ABSTRACT

Title of dissertation: AFFECTING CHILDREN AND

THE EFFECT OF CHILDREN

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In the first half of my dissertation, I estimate the causal effect of a first child on female labor supply. This is a difficult task given the endogeneity of the fertility decision. Ideally, this question could be answered by running a social experiment where women are randomly assigned children or not. Using field data from the National Survey of Family Growth (NSFG), I mimic this hypothetic experiment by focusing on a sample of women that sought help to become pregnant. After a certain period since they started receiving help, only some of these women are successful. In this instance, fertility appears to be exogenous to labor supply in that pre-treatment labor supply is uncorrelated with subsequent fertility. Using this strategy, I estimate that having a first child younger than a year old reduces female labor supply by 26.3 percentage points. These estimates are close to OLS

and fixed-effects estimates obtained from a panel data constructed from the NSFG. They are also

close to OLS estimates obtained using similarly defined samples from the 1980 and 1990 Censuses.

The second part of my dissertation explores the problem of an educational authority who decides his revelation policy about students educational attainments in order to maximize mean educational achievement. Incentives in an educational context are different from those in the marketplace. Schools cannot pay students to motivate them to attain higher levels of education. However, there is still a role for incentives. Since students care about which signal they can get from the school (pass/fail, GPA), the school has a tool to influence students' behavior. Using a theoretical model, I explore the optimal way to use this tool, i.e., the optimal way to reveal educational achievements. I find that this optimal revelation policy is dependent on the distribution of students with respect to ability. I show that this optimal scheme could be: a) classify individuals

in two groups and just reveal this information, b) reveal all information, c) set a critical standard and group all individuals together below this level and provide full information about students' productivity above it.

# AFFECTING CHILDREN AND THE EFFECT OF CHILDREN

by

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Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of Doctor of Philosophy 2006

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## ACKNOWLEDGMENTS

I am grateful to my parents Roberto and Tatana for their continuous support (and flexible credit line).

To my friends in College Park, especially Eugenio, Diego and Facundo, for showing me how to survive life as a graduate student having a good time in the meantime. To my friends and family in Argentina for having tried to teach me, thought sometimes unsuccessfully, the tricks to just enjoy and appreciate life.

Finally, I am especially thankful to my advisors. To William Evans for all his help during the day to day work of writing a paper but in particular, for giving me confidence in my work. To Seth Sanders for teaching me that there are no bizarre ideas and that the best question to explore is the one that we are eager to know the answer. To Lawrence Ausubel for challenging my arguments and in this way helping me to improve them. Lastly, to Mark Duggan for showing me that inspiration is not enough and that hard work really pays off.

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## Chapter 1

## Introduction

The first half of this dissertation explores how women adjust their employment decisions to the arrival of a child. Estimating the effect of fertility on female labor supply has been a long standing question in economics. In fact, hundreds of studies have aimed to answer this question and reported estimates of this effect. However, Browning (1992) in his survey of this topic expresses criticism about the state of the literature saying that "although we have a number of robust correlations there are few credible inferences that can be drawn from them".

Trying to overcome this criticism, two strategies have been proposed that exploit exogenous changes in family size. The first (Rosenzweig and Wolpin, 1980; Bronars and Grogger, 1994; and Jacobsen, Pearce and Rosenbloom, 1999) used the fact that twins in the first birth represent an exogenous change in family size in order to estimate the effect of having a second child. The second (Angrist and Evans, 1998) exploited parental preferences for mixed-sex siblings in order to estimate the effect of a third or higher order child.

Still, the question of what is the effect of a first child on female labor supply has not been addressed with a strategy that deals with the endogeneity of fertility. Ideally, this question could be answered by running a social experiment where childless women are randomly assigned children (the treatment group) or not (the control group). In this paper, I use field data that mimics this hypothetic experiment by focusing on a sample of women that sought help to become pregnant. After certain period since they started receiving help, some of these women are successful and have a child while others do not.

In this instance, I find that fertility appears to be exogenous to labor supply as pre-treatment labor supply is uncorrelated with subsequent fertility. Using the exogenous assignment of children to women via infertility treatments as an identifying strategy, I estimate that having a first child younger than one year-old reduces female labor force participation by 26.3 percentage points.

These estimates are close to OLS estimates obtained using similarly defined samples from the 1980 and 1990 Censuses. The results also indicate that the estimated short-term impact of fertility on female labor supply decreased 40 percent during the 1980 to 1990 period.

The second part of the dissertation analyzes the problem of an educational authority that decides how to optimally reveal information about students productivity in order to induce students to improve their educational attainments. In essence, schools and universities have valuable information about students performance and have to decide how to optimally reveal this information.

