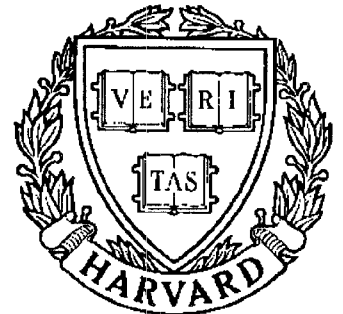


# TECHNICAL RESEARCH REPORT



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## **Design of a Crank-and-Rocker Driven Pantograph: A Leg Mechanism for the University of Maryland's 1991 Walking Robot**

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**Design of a Crank-and-Rocker Driven Pantograph:  
A Leg Mechanism for the University of Maryland's 1991 Walking Robot**

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## ABSTRACT

The National Aeronautics and Space Administration has been funding university research into flexible, six-legged walking machines capable of exploring alien terrain. This research has led to a progressive new look at standard four bar mechanisms.

Four bar mechanisms, by definition, consist of a crank link, coupler link, rocker link, and fixed (ground) link. The passive role of the coupler link, in traditional four bar mechanisms, can be reversed so that the coupler becomes the transmission link. This is achieved by replacing the coupler with an oblique triangular link. The internal angles of the modified coupler can be varied to create an array of continuous, ovoid paths at the disjointed vertex of the triangle.

Attaching a pantograph mechanism to the modified coupler's trace point amplifies and translates the ovoid path. The combination of these two mechanisms provides a stable, one degree of freedom walking path which emulates that of humans. This mechanism can therefore be used as a robotic leg.

A second degree of freedom is obtained by attaching an adjacent link to the pantograph mechanism which raises or lowers the walking path without effecting its shape or magnitude. This is used for climbing and maneuvering amidst rugged terrain.

## NOMENCLATURE

k	Rotational Spring Constant (N-m/deg)
$\alpha$	Fixed Coupler Angle (degrees)
$\theta$	Lifter Gear-Link Angle (degrees)

$\mu$  Transmission Angle (degrees)

## INTRODUCTION

The research and development of walking machines is progressing rapidly. There is an ever growing need for controllable, terrain adaptive machines capable of planetary exploration. In addition, the nuclear industry is interested in developing walking machines to be used in radioactive zones and other hostile environments. In order to perform these tasks, a walking machine must be completely autonomous or remotely controlled.

Walking machines usually have four to eight legs and may have as many as eighteen active degrees of freedom (DOF) [1]. The number of legs and the number of DOF depend on the robot's required tasks and walking environment. For example, a major design objective for the Ohio State Hexipod, was to investigate analysis and control methods for walking and exploring. It was important, therefore, to construct a machine with many DOF [2]. Consequently, the Hexipod has six legs and eighteen DOF [3,4]. The number of DOF closely correlates to the overall flexibility of a walking machine, however, complex software and controls are needed, and walking is usually quite slow.

The University of Maryland's 1991 walking robot adopted a six legged, three DOF design. Each leg has two DOF and is independently controlled. This provides adequate flexibility and fast locomotion, and simplifies the hardware and software design. Since each leg is independently controlled, the overall flexibility of the walking machine is increased. Designs which mechanically link all of the legs require much less control, but are not as adaptive or flexible.

The robotic leg presented in this paper was designed for the

University of Maryland's 1991 walking robot, in order to compete in the Fifth Annual Walking Machine Decathlon. This competition is a joint effort between Colorado State University at Fort Collins, and the University of Maryland at College Park. The competition objective is to provide the opportunity for undergraduate students to apply their engineering knowledge toward the hands-on design and construction of a walking machine, and to promote the advancement and further development of robotics technology. This year, eight universities throughout the United States and Canada participated in the competition. The competition rules define a walking machine as a mobile, terrain adaptive system with eight or less articulated legs, which can perform defined tasks in static or dynamic environments. The competition events ranged from a straight line dash to stair climbing, and included several autonomous events.

In accordance with these rules, a group of undergraduates from the Mechanical and Electrical Engineering Departments at the University of Maryland, challenged themselves to design and construct a walking machine. This paper presents the details of this year's leg design.

