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ABSTRACT

Efficient lumber utilization at the saw has become a key issue in the woodworking industry. This is because of shrinking supply and increasing raw material prices. In this paper, formulation of the cross-cut first method of cutting defects out of lumber, as a one-dimensional stock cutting problem is discussed. A Monte-Carlo simulation method has been used for generating boards of a given grade. This simulation greatly aids in comparing alternate solution procedures proposed in the paper. To facilitate real-time application, a fast heuristic for the digital computer is introduced. This is followed by a discussion of cutting length priority allocation. The heuristic solution is compared with the optimal solution obtained using Kolesar's knapsack algorithm.

1. INTRODUCTION

Wood has always been a very valuable material for a variety of constructional uses. It is easily worked upon with tools and machines. It has a large variety of species that provides a wide range of physical characteristics.

To some extent, wood has been replaced by many newer materials. But, at the same time, other new wood products have been added to the forest industry as a result of scientific research. These include particle board, hardboard and nonwoven wood cellulose fiber-based disposable products. The increasing demand for wood products will intensify the need for better utilization of what was formerly considered as unavoidable waste.

The annual consumption of wood in the United States is about 398 million cubic meters [1]. Two-thirds of this consumption is in the housing and nonresidential construction industries. Another third goes into manufacture of paper and related products.

Wood is one of the least recycled of our resources. Only proper planning and forest conservation measures can ensure a never ending supply of wood. It is a natural resource and as such it will tend to become precious with increasing demand. Hence, methods to improve utilization of this scarce resource play an invaluable role in the survival and flourishing of the woodworking industry. One of the main areas open for improvement in utilization is the cutting process used for removing defects from boards. With increasing usage of automated cutting saws, there is a pressing need for procedures that can help in fast and efficient decision making at the saw. The evaluation of alternate solution strategies involves considerable expense of material and time. Hence computer simulation provides an efficient method of evaluating such alternatives.

The remainder of this paper is organized as follows. First, a brief note on the basics of lumber processing is presented. This is followed by a survey of previous work on yield optimization found in the literature. In the following section, we formulate the one-dimensional stock cutting problem, followed by the solution procedure using Kolesar's knapsack algorithm. A time-efficient heuristic is next introduced for the solution of this one-dimensional problem. The heuristic is further modified to include cutting priorities in the solution procedure. The algorithmic and heuristic solutions are compared in terms of yield and other factors, followed by conclusion and recommendations for further work.

2. A BRIEF NOTE ON THE WOODWORKING INDUSTRY

2.1 Drying of Lumber

When first sawn from the log, green lumber contains a large proportion of water. In case of hardwoods, this makes the green lumber unfit for immediate commercial use. The process of removing moisture from wood is termed as drying. The two common methods of drying lumber are air drying and kiln drying, the latter being faster. The desirable moisture content of lumber for furniture is approximately 6 percent.

2.2 Grading of Lumber

Hardwoods are graded visually - each individual piece is inspected. Except in the case of selects, grading is based on the poorer side of the inspected piece. The grader visualizes on the surface of the board a number of defect-free rectangles known as cuttings. The number of cutting units in each rectangle is computed by multiplying the length of the rectangle in feet by its width in inches. The grader also computes the number of cutting units in the board if it were defect free. The grade allocation is then made based on grading specifications. In the USA, for example, these include the percentage area of the board that must be in clear cuttings of specified minimum dimensions. As a rule of thumb, the better the grade, the fewer the number of defects and the longer the clear sections available on a board. The following is a general guide on various hardwood grades [2]:

Hardwood grades:

First - is 91% clear.

Second - is 83-1/3% clear.

Select - can be cut into two-feet lengths that are 91% clear.

Number 1 Common - is 66-2/3% clear.

Number 2 Common - is 50% clear.

It may be noted that first and second are usually combined as one grade FAS. The percentage of first required in the combined grade varies from 20 to 40 percent. Also, the percentages mentioned above refer to the percentage clear area of the board that must be in clear cuttings of specified minimum dimensions. A more detailed discussion of lumber grades can also be found in [3].

2.3 Wood Defects

These are imperfections that reduce the strength in case of softwoods or affect the appearance in case of hardwoods. Some of the common defects include knots, checks, warp, wane, etc.

2.4. Units of Measure

Boards and planks are sold by the board-foot. A board-foot is equivalent to the volume of a board one inch nominal thickness, one foot long and one foot nominal width (actual may vary).

3. PREVIOUS WORK IN YIELD OPTIMIZATION

The one-dimensional problem of cutting a length of material into smaller lengths, so as to maximize utilization of material, gives rise to the well-known knapsack problem. An extension of the problem along the width of the material results in the two-dimensional stock cutting problem. Both of these problems have been solved using linear programming [4], dynamic programming [5,6] or the tree search method [6,7,8]. In this paper, we will consider the knapsack formulation as applied to the one-dimensional stock cutting problem in the furniture industry. This problem arises in the removal of defects from lengths of lumber to produce defect-free parts of specified length and width. The objective here is to maximize the yield or the value of the pieces cut and to develop a practical, time-efficient solution procedure that can be used to solve this problem on a real-time basis.

Several research-oriented techniques for this problem have been reported in the literature using heuristics [9,10,11,12,13,14]. Some of these methods have been based on an exhaustive search of feasible solutions [10,11,13,14]. In other cases, weighting factors are used to confine the search to a small subset of feasible solutions [9,12]. The major drawback of these methods has not been in the quality of the solutions produced, but in the practical application of the method. For example, the CROMAX program [11] requires 5 minutes or more of solution time to process an eight feet board (on UNIVAC 1100/80). The actual processing time, however, for a similar size board on an automated cutting saw is desired to be in the order of a few seconds. This difference in the computational versus the physical processing times is a serious disadvantage in a real-time application of the method, as it imposes considerable machine idle time.

Another drawback of these methods, as in the case of the yield program [9], is in the nature of the solution obtained. The program specifies the size and location of rectangular clear areas on the board with respect to the lower left corner of the board, in a way that maximizes yield. In order to cut these clear areas from the board, the cut-off or the rip-first method (explained in section 4.1) may have to be used, depending on the location of the clear areas. The program, however, does not consider the cutting of tickets (see Section. 9) on these clear areas. Also, the ticket dimensions are not considered in determining the clear area sizes and locations. The yield in [9] is computed as the ratio of the total clear area to the total area of the board. Hence such a program can only be used to determine the maximum cutting yield for a particular grade. The yield after the tickets are cut on these clear areas will be lower than the maximum yield so determined and will depend on the tickets to be cut and the cutting strategy.

4. METHODOLOGY

In this section, the cut-off-first method of processing, which is a one-dimensional stock cutting problem, is formulated as a knapsack problem. The application of the Monte-Carlo technique to simulate a board of a given grade is discussed. A fast heuristic to obtain a near optimal solution to the problem, on a real-time basis, is introduced. The algorithmic and heuristic solutions are compared in terms of yield and volume of wood cut for various ticket lengths.

4.1 Problem Background and Statement

The wood that comes to the shop floor is in the form of rectangular stock of uniform thickness. The length and width, however, vary randomly from one board to the other. Boards, depending on grade, have a number of defects of various shapes and sizes. Defects have random locations on the boards. The ends of a board are trimmed to remove checks and the width is ripped to remove rough edges and achieve parallelism. The defects, for reasons of strength and appearance, should be removed from the board by a cutting process. The remaining clear wood (later termed as "clear section") is further subjected to a cutting process to produce parts of specified dimensions and quantities as called for by the tickets. The cutting process, together with the grade of wood used, determines the yield of the process. Yield here is taken as the ratio of the volume of parts cut to the volume of raw wood processed.

Two methods are mainly used for cutting defects out of boards:

(i) cut-off-first

(ii) rip-first

In the first method, the defects are cut out by saw cuts that are perpendicular to the length of the board. The portion of wood between two clear sections constitutes waste. These clear sections may either be ripped to get parts of smaller width, or they may be glued-up to make parts of larger width.

In the second method, defects are ripped out by saw cuts parallel to the length of the board. The strips so produced are subjected to a cut-off operation as in (i) to obtain parts of required length. Further ripping or gluing may be necessary to get the finished dimensions of a part.

In either case, the basic objective is to produce parts of specified length, width and thickness. At this stage of operation the parts to be produced are always rectangular in shape and have uniform thickness. The two alternate methods of removing defects from boards are illustrated in Figures 1(a) and 1(b).

In this paper only the formulation and solution of the first method, namely the cut-off-first, is considered. Here, saw cuts are assumed to be perpendicular to the board length, regardless of the shape of the defects. The problem then is to determine the positions of the saw cuts along the length of the board, taking into account the ticket lengths, so that yield is maximized.

4.2 Simulation of Boards

In order to evaluate different solution strategies in an inexpensive and efficient manner, a simulation approach for generating boards of a given grade had to be adapted. This was done using the well-known Monte-Carlo method. The simulation of boards has certain distinct advantages. It can be used to evaluate the effect of different variables such as, ticket length, lumber grade and solution strategy on yield. It can also be used to generate the same set of boards repeatedly, hence comparisons of different solution strategies can be made quantitatively. The effect of mixing two or more grades in pre-specified proportions can also be evaluated. The savings in time and material required for such evaluations are considerable.

