0.09	-0.34	-0.16	-0.59	-0.09	-0.10	-0.03	-0.07	-0.13
4	.5	2	8	8.91	4.60	6.26	2.64	--	--	5.25	1.53	2.58	0.39	--	--	2.84	0.13	0.28	0.05	--	--
4	.5	2.5	8	10.00	4.53	5.04	2.17	--	--	3.17	0.69	0.73	0.05	--	--	0.18	0.01	0.003	-0.09	--	--
4	.7	2	8	2.51	0.80	0.69	0.19	--	--	0.83	0.13	0.03	0.13	--	--	0.10	-0.08	-0.07	0.02	--	--
4	.7	2.5	8	2.37	0.96	0.37	-0.06	--	--	0.28	0.13	0.05	-0.04	--	--	0.01	0.06	-0.04	0.09	--	--
8	.5	2	8	-8.54	-6.20	0.89	0.23	--	--	-5.63	-3.02	-0.03	-0.06	--	--	-2.70	-0.49	0.01	0.11	--	--
8	.5	2.5	8	-3.06	-0.66	1.53	-0.02	--	--	0.02	-0.05	0.03	-0.01	--	--	0.03	-0.05	-0.06	-0.04	--	--
8	.7	2	8	-12.12	-9.08	0.15	0.38	--	--	-7.43	-3.87	0.05	0.04	--	--	-4.23	-0.97	0.03	-0.004	--	--
8	.7	2.5	8	-4.75	-3.14	0.26	0.09	--	--	-2.06	-1.09	0.10	-0.13	--	--	-0.45	-0.12	0.03	-0.04	--	--

Table B6b: Standard Error of τ_p, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.1185	.1287	.2329	.1674	--	--	.0999	.0966	.1576	.0974	--	--	.0879	.0690	.1034	.0646	--	--
4	.5	2.5	4	.1336	.1385	.2228	.1515	--	--	.1137	.0936	.1308	.0846	--	--	.0874	.0624	.0836	.0591	--	--
4	.7	2	4	.1045	.1118	.1541	.1284	--	--	.0840	.0771	.1165	.0765	--	--	.0664	.0549	.0763	.0514	--	--
4	.7	2.5	4	.1439	.1152	.1505	.1127	--	--	.0851	.0703	.0973	.0685	--	--	.0637	.0489	.0642	.0469	--	--
8	.5	2	4	.2677	.2357	.2269	.1689	--	--	.2244	.1729	.1404	.0893	--	--	.1788	.1340	.0892	.0611	--	--
8	.5	2.5	4	.2108	.1970	.1865	.1392	--	--	.1600	.1069	.1131	.0812	--	--	.0999	.0696	.0734	.0561	--	--
8	.7	2	4	.2281	.2192	.1811	.1214	--	--	.2103	.1751	.1100	.0720	--	--	.1890	.0967	.0702	.0490	--	--
8	.7	2.5	4	.2007	.1860	.1478	.1122	--	--	.1502	.1081	.0947	.0652	--	--	.0780	.0589	.0565	.0452	--	--
4	.5	2	6	.1251	.1091	.1880	.1240	.3120	.2480	.0839	.0644	.1018	.0751	.1856	.1371	.0564	.0451	.0680	.0527	.1195	.0916
4	.5	2.5	6	.1231	.1011	.1606	.1165	.2388	.1731	.0730	.0628	.0868	.0712	.1339	.1044	.0502	.0444	.0584	.0499	.0897	.0717
4	.7	2	6	.0999	.0891	.1296	.1031	.2457	.2086	.0667	.0554	.0844	.0616	.1644	.1095	.0458	.0384	.0540	.0431	.1008	.0704
4	.7	2.5	6	.1019	.0864	.1135	.0959	.2053	.1570	.0602	.0539	.0690	.0583	.1105	.0844	.0413	.0377	.0470	.0410	.0703	.0563
8	.5	2	6	.1935	.1775	.1956	.1213	.2807	.2336	.1133	.0833	.0949	.0732	.1772	.1288	.0683	.0485	.0628	.0515	.1198	.0849
8	.5	2.5	6	.1520	.1275	.1392	.1133	.2294	.1956	.0834	.0681	.0807	.0695	.1321	.1000	.0477	.0431	.0560	.0490	.0830	.0690
8	.7	2	6	.1537	.1603	.1309	.0988	.2460	.2065	.0980	.0767	.0779	.0604	.1638	.1177	.0661	.0427	.0502	.0421	.0979	.0672
8	.7	2.5	6	.1265	.1232	.1107	.0932	.1821	.1488	.0738	.0603	.0656	.0573	.1048	.0836	.0440	.0375	.0454	.0406	.0670	.0546
4	.5	2	8	.1576	.1397	.2202	.1466	--	--	.1189	.0874	.1198	.0769	--	--	.0774	.0547	.0680	.0517	--	--
4	.5	2.5	8	.1607	.1468	.1919	.1486	--	--	.1206	.0763	.0938	.0725	--	--	.0622	.0504	.0588	.0497	--	--
4	.7	2	8	.1229	.1037	.1328	.0986	--	--	.0847	.0626	.0773	.0601	--	--	.0575	.0433	.0518	.0421	--	--
4	.7	2.5	8	.1161	.0973	.1155	.0936	--	--	.0712	.0590	.0673	.0579	--	--	.0488	.0411	.0472	.0408	--	--
8	.5	2	8	.2455	.1694	.1755	.1150	--	--	.1408	.0954	.0904	.0703	--	--	.0932	.0567	.0618	.0496	--	--
8	.5	2.5	8	.1580	.1194	.1445	.1100	--	--	.0885	.0704	.0815	.0692	--	--	.0596	.0491	.0565	.0483	--	--
8	.7	2	8	.2579	.1745	.1298	.0943	--	--	.1799	.1075	.0726	.0583	--	--	.0961	.0510	.0494	.0410	--	--
8	.7	2.5	8	.1226	.1226	.1081	.0917	--	--	.0668	.0668	.0658	.0570	--	--	.0413	.0413	.0457	.0401	--	--

