

ABSTRACT

Title of Document: ESSAYS ON SELF-EMPLOYMENT AND
ENTREPRENEURSHIP.

Alejandro Gabriel Rasteletti, Ph.D., 2009.

Directed By: Prof. John Haltiwanger and Prof. John Shea,
Department of Economics

This dissertation consists of three chapters studying different issues related to self-employment and entrepreneurship. The first chapter studies the effects of labor market frictions and credit constraints in an economy with self-employment. Two types of self-employed workers emerge in the model: (i) entrepreneurs and (ii) workers using self-employment as a stopgap. I show that labor market frictions generate a motive not to transition into self-employment, by making self-employment a choice that takes time to reverse. At the aggregate level, these frictions also reduce the average size of entrepreneurs' businesses. Meanwhile, even if credit constraints are of particular importance for entrepreneurs, they also affect the stopgap self-employed. When credit constraints are tighter, fewer vacancies are posted, which increases the number of workers using self-employment as a stopgap in equilibrium.

In the second chapter, I use data from the PSID to study the characteristics of workers using self-employment as a stopgap while searching for another job, vis-à-vis those of other self-employed workers. The data reveals that stopgap self-employment is relatively

high among young workers and those who experienced unemployment. Furthermore, the probability of entering self-employment increases monotonically with wealth for those not using self-employment as a stopgap, while it has an inverted U shape for those using self-employment as a stopgap. I also find that being unemployed increases the probability of becoming stopgap self-employed, but has no effect on the probability of becoming self-employed for other reasons.

The third chapter examines the impact of exogenous technological growth on entrepreneurship and unemployment. The model developed in that chapter predicts that in the absence of labor market frictions, technological growth has an effect on entrepreneurship if and only if it affects an entrepreneur's capacity to manage workers. When labor market frictions are present, technological growth may have a positive or negative impact on entrepreneurship and unemployment. The desirable outcome of an increase in the rate of technological growth enhancing entrepreneurship and dampening unemployment is more likely to be obtained when the interest rate does not increase significantly with growth, technological change is disembodied, and growth enhances entrepreneurial ability at managing workers.

ESSAYS ON SELF-EMPLOYMENT AND ENTREPRENEURSHIP.

By

Alejandro Gabriel Rasteletti

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Ph.D.
2009

Advisory Committee:
Professor John Haltiwanger, Co-Chair
Professor John Shea, Co-Chair
Professor Boragan Aruoba
Professor Pablo D'Erasmus
Professor Katharine Abraham

© Copyright by
Alejandro Gabriel Rasteletti
Ph.D.

Acknowledgements

This thesis has benefited greatly from the advice and support of my advisors John Haltiwanger and John Shea. They have both given me plenty of freedom to explore ideas, while at the same time giving me directions that have helped me getting a better sense of the ideas I was exploring. Without their help and support the quality of this dissertation would have not been the same. I also want to further thank professor Shea for the excellent editorial comments he provided while I was writing this dissertation.

I would also like to thank Prof. Boragan Aruoba. Besides giving very good feedback on my research, he also helped me solving some problems I faced while writing the codes for solving numerically the model on chapter 1 of this dissertation.

A special thank to the Department of Economics at the University of Maryland for the financial support given throughout the program, and for holding the Econ709 brownbag seminars. The comments received in the presentations of my work helped my research move forward.

My work has also benefited from discussions held with researchers at the International Labor Organization in Geneva, where I spent a summer as a research intern. I would like to thank Duncan Campbell for giving me this incredible research internship opportunity.

Finally, I would like to thanks my co-authors José Plehn-Dujowich and Dunli Li. I have learnt a lot from discussing ideas with them while writing the third chapter of this

dissertation.

Table of Contents

<u>Acknowledgements</u>	ii
<u>Table of Contents</u>	iv
<u>List of Tables</u>	v
<u>List of Figures</u>	vi
<u>Chapter 1: Self-Employment, Labor Market Frictions and Credit Constraints</u>	1
<u>Introduction</u>	1
<u>Model</u>	6
<u>Workers' Problem</u>	7
<u>Corporations' Problem</u>	15
<u>Equilibrium</u>	17
<u>Characterization of Optimal Behavior</u>	21
<u>Transitions into Self-Employment</u>	21
<u>Transitions out of Self-Employment</u>	23
<u>Wealth Accumulation</u>	26
<u>General Equilibrium Implications</u>	27
<u>Numerical Solution</u>	29
<u>Calibration</u>	29
<u>Worker's Decision Rules</u>	33
<u>General Equilibrium Results</u>	39
<u>Conclusions</u>	51
<u>Chapter 2: Stopgap Self-Employment: Evidence from the PSID</u>	53
<u>Introduction</u>	53
<u>Data and Descriptive Statistics</u>	56
<u>Data Description</u>	56
<u>Definition of Self-Employment</u>	57
<u>Descriptive Statistics of Self-Employed Workers</u>	58
<u>Stopgap and Other Self-Employed Workers</u>	59
<u>Transitions into Self-Employment</u>	66
<u>Transitions out of Self-Employment</u>	72
<u>Conclusions</u>	75
<u>Chapter 3: A Theory on the Impact of Technological Change on Entrepreneurship and Unemployment</u>	77
<u>Introduction</u>	77
<u>Overview of the Model</u>	83
<u>Frictionless Labor Markets</u>	96
<u>Disembodied Technological Change</u>	96
<u>Embodied Technological Change</u>	99
<u>Labor Market Frictions</u>	102
<u>Disembodied Technological Change</u>	103
<u>Embodied Technological Change</u>	115
<u>Conclusions</u>	123
<u>Appendices</u>	125
<u>Bibliography</u>	172

List of Tables

<u>Table 1.1:</u> <u>Parameter Values</u>	31
<u>Table 1.2:</u> <u>Data and Model Moments</u>	33
<u>Table 1.3:</u> <u>Transitions into Self-Employment by Unmatched Workers without Business Projects: Wealth Thresholds</u>	35
<u>Table 1.4:</u> <u>Transitions into Self-Employment by Workers With Business Projects</u>	37
<u>Table 1.5:</u> <u>Proportions of Workers using Self-Employment as a Stopgap</u>	41
<u>Table 1.6:</u> <u>Aggregate Wealth</u>	41
<u>Table 1.7:</u> <u>Wages and Interest Rates</u>	42
<u>Table 1.8:</u> <u>Aggregate, Corporate and Self-Employed Outputs</u>	43
<u>Table 1.9:</u> <u>Labor Force Participation</u>	44
<u>Table 1.10:</u> <u>Unemployment: Rates and Average Durations</u>	46
<u>Table 3.1:</u> <u>The Mechanisms by which Technological Growth Affects Entrepreneurship</u>	95
<u>Table 3.2:</u> <u>The Mechanisms by which Technological Growth Affects Unemployment</u>	96
<u>Table A.1:</u> <u>Self-Employment Rates for Different Groups of Workers</u>	136
<u>Table A.2:</u> <u>Descriptive Statistics</u>	137
<u>Table A.3:</u> <u>Probability of Leaving Self-Employment By Next Year</u>	138
<u>Table A.4:</u> <u>Overlap in Definitions of Stopgap Self-Employment</u>	139
<u>Table A.5:</u> <u>Proportion of Self-Employed Workers Using Self-Employment as a Stopgap</u>	140
<u>Table A.6:</u> <u>Descriptive Statistics for the Self-Employed</u>	141
<u>Table A.7:</u> <u>Proportions of Self-Employed Workers in Different Occupations</u>	142
<u>Table A.8:</u> <u>Proportions of Self-Employed Workers in Different Industries</u>	143
<u>Table A.9:</u> <u>Probability of Becoming Self-Employed</u>	144
<u>Table A.10:</u> <u>Effect of Local Unemployment Rates on the Probability of Becoming Self-Employed</u>	145
<u>Table A.11:</u> <u>Marginal Effect of Wealth on the Probability of Becoming Self-Employed</u>	146
<u>Table A.12:</u> <u>Effect of Wealth on the Probability of Becoming Self-Employed for Workers Who Experience and Did not Experience Unemployment</u>	147
<u>Table A.13:</u> <u>Probability of Becoming Self-Employed – Instrumental Variable Approach</u>	148
<u>Table A.14:</u> <u>Proportion of Self-Employed Workers Leaving Self-Employment By Next Year</u>	150
<u>Table A.15:</u> <u>Probability of Leaving Self-Employment By Next Year</u>	151
<u>Table A.16:</u> <u>Probability of Leaving Self-Employment By Next Year Conditional on Type of Self-Employment</u>	152
<u>Table A.17:</u> <u>Probability of Transitioning To Employment, Unemployed or Out of the Labor Force By Next Year</u>	153

List of Figures

<u>Figure 1.1: Unmatched Worker's Problem</u>	11
<u>Figure 1.2: Matched Worker's Problem</u>	15
<u>Figure 1.3: Wealth Distribution - PSID</u>	48
<u>Figure 1.4: Wealth Distribution - Model</u>	49
<u>Figure 1.5: Spike Close-Up - Model</u>	49
<u>Figure 1.6: Rates – Matched Workers without a Business Project</u>	50
<u>Figure 3.1: The Entrepreneurship and Job Creation Curves when the Labor Market is Frictionless</u>	88
<u>Figure 3.2: The Entrepreneurship and Job Creation Curves with Labor Market Frictions</u>	89
<u>Figure 3.3: The Impact of Technological Growth on Entrepreneurship when only the Capitalization Effect is Present and $R'(g) > 1$</u>	91
<u>Figure 3.4: The Impact of Technological Growth on Entrepreneurship when only the Firm Size Effect is Present and $A'(g) > 0$</u>	92
<u>Figure 3.5: The Impact of Technological Growth on Entrepreneurship when only the Employment Duration Effect is Present</u>	93
<u>Figure 3.6: The Impact of Technological Growth on Unemployment</u>	94
<u>Figure 3.7: The Impact of Technological Growth on Entrepreneurship when only the Firm Size Effect is Present and $A'(g) > 0$</u>	111
<u>Figure A.1: Distributions Over Self-Employment Duration</u>	154
<u>Figure A.2: Probability of Becoming Self-Employed As a Function of Wealth</u>	155
<u>Figure A.3: Probability of Becoming Self-Employed As a Function of Wealth and Unemployment</u>	157
<u>Figure A.4: Probability of Becoming Stopgap Self-Employed As a Function of Wealth - Instrumental Variable Approach</u>	159

Chapter 1: Self-Employment, Labor Market Frictions and Credit Constraints

1. Introduction

This dissertation chapter studies the effects of labor market frictions and credit constraints in a general equilibrium setting where self-employment is one of the possible economic activities. Allowing for self-employment in a general equilibrium model is important for several reasons. First, a non-trivial fraction of workers in the U.S. economy are self-employed. According to estimates by the U.S. Census Bureau, over 14 million workers were self-employed in 2008, accounting for about 10% of total employment. Second, a large fraction of the U.S. net wealth is in the hands of self-employed workers. Using the Survey of Consumer Finances, Cagetti and De Nardi (2006) estimate that self-employed workers in US economy (11.1% of their sample) hold about 39% of the U.S. economy's total net worth. Self-employed business owners (7.6% of their sample) account for 33% of the total wealth. For these and other reasons, it is important to understand how the behavior of self-employed workers is affected by the economic environment they operate in.

This dissertation chapter focuses on the effects of credit constraints and labor market frictions on the behavior of self-employed workers as well as on the aggregate implications of these frictions. The reason for focusing on these two frictions is that the empirical literature has often highlighted them as having important effects on the behavior of both the self-employed and the self-employed-to-be. With respect to credit

frictions, Cagetti and De Nardi (2006) report that 18% of self-employed business owners report having been turned down for credit and 29% of them use their own personal assets as collateral to finance their business. Several studies have also showed that credit constraints affect workers' decisions to become entrepreneurs.¹ With respect to labor market conditions, Evans and Leighton (1989) find that being unemployed increases the probability of becoming self-employed, while Blanchflower and Oswald (1991) find that higher regional unemployment rates also increase the probability of entering self-employment.

The emphasis given in the empirical literature to the effects of credit and labor market frictions on self-employment suggests that there are two types of self-employment. For some workers self-employment is an entrepreneurial activity. For others, self-employment is a stopgap. By introducing labor market frictions, the model developed in this dissertation chapter can generate both entrepreneurial and stopgap self-employment, which allows one to study how labor market frictions and credit constraints affect each type of self-employed worker. The model can also explain some of the observed differences in behavior between entrepreneurs and workers using self-employment as a stopgap documented in Rasteletti (2009b). These differences include: 1) workers using self-employment as a stopgap have very short self-employment spells; while 60% of new stopgap self-employed end their self-employment within the first year, only 23% of new entrepreneurs do so. 2) After controlling for workers' characteristics, labor income is lower for stopgap self-employed workers. Rasteletti (2009b) also finds that workers'

¹ Holtz-Eakin et al. (1994), Blanchflower and Oswald (1998) and Evans and Jovanovic (1989) find evidence of a positive correlation between wealth and entry into self-employment. More recently, Hurst and Lusardi (2004) find this positive relation is present only among the very rich.

wealth and individual labor market histories play an important role in explaining differences in the probabilities of workers transitioning into self-employment. While having more wealth increases the probability of becoming an entrepreneur, it reduces the probability of becoming self-employed as a stopgap. Rasteletti (2009b) also finds that being unemployed increases the probability of becoming stopgap self-employed, while having no significant effect on the probability of becoming an entrepreneur.

This dissertation chapter relates to the theoretical literature studying the effects of borrowing constraints on entrepreneurs.² The papers in this literature do not include labor market frictions, which leaves them unable to study workers using self-employment as a stopgap or the effects of borrowing constraints on this group of workers. These models are also unable to study how labor market frictions affect the behavior of entrepreneurs. My work on this chapter also relates to the literature studying the effect of labor market frictions on workers' behavior. This chapter differs from most of these models by allowing workers to become self-employed and search for a job while self-employed.³ By combining self-employment, credit constraints and labor market frictions, the model developed in this dissertation chapter can answer several questions that have not been addressed by either literature. Some of these questions are: How do labor market frictions affect workers' decision to transition into and out of self-employment in the presence of credit constraints? Does the interaction of labor market frictions and credit constraints on

² The most widely cited papers in this literature are Cagetti and De Nardi (2006), Quadrini (2000) and Buera (2006).

³ To my knowledge, there has been only one paper (Rissman, 2003) studying the effect of labor market frictions on self-employed workers' behavior. She examines a partial equilibrium model based on Mortensen (1970), where workers' only decision is whether to search for a job while unemployed or self-employed. Her model is relevant to studying transitions into self-employment by poor workers, but it is not suited for studying interactions between labor market frictions, credit constraints and self-employment given that workers cannot accumulate assets in her model.

entrepreneurs generate changes in workers' saving behavior? If so, what are the effects on capital supply and interest rates? And do these interactions have important aggregate implications?

The model developed in this dissertation chapter unveils some rich interactions between labor market frictions, credit constraints and self-employment that have implications not only for worker's decision rules but also for some key economic aggregates. One important effect of these interactions is on the decision rule for transitions into self-employment. Labor market frictions generate a motive not to transition into self-employment, which I call fear of failure. In the presence of labor market frictions, the worker realizes that if his business fails in the future, he will have to spend time searching for a job, which implies a cost in forgone income. Forward looking workers take this future cost into account at the time of making their decision on whether to become an entrepreneur. As found in other theoretical papers, the presence of credit constraints generates policy rules for transitions into entrepreneurship that are characterized by a wealth threshold property. That is, workers become entrepreneurs only if their financial wealth is high enough. The fear of failure motive increases the levels of these wealth thresholds, which might lead to a reduction in the proportion of entrepreneurs in the economy if labor market frictions are severe enough.

Two general equilibrium findings are worth highlighting. First, tighter credit constraints on entrepreneurs increase the proportion of workers using self-employment as a stopgap and the duration of their self-employment spells. When credit constraints on entrepreneurs are tightened, the entrepreneur's lower access to credit results in a reduction of both the output of their businesses and their income levels. Entrepreneurs then decide

to cut their savings and accumulate less wealth. As a consequence, the equilibrium aggregate wealth is lower and the interest rate is higher. This increase in the interest rate reduces the profits of firms in the corporate sector, who now decide to post fewer vacancies. Having fewer vacancies makes exiting self-employment more difficult and leads to an increase in the proportion of workers using self-employment as a stopgap and an increase in the duration of the self-employment spell.

The other important general equilibrium finding is that more severe labor market frictions both increase the relative size of the self-employed sector and reduce the average productivity of self-employed businesses. The average productivity decreases for two reasons. First, more workers use self-employment as a stopgap. In general, these workers are less productive than entrepreneurs, so the increase in their number reduces the average productivity of self-employed workers. Second, average productivity of the businesses of entrepreneurs also decreases. This decrease is mainly due to a lower access to credit by entrepreneurs, which originates from their lower capital holdings. Entrepreneurs hold lower levels of wealth because more severe labor market frictions reduce the equilibrium interest rate. This reduction in the interest rate originates both from a shift to the right of the supply of capital (workers save more out of precaution) and from a shift to the left of the demand for capital (the lower number of firm-worker matches reduces the demand for capital). The main (dominant) effect is the capital demand from firms, which reduces the interest rate and saving. This is offset somewhat by the increased precautionary saving of workers, which increases saving and further reduces interest rates. Overall, the reduction in saving is driven by the dominant effect of lower capital demand. The lower wealth holdings in steady state result in workers

entering entrepreneurship with lower levels of wealth. This reduces their access to credit and therefore the average productivity of new entrepreneurs. The lower interest rate also leads existing entrepreneurs to save less, and therefore the businesses of entrepreneurs grow at a slower rate. The lower initial productivity of new entrepreneurs and the lower growth rate of existing entrepreneurs explains the decrease in the entrepreneurs' productivity.

The rest of the chapter is organized as follows: In the next section, I develop a general equilibrium model that is used to analyze the effects of credit constraints and labor market frictions on worker behavior as well as the aggregate implications of these frictions. The model is a directed search model in which workers have a career choice to make, face labor market frictions if they decide to search for a job at a firm, and also face credit constraints if self-employed. In section 3, I analyze some of the implications of the model and characterize certain features of the equilibrium. In section 4, I calibrate the model to study numerically the effects of labor market frictions and credit constraints on worker behavior and on some key aggregates. Conclusions are presented in section 5.

2. *Model*

In this section, I develop a two sector general equilibrium model to study the effects of labor market frictions and credit constraints in an economy where self-employment is one of the possible economic activities. The economy consists of a continuum of workers, with measure one, and a continuum of potential corporations, with measure $m \gg 1$. Production in the economy takes place either in the corporate sector or in the non-corporate sector, with the non-corporate sector being comprised of the businesses of the

self-employed workers. Firms in the different sectors differ both in the technology used for production as well as in their access to the capital market. While corporations have perfect access to the capital market, self-employed workers only have partial access. Workers and firms in the corporate sector come together via search. Firms willing to hire a worker need to post a vacancy and workers that decide to search for a job have to apply to one of the vacancies posted by firms. This search process is similar to that in Acemoglu and Shimer (1999). The problems faced by workers and firms in the corporate sector are described below.

2.1. Workers' Problem

Agents make choices in order to maximize their expected discounted value of lifetime utility

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to an intertemporal budget constraint, a no-borrowing constraint, a wealth allocation constraint, a job search technology and a production technology. All these constraints are specified below. In expression (1), E_0 is the expectation operator as of time 0, β is the time discount factor, c_t and l_t represent worker's consumption and hours worked at time t , and $u(\cdot)$ is a strictly increasing and strictly concave utility function for all positive levels of consumption. If $c_t \leq 0$, $u(c_t, l_t) = -\infty$. The utility function also satisfies the Inada Conditions, that is $\lim_{c_t \rightarrow 0} \frac{\partial u(c_t, l_t)}{\partial c_t} = \infty$, and $\lim_{c_t \rightarrow \infty} \frac{\partial u(c_t, l_t)}{\partial c_t} = 0$.

Workers can either work full time ($l_t = 1$) or not work at all ($l_t = 0$). Leisure increases

utility, which implies that $u(c_t, 0) > u(c_t, 1)$ for all $c_t > 0$.

The decisions workers need to make at any point in time depend on whether they are currently working for a firm in the corporate sector or not. I name the former matched workers, and the latter unmatched workers. Unmatched workers need to search for a job in order to become matched. Matched workers can become unmatched either endogenously by deciding to quit their job, or through exogenous separation. Besides being matched to a firm or unmatched, workers can possess a business project or not. I use the variable e_t to capture whether the agent has a business project ($e_t = 1$) or not ($e_t = 0$). I assume that the arrival of a business project is a stochastic process. Losses of business projects can happen stochastically or can be a consequence of agents' decisions. Endogenous losses of business projects happen when an unmatched worker with a business project decides to accept a job at a firm, or when a self-employed worker with a business project decides not to work.

2.1.1. Unmatched Workers

Workers that are unmatched in a given period have to decide whether to be self-employed and whether to search for a job at a firm in the corporate sector. If an unmatched worker is not self-employed, he has no labor income in the current period, but he enjoys leisure time. If a worker decides to be self-employed in the current period, he has positive labor income, but he does not enjoy leisure. In what follows, I label unmatched workers who are not self-employed but who search for a job as "unemployed", while unmatched workers who are not self-employed and who do not search are labeled "not in the labor force". Workers who are self-employed may search

for a job at a firm. I label workers who are self-employed and not searching for a job "entrepreneurs" and those workers who are self-employed and searching for a job "stopgap self-employed".

If an unmatched worker decides to search for a job at a firm, he also needs to decide which of the posted vacancies to target. Vacancies differ only on wages paid, and wages remain constant for the whole duration of the worker-firm match. Applying to a vacancy has no cost for workers, but requires a commitment to accept the job and work for the firm for at least one period, in case the job is offered to them. For this reason, workers can only apply to one vacancy at a time. I name $\omega \in \Omega_t$ the particular vacancy the agent targets, where Ω_t is the set of all vacancies posted at time t , which is public information.

Given the existence of a continuum of workers, workers cannot coordinate their applications to vacancies and at a point in time several workers can apply to the same vacancy. Firms cannot differentiate among workers and choose randomly among applicants when they get more than one. Therefore, there is no guarantee that the worker will be able to get his target job. The probability of getting the job depends on the number of applicants to that same vacancy. I name the number of workers applying to a given vacancy queue length, and represent it as $q(\omega) \in [0, \infty]$. The probability of getting a particular job is then a function of its queue. I call this probability $\chi[q(\omega)]$. Workers do not know queue lengths before applying to a vacancy and therefore need to form expectations about them. I call these expectations $q^e(\omega)$. Given that $q(\omega)$ is unknown, the probability of getting particular jobs is also unknown to workers.

If an unmatched worker decides to be self-employed in the current period, he has

access to a production technology that is captured in a production function $g(k_t, e_t)$ where k_t is the amount of physical capital used in production. Capital depreciates at a rate δ per period. The amount of output a self-employed worker can produce also depends on whether the agent has a business project or not. For self-employed workers without a business project ($e_t = 0$), output is independent of physical capital. That is, $g(k_t, 0) = b > 0$. For self-employed agents with a business project ($e_t = 1$), output is a strictly increasing function of k_t . Self-employed workers with a business project can produce at least as much as self-employed workers without a business project, that is $g(k_t, 1) \geq g(k_t, 0) = b$. Furthermore, this inequality is strict provided that $k_t > 0$. Finally, the production function is strictly increasing and strictly concave in capital, with $\lim_{k_t \rightarrow 0} \frac{\partial g(k_t, 1)}{\partial k_t} = \infty$ and $\lim_{k_t \rightarrow \infty} \frac{\partial g(k_t, 1)}{\partial k_t} = 0$.

Self-employed workers also need to decide how much physical capital to use in their businesses. They can either use their own capital for production or they can borrow it in the financial market at a price r per period. Self-employed workers face a constraint on how much capital they can borrow, with the amount that can be borrowed being proportional to the worker's financial wealth. Workers can also lend all or part of their wealth in the financial market at the real interest rate r .

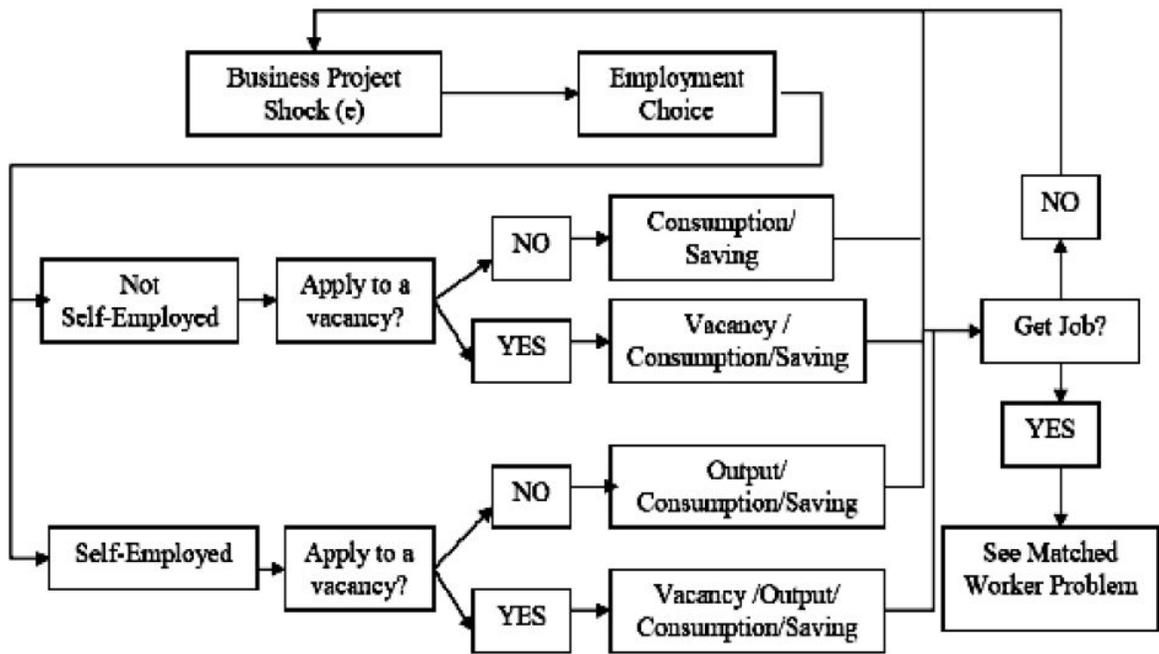
Finally, all unmatched workers decide how much to consume and how much to save in every period. Given consumption and production decisions, financial wealth evolves as follows: $a_{t+1} = g(k_t, e_t) - (r + \delta)k_t + (1 + r)(a_t - c_t)$.⁴ I also assume that workers

⁴ Note that for all unmatched workers without a business project the optimal $k_t=0$. This implies that for self-employed workers without a business project financial wealth evolution is given by $a_{t+1}=(1+r)[a_t-c_t]+b$, and for those workers unemployed or out of the labor force, financial wealth

must hold non-negative wealth at every point in time. That is, $a_t \geq 0$, for all t .

The timing of events and decisions made during the period is as follows: at the beginning of the period the agent receives a business project shock. Given all previous information and the realization of the business project, the unmatched worker has to decide whether to be self-employed and whether to apply for a job at a firm or not. If he decides to apply for a job, he has to decide which vacancy to target. Next, the worker decides how much to consume and, if self-employed, how much to produce. At the end of the period, if the agent is matched to a job, he becomes a worker. If not, he remains unmatched. Figure 1.1 below summarizes the timing of events.

FIGURE 1.1: Unmatched Worker's Problem.



Given the timing of events and the assumptions made above, the value of being an

evolution is given by $a_{t+1}=(1+r)[a_t-c_t]$.

unmatched worker $U(a_t, e_t)$ equals the maximum of the value of being out of the labor force $N^{NS}(a_t, e_t)$, the value of being unemployed $N^S(a_t, e_t)$, the value of being self-employed and searching for a job $S^S(a_t, e_t)$ and the value of being self-employed and not searching for a job $S^{NS}(a_t, e_t)$. That is to say,

$$(2) \quad U(a_t, e_t) = \max(N^S(a_t, e_t), N^{NS}(a_t, e_t), S^S(a_t, e_t), S^{NS}(a_t, e_t))$$

The value of being out of the labor force is the solution to the following functional equation

$$(3) \quad N^{NS}(a_t, e_t) = \max_{c_t} \{u(c_t, 0) + \beta E_t[U(a_{t+1}, e_{t+1})]\}$$

subject to

$$a_{t+1} = (1 + r)[a_t - c_t]$$

$$a_t \geq 0$$

The value of being unemployed is the solution to the following functional equation

$$(4) \quad N^S(a_t, e_t) = \max_{c_t, w_{t+1} \in \Omega_t} \left\{ \begin{array}{l} u(c_t, 0) + \beta(1 - \chi[q^e(\omega)])E_t[U(a_{t+1}, e_{t+1})] \\ + \beta\chi[q^e(\omega)]E_t[W(w_{t+1}, a_{t+1}, 0)] \end{array} \right\}$$

subject to the same constraints as in (3). $W(w_{t+1}, a_{t+1}, 0)$ is the value of being matched to a firm and being paid a wage w_{t+1} . One important feature to notice in my definition of the value function for $N^S(\cdot)$ is that the value of being matched to a firm is $W(w_{t+1}, a_{t+1}, 0)$ and not $W(w_{t+1}, a_{t+1}, e_{t+1})$. This comes from my assumption that unemployed and self-employed agents lose their business projects when they accept a job

at a firm.

Similarly, the value of being self-employed and searching for a job is the solution to the following functional equation:

$$(5) \quad S^S(a_t, e_t) = \max_{c_t, k_t, w_{t+1}^N \in \Omega_t} \left\{ u(c_t, 1) + \beta(1 - \chi[q^e(\omega)])E_t[U(a_{t+1}, e_{t+1})] \right. \\ \left. + \beta\chi[q^e(\omega)]E_t[W(w_{t+1}, a_{t+1}, 0)] \right\}$$

subject to

$$a_{t+1} = g(k_t, e_t) - (r + \delta)k_t + (1 + r)(a_t - c_t)$$

$$k_t \leq \lambda a_t$$

$$a_{t+1} \geq 0$$

where $\lambda \geq 1$ is a parameter capturing how much self-employed workers with a business project can borrow to rent physical capital.⁵

The value of being self-employed and not searching for a job is the solution to the functional equation

$$(6) \quad S^{NS}(a_t, e_t) = \max_{c_t, k_t} \{u(c_t, 1) + \beta E_t[U(a_{t+1}, e_{t+1})]\}$$

subject to the same constraints as in (5).

Given the value functions described above, an unmatched worker's optimal decision on whether to be out of the labor force, unemployed, stopgap self-employed, or an entrepreneur is given by

$$d^u(a_t, e_t) = \begin{cases} \text{Be out of the labor force if } U(a_t, e_t) = N^{NS}(a_t, e_t). \\ \text{Be unemployed if } U(a_t, e_t) = N^S(a_t, e_t). \\ \text{Be stopgap self – employed if } U(a_t, e_t) = S^S(a_t, e_t). \\ \text{Be an entrepreneur if } U(a_t, e_t) = S^{NS}(a_t, e_t). \end{cases}$$

⁵ The constraint $k_t \leq \lambda a_t$ is consistent with self-employed workers having to put up collateral in order to be able to borrow. While solving the model numerically in section 4, the benchmark value of λ is 1.44. I also analyze an extreme case where workers cannot borrow at all ($\lambda = 1$) and a case with a less stringent borrowing constraint ($\lambda = 1.8$).

The policy functions for consumption, capital utilization and wage targeting are given by $c^U(a, e)$, $k^U(a, e)$, $w^U(a, e)$ respectively, and are implied by the solutions to the value function that maximizes equation (2).⁶

2.1.2. Matched Workers

Workers that are matched to a firm cannot search on the job. Under this assumption, a matched worker has only two decisions. After observing his business project shock, a matched worker decides whether to quit in order to enter self-employment, unemployment or being out of the labor force. After this decision is made, a matched worker decides how much to consume. At the end of the period, workers and firms can be exogenously separated. Exogenous separations occur with probability s each period. The timing of events and decisions faced by matched workers is summarized in figure 1.2 below.

The value function for a matched worker who does not quit can be written as follows:

$$(7) \quad W(w, a_t, e_t) = \max_{c_t} \left\{ u(c_t, 1) + \beta(1 - s)E_t \max [W(w, a_{t+1}, e_{t+1}), U(a_{t+1}, e_{t+1})] \right. \\ \left. + \beta s E_t [U(a_{t+1}, e_{t+1})] \right\}$$

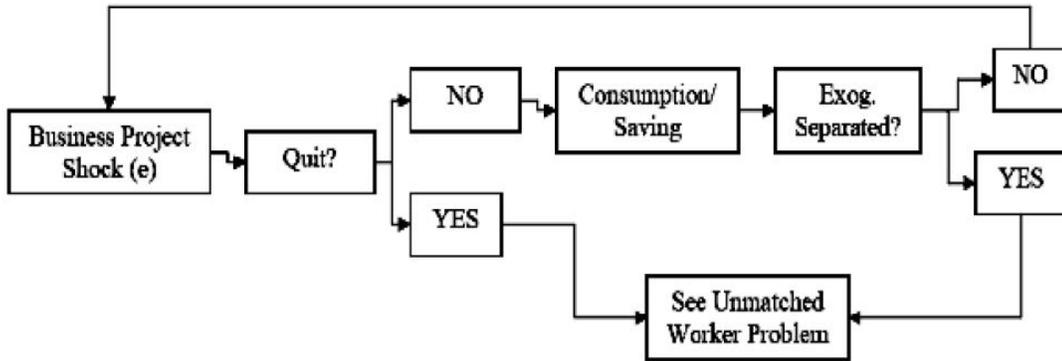
subject to

$$a_{t+1} = (1 + r)[a_t - c_t] + w$$

$$a_{t+1} \geq 0$$

⁶ If the maximum value in equation (2) corresponds to the value of being a self-employed worker not looking for a job, or to the value of being out of the labor force, $w^U(a, e) = \emptyset$. Similarly, if the worker optimally chooses unemployment or not being in the labor force, $k^U(a, e) = 0$.

FIGURE 1.2: Matched Worker's Problem.



The policy function for consumption for a matched worker that decides not to quit is represented by $c^M(w, a, e)$. His optimal decision on whether to remain matched to the firm or quit is given by

$$d^M(w_t, a_t, e_t) = \begin{cases} \text{Remain matched if } W(w_t, a_t, e_t) > U(a_t, e_t). \\ \text{Quit, otherwise.} \end{cases}$$

Having described the problems faced by matched and unmatched workers, I now describe the problem faced by firms in the corporate sector.

2.2. Corporations' Problem

Firms in the corporate sector can either be matched to a worker or unmatched. If a firm is unmatched and wants to produce, it must first hire a worker. To be able to hire a worker, unmatched firms need to post a vacancy, which has a cost c . Vacancies are posted with a wage, which the firm commits to pay for the duration of the match. Due to labor market frictions, there is no guarantee that posting a vacancy will result in the hiring of a worker. The probability of filling the vacancy depends on the number of applicants the vacancy attracts, that is, on the queue length $q(\omega)$. The probability of

filling a vacancy is given by $\eta[q(\omega)]$. I further assume that $\eta(0) = 0$, $\eta(\infty) = 1$ and $\frac{\partial \eta(\cdot)}{\partial q(\omega)} > 0$. Firms do not know ex-ante the length of the queue their vacancies will generate, so they form expectations about queues.

Once a firm finds a worker, it has access to a production technology $f(k_t)$, where k_t is the physical capital used in production. Firms can rent capital at a rental price per period of r , which is determined in equilibrium. The firm's production technology is increasing and concave in physical capital and satisfies Inada conditions.

The value of an unmatched firm, J^U , can be written as

$$(8) \quad J^U = \max\left(0, -c + \frac{1}{1+r} \max_w \{\eta[q^e(\omega)]J^M(w) + (1 - \eta[q^e(\omega)])J^U\}\right)$$

where $J^M(w)$ is the value of a newly matched firm paying a wage w .

Matched firms decide how much physical capital to rent each period and remain matched until the worker quits or the match is exogenously terminated. The value of a newly matched firm paying a wage w is

$$(9) \quad J^M(w) = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t [D_t(w)\pi(w) - (1 - D_t(w))J^U]$$

where $\pi(w) = \max_{k^F} \{f(k^F) - w - (r + \delta)k^F\}$ is the maximized per period profit of an active firm, and $D_t(w)$ is the probability that a match formed with wage w will still be alive after t periods. In turn, $D_t(w) \equiv (1 - s)^{t-1} (\int \omega^e(a_0) [\prod_{j=1}^t (1 - \phi_j^e(a_0))] da_0)$ where $\omega^e(a_0)$ represents the firm's beliefs on the distribution of applicants' wealth at the

time matched are formed and $\phi_j^e(a_0)$ captures the probability that a previously unmatched worker with initial assets a_0 will quit the match after j periods.⁷ $\phi_j^e(\cdot)$ is a function of the saving behavior of matched workers; the probability of getting a favorable business project shock; and the decision rule of matched households mapping current assets and business project state into the decision to quit or not quit.

Finally, I assume that there is free entry of firms. Firms enter the market until the point where no firm has an incentive to enter. That is, $J^U = 0$.

In what remains of this dissertation chapter, I limit my analysis to stationary rational expectations equilibria. This implies two things: first, the distribution of unmatched workers over asset holdings and business project shocks and the distribution of matched workers over asset holdings, wages and business project shocks are time invariant. I refer to these two distributions as $\Phi^U(a, e)$ and $\Phi^M(w, a, e)$, respectively. Secondly, agents' and firms' beliefs about queue lengths as well as firms' expectations of employment durations and wealth distribution of applicants are correct in equilibrium. To assure trembling hand perfection, I also require that beliefs about queue lengths and employment duration are correct along out-of-equilibrium paths.

