Abstract

Title of dissertation:	A STOCHASTIC EQUILIBRIUM MODEL FOR THE NORTH AMERICAN NATURAL GAS MARKET
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This dissertation is an endeavor in the field of energy modeling for the North American natural gas market using a mixed complementarity formulation combined with the stochastic programming.

The genesis of the stochastic equilibrium model presented in this dissertation is the deterministic market equilibrium model developed in [Gabriel, Kiet and Zhuang, 2005]. Based on some improvements that we made to this model including proving new existence and uniqueness results, we present a multistage stochastic equilibrium model with uncertain demand for the deregulated North American natural gas market using the recourse method of the stochastic programming. The market participants considered by the model are pipeline operators, producers, storage operators, peak gas operators, marketers and consumers. Pipeline operators are described with regulated tariffs but also involve "congestion pricing" as a mechanism to allocate scarce pipeline capacity. Marketers are modeled as Nash-Cournot players in sales to the residential and commercial sectors but price-takers in all other as-

pects. Consumers are represented by demand functions in the marketers' problem. Producers, storage operators and peak gas operators are price-takers consistent with perfect competition. Also, two types of the natural gas markets are included: the long-term and spot markets.

Market participants make both high-level planning decisions (first-stage decisions) in the long-term market and daily operational decisions (recourse decisions) in the spot market subject to their engineering, resource and political constraints, resource constraints as well as market constraints on both the demand and the supply side, so as to simultaneously maximize their expected profits given others' decisions. The model is shown to be an instance of a mixed complementarity problem (MiCP) under minor conditions. The MiCP formulation is derived from applying the Karush-Kuhn-Tucker optimality conditions of the optimization problems faced by the market participants. Some theoretical results regarding the market prices in both markets are shown.

We also illustrate the model on a representative, sample network of two production nodes, two consumption nodes with discretely distributed end-user demand and three seasons using four cases.

A STOCHASTIC EQUILIBRIUM MODEL FOR THE NORTH AMERICAN NATURAL GAS MARKET

by

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To my parents.

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I am grateful to my parents and my husband for their understanding and love. Without them, it would be much difficult for me to finish this work.

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Chapter 1

Introduction

1.1 Overview of the Industry

Natural gas is one of the cleanest, safest and most reliable source of energy and the natural gas industry is an important segment of the U.S. economy. With more than a million employees in North America alone, the natural gas market continues to grow due to ever-increasing opportunities from exploration and production, to marketing and trading, to transportation and consumption [72].

The natural gas market in the United States has undergone significant changes recently due to a variety of factors such as the restructuring of the gas and power industries. Since the passage of the Natural Gas Policy Act (NGPA) in 1978, the industry has been in a transition from a regulated market to a deregulated one fostering more market competition. Federal Energy Regulatory Commission (FERC) Order 636 issued in April 1992 ordered interstate natural gas pipelines to unbundle gas sales, transportation and storage and converted interstate gas pipelines to open access transporters. Since that time, these agents have undertaken sole roles in the market, competed with each other noncooperatively and acted relatively independently. After years of attempts at market deregulation, the industry has become much more open to choice and competition and therefore more efficient. As a result, the popularity of natural gas use has skyrocketed and its growth is expected to continue [10].

Although the industry has been studied and deregulated for more than two decades and plenty of lessons have been learned, the deregulated gas market has not evolved to its final form [78]. Deregulation of the energy market is still an area requiring more research and applications to make corresponding theories and practice more mature. The method of *mathematical programming* is one of the most powerful tools available to assist either private or public sector in decision making. Mathematical programming is the study and use of optimization models, which minimize or maximize a real function of real or integer variables, subject to constraints on the variables. Generally, optimization models not only can generate operational level planning for individual operators, they can also help policy makers capture a big picture of the industry. In a particular restructuring natural gas market, market participants such as gas producers, gas shippers, etc., would like to know the other agent's adaptive reaction to the new market, such as production planning or investment guidance, to remain competitive with the other agents. Policy makers, rather than using price controls, are concerned with how to guide the market in a direction to promote the desired level of competition, how to make certain the competition is accessible to every participant or how to achieve in energy market efficiency.

More importantly, the natural gas industry is far from static and deterministic. The inherent uncertainty typically stems from natural stochastic phenomena, the international economic condition, energy-environmental requirements and the functioning of the domestic economic system [55]. In addition, although the North American interstate natural gas pipeline system is generally a safe mode to transport the natural gas, the reliability of the system has been challenged since terrorist attacks in 2001. Enhanced security thus provides the motivation for how to keep the system running at minimal cost and greatest reliability given these uncertain events. Consequently, the question of how to treat the uncertainties becomes meaningful and necessary in the natural gas industry.

All of the answers to aforementioned questions can be found through specially designed optimization models. Optimization techniques have been applied in the industry to address many of these problems including linear programming (e.g., [2]), nonlinear programming (e.g., [13]), stochastic programming (e.g., [9, 39]) and market equilibrium modeling (e.g., [7, 33, 35]). Following the work by Gabriel et al. [35], this dissertation is a study of the deregulated natural gas market under uncertainty using an extension to optimization models and the notion of market equilibria.

1.2 Game Theory

When it comes to market equilibria, one will naturally consider game theory. Game theory is a formal study of multi-person decision making. It has two high-level branches: *noncooperative game theory* and *cooperative game theory*. A noncooperative game is a game in which each player pursues his or her own interests which are partly conflicting with others in the absence of an ability to make binding agreements. Cooperative games that we do not discuss in this dissertation are where such agreements are enforced. Noncooperative game theory is concerned with the analysis of strategic choices while cooperative game theory focuses on the achievement and management of a game coalition.

The fundamental unit of the analysis in game theory is the players' strategic interdependence. The subject has been extensively applied to many fields such as economics, politics, finance and computer science whenever the strategic interaction is present. The application of game theory in the economics can be dated back to 1944 when the book *Theory of Games and Economic Behavior* by von Neumann and Morgenstern [76] was published. Game theory is especially indispensable for the analysis of oligopolistic markets where there is more than one but still not many agents [54]. Game theory is less useful to the market analysis when the market operators under perfect competition or monopoly with no strategic interactions.

The natural gas industry is characterized by imperfect competition. For example, the supply side of the European natural gas market has an oligopolistic structure [7, 17, 44]. As for the North American gas industry after decades' efforts of deregulation, gas producers are considered price-takers due to the great number of producers each owning only a small share of the production. However, imperfect competition could exist in other aspects of the market, such as the local distribution companies (LDCs). According to Energy Information Administration website [21], as of December 2004, apart from regions of District of Columbia, New Jersey, New Mexico, New York, Pennsylvania and West Virginia, residential consumers in other places are under imperfect competition of various levels from LDCs. Despite the fact that LDCs could be monopolists in the consumption regions they serve, game-theoretic models are used to describe these imperfectly competing agents in this dissertation.

In terms of the symmetry of the roles of the players take in the game, we can distinguish between at least two equilibrium solutions: the Nash equilibrium solution [58] and Stackelberg equilibrium solution [71]. Formal mathematical definitions for these two solution concepts are available in [4]. In a nutshell, in a Nash equilibrium, one player cannot improve his/her outcome by altering his/her decision unilaterally assuming players act simultaneously and no single player dominates the decision process. When one considers application of the Nash equilibrium in the production/distribution models, two types of models are abundantly studied in the literature: the Nash-Cournot model of quantity competition and the Nash-Bertrand model of price competition. A Nash-Cournot production model concerns a number of firms, each setting its production level so as to maximize its own profit given that the production of the other firms remains constant. As opposed to choice of production, firms competing in a Nash-Bertrand model choose the prices for their single output based on maximizing their profit in the Nash manner. A Stackelberg equilibrium is one that involves a hierarchy in the decision making. In such a game, it is assumed that one player, the *leader*, declares his strategy first and enforces it on the other players, referred to as *followers*. The OPEC oil-cartel versus the fringe of non-member producers is a good example of such a game in the energy industry. Likewise, Stackelberg equilibrium could also lead to the concepts of the Stackelberg-Cournot and Stackelberg-Bertrand models. Compared with the Nash equilibrium concept, the application of the Stackelberg-Cournot and Stackelberg-Bertrand models are limited. Studies that report on the modeling, solution properties and algorithms include [14, 29, 69, 73].

Further, in terms of the importance of the order in which decisions are made, a game could be *static* or *dynamic*. In spite of the fact that many criteria can be used to distinguish static and dynamic games, what they have in common is the role that time plays in the game. In this dissertation, a game is static if the players act only once and independently of each other (e.g., [56]); a game is considered dynamic when the decision making involves multiple time periods. In this sense, a Nash game can be static (e.g., [66]) or dynamic (e.g., [44, 56]).

The central concept in dynamic games is the information structure which describes type and amount of information available to players. Usually three types of structure may be distinguished: the closed-loop, feedback, and open-loop information structures. In the first case, the players' decisions are based on all the available information about the past state and the previous moves made by the other players; in the second case, the players use a Markovian strategy based on the observation of the current state; in the last case, the only information available for the players is related to the stage. The equilibrium solutions calculated by using the three information structure are referred to as closed-loop, feedback and open-loop solutions, respectively. The feedback and open-loop solutions are contained within the set of closed-loop solutions. Due to the computational complexity, the applications of the closed-loop and feedback solutions to large-scale models are not available in the literature of mathematical programming. Unfortunately, open-loop equilibria are generally not subgame-perfect [68]. This is often viewed as a major drawback of this type of equilibrium. However, because of its simplicity, open-loop strategies are often used as a benchmark for analysis of other strategic dynamic equilibria. Applications of the open-loop solution in the energy industry include [44, 45, 66]. Introduced by Haurie, Zaccour and Smeers [45], *S-adapted open-loop* structure, where S stands for *samples* is a variation of open-loop structure designated for the stochastic dynamic games. In this case, the players make decisions based on their observation of the the stage and the random outcomes on the scenario tree (defined in Section 4.2). It lies halfway between the completely adaptive closed-loop and the completely nonadaptive open-loop.

The models presented in this dissertation treat market participants as game players in a multistage thus dynamic, noncooperative game. Some players are price takers as in a perfect competition environment; others compete in an open-loop, Nash-Cournot fashion. All players have symmetric roles and make decisions simultaneously. The overall equilibrium is computed as a (S-adapted, if stochastic) open-loop Nash equilibrium via a variational inequality formulation to be introduced.

1.3 NCP/VI and Stochastic Programming

The fields of mathematical programming and economic theory are closely interwoven. One of the spotlights in the academic and professional communities is how to compute economic and game theoretic equilibria by mathematical programming. Three major methods contributing to the computation of economic equilibria have emerged: fixed point theory (or homotopy-based) methods, nonlinear optimization, and nonlinear complementary/variational inequality problem (NCP/VI) theory. Neither the fixed point theory nor the nonlinear optimization provides satisfactory generality or computational efficiency for solving large-scale equilibrium problems. However, NCP/VI has been shown, both theoretically and practically, to be a promising candidate for computing large-scale equilibrium problems [42].

As a result of almost four decades of research, the subject of NCP/VI has become a well-established and fruitful discipline within mathematical programming. NCP/VI theory is now an important mathematical method used by many researchers who study equilibrium of economic systems. It has been verified both theoretically and with applications that the NCP/VI format has significant advantages in computing an economic equilibrium compared to general optimization methods [27, 42]. It is well known that the problem of a Nash Equilibrium can be formulated as a variational inequality. While a Stackelberg game is not known to be a compementarity problem, but a mathematical program with equilibrium constraints (MPEC).

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. "Stochastic is opposed to deterministic and means that some data are random, whereas programming refers to the fact that various parts of the problem can be modeled as linear or nonlinear mathematical programs" [6]. From the perspective of mathematical programming, a decision of deterministic considerations is different from one for an uncertain environment. Using a decision based on deterministic conditions in an uncertain situation will possibly lead to invalid results. Due to the fact that real world problems almost invariably include some unknown parameters, stochastic programming has been widely applied in a variety of areas including agriculture, energy, finance, scheduling, transportation, etc. It has also been extensively used in energy models over the years. The regulated electricity market is the most studied area which uses stochastic programming. In contrast to the large volume of models relating to electricity investments and operations, the application of stochastic optimization equilibrium models to the oil and gas markets is more limited [78].

The counterpart of a market equilibrium model in stochastic programming is a stochastic equilibrium model. Apart from the advantages an equilibrium model has, a stochastic equilibrium model will use probability distributions to consider the range of possible contingencies and then provide a set of strategies dealing with different situations for decision makers rather than a single decision for the simplified reality.

Naturally, a new area has evolved — how to solve a stochastic equilibrium problem by NCP/VI, which has not yet been well studied. Despite its significance, few works have been published on it so far. Haurie et al. [44] proposed a stochastic dynamic Nash-Cournot model of imperfect competition for studying the contracts in the European gas market, and used variational inequalities as the computational technique. Using the same computational technique, De Wolf and Smeers [14] considered a Stackelberg-Nash-Cournot equilibrium model with a numerical illustration of the European gas market. However, such an application in the North American natural gas market, which distinguishes itself from the European market both in regulatory and operational aspects, has not appeared. Moreover, all the completed works of stochastic NCP/VI of the natural gas market have not taken into account much detail of agents beyond demand and supply sides. This dissertation establishes a detailed stochastic NCP/VI model aiming at filling the blank of such an application to the North American natural gas market. Besides its academic innovation, such study would shed light on the evolution of the restructuring natural gas market.

1.4 Energy Modeling Activities

In general, there are two foci of natural gas optimization models: optimization of gas operations for a particular entity (gas marketer, utility, etc.) and computation of market equilibrium prices, flows and quantities. The latter is often accomplished by solving an appropriate optimization problem or sequence of optimization problems. They are referred to as operation and market (equilibrium) models, respectively. Since the model to be developed is a market equilibrium model, we concentrate below on the market-centric gas models but mention that [2, 13, 39] are samples of approaches for operation models.

Compared with operation models that focus on operational aspects, market equilibrium models are of particular importance to policy makers in that such models give insights concerning the trading prices and quantities of natural gas while generating more comprehensive and higher level information relevant to the entire market. An equilibrium model also facilitates the forecast of the changes in the level of social welfare that would be caused by a change in market conditions such as an improvement in technology or a new government tax policy. Since the oil embargo of the early 1970s, the U.S. energy community has done extensive mathematical modeling to analyze various energy issues and develop a national energy policy. Among these efforts, market equilibrium models have played an important role. Some examples of large-scale equilibrium models for the U.S. energy industry are the Project Independence Evaluation System (PIES) in the 1970s [47]; the Intermediate Future Forecasting System (IFFS) around 1980 [57]; and the National Energy Modeling System (NEMS) in the 1990s [33].

We concentrate on reviewing works for the North American and the European natural gas market, both of which have received considerable attention over the years. There have been a variety of modeling efforts for the European natural gas market since 1970s. One of the early market models was the peak-load pricing and investment model for the domestic gas market in Great Britain [74]. It was based on maximizing the social welfare function. Later, Haurie et al. [44] built a stochastic dynamic Nash-Cournot model for considering long-term gas contracts and applied their model to the European gas market. The model only considered oligopolistic producers and end-users represented by inverse demand functions. The market equilibrium was achieved by simultaneous choice of production by producers so as to maximize their expected profits. They showed that their problem was an instance of an NCP/VI. De Wolf and Smeers [14] considered a similar Nash-Cournot problem for the European gas market from the stochastic and Stackelberg aspect. Stackelberg problems can be described as special cases of mathematical programs with equilibrium constraints (MPEC) [26], which generalizes NCP/VIs. Lastly, GASTALE [7, 17] is a recent work modeling the European gas market given a successive oligopoly setting of two layers of imperfectly competitive suppliers. Producers, consumers, storage operators and a transmission system operator are considered in an equilibrium context described by a complementarity problem.

In North America, O'Neill et al. in [61] presented a network optimization model depicting the interstate pipeline system using a linearization scheme to handle the nonlinear relationships between gas flows and pressure in pipelines, compressors, or valves. GRIDNET [8] is an example of a generalized network optimization model for gas that contains very detailed data on pipelines and gas transactions from the gas marketing company's perspective. The Natural Gas Transmission and Distribution Module (NGTDM) and Oil and Gas Supply Module (OGSM) are two modules related to the natural gas market in NEMS [18, 19]. NGTDM derives natural gas production and end-user prices and flow patterns for movements of the natural gas through the regional interstate network. OGSM produces forecast of drilling investments for exploration and production for domestic crude oil and natural gas using the wellhead natural gas prices supplied by NGTDM and petroleum product prices developed by the Petroleum Market Model (PMN), a component of NEMS. NGTDM and OGSM jointly project the regional production and wellhead prices for the natural gas market [18, 19]. Note that OGSM or NGTDM alone is not a natural gas market model per se.

The Gas Systems Analysis Model (GSAM) is a large-scale modular model of the North American natural gas market developed at ICF consulting. The model is based on the notion of maximizing the social welfare function resulting in a large-scale nonlinear program. A successive linear programming strategy is employed to solve the overall nonlinear problem. GSAM has been used in a variety of industry and regulatory studies. Perhaps the most unique feature of GSAM as compared to other market equilibrium models is its database of over 17,000 natural gas production reservoirs each with approximately 200 variables. This makes the model free from assumptions regarding the functional forms for the supply curves. GSAM's time horizon spans 23 years (1998-2020) with each year segmented into four gas seasons and four demand sectors. The supply and demand sides are tied together by a gas network composed of 46 nodes and 79 transportation links. A storage reservoir database of over 500 storage sites as well as regional peak-shaving options (LNG, propane/air) complements the network [34].

Gabriel et al. [35] presented and analyzed a market equilibrium model that can be applied to a natural gas marketplace resulted from the restructuring of the industry and showed that the equilibrium model is an instance of a mixed nonlinear complementarity problem (NCP). As far as is known,it is the first time in the literature that NCP/VI formulations were used to model a natural gas market with such a market structure. Compared with GSAM, this model considered oligopolistic marketers competing in the Nash-Cournot manner, which is not usually seen in the models developed for the North American natural gas market. Based on the U.S. national pipeline grids, this model was used in [36, 37] to measure the market power under different economic scenarios in terms of the changes in the equilibrium prices and quantities. A detailed description for this model is presented in Chapter 3.

None of the preceding models for the North American natural gas market accounted for the uncertain factors when they were initially developed although they could be used for the analysis of issues involving uncertainty using techniques such as sensitivity analysis. However, Wallace showed in [77] that sensitivity analysis does not deliver good candidate solutions for an optimization problem with uncertain parameters. In order to capture the uncertainty in the real market, this dissertation provides a stochastic market model based on the market framework proposed by [35]. Unlike previous models, we distinguish two types of natural gas markets in this new model: the long-term market and the spot market, which is not generally considered as part of a natural gas market model. Market participants make deterministic long-term planning decision in the long-term market and respond to the market uncertainty in the spot market. The new model will be presented in Chapter 4.

As another direction of [35], Kiet proposed a market equilibrium model using micro-level data to develop the demand functions, supply functions or other elements in the gas market that do not have a closed form [50].

1.5 Contributions and Organization of Dissertation

The main contributions of this dissertation include:

- To our knowledge, it is the first detailed stochastic equilibrium model applicable to the deregulated natural gas market in North America formulated as an NCP/VI;
- It mathematically establishes new and improved existence and uniqueness results of the model presented in [35], which is the genesis of this dissertation;
- It explores the theoretical and computational aspects of the stochastic NCP/VI, which have been relatively unstudied to date;
- It investigates specific approaches for performing stochastic equilibrium programming [45] in the natural gas industry.

It is important to note that this dissertation does not directly model the nonconvexities[1] in the natural gas market. This is done on purpose since the convexity assumptions facilitate to establish useful properties for the model and alleviate the computational burdens. Such modeling assumptions have also been used by many others described in this section, although it is known that simplification of the nonconvexities will result in a situation of market imbalance [62]. Moreover, a NCP/VI model combined with integer variables represents a rather unstudied and challenging area and have not appeared extensively in the literature. However, there has been some work in integer-constrained LCPs (linear complementarity problems), for example, [12, 38, 64].

[1]Typical examples of nonconvexities include the nonlinear relationship between flows through an arc and the pressures at the terminal nodes of the arc [61], the discrete decisions on whether to invest in a new project or not and the fixed cost in the cost functions.

The rest of this dissertation is organized as follows. Chapter 2 introduces the industry background emphasizing the outcomes of the industry deregulation. A simplified market network is proposed for the models presented in the later chapters. This model framework gives very comprehensive considerations to the components of the gas supply chain. Traditionally, the focus of the market equilibrium models is restricted to the interaction between producers and consumers located at the two ends of the supply chain. However, this dissertation incorporates intermediate agents located in the middle of the supply chain, i.e., regulated pipeline operators, storage operators and peak gas operators as independent players maximizing their expected profits, respectively.

Chapter 3 describes a deterministic equilibrium model denoted D-NGEM, which appeared in [35], and establishes new existence and uniqueness results for the model. Based on the mixed complementarity formulation derived from the KKT and market-clearing conditions, we present new conditions regarding existence and uniqueness results for the model. These conditions are more easily verified compared with those presented in [35] in that the new conditions are requirements on the inputs (i.e., the (marginal) cost functions and the (marginal) revenue functions for the marketers) of the model rather than the outputs as previously.

Chapter 4 presents a stochastic equilibrium model denoted S-NGEM, which is an extension of the deterministic model discussed in the previous chapter. This stochastic equilibrium model takes advantage of the recourse method provided by stochastic programming and sets up two types of decision variables faced by all players, i.e., the long-term market and spot market decisions, corresponding to the first-stage and recourse variables in the recourse method, respectively. The long-term market decisions are concerned with high level long-term planning. The spot market decisions are the responses to the market uncertainty in order to compensate for any adverse effects from the first-stage decisions. The overall objective is to maximize the expected profits earned based on both types of decisions. This model is shown to be an instance of an NCP/VI under minor assumptions. It is interesting to note that we establish a relationship between the prices for the long-term and spot markets. That is, it is generally true (except the storage gas market) that when the production activities (i.e., pipeline flows, wellhead production rates and peak gas production rates) are positive for the long-term market and for all possible random outcomes in the spot market, the corresponding price (the pipeline congestion fee, the wellhead price and the peak gas price) for the long-term market is equal to the expected spot market price.

Using GAMS/PATH [28], we provide the numerical results for a sample network with three seasons, eleven players and 64 demand scenarios in Chapter 5. Lastly, we conclude this dissertation in Chapter 6 with a summary and some recommendations for the future work.

Chapter 2

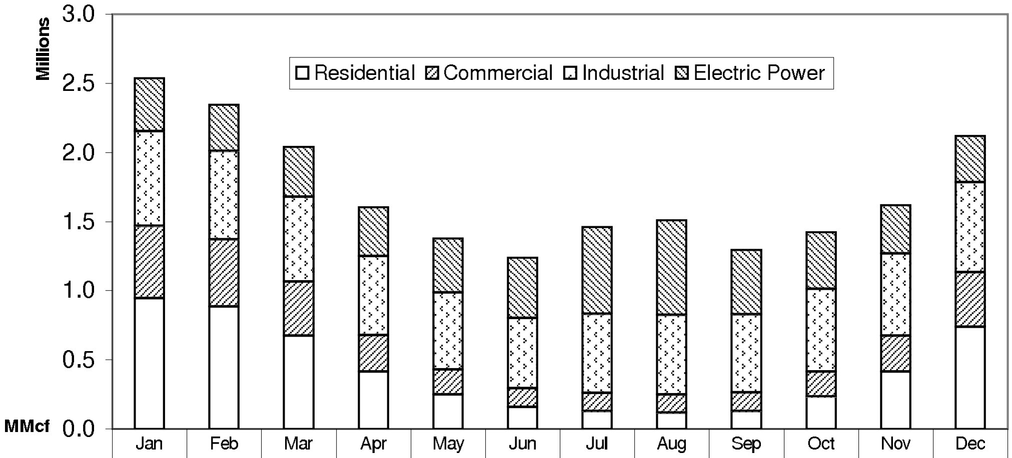
A Simplified Market Structure

In this chapter, we discuss a simplified market structure which is adopted by two equilibrium models for the North American natural gas market, that is, models D-NGEM and S-NGEM presented in Chapter 3 and Chapter 4, respectively. This framework takes into account several important market characteristics including demand seasonality, the emergence of new market participants and a variety of market contracts as a result of the market deregulation.

2.1 Demand Seasonality

Natural gas demand is highly seasonal with higher demand during the winter partially due to the fact it is used for heat in residential and commercial settings. We examine this inherent property of the market using data published by the U.S. Energy Information Administration (EIA). Figure 2.1 compares the U.S. monthly gas consumption in 2003 by sector. As shown in the figure, the consumption in the residential and commercial sectors is significantly higher in the winter than the summer. The industrial consumption is relatively constant throughout the year. On the contrary, electric power sector exhibits the strongest demand in the summer. Summing over four sectors, the seasonality of the consumption for the natural gas is apparent.

Such a trend can be easily found in other years as well.



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Electric Power	382,443	334,698	361,243	352,164	394,021	435,598	630,270	683,513	468,510	408,817	348,129	335,810
Industrial	686,025	640,202	615,171	574,328	556,416	508,348	572,719	577,497	561,221	601,231	595,609	650,261
Commercial	522,463	487,178	390,806	263,067	181,047	137,575	132,219	130,824	136,613	181,260	259,504	394,103
Residential	945,744	884,233	674,581	414,062	247,501	157,262	126,386	115,784	128,579	231,574	413,718	738,775

Source: EIA

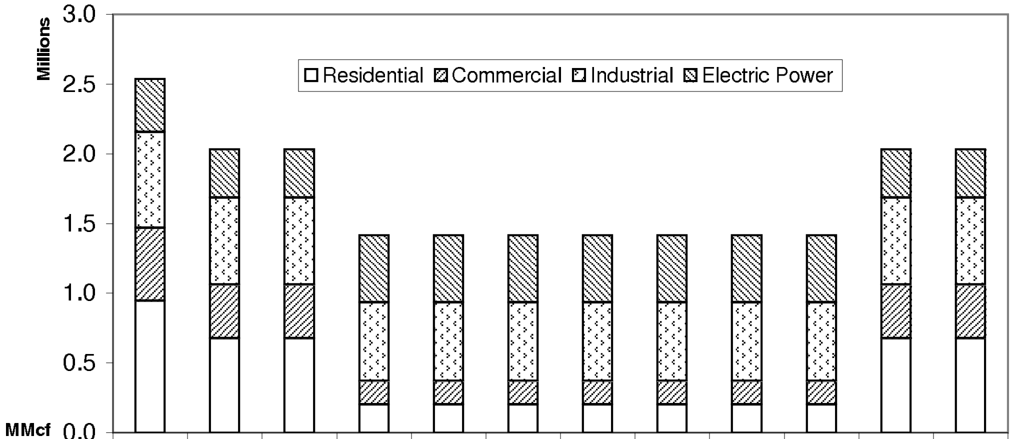
Figure 2.1: U.S. 2003 Monthly Natural Gas Consumption by Sector

Given the demand seasonality, we specify three seasons for a year, indexed by s to approximate this variability; $days_s$ is the number of days in season s. The year is divided as follows:

- s=1, low demand season, April-October, $days_1 = 214$;
- s=2, high demand season, November - March excluding January, $days_2 = 120$;
- s=3, peak demand season, January, $days_3 = 31$.

Using the seasons defined above, we average the monthly consumption within the three seasons in Figure 2.1. For example, the average monthly consumption in season 1 is the sum of monthly consumption from April to October divided by 7, the number of months in season 1. The result of the averaging is shown in Figure

2.2, which reduces the demand levels to three thus alleviating the modeling and computational complexity.



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Electric Power	382,443	344,970	344,970	481,842	481,842	481,842	481,842	481,842	481,842	481,842	344,970	344,970
Industrial	686,025	625,311	625,311	564,537	564,537	564,537	564,537	564,537	564,537	564,537	625,311	625,311
Commercial	522,463	382,898	382,898	166,086	166,086	166,086	166,086	166,086	166,086	166,086	382,898	382,898
Residential	945,744	677,827	677,827	203,021	203,021	203,021	203,021	203,021	203,021	203,021	677,827	677,827

Figure 2.2: Seasonally Averaged U.S. 2003 Monthly Consumption by Sector

2.2 Market Participants

The structure of the natural gas industry has changed dramatically over the past two and a half decades since the Natural Gas Policy Act (NGPA) passed by Congress in 1978. This act ended federal control of the wellhead price of gas as of January 1, 1985. Later in the mid 1980's, the Federal Energy Regulatory Commission (FERC) Orders 436 and 500 - the latter often referred to as the Open Access Order - encouraged pipelines to open access, non-discriminatory transportation services so end-users could contract directly with producers for gas supply. The Natural Gas Wellhead Decontrol Act (NGWDA) in 1989 fully lifted all controls on the wellhead prices. FERC Order 636 in 1992 required interstate pipelines to unbundle gas sales, transportation and storage to ensure that transportation was equally available to all [10].

In the restructurd market, producers, processing companies, pipeline companies, storage operators, marketers, local distribution companies (LDC) and liquified natural gas (LNG) suppliers make up the main parts of the natural gas industry. The existence of natural gas marketers who can serve as a middle-man between the buyer and the seller of gas to facilitate the movement of gas, is one of the major differences in the current structure of the market as compared to previously.

For the modeling purpose, following Gabriel et al. [35], the natural gas market in North America is simplified to the following agents:

- *Consumers* — indexed by $k \in K$, that exhibit demand for the natural gas. There is the demand by sector: *residential*, *commercial*, *industrial*, and *electrical power*, indexed by $k = 1, 2, 3$ and 4, respectively. There are no optimization problems for the consumers. In the model D-NGEM, they are instead represented by demand functions. However, in the model S-NGEM, only residential and commercial sectors are represented by demand functions. Industrial and electric power sectors have predetermined demand in different time periods.

- *Pipeline operators* — indexed by $a \in A$, that own the physical pipelines designed to move the gas from the wellhead in the production area to the city-gate in the consumption area via both long and short haul. The transportation rates of the pipeline use are regulated by FERC in America. Therefore, pipeline operators are assumed to be price-takers in the transportation market.

- *Production operators* — indexed by $p \in P$, also referred to as producers, that own the gas that is produced at a well and concern themselves with exploration and production activities. There are over 8,000 producers of natural gas in the United States [59]. Therefore, producers in our models are assumed price-takers given the small percentage of reserves that typically each producer holds in North America.

- *Storage operators* — indexed by $r \in R$, that inject natural gas into storage sites (depleted reservoirs, salt caverns, etc.) in the off-season (season 1) and extract gas to consumption market when the demand is high (seasons 2 and 3), typically the winter months. As of August 2004, there are about 120 natural gas storage operators in the United States, with control over 400 active underground storage facilities [25]. The storage facilities are regulated by the FERC or the state depending on its service scope. For this reason, storage operators are price-takers in the two models.

- *Peak gas operators* — indexed by $p \in P$, that sell peak gas (LNG or propane-air mixtures, collectively refereed to as "peak gas") to marketers during the peak season (season 3) when the peak demands are not adequately supplied by the pipeline or storage gas. Despite the forecast of the rapid growth of the LNG imports in the next few years in Annual Energy Outlook 2004 by EIA [20], the peak gas only serves a small amount of demand in the current North American market. According to the data available on EIA website, the LNG withdrawal in 2003 was 67,543 million cubit feet (MMcf). Compared with the total consumption for the natural gas of 22 trillion cubic feet (Tcf), the LNG supply was rather insignificant. Thus the peak gas operators are modeled as price-takers in our models.

- *Marketers/shippers* — indexed by $m \in M$, re-sellers of the gas that contract with pipeline companies, production operators, storage operators, and peak gas operators to procure the natural gas and sell it to end users in the residential, commercial, industrial, and electrical sectors. As modeled, the marketers are the only access of gas to end-users and thus are more likely to possess market power. Therefore, we consider oligopolistic marketers in the models. In the model D-NGEM, marketers are Nash-Cournot players for all the four demand sectors. While in the model S-NGEM, marketer are Nash-Cournot players in the residential and commercial sectors but price-takers in the industrial and electric power sectors. This change is under the consideration that large buyers in the industrial and electric power sectors are able to get discounts on large orders from marketers[1]. Large buyers could even have the market power to set the prices. Such a market is referred to as an oligopsony.

We note that in our nomenclature, we do not distinguish between marketers and shippers. In fact, marketers procure the natural gas from some source (producers, storage operators, etc.) and supply it to the end users in the four consumption sectors. Shippers can have a similar role to marketers except that they themselves may also be end users such as a local distribution company supplying natural gas to residential end users. Structurally, they have similar optimization problems; hereafter "marketer" will refer to either a marketer or a shipper unless specified otherwise.

Storage gas and peak gas will be used to handle peak demands not adequately supplied by the pipeline gas during the peak season. In general, peak gas or storage gas is expensive.

[1]Based on the data available at EIA, the average consumption per consumer in the residential sector was 394, 369, 377 and 362 thousand cubic fee (Mcf) for years 2000 - 2003, respectively. In the commercial sector, the average consumption per custom was 635, 605, 621 and 624 Mcf, respectively. The average consumption per industrial consumer was 36,968, 33,840, 36,458 and 34,747 Mcf, respectively.

For modeling purposes, the whole market is divided into several submarkets: pipeline, production, storage, peak gas and marketer submarkets. Each agent described above operates in the submarket to which it belongs. It is reasonable to assume that agents in the same submarket have the same competition pattern. For example, if the marketer submarket is oligopolistic, all the marketers belonging to it compete with each other in a Cournot-Nash or Bertrand manner. A noncooperative game theory model may be appropriate for this case.

2.3 Natural Gas Network

In terms of the natural gas network structure, there are a set of regions for production or consumption denoted as N. We also denote the sets for production and consumption regions as PN and CN, respectively, satisfying two conditions $PN \cup CN = N$ and $PN \cap CN = \emptyset$. In the case that a region physically exhibits both production and consumption behaviors, we can conceptually divide it into two regions, production and consumption, in the network. One possible benchmark for region division is the census regions. Examples include Oil and Gas Supply Module (OGSM) [18] and Natural Gas Transmission and Distribution Model (NGTDM) [19], both of which are components of the National Energy Modeling System (NEMS). The work by Gabriel et al in [37] adapts the same region map as NGTDM.

- ***Production Regions***

  Multiple producers are allowed per production region where one producer has a unique production location. The set of producers located at a production region $n \in PN$ is denoted by C^n. Therefore, $\{\cdots, C^n, \cdots\}$ forms a partition of the set C. The mathematical operator $n^c(c)$ calculates the location of producer c. In terms of the network structure, each production region is linked to the consumption regions (either to marketers or to storage operators) corresponding to physical interstate pipelines or pipeline aggregates either of which can accrue losses.

- ***Consumption Regions***

  The consumption regions encompass the groups of consumers, storage operators, peak gas operators and marketers. Multiple participants are allowed for each group while every participant has one unique location. The sets R^n, P^n and M^n are groups of storage operators, peak gas operators and marketers, respectively, located at consumption region $n \in CN$. Thus, $\{\cdots, R^n, \cdots\}$, $\{\cdots, P^n, \cdots\}$ and $\{\cdots, M^n, \cdots\}$ are partitions of sets R, P and M, respectively.

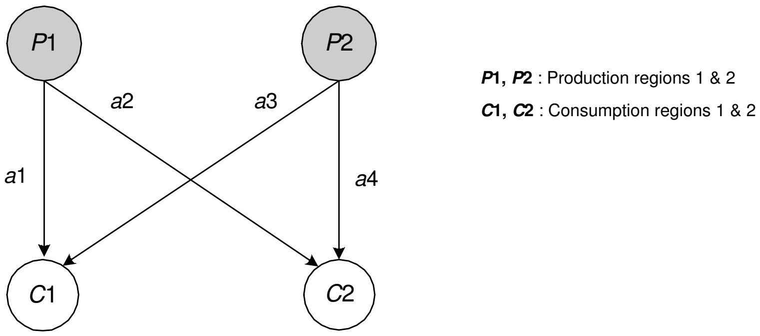
Connecting each of these production and consumption regions is a set of directed pipeline arcs A. A specific arc $a \in A$ represents an abstraction of a pipeline, pipeline segment, or pipelines measuring the flow between these production and consumption regions. $A(n)$ is the set of arcs connected to the node $n \in N$. If n is a consumption region, then $A(n)$ is also the set of arcs available to marketers and storage operators located at node n. Such a simplification of the network represents a transportation network per se. A more straightforward alternative is a transshipment network. Section 3.1.1 explains why a transportation network is chosen for the pipeline network.

Figure 2.3 is an illustration for a simplified network with interaction between market participants during three seasons. Geographically, there are four regions in the sample network, $N = \{P1, P2, C1, C2\}$ as shown in Figure 2.3 (a). The two shaded ones on the top represent two production nodes $PN = \{P1, P2\}$, each of which can have one or more producers as shown in Figure 2.3 (b). On the bottom, there are two consumption regions $CN = \{C1, C2\}$, where marketers, storage operators, peak gas operators and consumers are co-located, as shown in Figure 2.3 (b). The production and consumption regions are connected by a set of directed arcs $A = \{a1, a2, a3, a4\}$ as shown in Figure 2.3 (a). The sets of arcs available to regions $P1$, $P2$, $C1$ and $C2$ are, respectively, $A(P1) = \{a1, a2\}$, $A(P2) = \{a3, a4\}$, $A(C1) = \{a1, a3\}$ and $A(C1) = \{a2, a4\}$.

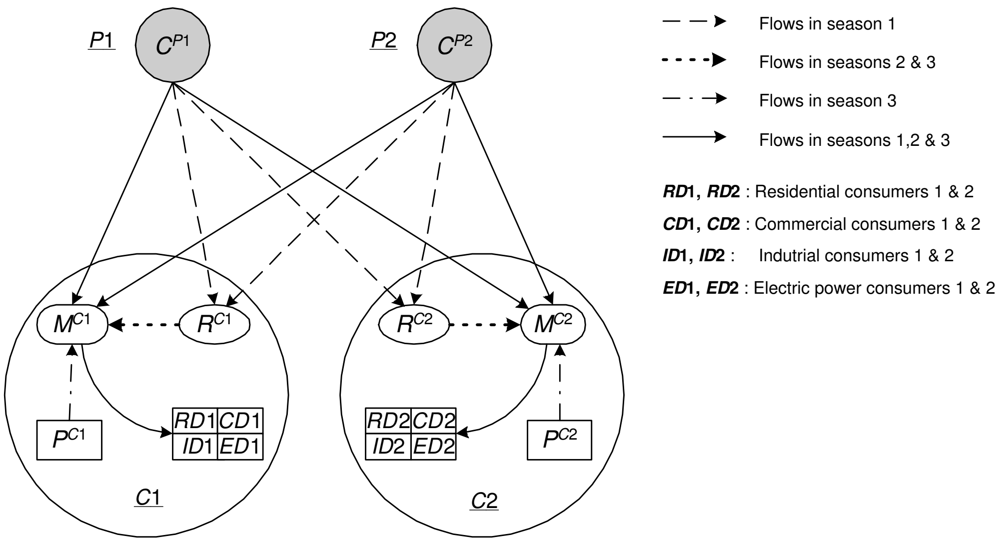
The nodes in Figure 2.3 (b) symbolize a group of operators; the number of operators located on each node could be one or more than one. The set of producers located at regions $P1$ and $P2$ are denoted by C^{P1} and C^{P2}, respectively. Marketers and storage operators located at the same consumption regions share the same set of arcs, e.g., for consumption region $C1$, the set of available arcs is $A(C1) = \{a1, a3\}$. Marketers ($m \in M^{C1}$ or $m \in M^{C2}$) procure the gas from producers in all three seasons. Storage operators ($r \in R^{C1}$ or $r \in R^{C2}$) obtain the gas from producers in season 1 and inject it into the storage for later use. Within the consumption regions, local marketers are the only access to gas for consumers; storage operators supply the gas to the local marketers in seasons 2 and 3; peak gas operators ($p \in P^{C1}$ or $p \in P^{C1}$) supply gas to marketers in the peak demand season, season 3. Note that these assumptions are for modeling purposes but are reasonable approximations of reality.

2.4 Contracts and Market

In this section, we discuss how the models in this dissertation considers the uncertainty inherent in the gas market and the means of contracting designed to protect market participants against risk.



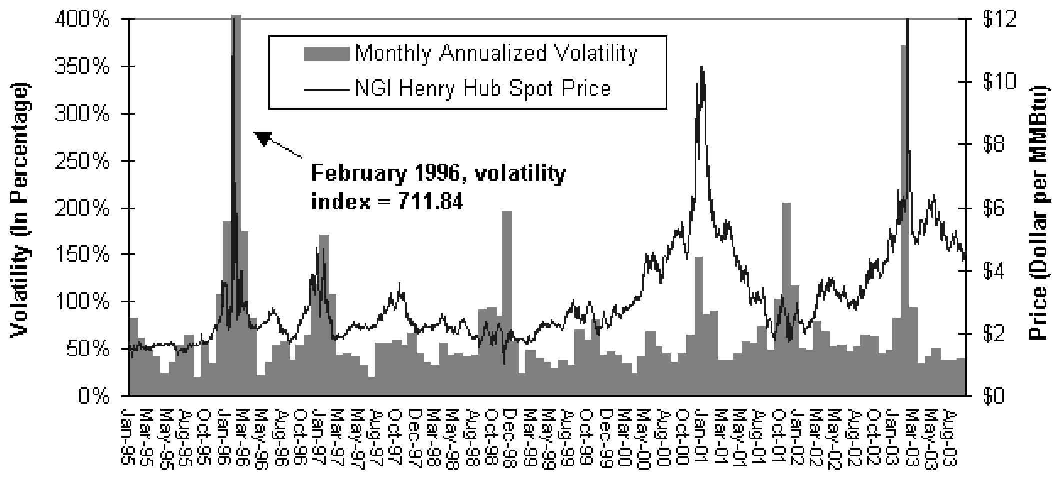
(a) Sample Network by Region



(b) Sample Network by Participant

Figure 2.3: Sample Network

It is well known that the natural gas industry is far from static and deterministic in that the fluctuation of the demand and prices over time are hard to predict. The factors that influence the demand for natural gas include weather changes, storage levels, market information as well as the economy, in general all of which are hard to predict. Figure 2.4 from EIA [23] shows this market uncertainty in terms of an index "price volatility", measured by the day-to-day percentage difference in the price of the gas. The figure examines daily spot market prices and the corresponding price volatility index at the Henry Hub market center in Louisiana from January 1995 through October 2003. It is noticeable that the spot market prices are subject to seasonal changes. The peak demand in the winter season is usually accompanied by high and often volatile prices because the natural gas supply has less flexibility to respond to the surging demand caused by the cold winter months.



Source: Daily spot prices—NGI's Daily Gas Price Index, Intelligence Press, Inc.; Monthly Volatility Index—derived by Energy Information Administration, Office of Oil and Gas.

Figure 2.4: Natural Gas Spot Market Prices and Volatility (1995-2003)

Consequently, finding an appropriate trade-off between gas price and supply assurance becomes a crucial question to the industry participants since gas suppliers do not want to commit to a low price over a long period of time while buyers do not want to commit to a high price over an extended period of time. Because of this relationship there are two types of gas markets that we concentrate on:

- ***Spot Market***

  As a result of deregulation, the spot market is "a market in which natural gas is bought and sold for immediate or very near-term delivery, usually for a period of 30 days or less" [24]. The spot market contracts are used to take advantage of market imbalance conditions and maintain market flexibility. Price competition is a dominant characteristic of the spot market. Spot market prices reveal the short-term supply and demand characteristics of the market. The Henry Hub in southern Louisiana, where more than 180 customers regularly conduct business through 14 interconnecting pipeline system and high-deliverability salt storage cavern facility according to EIA [22], is the most active and publicized spot market in North America.

- ***Long-Term Market***

  The long-term market consists of buying and selling natural gas under contract for at least one month in advance. Prices of the long-term market are negotiated between the buyer and seller and often an index is used as a benchmark. Because the long term market is less volatile than the spot market, assurance of supply is the major advantage of this market. The long-term contracts are the traditional ones in the natural gas market. Nevertheless, the importance of these contracts has been lessened as a result of deregulation of the industry.

In addition, based on a consumer's demand profile, there are three main types of physical trading contracts: swing contracts, baseload contracts and firm contracts.

Swing (or "Interruptible") contracts are usually short-term contracts. Under this contract, both the buyer and seller agree that neither party is obligated to deliver or receive the exact volume specified. Either party can terminate the contract with short or no notice. The buyer will generally pay only commodity charges (a unit charge of gas delivered to the buyers) when the gas is delivered. A swing contract has the lowest priority of all contracts.

Best-efforts contracts are similar to swing contracts. Neither the buyer nor seller is obligated to deliver or receive the exact volume specified. Interruption of service is allowable on short or no notice. However, it is agreed that both parties will attempt to fulfill the contract on a "best-efforts" basis.

Firm contracts provide service on a guaranteed basis. Unlike swing and best-effots contracts, there is legal recourse available to either party if the other party fails to fulfill its obligation under the agreement. The buyer will generally pay a reservation charge[2] and a commodity charge. A firm contract has the highest priority among all other contracts.

Based on the above analysis, the market structure depicted in Figure 2.3(b) can be understood as an abstract of the long-term market with firm contracts. In this dissertation, we assume a similar structure for the spot market: marketers buy gas from producers in all three seasons; storage operators inject the gas in the first season and then extract the gas to marketers in seasons 2 and 3; peak gas operators supply gas to marketers in the peak demand season. An important aspect of the spot market modeled in the dissertation is that all activities taken place in the spot market are merely committed to a season. Participants have the flexibility to make adjustment to their activity levels every season.

This dissertation develops an equilibrium model for an abstraction of the natural gas market which captures the market structure and contracts features involved

[2]In this dissertation, reservation charge is modeled as a charge per unit of capacity reserved on a pipeline by firm contracts.

in both the spot market and long-term markets. Thus, the results generated by the model can serve as references for policy makers or other market participants.

Chapter 3 of the dissertation incorporates a deterministic market equilibrium model, denoted D-NGEM, designed to simulate the long-term natural gas market. The span of the time period considered as “long-term” in this context is a medium-term horizon (one to three years) since capacity expansion decisions are not considered. The contract forms considered by this model can be understood as “firm contracts” in practice since market volatility is not a consideration of the model. Market participants can use this model to do long-term planing for their sales or purchases for the three seasons of every year at the beginning of the time.

Based on the deterministic model framework, a new stochastic equilibrium model, denoted S-NGEM is developed in Chapter 4. This stochastic model aids market participants in planning the sales or purchases under uncertain circumstances by using techniques of stochastic programming. It reflects the contractual features of not only the long-term market but also the spot market, where consumer demand over the time horizon is subject to one or multiple probabilistic distributions. The long-term and spot markets in the model are abstract generalizations of reality. First, the long-term market provides planning level contracts which must be made at the beginning of the time horizon and the spot market contracts are available for delivery at the beginning of each season. We believe that these are reasonable approximations due to the complexity of the problem and the limits on the computational capability. The model could be re-formulated to better approximate reality, where the long-term and spot market contracts are available at any time. In an extreme case when the number of long-term trading periods tends to infinity, the market equilibrium tends to the perfect competition solution [1]. Secondly, since there is no legal recourse and reservation charges associated with them, the best-efforts and swing contracts are considered as spot market contracts in the model; the firm contracts belong to the long-term market where a reservation charge is required.

Figure 2.5 demonstrates the relationship between these two models using a one-year time period. The horizontal axis represents the annual seasons and the vertical axis represents consumption rates. For each season, there is a white and a shaded block corresponding to the consumption in the long-term and the spot markets, respectively. The height of these blocks represents the level of the consumption rates. Consumption rates in the spot market are subject to probability distributions, e.g., an exponential distribution in season 1 and a discrete distribution of two realizations in season 2 as shown in the figure.

In the deterministic model in Chapter 3, all consumption from the four sectors are assumed to be deterministic and is met by long-term market supplies. By contrast, the stochastic model in Chapter 4 determines the consumption rate served both by the long-term and spot markets. The part of consumption rates met by long-term market contracts is determined at the beginning of the time horizon while those served by the spot market depends on the realizations of the possible outcomes of the market uncertainty. For example, in Figure 2.5, two realizations A and B in season 2 give rise to two levels of rates, a and b.

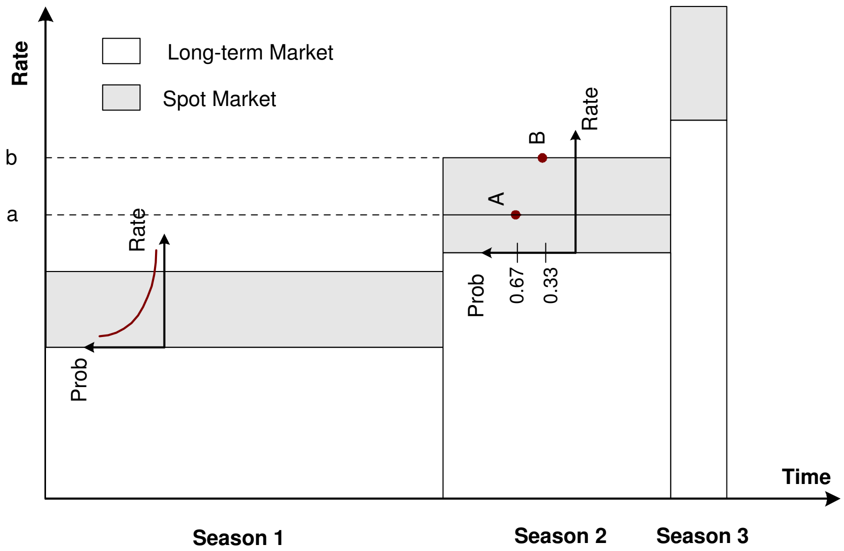


Figure 2.5: Deterministic and Stochastic Models

2.5 Conclusions

In this chapter, we discussed how the market characteristics emerging after deregulations, especially the existence of storage operators, marketeers and the spot market, are captured by the models D-NGEM and S-NGEM presented in the following chapters.

Chapter 3

A Deterministic Model D-NGEM

In this chapter, we first present a deterministic equilibrium model initially developed in [35], based on the market structure depicted in Section 2.3. This market model was denoted NGMEP in [35]. Next, we reformulate the model NGMEP as a mixed complementarity problem (MiCP) which is different from the one shown in [35] in that the market prices (wellhead prices, storage gas prices and peak gas prices) are defined to be nonnegative instead of free, so that new existence and uniqueness results are able to be developed by making mild assumptions on the marginal cost and revenue functions thus releasing the restrictions on price variables as imposed in [35].

For the purpose of this dissertation, we rename the model NGMEP as D-NGEM, where D stands for *deterministic*, as opposed to the S standing for *stochastic* in Chapter 4. This chapter is organized as follows: Section 3.1 discusses the components of the model D-NGEM and the conditions when these components are equivalent to MiCPs or NCPs. Sections 3.2 and 3.3, respectively, show the existence and uniqueness conditions for model D-NGEM based on the MiCP formulation developed in Section 3.1. Numerical demonstrations of the model D-NGEM can be found in [36] and [37], in which the model was run to predict gas consumption and prices in 2008 under different economic scenarios for the North American natural gas industry including the continental U.S. and Canada connected by 132 major interstate pipelines.

For the sake of establishing existence of an equilibrium solution, assumptions made in this chapter include:

- The cost functions of producers, storage operators and peak gas operators are convex, continuously differentiable and strictly increasing;
- For each $l \geq 0$, the end-user inverse demand functions $\theta(l)$ are nonnegative, continuously differentiable and non-increasing and satisfy the following inequality: $\theta'(l) + l\theta''(l) \leq 0$;
- Assumption 3.2.1: The pipeline congestion price is bounded below when the corresponding pipeline flow is zero.

All assumptions but the last item are common practice in developing existence results for an equilibrium solution. See [30, 41, 56, 60, 69], to name but a few. Assumption 3.2.1 is in place due to the lack of a cost function in the pipeline operators' objective function. Beside these preceding assumptions, if the marginal cost functions are accordingly strictly increasing or strictly monotone, the uniqueness of an equilibrium solution follows.

3.1 Model D-NGEM

Before getting into details of model D-NGEM, we introduce two important concepts used extensively in this dissertation.

Definition 3.1.1. Let X be a nonempty subset of $\mathbb{R}^n$ and F be a mapping from $\mathbb{R}^n \to \mathbb{R}^n$. The *variational inequality problem*, denoted $\mathrm{VI}(X, F)$ is to find a vector

$x^* \in X$ such that

$$F(x^*)^T(y - x^*) \geq 0 \qquad \text{for all } y \in X \tag{3.1.1}$$

It is well known that an important special case of VI(X, F) is the nonlinear complementarity problem NCP(F) [26]:

Definition 3.1.2. Let F be a mapping from $\mathbb{R}^n$ into itself. The *nonlinear complementarity problem*, denoted by NCP(F) is to find a vector $x \in \mathbb{R}^n$ such that

$$0 \leq x \perp F(x) \geq 0 \tag{3.1.2}$$

The notation "$\perp$" is used extensively in the dissertation to indicate a complementarity relation. It implies in the above equation that in addition to the stated inequalities, i.e., $x \geq 0$ and $F(x^*) \geq 0$, the equation $x^T F(x) = 0$ also holds. When $F(x)$ is an affine function of x, the problem NCP(F) reduces to a *linear complementarity problem*, abbreviated by LCP. A generalization of the NCP is the mixed complementarity problem, abbreviated as MiCP.

Definition 3.1.3. Let G and H be two mappings from $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+$ into $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}_+$, respectively. The *mixed nonlinear complementarity problem*, denoted by MiCP(G, H) is to find a pair of vectors (u, v) belonging to $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+$ such that

$$G(u, v) = 0, \quad u \text{ free}$$
$$0 \leq v \perp H(u, v) \geq 0.$$

In Definition 3.1.2, 3.1.3 and throughout the dissertation, $\mathbb{R}^n_+$ denotes the nonnegative orthant of $\mathbb{R}^n$. Both the (mixed) nonlinear complementarity problems and the variational inequality problems are related to each other. Theorem 3.1.1 and Theorem 3.1.2 establish the relations. See the work by Harker and Pang [42] for more details on the relations between the NCP, MiCP and VI problem.

Theorem 3.1.1. [26] *Let F be a mapping from $\mathbb{R}^n$ into itself. A vector x solves the* $\mathrm{VI}(\mathbb{R}^n_+, F)$ *if and only if x solves the* $\mathrm{NCP}(F)$.

Theorem 3.1.2. [26] *Let G and H be two mappings from $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+$ into $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}_+$, respectively. A vector x solves the VI($\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+, F$), where $F^T = (G^T, H^T)$ if and only if x solves the* $\mathrm{MiCP}(G, H)$.

In general, theorems designed for VI are applicable to NCP and MiCP. For the purposes of the dissertation, we refer VI, NCP and MiCP to as NCP/VI collectively.

In what follows, we discuss the model D-NGEM, which consists of optimization problems for market participants: pipeline operators, producers, storage operators, peak gas operators and marketers, and corresponding market-clearing conditions for sub-markets, in particular, transportation, production, storage gas and peak gas markets described in Section 2.2. The collection of optimization problems of each type of players are denoted as (PL), (PR), (ST), (PG) and (MK) for pipeline operators, producers, storage operators, peak gas operators and marketers, respectively. In the model D-NGEM, pipeline operators, producers, storage operators and peak gas operators are price-takers while marketers are Nash-Cournot players in the marketers' market, i.e., selling gas to the four end-user sectors. Therefore, (PL), (PR), (ST) and (PG) can also be formulated as optimization problems. Mathematically, a combined optimization formulation for these non-strategic players is equivalent to the separate optimization problems for each of them. However, a combined formulation would cause a lot of cancellation of terms since one player's expense is another's revenue. For this reason, separate optimization problems for the non-strategic players are adapted by this work so that the insights into the interactions between the players can be easily obtained from the problem solutions. On the other hand, in general, there is no corresponding optimization problem immediately available for problem (MK)[1]. Thus, it is shown to be an instance of an MiCP. Lastly, we show that model D-NGEM is an MiCP in Section 3.1.6.

Note that discounted revenues and costs are not considered for clarity of presentation as well as given the short timeframe involved. The units for the gas volume, rate and price are million cubic feet (MMcf), million cubic feet per day (MMcf/d) and $/Mcf. The objective functions for the players are in thousand dollars.

All variables and data used in model D-NGEM are organized in Table 3.1 by market agent. Endogenous variables are decision variables and multipliers to the optimization problems for individual players. Exogenous variables are market prices determined by market-clearing conditions. Data are independent inputs for the model.

Table 3.1: Variables and Data for Model D-NGEM

Problems	**Endogenous Variables**		**Exogenous Variables**	**Data**
	Decision Variables	**Multipliers**		
(PL)	f_{asy}	ρ_{asy}	τ_{asy}	$\overline{f}_a$
(PR)	q_{csy}	λ_{csy}, μ_c	π_{nsy}	$\overline{q}_c$, $prod_c$, $c_c^{PR}(\cdot)$
(ST)	g_{ary}, x_{rsy}	δ_{ry}, ω_{rsy}, ξ_{ry}, ζ_{ry}	γ_{nsy}	$\overline{x}_r$, $\overline{g}_r$, $\overline{k}_r$, τ_{asy}^{reg}, $c_r^{ST}(\cdot)$
(PG)	w_{py}	σ_{py}	β_{ny}	$\overline{w}_p$, $c_p^{PG}(\cdot)$
(MK)	l_{kmsy}, h_{amsy}, u_{msy}, v_{my}	ϕ_{msy}	τ_{asy}, π_{nsy}, γ_{nsy}, β_{ny}	τ_{asy}^{reg}

[1]Hashimoto [31] points out that in special cases, such as affine demand and supply functions, a Cournot equilibrium on a transportation network can be calculated by solving a single optimization problem. Applications of Hashimoto's contribution include [7, 46].

3.1.1 Pipeline Operator

In practice, an interstate pipeline gathers natural gas from production regions then transports the gas to consumption areas. It is common that one pipeline can serve more than one production area or consumption area. Similar network applications can be found in *traffic assignment problems* or *spatial price equilibrium models* [67]. A straightforward representation for such a network is an arc-path incidence matrix along with path variables [41, 42], which identifies whether a path flow traverses a physical arc or not in the network. An example can be found in [41] by Harker. However, this method is not computationally efficient for large-scale networks due to the need for a great number of path variables between production and consumption nodes. Alternatively, and without loss of generality, we assume there is only one consumption node denoted $n_1(a)$ and one production node $n_2(a)$ at either end of an *arc* a, which is an abstraction of pipelines actually connecting $n_2(a)$ and $n_1(a)$. The process of generating such an "arc" involves the breakdown of pipelines by consumption and production areas and then the aggregation of pipeline segments without violating actual pipeline capacities. The reference [37] contains a concrete example explaining the process. In fact, such abstraction requires a great amount of effort for data analysis before actually solving the model so as to lessen the computational burdens. We also assume that just one arc is considered for each pipeline operator. More generally, we would solve a corresponding problem which sums the objective function terms for each operator and includes the corresponding constraints for all the arcs in the network as was done in [37].