## DESIGN CONSTRAINTS

Kinematics was the first constraint considered in this leg design. The leg was designed to have an ovoid walking path in order to minimize the "slamming" effect caused by a robot's inertial forces during normal walking. This effect is highly pronounced in designs employing a circular kinematic path. The stride length of the leg (the major diameter of the walking path) was an additional kinematic constraint, particularly in designing the leg to climb stairs and maneuver over obstacles.

The number of degrees of freedom was a constraint pertaining to maximizing the flexibility of the robot while minimizing its weight and complexity. This constraint was closely related to the number of motors used.

The number of motors was limited to two per leg to minimize weight and simplify design. Increasing the number of motors often makes a design more flexible, but requires more complex hardware and software for reliable control.

## MODIFIED FOUR BAR MECHANISM

### Operation

The modified four bar mechanism shown in figure 1 is defined by links "AP" (crank), "BQ" (rocker), "ABC" (coupler), and "PQ" (ground). A motor and worm gear combination turns the crank link. As the crank rotates, a pendulum path is created by the rocker link. The crank and rocker links are connected to a triangular coupler link, which integrates the kinematic paths of the crank and the rocker, creating the ovoid path at point "C". The size and shape of the ovoid transmission curve is defined by the internal angles of the modified coupler link.

### Design Information

Figure 1 also shows the transmission angle " $\mu$ ", the internal angles of the coupler, and the dimensions of each link. As in traditional four bar mechanisms, the crank is the shortest link. The fourth link is ground and is defined by a support structure. It can be shown that,

$$AP + BQ < AB + PQ \quad (1)$$

in adherence to the rules of kinematics [4]. The transmission angle " $\mu$ " varies as the crank rotates. The link dimensions shown keep angle " $\mu$ " as close to 90 degrees as possible [5] such that,

$$|\mu \text{ min., max.} - 90^\circ| < 40^\circ \quad (2)$$

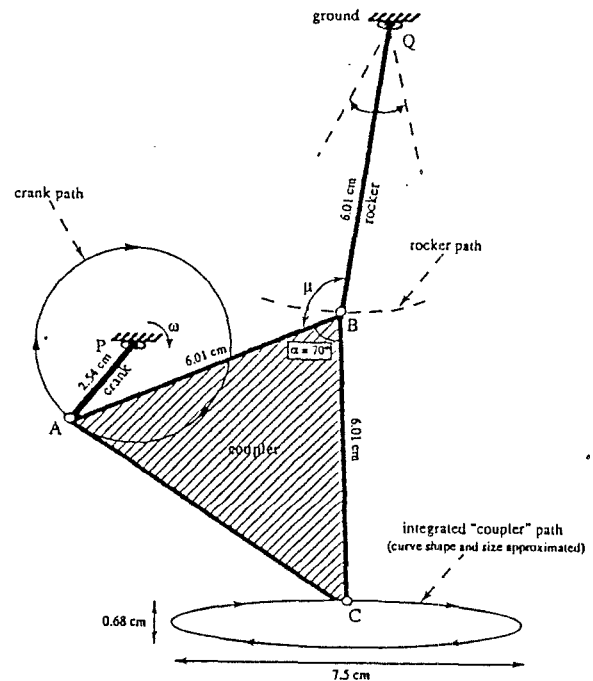


Figure 1. The Modified Four Bar Mechanism

Links AB, BQ, and BC are all equal, providing a symmetric coupler-point curve at point "C". The dimensions of links "AP", "AB", "BQ", and "PQ" were proportioned in such a way that the transmission angle deviates equally from 90 degrees [6, 7].

Angle " $\alpha$ " is a fixed angle of 70 degrees. Choosing angle " $\alpha$ " less than 70 degrees, produces a coupler path that is more circular. This implies that the crank path is integrated dominantly to the rocker path. Choosing angle " $\alpha$ " greater than 90 degrees, likewise produces a narrower, more pendular coupler path.

The major diameter of the ovoid coupler path is 7.5 cm, while the minor diameter is 0.68 cm. The coupler path is symmetric with respect to its minor axis; therefore, the walking motion is uniform in both forward and reverse directions. The coupler path is, however, not symmetric with respect to its major axis.

This modified four bar mechanism creates a complex transmission path with one motor and one degree of freedom.

Other designs which utilized this path required two prismatic joints and two motors.