2.3. Equilibrium

2.3.1. Definition of Equilibrium

A stationary rational expectations equilibrium consists of a set of value functions J^U ,

⁷ Note that since e_t is reset to zero when self-employed or unemployed workers take a job at a firm, the initial value of e is irrelevant to quit behavior, so the firm only needs to integrate over initial assets.

$J^M(w)$, $U(a, e)$ and $W(w, a, e)$; a set of decision rules $w^U(a, e)$, $k^U(a, e)$, $c^U(a, e)$, $d^U(a, e)$, $c^M(w, a, e)$, $d^M(w, a, e)$ and $k^F(w)$; a set of wage offers Ω , queues $q(\omega)$, an interest rate r , and a set of distribution functions $\Phi^U(a, e)$ and $\Phi^M(w, a, e)$, such that:

1. $J^U = 0$ (free entry condition).
2. All wage offers in Ω solve the Bellman equation (8) given $J^M(w)$, J^U , c and r (optimal posting).
3. $k^F(w) = \text{argmax}\{f(k) - w - (r + \delta)k\}$.
4. $J^M(w)$ and $k^F(w)$ satisfy the Bellman equation (9) given $D_t(w)$ and J^U .
5. The decision rules $w^U(a, e)$, $k^U(a, e)$, $c^U(a, e)$, $d^U(a, e)$, $d^M(w, a, e)$ and the value functions $U(a, e)$, $N^S(a, e)$, $N^{NS}(a, e)$, $S^S(a, e)$, $S^{NS}(a, e)$ and $W(w, a, e)$ solve the Bellman equations (2) through (7).
6. $q^e(\omega) = q(\omega)$, $\omega^e(a_0) = \omega(a_0)$ and $\phi_j^e(x_0) = \phi_j(x_0)$ (rational expectations).
7. $\Phi^U(a, e)$ and $\Phi^M(w, a, e)$ are the time-invariant distributions resulting from optimal behavior.
8. $\int \int a \Phi^U(a, e) da de + \int \int \int a \Phi^M(w, a, e) dw da de = k^F(w) \int \int \Phi^M(w, a, e) dw da de + \int \int k^U(a, 1) \Phi^U(a, 1) da$ (capital market clearing).

In the next section I show the existence of a unique stationary rational expectations equilibrium.

2.3.2. Existence Of Equilibrium

In order to show the existence of a unique stationary equilibrium I introduce some important lemmas. First, the assumptions of directed search and rational expectations allow me to derive the following:

LEMMA 1: All vacancies paying a given wage w have the same queue length in equilibrium. The higher the wage posted, the higher the queue attracted in equilibrium.

PROOF: The opposite cannot be true. If two jobs with the same wage offer had different queue lengths in equilibrium, agents would redirect their search toward the job with the shorter queue, increasing their probability of getting the job. Similarly, if two jobs had the same queue but one had a higher wage, agents would redirect their search towards the job with the higher wage, since the probabilities of getting the job are identical.

This lemma allows us to replace $q(\omega)$ with $q(w)$. From now on I will refer to the firm as posting vacancies or wages interchangeably.

LEMMA 2: Given $D_t(w)$, all posted wages $w \in \Omega$ satisfy

$$(10) \quad \eta[q(w)]\pi(w) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) = 0$$

PROOF: Substituting the free entry condition $J^U=0$ into the Bellman equations (8) and (9)

and using rational expectations I find that

$$J^M(w) = \frac{c(1+r)}{\eta[q(w)]}$$

$$J^M(w) = \pi(w) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w)$$

Combining these two equations one gets equation (10).

This lemma is important because it states that in equilibrium, queue lengths to different wages are determined endogenously as a function of the market interest rate and firm's expectations on match durations.

LEMMA 3: The value functions $U(\cdot)$ and $W(\cdot)$ exist and are continuously increasing in a and e . $W(\cdot)$ is strictly increasing in w .

PROOF: See appendix.

LEMMA 4: A stationary rational expectations equilibrium always exists.

PROOF: See appendix.

3. Characterization Of Optimal Behavior

3.1. Transitions Into Self-Employment

Depending on parameter values, two different types of entry into self-employment can occur in equilibrium. These differ on whether the newly self-employed worker searches for a job at a firm while self-employed or not. I define entrepreneurial entry into self-employment as entry by workers who do not search for a job at a firm while self-employed, and stopgap entry as that by workers who search for a job at a firm while self-employed.

Workers entering self-employment with an entrepreneurial motive have different characteristics than those entering with a stopgap motive. Under certain parameter values, entrepreneurial entry is only observed among workers who have a business project. Furthermore, in the case of matched workers with a business project, entrepreneurial entry is observed only among relatively rich workers.⁸ More specifically, there exists a threshold level of assets $\underline{a}^w(w, 1)$ such that a matched worker with a business project decides to quit his job and becomes self-employed only if $a > \underline{a}^w(w, 1)$.⁹ This result hinges upon the credit constraint assumption. Self-employed agents with low assets are forced to operate their business at an inefficiently low scale, causing income while self-employed with a business project to be too low compared to wages paid by firms. For this

⁸ This statement is not true if the disutility from working is too high and/or the productivity of self-employed workers with a business project is relatively low. If the disutility from working is too high, only poor workers work. Rich workers choose to be out of the labor force. If the productivity of the self-employed worker with a business project is relatively low, all entries into self-employment are stopgap entries.

⁹ I am unable to make this statement in a form of a lemma because the matched and unmatched workers' value functions are not differentiable everywhere. Given that $\lim_{a_t \rightarrow 0} W(w_t, a_t, 1) > \lim_{a_t \rightarrow 0} U(a_t, 1)$ and given that both $U(a_t, 1)$ and $W(w_t, a_t, 1)$ are continuous functions, if $\frac{\partial U(a_t, 1)}{\partial a_t} > \frac{\partial W(w_t, a_t, 1)}{\partial a_t}$ are uniquely defined over a relevant range, there exists a threshold level of assets $\underline{a}^w(w, 1)$ such that the matched worker decides to become self-employed only if $a > \underline{a}^w(w, 1)$. This result is observed in the numerical simulations of the model.

reason, matched workers with low assets decide to stay at the firm even if they have a business project. Self-employed workers with higher assets can operate their businesses at a more efficient level, which increases their business income and makes self-employment more attractive.

Stopgap entry into self-employment is observed among poor unmatched workers.¹⁰

This result hinges upon the credit constraint and the assumption that $\lim_{c_t \rightarrow 0} u(c_t, l_t) = -\infty$, which guarantees that $\lim_{a_t \rightarrow 0} S^S(a_t, e_t) > \lim_{a_t \rightarrow 0} N^S(a_t, e_t) = -\infty$. In the case

of unmatched workers without a business project, given that both $N^S(a_t, e_t)$ and

$S^S(a_t, e_t)$ are continuous functions, if $\frac{\partial N^S(a_t, e_t)}{\partial a_t} > \frac{\partial S^S(a_t, e_t)}{\partial a_t}$ in a relevant range,¹¹

(assuming these derivatives are uniquely defined over the relevant range), there exists a

threshold level of assets $\underline{a}^u(0)$ such that an unmatched worker without a business project

decides to become stopgap self-employed if and only if $a < \underline{a}^u(0)$, and searches for a job

while unemployed otherwise. Under certain parameter values, stopgap self-employment

can also be observed among poor unmatched workers with a business project. Given the

presence of borrowing constraints, sufficiently poor self-employed workers with a

business project have a relatively low labor income compared to the wages offered by

firms. For these workers to have a higher self-employment income in the future, an

increase in own wealth is required, which requires low levels of current consumption. If

consumption has to be relatively low for several periods in order for labor income while

self-employed to reach the level of wages paid by corporations, poor workers might find

it optimal to search for a job at a firm, even if they have a business project. There exists

¹⁰ This statement is true only if the disutility from working is high enough. If the disutility from working is low enough, even a relatively rich unmatched worker will use self-employment as a stopgap rather than searching while unemployed.

¹¹ This property holds in numerical simulations for low asset levels.

then a wealth threshold $\underline{a}^u(1)$ such that an unmatched worker with a business project decides to use self-employment as a stopgap if and only if $a < \underline{a}^u(1)$, and enters self-employment as an entrepreneur (i.e. does not search for a job at a firm) otherwise.¹²

Besides generating stopgap self-employment, labor market frictions also discourage matched workers with a business project from becoming entrepreneurs. In the presence of labor market frictions, the worker realizes that if his business fails in the future, he will have to spend time searching for a job, which implies a cost in forgone income. Forward looking workers realize this future cost, taking it into account at the time of making their decision on whether to transition into self-employment. The fact that workers who search for a job obtain their target job with probability less than one reduces the value of being an unmatched worker searching for a job at a firm. This effect can lead some matched workers not to enter self-employment in the first place. I call this reason not to enter self-employment fear of failure.¹³ I choose this name because, if workers voluntarily select themselves into self-employment, the option to search for a job at a firm only has a positive shadow value when the self-employed worker loses his business project. This fear of failure also translates into higher wealth thresholds for transitioning into entrepreneurship, as workers insure themselves against the probability of becoming stopgap self-employed.

3.2. Transitions Out of Self-Employment

Three types of transitions out of self-employment can occur: exit from the labor force,

¹² For this to happen it has to be the case that over a relevant range $\frac{\partial S^S(a_{t,1})}{\partial a_t} > \frac{\partial S^{NS}(a_{t,1})}{\partial a_t}$ (where uniquely defined). This property holds in numerical simulations for low asset levels.

¹³ A mathematical discussion of the fear of failure is presented in the appendix.

transition to unemployment and transition into jobs at a firm in the corporate sector. Exits from the labor force are observed among very rich agents. For these workers leisure yields a higher utility than the extra income from working full time, either at their business or at a firm. Transitions into unemployment happen if $S^S(a_t, e_t) < N^S(a_t, e_t)$ and $N^S(a_t, e_t) > N^{NS}(a_t, e_t)$. This type of transition out of self-employment is observed among relatively rich agents who lose their business project.

To understand transitions into jobs at a firm in the corporate sector by self-employed workers one needs to analyze first whether a self-employed worker searches for a job at a firm, and then, if he does, which wage does he target. The wage application decision is important because the probability of getting a job, and therefore the probability of transitioning out of self-employment, depends on the chosen wage. With respect to whether self-employed workers apply to a job, their decision is made by comparing the value of searching and not searching. If the income of self-employed workers without a business project is low enough compared to the wages offered by firms in equilibrium, this group will always search. For self-employed workers with business projects, the decision on whether to search might depend on their asset holdings, given the existence of credit constraints that prevent some self-employed workers from operating their businesses at an efficient scale.¹⁴

With respect to optimal wage application, given that in the model jobs differ only in the wages paid, a worker would prefer having a high wage job. However, high paying

¹⁴ It is important to note that the wealth threshold below which a self-employed worker with a business project decides to search for a job is not given by the level of assets that makes labor income of the self-employed worker equal to $w^U(a_t, e_t)$. Workers might not look for a job, even if their labor income as self-employed is lower than $w^U(a_t, e_t)$. The reason is that there is an option value of remaining self-employed, given the assumption that a worker with a business project loses his project if he transitions into employment at a firm. This option value is higher, the harder it is to come across business projects and the higher the self-employed worker's wealth (given that higher wealth leads to higher self-employment income).

jobs are harder to get in equilibrium. Therefore, applying to a high wage job is more risky than applying to a low wage job in the sense that a high wage job is less likely to be obtained. How much risk a worker will be willing to take when applying for a job depends on his wealth and on the coefficient of absolute risk aversion.

LEMMA 5: If the utility function presents decreasing absolute risk aversion (DARA),

$\frac{\partial w^U(a_t, e_t)}{\partial a_t} > 0$. For increasing absolute risk aversion (IARA), $\frac{\partial w^U(a_t, e_t)}{\partial a_t} < 0$. Finally,

constant absolute risk aversion (CARA) implies $\frac{\partial w^U(a_t, e_t)}{\partial a_t} = 0$.

The impact of wealth on wage application decisions is of particular importance because it is the basis on which firms form expectations of employment duration. For DARA or IARA, given the realization of the worker's business project shock, there is a one-to-one relation between asset holdings and wage applications. This is not true for CARA given that the wage application does not depend on wealth. In this case, firms' expected employment duration is given by the population average employment duration over subgroups of workers with a business project and without a business project.

The model also has predictions on how unmatched workers searching for a job change their wage application decision as their search duration increases. For these workers being unmatched is a relatively bad option. If they are impatient enough, their asset holdings decrease. Lower assets lead agents to apply for lower wages assuming DARA utility. This behavior generates a positive relation between search duration and the probability of finding a job.

3.3. Wealth Accumulation

Studying workers' saving decision is important not only because asset holdings play an important role in the decisions to become and remain self-employed, but also because the economy's behavior in general equilibrium depends crucially on the equilibrium interest rate, which is a function of the wealth distribution.

There are five forces that influence workers' decision on how much to save. The first three forces are present in most models with a saving decision while the last two forces are specific to dynamic career choice models with a constraint on borrowing. The first force is related to a permanent income motive for saving, with agents saving more in periods when labor income is high. The second force is related to a precautionary motive for saving. If the agent's utility function has a positive third derivative, workers save in order to afford higher levels of consumption in case they become exogenously separated or lose their business project. The third force is impatience. Workers have incentives to reduce or increase their asset holding over time depending on whether the discount factor β is smaller or greater than $(1+r)^{-1}$.

The fourth force, only present among constrained self-employed workers with a business project, is a higher return on savings. While for all other workers the return on savings is given by the interest rate, for constrained self-employed workers with a business project the return on saving is given by the marginal product of physical capital, which is higher than the market interest rate.

The last force is related to the possibility of becoming self-employed in the future. Agents working for a firm who get a business project will not enter self-employment immediately if their assets are low. Instead, they remain at a firm until they have saved

enough to start their business at a sufficiently big scale. This behavior implies a loss in lifetime income, so workers save in order to reduce the time spent out of self-employment when having a business project.

Given all the different forces affecting savings, interactions among these forces emerge. One of the more important interactions is between the permanent income motive and the return on savings. Constrained self-employed agents with a business project have a relatively high marginal product of capital, which leads them to save more. If the wealth of the self-employed worker with a business project is relatively high, his labor income is also relatively high compared to the wages paid by corporations. The permanent income motive then creates further incentives to increase savings. For these two reasons credit constrained self-employed agents tend to have very high saving rates.

3.4. General Equilibrium Implications

The equilibrium interest rate, which is affected by the wealth distribution, is an important indicator of the properties of the equilibrium because it affects the behavior of both matched and unmatched workers as well as the behavior of firms in the corporate sector. First, interest rates affect the size of corporate firms as well as the size of the businesses owned by self-employed workers with a business project. In the case of firms in the corporate sector, the optimal capital demand equates the marginal product of capital to the rental price. The lower the interest rate, the more capital is rented by firms and the more output is produced. In the case of self-employed workers with a business project, a lower interest rate reduces the return to lending compared to the return to investing in their own businesses, which leads self-employed workers to increase the

optimal size of their businesses.

Interest rates also affect the number of corporations that decide to enter the market, which has implications for the set of wages that are offered in equilibrium and the queue lengths associated with different wages. To see these effects more clearly, recall that wages offered in equilibrium have to satisfy the following equation:

$$\eta[q(w)]\pi(w) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) = 0$$

If the interest rate decreases, $c(1+r)$ decreases and, for any given wage w , $\pi(w)$ increases. For the equation above to be satisfied, $\eta[q(w)]$ and/or $\left(\frac{1}{1+r}\right)^t D_t(w)$ need to decrease in equilibrium. If $D_t(w)$ is relatively insensitive to the interest rate¹⁵, the queue associated with the wage w needs to decrease in equilibrium. For queue lengths to decrease, more firms need to enter the market, which causes workers applying to a wage w to be spread over more firms in equilibrium. Given this reduction in queue lengths, jobs paying a wage w become easier to get. On the other hand, this effect implies that vacancies posting a wage w become more difficult to fill. Therefore, a lower interest rate results in risk being shifted from workers to firms, which is efficient given that firms are risk neutral and workers are risk averse.

This reduction in the risk of applying to a certain wage has two additional effects. First, it encourages workers to apply to higher wages, which affects the set of wages offered in equilibrium. In particular, both the lowest and highest wages offered are now higher. Second, as argued above, one of the reasons matched workers might not become self-employed is the fear of failure, resulting from the fact that it takes time for

¹⁵ In the numerical simulation of the model we find that expected match durations in equilibrium are largely insensitive to the interest rate.

unsuccessful self-employed workers to find a job at a firm. Given that jobs become easier to get when the interest rate is lower, the fear of failure motive becomes weaker when the interest rate decreases. This increases incentives for matched workers with a business project to enter self-employment.

4. Numerical Solution

4.1. Calibration

To better understand how labor market frictions and credit constraints affect workers' behavior and the general equilibrium, the model is solved numerically in this section. To do this, one needs to choose the duration of the time period, the functional forms of the utility function, the production function and the matching function, as well as other parameters of the model. Most parameter values used are either obtained from previous empirical studies or are values commonly used in previous literature. Parameters that cannot be chosen in this fashion are calibrated to match certain moments of the U.S. data.

To account for the short duration of some observed unemployment and self-employment spells, the duration of the time period is one month. The worker's utility function is assumed separable in consumption and leisure and has the following form:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - D l_t$$

Remember that the worker either works full-time or does not work, so l_t must be either 0 or 1.

The production functions for corporations and self-employed businesses are:

$$q_t^{corp} = A k_t^\alpha$$

$$q_t^{se} = b + e_t B k_t^\mu$$

respectively. Note that for a self-employed worker without a business project $e_t=0$ and the production function is independent of the physical capital stock.

For the probability of a firm finding a worker I use the urn-ball matching function

$$\eta[q(w)] = 1 - \exp(-\gamma q(w))$$

I choose this functional form because it emerges in the limit as the number of workers applying to a wage w and the number of vacancies posting that wage approach infinity, keeping $q(w)$ fixed. This function assumes that workers looking for a job with wage w apply to all vacancies posting that wage with the same probability, without coordinating their applications. When only coordination frictions are present, γ is always one. Having a value of γ smaller than one allows me to introduce extra matching frictions into the model.

Having specified the functional forms for the utility, production and matching functions, there are fifteen parameters to be chosen: two parameters related to the utility function (σ and D), a discount factor (β), five parameters related to the production functions (A , α , b , B , μ), one parameter in the matching function (γ), one parameter for exogenous separations (s), one parameter for the cost of posting a vacancy (c), one parameter for the depreciation rate (δ), one parameter for the borrowing constraint (λ) and two parameters for the transition matrix for the business project shock. I call the probability of getting a business project shock pr^{get} and the probability of losing a business project pr^{lose} .

The coefficient of relative risk aversion σ used is 1.5. This is a value in the range commonly used in numerical simulations of similar models. It is also the value used by

Cagetti and De Nardi (2005), and is close to that estimated by Attanasio et al. (1999). The discount factor β is set at 0.994, which implies an annual discount of 0.95, a value commonly used in numerical simulations of similar models.

Both the cost of posting a vacancy (c) and A , a parameter describing the state of technology for firms in the corporate sector, are normalized to 1. The parameter α , which captures the income share in the corporate sector that goes to capital, is set to $1/3$, a value commonly used in the literature and consistent with findings by Gollin (2002). The depreciation rate for capital is set to 0.66%. This value corresponds to a yearly depreciation rate of 7%, which is consistent with the depreciation of machinery estimated by the Bureau of Economic Analysis (2003). The probability of exogenous separation s is set to 2.6% per period, as estimated by Davis et al. (2008). Finally, the parameter capturing the credit constraint on entrepreneurs, λ , is set at 1.44 to match the estimate of Evans and Jovanovic (1989).

The remaining seven parameters (D , b , B , μ , γ , pr^{get} and pr^{lose}) are calibrated to match some moments in the data. The selected moments to match are calculated from a sample of workers in the Panel Study of Income Dynamics (PSID) used in Rasteletti (2009b), that merges data from the years 1989 and 1994. The two parameters capturing the probability of workers obtaining and losing their business projects (pr^{get} and pr^{lose}) are selected to match the proportion of entrepreneurs in the data and the average duration of an entrepreneur's business in the data. In the PSID sample, 7.28% of all workers in the labor force are entrepreneurs and the average duration of an entrepreneur's business is 4.72 years.¹⁶ The parameter γ , which captures the severity of labor market frictions, is

¹⁶ Identifying entrepreneurs in the data is not a simple task. Most empirical studies define entrepreneurs either as business owners or as self-employed workers. Given that both empirical definitions of

calibrated so that the median duration of an unemployment spell is two months. This is the median unemployment duration in the PSID sample.

The labor income of a self-employed worker without a business project using self-employment as a stopgap, b , is calibrated so that in equilibrium the ratio of the mean labor income of workers using self-employment as a stopgap to the mean labor income of matched workers equals the same ratio in the data. In the PSID sample used in Rasteletti (2009b) this ratio is 0.65. The parameter D , which captures the disutility from working, is calibrated to match the proportion of workers in stopgap self-employment. Rasteletti (2009b) finds that approximately 2.27% all workers in the labor force use self-employment as a stopgap. Finally, the remaining parameters (B and μ) are calibrated to match the proportion of unemployed workers in the economy (6.11%) and the ratio of the average wealth of entrepreneurs to the average wealth of matched workers (6.21).

Table 1.1 summarizes the parameters used in the numerical solution of the model while Table 1.2 shows the moments generated by the model vis-a-vis the data moments targeted. The rest of this section is organized as follows: I first do a partial equilibrium analysis of how workers' decisions change when facing different labor market frictions and different borrowing constraints. While doing this I keep the interest rate and the queues associated with the different wages posted by firms fixed. Next, I look at the general equilibrium implications of labor market frictions and borrowing constraints.

entrepreneurship have problems (not all business owners or self-employed workers are entrepreneurs), some studies prefer to define entrepreneurs as self-employed business owners. This is the definition used in Rasteletti (2009) and in this paper to identify entrepreneurs. All other self-employed workers are treated as being in the stopgap group.

TABLE 1.1: Parameter Values

Fixed Parameters		Calibrated Parameters	
Parameter	Value	Parameter	Value
β	0.994	D	0.25
σ	1.5	b	2.5217
c	1	B	0.18
A	1	μ	0.70
α	1/3	γ	0.60
δ	0.0066	pr^{get}	0.0014
s	0.026	pr^{lose}	0.0177
λ	1.44		

4.2. Workers' Decision Rules

To analyze the effects of credit constraints and labor market frictions on workers' behavior, I fix the interest rate at 3.2%,¹⁷ and assume that the queues associated with wages are those implied by equation (10), assuming that firms believe that workers remain matched until they get a business project or are separated exogenously. We do this in order to identify how labor market frictions and credit constraints affect workers' behavior, without confounding these impacts with the effects of changes in the interest rate or in the wage offer distribution. To better understand the effect of labor market frictions, in this section I sometimes refer to the behavior of workers in a special case with no labor market frictions. In this special case workers always get the job they apply

¹⁷ This is the interest rate that emerges in equilibrium in the benchmark calibration.

to and firms always fill their vacancies in equilibrium. This case is explained in more detail in appendix I.4.

TABLE 1.2: Data and Model Moments

	MODEL	DATA
LABOR FORCE (L.F.) COMPOSTION		
Proportion of Entrepreneurs in L.F.	7.28	7.28
Proportion of Stopgap Self-Employed in L.F.	2.30	2.37
Proportion of Unemployed in L.F.	6.37	6.11
DURATIONS		
Mean Duration Entrepreneurs	4.72 yrs	4.72 yrs
Duration Unemployment (Median)	2 mo.	2 mo.
LABOR INCOME (L.I.)		
Ratio Mean L.I. Stopgap to Mean L.I. Matched Workers	0.65	0.65
WEALTH		
Ratio Mean Wealth Entrepreneurs to Mean Wealth Matched Workers	6.44	6.21

Transitions into self-employment can happen among workers with or without a business project. In my simulations, transitions into self-employment by workers without a business project only happen in equilibrium among relatively poor unmatched

workers.¹⁸ Relatively rich unmatched workers without a business project choose to search while unemployed, in order to enjoy leisure. The wealth thresholds below which an unmatched worker without a business project prefers self-employment to unemployment depend on the extent of labor market frictions and on the severity of credit constraints faced by self-employed workers. A summary of the different thresholds for different combinations of labor market frictions and credit constraints is presented in Table 1.3.

TABLE 1.3: Transitions into Self-Employment by Unmatched Workers without Business Project: Wealth Thresholds.

Credit Cons. \ L.M. Fric.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
None	Never	Never	Never
Benchmark ($\gamma=0.60$)	< 131.5209 (2.82 yrs)	< 134.3614 (2.89 yrs)	< 137.3584 (2.95 yrs)
Intermediate ($\gamma=0.30$)	< 142.7032 (3.06 yrs)	< 147.1181 (3.16 yrs)	< 152.7905 (3.28 yrs)
High ($\gamma=0.15$)	< 169.4828 (3.64 yrs)	< 171.9853 (3.69 yrs)	< 174.9970 (3.76 yrs)

Note: Numbers in parenthesis normalize the wealth thresholds by the maximum annual labor income if matched. Workers below the wealth threshold use self-employment as a stopgap. $\gamma = x$ implies that the job finding rate cannot be higher than x .

¹⁸ Out of equilibrium, transitions into self-employment can also happen among matched workers without a business project currently receiving a wage below a certain wage threshold. This finding depends critically on my assumption that matched workers are not allowed to search for a job. Matched workers without a business project receiving a low wage find it optimal to separate from firms to become self-employed and search for a job paying a higher wage. I do not observe this type of transition in equilibrium, given that workers always apply to wages above the threshold for this type of endogenous separation.

Several results are worth highlighting. First, when labor market frictions are not present, unmatched workers without a business project never transition to self-employment. If workers are exogenously separated from their jobs they immediately start working for another firm in the corporate sector, given the low labor income if they become self-employed without a business project. Therefore, stopgap self-employment cannot be present unless labor market frictions are present. Secondly, the wealth threshold below which an unmatched worker without a business project transitions into self-employment is higher, the more severe the labor market frictions. Thirdly, the wealth threshold below which an unmatched worker without a business project transitions into self-employment is higher, the more restrictive the borrowing constraints on the self-employed. These last two effects are a consequence of a permanent income effect. For unmatched workers without a business project, permanent income is higher when labor market frictions are lower and borrowing constraints are less restrictive.¹⁹ Given this higher permanent income, unmatched workers without a business project find it optimal to consume leisure at some levels of assets for which they would choose stopgap self-employment under stricter labor market or borrowing frictions.

With respect to transitions into self-employment by workers with business projects, credit constraints generate wealth thresholds below which a matched worker with a business project does not transition into self-employment. This is because the labor income while self-employed is lower than the wage obtained at the firm if the worker's wealth is low, since low wealth forces entrepreneurs to operate at an inefficiently small

¹⁹ Note that relaxing borrowing constraints does not immediately benefit stopgap entrepreneurs without a business project. For these workers current self-employment income equals b , and therefore is independent of physical capital. Their permanent income is higher because their future labor income is higher when they do get a business project.

scale. Unlike credit constraints, labor market frictions alone do not deter entry by self-employed workers with a business project, unless they are very strong. However, mild labor market frictions do increase the wealth thresholds when there are credit constraints on the self-employed. This result stems from the worker's fear of losing the business project and remaining unmatched for several time periods, which I refer to as the fear of failure effect. The thresholds for entry for different combinations of credit constraints and labor market frictions are shown in table 1.4 below.

TABLE 1.4: Transition into Self-Employment by Workers with Business Project.

Cred.Con \ L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
None	> 12.34 (0.27 yrs)	> 15.39 (0.33 yrs)	> 23.04 (0.49 yrs)
Benchmark ($\gamma=0.60$)	> 12.51 (0.27 yrs)	> 15.70 (0.34 yrs)	> 23.60 (0.51 yrs)
Intermediate ($\gamma=0.30$)	> 12.79 (0.27 yrs)	> 16.05 (0.35 yrs)	> 24.11 (0.52 yrs)
High ($\gamma=0.15$)	> 13.27 (0.28 yrs)	> 16.97 (0.36 yrs)	> 24.70 (0.53 yrs)

Note: Numbers in parenthesis normalize the wealth thresholds by the maximum annual labor income if matched. $\gamma = x$ implies that the job finding rate cannot be higher than x .

Several findings are worth highlighting. Firstly, in the absence of labor market frictions and borrowing constraints, all workers in the labor force that get a business project decide to become self-employed, and those without a business project decide to work for a firm in the corporate sector, as long as their wealth is below the leisure threshold. Otherwise they choose to be out of the labor force. Secondly, the entry threshold for matched

workers with a business project is highly sensitive to the extent of credit constraints. In the benchmark case with $\gamma=0.60$, the wealth threshold for transitioning into self-employment is increased by 50.32% when λ changes from 1.44 to 1. This is because $\lambda=1$ forces new self-employed workers to finance all capital used for production out of own financial wealth. Therefore, when $\lambda =1$, matched workers with a business project require a higher level of wealth to enter self-employment, in order not to experience a drop in labor income upon entry.

Thirdly, when labor market frictions are initially at a low level the wealth thresholds for entry into self-employment are not highly sensitive to small changes in the extent of labor market frictions. Changes in the extent of labor market frictions have a much bigger effect on the wealth threshold for transitioning into entrepreneurship when they are at a higher initial level. In the benchmark case when $\gamma =0.60$ and $\lambda=1.44$, the wealth threshold for matched workers with a business project to transition into self-employment only increases by 2.29% when γ is reduced by half to 0.30. When labor market frictions are further reduced by half to 0.15, the wealth threshold now increases by 5.73%, about two and a half times higher than the previous value. This reflects the increase in the fear of failure motive. When $\gamma =0.60$, it takes about two months on average for a worker to find a job given their optimal wage application. Given this short search spell and the fact that self-employed workers have a positive labor income while searching, the fear of failure motive is relatively very weak and the effect of labor market frictions on the wealth threshold for transitioning into self-employment is very small. When $\gamma =0.15$, it takes about eight months on average for a worker to find a job given their optimal wage application. The fear for failure motive is now more important, generating a higher

increase in the wealth threshold for transitions into self-employment.²⁰

Three types of transitions out of self-employment occur in the current calibration of the model. First, self-employed workers with very high wealth, with or without a business project, exit the labor force to enjoy leisure. Second, relatively rich self-employed workers leave self-employment to search for a job while unemployed after losing their business project. Third, transitions out of self-employment directly into employment at firms occur among self-employed workers who choose to search for a job at a firm. In my simulations, I find that in equilibrium all self-employed workers without a business project search for a job while those with a business project do not.

4.3. General Equilibrium Results

4.3.1. Implications of Credit Constraints on Entrepreneurs

For relatively mild labor market frictions, changes in the tightness of credit constraints do not affect the proportion of entrepreneurs in the economy. This result is due to the fact that in equilibrium all workers have asset holdings that are higher than the wealth thresholds for entry into self-employment by workers with a business project. Therefore, whenever a worker working for corporation gets a business project, he quits his job to become an entrepreneur. For this reason, the proportion of entrepreneurs in the economy is insensitive to the tightness of credit constraints.

Even if credit constraints do not prevent workers from becoming entrepreneurs, they do prevent the majority of entrepreneurs from running their business at an efficient scale.

²⁰ Remember that here we are only doing a partial equilibrium analysis, keeping the interest rate and the wage offer distribution fixed. In general equilibrium, the increase in the labor market frictions leads to a lower interest rate and higher wages offered. These higher wages increase the opportunity cost of not having a job at a corporation, which further increases the wealth threshold for transition into entrepreneurship.

Table 1.8 below shows that the average output of entrepreneurs is highly sensitive to the extent of credit constraints. Compared to the benchmark case with $\gamma=0.60$ and $\lambda=1.44$, the average output of a self-employed worker is 29.22% lower when entrepreneurs have no access to credit ($\lambda =1$).

Interestingly, table 1.5 shows that tighter credit constraints on entrepreneurs increase the proportion of workers using self-employment as a stopgap, even though credit constraints do not affect this group of self-employed workers directly. This result emerges from two effects. First, when studying the effect of credit constraints on the decision of unmatched workers to be unemployed or to use self-employment as a stopgap, we observed in the previous section that the wealth threshold below which workers decide to use self-employment as a stopgap is increased when credit constraints are tightened due to a permanent income effect. All else equal, this increase in thresholds tends to increase the proportion of workers using self-employment as a stopgap. The second effect comes from changes in the behavior of firms both in the corporate and non-corporate sector. When credit constraints on entrepreneurs are tightened, self-employed workers produce less and accumulate lower levels of wealth, which reduces aggregate wealth (see Table 1.6). As a consequence of lower aggregate wealth the equilibrium interest rate increases, which reduces profits of firms in the corporate sector. This reduction of profits results in fewer vacancies being posted at all wage levels and reduces the exit rate from stopgap self-employment, which leads to an increase in the proportion of workers using self-employment as a stopgap.

TABLE 1.5: Proportion of Workers Using Self-Employment as a Stopgap.

Cred.Cons. L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
None	0	0	0
Benchmark ($\gamma=0.60$)	2.24	2.30	2.48
Intermediate ($\gamma=0.30$)	6.03	6.22	6.67
High ($\gamma=0.15$)	12.99	13.40	13.99

Note: $\gamma = x$ implies that the job finding rate cannot be higher than x .

TABLE 1.6: Aggregate Wealth.

Cred.Cons. L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
None	235.1648	204.1200	174.4622
Benchmark ($\gamma=0.60$)	213.6409	194.6207	173.7297
Intermediate ($\gamma=0.30$)	207.6407	189.0099	167.8621
High ($\gamma=0.15$)	199.6007	178.4287	157.7756

Note: $\gamma = x$ implies that the job finding rate cannot be higher than x .

The higher aggregate wealth and lower interest rate in equilibrium when credit constraints on entrepreneurs are relaxed have some important implications for other key aggregates, in particular wage offers and aggregate output. With respect to wages, a lower interest rate increases wages offered by corporations and increases the probability of a worker getting a job at any given wage, due to the increase in vacancies posted by

firms at a given wage. When interest rates are lower, workers searching for a job apply to higher wages given the reduction in the associated risk, which results in workers having higher wages in equilibrium (see table 1.7).

TABLE 1.7: Wages and Interest Rate.

Cred.Cons. L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
Interest Rate			
Benchmark ($\gamma=0.60$)	3.12	3.18	3.42
Intermediate ($\gamma=0.30$)	2.98	3.05	3.29
High ($\gamma=0.15$)	2.62	2.74	3.05
Wages			
Benchmark ($\gamma=0.60$)	3.8553	3.8464	3.8094
Intermediate ($\gamma=0.30$)	3.8778	3.8670	3.8286
High ($\gamma=0.15$)	3.9423	3.9207	3.8671

Note: $\gamma = x$ implies that the job finding rate cannot be higher than x .

With respect to the effect of labor market frictions on output, the decrease in the equilibrium interest rate resulting from more severe labor market frictions leads firms in the corporate sector to rent more capital for production, which increases output per firm (see table 1.8).

TABLE 1.8: Aggregate, Corporate and Self-Employed Output.

Cred.Cons. L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
Aggregate Output			
Benchmark ($\gamma=0.60$)	6.7383	6.4317	6.0504
Intermediate ($\gamma=0.30$)	6.6251	6.3114	5.9022
High ($\gamma=0.15$)	6.4377	6.0961	5.6952
Proportion of Total Output Produced by Corporations			
Benchmark ($\gamma=0.60$)	71.89	76.24	81.76
Intermediate ($\gamma=0.30$)	70.00	74.29	79.63
High ($\gamma=0.15$)	66.46	70.65	75.90
Average Corporate Output			
Benchmark ($\gamma=0.60$)	5.8490	5.8351	5.7765
Intermediate ($\gamma=0.30$)	5.8870	5.8680	5.8074
High ($\gamma=0.15$)	5.9827	5.9495	5.8683
Average Self-Employed Output			
Benchmark ($\gamma=0.60$)	25.2331	20.1908	14.2913
Intermediate ($\gamma=0.30$)	25.2085	20.1238	14.1941
High ($\gamma=0.15$)	25.1492	19.9326	14.0129

Note: $\gamma=x$ implies that the job finding rate cannot be higher than x .

This increase in the productivity of firms in the corporate sector combined with the

increase in productivity of self-employed workers leads to an increase in aggregate output when credit constraints are relaxed. In the case $\gamma = 0.60$, going from a situation where entrepreneurs are not allowed to borrow ($\lambda = 1$) to one where $\lambda = 1.44$ increases aggregate output by 6.3%. Interestingly, this increase in aggregate output happens despite a decrease in the fraction of workers employed due to the larger proportion of wealthier workers who choose leisure over employment (see Table 1.9).

TABLE 1.9: Labor Force Participation.

Cred.Cons. \ L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
None	89.43	91.04	93.42
Benchmark ($\gamma=0.60$)	93.28	94.43	95.99
Intermediate ($\gamma = 0.30$)	93.73	94.81	96.10
High ($\gamma = 0.15$)	94.15	95.29	97.29

Note: $\gamma = x$ implies that the job finding rate cannot be higher than x .

One of the main differences between the quantitative results in this chapter and the ones in Cagetti and DiNardi (2006) is that more severe borrowing constraints do not affect the number of entrepreneurs in the economy in my model, while they reduce the number of entrepreneurs in Cagetti and DeNardi's baseline calibration.²¹ This difference is mainly due to the fact that they do a better job at matching the wealth distribution. In my

²¹ I only compare my results to Cagetti and DeNardi (2006) because their model and motivation are relatively similar to mine. The main difference of their paper with mine, besides the fact that their model does not include labor market frictions, is the way credit constraints are modelled. While in my model credit constraints are exogenous, in their model the borrowing constraints are endogenously determined. They originate from an assumption of imperfect enforceability of contracts.

calibration, all workers have asset holdings that are higher than the wealth thresholds for entry into self-employment by workers with a business project. Therefore, the proportion of entrepreneurs in the economy is insensitive to the severity of the borrowing constraints. Besides this difference, most other qualitative results are similar. We both find that more severe borrowing constraints reduce the average firm output, the aggregate capital stock and the interest rate in equilibrium. Quantitative results are difficult to compare due the differences in model assumptions and calibration.

