INSTITUTE FOR SYSTEMS RESEARCH

THESIS REPORT

Master's Degree

Enumeration and Automatic Sketching of Epicyclic-Type Automatic Transmission Gear Trains

by G. Chatterjee
Advisor: L-W. Tsai

M.S. 93-15

The Institute for Systems Research is supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), the University of Maryland, Harvard University, and Industry

Abstract

Title of Thesis:	Enumeration and Automatic Sketching of Epicyclic-Type Automatic Transmission Gear Trains
Name of degree candidate:	Goutam Chatterjee
Degree and Year:	Master of Science, 1993
Thesis directed by:	Dr. Lung-Wen Tsai, Professor, Department of Mechanical Engineering, and Institute for Systems Research

The automotive transmission maintains a proper equilibrium between the power and torque produced by the engine and those demanded by the drive wheels. Most automatic transmissions employ some kind of epicyclic gear mechanisms to achieve the above purpose. The first step in the design process of such a mechanism involves finding the configuration that provides the correct speed ratios, and meets other dynamic and kinematic requirements. In this work, the kinematic structural characteristics of epicyclic gear mechanisms have been identified, and a methodology is formulated to systematically enumerate all possible configurations of such mechanisms. This is achieved through representation of the mechanisms by graphs and their storage in the computer as vertex-to-vertex adjacency matrices. Some of the structural characteristics of the mechanisms that arise out of its functional requirements are taken into account during the

enumeration process. This helps limit the number of graphs at any stage of the enumeration procedure. Graphs of mechanisms with up to nine links have been generated using this methodology.

The representation of mechanisms by graphs precipitates the need of a methodology for reverse transformation, that is, for constructing the mechanisms from graphs. To accomplish this, a mechanism is discretized into Fundamental Geared Entities. Further, these geared entities are shown to be a conglomeration of five primitives; namely, carrier, sun, ring, the single planet, and the multiple planet gears. An algorithm is formulated to create these entities from the graph representation by using the primitives. These entities are then connected together to form the mechanism.

ENUMERATION AND AUTOMATIC SKETCHING OF EPICYCLIC-TYPE AUTOMATIC TRANSMISSION GEAR TRAINS

by

Goutam Chatterjee

Thesis submitted to the Faculty of the Graduate School
of The University of Maryland in partial fulfillment
of the requirements for the degree of
Master of Science
1993

Advisory Committee:

Professor Lung-Wen Tsai, Chairman/Advisor
Professor Edward Magrab
Professor Donald Barker

Dedication

To my parents

Acknowledgements

I would like to sincerely thank Dr. Lung-Wen Tsai for his continuous guidance during the course of my thesis work. Most of the ideas presented in this thesis were initiated during the long discussions that I had with him. Among his numerous other qualities the one that I appreciate the most is his availability to students. I would like to thank Dr. Magrab for his helpful suggestions and comments. He spent a considerable amount of time and effort to ensure that my thesis is not only technically correct, but also grammatically so. I would like to thank Dr. Barker for serving on my advisory committee. It was Dr. Barker who introduced me to PHIGS graphics package.

Thanks are also due to the Department of Mechanical Engineering, University of Maryland, College Park, for providing me financial support in the form of a Teaching Assistantship. Financial support for this work was also provided in part by the U.S. Department of Energy under Grant DEF 05 - 88 ER 13977.

My thanks go to several others who provided both technical and non-technical help towards the completion of this work. This includes SRC Open Sun Lab staff, my supportive roommates and officemates among others. Special mention is made of two of my senior colleagues Shuvankar Ghosh and Rajesh Ratnakar for sharing their experiences with me. The friendly advice and comments of Atul Kalia and Diganta Das are also hereby acknowledged.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

The design of a mechanism begins with structural synthesis followed by kinematic and dynamic analysis. The most difficult phase in the design process is the conceptual phase where one has to decide the basic structure of a mechanism, i.e., the number of links and the number and type of joints connecting them, to satisfy certain functional requirements. Traditionally, this has been achieved by the designer's intuition and ingenuity. However, this approach has one drawback. It does not guarantee that all the potential machanisms, which could have satisfied the functional requirements, have been evaluated and that the best candidate has been selected. Beginning in the early seventies, efforts were undertaken to overcome this drawback by systematically generating mechanisms for a particular use.

This thesis deals with the systematic enumeration and proper functional representation of epicyclic-gear-type automatic transmissions found in automobiles.

Methods of systematically enumerating the kinematic structures of mechanisms have been developed by applying graph theory to the enumeration procedure. This chapter reviews the method of representing the kinematic structure of a mechanism by graphs, introduces the concept of graph isomorphism and describes some methodologies for identifying non-isomorphic graphs. Then fundamental rules associated with the graphs of epicyclic gear trains (EGTs) as identified by Buschbaum and Freudenstein (1970), are explained. Finally, the scope and organization of this thesis are outlined.

1.1 Background

1.1.1 Graph Representation

A graph (Harary, 1969) consists of a set of points together with a set of unordered pairs of these points. Thus, a graph may be visualized as a set of points or vertices in space interconnected by lines or edges. An edge is said to be incident on a vertex if the vertex is one of the end vertices constituting that edge. A line merely carries the information that a pair of vertices are adjacent and hence its geometry is of no significance. For the purpose of analysis and storage in a digital computer, a graph can be represented by a matrix. There are several matrix representations of a graph, with the vertex-vertex adjacency matrix being the most commonly used. The vertex-vertex adjacency matrix of a graph of n vertices numbered from 1 to n, is an $n \times n$ matrix with its elements A(i,j) defined as follows

$$A(i,j) = \begin{cases} 1 & \text{if vertex } i \text{ is connected to vertex } j, \\ 0 & \text{otherwise (including } i = j). \end{cases}$$

The graph representation of a mechanism essentially ignores the dimensions of the mechanism and only retains information about its number of links and type of joints connecting them. In the graph representation of a mechanism, links are represented by vertices and joints by edges. Since an edge connects only two vertices, only binary joints can be represented through graph. Therefore a trinary joint is represented as two binary joints, a quarternary joint as three binary joints and so on. The type of joints between links can be stored in the graph representation by labeling or coloring the edges.

A *walk* in a graph is an alternating sequence of vertices and edges, beginning and ending with vertices such that each edge in the sequence is incident on the vertices immediately preceding and succeeding it. A walk is a *path* if all the vertices in the sequence are distinct. If each vertex in a walk is distinct except for the beginning and the end vertices it is called a *circuit*. A graph is said to be connected if there exist one or more paths between any two of its vertices. A *tree* is a connected graph with no circuits. Any two vertices of a tree are connected by exactly one path. The subgraph of a graph is obtained by removal of one or more vertices and edges from the graph. The removal of a vertex implies the removal of all the edges incident to the vertex. A component of a graph is the maximally connected subgraph.

An articulation point in a graph is a vertex, the removal of which increases the number of components. A mechanism whose graph has one or more articulation points is called a *fractionated mechanism* because it can be separated into two or more independent fractions by breaking the links corresponding to the articulation points.

A graph is said to be rooted if one of its vertices is marked distinctly from the others. From the mechanism point of view the root represents a unique link usually the base link (or the frame), in the mechanism. Since the enumeration of rooted graph is easier, one should always try to find a unique link in the class of mechanisms one wants to generate.

Two graphs are isomorphic if there exists a one-to-one correspondence between their vertices and edges which preserves the incidence and labeling. The adjacency matrix of two isomorphic graphs can be different depending on the numbering of their vertices. However, the adjacency matrix of any one of the isomorphic graphs can be made the same as the other by pre and post multiplying it by a matrix B and its transpose. The matrix B has the following characteristics.

(i) All the elements of matrix B are either 1 or 0.

(ii) The sum of the elements in any row or column is 1.

Thus, it follows from linear algebra that the adjacency matrices of two isomorphic graphs will have the same characteristic polynomials (Uicker and Raicu, 1975). However, the converse is not true. Therefore, although the use of characteristic polynomials has proved to be effective in the identification of isomorphic graphs, quite a few examples to the contrary can be found in the papers by Mruthyunjaya and Balasubramanian (1987), and Sohn and Freudenstein(1986).

A reliable method for identifying isomorphism is to develop a unique code for each graph, such that two isomorphic graphs will have the same code while two non-isomorphic graphs will have different codes. This usually involves finding a way of uniquely numbering the vertices. For example, in the *degree code* (Tang and Liu, 1988) the vertices with higher degrees are numbered first. Then permutation is carried out among the vertices of the same degrees. For each permutation, the number D formed by concatenating the elements to the right of the main diagonal of the adjacency matrix is compared with the others. The permutation for which this D is maximum gives the canonical sequence for numbering the vertices. This maximized D is the degree code for the graph.

1.1.2 Epicyclic Gear Trains

Definition

Epicyclic gear train (EGT) is defined (Buchsbaum and Freudenstein, 1970) as a geared kinematic chain containing only revolute and geared joints and conforming to the following rules.

R1: The mechanism shall obey the general degrees of freedom equation, i.e. no special proportions are required to ensure the mobility of an epicyclic gear train; its joints are binary and its graph planar.

R2: In the graph of an EGT, there shall be no circuit which has zero or negative degrees of freedom.

R3: The rotatability of all links shall be unlimited. Mechanisms with partial mobility or with partially locked structure shall be excluded.

R4: Each gear must have a turning pair on its axis, and each link in a gear train must have at least one turning pair in order to maintain a constant center distance between each gear pair.

Fundamental Characteristics

In the graph of an EGT, thick edges represent gear joints between links while thin edges represent revolute joints. Using the above rules it has been shown that the graph of an EGT of v links will possess the following characteristics (Buchsbaum and Freudenstein, 1970):

F1 The graph has v vertices, (v-1) turning pair edges and (v-1-F) geared edges, where F is the dof of the EGT.

F2 The subgraph obtained by deleting all the geared edges is a tree.

F3 Any geared edge added to the tree forms a fundamental circuit having one geared edge and several turning pair edges.

F4 The number of fundamental circuits equals the number of geared edges.

F5 Each turning-pair edge can be characterized by labeling it with different symbols a, b, c, etc., which identifies the location of its joint axis in space.

F6 All thin edges having the same label must form a tree.

F7 The differential dof of any circuit must be at least equal to one; for a fundamental circuit it is equal to the number of vertices in the circuit minus two.

F8 In each fundamental circuit there is one vertex, called the *transfer vertex*, such that all edges on one side of the transfer vertex have the same label and edges at opposite side of the transfer vertex have a different label.

F9 All vertices must have at least one incident edge, which represents a turning pair.

Though any graph representing an EGT will have these characteristics, a graph having these characteristics may not necessarily conform to rules R1 to R4.

Fig. 1.1(a) shows an EGT of 5 links. Fig. 1.1(c) depicts the corresponding graph representation. Fig. 1.1(b) shows another mechanism having the same graph representation. The two mechanisms have the same graph representation because the graph of an epicyclic gear train does not contain sufficient information to distinguish between external and internal gear pairs.

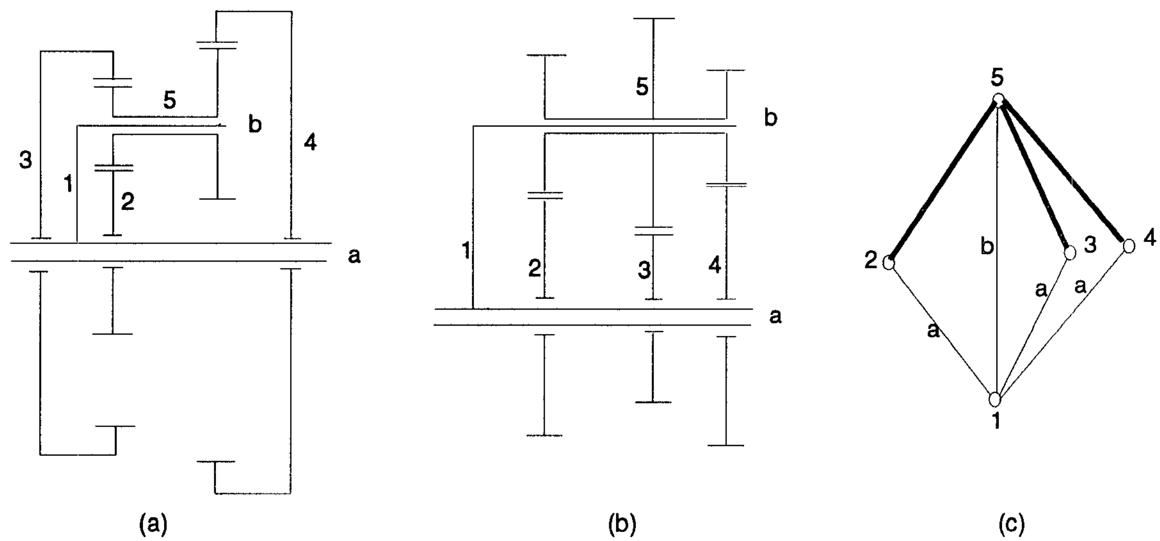


Figure 1.1 Epicyclic Gear Trains of 5 links: (a,b) functional representations, (c) graph representation for (a) and (b).

Rotation and Displacement Graphs

A rotation graph is a subgraph obtained from the graph of an EGT by deleting all the thin edges and those vertices that are not incident by any geared edge, and by labeling the geared edges with the associated transfer vertices. It has been shown that two EGTs will have the same set of rotational displacement equations if their rotation graphs are isomorphic (Freudenstein, 1979).

The displacement graph was originally defined by Freudenstein as a graph obtained by labeling each vertex of a rotation graph with the label of the edge connecting the vertex to the corresponding transfer vertex. Two EGTs will have the same set of linear displacement equations if their displacement graphs are isomorphic (Freudenstein, 1979). However, in this thesis we will consider any graph of an EGT possessing the fundamental characteristics listed before as a displacement graph, or simply a graph of an EGT.

It has already been mentioned in Section 1.1.1 that a graph can only represent binary joints. This, however, creates a problem in uniquely representing a mechanism. This is because if links A, B and C form a trinary joint one can represent it as two binary joints formed between links A & B and B & C, or between links A & C and C & B. For example, the mechanism shown in Fig. 1.2(a) can be reconfigured into the mechanism shown in Fig. 1.2(c) by rearranging the revolute joints among its coaxial links. Though these two mechanisms appear to be structurally non-isomorphic (Ravisankar and Mruthyunjaya, 1985), they are kinematically equivalent, and for the purpose of structural synthesis are considered the same. The graph representations of the two mechanisms are shown in Figs. 1.2(b) and (d) respectively. The two graphs are mathematically non-isomorphic. The graph in Fig. 1.2(d) can be formed from the graph of Fig. 1.2(b), if the thin edge joining the vertices 1 and 2 in the latter is replaced by a thin edge of the same label joining vertices 2 and 3. This method of creating mathematically non-isomorphic graphs representing kinematically equivalent EGTs by replacing a thin edge by another thin edge of the same label is known as vertex selection. This further complicates the issue of isomorphism and leads to the definition of pseudoisomorphism (Tsai and Lin, 1989). Two graphs are said to be pseudoisomorphic if they become isomorphic under single or repeated application of vertex selection. The problem of pseudoisomorphism can be averted by imposing some rules that result in unique arrangement of the edges of the same label. Such a displacement

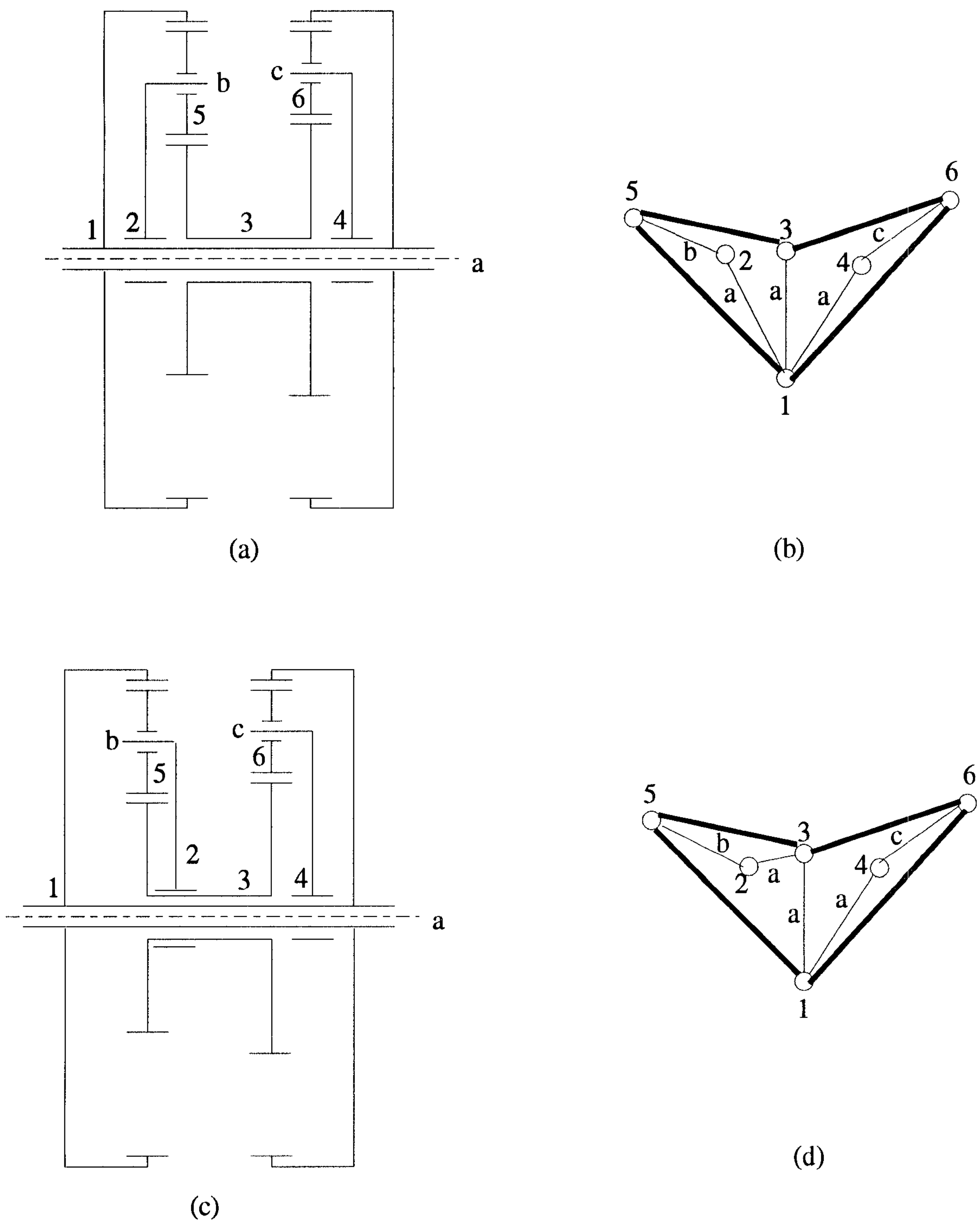


Figure 1.2 Epicyclic Gear Trains of 6 links: (a,c) functional representations, (b,d) Corresponding graph representations.

graph is called a canonical displacement graph, or canonical graph (Tsai, 1988).

1.1.3 Literature Review

After Buchsbaum and Freudenstein's pioneering work, which laid down the ground rules for EGTs, a considerable amount of work has been done in the area. Ravisankar and Mruthyunjaya (1985) computerized the method developed by Buchsbaum and Freudenstein and enumerated graphs of one degree-of-freedom (dof) EGTs with up to 6 links. Tsai (1987) developed a simpler method of generating graphs of one-dof EGTs in which geared edges do not form a loop. Tsai and Lin (1989) later extended this method to enumerate non-fractionated two-dof EGTs. Hsu (1992) developed alternative methods of enumerating epicyclic gear trains and has shown that all 6-link, one-dof EGTs can be generated from 81 displacement graphs. Kim and Kwak (1990) used edge permutation groups to identify isomorphic graphs and used Tsai's method (Tsai, 1987) of enumeration to derive one-dof EGTs with up to 7 links. They showed that there are 642 displacement graphs from which all the 7-link EGTs can be derived.

It is evident that the literature in this area contains a significant number of papers addressing the issue of graph enumeration. However very little work has been done in the area of constructing mechanisms from graphs; i.e., specifying the ground, input and output links to create useful mechanisms from graphs. Notable is the paper by Olson, et al. (1991) which addressed this issue and analyzed EGTs of 5 links. The other area that has attracted little attention is the sketching of

EGTs from their graphs. As mentioned in section 1.1.2 there isn't any one-to-one correspondence between the graphs and the mechanisms since the graphs do not differentiate between the external and internal gear pairs. Hoeltzel et al. (1990) gave an algorithm for sketching the mechanisms assuming that all the gear pairs are external. Thus, the process of sketching the mechanism from a graph is still largely dependent on the designer's ingenuity.

1.2 Scope of the Thesis

The power train (See Fig. 1.3) that transfers power from the engine crankshaft to the drive wheel of an automobile, consists of a transmission unit, a final reduction unit, and a differential. The transmission unit is a speed ratio change unit that maintains a proper balance between the speed and torque capabilities of the engine and the speed and torque demanded by the drive wheel. The ratio of the speed of the input shaft (which brings power from the engine) to that of the output shaft is called *speed ratio* or the *reduction ratio.* The transmission unit can be of two types - manual and automatic. The main component of a manual transmission unit is its gear train, which has its gears mounted on parallel non-revolving shafts. Such type of gear boxes are called *layshaft* or *countershaft* type of gear boxes. The changing of speed ratio is controlled by the driver.

Tsai, et al. (1988) have shown that unlike manual transmission units most of the gear boxes of the automatic transmission units use EGTs[1] to achieve the

[1]A few using countershaft type of arrangement is also available in the market. A hybrid

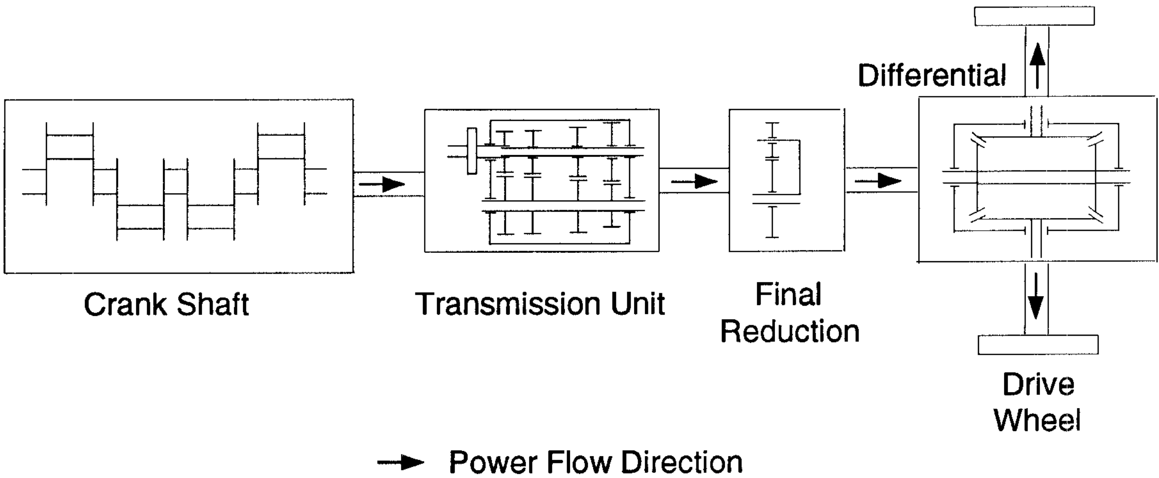


Figure 1.3 Components of a power train of a typical manual transmission.

desired reductions. Some of them use one-dof EGTs. Others use fractionated mechanisms, each fraction of which is a one-dof EGT. They have also identified some of the structural characteristics required by an EGT to qualify for automatic transmissions, and have shown that of the many non-isomorphic graphs of the six links only six graphs can be used in automatic transmission gear boxes. Henceforth, we will call the mechanism formed by an EGT and the casing of an automatic transmission gear box an *Epicyclic Gear Mechanism* (EGM). An EGM typically has two dofs.

The objectives of this thesis are to

- develop a convenient canonical graph representation for EGMs
- identify structural characteristics of EGMs and translate them into the language of graph representation.

type using a combination of both is also possible

- develop a methodology that takes into account the above characteristics and systematically enumerates the EGMs

- develop a systematic method of sketching the functional schematics of EGMs.

1.3 Organization of the Thesis

Chapter 2 gives a brief description of a typical automotive automatic transmission units and categorizes the gear trains used in most transmissions. Chapter 3 identifies the structural characteristics required of an EGM and develops a canonical graph representation for them. Chapter 4 introduces the idea of prelabeling and formulates a systematic method to generate these EGMs. Chapter 5 describes a methodology for drawing the functional schematics of these EGMs. A commercially available software program called PHIGS[2] has been used as a graphics aid for this purpose. Chapter 6 tabulates the results, summarizes this work and suggests some future extensions.

Appendix B contains all the 6 and 7 links EGMs. Appendix C contains all possible 8-link EGMs, while Appendix D contains the 9-link EGMs. In addition Appendices B, C, and D also give the functional representation of some of the 6, 7, 8 and 9-link EGMs.

[2]PHIGS is an acronym for Programmer's Hierarchical Interactive Graphics System. PHIGS standard is an international graphics programming standard developed by the International Standards Organization (ISO) and the American National Standards Institute (ANSI).

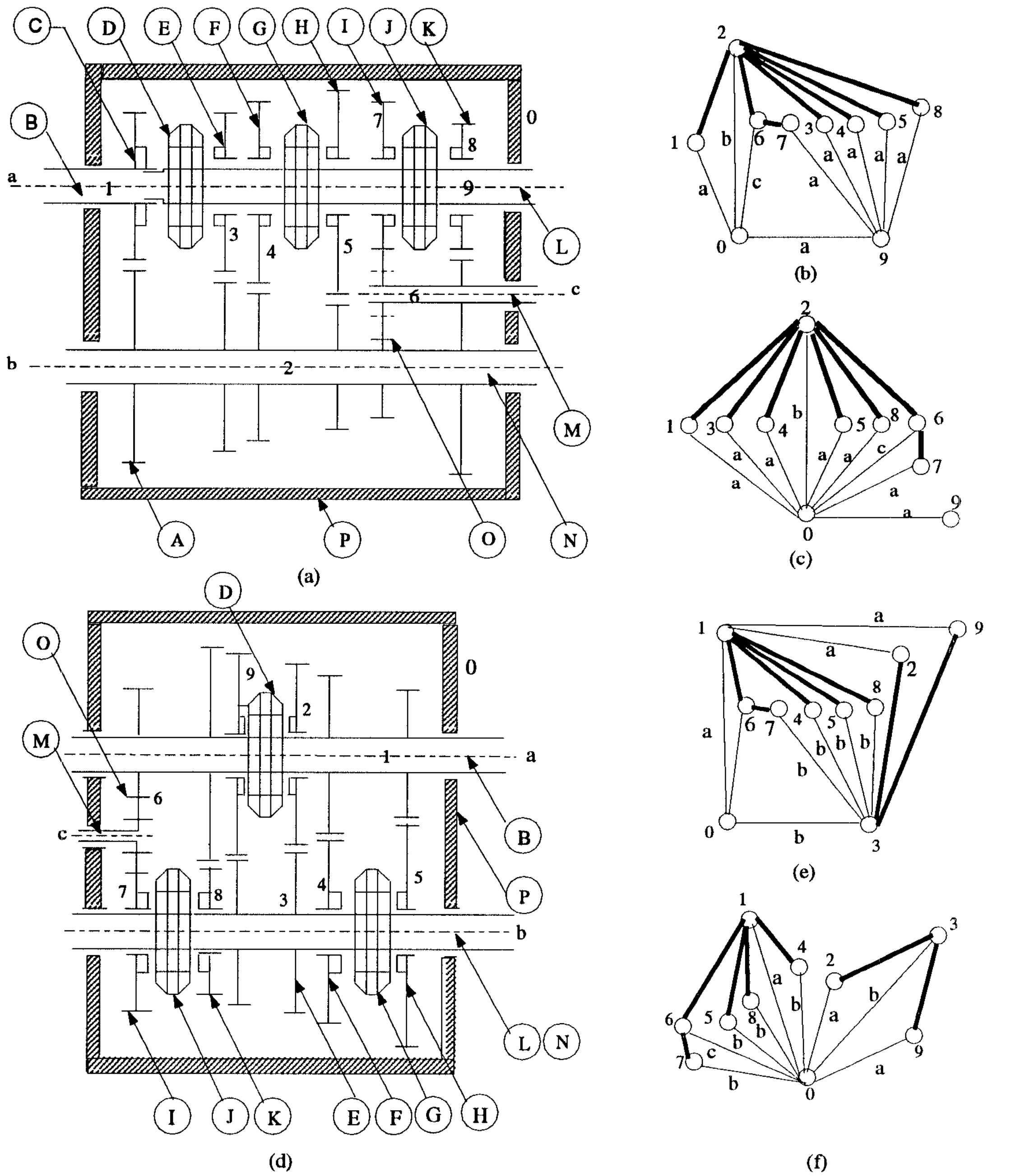
Chapter 2

Transmissions

Automotive transmissions can be broadly classified as manual and automatic. Both types of transmissions are discussed in this chapter, with the automatic transmission being discussed in more detail. Most of the materials included in this chapter have been compiled from published work (Larew, 1966; Levai, 1966; Erjavek 1990; Gotts, 1991). What is new is the study of the gear trains used in various types of transmissions from the viewpoint of their kinematic structures and graph representations.

2.1 Manual Transmissions

A manual transmission unit typically consists of a rotating clutch, a system of gears and gear synchronizers. The gears are mounted on parallel non-revolving shafts. The clutch is used to engage and disengage the gear system from the engine. When the clutch is engaged power flows from the engine to the gear



LEGEND

A. Counter Input Gear
B. Input Shaft
C. Input Gear
D. Third/Fourth Gear Synchronizer
E. Driven Third Gear
F. Driven Second Gear
G. First/Second Gear Synchronizer
H. Driven First Gear
I. Driven Reverse Gear
J. Reverse/Fifth Gear Synchronizer
K. Driven Fifth Gear
L. Output Shaft
M. Intermediate Shaft
N. Counter Shaft
O. Reverse Idler Gear
P. Casing

Figure 2.1 Manual transmissions: (a) five-speed rear wheel drive , (d) five-speed front wheel drive (Transaxle), (b) and (c) graph epresentations of (a), (e) and (f), graph representation of (d).

system. Figs. 2.1(a) and (d) show the typical arrangement of gears in a rear wheel drive and a front wheel drive transmission, respectively. In Fig. 2.1(a) the input and output shafts are coaxial. One of the gears (link no. 1) is permanently keyed to the input shaft and serves as the input link for all reductions. The transmission unit of a front wheel drive as shown in Fig. 2.1(d) is sometimes referred to as a transaxle, since the output shaft is offset from the input shaft. Hence, there is no direct drive. To engage a gear with a revolving shaft (input or output) both of them have to be brought to the same speed. This is done with the help of the synchronizer. The reverse gear is obtained with the help of an idler that rotates about a third (intermediate) axis. The transmissions described above are the fully synchronized constant mesh type transmissions and are found in all modern automobiles. The term constant mesh signifies that all the gears are in constant mesh with each other. In some cases the idler gear for reverse is not in constant mesh and reverse is obtained by shifting gears rather than with the help of a synchronizers.

To sketch the graph representations of the gear trains we consider the gear-box to be in the neutral position. This implies that none of the synchronizers are engaged with the gears and there is no output from the gear-box. The graph representations of the gear trains are depicted in Figs. 2.1(b) and (e), respectively. We apply vertex selection (see page 9 of Section 1.1.2) to these graphs to bring out special features of the mechanisms they represent. Kinematically this won't make any difference. The resulting graphs are shown in Figs. 2.1(c) and (f).

From the graph of Fig. 2.1(c) it is clear that the mechanism of Fig. 2.1(a) is a fractionated two-dof mechanism. The vertex No. "0" represents the casing, which is also the transfer vertex for all the gear pairs. The various reductions are obtained by engaging one of the links 1, 3, 4, 5, 7, and 8 to the output shaft (link 9) through the synchronizer. Thus, from the kinematic point of view the gear train of a manual transmission has the following features.

(i) The gear train has only one carrier, which is the casing. The carrier usually has three axes, one of which carries the idler for the reverse gear.

(ii) Different speed ratios are obtained by engaging an appropriate gear with the input or the output shaft by means of synchronizers. When no gear is engaged either with the input or the output shaft, the gear train is said to be in a neutral position and no power flows through the gear train. Also, for direct drive, the input and output shafts are coupled to each other and no power flows through any of the gears.

(iii) For rear wheel drive vehicles, the input and output shafts are usually coaxial. For front wheel drive vehicles, the input and output shafts are located at two different axes of the carrier.

2.2 Automatic Transmissions

The main components in a typical automatic transmission (see Fig. 2.2) are torque converter, epicyclic gear train(s), and clutch control system. There ex-

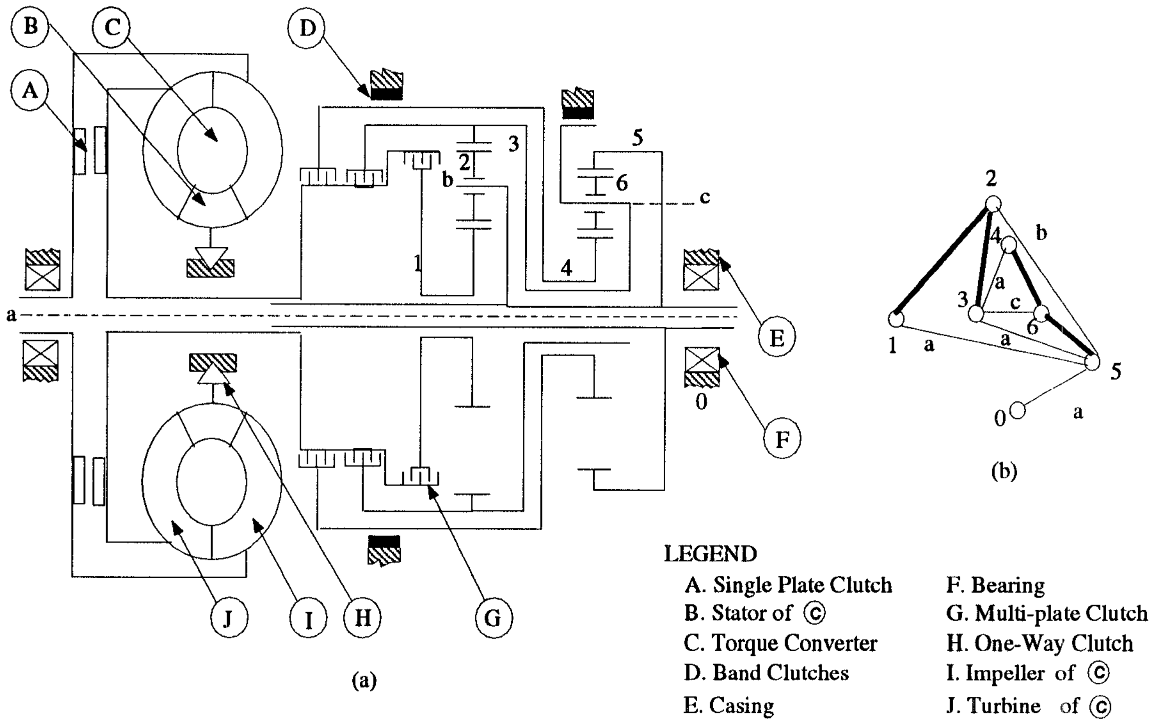


Figure 2.2 Automatic Transmission: (a) Schematic, (b) Graph Representation of the EGT.

ists another type of automatic transmissions that uses the layshaft type gear arrangement used in manual transmissions. This type of layshaft transmission is discussed at the end of this section.

2.2.1 Torque Converter

The torque converter consists of an impeller, a turbine and one or more stators. Part C in Fig. 2.2(a) is the schematic representation of a torque converter. The impeller gets power from the engine and therefore imparts motion to the fluid contained in it. The fluid escapes through its outer circumference and enters the turbine. The fluid leaves the turbine at the inner circumference of the blades and enters the impeller again through the stator blades. The stator blades redirect the flow from the turbine to the impeller blades to achieve a torque multiplication. This process continues until the impeller and the turbine are rotating at the same speed. When torque multiplication is not needed the stator free-wheels with the turbine and the torque convertor is reduced to a hydraulic coupling. The torque converter allows the engine to start from rest with a torque multiplication.

2.2.2 Epicyclic Gear Train

The power from the converter enters the EGT through a system of clutches. In Fig. 2.2(a) the EGT is formed by links 1, 2, 3, 4, 5 and 6. The EGT itself has one-dof. The EGT is supported by bearings housed in the casing. The resulting EGM formed is thus a fractionated two-dof mechanism in which the EGT as a whole can rotate freely with respect to the casing. The graph representation of the EGT along with the casing is shown in Fig. 2.2(b). The various speed ratios are obtained by clutching different links of the EGT to the output shaft of the torque converter and by fixing different links to the casing. These two actions are achieved by rotating clutches and band clutches, respectively. The clutches are usually controlled by hydraulically actuated mechanisms, and change of speed ratios are achieved without interruption of power flow by allowing slippage in the clutches. The epicyclic gear train of an automatic transmission has the following features.

(i) There can be more than one carrier that are not fixed in space

(ii) The same output link is used in all reductions.

(iii) The EGM is a fractionated two-dof mechanism in which the EGT can rotate freely with respect to the casing.

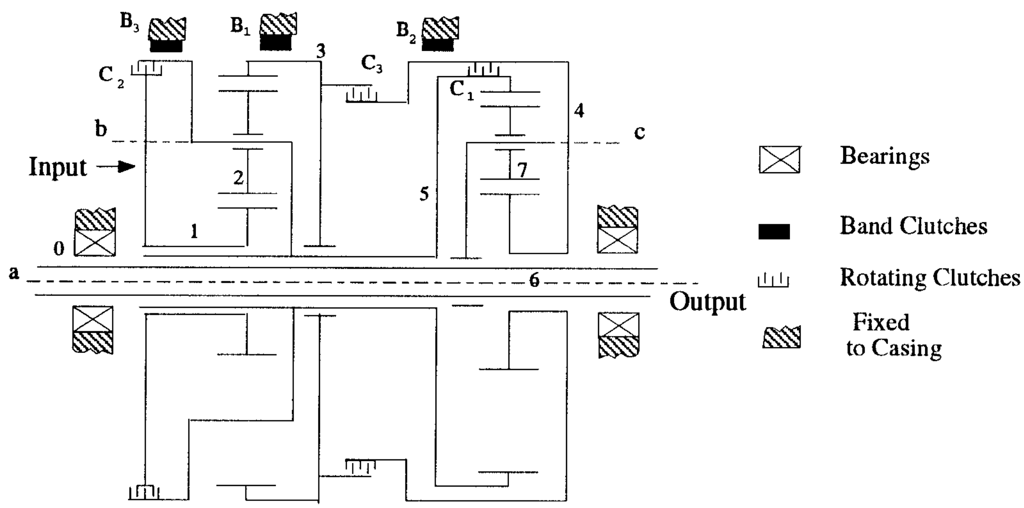
(iv) A desired speed ratio is typically obtained by clutching one link of the EGT to the power source and another to the casing. In all reductions, other than the direct drive, one of the links is fixed on to the casing and another is connected to the power source. In case of direct drive, none of the links of the EGT is fixed to the casing, and two of the links are connected to the power source. Hence, the EGT rotates as a rigid body.