This work belongs to a new literature that analyzes the educational process taking into account how students react to the way that information about their productivity is revealed to the marketplace. Costrell (1994) and Betts (1998) analyzed the optimal way to set a standard in a setting where the educational authority classifies students in two groups. Costrell (1994) also analyzed whether it is optimal to classify students in only two groups or rather to disclose all information about students' educational levels.

This paper contributes to this new literature analyzing the more real and interesting case in which the educational authority can choose into how many groups he wants to classify students (potentially infinite) and the standards for each group. A theoretical model is presented in which the educational authority enjoys a great deal of flexibility about how to optimally reveal information about students' attainments.

Using this model I find that the optimal revelation policy is dependent on how students are distributed with respect to ability or other personal characteristic that influence their optimal effort decisions. In particular, the optimal scheme could be: a) classify individuals in two groups and just reveal this information, b) reveal all information, c) set a critical standard and group all individuals together below this level and provide full information about students' productivity above it.

## Chapter 2

The effect of a first child on female labor supply: evidence from women in fertility treatments

#### 2.1 Introduction

Estimating the effect of fertility on female labor supply has been a long standing question in economics. Knowing how families optimize their labor supply decisions to the arrival of a child is important for several reasons. First, it is interesting to know how much of the increase in female labor supply during the postwar period can be explained by delayed childbearing and reduced fertility (Goldin, 1990). Second, some researchers believe that the interruption of work due to childbearing is responsible for a significant fraction of the female-male wage gap (Goldin and Polachek, 1987; Fuchs, 1989; Gronau, 1988; Korenman and Neumark, 1992) and the size of the impact of childbearing on female labor supply is an important variable in this calculation. Third, if declines in labor supply after childbearing correspond to increases in child care time, then knowing the effect of childbearing on female labor supply will provide information about time inputs invested in the child (Stafford, 1987; Blau and Grossberg, 1992). Finally and above all, economists have been interested in this question from a basic desire on knowing the quantitative importance of different determinants of female labor supply.

Given the importance of this topic, it is not surprising that hundred of published studies have examined the relationship between fertility and female labor supply. However, as Browning (1992) notes in his literature review on this topic, "Although we have a number of robust correlations, there are very few credible inferences that can be drawn from them." The key problem faced by researchers is that there are theoretical reasons to believe that the fertility decision may be endogenous and therefore the strong negative correlations found between different measures of fertility and female labor supply cannot be interpreted as evidence of causal effects.

In trying to overcome the type of criticism highlighted by Browning (1992), two strategies have been proposed that exploit exogenous changes in family size in order to estimate the effect of fertility on female labor supply. The first (Rosenzweig and Wolpin, 1980; Bronars and Grogger, 1994; Jacobsen, Pearce and Rosenbloom, 1999), used the fact that twins in the first birth represent an exogenous change in family size in order to estimate the effect of having a second child. The second (Angrist and Evans, 1998), exploited parental preferences for mixed-sex siblings in order to estimate the effect of a third or higher order child.

Still, the question of what is the effect of a first child on female labor supply has not been addressed with a strategy that convincingly tackles the problem of the endogeneity of fertility. It could be argued that the effect of having a first child is the most important one, given that it applies to nearly all women, whereas identifying the effect of having a second or higher order child only applies to a smaller subset of women.<sup>1</sup>

Ideally, this question could be answered by running a hypothetical social experiment where childless women are randomly assigned children or not. After the assignment of children, participation rates across the treated and control groups could be compared in order to estimate the causal effect of having a first child on female labor supply.

This paper focuses on a situation that mimics this hypothetic experiment. In particular, I construct a sample of childless women that sought help to get pregnant via fertility treatments. At the time of seeking help to get pregnant, all of them wanted to have a child but after a certain period some of these women were successful and give birth to a child while others did not. Then, I compare the labor participation of women that were "treated" (i.e., gave birth to a child) with those who were not.<sup>2</sup>

The contribution of this paper is that while the twins and the preference for mixed siblings strategies, under certain conditions, can be used to identify the effect of a second or higher order child, the estimation strategy pursued here is able to identify the effect of a first child on female

<sup>&</sup>lt;sup>1</sup>In the 1990 Census, among women aged 45 to 55, 89.0 percent of them had at least one child, whereas 78.3 percent had at least two and 50.4 percent had at least three children.

<sup>&</sup>lt;sup>2</sup>Along the paper "treatment" refers to having a child.

labor supply.

The proposed strategy tackles the potential problem of fertility being an endogenous variable as all women wanted to have a child at the time when they sought help for the first time. However, early success in fertility treatments is not expected to be completely random. Still, I provide several pieces of evidence that suggest that this strategy is consistently estimating the parameter of interest. First, following Heckman and Hotz (1989), I find that pre-treatment labor supply is uncorrelated with subsequent fertility. Second, estimates are very robust to the set of covariates added to the main regression. Third, observable characteristics of the sample of women that sought help to get pregnant while childless are quite similar to women that have their first child while aged 19 to 38 years old.

