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# Special Feature of Precedence Network Charts

By

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## Y.1 INTRODUCTION

One of the unique features widely used in precedence networks is that of Hammock activities. They are used to fill the time span between other "normal" activities since their duration cannot be calculated or estimated at the initial stage of project planning. Therefore, hammocks are neglected during the time analysis of a project. When the time analysis is finished, hammock activities are allocated durations based on the early and late times of their preceding and succeeding activities. For this reason it is necessary that both ends of hammock activities be linked to normal activities.

Hammocks can play a useful role in project management. Typically, they have been used to denote usage of equipment needed for a particular chain of activities (e.g. a load lifting device) without predetermining the estimated time the equipment must be present on site. Similarly, it may be required to pickup the cost of a complete section of a project, or more usually of some background cost related to a section. Such background costs could arise from storage, supervision, etc. and can be allocated to a hammock activity. Also, for high management level reporting, hammocks are used to represent collectively a sequence of consecutive normal activities, all of them forming the task of one department or relating to the same cost center.

Over the past few years the use of hammocks has become popular and in Europe most computer software on project planning can now treat them as part of the whole project analysis process. In the U.S.A. a search was conducted by the author aiming to identify how many systems can handle hammock activities and in what particular way. In response it was found that of 8 systems only one is considering handling hammock activities in the near future, in an

undisclosed manner. Nonetheless, some confusion still exists among hammock users, related to the procedure that must be used to calculate their durations after the normal time analysis is performed. Therefore, there is a need to develop an algorithm which should never fail to leave the project unaffected and its timing undistorted by the introduction and handling of hammocks. Such an algorithm is proposed in this paper. The paper reviews the methods applied by three systems[1,2,3] widely used in the U.K. and discusses the potential drawbacks related to the misinterpretation of time analysis results when hammocks are included in a project plan. The proposed algorithm is also incorporated in a computerized network time analysis application. Despite an extended literature search no reference was found on the timing of hammock activities, although they were formally introduced in a form of special activities by the British Standards which suggest several potential applications of hammocks in project planning[5].

In the following sections a formal definition of hammocks will be discussed. The drawbacks of the current handling methods will be reviewed and finally the suggested algorithm together with the computerized application will be described.

## Y.2 DEFINITION OF HAMMOCK ACTIVITIES

Formally a hammock activity can be defined as follows[4]: "An activity joining two specified events which may be regarded as spanning two or more activities; its duration is initially unspecified and is only determined by the difference between the start and the finish times of the events concerned".

The definition is rather vague and some comments can be made on it.

(i) The definition is implicitly referring to the use of hammock activities in arrow networks only. It does not cater for precedence networks although the latter have attracted the attention of most project planners and have become common practice over the last five years. In fact, this paper concentrates on hammocks and their treatment within precedence (else known as activity-on-node) diagrams.

(ii) There is no specific reason to have a hammock span two or more activities. In some practical applications spanning even one activity is perfectly legitimate without harming the concept of "hammocking" as defined above.

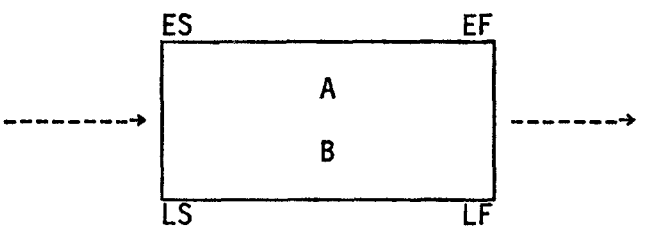
(iii) The main ambiguity of the above definition lies in the calculation of the duration of hammock activities. Most systems [1,2,3] studied consider arbitrarily early times when it comes to calculate hammock durations, thus causing severe timing problems in certain cases as seen later.

In another reference to hammock activities found in the British Standards, Guide to Resource Analysis and Cost Controls[5], hammocks are recommended for collective costing purposes. An example is also illustrated, but again no specific method or procedure is described for the calculation of hammock durations. In any case, it should be kept in mind that the duration of a hammock must be calculated in a way that it does not affect the free and total floats of any other activities linked to the path between the start and the finish of that hammock.

## Y.3 TIME ANALYSIS OF HAMMOCK ACTIVITIES

Precedence network diagrams (PND) are used exclusively because of their numerous advantages over arrow diagrams[6]. To mention a few, PND present planning charts are easy to be understood by non-experts, basically because of their similarity to bar charts. Another advantage of using PND is that they provide the starting and finishing times related to the activity blocks themselves as opposed to events used to separate activities in arrow networks. Hence, there is no coincidence with the start and finish times of adjacent activities, nor any confusion with the timing of the immediate predecessors and successors of any given activity. Finally, a number of extra types of relationships can be employed between adjacent activities[4] thus increasing the flexibility in accurately demonstrating the logical interactions between different tasks. Delays, or waiting times (lags), can also be allocated to logical links. It is not surprising therefore that 70% of currently drawn networks are in precedence form[6].

Following are the notations and symbols used in this ~~chapter~~ paper:




where:

A = Activity identity

B = Activity duration

ES = Early start

EF = Early finish

LS = Late start

LF = Late finish

L = Lag time allocated to a precedence arrow

Also, referring to hammock activities:

$D_E$ = Duration calculated based on early dates

$D_L$ = Duration calculated based on late dates

and finally:

---------→ = denotes a precedence arrow between activities

----|| --→ = denotes the critical path

----L----→ = Lag time allocated to a precedence arrow

In Figure 1, a hammock activity H has been linked to two "ordinary" activities $A_1$ and $A_{n+1}$, thus spanning a chain of n activities in a precedence network diagram. The following assumptions are made:

(a) The precedence relationships between the hammock and its predecessor and successor are finish-to-start[4].

(b) Activities $A_1$ and $A_{n+1}$ are "ordinary" activities in the sense that durations have been estimated and allocated to them.

(c) There is only one precedence arrow leading to the hammock and only one stemming from it.

Assumption (a) can be easily relaxed with the introduction of any permissible type of relationship. The mathematical analysis following

below can be adapted accordingly. Assumption (c) may also be relaxed by adding more predecessors and/or successors to the hammock, although this is rather unusual in practice. Multiple links to and from the hammock would add some more unnecessary complexity to the issue. The nature of the neighbouring activities $A_1$ and $A_{n+1}$, however, has to be such that the forward and backward scheduling may result in a set of specific times, ES, LS, EF, LF, for each one of them. Otherwise, the problem would have more than one solution.