Table B6c: Percent Bias of τ_p, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	5.08	3.08	-3.40	-4.89	--	--	3.91	1.27	-3.55	-1.88	--	--	2.91	0.29	-1.83	-0.31	--	--
4	.5	2.5	4	5.52	2.62	-3.11	-3.14	--	--	2.94	0.53	-3.25	-0.91	--	--	1.31	0.02	-1.03	-0.67	--	--
4	.7	2	4	2.19	0.72	-2.45	-2.90	--	--	1.36	0.11	-2.76	-1.21	--	--	0.84	-0.10	-0.66	-0.75	--	--
4	.7	2.5	4	2.14	0.60	-4.03	-4.38	--	--	0.63	0.11	-0.99	-1.01	--	--	0.09	-0.04	-0.73	-0.18	--	--
8	.5	2	4	-11.44	-11.10	-1.81	-0.45	--	--	-9.61	-7.93	-0.36	-0.16	--	--	-6.82	-3.45	-0.49	0.09	--	--
8	.5	2.5	4	-7.67	-7.24	-2.56	-1.17	--	--	-4.54	-2.94	-0.04	0.33	--	--	-2.09	-0.93	0.14	-0.14	--	--
8	.7	2	4	-11.12	-11.37	-2.41	-1.41	--	--	-7.58	-7.81	-0.51	-0.15	--	--	-5.06	-3.90	-0.13	-0.04	--	--
8	.7	2.5	4	-7.20	-6.61	-1.68	-0.78	--	--	-4.32	-3.15	-1.00	-0.26	--	--	-2.34	-1.30	0.22	0.06	--	--
4	.5	2	6	1.91	0.21	0.03	-0.33	-0.96	-1.58	0.69	0.11	-0.40	0.07	-0.76	-0.44	0.22	-0.001	0.02	-0.04	-0.04	-0.27
4	.5	2.5	6	1.00	0.16	-0.72	-0.12	-1.24	-0.22	0.20	0.12	-0.09	0.05	-0.14	-0.17	0.02	-0.01	0.01	-0.10	-0.20	-0.10
4	.7	2	6	0.53	-0.08	-0.11	-0.01	-2.78	-2.30	0.04	0.01	-0.36	0.04	-2.49	-0.67	-0.08	-0.01	0.10	-0.13	-0.52	-0.14
4	.7	2.5	6	0.08	-0.15	-0.32	-0.35	-2.39	-1.47	-0.06	-0.05	-0.14	-0.12	-1.06	-0.34	0.04	-0.06	0.06	0.03	-0.24	0.01
8	.5	2	6	-9.12	-7.81	0.45	-0.09	-2.62	-1.78	-5.31	-3.26	0.11	0.18	-1.28	0.21	-2.57	-0.60	-0.10	-0.13	-0.45	-0.08
8	.5	2.5	6	-5.00	-3.83	0.08	0.45	-0.52	-1.36	-1.50	-0.77	0.20	-0.08	-0.06	0.32	-0.07	0.02	0.04	-0.07	-0.38	-0.16
8	.7	2	6	-10.01	-8.01	0.03	-0.02	-6.30	-5.32	-4.99	-2.39	0.24	-0.15	-3.30	-1.71	-2.24	-0.56	0.07	0.32	-0.45	-0.08
8	.7	2.5	6	-5.59	-4.46	-0.08	-0.15	-1.76	-0.87	-1.68	-0.75	-0.10	0.06	-0.20	-0.15	-0.59	-0.09	-0.12	0.06	-0.12	-0.09
4	.5	2	8	8.91	4.60	-1.00	-2.15	--	--	5.25	1.53	-0.41	-0.51	--	--	2.84	0.13	-0.09	0.03	--	--
4	.5	2.5	8	10.00	4.53	-0.39	-1.23	--	--	3.17	0.69	0.28	-0.20	--	--	0.18	0.01	0.002	-0.19	--	--
4	.7	2	8	2.51	0.80	0.02	-0.14	--	--	0.83	0.13	-0.12	-0.12	--	--	0.10	-0.08	0.03	-0.06	--	--
4	.7	2.5	8	2.37	0.96	-0.41	0.20	--	--	0.28	0.13	-0.09	-0.03	--	--	0.01	0.06	-0.02	-0.01	--	--
8	.5	2	8	-8.54	-6.20	0.10	0.25	--	--	-5.63	-3.02	-0.09	-0.07	--	--	-2.70	-0.49	0.08	0.11	--	--
8	.5	2.5	8	-3.06	-0.66	-0.64	-0.20	--	--	0.02	-0.05	0.09	-0.14	--	--	0.03	-0.05	-0.05	-0.04	--	--
8	.7	2	8	-12.12	-9.08	-0.20	0.14	--	--	-7.43	-3.87	-0.01	-0.08	--	--	-4.23	-0.97	-0.02	0.05	--	--
8	.7	2.5	8	-4.75	-3.14	0.19	-0.08	--	--	-2.06	-1.09	0.10	-0.16	--	--	-0.45	-0.12	-0.02	0.04	--	--

Table B6d: Standard Error of τ_p, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.1185	.1287	.8576	.7136	--	--	.0999	.0966	.5281	.3424	--	--	.0879	.0690	.3041	.1890	--	--
4	.5	2.5	4	.1336	.1385	.8594	.7382	--	--	.1137	.0936	.5171	.3314	--	--	.0874	.0624	.2885	.2247	--	--
4	.7	2	4	.1045	.1118	.7424	.6240	--	--	.0840	.0771	.4892	.3297	--	--	.0664	.0549	.2677	.2023	--	--
4	.7	2.5	4	.1439	.1152	.7799	.7228	--	--	.0851	.0703	.4441	.3455	--	--	.0637	.0489	.2788	.2197	--	--
8	.5	2	4	.2677	.2357	.4500	.3233	--	--	.2244	.1729	.2587	.1799	--	--	.1788	.1340	.1700	.1225	--	--
8	.5	2.5	4	.2108	.1970	.4865	.3552	--	--	.1600	.1069	.2648	.2033	--	--	.0999	.0696	.1721	.1385	--	--
8	.7	2	4	.2281	.2192	.4909	.3477	--	--	.2103	.1751	.2712	.1965	--	--	.1890	.0967	.1779	.1316	--	--
8	.7	2.5	4	.2007	.1860	.4551	.3813	--	--	.1502	.1081	.2846	.2110	--	--	.0780	.0589	.1776	.1441	--	--
4	.5	2	6	.1251	.1091	.3611	.2568	.4718	.3567	.0839	.0644	.2069	.1497	.2998	.2077	.0564	.0451	.1345	.1045	.1936	.1392
4	.5	2.5	6	.1231	.1011	.3680	.2730	.4314	.3406	.0730	.0628	.2084	.1681	.2618	.1975	.0502	.0444	.1397	.1178	.1783	.1363
4	.7	2	6	.0999	.0891	.3474	.2778	.3862	.3774	.0667	.0554	.2195	.1594	.2868	.2260	.0458	.0384	.1407	.1108	.1983	.1491
4	.7	2.5	6	.1019	.0864	.3586	.2929	.4119	.3657	.0602	.0539	.2144	.1739	.2562	.2079	.0413	.0377	.1464	.1227	.1775	.1418
8	.5	2	6	.1935	.1775	.3140	.2150	.4133	.3420	.1133	.0833	.1692	.1283	.2835	.1907	.0683	.0485	.1138	.0903	.1861	.1306
8	.5	2.5	6	.1520	.1275	.3066	.2304	.4036	.3596	.0834	.0681	.1757	.1454	.2542	.1867	.0477	.0431	.1212	.1024	.1626	.1291
8	.7	2	6	.1537	.1603	.3048	.2305	.4468	.3903	.0980	.0767	.1853	.1409	.2977	.2373	.0661	.0427	.1216	.0984	.1878	.1397
8	.7	2.5	6	.1265	.1232	.3182	.2489	.3992	.3608	.0738	.0603	.1834	.1537	.2551	.1996	.0440	.0375	.1279	.1085	.1635	.1342
4	.5	2	8	.1576	.1397	.3524	.2473	--	--	.1189	.0874	.1838	.1226	--	--	.0774	.0547	.1061	.0827	--	--
4	.5	2.5	8	.1607	.1468	.3118	.2342	--	--	.1206	.0763	.1622	.1317	--	--	.0622	.0504	.1069	.0916	--	--
4	.7	2	8	.1229	.1037	.2666	.2008	--	--	.0847	.0626	.1512	.1214	--	--	.0575	.0433	.1028	.0844	--	--
4	.7	2.5	8	.1161	.0973	.2636	.2111	--	--	.0712	.0590	.1529	.1303	--	--	.0488	.0411	.1059	.0922	--	--
8	.5	2	8	.2455	.1694	.2219	.1603	--	--	.1408	.0954	.1232	.0984	--	--	.0932	.0567	.0852	.0692	--	--
8	.5	2.5	8	.1580	.1194	.2176	.1739	--	--	.0885	.0704	.1264	.1087	--	--	.0596	.0491	.0883	.0761	--	--
8	.7	2	8	.2579	.1745	.2225	.1629	--	--	.1799	.1075	.1251	.1008	--	--	.0961	.0510	.0859	.0712	--	--
8	.7	2.5	8	.1596	.1226	.2098	.1844	--	--	.0871	.0668	.1282	.1097	--	--	.0529	.0413	.0885	.0772	--	--