The simulation of boards, of a specified grade, was carried out using seven variables (Fig.2):

1. Board length (L)
2. Board width (W)
3. Number of defects (N)
4. Relative position of the center of the defect along the board length ($X' = x/L$)
5. Relative position of the center of the defect along the board width ($Y' = y/L$)
6. Defect length (a)
7. Defect width (b)

The left lower corner of the board was chosen as the origin with the +x axis along the board length and the +y axis along the board width. The defects, regardless of their shape, were treated as rectangular. This was done by enclosing the contour of the defects in a rectangle with edges parallel to the edges of the board. This does not affect the yield of a board, since the saw cuts are always parallel to the edges of the board. Also, since boards have defects on both sides, the defect coordinates were measured with respect to one corner of the board. This, in effect, tranfers all the defects to one face of the board. As defined above, the relative position of a defect (X',Y') is defined as the ratio of the actual coordinate (x or y) to the board length or width, respectively. The choice of the relative position as a variable ensures that a simulated defect stays inside the boundary of the simulated board. Actual data collected for 160 boards of No.1 common and 200 boards of No.2 common grades for the seven variables were used to generate simulated boards. This simulation procedure can be used to generate No.1 and No.2 common grades given the actual frequency distributions for the variables. A typical graph showing the comparison of frequency distributions for the actual and simulated values for one of the variables is shown in Fig. 3. The simulation model was validated using the chi-squared test [15]. The test was used to check if the distribution of the seven variables for the simulated boards agreed with the actual distributions for these same variables. The chi-square value (X^2) for a given distribution is given by,

$$X^2 = \sum_{j=1}^{J} (F_{aj} - F_{sj})^2 / F_{sj} \tag{1}$$

Here, F_{aj} is the actual frequency and F_{sj} is the simulated frequency in a given interval j. The summation is over the total number of intervals J into which the variable range has been subdivided. The chi-squared values for the actual and simulated distributions agreed well for a 5% significance level [table 1]. The first X^2 column of the table lists the chi-squared values for the simulated distributions. The second column lists the maximum chi-squared values taken from a standard table [15] for a 95% probability that the two distributions agree. The chi-squared values for the simulated distributions can be seen to be substantially less than the recommended values in the second column. Hence, the "goodness of fit" of the two distributions is considered to be satisfactory.

Also, the clear section lengths on the simulated boards and their yield closely agreed with the figures for actual boards of the simulated grade.

A plot of simulated boards made using AUTOCAD [16], a computer-aided drafting package, is shown in Fig. 4. As mentioned before, the simulated boards do not distinguish between defects that lie on the top or bottom surface of the board.

4.3 Problem Formulation

The one-dimensional stock cutting problem is to determine the ticket lengths and quantities to be cut on a clear section to maximize material utilization or yield. However, practical issues, such as ticket priority or value of the pieces cut, give rise to other objectives as well. These are (a) maximization of total value of pieces cut on a clear section (b) maximization of volume of wood cut for tickets with priority.

To formulate the problem of maximization of yield, let:

SL = Length of clear section

W = Width of clear section

k = Ticket number (k = 1,...,K)

TL(k) = Ticket length for ticket k

BFC_k = Cumulative volume of wood cut for ticket k including the section under consideration

BF_k = Volume requirement of ticket k

I_k = Maximum number of integer pieces that can be cut for ticket k on the clear section of length SL, i.e., I_k = Int (SL/TL(k))

L_{ki} = Length of material (wood) required to accomodate i pieces of ticket k ($i = 1,\ldots,I_k$)

X_{ki} = Binary variable: 1 if L_{ki} is cut; 0 otherwise

The objective f is to maximize the total length cut on a clear section:

$$\text{maximize } f = \sum_{k=1}^{K} \sum_{i=1}^{I_k} L_{ki} \; X_{ki} \tag{2}$$

subject to

$$\sum_{k=1}^{K} \sum_{i=1}^{I_k} L_{ki} X_{ki} \le SL \tag{3}$$

$$BFC_k \le BF_k \qquad (k=1,\ldots,K) \tag{4}$$

$$X_{ki} = 0 \text{ or } 1, (i=1,\ldots,I_k) \quad (k=1,\ldots,K) \tag{5}$$

4.4 Algorithmic Solution

Equations (2)-(5) constitute a (0-1) integer linear programming problem that can be solved by a number of algorithms. The number of binary variables involved depends on the number of tickets and the clear section length. In general, the number of binary variables n is given by the summation of the maximum number of integer pieces that can be cut for the K tickets.

$$n = \sum_{k=1}^{K} \mathrm{Int}(SL/TL(k)) \tag{6}$$

Hence, the number of binary variables is a function of section and ticket lengths. For No. 1 common grade boards, the number of variables involved with ten tickets is usually between 20 and 40. The clear section length range is from 16" to 143". The ticket lengths are assumed to be in the range of 16" to 80". The number of variables increases rapidly with increasing number of tickets or section length. This adversely affects the solution time. However, this problem can be formulated as a knapsack problem in which the weight and value are synonymous. This fact can be taken advantage of to solve this problem with an algorithm that is faster than general integer programming algorithms. One such algorithm was proposed by Kolesar [7].

In the above formulation, an integer number of cuts of a ticket length are treated as the items of the knapsack problem. The basic assumption is that every clear cut (the clear area in between any two successive saw cuts) is used to produce only one of the k tickets. In other words, tickets having same length but different width are not cut on the same clear cut. The weight and value of the items are synonymous with the length of cut. For example, if it is possible to cut three pieces of a ticket of length 16" on a clear section, the weight and value of the three items corresponding to this ticket would be 16, 32 and 48. The main advantage of this formulation is that the number of items in the objective function and the constraints is kept to a minimum, thereby improving the solution time. In using this formulation with the simulated boards, the clear sections on a board were solved one at a time for the tickets to be cut. Whenever the section width exceeds the part width, the parts are assumed to be ripped from the clear section. In contrast, if the part width exceeds the section width, the parts are assumed to be produced by gluing-up sections of same length. The cumulative total width cut for each ticket was used to keep track of the cumulative volume of wood cut for each ticket. The constraint of ticket requirements was thus included in the solution process. Whenever a ticket requirement was satisfied, it was replaced by a new ticket. A flowchart for the simulation of the cut-off-first method using Kolesar's algorithm is shown in Figure 5.

In the following paragraphs, the steps of the algorithm as it applies to the above formulation are presented. A general discription of this algorithm, as it applies to a standard knapsack problem, may be found in [7].

The Kolesar's algorithm uses the branch-and-bound technique. The algorithm proceeds by repeatedly partitioning the class of all feasible solutions into smaller subclasses in such a way that ultimately an optimal solution is reached. This partitioning or branching is done so that each feasible solution belongs to exactly one subclass. For each subclass or node j, an upper bound B(j) to the maximum value of the objective function is computed. Based on this upper bound, further branching is carried out. Next, upper bounds are calculated for the new branches. The process is repeated until a feasible solution is obtained that gives a value to the objective function greater than the value for all previous solutions.

The following provides the definition of certain terms required for the understanding of the algorithm:

<u>Feasible Solution</u>: A collection of X_{ki} satisfying equations (3) - (5).

<u>Item</u>: Length of cut equal to an integer multiple of the ticket length. A

length of cut is considered as an item only when the length of cut is less than the clear section length.

ϕ: The symbol for null or empty set.

Included Items: Set of items explicitly included in the solutions at a node j, denoted by I(j).

Excluded Items: Set of items explicitly excluded from solutions contained in node j, denoted by E(j).

Free Items: Items that do not belong to I(j) or E(j) and have not yet been specifically assigned. This set is denoted by F(j). When this set is empty at a node, the node contains only one solution and further branching from that node is impossible.

Terminal Node: A node that has no branches emanating from it.

The items are first ordered in descending order of magnitude. The feasibility of the solution at the node is checked. If constraint (3) is violated, no further computations are made for this node. Otherwise, the upper bound at a node is calculated by relaxing the constraint (3) for the free items. The items included at this node are cut first. Then all free items in the ranked list are included one at a time until (3) is exactly satisfied or until there are no more free items. The objective function at this stage is the upper bound for this node.

The branching operation is carried out based on two decisions. The first decision is to select the node from which the branching is to be done. In the branch-and-bound method, the node with the highest bound is selected for branching. The second decision is to select the item that will be included at the first and excluded at the second of the two descendent nodes. This selection is arbitrary and the free item that is highest in order in the ranked list is selected.

The following steps outline the operations and decisions involved in the algorithm:

Stage 1:

(a) Test the nontrivial feasibility of the problem by verifying for at least one index ki ($k=1,\ldots,K$; $i=1,\ldots,I_k$)

$$L_{ki} \leq SL \tag{7}$$

If the problem is nontrivially feasible, proceed to (b). If not, stop.

(b) If $\sum_{k=1}^{K} \sum_{i=1}^{I_k} L_{ki} \leq SL$, then all the items may be cut and the problem is trivial. If not, proceed to (c).

(c) Rank the items in decreasing magnitude.

(d) For node #1 set $B(1) = \phi$, $I(1) = \phi$, $E(1) = \phi$. Proceed to Stage 2.

Stage 2:

(a) Find the terminal node with the largest value of $B(j)$. This is the node from which further branching is done.