Using the exogenous assignment of children to women via infertility treatments as an identifying strategy, I estimate that having a first child younger than one year old reduces female labor supply by 26.3 percentage points. These estimates are close to OLS and fixed-effects estimates obtained from a panel data constructed from the NSFG. They are also close to OLS estimates obtained using similarly defined samples from the 1980 and 1990 Censuses. This finding is important because almost all studies that take into account the endogeneity of the fertility decision provide much smaller estimated impacts than those studies that assume exogenous fertility. Finally, I provide evidence of an important reduction in the estimated short-term impact of childbearing on female labor supply of around 40 to 50 percent during the 1980 to 1990 period.

#### 2.2 Previous research

This paper is related to several strands of economic literature. First, it belongs to a vast literature focusing on the effect of fertility on female labor supply. Second, its results shed light on a number of studies that have tried to explain the postwar rise in female labor supply. Third, it is related to a line of research that tries to establish the effect of childbearing related withdrawals from the labor market on females' wages and earnings. Lastly, it is also linked with a strand of the literature focusing on how maternal work affects children outcomes.

The attractiveness of the question of the effect of fertility on female labor supply is exposed in the long list of studies that have focused this issue. These studies can be classified into four groups depending on how they have tackled the problems of the endogeneity of the fertility decision. The first group is illustrated by the studies of Gronau (1973), Heckman (1974) and Heckman and Willis (1977), all who assumed that fertility was exogenous and established a strong negative correlation between female labor supply and fertility.

A second group of studies acknowledged the endogeneity of the fertility decision and tried to deal with this problem by estimating simultaneous equation models (Cain and Dooley, 1976; Schultz, 1977; Fleisher and Rhodes, 1979). The disadvantage of this approach is that it is difficult to find plausible exclusion restrictions that could identify the underlying structural parameters.

A third group of studies incorporated actual fertility as a regressor but added the lagged dependent variable in an effort of controlling for unobserved heterogeneity across women. Nakamura and Nakamura (1992) recommended this approach and it has been used by a number of authors (Lehrer, 1992; Even, 1987). Although adding the lagged dependent variable can be useful to control for unobserved heterogeneity, still it does not tackle the problem of the endogeneity of the fertility decision.

Finally, a fourth group of studies tackled the endogeneity of the fertility variable by exploiting exogenous sources of variation in family size. Rosenzweig and Wolpin (1980) first used this strategy comparing labor supply of mothers having twins on their first birth with those that had a single child. Subsequent studies by Bronars and Grogger (1994) and Jacobsen, Pearce and Rosenbloom (1998) employed the same strategy but managed to get more precise estimates by developing an algorithm to detect twin births using Census data.

In the same spirit as the twins studies mentioned above, Angrist and Evans (1998) exploited the fact that parents prefer mixed-sex siblings in order to estimate the effect of a third or higher order child on female labor supply. For a sample of couples with at least two children they instrumented further childbearing (i.e., having more than two children) with a dummy for whether the sex of the second child matches the sex of the first. As sex mix is virtually random, this

strategy identifies the effect of a third or higher order child.

My work is most similar to this last mentioned group of studies as I use the fact that the biology of reproduction is intrinsically stochastic in order to identify exogenous changes in fertility. Still, there are two main differences between these studies and my paper. First, I estimate the effect of a first child on female labor supply whereas these other studies estimates the effect of a second or higher order child. Second, while the mentioned group of studies have used instruments for fertility and then computed 2 SLS estimates, I tackle the endogeneity of fertility by focusing on a sample of women for which fertility is plausibly exogenous and then estimate the impact by just using OLS.

As noted above, my work can shed light in other strands of the literature which I will briefly summarize next. To start with, there are a number of studies that have focused on the remarkable increase in female labor supply during the postwar. Mincer (1962) argued that 90 percent of the increase in female employment in the postwar years can be attributed to an increase in demand. On the other side, Goldin (1990) argues that about half of the change in female labor supply can be explained by shifts in supply. Given that fertility declined strongly after the baby boom, my work can shed some light in how much of the increase in female labor supply could be explained by reduced childbearing in this period.

A second related strand of the literature examines the deleterious effect of childbearing on mothers' wages. Papers by Hill (1979), and Goldin and Polachek (1987) presented evidence that the majority of the negative association between children and wages disappear once detailed measures of past employment are included suggesting that there is an indirect reducing effect of children on wages through a decrease in human capital accumulation via on-the-job investment. However, Korenman and Neumark (1992) presented suggestive evidence that in addition to the indirect effect, there may be a direct negative effect of children on wages. Victor Fuchs (1989) goes a step further arguing that "the greatest barrier to economic equality (between men and women) is children."