In precedence networks, the total float of activities can be defined as the difference between their starting or finishing times:

$$TF = LF - EF \tag{1}$$

or $$TF = LS - ES \tag{2}$$

Referring to activities $A_1$ and $A_{n+1}$ respectively, their total floats are

$$TF_1 = LF_1 - EF_1 \tag{3}$$

and $$TF_{n+1} = LS_{n+1} - ES_{n+1} \tag{4}$$

During the forward run the earliest start time of the hammock can be determined unambiguously as:

$$ES_H = EF_1 + L_p \tag{5}$$

Similarly, during the backward run, the latest finish time of the hammock can be determined as

$$LF_H = LS_{n+1} - L_s \tag{6}$$

However, the calculation of its latest start and earliest finish time, hence its duration, can be performed following one of two ways:

(a) To assume the earliest finish time of the hammock based on the earliest start time of its successor and the lag between them; i.e.

$$EF_H = ES_{n+1} - L_s \tag{7}$$

Hence, the difference between the earliest finish time and the earliest start time (which is already uniquely identified) represents the duration of the hammock $D_E$ which is based on its early times:

$$D_E = EF_H - ES_H \tag{8}$$

This, in turn, allows for the calculation of the latest start time by simply subtracting the duration of the hammock from its latest finished time (already determined uniquely)

$$LS_H = LF_H - D_E \tag{9}$$

(b) To assume the latest start time of the hammock based on the latest finish time of its predecessor and the lag between them; i.e.

$$LS_H = LF_1 + L_P \tag{10}$$

Hence, the difference between the latest start time and the latest finish time (which is already uniquely identified) represents the duration of the hammock, $D_L$, which is based on its late dates:

$$D_L = LF_H - LS_H \tag{11}$$

This, in turn, allows for the calculation of the earliest finish time by simply adding the duration of the hammock to its earliest start time (already determined uniquely)

$$EF_H = ES_H + D_L \tag{12}$$

The resulting durations $D_E$, $D_L$ can be different depending on the location of the hammock in the network diagram. In fact, it can be proved that the difference of these two durations is equal to the difference of the total floats of the immediate predecessor and successor of the hammock:

$$D_E - D_L = TF_1 - TF_{n+1} \tag{13}$$

To prove this let us start with the definitions of $D_E$ and $D_L$, eq (6) and (9) respectively:

$$D_E - D_L = (EF_H - ES_H) - (LF_H - LS_H)$$

or, based on (7) and (10):

$$D_E - D_L = (ES_{n+1} - L_s - ES_H) - (LF_H - LF_1 - L_p).$$

which yields

$$D_E - D_L = (LF_1 - EF_1) - (LS_{n+1} - ES_{n+1})$$

or, using (3) and (4), eq (13) is derived.

Note that if $TF_1$ and $TF_{n+1}$ are equal, both procedures (a) and (b) described above, yield the same duration, i.e.

$$D_E = D_L \iff TF_1 = TF_{n+1} \tag{14}$$

In this particular case, either procedure can be used in order to compile the duration of a hammock, without any ambiguity induced.

In a general case, however, one would expect the total floats of $A_1$ and $A_{n+1}$ to be different. Hence, there is a need to identify the particular solution which will not affect the timing of some "ordinary" activities of the network.

Let us assume that activities $A_1$ and $A_{n+1}$ are such that

$$TF_1 \geq TF_{n+1} \tag{15}$$

The adoption of $D_E$ and $D_L$ will be examined separately and the implications will be evaluated with respect to the planner's desire not to affect the timing of the rest of the project by the introduction of hammocks.

### Y.3.1 Early Dates

Let us first examine the adoption of $D_E$ as a potential solution to the problem.

Note that under the power of (13), eq (15) indicates that:

$$D_L \leq D_E \tag{16}$$

The total float of $A_1$ can be written as

$$TF_1 = LF_1 - EF_1 \tag{17}$$

In this case it is reminded that the latest start time of the hammock is derived as $LS_H = LF_H - D_E$ (see eq (7)) hence, due to the backward run scheduling rules, the latest finish time of $A_1$ can be equal or smaller than the difference $LS_H - L_p$:*

$$LF_1 \leq LS_H - L_p \tag{18}$$

Introducing K as a positive quantity or zero

$$K \geq 0 \tag{19}$$

Equation (16) can be written as

$$LF_1 = LS_H - L_p - K \tag{20}$$

The combination of (19) and (5) into (17) yields:

$$TF_1 = (LS_H - L_p - K) - (ES_H - L_p),$$

or $$TF_1 = LS_H - ES_H - K \tag{21}$$

Based on (9) and (8), (21) can be written as

$$TF_1 = (LF_H - D_E) - (EF_H - D_E) - K$$

*During a backward run the latest finish time of an activity is always assumed to be the smallest of the differences of all latest start time of its successors minus any present lags on the respective precedence arrows.

or $$TF_1 = LF_H - EF_H - K \quad (22)$$

Note in this case that (21) and (22) indicate that the hammock has a total float greater than that of its predecessor.

Finally, based on (6) and (7), (22) yields:

$$TF_1 = (LS_{n+1} - L_s) - (ES_{n+1} - L) - K$$

or $$TF_1 = LS_{n+1} - ES_{n+1} - K$$

or $$TF_1 = TF_{n+1} - K \quad (23)$$

indicating that

$$TF_1 \leq TF_{n+1} \quad (24)$$

which is in contradiction to the assumption made earlier in (15):

$$TF_1 \geq TF_{n+1}$$

The conflict is removed only for K=0, i.e.

for $TF_1 = TF_{n+1}$

as expected from the analysis presented at the end of the previous section.

Y.3.2 Late Dates

Let us now examine the alternative duration $D_L$ as a potential solution to the problem.