(b) Test if at the current node j, $F(j) = \phi$. If so, an optimal solution is given by the items contained in $I(j)$. If not, select a new free item i*, highest in order from the ranked list for further branching.

Stage 3:

(a) Set $j = j+1$, $I(j) = I(j-1)$, $E(j) = E(j-1) \cup (i^*)$. This means that for even nodes, the items included in the parent node are carried over, and the item i* is added to the list of excluded items. Proceed to (c).

(b) Set $j = j+1$, $I(j) = I(j-1) \cup (i^*)$, $E(j) = E(j-1)$. This means that for odd nodes, the item i* is added to the list of included items, and the items excluded from the parent node are carried over. Proceed to (c)

(c) Test the feasibility of the solutions contained in node j by verifying if the sum of the items included exceeds the section length. If infeasible, set B(j)= -1000. Otherwise, compute the upper bound B(j) by first including the items in I(j). Then proceed in sequence to include the free items, one at a time, until the total length of the items is exactly equal to the section length or all free items are exhausted.

The summation of the item lengths is B(j). If the node is even, go to 3(b). Otherwise, go to Stage 2.

Example:

The operation of the algorithm is illustrated with a simple example. We consider a clear section of length, SL = 60", and two tickets of lengths, TL(1) = 28" and, TL(2) = 25". The maximum number of pieces that can be cut for tickets 1 and 2 is $I_k = 2$ (k = 1,2). The item lengths for ticket 1 are 28" and 56". Item lengths for ticket 2 are 25" and 50".

Stage 1:

(a) Since at least one item length is less than the section length, the problem is nontrivially feasible.

(b) Sum of the item lengths 56 + 50 + 28 + 25 > 60. Proceed to (c).

(c) The list of items ordered by magnitude is 56, 50, 28 and 25.

(d) Set B(1) = ϕ, I(1) = ϕ, E(1) = ϕ. Proceed to stage 2.

Stage 2:

(a) Since only node #1 exists, branching will be from this node.

(b) None of the four items has yet been assigned. Hence, the set of free items is F(1)= {1,2,3,4}. Select item #1 of length 56 as i* for further branching.

Stage 3:

(a) $j = j+1 = 1+1 = 2$, $I(2) = \phi$, $E(2) = \{1\}$, $F(2) = \{2,3,4\}$. Proceed to (c).

(c) Since there are no included items at node #2, the upperbound is found by relaxing the length constraint for the free items.

$B(1) = 50 + 10 = 60$.

Since j is even, proceed to 3(b).

(b) $j = j+1 = 2+1 = 3$, $I(3) = \{1\}$, $E(3) = \phi$, $F(3) = \{2,3,4\}$. Proceed to Stage 2.

The results of repeated application of the algorithm results in the solution tree of Fig. 6.

In the program runs used to evaluate the formulation, six tickets were considered with 250 boards (see Table 2). The average solution time per board was about one minute on an IBM compatible PC with an 8088-1 processor. The solution time mentioned above includes simulated board generation time.

The above formulation cannot be used on a real-time basis since the board processing time on an automated cut-off saw is of the order of a few seconds. The solution time depends on the number of nodes to be examined and increases exponentially with the number of nodes. In order to drive the saw using the results derived on a real-time basis, it would be desirable to have a solution that compares favorably with the algorithmic solution and is considerably faster. This was achieved by developing a simple heuristic, explained in the next section, to solve the one-dimensional stock cutting problem.

4.5 Heuristic Solution

The heuristic solution is derived by an exhaustive search of a small subset of the feasible solutions. The solution procedure is as follows:

Step 1: The tickets are considered one at a time, and the maximum number of pieces that can be cut on the clear section is computed for feasible tickets. A ticket is infeasible if the section length is shorter than the ticket length.

Step 2: Using the maximum number of pieces, the yield for each ticket when cut on the clear section is computed.

Step 3: The ticket with maximum yield is chosen as the solution. One piece of this ticket is cut on the clear section.

Step 4: The section length is reduced by the length of the ticket cut in step 3. If all the ticket lengths are longer than the revised section length, the solution process is complete. Otherwise, go to step 1.

The flowchart for the heuristic is shown in Fig. 7.

In the heuristic, the section length is optimized in several stages. The main advantage of this approach is that as the section length decreases at each stage, the number of feasible tickets that can be cut also decreases. This speeds up the solution process at each stage making this method very fast and efficient.

4.6 Inclusion of Ticket Priorities

The demand for clear cuts of different lengths is usually not uniform. In general, longer cuts are preferred over shorter cuts. This, to some extent, depends on the products to be fabricated. The table-top of a large conference-room table, for example, would require longer cuts than a small chair. Longer cuts are usually obtained by using better grades which are comparatively costlier.

In addition, the variation of volume of wood cut as a function of ticket length is shown in Fig. 8. The output of a simulation run using the heuristic for 1000 boards of No. 1 common grade with ticket lengths ranging from 16" to 74.5" was used to plot this figure. Short tickets can be cut on short as well as long clear sections. In contrast, long tickets cannot be cut on sections shorter than the ticket length. As a consequence, short tickets have a better chance of being cut compared to longer tickets. This appears to be the reason for the general trend of decreasing volume cut with increasing ticket length. In general, the cutting process will be biased toward shorter ticket lengths. This is true whenever the difference of length between short and long tickets is considerable as in Fig. 8.

The above discussion brings out the need for including priorities in the solution process. Priorities, to a limited extent, help in controlling the volume of wood cut for different ticket lengths. Hence, it may be desirable to obtain a certain proportion of long cuts from a lower lumber grade at the same time with shorter cuts. One way of achieving this is by using priorities. One method of allocating priorities is discussed below.

First, a scale of priorities was developed. The scale ranges from 0 to 11 in which 0 indicates normal or no priority and 11 indicates topmost priority. Priority 11 tickets are cut on the first available clear section, that is longer than the ticket length, regardless of yield. In using the priorities, the yield of individual tickets is artificially boosted by a factor F. This factor is related to the priority P by the relation,

$$F = e^{CP} \tag{8}$$

where e is the base of natural logarithms, P=0,...,10 and C is a constant (C=0.35, in this study). The boosted yield is used as the criterion to select the ticket to be cut. Thus, a ticket with a low yield may still be cut if its priority and hence the associated factor are comparatively large. This method of ticket selection results in a loss of yield. This is because, the ticket with the highest actual yield may not have the highest boosted yield. Thus the priorities are indirectly associated with increasing level of yield sacrifices. Here, we associate priority with yield sacrifice because yield is generally used as a performance measure for the rough-mill. Some other measure such as the value of pieces cut could also be used in allocating priorities. Using this framework, the general solution procedure using priorities is described below:

Step 1: The tickets are considered one at a time and the yield for each feasible ticket is calculated as before.

Step 2: The factor F for each feasible ticket is calculated using equation (8). The actual yield for each ticket is multiplied by the corresponding factor to get the boosted yield.

Step 3: The boosted yields of Step 2 are sorted in order of decreasing magnitude.

Step 4: From the above list the ticket highest in order is selected as the solution. If none of the tickets meets the above criteria, there is no feasible solution.

The flow chart of Figure 9 illustrates the above solution procedure. The results of a run made using ten tickets is shown in Figure 10. In this run, the shortest ticket 16" had a priority of 1 and the longest 80" had a priority of 10. The intermediate tickets had priorities 2 to 10 in the order of increasing ticket length. A set of 300 simulated boards was used for the two runs. The graph illustrates the significant increase in volume cut for long tickets when they are given high priority.

To implement this concept, the difference between requirement and production (i.e., demand versus supply) was used as the measure to allocate priorities. The priorities for the tickets were reallocated after solving each section. The tickets were ordered in the decreasing order of magnitude of the difference between requirement and production. Then the priorities were reallocated, and the highest priority was given to the ticket with the largest difference. This dynamic priority allocation ensures a close match between production and requirement by using the current ticket volumes cut. The output of a simulation run for 400 boards using the above system of priority allocation is shown in Figure 11. The match between production and requirement is indicated by the match between the slopes of corresponding sections of the two curves. If the simulation were continued for a very large volume of boards and tickets, the slopes of corresponding sections of the two curves would become equal. This means that all tickets will be completed at about the same time, which is desirable. However, it is important to note that the match between requirement and production is always achieved at the cost of yield.

5. COMPARISON AND DISCUSSION OF RESULTS

In the program runs used to evaluate the heuristic, the same 250 simulated boards used in the knapsack formulation were regenerated and used.

The tickets were also identical in the two cases. The yield obtained by using the heuristic compared very favorably with that of the algorithmic solution which gave a maximum yield of approximately 79% for No. 1 common grade. The difference in yield level for the two methods was about 2%. The results of a run using six tickets and 250 identical boards, for the algorithmic and heuristic methods, is presented in Table. 2 for the purpose of comparison. A drop of as much as 10% in yield may be expected if dynamic priority allocation is used in the solution process.

The average solution time for the heuristic (without priorities) was approximately 2 seconds per board, including the board generation time, against 55 seconds per board for the algorithmic solution on an IBM-AT compatible with an 8088-1 processor. The solution time would further be reduced since board generation time is eliminated during the actual processing of boards on a saw. The reduced solution time makes possible a realtime application of the method.