The pipeline operator in charge of arc a faces a linear programming problem $(\widetilde{PL})$ as shown below. The rates τ_{asy} are exogenous but are variables in the overall equilibrium problem D-NGEM. They are determined by the market-clearing conditions (3.1.12) and (3.1.13). Equation (3.1.3) describes the objective function which sums terms over all seasons s and years y (as is done for the other optimization problems to be presented). Constraints (3.1.4) are the upper bounds on the arc flows with the lower bounds being zeros.

The optimization problem $(\widetilde{PL})$ is used to simulate the actions of a regulated pipeline operation which must provide pipeline services to anyone that demands it at the regulated rate of τ^{reg}_{asy}, instead of the marginal cost, within the physical capacity. The regulated rate, which is usually determined by governmental administration (e.g., Federal Energy Regulatory Commission (FERC) in the U.S.), should recover the transmission costs incurred by the pipeline operator. When the actual demand for the pipeline service exceeds the pipeline capacity, the pipeline operator would need an endogenous decision-making process to decide how to ration the capacity. Problem $(\widetilde{PL})$ simulates the charging of a flow premium τ_{asy} to purchasers of the pipeline services. This premium is determined by the market-clearing conditions (3.1.12) and (3.1.13). When the flow is positive, the term τ_{asy} corresponds to the shadow price of the capacity constraint (3.1.4) divided by the number of days in the season (i.e., $days_s$). Alternative formulations for different purposes are possible. Reference [35] provides comparisons for some of them.

$$(\widetilde{PL}) \quad \max \quad \sum_{y\in Y}\sum_{s\in S} days_s \tau_{asy} f_{asy} \tag{3.1.3}$$

$$\text{s.t.} \quad f_{asy} \le \overline{f}_a \qquad (\rho_{asy} \ge 0) \qquad \forall s, y \tag{3.1.4}$$

$$0 \le f_{asy} \qquad \forall s, y$$

Note that dual variables are presented besides the associated constraints, e.g., $\rho_{asy} \ge 0$ for constraints (3.1.4). Given that pipeline operators have independent decision variables and separate constraints, summing up $(\widetilde{PL})$ for all $a \in A$ gives rise to a problem denoted (PL), which represents the optimization problem for the pipeline market. It is known that the problem (PL) is equivalent to problems $(\widetilde{PL})$ for all $a \in A$.

$$(PL) \quad \max \quad \sum_{a\in A}\sum_{y\in Y}\sum_{s\in S} days_s \tau_{asy} f_{asy} \tag{3.1.5}$$

$$\text{s.t.} \quad f_{asy} \le \overline{f}_a \qquad (\rho_{asy} \ge 0) \qquad \forall a, s, y \tag{3.1.6}$$

$$0 \le f_{asy} \qquad \forall a, s, y$$

Since this is a linear program, the KKT conditions are both necessary and sufficient for optimality [5] and are shown in (3.1.7) and (3.1.8). The notation "$\perp$" in the KKT conditions signify the complementarity between constraints and associated dual variables. The KKT conditions to problem (PL) are:

$$0 \le -days_s \tau_{asy} + \rho_{asy} \perp f_{asy} \ge 0 \qquad \forall a, s, y \tag{3.1.7}$$

$$0 \le \overline{f}_a - f_{asy} \perp \rho_{asy} \ge 0 \qquad \forall a, s, y \tag{3.1.8}$$

Clearly, (3.1.7) and (3.1.8) have a mathematical structure of an LCP. Let us define

$$v^{PL} \equiv \begin{pmatrix} f_{asy} & (\forall a, s, y) \\ \rho_{asy} & (\forall a, s, y) \end{pmatrix} \text{ and} \tag{3.1.9}$$

$$H^{PL}(v^{PL}) \equiv \begin{pmatrix} -\, days_s \tau_{asy} + \rho_{asy} \quad (\forall a, s, y) \\ \overline{f}_a - f_{asy} \quad (\forall a, s, y) \end{pmatrix} \tag{3.1.10}$$

Definitions (3.1.9) and (3.1.10) allow the KKT conditions (3.1.7) and (3.1.8) to be expressed equivalently as

$$0 \le v^{PL} \perp H^{PL}(v^{PL}) \ge 0 \tag{3.1.11}$$

Market-clearing conditions, shown below as (3.1.12) and (3.1.13), are used to enforce an equilibrium. They require that, at the equilibrium prices, the aggregate supply of pipeline service ($days_1 f_{a1y}$) equal the aggregate demand for it ($\sum_{r\in R(n_1(a))} days_1 g_{ary}$ in season 1 and $\sum_{m\in M(n_1(a))} days_1 h_{am1y}$). If excessive supply or demand existed at the going prices, the market could not be at a point of equilibrium. Equation (3.1.12) and (3.1.13) represent"derived demand" equations [30] as opposed to explicit demand functions. Variables τ_{asy}, the market equilibrium prices, are enforced to be dual variables to these market-clearing conditions. In order to have an NCP/VI formulation, τ_{asy} are set to be free such that a mixed complementarity problem can be derived.

Market-clearing conditions for the pipeline or transportation market are:

$$days_1 f_{a1y} - \sum_{r\in R(n_1(a))} days_1 g_{ary} - \sum_{m\in M(n_1(a))} days_1 h_{am1y} = 0$$

$$(\tau_{a1y} \text{ free}) \qquad \forall a, y \qquad (3.1.12)$$

$$days_s f_{asy} - \sum_{m\in M(n_1(a))} days_s h_{amsy} = 0 \qquad (\tau_{asy} \text{ free}) \quad \forall a, s = 2, 3, y \qquad (3.1.13)$$

In general, τ_{asy} is a positive penalty incurred by downstream operators when a particular pipeline a is full. If τ_{asy} is negative, we can explain such a value as a rebate for these downstream operators. Interestingly, as shown in Theorem 3.1.3, a negative τ_{asy} does not occur unless f_{asy} is zero. In other words, no one in the market actually gets any rebates.

Theorem 3.1.3. *For a particular pipeline arc $a \in A$ in season s year y,*

(1) if $f_{asy} = 0$ then $\tau_{asy} \leq 0$;

(2) if $f_{asy} > 0$ then $\tau_{asy} \geq 0$;

(3) if $\tau_{asy} < 0$ *then* $f_{asy} = 0$.

Proof. (1) When $f_{asy} = 0$, we must have $\tau_{asy} \leq \rho_{asy}/days_s$ by (3.1.7) and $\rho_{asy} = 0$ by (3.1.8). Hence $\tau_{asy} \leq 0$ implied by $\rho_{asy} = 0$.

(2) When $f_{asy} > 0$, by complementarity, we deduce from (3.1.7) that $\tau_{asy} = \rho_{asy}/days_s$. By definition, $\rho_{asy} \geq 0$. Therefore, $\tau_{asy} \geq 0$.

(3) The result is contrapositively true from (2) and the fact that $f_{asy} \geq 0$. □

3.1.2 Producer

The producers are modeled as price-takers in a perfect competition environment given the small percentage of reserves that typically each producer holds in North America. We denote $(\widetilde{PR})$ for the optimization problems faced by producer c. Each production company $c \in C$ located at production node $n \in PN$ is modeled to choose gas production rates q_{csy} so as to maximize its net profit, which is the difference between seasonal revenue $(days_s \pi_{n^c(c)sy} q_{csy})$ and seasonal costs $(days_s c_c^{PR}(q_{csy}))$, summed over the time horizon.

The terms $\pi_{n^c(c)sy}$ in the objective function of $(\widetilde{PR})$, derived from market-clearing conditions at production node $n \in PN$, are production or wellhead prices for the node where producer c is located, and are exogenous to $(\widetilde{PR})$ but a variable for D-NGEM. Wellhead price $\pi_{n^c(c)sy}$ is confined to be a nonnegative price, which differs from model NGMEP in [35] where $\pi_{n^c(c)sy}$ was defined free. Similar changes are made to the storage gas price γ_{nsy} and peak gas price β_{ny} to be presented since the nonnegative price gives a better approximation to reality. The mathematical operator $n^c(c)$ specifies the location of producer c, s and y are for season and year, respectively. Furthermore, we suppose that the cost functions $c_c^{PG}(\cdot)$ are convex and continuously differentiable in order to derive sufficient KKT conditions from problem $(\widetilde{PR})$. Similar assumptions are made on cost functions of storage operators in Section 3.1.3 and peak gas operators in Section 3.1.4. In addition, cost functions are assumed not to vary with seasons and years in the time horizon considered by the model. Constraint (3.1.15) stipulates that the production capacity is fixed and cannot be easily expanded within the medium-term (one to three years). Constraint (3.1.16) states that the total volume of gas produced in the time horizon must not exceed the production forecast of $prod_c$. It is an approximation to the very complicated spatial and temporal dependencies that can exist, examples of which can be found in [11, 32, 34]. Note that constraint (3.1.16) links decisions from different seasons and years, making the equilibrium model D-NGEM nonseparable.

$$
\begin{aligned}
(\widetilde{PR}) \quad \max \quad & \sum_{y\in Y}\sum_{s\in S} days_s\big(\pi_{n^c(c)sy}q_{csy} - c_c^{PR}(q_{csy})\big) && && (3.1.14)\\
\text{s.t.} \quad & q_{csy} \le \overline{q}_c && (\lambda_{csy}\ge 0) \quad \forall s,y && (3.1.15)\\
& \sum_{y\in Y}\sum_{s\in S} days_s q_{csy} \le prod_c && (\mu_c \ge 0) && (3.1.16)\\
& 0 \le q_{csy} && \forall s,y &&
\end{aligned}
$$

Given that producers are price-takers and hence there is no interaction between decisions made by different individuals, the problems $(\widetilde{PR})$ for all producers $c \in C$ can be equivalently simplified into one larger optimization problem (PR) as shown below.

$$(PR) \quad \max \quad \sum_{c\in C}\sum_{y\in Y}\sum_{s\in S} days_s\big(\pi_{n^c(c)sy}q_{csy} - c_c^{PR}(q_{csy})\big) \tag{3.1.17}$$

$$\text{s.t.} \quad q_{csy} \le \overline{q}_c \qquad (\lambda_{csy} \ge 0) \quad \forall c, s, y \tag{3.1.18}$$

$$\sum_{y\in Y}\sum_{s\in S} days_s q_{csy} \le prod_c \qquad (\mu_c \ge 0) \tag{3.1.19}$$

$$0 \le q_{csy} \qquad \forall c, s, y$$

Given that the cost functions are convex and the constraints are affine, the KKT conditions (3.1.20) - (3.1.22) are equivalent to solving (PR) [5]. The KKT conditions to (PR) are:

$$0 \le days_s\Big(-\pi_{n^c(c)sy} + \frac{d\big(c_c^{PR}(q_{csy})\big)}{d(q_{csy})} + \mu_c\Big) + \lambda_{csy} \perp q_{csy} \ge 0 \quad \forall c, s, y \tag{3.1.20}$$

$$0 \le \overline{q}_c - q_{csy} \perp \lambda_{csy} \ge 0 \qquad \forall c, s, y \tag{3.1.21}$$

$$0 \le prod_c - \sum_{y\in Y}\sum_{s\in S} days_s q_{csy} \perp \mu_c \ge 0 \qquad \forall c \tag{3.1.22}$$

For ease of presentation, hereafter we use MC_{csy}^{PR} to denote the marginal cost functions, that is,

$$MC_{csy}^{PR} \equiv \frac{d\big(c_c^{PR}(q_{csy})\big)}{d(q_{csy})}, \quad \forall c, s, y \tag{3.1.23}$$

In light of the mathematical structure of these KKT conditions, we define

$$v^{PR} \equiv \begin{pmatrix} q_{csy} \quad (\forall c, s, y) \\ \lambda_{csy} \quad (\forall c, s, y) \\ \mu_c \quad (\forall c) \end{pmatrix} \text{ and} \tag{3.1.24}$$

$$H^{PR}(v^{PR}) \equiv \begin{pmatrix} days_s\big(- \pi_{n^c(c)sy} + MC^{PR}_{csy} + \lambda_{csy} \quad (\forall c, s, y) \\ \overline{q}_c - q_{csy} \quad (\forall c, s, y) \\ prod_c - \sum_{y \in Y} \sum_{s \in S} days_s q_{csy} \quad (\forall c) \end{pmatrix} \tag{3.1.25}$$

Definitions of (3.1.24) and (3.1.25) allow the KKT conditions (3.1.20)-(3.1.22) to be expressed equivalently as an NCP:

$$0 \leq v^{PR} \perp H^{PR}(v^{PR}) \geq 0 \tag{3.1.26}$$

The following theorem, which shows that with positive cost functions, a nonzero production rate leads to a nonzero market price, is developed for the needs of the existence and uniqueness results shown in Section 3.1.6. Prior to the theorem, an important assumption requiring that the marginal cost, the cost of producing one more unit of a good, be positive when the production rate is positive, is presented.

Assumption 3.1.1. *Given s and y, the marginal cost of producer $c \in C$ satisfies the following:*

$$MC^{PR}_{csy} > 0, \quad \text{if } q_{csy} > 0. \tag{3.1.27}$$

Theorem 3.1.4. *Assume Assumption 3.1.1 holds for all producers located at a production node n, i.e., $\forall c \in C^n$. If there exists a producer $c \in C^n$ who has a positive production rate, i.e., $q_{csy} > 0$, then the production price at production node n is positive too, that is, $\pi_{n^c(c)sy} > 0$.*

Proof. From (3.1.20), with positive q_{csy}, we must have

$$\pi_{n^c(c)sy} = MC^{PR}_{csy} + \mu_c + \lambda_{csy}/days_s.$$

By definition, λ_{csy} and μ_c are nonnegative. The term MC^{PR}_{csy} is positive by Assumption 3.1.1. Thus, $\pi_{n^c(c)sy}$ must be positive. $\square$

A general example of a cost function satisfying the above assumption is when $c^{PR}_c(\cdot)$ is increasing with q_{csy}. In particular, examples of cost functions which satisfy this assumption are when (1) $c^{PR}_c(\cdot)$ is a affine function with positive slope; and (2) $c^{PR}_c(\cdot)$ is quadratic with positive coefficients. Such an assumption is quite plausible for most of the production industry and a reasonable approximation to the production activities in that producing at a higher rate would require more resources.

The market-clearing conditions (3.1.28) and (3.1.29) for production market state that the aggregate supply of gas production at a node equals the aggregate amount sent out to either storage operators and marketers in season 1 or just marketers in seasons 2 and 3. The market-clearing conditions for the producer's market are as follows:

$$\sum_{c \in C^n} days_1 q_{c1y} = \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g_{ary} + \sum_{m \in M(n_1(a))} days_1 h_{am1y} \right) \quad \forall n \in PN, y \qquad (3.1.28)$$

$$\sum_{c \in C^n} days_s q_{csy} = \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_s h_{amsy} \quad \forall n \in PN, s = 2, 3, y \qquad (3.1.29)$$

Note that the conditions of the nonnegative market prices, that is, $\pi_{nsy} \geq 0, \forall n, s, y$ are not incorporated in the model yet. In order to do so, we construct equations (3.1.30) and (3.1.31) where production prices π_{nsy} are dual variables to the modified market-clearing conditions for the production market implying that when the supply exceeds the demand, the market price must be zero. Theorem 3.1.5 shows the validation for such association.

$$0 \leq \sum_{c \in C^n} days s_1 q_{c1y} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days s_1 g_{ary} + \sum_{m \in M(n_1(a))} days s_1 h_{am1y} \right)$$
$$\perp \pi_{n1y} \geq 0 \qquad \forall n \in PN, y \tag{3.1.30}$$

$$0 \leq \sum_{c \in C^n} days_s q_{csy} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_s h_{amsy} \perp \pi_{nsy} \geq 0$$
$$\forall n \in PN, s = 2, 3, y \tag{3.1.31}$$

Theorem 3.1.5. *If Assumption 3.1.1 holds for all $c \in C$, then system PR-MCC is equivalent to system PR-MCC-NCP, where*

$$PR\text{-}MCC \equiv \begin{cases} NCP(3.1.26) \\ (3.1.28) - (3.1.29) \\ \pi_{nsy} \geq 0 \quad \forall n \in PN, s, y \end{cases} \tag{3.1.32}$$

$$PR\text{-}MCC\text{-}NCP \equiv \begin{cases} NCP(3.1.26) \\ (3.1.30) - (3.1.31) \end{cases} \tag{3.1.33}$$

Proof. By construction, any solution satisfying *PR-MCC* also satisfies *PR-MCC-NCP*. Therefore, we must show that every solution to *PR-MCC-NCP* will be a solution to *PR-MCC*. Suppose the contrary that there exists a solution satisfying *PR-MCC-NCP* such that when $s = 1$, for some $n \in PN, y$:

$$0 < \sum_{c \in C^n} days s_1 q_{c1y} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days s_1 g_{ary} + \sum_{m \in M(n_1(a))} days s_1 h_{am1y} \right)$$
$$\text{and } \pi_{n1y} = 0 \tag{3.1.34}$$

or when $s = 2$ or 3, for some $n \in PN, y$

$$0 < \sum_{c \in C^n} days_s q_{csy} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_1 h_{amsy} \text{ and } \pi_{nsy} = 0 \quad (3.1.35)$$

From (3.1.34)-(3.1.35), it must follow that for some $n \in PN, s, y$

$$0 < q_{csy}, \ \exists c \in C^n \quad \text{and} \quad \pi_{nsy} = 0 \qquad (3.1.36)$$

However, $0 < q_{csy}$ for some $c \in C^n$ in (3.1.36), by Theorem 3.1.4 implies that $\pi_{nsy} > 0$ for the location of producer c is node n. However, this contradicts $\pi_{nsy} = 0$ in (3.1.36). Consequently, every solution of *PR-MCC-NCP* is also a solution to *PR-MCC*. This completes the proof. □

Alternatively, we could let free prices π_{nsy} associate with (3.1.28) and (3.1.29) to construct a mixed complementarity formulation as was done for the transportation market in Section 3.1.1 as well as in [35]. Assumption 3.1.1 ensures that $\pi_{asy} > 0$ when $q_{csy} > 0$ for some $c \in C^n$. Without further assumptions, π_{nsy} could be less than zero only when $q_{csy} = 0$ for all $c \in C^n$, which, however, means that this production node does not play a role in the market equilibrium in question and hence can be left out. However, an NCP formulation of market-clearing conditions for the transportation market is impossible since there are no such cost functions in the objective function for the pipeline operators to make restrictions on.

3.1.3 Storage Operator

The storage market is modeled as perfectly competitive where the individual storage operators each pursues maximum net profits. As such, for the sake of simplification, a collective optimization problem (ST) is used to present all operators in the storage market which sums objective functions for individual operators and includes the corresponding constraint sets. For a storage operator $r \in R$ located at $n \in CN$, he/she decides on the amount injected into the reservoir from arc a available to him/her in season 1, denoted g_{ary}, and the amount extracted in season 2 and 3, denoted x_{rsy}. The revenues are the sales income of extraction over the time horizon, that is the term $(days_s \gamma_{n^c(c)sy} x_{rsy})$ summed over high demand seasons, (i.e., seasons 2 and 3) and years, where $\gamma_{n^r(r)sy}$ is the nonnegative market prices for storage gas. The costs are assumed to be incurred just in season 1 and consist of commodity costs, transportation charges and injection costs for the amount delivered. Commodity costs, shown as $(\sum_{a \in A(n)} days_1 \pi_{n_2(a)1y} g_{ary})$ in (3.1.37) are the expenses of purchasing the gas from producers, which in contrast are part of revenues for producers. Transportation charges for using the pipeline services include regulated charges $(\sum_{a \in A(n^r(r))} days_1 \tau^{reg}_{a1y} g_{ary})$ and congestion charges $(\sum_{a \in A(n^r(r))} days_1 \tau_{a1y} g_{ary})$ (see Section 3.1.1). As in the case of $c^{PR}_c(\cdot)$ in Section 3.1.2, the injection cost function $(c^{ST}_r(\sum_{a \in A(n^r(r))} g_{ary}))$ is assumed convex and continuously differentiable and not to vary with seasons and years.

In constraint (3.1.38), there are two loss factors, $loss_r$ for storage injection and $loss_a$ for pipeline transmission. The term $loss_r \in (0, 1)$ for storage operator $r \in R$ accounts for fueling the compressors so that $(1 - loss_r)$ is the effective injection amount. In a similar vein, $loss_a \in (0, 1)$ for pipeline arc a takes into account the compression fuel by the pipeline company as well as any gas lost to pipeline cracks. Constraint (3.1.38) stipulates that the aggregate extraction for the year equals the total injection after losses. Such balancing of the working gas is just one way to model these activities. In [39], alternative approaches were considered. We could also allow the injected gas remained in the storage for the future use in the following years. This can be done by adding a type of new variables for the leftover gas at the end of each year and making appropriate modification to the constraint (3.1.38). However, Assumption 3.1.2, the positive marginal cost functions assumption, ensures that all injected gas will be cleared at the end of each year by Theorem 3.1.7. Therefore, constraint (3.1.38) suffices for our purposes.

Constraints (3.1.39) and (3.1.40) provide upper bounds on the extraction and injection rates, respectively. Constraint (3.1.41) states the upper bound on the working gas volume.

Assuming that $c_r^{ST}(\cdot)$ is convex and continuously differentiable, the KKT conditions presented from (3.1.42) to (3.1.47) are necessary and sufficient for optimality of (ST) since the objective function is concave and the feasible region is polyhedral [5].

$$(ST) \quad \max \quad \sum_{r\in R}\sum_{y\in Y}\Bigg\{\sum_{s=2,3} days_s \gamma_{n^r(r)sy} x_{rsy} - days_1\Big[c_r^{ST}\Big(\sum_{a\in A(n^r(r))} g_{ary}\Big) + \sum_{a\in A(n^r(r))}\big(\tau_{a1y}+\tau_{a1y}^{reg}+\pi_{n2(a)1y}\big)\, g_{ary}\Big]\Bigg\} \tag{3.1.37}$$

$$\text{s.t.} \quad days_1 \sum_{a\in A(n^r(r))} g_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x_{rsy} = 0 \qquad (\delta_{ry} \text{ free}) \quad \forall r,y \tag{3.1.38}$$

$$x_{rsy} \le \overline{x}_r \qquad (\omega_{rsy}\ge 0) \quad \forall r, s=2,3,y \tag{3.1.39}$$

$$\sum_{a\in A(n^r(r))} g_{ary} \le \overline{g}_r \qquad (\xi_{ry}\ge 0) \quad \forall r,y \tag{3.1.40}$$

$$\sum_{s=2,3} days_s x_{rsy} \le \overline{k}_r \qquad (\zeta_{ry}\ge 0) \quad \forall r,y \tag{3.1.41}$$

$$0 \le g_{ary}, \forall a\in A(n^r(r)), x_{r2y}, x_{r3y} \qquad \forall r,y$$

The KKT conditions to (ST) are:

$$0 \le days_s\big(-\gamma_{n^r(r)sy}+\delta_{ry}+\zeta_{ry}\big)+\omega_{rsy} \perp x_{rsy} \ge 0 \quad \forall r, s=2,3,y \tag{3.1.42}$$

$$0 \le days_1 \Big(\tau_{a1y} + \tau^{reg}_{a1y} + \pi_{n_2(a)1y} + \frac{\partial c^{ST}_r \Big(\sum\limits_{a \in A(n^r(r))} g_{ary} \Big)}{\partial g_{ary}} - \delta_{ry}(1 - loss_a)(1 - loss_r) \Big)$$

$$+ \xi_{ry} \perp g_{ary} \ge 0 \qquad \forall a \in A(n^r(r)), r, y \qquad (3.1.43)$$

$$0 = days_1 \sum_{a \in A(n^r(r))} g_{ary}(1 - loss_a)(1 - loss_r) - \sum_{s=2,3} days_s x_{rsy} \quad (\delta_{ry} \text{ free}) \quad \forall r, y$$

$$(3.1.44)$$

$$0 \le \overline{x}_r - x_{rsy} \perp \omega_{rsy} \ge 0 \qquad \forall r, s = 2, 3, y \qquad (3.1.45)$$

$$0 \le \overline{g}_r - \sum_{a \in A(n^r(r))} g_{ary} \perp \xi_{ry} \ge 0 \quad \forall r, y \qquad (3.1.46)$$

$$0 \le \overline{k}_r - \sum_{s=2,3} days_s x_{rsy} \perp \zeta_{ry} \ge 0 \quad \forall r, y \qquad (3.1.47)$$

An important property as shown below regarding the cost functions of the storage operators is needed for the further analysis. We first use MC^{ST}_{ary} to denote the marginal cost functions for storage operators.

$$MC^{ST}_{ary} \equiv \frac{\partial c^{ST}_r \big(\sum\limits_{a \in A(n^r(r))} g_{ary} \big)}{\partial g_{ary}} \quad \forall a \in A(n^r(r)), r, y \qquad (3.1.48)$$

Assumption 3.1.2. *Given $a \in A(n^r(r))$, s and y, the marginal cost function of storage operator $r \in R$ satisfies the following condition:*

$$MC^{ST}_{ary} > 0, \quad \text{if } g_{ary} > 0. \qquad (3.1.49)$$

Theorem 3.1.6 shows that under Assumption 3.1.2 the storage prices are positive provided that there exists a positive extraction rate at the same location .

Theorem 3.1.6. *Suppose that Assumption 3.1.2 holds for all storage operators located at consumption node n. If there exists a storage operator $r \in R^n$ who has a positive extraction rate in season s, i.e., $x_{rsy} > 0$, then the storage gas price for that season at node n is positive too, that is, $\gamma_{n^r(r)sy} > 0$.*

Proof. By (3.1.44), a positive extraction rate implies a positive injection rate in season 1. Consequently, by (3.1.43), it follows that for some $a \in A(n^r(r))$,

$$\delta_{ry}(1 - loss_a)(1 - loss_r) = \tau_{a1y} + \tau_{a1y}^{reg} + \pi_{n_2(a)1y} + MC_{ary}^{ST} + \frac{\xi_{ry}}{days_1}$$

Among terms in the above equation, $MC_{ary}^{ST} > 0$ because of Assumption 3.1.2; $f_{a1y} > 0$ by market-clearing condition (3.1.12) showing by Theorem 3.1.3 $\tau_{a1y} \geq 0$; $\pi_{n_2(a)1y} \geq 0$ by definition; τ_{a1y}^{reg} is a predetermined positive parameter. Hence, δ_{ry} must be positive. By complementarity, we also have $\gamma_{n^r(r)sy} = \delta_{ry} + \zeta_{ry} + \omega_{rsy}/days_s$ from (3.1.42). Both ζ_{ry} and ω_{rsy} are nonnegative by their definitions. Thus $\gamma_{n^r(r)sy} > 0$. □

We note that KKT conditions to (ST) could be expressed as an instance of a pure NCP if (3.1.44) could be substituted for the following:

$$0 \leq days_1 \sum_{a \in A(n^r(r))} g_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x_{rsy} \perp \delta_{ry} \geq 0 \quad \forall r, y \tag{3.1.50}$$

Theorem 3.1.7 shows the validation for such substitution when the exogenous storage price $\gamma_{n^r(r)sy} \geq 0$. In the overall NCP to be described, this nonnegativity is enforced. For ease of presentation, we define *ST-NCP* the new system consisting of (3.1.42)-(3.1.43), (3.1.45)-(3.1.47) and (3.1.50). That is,

$$ST\text{-}NCP \equiv \begin{cases} (3.1.42) - (3.1.43), (3.1.45) - (3.1.47) \\ (3.1.50) \end{cases} \tag{3.1.51}$$

Theorem 3.1.7. *Suppose Assumption 3.1.2 is in force for all* $r \in R$*. Considering model* D-NGEM *in its entirety, the KKT conditions to* (ST)*, (3.1.42)-(3.1.47), are equivalent to system ST-NCP.*

Proof. First, we show that any solution of (3.1.42)-(3.1.47) is a solution of *ST-NCP*. It suffices to show that $\delta_{ry} \geq 0$ is always true for all r, y in (3.1.42)-(3.1.47). We have two cases for discussion:

Case 1: For a storage operator r, if $\sum_{a \in A(n^r(r))} g_{ary}(1 - loss_a)(1 - loss_r) = \sum_{s=2,3} days_s x_{rsy} = 0$, which, by (3.1.45) and (3.1.47), implies respectively, that $\omega_{rsy} = 0$ and $\zeta_{ry} = 0$, then it must follow that $\delta_{ry} \geq 0$ by (3.1.42) and the premise of the nonnegativity of $\gamma_{n^r(r)sy}$.

Case 2: For a storage operator r, if $\sum_{a \in A(n^r(r))} g_{ary}(1 - loss_a)(1 - loss_r) = \sum_{s=2,3} days_s x_{rsy} > 0$, then there must exist some $a \in A(n^r(r))$ such that $g_{ary} > 0$ so that following the proof for Theorem 3.1.6, we know that δ_{ry} must be positive. This completes the first part of the proof.

Now we show that any solution of *ST-NCP* is a solution of (3.1.42)-(3.1.47). Assume the contrary. This can only be the case if a solution of *ST-NCP* exists with $days_1 \sum_{a \in A(n^r(r))} g_{ary}(1 - loss_a)(1 - loss_r) - \sum_{s=2,3} days_s x_{rsy} > 0$ for some r, y, which, by (3.1.50), implies that $\delta_{ry} = 0$. Further, we have $days_1 \sum_{a \in A(n^r(r))} g_{ary}(1 - loss_a)(1 - loss_r) > 0$, by the nonnegativity of x_{rsy}. It follows that $g_{ary} > 0$ for some $a \in A(n^r(r)), r$ and $y \in Y$. Hence, by (3.1.43), $days_1\Big(\tau_{a1y} + \tau^{reg}_{a1y} + \pi_{n_2(a)1y} + MC^{ST}_{ary} - \delta_{ry}(1 - loss_a)(1 - loss_r)\Big) + \xi_{ry} = 0$ for that a, r and y, which cannot hold unless $MC^{ST}_{ary} = 0$. However, this contradicts Assumption 3.1.2. Therefore, the KKT conditions to problem (ST) are equivalent to system *ST-NCP*. $\square$

Theorem 3.1.7 states that, in order to get maximum profits, storage operators will not leave any injected gas that has cost them a positive amount unsold at the end of the year. As a result, *ST-NCP* is equivalent to an NCP as follows:

$$0 \le v^{ST} \perp H^{ST}(v^{ST}) \ge 0 \tag{3.1.52}$$

where

$$v^{ST} \equiv \begin{pmatrix} x_{rsy} \quad (\forall r, s=2,3,y) \\ g_{ary} \quad (\forall a \in A(n^r(r)), r, y) \\ \delta_{ry} \quad (\forall r, y) \\ \omega_{rsy} \quad (\forall r, s=2,3,y) \\ \xi_{ry} \quad (\forall r, y) \\ \zeta_{ry} \quad (\forall r, y) \end{pmatrix} \text{ and} \tag{3.1.53}$$

$$H^{ST}(v^{ST}) \equiv \begin{pmatrix} days_s\big(-\gamma_{n^r(r)sy} + \delta_{ry} + \zeta_{ry}\big) + \omega_{rsy} \quad (\forall r, s=2,3,y) \\ days_1\Big(\tau_{a1y} + \tau^{reg}_{a1y} + \pi_{n_2(a)1y} + MC^{ST}_{ary} \\ \qquad - \delta_{ry}(1-loss_a)(1-loss_r)\Big) + \xi_{ry} \\ \qquad\qquad (\forall a \in A(n^r(r)), r, y) \\ days_1 \sum_{a \in A(n^r(r))} g_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x_{rsy} \\ \qquad\qquad (\forall r, y) \\ \overline{x}_r - x_{rsy} \quad (\forall r, s=2,3,y) \\ \overline{g}_r - \sum_{a \in A(n^r(r))} g_{ary} \quad (\forall r, y) \\ \overline{k}_r - \sum_{s=2,3} days_s x_{rsy} \quad (\forall r, y) \end{pmatrix} \tag{3.1.54}$$

The market-clearing conditions (3.1.55) for the storage gas market states that the aggregate supply of storage gas at a consumption node ($\sum_{r \in R^n} days_s x_{rsy}$)

equals the aggregate amount demanded by marketers located at the same node ($\sum_{m \in M^n} days_s u_{msy}$). The market-clearing conditions for the storage market are:

$$\sum_{r \in R^n} days_s x_{rsy} = \sum_{m \in M^n} days_s u_{msy} \qquad \forall n \in CN, s = 2, 3, y \tag{3.1.55}$$

The condition for the nonnegativity of storage gas prices γ_{nsy} has not been stated as part of the model. In order to do so, we make γ_{nsy} the dual variables to the revised market-clearing conditions (3.1.55) as was done for the production market.

$$0 \leq \sum_{r \in R^n} days_s x_{rsy} - \sum_{m \in M^n} days_s u_{msy} \perp \gamma_{nsy} \geq 0 \qquad \forall n \in CN, s = 2, 3, y \tag{3.1.56}$$

The following theorem shows that (3.1.56) is equivalent to a system of (3.1.55) and $\gamma_{nsy} \geq 0$, $\forall n \in CN, s = 2, 3, y$ when considered in the context of *ST-NCP*.

Theorem 3.1.8. *If Assumption 3.1.2 holds for all $r \in R$, then system ST-MCC is equivalent to system ST-MCC-NCP, where*

$$\textit{ST-MCC} \equiv \begin{cases} NCP(3.1.52) \\ (3.1.55) \\ \gamma_{nsy} \geq 0, \quad \forall n \in CN, s = 2, 3, y \end{cases} \tag{3.1.57}$$

$$\textit{ST-MCC-NCP} \equiv \begin{cases} NCP(3.1.52) \\ (3.1.56) \end{cases} \tag{3.1.58}$$

Proof. By construction, any solution satisfying *ST-MCC* also satisfies *ST-MCC-NCP*. Therefore, we must show that every solution to *ST-MCC-NCP* will be a solution to *ST-MCC*. Suppose the contrary that there exists a solution satisfying *ST-MCC-NCP* such that for some $n \in CN, s = 2, 3, y$:

$$0 < \sum_{r \in R^n} days_s x_{rsy} - \sum_{m \in M^n} days_s u_{msy} \text{ and } \gamma_{nsy} = 0 \tag{3.1.59}$$

From (3.1.59), it must follow that for some $n \in CN, s = 2, 3, y \in Y$:

$$0 < x_{rsy}, \ \exists r \in R^n \quad \text{and} \quad \gamma_{nsy} = 0 \tag{3.1.60}$$

However, $0 < x_{rsy}$ for some $r \in R^n$ in (3.1.60), by Theorem 3.1.6 implies that $\gamma_{nsy} > 0$ for the location of storage operator r is node n. However, this contradicts $\gamma_{nsy} = 0$ in (3.1.60). Consequently, every solution of *ST-MCC-NCP* is also a solution to *ST-MCC*. This completes the proof. □

3.1.4 Peak Gas Operator

The peak gas operator $p \in P$ at consumption node n supplies to marketers "peak gas", either LNG or propane/air mixtures, to service peak demand in the highest demand time of season 3. The peak gas market is assumed to be under perfect competition. Therefore, we label (PG) for the optimization problem for all perfectly competing peak gas operators $p \in P$ without spelling out individual problems for them for the reason stated in earlier sections. Peak gas operators are modeled as choosing the production rates w_{py} taking the prices of peak gas, $\beta_{n^p(p)y}$ as given, so as to maximize the net profits, which is the difference between yearly sales income $(days_3 \beta_{n^p(p)y} w_{py})$ and production costs $(days_3 c_p^{PG}(w_{py}))$ summed over years as shown in (3.1.61). We assume that the cost function $c_p^{PG}(\cdot)$ convex and continuously differentiable in order to have a convex program. Constraint (3.1.62) sets an upper bound on the decision variables w_{py}. Since this is a convex program, the KKT conditions shown in (3.1.63) and (3.1.64) are equivalent to solving the problem (PG).

$$(PG) \quad \max \quad \sum_{p\in P}\sum_{y\in Y} days_3\big(\beta_{n^p(p)y}w_{py} - c_p^{PG}(w_{py})\big) \tag{3.1.61}$$

$$\text{s.t.} \quad w_{py} \le \overline{w}_p \qquad (\sigma_{py}) \qquad \forall p,y \tag{3.1.62}$$

$$0 \le w_{py} \qquad \forall p,y$$

The KKT conditions to (PG) are:

$$0 \le days_3\Big(-\beta_{n^p(p)y} + \frac{d\big(c_p^{PG}(w_{py})\big)}{d(w_{py})}\Big) + \sigma_{py} \perp w_{py} \ge 0 \quad \forall p,y \tag{3.1.63}$$

$$0 \le \overline{w}_p - w_{py} \perp \sigma_{py} \ge 0 \qquad \forall p,y \tag{3.1.64}$$

Hereafter, for brevity, the marginal cost functions of peak gas operators are denoted as MC_{py}^{PG}. That is,

$$MC_{py}^{PG} \equiv \frac{d\big(c_p^{PG}(w_{py})\big)}{d(w_{py})}, \quad \forall p,y \tag{3.1.65}$$

Similar to the previous analysis, the KKT conditions for all peak gas operators $p \in P$ can be expressed equivalently as an NCP:

$$0 \le v^{PG} \perp H^{PG}(v^{PG}) \ge 0 \tag{3.1.66}$$

where

$$v^{PG} \equiv \begin{pmatrix} w_{py} & (\forall p,y) \\ \sigma_{py} & (\forall p,y) \end{pmatrix} \text{ and} \tag{3.1.67}$$

$$H^{PG}(v^{PG}) \equiv \begin{pmatrix} days_3\Big(- \beta_{n^p(p)y} + MC^{PG}_{py}\Big) + \sigma_{py} \quad (\forall p, y) \\ \overline{w}_p - w_{py} \quad (\forall p, y) \end{pmatrix} \tag{3.1.68}$$

Assuming positive cost function in Assumption 3.1.3, Theorem 3.1.9 establishes a relationship between production rates w_{py} and market prices $\beta_{n^p(p)y}$.

Assumption 3.1.3. *Given y, the cost function of peak gas operator $p \in P$ satisfies the following condition:*

$$MC^{PG}_{py} > 0, \quad \text{if } w_{py} > 0, \forall y. \tag{3.1.69}$$

Theorem 3.1.9. *Suppose that all the peak gas operators located at a consumption node n satisfy Assumption 3.1.3. If there exists a peak gas operator $p \in P^n$ who has a positive production rate, i.e., $w_{py} > 0$, then the peak gas price for the same time period at node n is positive too, that is, $\beta_{n^p(p)y} > 0$.*

Proof. From (3.1.63), with positive w_{py}, we have $\beta_{n^p(p)y} = MC^{PG}_{py} + \sigma_{py}/days_3$, which, by Assumption 3.1.3 implies the conclusion. □

The market-clearing conditions for the peak gas market stipulate that the total supply of the peak gas at a consumption node ($\sum_{p\in P^n} days_3 w_{py}$) equals the total demand of peak gas from marketers located at the same consumption node ($\sum_{m\in M^n} days_3 v_{my}$). The market-clearing conditions for the peak gas market are:

$$\sum_{p\in P^n} days_3 w_{py} = \sum_{m\in M^n} days_3 v_{my} \qquad \forall n \in CN, y \tag{3.1.70}$$

Similar to the previous discussion, we incorporate the nonnegative peak gas prices β_{ny} into the equilibrium model D-NGEM by the following NCP formulation:

$$0 \leq \sum_{p \in P^n} days_3 w_{py} - \sum_{m \in M^n} days_3 v_{my} \perp \beta_{ny} \geq 0 \qquad \forall n \in CN, y \tag{3.1.71}$$

Equation (3.1.71) is shown to be equivalent to the original formulation in Theorem 3.1.10 under Assumption 3.1.3.

Theorem 3.1.10. *If Assumption 3.1.3 holds for all $p \in P$, then system PG-MCC, is equivalent to system PG-MCC-NCP, where*

$$PG\text{-}MCC \equiv \begin{cases} NCP(3.1.66) \\ (3.1.70) \\ \beta_{ny} \geq 0, \forall n \in CN, y \end{cases} \tag{3.1.72}$$

$$PG\text{-}MCC\text{-}NCP \equiv \begin{cases} NCP(3.1.66) \\ (3.1.71) \end{cases} \tag{3.1.73}$$

Proof. By construction, any solution satisfying *PG-MCC* also satisfies *PG-MCC-NCP*. Therefore, we must show that every solution to *PG-MCC-NCP* will be a solution to *PG-MCC*. Suppose the contrary that there exists a solution satisfying *PG-MCC-NCP* such that for some $n \in CN, y$:

$$0 < \sum_{p \in P^n} days_3 w_{py} - \sum_{m \in M^n} days_3 v_{my} \text{ and } \beta_{ny} = 0 \tag{3.1.74}$$

From (3.1.74), it must follow that for some $n \in CN, y$:

$$0 < w_{py},\ \exists p \in P^n \quad \text{and} \quad \beta_{ny} = 0 \tag{3.1.75}$$

However, $0 < w_{py}$ for some $p \in P^n$ in (3.1.75), by Theorem 3.1.9 implies that $\beta_{ny} > 0$ for the location of peak gas operator r is node n. However, this contradicts $\beta_{ny} = 0$ in (3.1.75). Consequently, every solution of *PG-MCC-NCP* is a solution to *PG-MCC*. This completes the proof. $\square$

3.1.5 Marketer

Unlike participants described above, marketers are modeled as Nash-Cournot players in the "marketer" market while price-takers in other markets. The corresponding optimization problem for a marketer $m \in M$ is denoted $(\widetilde{MK})$ as shown below. Marketer $m \in M^n$ located at consumption node $n \in CN$, competes against other marketers at the same location (denoted $-m(n) \in M^n$) by determining the daily amount purchased from producers, storage operators and peak gas operators, denoted h_{amsy}, u_{msy} and v_{my}, respectively, and the amount sold to four end-user sectors $k \in K$, denoted by l_{kmsy}. All marketers at node $n \in CN$ face the same inverse demand functions $\theta_{kn^m(m)sy}(\cdot)$ corresponding to sector k, season s, year y at node $n^m(m)$, as shown in (3.1.76). As opposed to price-takers accepting market prices determined by "derived demand" equations, marketers can influence the market prices by varying their share of supply through the explicit inverse demand functions $\theta(l_{kmsy} + l^*_{k(-m(n))sy})$, where the notation "$*$" in $l^*_{k(-m(n))sy}$ indicates the "optimal" solutions from the other marketers, appearing in their objective functions so as to maximize net profits. The inverse demand functions $\theta(\cdot)$ are assumed to be nonnegative since if the natural gas prices for consumers were below zeros, no operator would make profits by providing the natural gas. Other marketers' supply shares $l^*_{k(-m(n))sy}$ are exogenous to marketer m, but variables in the large, overall model D-NGEM. The seasonal revenue is $(days_s \theta_{kn^m(m)sy}(\cdot) l_{kmsy})$ earned from sector k by marketer $m \in M$ at node $n \in CN$. The costs include commodity costs (i.e., $days_s \pi_{n_2(a)sy} h_{amsy}$, $days_s \gamma_{n^m(m)sy} u_{msy}$ or $days_s \beta_{n^m(m)y} v_{my}$ depending on the gas sources) and transportation charges which are the same as storage operators. The rates for commodity and transportation costs are fixed in $(\widetilde{MK})$ since marketers are price-takers in the transportation, production, storage, peak gas markets.

Constraints (3.1.77), (3.1.78) and (3.1.79) (for seasons 1,2 and 3, respectively) state that the amount available to end-users and the amount procured from upstream sides by the marketers must be consistent during all seasons.

Provided that the only nonlinear terms $\theta_{kn^m(m)sy}(l_{kmsy} + l^*_{k(-m(n))sy})l_{kmsy}$ in $(\widetilde{MK})$ are concave, the KKT conditions shown in (3.1.80)-(3.1.86) are both necessary and sufficient for solving $(\widetilde{MK})$. To this end, typical assumptions are that $\theta_{knsy}(\cdot)$ is a continuously differentiable and nonincreasing function and satisfies the inequality: $\theta'_{kn^m(m)sy}(l_{kmsy}) + l_{kmsy}\theta''_{kn^m(m)sy}(l_{kmsy}) \leq 0$, for all $l_{kmsy} \geq 0$. This condition was also used in [60] and [69] for the oligopolistic market. Murphy et al. in [56] improve this condition by showing an easily verified condition of nonincreasing of $\theta_{kn^m(m)sy}(l_{kmsy})$ and concavity of $l_{kmsy}\theta_{kn^m(m)sy}(l_{kmsy})$. Examples of inverse demand functions which satisfy this condition are when: (1) $\theta(\cdot)$ is affine and non-increasing; and (2) $\theta(\cdot)$ is concave and non-increasing.

$$
\begin{aligned}
(\widetilde{MK}) \quad \max \quad & \sum_{y \in Y} \Bigg[\sum_{k \in K} \sum_{s \in S} days_s \theta_{kn^m(m)sy}\Big(l_{kmsy} + l^*_{k(-m(n))sy}\Big) l_{kmsy} \\
& - \sum_{s \in S} \sum_{a \in A^m(m)} days_s \big(\tau_{asy} + \tau^{reg}_{asy} + \pi_{n_2(a)sy}\big) h_{amsy} \\
& - \sum_{s=2,3} days_s \gamma_{n^m(m)sy} u_{msy} - days_3 \beta_{n^m(m)y} v_{my} \Bigg]
\end{aligned} \tag{3.1.76}
$$

$$
\text{s.t.} \quad days_1 \Big(\sum_{a \in A(n^m(m))} (1 - loss_a) h_{am1y} - \sum_{k \in K} l_{km1y} \Big) = 0
$$

$$
(\phi_{m1y}) \quad \forall y \tag{3.1.77}
$$

$$days_2\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h_{am2y}+u_{m2y}-\sum_{k\in K}l_{km2y}\Big)=0$$

$$(\phi_{m2y}) \quad \forall y \qquad (3.1.78)$$

$$days_3\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h_{am3y}+u_{m3y}+v_{my}-\sum_{k\in K}l_{km3y}\Big)=0$$

$$(\phi_{m3y}) \quad \forall y \qquad (3.1.79)$$

$$0\le l_{kmsy} \qquad \forall k,s,y$$

$$0\le h_{amsy} \qquad \forall a\in A(n^m(m)),s,y$$

$$0\le u_{msy} \qquad \forall s=2,3,y$$

$$0\le v_{my} \qquad \forall y$$

The KKT conditions to $(\widetilde{MK})$ are:

$$0\le -days_s\Big[\frac{\partial\theta_{kn^m(m)sy}(l_{kmsy}+l^*_{k(-m(n))sy})}{\partial l_{kmsy}}l_{kmsy}-\theta_{kn^m(m)sy}(l_{kmsy}+l^*_{k(-m(n))sy})\Big]$$

$$+\,days_s\phi_{msy}\perp l_{kmsy}\ge 0 \quad \forall k,s,y \qquad (3.1.80)$$

$$0\le days_s\big[\tau_{asy}+\tau^{reg}_{asy}+\pi_{n_2(a)sy}-(1-loss_a)\phi_{msy}\big]\perp h_{amsy}\ge 0$$

$$\forall a\in A(n^m(m)),s,y \qquad (3.1.81)$$

$$0\le days_s\big(\gamma_{n^m(m)sy}-\phi_{msy}\big)\perp u_{msy}\ge 0 \qquad s=2,3,\forall y \qquad (3.1.82)$$

$$0\le days_3\big(\beta_{n^m(m)y}-\phi_{m3y}\big)\perp v_{my}\ge 0 \qquad \forall y \qquad (3.1.83)$$

$$0 = days_1\Big[\sum_{a\in A(n^m(m))}(1-loss_a)h_{am1y} - \sum_{k\in K} l_{km1y}\Big]$$

$$(\phi_{m1y} \text{ free}) \quad \forall y \qquad (3.1.84)$$

$$0 = days_2\Big[\sum_{a\in A(n^m(m))}(1-loss_a)h_{am2y} + u_{m2y} - \sum_{k\in K} l_{km2y}\Big]$$

$$(\phi_{m2y} \text{ free}) \quad \forall y \qquad (3.1.85)$$

$$0 = days_3\Big[\sum_{a\in A(n^m(m))}(1-loss_a)h_{am3y} + u_{m3y} + v_{my} - \sum_{k\in K} l_{km3y}\Big]$$

$$(\phi_{m3y} \text{ free}) \quad \forall y \qquad (3.1.86)$$

For brevity, hereafter we use MR_{kmsy} to denote the marginal revenue functions for marketers. That is,

$$MR_{kmsy} \equiv \frac{\partial\theta_{kn^m(m)sy}(l_{kmsy}+l^*_{k(-m)sy})}{\partial l_{kmsy}} l_{kmsy} + \theta_{kn^m(m)sy}(l_{kmsy}+l^*_{k(-m)sy})$$

$$\forall k,m,s,y \qquad (3.1.87)$$

Note that there is not an optimization problem that can simply represent the problems $(\widetilde{MK})$ for all $m \in M$ as is done for pipeline operators, producers, storage operator and peak gas operators due to marketers' oligopolistic behavior aspects. Still, it is well known that the Nash equilibrium problem can be formulated as an NCP/VI dating back to the early paper by Lions and Stampacchia [43]. Therefore, in this dissertation, we adopt the NCP/VI approach for the whole marketers' market problem (MK). In light of their mathematical structure, including the KKT conditions for all marketers $m \in M$ results in an MiCP as follows:

$$\begin{aligned} &G^{MK}(u^{MK}, v^{MK}) = 0, \quad u^{MK} \text{ free} \\ &0 \le v^{MK} \perp H^{MK}(u^{MK}, v^{MK}) \ge 0 \end{aligned} \qquad (3.1.88)$$

where

$$u^{MK} \equiv \begin{pmatrix} \phi_{msy} \quad (\forall m, s, y) \end{pmatrix} \tag{3.1.89}$$

$$v^{MK} \equiv \begin{pmatrix} l_{kmsy} \quad (\forall k, m, s, y) \\ h_{amsy} \quad (\forall a \in A(n^m(m)), m, s, y) \\ u_{msy} \quad (s = 2, 3, \forall m, y) \\ v_{my} \quad (\forall m, y) \end{pmatrix} \tag{3.1.90}$$

$$G^{MK}(u^{MK}, v^{MK}) \equiv \begin{pmatrix} days_1 \Big[\sum_{a \in A(n^m(m))} (1 - loss_a) h_{am1y} - \sum_{k \in K} l_{km1y} \Big] \\ (\forall m, y) \\ days_2 \Big[\sum_{a \in A(n^m(m))} (1 - loss_a) h_{am2y} + u_{m2y} \\ - \sum_{k \in K} l_{km2y} \Big] \quad (\forall m, y) \\ days_3 \Big[\sum_{a \in A(n^m(m))} (1 - loss_a) h_{am3y} + u_{m3y} + v_{my} \\ - \sum_{k \in K} l_{km3y} \Big] \quad (\forall m, y) \end{pmatrix} \tag{3.1.91}$$

$$H^{MK}(u^{MK}, v^{MK}) \equiv \begin{pmatrix} - days_s MR_{kmsy} + days_s \phi_{msy} \quad (\forall k, m, s, y) \\ days_s \Big[\tau_{asy} + \tau_{asy}^{reg} + \pi_{n_2(a)sy} - (1 - loss_a) \phi_{msy} \Big] \\ \Big(\forall a \in A(n^m(m)), m, s, y \Big) \\ days_s \Big(\gamma_{n^m(m)sy} - \phi_{msy} \Big) \quad (s = 2, 3, \forall m, y) \\ days_3 \Big(\beta_{n^m(m)y} - \phi_{m3y} \Big) \quad (\forall m, y) \end{pmatrix} \tag{3.1.92}$$

3.1.6 NCP/VI Formulation of Model D-NGEM

From Sector 3.1.1 to Section 3.1.5, we presented all components of the model D-NGEM. In this section, we show that how model D-NGEM is equivalent to an MiCP. We first define the model D-NGEM mathematically as follows.

Definition 3.1.4. The model D-NGEM is a system composed of optimization problems (PL), (PR), (ST), (PG) and $(\widetilde{MK}), \forall m \in M$, market-clearing conditions (3.1.12) - (3.1.13), (3.1.28) - (3.1.29), (3.1.55) and (4.3.114) as well as nonnegative market price conditions, i.e., $\pi_{nsy} \geq 0, \forall n \in PN, s, y$; $\gamma_{nsy} \geq 0, \forall n \in CN, s = 2, 3, y$; $\beta_{ny} \geq 0, \forall n \in CN, y$. That is,

$$\text{D-NGEM} \equiv \begin{cases} (PL); (PR); (ST); (PG); (\widetilde{MK}), \forall m \in M, \\ (3.1.12) - (3.1.13); (3.1.28) - (3.1.29); (3.1.55); (3.1.70) \\ \pi_{nsy} \geq 0, \forall n \in PN, s, y \\ \gamma_{nsy} \geq 0, \forall n \in CN, s = 2, 3, y \\ \beta_{ny} \geq 0, \forall n \in CN, y \end{cases} \tag{3.1.93}$$

The KKT conditions to these optimization problems have been converted to NCPs, that is (3.1.11), (3.1.26), (3.1.52), (3.1.66) and (**??**), assuming cost functions are convex and continuously differentiable and the marginal costs and revenues are positive in the positive orthant. Under the same assumptions, market-clearing conditions were shown to in a format of an NCP with the exception of market-clearing conditions for the transportation market, which are shown to be a MiCP. Given that its components are either NCP or MiCP, Theorem 3.1.11 shows that the model D-NGEM is an MiCP per se. First, define some new terms for the market-clearing conditions.

$$(u^{MCC}) \equiv \big(\tau_{asy} \quad (\forall a, s, y) \quad \big) \quad , \tag{3.1.94}$$

$$(v^{MCC}) \equiv \begin{pmatrix} \pi_{nsy} \quad (\forall n \in PN, s, y) \\ \gamma_{nsy} \quad (\forall n \in CN, s = 2, 3, y) \\ \beta_{ny} \quad (\forall n \in CN, y) \end{pmatrix}, \tag{3.1.95}$$

$$G^{MCC} \equiv \begin{pmatrix} days_1 f_{a1y} - \sum_{r \in R(n_1(a))} days_1 g_{ary} - \sum_{m \in M(n_1(a))} days_1 h_{am1y} \quad (\forall a, y) \\ days_s f_{asy} - \sum_{m \in M(n_1(a))} days_s h_{amsy} \quad (\forall a, s = 2, 3, y) \end{pmatrix} \text{ and} \tag{3.1.96}$$

$$H^{MCC} \equiv \begin{pmatrix} \sum_{c \in C^n} days_1 q_{c1y} \\ \qquad - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g_{ary} + \sum_{m \in M(n_1(a))} days_1 h_{am1y} \right) \\ \qquad\qquad (\forall n \in PN, y) \\ \sum_{c \in C^n} days_s q_{csy} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_1 h_{amsy} \\ \qquad\qquad (\forall n \in PN, s = 2, 3, y) \\ \sum_{r \in R^n} days_s x_{rsy} - \sum_{m \in M^n} days_s u_{msy} \quad (\forall n \in CN, s = 2, 3, y) \\ \sum_{p \in P^n} days_3 w_{py} - \sum_{m \in M^n} days_3 v_{my} \quad (\forall n \in CN, y) \end{pmatrix} \tag{3.1.97}$$

Theorem 3.1.11. *Let*

$$u^T \equiv \big[(u^{MK})^T \quad (u^{MCC})^T \big];$$

$$v^T \equiv \big[(v^{PL})^T \quad (v^{PR})^T \quad (v^{ST})^T \quad (v^{PG})^T \quad (v^{MK})^T \quad (v^{MCC})^T \big];$$

$$G^T(u, v) \equiv \big[(G^{MK})^T \quad (G^{MCC})^T \big];$$

$$H^T(u, v) \equiv \big[(H^{PL})^T \quad (H^{PR})^T \quad (H^{ST})^T \quad (H^{PG})^T \quad (H^{MK})^T \quad (H^{MCC})^T \big].$$

Suppose that Assumptions 3.1.1, 3.1.2 and 3.1.3 hold for all $c \in C$, $r \in R$ and $p \in P$, respectively. Model D-NGEM is equivalent to an MiCP, denoted D-NGEM-MiCP(G, H) where

$$\begin{aligned} &G(u, v) = 0 \quad u \textit{ free} \\ &0 \leq v \perp H(u, v) \geq 0 \end{aligned} \tag{3.1.98}$$

Proof. Following the definition for MiCP, by Theorems 3.1.5, 3.1.7, 3.1.8 and 3.1.10, it is trivial to show the results. □

3.2 Existence Results

In what follows, we provide existence results for the model D-NGEM. In light of the relationship between the MiCP and VI demonstrated in Theorem 3.1.1, the D-NGEM-MiCP(G, H) can also be written as a VI denoted D-NGEM-VI$(\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+, F)$, where $F^T = (G^T, H^T)$ and the values of n_1 and n_2 depend on the actual size of the problem. Theorem 3.2.1 shown below is a well-known existence result for VI problems. The existence results will be established in this sector are bases on this Theorem as well.

Theorem 3.2.1. [16, 43] *Let X be a nonempty, compact and convex subset of $\mathbb{R}^n$ and let F be a continuous mapping from X into $\mathbb{R}^n$. Then, there exists a solution to the problem* VI(X, F).

Clearly, the functions G and H in MiCP (3.1.98) are continuous given the functional form and the earlier assumptions that the cost functions were continuously differentiable. The corresponding set X for MiCP (3.1.98) is certainly nonempty, convex and closed, since all variables are constrained to be nonnegative or free.

Thus, to invoke Theorem 3.2.1, it suffices to show that the ground set X is bounded. Lemma 3.2.1 and Lemma 3.2.2 as follows show that under suitable assumptions, all variables for D-NGEM-MiCP(G, H) are bounded.

Assumption 3.2.1. *All the pipeline congestion fees τ_{asy} are bounded below when $f_{asy} = 0$.*

Assumption 3.2.1 relaxes Assumption 1 in [35]. In particular, prices π_{cst}, γ_{nsy} and β_{ny} were shown to be bounded below in Section 3.1 using assumptions relative to cost functions instead of direct assumptions made in [35]. However, such relaxation cannot be done to pipeline price τ_{asy} due to the fact that there is not a cost function in the problem (PL). These prices including τ_{asy} will be shown to be bounded above in Lemma 3.2.1. In fact, Assumption 3.2.1 along with Lemma 3.2.1 plays the same role as Assumption 1 in [35].

