Technical Research Report

SYSTEMS RESEARCH CENTER

Supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), Industry and the University

Formation of Manufacturing Cells: An Algorithm for Minimizing the Inter-Cell Traffic

by G. Harhalakis, J. Hilger, R. Nagi and J.M. Proth

TR 89-66

FORMATION OF MANUFACTURING CELLS :

AN ALGORITHM FOR MINIMIZING THE INTER-CELL TRAFFIC

G. HARHALAKIS[(1)], J. HILGER[(2)], R. NAGI[(1)], J.M. PROTH[(2)]

(1) Department of Mechanical Engineering and Systems Research Center, University of Maryland, USA.

(2) Projet SAGEP, INRIA-LORRAINE, BP 239, 54506 VANDOEUVRE Cedex, FRANCE.

ABSTRACT :

In this paper, we propose a parametrized algorithm to decompose a manufacturing system into manufacturing cells. The objective is to minimize the inter-cell traffic.

The algorithm is based on a proximity index defined between any two machines and which is conversely proportional to the intensity of the traffic between these machines. We compute a density for each machine. This density is defined as the number of machines close to the considered machine in the sense of the previous index. We then group into cells the machines which are in the same high density domains. We finally associate a family of parts to each of the previous cells.

A numerical example illustrates this approach.

KEYWORDS : Group Technology, Manufacturing Cells, Part Families, Cross-Decomposition, Traffic Minimization, Layout.

1. INTRODUCTION

We consider a job-shop [illegible]posed of machines and the set of part types this job-shop manufactures. A wei[illegible] is associated to each part type. It represents the proportion of parts of this type which have to be manufactured. Many works have been done to group the machines into cells and the parts into part families in order to improve the production management by splitting scheduling problems into smaller scheduling sub-problems or to reduce the production cost by designing an effective production system layout. Some examples of these improvements can be found in particular in [1], [4], [5], [6], [7], [9], [10], [11], [12].

The problem of partitioning the set of machines into a set of cells and, simultaneously, the set of parts into a set of part families is known as a cross-decomposition process. Various approaches, involving more or less human expertise, are available in particular in [2], [3], [8]. Some of them are only supports to human decision and do not take into account criteria. Amongst the approaches which aim to optimize some quantitative criteria, let us refer in particular to [2] and [3]. In these papers, authors aim to solve the cross-decomposition problem and to establish a one-to-one relationship between the set of cells and the set of part families in order to both maximize the number of operations performed on the parts inside the related cell and to minimize the number of operations performed on the parts outside the related cell. The goal of this approach is not to minimize the inter-cell traffic, but usually leads to a quite small value of this parameter, depending on the number of cells obtained.

Approaches which try to directly minimize the inter-cell traffic seem to be more convenient. In particular, [10] proposes a twofold heuristic algorithm which works as follows : the first step is a bottom-up aggregation procedure which minimizes the inter-cell traffic while the second step is an improvement procedure in which the significance of a machine to a cell is validated.

The following algorithm also tries to minimize the inter-cell traffic, but uses a totally different procedure to reach this goal.

The paper is organized as follows. In section 2, we provide the definitions. Section 3 is devoted to the presentation of the cell formation algorithm and to the discussion concerning the expected results and the convergence. Section 4 provides a second algorithm to compute the part families. This algorithm is based on the machine cells obtained just before. In section 5, we remember some criteria usually used to evaluate the cross-decomposition. Section 5 proposes some numerical examples which are discussed.