(v) The output and input links are coaxial with each other.

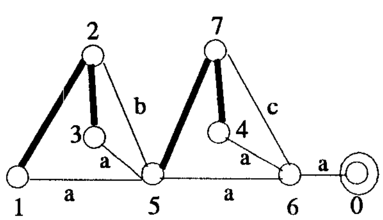
Some transmissions employ fractionated epicyclic gear trains. These are formed by connecting two one-dof EGTs with a common link. In such cases either two links have to be fixed to the casing by band clutches or two links, one from each EGT, have to be connected by clutches in order to get a reduction ratio. An example of such a gear train is shown in Fig. 2.3. The clutching sequence is also shown in the same figure and where an X indicates that the corresponding clutch is activated for the speed reduction given in the first column.

2.2.3 Automatic Layshaft Transmissions.

The gear train of an automatic layshaft transmission (ALT) has the same features as that of a manual layshaft transmission. Here, also, the different speed



(a)



(b)

RANGE	C_1	C_2	C_3	B_1	B_2	B_3
FIRST				X	X	
SECOND	X			X		
THIRD		X			X	
FOURTH	X	X				
REVERSE			X			X

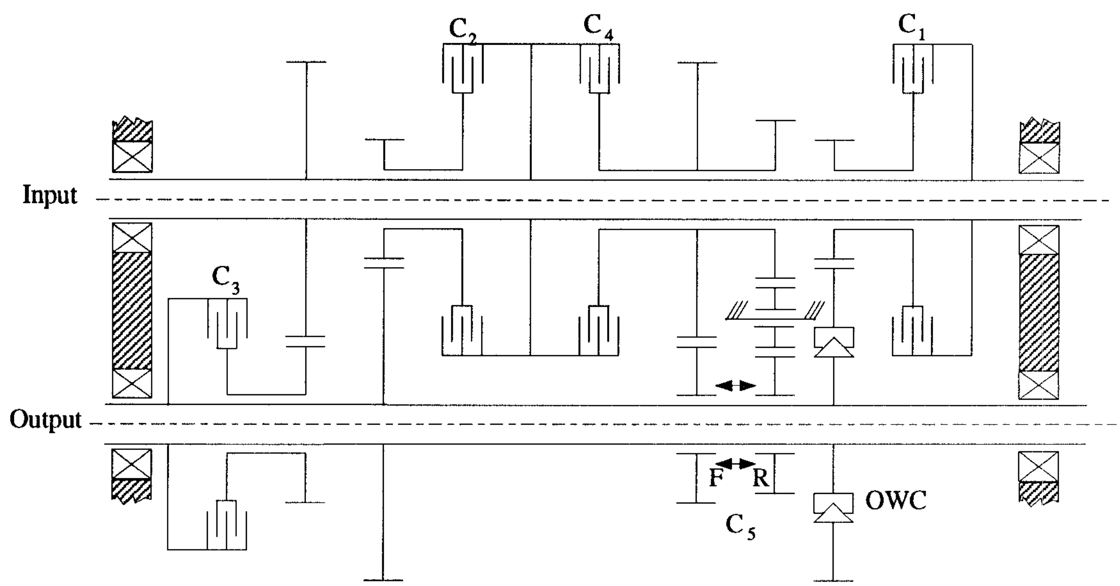
(c)

Figure 2.3 Fractionated EGT employed in an automatic transmission: (a) functional representation, (b) graph representation, (c) clutching Sequence.

ratios are obtained by engaging different gears of the gear train with the input and/or output shafts. The difference lies in the action being performed automatically without any driver's efforts. In this respect there are two types of ALT (Gotts, 1991). One of them retains all the features of a manual layshaft transmission, including its synchronizers, and only automates the actuations of these mechanisms. These are called automated layshaft transmissions. The other type uses multidisc clutches instead of synchronizers for each gear to be connected to the input or output shaft. These are true ALTs since they allow for a change of speed ratio without interruption of power flow. This type of transmission sometimes requires more than two parallel shafts to provide the required number of forward speeds. Fig. 2.4 shows an ALT that provides four forward speeds and one reverse speed. The dog clutch is engaged manually, but only once at the start of the motion, to set the transmission either in the forward mode or in the reverse mode.

2.2.4 Hybrid Transmissions

A combination of EGT and layshaft type arrangement results in a hybrid transmission system. The casing is connected to a one-dof EGT through a revolute joint. The input, output, and the fixed links of the EGT are coaxial with the axis of this revolute joint. The casing also serves as the carrier for a layshaft type arrangement. Therefore if a link of the layshaft type arrangement is connected to any of the above mentioned coaxial links through a shaft (see Fig. 2.5(a)),



(a)

Range	Ratio	C_1	C_2	C_3	C_4	C_5	OWC
1	2.38	X				F	X
2	1.56	X	X			F	
3	1.03	X		X		F	
4	.77	X			X	F	
5	1.95				X	R	

(b)

Note :
1. C_5 is a Dog Clutch
2. OWC - One Way Clutch

Figure 2.4 Honda four speed automatic layshaft transmission: (a) functional representation, (b) clutching sequence.

the resulting mechanism is a three-dof fractionated mechanism. The graph of the mechanism is shown in Fig. 2.5(b). The graph shown in Fig. 2.5(c) is obtained after the application of vertex selection to the graph of Fig 2.5(b). From the graph it is clear that the mechanism is indeed fractionated since it has two articulation points (vertices 1 and 0).

One of the fractions represents the layshaft type of arrangement that has one link fixed (casing represented by vertex no. 0). Therefore after engaging the dog clutch, if one fixes any of the links of the other fraction (other than that represented by the articulation point i.e. vertex 1) the mechanism will require one input to give a unique output. The output can be tapped from link 1 or 7 by appropriate engagement of the dog clutch. Also, if two of the links of the EGT are engaged with the input, the EGT will act as a locked structure and the mechanism is reduced to a simple layshaft type structure with link 1 as the input link.

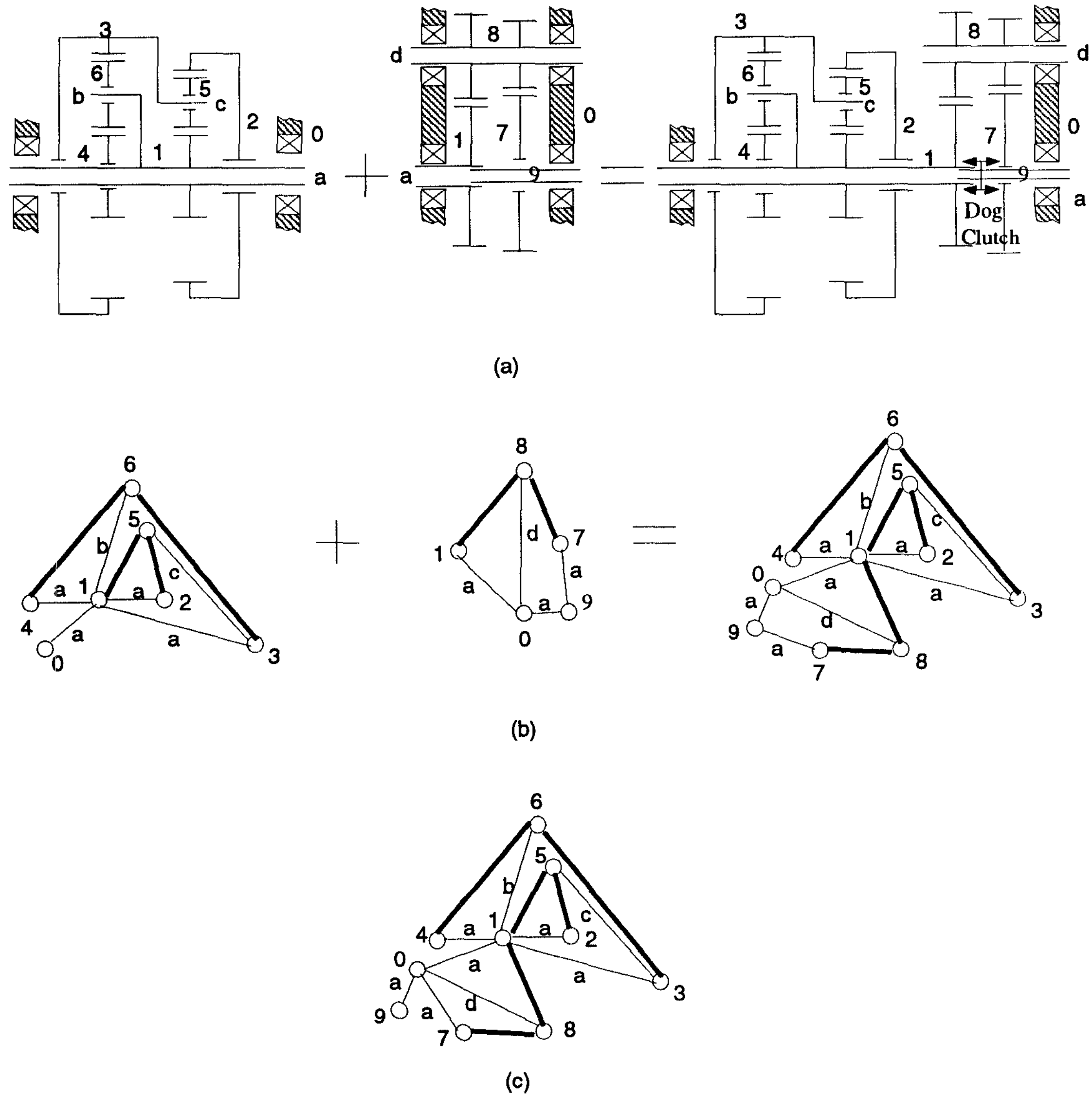


Figure 2.5 Hybrid Type Automatic Transmission: (a) Functional Representation, (b) Graph Representation, (c) Graph representation after application of vertex selection.

Chapter 3

Canonical Graph Representations and Structural Characteristics

In this chapter we will concentrate only on EGMs. These, as one may recall, are mechanisms, each consisting of a non-fractionated one-dof EGT and its casing.

3.1 Canonical Graph Representation

An EGM typically consists of a one-dof EGT supported by the casing on one axis, which results in a fractionated two-dof mechanism. Fig. 3.1(a) shows an EGM employing the Simpson gear train as the multi-speed reduction unit. The graph of the Simpson gear train along with the casing is shown in Fig. 3.1(b). Tsai et al.

(1988) after examining existing transmissions found out that only coaxial links of an EGT are used as input, output or fixed links. These coaxial links are mounted on concentric bearings that are housed in the casing. It's obvious that the input, output and the fixed links have to be coaxial, otherwise one of the links will have its axis moving in space and it won't be possible to connect it either to the output shaft of the torque converter or to the final reduction unit. In an EGM the output link is never changed. The desired reduction ratios are obtained by changing the input and the fixed links. Also, it is always possible to achieve a direct drive by locking all the links in the EGT together such that they rotate as a single link. Thus if N_e links of an EGT are coaxial, then it is possible to get (N_e - 1)(N_e - 2) + 1 number of speed reductions. Therefore if the desired number of speed reductions is N_r, then N_r will be limited by the equation

$$N_r \leq (N_e - 1)(N_e - 2) + 1 \tag{3.1}$$

For example, if a four-speed-reduction (four forward and one reverse speed) unit is desired the EGT must have a minimum of four coaxial links.

The casing of an EGM is a unique link in its kinematic structure. Therefore, we will take advantage of this fact and introduce a canonical graph to represent the EGM. In such a representation the vertex representing the casing will be marked as the root of the graph. Recall that when there are three or more coaxial links in a mechanism, the joints connecting these coaxial links can be rearranged without affecting the functionality of the mechanism (Tsai, 1988). Among various arrangements of the coaxial joints there exists a unique configuration such that

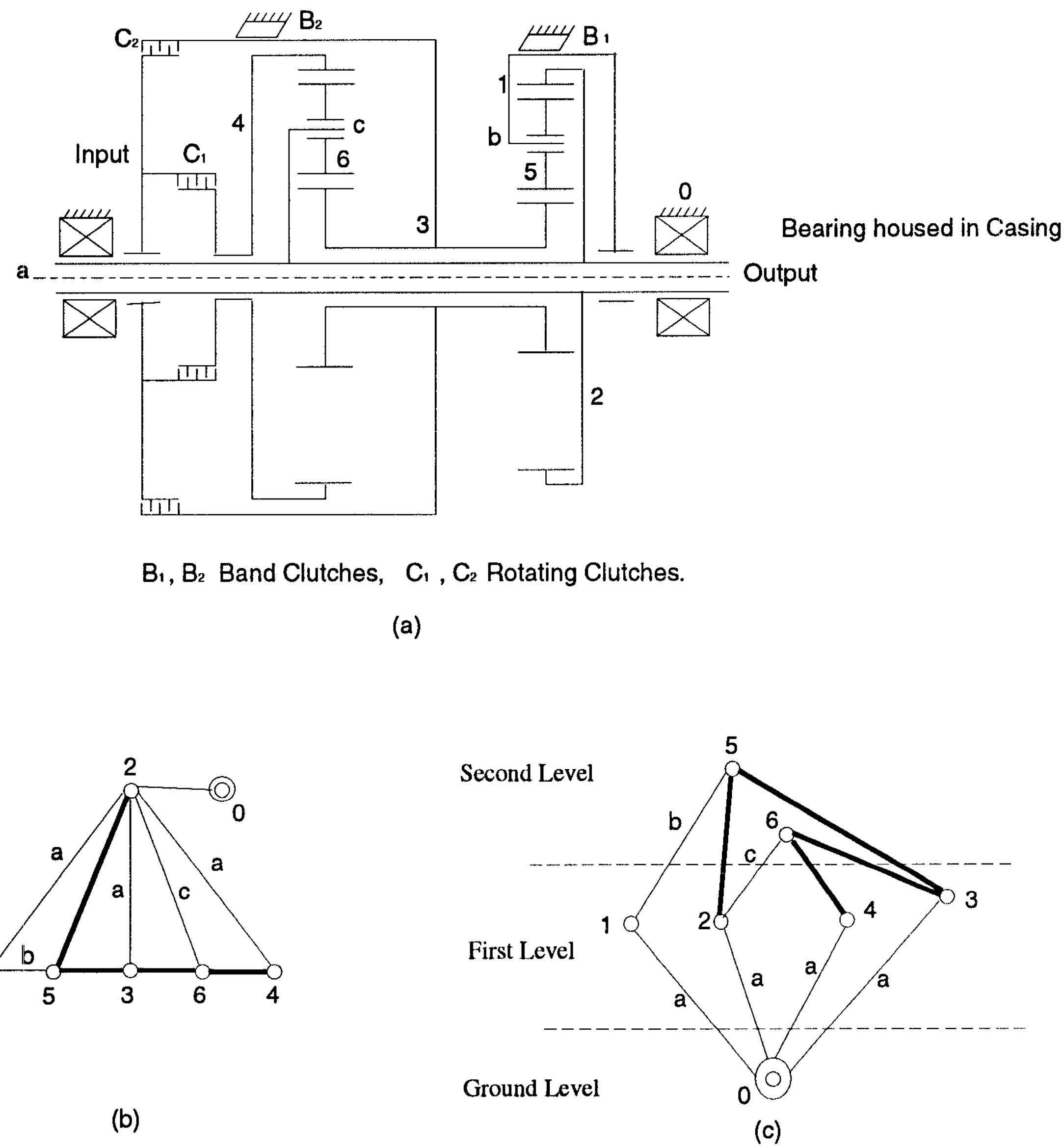


Figure 3.1 EGM employing Simpson gear set: (a) functional representation, (b) conventional graph represntation, (c) canonical graph representation.

all the thin edged paths originating from the root and ending at all the other vertices will have distinct edge labels. This unique graph representation is called the canonical graph representation. Using canonical graph representation, the vertices can be divided into several levels. The casing is denoted as the *ground* level vertex. A vertex that is connected by only one thin edge to the root is defined as a *first* level vertex. A vertex that is connected to the root by two thin edges is defined as the *second* level vertex and so on. Thus, each vertex at any particular level is connected to exactly one vertex at the immediate preceding level by a thin edge. All thin edges having the same label[1] must have one common lower level vertex. The canonical graph representation of the Simpson gear set shown in Fig. 3.1(a), is shown in Fig. 3.1(c). The canonical graph representation helps overcome the problem of pseudo-isomorphism explained in Section 1.1.2. Henceforth, all the vertices at a particular level that are connected to a lower level vertex by thin edges of the same label, will be referred to as members of a *family*.

3.2 Structural Characteristics

This section discusses the structural characteristics of an EGM and their manifestation in the canonical graph representation. The canonical graph by virtue

[1]Note the difference between *label* and *level*. A *label* denotes the location of the axis of a link in space while the word *level* denotes the location of a link in the kinematic chain relative to the casing.

of its definition has one special feature, i.e.

C1: All the thin edges of the same label should be incident to a common lower level vertex.

Since an EGM is virtually a two-dof EGT with a fractionated link, it should also conform to the rules R1 to R4 (Section 1.1.2) that apply to EGTs. Consequently, all the fundamental characteristics of a graph of an EGT as described in Section 1.1.2 also apply to the canonical graph. These are described in section 3.2.1 under the heading *General Characteristics.* An EGM, besides possessing the characteristics of an EGT, also has its own specific characteristics because it is a mechanism that performs some special functions. These characteristics and their expressions in canonical graph representation are discussed in Sections 3.2.2 and 3.2.4.

3.2.1 General Characteristics

The canonical graph of an EGM has no articulation point. It possesses the following characteristics :

C2: If there are n vertices in the canonical graph of an EGM then it must have $n-1$ thin edges and $n-1-F$ geared edges, where F is the number of dof of the EGM. In this paper we shall limit ourselves to $F=2$ mechanisms.

C3: The subgraph formed by removing the geared edges is a tree.

C4: A geared edge can *only* be incident with one of the following pairs of vertices.

(a) Two vertices at the same level, provided that they are connected to the same lower level vertex by thin edges of different labels.

(b) Two vertices at adjacent levels, if they are connected by a path of exactly three thin edges having two different labels.

(c) Two vertices one at level k and another at level $k-2$, if there is a path of exactly two thin edges between them. For every vertex at level k there is only one vertex at level $k-2$, to which it can be connected by a geared edge.

The characteristics C2 and C3 are just restatements of F1 and F2. The characteristic C4 follows from fundamental characteristics F3 and F8.

3.2.2 Coaxial Links

The first level vertices in the canonical graph of an EGM represent potential candidates for the input, output or fixed links. These links are connected to the casing by coaxial revolute joints. None of the links of an EGM should be connected to the casing by a gear joint[2]. Therefore,

[2]Note that it is possible to have one of the links of an EGT permanently fixed to the casing. This, however, will reduce the flexibility of obtaining more speed ratios from the gear train.

C5: The first level vertices are connected to the root by thin edges of the same label.

C6: No geared edge can be incident to the root.

C7: If N_r is the number of required speed reductions (including reverse), then the number of first level vertices N_e in the canonical graph must satisfy Eq.(3.1).

Fig. 3.1(c) shows the canonical graph representation of the Simpson gear set shown in Fig. 3.1(b). There are four first level vertices and two second level vertices. The first level vertices represent the potential input, output or fixed links of the EGM. The second level vertices represent the planet gears. There are no vertices located at a level higher than the second level. A review of the work of Larew (1966), Levai (1966), Gott (1991), and Tsai, et al. (1988) has not revealed a single automatic transmission gear box having a link located on the third or higher levels. However, since no physical reason can be found for this observation, it will not be considered a structural characteristic of such gearboxes.

3.2.3 Locked Chains

A set of links forming a part of a mechanism is said to be locked if they undergo no relative motion when the mechanism is in operation and, hence, can be replaced by one link without altering the functional characteristics of the mechanism. Rule

R3 (Section 1.1.2) states that a geared kinematic chain having a partially locked structure cannot be considered as an EGT. However, mechanisms obeying the fundamental characteristics F1 through F9 do not necessarily satisfy this rule.

Consider the canonical graph of an EGM shown in Fig. 3.2(a) that satisfies all the fundamental characteristics. However, the subgraph formed by removing vertices 0, 1, 2, 3, and 4 from the graph represents a kinematically locked chain. This is because the subgraph has 5 vertices but 4 geared edges. Fig. 3.2(c) shows a graph formed by replacing vertices 5, 6, 7, 8, and 9 in Fig. 3.2(a) by a vertex 5, that will perform the same functions.

C8: An EGM contains a locked chain if there exists a subgraph of p verices in the graph of the EGM such that

(a) the transfer vertex of each of the geared edges in the subgraph lies in the subgraph, and

(b) the number of geared edges in the subgraph is more than $p - 2$.

A methodology is described in the next chapter that prevents the generation of EGM with locked chains during the enumeration process.

3.2.4 Redundant Links

The other desired feature for an EGM to qualify for automatic transmission is that it should not have any *redundant* link.

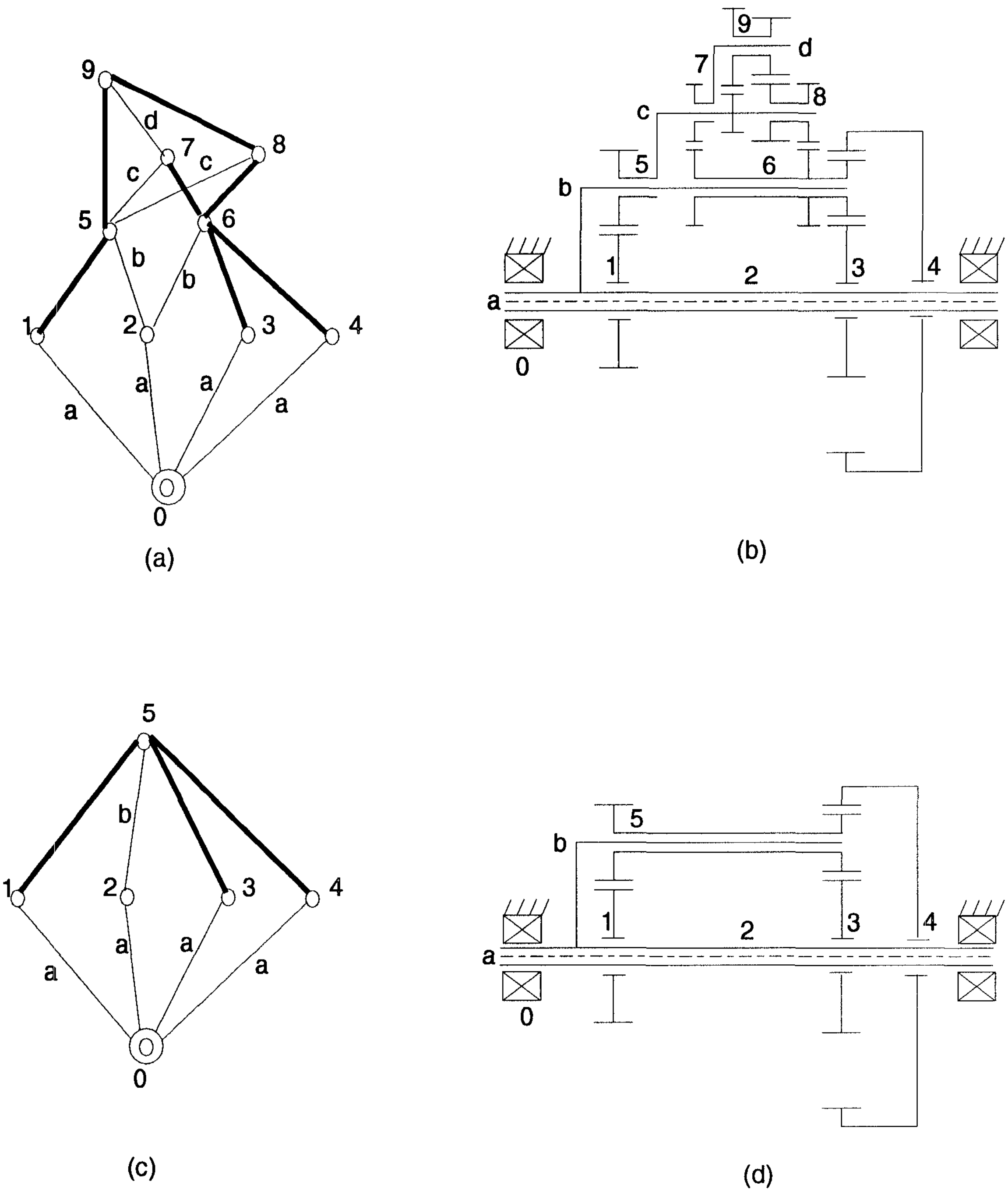


Figure 3.2 : (a) A canonical graph containing a locked chain, (b) functional representation of the mechanism, (c) graph obtained by removal of locked chain, (d) functional representation of the resulting mechanism.

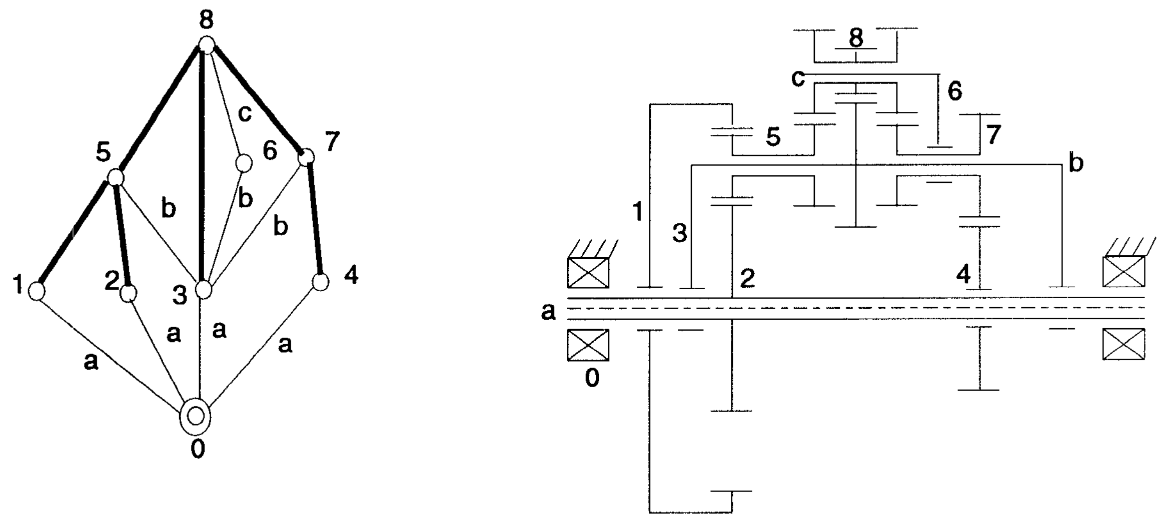
Definition of redundant links - A link is said to be redundant, if it is never used as an input, output or a fixed link, and the removal of such a link does not change the degrees of freedom of the EGT. Such a link will not carry power during the operation of the mechanism at any of its reductions.

A one-dof EGT communicating, that is, giving and taking power, with the external environment requires at least an input, an output and a reaction (fixed) link. Thus, to function effectively it requires at least three ports of communication with the external environment. Similarly, a two degrees of freedom EGT requires at least four ports of communication, a three degrees of freedom EGT requires five ports of communication, and so on.

In the canonical graph representation of an EGM if there exists a subgraph that represents an n-dof EGT, then it must have at least $n+2$ ports of communication with the external environment. Otherwise, those links that can not interact with the external environment would be redundant. The external environment includes both the rest of the gear train and the outside world. Two things are to be noted here.

1. The subgraph of a canonical graph represents an EGT if and only if the carrier of any gear pair within the subgraph is also a member of the subgraph. For example, the subgraph formed from the canonical graph of Fig. 3.3(a) by deleting vertices 0, 1, 2, 4, and 6 does not represent an EGT.

2. Some of the ports of communication at a first glance may not be obvious. For example, in the graph shown in Fig. 3.3(a), let vertex 1 represent the



(a) (b)

Figure 3.3 Canonical graph of an EGM with a binary carrier represented by vertex no. 6.

input link, vertex 2 the fixed link, and vertex 4 the output link. Consider the subgraph formed by deleting vertices 0, 1, 2, 4, 5, and 7. It appears as if the subgraph has only two ports of communication, namely 3 and 8. However, the carrier represented by vertex 6 is also a port of communication. This is because vertex 6 is the transfer vertex of the geared pair connecting vertices 7 and 8, and vertex 7 is external to the subgraph. Therefore, the subgraph has actually three ports of communication. One may note that the removal of vertex 6 from the graph of Fig 3.3(a) does change the dof of the mechanism.

From the above observations we will derive several conditions that the canonical graph of an EGM must satisfy. These conditions are necessary, but not

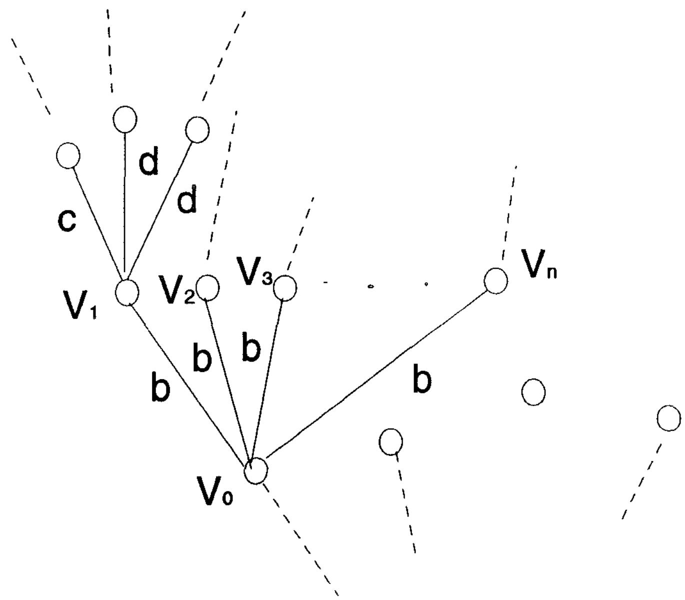


Figure 3.4 A tree obtained by removing the geared edges from a canonical graph

sufficient. The importance of these conditions lies in the fact that they simplify the enumeration procedure to be described in the next chapter, and drastically reduce the number of graphs with redundant links. The remaining few can be weeded out by inspection or by a methodology described in the next chapter.

Condition 1:

Consider the branches of a tree of a canonical graph emanating from vertex V_0 as shown in Fig. 3.4. Let the level immediately above and arising from V_0 contain n vertices V_1,$V_2 \ldots V_n$ that belong to one family, i.e. they are connected to V_0 with thin edges of the same label. Let the branches emanating from these vertices have N_1,$N_2 \ldots N_n$ vertices, respectively. Now the branches emanating from vertex V_1, along with V_1, form a subgraph that represents an EGT, because a geared edge joining any two vertex of the subgraph will have its transfer vertex contained in the subgraph. Let all such subgraphs be named G_1,$G_2 \ldots G_n$, and let them have F_1,$F_2 \ldots F_n$ degrees-of-freedom, respectively. Then G_i will have N_i - F_i geared edges and will require at least $F_i + 2$ ports of communication. One port of communication is the vertex V_i itself. The rest of the ports of communication will communicate through gear edges with any of the vertices V_0 to V_n other than V_i. A geared edge coming from any port of G_i cannot connect to any vertex other than V_0 to V_n of the EGM.

Next consider the subgraph G_0 formed by vertices V_0 to V_n and the branches emanating from V_1 to V_n. The minimum number of geared edges that G_0 should have in order that none of the links represented by vertices in the subgraphs G_1,$G_2 \ldots G_n$, is rendered redundant can be calculated by summing the number of geared edges in each subgraph, and the number of geared edges required by each subgraph to maintain the minimum number of ports of communication. Thus the number of geared edges in G_0 should be at least

$$\sum_{i=1}^{n}(N_i + 1 - F_i - 1) + \sum_{i=1}^{n}(F_i + 1) = \sum_{i=1}^{n} N_i + n \tag{3.2}$$

The number of geared edges in a subgraph representing an EGT must be less than the number of vertices by 2 or more, otherwise it will be locked. Since G_0 has $\sum_{i=1}^{n} N_i + n + 1$ vertices, this condition can not be satisfied for the above case. Therefore, not all the members of a family can give rise to branches. If one of the members does not give rise to any branch, then the number of vertices in

G_0 will be $\sum_{i=1}^{n-1} N_i + n + 1$ and the minimum number of geared edges required to prevent redundancy will be at least $\sum_{i=1}^{n-1} N_i + n - 1$. Thus, if one of the members does not give rise to any branch, the above condition is satisfied. Also, if more than one member of a family does not give rise to any branch the above condition can be satisfied.

Let V_1,$V_2 \ldots V_n$ represent the first level vertices, and vertex V_0 represents the root of the EGM. Since an EGM has exactly two degrees of freedom, the number of geared edges must be less than the number of vertices by 3. Applying this condition and following the above logic one can prove that there must be at least two vertices in the first level which should not give rise to any branch.

Thus, we get the following two conditions.

C9: For those vertices located at the higher levels, there must be at least one member in a family that does not give rise to any branch.

C10: There must be at least two vertices in the first level that do not give rise to any branch.

As a special case of Condition 1, a vertex cannot give rise to any branch if it is the only vertex in the family.

Condition 2:

The subgraph formed by vertices 3, 6, and 4 in Fig. 3.5 represents a one-dof EGT. Hence it should have at least three ports of communication. But it has only two ports of communication, namely vertices 3 and 4, and the removal of vertex 6 does not change the dof of the mechanism. Therefore, vertex 6 represents a link that is redundant and can be removed. This is generally true for any link that is not used as the input, output or fixed link and is connected by only one revolute joint and one gear joint. Thus,

C11: If a vertex is not located at the first level and is incident by only one thin edge, then it must be incident by at least two geared edges.

C12: If a vertex is located at the first level and is incident by only one thin edge, it must be incident by at least one geared edge.

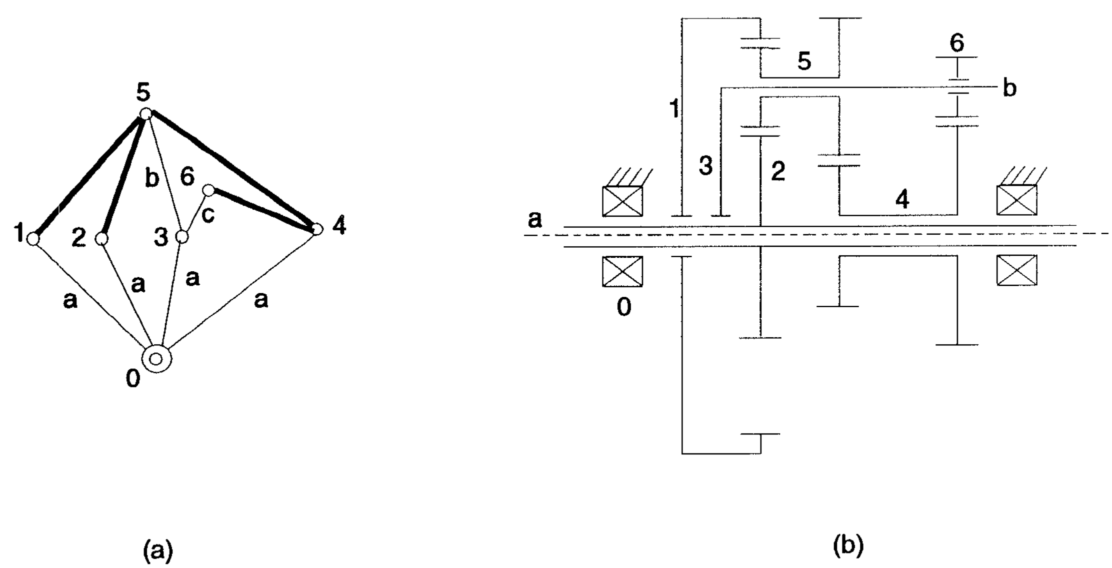


Figure 3.5 : (a) A canonical graph containing a vertex (vertex no. 6) representing a redundant link, (b) functional representation of the mechanism.

Condition 3:

Consider the graph shown in Fig. 3.6(a). The subgraph formed by vertices 3, 5, 6, 7 and 8 represents a two-dof EGT. However, it has only three ports of communication, namely 3, 5, and 7. Therefore, the links represented by vertices 6 and 8 are redundant. Note that the vertex 6 represents a binary vertex that is not connected to any other vertex by a geared edge. According to Condition C9, a binary vertex of this kind can only occur at the penultimate level of a branch. If the vertex at the higher level that is connected to the binary vertex is incident by two geared edges, then there are two possible ways of connecting them. These two ways are shown in Figs. 3.6(a) and (c). In both these cases the binary vertex, and the higher level vertex that is incident to it are redundant. In Fig. 3.6(c) links represented by vertices 5 and 7 are redundant because the subgraph formed by vertices 1, 5, 6 and 7 represents a one-dof EGT that has only two ports of communication. Thus,

C13: If a binary vertex is not at the first level then of the two vertices that it connects, the higher level vertex must be incident by more than two geared edges.

Fig. 3.3 shows the graph of an EGM that has a binary vertex, but no redundant links. In this case the higher level vertex (vertex No. 8) that is connected to the binary vertex 6 is incident by three geared edges.