The simulation and formulation were also used to evaluate the sensitivity of the yield with varying ticket lengths. This variation is shown in Fig. 12. To evaluate this variation, ten runs were made with the ticket length held constant for each run. This was done by confining the ticket lengths to a narrow range. Each of the ten runs shown was made using ten tickets with ticket lengths $\pm$ 3" about the mean value. A set of 250 identical boards was used for each run. As indicated in the graph, the yield decreases with increasing ticket length. This is a natural consequence of the fact that as the ticket length increases, more clear sections are left unutilized because of the section length constraint. In fact, a good level of yield could be obtained when the tickets used covered the whole range from the shortest to the longest (16" to 80").

Fig. 12 also brings out another important fact of practical significance. The yield obtains its higest value when the shortest tickets are used. In other words, the solutions obtained using the algorithm or the heuristic are biased toward the shortest tickets. This results in more volume of wood being cut for the shortest tickets. This is a serious disadvantage since, longer cuts may be occasionally preferred over shorter cuts. This bias also has an adverse effect on value optimization, since longer cuts usually have a higher monetary value. This brings out the fact that inclusion of priority or weighting factor for the tickets is mandatory before these algorithms can be implemented.

The practical implementation of these algorithms, typically, involves executing the optimization program on the saw computer or on a PC external to the saw. The input to the program includes board dimensions and x-coordinates of the defects. To accomplish this input a 'computer vision' system or an equivalent device, though theoretically not required, is a practical necessity. The output of the program is in the form of x-coordinates where the saw cuts are to be made on the board. In the case of automated saws, this output can be used directly to actuate the saw. In the case of manually operated saws, this output can be used to display a drawing of the board with the position of saw cuts indicated. This drawing can be used by the operator to make the required cuts.

One drawback of the approach presented here is that it does not take into account some of the "hard constraints" encountered in a practical application. These include constraints that may not be violated at any cost. As applied to the lumber industry, these may include:

(a) parts of fixed width to be produced by a single rip,

(b) parts that may be required on priority to be cut at any cost of yield or value.

The approach, as presented, does not consider the rips available on a board. Strips of small width, produced during the ripping operation are salvaged by a sawing operation. In some cases, it may be possible to achieve a better yield by first ripping out the defect and then using the cut-off operation. This becomes clear when we consider the extreme case when a narrow defect has a length comparable to the board length. In case of ripfirst method, once the defect is ripped, the approach as presented can be used without any change.

6. CONCLUSION

In this paper we have discussed the formulation and solution of the cut-off first method of removing defects from lumber, as a one-dimensional stock cutting problem. Use of simulation to evaluate different solution strategies has been found to result in considerable savings in material and time. Developing an efficient heuristic for this problem is a first step toward solving the two-dimensional stock cutting problem on a real-time basis. Use of priorities provides an important means of controlling the saw output for varying requirements.

The cut-off-first method of removing defects gives a reasonably good yield in most cases. However, in cases where the defect length is very large, excessive wastages can result. Hence, by considering the rips available on a board it may be possible to get better yields. This is one of the objectives of our current work.

The implementation of the work described in this paper involves capital investment with savings in raw material and processing time. In contrast, the manual operation involves wastage in material and increased processing time. The choice of process will have to be based on an economic analysis of process alternatives. This aspect will be covered as part of our future work.

7. ACKNOWLEDGEMENTS

The work reported here was supported by the Maryland Industrial Partnerships (MIPS) program of the Engineering Research Center, and the Statton Furniture Manufacturing Company in Hagerstown, Maryland. This support is gratefully acknowledged. Our special thanks go to Mr. Dan Moore from Statton Furniture Manufacturing Company for his many helpful suggestions and comments during the course of the project.

8. REFERENCES

1. Groneman, C.H., "General Woodworking," McGraw-Hill, 1976, pp. 2-9.
2. Wolansky, W.D., "Woodworking Made Easy," Charles Scribner's Sons, New York, 1972, pp. 1-9.
3. "Wood Handbook 72," Forest Products Laboratory, U.S Department of Agriculture, Washington D.C.
4. Gilmore, P.C., and Gomory, R.E.,"A Linear Programming Approach to the Cutting Stock Problem," Opns.Res., Vol. 11, 1963, pp. 863887.
5. Hahn, S.G.,"On the Optimal Cutting of Defective Sheets," I.B.M. New York Scientific Center, Report No. 320-2916, 1967.
6. Sarin, S.C.,"Two-dimensional Stock Cutting Problems and Solution Methodologies," Trans. of ASME., Journal of Engineering for Industry., Vol. 105, No. 3, Aug. 83, pp. 155-160.
7 Kolesar, P.J., "A Branch and Bound Algorithm for the Knapsack Problem," Manage. Sci., vol. 13, 1967, pp. 723-735.
8. Christofides, N.,and Whitlock, C.,"An Algorithm for Twodimensional Cutting Problem," Opns.Res., Vol. 25, No. 1, 1977, pp. 30-44.
9. Wodzinski, C., and Hahm, E., "A Computer Program to Determine Yields of Lumber," USDA Forest Products Laboratory unnumbered Research Paper, 1966.
10. Giese, P.J., and McDonald, K.A., "OPTYLD - A Multiple Rip-First Computer Program to Maximize Cutting Yields," Forest Products Laboratory Research Paper FPL 412, 1982.
11. Giese, P.J., and Danielson, J.D.,"CROMAX - A Crosscut-First Computer Simulation Program to Determine Cutting Yield," Forest Products Laboratory General Technical Report FPL-38, 1983.

12. Hallock, H., and Giese, P., "Does Gang Ripping Hold the Potential for Higher Clear Cutting Yields," Forest Products Laboratory Research Paper FPL 369, 1980.

13. McDonald, K.A., Giease, P.J., and Woodfin, R.O., "Maximum Cutting Yields for 6/4 Ponderosa Pine Shop Lumber," Forest Products Laboratory Research Paper FPL 437, 1983.

14. Hallock, H., and Giese, P., "Cutting Yields from Standard Hardwood Lumber Grades when Gang Ripping," USDA Forest Service Research Paper FPL 370.

15. Kreyszig, E., Chapter. 18, "Advanced Engineering Mathematics," Second Edition, John Wiley and Sons, New York, 1967, pp. 817-819.

16. Berghauser, T.W., and Schlieve, P.L., "The Illustrated AutoCAD Book," Wordware Publishing, Inc., Texas, 1987.

17. "Selected ASTM Engineering Materials Standards," American Society for Testing Materials, Philadelphia, PA, 1956, pp. 341-343.

9. APPENDIX

GLOSSARY OF COMMONLY USED TERMS IN WOODWORKING [17]

Annular rings: The growth layer put on in a single growth year.

Checks: A separation along the grain, the greater part of which occurs across the rings of annular growth.

Grade: The designation of the quality of wood.

Grading rules: Quality specifications used in the classification of lumber.

Grain: The direction, size, arrangement, appearance or quality of the fibers in wood.

Green lumber: Lumber which has been sawed from timber but not dried.

Hardwoods: Generally, one of the botanical groups of trees that have broad leaves in contrast to the conifers. The term has no reference to the actual hardness of wood.

Kerf: The cut made by a saw.

Kiln drying: The process of drying lumber in a drying room or kiln; faster than air drying.

Knot: That portion of a branch which has become incorporated in the body of a tree.

Laminate: To glue wood in layers as in the making of plywood.

Lumber: Sawed wood.

Rip: To saw or split lumber with the grain.

Rough mill: The area of fabrication that involves cutting boards into rough size lengths and gluing-up if necessary. Finishing the parts to proper dimensions and contouring is done in a different area.

Softwoods: Generally, one of the botanical groups of trees that in most cases have needle or scalelike leaves; the conifers; also the wood produced by such trees. The term has no reference to the actual hardness of wood.

Sound knot: A knot which is solid across its face and which is as hard as the surrounding wood.

Shake: A separation along the grain, the greater part of which occurs between the rings of annual growth.

Ticket: A shop-floor document that specifies the rough, finished dimensions of a part and the quantity required. It is also accompanied by a set of shop orders required for the fabrication of the part at the various work centers.

Wane: Bark or lack of wood from any cause by shrinkage or swelling.

Warp: A distortion in wood caused by shrinkage or swelling.

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VARIABLE	χ^2	$\chi^2_{(MAX)}$
LENGTH	1.673	26.296
WIDTH	3.388	33.924
NUMBER OF DEFECTS	1.252	18.307
X' (x/L)	2.880	53.358
Y' (y/L)	3.225	49.758
a	2.867	42.557
b	0.976	15.507

TABLE. 1 Comparison of Chi-Square Values for the Simulation Variables.

	BOARD FEET CUT	
TICKET LENGTH	ALGORITHM	HEURISTIC
16.75	277.52	240.32
24.75	265.48	160.11
34.75	234.31	441.63
45.125	149.61	116.66
62.25	76.61	41.98
82.125	19.74	14.75
RAW VOLUME OF WOOD	1406.24	1406.24
TOTAL VOLUME CUT (BOARD FEET)	1023.29	1015.45
YIELD (%)	72.76	72.21
SOLUTION TIME (SEC/BOARD)	55	2

TABLE. 2 Comparison of Algorithmic and Heuristic Solutions.