Starting again with $TF_1$ it can be seen that:

$$TF_1 = LF_1 - EF_1 \quad (25)$$

In this case, it is reminded that the earliest finish time of the hammock is derived as $EF_H = ES_H + D_L$ (see eq (10)) hence, due to the forward run scheduling rules, the earliest start time of $A_{n+1}$ can be equal or greater than the sum $EF_H + L_S$:*

$$ES_{n+1} \geq EF_H + L_S \tag{26}$$

Introducing N as a positive quantity or zero

$$N \geq 0 \tag{27}$$

(24) can be written as

$$ES_{n+1} = EF_H + L_S + N \tag{28}$$

The combination of (10) and (5) into (25) yields

$$TF_1 = (LS_H - L_p) - (ES_H - L_p)$$

or $$TF_1 = LS_H - ES_H \tag{29}$$

Based on (11) and (12), (29) can be written as

$$TF_1 = (LF_H - D_L) - (EF_H - D_L)$$

or $$TF_1 = LF_H - EF_H \tag{30}$$

* During a backward run the latest finish time of an activity is always assumed to be the smallest of the differences of all latest start times of its successors minus any present lags on the respective precedence arrows.

Finally, based on (6) and (28), (30) can be written as

$$TF_1 = (LS_{n+1} - L_s) - (ES_{n+1} - L_s - N)$$

or $$TF_1 = LS_{n+1} - ES_{n+1} + N$$

or $$TF_1 = TF_{n+1} + N \tag{31}$$

indicating that

$$TF_1 \geq TF_{n+1} \tag{32}$$

which is in accordance to the assumption made earlier, in (15):

$$TF_1 \geq TF_{n+1}$$

The equal sign of (32) holds for N=0, as expected.

The conclusion of the analysis presented above is that for $TF_1 > TF_{n+1}$ the solution which does not distort the timing of the predecessor of the hammock is the one based on its latest times, i.e. $D_L$, whose value happens to be smaller than $D_E$.

Following a similar analysis it can be proved that for

$$TF_1 \leq TF_{n+1} \tag{33}$$

the solution not affecting the timing of the successor of the hammock is the one based on its earliest times, i.e. $D_E$, whose value in this case happens to be smaller than $D_L$.

It may also be seen that the quantities K and N introduced earlier are equal to the difference of the total floats of $A_1$ and $A_{n+1}$.

Consequently, adopting the greatest of $D_E$, $D_L$, as a potential solution of the problem, results in incorporating the difference $|TF_1 - TF_{n+1}|$ to the duration of the hammock. Thus, either the predecessor or the successor of the hammock, whichever had the greatest total float, will be deprived of a portion of its total float, equal to $|TF_1 - TF_{n+1}|$.

As will be seen in the following section, the practical implications of the arbitrary rule to always adopt $D_E$, endorsed by most users[1,2,3] may cause severe distortions. As severe as transforming a non-critical predecessor (and in turn some of its predecessors) to a critical activity.

In conclusion, the following methodology is suggested:

(i) To compile the total floats of the predecessor and the successor of the hammock $TF_1$ and $TF_{n+1}$ respectively as a result of the time analysis process.

(ii) If $TF_1 = TF_{n+1}$, it follows that $D_E = D_L$, hence either solution is acceptable.

If $TF_1 > < TF_{n+1}$, it follows that $D_E > < D_L$, hence the smallest of $D_E$, $D_L$ is only acceptable.

In general, for $TF_1 > < TF_{n+1}$, assign $D_H = \min(D_E, D_L)$

## Y.5 PRACTICAL APPLICATIONS OF USING HAMMOCKS

In the following section some numerical applications of the proposed algorithm are presented. The implications of the adoption of early dates as a "rule of thumb" are demonstrated. Finally the structure of the software developed for the application of the algorithm is presented together with sample results.

A. Hammocks Linked Between Two Ordinary Activities with Equal Total Floats:

For the sake of simplicity an absolute time scale is being used in the following examples. This also helps in avoiding any ambiguities induced by real time periods (e.g. actual time calendar dates) due to holidays, weekends, etc.

Let us consider an example of a small project as shown in Figure $2^7$ in which a hammock activity H has been linked between two critical activities A1 and A7. The duration and the early finish and late start times of the hammock are not known initially. During the forward time run the early times of all other normal activities are calculated and consequently the early start time of the hammock H can be compiled. The early start time of it is 9 because the early finish of activity A1 is 8 time units and there is no time lag on the finish-to-start relationship between activities A1 and H. Note the convention of starting an activity at the beginning of a time period (eg morning of a day) and finishing it at the end of it.

During the backward time run the late times of all other normal activities are calculated and consequently the late finish time of the hammock H can be compiled. The late finish of H is 35 time units because the late start of activity A7 is 36 time units and there is no lag on the finish-to-start relationship between H and A7. If some lag is present then the lag is subtracted from the late start time of the succeeding activity.

Note, that the early start and late finish times of the hammock are

uniquely and unambiguously identified, while the calculation of the late start and early finish time of it can be performed following one of two ways, as described in the theoretical analysis.

The application of either method to calculate the late start and early finish times of the hammock of the network in Figure 2 yields the same duration. Indeed, by the application of the method based on late times the late start time of H is 9 time units since the late finish time of the activity A1 is 8 time units. Therefore, the duration of the hammock based on late times spans from the beginning of period 9 to the end of period 35 i.e. $D_L$ = 27 time units. Consequently, the early finish time of H is determined by adding the duration $D_L$ to the early start time of it which gives 35 time units also. Applying the method based on early times the early finish time of H is calculated based on its successor's early start time, which yields 35. Now, since the early start time of the hammock is 9 time units (already determined uniquely), the duration of it turns out to be $D_E$ = 27 time units as previously. Consequently, the late start time of H becomes again 9 time units by subtracting 27 time periods from the end of period 35.

The flow chart of a software that the reader may wish to develop to handle hammocks as described in the theoretical part is shown in Figure 8. The tabular results of the time analysis as produced by the system are shown in Figure 3. The bar chart obtained from the network in Figure 2 is shown in Figure 4.

B. Hammocks Linked Between Two Ordinary Activities with Unequal Total Floats.

Part of the project shown in Figure 2 is repeated in Figure 5 together with a hammock activity H. The only difference is that this time the predecessor of the hammock activity H is a non critical activity A3 and the successor is a critical activity A7. As in the previous example the early start and late finish times are identified uniquely based on the early finish time of activity A3 and the late start time of activity A7 respectively. Applying the method based on late times, the late start of the hammock activity comes to 33 time units, producing a duration $D_L$ = 3 time units as the difference between 33 and 35. Hence, the early finish time of the hammock can be determined by adding 3 time units to its early start, which yields 21.