## PANTOGRAPH MECHANISM

The desired stride length was 15 cm. This distance was chosen so that the robot could safely maneuver amid small to medium sized obstacles such as rocks and trenches. This was also an appropriate constraint for insuring dynamic and static stability of the robot. Increasing stride length increases vibration and also decreases the robot body's region of stability.

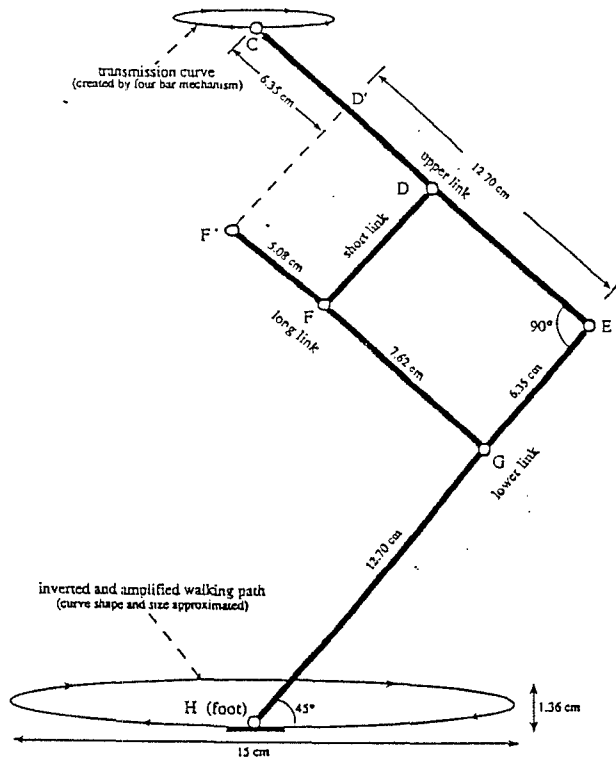


Figure 2. The Pantograph Mechanism

### Pantograph Operation

The 7.5 cm coupler path, produced by the four bar mechanism, requires an amplification factor of two in order to achieve the desired 15 cm stride length. The pantograph mechanism shown in figure 2 is defined by links "CDE" (upper link), "HGE" (lower link), "F'FG" (long link), and "DF" (short link). This mechanism acts as a mechanical amplifier; when the coupler-point is attached to point "C" in the orientation shown, the coupler path is translated, inverted, and magnified by a factor of two at point "H" (the foot). During normal walking the angle between the lower link and the ground is 45 degrees at the center of the stride. This maintains a horizontal walking path.

### Design Information

The pantograph mechanism was used to magnify the

coupler path to produce a stride length of 15 cm, and to provide a means of supporting the robot body and computer hardware. The combination of the four bar and pantograph mechanisms provides the first degree of freedom for this robotic leg.

Pantograph mechanisms act as mechanical amplifiers for kinematics as well as static forces, therefore, link materials and joint bearings were designed to withstand the amplified forces. According to kinematic laws, the force at point "F" (figure 2) is three times that of point "H", and the force at point "C" is two times that of point "H". The amplifications at points "C" and "F" are given by the following relations,

$$\text{Point "C" to point "H"} = (D'E)/(CD') = 2 \quad (3)$$

$$\text{Point "F" to point "H"} = (CE)/(CD') = 3 \quad (4)$$

The dimensions of each link in the pantograph mechanism were determined using similar kinematic relations. Point "F" is attached to a support structure via the leg lift mechanism, and is kept stationary during normal walking.

## LEG LIFT MECHANISM

The ovoid walking path required another degree of freedom in order to climb steep slopes and avoid obstacles. The second degree of freedom is achieved by a leg lift mechanism, capable of changing the leg height as well as the stride length. The leg lift mechanism is defined by the pinion gear and lifter gear-link attached to point "F" as shown in figure 3.