Lemma 3.2.1. *If Assumption 3.2.1 holds, all the prices in* D-NGEM-MiCP(G, H) *are bounded. That is, there exists a positive scalar Δ such that*

a. $\tau_{asy} \in [-\Delta, \Delta], \forall a, s, y$

b. $\pi_{nsy} \in [0, \Delta], \forall n \in PN, s, y$

c. $\gamma_{nsy} \in [0, \Delta], s = 2, 3, \forall n \in CN, y$

d. $\beta_{ny} \in [0, \Delta], \forall n \in CN, y$

Proof. Because the cost functions $c_c^{PR}(\cdot)$, $c_r^{ST}(\cdot)$ and $c_p^{PG}(\cdot)$ and the revenue function $\theta(\cdot)l$ were assumed continuously differentiable in Section 3.1, the marginal costs, that is, MC_{csy}^{PR}, MC_{ary}^{ST} and MC_{py}^{PG} and marginal revenues MR_{kmsy} are continuous. Also, it is well known that if a function f is continuous on a bounded and closed (i.e., compact) set S, f is bounded on S as a result of *Weirstrass's Theorem* [5]. The following statements are true:

$$a.\quad MC^{PR}_{csy} \text{ is bounded over } q_{csy} \in [0, \overline{q}_c], \forall c, s, y, \tag{3.2.1}$$

$$b.\quad MC^{ST}_{rsy} \text{ is bounded over } g_{ary} \in [0, \overline{g}_r], \forall a \in A(n^r(r)), r, y, \tag{3.2.2}$$

$$c.\quad MC^{PG}_{py} \text{ is bounded over } w_{py} \in [0, \overline{w}_p], \forall p, y, \text{ and} \tag{3.2.3}$$

$$d.\quad MR_{kmsy} \text{ is bounded over } l_{kmsy} \in [0, L], \forall k, m, s, y, \text{where } L \text{ is some nonnegative scalar.} \tag{3.2.4}$$

Clearly, the prices π_{nsy}, γ_{nsy} and β_{ny} are bounded below by zero by definition and prices τ_{asy} are bounded below when $f_{asy} = 0$ and $f_{asy} > 0$ by Assumption 3.2.1 and Theorem 3.1.3, respectively. It suffices to show that these price variables are bounded above.

First consider the production prices π_{nsy} at production node n. Suppose all producers located at node n, that is, $c \in C^n$ have $q_{csy} = 0$ for all s, y. By (3.1.21) and (3.1.22), we obtain that $\lambda_{csy} = 0$ and $\mu_c = 0$. Further, we have $\pi_{n^c(c)sy} \leq MC^{PR}_{csy}$ from (3.1.20), which implies π_{nsy} is bounded above because of the statement (3.2.1). Second, given s, y, if some producer $c \in C^n$ has $q_{csy} > 0$, we consider two cases implied by the market-clearing conditions (3.1.28) and (3.1.29): 1) $h_{amsy} > 0$ for some marketer m located at consumption node $n_1(a)$, where $a \in A(n)$; and thus implied by (3.1.77)-(3.1.79), $l_{kmsy} > 0$ for some k of that marketer m; 2) when $s = 1$, $g_{ary} > 0$ for some storage operator located at consumption node $n_1(a)$, where $a \in A(n)$; and thus $x_{rsy} > 0$ for that r, where $s = 2$ or 3. In the first case, by (3.1.80) and $l_{kmsy} > 0$, it must follow that for some a, k, s, y

$$MR_{kmsy} = \phi_{msy} \tag{3.2.5}$$

where ϕ_{msy} is bounded above because MR_{kmsy} is bounded via (3.2.4). Meanwhile, by (3.1.81) and $h_{amsy} > 0$, we see that $\tau_{asy} + \tau^{reg}_{asy} + \pi_{n_2(a)sy} = \phi_{msy}(1 - loss_a)$, which implies that the production prices $\pi_{n_2(a)sy}$, where $n_2(a)$ refers to where producer c is located, and τ_{asy} are bounded above because ϕ_{msy} is bounded. Therefore, π_{nsy} for the node where producer c is located must be bounded above.

In the second case for proving π_{nsy} is bounded above we see that $x_{rsy} > 0$ when $s = 2$ or 3 implies that $u_{msy} > 0$ for some marketer m located at the same node where storage operator r is, by market-clearing condition (3.1.55). By (3.1.78) or (3.1.79) depending on the actual season in question, $u_{msy} > 0$ implies that $l_{kmsy} > 0$ for some k, which indicates ϕ_{msy} is bounded via Assumption (3.2.4). From (3.1.82) and $u_{msy} > 0$, we know that $\gamma_{n^m(m)sy} = \phi_{msy}$, which means that $\gamma_{n^m(m)sy}$ for the node where storage operator r and marketer m are co-located is bounded. Also, by (3.1.42) and $x_{rsy} > 0$, it follows that δ_{ry} for that storage operator r is bounded above since $\gamma_{n^r(r)sy}$ is bounded and ω_{rsy} and ζ_{ry} are nonnegative. Further by (3.1.43) and $g_{ary} > 0$, we see that

$$\delta_{ry}(1 - loss_a)(1 - loss_r) = \tau_{a1y} + \tau^{reg}_{a1y} + \pi_{n_2(a)1y} + MC^{ST}_{ary} + \frac{\xi_{ry}}{days_1} \tag{3.2.6}$$

In the above equation, because δ_{ry} is bounded above and ξ_{ry} is bounded below, MC^{ST}_{ary} is bounded and $\gamma_{n^r(r)sy}$ is bounded, then $\pi_{n_2(a)1y}$ and τ_{a1y} must be bounded above. In other words, the production price for node $n_2(a)$ or the production node in question n is bounded above. Therefore, the production prices π_{nsy} are bounded above for all $n \in PN, s, y$.

Second, we consider the pipeline prices τ_{asy} for arc a in season s and year y. We already showed τ_{asy} bounded below, now we show bounded above. Consider the case when $f_{asy} = 0$. In this case, by complementarity, (3.1.8) requires $\rho_{asy} = 0$. We see by (3.1.7) that $days_s \tau_{asy} \leq \rho_{asy}$. Thus, τ_{asy} is bounded above. When $f_{asy} > 0$, by market-clearing conditions (3.1.12) and (3.1.13), we consider two cases 1) when $s = 1$, $g_{ary} > 0$ for some storage operator r located at node $n_1(a)$ or 2) $h_{amsy} > 0$ for some marketers located at node $n_1(a)$. Following the two cases discussed above when $\pi_{nsy} > 0$, it is not difficult to deduce that τ_{asy} is bounded above. Therefore, τ_{asy} is bounded above for all a, s, y.

As for storage gas prices, γ_{nsy} was shown to be bounded above when some $g_{ary} > 0$ in the discussion of the boundness of the production price π_{nsy}. We now show the boundness results when $g_{ary} = 0$ for all $a \in A(n), y$ and all storage operators $r \in R^n$. Consider a consumption node $n \in CN$. By (3.1.43), we have

$$\delta_{ry}(1 - loss_a)(1 - loss_r) \leq \tau_{a1y} + \tau_{a1y}^{reg} + \pi_{n_2(a)1y} + MC_{ary}^{ST} + \frac{\xi_{ry}}{days_s} \qquad \forall a \in A(n), r \in R^n, y \tag{3.2.7}$$

which implies δ_{ry} is bounded above because: 1) τ_{a1y} has shown to be bounded; 2) τ_{a1y}^{reg} is a positive input; 3) $\pi_{n_1(a)1y}$ has shown to be bounded; 4) MC_{ary}^{ST} is bounded via (3.2.2); 5) by (3.1.44), (3.1.45), (3.1.46) and (3.1.47), we obtain that $x_{rsy} = 0$, $\omega_{rsy} = 0$ showing that $\xi_{ry} = 0$ and $\zeta_{ry} = 0$, respectively. Further, by (3.1.42), $\gamma_{n^r(r)sy} \leq \delta_{ry}$, which implies that $\gamma_{n^r(r)sy}$ is bounded above by δ_{ry}. Thus γ_{nsy} is bounded above when $g_{ary} = 0$ for all storage operators $r \in R^n$ and all arcs $a \in A(n)$ in all year y. Next, suppose $g_{ary} > 0$ for some storage operators $r \in R^n$ and some arc $a \in A(n)$. Therefore, γ_{nsy} is bounded above for all $n, s = 2, 3, y$.

In terms of the peak gas prices β_{ny} for a consumption node $n \in CN$, if all peak gas operators located at node n have $w_{py} = 0$, the corresponding $\sigma_{py} = 0$ by (3.1.64). Therefore by (3.1.63), $\beta_{n^p(p)y} \leq MC_{py}^{PG}$ thus β_{py} is bounded by statement (3.2.3). If $w_{py} > 0$ for some peak gas operator $p \in P^n$, then by (3.1.70) and (3.1.79), there exists at least one marketer $m \in M^n$ who has $v_{my} > 0$ and thus the corresponding $l_{km3y} > 0$ for some k. $l_{km3y} > 0$ implies that ϕ_{m3y} is bounded above by statement (3.2.4) as shown in (3.2.5). Also, $v_{my} > 0$ implies $\beta_{ny} = \phi_{m3y}$ by (3.1.83). Thus β_{ny} are bounded above by ϕ_{m3y} when $w_{py} > 0$. Therefore, β_{ny} is bounded above for all n, y. □

Lemma 3.2.2. [35] *If all prices (τ_{asy}, π_{nsy}, γ_{nsy}, β_{ny}) are bounded, all the variables in* D-NGEM-MiCP(G, H) *are bounded.*

Proof. In [35], it was showed that all variables for optimization problems (PL), (PR), (ST) and (PG) were bounded.

As for the problem (MK), h_{asmy}, u_{msy} and v_{my} were shown to be bounded in [35]. We see via (3.1.84), (3.1.85), (3.1.86) and the fact that the h_{amsy}, u_{msy} and v_{my} have each shown to be bounded, that the variables l_{kmsy} are bounded. Multiplier variables ϕ_{msy} are bounded below by MR_{kmsy} via (3.1.80) as well as (3.2.4). Equation (3.1.80), (3.1.81) or (3.1.82) shows that ϕ_{msy} is bounded above by appropriate prices shown or assumed (for τ_{asy}) bounded. This completes the proof showing that all the variables are bounded. □

Using Theorem 3.2.1, Lemmas 3.2.1 and 3.2.2, we obtain the following existence result.

Theorem 3.2.2. *If Assumptions (3.2.1) holds, then there exists a solution to D-NGEM-MiCP(G, H).*

The conclusion of Theorem 3.2.2 is straightforward based on the previous analysis. Moreover, if Theorem 3.1.11 holds, Theorem 3.2.2 shows that a solution to the model D-NGEM exists. Specifically, if Assumptions 3.1.1, 3.1.2, 3.1.3 and 3.2.1 are in force, a solution to the model D-NGEM always exists.

3.3 Uniqueness Results

In this section, we consider the uniqueness of solutions to D-NGEM. Two definitions regarding monotonicity are introduced first.

Definition 3.3.1. [26] The mapping $F : R^n \rightarrow R^n$ is said to be

a. *monotone over* X, if

$$[F(x) - F(y)]^T(x - y) \geq 0 \quad \forall x, y \in X; \tag{3.3.1}$$

b. *strictly monotone over* X, if

$$[F(x) - F(y)]^T(x - y) > 0 \quad \forall x, y \in X, x \neq y. \tag{3.3.2}$$

Among these properties, it is clear that every strictly monotone function must be a monotone function but not necessary the reverse. More generally, if F is continuously differentiable, then the various monotonicity properties of F are related to the positive semi-definiteness or positive definiteness of the Jacobian matrix $\nabla F(x)$ [63].

In general, VI(X, F) can have more than one solution. However, if F is strictly monotone, then VI(X, F) can have at most one solution as shown in the following result.

Theorem 3.3.1. [26] *If F is strictly monotone on X, then the problem* VI(X, F) *has at most one solution.*

Theorem 3.3.1 does not guarantee the existence of a solution to problem VI(X, F). However, it can be used for uniqueness result given existence results such as Theorem 3.2.2. The following theorem shows the uniqueness conditions to the model D-NGEM-MiCP(G, H).

Theorem 3.3.2. *If Theorem 3.2.2 holds, and*

a. $MC^{PR}_{csy}(\cdot)$ *are strictly increasing functions over the nonnegative orthant for all* $c \in C, s \in S$ *and* $y \in Y$,

b. $MC^{PG}_{py}(\cdot)$ *are strictly increasing functions over the nonnegative orthant line for all* $p \in P$ *and* $y \in Y$,

c. $MC^{ST}_{ry}(\cdot) \equiv \left[\cdots, MC^{ST}_{ary}(\cdot), \cdots\right]^T$, *where* $a \in A(n^r(r))$, *are strictly monotone functions over nonnegative orthant for all* $r \in R$ *and* $y \in Y$,

d. $-MR_{knsy}(\cdot) \equiv \left[\cdots, -MR_{kmsy}(\cdot), \cdots\right]^T$, *where* $m \in M^n$, *are strictly monotone functions over nonnegative orthant for all* $k \in K$, $n \in CN$, $s \in S$ *and* $y \in Y$,

then D-NGEM-MiCP(G, H) *has a unique solution.*

Proof. Theorem 3.2.2 ensures a solution to *D-NGEM-MiCP*(G, H). For the uniqueness of the solution, it is known that $F^T = (G^T \quad H^T)$ being a strictly monotone function will suffice by Theorem 3.3.1. Our immediate goal is to show that $[F(x) - F(y)]^T(x - y) > 0$ for $x \neq y$. For brevity, we use "Δ" to denote the differences between x and y so that we have the following:

$$
x - y \equiv \begin{pmatrix} \Delta u^{MCC} \\ \Delta v^{PL} \\ \Delta v^{PR} \\ \Delta v^{ST} \\ \Delta v^{MK} \\ \Delta u^{MK} \\ \Delta v^{MCC} \end{pmatrix} \equiv \left(\begin{array}{ll}
\left.\Delta\tau_{asy} \quad (\forall a, s, y)\right\} & (\Delta u^{MCC}) \\
\left.\begin{array}{l} \Delta f_{asy} \quad (\forall a, s, y) \\ \Delta\rho_{asy} \quad (\forall a, s, y) \end{array}\right\} & (\Delta v^{PL}) \\
\left.\begin{array}{l} \Delta q_{csy} \quad (\forall c, s, y) \\ \Delta\lambda_{csy} \quad (\forall c, s, y) \\ \Delta\mu_{c} \quad (\forall c) \end{array}\right\} & (\Delta v^{PR}) \\
\left.\begin{array}{l} \Delta x_{rsy} \quad (\forall r, s, y) \\ \Delta g_{ary} \quad (\forall a \in A(n^{r}(r)), r, y) \\ \Delta\delta_{ry} \quad (\forall r, y) \\ \Delta\omega_{rsy} \quad (s = 2, 3, \forall r, y) \\ \Delta\xi_{ry} \quad (\forall r, y) \\ \Delta\zeta_{ry} \quad (\forall r, y) \end{array}\right\} & (\Delta v^{ST}) \\
\left.\begin{array}{l} \Delta w_{py} \quad (\forall p, y) \\ \Delta\sigma_{py} \quad (\forall p, y) \\ \Delta l_{kmsy} \quad (\forall k, m, s, y) \\ \Delta h_{amsy} \quad (\forall a \in A(n^{m}(m)), m, s, y) \\ \Delta u_{msy} \quad (s = 2, 3, \forall m, y) \\ \Delta v_{my} \quad (\forall m, y) \end{array}\right\} & (\Delta v^{MK}) \\
\left.\Delta\phi_{msy} \quad (\forall m, s, y)\right\} & (\Delta u^{MK}) \\
\left.\begin{array}{l} \Delta\pi_{nsy} \quad (\forall n \in PN, s, y) \\ \Delta\gamma_{nsy} \quad (\forall n \in CN, s, y) \\ \Delta\beta_{ny} \quad (\forall n \in CN, y) \end{array}\right\} & (\Delta v^{MCC})
\end{array} \right)
$$

Spelling out all terms for $[F(x)-F(y)]^T(x-y)$, what we obtain is as follows:

$$
\begin{aligned}
[F(x)-F(y)]^T(x-y) =& \big[G^{MCC}(x)-G^{MCC}(y)\big]^T \Delta u^{MCC} \\
&+ \big[H^{PL}(x)-H^{PL}(y)\big]^T \Delta v^{PL} \\
&+ \big[H^{PR}(x)-H^{PR}(y)\big]^T \Delta v^{PR} \\
&+ \big[H^{ST}(x)-H^{ST}(y)\big]^T \Delta v^{ST} \\
&+ \big[H^{PG}(x)-H^{PG}(y)\big]^T \Delta v^{PG} \\
&+ \big[H^{MK}(x)-H^{MK}(y)\big]^T \Delta v^{MK} \\
&+ \big[G^{MK}(x)-G^{MK}(y)\big]^T \Delta u^{MK} \\
&+ \big[H^{MCC}(x)-H^{MCC}(y)\big]^T \Delta v^{MCC}
\end{aligned}
$$

Based on the definitions in Section 3.1, individual terms in the above equality are laid out in the following detail:

$$
\begin{aligned}
&\big[H^{PL}(x)-H^{PL}(y)\big]^T \Delta v^{PL} \\
&\quad= \sum_{a,s,y} (-days_s \Delta\tau_{asy} + \Delta\rho_{asy})\Delta f_{asy} - \sum_{a,s,y} \Delta f_{asy}\Delta\rho_{asy} \\
&\quad= \sum_{a,s,y} -days_s \Delta\tau_{asy}\Delta f_{asy}
\end{aligned}
\tag{3.3.3}
$$

$$
\begin{aligned}
&\big[H^{PR}(x)-H^{PR}(y)\big]^T \Delta v^{PR} \\
&\quad= \sum_{c,s,y} \Big(days_s\big(-\Delta\pi_{n^c(c)sy} + \Delta MC^{PR}_{csy} + \Delta\mu_c\big) + \Delta\lambda_{csy}\Big)\Delta q_{csy} \\
&\qquad - \sum_{c,s,y} \Delta q_{csy}\Delta\lambda_{csy} - \sum_{c,s,y} days_s \Delta q_{csy}\Delta\mu_c \\
&\quad= \sum_{c,s,y} days_s\big(-\Delta\pi_{n^c(c)sy} + \Delta MC^{PR}_{csy}\big)\Delta q_{csy}
\end{aligned}
\tag{3.3.4}
$$

$$
\begin{aligned}
&\big[H^{ST}(x) - H^{ST}(y)\big]^T \Delta v^{ST} \\
&\quad = \sum_{r,s=2,3,y} \Big(days_s\big(- \Delta\gamma_{n^r(r)sy} + \Delta\delta_{ry} + \Delta\zeta_{ry}\big) + \Delta\omega_{rsy}\Big)\Delta x_{rsy} \\
&\qquad + \sum_{a\in A(n^r(r)),r,y} \Big(days_1\big(\Delta\tau_{a1y} + \Delta\pi_{n_2(a)1y} + \Delta MC^{ST}_{ary} \\
&\qquad - \Delta\delta_{ry}(1-loss_a)(1-loss_r)\big) + \Delta\xi_{ry}\Big)\Delta g_{ary} \\
&\qquad + \sum_{r,y} \Big(days_1 \sum_{a\in A(n^r(r))} \Delta g_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s \Delta x_{rsy}\Big)\Delta\delta_{ry} \\
&\qquad - \sum_{r,s=2,3,y} \Delta x_{rsy}\Delta\omega_{rsy} - \sum_{a\in A(n^r(r)),r,y} \Delta g_{ary}\Delta\xi_{ry} - \sum_{r,s=2,3,y} days_s \Delta x_{rsy}\Delta\zeta_{ry} \\
&\quad = - \sum_{r,s=2,3,y} days_s \Delta\gamma_{n^r(r)sy}\Delta x_{rsy} \\
&\qquad + \sum_{a\in A(n^r(r)),r,y} days_1 \Big(\Delta\tau_{a1y} + \Delta\pi_{n_2(a)1y} + \Delta MC^{ST}_{ary}\Big)\Delta g_{ary}
\end{aligned}
\tag{3.3.5}
$$

$$
\begin{aligned}
&\big[H^{PG}(x) - H^{PG}(y)\big]^T \Delta v^{PG} \\
&\quad = \sum_{p,y} \big(- days_3(\Delta\beta_{n^p(p)y} + \Delta MC^{PG}_{py}) + \Delta\sigma_{py}\big)\Delta w_{py} \\
&\qquad - \sum_{p,y} \Delta w_{py}\Delta\sigma_{py} \\
&\quad = \sum_{p,y} days_3\big(- \Delta\beta_{n^p(p)y} + \Delta MC^{PG}_{py}\big)\Delta w_{py}
\end{aligned}
\tag{3.3.6}
$$

$$
\begin{aligned}
&\big[H^{MK}(x) - H^{MK}(y)\big]^T \Delta v^{MK} + \big[G^{MK}(x) - G^{MK}(y)\big]^T \Delta u^{MK} \\
&\quad = \sum_{k,m,s,y} days_s\big(- \Delta MR_{kmsy} + \Delta\phi_{msy}\big)\Delta l_{kmsy} \\
&\qquad + \sum_{a\in A(n^m(m)),m,s,y} days_s\big(\Delta\tau_{asy} + \Delta\pi_{n_2(a)sy} - (1-loss_a)\Delta\phi_{msy}\big)\Delta h_{amsy} \\
&\qquad + \sum_{s=2,3,m,y} days_s\big(\Delta\gamma_{n^m(m)sy} - \Delta\phi_{msy}\big)\Delta u_{msy} \\
&\qquad + \sum_{m,y} days_3\big(\Delta\beta_{n^m(m)y} - \Delta\phi_{m3y}\big)\Delta v_{my} \\
&\qquad + \sum_{a\in A(n^m(m)),m,s,y} days_s(1-loss_a)\Delta h_{amsy}\Delta\phi_{msy} \\
&\qquad + \sum_{m,s=2,3,y} days_s\Delta u_{msy}\Delta\phi_{msy} \\
&\qquad + \sum_{m,y} days_s\Delta v_{my}\Delta\phi_{my} - \sum_{k,m,s,y} days_s\Delta l_{kmsy}\Delta\phi_{msy} \\
&\quad = \sum_{k,m,s,y} -days_s\Delta MR_{kmsy}\Delta l_{kmsy} \\
&\qquad + \sum_{a\in A(n^m(m)),m,s,y} days_s\big(\Delta\tau_{asy} + \Delta\pi_{n_2(a)sy}\big)\Delta h_{amsy} \\
&\qquad + \sum_{s=2,3,m,y} days_s\Delta\gamma_{n^m(m)sy}\Delta u_{msy} + \sum_{m,y} days_3\Delta\beta_{n^m(m)y}\Delta v_{my}
\end{aligned}
\tag{3.3.7}
$$

$$
\begin{aligned}
&\left[G^{MCC}(x) - G^{MCC}(y)\right]^T \Delta u^{MCC} + \left[H^{MCC}(x) - H^{MCC}(y)\right]^T \Delta v^{MCC} \\
&\quad = \sum_{a,s,y} days_s \big(\Delta f_{asy} - \Delta h_{amsy}\big) \Delta\tau_{asy} - \sum_{a,r\in R(n_1(a)),y} days_1 \Delta g_{ary} \Delta\tau_{a1y} \\
&\qquad + \sum_{n\in PN,s,y} days_s \big(\sum_{c\in C^n} \Delta q_{csy} - \sum_{a\in A(n),m\in M(n_1(a))} \Delta h_{amsy} \big) \Delta\pi_{nsy} \\
&\qquad - \sum_{n\in PN,y} days_1 \big(\sum_{a\in A(n),r\in R(n_1(a),} \Delta g_{ary} \big) \Delta\pi_{nsy} \\
&\qquad + \sum_{n\in CN,s=2,3,y} days_s \big(\sum_{r\in R^n} \Delta x_{rsy} - \sum_{m\in M^n} \Delta u_{msy} \big) \Delta\gamma_{nsy} \\
&\qquad + \sum_{n\in CN,y} days_3 \big(\sum_{p\in P^n} \Delta w_{py} - \sum_{m\in M^n} \Delta v_{my} \big) \Delta\beta_{ny}
\end{aligned} \tag{3.3.8}
$$

Among those terms in the above equality, we can permute them and have:

$$
\begin{aligned}
&- \sum_{a,s,y} days_s \big(\Delta h_{amsy}\big) \Delta\tau_{asy} \\
&\quad = - \sum_{a\in A(n^m(m)),m,s,y} days_1 \Delta h_{amsy} \Delta\tau_{asy} \qquad \big(\because \sum_{a} = \sum_{a\in A(n^m(m)),m} \big)
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{a,r\in R(n_1(a)),y} days_1 \Delta g_{ary} \Delta\tau_{a1y} \\
&\quad = - \sum_{a\in A(n^r(r)),r,y} days_1 \Delta g_{ary} \Delta\tau_{a1y} \qquad \big(\because \sum_{a,r\in R(n_1(a)))} = \sum_{a\in A(n^r(r)),r} \big)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n\in PN,s,y} days_s \big(\sum_{c\in C^n} \Delta q_{csy} \big) \Delta\pi_{nsy} \\
&\quad = \sum_{n\in PN,s,y} days_s \big(\sum_{c\in C^n} \Delta q_{csy} \Delta\pi_{n^c(c)sy} \big) \quad \big(\because \Delta\pi_{nsy} = \Delta\pi_{n^c(c)sy}, \forall c \in C^n \big) \\
&\quad = \sum_{c,s,y} days_s \big(\Delta q_{csy} \Delta\pi_{n^c(c)sy} \big) \qquad \big(\because \sum_{n\in PN} \sum_{c\in c(n)} = \sum_{c\in C} \big)
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{n \in PN, s, y} days_s \big(\sum_{a \in A(n), m \in M(n_1(a))} \Delta h_{amsy} \big) \Delta \pi_{nsy} \\
&\quad = - \sum_{n \in PN, s, y} days_s \big(\sum_{a \in A(n), m \in M(n_1(a))} \Delta h_{amsy} \Delta \pi_{n_2(a)sy} \big) \\
&\qquad\qquad\qquad\qquad \big(\because \Delta \pi_{nsy} = \Delta \pi_{n_2(a)sy}, \forall a \in A(n), n \in PN \big) \\
&\quad = - \sum_{s,y} days_s \big(\sum_{n \in PN, a \in A(n), m \in M(n_1(a))} \Delta h_{amsy} \Delta \pi_{n_2(a)sy} \big) \\
&\quad = - \sum_{s,y} days_s \big(\sum_{a \in A, m \in M(n_1(a))} \Delta h_{amsy} \Delta \pi_{n_2(a)sy} \big) \qquad \big(\because \sum_{n \in PN} \sum_{a \in A(n)} = \sum_{a \in A} \big) \\
&\quad = - \sum_{s,y} days_s \big(\sum_{m \in M, a \in A(n^m(m))} \Delta h_{amsy} \Delta \pi_{n_2(a)sy} \big) \\
&\quad = - \sum_{a \in A(n^m(m)), m, s, y} days_s \big(\Delta h_{amsy} \Delta \pi_{n_2(a)sy} \big)
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{n \in PN, y} days_1 \big(\sum_{a \in A(n), r \in R(n_1(a))} \Delta g_{ary} \big) \Delta \pi_{nsy} \\
&\quad = - \sum_{n \in PN, y} days_1 \big(\sum_{a \in A(n), r \in R(n_1(a))} \Delta g_{ary} \Delta \pi_{n_2(a)sy} \big) \\
&\qquad\qquad\qquad\qquad \big(\because \Delta \pi_{nsy} = \Delta \pi_{n_2(a)sy}, \forall a \in A(n), n \in PN \big) \\
&\quad = - \sum_{y} days_1 \big(\sum_{a \in A(n), n \in PN, r \in R(n_1(a))} \Delta g_{ary} \Delta \pi_{n_2(a)sy} \big) \\
&\quad = - \sum_{y} days_1 \big(\sum_{a, r \in R(n_1(a))} \Delta g_{ary} \Delta \pi_{n_2(a)sy} \big) \qquad \big(\because \sum_{n \in PN} \sum_{a \in A(n)} = \sum_{a \in A} \big) \\
&\quad = - \sum_{y} days_1 \big(\sum_{a \in A(n^r(r)), r} \Delta g_{ary} \Delta \pi_{n_2(a)sy} \big) \qquad \big(\because \sum_{a, r \in R(n_1(a)))} = \sum_{a \in A(n^r(r)), r} \big) \\
&\quad = - \sum_{a \in A(n^r(r)), r, y} days_1 \big(\Delta g_{ary} \Delta \pi_{n_2(a)sy} \big)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n\in CN,s=2,3,y} days_s\big(\sum_{r\in R^n}\Delta x_{rsy}-\sum_{m\in M^n}\Delta u_{msy}\big)\Delta\gamma_{nsy}\\
&\quad=\sum_{n\in CN,s=2,3,y} days_s\big(\sum_{r\in R^n}\Delta x_{rsy}\Delta\gamma_{n^r(r)sy}-\sum_{m\in M^n}\Delta u_{msy}\Delta\gamma_{n^m(m)sy}\big)\\
&\qquad\qquad\big(\because \Delta\gamma_{nsy}=\Delta\gamma_{n^r(r)sy},\forall r\in R^n;\Delta\gamma_{nsy}=\Delta\gamma_{n^m(m)sy},\forall m\in M^n\big)\\
&\quad=\sum_{s=2,3,y} days_s\sum_{n\in CN}\big(\sum_{r\in R^n}\Delta x_{rsy}\Delta\gamma_{n^r(r)sy}-\sum_{m\in M^n}\Delta u_{msy}\Delta\gamma_{n^m(m)sy}\big)\\
&\quad=\sum_{s=2,3,y} days_s\big(\sum_{r}\Delta x_{rsy}\Delta\gamma_{n^r(r)sy}-\sum_{m}\Delta u_{msy}\Delta\gamma_{n^m(m)sy}\big)\\
&\qquad\qquad\qquad\qquad\big(\because \sum_{n\in CN}\sum_{r\in R^n}=\sum_{r\in R};\ \sum_{n\in CN}\sum_{m\in M^n}=\sum_{m\in M}\big)\\
&\quad=\sum_{s=2,3,r,y} days_s\Delta x_{rsy}\Delta\gamma_{n^r(r)sy}-\sum_{s=2,3,m,y} days_s\Delta u_{msy}\Delta\gamma_{n^m(m)sy}
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n\in CN,y} days_3\big(\sum_{p\in P^n}\Delta w_{py}-\sum_{m\in M^n}\Delta v_{my}\big)\Delta\beta_{ny}\\
&\quad=\sum_{p,y} days_s\Delta w_{py}\Delta\beta_{n^p(p)y}-\sum_{m,y} days_s\Delta v_{my}\Delta\beta_{n^p(p)y}\\
&\qquad\qquad\qquad\big(\because \sum_{n\in CN}\sum_{p\in P^n}=\sum_{p\in P};\ \sum_{n\in CN}\sum_{m\in M^n}=\sum_{m\in M}\big)
\end{aligned}
$$

In summary, we have the following:

$$
\begin{aligned}
&\left[G^{MCC}(x) - G^{MCC}(y)\right]^T \Delta u^{MCC} + \left[H^{MCC}(x) - H^{MCC}(y)\right]^T \Delta v^{MCC} \\
&\quad = \sum_{a,s,y} days_s \Delta f_{asy} \Delta \tau_{asy} - \sum_{a \in A(n^m(m)),m,s,y} days_1 \Delta h_{amsy} \Delta \tau_{asy} \\
&\qquad - \sum_{a \in A(n^r(r)),r,y} days_1 \Delta g_{ary} \Delta \tau_{a1y} + \sum_{c,s,y} days_s \big(\Delta q_{csy} \Delta \pi_{n^c(c)sy}\big) \\
&\qquad - \sum_{a \in A(n^m(m)),m,s,y} days_s \big(\Delta h_{amsy} \Delta \pi_{n_2(a)sy}\big) \\
&\qquad - \sum_{a \in A(n^r(r)),r,y} days_1 \big(\Delta g_{ary} \Delta \pi_{n_2(a)sy}\big) \\
&\qquad + \sum_{s=2,3,r,y} days_s \Delta x_{rsy} \Delta \gamma_{n^r(r)sy} - \sum_{s=2,3,m,y} days_s \Delta u_{msy} \Delta \gamma_{n^m(m)sy} \\
&\qquad + \sum_{p,y} days_s \Delta w_{py} \Delta \beta_{n^p(p)y} - \sum_{m,y} days_s \Delta v_{my} \Delta \beta_{n^p(p)y}
\end{aligned} \tag{3.3.9}
$$

Summing up (3.3.3)-(3.3.9), we have

$$
\begin{aligned}
&[F(x) - F(y)]^T (x - y) \\
&\quad = \sum_{c,s,y} days_s \big(\Delta MC^{PR}_{csy}\big) \Delta q_{csy} + \sum_{a \in A(n^r(r)),r,y} days_1 \big(\Delta MC^{ST}_{ary}\big) \Delta g_{ary} \\
&\qquad + \sum_{p,y} days_3 \big(\Delta MC^{PG}_{py}\big) \Delta w_{py} - \sum_{k,m,s,y} days_s \big(\Delta MR_{kmsy}\big) \Delta l_{kmsy}
\end{aligned}
$$

By assumptions (a) and (b), we must, respectively, have

$$
\begin{aligned}
&\big(\Delta MC^{PR}_{csy}\big)\Delta q_{csy} > 0, \forall c, s, y \Longrightarrow \sum_{c,s,y} days_s \big(\Delta MC^{PR}_{csy}\big)\Delta q_{csy} > 0 \\
&\big(\Delta MC^{PG}_{py}\big)\Delta w_{py} > 0, \forall p, y \Longrightarrow \sum_{p,y} days_3 \big(\Delta MC^{PG}_{py}\big)\Delta w_{py} > 0
\end{aligned}
$$

By the strict monotonicity assumption (c), the following must be true:

$$
\begin{aligned}
\sum_{a\in A(n^r(r))} \big(\Delta MC^{ST}_{ary}\big)\Delta g_{ary} > 0, \forall r, y &\Longrightarrow \sum_{a\in A(n^r(r))} days_1 \big(\Delta MC^{ST}_{ary}\big)\Delta g_{ary} > 0, \forall r, y \\
&\Longrightarrow \sum_{a\in A(n^r(r)),r,y} days_1 \big(\Delta MC^{ST}_{ary}\big)\Delta g_{ary} > 0
\end{aligned}
$$

By the strict monotonicity assumption (d), the following must hold:

$$
\begin{aligned}
&-\sum_{m\in M^n} \big(\Delta MR_{kmsy}\big)\Delta l_{kmsy} > 0, \forall k, n \in CN, s, y \\
&\qquad \Longrightarrow -\sum_{k,m\in M^n,n\in CN,s,y} days_s \big(\Delta MR_{kmsy}\big)\Delta l_{kmsy} > 0 \\
&\qquad \Longrightarrow -\sum_{k,m,s,y} days_s \big(\Delta MR_{kmsy}\big)\Delta l_{kmsy} > 0
\end{aligned}
$$

Hence, $[F(x) - F(y)]^T(x - y) > 0$, which implies that F is a strictly monotone function, and thus MiCP(G, H) has a unique solution. $\square$

3.4 Conclusions

We discussed a deterministic equilibrium model for the natural gas market D-NGEM, which includes optimization problems for market participants, some strategic, some not, as well as market-clearing conditions for different markets where these participants are located. with reasonable assumptions on the marginal cost/revenue functions, an MiCP formulation, denoted D-NGEM-MiCP, are derived from the model D-NGEM. This MiCP formulation is different from the one in [35] in that market-clearing conditions for production, storage and peak gas markets are presented in a format of an NCP instead of an MiCP in [35]. Furthermore, we relaxed the assumption of bounded prices in [35] using the fact that marginal costs and revenues are bounded assuming that cost and revenue functions are continuously differentiable, respectively. Lastly, based on these enhancements, we provided theoretical analysis regarding the existence and uniqueness of model D-NGEM.

Chapter 4

A Stochastic Model S-NGEM with Recourse Method

In this chapter, we present a stochastic extension to model D-NGEM discussed in Chapter 3 using the recourse method of stochastic programming. The new model, denoted S-NGEM, where S stands for *stochastic*, is a multistage stochastic equilibrium model of a finite number of scenarios with random demand. It captures several important market characteristics not considered by Model D-NGEM, in particular, the important roles of the spot market which emerges with deregulation to handle market imbalances caused by random factors.

This Chapter is organized as follows. Two commonly used methods of stochastic programming are introduced in Section 4.1. Section 4.2 discusses the concept of a scenario tree and associated notation relative to the modeling of the recourse decisions for the spot market. In Section 4.3, we present the formulation for model S-NGEM in terms of optimization problems faced by each type of participants and a series of market-clearing conditions and develop some mathematical properties in order to derive its NCP/VI formulation from applying KKT conditions to these optimization problems.

4.1 Stochastic Programming

Stochastic programming is a generalization of nonlinear programming, whose goal is to find some policy that is feasible for all (or almost all) the possible data instances. For example, one possible use of stochastic programming is to maximize or minimize the expectation of some function of the decisions and the random variables, taking advantage of the fact that probability distributions governing the data are known or can be estimated, i.e., the recourse method. The general method for solving stochastic problems is to formulate deterministic equivalents to the constraints and the objective functions, and to solve the resulting mathematical program with appropriate algorithms.

4.1.1 Recourse Method

A widely applied and studied stochastic programming model is a two-stage recourse program. In the first stage, the decision maker takes some action, after which a random event occurs affecting the outcome of the first-stage decision; in the second stage, a *recourse* decision can then be made to compensate for any bad effects that might have been experienced as a result of the first-stage decision. In particular, two-stage recourse programs seek to minimize the cost of the first-stage decision plus the expected cost of the second-stage recourse decision. The following is a general example of two-stage stochastic linear program with recourse [6].

$$\begin{aligned} \min \quad & c^T x + E_\omega Q(x, \omega) \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{4.1.1}$$

$$\text{where} \begin{cases} Q(x,\omega) = \min \; q(\omega)^T y(\omega) \\ \text{s.t.} \quad T(\omega)x + W(\omega)y(\omega) = h(\omega) \end{cases} \tag{4.1.2}$$

The first linear program (4.1.1) minimizes the first-stage direct costs, $c^T x$ plus the expected recourse cost, $Q(x, \omega)$, over all of the possible scenarios while meeting the *first-stage constraints*, $Ax = b$. The recourse cost depends both on x, the *first-stage decision*, and on the random event, indexed by ω. The second linear programming (4.1.2) describes how to choose $y(\omega)$ (a *second-stage/recourse variable*, a different decision for each random ω). It minimizes the cost $q^T y$ subject to some *second-stage/recourse constraints*, $T(\omega)x + W(\omega)y(\omega) = h(\omega)$. These constraints can be thought of as requiring some action to correct the system after the random event occurs.

When ω represents an index for a discrete random variable, (4.1.1) and (4.1.2) are equivalent to a deterministic linear program as follows:

$$\begin{aligned} \min \quad & c^T x + \sum_{i=1}^{N} p_i d_i^T y_i \\ \text{s.t.} \quad & Ax = b \\ & T_i x + W_i y_i = h_i \qquad i = 1, \cdots, N \\ & x > 0 \\ & y_i \geq 0, \forall i \end{aligned} \tag{4.1.3}$$

where N is the number of scenarios associated with random event ω and p_i is the probability associated with the occurrence of scenario i.

For cases where ω represents continuous random variable(s), such deterministic equivalents are also available, though the form of the deterministic equivalent usually depends on the problem itself as well as the probability distributions involved and it is not always easy to find a general form. In this dissertation, the random variables considered by model S-NGEM are assumed discrete. However, it is common practice to approximate continuous distribution with a discrete distribution in decision and risk analysis. The bracket median approaches, extended Pearson-Tukey method and extended Swanson-Megill method are examples of well-known approximation methods. Studies on the relative performance of difference discrete-distribution approximations include [49, 48, 70].

Rather than a one-shot decision from a deterministic model, the optimal policy from such a model is a single first-stage policy and a collection of recourse decisions defining which second-stage action should be taken in response to each random outcome. Figure 2.5 in Chapter 2 has illustrated the difference between models D-NGEM and S-NGEM in this sense.

But most practical decision problems, including what is covered in the dissertation, involve a sequence of decisions that react to outcomes that evolve over time. A *multi-stage stochastic programming* approach, an extension to two-stage stochastic programming in terms of the number of decision time periods involved, is developed to deal with these situations. In general, the decision variables and constraints for multistage stochastic programs can still be broken down into the first-stage decisions and constraints that have nothing to do with the random event, ω and the recourse decisions and constraints that depend on each random outcome ω.

4.1.2 Chance-Constraint Method

An alternative stochastic modeling approach is based on the notion of *chance-constraints*, which does not require that decisions are feasible for every outcome of the random parameters, but instead requires feasibility with at least some given probability [75]. Consider the following linear programming:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \geq b_i, \qquad i = 1, \cdots, m \\ & x \geq 0 \end{aligned} \tag{4.1.4}$$

where $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $x \in \mathbb{R}^n$. Suppose that either a_{ij} or b_i is random, then the associated probabilistic form is:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & P\Big(\sum_{j=1}^{n} a_{ij} x_j \geq b_i\Big) \geq \alpha_i, \qquad i = 1, \cdots, m \\ & x \geq 0 \end{aligned} \tag{4.1.5}$$

where α_i is a "satisfaction" or "reliability" level with $\alpha_i \in [0, 1]$; and the symbol $P(\cdot)$ means the *probability* of $(\cdot)$.

The main task of solving this chance-constraints problem is to find a deterministic equivalent to (4.1.5). The deterministic equivalent will, in general, be a nonlinear program depending on what is random $(a_{ij}, b_i$ or $c)$ as well as the particular probability distribution and independence assumptions.

As an example to (4.1.5), suppose b_i is random and its cumulative distribution function (CDF) is given by $F_{b_i}(x)$. With the assumption that $F_{b_i}^{-1}(x)$ exists, a chance constraint to (4.1.5) for some i is equivalent to

$$\begin{aligned} P(b_i \leq \sum_{j=1}^{n} a_{ij} x_j) \geq \alpha_i & \Longleftrightarrow F_{b_i}(\sum_{j=1}^{n} a_{ij} x_j) \geq \alpha_i \\ & \Longleftrightarrow \sum_{j=1}^{n} a_{ij} x_j \geq B_\alpha = F_{b_i}^{-1}(\alpha_i) \end{aligned}$$

where B_α is the smallest value such that $F_{b_i}(B_\alpha) = \alpha_i$ [75].

4.1.3 The Value of Information and Stochastic Solution

There are two key numbers in stochastic programming measuring its performance, the *Expected Value of Perfect Information* ($EVPI$) and the *Value of the Stochastic Solution* (VSS) .

Following [6], we introduce several solution values related to these two concepts. To be consistent with the profit-maximizing problems presented in this dissertation, we use maximizing-programs for illustrative purposes. Suppose there is a stochastic program as follows,

$$RP \equiv \max_{x \in S} \; E_{\boldsymbol{\xi}} z(x, \boldsymbol{\xi}) \tag{4.1.6}$$

where the set S is the feasible region of the decision variable x; symbol E is the expectation sign; $\boldsymbol{\xi}$ is the (vector of) random variable whose realizations correspond to the various scenarios ξ. Assuming there exists at least one feasible solution to it, a solution of (4.1.6) is denoted x^*. The optimal value of (4.1.6) is known in the literature as the *here-and-now* solution, denoted RP.

Assuming that we somehow have perfect information about the future knowing each realization of $\boldsymbol{\xi}$ in advance, then we are able to find the corresponding optimal solutions for each scenario ξ. The expected value of these optimal values is known as the *wait-and-see* solution, denoted WS.

$$WS \equiv E_{\boldsymbol{\xi}}\big[\max_{x \in S} \; z(x, \boldsymbol{\xi})\big] \tag{4.1.7}$$

We define the $EVPI$ equal to the WS less the RP as shown in (4.1.8). The $EVPI$ measures the maximum amount a decision maker would pay in return for the complete information about the future.

$$EVPI \equiv WS - RP \tag{4.1.8}$$

Due to computational difficulties to solve (4.1.6), a natural alternative is to solve (4.1.6) by replacing all random variables by their expected values, which is

$$EV \equiv \max_{x \in S} \; z(x, \overline{\xi}) \tag{4.1.9}$$

where $\overline{\xi} = E(\boldsymbol{\xi})$ is the expectation of the random variable $\boldsymbol{\xi}$. The optimal solution to (4.1.9) is denoted as $\overline{x}(\overline{\xi})$. Using this solution to the original stochastic problem (4.1.6), we have the *expected value of using the EV solution* (EEV) as follows.

$$EEV \equiv E_{\boldsymbol{\xi}}\big(z(\overline{x}(\overline{\xi}), \boldsymbol{\xi})\big) \tag{4.1.10}$$

The difference between RP and EEV is defined the *value of stochastic solution*, denoted VSS. The VSS measures the cost of using the expectation of the uncertainty thus ignoring the stochastic elements in the decision making process.

$$VSS \equiv RP - EEV \tag{4.1.11}$$

The following relationship between WS, RP and EEV has been established for both linear and nonlinear stochastic programming [52, 53]. According to (4.1.12), the $EVPI$ and VSS must be nonnegative values.

$$EEV \leq RP \leq WS \tag{4.1.12}$$

Using the concepts presented above, we calculate the WS, RP, and EEV in the context of an equilibrium model in Chapter 5 of the numerical analysis for the stochastic equilibrium model to be presented in this chapter. It turns out that the relationship presented in (4.1.12) does not hold in terms of the expected profits evaluated by the equilibrium prices and quantities for most of the individual players.

4.2 A Scenario Tree

In model S-NGEM, market participants face two types of decisions: long-term market decisions and spot market decisions. Long-term market decisions have to be made before random outcomes can be observed. The span for the so-called "long-term" is the time horizon of the model, which is usually one to three years. Market participants plan on how much to produce or purchase for each season over the entire time horizon at the beginning of the time considered, referred to as time "0". When the outcome of the random elements is observed, market participants make decisions for the spot market, which can be adjusted every season depending on the actual outcome of the randomness. We assume in the model S-NGEM the possible outcomes for each season cannot be observed until that season begins. In other words, we cannot anticipate what would happen in the future. The goal of the participants is to maximize the profits earned from the long-term market plus the expected profits earned from the spot market. The long-term market decisions can be thought of as the "first-stage decisions" in the recourse method. While the spot market is a multi-stage "recourse" to compensate for any bad effects that might have been caused by all previous decisions. All the decisions for the spot market are "recourse decisions", which actually involve more than one time period. In this section, we introduce new notation regarding the discrete random events related to the first-stage and recourse decision making for model S-NGEM.

A sample scenario tree for the demand is presented in Figure 4.1. The time horizon involved in the event tree is one year with three seasons. We assume the demand for each season has two possible levels, high ($D^{s,y} = 2$) or low ($D^{s,y} = 1$). Hence, there are 8 scenarios, each of which can be identified as a branch in the event tree, e.g., scenario 5 is the branch composed of nodes 2, 5 and 11. The total number of the possible realizations of the randomness of this example is 14, shown respectively as nodes 1 to 14 in Figure 4.1. These events are hereafter referred to as $N1$ to $N14$, respectively. A special node, node 0 is the root of the event tree. This is the time point when the long-term decisions (or first-stage decisions) are made therefore it is not a part of the scenario set I. Spot market decisions (or recourse decisions) are made in response to each random event represented by nodes 1 to 14. The bold number assigned under each node is the probability of occurrence of the random event represented by that node.

First, we define sets for random elements. Let I denote the set of possible realizations of the random events in the scenario tree, indexed by i. Also, let $I^{s,y}$ denote the set of possible realizations of the random events related to the season s in year y, indexed by $i^{s,y}$. Naturally, $\{I^{1,1}, \cdots, I^{s,y}, \cdots\}$ forms a partition of I. In the example, we have $I = \{N1, \cdots, N14\}$ and three subsets of I, that is, $I^{1,1} = \{N1, N2\}$, $I^{2,1} = \{N3, N4, N5, N6\}$ and $I^{3,1} = \{N7, N8, N9, N10, N11, N12, N13, N14\}$ for seasons 1, 2 and 3, respectively.

Next, let $\eta(i^{s,y})$ denote the probability of occurrence of the random event $i^{s,y} \in I^{s,y}$, with $\sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) = 1$, where $\eta(i^{s,y}) > 0$ for all $i^{s,y} \in I$ assumed. For example, the probabilities of the possible realizations in season 2 are respectively, 0.2 for nodes 3 and 4; 0.18 for node 5 and 0.42 for node 6. The sum of the probabilities of these four occurrences is 1.

Let $\psi(i^{s,y})$ denote the unique immediate predecessor of $i^{s,y} \in I^{s,y}$ in the event tree; let $\Psi(i^{s,y})$ denote the unique immediate predecessor of $\psi(i^{s,y})$ in the event tree. By an abuse of language, sometimes $\psi(i^{s,y})$ and $\Psi(i^{s,y})$ are referred to as their singleton elements. Let $PD(i^{s,y})$ denote the set consisting of all the predecessors of $i^{s,y} \in I^{s,y}$ inclusive of $i^{s,y}$ in the event tree. For instance, nodes 7 and 8 have the same predecessor, node 3, that is, $\psi(N7) = \psi(N8) = \{N3\}$. Similarly, nodes 9 and 10 share a predecessor, node 4, that is, $\psi(N9) = \psi(N10) = \{N4\}$.

Furthermore, the predecessor of the predecessors for nodes 7-10 is node 1, i.e., $\Psi(N7) = \Psi(N8) = \Psi(N9) = \Psi(N10) = \{N1\}$. The predecessor set of node 7 is $PD(N7) = \{N1, N3, N7\}$.

In addition, let $SC(i^{s,y})$ denote the set consisting of the all the successors of $i^{s,y} \in I^{s,y}$ inclusive of $i^{s,y}$ in the event tree; let $ISC(i^{s,y})$ denote the set of the immediate successors of $i^{s,y} \in I^{s,y}$ in the event tree; and let $IISC(i^{s,y})$ denote the set of the immediate successors for $\forall \widetilde{i}^{s,y} \in ISC(i^{s,y})$, where $\widetilde{i}^{s,y}, i^{s,y} \in I^{s,y}$ in the event tree. Considering node 1 in Figure 4.1, we have three sets related to successors, $ISC(N1) = \{N3, N4\}$, $IISC(N1) = \{N7, N8, N9, N10\}$ and $SC(N1) = \{N1, N3, N4, N7, N8, N9, N10\}$.

In this example, the long-term market decisions are made at node 0. Players decide on the actions taken for the following seasons, seasons 1, 2 and 3, respectively. From season 1 on, players make spot market decisions in response to each random event. Players are aware of the time stages and the random events they stick to. But the decisions taken by the others at any time stages are beyond the players' knowledge.

The description of the scenario tree relative to the predecessors and successors aims to link separate scenarios of the same history together. However, a formulation based on the enumeration of all random outcomes can become quite cumbersome as the time horizon increases. In order to alleviate the computational complexity involved in this type of problems, [6] presented a simpler formulation which decomposes the problem into separate problems for each scenarios and then add nonanticipativity constraints to link the separate scenarios. Other methods include an approach proposed in [15] regarding how to eliminate irrelevant scenarios involved to lessen the computation burden.

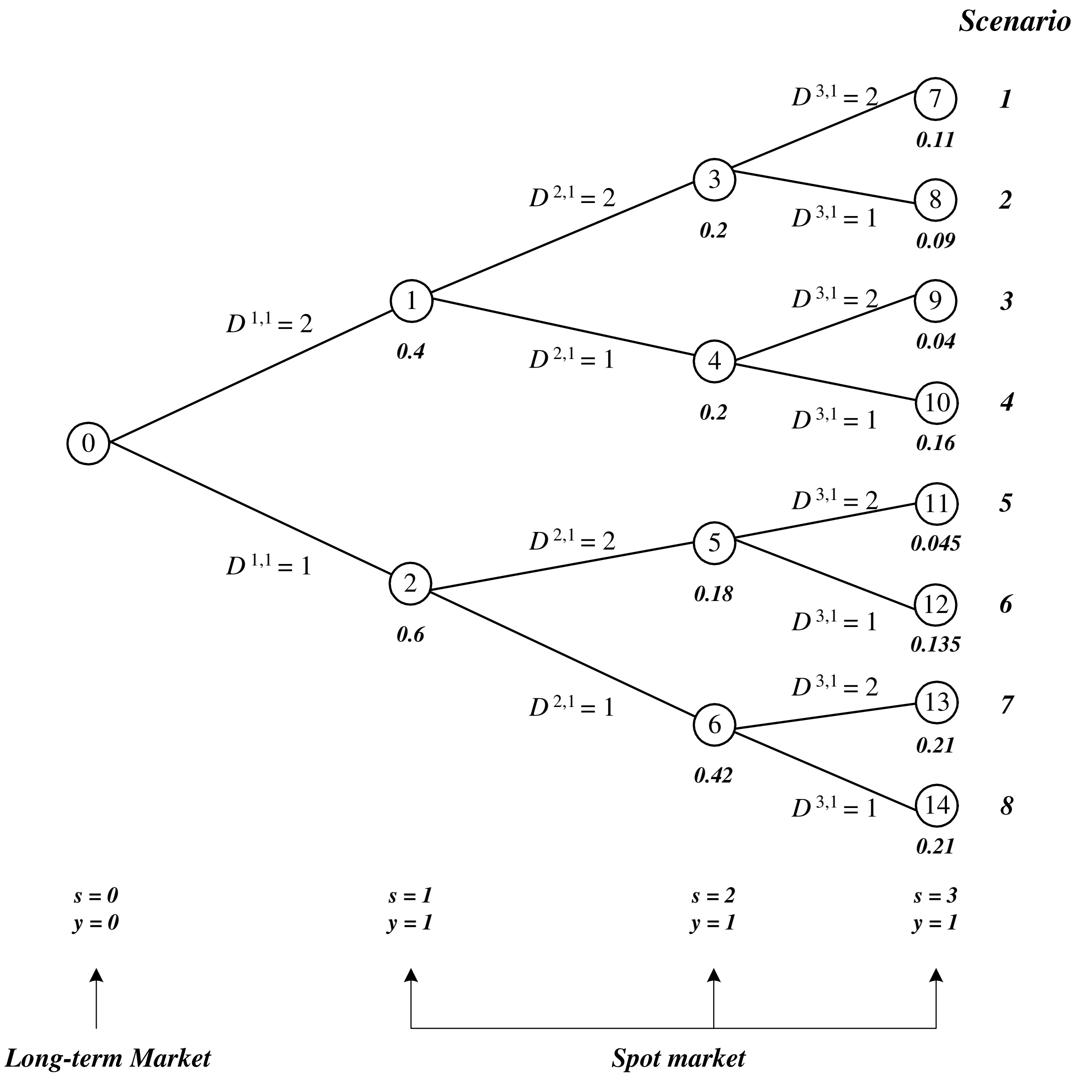


Figure 4.1: Scenario Tree of Three Time Periods

4.3 Model S-NGEM

Model S-NGEM is a multistage stochastic equilibrium model of a finite number of scenarios developed based on the market structure considered for model D-NGEM. Both models have the same cost functions, that is, $c_c^{PR}(\cdot)$ for producers, $c_r^{ST}(\cdot)$ for storage operators and $c_p^{PG}(\cdot)$ for peak gas operators. All the cost functions are assumed convex and continuously differentiable so that KKT conditions are equivalent to the original optimization problems. In addition, all the input data for model D-NGEM, such as pipeline capacity ($\overline{f}_a$) and production capacity ($\overline{q}_c$), are still relevant to model S-NGEM (refer to Table 4.2).

The types of market participants modeled in model S-NGEM are the same as those incorporated in model D-NGEM, in particular, pipeline operators, producers, storage operators, peak gas operators and marketers. The objectives for all these participants are to maximize the sum of net profits of long-term decisions and expected net profits of spot market decisions subject to capacity and technical restrictions for all scenarios. The optimization problems composed of model S-NGEM are expressed in an extensive form of a stochastic program which explicitly describes the recourse decision variables for all scenarios. As an alternative, Birge and Louveaux propose in [6] a formulation with *nonanticipativity constraints* for multi-stage stochastic problems in order to alleviate the exponential growth of the size of the formulation as the time horizon increases.

As is done in model D-NGEM, all market participants but marketers are assumed as price-takers in a perfect competition environment. Generally, solving the individual problem for each price-takers is equivalent to solving a collective problem which sums the objective functions for all and include the corresponding constraints. Therefore, for brevity, only the collective problems for each type of price-taking participants, denoted (PL^S), (PR^S), (ST^S) and (PG^S), are presented in the following sections. The superscribes of "S" in the notations stand for *stochastic*, distinguishing the notations in this chapter from their deterministic counterparts in Chapter 3.

In model S-NGEM, marketers are modeled as Nash-Cournot players for residential and commercial sectors while price-takers for industrial and electric power demand sectors. The optimization problem for the individual marketer is denoted $\widetilde{MK}^S$. Because of the imperfect competition between marketers, a single aggregate optimization problem for all marketers is not available. A commonly considered approach for solving Nash equilibrium problems is via NCP/VIs.

The industrial and electric power demand sectors are not modeled to be under the market power of the marketers in model S-NGEM. These two sectors are able to specify the amount of consumption desired throughout the time horizon in the model. In contrary, residential and commercial consumption is still controlled by marketers via inverse demand functions, denoted $\theta^1_{knsy,i^{s,y}}$, which vary with the random event i, as opposed to just θ_{knsy} in model D-NGEM.

The market-clearing conditions for both the long-term and spot markets integrates all optimization problems together and forms the equilibrium model S-NGEM, which is shown to be an MiCP under assumptions of positive marginal costs with positive production.

We organize the assumptions and theorems presented in this chapter in Table 4.1 in terms of market participants and theorem types. Theorems-Type 1, without additional assumptions, are those that establish the price relationship for the long-term and spot markets. They are relatively isolated from the other theorems. Generally speaking, these theorems show that if there is positive activity (e.g., pipeline flow, production), then the equilibrium long-term market price equals the expected spot market price. The assumptions made in this chapter are relative to the marginal cost functions for producers, storage operators and peak gas operators. They are very similar to those in Chapter 3. These assumptions can be described briefly as positive marginal costs with positive production, where the "production" could be production rates for producers, extraction rates for storage operators or production rates for the peak gas operators depending on the context. Theorems-Type 2 state that with positive production, the market prices are always positive, followed by Theorems-Type 3, which directly lead to Theorem 4.3.14, show how to convert the market-clearing conditions to instances of MiCPs or NCPs. Theorem 4.3.14 shows that the model S-NGEM as a whole is an instance of an MiCP.