2. DEFINITIONS

We call $\mathcal{M} = \{M_1, M_2, ..., M_m\}$ the set of machines which compose the job-shop and $\mathcal{P} = \{P_1, ..., P_p\}$ the set of part types which have to be manufactured using the job-shop. α_i, $i = 1, ..., p$, is the proportion of parts type P_i to be manufactured. Thus, $\alpha_1 + \alpha_2 + ... + \alpha_p = 1$. Each part type P_i is specified by its manufacturing process, which is the sequence of machines each part has to visit to be completed along with the time it spends on each machine of the sequence. Because times are useless in this approach, we restrict the definition of the manufacturing process of P_i to the sequence of machines it has to visit to be completed :

$$G(P_i) = \{M^i_1, M^i_2, ..., M^i_{k_i}\}$$

where

$$M^i_s \in \mathcal{M} \text{ for } s = 1, 2, ..., k_i$$

The traffic between $M_s \in \mathcal{M}$ and $M_r \in \mathcal{M}$ is defined as follows :

$$t(M_s, M_r) = \sum_{i=1}^{p} \alpha_i n_i \tag{1}$$

where n_i is the number of times the sub-sequences (M_s, M_r) or (M_r, M_s) are included in $G(P_i)$. We assume that :

$$t(M_s, M_s) = +\infty \quad \text{for } s = 1, ..., p$$

To illustrate this definition, we consider figure 1 which represents a job-shop composed of three machines and able to manufacture three types of parts P_1, P_2 and P_3 respectively in proportions 30 %, 20 % and 50 %.

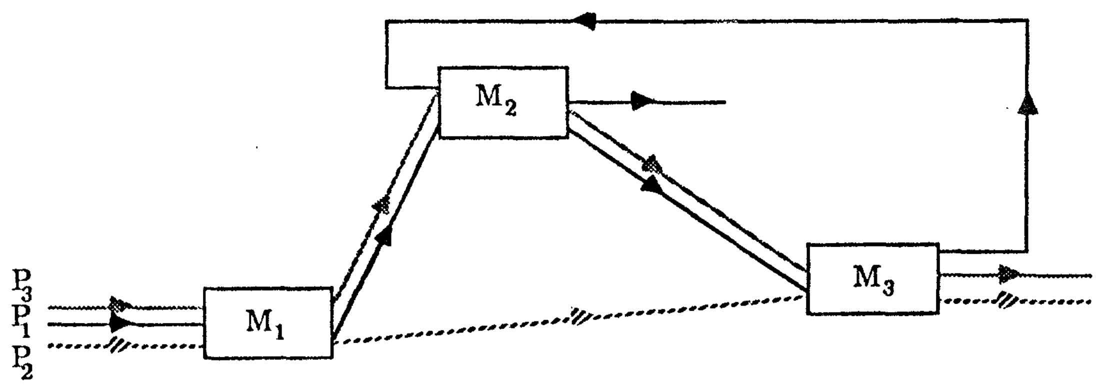

Figure 1 : An example of inter-machine traffic

Applying relation (1), we obtain :

$$t(M_1, M_2) = 0.3 \times 1 + 0.5 \times 1 = 0.8$$
$$t(M_1, M_3) = 0.2 \times 1$$
$$t(M_2, M_3) = 0.3 \times 2 + 0.5 \times 1 = 1.1$$

Observe that traffic is a symetric measure, i.e. :

$$t(M_1, M_2) = t(M_2, M_1)$$
$$t(M_1, M_3) = t(M_3, M_1)$$
$$t(M_2, M_3) = t(M_3, M_2)$$

Starting from the traffic between $M_s \in \mathcal{M}$ and $M_r \in \mathcal{M}$, we define the proximity index between these machines as :

$$I(M_s, M_r) = 1 \;/\; [1 + t(M_s, M_r)] \qquad (2)$$

Note that :

a. $I(M_s, M_r) = 1$ if $t(M_s, M_r) = 0$: the maximal value of the proximity index is obtained when there is no traffic between the considered machines.

b. $I(M_s, M_r) \to 0$ if $t(M_s, M_r) \to +\infty$: the higher the traffic, the closer the machines in the sense of the proximity index **which is not a distance.**

In particular :

$I(M_s, M_s) = 0$ for $s = 1, \ldots, p$

We finally introduce a density δ_η assigned to each machine of $\mathcal{M}$. It is defined as follows :

> Let $M^* \in \mathcal{M}$. The density of M^*, called $\delta_\eta(M^*)$, is the number of machines $M \in \mathcal{M}$ such that $I(M, M^*) \leq \eta$. $\eta > 0$ is a parameter which value is provided by the users. (3)

Because $I(M^*, M^*) = 0$ for $M^* \in \mathcal{M}$, $\delta_\eta(M^*) \geq 1$.