More recently, Miller (2005) used biological fertility shocks in order to estimate the impact

of delay of childbearing on wages, hours worked and earnings. One of the instruments that she used for women's age at first birth is the lag between reporting having sex without contraception and their first birth. In a sense, we exploit the same type of fertility shock (women that want to have a baby cannot decide to conceive it right away). Still, our studies differ in a number of dimensions. Most importantly, while I aim to estimate the impact of childbearing on female labor supply, Miller was primarily concerned about how changes in the age at first birth impact long-run earnings and future wages.<sup>3</sup>

Finally, a third related line of research focus on the effect of maternal work on children outcomes. Some evidence has been presented suggesting that employment when children are young may harm their cognitive abilities (Blau and Grossberg, 1992; Stafford, 1987; Desai et al., 1989). Again, in order to gauge the relative importance of this effect, we need to know how fertility impacts female labor supply.

## 2.3 Background: the reproductive process and infertility

Reproduction is a very delicate process that requires the correct functioning of the male and female reproduction systems as well as ideally timed sexual intercourse. Conception takes place when a motile sperm from the man burrows into an egg (ovum) from the woman and fertilizes it. Fertilization occurs in one of the Fallopian tubes and the fertilized egg starts dividing itself as it travels through the Fallopian tube towards the uterus. There, it will settle in the lining of the uterus and hopefully it will grow until the baby is born.

Healthy couples having intercourse regularly have only a 20 percent chance of conceiving during a month. This implies that around 26 percent of healthy couples will not have conceived after 6 months of unprotected sex, and this number falls to around 7 percent after 12 months.

<sup>&</sup>lt;sup>3</sup>Our studies also differ in the following ways. First, my empirical strategy consists in selecting a sample of women for which fertility is presumably exogenous while Miller used the delay shock as an instrument for fertility. Second, I use the NSFG Cycle 5 as my main dataset where Miller used the NLSY 1979. Finally, we differ in the way that women trying to conceive are identified (I identify women seeking help to get pregnant while Miller identified women that report having sex without contraception).

Given these facts, couples are recommended to start receiving infertility testing and treatment only after 6 to 12 months of trying to conceive without success. Moreover, the medical community defines a couple as infertile if they have not conceived after 12 months of unprotected sex.<sup>4</sup> The National Center for Health Statistics estimated that in the United States in 1995 there were 2.1 million of infertile married couples in reproductive age and 6.1 million of women aged 15-44 had an impaired ability to have children.<sup>5</sup>

The causes for infertility are typically grouped into four categories: a) tubal blockage or endometriosis, b) ovulation problems, c) male (sperm) problems and d) unexplained infertility. Tubal blockage is diagnosed in the presence of certain adhesions or scars that prevent the sperm or the egg to move through the Fallopian tubes. It can also be the case that the tubes are swollen or full of fluid and this prevents communication through the tubes. Depending on the severity of the case, the problem can be fixed using a laparoscope or tubal surgery. In any case, if a pregnancy has not taken place after 12 months following tubal surgery, in vitro fertilization should be considered.

Endometriosis is a condition where the lining of the uterus extends outside the uterus into the ovaries, the back of the uterus and the ligaments that support the uterus. When treating this disease, the goal is to remove the endometriosis by cutting or burning it away and leaving as much normal ovary tissue as possible. Moderate endometriosis can be treated using a laparoscope but more severe cases require surgery.

In almost half of the infertility cases, there is a male contribution to the problem. Samples of sperm are analyzed in order to check whether the volume, motility, morphology and concentration are normal. For around 90 percent of male problems, there are no effective treatments.

Finally, in approximately 15-20 percent of the cases, no obvious problem is found and a diagnosis of unexplained infertility is made. About 60 percent of couples with unexplained infertility will conceive in the next 3 years without any treatment at all.

Medical researchers have identified a number of factors (besides the conditions mentioned <sup>4</sup>The WHO defines a couple as infertile if they have not been able to conceive after 24 months of unprotected

sex.

<sup>5</sup>Source: http://www.cdc.gov/nchs/fastats/fertile.htm.

above) that affects the prognosis of a couple. Female's age, education, smoking status, consumption of recreational drugs and obesity as well as sexual frequency are important predictors of the probability of conception.<sup>6</sup>

Given the stochastic nature of the reproduction process, physicians usually start treatment with simple and cheap procedures (like advice and testing) and only start employing more invasive and expensive procedures as the simple treatments prove unsuccessful. As an example of this optimal sequential strategy, physicians typically only recommend in vitro fertilization methods after all other choices have been tried or if they strongly believed that those techniques will be unsuccessful.