Table 4.1: Theorems in Chapter 4

Participants	**Theorems** Type 1	**Assumptions**	**Theorems** Type 2	**Theorems** Type 3	**Theorems**
PL	4.3.1				4.3.2
PR	4.3.3	4.3.1	$\Longrightarrow$ 4.3.4	$\Longrightarrow$ 4.3.5	$\Longrightarrow$ 4.3.14
ST	4.3.6	4.3.2	$\Longrightarrow$ 4.3.7	$\Longrightarrow$ 4.3.8	
PG	4.3.9	4.3.3	$\Longrightarrow$ 4.3.10	$\Longrightarrow$ 4.3.11	
Industrial and Electric Power Sectors		4.3.1 4.3.2 4.3.3	$\Longrightarrow$ 4.3.12	$\Longrightarrow$ 4.3.13	

Based on the nomenclature for variables in model D-NGEM, we distinguish the first-stage and recourse variables in model S-NGEM by superscripts "0" and "1", respectively. For example, the production rate for producer c in season s of year y is denoted q_{csy} in model D-NGEM. Thus, we use q^0_{csy} and $q^1_{csy,i^{s,y}}$ in model S-NGEM to denote the production rates for the first-stage and recourse decisions, respectively. Note that the recourse variable $q^1_{csy,i^{s,y}}$ has one more dimension, $i^{s,y}$ which belongs to the random event set $I^{s,y}$ for season s in year y, than the corresponding first-stage variable q^0_{csy} because the value of $q^1_{csy,i^{s,y}}$ depends on the actual realization of the random events. The season s and year y associated with the random event $i^{s,y}$ should synchronize with the season and year indices of the variable in question. In general, the indices s and y determine the random event $i^{s,y}$. In the case that the time point (s and y) associated with the random event i is unclear, we use two operators $s(i)$ and $y(i)$ to calculate the season and year relative to i. For example, in equation (4.3.20), the variables $q^1_{csy,i^{s,y}}$ are summed over a time period of more than one season so that to be more accurate, they are re-written as $q^1_{cs(i)y(i),i}$, summed over i subject to a certain condition.

Similar to Table 3.1, Table 4.2 organizes the variables and data used in model S-NGEM by market agent. Endogenous variables include decision variables and multipliers for constraints. Exogenous variables in our case are market prices determined by market-clearing conditions. The table also distinguishes the long-term and spot market variables in a pair of rows. All the variables including multipliers related to the long-term market are superscripted by "0", while those for the spot market are superscripted by "1".

Note that discounted revenues and costs are not considered for clarity of presentation as well as given the short timeframe involved. The units for the gas volume, rate and price are million cubic feet (MMcf), million cubic feet per day (MMcf/d) and \$/Mcf. The objective functions for the players are in thousands of dollars.

All input data for model D-NGEM are also used by model S-NGEM. However, the reservation charges RC^0_{asy} are specific to the model S-NGEM. Besides, a new type of data in model S-NGEM is the description of the randomness, i.e, the information conveyed by the scenario tree introduced in Section 4.2. This includes $\eta_{i^{s,y}}$ explicitly shown in the table, and scenario sets and predecessor and successor sets that are not present in the table.

Table 4.2: Variables and Data for Model S-NGEM

Problems		Endogenous Variables		Exogenous	Data
		Decision Variables	**Multipliers**	**Variables**	
(PL^S)	L*	f^0_{asy}		τ^0_{asy}	$\eta(i^{s,y})$, $\overline{f}_a$
	S**	$f^1_{asy,i^{s,y}}$	$\rho^1_{asy,i^{s,y}}$	$\tau^1_{asy,i^{s,y}}$	
(PR^S)	L	q^0_{csy}		π^0_{nsy}	$\eta(i^{s,y})$, $\overline{q}_c$, $prod_c$, $c^{PR}_c(\cdot)$
	S	$q^1_{csy,i^{s,y}}$	$\lambda^1_{csy,i^{s,y}}$, $\mu^1_{c,i^{3,\lvert Y\rvert}}$	$\pi^1_{nsy,i^{s,y}}$	
(ST^S)	L	g^0_{ary}, x^0_{rsy}	δ^0_{ry}	γ^0_{nsy}	$\eta(i^{s,y})$, $\overline{x}_r$, $\overline{g}_r$, $\overline{k}_r$, $c^{ST}_r(\cdot)$, τ^{reg}_{asy}, RC^0_{asy}
	S	$g^1_{ary,i^{1,y}}$, $x^1_{rsy,i^{s,y}}$	$\delta^1_{ry,i^{3,y}}$, $\omega^1_{rsy,i^{s,y}}$, $\xi^1_{ry,i^{1,y}}$, $\zeta^1_{ry,i^{3,y}}$	$\gamma^1_{nsy,i^{s,y}}$	
(PG^S)	L	w^0_{py}		β^0_{ny}	$\eta(i^{s,y})$, $\overline{w}_p$, $c^{PG}_p(\cdot)$
	S	$w^1_{py,i^{3,y}}$	$\sigma^1_{py,i^{3,y}}$	$\beta^1_{ny,i^{3,y}}$	
(MK^S)	L	l^0_{kmsy}, h^0_{amsy}, u^0_{msy}, v^0_{my}	ϕ^0_{msy}	τ^0_{asy}, π^0_{nsy}, γ^0_{nsy}, β^0_{ny}	$\eta(i^{s,y})$, τ^{reg}_{asy}, RC^0_{asy}
	S	$l^1_{kmsy,i^{s,y}}$, $h^1_{amsy,i^{s,y}}$, $u^1_{msy,i^{s,y}}$, $v^1_{my,i^{3,y}}$	$\phi^1_{msy,i^{s,y}}$	$\tau^1_{asy,i^{s,y}}$, $\pi^1_{nsy,i^{s,y}}$, $\gamma^1_{nsy,i^{s,y}}$, $\beta^1_{ny,i^{3,y}}$	

**: Long-term decision variables; **: Spot market decision variables.*

4.3.1 Pipeline Operator

The problem for the pipeline market is denoted (PL^S) as shown below. There are two types of decision variables for pipeline operators, f^0_{asy}, the arc flow in the long-term market, and $f^1_{asy,i^{s,y}}$, the arc flow for the spot market under different random outcomes. The objective function as shown in (4.3.1) is a summation of two parts: the congestion fee income of the long-term market, which is similar to the objective function (3.1.5) of (PL) in D-NGEM, and the expected congestion fee income of the spot market. As τ_{asy} in D-NGEM, the terms τ^0_{asy} and $\tau^1_{asy,i^{s,y}}$, exogenous to (PL^S) but a variable in S-NGEM, are derived from the market-clearing conditions (4.3.16) and (4.3.17). They represent the congestion fee for the pipeline in the long-term and spot markets, respectively. The term $\eta(i^{s,y})$ is the probability associated with the occurrence of realization $i^{s,y}$. The expected profits for pipeline a from the spot market are $days_s \eta(i^{s,y})\tau^1_{asy,i^{s,y}} f^1_{asy,i^{s,y}}$ summed over all possible outcomes of the random perturbation in the time horizon. Constraints (4.3.2) are the upper bound on the arc flows for all realizations with the lower bounds being zero.

$$
\begin{aligned}
(PL^S) \quad \max \quad & \sum_{a\in A}\sum_{y\in Y}\sum_{s\in S} days_s\big[\tau^0_{asy} f^0_{asy} + \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\tau^1_{asy,i^{s,y}} f^1_{asy,i^{s,y}}\big] && (4.3.1)\\
\text{s.t.} \quad & f^0_{asy} + f^1_{asy,i^{s,y}} \le \overline{f}_a \qquad (\rho^1_{asy,i^{s,y}} \ge 0) \quad \forall a, s, i^{s,y}, y && (4.3.2)\\
& 0 \le f^1_{asy,i^{s,y}}, \forall i^{s,y}, f^0_{asy} \qquad \forall a, s, y && (4.3.3)
\end{aligned}
$$

The KKT conditions are both necessary and sufficient for the optimality of the problem because (PL^S) is a linear programming. The KKT conditions to (PL^S) are:

$$0 \le -days_s \tau^0_{asy} + \sum_{i^{s,y}\in I^{s,y}} \rho^1_{asy,i^{s,y}} \perp f^0_{asy} \ge 0 \qquad \forall a, s, y \qquad (4.3.4)$$

$$0 \le -\eta(i^{s,y}) days_s \tau^1_{asy,i^{s,y}} + \rho^1_{asy,i^{s,y}} \perp f^1_{asy,i^{s,y}} \ge 0 \qquad \forall a, i^{s,y}, s, y \qquad (4.3.5)$$

$$0 \le \overline{f}_a - f^0_{asy} - f^1_{asy,i^{s,y}} \perp \rho^1_{asy,i^{s,y}} \ge 0 \qquad \forall a, i^{s,y}, s, y \qquad (4.3.6)$$

Naturally, (4.3.4) - (4.3.6) have a mathematical structure of an NCP. Let us define

$$v^{PL^S} \equiv \begin{pmatrix} f^0_{asy} & (\forall a, s, y) \\ f^1_{asy,i^{s,y}} & (\forall a, i^{s,y}, s, y) \\ \rho^1_{asy,i^{s,y}} & (\forall a, i^{s,y}, s, y) \end{pmatrix} \text{ and} \qquad (4.3.7)$$

$$H^{PL^S}(v^{PL^S}) \equiv \begin{pmatrix} -\, days_s \tau^0_{asy} + \displaystyle\sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}} \quad (\forall a, s, y) \\ -\, \eta(i^{s,y}) days_s \tau^1_{asy,i^{s,y}} + \rho^1_{asy,i^{s,y}} \quad (\forall a, i^{s,y}, s, y) \\ \overline{f}_a - f^0_{asy} - f^1_{asy,i^{s,y}} \quad (\forall a, i^{s,y}, s, y) \end{pmatrix} \tag{4.3.8}$$

Definitions (4.3.7) and (4.3.8) allow the KKT conditions (4.3.4) - (4.3.6) to be expressed equivalently as

$$0 \leq v^{PL^S} \perp H^{PL^S}(v^{PL^S}) \geq 0 \tag{4.3.9}$$

We establish a relationship between the congestion fees for the long-term market and the spot market in Theorem 4.3.1. This theorem states that for any pipeline arc with a positive flow in a given season and year for the long-term market, the corresponding long-term congestion fee for that pipeline (τ^0_{asy}) is greater or equal to the expected congestion fees for the spot market in that season ($\sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}}$). In contrast, when some pipeline has positive arc flows under all random outcomes in some season, the congestion fee for the long-term market in that season has to be less or equal to the expected congestion fee in the same season. Similar relations are also established for producers, storage operator and peak gas operators to be presented in the following sections.

Theorem 4.3.1. *For any pipeline arc $a \in A$ in seasons s of years y,*

a) if the long-term pipeline flow $f^0_{asy} > 0$, then the long-term market congestion fee is greater than or equal to the expected spot market congestion fee, that is, $\tau^0_{asy} \geq \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}}$;

b) if the spot market pipeline flow $f^1_{asy,i^{s,y}} > 0, \forall i^{s,y}$, then the long-term market congestion fee is less than or equal to the expected spot market congestion fee, that is, $\tau^0_{asy} \leq \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}}$;

c) if the long-term pipeline flow $f^0_{asy} > 0$ and spot market pipeline flow $f^1_{asy,i^{s,y}} > 0, \forall i^{s,y}$, then the long-term market congestion fee is equal to the expected spot market congestion fee, that is, $\tau^0_{asy} = \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}}$.

Proof. a) By (4.3.4), if $f^0_{asy} > 0$, we see that

$$days_s \tau^0_{asy} = \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}} \tag{4.3.10}$$

By (4.3.5), it follows that

$$\eta(i^{s,y}) days_s \tau^1_{asy,i^{s,y}} \leq \rho^1_{asy,i^{s,y}}, \forall i^{s,y} \tag{4.3.11}$$

Summing this inequality for all $i^{s,y} \in I^{s,y}$, we have

$$\begin{aligned} days_s \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}} &\leq \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}} \\ &= days_s \tau^0_{asy} && \text{by (4.3.10)} \end{aligned}$$

Therefore, $\tau^0_{asy} \geq \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}}$ when $f^0_{asy} > 0$.

b) By equation (4.3.5), if $f^1_{asy,i^{s,y}} > 0, \forall i^{s,y}$, it follows that

$$\eta(i^{s,y}) days_s \tau^1_{asy,i^{s,y}} = \rho^1_{asy,i^{s,y}}, \forall i^{s,y} \tag{4.3.12}$$

The summation of the above equation over $i^{s,y} \in I^{s,y}$ gives

$$days_s \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}} = \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}} \tag{4.3.13}$$

Also by (4.3.4), we see that

$$\begin{aligned} days_s \tau^0_{asy} &\leq \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}} \\ &= days_s \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \tau^1_{asy,i^{s,y}} && \text{by (4.3.12)} \end{aligned}$$

Clearly, the conclusion follows.

c) The conclusion is evident from parts (a) and (b). □

An equilibrium is enforced by market-clearing conditions (4.3.14) – (4.3.17), which state that the supply of pipeline gas is equal to the demand for all circumstances. Equations (4.3.14) and (4.3.15), which are similar to conditions (3.1.12) and (3.1.13) in model D-NGEM, are market-clearing conditions for the equilibrium in the long-term market. Besides, Equations (4.3.16), for season 1 when the storage injection denoted by variable $g^1_{ary,i^{1,y}}$ is present, and (4.3.17), for seasons 2 and 3, are market-clearing conditions for the spot market under different random outcomes, whose dual variables $\tau^1_{asy,i^{s,y}}$ are used to measure the congestion fees for each $i^{s,y} \in I$ accordingly.

$$days_1 f^0_{a1y} = \sum_{r \in R(n_1(a))} days_1 g^0_{ary} + \sum_{m \in M(n_1(a))} days_1 h^0_{am1y}$$
$$\left(\tau^0_{a1y} \text{ free}\right) \quad \forall a, y \tag{4.3.14}$$

$$days_s f^0_{asy} = \sum_{m \in M(n_1(a))} days_s h^0_{amsy} \qquad \left(\tau^0_{asy} \text{ free}\right) \quad \forall a, s = 2, 3, y \tag{4.3.15}$$

$$days_1 f^1_{a1y,i^{1,y}} = \sum_{r \in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} + \sum_{m \in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}}$$

$$\left(\tau^1_{a1y,i^{1,y}} \text{ free}\right) \quad \forall a, i^{1,y}, y \qquad (4.3.16)$$

$$days_s f^1_{asy,i^{s,y}} = \sum_{m \in M(n_1(a))} days_s h^1_{amsy,i^{s,y}}$$

$$\left(\tau^1_{asy,i^{s,y}} \text{ free}\right) \quad \forall a, i^{s,y}, s = 2, 3, y \quad (4.3.17)$$

Note that the congestion fees, τ^0_{asy} and $\tau^1_{asy,i^{s,y}}$ associated with market-clearing conditions (4.3.14) - (4.3.17), are all free. A negative congestion fee can be interpreted as an incentives for pipeline users. However, as shown in Theorem 4.3.2, the congestion fees would not take negative values unless the arc flows are zeros for both the long-term and spot markets. Also, whenever the arc flow is positive, the corresponding congestion fee is nonnegative. This means that the total congestion cost, a product of the pipeline flow and the congestion fees, is always zero when a negative congestion fee is present. Being a portion of the costs for storage operators and marketers, a zero congestion cost certainly has no influence on the equilibrium solution.

Theorem 4.3.2. *For a pipeline $a \in A$ in season s of year y, the following statements are true:*

(a) if the long-term pipeline flow $f^0_{asy} = 0$ and the spot market pipeline flow $f^1_{asy,i^{s,y}} = 0$ for a random outcome $i^{s,y} \in I^{s,y}$, then the long-term market congestion fee $\tau^0_{asy} \le 0$ and the spot market congestion fee $\tau^1_{asy,i^{s,y}} \le 0$ for that random outcome $i^{s,y}$;

(b) if the long-term pipeline flow $f^0_{asy} > 0$ then the long-term market congestion fee $\tau^0_{asy} \ge 0$;

(c) *if the spot market pipeline flow* $f^1_{asy,i^{s,y}} > 0$ *for a random event* $i^{s,y} \in I^{s,y}$, *then the spot market congestion fee* $\tau^1_{asy,i^{s,y}} \geq 0$ *for that* $i^{s,y}$;

(d) *if the long-term market pipeline flow* $\tau^0_{asy} < 0$ *then the long-term market congestion fee* $f^0_{asy} = 0$.

(e) *if the spot market pipeline flow* $\tau^1_{asy,i^{s,y}} < 0$ *for a random event* $i^{s,y} \in I^{s,y}$, *then the spot market congestion fee* $f^1_{asy,i^{s,y}} = 0$ *for that* $i^{s,y}$.

Proof. (a) When $f^0_{asy} = 0$ and $f^1_{asy,i^{s,y}} = 0$ for an $i^{s,y}$, we must have $\rho^1_{asy,i^{s,y}} = 0$ for that $i^{s,y}$ by (4.3.6). Also, (4.3.4) and (4.3.5) imply that $\tau^0_{asy} \leq \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}}/days_s$ and $\tau^1_{asy,i^{s,y}} \leq \rho^1_{asy,i^{s,y}}/[days_s \eta(i^{s,y})]$. This shows that $\tau^0_{asy} \leq 0$ and $\tau^1_{asy,i^{s,y}} \leq 0$ for that random outcome $i^{s,y}$ because $\rho^1_{asy,i^{s,y}} = 0$.

(b) When $f^0_{asy} > 0$, by complementarity, we deduce from (4.3.4) that $\tau^0_{asy} = \sum_{i^{s,y} \in I^{s,y}} \rho^1_{asy,i^{s,y}}/days_s$, which by definition is nonnegative. Therefore, $\tau^0_{asy} \geq 0$.

(c) When $f^1_{asy,i^{s,y}} > 0$, by complementarity, we deduce from (4.3.5) that $\eta(i^{s,y})\tau^1_{asy,i^{s,y}} = \rho^1_{asy,i^{s,y}}/days_s$ where $\rho^1_{asyi^{s,y}} \geq 0$ and $\eta(i^{s,y}) > 0$ by definition. Therefore, $\tau^1_{asy,i^{s,y}} \geq 0$.

(d), (e) are contrapositives of statements (b) and (c), respectively. Therefore the conclusions follow. □

4.3.2 Producer

The production market's problem is denoted by (PR^S) It aggregates all producers $c \in C$ given the assumption of perfect competition in the marketplace for producers. Producers make decisions on how much to produce in the long-term market, denoted

q^0_{csy} i.e., the first-stage decision, and the spot market, denoted $q^1_{csy,i^{s,y}}$ i.e., the recourse decision. The nodal production prices faced by producers are denoted $\pi^0_{n^c(c)sy}$ and $\pi^1_{n^c(c)sy,i^{s,y}}$ for the long-term market and spot market, respectively. Both $\pi^0_{n^c(c)sy}$ and $\pi^1_{n^c(c)sy,i^{s,y}}$, exogenous for (PR^S) but overall variables in model S-NGEM, are derived from the market-clearing conditions (4.3.36) - (4.3.39) for the production market, by Theorem 4.3.5. Therefore, in the objective function (4.3.18) of (PR^S), the first term $days_s \pi^0_{n^c(c)sy} q^0_{csy}$ is the long-term seasonal gross income; the second term $\eta(i^{s,y}) days_s \pi^1_{n^c(c)sy,i^{s,y}} q^1_{csy,i^{s,y}}$ summed over all the possible outcomes $i^{s,y} \in I^{s,y}$ in season s and year y is the expected seasonal gross income in the spot market. The last term in the objective function $c^{PR}_c(q^0_{csy} + q^1_{csy,i^{s,y}})$ approximates the aggregate production costs incurred in the two markets, where $(q^0_{csy} + q^1_{csy,i^{s,y}})$ is the total production rate for random outcome $i^{s,y}$ in season s and year y. By $(q^0_{csy} + q^1_{csy,i^{s,y}})$, we assume that the long-term and spot market production rates are non-discriminative and additive, which is reasonable because the natural gas is generally considered as a homogeneous product. Similar assumptions can be found in the storage operator's problem (ST^S) and the peak gas operator's problem (PG^S). Alternatively, but more restrictively, we could separately write the costs as $c^{PR}_c(q^0_{csy}) + c^{PR}_c(q^1_{csy,i^{s,y}})$, which assumes the production costs for the long-term and spot markets are additive.

Constraints (4.3.19) stipulate upper bounds on the production capacity with the lower bounds being zero. Constraints (4.3.20) state that, for each scenario, the total gas produced to both markets in the time horizon cannot exceed the production forecast $prod_c$. The term $\sum_{y \in Y} \sum_{s \in S} days_s q^0_{csy}$ in constraints (4.3.20) is the total gas produced over time according to the long-term contracts. The total gas produced over the time horizon under each scenario is summed in the term $\sum_{\tilde{i} \in PD(i^{3,|Y|})} days_s q^1_{cs(\tilde{i})y(\tilde{i}),\tilde{i}}$, where the set $PD(i^{3,|Y|})$ keeps track of all random outcomes belonging to the same scenarios; the operators $s(i)$ and $y(i)$ point to the season and year associated with the realization i, respectively.

As in the model D-NGEM, the cost function $c_C^{PR}(\cdot)$ is assumed convex and continuously differentiable. Therefore, given that the constraints are affine, the KKT conditions are both necessary and sufficient to solving (PR^S) [5].

$$
\begin{aligned}
(PR^S) \quad \max \quad & \sum_{c\in C}\sum_{y\in Y}\sum_{s\in S} days_s \Big\{ \pi^0_{n^c(c)sy} q^0_{csy} \\
& + \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) \big[\pi^1_{n^c(c)sy,i^{s,y}} - c_c^{PR}(q^0_{csy} + q^1_{csy,i^{s,y}}) \big] \Big\} && (4.3.18) \\
\text{s.t.} \quad & q^0_{csy} + q^1_{csy,i^{s,y}} \le \overline{q}_c \qquad (\lambda^1_{csy,i^{s,y}} \ge 0) \quad \forall c, i^{s,y}, s, y && (4.3.19) \\
& \sum_{y\in Y}\sum_{s\in S} days_s q^0_{csy} + \sum_{\widetilde{i}\in PD(i^{3,|Y|})} days_s q^1_{cs(\widetilde{i})y(\widetilde{i}),\widetilde{i}} \le prod_c \\
& \qquad (\mu^1_{c,i^{3,|Y|}} \ge 0) \quad \forall c, i^{3,|Y|} && (4.3.20) \\
& 0 \le q^1_{csy,i^{s,y}}, \forall i^{s,y}, q^0_{csy} \qquad \forall c, s, y
\end{aligned}
$$

The KKT conditions to (PR^S) are:

$$
\begin{aligned}
& 0 \le days_s \Big[-\pi^0_{n^c(c)sy} + \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) \frac{\partial\big(c_c^{PR}(q^0_{csy} + q^1_{csy,i^{s,y}})\big)}{\partial(q^0_{csy})} \Big] + \sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}} \\
& \qquad + days_s \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} \perp q^0_{csy} \ge 0 \qquad \forall c, s, y && (4.3.21) \\
& 0 \le days_s \Big[-\eta(i^{s,y}) \pi^1_{n^c(c)sy,i^{s,y}} + \eta(i^{s,y}) \frac{\partial\big(c_c^{PR}(q^0_{csy} + q^1_{csy,i^{s,y}})\big)}{\partial(q^1_{csy,i^{s,y}})} \Big] + \lambda^1_{csy,i^{s,y}} \\
& \qquad + days_s \sum_{\widetilde{i}\in SC(i^{s,y})\bigcap I^{3,|Y|}} \mu^1_{c,\widetilde{i}} \perp q^1_{csy,i^{s,y}} \ge 0 \qquad \forall c, i^{s,y}, s, y && (4.3.22) \\
& 0 \le \overline{q}_c - q^0_{csy} - q^1_{csy,i^{s,y}} \perp \lambda^1_{csy,i^{s,y}} \ge 0 \qquad \forall c, i^{s,y}, s, y && (4.3.23) \\
& 0 \le prod_c - \Big(\sum_{y\in Y}\sum_{s\in S} days_s q^0_{csy} + \sum_{\widetilde{i}\in PD(i^{3,|Y|})} days_s q^1_{cs(\widetilde{i})y(\widetilde{i}),\widetilde{i}} \Big) \perp \mu^1_{c,i^{3,|Y|}} \ge 0 \\
& \qquad \forall c, i^{3,|Y|} && (4.3.24)
\end{aligned}
$$

For ease of presentation, hereafter we use $MC^{PR^S,0}_{csy,i^{s,y}}$ and $MC^{PR^S,1}_{csy,i^{s,y}}$ to denote the marginal cost functions in the long-term and spot markets, respectively. In particular,

$$MC^{PR^S,0}_{csy,i^{s,y}} \equiv \frac{\partial\Big(c^{PR}_c(q^0_{csy} + q^1_{csy,i^{s,y}})\Big)}{\partial(q^0_{csy})}, \forall c, i^{s,y}, s, y \tag{4.3.25}$$

$$MC^{PR^S,1}_{csy,i^{s,y}} \equiv \frac{\partial\Big(c^{PR}_c(q^0_{csy} + q^1_{csy,i^{s,y}})\Big)}{\partial(q^1_{csy,i^{s,y}})}, \forall c, i^{s,y}, s, y \tag{4.3.26}$$

Algebraically, given q^0_{csy} and $q^1_{csy,i^{s,y}}$, $MC^{PR^S,0}_{csy}$ and $MC^{PR^S,1}_{csy,i^{s,y}}$ have the same values, that is

$$MC^{PR^S,0}_{csy,i^{s,y}} = MC^{PR^S,1}_{csy,i^{s,y}} \tag{4.3.27}$$

In light of the mathematical structure of the KKT conditions (4.3.21) - (4.3.24), we define

$$v^{PR^S} \equiv \begin{pmatrix} q^0_{csy} \quad (\forall c, s, y) \\ q^1_{csy,i^{s,y}} \quad (\forall c, i^{s,y}, s, y) \\ \lambda^1_{csy,i^{s,y}} \quad (\forall c, i^{s,y}, s, y) \\ \mu^1_{c,i^{3,|Y|}} \quad (\forall c, i^{3,|Y|}) \end{pmatrix} \text{ and} \tag{4.3.28}$$

$$H^{PR^S}(v^{PR^S}) \equiv \begin{pmatrix} days_s\Big(-\pi^0_{n^c(c)sy} + MC^{PG^S,0}_{csy,i^{s,y}}\Big) + \sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}} \\ \qquad + days_s \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} \quad (\forall c, s, y) \\ days_s\Big[-\eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} + \eta(i^{s,y})MC^{PG^S,1}_{csy,i^{s,y}}\Big] + \lambda^1_{csy,i^{s,y}} \\ \qquad + days_s \sum_{\widetilde{i}\in SC(i^{s,y})\bigcap I^{3,|Y|}} \mu^1_{c,\widetilde{i}} \quad (\forall c, i^{s,y}, s, y) \\ \overline{q}_c - q^0_{csy} - q^1_{csy,i^{s,y}} \quad (\forall c, i^{s,y}, s, y) \\ prod_c - \Big(\sum_{y\in Y}\sum_{s\in S} days_s q^0_{csy} + \sum_{\widetilde{i}\in PD(i^{3,|Y|})} days_s q^1_{cs(\widetilde{i})y(\widetilde{i}),\widetilde{i}}\Big) \\ \qquad (\forall c, i^{3,|Y|}) \end{pmatrix}$$

(4.3.29)

Definitions of (4.3.28) and (4.3.29) allow the KKT conditions (4.3.21)-(4.3.24) for all producers $c \in C$ to be expressed equivalently as an NCP:

$$0 \leq v^{PR^S} \perp H^{PR^S}(v^{PR^S}) \geq 0 \tag{4.3.30}$$

Theorem 4.3.3 shows the relationship between the long-term market prices (π^0_{nsy}) and the expected spot market prices ($\sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y})\pi^1_{nsy,i^{s,y}}$) for the production market, similar to those established in Theorem 4.3.1 for the transportation market.

Theorem 4.3.3. *For a production node $n \in PN$, given q^0_{csy} and $q^1_{csy,i^{s,y}}$, if there exists some producer $c \in C^n$ such that*

(a) if the long-term production rate $q^0_{csy} > 0$, then the long-term production price is greater than or equal to the expected spot market price, that is, $\pi^0_{n^c(c)sy} \geq \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}}$; and likewise if

(b) if the spot market production rate $q^1_{csy,i^{s,y}} > 0, \forall i^{s,y} \in I^{s,y}$, then the long-term production price is less than or equal to the expected spot market price, that is, $\pi^0_{n^c(c)sy} \leq \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}}$;

(c) if the long-term production rate $q^0_{csy} > 0$ and the spot market production rate $q^1_{csy,i^{s,y}} > 0, \forall i^{s,y} \in I^{s,y}$, then the long-term production price is equal to the expected spot market price, that is, $\pi^0_{n^c(c)sy} = \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}}$.

Proof. (a) By (4.3.21), if $q^0_{csy} > 0$, we see that

$$\pi^0_{n^c(c)sy} = \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,0}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \tag{4.3.31}$$

Meanwhile, it follows by (4.3.22) that

$$\eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} \leq \eta(i^{s,y}) MC^{PR^S,1}_{csy,i^{s,y}} + \sum_{\widetilde{i}\in SC(i^{s,y})\bigcap I^{3,|Y|}} \mu^1_{c,\widetilde{i}} + \frac{\lambda^1_{csy,i^{s,y}}}{days_s}, \quad \forall i^{s,y} \tag{4.3.32}$$

Summing the above inequality for all $i^{s,y} \in I^{s,y}$, we have

$$\begin{aligned}
\sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} &\leq \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,1}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,\widetilde{i}} \\
&\quad + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \\
&= \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,0}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,\widetilde{i}} \\
&\quad + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \qquad \text{by (4.3.27)} \\
&= \pi^0_{n^c(c)sy} \qquad \text{by (4.3.31)}
\end{aligned}$$

Therefore, when $q^0_{csy} > 0$, $\pi^0_{n^c(c)sy} \geq \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\pi^1_{csy,i^{s,y}}$.

(b) By (4.3.22), if $q^1_{csy,i^{s,y}} > 0, \forall i^{s,y}$, it follows that

$$\eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} = \eta(i^{s,y}) MC^{PR^S,1}_{csy,i^{s,y}} + \sum_{\widetilde{i}\in SC(i^{s,y})\bigcap I^{3,|Y|}} \mu^1_{c,\widetilde{i}} + \frac{\lambda^1_{csy,i^{s,y}}}{days_s}, \quad \forall i^{s,y} \tag{4.3.33}$$

Summing the above equations over $i^{s,y} \in I^{s,y}$ results in the following:

$$\sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} = \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,1}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,\tilde{i}} + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \tag{4.3.34}$$

Also by (4.3.21), we see that

$$\begin{aligned}
\pi^0_{n^c(c)sy} &\leq \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,1}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \\
&= \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,0}_{csy,i^{s,y}} + \sum_{i^{3,|Y|}\in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} + \frac{\sum_{i^{s,y}\in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s} \\
&\qquad \text{by (4.3.27)} \\
&= \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}} \qquad \text{by (4.3.34)}
\end{aligned}$$

Clearly, when $q^1_{csy,i^{s,y}} > 0, \forall i^{s,y}$, $\pi^0_{n^c(c)sy} \leq \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\pi^1_{n^c(c)sy,i^{s,y}}$ holds.

(c) The conclusion follows immediately from (a) and (b). □

A key in the proof of Theorem 4.3.6 is that the long-term production rate q^0_{csy} and the spot market production rate $q^1_{csy,i^{s,y}}$ are assumed additive. For example, the cost function $c^{PR}_c(q^0_{csy} + q^1_{csy,i^{s,y}})$ based on this assumption leads to the fact that $MC^{PR^S,0}_{csy,i^{s,y}} = MC^{PR^S,1}_{csy,i^{s,y}}$ as stated in equation (4.3.27). Also, this assumption ensures that both q^0_{csy} and $q^1_{csy,i^{s,y}}$ are interchangeable in the capacity constraints (4.3.19) and (4.3.20). Without this assumption, the price relationship shown in Theorem 4.3.3 is not necessarily valid.

In order to derive an NCP/VI formulation, as is done in Chapter 3, assumptions of positive marginal costs with positive production are assumed for the model S-NGEM as well, e.g., Assumption 4.3.1 as shown below is the one for the producers.

Assumption 4.3.1. *Given q^0_{csy} and $q^1_{csy,i^{s,y}}$, the following statement holds producer $c \in C$ in season s, year y:*

$$MC^{PR^S,0}_{csy,i^{s,y}} = MC^{PR^S,1}_{csy,i^{s,y}} > 0, \quad \text{when } q^0_{csy} + q^1_{csy,i^{s,y}} > 0, \quad \forall i^{s,y} \tag{4.3.35}$$

Theorem 4.3.4 states that with Assumption 4.3.1 in force, the long-term market price is greater than zero when the production rate is positive. Similar theorems are also shown for the storage operators and peak gas operators. These sorts of theorems play a key role in the formation of the NCP/VI formulation with market-clearing conditions that has complementary nonnegative market prices.

Theorem 4.3.4. *Suppose that Assumption 4.3.1 holds for all producers located at a production node n. If there exists a producer $c \in C^n$ such that*

(a) their production rate in the long-term market is positive, that is, $q^0_{csy} > 0$, then so is the long-term production price at node n, that is, $\pi^0_{n^c(c)sy} > 0$;

(b) their production rate in the spot market is positive, that is, $q^1_{csy,i^{s,y}} > 0$, then so is the spot market production price at node n, that is, $\pi^1_{n^c(c)sy,i^{s,y}} > 0$.

Proof. (a) From (4.3.21), with positive q^0_{csy}, we must have

$$\pi^0_{n^c(c)sy} = \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) MC^{PR^S,0}_{csy,i^{s,y}} + \sum_{i^{3,|Y|} \in I^{3,|Y|}} \mu^1_{c,i^{3,|Y|}} + \frac{\sum_{i^{s,y} \in I^{s,y}} \lambda^1_{csy,i^{s,y}}}{days_s}$$

By definition, in the above equation, $\lambda^1_{csy,i^{s,y}}$ and $\mu^1_{c,i^{3,|Y|}}$ are nonnegative and $\eta(i^{s,y})$ is positive. The term $MC^{PR^S,0}_{csy,i^{s,y}}$ is positive by Assumption 4.3.1. Thus, $\pi^0_{n^c(c)sy}$ must be positive.

(b) From (4.3.22), with positive $q^1_{csy,i^{s,y}}$, we must have

$$\pi^1_{n^c(c)sy,i^{s,y}} = MC^{PR^S,1}_{csy,i^{s,y}} + \frac{\sum_{\widetilde{i}\in SC(i^{s,y})\bigcap I^{3,|Y|}} \mu^1_{c,\widetilde{i}}}{\eta(i^{s,y})} + \frac{\lambda^1_{csy,i^{s,y}}}{\eta(i^{s,y})days_s}$$

For the same reasons in part (a), $\pi^1_{n^c(c)sy,i^{s,y}}$ must be positive. □

The market equilibrium is enforced by the market-clearing conditions. Equations (4.3.36), for season 1, and (4.3.37) for seasons 2 and 3, are market-clearing conditions for the long-term production market. Analogously, equations (4.3.38) for season 1 and (4.3.39) for seasons 2 and 3 are market-clearing conditions for the spot market. The market-clearing conditions to the production market are as follows:

$$\sum_{c\in C^n} days_1 q^0_{c1y} = \sum_{a\in A(n)} \left(\sum_{r\in R(n_1(a))} days_1 g^0_{ary} + \sum_{m\in M(n_1(a))} days_1 h^0_{am1y} \right) \quad \forall n \in PN, y \qquad (4.3.36)$$

$$\sum_{c\in C^n} days_s q^0_{csy} = \sum_{a\in A(n)} \sum_{m\in M(n_1(a))} days_s h^0_{amsy} \quad \forall n \in PN, s = 2,3, y \qquad (4.3.37)$$

$$\sum_{c\in C^n} days_1 q^1_{c1y,i^{1,y}} = \sum_{a\in A(n)} \left(\sum_{r\in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} + \sum_{m\in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}} \right) \quad \forall n \in PN, i^{1,y}, y \qquad (4.3.38)$$

$$\sum_{c\in C^n} days_s q^1_{csy,i^{s,y}} = \sum_{a\in A(n)} \sum_{m\in M(n_1(a))} days_s h^1_{amsy,i^{s,y}} \quad \forall n \in PN, i^{s,y}, s = 2,3, y \quad (4.3.39)$$

Equations (4.3.40) - (4.3.41) amend the marketing-clearing conditions presented in (4.3.36) - (4.3.39) by relaxing the equalities to inequalities which imply that the total supply is greater than or equal to the total demand. The nonnegative market prices π^0_{nsy} and $\pi^1_{nsy,i^{s,y}}$ are the dual variables to the corresponding market-clearing conditions, for instance, $\pi^1_{n1y,i^{1,y}}$ is the dual price to the (4.3.42), the market-clearing condition for season 1 in the spot market.

$$0 \leq \sum_{c\in C^n} days_1 q^0_{c1y} - \sum_{a\in A(n)} \left(\sum_{r\in R(n_1(a))} days_1 g^0_{ary} + \sum_{m\in M(n_1(a))} days_1 h^0_{am1y} \right)$$
$$\perp \pi^0_{n1y} \geq 0 \quad \forall n \in PN, y \qquad (4.3.40)$$

$$0 \leq \sum_{c\in C^n} days_s q^0_{csy} - \sum_{a\in A(n)} \sum_{m\in M(n_1(a))} days_s h^0_{amsy} \perp \pi^0_{nsy} \geq 0$$
$$\forall n \in PN, s = 2,3, y \quad (4.3.41)$$

$$0 \leq \sum_{c\in C^n} days_1 q^1_{c1y,i^{1,y}} - \sum_{a\in A(n)} \left(\sum_{r\in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} + \sum_{m\in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}} \right)$$
$$\perp \pi^1_{n1y,i^{1,y}} \geq 0 \quad \forall n \in PN, i^{1,y}, y \qquad (4.3.42)$$

$$0 \leq \sum_{c\in C^n} days_s q^1_{csy,i^{s,y}} - \sum_{a\in A(n)} \sum_{m\in M(n_1(a))} days_s h^1_{amsy,i^{s,y}} \perp \pi^1_{nsy,i^{s,y}} \geq 0$$
$$\forall n \in PN, i^{s,y}, s = 2,3, y \quad (4.3.43)$$

Theorem 4.3.5 validates that, in the entirety of the model S-NGEM, (4.3.36) - (4.3.39) along with nonnegative π^0_{nsy} and $\pi^1_{nsy,i^{s,y}}$ are equivalent to (4.3.40) - (4.3.43). With this result, the market-clearing conditions properly coupled with nonnegative prices are an NCP/VI per se.

Theorem 4.3.5. *If Assumption 4.3.1 holds for all $c \in C$, then the system S-PR-MCC is equivalent to the system S-PR-MCC-NCP, where*

$$\textit{S-PR-MCC} \equiv \begin{cases} NCP(4.3.30) \\ (4.3.36) - (4.3.39) \\ \pi^0_{nsy} \geq 0, \quad \forall n \in PN, s, y \\ \pi^1_{nsy,i^{s,y}} \geq 0, \forall i^{s,y}, n \in PN, s, y \end{cases} \qquad (4.3.44)$$

$$S\text{-}PR\text{-}MCC\text{-}NCP \equiv \begin{cases} NCP(4.3.30) \\ (4.3.40)-(4.3.43) \end{cases} \tag{4.3.45}$$

Proof. By construction, any solution satisfying *S-PR-MCC* also satisfies *S-PR-MCC-NCP*. Therefore, we must show that every solution to *S-PR-MCC-NCP* will be a solution to *S-PR-MCC* as well. Suppose the contrary, that there exists a solution satisfying *S-PR-MCC-NCP* such that when $s = 1$ for some $n \in PN, y$:

$$0 < \sum_{c \in C^n} days_1 q^0_{c1y} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g^0_{ary} + \sum_{m \in M(n_1(a))} days_1 h^0_{am1y} \right) \text{ and } \pi^0_{n1y} = 0 \tag{4.3.46}$$

or when $s = 2$ or 3, for some $n \in PN, y$:

$$0 < \sum_{c \in C^n} days_s q^0_{csy} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_s h^0_{amsy} \text{ and } \pi^0_{nsy} = 0 \tag{4.3.47}$$

or when $s = 1$, for some $i^{s,y}, n \in PN, y$:

$$0 < \sum_{c \in C^n} days_1 q^1_{c1y,i^{1,y}} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} + \sum_{m \in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}} \right) \text{ and } \pi^1_{n1y,i^{1,y}} = 0 \tag{4.3.48}$$

or when $s = 2$ or 3, for some $i^{s,y}, n \in PN, y$:

$$0 < \sum_{c \in C^n} days_s q^1_{csy,i^{s,y}} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_s h^1_{amsy,i^{s,y}} \text{ and } \pi^1_{nsy,i^{s,y}} = 0 \tag{4.3.49}$$

It must respectively follow from (4.3.46)-(4.3.47) and (4.3.48)-(4.3.49) that for some $n \in PN, y$,

$$0 < q^0_{csy} \quad \exists c \in C^n \quad \text{and} \quad \pi^0_{nsy} = 0 \tag{4.3.50}$$

or for some $i^{s,y}, n \in PN, y$,

$$0 < q^1_{csy,i^{s,y}} \quad \exists c \in C^n \quad \text{and} \quad \pi^1_{nsy,i^{s,y}} = 0 \tag{4.3.51}$$

However, by Theorem 4.3.4, it follows from $0 < q^0_{csy}, \exists c \in C^n$ and $0 < q^1_{csy,i^{s,y}}, \exists c \in C^n$ that $\pi^0_{nsy} > 0$ and $\pi^1_{nsy,i^{s,y}} > 0$, respectively, for producer c is located at node n. This contradicts (4.3.50) and (4.3.51), respectively. In other words, none of the cases presented in (4.3.46) - (4.3.49) are possible. Consequently, every solution of *S-PR-MCC-NCP* is also a solution to *S-PR-MCC*. This completes the proof. $\square$

4.3.3 Storage Operator

The storage market is modeled as a perfectly competitive market. ST^S as shown below is the optimization problem for the storage operators. The objective of problem (ST^S) is to maximize the profits gained in the long-term market plus expected net profits, expected income minus expected costs, gained both in the spot market.

The incomes for storage operators come from the sales of the gas to marketers. Since it is a perfectly competitive market, the storage operators have no power to influence the prices for the storage gas, that is, $\gamma^0_{n^r(r)sy}$ and $\gamma^1_{n^r(r)sy,i^{s,y}}$ are exogenous to problem (ST^S). But the storage operators decide on the gas rates (x^0_{rsy} and $x^1_{rsy,i^{s,y}}$) sold to the marketers in seasons 2 and 3.

The costs for the storage operators fall into three parts: commodity charges, transportation costs and production costs. The commodity charge rates, $\pi^0_{n_2(a)1y}$

and $\pi^1_{n_2(a)1y,i^{s,y}}$, are fixed for problem (ST^S). Storage operators pay the pipeline charges, including regulated charges (τ^{reg}_{a1y}), congestion charges $(\tau^0_{a1y}$ and $\tau^1_{a1y,i^{1,y}})$ and reservation charges (RC^0_{asy}) for firm service. Unlike a fixed charge, the reservation charge, RC^0_{asy} is a charge for per unit shipped by firm contracts. The production cost is approximated by a cost function $c^{ST}_r(\cdot)$ which is a function of the total extraction rates in the long-term and spot markets, i.e., $x^0_{rsy} + x^1_{rsy,i^{1,y}}$. This setting of the cost functions differs from the problem (ST) in model D-NGEM in Chapter 3, where the storage cost functions take the injection rates as arguments. These two settings of the cost functions are in essence the same because injection and extraction rates are related to one another by mass balance constraints. Nevertheless, in model D-NGEM, all costs are incurred in season 1 and thus there is no explicit cost relevant to the gas extraction sent to the marketers; while the new cost functions aim to build a direct cost connection between storage operators and marketers.

One type of constraints needed for the problem (ST^S) is the mass balance constraint, whose counterpart in model D-NGEM is constraint (3.1.38), which stipulates that the total extraction should be equal to the total injection after taking into account appropriate losses. However, in order to establish further theoretical results for model S-NGEM, the mass balance constraint is relaxed as shown in constraints (4.3.53) and (4.3.54), which require that the total exaction be less or equal to the total injection after pipeline and storage losses for each storage operator $r \in R$ in a year. As a result, under some circumstance, storage operators could have gas left unsold at the end of the year. Although, in practice, unsold gas is passed on to the next year's inventory, the model S-NGEM currently leaves the leftover gas at the end of each year out of the system. However, this is more of an abstract question since all the numerical results presented in Chapter 5 show that storage operators cleared their gas reservoirs in all instances. One possibility is that model S-NGEM has some mechanism to clear the storage gas, which are unknown to us so far.

Constraints (4.3.55) and (4.3.56) stipulate upper bounds on the injection and exaction rate, which are nonnegative variables. Constraints (4.3.57) impose the maximum volume of the working gas for the storage facilities owned by each storage operator.

$$
\begin{aligned}
(ST^S) \quad \max \quad & \sum_{r\in R}\sum_{y\in Y}\Bigg\{\sum_{s=2,3} days_s\Big(\gamma^0_{n^r(r)sy}x^0_{rsy} + \sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})\gamma^1_{n^r(r)sy,i^{s,y}}x^1_{rsy,i^{s,y}}\Big) \\
& - days_1\Bigg[\sum_{a\in A(n^r(r))}\left(\tau^0_{a1y}+\tau^{reg}_{a1y}+\pi^0_{n_2(a)1y}+RC^0_{asy}\right)g^0_{ary} \\
& + \sum_{i^{1,y}\in I^{1,y}}\eta(i^{1,y})\sum_{a\in A(n^r(r))}(\tau^1_{a1y,i^{1,y}}+\tau^{reg}_{a1y}+\pi^1_{n_2(a)1y,i^{s,y}})g^1_{ary,i^{1,y}}\Bigg] \\
& - \sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})c^{ST}_r\Big(x^0_{rsy}+x^1_{rsy,i^{s,y}}\Big)\Bigg\} \qquad (4.3.52)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & days_1\sum_{a\in A(n^r(r))} g^0_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x^0_{rsy} \ge 0 \\
& \qquad (\delta^0_{ry}\ge 0) \quad \forall r,y \qquad (4.3.53)
\end{aligned}
$$

$$
\begin{aligned}
& days_1\sum_{a\in A(n^r(r))} g^1_{ary,\Psi(i^{3,y})}(1-loss_a)(1-loss_r) - days_2 x^1_{r2y,\psi(i^{3,y})} \\
& \quad - days_3 x^1_{r3y,i^{3,y}} \ge 0 \quad (\delta^1_{ry,i^{3,y}}\ge 0) \quad \forall i^{3,y},r,y \qquad (4.3.54)
\end{aligned}
$$

$$
x^0_{rsy}+x^1_{rsy,i^{s,y}} \le \overline{x}_r \qquad (\omega^1_{rsy,i^{s,y}}\ge 0) \quad \forall i^{s,y},r,s=2,3,y \qquad (4.3.55)
$$

$$
\sum_{a\in A(n^r(r))}(g^0_{ary}+g^1_{ary,i^{1,y}}) \le \overline{g}_r \quad (\xi^1_{ry,i^{1,y}}\ge 0) \quad \forall i^{1,y},r,y \qquad (4.3.56)
$$

$$
\begin{aligned}
& \sum_{s=2,3} days_s x^0_{rsy} + days_2 x^1_{r2y,\psi(i^{3,y})} + days_3 x^1_{r3y,i^{3,y}} \le \overline{k}_r \\
& \qquad (\zeta^1_{ry,i^{3,y}}\ge 0) \quad \forall i^{3,y},r,y \qquad (4.3.57)
\end{aligned}
$$

$$
0 \le g^0_{ary}, \forall a\in A(n^r(r)), x^0_{r2y}, x^0_{r3y} \qquad \forall r,y
$$

$$
0 \le g^1_{ary,i^{1,y}}, \forall a\in A(n^r(r)), x^1_{r2y,i^{2,y}}, x^1_{r3y,i^{3,y}} \qquad \forall i,r,y
$$

Assuming that the cost functions $c_r^{ST}(\cdot)$ are convex and continuously differentiable, the KKT conditions are both necessary and sufficient for solving the problem ST^S since it has a concave objective funtion being maximized and the constraint set is polyhedral [5]. The KKT conditions to (ST^S) are:

$$0 \leq days_s\Big[-\gamma^0_{n^r(r)sy} + \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\frac{\partial c_r^{ST}\big(x^0_{rsy} + x^1_{rsy,i^{s,y}}\big)}{\partial x^0_{rsy}} + \delta^0_{ry} + \sum_{i^{3,y}\in I^{3,y}} \zeta^1_{ry,i^{3,y}}\Big]$$
$$+ \sum_{i^{s,y}\in I^{s,y}} \omega^1_{rsy,i^{s,y}} \perp x^0_{rsy} \geq 0 \quad \forall r, s = 2, 3, y \qquad (4.3.58)$$

$$0 \leq days_2\Big[-\eta(i^{2,y})\gamma^1_{n^r(r)2y,i^{2,y}} + \eta(i^{2,y})\frac{\partial c_r^{ST}\big(x^0_{r2y} + x^1_{r2y,i^{2,y}}\big)}{\partial x^1_{r2y,i^{2,y}}} + \sum_{\widetilde{i}\in ISC(i^{2,y})} \delta^1_{ry,\widetilde{i}}$$
$$+ \sum_{\widetilde{i}\in ISC(i^{2,y})} \zeta^1_{ry,\widetilde{i}}\Big] + \omega^1_{r2y,i^{2,y}} \perp x^1_{r2y,i^{2,y}} \geq 0 \qquad \forall i^{2,y}, r, y \qquad (4.3.59)$$

$$0 \leq days_3\Big[-\eta(i^{3,y})\gamma^1_{n^r(r)3y,i^{3,y}} + \eta(i^{3,y})\frac{\partial c_r^{ST}\big(x^0_{r3y} + x^1_{r3y,i^{3,y}}\big)}{\partial x^1_{r3y,i^{3,y}}} + \delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}}\Big]$$
$$+ \omega^1_{r3y,i^{3,y}} \perp x^1_{r3y,i^{3,y}} \geq 0 \quad \forall i^{3,y}, r, y \qquad (4.3.60)$$

$$0 \leq days_1\Big[\tau^0_{a1y} + \tau^{reg}_{a1y} + \pi^0_{n_2(a)1y} + RC^0_{a1y} - \delta^0_{ry}(1-loss_a)(1-loss_r)\Big]$$
$$+ \sum_{i^{1,y}\in I^{1,y}} \xi^1_{ry,i^{1,y}} \perp g^0_{ary} \geq 0 \qquad \forall a \in A(n^r(r)), r, y \qquad (4.3.61)$$

$$0 \leq days_1\Big[\eta(i^{1,y})\big(\tau^1_{a1y,i^{1,y}} + \tau^{reg}_{a1y} + \pi^1_{n_2(a)1y,i^{1,y}}\big)$$
$$- \sum_{\widetilde{i}\in IISC(i^{1,y})} \delta^1_{ry,\widetilde{i}}(1-loss_a)(1-loss_r)\Big] + \xi^1_{ry,i^{1,y}} \perp g^1_{ary,i^{1,y}} \geq 0$$
$$\forall a \in A(n^r(r)), i^{1,y}, r, y \qquad (4.3.62)$$

$$0 \leq days_1 \sum_{a\in A(n^r(r))} g^0_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x^0_{rsy} \perp \delta^0_{ry} \geq 0$$
$$\forall r, y \qquad (4.3.63)$$

$$0 \leq days_1 \sum_{a\in A(n^r(r))} g^1_{ary,\Psi(i^{3,y})}(1-loss_a)(1-loss_r) - days_2 x^1_{r2y,\psi(i^{3,y})}$$
$$-days_3 x^1_{r3y,i^{3,y}} \perp \delta^1_{ry,i^{3,y}} \geq 0 \quad \forall i^{3,y}, r, y \qquad (4.3.64)$$

$$0 \le \overline{x}_r - x^0_{rsy} - x^1_{rsy,i^{s,y}} \perp \omega^1_{rsy,i^{s,y}} \ge 0 \qquad \forall i^{s,y}, r, s = 2,3, y \quad (4.3.65)$$

$$0 \le \overline{g}_r - \sum_{a \in A(n^r(r))} (g^0_{ary} + g^1_{ary,i^{1,y}}) \perp \xi^1_{ry,i^{1,y}} \ge 0 \qquad \forall i^{1,y}, r, y \quad (4.3.66)$$

$$0 \le \overline{k}_r - \sum_{s=2,3} days_s x^0_{rsy} - days_2 x^1_{r2y,\psi(i^{3,y})} - days_3 x^1_{r3y,i^{3,y}} \perp \zeta^1_{ry,i^{3,y}} \ge 0$$

$$\forall i^{3,y}, r, y \quad (4.3.67)$$

For brevity, hereafter, we use two simplified terms $MC^{ST^S,0}_{rsy,i^{s,y}}$ and $MC^{ST^S,1}_{rsy,i^{1,y}}$ to denote the marginal cost functions in the long-term and spot markets, respectively. That is,

$$MC^{ST^S,0}_{rsy,i^{s,y}} \equiv \frac{\partial c^{ST}_r \left(x^0_{rsy} + x^1_{rsy,i^{s,y}} \right)}{\partial x^0_{rsy}} \quad (4.3.68)$$

$$MC^{ST^S,1}_{rsy,i^{s,y}} \equiv \frac{\partial c^{ST}_r \left(x^0_{rsy} + x^1_{rsy,i^{s,y}} \right)}{\partial x^1_{rsy,i^{s,y}}} \quad (4.3.69)$$

As is done in Section 4.3.2, given x^0_{rsy} and $x^1_{rsy,i^{s,y}}$, it is true that the marginal cost functions for the two markets are the same, i.e.,

$$MC^{ST^S,0}_{rsy,i^{s,y}} = MC^{ST^S,1}_{rsy,i^{s,y}} \quad (4.3.70)$$

Naturally, the KKT conditions to problem (ST^S) are equivalent to an NCP as follows:

$$0 \le v^{ST^S} \perp H^{ST^S}(v^{ST^S}) \ge 0 \quad (4.3.71)$$

where

$$v^{ST^S} \equiv \begin{pmatrix} x^0_{rsy} \quad (\forall r, s = 2, 3, y) \\ x^1_{rsy,i^{s,y}} \quad (\forall i^{s,y}, r, s = 2, 3, y) \\ g^0_{ary} \quad (\forall a \in A(n^r(r)), r, y) \\ g^1_{ary,i^{1,y}} \quad (\forall a \in A(n^r(r)), i^{1,y}, r, y) \\ \delta^0_{ry} \quad (\forall r, y) \\ \delta^1_{ry,i^{3,y}} \quad (\forall i^{3,y}, r, y) \\ \omega^1_{rsy,i^{s,y}} \quad (\forall i^{s,y}, r, s = 2, 3, y) \\ \xi^1_{ry,i^{1,y}} \quad (\forall i^{1,y}, r, y) \\ \zeta^1_{ry,i^{3,y}} \quad (\forall i^{3,y}, r, y) \end{pmatrix} \text{ and} \tag{4.3.72}$$

$$
H^{ST^S}(v^{ST^S}) \equiv \left(\begin{array}{l}
days_s\big[-\gamma^0_{n^r(r)sy} + \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y}) MC^{ST^S,0}_{rsy,i^{s,y}} + \delta^0_{ry} + \sum_{i^{3,y}\in I^{3,y}} \zeta^1_{ry,i^{3,y}}\big] \\
\qquad\qquad + \sum_{i^{s,y}\in I^{s,y}} \omega^1_{rsy,i^{s,y}} \quad (\forall r, s=2,3,y) \\
days_2\big[-\eta(i^{2,y})\gamma^1_{n^r(r)2y,i^{2,y}} + \eta(i^{2,y}) MC^{ST^S,1}_{rsy,i^{2,y}} \\
\qquad\qquad + \sum_{\widetilde{i}\in ISC(i^{2,y})} (\delta^1_{ry,\widetilde{i}} + \zeta^1_{ry,\widetilde{i}})\ \big] + \omega^1_{r2y,i^{2,y}} \quad (\forall i^{2,y}, r, y) \\
days_3\big[-\eta(i^{3,y})\gamma^1_{n^r(r)3y,i^{3,y}} + \eta(i^{3,y}) MC^{ST^S,1}_{rsy,i^{3,y}} \\
\qquad\qquad + \delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}}\big] + \omega^1_{r3y,i^{3,y}} \quad (\forall i^{3,y}, r, y) \\
days_1\Big[\tau^0_{a1y} + \tau^{reg}_{a1y} + \pi^0_{n_2(a)1y} + RC^0_{asy} - \delta^0_{ry}(1-loss_a)(1-loss_r)\Big] \\
\qquad\qquad + \sum_{i^{1,y}\in I^{1,y}} \xi^1_{ry,i^{1,y}} \quad (\forall a \in A(n^r(r)), r, y) \\
days_1\Big[\eta(i^{1,y})\big(\tau^1_{a1y,i^{1,y}} + \tau^{reg}_{a1y} + \pi^1_{n_2(a)1y,i^{1,y}}\big) \\
\qquad\qquad - \sum_{\widetilde{i}\in IISC(i^{1,y})} \delta^1_{ry,\widetilde{i}}(1-loss_a)(1-loss_r)\Big] + \xi^1_{ry,i^{1,y}} \\
\qquad\qquad\qquad \forall a \in A(n^r(r)), i^{1,y}, r, y \\
days_1 \sum_{a\in A(n^r(r))} g^0_{ary}(1-loss_a)(1-loss_r) - \sum_{s=2,3} days_s x^0_{rsy} \quad (\forall r, y) \\
days_1 \sum_{a\in A(n^r(r))} g^1_{ary,\Psi(i^{3,y})}(1-loss_a)(1-loss_r) \\
\qquad\qquad - days_2 x^1_{r2y,\psi(i^{3,y})} - days_3 x^1_{r3y,i^{3,y}} \quad (\forall i^{3,y}, r, y) \\
\overline{x}_r - x^0_{rsy} - x^1_{rsy,i^{s,y}} \quad (\forall i^{s,y}, r, s=2,3,y) \\
\overline{g}_r - \sum_{a\in A(n^r(r))} (g^0_{ary} + g^1_{ary,i^{1,y}}) \quad (\forall i^{1,y}, r, y) \\
\overline{k}_r - \sum_{s=2,3} days_s x^0_{rsy} - days_2 x^1_{r2y,\psi(i^{3,y})} - days_3 x^1_{r3y,i^{3,y}} \quad (\forall i^{3,y}, r, y)
\end{array}\right) \tag{4.3.73}
$$

The first property we show for problem (ST^S) is the price relationship between the long-term and spot markets. This is accomplished by Lemma 4.3.1 and Theorem 4.3.6.

