"Acquired" Equals Addition? Associating Verbs with Arithmetic Operations Impacts Word Problem Performance

Joshua Benjamin Jaffe, Troy Gharibani, and Donald Joseph Bolger

ABSTRACT— Successful word problem performance often requires understanding the linguistic relations between characters and objects. However, the keyword method promotes associating specific words with mathematical operations while neglecting the situational context. Research has thoroughly investigated the detrimental effects of individuals associating relational terminology (e.g., "more") with mathematical operations (e.g., "addition"). The current study expands upon this line of research by examining whether undergraduate students associate verbs with mathematical operations and if verbal associations affect word problem performance. Similar to relational terminology, the participants associated verbs with operations, which significantly impacted performance. The educational implications are discussed.

Word problems (WP), numerical problems that integrate non-mathematical language, are at the intersection of two domains in education: mathematics and language. Classified as mathematical problems and predominantly taught through mathematics and science courses, empirical evidence has suggested some classrooms place more emphasis on the numerical and calculation components than on the linguistic component (Chapman, 2006; Depaepe, de Corte, & Verschaffel, 2010). This has led to teachers emphasizing (Chapman, 2006; Rosales, Vicente, Chamoso, Muñez, & Orrantia, 2012), and students using (Hegarty, Mayer, & Monk, 1995), suboptimal strategies to understand the WP narrative.

The keyword method, a popular classroom strategy, often impedes individuals from gaining an appropriate understanding of the linguistic component (Karp, Bush, & Dougherty, 2019). Rather than identifying the semantic relationships between characters and objects, individuals who use this method mentally highlight the word(s) that they believe represents the mathematical operation (Briars & Larkin, 1984). This approach involves individuals associating specific words with specific operations. For instance, individuals may associate "more" or "altogether" with addition, and "less" with subtraction, although these words can represent either operation depending on their relationship with the characters and objects.

The usage of this method may be a result of both classroom instruction (Chapman, 2006; Pearce, Bruun, Skinner, & Lopez-Mohler, 2013) and the problems students encounter (Verschaffel, Greer, & de Corte, 2000). Verschaffel, Schukajlow, Star, and van Dooren (2020) suggest that solvers may struggle to inhibit their operational associations because curricula provide students with problems where the keyword method regularly produces correct answers. Research involving the keyword method has predominantly investigated relational terminology (e.g., more, less, and fewer) due to the consensus and strong association with the perceived mathematical operation. Empirical studies have consistently identified the detrimental effects of using this method with relational terminology problems (Hegarty et al., 1995; Hegarty, Mayer, & Green, 1992; van der Schoot, Bakker Arkema, Horsley, & van Lieshout, 2009). However, similar studies have not been conducted with verbs (e.g., received, took, and gave).

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The current study further develops WP research to examine whether individuals form operational associations with verbs and if verbal associations affect WP performance. First, many WPs do not contain relational terminology but contain verbs. For instance, Change problems—John has six apples. Mary gave John five apples. How many apples does John have now? (Riley, Greene, & Heller, 1984) exclusively use verbs. Therefore, it is essential to understand if individuals form verbal associations with mathematical operations. Second, theoretical frameworks have indirectly suggested that individuals can form operational associations with verbs. Reusser's (1990) Situation Problem-Solver model posits that mentally representing the WP narrative from the perspective of the main character can enhance an individual's situational understanding. This model, coupled with the keyword method, suggests that solvers may associate verbs with the main character performing the action (e.g., "received" could be interpreted as the main character receiving objects, resulting in "received" being associated with addition). Lastly, WP intervention studies focused on relational terminology problems have shown to significantly improve WP performance by helping students understand the situational importance of the narrative rather than using the keyword method (Fuchs, Fuchs, Seethaler, & Craddock, 2019; Schumacher & Fuchs, 2012). These types of interventions are only possible by first understanding that individuals form operational associations with keywords. If similar associations are seen with verbs, similar interventions could remediate the negative effects of associating verbs with mathematical operations and subsequently improve WP performance.

The present study parallels relational terminology studies (e.g., Hegarty et al., 1992, 1995; Kelly, Davis, Mousley, & Lang, 2003; Lewis & Mayer, 1987) by examining verbal associations with an undergraduate population. The purpose of this study is to identify if students, who have had years of educational exposure to WPs, form verbal associations with operations and examine what effect this may have on performance.