#### 2.4 Data

This paper uses data from the National Survey of Family Growth (NSFG), a survey conducted by the National Center for Health Statistics in 6 cycles (1973, 1976, 1982, 1988, 1995 and 2002). Cycles 1 to 5 were conducted at the homes of a national sample of women 15-44 years old, in the civilian non-institutionalized population of the U.S. Cycle 6 also sampled men 15-44 years old. The main purpose of these surveys was to provide reliable national data on marriage, divorce, contraception, infertility and the health of women and infants in the U.S.

The NSFG Cycle 5 was chosen because it provides information about births, pregnancies, infertility services, demographic characteristics and in particular the complete work history for each individual.<sup>7</sup> For the empirical strategy pursued in this paper, I need information about the month in which each woman sought help to get pregnant. Fortunately, this information is provided by this survey. Other important variables included are age, race, ethnicity, educational attainment, school enrollment and smoking history. The survey also reports data on each full-time and part-time employment spell.

<sup>&</sup>lt;sup>6</sup>Sources: Dunson et all (2004) and Baird and Wilcox (1985).

<sup>&</sup>lt;sup>7</sup>Other cycles included all needed information except from monthly employment status for each woman. I cannot run this analysis without this information as I compare employment 21 months after each women sought help to become pregnant.

The NSFG Cycle 5 employs a multi-stage sampling design with an over-sample of Hispanic and black women. It took place between January and October, 1995 and the overall response rate was 79 percent. A total of 10,847 women were interviewed.

Data on fertility and employment is collected retrospectively. Though it is recognized the limitations of this type of design, Teachman, Tedrow and Crowder (1998) found the NSFG Cycle 5 to be of high quality. They concluded that the employment information matches CPS data reasonably well, although the data on employment spells has not been validated using external records.

## 2.5 Empirical strategy, parameter of interest and sample construction

## 2.5.1 Empirical strategy

An ideal social experiment aimed to estimate the causal effect of childbearing on female labor supply would recruit women that wanted to have a child and then assign a child to a group of women (treated individuals) while not assigning a child to a second group (controls).<sup>8</sup> In the real world we cannot run this type of experiment. However, given the stochastic nature of conception, we can attempt to mimic it. To start with, we need a group of women that wants to conceive a baby. Second, some of these women should receive babies in a way that is uncorrelated to baseline employment. Third, we need to observe female labor supply for both groups of women after the assignment of babies.

I aim to mimic this ideal social experiment and fulfill the three mentioned conditions by focusing on the following situation. I construct a sample of women that sought help to have a child while childless (called the HELP sample). As women in this sample are starting fertility treatments at different points in time, I normalize time by the month that they sought help for the very first time, denoted as month 0. Next, I classify these women depending on whether they had already given birth to a child in month 21 and in this way I get two groups of women: treated

<sup>&</sup>lt;sup>8</sup>To be precise, this experiment will estimate the effect of having a child on female labor supply for women that wanted to have a child, not for all women.

and controls. Finally, I compare employment rates of these two groups of women in month 21 and in this way I estimate the causal effect of having a first child younger than a year on female labor supply.

I choose to compare employment in month 21 instead of other months for several reasons. First, at this point in time almost all babies born are younger than a year old (97 percent of them) making more precise the definition of the treatment effect. Second, using a longer horizon will allow some women to have additional children complicating the analysis. Third, as time in treatment increases, women who are unsuccessful conceiving may start adopting. Finally, looking at this shorter time span, it is more plausible that women receive similar types of infertility treatments (e.g., in vitro fertilization treatments are typically not considered an option in the first 12 months after seeking help to get pregnant).

Following this strategy, I tackle the endogeneity problem given that all the women in the HELP sample wanted to have children. Still, in order to obtain consistent estimates of the effect of childbearing on female labor supply it is needed that the assignment process of children is uncorrelated to baseline female labor supply. Clearly, this assumption is untestable. However, following Heckman and Hotz (1989), I provide evidence of the plausibility of this assumption in section 2.7, where I test whether pre-treatment labor supply is correlated with subsequent fertility.

A potential problem of this empirical strategy arises if women in the control group adopt a child or start cohabitating or marry an individual with children. In the treatment evaluation literature this is denoted as "substitution bias" and represents a situation where individuals in the control group receive close substitutes for the treatment in question (see Heckman and Smith (1995), pages 22-24). In the context of this paper, treatment is having a natural birth and a close substitute is adopting a child (or start having a step child). Even though substitution bias can be

<sup>&</sup>lt;sup>9</sup>Focusing on month 21 there are only 6 women that had 2 babies by this time and only one of them had twice one baby (the other 5 women gave birth to twins).

a real problem in certain social experiments, it is not in this case.<sup>10</sup> Only 2.7 percent of women in the control group adopt or start having a step child in the 21 months after they seek help to become pregnant (and only 0.5 percent in the treatment group).

