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PERFORMANCE OF RS-BCH CONCATENATED CODES AND BCH SINGLE-STAGE CODES ON AN INTERFERENCE SATELLITE CHANNEL

by

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ABSTRACT

A model is developed and analyzed for the study of interference on satellite channels. It is utilized in performance evaluation of RS-BCH concatenated codes and BCH single-stage codes on a satellite channel corrupted by co-channel interference. A coherent phase-shift keyed (CPSK) system is assumed. Results obtained utilize earlier work on performance analysis of an m-phase CPSK system in the presence of random Gaussian noise and non-Gaussian interference.

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Earlier work on performance evaluation of concatenated codes on an equierror channel is also utilized. Our model incorporates features that account for the burst behavior of the interference sources. Results show that the use of RS-BCH concatenated codes provide significant performance improvement over the case of no coding as well as over that of single-stage BCH codes. In addition, the use of single-stage BCH codes is shown to provide performance improvement over no coding.

SECTION 1

1. INTRODUCTION

The ever increasing demand for satellite communications capacity is motivating the search for techniques to enhance the utilization efficiency of the preferred geostationary orbit. Since this orbit is a naturally fixed resource, the search clearly points to the need for reducing intersatellite spacing which manifests intersatellite interference such as co-channel and adjacent channel interference. The problem of modeling non-Gaussian interference on a satellite channel or at an earth station has been considered in previous works [1-6]. Prabhu analyzed the performance of an m-phase CPSK system in the presence of additive random Gaussian noise and co-channel interference [1]. Goldman determined the statistical properties of an n-dimensional Gaussian random vector plus the sum of M vectors having random amplitudes and independent, arbitrary orientations in n-dimensional space. Goldman's results are directly applicable to CPSK systems operating in the presence of M co-channel interferers modeled by a sum of constant amplitude sinusoids with independent, uniformly distributed phase angles [2]. The results of Prabhu and Goldman are useful because they permit performance evaluation of any given CPSK system when the received signal is corrupted by both interference and random Gaussian noise. Those results are limited, however, in that they did not address the question of performance evaluation under system enhancements, which may be required or preferred, in order to achieve reliable point-to-point communications when interference is present. For example, error-control codes provide an attractive and effective alternative to high signal power, which is expensive and not always available.

There are two fundamentally different types of error-control codes: algebraic block codes, and probabilistic tree codes. Of the many classes of these codes, single-stage BCH block codes and RS-BCH concatenated codes are considered here. Single-stage BCH codes are selected for two reasons. First, a simply implemented decoding algorithm for BCH codes was devised by Peterson [7] and later made more powerful by Berlekamp [8] and Massey [9]. Second, BCH codes provide satisfactory, although not necessarily optimal, inner codes for concatenated codes. The outer codes of the concatenated codes we investigate are Reed-Solomon (RS) codes which are non-binary BCH codes. RS-BCH concatenated codes are selected, rather than RS-tree concatenated codes, because performance analysis is simpler. In fact, we can utilize directly Forney's model for performance evaluation of concatenated codes on an equierror channel [10]. Forney originated the idea of concatenated coding in which the encoder consists of two encoders placed back-to-back, and the decoder consists of two decoders placed back-to-back. This idea permits the construction of very long codes for which the probability of error on a memoryless channel decreases exponentially, while decoder complexity increases only as a small power of the block length.

This paper combines the approaches of Prabhu and Forney by addressing the application of RS-BCH concatenated codes and BCH single-stage codes in a binary CPSK system to combat co-channel interference plus additive random Gaussian noise on a satellite channel. The results obtained here provide simple expressions with which to evaluate the performance of the selected codes in the given system. They also permit the incorporation of burst interference behavior in the model following the approach developed in [11 - 14].

This paper is organized as follows:

> In Section 2, the model for co-channel interference is presented. Section 2 also provides the performance analysis for single-stage BCH codes and RS-BCH concatenated codes on a satellite channel with co-channel interference. Both fixed and random interference are considered. In Section 3, performance results for no coding, single-stage BCH coding, and RS-BCH concatenated coding are presented and compared. In Section 4, conclusions are drawn and future applications of the model are suggested.

SECTION 2

2. MODEL

CPSK is an efficient technique for trading bandwidth for signal-to-noise ratio (SNR) and is, therefore, well suited for satellite communications systems. The performance of CPSK systems traditionally has been investigated for the additive random Gaussian noise channel. Here we consider the use of binary CPSK modulation on a satellite channel that is corrupted by both additive random Gaussian noise and interference. The interference imparts memory to the channel. We assume that the memory has duration no longer than one symbol, where a symbol is a suitably long string of bits, so that interference results in a burst of bit errors no longer than one symbol. Coding techniques can then be utilized to improve system performance.

Assume also that the interference is due to M co-channel interferers. The total interference from the co-channel interfering signals is modeled by a sum of constant amplitude sinusoids with independent, uniformly distributed phase angles [2]. Let K be the total interference level defined as the sum of the powers of the individual interferers, i.e.,

$$K = \sum_{i=1}^{M} I_i \quad , \tag{1}$$

where I_i = power in the ith co-channel interfering signal, and M = total number of co-channel interferers.

M will be considered fixed initially. Thus the total interference level will be constant at a value K during a symbol. For such a constant interference level, we will be able to calculate the probability of decoding error. Then by considering the statistics according to which the interferers are transmitting or are silent (burstiness), we can obtain average unconditional error probability performance.

Prabhu provides a theoretical analysis of the performance of an m-phase CPSK system operating in the presence of random Gaussian noise and co-channel interference [1]. In his analysis, a symbol is defined as a bit. Then the channel bit error rate for a fixed level of interference K is computed at specific values of SNR. Results from Prabhu for a binary CPSK system appear in Figure 1. SNR is defined as the signal power per bit per noise spectral density. In Section 3, we show the relationship between this parameter and the more traditional E_b/N_o, the energy bit per noise spectral density.

The expression from which these results were obtained are rather complex and their subsequent use in the error control code analysis would lead to unnecessary mathematical complication. To avoid this, we choose instead to curve-fit these results by the use of a much simpler analytical expression. Define,

Pe = Pr [bit error due to interference and noise],

S = received signal power,

σ^2 = noise power,

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$\frac{S}{\sigma^2}$ = signal-to-noise ratio (SNR),

R = code rate for single-stage BCH code or inner code rate for RS-BCH concatenated code,

K = total interference level as defined in expression (1),

$\frac{S}{K}$ = signal-to-interference ratio (SIR).

Let, $$Pe = 10^{-10^{C}}, \tag{2}$$

where, $$C = (C_1 + C_2\,[\sigma^2/(RS) + K/S]^{-1})$$

We use mathematical expression (2) for Pe and curve-fit Prabhu's results to it to obtain the values of C_1 and C_2. Expression (2) was selected over numerous other attempts because it provides less curve-fitting error than do the other expressions tried. Furthermore, it contains the critical channel parameters.

The parameters C_1 and C_2 are constants that depend on the value of S/K. The values of C_1 and C_2 for the six practically representative values of S/K in Figure 1 appear in Table 1. By changing the values of C_1 and C_2 according to this table, curve-fitting error remains negligible. Figures 2 and 3 provide curves for S/K versus C_1 and S/K versus C_2, respectively, from which C_1 and C_2 can be determined for values of S/K not specified in Figure 1.

TABLE 1 2. Model in

Table 1. Values of C_1 and C_2

SIR	C_1	C_2
5	-3.423477423	1.427910383
8	-0.719464175	0.300528318
10	-0.1546594694	0.1414808693
15	0.1762136293	0.0600323776
20	0.359482065	0.0262154118
∞	0.4057137519	0.0315152725

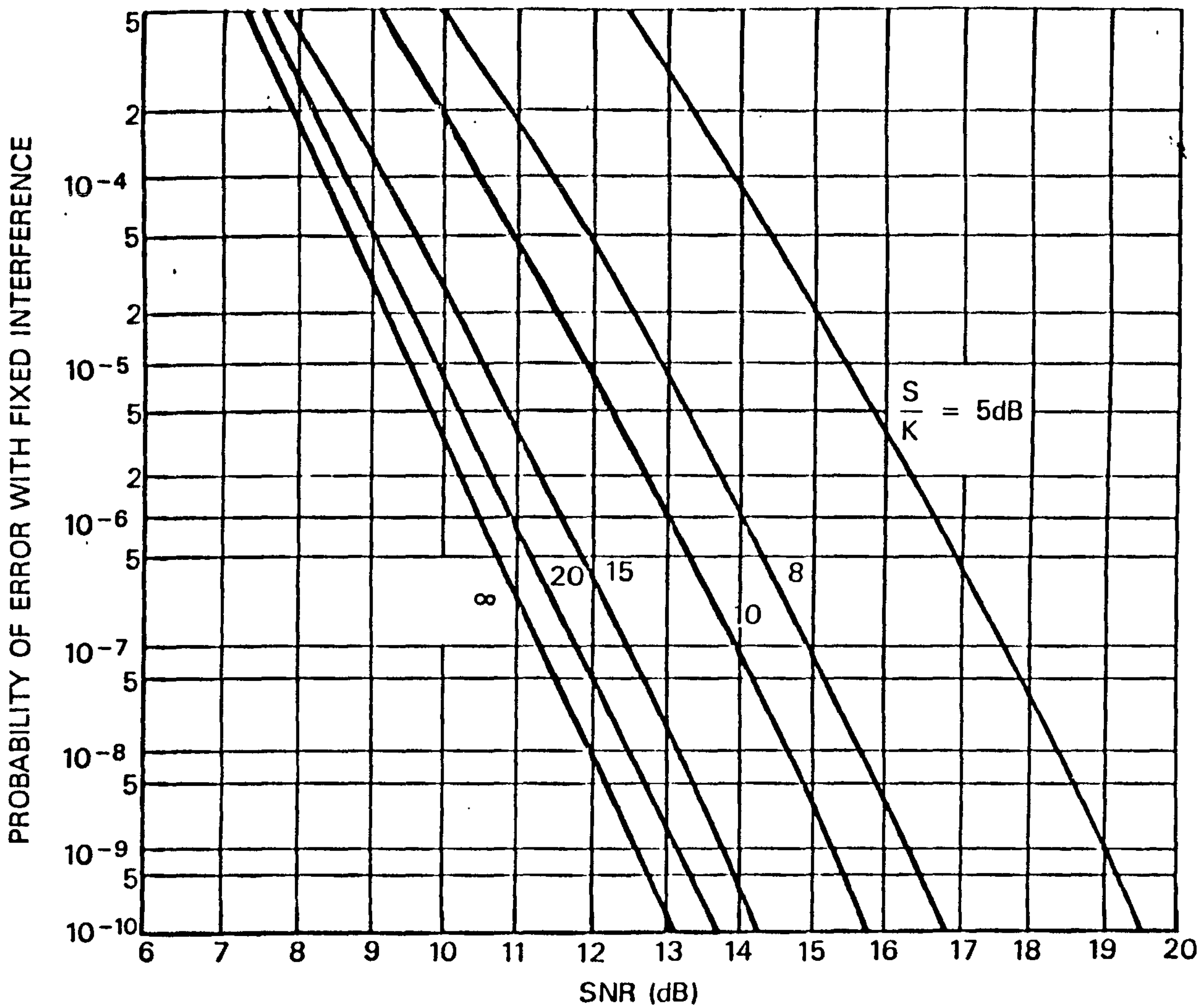


FIGURE 1. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS. SNR FOR NO CODING

(8)