Table B7a: Percent Bias of Φ_{11}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	64.91	45.85	40.64	13.32	--	--	48.59	19.25	5.28	0.99	--	--	35.66	5.52	-3.55	0.06	--	--
4	.5	2.5	4	93.88	48.90	39.26	14.65	--	--	47.95	6.55	0.88	2.22	--	--	18.23	1.23	-2.45	-1.24	--	--
4	.7	2	4	34.73	14.67	22.14	8.65	--	--	19.27	3.69	0.36	0.04	--	--	9.94	-0.11	1.81	-0.24	--	--
4	.7	2.5	4	45.13	11.73	13.95	4.17	--	--	12.03	3.22	0.68	2.94	--	--	1.59	-1.58	0.12	0.68	--	--
8	.5	2	4	-49.98	-59.38	3.80	-0.96	--	--	-59.03	-55.79	0.39	0.06	--	--	-58.98	-32.04	1.42	0.49	--	--
8	.5	2.5	4	-49.49	-52.75	4.64	-0.99	--	--	-41.78	-30.41	-2.15	-0.14	--	--	-21.95	-8.23	1.26	-0.09	--	--
8	.7	2	4	-54.98	-66.83	-1.77	-0.57	--	--	-61.90	-62.90	0.71	-0.90	--	--	-65.01	-45.28	0.46	0.36	--	--
8	.7	2.5	4	-61.76	-64.22	-0.21	-0.70	--	--	-57.97	-42.26	-2.62	-1.78	--	--	-35.60	-20.97	1.04	1.13	--	--
4	.5	2	6	19.52	2.07	8.67	-1.02	-1.33	-5.67	8.78	0.06	-1.56	0.50	-2.75	-0.89	2.11	-0.96	-0.26	0.23	-0.94	-0.59
4	.5	2.5	6	13.50	3.17	5.26	0.57	0.79	1.11	1.58	-1.17	0.91	-0.32	0.01	-0.48	0.64	-0.34	0.48	-0.45	0.54	0.18
4	.7	2	6	4.35	0.55	0.10	-0.05	-14.33	-6.94	-0.73	0.51	-1.58	0.06	-10.77	-3.23	0.003	0.12	-0.23	-0.07	-3.30	-0.31
4	.7	2.5	6	0.55	-0.56	1.18	-0.70	-10.45	-4.78	-0.96	1.23	0.82	0.75	-3.27	-1.42	-0.39	0.51	-0.33	0.27	-1.91	-0.37
8	.5	2	6	-52.35	-46.81	1.17	0.91	-0.41	-0.01	-38.87	-20.79	-0.49	0.51	-1.92	-0.55	-19.61	-4.82	-0.46	0.08	-0.44	-0.91
8	.5	2.5	6	-34.84	-27.37	2.18	1.22	3.97	0.73	-13.31	-8.23	0.26	-1.19	0.20	-0.41	-2.69	-1.32	0.37	-1.00	-0.02	-1.03
8	.7	2	6	-60.88	-50.79	1.63	-0.20	-9.66	-10.27	-43.01	-18.29	0.08	0.58	-6.34	-2.36	-22.30	-6.45	0.19	-0.11	-1.39	0.11
8	.7	2.5	6	-46.16	-39.00	-1.82	-0.21	-1.02	-2.52	-20.12	-9.13	0.29	-0.31	0.09	-0.30	-7.43	-1.52	-0.02	-0.06	0.50	-0.32
4	.5	2	8	45.03	23.22	18.84	3.99	--	--	32.00	8.59	6.87	0.09	--	--	18.24	1.80	0.67	0.46	--	--
4	.5	2.5	8	68.75	31.80	24.90	7.24	--	--	22.47	5.17	3.67	-1.13	--	--	2.11	-0.01	-0.27	0.30	--	--
4	.7	2	8	13.85	3.41	2.32	-0.80	--	--	4.37	0.67	0.08	-0.56	--	--	0.73	-0.59	0.87	-0.42	--	--
4	.7	2.5	8	15.10	6.13	-0.49	-0.78	--	--	2.21	0.46	-1.48	0.13	--	--	0.41	-0.24	0.38	-0.10	--	--
8	.5	2	8	-27.89	-23.38	1.34	-0.53	--	--	-21.60	-10.05	-1.44	-0.89	--	--	-9.61	-3.01	-0.69	-0.01	--	--
8	.5	2.5	8	-12.40	-2.74	3.30	-1.48	--	--	0.28	0.28	0.74	0.12	--	--	-0.04	-1.02	-0.60	-0.34	--	--
8	.7	2	8	-46.54	-39.50	0.94	0.36	--	--	-37.21	-20.58	-0.42	-0.59	--	--	-21.42	-5.12	-0.15	0.01	--	--
8	.7	2.5	8	-28.04	-16.76	-0.58	0.14	--	--	-11.46	-4.51	-0.10	-0.02	--	--	-4.21	-0.79	0.11	0.05	--	--