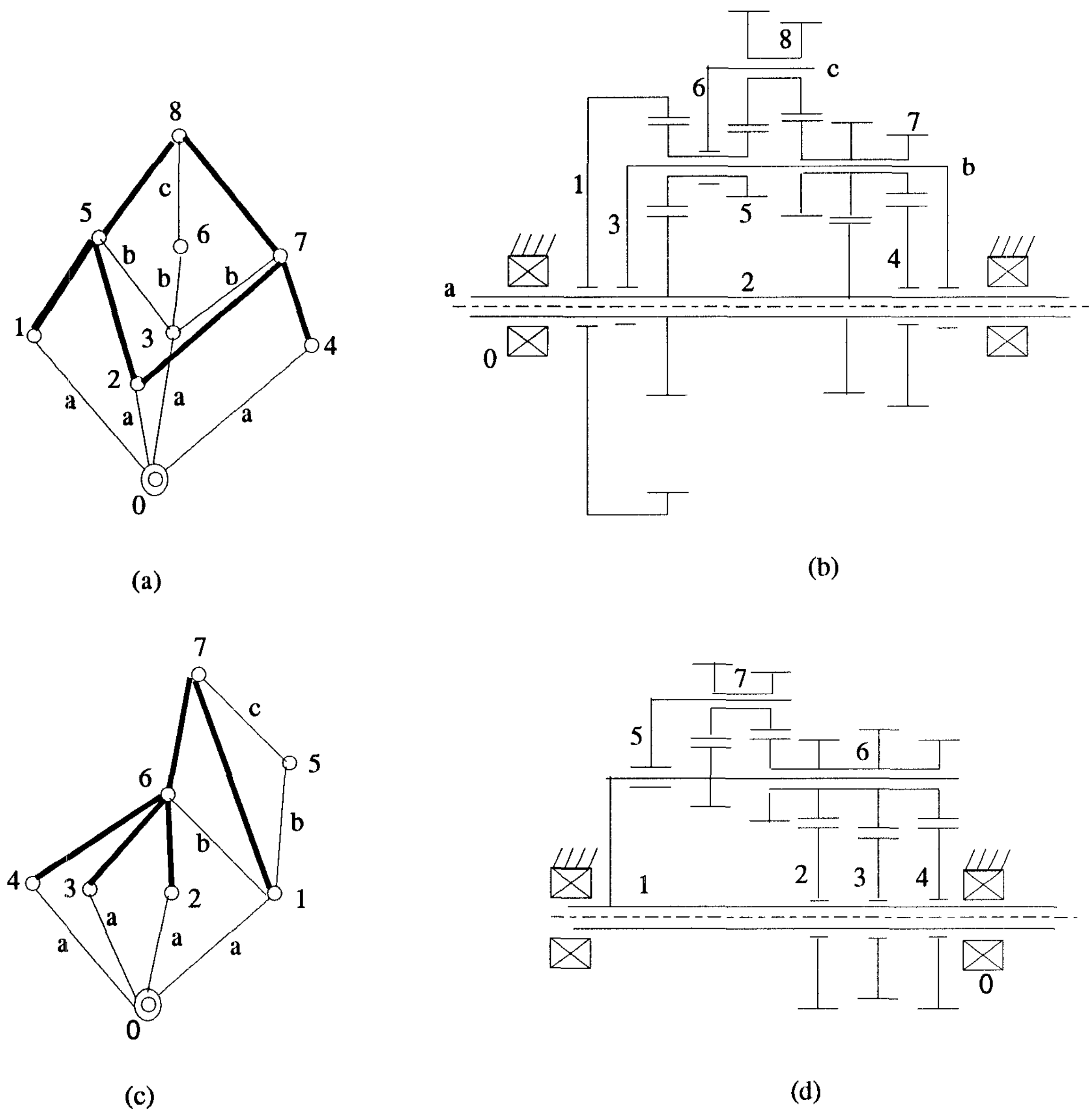


Figure 3.6(a) A canonical graph containing vertices (No. 6 & 8) representing redundant links, (b) functional representation of the mechanism.

Chapter 4

Graph Enumeration

Most combinatorial enumeration procedures such as the graph enumeration procedures that require the enumeration of all possible solutions satisfying certain constraints, are done through the process of generating and testing. The procedure is thus divided into two parts (Hayes-Roth et al., 1983): a generator of all possible solutions and a tester that selects only those solutions that meet the constraints. For example, in order to generate canonical graphs of n vertices that represent fractionated two-dof freedom EGMs, we could first generate all the graphs representing one-dof EGTs with $n-1$ links. Then from Eq. 3.1 we could calculate the minimum number of coaxial links required. Using it as a criterion some of the graphs of one-dof freedom EGTs could be pruned. The remaining graphs could then be converted into canonical graphs by using the vertices representing the coaxial links as the first level vertices and adding an extra vertex as the root to represent the casing. Finally, we could have chosen those canonical graphs that satisfy the characteristics C9 to C12 mentioned in Chapter 3. This method was used by Lin and Tsai(1989) for the creation of robotic wrist mechanisms.

The advantage of the above process is that we don't have to think of any new generating method since more than one method for enumerating graphs of EGTs exist. However, only graphs with up to 7 vertices for EGTs have been enumerated (Tsai, 1987; Kim and Kwak, 1991). Also the method used by Kim and Kwak does not generate graphs in which geared edges form a closed loop. This section presents a method to enumerate canonical graphs directly. The direct enumeration of canonical graphs is inherently more efficient because of the following two reasons.

1. The canonical graph has a unique vertex - the root, with reference to which other vertices are divided into several levels. Thus, there is already some arrangement among the vertices. One can therefore think of obtaining a unique arrangement of vertices by adding some rules. This can then be used to develop a unique code for each graph that will serve as a reliable tool for an isomorphism test.

2. As mentioned in Section 3.1, the canonical graph by requiring thin edges of the same label to be arranged in a particular manner prevents the creation of pseudo-isomorphic graphs.

The enumeration procedure described below uses a hierarchial generating and testing technique. A part of the canonical graph is generated at each step, and a test is carried out to prune out those solutions that will not give rise to canonical graphs with the desired characteristics. The generator in any graph enumeration procedure always gives rise to isomorphic solutions. Therefore, after completion of each step isomorphic solutions must be identified and eliminated. This is done as mentioned above by developing a unique code for each solution and comparing them. An important issue in using a generating and testing technique is the distribution of knowledge between the generator and tester. The generator produces solutions satisfying some of the constraints. The tester then selects those solutions that satisfy the rest of the constraints. By distribution of knowledge we mean the division of constraints between the tester and the generator. Usually, putting more knowledge in the generator results in a more efficient procedure.

The following observations are made before formulating an efficient enumeration procedure.

1. Most of the characteristics described in Chapter 3 are applicable to the tree of a canonical graph, rather than the canonical graph as a whole.

2. The definition of the canonical graph requires the edge labels to be distributed in a particular way.

3. Characteristic C4, which prescribes the allowable geared edge connections, presumes the existence of a labeled tree.

Therefore, it follows that an efficient enumeration procedure can be achieved if it is divided into two phases. In the first phase labeled trees that will give rise to admissible canonical graphs are enumerated. In the second phase geared edges are added to these trees to create the canonical graphs. Each of these phases has various steps. These two phases are described in Sections 4.1 and 4.3, respectively. Section 4.2 lists the procedure for identifying isomorphic graphs. Though the isomorphism test is carried out at the end of each step involved in the two phases, the procedure for it is described in a separate section to show how it relates to the overall enumeration procedure.

4.1 Enumeration of Trees

The characteristics C1, C2, C5 to C8, C9 and C10 are used to formulate the procedure for enumerating trees with n vertices.

Step 1. From C7 calculate the minimum number of coaxial links required. This gives the minimum number of vertices that a tree should have at the first level.

Step 2. Distribute the remaining vertices into various levels. While making such a distribution, remember that all levels other than the highest level must have at least 2 vertices in order to satisfy C9.

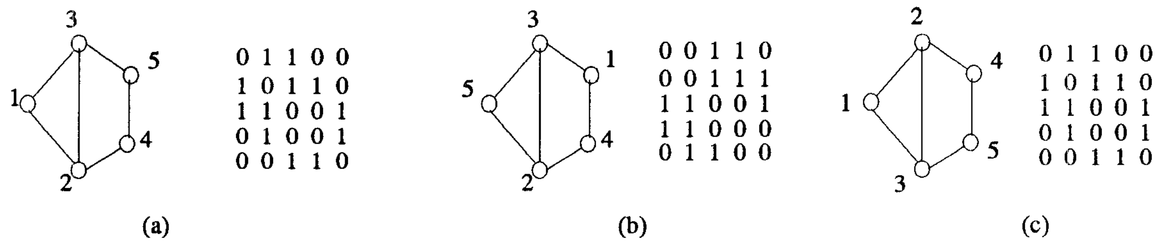
Step 3. Divide the vertices at each level into families (see Section 3.1). At all levels other than the highest level there must be at least one family with two or more members in order to satisfy C9. One must note that the distribution of vertices into families is same as the problem of partitioning of integers. An algorithm to this end can be easily developed by using the concept of generating functions, explained in the book by Liu (1968).

Step 4. Start adding the vertices of the first level. According to C4 all the vertices at the first level should be connected to the root with thin edges of the same label.

Step 5. Next connect the vertices of the second level to the first level vertices. According to C10, two of the vertices at the first level should not give rise to any branch. The definition of a canonical graph requires the members of the same family to be connected to the same lower level vertex by edges of the same label. Therefore, while adding the second-level vertices, add one family at a time. Start with the family that has the lagest number of members. If a family of vertices is added in all possible ways a lot of isomorphic graphs would be generated. To reduce the number of isomorphic graphs we digress and introduce the concept of graph automorphism and similar vertices.

4.1.1 Graph Automorphism

Consider the graph shown in Fig. 4.1. The edges of the graph are unlabeled and its vertices are numbered. If we permute the numbering of the vertices,



Note : The matrix beside each graph is the link to link adjacency matrix.

Figure 4.1 Graphs in (a) and (b) are isomorphic but not automorphic, Graphs in (a) and (c) are automorphic.

isomorphic graphs are produced. Most of these isomorphic graphs have their corresponding vertices numbered differently. However, some specific permutations produce graphs whose corresponding vertices bear the same number as the original one. These graphs are called *automorphic graphs.* For example, if we number the vertices 1, 2, 3, 4, and 5 of the graph in Fig. 4.1(a) as 1, 3, 2, 5, and 4, the resulting graph as shown in Fig. 4.1(c) is automorphic. The permutation in this case is denoted by (1)(2,3)(4,5). Elements 2 and 3 are said to form a cycle of length 2, and element 1 a cycle of length 1. All such permutations that produce automorphic graphs form a group (Liu, 1968). Each of these permutations is refered to as a *member* of the group. The application of any of the members from the group won't alter the adjacency matrix of the graph in any way. If two vertices p and q are contained in the same cycle of any member of such a permutation group, then vertices p and q are said to be similar (Yan and Hwang, 1991). Thus, one can divide the vertices of a graph into classes by putting the similar vertices together. If one wants to add a particular property to any vertex, then there can be one choice from a class of similar vertices since any choice is as good as the other. However, once the property has been assigned to a vertex, the similarity among vertices is destroyed. The members of the permutation group, which contain that vertex (to which the property has been assigned) in cycles of lengths greater than 1, has to be removed from the group since their applications no longer produce automorphic graphs. Therefore, one has to derive new classes of similar vertices from the remaining members in the permutation group. This method has been given in the paper by Yan and Hwang (1991) where they used it to assign various properties to links and joints of mechanisms.

In our case, all the edges of a tree are labeled and the labeling is arbitrary, i.e., if we change all the edges with label a to label b and vice-versa, the structural topology of a mechanism won't change. Therefore, after applying a permutation to the vertices, we also have to permute the edge labels in all possible ways and check if the resulting adjacency matrix becomes identical with the original one. If it does, then we can identify similar vertices from the permutation applied. However, this method of finding permutation group and similar vertices is time consuming. The addition of a family or families with the same number of members will not only require the reorganization of existing vertices into classes, but also the creation of permutation groups for the vertices being added.

Therefore, some very simple rules are prescribed here to identify some of the similar vertices. These rules won't prevent the generation of isomorphic graphs completely, but will reduce their number drastically. These rules are

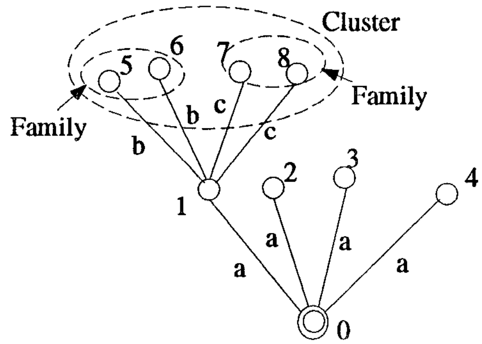


Figure 4.2 A tree showing similar vertices.

S1: Vertices belonging to the same family and incident by only one thin edge and no other edges are similar. For example, vertices 2, 3 and 4 in Fig. 4.2 are similar.

S2: Families that have the same number of members and are connected to the same lower level vertex form a *cluster*. If none of the vertices in a cluster is incident by more than one thin edge, then all the vertices in the cluster are similar. For example, vertices 5, 6, 7 and 8 in Fig. 4.2 are similar. If a vertex is incident by more than one edge, then the family of vertices to which that vertex belongs becomes dissimilar, while the rest of the families in the cluster remain similar.

Whenever a choice for a vertex is to be made for adding a vertex, or a family of vertices, choose only one vertex out of a class of similar vertices.

The addition of vertices at the higher levels should proceed in the same way as the second level vertices. The only difference is that C9 is applicable instead of C10, i.e., at least one member in each family of vertices should not give rise to any branches. A test to eliminate isomorphic graphs is carried out after completing the addition of vertices at each level.

The example given below demonstrates the above enumeration method. In this example, trees that can give rise to admissible canonical graphs which represent 9-link EGMs, capable of providing four speed reductions, are enumerated.

Step 1. There are nine links. The minimum number of coaxial links required is four. Therefore, the first level should have at least four vertices.

Step 2. The remaining vertices can be divided into levels as shown in Table 4.1(a).

Step 3. The vertices at each level are further divided into families as shown in Table 4.1(b).

Step 4. This step and the next one are demonstrated by choosing two distributions from Table 4.1(b). One of the distributions chosen is shown in Fig. 4.3(a). Fig. 4.3(b) shows the tree formed after addition of the first level vertices. The corresponding adjacency matrix is shown in Fig. 4.3(c). The adjacency matrix has all its elements zero except for those in the rows or columns corresponding to the root. Vertices 1, 2, 3 and 4 are similar according to S1.

Step 5. In the second level there is only one family with two members. There is only one choice of vertex in the first level to which this family can be connected since all the vertices at the first level are similar. The resulting

Table 4.1 Distribution of vertices of the graphs representing 9-link EGM into levels and families: (a) level distribution, (b) family distribution.

Level I	7	6	5	5	4	4	4
Level II	1	2	3	2	4	3	2
Level III				1		1	2

(a)

<table>
<tr><th>Levels</th><th>Families</th><th>Levels</th><th>Families</th><th>Levels</th><th>Families</th></tr>
<tr><td>6</td><td>6</td><td>5</td><td>5</td><td>5</td><td>5</td></tr>
<tr><td rowspan="3">2</td><td>2</td><td rowspan="3">3</td><td>3</td><td rowspan="3">2</td><td rowspan="3">2</td></tr>
<tr><td rowspan="2">1,1</td><td>2,1</td></tr>
<tr><td>1,1,1</td></tr>
<tr><td></td><td></td><td></td><td></td><td>1</td><td>1</td></tr>
</table>

<table>
<tr><th>Levels</th><th>Families</th><th>Levels</th><th>Families</th><th>Levels</th><th>Families</th></tr>
<tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td></tr>
<tr><td rowspan="5">4</td><td>4</td><td rowspan="3">3</td><td>3</td><td rowspan="3">2</td><td rowspan="3">2</td></tr>
<tr><td>2,2</td><td rowspan="2">2,1</td></tr>
<tr><td>3,1</td></tr>
<tr><td>2,1,1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="2">2</td><td>2</td></tr>
<tr><td>1,1,1,1</td><td>1,1</td></tr>
</table>

(b)

Level	0	I	II	III	
Family	1	4	2	1	1

(a)

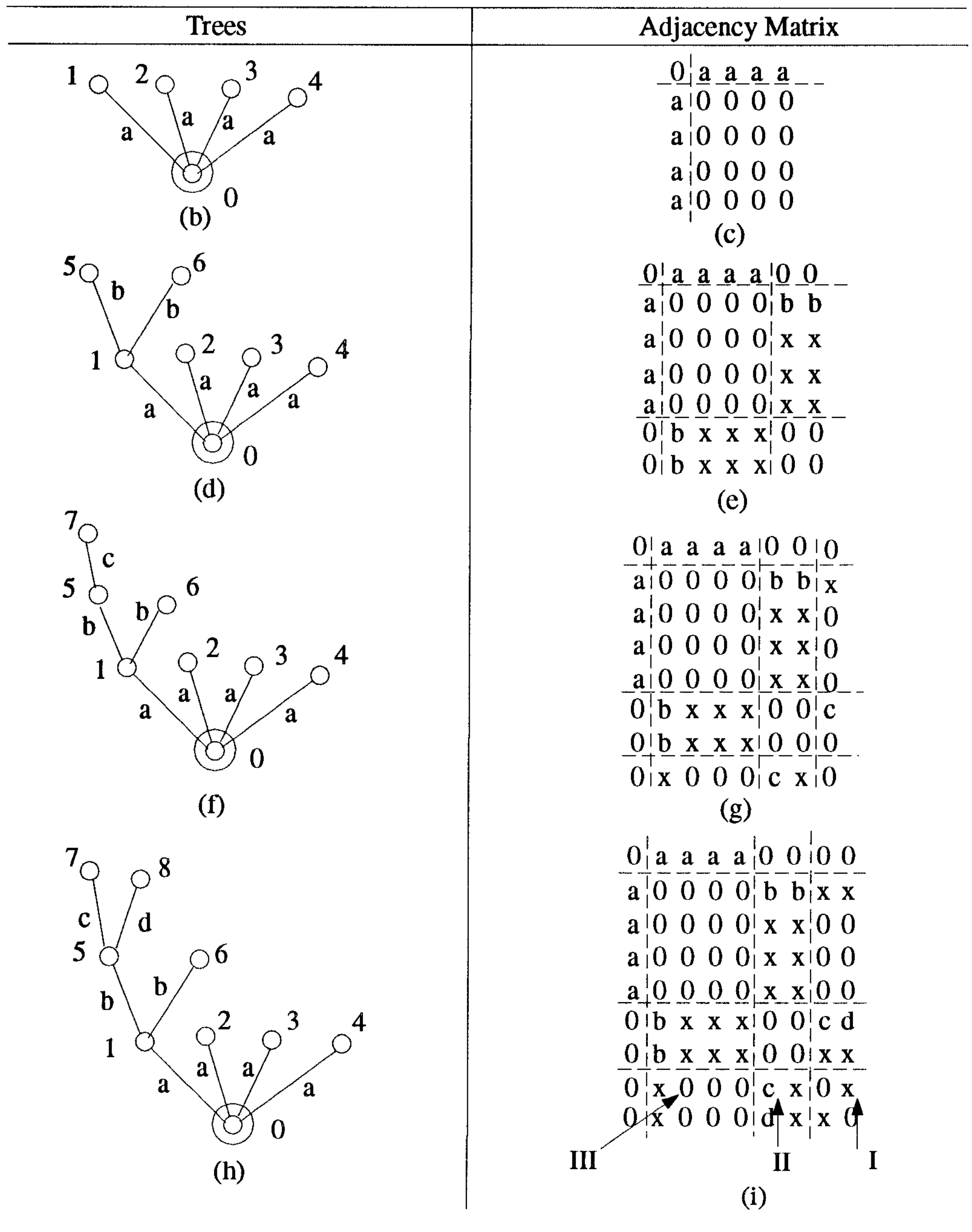


Figure 4.3 Enumeration of trees for a given family and level distribution: (a) level and family distribution, (b) and (c) addition of first level vertices, (d) and (e) addition of second level vertices, (f) - (i) addition of third level vertices.

tree is shown in Fig. 4.3(d). In terms of the adjacency matrix this would mean the addition of two rows and columns to the matrix of Fig. 4.3(c). The resulting matrix is shown in Fig. 4.3(e). All the elements of the new rows and columns are zero except for element number[1] 1. However, some of the zero labeled elements will be converted to g when geared edges are added. Potential geared edge connections can be found from C4. The corresponding elements, instead of being set to zero, are therefore set to x to facilitate the process of geared edges addition in the second phase. The elements A_{56} and A_{65} are set to zero because there can be no geared edge between vertices of the same family according to C4(a). All other elements are set to x since they satisfy C4(b). This completes the addition of vertices at the second level.

The third level contains two families each having one member. Therefore, any one of them can be added first. The first family can be added in only one way (see Fig. 4.3(f)) since vertices 5 and 6 are similar. The second family can also be added in only one way since one of the vertices at the second level cannot give rise to any branch according to C9. The dashed lines across the adjacency matrices divide them into various sub-matrices, each containing information about a particular type of interactions. For example, submatrix I in Fig. 4.3(i) represents the interaction within the

[1] We are following the convention of C programming language in indexing the elements of rows and columns. The indexing starts with the number 0.

third level vertices, submatrix II the interaction between the third and second level vertices, and submatrix III the interaction between the third and first level vertices. Whether an element of the submatrices I, II, and III can be converted into x or not can be determined by applying C4(a), C4(b), and C4(c), respectively.

A second chosen distribution is shown in Fig. 4.4. The process of enumeration of the trees from the distribution is illustrated in the figure itself. There are quite a few isomorphic trees. This is because of the presence of a large number of families having the same number of members.

Before ending this section on enumeration of trees we note that the generator generates only those trees that have the desired characteristics. The tester has only to identify the isomorphic graphs whose number has been reduced by incorporating some rules in the generator.

4.2 Isomorphism

The issue of developing unique code for identifying isomorphic graphs has been addressed by many authors (Tang and Liu, 1988; Ambedkar and Agrawal, 1987). Most of these papers dealt with graphs whose edges are not labeled. The labeling of edges of a graph has both its advantages and disadvantages. On one hand, it divides the vertices into classes that we have already named as families and, therefore, introduces some amount of ordering among the vertices. On the other

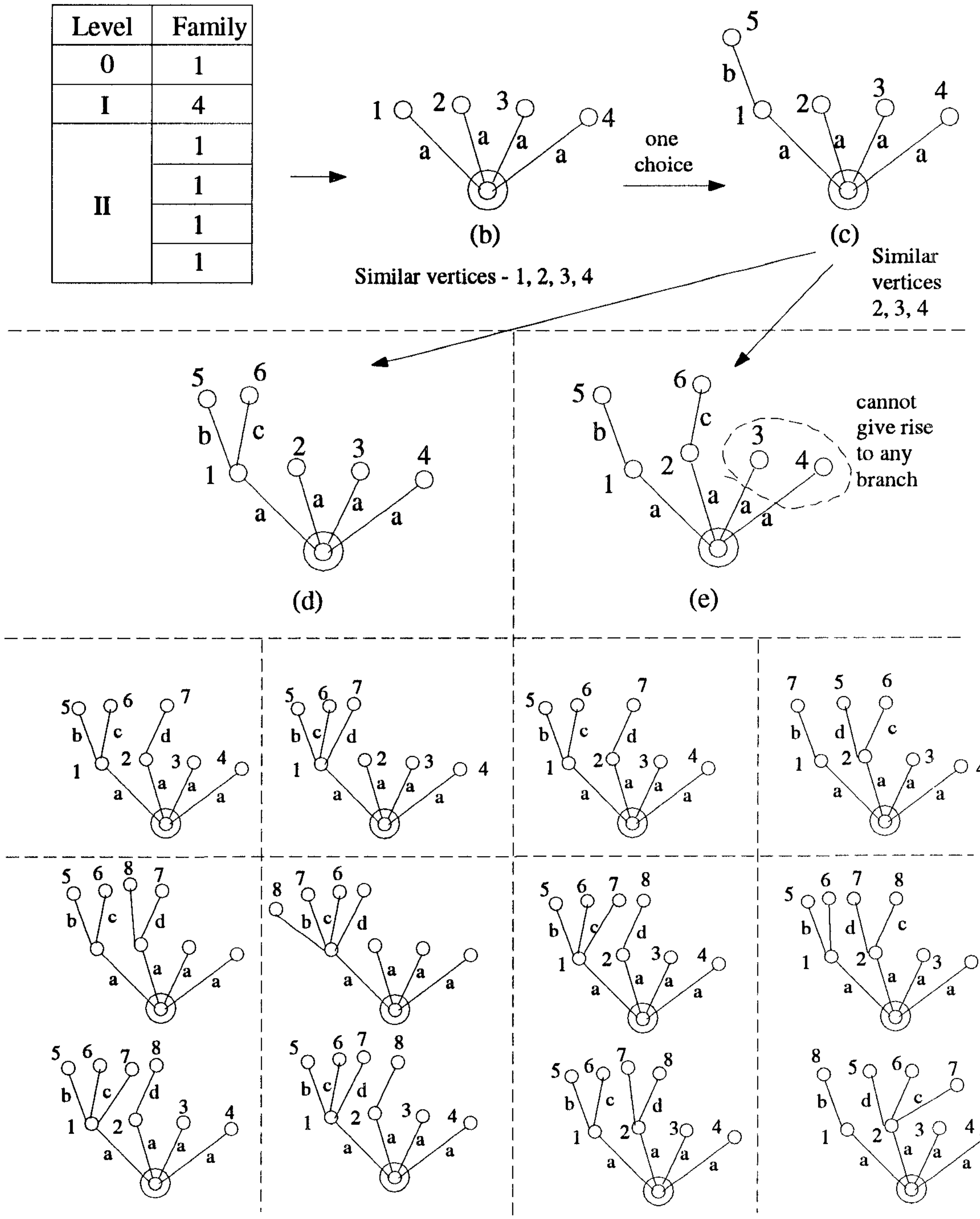
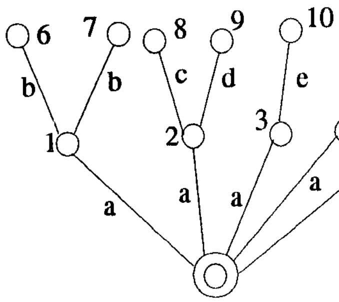


Figure 4.4 Enumeration of trees for a given family and level distribution.



(a)

Level	Family	Cluster	Priority
0	(0)	(0)	(0)
I	(1,2,3,4,5)	(1,2,3,4,5)	(1)(2)(3)(4,5)
II	(6,7)(8)(9)(10)	(6,7)(8,9)(10)	(6,7)(8,9)(10)

(b)

Figure 4.5 A tree having its vertices numbered according to their priority.

hand, since the labels being arbitrary they have to be permuted in all possible ways in order to detect isomorphism. Some papers have presented graph representations (Olson et al. 1991) that obviate the need to explicitly represent the labels of the revolute edges in the adjacency matrices. This thesis achieves the above objective by proposing four simple rules.

I1 Vertices at the lower levels should have higher priority than those at higher levels. For example, vertices 1, 2, 3, 4, and 5 in Fig. 4.5 have higher priority than vertices 6, 7, 8, 9, and 10. Hereafter, whenever we say that a vertex has a higher priority than another it means that the former is numbered lower than the latter.

I2 Members of a family such as vertices 1, 2, 3, 4, and 5 in Fig. 4.5, should be consecutively numbered.

I3 All members of the families that belong to the same level and have the same number of members should be consectively numbered. For example, vertices 8, 9, and 10 in Fig. 4.5 are consecutively numbered.

I4 Families that have more members, have higher priority than those having less. Vertices 6 and 7 in Fig. 4.5 have higher priority than vertices 8, 9 or 10.

If we number the vertices following the above rules, then there is no need to explicitly label the edges. One can uniquely determine the labels of the edges from the level and family distributions.

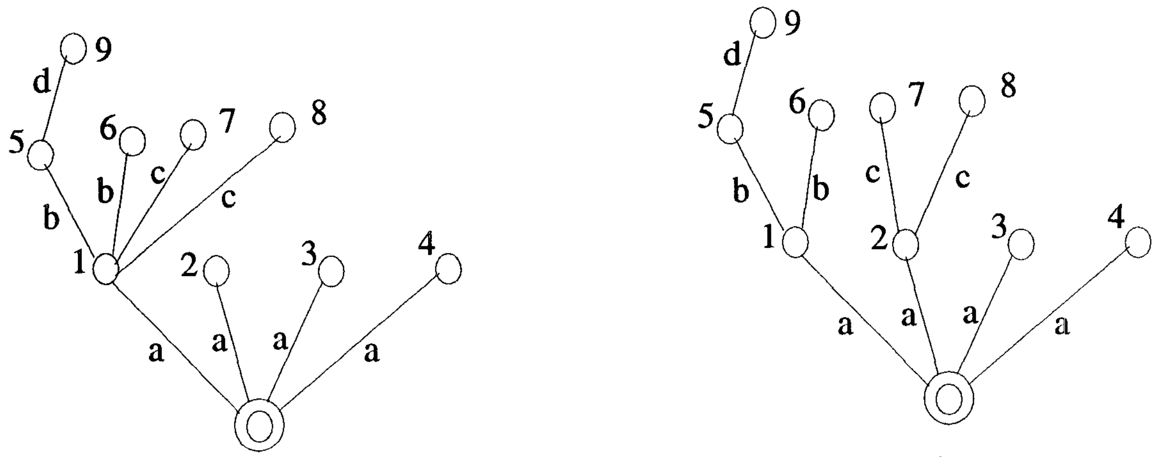
Next we propose a set of rules that decide the priorities of vertices within a family and the priority of a family of vertices over another having the same number of vertices. These rules should be applied successively in the order stated below and should not alter the priorities already decided by the application of previous rules. These rules are:

I5 Members of a cluster, e.g., vertices 8 and 9 in Fig. 4.5, should be consecutively numbered.

I6 If two clusters are of the same type[2], then the one with more families has a higher priority. For example, the cluster formed by vertices 8 and 9 in Fig. 4.5 has a higher priority than the cluster formed by vertex 10.

I7 A vertex in a family precedes another vertex if it gives rise to more families with a higher number of members. For example, in Fig. 4.5 vertex 1 precedes vertex 2, which in turn precedes vertex 3.

[2]Two clusters are of the same type if the number of members in a family of one is same as in that of the other.



(a) (b)

Figure 4.6 Trees having their vertices numbered in order of their priority.

I8 To determine the order of families in a cluster, the vertices of the families are compared. If the highest priority vertex of family A gives rise to more families with a higher number of members than that of family B, then A precedes B. If the precedence cannot be determined by comparing the highest priority vertices of the two families then the next priority vertices are compared. For example, vertices 5 and 6 precede vertices 7 and 8 in Fig. 4.6(a).

I9 The priority of a cluster over another is determined similarly. Vertices 5 and 6 precedes vertices 7 and 8 in Fig. 4.6(b).

To develop the code all labels of the thin edges are replaced by 1 in the adjacency matrix. Then, the vertices are permuted to maximize the number formed by concatenating the elements of the upper triangular matrix (of the adjacency matrix), starting from the leftmost element of the topmost row and moving along row by row downwards. This maximized number, along with the level and family distribution gives a unique code for isomorphic trees. Since the permutation of similar vertices does not change the adjacency matrix, only dissimilar vertices of a family that have the same priority are permuted. Families in a cluster that have the same priority and all of whose vertices are not similar are permuted en bloc. Similarly, clusters of same priority are permuted en bloc.

The isomorphism test is applied after completing the addition of vertices at each level. Rules I1 to I6 are applied to arrange the vertices in the level that has just been added, whereas rules I6 to I9 are applied to arrange the vertices in the immediately preceding level.

4.3 Enumeration of EGMs

The adjacency matrices that have been enumerated until now have some of their elements labeled x. The addition of geared edges means converting some of these x's to g's, and the rest to zeroes. The number of geared edges to be added can be calculated from C1. Before we proceed further in formulating a methodology for addition of geared edges, we describe an algorithm to find the transfer vertex associated with each gear pair, and study the interaction among fundamental circuits formed by the addition of geared edges.

4.3.1 Locating the Transfer Vertex

Consider the tree and its adjacency matrix shown in Fig. 4.7. Suppose, we connect a geared edge between two vertices 7 and 6. To find the associated transfer vertex we scan the row corresponding to the higher level vertex (vertex 7 in this case). If both the vertices are at the same level, then we scan the row corresponding to any of the vertices. The column number corresponding to the first non-zero element that is not an 'x' or a 'g' gives the number of the transfer vertex. In case of the above example element no. 5 of row 7 gives the number of the transfer vertex. A fundamental circuit is characterized by the two end vertices of a geared edge and the associated transfer vertex. Therefore, once the transfer vertex is known, the fundamental circuit is in effect known.

4.3.2 Interaction Among Fundamental Circuits

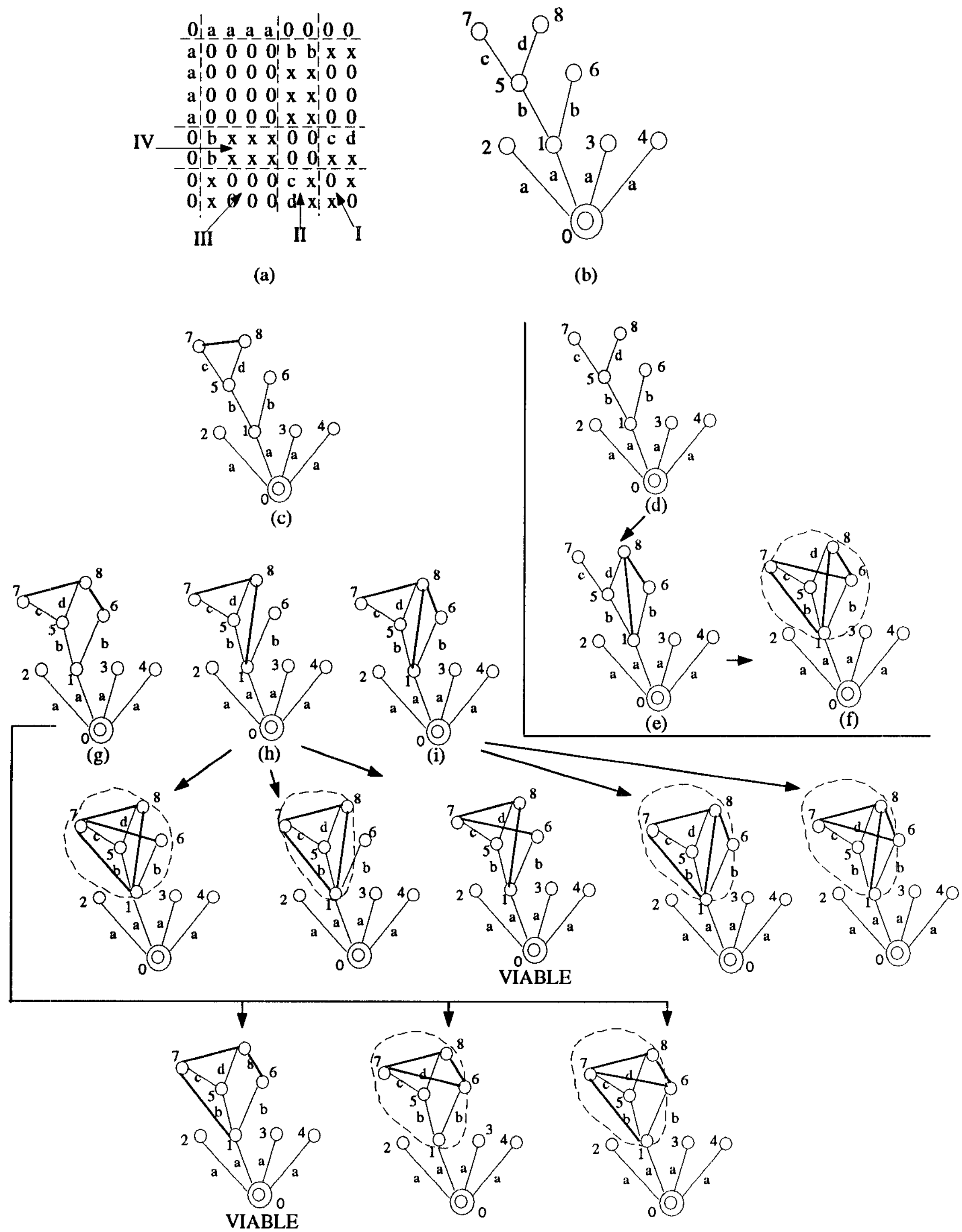
The three vertices that characterize a fundamental circuit form a simple one-dof EGT with three links. For example, if we connect the vertices 7 and 8 of the tree shown in Fig. 4.7(b) by a geared edge, then vertices 5, 7, and 8 that characterize the fundamental circuit, form a simple one-dof EGT. In order to keep track of interactions among fundamental circuit we construct a matrix whose column number corresponds to the vertex number. In the first row of this matrix we mark the elements that correspond to the characteristic vertices of a fundamental circuit by a label, say 1. The matrix is shown in Fig 4.8(a). When another geared edge is added, the newly formed fundamental circuit can have the following relationships with the existing one.

1. It can have two of its vertices in common with the existing fundamental circuit. The two fundamental circuits constitute a one-dof EGT. In the first row of the above matrix we mark the element corresponding to the non-common vertex with the same label as the other elements of the row. For example, in Fig. 4.7(e) the fundamental circuit formed by the addition of the geared edge connecting vertices 6 and 8 has two of its characteristic vertices (5 and 8) in common with the previous one. Therefore, the first row in the matrix is modified as shown in Fig. 4.8(b).

2. It can share only one of its vertices with the existing fundamental circuit. In this case they form a fractionated two-dof EGT with the common vertex as the articulation point. For this case we add a new row to the above matrix and label the elements corresponding to the characteristic vertices of the fundamental circuit that is being added with a new label, say 2.

In general if we consider a graph in which k geared edges have been added, then the fundamental circuit formed due to the addition of the $k+1$ geared edge can have the following relationships with the existing subgraphs.

1. It can have two of its vertices in common with a subgraph representing a one-dof EGT. In this case the resulting subgraph also represents a one-dof EGT.



Note : The vertices inside a dashed line envelope represent links that are locked.

Figure 4.7 Step by step enumeration of EGMs from a tree - Part I.

Vertices	1	2	3	4	5	6	7	8
					1		1	1

(a)

Vertices	1	2	3	4	5	6	7	8
					1	1	1	1

(b)

Figure 4.8 Matrices to keep track of the interaction among fundamental circuits.

2. It can have one of its vertices in common with a subgraph representing a one-dof EGT. The resulting subgraph forms a fractionated two-dof EGT with the common vertex as the articulation point.

3. It can have two vertices in common with a two-dof fractionated EGT, one vertex in common with each fraction. The resulting subgraph forms a two-dof non-fractionated EGT. In this case we also add a new row and mark the proper elements with a new label.

4. It can have three of its vertices in common with a subgraph representing two-dof EGT. The resulting subgraph forms a one-dof EGT. For such cases all the rows corresponding to the subgraph should be collapsed into one and all the elements in the row that corresponds to the subgraph should be marked with the same label.