FIGURE 2

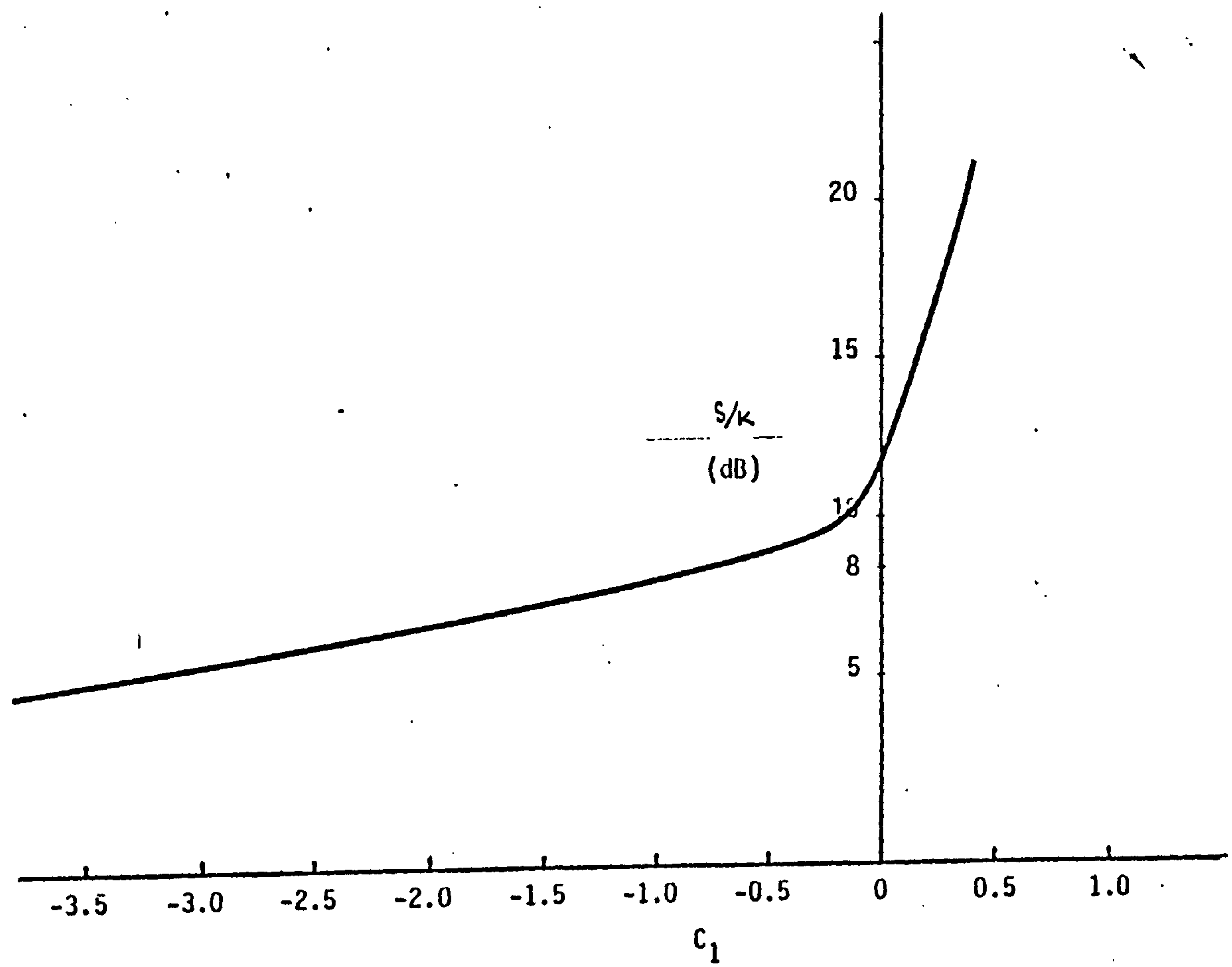


FIGURE 2. S/K vs C_1

FIGURE 3

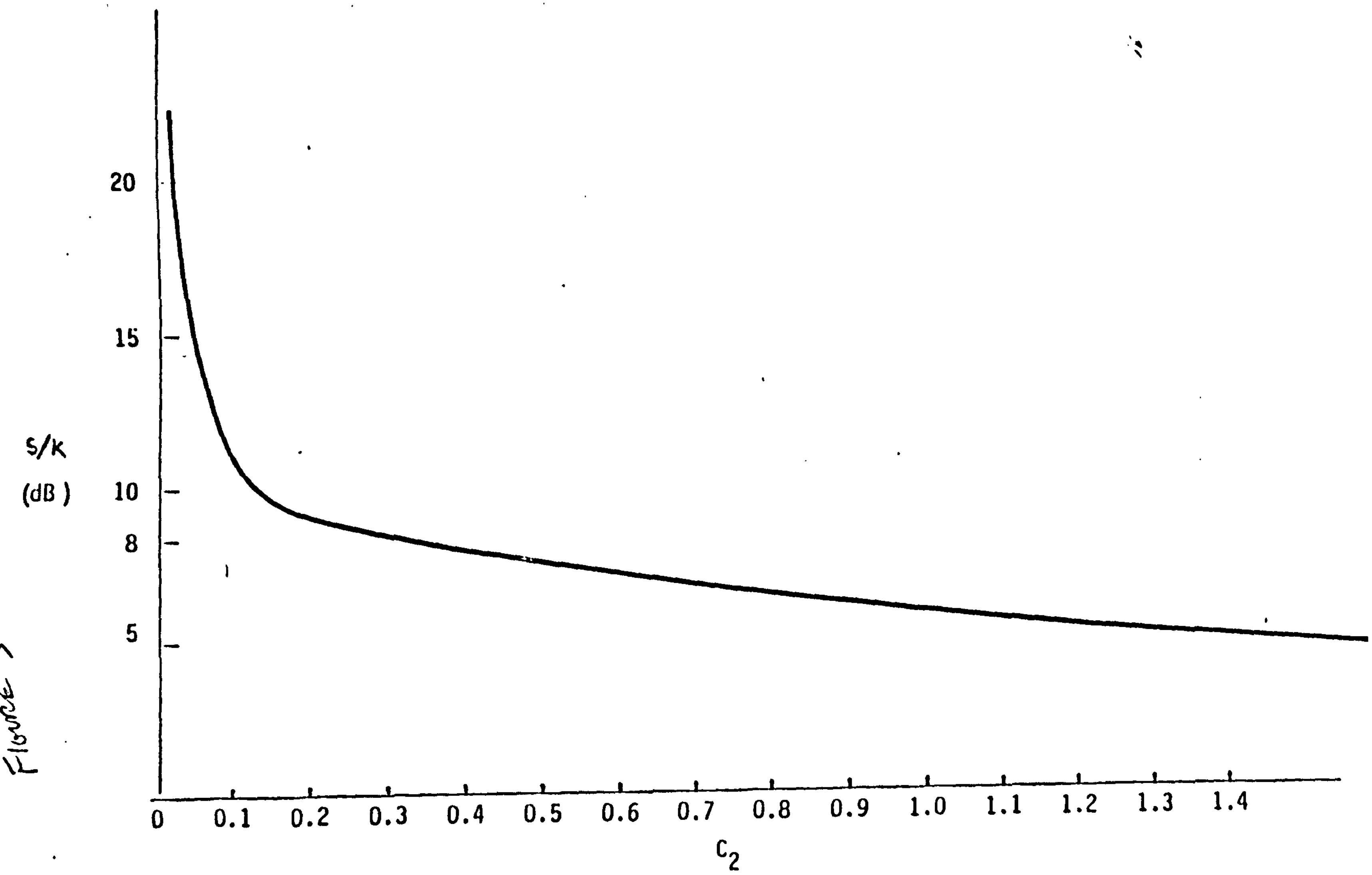


FIGURE 3. S/K vs C_2

The parameter R in expression (2) equals 1.0 for curve-fitting since Prabhu's results hold only for the uncoded case. R affects SNR but not SIR because R is a normalization factor which depends on the choice of codes investigated. These codes will be discussed in detail in Section 3.

We now assume hard decisions are made and that the channel is equierror with Pe as in expression (1). Thus the modulator, demodulator, and detector are considered to be part of a discrete memoryless channel with q inputs, q outputs, and average bit probability Pe in one use of the channel.

Consider the use of binary signaling and the use of a single-stage BCH block code to improve system performance. Assume that the code is primitive, the decoder does errors-only (as opposed to errors-erasures) decoding, and the correction process does not add errors when more than the correctable number of errors per code word is present.

Earlier we assumed that the channel memory has duration no longer than a symbol. For a single-stage BCH code we can let a symbol be a bit. Then the bit error probability due to co-channel interference and noise, Pe, can be utilized along with the equierror channel model in order to compute the probability of decoder error with fixed interference for single-stage BCH codes. Define,

P(IE/K) = Pr [code word error at the output of the decoder given interference level K].

Then,

$$\Pr(IE/K) = \sum_{L=T+1}^{N} \binom{N}{L} (Pe)^{L} (1-Pe)^{N-L} \quad , \tag{3}$$

where:

Pe is given by expression (2),

N = total number of bits in a code word, and

T = bit error correction capability of code.

The channel assumptions also permit the use of Forney's results for concatenated codes on an equierror channel [10]. Assume that we use a concatenated code in which the inner code is a primitive BCH block code and the outer code is an RS block code. Let the number of bits per inner code word equal the number of bits per outer code symbol. Assume that both inner and outer decoders do errors-only (as opposed to errors-erasures) decoding. Also assume that neither the error correction process of the outer nor that of the inner decoder adds errors when more than the correctable number of errors per respective code word is present. For a single-stage code, a bit was considered to be a symbol or a character. A bit takes on an identity only when we bring the outer decoder into the picture. Now a symbol is an inner code word. Therefore, a symbol error refers to the case where an inner code word, i.e., a sequence of bits, is in error. This definition of symbol error would provide pessimistic results when considering only the inner decoder or a single-stage decoder. To compute probability of decoder error with fixed interference for RS-BCH concatenated codes, define,

$Pr(OE/K)$ = Pr [outer code word error at the output of the outer decoder given interference level K].

Then,

$$Pr(OE/K) = \sum_{i=t+1}^{n} \binom{n}{i} [Pr(IE/K)]^i \, [1-Pr(IE/K)]^{n-i} \quad , \tag{4}$$

where,

$Pr(IE/K)$ = Pr [inner code word error at the output of the inner decoder given interference level K] as given by expression (3),

n = total number of symbols in an outer code word,

and

t = symbol error correction capability of outer code.

Thus far we have considered the case of fixed interference level K. The approach to modeling random, i.e., bursty, interference and to applying error-control coding to correct errors due to such interference was inspired by recent work done by Pursley et. al. [11, 12] and Ephremides et. al. [13, 14]. For the case of random interference, the error statistics vary from symbol to symbol, although they remain constant for the duration of a particular symbol. The error statistics vary because a different number of interferers is present per inner code word for concatenated codes or per bit for single-stage codes. To account for these differences, we average the error statistics by weighting the conditional probability of decoding error. The probability of decoding error with random interference for single-stage BCH codes is computed in this way by combining Pr[IE/K] from expression (3) with the probability distribution of the level of interference, Pr(K). Define,

Pr(IE) = Pr [code word error at the output of the decoder], i.e., the unconditional, average probability of decoding error.

$$Pr(IE) = \sum_K Pr(K)\ Pr(IE/K). \tag{5}$$

The probability of decoding error with RS-BCH concatenated coding and random interference is computed similarly. Define,

$$Pr(OE) = Pr\ [\text{outer code word error at the output of the outer decoder}]$$

$$= \sum_K Pr(K)\ Pr(OE/K), \tag{6}$$

where Pr(OE/K) is given by expression (4), and Pr(K) is defined as the probability distribution of the level of interference, K. Although Pr(K) can be any probability distribution, we assume the uniform distribution for performance evaluation. This choice ideally must be motivated by the burstiness profile of the interfering users, including any multiple access protocol constraints that may be present.

3. PERFORMANCE EVALUATION

Performance results are computed for the single-stage BCH and RS-BCH concatenated codes appearing in Table 2. These codes are considered to be the best errors-only, single-stage BCH codes and RS-BCH concatenated codes because they provide a given performance with less error correction capability and a comparatively shorter block length than other codes with the same code rate in their respective classes [10]. Each of these codes provides probability of decoder error with no interference, i.e., Gaussian noise only, equal to approximately 10^{-6} when the channel bit error probability equals 10^{-2}. This is the reason that R is a normalization parameter that affects only S/σ^2 in the model of interference.

