Stabilization of Networked Control Systems: Designing Effective Communication Sequences

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TR 2004-45

STABILIZATION OF NETWORKED CONTROL SYSTEMS: DESIGNING EFFECTIVE COMMUNICATION SEQUENCES [1]

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Abstract: This paper discusses the stabilization of a networked control system (NCS) whose sensors and actuators exchange information with a remote controller over a shared communication medium. Access to that medium is governed by a pair of periodic communication sequences. Under the model utilized here, the controller and plant handle communication disruptions by "ignoring" (in a sense to be made precise) sensors and actuators that are not actively communicating. It is shown that under mild conditions, there exist periodic communication sequences that preserve the reachability and observability of the plant, leading to a straightforward design of a stabilizing feedback controller.

Keywords: Reachability, Observability, Stabilization, Linear systems, Communication networks, Communication control applications

1. INTRODUCTION

The progress in digital computation and communication has enabled the development of large-scale distributed control systems in which multiple sensors, actuators and feedback controllers exchange information through a shared communication medium. In such *networked control systems* (NCS), the medium's communication capacity is limited and must be allocated to sensors, actuators, and controllers. As a consequence, various communication constraints such as network-induced delays (Nilsson, 1998; Zhang *et al.*, 2001; Lian *et al.*, 2002), data rate limitations (Wong and Brockett, 1997; Wong and Brockett, 1999; Tatikonda *et al.*, 1998; Nair and Evans, 2000), quantization effects (Brockett and Liberzon, 2000; Elia and Mitter, 2001; Ishii and Francis, 2002), and medium access constraints (Brockett, 1995; Hristu-Varsakelis and Morgansen, 1999; Walsh and Ye, 2001; Branicky *et al.*, 2002) all present potential problems whose effects on closed-loop performance and controller design must be understood and dealt with.

The focus of this paper is the stabilization of NCS under the medium access constraints. In this setting, only some of the sensors and actuators of a plant can access the shared medium and exchange information with a remote controller at any one time, while others must wait. In contrast to traditional control systems, the stabilization of NCS under medium access constraints involves designing not only the controller but also a medium access strategy. One possibility is to let the order

[1] This work was supported by the National Science Foundation under Grant No. EIA0088081 and by ARO ODDR&E MURI01 Grant No. DAAD19-01-1-0465, (Center for Communicating Networked Control Systems, through Boston University). Corresponding author: D. Hristu-Varsakelis. Tel: +1-301-405-5283, Fax: +1-301-314-9477. E-mail: hristu@umd.edu

of medium access be specified in advance by a periodic *communication sequence* (Brockett, 1995) and then design a feedback controller that stabilizes the system under that communication sequence. Under that formulation, the problem of designing a constant gain controller that stabilizes a linear plant is NP-hard. Solutions may be obtained via simulated annealing (Hristu-Varsakelis and Morgansen, 1999) but the computational burden is high and the existence of a solution is not guaranteed.

Another possibility is to first design a controller that stabilizes the NCS absent any communication constraints and then look for medium access strategies that preserve stability. Two types of medium access strategies have been proposed in this case, termed "static access scheduling" and "dynamic access scheduling". Under static access scheduling, the order of medium access for the sensors and actuators is determined off-line (in terms of a "periodic communication sequence") and remain fixed over time; under dynamic access scheduling, the medium access is determined "on-line" by a feedback-based arbitration policy. The existence and design of periodic communication sequences is studied in (Hristu-Varsakelis, 2001) and (Branicky *et al.*, 2002), where the schedulability of NCS under static scheduling is checked by the rate monotonic (RM) rule. Examples of dynamic access scheduling policies include MEF-TOD (Walsh and Ye, 2001) and CLS-ϵ (Hristu-Varsakelis and Kumar, 2002).