4.3.2. Implications of Labor Market Frictions

Changes in the severity of labor market frictions do not affect the proportion of entrepreneurs in the economy. This result is due to the fact that in equilibrium all workers have asset holdings that are higher than the wealth thresholds for entry into self-employment by workers with a business project. Therefore, whenever a worker working for corporation get a business project, he quits his job to become an entrepreneur. For this reason, the proportion of entrepreneurs in the economy is insensitive to the severity of labor market frictions. Labor market frictions have a much bigger impact on the proportion of workers using self-employment as a stopgap. Table 1.5 shows that starting from the benchmark case with $\gamma = 0.60$ and $\lambda = 1.44$, reducing γ to 0.30 increases the proportion of workers using self-employment as a stopgap from 2.30% to 6.22%. The proportion of unemployed workers also increases, as do the durations of unemployment and stopgap self-employment spells (see Table 1.10).

I find that more severe labor market frictions lead to a decrease in both the interest rate and the aggregate wealth (see table 1.6 and 1.7). The interest rate decreases both because

the supply of capital shifts to the right due to workers saving more out of precaution, and because the lower number of firm-worker matches shifts the demand for capital to the left. With respect to aggregate wealth, there are two opposing forces at work. On the one hand, the fact that labor market frictions are more severe creates incentives to save more out of precaution. On the other hand, the reduction in the demand for capital reduces interest rates, which reduces workers incentive to save. I find that the latter effect dominates, leading to a reduction in the aggregate wealth in the economy. The lower interest rate also generates higher wage offers and more vacancies being posted, which partially offsets the impact of higher unemployment risk introduced by the stronger labor market frictions.

TABLE 1.10: Unemployment Rates and Average Durations.

Cred.Cons. L.M.Frict.	Low ($\lambda=1.80$)	Benchmark ($\lambda=1.44$)	Tighest ($\lambda=1$)
Unemployment Rate			
Benchmark ($\gamma=0.60$)	7.66	6.22	4.60
Intermediate ($\gamma=0.30$)	7.92	6.59	5.11
High ($\gamma=0.15$)	8.21	6.93	5.07
Average Search Duration			
Benchmark ($\gamma=0.60$)	2.25 months	2.26 months	2.30 months
Intermediate ($\gamma=0.30$)	4.42 months	4.47 months	4.55 months
High ($\gamma=0.15$)	8.83 months	8.87 months	8.94 months

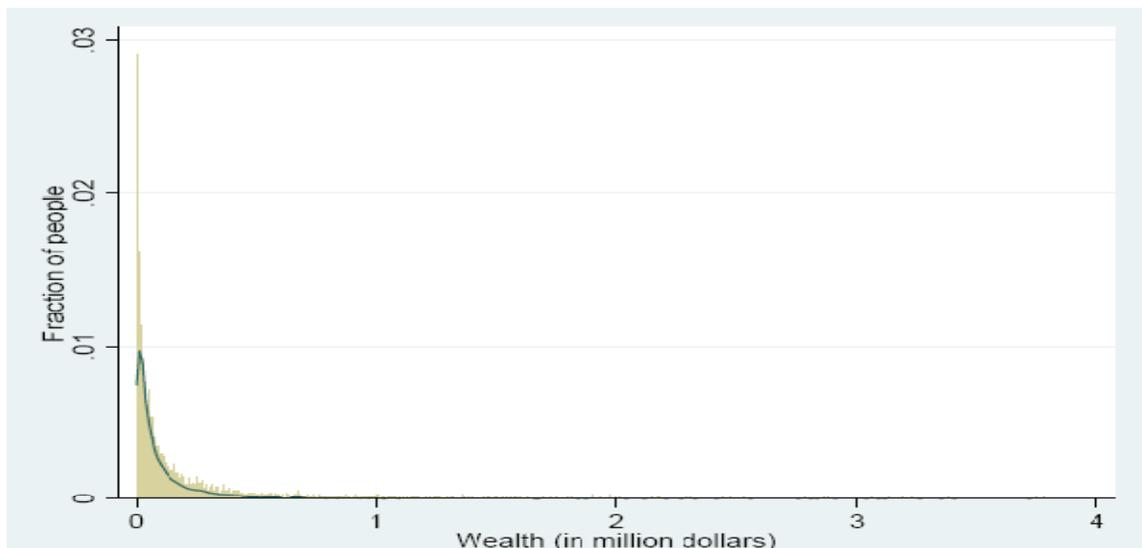
Note: $\gamma = x$ implies that the job finding rate cannot be higher than x .

As one might expect, when labor market frictions become more severe, aggregate output decreases. Changes in labor market frictions also have some interesting implications both for the composition of aggregate output as well as for the productivity of firms in the corporate and non-corporate sector (see Table 1.8). More severe labor market frictions lead to a decrease of total output in the corporate sector and to an increase of output in the non-corporate sector. The reduction of the corporate sector output is due to the fact that there are fewer firm-worker matches in equilibrium, while the increase in total output in the non-corporate sector is due to the increase in the number of workers using self-employment as a stopgap. Labor market frictions also have implications for the productivity of firms in both sectors. The lower interest rate in equilibrium increases the amount of capital firms rent, which increases the average output of firms in the corporate sector. However, the average output of self-employed workers decreases when labor market frictions increase. This reduction in average output is not only a consequence of the increased number of workers using self-employment as a stopgap, but is also due to a reduction in the average productivity of the businesses of entrepreneurs. The reason for the lower productivity of entrepreneurs is twofold. First, the reduction of wealth holdings in steady state results in workers entering entrepreneurship with lower levels of wealth, which reduces the average productivity of new entrepreneurs. Secondly, the lower interest rate also leads existing entrepreneurs to save less, and therefore the businesses of entrepreneurs grow at a slower rate. The lower initial productivity of new entrepreneurs and the lower growth rate of existing entrepreneurs explain the decrease in the entrepreneurs' productivity.

4.3.3. Wealth Distribution

The main problem with the calibrated version of the model is that it fails to reproduce the wealth distribution observed in the data. The calibrated model produces a very large spike in the wealth distribution, which is not observed in the data. Figure 1.3 presents the wealth distribution observed in the PSID sample used to calibrate the model. The figure shows that the wealth distribution presents a spike at a wealth level close to zero. The model fails to reproduce this fact. Figure 1.4 presents the wealth distribution obtained using the baseline calibration of the model.²² As can clearly seen, the wealth

Figure 1.3. Wealth Distribution - PSID



NOTE: The distribution is plotted conditional on wealth being positive. The PSID sample only includes households heads aged 21-60.

²² The properties of the wealth distribution for other reasonable calibrations of the model present similar features.

Figure 1.4. Wealth Distribution - Model

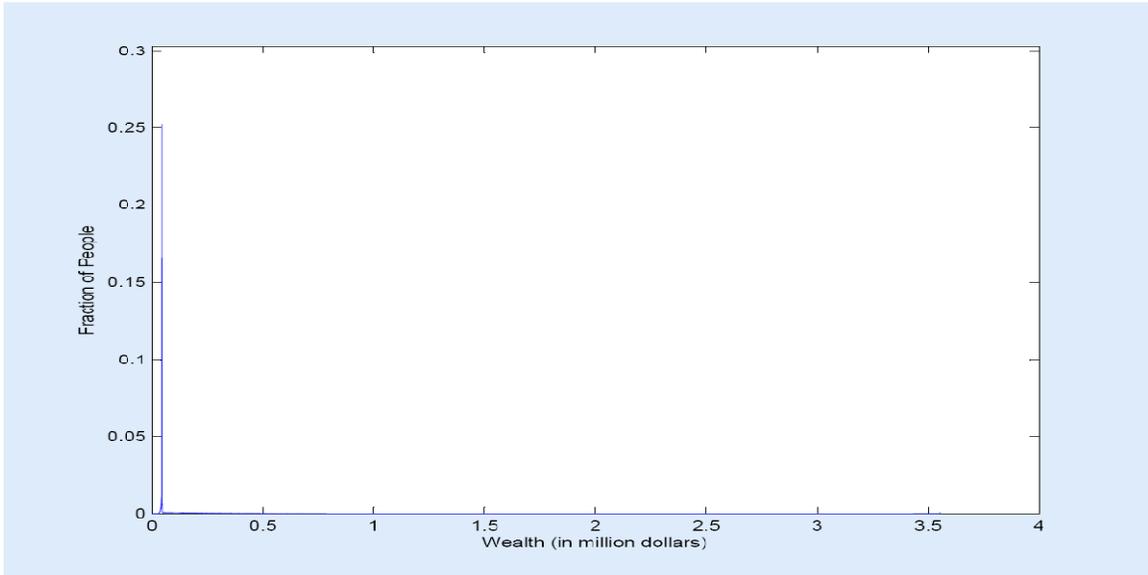
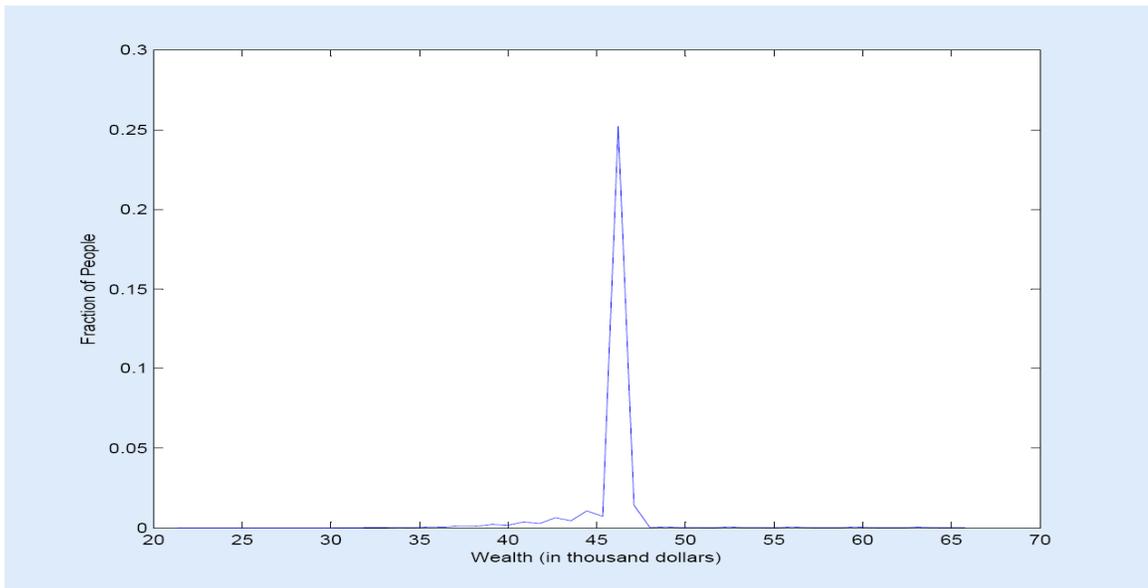


Figure 1.5. Spike Close-up - Model

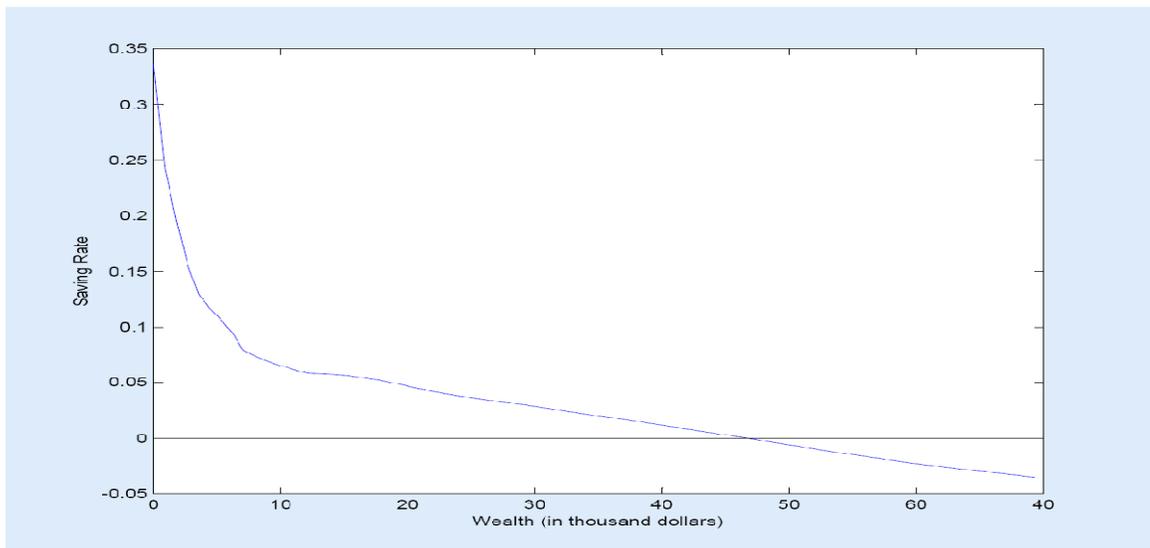


NOTE: This graph plots the portion of the economy's wealth distribution for households between 20,000 and 70,000.

distribution generated by the model presents a very large spike at \$47,000. This spike is a result of the characteristics of the saving function of the matched workers without a

business project, who represent about 84% of workers in the economy. Figure 1.6 plots the saving rate for an unmatched worker receiving the median wage in the economy.²³ Notice that the saving rates are positive for workers with wealth below \$47,000 and negative for workers with wealth above \$47,000. This then explains the peak of the wealth distribution at \$47,000. The existence of agents with wealth level below the peak is mainly explained by the presence of unmatched workers without a business project, who have negative saving rates throughout the wealth domain. Given the relatively short duration of unemployment and stopgap self-employment spells and the high saving rates of unmatched workers without a business project, the calibrated model fails to generate a relatively high fraction of workers with wealth holdings close to zero, as observed in the data.

Figure 1.6. Saving Rates – Matched Workers without a Business Project.



NOTE: The wage used in the graph equals the median wage in the economy.

²³ For the wages observed in the economy in equilibrium, the location of the saving rate is barely affected by wage level.

5. *Conclusions*

The model developed in this dissertation chapter generates rich interactions between labor market frictions, credit constraints and self-employment that have implications for worker decision rules as well as some key aggregate variables. The presence of labor market frictions and credit constraints generates both entrepreneurial and stopgap self-employment, and affects workers' decision to become self-employed. In particular, labor market frictions create a motive not to enter self-employment, which I label fear of failure. In the model, the fear of failure is based on the probability that an entrepreneur loses the business project and experiences a spell of unemployment. While this is not costless, a two or three month spell of unemployment is unlikely to be a major cost to the worker. However, there are other costs not included in this model that might be more important. For example, workers may have to disrupt their career path and human capital accumulation to become entrepreneurs. If workers have to re-enter at or close to the bottom of the career ladder when they return to the corporate sector as a worker, the fear of failure may play a much more important role.

The model also reveals that labor market frictions and credit constraints on entrepreneurs have important general equilibrium implications. Tighter credit constraints on entrepreneurs not only reduce the total output produced by the businesses of entrepreneurs, but also increase the proportion of workers using self-employment as a stopgap as well as the duration of the stopgap self-employment spells. Meanwhile, more severe labor market frictions not only reduce aggregate output and increase the number of workers who are unemployed or using self-employment as a stopgap, but also lead to a reduction in the average productivity of entrepreneurs.

While both the theoretical and numerical findings show the importance of analyzing the interactions of labor market frictions, credit constraints and self-employment in a general equilibrium setting, one of the limitations of the existing model is that it cannot generate a wealth distribution that resembles that in the US data. This is partly due to the strength of the precautionary saving motive, which leads relatively poor workers to have high saving rates while employed at firms to avoid finding themselves in the stopgap state in the future. As a result, the wealth distribution that emerges in the stationary equilibrium does not have many poor workers. This is a standard result in incomplete markets models where workers face uninsurable idiosyncratic risk. Researchers have solved this problem by modeling different types of government insurance programs, which reduce the strength of the precautionary saving motive. Introducing some type of insurance into my model to obtain a more realistic wealth distribution is left for future work. However, as long as this insurance is not complete, the interactions of credit constraints, labor market frictions and self-employment analyzed in this chapter are likely to remain relevant for explaining the main features of the equilibrium.

Chapter 2: Stopgap Self-Employment: Evidence from the PSID.

1. Introduction

Economists studying the reasons why workers become self-employed have traditionally argued that some workers are "pulled" into self-employment while others are "pushed" into it (Parker 2004, Dawson et al. 2009). Workers pulled into self-employment are those who choose to become self-employed due to the pecuniary and/or non-pecuniary benefits of being self-employed.²⁴ Workers pushed into self-employment are those who become self-employed after failing to find a paid job working for others. If workers become self-employed for different reasons, one would then expect to observe important differences in behavior between different groups of self-employed workers. Workers pushed into self-employment should have relatively short self-employment spells, using self-employment as a stopgap until they find a job working for others. On the other hand, workers pulled into self-employment should have longer self-employment spells.

Despite the emphasis given in the literature to the fact that workers become self-employed for different reasons, most of the empirical work on self-employment does not take into account the different reasons why workers are self-employed. All self-employed workers are usually grouped together and their characteristics and behavior are compared to those working for others. Not taking into account the different reasons why workers

²⁴ The most commonly mentioned non-pecuniary reasons to become self-employed are time flexibility, job satisfaction, the value of being your own boss and the prestige associated with owning a business.

are self-employed could lead to misleading conclusions or results that are hard to interpret, especially when the researcher is interested in studying a particular subgroup of self-employed workers. The empirical literature on entrepreneurship presents a good example of this point. Several studies consider a worker to be an entrepreneur if the worker is self-employed.²⁵ Now, if some workers are using self-employment as a stopgap, some estimates of interest might be biased given the presence of a subgroup of workers that behave differently than the group of interest. For example, if workers using self-employment as a stopgap have short self-employment spells, the inclusion of stopgap self-employed workers will result in higher estimates of business failure rates among entrepreneurs.

This dissertation chapter revisits results in the empirical literature in self-employment, being careful to treat workers using self-employment as a stopgap separately from workers that are self-employed for other reasons. Particular attention is paid to the group of workers using self-employment as a stopgap, given that little is known about them. My first task is to distinguish in the data workers using self-employment as a stopgap from those that are self-employed for other reasons. Previous empirical work treats self-employment as uniform in part because most surveys don't ask workers why they chose self-employment. One of the contributions of this chapter is to introduce three different working definitions of stopgap self-employment. The first definition follows Rasteletti (2009a), who defines a worker as stopgap self-employed if he searches for a job while self-employed. The second definition is based on business ownership. Stopgap self-employed workers are those who report being self-employed but not owning a business.

²⁵ Studies using self-employment as a proxy for entrepreneurship include Blanchflower and Oswald (1998), Evans and Leighton (1989), and Hamilton (2000), among others.

The third definition combines the two previous definitions, defining a worker as stopgap self-employed if he is either searching for a job or reports not owning a business. Given these definitions, the Panel Study of Income Dynamics (PSID) has all the relevant information needed to classify self-employed workers into those using self-employment as stopgap and those that are self-employed for other reasons.

My findings on stopgap self-employment are similar regardless of which measure I use: 1) Stopgap self-employment prevails mostly among young workers and among those who experience unemployment. 2) Stopgap self-employment is also relatively high among minorities, women and workers with low educational achievement. 3) Workers using self-employment as a stopgap have very short self-employment spells. 4) An important fraction of stopgap self-employed workers work as laborers and in service related occupations.

I also find liquidity constraints, labor market conditions and workers' labor market histories do not affect all self-employed workers equally. While having more wealth increases the probability of becoming self-employed for workers that do not use self-employment as a stopgap, the effect of wealth on the probability of entering stopgap self-employment presents an inverted U shape, with the peak at a relatively low wealth levels. Workers' labor market histories, meanwhile, are only relevant for stopgap self-employment. More specifically, being unemployed increases the probability of becoming stopgap self-employed, but has no effect on the probability of becoming self-employed for other reasons.

Furthermore, I find that workers using self-employment as a stopgap are far more

likely to leave self-employment. Regardless of whether the worker is using self-employment as a stopgap, self-employed workers that have spent more years in self-employment are less likely to leave self-employment. Interestingly, wealth seems to have no effect on the probability of leaving stopgap self-employment.

The rest of the chapter is organized as follows: in the next section, I describe the dataset and discuss my definitions of self-employment and stopgap self-employment. I also present descriptive statistics to characterize how self-employed workers differ from other workers, and how stopgap self-employed workers differ from other self-employed workers. In section 3, I study transitions into self-employment, both as a stopgap and for other reasons, paying particular attention to the effects of wealth and labor market conditions on workers' decision to transition into self-employment. In section 4, I analyze transitions out of self-employment. Finally, I present conclusions in section 5.

2. Data and Descriptive Statistics

2.1. Data Description

The data used for this study are drawn from the Panel Study of Income Dynamics (PSID). The PSID is a large panel data set that has followed a fixed set of workers and their spin-offs since 1968. Until 1997 surveys were conducted on a yearly basis. Since 1997, surveys have been done every other year. An important advantage of the PSID over other data sets is that it has information on workers' financial wealth. The PSID started asking questions on wealth in 1984. For the next fifteen years, wealth questions were

asked every 5 years, and since 1999 they have been included in every survey. The samples I examine in this study are centered on the years 1989 and 1994. The reason for choosing these years is two-fold. First, data on wealth is available in those two years. Second, if years after 1997 were used, much of the high frequency dynamics in employment decisions would disappear given that data is collected only biannually.

Following Hurst and Lusardi (2004), I merge data centered on 1989 and 1994 in order to get an adequate sample size. I restrict the sample to male household heads aged 21-60 to reduce variation in labor force participation. I also drop observations on farmers, farm laborers or farm managers because agricultural self-employment has different features than other forms of self-employment. The resulting sample contains 9,808 observations, with 57.62% of them centered around 1994.

2.2. Definition of Self-Employment

Defining self-employment and identifying self-employed workers is not trivial. The International Conference of Labour Statisticians (ICLS), organized by the International Labour Organization (ILO), defines self-employment jobs as those where the remuneration is directly dependent upon the profits derived from the goods and services produced. Self-employed workers are those who hold a self-employment job. There are two points worth highlighting about this definition. First, it allows workers with multiple jobs to be both self-employed and in paid-employment. Second, this definition excludes from self-employment those workers who receive a salary from their own business, given that remuneration is then not directly dependent upon the profits.

The definition of self-employment used in this dissertation chapter departs from the

ICLS definition on the two points mentioned above. A worker is considered to be self-employed if and only if he reports being self-employed in his main job. This definition does not allow workers to be both self-employed and in paid-employment. Workers who have a second self-employment job are not considered to be self-employed if their primary job is in paid-employment. Whether a job is a main job or a secondary job is self-assessed by the worker. My definition also departs from the ICLS definition in that it allows workers who receive a wage or salary from their business to be considered self-employed. In the sample, 42.11% of those reporting being self-employed on their main job also report receiving a salary in that job. Overall, 11.44% of the workers in my sample report being self-employed in their main job at the time of the interview.

2.3. Descriptive Statistics of Self-Employed Workers

Before comparing workers using self-employment as a stopgap and workers that are self-employed for other reasons, I present some descriptive statistics for all self-employed workers and compare them with the characteristics of those working for others. The definitions of the variables used below are presented in the appendix.

Consistent with Evans and Leighton (1989) and others, self-employed workers tend to be older and more educated on average, with a predominance of married males (see Table A2 in the appendix). Self-employment is particularly low among blacks, the unmarried and very young workers (see Table A1). Self-employed workers also tend to be wealthier and have a higher hourly labor income on average (see Table A2). While looking at means can be misleading given the skewness of the labor income and wealth

distributions, the self-employed also have higher median wealth and labor income.²⁶

Another important feature of the data is that dispersion in hourly labor income and wealth is much higher for self-employed workers than for other workers (see the 90-10th percentile differentials in Table A2). The standard deviation of hourly income for self-employed workers is over three and a half times higher than among workers in paid-employment. For wealth, the standard deviation among self-employed workers is almost four and a half times higher than that among workers in paid-employment. Given the high dispersion in labor income and wealth among self-employed workers, treating the self-employed as a homogenous group might not be a good idea. In what follows, I study workers using self-employment as a stopgap separately from other self-employed workers.

2.4. Stopgap and Other Self-Employed Workers

2.4.1. Definitions of Stopgap Self-Employment

Stopgap self-employment refers to the use of self-employment as a temporary substitute for other (more desired) types of employment. Even if the theoretical notion of stopgap self-employment is straightforward, empirically distinguishing workers using self-employment as a stopgap from other self-employed workers is not. Given that there is no uncontroversial way of identifying stopgap self-employed workers in the data, I use three different working definitions of stopgap self-employment. The first definition is related to job search while the second is related to business ownership. The third

²⁶ One needs to be careful when interpreting differences in reported labor income between those working for firms and self-employed workers. Most who work for firms do not report their firm's contributions to health insurance, retirement plans and social security. Self-employed workers do not receive these benefits.

definition is a combination of the previous two.

According to the first definition, a worker is stopgap self-employed if he is currently self-employed and searching for a job. This definition corresponds to the one used in the theoretical work of Rasteletti (2009a). To identify whether a worker is looking for a job, I use the PSID question "Have you been looking for another job during the past four weeks?" If the answer to this question by a self-employed worker is yes, I classify him as a stopgap self-employed. In my sample, 6.91% of all self-employed workers are stopgap self-employed according to this definition. This is likely to be an underestimate of the extent of stopgap self-employment, because it does not include discouraged workers. These workers are willing to work for others, but are not currently searching because they perceive a low probability of finding a job.

According to our second definition, a worker is stopgap self-employed if he is currently self-employed and reports not owning a business. To identify business owners I use the PSID question "Did you (or anyone else in the family there) own a business or have a financial interest in any business enterprise?" If the worker reports not owning a business, I classify him as a stopgap self-employed. In my sample, 16.90% of all self-employed workers are stopgap self-employed according to this definition. The estimated extent of stopgap self-employment is thus over twice as large using this definition, compared to the definition based on job search.

Business ownership has been used to identify self-employed workers in the previous empirical literature on entrepreneurship, with entrepreneurs being defined as self-

employed business owners.²⁷ In the PSID data, however, many self-employed workers declare not owning a business. I use this difference in self-perception to distinguish among the self-employed. This is sensible strategy for two reasons: First, introspectively, it seems likely that a worker using self-employment as a stopgap would be less likely to consider his income generating activity a business. Secondly, if one takes the subsample of self-employed workers and runs a probit model to estimate the likelihood of remaining self-employed for at least one more year controlling for worker characteristics and a dummy variable for self-reported business ownership, the coefficient of this dummy variable is positive and highly significant.²⁸ This result seems to imply that self-employed workers who report not owning a business are less attached to self-employment, something one would expect from workers using self-employment as a stopgap.

The third definition of stopgap self-employment is a combination of the previous two. A worker is stopgap self-employed if he is currently self-employed and searching for a job or if he is currently self-employed and reports not owning a business. According to this definition of entrepreneurship, 21.07% of the self-employed workers in the sample are stopgap self-employed. There is some overlap between the first two definitions of stopgap self-employment. Of all workers that are stopgap self-employed according to the job-search definition, 39.74% of them are also stopgap self-employed according to the business-ownership definition. For workers that have been self-employed for less than a year, this percentage increases to 47.61%. The overlap is much smaller when focusing on

²⁷ This the definition of entrepreneurship is used by Cagetti and De Nardi (2006). Their entrepreneur also has to have an active management role in the business he owns. Information on active management is not available in the PSID for the years studied.

²⁸ The estimates of two probits, one for all self-employed workers and other for self-employed workers who have spent less than a year in self-employment, are reported in table A3 in the appendix.

workers that are stopgap self-employed according to the business ownership definition. This is not surprising given that there are two and a half times more stopgap self-employed according to the business-ownership definition. Of all workers that are stopgap self-employed according to the business-ownership definition, 16.25% of them are also stopgap self-employed according to the job-search definition. For workers that have been self-employed for less than a year, this percentage increases to 21.11%. Table A4 in the appendix shows the degree of overlap for workers with different characteristics, and presents the correlation between the indicator variables for the two definitions for various subsamples. The correlation for the sample of all self-employed is 0.1660, which is statistically different from zero.

A notable feature of these definitions is that whether a worker is stopgap self-employed or self-employed for other reasons might change over time. Some workers might initially use self-employment as a stopgap, only to realize later that self-employment is a good opportunity for them. Once they stop looking for a job or start reporting they own a business, they become self-employed for other reasons according to my definitions. Similarly, workers might initially be drawn to self-employment by some of its pecuniary or non-pecuniary attributes, only to realize later that self-employment is not appropriate for them. These workers are initially not stopgap self-employed. Once these workers start looking for a job working for others or report not owning a business, they become stopgap self-employed. Depending on the definition of stopgap self-employment, 7.17% (9.70%, 13.81%) of surviving self-employed workers transition from one type of self-employment to the other, using the job search (business-ownership and combined) definitions of stopgap self-employment, respectively. This convention for

reporting figures obtained from the three definitions will be used in the rest of the chapter. Transitions from one group of self-employed to the other are relatively more common among the initially stopgap self-employed, as 82.92% (55.14%, 55.58%) of those initially stopgap self-employed who remain self-employed next year transition to non-stopgap status, while the opposite figures are only 4.30% (5.56% and 8.43%) for those initially in non-stopgap self-employment. Transitions are more common among self-employed workers in their first year in self-employment, of whom 11.65% (17.24% and 18.66%) change groups. For those who have spent more than a year in self-employment, only 5.80% (13.22% and 13.24%) change groups conditional on remaining self-employed.

2.4.2. Descriptive Statistics

Regardless of the definition used, stopgap self-employment is more prevalent among newly self-employed workers. Of all workers using self-employment as a stopgap, 51% (47% and 46%) have been self-employed for a year or less, and 71% (70% and 68%) have been self-employed for 2 years or less, using the job search (business-ownership and combined) definitions of stopgap self-employment, respectively. Meanwhile, the proportion of other self-employed workers that have been self-employed for a year or less is only 27% (25% and 24%), while the proportion self-employed for less than two years is 40% (37% and 36%). The kernel densities for years spent in self-employment are presented in Figure A1.

These differences are also reflected in the survival rate of newly self-employed workers, defined as those not self-employed in the previous year. While only 35.52%

(30.55% and 34.92%) of newly self-employed workers in the stopgap group remain self-employed in the following year, 67.22% (74.42% and 76.59%) of other newly self-employed remain self-employed in the following year. This difference becomes bigger by the second year. While only 21.39% (18.04% and 21.17%) of newly stopgap self-employed workers remain self-employed for two years or more, 52.71% (59.30% and 61.51%) of other newly self-employed workers remain self-employed for two years or more.

The stopgap self-employment rate is particularly high among newly self-employed workers who experienced an unemployment spell before becoming self-employed. I find that 12.70% (28.65% and 35.31%) of all newly self-employed workers who experienced unemployment immediately before becoming self-employed use self-employment as a stopgap (see Table A5). Meanwhile, 29.49% (20.44% and 19.53%) of newly stopgap self-employed workers experienced unemployment before becoming self-employed, while only 6.54% (5.88% and 5.30%) of other newly self-employed workers experienced unemployment before entering self-employment. These differences are consistent with theories suggesting that some workers are pushed into self-employment.

Relatively high rates of stopgap self-employment are also observed among blacks, singles and very young workers (see Table A5). Stopgap self-employment is also relatively high among high school drop-outs. It is interesting to note that the groups more likely to use self-employment as a stopgap also have low overall rates of self-employment.

Stopgap self-employed workers have lower hourly income and lower wealth than other

self-employed workers (see Table A6). They also work fewer hours, which might reflect their lower commitment to the businesses they are running.

2.4.3. Occupations and Industries

There are important differences in occupation and industry affiliation between workers that use self-employment as a stopgap and those who are self-employed for other reasons. Table A7 reports the proportions of workers in different occupation groups, conditional on being stopgap self-employed or being self-employed for other reasons. The main difference in the distribution over occupations is for managers and administrators. While 41% (44% and 45%) of those who are self-employed for other reasons report being a manager or an administrator, only 24% (15% and 18%) of those using self-employment as a stopgap report being in that occupational group. Another important difference is for laborers, who comprise 11% (9% and 10%) of those using self-employment as a stopgap, but only around 4% of those self-employed for other reasons.

With respect to industry, all definitions suggest that there are relatively few stopgap workers in manufacturing (see Table A8). The job search definition suggests that there is a relatively high proportion of stopgap self-employed workers in agriculture, forestry, and fisheries as well. This result is not obtained using the other two definitions. Meanwhile, the business ownership and combined definitions suggest a relatively low proportion of stopgap self-employed workers in wholesale trade and a relatively high proportion in professional and related services. These results are not obtained using the job-search definition.

3. *Transitions Into Self-Employment*

Worker transitions into self-employment have received plenty of attention in the literature given their relevance for business creation. Two questions that have been widely studied are: 1) how do individual labor market histories and labor market conditions affect workers' decision to become self-employed? and 2) how does access to credit affect workers' decision to become self-employed? With respect to the first question, Evans and Leighton (1989) find that being unemployed increases the probability of becoming self-employed. Blanchflower and Oswald (1991) find that higher regional unemployment rates also increase the probability of entering self-employment. Rissman (2003) finds that young workers are more likely to enter self-employment in recessions and leave in booms. With respect to access to credit, researchers have usually looked at the effect of workers' wealth on the decision to become self-employed. A positive effect of wealth on the probability of becoming self-employed is usually interpreted as credit constraints preventing workers from starting their businesses. Several studies have found evidence that higher levels of wealth increase the probability of transitioning into self-employment.²⁹ More recently, Hurst and Lusardi (2003) found that this positive relation is only present among the very rich.

This section differs from previous studies in the literature in that I study transitions into stopgap self-employment separately from other transitions into self-employment. To study transitions into self-employment, I use the sample of all agents who are not self-employed in 1989 or 1994. I first estimate a probit model of the probability of observing

²⁹ Studies finding a positive effect of wealth on the probability of transitioning into self-employment include Holtz-Eakin, Joulfaian, and Rosen (1994), Blanchflower and Oswald (1998), Evans and Jovanovic (1989), and Evans and Leighton (1989), among others.

a worker transitioning into self-employment in the following year. This is the type of model usually estimated in the literature. I then estimate separate probit models of the probability of transitioning into stopgap self-employment and self-employment for other reasons. This allows me to study conditions under which workers are more likely to enter self-employment with a stopgap motive or for other reasons. Given that the effects of wealth and unemployment duration on entry into self-employment might be non-linear, I include these two variables as fifth-order polynomials.³⁰ The estimated average marginal effects implied by these different probits are presented in table A9.³¹

Several findings are worth highlighting. First, being black has no significant effect on the probability of becoming stopgap self-employed, while it decreases the probability of becoming self-employed for other reasons, although the latter effect is not significant for the combined definition of stopgap self-employment. Second, having higher labor income reduces the probability of transitions into stopgap self-employment. Thirdly, as found by other papers in the literature, having been self-employed previously has a highly significant positive effect on the probability of transitioning into both types of self-employment.

Distinguishing workers using self-employment as a stopgap from other self-employed workers allows me to uncover some interesting effects of unemployment on the

³⁰ I use a third order polynomial in wealth when estimating transitions into stopgap self-employment to increase precision of my estimates given the small number of such transitions. Results are qualitatively similar if I use a fifth order polynomial.

³¹ In this paper marginal effects are calculated for each individual, and then averaged across individuals. The marginal effect of a variable X is calculated in the following way: for each individual the estimated probability is calculated given the worker's characteristics. Then, a new probability is calculated for each individual by marginally increasing the value taken by X. The difference between these two probabilities is then calculated and divided by the increment in X. If X is entered as a second or higher order polynomial, the second probability is calculated by simultaneously increasing all terms in the X polynomial.

probability of becoming self-employed. I find that being unemployed has a positive and highly significant effect on the probability of transitioning into stopgap self-employment, but a smaller and insignificant effect on other transitions into self-employment. These findings suggest that the positive effect of being unemployed on the probability of becoming self-employed found in previous literature is mainly driven by workers transitioning into stopgap self-employment. I also find that receiving unemployment compensation reduces the probability of transitioning into both types of self-employment, although the effect is not significant for transitions into stopgap self-employment when the job search definition is used, and for transitions into non-stopgap self-employment when the business ownership and combined definitions are used.

Some interesting effects appear when studying the effects of labor market conditions on transitions into self-employment. Given that the PSID does not have information on county level unemployment for 1994, we use the subsample of 1989 observations to look at how the county unemployment rate affects workers' decision to become self-employed.³² Looking at the effects of county level unemployment rates on the probability of becoming self-employed is important because different regional unemployment rates might affect how difficult it is for an unemployed worker to find a new job. All else equal, one would expect that the higher the local unemployment rate, the higher the probability of transitioning into stopgap self-employment. I re-estimate the probit model used before, but now including two new control variables: the county unemployment rate and the county unemployed rate interacted with a dummy for being unemployed.

³² While county level unemployment rates for 1994 are relatively easy to obtain, one cannot construct county level unemployment rates using the public release of the PSID because the PSID has suppressed information on the county of residence.

Contrary to what one might expect, I find that the interaction term for being unemployed and the unemployment rates at the county level has a negative effect on the probability of becoming self-employed, although the effect is not significant (see Table A10 in the appendix). This negative effect might reflect the fact that workers become less willing to enter self-employment, given that it might take them longer to transition out of self-employment when the unemployment rate is higher.