The parameter S/σ^2 in expression (2) is defined as the signal power per bit per noise spectral density. If we consider the case of no coding, S/σ^2 is the signal power per information bit per noise spectral density, and the relationship between S/σ^2 and E_b/N_o can be derived. E_b/N_o is defined as the energy per information bit per noise spectral density. For the uncoded case,

$$\frac{E_b}{N_o} = \frac{(Ac^2/2)\ \tau}{N\ /\ BW} \tag{7}$$

TABLE 2

Reduce original to 95%

TABLE III.1 "BEST" SINGLE STAGE BCH AND RS-BCH CONCATENATED CODES

CODE RATE	RS-BCH CONCATENATED CODE (n,k), (N,K) *	BCH CODE (N,K)
1.0	No Coding	No Coding
0.3	(9,5), (7,4)	(30,10)
0.4	(28,24), (15,7)	(94,38)
0.5	(31,23), (31,21)	(112,56)
0.6	(63,53), (63,45)	(230,138)
0.7	(123,115), (255,191)	(784,549)
0.75	(286,272), (511,403)	(1672,1254)

*A specific RS-BCH concatenated code is identified as an "(n,k), (N,K) code" where the first code is the outer RS code, and the following code is the inner BCH code.

where,

$Ac^2/2$ = carrier power = S ,

N = total noise power entering the demodulator = σ^2,

τ = bit duration,

BW = bandwidth.

Assume that BW = $\frac{1}{\tau}$. Then,

$$E_b/N_o = S/\sigma^2 \tag{8}$$

For the coded case, $R(S/\sigma^2)$ is the signal power per transmitted bit per noise spectral density which is present at the input to the decoder. Likewise, $R(E_b/N_o)$ is the energy per transmitted bit per noise spectral density, which is present at the input to the decoder, and the following relationship holds:

$$R(E_b/N_o) = R\ (S/\sigma^2). \tag{9}$$

Note that when no coding is present, R = 1.0. For R < 1.0, $S/\sigma^2 > R\ (S/\sigma^2)$ because, for the same signal power, an increased number of bits must be transmitted for the coded case. At the output of the decoder, only information bits are present, and S/σ^2 is the correct parameter to account for the signal power per bit per noise spectral density. In this paper we present computed results for probability of error at the output of the decoder versus S/σ^2.

Throughout this section, specific codes are identified by code rate and class. We begin performance evaluation by computing the probability of decoder error with fixed interference for single-stage BCH codes. To do so we utilize Pr(IE/K) from expression (3). Results for the rate 0.3, 0.4, 0.5, and 0.7 BCH codes of Table 1 appear in Figures 4, 6, 8, and 10, respectively. In these figures, the relationship between Pr(IE/K), values of SNR ranging 8 dB to 20 dB, and five values of S/K ranging from 5 dB to infinity (i.e., only Gaussian noise present on the channel) can be observed. The horizontal axis represents SNR in decibels, and the vertical axis represents Pr(IE/K) in $\log_{10}$ ($-\log_{10}$) scale. This scale is used in Figures 4 through 13 and is due to the form of the probability of bit error, Pe, as given in expression (2). Results for the rate 0.6 and 0.75 BCH codes and RS-BCH concatenated codes are available but not presented here for brevity.

Corresponding values of the probability of error with fixed interference for RS-BCH concatenated codes, Pr(OE/K), are calculated from expression (4) and results for the rate 0.3, 0.4, 0.5, and 0.7 RS-BCH concatenated codes of Table 1 appear in Figures 5, 7, 9, and 11, respectively. The values of probability of error with random interference for single-stage BCH codes, Pr(IE), are calculated from expression (5) and for RS-BCH concatenated codes, Pr(OE), from expression (6). The level of interference, K, is assumed to be uniformly distributed. These results appear in Figures 12 through 15 for the rate 0.3, 0.4, 0.5 and 0.7 codes of Table 1.

The effectiveness of RS-BCH concatenated codes and single-stage BCH codes on an interference-plus-noise satellite channel can be demonstrated by analyzing and comparing computed performance results. For example, Table 3 compares

FIGURE 4

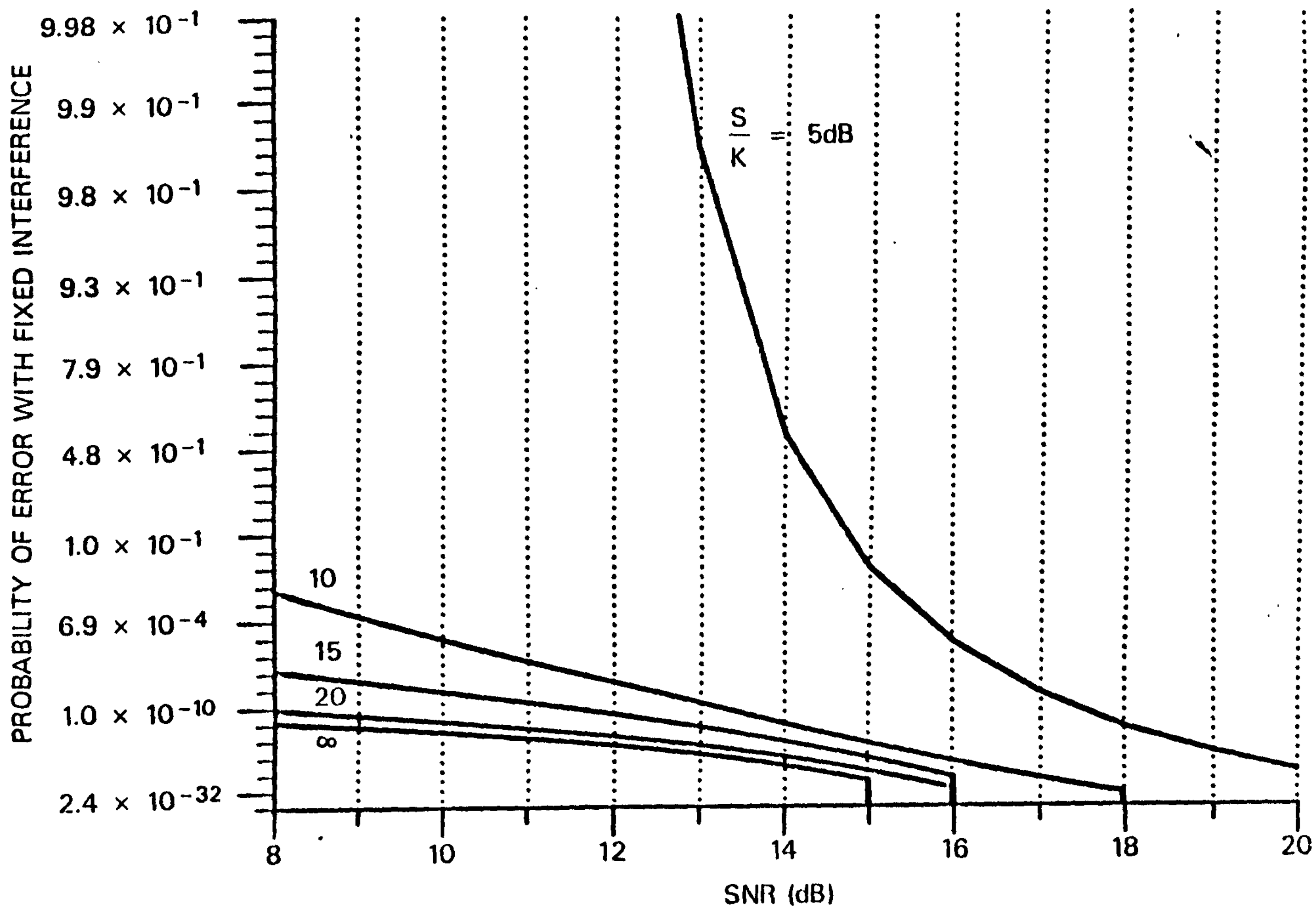


FIGURE 2. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS. SNR FOR THE RATE 0.3 BCH CODE