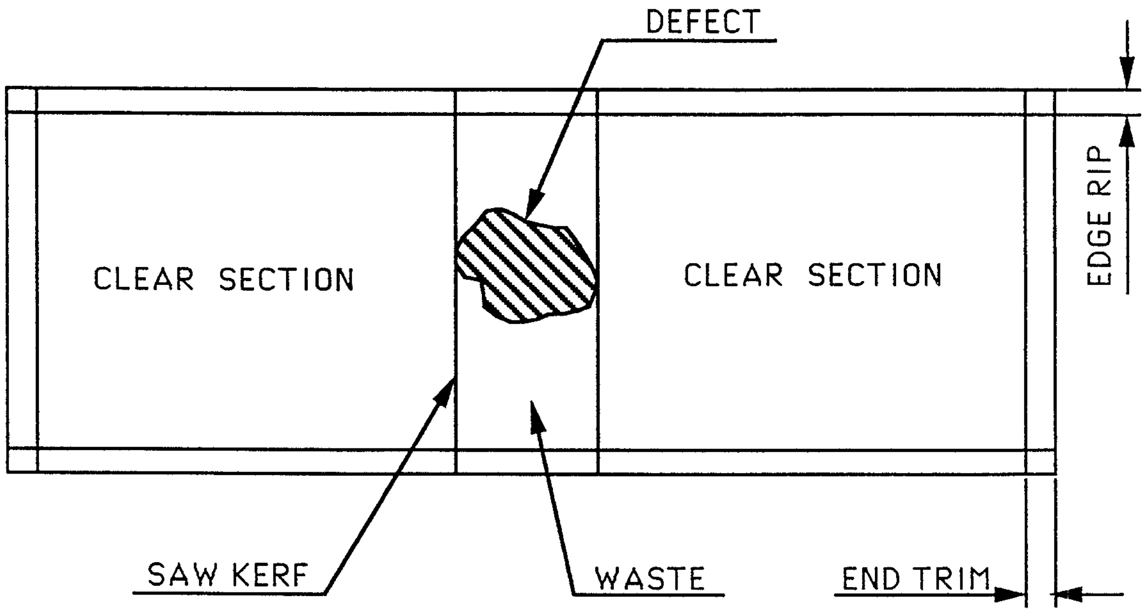

FIG. 1(A) Cut-Off-First

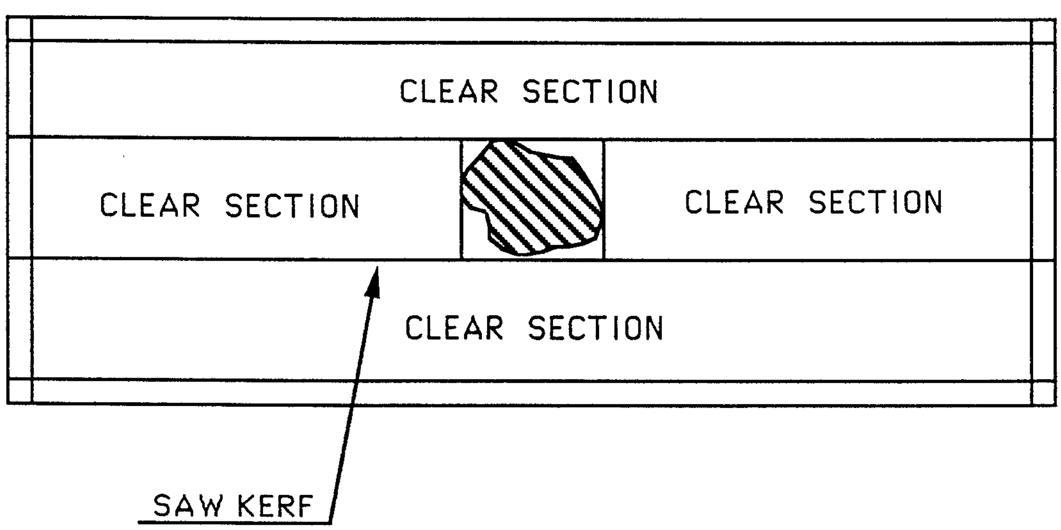

FIG. 1(B) Rip-First

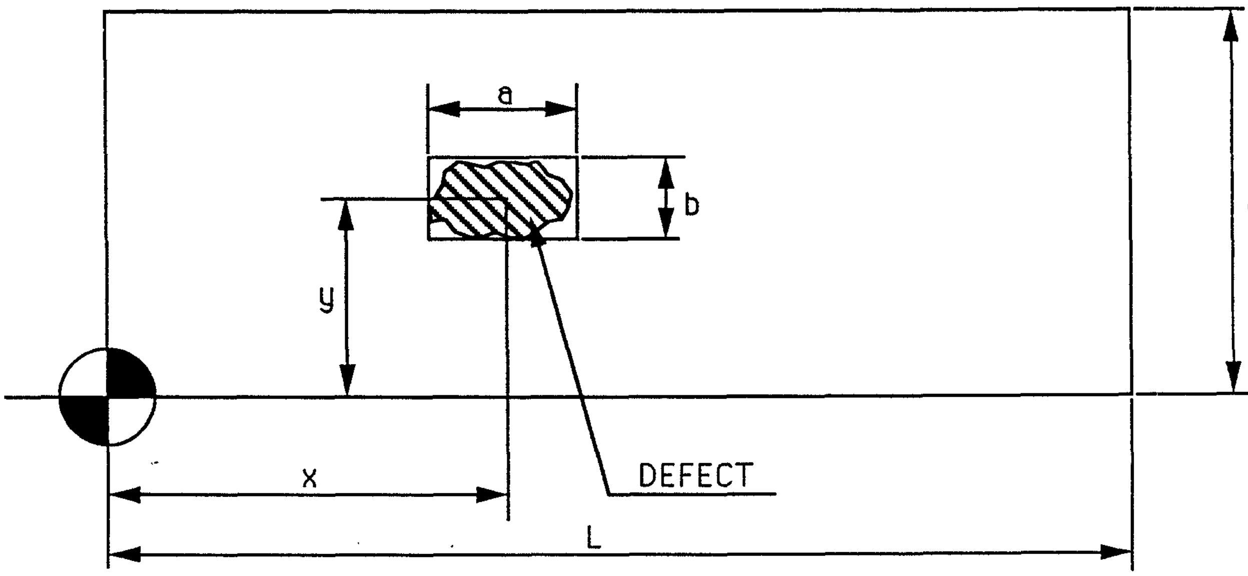

FIG. 2 Variables Used in Monte-Carlo Simulation.

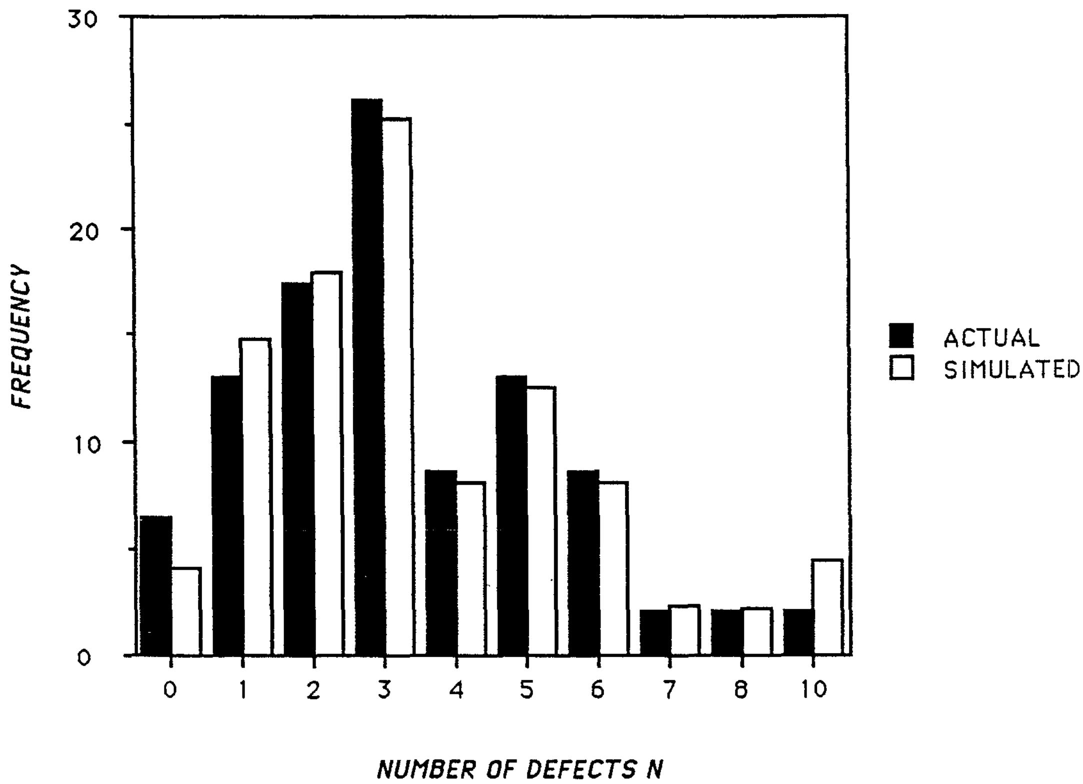

FIG. 3 Comparison of Actual and Simulated Frequency Distributions for the Variable N.