Lemma 4.3.1. *Considering a particular storage operator r in season s=2 or 3 of year y, if the long-term extraction rate $x^0_{rsy} > 0$, then $\delta^0_{ry} > \sum_{i^{3,y}\in I^{3,y}} \delta^1_{ry,i^{3,y}}$.*

Proof. If $x^0_{rsy} > 0$, then by (4.3.63), there exists some $a \in A(n^r(r))$ such that $g^0_{ary} > 0$, which by (4.3.61) implies that:

$$\delta^0_{ry} = \frac{\tau^0_{a1y} + \tau^{reg}_{a1y} + \pi^0_{n_2(a)1y} + RC^0_{a1y} + \frac{\sum_{i^{1,y}\in I^{1,y}} \xi^1_{ry,i^{1,y}}}{days_1}}{(1-loss_a)(1-loss_r)} \tag{4.3.74}$$

On the other hand, by (4.3.62), we see that for all $a \in A(n^r(r))$

$$\sum_{\tilde{i}\in IISC(i^{1,y})} \delta^1_{ry,\tilde{i}} \leq \frac{\eta(i^{1,y})\tau^1_{a1y,i^{1,y}} + \eta(i^{1,y})\tau^{reg}_{a1y} + \eta(i^{1,y})\pi^1_{n_2(a)1y,i^{1,y}} + \frac{\xi^1_{ry,i^{1,y}}}{days_1}}{(1-loss_a)(1-loss_r)} \qquad \forall i^{1,y} \tag{4.3.75}$$

Summing the above equation for all $i^{1,y} \in I^{1,y}$ and recall that $\sum_{i^{3,y}\in I^{3,y}}(\cdot) = \sum_{i^{1,y}\in I^{1,y}} \sum_{\tilde{i}\in IISC(i^{1,y})}(\cdot)$, we have

$$\begin{aligned}
\sum_{i^{3,y}\in I^{3,y}} \delta^1_{ry,i^{3,y}} &\leq \frac{\sum_{i^{1,y}\in I^{1,y}} \eta(i^{1,y})\left(\tau^1_{a1y,i^{1,y}} + \pi^1_{n_2(a)1y,i^{1,y}} + \tau^{reg}_{a1y}\right) + \frac{\sum_{i^{1,y}\in I^{1,y}} \xi^1_{ry,i^{1,y}}}{days_1}}{(1-loss_a)(1-loss_r)} \\
&\leq \frac{\tau^0_{a1y} + \pi^0_{n_2(a)1y} + \tau^{reg}_{a1y} + \frac{\sum_{i^{1,y}\in I^{1,y}} \xi^1_{ry,i^{1,y}}}{days_1}}{(1-loss_a)(1-loss_r)} \\
&\qquad\qquad \text{by Theorems 4.3.1 and 4.3.3} \\
&< \delta^0_{ry} \qquad\qquad \text{by (4.3.74)}
\end{aligned}$$

Because $g^0_{ary} > 0$ for some $a \in A(n^r(r))$, we deduce that $q^0_{c1y} > 0$ for some $c \in C^{n_2(a)}$ by (4.3.36) and $f^0_{a1y} > 0$ by (4.3.14), both of which further imply that $\pi^0_{n^c(c)1y} \geq \sum_{i^{1,y} \in I^{1,y}} \eta(i^{1,y}) \pi^1_{n^c(c)1y,i^{1,y}}$, where $n^c(c)$ refers to the node where c is located, i.e., $n_2(a)$ in this case; and $\tau^0_{a1y} \geq \sum_{i^{1,y} \in I^{1,y}} \eta(i^{1,y}) \tau^1_{a1y,i^{1,y}}$ by Theorems 4.3.3 and 4.3.1, respectively. Furthermore, given that RC^0_{asy} is a predetermined positive rate, the conclusion follows $\delta^0_{ry} > \sum_{i^{3,y} \in I^{3,y}} \delta^1_{ry,i^{3,y}}$. □

Theorem 4.3.6. *If there exists a storage operator r in season s=2 or 3 year y, such that if the long-term extraction rate $x^0_{rsy} > 0$, then the price in the long-term market is greater than the expected one in the spot market, that is, $\gamma^0_{n^r(r)sy} > \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) \gamma^1_{n^r(r)y,i^{s,y}}$;*

Proof. First, by (4.3.63), the premise of some $x^0_{rsy} > 0$ implies that some $g^0_{ary} > 0$ which indicates, by Lemma 4.3.1, that $\delta^0_{ry} > \sum_{i^{3,y} \in I^{3,y}} \delta^1_{ry,i^{3,y}}$. In addition, by (4.3.58), if some $x^0_{rsy} > 0$, we see that

$$\gamma^0_{n^r(r)sy} = \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) MC^{ST^S,0}_{rsy,i^{s,y}} + \delta^0_{ry} + \sum_{i^{3,y} \in I^{3,y}} \zeta^1_{ry,i^{3,y}} + \frac{\sum_{i^{s,y} \in I^{s,y}} \omega^1_{rsy,i^{3,y}}}{days_s} \tag{4.3.76}$$

From (4.3.59) for the season 2, we see that

$$\eta(i^{2,y}) \gamma^1_{n^r(r)2y,i^{2,y}} \leq \eta(i^{2,y}) MC^{ST^S,1}_{r2y,i^{2,y}} + \sum_{\tilde{i} \in ISC(i^{2,y})} \left(\delta^1_{ry,\tilde{i}} + \zeta^1_{ry,\tilde{i}} \right) + \frac{\omega^1_{rsy,i^{2,y}}}{days_2}, \quad \forall i^{2,y} \tag{4.3.77}$$

Similarly, summing this inequality for all $i^{2,y} \in I^{2,y}$, we have

$$\sum_{i^{2,y} \in I^{2,y}} \eta(i^{2,y}) \gamma^1_{n^r(r)2y,i^{2,y}} \leq \sum_{i^{2,y} \in I^{2,y}} \eta(i^{2,y}) MC^{ST^S,1}_{r2y,i^{2,y}} + \sum_{i^{3,y} \in I(3,y)} \left(\delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}} \right) + \sum_{i^{2,y} \in I^{2,y}} \frac{\omega^1_{rsy,i^{2,y}}}{days_2} \tag{4.3.78}$$

Also, by (4.3.60) for season 3, it follows that

$$\eta(i^{3,y})\gamma^1_{n^r(r)3y,i^{3,y}} \leq \eta(i^{3,y})MC^{ST^S,1}_{r3y,i^{3,y}} + \delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}} + \frac{\omega^1_{r3y,i^{3,y}}}{days_3}, \quad \forall i^{3,y} \tag{4.3.79}$$

Summing this inequality for all $i^{3,y} \in I^{3,y}$, we have

$$\begin{aligned}\sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\gamma^1_{n^r(r)3y,i^{3,y}} \leq \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})MC^{ST^S,1}_{r3y,i^{3,y}} + \sum_{i^{3,y}\in I^{3,y}} (\delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}})\\ + \sum_{i^{3,y}\in I^{3,y}} \frac{\omega^1_{r3y,i^{3,y}}}{days_3}\end{aligned} \tag{4.3.80}$$

Combining equations (4.3.78) and (4.3.80) results in one equation as follows for $s = 2, 3$:

$$\begin{aligned}\sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})\gamma^1_{n^r(r)sy,i^{s,y}} \leq & \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})MC^{ST^S,1}_{rsy,i^{s,y}} + \sum_{i^{3,y}\in I^{3,y}} (\delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}})\\ & + \sum_{i^{s,y}\in I^{s,y}} \frac{\omega^1_{rsy,i^{s,y}}}{days_s} \qquad (4.3.81)\\ = & \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})MC^{ST^S,0}_{rsy,i^{s,y}} + \sum_{i^{3,y}\in I^{3,y}} (\delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}})\\ & + \sum_{i^{s,y}\in I^{s,y}} \frac{\omega^1_{rsy,i^{s,y}}}{days_s} \qquad \text{by (4.3.70)}\\ < & \sum_{i^{s,y}\in I^{s,y}} \eta(i^{s,y})MC^{ST^S,0}_{rsy,i^{s,y}} + \delta^0_{ry} + \sum_{i^{3,y}\in I^{3,y}} \zeta^1_{ry,i^{3,y}}\\ & + \sum_{i^{s,y}\in I^{s,y}} \frac{\omega^1_{rsy,i^{s,y}}}{days_s} \qquad \text{by Lemma 4.3.1}\\ = & \gamma^0_{n^r(r)sy} \qquad \text{by (4.3.76)}\end{aligned}$$

This completes the proof. □

The above theorem shows that as long as the extraction rate in the long-term market is positive, the long-term market price is always greater than the expected spot market price. This is unlike what was done in Theorems 4.3.1, 4.3.3 for pipeline operators, producers and Theorem 4.3.9 to be presented for peak gas operators, respectively. As we can see from the proof, this difference attributes to the existence of reservation charges, RC^0_{asy} for the long-term market, which are not present in problems (PL^S), (PR^S) and (PG^S).

Next, an important assumption regarding the marginal cost functions of the storage operators is presented as follows.

Assumption 4.3.2. *The marginal cost functions of storage operator $r \in R$ meet the following conditions in seasons 2 and 3 for all years:*

$$MC^{ST^S,0}_{rsy,i^{s,y}} = MC^{ST^S,1}_{rsy,i^{s,y}} > 0, \quad \text{when } x^0_{rsy} + x^1_{rsy,i^{s,y}} > 0, \quad \forall i^{s,y} \tag{4.3.82}$$

When positive marginal cost functions with positive extraction rates are assumed (Assumption 4.3.2), Theorem 4.3.7 shows for the problem (ST^S) that positive extraction rates result positive market prices for both the long-term and spot markets.

Theorem 4.3.7. *Suppose that Assumption 4.3.2 holds for all storage operators located at a consumption node n. If there exists a storage operator $r \in R^n$ who*

(a) has a positive long-term extraction rate in season s, year y, i.e., $x^0_{rsy} > 0$, then the corresponding long-term storage gas price for that season at node n is positive too, that is, $\gamma^0_{n^r(r)sy} > 0$;

(b) has a positive extraction rate in season s year y in the spot market, i.e., $x^1_{rsy,i^{s,y}} > 0$, for some $i^{s,y}$, then the corresponding spot market storage gas price is positive too, that is, $\gamma^1_{n^r(r)sy,i^{1,y}} > 0$ for that $i^{s,y}$.

Proof. (a) When $x^0_{rsy} > 0$, by (4.3.58), we see that

$$\gamma^0_{n^r(r)sy} = \sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) MC^{ST^S,0}_{rsy,i^{s,y}} + \delta^0_{ry} + \sum_{i^{3,y} \in I^{3,y}} \zeta^1_{ry,i^{3,y}} + \frac{\sum_{i^{s,y} \in I^{s,y}} \omega^1_{rsy,i^{3,y}}}{days_s} \quad (4.3.83)$$

where $\sum_{i^{s,y} \in I^{s,y}} \eta(i^{s,y}) MC^{ST^S,0}_{rsy,i^{s,y}}$ is positive because of Assumption 4.3.2 and that fact that $\eta(i^{s,y}) > 0$; $\delta^0_{ry} > 0$ as shown in Lemma 4.3.1 as $x^0_{rsy} > 0$; $\zeta^1_{ry,i^{3,y}}$ and $\omega^1_{rsy,i^{s,y}}$ are nonnegative as defined. Therefore, $\gamma^0_{n^r(r)sy} > 0$.

(b) When $x^1_{rsy,i^{s,y}} > 0$, by (4.3.59) for season 2 we see that

$$\eta(i^{2,y}) \gamma^1_{n^r(r)2y,i^{2,y}} = \eta(i^{2,y}) MC^{ST^S,1}_{rsy,i^{2,y}} + \sum_{\widetilde{i} \in ISC(i^{2,y})} \left(\delta^1_{ry,\widetilde{i}} + \zeta^1_{ry,\widetilde{i}} \right) + \frac{\omega^1_{rsy,i^{2,y}}}{days_2} \qquad \forall i^{2,y} \quad (4.3.84)$$

or by (4.3.60) for season 3, we see that

$$\eta(i^{3,y}) \gamma^1_{n^r(r)3y,i^{3,y}} = \eta(i^{3,y}) MC^{ST^S,1}_{rsy,i^{3,y}} + \delta^1_{ry,i^{3,y}} + \zeta^1_{ry,i^{3,y}} + \frac{\omega^1_{r3y,i^{3,y}}}{days_3}, \quad \forall i^{3,y} \quad (4.3.85)$$

In both equations, $\delta^1_{ry,i^{3,y}}$, $\zeta^1_{ry,i^{3,y}}$ and $\omega^1_{rsy,is,y}$ are nonnegative by default; $\eta(i^{s,y}) MC^{ST^S,1}_{rsy,i^{s,y}}$ is positive. Given that $\eta(i^{s,y}) > 0, \forall s = 2, 3, y$, $\gamma^1_{n^r(r)sy,i^{1,y}} > 0$ must hold for either season 2 or 3. This completes the proof. □

Equations (4.3.86) and (4.3.87) are market-clearing conditions for the storage gas market. Equation (4.3.86) states that in the long-term market, the total gas supplied by storage operators located at a consumption node ($\sum_{r \in R^n} days_s x^0_{rsy}$) should be equal to the total amount demanded by marketers at the same location for each high demand season in each year ($\sum_{m \in M^n} days_s u^0_{msy}$). Also, (4.3.87) enforces such a relation for each possible random outcome $i^{s,y} \in I^{s,y}$, where $s = 2, 3$.

$$\sum_{r \in R^n} days_s x^0_{rsy} = \sum_{m \in M^n} days_s u^0_{msy} \qquad \forall n \in CN, s = 2, 3, y \tag{4.3.86}$$

$$\sum_{r \in R^n} days_s x^1_{rsy,i^{s,y}} = \sum_{m \in M^n} days_s u^1_{msy,i^{s,y}} \qquad \forall i^{s,y}, n \in CN, s = 2, 3, y \tag{4.3.87}$$

Next, the two inequalities in (4.3.88) and (4.3.89) relax these original market-clearing conditions so that the supply of the storage gas by storage operators could exceed the demand for it by marketers.

$$0 \le \sum_{r \in R^n} days_s x^0_{rsy} - \sum_{m \in M^n} days_s u^0_{msy} \perp \gamma^0_{nsy} \ge 0 \qquad \forall n \in CN, s = 2, 3, y \tag{4.3.88}$$

$$0 \le \sum_{r \in R^n} days_s x^1_{rsy,i^{s,y}} - \sum_{m \in M^n} days_s u^1_{msy,i^{s,y}} \perp \gamma^1_{nsy,i^{s,y}} \ge 0 \qquad \forall i^{s,y}, n \in CN, s = 2, 3, y \tag{4.3.89}$$

Coupled with nonnegative prices γ^0_{rsy} and $\gamma^0_{rsy,i^{s,y}}$ as dual variables to the market-clearing condition relaxations, new market-clearing conditions (4.3.88) and (4.3.89) are shown to be equivalent to (4.3.86) and (4.3.87) with nonnegative market prices by Theorem 4.3.8.

Theorem 4.3.8. *If Assumption 4.3.2 holds for all $r \in R$, then the system S-ST-MCC is equivalent to the system S-ST-MCC-NCP, where*

$$\text{S-ST-MCC} \equiv \begin{cases} NCP(4.3.71) \\ (4.3.86) - (4.3.87) \\ \gamma^0_{nsy} \ge 0, \quad \forall n \in PN, s = 2, 3, y \\ \gamma^1_{nsy,i^{s,y}} \ge 0, \forall i^{s,y}, n \in PN, s = 2, 3, y \end{cases} \tag{4.3.90}$$

$$S\text{-}ST\text{-}MCC\text{-}NCP \equiv \begin{cases} NCP(4.3.71) \\ (4.3.88) - (4.3.89) \end{cases} \tag{4.3.91}$$

Proof. By construction, any solution satisfying *S-ST-MCC* also satisfies *S-ST-MCC-NCP*. Therefore, we must show that every solution to *S-ST-MCC-NCP* will be a solution to *S-ST-MCC*. Suppose the contrary that there exists a solution satisfying *S-ST-MCC-NCP* such that for some $n \in CN, s = 2, 3, y$,

$$0 < \sum_{r \in R^n} days_s x^0_{rsy} - \sum_{m \in M^n} days_s u^0_{msy} \text{ and } \gamma^0_{nsy} = 0 \tag{4.3.92}$$

or for some $i^{s,y}, n \in CN, s = 2, 3, y$,

$$0 < \sum_{r \in R^n} days_s x^1_{rsy,i^{s,y}} - \sum_{m \in M^n} days_s u^1_{msy,i^{s,y}} \text{ and } \gamma^1_{nsy,i^{s,y}} = 0 \tag{4.3.93}$$

From (4.3.92) and (4.3.93), it must follow that for some $n \in CN, s = 2, 3, y$,

$$0 < x^0_{rsy} \quad \exists r \in R^n \quad \text{and} \quad \gamma^0_{nsy} = 0 \tag{4.3.94}$$

or for some $i^{s,y}, n \in CN, s = 2, 3, y$,

$$0 < x^1_{rsy,i^{s,y}} \quad \exists r \in R^n \quad \text{and} \quad \gamma^1_{nsy,i^{s,y}} = 0 \tag{4.3.95}$$

However, either (4.3.94) or (4.3.95) contradicts Theorem 4.3.7. Consequently, every solution of *S-ST-MCC-NCP* is a solution to *S-ST-MCC*. This completes the proof. □

4.3.4 Peak Gas Operator

The peak gas market is assumed to operate under perfect competition as was the case in the pervious market studied. Problem (PG^S) as shown below, which aggregates optimization problems for all peak gas operators in the market, is the optimization problem for the peak gas market. Given that we model this market as serving the rest of the market only in season 3, problem (PG^S) just takes into account those realizations that could occur in season 3, that is $i^{3,y} \in I^{3,y}, \forall y$. The objective for the peak gas operator is to maximize the expected net profits as shown in (4.3.96), subject to the capacity constraints (4.3.97). In the objective function, $\beta^0_{n^p(p)y}$ and $\beta^1_{n^p(p)y,i^{3,y}}$, where $n^p(p)$ specifies the node where peak gas operator p is located, represent, respectively, the market prices faced by peak gas operator p and are exogenous to the problem (PG^S) but an overall variable for model S-NGEM. The seasonly supply of peak gas operator p is denoted by w^0_{py} and $w^1_{py,i^{3,y}}$ for the long-term and spot market, respectively. The cost function for peak gas operator p, $c^{PG}_p(\cdot)$, assumed convex and continuously differentiable, is a function of the total daily production rates, $w^0_{py} + w^1_{py,i^{3,y}}$. Constraints (4.3.97) require that the daily supply rates of peak gas cannot exceed the supply capacity.

$$
\begin{aligned}
(PG^S) \quad \max \quad & \sum_{p\in P}\sum_{y\in Y} days_3\Big\{\beta^0_{n^p(p)y} w^0_{py} + \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})[\beta^1_{ny,i^{3,y}} w^1_{py,i^{3,y}} \\
& - c^{PG}_p(w^0_{py} + w^1_{py,i^{3,y}})]\Big\} && (4.3.96)\\
\text{s.t.} \quad & w^0_{py} + w^1_{py,i^{3,y}} \le \overline{w}_p \qquad (\sigma^1_{py,i^{3,y}} \ge 0) \qquad \forall i^{3,y}, p, y && (4.3.97)\\
& 0 \le w^1_{py,i^{3,y}}, \forall i^{3,y}, w^0_{py} \qquad \forall p, y
\end{aligned}
$$

Given the convexity of the problem (PG^S) and the linearity of the constraint functions, the KKT conditions are both necessary and sufficient for the optimality [5] of the problem. The KKT conditions for problem (PG^S) are:

$$0 \leq days_3 \Big[-\beta^0_{n^p(p)y} + \sum_{i^{3,y} \in I^{3,y}} \eta(i^{3,y}) \frac{\partial \big(c_p^{PG}(w^0_{py} + w^1_{py,i^{3,y}}) \big)}{\partial (w^0_{py})} \Big] + \sum_{i^{3,y} \in I^{3,y}} \sigma^1_{py,i^{3,y}} \perp w^0_{py} \geq 0 \qquad \forall p, y \qquad (4.3.98)$$

$$0 \leq \eta(i^{3,y}) days_3 \Big[-\beta^1_{n^p(p)y,i^{3,y}} + \frac{\partial \big(c_p^{PG}(w^0_{py} + w^1_{py,i^{3,y}}) \big)}{\partial (w^1_{py,i^{3,y}})} \Big] + \sigma^1_{py,i^{3,y}} \perp w^1_{py,i^{3,y}} \geq 0 \qquad \forall i^{3,y}, p, y \qquad (4.3.99)$$

$$0 \leq \overline{w}_p - w^0_{py} - w^1_{py,i^{3,y}} \perp \sigma^1_{py,i^{3,y}} \geq 0 \qquad \forall i^{3,y}, p, y \qquad (4.3.100)$$

As in the case of producer and storage operator, we use two simplified terms to represent the marginal cost functions for the long term and spot markets. In particular,

$$MC^{PG^S,0}_{py,i^{3,y}} \equiv \frac{\partial \big(c_p^{PG}(w^0_{py} + w^1_{py,i^{3,y}}) \big)}{\partial (w^0_{py})} \qquad (4.3.101)$$

$$MC^{PG^S,1}_{py,i^{3,y}} \equiv \frac{\partial \big(c_p^{PG}(w^0_{py} + w^1_{py,i^{3,y}}) \big)}{\partial (w^1_{py,i^{3,y}})} \qquad (4.3.102)$$

Given w^0_{py} and $w^1_{py,i^{3,y}}$, the two marginal costs have the same values, that is,

$$MC^{PG^S,0}_{py,i^{3,y}} = MC^{PG^S,1}_{py,i^{3,y}} \qquad (4.3.103)$$

Similar to the previous analysis, the KKT conditions for all peak gas operators $p \in P$ can be expressed equivalently as

$$0 \leq v^{PG^S} \perp H^{PG^S}(v^{PG^S}) \geq 0 \qquad (4.3.104)$$

where

$$v^{PG^S} \equiv \begin{pmatrix} w^0_{py} \quad (\forall p, y) \\ w^1_{py,i^{3,y}} \quad (\forall i^{3,y}, p, y) \\ \sigma^1_{py,i^{3,y}} \quad (\forall i^{3,y}, p, y) \end{pmatrix} \text{ and} \tag{4.3.105}$$

$$H^{PG^S}(v^{PG^S}) \equiv \begin{pmatrix} days_3\Big[-\beta^0_{n^p(p)y} + \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y}) MC^{PG^S,0}_{py,i^{3,y}}\Big] \\ \qquad + \sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}} \quad (\forall p, y) \\ \eta(i^{3,y}) days_3\Big[-\beta^1_{n^p(p)y,i^{3,y}} + MC^{PG^S,1}_{py,i^{3,y}}\Big] \\ \qquad + \sigma^1_{py,i^{3,y}} \quad (\forall i^{3,y}, p, y) \\ \overline{w}_p - w^0_{py} - w^1_{py,i^{3,y}} \quad (\forall i^{3,y}, p, y) \end{pmatrix} \tag{4.3.106}$$

Similar to the previous analysis, the following theorem shows relationship between the long-term market prices and spot market prices (ω^0_{py}) for the peak gas market: when the production rate for the long-term market is positive, the long-term market price (β^0_{ny}) is greater than or equal to the expected spot market prices ($\sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}}$). Also, when the production rates for all realizations in some season are positive, the expected spot market prices for peak gas is greater or equal to the long-term market price.

Theorem 4.3.9. *Considering a consumption node $n \in CN$, if there exists a peak gas operator $p \in P^n$ such that*

(a) if the long-term production rate $w^0_{py} > 0$, then the long-term peak gas price is greater than or equal to the expected spot market price, that is, $\beta^0_{n^p(p)y} \geq \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}}$;

(b) if the spot market production rate $w^1_{py,i^{3,y}} > 0, \forall i^{3,y}$, then the long-term peak gas price is less than or equal to the expected spot market price, that is, $\beta^0_{n^p(p)y} \leq \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}}$;

(c) *if the long-term production rate* $w^0_{py} > 0$ *and the spot market production rate* $w^1_{py,i^{3,y}} > 0, \forall i^{3,y}$, *then the long-term peak gas price is equal to the expected spot market price, that is,* $\beta^0_{n^p(p)y} = \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}}$.

Proof. (a) By (4.3.98), if $w^0_{py} > 0$, we see that

$$\beta^0_{n^p(p)y} = \sum_{i^{3,y}\in I^{s,y}} \eta(i^{3,y}) MC^{PG^S,0}_{py} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \tag{4.3.107}$$

Also, by (4.3.99), it follows for all $i^{3,y} \in I^{3,y}$ that

$$\eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}} \le \eta(i^{3,y}) MC^{PG^S,1}_{py,i^{s,y}} + \frac{\sigma^1_{py,i^{3,y}}}{days_3} \tag{4.3.108}$$

Summing this inequality over all $i^{3,y} \in I^{3,y}$, we have

$$\begin{aligned}
\sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}} &\le \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y}) MC^{PG^S,1}_{py,i^{s,y}} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \\
&= \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y}) MC^{PG^S,0}_{py} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \\
&\qquad\qquad\qquad\qquad \text{by (4.3.103)} \\
&= \beta^0_{n^p(p)y} \qquad\qquad \text{by (4.3.107)}
\end{aligned}$$

Therefore, the conclusion follows.

(b) By equation (4.3.99), if $w^1_{py,i^{s,y}} > 0, \forall i^{s,y}$, it follows that

$$\eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}} = \eta(i^{3,y}) MC^{PG^S,1}_{py,i^{s,y}} + \frac{\sigma^1_{py,i^{3,y}}}{days_3}, \quad \forall i^{3,y} \tag{4.3.109}$$

Summing the above equations over all $i^{s,y} \in I^{s,y}$ results in the following:

$$\sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}} = \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y}) MC^{PG^S,1}_{py,i^{s,y}} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \quad (4.3.110)$$

Also by (4.3.98), we see that

$$\begin{aligned}
\beta^0_{n^p(p)y} &\leq \sum_{i^{3,y}\in I^{s,y}} \eta(i^{3,y}) MC^{PG^S,0}_{py,i^{3,y}} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \\
&= \sum_{i^{3,y}\in I^{s,y}} \eta(i^{3,y}) MC^{PG^S,1}_{py,i^{3,y}} + \frac{\sum_{i^{3,y}\in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} && \text{by (4.3.103)} \\
&= \sum_{i^{3,y}\in I^{3,y}} \eta(i^{3,y})\beta^1_{n^p(p)y,i^{3,y}} && \text{by (4.3.110)}
\end{aligned}$$

(c) The conclusion follows immediately from parts (a) and (b).

This completes the proof. □

The condition of positive marginal cost functions under positive production rates are presented in Assumption 4.3.3, based on which, a relationship between the peak gas production rates and the peak gas market prices for both the long-term and spot markets is established in Theorem 4.3.10.

Assumption 4.3.3. *Peak gas operator p's marginal cost function satisfies the following condition for all years:*

$$MC^{PG^S,0}_{py,i^{3,y}} = MC^{PG^S,1}_{py,i^{3,y}} > 0, \quad \text{when } w^0_{py} + w^1_{py,i^{3,y}} > 0 \quad \forall i^{3,y}. \quad (4.3.111)$$

Theorem 4.3.10. *Suppose that the marginal cost functions of all the peak gas operators located at a consumption node n satisfy Assumption 4.3.3. If there exists a peak gas operator $p \in P^n$,* *(a) who has a positive production rate in the long-term market, i.e., $w^0_{py} > 0$, then the corresponding peak gas price at node n is positive too, that is, $\beta^0_{n^p(p)y} > 0$;*

(b) who has a positive production rate in the spot market, i.e., $w^1_{py,i^{3,y}} > 0$ for some $i^{3,y}$, then the corresponding peak gas price at node n is positive too, that is, $\beta^1_{n^p(p)y,i^{3,y}} > 0$.

Proof. (a) From (4.3.98), with positive w^0_{py}, we have

$$\beta^0_{n^p(p)y} = \sum_{i^{3,y} \in I^{s,y}} \eta(i^{3,y}) MC^{PG^S,0}_{py} + \frac{\sum_{i^{3,y} \in I^{3,y}} \sigma^1_{py,i^{3,y}}}{days_3} \tag{4.3.112}$$

which, by Assumption 4.3.3 implies that $\beta^0_{n^p(p)y} > 0$.

(b) From (4.3.99), with positive $w^1_{py,i^{3,y}}$ for some $i^{3,y}$, we have

$$\beta^1_{n^p(p)y,i^{3,y}} = MC^{PG^S,1}_{py,i^{3,y}} + \frac{\sigma^1_{py,i^{3,y}}}{\eta(i^{3,y}) days_3}, \quad \exists i^{3,y} \tag{4.3.113}$$

which, by Assumption 4.3.3 implies that $\beta^1_{n^p(p)y,i^{3,y}} > 0$ for that $i^{s,y}$. □

Equations (4.3.114) and (4.3.115) are market-clearing conditions for the peak gas market for the long-term and spot markets, respectively. Both (4.3.114) and (4.3.115) state that the total peak gas supplied by peak gas operators located at a consumption node ($\sum_{p \in P^n} days_3 w^0_{py}$ and $\sum_{p \in P^n} days_3 w^1_{py,i^{3,y}}$) should equal to the total peak gas demanded by marketers at the same node ($\sum_{m \in M^n} days_3 v^0_{my}$ and $\sum_{m \in M^n} days_3 v^1_{my,i^{3,y}}$).

$$\sum_{p \in P^n} days_3 w^0_{py} = \sum_{m \in M^n} days_3 v^0_{my} \qquad \forall n \in CN, y \tag{4.3.114}$$

$$\sum_{p \in P^n} days_3 w^1_{py,i^{3,y}} = \sum_{m \in M^n} days_3 v^1_{my,i^{3,y}} \qquad \forall i^{3,y}, n \in CN, y \tag{4.3.115}$$

In (4.3.116) and (4.3.117), the market-clearing conditions (4.3.114) and (4.3.115) and the corresponding market prices β^0_{ny} and $\beta^1_{ny,i^{3,y}}$ are constructed as an NCP. Theorem 4.3.11 shows that this NCP is equivalent to the original formulation.

$$0 \leq \sum_{p \in P^n} days_3 w^0_{py} - \sum_{m \in M^n} days_3 v^0_{my} \perp \beta^0_{ny} \geq 0 \qquad \forall n \in CN, y \tag{4.3.116}$$

$$0 \leq \sum_{p \in P^n} days_3 w^1_{py,i^{3,y}} - \sum_{m \in M^n} days_3 v^1_{my,i^{3,y}} \perp \beta^1_{ny,i^{3,y}} \geq 0 \quad \forall i^{3,y}, n \in CN, y \tag{4.3.117}$$

Theorem 4.3.11. *If Assumption 4.3.3 holds for all $p \in P$, then the system S-PG-MCC is equivalent to the system S-PG-MCC-NCP, where*

$$\textit{S-PG-MCC} \equiv \begin{cases} NCP(4.3.104) \\ (4.3.114) - (4.3.115) \\ \beta^0_{ny} \geq 0, \quad \forall n \in PN, y \\ \beta^1_{ny,i^{3,y}} \geq 0, \quad \forall i^{s,y}, n \in PN, y \end{cases} \tag{4.3.118}$$

$$\textit{S-PG-MCC-NCP} \equiv \begin{cases} NCP(4.3.104) \\ (4.3.116) - (4.3.117) \end{cases} \tag{4.3.119}$$

Proof. By construction, any solution satisfying *S-PG-MCC* also satisfies *S-PG-MCC-NCP*. Therefore, we must show that every solution to *S-PG-MCC-NCP* will be a solution to *S-PG-MCC*. Suppose the contrary that there exists a solution satisfying *S-PG-MCC-NCP* such that for some $n \in CN, y$:

$$0 < \sum_{p \in P^n} days_3 w^0_{py} - \sum_{m \in M^n} days_3 v^0_{my} \;\text{ and }\; \beta^0_{ny} = 0 \tag{4.3.120}$$

or for some $i^{s,y}, n \in CN, y$:

$$0 < \sum_{p \in P^n} days_3 w^1_{py,i^{3,y}} - \sum_{m \in M^n} days_3 v^1_{my,i^{3,y}} \quad \text{and} \quad \beta^1_{ny,i^{3,y}} = 0 \tag{4.3.121}$$

From (4.3.120) and (4.3.121), it must follow that for some $n \in CN, y$:

$$0 < w^0_{py}, \exists p \in P^n \quad \text{and} \quad \beta^0_{ny} = 0 \tag{4.3.122}$$

or for some $i^{s,y}, n \in CN, y$:

$$0 < w^1_{py,i^{3,y}}, \exists p \in P^n \quad \text{and} \quad \beta^1_{ny,i^{3,y}} = 0 \tag{4.3.123}$$

However, $0 < w^0_{py}, \exists p \in P^n$ in (4.3.122) and $0 < w^1_{py,i^{3,y}}, \exists p \in P^n$ in (4.3.123) result in $\beta^0_{ny} > 0$ and $\beta^1_{ny,i^{3,y}} > 0$ respectively by Theorem 4.3.10, both of which contradict $\beta^0_{ny} = 0$ and $\beta^1_{ny,i^{3,y}} = 0$ in (4.3.122) and (4.3.123), respectively. Consequently, every solution of *S-PG-MCC-NCP* is also be a solution to *S-PG-MCC*. This completes the proof. $\square$

4.3.5 Marketer

Problem $\widetilde{MK}^S$ as shown below is the optimization problem faced by marketer m. Unlike the model D-NGEM in which marketers have the power to influence the end-use prices of all the four consumer sectors, marketers in model S-NGEM are assumed to be Nash-Cournot players only in the residential and commercial sectors given the fact that the demand in these two sectors is highly seasonal and subject to factors hardly predictable. They have power to influence these sectors via the inverse demand functions $\theta_{kn^m(m)sy}(l^1_{kmsy,i^{s,y}} + l^{1*}_{k(-m(n))sy,i^{s,y}})$, where $l^1_{kmsy,i^{s,y}}$ is the market supply served by marketer m and $l^{1*}_{k(-m(n))sy,i^{s,y}}$ is "optimal" supply served by other marketers located at the same node n as m. As θ_{knsy} in model D-NGEM, $\theta^1_{knsy,i^{s,y}}$ is typically assumed continuously differentiable, nonincreasing and nonnegative on the nonnegative orthant. Conversely, the industrial and electric power sectors, whose demand are less likely impacted by seasonal factors and are relatively constant throughout the year, use firm contracts extensively in order to ensure the supply assurance. Therefore, in model S-NGEM, marketers are assumed to be price-takers in the industrial and electric power sectors. In addition, the industrial and electric power sectors are modeled to have options to use both firm contracts (in the long-term market) and swing/best-efforts contracts (in the spot market) with pipeline operators. In the long-term market, marketers are obligated to supply industrial and electric power sectors with gas at predetermined rates, denoted D^0_{knsy} over the time horizon and model-determined prices $\Theta^0_{knsy}, k = 3, 4$ and RC^0_{asy} for firm pipeline service. The spot market is then used by the two sectors to obtain the extra gas demand denoted $D^1_{knsy,i^{s,y}}$ under different scenarios, where the market prices for these market are denoted $\Theta^1_{knsy,i^{s,y}}, k = 3, 4$. In this sense, marketers are not modeled to have market power over the industrial and electric demand any more and thus the end-use prices for industrial and electric sectors Θ^0_{knsy} and $\Theta^1_{knsy,i^{s,y}}, k = 3, 4$ are determined by market-clearing conditions rather than the inverse demand functions.

The objective for marketer m is to maximize the expected net profits, which is the difference between expected incomes and expected costs. The income for marketer m is composed of sales to the industrial and electric power sectors in the long-term market and the expected sales to all sectors in the spot market. The costs can be broken down into two categories: commodity charges and pipeline charges both in the long-term and spot markets. The commodity charges are what marketers pay for the value of the gas and are exogenous to the problem $\widetilde{MK}^S$. Depending on the source of the gas, the commodity charge rates are $\pi^0_{n_2(a)sy}$, $\gamma^0_{n^m(m)sy}$ or $\beta^0_{n^m(m)y}$ in the long-term market and $\pi^1_{n_2(a)sy,i^{s,y}}$, $\gamma^1_{n^m(m)sy,i^{s,y}}$ or $\beta^1_{n^m(m)y,i^{3,y}}$ in the spot market, respectively. The pipeline charges include pipeline regulated charges (τ^{reg}_{asy} per Mcf) whenever the pipeline is used, congestion fees (τ^0_{asy} or $\tau^1_{asy,i^{s,y}}$ per Mcf) whenever using a full pipeline and reservation charges (RC^0_{asy} per Mcf) for the firm service.

Constraints (4.3.125) - (4.3.130) state that the amount of gas marketer m can supply the consumers should be equal to the total amount they purchase from the producers, storage operators or peak gas operators. Constraints (4.3.125) - (4.3.127) enforce such a balance for the long-term market and constraints (4.3.128) - (4.3.130) are for spot market.

$$
\begin{aligned}
(\widetilde{MK}^S) \quad \max \quad & \sum_{y\in Y}\Bigg\{\sum_{s\in S} days_s\Big[\sum_{k=3,4}\Theta^0_{kn^m(m)sy}l^0_{kmsy} \\
&+\sum_{k=1,2}\sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})\theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}}+l^{1*}_{k(-m(n))sy,i^{s,y}}\big)l^1_{kmsy,i^{s,y}} \\
&+\sum_{k=3,4}\sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})\Theta^1_{kn^m(m)sy,i^{s,y}}l^1_{kmsy,i^{s,y}} \\
&-\sum_{s\in S}\sum_{a\in A(n^m(m))} days_s\Big[\big(\tau^0_{asy}+\tau^{reg}_{asy}+\pi^0_{n_2(a)sy}+RC^0_{asy}\big)h^0_{amsy} \\
&+\sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})\big(\tau^1_{asy,i^{s,y}}+\tau^{reg}_{asy}+\pi^1_{n_2(a)sy,i^{s,y}}\big)h^1_{amsy,i^{s,y}}\Big] \\
&-\sum_{s=2,3} days_s\Big[\gamma^0_{n^m(m)sy}u^0_{msy}+\sum_{i^{s,y}\in I^{s,y}}\eta(i^{s,y})\gamma^1_{n^m(m)sy,i^{s,y}}u^1_{msy,i^{s,y}}\Big] \\
&-days_3\Big[\beta^0_{n^m(m)y}v^0_{my}+\sum_{i^{3,y}\in I^{3,y}}\eta(i^{3,y})\big(\beta^1_{n^m(m)y,i^{3,y}}\big)v^1_{my,i^{3,y}}\Big]\Bigg\}
\end{aligned}
\tag{4.3.124}
$$

$$
\text{s.t.} \quad days_1\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h^0_{am1y}-\sum_{k=3,4}l^0_{km1y}\Big)=0 \qquad (\phi^0_{m1y}) \quad \forall y \tag{4.3.125}
$$

$$
days_2\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h^0_{am2y}+u^0_{m2y}-\sum_{k=3,4}l^0_{km2y}\Big)=0 \qquad (\phi^0_{m2y}) \quad \forall y \tag{4.3.126}
$$

$$days_3\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h^0_{am3y}+u^0_{m3y}+v^0_{my}-\sum_{k=3,4}l^0_{km3y}\Big)=0$$

$$(\phi^0_{m3y})\quad \forall y \qquad (4.3.127)$$

$$days_1\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h^1_{am1y,i^{1,y}}-\sum_{k\in K}l^1_{km1y,i^{1,y}}\Big)=0$$

$$(\phi^1_{m1y,i^{1,y}}\text{ free})\quad \forall i^{1,y},y \qquad (4.3.128)$$

$$days_2\Big(\sum_{a\in A(n^m(m)}(1-loss_a)h^1_{am2y,i^{2,y}}+u^1_{m2y,i^{2,y}}$$

$$-\sum_{k\in K}l^1_{km2y,i^{2,y}}\Big)=0\quad(\phi^1_{m2y,i^{2,y}}\text{ free})\quad \forall i^{2,y},y \quad (4.3.129)$$

$$days_3\Big(\sum_{a\in A(n^m(m))}(1-loss_a)h^1_{am3y,i^{3,y}}+u^1_{m3y,i^{3,y}}+v^1_{my,i^{3,y}}$$

$$-\sum_{k\in K}l^1_{km3y,i^{3,y}}\Big)=0\quad(\phi^1_{m3y,i^{3,y}}\text{ free})\quad \forall i^{3,y},y \quad (4.3.130)$$

$$0\le l^1_{kmsy,i^{s,y}},\forall i^{s,y},l^0_{kmsy} \qquad \forall k,s,y$$

$$0\le h^1_{amsy,i^{s,y}},\forall i^{s,y},h^0_{amsy} \qquad \forall a\in A(n^m(m),s,y$$

$$0\le u^1_{msy,i^{s,y}},\forall i^{s,y},u^0_{msy} \qquad \forall s=2,3,y$$

$$0\le v^1_{my,i^{3,y}}\forall i^{3,y},v^0_{my} \qquad \forall y$$

Problem $(\widetilde{MK}^S)$ is a convex program provided that the only nonlinear term, the revenue function $\theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}}+l^{1*}_{k(-m(n))sy,i^{s,y}}\big)\cdot l^1_{kmsy,i^{s,y}}$ in the objective function (4.3.124) is concave. The conditions of $\theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}}+l^{1*}_{k(-m(n))sy,i^{s,y}}\big)\cdot l^1_{kmsy,i^{s,y}}$ being concave was discussed in Section 3.1.5. Given the problem $(\widetilde{MK}^S)$ is a convex programming, the KKT conditions are both necessary and sufficient for optimality [5]. The KKT conditions are:

$$0\le -\Theta^0_{kn^m(m)sy}+\phi^0_{msy}\perp l^0_{kmsy}\ge 0 \qquad \forall k=3,4,s,y \qquad (4.3.131)$$

$$0 \le \eta(i^{s,y})\Big[-\frac{\partial \theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}} + l^{1*}_{k(-m(n))sy,i^{s,y}}\big)}{\partial l^1_{kmsy,i^{s,y}}} l^1_{kmsy,i^{s,y}} - \theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}} + l^{1*}_{k(-m(n))sy,i^{s,y}}\big)\Big] + \phi^1_{msy,i^{s,y}} \perp l^1_{kmsy,i^{s,y}} \ge 0 \quad \forall i^{s,y}, k = 1, 2, s, y \qquad (4.3.132)$$

$$0 \le -\eta(i^{s,y})\Theta^1_{kn^m(m)sy,i^{s,y}} + \phi^1_{msy,i^{s,y}} \perp l^1_{kmsy,i^{s,y}} \ge 0 \quad \forall i^{s,y}, k = 3, 4, s, y \qquad (4.3.133)$$

$$0 \le \tau^0_{asy} + \tau^{reg}_{asy} + \pi^0_{n2(a)sy} + RC^0_{asy} - (1 - loss_a)\phi^0_{msy} \perp h^0_{amsy} \ge 0 \quad \forall a \in A(n^m(m)), s, y \qquad (4.3.134)$$

$$0 \le \gamma^0_{n^m(m)sy} - \phi^0_{msy} \perp u^0_{msy} \ge 0 \qquad s = 2, 3, \forall y \qquad (4.3.135)$$

$$0 \le \beta^0_{n^m(m)y} - \phi^0_{m3y} \perp v^0_{my} \ge 0 \qquad \forall y \qquad (4.3.136)$$

$$0 \le \eta(i^{s,y})(\tau^1_{asy,i^{s,y}} + \tau^{reg}_{asy} + \pi^1_{n2(a)sy,i^{s,y}}) - (1 - loss_a)\phi^1_{msy,i^{s,y}} \perp h^1_{amsy,i^{s,y}} \ge 0 \quad \forall a \in A(n^m(m)), i^{s,y}, s, y \qquad (4.3.137)$$

$$0 \le \eta(i^{s,y})\gamma^1_{n^m(m)sy,i^{s,y}} - \phi^1_{msy,i^{s,y}} \perp u^1_{msy,i^{s,y}} \ge 0 \qquad \forall i^{s,y}, s = 2, 3, y \qquad (4.3.138)$$

$$0 \le \eta(i^{3,y})\beta^1_{n^m(m)y,i^{3,y}} - \phi^1_{m3y,i^{3,y}} \perp v^1_{my,i^{3,y}} \ge 0 \qquad \forall i^{3,y}, y \qquad (4.3.139)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a)h^0_{am1y} - \sum_{k=3,4} l^0_{km1y} \qquad (\phi^0_{m1y} \text{ free}) \quad \forall y \qquad (4.3.140)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a)h^0_{am2y} + u^0_{m2y} - \sum_{k=3,4} l^0_{km2y} \qquad (\phi^0_{m2y} \text{ free}) \quad \forall y \qquad (4.3.141)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a)h^0_{am3y} + u^0_{m3y} + v^0_{my} - \sum_{k=3,4} l^0_{km3y} \qquad (\phi^0_{m3y} \text{ free}) \quad \forall y \qquad (4.3.142)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a)h^1_{am1y,i^{1,y}} - \sum_{k \in K} l^1_{km1y,i^{1,y}} \qquad (\phi^1_{m1y,i^{s,y}} \text{ free}) \quad \forall i^{1,y}, y \qquad (4.3.143)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a) h^1_{am2y,i^{2,y}} + u^1_{m2y,i^{2,y}} - \sum_{k \in K} l^1_{km2y,i^{2,y}}$$

$$(\phi^1_{m2y,i^{2,y}} \text{ free}) \quad \forall i^{2,y}, y \qquad (4.3.144)$$

$$0 = \sum_{a \in A(n^m(m))} (1 - loss_a) h^1_{am3y,i^{3,y}} + u^1_{m3y,i^{3,y}} + v^1_{my,i^{3,y}} - \sum_{k \in K} l^1_{km3y,i^{3,y}}$$

$$(\phi^1_{m3y,i^{3,y}} \text{ free}) \quad \forall i^{3,y}, y \qquad (4.3.145)$$

In light of their mathematical structure, including the KKT conditions for all marketers $m \in M$ results in an MiCP as follows:

$$\begin{aligned} &G^{MK^S}(u^{MK^S}, v^{MK^S}) = 0, \quad u^{MK^S} \text{ free} \\ &0 \leq v^{MK^S} \perp H^{MK^S}(u^{MK^S}, v^{MK^S}) \geq 0 \end{aligned} \qquad (4.3.146)$$

where

$$v^{MK^S} \equiv \begin{pmatrix} l^0_{kmsy} \quad (\forall k = 3,4,m,s,y) \\ l^1_{kmsy,i^{s,y}} \quad (\forall i^{s,y}, k, m, s, y) \\ h^0_{amsy} \quad (\forall a \in A(n^m(m)), m, s, y) \\ u^0_{msy} \quad (\forall m, s = 2,3, y) \\ v^0_{my} \quad (\forall m, y) \\ h^1_{amsy,i^{s,y}} \quad (\forall a \in A(n^m(m)), i^{s,y}, m, s, y) \\ u^1_{msy,i^{s,y}} \quad (\forall i^{s,y}, m, s = 2,3, y) \\ v^1_{my,i^{3,y}} \quad (\forall i^{3,y}, m, y) \end{pmatrix} \qquad (4.3.147)$$

$$H^{MK^S}(v^{MK^S}) \equiv \begin{pmatrix} -\Theta^0_{kn^m(m)sy} + \phi^0_{msy} \quad (\forall k = 3,4,s,y) \\ \eta(i^{s,y})\Big[-\dfrac{\partial\theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}} + l^{1*}_{k(-m(n))sy,i^{s,y}}\big)}{\partial l^1_{kmsy,i^{s,y}}} l^1_{kmsy,i^{s,y}} \\ \qquad -\theta^1_{kn^m(m)sy,i^{s,y}}\big(l^1_{kmsy,i^{s,y}} + l^{1*}_{k(-m(n))sy,i^{s,y}}\big)\Big] + \phi^1_{msy,i^{s,y}} \\ \hfill (\forall i^{s,y}, k = 1,2,s,y) \\ -\eta(i^{s,y})\Theta^1_{kn^m(m)sy,i^{s,y}} + \phi^1_{msy,i^{s,y}} \quad (\forall i^{s,y}, k = 3,4,s,y) \\ \tau^0_{asy} + \tau^{reg}_{asy} + \pi^0_{n_2(a)sy} + RC^0_{asy} - (1 - loss_a)\phi^0_{msy} \\ \hfill (\forall a \in A(n^m(m)), s, y) \\ \gamma^0_{n^m(m)sy} - \phi^0_{msy} \quad (s = 2,3,\forall y) \\ \beta^0_{n^m(m)y} - \phi^0_{m3y} \quad (\forall y) \\ \eta(i^{s,y})(\tau^1_{asy,i^{s,y}} + \tau^{reg}_{asy} + \pi^1_{n_2(a)sy,i^{s,y}}) - (1 - loss_a)\phi^1_{msy,i^{s,y}} \\ \hfill (\forall a \in A(n^m(m)), i^{s,y}, s, y) \\ \eta(i^{s,y})\gamma^1_{n^m(m)sy,i^{s,y}} - \phi^1_{msy,i^{s,y}} \quad (\forall i^{s,y}, s = 2,3,y) \\ \eta(i^{3,y})\beta^1_{n^m(m)y,i^{3,y}} - \phi^1_{m3y,i^{3,y}} \quad (\forall i^{3,y}, y) \end{pmatrix} \tag{4.3.148}$$

$$u^{MK^S} \equiv \begin{pmatrix} \phi^0_{msy} \quad (\forall m,s,y) \\ \phi^1_{msy,i^{s,y}} \quad (\forall i^{s,y}, m, s, y) \end{pmatrix} \text{ and} \tag{4.3.149}$$

$$G^{MK^S}(u^{MK^S}) \equiv \begin{pmatrix} \sum_{a \in A(n^m(m))} (1 - loss_a) h^0_{am1y} - \sum_{k=3,4} l^0_{km1y} \quad (\forall y) \\ \sum_{a \in A(n^m(m))} (1 - loss_a) h^0_{am2y} + u^0_{m2y} - \sum_{k=3,4} l^0_{km2y} \quad (\forall y) \\ \sum_{a \in A(n^m(m))} (1 - loss_a) h^0_{am3y} + u^0_{m3y} + v^0_{my} - \sum_{k=3,4} l^0_{km3y} \\ (\forall y) \\ \sum_{a \in A(n^m(m))} (1 - loss_a) h^1_{am1y,i^{1,y}} - \sum_{k \in K} l^1_{km1y,i^{1,y}} \quad (\forall i^{1,y}, y) \\ \sum_{a \in A(n^m(m))} (1 - loss_a) h^1_{am2y,i^{2,y}} + u^1_{m2y,i^{2,y}} - \sum_{k \in K} l^1_{km2y,i^{2,y}} \\ (\forall i^{2,y}, y) \\ \sum_{a \in A(n^m(m))} (1 - loss_a) h^1_{am3y,i^{3,y}} + u^1_{m3y,i^{3,y}} + v^1_{my,i^{3,y}} \\ - \sum_{k \in K} l^1_{km3y,i^{3,y}} \quad (\forall i^{3,y}, y) \end{pmatrix} \tag{4.3.150}$$

It is well known that MiCP (4.3.146) is equivalent to a collection, denoted (MK), of the optimization problems for all the marketers who are strategic players in an imperfect competition setting [26].

All theoretical results shown in this section consider model S-NGEM in its entirety with previous results holding. For brevity, we do not state them explicitly for each result. In Theorem 4.3.12, we show that, for the industrial and electric power sectors, if a sector located at node n receives a positive supply of gas from some marketer, the sectoral end-use prices for node n must be positive. We first present two useful lemmas.

Lemma 4.3.2. *Suppose that Assumptions 4.3.1, 4.3.2 and 4.3.3 hold for all producers $c \in C$, storage operators $r \in R$ and peak gas operators $p \in P$, respectively. For a marketer m in the long-term market,*

(a) if $h^0_{amsy} > 0$ for some $a \in A(n^m(m))$, then $\phi^0_{msy} > 0$;

(b) if $u^0_{msy} > 0$, then $\phi^0_{msy} > 0$ when $s = 2, 3$;

(c) if $v^0_{my} > 0$, then $\phi^0_{m3y} > 0$.

Proof. a) First, $h^0_{amsy} > 0$ via (4.3.134) implies that

$$\phi^0_{msy} = \frac{\tau^0_{asy} + \tau^{reg}_{asy} + \pi^0_{n_2(a)sy} + RC^0_{asy}}{1 - loss_a} \tag{4.3.151}$$

where $(1 - loss_a)$ is positive, since it does not make sense in reality to have the loss factor $loss_a$ greater than or equal to 1.

In addition, $h^0_{amsy} > 0$ implies that $q^0_{csy} > 0$ for some producer c located at node $n_2(a)$ by (4.3.36) in season 1 or (4.3.37) in seasons 2 or 3. Further, by Theorem 4.3.4, we see that when $q^0_{csy} > 0$, $\pi^0_{n^c(c)sy} > 0$, i.e., $\pi^0_{n_2(a)sy} > 0$ for node $n_2(a)$ is where producer c is located. Also, $h^0_{amsy} > 0$ implies that $f^0_{asy} > 0$ by (4.3.14) or (4.3.15) showing that $\tau^0_{asy} \geq 0$ by Theorem 4.3.2. Given that $\pi^0_{n_2(a)sy} > 0$, $\tau^0_{asy} \geq 0$ and the fact that τ^{reg}_{asy} and RC^0_{asy} are all positive, $\phi^0_{msy} > 0$ must hold in (4.3.151).

b) $u^0_{msy} > 0$ via (4.3.135) implies that

$$\phi^0_{msy} = \gamma^0_{n^m(m)sy} \tag{4.3.152}$$

By the market-clearing condition (4.3.86), $u^0_{msy} > 0$ also implies that $x^0_{rsy} > 0$ for some storage operator located where marketer m is. This further indicates that the storage price for the node is positive, that is, $\gamma^0_{n^m(m)sy} > 0$ by Theorem 4.3.7. Therefore ϕ^0_{msy} in (4.3.152) must be positive.

c) In this case, via (4.3.136), $v^0_{my} > 0$ implies that

$$\phi^0_{m3y} = \beta^0_{n^m(m)y} \tag{4.3.153}$$

On the other hand, by the market-clearing condition (4.3.114), $v^0_{my} > 0$ implies that $w^0_{py} > 0$ for some peak gas operator p located where marketer m is. By Theorem 4.3.10, this further shows that the peak gas price for this node is positive, that is, $\beta^0_{n^m(m)y} > 0$. Hence $\phi^0_{m3y} > 0$ in (4.3.153). This completes the proof. $\square$

Lemma 4.3.3. *Suppose that Assumptions 4.3.1, 4.3.2 and 4.3.3 hold. For a marketer m in the spot market,*

(a) if $h^1_{amsy,i^{s,y}} > 0$ for some $a \in A(n^m(m))$, then $\phi^1_{msy,i^{s,y}} > 0$;

(b) if $u^1_{msy,i^{s,y}} > 0$, then $\phi^1_{msy,i^{s,y}} > 0$;

(c) if $v^1_{my,i^{3,y}} > 0$, then $\phi^1_{m3y,i^{3,y}} > 0$.

Proof. a) First, $h^1_{amsy,i^{s,y}} > 1$ via (4.3.137) implies that

$$\phi^1_{msy,i^{s,y}} = \eta(i^{s,y})\frac{\tau^1_{asy,i^{s,y}} + \tau^{reg}_{asy} + \pi^1_{n_2(a)sy,i^{s,y}}}{1 - loss_a} \tag{4.3.154}$$

In addition, $h^1_{amsy,i^{s,y}} > 0$ implies that $q^1_{csy,i^{s,y}} > 0$ for some producer located at node $n_2(a)$ by the market-clearing conditions (4.3.38) or (4.3.39). Further, by Theorem 4.3.4, we see that when $q^1_{csy,i^{s,y}} > 0$, $\pi^1_{n^c(c)sy,i^{s,y}} > 0$, i.e., $\pi^1_{n_2(a)sy,i^{s,y}} > 0$ for $n^c(c)$ and $n_2(a)$ both refer to the same production node where producer c is.

Also, $h^1_{amsy,i^{s,y}} > 0$ shows that $f^1_{asy,i^{s,y}} > 0$ by market-clearing conditions (4.3.16) or (4.3.17), showing that $\tau^1_{asy,i^{s,y}} \geq 0$ by Theorem 4.3.2. Given that $\pi^1_{n_2(a)sy,i^{s,y}} > 0$, $\tau^1_{asy,i^{s,y}} \geq 0$, and the fact that $\eta(i^{s,y})$ and τ^{reg}_{asy} are positive, $\phi^1_{msy,i^{s,y}} > 0$ must hold in (4.3.154).

b) $u^1_{msy,i^{s,y}} > 0$ via (4.3.138) implies that

$$\phi^1_{msy,i^{s,y}} = \eta(i^{s,y})\gamma^1_{n^m(m)sy,i^{s,y}} \tag{4.3.155}$$

By the market-clearing condition (4.3.87), $u^1_{msy,i^{s,y}} > 0$ also implies that $x^1_{rsy,i^{s,y}} > 0$ for some storage operator r located where marketer m is. Thus by Theorem 4.3.7, the storage price for the node is positive, that is, $\gamma^1_{n^r(r)sy,i^{s,y}} > 0$, i.e., $\gamma^1_{n^m(m)sy,i^{s,y}} > 0$ for in this case, storage operator r and marketer m are located at the same consumption node. Therefore $\phi^1_{msy,i^{s,y}}$ in (4.3.155) must be positive due to the fact that $\eta(i^{s,y}) > 0$.

c) In this case, via (4.3.139), $v^1_{my,i^{3,y}} > 0$ implies that

$$\phi^1_{m3y,i^{s,y}} = \eta(i^{3,y})\beta^1_{n^m(m)y,i^{3,y}} \tag{4.3.156}$$

By market-clearing condition (4.3.115), $v^1_{my,i^{3,y}} > 0$ implies that $w^1_{py,i^{3,y}} > 0$ for some peak gas operator p located where marketer m is. Thus by Theorem 4.3.10, the peak gas price for this node is positive, that is, $\beta^1_{n^m(m)y,i^{3,y}}$. Hence $\phi^1_{m3y,i^{3,y}}$ in (4.3.156) must be positive given that $\eta(i^{3,y}) > 0$. This completes the proof. $\square$

The following theorem, Theorem 4.3.12 shows that the end-user prices for the industrial and electric power sectors, Θ^0_{knsy} and $\Theta^1_{knsy,i^{s,y}}$ are positive when they receive positive supply from marketers. Note that the end-user prices for the residential and commercial sectors are positive in that they are determined by the inverse demand functions, which are assume nonincreasing and nonnegative on the nonnegative orthant.