FIGURE 5

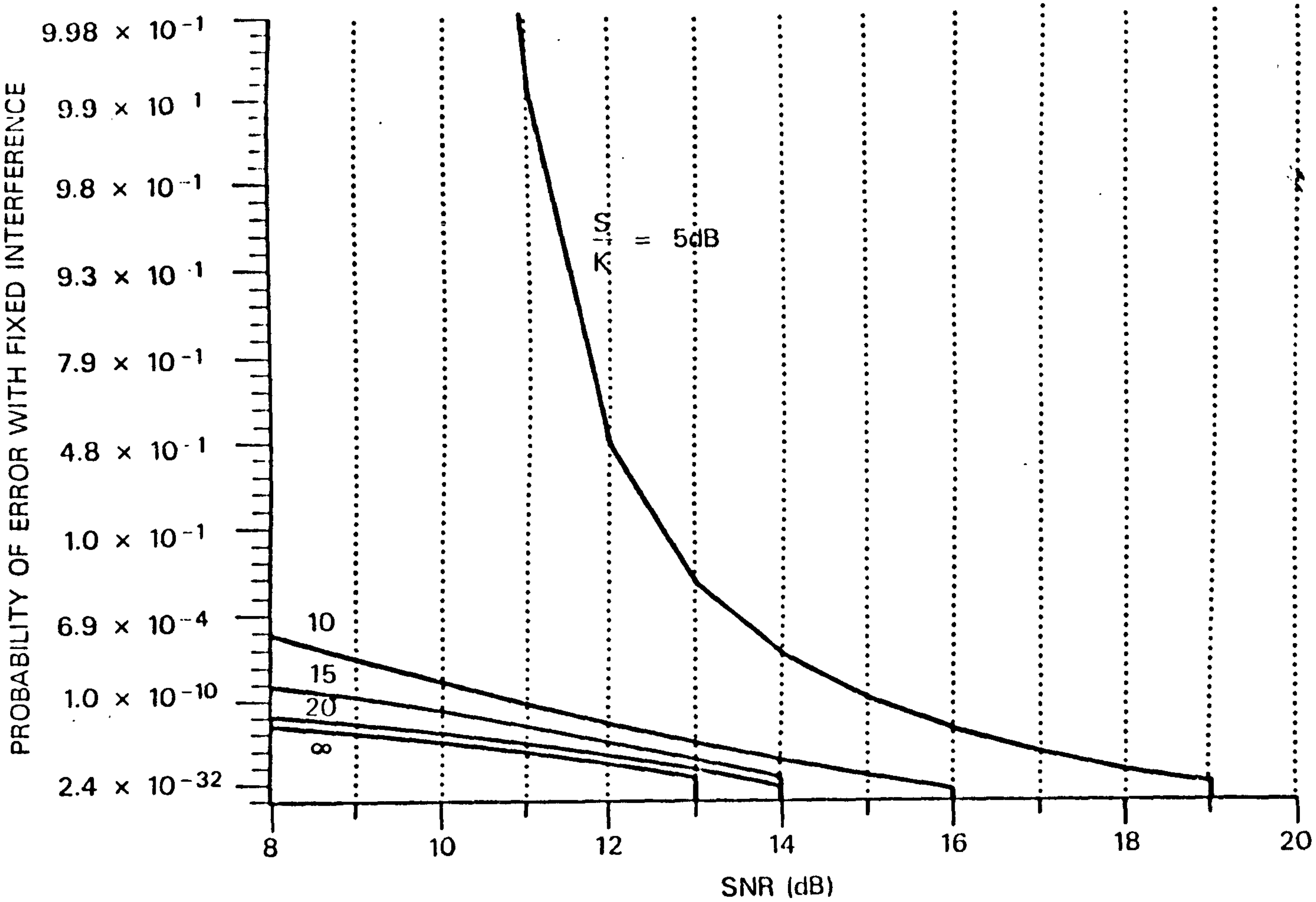
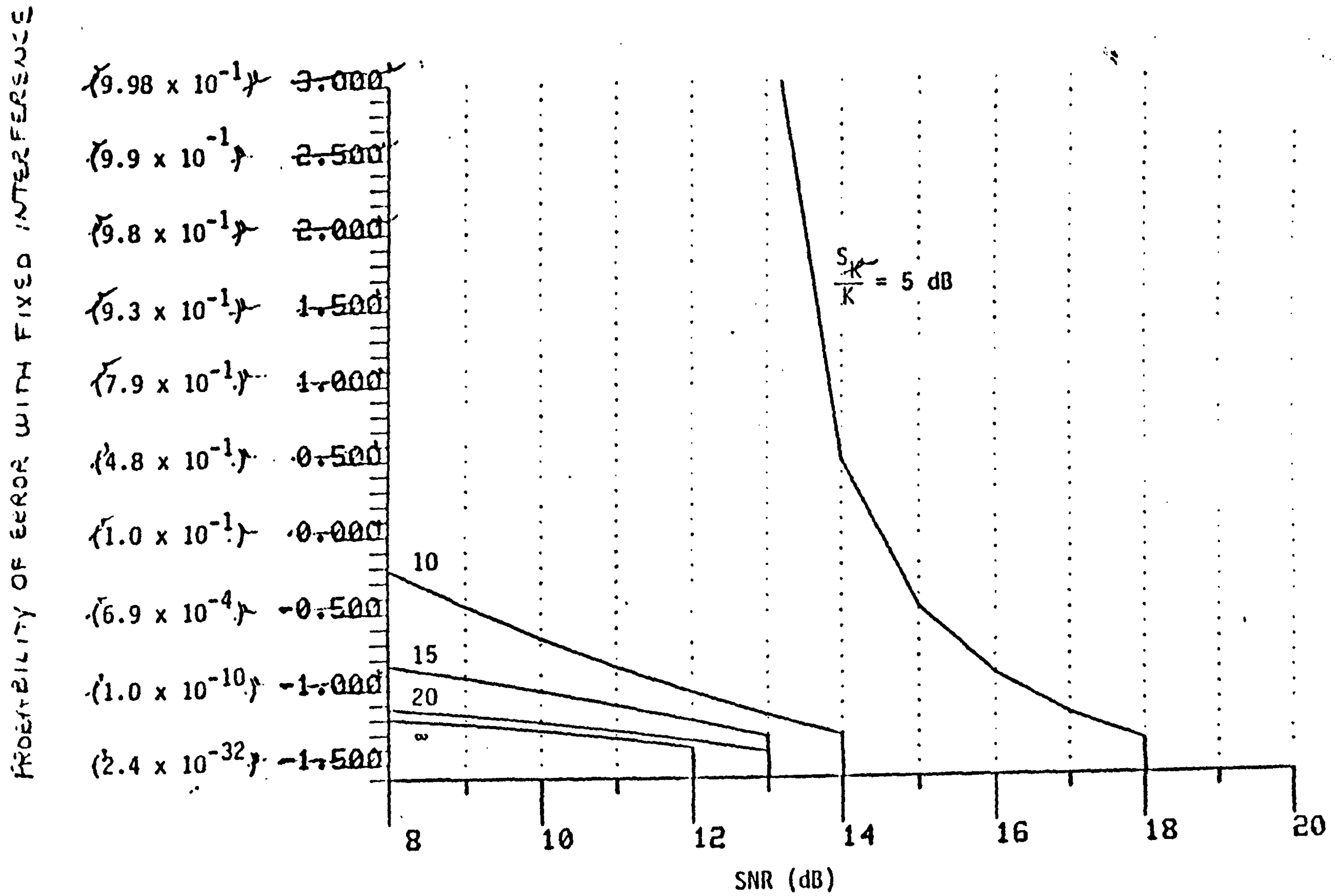
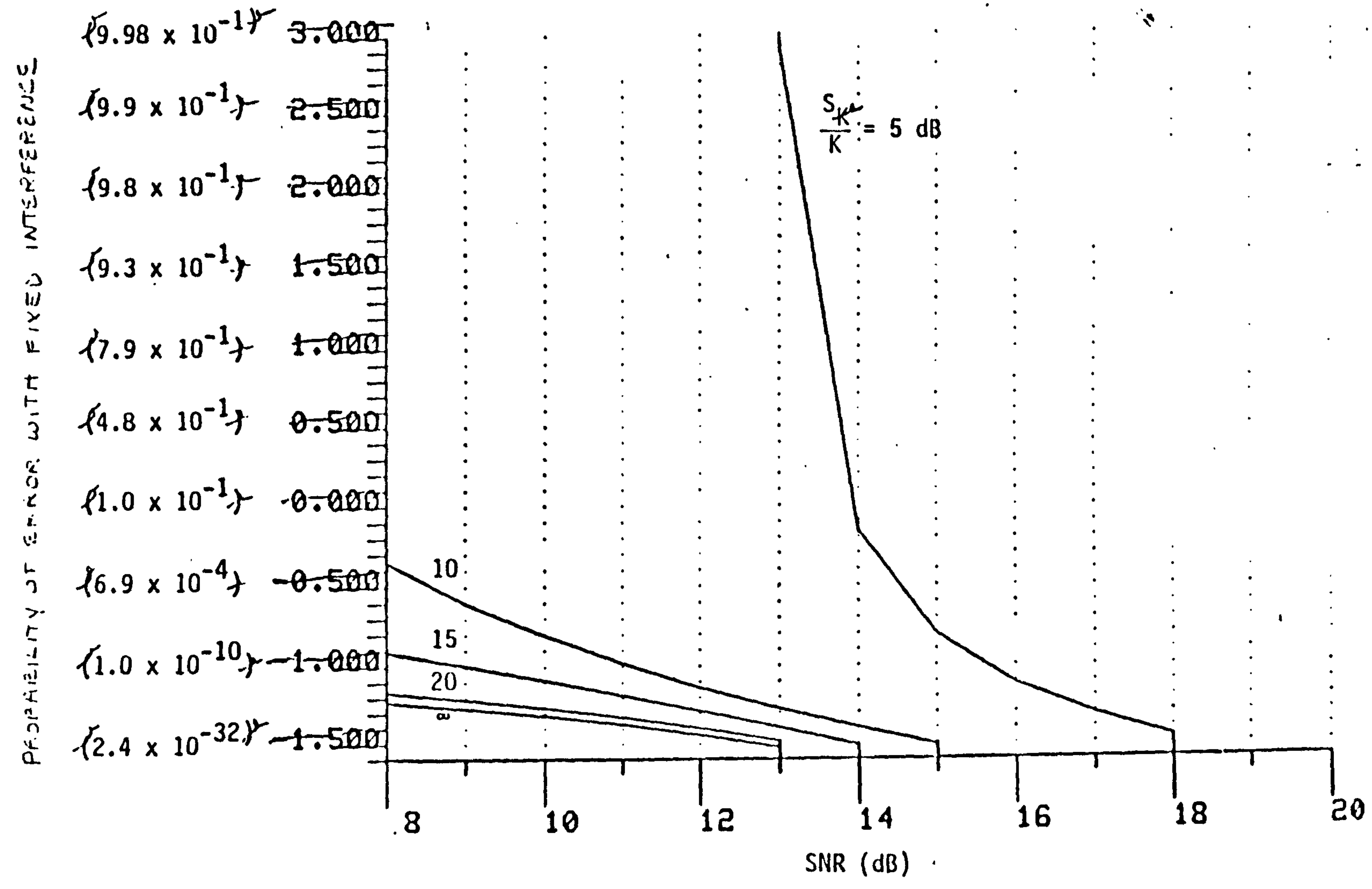


FIGURE 3. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS. SNR FOR THE RATE 0.3 RS-BCH CONCATENATED CODE



~~FIGURE E.2. Pr(OE/K) VS SNR FOR THE (94, 38) BCH CODE~~

Figure 6. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS SNR FOR THE RATE 0.4 BCH Code



~~FIGURE VII.2. Pr(OE/K) VS SNR FOR THE (28, 24), (15, 7)~~-CONCATENATED-CODE

Figure 7. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS SNR FOR THE RATE 0.4 RS-BCH Concatenated Code

FIGURE 8

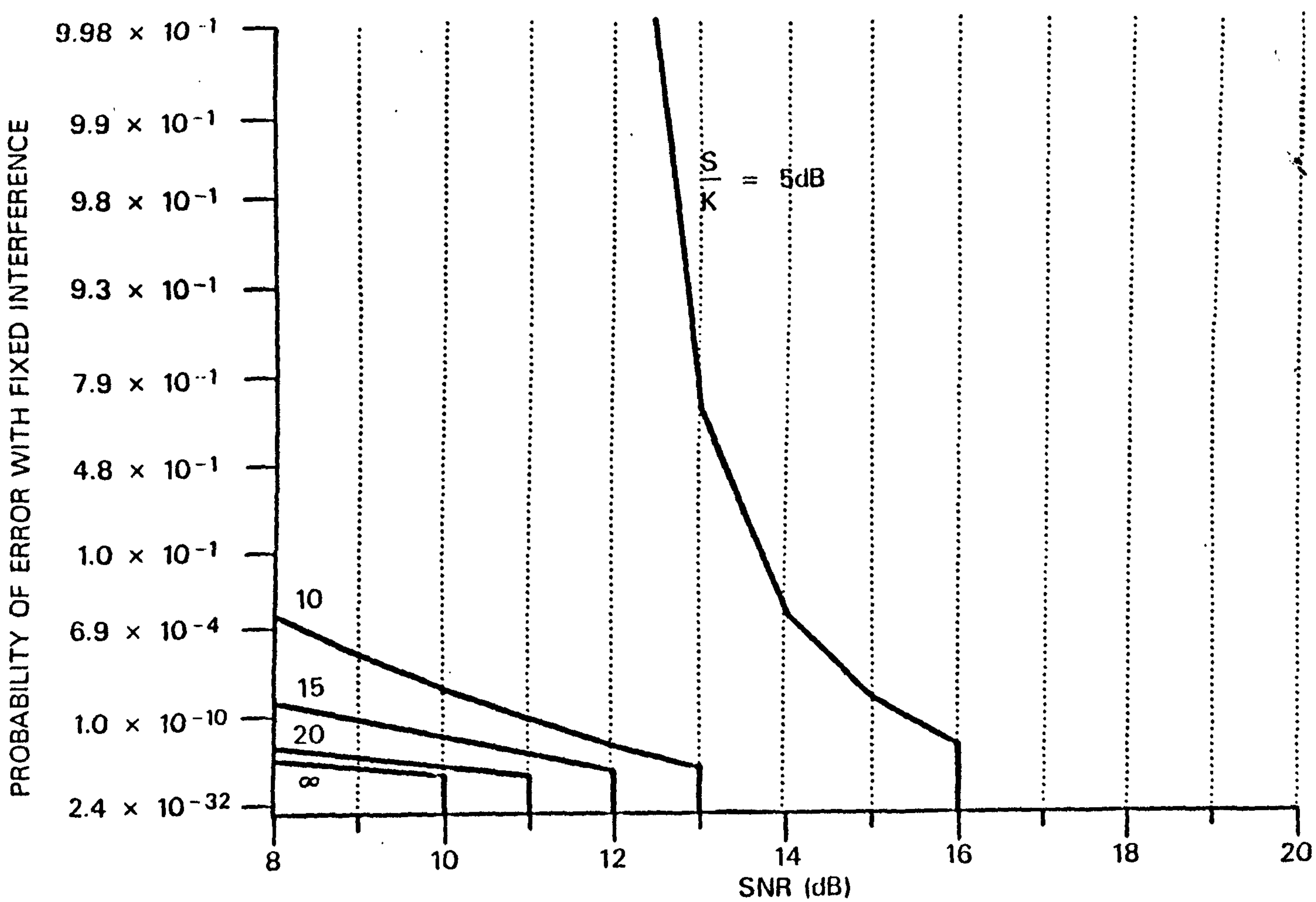


FIGURE 4. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS. SNR FOR THE RATE 0.5 BCH CODE