5. A new fundamental circuit formed cannot have three of its vertices in common with a subgraph representing a one-dof EGT, otherwise the mechanism

will be locked.

The above observations will be used to keep a track of the subgraphs that are being formed, and to prevent the occurrence of locked chains. It is apparent that there can be more relationships between a newly added fundamental circuit and the existing subgraphs. However, if we follow the enumeration procedure described below and limit ourselves to a small number of links, say ten, no more relationships are necessary.

4.3.3 Similar Edges

Consider the tree shown in Fig. 4.7. The similar vertices in the tree can be identified by applying S1 and S2 as defined in Section 4.1.1. Because of the similarity among vertices some of the candidate geared edges (represented by label x in the adjacency matrix) are similar. Some of these sets of similar geared edges can be identified by the application of the rule given below.

S3 When several geared edges connect a common vertex to a set of similar vertices, they form a similar edge set.

4.3.4 Addition of Geared Edges

The methodology for addition of geared edges is introduced through an example. Consider again the tree shown in Fig. 4.7(b). The number of geared edges to be added is six.

Step 1. First add geared edges to connect vertices at the highest level. This means submatrix I of the adjacency matrix shown in Fig. 4.7(a) is under consideration. First add geared edges to one vertex, then to the next and so on. For this instance there are two provisions - one of adding a geared edge and the other of not adding any, as shown in Figs. 4.7(c) and (d), respectively.

Step 2. Next add geared edges that connect the highest level vertices to vertices at the lower levels, i.e., relabel the elements with label x in submatrices II and III. Since this is the last chance of adding geared edges to the highest level vertices, calculate the minimum number of geared edges to be added to these vertices to satisfy C11. Add the geared edges first to one vertex in all possible ways, starting from the minimum required to maximum possible. For example, in Fig. 4.7(d) vertex 8 should have a minimum of two geared edges incident on to it. The maximum number of geared edges that can be added to it is also two. Therefore, there can be only one way of adding the geared edges to vertex 8, which is shown in Fig. 4.7(e). Whenever a geared edge is added, check for the possibility of forming a locked chain following the methodology described previously. For example, addition of two geared edges to vertex 7 of the graph in Fig. 4.7(e) results in a locked chain as shown in Fig. 4.7(f). After the addition of geared edges to submatrices II and III is complete, a test of isomorphism is carried out before adding geared edges to submatrices IV, i.e., between the second and

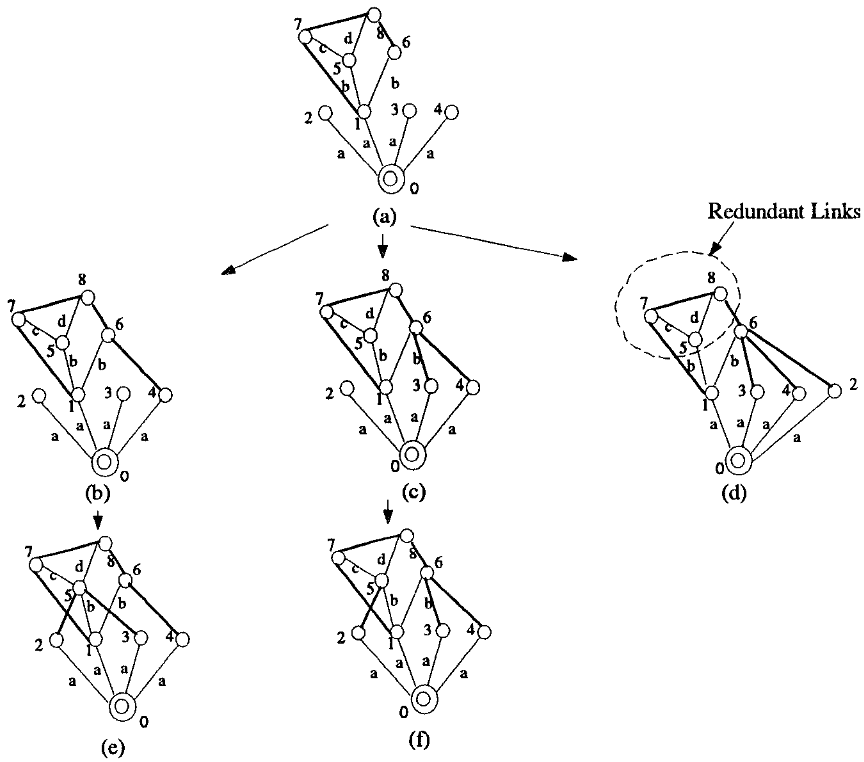


Figure 4.9 Step by step enumeration of EGMs from a tree - Part II.

first level vertices. The priority of vertices is decided according to some rules described later in this section under the heading isomorphism. After the isomorphism test only one viable gear train remains, which is redrawn in Fig. 4.9(a).

Step 3. Since, no geared edge connecting vertices at the second level is possible the next step is to add geared edges between the second and first level vertices. For this purpose the previous step is repeated. The difference is that the minimum number of geared edges to be added to the first level vertices is also to be taken into consideration. Also note that the geared edges that can be connected to vertex 6 are all similar according to S3.

Therefore, only three choices are possible as shown in Figs. 4.9(b), (c), and (d). At the end of enumeration procedure three EGMs are formed. They are shown in Fig. 4.9(d), (e), and (f). The one shown in Fig. 4.9(d) has redundant links. This is because the subgraph formed by vertices 1, 5, 6, 7 and 8 represents a one-dof EGT that has only two ports of communication, i.e., vertices 1 and 6. The generation of such graphs can be prevented if we ensure that every subgraph (formed at the end of step II) that represents an n-dof EGT has n + 2 ports of communication. The subgraphs and their dof can be obtained from the matrix that has been developed to prevent the occurrence of locked chains. However, such a verification is not required when geared edges are added to connect the highest level vertices to the lower level vertices, since conformation to C11 and C12 ensures that there will be no redundant links.

Thus, the method of adding geared edges can be formulated as follows.

Step I First add geared edges connecting vertices at the highest level. For every geared edge that is being added check whether the addition of the geared edge results in a locked chain by the method described above. If it does, then set the label x that corresponds to the geared edge in the adjacency matrix to zero.

Step II Next add geared edges from the highest level to the lower levels. Before doing this calculate the minimum number of geared edges to be added to each of the vertices at the highest level from C11. If the highest level is the second level, then the minimum number of geared edges to be incident on a first level vertex as given in C12 should also be taken into account. Care should be taken that the total number of geared edges to be added does not exceed that given by C1. As before, check for locked chains for every geared edge added.

Step III Repeat steps I and II for the next lower level vertices, i.e, the vertices that are at one level immediately below the highest level. The only thing to be checked, after completing the addition of a geared edge, is the presence of any redundant links as described in the above example.

Repeat step III until the second level is reached.

Isomorphism

A test to identify isomorphic graphs is performed at the end of each step. To do this we extend the procedure described in Section. 4.2. The rules to identify the priority of vertices are given below. They, however, should not alter the priority set by rules I1 to I9. Also, the rules should be applied in the order given below and should not alter the arrangement set by the previous rules.

I10 The vertex that is connected to vertices at two levels above it with more geared edges has the highest priority.

I11 Among vertices of same priority in a family, the vertex that is connected to vertices at one level above it with more geared edges is given higher priority.

I12 Among vertices of same priority in a family, the vertex that is connected to vertices at the same level with more geared edges is given higher priority.

I13 Among vertices of same priority in a family, the vertex that is connected to vertices at lower levels with more geared edges is given higher priority.

These rules are prescribed to ensure that the priority set due to the application of isomorphism test at the end of each step of geared edge addition, is not altered upon the application of the same test at the end of next step.

To develop a code for the graph, the g's in the adjacency matrix are replaced by 2's. Then, the vertices are permuted to maximize the number formed by concatenating the elements of the upper triangular matrix as described in page 60.

Chapter 5

Functional Representation

The graph representation of a mechanism helps in enumerating the basic configuration of a mechanism and stores all the relevant topological data. However, it does not aid the designer in visualizing how a mechanism looks and how it will work. This is a big handicap for those designers who are not familiar with graph representation, and can prevent a smooth flow from the type-synthesis phase to the dimensional synthesis phase. A search through the literature has shown that very little work has been done in the area of sketching the functional representation of a mechanism from its graph representation, especially in the area of EGTs.

The functional representation of an EGT, unlike those of other planar kinematic chains, has traditionally been sketched by using a sidewise orthographic projection. While sketching the functional schematic of an EGT the following points, some of which have been mentioned in the previous chapters, should be

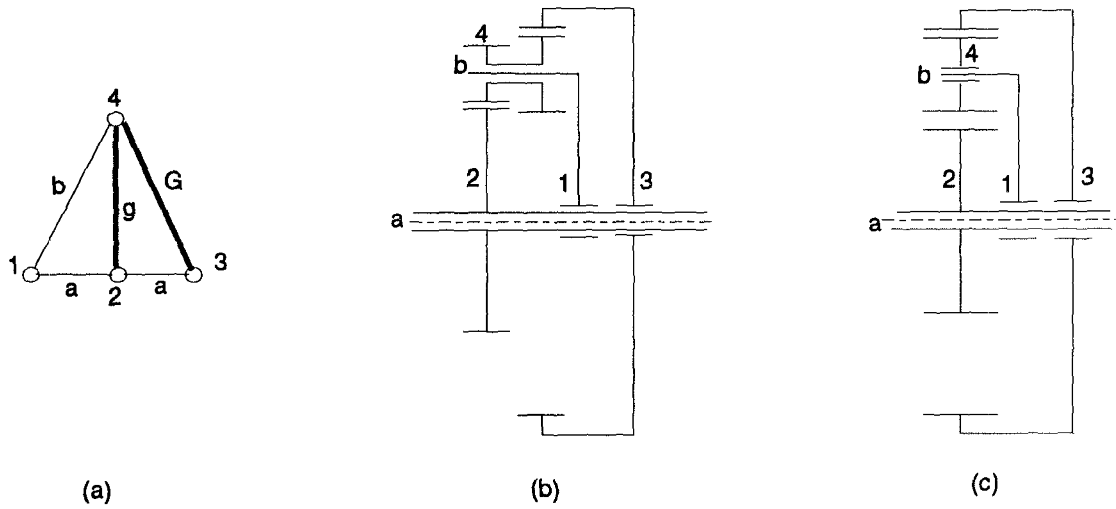


Figure 5.1 Epicyclic gear train of 3 links: (a) graph representation, (b) functional representation with three independent gear diameters, (c) functional representation with only two independent gear diameters.

kept in mind.

- There isn't any one-to-one correspondence between the graph of an EGT and its functional representation, because the graph does not store any information about the type of gear pairs.

- The coaxial links in an EGT actually form multiple joints. Therefore, the joints between a set of coaxial links can be arbitrarily rearranged without changing the functional characteristics of a mechanism.

- One of the desired features of any functional schematic of an EGT is the absence of any crossing (or interference) between links. However, Chieng and Hoeltzel (1990) observed that "this constraint alone cannot provide any control over the numerical solution of the problem and hence the problem is ill constrained".

- Since the vertices of a graph do not carry information about the shape of links, it is possible to draw structures that are less generalized, and put additional constraint on the dimensions of certain links. For example, the functional schematic shown in Figs. 5.1(b) and (c) has the same graph representation as that shown in Fig. 5.1(a). However the EGT of Fig. 5.1(c) must have all its gears of the same module and the diameters of the gears are not independent. Therefore, during dimensional synthesis the designer can choose only two parameters (diameters) for the mechanism shown in Fig. 5.1(c) while three can be chosen for the mechanism shown in Fig. 5.1(b).

Fig. 5.2(a) shows the canonical graph representation of the EGM shown in Fig. 5.2(b). It has been mentioned in Section 3.2.2 that all existing EGMs studied have their links distributed only up to the second level. This mechanism is no exception. Therefore in what follows we shall, for the purpose of sketching, restrict ourselves only to those graphs that have at most two levels of vertex distribution. In the following sections, first the desired features of a *good* functional representation of an EGM is discussed. Next, the interactions among the various types of links of an EGM are studied. A method of assigning the gear type (external or internal) to the gear edge is described. The graph of an EGM is decomposed into subgraphs having certain characteristics. We call mechanisms

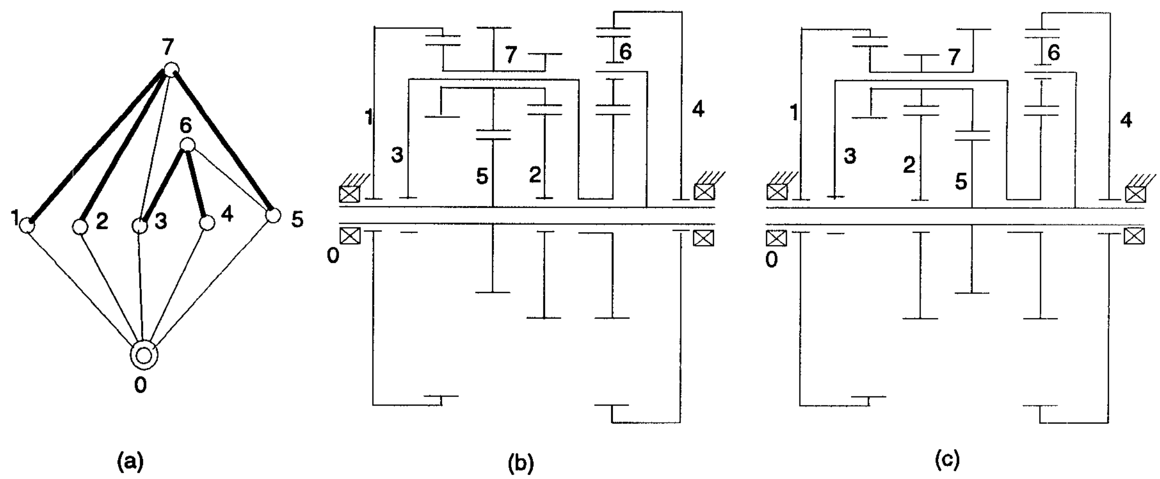
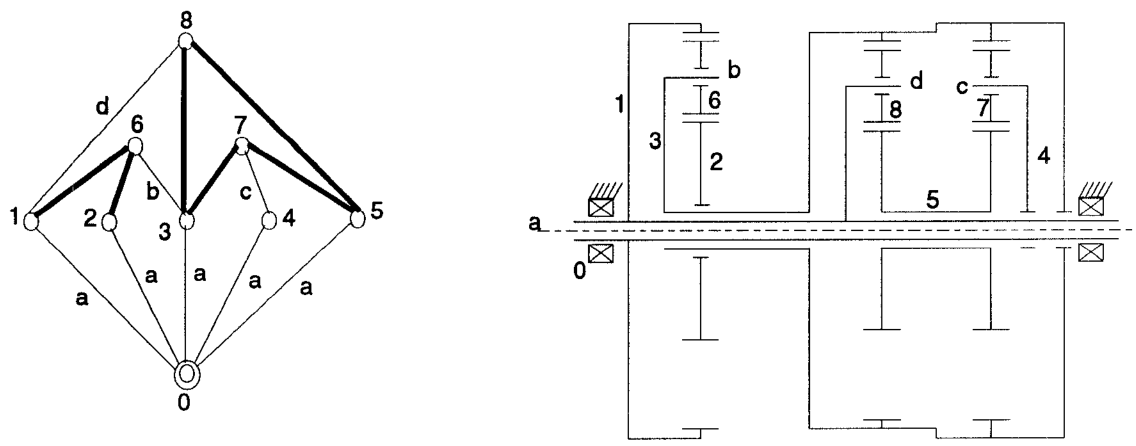


Figure 5.2 Epicyclic gear mechanism of 8 links: (a) canonical graph representation, (b) functional representation showing an inaccessible link, (c) functional representation having all links accessible.

represented by these subgraphs - *fundamental geared entities* (FGEs). Based on these, some primitives are developed which form the building blocks of the FGEs. Subsequently, a methodology for generating the functional schematic of an EGM by welding (joining) the primitives into an FGE and then the FGEs into an EGM is formulated. The chapter ends with a discussion on the data structure for storing the information regarding the functional schematic of an EGM, and a method of producing the same on the computer screen.

5.1 Desired Features

The absence of crossing links, which is an important requirement of an EGT, should be observed in sketching an EGM. In addition there are other desired



(a) (b)

Figure 5.3 Epicyclic gear mechanism of 8 link: (a) graph representation, (b) functional representation showing coaxial shafts and overhead connection.

features specific to the functional representation of an EGM that are enumerated below.

Accessibility - The first level links of an EGM are used as input, output or fixed links. The link chosen as the output link is to be permanently connected to the final reduction unit of a transmission. The other links are connected either to the input shaft through rotating clutches or to the casing through band clutches, depending on the clutching condition. Therefore, it is necessary that, in the functional schematic of an EGM, these links be arranged in such a way that they remain accessible and hence can be connected to other elements of the transmission as, and when, required. For example, link 2 in the mechanism shown in Fig. 5.2(b) is inaccessible. It won't be possible to attach any clutch to this link. However, if the joints among the coaxial links are rearranged as shown in Fig. 5.2(c), the accessibility is not impaired.

Few Coaxial Shafts - Link No. 1 of the functional schematic shown in Fig. 5.3(b) has a carrier and a ring gear connected[1] to each other by a shaft. This shaft is coaxial with another shaft, which connects the carrier and the ring gear of link 3. From a manufacturing point of view coaxial shafts are undesirable as tolerance limits on such shafts are too rigid. Therefore, their number should be kept as low as possible. This may result in a compromise with some other respect as explained in what follows .

Low Inertia - Link No. 3 of the functional schematic shown in Fig. 5.3(b) has two ring gears attached edge to edge. Such connection will be referred to as overhead connection. This way one can reduce the number of coaxial shafts. However, this results in a high moment of inertia of the link. It also requires more material in its manufacture. Thus one has to strike a balance between the two desirable features. In Section 5.5 a method of maintaining a proper balance between coaxial shafts and overhead connection is described. For an EGM of n links the maximum number of such connections (coaxial shaft connections and overhead connections) is equal to the greatest even number

[1]The difference between the words *connections* and *joints* as used here are: two or more members such as gears or carriers are said to be *connected* when they form a link with no relative motion between them, whereas two links are said to be *joined* when they can have specific relative motion with respect to each other.

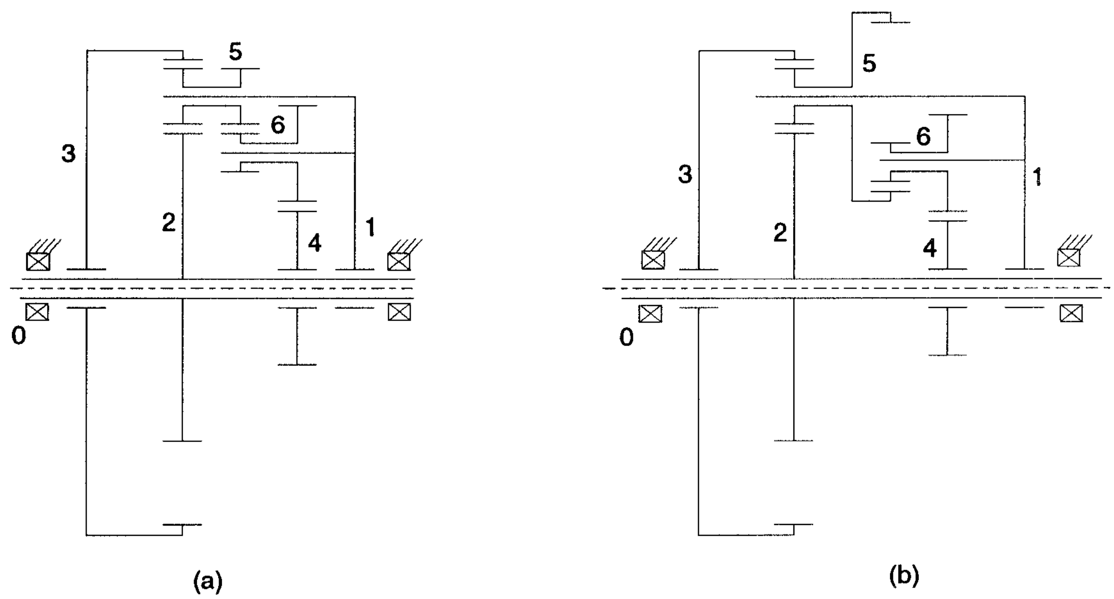


Figure 5.4 Epicyclic gear mechanism of 7 links having (a) external planet gear pair, (b) internal planet gear pair.

that is less than or equal to $(n-5)$. The proof of this statement is given in Appendix A.

It should be noted that the accessibility of a link is a more desirable feature than the other two. Therefore, the latter two are overruled whenever the accessibility of a link is in question.

5.2 Assignment of gear type

The second level vertices of a canonical graph represent planet gears or multiple planet gears of an EGM. A gear edge connecting a second-level vertex to a first-

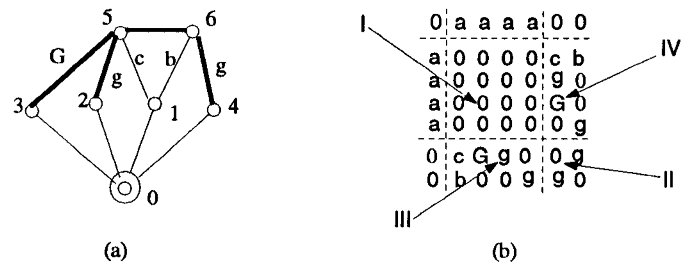


Figure 5.5 : (a) Canonical Graph Representation of the Epicyclic Gear Mechanism shown in Fig. 5.4(a), (b) Link-to-Link Adjacency Matrix of (a).

level vertex represents a planet-to-sun or planet-to-ring gear pair depending on whether the gear pair is external or internal. A geared edge connecting two vertices of the second level represents a planet-to-planet interaction. Hence, we will call the gear edge connecting two such vertices as *planet-to-planet gear edge*. Theoretically a gear pair between two planets can be either external or internal. For example, the mechanism of Fig. 5.4(a) has the same features as that of Fig. 5.4(b) except the joint connecting the two planet gears has an external gear mesh in Fig. 5.4(a) and of the internal gear mesh in Fig. 5.4(b). However, for all the EGMs studied, only external gear meshes between planets have been found. Therefore to keep the sketching algorithm simple, we shall assume that a geared-edge connecting two vertices at the second level represents an external gear pair. In the adjacency matrix, an internal gear pair will be represented by a upper case G and an external gear pair will be represented by a lower case g. Fig. 5.5(b) shows the adjacency matrix for the canonical graph representation shown in

Fig. 5.5(a)). If we remove the first row and the first column corresponding to the root, we can divide the remaining matrix into four sub-matrices marked I, II, III, and IV in Fig. 5.5(b) according to the vertex level distributions. Since the adjacency matrix is symmetrical, the sub-matrices III and IV are transposes of each other. All elements in the sub-matrix I are zero, since there can be no connections among the first level vertices. The sub-matrix II gives the interaction among the second level vertices. The sub-matrix III represents the interaction between the first and the second level vertices. Therefore, only those elements of sub-matrix III that have the label g can be reassigned the label G. Of course, corresponding changes have to be made in sub-matrix IV.

After the assignments of internal and external gear pairs, some of the graphs generated will be isomorphic. To check for isomorphism we can extend the procedure developed in the previous chapter. We can add one more rule to the rules of numbering the verices by assigning the vertex with the larger number of external gear edges to have higher priority.

5.3 Fundamental Geared Entities

In this section, we shall divide the graph of an EGM into several FGEs. This is accomplished by using the planet gears as the dividing elements. The planet gears can either be a rigid link by itself or several planets connected together as a planet train. From the graph point of view, the second-level vertices either

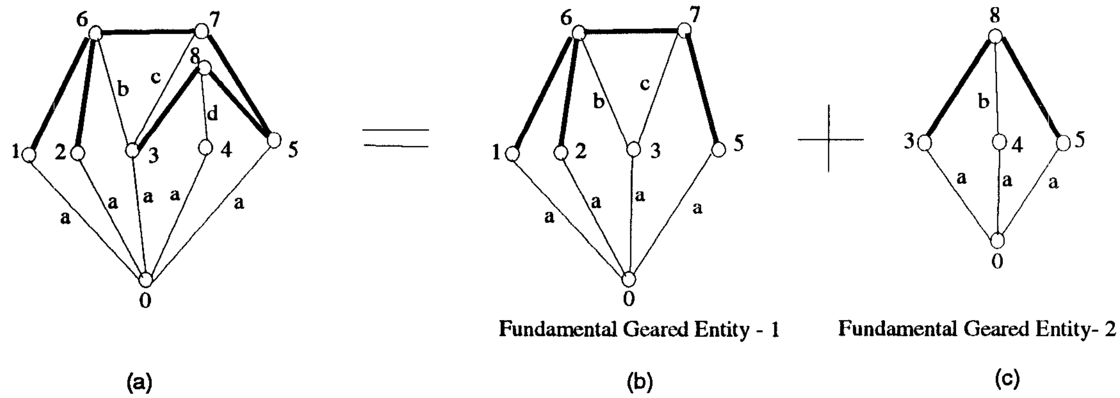


Figure 5.6 : (a) Canonical graph of an EGM, (b,c) subgraphs representing fundamental geared entities.

become isolated or form several second-level vertex-chains after the removal of all the lower level vertices. For example, in the graph shown in Fig. 5.6(a) vertices 6 and 7, along with the geared edge connecting, them forms a double planet train, while vertex 8 represents a single planet. The mechanism represented by a subgraph formed by an isolated second-level vertex or chain of a second-level vertices, and all the lower level vertices connected to them by geared or revolute edges, is called a *fundamental geared entity.* The links corresponding to the vertices of a second-level vertex-chain do not interact directly with those of the others. Figs. 5.6(b) and (c) show two subgraphs formed from the graph of Fig. 5.6(a), each of which represents an FGE. Each of these subgraphs is associated with a second-level isolated vertex (or a vertex-chain), and is formed by deleting all the other second-level vertices or vertex-chain and the first-level vertices that are not directly connected to the second-level vertex (or vertex chain) under consideration. While drawing the functional representation of an

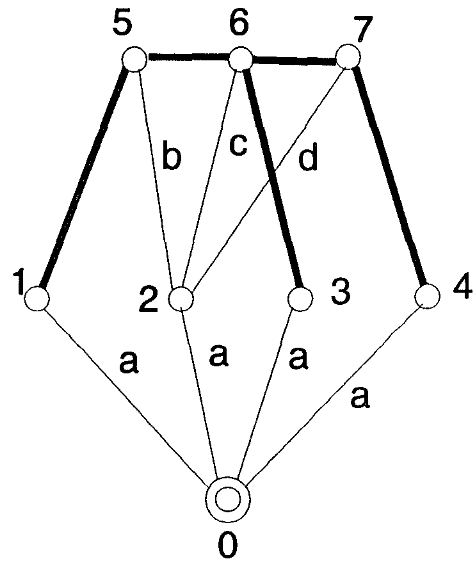
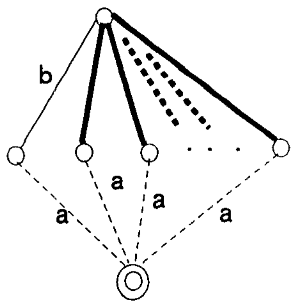


Figure 5.7 Graph of an FGE having three planets in a chain.

EGM, *one should draw those links belonging to one FGE of an EGM together and then join the common links of the FGEs by means of coaxial shafts or overhead connections.*

Fig. 5.7 shows an FGE having three planets. Note that this FGE can itself serve as an EGM since it has four links at the first level. FGEs and EGMs that have more than two planets in a chain are not very practical and will be excluded from further consideration. In terms of the adjacency matrix of an EGM (see Fig. 5.5(b)), this means that at most one element in each row of sub-matrix II can be labeled g.

The FGEs can now be categorized into two general types, one containing only one planet and the other containing two meshing planets. Their graphs are shown in Fig. 5.8(a) and Fig. 5.9(a), respectively. The general form of a *single-planet FGE* as it appears in the functional schematic of a mechanism is



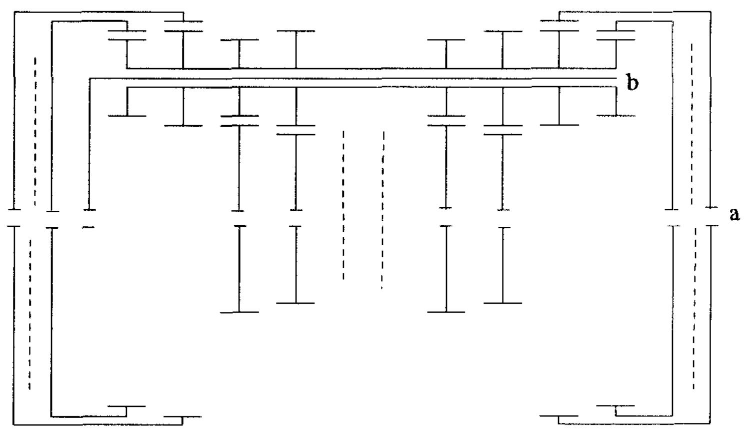


Figure 5.8 General Form of Single-Planet FGE.

shown in Fig. 5.8(b). The ring gears can occur only at either end of the FGE. The general form of the functional schematic of a *double-planet FGE* is shown in Fig. 5.9(b). It appears to be complicated because of the presence of a ring gear in the middle of the FGE. If there is no sun gear meshed with the planet at the higher label (axis c), then the ring gear can be drawn as shown in Fig. 5.9(c). Thus, there are two general forms of the second-type FGEs. One may note that the restriction we have imposed on the position of the ring gears (joined to the planet at its lower label) as shown in Fig. 5.9(b) is artificial. Clearly, one can also place the ring gears at the end by offsetting the lower label (axis *b*) of the carrier (see Fig. 5.9(d)). This, however, leads to too many hidden lines in a 2D representation and therefore is avoided. Such an artificial restriction however may not permit the achievement of the desired features (of functional representation) in some cases.

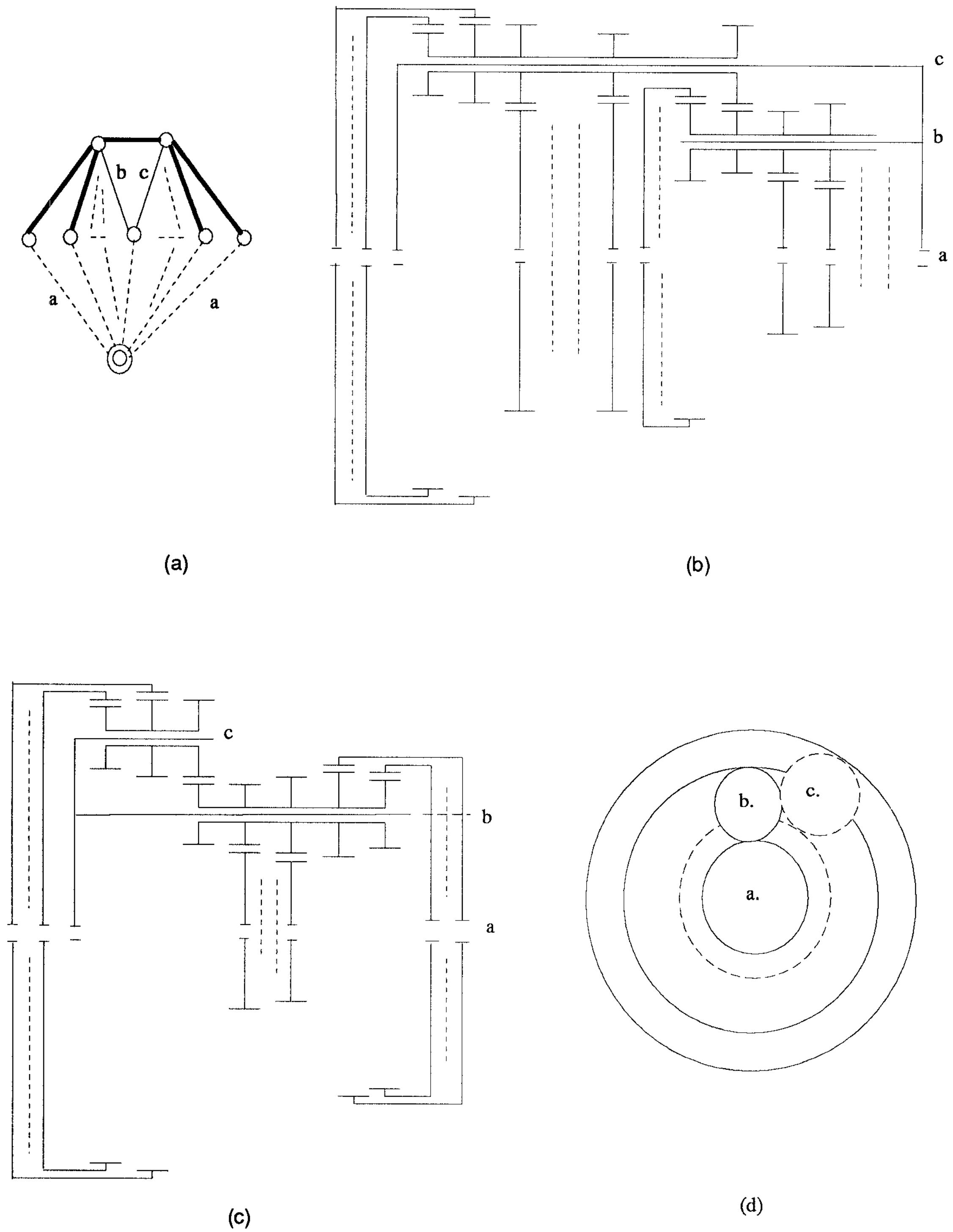


Figure 5.9 General Forms of Double-Planet FGE.

5.4 Primitives

An FGE consists of only one carrier, which carries the planet(s) that interact with the sun and ring gear of the first level. Therefore, it is natural that the functional schematic of an FGE can be sketched by using four primitives : one each for depicting the sun, the ring, the planet and the carrier. The dimensions that characterize these primitives are decided by the computer based on the arrangement of the primitives in the FGE. A methodology to decide the arrangement of the primitives is described in a Section 5.5. However, since the computer does not have any aesthetic sense, it will be given some proportions to begin with. These are described under specific primitives in the following sections.

5.4.1 Carrier

The carrier primitive is shown in Fig. 5.10(a). All the planets in an FGE are to be supported on the higher level axes of a carrier. The height of the axis b from the center of the carrier (line a) is introduced as a constant in the computer. We will call this constant the *reference value*. For the case of a double planet, the ratio of the distance between two higher level axes b and c, to the reference value is also defined. Whenever an axis corresponding to a new label is to be created the computer creates a variation about the reference value to define its height. The number of axes, the options of whether the carrier should face left or right, and the other parameters are decided by the program. The coordinates of the numbered points 1, 2, 3 etc. as shown in Fig. 5.10(a) are expressed as functions of the reference parameter with respect to the coordinates of the local origin of the primitive.

5.4.2 Sun Gear

The sun gear primitive is shown in Fig. 5.10(b). The parameters characterizing the primitive are shown in the same figure. Some of these parameters are defined as constants in the program as they don't affect any of the desirable features of the functional representation. They are defined in proportion to the above mentioned reference value. These parameters are the *face width* of the gear, the *gap between the teeth* of two meshing gear, the *hub* width and the *nominal hub diameter*. Another proportion that needs to be defined is the ratio of the radius of a sun gear to that of the meshing planet. The computer always uses a pre-specified ratio and creates a small variation about this ratio while sketching the structure. In the graph of an FGE this primitive appears as a first level vertex incident by a geared edge labelled g.

5.4.3 Ring Gear

The ring gear primitive and the parameters characterizing it are shown in Fig. 5.10(c). Some parameters defined for the previous primitive also apply here. In addition, the *gear radius*, the *ratio of arm radius to gear radius*, and the *nominal edge span* as a proportion to the reference value are also defined. The computer, how-

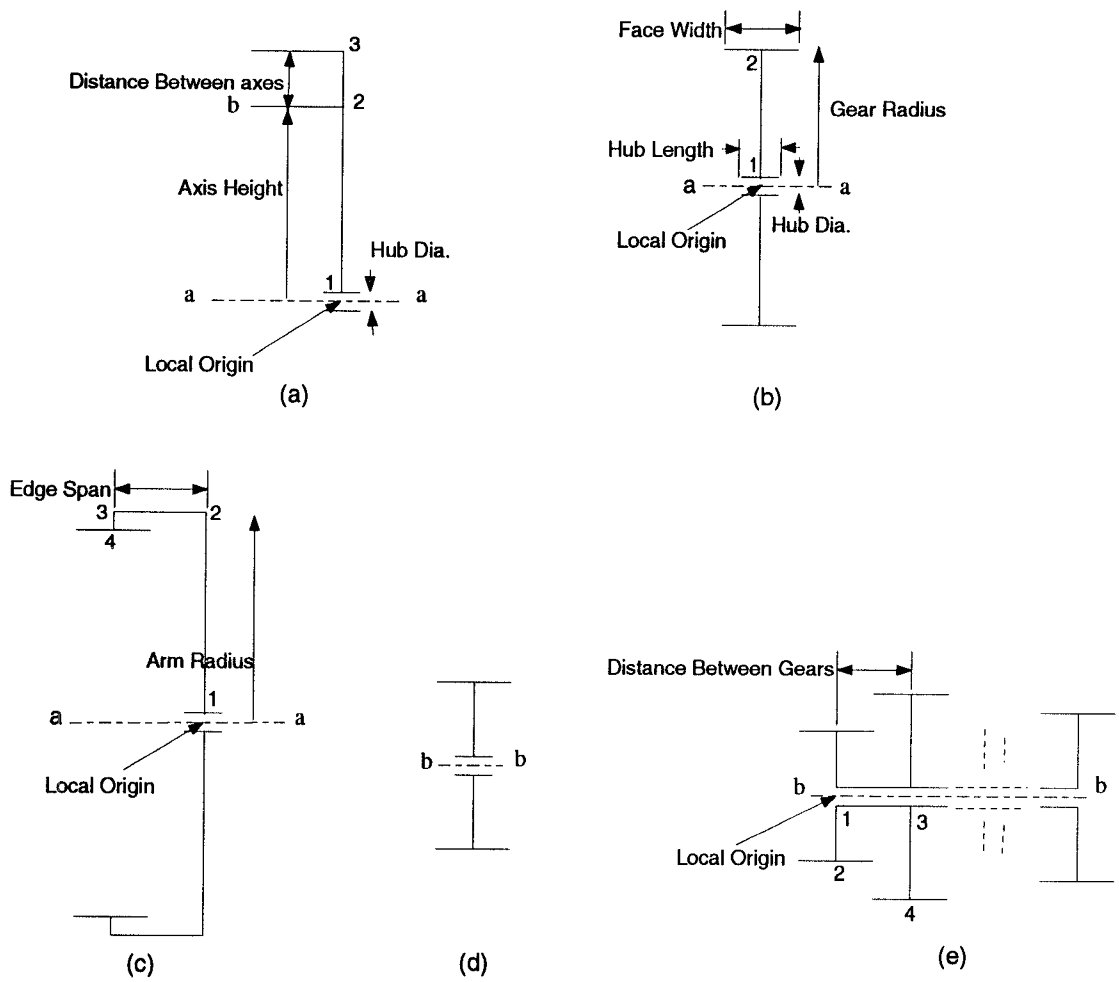


Figure 5.10 Primitives: (a)carrier, (b) sun gear, (c) ring gear, (d) single-planet gear, (e) multiple planet gear.