3. THE ALGORITHM

This section is devoted to the presentation of the algorithm, to the proof of its convergence, and to some remarks about the parameters controlling the algorithm.

3.1. Presentation of the algorithm

We have to define a supplementary integer parameter h, called **partitioning parameter**. The definition of the cells of machines is based on this parameter. The value of h lies between the maximal and the minimal values of the densities, i.e. :

$$\min_{M \in \mathcal{M}} \delta_\eta (M) \le h \le \max_{M \in \mathcal{M}} \delta_\eta (M)$$

Starting from the previous definitions, the algorithm used to define the cells is the following :

1. Introduce η.
2. Compute δ_η (M) for all $M \in \mathcal{M}$ (see relations (1), (2) and (3)).
3. Compute :

$$\min = \min_{M \in \mathcal{M}} \delta_\eta (M)$$

$$\max = \max_{M \in \mathcal{M}} \delta_\eta (M)$$

4. Choose $h \in$ [min, max].
5. Compute $R = \{ M / M \in \mathcal{M} \text{ and } \delta_\eta (M) > h \}$.
6. Set r = 0.
7. While $R \neq \emptyset$ do
    - 7.1. r = r + 1.
    - 7.2. $M_M = \max_{M \in R} \delta_\eta (M)$
    - 7.3. $R = R \setminus \{M_M\}$.
    - 7.4. $C_r = \{M_M\}$.
    - 7.5. $E(M_M) = \{ M / M \in R \text{ and } I(M, M_M) \le \eta \}$.
    - 7.6. While $E(M_M) \neq \emptyset$ do
        - 7.6.1. Choose $M_0 \in E(M_M)$ at random.
        - 7.6.2. $R = R \setminus \{M_0\}$.
        - 7.6.3. $C_r = C_r \cup \{M_0\}$.
        - 7.6.4. $E(M_0) = \{ M / M \in R \text{ and } I(M, M_0) \le \eta \}$.
        - 7.6.5. $E(M_M) = (E(M_M) \cup E(M_0)) \setminus \{M_0\}$.

8. Done.

As outputs of the algorithm we obtain :

- the number of machine cells which is the value of r,
- the machine cells C_i, $i = 1, 2, ..., r$,
- the set H of machines which do not belong to a cell, i.e.

$$H = \mathcal{M} \setminus \bigcup_{i=1}^{r} C_i$$

3.2. Convergence

The following result holds.

RESULT :

The algorithm presented in section 3.1. converges.

PROOF :

a. At each step of the loop controlled by $E(M_M)$ (innerloop), card (R) decreases from one unit. Furthermore, $E(M_M)$ is always included in R. As a consequence, $E(M_M)$ becomes empty after a finite number of steps and the innerloop ends.

b. We now consider the loop controlled by R (external loop). At each step, card (R) decreases at least from one unit. Then, R becomes empty after a finite number of steps and the external loop ends after a finite number of steps.

E.O.Q.

3.3. Remarks

The result obtained by applying the previous algorithm strongly depends on parameters η and h.

When h increases the set of machines which do not belong to a cell increase. The number of cells tends to increase with h. If h is equal to its maximal value, all the machines remain unclassified. If h is equal to its minimal value, all the machines, except the ones with the minimal density, are grouped into cells.

The choice of η is determinant for the quality of the solution.

If η is less than $\min_{M_s, M_r \in \mathcal{M}} I(M_s, M_r)$ where $r \neq s$, then $\delta_\eta(M) = 1$ whatever $M \in \mathcal{M}$ may be.

In that case, h = min = max and all the machines remain unclassified.

If $\eta > \max_{M_s, M_r \in \mathcal{M}} I(M_s, M_r)$, then $\delta_\eta(M) = \text{card}(\mathcal{M})$ whatever $M \in \mathcal{M}$ may be.