Finally, I look at the effect of wealth on the probability of becoming self-employed. Previous researchers have focused on wealth to study the effects of liquidity constraints on the probability that a worker starts a business. However, one would expect liquidity constraints to play different roles for workers transitioning into entrepreneurship and for workers transitioning into stopgap self-employment. For workers transitioning into stopgap self-employment, wealth might be important not because workers do not have sufficient liquidity to start their business projects, but because the lack of financial resources might force workers to choose self-employment over unemployment. These differences in the effect of wealth are confirmed by the data (see Figure A2). The probability that a worker enters self-employment for other reasons is monotonically increasing in wealth (marginal effects at different quintiles of the wealth distribution are presented on Table A11). Meanwhile, the probability of entering stopgap self-employment as a function of wealth has an inverted-U shape, peaking at \$95,000 (\$73,000 and \$64,440) for the search (business ownership and combined) definition. These peaks are located at the 61st (54th and 51st) percentiles of the wealth distribution including all workers. When looking at the wealth distribution of self-employed workers only, these peaks are located at the 33th (28th and 25th) percentiles of the wealth

distribution. To better understand the reason behind the inverted U-shape, I ask whether wealth plays different roles for workers who experienced an unemployment spell and workers who did not. This question is relevant because the U-shape could be a consequence of a composition effect, given that 30 to 40% of workers who become stopgap self-employed experienced unemployment before transitioning into self-employment. Therefore, I re-estimate the model interacting a third order polynomial on worker's wealth with a dummy capturing whether the worker experienced an unemployment spell prior to self-employment. The marginal effects obtained from these probits are reported in Table A12 in the appendix. This estimation reveals that the inverted U-shape of the impact of wealth on the probability of stopgap self-employment is due largely to people who experienced unemployment (see Figure A3). For all definitions of stopgap self-employment the probability of becoming stopgap self-employed has an inverted U-shape in wealth for those workers who experienced unemployment between the two interview periods.³³ For workers who did not experience unemployment, the probability of becoming stopgap self-employed is monotonically decreasing in wealth for the business ownership and combined definition, and decreasing over most of the wealth distribution for the job search definition.

So far we have treated wealth as exogenous. In fact, wealth can be endogenous with respect to occupational choice, due to its possible correlation with preferences or unobserved ability. The endogeneity of wealth could bias the estimated effects of wealth on the probability of transitioning into different types of self-employment. This

³³ The probability of becoming stopgap self-employed for workers who experienced unemployment peaks at \$97,000 (\$36,500 and \$40,000) for the search (business ownership and combined) definition. The probability of becoming stopgap self-employed for workers who did not experience unemployment peaks at \$82,000 for the search definition.

endogeneity problem is more likely to be present in transitions into non-stopgap self-employment. Buera (2003) argues that in the presence of borrowing constraints, workers that want to become entrepreneurs will accumulate more wealth while working so that they can start their businesses. If this is true, wealth will then be correlated with unobserved variables that make workers more likely to become an entrepreneur, creating a bias in the estimates of wealth effects. To get around this problem, we use an instrumental variable approach. The instrumental variable used is an estimated measure of regional changes in house prices, introduced by Hurst and Lusardi (2004). This variable is constructed in the following way: first, I generate a variable capturing the change in home value of non-movers over the periods 1985-88 and 1990-93. These home values are self-reported in the PSID. I then regress this variable on household characteristics (age, education, race, gender, marital status, family size, income, employment status, and initial house value), region dummies and state economic conditions (state gross domestic product per capita in 1985 and 1990, growth rate of state GDP per capita between 1985 and 1988 and between 1990 and 1993, and the state unemployment rate in levels from 1985 to 1988 and from 1990 to 1993).³⁴ The region dummies capture regional variation in growth in house prices controlling for changes in the state economic conditions. A new variable capturing differences in regional housing value appreciation is created by assigning to each household the estimated coefficient on the region dummy for the state they live in. This new variable is used as the instrument for wealth.

³⁴ Hurst and Lusardi include the region dummies and state economic conditions to control for the fact that house prices are not purely exogenous, but rather be affected by some of the same factors that affect household decision to become self-employed.

To make sure that the inverted U shape for the effect of wealth on the probability of becoming stopgap self-employed is not a consequence of the endogeneity problem, I now estimate a linear probability model including all the same controls as in the previous probit model but instrumenting the third order polynomial in wealth with a third order polynomial in regional house appreciation. The estimates are reported in Table A13 in the appendix. The estimated effect of wealth on the probability of transitioning into stopgap self-employment maintains its inverted U shape (see Figure A4), but this shape is not statistically significant. More specifically, the predicted probability is never different from zero. This might be due to a relatively weak first-stage while implementing the instrumental variable estimation (see R^2 of the first-stage regression on Table A13).

4. Transitions Out Of Self-Employment

In the PSID sample used, the proportion of self-employed workers that leaves self-employment from one year to the next is 20.52%. This proportion differs greatly among those who have spent more or less than one year in self-employment. For those who have spent more than a year in self-employment, the proportion leaving is 15.54%. For those self-employed for less than a year, the proportion that leaves is over two times higher, at 33.95%. As one might expect, the proportion of workers leaving self-employment is particularly high among those using self-employment as a stopgap (see Table A14). The proportion of stopgap self-employed workers that leaves self-employment is between three and five times higher than that of other self-employed workers, depending on the definition used. A large difference in exit rates between stopgap and other self-employed

workers is observed both among newly self-employed workers and workers who have spent more than a year in self-employment.

To study transitions out of self-employment in greater detail I estimate two sets of probit models of the probability that a currently self-employed worker will leave self-employment within a year. In the first set, the probits are estimated using the sample of all self-employed workers, and a dummy variable is included to capture whether the worker is stopgap self-employed. Given the relatively high exit rate of self-employed workers during their first year in self-employment, I also estimate separate probits for those who have spent more or less than a year in self-employment. Next, I estimate separate probits for the samples of stopgap and non-stopgap self-employed workers. This allows other variables to have different effects for the stopgap and non-stopgap self-employed. Given that the effect of time spent in self-employment might be non-linear, this variable is entered as a fifth-order polynomial in all probits.

The estimated average marginal effects for the first set of regressions are presented in Table A15. As one might expect, using self-employment as a stopgap has a large positive and highly significant effect on the probability of leaving self-employment. Exits from self-employment seem not to be affected by educational achievement, except for newly self-employed high-school graduates. Meanwhile, being married, being older and working longer hours reduce the probability of leaving self-employment for self-employed workers with more than a year in self-employment. Finally, time spent in the current self-employment spell has a highly significant negative effect on the probability of leaving self-employment in the sample of all self-employed workers, but positive and significant in the sample of with self-employed workers with more than a year in self-

employment.

Estimating different probits for the stopgap and nonstop self-employed does not shed much light on transitions out of self-employment since few coefficients are statistically significant (See Table A16). In the case of workers using self-employment as a stopgap, no clear pattern emerges. Most age groups dummies have a negative effect, with a few being significant. Being black decreases the probability of leaving self-employment according to the job search definition but increases it according to the business ownership definition. Other individual characteristics might fail to have significant effects simply because transitions are calculated over a one-year period. In the sample, the median duration of an unemployment spell is nine weeks. A year seems to be enough time for most stopgap self-employed workers to find a job working for others. In the case of other self-employed workers, hours worked and being married also have a negative and significant effect. Time spent in self-employment has a negative effect, although it is only significant when using the business ownership definition.

Finally, I look at the determinants of whether exiting self-employed workers transition into paid-employment, unemployment or out-of-the-labor-force. I estimate different probit models of the probability that a self-employed worker transitions into paid-employment, unemployment or out of the labor force, conditional on being self-employed. All these models have a dummy variable capturing whether the worker is stopgap self-employed.³⁵ The estimated average marginal effects are presented in Table A17. Interestingly, being stopgap self-employed has a large positive effect on the

³⁵ One could also estimate separate probits for currently stopgap and non-stopgap self-employed workers leaving self-employment within a year. The problem is that the resulting samples from this partition become too small, rendering all coefficients insignificant.

probability of transitioning into paid-employment and insignificant effects on the probability of transitioning into unemployment. Given that stopgap self-employed workers are far more likely to leave self-employment, the non-significance of the stopgap dummy for transitions into unemployment seems to imply that stopgap self-employed workers are less willing than other self-employed workers to become unemployed. This result is consistent with the notion that the stopgap self-employed use self-employment as an employment option to avoid unemployment. Interestingly, being stopgap self-employed increases the probability of exiting the labor force, although this effect is not significant when the job search definition is used.

5. Conclusions

This chapter uses data from the PSID to revisit existing results in the empirical literature on self-employment, treating workers using self-employment as a stopgap separately from workers that are self-employed for other reasons. The main features of stopgap self-employment unveiled in this chapter are the following: 1) Stopgap self-employment is most common among young workers, those who have experienced recent unemployment, minorities, women and workers with low educational achievement. 2) Workers using self-employment as a stopgap have relatively short self-employment spells. 3) Workers in stopgap self-employment tend to have lower hourly labor income and to work fewer hours. 4) An important fraction of stopgap self-employed workers work as laborers and in service related occupations. This chapter also discovers different patterns of transition into and out of self-employment for these different types of self-employed workers. For those not using self-employment as a stopgap, the probability of entering self-

employment increases monotonically with wealth. On the other hand, for those using self-employment as a stopgap, the probability of entering self-employment has an inverted U shape as a function of wealth, with the peak at a relatively low wealth level. I also find that workers' labor market histories are only relevant for stopgap self-employment. More specifically, being unemployed increases the probability of becoming stopgap self-employed, but has no effect on the probability of becoming self-employed for other reasons.

This dissertation chapter should serve as a reminder to researchers that not all workers are self-employed for the same reasons. Estimates of the effects of certain variables of interest on the behavior of the self-employed or the self-employed-to-be are an average of the effects on different types of self-employed workers. This averaging can create biases that hide the real effects of interest. This dissertation chapter suggests that one should distinguish effects on workers using self-employment as a stopgap from effects on other self-employed workers. More work should be devoted to finding new and better ways of distinguishing different types of self-employed workers, in order to get more precise estimates of effects of interest.

Chapter 3: A Theory on the Impact of Technological Change on Entrepreneurship and Unemployment.

Joint Work with Jose Plehn-Dujowich and Dunli Li.

1. Introduction

Technological growth, entrepreneurship, and unemployment influence each other in numerous ways, forming a trio of inter-related components, yet the literature has traditionally emphasized the endogenous determination of one or two components of this trio, and the exogenous impact of one component on another, without taking into account the third. Consider the impact of entrepreneurship on growth. Endogenous growth theory suggests that entrepreneurship is an important determinant of growth. Such models predict or assume that an increase in the resources devoted toward innovation and R&D mechanically lead to higher growth, implying positive correlation between entrepreneurship and growth (Aghion and Howitt, 1992; Grossman and Helpman, 1991; Segerstrom, 1991, 1998; Romer, 1990; Jones, 1995); entrepreneurship is the means by which to launch, but not sustain, the economy, such that eventually it ceases altogether (Peretto, 1998, 1999a); and the growth rate and rate of entry may be positively or negatively correlated as the economy evolves over time (Peretto, 1999b). Next consider the impact of growth on unemployment. An increase in growth leads to a decrease in unemployment when technological change is disembodied (Pissarides, 1990); or an increase in unemployment when technological change is embodied (Aghion and Howitt, 1994). Finally, Fonseca et al. (2001) study the endogenous determination of entrepreneurship and unemployment, to find that the two are negatively related.

We argue that the three components of technological growth, entrepreneurship, and unemployment should not be studied in isolation or in pairs because doing so may engender a misleading over-simplification. Indeed, we find that these important results in the literature concerning the impact of growth on unemployment no longer necessarily hold when one incorporates entrepreneurship; and the result concerning the relationship between entrepreneurship and unemployment no longer necessarily holds when one incorporates growth. Whereas, ideally, a theory should have all three components be endogenous, we pursue the first step towards integration in a model with exogenous growth that, nonetheless, attempts to include hints of endogenous growth theory.

By developing a unified framework incorporating occupational choice and different types of labor markets and exogenous technological change, this paper provides a comprehensive taxonomy of the various mechanisms by which technological growth affects entrepreneurship and unemployment. The model is constructed as follows. There is a continuum of agents that choose between being a worker or an entrepreneur. Entrepreneurs create and manage jobs, while workers occupy the jobs created by entrepreneurs. Agents differ in terms of the number of workers they can manage if they become entrepreneurs; however, entrepreneurial ability does not affect an agent's productivity if he becomes a worker. In equilibrium, agents with sufficiently high ability become entrepreneurs, while those below a threshold level of ability become workers. We consider two types of exogenous technological change, disembodied and embodied. With disembodied technological change, the productivity of all jobs increases with the growth rate. With embodied technological change, only the productivity of new jobs increases with the growth rate, giving rise to creative destruction, the process by which

entrepreneurs destroy existing jobs in order to implement new technologies. Furthermore, we make two assumptions that are reduced-form representations of general equilibrium outcomes: we allow for the possibilities that technological growth augments the effective discount rate and enhances entrepreneurial ability at managing workers.

Given that the decision to become an entrepreneur may depend on the characteristics of the labor market, we analyze both forms of technological change under two different labor market structures. First, we consider the case of perfectly competitive labor markets. Workers can move freely between jobs, at no cost. Price competition among firms for workers guarantees that all workers are paid the same wage and that all entrepreneurs can hire all the workers they desire at the equilibrium wage. In this case, there are no unemployed agents or unfilled vacancies in equilibrium. Second, we analyze the case in which there are search frictions in the labor market. The allocation of jobs to workers takes place according to a process of search and matching, and unemployment can result in equilibrium. In order to find workers, entrepreneurs post vacancies; and in order to find jobs, workers engage in random search.

Technological growth has an impact on entrepreneurship via three mechanisms: the capitalization, firm size, and employment duration effects, relating to how growth affects the effective discount rate, entrepreneurial ability (which determines the optimal size of a firm), and the optimal duration of a job, respectively. The impact of these mechanisms on entrepreneurship differs depending on whether the labor market is frictionless. In the absence of labor market frictions, an increase in growth dampens entrepreneurship if and only if growth enhances entrepreneurial ability at managing workers, i.e. the firm size effect is present. Once labor market frictions are introduced, however, this result no

longer holds. The impact of an increase in technological growth on entrepreneurship depends on how the three mechanisms interact. While a positive capitalization effect enhances entrepreneurship, positive firm size and employment duration effects tend to dampen entrepreneurship under reasonable parameter restrictions. The overall effect thereby depends on the relative strength of each of the three competing forces.

The introduction of labor market frictions also allows us to study the impact of technological growth on unemployment, which operates via three mechanisms. First, there is the composition effect: by affecting an agent's decision to become an entrepreneur, growth influences the number of workers relative to entrepreneurs. All else being equal, the smaller is the proportion of agents that are entrepreneurs, the higher is the unemployment rate. Second, there is the job creation effect: a change in the proportion of entrepreneurs affects job creation, which in turn influences the probability that a worker finds a job. Given the occupational choice decision and optimal duration of a job, the more difficult it is for workers to find jobs, the higher is the unemployment rate. Third, there is the creative destruction effect: a higher growth rate decreases the optimal duration of a job, which tends to increase unemployment.

To summarize, technological growth may have a positive or negative impact on entrepreneurship and unemployment, in such a manner that any combination is possible. Overall, an increase in growth is more likely to enhance entrepreneurship and dampen unemployment when the interest rate does not increase significantly with growth, technological change is disembodied, and growth enhances entrepreneurial ability at managing workers.

The most influential theoretical works studying the effect of technological growth on

unemployment in an economy with labor market frictions are those of Pissarides (1990), Aghion and Howitt (1994), and Mortensen and Pissarides (1998). These models differ from ours in that they do not include entrepreneurship, the endogenous determination of firm size, or the equilibrium effect of growth on the interest rate; thus, by not having an occupational choice decision or agents managing workers, the composition, job creation, and firm size effects are absent. Pissarides (1990) presents a model wherein the productivity of all existing jobs increases at the rate of technological change (i.e., it is disembodied). In this setting, unemployment is a decreasing function of growth: an increase in the growth rate reduces the effective discount rate, which leads to an increase in job creation and a reduction in unemployment. In Aghion and Howitt (1994), only the productivity of new jobs increases at the rate of technological change (i.e., it is embodied), capturing the Schumpeterian notion of creative destruction that technological change renders jobs obsolete. In this setting, unemployment is an increasing function of growth: given the obsolescence of jobs, firms anticipate that jobs have a shorter life, which reduces job creation and increases unemployment. Pissarides and Vallanti (2007) present a model in which both embodied and disembodied technological change take place simultaneously. The impact of growth on unemployment is undetermined, but the authors find that in a calibrated version of their model, the negative effect of TFP growth on unemployment observed in the data is inconsistent with embodied technology. The reason for this result is that when technology is embodied, the capitalization effect has a much smaller quantitative impact on unemployment than the creative destruction effect. Finally, Mortensen and Pissarides (1998) criticize the Aghion and Howitt (1994) assumption, also made in Pissarides and Vallanti (2007), that jobs need to be destroyed in

order for firms to implement new technologies. The authors present a model in which technology can be updated, at some cost, without destroying jobs. The authors find that, depending on the cost of updating technology, higher growth can lead to either higher or lower unemployment: it is only in the case when updating technology is too expensive that higher growth leads to higher unemployment.

When one takes into account the additional mechanisms we identified by which technological growth affects unemployment, some of the key results in these and related papers no longer hold. Entrepreneurship gives rise to the composition effect of technological growth on unemployment; and the firm size effect operating via the impact of growth on entrepreneurial ability. The consequences of these effects can lead to reversals in common wisdom. Specifically, contrary to Pissarides (1990), we find that unemployment does not necessarily decline in response to an increase in the rate of disembodied technological change; contrary to Aghion and Howitt (1994), unemployment does not necessarily rise in response to an increase in the rate of embodied technological change; and contrary to Fonseca et al. (2001), entrepreneurship and unemployment are not necessarily negatively related when one takes into account their responses to a change in the growth rate.

The rest of the paper is organized as follows. In section 2, we provide an overview of the model, discuss the mechanisms at work, and present figures that illustrate some of our main results. In section 3, we study in greater detail how technological change affects entrepreneurship in an economy with a frictionless labor market. In section 4, we perform a similar analysis in an economy with labor market frictions, and study how unemployment is affected by the rate of technological growth. Finally, section 5

concludes.

2. Overview of the Model

The economy consists of a continuum of infinitely-lived agents, with their mass normalized to one. Following Fonseca et al. (2001), at any point in time, an agent chooses to become an entrepreneur who creates and manages jobs, or a worker occupying a job created by an entrepreneur. The number of workers an agent can manage is a function of his entrepreneurial ability α which is drawn from the distribution $F(\alpha)$ with the support $[0, \alpha^{max}]$. To become an entrepreneur, an agent pays the business start-up cost $p(t)K$, where $p(t) = e^{gt}$, t denotes the date, and g is the exogenous rate of technological growth. After paying the start-up cost, entrepreneurs post vacancies to hire workers. The cost of posting a vacancy is $p(t)c$. Once a job has been filled, the entrepreneur has access to a production technology that is a linear function of labor up to the maximum number of workers the entrepreneur can manage. If the entrepreneur has more workers than the maximum, the extra workers are unproductive. The worker remains with the entrepreneur until the entrepreneur closes the job, the worker quits the job, or the job is exogenously destroyed. The exogenous destruction of jobs follows a Poisson process with arrival rate δ . When a worker is employed, he earns a wage $w(t)$. If a worker is unemployed, he has access to a home production technology with which he produces $p(t)b$. Neither entrepreneurs nor workers can save, such that consumption equals labor income.

The impact of technological growth on entrepreneurship depends on the nature of the labor market and the type of technological change. With respect to the nature of the labor

market, we study two cases. First, we consider perfectly competitive labor markets. Workers can move freely between jobs at no cost. Price competition among firms for workers guarantees that all workers are paid the same wage and that all entrepreneurs can hire all the workers they desire at the equilibrium wage. Thus, there are no unemployed agents or unfilled vacancies in equilibrium. Second, we consider search frictions in the labor market. The allocation of jobs to workers takes place according to a process of search and matching, and unemployment can result in equilibrium. In the absence of growth, this case corresponds to the model in Fonseca et al. (2001).

With respect to the type of technological change, we study two cases. First, we consider disembodied technological change: entrepreneurs update the technology used for production in all existing jobs immediately at no cost. Second, we consider embodied technological change: entrepreneurs destroy existing jobs in order to implement new technologies. The main difference between the two cases is that while the productivity of all jobs increases at the rate g in the disembodied case, only the productivity of new jobs increases at the rate g in the embodied case.

Finally, to capture the ways in which the rate of technological growth affects the interest rate and entrepreneurial ability at managing workers, we introduce the following two assumptions that are reduced-form representations of outcomes that would arise in a general equilibrium model of endogenous growth:

Assumption 1: Along the balanced growth path, the interest rate is $r = R(g)$, where $R(g)$ is a differentiable function.

Assumption 2: Along the balanced growth path, the maximum number of workers an entrepreneur with ability α can manage is $A(g)$, where $A(g)$ is a differentiable function.

Before analyzing the four cases with or without labor market frictions and with embodied or disembodied technological change, we summarize the general structure of the model and advance some intuition of the results. Throughout the paper, we focus on the impact of technological change on entrepreneurship and unemployment along the balanced growth path.

Each of the four cases can be summarized by a system of two equations in two unknowns. First, using the same terminology as in Fonseca et al. (2001), there is the entrepreneurship equation, reflecting the agent's occupational choice decision. Denote the value of being a worker by $W(t)$ and the value of being an entrepreneur by $S(\alpha, t)$. The value of being an entrepreneur can also be expressed in terms of the value of opening a vacancy $V(t)$: $S(\alpha, t) = A(g)V(t) - p(t)K$. We will show that $V(t)$ and $W(t)$ are independent of α , such that the occupational choice decision is governed by a reservation entrepreneurial ability $\hat{\alpha}$: an agent with ability becomes an entrepreneur if $\alpha \geq \hat{\alpha}$, otherwise he becomes a worker. The reservation ability satisfies

$$(3.1) \quad \hat{\alpha} = \frac{W(t) + p(t)K}{A(g)V(t)}$$

Second, there is the job creation equation, which ensures that the number of employed workers, entrepreneurs, and unemployed agents do not change over time. To derive this equation, first note that the proportion of employed and unemployed workers can be expressed as

$$(3.2) \quad u + n = F(\hat{\alpha})$$

where n is the proportion of employed workers, u is the proportion of unemployed agents, and $1 - F(\hat{\alpha})$ is the proportion of entrepreneurs. Meanwhile, the number of jobs created by entrepreneurs is

$$(3.3) \quad n + v = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

where n is the number of filled jobs and v is the number of vacant jobs. Without labor market frictions, we have $u = v = 0$. The job creation equation is obtained by combining equations (3.2) and (3.3):

$$(3.4) \quad F(\hat{\alpha}) = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

With labor market frictions, to ensure that the proportions of employed workers and entrepreneurs do not change of time, we require that the number of workers entering unemployment equals the number of workers leaving unemployment. We will show that for this condition to be satisfied, the following equation must hold:

$$(3.5) \quad \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha F(\hat{\alpha}) = \frac{\delta\theta + [1 - \exp(-\delta T^*)]\theta q(\theta)}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

where θ is the ratio of vacancies to unemployment, $q(\theta)$ is the rate at which vacancies are filled, and T^* is the optimally chosen age at which to destroy a job. When technological change is disembodied, T^* is infinite; and when it is embodied, T^* is finite. Finally, we will show that the proportion of unemployed workers in the steady state is given by

$$(3.6) \quad u = \frac{\delta F(\hat{\alpha})}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

Figures 3.1 and 3.2 are graphical representations of the entrepreneurship and job creation equations for each of the four cases. The entrepreneurship curve is always upward sloping.³⁶ Meanwhile, the job creation curve does not have the same slope in all cases. While the job creation curve is horizontal in a frictionless labor market, it is downward sloping with labor market frictions.³⁷ Whether technological change is embodied or disembodied does not affect the proportion of entrepreneurs with frictionless labor markets. Keeping wages fixed, the value of being an entrepreneur is greater under disembodied (relative to embodied) technological change given that the employment duration is higher. However, as the number of entrepreneurs increases, the number of unfilled vacancies increases accordingly, leading to a rise in wages, which in turn reduces the value of being an entrepreneur and therefore the number of agents that become entrepreneurs in equilibrium.

With labor market frictions, the number of entrepreneurs is usually lower with embodied technological change, but it could be higher if the equilibrium ratio of vacancies to unemployment is low enough (see Figure 3.2).³⁸ As in the case with

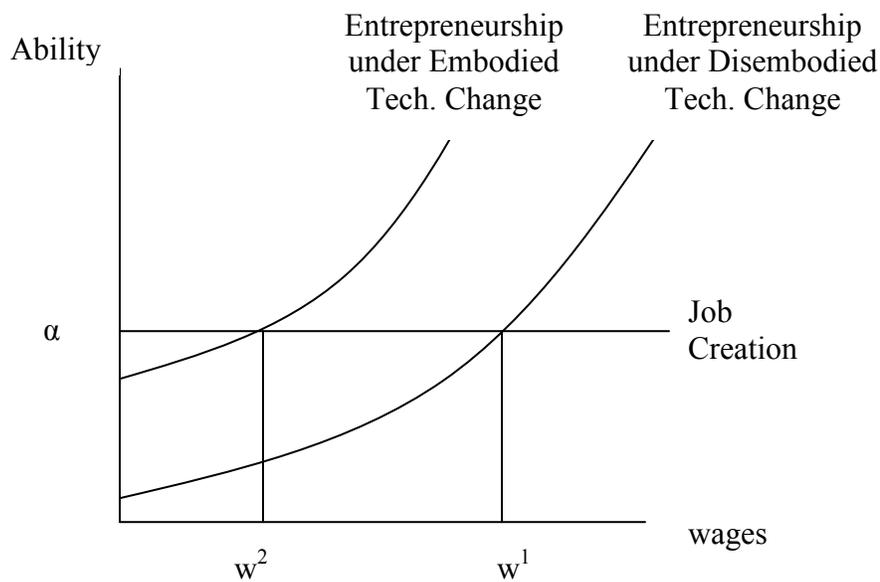
³⁶ In the case with frictionless labor markets, the entrepreneurship curve is increasing in wages because higher wages increase the value of being a worker and reduce the value of being an entrepreneur. This then leads to an increase in the reservation ability. In the case with labor market frictions, a second effect is present. We show later in this paper that wages are an increasing function of θ , the ratio of vacancy to unemployment or labor market tightness. Therefore, for wages to be higher, the labor market tightness also has to be higher. Now, when labor market tightness is higher, it becomes more difficult for entrepreneurs to fill their vacancies, while it becomes easier for unemployed workers to find jobs. Therefore, the value of being an entrepreneur is reduced and the value of being unemployed increases. This second effect leads to a further increase in the reservation ability.

³⁷ To understand why the job creation curve is downward sloping when labor market frictions are present, one must first understand the relation between wages and θ , the ratio of vacancy to unemployment or labor market tightness. We show below that wages are increasing in labor market tightness. Now, when the reservation ability increases, there are fewer jobs for more workers in equilibrium. This then leads to a decrease in labor market tightness and a reduction in wages paid. This change in labor market tightness is not present in the case with frictionless labor markets because the unemployment rate is always zero. This explains why the job creation curve is horizontal when labor markets are frictionless.

³⁸ When vacancies are scarce, being a worker in an economy where jobs are endogenously destroyed is less attractive to the agent because he may spend a long period of time unemployed if he is laid off. This makes being an entrepreneur more attractive as compared to the case when jobs are not endogenously destroyed

frictionless labor markets, wages are higher with disembodied technological change: ceteris paribus, being an entrepreneur is more profitable when technological change is disembodied, which increases the number of vacancies, leading to higher wages compared to the case with embodied technological change.

Figure 3.1: The Entrepreneurship and Job Creation Curves when the Labor Market is Frictionless

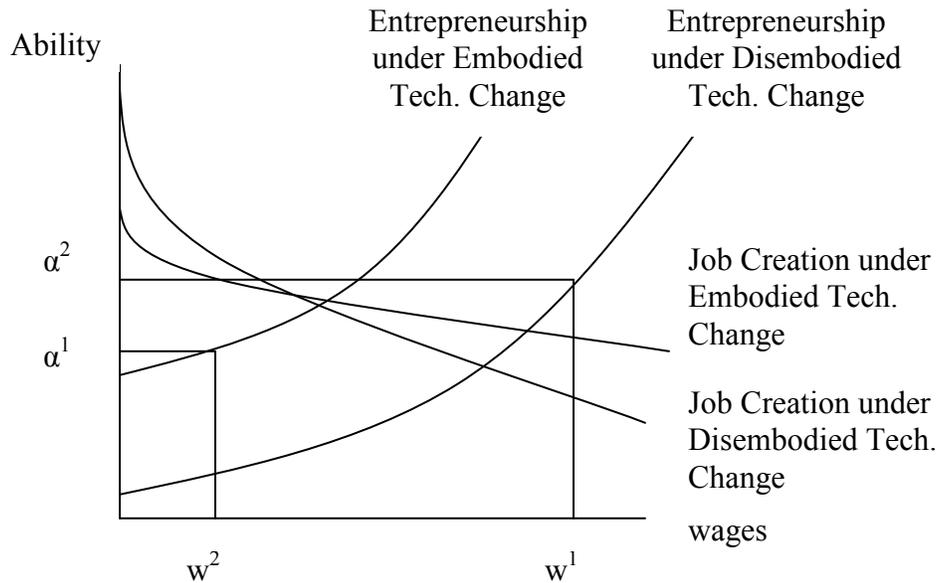


Technological growth affects entrepreneurship via three mechanisms. First, there is the capitalization effect, which operates through the effective discount rate $R(g) - g$. Second, there is the firm size effect, which operates through changes in the number of worker an entrepreneur can manage, namely $A(g)$. Finally, there is the employment duration effect, which operates through the optimal duration of a job T^* and is only

(i.e., when technological change is disembodied). Therefore, when the ratio of vacancies to unemployment is very low in equilibrium, the number of entrepreneurs can be higher in the case with embodied technological change.

present when technological change is embodied.

Figure 3.2: The Entrepreneurship and Job Creation Curves with Labor Market Frictions



With frictionless labor markets, the proportion of entrepreneurs is solely determined by the position of the job creation curve. Given that neither the capitalization effect nor the employment duration effect influences the location of the job creation curve, it is only the firm size effect that matters. Therefore, technological growth affects the number of entrepreneurs if and only if growth affects the number of worker an entrepreneur can manage, i.e. $A'(g) \neq 0$. If technological growth enhances entrepreneurial ability at managing workers, i.e. $A'(g) > 0$, then a higher growth rate shifts the job creation curve upward, leading to a reduction in the number of entrepreneurs. The opposite occurs when $A'(g) < 0$.

Labor market frictions introduce numerous complications. The number of

entrepreneurs is no longer determined by the position of one of the curves. In some cases the overall effect of technological growth on entrepreneurship can be easily determined, while in others the overall effect depends on how the elasticity of the entrepreneurship curve compares to that of the job creation curve. To illustrate the three mechanisms in the presence of labor market frictions, we shut down two at a time to study how the third operates.

Suppose only the capitalization effect is present, i.e. technological change is disembodied and entrepreneurial ability is independent of growth, $A'(g) = 0$. Then the capitalization effect does not influence the location of the the job creation curve. Suppose the agent's effective discount rate $R(g) - g$ is increasing in the growth rate, $R'(g) > 1$. Then an increase in growth makes increases the effective discount rate. This makes being an entrepreneur less attractive given that an entrepreneur has to incur initial sunk costs. The entrepreneurship curve thereby shifts to the left, decreasing the number of entrepreneurs (see Figure 3.3)³⁹ The opposite occurs when $R'(g) < 1$.

Suppose only the firm size effect is present, i.e. technological change is disembodied and the effective discount rate is independent of growth, $R'(g) = 1$. If $A'(g) > 0$, an increase in the rate of technological growth shifts the entrepreneurship curve to the right and the job creation curve upward. The opposite is true when $A'(g) < 0$. The entrepreneurship curve shifts to the right when $A'(g) > 0$ because, all else equal, the higher $A(g)$ is, the more workers an entrepreneur can manage. This increases the value of being an entrepreneur and reduces the threshold ability for any given level of labor

³⁹ Figure 3 (as well as in the remaining figures in this paper) has labor market tightness in the horizontal axis, and not wages as in Figure 1 and 2. Having labor market tightness is more convenient because wages are a function of labor market tightness in equilibrium when labor market frictions are present. We use wages instead in Figure 1 and 2 because labor market tightness is not defined in the frictionless labor market case. This is because both vacancies and unemployment are zero, so their ratio is not defined.

market tightness. The job creation curve shifts upward when $A'(g) > 0$ because, for any given reservation ability, entrepreneurs now create more jobs. This leads to higher levels of vacancies, which explains the direction of the shift. Given that both curves shift in different directions, the overall effect of the impact of an increase in technological growth on entrepreneurship cannot be determined unless parameters values are assumed. The overall effect depends on the elasticities of the entrepreneurship and job creation curves together with the extent to which a change in entrepreneurial ability $A(g)$ causes shifts in both curves. The greater is the elasticity of the entrepreneurship curve relative to that of the job creation curve, the more likely it is that the number of entrepreneurs decreases in response to an increase in growth (see Figure 3.4).

Figure 3.3: The Impact of Technological Growth on Entrepreneurship when only the Capitalization Effect is Present and $R'(g) > 1$.

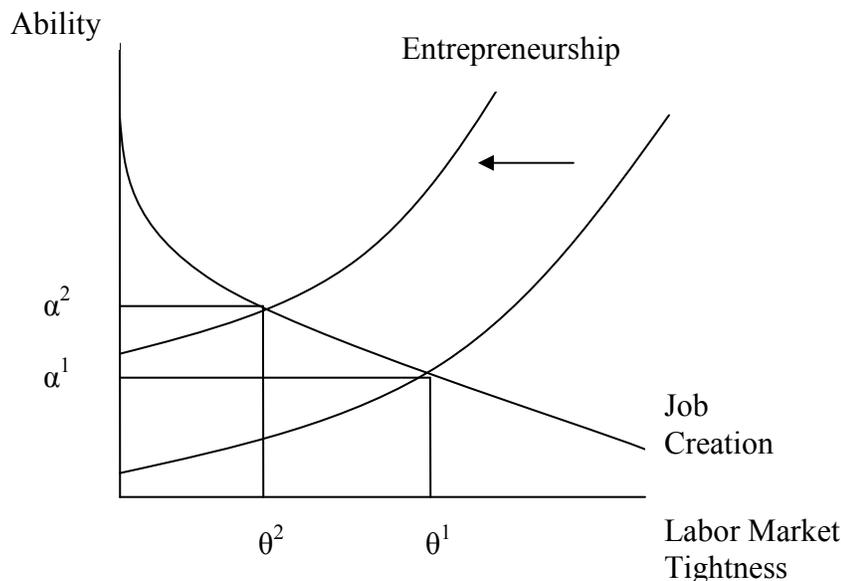
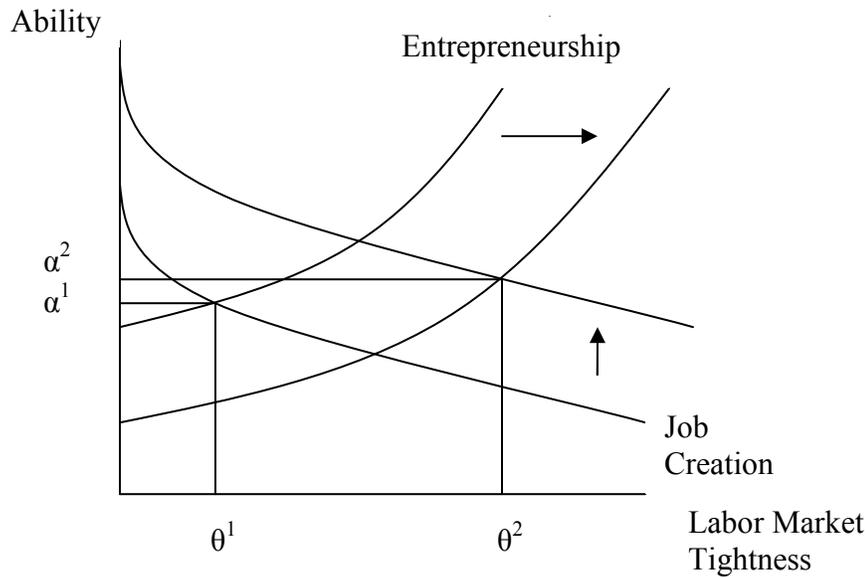


Figure 3.4: The Impact of Technological Growth on Entrepreneurship when only the Firm Size Effect is Present and $A'(g) > 0$.

