

## ABSTRACT

Title of dissertation:      THE ROLE OF INFORMATION  
IN PRIVATE VALUE AUCTIONS

Svetlana Pivovarova  
Doctor of Philosophy, 2019

Dissertation directed by:   Professor Andrew Sweeting  
Department of Economics

This dissertation studies the role that the information available to the participants of private value procurement auctions prior to the auction has on the equilibrium auction outcomes. Chapters 1 and 2 present two different models in which the private type of one of the participants is persistent over time and can be informative to her competitors. Chapter 3 looks at the effects of a policy change making the information about upcoming procurement auctions more easily available has on the entry of different types of firms in the auctions.

In Chapter 1 I build and estimate a model of repeated asymmetric first price auction in which one of the bidders has a persistent private type, all bidders are backward looking, and all bids are made public after the auction. In particular, I show that the standard model without a binding reserve price misestimates expected procurement costs by 2-14% compared to my model, and withholding past bid information from auction participants can reduce expected procurement costs by up to 11%. These results are relevant for the estimation of both US highway

procurement auctions since all of the states' Departments of Transportation publish full auction results online.

In Chapter 2 I look at a theoretical model of repeated asymmetric first price auction in which all bids are made public between the auctions, one of the bidders has a persistent private type and is forward-looking. I show that a strictly monotonic equilibrium would not exist in this game, and provide an example of a partially pooling equilibrium in which the bidder with persistent type forgoes profits in the earlier period to withhold the information from her competitors in future periods.

Chapter 3 studies the effect that the changes in public procurement rules in Russia had on participation and bidding in regional gasoline procurement auctions. In particular, I look at the difference in changes of entry and bidding patterns for large and small firms after the information about upcoming auctions became more easily available in Jan 2011. I show that the larger firms who have stations both outside and inside of the studied region enter more auctions and bid more aggressively, while local firms who only have stations inside the region do not change entry patterns and bid less aggressively. I associate these changes to the differential changes in entry costs for the different types of firms, and confirm this intuition by comparing the structural estimates of entry costs between firm types and time periods.

THE ROLE OF INFORMATION  
IN PRIVATE VALUE AUCTIONS

by

Svetlana Pivovarova

Dissertation submitted to the Faculty of the Graduate School of the  
University of Maryland, College Park in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
2019

Advisory Committee:  
Professor Andrew Sweeting, Chair  
Professor Daniel Vincent  
Professor Emel Filiz Ozbay  
Professor Guido Kuersteiner  
Professor Lori Lynch

© Copyright by  
Svetlana Pivovarova  
2019



## Acknowledgments

I would like to express my deepest gratitude to all the people who made this dissertation possible.

First and foremost, my advisor Andrew Sweeting for the support and guidance all the way through the process. I am also grateful to Daniel Vincent for valuable theoretical insights, and the members of my dissertation committee Emel Filiz Ozbay, Guido Kuersteiner, and Lori Lynch for their time and useful comments.

I would not have completed this dissertation without the support of my economist friends and non-economist friends. I would like to thank Ekaterina Khmelnitskaya and Irina Kirysheva for the emotional support, numerous proofreads, and informative conversations. I have also been supported by two dissertation writing groups and would like to thank all of the participants who helped me to keep going.

# Table of Contents

Acknowledgements	ii
List of Tables	v
List of Figures	vi
1 Autocorrelated Costs and Information in Repeated First Price Procurement Auctions	1
1.1 Introduction	1
1.2 Data and institutional environment	8
1.3 Model	17
1.3.1 Setup	17
1.3.2 Optimal bidding and equilibrium	18
1.3.3 Identification	20
1.4 Structural estimation	21
1.4.1 Parametric estimation of bid distributions	22
1.4.2 Estimation of cost distributions	27
1.4.3 Bid function example	32
1.5 Counterfactual analysis	40
1.5.1 Solving for the counterfactual equilibrium and the optimal reserve price	41
1.5.2 The standard model	45
1.5.3 The public signal model	46
1.5.4 The naïve auctioneer model	48
1.5.5 The informed auctioneer model	48
1.6 Conclusions	50
1.7 Appendix	52
1.7.1 Sample selection and variable construction	52
1.7.2 Construction of the bid residual	52
1.7.3 Reduced form regressions with average bid as a signal	53
1.7.4 Gamma-Weibul log-likelihood function	54
1.7.5 Numerical integration details	55
1.7.6 Typical auction documentation published before and after the auction	55

2	Partial pooling in repeated auctions with autocorrelated costs	58
2.1	Introduction	58
2.2	Model	60
2.2.1	Second period solution	61
2.2.2	First period problem	63
2.3	Example: Beta distribution family	67
2.4	Partially pooling solution for the Beta distribution family	75
2.5	Conclusions	81
3	The role of information in repeated procurement auctions: monitoring and entry	83
3.1	Introduction	83
3.2	Data and institutional environment	89
3.2.1	North subsample	93
3.3	Reduced form strategy and results	96
3.4	Estimation of entry costs	102
3.5	Conclusions	107
3.6	Appendix	109
3.6.1	Choice of subsample	109
3.6.2	Summary statistics	110
3.6.3	Consistency of demand over time and municipalities	113
3.6.4	Robustness to potential biases in data	115
	Bibliography	117



## List of Tables

1.1	Auction summary statistics: Bridge auctions . . . . .	13
1.2	Auction summary statistics: Pavement auctions . . . . .	13
1.3	Bridge projects with bid residual signal . . . . .	15
1.4	Pavement projects with bid residual signal . . . . .	15
1.5	Bridge projects with market level bid residual signal . . . . .	16
1.6	Pavement projects with market level bid residual signal . . . . .	16
1.7	Gamma-Weibull estimates: Bridge projects . . . . .	23
1.8	Gamma-Weibull estimates: Pavement projects . . . . .	24
1.9	Cost distribution estimates under different information regimes . . . .	41
1.10	Counterfactual estimation results for the Standard model, $n_f = 4$ . .	45
1.11	Counterfactual estimation results for the Public signal model, $n_f = 4$	47
1.12	Counterfactual estimation results for the Naïve auctioneer model, $n_f$ = 4 . . . . .	49
1.13	Counterfactual estimation results for the Informed auctioneer model, $n_f = 4$ . . . . .	49
1.14	Construction of the bid residual . . . . .	53
1.15	Bridge projects with average bid signal . . . . .	53
1.16	Pavement projects with average bid signal . . . . .	54
3.1	Wining bid and winning rebate naive regressions . . . . .	97
3.2	Possibility of entry and entry by small and big firms . . . . .	98
3.3	All bids . . . . .	100
3.4	Probability of entry . . . . .	101
3.5	Probability of winning conditional on entry . . . . .	102
3.6	Estimated entry costs for large and small firms . . . . .	107
3.7	Summary statistics: North subsample . . . . .	110
3.8	Summary statistics: North subsample before Jan 1, 2011 . . . . .	111
3.9	Summary statistics: North subsample after Jan 1, 2011 . . . . .	111
3.10	Summary statistics by number of participating bidders . . . . .	112
3.11	Without municipalities with partial data . . . . .	115
3.12	Without municipalities with partial data and “federal” firms . . . .	116

## List of Figures

1.1	Estimated conditional bid distribution for regular bidders in the bridge market . . . . .	26
1.2	Estimated conditional bid distribution for fringe bidders in the bridge market . . . . .	26
1.3	Estimated conditional bid distribution for regular bidders in the pavement market . . . . .	27
1.4	Estimated conditional bid distribution for fringe bidders in the pavement market . . . . .	27
1.5	Marginal pdf's of regular bidder costs for bridge projects (left) and pavement projects (right) . . . . .	29
1.6	Marginal pdf's of fringe bidder costs for bridge projects (left) and pavement projects (right) . . . . .	29
1.7	Conditional and marginal pdf's of regular bidder costs . . . . .	29
1.8	Conditional distribution of regular bidder costs in the pavement market	30
1.9	Conditional distribution of regular bidder costs in the pavement market	31
1.10	Bid functions for fringe bidder, low signal, $n_f = 1$ . . . . .	33
1.11	Estimated bidding functions for $n_f = 1$ . . . . .	36
1.12	Bid functions for fringe bidders, low signal, $n_f = 4$ . . . . .	37
1.13	Estimated bidding functions for $n_f = 4$ . . . . .	39
1.14	Typical project description and bids: Bridge project . . . . .	56
1.15	Typical project description and bids: Pave project . . . . .	57
2.1	Bidding functions for different values of $\alpha$ . . . . .	72
2.2	Bidding functions for different values of $\alpha$ . . . . .	73
2.3	Second round bidding functions for $b_1 = 1$ , $\alpha = 0.5$ , and several values of $\hat{c}$ . . . . .	76
2.4	Expected profits of the regular bidder in second round auction, $\alpha = 0.5$ , $n = 1$ . . . . .	77
2.5	Second round bidding functions for $b_1 = 1$ , $\alpha = 0.5$ , 2 fringe bidders, and several values of $\hat{c}$ . . . . .	80
2.6	Expected profits of the regular bidder in second round auction with two fringe bidders: $\hat{c}^* = 0.0358$ . . . . .	81

2.7	Expected profits of the regular bidder in second round auction with three fringe bidders: $\hat{c}^* = 0.0656$ . . . . .	81
2.8	Expected profits of the regular bidder in second round auction with three fringe bidders: $\hat{c}^* = 0.2666$ . . . . .	82
3.1	Krasnoturyinsk police station procurement . . . . .	91
3.2	Krasnoturyinsk police station: variation in area of service . . . . .	94
3.3	Kernel estimates of reserve price distribution before and after the policy change . . . . .	105
3.4	Kernel estimates of relative bid distributions for small and large firms	105
3.5	Kernel estimates of relative costs distributions for small and large firms	106
3.6	Subsample definition . . . . .	109
3.7	Volume demanded: North subsample . . . . .	113
3.8	Volume demanded: North subsample by municipality . . . . .	114

# Chapter 1: Autocorrelated Costs and Information in Repeated First Price Procurement Auctions

## 1.1 Introduction

In this chapter I study the effect of transparency in repeated procurement auctions on bidder behavior and optimal auction outcomes. The practice of publishing the results of previous public procurement auctions provides additional information on the rival bidders costs in markets where costs are correlated over time. I extend the standard framework for the estimation of firm costs in an asymmetric first price auction to account for the presence of past bid information. Using data on two types of highway procurement auctions in Oklahoma I show that firms account for available information on past bids when the competitor bids are correlated over time. In such markets all firms bid more aggressively when a low bid from the leading firm was observed in the past, and less aggressively when the high bid was observed. Increasing competition diminishes, but does not cancel out these effects. In my counterfactual analysis I show that the standard model without a binding reserve price misestimates expected procurement costs by 2-14%, and withholding past bid information from auction participants can reduce expected procurement costs by

up to 11%, and the leading firm markup by up to 3%. Setting an optimal reserve price reduces expected procurement costs by 9% in the standard model, and by up to additional 7% if the past bids are observed by the firms. Withholding past bid information in an auction with optimal reserve price does not change the expected cost of procurement significantly, but reduces the markup of the leading firm by 1%.

Transparency is a common requirement for public procurement auctions. In practice this means that most of the government bodies conducting auctions publish extensive information both before and after the auction. For example, Departments of Transportation of all fifty US states publish auction results within several days after the winner is determined. Full bid tabulations including the identities and bids of all participating firms are published in most states, with a few remaining states publishing only the identities and the bids of the winner and the runner-up in each auction. At the same time auctions conducted by the private parties are usually not so transparent. Most of the private company procurement policies protect the identity of contractors, and Ebay has introduced “private auctions” in which bidders can only see their own bid and losing or winning status both during and after the auction. In both of these cases the identities of the bidders and their bids are known either only to the auctioneer, or to the auctioneer and the direct auction participants. The transparency of public procurement auctions is usually motivated by preventing corruption and general government accountability, while the non-transparency of the private auctions is motivated by personal and corporate privacy, and preventing possible collusion by bidders. However, in the repeated auction setting the choice between releasing and concealing the information about past auction may have

additional consequences. In this chapter I argue that transparency can have a direct effect on the outcomes of a set of repeated private value auctions through the channels not related to corruption and collusion concerns since it can provide valuable information to auction participants and promote competition from weaker bidders.

Consider a typical highway procurement auction market. The firms in this market compete repeatedly to fulfill contracts for similar projects. In this setting, the firm's bidding history might be informative to its competitors if firm's costs of completing these projects are serially correlated. The serial correlation can be caused by overlapping timeline of completing the projects, or by the costs structure including some of the slower changing costs, for example labor costs which are only renegotiated once a year, long term supplier costs, etc. Previous work has tried to control for the serial correlation in firm costs using observable firm backlog. But backlog is likely to be an insufficient representation of the serial correlation of firm's costs since it reflects only some of the reasons for persistent costs, provides additional information only for the winner of the last auction, and is in general less informative than the past bids submitted by the firm.

In this chapter I estimate the effect of the past bid information on the behavior of firms in the current auction. I develop a model of a first price procurement auction with private values in which all participating firms can observe an informative signal about the past period costs of one of the firms. Auction participants update their beliefs of the rival's costs upon observing the signal and adjust their bidding strategies accordingly. I use the parametric structural model introduced by [Athey](#),

[Levin, and Seira \(2011\)](#) for the first price auctions with unobserved heterogeneity, and introduce past bid information as an additional observable auction characteristic to estimate the underlying firm characteristics in two highway procurement markets in the state of Oklahoma. I compare the market for bridge construction and repair contracts, with fulfillment times ranging from 3 to 12 months and the bids of the leading firms are strongly correlated over time, and the market for pavement contracts, in which a typical project takes 1 to 2 months to complete and the bids of the leading firms are not significantly correlated over time.

I find that releasing the information about past bids significantly changes the behavior of all firms participating in the bridge auctions, and almost does not distort the behavior of the firms participating in the pavement auctions. The results of the model estimation show that when the past bid information factors into the firm bidding behavior, the firms submit lower bids for any draw of their own costs if they have observed a signal about low competitor costs, and higher bids for the own costs below some threshold value if they have observed a signal about high competitor costs.

To quantify the consequences of the auction transparency for the auctioneer, I compute several counterfactuals. The first scenario corresponds to the standard asymmetric first price auction model in which neither the auctioneer nor the participating firms account for the past bid information. The other scenarios simulate the outcome for the case when either only the bidders, only the auctioneer, or both the bidders and the auctioneer have access to and account for the available past bid information. I show that ignoring the informational effect significantly changes both

the optimal reserve price and the expectations of the auctioneer with regards to the price of the contract compared to the standard model. However, if the auctioneer can set an optimal reserve price, additional benefits of withholding this information from the bidders are small, which argues in favor of the existing practice of full transparency.

To my knowledge, the effect of the past bid information on the current auction has not been studied empirically. However, there are several categories of papers that are methodologically and theoretically related to this question. In particular, this chapter is related to the limited empirical literature on dynamic auctions originating from [Jofre-Bonet and Pesendorfer \(2007\)](#). In these papers a discrete event, such as winning an auction or participating in an auction, changes an observed dynamic firm-level state variable, providing the link between time periods. The first group of these papers focuses on the effect of firm backlog and capacity constraints in repeated auction markets. The firms can participate in auctions each period, but, similarly to the case studied in this chapter, it takes them longer than one period to fulfill most of the contracts. This means that a firm already burdened by sufficient workload might chose not to participate in the auction or bid less aggressively in the current period. In the original paper by [Jofre-Bonet and Pesendorfer \(2007\)](#) regular bidders take into account the effect of increased backlog in case of winning the current auction when submitting a bid in the current auction. They show that an increase in capacity utilization significantly increases the costs of regular bidders. [Balat \(2013\)](#) extends this model to add unobserved auction heterogeneity and endogenous firm participation. [Saini \(2012\)](#) and [Jeziorski and Krasnokutskaya \(2016\)](#) extend the



model to include optimal scheduling and subcontracting respectively. Second group of papers, for example [Tiererova \(2013\)](#) and [Groeger \(2014\)](#), use firm participation and winning dynamics in a learning-by-doing model, showing that firms who have experience in preparing bids have lower participation costs and firms who have experience in performing the contracts have lower anticipated costs and would bid more aggressively.

Another related subject of empirical auctions literature is sequential auctions with complementarities or substitutabilities across units. [Kong \(2016\)](#) studies sequential auctions for oil and gas tracts let in adjacent pairs, and finds that both synergy and value affiliation are present in these auctions, with affiliation being responsible for the allocation patterns. [Donna and Espín-Sánchez \(2018\)](#) study water auctions in which the units sold can be complements or substitutes depending on the auction timing. They find that firms show different pricing and participation patterns depending on the relationship between objects sold in sequential auctions: when units are complements, one bidder wins all units by paying a high price for the first unit and deters entry by competitors; when units are substitutes, different bidders win the units with positive probability. [De Silva et al. \(2005\)](#) show in a reduced form study of Oklahoma DoT auctions that past winners are both more likely to participate in and win future auctions in the same geographical location, and attribute this fact to production synergy.

Finally, [Somaini \(2011\)](#) shows that the model with affiliated private firm costs describes the Michigan DoT procurement auctions better than a model with standard private costs. In his model the firms receive correlated private signals about

their own costs prior to the auction.

Some theoretical and experimental literature addresses the effect of information disclosure in auctions with persistent bidder valuations or costs more directly. However only partial and special case solutions of the problem exist, and it is not clear if an equilibrium in a fully dynamic model with persistent firm characteristics would always exist. [Martínez-Pardina \(2006\)](#) looks at the auctions in which one of the firm's valuations is common knowledge prior to the auction. She finds the equilibrium of this game, including the mixed equilibrium strategy of the player with publicly known valuation, and shows that revealing one of the bidders' valuations serves as a random reserve price in the auction. She shows that for a particular set of symmetric distributions it is beneficial for the auctioneer to commit to revealing one of the bidder's valuations ex ante, though this might hurt the auctioneer ex post. [Landsberger et al. \(2001\)](#) study first price auctions in which the ranking of the valuations is common knowledge. They show that bidders bid more aggressively when this information is provided, and hence the auctioneer is better off providing this information. [Fang and Morris \(2006\)](#) consider a first price auction with two bidders and discrete valuations, in which competitors can receive costly signals about each others valuations. In this very limited setting they also show that the auctioneer is better off providing very precise signals about the bidder valuations to all players. [Tu \(2006\)](#) shows that announcing the winning bid is the most beneficial disclosure policy for an auctioneer in a first price auction with two bidders and uniformly distributed valuations. He also provides an example of a disclosure policy that would lead to non-existence of non-decreasing equilibrium. [Thomas \(2010\)](#)

shows in a discrete type two bidder model that the auctioneer is better off either revealing no information at all, or all information available about the past auction. [Bergemann and Hörner \(2018\)](#) look at an infinitely repeated first price auction with perfectly persistent values and show that minimal information disclosure is good for the auctioneer. However they note that if the values are not perfectly persistent this result might cease to hold. Finally experimental papers by [Andreoni et al. \(2007\)](#), [Cason et al. \(2011\)](#), and [Dufwenberg and Gneezy \(2002\)](#), show that players learn, play pooling equilibria in the discrete values case, and in general take into account the past bid information released by the auctioneer.

This chapter provides an additional argument in support of the importance of information disclosure regimes, and a simple approach for estimating the effects of information disclosure in repeated procurement auction setting.

## 1.2 Data and institutional environment

I use the data on procurement auctions held by Oklahoma Department of Transportation (DoT) between April 2000 and August 2003. In this period of time the Oklahoma DoT held between 15 and 70 auctions per month with total budget of 8.5 to 65.5 million dollars. The timeline of a typical auction starts about a year before the letting date when the preliminary plan of works is published. More detailed auction documentation describing the projects is published one month in advance. Firms can purchase full project documentation, also known as a “plan”, for a small fee ( $\sim \$20$ ), and the list of planholders is publicly available and regularly

updated until the contract letting date. The firms that want to participate in the auction must be prequalified<sup>1</sup> and obtain the project documentation. After that a firm can submit a sealed bid at any day between the last month's letting date and current month's letting date. All of the Oklahoma DoT auctions are first price low bid auctions, i.e. all submitted bids for a given auction are opened at the same time, the firm with the lowest submitted bid wins and pays her own bid. All auctions scheduled for the month are closed at the same letting date. After all bids are opened and auction winners determined, the auction results and detailed bid tabulations including names of all participating firms are published on the Oklahoma DoT website. Since April 2000 Oklahoma DoT also publishes the engineer's estimate for the project with the detailed break-down of the estimate in the planning documentation<sup>2</sup> ahead of time. De Silva et al. (2008) show that the publication of the engineer's estimate has significantly changed the bidding behavior of both the incumbents and the entrants suggesting that firms in the Oklahoma DoT procurement market pay attention to the publicly released information. To avoid conflating the effect of information available in the engineer's estimate and the information available in the rival bids, I would only use the data after April 2000.

There are six categories of works a firm may prequalify for in the state of Oklahoma. I concentrate on the paving (category C) and bridge (category D) works since they are the two most prevalent types of works both by the number of firms participating and the number of auctions held. Bridge and pavement works can

---

<sup>1</sup>This requires filling in a simple application form, and undergoing a financial audit.

<sup>2</sup>Figures 1.14 and 1.15 in Appendix 1.7.6 provide an example of publicly available information before and after the auction letting date.

be viewed as separate markets. The firms performing bridge works and pavement projects in general do not overlap and specialize only in one type of works. Though about a third of the firms have participated in the auctions of both types during the observed time period, only 19 out of 136 have won auctions in both markets, and only one of them won more than 10 auctions in each of the markets. There are several distinctions between pavement and bridge works markets that are related to the subject of this chapter, namely to the way the firms' costs are correlated over time, and how much additional information about the rivals' costs can be inferred from her past bids. First of the prominent differences between the bridge works and pavement contracts auctioned is the average duration of estimated work. A typical bridge repair project (see Figure 1.14 in Appendix 1.7.6) takes 3-12 months and is geographically concentrated while a typical pavement project (see Figure 1.15 in Appendix 1.7.6) takes 1-2 months to complete and sometimes covers long distances. This distinction can affect the importance of the past bids information for the current bidding. Both types of auctions are conducted once a month, and it is more likely that the bids submitted for the bridge works auctions would be informative about the next month's costs since most of the time the bids on a bridge project would include the anticipated costs of operation in the next month. Second, the pavement works are more consistent and depend on certain inputs, which makes contract costs largely depend on the costs of the materials which are known to all market participants and in the state of Oklahoma are updated every month with the baseline price publicly available in the "Asphalt Binder Price Index". At the same time bridge works have more variety and often include, among other things, the cost

of highly skilled labor which can vary between firms and is less public, while at the same time being consistent over time. Finally, the work on pavement contracts is more consistent with the standard measure of backlog used in the literature. It is reasonable to assume that a firm performing a pavement contract paves some fixed proportion of contracted surface each month. At the same time the tasks involved in bridge works are less consistent and need to be performed in a particular order, making the strain on the firm capacity uneven over the duration of the contract, and the backlog a less informative measure of the possible shifts in firm costs. All of these factors make it more likely that the firms in the bridge works market would pay closer attention to the past bids of their competitors than the firms in the pavement market.

For tractability, I would aggregate all of the past bids information into one observable signal. To do that first I select one “regular” firm in each market, and summarize all bids submitted by the regular firm in one month into a one-dimensional signal. In both markets the regular firm is the firm that participates in the largest number of auctions: 203 out of 373, or more than 54%, in the bridge works market, and 117 out of 397, or around 30%, in the pavement auctions.<sup>3</sup>

Second, as regular bidders participate in multiple auctions per month, I would further aggregate the past month bid information into a one-dimensional signal I would call an average bid residual.<sup>4</sup> The bid residual is constructed as a predicted

---

<sup>3</sup>In both markets there are close “second contenders” with 188 and 113 appearances respectively, with no further close contenders. Though both of the two top firms are most likely providing an important information with their bids, to abstract from the strategic interaction between them, I would consider the “second contenders” to be fringe bidders.

<sup>4</sup>I also use the average bid submitted in the past month for robustness checks. Though the average bid is a more straightforward choice for a signal, it does not capture possible auction

residual from an OLS regression of the submitted bid on the observable characteristics of the past auction. The main benefit of using the average bid residual instead of the average bid, is that the residual makes the signals from different auctions comparable regardless of the project size, location, number of bidders etc. The details of the bid residual constructions can be found in Appendix 1.7.2.

Tables 1.1 and 1.2 show the summary statistics for the bridge and pavement auctions in the sample. There are 373 bridge auctions with 1556 bids from 81 firms in the sample, and 397 pavement auctions with 1441 bids from 97 firms. Details of sample constructions can be found in Appendix 1.7.1. There are several notable differences between the two markets. First, the pavement contract are almost five times larger in size on average. Second, even though in both markets the regular bidder skips auction participation only in 5 time periods, the pavement regular bidder participates in less auctions per month on average. This can make the information contained in her past month bid less reliable and less useful to the fringe bidders. The regular bidder in the pavement market submits 8.5% of all observed bids, and the past bid information is available for 97% of the auctions the regular bidder is participating in. The regular bidder in the bridge market submits about 10% of all observed bids. In 93% of the auctions with regular bidder participation the past bid information is available for the regular bidder.

As a first step to establish the relationship between the past bid signal and

---

heterogeneity not observed by the researcher, but observed by all participants in the market. It is also harder to interpret: while a negative bid residual signals about lower than average costs, and a positive bid residual signals about higher than average costs, we would need additional information about bid averages to interpret the value of the average bid signal.

Table 1.1: Auction summary statistics: Bridge auctions

<b>Variable</b>	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Min</b>	<b>Max</b>
Number of auctions per month	41	9.1	4.84	3	20
Number of regular bidder participations per month	41	4.59	3.84	0	16
Engineer's estimate (\$000)	373	546.77	1278.89	7.9	18664.31
Number of plan holders.	373	6.83	2.65	2	19
Regular bidder participation	373	.69	.46	0	1
Number of fringe bidders	373	3.48	1.59	1	12
Lagged average bid (% of Eng. est.)	340	1.01	.13	.8	1.47
Lagged bid residual	340	-.02	.1	-.35	.32
Winning bid (% of Eng. est.)	373	.92	.14	.7	1.49
Regular bidder bid (% of Eng. est.)	188	.99	.17	.73	1.85
Bid (% of Eng. est.)	1556	1.05	.24	.7	2.44

Table 1.2: Auction summary statistics: Pavement auctions

<b>Variable</b>	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Min</b>	<b>Max</b>
Number of auctions per month	41	9.68	7.18	2	37
Number of regular bidder participations per month	41	2.76	2.4	0	11
Engineer's estimate (\$000)	397	2521.61	4407.27	24.25	32487.49
Number of plan holders.	397	6.89	4.57	2	25
Regular bidder participation	397	.39	.49	0	1
Number of fringe bidders	397	3.24	1.76	1	11
Lagged average bid (% of Eng. est.)	383	.99	.14	.69	1.32
Lagged bid residual	383	0	.17	-.44	.32
Winning bid (% of Eng. est.)	397	.94	.13	.71	1.48
Regular bidder bid (% of Eng. est.)	113	1.02	.15	.78	1.57
Bid (% of Eng. est.)	1441	1.02	.17	.32	1.95

current firm bids, I run a number of OLS regressions of the form:

$$B_{ti} = \alpha + \beta \overline{X}_t + \gamma S_{t-1} + \delta \overline{Y}_{it}, \quad (1.1)$$

where  $B_{ti}$  is the bid firm  $i$  submits at time  $t$ ,  $\overline{X}_t$  are auction characteristics such as the number of bidders, project size, project location etc.,  $\overline{Y}_{it}$  are firm-specific characteristics at time  $t$  such as backlog and distance to the project, and  $S_{t-1}$  is the



past bid signal with  $\gamma$  being the coefficient of interest.

Tables 1.3 and 1.4 present the results of these regressions using past month's average bid residual of the regular bidder as an observable signal. All specifications include project size and location fixed effects, second specification in both tables includes firm fixed effects <sup>5</sup>. The results suggest that both the bids of the regular bidder and the bids of the fringe bidders positively and significantly depend on the value of the signal in the bridge market. At the same time there is no relation between the value of the signal and current bids of either type of bidders in the pavement market. The bids of the pavement regular bidder are significantly related to the bidder backlog, while there is no significant dependence between bids and backlog of the bridge regular bidder. In addition the participation of the regular bidder significantly reduces the bids submitted by fringe bidders in both markets, which might signify that the fringe bidders are aware of the regular bidder status in both markets.

The correlation between lagged bid residual of the regular bidder and current bids of all market participants might be a result of a general time trend of the costs of all firms on the market. To control for this I construct an additional observed signal, a lagged market bid residual, which aggregates the bids of all fringe bidders observed in the past month. Tables 1.5 and 1.6 present the results of the regressions controlling for this additional signal. As with previous results, the firms on the bridge market pay attention to the signal from the regular bidder, and only the

---

<sup>5</sup>Tables 1.15 and 1.16 in Appendix 1.7.3 contain results of the similar regressions for average bid of the regular bidder in past month as an observable signal. Since the results of these two sets of regressions are qualitatively the same, and it is easier to interpret and model the average bid residual, I would use it as the signal for the rest of the chapter.