In that case, the result is the same than the previous one. Thus η has to be carefully chosen between the minimal and the maximal value of the index.

4. COMPUTATION OF PART FAMILIES

At this point of the process, we have some machine cells $C_1, \ldots, C_r$ as well as a set of unclassified machines called R. In the following, R will be considered as the $(r+1)$-th cell and denoted C_{r+1}. We want to find $r+1$ part families $D_1, D_2, \ldots, D_{r+1}$ and a one-to-one relationship between the set of cells and the set of part families such that the number of operations performed inside C_i on the parts of D_i is maximal and the operations performed outside C_i on the same part is minimal.

Let $A = [a_{ij}]$, $i = 1, \ldots, m$; $j = 1, \ldots, p$ a matrix such that a_{ij} is the number of operations performed on a P_j-type part using M_i. We assume that A is normalized, i.e. all the elements of A have been divided by the greatest of them. Thus $0 \leq a_{ij} \leq 1$ for $i = 1, \ldots, m$; $j = 1, \ldots, p$.

For each part type P_j, we compute, for $k = 1, 2, \ldots, r+1$

$$W_k(P_j) = \lambda \sum_{i \in C_k} b_{ij} + (1-\lambda) \sum_{i \notin C_k} (1 - b_{ij}) \tag{4}$$

where $\lambda \in [0, 1]$ is chosen by the user.

Let :

$$W_{k^*}(P_j) = \operatorname*{Max}_{k=1,\ldots,r+1} W_k(P_j)$$

We assign the part type P_j to D_{k^*}.

At the end of the process, it is possible to find that some of the part families $D_1, D_2, \ldots, D_{r+1}$ are empty. This kind of approach has already been used in particular in [2] and [3].

5. EVALUATION

The cross-decomposition approach presented in this paper is twofold. It first leads to machine cells, and this first stage is based on the minimization of the inter-cell traffic. Starting from the set of machine cells, the second stage provides part families by minimizing the sum of the number of operations performed on the part types inside their related cell and the number of machines belonging to their non-related cells and which are not used to manufacture the part types. Thus, two different criteria are used successively, and we need some global criteria to evaluate the cross-decomposition.

We use the three global criteria proposed in [10] :

a. Global Efficiency (GL.E.)

It is the ratio of the total number of operations that are performed within related cells to the global number of operations in the system.

According to the previous notations :

$$GL.E. = \sum_{(i,j) \in E} a_{ij} \Big/ \sum_{i=1}^{m} \sum_{j=1}^{p} a_{ij}$$

where $E = \{ (i, j) \ / \ M_i \in C_k, P_j \in D_k \text{ for } k \in (1, 2, \dots r+1) \}$

The closer GL.E. is to 1, the better the result.

b. Group Efficiency (GR.E.)

It is the ratio of :

- the difference between the maximal number of non-related cells that could be visited and the total number of non-related cells that are actually visited by the parts ;

and

- the maximal number of non-related cells that could be visited.

The closer GR.E. is to 1, the better the result.

c. Group Technology Efficiency (G.T.E.)

It is the ratio of:

- the difference between the maximal amount of inter-cells traffic and the amount of actual inter-cell traffics ;

and

- the maximal amount of inter-cell traffic.

If k_j is the length of a P_j type part manufacturing process, the maximal amount of inter-cell traffic is $k_j - 1$.

The closer G.T.E. to 1, the better the result.

Remark :

It is possible to generalize these criteria by taking into account some weights attached to the part types. For instance, the weight attached to P_j could be the ratio of P_j type parts we have to manufacture on the average.

These criteria will be used in the numerical example presented in the next

6. AN EXAMPLE

To illustrate the previous approach, we use the same example as the one introduced in [10].

The manufacturing processes are given in table 1.