Table B7b: Standard Error of Φ_{11}, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.6796	.6484	.7513	.6319	--	--	.4748	.4299	.4660	.3867	--	--	.3437	.2997	.3219	.2739	--	--
4	.5	2.5	4	.7527	.6841	.7472	.6281	--	--	.5178	.4145	.4433	.3714	--	--	.3677	.2836	.3135	.2649	--	--
4	.7	2	4	.5874	.5414	.6283	.5578	--	--	.4024	.3593	.4006	.3440	--	--	.2928	.2552	.2817	.2422	--	--
4	.7	2.5	4	.6197	.5548	.6028	.5281	--	--	.3995	.3358	.3824	.3333	--	--	.2828	.2332	.2656	.2337	--	--
8	.5	2	4	.4971	.4065	.5071	.4245	--	--	.4184	.3054	.3093	.2525	--	--	.3432	.2916	.2130	.1769	--	--
8	.5	2.5	4	.4373	.3903	.4835	.3894	--	--	.3897	.2849	.2854	.2369	--	--	.2907	.2113	.1959	.1660	--	--
8	.7	2	4	.4491	.3445	.4412	.3664	--	--	.4070	.3178	.2734	.2252	--	--	.3565	.2121	.1792	.1600	--	--
8	.7	2.5	4	.4119	.3293	.4040	.3497	--	--	.2957	.2522	.2450	.2116	--	--	.2234	.1813	.1679	.1510	--	--
4	.5	2	6	.3483	.3025	.3618	.2691	.4428	.3184	.2427	.1774	.2065	.1693	.2517	.1940	.1611	.1239	.1425	.1190	.1650	.1323
4	.5	2.5	6	.3583	.2741	.3365	.2649	.3648	.2969	.2096	.1683	.1969	.1625	.2210	.1745	.1452	.1187	.1359	.1152	.1493	.1230
4	.7	2	6	.2865	.2396	.2719	.2396	.3100	.2615	.1908	.1530	.1734	.1443	.2244	.1584	.1326	.1057	.1190	.1019	.1372	.1083
4	.7	2.5	6	.2839	.2325	.2686	.2291	.2812	.2448	.1742	.1466	.1652	.1421	.1728	.1407	.1186	.1010	.1132	.0999	.1165	.0980
8	.5	2	6	.3232	.2509	.2778	.2053	.3486	.2857	.2085	.1424	.1570	.1261	.2304	.1684	.1371	.1005	.1051	.0880	.1510	.1154
8	.5	2.5	6	.2848	.2187	.2491	.2001	.3307	.2896	.1711	.1340	.1447	.1200	.1989	.1533	.1126	.0920	.0998	.0856	.1271	.1063
8	.7	2	6	.2337	.2129	.2094	.1718	.3008	.2496	.1730	.1329	.1275	.1068	.1996	.1646	.1217	.0828	.0857	.0749	.1208	.0941
8	.7	2.5	6	.2104	.1864	.1975	.1663	.2747	.2280	.1527	.1170	.1175	.1036	.1602	.1275	.0945	.0776	.0829	.0733	.1020	.0873
4	.5	2	8	.2910	.2528	.3003	.2223	--	--	.2041	.1555	.1778	.1290	--	--	.1326	.1012	.1080	.0911	--	--
4	.5	2.5	8	.3160	.2760	.2915	.2158	--	--	.1936	.1423	.1549	.1267	--	--	.1121	.0945	.1022	.0892	--	--
4	.7	2	8	.2300	.1956	.2092	.1755	--	--	.1515	.1188	.1256	.1089	--	--	.1018	.0821	.0876	.0769	--	--
4	.7	2.5	8	.2275	.1878	.1975	.1711	--	--	.1341	.1130	.1180	.1065	--	--	.0920	.0784	.0844	.0759	--	--
8	.5	2	8	.2854	.2083	.2379	.1825	--	--	.1724	.1364	.1307	.1122	--	--	.1405	.0914	.0914	.0810	--	--
8	.5	2.5	8	.2358	.1998	.2174	.1747	--	--	.1506	.1240	.1277	.1122	--	--	.1020	.0862	.0888	.0784	--	--
8	.7	2	8	.2460	.2000	.1979	.1577	--	--	.2089	.1356	.1119	.0983	--	--	.1151	.0807	.0773	.0696	--	--
8	.7	2.5	8	.1767	.1767	.1706	.1545	--	--	.1105	.1105	.1066	.0965	--	--	.0736	.0736	.0747	.0679	--	--

Table B7c: Percent Bias of Φ_{11}, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-36.47	-27.61	-18.15	-1.81	--	--	-42.36	-19.83	-15.24	-3.03	--	--	-35.33	-9.16	-8.07	1.43	--	--
4	.5	2.5	4	-39.49	-22.90	-15.66	0.85	--	--	-29.26	-6.86	-8.30	1.95	--	--	-16.76	-2.39	0.01	-1.58	--	--
4	.7	2	4	-15.63	-8.43	13.31	8.41	--	--	-21.40	-1.42	-4.19	-1.52	--	--	-17.74	0.45	-2.01	-1.21	--	--
4	.7	2.5	4	-11.19	1.13	11.10	8.72	--	--	-6.14	1.65	-1.53	-2.69	--	--	-2.81	-2.05	0.85	-0.05	--	--
8	.5	2	4	89.28	89.57	1.57	0.39	--	--	85.87	64.32	1.20	0.41	--	--	74.34	33.07	0.45	1.09	--	--
8	.5	2.5	4	124.0	98.18	1.42	1.31	--	--	84.17	47.87	1.14	-1.13	--	--	39.63	10.03	0.95	0.02	--	--
8	.7	2	4	85.50	86.95	13.37	4.21	--	--	86.89	78.64	1.27	-0.05	--	--	83.68	58.28	-0.44	-0.64	--	--
8	.7	2.5	4	134.3	121.2	10.48	0.53	--	--	107.6	76.80	0.06	-2.24	--	--	67.29	40.48	0.66	0.40	--	--
4	.5	2	6	-28.21	-7.68	-8.06	-0.57	16.41	5.44	-10.67	-1.45	-1.67	0.45	2.94	0.39	-1.37	-1.58	-0.05	-0.40	0.27	-0.38
4	.5	2.5	6	-14.14	-3.35	-6.02	-0.08	8.74	1.47	-4.08	-0.20	-1.23	0.26	-1.71	-0.38	-0.39	-0.59	-0.02	-0.59	1.09	1.38
4	.7	2	6	-14.12	-3.70	-3.37	1.42	44.01	18.12	-2.60	0.38	1.08	0.32	26.74	7.17	0.39	0.78	-0.22	-0.60	7.64	0.65
4	.7	2.5	6	-2.65	-0.09	-2.64	-3.45	38.23	15.45	0.28	-0.87	-0.91	0.47	13.38	0.02	0.38	0.36	-0.13	-0.65	3.28	0.12
8	.5	2	6	57.39	39.55	-1.19	-0.72	13.23	1.91	40.79	18.93	-2.13	0.06	2.38	-0.25	20.04	4.01	-0.38	-0.20	0.68	-0.23
8	.5	2.5	6	53.90	38.25	-0.37	-1.36	4.61	1.80	20.31	9.83	0.55	0.37	0.17	-0.44	2.76	-0.03	-0.05	0.24	-0.82	-0.21
8	.7	2	6	69.22	52.80	-1.55	-2.41	29.88	14.99	53.11	20.28	1.30	0.78	17.29	6.74	31.51	7.75	-0.29	-0.49	5.08	0.33
8	.7	2.5	6	87.39	62.57	-2.40	-1.82	18.63	7.31	39.94	17.07	1.42	-1.13	1.16	0.64	16.27	1.57	0.62	-1.06	0.31	-0.13
4	.5	2	8	-56.90	-35.94	-28.48	-12.04	--	--	-33.02	-10.63	-9.81	-2.82	--	--	-17.71	-1.79	-1.18	-0.38	--	--
4	.5	2.5	8	-51.81	-27.29	-20.27	-9.32	--	--	-17.37	-4.54	-2.66	-0.06	--	--	-1.84	-0.41	-0.76	-0.53	--	--
4	.7	2	8	-28.43	-14.17	-8.13	-2.68	--	--	-9.77	-0.89	-2.04	-1.05	--	--	-1.16	-1.21	-0.75	0.24	--	--
4	.7	2.5	8	-18.42	-11.20	-0.40	-1.03	--	--	-3.29	-1.85	-0.80	-1.66	--	--	0.14	-0.17	-0.08	0.15	--	--
8	.5	2	8	31.83	20.49	-2.41	-2.04	--	--	22.09	8.75	-0.36	-0.07	--	--	11.33	2.14	-0.25	0.65	--	--
8	.5	2.5	8	13.80	3.04	-4.24	-0.81	--	--	-0.04	-1.42	0.68	-0.10	--	--	-0.22	-0.12	0.06	-0.61	--	--
8	.7	2	8	60.74	43.09	0.14	-0.89	--	--	53.19	23.85	-0.61	-0.23	--	--	32.24	5.72	0.03	0.35	--	--
8	.7	2.5	8	66.28	32.92	-0.50	-1.26	--	--	26.70	6.59	-1.56	-0.97	--	--	11.05	0.50	0.31	0.28	--	--