It should be noted that most of the medium access strategies proposed in previous works have focused only on NCS whose dynamics are "block-diagonal", in the sense that they consist of collections of sub-systems that are uncoupled in the absence of communication constraints. That is the case, for example, with (Hristu-Varsakelis and Kumar, 2002; Hristu-Varsakelis, 2001; Branicky *et al.*, 2002). Other analyses (Walsh and Ye, 2001; Ye, 2000), are attached to very conservative stability criteria. Furthermore, previous works have often assumed zero order holding (ZOH) at the receiver side of a communication medium: whenever an actuator or sensor fails to access the medium, the value stored in a ZOH will be fed to the plant or controller. In this work, a new communication protocol which foregoes the use of ZOH is proposed. Instead, when an actuator or sensor fails to access the medium *zero* will be fed into the plant or controller. It will be shown that this protocol leads to a simpler but more powerful model for NCS which enables one to jointly design a static access scheduling strategy and a stabilizing feedback controller. In particular, the design method presented in this paper can be used with a more general class of NCS, which have "fully coupled" dynamics.

The remainder of this paper is organized as follows: Section 2 shows that a linear plant subject to medium access constraints can be modeled as a linear time-varying (LTV) system of the same dimension. Section 3 proves that, under mild conditions, there exist periodic communication sequences that preserve the plant's reachability and observability. An algorithm is given for identifying such communication sequences. In Section 4, an observer-based controller is designed (using standard linear systems theory) to stabilize an NCS at a desired decay rate. An example is given in Section 5.

2. MODELING MEDIUM ACCESS CONSTRAINTS

Consider a NCS in which a plant is controlled by a remote controller via a shared communication medium. Let the dynamics of the plant be given by the discrete-time linear time-invariant (LTI) system

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + B\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= C\boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$, $\boldsymbol{u} = [u_1, \cdots, u_m]^T \in \mathbb{R}^m$, and $\boldsymbol{y} = [y_1, \cdots, y_p]^T \in \mathbb{R}^p$ are the states, inputs, and outputs of the plant. Each output y_i is associated with a corresponding sensor while each input u_i is connected to an actuator.

Suppose that the communication medium connecting the sensors and the controller has w_σ *output channels*, where $(1 \le w_\sigma < p)$. At any one time, only w_σ of the p sensors can access these channels to send their output to the controller, while others have to wait. Likewise, the plant's m actuators share w_ρ *input channels*, with $(1 \le w_\rho < m)$, to receive control signals from controller; only w_ρ of the m actuators can access the input channels at any time. An NCS having this configuration is illustrated in Fig. 1 where the "open" or "closed" status of the switches indicates the medium access status of the sensors or actuators. The vectors $\bar{\boldsymbol{y}} \in \mathbb{R}^p$, $\bar{\boldsymbol{u}} \in \mathbb{R}^m$ are the input and output of the controller, respectively. Those will generally differ from the plant output and input $y(k), u(k)$ due to the communication constraint discussed above. The "communication sequences" (to be defined shortly), $\boldsymbol{\rho}(k)$ and $\boldsymbol{\sigma}(k)$, govern the medium access to the input and output channels at time k.

For all $i = 1, \cdots, p$, let the binary-valued function $\sigma_i(k)$ denote the medium access status of sensor i at time k, i.e. $\sigma_i(k) : \mathbb{Z} \mapsto \{0, 1\}$, where 1 means "accessing" and 0 means "not accessing". The instantaneous medium access status of the p sensors at time k can be represented by the p-

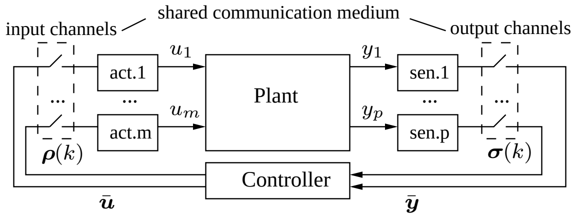

Fig. 1. A Networked Control System

to-w_σ *"communication sequence"* (Brockett, 1995; Hristu-Varsakelis and Kumar, 2002)

$$\boldsymbol{\sigma}(k) = [\sigma_1(k), \cdots, \sigma_p(k)]^T$$

Definition 1. An M-to-N communication sequence is the map, $\boldsymbol{\sigma}(k) : \mathbb{Z} \mapsto \{0,1\}^M$, satisfying $\|\boldsymbol{\sigma}(k)\|^2 = N, \forall k$.