PARTS \ MACHINES	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
01	2	.	.	.	.	.	.	.	3	.	.	1	.	.	.	.	.	4	.	5
02	.	3	2	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
03	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	3	2
04	.	3	1	.	.	.	.	.	.	4	2	.	.	.	.	.	.	.	.	.
05	.	.	.	1	.	3	4	.	.	.	.	.	.	.	2	.	.	.	.	.
06	.	.	.	.	5	.	.	.	.	.	1	.	.	2	.	3	4	.	.	.
07	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	2	3	.	.	.
08	.	.	.	5	.	.	3	.	4	.	.	.	2	.	1	.	.	.	.	.
09	4	.	.	.	.	.	.	.	2	.	3	5	.	.	.	.	.	1	.	.
10	.	.	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.	.	1	2
11	.	.	3	.	.	.	.	.	.	.	1	.	.	2	.	.	.	.	.	.
12	5	.	.	.	3	.	.	.	1	.	.	4	.	.	.	.	.	2	.	.
13	.	.	.	.	.	1	2	.	.	.	.	.	.	.	3	.	4	.	.	.
14	3	4	.	.	.	.	.	1	.	2	.	.	.	.	.	.	.	.	.	.
15	.	.	.	.	.	.	.	.	.	.	.	.	1	2	.	3	4	.	.	.
16	.	.	.	.	.	3	2	.	.	.	.	.	.	.	1	.	.	.	4	.
17	2	.	.	.	.	.	.	.	1	.	.	3	.	.	.	.	.	.	.	.
18	.	.	.	.	.	.	.	1	.	4	.	.	.	.	.	.	.	.	2	3
19	.	2	1	.	4	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.
20	3	.	.	.	.	.	.	.	.	2	.	4	.	.	.	.	.	1	.	.

Table 1 : Manufacturing Processes

All the products have the same weight 0.05.

The values of the parameters we choosed are the following :

$\lambda = 0.7$, $\eta = 0.95$ and $h = 1$

The result is given in table 2.

MACHINES

	06	07	15	01	02	03	09	10	11	12	14	16	17	18	08	19	20	04	05	13
05	3	4	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.
13	1	2	3	.	.	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.
16	3	2	1	.	.	.	.	.	.	.	.	.	.	.	.	4	.	.	.	.
01	.	.	.	2	.	.	3	.	.	1	.	.	.	4	.	.	5	.	.	.
02	.	.	.	.	3	2	.	.	1	.	.	.	.	.	.	.	.	.	.	.
04	.	.	.	.	3	1	.	4	2	.	.	.	.	.	.	.	.	.	.	.
P 06	.	.	.	.	.	.	.	.	1	.	2	3	4	.	.	.	.	.	5	.
A 09	.	.	.	4	.	.	2	.	3	5	.	.	.	1	.	.	.	.	.	.
R 11	.	.	.	.	.	3	.	.	1	.	2	.	.	.	.	.	.	.	.	.
T 12	.	.	.	5	.	.	1	.	.	4	.	.	.	2	.	.	.	.	3	.
S 17	.	.	.	2	.	.	1	.	3	.	.	.	.	.	.	.	.	.	.	.
20	.	.	.	3	.	.	.	2	.	4	.	.	.	1	.	.	.	.	.	.
03	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	3	2	.	.	.
10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	1	2	.	.	.
14	.	.	.	3	4	.	.	2	.	.	.	.	.	.	1	.	.	.	.	.
18	.	.	.	.	.	.	.	4	.	.	.	.	.	.	1	2	3	.	.	.
07	.	.	.	.	.	.	.	.	.	.	.	2	3	.	.	.	.	.	1	.
08	.	3	1	.	.	.	4	.	.	.	.	.	.	.	.	.	.	5	.	2
15	.	.	.	.	.	.	.	.	.	.	2	3	4	.	.	.	.	.	.	1
19	.	.	.	.	2	1	.	.	3	.	.	.	.	;	.	.	.	.	4	.