Table 1.3: Bridge projects with bid residual signal

	Full sample (1)	Full sample (2) <sup>†</sup>	Full sample ID FE (3)	Regular bidder (4)	Fringe bidders
N (fringe) bidders	−0.0055* (0.0033)	−0.0074** (0.0033)	−0.0064* (0.0033)	−0.0168** (0.0072)	−0.0066* (0.0036)
Distance	0.0002*** (0.0000)	0.0002*** (0.0000)	0.0001 (0.0001)	−0.0001 (0.0002)	0.0002*** (0.0000)
Backlog	0.0017 (0.0217)	−0.0136 (0.0219)	0.0411* (0.0246)	0.0521 (0.0532)	−0.0203 (0.0238)
Lagged bid residual		0.2020*** (0.0559)	0.1580*** (0.0546)	0.2488** (0.1154)	0.1971*** (0.0618)
Regular participation dummy	−0.0357** (0.0141)	−0.0356** (0.0148)	−0.0082 (0.0159)		−0.0353** (0.0154)
<i>N</i>	1556	1428	1410	181	1246
adj. <i>R</i> <sup>2</sup>	0.1220	0.1373	0.2540	0.1826	0.1268

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.

Table 1.4: Pavement projects with bid residual signal

	Full sample (1)	Full sample (2) <sup>†</sup>	Full sample ID FE (3)	Regular bidder (4)	Fringe bidders
N (fringe) bidders	−0.0107*** (0.0024)	−0.0114*** (0.0024)	−0.0107*** (0.0026)	−0.0002 (0.0125)	−0.0119*** (0.0025)
Distance	0.0000 (0.0000)	0.0000 (0.0000)	−0.0000 (0.0001)	−0.0000 (0.0002)	0.0000 (0.0000)
Backlog	−0.0260* (0.0149)	−0.0230 (0.0154)	0.0204 (0.0184)	−0.1713** (0.0687)	−0.0174 (0.0159)
Lagged bid residual		0.0085 (0.0257)	0.0126 (0.0253)	0.0668 (0.0821)	−0.0004 (0.0270)
Regular participation dummy	−0.0409*** (0.0093)	−0.0388*** (0.0103)	−0.0256** (0.0121)		−0.0398*** (0.0104)
<i>N</i>	1441	1378	1356	110	1268
adj. <i>R</i> <sup>2</sup>	0.1475	0.1457	0.2183	0.2258	0.1441

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.

regular bidder pays attention to the rest of the market. On the other hand, the firms on the pavement market pay no attention to the regular bidder signal, but make the current bids significantly correlated with the observed market average residual from past month, with the regular bidder basing a larger proportion of her bid on this signal.

The reduced form evidence suggests that the behavior of the regular bidder

Table 1.5: Bridge projects with market level bid residual signal

	Full sample (1)	Full sample ID FE (2) <sup>†</sup>	Regular bidder (3)	Fringe bidders (4)
N (fringe) bidders	−0.0060* (0.0034)	−0.0061* (0.0034)	−0.0235*** (0.0078)	−0.0052 (0.0038)
Distance	0.0002*** (0.0000)	−0.0000 (0.0001)	−0.0004 (0.0004)	0.0002*** (0.0000)
Backlog	−0.0088 (0.0220)	0.0419* (0.0247)	0.0779 (0.0543)	−0.0171 (0.0240)
Lagged bid residual	0.2069*** (0.0578)	0.1583*** (0.0560)	0.2790** (0.1151)	0.1983*** (0.0641)
Lagged market bid residual	0.0292 (0.0994)	0.1068 (0.0949)	0.4342** (0.2067)	−0.0021 (0.1095)
Regular participation dummy	−0.0365** (0.0159)	−0.0064 (0.0170)		−0.0307* (0.0168)
<i>N</i>	1428	1410	181	1246
adj. <i>R</i> <sup>2</sup>	0.1360	0.2558	0.1871	0.1275

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.

Table 1.6: Pavement projects with market level bid residual signal

	Full sample (1)	Full sample ID FE (2) <sup>†</sup>	Regular bidder (3)	Fringe bidders (4)
N (fringe) bidders	−0.0107*** (0.0025)	−0.0107*** (0.0027)	−0.0063 (0.0126)	−0.0112*** (0.0026)
Distance	0.0000 (0.0000)	−0.0000 (0.0001)	−0.0004 (0.0003)	0.0000 (0.0000)
Backlog	−0.0171 (0.0154)	0.0217 (0.0184)	−0.1923*** (0.0665)	−0.0102 (0.0160)
Lagged bid residual	−0.0587* (0.0356)	−0.0474 (0.0355)	−0.0935 (0.1095)	−0.0567 (0.0375)
Lagged market bid residual	0.1972*** (0.0711)	0.1828** (0.0709)	0.5201** (0.2155)	0.1687** (0.0749)
Regular participation dummy	−0.0308** (0.0120)	−0.0291** (0.0133)		−0.0340*** (0.0126)
<i>N</i>	1378	1356	110	1268
adj. <i>R</i> <sup>2</sup>	0.1551	0.2206	0.2948	0.1523

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.

in the bridge works market shows significant autocorrelation, and the fringe bidders take this into account. At the same time, there is no significant autocorrelation in the pavement regular bidder behavior. In my further analysis I would use this difference between the two markets to contrast the effects of information disclosure on the auction outcomes.

## 1.3 Model

In this section I describe the model of an asymmetric first price procurement auction with one regular and  $n$  fringe bidders in which a public informative signal about the costs of the regular bidder is released to all bidders prior to the auction. I show that after the signal is observed by all auction participants the model becomes equivalent to a standard asymmetric first price auction, and its identification follows the results of [Guerre et al. \(2000\)](#), and [Flambard and Perrigne \(2006\)](#).

### 1.3.1 Setup

There are two types of risk neutral bidders participating in a first price sealed bid procurement auction. The signal  $s \sim S(0, \sigma_s^2)$  is released publicly before the auction. This signal is informative of the regular bidder's costs of completing the contract in the sense that after receiving the signal all auction participants know that the regular bidder would draw her costs from a conditional distribution  $F(\cdot|s)$ , and  $F(\cdot|s') \succeq_{FOSD} F(\cdot|s)$  for any  $s' \geq s$ <sup>6</sup>. There are  $n$  fringe bidders independently drawing costs from a distribution  $Z(\cdot)$  which does not depend on the signal.

The regular bidder submits a bid  $b$ , and each fringe bidder  $i \in \overline{1..n}$  submits a bid  $b^i$  taking into account their own cost draw, the value of the signal observed, and the number of participants in the auction.<sup>7</sup>

---

<sup>6</sup>Similar condition arises in the models of auctions with affiliated values. However the models with affiliated or public values assume that receiving a public signal changes the firms' beliefs about her own costs or valuations, and not the beliefs about the competitors' costs or valuations.

<sup>7</sup>The number of fringe participant is treated as an exogenous auction characteristic in this model, however considering an endogenous entry model would be a logical model extension.

As I would show below, after the signal is observed, and given some regularity assumptions on the equilibrium bidding strategies and the underlying cost distributions, the problem reduces to a standard asymmetric first price auction with the regular bidder drawing costs from the distribution  $F(\cdot|s)$ , and fringe bidders independently drawing costs from  $Z(\cdot)$ . Hence, following [Maskin and Riley \(2000a\)](#) and [Maskin and Riley \(2000b\)](#), a unique set of monotone equilibrium bidding functions exists. However the following assumptions are useful for identification purposes.

**A1.1.** The regular bidder bids according to a bidding function  $\beta(c, s)$ , and each of the fringe bidders bids according to a bidding function  $\zeta(c^i, s)$  in equilibrium.

**A1.2.**  $\beta(c, s)$  is a monotonically increasing function of both arguments, and a function  $\beta^{-1}(b, s)$ , such that  $\beta(\beta^{-1}(b, s), s) = b$  exists for any value of  $s$ .

**A1.3.**  $\zeta(c^i, s)$  is a monotonically increasing function of both arguments, and a function  $\zeta^{-1}(b^i, s)$ , such that  $\zeta(\zeta^{-1}(b^i, s), s) = b^i$  exists for any value of  $s$ .

### 1.3.2 Optimal bidding and equilibrium

To find the optimal bidding strategies of both types of bidders one would solve for the Bayesian-Nash Equilibrium (BNE) in an asymmetric first price procurement auction for the cost distributions  $F(\cdot|s)$  for the single regular bidder, and  $Z(\cdot)$  for  $n$  fringe bidders.

To show this, consider the regular bidder problem:

$$(b - c)(1 - Z(\zeta^{-1}(b, s))^n) \rightarrow \max_b \quad (1.2)$$

Similarly to the standard first price auction problem, the first order condition is:

$$1 = \frac{n(b-c)z(\zeta^{-1}(b, s))\zeta_1^{-1}(b, s)}{1 - Z(\zeta^{-1}(b, s))}, \quad (1.3)$$

where  $\zeta_1^{-1}(\cdot, s)$  is the derivative of  $\zeta^{-1}$  with respect to the first argument.

Assuming there are  $n$  fringe bidders in the auction the  $i$ 'th fringe bidder's problem is:

$$(b^i - c^i)(1 - Z(\zeta^{-1}(b^i, s)))^{n-1}(1 - F(\beta^{-1}(b^i, s)|s)) \rightarrow \max_{b^i} \quad (1.4)$$

With the first order condition:

$$1 = \frac{(n-1)(b^i - c^i)z(\zeta^{-1}(b^i, s))\zeta_1^{-1}(b^i, s)}{1 - Z(\zeta^{-1}(b^i, s))} + \frac{(b^i - c^i)f(\beta^{-1}(b^i, s)|s)\beta_1^{-1}(b^i, s)}{1 - F(\beta^{-1}(b^i, s)|s)}, \quad (1.5)$$

where  $\beta_1^{-1}(\cdot, b_{t-1})$  is the derivative of  $\beta^{-1}$  with respect to the first argument.

The pair of functions  $\beta(c, s)$  and  $\zeta(c^i, s)$  satisfying first order conditions (1.3) and (1.5) constitute a BNE of this game. Since the firms ex post payoffs are linear in their own type (and hence are supermodular in own and competitor type), and the firm costs are privately and independently drawn from the distributions with finite support and a common upper endpoint with a positive mass for each firm type, according to Maskin and Riley (2000b), Maskin and Riley (2003), and Maskin and Riley (2000a) this equilibrium would be unique and monotonic in  $c$  for the regular bidder or  $c^i$  for the fringe bidders.

### 1.3.3 Identification

In order to identify the unobserved costs of regular and fringe bidders using the methodology introduced by [Guerre et al. \(2000\)](#) (GPV), the relationship between the distribution of bids and the distribution of costs for each of the bidders should be established. In this section I would show that the standard relationship between the observed bid distributions and unobserved cost distributions, allowing to eliminate the bid function derivatives from equations (1.3) and (1.5), hold for this model. However, the distributions of bids conditional on the signal should be used instead of marginal bid distributions for both regular and fringe bidders.

**Proposition 1.1.** The following relationships between the distributions of costs  $F(\cdot|s)$ ,  $Z(\cdot)$ , and conditional distributions of bids  $G^R(\cdot|s)$ ,  $G^F(\cdot|s)$ , hold if the firms bid according to equilibrium bidding functions  $\beta(c, s)$  and  $\zeta(c^i, s)$ :

1. For the regular bidder:

$$F(\beta^{-1}(y, s)|s) = G^R(y|s), \text{ and}$$

$$f(\beta^{-1}(y, s)|s)\beta_1^{-1}(y, s) = g^R(y|s)$$

2. For the fringe bidder:

$$Z(\zeta^{-1}(y, s)) = G^F(y|s), \text{ and}$$

$$z(\zeta^{-1}(y, s))\zeta_1^{-1}(y, s) = g^F(y|s)$$

Taking into account the relationships in Proposition 1.1 the system of first order conditions (1.3) and (1.5) can be rewritten as:

$$\begin{aligned} c &= b - \left[ \frac{ng^F(b|s)}{1 - G^F(b|s)} \right]^{-1}, \\ c^i &= b^i - \left[ \frac{(n-1)g^F(b^i|s)}{1 - G^F(b^i|s)} + \frac{g^R(b^i|s)}{1 - G^R(b^i|s)} \right]^{-1}. \end{aligned} \tag{1.6}$$

This allows the identification of the costs of regular and fringe bidders using the observed values of bids, signals, and other auction characteristics.

## 1.4 Structural estimation

I use the parametric structural model introduced by [Athey et al. \(2011\)](#) for first price auctions with unobserved heterogeneity to estimate the probability distributions of observed bids. I introduce past bid signal as an observable auction characteristic, and assume the common parametric functional form of the Gamma-Weibull distribution for the joint distribution of bids of regular and fringe firms in both the bridge and the pavement market. Using a parametric method for the estimation of bid distributions helps to alleviate the small size of the data for each of the realizations of the signal. I use the system of first order conditions (2.1) to calculate pseudo costs, and estimate the conditional distribution of costs for the regular bidders, and the (marginal) distribution of costs for the fringe bidders nonparametrically in both markets. Finally, to illustrate the results of the last estimation step, I approximate the bidding functions for a particular set of auction characteristics and contrast the bidding function estimates resulting from the estimation of the



standard model and the model with the past bid signal.

#### 1.4.1 Parametric estimation of bid distributions

To account for the observed value of the past bid signal explicitly I use a semi-parametric approach. First I estimate the conditional bid distributions of regular and fringe bidders parametrically, accounting for the unobserved auction heterogeneity.

The bid distributions of regular and fringe bidders are estimated using a Gamma-Weibull distribution families:

$$G^j(b^j, b_{t-1}) = 1 - \exp(-u \cdot (\frac{b_j}{\lambda_j})^{\rho_j}), \quad (1.7)$$

where  $j = R, F$  for regular or fringe bidder, and the scale parameters  $\lambda_j$  and the shape parameters  $\rho_j$  are of the form:

$$\begin{aligned} \ln(\lambda_j) &= \beta_0^j + \beta_j X_1^j \\ \ln(\rho_j) &= \gamma_0^j + \gamma_j X_2^j \\ u &\sim \text{Gamma}(\frac{1}{\theta}, \theta) \\ \ln(\theta) &= \theta_0 \end{aligned} \quad (1.8)$$

In particular,  $X_1^j$  and  $X_2^j$  are the observable auction or firm characteristics.  $X_1^R$  contains the number of fringe bidders (number of regular bidders is = 1 and omitted for multicollinearity), and the past bid signal.  $X_2^R$  contains the number of

Table 1.7: Gamma-Weibull estimates: Bridge projects

	Regular			
	Scale			
Constant	2.1816***	(0.1379)	2.2205***	(0.1270)
N (fringe) bidders	0.0255	(0.0311)	0.0388	(0.0307)
Lag av bid residual	1.5825*	(0.8304)		
	Shape			
Constant	−0.0154	(0.0341)	−0.0332	(0.0354)
N (fringe) bidders	−0.0099	(0.0062)	−0.0075	(0.0063)
Backlog	0.0765*	(0.0455)	0.0481	(0.0442)
Distance	0.0817	(0.1519)	0.1101	(0.1612)
Lag av bid residual	0.3864***	(0.0929)		
	Fringe			
	Scale			
Constant	1.9291***	(0.0589)	1.9131***	(0.0563)
N (fringe) bidders	−0.0084	(0.0114)	−0.0085	(0.0113)
Regular dummy	0.2813***	(0.0431)	0.2763***	(0.0429)
Lag av bid residual	−0.2615	(0.3257)		
	Shape			
Constant	0.0812***	(0.0234)	0.0700***	(0.0237)
N (fringe) bidders	−0.0033	(0.0055)	−0.0017	(0.0056)
Regular dummy	−0.0527***	(0.0180)	−0.0612***	(0.0181)
Lag av bid residual	0.3400***	(0.0877)		
	Heterogeneity			
$\ln(\theta)$	−0.0098	(0.0888)	0.0298	(0.0881)

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Asymptotic standard errors in parentheses.

fringe bidders, backlog, distance to the project, and the past bid signal.  $X_1^F$  and  $X_2^F$  have the same structure which contains the number of fringe competitors, regular bidder participation dummy, and the past bid signal.<sup>8</sup> The estimation for bridge and pavement projects is performed separately.

Table 1.7 presents parameter estimates for the bridge projects with the lagged average bid residual included in  $X_1$  and  $X_2$  (the unrestricted model) on the left, and no lagged information (the restricted model) on the right. The lagged average

<sup>8</sup>Versions with backlog and distance for fringe bidders were estimated and do not change the estimates much while hurting the standard errors. Versions with  $\ln(\theta)$  depending on past bid signal  $s_{t-1}$  were also estimated and do not change core results.

Table 1.8: Gamma-Weibull estimates: Pavement projects

	Regular	
	Scale	
Constant	2.5248*** (0.1739)	2.4943*** (0.1306)
N (fringe) bidders	−0.0182 (0.0408)	−0.0106 (0.0400)
Lag av bid residual	−0.0392 (0.8240)	
	Shape	
Constant	0.0986*** (0.0343)	0.1006*** (0.0342)
N (fringe) bidders	−0.0280*** (0.0078)	−0.0273*** (0.0076)
Backlog	−0.1455*** (0.0554)	−0.1590*** (0.0518)
Distance	0.1232 (0.1688)	0.1224 (0.1676)
Lag av bid residual	0.0520 (0.0732)	
	Fringe	
	Scale	
Constant	2.2441*** (0.0617)	2.2333*** (0.0548)
N (fringe) bidders	0.0057 (0.0100)	0.0068 (0.0099)
Regular dummy	0.1134** (0.0512)	0.1147** (0.0511)
Lag av bid residual	−0.0460 (0.1997)	
	Shape	
Constant	0.0917*** (0.0165)	0.0927*** (0.0165)
N (fringe) bidders	−0.0190*** (0.0037)	−0.0192*** (0.0037)
Regular dummy	−0.0578*** (0.0148)	−0.0592*** (0.0147)
Lag av bid residual	−0.0484 (0.0406)	
	Heterogeneity	
$\ln(\theta)$	0.0236 (0.0949)	0.0226 (0.0950)

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Asymptotic standard errors in parentheses.

bid residual has a large and statistically significant (at a 10% level and 1% level accordingly) effect on both the scale and the shape parameter of the regular bidder bid distribution. The likelihood ratio test rejects the restricted model with p-value less than 0.0001. Table 1.8 presents parameter estimates for the pavement projects. The lagged average bid residual does not have a significant effect on any of the estimates, and in this case the likelihood ratio test does not reject the restricted model with p-value 0.222. It is also worth noting that the competition between fringe bidders does not have a significant role in the estimated bid distributions for the bridge projects, but does enter the estimated bid distributions for the pavement projects. Also the backlog is more important than the lagged information in the pavement projects, and does not appear to play a role for the bidding in bridge projects.

To contextualize the effect that the estimated coefficients for the value of the past bid signal has on the bid distributions, I plot the estimated bid distributions for each market and each bidder type conditional on the mean values of all other observable auction and firm characteristics, and the full range of the value of the lagged average bid residual in Figures 1.1 through 1.4.

Figures 1.1 and 1.2 present the conditional distributions for the regular and fringe bidders in bridge market with the left panel presenting a 3D view, and the right panel tracing the mean and variance of the estimated distribution in the bid-signal space. For both of the bidder types mean value of the distribution is increasing with the signal, the variance of the regular bidder bid distribution is decreasing with the signal, and the variance of the fringe bidder bid distribution increases slightly with

Figure 1.1: Estimated conditional bid distribution for regular bidders in the bridge market

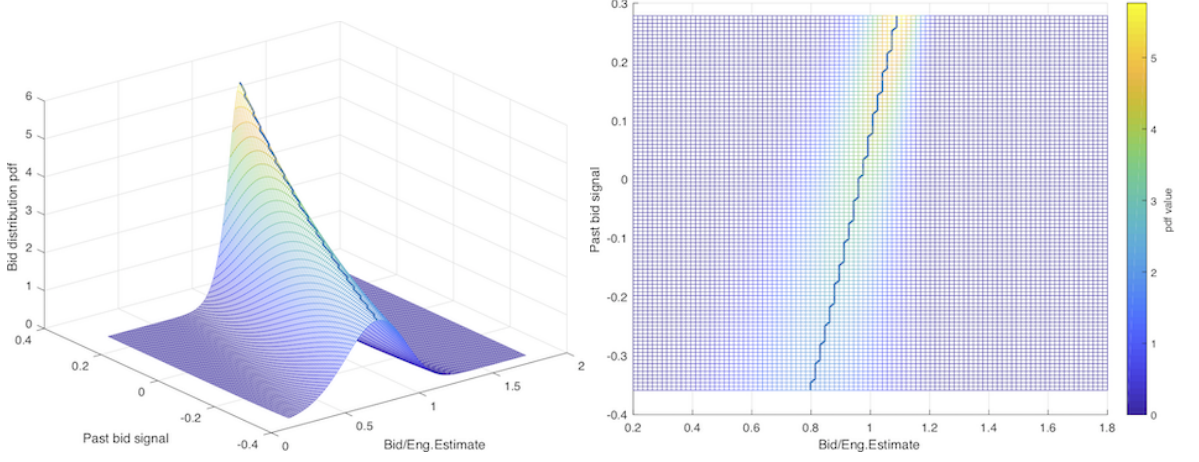
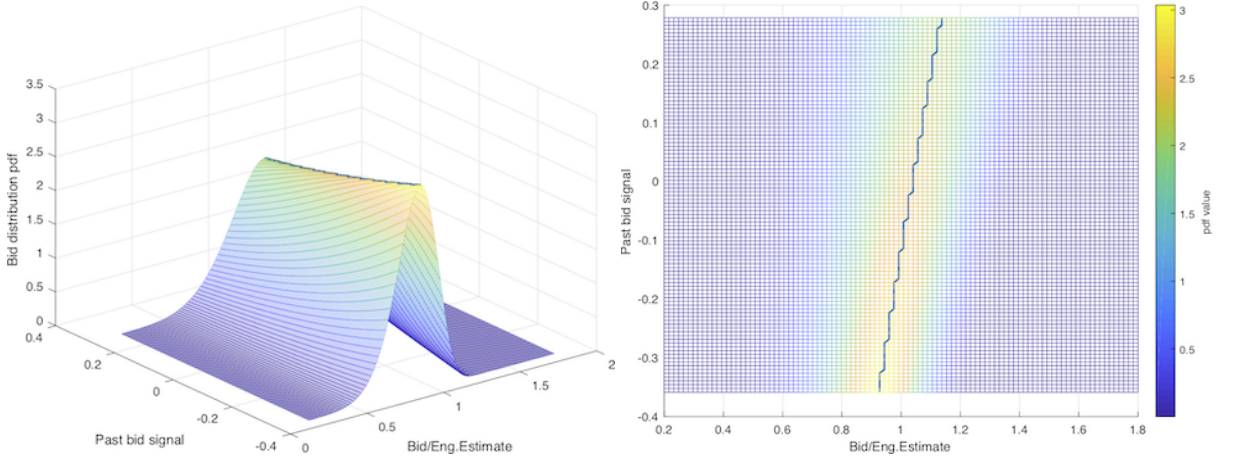


Figure 1.2: Estimated conditional bid distribution for fringe bidders in the bridge market



the signal. Figures 1.3 and 1.4 present the conditional distributions of the regular and fringe bidders in the pavement market. The estimated distributions vary much less with the value of the past bid signal than in the bridge market. The mean of the regular bidder bid distribution increases slightly with the value of the signal, the mean of the fringe bidder distribution does not change significantly. The variance for both the regular and the fringe bidders stays roughly the same regardless of the value of the signal.

Figure 1.3: Estimated conditional bid distribution for regular bidders in the pavement market

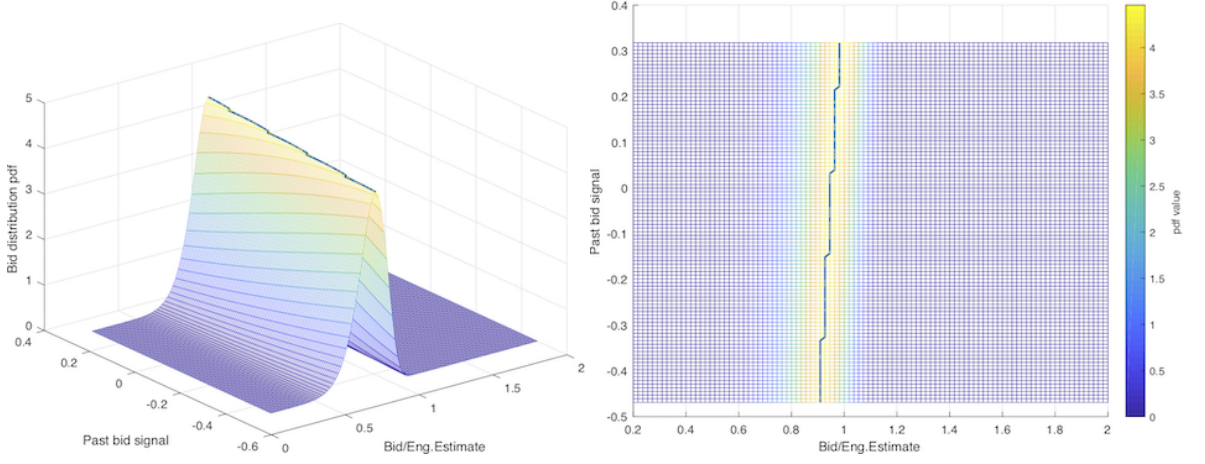
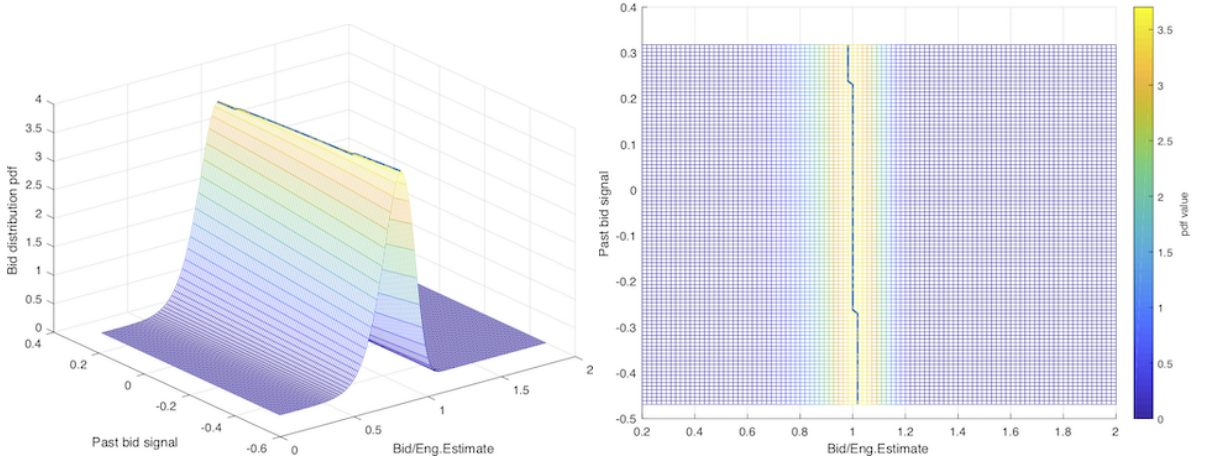


Figure 1.4: Estimated conditional bid distribution for fringe bidders in the pavement market



#### 1.4.2 Estimation of cost distributions

After obtaining the conditional bid distributions for all groups of bidders, I can use (2.1) to calculate pseudo costs for each observed bid and estimate the corresponding cost distributions nonparametrically. I use the standard kernel density estimator (1.9) with an optimal bandwidth  $h = 1.06\hat{\sigma}n^{-1/5}$  for all marginal density estimates and the one-step conditional density estimator (1.10) with the optimal bandwidth

matrix  $[h, h_s] = [\hat{\sigma}n^{-1/6}, \hat{\sigma}_sn^{-1/6}]$  for the estimation of regular bidder conditional distribution of costs. In both cases I use the Gaussian kernel with boundary correction to allow for common support of costs in the further counterfactual analysis.

$$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^n K_h(x - X_i) \quad (1.9)$$

$$\hat{f}(x|s) = \frac{\sum_{i=1}^n K_h(x - X_i) K_{h_s}(s - S_i)}{\sum_{i=1}^n K_{h_s}(s - S_i)} \quad (1.10)$$

Figures 1.5 and 1.6 show the difference between the marginal cost distribution estimates within the standard model and the model accounting for the presence of the past bid signal in the first step of the estimation procedure. For both regular and fringe bidders there is no difference in these estimates for the firms in the pavement market. In the bridge market, the marginal distribution of costs for the regular bidder is slightly less precise when we take the past bid signal into account in the estimation procedure, since some of the observed bid variation would be associated with the signal variation. The marginal distribution of costs for the fringe bidder is slightly more precise. In general, there is no big difference in the estimates of the marginal distribution of costs of both the regular and fringe bidders. In addition the estimated marginal cost distributions from different models lie within the 95% confidence interval of each other for both types of the bidders. This changes, however, if we look at the conditional distributions of regular bidder costs for different values of the signal.