An output, say $y_i(k)$, is available to the controller only when sensor i is accessing the communication medium, i.e. $\sigma_i(k) = 1$. At other times ($\sigma_i(k) = 0$) it is assumed that a *zero* value will be used by the controller for that sensor to generate the control signals, while the actual output $y_i(k)$ will be ignored due to its being unavailable. Let $\bar{y}_i(k)$ denote the output signal available to the controller at time k. Based on the above protocol,

$$\bar{y}_i(k) = \sigma_i(k) \cdot y_i(k); \qquad (2)$$

for all $i = 1, 2 \cdots p$. It will be convenient to define the *matrix form* of a communication sequence $\boldsymbol{\eta}(k)$, by

$$M_\eta(k) \triangleq diag(\boldsymbol{\eta}(k)).$$

Now let $\bar{\boldsymbol{y}} = [\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_p]^T$, so that

$$\bar{\boldsymbol{y}}(k) = M_\sigma(k) \cdot \boldsymbol{y}(k) \qquad (3)$$

Similarly, whenever actuator j loses its access to the communication medium, the control signal generated at the controller for that actuator will be unavailable to and hence ignored by the plant. Instead, the plant sets $u_j = 0$ until actuator j regains medium access. The medium access status of the plant's m actuators is represented by an m-to-w_ρ communication sequence $\boldsymbol{\rho}(k)$ (Def. 1). Let $\bar{\boldsymbol{u}} = [\bar{u}_1, \cdots \bar{u}_m]^T$ denote the signal generated by the controller, and let $\boldsymbol{u}(k)$ be the input signals actually used by the plant, under the above protocol. Then,

$$\boldsymbol{u}(k) = M_\rho(k)\bar{\boldsymbol{u}}(k) \qquad (4)$$

In (3) and (4) the effect of medium access constraints at the plant's output and input sides has been captured by the time varying linear maps $M_\sigma(k)$ and $M_\rho(k)$. From the controller's viewpoint, the plant acts as a time-varying system with $\bar{\boldsymbol{u}}$ as its input and $\bar{\boldsymbol{y}}$ as its output (see Fig. 2).

From (1)-(4), the dynamics of that system are:

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + BM_\rho(k)\bar{\boldsymbol{u}}(k), \\ \bar{\boldsymbol{y}}(k) &= M_\sigma(k)C\boldsymbol{x}(k), \end{aligned} \qquad (5)$$

which is the system to be stabilized by the controller. We will refer to (5) as the "extended plant". Notice that the state of the extended plant coincides with that of the original plant. Hence the NCS can be stabilized by designing a feedback controller that stabilizes the extended plant.

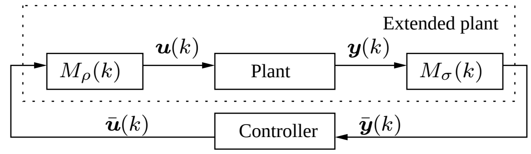

Fig. 2. The extended plant

Remark 1. In previous works, e.g. (Brockett, 1995; Hristu-Varsakelis and Morgansen, 1999), it is often assumed that there is a zero-order-hold (ZOH) at the receiving side of a communication medium. The use of ZOH significantly increases the system's complexity because it introduces time-varying delays to the closed-loop dynamics (see, for example, the "extensive form" in (Hristu-Varsakelis and Morgansen, 1999) and other similar constructions). Here, the communication protocol "ignores" outdated inputs or outputs, and as a result the effect of medium access constraints can be modeled as cascading the original plant with a pair of communication sequences (in their matrix form). As will be shown in the sequel, this model leads to a straightforward and complete solution to the stabilization problem.

3. COMMUNICATION SEQUENCES THAT PRESERVE REACHABILITY AND OBSERVABILITY

In order to design a feedback controller that stabilizes the NCS, it is helpful to first study how the choice of $M_\rho(\cdot)$ and $M_\sigma(\cdot)$ (equivalently $\boldsymbol{\rho}(\cdot)$ and $\boldsymbol{\sigma}(\cdot)$) affects the reachability and observability of (5).