Table B7d: Standard Error of Φ_{11}, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.6945	.6049	.6132	.5727	--	--	.5470	.4137	.4212	.3767	--	--	.4034	.3001	.3139	.2769	--	--
4	.5	2.5	4	.7049	.5747	.6030	.5650	--	--	.5242	.3952	.4155	.3733	--	--	.3680	.2773	.3173	.2653	--	--
4	.7	2	4	.8140	.7591	.8675	.7578	--	--	.7116	.5586	.6010	.4943	--	--	.5089	.4028	.4425	.3454	--	--
4	.7	2.5	4	.8732	.7659	.8309	.7514	--	--	.6361	.5360	.5806	.4596	--	--	.4792	.3723	.4181	.3317	--	--
8	.5	2	4	.5534	.5675	.4953	.4123	--	--	.4032	.3557	.3194	.2520	--	--	.2767	.2728	.2111	.1769	--	--
8	.5	2.5	4	.6265	.5922	.4665	.3882	--	--	.4096	.3474	.2872	.2359	--	--	.2690	.2128	.1972	.1665	--	--
8	.7	2	4	.5790	.5913	.6394	.5273	--	--	.3993	.4322	.4201	.3240	--	--	.3045	.3251	.2887	.2191	--	--
8	.7	2.5	4	.6741	.6701	.6028	.4993	--	--	.4731	.4642	.3937	.2949	--	--	.3625	.3212	.2599	.2128	--	--
4	.5	2	6	.3298	.2825	.3376	.2736	.3900	.3307	.2399	.1779	.2072	.1717	.2462	.1949	.1676	.1236	.1447	.1187	.1667	.1332
4	.5	2.5	6	.3248	.2607	.3362	.2632	.3712	.2902	.2071	.1700	.1952	.1660	.2170	.1729	.1431	.1192	.1348	.1157	.1509	.1220
4	.7	2	6	.4297	.3514	.3986	.3538	.4448	.4061	.3134	.2380	.2953	.2169	.3226	.2692	.2240	.1640	.1940	.1508	.2340	.1803
4	.7	2.5	6	.4222	.3398	.3955	.3283	.4670	.4216	.2872	.2215	.2615	.2074	.3110	.2420	.1961	.1545	.1783	.1465	.2084	.1657
8	.5	2	6	.3264	.2654	.2728	.2038	.3396	.2793	.2039	.1590	.1495	.1248	.2183	.1683	.1323	.1023	.1045	.0879	.1512	.1167
8	.5	2.5	6	.3486	.2761	.2504	.1972	.3476	.2817	.2101	.1450	.1462	.1217	.1991	.1542	.1140	.0931	.1000	.0863	.1286	.1064
8	.7	2	6	.3048	.3317	.3610	.2583	.4444	.3901	.2462	.2089	.2206	.1640	.3198	.2540	.1839	.1370	.1439	.1153	.2212	.1619
8	.7	2.5	6	.4003	.3633	.3213	.2512	.4325	.3620	.2609	.2011	.1977	.1567	.2821	.2225	.1709	.1245	.1384	.1115	.1829	.1469
4	.5	2	8	.2321	.2104	.2673	.1974	--	--	.2064	.1636	.1625	.1279	--	--	.1361	.0983	.1074	.0903	--	--
4	.5	2.5	8	.1991	.2233	.2208	.2135	--	--	.1752	.1361	.1488	.1278	--	--	.1126	.0942	.1021	.0889	--	--
4	.7	2	8	.3299	.2682	.3343	.2579	--	--	.2465	.1946	.2105	.1680	--	--	.1750	.1285	.1434	.1175	--	--
4	.7	2.5	8	.3270	.2667	.3188	.2783	--	--	.2243	.1762	.1990	.1606	--	--	.1565	.1254	.1386	.1158	--	--
8	.5	2	8	.3002	.2355	.2316	.1771	--	--	.1781	.1409	.1332	.1139	--	--	.1195	.0950	.0922	.0811	--	--
8	.5	2.5	8	.2759	.2057	.2025	.1765	--	--	.1503	.1228	.1283	.1117	--	--	.1028	.0867	.0893	.0783	--	--
8	.7	2	8	.3350	.3302	.3078	.2344	--	--	.2620	.1942	.1816	.1457	--	--	.1651	.1237	.1237	.1039	--	--
8	.7	2.5	8	.3918	.3122	.2715	.2236	--	--	.2302	.1728	.1663	.1407	--	--	.1525	.1153	.1195	.1027	--	--