## 2.5.2 Parameter of interest

following random assignment.

In this study, the parameter of interest is the average impact of having a first child younger than 12 months on female labor supply for women that want to have a child. It is important to note that it does not give an estimate of the effect of having a first child for women whose child is unwanted. All the same, the parameter of interest that I am estimating applies to a fairly large population. Henshaw (1998) using data from the NSFG Cycle 5 found that 69.8 percent of births were intended for women aged 15-44 years old in 1994.

Throughout this study, I focus only on the short-term effects of having a first child (i.e., the estimated effect of having a child younger than one year old). It is clear that there are other treatment effects that are worthy of attention. However, for reasons already discussed, the strategy employed in this study is best suited to estimate this treatment effect.

Finally, an estimate of the impact of the first child younger than a year old is important for a number of reasons. First, as mentioned above, this effect will apply to a much wider population than estimates that focus the effect of a second or higher order child. Second, there is consensus that the short-term effects of childbearing are substantially bigger than the longer-term effects (Browning, 1992). Then, knowing the short-term effects is useful as it gives an upper bound for these longer-term effects. Third, Shapiro and Mott (1991) provide strong evidence that labor force status following the first birth is an important predictor of lifetime work experience. This implies that changes in the estimated short-term impact of having a first child on female labor supply could be predicting a substantial change in overall lifetime work experience for women. Finally, using this empirical strategy I can compare the estimated impacts obtained when tackling the  $10 \, \mathrm{m}$  lifetime to the experimental evaluation of the training program JTPA, Heckman and Smith (1995) noted that 32 percent of control group members self-reported receiving training from other sources over the 18 months

endogeneity problem (i.e., using the HELP sample) with estimates from strategies that do not tackle this problem (e.g., OLS on Census data).

## 2.5.3 Sample construction

The main sample used in this paper (the HELP sample) includes childless women who sought help to become pregnant while aged 19 to 38 years old.<sup>11</sup> Women that sought help less than 21 months before the interview are dropped from the HELP sample, because it is not possible to observe their child and labor status at this key time. Finally, I drop one observation for which there is a missing value for one of the variables used in the analysis.

Table 2.1 presents the algorithm employed in order to construct the HELP sample. This table shows that only 499 observations are included in the empirical analysis, a fact that may seem as an important limitation for this study. However, as shown in section 2.6, I precisely estimate the relevant coefficient of the effect of having a first child on female labor supply.

The basic empirical strategy of this paper consists on comparing women in the HELP sample that already had a baby by month 21 with those who did not. To identify these two groups of women, an indicator called *AnyChildren21* is defined that equals to 1 if the woman had a baby by month 21. In this setting, women from the HELP sample for whom *AnyChildren21* equals to 1 correspond to women that were "treated" and those for whom *AnyChildren21* equals to 0 correspond to the "control" or comparison group.<sup>12</sup>

Descriptive statistics for women in the treatment and control groups are presented in Table 2.2. In the NSFG Cycle 5, respondents were asked about all their employment spells information which I use in order to construct three employment variables. The variables *Employed21* and *Employed0* are dummy variables that equal to 1 if the individual was employed in months 21 and 0 1 I drop women younger than 19 years old at the time when they first sought help to become pregnant because work information is only reported since the women turned 18 and I want to know employment status one year before seeking help to become pregnant.

<sup>12</sup>For easy of exposition, along the paper I will denote as treated women to those that had a baby by month 21 whereas women that did not have a baby by month 21 are referred as "controls" (even though in the evaluation literature the word "controls" is used for subjects not receiving treatment in an experiment).

respectively. Similarly, *Employed\_12* represents labor status in month -12 (i.e., 12 months before the woman sought help for the first time).

It is important to note that while employment rates in month 0 and -12 are similar between treated and control women, employment rates differ by 25.3 percentage points in month 21. Moreover, observable characteristics in month 0 between treated and control women are quite similar. The only exception is that the control group has a much lower fraction of Hispanic women than the treatment group, but it is important to point out that there are only 34 Hispanic women in the HELP sample. Finally, the treated group tends to include individuals that are slightly younger, more educated and a lower fraction of them were smoking when sought help to get pregnant for the first time.<sup>13</sup>

A potential caveat for the strategy pursued in this paper is that, as typically is the case in social and medical experiments, the population involved in the experiment may not be representative of the whole population. In order to gauge the potential severity of this problem Table 2.3 compares descriptive statistics of women in the HELP sample with those of women in the NSFG that had at least a child. For women in the HELP sample, time-varying variables are measured at the time they first sought help to get pregnant where for women that had at least a child these variables are measured at the time of first birth. In the last column of Table 2.3 statistics are presented for the set of women in the NSFG that had their first child while being 19 to 38 years old (as these age requirements were used to construct the HELP sample). Table 2.3 shows that women in the HELP sample tend to be older, more educated, have higher employment rates and have higher smoking rates, while a lower proportion of them are Hispanic or black, as compared to women from the NSFG that were aged 19 to 38 when they had their first child. Still, basic statistics for the HELP sample are not very different from those women in the NSFG that had a first child while being 19 to 38 years old.