FIGURE 9

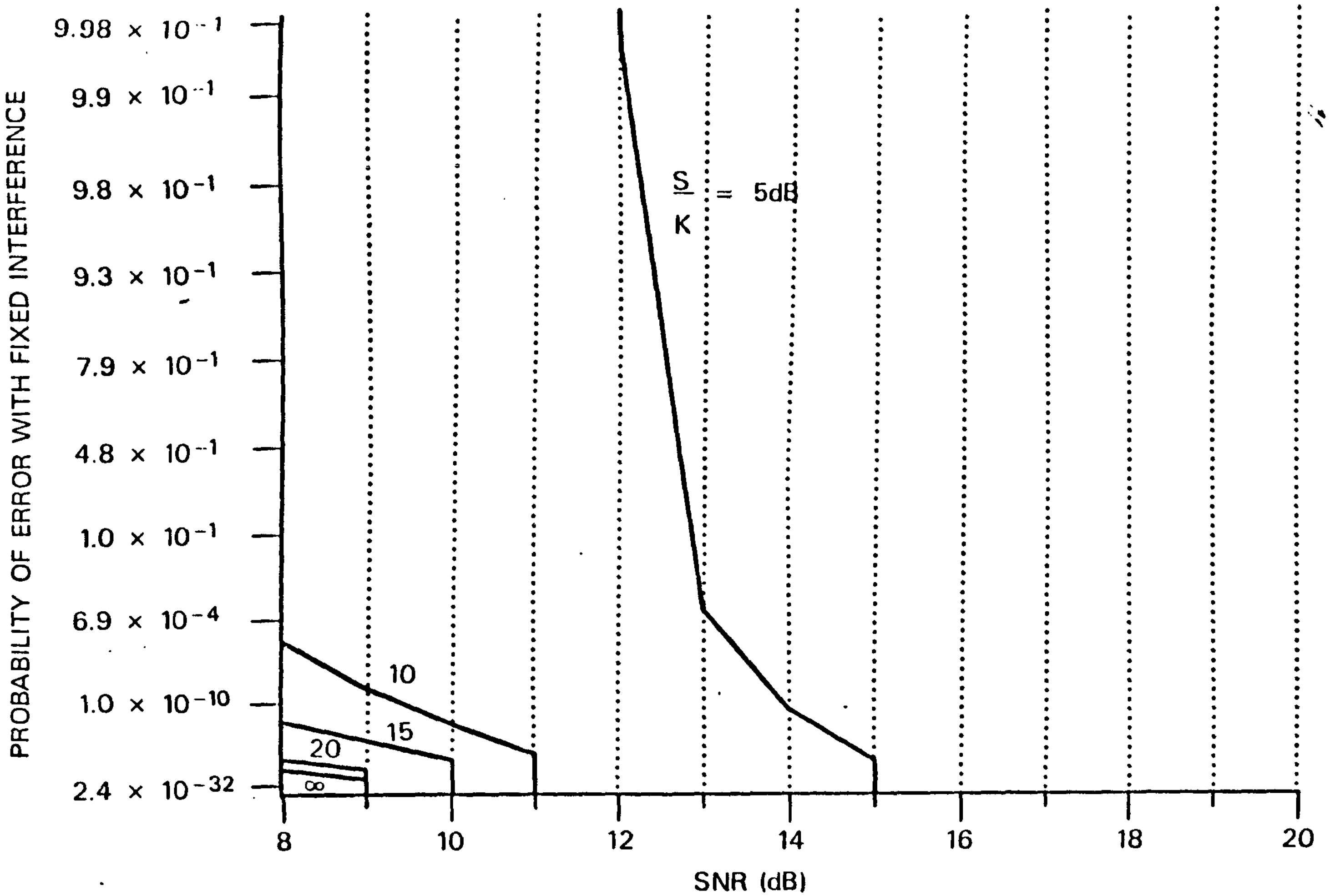
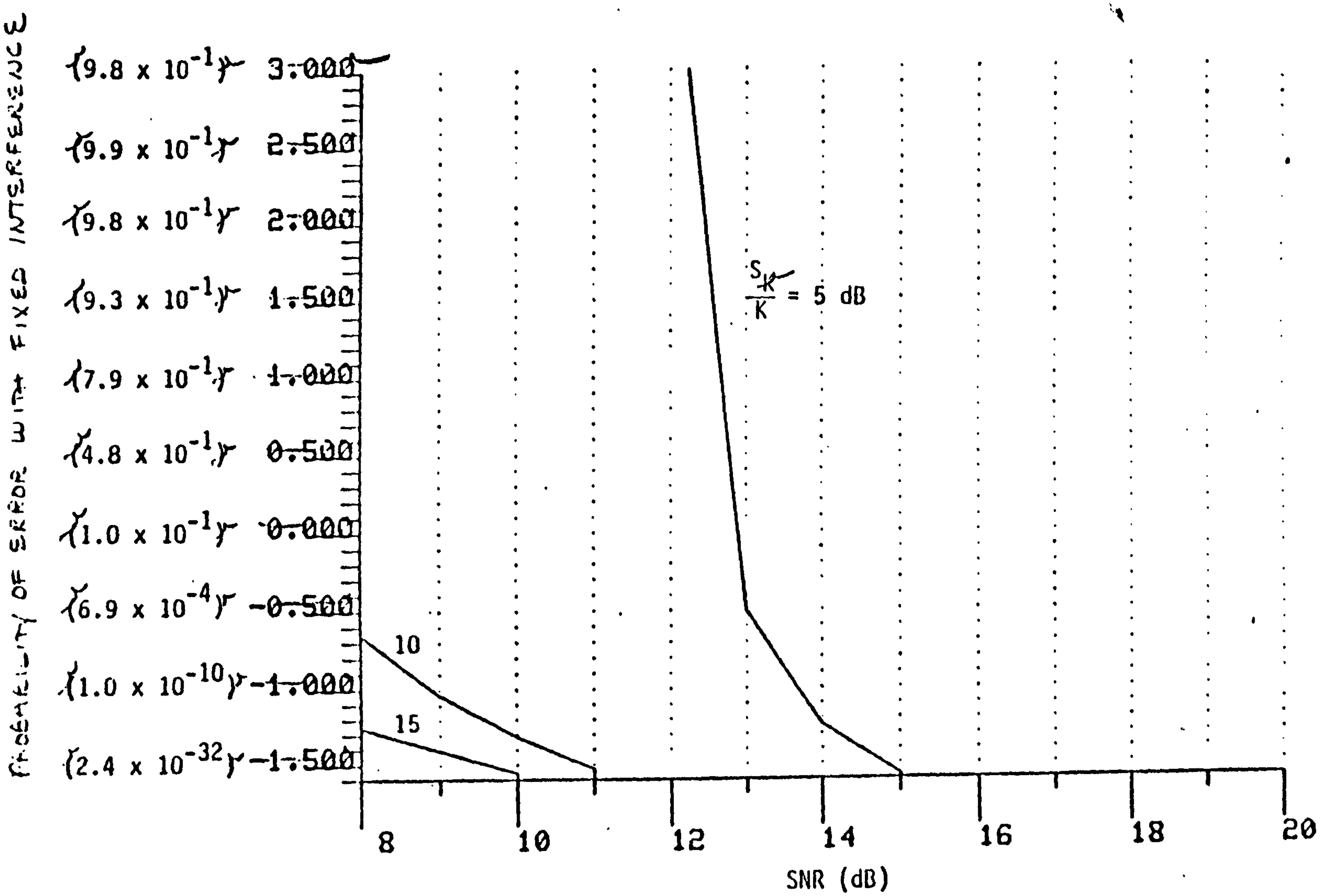
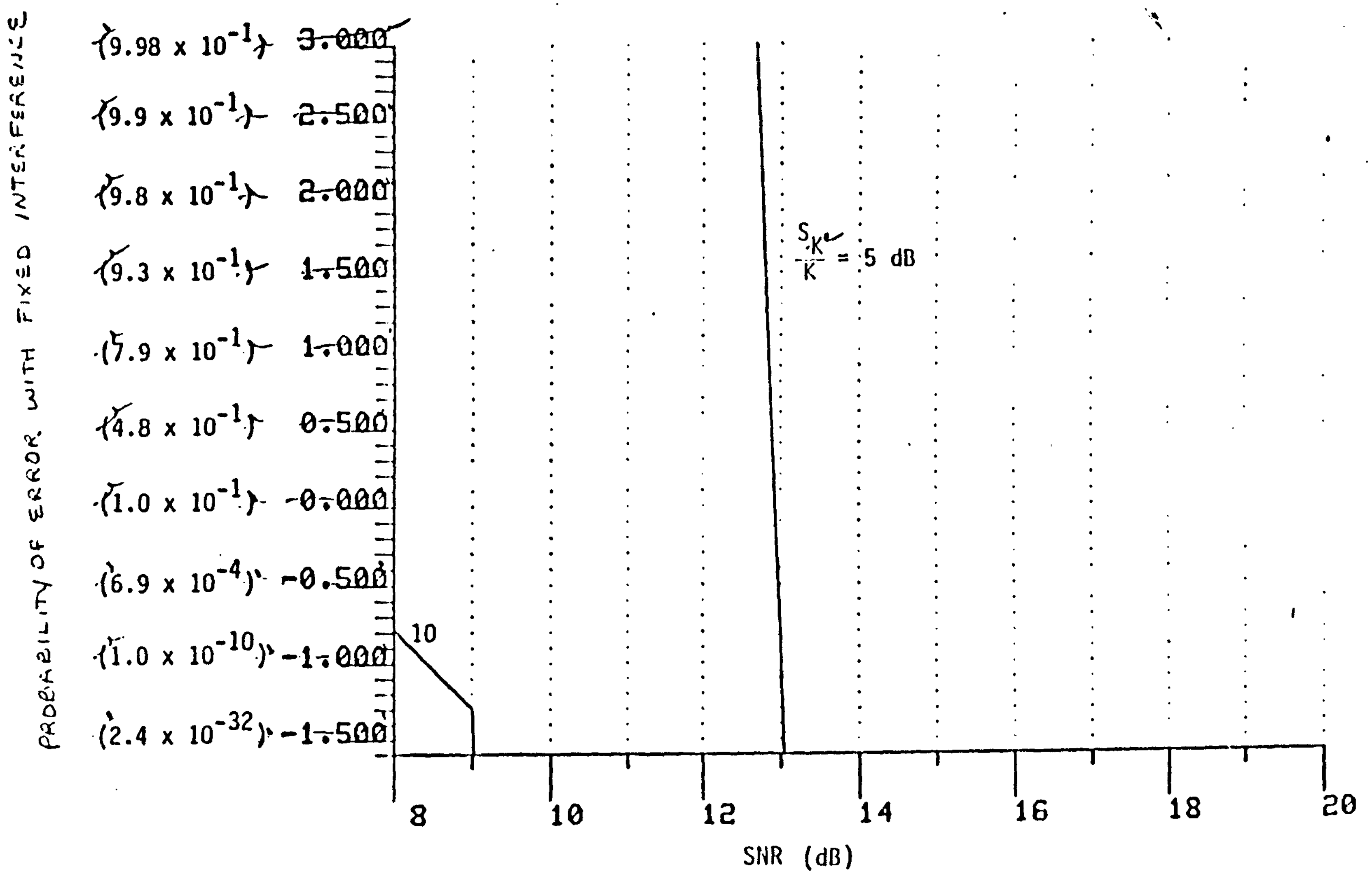


FIGURE 5. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS. SNR FOR THE RATE 0.5 RS-BCH CONCATENATED CODE



~~FIGURE E.5. Pr(OE/K) VS SNR FOR THE (784, 549) BCH CODE~~

Figure 10, PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS SNR FOR THE RATE 0.7 BCH code



~~FIGURE VII.5. Pr(OE/K) VS SNR FOR (123, 115), (255, 191) CONCATENATED CODE~~

Figure 11. PROBABILITY OF ERROR WITH FIXED INTERFERENCE VS SNR FOR THE RATE 0.7 RS-BCH Concatenated Code