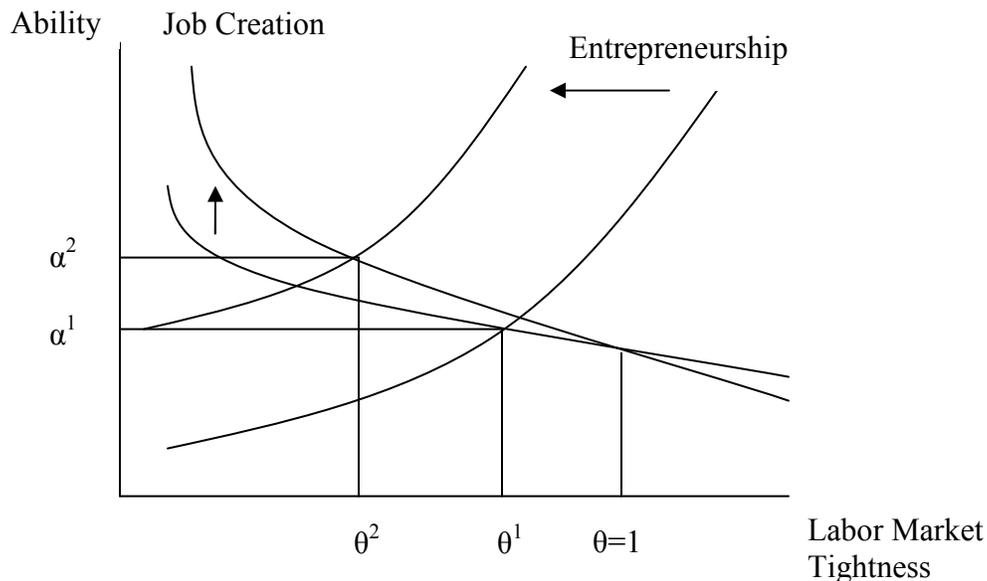


Suppose only the employment duration effect is present, i.e. technological change is embodied, the effective discount rate is independent of growth, $R'(g) = 1$, and entrepreneurial ability is independent of growth, $A'(g) = 0$. An increase in the rate of technological growth shifts the entrepreneurship curve inward because, for any given level of labor market tightness, the increase in the rate of technological growth reduces the optimal job destruction age, which in turns reduces the value of being an entrepreneur. Meanwhile, the job creation curve rotates around $\theta = 1$ when the optimal duration decreases, with the new job creation curve being above (below) the previous one if $\theta < 1$ ($\theta > 1$, respectively).⁴⁰ These changes in the entrepreneurship and job creation curves imply that an increase in technological growth has a negative impact on

⁴⁰ The reason why the job creation curve rotates around $\theta = 1$ is explained on page 119, in the second paragraph after proposition 7.

entrepreneurship when the ratio of vacancies to unemployment is less than one (see Figure 3.5). When the ratio of vacancies to unemployment is greater than one, the impact of enhanced growth on entrepreneurship depends once again on the elasticities of the entrepreneurship and job creation curves together with the extent to which a change in entrepreneurial ability $A(g)$ causes shifts in both curves.

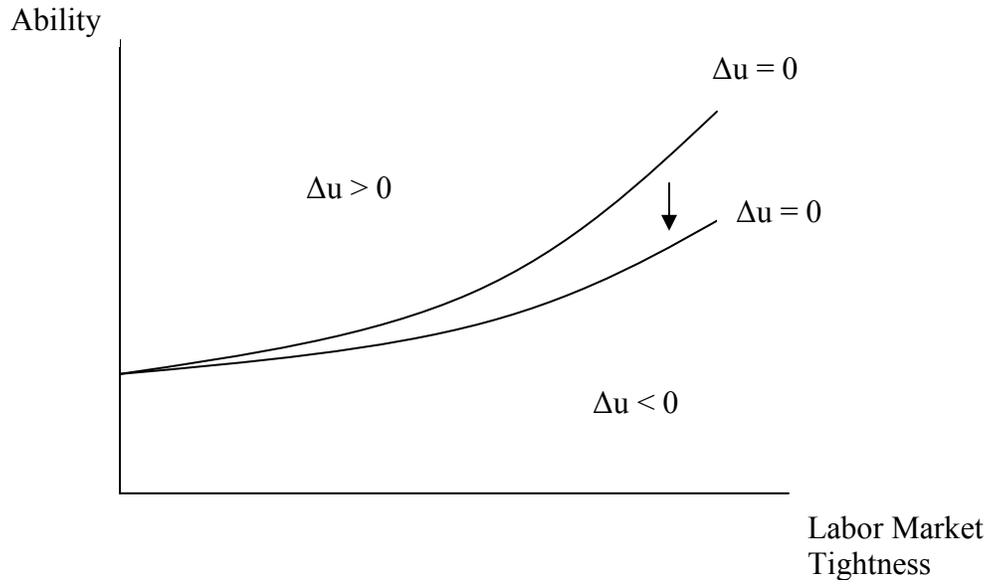
Figure 3.5: The Impact of Technological Growth on Entrepreneurship when only the Employment Duration Effect is Present



We next provide an overview of the ways in which technological growth affects unemployment. According to equation (6), unemployment is sensitive to changes in the threshold level of ability beyond which agents become entrepreneurs, labor market tightness θ , and the optimal duration of a job T^* . The impact of growth on unemployment that operates via changes in $\hat{\alpha}$ is labeled the composition effect, that which operates via changes in θ the job creation effect, and that which operates via changes in T^* the

creative destruction effect. Figure 3.6 illustrates the three effects.

Figure 3.6: The Impact of Technological Growth on Unemployment



The curve $\Delta u = 0$ consists of the combinations of the reservation ability and labor market tightness that can sustain the current level of unemployment, given the optimal duration of a job. An increase in the growth rate shifts the $\Delta u = 0$ curve down if technological change is embodied.⁴¹ Starting from a point on the $\Delta u = 0$ curve, both an increase in the reservation ability (moving upward along the vertical axis) and a decrease in labor market tightness (moving left along the horizontal axis) lead to an increase in unemployment. A decrease in the job destruction age also makes a hike in unemployment more likely in response to an increase in growth.

⁴¹ When the technological change is embodied, an increase in the growth rate leads to a decrease in the optimal job destruction age. If the labor market tightness and the reservation ability were to remain constant, unemployment would increase in equilibrium. For unemployment to remain constant, either the reservation ability has to decrease or the labor market tightness has to raise. Both options cause the $u=0$ to shift down.

To summarize, our model suggests that technological growth may affect entrepreneurship and unemployment in any fashion. Table 3.1 and 3.2 provide an overview of the mechanisms by which the growth rate affects entrepreneurship and unemployment. In the absence of the firm size effect, entrepreneurship and unemployment tend to move in opposite directions. For example, an increase in the number of entrepreneurs leads to an increase in labor market tightness, which reduces unemployment. Entrepreneurship and unemployment move in the same direction if there is a sufficiently strong firm size effect. When the firm size effect is strong enough, a decrease in the number of entrepreneurs no longer immediately translates into a decrease in labor market tightness: because entrepreneurs can create more jobs when growth enhances entrepreneurial ability, $A'(g) > 0$, labor market tightness can increase even if the number of entrepreneurs is lower. This scenario is also likely when the job creation curve is very elastic.

Table 3.1: The Mechanisms by which Technological Growth Affects Entrepreneurship.

Effects Relating to the Impact of Technological Growth on Entrepreneurship		
<u>Name</u>	<u>Source</u>	<u>Effect Positive if</u>
Capitalization	Effective discount rate	Effective Discount Rate is Increasing in Growth
Firm Size	Entrepreneurial ability	Growth Enhances Entrepreneurial Ability
Employment Duration	Optimal Job Destruction Age	Never

Table 3.2: The Mechanisms by which Technological Growth Affects Unemployment.

Effects Relating to the Impact of Technological Growth on Unemployment		
<u>Name</u>	<u>Source</u>	<u>Effect Positive if</u>
Composition	Changes in Labor Force Composition	Growth Decreases Entrepreneurship
Job Creation	Changes in Labor Market Tightness	Growth Decreases Labor Market Tightness
Creative Destruction	Optimal Job Destruction Age	Technological Growth is Embodied

We now proceed with studying analytically in detail the impact of technological growth on entrepreneurship and unemployment when labor markets are frictionless. We then examine the cases in which there are labor market frictions.

3. Frictionless Labor Markets

3.1. Disembodied Technological Change

An individual chooses the occupation that maximizes the present discounted value of future income. An agent with entrepreneurial ability α can either become an entrepreneur that creates $\alpha A(g)$ jobs, or a worker that gets the wage $w(t)$. In the entrepreneurial occupation, the expected payoff to the individual is $\alpha A(g)V(t) - p(t)K$, which equals the total expected profit of creating $\alpha A(g)$ new jobs minus the business start-up cost $p(t)K$. The payoff of becoming a worker is $W(t)$. Entrepreneurs are therefore those whose ability satisfies the inequality

$$(3.7) \quad \alpha A(g)V(t) - p(t)K \geq W(t)$$

The value of being a worker solves the asset pricing equation

$$(3.8) \quad R(g)W(t) = w(t) + \dot{W}(t)$$

where a dot on top of a value function refers to the derivative with respect to time. The value of creating a job is given by

$$(3.9) \quad V(t) = J(t) - p(t)c$$

where $J(t)$ is the value to the entrepreneur of a filled job position. Jobs are subject to separation shocks, which follow a Poisson process with arrival rate δ . Given these exogenous separations shocks, $J(t)$ is the solution to the following asset pricing equation:

$$(3.10) \quad R(g)J(t) = p(t)y - w(t) - [J(t) - V(t)] + \dot{J}(t)$$

where $p(t)y$ is the output generated from a match.

In what follows, we focus our attention on the balanced growth path, along which $w(t)$ grows at the rate g , as do $J(t)$ and $W(t)$. Thus, we have that $J(t) = p(t)J$, $W(t) = p(t)W$, and $w(t) = p(t)w$, where

$$(3.11) \quad W = \frac{w}{R(g) - g}$$

and

$$(3.12) \quad J = \frac{y - w - \delta c}{R(g) - g}$$

The value of creating a job can thereby be written as

$$(3.13) \quad V = \frac{y - w - \delta c}{R(g) - g} - c$$

Given that V and W are independent of α , the occupational choice decision is governed by a reservation entrepreneurial ability $\hat{\alpha}$, such that an agent with ability α becomes an entrepreneur if $\alpha \geq \hat{\alpha}$, otherwise he becomes a worker. The reservation ability satisfies

$$(3.14) \quad \hat{\alpha} = \frac{W + K}{A(g)V}$$

Replacing equations (3.11) and (3.13) into equation (3.14), we find that

$$(3.15) \quad \hat{\alpha} = \frac{w + (R(g) - g)K}{A(g)[y - w - (\delta + R(g) - g)c]}$$

For the labor market to be in equilibrium, the number of workers in the economy $F(\hat{\alpha})$ must equal the number of jobs created by entrepreneurs, $[1 - F(\hat{\alpha})]A(g)E(\alpha|\alpha \geq \hat{\alpha})$.

The labor market clearing condition can then be written as

$$(3.16) \quad F(\hat{\alpha}) = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

Note that the system of equations that characterize the equilibrium is block recursive. Equation (16) uniquely determines the threshold ability $\hat{\alpha}$. Knowing $\hat{\alpha}$, equation (15) uniquely determines the wage w .⁴² The following proposition describes the impact of growth on entrepreneurship.

PROPOSITION 1: Suppose technological growth is disembodied and the labor market is frictionless. An increase in the growth rate reduces the number of entrepreneurs if and only if growth enhances entrepreneurial ability, i.e. $A'(g) > 0$.

PROOF: Immediate from (16).

Technological growth dampens the extent of entrepreneurship if and only if an increase in growth enhances entrepreneurial ability at managing workers. The intuition of this result follows from equation (16). When the labor market is frictionless, there are no

⁴² Specifically, we have that $w = \frac{\hat{\alpha}}{1+\hat{\alpha}}[y - (\delta + R(g) - g)c] - \frac{R(g)-g}{1+\hat{\alpha}}K$.

unemployed workers or unfilled vacancies in equilibrium. The number of entrepreneurs is then given by the minimum number of entrepreneurs that are needed to create enough jobs for the workers in the economy. The sign of $A'(g)$ plays an important role in determining the impact of technological growth on entrepreneurship because it captures whether more or less entrepreneurs are needed to hire all the workers when growth is augmented. Given that fewer entrepreneurs are needed to hire the workers when $A'(g) > 0$, the number of entrepreneurs in equilibrium decreases in response to an increase in growth. The opposite occurs when $A'(g) < 0$. If $A'(g) = 0$, then changes in the growth rate have no effect on the number of entrepreneurs. Finally, note that to obtain this result, no restrictions on the interest rate $R(g)$ are required.

3.2. Embodied Technological Change

When technological change is embodied, only the productivity of new jobs grows at the exogenous rate g . Once a job is created, its productivity remains fixed for the duration of the match. Given that labor markets are competitive, wages paid by entrepreneurs follow the course of productivity growth very closely.⁴³ Technological change causes wages to grow over time, even if the productivity of the job remains fixed. As a result, the match eventually becomes unprofitable for the entrepreneur. Once that point is reached, the entrepreneur destroys the job and lays off the worker.

The difference between embodied and disembodied technological change is that now the value of being a worker might depend on the date the job was created, given that not all jobs are equally productive. The value to a worker of being employed at time t in a job

⁴³ Entrepreneurs do not pay workers their marginal product because they need to recuperate the business start-up cost and cover the opportunity cost of being an entrepreneur.

created at time τ is the solution to the following functional equation:

$$(3.17) \quad R(g)W(t, \tau) = \max\{w(t, \tau) + \delta[W(t, t) - W(t, \tau)] + \dot{W}(t, \tau), R(g)W(t, t)\}$$

where $w(t, \tau)$ is the wage paid at time t to a worker that joined the entrepreneur at time τ .

The max operator indicates that workers can move instantaneously at no cost between jobs. This assumption implies that workers must be indifferent between all jobs. That is,

$W(t, \tau) = W(t, t)$, which in turn implies that wages in all jobs have to be equal, i.e.

$w(t, \tau) = w(t, t)$. Equation (17) can then be written as

$$(3.18) \quad R(g)W(t) = w(t) + \dot{W}(t)$$

As before, the value of being an entrepreneur is given by

$$S(\alpha, t) = A(g)V(t) - p(t)K$$

and the value of having a vacant job can be written as

$$(3.19) \quad V(t) = J(t, t) - p(t)c$$

where $J(t, t)$ is the value to an entrepreneur of a recently filled job. The value to an entrepreneur of a filled job at date t which was created at time τ , $J(t, \tau)$, is given by

$$(3.20) \quad R(g)J(t, \tau) = \max\{p(\tau)y - w(t, \tau) + \delta[V(t) - J(t, \tau)] + \dot{J}(t, \tau), R(g)V(t)\}$$

where $p(\tau)y$ is the output generated from a match created at time τ .

Finally, we specify how entrepreneurs choose the age at which a job is destroyed. The value to the entrepreneur of a filled job position can be written as

$$(3.21) \quad J(t, \tau) = \max_T \left\{ \begin{array}{l} \int_t^{t+T} [p(\tau)y - w(t, \tau)] e^{-(R(g)+\delta)(s-t)} ds \\ + \int_t^{t+T} [1 - e^{-\delta(s-t)}] V(t) e^{-(R(g))(s-t)} ds + V(t+T) e^{-(R(g)+\delta)(t+T)} \end{array} \right\}$$

where the first term captures the expected present discounted value of future profits

generated by a filled job, while the last two terms capture the expected continuation value

of a vacant job.⁴⁴

In what follows, we focus our attention on the balanced growth path. Given that wages and asset prices grow at the same rate along the balanced growth path, the value of being a worker and the value of posting a vacancy can be written as

$$(3.22) \quad R(g)W = w + gW$$

$$(3.23) \quad V = J(0) - c$$

The problem of finding the optimal age at which to destroy a job, T^* , can be expressed as

$$(3.24) \quad J = \max_T \left\{ \begin{array}{l} \int_0^T [p(-\tau)y - w(-\tau)]e^{-(R(g)+\delta)t} dt \\ + \int_0^T [1 - e^{-\delta t}]Ve^{-R(g)t} dt + V(T)e^{-(R(g)+\delta)(T)} \end{array} \right\}$$

where $p(-\tau)$ and $w(-\tau)$ represent the productivity and the wage paid by a job τ created periods ago.

The optimal duration of a job is a decreasing function of the growth rate g . This is so because while the expected present discounted value of future profits generated by a filled job (the first term in equation (3.24) is decreasing in g , the expected continuation value of a vacant job (the last two terms in equation (3.24) is increasing in g .

As before, two equations play a key role in the solution of the model. First, for the labor market to be in equilibrium, the number of workers in the economy $F(\hat{\alpha})$ must equal the number of jobs created by entrepreneurs, $[1 - F(\hat{\alpha})]A(g)E(\alpha|\alpha \geq F(\hat{\alpha}))$. The labor market clearing condition is therefore the same as in the case with disembodied technological change:

⁴⁴ When the optimal age at which a job is destroyed is infinity, the value to the entrepreneur of a filled job position in the case with embodied technological change coincides with the value to the entrepreneur of a filled job position in the case with disembodied technological, which was defined in equation (3.10)

$$(3.25) \quad F(\hat{\alpha}) = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

Second, there is the occupational choice decision, which is characterized by the reservation entrepreneurial ability $\hat{\alpha}$, such that an agent with ability α becomes an entrepreneur if $\alpha \geq \hat{\alpha}$, otherwise he becomes a worker. The reservation ability satisfies

$$(3.26) \quad \hat{\alpha} = \frac{W + K}{A(g)V}$$

The following proposition describes the impact of technological growth on entrepreneurship.

PROPOSITION 2: Suppose technological growth is embodied and the labor market is frictionless. An increase in the growth rate reduces the number of entrepreneurs if and only if growth enhances entrepreneurial ability, i.e. $A'(g) > 0$.

PROOF: Immediate from (3.25).

As in the case with disembodied technological change, growth dampens entrepreneurship if and only if an increase in growth enhances entrepreneurial ability at managing workers. The same intuition follows through, so it is not repeated.

4. Labor Market Frictions

Suppose the labor market is burdened by frictions that prevent entrepreneurs from finding workers and workers from finding jobs. In order to find workers, entrepreneurs post vacancies and workers search for jobs. Only unemployed workers can search for jobs and the search process is random in the sense that they cannot target their search to a

particular vacancy. The cost to entrepreneurs of posting a vacancy is $p(t)c$ per vacancy. While considering whether to post a vacancy, entrepreneurs compare the expected future profit with the cost of posting a vacancy.

The number of jobs that are filled at a given point in time is summarized by a matching function $m(v, u)$, where v is the number of vacancies and u is the number of unemployed workers, both expressed in terms of the fixed labor force. The matching function $m(\cdot)$ is increasing, concave, and homogeneous of degree 1 in vacancies and unemployment. Once a match has been formed, wages paid by entrepreneurs to workers are determined via Nash bargaining, where the worker's bargaining power is denoted by β . Wages can be recontracted at any point in time, at no cost. Entrepreneur-worker matches are subject to separation shocks, which follow a Poisson process with arrival rate δ . Matches can also be endogenously destroyed if the match is no longer profitable.

4.1. Disembodied Technological Change

The value of becoming an entrepreneur at time t for an agent with entrepreneurial ability α , $S(\alpha, t)$, is given by

$$(3.27) \quad S(\alpha, t) = A(g)V(t) - p(t)K$$

where $V(t)$ is the value of creating a new vacancy at date t and $p(t)K$ is the cost of starting a business. $V(t)$ is the solution to the following asset pricing equation:

$$(3.28) \quad R(g)V(t) = q(\theta)[J(t) - V(t)] - p(t)c + \dot{V}(t)$$

where $\theta \equiv (v/u)$ is labor market tightness, $q(\theta) \equiv m(v, u)/v$ is the probability of filling the vacancy, $p(t)c$ is the cost of posting a vacancy, and $J(t)$ is the value to an entrepreneur of having a filled job at time t . $J(t)$ solves the asset pricing equation

$$(3.29) \quad R(g)J(t) = \max\{p(t)y - w(t) + \delta(V(t) - J(t)) + \dot{J}(t), R(g)V(t)\}$$

where $p(t)y$ is the output generated from a match, $w(t)$ is the wage paid at time t to the worker, and the max operator in equation (29) captures the fact that the job is destroyed once the value of the filled job falls below that of a vacant job. The value of being a worker, $W(t)$, is the solution to the following asset pricing equation:

$$(3.30) \quad R(g)W(t) = \max\{w(t) - [W(t) - U(t)] + \dot{W}(t), R(g)U(t)\}$$

The max operator in equation (3.30) allows the worker to quit his job to become unemployed. Finally, the value of being unemployed, $U(t)$, solves

$$(3.31) \quad R(g)U(t) = p(t)b + q(\theta)[W(t) - U(t)] + \dot{U}(t)$$

As before, given that the value of being an entrepreneur is increasing in ability α , while the value of being a worker is independent of α , the choice of whether to become an entrepreneur can be characterized by a reservation entrepreneurial ability $\hat{\alpha}$. That is, an agent with entrepreneurial ability α becomes an entrepreneur if $\alpha \geq \hat{\alpha}$, otherwise he becomes a worker. The reservation ability is given by

$$(3.32) \quad \hat{\alpha} = \frac{U(t) + p(t)K}{A(g)V(t)}$$

One can show that $\frac{\partial U(\theta)}{\partial \theta} > 0$ and $\frac{\partial V(\theta)}{\partial \theta} < 0$, which by equation (32) implies that the

extent of entrepreneurship is decreasing in labor market tightness, $\frac{\partial \hat{\alpha}}{\partial \theta} > 0$. Intuitively,

when labor market tightness is higher, it becomes more difficult for entrepreneurs to fill their vacancies, while it becomes easier for unemployed workers to find jobs. Therefore, more agents become workers and fewer become entrepreneurs in response to an increase in labor market tightness.

In what follows, we focus our attention on the balanced growth path, along which labor

market tightness θ is constant. Given that θ is fixed, wages are set via Nash bargaining, and labor productivity and all costs grow at the same rate g , one can show that $J(t)$, $V(t)$, $W(t)$ and $U(t)$ also grow at the rate g . The implications are that these value functions can be written as $J(t) = p(t)J$, $V(t) = p(t)V$, $W(t) = p(t)W$, and $U(t) = p(t)U$. Hence, along the balanced growth path, equations (3.28), (3.29), (3.30), and (3.31) can be written as

$$(3.33) \quad (R(g) - g)V = q(\theta)[J - V] - c$$

$$(3.34) \quad (R(g) - g)J = \max\{y - w + \delta(V - J), (R(g) - g)V\}$$

$$(3.35) \quad (R(g) - g)W = \max\{w - \delta[W - U], (R(g) - g)U\}$$

$$(3.36) \quad (R(g) - g)U = b + q(\theta)[W - U]$$

4.1.1. Labor Market Flows

Given the reservation ability $\hat{\alpha}$, the number of entrepreneurs is given by $1 - F(\hat{\alpha})$. Because entrepreneurs find it optimal to create jobs up to their entrepreneurial ability, the total number of jobs created is

$$(3.37) \quad n + v = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

where n is the number of filled jobs and v is the number of vacant jobs. The unemployment rate is the difference between the total number of workers minus the number of filled jobs:

$$(3.38) \quad u = F(\hat{\alpha}) - n$$

To determine how employment evolves, one can examine employment from the entrepreneur's or worker's perspective. From the entrepreneur's perspective, given that

vacancies are filled with probability $q(\theta)$ and filled jobs are destroyed with probability δ , employment evolves according to

$$(3.39) \quad \dot{n} = q(\theta) \left[A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - n \right] - \delta n$$

where the term in brackets is the number of vacancies. In terms of the worker's transition rates, given that unemployed workers find jobs with probability $\theta q(\theta)$ and employed workers lose their jobs with probability δ , employment evolves according to

$$(3.40) \quad \dot{n} = \theta q(\theta) [F(\hat{\alpha}) - n] - \delta n$$

where the term in brackets is the number of unemployed workers.

Combining equations (39) and (40), and evaluating n at its steady-state value, one can show that along a balanced growth path the following relation between reservation ability and labor market tightness is satisfied:

$$(3.41) \quad \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) = \frac{\delta \theta + \theta q(\theta)}{\delta + \theta q(\theta)}$$

Finally, replacing the steady-state value of employment into equation (38), one can show that the proportion of unemployed workers in the steady state is given by

$$(3.42) \quad u = \frac{\delta F(\hat{\alpha})}{\delta + \theta q(\theta)}$$

4.1.2. Equilibrium

Equilibrium can be summarized by a system of two equations in two unknowns.

These equations are:

$$(3.43) \quad \hat{\alpha} = \frac{U(\theta) + K}{A(g)V(\theta)}$$

$$(3.44) \quad \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) = \frac{\delta\theta + \theta q(\theta)}{\delta + \theta q(\theta)}$$

where $U(\theta)$ and $V(\theta)$ are

$$(3.45) \quad U(\theta) = \frac{b}{R(g) - g} + \frac{q(\theta)[y - b + c]}{(R(g) - g)[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\theta q(\theta)]}$$

$$(3.46) \quad V(\theta) = \frac{(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\theta q(\theta)]c}{(R(g) - g)[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\theta q(\theta)]}$$

The unknowns in the system of equations are labor market tightness θ and the entrepreneurial ability threshold $\hat{\alpha}$. With knowledge of θ and $\hat{\alpha}$, the following are uniquely determined: the number of entrepreneurs $1 - F(\hat{\alpha})$, the number of employed workers $1 - F(\hat{\alpha}) - u$, and total unemployment (3.42). This system of two equations can be represented graphically as in Fonseca et al. (2001). One can show that equation (43), which we refer to as the entrepreneurship curve, is upward sloping in θ - $\hat{\alpha}$ space, while equation (3.34), which we refer to as the job creation curve, is downward sloping in θ - $\hat{\alpha}$ space (see Figure 3.3).⁴⁵

With labor market frictions and disembodied technological change, growth affects entrepreneurship via two mechanisms. The first mechanism operates through the entrepreneur's effective discount rate $R(g)-g$. The impact of growth on the effective discount rate is relevant because the costs of creating a job are paid initially, while the benefits accrue in the future. Aghion and Howitt (1994) and Mortensen and Pissarides (1999) find a similar mechanism while studying the effect of growth on unemployment, which they label the capitalization effect of growth. Faster technological progress has both a direct effect on the effective discount rate and an indirect effect through changes in

⁴⁵ The intuition for the slopes of these curves can be found in footnotes 33 and 34.

the interest rate. While the direct effect reduces the effective discount rate, the indirect effect can either increase or decrease the effective discount rate depending on the sign of $R'(g)$. The net effect is such that a higher rate of technological growth increases the effective discount rate if and only if $R'(g) > 1$.

The second mechanism operates via the ability of entrepreneurs to create jobs, which we label the firm size effect of growth. Recall that the maximum number of workers an entrepreneur can manage is $\alpha A(g)$. Hence, all else being equal, the greater is $A(g)$, the higher is the value of being an entrepreneur. However, an increase in $A(g)$ might not result in more entrepreneurship because when $A(g)$ is greater, each entrepreneur creates more jobs, which leads to higher wages in equilibrium.

The following proposition derives the ways in which entrepreneurship and unemployment depend on the exogenous growth rate when the firm size effect is absent.

PROPOSITION 3: Suppose technological growth is disembodied, there are labor market frictions, and the firm size effect is absent, i.e. entrepreneurial ability is independent of growth, $A'(g)=0$. An increase in the growth rate reduces the number of entrepreneurs and increases the number of unemployed workers if and only if the effective discount rate is increasing in growth, $R'(g) > 1$.

PROOF: See Appendix.

Whether $R'(g)$ is smaller or greater than 1 is crucial because it determines the direction of the capitalization effect of growth. When $R'(g) > 1$, the effective discount rate is higher the greater is the growth rate. This makes creating jobs less profitable, decreasing the

value of being an entrepreneur.

The impact of technological growth on entrepreneurship and unemployment is illustrated in Figure 3.3, which considers the case in which $R'(g) > 1$. From equation (43), we infer that a higher growth rate decreases the threshold ability $\hat{\alpha}$ at all levels of labor market tightness θ (i.e., it shifts the entrepreneurship curve up) if and only if $R'(g) > 1$. Meanwhile, the growth rate does not influence the job creation curve. Therefore, when the growth rate is elevated, $\hat{\alpha}$ is higher and θ is lower if and only if $R'(g) > 1$.

When the effective discount rate is increasing in growth, $R'(g) > 1$, the unemployment rate is increasing in growth due to two effects that operate in the same direction. First, because fewer workers become entrepreneurs (in response to the increase in g when $R'(g) > 1$), the increase in the number of agents becoming workers increases unemployment since there are more agents searching for jobs. We previously labeled this mechanism the composition effect, which is reflected in equation (42) by a higher $F(\hat{\alpha})$. The second reason why the unemployment rate is higher is that the decrease in the number of entrepreneurs (brought about by the increase in g when $R'(g) > 1$) decreases job creation, so it becomes more difficult for workers to find jobs. We previously labeled this mechanism the job creation effect, which is reflected in equation (42) by a lower $\theta q(\theta)$.

The following proposition derives the ways in which entrepreneurship depends on the exogenous growth rate when the capitalization effect is absent. To simplify the exposition, define $\varepsilon(\hat{\alpha}, \theta)|_E$ and $\varepsilon(\hat{\alpha}, \theta)|_{JC}$ as the elasticities with respect to labor market tightness of the reservation ability along the entrepreneurship and job creation curves, respectively; and define $\bar{\alpha} \equiv E(\alpha | \alpha \geq \hat{\alpha})$ as the average ability of an entrepreneur.

PROPOSTION 4: Suppose technological growth is disembodied, there are labor market frictions, and the capitalization effect is absent, i.e. the effective discount rate is independent of growth, $R'(g)=1$. The number of entrepreneurs is decreasing in the growth rate g if growth enhances entrepreneurial ability, $A'(g)>0$, and

$$(3.47) \quad \frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} > \frac{\hat{\alpha}^2}{(1 - F(\hat{\alpha}))\bar{\alpha}} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})}$$

or if growth dampens entrepreneurial ability, $A'(g)<0$, and

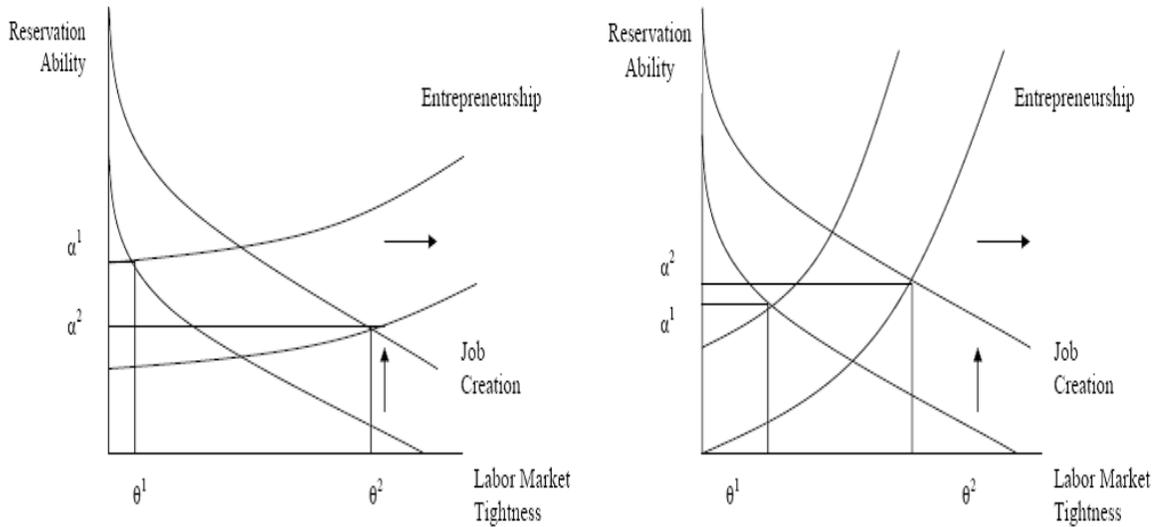
$$(3.48) \quad \frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} < \frac{\hat{\alpha}^2}{(1 - F(\hat{\alpha}))\bar{\alpha}} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})}$$

Otherwise, the number of entrepreneurs is increasing in the growth rate g .

PROOF: See Appendix.

The elasticities of the entrepreneurship and job creation curves are relevant because a change in the growth rate has direct and indirect effects that operate in opposite directions. When technological growth enhances entrepreneurial ability, $A'(g)>0$, the value of being an entrepreneur increases at all levels of labor market tightness θ because entrepreneurs can now manage more workers in response to an increase in growth. However, the increase in the number of jobs leads to a reduction in unemployment, which has a negative effect on entrepreneurship both through higher wages and a lower probability of filling vacancies. Whether the direct or indirect effect dominates depends on how sensitive job creation and entrepreneurship are to labor market tightness. The more sensitive the decision to become an entrepreneur is to labor market tightness, the stronger is the indirect effect and the more likely it becomes that the number of entrepreneurs is reduced when the growth rate increases (see Figure 3.7).

Figure 3.7: The Impact of Technological Growth on Entrepreneurship when only the Firm Size Effect is Present and $A'(g) > 0$.



A simple back of the envelope calculation suggests that the impact of technological growth on entrepreneurship is negative when the capitalization effect is absent. Suppose $A'(g) > 0$, entrepreneurial ability is uniformly distributed and 10% of agents in the economy are employers, i.e. $\lambda = 0.1$. Without labor market frictions, to obtain full employment, each entrepreneur must hire 9 workers on average. Suppose that when there are labor market frictions, the average ability of an entrepreneur is $\bar{a} = 9.5$ and the reservation ability is $\bar{a} = 9$. Then, we have that term on the right hand side of inequality (47) is $(9^2 / (0.1 * 9.5)) + 1 = 85.26^{46}$. Therefore, for reasonable parameter values and entrepreneurial ability distribution, the elasticity of the reservation ability along the

⁴⁶ Note that $\frac{d\alpha}{d\theta} < 0$ when the distribution is uniform and the minimum of the support of the distribution function equals 0.

entrepreneurship curve $\varepsilon(\hat{\alpha}, \theta)|_E$ has to be considerably larger than that along the job creation curve $\varepsilon(\hat{\alpha}, \theta)|_{JC}$ for higher growth to enhance entrepreneurship.

The following proposition derives the way in which unemployment depends on the exogenous growth rate when the capitalization effect is absent. For convenience, let $\mu(\theta) \equiv \theta q(\theta)$ denote the probability that an unemployed worker finds a job.

PROPOSITION 5: Suppose technological growth is disembodied, there are labor market frictions, and the capitalization effect is absent, i.e. the effective discount rate is independent of growth, $R'(g)=1$. The number of unemployed workers is decreasing in the growth rate g if growth enhances entrepreneurial ability, $A'(g)>0$, and

$$(3.49) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \varepsilon(\hat{\alpha}, \theta)|_{JC} + A(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \varepsilon(\hat{\alpha}, \theta)|_E$$

$$< \frac{A(g)\mu'(\theta)\theta}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha} + \left(1 + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

or if growth dampens entrepreneurial ability, $A'(g)<0$, and

$$(3.50) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \varepsilon(\hat{\alpha}, \theta)|_{JC} + A(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \varepsilon(\hat{\alpha}, \theta)|_E$$

$$> \frac{A(g)\mu'(\theta)\theta}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha} + \left(1 + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

Otherwise, the number of unemployed workers is increasing in the growth rate g .

PROOF: See Appendix.

Condition (49) is easier to interpret when expressed in term of equilibrium changes in the reservation ability and labor market tightness:

$$(3.51) \quad \frac{d\hat{\alpha}}{dg} < \frac{\delta F(\hat{\alpha})\mu'(\theta) d\theta}{[\delta + \mu(\theta)]^2 dg}$$

The left hand side is the composition effect and the right hand side is the job creation effect. One can show that labor market tightness is increasing in the growth rate, $\frac{d\theta}{dg} > 0$, when growth enhances entrepreneurial ability, $A'(g) > 0$. If the extent of entrepreneurship is increasing in growth, $\frac{d\hat{\alpha}}{dg} < 0$, then condition (3.51) is immediately satisfied. This arises because, in this scenario, the composition and job creation effects operate in the same direction. The increase in the number of entrepreneurs reduces the number of workers, which in turn reduces unemployment at any level of labor market tightness. The increase in labor market tightness also reduces unemployment since it becomes easier for workers to find jobs. It is only in the case when the composition and job creation effects operate in different directions (for example, as arises when $\frac{d\hat{\alpha}}{dg} > 0$) that further conditions are required for growth to have a negative effect on unemployment. When $\frac{d\hat{\alpha}}{dg}$ and $\frac{d\theta}{dg}$ are both positive, the increase in the number of workers raises unemployment but the increase in labor market tightness reduces unemployment. The condition (49) limits how much entrepreneurship can change in comparison to labor market tightness, to ensure a decrease in unemployment in response to an increase in growth.

The following proposition derives the ways in which entrepreneurship and unemployment depend on the growth rate when both the capitalization and firm size effects are present.

PROPOSITION 6: Suppose technological growth is disembodied and there are labor market frictions. The number of entrepreneurs is decreasing in the growth rate g if growth

enhances entrepreneurial ability, $A'(g) > 0$, and

$$(3.52) \quad \frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} > \Psi \left(\frac{\hat{\alpha}^2}{(1 - F(\hat{\alpha}))\bar{\alpha}} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right)$$

or if growth dampens entrepreneurial ability, $A'(g) < 0$, and

$$(3.53) \quad \frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} < \Psi \left(\frac{\hat{\alpha}^2}{(1 - F(\hat{\alpha}))\bar{\alpha}} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right)$$

where $\Psi \equiv 1 - \frac{(R'(g)-1)[\hat{\alpha}A(g)c+b+K]}{\hat{\alpha}(A'(g)[(1-\beta)q(\theta)[y-b]-[R(g)-g+\delta+\beta\mu(\theta)]c)}$. Otherwise, the number of entrepreneurs is increasing in the growth rate g .

The number of unemployed workers is decreasing in g if $A'(g) > 0$ and

$$(3.54) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \varepsilon(\hat{\alpha}, \theta)|_{JC} + A(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \varepsilon(\hat{\alpha}, \theta)|_E$$

$$< \frac{A(g)\mu'(\theta)\theta}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha} + \left(1 + \Psi \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

or if $A'(g) < 0$ and

$$(3.55) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \varepsilon(\hat{\alpha}, \theta)|_{JC} + A(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \varepsilon(\hat{\alpha}, \theta)|_E$$

$$> \frac{A(g)\mu'(\theta)\theta}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha} + \left(1 + \Psi \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

Otherwise, the number of unemployed workers is increasing in g .