Figure 2.1 compares the age distribution of women in the NSFG that gave birth when aged 19 to 38 years old to the age distribution of women in the HELP sample. This figure shows that 

13 In section 2.7 I explore more deeply which variables predict fertility by month 21.

the difference in mean age across these two groups is driven primarily by the group of women aged 19 to 21. This difference can be explained by the fact that some women in the NSFG group are having unplanned children and also by the fact that really young women would tend to delay their decision to seek help to get pregnant.<sup>14</sup>

#### 2.6 Results

This section presents the main results of the empirical analysis. In essence, I will compare participation rates in month 21 for treated and control women in the HELP sample. The econometric model is represented by this simple OLS equation (remember that *AnyChildren21* equals to 1 for treated women):

$$Employed21_i = \alpha + \beta \ AnyChildren21_i + \gamma \ X_i + u_i$$

The vector of covariates include black and Hispanic dummies, an indicator for insurance coverage of infertility treatments, year in which they sought help for the first time and the following variables measured in month 0: age, smoking status and years of education.

In order to gauge the potential importance of the problem of not having information of certain variables that may be simultaneously correlated with the probability of conception and labor supply, I run a number of regressions including separate sets of covariates. If the results were sensitive to the set of covariates added to the regression, this would raise some doubts about whether the identification strategy is consistently estimating the parameter of interest. Table 2.4 presents these regressions results.

percentage point or 4 percent of the estimated impact.

Women that have a child not only decide whether to have a job or not (the extensive margin) but also how many hours to work (the intensive margin). Unfortunately the NSFG does not provide retrospective information on hours worked for women in the sample. Still, it provides information about whether the individual was working full-time or part-time and also the availability of maternity leave. Then, defining work status in four categories (full-time, part-time, maternity leave and no job), Table 2.5 provides data on how work status across these four groups is affected by the fertility status of each woman.<sup>15</sup>

This table shows that although the fraction of women not holding a job is 25.3 percentage points higher for women that had a child in month 21, there is a larger impact on the fraction of women that work full-time (it is 41.7 percentage points lower for women having a baby). Also as expected, the fraction of women on maternity leave is 12.5 percentage points higher for women than had a child in month 21 while the fraction of women working part-time is 3.8 percentage points higher.<sup>16</sup>

Table 2.6 presents multinomial logit regression results of having a first child on work status. The dependent variable has the same four categories mentioned above (full-time, part-time, maternity leave and no job). Marginal effects of increasing *AnyChildren21* from 0 to 1 are presented. Results are similar to those obtained by the comparison of controls and treated groups in Table 2.5 though the estimated impact of having child on not having a job and working full time are (in absolute value) larger, while the estimated impact on maternity leave is slightly smaller.

<sup>&</sup>lt;sup>15</sup>Throughout the paper, "work status" refers to the classification of women with respect to these four status (full-time, part-time, maternity leave and no job) whereas "employment" denotes to the classification of women with respect to whether they hold a job or not.

<sup>&</sup>lt;sup>16</sup>The 0.4 percent of women in the control group that were on maternity leave corresponds to women that by month 21 still did not have a child but were pregnant.

## 2.7 Robustness of the empirical strategy

This section explores the robustness of the empirical strategy pursued. First, I try to identify which covariates can predict treatment and how much of the variation in the fertility variable is explained by these variables. Second, following Heckman and Hotz (1989), I test whether there are pre-treatment differences in the outcome variable (labor market participation) between the treated and control groups. Finally, I check how robust the results are to changes in the specification of the econometric model such as adding a dummy variable for whether the woman was pregnant or not in month 21.

To start with, I explore which variables in the dataset predict early fertility success in the HELP sample. If I was really running a social experiment, this step would have been seen as a way of testing whether the randomization mechanism involved in the experiment attained the goal of balancing the different covariates in the treatment and control groups. In the case of the empirical strategy followed, given that the medical literature documents a number of variables that predict success rates of fertility attempts (for example, female's age), I just try to identify which these variables are and how much variation in the fertility variable they can explain. In order to do so, I regress AnyChildren21 on the set of covariates used in the analysis of the previous section.