METHODS

Participants

A total of 88 participants were recruited to participate in this study. All participants were recruited through undergraduate human development courses at the University of Maryland College Park. Participation in this study was voluntary and all participants were compensated with course extra credit. Of the original sample, four participants were removed due to abnormally high error rates (less than 50% accuracy). Overall, 84 participants (*Mage* = 20.16, *SD* = 2.30; 67 Female) were included for analysis. Demographics were self-reported as 58% White, 25% Asian, 7% Hispanic, 5% Black/African-American, and 5% as multiple racial identities. All participants reported English as their native language. About 68% of participants indicated that they completed either a college-level course in high school (e.g., Advanced Placement or International Baccalaureate) or a college course as their highest-level mathematics class.

The ethics committee at the University of Maryland College Park approved the current research project. All participants gave informed written consent prior to participating in the study.

Table 1
Example of word problems and verbs used in the study

Example of word problems
1) John has 35 apples. John acquired 23 apples from Mary. How many apples does John have now?
2) John has 64 toys. Mary received 27 toys from John. How many toys does John have now?
Verbs used in word problems
Acquired, commandeered, confiscated, forfeited, gave, got, handed, passed, received, relinquished, surrendered, took

Procedure

Participants completed the study during the 2021 spring and fall semesters online via Qualtrics XM survey software (Qualtrics, Provo, UT). First, participants completed 96 one-step WPs designed for the study (one problem was excluded from the analysis due to technical issues). According to Riley et al.'s (1984) categorization criteria, problems were classified as either Change 1 WPs or Change 2 WPs. All problems contained two double-digit numbers and all problems resulted in a double-digit resultant. Overall, 12 verbs were used in the study, with each verb being used in eight problems (four addition problems and four subtraction problems; Table 1). The problem order was randomized.

After completing the WPs, participants were presented with each verb and instructed to self-report if they comprehended the verb and if they associated the verb with representing an addition problem, or subtraction problem, or had no operational judgment toward the verb. Problems and self-reports where the participants reported they did not comprehend the verb were excluded from the analysis.

RESULTS

The results were broken down into three sections. First, we provided descriptive statistics for the WPs and self-report verbal associations. Next, we conducted a chi-square goodness of fit test to examine the incongruent errors (i.e., errors

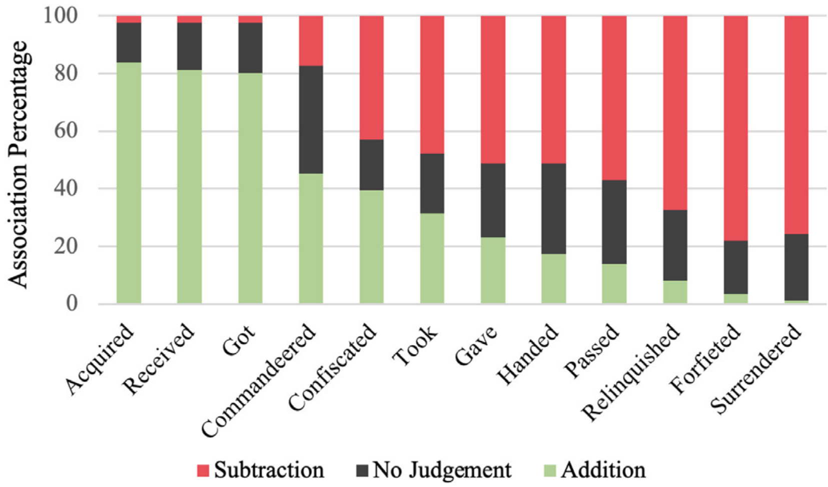

Fig. 1. Self-reported associations between verbs and mathematical operations.

where solvers used the opposite operation), and determine if they were more likely to occur on problems where the verbal association is consistent with a problem's operation compared with when the verbal association is inconsistent with a problem's operation. Lastly, paired sample t-tests were used to examine performance and response times for consistent verbal association problems compared with inconsistent verbal association problems.