PROOF: See Appendix.

The proposition is similar in format to the one obtained when only the firm size effect is present. Once we allow for the capitalization effect to be present, it makes having a positive impact of growth on entrepreneurship less (more) likely when the effective discount rate is increasing (decreasing) in the growth rate, $R'(g) > (<) 1$.

Similarly, a negative (positive) capitalization effect makes having a negative (positive) impact of growth on unemployment less (more) likely. The intuition is that a negative capitalization effect reduces the value of being an entrepreneur when $R'(g) > 1$, which translates into lower entrepreneurship and higher unemployment.

4.2. Embodied Technological Change

When technological change is embodied, only the productivity of new jobs grows at the exogenous rate g . Once a job is created, its productivity remains constant for the duration of the match. The value of being an entrepreneur is still given by $S(\alpha, t) = A(g)V(t) - p(t)K$ and the value of creating a new vacancy at date t is the solution to the following asset pricing equation:

$$(3.56) \quad R(g)V(t) = q(\theta)[J(t, t) - V(t)] - p(t)c + \dot{V}(t)$$

where $J(t, \tau)$ is the value at time t of a filled job that was created at time τ . $J(t, \tau)$ solves the asset pricing equation

$$(3.57) \quad R(g)J(t, \tau) = \max\{p(\tau)y - w(t, \tau) + \delta(V(t) - J(t, \tau)) + \dot{J}(t, \tau), R(g)V(t)\}$$

where $p(\tau)y$ is the output produced by a worker that was matched to the entrepreneur at time τ , and $w(t, \tau)$ is the wage paid at time t to that same worker. The max operator in equation (57) captures the fact that the job is destroyed once the value of the filled job is lower than that of having a vacant job.

The value to a worker of being employed in a job that started producing at time τ solves

$$(3.58) \quad R(g)W(t, \tau) = \max\{w(t, \tau) - \delta(W(t, \tau) - U(t)) + \dot{W}(t, \tau), R(g)U(t)\}$$

The max operator in equation (3.58) allows the worker to quit his job to become unemployed, which has value $U(t)$. The value of being unemployed solves

$$(3.59) \quad R(g)U(t) = p(t)b + q(\theta)[W(t, t) - U(t)] + \dot{U}(t)$$

All jobs are eventually destroyed, either exogenously or endogenously. Endogenous destruction occurs because wages are determined via Nash bargaining, taking into account the employer's and worker's options to continue searching. Since the wages of new jobs grow over time due to technological change, the value of searching for a new job also rises over time. This causes wages in existing jobs to grow over time.⁴⁷

Entrepreneurs choose the age at which to destroy a job by maximizing the value of a job position with respect to job duration. The maximal value of a job position can be written as

$$(3.60) \quad J(t, \tau) = \max_T \left\{ \begin{array}{l} \int_t^{t+T} [p(\tau)y - w(t, \tau)] e^{-(R(g)+\delta)(s-t)} ds \\ + \int_t^{t+T} [1 - e^{-\delta(s-t)}] V(t) e^{-(R(g))(s-t)} ds + V(t+T) e^{-(R(g)+\delta)(t+T)} \end{array} \right\}$$

where the first term captures the expected present discounted value of future profits generated by a filled job, while the last two terms capture the expected continuation value of a vacant job.⁴⁸

As before, an agent with ability α becomes an entrepreneur if $\alpha \geq \hat{\alpha}$, otherwise he becomes a worker, where

$$(3.61) \quad \hat{\alpha} = \frac{U(t) + p(t)K}{A(g)V(t)}$$

We once again focus on the balanced growth path, along which we have $J(t, \tau) = p(t)J(-\tau)$, $V(t) = p(t)V$, $W(t, \tau) = p(t)W(-\tau)$, and $U(t) = p(t)U$. $J(-\tau)$ is the value along the balanced growth path of a job position that was created τ periods ago. Similarly,

⁴⁷ One can show that, along the balanced growth path, wages are established according to $w(t, \tau) = p(\tau)y + p(t)[(1 - \beta)b + \beta\theta c + [\theta - 1]\beta(r - g)V]$.

⁴⁸ When the optimal age at which a job is destroyed is infinity, the value to the entrepreneur of a filled job position in the case with embodied technological change coincides with the value to the entrepreneur of a filled job position in the case with disembodied technological, which was defined in equation (3.29).

$W(-\tau)$ is the value along the balanced growth path of being a worker in a job that was created τ periods ago. Equations (56) through (59) can then be written as

$$(3.62) \quad (R(g) - g)V = q(\theta)[J - V] - c$$

$$(3.63) \quad R(g)J(-\tau) = \max \left\{ p(-\tau)y - w(-\tau) + \delta(V - J(-\tau)) + \frac{d[p(t)J(-\tau)]}{dt}, R(g)V \right\}$$

$$(3.64) \quad R(g)W(-\tau) = \max \left\{ w(-\tau) - \delta(W(-\tau) - U) + \frac{d[p(t)W(-\tau)]}{dt}, R(g)U \right\}$$

$$(3.65) \quad (R(g) - g)U = b + q(\theta)[W(0) - U]$$

where $p(-\tau)$ is the productivity of a job that was created τ periods ago and $w(-\tau)$ is the wage paid to a worker that was matched to the entrepreneur τ periods ago.

Finally, the problem of solving for the optimal duration of a job T^* can be expressed as

$$(3.66) \quad J = \max_T \left\{ \begin{array}{l} \int_0^T [p(\tau)y - w(t, \tau)]e^{-(R(g)+\delta)t} dt \\ + \int_0^T [1 - e^{-\delta t}]Ve^{-R(g)t} dt + V(T)e^{-(R(g)+\delta)(T)} \end{array} \right\}$$

The optimal duration of a job is decreasing in the growth rate g : the expected present discounted value of future profits generated by a filled job (the first term in equation (66)) is decreasing in g , but the expected continuation value of a vacant job (the last two terms in equation (66)) is increasing in g .

4.2.1. Labor Market Flows

The difference with respect to the case in which technology is disembodied arises from flows into unemployment. When technological change is disembodied, jobs are destroyed only when they experience an exogenous destruction shock. When technological change

is embodied, jobs are also destroyed when they reach their age of obsolescence.

The total number of jobs created is

$$(3.67) \quad n + v = A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

and the unemployment rate is

$$(3.68) \quad u = F(\hat{\alpha}) - n$$

The evolution of employment can be written from the entrepreneur's perspective:

$$\dot{n} = q(\theta) \left[A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - n \right] - \delta n - \exp(-\delta T^*) q(\theta) \left[A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - n \right]$$

where the first term is the number of vacancies that are filled (which equals the number of vacancies times the vacancy filling rate), the second term is the number of jobs that are exogenously destroyed, and the last term is the number of jobs that are endogenously destroyed. This last expression can be re-stated as

$$(3.69) \quad \dot{n} = (1 - \exp(-\delta T^*)) q(\theta) \left[A(g) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - n \right] - \delta n$$

In terms of a worker's transition rates, the evolution of employment can be written

$$\dot{n} = \theta q(\theta) [F(\hat{\alpha}) - n] - \delta n - \exp(-\delta T^*) \theta q(\theta) [F(\hat{\alpha}) - n]$$

where the first term is the number of workers that find a job (which equals the number of unemployed workers times the job finding rate). The second and third terms are the number of workers that exogenously and endogenously lose their jobs, respectively. This last expression can be re-stated as

$$(3.70) \quad \dot{n} = (1 - \exp(-\delta T^*)) \theta q(\theta) [F(\hat{\alpha}) - n] - \delta n$$

Combining equations (69) and (<70), and evaluating at the steady state, we find

$$(3.71) \quad \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) = \frac{\delta\theta + [1 - \exp(-\delta T^*)]\theta q(\theta)}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

Finally, replacing the steady state level of employment into equation (68), one can show that the proportion of unemployed workers in the steady state is given by\

$$(3.72) \quad u = \frac{\delta F(\hat{\alpha})}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

4.2.2. Equilibrium

As before, two equations play a key role in the solution of the model. The two equations are:

$$(3.73) \quad \hat{\alpha} = \frac{U(\theta) + K}{A(g)V(\theta)}$$

$$(3.74) \quad \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) = \frac{\delta\theta + [1 - \exp(-\delta T^*)]\theta q(\theta)}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

With embodied technological change, growth affects entrepreneurship via three mechanisms. The first two mechanisms are the capitalization effect and the firm size effect, which were also present with disembodied technological change. The third mechanism operates via changes in the optimal duration of a job, labeled the employment duration effect. As the growth rate increases, the average duration of a job becomes shorter. All else being equal, this reduces the value of opening vacancies and thereby the value of becoming an entrepreneur.

The following proposition derives the ways in which entrepreneurship and unemployment depend on the exogenous growth rate when the capitalization and firm size effects are absent.

PROPOSITION 7: Suppose technological growth is embodied, there are labor market frictions, the capitalization effect is absent, i.e. the effective discount rate is independent of growth, $R'(g)=1$, and the firm size effect is absent, i.e. entrepreneurial ability is independent of growth, $A'(g)=0$, and the initial labor market tightness satisfies $\theta < 1$. The number of entrepreneurs is decreasing in the growth rate g and the number of unemployed workers is increasing in g .

PROOF: See Appendix.

The intuition of the proposition is illustrated graphically in Figure 3.5. With embodied technological change, a change in the growth rate affects not only the location of the entrepreneurship curve but also that of the job creation curve. As with frictionless labor markets, an increase in the growth rate reduces the optimal duration of a job, which shifts upward the entrepreneurship curve. According to equation (74), the job creation curve rotates around $\theta = 1$ when the optimal duration decreases, with the new job creation curve being above (below) the previous one if $\theta < 1$ ($\theta > 1$, respectively). By inspecting Figure 3.5, one notices that it is only in the region where $\theta < 1$ that an increase in the growth rate causes both the entrepreneurship and job creation curves to shift upward. This ensures that an increase in the growth rate decreases the number of entrepreneurs when $\theta < 1$. To be able to sign the impact of technological change on entrepreneurship when $\theta > 1$, further restrictions on the elasticities of the entrepreneurship and job creation curves are needed.

The result that the job creation curve rotates around $\theta = 1$ when the optimal job

duration of a job changes is based on the assumption that the matching function is homogenous of degree 1, as it implies that the probability a firm fills a vacancy is $q(\theta)$ and the probability of an unemployed worker finding a job is $\theta q(\theta)$; thus, for $\theta < 1$, the probability of filling a vacancy is higher than the probability of finding a job. Evaluating the steady-state value of employment n ($\dot{n} = 0$) from the entrepreneur's perspective, equation (69) implies a level of employment of

$$(3.75) \quad n = \frac{[1 - \exp(-\delta T^*)]q(\theta) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{\delta + [1 - \exp(-\delta T^*)]q(\theta)}$$

From the worker's perspective, the steady state level of employment implied by equation (70) is

$$(3.76) \quad n = \frac{[1 - \exp(-\delta T^*)]\theta q(\theta)F(\hat{\alpha})}{\delta + [1 - \exp(-\delta T^*)]\theta q(\theta)}$$

At the steady state, the levels of employment implied by the two equations must be the same. Now, if the reservation entrepreneurial ability $\hat{\alpha}$ and labor market tightness θ were to remain fixed at their original levels after the reduction in the optimal job destruction age, the new steady state level of employment implied by equation (76) would be lower than that implied by equation (75). This cannot be an equilibrium. For the two new implied levels of employment to be the same, $F(\hat{\alpha})$ has to increase and $\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$ has to decrease. This is achieved by an increase in the reservation ability $\hat{\alpha}$, which explains the rotation upward of the job creation curve when $\theta < 1$. The opposite occurs when $\theta > 1$.

The rotation of the job creation curve is important because it reflects an indirect effect of growth, which operates via changes in labor market tightness. For any given reservation ability, the rotation of the job creation curve leads to an increase (decrease) in

labor market tightness whenever $\theta < 1$ ($\theta > 1$, respectively). Thus, there are two forces operating at once when only the employment duration effect is present. On the one hand, the reduction of the job destruction age reduces the value of being an entrepreneur. On the other hand, the change in labor market tightness also affects the value of being an entrepreneur, with an increase in labor market tightness reducing the value of being an entrepreneur. When $\theta < 1$, the two forces reduce the value of being an entrepreneur, such that the number of entrepreneurs decreases in equilibrium. However, when $\theta > 1$, the two forces operate in different directions, such that the overall effect depends on which of the two forces dominates.

With regards to the impact of an increase in growth on unemployment, note that both the creative destruction effect, brought about by the reduction of the optimal duration of a job T^* , and the job creation effect, brought about by the decrease in labor market tightness θ , tend to increase unemployment. The composition effect operates in the same direction when $\theta < 1$, so unemployment is increasing in g when $\theta < 1$. However, the composition effect operates in the opposite direction than the creative destruction and job creation effects when $\theta > 1$. Further conditions are then required to determine whether unemployment increases or decreases in response to a rise in growth when $\theta > 1$.

Finally, one can generalize our results when the employment duration effect is present, but the conditions that emerge are very mathematically involved, so they are omitted. This is so because in some occasions the capitalization, firm size, and employment duration effects operate in different directions, such that the relative strength of each must be considered. Nevertheless, if the decrease in labor market tightness implied by the rotation of the job creation curve is sufficiently weak, then the presence of the creative

destruction effect reduces the likelihood that a higher rate of technological change increases entrepreneurship and decreases unemployment.

5. Conclusion

This paper proposed a simple theory to examine the impact of exogenous technical change on entrepreneurship. Our model unveiled three mechanisms through which growth affects entrepreneurship: the capitalization effect, which pertains to the impact of growth on the effective discount rate; the firm size effect, which concerns the impact of growth on entrepreneurial ability at managing workers; and the employment duration effect, which operates via the optimal duration of a job in light of technological obsolescence. We found that when the labor market is frictionless, only the firm size effect affects entrepreneurship: growth dampens entrepreneurship if and only if growth enhances entrepreneurial ability. When there are frictions in the labor market, all three effects influence entrepreneurship. Given that in some occasions the capitalization, firm size, and employment duration effects operate in different directions, the relative strength of each has to be considered in order to evaluate the overall impact of technological change on entrepreneurship.

Labor market frictions introduce unemployment in equilibrium, which allowed us to study the effect of exogenous technical change on unemployment. Our model unveiled three mechanisms through which technological change affects unemployment: the composition effect, which pertains to the occupational choice decision; the job creation effect, which is associated with labor market tightness; and the creative destruction effect, which operates via the endogenous destruction of jobs resulting from technological

change. Once again, because these effects may operate in different directions, the relative strength of each has to be considered in order to evaluate the overall impact of technological change on unemployment.

Our analysis suggests that technological growth may affect entrepreneurship and unemployment in almost any fashion. In the absence of a firm size effect, entrepreneurship and unemployment tend to move in opposite directions. For example, an increase in the number of entrepreneurs leads to an increase in labor market tightness, which reduces unemployment. For entrepreneurship and unemployment to move in the same direction, there must be a sufficiently strong firm size effect: when this is so, a decrease in the number of entrepreneurs no longer necessarily leads to a decrease in labor market tightness.

Future theoretical research should aim toward deriving from first principles our reduced-form assumptions relating to the impact of growth on the effective discount rate and entrepreneurial ability at managing workers. Doing so might shed light on the relative strengths of some mechanisms identified in this paper. Our model also takes technological growth to be exogenous: the challenge lies with formulating a single tractable framework within which entrepreneurship, unemployment, and growth are all endogenous.

Appendices

I. Appendices to Chapter 1.

I.i. Proof Lemma 3

LEMMA 3. The value functions $U(\cdot)$ and $W(\cdot)$ exist and are continuously increasing in a and e . $W(\cdot)$ is strictly increasing in w .

Proof

Rewrite the mapping defined by equations (2) and (7) as $(W^{t+1}, U^{t+1}) = T(W^t, U^t)$. Set $(W^0, U^0) = 0$. Given that one is maximizing continuous functions over a compact set, the theorem of the maximum implies that (W^1, U^1) are continuous functions. Iterating on the operator T one can show that the functions (W^{t+1}, U^{t+1}) are also continuous. Then T maps functions from the set of continuous functions into itself. It can also be shown that the operator T satisfies Blackwell's monotonicity and discounting sufficient conditions for a contraction. Then, the Contraction Mapping Theorem implies that the functions $U(\cdot)$ and $W(\cdot)$ exist, they are unique and continuous.

Let V be the space of pairs of functions $(U(a_t, e_t), W(w, a_t, e_t))$ with the property that $U(\cdot)$ and $W(\cdot)$ are nondecreasing in a . Define V' as the set of strictly increasing pairs of such functions. To prove that $S(\cdot)$ and $W(\cdot)$ are strictly increasing in a , I need to show that $T(V) \subseteq V'$. For any $a'_t > a_t$

$$W(w, a'_t, e_t) =$$

$$\max_{0 \leq a_{t+1} \leq (1+r)[a'_t + w_t]} \left\{ \begin{array}{l} u\left(a'_t + w_t - \frac{a_{t+1}}{1+r}, 1\right) + \beta s E_t[U(a_{t+1}, e_{t+1})] \\ + \beta(1-s) E_t \max[W(w, a_{t+1}, e_{t+1}), U(a_{t+1}, e_{t+1})] \end{array} \right\} >$$

$$\max_{0 \leq a_{t+1} \leq (1+r)[a_t + w_t]} \left\{ u\left(a_t + w_t - \frac{a_{t+1}}{1+r}, 1\right) + \beta s E_t[U(a_{t+1}, e_{t+1})] \right. \\ \left. + \beta(1-s) E_t \max[W(w, a_{t+1}, e_{t+1}), U(a_{t+1}, e_{t+1})] \right\} = \\ W(w, a_t, e_t)$$

A similar argument can be used to show that $U(a_t, e_t)$ is also strictly increasing in a_t .

Proofs for the remaining propositions in the lemma are very similar to the one above and are therefore omitted.

I.ii. Proof Lemma 4

LEMMA 4. There always exists an equilibrium.

Proof

This proof follows closely the proof of proposition 1 in Acemoglu and Shimer (1999). As in their proof, I take three steps. In step 1, I show that any equilibrium solves a constrained problem to be specified. In step 2, I show that any allocation that solves the constrained problem specified in step 1 is part of an equilibrium. In step 3, I show that an equilibrium always exist.

Step 1

Let $\{J^U, J^M(w), W(w, a, e), D_t(w), r, w^U(a, e), k^F, q(w)\}$ be part of an equilibrium.

Then, for any set (a, e) for which an unmatched worker finds it optimal to search for a job, $w^U(a, e), k^F, q(w)$ solve

$$(A1) \quad S^S(a_t, e_t) = \max_{k^F, w, q} \left\{ u(c_t, l_t) + \beta \chi[q(w)] W(w, a_{t+1}, 0) \right. \\ \left. + \beta(1 - \chi[q(w)]) E_t[U(a_{t+1}, e_{t+1})] \right\}$$

subject to

$$(A2) \quad \eta[q(\omega)]\pi(\omega) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) = 0$$

and

$$(A3) \quad w \geq b$$

To see this, take a set (a, e) for which an unmatched worker finds it optimal to search for a job. Let c^* , k^* and $w^* \in \Omega$ be the optimal actions given the state. Optimality implies

$$S = S^S(a_t, e_t) = \max_{k^F, w, q} \left\{ \begin{array}{l} u(c^*, l_t) + \beta \chi[q(w^*)]W(w^*, a_{t+1}, 0) \\ + \beta(1 - \chi[q(w^*)])E_t[U(a_{t+1}, e_{t+1})] \end{array} \right\}$$

with $a_{t+1} = g(k^*, e_t) - (r + \delta)k^* + (1+r)(a_t - c_t)$. Now, suppose I force the agent with the same (a, e) to apply to a wage $w \neq w^*$. If I allow all other control variables to be chosen optimally, (given wage application) and label these with a double star

$$(A4) \quad S \geq \left\{ \begin{array}{l} u(c^{**}, l_t) + \beta \chi[q(w^*)]W(w^*, a_{t+1}, 0) \\ + \beta(1 - \chi[q(w^*)])E_t[U(a_{t+1}, e_{t+1})] \end{array} \right\}$$

For firms posting such wage w , it is true from the free entry condition that

$$(A5) \quad \eta[q(\omega)]\pi(\omega) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) = 0$$

Now, if $\{w, k^{**}, q^{**}, c^{**}\}$ were to yield a higher value for the same (a, e) , That is,

$$(A6) \quad S < \left\{ \begin{array}{l} u(c^{**}, l_t) + \beta \chi[q^{**}]W(w^*, a_{t+1}, 0) \\ + \beta(1 - \chi[q^{**}])E_t[U(a_{t+1}, e_{t+1})] \end{array} \right\}$$

equations (A4) and (A6) would then imply that $\chi[(q(w))] < \chi(q)$ and $q(w) > q$. But if that is true, then, $\eta(q) < \eta[(q(w))]$ which together with equation (A5) implies

$$\eta[q(\omega)]\pi(\omega) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) < 0$$

Therefore, $\{k^{**}, w, q^{**}, c^{**}\}$ cannot be part of an equilibrium.

Step 2

I now show that any allocation that solves the constrained problem described by equation (A1), (A2) and (A3) is part of an equilibrium. Let $\{w^*, q^*, c^*\}$ be the solution to the unmatched worker who decides to search and faces a state (a, e)

$$S = S(a_t, e_t) = u(c_t^*, l_t) + \beta\chi[q^*]E_t W(w^*, a_{t+1}, e_{t+1}) + \beta(1 - \chi[q^*])E_t U(a_{t+1}, e_{t+1})$$

Define k^* as

$$(A7) \quad \eta[q(\omega)](f(k^*) - w - rk^*) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w) - c(1+r) = 0$$

By construction, $\{w^*, q^*, c^*, k^*\}$ is part of an equilibrium.

Next, define $Q(w)$ as the solution to

$$S = S(a_t, e_t) = u(c_t^*, l_t) + \beta\chi[Q(w)]E_t W(w^*, a_{t+1}, e_{t+1}) + \beta(1 - \chi[Q(w)])E_t U(a_{t+1}, e_{t+1})$$

and let $Q(w) = 0$ if no solution exists. Now suppose one can find a set

$\{w', Q(w'), c^*, k'\}$ such that

$$(A8) \quad \eta[Q(w')](f(k^*) - w' - rk^*) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w') - c(1+r) > 0$$

For such a triple, $Q(w') > 0$. Now define q' as

$$(A9) \quad \eta[q'](f(k') - w' - rk') \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t D_t(w') - c(1+r) = 0$$

Obviously $q' < Q(w')$. Also,

$$u(c_t^*, l_t) + \beta\chi[q']E_t W(w', a_{t+1}, e_{t+1}) + \beta(1 - \chi[q'])E_t U(a_{t+1}, e_{t+1}) > S$$

Given that $\{w', q', c^*, k'\}$ satisfies all the constraints and yields a higher value for the searcher than $\{w^*, q^*, c^*, k^*\}$, the latter cannot have been a solution to the constrained

problem. In the solution to the constrained problem, the profit condition is satisfied with equality and therefore, the subset is part of an equilibrium.

Step 3

To show existence of an equilibrium I need to consider two particular cases. These cases depend critically on the maximum wage a firm would ever consider offering. If the firm assumes that the worker will never choose to endogenously terminate a match and can hire a worker with certainty next period, the present discounted value of future profits by posting a wage w is given by

$$J = \eta[q(w)](f(k^*) - w - rk^*) \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t - c(1+r) = 0$$

where k^* is determined by $f'(k^*)=r+\delta$. Free entry condition implies $J=0$ which results in a maximum wage of that is implicitly defined by $\hat{w} = f(k^*) - rk^* - c(r + \delta)/\eta[q(\hat{w})]$ and

$$f(k^*) - rk^* - (r + \delta)c .$$

Case 1: $b \geq \hat{w}$

The maximum wage a firm would be willing to post is too low for workers to apply to them. No firm enters the market resulting in $\Omega=\emptyset$. Firms have no incentives to deviate. For wages lower than b no one applies, so firms' profit equal $-c$ if they decide to enter. For any wage $w > b \geq \hat{w}$ expected profits are negative by construction.

Given the properties of the production function available to the unmatched workers, the Markov property for the productivity and the business project shocks, and the fact

that the unmatched worker value function exists and is well-defined

($U(a_t, e_t) = \max(S^{NS}(a_t, e_t), N^{NS}(a_t, e_t))$), the distributions $\Phi^U(a, e)$ and $\Phi^M(w, a, e)$ are stationary (with $\Phi^M(w, a, e) = 0$ for any (w, a, e)) and an equilibrium exists.

Case 2: $b < \hat{w}$

It can be easily shown that no worker applies to a wage below b . Then $w \in [b, \hat{w}]$.

Then, the constraint set in the unmatched worker problem is compact. Given that the unmatched worker value function is continuous, the constrained problem has a solution. By step 2, I know that the solution is part of an equilibrium.

Depending on parameter values, two different types of equilibria can emerge. One with only unmatched workers. The other with both matched and unmatched workers. In the one with unmatched workers only, $\Omega = \emptyset$. The equilibrium will have the same features as case 1 above.

In cases where unmatched workers decide to search for a job, one can show that $\Omega = \emptyset$ cannot be an equilibrium. Since $W(w, a, 0) > U(z, a, 0)$, as long as there are agents without business projects, workers will apply to vacancies offering a wage greater than b . I need to check whether a firm has incentives to deviate and post wages greater than b . The deviating firm gets an infinite amount of applicants, which implies that it gets matched to a worker with probability 1. If the deviating firm posts a wage smaller than $f(k^*) - rk^* - (r + \delta)c$, it makes a strictly positive profit and the deviation is profitable.

Given the properties of the self-employed and the firm's production functions, the properties of the productivity and business project shocks, and that in equilibrium $\beta(1 + r) < 1$ given that the matched and unmatched workers value functions are super-

martingales, the distributions $\Phi^U(a, e)$ and $\Phi^M(w, a, e)$ are stationary and an equilibrium exists.

I.iii. Fear of Failure

To understand the fear of failure effect more clearly, suppose there are states in which an unmatched worker finds it optimal to search for a job. Define the set of all such states as Δ . This implies that, for $(a_t, e_t) \in \Delta$, the value of being working for a corporation next period must be higher than the expected value of being self-employed next period, since searching workers must commit to accepting a job if offered. That is,

$W(w^U(a_t, e_t), a_{t+1}, 0) > E_t[U(a_{t+1}, e_{t+1})]$, since otherwise the unmatched worker would not look for a job at a firm. Now, if for these states the unmatched worker chooses to search while self-employed over searching while unemployed and if

$\chi(q[w^U(a_t, e_t)]) < 1$, it must be the case that

$$\begin{aligned} S^S(a_t, e_t) &= \max_{c_t, k_t, w_{t+1} \in \Omega_t} \{u(c_t, 1) + \beta(1 - \chi[q^e(w)])E_t[U(a_{t+1}, e_{t+1})] \\ &\quad + \beta(\chi[q^e(w)][W(w, a_{t+1}, 0)])\} \\ &< u(c^U(a_t, e_t), 1) + \beta[W(w^U(a_t, e_t), a_{t+1}^U(a_t, e_t), 0)] \equiv S^{SC}(a_t, e_t) \end{aligned}$$

where $a_{t+1}^U(a_t, e_t) = g(k^U(a_t, e_t), e_t) + (1+r)[a_t - c^U(a_t, e_t)] - rk^U(a_t, e_t)$.

$S^{SC}(a_t, e_t)$ is the value of being a self-employed worker this period, with a guaranteed job at a firm that starts next period and pays a wage $w^U(a_t, e_t)$, and with assets next period equal to $a_{t+1}^U(a_t, e_t)$.⁴⁹ Similarly, if for $(a_t, e_t) \in \Delta$ the unmatched worker chooses to

⁴⁹ $S^{SC}(a_t, e_t)$ is introduced to capture the cost of not getting the targeted job with probability one, abstracting from other implications for optimal behavior of having a frictionless labor market. This is why $w^U(\cdot)$, $c^U(\cdot)$ and $k^U(\cdot)$ are used in the definition of $S^{SC}(a_t, e_t)$. If one removes labor market frictions and allows workers to maximize over all choice variables, differences between $S^S(a_t, e_t)$ and $S^{SC}(a_t, e_t)$ come not only from changes in the probability of getting a job, but also from changes in the choice variables. In

search while unemployed over searching while self-employed, $N^S(a_t, e_t) < N^{SC}(a_t, e_t)$, where $N^{SC}(a_t, e_t)$ is the value of being an unemployed worker this period, with a guaranteed job at a firm that starts next period and pays a wage $w^U(a_t, e_t)$, and initial assets next period equal to $a_{t+1}^U(a_t, e_t)$. Now, define

$$U^{NF}(a_t, e_t) = \max(N^{SC}(a_t, e_t), N^{NSNF}(a_t, e_t), S^{SC}(a_t, e_t), S^{NSNF}(a_t, e_t))$$

where

$$N^{NSNF}(a_t, e_t) = u(c^U(a_t, e_t), 0) + \beta E_t[U^{NF}(a_{t+1}^U(a_t, e_t), e_{t+1})]$$

$$S^{NSNF}(a_t, e_t) = u(c^U(a_t, e_t), 1) + \beta E_t[U^{NF}(a_{t+1}^U(a_t, e_t), e_{t+1})]$$

$U^{NF}(a_t, e_t)$ is then the value of being unmatched when the fear of failure is not present.

Its definition guarantees that whenever an unmatched worker searches for a job (either in the current or future periods), he always gets the targeted job with probability one.

The fear of failure then prevent currently matched workers for whom $U^{NF}(a_t, 1) > W(w, a_t, 1) > U(a_t, 1)$ to enter self-employment. These workers would be willing to become self-employed if they were guaranteed getting a paid job in the future with certainty, after just one period of search. They choose to remain matched due to the existence of search frictions in the labor market.

I.iv. Frictionless Economy

In the frictionless economy, unmatched workers can instantly move back to paid employment, if they decide to do so. To allow for this possibility, I must modify my model so that workers learn immediately whether they get their target job. In addition, to ensure that workers get their target job with probability one, I set $\gamma = \infty$, so that

particular, if one removes labor market frictions, workers would only apply to the highest wage offered.

$\chi[q(w)] = 1$. To eliminate all other frictions, I further assume that workers can coordinate which vacancies they apply for and that firms can also coordinate on which ones will post vacancies. Under these assumptions, workers only apply to jobs paying the highest wage in equilibrium, and $q(w^{max}) = 1$ and $q(w) = 0$ if $w < w^{max}$. This implies that $\chi [q(w^{max})] = 1$ and $[q(w)] = 0$ if $w < w^{max}$. Finally, given that posting a vacancy is costly and that workers only apply to w^{max} , the set of all wages posted is $\Omega_t = \{w^{max}\}$.

In this new setting the value function of an unmatched worker becomes

$$U(a_t, e_t) = \max(N(a_t, e_t), S(a_t, e_t))$$

where

$$N(a_t, e_t) = \max_{c_t} \{u(c_t, 0) + \beta E_t [\max(U(a_{t+1}, e_{t+1}), W(a_{t+1}, 0))]\}$$

subject to

$$a_{t+1} = (1 + r)[a_t - c_t]$$

$$a_{t+1} \geq 0$$

And

$$S(a_t, e_t) = \max_{c_t, k_t} \{u(c_t, 1) + \beta E_t [\max(U(a_{t+1}, e_{t+1}), W(a_{t+1}, 0))]\}$$

subject to

$$a_{t+1} = g(k_t, e_t) - (r + \delta)k_t + (1 + r)[a_t - c_t]$$

$$k_t \leq \lambda a_t$$

$$a_{t+1} \geq 0$$

The value functions of a matched workers can now be written as

$$W(a_t, e_t) = \max_{c_t} \{u(c_t, 1) + \beta E_t [\max(W(a_{t+1}, e_{t+1}), U(a_{t+1}, e_{t+1}))]\}$$

subject to

$$a_{t+1} = (1 + r)[a_t + w^{max} - c_t]$$

$$a_{t+1} \geq 0$$

II. Appendices to Chapter 2.

II.i. Variables Definitions

WEALTH: The measure of wealth used includes the value of businesses owned, deposits in checking and saving accounts, the value of stocks, mutual funds, or investment trusts (including stocks in IRA's), value on wheels (cars, trucks, etc.), real estate and home equity.

LABOR INCOME: The measure of labor income used includes the worker's wage income, bonuses, overtime and commissions, the income from professional practice or trade as well as the labor part of farm income, business income, market gardening income and roomers and boarders income.

AVERAGE LABOR INCOME: Average of the labor income in the 5 years previous to the interview date. If data for the worker's labor income is not available for any of the previous 5 years, the average is calculated using data on the years for which labor income is available.

AVERAGE LABOR INCOME IN S.E (Self-Employment): Similar to average labor income, but only averaging the labor income in those years when the worker is self-employed.

AVERAGE LABOR INCOME IN P.E (Paid-Employment): Similar to average labor income, but only averaging the labor income in those years when the worker is in paid-employment.

HOURLY INCOME: I measure this as hourly labor earnings. The PSID asks different questions to salaried workers, workers paid by the hour and workers paid in other forms. All questions ask how much would the worker earn for an extra hour of regular work time.

EVER SELF-EMPLOYED: Dummy variable capturing whether the worker was ever self-employed in the 5 years previous to the interview date.

UNEMPLOYED BETWEEN INTERVIEWS: Dummy variable capturing whether the worker was ever unemployed in the year previous to the interview date.

UNEMPLOYED IN THE 3 YEARS BEFORE LAST: Dummy variable capturing whether the workers was ever unemployed in the 3 years previous to a year before the interview date.

WEEKS UNEMPLOYED: Number of weeks unemployed in the year prior to the interview date.

II.ii. Tables

TABLE A1: Self-Employment Rates for Different Groups of Workers.

GROUP	Proportion Self-Employed
Married (N=6,573)	14.47 (0.58)
Not Married (N=1,528)	9.56 (0.96)
White (N=5,059)	14.31 (0.42)
Black (N=2,040)	4.22 (0.74)
High School Drop-out (N= 1,247)	11.85 (0.65)
High School Grad (N=2,497)	10.06 (0.75)
College Drop-out (N=1,704)	14.65 (1.06)
College Grad (N=1,706)	16.41 (1.01)
21-25 (N=633)	7.42 (1.43)
26-30 (N=1,210)	9.20 (0.62)
31-35 (N=1,548)	10.02 (0.92)
36-40 (N=1,421)	13.52 (1.11)
41-45 (N=1,065)	14.60 (1.28)
46-50 (N=660)	18.14 (1.73)
51-55 (N=419)	19.41 (2.32)
56-60 (N=339)	19.83 (2.47)

NOTE: This table shows the proportion of workers that are self-employed, conditional on having a particular characteristic. Standard errors are reported in parenthesis.

TABLE A2: Descriptive Statistics.

VARIABLES	Working for Others (N = 6,456)	Self-Employed (N=864)	p-value of difference
Age	38.30	41.63	< .01
Dummy: Married	0.76	0.83	< .01
Dummy: White	0.88	0.95	< .01
Dummy: Black	0.10	0.03	< .01
Hours Worked per Week	44.72	47.31	< .01
Dummy: Education			
High School Drop-Out	0.14	0.13	.44
High School Grad	0.33	0.24	< .01
College Drop-Out	0.23	0.26	.22
College Grad	0.30	0.37	< .01
Hourly Income			
Mean	19.11	36.28	< .01
10 th Percentile	7.80	7.6	< .01
50 th Percentile	16.19	21.6	< .01
90 th Percentile	31.15	74.42	< .01
Wealth			
Mean	104,941.5	485,037.2	< .01
10 th Percentile	800	13,000	< .01
50 th Percentile	54,500	204,000	< .01
90 th Percentile	289,200	1,001,500	< .01

NOTE: All dollar figures are expressed in 1994 dollars.

TABLE A3: Probability of Leaving Self-Employment By Next Year

	All Self-Employed	Newly Self-Employed
Year 1994	-3.173	-0.8387
Business Owner	-24.994**	-23.574**
Married	-12.264**	-8.835
Black	8.622	14.893
High School Grad	-7.450	-25.615*
College Drop-Out	3.559	12.280
College Grad	1.047	-6.215
Hours Week	-0.284	-0.090
Labor Income/10,000	-0.067*	0.495
Age 26-30	-23.805*	-28.333
Age 31-35	-24.980	-23.625
Age 36-40	-19.447	-13.372
Age 41-45	-26.426*	-27.787
Age 46-50	-22.347*	-40.264
Age 51-55	-28.961**	-34.418
Age 56-60	-13.222	0.8622
Duration in S.E.	-1.506*	-
Wealth /100,000	0.167	-0.374
Observations	810	237
Pseudo-R ²	0.3063	0.2326

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Duration in self-employment is included as a fifth order polynomial. Newly self-employed refers to those workers who have been self-employed for less than a year. * and ** means that the effects are significant at a 5 and 1% significance.