Conversely, applying the method based on early times the early finish time of the hammock is 35 time units (based on its successor's early start), which produces a duration of $D_E$ = 17 time units for the hammock. Hence, the late start time of the hammock is compiled and displayed as 19.

The adoption of the results generated from the early dates, endorsed by all three systems in reference indicates that one has to reconsider the late finish time of activity A3 which no longer can be 32, as its succeeding activity H must start at 19 time units in order not to delay the duration of the whole project. If the late start time of activity H is indeed 19 then one has to update the late start and finish times of activity A3, which become 18 and 13 respectively, hence the late finish

and late start times of activity A2 must in turn be updated to 12 and 9 respectively. Consequently, the timing of the project indicates that activities A2 and A3 have become critical. A better illustration of the implications is given in the bar chart of Figure 6. The duration of the hammock based on early dates $D_E = 17$ no longer allows activities A2 to share any total float with activity A4. It is all left to activity A4 which is still non critical.

Alternatively, the adoption of the results based on late dates in this case means that the hammock activity H does not affect the "behaviour" of its predecessors. The hammock simply fills the time span between its predecessor and its successor. The results produced by the system based on the proposed algorithm are tabulated in Figure 7.

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## Captions of Figures

Figure 1 : A hammock activity H linked between two incidental "ordinary" activities $A_1$ and $A_{n+1}$ in a precedence network

Figure 2 : A sample project illustrated in precedence form with a hammock activity linked between ordinary activities with equal total floats

Figure 3 : Tabulation of the time analysis results of the network presented in Figure 2

Figure 4 : Barchart of the network presented in Figure 2

Figure 5 : Part of the sample project illustrated in Figure 2 with a hammock linked between a non-critical predecessor and a critical successor

Figure 6 : Barchart of the network presented in Figure 5

Figure 7 : Tabulation of the time analysis results of the network presented in Figure 5

Figure 8 : Flowchart of a suggested computerized algorithm

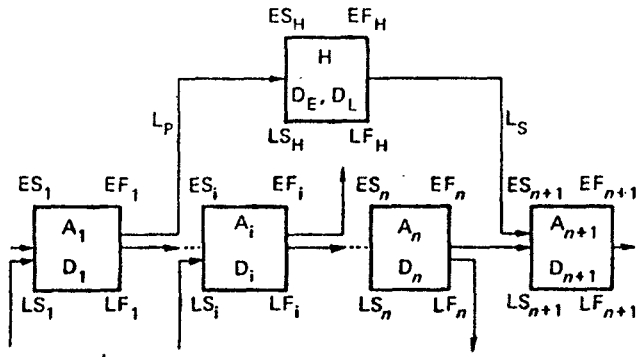



*Figure 2. Hammock activity, H, linked between two incidental 'ordinary' activities, $A_1$ and $A_{n+1}$, in a precedence network*

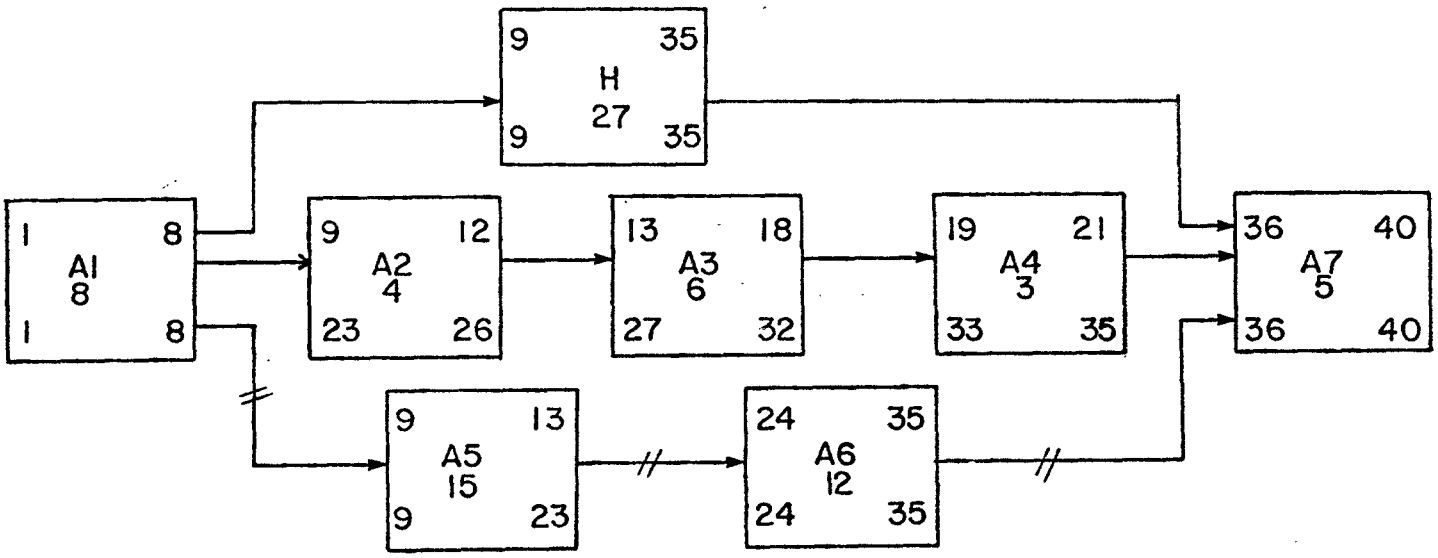
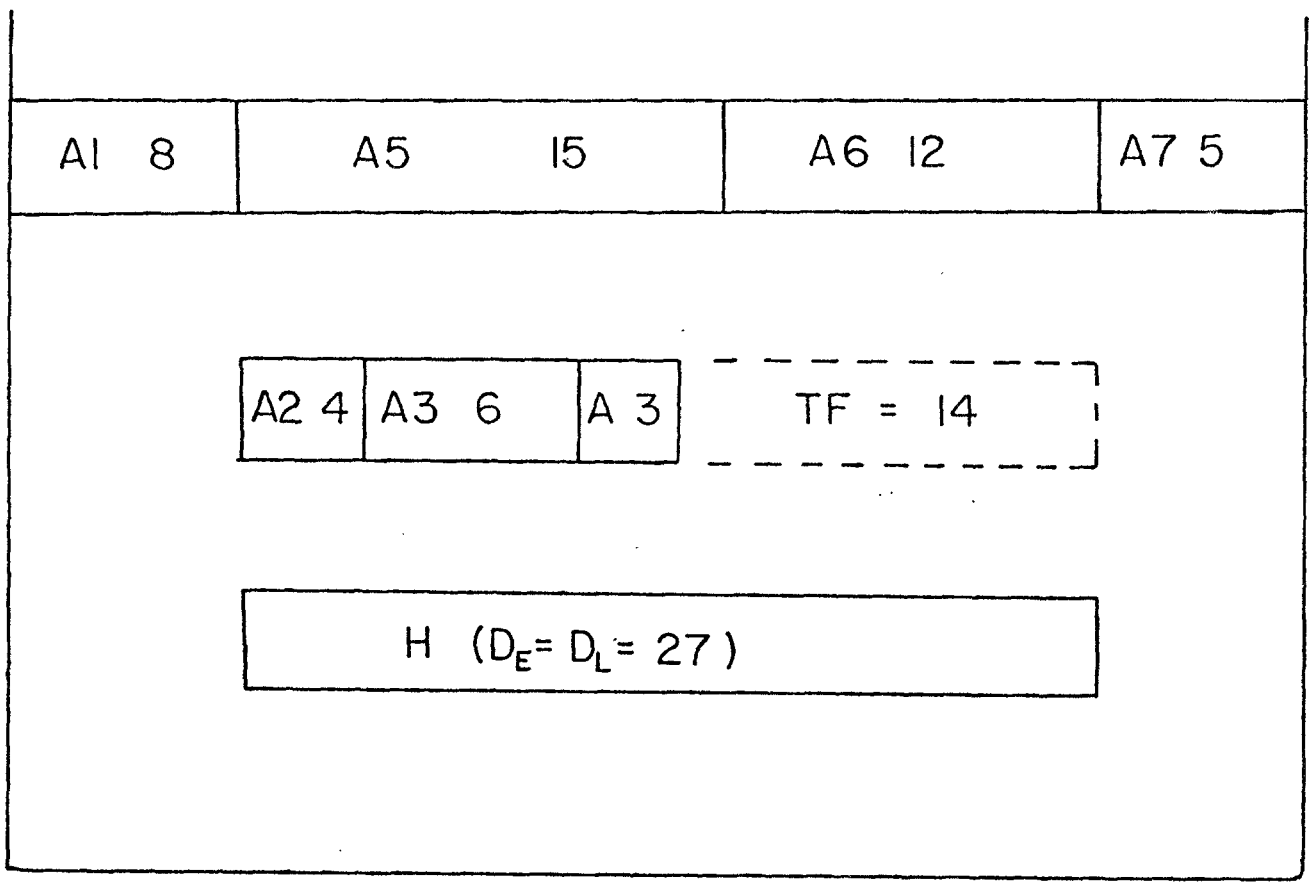