FIGURE 12

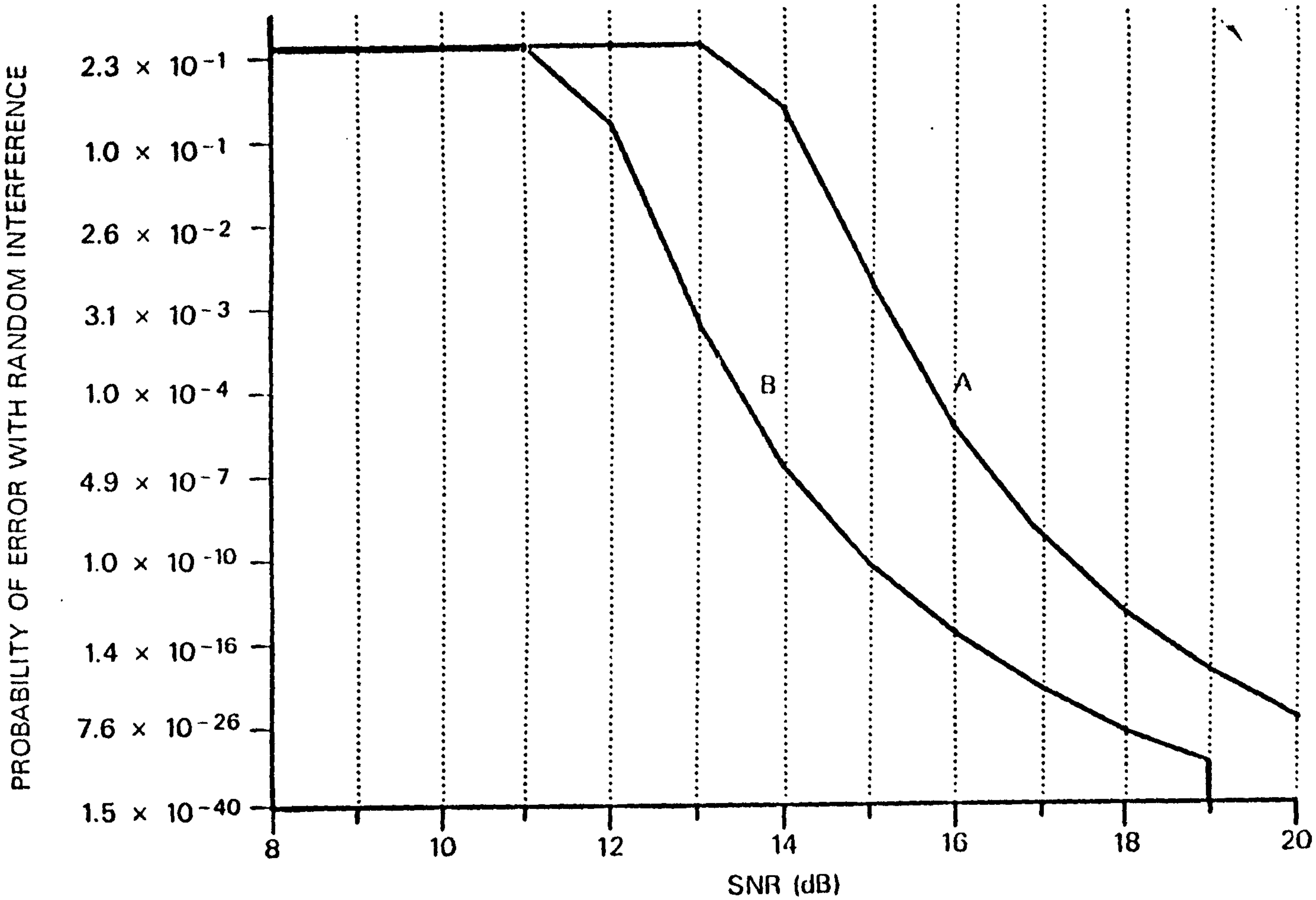


FIGURE 6. PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS. SNR FOR (A) THE RATE 0.3 BCH CODE AND (B) THE RATE 0.3 RS-BCH CONCATENATED CODE

FIGURE 13

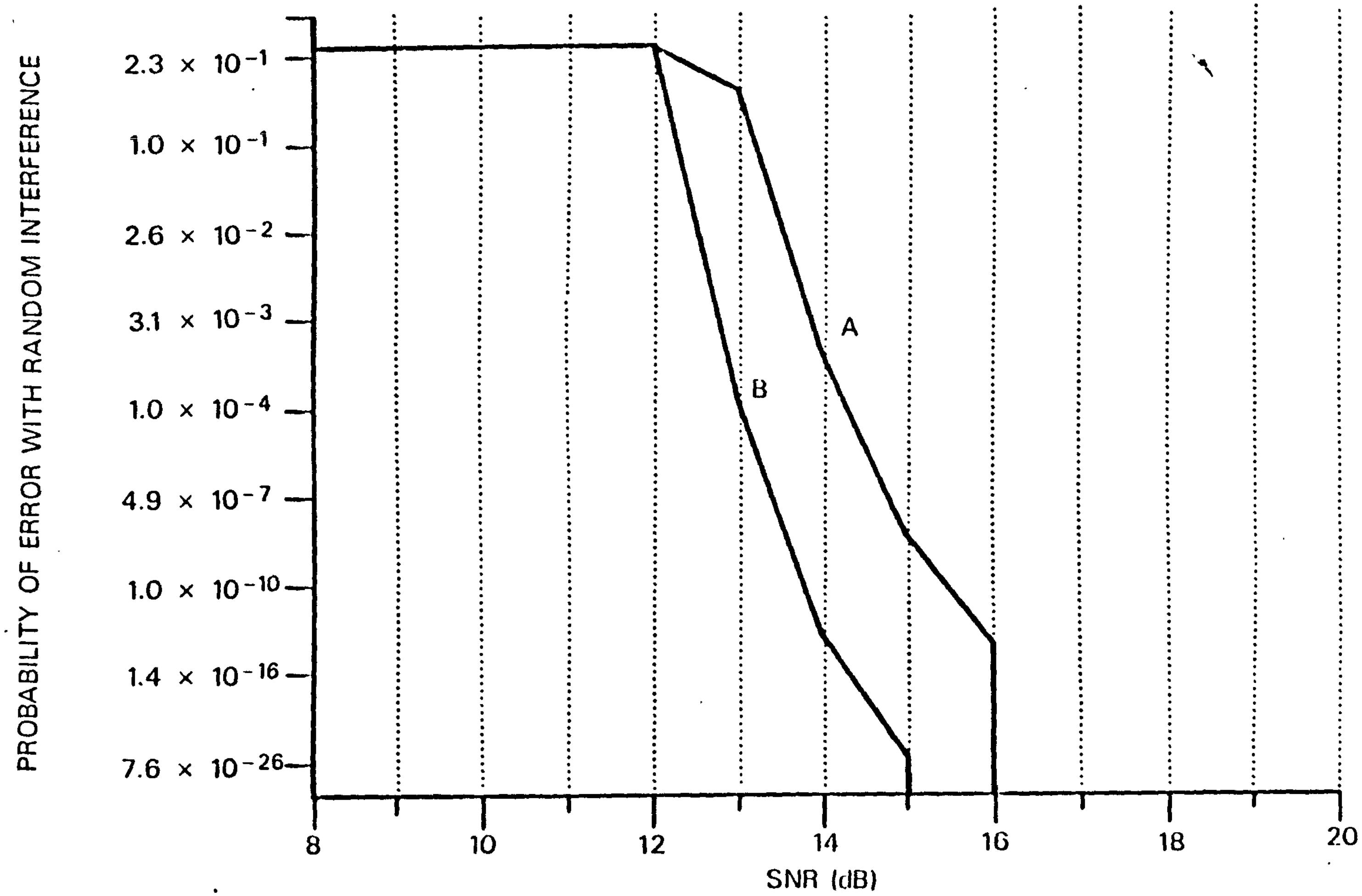
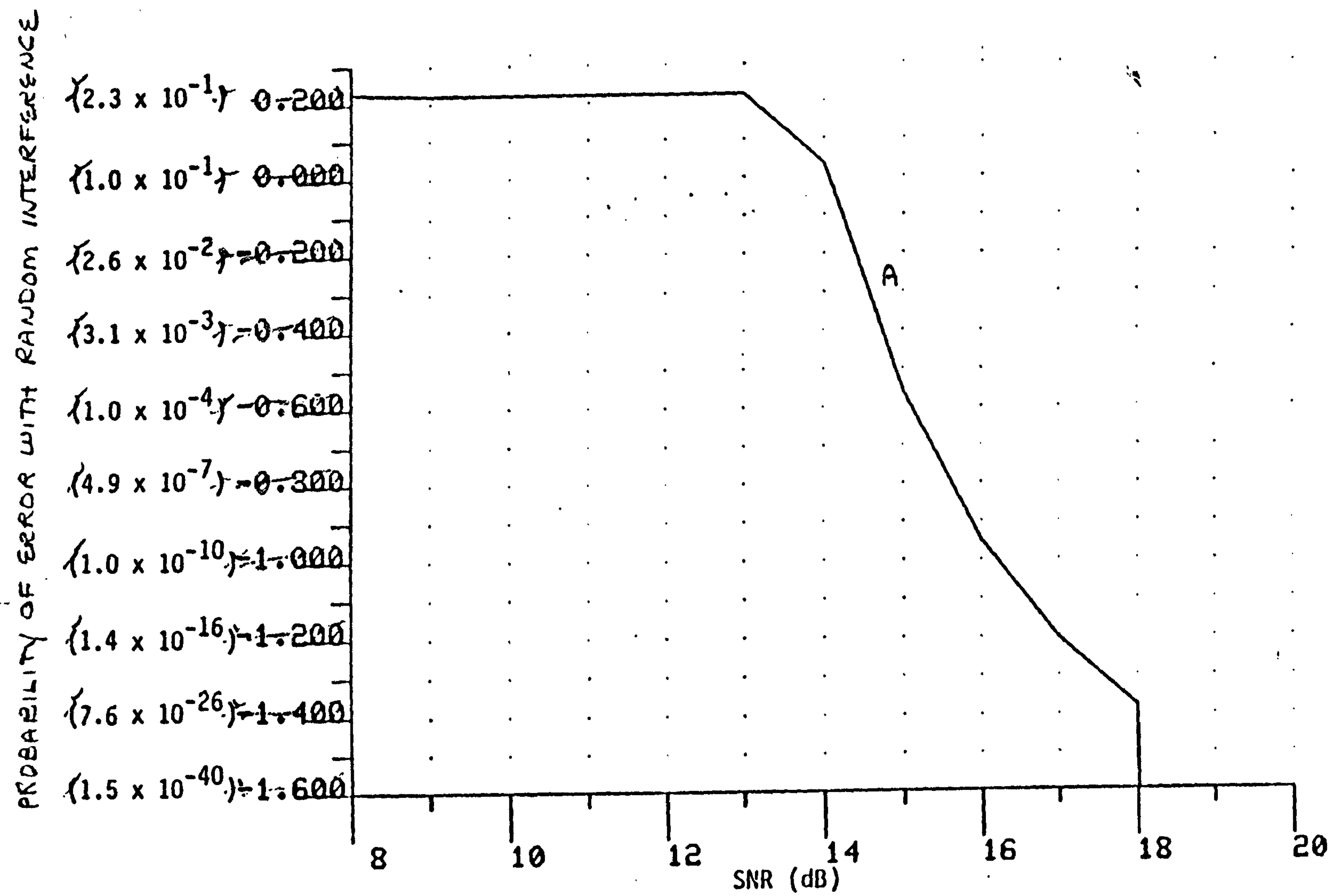
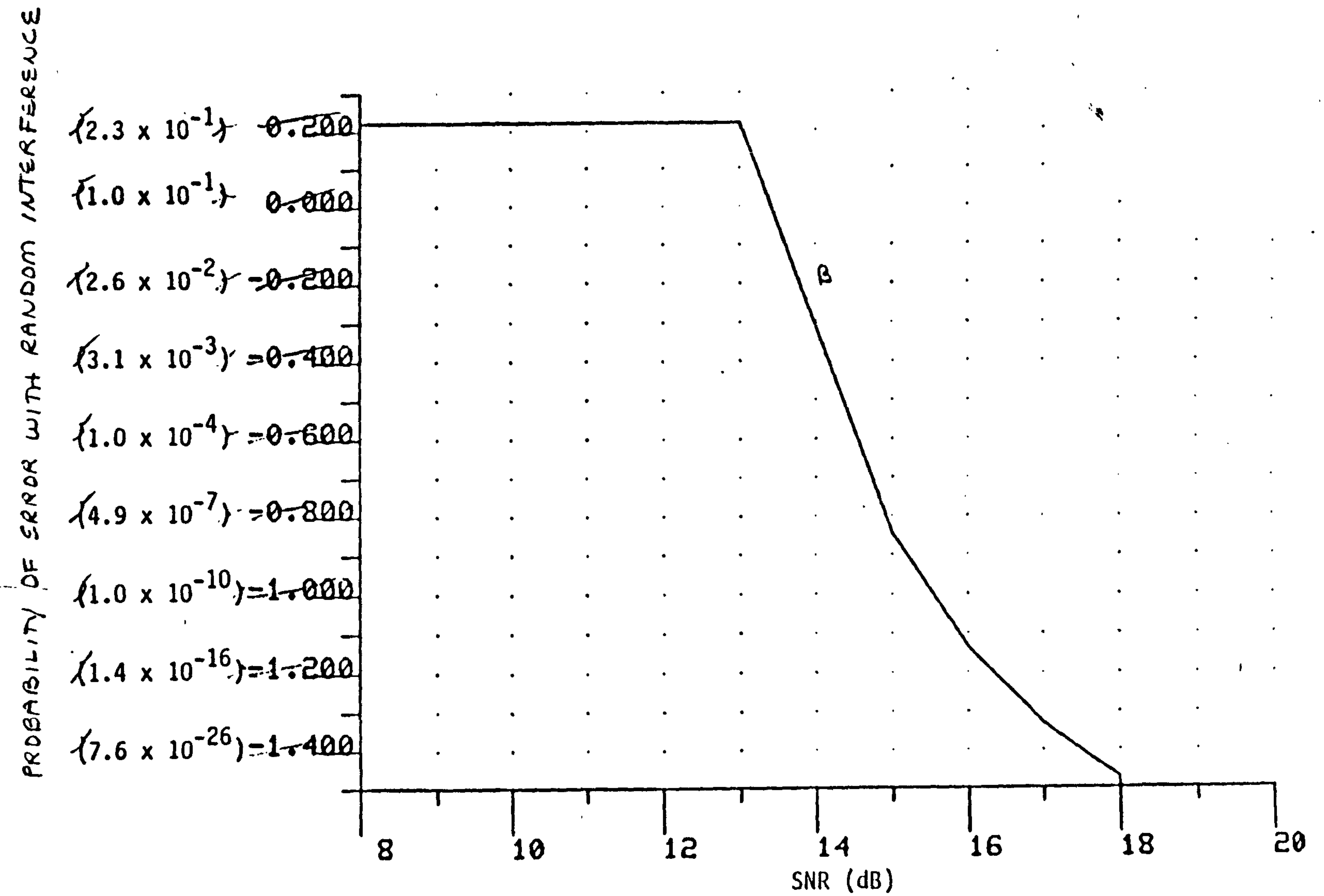