Table 2 : Result with $\eta = 0.95$, $h = 1$ and $\lambda = 0.7$

The values of the global criteria are the following :

GL.E. $\simeq 0.74$

GR.E. $\simeq 0.78$

G.T.E. $\simeq 0.73$

Remember that, in this example, $C_4 = \{M_4, M_5, M_{13}\}$ is he set of unclassified machines. Consequently, $D_4 = \{P_7, P_8, P_{15}, P_{19}\}$ is he set of unclassified product types. If we restrict ourselves to $\{C_1, C_2, C_3\}$ and $\{D_1, D_2, D_3\}$ when computing the global criteria, we obtain the following values :

GL.E. $\simeq$ 0.88

GR.E. $\simeq$ 0.84

G.T.E. $\simeq$ 0.90

It is possible to obtain better results by modifying the values of the parameters, and in particular by increasing the value of λ.

7. CONCLUSION

The main objective of the previous algorithm is to minimize the inter-cell traffic, and the definition of the part families are based on the cells obtained as a result of the first stage.

This algorithm can be improved in three directions. The first one consists in giving the possibility of limiting the number of machines in the cells and the number of part types in the part families. The second one consists of assigning the unclassified machines to the machine cells after the end of the first stage (i.e. after obtaining the cells). Finally, the last direction could be the introduction of an initial stage consisting of defining the machines and the part types which can be eliminate "a priori" from the computation. It is for instance the case of the machines which are used together by a wide percentage of part types. It is also the case of the parts types which have to pass through a great quantity of machines in order to be manufactured. There is no difficulty to introduce these changes in the computer program which apply the heuristic algorithm presented in this paper.

REFERENCES

[1] ASKIN R. and SUBRAMANIAN S.B.,
"A Cost-Based Heuristic for Group Technology Configuration", International Journal of Production Research, 25, 1, pp. 101-113, 1987.

[2] GARCIA H. and PROTH J.M.,
"A New Cross-Decomposition Algorithm : the GPM. Comparison with the Bond Energy Method", Control and Cybernetics, n° 2, vol. 15, pp. 115-165, 1986.

[3] GARCIA H. and PROTH J.M.,
"Group Technology in Production Mangement : the Short Horizon Planning Level", Applied Stochastic Models and Data Analysis, n° 1, pp. 25-34, 1985.

[4] KING J.R.,
"Machine-Component Group Formation in Group Technology", OMEGA The International Journal of Management Science, 8, 2, pp. 193-199, 1979.

[5] KUMAR R.K., KUSIAK A. and VANNELLI A.,
"Grouping of Parts and Components in Flexible Manufacturing Systems", European Journal of Operations Research, 24, pp. 387-397, 1986.

[6] KUSIAK A.,
"The Part Families Problem in Flexible Manufacturing Systems", Annals of Operations Research, 3, pp. 279-300, 19855.

[7] Mc AULEY J.,
"Machine Grouping for Efficient Production", The Production Engineer, pp. 53-57, Feb. 1972.

[8] Mc CORMICK W.T., SCHWEITZER P.J. and WHITE T.E.,
"Problem Decomposition and Data Reorganization by a Cluster Technique", Operations Research, 20, 1972.

[9] HARHALAKIS G., HILLION H.P. and PROTH J.M.,
"Preliminary Design of Manufacturing Systems", proposed for publication ~~to~~ the International Journal of Operations Research.

[10] HARHALAKIS G., NAGI R. and PROTH J.M.,
"An Efficient Heuristic in Manufacturing Cell Formation for Group Technology Applications", proposed for publication to the Journal of ~~Engineering Costs and Production Economics~~. Production Research.

[11] PORTMANN M.C. and PROTH J.M.,
"Spatial and Temporal Decomposition Methods in Production Management", International Conference on Computer Integrated Manufacturing, CMP/CIM Computer Society of theIEEE, May 23-25, 1988.

[12] PORTMANN M.C. and PROTH J.M.,
"A Cross-Decomposition Method for Layout Systems and Scheduling Problem", Third International Conference on CAD/CAM, Robotics and Factories of the Future, CARS and FOF Conference and Exhibits, August 14-17, 1988, Southfield, Mich.