Figure 2. Harhalakis

T A B L E  O F  R E S U L T S (ACTIVITIES)

| ACTI# | DESCRIPTION OF JOBS | DURATION | START | | FINISH | | FLOATS | |
|---|---|---|---|---|---|---|---|---|
| | | | EARL | LATE | EARL | LATE | TOT | FREE |
| 1 | A1 | 8 | 1 | 1 | 8 | 8 | 0 | 0 |
| 2 | A2 | 4 | 9 | 23 | 12 | 26 | 14 | 0 |
| 3 | A3 | 6 | 13 | 27 | 18 | 32 | 14 | 0 |
| 4 | A4 | 3 | 19 | 33 | 21 | 35 | 14 | 14 |
| 5 | HAMM | 27 | 9 | 9 | 35 | 35 | 0 | 0 |
| 6 | A5 | 15 | 9 | 9 | 23 | 23 | 0 | 0 |
| 7 | A6 | 12 | 24 | 24 | 35 | 35 | 0 | 0 |
| 8 | A7 | 5 | 36 | 36 | 40 | 40 | 0 | 0 |

Figure 3 Harhalakis




**Key:**

A1 to A7 = Activity Identities

TF = Total float

$D_E$ = Duration Based on Early Times

$D_L$ = Duration Based on Late Times

Figure 4. Harhalakis

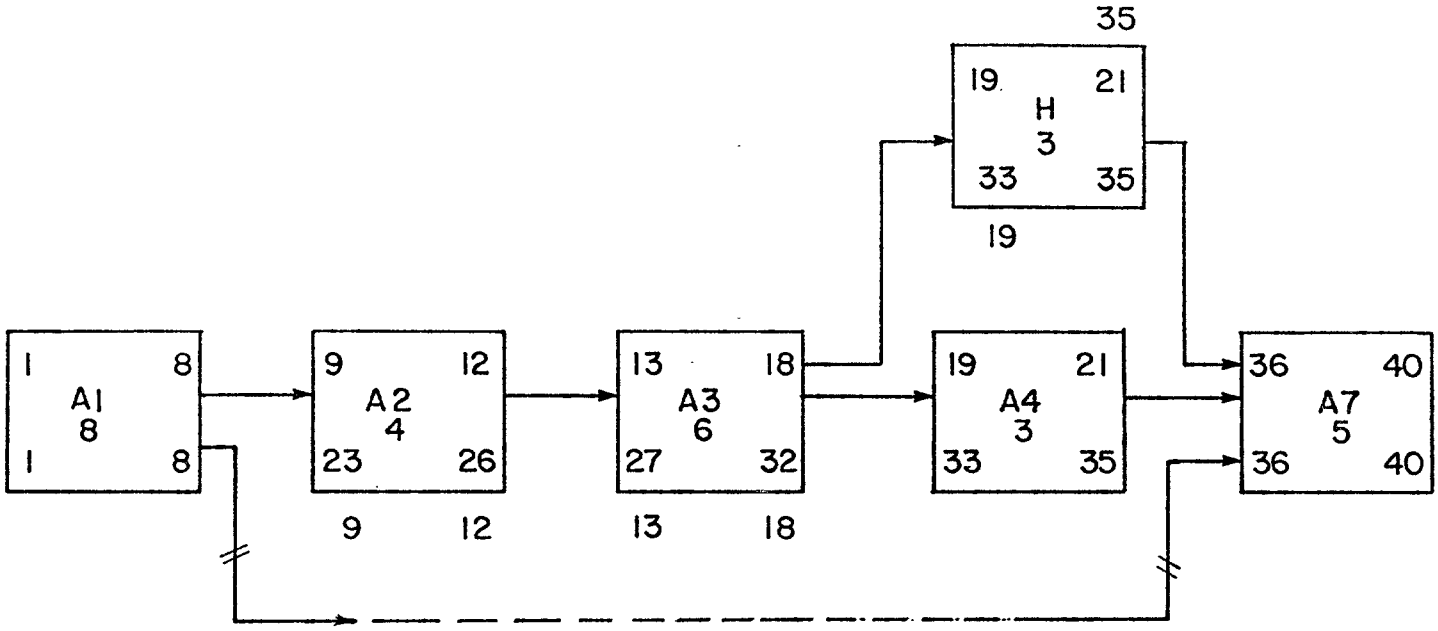
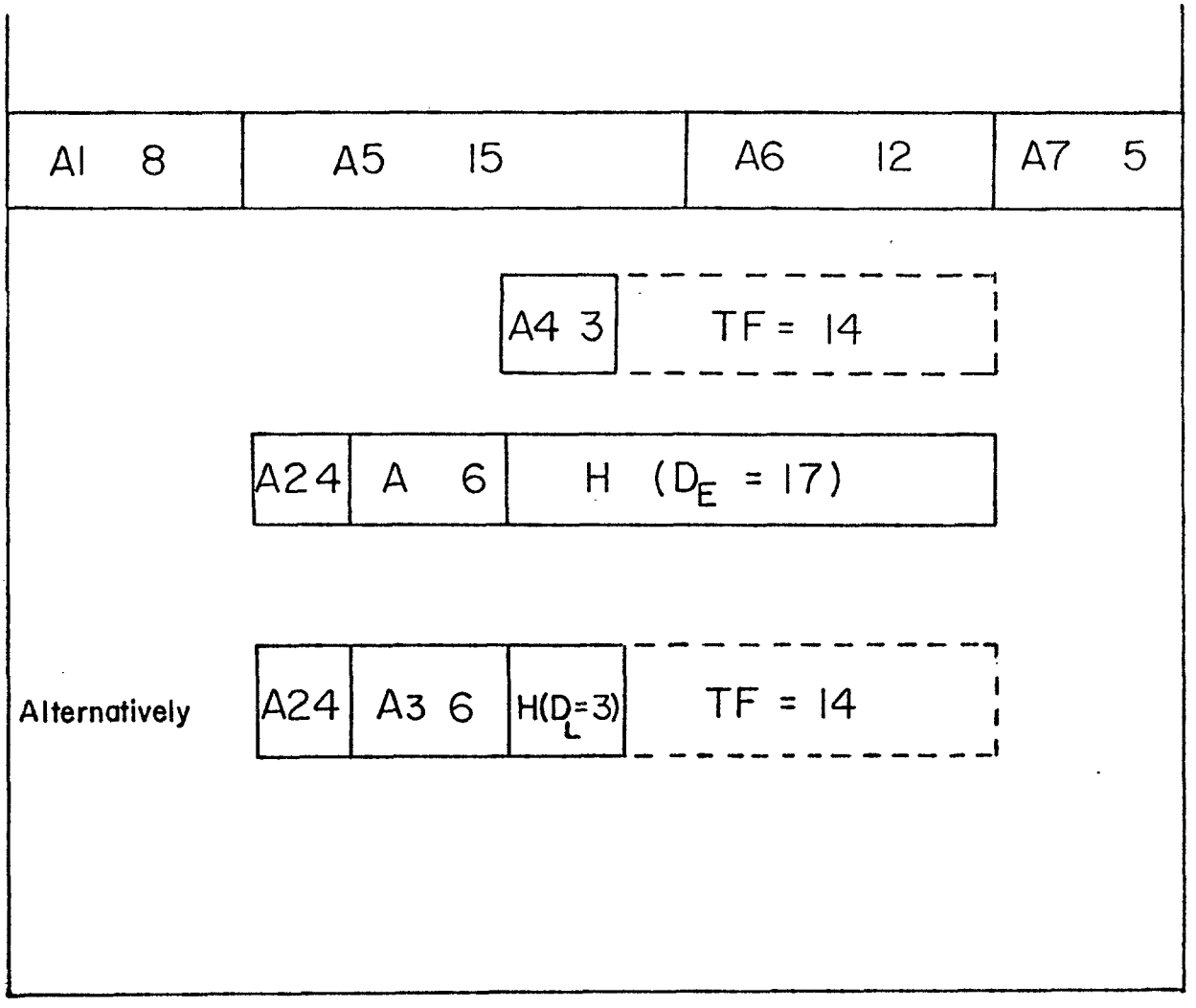