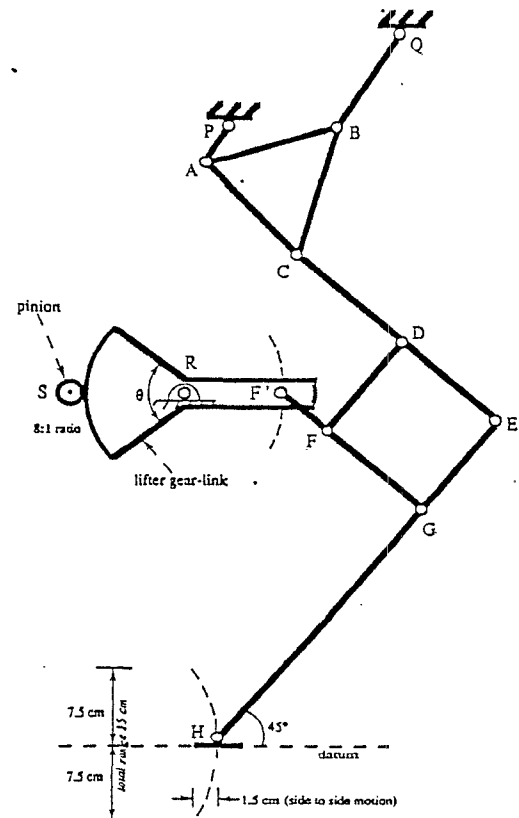


Figure 3. The Leg Lift Mechanism

### Lifter Operation

The lifter motor turns a worm gear combination which drives the lifter pinion. The lifter gear then rotates, causing the pantograph mechanism to compress or expand, depending on the direction of rotation. The leg extends and compresses 7.5 cm from the datum at the foot during normal walking. This results in a total lift range of 15 cm, sufficient to clear small to medium sized obstacles and maneuver within rough landscapes.

### Design Information

There is an 8 to 1 torque gain between the gear-link and pinion. This high gain was needed to effect the amplified forces at point "F" (figures 2 and 3). Modeling 22.41 N (50 lb) of force at the foot of the leg, results in 667.23 N (150 lb) of force at point "F". The 8 to 1 gear ratio, combined with a worm gear combination, allows the use of a small motor.

The lifter gear-link was designed with an angle " $\theta$ " of 45 degrees to produce the 15 cm lift range. The non-linear side to side motion of the foot during lifting was held within a 1.5 cm range. The non-linearity of the lifting motion was minimized to prevent the foot from moving into obstacles during lifting. Linear leg lift mechanisms were considered but were slow and cumbersome. Angle " $\theta$ " is given by the following relation (through several iterations),

$$\sin(\theta/2) = 5[1 - \cos(\theta/2)] \quad (5)$$

Equation (5) incorporates the 1.5 cm restriction on side to side foot motion.

## SUPPORT STRUCTURE

The crank-and-rocker, pantograph, and leg lift mechanisms are supported between two rectangular plates as shown in figure 4 (top plate removed). These plates provide the ground attachments for the crank and rocker links at points "P" and "Q", and for the lifter mechanism at points "R" and "S". The plates also provide a convenient means for mounting the entire leg assembly to the robot body, and protect the leg links from external objects which could damage or bind the moving links during operation.

The motor and gearbox combinations of the lifter and four bar mechanisms are mounted outside the plates to avoid mechanical interference. Motors and gearboxes can be mounted on either side of the two plates, depending on their orientation on the robot body. Three legs have a right hand orientation, and the remaining three have a left hand orientation for this design.

The two support plates are rigidly connected by four support columns that are bolted together between the plates. The complete leg assembly weighs 5.58 kg (12.3 lbs). This includes motors and gearboxes.

## MATHEMATICAL MODELING AND ENGINEERING ANALYSIS

Dynamic analysis using DADS [8] computer software provided insight into link forces, torques, displacements,

velocities, and accelerations during normal walking and climbing. These data were used to design against stress failure and to select appropriate motors and bearings for the legs. This analysis also prompted an idea to connect a torsion spring to point "Q" on the rocker link. This modification is shown to reduce the required motor torque by approximately 40%. The result is a smoother torque vs. time curve (smoother walking motion) and a reduction in motor weight and size.