Overall, participants exhibited 91.0% accuracy ($SD = 0.08$) and had an average response time of 12.95 s ($SD = 3.26$). Focusing on the 9.0% of problems that elicited incorrect responses, 53.5% were miscalculation errors (i.e., all incorrect responses that were not incongruent errors; e.g., $10 + 4 = 19$) and 46.5% were incongruent errors. Regarding self-reports, 75.6% of all responses indicated an operational association toward a verb (i.e., addition or subtraction). 24.4% of responses indicated no operational judgment toward a verb. Prior to analysis, the researchers developed categories to classify verbs: consensus and non-consensus. In consensus, verbs over 60% of participants associate the verb with one operation and less than 10% associate the verb with the opposite operation. Non-consensus verbs did not reach this threshold. Overall, three verbs were categorized as a consensus for addition (i.e., got, received, and acquired), three verbs were categorized as a consensus for subtraction (i.e., relinquished, surrendered, and forfeited), and the remaining six verbs were categorized as non-consensus (Figure 1).

A chi-square goodness of fit test was conducted to determine whether verbal associations could predict the type of incongruent errors above chance. To perform this test, incongruent errors were broken down into two categories: (a) incongruent errors consistent with the perceived operation (e.g., an individual associate "acquired" with addition, the problem used acquired in a subtraction problem, the individual used addition as the operation) and (b) incongruent errors inconsistent with the perceived operation (e.g., an individual associate "acquired" with addition, the problem used acquired in an addition problem, the individual used subtraction as the operation). Incongruent errors where participants indicated they did not associate the verb with an operation were excluded from this analysis. This allowed us to focus on examining the effects of associating verbs with operations. For this analysis, all incongruent errors were matched with each individual's self-reports and categorized. Overall, the test indicated that errors did differ by category, $X^2(1) = 10.86, p < .001$. The number of errors consistent with the perceived operation was significantly above chance, suggesting that verbal associations could predict the type of incongruent errors (Figure 2).

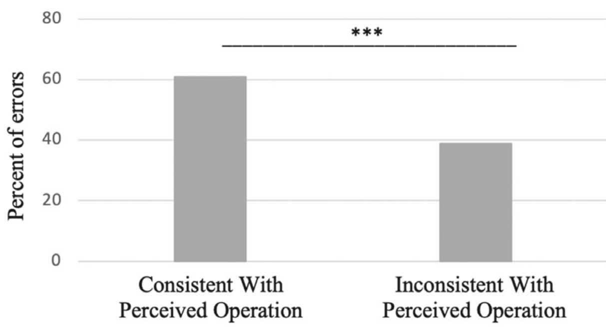

Fig. 2. Percentage of incongruent errors by category. *** Indicates $p < .001$.

Paired sample t-tests were used to compare accuracy and response times between consistent verbal association problems and inconsistent verbal association problems. Paralleling relational terminology work (e.g., Hegarty et al., 1992, 1995), this analysis focused on verbs that were categorized as having an operational consensus. This

Table 2
Categorization of consensus—verb word problems by verb and arithmetic operation

Category one (Consistent)	*Category two (Inconsistent)*
Acquired—Addition problem	Acquired—Subtraction problem
Received—Addition	Received—Subtraction
Got—Addition	Got—Subtraction
Relinquished—Subtraction	Relinquished—Addition
Forfeited—Subtraction	Forfeited—Addition
Surrendered—Subtraction	Surrendered—Addition

allowed us to separate questions into two categories by using the participants' overall verbal association consensus as guidelines. Category one consists of problems where the verbal association is consistent with the problem's operation. Category two consists of problems where the verbal association is inconsistent with the problem's operation (see Table 2 for the breakdown of each category). The accuracy results suggest that individuals performed significantly better on consistent verbal association problems ($M = 0.92$, $SD = 0.09$) compared with inconsistent verbal association problems ($M = 0.89$, $SD = 0.12$), $t(83) = 2.54$, $p < .05$. An examination of response times identified no significant differences between consistent verbal association problems ($M = 12.83$, $SD = 3.38$) and inconsistent verbal association problems ($M = 13.01$, $SD = 3.19$), $t(83) = 0.86$, $p = .39$ (Figure 3).