Table B8c: Percent Bias of $\Delta\kappa$, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)						Intercept heterogeneity (location;$\Delta\tau$)					
p	φ	$\Delta\kappa$	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-8.46	-7.46	-9.07	-0.87	--	--	-8.12	-4.07	0.91	0.85	--	--	-5.76	-0.04	1.13	-0.23	--	--
4	.5	2.5	4	-11.54	-4.94	-4.61	-0.08	--	--	-6.75	-1.19	0.50	-0.28	--	--	-1.18	0.32	0.02	0.38	--	--
4	.7	2	4	8.65	-0.38	-2.68	-1.96	--	--	7.90	0.19	1.19	-0.03	--	--	3.44	0.63	0.41	0.87	--	--
4	.7	2.5	4	3.15	-1.30	-1.31	1.41	--	--	0.98	-0.11	-0.43	-0.10	--	--	0.82	0.25	0.25	-0.22	--	--
8	.5	2	4	-1.48	1.96	-2.89	-1.49	--	--	-6.72	-1.91	-0.28	-1.18	--	--	-11.37	-2.47	-0.09	-0.23	--	--
8	.5	2.5	4	-12.49	-8.07	-0.46	0.07	--	--	-10.24	-4.95	-0.25	-0.69	--	--	-4.68	-0.41	-0.25	0.01	--	--
8	.7	2	4	-22.41	-17.47	-7.92	-1.94	--	--	-33.87	-24.26	-0.86	-0.51	--	--	-37.64	-18.87	0.03	0.32	--	--
8	.7	2.5	4	-37.00	-29.89	-2.69	-1.13	--	--	-32.24	-19.30	-0.48	-1.35	--	--	-18.77	-9.50	0.26	-0.27	--	--
4	.5	2	6	0.45	0.66	-0.99	-0.03	-7.53	-1.76	-1.08	0.03	-0.08	-0.12	-1.69	0.27	-0.50	0.36	0.06	0.03	-0.05	0.23
4	.5	2.5	6	-1.32	-0.30	-0.40	-0.41	-4.01	-1.45	-0.26	-0.38	-0.09	0.02	0.36	0.26	-0.09	-0.02	0.02	-0.02	-0.04	-0.31
4	.7	2	6	4.19	1.02	-0.36	-0.50	-23.96	-9.33	0.28	-0.35	-0.28	0.12	-12.90	-3.33	0.39	-0.19	-0.11	-0.28	-4.37	-0.50
4	.7	2.5	6	-0.13	1.42	0.06	0.73	-11.08	-5.02	-0.01	0.58	-0.15	0.06	-3.87	-0.19	-0.37	0.07	-0.04	-0.03	-1.06	0.07
8	.5	2	6	5.21	8.34	-1.50	-0.40	-6.36	-1.63	0.55	2.77	0.27	0.10	-0.38	-0.37	-0.30	0.24	0.09	0.31	-0.28	0.17
8	.5	2.5	6	-2.11	-0.60	-0.74	-0.91	-1.84	-1.14	-2.12	-0.88	-0.25	0.10	-0.83	-0.15	-0.52	-0.12	0.05	-0.05	0.52	-0.42
8	.7	2	6	-13.57	-6.37	-0.90	-0.52	-14.76	-7.52	-15.85	-4.97	-0.94	0.28	-8.33	-3.51	-11.40	-2.81	0.30	-0.09	-2.14	0.30
8	.7	2.5	6	-18.50	-13.20	-0.26	-0.10	-7.19	-2.98	-10.03	-4.26	0.09	0.20	-0.57	0.20	-4.28	-0.42	-0.09	-0.02	-0.13	-0.12
4	.5	2	8	7.51	2.78	-0.01	1.76	--	--	-0.24	0.74	-0.90	1.23	--	--	-0.69	0.03	-0.09	-0.19	--	--
4	.5	2.5	8	-1.29	-0.82	-1.84	-0.07	--	--	-0.46	-0.02	-1.07	0.30	--	--	-0.02	-0.08	-0.21	0.26	--	--
4	.7	2	8	12.21	3.80	0.64	-0.24	--	--	3.16	0.87	0.11	-0.41	--	--	0.35	0.04	0.02	-0.11	--	--
4	.7	2.5	8	2.80	2.03	-0.27	-0.19	--	--	0.72	0.21	-0.09	0.24	--	--	-0.02	0.04	0.02	-0.04	--	--
8	.5	2	8	-1.07	0.42	-0.96	-0.48	--	--	-0.91	0.94	-0.002	0.48	--	--	-0.70	-0.40	-0.30	0.17	--	--
8	.5	2.5	8	-0.32	0.37	0.48	0.21	--	--	-0.10	-0.01	0.10	0.12	--	--	-0.22	0.12	-0.01	-0.13	--	--
8	.7	2	8	-19.90	-11.94	-0.91	0.29	--	--	-20.06	-8.36	-0.14	-0.16	--	--	-11.73	-1.93	-0.04	-0.03	--	--
8	.7	2.5	8	-17.63	-8.20	0.19	-0.25	--	--	-7.09	-1.51	-0.21	0.25	--	--	-2.88	-0.30	-0.22	-0.35	--	--

Table B8d: Standard Error of Δκ, Class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.7072	.5986	.6211	.4378	--	--	.5645	.3736	.3486	.2463	--	--	.3901	.2438	.2109	.1702	--	--
4	.5	2.5	4	.7053	.5291	.5309	.3968	--	--	.4488	.2904	.2860	.2351	--	--	.2715	.1903	.1886	.1661	--	--
4	.7	2	4	.7060	.5909	.5883	.4561	--	--	.5702	.3781	.3779	.2791	--	--	.3731	.2507	.2539	.1912	--	--
4	.7	2.5	4	.7030	.5684	.5365	.4367	--	--	.4311	.3247	.3281	.2637	--	--	.2844	.2204	.2233	.1837	--	--
8	.5	2	4	.6308	.5810	.4458	.3673	--	--	.5322	.4428	.2580	.2114	--	--	.4087	.3099	.1699	.1480	--	--
8	.5	2.5	4	.5613	.5210	.3949	.3428	--	--	.4107	.3085	.2308	.2121	--	--	.2656	.1872	.1586	.1494	--	--
8	.7	2	4	.5890	.5285	.5206	.3906	--	--	.5136	.4806	.3038	.2359	--	--	.4445	.3262	.2022	.1623	--	--
8	.7	2.5	4	.5854	.5377	.4499	.3890	--	--	.4264	.3752	.2871	.2329	--	--	.2886	.2428	.1829	.1639	--	--
4	.5	2	6	.3236	.2447	.2884	.1992	.3996	.2776	.1946	.1332	.1471	.1230	.2058	.1607	.1190	.0910	.0993	.0859	.1292	.1086
4	.5	2.5	6	.2574	.1994	.2400	.1900	.2755	.2266	.1412	.1214	.1281	.1175	.1462	.1318	.0932	.0850	.0888	.0826	.1000	.0922
4	.7	2	6	.3476	.2516	.2848	.2335	.4345	.3637	.2188	.1578	.1869	.1383	.3012	.2035	.1466	.1064	.1201	.0961	.1888	.1312
4	.7	2.5	6	.2934	.2249	.2499	.2122	.3595	.3076	.1736	.1405	.1551	.1310	.2139	.1644	.1162	.0969	.1046	.0921	.1373	.1101
8	.5	2	6	.4211	.3179	.2384	.1882	.3273	.2612	.2518	.1654	.1328	.1138	.1831	.1474	.1450	.0981	.0900	.0798	.1208	.0995
8	.5	2.5	6	.2898	.2437	.2013	.1798	.2437	.2202	.1714	.1298	.1196	.1110	.1381	.1238	.0916	.0805	.0828	.0783	.0903	.0857
8	.7	2	6	.3491	.3563	.2864	.2069	.4184	.3332	.2707	.1984	.1645	.1264	.2897	.2101	.1876	.1127	.1058	.0886	.1758	.1216
8	.7	2.5	6	.3478	.3157	.2368	.1979	.3454	.2822	.2117	.1541	.1401	.1223	.1936	.1569	.1243	.0924	.0966	.0864	.1246	.1036
4	.5	2	8	.3400	.2505	.3137	.2068	--	--	.2355	.1578	.1728	.1189	--	--	.1458	.0902	.0912	.0802	--	--
4	.5	2.5	8	.2746	.2384	.2327	.1958	--	--	.1696	.1223	.1310	.1134	--	--	.0886	.0810	.0829	.0790	--	--
4	.7	2	8	.3183	.2350	.2592	.2016	--	--	.2016	.1452	.1511	.1266	--	--	.1304	.0958	.1021	.0886	--	--
4	.7	2.5	8	.2756	.2218	.2339	.2045	--	--	.1579	.1307	.1369	.1229	--	--	.1050	.0911	.0952	.0871	--	--
8	.5	2	8	.3343	.2439	.2030	.1727	--	--	.1802	.1381	.1192	.1078	--	--	.1286	.0885	.0831	.0762	--	--
8	.5	2.5	8	.2100	.1852	.1886	.1709	--	--	.1217	.1120	.1137	.1079	--	--	.0836	.0785	.0794	.0761	--	--
8	.7	2	8	.3827	.3061	.2483	.1922	--	--	.2967	.1963	.1386	.1183	--	--	.1661	.1035	.0951	.0836	--	--
8	.7	2.5	8	.3339	.2559	.2094	.1878	--	--	.1806	.1414	.1280	.1179	--	--	.1156	.0894	.0900	.0835	--	--