Figure 1.7 presents the results of the estimation of the regular bidder cost

Figure 1.5: Marginal pdf's of regular bidder costs for bridge projects (left) and pavement projects (right)

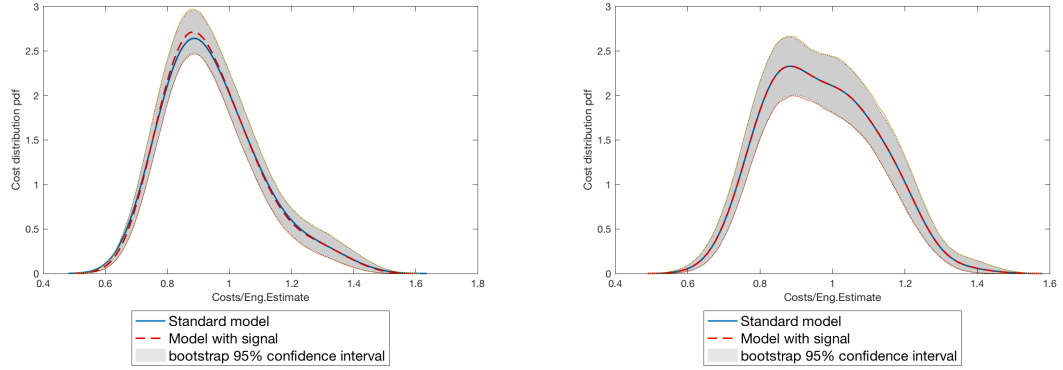


Figure 1.6: Marginal pdf's of fringe bidder costs for bridge projects (left) and pavement projects (right)

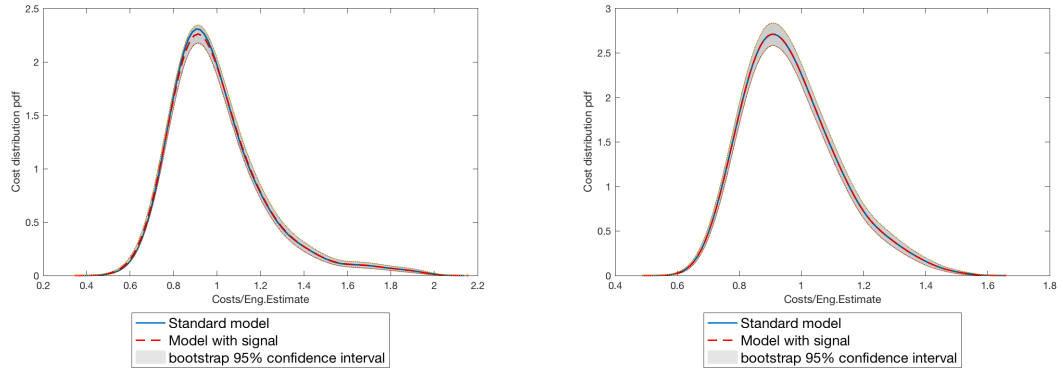


Figure 1.7: Conditional and marginal pdf's of regular bidder costs

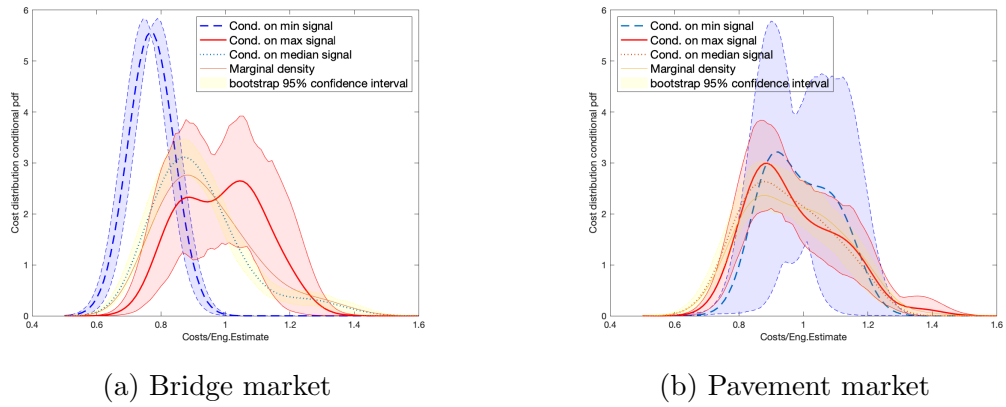
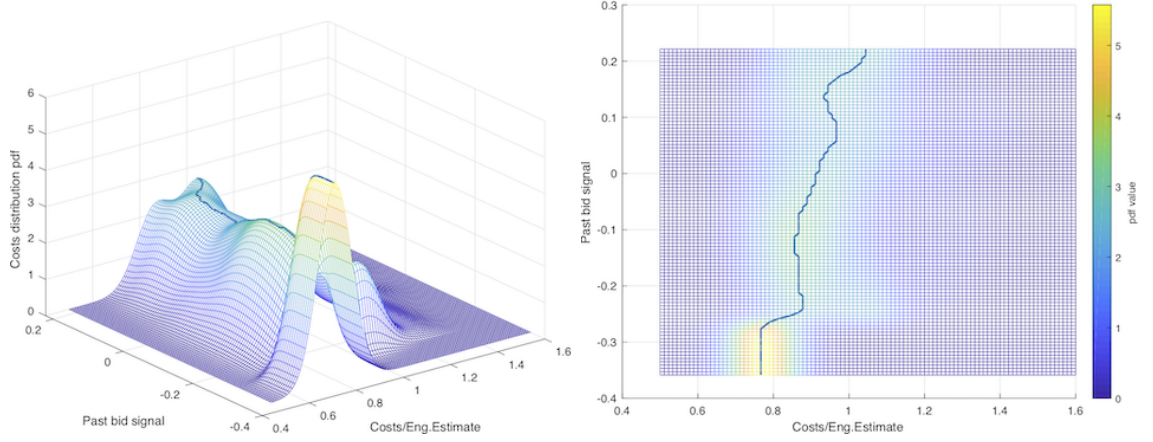




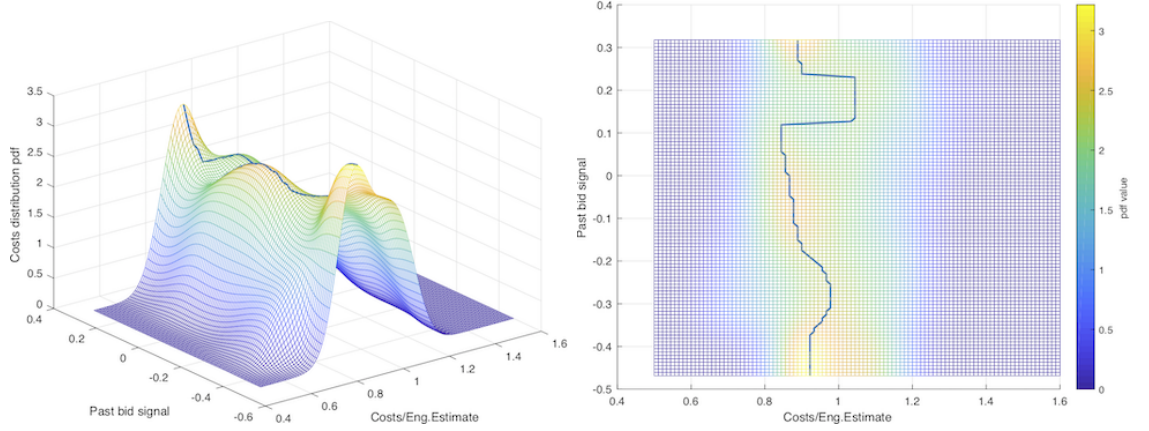
Figure 1.8: Conditional distribution of regular bidder costs in the pavement market



distribution conditional on several selected values of the signal observed in the data along with the marginal distribution of costs and the 95% confidence intervals for each of the estimates. The estimates for the bridge projects are presented on the left panel. For this market, though the cost distribution conditional on the median value of the signal is close to the marginal distribution estimate from the standard model, conditioning on the extreme values of the signal produces drastically different cost distribution estimates. In particular, the distribution conditional on the low value of the signal is significantly more precise and has a lower mean, while the distribution conditional on the high value of the signal is less precise and gives higher weight for high cost realizations in the current period. The estimates for the paving projects are presented on the right panel. The estimated conditional cost distributions for different values of the signal are closer together than similar distribution estimates for the bridge projects, have comparable means, and can not be differentiated from each other with the 95% confidence.

Figures 1.8 and 1.9 present the full range of the estimated conditional distri-

Figure 1.9: Conditional distribution of regular bidder costs in the pavement market



butions of costs for the regular bidders in the bridge and the pavement market. The panels on the left present the 3D view of the conditional distribution map, and the panels on the right trace the maximum of conditional probability distribution function (which in several cases does not represent the true mean due to dual peaked pdf estimate) and the variance in the cost-signal space. The estimated conditional distribution of costs of the regular bridge bidder is visibly more precise at the low end of the signal, and the mean does not shift between the minimum of the signal value -0.36 and -0.27. There is a sharp jump in the variance of the estimated distribution and a steady increase of the distribution mean after that. At the same time the estimated conditional distribution of costs of the regular pavement bidder is roughly equally precise for all values of the signal.

As the following bidding function example and the series of counterfactuals show, the difference in the conditional cost distributions leads to a sufficient difference in expected bidder behavior, and, as a result, expected procurement costs and the optimal behavior for the auctioneer.

### 1.4.3 Bid function example

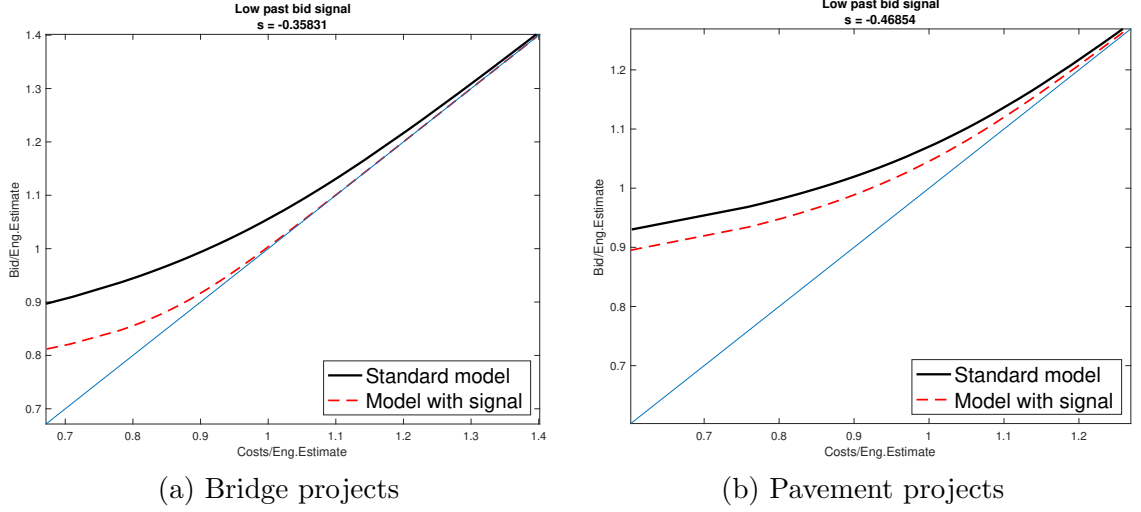
After obtaining the estimates of the conditional cost distributions for the regular bidders and the marginal cost distribution estimates for the fringe bidders one can attempt to solve a generic asymmetric first price auction for a given number of fringe bidders numerically using (1.3) and (1.5) with the appropriate boundary conditions. However it is useful to first look at a simpler approximation of the bidding functions using the parametric estimates of the bid distributions I have already established to inverse (2.1). This section provides an example of bidding function estimates for regular and fringe bidders in both the bridge and the pavement market following this strategy. In each case I estimate the bidding functions for the auctions with one regular and one fringe bidder, the mean values of backlog and distance for the regular bidder in the given market, and several different values of the signal value:

$$\begin{aligned}
X_1^R &= [1, s_{t-1}]; \\
X_2^R &= [1, \overline{\text{Backlog}_R}, \overline{\text{Distance}_R}, s_{t-1}]; \\
X_1^F &= X_2^F = [0, 1, s_{t-1}].
\end{aligned} \tag{1.11}$$

To obtain the bidding function estimates I solve the equations:

$$\begin{aligned}
c &= b - \left[ \frac{ng^F(b|s)}{1 - G^F(b|s)} \right]^{-1}, \\
c^i &= b^i - \left[ \frac{(n-1)g^F(b^i|s)}{1 - G^F(b^i|s)} + \frac{g^R(b^i|s)}{1 - G^R(b^i|s)} \right]^{-1},
\end{aligned} \tag{1.12}$$

Figure 1.10: Bid functions for fringe bidder, low signal,  $n_f = 1$



which we used to identify the costs earlier, for  $b$  and  $b^i$  using the parametric distribution of the bidding function and a sufficient number of random cost draws from the distributions estimated earlier.

Figure 1.10 presents the estimated bidding functions of the fringe bidders after observing a low value of the past bid signal in the bridge market in the top panel, and in the pavement market in the bottom panel. The solid line on each panel represents the bidding function approximated using the standard model, while the dashed lines represent the bidding functions approximated with the observed signal included in the model at every estimation stage. In both of the markets the fringe bidders bid more aggressively if the low signal is observed, yet the fringe bidder would reduce the bid by up to 10% if the low signal is observed in the bridge market, and only by roughly 4% in the pavement market.

Similar patterns hold for the regular bidders and different values of the signal. Figure 2.1 presents a larger set of the bidding function estimates for different types

of bidders in both the bridge and pavement markets. On each figure the panels on the left show the bidding functions for the extreme values of the signal observed prior to the auction: minimal observed signal at the top, and maximal observed signal at the bottom. The right panels show the bidding function estimates for the median observed value of the signal, which by construction is close to 0. Since the standard model does not take the value of the signal into account the solid lines representing the bidding functions derived from it are the same for the market-bidder type pair regardless of the value of the signal.

The top row of figures, [1.11a](#) and [1.11b](#), show the bidding function approximation for the regular bidders. In the case of one fringe competitor, the regular bidder strategy approximation differs significantly at lower cost values in the bridge market when the value of the past bid signal is included in the estimation. As expected, the regular bidder bids more aggressively if a low value of the signal was observed, and less aggressively if a high value of the signal was observed. At the same time there is no significant difference between the approximations of the regular bidder strategy under different values of the signal being released in the pavement market.

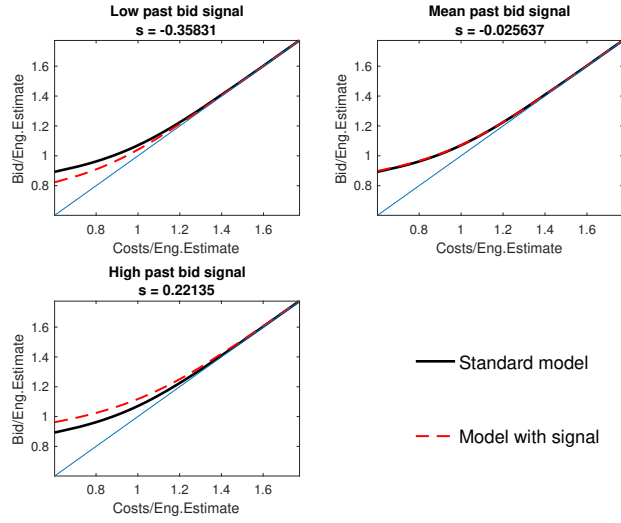
The same pattern is true for the fringe bidders and can be seen in figures [1.11c](#) and [1.11d](#). The only participating fringe bidder bids more aggressively upon observing a low signal about the regular bidder costs, and less aggressively upon observing a high signal in the bridge market. And though there is a slight difference in the fringe bidder strategy approximations for different signals observed in the pavement market, they are much smaller. The reaction of the fringe bidders to the signal is stronger than the reaction of the regular bidder. This can be explained by

the fact that the signal brings less new information to the regular bidder than to the fringe bidder, i.e. the variation between possible cost draws for the fringe bidder are more important for the regular bidder than the shift in the fringe bidder bidding function in response to the signal being released.

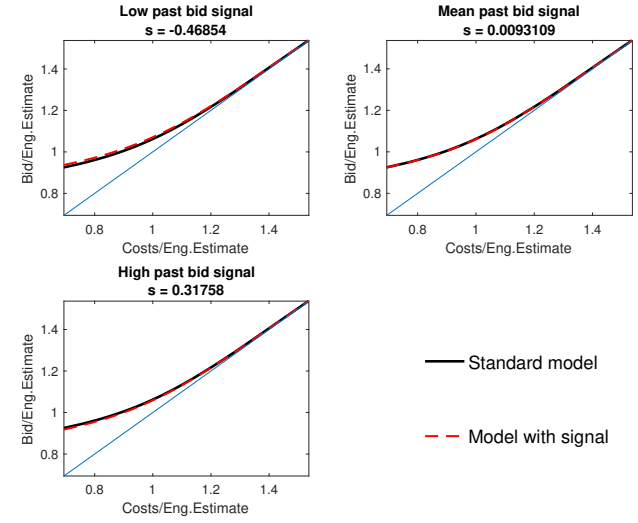
The effect that the past bid signal has on the bidding behavior of both the regular and the fringe bidders diminishes with competition. Figure 1.12 shows the estimated bidding functions of the fringe bidders conditional on the low value of the signal being observed when four fringe bidders participate in the auction, which is close to the level of competition in an average auction in both markets. With higher competition from the other fringe bidders the fringe bidder in bridge market, depicted on the left panel, would reduce her bid by approximately 5% when her costs are low compared to the 10% reduction when she is a sole fringe bidder in the auction. The fringe bidders in the pavement market, in the Figure 1.12b, do not change their bidding strategy conditional on observing a low past bid signal.

Figure 1.11: Estimated bidding functions for  $n_f = 1$

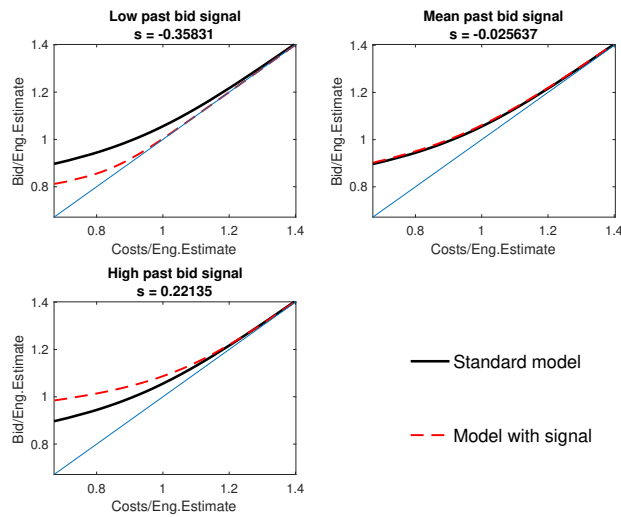
(a) Bridge market, regular bidder



(b) Pavement market, regular bidder



(c) Bridge market, fringe bidder



(d) Pavement market, fringe bidder

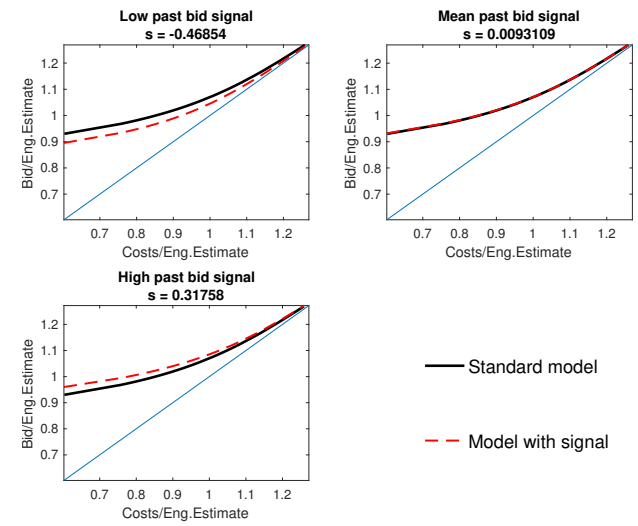


Figure 1.12: Bid functions for fringe bidders, low signal,  $n_f = 4$

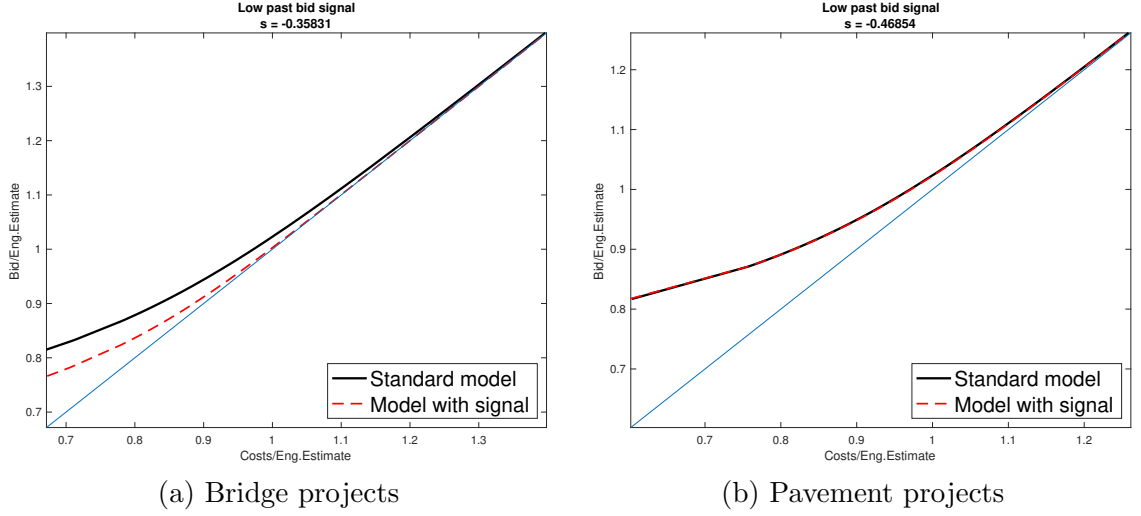


Figure 1.13 shows a set of estimated bidding functions for the auction with four fringe bidders conditional on several different values of the signal being observed in both markets. It shows that in the bridge market (Figures 1.13a and 1.13c) the effect of the past bid information also diminishes with competition for the regular bidder, and a high value of the signal being observed, yet is still present, especially for lower cost draws. The effect of the signal on the bidding function in the pavement market (Figures 1.13b and 1.13d) disappears.

To summarize, in an average auction firms change their bidding strategies depending on the value of the signal observed in the market with serially correlated bids, but not in the market with independent bids. When the past bid information matters, both regular and fringe firms bid more aggressively when a low bid signal is observed, and less aggressively when a high signal is observed.

Fringe bidders estimated bid functions are affected more strongly by the presence of the past bid signal in the model than the regular bidders. This can be ex-

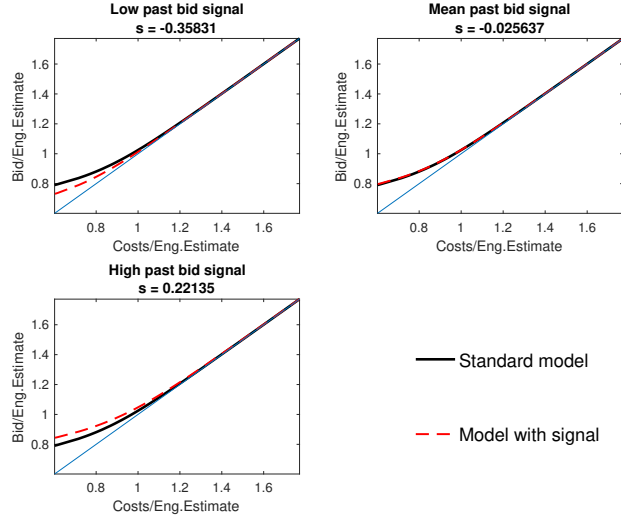


plained by the fact the the regular bidders receive less additional information with the signal, as they only have to take into account the deviation in the fringe bidders bidding strategies, while the fringe bidders take into account both the change in competitor bidding, and the change in their beliefs about the regular bidder costs.

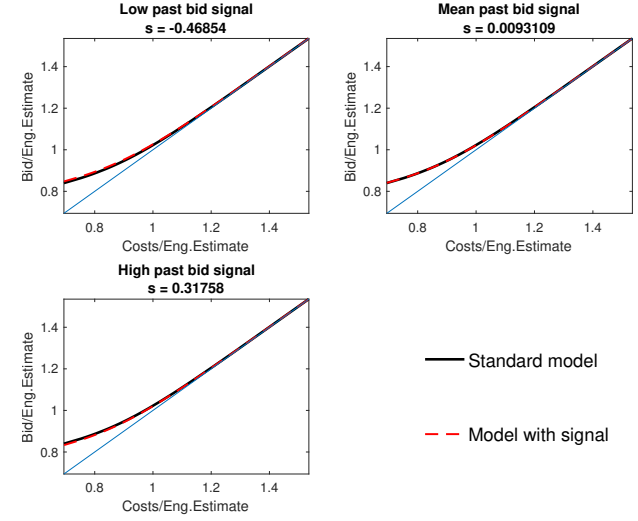
The corrections to the bidding strategies are larger for lower cost draws. Since both fringe and regular firms with lower cost draws would have higher probability of winning, they have both more incentives to adjust upwards when a high signal is observed, and more room to adjust when a low signal is observed. Finally, all of these effects get smaller when competition grows, but do not completely disappear.

Figure 1.13: Estimated bidding functions for  $n_f = 4$

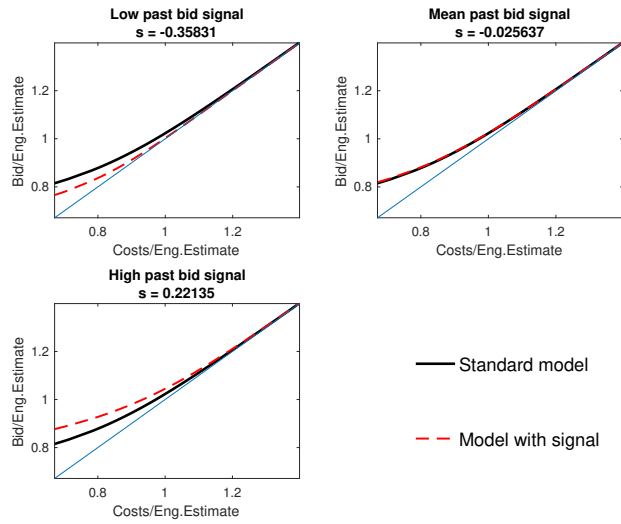
(a) Bridge market, regular bidder



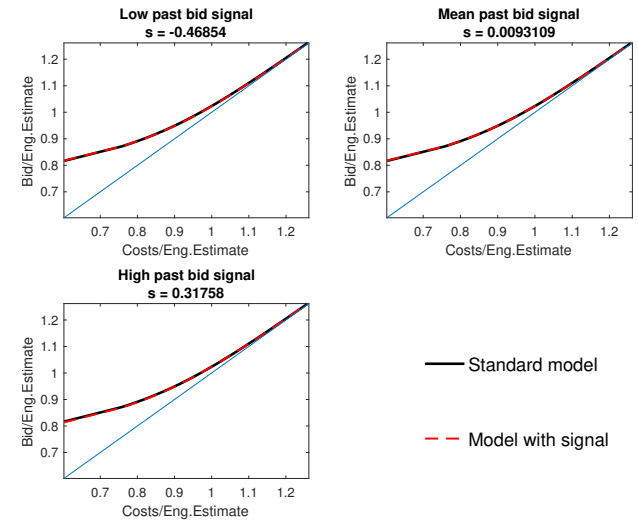
(b) Pavement market, regular bidder



(c) Bridge market, fringe bidder



(d) Pavement market, fringe bidder



## 1.5 Counterfactual analysis

The differences in the bidding function estimates shown in the previous section are sufficient to imply that the different access to the past bid information may lead to large differences in the auction outcomes. In this section I use the estimates of the cost distributions of all players to solve asymmetric first price auctions under four different information regimes which I would refer to as the standard model, the public signal model, the naïve auctioneer model, and the informed auctioneer model. In addition I allow the auctioneer to set a binding reserve price and find the optimal auction outcomes in terms of minimal expected procurement costs for each of the four information regimes.

The difference between the four information regimes is summarized in Table [1.9](#). Each of the models is characterized by whether the auctioneer and/or the fringe bidders have access to the past bid information and internalize it in their optimization decisions. If the fringe bidders have access to the past bid information they would use the true cost distribution conditional on the value of the signal as their belief about the regular bidder costs when solving for the optimal bidding function. If the fringe bidders have no access to the past bid information they would use the marginal cost distribution as their belief about the regular bidder. In the same way if the auctioneer observes the past bid information she would use the true conditional cost distribution as her belief about the regular bidder costs. The standard model assumes that neither the auctioneer nor the fringe bidders have access to the past bid information, the public signal model assumes that both the auctioneer and

the fringe bidders have access to this information. The naïve auctioneer and the informed auctioneer models assume in turn that either the auctioneer or the fringe bidders do not use the past bid information in their decision-making while the other party does.

Table 1.9: Cost distribution estimates under different information regimes

		Fringe bidders	
		Uninformed, Marginal distribution	Informed, Conditional distribu- tion
Auctioneer	Uninformed, Marginal distribution	Standard model	Naïve auctioneer
	Informed, Conditional distribu- tion	Informed auctioneer	Public signal

### 1.5.1 Solving for the counterfactual equilibrium and the optimal reserve price

In the structural estimation part of this chapter I have implicitly assumed that the Oklahoma DoT auctions are run without a binding reserve price, and that the observed bids are generated by either the standard model or the public signal model.