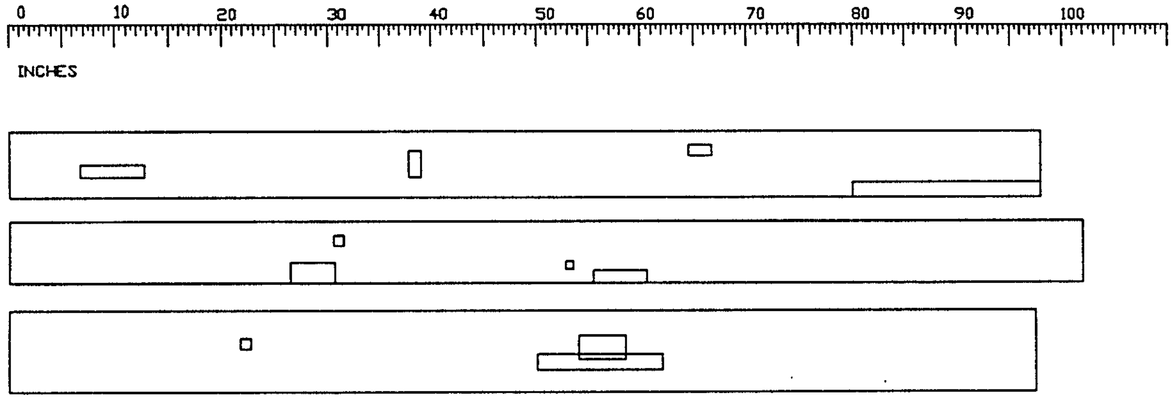

FIG. 4 Plots of Simulated Boards.

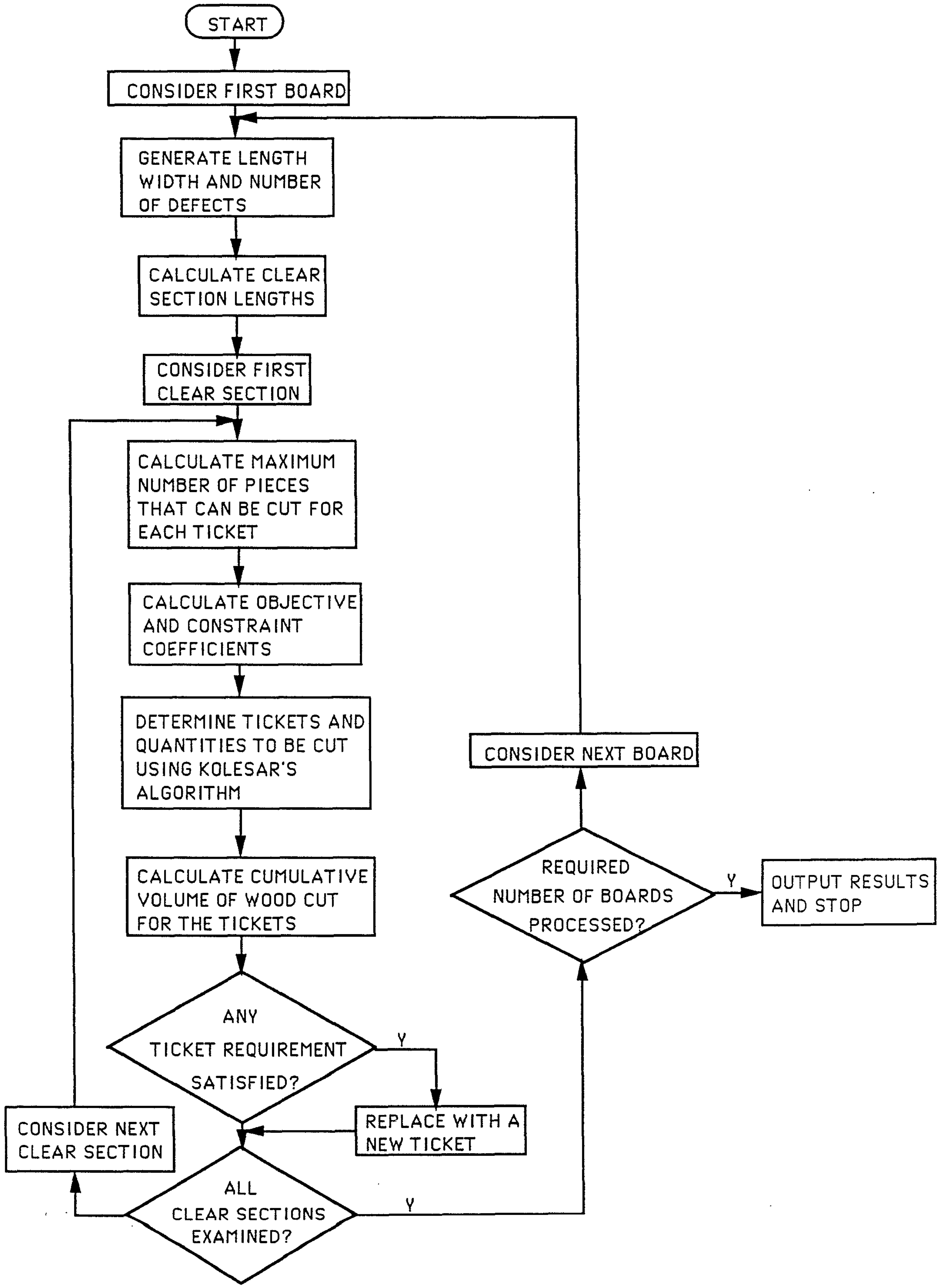

FIG. 5 Algorithmic Solution Procedure Using Kolesar's Method.

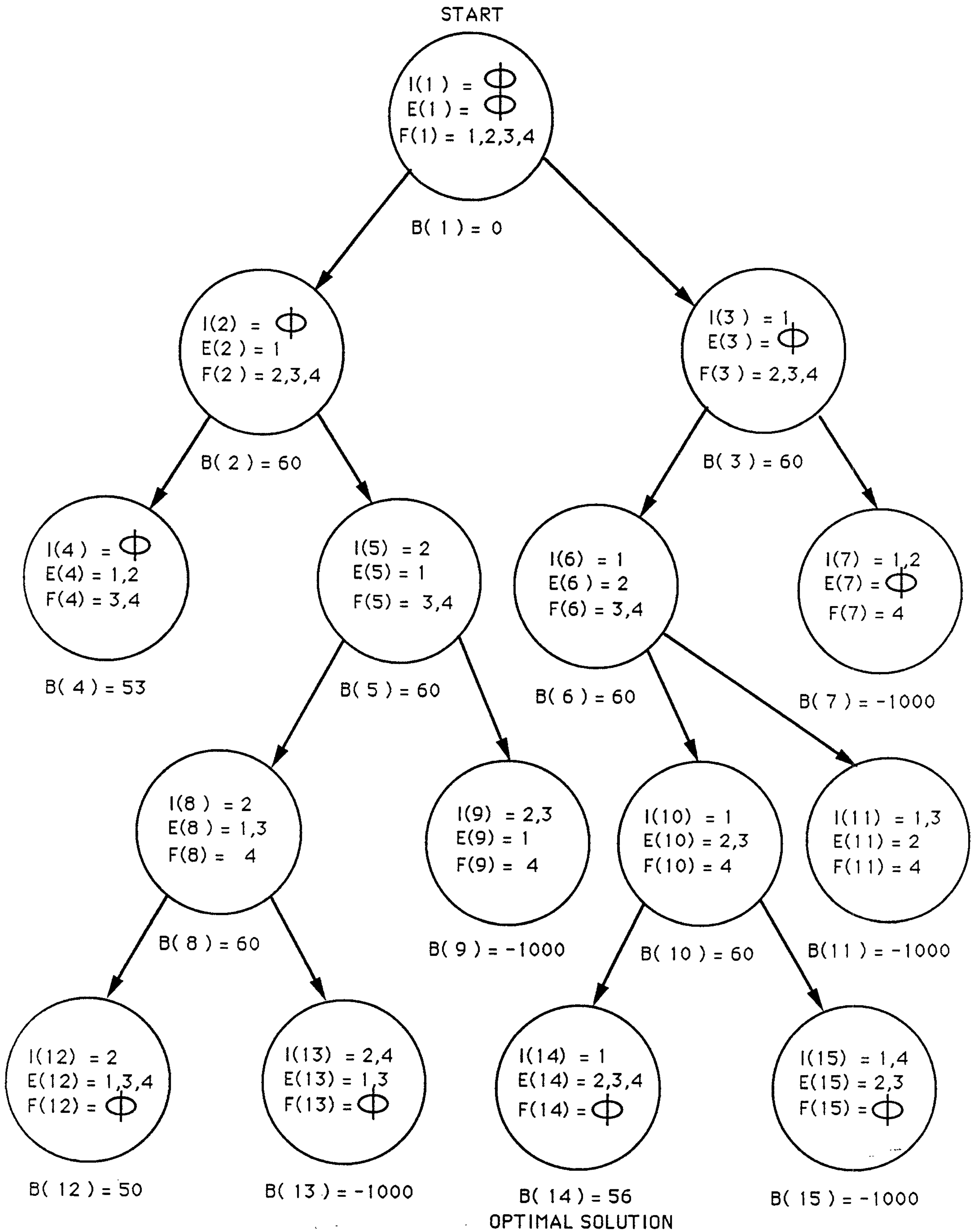

FIG. 6 Solution Tree for the Example Using Kolesar's Algorithm.