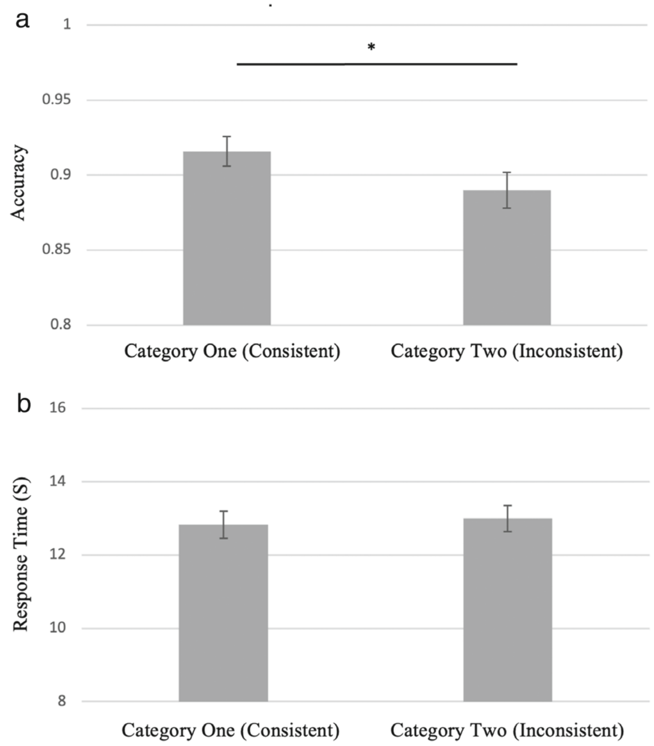

Fig. 3. (a) Accuracy and (b) response time results for consensus verb word problems. * Indicates $p < .05$.

DISCUSSION

The current study showed that individuals associate verbs with mathematical operations. Half of the verbs were categorized as receiving an operational consensus. Supporting Reusser's (1990) Situation Problem-Solver model, verbs that involve the action of taking possession (e.g., received and acquired) were generally reported to represent addition while verbs that involve losing possession (e.g., forfeited and surrendered) were reported to represent subtraction.

Examination of the incongruent errors and consensus verb WPs suggest that verbal associations may impact performance. Participants made significantly more incongruent errors that were consistent with their perceived operation and showed stronger performance on consistent verbal association problems. There were no response time differences for the consensus verb WPs which may suggest some usage of the keyword method (Hegarty et al., 1992). The keyword method promotes the identification of specific words rather than understanding the situational narrative. Therefore, no time differences would be expected between consistent and inconsistent problems.

Associating words with mathematical operations can have a detrimental impact on performance (Hegarty et al., 1995). This has led researchers and practitioners to develop lessons that dissuade this strategic approach (Fuchs et al., 2019). However, these lessons can only be implemented by first understanding the operational associations students possess. Powell, Namkung, and Lin (2022) argue that all teachers should be aware of the connections students make between words and operations. With this knowledge, teachers can customize lessons to focus on the structure of the problem and present problems that cannot be solved correctly by using the keyword method. This study helped identify operational associations beyond relational terminology. While the participants exhibited strong accuracy on all WPs, approximately half of the incorrect responses were classified as incongruent errors, with a significant proportion being consistent with their verbal operational associations.

Overall, the present study exhibited promising results. However, it does come with limitations and warrants further research. First, more structured methods could be used to examine verbal operational associations, rather than a three-choice forced response. Structured interviews could give valuable insight into why individuals associate verbs with operations. Second, while the results suggest some usage of the keyword method, eye-tracking data or a think-out-loud protocol could help us further understand how individuals process verb WPs. Further, future research must investigate verbal associations in younger populations and if interventions can help suppress these associations. While this study was conducted with research restrictions due to COVID-19, we will continue to examine this construct and encourage future researchers to as well.

CONCLUSION

WPs play an important role in school curricula. Therefore, it is essential to understand all challenges students face. Decades of keyword method research have shown the detrimental impacts of associating relational terminology with operations and how appropriate lessons can remediate the usage of this suboptimal strategy (Hegarty et al., 1995; Schumacher & Fuchs, 2012). While further research is needed, the current study suggests that individuals may be able to use the keyword method beyond relational terminology problems.

Conflict of Interest

The authors have stated explicitly that there are no conflicts of interest in connection with this article.

Data Availability Statement

Data and supplementary materials are available on the Open Science Framework (OSF) at https://osf.io/zyq7e/?view_only=7578fd79ebc14515bf361db0f4d20852.

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