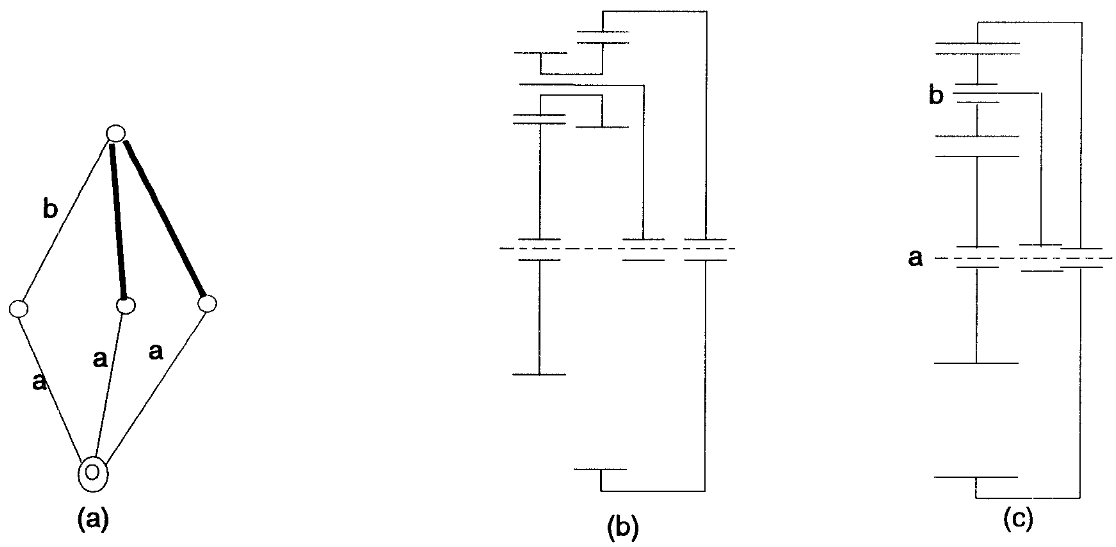


Figure 5.11 Simple lanetary ear et

ever, may override these specifications under certain circumstances that will be described later. In the case of a ring-planet pair the program also has to decide the orientation of the primitive.

5.4.4 Planet Gear

The planet consists of one or more gears attached to each other side by side as shown in Fig. 5.10(d) and (e). In the graph representation of an FGE it appears as a second-level vertex or vertex chain. The number of gears in the planet is determined by the number of geared edges incident on the vertex. In this case the distance between planet gears are defined in proportion to the reference value.

5.4.5 Simple Planetary Gear Set

Figure 5.11(a) shows the graph of a simple planetary gear set. A simple planet FGE consisting of only a ring gear, a sun gear, and a carrier as shown in Fig. 5.11(b). The structure can be made more compact as shown in Fig. 5.11(c), and is called a simple planetary set. It is introduced as an additional primitive because most automotive automatic transmission gear-trains contain at least one such set. Thus, whenever a subgraph is of the form shown in Fig. 5.11(a), the primitive shown in Figs. 5.11(b) or 5.11(c) will be used to sketch its functional representation. The parameters for this primitive are not illustrated separately because they are the same as those shown in Figs. 5.10(a), (b) and (c), except that the sum of the radii of two planets and a sun gear is equal to that of the ring gear.

5.5 Connecting the FGEs

The remaining task now is to find the sequence of links in the first level of an FGE so that when the common links of two or more FGEs are connected there is no crossing of links. It has already been mentioned that the common links of the FGEs can be connected either by shafts or by an overhead connection. This results in many possible types of connections and link arrangements. Therefore, to reduce the number of choices we will forego the overhead connections. One may argue that this will lead to too many coaxial shafts, and hence, one of the desirable features of a functional schematic will not be achieved. To achieve the desirable features we will present a method of converting some of the coaxial shafts to overhead connections later.

The general form of a single planet FGE is again shown in Fig. 5.12(a). This time certain points along the axis of the FGE are labeled with letters. These points are those that can be connected by shafts to similar points of other FGEs. Such points will be called *welding* points. The left most or the right most ring gear can have two welding points: one by coaxial shafts and the other by an overhead connection as shown by the dashed lines labeled as (pqtR) in Fig. 5.12. Such a construction is permissible because it does not hamper the accessibility to any other points. In the same manner the carrier can also have two welding points: one on the left-hand-side and the other on the right-hand-side (line mc) of the shaft as shown in Fig. 5.12(a). Therefore, the welding points along the axis can be represented as R r r .. c .. s s .. c' .. r r R', where the r's represent the welding point of the ring gears, the c's that of the carrier and the s's that of the sun gears. The R represents the ring gear that has two welding points. Simalarly, the welding points of the two double planets FGEs shown in Fig. 5.12(b) and (c) can be represented as R r r .. c .. s s c' .. r r c" R' and R r r .. c .. s s s ..r r .. c' .. s s c" R', respectively.

The link numbers corresponding to sun gears joined to the same planet of an FGE can be permuted among each other because of symmetry. The same thing can be done for the ring gears that are joined to the same planet. Fig. 5.13(a) shows a canonical graph with geared edges properly labeled. It can be divided

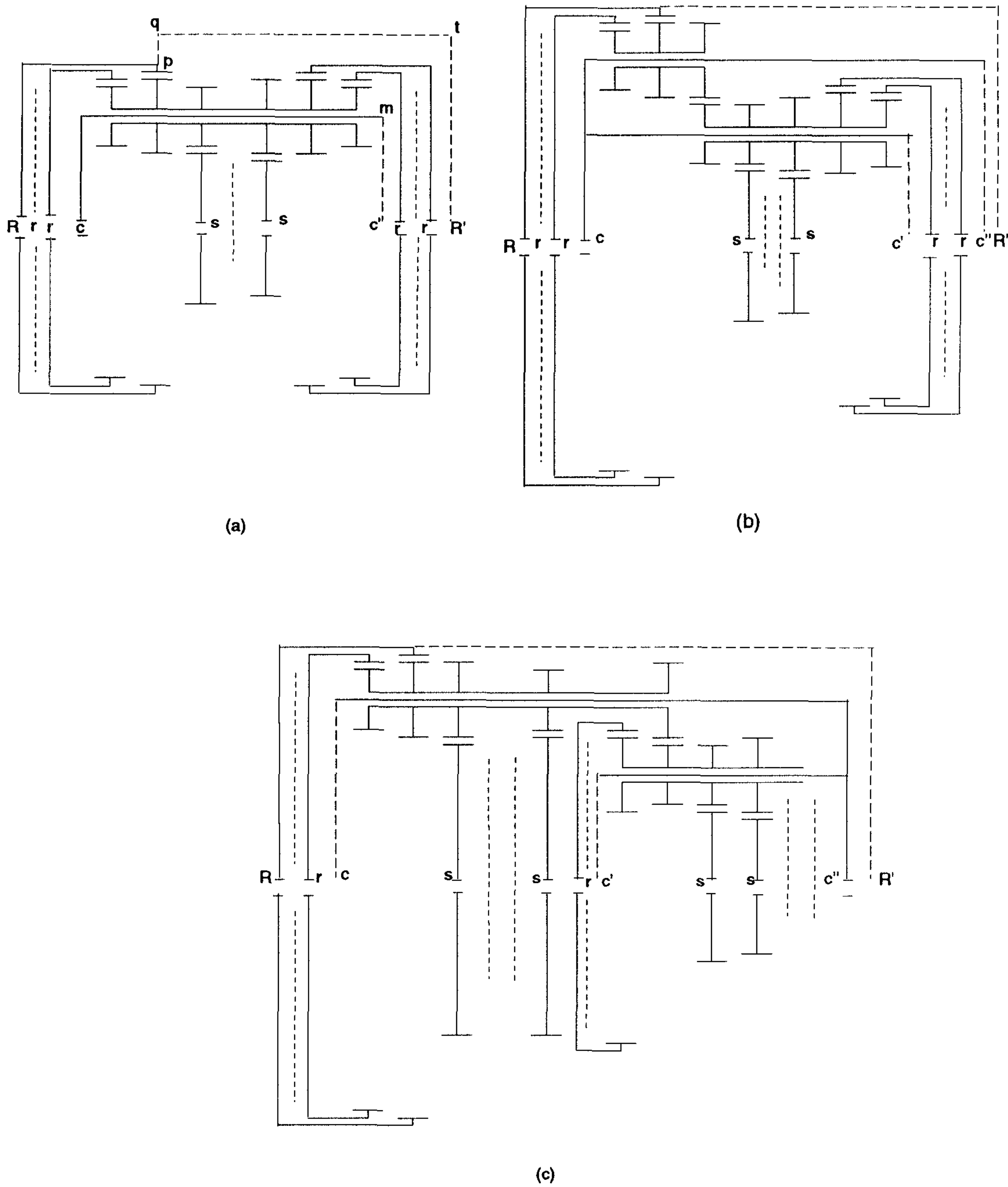


Figure 5.12 Welding points of (a) single-planet FGE, (b) and (c) double-planet FGE.

into three subgraphs, each of which represents an FGE as shown in Fig. 5.13(b). Each FGE consists of only one carrier, one ring gear, and one sun gear. Each ring gear and carrier has two welding points. The welding points of the FGEs can be arranged graphically as shown in Fig. 5.13(c). The welding points are labeled by the number of the links. The welding points belonging to the same FGE are joined by edges, which are called *primary edges*. Edges connecting two welding points of two different FGEs are called *secondary edges*. A secondary edge can only connect two welding points of the same label. The label of a secondary edge is the same as those of its end points. A secondary edge between two points represents a shaft connection between the two links. The secondary edges should be drawn according to the following rules.

M1 The secondary edges should be contained in the half space below the line *ab* formed by the primary edges as shown in Fig. 5.13(d). This restriction is necessary because we are only considering shaft connections.

M2 Any two welding points of the same label should be connected by a path consisting of either secondary edges of the same label or several primary edges or a combination of both.

M3 There should not be a circuit formed by only secondary edges of the same label and primary edges.

M4 A secondary edge should not intersect or coincide with another secondary or primary edge except at its end points.

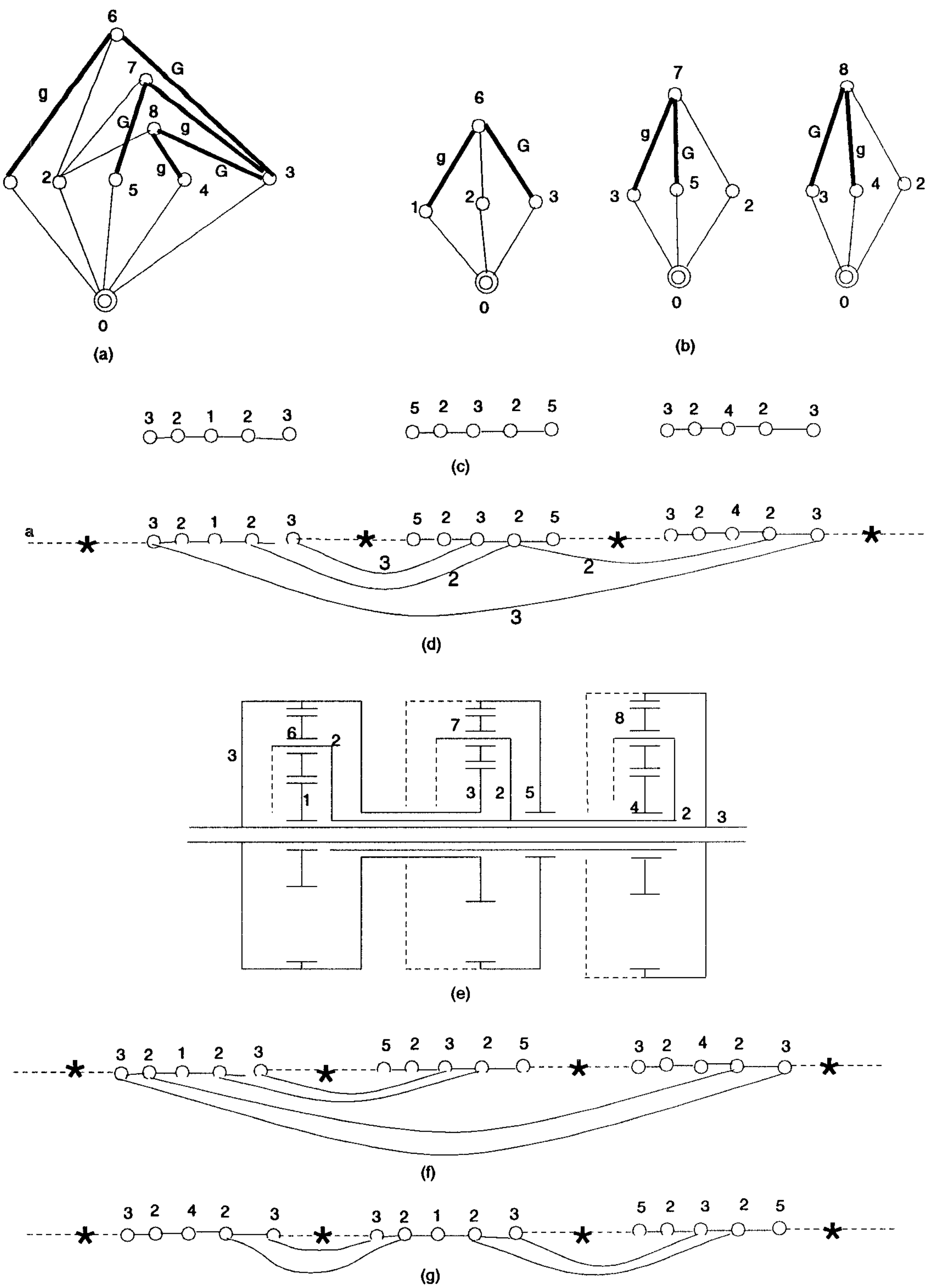


Figure 5.13 Constructing the Kinematic Structure from Primitives.

After all the above rules are satisfied, a test for accessibility of links should be carried out. A welding point in Fig. 5.13(d) is accessible if one can draw a secondary edge from that welding point to any one of the points marked by an asterisk without tracing on or crossing any edge. Otherwise, the point is inaccessible. A link is inacessible if all its welding points are inaccessible. For example, the way the secondary edges has been drawn in Fig. 5.13(d), welding point 1 is inaccessible. Fig. 5.13(e) shows the corresponding connections in the mechanism. It is evident that link 1 is indeed inaccessible. However, if one connects the FGEs in the way shown in Fig. 5.13(f) instead, link 1 becomes accessible. Although this serves the purpose, a pair of connections between two non-adjacent FGEs means two long coaxial shafts are needed. This should be avoided, if possible. In this case, it can be avoided by arranging the FGEs as shown in Fig. 5.13(g).

Based on the above discussion the following algorithm is proposed to find a proper arrangement of the FGEs and that of the links within the FGEs. The first two steps describe how the first FGE should be constructed. The rest of the steps describe how the subsequent FGEs should be constructed.

Step 1 Draw the welding points of an FGE that has only two of its links connected to other units. There is at least one such unit in any EGM. If there is more than one such unit, choose any.

Step 2 For each link that is connected to other FGEs choose one welding point. Place these points adjacent to each other, otherwise the welding point that lies between becomes inaccessible. The only welding point that can be trapped between is the one belonging to the carrier, provided the other one is accessible.

Step 3 Place another FGE to the right of the preceding partially completed mechanism schematic. Select the one that has the maximum number of links to be connected to the preceding primary units. If there is more than one choice, choose any. Henceforth, we will refer to an FGE that is being added as the current FGE and its welding points as current welding points. An FGE that has just been added in a partially completed mechanism schematic will be referred to as the preceding FGE.

Step 4 For every current welding point, choose a corresponding welding point in the preceding FGE to which a connection can be made without crossing edges. If more than one such welding point is found choose the right-most among them. If no such welding point is found, then alter the arrangement of the welding points in the preceding FGEs. While altering the arrangement of the welding points in the preceding FGEs it should be kept in mind that the general form of the FGE as described previously should not be changed. Also, permutations can be done only among those welding points that are either connected to the current FGE or not yet connected. Those belonging to the partially connected mechanism cannont be permuted. If this does not work, make a different choice of the preceding FGE and repeat Step 3.

Step 5 If a and b are two current welding points to be connected to the preceding FGE, then a should be placed to the left of b if the corresponding welding point a in the preceding FGE is located to the right of b (in the preceding FGE). If this is not possible, alter the choice of the preceding welding points.

Step 6 The first connection from a current FGE to the preceding FGE won't make any welding point inaccessible. However, any successive connections could. In that case check whether the corresponding link becomes inaccessible. If it is so, go to Step 4 and choose a different welding point. If this does not work, follow the permutation procedure given in Step 4. First permute the welding points of the preceding FGEs and then change the sequence of the FGEs themselves.

After the secondary edge connection is made so that rules M1 to M4 are satisfied, some of the welding points that are not connected by secondary edges can be removed if a corresponding welding point of the same label exists in the other FGEs. Once the above operations are complete the EGM can be sketched by mapping the welding points to the primitives of the links to which they belong. We note that among several secondary edges connecting two FGEs, the topmost edge represents the outer most shaft in a set of coaxial link. For example, functional schematic developed by mapping the welding points in Fig. 5.14(a) is

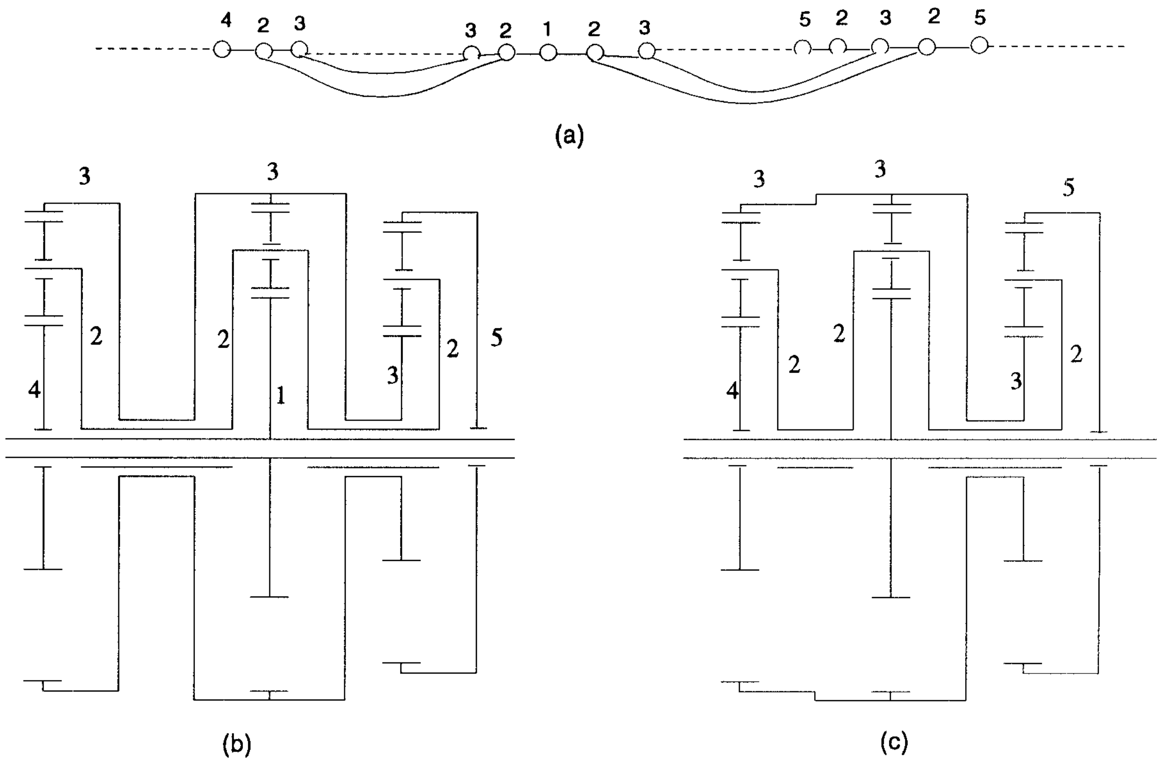


Figure 5.14 Constructing the Functional Schematic.

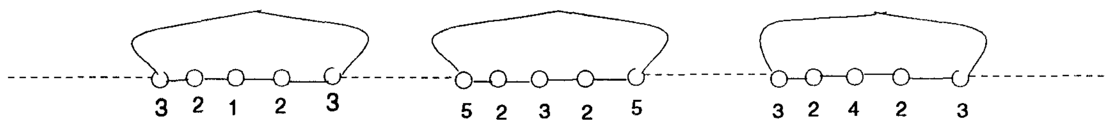


Figure 5.15 Arrangement of Connection Point to Sketch Both Overhead and Shaft Connections.

shown in Fig. 5.14(b). Some of the shaft connections between adjacent units can be converted into overhead connections as shown in Fig. 5.14(c). This can be done only if all of the following conditions are satisfied.

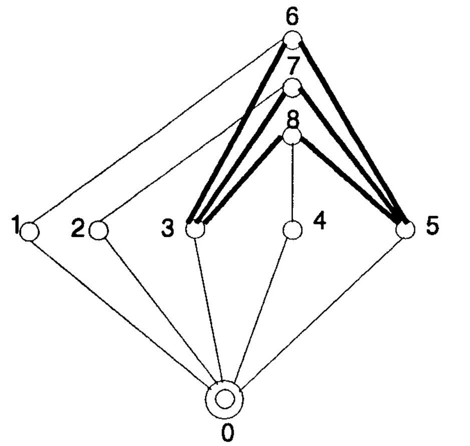
(i) the welding points are adjacent to each other,

(ii) they belong to a ring or a carrier,

(iii) there is one more welding point of the same label in either of the primitives.

Till now we have only discussed shaft connections and converting shaft connections of adjacent FGEs to overhead connections. However, the method discussed above can easily be generalized to include all possible overhead connections by arranging the welding points of the primitives as shown in Fig. 5.15, and then making the secondary-edge connections so that they satisfy rules M2 to M4. This however, will result in complicated connection that won't be easy to sketch in practice. It is also not necessary if we limit ourselves to a small number, of FGEs, say four.

We end this section by citing an EGM that will always have one inaccessible link no matter how the FGEs are connected. The example is shown in Fig. 5.16.



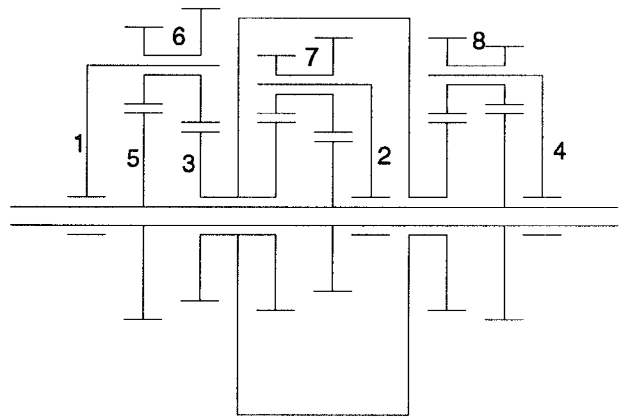


Figure 5.16 An EGM that cannot be constructed without an inaccessible link.

Since an inaccessible link cannot be used as an input, output or a fixed link , some of the links of such mechanisms become redundant.

5.6 Display of the Functional Schematic

The functional schematic of an EGM is stored in the computer as a network of structures. For example, each of the primitives is stored as a structure. These structures contain not only the coordinates of the points that define the dimensions of the primitives, but also the pointers to other primitives to which they are connected. Each of the primitives is created in the local coordinate system of an FGE to which it belongs. Then the FGEs are translated to occupy their relative position with respect to each other in the global coordinate system. The lines corresponding to shaft connections are then created. The hub diameter of the gears and carriers are adjusted so that they don't interfere with the shafts. To display the structure on to the computer screen, graphics package called PHIGS is used. The PHIGS window is a square window with its sides of unit length. The lower left corner has the coordinate of (0,0) and the upper right corner has the coordinate (1,1). Therefore, the structure before being displayed is first transformed from the World Coordinate System to the Screen Coordinate System and then scaled appropriately to fit the window.

Chapter 6

Results, Discussion and Suggestions for Future work

6.1 Results and Discussions

This work is divided into two parts. In the first part the structural characteristics of epicyclic gear mechanisms (EGMs) that are commonly used in automatic transmissions to obtain various speed ratios have been identified from the view point of kinematics. A canonical graph representation for this type of mechanisms has been defined. A methodology to systematically enumerate these graphs has been developed and illustrated through various examples. The results are tabulated in Table 6.1. The table gives the number of graphs before the assignment of external and internal gear pairs. Detailed results are given in Appendices

Table 6.1 Table of the number of graphs enumerated for EGMs with up to 9 links.

No. of Links	No. of Graphs
6	1
7	7
8	22
9	157

B, C and D. There is only one graph for 6-link EGMs[1], which is in agreement with the result given by Tsai et al. (1988). There are 7 graphs for 7-link EGMs, which is one more than that given in the same paper. This is because that paper has excluded those graphs in which the geared edges form a closed loop. The verification for completeness of the set of graphs enumerated for 8-link EGMs has been accomplished in an indirect way. From the set of graphs of 7-link EGTs generated by Kim and Kwak (1991), those that qualify for automatic transmissions were selected. A total of 20 such graphs were extracted from their paper which is less than the 22 given in Table. 6.1. The reason is that there are exactly 2 graphs of 8-link EGMs (Figs. C.1(s) and C.2(j)) that have geared edges forming a loop, and those graphs cannot be generated by the method of Kim and Kwak. The completeness of the set of graphs for 9-link EGMs cannot be confirmed since no published results exist for 8-link EGTs.

[1]An n-link EGM contains a $(n-1)$ link EGT and the casing of a transmission.

In the second part of this thesis a method to sketch the functional schematic of a mechanism from its graph representation has been formulated. Using this method the functional schematic of a fairly large class of mechanisms can be sketched automatically. At present, a C-language program has been developed to sketch those EGMs in which each FGE contains only one planet gear. Fig. 6.1(b) shows an 11-link EGM that is sketched from the graph representations shown in Fig. 6.1(a) using this program.

Fig. B.1 in Appendix B lists the graphs of 6 and 7-link EGMs that have single-planet FGEs. These graphs have their geared edges labeled as internal or external. The functional representation of the corresponding EGM has been sketched adjacent to each of the graphs. Fig. B.2 shows a graph whose geared edge hasn't been labeled yet, and which represents a 7-link EGM containing a double planet FGE. There is only one such graph for 7-link EGM.

Column 1 of Fig. C.1 shows the graphs of 8-link EGMs that are made up of single-planet FGEs. These graphs are shown before the types of gear meshes are assigned. The graphs in Column 2 are formed after assigning one of the many possible distributions of external and internal gear meshes to the graphs of Column 1. Column 3 shows the corresponding functional schematic. Fig. C.2 shows the remaining graphs of 8-link EGMs. Fig. D.1 depicts the same thing as Fig. C.1, but for those 9-link EGMs that contain three single-planet FGEs. The adjacency matrices of the remaining graphs of 9-links are tabulated in Table D.1.

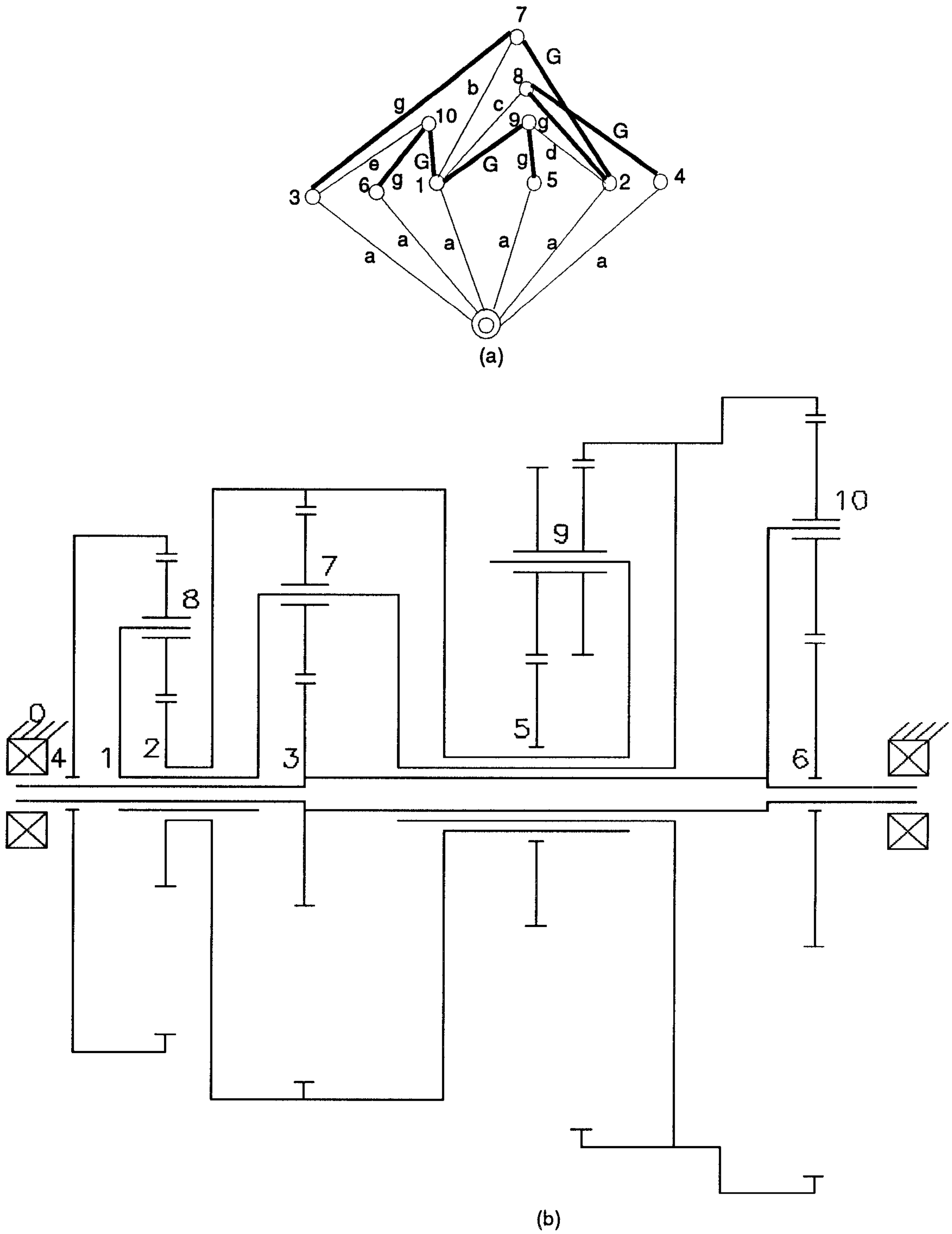


Figure 6.1 An 11-link EGM: (a) Graph Representation (b) Functional Representation.

6.2 Future Work

In this thesis a systematic procedure for the synthesis of the kinematic structure of EGMs is presented. However, as mentioned at the beginning of the introduction, this is only the first phase in the overall synthesis procedure. Therefore, more work can be done on the EGMs that have been enumerated. The following suggestions are made.

- Perform dimensional synthesis of the EGMs beginning with the identification of different clutching sequences for each EGM.
- Find the optimized gear ratios (Mogalapalli et. al., 1993) and the number of teeth to achieve a set of desired speed (reduction) ratios.
- Do the power flow, torque and force analysis of the mechanisms following the procedure formulated by Pennestri and Freudenstein (1990).
- Finally, perform strength analyses to decide upon the dimensions of the gear, shafts, bearings, etc.
- Achieve an optimal design for the overall transmission system by integrating all the above steps.
- Generalize the method of sketching the functional schematics to include EGMs having other types of fundamental entities, such as the one shown in Fig. 5.7.

Appendix A

Interactions Among FGEs

The maximum number of shafts and overhead connections in an EGM of n-link should be equal to the greatest even number that is less than or equal to $(n-5)$. This can be proved as follows.

Let an EGM contains k FGEs having $N_1, N_2 \ldots N_k$ links, respectively. The minimum value of N_i is five. Since an FGE is a two-dof gear train according to F1 the i^{th} FGE will have $N_i - 3$ gear pairs. Therefore, the total number of gear pairs in the EGM is $\sum_{i=1}^{k}(N_i - 3)$. Since the EGM is also a two-dof mechanism the total number of links that it can have is $\sum_{i=1}^{k}(N_i - 3) + 3$. Thus, if the number of links in the EGM is n, then n must satisfy the equation

$$n = \sum_{i=1}^{k} N_i - 3k + 3 \tag{A.1}$$

where $N_i \leq 5$, $n \geq 6$ and $k \geq 1$

However, if we sum up the links of the FGEs that constitute the EGM the number of links are $\sum_{i=1}^{k} N_i$. Therefore, while constructing the EGM one has to

No. of Links	6	7	8	9	10	11	12
Maximum No. of FGEs	1	2	2	3	3	4	4

Table A.1 Table of the maximum number of FGEs versus the number of links in EGMs.

reduce the number of links by $\sum_{i=1}^{k} N_i - n = 3(k-1)$ links. This can be done by connecting (rigidly) $3(k-1)$ links to other links among the FGEs. Since the root of all the FGEs are combined together while constructing the EGM, only $3(k-1)$ - $(k-1) = 2(k-1)$ links need to be connected by means of shaft or overhead connections. Thus, to find the maximum number of shaft and overhead connections we have to find the maximum number of FGEs that an EGM of n links can contain. If we set $k = 2$ in Eq. (A.1), then the minimum value of n is obtained when both N_1 and N_2 are equal to five, the minimum permissible number for an FGE. The corresponding minimum value of n is seven. Thus, all EGMs having less than 7 links, can have only one FGE. Similarly, if we set k to 3 then the minimum value of n is 9 and hence all EGMs with links less than 9 can have atmost two FGEs. The maximum number of FGEs is listed against the number of links in an EGM in Table A.1.

From the table it is clear that the maximum number of FGEs for an EGM of n-link is equal to $n/2 - 2$ when n is even and $(n+1)/2 - 2$ when n is odd. Therefore, the maximum number of shaft and overhead connections in an EGM of n-link is $n - 6$ when n is even and $n - 5$ when n is odd.

Appendix B

Six and Seven-Link EGMs

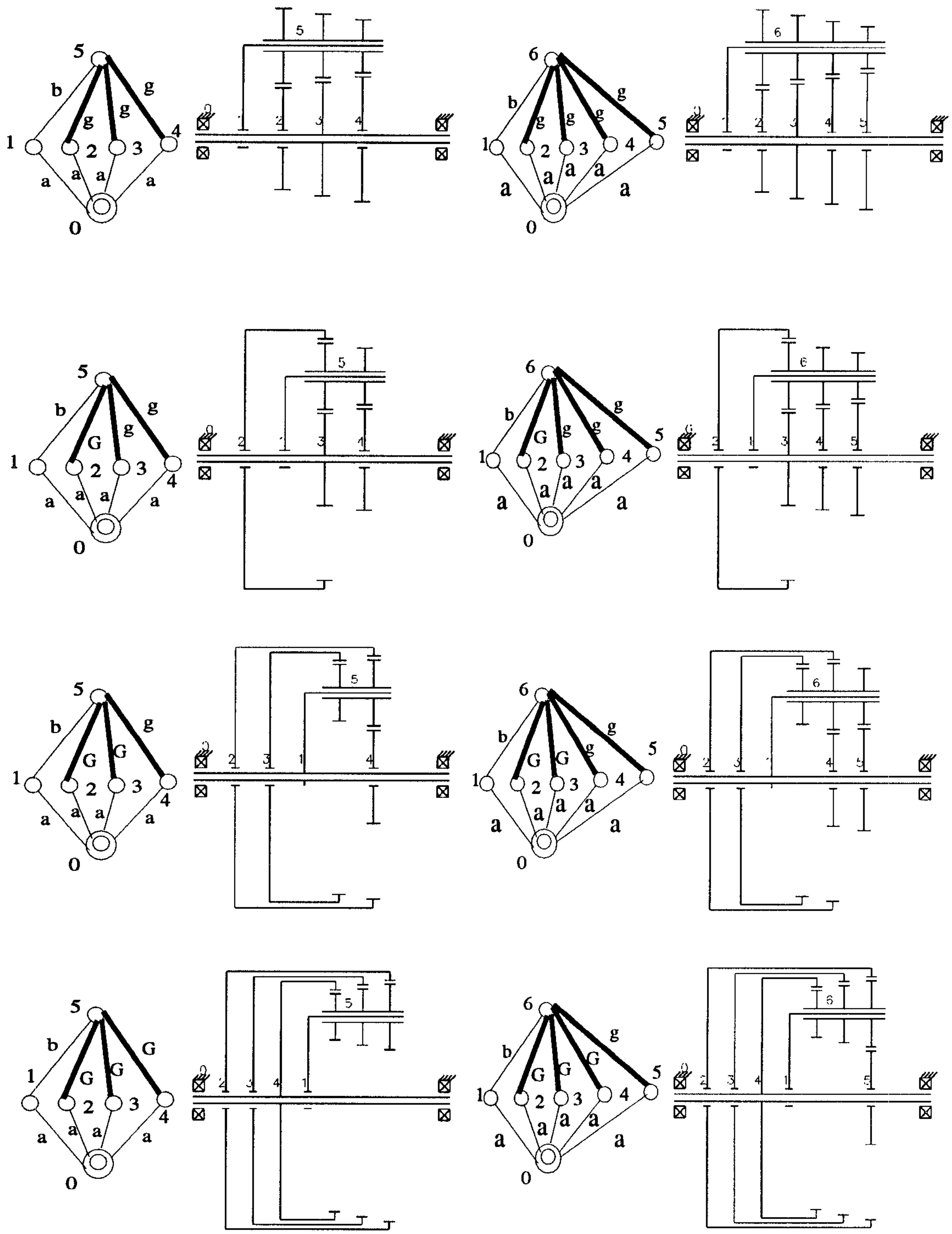


Figure B.1 Six and seven-link EGMs that have only single-planet FGEs. Columns 1 and 3: Graph representation, Columns 2 and 4: Functional representation.