Figure 5. Harhalakis




Key: A1 to A7 = Activity Identities

TF = Total float

$D_E$ = Duration Based on Early Times

$D_L$ = Duration Based on Late Times

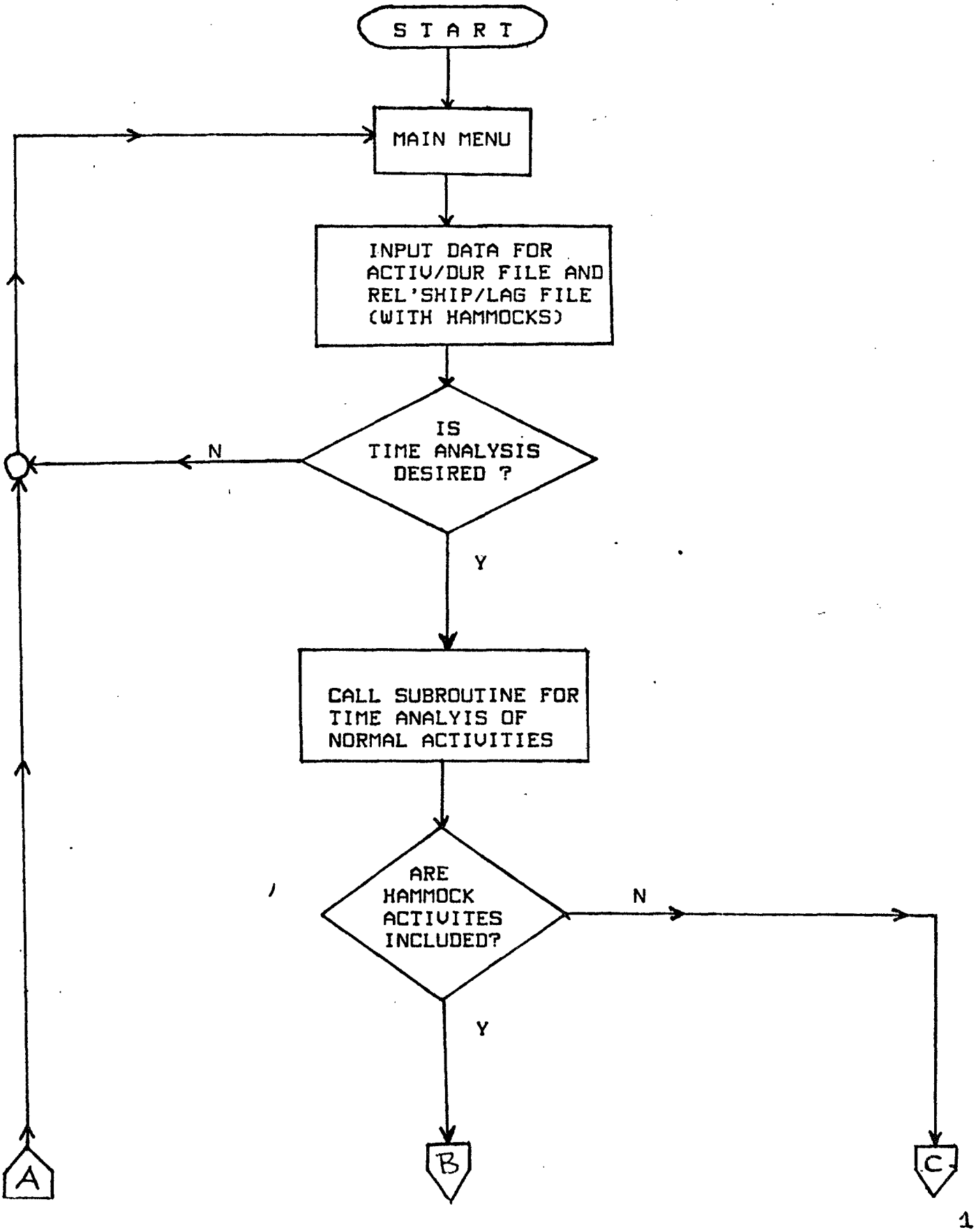
Figure 6. Harbalakis

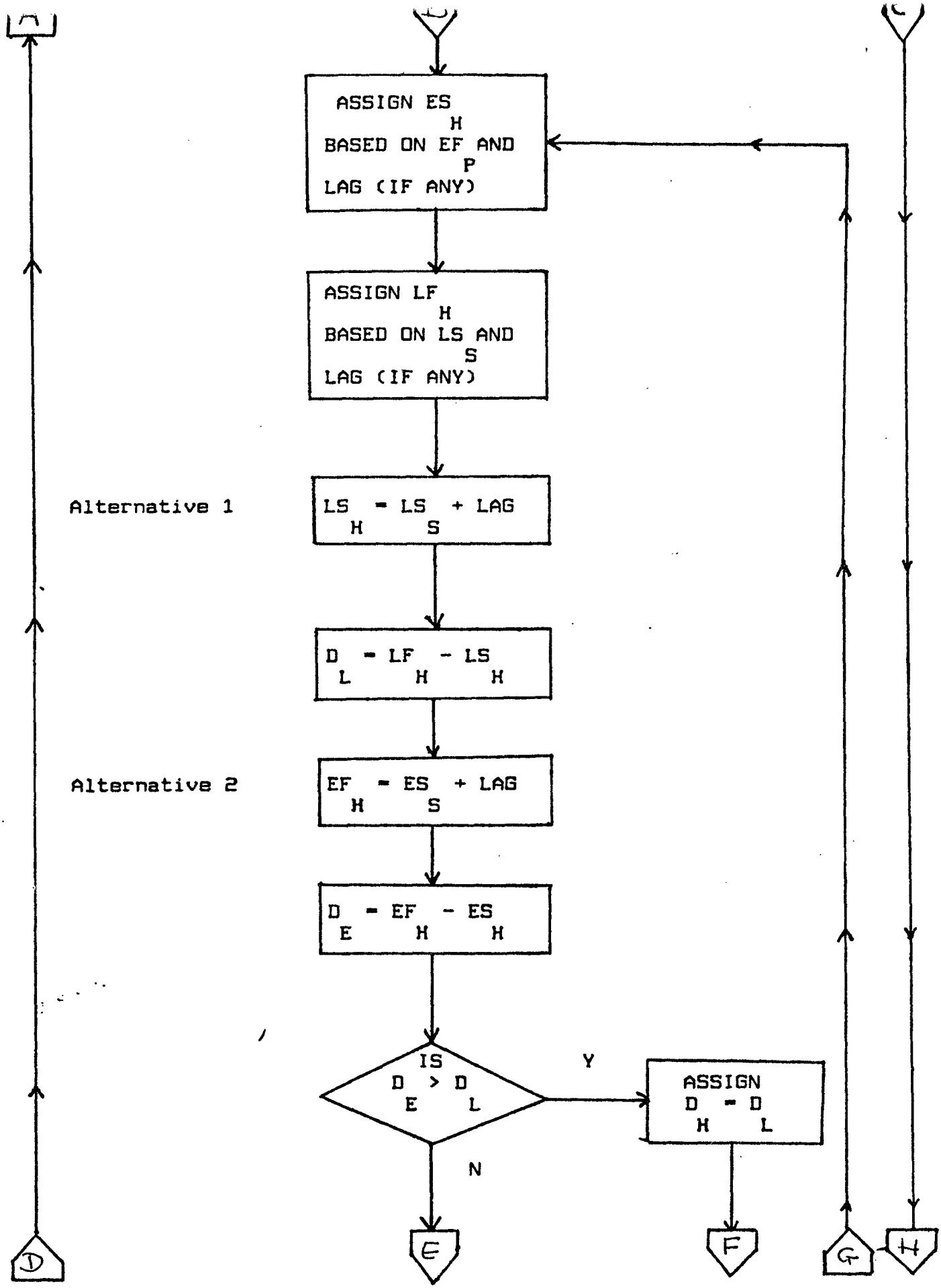
T A B L E  O F  R E S U L T S (ACTIVITIES)