Introducing a binding reserve price would change the system of equations defining the BNE of the game to:

$$\begin{aligned}
\frac{nz(\zeta^{-1}(b))\zeta'^{-1}(b)}{1 - Z(\zeta^{-1}(b))} &= \frac{1}{b - \beta^{-1}(b)}, \\
\frac{(n-1)z(\zeta^{-1}(b^i))\zeta'^{-1}(b^i)}{1 - Z(\zeta^{-1}(b^i))} + \frac{f(\beta^{-1}(b^i))\beta'^{-1}(b^i)}{1 - F(\beta^{-1}(b^i))} &= \frac{1}{b^i - \zeta^{-1}(b^i)},
\end{aligned} \tag{1.13}$$

with the boundary conditions for the bidding functions:

$$\begin{aligned}\beta^{-1}(R) &= \zeta^{-1}(R) = R, \\ \beta^{-1}(\underline{b}) &= \zeta^{-1}(\underline{b}) = \underline{c}\end{aligned}\tag{1.14}$$

Since the closed form solution of this system of differential equations does not exist in the general case, and both of the equations are not well defined at the boundary points making standard numerical methods for solving the systems of differential equations less reliable, polynomial approximation methods are usually used to solve this type of problem. In particular, I use the method described in [Hubbard et al. \(2013a\)](#) adjusted for the procurement auctions and the presence of a binding reserve price.

This method involves approximating the inverse bidding functions by Chebyshev polynomials and use the MPEC approach to solve the following problem for the auction with reserve price  $R$ ,  $n$  fringe bidders, and one regular bidder, assuming

common cost distribution support  $[\underline{c}, \bar{c}]$  for both types of bidders<sup>9</sup>:

$$\begin{aligned}
& \min_{\substack{\beta^{-1}(b), \\ \zeta^{-1}(b); \underline{b}}} [FOC_R^2 + FOC_F^2] \\
\text{s.t.} \quad & \beta^{-1}(R) = \zeta^{-1}(R) = R & \text{(a) Left boundary conditions} \\
& \beta^{-1}(\underline{b}) = \zeta^{-1}(\underline{b}) = \underline{c} & \text{(b) Right boundary conditions} \\
& z(\underline{c})\zeta'^{-1}(\underline{b}) = \frac{1}{\underline{b} - \underline{c}} & \text{(c) Right boundary condition for regular bidder FOC} \\
& f(\underline{c})\beta'^{-1}(\underline{b}) + (n-1)z(\underline{c})\zeta'^{-1}(\underline{b}) = \frac{1}{\underline{b} - \underline{c}} & \text{(d) Right boundary condition for fringe bidder FOC} \\
& \beta'^{-1}(R) = \zeta'^{-1}(R) = \frac{n+1}{n} & \text{(e) Left boundary condition for regular and fringe bidder FOCs} \\
& \beta'^{-1}(b) \geq 0, \zeta'^{-1}(b) \geq 0 \quad \forall b \in [\underline{b}, R] & \text{(f) Monotonicity} \\
& b \geq \beta^{-1}(b), b \geq \zeta^{-1}(b) \quad \forall b \in [\underline{b}, R] & \text{(g) Rationality}
\end{aligned} \tag{1.15}$$

where:

$$FOC_R^2 = \left[ \frac{n(b - \beta^{-1}(b))z(\zeta^{-1}(b))\zeta'^{-1}(b)}{1 - Z(\zeta^{-1}(b))} - 1 \right]^2, \tag{1.16}$$

and

$$FOC_F^2 = \sum_{i=1}^n \left[ \frac{(n-1)(b^i - \zeta^{-1}(b^i))z(\zeta^{-1}(b^i))\zeta'^{-1}(b^i)}{1 - Z(\zeta^{-1}(b^i))} + \frac{((b^i - \zeta^{-1}(b^i))f(\beta^{-1}(b^i))\beta'^{-1}(b^i))}{1 - F(\beta^{-1}(b^i))} - 1 \right]^2 \tag{1.17}$$

Conditions (15.c) - (15.e) introduced in [Fibich et al. \(2002\)](#) apply LHospitals

rule to estimate the first order conditions for each bidder type at the boundary

---

<sup>9</sup>Though only conditions (a) and (b) are necessary, conditions (c), (d), and (e) hold if the FOCs are satisfied, and conditions (f) and (g) are assumed for the existence of PBNE in the asymmetric auction game. Including these conditions improves the quality of the estimates.

points. Conditions (15.f) and (15.g) define the desirable properties of the solution and are applied at each approximation point. The KNITRO solver with the help of the NEOS<sup>10</sup> server is used to handle the nonlinear nature and the large number of the constraints.

After obtaining the estimates for the inverse bidding functions, the expected price of the contract is calculated as:

$$\mathbb{E}P(R) = \int_{\underline{b}}^R x d[1 - (1 - F(\zeta_R^{-1}(x)))(1 - Z(\beta_R^{-1}(x)))^n], \quad (1.18)$$

which can be rewritten in terms of the expected probability of zero bidders submitting a bid,  $\Phi(x) = (1 - F(\zeta_R^{-1}(x)))(1 - Z(\beta_R^{-1}(x)))^n$ :

$$\mathbb{E}P(R) = \int_{\underline{b}}^R \Phi(x) dx - R\Phi(R) + \underline{b}. \quad (1.19)$$

Since lowering the reserve price down to the lower bound of the cost support,  $\underline{c}$ , would bring the expected price of the contract to zero, it is important to account for the potential harm that holding a void auction can cause the auctioneer. To do this I introduce a penalty for not determining the contractor in the current round of auctions. I assume that the auctioneer loses twice the value of the engineer's estimate if the auction fails<sup>11</sup>, and hence the auctioneer the expected cost of procurement

---

<sup>10</sup>See Czyzyk et al. (1998), Dolan (2001), and Gropp and Moré (1997) for details on the NEOS server; Byrd et al. (2006) for using KNITRO for nonlinear optimization.

<sup>11</sup>Different penalty coefficients were also tested. The results under different values of the penalty maintain the same order, yet call for less realistic values of the optimal reserve price.

Table 1.10: Counterfactual estimation results for the Standard model,  $n_f = 4$

Reserve price	Expected price	Probability of sale	Expected cost	Exp. markup of regular bidder	Exp. markup of fringe bidder
Bridge projects					
None	1.08	1.00	1.08	4.0%	2.3%
0.99	0.93	0.98	0.97	2.3%	1.6%
Pavement projects					
None	1.07	1.00	1.07	3.4%	2.6%
0.99	0.94	0.99	0.97	2.0%	1.4%

\*All reserve prices are optimal reserve prices

calculated as:

$$\mathbb{E}C(R) = \mathbb{E}P(R) + 2\Phi(R) \quad (1.20)$$

For each of the information regimes I search for the optimal reserve price by solving the auctions over a finite grid of candidate reserve prices, calculating the values of expected contract price and the probability of void auction, and finding the minimum of  $\mathbb{E}C(R)$  over this grid.

### 1.5.2 The standard model

I present the results of the counterfactual simulations starting with the benchmark auction outcomes predicted by the standard model in Table 1.10. For each market the first row shows the simulated outcomes without a binding reserve price, and the second row shows an optimal reserve price which minimizes the expected cost of procurement and the simulated auction outcomes under this reserve price. Under the assumptions of the standard model bridge and pavement market out-



comes are very similar both with and without the optimal reserve price, with the exception of expected firm markups. Setting an optimal reserve price saves 10% of the engineer’s estimate in the expected costs of procurement in both markets. The optimal reserve price is the same in both markets, so the auctioneer can set the same optimal reserve price for both types of auctions.<sup>12</sup> Finally, the bidder asymmetry, as measured by the difference in expected markups, is more prominent in the bridge market.

### 1.5.3 The public signal model

Accounting for the presence of the past bid information changes some of the key model predictions and highlights the difference between the two markets. The results of the public signal model simulation are presented in Table 1.11. The simulated auction outcomes in two markets are still similar if there is no binding reserve price, but diverge significantly when the optimal reserve price comes into consideration. In particular, the optimal reserve price in the bridge market varies depending on the signal observed, with the lowest optimal reserve of 92% of the engineer’s estimate being set when the minimal signal is observed. The optimal reserve price in the pavement market stays largely the same regardless of the value of the signal. As a consequence, the simulated auction outcomes under optimal reserve are vastly different between the bridge and the pavement market, with both expected price of the contract and expected procurement costs being much lower

---

<sup>12</sup>Here and throughout I present the counterfactual results up to second decimal due to the limited precision of the grid search procedure.

Table 1.11: Counterfactual estimation results for the Public signal model,  $n_f = 4$

Reserve price	Expected price	Probability of sale	Expected cost	Exp. markup of regular bidder	Exp. markup of fringe bidder
Bridge projects					
None	$1.06^{\dagger} - 1.22^{\ddagger}$	1.00	$1.06^{\dagger} - 1.22^{\ddagger}$	$6.9\%^{\dagger} - 3.7\%^{\ddagger}$	$2.1\%^{\dagger} - 2.5\%^{\ddagger}$
$0.92^{\dagger} - 0.99^{\ddagger}$	$0.88^{\dagger} - 0.93^{\ddagger}$	$0.99^{\dagger} - 0.98^{\ddagger}$	$0.90^{\dagger} - 0.97^{\ddagger}$	$3.4\%^{\dagger} - 1.2\%^{\ddagger}$	$0.9\%^{\dagger} - 1.6\%^{\ddagger}$
Pavement projects					
None	$1.07^{\dagger} - 1.19^{\ddagger}$	1.00	$1.07^{\dagger} - 1.19^{\ddagger}$	$2.6\%^{\dagger} - 2.3\%^{\ddagger}$	$2.5\%^{\dagger} - 2.2\%^{\ddagger}$
$0.99^{\dagger} - 0.98^{\ddagger}$	$0.93^{\dagger} - 0.92^{\ddagger}$	$0.99^{\dagger} - 0.98^{\ddagger}$	$0.97^{\dagger} - 0.96^{\ddagger}$	$1.1\%^{\dagger} - 1.5\%^{\ddagger}$	$1.4\%^{\dagger} - 1.3\%^{\ddagger}$

\*All reserve prices are optimal reserve prices ;  $^{\dagger}$  Conditional on minimum realization of the signal;  $^{\ddagger}$  Conditional on maximum realization of the signal.

when a low value of the signal is observed.

Setting an optimal reserve is even more important within the public signal model and brings between 16% and 25% of savings in expected costs in the bridge market, and 10% to 23% in the pavement market.

The asymmetry between bidders in the bridge market is even more prominent under the public signal model, especially when no binding reserve price is set. At the same time, the bidders in the pavement market get closer expected markups in the public signal model than in the standard model.

Finally, using the standard model to simulate the auction outcomes when the public signal game is played in reality would lead to a misestimation of expected procurement costs by up to 14% without a binding reserve price, and up to 7% with the optimal reserve price.

#### 1.5.4 The naïve auctioneer model

Since the optimal reserve price changes depending on the value of the signal in the bridge market, failing to account for this information in setting the optimal reserve price should lead some losses in the expected procurement cost. Comparing the results of the naïve auctioneer model simulation in Table 1.12 with the results in Table 1.11 shows that the auctioneer would lose up to 3% in expected procurement costs or 4% in expected contract price by setting a sub-optimal reserve price in the bridge market. The fact that the auctioneer does not use all of the information available also brings additional markups to all firms in the market with the low-signal regular bidder gaining additional 0.8% compared to the public signal model, and the fringe bidders gaining between 0.5% and 0.1% depending on the value of the signal. At the same time the main outcomes of the pavement auctions, including expected firm markups do not change significantly between the public signal and the naïve auctioneer model.

#### 1.5.5 The informed auctioneer model

Finally, I estimate the cost of information transparency in each of the markets by calculating the outcomes of the informed auctioneer model. The results of this model are presented in Table 1.13.

Concealing the past bid information from the fringe bidders reduces the variation in the expected costs if no binding reserve price is set in either of the markets.

Table 1.12: Counterfactual estimation results for the Naïve auctioneer model,  $n_f = 4$ 

Reserve price	Expected price	Probability of sale	Expected cost	Exp. markup of regular bidder	Exp. markup of fringe bidder
Bridge projects					
None	$1.06^{\dagger} - 1.22^{\ddagger}$ (1.08)	1.00 (1.00)	$1.06^{\dagger} - 1.22^{\ddagger}$ (1.08)	$6.9\%^{\dagger} - 3.7\%^{\ddagger}$	$2.1\%^{\dagger} - 2.5\%^{\ddagger}$
0.99	$0.92^{\dagger} - 0.93^{\ddagger}$ (0.93)	$1.00^{\dagger} - 0.98^{\ddagger}$ (0.98)	$0.93^{\dagger} - 0.97^{\ddagger}$ (0.97)	$4.2\%^{\dagger} - 1.2\%^{\ddagger}$	$1.2\%^{\dagger} - 1.7\%^{\ddagger}$
Pavement projects					
None	$1.07^{\dagger} - 1.19^{\ddagger}$ (1.07)	1.00 (1.00)	$1.07^{\dagger} - 1.19^{\ddagger}$ (1.07)	$2.6\%^{\dagger} - 2.3\%^{\ddagger}$	$2.5\%^{\dagger} - 2.2\%^{\ddagger}$
0.99	$0.93^{\dagger} - 0.93^{\ddagger}$ (0.94)	$0.99^{\dagger} - 0.99^{\ddagger}$ (0.99)	$0.97^{\dagger} - 0.97^{\ddagger}$ (0.97)	$1.1\%^{\dagger} - 1.5\%^{\ddagger}$	$1.4\%^{\dagger} - 1.3\%^{\ddagger}$

\*All reserve prices are optimal reserve prices ;  $^{\dagger}$  Conditional on minimum realization of the signal;  $^{\ddagger}$  Conditional on maximum realization of the signal; Auctioneer-predicted numbers in parenthesis.

Table 1.13: Counterfactual estimation results for the Informed auctioneer model,  $n_f = 4$ 

Reserve price	Expected price	Probability of sale	Expected cost	Exp. markup of regular bidder	Exp. markup of fringe bidder
Bridge projects					
None	$1.06^{\dagger} - 1.11^{\ddagger}$	1.00	$1.06^{\dagger} - 1.11^{\ddagger}$	$3.9\%^{\dagger} - 4.1\%^{\ddagger}$	$2.3\%^{\dagger} - 2.3\%^{\ddagger}$
$0.91^{\dagger} - 0.99^{\ddagger}$	$0.87^{\dagger} - 0.93^{\ddagger}$	$0.91^{\dagger} - 0.98^{\ddagger}$	$0.91^{\dagger} - 0.97^{\ddagger}$	$1.4\%^{\dagger} - 2.3\%^{\ddagger}$	$1.0\%^{\dagger} - 1.6\%^{\ddagger}$
Pavement projects					
None	$1.08^{\dagger} - 1.10^{\ddagger}$	1.00	$1.08^{\dagger} - 1.10^{\ddagger}$	$3.5\%^{\dagger} - 3.7\%^{\ddagger}$	$2.6\%^{\dagger} - 2.6\%^{\ddagger}$
$0.99^{\dagger} - 0.99^{\ddagger}$	$0.93^{\dagger} - 0.93^{\ddagger}$	$0.99^{\dagger} - 0.99^{\ddagger}$	$0.97^{\dagger} - 0.96^{\ddagger}$	$2.0\%^{\dagger} - 1.8\%^{\ddagger}$	$1.4\%^{\dagger} - 1.4\%^{\ddagger}$

\*All reserve prices are optimal reserve prices ;  $^{\dagger}$  Conditional on minimum realization of the signal;  $^{\ddagger}$  Conditional on maximum realization of the signal.

In the bridge market it also reduces the low-signal regular bidder markup by 3-4% without the binding reserve price, or 1-2% with the optimal reserve price. It also reduces the expected procurement costs by 1% compared to the public signal model.

There are no notable differences between the outcomes of the informed auctioneer and the public signal models for the pavement market.

## 1.6 Conclusions

This chapter studies the effect of the availability of information about past auctions on the current auction in two highway procurement markets. I show that the information about the past bids of the participants has a significant effect on their bidding behavior in the market for bridge construction and repair projects where bids, and, consequently, contract fulfillment costs are correlated across time. In particular, all bidders bid more aggressively if the signal about low cost of one of the bidders in the previous period was received, and less aggressively if the high signal was received. There is no significant effect of the past bid information on bidder behavior in the market for pavement projects where there is no correlation between firm bids and construction costs over time.

I also show that it is beneficial for the auctioneer to account for the existence of the dynamic nature of the market when setting the optimal reserve price. Using the standard model to calculate the optimal reserve price, or in other words, being the “naïve auctioneer” would bring a loss of 2-3% of the engineer’s estimate in expected cost of procurement and up to 4% of the engineer’s estimate in expected contract price in the markets with autocorellated costs.

Throughout the chapter I assume that all players are myopic, and that there is no endogenous entry to the auction. Relaxing these assumptions might change the conclusions of this chapter. If the players are not myopic, a forward looking regular firm might reduce the spread of its bids to conceal the information about its costs and have an information advantage in the next period. This would reduce the

effect of the past bid information on current bids. On the other hand, if we allow for endogenous entry, a low cost signal would deter entry by the fringe bidders, reducing the gains of releasing the information for the auctioneer. At the same time the high cost signal would encourage entry, and reduce the auctioneer's losses of releasing the information.

## 1.7 Appendix

### 1.7.1 Sample selection and variable construction

I started with 1181 bridge and pavement auctions in Oklahoma from April 2000 to August 2003. I dropped 209 auctions with less than two recorded bidders, and 76 auctions in which no winner was determined by the Oklahoma DoT. In addition, I dropped 19 auctions in which the winning bid was either too high (higher than 150% of the engineer's estimate), or too low (less than 40% of the engineer's estimate), and 8 auctions in which the maximum submitted bid is higher than 250% of the engineer's estimate. Since the engineer's estimate was publicly available to bidders before the auction, and the winning bids are within reason, these high bids do not look strategic, and dropping just the extreme bids without dropping the full auction data would distort the estimates.

### 1.7.2 Construction of the bid residual

I use the lagged average bid residuals in my estimates. The bid residuals are constructed from an OLS regression of normalized bids on the set of observed auction and bidder characteristics including number of plan holders, auction date, size quintile based on the engineer's estimate, project location, and the bidder's backlog and distance to the project.

Table 1.14: Construction of the bid residual

Variables	Bid divided by eng. estimate
Number of planholders	0.0072***
	0.0016
Backlog	0.0296*
	0.0167
Distance	0.0002***
	0.0001
$N$	5537
adj. $R^2$	0.1791

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively.

Standard errors in parentheses.

Firm ID, date, project size, and project type fixed effects included

### 1.7.3 Reduced form regressions with average bid as a signal

Table 1.15: Bridge projects with average bid signal

	Full sample (1)	Full sample (2) <sup>†</sup>	Full sample ID FE (3)	Regular bidder (4)	Fringe bidders
N (fringe) bidders	−0.0055* (0.0033)	−0.0074** (0.0033)	−0.0061* (0.0033)	−0.0176** (0.0071)	−0.0066* (0.0036)
Distance	0.0002*** (0.0000)	0.0002*** (0.0000)	0.0001 (0.0001)	−0.0001 (0.0002)	0.0002*** (0.0000)
Backlog	0.0017 (0.0217)	−0.0153 (0.0217)	0.0418* (0.0245)	0.0641 (0.0524)	−0.0230 (0.0237)
Lagged average bid		0.2339*** (0.0426)	0.1986*** (0.0417)	0.2950*** (0.0944)	0.2298*** (0.0466)
Regular participation dummy	−0.0357** (0.0141)	−0.0273* (0.0148)	−0.0030 (0.0158)		−0.0272* (0.0154)
$N$	1556	1428	1410	181	1246
adj. $R^2$	0.1220	0.1475	0.2618	0.2054	0.1366

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.



Table 1.16: Pavement projects with average bid signal

	Full sample (1)	Full sample (2) <sup>†</sup>	Full sample ID FE (3)	Regular bidder (4)	Fringe bidders
N (fringe) bidders	-0.0107*** (0.0024)	-0.0115*** (0.0024)	-0.0108*** (0.0026)	-0.0003 (0.0125)	-0.0120*** (0.0025)
Distance	0.0000 (0.0000)	0.0000 (0.0000)	-0.0000 (0.0001)	-0.0000 (0.0002)	0.0000 (0.0000)
Backlog	-0.0260* (0.0149)	-0.0234 (0.0153)	0.0196 (0.0184)	-0.1835*** (0.0679)	-0.0175 (0.0158)
Lagged average bid		0.0264 (0.0308)	0.0247 (0.0302)	0.0550 (0.0971)	0.0217 (0.0323)
Regular participa- tion dummy	-0.0409*** (0.0093)	-0.0393*** (0.0103)	-0.0255** (0.0120)		-0.0406*** (0.0104)
<i>N</i>	1441	1378	1356	110	1268
adj. <i>R</i> <sup>2</sup>	0.1475	0.1461	0.2186	0.2232	0.1444

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses. All models include contract size and location fixed effects; † includes firm fixed effects.

#### 1.7.4 Gamma-Weibul log-likelihood function

The Gamma-Weibul log-likelihood function for a given auction is:

$$\begin{aligned}
\ln(\mathcal{L}_t) = & (n_{Rt} + n_{Ft})\ln(\theta) + \ln\left[\Gamma\left(\frac{1}{\theta} + n_{Rt} + n_{Ft}\right)\right] - \ln\left[\Gamma\left(\frac{1}{\theta}\right)\right] + \\
& + \sum_{i=1}^{n_{Rt}+n_{Ft}} \ln\left[\frac{\rho_{it}}{\lambda_{it}}\left(\frac{b_{it}}{\lambda_{it}}\right)^{\rho_{it}-1}\right] - \\
& - \left(\frac{1}{\theta} + n_{Rt} + n_{Ft}\right)\ln\left[1 + \theta \sum_{i=1}^{n_{Rt}+n_{Ft}} \left(\frac{b_{it}}{\lambda_{it}}\right)^{\rho_{it}}\right],
\end{aligned} \tag{1.21}$$

where  $n_{Rt}$  is the number of regular bidders (in my application is equal to 0 or 1),

and  $n_{Ft}$  is the number of fringe bidders.

### 1.7.5 Numerical integration details

I use alternative extended Simpson's rule to calculate the expected price of the contract:

$$\int_a^b f(x) = \frac{b-a}{28N} [17f(x_0) + 59f(x_1) + 43f(x_2) + 49f(x_3) + 48 \sum_{i=4}^{N-4} f(x_i) + 49f(x_{N-3}) + 43f(x_{N-2}) + 59f(x_{N-1}) + 17f(x_N)] \quad (1.22)$$

The integration is performed in the bid space since values of the functions  $F(\zeta^{-1}(b))$  and  $Z(\beta^{-1}(b))$  on the uniform  $b$ -grid are calculated in the process of numerically solving the auction at hand. The approximate value of expected price is calculated as:

$$\mathbb{E}P(R) = \frac{R - \underline{b}}{28N} [17\Phi(b_0) + 59\Phi(b_1) + 43\Phi(b_2) + 49\Phi(b_3) + 48 \sum_{i=4}^{N-4} \Phi(b_i) + 49\Phi(b_{N-3}) + 43\Phi(b_{N-2}) + 59\Phi(b_{N-1}) + 17\Phi(b_N)] - R\Phi(R) + \underline{b}, \quad (1.23)$$

where  $R$  is the reserve price,  $\underline{b}$  is the minimal calculated bid, and  $\Phi(b) = (1 - F(\zeta^{-1}(b)))(1 - Z(\beta^{-1}(b)))^n$ , and  $n$  is the number of participating fringe bidders.

### 1.7.6 Typical auction documentation published before and after the auction

Figure 1.14: Typical project description and bids: Bridge project

(a) Short Form

```

-----
                                FEDERAL PROJECT
CALL ORDER   : 045                CONTRACT ID : 020321
LETTING DATE : June 20, 2002      PCN          : 1804604
LETTING TIME : 10:30 A.M.
COUNTIES: CHOCTAW                PROJECTS    : BRF-112C(26) CO
                                   DBE/WBE GOALS: 10.00
                                   GUARANTY   : 5% OF BID TOTAL
                                   PLAN PRICE: $ 4.34
                                   RES. ENG.:
                                   CALENDAR DAYS : 90
                                   LIQ. DAM: $ 500

CONTRACT DESCRIPTION:
BRIDGE AND APPROACHES
COUNTY BRIDGE OVER OWL CREEK, 1.1 MILES SOUTH
AND 2.7 MILES EAST OF GRANT.
PROJECT LENGTH = 0.264
-----

```

(b) Long Form

```

-----
                                FEDERAL PROJECT                                PAGE 5
CALL ORDER   : 045                CONTRACT ID : 020321
LETTING DATE : June 20, 2002      PCN          : 1804604
LETTING TIME : 10:30 A.M.         PROJECTS    : BRF-112C(26) CO
COUNTIES    : CHOCTAW            DBE GOALS   : 10.00
                                   GUARANTY   : 5% OF BID TOTAL
                                   PLAN PRICE: $ 4.34
                                   CALENDAR DAYS : 90
                                   RES. ENG.:
                                   LIQ. DAM.: $ 500

CONTRACT DESCRIPTION:
BRIDGE AND APPROACHES COUNTY BRIDGE OVER OWL CREEK, 1.1
MILES SOUTH AND 2.7 MILES EAST OF GRANT. PROJECT LENGTH = 0.
264
-----

```

ITEM NO.	DESCRIPTION	QUANTITY	UNITS
SECTION NO. 0001 E000-ROADWAY			
201 0102	CLEARING AND GRUBBING	1.000	LSUM
202(G) 0185	EARTHWORK	1.000	LSUM
205 4229	TYPE A-SALVAGED TOPSOIL	1.000	LSUM
227 0100	TEMPORARY SILT DIKE	3,449.000	LF
230(A) 2806	SOLID SLAB SODDING	5,789.000	SY
233(A) 2817	VEGETATIVE MULCHING	2.400	AC
403(E) 0225	TRAFFIC BOUND SURFACE COURSE TYPE E	710.000	TON
509(D) 0325	CLASS C CONCRETE	15.000	CY
SECTION NO. 0002 X081- BRIDGE 'A'			
501(B) 1307	SUBSTRUCTURE EXCAVATION COMMON	48.000	CY
501(D) 6353	UNCLASSIFIED BACKFILL	134.000	CY
503(B) 6144	PRESTRESSED CONCRETE DOUBLE TEE	272.500	LF
504(E) 1380	CONCRETE RAIL (TR1)	176.000	LF
509(A) 1326	CLASS AA CONCRETE	37.600	CY
509(B) 1328	CLASS A CONCRETE	51.800	CY
511(A) 1332	REINFORCING STEEL	11,920.000	LB
514(A) 6010	PILES, FURNISHED (HP 10X42)	340.000	LF
514(B) 6292	PILES, DRIVEN (HP 10X42)	340.000	LF
601(A-1) 1353	TYPE I-A PLAIN RIPRAP	470.000	TON
601(A-2) 1355	TYPE I-A FILTER BLANKET	110.000	TON

(c) Submitted bids

PAGE : 045 -1

TABULATION OF BIDS

CALL ORDER : 045

Figure 1.15: Typical project description and bids: Pave project

(a) Short Form

```

-----
STATE PROJECT
CALL ORDER : 050 CONTRACT ID : 020224
LETTING DATE : June 20, 2002 PCN : 1825604
LETTING TIME : 10:30 A.M.
COUNTIES: GARVIN

PROJECTS : CR-125C(58)
DBE/WBE GOALS: 0.00
GUARANTY : 5% OF BID TOTAL
PLAN PRICE: $ 1.63
RES. ENG.:
CALENDAR DAYS : 30
LIQ. DAM: $ 500

CONTRACT DESCRIPTION:
RESURFACE (ASPHALT)
COUNTY ROAD BEGIN 10.0 MILES SOUTH OF THE SH-19/SH-133 JCT.,
EXTEND NORTH
PROJECT LENGTH= 5.978 MILES
FLEXIBLE ESALS= 0.47 MILLION
ASPHALT GRADE= PG 84-22 OK

```

(b) Long Form

```

-----
STATE PROJECT PAGE 7
CALL ORDER : 050 CONTRACT ID : 020224
LETTING DATE : June 20, 2002 PCN : 1825604
LETTING TIME : 10:30 A.M. PROJECTS : CR-125C(58)
COUNTIES : GARVIN GUARANTY : 5% OF BID TOTAL
PLAN PRICE : 1.63
CALENDAR DAYS : 30
RES. ENG.:
LIQ. DAM.: $ 500

CONTRACT DESCRIPTION:
RESURFACE (ASPHALT) COUNTY ROAD BEGIN 10.0 MILES SOUTH OF
THE SH-19/SH-133 JCT., EXTEND NORTH PROJECT LENGTH= 5.978
MILES FLEXIBLE ESALS= 0.47 MILLION ASPHALT GRADE= PG 84-22
OK

```

ITEM NO.	DESCRIPTION	QUANTITY	UNITS
SECTION NO. 0001 I000-ROADWAY			
202(G) 0185	EARTHWORK	1.000	LSUM
210 0121	OBLITERATING ABANDONED ROAD	3.000	STA
230(A) 2806	SOLID SLAB SODDING	7,014.000	SY
307(A) 4233	LIME	1,086.000	TON
307(B) 4234	6" LIME TREATED SUBGRADE	67,021.000	SY
311(C) 0153	PROCESS EXISTING BASE & SURFACE, METHOD C	211.640	STA
403(E) 0225	TRAFFIC BOUND SURFACE COURSE TYPE E	436.000	TON
407 0250	TACK COAT	1,271.000	GAL
408 5774	PRIME COAT	20,901.000	GAL
411(B) 6530	(SP) ASPHALT CONCRETE TYPE B (PG 64-22 OK)	13,600.000	TON
509(B) 0321	CLASS A CONCRETE	20.130	CY
509(D) 0325	CLASS C CONCRETE	10.000	CY
511(A) 0332	REINFORCING STEEL	2,186.000	LB
601(D) 1390	TYPE IV GROUTED RIPRAP	20.000	SY
619(B) 0291	REMOVAL OF HEADWALL	3.000	EA
850(A) 8110	SHEET ALUMINUM SIGNS	253.000	SF
851(D) 8597	FLANGE CHANNEL POSTS (4 LB/FT)	805.000	LF
853 9030	DELINEATORS (TYPE 1, CODE 3)	22.000	EA
855(A) 8812	TRAFFIC STRIPE (PLASTIC) (4" WIDE)	118,365.000	LF
880(J) 8905	CONSTRUCTION TRAFFIC CONTROL	1.000	LSUM

(c) Submitted bids

TABULATION OF BIDS

```

CALL ORDER : 050 CONTRACT ID : 020224 COUNTIES : GARVIN
CONTRACT DESCRIPTION: 12 10:30 A.M. DISTRICT : 32
RESURFACE (ASPHALT) CONTRACT TIME : 90 CALENDAR DAYS
COUNTY ROAD BEGIN 10.0 MILES SOUTH OF TREASH-19/SH-133 JCT., PROJECT(S) : CR-125C(58)
EXTEND NORTH
PROJECT LENGTH= 5.978 MILES
FLEXIBLE ESALS= 0.47 MILLION
ASPHALT GRADE= PG 84-22 OK

SET-ASIDE :
VENDOR RANKING :

```

RANK	VENDOR NO./NAME	TOTAL BID	% OVER LOW BID	% OVER EST
0	-EST- ENGINEER'S ESTIMATE	\$ 1,095,278.50	118.2067%	100.0000%
1	731452852 OVERLAND CORPORATION	\$ 926,579.35	100.0000%	84.5976%
2	581401463 APAC-OKLAHOMA, INC.	\$ 976,743.31	105.4139%	89.1776%
3	730626847 THE CUMMINS CONST. CO., INC.	\$ 995,425.75	107.4302%	90.8833%
4	730574091 BROCE CONSTRUCTION CO., INC.	\$ 1,135,440.50	122.5411%	103.6668%

## Chapter 2: Partial pooling in repeated auctions with autocorrelated costs

### 2.1 Introduction

In chapter 1 I have limited my analysis to the backward-looking model of repeated procurement auctions in which the bidder with autocorrelated costs does not take into account the effect her behaviour in the current auction will have on her future profits. This assumption is limiting and most likely does not reflect the real behavior of bidders in repeated auctions. In this chapter I study a simple model of repeated procurement auctions in which the bidder with autocorrelated costs is forward-looking. When she chooses the optimal strategy in the first round, she is balancing the effects her bid would have on the current auction and on the expectations of other bidders in the next auction. I show that in the model with exogenous entry the forward looking bidder would always bid higher than in the standard model. Moreover, an equilibrium in strictly monotonic bidding strategies does not exist in this game. I also provide an example of a partially pooling equilibrium with the forward looking bidder pooling towards the weak position (high costs).<sup>1</sup>

---

<sup>1</sup>Following this result, identification of the forward looking dynamic model from the data used in Chapter 1 would not have been possible.