Theorem 4.3.12. *Suppose Assumptions 4.3.1, 4.3.2 and 4.3.3 are in force for all* $c \in C$*,* $r \in R$ *and* $p \in P$*, respectively. For the industrial and electric power sectors* ($k = 3, 4$)*,*

(a) if sector k *receives a positive supply from some marketer* m *in the long-term market, that is,* $l^0_{kmsy} > 0$*, then the corresponding sectoral end-use price is positive too, that is,* $\Theta^0_{kn^m(m)sy} > 0$*;*

(b) if sector k *receives a positive supply from some marketer* m *in the spot market, that is,* $l^1_{kmsy,i^{s,y}} > 0$*, then the corresponding sectoral end-use price is positive too, that is,* $\Theta^1_{kn^m(m)sy,i^{s,y}} > 0$*.*

Proof. (a) First, by (4.3.131), $l^0_{kmsy} > 0$ implies that $\Theta^0_{kn^m(m)sy} = \phi^0_{msy}$. By the mass balance constraints (4.3.140) - (4.3.142), $l^0_{kmsy} > 0$ also implies, respectively, that at least one of h^0_{amsy}, u^0_{msy} and v^0_{my} is positive, and Lemma 4.3.2 shows that $\phi^0_{msy} > 0$. Therefore, $l^0_{kmsy} > 0$ implies that $\Theta^0_{kn^m(m)sy} > 0$.

(b) By (4.3.133), $l^1_{kmsy,i^{s,y}} > 0$ implies that $\eta(i^{s,y})\Theta^1_{kn^m(m)sy,i^{s,y}} = \phi^1_{msy,i^{s,y}}$. By the mass balance constraints (4.3.143) - (4.3.145), $l^1_{kmsy,i^{s,y}} > 0$ also implies, respectively, that at least one of $h^1_{amsy,i^{s,y}}$, $u^1_{msy,i^{s,y}}$ and $v^1_{my,i^{3,y}}$ is positive, and Lemma 4.3.3 shows that $\phi^1_{msy,i^{s,y}} > 0$. Therefore, $l^1_{kmsy,i^{s,y}} > 0$ implies that $\Theta^1_{kn^m(m)sy,i^{s,y}} > 0$ given that fact that $\eta(i^{s,y})$ is positive. □

Since the inverse demand functions for industrial and electric power sectors are not in the objective function of problem $\widetilde{MK}^S$, we need a new mechanism to establish a relationship between the equilibrium consumption and prices. Thus, market-clearing conditions (4.3.157) and (4.3.158) are introduced. These conditions state that the total supply of gas from the marketers ($days_s \sum_{m \in M^n} l^0_{kmsy}$ and $days_s \sum_{m \in M^n} l^1_{kmsy,i^{s,y}}$) should be equal to the volume demanded by the end-users ($days_s D^0_{knsy}$ and $days_s D^1_{knsy,i^{s,y}}$) in the long-term and spot markets, respectively, are used to enforce such an equilibrium.

$$days_s \sum_{m \in M^n} l^0_{kmsy} = days_s D^0_{knsy} \qquad \forall k = 3, 4, n \in CN, s, y \qquad (4.3.157)$$

$$days_s \sum_{m \in M^n} l^1_{kmsy,i^{s,y}} = days_s D^1_{knsy,i^{s,y}} \qquad \forall k = 3, 4, i^{s,y}, n \in CN, s, y \qquad (4.3.158)$$

In order to have an MiCP formulation, the above two equations need to be amended into inequalities as shown in (4.3.159) and (4.3.160). The corresponding dual variables are the prices Θ^0_{knsy} and $\Theta^1_{knsy,i^{s,y}}$.

$$0 \leq days_s \Big(\sum_{m \in M^n} l^0_{kmsy} - D^0_{knsy} \Big) \perp \Theta^0_{knsy} \geq 0$$
$$\forall k = 3, 4, n \in CN, s, y \qquad (4.3.159)$$

$$0 \leq days_s \Big(\sum_{m \in M^n} l^1_{kmsy,i^{s,y}} - D^1_{knsy,i^{s,y}} \Big) \perp \Theta^1_{knsy,i^{s,y}} \geq 0$$
$$\forall k = 3, 4, i^{s,y}, n \in CN, s, y \qquad (4.3.160)$$

Theorem 4.3.13 shows how the new system of (4.3.159) and (4.3.160) is equivalent to (4.3.157) and (4.3.158) with nonnegative Θ^0_{knsy} and $\Theta^1_{knsy,i^{s,y}}$.

Theorem 4.3.13. *Suppose Assumptions 4.3.1, 4.3.2 and 4.3.3 are in force for all $c \in C$, $r \in R$ and $p \in P$, respectively. The system S-CM-MCC is equivalent to the* *system S-CM-MCC-NCP, where*

$$S\text{-}CM\text{-}MCC \equiv \begin{cases} MiCP(4.3.146) \\ (4.3.157) - (4.3.158) \\ \Theta^0_{knsy} \geq 0, \forall k = 3,4, n \in CN, s, y \\ \Theta^1_{knsy,i^{s,y}} \geq 0, \forall i^{s,y}, k = 3,4, n \in CN, s, y \end{cases} \tag{4.3.161}$$

$$S\text{-}CM\text{-}MCC\text{-}NCP \equiv \begin{cases} MiCP(4.3.146) \\ (4.3.159) - (4.3.160) \end{cases} \tag{4.3.162}$$

Proof. By construction, any solution to *S-CM-MCC* is also a solution to *S-CM-MCC-NCP*. It suffices to show that any solution to *S-CM-MCC-NCP* is also a solution to *S-CM-MCC*. Suppose there exists a solution to S-CM-MCC-NCP such that for some $k \in \{3,4\}, n \in CN, s, y$:

$$0 < days_s\Big(\sum_{m \in M^n} l^0_{kmsy} - D^0_{knsy}\Big) \;\text{ and }\; \Theta^0_{knsy} = 0 \tag{4.3.163}$$

or for some $i^{s,y}, k \in \{3,4\}, n \in CN, s, y$:

$$0 < days_s\Big(\sum_{m \in M^n} l^1_{kmsy,i^{s,y}} - D^1_{knsy,i^{s,y}}\Big) \;\text{ and }\; \Theta^1_{knsy,i^{s,y}} = 0 \tag{4.3.164}$$

This implies that for some $k \in \{3,4\}, n \in CN, s, y$:

$$0 < l^0_{kmsy} \;\text{ and }\; \Theta^0_{knsy} = 0 \tag{4.3.165}$$

or for some $i^{s,y}, k \in \{3,4\}, n \in CN, s, y$:

$$0 < l^1_{kmsy,i^{s,y}} \;\text{ and }\; \Theta^1_{knsy,i^{s,y}} = 0 \tag{4.3.166}$$

By Theorem 4.3.12, $0 < l^0_{kmsy}$ in (4.3.165) implies that $\Theta^0_{knsy} > 0$. This is a contradiction to $\Theta^0_{knsy} = 0$ in (4.3.165). Also, by Theorem 4.3.12, $0 < l^1_{kmsy,i^{s,y}}$ in (4.3.166) implies that $\Theta^1_{knsy,i^{s,y}} > 0$, which is a contradiction to $\Theta^1_{knsy,i^{s,y}} = 0$ in (4.3.166).

Therefore, any solution to *S-CM-MCC* is also a solution to *S-CM-MCC-NCP*. This completes the proof. □

4.3.6 NCP/VI formulation of Model S-NGEM

In this section, we construct an MiCP, which is equivalent to model S-NGEM based on the previous analysis of its components. We first define the equilibrium model S-NGEM mathematically.

Definition 4.3.1. The model S-NGEM is a system composed of optimization problems PL^S, PR^S, ST^S, PG^S and $\widetilde{MK}^S, \forall m \in M$, market-clearing conditions (4.3.14) - (4.3.17), (4.3.36) - (4.3.39), (4.3.86) - (4.3.87), (4.3.114) - (4.3.115) and (4.3.157) - (4.3.158) as well as nonnegative market price conditions, i.e., $\pi^0_{nsy} \geq 0, \forall n \in PN, s, y$; $\pi^1_{nsy,i^{s,y}} \geq 0, \forall i^{s,y}, n \in PN, s, y$; $\gamma^0_{nsy} \geq 0, \forall n \in CN, s = 2, 3, y$; $\gamma^1_{nsy,i^{s,y}} \geq 0, \forall i^{s,y}, n \in CN, s = 2, 3, y$; $\beta^0_{ny} \geq 0, \forall n \in CN, y$; $\beta^1_{ny,i^{3,y}} \geq 0, \forall i^{3,y}, n \in CN, y$; $\Theta^0_{knsy} \geq 0, \forall k = 3, 4, n \in CN, s, y$; $\Theta^1_{knsy,i^{s,y}} \geq 0, \forall i^{s,y}, k = 3, 4, n \in CN, s, y$. That is,

$$
\text{S-NGEM} \equiv \begin{cases}
PL^S, PR^S, ST^S, PG^S, \widetilde{MK}^S, \forall m \in M \\
(4.3.14) - (4.3.17) \\
(4.3.36) - (4.3.39) \\
(4.3.86) - (4.3.87) \\
(4.3.114) - (4.3.115) \\
(4.3.157) - (4.3.158) \\
\pi^0_{nsy} \geq 0, \forall n \in PN, s, y \\
\pi^1_{nsy,i^{s,y}} \geq 0, \forall i^{s,y}, n \in PN, s, y \\
\gamma^0_{nsy} \geq 0, \forall n \in CN, s = 2, 3, y \\
\gamma^1_{nsy,i^{s,y}} \geq 0, \forall i^{s,y}, n \in CN, s = 2, 3, y \\
\beta^0_{ny} \geq 0, \forall n \in CN, y \\
\beta^1_{ny,i^{3,y}} \geq 0, \forall i^{3,y}, n \in CN, y \\
\Theta^0_{knsy} \geq 0, \forall k = 3, 4, n \in CN, s, y \\
\Theta^1_{knsy,i^{s,y}} \geq 0, \forall i^{s,y}, k = 3, 4, n \in CN, s, y
\end{cases} \tag{4.3.167}
$$

In previous sections, we found NCP/VI equivalents to the components of the model S-NGEM with appropriate assumptions in Theorems 4.3.5, 4.3.8, 4.3.11 and 4.3.13. Therefore, it is trivial to show that model S-NGEM is an instance of an MiCP. First, let us define:

$$
(u^{MCC^S}) \equiv \begin{pmatrix}
\tau^0_{asy} & (\forall a, s, y) \\
\tau^1_{asy,i^{s,y}} & (\forall a, i^{s,y}, s, y)
\end{pmatrix} \tag{4.3.168}
$$

$$
(v^{MCC^S}) \equiv \begin{pmatrix}
\pi^0_{nsy} \quad (\forall n \in PN, s, y) \\
\pi^1_{nsy,i^{s,y}} \quad (\forall i^{s,y}, n \in PN, s, y) \\
\gamma^0_{nsy} \quad (\forall n \in CN, s = 2, 3, y) \\
\gamma^1_{nsy,i^{s,y}} \quad (\forall i^{s,y}, n \in CN, s = 2, 3, y) \\
\beta^0_{ny} \quad (\forall n \in CN, y) \\
\beta^1_{ny,i^{3,y}} \quad (\forall i^{3,y}, n \in CN, y) \\
\Theta^0_{knsy} \quad (\forall k = 3, 4, n \in CN, s, y) \\
\Theta^1_{knsy,i^{s,y}} \quad (\forall i^{s,y}, k = 3, 4, n \in CN, s, y)
\end{pmatrix} \tag{4.3.169}
$$

$$
G^{MCC^S} \equiv \begin{pmatrix}
days_1 f^0_{a1y} - \sum_{r \in R(n_1(a))} days_1 g^0_{ary} - \sum_{m \in M(n_1(a))} days_1 h^0_{am1y} \\
\hfill (\forall a, y) \\
days_s f^0_{asy} - \sum_{m \in M(n_1(a))} days_s h^0_{amsy} \quad (\forall a, s = 2, 3, y) \\
days_1 f^1_{a1y,i^{1,y}} - \sum_{r \in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} \\
\qquad - \sum_{m \in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}} \quad (\forall a, i^{1,y}, y) \\
days_s f^1_{asy,i^{s,y}} - \sum_{m \in M(n_1(a))} days_s h^1_{amsy,i^{s,y}} \\
\hfill (\forall a, i^{s,y}, s = 2, 3, y)
\end{pmatrix} \tag{4.3.170}
$$

$$H^{MCC^S} \equiv \left(\begin{array}{l} \sum_{c \in C^n} days_1 q^0_{c1y} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g^0_{ary} \right. \\ \qquad \left. + \sum_{m \in M(n_1(a))} days_1 h^0_{am1y} \right) \quad (\forall n \in PN, y) \\ \sum_{c \in C^n} days_s q^0_{csy} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_1 h^0_{amsy} \\ \qquad (\forall n \in PN, s = 2, 3, y) \\ \sum_{c \in C^n} days_1 q^1_{c1y,i^{1,y}} - \sum_{a \in A(n)} \left(\sum_{r \in R(n_1(a))} days_1 g^1_{ary,i^{1,y}} \right. \\ \qquad \left. + \sum_{m \in M(n_1(a))} days_1 h^1_{am1y,i^{1,y}} \right) \quad (\forall i^{1,y}, n \in PN, y) \\ \sum_{c \in C^n} days_s q^1_{csy,i^{s,y}} - \sum_{a \in A(n)} \sum_{m \in M(n_1(a))} days_1 h^1_{amsy,i^{s,y}} \\ \qquad (\forall i^{s,y}, n \in PN, s = 2, 3, y) \\ \sum_{r \in R^n} days_s x^0_{rsy} - \sum_{m \in M^n} days_s u^0_{msy} \quad (\forall n \in CN, s = 2, 3, y) \\ \sum_{r \in R^n} days_s x^1_{rsy,i^{s,y}} - \sum_{m \in M^n} days_s u^1_{msy,i^{s,y}} \quad (\forall i^{s,y}, n \in CN, s = 2, 3, y) \\ \sum_{p \in P^n} days_3 w^0_{py} - \sum_{m \in M^n} days_3 v^0_{my} \quad (\forall n \in CN, y) \\ \sum_{p \in P^n} days_3 w^1_{py,i^{3,y}} - \sum_{m \in M^n} days_3 v^1_{my,i^{3,y}} \quad (\forall i^{s,y}, n \in CN, y) \\ \sum_{m \in M^n} days_s l^0_{kmsy} - days_s D^0_{knsy} \quad (\forall k = 3, 4, n \in CN, s, y) \\ \sum_{m \in M^n} days_s l^1_{kmsy,i^{s,y}} - days_s D^1_{knsy,i^{s,y}} \quad (\forall i^{s,y}, k = 3, 4, n \in CN, s, y) \end{array} \right) \tag{4.3.171}$$

Theorem 4.3.14 in the following shows that the model S-NGEM is an instance of an MiCP mathematically with Assumptions 4.3.1, 4.3.2 and 4.3.3 holding, respectively, for cost functions of all producer, storage operators and peak gas operators.

Theorem 4.3.14. *Let*

$$(u^S)^T \equiv \big[(u^{MK^S})^T \quad (u^{MCC^S})^T\big];$$
$$(v^S)^T \equiv \big[(v^{PL^S})^T \quad (v^{PR^S})^T \quad (v^{ST^S})^T \quad (v^{PG^S})^T \quad (v^{MK^S})^T \quad (v^{MCC^S})^T\big];$$
$$[G^S(u^S, v^S)]^T \equiv \big[(G^{MK^S})^T \quad (G^{MCC^S})^T\big];$$
$$[H^S(u^S, v^S)]^T \equiv \big[(H^{PL^S})^T \quad (H^{PR^S})^T \quad (H^{ST^S})^T \quad (H^{PG^S})^T \quad (H^{MK^S})^T \quad (H^{MCC^S})^T\big].$$

Suppose that Assumptions 4.3.1, 4.3.2 and 4.3.3 hold for all $c \in C$, $r \in R$ and $p \in P$, respectively. S-NGEM is equivalent to an MiCP, denoted S-NGEM-MiCP(G^S, H^S) where

$$\begin{aligned} &G^S(u^S, v^S) = 0 \quad u^S \text{ free} \\ &0 \le v^S \perp H^S(u^S, v^S) \ge 0 \end{aligned} \tag{4.3.172}$$

Proof. Following the definition for MiCP, by Theorems 4.3.5, 4.3.8 and 4.3.11, it is trivial to show the results. □

4.4 Conclusions

In this chapter, a stochastic equilibrium model for the natural gas market depicted in Chapter 2, S-NGEM is developed in an extensive form of stochastic programming. Model S-NGEM takes into account the long-term and spot markets of the gas industry. The long-term market modeled is featured by supply assurance. The decision made for the long-term market are first-stage variables. The spot market is characterized by market uncertainty. The spot market decisions are recourse variables. Assuming that the marginal cost functions are positive when the production is positive for producers, storage operators and peak gas operators, model S-NGEM is shown to be an instance of NCP/VI. The GAMS/PATH solver is thus appropriate for generating the numerical results presented in Chapter 5.

Chapter 5

Example Application of Model S-NGEM

In this chapter, we study numerical results for model S-NGEM using a sample network of two production nodes and two consumptions nodes with eleven players for a time horizon of one year. Our goal is to examine how the stochastic aspects of the market influence the market activities. We use a discretized random demand following Haurie et al. [45] and De Wolf and Smeers [14].

A base case was calibrated using the data publicly available at www.eia.doe.gov. The values used are not unrepresentative of actual supply and demand conditions for a small network. In particular, residential and commercial sectors exhibit strong seasonality in terms of prices and consumption rates; the industrial consumption is relative stable throughout the year; the electric power sector has a higher demand in the summer season; and the end-user industrial and electric power prices are generally lower than the other two sectors. Varying the parameters relevant to the probability distribution of the end-user demand resulted in two cases (case 1 and case 2). A third case, case 3 representing a perfect competition market where all market players were price-takers was also considered. These cases, arranged in increasing order of the end-user consumption, from low to high, were case 1, base case, case 2 and case 3. We compare the changes in the equilibrium prices and quantities for all agents in these three cases as opposed to the base case. We also calculate the expected producer and consumer surplus for comparison purposes.

5.1 Data Set

This section presents the data used for the model S-NGEM. All the data to be presented were realistic in terms of order of magnitude for small regions but not real data per se. First, Section 5.1.1 discusses the composition of a sample network adapted for the case studies. Next, in Section 5.1.2, we discuss the parameters, including cost functions, production capacities, pipeline capacities and charges, which were taken to be deterministic factors for the model. Lastly, in Section 5.1.3, we present the stochastic parameters, i.e., the coefficients for the demand functions for the residential and commercial sectors and the spot market demand for the industrial and electric power sectors.

5.1.1 Sample Network

The sample network has the following elements, which are illustrated in Figures 5.1 and 5.2.

- Two production nodes, denoted $pn1$ and $pn2$;
  - One producer located at each production node, denoted $C1$ and $C2$, respectively;
- Two consumption nodes, denoted $cn1$ and $cn2$;
  - Two storage operators, denoted $R1$ and $R2$, located respectively at nodes $cn1$ and $cn2$;

- Two peak gas operators, denoted $P1$ and $P2$, located respectively at nodes $cn1$ and $cn2$;
- Four marketers, denoted $M1$, $M2$, $M3$ and $M4$: $M1$ and $M2$ located at node $cn1$, $M3$ and $M4$ located at node $cn2$;
- Four consumption sectors located at each consumption node;
  * Two residential sectors, denoted $RD1$ and $RD2$, located respectively at nodes $cn1$ and $cn2$;
  * Two commercial sectors, denoted $CD1$ and $CD2$, located respectively at nodes $cn1$ and $cn2$;
  * Two industrial sectors, denoted $ID1$ and $ID2$, located respectively at nodes $cn1$ and $cn2$;
  * Two electric power sectors, denoted $ED1$ and $ED2$, located respectively at nodes $cn1$ and $cn2$;

- Four pipeline arcs connecting the production and consumption nodes, denoted $a1$, $a2$, $a3$ and $a4$.

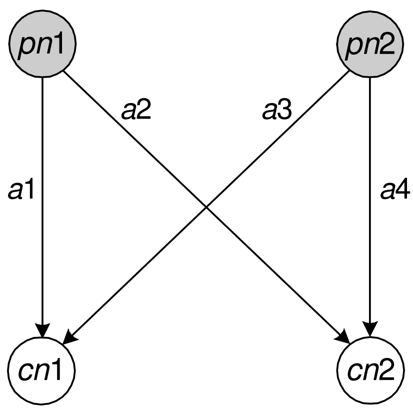


Figure 5.1: Sample Network Structure

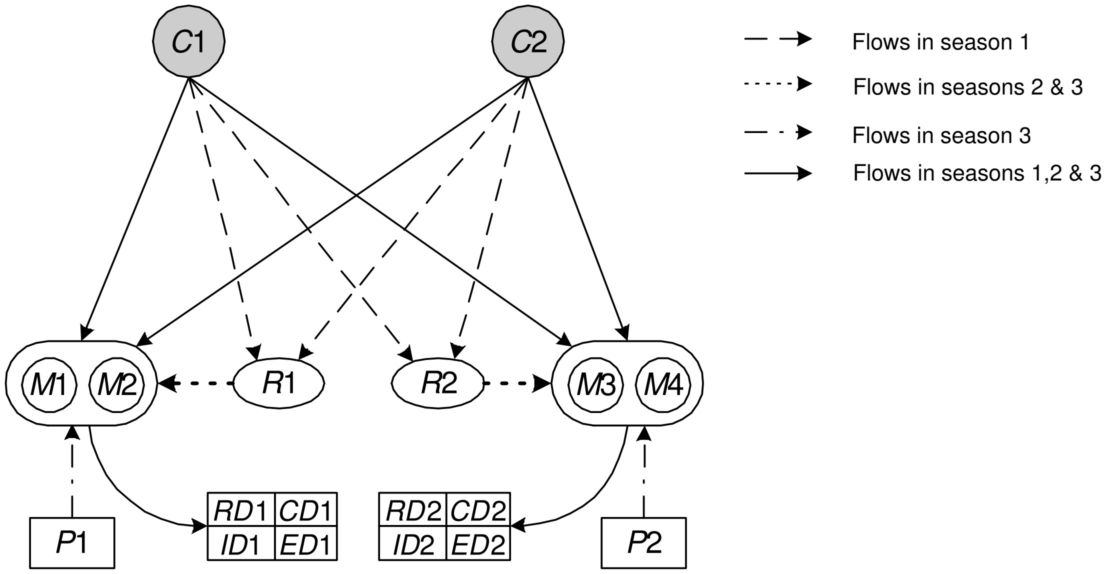


Figure 5.2: Sample Network Elements

5.1.2 Deterministic Parameters

The cost functions $c_c^{PR}(\cdot)$, $c_r^{ST}(\cdot)$ and $c_p^{PG}(\cdot)$ respectively for the producers, storage operators and peak gas operators were taken to be convex and quadratic, thus of the form $\alpha_1 x + \frac{1}{2}\alpha_2 x^2$, where x was the quantity in question and α_1, α_2 the coefficients. Table 5.1 presents these coefficients as well as a variety of capacity values used. It is clear that the pair of producers, storage operators and peak gas operators are taken to be identical.

Table 5.1: Data for Producers, Storage Operators and Peak Gas Operators

Participants	α_1	α_2	$\overline{q}_c$ (MMcf/d)	$loss_r$	$\overline{x}_r$ (MMcf/d)	$\overline{g}_r$ (MMcf/d)	$\overline{k}_r$ (MMcf)	$\overline{w}_p$ (MMcf/d)
$C1$, $C2$	0.003	0.0018	2500	-	-	-	-	-
$R1$, $R2$	0.002	0.002	-	0.01	500	500	50000	-
$P1$, $P2$	0.5	0.035	-	-	-	-	-	200

Table 5.2 shows the set of inputs for the four pipelines in the sample network, including the pipeline capacity, loss factors, regulated pipeline charges and

reservation charges.

Table 5.2: Data for Pipelines

Pipelines	$\overline{f}_a$ (MMcf/d)	$loss_a$	$\tau^{reg}_{asy}, \forall s, y$ ($/Mcf)	$RC^0_{asy}, \forall s, y$ ($/Mcf)
Arc a_1	2000	0.01	0.15	0.25
Arc a_2	900	0.02	0.15	0.25
Arc a_3	900	0.02	0.15	0.25
Arc a_4	2000	0.01	0.15	0.25

The long-term industrial and electric power consumption, denoted D^0_{knsy} when $k = 3, 4$ in the model, are shown below in Table 5.3. Although model S-NGEM allows the parameters D^0_{knsy} to vary by season, for simplicity, we set the long-term demand constant throughout the year, e.g. the long-term demand for the industrial sector at node $cn1$ was 700 MMcf/d for all the three seasons.

Table 5.3: Long-term Demand for Industrial and Electric Power Sectors (MMcf/d)

	Industrial Demand			**Electric Power Demand**		
	Season 1	Season 2	Season 3	Season 1	Season 2	Season 3
Node $cn1$	700	700	700	420	420	420
Node $cn2$	650	650	650	400	400	400

5.1.3 Stochastic Parameters

The end-user spot market demand was the only random element in the case study. At a consumption node, for each season, there were two possible random outcomes in the demand level, high or low. Therefore, eight scenarios occurred after three seasons in each node as shown below in Table 5.4, which resulted in 64 scenarios for the time horizon in the market of two consumption nodes assuming the random demand fluctuations at the two nodes were independent. Table 5.4 shows alternatives to the scenario tree presented in Figure 4.1. For simplicity, we also assumed that the four demand sectors at the same consumption node were in the same state of demand, either high demand or low demand, at the same time. For instance, if one consumption node has a high level of demand, all demand sectors at this location would have a high demand.

Table 5.4: Random Outcomes for Demand Levels

(a) Node $cn1$

Season 1	**Season 2**	**Season 3**
Low	Low	Low
		High
	High	Low
		High
High	Low	Low
		High
	High	Low
		High

(b) Node $cn2$

Season 1	**Season 2**	**Season 3**
Low	Low	Low
		High
	High	Low
		High
High	Low	Low
		High
	High	Low
		High

The demand functions for the residential and commercial sectors were of the form $A - By$ where y was the price in question and A, $-B$ were the intercept and the slope values, respectively. The uncertainty in the demand allow for the fact that both A and B could be random. However, in all the cases studied, we assumed that the intercept A was a random variable while the slope $-B$ was deterministic. In other words, the demand functions of the same sector at the same node and same season were parallel to each other in different scenarios. The values of A are shown in later sections of the case study; the values of B are shown in Table 5.5.

With the sample network and the scenario trees defined in Table 5.4, a linear

Table 5.5: Slopes of Linear Demand Functions

	Residential Demand			Commercial Demand		
	Season 1	Season 2	Season 3	Season 1	Season 2	Season 3
Node $cn1$	-35	-60	-90	-38	-65	-85
Node $cn2$	-31	-57	-85	-35	-60	-82

complementarity problem (LCP) with 6,186 variables resulted, 142 of which were first stage variables and the remaining 6,044 recourse variables. Note that using the same network, model D-NGEM would only result in an LCP of 184 variables. Clearly, the number of the variables in the problem would increase exponentially when we consider a more complicated example thus giving rise to a great deal of computational difficulties. The problem was solved using GAMS/PATH software (www.gams.com, [28]) on a PC computer with a 2.26 GHz Intel®Pentium®4 Processor and 1.0GB of memory. The typical CPU time used by GAMS/PATH software ranged from 5 seconds to 20 seconds. About two minutes were needed to read input from an EXCEL file and write the output to another EXCEL file.

5.2 Numerical Results

5.2.1 Base Case

In all cases studied in this chapter, nodes $cn1$ and $cn2$ were identical in terms of the probabilities associated with the corresponding random events. Therefore, we used Table 5.6 to assign probabilities to the random demand levels for the base case regardless of which node was under consideration. There were two random events, high demand and low demand for each season. As a result, eight scenarios were present for the time horizon. Table 5.6 is another expression of the scenario tree introduced in Section 4.2. The first columns for each season list all the possible random realizations indexed by $i^{s,y}$ in terms of demand levels. The last columns for each season show the values for $\eta(i^{s,y})$, the probability of the occurrence of event $i^{s,y}$. The second columns for seasons 2 and 3 show the conditional probability of the occurrence of event $i^{s,y}$. For example, the high demand level in season 2 would occur after the high demand of season 1 with a probability of 0.48 while the chance that this event would occur given a high demand in season 1 was 0.8. Note that among the two outcomes in each season, the high demand outcomes were more likely than low demand outcomes in terms of associated probabilities.

Table 5.6: Base Case — Random Outcomes and Associated Probabilities for Nodes $cn1$ and $cn2$

<table>
<tr><th colspan="2">Season 1</th><th colspan="3">Season 2</th><th colspan="3">Season 3</th><th rowspan="2">Scenarios</th></tr>
<tr><th>Event</th><th>$\eta(i^{1,1})$</th><th>Event</th><th>$\eta(i^{2,1}|i^{1,1})$</th><th>$\eta(i^{2,1})$</th><th>Event</th><th>$\eta(i^{3,1}|i^{2,1})$</th><th>$\eta(i^{3,1})$</th></tr>
<tr><td rowspan="4">Low</td><td rowspan="4">0.4</td><td rowspan="2">Low</td><td rowspan="2">0.2</td><td rowspan="2">0.08</td><td>Low</td><td>0.3</td><td>0.024</td><td>Scenario 1</td></tr>
<tr><td>High</td><td>0.7</td><td>0.056</td><td>Scenario 2</td></tr>
<tr><td rowspan="2">High</td><td rowspan="2">0.8</td><td rowspan="2">0.32</td><td>Low</td><td>0.3</td><td>0.096</td><td>Scenario 3</td></tr>
<tr><td>High</td><td>0.7</td><td>0.224</td><td>Scenario 4</td></tr>
<tr><td rowspan="4">High</td><td rowspan="4">0.6</td><td rowspan="2">Low</td><td rowspan="2">0.2</td><td rowspan="2">0.12</td><td>Low</td><td>0.3</td><td>0.036</td><td>Scenario 5</td></tr>
<tr><td>High</td><td>0.7</td><td>0.084</td><td>Scenario 6</td></tr>
<tr><td rowspan="2">High</td><td rowspan="2">0.8</td><td rowspan="2">0.48</td><td>Low</td><td>0.3</td><td>0.144</td><td>Scenario 7</td></tr>
<tr><td>High</td><td>0.7</td><td>0.336</td><td>Scenario 8</td></tr>
</table>

When jointly considering nodes $cn1$ and $cn2$ as a whole, the elements of random outcomes $I^{s,y}$ and the values of $\eta(i^{s,y})$ changed. Each random event belonging to I now corresponds to the demand level of consumption nodes $cn1$ and $cn2$ occurring at the same time. As a result, we see $8^2 = 64$ scenarios on the new scenario tree as shown in Table 5.7. This tree described all random events for three seasons used for the base case. Because of the assumption of independence between the random demand between the two consumption nodes, the product of the probabilities of the demand level for each node could be used. For example, the chance that node $cn1$ had a high demand and node $cn2$ had a low demand in season 1 was 0.24, which was the product of probabilities of one event that node $cn1$ had high demand (0.4) and the other event that node $cn2$ had low demand (0.6).

Table 5.7: Scenario Description for Base Case

Season 1			**Season 2**			**Season 3**			**Scenarios**
$cn1$	$cn2$	$\eta(i^{1,1})$	$cn1$	$cn2$	$\eta(i^{2,1})$	$cn1$	$cn2$	$\eta(i^{3,1})$	
Low	Low	0.16	Low	Low	0.0064	Low	Low	0.000576	Scenario 1
						Low	High	0.001344	Scenario 2
						High	Low	0.001344	Scenario 3
						High	High	0.003136	Scenario 4
			Low	High	0.0256	Low	Low	0.002304	Scenario 5
						Low	High	0.005376	Scenario 6
						High	Low	0.005376	Scenario 7
						High	High	0.012544	Scenario 8
			High	Low	0.0256	Low	Low	0.002304	Scenario 9
						Low	High	0.005376	Scenario 10
						High	Low	0.005376	Scenario 11
						High	High	0.012544	Scenario 12
			High	High	0.1024	Low	Low	0.009216	Scenario 13
						Low	High	0.021504	Scenario 14
						High	Low	0.021504	Scenario 15
						High	High	0.050176	Scenario 16
Low	High	0.24	Low	Low	0.0096	Low	Low	0.000864	Scenario 17
						Low	High	0.002016	Scenario 18
						High	Low	0.002016	Scenario 19
						High	High	0.004704	Scenario 20
			Low	High	0.0384	Low	Low	0.003456	Scenario 21
						Low	High	0.008064	Scenario 22
						High	Low	0.008064	Scenario 23
						High	High	0.018816	Scenario 24
			High	Low	0.0384	Low	Low	0.003456	Scenario 25
						Low	High	0.008064	Scenario 26
						High	Low	0.018816	Scenario 27
						High	High	0.018816	Scenario 28
			High	High	0.1536	Low	Low	0.013824	Scenario 29
						Low	High	0.032256	Scenario 30
						High	Low	0.032256	Scenario 31
						High	High	0.075264	Scenario 32

Table 5.7: (Continued)

Season 1			Season 2			Season 3			Scenarios
$cn1$	$cn2$	$\eta(i^{1,1})$	$cn1$	$cn2$	$\eta(i^{2,1})$	$cn1$	$cn2$	$\eta(i^{3,1})$	
High	Low	0.24	Low	Low	0.0096	Low	Low	0.000864	Scenario 33
						Low	High	0.002016	Scenario 34
						High	Low	0.002016	Scenario 35
						High	High	0.004704	Scenario 36
			Low	High	0.0384	Low	Low	0.003456	Scenario 37
						Low	High	0.008064	Scenario 38
						High	Low	0.008064	Scenario 39
						High	High	0.018816	Scenario 40
			High	Low	0.0384	Low	Low	0.003456	Scenario 41
						Low	High	0.008064	Scenario 42
						High	Low	0.008064	Scenario 43
						High	High	0.018816	Scenario 44
			High	High	0.1536	Low	Low	0.013824	Scenario 45
						Low	High	0.032256	Scenario 46
						High	Low	0.032256	Scenario 47
						High	High	0.075264	Scenario 48
High	High	0.36	Low	Low	0.0144	Low	Low	0.001296	Scenario 49
						Low	High	0.003024	Scenario 50
						High	Low	0.003024	Scenario 51
						High	High	0.007056	Scenario 52
			Low	High	0.0576	Low	Low	0.005184	Scenario 53
						Low	High	0.012096	Scenario 54
						High	Low	0.012096	Scenario 55
						High	High	0.028224	Scenario 56
			High	Low	0.0576	Low	Low	0.005184	Scenario 57
						Low	High	0.012096	Scenario 58
						High	Low	0.012096	Scenario 59
						High	High	0.028224	Scenario 60
			High	High	0.2304	Low	Low	0.020736	Scenario 61
						Low	High	0.048384	Scenario 62
						High	Low	0.048384	Scenario 63
						High	High	0.112896	Scenario 64

Next, we discuss how the values of A, the intercepts of the demand functions, vary randomly. Tables 5.8 and 5.9 present the values of A under different random events for the residential and commercial sectors, respectively. More precisely, the "low" or "high" demand in the residential and commercial sectors referred to the relative levels of the values of A in the demand functions rather than the actual consumption obtained by running the model. The values of A for the "high" demand levels were universally higher than those in the corresponding "low" demand levels as shown in Tables 5.8 and 5.9. As we can see from results shown in Section 5.2.3, a higher value of A in the demand function did not necessarily lead to a higher consumption at equilibrium point.

Table 5.8: Base Case — Intercepts of Residential Demand Functions

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	349.6	355.2	Low	912	960	Low	1380	1343.2
						High	1650	1679
			High	1064	1100	Low	1500	1460
						High	1725	1606
High	410.4	384.8	Low	893	940	Low	1350	1314
						High	1695	1635.2
			High	1092.5	1170	Low	1425	1387
						High	1875	1722.8

Table 5.9: Base Case — Intercepts of Commercial Demand Functions

Season 1			Season 2			Season 3		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	329	329	Low	720	768	Low	1104	1012
						High	1380	1210
			High	825	880	Low	1200	1100
						High	1344	1265
High	371	367.5	Low	697.5	744	Low	1080	990
						High	1308	1309
			High	862.5	920	Low	1140	1045
						High	1440	1342

As modeled, the spot market consumption for the industrial and electric power sectors are predetermined stochastic elements subject to certain probability distributions. Tables 5.10 and 5.11 show the the values of $D^1_{knsy,i^{s,y}}, \forall k = 3, 4$, the spot market demand for the industrial and electric power sectors. When the outcomes were called "high" in these two tables, the associated $D^1_{knsy,i^{s,y}}$ had a greater values than those associated with random outcomes of low demand.

Table 5.10: Base Case — Industrial Demand in Spot Market (MMCf/d)

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	14	19.5	Low	35	39	Low	56	58.5
						High	105	97.5
			High	63	65	Low	56	58.5
						High	91	117
High	28	32.5	Low	35	39	Low	56	58.5
						High	77	97.5
			High	63	65	Low	56	58.5
						High	126	130

Table 5.11: Base Case — Electric Power Demand in Spot Market (MMCf/d)

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	105	100	Low	63	60	Low	8.4	16
						High	21	32
			High	126	140	Low	8.4	16
						High	25.2	40
High	189	140	Low	58.8	80	Low	8.4	16
						High	33.6	60
			High	142.8	120	Low	8.4	16
						High	42	80

The numerical results for the base case are presented in Tables B.1 to B.20 in Appendix B. Table B.1 shows the seasonal flow rates of pipeline arcs in the long-term and spot markets. For example, in the long-term market, the gas flow rates carried by arc $a1$ were 1,131.1 MMcf/d, 1,131.31 MMcf/d and 1,017.92 MMcf/d for seasons 1,2 and 3, respectively. Note that there was no flow along arcs $a2$ and $a3$ in the long-term market. The spot market had 64 possible scenarios as established in Table 5.7. For instance, in scenario 1, the flow rates along arc $a1$ was 592.06 MMcf/d, 686.46 MMcf/d and 809.33 MMcf/d for seasons 1, 2 and 3, respectively. Table B.2 shows the seasonal congestion fees for each pipeline arc in the long-term and spot markets. Note that the congestion fees for arc $a3$ for season 1 in the long-term market were negative, -0.02\$/Mcf; meanwhile, the corresponding pipeline flow rates were zero. This fact was consistent with Theorem 4.3.2, which stated that when the congestion fee $\tau^0_{asy,i^s,y}$ is less than zero, then the pipeline flow is $f^1_{asy,i^s,y}$ zero. Thus negative fees are unimportant. In the table, some congestion charges were shown as \$(0.00), which were actually the results of rounding off negative values very close to zeros. For example, the congestion fee charged for arc $a1$ in season 1 of scenario 1, shown as \$(0.00) in Table B.2, was accurately -2.7E-13, which can be considered as zero instead of a negative number taking into account the solver tolerance.

Tables B.3 and B.4 present the seasonal production rates and prices for producers in both long-term and spot markets. Producers had positive production rates over the time horizon for both markets. Also, it can be verified from Table B.8 that the long-term production prices were equal to the expected spot market prices, which was in accordance with Theorem 4.3.3.

Table B.5 shows the storage injection and extraction rates. The injection rates are shown in the columns under label "Season 1", followed by columns for extraction rates in Seasons 2 and 3. In this case, storage operators did not have long-term activities. The storage operators served the spot market for all random outcomes. Table B.6 shows the storage gas prices faced by marketers. The storage gas prices were always higher than production prices because of the pipeline transportation charges and the positive marginal cost functions $MC^{ST^S,1}_{rsy,i^s,y}$.

Tables B.7 and B.8 show the production activities and the market prices for the peak gas operators, respectively. Peak gas operators $p1$ and $p2$ contracted to supply the markets with gas at 112.26 MMcf/d and 112.03 MMcf/d, respectively. In only three scenarios 32, 48 and 64, the peak gas operators supplied gas for the spot market. As an example of Theorem 4.3.9, we note that the expected price for peak gas in the spot market at node $cn1$ was \$4.33/Mcf, which was less than the corresponding long-term market price \$4.46/Mcf when the long-term supply of peak gas was positive, i.e., 112.26 MMcf/d as shown in Table B.7.

Given a random event, if both the storage and peak gas served the market, the prices for these two types of gas must be the same, e.g., prices for storage gas and peak gas were both \$4.45/Mcf for node $cn2$ in scenario 32 or \$4.46/Mcf for node $cn1$ in scenario 60. This fact follows from the KKT conditions (4.3.138) and (4.3.139) and implies that storage gas and peak gas were substitutions goods for the marketers.

Tables B.9 and B.10 summarize the consumption rates and end-user wholesale prices for the residential sector, respectively. Tables B.11 and B.12 are the equivalent tables for the commercial sectors. These two sectors were under the market power of marketers and were not modeled with contracted long-term demand as opposed to the industrial and electric power sectors.

Tables B.13 and B.14 show the consumption rates and end-user wholesale prices for the industrial sector, respectively. Whereas tables B.15 and B.16 are for the electric power sector. The long-term consumption for these two sectors was determined by the input of the data presented in Table 5.3. The spot market consumption was based on data in Tables 5.10 and 5.11. Note that the industrial and electric power sectors at the same locations had identical end-user prices. This can be explained by the KKT conditions (4.3.131) and (4.3.132). Taking the long-term market as an example, from (4.3.131), we see that for some marketer m, when $l^0_{kmsy} > 0$, it must follow that $\Theta^0_{kn^m(m)sy} = \phi^0_{msy}$, where ϕ^0_{msy} is independent of the index k. Thus, $\Theta^0_{3n^m(m)sy} = \Theta^0_{4n^m(m)sy}$ as long as $l^0_{3msy} > 0$ and $l^0_{4msy} > 0$, which was universally true for all four marketers as shown in Tables B.17 - B.20. Similar reasoning can be used to explain the identical spot market prices of industrial and electric power sectors at the same location.

Also, we see that the end-user wholesale prices for residential and commercial sectors were higher than those for industrial and commercial sectors. These differences were caused by the market power that marketers had in the residential and commercial sectors. In fact, in case 3 when the market power was lifted, the differences between these sectoral prices disappeared further validating this line of reasoning.

5.2.2 Case 1

In this case, the probabilities for high demand were exchanged with those for low demand so that the low demand outcomes were more favorable. As shown in Table 5.12, for example, the high demand in season 2 had a chance of 0.2, instead of 0.8 in the base case. All other data remained the same as in the base case. Generally speaking, for both the input or output, case 1 represented a lower demand market with less consumption and lower market prices.

Table 5.12: Case 1 — Random Outcomes and Probabilities for Nodes $cn1$ and $cn2$

<table>
<tr><th colspan="2">Season 1</th><th colspan="3">Season 2</th><th colspan="3">Season 3</th></tr>
<tr><th>Event</th><th>$\eta(i^{1,1})$</th><th>Event</th><th>$\eta(i^{2,1}|i^{1,1})$</th><th>$\eta(i^{2,1})$</th><th>Event</th><th>$\eta(i^{3,1}|i^{2,1})$</th><th>$\eta(i^{3,1})$</th></tr>
<tr><td rowspan="4">Low</td><td rowspan="4">0.6</td><td rowspan="2">Low</td><td rowspan="2">0.8</td><td rowspan="2">0.48</td><td>Low</td><td>0.7</td><td>0.336</td></tr>
<tr><td>High</td><td>0.3</td><td>0.144</td></tr>
<tr><td rowspan="2">High</td><td rowspan="2">0.2</td><td rowspan="2">0.12</td><td>Low</td><td>0.7</td><td>0.084</td></tr>
<tr><td>High</td><td>0.3</td><td>0.036</td></tr>
<tr><td rowspan="4">High</td><td rowspan="4">0.4</td><td rowspan="2">Low</td><td rowspan="2">0.8</td><td rowspan="2">0.32</td><td>Low</td><td>0.7</td><td>0.224</td></tr>
<tr><td>High</td><td>0.3</td><td>0.096</td></tr>
<tr><td rowspan="2">High</td><td rowspan="2">0.2</td><td rowspan="2">0.08</td><td>Low</td><td>0.7</td><td>0.056</td></tr>
<tr><td>High</td><td>0.3</td><td>0.024</td></tr>
</table>

Table 5.22 examines the expected profits (i.e., the optimal values of the objective functions) for each player in the different cases and the respective percentage differences as opposed to the base case. We see that all players in case 1 were worse off in terms of expected profits, especially the storage operators whose profits dropped off by more than 40%. Additionally, the last row of the table shows the expected producer surplus, which is the sum of the expected profits of all players. The producer surplus reduced by -13.14% compared with the base case. Also, Table 5.23 compares the expected consumer surplus in terms of the residential and commercial demand sectors. The consumer surplus for the industrial and electric power

sectors are not shown in this table since we do not have explicit demand functions for these two demand sectors in the model. Because this case represented a low consumption scenario, it is reasonable to observe decreases in the consumer surplus for the residential and commercial sectors.

Next, Table 5.13 compares the results of case 1 with those of the base case in the aspects of equilibrium quantities and market prices faced by market suppliers (i.e., producers, storage operators and peak gas operators) and consumers (i.e., four demand sectors) in both the long-term and spot markets. Activities (the rates of gas bought and sold) directly related to marketers, who are the middlemen between suppliers and consumers, can be inferred from what was shown for the suppliers and consumers and thus were not compared in the table. The "rates" in the table, on the supply side, are production rates (q^0_{csy}, $q^1_{csy,i^{s,y}}$) for producers $C1$, $C2$; injection rates (g^0_{ary}, $g^1_{ary,i^{1,y}}$) in season 1 and extraction rates (x^0_{rsy}, $x^1_{rsy,i^{s,y}}$) in seasons 2 and 3 for storage operators $R1$ and $R2$; peak gas production rates (w^0_{py}, $w^1_{py,i^{3,y}}$) for peak gas operators $P1$ and $P2$. For the end-user side, "rates" refer to consumption rates. Also, on the supply side, the "prices" refer to production prices (π^0_{nsy}, $\pi^1_{nsy,i^{s,y}}$) in the row for producers, storage gas prices (γ^0_{nsy}, $\gamma^1_{nsy,i^{s,y}}$) for storage operators and peak gas prices (β^0_{ny}, $\beta^1_{ny,i^{3,y}}$) for peak gas operators. For the four consumption sectors, "prices" are end-user prices.

The top part of Table 5.13 compares the long-term rates and prices of case 1 with the base case. We use "same", "up" or "down" to explain whether the results in case 1 were the same as, greater than or less than their counterparts in the base case. The second part of the table summarizes the differences in the spot market for all outcomes between these two cases. For brevity, the word same/up/down used in this part means, respectively, same/greater/less for <u>all</u> random outcomes. For instance, a "down" means that the value from case 1 was less than the base case for all random outcomes. For mixed results, we would specify which case dominates. The

Table 5.13: Case 1 v.s. Base Case — Overview

Long-Term Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Same	Same	Up	Down	Down	Down
$R1$, $R2$	Same	Same	Same	-	Down	Down
$P1$, $P2$	-	-	Down	-	-	Down
$RD1$, $RD2$	-	-	-	-	-	-
$CD1$, $CD2$	-	-	-	-	-	-
$ID1$, $ID2$	Same	Same	Same	Down	Down	Down
$ED1$, $ED2$	Same	Same	Same	Down	Down	Down
Spot Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Down	Up	Up	Down	Up	Up
$R1$, $R2$	Down	Down	Down	-	Up	Up
$P1$, $P2$	-	-	Up	-	-	Up
$RD1$, $RD2$	Up	Down	Down	Down	Up	Up
$CD1$, $CD2$	Up	Down	Down	Down	Up	Up
$ID1$, $ID2$	Same	Same	Same	Down	Up	Up
$ED1$, $ED2$	Same	Same	Same	Down	Up	Up
Total end-user consumption: Down						

total consumption shown in the last row of the table was the aggregate consumption volume of the four sectors over the three seasons. Similar tables were also used in Sections 5.2.3 and 5.2.4 to compare cases 2 and 3 with the base case.

The activities of players in the long-term market did not change much. The aggregate gas produced in each season for the long-term market was the same because the long-term demand D^0_{kmsy} as shown in Table 5.3 was unchanged. However, producers produced more while peak gas operators produced less in season 3 although the total was the same. Storage operators remained nonactive. Meanwhile, seasonal long-term market prices were lower both on the supply and demand sides.

For all scenarios in the spot market, producers produced less in season 1 while more in seasons 2 and 3. Storage operators injected less gas in season 1 and thus less gas was extracted in seasons 2 and 3. Peak gas operators had more peak gas supply in season 3. The residential and commercial sectors had higher consumption rates in season 1 but lower in seasons 2 and 3. However, the total consumption volume over seasons decreased for both sectors. The consumption for the industrial and electric power sectors was the same as in the base case.

In terms of the prices in the spot market, wellhead prices were lower in season 1 but higher in seasons 2 and 3; storage gas prices were higher for both seasons 2 and 3; peak gas prices were higher; thus it was reasonable to see that four demand sectors had lower prices in season 1 and higher prices in seasons 2 and 3.

Furthermore, Table 5.14 shows the percentage differences of the expected rates and prices in the spot market between case 1 and the base case. It is generally true that expected rates and prices decreased from the base case. The rates for peak gas were the exception showing 6.62% and 34.45% increases for peak gas operators $p1$ and $p2$, respectively. However,the absolute differences the two numbers representing were only 0.05 MMcf/d and 0.21MMcf/d, respectively.

Table 5.14: Case 1 v.s. Base Case — Expected Rates and Prices in Spot market

Participants	Expected Rates			Expected Prices		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$	-9.90%	-14.08%	-12.11%	-3.61%	-6.78%	-6.15%
$C2$	-9.00%	-12.50%	-12.11%	-3.61%	-6.41%	-6.53%
$R1$	-23.92%	-26.98%	-18.97%	-	-6.96%	-6.31%
$R2$	-22.41%	-24.45%	-19.51%	-	-6.79%	-4.48%
$P1$	-	-	6.62%	-	-	-8.11%
$P2$	-	-	34.45%	-	-	-7.96%
$RD1$	-3.02%	-12.26%	-13.72%	-3.26%	-9.82%	-10.27%
$RD2$	-0.86%	-12.62%	-9.94%	-2.25%	-10.12%	-8.31%
$CD1$	-1.76%	-12.74%	-11.41%	-2.83%	-9.44%	-8.72%
$CD2$	-1.49%	-12.53%	-13.06%	-2.71%	-9.54%	-9.49%
$ID1$	-12.50%	-30.04%	-25.97%	-3.45%	-6.96%	-6.31%
$ID2$	-9.52%	-26.88%	-29.09%	-3.44%	-6.79%	-6.34%
$ED1$	-10.81%	-38.14%	-46.55%	-3.45%	-6.96%	-6.31%
$ED2$	-6.45%	-31.53%	-47.49%	-3.44%	-6.79%	-6.43%

5.2.3 Case 2

In this case, we increased the demand for all high demand outcomes from the base case. In particular, the demand intercept values of A of the residential and commercial demand functions were increased by a certain amount for the high demand outcomes. Also, the spot market demand $D^1_{knsy,i^s,y}$ for the industrial and electric power sectors was doubled all high demand outcomes. The other data remained unchanged from the base case.

Tables 5.15 and 5.16 present the values of A used for the residential and commercial sectors in this case. Those underlined values in the tables increased by approximately 5% to 20% of their counterparts in the base case. Tables 5.17 and 5.18 show the spot market demand values of $D^1_{knsy,i^s,y}$ used for industrial and electric power sectors in this case. Those underlined in the tables were double of their counterpart in the base case. Generally speaking, this case represents a higher demand scenario as opposed to the base case.

Table 5.15: Case 2 — Intercepts of Residential Demand Functions

<table>
<tr><th colspan="3">Season 1</th><th colspan="3">Season 2</th><th colspan="3">Season 3</th></tr>
<tr><th>Event</th><th>Node $cn1$</th><th>Node $cn2$</th><th>Event</th><th>Node $cn1$</th><th>Node $cn2$</th><th>Event</th><th>Node $cn1$</th><th>Node $cn2$</th></tr>
<tr><td rowspan="4">Low</td><td rowspan="4">349.6</td><td rowspan="4">355.2</td><td rowspan="2">Low</td><td rowspan="2">912</td><td rowspan="2">960</td><td>Low</td><td>1380</td><td>1343.2</td></tr>
<tr><td>High</td><td><u>1800</u></td><td><u>1898</u></td></tr>
<tr><td rowspan="2">High</td><td rowspan="2"><u>1178</u></td><td rowspan="2"><u>1200</u></td><td>Low</td><td>1500</td><td>1460</td></tr>
<tr><td>High</td><td><u>1950</u></td><td><u>1752</u></td></tr>
<tr><td rowspan="4">High</td><td rowspan="4"><u>440.8</u></td><td rowspan="4"><u>399.6</u></td><td rowspan="2">Low</td><td rowspan="2">893</td><td rowspan="2">940</td><td>Low</td><td>1350</td><td>1314</td></tr>
<tr><td>High</td><td><u>1890</u></td><td><u>1810.4</u></td></tr>
<tr><td rowspan="2">High</td><td rowspan="2"><u>1235</u></td><td rowspan="2"><u>1340</u></td><td>Low</td><td>1425</td><td>1387</td></tr>
<tr><td>High</td><td><u>2250</u></td><td><u>1985.6</u></td></tr>
</table>

In terms of the expected profits, it is reasonable to see from Table 5.22 that

Table 5.16: Case 2 — Intercepts of Commercial Demand Functions

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	329	329	Low	720	768	Low	1104	1012
						High	1560	1320
			High	900	960	Low	1200	1100
						High	1488	1430
High	392	385	Low	697.5	744	Low	1080	990
						High	1416	1518
			High	975	1040	Low	1140	1045
						High	1680	1584

Table 5.17: Case 2 — Industrial Demand in Spot Market(MMcf/d)

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	14	19.5	Low	35	39	Low	56	58.5
						High	210	195
			High	126	130	Low	56	58.5
						High	182	234
High	56	65	Low	35	39	Low	56	58.5
						High	154	195
			High	126	130	Low	56	58.5
						High	252	260

Table 5.18: Case 2 — Electric Power Demand in Spot Market(MMCf/d)

Season 1			**Season 2**			**Season 3**		
Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$	Event	Node $cn1$	Node $cn2$
Low	105	100	Low	63	60	Low	8.4	16
						High	42	64
			High	252	280	Low	8.4	16
						High	50.4	80
High	378	280	Low	58.8	80	Low	8.4	16
						High	67.2	120
			High	285.6	240	Low	8.4	16
						High	84	160

all players in case 2 were doing better in their expected profits. In particular, both storage operators had their profits doubled. The fact of increased profits for players can be explained as a result of the high demand market with higher consumption and prices. Also, as shown in Table 5.23, the consumer surplus in all the residential and commercial sectors increased leading to a total increase by 17.32% as opposed to the base case.

Similar to what was done done for case 1, Table 5.19 compares case 2 with the base case in terms of equilibrium rates and market prices for both the supply and demand sides of the market. For the most part, case 2 represented a high demand market where the consumption rates and prices were all higher.

The long-term consumption rates remained the same since no data in Table 5.3 were varied. The long-term demand was supplied by producers and peak gas operators. There were no storage activities in the long-term market. We also observed that all the long-term prices went up, which is the opposite of case 1.

The changes in the spot market were more complicated than case 1. With increased intercepts of the demand functions, we see significant increases in the end-user consumption rates for the residential and commercial sectors in most cases, accordingly accompanied by increased market prices. However, in some cases, the rates reduced and so did the corresponding prices while in others, reduced rates came with increased prices. On the other hand, the increases in the industrial and electric demand were determined by the parameter changes shown in Tables 5.17 and 5.18; the corresponding prices for the four sectors were up for most random outcomes. It was generally true that for each random outcome, the changes of prices were determined by the upstream prices. For example, if the wellhead price increased, then so did the downstream, including storage gas, peak gas and end-user prices. Lastly, we noticed that the aggregate end-user consumption of the four sectors across seasons was higher than the base case except scenario 1 where the demand levels were low for three seasons in both nodes.

Further, in order to have more insight, we compared the expected spot market rates and prices of case 2 with the base case as shown in Table 5.20. For suppliers, both the rates and market prices increased significantly. Note that although expected production rates of peak gas increased by over 2000%, the absolute values increased by only 15 MMcf/d or so. Except the commercial sectors, $CD1$ and $CD2$ experienced consumption decreases by insignificant percentages, -0.16% and -0.29%, respectively in season 1, we see significant increases in the expected end-user consumption rates. Also, the expected end-user prices exhibited universal increases.