3.1 Reachability

Suppose that $\boldsymbol{x}(0) = 0$, and let the extended plant (5) evolve from $k = 0$ to $k = k_f$. Then

$$\boldsymbol{x}(k_f) = R \cdot \left[\, \bar{\boldsymbol{u}}(0) \;\; \bar{\boldsymbol{u}}(1) \;\; \cdots \;\; \bar{\boldsymbol{u}}(k_f - 1) \,\right]^T$$

where

$$R = [A^{k_f-1}BM_\rho(0),\, A^{k_f-2}BM_\rho(1), \\ \cdots,\; BM_\rho(k_f - 1)] \qquad (6)$$

The extended plant (5) is reachable in $[0, k_f]$ iff $rank(R) = n$. Notice that for each k, $\boldsymbol{\rho}(k)$ is an m-dimensional vector consisting of w_ρ ones and $m - w_\rho$ zeros. Hence, at each step k, $M_\rho(k)$ has the effect of "selecting" w_ρ columns from the m

columns of the term $A^{k_f-k-1}B$ on the RHS of (6). The matrix R will have full rank if the $k_f \cdot w_\rho$ columns that $M_\rho(k)$ selects (for $k = 0, \cdots, k_f - 1$) contain n linearly independent columns.

Theorem 1. Suppose that A is invertible and the pair (A, B) is reachable. For any integer $1 \le w_\rho < m$, there exists an m-to-w_ρ communication sequence $\boldsymbol{\rho}(\cdot)$ and an integer $k_f \le \left\lceil \frac{n}{w_\rho} \right\rceil \cdot n$, such that the extended plant (5) is reachable in $[0, k_f]$.

PROOF. First, consider the worst-case scenario where $w_\rho = 1$. The theorem holds if for $k_f = n^2$, it is always possible to design a m-to-1 communication sequence $\boldsymbol{\rho}(k)$, such that the sequence $M_\rho(k)$ can select n independent columns (RHS of (6)) by selecting one column from the matrix $A^{k_f-k-1}B$ at each step $k = 0, \cdots k_f - 1$.

Let $\Gamma_i = \left[A^{ni}B, A^{ni+1}B, \cdots, A^{ni+n-1}B \right]$. The matrix Γ_i has rank n for all $i = 0, \cdots n-1$ because A is invertible and (A, B) is reachable. Now let $\gamma_i^0, \cdots, \gamma_i^{n-1}$ be any n linearly independent columns from Γ_i and let L_i be the set $\{\gamma_i^0, \cdots, \gamma_i^{n-1}\}$. Consider the following algorithm

(1) Let $L = L_0$.
(2) Replace γ_0^1 in L by a column from L_1 while keeping the rank of L being n. Such a replacement can always be found because $rank(L_1) = n$.
(3) For $i = 2, \cdots, n-1$, replace γ_0^i in L by a column from L_i while keeping the rank of L.

The resulting L has one column from each Γ_i $(i = 0, \cdots, n-1)$ and has rank n. The above algorithm ensures that it is possible to select n linearly independent columns as long as one can select one column from each Γ_i. But on the RHS of (6), $M_\rho(\cdot)$ can actually select n columns from each Γ_i. Hence there always exists an $M_\rho(\cdot)$ that selects n independent columns in at most n^2 steps.

Now consider the less restrictive case $w_\rho > 1$. The sequence $M_\rho(\cdot)$ can select at least w_ρ independent columns from each Γ_i $(i = 0, 1, \cdots)$. Using a similar algorithm as in the single-channel case (replacing w_ρ columns in L at each step), one can design an m-to-w_ρ communication sequence $\boldsymbol{\rho}$ such that $M_\rho(\cdot)$ selects n independent columns from the Γ_i's for $i = 0, 1, \cdots, \left\lceil \frac{n}{w_\rho} \right\rceil - 1$ in at most $\left\lceil \frac{n}{w_\rho} \right\rceil \cdot n$ steps. □

Remark 2. The bound for k_f in Theorem 1 is conservative. Notice that for each k, $\boldsymbol{\rho}(k)$ can only have $\binom{m}{w_\rho}$ possible values. Thus it is possible to find the minimum k_f by searching over all possible communication sequences $\boldsymbol{\rho}(\cdot)$ in the interval $k = \left[0, \left\lceil \frac{n}{w_\rho} \right\rceil \cdot n\right]$.

Remark 3. The invertibility of A is necessary in Theorem 1. A counter-example is:

$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$. Here, (A, B) is reachable but there is no 3-to-1 communication sequence $\boldsymbol{\rho}(k)$ such that the extended plant is reachable in $[0, k_f]$ for any k_f. Of course, the invertibility of A is guaranteed if A is obtained by discretizing a continuous-time plant.