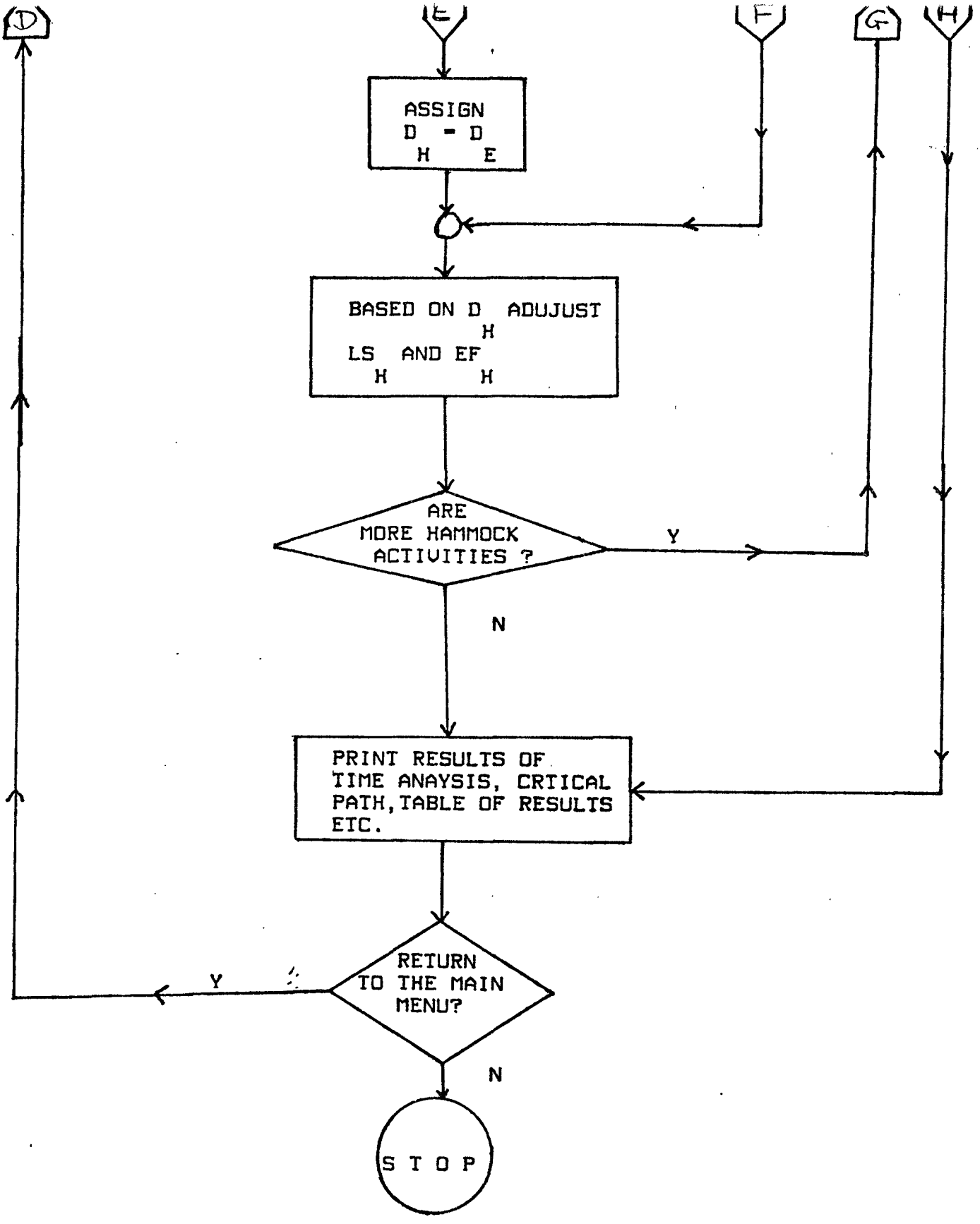
| ACTI# | DESCRIPTION OF JOBS | DURATION | START | | FINISH | | FLOATS | |
|---|---|---|---|---|---|---|---|---|
| | | | EARL | LATE | EARL | LATE | TOT | FREE |
| 1 | A1 | 8 | 1 | 1 | 8 | 8 | 0 | 0 |
| 2 | A2 | 4 | 9 | 23 | 12 | 26 | 14 | 0 |
| 3 | A3 | 6 | 13 | 27 | 18 | 32 | 14 | 0 |
| 4 | A4 | 3 | 19 | 33 | 21 | 35 | 14 | 14 |
| 5 | HAMM | 3 | 19 | 33 | 21 | 35 | 14 | 14 |
| 6 | A5 | 15 | 9 | 9 | 23 | 23 | 0 | 0 |
| 7 | A6 | 12 | 24 | 24 | 35 | 35 | 0 | 0 |
| 8 | A7 | 5 | 36 | 36 | 40 | 40 | 0 | 0 |

Figure 7 Harbenlake is

Figure 8. Flowchart of ~~the~~ suggested computerized algorithm




ASSIGN ES H BASED ON EF AND P LAG (IF ANY)
ASSIGN LF H BASED ON LS AND S LAG (IF ANY)
Alternative 1
LS H = LS S + LAG
D L = LF H - LS H
Alternative 2
EF H = ES S + LAG
D E = EF H - ES H
IS D E > D L
Y
N
ASSIGN D H = D L
D
E
F
G
H


D
E
F
G
H
ASSIGN
D - D
H E
BASED ON D ADUJUST
H
LS AND EF
H H
ARE
MORE HAMMOCK
ACTIVITIES ?
Y
N
PRINT RESULTS OF
TIME ANAYSIS, CRTICAL
PATH, TABLE OF RESULTS
ETC.
RETURN
TO THE MAIN
MENU?
Y
N
S T O P