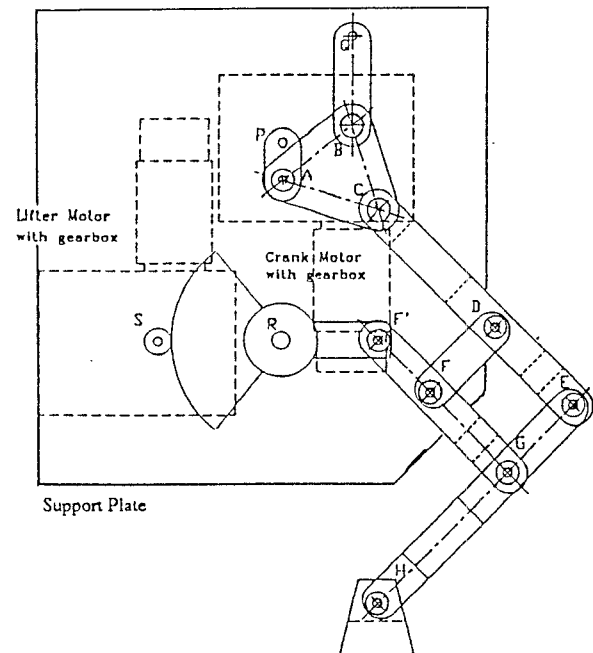


Figure 4. The Complete Leg Assembly With Top Support Plate Removed.

The rotational spring constant " $k$ " (N-m/deg) was determined by trial and error with the DADS software. Graphed functions of torque vs. time were analyzed during normal walking, and different " $k$ " values were input until the torque was evenly distributed through the crank cycle. The design value of " $k$ " was found to be 0.116 N-m/deg (1.02 lb-in/deg).

Figure 5 shows torque curves through one crank cycle (at 60 rev/min). Curve "A" shows the torque curve for the crank with 177.92 N (40 lbs) of force at the foot of the leg. A force of 177.92 N (40 lbs) was used as an approximation for the foot force during normal walking. Under these conditions, the maximum crank torque is approximately 3.95 N-m (35 lb-in). Curve "B" in figure 5 shows the torque curve of the crank with 177.92 N (40 lbs) of force at the foot and a torsion spring connected to the rocker link. The addition of the torsion spring decreased the crank torque and produced a more evenly distributed torque curve. The maximum value of the crank torque is 2.37 N-m (21 lb-in) when a torsion spring is used.

The crank torque was also analyzed during stair climbing. In this case, a force of 226.89 N (60 lbs) was assumed at the foot since more force exists on the foot when climbing an inclined surface. Curve "B" shows the torque curve of the crank for 226.89 N (60 lbs) of force on the foot with a torsion spring on the rocker link. Curve "A" in figure 6 is the same as curve

"A" in figure 5 and is used for comparison. Comparison of Curves "A" and "B" (figure 6) shows the advantage of using the torsion spring. This analysis shows that approximately the same crank torque is required for normal walking (40 lbs at the foot) without the torsion spring, as for climbing stairs (60 lbs at the foot) with the spring.

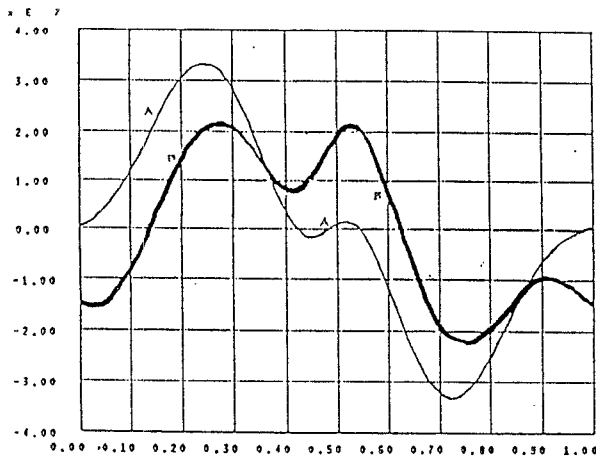


Figure 5. Torque vs. Time Curves for the Crank (units in cgs system). Curve A: 177.92 N of force at foot with no torsion spring. Curve B: 177.92 N of force at foot with a torsion spring.

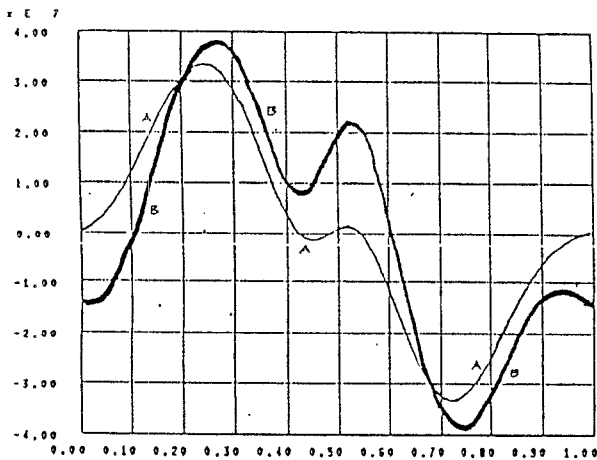


Figure 6. Torque vs. Time Curves for the Crank (units in cgs system). Curve A: 177.92 N of force at foot with no torsion spring. Curve B: 226.89 N of force at foot with a torsion spring.