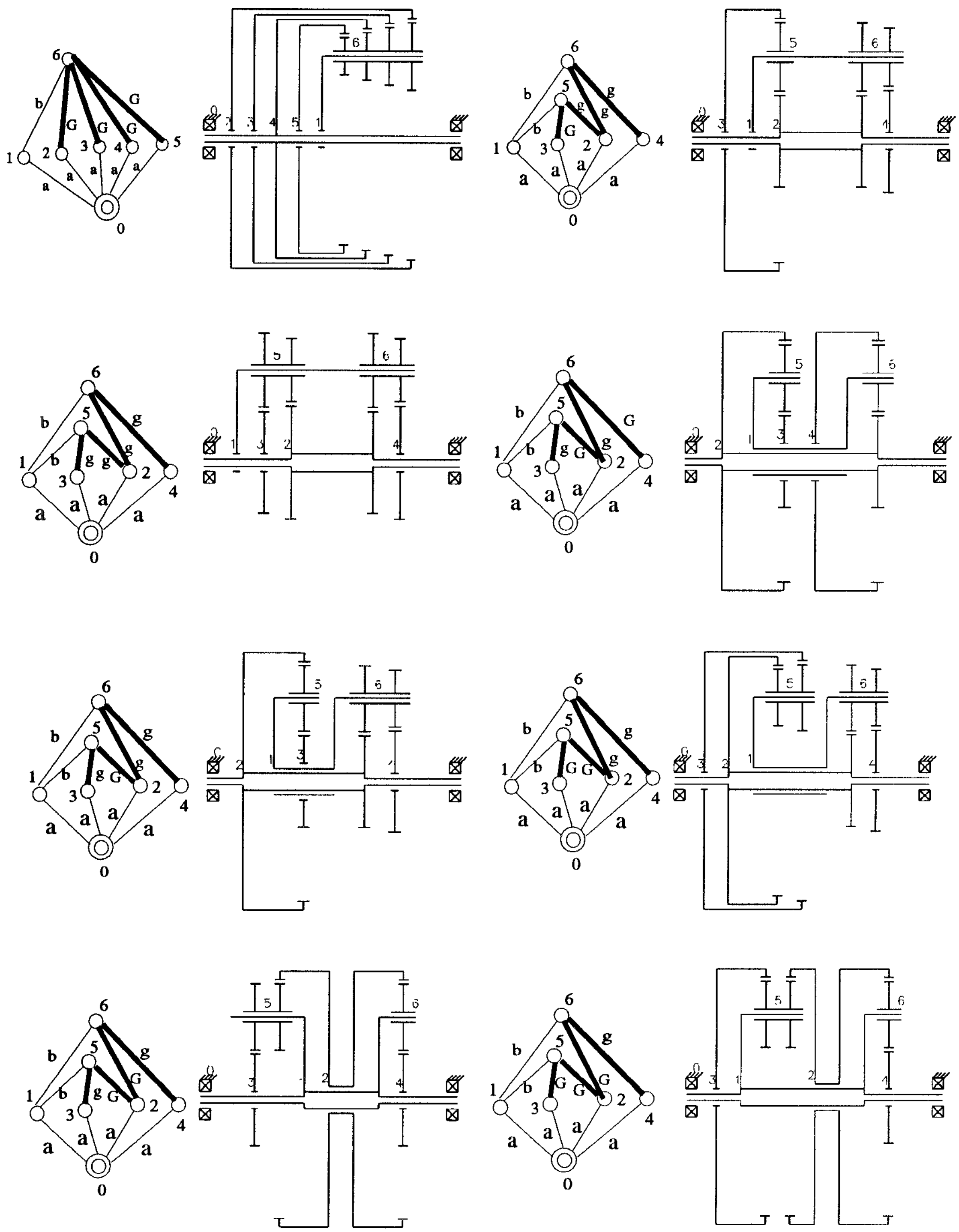


Figure B.1 (contd.)

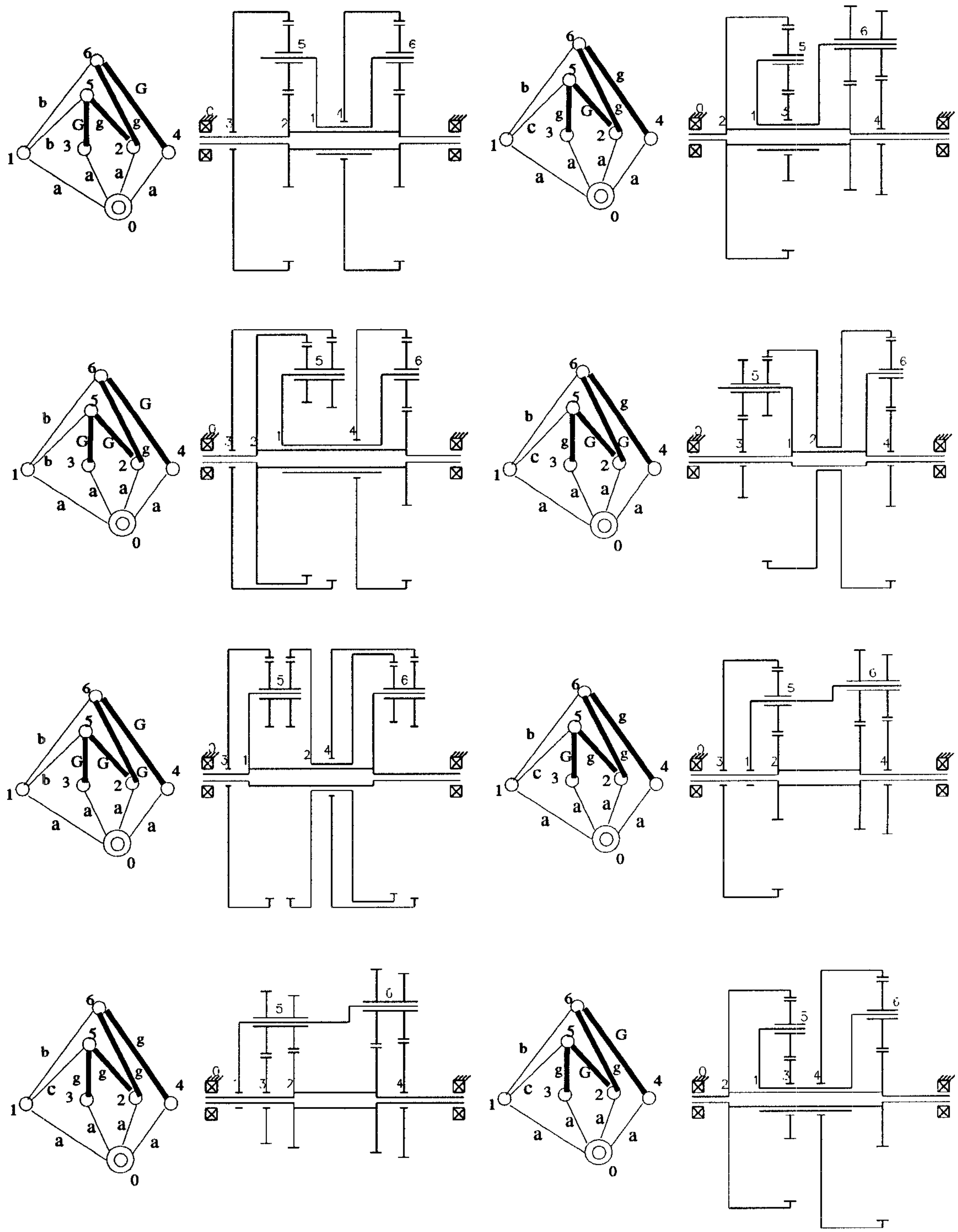


Figure B.1 (contd.)

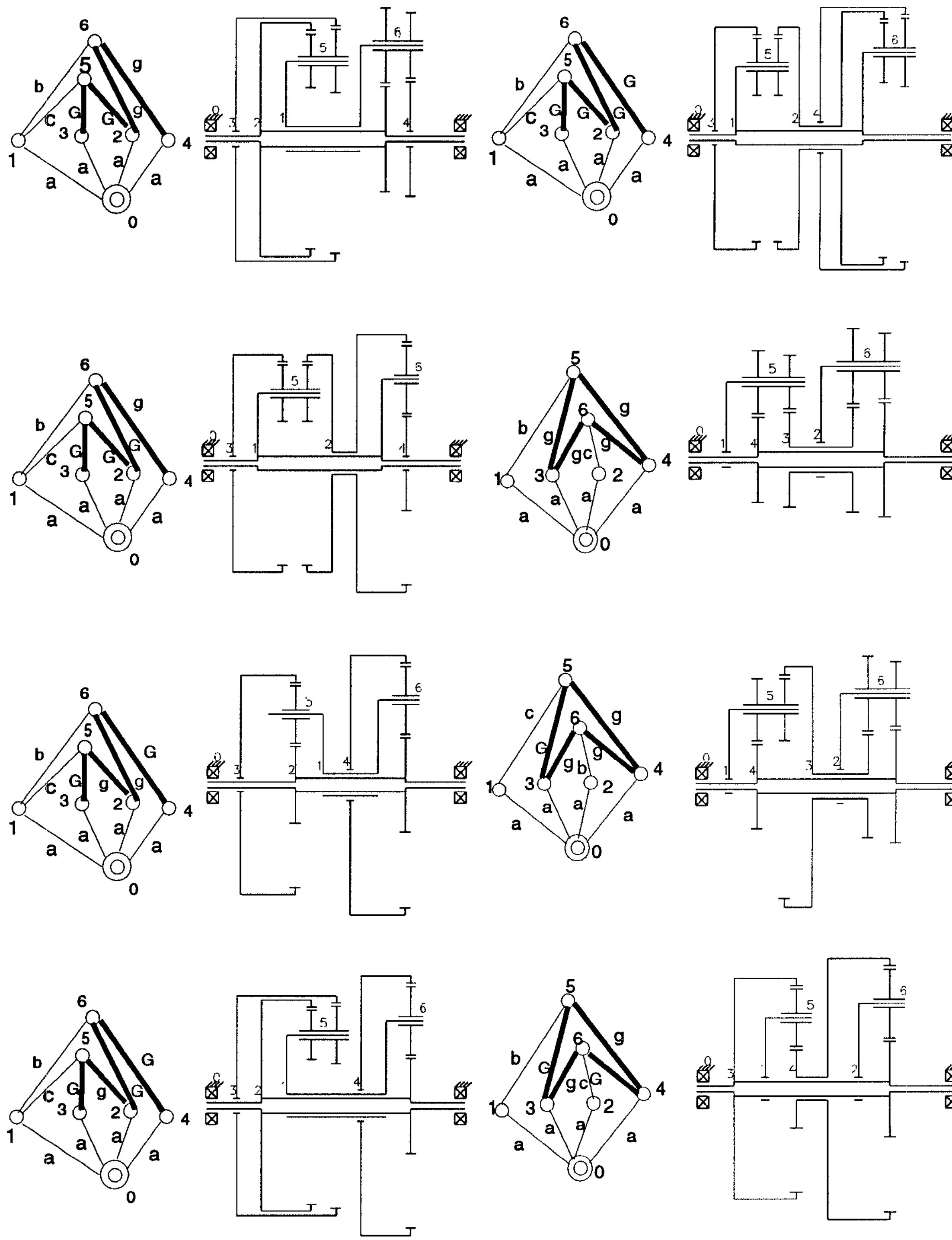


Figure B.1 (contd.)

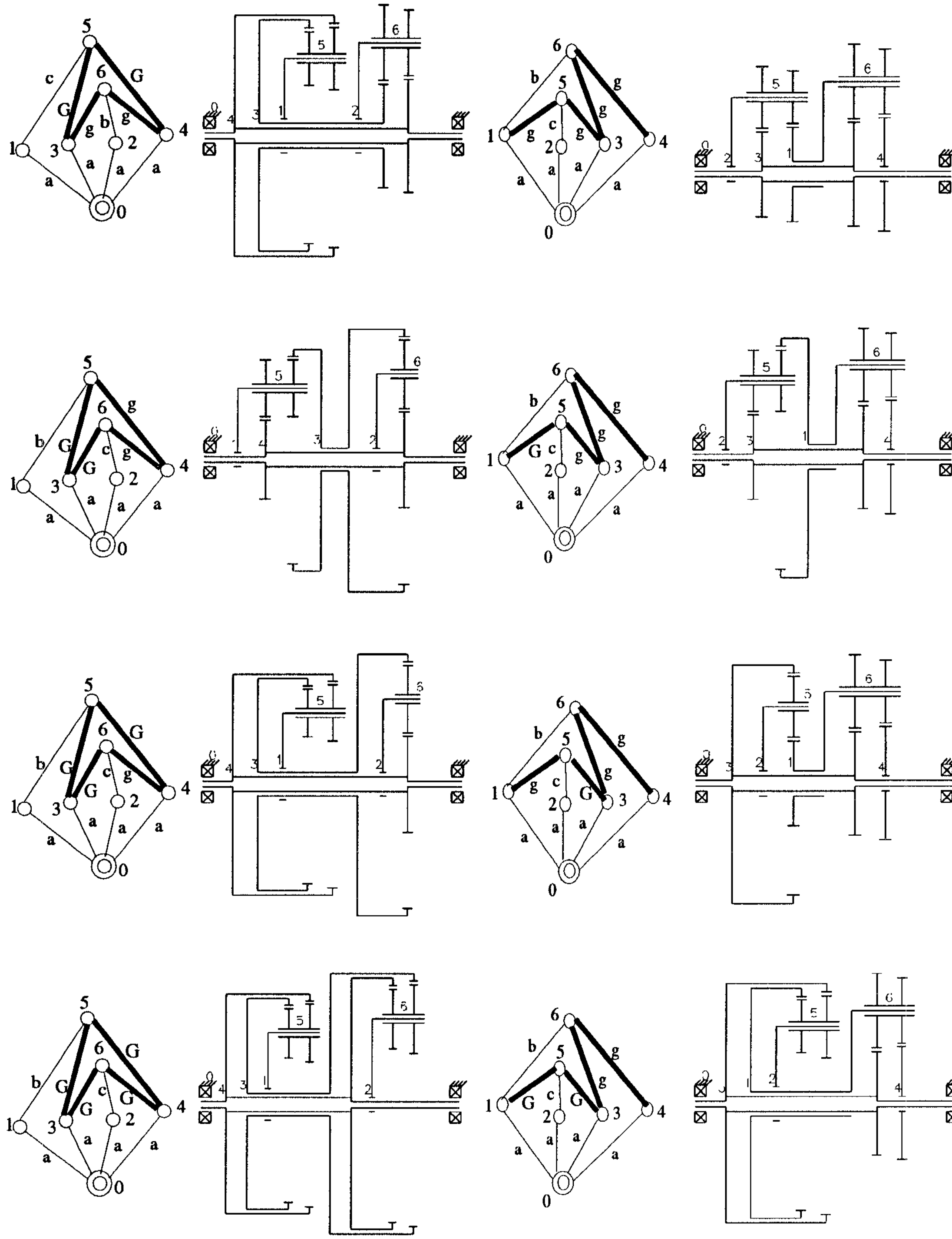


Figure B.1 (contd.)

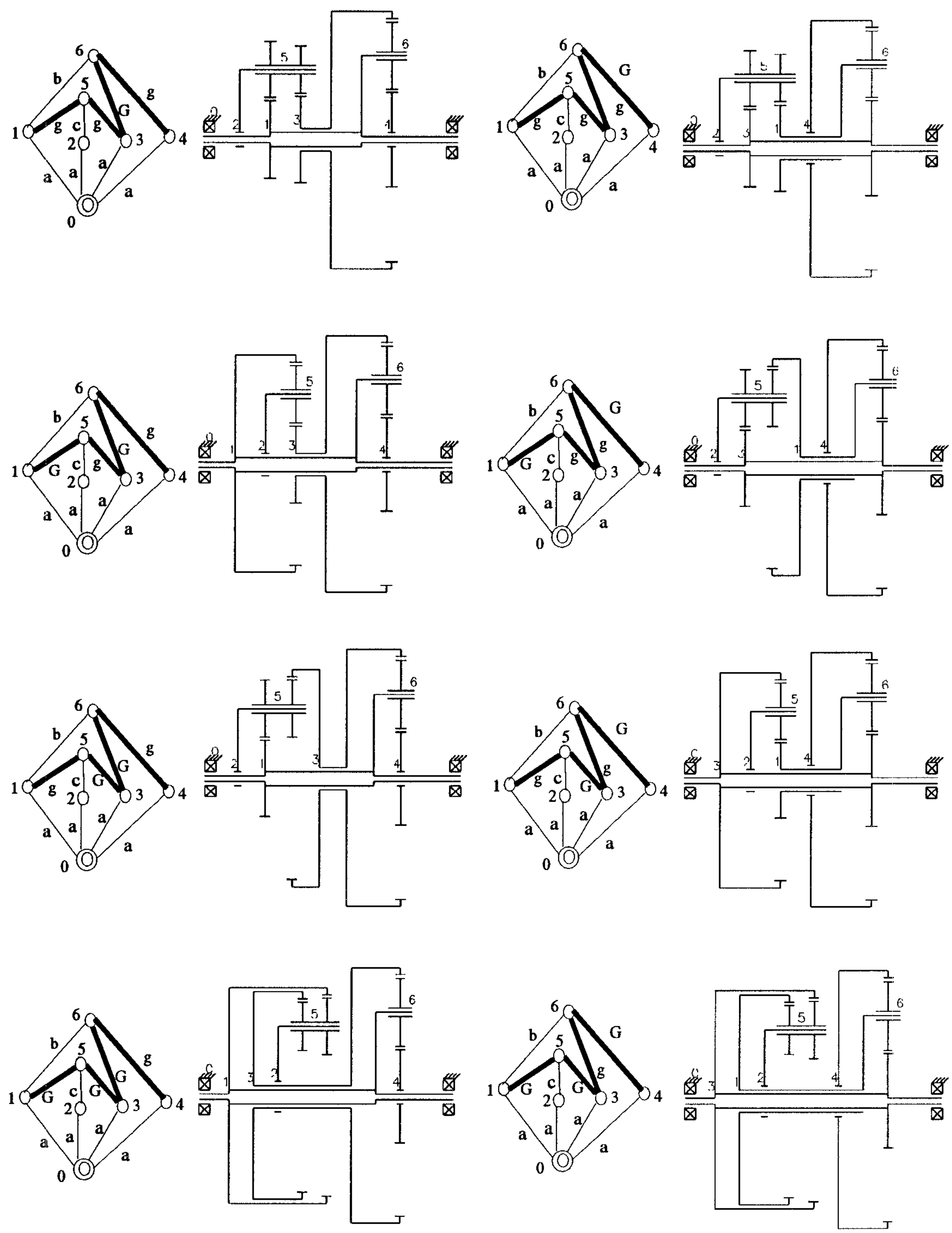


Figure B.1 (contd.)

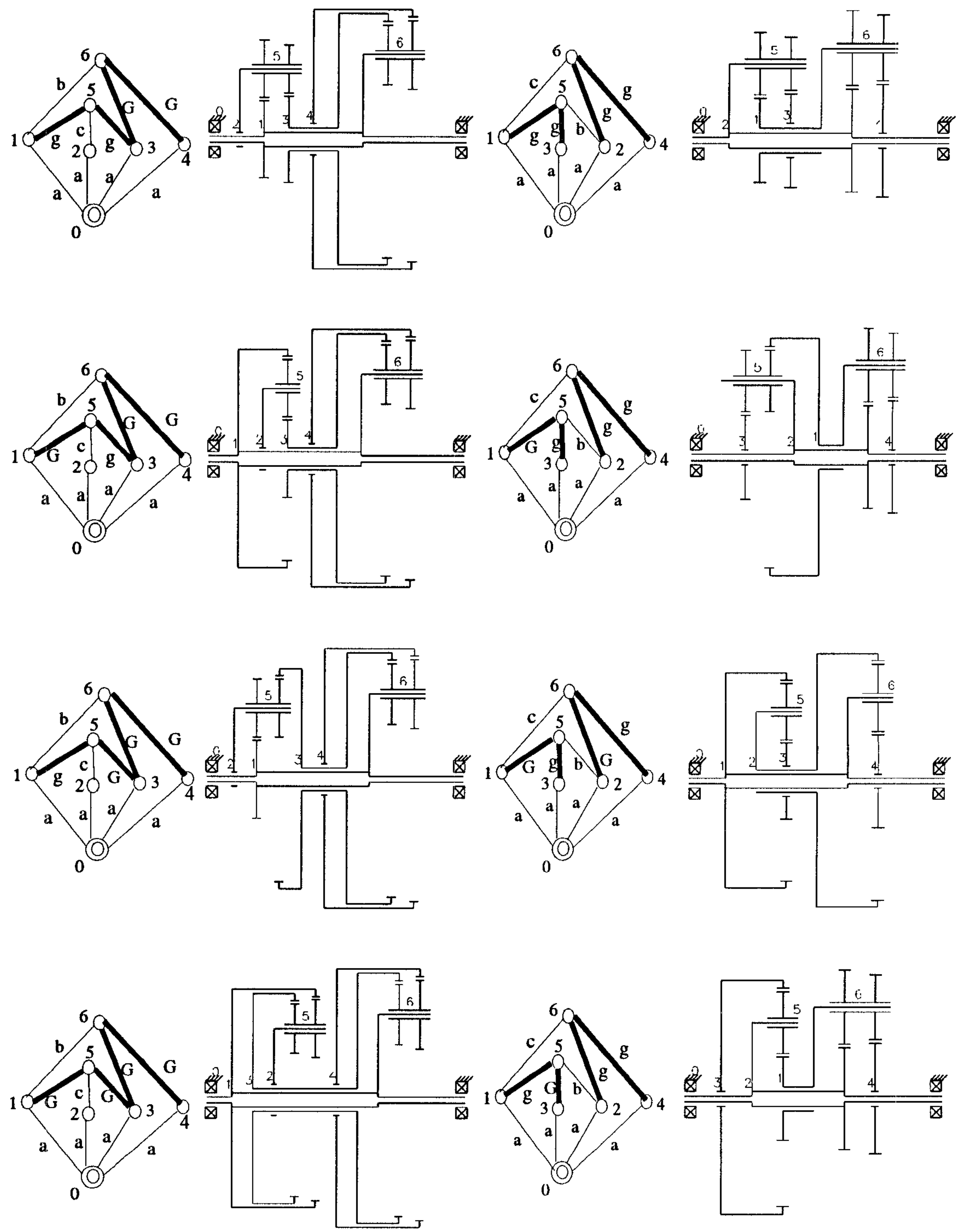


Figure B.1 (contd.)

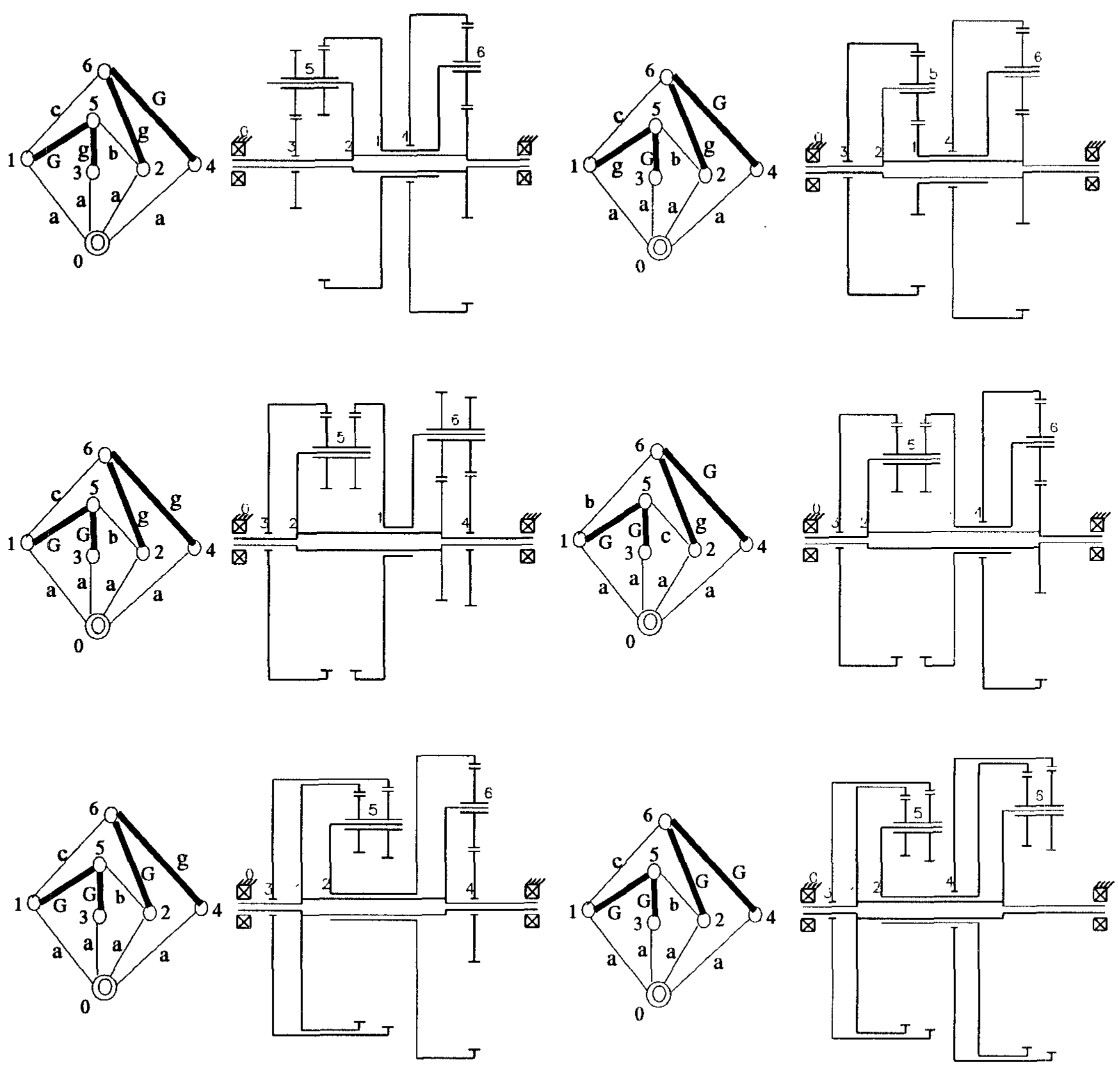


Figure B.1 (contd.)

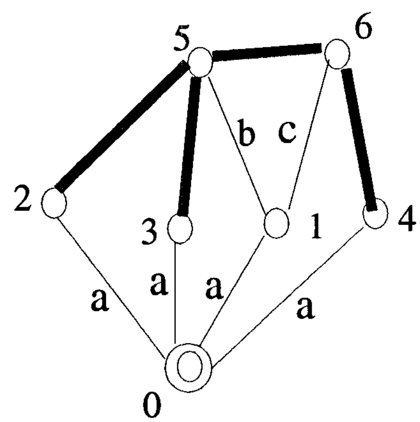


Figure B.2 Graph of a 7-link EGM made up of double-planet FGEs.

Appendix C

Eight-Link EGMs

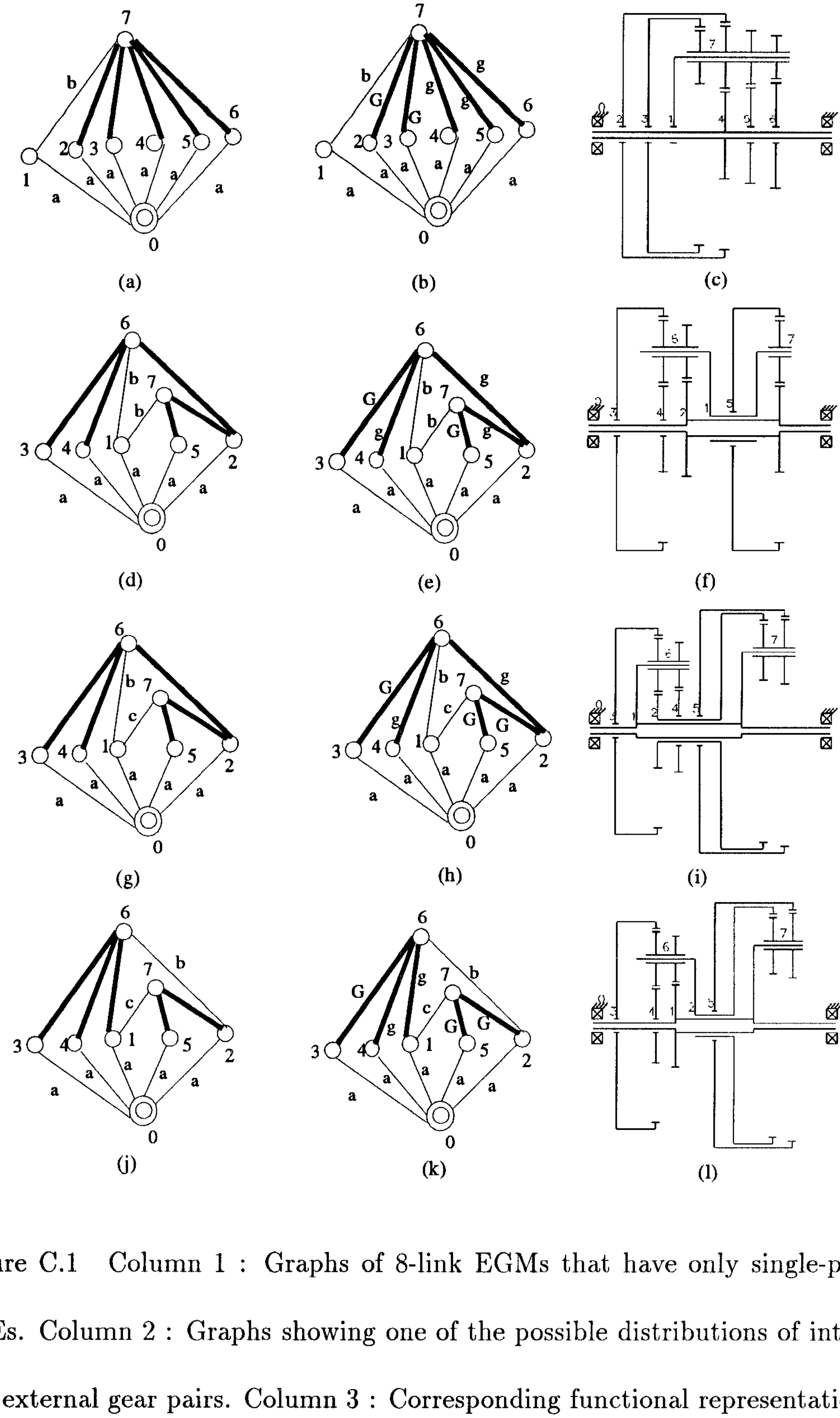


Figure C.1 Column 1 : Graphs of 8-link EGMs that have only single-planet FGEs. Column 2 : Graphs showing one of the possible distributions of internal and external gear pairs. Column 3 : Corresponding functional representation to Column 2.

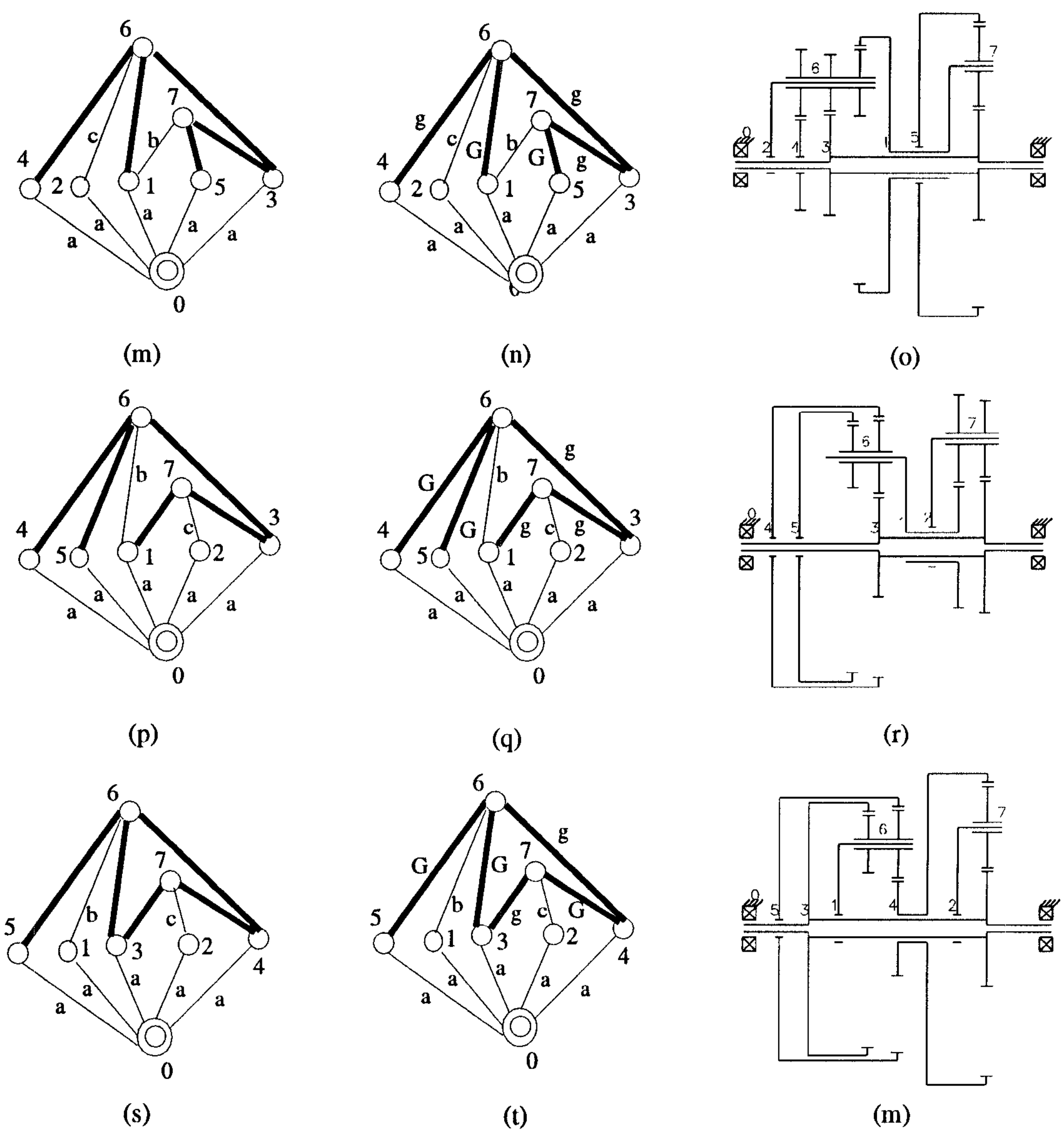


Figure C.1 (contd.)

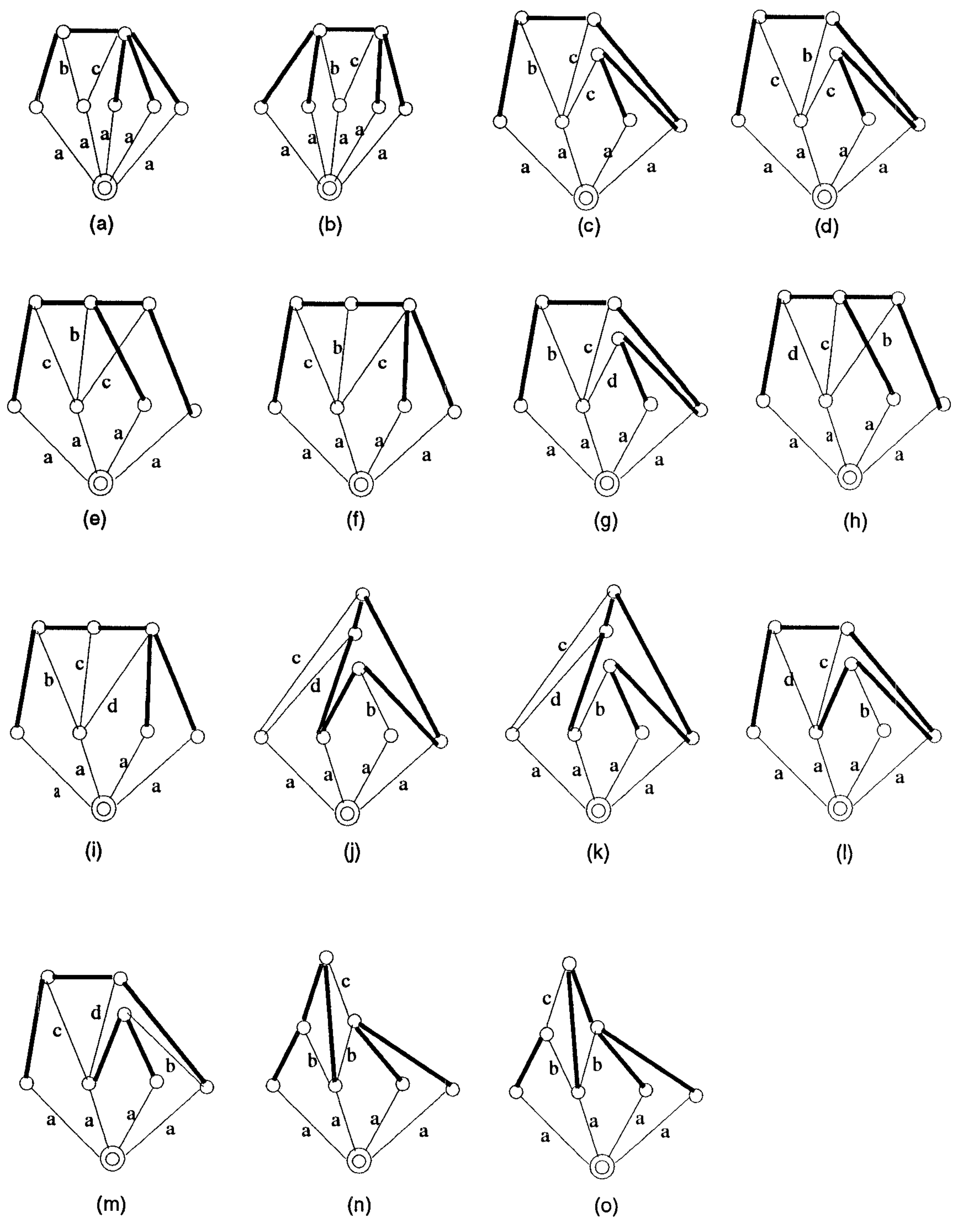


Figure C.2 Remaining graphs of 8-link EGMs.