This result expands related results of a number of papers studying repeated auctions with perfectly persistent types and auctions with resale. Literature on auctions with resale can provide useful intuition and reference for the model presented in this chapter since the participants of such auctions also care about the way they would be perceived by their competitors after the initial auction. Similar results appear in a number of papers on auctions with resale. In particular, [Haile \(2000\)](#) shows that there is partial pooling at reserve price in auctions with resale if the bidders get a noisy signal of their true type at the time of the auction. [Hafalir and Krishna \(2008\)](#) note that if a losing bid is revealed after an asymmetric first price auction “there is no nondecreasing equilibrium with (partial) pooling” in an auction with resale.

Existing repeated auctions literature mostly focuses on persistent discrete player types. [Ding et al. \(2010\)](#) study repeated first price auctions with stable discrete types and show that there would be “signal jamming”, the practice of the strong (high value) bidder bidding as the weak bidder, in equilibrium. [Bergemann and Hörner \(2018\)](#) provide more general results for the case of persistent discrete types. They show that “a low-revenue pooling equilibrium might exist” if any bid information is released between auctions, and in case of full bid disclosure “the existence of a pooling equilibrium rules out the possibility of a separating equilibrium”. [Kannan \(2012\)](#) show that in the setting of procurement auctions with  $n$  bidders having binary types there exists a semipulling equilibrium in which some low-cost suppliers bid as high-cost suppliers in the first period.

In this chapter I show that a similar result holds in case of autocorrelated

continuous types. In particular I show that if only one of the bidders has autocorrelated continuous costs in a series of two procurement auctions, no strictly monotone (separating) equilibrium exists, and if an equilibrium exists, this bidder would be pooling at the high cost, “pretending” to be a weaker bidder. To obtain this result I combine the methodology used in the auctions with resale literature with several results on comparative statics in asymmetric first price auctions from [Lebrun \(1998\)](#) and [de Castro and de Frutos \(2010\)](#). I also describe how a partially pooling equilibrium can be constructed using a particular family of type distributions as an example.

## 2.2 Model

Consider a sequence of two sealed bid first price procurement auctions with  $(n + 1)$  bidders participating in each auction. One of the bidders, which I would call a regular bidder, has autocorrelated costs. In the first period her costs are drawn from a distribution  $F_1(c)$  with support  $[\underline{c}, \bar{c}]$ , and in the second period her costs are drawn from a distribution  $F_2(c|c_1)$  where  $c_1$  is the realization of her first period costs, and  $F_2(c|c_1)$  has the same support as  $F_1(c)$ . Both the first round and the second round cost distributions are differentiable with the corresponding probability density functions  $f_1(c)$  and  $f_2(c|c_1)$ . The second period distribution is “positively related” to the first period costs, in particular, the following assumption holds:<sup>2</sup>

---

<sup>2</sup>Note that this assumption also implies first order stochastic dominance ordering.

**A2.1.**  $F_2(c|c_1)$  satisfies monotone hazard rate properly with respect to  $c_1$ :

$$\frac{f_2(c|c'_1)}{1 - F_2(c|c'_1)} < \frac{f_2(c|c_1)}{1 - F_2(c|c_1)} \quad \forall c'_1 > c_1$$

The other  $n$  bidders, which I would call fringe bidders, draw their costs independently from a distribution  $Z(c)$  with support  $[\underline{c}, \bar{c}]$  and a probability density function  $z(c)$  before each of the auctions. Cost realizations of each of the fringe bidders are independent across time as well as across bidders. All bids submitted in the first auction are common knowledge before the second auction starts.

Upon observing the first round bids the fringe bidders would form new expectations about the regular bidder type  $\hat{F}(c|b_1)$ , where  $b_1$  is the regular bidder bid in the first period. If the bidding strategies in the first periods are strictly monotone these beliefs are true and  $\hat{F}(c|b_1) = F(c|c_1)$ . I would assume that  $\hat{F}(c|b_1)$  also satisfies the monotone hazard rate with respect to  $b_1$ .<sup>3</sup>

### 2.2.1 Second period solution

In the second period, after  $b_1 = y$  is observed, the problem reduces to an asymmetric first price auction where, from the perspective of the fringe bidders the regular bidder draws costs from  $\hat{F}_2(c_2|y)$ , and the fringe bidder draws costs from  $Z(c)$ . The exact shape of  $\hat{F}_2(c_2|y)$  depends on whether the fringe bidders can perfectly identify the costs of the regular bidder from the first period, but from a perspective of the fringe bidders it is a well-defined distribution. This means that in

---

<sup>3</sup>This implies that only non-decreasing bidding strategies can constitute an equilibrium in the first round, yet they don't have to be strictly increasing.



the second period there exists a unique monotone equilibrium  $\{\beta_R^2(c_2, y); \beta_F^2(c, y)\}$  with the corresponding inverse bidding functions  $\{\phi_R^2(b, y); \phi_F^2(b, y)\}$ . Using [Lebrun \(1998\)](#) and [de Castro and de Frutos \(2010\)](#) the following comparative statics result holds when  $\hat{F}_2(c_2|y)$  satisfies the monotone hazard rate assumption ([A2.1](#)):

**Proposition 2.1.** For any  $y' > y$ , and for any  $c$  and  $b$ :

$$\begin{aligned}\beta_F^2(c, y') &> \beta_F^2(c, y), \\ F_2(\phi_R^2(b, y')|y') &< F_2(\phi_R^2(b, y)|y).\end{aligned}\tag{2.1}$$

Proposition [2.1](#) implies that fringe bidders would bid more aggressively after observing a lower bid from the regular bidder regardless of their types. And even though we can not establish the same order of bidding functions for the regular bidder, we know that her distribution of second round bids would follow a first order stochastic dominance with respect to her first period bids. Using the first part of proposition [2.1](#) also allows to provide useful comparative statics result for the second round payoffs of the regular bidder:

**Proposition 2.2.** If  $\hat{F}_2(c_2|y)$  satisfies the monotone hazard rate assumption expected profit of the regular bidder in the second period is an increasing function of her first period bid.

*Proof.* The interim second round expected profit of the regular bidder is:

$$\Pi_2^*(y) = \max_{b_2} (b_2 - c_2)(1 - Z(\phi_F^2(b_2, y)))$$

By proposition 2.1:

$$Z(\phi_F^2(b_2, y')) < Z(\phi_F^2(b_2, y)) \quad \forall y' > y,$$

hence:

$$(b_2 - c_2)(1 - Z(\phi_F^2(b_2, y'))) > (b_2 - c_2)(1 - Z(\phi_F^2(b_2, y))) \quad \forall y' > y, \quad \forall b_2$$

And  $\Pi_2^*(y') > \Pi_2^*(y)$ . □

### 2.2.2 First period problem

First, assume that there exists an equilibrium in strictly increasing bidding strategies in the first round auction  $\{\beta_R^1(c); \beta_F^1(c)\}$ , with corresponding inverse bidding functions  $\{\phi_R^1(c); \phi_F^1(c)\}$ . Then the equilibrium condition for the fringe bidder follows that of the standard asymmetric first price auction:

$$b - \phi_F^1(b) = \left[ \frac{f_1(\phi_R^1(b))}{1 - F_1(\phi_R^1(b))} \frac{\partial \phi_R^1(b)}{\partial b} + \frac{(n-1)z(\phi_F^1(b))}{1 - Z(\phi_F^1(b))} \frac{\partial \phi_F^1(b)}{\partial b} \right]^{-1} \quad (2.2)$$

However The regular bidder problem for the first round auction would include expected second period profits, since they also depend  $b_1$ . Assuming no intertemporal discounting, the regular bidder expected profits are:

$$\Pi(b_1) = [b_1 - c_1][1 - Z(\phi_F^1(b_1))]^n + \mathbb{E}_{c_1, b_1} \Pi_2^*(b_1). \quad (2.3)$$

The set of first order conditions defining the BNE of the game is:

$$\begin{aligned} b - \phi_F^1(b) &= \left[ \frac{f(\phi_R^1(b))}{1 - F(\phi_R^1(b))} \frac{\partial \phi_R^1(b)}{\partial b} + \frac{(n-1)z(\phi_F^1(b))}{1 - Z(\phi_F^1(b))} \frac{\partial \phi_F^1(b)}{\partial b} \right]^{-1} \\ (1 - Z(\phi_F^1(b)))^n - n(b - \phi_R^1(b))z(\phi_F^1(b))(1 - Z(\phi_F^1(b)))^{n-1} \frac{\partial \phi_F^1(b)}{\partial b} \\ &+ \frac{\partial \mathbb{E}_{c,b} \Pi_2^*(b)}{\partial b} = 0 \end{aligned} \quad (2.4)$$

With standard boundary conditions:

$$\begin{aligned} \phi_F^1(\bar{b}) &= \phi_R^1(\bar{b}) = \bar{c}, \\ \bar{b} &= \bar{c}, \text{ and} \\ \phi_F^1(\underline{b}) &= \phi_R^1(\underline{b}) = \underline{c} \end{aligned} \quad (2.5)$$

**Proposition 2.3.** The pair of strictly increasing functions  $\phi_F^1(b)$  and  $\phi_R^1(b)$  satisfying conditions (2.4) and (2.5) at any point of their support  $[\underline{b}, \bar{c}]$  does not exist.

*Proof.* Following [Hubbard et al. \(2013b\)](#) and [Fibich et al. \(2002\)](#), several additional boundary conditions should be added, noting that the first order conditions (2.4) should hold at  $\underline{b}$  and  $\bar{b}$  as well as at all interior points. Evaluating these conditions at boundary points gives:

$$\begin{aligned}
f(\underline{c}) \frac{\partial \phi_R^1(\underline{b})}{\partial b} + (n-1)z(\underline{c}) \frac{\partial \phi_F^1(\underline{b})}{\partial b} &= \frac{1}{\underline{b} - \underline{c}}, \\
nz(\underline{c})(\underline{b} - \underline{c}) \frac{\partial \phi_F^1(\underline{b})}{\partial b} &= 1 - \frac{\partial \mathbb{E}_{c,b} \Pi_2^*(\underline{b})}{\partial b}, \\
\frac{\partial \phi_F^1(\bar{b})}{\partial b} &= 1 + \frac{1}{n}, \text{ and} \\
\frac{\partial \mathbb{E}_{c,b} \Pi_2^*(\bar{b})}{\partial b} &= 0.
\end{aligned} \tag{2.6}$$

L'Hospital rule is used to derive the third condition:

$$\begin{aligned}
\lim_{b \rightarrow \bar{b}} (b - \phi_F^1(b)) \left[ \frac{f(\phi_R^1(b))}{1 - F(\phi_R^1(b))} \frac{\partial \phi_R^1(b)}{\partial b} + \frac{(n-1)z(\phi_F^1(b))}{1 - Z(\phi_F^1(b))} \frac{\partial \phi_F^1(b)}{\partial b} \right] &= \\
f(\bar{c}) \frac{\partial \phi_R^1(\bar{b})}{\partial b} \lim_{b \rightarrow \bar{b}} \frac{b - \phi_F^1(b)}{1 - F(\phi_R^1(b))} + (n-1)z(\bar{c}) \frac{\partial \phi_F^1(\bar{b})}{\partial b} \lim_{b \rightarrow \bar{b}} \frac{b - \phi_F^1(b)}{1 - Z(\phi_F^1(b))} &= \\
n(1 - \frac{\partial \phi_R^1(\bar{b})}{\partial b}) &= 1
\end{aligned} \tag{2.7}$$

However in the presence of additional second round profits the first order condition for the regular bidder can only hold is the derivative of expected profit at the upper boundary is equal to zero. This contradicts Proposition 2.2 implying that the equilibrium in monotonic strategies does not exist in this game. Since the proof of Proposition 2.4 did not depend on the existence of a monotone bidding function for the regular bidder, this also implies that if an equilibrium exists, all bids submitted by the regular bidder are higher than those of the standard equilibrium, and any pooling should happen at the higher end of the cost distribution.  $\square$

Let  $\beta_R(c)$  and  $\phi_R(c)$  be the equilibrium bidding and inverse bidding functions of the regular bidder in the standard asymmetric first price auction with own cost distribution of  $F_1(c)$  and the fringe bidder cost distribution of  $Z(c)$ . Then the

following proposition is true.

**Proposition 2.4.** The regular bidder overbids in the first period of the repeated auction model compared to the standard auction model:

$$\beta_R^1(c) > \beta_R(c) \quad \forall c \in (\underline{c}, \bar{c}).$$

*Proof.* In the standard model the regular bidder solves the problem

$$\Pi_1^*(b_1) = \max_{b_1} [b_1 - c_1] [1 - Z(\phi_F^1(b_1))]^n$$

with  $\frac{\partial \Pi_1(b_1)}{\partial b_1} = 0$ . In the first round of the repeated auction the regular bidder solves the problem

$$\Pi^*(b_1) = \max_{b_1} \{ [b_1 - c_1] [1 - Z(\phi_F^1(b_1))]^n + \mathbb{E}_{c_1, b_1} \Pi_2^*(b_1) \}$$

with  $\frac{\partial \Pi_1(b_1)}{\partial b_1} + \frac{\partial \mathbb{E}_{c_1, b_1} \Pi_2^*(b_1)}{\partial b_1} = 0$ . Following proposition 2.2 the second term in this equation is greater than zero, which means that the first term must be less than zero and the solution of the first round of the repeated game must lie on the right from the solution of the standard game. Or, in other words,  $\beta_R^1(c) > \beta_R(c)$ .  $\square$

Propositions 2.3 and 2.4 combined together let me state the following main result:

**Proposition 2.5.** The equilibrium in strictly increasing bidding strategies  $\{\beta_R^1(c); \beta_F^1(c)\}$  does not exist in the first stage game. If an equilibrium of the first stage game exists,

the regular bidder would be at least partially pooling at the high type.

### 2.3 Example: Beta distribution family

Since no analytic solution for the general problem can be found, it is useful to consider a tractable example showing both the construction of the partially pooling equilibrium in the first round auction and the consequences of using monotonic functions solving (2.4) and (2.5) numerically instead of true equilibrium bidding functions. In this section I would assume that in the first period both types of bidders are symmetric with costs independently drawn from a uniform distribution with support  $[0,1]$ . In the second period the regular bidder draws her costs from a distribution which depends on her first period costs. In particular, I would consider a family of Beta distributions with the first shape parameter equal to 1, and the second shape parameter equal to  $\frac{\alpha}{c_1}$ ,  $Beta(1, \frac{\alpha}{c_1})$ . The fringe bidders draw their costs in the second period from the same uniform distribution with support  $[0,1]$  independently of their first period costs, or any of the competitor costs.

The CDF and PDF of the second period regular bidder costs are:

$$\begin{aligned} F_2(c, \frac{\alpha}{c_1}) &= 1 - (1 - c)^{\frac{\alpha}{c_1}}, \\ f_2(c, \frac{\alpha}{c_1}) &= \frac{\alpha}{c_1} (1 - c)^{\frac{\alpha}{c_1} - 1}. \end{aligned} \tag{2.8}$$

This distribution family has several useful properties in relation to my model setup:

- It has support  $[0,1]$  regardless of the value of the parameters;

- It has a clear stochastic ordering with respect to both  $c_1$  and  $\alpha$ ;
- For any invertible bid in the first period it allows to solve a standard asymmetric first price procurement auction that happens in the second period analytically.

**Proposition 2.6.**  $F_2(c, \frac{\alpha}{c_1}) = 1 - (1 - c)^{\frac{\alpha}{c_1}}$  satisfies the monotone hazard rate assumption

*Proof.* Note that the hazard rate conditional on a realization of first price costs  $c_1 = y$  for the Beta distribution function is:

$$\frac{f_2(c|y)}{1 - F_2(c|y)} = \frac{\alpha}{y(1 - c)}$$

and since  $c \in [0, 1]$ ,

$$\frac{\alpha}{y'(1 - c)} < \frac{\alpha}{y(1 - c)} \quad \forall y' > y \text{ and } \forall c \in [0, 1].$$

□

**Proposition 2.7.** In an asymmetric first price auction with one regular bidder drawing her costs from the distribution  $Beta(1, \frac{\alpha}{c_1})$  and  $n$  fringe bidders drawing their costs from the distribution  $U(0, 1)$  there exists an equilibrium in strictly increasing strategies. The inverse bidding functions composing the equilibrium are:

$$\begin{aligned}
\phi_R^2(b) &= b \frac{n+1}{n} - \frac{1}{n}, & \text{for the regular bidder,} \\
\phi_F^2(b) &= b \frac{n+\alpha/y}{\alpha/y+n-1} - \frac{1}{\alpha/y+n-1}, & \text{for the fringe bidder}
\end{aligned} \tag{2.9}$$

*Proof.* With the regular bidder drawing her costs from  $Beta(1, \frac{\alpha}{c_1})$ , and fringe bidders drawing their costs from  $U(0, 1)$  the first order conditions for the second period auction simplify to:

$$\begin{aligned}
(b - \phi_F^2(b)) \left[ \frac{\alpha}{y(1 - \phi_R^2(b))} \frac{\partial \phi_r^2(b)}{\partial b} + \frac{(n-1)}{1 - \phi_F^2(b)} \frac{\partial \phi_F^2(b)}{\partial b} \right] &= 1 \\
(b - \phi_R^2(b)) \frac{n}{1 - \phi_F^2(b)} \frac{\partial \phi_F^2(b)}{\partial b} &= 1
\end{aligned} \tag{2.10}$$

Plugging in the candidate solutions in the system of first order conditions above verifies the statement of the proposition.  $\square$

This means that, assuming that the fringe bidder's expectations about the regular bidder's bidding in the first round are  $\phi_R^1(b, y)$ , the expected second period profit of the regular bidder can be written as (with the realization of  $c_1 = y$ ):

$$\begin{aligned}
\mathbb{E} \Pi_2 &= \int_0^1 \frac{1-b_2}{n} \frac{(1-b_2)(\alpha/\phi_R^1(b_1) + n)}{\alpha/\phi_R^1(b_1) + n - 1} \left[ \frac{\alpha}{y} (1-c_2)^{\frac{\alpha}{y}-1} \right] dc_2 = \\
&= \int_0^1 \frac{1}{n} \left( \frac{n(1-c_2)}{n+1} \right)^2 \frac{\alpha/\phi_R^1(b_1) + n}{\alpha/\phi_R^1(b_1) + n - 1} \left[ \frac{\alpha}{y} (1-c_2)^{\frac{\alpha}{y}-1} \right] dc_2 = \\
&= \frac{n}{(n+1)^2} \frac{n\phi_R^1(b) + \alpha}{(n-1)\phi_R^1(b) + \alpha} \frac{\alpha}{y} \int_0^1 (1-c_2)^{\alpha/y+1} dc_2 = \\
&= \frac{n}{(n+1)^2} \frac{\phi_R^1(b)}{\alpha + 2y} \frac{n\phi_R^1(b) + \alpha}{(n-1)\phi_R^1(b) + \alpha}
\end{aligned} \tag{2.11}$$

In the first round the regular bidder solves the problem:



$$(b_1 - c_1)(1 - \phi_F^1(b_1))^n + \frac{n}{(n+1)^2} \frac{\phi_R^1(b)}{\alpha + 2c_1} \frac{n\phi_R^1(b) + \alpha}{(n-1)\phi_R^1(b) + \alpha} \rightarrow \max_{b_1} \quad (2.12)$$

And the set of equations defining the equilibrium inverse bidding functions in the first period is:

$$\begin{aligned} (b - \phi_F^1(b)) \left( \frac{(n-1)\partial\phi_F^1(b)/\partial b}{1 - \phi_F^1(b)} + \frac{\partial\phi_R^1(b)/\partial b}{1 - \phi_R^1(b)} \right) &= 1 \\ (1 - \phi_F^1(b))^n - n(1 - \phi_F^1(b))^{n-1}(b - \phi_R^1(b))\phi_F^1(b) + \frac{\partial \mathbb{E} \Pi_2(b)}{\partial b} &= 0 \end{aligned} \quad (2.13)$$

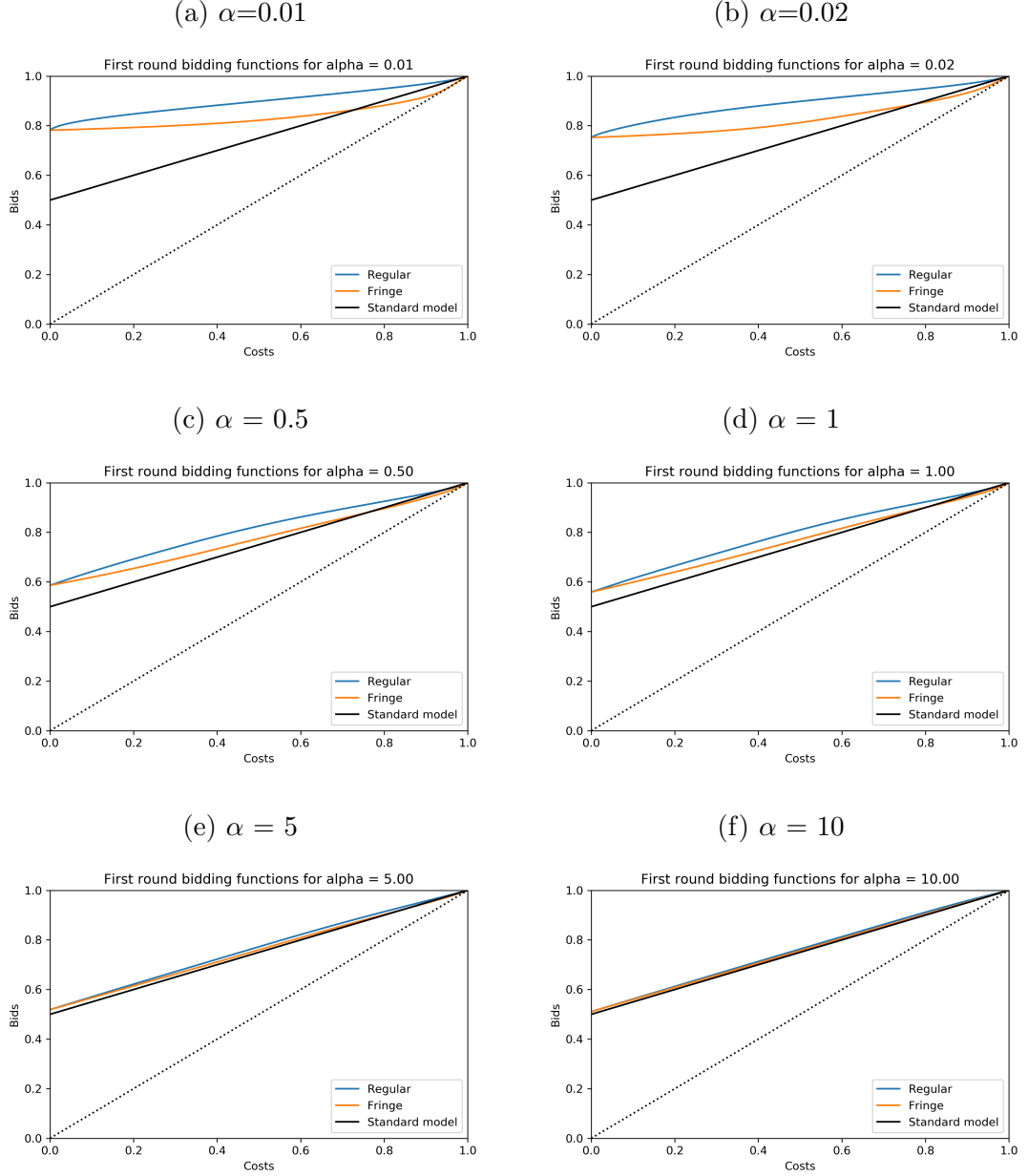
I proceed to solve the system of equations (2.13) numerically using the standard [Hubbard et al. \(2013b\)](#) methodology described in Chapter 1 for various values of  $\alpha$  and  $n$ . Figure 2.1 shows the resulting bidding functions for  $n = 1$  along with the solution of a symmetric auction with standard uniform cost distributions for reference. The regular bidder always bids higher than both the fringe bidder and the standard symmetric bidder. The higher is the value of  $\alpha$ , the lower is the correlation between first period and second period costs of the regular bidder, and the closer are the first round simulated bidding functions to the standard one-shot symmetric auction.