FIGURE 7. PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS. SNR FOR (A) THE RATE 0.5 BCH CODE AND (B) THE RATE 0.5 RS-BCH CONCATENATED CODE



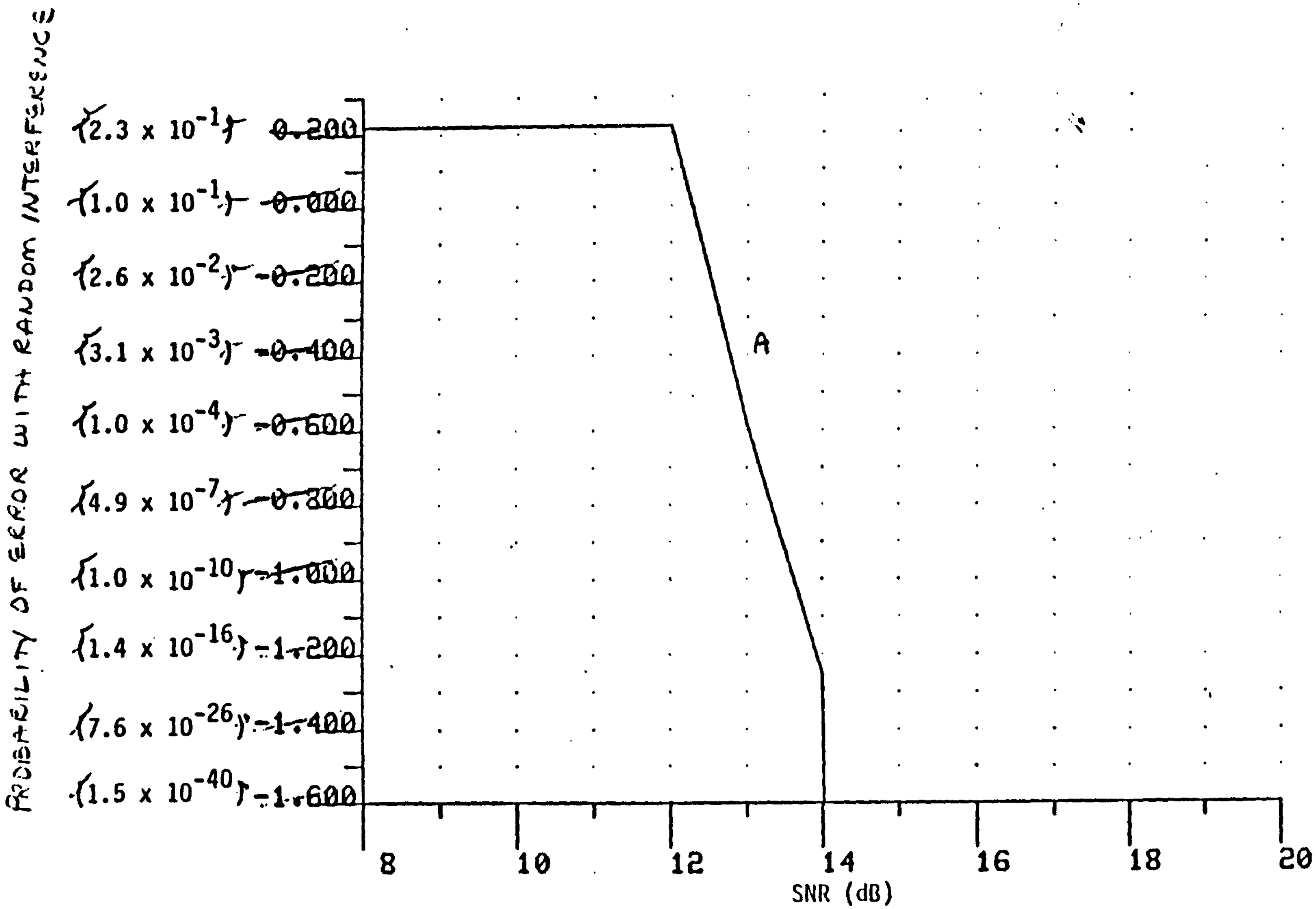
~~FIGURE E.8. Pr(OE) VS SNR FOR THE (94, 38) BCH CODE~~

Figure 14.(A) PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS SNR FOR THE RATE 0.4 BCH Code



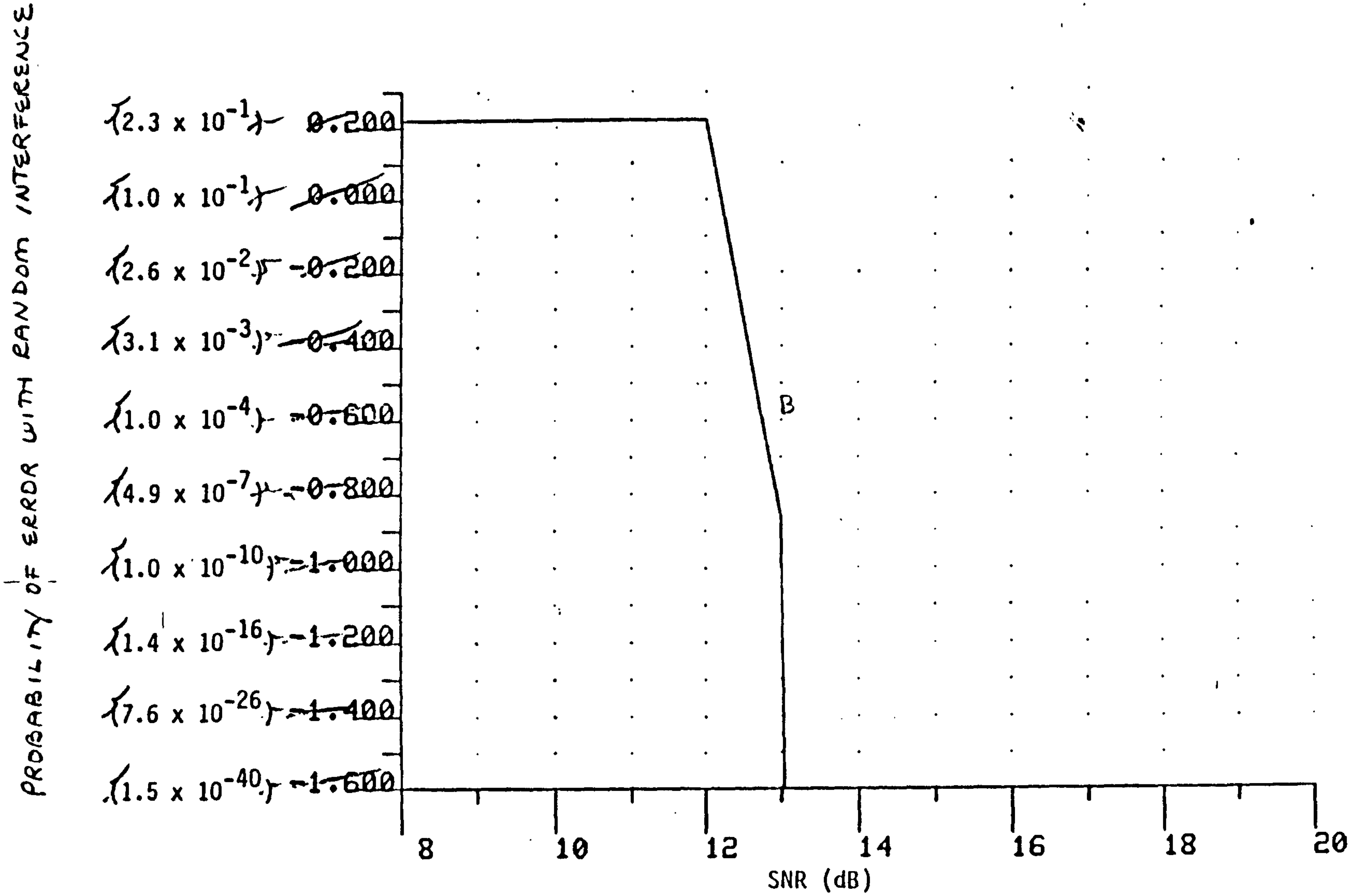
~~FIGURE VII.8. Pr(OE) VS SNR FOR THE (28, 24), (15, 7) CONCATENATED CODE~~

Figure 14.(8) PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS SNR FOR THE RATE 0.4 RS-BCH Concatenated Code



~~FIGURE E.11.~~ Pr(OE) VS SNR FOR THE (784, 549) BCH CODE

Figure 15.(A) PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS SNR FOR THE RATE 0.7 BCH Code



~~FIGURE VII.11. Pr (OE) VS SNR FOR THE (123, 115), (255, 191) CONCATENATED CODE~~

Figure 15,(B) PROBABILITY OF ERROR WITH RANDOM INTERFERENCE VS SNR FOR THE RATE 0.7 RS-BCH Concatenated Code

TABLE 3

TABLE 2. PROBABILITY OF ERROR WITH FIXED INTERFERENCE WITH RATE 0.3 RS-BCH CONCATENATED CODING, RATE 0.3 BCH CODING, AND NO CODING

SNR (dB)	SIR (dB)	Pr(OE/K) Rate 0.3 Concatenated Code	Pr(IE/K) Rate 0.3 BCH Code	No Coding
17	5	0.331×10^{-20}	0.365×10^{-8}	5.0×10^{-7}
14	10	0.168×10^{-22}	0.166×10^{-12}	1.0×10^{-7}
13	15	0.166×10^{-22}	0.336×10^{-13}	1.0×10^{-8}
10	20	0.216×10^{-15}	0.619×10^{-12}	1.0×10^{-5}
12	Infinity	0.268×10^{-23}	0.232×10^{-16}	1.0×10^{-8}

the probability of error with fixed interference performance results of the rate 0.3 RS-BCH concatenated code, the rate 0.3 BCH code, and no coding. For the operating parameters selected, the rate 0.3 concatenated code improves performance by a factor ranging from 10^{10} to 10^{16} over no coding and by a factor ranging from 10^3 to 10^{12} over the same rate BCH code. Results vary depending upon operating parameters. However, in all cases, except at low values of SNR for each SIR, performance improves significantly when concatenated coding is employed. At low values of SNR for each SIR, the inner decoder for the concatenated code frequently fails to properly decode and, thus, the probability of inner codeword error at the output of the inner decoder, Pr(IE/K), will be high. This has the effect of making the probability of outer code word error at the output of the outer decoder, Pr(OE/K), higher. Table 3 also shows that single-stage BCH codes improve performance by a factor ranging from 10^2 to 10^9 over no coding.