Appendix D

Nine-Link EGMs

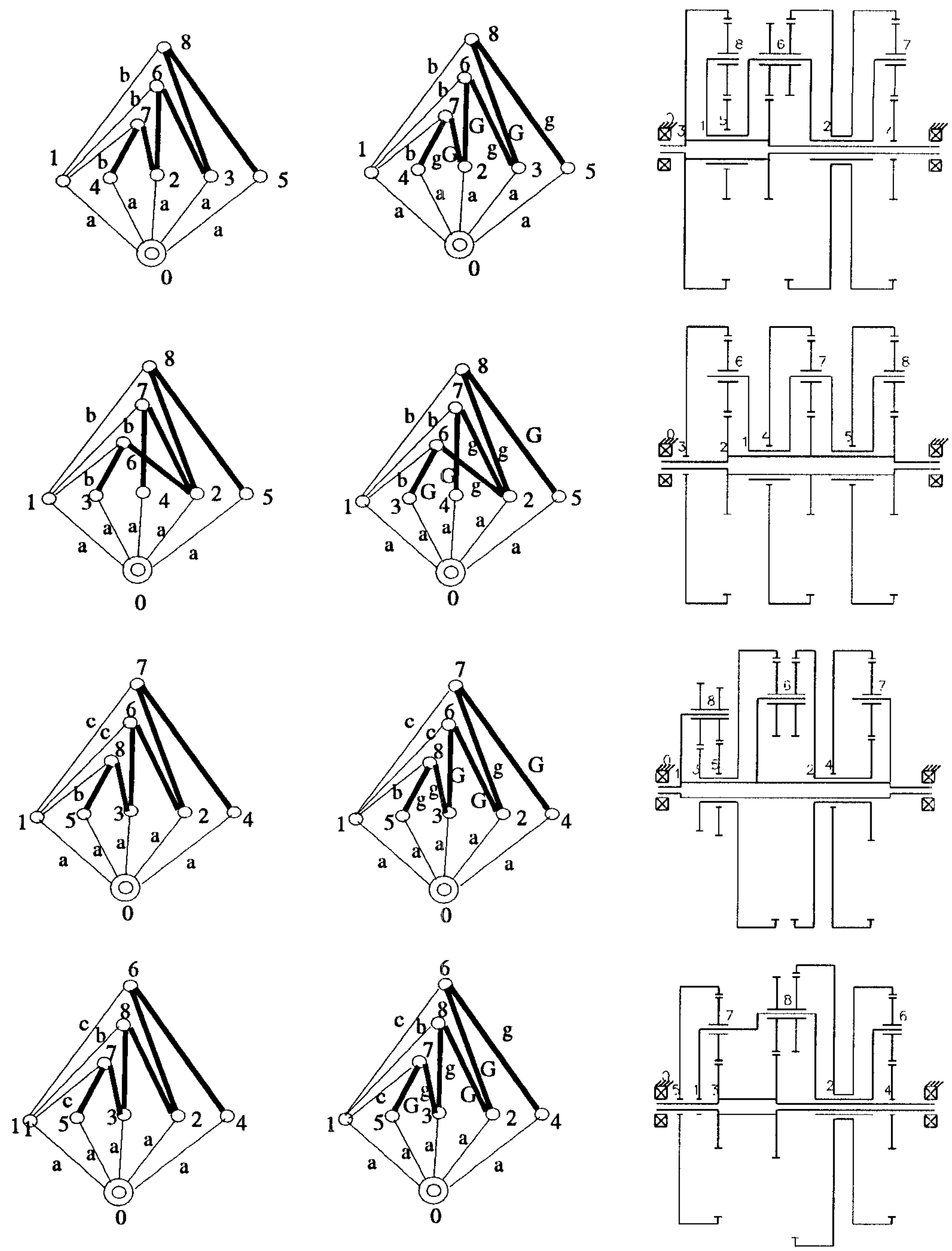


Figure D.1 Column 1 : Graphs of 9-link EGMs that have three single-planet FGEs. Column 2 : Graphs showing one of the possible distributions of internal and external gear pairs. Column 3 : Corresponding functional representations to Column 2.

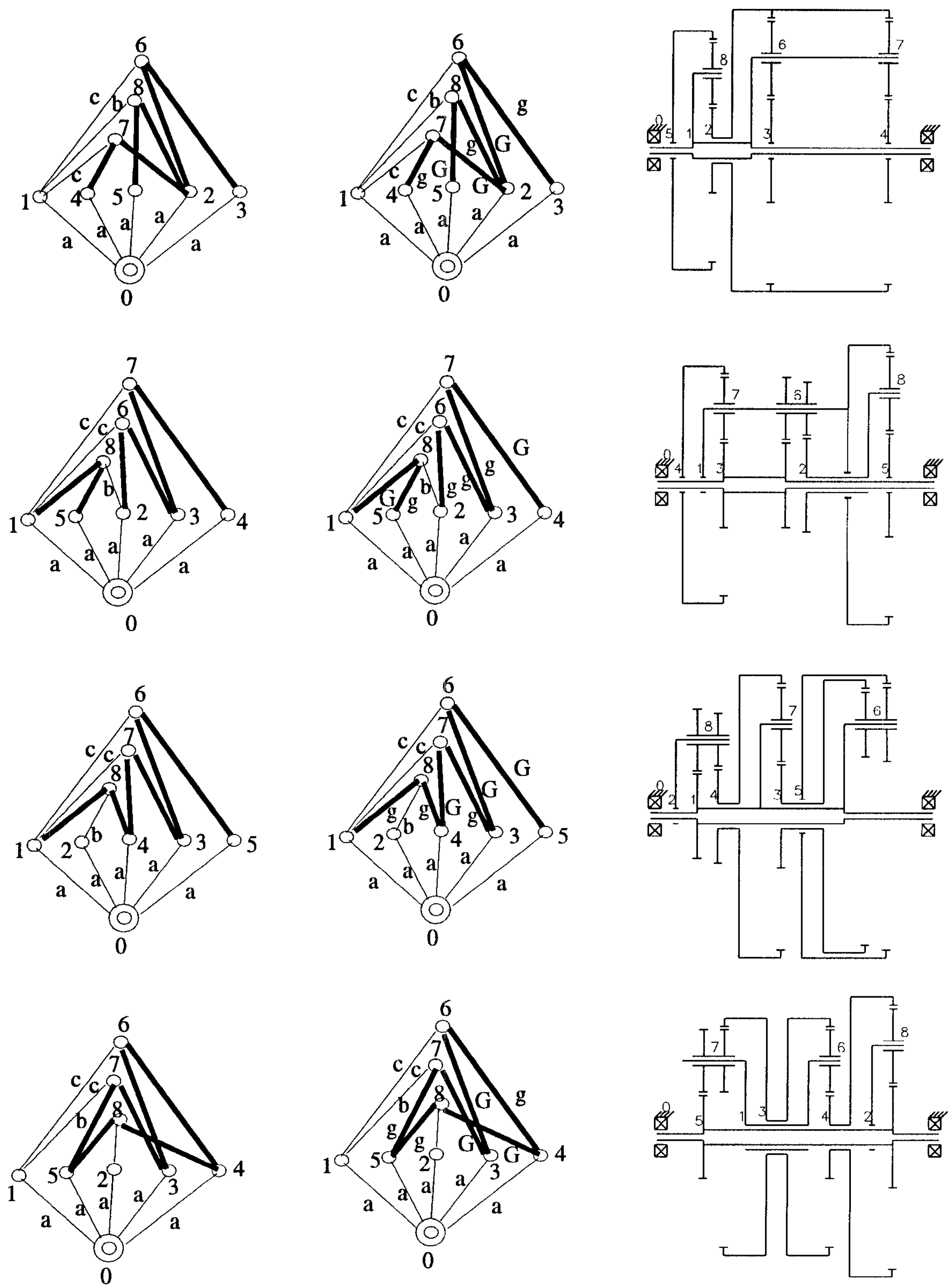


Figure D.1 (contd.)

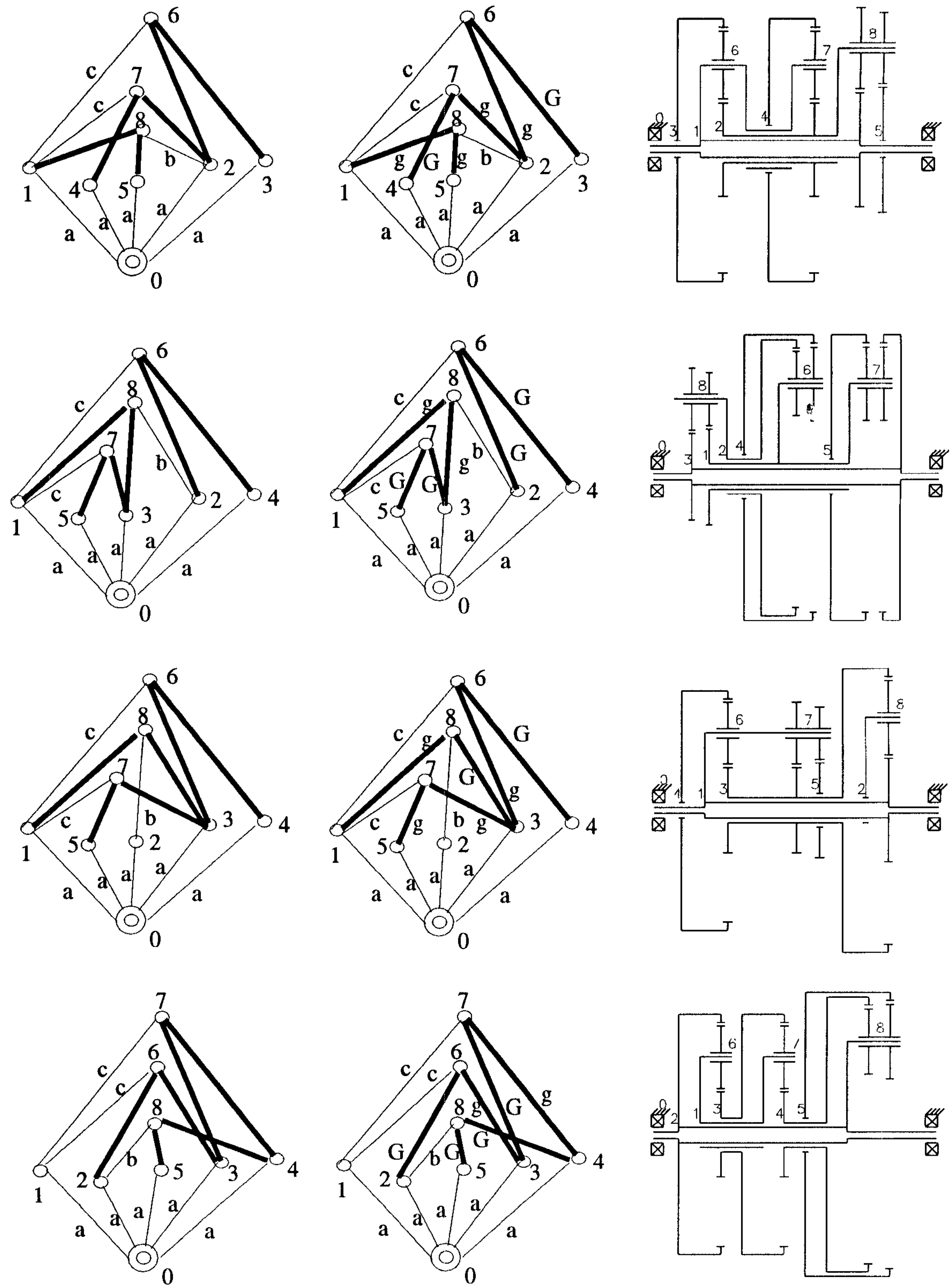


Figure D.1 (contd.)

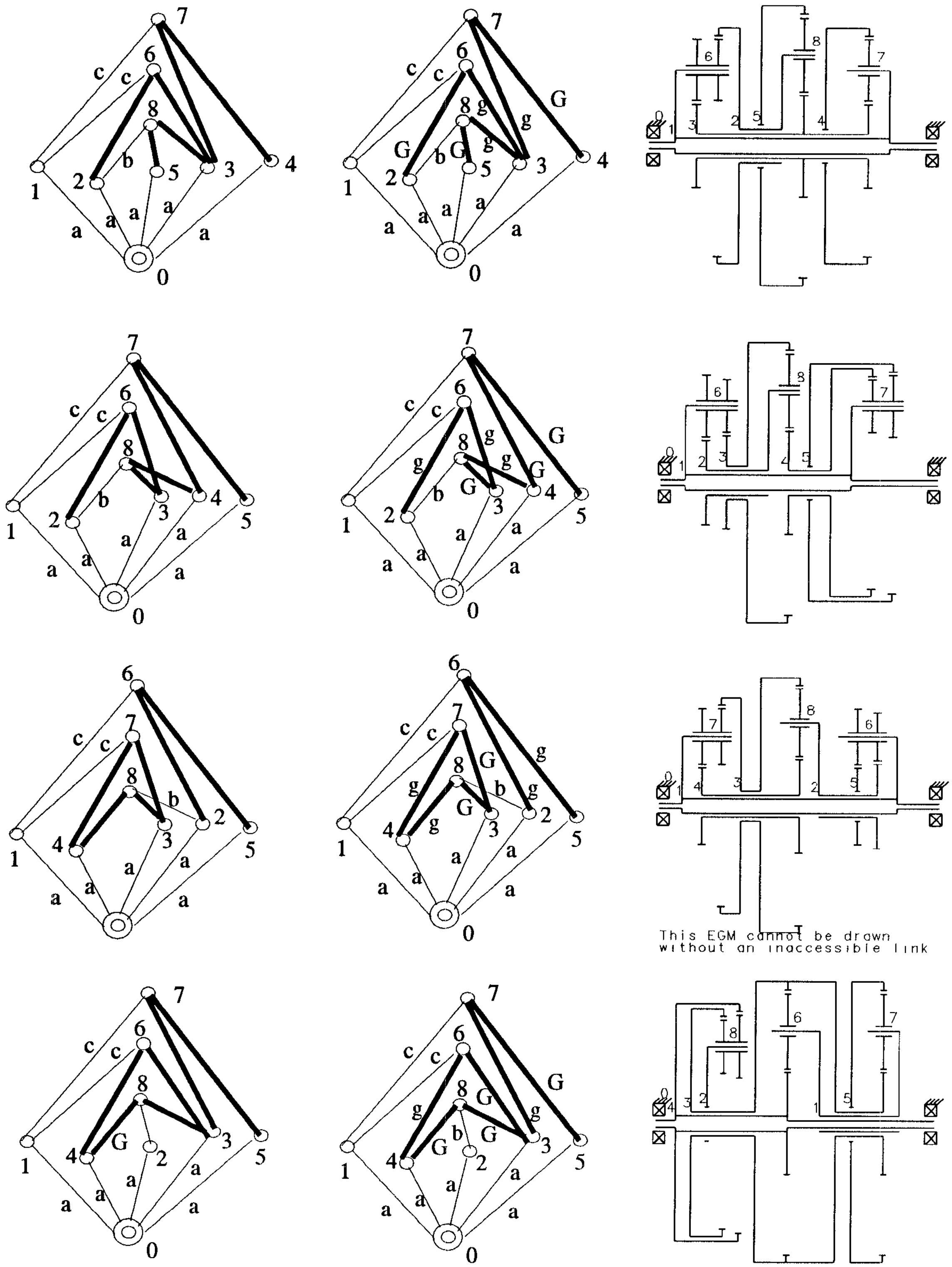


Figure D.1 (contd.)

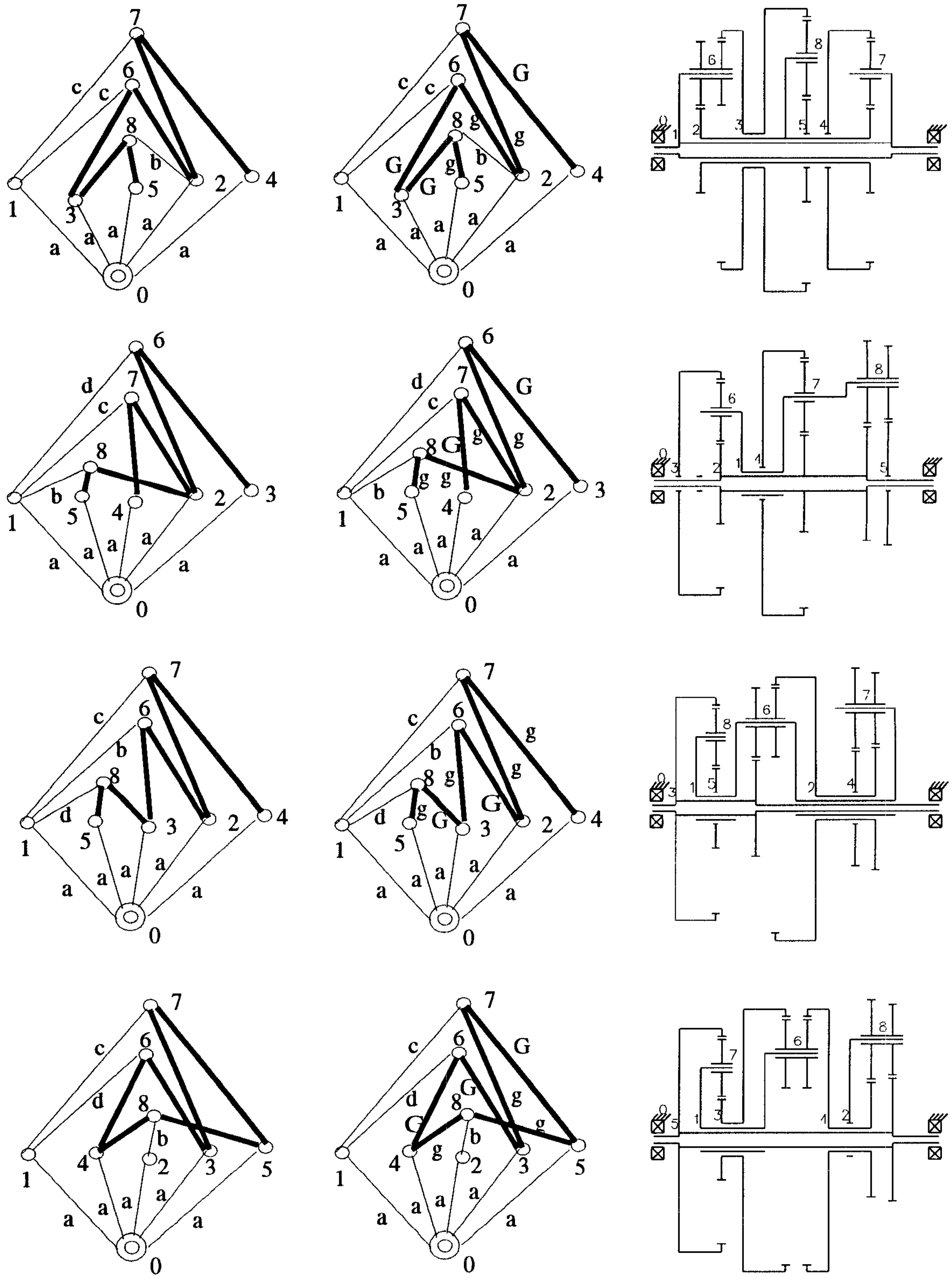


Figure D.1 (contd.)

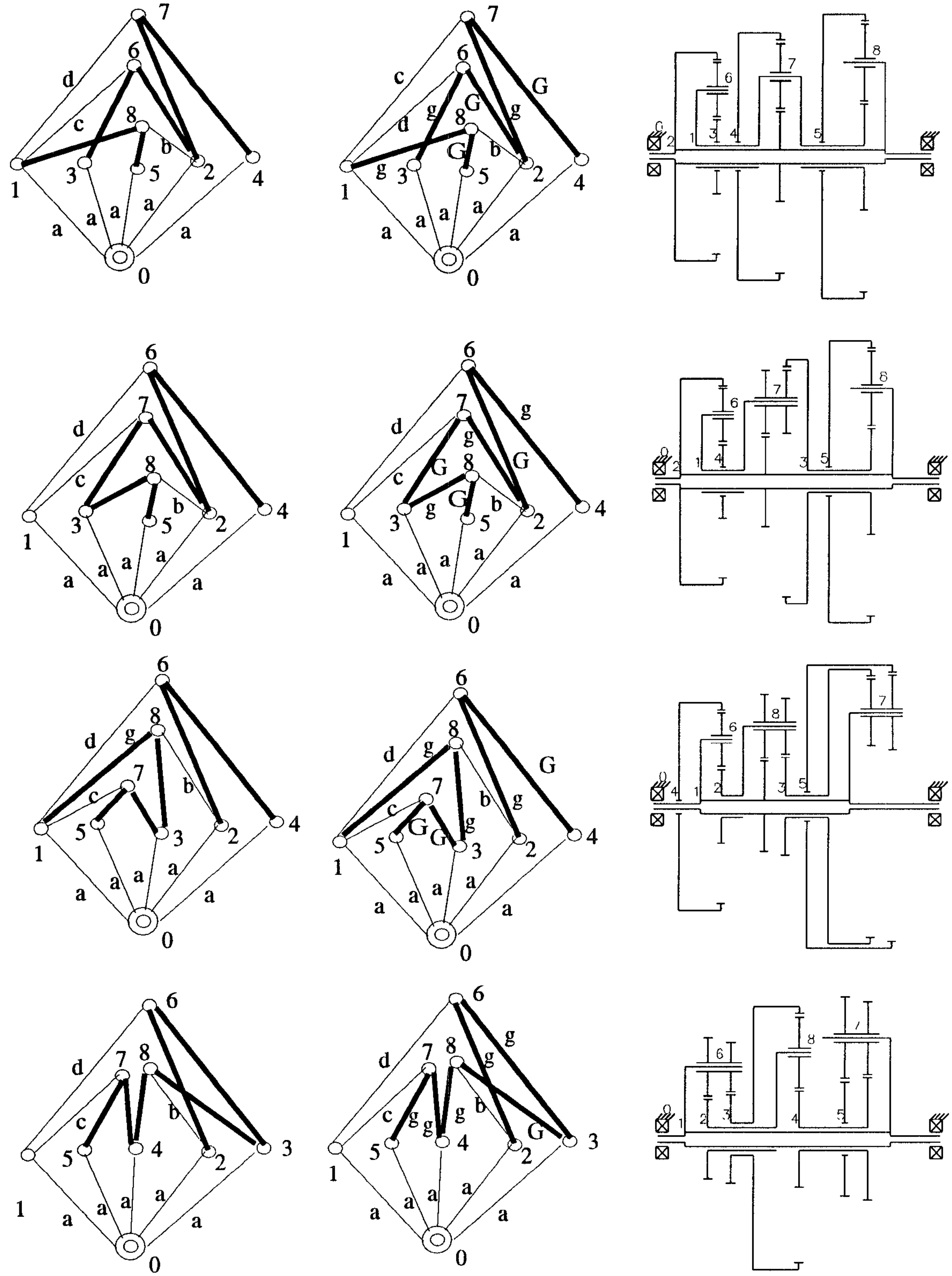


Figure D.1 (contd.)

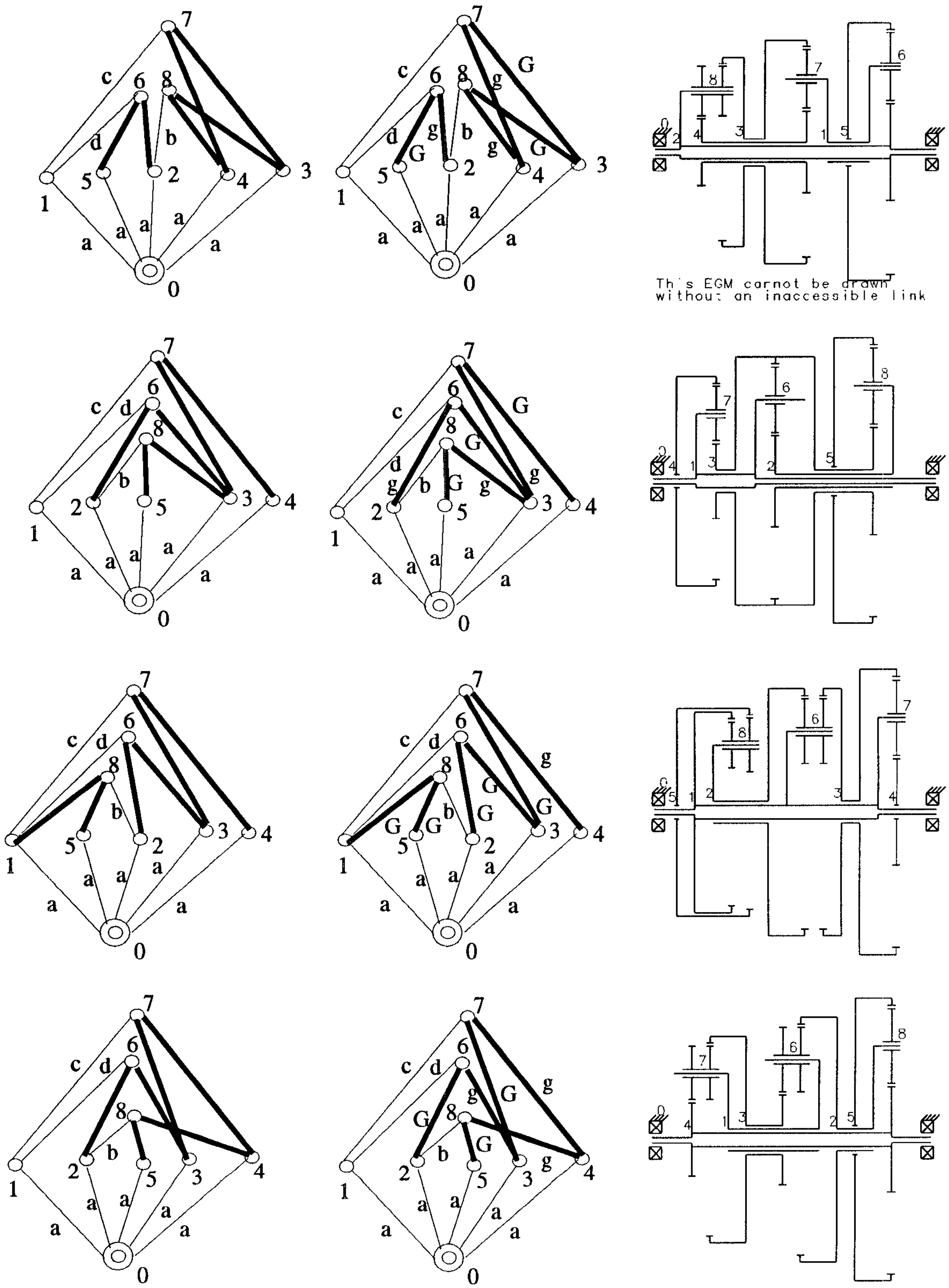


Figure D.1 (contd.)

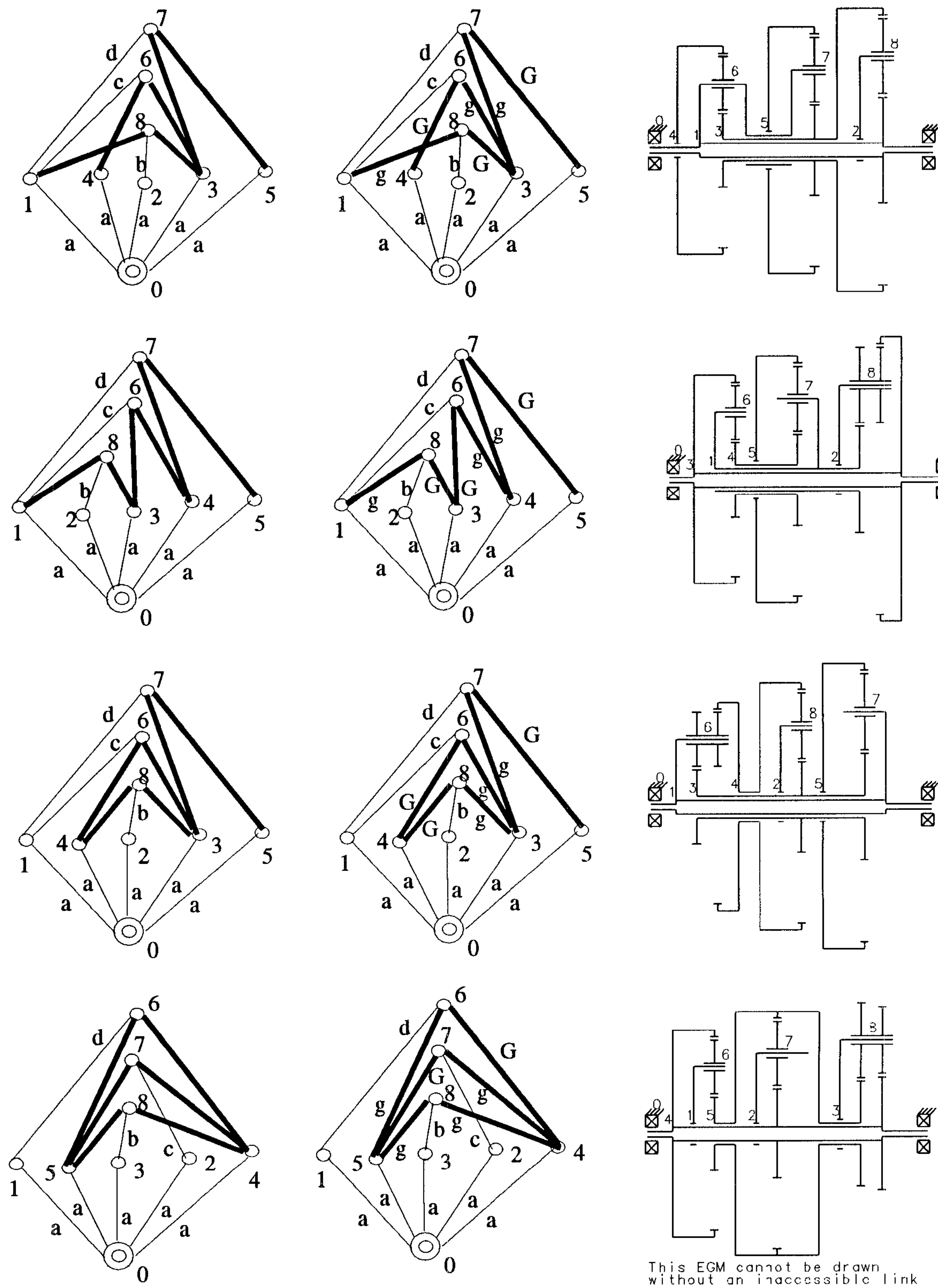


Figure D.1 (contd.)

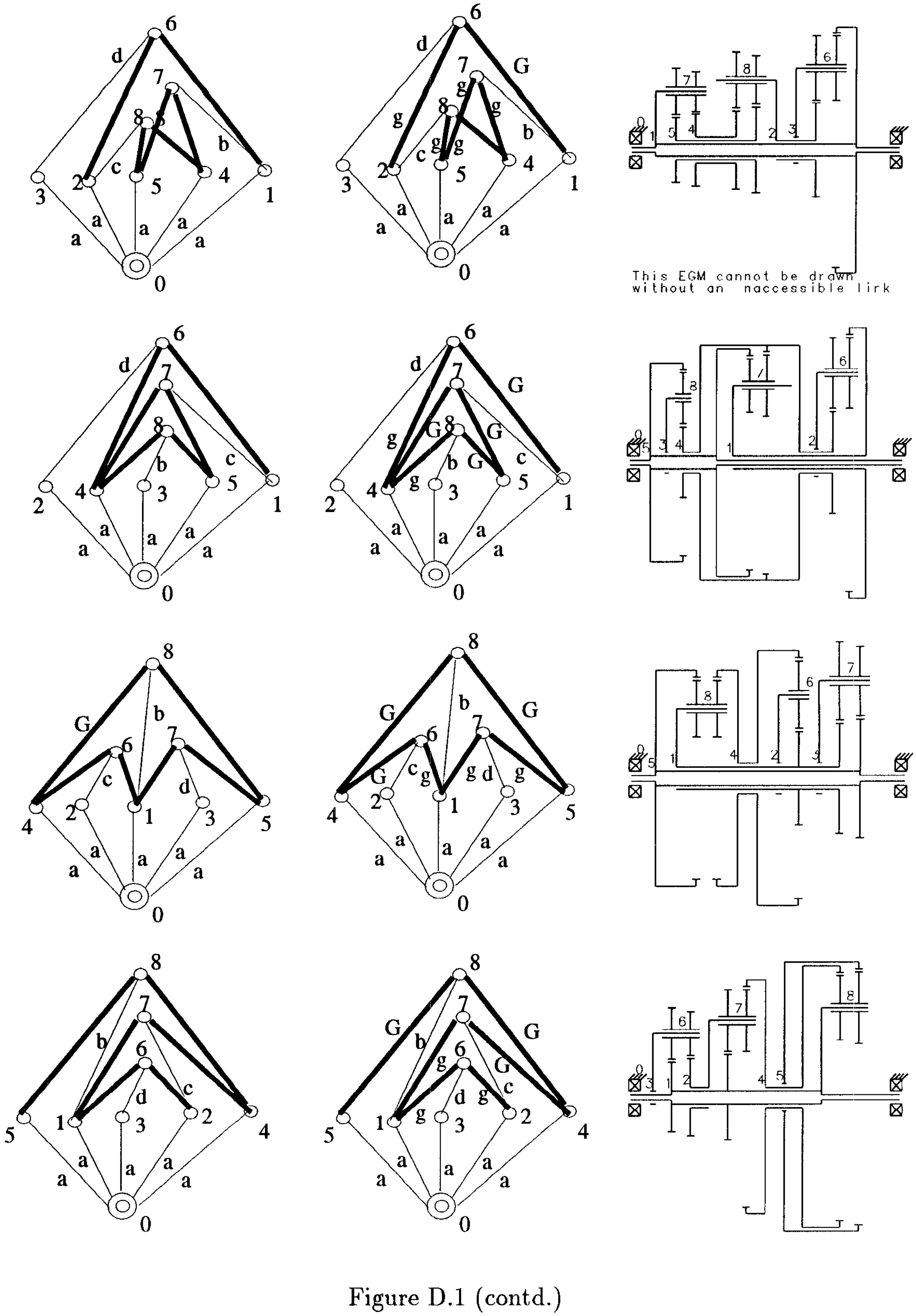


Figure D.1 (contd.)

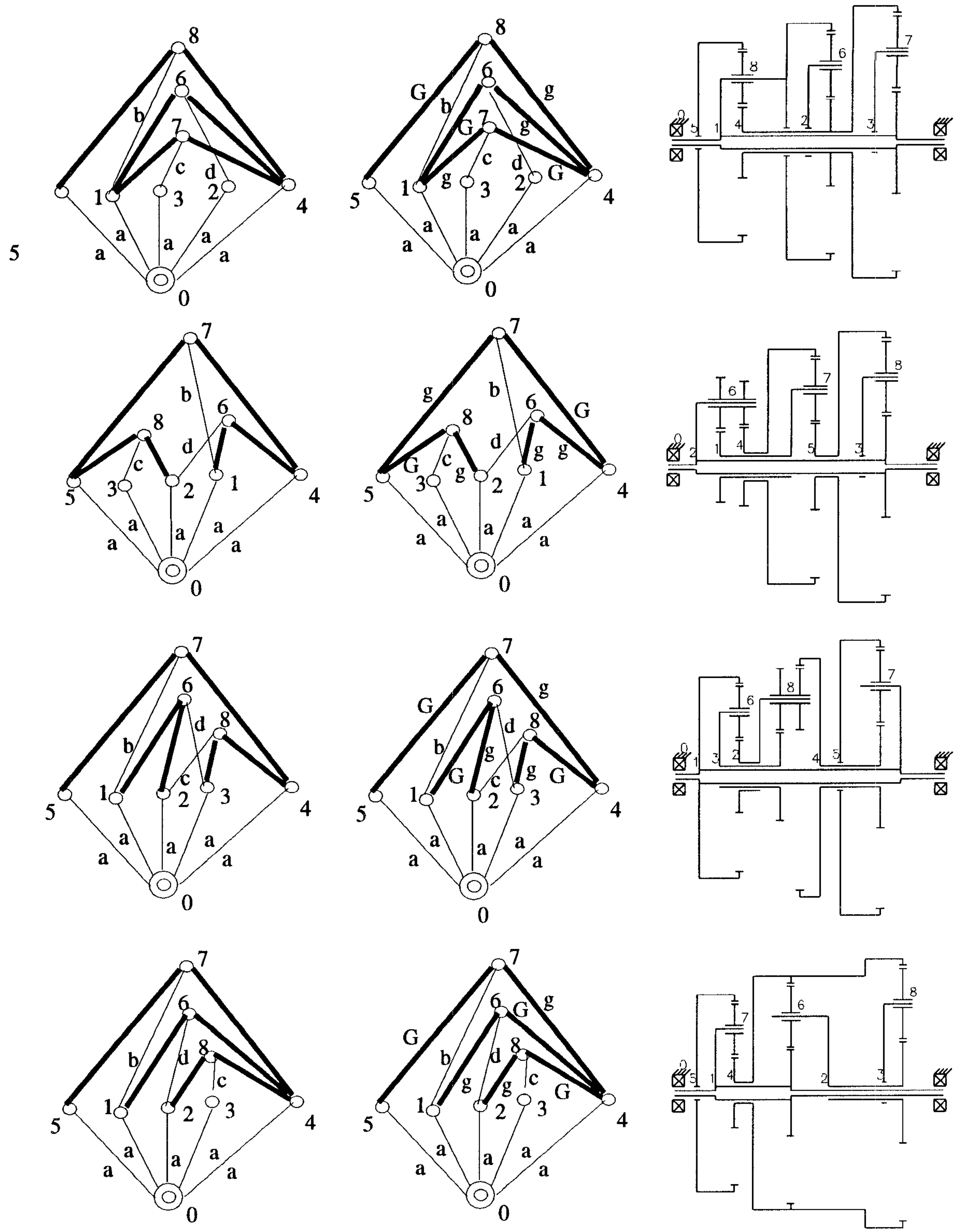


Figure D.1 (contd.)