Table B9a: Percent Bias of φ, Class 1

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	57.54	33.57	22.54	4.32	--	--	47.96	15.87	8.41	0.82	--	--	36.32	5.81	1.60	-0.22	--	--
4	.5	2.5	4	51.99	22.49	15.74	2.62	--	--	27.93	4.56	3.60	-0.11	--	--	12.41	0.88	.0001	0.17	--	--
4	.7	2	4	18.26	5.79	2.24	-0.76	--	--	12.43	0.49	-1.02	-0.22	--	--	7.63	-0.77	-0.13	-0.004	--	--
4	.7	2.5	4	14.27	2.62	0.44	-0.48	--	--	3.37	0.12	-0.57	0.10	--	--	0.41	-0.20	-0.11	0.003	--	--
8	.5	2	4	-83.74	-75.77	0.30	-0.25	--	--	-81.58	-62.51	-0.13	0.18	--	--	-71.48	-31.47	0.04	-0.30	--	--
8	.5	2.5	4	-67.35	-56.04	1.74	-0.13	--	--	-48.15	-27.01	-0.96	0.23	--	--	-22.53	-6.44	0.21	0.08	--	--
8	.7	2	4	-86.13	-81.21	-4.85	-0.97	--	--	-84.36	-68.86	-0.55	-0.06	--	--	-74.40	-41.25	0.17	-0.07	--	--
8	.7	2.5	4	-75.51	-63.29	-3.13	-0.25	--	--	-56.65	-33.02	-0.58	-0.16	--	--	-30.71	-14.79	-0.03	0.07	--	--
4	.5	2	6	23.41	4.23	7.49	0.45	-9.46	-4.23	9.08	0.69	0.56	0.13	-3.22	-0.30	1.85	0.19	0.23	0.07	-0.20	-0.06
4	.5	2.5	6	9.39	1.35	3.04	0.30	-3.08	0.23	1.84	-0.06	0.19	-0.02	0.75	0.12	0.38	0.06	-0.06	0.08	-0.11	-0.36
4	.7	2	6	3.54	-0.11	0.18	-0.01	-21.27	-8.73	-0.42	-0.22	-0.57	-0.17	-14.90	-3.06	-0.32	-0.28	0.004	0.06	-4.68	-0.53
4	.7	2.5	6	-0.34	-0.52	0.31	0.04	-12.50	-5.33	-0.40	0.21	0.10	-0.02	-4.50	-0.54	-0.26	-0.12	-0.04	0.03	-1.32	-0.14
8	.5	2	6	-57.91	-44.44	0.76	0.30	-8.57	-2.02	-39.97	-19.36	0.25	0.05	-2.42	-0.11	-20.14	-4.35	-0.02	0.06	-0.70	0.01
8	.5	2.5	6	-33.63	-24.06	0.95	0.09	-0.76	-0.58	-11.38	-6.33	0.37	-0.07	-0.04	0.05	-1.75	-0.37	0.01	-0.11	0.13	-0.03
8	.7	2	6	-66.43	-48.38	-0.45	-0.13	-18.13	-10.72	-43.64	-17.04	-0.34	-0.08	-10.68	-3.81	-23.55	-5.67	0.12	0.05	-2.61	0.01
8	.7	2.5	6	-43.77	-31.21	-0.06	0.04	-7.38	-3.03	-17.81	-7.11	-0.15	0.04	-1.02	-0.27	-6.30	-0.83	-0.05	0.04	-0.22	-0.11
4	.5	2	8	55.51	28.66	21.94	7.86	--	--	32.62	9.21	8.05	1.54	--	--	17.75	1.16	1.14	-0.06	--	--
4	.5	2.5	8	46.79	21.54	14.76	5.41	--	--	15.15	3.39	1.85	0.28	--	--	1.11	0.01	0.03	-0.01	--	--
4	.7	2	8	11.82	3.57	2.03	0.05	--	--	3.76	0.46	0.002	0.09	--	--	0.39	-0.08	-0.001	-0.01	--	--
4	.7	2.5	8	7.46	2.90	0.40	-0.08	--	--	1.06	0.28	-0.10	0.07	--	--	0.04	0.01	-0.08	0.003	--	--
8	.5	2	8	-33.10	-22.97	1.67	0.21	--	--	-23.44	-10.04	-0.06	-0.09	--	--	-11.36	-2.51	0.06	0.13	--	--
8	.5	2.5	8	-10.02	-2.23	2.98	-0.05	--	--	-0.41	-0.02	-0.03	-0.02	--	--	0.11	-0.11	-0.02	-0.10	--	--
8	.7	2	8	-54.24	-37.70	-0.37	0.21	--	--	-40.95	-19.21	0.03	-0.08	--	--	-23.89	-4.78	-0.02	-0.03	--	--
8	.7	2.5	8	-28.21	-14.57	0.17	-0.15	--	--	-11.44	-3.61	0.10	-0.08	--	--	-3.69	-0.44	-0.10	0.01	--	--

Table B9b: Empirical Standard Error of φ (Equal for Both Classes)

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	.1744	.1774	.1510	.0784	--	--	.1863	.1582	.1169	.0360	--	--	.1868	.1064	.0594	.0184	--	--
4	.5	2.5	4	.1765	.1536	.1289	.0554	--	--	.1795	.0978	.0744	.0214	--	--	.1380	.0513	.0358	.0150	--	--
4	.7	2	4	.1414	.1423	.1193	.0625	--	--	.1383	.0968	.0844	.0295	--	--	.1139	.0581	.0488	.0190	--	--
4	.7	2.5	4	.1320	.1162	.0987	.0393	--	--	.1249	.0612	.0568	.0203	--	--	.0694	.0388	.0376	.0126	--	--
8	.5	2	4	.0907	.1293	.1283	.0497	--	--	.1308	.1929	.0724	.0244	--	--	.1726	.1989	.0378	.0157	--	--
8	.5	2.5	4	.1496	.1802	.0862	.0342	--	--	.2038	.1849	.0431	.0184	--	--	.1811	.1027	.0252	.0121	--	--
8	.7	2	4	.1230	.1558	.1262	.0551	--	--	.1619	.2264	.0664	.0234	--	--	.1958	.2451	.0364	.0156	--	--
8	.7	2.5	4	.1727	.2246	.1041	.0438	--	--	.2412	.2257	.0456	.0186	--	--	.2294	.1674	.0239	.0123	--	--
4	.5	2	6	.1683	.0893	.1068	.0346	.1581	.1088	.1301	.0384	.0507	.0151	.1134	.0518	.0756	.0197	.0238	.0110	.0614	.0323
4	.5	2.5	6	.1186	.0496	.0689	.0278	.1235	.0758	.0600	.0190	.0278	.0109	.0603	.0318	.0293	.0138	.0158	.0075	.0361	.0200
4	.7	2	6	.1162	.0657	.0747	.0270	.2001	.1404	.0758	.0282	.0353	.0155	.1848	.0890	.0446	.0198	.0236	.0097	.1154	.0470
4	.7	2.5	6	.0828	.0397	.0530	.0187	.1609	.1090	.0385	.0196	.0247	.0109	.1055	.0473	.0258	.0117	.0158	.0080	.0604	.0244
8	.5	2	6	.1723	.1855	.0752	.0230	.1659	.1211	.1964	.1641	.0312	.0132	.1118	.0466	.1688	.0860	.0197	.0097	.0586	.0310
8	.5	2.5	6	.1741	.1644	.0471	.0182	.1189	.0707	.1260	.0969	.0210	.0101	.0513	.0290	.0560	.0204	.0138	.0069	.0319	.0197
8	.7	2	6	.1893	.2465	.0690	.0244	.2050	.1639	.2429	.2066	.0339	.0124	.1759	.1014	.2169	.1213	.0191	.0094	.0910	.0341
8	.7	2.5	6	.2201	.2122	.0446	.0190	.1464	.0902	.1878	.1239	.0192	.0099	.0649	.0340	.1178	.0437	.0132	.0071	.0303	.0182
4	.5	2	8	.1689	.1732	.1511	.1017	--	--	.1835	.1195	.1102	.0483	--	--	.1612	.0470	.0437	.0077	--	--
4	.5	2.5	8	.1908	.1652	.1310	.0845	--	--	.1546	.0747	.0541	.0224	--	--	.0507	.0173	.0124	.0056	--	--
4	.7	2	8	.1211	.0810	.0592	.0200	--	--	.0805	.0388	.0296	.0110	--	--	.0354	.0147	.0160	.0076	--	--
4	.7	2.5	8	.1043	.0677	.0439	.0133	--	--	.0484	.0248	.0169	.0080	--	--	.0225	.0108	.0127	.0055	--	--
8	.5	2	8	.1827	.1644	.0525	.0153	--	--	.1699	.1251	.0218	.0095	--	--	.1349	.0691	.0150	.0064	--	--
8	.5	2.5	8	.1114	.0629	.0702	.0118	--	--	.0356	.0214	.0162	.0071	--	--	.0179	.0100	.0105	.0049	--	--
8	.7	2	8	.2097	.2353	.0457	.0158	--	--	.2335	.2105	.0215	.0093	--	--	.2204	.1201	.0144	.0064	--	--
8	.7	2.5	8	.2005	.1738	.0337	.0116	--	--	.1576	.0973	.0155	.0074	--	--	.0940	.0368	.0103	.0047	--	--