Finally, one must ensure "l-step reachability" for the extended plant.

Definition 2. (Rugh, 1996) A discrete-time linear system is called l-step reachable (observable) if l is a positive integer such that the system is reachable (observable) on $[i, i+l]$ for any i.

Theorem 2. Suppose that A is invertible and the pair (A, B) is reachable. For any integer $1 \le w_\rho < m$, there exist integers $l, N > 0$ and an **N-periodic**[2] m-to-w_ρ communication sequence $\boldsymbol{\rho}(\cdot)$ such that the extended plant (5) is l-step reachable.

PROOF. From Theorem 1, there exists an integer k_f and a communication sequence $\boldsymbol{\rho}(k)$ such that the extended plant is reachable in $[0, k_f]$. Hence the sequence $M_\rho(k)$ can select n independent columns from the matrices $A^{k_f-k-1}B$ during $k = 0, \cdots, k_f - 1$. Now let $N = k_f$ and extend $\boldsymbol{\rho}(k)$ for $k \ge k_f$ by setting $\boldsymbol{\rho}(k) = \boldsymbol{\rho}(k+N)$, $\forall k$. Because A is invertible, the N-periodic sequence $M_\rho(k)$ will select n independent columns in every interval $k = [jN, (j+1)N - 1]$ $(j = 0, 1, 2 \cdots)$. Now let l be any integer greater than or equal $2N - 1$, then for all $i \ge 0$ there always exists an integer $j \ge 0$, such that $[jN, (j+1)N-1] \in [i, i+l]$. Hence the periodic sequence $M_\rho(k)$ will select n independent columns in $[i, i+l]$ for all i. Thus the extended plant is l-step reachable under the N-periodic communication sequence $\boldsymbol{\rho}(k)$. □

3.2 Observability

The duality of reachability and observability suggests using the same techniques as in Section 3.1 to examine the observability of the extended plant (5) . Note that the extended plant (5) is observable in $[0, k_f]$ iff

[2] A communication sequence $\boldsymbol{\sigma}(\cdot)$ is called N-periodic if $\boldsymbol{\sigma}(k) = \boldsymbol{\sigma}(k+N)$ for all k.

$$rank\left(\begin{bmatrix} M_\sigma(0)C \\ M_\sigma(1)CA \\ \vdots \\ M_\sigma(k_f-1)CA^{k_f-1} \end{bmatrix}\right) = n \qquad (7)$$

At each step k, $M_\sigma(k)$ selects w_σ rows from CA^k. By switching column manipulations to row manipulations, all the arguments used in Section 3.1 apply in this section. The following theorems are stated without their proofs.

Theorem 3. Suppose that A is invertible and the pair (A, C) is observable. For any integer $1 \le w_\sigma < p$, there exists a p-to-w_σ communication sequence $\boldsymbol{\sigma}(\cdot)$ and an integer $k_f \le \left\lceil \frac{n}{w_\sigma} \right\rceil \cdot n$, such that the extended plant (5) is observable in $[0, k_f]$.

Theorem 4. Suppose that A is invertible and the pair (A, C) is observable. For any integer $1 \le w_\sigma < p$, there exist integers $l, N > 0$ and an N-periodic p-to-w_σ communication sequence $\boldsymbol{\sigma}(\cdot)$ such that the extended plant (5) is l-step observable.

4. STABILIZATION OF NCS

From linear systems theory it is known that the LTV extended plant can be stabilized via output feedback if it is l-step reachable and l-step observable. The feedback controller consists of a state observer followed by a time varying feedback gain $K(k)$ (shown in Fig. 3).