The effect of higher-rate RS-BCH concatenated codes and BCH codes at SNR of 9 dB and SIR of 10 dB is shown in Table 4. These results indicate that, for code rates above 0.4, performance improves by a factor ranging from 10^3 to 10^7 per 0.1 increase in code rate for RS-BCH concatenated codes and by a factor ranging from 10^2 to 10^6 per 0.1 increase in code rate for BCH codes. This behavior follows from the fact that increasing the coding rate, R, has the effect of increasing the ratio of signal power per information symbol to noise spectral density, thereby reducing the probability of bit error, Pe, given in expression (2). Thus, as the coding rate increases, the probability of error with fixed interference decreases [15]. This result is based on constant signal power per transmitted bit. In support of this conclusion, Pr(OE/K) with rate 0.5 RS-BCH concatenated coding is $\leq 10^{-38}$ for each SNR and SIR combination considered in Table 3. Likewise, Pr(IE/K) with rate 0.5 BCH coding is $\leq 10^{-38}$ for each SNR and SIR considered in Table 3 with the exception of SNR = 10 dB and SIR = 20 dB. For this combination, Pr(IE/K) = 0.488×10^{-20}. This example shows that higher-rate RS-BCH concatenated codes and higher-rate BCH codes improve probability of error with fixed interference performance at each SIR considered. The results also indicate that the amount of performance improvement due to increases in RS-BCH concatenated code rate is greater than that due to equal increases in BCH code rate.

Due to computational limitations, when Pr(OE/K) $\leq 10^{-38}$, Pr(OE/K) is assigned a value of 0.00, and when (1.00 - Pr(OE/K) $\geq$ (1.00 - 10^{-38}), Pr(OE/K) is assigned the value of 1.00. The same computational limitation hold for Pr(IE/K), Pr(OE), and Pr(IE). These limitations are more than adequate for all practical applications of the model presented here.

For code rates below 0.4, the above conclusions do not hold. For example, Pr(OE/K) for the rate 0.4 (28,24), (15,7) RS-BCH concatenated code is worse than Pr(OE/K) for the rate 0.3 (9,5), (7,4) code for SNR = 9 dB and SIR = 10 dB as shown in Table 4. This result is not surprising because the SNR is so low for the given interference level, that the rate 0.47 inner decoder of the rate 0.4 concatenated code fails to properly decode more frequently than the rate 0.57 inner decoder of the rate 0.3 concatenated code. Thus, the rate 0.47 inner decoder passes on a higher Pr(IE/K) to the outer decoder than does the rate 0.57 inner decoder. Even though the outer code rate for the inner decoder of the rate 0.4 concatenated code is greater than that of the rate 0.3 concatenated code, the higher Pr(IE/K) passed on by the inner decoder of the rate 0.4 concatenated code is not fully offset, thus yielding the result stated and

TABLE 4

TABLE 3. PROBABILITY OF ERROR WITH FIXED INTERFERENCE WITH RS-BCH CONCATENATED CODING AND BCH CODING AT SELECTED OPERATING PARAMETERS

Code Rate	SNR = 9dB, SIR = 10dB		SNR = 9dB, SIR = Infinity	
	Pr(OE/K) RS-BCH Concatenated Code	Pr(IE/K) BCH Code	Pr(OE/K) RS-BCH Concatenated Code	Pr(IE/K) BCH Code
1.0	0.6×10^{-5}	0.6×10^{-5}	0.3×10^{-6}	0.3×10^{-6}
0.3	0.164×10^{-5}	0.123×10^{-2}	0.459×10^{-15}	0.182×10^{-12}
0.4	0.102×10^{-3}	0.147×10^{-2}	0.321×10^{-18}	0.123×10^{-16}
0.5	0.224×10^{-9}	0.242×10^{-4}	0.118×10^{-32}	0.607×10^{-20}
0.6	0.146×10^{-12}	0.684×10^{-6}	$\leq 10^{-38}$	0.950×10^{-26}
0.7	0.154×10^{-19}	0.124×10^{-10}	$\leq 10^{-38}$	$\leq 10^{-38}$
0.75	$\leq 10^{-38}$	0.192×10^{-16}	$\leq 10^{-38}$	$\leq 10^{-38}$

explained above. As another example, observe from Table 4 that the rate 0.3 BCH code provides slightly better performance than the rate 0.4 BCH code for SNR = 9 dB and SIR = 10 dB. This is because the curve-fitting results are not as accurate at error rates around 10^{-2}. However, the curve-fitting error is relatively small and should have no significant impact in this region.

The case in which only Gaussian noise is present on the satellite channel is shown in Figures 4 through 9 as the S/K = infinity curve. It may be noted that this is the lowest curve, which means that performance of both single-stage BCH codes and RS-BCH concatenated codes is better for the Gaussian noise-only case than for the Gaussian noise-plus-interference case. In Figures 10 and 11, the curve for S/K = infinity does not appear because probability of error with fixed interference $\leq 10^{-38}$. Table 4 provides a comparison of the performance of RS-BCH concatenated codes with single-stage BCH codes of equal rate at SNR of 9 dB and SIR of infinity. This table shows that RS-BCH concatenated codes provide better performance on a Gaussian noise-only channel than do single-stage BCH codes of equal rate. For the selected concatenated codes with rates greater than 0.5, Pr(OE/K) $\leq 10^{-38}$ when only Gaussian noise is present on the channel, so a meaningful comparison cannot be made. Notice also that for the selected RS-BCH concatenated codes at these operating parameters, performance steadily improves with increasing code rate. This suggests that the effect of the inner decoder rate decreases as SIR increases. Likewise, for single-stage BCH codes at these operating parameters, performance improves with each increase in code rate. This implies that the curve-fitting error diminishes as SIR increases. This trend of improving performance as the code rate increases does not continue forever as evidenced by the case of no coding, i.e., code rate R = 1. For the operating parameters of SNR = 9dB and SIR =

infinity, no coding provides worse performance than does any of the selected codes with R < 1. This is explained by recalling that the normalization effect of R is based on the codes selected for investigation. Forney found no RS-BCH concatenated codes with rates above 0.75, which are included in the set of codes considered here, i.e., codes that provide probability of decoder error = 10^{-6} when the channel bit error probability = 10^{-2} on a noise only channel.

A final observation from Table 4 is that performance results are very good because we considered high values for SNR. We did so because the results of Prabhu to which we curve-fit did not extend below SNR of 8dB. Therefore, no judgement of curve-fitting error could be made below this SNR.

Probability of error with random interference performance results appear in Figures 12 through 16 for the rate 0.3, 0.4, 0.5, and 0.7 RS-BCH concatenated codes and BCH codes of Table 2. The level of interference, K, is assumed to be uniformly distributed. From the B curves in Figures 12 through 16, the effect of RS-BCH concatenated code rates on Pr(OE) can be observed. At the lower values of SNR for each SIR, where Pr(OE/K) is close to 1.0 for the uncoded case, Pr(OE) remains constant. Pr(OE) begins to drop at values of SNR between 11 dB and 13 dB. From these figures, it is evident that Pr(OE) drops faster as the coding rate increases. As shown by the A curves in Figures 12 through 16, Pr(IE) for single-stage BCH codes also drops faster with increasing coding rate.

Table 5 supports these observations with a comparison of the probability of error with random interference performance results for the codes of Table 2 at SNR of 13dB and at SNR of 14dB. Results at SNR of 13dB indicate that

TABLE 5 PROBABILITY OF ERROR WITH RANDOM INTERFERENCE WITH RS-BCH CONCATENATED CODES AND BCH CODES AT SELECTED OPERATING PARAMETERS

CODE RATE	SNR = 14 dB		SNR = 13 dB	
	Pr(CE) RS-BCH CONCATENATED CODE	Pr(IE) BCH CODE	Pr(OE) RS-BCH CONCATENATED CODE	Pr(IE) BCH CODE
0.3	0.756×10^{-6}	0.133×10^{0}	0.165×10^{-2}	0.246×10
0.4	0.883×10^{-2}	0.112×10^{0}	0.249×10^{0}	0.250×10
0.5	0.237×10^{-13}	0.485×10^{-3}	0.104×10^{-3}	0.165×10^{0}
0.6	0.135×10^{-20}	0.752×10^{-7}	0.132×10^{-4}	$0.300 \times 10^{-}$
0.7	$\leq 10^{-38}$	0.172×10^{-17}	0.542×10^{-7}	0.16[illegible] $\times 10^{-}$
0.75	$\leq 10^{-32}$	0.132×10^{-21}	0.544×10^{-19}	$0.882 \times 10^{-}$

RS-BCH concatenated codes with rates above 0.4 improve performance by a factor ranging from 10^1 to 10^{12} per 0.1 increase in code rate. BCH codes with rates above 0.4 also improve performance at SNR of 13dB; however, only by a factor ranging from 10^1 to 10^4. As was true for fixed interference, these results are due to the increase in the ratio of signal power per information symbol to noise spectral density caused by an increase in the code rate R.

Because the inner code of the rate 0.3 RS-BCH concatenated code has a higher rate than that of the rate 0.4 RS-BCH concatenated code, the Pr(OE) performance is better for the rate 0.3 concatenated code. Pr(IE) performance is better for the rate 0.3 BCH code than for the rate 0.4 BCH code. This slight difference is due to the curve-fitting error that exists at practically unimportant error rates around 10^0. A curve-fitting error this small has no significance in this region.

As the SNR increases, the Pr(OE) and Pr(IE) performance of RS-BCH concatenated codes and single-stage BCH codes, respectively, improves as expected. This result becomes evident in Table 5 by comparing the SNR = 13dB and SNR = 14dB entries for a given code at a given rate. For example, Pr(OE) performance of the rate 0.6 RS-BCH concatenated code at SNR = 14dB improves by a factor of 10^{16} over that for the same code at SNR = 13dB. For the rate 0.6 single-stage BCH code, Pr(IE) performance at SNR = 14dB improves by a factor of 10^6 over that at SNR = 13dB. Table 5 also reveals that the inner decoder rate of RS-BCH concatenated codes has greater impact on Pr(OE) as SNR increases. This effect is demonstrated by noting that at SNR = 13dB, the rate 0.3 RS-BCH concatenated code provides Pr(OE) performance that is a factor of 10^2 better than that for the rate 0.4 RS-BCH concatenated code. At SNR = 14dB, the Pr(OE) performance difference rises to a factor of 10^4. The rate 0.3 single-stage BCH code Pr(IE) performance at SNR = 14dB is slightly worse than that for the rate 0.4 single-stage BCH code. This indicates that the curve-fitting error reduces as the SNR increases.

Because the B curves lie to the left of the A curves in Figures 12 through 16, we can see that probability of error with random interference performance results are better for RS-BCH concatenated codes than for single-stage BCH codes of equal rate when the distribution of K is uniform. The results of Table 5 support this observation for both SNR = 13dB and SNR = 14dB. For SNR = 13dB, RS-BCH concatenated codes provide performance improvement up to a factor of 10^{12} over single-stage BCH codes of equal rate. Recall that the parameter SNR used in this section is the same as E_b/N_o.

SECTION 4

4. CONCLUSIONS AND SUGGESTIONS FOR FUTURE ANALYSIS

The results show that when RS-BCH concatenated coding is used on an interference satellite channel, performance is significantly better than when no coding is employed. In addition, for the selected codes, higher rate RS-BCH concatenated codes provide better performance than lower rate RS-BCH concatenated codes on both the noise-only channel and the interference-plus-noise channel. This performance improvement is greatest on the noise-only channel. RS-BCH concatenated codes also provide better all-around performance on an interference channel than do single-stage BCH codes of equal rate.

The results presented in this paper are preliminary in that they depend on assumptions made in the performance model. However, this model does treat interference in a realistic fashion. By modifying certain assumptions, additional applications of the model can be made to provide a more comprehensive set of results. For example, performance results for single-stage RS codes can be obtained and compared with those generated here to provide an assessment of the effectiveness of these codes versus that of RS-BCH concatenated codes. Results can also be obtained for concatenated codes where the inner code is a convolutional code and the outer code is an RS code. Interleaving results could likewise be examined and compared.

Another example could be to use RS-BCH concatenated codes and erasures-and-errors decoding. The presented performance model lends itself to extension for incorporating this decoding strategy in it. Extensive treatment of such decoding is provided by Forney [10].

The performance model could also be extended to include interference level probability distributions which are other than uniform.

In addition, interference which is different from co-channel interference could be considered. One example is the case of frequency-hopping, spread-spectrum signaling used with QPSK modulation. In this case, the interference takes the form of a variable number of other users transmitting in the same frequency bin at the same time. This topic is thoroughly discussed in the references [11-14].

A final suggestion for future analysis would be to investigate implementation techniques for the concatenated and single-stage codes discussed here. As part of this effort, incorporation of the implementations into existing and future systems could be considered.

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