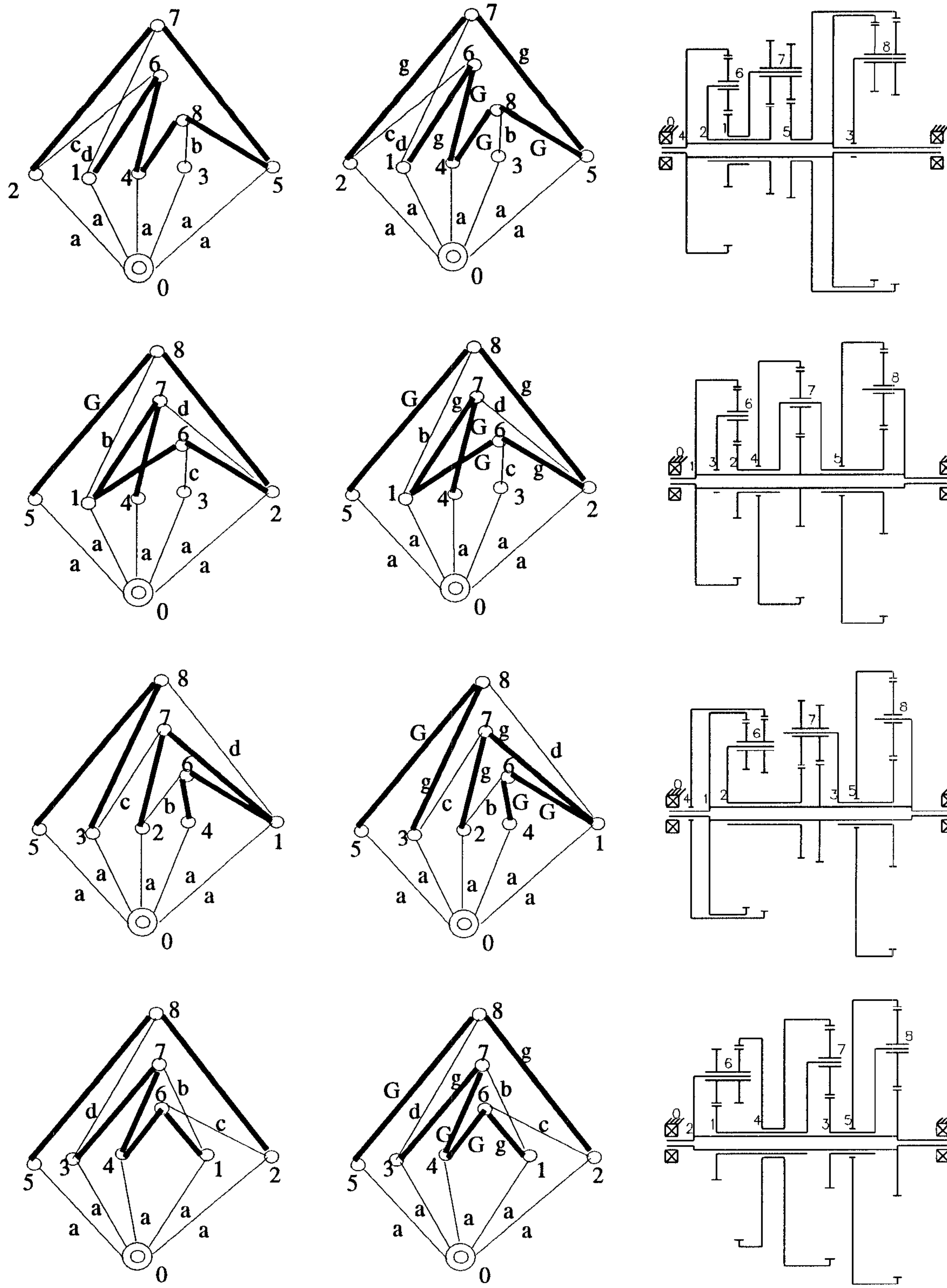


Figure D.1 (contd.)

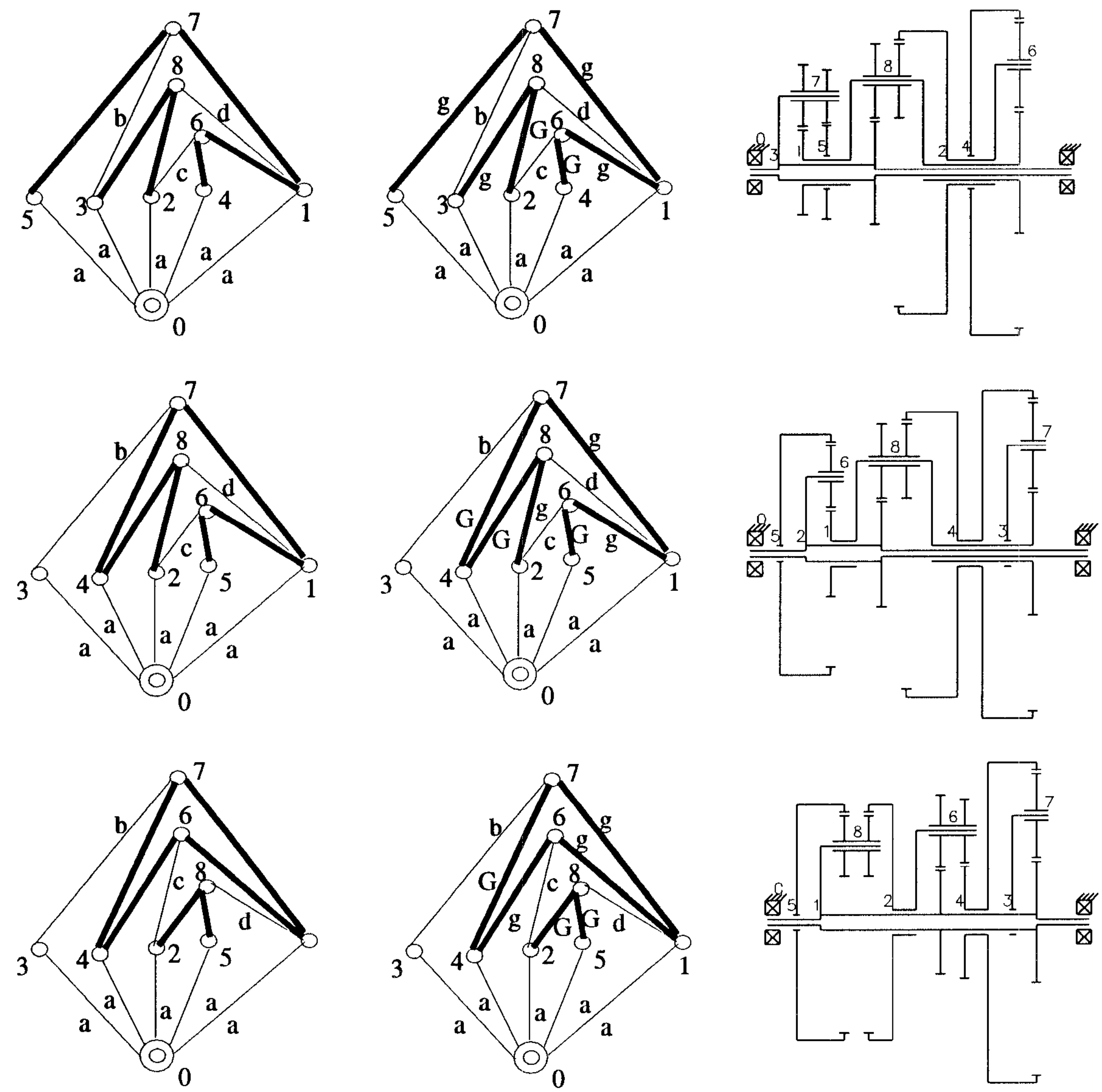


Figure D.1 (contd.)

Table D.1 Adjacency matrices of the remaining graphs of 9-link EGMs

$$\begin{matrix} 0&a&a&a&a&a&a&a&0\\ a&0&0&0&0&0&0&0&b\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&g&g&g&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&b\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&0&0&0&0\\ 0&b&g&0&0&g&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&b\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&g&0&0&0\\ 0&b&g&0&0&0&g&0&0 \end{matrix}$$

$$\begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&c\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&0&0&0&0\\ 0&c&g&0&0&g&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&c\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&g&0&0&0\\ 0&c&g&0&0&0&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&c&b\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&c&g&g&g&g&0&0&g\\ 0&b&0&0&0&0&g&g&0 \end{matrix}$$

$$\begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&c\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&b&g&g&g&0&0&0&g\\ 0&c&0&0&0&g&g&g&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&g&c\\ a&0&0&0&0&0&0&b&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&g&b&g&g&0&0&0&0\\ 0&c&g&0&0&g&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&g&c\\ a&0&0&0&0&0&0&b&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&g&b&g&g&g&0&0&0\\ 0&c&g&0&0&0&g&0&0 \end{matrix}$$

$$\begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&g&b\\ a&0&0&0&0&0&0&c&0\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&g&c&g&g&g&0&0&0\\ 0&b&0&g&0&0&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&g&b\\ a&0&0&0&0&0&0&c&0\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ a&0&0&0&0&0&0&0&g\\ 0&g&c&g&g&0&0&0&0\\ 0&b&0&g&0&g&g&0&0 \end{matrix} \qquad \begin{matrix} 0&a&a&a&a&a&a&0&0\\ a&0&0&0&0&0&0&b&0\\ a&0&0&0&0&0&0&0&c\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&g\\ a&0&0&0&0&0&0&g&0\\ a&0&0&0&0&0&0&0&g\\ 0&b&0&g&g&g&0&0&0\\ 0&0&c&g&g&0&g&0&0 \end{matrix}$$

```
0 a a a a a a 0 0    0 a a a a a a 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 0 b g    a 0 0 0 0 0 0 b 0    a 0 0 0 0 0 b b c
a 0 0 0 0 0 0 0 c    a 0 0 0 0 0 0 0 c    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g g    a 0 0 0 0 0 0 g g    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g g    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    0 b g g g 0 0 0 g
0 b 0 g g g g 0 0    0 b 0 g g g g 0 0    0 b 0 0 0 g 0 0 g
0 g c g 0 0 0 0 0    0 0 c g g 0 0 0 0    0 c 0 0 0 0 g g 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g
0 b g g 0 0 0 0 g    0 b g 0 0 0 0 0 g    0 b g g 0 0 0 0 g
0 b 0 0 g g 0 0 g    0 b 0 g 0 0 0 0 g    0 b 0 0 g 0 0 0 g
0 c 0 0 0 0 g g 0    0 c 0 0 g g g g 0    0 c 0 0 0 g g g 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c
a 0 0 0 0 0 g g 0    a 0 0 0 0 0 g g 0    a 0 0 0 0 0 0 g g
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0
0 b g 0 0 0 0 0 g    0 b g g 0 0 0 0 g    0 b 0 g g 0 0 0 g
0 b g g g 0 0 0 0    0 b g 0 g 0 0 0 0    0 b g 0 0 g 0 0 0
0 c 0 0 0 g g 0 0    0 c 0 0 0 g g 0 0    0 c g 0 0 0 g 0 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c    a 0 0 0 0 0 b b c
a 0 0 0 0 0 g g 0    a 0 0 0 0 0 0 g g    a 0 0 0 0 0 0 g g
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0
0 b g 0 0 0 0 0 g    0 b 0 g 0 0 0 0 g    0 b 0 g 0 0 0 0 g
0 b g g 0 0 0 0 0    0 b g 0 g 0 0 0 0    0 b g 0 g g 0 0 0
0 c 0 0 g g g 0 0    0 c g 0 0 g g 0 0    0 c g 0 0 0 g 0 0
```

Table D.1 (contd.)

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	b	d
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	c	g	g	0	0	0	g	g
0	b	0	0	g	0	g	0	0
0	d	0	0	0	g	g	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	d	b
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	c	0	0	0	0	0	g	g
0	d	g	g	g	0	g	0	0
0	b	0	0	0	g	g	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	b	d
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
a	0	0	0	0	0	0	0	g
0	c	0	0	0	0	0	g	g
0	b	g	g	0	0	g	0	0
0	d	0	0	g	g	g	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	d	b
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	c	g	0	0	0	0	g	g
0	d	0	g	g	0	g	0	0
0	b	0	0	0	g	g	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	d	c	b
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
a	0	0	0	0	0	0	0	g
0	d	g	0	0	0	0	g	0
0	c	0	g	0	0	g	0	0
0	b	g	0	g	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	d	c	b
a	0	0	0	0	0	0	g	g
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	0	g
0	d	0	g	g	0	0	g	0
0	c	g	0	0	0	g	0	0
0	b	g	0	0	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	d	b
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	c	g	g	0	0	0	g	0
0	d	0	0	g	0	g	0	0
0	b	g	0	0	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	d	0
a	0	0	0	0	0	0	0	b
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	0	g	0
0	c	0	g	g	0	0	g	0
0	d	0	0	0	g	g	0	0
0	0	b	g	g	0	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	c	d	0
a	0	0	0	0	0	g	0	b
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	c	g	g	0	0	0	g	0
0	d	0	0	g	0	g	0	0
0	0	b	g	0	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	d	c	g
a	0	0	0	0	0	g	0	b
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
a	0	0	0	0	0	0	0	g
0	d	g	0	0	0	0	g	0
0	c	0	g	0	0	g	0	0
0	g	b	0	g	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	d	c	0
a	0	0	0	0	0	g	0	b
a	0	0	0	0	0	0	g	g
a	0	0	0	0	0	0	0	g
a	0	0	0	0	0	0	0	g
0	d	g	0	0	0	0	g	0
0	c	0	g	0	0	g	0	0
0	0	b	g	g	g	0	0	0

0	a	a	a	a	a	0	0	0
a	0	0	0	0	0	d	c	g
a	0	0	0	0	0	g	0	b
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	d	g	g	0	0	0	g	0
0	c	0	0	g	0	g	0	0
0	g	b	0	0	g	0	0	0

Table D.1 (contd.)

```
0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 d c 0    a 0 0 0 0 0 d c g    a 0 0 0 0 0 c d 0
a 0 0 0 0 0 g 0 b    a 0 0 0 0 0 0 0 b    a 0 0 0 0 0 0 0 b
a 0 0 0 0 0 0 g g    a 0 0 0 0 0 g 0 g    a 0 0 0 0 0 g 0 g
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g g
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g
0 d g 0 g 0 0 g 0    0 d 0 g 0 0 0 g 0    0 c 0 g 0 0 0 g 0
0 c 0 g 0 0 g 0 0    0 c 0 0 g 0 g 0 0    0 d 0 0 g 0 g 0 0
0 0 b g 0 g 0 0 0    0 g b g 0 g 0 0 0    0 0 b g g g 0 0 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 d c g    a 0 0 0 0 0 d c g    a 0 0 0 0 0 d c 0
a 0 0 0 0 0 0 0 b    a 0 0 0 0 0 0 0 b    a 0 0 0 0 0 0 0 b
a 0 0 0 0 0 0 g g    a 0 0 0 0 0 g 0 g    a 0 0 0 0 0 g 0 g
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g g
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0
0 d 0 0 g g 0 g 0    0 d 0 g g 0 0 g 0    0 d 0 g 0 g 0 g 0
0 c 0 g 0 0 g 0 0    0 c 0 0 0 g g 0 0    0 c 0 0 g 0 g 0 0
0 g b g 0 0 0 0 0    0 g b g 0 0 0 0 0    0 0 b g g 0 0 0 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a a 0 0 0
a 0 0 0 0 0 c d g    a 0 0 0 0 0 c d 0    a 0 0 0 0 0 b b g
a 0 0 0 0 0 0 g b    a 0 0 0 0 0 0 g b    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 g    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0
0 c 0 g g 0 0 g 0    0 c 0 g g 0 0 g 0    0 b g g g 0 0 0 c
0 d g 0 0 0 g 0 0    0 d g 0 0 0 g 0 0    0 b 0 0 0 g 0 0 g
0 g b 0 0 g 0 0 0    0 0 b g 0 g 0 0 0    0 g 0 0 0 0 c g 0

0 a a a a a 0 0 0    0 a a a a a 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 0 b b g    a 0 0 0 0 0 b b g    a 0 0 0 0 c c b b
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 g 0 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 0 g
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    0 c g 0 0 0 0 g g
0 b g 0 0 0 0 0 c    0 b g g 0 0 0 0 c    0 c 0 g 0 0 0 g 0
0 b 0 g g g 0 0 g    0 b 0 0 g g 0 0 g    0 b 0 0 0 g g 0 0
0 g 0 0 0 0 c g 0    0 g 0 0 0 0 c g 0    0 b 0 0 g g 0 0 0
```

Table D.1 (contd.)

```
0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 c c b b    a 0 0 0 0 b b c c    a 0 0 0 0 b b c c
a 0 0 0 0 0 g 0 0    a 0 0 0 0 g g 0 0    a 0 0 0 0 g 0 g 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g
0 c 0 0 0 0 0 g g    0 b g 0 0 0 0 g 0    0 b g 0 0 0 0 0 g
0 c g g 0 0 0 g 0    0 b g 0 0 0 0 0 g    0 b 0 g 0 0 0 g 0
0 b 0 0 0 g g 0 0    0 c 0 g 0 g 0 0 0    0 c g 0 0 0 g 0 0
0 b 0 0 g g 0 0 0    0 c 0 0 g 0 g 0 0    0 c 0 0 g g 0 0 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b c c    a 0 0 0 0 b b b c    a 0 0 0 0 b b b c
a 0 0 0 0 0 g g 0    a 0 0 0 0 g 0 0 0    a 0 0 0 0 g 0 g 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
0 b 0 0 0 0 0 g g    0 b g 0 0 0 0 0 g    0 b g 0 0 0 0 0 g
0 b g g 0 0 0 0 0    0 b 0 g 0 0 0 0 g    0 b 0 g 0 0 0 0 g
0 c g 0 0 g 0 0 0    0 b 0 0 g 0 0 0 g    0 b g 0 g 0 0 0 0
0 c 0 0 g g 0 0 0    0 c 0 0 0 g g g 0    0 c 0 0 0 g g 0 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b c d    a 0 0 0 0 b b c d    a 0 0 0 0 b b c d
a 0 0 0 0 g 0 0 g    a 0 0 0 0 0 g 0 0    a 0 0 0 0 g 0 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g
0 b g 0 0 0 0 g 0    0 b 0 0 0 0 0 g g    0 b g 0 0 0 0 g g
0 b 0 g 0 0 0 g 0    0 b g g 0 0 0 g 0    0 b 0 g 0 0 0 g 0
0 c 0 0 0 g g 0 0    0 c 0 0 0 g g 0 0    0 c 0 0 0 g g 0 0
0 d g 0 g 0 0 0 0    0 d 0 0 g g 0 0 0    0 d 0 0 g g 0 0 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b c d    a 0 0 0 0 b b c d    a 0 0 0 0 b b c d
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 g 0 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g    a 0 0 0 0 0 0 0 g
0 b 0 0 0 0 0 g g    0 b 0 0 0 0 0 g g    0 b g 0 0 0 0 g 0
0 b g 0 0 0 0 g 0    0 b g 0 0 0 0 g 0    0 b 0 g 0 0 0 g 0
0 c 0 g 0 g g 0 0    0 c 0 0 0 g g 0 0    0 c 0 0 0 g g 0 g
0 d 0 0 g g 0 0 0    0 d 0 g g g 0 0 0    0 d 0 0 g 0 0 g 0
```

Table D.1 (contd.)

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	d
a	0	0	0	0	0	0	g	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
0	b	0	g	0	0	0	g	0
0	b	0	0	g	0	0	0	g
0	c	g	0	0	g	0	0	0
0	d	g	0	0	0	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	d
a	0	0	0	0	g	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	0	g
0	b	g	0	0	0	0	g	0
0	b	g	0	0	0	0	0	g
0	c	0	g	0	g	0	0	0
0	d	0	0	g	0	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	d	c
a	0	0	0	0	g	0	g	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	0	g
0	b	g	0	0	0	0	0	g
0	b	0	g	0	0	0	g	0
0	d	g	0	0	0	g	0	0
0	c	0	0	g	g	0	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	d	c
a	0	0	0	0	g	0	0	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
0	b	g	g	0	0	0	g	0
0	b	0	0	g	0	0	0	g
0	d	0	0	0	g	0	0	g
0	c	0	0	0	0	g	g	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	d	c
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	g	0
0	b	0	g	0	0	0	0	g
0	d	0	0	g	g	0	0	g
0	c	0	0	0	0	g	g	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	d	c
a	0	0	0	0	0	g	g	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	0	g
0	b	0	0	0	0	0	g	g
0	b	g	g	0	0	0	0	0
0	d	g	0	0	g	0	0	0
0	c	0	0	g	g	0	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	d
a	0	0	0	0	g	g	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	0	g
0	b	g	0	0	0	0	g	0
0	b	g	g	0	0	0	0	0
0	c	0	0	0	g	0	0	g
0	d	0	0	g	0	0	g	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	d
a	0	0	0	0	0	g	0	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
0	b	0	g	0	0	0	g	0
0	b	g	0	g	0	0	0	0
0	c	0	0	0	g	0	0	g
0	d	g	0	0	0	0	g	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	0
a	0	0	0	0	0	0	0	d
a	0	0	0	0	g	0	0	g
a	0	0	0	0	0	g	0	g
0	b	0	g	0	0	0	g	0
0	b	0	0	g	0	0	g	0
0	c	0	0	0	g	g	0	0
0	0	d	g	g	0	0	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	0
a	0	0	0	0	g	0	0	d
a	0	0	0	0	0	g	0	g
a	0	0	0	0	0	0	0	g
0	b	g	0	0	0	0	g	0
0	b	0	g	0	0	0	g	0
0	c	0	0	0	g	g	0	0
0	0	d	g	g	0	0	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	0	0	0	d
a	0	0	0	0	g	0	0	g
a	0	0	0	0	0	g	0	0
0	b	0	g	0	0	0	g	0
0	b	0	0	g	0	0	g	0
0	c	0	0	0	g	g	0	0
0	g	d	g	0	0	0	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	g	0	0	d
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	0	g
0	b	g	0	0	0	0	g	0
0	b	0	g	0	0	0	g	0
0	c	0	0	0	g	g	0	0
0	g	d	0	g	0	0	0	0

Table D.1 (contd.)

```
0 a a a a 0 0 0 0     0 a a a a 0 0 0 0     0 a a a a 0 0 0 0
a 0 0 0 0 c b d e     a 0 0 0 0 d c e b     a 0 0 0 0 d c e b
a 0 0 0 0 0 g 0 0     a 0 0 0 0 g 0 0 0     a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 g 0     a 0 0 0 0 0 0 g 0     a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 0 g
0 c 0 0 0 0 g g g     0 d g 0 0 0 g g 0     0 d 0 0 0 0 g g 0
0 b g 0 0 g 0 0 0     0 c 0 0 0 g 0 0 g     0 c 0 0 0 g 0 0 g
0 d 0 g 0 g 0 0 0     0 e 0 g 0 g 0 0 0     0 e g g 0 g 0 0 0
0 e 0 0 g g 0 0 0     0 b 0 0 g 0 g 0 0     0 b 0 0 g 0 g 0 0

0 a a a a 0 0 0 0     0 a a a a 0 0 0 0     0 a a a a 0 0 0 0
a 0 0 0 0 e c d b     a 0 0 0 0 d c e b     a 0 0 0 0 c b 0 0
a 0 0 0 0 g g 0 0     a 0 0 0 0 0 g 0 g     a 0 0 0 0 0 0 d e
a 0 0 0 0 0 0 g 0     a 0 0 0 0 0 0 g 0     a 0 0 0 0 g 0 g 0
a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 g 0 g
0 e g 0 0 0 0 g 0     0 d 0 0 0 0 g g 0     0 c 0 g 0 0 g 0 0
0 c g 0 0 0 0 0 g     0 c g 0 0 g 0 0 0     0 b 0 0 g g 0 0 0
0 d 0 g 0 g 0 0 0     0 e 0 g 0 g 0 0 0     0 0 d g 0 0 0 0 g
0 b 0 0 g 0 g 0 0     0 b g 0 g 0 0 0 0     0 0 e 0 g 0 0 g 0

0 a a a a 0 0 0 0     0 a a a a 0 0 0 0     0 a a a a 0 0 0 0
a 0 0 0 0 g 0 e d     a 0 0 0 0 g 0 b c     a 0 0 0 0 d c e 0
a 0 0 0 0 c b g 0     a 0 0 0 0 e d 0 0     a 0 0 0 0 0 0 0 b
a 0 0 0 0 0 g 0 0     a 0 0 0 0 0 g g 0     a 0 0 0 0 0 g 0 g
a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 g g
0 g c 0 0 0 g 0 0     0 g e 0 0 0 g 0 0     0 d 0 0 0 0 g g 0
0 0 b g 0 g 0 0 0     0 0 d g 0 g 0 0 0     0 c 0 g 0 g 0 0 0
0 e g 0 0 0 0 0 g     0 b 0 g 0 0 0 0 g     0 e 0 0 g g 0 0 0
0 d 0 0 g 0 0 g 0     0 c 0 0 g 0 0 g 0     0 0 b g g 0 0 0 0

0 a a a a 0 0 0 0     0 a a a a 0 0 0 0     0 a a a a 0 0 0 0
a 0 0 0 0 d e c 0     a 0 0 0 0 d c e g     a 0 0 0 0 d e c g
a 0 0 0 0 0 g 0 b     a 0 0 0 0 0 0 0 b     a 0 0 0 0 0 g 0 b
a 0 0 0 0 0 0 g g     a 0 0 0 0 0 g 0 g     a 0 0 0 0 0 0 g 0
a 0 0 0 0 0 0 0 g     a 0 0 0 0 0 0 g 0     a 0 0 0 0 0 0 0 g
0 d 0 0 0 0 g g 0     0 d 0 0 0 0 g g 0     0 d 0 0 0 0 g g 0
0 e g 0 0 g 0 0 0     0 c 0 g 0 g 0 0 0     0 e g 0 0 g 0 0 0
0 c 0 g 0 g 0 0 0     0 e 0 0 g g 0 0 0     0 c 0 g 0 g 0 0 0
0 0 b g g 0 0 0 0     0 g b g 0 0 0 0 0     0 g b 0 g 0 0 0 0
```

Table D.1 (contd.)

```
0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b b 0    a 0 0 0 0 b b b 0    a 0 0 0 0 b b b 0
a 0 0 0 0 0 g g 0    a 0 0 0 0 g g 0 0    a 0 0 0 0 g g 0 0
a 0 0 0 0 g 0 0 0    a 0 0 0 0 g 0 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 g 0 0 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
0 b 0 g g 0 0 0 c    0 b g g 0 0 0 0 c    0 b g 0 0 0 0 0 c
0 b g 0 0 0 0 0 g    0 b g 0 0 0 0 0 g    0 b g g 0 0 0 0 g
0 b g 0 0 0 0 0 g    0 b 0 0 g 0 0 0 g    0 b 0 0 g 0 0 0 g
0 0 0 0 0 c g g 0    0 0 0 0 0 c g g 0    0 0 0 0 0 c g g 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b b 0    a 0 0 0 0 b b b 0    a 0 0 0 0 b b b g
a 0 0 0 0 g 0 g 0    a 0 0 0 0 0 g g 0    a 0 0 0 0 0 g g 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 g 0 0 0    a 0 0 0 0 g 0 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 0 g 0
0 b g 0 0 0 0 0 c    0 b 0 g 0 0 0 0 c    0 b 0 g 0 0 0 0 c
0 b 0 g g 0 0 0 g    0 b g 0 g 0 0 0 g    0 b g 0 0 0 0 0 g
0 b g 0 0 0 0 0 g    0 b g 0 0 0 0 0 g    0 b g 0 g 0 0 0 0
0 0 0 0 0 c g g 0    0 0 0 0 0 c g g 0    0 g 0 0 0 c g 0 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b b g    a 0 0 0 0 b b b g    a 0 0 0 0 b b b g
a 0 0 0 0 g 0 g 0    a 0 0 0 0 g 0 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
0 b g 0 0 0 0 0 c    0 b g 0 0 0 0 0 c    0 b 0 0 0 0 0 0 c
0 b 0 g 0 0 0 0 g    0 b 0 g 0 0 0 0 g    0 b g g 0 0 0 0 g
0 b g 0 g 0 0 0 0    0 b 0 0 g 0 0 0 g    0 b 0 0 g 0 0 0 g
0 g 0 0 0 c g 0 0    0 g 0 0 0 c g g 0    0 g 0 0 0 c g g 0

0 a a a a 0 0 0 0    0 a a a a 0 0 0 0    0 a a a a 0 0 0 0
a 0 0 0 0 b b c g    a 0 0 0 0 b b c g    a 0 0 0 0 b b c g
a 0 0 0 0 0 g g 0    a 0 0 0 0 g 0 g 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 g 0 0 0    a 0 0 0 0 0 g 0 0    a 0 0 0 0 0 g 0 0
a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0    a 0 0 0 0 0 0 g 0
0 b 0 g 0 0 0 0 d    0 b g 0 0 0 0 0 d    0 b 0 0 0 0 0 g d
0 b g 0 0 0 0 0 g    0 b 0 g 0 0 0 0 g    0 b g g 0 0 0 0 g
0 c g 0 g 0 0 0 0    0 c g 0 g 0 0 0 0    0 c 0 0 g g 0 0 0
0 g 0 0 0 d g 0 0    0 g 0 0 0 d g 0 0    0 g 0 0 0 d g 0 0
```

Table D.1 (contd.)

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	g	d
0	b	0	g	0	0	0	0	g
0	c	0	0	g	g	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
0	b	0	0	0	0	0	g	d
0	b	g	0	0	0	0	0	g
0	c	0	g	g	g	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	0	g	0
0	b	g	g	0	0	0	0	d
0	b	0	0	0	0	0	g	g
0	c	0	0	g	0	g	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	0	d
0	b	0	g	0	0	0	g	g
0	c	0	0	g	0	g	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	c	g
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	0	g	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	0	d
0	b	0	0	0	0	0	g	g
0	c	0	g	g	0	g	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	0	g
a	0	0	0	0	0	0	c	0
a	0	0	0	0	g	0	g	0
a	0	0	0	0	0	g	g	0
0	b	0	g	0	0	0	0	d
0	b	0	0	g	0	0	0	g
0	0	c	g	g	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	0	g
a	0	0	0	0	g	0	c	0
a	0	0	0	0	0	g	g	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	0	d
0	b	0	g	0	0	0	0	g
0	0	c	g	g	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	0	g
a	0	0	0	0	0	g	c	0
a	0	0	0	0	g	0	g	0
a	0	0	0	0	0	0	g	0
0	b	0	g	0	0	0	0	d
0	b	g	0	0	0	0	0	g
0	0	c	g	g	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	g
a	0	0	0	0	0	0	c	0
a	0	0	0	0	0	g	g	0
a	0	0	0	0	g	0	0	0
0	b	0	0	g	0	0	0	d
0	b	0	g	0	0	0	0	g
0	g	c	g	0	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	g
a	0	0	0	0	0	0	c	0
a	0	0	0	0	g	0	g	0
a	0	0	0	0	0	g	0	0
0	b	0	g	0	0	0	0	d
0	b	0	0	g	0	0	0	g
0	g	c	g	0	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	g
a	0	0	0	0	g	0	c	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	0	g	0
0	b	g	0	0	0	0	0	d
0	b	0	g	0	0	0	0	g
0	g	c	0	g	0	0	0	0
0	g	0	0	0	d	g	0	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	g
a	0	0	0	0	0	g	c	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	0	g	0
0	b	0	g	0	0	0	0	d
0	b	g	0	0	0	0	0	g
0	g	c	0	g	0	0	0	0
0	g	0	0	0	d	g	0	0

Table D.1 (contd.)

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
0	b	g	g	0	0	0	d	c
0	b	0	0	g	0	0	0	g
0	g	0	0	0	d	0	0	g
0	0	0	0	0	c	g	g	0

0	a	a	a	a	0	0	0	0
a	0	0	0	0	b	b	g	0
a	0	0	0	0	g	0	0	0
a	0	0	0	0	0	g	0	0
a	0	0	0	0	0	g	0	0
0	b	g	0	0	0	0	d	c
0	b	0	g	g	0	0	0	g
0	g	0	0	0	d	0	0	g
0	0	0	0	0	c	g	g	0

Table D.1 (contd.)

Bibliography

Ambedkar, A.G., and Agrawal, V.P., 1987, "Canonical Numbering of Kinematic Chains and Isomorphism Problem: MIN Code," *Journal of Mechanism and Machine Theory*, Vol. 22, No. 5, pp. 453-461.

Buchsbaum, F., and Freudenstein, F., 1970, "Synthesis of Kinematic Structure of Geared Kinematic Chains and Other Mechanisms," *Journal of Mechanisms and Machine Theory*, Vol. 5, pp. 357-392.

Chang , S.L., and Tsai, L.W., 1990, "Topological Synthesis of Articulated Gear Mechanisms," *IEEE Transaction on Robotics and Automation*, Vol. 6, No. 1, pp. 97-103.

Chieng, W.H., and Hoeltzel, D.A., 1990, "A Combinatorial Approach for the Automatic Sketching of Planar Kinematic Chains and Epicyclic Gear Trains," *ASME Journal of Mechanical Design*, Vol. 112, pp. 6-15.

Erjavek, J., 1990, *Manual Transmissions and Transaxles*, Harcourt Brace Jovanovich, New York.

Freudenstein, F., 1971, "An Application of Boolean Algebra to the Motion of Epicyclic Drives," *ASME Journal of Engineering for Industry*, Vol. 93, pp. 176-182.

Gott, P. G., 1991, *Changing Gears*, Society of Automotive Engineeers, Inc., Warrendale, Pennsylvania.

Hall, A.S., and Yan, H.S., 1982, "Linkage Characteristic Polynomials: Assembly Theorems, Uniqueness," *ASME Journal of Mechanical Design*, Vol. 104, pp. 11-20.

Harary, F., 1969, *Graph theory*, Addison-Wesley, Reading, Massachusetts.

Hayes-Roth, F., Waterman, D.A., and Lenat, D.B., 1983 *Building Expert Sustems*, Reading, Massachusetts.

Hsu, C.H., and Lam, K.T., 1992, "A New Graph Representation for the Automatic Kinematic Analysis of Planetary Spur-Gear Trains," *ASME Journal of Mechanical Design*, Vol. 114, pp. 196-200.

Hsu, C.H., 1992 "An Application of Generalized Kinematic Chains to The Structural Synthesis of Non-Fractionated Epicyclic Gear Trains," *Proceeding of the* 22nd *Biennial ASME Mechanisms Conference*, Scottsdale, Arizona, DE-Vol. 46, pp. 451-458.

Kim, J.T., and Kwak, B.M., 1990, "Application of Edge Permutation Group to Structural Synthesis of Epicyclic Gear Trains," *Journal of Mechanisms and Machine Theory*, Vol. 25, No. 5, pp. 563-574.

Kim, J.T., and Kwak, B.M., 1992, "An Algorithm of Topological Ordering for Unique Representation of Graphs," *ASME Journal of Mechanical Design*, Vol. 114, pp. 103-108.

Larew, W. B., 1966, *Automatic Transmissions*, Chilton Book Company, Radnor, Pennsylvania.

Levai, Z., 1964, *Theory of Epicyclic Gears and Epicyclic Change-Speed Gears*, Doctoral Dissertation, Budapest.

Lin, C.C., and Tsai, L.W., 1989, "The Development of an Atlas of Bevel-Gear-Type Spherical Wrist Mechanisms," *Proceeding of the* 1^{st} *National Conference on Applied Mechanisms and Robotics*, Paper No. 89-AMR-7A-6, Cincinati, Ohio.

Liu, C.T., *Introduction to Combinatorial Mathematics*, Mcgraw-Hill, New York.

Mogalapalli, S.N., Magrab, E.B., and Tsai, L.W., 1993, "A Cad System for the Optimization of Gear Ratios for Automotive Automatic Transmissions," SAE Technical Paper # 930675, *SAE International Congress and Exposition*, Detroit, Michigan.

Mruthyunjaya, T.S., and Balasubramanian, H.R., 1987, "In Quest of a Reliable and Efficient Computational Test for Detection of Isomorphism in Kinematic Chains," *Journal of Mechanisms and Machine Theory*, Vol. 22, No. 2, pp. 131-139.

Mruthyunjaya, T.S., and Ravisankar, R., 1985, "Computerized Synthesis of the Structure of Geared Kinematic Chains," *Journal of Mechanisms and Machine Theory*, Vol. 20, pp. 367-387.

Olson, D.G., Erdman, A.G., and Riley, D.R., 1991, "Toplogical Analysis of Single-Degree-of-Freedom Planetary Gear Trains," *ASME Journal of Mechanical Design*, Vol. 113, pp. 10-16.

Pennestri, E., and Freudenstein, F., 1990, "A Systematic Approach to Power-Flow and Static-Force Analysis in Epicyclic Spur-Gear Trains," *Proceeding of the* 21^{st} *Biennial ASME Mechanisms Conference*, Chicago, Illinois, DE-Vol. 26, pp. 63-70.

Sohn, W., and Freudenstein, F., 1986, "An Application of Dual Graphs to the Automatic Creation of the Kinematic Structure of Mechanisms," *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 108, pp. 392-398.

Tang, C.S. and Liu, T., 1988, "Degree Code - A New Mechanisms Identifier," *Proceeding of the* 20^{th} *Biennial ASME Mechanisms Conference*, Kissimmee, Florida, DE-Vol. 15-1, pp. 147-151.

Tsai, L.W., 1987, "An Application of Linkage Characteristic Polynomial to the Topological Synthesis of Epicyclic Gear Trains," *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 109, No. 3, pp. 329-336.

Tsai, L.W., 1988, "The Kinematics of Spatial Robotic Bevel-Gear Trains," *IEEE Journal of Robotics and Automation*, Volume 4, pp. 150-156.

Tsai, L.W., Maki, E.R., Liu, T., and Kapil, N.G., 1988, "The Categorization of Planetary Gear Trains for Automatic Transmissions According to Kinematic Topology," SAE Technical Paper # 885602, *Proceeding of the XXII FISTA Congress and Exposition*, Vol. 1., pp. 1513-1512.

Tsai, L.W., and Lin, C.C. 1989, "The Creation of Non-Fractionated Two-Degree-of-Freedom Epicyclic Gear Trains," *ASME J. of Mechanisms, Transmissions, and Automation in Design*, Vol. 111, pp. 524-529.

Uicker, J.J.,Jr., and Raicu, A., 1975, "A Method for the Identification and Recognition of Equivalence of Kinematic Chains," *Journal of Mechanisms and Machine Theory*, Vol. 10, No. 5-B, pp. 375-381.

Yan, H.S., and Hwang, Y.W., 1991, "The Specialization of mechanisms," *Mechanisms and Machine Theory*, Vol. 26, No. 6, pp. 541-551.