However we can also see that the simulated bidding function do not constitute an equilibrium by finding best response functions to each of the proposed solutions. In particular, plugging in the simulated  $\phi_F^1(b)$  and  $\phi_R^1(b)$  into the regular bidder problem for  $n = 1$  and numerically finding optimal  $b_1$  that solves:

$$(b_1 - c_1)(1 - \phi_F^1(b_1)) + \frac{\phi_R^1(b_1) + \alpha}{4(\alpha + 2c_1)} \rightarrow \max_{b_1} \quad (2.14)$$

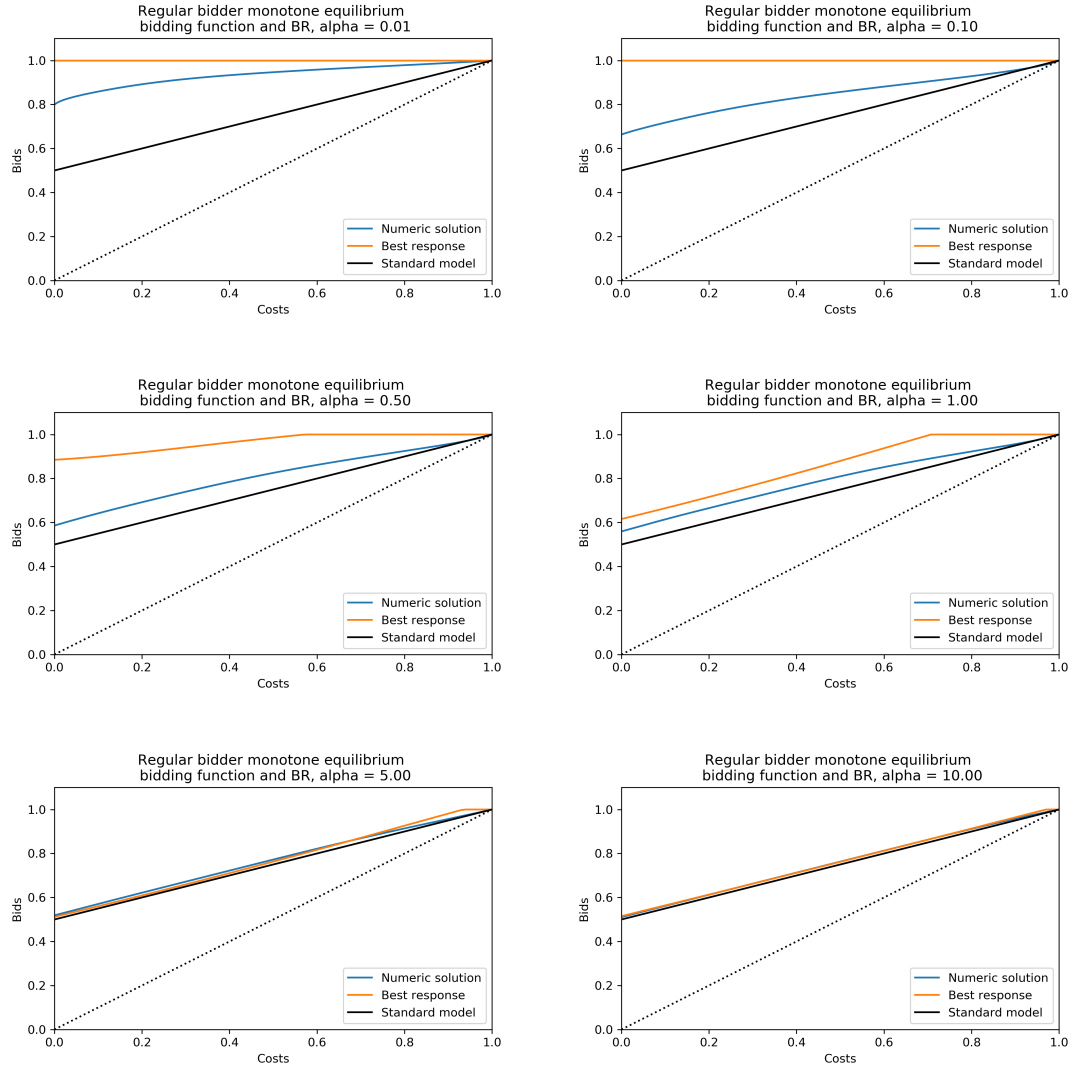
for each  $c_1$  shows that the assumption of monotone first round bidding strategies is violated, especially for low values of  $\alpha$ . Figure 2.2 shows the regular bidder best response functions along with the simulated monotone bidding functions for various values of  $\alpha$ . For values below 0.3 the best response function is fully pooling at  $\phi_R^1(b) = \bar{b} = \bar{c} = 1$ . At  $\alpha = 0.5$  there is some partial pooling at the right end of the best response function. The pooling interval reduces with  $\alpha$  but is present even for high values.

Figure 2.1: Bidding functions for different values of  $\alpha$



Theoretical intuition for this result can be obtained by explicitly applying the [Hubbard et al. \(2013b\)](#) and [Fibich et al. \(2002\)](#) boundary conditions and evaluating (2.13) at points  $\underline{b}$  and  $\bar{b} = 1$  and taking into account the boundary conditions for the inverse bidding functions ( $\phi_F^1(\underline{b}) = \phi_R^1(\underline{b}) = \underline{c} = 0$ , and  $\phi_F^1(1) = \phi_R^1(1) = 1$ ).

Figure 2.2: Bidding functions for different values of  $\alpha$



Evaluated at the lower boundary, (2.13) would provide additional conditions for the shape of  $\phi_F^1(\underline{b})$  and  $\phi_R^1(\underline{b})$  at the lower boundary:

$$\begin{aligned} (\underline{b} - 0)\phi_R^{1'}(\underline{b}) - (1 - 0) &= 0 \\ (\underline{b} - 0)\phi_F^{1'}(\underline{b}) - (1 - 0) + \frac{\phi_R^{1'}(\underline{b})}{4(\alpha + 2 * 0)} &= 0, \end{aligned} \tag{2.15}$$

or:

$$\begin{aligned} \phi_R^{1'}(\underline{b}) &= \frac{1}{\underline{b}} \\ \phi_F^{1'}(\underline{b}) &= \frac{1}{\underline{b}} - \frac{1}{4\alpha\underline{b}^2}, \end{aligned} \tag{2.16}$$

Evaluated at the upper boundary, however, (2.13) shows that there should be some pooling by the regular bidder, which contradicts our initial assumption about the monotonicity of the equilibrium:

$$\begin{aligned} (1 - 1)\phi_R^{1'}(\bar{b}) - (1 - 1) &= 0 \\ (1 - 1)\phi_F^{1'}(\bar{b}) - (1 - 1) + \frac{\phi_R^{1'}(\bar{b})}{4(\alpha + 2)} &= 0 \end{aligned} \tag{2.17}$$

We can use L'Hospital rule to estimate the first equation in (2.17) providing additional condition on  $\phi_R^{1'}(\bar{b}) = 2$  or  $\phi_R^{1'}(\bar{b}) = 0$ , however the second equation can only hold if  $\phi_R^{1'}(\bar{b}) = 0$ .

## 2.4 Partially pooling solution for the Beta distribution family

In this section I describe the algorithm for finding a partially pooling solution for a selected Beta distribution. In particular I assume that  $\alpha = 0.5$ , and in the first period the regular bidder bids according to some monotonic function  $\beta_R^1(c)$  up to a cut-off point  $\hat{c}$  and 1 after this cut-off point. Since the first round bidding function of the regular bidder is not invertible at 1, the beliefs of the fringe bidder in the second round have to be corrected to solve the second round auction appropriately.<sup>4</sup>

$$\hat{F}_2(c|b_1) = \begin{cases} 1 - (1 - c)^{\alpha/\phi_R^1(b_1)}, & \text{if } b_1 < 1; \\ \frac{\int_{\hat{c}}^1 (1 - (1 - c)^{\alpha/y}) dy}{1 - \hat{c}}, & \text{if } b_1 = 1. \end{cases} \quad (2.18)$$

Despite the changes in the fringe bidder beliefs about the regular bidder costs, the second round auction is still a standard asymmetric first price procurement auction and a unique monotonic equilibrium exists for any given value of  $b_1$ . Moreover, for any  $c_1 = y < \hat{c}$  and  $b_1 < 1$  the solution of the auction is still described by (2.9). When  $b_1 = 1$  the solution of the second round auction satisfies the following system of first order conditions:

$$\begin{aligned} (b - \phi_F^2(b)) \left[ \frac{(n - 1)\phi_R^{2'}(b)}{1 - \phi_F^2(b)} + H_R(\phi_R^2(b)) \right] &= 1 \\ (b - \phi_R^2(b)) \frac{n\phi_F^{2'}(b)}{1 - \phi_F^2(b)} &= 1 \end{aligned} \quad (2.19)$$

where  $H_R(c)$  is the hazard ratio for the regular bidder conditional on  $b_1 = 1$ .

---

<sup>4</sup>The analytical expression for the integral in (2.18) involves the exponential integral special function and is not very useful. I would use the integral form in further derivations and a numerical approximation for the simulated solutions.

Figure 2.3 shows the numerical solutions for this system of equations for different values of  $\hat{c}$ ,  $\alpha = 0.5$ , and  $n = 1$ . It is clear that for high values of  $\hat{c}$  both bidding functions are increasing with  $\hat{c}$ , but the relation is less clear for the lower values of  $\hat{c}$ . Figure 2.4 shows the expected profits of the regular bidder in the second round auction conditional on  $b_1 = 1$  as a function of  $\hat{c}$ . As we can see it is not a monotone function of  $\hat{c}$  and it is minimized at  $\hat{c} \approx 0.5$ .

Figure 2.3: Second round bidding functions for  $b_1 = 1$ ,  $\alpha = 0.5$ , and several values of  $\hat{c}$

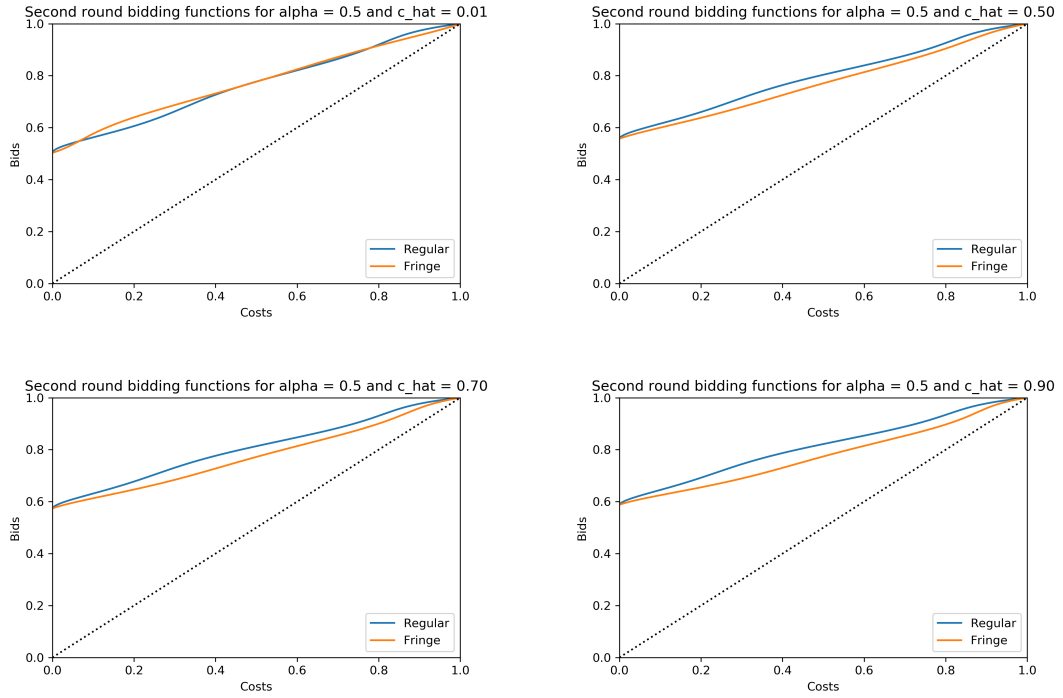
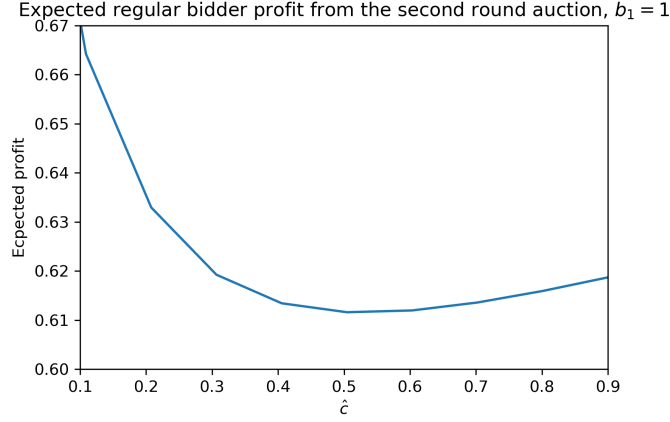


Figure 2.4: Expected profits of the regular bidder in second round auction,  $\alpha = 0.5$ ,  $n = 1$



In the first round the regular bidder would balance the profits from the first round auction with the expected profits from the second round auction and is unlikely to choose a low value of  $\hat{c}$ . So we might expect the optimal cut-off value to be above 0.5.

To solve the first round problem, first assume that the regular bidder bids according to some monotonic function  $\beta_R^1(c)$  if her first period costs are less than the cut-off value  $\hat{c}$ , and 1 if her costs are greater or equal than  $\hat{c}$ .

**Proposition 2.8.** If the regular bidder is playing the partially pooling strategy, the fringe bidders would play a strictly increasing strategy in the equilibrium.

*Proof.* Assume that the regular bidder is playing a partially pooling strategy with the strictly increasing inverse bidding function  $\phi_R^1(b)$ ,  $\phi_R^1(1) = \hat{c}$ ,  $\phi_R^1(0) = 0$ , and bids 1 whenever her costs are above  $\hat{c}$ . If  $n > 1$  the fringe bidders would have no incentives to play a partially pooling strategy, since they will be competing with other fringe bidders who don't value the future and have incentives to undercut the



bid  $b = 1$  whenever their cost draw is below 1. If  $n = 1$  the expected profits of the fringe bidder bidding some  $b < 1$  are:

$$\Pi_F(b, c) = (b - c)(1 - \hat{c} + (1 - \phi_R^1(b))\hat{c}),$$

and, if the ties are decided by a fair coin flip, her expected profits when bidding 1 are:

$$\Pi_F(1, c) = (1 - c)\frac{1 - \hat{c}}{2}$$

Since for any  $\hat{c} > 0$ :

$$\Pi_F(b, c) = (b - c)(1 - \phi_R^1(b)\hat{c}) \geq (b - c)(1 - \hat{c}^2)$$

the fringe bidder chooses to play a strictly monotonic strategy whenever:

$$\frac{b - c}{1 - c} > \frac{1}{2(1 + \hat{c})},$$

the left hand side is a decreasing function of  $c$  and for any  $b$  there exists a threshold  $\hat{c}_F$  such that:

$$\hat{c}_F = \frac{2b(1 + \hat{c}) - 1}{1 + 2\hat{c}}$$

such that the fringe bidder prefers the mixed strategy whenever  $c \geq \hat{c}_F$ . At  $b = 1$  this means that  $\hat{c}_F = 1$  and the fringe bidder plays a strictly increasing bidding strategy in the equilibrium. □

If the fringe bidder is playing a strictly increasing equilibrium strategy  $\phi_F^1(b)$ , expected profits of the regular bidder are:

$$\Pi_R(b, \hat{c}) = \begin{cases} \Pi_2^*(\hat{c}), & \text{if } b = 1; \\ (b - c)(1 - \phi_F^1(b))^n + \Pi_2(b), & \text{if } b_1 < 1. \end{cases} \quad (2.20)$$

where  $\Pi_2^*(\hat{c})$  is the expected second round profit from the pooling signal shown in figure (2.4), and  $\Pi_2(b)$  is the expected second round profit from signal  $\phi_R^1(b) = c$ .

Equilibrium threshold value  $\hat{c}$  is defined by equality:

$$\Pi_2^*(\hat{c}) = \mathbb{E} \Pi_2(1) = \frac{n}{(n+1)^2} \frac{\phi_R^1(b)}{\alpha + 2\phi_R^1(b)} \frac{n\phi_R^1(b) + \alpha}{(n-1)\phi_R^1(b) + \alpha} \quad (2.21)$$

In case of  $n = 1$ :

$$\Pi_2^*(\hat{c}) = \Pi_2(1) = \frac{\phi_R^1(b) + 0.5}{4(0.5 + 2\phi_R^1(b))} = \frac{\hat{c} + 0.5}{4(0.5 + 2\hat{c})}. \quad (2.22)$$

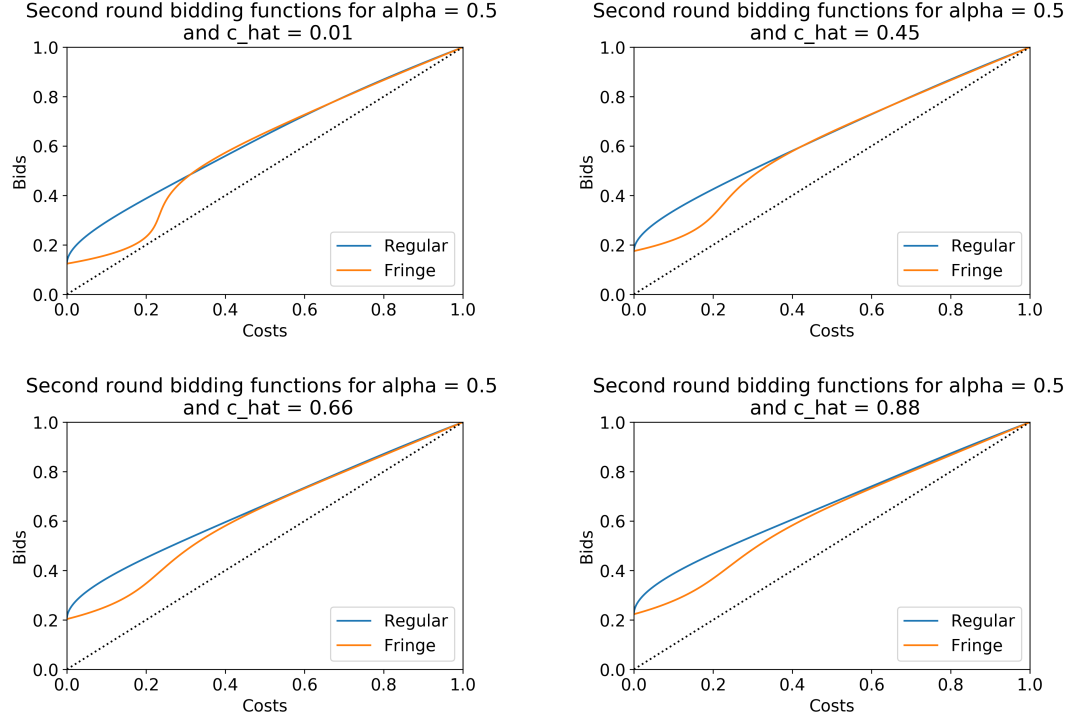
However in this example the right hand side of equation (2.22) is maximized at  $\hat{c} = 0$  with the value 0.25, which is below the minimum value of  $\Pi_2^*(\hat{c})$ . Hence no equilibrium exists in an auction with one fringe bidder. <sup>5</sup>

A partially pooling equilibrium would exist if more than one fringe bidder is participating in the auctions. Similar solution strategy can be used to find the threshold values of  $\hat{c}_n$  for higher values of  $n$ . For example, Figure 2.5 shows numerical solutions of the second round auction for selected values of  $\alpha$  and  $n = 2$ .

---

<sup>5</sup>Alternatively, we might assume that the ties are always broken in favor of the fringe bidder. Then multiple equilibria with both the regular and the fringe bidder bidding  $\bar{b} = 1 - \sqrt{\Pi_2^*(0)}$  would exist in this game.

Figure 2.5: Second round bidding functions for  $b_1 = 1$ ,  $\alpha = 0.5$ , 2 fringe bidders, and several values of  $\hat{c}$



Applying condition (2.21) for these solutions and  $n = 2$ , graphically shown in Figure 2.6, would provide the value of  $\hat{c}_2^* = 0.0358$ . As shown in Figures 2.7 and 2.8 the solutions for three and seven fringe bidders are  $\hat{c}_3^* = 0.0656$ , and  $\hat{c}_7^* = 0.2666$ .

To finish constructing a partially pooling equilibrium one would solve the following system of differential equations numerically:

$$\begin{aligned}
 (1 - \phi_F^1(b))^n - n\phi_F^{1'}(b)(b - c)(1 - \phi_F^1(b))^{n-1} + \frac{\partial \Pi_2(b)}{\partial b} &= 0, \\
 (b - \phi_F^1(b)) \left[ \frac{(n-1)\phi_R^{1'}(b)}{1 - \phi_F^1(b)} + \frac{\phi_R^{1'}(b)}{1 - \phi_R^1(b)} \right] &= 1, \\
 \phi_F^1(b) = \phi_R^1(b) &= 0, \\
 \phi_F^1(1) = 1, \phi_R^1(1) &= \hat{c}.
 \end{aligned} \tag{2.23}$$

Though the unique solution to such system exists, the fact that the system is

not fully fixed at the lower end of support, and the functions we are seeking have different supports make standard solution numerical solution methods unstable.

Figure 2.6: Expected profits of the regular bidder in second round auction with two fringe bidders:  $\hat{c}^* = 0.0358$

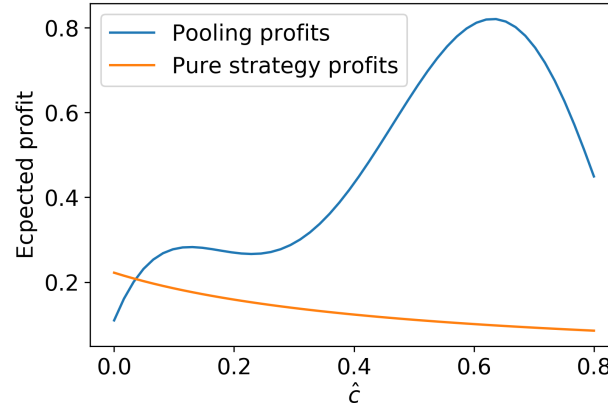
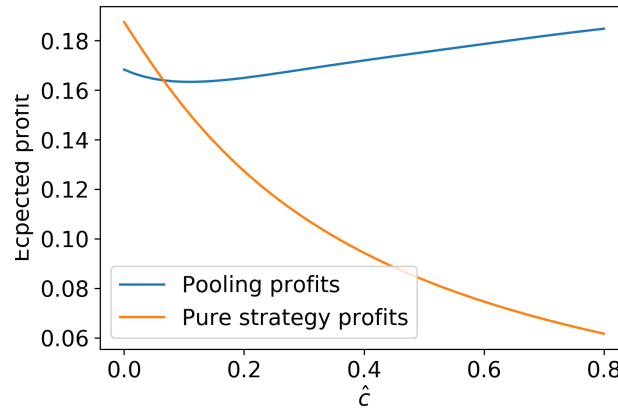


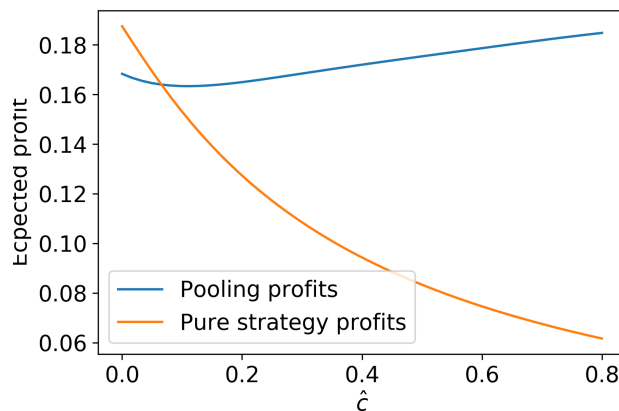
Figure 2.7: Expected profits of the regular bidder in second round auction with three fringe bidders:  $\hat{c}^* = 0.0656$



## 2.5 Conclusions

In this chapter I have studied a simple two-period model of repeated procurement auction with autocorrelated costs and forward-looking bidders. I have shown

Figure 2.8: Expected profits of the regular bidder in second round auction with three fringe bidders:  $\hat{c}^* = 0.2666$



that in this model the equilibrium in strictly increasing strategies does not exist in the first time period. In particular, the bidder with autocorrelated type would have incentives to hide her true type in the earlier period partially pool at her weakest type in equilibrium.

I have also provided equilibrium solutions for several examples of two-period auction games with mostly symmetric players, and shown that the bidder with persistent type would pool towards the higher type even if the competition is high.

This result provides an extension of previously existing literature on repeated auctions with persistent bidder types, and can inform future work concerning auction transparency and estimation of data from repeated auctions.

## Chapter 3: The role of information in repeated procurement auctions: monitoring and entry

### 3.1 Introduction

On Jan 1, 2011, Russian public procurement system experienced a change in the rules of publishing information. Before this date the information was published in a decentralized manner, separated into municipal (published mostly offline or on the municipal administration website), regional (published on regional procurement websites), and federal (published on the federal procurement website). Since Jan 1, 2011, all the information is published on the federal website in a unified format.

This shift makes acquiring information about upcoming and past auctions cheaper both for the firms considering entering and for the regulators monitoring the procurement system for the cases of possible corruption. From the point of view of policy makers increased information should increase competition and reduce procurement costs through both of these channels. However in a repeated auction setting releasing information about past auctions might have adverse effects on competition both by facilitating collusion and by providing incumbents with additional instruments to deter entry. Although better information about upcoming auctions

induces entry, information about past auctions allows bidders to coordinate, and may discourage entry by weaker bidders if they observe a strong incumbent in similar past auctions.

I suggest that in a repeated auction framework firms make not only entry and bidding decisions, but also monitoring decisions in each period of time. If a firm wants to participate in an auction it has to acquire information about upcoming auctions. It can also get information about past auctions' outcomes and use this information while making entry and bidding decisions. Increase of information transparency lowers monitoring costs, the costs of acquiring both types of information. The existence of third-party firms that specialize in collecting information about procurement auctions and selling it to potential entrants can serve as an anecdotal evidence of existence of monitoring costs in the system.

In this chapter I study the effect of increased information transparency on entry and bidding decisions in repeated public procurement auctions for gasoline service contracts in one of the Russian regions. I use the difference in the role of monitoring for big and small firms and the variation in their entry and bidding decisions to provide some reduced form evidence of the ambivalent role of increasing information transparency. I also suggest an (incomplete) structural model showing the different effects that the policy change had on the different types of firms participating in this market. The results of the estimation of a static model with selective costly entry suggest that the new information policy changed entry costs benefiting the large federal firms and potentially decreasing competition. However the resulting estimates of cost distributions before and after the policy change are significantly

different for the same types of firms suggesting that a more comprehensive model incorporating forward-looking behavior by some or all firms is needed to reflect the data.

Related literature can be separated into four main groups: evaluation of information disclosure policy for procurement auctions; endogenous entry in one-shot auctions; bidding and entry behavior in repeated auctions with no entry costs; and theoretical models of repeated auctions with monitoring. [Coviello and Mariniello \(2014\)](#), [Leslie and Zoido \(2011\)](#), [Ohashi \(2009\)](#), [De Silva et al. \(2008\)](#) evaluate the effects of various rules of disclosing information about procurement auctions in Italy, Argentina, Japan and USA. [Ohashi \(2009\)](#) and [De Silva et al. \(2008\)](#) study the effects of providing more precise information about the costs of performing a public work on entry and bidding behavior. [Ohashi \(2009\)](#) finds both an increase in number of entrants and decrease in price of contracts due to improved information; [De Silva et al. \(2008\)](#) find reduced information asymmetries between “entrants” and “incumbents”, but no proof of increased overall entry. Since public works (such as construction, repair, gardening etc.) are associated with common costs for all firms, the policies studied in both papers are aimed at reducing ex ante information asymmetries between different types of bidders, and between the buyers and the bidders. In my chapter I would focus on auctions for a simple product (gasoline) with well defined characteristics which I assume to be private cost auctions. Yet these papers provide useful insights in possible reduced form techniques (differences-in-differences) as well as an example of ambiguous results of increasing information transparency on auction outcomes. Papers by [Coviello and Mariniello \(2014\)](#), and



[Leslie and Zoido \(2011\)](#) are studying private value auctions and are focused on the effect of publicity on auction outcomes. [Coviello and Mariniello \(2014\)](#) use a regression discontinuity design to show that entry and winning rebate are lower in auctions with reserve prices below 500,000 euros which are publicized on local bulletin boards in Italian municipalities than for auctions with reserve prices above 500,000 euros which are publicized in a number of regional newspapers. [Leslie and Zoido \(2011\)](#) study the effect of “informational entrepreneurs”, the third-party firms that sell bundled auction announcements to potential bidders, on auction outcomes in Buenos Aires. They show that firms that buy information services enter into auctions with less competitors, and as a consequence bid less aggressively, but the appearance of informational entrepreneurs reduces costs of procurement by 2.9%.

The second group of related papers models one-shot auctions with costly entry and estimates the entry costs. [Krasnokutskaya and Seim \(2011\)](#), and [Athey et al. \(2011\)](#) use a model of non-selective endogenous entry suggested by [Levin and Smith \(1994\)](#) (LS). It is assumed that potential entrants do not know their private values (or costs in the case of procurement auctions) before making an entering decision. Both papers model two types of firms differing in the distribution of their values either because of the difference in the size of the firms or because of bid preferences provided by the auctioneer. The empirical parts of the papers focus on highway procurement and timber auctions in various US states respectively. In both types of auctions only general information about the auctioned work contract or tract is publicly released and firms have to pay a small fee to acquire further information. This allows to observe the number of “planholders” or firms choosing to “cruise the

tract” and assume that all of these firms are potential entrants in the auction game, while firms that submit bids are actual entrants. Yet the “planholders” represent an already refined group of firms that chose to incur costs of acquiring further information, and in the logic of my chapter are firms that made a positive monitoring decision, paid the monitoring cost, and as a result of this monitoring know that they are eligible to take part in the auction. [Hubbard and Paarsch \(2009\)](#) model and parametrically simulate the consequences of a bid preference program with selective entry following [Samuelson \(1985\)](#) (S) model. Under Samuelson assumption potential entrants learn their private values before making the entry decision so that only firms with valuations higher than a certain threshold (or costs lower than threshold in case of procurement auctions) are participating in the auction. [Roberts and Sweeting \(2010\)](#) model and parametrically estimate partially selective entry allowing potential entrants to observe signals affiliated with their private valuations prior to making entry decision. [Li and Zheng \(2012\)](#) suggest a method of discriminating between selective and non-selective entry models and provide evidence in favor of selective entry in timber auctions.