Table 5.19: Case 2 v.s. Base Case — Overview

Long-Term Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Same	Same	Down	Up	Up	Up
$R1$, $R2$	Same	Same	Same	-	Up	Up
$P1$, $P2$	-	-	Up	-	-	Up
$RD1$, $RD2$	-	-	-	-	-	-
$CD1$, $CD2$	-	-	-	-	-	-
$ID1$, $ID2$	Same	Same	Same	Up	Up	Up
$ED1$, $ED2$	Same	Same	Same	Up	Up	Up
Spot Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Up	Mostly Up	Mostly Up	Up	Mostly Up	Mostly Up
$R1$, $R2$	Up	Up	Up	-	Mostly Up	Mostly Up
$P1$, $P2$	-	-	Up	-	-	Up
$RD1$, $RD2$	Mostly Up	Mostly Up	Mostly Up	Up	Mostly Up	Mostly Up
$CD1$, $CD2$	Mostly Up	Mostly Up	Mostly Up	Up	Mostly Up	Mostly Up
$ID1$, $ID2$	By Table 5.17			Up	Mostly Up	Mostly Up
$ED1$, $ED2$	By Table 5.18			Up	Mostly Up	Mostly Up
Total end-user consumption: Up except Scenario 1						

Table 5.20: Case 2 v.s. Base Case — Expected Rates and Prices in Spot market

Participants	**Expected Rates**			**Expected Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$	23.19%	24.30%	10.22%	8.45%	10.74%	17.28%
$C2$	23.26%	22.30%	9.19%	9.32%	10.75%	17.33%
$R1$	20.45%	17.53%	26.90%	-	10.56%	18.20%
$R2$	19.30%	16.86%	25.00%	-	10.66%	16.57%
$P1$	-	-	2019.87%	-	-	17.13%
$P2$	-	-	2472.47%	-	-	15.55%
$RD1$	2.86%	9.90%	10.15%	6.32%	10.20%	13.90%
$RD2$	-0.16%	10.26%	7.17%	4.73%	10.43%	11.60%
$CD1$	0.49%	9.06%	5.39%	5.81%	9.91%	12.15%
$CD2$	-0.29%	9.16%	9.56%	5.44%	9.94%	13.41%
$ID1$	75.00%	87.94%	81.71%	8.91%	10.56%	18.20%
$ID2$	71.43%	87.10%	82.64%	8.90%	10.66%	16.71%
$ED1$	72.97%	90.10%	90.41%	8.91%	10.56%	18.20%
$ED2$	67.74%	87.80%	89.89%	8.90%	10.66%	16.71%

5.2.4 Case 3

This case studied a perfectly competitive market where all players were price-takers including marketers, based on data of the base case.

In terms of expected profits, all players but the marketers were better off, especially the storage operators whose profits increased by 370.84% and 342.10%, respectively. Without market power, the marketers had zero profits. The drop in the marketers' profits shifted to other players' profits and the consumer surplus. Also, we observed dramatic increases in the consumer surplus for the residential and commercial demand sectors. These increases included the deadweight loss recovered by lifting imperfect competition from the market.

Table 5.21 presents the differences between case 3 and the base case. Similar to what was observed for cases 1 and case 2, the rates in the long-term market were relatively stable while the corresponding prices either on the supply side or demand side rose. In the spot market, the supply rates all increased with increased supply prices since the setup of cost functions, i.e., positive marginal costs, implied that the more produced, the more expensive the product. The consumption rates increased for the residential and commercial sectors, where marketers could not exert their market power any more. As a result, the corresponding end-user prices dropped for the two sectors. However, the industrial and electric power sectors suffered price increases because of the increase of the upstream prices. Note that after lifting the market power on the residential and commercial sectors, price discrimination disappeared and thus the four sectors faced common end-user prices.

Table 5.21: Case 3 v.s. Base Case — Overview

Long-Term Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Same	Same	Down	Up	Up	Up
$R1$, $R2$	Same	Same	Same	N/A	Up	Up
$P1$, $P2$	-	-	Up	-	-	Up
$RD1$, $RD2$	-	-	-	-	-	-
$CD1$, $CD2$	-	-	-	-	-	-
$ID1$, $ID2$	Same	Same	Same	Up	Up	Up
$ED1$, $ED2$	Same	Same	Same	Up	Up	Up
Spot Market						
Participants	**Rates**			**Prices**		
	Season 1	**Season 2**	**Season 3**	**Season 1**	**Season 2**	**Season 3**
$C1$, $C2$	Up	Up	Up	Up	Up	Up
$R1$, $R2$	Up	Up	Up	-	Up	Up
$P1$, $P2$	-	-	Up	-	-	Up
$RD1$, $RD2$	Up	Up	Up	Down	Down	Down
$CD1$, $CD2$	Up	Up	Up	Down	Down	Down
$ID1$, $ID2$	Same	Same	Same	Up	Up	Up
$ED1$, $ED2$	Same	Same	Same	Up	Up	Up
Total end-user consumption: Up						

Table 5.22: Expected Profits for Players (Million Dollars)

Participants	Base Case	Case 1	Case 2	Case 3
$C1$	1,191.8	1,081.8/-9.23%	1,456.4/22.20%	1,619.3/35.87%
$C2$	1,185.3	1,077.3/-9.11%	1,459.9/23.16%	1,617.9/36.49%
$R1$	8.0	4.6/-43.22%	17.5/117.18%	38.0/371.84%
$R2$	8.6	5.0/-41.67%	17.4/101.88%	38.0/342.10%
$P1$	6.9	6.0/-12.93%	10.2/46.48%	14.8/114.10%
$P2$	6.9	6.0/-13.17%	9.8/42.93%	14.3/107.82%
$M1$	399.4	324.0/-18.88%	469.7/17.60%	0/-100%
$M2$	399.4	324.0/-18.88%	469.7/17.60%	0/-100%
$M3$	458.1	376.0/-17.91%	536.3/17.07%	0/-100%
$M4$	458.1	376.0/-17.91%	536.3/17.07%	0/-100%
Producer Surplus[a]	4,122.5	3.580.8/-13.14%	4,983.1/20.87%	3,342.4/-18.92%

[a]The pipeline operators' profits are not included since their costs are not modeled in (PL).

Table 5.23: Expected Consumer Surplus (Million Dollars)

Participants	Base Case	Case 1	Case 2	Case 3
$RD1$	525.9	423.1/-19.55%	628.4/19.48%	1,031.7/96.16%
$RD2$	602.5	496.6/-17.57%	706.6/17.29%	1,204.3/99.89%
$CD1$	272.8	224.9/-17.57%	311.0/13.99%	504.5/84.92%
$CD2$	313.7	255.4/-18.56%	365.9/16.67%	596.4/90.14%
Consumer Surplus[a]	1,714.9	1,400.0/-18.36%	2,011.9/17.32%	3,336.8/94.58%

[a]The surplus of the industrial and electric power sectors are not included.

5.3 VSS and $EVPI$

Several concepts related to the performance of stochastic programming were presented in Section 4.1.3. Here, we use these concepts in the context of a stochastic equilibrium model to quantitatively evaluate the advantage of using a stochastic solution, from both the market and the individual players' perspectives, as opposed to a deterministic one. In particular, we explore the WS, RP and EEV for the four cases presented above. In fact, these four cases themselves indeed are examples of the here-and-now solution concept. Thus, the values of the RP for the players' profit and consumer surplus, can be found in Tables 5.22 and 5.23, respectively. In what follows, the calculations for WS and EEV are presented.

First, we define $z^i(x, \xi)$ the profit/consumer surplus for player/demand sector i choosing the decision x under scenario ξ. The choice of x is obtained by solving an overall stochastic equilibrium problem for various players, which is formulated as an MiCP as shown in Theorem 4.3.14. This differs from what was discussed in Section 4.1.3 for the stochastic programming, where the x is the result of solving an optimization problem.

In the wait-and-see case, it is assumed that all players have same access to the information about the future. As a result, we solve an overall MiCP for each scenario ξ, i.e., *S-NGEM-MiCP*(G^S, H^S, ξ) (see Theorem 4.3.14 for the definition), where ξ is treated as a parameter. The solution for each of these MiCP is denoted $\overline{x}(\xi)$, which implies that the WS for player i is

$$WS^i \equiv E_{\boldsymbol{\xi}}\big[z^i(\overline{x}(\boldsymbol{\xi}), \boldsymbol{\xi})\big] \tag{5.3.1}$$

Also, we can use the expected value $\overline{\xi}$ of the random variable random $\boldsymbol{\xi}$ to solve an alternative problem. Thus, the resulting MiCP is *S-NGEM-MiCP*($G^S, H^S, \overline{\xi}$), whose solution is $\overline{x}(\overline{\xi})$. Thus the EEV for player i is

$$EEV^i \equiv E_{\boldsymbol{\xi}}\big[z^i(\overline{x}(\overline{\xi}), \boldsymbol{\xi})\big] \tag{5.3.2}$$

Based on the preceding descriptions for the WS, RP and EEV, Tables 5.24 - 5.27 show these three values for the four cases as well as $EVPI$ and VSS by market agent. From these tables, we observe that the $EVPI$ and VSS have negative values for some agents in various cases. In general, this means that the relationship shown in (4.1.12) (i.e., $EEV \leq RP \leq WS$) does not hold in equilibrium problems. However, these negative values are very small as compared with EEV, RP or WS. Some of them might be numerical errors caused by the solver's tolerance. Concentrating on the value for the stochastic solution, VSS, we further observe that

- Marketers and the producer surplus have positive VSS in the four cases. These values for VSS are relatively significant as compared with the VSS for the other players. Thus, the stochastic solutions are favorable for marketers and the entire market. Note that the marketers are the only players in the market who explicitly face the random demand in their objective functions and thus this corresponds best to the case of "regular stochastic programming" with just one optimization problem.

- In general, the consumer surplus for the residential and commercial sectors have positive VSS in the four cases, except for the commercial demand sector $CD1$ in case 3, which has very small negative VSS of \$-0.048 million as opposed to a here-and-now solution of \$504.5 million. Thus, the stochastic solution is generally advantageous to the residential and commercial consumers.

- Producer $C2$, storage operator $R1$ and peak gas operator $P2$ have all positive VSS for the four cases. While producer $C1$, storage operator $R2$ and peak gas operator $P1$ have mixed values for the VSS. Therefore, a stochastic solution is not necessary a better choice for these players, who see the random demand implicitly via the market-clearing conditions and do not have as much control over it as for example, the marketers.

We learn from these values of the VCC that a stochastic solution for an equilibrium model generally improves the profits of the players as well as the consumer surplus. However, the observation of the negative VSS cannot be explained accurately without further investigation.

In terms of the $EVPI$, it seems that the perfect information does not bring significant returns to the players in that the values of $EVPI$ are generally insignificant as opposed to the values of RP or WS and some players (e.g., producers $C1$ and $C2$) even suffered losses as a result of the perfect information. Since this dissertation is concerned with the stochastic solution rather than the perfect information, we do not elaborate on the details regarding the $EVPI$ for the various agents. Further research on these phenomena is certainly desired.

Table 5.24: Base Case — EEV, RP, WS (Million Dollars)

Participants	EEV	RP	WS	$EVPI$	VSS
$C1$	1,192.1	1,191.8	1,190.8	-1.0	-0.3
$C2$	1,180.5	1,185.3	1,184.8	-0.5	4.8
$R1$	8.0	8.0	8.4	0.4	0.0
$R2$	9.0	8.6	9.0	0.4	-0.4
$P1$	7.0	6.9	6.9	0.0	-0.056
$P2$	6.8	6.9	6.9	0.0	0.1
$M1$	395.0	399.4	399.5	0.1	4.4
$M2$	395.0	399.4	399.5	0.1	4.4
$M3$	454.5	458.1	458.2	0.1	3.6
$M4$	454.5	458.1	458.2	0.1	3.6
Producer Surplus	4,102.4	4,122.5	4,122.2	-0.3	20.10
$RD1$	520.2	525.9	526.1	0.2	5.7
$RD2$	598.2	602.5	602.6	0.1	4.3
$CD1$	269.9	272.8	273.0	0.2	2.9
$CD2$	310.9	313.7	313.8	0.1	2.8
Consumer Surplus	1,699.2	1,714.9	1,715.5	0.6	15.7

Table 5.25: Case 1 — EEV, RP, WS (Million Dollars)

Participants	EEV	RP	WS	$EVPI$	VSS
$C1$	1,082.3	1,081.8	1,081.4	-0.4	-0.5
$C2$	1,074.1	1,077.3	1,076.8	-0.5	3.2
$R1$	4.3	4.6	4.7	0.1	0.3
$R2$	5.0	5.0	5.1	0.1	0.0
$P1$	6.1	6.0	6.0	0.0	-0.05
$P2$	6.0	6.0	6.0	0.0	0.0
$M1$	320.5	324.0	324.2	0.2	3.5
$M2$	320.5	324.0	324.2	0.2	3.5
$M3$	373.4	376.0	376.3	0.3	2.6
$M4$	373.4	376.0	376.3	0.3	2.6
Producer Surplus	3,565.6	3,580.7	3,581.0	0.3	15.5
$RD1$	418.6	423.1	423.4	0.3	4.5
$RD2$	493.3	496.6	497.0	0.4	3.3
$CD1$	222.4	224.9	225.1	0.2	2.5
$CD2$	253.4	255.4	255.7	0.3	2.0
Consumer Surplus	1,387.7	1,400.0	1,401.2	1.2	12.3

Table 5.26: Case 2 — EEV, RP, WS (Million Dollars)

Participants	EEV	RP	WS	$EVPI$	VSS
$C1$	1,428.8	1,456.4	1,454.4	-2.0	27.6
$C2$	1,414.2	1,459.9	1,455.6	-4.3	45.7
$R1$	11.6	17.5	21.6	4.1	5.9
$R2$	13.0	17.4	21.8	4.4	4.4
$P1$	8.4	10.1	10.6	0.5	1.7
$P2$	8.3	9.8	10.3	0.5	1.5
$M1$	467.0	469.7	469.4	-0.3	2.7
$M2$	467.0	469.7	469.4	-0.3	2.7
$M3$	533.0	536.3	535.6	-0.7	3.3
$M4$	533.0	536.3	535.6	-0.7	3.3
Producer Surplus	4,884.3	4,983.1	4,984.3	1.2	98.8
$RD1$	623.7	628.4	628.0	-0.4	4.7
$RD2$	701.9	706.6	705.8	-0.8	4.7
$CD1$	310.3	311.0	310.8	-0.2	0.7
$CD2$	364.1	365.9	365.4	-0.5	1.8
Consumer Surplus	2,000.0	2,011.9	2,010.0	-1.9	11.9

Table 5.27: Case 3 — EEV, RP, WS (Million Dollars)

Participants	EEV	RP	WS	$EVPI$	VSS
$C1$	1,625.0	1,619.3	1,620.2	0.9	-5.7
$C2$	1,600.9	1,617.9	1,617.5	-0.4	17.0
$R1$	35.8	38.0	39.2	1.2	2.2
$R2$	38.3	38.0	39.4	1.4	-0.3
$P1$	14.0	14.8	15.1	0.3	0.8
$P2$	13.7	14.3	14.6	0.3	0.6
$M1$	0.0	0.0	0.0	0.0	0.0
$M2$	0.0	0.0	0.0	0.0	0.0
$M3$	0.0	0.0	0.0	0.0	0.0
$M4$	0.0	0.0	0.0	0.0	0.0
Producer Surplus	3,327.7	3,342.4	3,346.0	3.7	14.6
$RD1$	1,028.8	1,031.7	1,031.1	-0.6	2.9
$RD2$	1,198.7	1,204.3	1,203.4	-0.9	5.6
$CD1$	504.6	504.5	504.2	-0.3	-0.05
$CD2$	593.2	596.4	595.9	-0.5	3.2
Consumer Surplus	3,325.3	3,336.9	3,334.6	-2.3	11.6

5.4 Conclusions

A sample complementarity problem of 6,186 variables as well as three variations was used to validate the model S-NGEM and related theorems established in Chapter 4.

From the base case, case 1 and case 2, we learned that the greater the consumption is, the greater the expected profits for suppliers and the higher the expected end-user prices for four demand sectors. Also, case 3 showed the influence of market power of imperfect competing marketers on the market. We also observed some results concerning the properties of the VSS and $EVPI$ established for "regular" stochastic programming with one optimization problem.

Among market players, storage operators' profits changed most dramatically. The levels of storage gas varied greatly with these cases so as to lighten the congestion in pipelines in wintertime and buffer the impact of demand changes on the market. In addition, recalling that in Section 4.3.3 for storage operators in model S-NGEM, the mass balance constraints (4.3.53) and (4.3.54) are relaxed as inequalities for modeling purposes, we note that the two constraints held as equalities for the four cases presented.

Chapter 6

Summary and Future Work

One of the main thrusts of this dissertation is an endeavor in the field of energy modeling for the North American natural gas market using an NCP/VI formulation combined with the stochastic programming. To our knowledge, a model with as much detail as the model S-NGEM using the NCP/VI format and stochastic programming for modeling equilibrium activities has not appeared before. We anticipate that such a model will be of use to both public and private sector concerns.

As part of the process of building a stochastic model, we first described a deterministic model D-NGEM, which is an enhancement of the equilibrium model initially developed in [35] in that free market prices (i.e., π_{nsy}, γ_{nsy} and β_{ny}) are equivalently replaced by nonnegative prices under minor assumptions. The improvement on the existence and uniqueness results was also presented. However, since it does not take into account the uncertain aspects inherent in the industry, model D-NGEM would not provide a satisfactory solution to an uncertainty environment. To this end, a new model S-NGEM emphasizing the decision making under uncertainty is developed using stochastic programming techniques. In general, model S-NGEM is a stochastic extension of model D-NGEM and model D-NGEM can be considered as a special case of model S-NGEM.

Model S-NGEM is composed of separate optimization problems that maximize the expected profits for pipeline operators, producers, storage operators, peak gas operators and Nash-Cournot marketers as well as market-clearing conditions. Besides the long-term decisions incorporated by model D-NGEM, model S-NGEM also considers decisions regarding how to react to the uncertainty. In particular, after they have observed the realizations of the randomness, players make recourse decisions in the spot market to compensate for any adverse effects that might have been experienced as a result of the long-term decisions. This model generates a set of strategies or policies in response to difference scenarios resulting from the uncertainty. The model was shown to be a instance of an MiCP under minor assumptions. These assumptions require that the cost functions are convex and continuously differentiable and that the marginal cost functions are positive in the positive orthant.

We illustrate the model S-NGEM on a sample network of two production and consumption nodes, respectively and four connecting pipeline arcs. The end-user demand for the four demand sectors is taken as random subject to a discrete probability distribution. A base case was calibrated using the right magnitude for this small network based on the data on consumption, wellhead prices and end-user prices publicized on the EIA website. Varying the distribution of the end-user demand resulted in two comparison cases, one representing a relatively high demand scenario, the other for a low demand scenario as opposed to the base case. A third case of perfectly competing marketers was also presented to show the influence of market power in the equilibrium prices and quantities. We compared the changes in the network activities, the expected profits for all agents, the prices and consumption for the four demand sectors, and the overall producer and consumer surplus. EVPI and VSS were also presented.

Recall that in Chapter 5, model S-NGEM resulted in over 6,000 variables with only a four-node sample network, a simple two-realization seasonal demand and a time horizon of three seasons. This is far too simple to be a real application. The increase in the time horizon or the number of scenarios associated with uncertain seasonal demand will certainly cause great computational difficulties given the larger number of variables if treated as is. Therefore, an important future direction of the current work is to develop efficient algorithms for this stochastic equilibrium model formulated as an MiCP. Algorithms for a stochastic NCP/VI are still in infant stage. The existing studies include [40] that used a sample-path method, a simulation-based scheme to solve stochastic variational inequalities and [3] that analyzed the stochastic NCP/VI using a quasi-Monte-Carlo simulation technique and relied on the gradient information at sample solution points.

In the numerical results, the concept of the VSS is presented, showing that a stochastic solution (RP) was better than a solution obtained using the expectation of the uncertainty (EEV) in terms of the total producer surplus and consumer surplus. However, this quantity failed to measure the advantage of a stochastic solution in terms of the individual players. Theoretically, we need to establish a mechanism similar to what has been done for stochastic programming showing the value of the stochastic solution in a market equilibrium model.

As opposed to the recourse method, another method in stochastic programming is the chance-constraint method, which, however, was not adopted in this dissertation. Some results using the chance-constraint method based on the model D-NGEM have been obtained and presented at the INFORMS annual meeting in Denver in 2004. Due to time limitations, this part of work was not included here. Further exploration of the corresponding application of this method for the equilibrium model in the future is desired. Last but not least, establishing the existence and uniqueness results for the model S-NGEM is another future research item.

Appendix A

Notation

Acronyms

D-NGEM	Deterministic natural gas equilibrium model presented in Chapter 3
D-NGEM-MiCP	MiCP formulation for model D-NGEM
D-NGEM-VI	VI formulation for model D-NGEM
LCP	Linear complementarity problems
LNG	Liquified natural gas
MiCP	Mixed complementarity problems
MCC	Market clearing conditions
MK	Marketer(s)
NCP	Nonlinear complementarity problems
PG	Peak gas operator(s)
PL	Pipeline operator(s)
PR	Producer(s)
S-NGEM	Stochastic natural gas equilibrium model presented in Chapter 4
S-NGEM-MiCP	MiCP formulation for model S-NGEM
ST	Storage operator(s)
VI	Variational inequalities problems

Problem Classes

D-NGEM	The deterministic natural gas equilibrium model presented in Chapter 3
D-NGEM-MiCP	The MiCP formulation for model D-NGEM
D-NGEM-VI	The VI formulation for model D-NGEM
(MK)	The collection of optimization problems $(\widetilde{MK})$ for all $m \in M$ in model D-NGEM
$(\widetilde{MK})$	Optimization problems of the individual marketers in model D-NGEM
(MK^S)	The collection of optimization problems $(\widetilde{MK}^S)$ for all $m \in M$ in model S-NGEM
$(\widetilde{MK}^S)$	Optimization problems of the individual marketers in model S-NGEM
(PG)	The optimization problem for all peak gas operators in model D-NGEM
$(\widetilde{PG})$	Optimization problems for the individual peak gas operators in model D-NGEM
(PG^S)	The optimization problem for all peak gas operators in model S-NGEM
$(\widetilde{PG}^S)$	Optimization problems for the individual peak gas operators in model S-NGEM
PG-MCC	The system of the problem (PG), market-clearing conditions for peak gas market and the corresponding nonnegative prices β_{ny} in model D-NGEM
PG-MCC-NCP	The NCP formulation for system *PG-MCC* in model D-NGEM
(PL)	The optimization problem for all pipeline operators in model D-NGEM
$(\widetilde{PL})$	Optimization problems for the individual pipeline operators in model D-NGEM
(PL^S)	The optimization problem for all pipeline operators in model S-NGEM
$(\widetilde{PL}^S)$	Optimization problems for the individual pipeline operators in model S-NGEM
(PR)	The optimization problem for all producers in model D-NGEM

(To be continued)

Problem Classes (Cont'd)

$(\widetilde{PR})$	Optimization problems for the individual producers in model D-NGEM
(PR^S)	The optimization problem for all producers in model S-NGEM
$(\widetilde{PR}^S)$	Optimization problems for the individual producers in model S-NGEM
PR-MCC	The system of the problem (PR), market-clearing conditions for the production market and the corresponding nonnegative prices π_{nsy} in model D-NGEM
PR-MCC-NCP	The NCP formulation for system *PR-MCC* in model D-NGEM
S-CM-MCC	The system of the the problem $(\widetilde{MK}^S)$ for all $m \in M$, market-clearing conditions for the industrial and electric power sectors and the corresponding nonnegative prices Θ^0_{knsy} and $\Theta^1_{knsy,i^{s,y}}$ in model S-NGEM
S-CM-MCC-NCP	The NCP formulation for system *S-CM-MCC* in model S-NGEM
S-NGEM	The stochastic natural gas equilibrium model presented in Chapter 4
S-NGEM-MiCP	The MiCP formulation for model S-NGEM
S-PG-MCC	The system of the problem (PG^S) , market-clearing conditions for peak gas market and the corresponding nonnegative prices β^0_{ny} and $\beta^1_{ny,i^{3,y}}$ in model S-NGEM
S-PG-MCC-NCP	The NCP formulation for *S-PG-MCC* in model S-NGEM
S-PR-MCC	The system of the problem (PR^S), market-clearing conditions for the production market and the corresponding nonnegative prices π^0_{nsy} and $\pi^1_{nsy,i^{s,y}}$ in model S-NGEM
S-PR-MCC-NCP	The NCP formulation for *S-PR-MCC* in model S-NGEM
S-ST-MCC	The system of the problem (ST^S), market-clearing conditions for the storage gas market and the corresponding nonnegative prices γ^0_{nsy} and $\gamma^1_{nsy,i^{s,y}}$ in model S-NGEM
S-ST-MCC-NCP	The NCP formulation for system *S-ST-MCC* in model S-NGEM
(ST)	The optimization problem for all storage operators in model D-NGEM
$(\widetilde{ST})$	Optimization problems for the individual storage operators in model D-NGEM
(ST^S)	The optimization problem for all storage operators in model S-NGEM

(To be continued)

Problem Classes (Cont'd)

$(\widetilde{ST}^S)$	Optimization problems for the individual storage operators in model S-NGEM
ST-MCC	The system of the problem *ST-NCP*, market-clearing conditions for the storage gas market and the corresponding nonnegative prices γ_{nsy} in model D-NGEM
ST-MCC-NCP	The NCP formulation for system *ST-MCC* in model D-NGEM
ST-NCP	The NCP formulation for problem (ST) in model D-NGEM

Data in Models D-NGEM and S-NGEM

$days_s$	The number of days in season s
D^0_{knsy}	The long-term consumption rates of industrial and electric power sectors located at node n in season s of year y, (MMcf/day)
$D^1_{knsy,i^{s,y}}$	The spot market consumption rates of industrial and electric power sectors located at node n in season s of year y under random event $i^{s,y}$, (MMcf/day)
$\overline{f}_a$	Positive pipeline capacity value of arc $a \in A$, (MMcf/day)
$\overline{g}_r$	Positive upper bound on injection rate for storage operator r, (MMcf/day)
$\overline{k}_r$	Positive upper bound on capacity of working gas storage volume for storage operator r, (MMcf)
$loss_a$	Loss along arc $a \in A$, $loss_a \in [0, 1)$
$loss_r$	Storage operator r loss factor, $loss_r \in [0, 1)$
$\overline{q}_c$	Positive upper bound on production rate for producer c, (MMcf/day)
$prod_c$	positive production forecast for producer c for the time horizon, (MMcf)
$\overline{w}_p$	Positive upper bound on peak gas rate for peak gas operator p, (MMcf/day)
$\overline{x}_r$	Positive upper bound on extraction rate for storage operator r, (MMcf/day)
RC^0_{asy}	Reservation charge rates for the firm service with pipeline company, (\$/Mcf)
$\eta(i^{s,y})$	The probability node $i^{s,y}$ on the event tree
τ^{reg}_{asy}	Positive regulated transportation rate (e.g., FERC rate) for season s and year y, (\$/Mcf)

Functions and Operations in Models D-NGEM and S-NGEM

$c^{PL}(\cdot)$	The nonnegative, continuously differentiable, pipeline operations cost function
$c_c^{PR}(\cdot)$	The nonnegative, continuously differentiable cost function for producer c
$c_p^{PG}(\cdot)$	The nonnegative, continuously differentiable cost function for peak gas operator p
$c_r^{ST}(\cdot)$	The nonnegative, continuously differentiable cost function for storage operator r
$n_1(a)$	Consumption node at the end of arc a
$n_2(a)$	Production node at the end of arc a
$n^c(c)$	The node where producer c is located
$n^m(m)$	The node where marketer m is located
$n^r(r)$	The node where storage operator r is located
$n^p(p)$	The node where peak gas operator p is located
$s(i)$	The season associated with random event i
$y(i)$	The year associated with random event i
$\theta_{knsy}(\cdot)$	The nonnegative, nonincreasing, continuously differentiable, inverse end-use demand function for consumption node n for season s and year y in model D-NGEM
$\theta^1_{knsy,i^{s,y}}(\cdot)$	the nonnegative, nonincreasing, continuously differentiable, inverse end-use demand function for residential and commercial sectors for consumption node n for season s and year y under random event $i^{s,y}$ in model S-NGEM
$\psi(i)$	The immediate predecessor set of random event i on the scenario tree
$\Psi(i)$	The immediate predecessor set the immediate predecessors of random event i on the scenario tree

Variables in Model D-NGEM

f_{asy}	Flow along arc $a \in A$ for season s and year y, (MMcf/day)
g_{ary}	Flow rate of gas for storage operators r from producer in season 1 along arc $a \in A(n^r(r))$, (MMcf/day)
h_{amsy}	Flow rate of gas shipped to marketer m from producers along arc $a \in A^m(m)$ in season s and year y, (MMcf/day)
l_{kmsy}	Rate of gas to demand sector k from marketer m in season s and year y, (MMcf/day)
q_{csy}	Production rate for producer c for season s and year y, (MMcf/day)
u_{msy}	Rate of storage gas shipped to marketer m for $s = 2, 3$ and year y, (MMcf/day)
v_{my}	Rate of peak gas shipped to marketer m for $s = 3$ and year y, (MMcf/day)
w_{py}	Rate of peak gas produced by peak gas operator p in year y, (MMcf/day)
x_{rsy}	Extraction rate by storage operator r for seasons $s = 2, 3$, and year y, (MMcf/day)
β_{py}	Peak gas price for node $n \in CN$, in year y, ($/Mcf)
γ_{nsy}	Storage market gas price for $n \in CN$, seasons $s = 2, 3$ and year y, ($/Mcf)
δ_{ry}	Multiplier for material balance constraint for storage operator r in year y, ($/Mcf)
ζ_{ry}	Multiplier for storage volume capacity constraint for storage operator r and year y, ($/Mcf)
λ_{csy}	Multiplier for production capacity constraint of producer c for season s and year y, ($/Mcf)
μ_c	Multiplier for production forecast constraint, ($/Mcf)
ξ_{rsy}	Multiplier for injection capacity constraint for storage operator r for $s = 2, 3$, and year y, ($1000/MMcf/day)
ω_{ry}	Multiplier for extraction capacity constraint for storage operator r for $s = 2, 3$, and year y, ($1000/MMcf/day)
π_{nsy}	Production price for node $n \in PN$ in season s and year y, ($/Mcf)
ρ_{asy}	Multiplier for pipeline capacity constraint for season s and year y, ($1000/MMcf/day)

(To be continued)

Variables in Model D-NGEM (Cont'd)

σ_{py}	Multiplier for capacity constraint for peak gas operator p in year y (\$1000/MMcf/day)
τ_{asy}	Transportation rates for arc a season s and year y, exogenous for pipeline, marketers and storage operators, but a variable in the overall equilibrium problem, (\$/Mcf)
ϕ_{msy}	Multiplier for gas balance constraint between four sectors for marketer m in season s and year y, (\$/Mcf)

Variables in Model S-NGEM

f^0_{asy}	Flow along arc $a \in A$ for season s and year y in the long-term market, (MMcf/day)
$f^1_{asy,i^{s,y}}$	Flow along arc $a \in A$ for season s and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
g^0_{ary}	Flow rate of gas to storage operators r from producer in season 1 along arc $a \in A(n^r(r))$, (MMcf/day)
$g^1_{ary,i^{1,y}}$	Flow rate of gas to storage operators r from producer in season 1 along arc $a \in A(n^r(r))$ in the spot market under random event $i^{s,y}$, (MMcf/day)
h^0_{amsy}	Flow rate of gas shipped to marketer m from producers along arc $a \in A^m(m)$ in season s and year y in the long-term market, (MMcf/day)
$h^1_{amsy,i^{s,y}}$	Flow rate of gas shipped to marketer m from producers along arc $a \in A^m(m)$ in season s and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
l^0_{kmsy}	Rate of gas to demand sector k from marketer m in season s and year y in the long-term market, (MMcf/day)
$l^1_{kmsy,i^{s,y}}$	Rate of gas to demand sector k from marketer m in season s and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
q^0_{csy}	Production rate for producer c for season s and year y in the long-term market, (MMcf/day)
$q^1_{csy,i^{s,y}}$	Production rate for producer c for season s and year y in the spot market under random event $i^{s,y}$, (MMcf/day)

(To be continued)

Variables in Model S-NGEM (Cont'd)

u^0_{msy}	Rate of storage gas shipped to marketer m for $s = 2, 3$ and year y in the long-term market, (MMcf/day)
$u^1_{msy,i^{s,y}}$	Rate of storage gas shipped to marketer m for $s = 2, 3$ and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
v^0_{my}	Rate of peak gas shipped to marketer m for $s = 3$ and year y in the long-term market, (MMcf/day)
$v^1_{my,i^{s,y}}$	Rate of peak gas shipped to marketer m for $s = 3$ and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
w^0_{py}	Rate of peak gas produced by peak gas operator p in year y in the long-term market, (MMcf/day)
$w^1_{py,i^{3,y}}$	Rate of peak gas produced by peak gas operator p in year y in the spot market under random event $i^{s,y}$, (MMcf/day)
x^0_{rsy}	Extraction rate by storage operator r for seasons $s = 2, 3$, and year y in the long-term market, (MMcf/day)
$x^1_{rsy,i^{s,y}}$	Extraction rate by storage operator r for seasons $s = 2, 3$, and year y in the spot market under random event $i^{s,y}$, (MMcf/day)
β^0_{py}	Peak gas price for node $n \in CN$, in year y in the long-term market, (\$/Mcf)
$\beta^1_{py,i^{3,y}}$	Peak gas price for node $n \in CN$, in year y in the spot market under random event $i^{s,y}$, (\$/Mcf)
γ^0_{nsy}	Storage market gas price for $n \in CN$, seasons $s = 2, 3$ and year y in the long-term market, (\$/Mcf)
$\gamma^1_{nsy,i^{s,y}}$	Storage market gas price for $n \in CN$, seasons $s = 2, 3$ and year y in the spot market under random event $i^{s,y}$, (\$/Mcf)
δ^0_{ry}	Multiplier for material balance constraint for storage operator r in year y in the long-term market, (\$/Mcf)
$\delta^1_{ry,i^{3,y}}$	Multiplier for material balance constraint for storage operator r in year y in the spot market under random event $i^{3,y}$, (\$/Mcf)
Θ^0_{knsy}	End-use prices for industrial and electric power sectors for node $n \in CN$, season s year y in the long-term market, (\$/Mcf)
$\Theta^1_{knsy,i^{s,y}}$	End-use prices for industrial and electric power sectors for node $n \in CN$, season s year y in the spot market under random event $i^{s,y}$, (\$/Mcf)
$\zeta^1_{ry,i^{3,y}}$	Multiplier for storage volume capacity constraint for storage operator r and year y in the spot market under random event $i^{3,y}$, (\$/Mcf)

(To be continued)

Variables in Model S-NGEM (Cont'd)

$\lambda^1_{csy,i^{s,y}}$	Multiplier for production capacity constraint of producer c for season s and year y in the spot market under random event $i^{s,y}$, (\$/Mcf)
$\mu^1_{c,i^{3,\lvert Y \rvert}}$	Multiplier for production forecast constraint in the spot market under random event $i^{3,\lvert Y \rvert}$, (\$/Mcf)
$\xi^1_{rsy,i^{1,y}}$	Multiplier for injection capacity constraint for storage operator r for $s = 2, 3$, and year y in the spot market under random event $i^{1,y}$, (\$1000/MMcf/day)
$\omega^1_{ry,i^{s,y}}$	Multiplier for extraction capacity constraint for storage operator r for $s = 2, 3$, and year y in the spot market under random event $i^{s,y}$, (\$1000/MMcf/day)
π^0_{nsy}	Production price for node $n \in PN$ in season s and year y in the long-term market, (\$/Mcf)
$\pi^1_{nsy,i^{s,y}}$	Production price for node $n \in PN$ in season s and year y in the spot market under random event $i^{s,y}$, (\$/Mcf)
$\rho^1_{asy,i^{s,y}}$	Multiplier for pipeline capacity constraint for season s and year y in the spot market under random event $i^{s,y}$, (\$1000/MMcf/day)
$\sigma^1_{py,i^{3,y}}$	Multiplier for capacity constraint for peak gas operator p in year y in the spot market under random event $i^{s,y}$, (\$1000/MMcf/day)
τ^0_{asy}	Transportation rates for arc a season s and year y in the long-term market, exogenous for pipeline, marketers and storage operators, but a variable in the overall model S-NGEM
$\tau^1_{asy,i^{s,y}}$	Transportation rates for arc a season s and year y in the spot market under random event $i^{s,y}$, exogenous for pipeline, marketers and storage operators, but a variable in the overall equilibrium problem, (\$/Mcf)
ϕ^0_{msy}	Multiplier for gas balance constraint between four sectors for marketer m in season s and year y in the long-term market, (\$/Mcf)
$\phi^1_{msy,i^{s,y}}$	Multiplier for gas balance constraint between four sectors for marketer m in season s and year y in the spot market under random event $i^{s,y}$, (\$/Mcf)

Sets

A	The set of pipeline arcs, $a \in A$
$A(n)$	The set of arcs connected to node $n \in N$
C	The set of producers, $c \in C$
C^n	The set of producers located at node $n \in PN$
CN	The set of consumption nodes, $CN \subset N$
I	The set of realizations of random perturbation
$I^{s,y}$	The set of realizations of random perturbation in season s of year y
$ISC(i)$	The set of immediate successors of random outcome i in the scenario tree
$IISC(i)$	The set of immediate successors of immediate successors of random outcome i in the scenario tree
M	The set of marketers, $m \in M$
M^n	The set of marketers located at node $n \in CN$
N	The set of nodes in the network, $n \in N$
P	The set of peak gas operators, $p \in P$
P^n	The set of peak gas operators located at node $n \in CN$
$PD(i)$	The set consisting of all the predecessors of random outcome i in the scenario tree, inclusive of i
PN	The set of production nodes, $PN \subset N$
R	The set of storage operators
R^n	The set of storage operators located at node $n \in CN$
S	The set of seasons under consideration, $S = \{1, 2, 3\}, s \in S$
$SC(i)$	The set consisting of all the successors of random outcome i in the scenario tree, inclusive of i
Y	The set of years under consideration, $y \in Y$
$\psi(i)$	The immediate predecessor (set) of random event i in the scenario tree
$\Psi(i)$	The immediate predecessor (set) the immediate predecessors of random event i in the scenario tree

Appendix B

Numerical Results for Base Case

Table B.1: Pipeline Flow Rates, f^0_{asy}, $f^1_{asy,i^{s},y}$ (MMcf/d)

Long-term Market											
Season 1				Season 2				Season 3			
Arc *a1*	Arc *a2*	Arc *a3*	Arc *a4*	Arc *a1*	Arc *a2*	Arc *a3*	Arc *a4*	Arc *a1*	Arc *a2*	Arc *a3*	Arc *a4*
1131.31	0.00	0.00	1060.61	1131.31	0.00	0.00	1060.61	1017.92	0.00	0.00	947.45

Spot Market												
Season 1				Season 2				Season 3				
Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4	
592.06	0.00	15.89	628.61	686.46	0.00	0.00	752.63	809.33	0.00	58.28	802.22	Scenario 1
								834.49	126.67	0.00	1052.55	Scenario 2
								982.08	0.00	278.14	764.90	Scenario 3
								982.08	152.91	237.63	990.67	Scenario 4
				663.39	123.11	0.00	877.64	881.59	0.00	0.00	965.11	Scenario 5
								839.27	177.68	13.45	1052.55	Scenario 6
								982.08	70.36	286.22	858.69	Scenario 7
								982.08	190.96	259.58	1007.16	Scenario 8
				802.29	0.00	122.00	730.61	899.64	0.00	136.13	813.75	Scenario 9
								934.25	134.36	64.60	1052.55	Scenario 10
								982.08	18.21	325.61	766.62	Scenario 11
								982.08	189.18	287.84	977.09	Scenario 12
				868.69	33.16	27.20	939.39	982.08	0.00	68.11	987.63	Scenario 13
								916.59	198.64	110.74	1052.55	Scenario 14
								982.08	114.57	342.72	846.85	Scenario 15
								982.08	235.17	316.08	995.32	Scenario 16
607.76	5.14	0.00	702.27	687.02	0.00	0.00	763.32	795.12	0.00	60.53	785.89	Scenario 17
								807.20	198.85	2.66	1052.55	Scenario 18
								974.65	0.00	274.59	749.55	Scenario 19
								982.08	179.90	216.48	1039.09	Scenario 20
				655.92	163.60	0.00	911.00	863.13	0.00	19.43	894.31	Scenario 21
								742.81	352.25	90.77	1052.55	Scenario 22
								982.08	35.55	290.69	819.05	Scenario 23
								959.05	314.91	267.88	1052.55	Scenario 24
				807.65	0.00	115.16	742.75	886.33	0.00	142.10	794.61	Scenario 25
								889.88	207.29	92.87	1052.55	Scenario 26
								982.08	4.94	318.17	760.66	Scenario 27
								982.08	217.07	271.31	1021.80	Scenario 28
				838.25	104.79	52.54	939.39	961.96	0.00	91.06	920.51	Scenario 29
								815.65	378.29	188.65	1052.55	Scenario 30
								982.08	80.31	343.26	811.69	Scenario 31
								967.35	351.44	312.25	1052.55	Scenario 32
656.38	0.00	104.10	604.07	672.83	6.19	0.00	769.07	811.84	0.00	25.16	837.80	Scenario 33
								803.74	162.14	0.00	1052.55	Scenario 34
								982.08	0.00	249.41	799.73	Scenario 35
								982.08	156.49	209.45	1022.47	Scenario 36
				645.53	145.38	0.00	882.10	861.28	19.74	0.00	971.74	Scenario 37
								815.17	204.85	16.49	1052.55	Scenario 38
								982.08	74.29	267.36	881.51	Scenario 39
								982.08	194.89	240.72	1029.98	Scenario 40
				850.21	0.00	146.33	753.71	870.55	0.00	84.63	836.46	Scenario 41
								882.39	157.37	36.04	1052.55	Scenario 42
								982.08	117.77	528.47	664.32	Scenario 43
								982.08	288.73	490.71	874.79	Scenario 44
				868.69	66.91	94.86	933.39	968.01	0.00	14.57	1002.99	Scenario 45
								874.90	212.15	82.86	1052.55	Scenario 46
								982.08	214.88	556.51	734.39	Scenario 47
								982.08	334.48	528.33	883.39	Scenario 48
695.63	0.00	50.33	696.69	687.73	0.00	0.00	771.27	797.96	0.00	50.89	798.34	Scenario 49
								797.60	211.39	5.58	1052.55	Scenario 50
								978.59	0.00	266.26	761.78	Scenario 51
								982.08	183.84	212.12	1047.42	Scenario 52
				655.40	168.43	0.00	915.36	865.97	0.00	12.44	904.11	Scenario 53
								735.86	362.12	93.67	1052.55	Scenario 54
								982.08	39.48	288.98	824.72	Scenario 55
								953.43	324.56	271.86	1052.55	Scenario 56
				855.60	0.00	156.87	748.51	857.34	0.00	107.96	800.05	Scenario 57
								855.40	212.89	64.27	1052.55	Scenario 58
								982.08	104.61	538.51	640.99	Scenario 59
								982.08	316.41	490.95	902.52	Scenario 60
				868.69	111.44	110.40	939.39	933.81	0.00	55.56	928.14	Scenario 61
								779.81	386.09	160.90	1052.55	Scenario 62
								982.08	180.80	563.08	693.40	Scenario 63
								982.08	408.46	507.03	979.42	Scenario 64

Table B.2: Pipeline Congestion Fees, τ^0_{asy}, $\tau^1_{asy,i^{s,y}}$ ($/Mcf)

Long-term Market											
Season 1				Season 2				Season 3			
Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4
$ -	$ -	$ (0.02)	$ -	$ 0.02	$ -	$ (0.00)	$ 0.03	$ 0.05	$ -	$ -	$ 0.03

Spot Market												
Season 1				Season 2				Season 3				
Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4	Arc a1	Arc a2	Arc a3	Arc a4	
$ (0.00)	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 1
								$ -	$ -	$ -	$ 0.00	Scenario 2
								$ 0.00	$ -	$ -	$ -	Scenario 3
								$ 0.01	$ -	$ -	$ -	Scenario 4
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 5
								$ -	$ -	$ -	$ 0.01	Scenario 6
								$ 0.01	$ -	$ -	$ -	Scenario 7
								$ 0.03	$ -	$ -	$ -	Scenario 8
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 9
								$ -	$ -	$ -	$ 0.01	Scenario 10
								$ 0.01	$ -	$ -	$ -	Scenario 11
								$ 0.03	$ -	$ -	$ -	Scenario 12
				$ 0.34	$ -	$ -	$ 0.61	$ 0.01	$ -	$ -	$ -	Scenario 13
								$ -	$ -	$ -	$ 0.05	Scenario 14
								$ 0.05	$ -	$ -	$ -	Scenario 15
								$ 0.13	$ -	$ -	$ -	Scenario 16
$ 0.00	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 17
								$ -	$ -	$ -	$ 0.00	Scenario 18
								$ -	$ -	$ -	$ -	Scenario 19
								$ 0.01	$ -	$ -	$ -	Scenario 20
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 21
								$ -	$ -	$ -	$ 0.02	Scenario 22
								$ 0.02	$ -	$ -	$ -	Scenario 23
								$ -	$ -	$ -	$ 0.05	Scenario 24
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 25
								$ -	$ -	$ -	$ 0.02	Scenario 26
								$ 0.02	$ -	$ -	$ -	Scenario 27
								$ 0.05	$ -	$ -	$ -	Scenario 28
				$ -	$ -	$ -	$ 1.45	$ -	$ -	$ -	$ -	Scenario 29
								$ -	$ -	$ -	$ 0.08	Scenario 30
								$ 0.08	$ -	$ -	$ -	Scenario 31
								$ -	$ -	$ -	$ 0.21	Scenario 32
$ (0.00)	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 33
								$ -	$ -	$ -	$ 0.00	Scenario 34
								$ 0.00	$ -	$ -	$ -	Scenario 35
								$ 0.01	$ -	$ -	$ -	Scenario 36
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 37
								$ -	$ -	$ -	$ 0.02	Scenario 38
								$ 0.02	$ -	$ -	$ -	Scenario 39
								$ 0.05	$ -	$ -	$ -	Scenario 40
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 41
								$ -	$ -	$ -	$ 0.02	Scenario 42
								$ 0.02	$ -	$ -	$ -	Scenario 43
								$ 0.05	$ -	$ -	$ -	Scenario 44
				$ 1.46	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 45
								$ -	$ -	$ -	$ 0.08	Scenario 46
								$ 0.08	$ -	$ -	$ -	Scenario 47
								$ 0.21	$ -	$ -	$ -	Scenario 48
$ 0.00	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 49
								$ -	$ -	$ -	$ 0.01	Scenario 50
								$ -	$ -	$ -	$ -	Scenario 51
								$ 0.02	$ -	$ -	$ -	Scenario 52
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 53
								$ -	$ -	$ -	$ 0.03	Scenario 54
								$ 0.03	$ -	$ -	$ -	Scenario 55
								$ -	$ -	$ -	$ 0.08	Scenario 56
				$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	Scenario 57
								$ -	$ -	$ -	$ 0.03	Scenario 58
								$ 0.03	$ -	$ -	$ -	Scenario 59
								$ 0.08	$ -	$ -	$ -	Scenario 60
				$ 1.06	$ -	$ -	$ 1.17	$ -	$ -	$ -	$ -	Scenario 61
								$ -	$ -	$ -	$ 0.12	Scenario 62
								$ 0.13	$ -	$ -	$ -	Scenario 63
								$ 0.32	$ -	$ -	$ -	Scenario 64

Table B.3: Production Rates, q^0_{csy}, $q^1_{csy,i^s,y}$ (MMcf/d)

Long-term Market						
Season 1		**Season 2**		**Season 3**		
Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	
1131.31	1060.61	1131.31	1060.61	1017.92	947.45	

Spot Market						
Season 1		**Season 2**		**Season 3**		
Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	
592.06	644.50	686.46	752.63	809.33	860.50	Scenario 1
				961.16	1052.55	Scenario 2
				982.08	1043.03	Scenario 3
				1134.99	1228.30	Scenario 4
		786.50	877.64	881.59	965.11	Scenario 5
				1016.95	1066.00	Scenario 6
				1052.44	1144.91	Scenario 7
				1173.04	1266.74	Scenario 8
		802.29	852.61	899.64	949.89	Scenario 9
				1068.61	1117.15	Scenario 10
				1000.29	1092.23	Scenario 11
				1171.25	1264.93	Scenario 12
		901.84	966.59	982.08	1055.73	Scenario 13
				1115.23	1163.30	Scenario 14
				1096.65	1189.57	Scenario 15
				1217.25	1311.40	Scenario 16
612.90	702.27	687.02	763.32	795.12	846.41	Scenario 17
				1006.05	1055.21	Scenario 18
				974.65	1024.14	Scenario 19
				1161.98	1255.57	Scenario 20
		819.52	911.00	863.13	913.74	Scenario 21
				1095.05	1143.32	Scenario 22
				1017.63	1109.74	Scenario 23
				1273.97	1320.43	Scenario 24
		807.65	857.91	886.33	936.70	Scenario 25
				1097.17	1145.42	Scenario 26
				987.02	1078.82	Scenario 27
				1199.15	1293.11	Scenario 28
		943.04	991.93	961.96	1011.57	Scenario 29
				1193.94	1241.21	Scenario 30
				1062.39	1154.96	Scenario 31
				1318.79	1364.80	Scenario 32
656.38	708.17	679.02	769.07	811.84	862.96	Scenario 33
				965.88	1052.55	Scenario 34
				982.08	1049.14	Scenario 35
				1138.57	1231.91	Scenario 36
		790.91	882.10	881.02	971.74	Scenario 37
				1020.02	1069.04	Scenario 38
				1056.37	1148.88	Scenario 39
				1176.97	1270.71	Scenario 40
		850.21	900.05	870.55	921.09	Scenario 41
				1039.76	1088.59	Scenario 42
				1099.85	1192.80	Scenario 43
				1270.81	1365.50	Scenario 44
		935.59	1028.26	968.01	1017.56	Scenario 45
				1087.06	1135.41	Scenario 46
				1196.96	1290.90	Scenario 47
				1316.56	1411.72	Scenario 48
695.63	747.02	687.73	771.27	797.96	849.23	Scenario 49
				1008.99	1058.13	Scenario 50
				978.59	1028.04	Scenario 51
				1165.91	1259.54	Scenario 52
		823.83	915.36	865.97	916.56	Scenario 53
				1097.98	1146.22	Scenario 54
				1021.56	1113.71	Scenario 55
				1277.98	1324.41	Scenario 56
		855.60	905.38	857.34	908.02	Scenario 57
				1068.29	1116.83	Scenario 58
				1086.69	1179.50	Scenario 59
				1298.49	1393.47	Scenario 60
		980.13	1049.80	933.81	983.71	Scenario 61
				1165.90	1213.45	Scenario 62
				1162.88	1256.47	Scenario 63
				1390.54	1486.45	Scenario 64

Table B.4: Production Prices, π^0_{nsy}, $\pi^1_{nsy,i^{s,y}}$ (\$/Mcf)

Long-term Market						
Season 1		Season 2		Season 3		
Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	
\$ 3.21	\$ 3.19	\$ 3.65	\$ 3.65	\$ 3.96	\$ 3.97	

Spot Market						
Season 1		Season 2		Season 3		
Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	Node *pn*1	Node *pn*2	
\$ 3.11	\$ 3.07	\$ 3.27	\$ 3.27	\$ 3.29	\$ 3.26	Scenario 1
				\$ 3.57	\$ 3.60	Scenario 2
				\$ 3.60	\$ 3.59	Scenario 3
				\$ 3.88	\$ 3.92	Scenario 4
		\$ 3.46	\$ 3.49	\$ 3.42	\$ 3.45	Scenario 5
				\$ 3.67	\$ 3.63	Scenario 6
				\$ 3.73	\$ 3.77	Scenario 7
				\$ 3.95	\$ 3.99	Scenario 8
		\$ 3.48	\$ 3.45	\$ 3.45	\$ 3.42	Scenario 9
				\$ 3.76	\$ 3.72	Scenario 10
				\$ 3.64	\$ 3.67	Scenario 11
				\$ 3.94	\$ 3.99	Scenario 12
		\$ 3.66	\$ 3.65	\$ 3.60	\$ 3.61	Scenario 13
				\$ 3.84	\$ 3.80	Scenario 14
				\$ 3.81	\$ 3.85	Scenario 15
				\$ 4.03	\$ 4.07	Scenario 16
\$ 3.14	\$ 3.18	\$ 3.28	\$ 3.29	\$ 3.27	\$ 3.23	Scenario 17
				\$ 3.65	\$ 3.61	Scenario 18
				\$ 3.59	\$ 3.55	Scenario 19
				\$ 3.93	\$ 3.97	Scenario 20
		\$ 3.51	\$ 3.55	\$ 3.39	\$ 3.35	Scenario 21
				\$ 3.81	\$ 3.77	Scenario 22
				\$ 3.67	\$ 3.71	Scenario 23
				\$ 4.13	\$ 4.09	Scenario 24
		\$ 3.49	\$ 3.46	\$ 3.43	\$ 3.39	Scenario 25
				\$ 3.81	\$ 3.77	Scenario 26
				\$ 3.61	\$ 3.65	Scenario 27
				\$ 3.99	\$ 4.04	Scenario 28
		\$ 3.74	\$ 3.70	\$ 3.57	\$ 3.53	Scenario 29
				\$ 3.98	\$ 3.94	Scenario 30
				\$ 3.75	\$ 3.79	Scenario 31
				\$ 4.21	\$ 4.17	Scenario 32
\$ 3.22	\$ 3.19	\$ 3.26	\$ 3.30	\$ 3.30	\$ 3.26	Scenario 33
				\$ 3.57	\$ 3.60	Scenario 34
				\$ 3.60	\$ 3.60	Scenario 35
				\$ 3.88	\$ 3.93	Scenario 36
		\$ 3.46	\$ 3.50	\$ 3.42	\$ 3.46	Scenario 37
				\$ 3.67	\$ 3.63	Scenario 38
				\$ 3.74	\$ 3.78	Scenario 39
				\$ 3.95	\$ 4.00	Scenario 40
		\$ 3.57	\$ 3.53	\$ 3.40	\$ 3.37	Scenario 41
				\$ 3.71	\$ 3.67	Scenario 42
				\$ 3.81	\$ 3.86	Scenario 43
				\$ 4.12	\$ 4.17	Scenario 44
		\$ 3.72	\$ 3.76	\$ 3.58	\$ 3.54	Scenario 45
				\$ 3.79	\$ 3.75	Scenario 46
				\$ 3.99	\$ 4.03	Scenario 47
				\$ 4.21	\$ 4.25	Scenario 48
\$ 3.29	\$ 3.26	\$ 3.28	\$ 3.30	\$ 3.27	\$ 3.24	Scenario 49
				\$ 3.65	\$ 3.61	Scenario 50
				\$ 3.60	\$ 3.56	Scenario 51
				\$ 3.93	\$ 3.98	Scenario 52
		\$ 3.52	\$ 3.54	\$ 3.39	\$ 3.36	Scenario 53
				\$ 3.81	\$ 3.77	Scenario 54
				\$ 3.67	\$ 3.71	Scenario 55
				\$ 4.14	\$ 4.09	Scenario 56
		\$ 3.58	\$ 3.54	\$ 3.38	\$ 3.34	Scenario 57
				\$ 3.76	\$ 3.72	Scenario 58
				\$ 3.79	\$ 3.83	Scenario 59
				\$ 4.17	\$ 4.22	Scenario 60
		\$ 3.80	\$ 3.80	\$ 3.52	\$ 3.48	Scenario 61
				\$ 3.93	\$ 3.89	Scenario 62
				\$ 3.93	\$ 3.97	Scenario 63
				\$ 4.34	\$ 4.38	Scenario 64

Table B.5: Storage Injection/Extraction Rates, g^0_{ary}, $g^1_{ary,i^{1},y}$, x^0_{rsy}, $x^1_{rsy,i^{s},y}$(MMcf/d)

Long-term Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
0.00	0.00	0.00	0.00	0.00	0.00	

Spot Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
192.25	204.48	218.10	236.69	456.42	467.26	Scenario 1
				456.42	467.26	Scenario 2
				456.42	467.26	Scenario 3
				456.42	467.26	Scenario 4
		225.79	248.55	426.67	421.36	Scenario 5
				426.67	421.36	Scenario 6
				426.67	421.36	Scenario 7
				426.67	421.36	Scenario 8
		228.66	244.32	415.56	437.74	Scenario 9
				415.56	437.74	Scenario 10
				415.56	437.74	Scenario 11
				415.56	437.74	Scenario 12
		238.38	259.05	377.93	380.72	Scenario 13
				377.93	380.72	Scenario 14
				377.93	380.72	Scenario 15
				377.93	380.72	Scenario 16
194.08	189.99	217.46	215.25	471.31	452.14	Scenario 17
				471.31	452.14	Scenario 18
				471.31	452.14	Scenario 19
				471.31	452.14	Scenario 20
		228.18	224.46	429.83	416.51	Scenario 21
				429.83	416.51	Scenario 22
				429.83	416.51	Scenario 23
				429.83	416.51	Scenario 24
		229.24	222.21	425.72	425.23	Scenario 25
				425.72	425.23	Scenario 26
				425.72	425.23	Scenario 27
				425.72	425.23	Scenario 28
		239.81	236.28	384.82	370.77	Scenario 29
				384.82	370.77	Scenario 30
				384.82	370.77	Scenario 31
				384.82	370.77	Scenario 32
181.42	185.08	200.86	212.02	449.86	431.53	Scenario 33
				449.86	431.53	Scenario 34
				449.86	431.53	Scenario 35
				449.86	431.53	Scenario 36
		210.93	221.69	410.90	394.11	Scenario 37
				410.90	394.11	Scenario 38
				410.90	394.11	Scenario 39
				410.90	394.11	Scenario 40
		210.90	214.71	411.00	421.10	Scenario 41
				411.00	421.10	Scenario 42
				411.00	421.10	Scenario 43
				411.00	421.10	Scenario 44
		223.43	227.08	362.50	373.23	Scenario 45
				362.50	373.23	Scenario 46
				362.50	373.23	Scenario 47
				362.50	373.23	Scenario 48
170.95	182.93	184.79	206.26	441.32	439.24	Scenario 49
				441.32	439.24	Scenario 50
				441.32	439.24	Scenario 51
				441.32	439.24	Scenario 52
		196.18	214.79	397.25	406.24	Scenario 53
				397.25	406.24	Scenario 54
				397.25	406.24	Scenario 55
				397.25	406.24	Scenario 56
		194.43	209.77	404.01	425.65	Scenario 57
				404.01	425.65	Scenario 58
				404.01	425.65	Scenario 59
				404.01	425.65	Scenario 60
		204.91	224.45	363.44	368.85	Scenario 61
				363.44	368.85	Scenario 62
				363.44	368.85	Scenario 63
				363.44	368.85	Scenario 64

Table B.6: Storage Gas Prices, γ^0_{nsy}, $\gamma^1_{nsy,i^{s,y}}$ (\$/Mcf)

Long-term Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
		$ 4.12	$ 4.12	$ 4.46	$ 4.44	