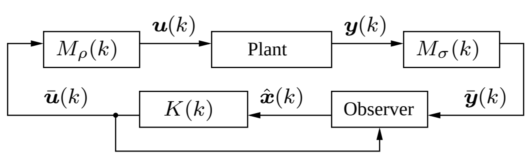

Fig. 3. Stabilization of NCS using an observer-based controller

Define $\bar{C}(k) = M_\sigma(k)C$, $\bar{B}(k) = BM_\rho(k)$; then the dynamics of the extended plant can be rewritten as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \bar{B}(k)\bar{\boldsymbol{u}}(k) \\ \bar{\boldsymbol{y}}(k) &= \bar{C}(k)\boldsymbol{x}(k) \end{aligned} \qquad (8)$$

For (8), define the grammians

$$\mathcal{W}_\alpha(k_0, k_f) \triangleq \sum_{j=k_0}^{k_f-1} \alpha^{4(k_0-j)} A^{k_0-j-1} \bar{B}(j)\bar{B}^T(j)(A^{k_0-j-1})^T$$

$$\mathcal{M}_\alpha(k_0, k_f) \triangleq \sum_{j=k_0}^{k_f-1} \alpha^{4(j-k_f+1)} (A^{j-k_0})^T \bar{C}^T(j)\bar{C}(j)A^{j-k_0}$$

Define a state observer for (8):

$$\begin{aligned} \hat{\boldsymbol{x}}(k+1) &= A\hat{\boldsymbol{x}}(k) + \bar{B}(k)\bar{\boldsymbol{u}}(k) + \\ &\qquad H(k)[\bar{\boldsymbol{y}}(k) - \hat{\boldsymbol{y}}(k)]; \\ \hat{\boldsymbol{y}}(k) &= \bar{C}(k)\hat{\boldsymbol{x}}(k) \end{aligned} \qquad (9)$$

The following theorem is a rephrasing of Theorem 29.5 in (Rugh, 1996) for the extended plant (8).

Theorem 5. Suppose that the extended plant (8) is l-step reachable, l-step observable, and that A is invertible. Then given a constant $\alpha > 1$ and $\eta > 1$ the feedback and observer gain

$$\begin{aligned} K(k) &= -\bar{B}^T(k)(A^{-1})^T \mathcal{W}_{\eta\alpha}^{-1}(k, k+l) \\ H(k) &= [(A^{-l})^T \mathcal{M}_{\eta\alpha}(k-l+1, \\ &\qquad k+1)A^{-l}]^{-1}(A^{-1})^T \bar{C}^T(k) \end{aligned}$$

are such that the extended plant (8) is uniformly exponentially stable with rate α under the feedback law $\bar{\boldsymbol{u}}(k) = K(k)\hat{\boldsymbol{x}}(k)$, where $\hat{\boldsymbol{x}}(k)$ is the state of the observer (9).

5. AN EXAMPLE

In this example, the plant is a 4th order unstable LTI system having 2 inputs and 2 outputs. The dynamics of the plant (1) are

$$A = \begin{bmatrix} 1 & 1/5 & 0 & 0 \\ 0 & 11/4 & 0 & 1/5 \\ 1 & 1/5 & 1/3 & 3/4 \\ 0 & -1 & 0 & 1/4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The plant's initial condition was set to $\boldsymbol{x}(0) = [3, 5, 7, 6]^T$. The communication medium had $w_\rho = w_\sigma = 1$. The input and output communication sequences were both 2-periodic with

$$\{\boldsymbol{\sigma}(0), \boldsymbol{\sigma}(1), \cdots\} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \cdots \right\}$$

$$\{\boldsymbol{\rho}(0), \boldsymbol{\rho}(1), \cdots\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \cdots \right\}$$

It easy to verify that under this pair of communication sequences, the extended plant is 7-step reachable and 7-step observable.

The extended plant is stabilized by the control schematic shown in Fig. 3. The observer gains $H(k)$ and the feedback gains $K(k)$ were calculated from the formulas in Theorem 5, where α and η were chosen as $\alpha = 2$, $\eta = 1.2$. The observer's initial condition was $\hat{\boldsymbol{x}}(0) = [1, 1, 3, 4]^T$. The state evolution is shown in Fig. 4.

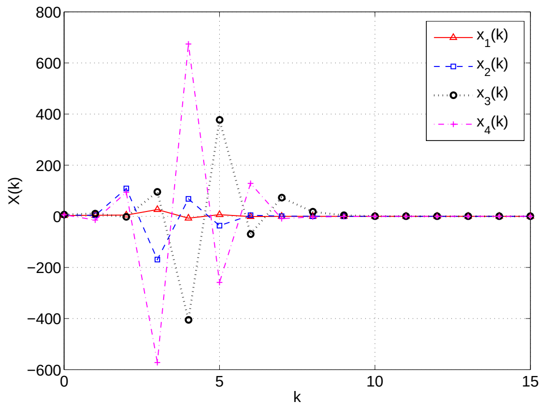

Fig. 4. State evolution for the closed-loop NCS under periodic communication.