Although assuming that there is at least some degree of selection in entry decision seems natural, the truncated nature of observed bids make identification of the entry costs and distribution of bids difficult. [Gentry and Li \(2014\)](#) show that if the number of potential entrants, number of bidders and full set of bids is observed the bounds on the entry costs and conditional distribution of values can be identified in the affiliated signal model. These bounds would collapse to point estimates if the variation in entry is continuous (there exists a continuous auction level instrument),

and if entry is non-selective (LS), but not if entry is fully selective (S).

In this chapter I am focusing on the impact of accumulated information and the repeated nature of procurement auctions on entry decisions. [Jofre-Bonet and Pesendorfer \(2000\)](#) and [Jofre-Bonet and Pesendorfer \(2007\)](#) (JBP) suggest a model of repeated procurement auction in which past actions taken by the firm accumulated in a state variable affect its bidding decisions in the current auction. Jofre-Bonet and Pesendorfer use firms' backlog as such state variable, [Balat \(2013\)](#) allows for more general intertemporal links in private costs, [Groeger \(2014\)](#) allows for entry costs that depend on the state variable. The estimation of JBP model relies on observing the state variable for all regularly participating firms and the distribution of bids conditional on the state variable.

Finally, as monitoring provides potential entrants with information about past auctions, I have to make certain assumptions on the way this information may be used. Papers by [Danak and Mannor \(2009\)](#), [Han et al. \(2009\)](#), [Figliozzi et al. \(2008\)](#) build and simulate models of repeated auctions with monitoring and provide some insight to deriving the expected upcoming auction characteristics with both perfectly and boundedly rational potential participants.

In this chapter I provide some evidence of ambiguous effect of increased information transparency on key auction outcomes (participation and costs of procurement) which might be attributed to the presence of asymmetric monitoring costs. I further suggest a model of repeated auction with monitoring and endogenous entry by building up on the JBP model of repeated procurement auction.

### 3.2 Data and institutional environment

I start with the data from approximately 7000 sealed bid and open bid gasoline procurement auctions in Sverdlovsk region of Russia conducted in 2008-2013<sup>1</sup>. Prior to January 2011 both calls for bids and auction protocols were publicized on different levels depending on financing features of the organization. Municipal procurements were published on either municipal or regional procurement website; regional procurements were published on regional websites; federal procurements were published separately on a centralized federal website. The information about auctions from 2008 to 2011 is obtained from a number of archived municipal administrations websites and the regional procurement website for Sverdlovsk region. Auctions after Jan 2011 are obtained from the centralized federal procurement website<sup>2</sup>.

Starting from January 2011 two changes to the procurement system were implemented. First, the information about procurements of any level is published on a centralized website, potentially decreasing monitoring efforts for all interested parties. Second, electronic applications for both sealed bid and open bid auctions were introduced, potentially reducing entry costs for all bidders.

A typical gasoline procurement auction is run to provide cars owned by a government or government-financed organization<sup>3</sup> with gasoline supplied through

---

<sup>1</sup>Sverdlovsk region is chosen for several reasons. It is consistently in top-10 Russian regions by gross regional product, has substantial population density so that consumer market for gasoline suppliers is of more importance than government contracts, yet has not many big federal buyers with potentially large bargaining power (unlike buyers in Moscow, St. Petersburg and Tatarstan), finally the pre-2011 data for the region is relatively well preserved.

<sup>2</sup>Data published on the federal website prior to Jan 2011 is currently not available, though might be used as a control grouped if ever obtained. This makes the set of buyers covered by the data after 2011 about 10% larger than those covered before 2011.

<sup>3</sup>Such as a local administration, school, or hospital.

existing gas stations in a particular locality. At least two weeks before the auction the procurer has to publish auction documentation which specifies area and duration of service, amount of gasoline demanded, the type of auction (open or sealed bid), and the place and time that this auction would take place in. An example of such procurement auction is described below.

A local police station buys 14300 liters of AI92 gasoline to be supplied over the second quarter of 2009 through gas stations within municipality borders. Figure 3.1 shows the procurer labeled with tax-payer id, area of service, and local gas stations labeled with green dots. There are 5 eligible bidders. The call for bids was published on April 2, 2009 on the regional website. It specified the duration of service (during second quarter), area of service (within municipality borders), reserve price 267670 RUB which is approximately 3% higher than the market price of this volume of AI92 at the time of auction, and the date and time by which bids should be received by the procurer (April 8, 18:00). Two bidders submitted their bids, but one of them was excluded from the auction since it had no stations in the service area. The only remaining bidder was announced the winner with the price 265980 (2.3% higher than the market price). On April 10 the protocol was published on the same website.

Figure 3.1: Krasnoturyinsk police station procurement



Sealed bid and open bid auctions run by standard rules with a public reserve price. In a sealed bid auction bidders submit their bids and supporting documents (such as safety certificates, list of stations in operation etc.) either in an envelope or electronically, the buyer opens the bid packages at a designated time, decides whether all bidders satisfy auction requirements and announces the bidder who submitted the lowest admitted bid the winner. In an open bid auction firms register to bid and submit supporting documents before the beginning of the auction, buyer decides and announces whether all bidders satisfy auction requirements. At the predefined time of the auction admitted bidders submit their descending bids either in person or electronically until no bidder is willing to submit another bid. The

last bid is announced the winning bid and the winner receives the amount of this bid as his payment<sup>4</sup>. Buyers have to conduct open bid auctions if the reserve price is above 500,000 RUB (~\$15,500 by 2011 exchange rate), they are free to choose between sealed bid and open bid auction if the reserve price is below this threshold<sup>5</sup>. Auction results are protocolled and published within three days after the auction and contain the information about bidder identities, winner identity, winning bid, and sometimes second and subsequent bids. Procurer has the right to exclude a bidder from the auction if certain documentation is not supplied or if bidder doesn't satisfy criteria of the contract (for example, if bidder doesn't have stations in the specified service area). Bidders in an open bid auction can apply but not submit any bids. The information about excluded bidders is published in the protocols. The information about non-active bidders is not necessarily published and in some cases the identity of a non-active bidder is unknown.

An important feature of the data is the variation in potential number of bidders across auctions in the same municipality and sometimes for the same buyer. Some buyers change the definition of area of service over time, providing exogenous variation in the number of potential bidders<sup>6</sup>. The aforementioned police station

---

<sup>4</sup>In practice open auctions have a discrete bid step initially equal to 5% of reserve price and reducing by 0.5% of reserve price each time there are no new bids in a given round. Here I would assume that the bids in open bid auctions are continuous and hence open bid first price auction can be treated as a second price sealed bid auction if there are two or more bidders present.

<sup>5</sup>No auction is necessary if the reserve price is below 50,000 RUB, or total quarterly spending on a given type of product is below 500,000 RUB, or if the buyer can prove that the purchased good is a product of local monopoly. These small purchases are rare in gasoline procurement and are not a part of the data set.

<sup>6</sup>The degree of freedom that definition of area of service and the reserve price gives to the buyer might be used for implicitly executing preferences that the buyer has over potential entrants (for corrupt or benevolent reasons). But for the purpose of this chapter I would ignore strategic considerations that the buyers might have.

defined are of service within city limits in 2008, first half of 2009, and after second half of 2011. It conducted four auction with area of service defined as “within 2 km radius” (blue circle on Figure 3.2) in 2009 cutting down the number of potential entrants to 3, and four auctions “within 4 km radius” in 2010 with 4 eligible bidders.

### 3.2.1 North subsample

Since defining the volume demanded, area of service and the number and identity of potential bidders for each auction is labor-intensive, I will be using a small subsample of 348 auctions in eight municipalities in the north of the region. Appendix 1 shows the geographic area covered by the subsample (top of Figure 3.6) and the locations of buyers and gas stations in the covered area and reasonable vicinity (bottom of Figure 3.6)<sup>7</sup>. There are 46 procurers from 8 municipalities in the sample, each conducting 1 to 28 auctions from 2009 to 2013. 13 of these procurers are “federal” organizations<sup>8</sup>.

One of the main concerns about the data is missing information for some procurers. To see if it affects my subsample I study the changes in volume of gasoline demanded over time (Appendix 3). Although it doesn’t seem to be an issue for the whole subsample since there seems to be no substantial break in demand patterns over the years(Figure 3.7), it affects municipality level data (Figure 3.8).

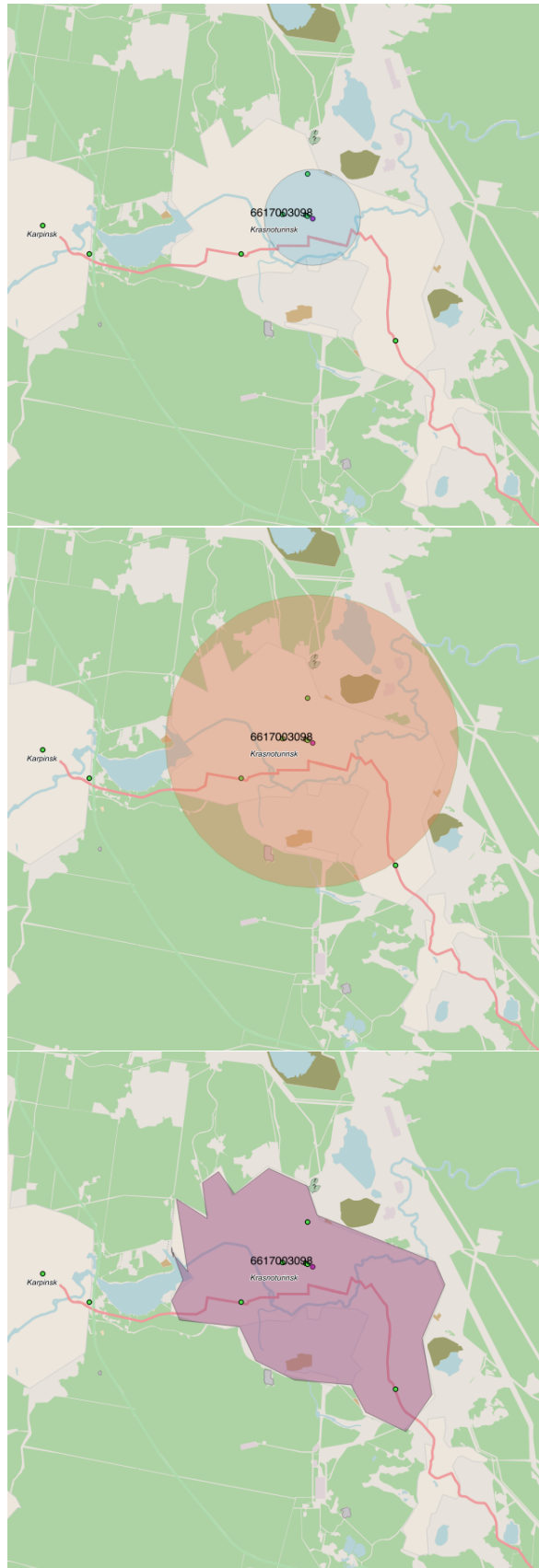
---

<sup>7</sup>This area consists of both small and medium municipalities and is separated from the rest of the region which makes definition of areas of service simpler. In further data collection process I will add municipalities moving south from the current subsample border.

<sup>8</sup>These include prisons, extra-territorial police units and tax agencies. The presence of federal buyers in the subsample is higher than in the full sample since prisons are clustered around two of the municipalities: Ivdal and Gari. Though if this adds a bias to my reduced form results, the bias should be in my favor since no big firms take part in “prison auctions”.



Figure 3.2: Krasnoturyinsk police station: variation in area of service



Two of the municipalities don't show up in the pre-2011 data at all, suggesting that they used alternative methods of publishing information, Ivdel is severely affected by the lack of pre-2011 federal data and has more auctions for higher volumes of gasoline documented after 2011. One municipality has no data after 2008 which may be explained by local administrative reforms.

Appendix 2 presents summary statistics for the subsample (Table 3.7), pre-2011 part of the subsample (Table 3.8) and post-2011 part of the subsample (Table 3.9). Some important features of the data include low participation rates (there is no auction with more than 4 bidders, and no auction with more than 3 active bidders), prevalence of sealed bid auctions for both pre- and post-2011 periods, and a shift to smaller auctions with better reserve prices closer to current market prices for the same bundles of gasoline at the time of auction after 2011. The shift to sealed bid auctions and to the smaller contracts can be linked since the choice of auction format depends on the reserve price. The shift to smaller contracts can be explained by reduced costs of conducting an auction and by the shift to fewer centralized auctions, for example Severouralsk administration used to buy gasoline for several city hospitals before 2011 and after 2011 hospitals run these auctions separately. In this chapter I would ignore possible strategic behavior of the buyers. Since the number of active bidders is low in the observed auctions I also look at the summary statistics by number of bidders (Table 3.10). There are less auctions with no entry after 2011<sup>9</sup>. Auctions with no entry have higher dispersion of reserve

---

<sup>9</sup>Although there is no zero-entry auctions in the subsample after 2011, they do exist in the full sample.

prices, but no other obvious differences from auction with nontrivial entry. Also there are significantly more open bid auctions among auctions with only one active bidder after 2011 which might be explained by higher visibility of “big efficient firm” presence in big auctions after the introduction of new information policy which scares off other potential entrants.

### 3.3 Reduced form strategy and results

In this section I estimate the results of policy change for two key auction outcomes: entry and price of the auction. I start by running naive OLS regressions of relative price of the procurement contract<sup>10</sup> and winning discount<sup>11</sup> on the policy dummy variable and the set of auction characteristics. The results of these regressions are presented in Table 3.1. They show that the mean relative price is higher when I control for the relative reserve price and the mean discount is lower after the introduction of the new information policy<sup>12</sup>. Or, in other words, the average cost of gasoline procurement goes up after the introduction of the new information policy.

---

<sup>10</sup>Relative price is equal to the ratio of winning bid to the average market price of the contract at the Sverdlovsk region station at the time of the auction.

<sup>11</sup>Winning discount is equal to the difference between relative start price and relative winning price.

<sup>12</sup>It also shows that the variation in relative reserve price explains most of the variation in final relative price, so if we take into account the changes in buyer behavior the estimation of policy results might be different since summary statistics in Tables 3.8 and 3.9 show that reserve prices are consistently set closer to market prices after 2011.

Table 3.1: Wining bid and winning rebate naive regressions

	Normalized contract price	Normalized contract price	% Discount
Intercept	1.0806*** (0.0178)	0.1953*** (0.0411)	0.0121 (0.0115)
After	-0.0082 (0.0100)	0.0125** (0.0063)	-0.0174*** (0.0065)
Open auction	0.0382*** (0.0128)	0.0055 (0.0081)	0.0022 (0.0083)
Duration of con- tract	-0.0001 (0.0001)	-0.0001* (0.0000)	0.0001* (0.0000)
N bidders	-0.0158** (0.0072)	-0.0193*** (0.0045)	0.0201*** (0.0047)
N products	0.0054 (0.0054)	0.0052 (0.0034)	-0.0052 (0.0035)
Volume	-0.0000 (0.0000)	0.0000 (0.0000)	-0.0000 (0.0000)
Normalized reserve price		0.8102*** (0.0362)	
Observations	321	321	321
R-squared	0.077	0.645	0.090

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses.

If firms have to pay monitoring costs to obtain information about auctions in our sample the policy would have different effect for small local firms and big firms that have stations both inside and outside the region. For local firms that have stations in one or several geographically close municipalities new policy means a switch to a new website that provides information about the same set of upcoming auctions. Although the quality of information provided might be slightly higher reducing monitoring costs this change should be relatively small. For bigger firms operating stations in several regions the new policy means a reduction in number of sources they have to monitor significantly reducing monitoring costs per auction.

The asymmetric reduction of monitoring costs should facilitate entry for big firms. As the monitoring efforts don't change for the small firms their entry is either unchanged, or reduced if they are faced by new competition from the big firms and chose to abandoned these auctions. The lower monitoring costs should also reduce the price in the auctions where big firms participate (because they are more likely to be an "additional entrant" and increase competition and because they tend to have lower costs of performing the contract), but not necessarily the prices in auctions where only small firms are present. Table 3.2 shows the presence of big and small firms in the markets defined by average auctions before and after 2011; and the entry decisions by small and big firms before and after 2011.

Table 3.2: Possibility of entry and entry by small and big firms

		Share of station in municipality	Entry
Big firms	Before	0.3080	0.2707
		(0.1268)	(0.4456)
	After	0.3136	0.5570
		(0.1480)	(0.4974)
Small firms	Before	0.2003	0.2731
		(0.0754)	(0.4466)
	After	0.1718	0.2447
		(0.0672)	(0.4302)

Standard deviations in parenthesis.

To estimate the effect of the policy on small and big firms I use the differences-in-differences approach. I estimate linear models of the following general form:

$$Outcome_{jit} = \alpha_0 + \alpha_1 P_t + \alpha_2 F_j + \alpha_3 P_t \times F_j + X'_{it} \beta + Z'_{ijt} \delta + u_{ijt},$$

where  $Outcome_{jit}$  is entry decision, bidding decision or probability of winning conditional on entry of firm  $j$  in auction  $i$  at time  $t$ ;  $P_t$  is a policy dummy equal to one after 2011 and to zero before 2011;  $F_j$  - firm type dummy equal to one for firms that have stations outside of Sverdlovsk region as well as inside the region, and to zero for firms that operate only inside the region;  $X_{it}$  is a set of auction characteristics (same for all firms);  $Z_{ijt}$  - a set of firm-specific auction characteristics (such as participation eligibility, number of stations in the defined area of service etc.)

Table 3.3 presents the results of OLS and differences-in-differences analysis for all observed bids. It shows that small firms bid less aggressively under the new information policy while big firms bid more aggressively. Table 3.4 presents the results of probability of entrance regressions. Since interpreting the coefficients associated with the interaction terms in non-linear models is somewhat fuzzy I focus on linear probability model for the differences-in-differences results. As predicted big firms enter significantly more often after 2011 than before 2011, while there is no significant effect on small firm entry. It is also worth noting that conditional on eligibility big firms entered significantly less often than small firms prior to 2011 (firm\_type coefficient in LPM-DD model in Table 3.4) and enter more often after 2011 (the sum of firm\_type and after×firm\_type coefficients in Table 3.4). Table 3.5 presents the results for probability of winning conditional on entry. It shows no significant differences in probability of winning for small vs big firms before and after the change of information policy. This means that we would see more big firms winning the auctions after 2011 - they have the same chance of winning the auction

if they entered and enter more auctions. These results are in line with my initial assumption about the effect that the new policy has on monitoring costs<sup>13</sup>.

Table 3.3: All bids

	All bids, OLS		All bids, DD	
	Normalized bid	% Discount	Normalized bid	% Discount
Intercept	0.1529*** (0.0383)	0.0599*** (0.0121)	0.1622*** (0.0379)	0.0672*** (0.0123)
After	0.0154*** (0.0056)	-0.0184** (0.0057)	0.0314*** (0.0070)	-0.0306*** (0.0072)
Large firm	0.0084* (0.0049)	-0.0107** (0.0050)	0.0381*** (0.0094)	-0.0334*** (0.0097)
After×Large firm			-0.0356*** (0.0096)	0.0270*** (0.0099)
Open auction	0.0011 (0.0067)	0.0039 (0.0069)	0.0010 (0.0067)	0.0043 (0.0069)
Normalized reserve price	0.8174*** (0.0313)		0.8011*** (0.0312)	
N pot. entrants	0.0036* (0.0020)	-0.0068*** (0.0020)	0.0027 (0.0020)	-0.0063*** (0.0020)
Stations share	-0.0243 (0.0154)	0.0280* (0.0159)	-0.0263* (0.0153)	0.0298* (0.0158)
Contract controls	Yes	Yes	Yes	Yes
Observations	519	519	519	519
R-squared	0.597	0.091	0.608	0.104

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses.

<sup>13</sup>These results hold if municipalities with partial data and “federal” buyers are left out of the sample, see Appendix 5.

Table 3.4: Probability of entry

	Probit	LPM	LPM-DD
Intercept	-1.7292*** (0.6369)	-0.1411 (0.2029)	0.0004 (0.2026)
After	0.3595*** (0.0923)	0.1325*** (0.0302)	0.0020 (0.0381)
Large firm	0.3124*** (0.0860)	0.1117*** (0.0283)	-0.0907*** (0.0461)
After $\times$ Large firm			0.2915*** (0.0528)
Open auction	-0.5695*** (0.1291)	-0.1743*** (0.0403)	0.0010 (0.0067)
Normalized reserve price	0.6200 (0.5109)	0.2153 (0.1617)	0.8011*** (0.0312)
N pot. entrants	-0.0858** (0.0398)	-0.0259* (0.0133)	0.0027 (0.0020)
Stations share	2.3417*** (0.3890)	0.8478*** (0.1297)	-0.0263* (0.0153)
Contract con- trols	Yes	Yes	Yes
Observations	1503	1503	1503
R-squared		0.121	0.138

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses.



Table 3.5: Probability of winning conditional on entry

	Probit	LPM	LPM-DD
Intercept	-1.4758 (1.2815)	0.0059 (0.4317)	-0.0130 (0.4326)
After	0.0709 (0.1697)	0.0025 (0.0600)	-0.0351 (0.0775)
Large firm	-0.6101*** (0.1645)	-0.2280*** (0.0573)	-0.2893*** (0.0985)
After $\times$ Large firm			0.0791 (0.1033)
Open auction	0.7850*** (0.2654)	0.2349*** (0.0825)	0.2335*** (0.0825)
Normalized reserve price	0.0329 (1.0329)	0.0097 (0.3489)	0.0497 (0.3518)
N pot. entrants	0.2081*** (0.0711)	0.0734*** (0.0242)	0.0823*** (0.0252)
Stations share	3.6337** (0.6300)	1.3133*** (0.2128)	1.3645*** (0.2172)
Contract controls	Yes	Yes	Yes
Observations	502	502	502
R-squared		0.119	0.120

\*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels respectively. Standard errors in parentheses.

### 3.4 Estimation of entry costs

To tackle the structural effect of the policy change on behavior of both large and small firms, I assume that all bidders must obtain information about the auction prior to the auction. A more centralized source of information would then reduce entry costs for all types of bidders. However, since I study a simple product which is sold by all auction participants in an open market, I would assume that all potential bidders know their true costs of fulfilling the project prior to entry, and the entry

costs consist of searching the auction, fulfilling the eligibility criteria, preparing bidding documentation etc. To estimate the cost distributions of large and small firms, and their entry costs before and after policy change, I use the [Samuelson \(1985\)](#) model of selective entry: the bidders would enter as long as their expected profits are higher than the entry costs, and each equilibrium bidding strategy includes a threshold value of costs such that bidders with higher costs do not enter the auction. I would also use the methodology of estimating truncated cost distributions of small and large firms based on [Guerre et al. \(2000\)](#) and further on [Flambard and Perrigne \(2006\)](#).

Assume that there are  $n_F$  small firms and  $n_R$  large firms who are eligible to participate in the auction. Small firms draw their costs independently from a distribution  $F_F(c)$ , and large firms draw their costs independently from a distribution  $F_R(c)$ . To enter into the auction, the small firm has to incur entry costs  $\kappa_F$ , and the large firm has to incur entry costs  $\kappa_R$ .

If all other bidders bid according to some monotonic strategies  $\phi_R(b)$  and  $\phi_F(b)$  the expected profits of bidder type  $i = F, R$  are:

$$\Pi_i(b) = (b - c)(1 - F_i(\phi_i(b)))^{n_i-1}(1 - F_j(\phi_j(b)))^{n_j}. \quad (3.1)$$

The bidder enters the auction if  $\Pi_i^*(c) \geq \kappa_i$ , and in equilibrium should choose her bid to follow the inverse bidding function  $\phi_i(b)$  and the set of standard first order conditions:

$$\frac{1}{b-c} = \frac{(n_i-1)f_i(\phi_i(b))\phi'_i(b)}{1-F_i(\phi_i(b))} + \frac{n_j f_j(\phi_j(b))\phi'_j(b)}{1-F_j(\phi_j(b))}, \quad i = F, R \quad (3.2)$$

As [Flambard and Perrigne \(2006\)](#) note, the distribution of bids we observe in this case is truncated, and if  $\hat{c}_i$  is the threshold entry value for the bidder of type  $i$  her bids and costs distributions have a following relation:

$$\begin{aligned} F_i(\phi_i(b))F_i(\hat{c}_i) &= \hat{G}_i(b), \\ f_i(\phi_i(b))\phi'_i(b)F_i(\hat{c}_i) &= \hat{g}_i(b) \end{aligned} \quad (3.3)$$

This means that it is enough to observe the distribution of bids and the entry probability  $\hat{F}_i(\hat{c})$  to be able to estimate both the distribution of costs and the entry costs of each type of bidder.

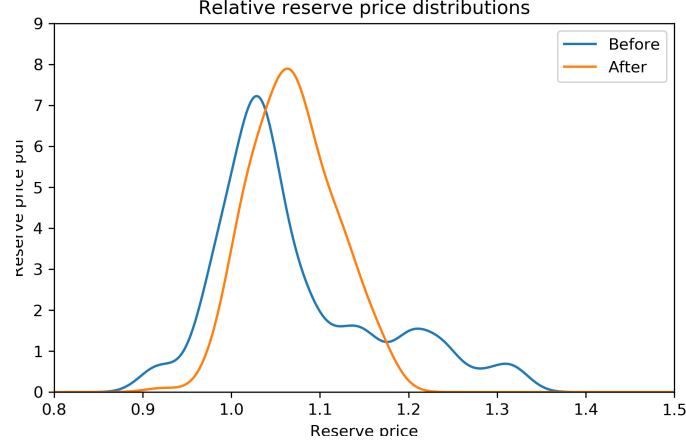
Since the distribution of reserve prices in my data changes over the time (shown on figure [3.3](#)), I start the estimation by calculating the entry probabilities for both types of bidders conditional on the reserve price:

$$\hat{F}_i(\hat{c}_i) = \frac{n_i}{\mathbb{E}(n_i|r)} = \frac{n_i}{\int_0^r n_i \hat{z}_r(p) dp}, \quad (3.4)$$

where  $n_i$  is the number of entrants in the auction and  $\hat{z}_r(p)$  is the nonparametrically estimated distribution of reserve price.

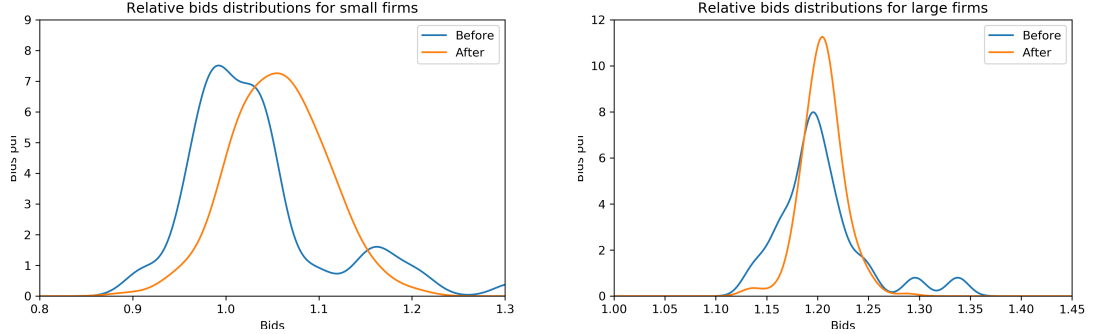
I then estimate the bid distributions for small and large firms before and after the policy change. Figure [3.4](#) shows the resulting estimates of these distributions. Both small and large firms bid lower on average after the policy change, though it is hard to say whether it is linked to the changes in the reserve prices or in entry

Figure 3.3: Kernel estimates of reserve price distribution before and after the policy change



costs.

Figure 3.4: Kernel estimates of relative bid distributions for small and large firms



Finally, I use the equations:

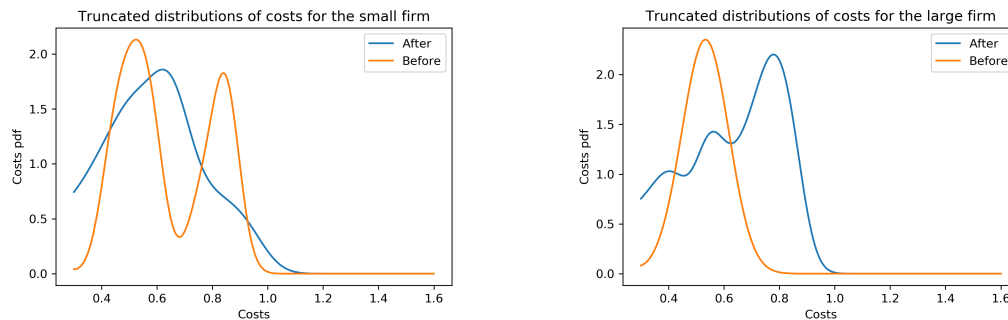
$$\frac{1}{b-c} = \frac{(n_i-1)\hat{g}_i(b)}{1-\hat{G}_i(b)} + \frac{n_j\hat{g}_j(b)}{1-\hat{G}_j(b)}, \quad i = F, R \quad (3.5)$$

to calculate cost estimates for each bidder and estimate the truncated cost distributions for both types of bidders before and after the policy change.