Spot Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
		$ 3.46	$ 3.45	$ 3.48	$ 3.44	Scenario 1
				$ 3.75	$ 3.79	Scenario 2
				$ 3.81	$ 3.77	Scenario 3
				$ 4.15	$ 4.11	Scenario 4
		$ 3.64	$ 3.68	$ 3.61	$ 3.63	Scenario 5
				$ 3.85	$ 3.89	Scenario 6
				$ 4.00	$ 3.96	Scenario 7
				$ 4.22	$ 4.18	Scenario 8
		$ 3.67	$ 3.63	$ 3.64	$ 3.60	Scenario 9
				$ 3.95	$ 3.99	Scenario 10
				$ 3.90	$ 3.86	Scenario 11
				$ 4.22	$ 4.18	Scenario 12
		$ 3.88	$ 3.89	$ 3.84	$ 3.80	Scenario 13
				$ 4.03	$ 4.07	Scenario 14
				$ 4.08	$ 4.04	Scenario 15
				$ 4.31	$ 4.26	Scenario 16
		$ 3.46	$ 3.47	$ 3.45	$ 3.42	Scenario 17
				$ 3.83	$ 3.87	Scenario 18
				$ 3.78	$ 3.74	Scenario 19
				$ 4.20	$ 4.16	Scenario 20
		$ 3.70	$ 3.74	$ 3.57	$ 3.54	Scenario 21
				$ 4.00	$ 4.04	Scenario 22
				$ 3.93	$ 3.89	Scenario 23
				$ 4.32	$ 4.37	Scenario 24
		$ 3.68	$ 3.64	$ 3.62	$ 3.58	Scenario 25
				$ 4.00	$ 4.04	Scenario 26
				$ 3.88	$ 3.84	Scenario 27
				$ 4.27	$ 4.23	Scenario 28
		$ 3.93	$ 3.97	$ 3.75	$ 3.72	Scenario 29
				$ 4.18	$ 4.22	Scenario 30
				$ 4.02	$ 3.98	Scenario 31
				$ 4.40	$ 4.45	Scenario 32
		$ 3.45	$ 3.48	$ 3.48	$ 4.45	Scenario 33
				$ 3.76	$ 3.80	Scenario 34
				$ 3.82	$ 3.78	Scenario 35
				$ 4.16	$ 4.12	Scenario 36
		$ 3.65	$ 3.69	$ 3.61	$ 3.64	Scenario 37
				$ 3.86	$ 3.90	Scenario 38
				$ 4.01	$ 3.97	Scenario 39
				$ 4.23	$ 4.19	Scenario 40
		$ 3.76	$ 3.72	$ 3.59	$ 3.55	Scenario 41
				$ 3.90	$ 3.94	Scenario 42
				$ 4.09	$ 4.05	Scenario 43
				$ 4.40	$ 4.36	Scenario 44
		$ 3.99	$ 3.95	$ 3.77	$ 3.73	Scenario 45
				$ 3.98	$ 3.73	Scenario 46
				$ 4.27	$ 4.22	Scenario 47
				$ 4.49	$ 4.44	Scenario 48
		$ 3.46	$ 3.49	$ 3.46	$ 3.42	Scenario 49
				$ 3.84	$ 3.88	Scenario 50
				$ 3.78	$ 3.75	Scenario 51
				$ 4.21	$ 4.17	Scenario 52
		$ 3.71	$ 3.75	$ 3.58	$ 3.54	Scenario 53
				$ 4.00	$ 4.04	Scenario 54
				$ 3.94	$ 3.90	Scenario 55
				$ 4.33	$ 4.37	Scenario 56
		$ 3.77	$ 3.73	$ 3.56	$ 3.53	Scenario 57
				$ 3.95	$ 3.99	Scenario 58
				$ 4.06	$ 4.02	Scenario 59
				$ 4.46	$ 4.41	Scenario 60
		$ 4.03	$ 4.03	$ 3.70	$ 3.67	Scenario 61
				$ 4.13	$ 4.17	Scenario 62
				$ 4.20	$ 4.16	Scenario 63
				$ 4.63	$ 4.58	Scenario 64

Table B.7: Peak Gas Production Rates, w^0_{py}, $w^1_{py,i^{3,y}}$ (MMcf/d)

Long-term Market						
Season 1		Season 2		Season 3		
Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	
				112.26	112.03	
Spot Market						
Season 1		Season 2		Season 3		
Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	
				0.00	0.00	Scenario 1
				0.00	0.00	Scenario 2
				0.00	0.00	Scenario 3
				0.00	0.00	Scenario 4
				0.00	0.00	Scenario 5
				0.00	0.00	Scenario 6
				0.00	0.00	Scenario 7
				0.00	0.00	Scenario 8
				0.00	0.00	Scenario 9
				0.00	0.00	Scenario 10
				0.00	0.00	Scenario 11
				0.00	0.00	Scenario 12
				0.00	0.00	Scenario 13
				0.00	0.00	Scenario 14
				0.00	0.00	Scenario 15
				0.00	0.00	Scenario 16
				0.00	0.00	Scenario 17
				0.00	0.00	Scenario 18
				0.00	0.00	Scenario 19
				0.00	0.00	Scenario 20
				0.00	0.00	Scenario 21
				0.00	0.00	Scenario 22
				0.00	0.00	Scenario 23
				0.00	0.00	Scenario 24
				0.00	0.00	Scenario 25
				0.00	0.00	Scenario 26
				0.00	0.00	Scenario 27
				0.00	0.00	Scenario 28
				0.00	0.00	Scenario 29
				0.00	0.00	Scenario 30
				0.00	0.00	Scenario 31
				0.00	0.78	Scenario 32
				0.00	0.00	Scenario 33
				0.00	0.00	Scenario 34
				0.00	0.00	Scenario 35
				0.00	0.00	Scenario 36
				0.00	0.00	Scenario 37
				0.00	0.00	Scenario 38
				0.00	0.00	Scenario 39
				0.00	0.00	Scenario 40
				0.00	0.00	Scenario 41
				0.00	0.00	Scenario 42
				0.00	0.00	Scenario 43
				0.00	0.00	Scenario 44
				0.00	0.00	Scenario 45
				0.00	0.00	Scenario 46
				0.00	0.00	Scenario 47
				1.72	0.66	Scenario 48
				0.00	0.00	Scenario 49
				0.00	0.00	Scenario 50
				0.00	0.00	Scenario 51
				0.00	0.00	Scenario 52
				0.00	0.00	Scenario 53
				0.00	0.00	Scenario 54
				0.00	0.00	Scenario 55
				0.00	0.00	Scenario 56
				0.00	0.00	Scenario 57
				0.00	0.00	Scenario 58
				0.00	0.00	Scenario 59
				0.76	0.00	Scenario 60
				0.00	0.00	Scenario 61
				0.00	0.00	Scenario 62
				0.00	0.00	Scenario 63
				5.64	4.54	Scenario 64

Table B.8: Peak Gas Prices, β^0_{ny}, $\beta^1_{ny,i^3,y}$ (\$/Mcf)

Long-term Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
				$ 4.46	$ 4.44	

Spot Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
				$ 4.43	$ 4.42	Scenario 1
				$ 4.43	$ 4.42	Scenario 2
				$ 4.43	$ 4.42	Scenario 3
				$ 4.43	$ 4.42	Scenario 4
				$ 4.43	$ 4.42	Scenario 5
				$ 4.43	$ 4.42	Scenario 6
				$ 4.43	$ 4.42	Scenario 7
				$ 4.43	$ 4.42	Scenario 8
				$ 4.43	$ 4.42	Scenario 9
				$ 4.43	$ 4.42	Scenario 10
				$ 4.43	$ 4.42	Scenario 11
				$ 4.43	$ 4.42	Scenario 12
				$ 4.15	$ 4.13	Scenario 13
				$ 4.15	$ 4.15	Scenario 14
				$ 4.16	$ 4.15	Scenario 15
				$ 4.31	$ 4.26	Scenario 16
				$ 4.43	$ 4.42	Scenario 17
				$ 4.43	$ 4.42	Scenario 18
				$ 4.43	$ 4.42	Scenario 19
				$ 4.43	$ 4.42	Scenario 20
				$ 4.43	$ 4.42	Scenario 21
				$ 4.33	$ 4.42	Scenario 22
				$ 4.31	$ 4.23	Scenario 23
				$ 4.43	$ 4.42	Scenario 24
				$ 4.43	$ 4.42	Scenario 25
				$ 4.35	$ 4.42	Scenario 26
				$ 4.26	$ 4.16	Scenario 27
				$ 4.35	$ 4.35	Scenario 28
				$ 4.03	$ 3.96	Scenario 29
				$ 4.25	$ 4.25	Scenario 30
				$ 4.02	$ 4.06	Scenario 31
				$ 4.40	$ 4.45	Scenario 32
				$ 4.43	$ 4.42	Scenario 33
				$ 4.43	$ 4.42	Scenario 34
				$ 4.43	$ 4.42	Scenario 35
				$ 4.43	$ 4.42	Scenario 36
				$ 4.43	$ 4.42	Scenario 37
				$ 4.21	$ 4.25	Scenario 38
				$ 4.43	$ 4.30	Scenario 39
				$ 4.32	$ 4.30	Scenario 40
				$ 4.43	$ 4.42	Scenario 41
				$ 4.26	$ 4.28	Scenario 42
				$ 4.43	$ 4.35	Scenario 43
				$ 4.43	$ 4.42	Scenario 44
				$ 4.02	$ 4.01	Scenario 45
				$ 4.07	$ 4.07	Scenario 46
				$ 4.27	$ 4.29	Scenario 47
				$ 4.49	$ 4.44	Scenario 48
				$ 4.43	$ 4.42	Scenario 49
				$ 4.43	$ 4.42	Scenario 50
				$ 4.43	$ 4.42	Scenario 51
				$ 4.43	$ 4.42	Scenario 52
				$ 4.23	$ 4.21	Scenario 53
				$ 4.22	$ 4.24	Scenario 54
				$ 4.16	$ 4.11	Scenario 55
				$ 4.33	$ 4.37	Scenario 56
				$ 4.25	$ 4.15	Scenario 57
				$ 4.18	$ 4.18	Scenario 58
				$ 4.29	$ 4.22	Scenario 59
				$ 4.46	$ 4.41	Scenario 60
				$ 3.84	$ 3.82	Scenario 61
				$ 4.18	$ 4.17	Scenario 62
				$ 4.20	$ 4.22	Scenario 63
				$ 4.63	$ 4.58	Scenario 64

Table B.9: Consumption Rates for Residential Sector (MMcf/d)

Spot Market						
Season 1		Season 2		Season 3		
Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	
156.35	169.54	469.62	508.85	711.39	700.44	Scenario 1
				694.83	904.50	Scenario 2
				871.27	681.63	Scenario 3
				850.86	886.41	Scenario 4
		462.34	593.55	703.51	767.52	Scenario 5
				688.74	850.03	Scenario 6
				860.05	749.00	Scenario 7
				846.62	833.78	Scenario 8
		562.53	501.94	781.54	691.23	Scenario 9
				763.11	893.32	Scenario 10
				915.85	676.56	Scenario 11
				896.82	882.63	Scenario 12
		554.15	585.49	769.87	758.19	Scenario 13
				758.02	839.80	Scenario 14
				905.12	744.40	Scenario 15
				891.70	829.18	Scenario 16
155.46	187.10	469.58	494.78	712.94	682.42	Scenario 17
				689.93	870.63	Scenario 18
				873.36	664.11	Scenario 19
				847.85	854.40	Scenario 20
		459.94	637.91	705.52	724.15	Scenario 21
				680.22	919.76	Scenario 22
				863.92	703.96	Scenario 23
				840.70	901.14	Scenario 24
		562.14	488.24	782.99	673.12	Scenario 25
				759.99	861.14	Scenario 26
				917.33	658.48	Scenario 27
				893.71	850.53	Scenario 28
		552.29	629.29	774.74	714.07	Scenario 29
				749.43	909.47	Scenario 30
				908.94	699.30	Scenario 31
				885.81	896.48	Scenario 32
194.15	167.14	457.49	507.71	691.12	700.18	Scenario 33
				674.31	904.01	Scenario 34
				900.60	681.00	Scenario 35
				880.46	886.04	Scenario 36
		449.35	593.24	683.57	766.84	Scenario 37
				668.41	849.71	Scenario 38
				889.61	748.59	Scenario 39
				876.18	833.37	Scenario 40
		578.04	498.66	734.71	694.19	Scenario 41
				716.25	896.32	Scenario 42
				1004.77	666.20	Scenario 43
				985.74	872.27	Scenario 44
		568.62	583.14	724.08	762.12	Scenario 45
				711.09	842.73	Scenario 46
				993.96	733.96	Scenario 47
				980.64	818.84	Scenario 48
192.49	185.42	456.86	494.23	692.63	682.13	Scenario 49
				669.61	870.32	Scenario 50
				902.93	663.71	Scenario 51
				877.41	853.99	Scenario 52
		446.96	637.61	685.21	723.86	Scenario 53
				659.90	919.46	Scenario 54
				893.49	703.55	Scenario 55
				870.26	900.72	Scenario 56
		577.65	484.96	736.15	676.07	Scenario 57
				713.14	864.15	Scenario 58
				1006.23	648.10	Scenario 59
				982.65	840.19	Scenario 60
		567.04	626.70	727.81	716.94	Scenario 61
				702.49	912.39	Scenario 62
				997.75	688.84	Scenario 63
				972.41	889.01	Scenario 64

Table B.10: End-User Prices for Residential Sector ($/Mcf)

Spot Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
$ 5.52	$ 5.99	$ 7.37	$ 7.91	$ 7.43	$ 7.56	Scenario 1
				$ 7.61	$ 9.11	Scenario 2
				$ 8.65	$ 7.78	Scenario 3
				$ 8.88	$ 9.32	Scenario 4
		$ 7.49	$ 8.89	$ 7.52	$ 8.15	Scenario 5
				$ 7.68	$ 8.89	Scenario 6
				$ 8.78	$ 8.36	Scenario 7
				$ 8.93	$ 9.08	Scenario 8
		$ 8.36	$ 8.04	$ 7.98	$ 7.67	Scenario 9
				$ 8.19	$ 9.24	Scenario 10
				$ 8.99	$ 7.84	Scenario 11
				$ 9.20	$ 9.37	Scenario 12
		$ 8.50	$ 9.03	$ 8.11	$ 8.26	Scenario 13
				$ 8.24	$ 9.01	Scenario 14
				$ 9.11	$ 8.42	Scenario 15
				$ 9.26	$ 9.14	Scenario 16
$ 5.55	$ 6.38	$ 7.37	$ 7.81	$ 7.41	$ 7.43	Scenario 17
				$ 7.67	$ 8.99	Scenario 18
				$ 8.63	$ 7.65	Scenario 19
				$ 8.91	$ 9.19	Scenario 20
		$ 7.53	$ 9.33	$ 7.49	$ 7.80	Scenario 21
				$ 7.78	$ 9.45	Scenario 22
				$ 8.73	$ 8.04	Scenario 23
				$ 8.99	$ 9.67	Scenario 24
		$ 8.36	$ 7.93	$ 7.97	$ 7.54	Scenario 25
				$ 8.22	$ 9.11	Scenario 26
				$ 8.97	$ 7.71	Scenario 27
				$ 9.24	$ 9.23	Scenario 28
		$ 8.53	$ 9.49	$ 8.06	$ 7.92	Scenario 29
				$ 8.34	$ 9.57	Scenario 30
				$ 9.07	$ 8.09	Scenario 31
				$ 9.32	$ 9.72	Scenario 32
$ 6.18	$ 6.07	$ 7.26	$ 7.93	$ 7.32	$ 7.56	Scenario 33
				$ 7.51	$ 9.12	Scenario 34
				$ 8.83	$ 7.79	Scenario 35
				$ 9.05	$ 9.33	Scenario 36
		$ 7.39	$ 8.89	$ 7.40	$ 8.15	Scenario 37
				$ 7.57	$ 8.90	Scenario 38
				$ 8.95	$ 8.37	Scenario 39
				$ 9.10	$ 9.09	Scenario 40
		$ 8.57	$ 8.09	$ 7.67	$ 7.64	Scenario 41
				$ 7.87	$ 9.21	Scenario 42
				$ 9.67	$ 7.96	Scenario 43
				$ 9.88	$ 9.49	Scenario 44
		$ 8.73	$ 9.07	$ 7.79	$ 8.21	Scenario 45
				$ 7.93	$ 8.98	Scenario 46
				$ 9.79	$ 8.54	Scenario 47
				$ 9.94	$ 9.26	Scenario 48
$ 6.23	$ 6.43	$ 7.27	$ 7.82	$ 7.30	$ 7.43	Scenario 49
				$ 7.56	$ 9.00	Scenario 50
				$ 8.80	$ 7.65	Scenario 51
				$ 9.08	$ 9.19	Scenario 52
		$ 7.43	$ 9.34	$ 7.39	$ 7.80	Scenario 53
				$ 7.67	$ 9.45	Scenario 54
				$ 8.91	$ 8.04	Scenario 55
				$ 9.16	$ 9.67	Scenario 56
		$ 8.58	$ 7.98	$ 7.65	$ 7.51	Scenario 57
				$ 7.91	$ 9.07	Scenario 58
				$ 9.65	$ 7.83	Scenario 59
				$ 9.91	$ 9.35	Scenario 60
		$ 8.76	$ 9.53	$ 7.75	$ 7.88	Scenario 61
				$ 8.03	$ 9.53	Scenario 62
				$ 9.75	$ 8.21	Scenario 63
				$ 10.03	$ 9.81	Scenario 64

Table B.11: Consumption Rates for Commercial Sector (MMcf/d)

Spot Market						
Season 1		Season 2		Season 3		
Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	Node $cn1$	Node $cn2$	
136.04	130.86	330.08	373.95	538.98	486.52	Scenario 1
				523.34	599.42	Scenario 2
				703.98	468.38	Scenario 3
				684.70	581.96	Scenario 4
		322.20	439.52	531.54	534.79	Scenario 5
				517.59	630.48	Scenario 6
				693.38	516.92	Scenario 7
				680.70	614.81	Scenario 8
		390.96	366.68	593.68	477.63	Scenario 9
				576.27	588.63	Scenario 10
				674.86	463.49	Scenario 11
				656.88	578.32	Scenario 12
		381.89	431.05	582.66	525.78	Scenario 13
				571.46	620.61	Scenario 14
				664.73	512.48	Scenario 15
				652.05	610.37	Scenario 16
135.08	152.61	330.04	357.17	540.44	473.25	Scenario 17
				518.71	660.91	Scenario 18
				705.95	455.59	Scenario 19
				681.86	645.25	Scenario 20
		319.60	463.76	533.44	503.23	Scenario 21
				509.54	673.97	Scenario 22
				697.04	483.75	Scenario 23
				675.11	656.01	Scenario 24
		390.54	350.29	595.05	464.28	Scenario 25
				573.32	651.76	Scenario 26
				676.26	450.15	Scenario 27
				653.95	641.52	Scenario 28
		379.87	454.69	587.25	493.50	Scenario 29
				563.35	664.04	Scenario 30
				668.33	479.25	Scenario 31
				646.49	651.51	Scenario 32
161.08	128.16	315.67	372.75	522.72	486.27	Scenario 33
				506.85	598.94	Scenario 34
				655.34	467.77	Scenario 35
				636.32	581.60	Scenario 36
		306.86	439.20	515.59	534.13	Scenario 37
				501.27	630.17	Scenario 38
				644.96	516.52	Scenario 39
				632.28	614.41	Scenario 40
		412.18	363.23	556.67	480.50	Scenario 41
				539.24	591.52	Scenario 42
				728.39	453.49	Scenario 43
				710.42	568.32	Scenario 44
		401.98	428.57	546.63	529.57	Scenario 45
				534.37	623.44	Scenario 46
				718.18	502.41	Scenario 47
				705.61	600.40	Scenario 48
159.27	150.71	314.98	356.59	524.15	472.97	Scenario 49
				502.41	660.61	Scenario 50
				657.54	455.20	Scenario 51
				633.45	644.86	Scenario 52
		304.26	463.45	517.14	502.95	Scenario 53
				493.24	673.68	Scenario 54
				648.62	483.35	Scenario 55
				626.69	655.60	Scenario 56
		411.76	346.84	558.03	467.13	Scenario 57
				536.30	654.66	Scenario 58
				729.78	440.15	Scenario 59
				707.51	631.55	Scenario 60
		400.26	451.96	550.16	496.27	Scenario 61
				526.24	666.86	Scenario 62
				721.77	469.16	Scenario 63
				697.83	644.30	Scenario 64

Table B.12: End-User Prices for Commercial Sector ($/Mcf)

Spot Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
$ 5.08	$ 5.12	$ 6.00	$ 6.57	$ 6.65	$ 6.41	Scenario 1
				$ 6.83	$ 7.45	Scenario 2
				$ 7.95	$ 6.63	Scenario 3
				$ 8.18	$ 7.66	Scenario 4
		$ 6.12	$ 7.34	$ 6.73	$ 6.89	Scenario 5
				$ 6.90	$ 7.74	Scenario 6
				$ 8.08	$ 7.11	Scenario 7
				$ 8.23	$ 7.93	Scenario 8
		$ 6.68	$ 6.69	$ 7.13	$ 6.52	Scenario 9
				$ 7.34	$ 7.58	Scenario 10
				$ 7.87	$ 6.69	Scenario 11
				$ 8.08	$ 7.70	Scenario 12
		$ 6.82	$ 7.48	$ 7.26	$ 7.00	Scenario 13
				$ 7.39	$ 7.86	Scenario 14
				$ 7.99	$ 7.16	Scenario 15
				$ 8.14	$ 7.98	Scenario 16
$ 5.10	$ 5.54	$ 6.00	$ 6.45	$ 6.63	$ 6.30	Scenario 17
				$ 6.89	$ 7.90	Scenario 18
				$ 7.93	$ 6.52	Scenario 19
				$ 8.21	$ 8.09	Scenario 20
		$ 6.16	$ 7.60	$ 6.71	$ 6.61	Scenario 21
				$ 6.99	$ 8.15	Scenario 22
				$ 8.03	$ 6.84	Scenario 23
				$ 8.29	$ 8.37	Scenario 24
		$ 6.68	$ 6.56	$ 7.12	$ 6.41	Scenario 25
				$ 7.37	$ 8.02	Scenario 26
				$ 7.86	$ 6.58	Scenario 27
				$ 8.12	$ 8.14	Scenario 28
		$ 6.85	$ 7.76	$ 7.21	$ 6.73	Scenario 29
				$ 7.49	$ 8.27	Scenario 30
				$ 7.95	$ 6.90	Scenario 31
				$ 8.21	$ 8.42	Scenario 32
$ 5.52	$ 5.20	$ 5.87	$ 6.59	$ 6.56	$ 6.41	Scenario 33
				$ 6.74	$ 7.45	Scenario 34
				$ 7.68	$ 6.64	Scenario 35
				$ 7.90	$ 7.66	Scenario 36
		$ 6.01	$ 7.35	$ 6.64	$ 6.90	Scenario 37
				$ 6.81	$ 7.74	Scenario 38
				$ 7.80	$ 7.12	Scenario 39
				$ 7.95	$ 7.93	Scenario 40
		$ 6.93	$ 6.75	$ 6.86	$ 6.48	Scenario 41
				$ 7.07	$ 7.54	Scenario 42
				$ 8.37	$ 6.81	Scenario 43
				$ 8.58	$ 7.83	Scenario 44
		$ 7.08	$ 7.52	$ 6.98	$ 6.96	Scenario 45
				$ 7.13	$ 7.82	Scenario 46
				$ 8.49	$ 7.29	Scenario 47
				$ 8.64	$ 8.10	Scenario 48
$ 5.57	$ 5.59	$ 5.88	$ 6.46	$ 6.54	$ 6.31	Scenario 49
				$ 6.80	$ 7.91	Scenario 50
				$ 7.65	$ 6.52	Scenario 51
				$ 7.94	$ 8.10	Scenario 52
		$ 6.05	$ 7.61	$ 6.62	$ 6.61	Scenario 53
				$ 6.90	$ 8.15	Scenario 54
				$ 7.76	$ 6.85	Scenario 55
				$ 8.02	$ 8.37	Scenario 56
		$ 6.93	$ 6.62	$ 6.85	$ 6.38	Scenario 57
				$ 7.10	$ 7.98	Scenario 58
				$ 8.36	$ 6.71	Scenario 59
				$ 8.62	$ 8.26	Scenario 60
		$ 7.11	$ 7.80	$ 6.94	$ 6.69	Scenario 61
				$ 7.22	$ 8.23	Scenario 62
				$ 8.45	$ 7.02	Scenario 63
				$ 8.73	$ 8.51	Scenario 64

Table B.13: Consumption Rates for Industrial Sector (MMcf/d)

Long-term Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
700.00	650.00	700.00	650.00	700.00	650.00	

Spot Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
14.00	19.50	35.00	39.00	56.00	58.50	Scenario 1
				56.00	97.50	Scenario 2
				105.00	58.50	Scenario 3
				105.00	97.50	Scenario 4
		35.00	65.00	56.00	58.50	Scenario 5
				56.00	117.00	Scenario 6
				105.00	58.50	Scenario 7
				105.00	117.00	Scenario 8
		63.00	39.00	56.00	58.50	Scenario 9
				56.00	97.50	Scenario 10
				91.00	58.50	Scenario 11
				91.00	97.50	Scenario 12
		63.00	65.00	56.00	58.50	Scenario 13
				56.00	117.00	Scenario 14
				91.00	58.50	Scenario 15
				91.00	117.00	Scenario 16
14.00	32.50	35.00	39.00	56.00	58.50	Scenario 17
				56.00	97.50	Scenario 18
				105.00	58.50	Scenario 19
				105.00	97.50	Scenario 20
		35.00	65.00	56.00	58.50	Scenario 21
				56.00	130.00	Scenario 22
				105.00	58.50	Scenario 23
				105.00	130.00	Scenario 24
		63.00	39.00	56.00	58.50	Scenario 25
				56.00	97.50	Scenario 26
				91.00	58.50	Scenario 27
				91.00	97.50	Scenario 28
		63.00	65.00	56.00	58.50	Scenario 29
				56.00	130.00	Scenario 30
				91.00	58.50	Scenario 31
				91.00	130.00	Scenario 32
28.00	19.50	35.00	39.00	56.00	58.50	Scenario 33
				56.00	97.50	Scenario 34
				77.00	58.50	Scenario 35
				77.00	97.50	Scenario 36
		35.00	65.00	56.00	58.50	Scenario 37
				56.00	117.00	Scenario 38
				77.00	58.50	Scenario 39
				77.00	117.00	Scenario 40
		63.00	39.00	56.00	58.50	Scenario 41
				56.00	97.50	Scenario 42
				126.00	58.50	Scenario 43
				126.00	97.50	Scenario 44
		63.00	65.00	56.00	58.50	Scenario 45
				56.00	117.00	Scenario 46
				126.00	58.50	Scenario 47
				126.00	117.00	Scenario 48
28.00	32.50	35.00	39.00	56.00	58.50	Scenario 49
				56.00	97.50	Scenario 50
				77.00	58.50	Scenario 51
				77.00	97.50	Scenario 52
		35.00	65.00	56.00	58.50	Scenario 53
				56.00	130.00	Scenario 54
				77.00	58.50	Scenario 55
				77.00	130.00	Scenario 56
		63.00	39.00	56.00	58.50	Scenario 57
				56.00	97.50	Scenario 58
				126.00	58.50	Scenario 59
				126.00	97.50	Scenario 60
		63.00	65.00	56.00	58.50	Scenario 61
				56.00	130.00	Scenario 62
				126.00	58.50	Scenario 63
				126.00	130.00	Scenario 64

Table B.14: End-Use Prices for Industrial Sector ($/Mcf)

Long-term Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
$ 3.65	$ 3.63	$ 4.12	$ 4.12	$ 4.46	$ 4.44	
Spot Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
$ 3.29	$ 3.25	$ 3.46	$ 3.45	$ 3.48	$ 3.44	Scenario 1
				$ 3.75	$ 3.79	Scenario 2
				$ 3.81	$ 3.77	Scenario 3
				$ 4.15	$ 4.11	Scenario 4
		$ 3.64	$ 3.68	$ 3.61	$ 3.63	Scenario 5
				$ 3.85	$ 3.89	Scenario 6
				$ 4.00	$ 3.96	Scenario 7
				$ 4.22	$ 4.18	Scenario 8
		$ 3.67	$ 3.63	$ 3.64	$ 3.60	Scenario 9
				$ 3.95	$ 3.99	Scenario 10
				$ 3.90	$ 3.86	Scenario 11
				$ 4.22	$ 4.18	Scenario 12
		$ 3.88	$ 3.89	$ 3.84	$ 3.80	Scenario 13
				$ 4.03	$ 4.07	Scenario 14
				$ 4.08	$ 4.04	Scenario 15
				$ 4.31	$ 4.26	Scenario 16
$ 3.33	$ 3.36	$ 3.46	$ 3.47	$ 3.45	$ 3.42	Scenario 17
				$ 3.83	$ 3.87	Scenario 18
				$ 3.78	$ 3.74	Scenario 19
				$ 4.20	$ 4.16	Scenario 20
		$ 3.70	$ 3.74	$ 3.57	$ 3.54	Scenario 21
				$ 4.00	$ 4.04	Scenario 22
				$ 3.93	$ 3.89	Scenario 23
				$ 4.32	$ 4.37	Scenario 24
		$ 3.68	$ 3.64	$ 3.62	$ 3.58	Scenario 25
				$ 4.00	$ 4.04	Scenario 26
				$ 3.88	$ 3.84	Scenario 27
				$ 4.27	$ 4.23	Scenario 28
		$ 3.93	$ 3.97	$ 3.75	$ 3.72	Scenario 29
				$ 4.18	$ 4.22	Scenario 30
				$ 4.02	$ 3.98	Scenario 31
				$ 4.40	$ 4.45	Scenario 32
$ 3.40	$ 3.37	$ 3.45	$ 3.48	$ 3.48	$ 3.45	Scenario 33
				$ 3.76	$ 3.80	Scenario 34
				$ 3.82	$ 3.78	Scenario 35
				$ 4.16	$ 4.12	Scenario 36
		$ 3.65	$ 3.69	$ 3.61	$ 3.64	Scenario 37
				$ 3.86	$ 3.90	Scenario 38
				$ 4.01	$ 3.97	Scenario 39
				$ 4.23	$ 4.19	Scenario 40
		$ 3.76	$ 3.72	$ 3.59	$ 3.55	Scenario 41
				$ 3.90	$ 3.94	Scenario 42
				$ 4.09	$ 4.05	Scenario 43
				$ 4.40	$ 4.36	Scenario 44
		$ 3.99	$ 3.95	$ 3.77	$ 3.73	Scenario 45
				$ 3.98	$ 4.02	Scenario 46
				$ 4.27	$ 4.22	Scenario 47
				$ 4.49	$ 4.44	Scenario 48
$ 3.48	$ 3.44	$ 3.46	$ 3.49	$ 3.46	$ 3.42	Scenario 49
				$ 3.84	$ 3.88	Scenario 50
				$ 3.78	$ 3.75	Scenario 51
				$ 4.21	$ 4.17	Scenario 52
		$ 3.71	$ 3.75	$ 3.58	$ 3.54	Scenario 53
				$ 4.00	$ 4.04	Scenario 54
				$ 3.94	$ 3.90	Scenario 55
				$ 4.33	$ 4.37	Scenario 56
		$ 3.77	$ 3.73	$ 3.56	$ 3.53	Scenario 57
				$ 3.95	$ 3.99	Scenario 58
				$ 4.06	$ 4.02	Scenario 59
				$ 4.46	$ 4.41	Scenario 60
		$ 4.03	$ 4.03	$ 3.70	$ 3.67	Scenario 61
				$ 4.13	$ 4.17	Scenario 62
				$ 4.20	$ 4.16	Scenario 63
				$ 4.63	$ 4.58	Scenario 64

Table B.15: Consumption Rates for Electric Power Sector (MMcf/d)

Long-term Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
420.00	400.00	420.00	400.00	420.00	400.00	
Spot Market						
Season 1		**Season 2**		**Season 3**		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
105.00	100.00	63.00	60.00	8.40	16.00	Scenario 1
				8.40	32.00	Scenario 2
				21.00	16.00	Scenario 3
				21.00	32.00	Scenario 4
		63.00	140.00	8.40	16.00	Scenario 5
				8.40	40.00	Scenario 6
				21.00	16.00	Scenario 7
				21.00	40.00	Scenario 8
		126.00	60.00	8.40	16.00	Scenario 9
				8.40	32.00	Scenario 10
				25.20	16.00	Scenario 11
				25.20	32.00	Scenario 12
		126.00	140.00	8.40	16.00	Scenario 13
				8.40	40.00	Scenario 14
				25.20	16.00	Scenario 15
				25.20	40.00	Scenario 16
105.00	140.00	63.00	80.00	8.40	16.00	Scenario 17
				8.40	60.00	Scenario 18
				21.00	16.00	Scenario 19
				21.00	60.00	Scenario 20
		63.00	120.00	8.40	16.00	Scenario 21
				8.40	80.00	Scenario 22
				21.00	16.00	Scenario 23
				21.00	80.00	Scenario 24
		126.00	80.00	8.40	16.00	Scenario 25
				8.40	60.00	Scenario 26
				25.20	16.00	Scenario 27
				25.20	60.00	Scenario 28
		126.00	120.00	8.40	16.00	Scenario 29
				8.40	80.00	Scenario 30
				25.20	16.00	Scenario 31
				25.20	80.00	Scenario 32
189.00	100.00	58.80	60.00	8.40	16.00	Scenario 33
				8.40	32.00	Scenario 34
				33.60	16.00	Scenario 35
				33.60	32.00	Scenario 36
		58.80	140.00	8.40	16.00	Scenario 37
				8.40	40.00	Scenario 38
				33.60	16.00	Scenario 39
				33.60	40.00	Scenario 40
		142.80	60.00	8.40	16.00	Scenario 41
				8.40	32.00	Scenario 42
				42.00	16.00	Scenario 43
				42.00	32.00	Scenario 44
		142.80	140.00	8.40	16.00	Scenario 45
				8.40	40.00	Scenario 46
				42.00	16.00	Scenario 47
				42.00	40.00	Scenario 48
189.00	140.00	58.80	80.00	8.40	16.00	Scenario 49
				8.40	60.00	Scenario 50
				33.60	16.00	Scenario 51
				33.60	60.00	Scenario 52
		58.80	120.00	8.40	16.00	Scenario 53
				8.40	80.00	Scenario 54
				33.60	16.00	Scenario 55
				33.60	80.00	Scenario 56
		142.80	80.00	8.40	16.00	Scenario 57
				8.40	60.00	Scenario 58
				42.00	16.00	Scenario 59
				42.00	60.00	Scenario 60
		142.80	120.00	8.40	16.00	Scenario 61
				8.40	80.00	Scenario 62
				42.00	16.00	Scenario 63
				42.00	80.00	Scenario 64

Table B.16: End-User Prices for Electric Power Sector ($/Mcf)

Long-term Market					
Season 1		Season 2		Season 3	
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2
$ 3.65	$ 3.63	$ 4.12	$ 4.12	$ 4.46	$ 4.44

Spot Market						
Season 1		Season 2		Season 3		
Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	Node *cn*1	Node *cn*2	
$ 3.29	$ 3.25	$ 3.46	$ 3.45	$ 3.48	$ 3.44	Scenario 1
				$ 3.75	$ 3.79	Scenario 2
				$ 3.81	$ 3.77	Scenario 3
				$ 4.15	$ 4.11	Scenario 4
		$ 3.64	$ 3.68	$ 3.61	$ 3.63	Scenario 5
				$ 3.85	$ 3.89	Scenario 6
				$ 4.00	$ 3.96	Scenario 7
				$ 4.22	$ 4.18	Scenario 8
		$ 3.67	$ 3.63	$ 3.64	$ 3.60	Scenario 9
				$ 3.95	$ 3.99	Scenario 10
				$ 3.90	$ 3.86	Scenario 11
				$ 4.22	$ 4.18	Scenario 12
		$ 3.88	$ 3.89	$ 3.84	$ 3.80	Scenario 13
				$ 4.03	$ 4.07	Scenario 14
				$ 4.08	$ 4.04	Scenario 15
				$ 4.31	$ 4.26	Scenario 16
$ 3.33	$ 3.36	$ 3.46	$ 3.47	$ 3.45	$ 3.42	Scenario 17
				$ 3.83	$ 3.87	Scenario 18
				$ 3.78	$ 3.74	Scenario 19
				$ 4.20	$ 4.16	Scenario 20
		$ 3.70	$ 3.74	$ 3.57	$ 3.54	Scenario 21
				$ 4.00	$ 4.04	Scenario 22
				$ 3.93	$ 3.89	Scenario 23
				$ 4.32	$ 4.37	Scenario 24
		$ 3.68	$ 3.64	$ 3.62	$ 3.58	Scenario 25
				$ 4.00	$ 4.04	Scenario 26
				$ 3.88	$ 3.84	Scenario 27
				$ 4.27	$ 4.23	Scenario 28
		$ 3.93	$ 3.97	$ 3.75	$ 3.72	Scenario 29
				$ 4.18	$ 4.22	Scenario 30
				$ 4.02	$ 3.98	Scenario 31
				$ 4.40	$ 4.45	Scenario 32
$ 3.40	$ 3.37	$ 3.45	$ 3.48	$ 3.48	$ 3.45	Scenario 33
				$ 3.76	$ 3.80	Scenario 34
				$ 3.82	$ 3.78	Scenario 35
				$ 4.16	$ 4.12	Scenario 36
		$ 3.65	$ 3.69	$ 3.61	$ 3.64	Scenario 37
				$ 3.86	$ 3.90	Scenario 38
				$ 4.01	$ 3.97	Scenario 39
				$ 4.23	$ 4.19	Scenario 40
		$ 3.76	$ 3.72	$ 3.59	$ 3.55	Scenario 41
				$ 3.90	$ 3.94	Scenario 42
				$ 4.09	$ 4.05	Scenario 43
				$ 4.40	$ 4.36	Scenario 44
		$ 3.99	$ 3.95	$ 3.77	$ 3.73	Scenario 45
				$ 3.98	$ 4.02	Scenario 46
				$ 4.27	$ 4.22	Scenario 47
				$ 4.49	$ 4.44	Scenario 48
$ 3.48	$ 3.44	$ 3.46	$ 3.49	$ 3.46	$ 3.42	Scenario 49
				$ 3.84	$ 3.88	Scenario 50
				$ 3.78	$ 3.75	Scenario 51
				$ 4.21	$ 4.17	Scenario 52
		$ 3.71	$ 3.75	$ 3.58	$ 3.54	Scenario 53
				$ 4.00	$ 4.04	Scenario 54
				$ 3.94	$ 3.90	Scenario 55
				$ 4.33	$ 4.37	Scenario 56
		$ 3.77	$ 3.73	$ 3.56	$ 3.53	Scenario 57
				$ 3.95	$ 3.99	Scenario 58
				$ 4.06	$ 4.02	Scenario 59
				$ 4.46	$ 4.41	Scenario 60
		$ 4.03	$ 4.03	$ 3.70	$ 3.67	Scenario 61
				$ 4.13	$ 4.17	Scenario 62
				$ 4.20	$ 4.16	Scenario 63
				$ 4.63	$ 4.58	Scenario 64

Table B.17: Supply by Marketer $M1$, l^0_{kmsy}, $l^1_{kmsy,i^{s,y}}$ (MMcf/d)

Long-Term Market											
Season 1				Season 2				Season 3			
*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1
0.00	0.00	350.00	210.00	0.00	0.00	350.00	210.00	0.00	0.00	350.00	210.00

Spot Market												
Season 1				Season 2				Season 3				
*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	
78.17	68.02	7.00	52.50	234.81	165.04	17.50	31.50	355.70	269.49	28.00	4.20	Scenario 1
								347.41	261.67	28.00	4.20	Scenario 2
								435.64	351.99	52.50	10.50	Scenario 3
								425.43	342.35	52.50	10.50	Scenario 4
				231.17	161.10	17.50	31.50	351.75	265.77	28.00	4.20	Scenario 5
								344.37	258.79	28.00	4.20	Scenario 6
								430.02	346.69	52.50	10.50	Scenario 7
								423.31	340.35	52.50	10.50	Scenario 8
				281.26	195.48	31.50	63.00	390.77	296.84	28.00	4.20	Scenario 9
								381.55	288.13	28.00	4.20	Scenario 10
								457.93	337.43	45.50	12.60	Scenario 11
								448.41	328.44	45.50	12.60	Scenario 12
				277.08	190.94	31.50	63.00	384.94	291.33	28.00	4.20	Scenario 13
								379.01	285.73	28.00	4.20	Scenario 14
								452.56	332.36	45.50	12.60	Scenario 15
								445.85	326.02	45.50	12.60	Scenario 16
77.73	67.54	7.00	52.50	234.79	165.02	17.50	31.50	356.47	270.22	28.00	4.20	Scenario 17
								344.97	259.36	28.00	4.20	Scenario 18
								436.68	352.97	52.50	10.50	Scenario 19
								423.93	340.93	52.50	10.50	Scenario 20
				229.97	159.80	17.50	31.50	352.76	266.72	28.00	4.20	Scenario 21
								340.11	254.77	28.00	4.20	Scenario 22
								431.96	348.52	52.50	10.50	Scenario 23
								420.35	337.55	52.50	10.50	Scenario 24
				281.07	195.27	31.50	63.00	391.50	297.52	28.00	4.20	Scenario 25
								380.00	286.66	28.00	4.20	Scenario 26
								458.66	338.13	45.50	12.60	Scenario 27
								446.86	326.98	45.50	12.60	Scenario 28
				276.14	189.93	31.50	63.00	387.37	293.63	28.00	4.20	Scenario 29
								374.72	281.68	28.00	4.20	Scenario 30
								454.47	334.17	45.50	12.60	Scenario 31
								442.91	323.24	45.50	12.60	Scenario 32
97.08	80.54	14.00	94.50	228.75	157.84	17.50	29.40	345.56	261.36	28.00	4.20	Scenario 33
								337.16	253.43	28.00	4.20	Scenario 34
								450.30	327.67	38.50	16.80	Scenario 35
								440.23	318.16	38.50	16.80	Scenario 36
				224.68	153.43	17.50	29.40	341.79	257.80	28.00	4.20	Scenario 37
								334.20	250.64	28.00	4.20	Scenario 38
								444.80	322.48	38.50	16.80	Scenario 39
								438.09	316.14	38.50	16.80	Scenario 40
				289.02	206.09	31.50	71.40	367.36	278.34	28.00	4.20	Scenario 41
								358.13	269.62	28.00	4.20	Scenario 42
								502.38	364.20	63.00	21.00	Scenario 43
								492.87	355.21	63.00	21.00	Scenario 44
				284.31	200.99	31.50	71.40	362.04	273.32	28.00	4.20	Scenario 45
								355.55	267.18	28.00	4.20	Scenario 46
								496.98	359.09	63.00	21.00	Scenario 47
								490.32	352.80	63.00	21.00	Scenario 48
96.24	79.63	14.00	94.50	228.43	157.49	17.50	29.40	346.32	262.08	28.00	4.20	Scenario 49
								334.80	251.20	28.00	4.20	Scenario 50
								451.46	328.77	38.50	16.80	Scenario 51
								438.71	316.72	38.50	16.80	Scenario 52
				223.48	152.13	17.50	29.40	342.61	258.57	28.00	4.20	Scenario 53
								329.95	246.62	28.00	4.20	Scenario 54
								446.74	324.31	38.50	16.80	Scenario 55
								435.13	313.35	38.50	16.80	Scenario 56
				288.82	205.88	31.50	71.40	368.08	279.02	28.00	4.20	Scenario 57
								356.57	268.15	28.00	4.20	Scenario 58
								503.12	364.89	63.00	21.00	Scenario 59
								491.33	353.75	63.00	21.00	Scenario 60
				283.52	200.13	31.50	71.40	363.91	275.08	28.00	4.20	Scenario 61
								351.25	263.12	28.00	4.20	Scenario 62
								498.88	360.88	63.00	21.00	Scenario 63
								486.20	348.91	63.00	21.00	Scenario 64

Table B.18: Supply by Marketer $M2$, l^0_{kmsy}, $l^1_{kmsy,i^s,y}$ (MMcf/d)

Long-Term Market											
Season 1				Season 2				Season 3			
*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1
0.00	0.00	350.00	210.00	0.00	0.00	350.00	210.00	0.00	0.00	350.00	210.00

Spot Market												
Season 1				Season 2				Season 3				
*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	*RD*1	*CD*1	*ID*1	*ED*1	
78.17	68.02	7.00	52.50	234.81	165.04	17.50	31.50	355.70	269.49	28.00	4.20	Scenario 1
								347.41	261.67	28.00	4.20	Scenario 2
								435.64	351.99	52.50	10.50	Scenario 3
								425.43	342.35	52.50	10.50	Scenario 4
				231.17	161.10	17.50	31.50	351.75	265.77	28.00	4.20	Scenario 5
								344.37	258.79	28.00	4.20	Scenario 6
								430.02	346.69	52.50	10.50	Scenario 7
								423.31	340.35	52.50	10.50	Scenario 8
				281.26	195.48	31.50	63.00	390.77	296.84	28.00	4.20	Scenario 9
								381.55	288.13	28.00	4.20	Scenario 10
								457.93	337.43	45.50	12.60	Scenario 11
								448.41	328.44	45.50	12.60	Scenario 12
				277.08	190.94	31.50	63.00	384.94	291.33	28.00	4.20	Scenario 13
								379.01	285.73	28.00	4.20	Scenario 14
								452.56	332.36	45.50	12.60	Scenario 15
								445.85	326.02	45.50	12.60	Scenario 16
77.73	67.54	7.00	52.50	234.79	165.02	17.50	31.50	356.47	270.22	28.00	4.20	Scenario 17
								344.97	259.36	28.00	4.20	Scenario 18
								436.68	352.97	52.50	10.50	Scenario 19
								423.93	340.93	52.50	10.50	Scenario 20
				229.97	159.80	17.50	31.50	352.76	266.72	28.00	4.20	Scenario 21
								340.11	254.77	28.00	4.20	Scenario 22
								431.96	348.52	52.50	10.50	Scenario 23
								420.35	337.55	52.50	10.50	Scenario 24
				281.07	195.27	31.50	63.00	391.50	297.52	28.00	4.20	Scenario 25
								380.00	286.66	28.00	4.20	Scenario 26
								458.66	338.13	45.50	12.60	Scenario 27
								446.86	326.98	45.50	12.60	Scenario 28
				276.14	189.93	31.50	63.00	387.37	293.63	28.00	4.20	Scenario 29
								374.72	281.68	28.00	4.20	Scenario 30
								454.47	334.17	45.50	12.60	Scenario 31
								442.91	323.24	45.50	12.60	Scenario 32
97.08	80.54	14.00	94.50	228.75	157.84	17.50	29.40	345.56	261.36	28.00	4.20	Scenario 33
								337.16	253.43	28.00	4.20	Scenario 34
								450.30	327.67	38.50	16.80	Scenario 35
								440.23	318.16	38.50	16.80	Scenario 36
				224.68	153.43	17.50	29.40	341.79	257.80	28.00	4.20	Scenario 37
								334.20	250.64	28.00	4.20	Scenario 38
								444.80	322.48	38.50	16.80	Scenario 39
								438.09	316.14	38.50	16.80	Scenario 40
				289.02	206.09	31.50	71.40	367.36	278.34	28.00	4.20	Scenario 41
								358.13	269.62	28.00	4.20	Scenario 42
								502.38	364.20	63.00	21.00	Scenario 43
								492.87	355.21	63.00	21.00	Scenario 44
				284.31	200.99	31.50	71.40	362.04	273.32	28.00	4.20	Scenario 45
								355.55	267.18	28.00	4.20	Scenario 46
								496.98	359.09	63.00	21.00	Scenario 47
								490.32	352.80	63.00	21.00	Scenario 48
96.24	79.63	14.00	94.50	228.43	157.49	17.50	29.40	346.32	262.08	28.00	4.20	Scenario 49
								334.80	251.20	28.00	4.20	Scenario 50
								451.46	328.77	38.50	16.80	Scenario 51
								438.71	316.72	38.50	16.80	Scenario 52
				223.48	152.13	17.50	29.40	342.61	258.57	28.00	4.20	Scenario 53
								329.95	246.62	28.00	4.20	Scenario 54
								446.74	324.31	38.50	16.80	Scenario 55
								435.13	313.35	38.50	16.80	Scenario 56
				288.82	205.88	31.50	71.40	368.08	279.02	28.00	4.20	Scenario 57
								356.57	268.15	28.00	4.20	Scenario 58
								503.12	364.89	63.00	21.00	Scenario 59
								491.33	353.75	63.00	21.00	Scenario 60
				283.52	200.13	31.50	71.40	363.91	275.08	28.00	4.20	Scenario 61
								351.25	263.12	28.00	4.20	Scenario 62
								498.88	360.88	63.00	21.00	Scenario 63
								486.20	348.91	63.00	21.00	Scenario 64

Table B.19: Supply by Marketer $M3$, l^0_{kmsy}, $l^1_{kmsy,i^{s,y}}$ (MMcf/d)

Long-Term Market											
Season 1				Season 2				Season 3			
*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2
0.00	0.00	325.00	200.00	0.00	0.00	325.00	200.00	0.00	0.00	325.00	200.00

Spot Market												
Season 1				Season 2				Season 3				
*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	
84.77	65.43	9.75	50.00	254.42	186.97	19.50	30.00	350.22	243.26	29.25	8.00	Scenario 1
								452.25	299.71	48.75	16.00	Scenario 2
								340.81	234.19	29.25	8.00	Scenario 3
								443.20	290.98	48.75	16.00	Scenario 4
				296.77	219.76	32.50	70.00	383.76	267.39	29.25	8.00	Scenario 5
								425.01	315.24	58.50	20.00	Scenario 6
								374.50	258.46	29.25	8.00	Scenario 7
								416.89	307.40	58.50	20.00	Scenario 8
				250.97	183.34	19.50	30.00	345.61	238.82	29.25	8.00	Scenario 9
								446.66	294.31	48.75	16.00	Scenario 10
								338.28	231.74	29.25	8.00	Scenario 11
								441.32	289.16	48.75	16.00	Scenario 12
				292.75	215.52	32.50	70.00	379.09	262.89	29.25	8.00	Scenario 13
								419.90	310.31	58.50	20.00	Scenario 14
								372.20	256.24	29.25	8.00	Scenario 15
								414.59	305.18	58.50	20.00	Scenario 16
93.55	76.30	16.25	70.00	247.39	178.58	19.50	40.00	341.21	236.63	29.25	8.00	Scenario 17
								435.31	330.45	48.75	30.00	Scenario 18
								332.05	227.79	29.25	8.00	Scenario 19
								427.20	322.63	48.75	30.00	Scenario 20
				318.95	231.88	32.50	60.00	362.08	251.61	29.25	8.00	Scenario 21
								459.88	336.99	65.00	40.00	Scenario 22
								351.98	241.87	29.25	8.00	Scenario 23
								450.57	328.00	65.00	40.00	Scenario 24
				244.12	175.14	19.50	40.00	336.56	232.14	29.25	8.00	Scenario 25
								430.57	325.88	48.75	30.00	Scenario 26
								329.24	225.08	29.25	8.00	Scenario 27
								425.26	320.76	48.75	30.00	Scenario 28
				314.64	227.34	32.50	60.00	357.04	246.75	29.25	8.00	Scenario 29
								454.74	332.02	65.00	40.00	Scenario 30
								349.65	239.63	29.25	8.00	Scenario 31
								448.24	325.75	65.00	40.00	Scenario 32
83.57	64.08	9.75	50.00	253.86	186.38	19.50	30.00	350.09	243.14	29.25	8.00	Scenario 33
								452.00	299.47	48.75	16.00	Scenario 34
								340.50	233.88	29.25	8.00	Scenario 35
								443.02	290.80	48.75	16.00	Scenario 36
				296.62	219.60	32.50	70.00	383.42	267.06	29.25	8.00	Scenario 37
								424.85	315.09	58.50	20.00	Scenario 38
								374.30	258.26	29.25	8.00	Scenario 39
								416.69	307.21	58.50	20.00	Scenario 40
				249.33	181.61	19.50	30.00	347.10	240.25	29.25	8.00	Scenario 41
								448.16	295.76	48.75	16.00	Scenario 42
								333.10	226.75	29.25	8.00	Scenario 43
								436.14	284.16	48.75	16.00	Scenario 44
				291.57	214.28	32.50	70.00	381.06	264.79	29.25	8.00	Scenario 45
								421.37	311.72	58.50	20.00	Scenario 46
								366.98	251.20	29.25	8.00	Scenario 47
								409.42	300.20	58.50	20.00	Scenario 48
92.71	75.35	16.25	70.00	247.11	178.30	19.50	40.00	341.06	236.49	29.25	8.00	Scenario 49
								435.16	330.31	48.75	30.00	Scenario 50
								331.85	227.60	29.25	8.00	Scenario 51
								426.99	322.43	48.75	30.00	Scenario 52
				318.80	231.72	32.50	60.00	361.93	251.47	29.25	8.00	Scenario 53
								459.73	336.84	65.00	40.00	Scenario 54
								351.77	241.68	29.25	8.00	Scenario 55
								450.36	327.80	65.00	40.00	Scenario 56
				242.48	173.42	19.50	40.00	338.04	233.56	29.25	8.00	Scenario 57
								432.08	327.33	48.75	30.00	Scenario 58
								324.05	220.07	29.25	8.00	Scenario 59
								420.10	315.77	48.75	30.00	Scenario 60
				313.35	225.98	32.50	60.00	358.47	248.14	29.25	8.00	Scenario 61
								456.20	333.43	65.00	40.00	Scenario 62
								344.42	234.58	29.25	8.00	Scenario 63
								444.51	322.15	65.00	40.00	Scenario 64

Table B.20: Supply by Marketer $M4$, l^0_{kmsy}, $l^1_{kmsy,i^{s,y}}$ (MMcf/d)

Long-Term Market												
Season 1				Season 2				Season 3				
*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	
0.00	0.00	325.00	200.00	0.00	0.00	325.00	200.00	0.00	0.00	325.00	200.00	
Spot Market												
Season 1				Season 2				Season 3				
*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	*RD*2	*CD*2	*ID*2	*ED*2	
84.77	65.43	9.75	50.00	254.42	186.97	19.50	30.00	350.22	243.26	29.25	8.00	Scenario 1
								452.25	299.71	48.75	16.00	Scenario 2
								340.81	234.19	29.25	8.00	Scenario 3
								443.20	290.98	48.75	16.00	Scenario 4
				296.77	219.76	32.50	70.00	383.76	267.39	29.25	8.00	Scenario 5
								425.01	315.24	58.50	20.00	Scenario 6
								374.50	258.46	29.25	8.00	Scenario 7
								416.89	307.40	58.50	20.00	Scenario 8
				250.97	183.34	19.50	30.00	345.61	238.82	29.25	8.00	Scenario 9
								446.66	294.31	48.75	16.00	Scenario 10
								338.28	231.74	29.25	8.00	Scenario 11
								441.32	289.16	48.75	16.00	Scenario 12
				292.75	215.52	32.50	70.00	379.09	262.89	29.25	8.00	Scenario 13
								419.90	310.31	58.50	20.00	Scenario 14
								372.20	256.24	29.25	8.00	Scenario 15
								414.59	305.18	58.50	20.00	Scenario 16
93.55	76.30	16.25	70.00	247.39	178.58	19.50	40.00	341.21	236.63	29.25	8.00	Scenario 17
								435.31	330.45	48.75	30.00	Scenario 18
								332.05	227.79	29.25	8.00	Scenario 19
								427.20	322.63	48.75	30.00	Scenario 20
				318.95	231.88	32.50	60.00	362.08	251.61	29.25	8.00	Scenario 21
								459.88	336.99	65.00	40.00	Scenario 22
								351.98	241.87	29.25	8.00	Scenario 23
								450.57	328.00	65.00	40.00	Scenario 24
				244.12	175.14	19.50	40.00	336.56	232.14	29.25	8.00	Scenario 25
								430.57	325.88	48.75	30.00	Scenario 26
								329.24	225.08	29.25	8.00	Scenario 27
								425.26	320.76	48.75	30.00	Scenario 28
				314.64	227.34	32.50	60.00	357.04	246.75	29.25	8.00	Scenario 29
								454.74	332.02	65.00	40.00	Scenario 30
								349.65	239.63	29.25	8.00	Scenario 31
								448.24	325.75	65.00	40.00	Scenario 32
83.57	64.08	9.75	50.00	253.86	186.38	19.50	30.00	350.09	243.14	29.25	8.00	Scenario 33
								452.00	299.47	48.75	16.00	Scenario 34
								340.50	233.88	29.25	8.00	Scenario 35
								443.02	290.80	48.75	16.00	Scenario 36
				296.62	219.60	32.50	70.00	383.42	267.06	29.25	8.00	Scenario 37
								424.85	315.09	58.50	20.00	Scenario 38
								374.30	258.26	29.25	8.00	Scenario 39
								416.69	307.21	58.50	20.00	Scenario 40
				249.33	181.61	19.50	30.00	347.10	240.25	29.25	8.00	Scenario 41
								448.16	295.76	48.75	16.00	Scenario 42
								333.10	226.75	29.25	8.00	Scenario 43
								436.14	284.16	48.75	16.00	Scenario 44
				291.57	214.28	32.50	70.00	381.06	264.79	29.25	8.00	Scenario 45
								421.37	311.72	58.50	20.00	Scenario 46
								366.98	251.20	29.25	8.00	Scenario 47
								409.42	300.20	58.50	20.00	Scenario 48
92.71	75.35	16.25	70.00	247.11	178.30	19.50	40.00	341.06	236.49	29.25	8.00	Scenario 49
								435.16	330.31	48.75	30.00	Scenario 50
								331.85	227.60	29.25	8.00	Scenario 51
								426.99	322.43	48.75	30.00	Scenario 52
				318.80	231.72	32.50	60.00	361.93	251.47	29.25	8.00	Scenario 53
								459.73	336.84	65.00	40.00	Scenario 54
								351.77	241.68	29.25	8.00	Scenario 55
								450.36	327.80	65.00	40.00	Scenario 56
				242.48	173.42	19.50	40.00	338.04	233.56	29.25	8.00	Scenario 57
								432.08	327.33	48.75	30.00	Scenario 58
								324.05	220.07	29.25	8.00	Scenario 59
								420.10	315.77	48.75	30.00	Scenario 60
				313.35	225.98	32.50	60.00	358.47	248.14	29.25	8.00	Scenario 61
								456.20	333.43	65.00	40.00	Scenario 62
								344.42	234.58	29.25	8.00	Scenario 63
								444.51	322.15	65.00	40.00	Scenario 64

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