6. CONCLUSIONS AND FUTURE WORK

This paper discussed the design of communication sequences and controller for stabilizing a linear NCS. It was shown that by "ignoring" the actuators and sensors that are not actively communicating with the plant and controller, the complexity of the joint controller/communication design problem becomes manageable. If A is invertible, there always exist periodic communication sequences such that the extended plant (5) retains the l-step reachability and l-step observability of the original plant (1). The availability of such sequences (which can be constructed using the algorithm given in the paper) makes it straightforward to design an accompanying output feedback controller that exponentially stabilizes the NCS at an arbitrary decay rate. The controller consists of a state observer followed by time varying feedback. Opportunities for future work include studying the continuous-time counterpart of this problem (where the requirement for invertibility of A can be removed) and investigating the selection of communication sequences in the context of robustness.

REFERENCES

Branicky, M. S., S. M. Phillips and W. Zhang (2002). Scheduling and feedback co-design for networked control systems. In: *Proceedings of the 41st IEEE Conference on Decision and Control*. pp. 1211–1217.

Brockett, R. W. (1995). Stabilization of motor networks. In: *Proceedings of the 34th IEEE Conference on Decision and Control*. pp. 1484–8.

Brockett, R. W. and D. Liberzon (2000). Quantized feedback stabilization of linear systems. *IEEE Transactions on Automatic Control* **45**(7), 1279–1289.

Elia, N. and S. K. Mitter (2001). Stabilization of linear systems with limited information. *IEEE Transactions on Automatic Control* **46**(9), 1384–1400.

Hristu-Varsakelis, D. (2001). Feedback control systems as users of a shared network: Communication sequences that guarantee stability. In: *Proceedings of the 40th IEEE Conference on Decision and Control*. Vol. 4. pp. 3631–6.

Hristu-Varsakelis, D. and K. Morgansen (1999). Limited communication control. *Systems and Control Letters* **37**(4), 193–205.

Hristu-Varsakelis, D. and P. R. Kumar (2002). Interrupt-based feedback control over a shared communication medium. In: *Proceedings of the 41st IEEE Conference on Decision and Control*. pp. 3223–8.

Ishii, H. and B. A. Francis (2002). *Limited Data Rate in Control Systems with Networks*. Springer. Berlin.

Lian, F., J. Moyne and D. Tilbury (2002). Network design consideration for distributed control systems. *IEEE Transactions on Control System Technology* **10**(2), 297–307.

Nair, G. N. and R. J. Evans (2000). Stabilization with data-rate-limited feedback: tightest attainable bounds. *Systems and Control Letters* **41**(1), 49–56.

Nilsson, J. (1998). Real-Time Control Systems with Delays. PhD thesis. Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Rugh, W. J. (1996). *Linear System Theory*. 2nd ed.. Prentice Hall. Upper Saddle River, New Jersey.

Tatikonda, S., A. Sahai and S. Mitter (1998). Control of LQG systems under communication constraints. In: *Proceedings of the 37th IEEE Conference on Decision and Control*. pp. 1165–1170.

Walsh, G. and H. Ye (2001). Scheduling of networked control systems. *IEEE Control Systems Magazine* **21**(1), 57–65.

Wong, W. S. and R. W. Brockett (1997). Systems with finite communication bandwidth constraints. i. state estimation problems.. *IEEE Transactions on Automatic Control* **42**(9), 1294–9.

Wong, W. S. and R. W. Brockett (1999). Systems with finite communication bandwidth constraints. ii. stabilization with limited information feedback.. *IEEE Transactions on Automatic Control* **44**(5), 1049–1053.

Ye, H. (2000). Research on Networked Control Systems. PhD thesis. Department of Mechanical Engineering, University of Maryland at College Park.

Zhang, W., M. S. Branicky and S. M. Phillips (2001). Stability of networked control systems. *IEEE Control System Magazine* **21**(1), 84–99.