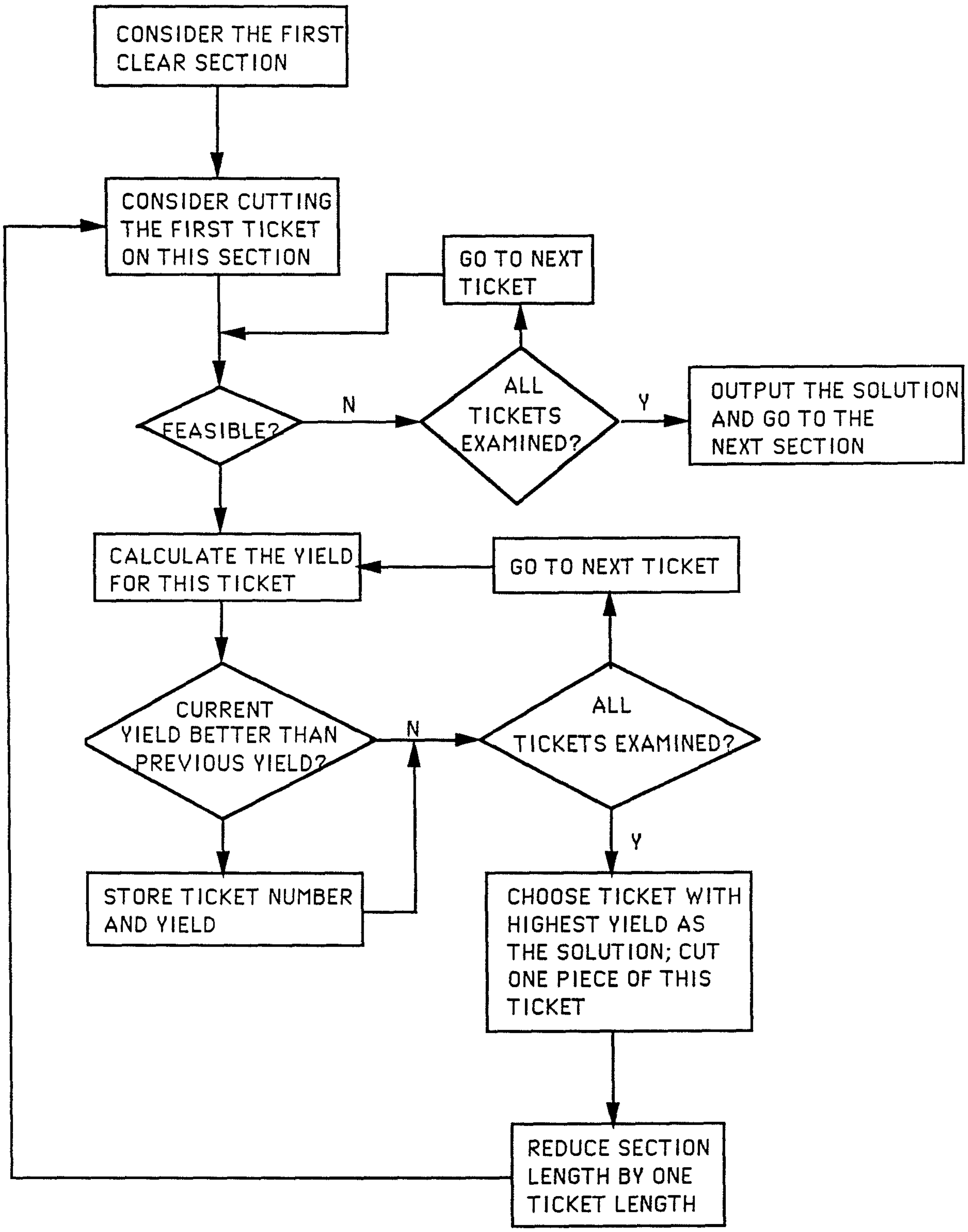

FIG. 7 Solution Procedure Using Heuristic Method.

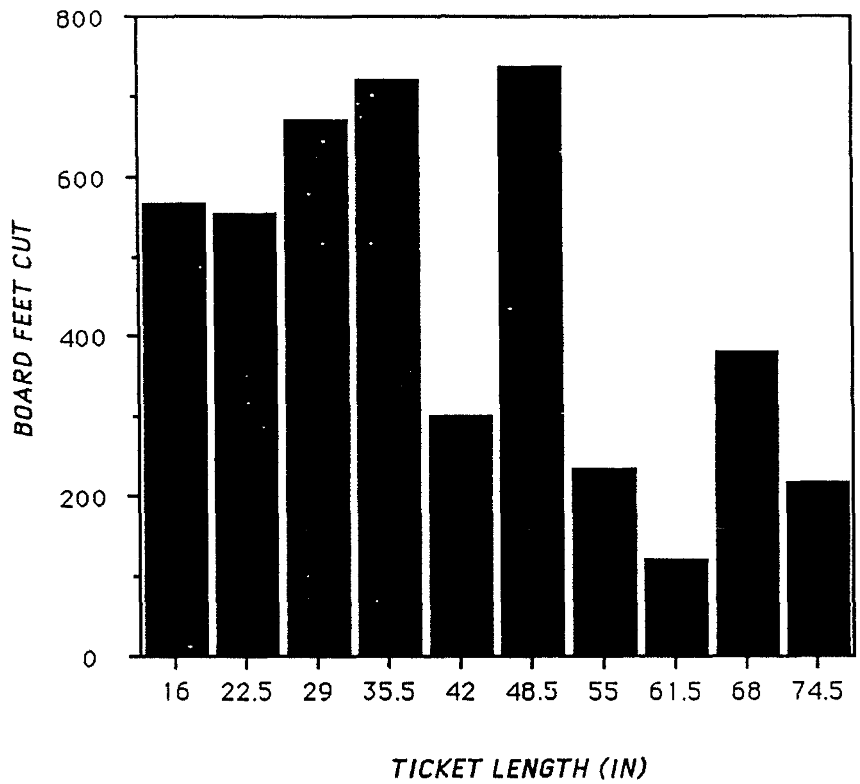

FIG. 8 Volume of Wood Cut as a Function of Ticket Length.

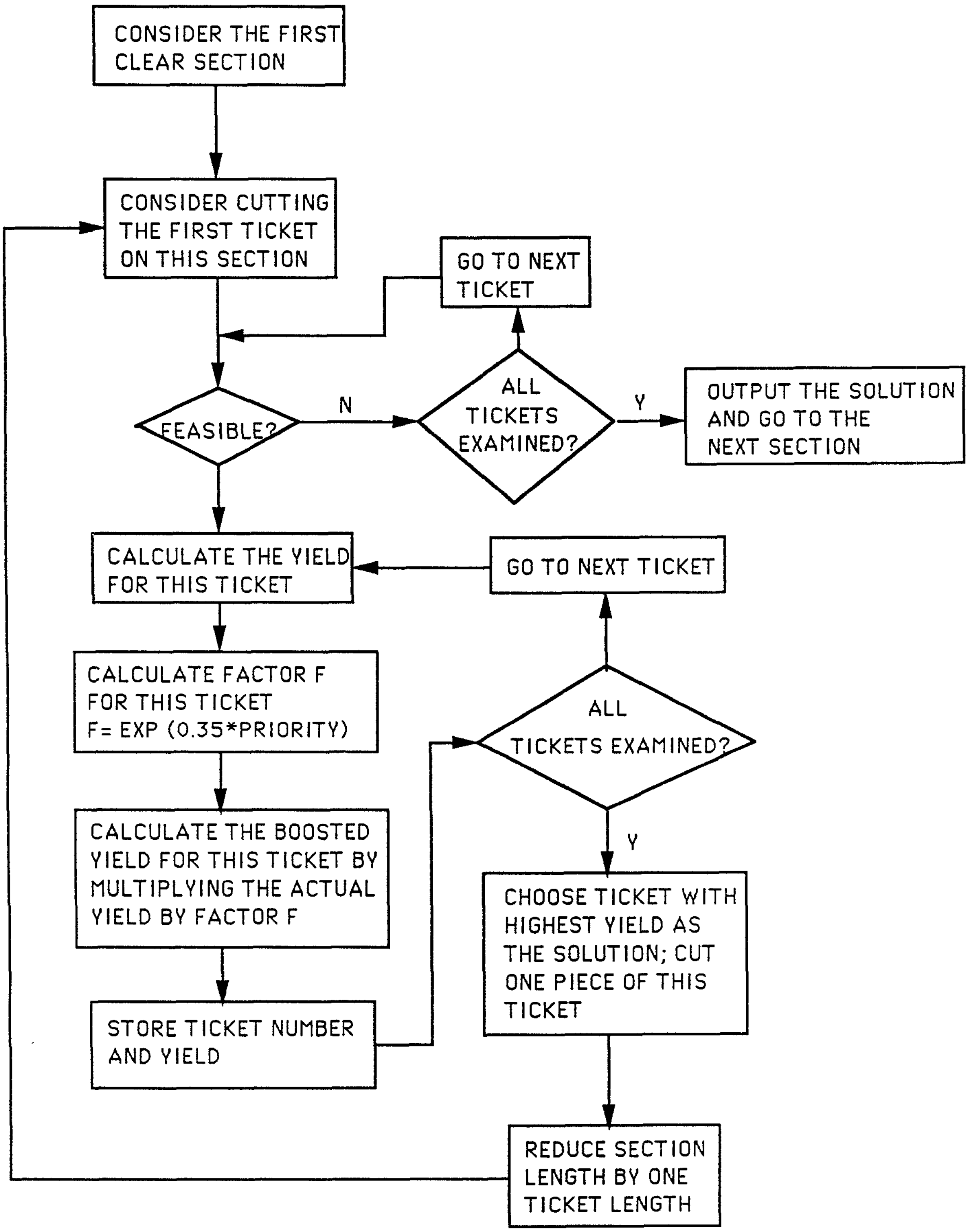

FIG. 9 Flowchart of Heuristic with Priorities.