TABLE A4: Overlap in Definitions of Stopgap Self-Employment

Conditional on being ...	Proportion of Job Search Stopgap who are Business Ownership Stopgap	Proportion of Ownership Stopgap who are Job Search Stopgap	CORRELATION
All Self-Employed (S.E.)	39.74 (N=77)	16.25 (N=175)	0.1660** (N=817)
Old S.E.	31.70 (N=32)	12.02 (N=77)	0.1302** (N=576)
Newly Self-Employed	47.61 (N=45)	21.11 (N=98)	0.1599 (N=246)
Newly S.E. – Unemployed	77.32 (N=22)	36.01 (N=35)	0.2992* (N=55)
Newly S.E.- Not Unemployed	32.84 (N=23)	14.22 (N=63)	0.0712* (N=192)
S.E. High School Drop-Out	58.11 (N=14)	24.68 (N=44)	0.2518* (N=116)
S.E. High School Grad	42.99 (N=18)	10.73 (N=51)	0.1487** (N=220)
S.E. College Drop-Out	28.79 (N=15)	11.71 (N=29)	0.1103** (N=207)
S.E. College Grad	35.13 (N=30)	17.58 (N=51)	0.1481** (N=267)
S.E. White	39.52 (N=52)	15.99 (N=126)	0.1635** (N=699)
S.E. Black	48.50 (N=24)	26.86 (N=43)	0.1788** (N=102)
S.E. Married	34.88 (N=52)	14.89 (N=121)	0.1513** (N=692)
S.E. Not Married	52.13 (N=25)	19.26 (N=109)	0.1674** (N=125)
S.E. Age 21-25	60.00 (N=9)	32.43 (N=14)	0.1283 (N=34)
S.E. Age 26-30	49.76 (N=10)	13.66 (N=27)	0.0871 (N=90)
S.E. Age 31-35	18.36 (N=15)	5.83 (N=36)	0.0346 (N=90)
S.E. Age 36-40	44.38 (N=16)	15.59 (N=33)	0.2059** (N=168)
S.E. Age 41-45	41.90 (N=9)	26.00 (N=20)	0.2656** (N=145)
S.E. Age 46-50	36.72 (N=11)	16.98 (N=21)	0.1672** (N=113)
S.E. Age 51-55	36.72 (N=5)	20.67 (N=11)	0.2965** (N=64)
S.E. Age 56-60	100.00 (N=2)	7.45 (N=13)	0.1592* (N=62)

Note: The second and third columns of this table shows the proportion of self-employed workers that are stopgap self-employed according to the job search and business ownership definition, conditional on satisfying the characteristics specified on the row and on being stopgap self-employed according to the stopgap definition specified in the column. The third column shows correlation between the variables conditional on satisfying the characteristic. The numbers in brackets are the number of observations used to calculate the proportions. Old self-employed refers to workers that have been self-employed for over a year. Newly self-employed are workers that have been self-employed for less than a year. ** denotes statistically significant at 1%, * statistically significant at 5%.

**TABLE A5: Proportion of Self-Employed Workers Using
Self-Employment as a Stopgap.**

Conditional on being ...	PROPORTION OF STOPGAP SELF-EMPLOYED		
	Search Definition	Bus. Ownership Definition	Combined Definition
Self-Employed (S.E) (N = 871)	6.916 (1.004)	16.909 (1.531)	21.077(1.644)
Old Self-Employed (N = 590)	4.688 (0.979)	12.368 (1.604)	15.571(1.736)
Newly Self-Employed (N= 286)	12.704(2.517)	28.656 (3.449)	35.312(3.625)
Newly S.E. - Unemployed(N= 65)	23.413(7.536)	50.306 (8.841)	55.606(8.731)
Newly S.E.- Not Unemployed(N= 220)	10.353(2.548)	23.903 (3.567)	30.857(3.864)
S.E. High School Drop-Out (N=129)	11.492(3.842)	27.058 (4.978)	31.872(5.222)
S.E. High School Grad (N=235)	4.076 (1.279)	16.328 (2.897)	18.65 (2.997)
S.E. College Drop-Out (N=218)	5.334 (1.722)	13.111 (2.801)	16.910(3.056)
S.E. College Grad (N=282)	8.435 (1.856)	16.863 (2.617)	22.335(2.853)
S.E. White (N=743)	6.757 (1.045)	16.507 (1.571)	20.625(1.690)
S.E. Black (N=110)	16.708(5.089)	30.175 (8.687)	38.780(8.875)
S.E. Married (N=723)	6.005 (0.969)	14.066 (1.504)	17.977(1.631)
S.E. Not Married (N=147)	11.275(3.453)	30.512 (4.913)	35.909(5.121)
S.E. Age 21-25 (N=36)	25.451(8.958)	39.051(10.243)	51.83(10.233)
S.E. Age 26-30 (N=193)	8.962 (3.633)	21.952 (5.178)	27.916(5.571)
S.E. Age 31-35 (N=150)	7.471 (2.331)	23.519 (4.353)	29.619(4.533)
S.E. Age 36-40 (N=199)	4.442 (1.536)	12.642 (2.866)	15.114(3.023)
S.E. Age 41-45 (N=152)	6.989 (2.708)	11.261 (3.095)	15.322(3.591)
S.E. Age 46-50 (N=121)	6.846 (2.592)	14.809 (3.821)	19.141(4.186)
S.E. Age 51-55 (N=68)	4.561 (2.591)	12.853 (4.512)	14.758(4.747)
S.E. Age 56-60 (N=63)	2.283 (1.637)	17.469 (5.198)	18.450(5.263)

Note: This table shows the proportion of self-employed workers using self-employment as a stopgap, conditional on having a particular characteristic. Old self-employed refers to workers that have been self-employed for over a year. Newly self-employed are workers that have been self-employed for less than a year. Standard errors are reported in parenthesis.

TABLE A6: Descriptive Statistics for the Self-Employed.

	Search Definition		Business Ownership Definition		Combined Definition	
	Stopgap (N = 77)	Others (N = 787)	Stopgap (N = 174)	Others (N = 689)	Stopgap (N = 215)	Others (N = 647)
Age	37.55	41.93**	39.40	42.07**	38.99	42.34**
Dummy: Married	0.72	0.84*	0.69	0.86**	0.71	0.86**
Dummy: White	0.93	0.95	0.93	0.96	0.93	0.96**
Hours Worked per Week	39.76	47.87*	38.28	49.16**	39.64	49.37**
Dummy: Education						
High School Drop-Out	0.21	0.12	0.20	0.11**	0.19	0.10**
High School Grad	0.14	0.25	0.23	0.24	0.21	0.25
College Drop-Out	0.20	0.27	0.20	0.27*	0.21	0.27*
College Grad	0.45	0.51	0.37	0.37	0.39	0.37
Self-Employment Duration						
Years Self-Employed (Median)	0	3*	1	3**	1	4**
Years Self-Employed <1	0.52	0.25*	0.46	0.23**	0.47	0.22**
Hourly Income						
Mean	20.00	30.58*	28.73	32.80	28.87	29.04
10 th Percentile	1.54	5.50*	5.25	5.21	5.14	5.51
50 th Percentile	12.86	17.30	14.71	17.73	14.42	17.15*
90 th Percentile	38.48	57.70*	42.00	60.00	42.00	60.00
Wealth						
Mean	177,004.6	508,116.8**	127,847.8	558,412.3**	116,572.1	459,408.8**
10 th Percentile	-423.6	9,120**	-3480	19,200**	0	20,000**
50 th Percentile	56,000	190,800**	32,250	227,000**	45000	236,520**
90 th Percentile	650,373.6	987,240**	487,000	1,062,000**	522,500	1,110,000**

Note: All dollar figures are expressed in 1994 dollars. ** and * indicate statistically significant differences between stopgap and other self-employed workers at 1% and 5% significance, respectively.

TABLE A7: Proportions of Self-Employed Workers in Different Occupations.

	Search Definition		Business Ownership Definition		Combined Definition	
	Stopgap (N = 77)	Others (N =794)	Stopgap (N = 175)	Others (N = 696)	Stopgap (N = 216)	Others (N = 655)
Professional and Technical Workers	0.27	0.22	0.29	0.21	0.29	0.21
Managers and Administrators	0.24	0.41**	0.15	0.44**	0.18	0.45**
Sales Workers	0.13	0.08	0.13	0.07	0.13	0.07
Clerical and Kindred Workers	0.02	0.01	0.03	0.01*	0.03	0.01
Craftsmen and Kindred Workers	0.17	0.17	0.23	0.16	0.21	0.16
Operatives, except Transport	0	0.01	0.01	0.01	0.01	0.01
Transport Equipment Operatives	0.06	0.02	0.03	0.02	0.03	0.02
Laborers, except Farm	0.11	0.04**	0.09	0.04**	0.10	0.03*
Service Workers	0.00	0.02	0.03	0.02	0.02	0.02

Note: ** and * indicate statistically significant differences between stopgap and other self-employed workers at 1% and 5% significance, respectively.

TABLE A8 Proportions of Self-Employed Workers in Different Industries.

	Search Definition		Business Ownership Definition		Combined Definition	
	Stopgap (N = 77)	Others (N =874)	Stopgap (N = 175)	Others (N = 696)	Stopgap (N = 216)	Others (N = 654)
Agriculture, Forestry, and Fisheries	0.12	0.04**	0.06	0.05	0.08	0.04
Construction	0.18	0.24*	0.26	0.23	0.24	0.24
Manufacturing	0	0.07*	0.04	0.07*	0.03	0.07*
Transportation/Com munications/Utilities	0.07	0.02	0.02	0.03	0.03	0.03
Wholesale and Retail Trade	0.20	0.20	0.11	0.21**	0.13	0.21**
Finance, Insurance, and Real Estate	0.09	0.06	0.08	0.06	0.08	0.06
Business and Repair Services	0.11	0.13	0.12	0.13	0.12	0.13
Personal Services	0	0.04	0.04	0.03	0.03	0.04
Entertainment and Recreation Services	0.04	0.02	0.03	0.02	0.04	0.02
Professional and Related Services	0.19	0.16	0.22	0.15	0.20	0.15*

Note: ** and * indicate statistically significant differences between stopgap and other self-employed workers at 1% and 5% significance, respectively.

TABLE A9: Probability of Becoming Self-Employed

	ALL SELF	STOPGAP			OTHER		
		Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	0.462	-0.374	-0.217	-0.222	0.740	0.773	0.732
Married	-1.420	-0.163	0.378	0.360	-1.404	-2.047**	-1.966**
Black	-0.680	0.553	-0.020	-0.270	-1.222*	-0.933*	-0.700
High School Grad	-0.253	0.187	0.296	0.274	-0.251	-0.210	-0.222
College Drop-Out	-2.126	-0.054	-0.303	-0.294	-1.817*	-1.137	-0.117
College Grad	-1.128	0.664	-0.415	-0.101	-1.520	-0.518	-0.638
Labor Income / 10,000	0.4503	-0.238	-0.178**	-0.199**	0.740	0.332	-0.332
Average Labor Income / 10,000	-0.404	0.156	-0.149	-0.110	0.450	-0.219	-0.224*
Ever Self-Employed	14.915**	5.619**	9.579**	10.047**	9.007**	4.878**	4.415**
Unemployed in 3 years Before Last	1.518	0.123	1.090*	1.281*	1.822*	0.285	0.099
Unemployed Between Interviews	22.454*	0.958	26.817**	28.034**	9.007	12.373	8.752
Received Unempl. Comp.	-2.755**	-0.485	-1.665*	-1.613*	-17.773*	-0.687	-0.816
Weeks Unemployed	-1.277	-0.074	-0.835	-0.925	-0.903	-0.815	-0.701
Wealth / 100,000	0.060	0.506	0.818	0.663	0.046	0.150	0.223
Observations	6,395	6,395	6,395	6,395	6,395	6,395	6,395
Pseudo-R ²	0.2272	0.3291	0.3263	0.3117	0.2216	0.1902	0.1226

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Wealth and weeks unemployed are both entered as a fifth order polynomial, expect for transitions into stopgap self-employment, where wealth is introduced as a third-order polynomial. Labor income is entered as a second order polynomial. Average labor income is calculated over the previous five years. ** denotes statistically significant at 1%, * statistically significant at 5%.

TABLE A10: Effect of Local Unemployment Rates on the Probability of Becoming Self-Employed.

	ALL SELF	STOPGAP			OTHER		
		Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Married	-0.765	0.195	-0.214	-1.110	-1.054	-1.308**	-0.236**
Black	0.028	3.193	0.539	-0.765	-0.461**	-1.921**	-0.032**
High School Grad	-1.689	-1.266	0.170	0.028	-1.130	-0.032	0.042
College Drop-Out	-0.465	3.436*	-1.015	-1.689	-0.754*	-0.415	-1.292
College Grad	-1.110	-3.022	-2.140	-0.465	-0.463	0.143	-0.247
Labor Income / 10,000	0.0708	0.070	0.160*	0.154	0.307	-0.250	-0.174
Average Labor Income / 10,000	-0.025	0.470	-0.529	-0.025	-0.131	0.163*	-0.203*
Ever Self-Employed	7.925**	13.239**	7.247**	7.925**	3.651**	0.736**	7.264**
Unemployed in 3 years Before Last	1.468*	1.042*	0.572	1.468*	1.388*	0.710*	0.900*
Unemployed Between Interviews	16.031*	4.551*	13.970**	16.031**	25.284*	5.156	7.850
Received Unempl. Comp.	-1.840**	-3.861**	-0.663	-1.840*	-1.048**	-0.390	-1.424
County Unempl. Rate	-0.288*	0.100*	-0.281*	-0.288*	-0.247	-0.053	-0.212
CountyUnempl.Rate*Dummy Unempl.	-0.767	-3.032	-0.532	-0.767	-0.376*	-0.237	-0.386
Weeks Unemployed	-1.110	-1.110	-0.778**	-0.458**	-0.991	-0.462	-0.364
Wealth / 100,000	-0.899	-0.899	-0.477	-0.956	0.467	0.023	0.188
Observations	2,560	2,560	2,560	2,560	2,560	2,560	2,560
Pseudo-R ²	0.2855	0.2855	0.3947	0.2855	0.3130	0.2909	0.2772

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Wealth and weeks unemployed are both entered as a fifth order polynomial, expect for transitions into stopgap self-employment, where wealth is introduced as a third-order polynomial. Labor income is entered as a second order polynomial. The average labor income is calculated over the previous five years. 5-year age group dummies are also included. ** denotes statistically significant at 1%, * statistically significant at 5%.

TABLE A11: Marginal Effect of Wealth on the Probability of Becoming Self-Employed

	ALL SELF	STOPGAP			OTHER		
		Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
All Workers	0.069	0.506	0.202	0.663	0.046	0.150	0.223
1 st Wealth Quintile	0.062	1.048	1.733	1.484	0.033	0.204	0.322
2 nd Wealth Quintile	0.039	0.891	1.615	1.368	0.033	0.150	0.223
3 rd Wealth Quintile	0.046	0.370	0.688	0.583	0.026	0.106	0.159
4 th Wealth Quintile	0.090	0.238	0.315	0.203	0.039	0.123	0.184
5 th Wealth Quintile	0.202	-0.067	-0.293	-0.353	0.105	0.172	0.234
Highest 5%	0.506	-0.058	-0.143	-0.148	0.258	0.268	0.318

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Wealth and weeks unemployed are both entered as a fifth order polynomial, expect for transitions into stopgap self-employment, where wealth is introduced as a third-order polynomial.

TABLE A12: Effect of Wealth on the Probability of Becoming Self-Employed for Workers Who Experience and Did not Experience Unemployment.

	ALL SELF	STOPGAP			OTHER		
		Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	0.562	-0.361	-0.199	-0.195	0.791	0.835	0.788
Married	-1.540	-0.162	0.361*	0.327**	-1.473	-2.132**	-2.031**
Black	-0.678	0.535	0.0158	-0.230	-1.233*	-0.931*	-0.700
High School Grad	-0.271	0.172	0.289	0.249	-0.252	-0.288	-0.279
College Drop-Out	-2.137	-0.068	-0.354	-0.354	-1.827*	1.175	-1.194
College Grad	-1.118	0.621	-0.620	-0.330	-1.493	-0.544	-0.649
Labor Income / 10,000	0.443*	-0.236	-0.087	-0.116	0.044	0.032	0.032
Average Labor Income / 10,000	-0.402	0.152	-0.081	-0.038	-0.355	-0.087	-0.222
Ever Self-Employed	15.02**	5.596**	9.947**	10.391**	9.119**	4.950**	4.477**
Unemployed in 3 years Before Last	1.490	0.116	1.224*	1.431	1.793*	0.267	0.096
Unemployed Between Interviews	22.518*	0.822	27.821**	28.902**	18.884	10.236	7.828
Received Unempl. Comp.	-2.850**	-0.485	-1.758*	-1.711	-2.134*	-0.781	-0.866
Weeks Unemployed	-1.305	-0.697	-0.838	-0.927	-0.950	-0.820	-0.716
Wealth Unemployed / 100,000	17.305	2.526	1.056	1.757	4.486	8.175	5.970
Wealth Employed / 100,000	-0.004	0.250	0.096	0.143	0.059	0.087	0.151
Observations	6,395	6,395	6,395	6,395	6,395	6,395	6,395
Pseudo-R ²	0.2396	0.3299	0.3177	0.2083	0.2251	0.1972	0.2083

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Wealth and weeks unemployed are both entered as a fifth order polynomial, expect for transitions into stopgap self-employment, where wealth is introduced as a third-order polynomial. Labor income is entered as a second order polynomial. The average labor income is calculated over the previous five years. -year age group dummies are also included. ** denotes statistically significant at 1%, * statistically significant at 5%.

TABLE A13: Probability of Becoming Self-Employed - Instrumental Variable Approach.

	STOPGAP			OTHER		
	Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	0.137	0.422	0.548	1.545	1.0688	0.806
Married	-0.365	-1.119	-1.462	-5.771	-3.609	-3.600
Black	0.121	0.442	0.570	3.956	2.276	2.203
High School Grad	-0.037	-0.072	-0.087	-1.038	-0.402	-0.798
College Drop-Out	-0.109	-0.331	-0.417	-3.875	-1.621	-2.045
College Grad	0.252	0.816	1.071	3.341	0.8993	0.089
Labor Income / 10,000	-0.359	-1.011	-1.305	4.456	2.2769	2.556
(Labor Income / 10,000) ²	0.015	0.039	0.050	-0.394	-0.135	-0.169
Average Labor Income / 10,000	0.143	0.339	0.433	-7.316	-4.517	-4.303
Ever Self-Employed	-0.252	-1.157	-1.277	-3.400	-2.553	-1.643
Unemployed in 3 years Before Last	0.140	0.537	0.693	3.621	0.8217	0.527
Unemployed Between Interviews	0.232	1.414	1.372	28.257	7.2939	6.223
Received Unempl. Comp.	0.012	0.103	0.177	-1.261	0.0723	-0.128
Weeks Unemployed	-0.051	-0.348	-0.414	-6.475	-1.343	-1.061
(Weeks Unemployed) ²	0.010	0.042	0.051	0.539	0.0896	0.063
(Weeks Unemployed) ³	-0.001	-0.002	-0.002	-0.018	-5.34e-4	5.67e-4
(Weeks Unemployed) ⁴	1.42e-5	3.233e-5	3.23e-5	2.40e-4	-5.63e-5	-7.65e-5
(Weeks Unemployed) ⁵	-1.1e-7	-3.53e-7	-4.3e-7	-1.10e-6	8.23e-7	9.59e-7
Wealth / 100,000	0.232	0.118	0.153	70.968	34.7878	31.965
(Wealth / 100,000) ²	0.012	-0.028	-0.037	-21.687	-11.231	-9.446
(Wealth / 100,000) ³	-0.051	0.007	0.009	1.570	1.0408	0.789
(Wealth / 100,000) ⁴				-0.036	-0.03	-0.020
(Wealth / 100,000) ⁵				2.87e-4	2.64e-4	1.70e-5
Observations	5,896	5,896	5,896	5,896	5,896	5,896

Note: 5-year age group dummies are also included. ** statistically significant at 1%, * statistically significant at 5%.

TABLE A13 (cont): Instrumental Variable Approach – First Stage Regression.

	WEALTH
Year 1994	0.058
Married	0.062**
Black	-0.274***
High School Grad	0.038
College Drop-Out	0.063
College Grad	0.177***
Labor Income / 10,000	-0.003
(Labor Income / 10,000) ²	0.002***
Average Labor Income / 10,000	0.200***
Ever Self-Employed	0.172***
Unemployed in 3 years Before Last	-0.015
Unemployed Between Interviews	-0.223
Received Unempl. Comp.	-0.092**
Weeks Unemployed	0.047
(Weeks Unemployed) ²	-0.002
(Weeks Unemployed) ³	3.16 e-5
(Weeks Unemployed) ⁴	6.05e-7
(Weeks Unemployed) ⁵	-1.19e-8
Regional House Value Appreciation	7.69e-7
(Regional House Value Appreciation) ²	-4.29e-10*
(Regional House Value Appreciation) ³	1.15e-14***
Observations	5,892
R ²	0.2294

Note: 5-year age group dummies are also included. *** denotes statistically significant at 1%, ** statistically significant at 5%, * statistically significant at 10%.

**TABLE A14: Proportion of Self-Employed Workers Leaving
Self-Employment By Next Year**

VARIABLES	Job Search Definition	Business Ownership Definition	Combined Definition
Stopgap	60.21 (N=77)	54.73 (N=175)	55.42 (N=216)
Others	17.57 (N=794)	13.56 (N=696)	11.20 (N=655)
Old Self-Emp in Stopgap	63.46 (N=32)	43.38 (N=77)	61.82 (N=98)
New Self-Emp in Stopgap	57.03 (N=45)	67.73 (N=98)	60.13 (N=118)
Old Self-Emp in Other	13.18 (N=559)	11.61 (N=514)	9.20 (N=493)
New Self-Emp in Other	30.53 (N=235)	20.50 (N=182)	18.32 (N=162)

NOTE: This table shows the proportion of self-employed workers leaving self-employment from one year to the next conditional on having a particular characteristic.

TABLE A15: Probability of Leaving Self-Employment By Next Year.

	ALL SELF-EMPLOYED			OLD SELF-EMPLOYED			NEW SELF-EMPLOYED		
	Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	1.662	-2.544	-3.374	2.087	-0.465	-1.822	5.926	-1.272	-0.905
Stoppag	11.482*	33.659**	30.689**	41.992**	50.871**	46.584**	4.413*	28.02**	24.818**
Married	-13.010	-11.688	-12.210	-18.333*	-16.700**	-17.545**	-9.017	-8.535	-8.859
Black	11.091*	8.311	8.667	7.912	3.071	3.627	15.922	14.230*	14.031
High School Grad	-11.033	-6.862	-7.209	1.447	5.748	5.149	-28.199*	-23.782*	-24.049*
College Drop-Out	-3.584	4.532	2.618	-2.685	2.596	-1.552	1.791	13.464	11.891
College Grad	-4.178	1.077	-1.481	-5.567	5.173	3.333	-13.070	-5.587	-8.91
Hours Week	-0.511**	-0.297*	-0.335	-0.632**	-0.294*	-0.345*	-0.267	-0.071	-0.104
Labor Income/10,000	0.009	-0.016	-0.062	-0.089	-0.211	-0.051	0.576	0.6715	0.205
Age 26-30	-23.650*	-23.512*	-23.027*	-24.005	-22.186*	-23.380*	-26.790	-24.071	-21.909
Age 31-35	-25.334*	-24.571*	-24.568*	-27.501*	-29.113*	-31.493*	-26.367	-20.935	-18.914
Age 36-40	-21.575*	-19.191	-19.580	-27.619*	-25.925*	-28.469*	-19.245	-11.966	-10.665
Age 41-45	-27.475*	-25.802*	-25.560*	-32.713*	-32.250**	-33.544**	-30.908	-22.188	-21.983
Age 46-50	-23.799*	-21.582*	-21.892	-25.783*	-25.180*	-27.294*	-41.118*	-29.273	-27.522
Age 51-55	-30.712*	-28.375*	-29.574**	-27.644*	-29.227*	-29.733**	-41.027*	-26.148*	-29.471
Age 56-60	-13.495	-11.306	-12.029	-25.576*	-24.542*	-25.893*	-6.151	3.303	1.932
Duration in S.E.	-0.080*	-0.0414*	0.321**	1.242**	0.5431**	1.163**	-	-	-
Wealth Stoppag/100,000	4.180	-1.648	0.877	-1.220	-2.485	-1.227	6.163	-3.275	1.377
Wealth Others/100,000	0.112	0.178	0.205	0.229	0.292	0.304	-0.596	-0.341	-0.292
Observations	810	810	810	573	573	573	237	237	237
Pseudo-R ²	0.2554	0.3132	0.3194	0.3627	0.4485	0.4628	0.1619	0.2337	0.2341

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Duration in self-employment is included as a fifth order polynomial. * and ** means that the effects are significant at 5 and 1%.

TABLE A16: Probability of Leaving Self-Employment By Next Year Conditional on Type of Self-Employment.

	ALL SELF	STOPGAP			OTHER		
		Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	0.280	12.586	-11.146	-8.699	5.923	5.017	3.669
Married	-0.39	22.791	-12.161	-8.736	-20.745**	-16.977*	-18.338*
Black	0.337*	-17.948**	13.914*	7.380	9.153	3.254	6.896
High School Grad	-0.392	-14.852	-14.682	-13.523	-7.949	1.864	1.760
College Drop-Out	0.370	-1.945	22.403	-0.985	0.106	1.903	2.374
College Grad	0.381	-0.921	6.394	1.458	-1.861	-1.339	-5.386
Hours Week	-0.011**	4.099**	0.081	-0.284	-0.597**	-0.549**	-0.517**
Labor Income/10,000	0.018	-43.959	-2.025	-0.154	-0.025	0.440	0.242
Age 26-30	-0.402*	-13.581**	-36.298*	-28.727	-16.245	-17.376	-17.818
Age 31-35	-0.368*	-39.509	-28.815	-22.085	-25.707*	-24.547*	-26.640*
Age 36-40	-0.376*	-26.364**	-25.423	-24.612	-20.278	-16.326	-17.467
Age 41-45	-0.445**	-37.778	-19.185	-7.001	-25.080	-28.198*	-29.622*
Age 46-50	-0.358*	-47.790**	-47.181**	-37.487*	-20.773	-15.129	-15.965
Age 51-55	-0.523**	12.586**	-56.092**	-50.729**	-26.725*	-23.660*	-20.986
Age 56-60	-0.429	22.791	-24.859	-19.052	-11.450	-4.822	-6.748
Duration in S.E.	-0.250**	24.155	5.516	5.388	-0.958	-1.395**	-1.120
Wealth/100,000	0.009	0.0333	-1.772	-2.001	0.1743	0.135	0.207
Observations	810	70	157	196	738	653	684
Pseudo-R ²	0.2423	0.4207	0.2279	0.1470	0.2739	0.2631	0.3326

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Duration in self-employment is included as a fifth order polynomial. * and ** means that the effects are significant at a 5 and 1% confidence level.

TABLE A17: Probability of Transitioning To Employment, Unemployed or Out of the Labor Force By Next Year.

	EMPLOYMENT			UNEMPLOYMENT			OUT OF LABOR FORCE		
	Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition	Search Definition	Ownership Definition	Combined Definition
Year 1994	0.238	-5.103	-6.160	0.091	2.494	1.474	0.176**	02.968*	3.055*
Stopgap	7.050*	39.310**	38.592**	-0.084	-5.492	-4.355	0.270	4.343**	3.537**
Male	-0.336	-5.322	-6.069	-0.087	-2.788	-2.900	-0.101	-3.186	-3.353
Black	0.280	3.332	3.681	0.085	2.331	1.835	-0.095	-1.192	-1.197
High School Grad	0.341	3.260	3.205	-0.124**	-6.197**	-6.709**	-0.092	-1.338	-1.454
College Drop-Out	-0.360	-2.183	-3.635	0.140**	5.055*	5.240*	0.066	0.125	-0.066
College Grad	0.395	4.519	1.698	-0.134	-4.069	-4.601	0.129	2.512	2.303
Hours Week	-0.006	0.128	0.106	0.003**	-0.294**	-0.247**	-0.001	0.014	0.006
Labor Income/10,000	0.015	0.120	0.089	-0.037**	-1.582*	-1.682**	-0.021	-0.554*	-0.543*
Age 26-30	-0.315	-15.281	-14.441	-0.104	-5.221	-4.578	-0.118	-2.101	-2.024
Age 31-35	-0.323**	-20.754**	-20.296*	0.222	2.085	3.196	0.143	0.564	0.787
Age 36-40	-0.324*	-17.773*	-17.892*	0.203	1.458	2.197	-0.084	-0.886	-1.102
Age 41-45	-0.348*	-20.167*	-19.270*	0.219	1.234	1.283	0.215	3.329	3.534
Age 46-50	-0.319*	-16.462*	-16.130	-0.091	-5.226	-5.241	0.195	1.155	1.326
Age 51-55	-0.420**	-21.222**	-21.800**	0.347	3.837	3.668	0.152	2.389	1.585
Age 56-60	-0.344	-14.832	-14.696	0.091	2.494	1.474	0.684**	12.560**	12.922**
Duration in S.E.	0.723**	0.487**	0.943**	-0.743**	-0.373**	-0.508**	-0.178	-0.095	-0.091
Wealth Stopgap/100,000	0.160	-2.431	-0.500	0.056	0.699	0.765	-0.434*	-0.244	-0.240
Wealth Others/100,000	0.009	0.239	0.277	-0.109	-1.654	-3.180	-0.005	-0.015	-0.018
Observations	810	810	810	810	810	810	810	810	810
Pseudo-R ²	0.1712	0.2558	0.2773	0.6738	0.6610	0.6625	0.3873	0.2731	0.3759

Note: The numbers reported are average marginal effects. Marginal effects are calculated for each individual, and then averaged across individuals. Duration in self-employment is included as a fifth order polynomial. * and ** means that the effects are significant at a 5 and 1% confidence level.

II.iii. Figures

FIGURE A1: Distributions Over Self-Employment Duration

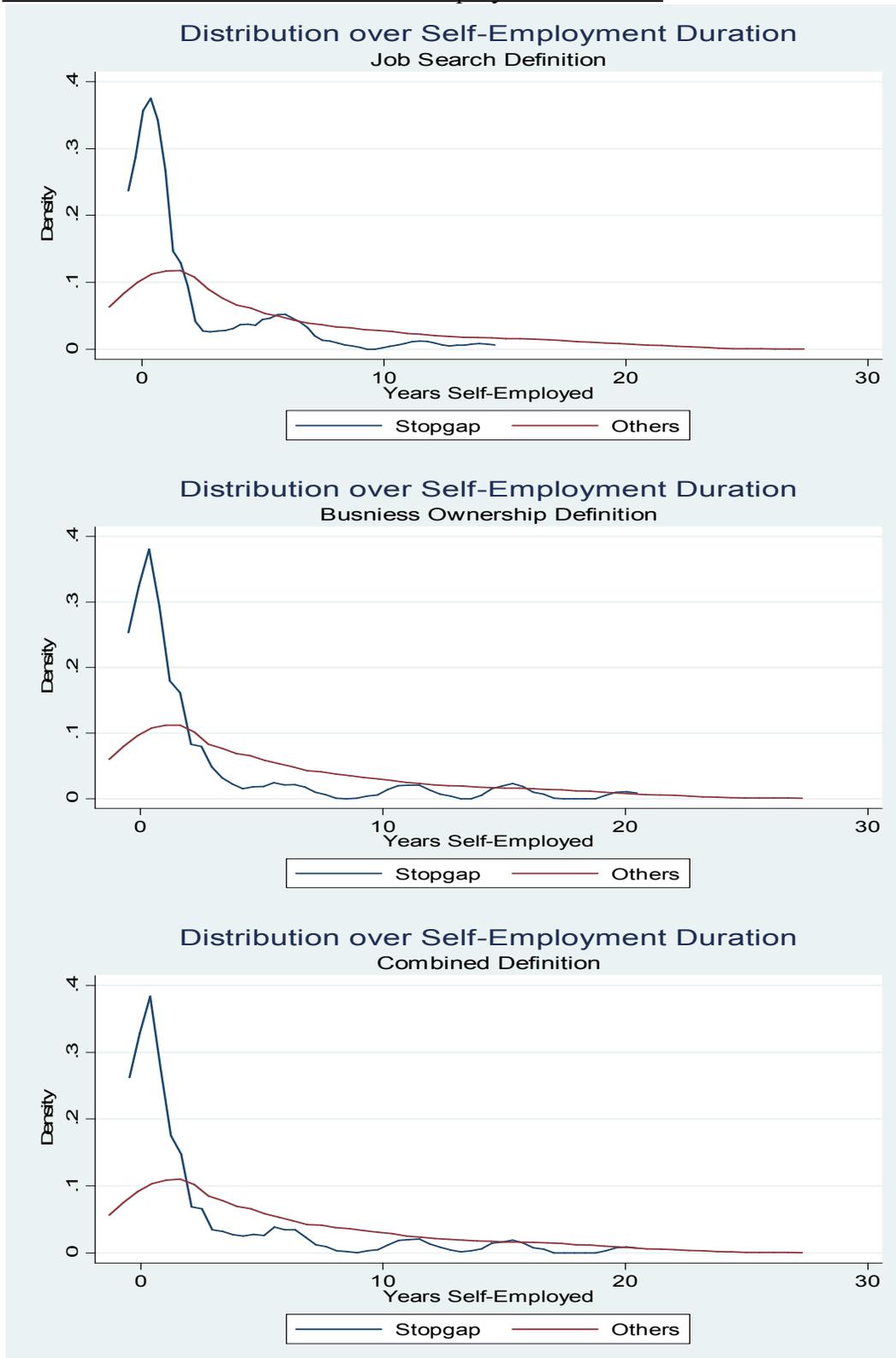
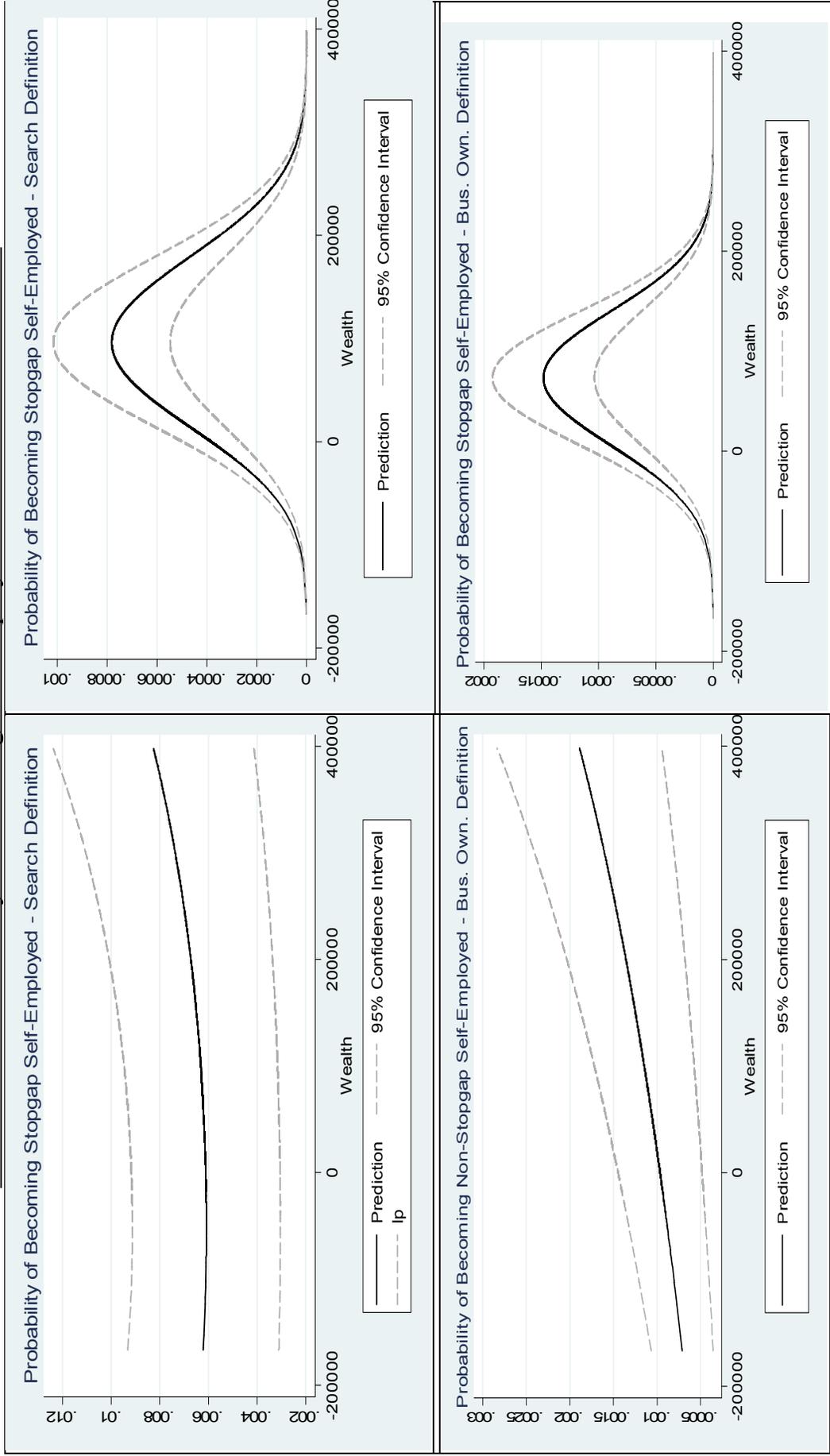
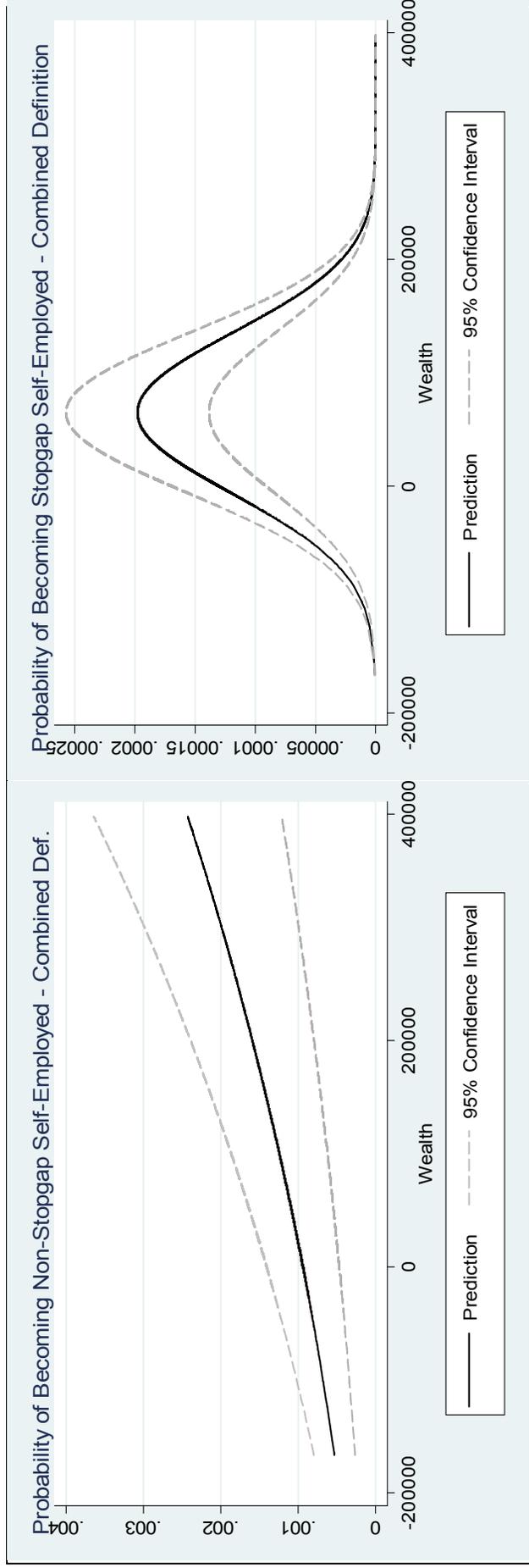


FIGURE A2: Probability of Becoming Self-Employed As a Function of Wealth



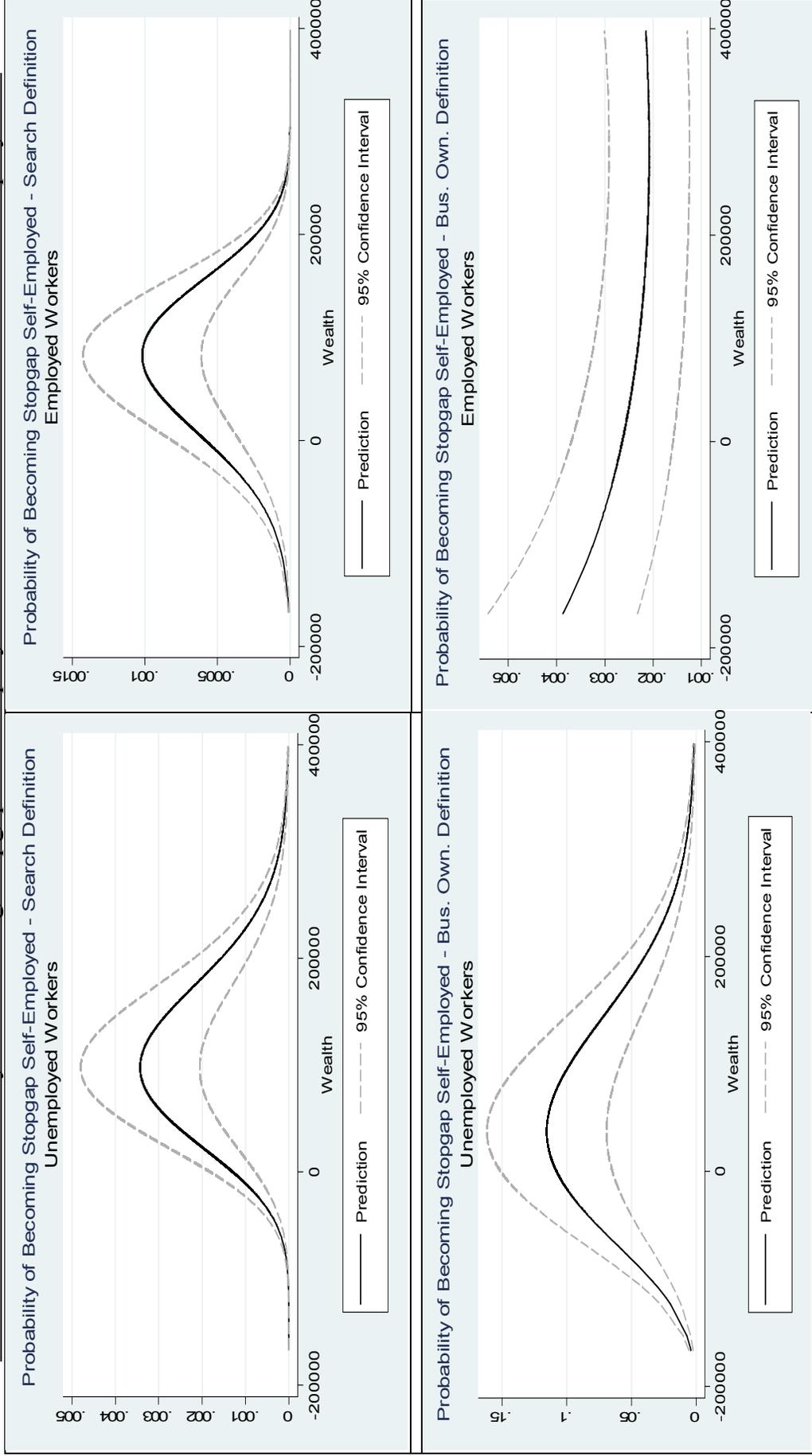
Note: All other controls included in the probit are evaluated at their mean values.