Table B9c: Percent Bias of φ, class 2

				N = 200						N = 500						N = 1000					
				Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)						Intercept heterogeneity (location;Δτ)					
p	φ	Δκ	lc	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5	τ_1;1	τ_1;1.5	$\tau_{1,p}$;1	$\tau_{1,p}$;1.5	τ_p;1	τ_p;1.5
4	.5	2	4	-57.54	-33.57	-22.54	-4.32	--	--	-47.96	-15.87	-8.41	-0.82	--	--	-36.32	-5.81	-1.60	0.22	--	--
4	.5	2.5	4	-51.99	-22.49	-15.74	-2.62	--	--	-27.93	-4.56	-3.60	0.11	--	--	-12.41	-0.88	0.00	-0.17	--	--
4	.7	2	4	-42.62	-13.52	-5.22	1.77	--	--	-29.01	-1.15	2.38	0.52	--	--	-17.80	1.80	0.30	0.01	--	--
4	.7	2.5	4	-33.29	-6.10	-1.02	1.13	--	--	-7.87	-0.27	1.33	-0.24	--	--	-0.96	0.46	0.26	-0.01	--	--
8	.5	2	4	83.74	75.77	-0.30	0.25	--	--	81.58	62.51	0.13	-0.18	--	--	71.48	31.47	-0.04	0.30	--	--
8	.5	2.5	4	67.35	56.04	-1.74	0.13	--	--	48.15	27.01	0.96	-0.23	--	--	22.53	6.44	-0.21	-0.08	--	--
8	.7	2	4	201.0	189.5	11.31	2.27	--	--	196.8	160.7	1.27	0.15	--	--	173.6	96.25	-0.40	0.16	--	--
8	.7	2.5	4	176.2	147.7	7.30	0.59	--	--	132.2	77.04	1.36	0.37	--	--	71.66	34.52	0.06	-0.17	--	--
4	.5	2	6	-23.41	-4.23	-7.49	-0.45	9.46	4.23	-9.08	-0.69	-0.56	-0.13	3.22	0.30	-1.85	-0.19	-0.23	-0.07	0.20	0.06
4	.5	2.5	6	-9.39	-1.35	-3.04	-0.30	3.08	-0.23	-1.84	0.06	-0.19	0.02	-0.75	-0.12	-0.38	-0.06	0.06	-0.08	0.11	0.36
4	.7	2	6	-8.27	0.26	-0.42	0.03	49.63	20.36	0.98	0.52	1.33	0.40	34.76	7.15	0.75	0.66	-0.01	-0.13	10.92	1.24
4	.7	2.5	6	0.79	1.21	-0.72	-0.09	29.16	12.43	0.94	-0.48	-0.24	0.04	10.51	1.26	0.60	0.28	0.10	-0.07	3.09	0.33
8	.5	2	6	57.91	44.44	-0.76	-0.30	8.57	2.02	39.97	19.36	-0.25	-0.05	2.42	0.11	20.14	4.35	0.02	-0.06	0.70	-0.01
8	.5	2.5	6	33.63	24.06	-0.95	-0.09	0.76	0.58	11.38	6.33	-0.37	0.07	0.04	-0.05	1.75	0.37	-0.01	0.11	-0.13	0.03
8	.7	2	6	155.0	112.9	1.06	0.30	42.30	25.02	101.8	39.75	0.78	0.18	24.92	8.89	54.95	13.23	-0.28	-0.11	6.08	-0.03
8	.7	2.5	6	102.1	72.81	0.14	-0.10	17.21	7.06	41.55	16.59	0.34	-0.09	2.39	0.62	14.70	1.95	0.12	-0.10	0.52	0.26
4	.5	2	8	-55.51	-28.66	-21.94	-7.86	--	--	-32.62	-9.21	-8.05	-1.54	--	--	-17.75	-1.16	-1.14	0.06	--	--
4	.5	2.5	8	-46.79	-21.54	-14.76	-5.41	--	--	-15.15	-3.39	-1.85	-0.28	--	--	-1.11	-0.01	-0.03	0.01	--	--
4	.7	2	8	-27.58	-8.33	-4.73	-0.12	--	--	-8.77	-1.07	-0.01	-0.22	--	--	-0.90	0.18	0.00	0.03	--	--
4	.7	2.5	8	-17.41	-6.78	-0.94	0.18	--	--	-2.47	-0.66	0.24	-0.17	--	--	-0.09	-0.02	0.19	-0.01	--	--
8	.5	2	8	33.10	22.97	-1.67	-0.21	--	--	23.44	10.04	0.06	0.09	--	--	11.36	2.51	-0.06	-0.13	--	--
8	.5	2.5	8	10.02	2.23	-2.98	0.05	--	--	0.41	0.02	0.03	0.02	--	--	-0.11	0.11	0.02	0.10	--	--
8	.7	2	8	126.5	87.96	0.86	-0.50	--	--	95.54	44.83	-0.07	0.19	--	--	55.74	11.15	0.06	0.06	--	--
8	.7	2.5	8	65.81	34.01	-0.40	0.35	--	--	26.69	8.43	-0.24	0.20	--	--	8.62	1.02	0.24	-0.02	--	--

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