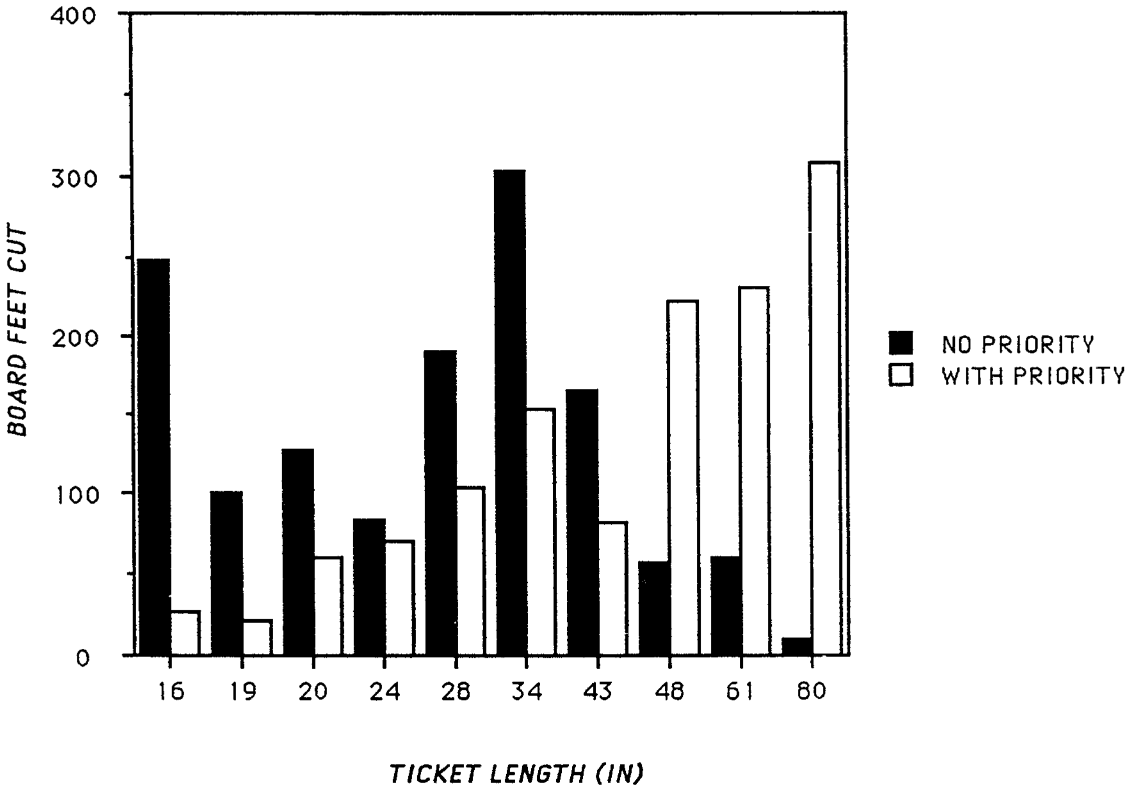

FIG. 10 Effect of Priority on Volume of Wood Cut.

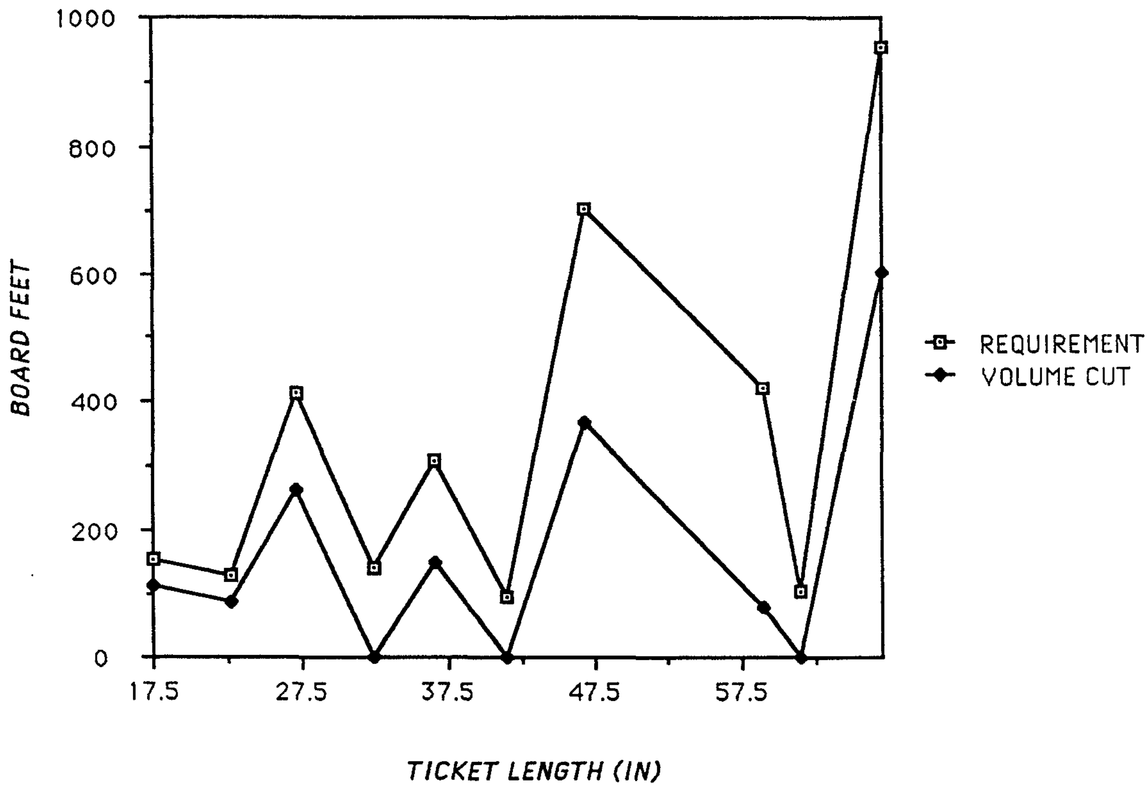

FIG. 11 Controlling Saw Output Using Priorities.

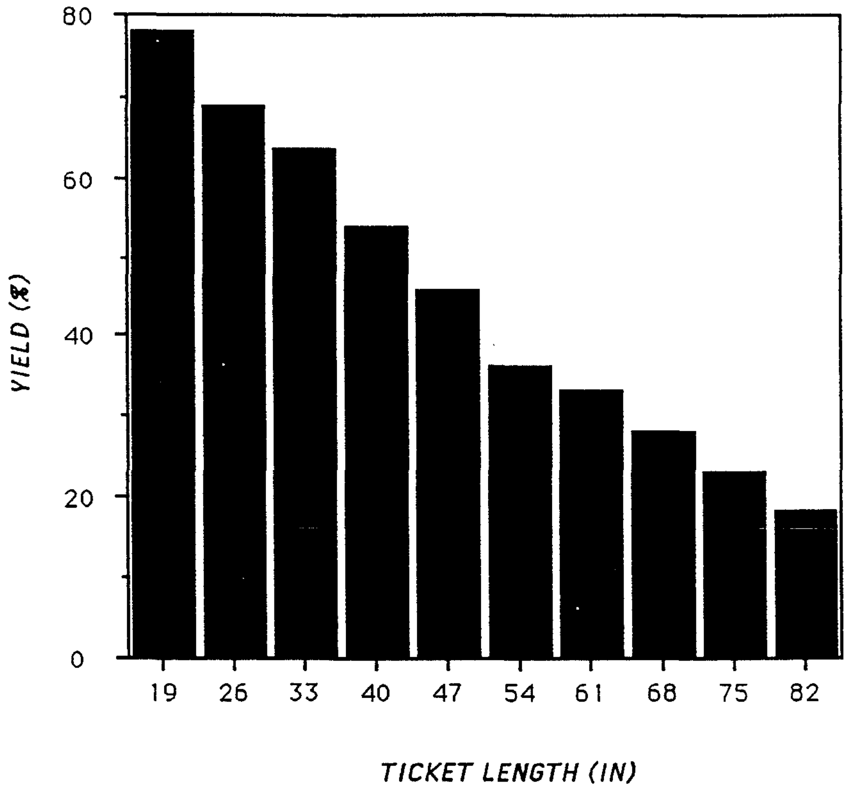

FIG. 12 Variation of Yield with Ticket Length.