The resulting estimates of the truncated cost distributions for small and large firms are shown on Figure 3.5. One can note that the large firm costs are visibly

smaller on average after the policy change. However the small firms cost distribution before the policy change is bimodal with one of the modes lower than the post change average costs of a small entrant, and the other one larger. This might indicate that three types of firms rather than two should be used, especially since the market structure can also be represented by three distinct types of firms (local, regional, and federal). However the difference in estimated cost distributions of the large firms might also indicate that a more comprehensive model accounting for forward-looking behavior of these firms is needed. Though it can be partially explained by smaller entry costs that allow large firms to enter even when expected profits are low, it might also be a sign of the large firms entering to prevent future competition or signal their current state to the small firms as in the model of Chapter 2. These results might also indicate the identification problems linked to a small sample size of the auctions before the policy change.

Figure 3.5: Kernel estimates of relative costs distributions for small and large firms



However if we trust the estimated cost distributions above, we can derive the estimates of entry costs for each type of firms before and after the policy change. To estimate the entry costs for each type of bidders, I assume that I observe the threshold bidder of each type before and after the change, and that the expected

Table 3.6: Estimated entry costs for large and small firms

	Large	Small
Before	0.000092	0.000016
After	0.000002	0.000033

profits at  $\hat{c}_i = \max c_i$  are the entry costs. The estimates shown in Table 3.6 suggest that the entry costs are relatively small (less than 0.01% of the market price of the contract), but were decreased by the policy change order of magnitude for the large firms, and increased for the small firms.

### 3.5 Conclusions

In this chapter I have looked at the effects of reducing costs of obtaining information about upcoming and past auctions for key auction outcomes. The information policy introduced in Jan 2011 in Russian public procurement had ambiguous effects. The reform had different effect on small and large firms: there is no significant effect on observed small firms entry, significant positive effect on large firms entry; large firms bid more aggressively after the introduction of new policy, while small firms bid less aggressively. Although it did increase participation rates especially for big firms that have stations in several Russian regions, it did not increase the overall competitiveness of bidding.

The asymmetry of the reaction of big firms and small firms to the reform is in line with the “monitoring costs” intuition: firms have to obtain information about upcoming and past auctions in order to participate and bid optimally. The new information policy should significantly reduce these costs for the big firms but not

for small firms.

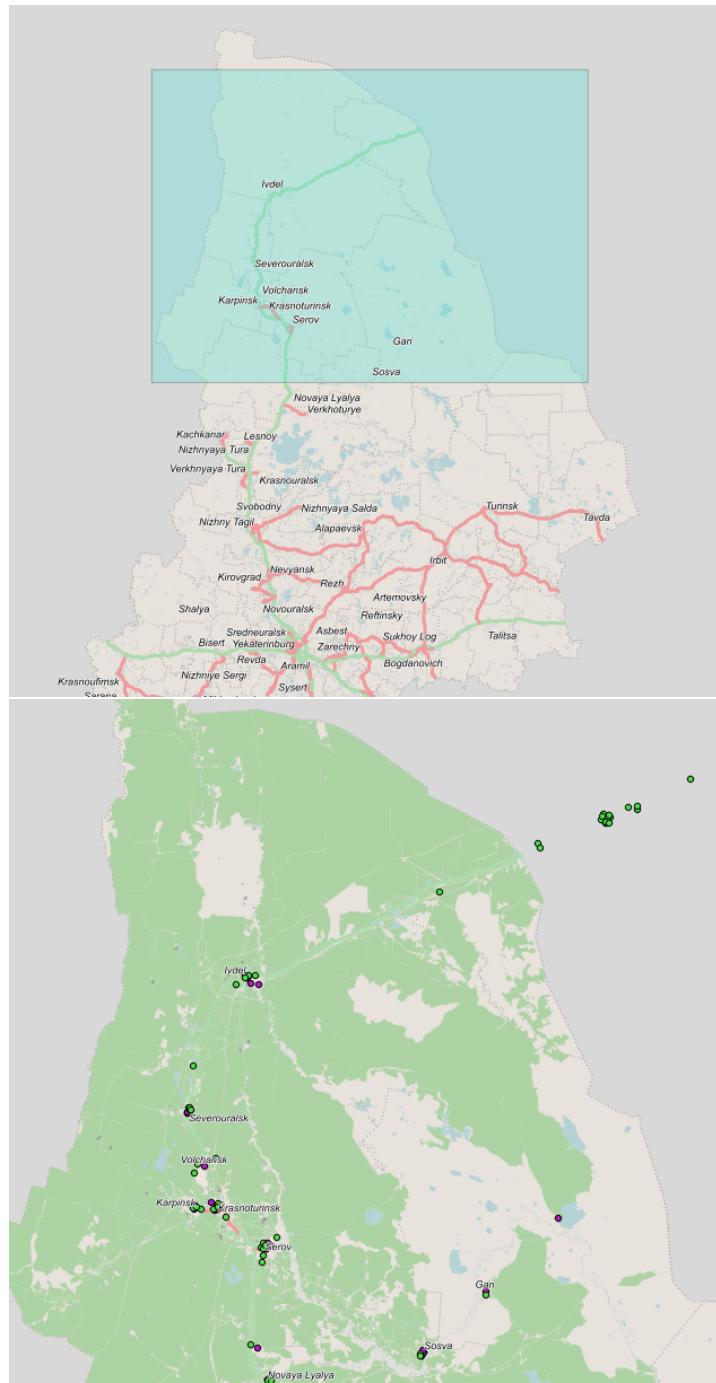
The existence of monitoring costs should be taken into account when policies stimulating entry into auctions are implemented. For example, reducing entry costs may be not sufficient to induce more entry if the costs of acquiring information stay the same. In order to assess the relevant importance of informational (monitoring costs) and non-informational (entry costs) barriers to entry we should be able to see how these costs factor in firms participation rate and bidding decisions.

Estimates of a static selective entry model show that the entry costs did decrease substantially for large firms, and increased for the small firms, potentially decreasing the overall competitiveness of the auctions. However the resulting estimates of cost distributions for both types of firms also change significantly over type, suggesting that a more comprehensive model capturing the dynamic considerations of both types of firms is needed to capture the full effects of the policy change.

## 3.6 Appendix

### 3.6.1 Choice of subsample

Figure 3.6: Subsample definition





### 3.6.2 Summary statistics

Table 3.7: Summary statistics: North subsample

Variable	Description	Mean	St.dev	Min	Max	Count
After	dummy: 0 before Jan 1 2011; 1 after	0.7155	0.4518	0	1	348
Federal	dummy: 1 for federal procurers (are not in “before”-database)	0.2069	0.4057	0	1	348
Auct_type	dummy: 1 for open bid auctions; 0 for sealed bid	0.2213	0.4157	0	0	348
Sprice	Reserve price in RUR	360992	329688	10000	2335800	348
Volume	Liters of gasoline procured	14246.27	14801.23	380	123702	348
nNprodtypes	number of types of gasoline and related products procured	1.6523	0.8367	1	7	348
Market_price	market price of bundle procured at the time of auction	337105	306585	9283	2369575	348
Rel_start_price	relative reserve price = (reserve price)/(market price)	1.0735	0.0772	0.3712	1.5401	348
Npot	potential number of entrants, firms that have stations in the area of service defined	4.1876	1.0267	2	7	348
Npart	number of active bidders who were accepted and submitted a bid	1.4454	0.6704	0	3	348
Rel_price	relative winning bid = (winning bid)/(market price)	1.0519	0.0683	0.5498	1.540	324
Rel_bid1	relative second bid where there were 2 or more bidders	1.0637	0.046	0.9228	1.2059	182

Table 3.8: Summary statistics: North subsample before Jan 1, 2011

<b>Variable</b>	<b>Mean</b>	<b>St.dev</b>	<b>Min</b>	<b>Max</b>	<b>Count</b>
Auct_type	0.3637	0.4835	0	0	99
Sprice	466191	469157	27000	2335800	99
Volume	22140	22568	1139	123702	99
Nprodtypes	2.1717	0.9902	2	7	99
Market_price	433487	434522	24227	2369574.60	99
Rel_start_price	1.0795	0.1267	0.3712	1.5401	99
Npot	4.2528	0.741	3	7	99
Npart	1.0404	0.7412	0	3	99
Rel_price	1.0621	0.1035	0.9112	1.540	75
Rel_bid1	1.0151	0.0431	0.9511	1.0759	8

Table 3.9: Summary statistics: North subsample after Jan 1, 2011

<b>Variable</b>	<b>Mean</b>	<b>St.dev</b>	<b>Min</b>	<b>Max</b>	<b>Count</b>
Federal	0.2892	0.4542	0	1	249
Auct_type	0.1647	0.3716	0	0	249
Sprice	319166	242758	10000	1357200	249
Volume	11108	8397	380	44800	249
Nprodtypes	1.4457	0.6645	1	4	249
Market_price	298785	227427	9283	1310800	249
Rel_start_price	1.0711	0.0445	0.9229	1.1776	249
Npot	4.1646	0.9883	2	7	249
Npart	1.6064	0.5659	1	3	249
Rel_price	1.0488	0.0532	0.5498	1.1657	249
Rel_bid1	1.0659	0.0450	0.9228	1.2059	174

Table 3.10: Summary statistics by number of participating bidders

	Npart	Auct_type	Nprodtypes	Rel_start_price	Volume	Npot	Rel_price	N
Before	0	0.3333 (0.4815)	2.3333 (1.3077)	1.0341 (0.1670)	21303.88 (2867.0)	3.5833 (0.7755)	NA	24
	1	0.4167 (0.4982)	2.00 (0.8251)	1.0965 (0.1168)	26623.48 (26112.59)	4.0208 (1.0208)	1.0840 (0.1085)	48
	2	0.3077 (0.4707)	2.3462 (0.9356)	1.0851 (0.0916)	14581.71 (12622.48)	3.9615 (0.9992)	1.0184 (0.0795)	26
	3	1	2	1.211	23490	4	1.1543	1
After	1	0.3491 (0.4789)	1.3397 (0.6155)	1.0707 (0.0425)	13608.83 (10392.73)	4.3396 (1.0680)	1.0579 (0.0482)	106
	2	0.0231 (0.1507)	1.5462 (0.7056)	1.0716 (0.0452)	9195.86 (5695.28)	4.0462 (1.0555)	1.0417 (0.0567)	130
	3	0.0 (0.0)	1.300 (0.4830)	1.0561 (0.0518)	9013.40 (5726.41)	5.100 (0.7379)	1.0320 (0.0473)	10

### 3.6.3 Consistency of demand over time and municipalities

Figure 3.7: Volume demanded: North subsample

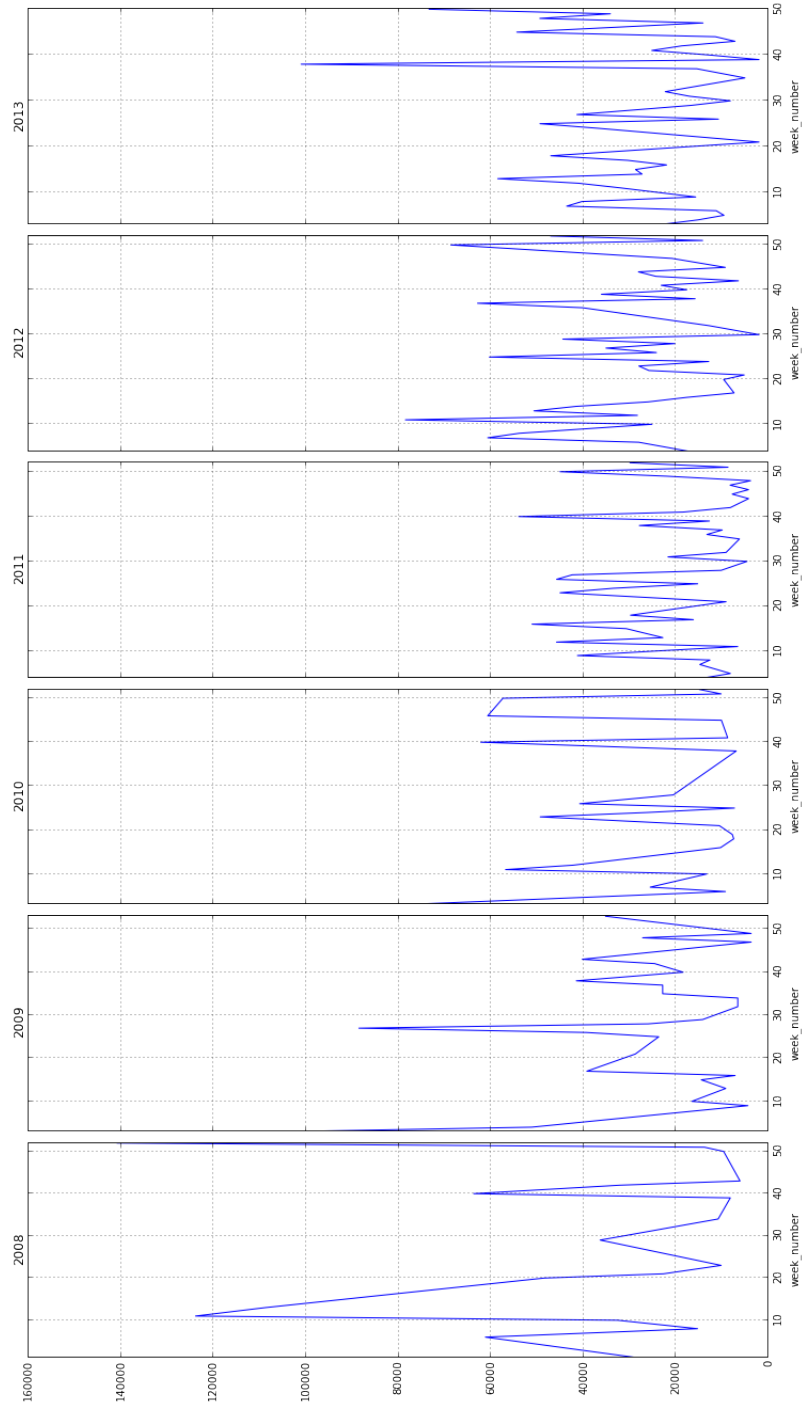
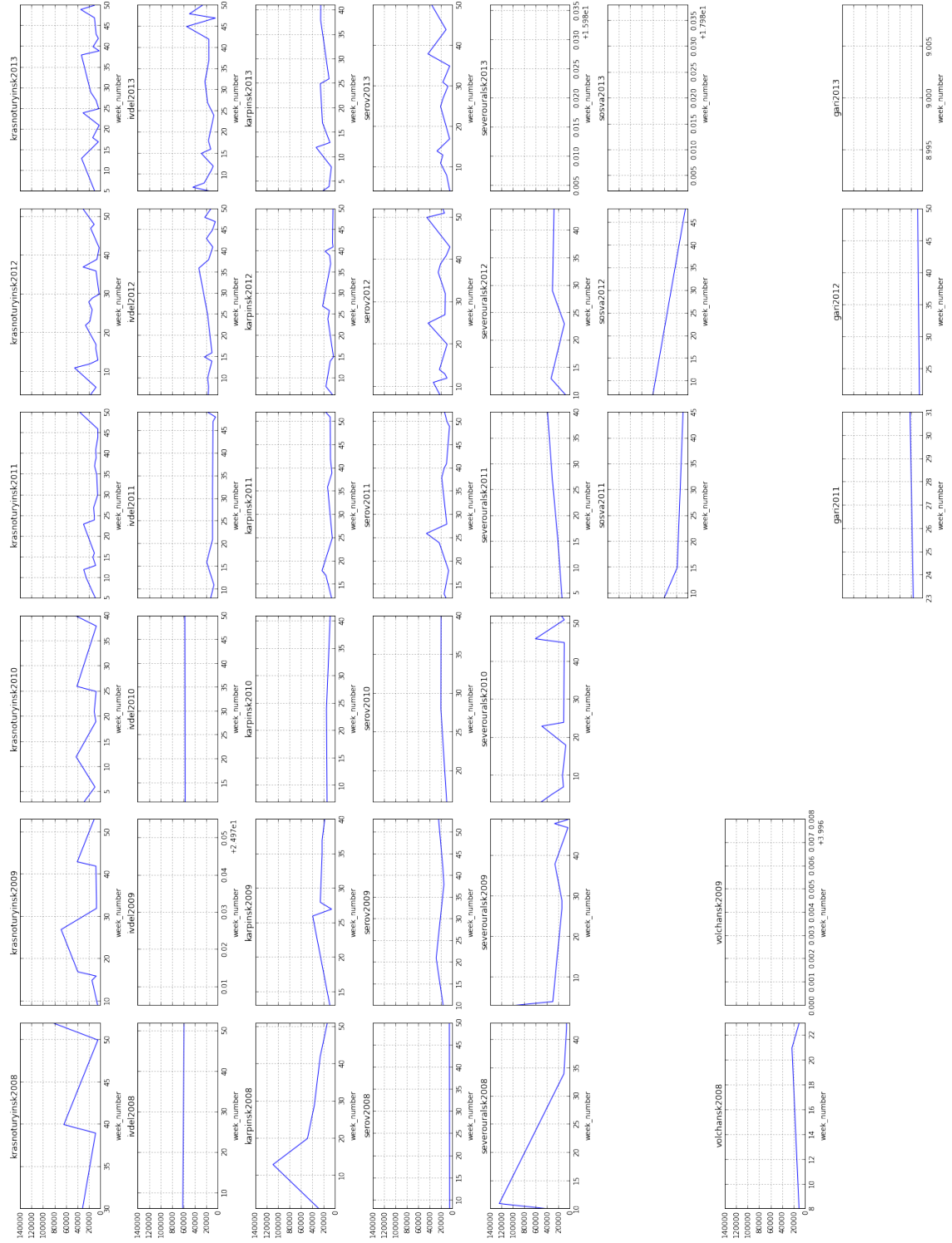


Figure 3.8: Volume demanded: North subsample by municipality



### 3.6.4 Robustness to potential biases in data

Table 3.11: Without municipalities with partial data

	Relative bid	Bid discount	Probability of entry	Probability of winning (conditional on entry)
Intercept	0.1224*** (0.0396)	0.0680*** (0.0148)	0.0088 (0.2077)	-0.1164 (0.4492)
After	0.0253*** (0.0082)	-0.0245*** (0.0084)	0.0021 (0.0400)	0.02 (0.0836)
Big_firm	0.0300*** (0.0097)	-0.0253** (0.0100)	-0.0868* (0.0476)	-0.2356** (0.1039)
After×Big_firm	-0.0293*** (0.0100)	0.0216** (0.0102)	0.2886*** (0.0540)	0.0149 (0.1075)
Open_bid	-0.0032 (0.0070)	0.0075 (0.0072)	-0.1729*** (0.0408)	0.2447*** (0.0844)
Rel_start_price	0.8324*** (0.0324)		0.2004 (0.1645)	0.0372 (0.3684)
Potential entrants	0.0041* (0.0021)	-0.0067*** (0.0021)	-0.0293** (0.0137)	0.0831*** (0.0254)
Stations_share	-0.002 (0.0180)	0.0066 (0.0185)	0.7992*** (0.1319)	1.3612*** (0.2185)
Contract controls	Yes	Yes	Yes	Yes
Observations	456	456	1446	484
R-squared	0.634	0.074	0.140	0.123

Table 3.12: Without municipalities with partial data and “federal” firms

	Relative bid	Bid discount	Probability of entry	Probability of winning (conditional on entry)
Intercept	0.1249*** (0.0442)	0.0613*** (0.0164)	-0.0226 (0.2128)	-0.3913 (0.4759)
After	0.0275*** (0.0088)	-0.0297*** (0.0090)	-0.0362 (0.0401)	0.0019 (0.0877)
Big_firm	0.0326*** (0.0102)	-0.0264** (0.0104)	-0.0880* (0.0465)	-0.2413** (0.1047)
After×Big_firm	-0.0358*** (0.0107)	0.0313*** (0.0109)	0.3760*** (0.0545)	0.0537 (0.1120)
Open_bid	-0.0039 (0.0074)	0.0088 (0.0075)	-0.1528*** (0.0416)	0.2372*** (0.0849)
Rel_start_price	0.8343*** (0.0367)		0.1544 (0.1680)	0.2616 (0.3940)
Potential entrants	0.0048* (0.0024)	-0.0067*** (0.0025)	-0.013 (0.0151)	0.0862*** (0.0275)
Stations_share	-0.0061 (0.0197)	0.0058 (0.0202)	0.8612*** (0.1393)	1.3716*** (0.2293)
Contract controls	Yes	Yes	Yes	Yes
Observations	350	350	1133	378
R-squared	0.644	0.099	0.194	0.151

## Bibliography

- Andreoni, J., Y. K. Che, and J. Kim (2007). Asymmetric information about rivals' types in standard auctions: An experiment. *Games and Economic Behavior* 59(2), 240–259.
- Athey, S., J. Levin, and E. Seira (2011). Comparing open and sealed bid auctions: Evidence from timber auctions. *Quarterly Journal of Economics* 126(1), 207–257.
- Balat, J. (2013). Highway Procurement and the Stimulus Package : Identification and Estimation of Dynamic Auctions with Unobserved Heterogeneity. *Working paper*.
- Bergemann, D. and J. Hörner (2018). Should first-price auctions be transparent? *American Economic Journal: Microeconomics* 10(3), 177–218.
- Byrd, R. H., J. Nocedal, and R. A. Waltz (2006). *Knitro: An Integrated Package for Nonlinear Optimization*, pp. 35–59. Boston, MA: Springer US.
- Cason, T. N., K. N. Kannan, and R. Siebert (2011). An Experimental Study of In-



- formation Revelation Policies in Sequential Auctions. *Management Science* 57(4), 667–688.
- Coviello, D. and M. Mariniello (2014). Publicity Requirements in Public Procurement: Evidence from a Regression Discontinuity Design. *Journal of Public Economics* 109, 76–100.
- Czyzyk, J., M. P. Mesnier, and J. J. Moré (1998). The NEOS Server. *IEEE Journal on Computational Science and Engineering* 5(3), 68 – 75.
- Danak, A. and S. Mannor (2009). Bidding efficiently in repeated auctions with entry and observation costs. *Proceedings of the 2009 International Conference on Game Theory for Networks, GameNets '09*, 299–307.
- de Castro, L. I. and M.-A. de Frutos (2010). How to translate results from auctions to procurements. *Economics Letters* 106(2), 115–118.
- De Silva, D. G., T. Dunne, A. Kankanamge, and G. Kosmopoulou (2008). The impact of public information on bidding in highway procurement auctions. *European Economic Review* 52(1), 150–181.
- De Silva, D. G., T. D. Jeitschko, and G. Kosmopoulou (2005). Stochastic synergies in sequential auctions. *International Journal of Industrial Organization* 23(3-4), 183–201.
- Ding, W., T. D. Jeitschko, and E. G. Wolfstetter (2010). Signal jamming in a sequential auction. *Economics Letters* 108(1), 58–61.

- Dolan, E. D. (2001). The neos server 4.0 administrative guide. Technical Memorandum ANL/MCS-TM-250, Mathematics and Computer Science Division, Argonne National Laboratory.
- Donna, J. D. and J.-A. Espín-Sánchez (2018). Complements and substitutes in sequential auctions: the case of water auctions. *The RAND Journal of Economics* 49(1), 87–127.
- Dufwenberg, M. and U. Gneezy (2002). Information disclosure in auctions: An experiment. *Journal of Economic Behavior and Organization* 48(4), 431–444.
- Fang, H. and S. Morris (2006). Multidimensional private value auctions. *Journal of Economic Theory* 126(1), 1–30.
- Fibich, G., A. Gaviols, and A. Sela (2002). Low and high types in asymmetric first-price auctions. *Economics Letters* 75(2), 283–287.
- Figliozi, M. a., H. S. Mahmassani, and P. Jaillet (2008). Repeated auction games and learning dynamics in electronic logistics marketplaces: Complexity, bounded rationality, and regulation through information. *Understanding Complex Systems* 2008, 137–175.
- Flambard, V. and I. Perrigne (2006). Asymmetry in procurement auctions: Evidence from snow removal contracts. *Economic Journal* 116(514), 1014–1036.
- Gentry, M. and T. Li (2014). Identification in Auctions With Selective Entry. *Econometrica* 82(1), 315–344.

- Groeger, J. R. (2014). A Study of Participation in Dynamic Auctions. *International Economic Review* 55(4), 1129–1154.
- Gropp, W. and J. J. Moré (1997). Optimization environments and the neos server. In M. D. Buhman and A. Iserles (Eds.), *Approximation Theory and Optimization*, pp. 167 – 182. Cambridge University Press.
- Guerre, E., I. Perrigne, and Q. Vuong (2000). Optimal Nonparametric Estimation of First-price Auctions. *Econometrica* 68(3), 525–574.
- Hafalir, I. and V. Krishna (2008). Asymmetric auctions with resale. *American Economic Review* 98(1), 87–112.
- Haile, P. A. (2000). Partial pooling at the reserve price in auctions with resale opportunities. *Games and Economic Behavior* 33(2), 231–248.
- Han, Z., R. Zheng, and V. H. Poor (2009). Repeated Auctions with Learning for Spectrum Access in Cognitive Radio Networks. *IEEE Transactions on Wireless Communications* 10(3), 890–901.
- Hubbard, T. P., R. Kirkegaard, and H. J. Paarsch (2013a). Using Economic Theory to Guide Numerical Analysis: Solving for Equilibria in Models of Asymmetric First-Price Auctions. *Computational Economics* 42(2), 241–266.
- Hubbard, T. P., R. Kirkegaard, and H. J. Paarsch (2013b). Using economic theory to guide numerical analysis: Solving for equilibria in models of asymmetric first-price auctions. *Computational Economics* 42(2), 241–266.

- Hubbard, T. P. and H. J. Paarsch (2009). Investigating bid preferences at low-price, sealed-bid auctions with endogenous participation. *International Journal of Industrial Organization* 27(1), 1–14.
- Jeziorski, P. and E. Krasnokutskaya (2016). Dynamic Auction Model with Subcontracting. *The RAND Journal of Economics* 47(4), 751–791.
- Jofre-Bonet, M. and M. Pesendorfer (2000). Bidding behavior in a repeated procurement auction: A summary. *European Economic Review* 44(4-6), 1006–1020.
- Jofre-Bonet, M. and M. Pesendorfer (2007). Estimation of a Dynamic Auction Game. *Econometrica* 71(5), 1443–1489.
- Kannan, K. N. (2012). Effects of information revelation policies under cost uncertainty. *information systems Research* 23(1), 75–92.
- Kong, Y. (2016). Sequential Auctions with Synergy and Affiliation. *Working paper*, 1–61.
- Krasnokutskaya, E. and K. Seim (2011). American Economic Association Bid Preference Programs and Participation in Highway Procurement Auctions Bid Preference Programs and Participation in Highway Procurement Auctions<sup>1</sup>. *American Economic Review* 101(6), 2653–2686.
- Landsberger, M., J. Rubinstein, E. Wolfstetter, and S. Zamir (2001). First-price auctions when the ranking of valuations is common knowledge. *Review of Economic Design* 6(3-4), 461–480.

- Lebrun, B. (1998). Comparative statics in first price auctions. *Games and Economic Behavior* 25(1), 97–110.
- Leslie, P. and P. Zoido (2011). Information Entrepreneurs and Competition in Procurement Auctions. *Working Paper*.
- Levin, D. and J. L. Smith (1994). Equilibrium in Auctions with Entry. *The American Economic Review* 84(3), 585–599.
- Li, T. and X. Zheng (2012). Information acquisition and/or bid preparation: A structural analysis of entry and bidding in timber sale auctions. *Journal of Econometrics* 168(1), 29–46.
- Martínez-Pardina, I. (2006). First-price auctions where one of the bidders’ valuations is common knowledge. *Review of Economic Design* 10(1), 31–51.
- Maskin, E. and J. Riley (2000a). Asymmetric Auctions. *The Review of Economic Studies* 67(3), 413–438.
- Maskin, E. and J. Riley (2000b). Equilibrium in Sealed High Bid Auctions. *The Review of Economic Studies* 67(3), 439–454.
- Maskin, E. and J. Riley (2003). Uniqueness of equilibrium in sealed high-bid auctions. *Games and Economic Behavior* 45(2), 395–409.
- Ohashi, H. (2009, June). Effects of Transparency in Procurement Practices on Government Expenditure: A Case Study of Municipal Public Works. *Review of Industrial Organization* 34(3), 267–285.

- Roberts, J. J. W. and A. Sweeting (2010). Entry and Selection in Auctions. *Nber 16650*.
- Saini, V. (2012). Endogenous asymmetry in a dynamic procurement auction. *RAND Journal of Economics* 43(4), 726–760.
- Samuelson, W. F. (1985). Competitive Bidding with Entry Costs. *Economic Letters* 17, 53–57.
- Somaini, P. (2011). Spatial Competition and Interdependent Costs in Highway Procurement. *Working paper*, 1–56.
- Thomas, C. J. (2010). Information revelation and buyer profits in repeated procurement competition. *Journal of Industrial Economics* 58(1), 79–105.
- Tiererova, L. (2013). A Dynamic Model of Bidder Learning in Procurement Auctions Job Market Paper. *Working paper*.
- Tu, Z. (2006). Why do we use Dutch Auction to Sell Flowers? Information Disclosure in Sequential auctions. *Working paper*, 1–42.