FIGURE A2: Probability of Becoming Self-Employed As a Function of Wealth (cont.)



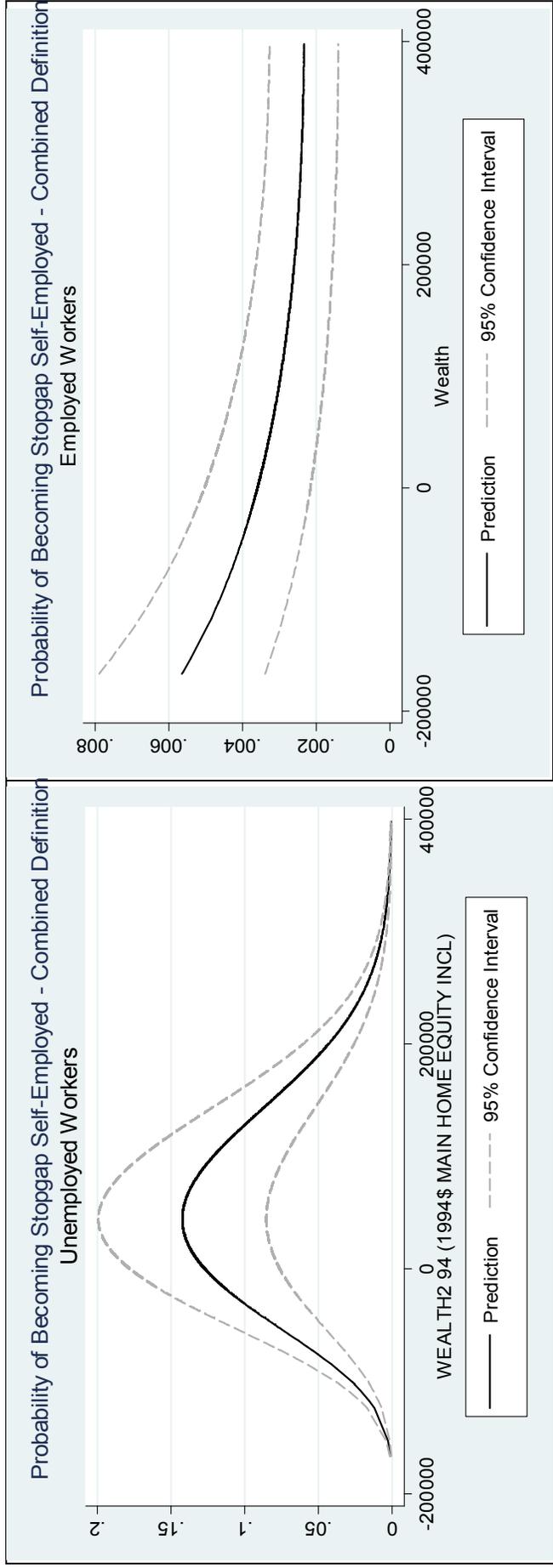
Note: All other controls included in the probit are evaluated at their mean values.

FIGURE A3: Probability of Becoming Stopgap Self-Employed As a Function of Wealth and Unemployment



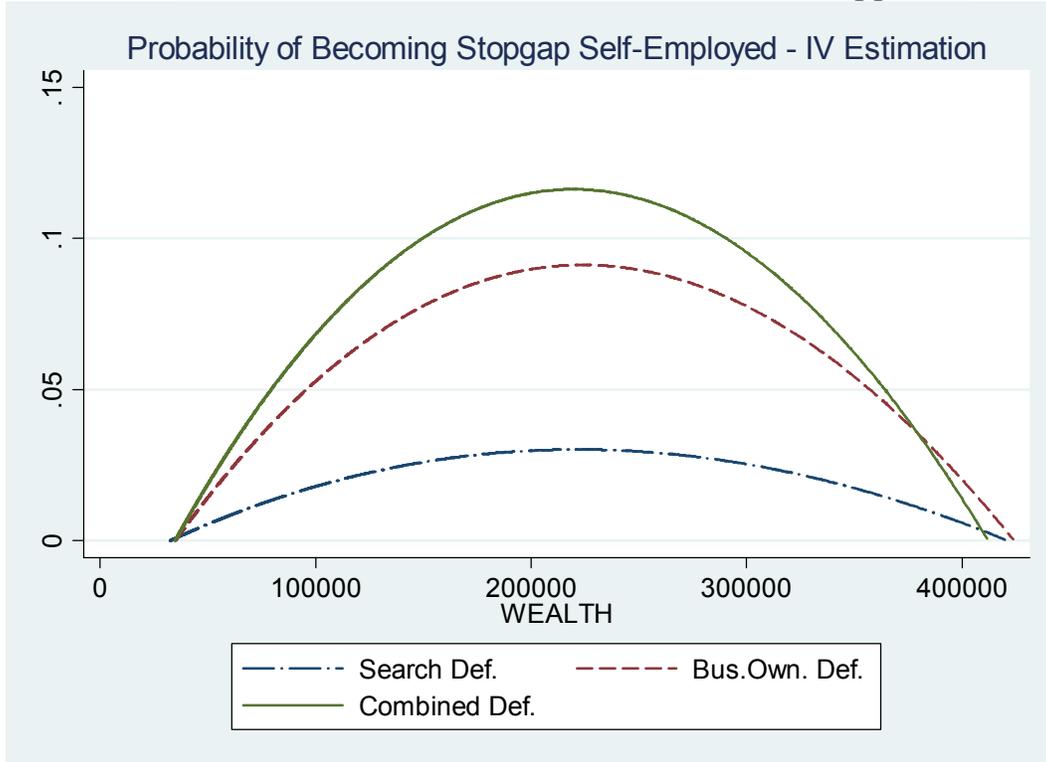
Note: All other controls included in the probit are evaluated at their mean values.

FIGURE A3: Probability of Becoming Stopgap Self-Employed As a Function of Wealth and Unemployment (cont.)



Note: All other controls included in the probit are evaluated at their mean values.

FIGURE A4: Probability of Becoming Stopgap Self-Employed As a Function of Wealth - Instrumental Variable Approach



Note: All other controls included in the probit are evaluated at their mean values.

III. Appendices to Chapter 3.

III.i. Proof of Proposition 3

One can show that, along the balanced growth path, the value of having a vacant job and the value of being unemployed can be expressed as

$$(A3.1) \quad V(\theta) = \frac{(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\theta q(\theta)]c}{(R(g) - g)[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\theta q(\theta)]}$$

$$(A3.2) \quad U(\theta) = \frac{b}{R(g) - g} + \frac{q(\theta)[y - b + c]}{(R(g) - g)[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\theta q(\theta)]}$$

After replacing equations (A3.1) and (A3.2) into equation (3.43), one can then define the

system of equations formed by equations (3.43) and (3.44) as

(A3.3)

$$H(\hat{\alpha}, \theta, g) = \hat{\alpha} - \frac{b[(1-\beta)q(\theta) + R(g) - g + \delta + \beta\mu(\theta)] + \beta\mu(\theta)[y - b + c] + (R(g) - g)K}{A(g)\{[(1-\beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}}$$

$$(A3.4) \quad G(\hat{\alpha}, \theta, g) = \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - \frac{\delta\theta + \mu(\theta)}{\delta + \mu(\theta)}$$

where $\mu(\theta) \equiv \theta q(\theta) = m(v, u)/u$. Given that we assumed that the matching function $m(v, u)$ is homogeneous of degree 1 in vacancies and unemployment, $\mu'(\theta) > 0$.

Comparative statics can be performed by total differentiation of equations (A3.3) and (A3.4) with respect to g :

$$(A3.5) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial g} + \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \frac{\partial \theta}{\partial g} + \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}$$

$$(A3.6) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial g} + \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \frac{\partial \theta}{\partial g} + \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}$$

In matrix notation, we have

$$(A3.7) \quad \begin{bmatrix} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{\alpha}}{\partial g} \\ \frac{\partial \theta}{\partial g} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} \\ -\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} \end{bmatrix}$$

Define $B \equiv \begin{bmatrix} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{bmatrix}$. The elements of B are the following:

$$(A3.8) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} = 1$$

$$(A3.9) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} = -\frac{U'(\theta)}{V(\theta)} + \frac{V'(\theta)[U(\theta) + (R(g) - g)K]}{[V(\theta)]^2} < 0$$

$$(A3.10) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} = \frac{-\hat{\alpha}A(g)F(\hat{\alpha}) - A(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} < 0$$

$$(A3.11) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} = -\frac{\delta\theta + \mu'(\theta)}{\delta + \mu(\theta)} - \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{[\delta + \mu(\theta)]^2} < 0$$

Equations (A3.8) through (A3.11) can be used to sign the slopes of the entrepreneurship and job creation curves:

$$(A3.12) \quad \left. \frac{d\theta}{d\hat{\alpha}} \right|_E = -\left(\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} / \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \right) > 0$$

$$(A3.13) \quad \left. \frac{d\theta}{d\hat{\alpha}} \right|_{JCE} = -\left(\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} / \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \right) < 0$$

Equations (A3.8) through (A3.11) also allow one to sign the determinant of B:

$$(A3.14) \quad \det(B) = \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} - \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} < 0$$

We can now use Cramer's rule to calculate the effect of growth on entrepreneurship:

$$(A3.15) \quad \frac{d\hat{\alpha}}{dg} = [\det(B)]^{-1} \begin{vmatrix} -\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{vmatrix}$$

where

$$(A3.16) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} = \frac{A'(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

and

$$(A3.17)$$

$$\begin{aligned} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} = & -(R(g) - g)c \frac{b[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\mu(\theta)] + \beta\mu(\theta)[y - b + c] + (R(g) - g)K}{A(g)\{[(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}^2} \\ & - \frac{(R'(g) - 1)[b + K]}{A(g)\{[(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}} \\ & + A'(g) \frac{b[(1 - \beta)q(\theta) + R(g) - g + \delta + \beta\mu(\theta)] + \beta\mu(\theta)[y - b + c] + (R(g) - g)K}{[A(g)]^2\{[(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}} \end{aligned}$$

Therefore, the equilibrium effect of growth on the reservation ability is given by

$$(A3.18) \quad \frac{d\hat{\alpha}}{dg} = \underbrace{[\det(B)]^{-1}}_{(-)} \left[\underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} \underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta}}_{(-)} - \underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} \underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta}}_{(-)} \right]$$

where a minus sign below a term implies it is negative and a question mark implies the term cannot be signed unless further assumptions are imposed.

Similarly, the equilibrium effect of growth on labor market tightness is given by

$$(A3.19) \quad \frac{d\theta}{dg} = \underbrace{[\det(B)]^{-1}}_{(-)} \left[- \underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}}}_{(-)} \underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} - \underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}}}_{(-)} \underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} \right]$$

When $A'(g) = 0$, equation (A3.16) implies that $\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} = 0$ and equation (A3.17) implies that $\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} < 0$ if and only if $R'(g) > 1$. Given these findings, equations

(A3.18) and (a3.19) imply that when $A'(g) = 0$, $\frac{d\hat{\alpha}}{dg} > 0$ and $\frac{d\theta}{dg} < 0$ if and only if $R'(g) > 1$.

To calculate the equilibrium effect of technological growth on unemployment, differentiate equation (3.42) to get

$$(A3.20) \quad \frac{du}{dg} = \underbrace{\frac{\delta f(\hat{\alpha})}{\delta + \mu(\theta)}}_{(+)} \underbrace{\frac{d\hat{\alpha}}{dg}}_{(?)} - \underbrace{\frac{\delta F(\hat{\alpha})\mu'(\theta)}{[\delta + \mu(\theta)]^2}}_{(+)} \underbrace{\frac{d\theta}{dg}}_{(?)}$$

Therefore, we find that $\frac{d\hat{u}}{dg} > 0$ if and only if $R'(g) > 1$.

III.ii. Proof of Proposition 4

Using the Cramer's rule, we showed in equation (A3.15) that, regardless of the assumptions made on $A'(g)$ and $R'(g)$, the effect of technological growth on entrepreneurship is:

$$\frac{d\hat{\alpha}}{dg} = [\det(B)]^{-1} \begin{vmatrix} -\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ -\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{vmatrix}$$

with $\det(B) < 0$. To be able to evaluate this effect when $R'(g)=1$, we need to evaluate the determinant on the right-hand-side of equation (A3.15). To do that, note that

$$(A3.21) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} = \hat{\alpha} \frac{A'(g)}{A(g)}$$

$$(A3.22) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} = \frac{A'(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$$

To obtain $\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta}$ and $\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta}$ first re-express equations (A3.12) through (A3.13) as

$$(A3.23) \quad \left. \frac{d\hat{\alpha}}{d\theta} \right|_E = - \left(\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} / \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \right) > 0$$

$$(A3.24) \quad \left. \frac{d\hat{\alpha}}{d\theta} \right|_{JC} = - \left(\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} / \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \right) < 0$$

Using equations (A3.8) and (A3.10) we get

$$(A3.25) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} = - \left. \frac{d\hat{\alpha}}{d\theta} \right|_E$$

$$(A3.26) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} = \frac{-\hat{\alpha}A(g)F(\hat{\alpha}) - A'(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} \left. \frac{d\hat{\alpha}}{d\theta} \right|_{JC}$$

According to equation (A3.15), $\frac{d\hat{\alpha}}{dg} > 0$ if and only if the determinant in equation (A3.15) is negative. When $R'(g) = 1$, this condition is satisfied if and only if

$$A'(g) \left. \frac{\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})} \frac{d\hat{\alpha}}{d\theta} \right|_E > \hat{\alpha} \frac{A'(g) - \hat{\alpha}A(g)F(\hat{\alpha}) - A'(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} \left. \frac{d\hat{\alpha}}{d\theta} \right|_{JC}$$

which can be simplified to

$$\frac{\frac{d\hat{\alpha}}{d\theta}|_E}{\frac{d\hat{\alpha}}{d\theta}|_{JC}} > -\frac{\hat{\alpha}^2}{\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)} - \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})}$$

if $A'(g) > 0$. Otherwise the inequality sign would be reversed.

Re-expressing in terms of elasticities, we obtain the following expression:

$$\frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} > \frac{\hat{\alpha}^2}{\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})}$$

which is the condition used in the lemma. Note that a reversed inequality signed is obtained if $A'(g) < 0$.

III.iii. Proof of Proposition 5

In equation (A3.20) we showed that equilibrium effect of technological growth on unemployment is given by

$$\frac{du}{dg} = \underbrace{\frac{\delta f(\hat{\alpha})}{\delta + \mu(\theta)}}_{(+)} \underbrace{\frac{d\hat{\alpha}}{dg}}_{(?)} - \underbrace{\frac{\delta F(\hat{\alpha})\mu'(\theta)}{[\delta + \mu(\theta)]^2}}_{(+)} \underbrace{\frac{d\theta}{dg}}_{(?)}$$

To be able to evaluate this effect when $R'(g)=1$, we need to evaluate $\frac{d\theta}{dg}$ when $R'(g)=1$.

To do that, remember that $\frac{d\theta}{dg}$ is given by

$$\frac{d\theta}{dg} = \underbrace{[\det(B)]^{-1}}_{(-)} \left[\underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}}}_{(-)} \underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} - \underbrace{\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}}}_{(-)} \underbrace{\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}}_{(?)} \right]$$

According to equation (A3.21) and (A3.22) $sign\left(\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}\right) = sign\left(\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}\right) > 0 (< 0)$

iff $A'(g) > 0$ ($A'(g) < 0$). Therefore $\frac{d\theta}{dg} > 0$ (< 0) iff $A'(g) > 0$ ($A'(g) < 0$).

Now, according to equation (A3.20) for $\frac{du}{dg}$ to be positive

$$(A3.27) \quad \frac{d\hat{\alpha}}{dg} > \frac{\delta F(\hat{\alpha})\mu'(\theta)}{f(\hat{\alpha})[\delta + \mu(\theta)]} \frac{d\theta}{dg}$$

Replacing the expressions for $\frac{d\hat{\alpha}}{dg}$ and $\frac{d\theta}{dg}$ into equation (A3.27) and multiplying both sides by $A(g)/A'(g)$ and re-arranging we get

$$(A3.28) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \frac{d\hat{\alpha}}{d\theta} \Big|_{JC} - A'(g) \frac{\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})} \frac{d\hat{\alpha}}{d\theta} \Big|_E \\ > - \frac{A(g)\mu'(\theta)}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha}^2 + \left(1 + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) \right]$$

The inequality sign should be reversed if $A'(g) < 0$.

Defining $\bar{\alpha} \equiv (1 - F(\hat{\alpha})) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$, the expression above can be repressed as

$$(A3.29) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \frac{d\hat{\alpha}}{d\theta} \Big|_{JC} - A'(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))F(\hat{\alpha})} \frac{d\hat{\alpha}}{d\theta} \Big|_E \\ > - \frac{A(g)\mu'(\theta)}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha}^2 + \left(1 + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

or in terms of elasticities

$$(A3.30) \quad \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \varepsilon(\hat{\alpha}, \theta) \Big|_{JC} - A'(g) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))F(\hat{\alpha})} \varepsilon(\hat{\alpha}, \theta) \Big|_E \\ > - \frac{A(g)\mu'(\theta)\theta}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha} + \left(1 + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right) \frac{\bar{\alpha}}{(1 - F(\hat{\alpha}))\hat{\alpha}} \right]$$

Remember that the inequality sign should be reversed if $A'(g) < 0$.

III.iv. Proof of Proposition 6

Using the Cramer's rule, we showed in equation (A3.15) that, regardless of the assumptions made on $A'(g)$ and $R'(g)$, the effect of technological growth on entrepreneurship is:

$$\frac{d\hat{\alpha}}{dg} = [\det(B)]^{-1} \begin{vmatrix} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{vmatrix}$$

with $\det(B) < 0$. To be able to evaluate this effect we need to evaluate the determinant on the right-hand-side of equation (A3.15). To do that, note that

(A3.31)

$$\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} = \hat{\alpha} \frac{A'(g)}{A(g)} \left(1 - \frac{(R'(g) - 1)[\hat{\alpha}A(g)c + b + K]}{\hat{\alpha}A(g)\{[(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}} \right)$$

and

$$(A3.32) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} = \frac{A'(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{\max}} \alpha dF(\alpha)$$

Using equations (A3.8) and (A3.10) we get

$$(A3.33) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} = - \left. \frac{d\hat{\alpha}}{d\theta} \right|_E$$

$$(A3.34) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} = \frac{-\hat{\alpha}A(g)F(\hat{\alpha}) - A'(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{\max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} \left. \frac{d\hat{\alpha}}{d\theta} \right|_{JC}$$

According to equation (A3.15), $\frac{d\hat{\alpha}}{dg} > 0$ if and only if the determinant in equation (A3.15)

is negative. This condition is satisfied if and only if

$$A'(g) \left. \frac{\int_{\hat{\alpha}}^{\alpha^{\max}} \alpha dF(\alpha)}{F(\hat{\alpha})} \frac{d\hat{\alpha}}{d\theta} \right|_E > \hat{\alpha} \frac{A'(g)}{A(g)} \left(1 - \frac{(R'(g) - 1)[\hat{\alpha}A(g)c + b + K]}{\hat{\alpha}A(g)\{[(1 - \beta)q(\theta)[y - b] - [R(g) - g + \delta + \beta\mu(\theta)]c\}} \right) * \\ * \left(\frac{-\hat{\alpha}A(g)F(\hat{\alpha}) - A'(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{\max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} \left. \frac{d\hat{\alpha}}{d\theta} \right|_{JC} \right)$$

which can be simplified to

$$\left. \frac{d\hat{\alpha}}{d\theta} \right|_E > -\Psi \left(\frac{\hat{\alpha}^2}{(1 - F(\hat{\alpha}))\bar{\alpha}} + \frac{\hat{\alpha}f(\hat{\alpha})}{F(\hat{\alpha})} \right)$$

where $\Psi \equiv 1 - \frac{(R'(g)-1)[\hat{\alpha}A(g)c+b+K]}{\hat{\alpha}(A'(g)[(1-\beta)q(\theta)[y-b]-[R(g)-g+\delta+\beta\mu(\theta)]c)}$ and $\bar{\alpha} \equiv \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$. A reversed inequality signed is obtained if $A'(g) < 0$.

In terms of elasticities, the expression above becomes

$$\frac{\varepsilon(\hat{\alpha}, \theta)|_E}{\varepsilon(\hat{\alpha}, \theta)|_{JC}} > \Psi \left(\frac{\hat{\alpha}^2}{(1-F(\hat{\alpha}))\bar{\alpha}} + \frac{\alpha f(\hat{\alpha})}{F(\hat{\alpha})} \right)$$

Note that a reversed inequality signed is obtained if $A'(g) < 0$.

To look at how unemployment reacts, remember that in equation (A3.20) we showed that equilibrium effect of technological growth on unemployment is given by

$$\frac{du}{dg} = \frac{\delta f(\hat{\alpha})}{\delta + \mu(\theta)} \underbrace{\frac{d\hat{\alpha}}{dg}}_{(?)}} - \frac{\delta F(\hat{\alpha})\mu'(\theta)}{[\delta + \mu(\theta)]^2} \underbrace{\frac{d\theta}{dg}}_{(?)}}$$

Therefore, for $\frac{du}{dg}$ to be positive $\frac{d\hat{\alpha}}{dg} > \frac{\delta F(\hat{\alpha})\mu'(\theta)}{f(\hat{\alpha})[\delta + \mu(\theta)]} \frac{d\theta}{dg}$. Substituting $\frac{d\hat{\alpha}}{dg}$ and $\frac{d\theta}{dg}$ in and

multiplying both sides by $A(g)/A'(g)$

$$\begin{aligned} & \frac{\hat{\alpha}}{\delta + \mu(\theta)} \left(\delta + \mu'(\theta) + \frac{\mu'(\theta)[\delta\theta + \mu(\theta)]}{\delta + \mu(\theta)} \right) \frac{d\hat{\alpha}}{d\theta} \Big|_{JC} - A'(g) \frac{\int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})} \frac{d\hat{\alpha}}{d\theta} \Big|_E \\ & > - \frac{A(g)\mu'(\theta)}{f(\hat{\alpha})[\delta + \mu(\theta)]} \left[\hat{\alpha}^2 \Psi + \left(1 + \hat{\alpha} \Psi \frac{f(\hat{\alpha})}{F(\hat{\alpha})} \right) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) \right] \end{aligned}$$

Defining $\bar{\alpha} \equiv (1 - F(\hat{\alpha})) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)$ and re-expressing in terms of elasticities, one gets the expression in the proposition. Remember that the inequality sign should be reversed if $A'(g) < 0$

III.v. Proof of Proposition 7

Define Eq (A3.73) and (A3.74) implicitly as

$$(3.35) \quad H(\hat{\alpha}, \theta, g) = \hat{\alpha} - \frac{U(\theta, g) + K}{A(g)V(\theta, g)}$$

$$(3.36) \quad G(\hat{\alpha}, \theta, g) = \frac{A(g)}{F(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha) - \frac{\delta\theta + T(\theta, g)\mu(\theta)}{\delta + T(\theta, g)\mu(\theta)}$$

where $T(\theta, g) \equiv [1 - \exp(-\delta T^*)]$. Note that $T_g(\theta, g) \equiv \frac{\partial T(\theta, g)}{\partial g} > 0$ and $T_\theta(\theta, g) \equiv$

$$\frac{\partial T(\theta, g)}{\partial \theta} > 0.$$

Comparative statics can be performed by total differentiation of equation (3.35) and (3.36) with respect to g

$$(A3.37) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial g} + \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \frac{\partial \theta}{\partial g} + \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g}$$

$$(A3.38) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial g} + \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \frac{\partial \theta}{\partial g} + \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g}$$

In matrix notation, we have

$$(A3.39) \quad \begin{bmatrix} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{\alpha}}{\partial g} \\ \frac{\partial \theta}{\partial g} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} \\ -\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} \end{bmatrix}$$

Define $B \equiv \begin{bmatrix} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{bmatrix}$. The elements of B are the following:

$$(A3.40) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} = 1$$

$$(A3.41) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} = -\frac{U'(\theta)}{A(g)V(\theta)} + \frac{V'(\theta)[U(\theta) + (R(g) - g)K]}{[V(\theta)]^2} < 0$$

$$(A3.42) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} = \frac{-\hat{\alpha}A(g)F(\hat{\alpha}) - A(g)f(\hat{\alpha}) \int_{\hat{\alpha}}^{\alpha^{max}} \alpha dF(\alpha)}{F(\hat{\alpha})^2} < 0$$

$$(A3.43) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} = -\frac{\delta\theta + T(\theta, g)\mu'(\theta) + T(\theta, g)\mu(\theta)}{\delta + T(\theta, g)\mu(\theta)}$$

$$-\frac{[\delta\theta + T(\theta, g)\mu(\theta)][T_\theta(\theta, g)\mu(\theta) + T(\theta, g)\mu'(\theta)]}{[\delta + T(\theta, g)\mu(\theta)]^2} < 0$$

Equations (A3.40) through (A3.43) can be used to sign the slopes of the entrepreneurship and job creation curves:

$$(A3.44) \quad \left. \frac{d\theta}{d\hat{\alpha}} \right|_E = -\left(\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} / \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \right) > 0$$

$$(A3.45) \quad \left. \frac{d\theta}{d\hat{\alpha}} \right|_{JCE} = -\left(\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} / \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \right) < 0$$

Equations (A3.40) through (A3.43) also allow one to sign the determinant of B:

$$(A3.46) \quad \det(B) = \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} - \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \hat{\alpha}} \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} < 0$$

We can now use Cramer's rule to calculate the effect of growth on entrepreneurship:

$$(A3.47) \quad \frac{d\hat{\alpha}}{dg} = [\det(B)]^{-1} \begin{vmatrix} -\frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial \theta} \\ -\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} & \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial \theta} \end{vmatrix}$$

where

$$(A3.48) \quad \frac{\partial H(\hat{\alpha}, \theta, g)}{\partial g} = -\frac{\partial U(\theta, g)/\partial g}{AV(\theta, g)} + \frac{U(\theta, g) + K}{[A(g)V(\theta, g)]^2} < 0$$

and

$$(A3.49) \quad \frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} = -\frac{T_g(\theta, g)\mu(\theta)}{\delta + T(\theta, g)\mu(\theta)} + \frac{[\delta\theta + T(\theta, g)\mu(\theta)]T_g(\theta, g)\mu(\theta)}{[\delta + T(\theta, g)\mu(\theta)]^2}$$

$$= -\frac{(1 - \theta)\delta T_g(\theta, g)\mu(\theta)}{[\delta + T(\theta, g)\mu(\theta)]^2}$$

Note that $\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} > 0$ if $\theta < 1$ and $\frac{\partial G(\hat{\alpha}, \theta, g)}{\partial g} < 0$ if $\theta > 1$. Therefore, $\frac{d\hat{\alpha}}{dg}$ is unambiguously

negative if $\theta < 1$. Using equation (3.72) one can use the results obtained so far to show

that unemployment increases when $\theta < 1$.

Bibliography

- Acemoglu, Daron, and Robert Shimer. 1999. "Efficient Unemployment Insurance." *Journal of Political Economy*, Vol. 107 (5), pp. 893-928.
- Aghion, Philippe and Peter Howitt (1992). "A Model of Growth through Creative Destruction," *Econometrica*, Vol. 60, No. 2, pp. 323-51.
- Aghion, Philippe and Peter Howitt (1994). "Growth and Unemployment," *The Review of Economic Studies*, Vol. 61, No 3, pp. 477-494.
- Attanasio, Orazio P, James Banks, Costas Meghir and Guglielmo Weber. 1999. "Humps and Bumps in Lifetime Consumption." *Journal of Business and Economic Statistics*, Vol. 17 (1), pp. 22-35
- Blanchflower, David G., and Andrew J. Oswald. 1991. "Self-Employment and Mrs. Thatcher's Enterprise Culture." Working Paper, no. 30, Center for Economic Performance.
- Blanchflower, David G., and Andrew J. Oswald. 1998. "What Makes an Entrepreneur." *Journal of Labor Economics*, Vol. 16 (1), pp. 26-60.
- Buera, Francisco. 2006. "Persistency of Poverty, Financial Frictions, and Entrepreneurship." Working Paper, Northwestern University.
- Bureau of Economic Analysis (BEA), U.S. Department of Commerce. 2003. "Fixed Reproducible Tangible Wealth in the United States, 1925--99." Washington, DC: U.S.

Government Printing Office, September.

Cagetti, Marco and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." *Journal of Political Economy*, Vol. 115 (5), pp. 835-870.

Davis, Steven, Jason Faberman, John Haltiwanger and Ian Rucker. 2008. "Adjusted Estimates of Worker Flows and Job Openings in Jolts." NBER Working Paper No 14137.

Dawson, Christopher, Andrew Heley, and Paul Latreille. 2009. "Why Do Individuals Choose Self-Employment." Discussion Paper, no. 3974, IZA.

Evans, David and Boyan Jovanovic. 1989. "An Estimated Model of Entrepreneurial Choice under Borrowing Constraints." *Journal of Political Economy*, Vol. 97 (4), pp. 808-827.

Evans, David S., and Linda S. Leighton. 1989. "Some Empirical Aspects of Entrepreneurship." *American Economic Review*, Vol. 79 (3), pp. 519-535.

Fonseca, Raquel, Paloma Garcia-Perez and Christopher A. Pissarides (2001). "Entrepreneurship, Start-Up Costs and Employment," *European Economic Review*, Vol. 45, No 4-6, pp. 692-705.

Grossman, Gene M. and Elhanan Helpman (1991). *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.

Hamilton, Barton H. 2000. "Does Entrepreneurship Pay? An Empirical Analysis of the Return to Self-Employment." *Journal of Political Economy*, Vol. 108 (3), pp. 604-631.

Jones, Charles I. (1995). "R&D-Based Models of Economic Growth," *Journal of Political*

Economy, Vol. 103, No. 4, pp. 759-84.

Holtz-Eakin, Douglas, David Joulfaian, and Harvey S. Rosen. 1994. "Sticking it Out: Entrepreneurial Survival and Liquidity Constraints." *Journal of Political Economy*, Vol. 102 (1), pp. 53-75.

Hurst, Erik and Annamaria Lusardi. 2004. "Liquidity Constraints, worker wealth and Entrepreneurship." *Journal of Political Economy*, Vol. 112 (2), pp. 319-347.

ILO (1993). "Fifteenth International Conference of Labour Statisticians, Report of the Conference." ICLS/15D.6 (Rev. 1). International Labour Office, Geneva 1993.

Gollin, Douglas. 2002. "Getting Income Shares Right." *Journal of Political Economy*, Vol. 110 (2), pp. 458-474.

Mortensen, Dale T. 1970. "Job Search, the Duration of Unemployment, and the Phillips Curve." *American Economic Review*, Vol. 60 (5), pp. 847-862.

Mortensen, Dale T. and Christopher A. Pissarides (1998). "Technological Progress, Job Creation and Job Destruction," *Review of Economic Dynamics*, Vol. 1, No 4, pp. 733-753.

Pissarides, Christopher A. (1990). *Equilibrium Unemployment Theory*. Cambridge, Massachusetts. The MIT Press.

Pissarides, Christopher A. (2000). *Equilibrium Unemployment Theory*. Second Edition. Cambridge, Massachusetts. The MIT Press.

Pissarides, Christopher A. and Giovanna Vallanti (2007). "The Impact of TFP Growth on

Steady-State Unemployment," *International Economic Review*, Vol. 48, No. 2, pp. 607-640.

Quadrini, Vincenzo. 2000. "Entrepreneurship, Saving, and Social Mobility." *Review of Economic Dynamics*, Vol. 3 (1), pp. 1--40.

Rasteletti, Alejandro. 2009a. "Self-Employment, Labor Market Frictions and Credit Constraints." Working Paper, University of Maryland.

Rasteletti, Alejandro. 2009b. "Stopgap Self-Employment: Evidence from the PSID." Working Paper, University of Maryland.

Rissman, Ellen R. 2003. "Self-Employment as an Alternative to Unemployment." Working Paper no. 2003-24, Fed. Reserve Bank Chicago.

Romer, Paul M. (1990). "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, pp. 71-102.

Segerstrom, Paul S. (1991). "Innovation, Imitation, and Economic Growth," *Journal of Political Economy*, Vol. 99, No. 4, pp. 807-27.

Segerstrom, Paul S. (1998). "Endogenous Growth without Scale Effects," *American Economic Review*, Vol. 88, No. 5, pp. 1290-1310.

U.S. Census Bureau, "Statistical Abstract of the United States: 2009." 128th Edition. Washington, DC, 2008; <http://www.census.gov/statab/www/>.