The kinematics of the walking path was also analyzed. Though the walking path is symmetric about the minor axis, it is not symmetric about the major axis. Therefore, a 180 degree rotation of the crank does not bring the foot from its furthest forward position to its furthest back position. Since the transition of weight occurs at these points, it is necessary to know the transformation from motor position to foot position. Using MATLAB software, an easily computable approximation to this transform was obtained. Figure 7 shows the kinematic paths of the coupler, pantograph "knee section", and the foot during normal walking. Note the

inverted and amplified coupler path "C" at the foot "H", and the symmetry of curves "C" and "H".

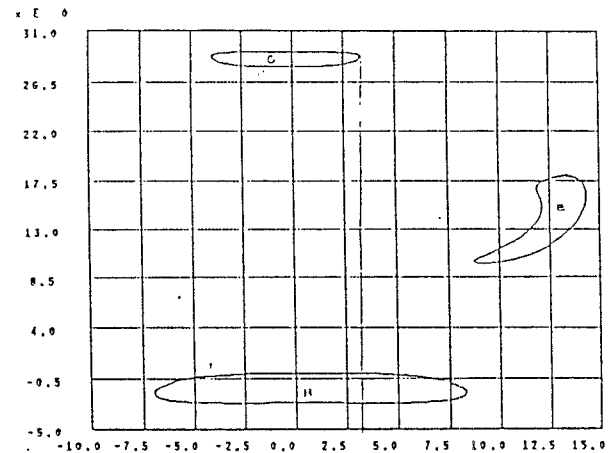


Figure 7. Coupler and Pantograph Kinematic Curves for Normal Walking.

## GEARBOXES

Each leg assembly harnesses a crank motor and gearbox and a lifter motor and gearbox. Both gearboxes use worm gear combinations to boost the motor torque to the required level.

### Crank Gearbox

The crank gearbox consists of a worm and a worm gear. The ratio between the gears is 35 to 1. The high speed, low torque motors drive the worm gear set producing a running torque of 6.63 N-m (58.7 lb-in) at the crank shaft. A gear mesh efficiency of 60% is included in this torque value. The crank shaft rotates 60 rev/min at its running torque.

A high ratio worm gear combination was used since it is non-backdrivable and translates power at right angles [9]. The vertical position of the motors (figure 4) required right angle transmission of this nature.

### Lifter Gearbox

The lifter gearbox is configured identically to the crank gearbox, however the gear ratio is 60 to 1. This, coupled with the 8 to 1 gear ratio between the lifter pinion and gear-link, gives an overall amplification of 480 to 1. A larger gear ratio was required for the lifter mechanism since the forces at the foot of the leg are amplified by a factor of three.

The running output torque of the lifter gearbox is 79.48 N-m (703.5 lb-in), accounting for 54% gear efficiency. This value is well beyond the needed value of 58.19 N-m (515 lb-in). The lifter pinion rotates at a speed of 4 rev/min at its running torque.

## MOTORS

Two identical 24 volt motors drive the crank and lifter, respectively. Each motor is powered using pulse width modulation for maximum power efficiency. Table 1 shows the motor specifications. The calculations for running torque were based on a 12 volt armature winding of the same model motor.

Table 1. Motor Specifications

Rated Voltage (V)	24
Rated Current (A)	13.8
Rated Speed (rpm)	3820
Stall Torque (N-m)	0.75

## SUMMARY

This leg design combines a four bar mechanism with a pantograph and leg lift mechanism. The leg assembly operates with two degrees of freedom, providing great flexibility. Structural integrity was assured through computer engineering analysis. Each leg assembly is an independent unit, designed to be computer controlled. These assemblies are compatible with many body, hardware, and software designs.

## ACKNOWLEDGEMENTS

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