Table 2.7 shows that, as documented in the medical literature, female's age is one of the most important predictors of fertility. In this linear probability model, an increase in one year in the age of a woman decreases her expected probability of having a child by 1.6 percentage points (the associated t-ratio is -2.67). Smoking, also documented in the medical literature as having an effect on fertility, is a significant predictor of fertility success. Finally, Hispanics and more educated women are also more likely to be successful.

Even though there are several variables that can predict treatment, it should be noted that the adjusted r-squared is only 4.3 percent and then there is much of the variation in the fertility variable that remains unexplained in this model.

Next, I tackle the issue of whether the significant differences in labor market participation between treated and control woman in month 21 can be interpreted as the effect of treatment or

rather just heterogeneity in labor market attachment between groups. This is an important test for the empirical strategy pursued in the paper. Before presenting the regression results it is useful to look at Figure 2.2 which shows employment rates of the treatment and control groups for months -12 to 21 (again, month 0 corresponds to the month in which each woman first sought help to get pregnant). Employment rates of both groups are quite similar for months -12 to 0 but they start diverging around month 3 and are far apart by month 21. The continuous decline in employment rates for the treated group corresponds to the fact that, as time goes by, different women are giving birth (but all of them had already given birth by month 21).

I check more systematically this difference by regressing employment status in month 0 (*Employed0*) on *AnyChildren21*. Several regressions are ran in which I control for different set of covariates in order to gauge the robustness of the results. These results are presented in Table 2.8.

The main result is that there are no statistically significant differences between the treated and control groups in labor market participation in month 0. This result is robust across a number of specifications in which different sets of covariates are added, giving additional evidence that there were no significant differences in employment across the treated and control groups when women sought help to become pregnant.

Similar regressions as those presented in Table 2.8 are ran using as dependent variable *Employed\_12* (employment dummy for month -12) instead of *Employed0*. Results are presented in Table 2.9. In these additional regressions I check whether there are differences in labor participation rates across the treated and control groups 12 months before women sought help to get pregnant. The results also give evidence in favor of the empirical strategy pursued. In all regressions the estimated coefficient for *AnyChildren21* is smaller (in absolute value) than 0.023 and the associated t-ratio are (in absolute value) between 0.38 and 0.59 depending on the specification.

Finally, I run a number of regressions in order to check whether the results are robust to changes in the specification. First, I re-run the regressions whose results were presented in Table 2.4 but adding a dummy for whether the woman was pregnant in month 21 (this variable is denoted *Pregnant21*). These results are presented in Table 2.10. The estimated impact is very similar to

the results reported in Table 2.4. Next, the main independent variable AnyChildren21 is replaced with another variable Own21 that equals to the number of children in month 21. Third, I replace AnyChildren21 with two dummy variables: OneChild21 and TwoChildren21 (that equals to 1 if the woman had one child and two children in month 21, respectively). Fourth, instead of running linear probability models of Employed21 on AnyChildren21, I run probit and logit models using the same set of variables as in Table 2.4. In all these cases the estimated impacts are very similar to those reported in section 2.6.<sup>17</sup>

## 2.8 Comparison to estimates from NSFG and Census data

In his survey of the effect of children in the household, Browning (1992) concluded that studies that take fertility as exogenous typically found significant larger impact of fertility on female labor supply than those that treat it as endogenous and estimate simultaneous equation models. Angrist and Evans (1998) provided further evidence about this argument as they reported that their 2SLS of the impact of having more than two children on female labor supply were statistically smaller than their OLS estimates. This section compares estimates obtained using the HELP sample with those from similarly defined samples but without restricting to women that sought help to get pregnant.

A problem faced in trying to replicate the HELP sample is that this dataset includes observations of fertility and labor supply for women that sought help to get pregnant at different points in time. This implies that in order to replicate the results from the HELP sample I should construct comparable data sets with observations for individuals at different points in time: i.e., a panel data or repeated cross-sections. Having this in mind, I compare estimates from the HELP sample to estimates from a panel data from the NSFG (in subsection 2.8.1) and to estimates from Census data for 1980 and 1990 (in subsection 2.8.2).

 $<sup>^{17}\</sup>mathrm{Results}$  are available from the author upon request.

## 2.8.1 Comparison to estimates from NSFG panel data

I construct a panel data from the NSFG Cycle 5 (called the NSFG panel data) following similar requirements to those used to construct the HELP sample. The unit of observation in this panel data is a woman-month. An observation is included in the NSFG panel data if the woman was aged 21 to 40 years old at that month, was childless or had children younger than a year old and was cohabitating or married.

As the HELP sample corresponds to a cross-section, in order to use the same source of variation when estimating both models, I construct a panel data set (called the HELP panel data) including for each individual in the HELP sample, observations for months -12 to 21 (remember that month 0 corresponds to when the individual first sought help to get pregnant). Again, the unit of observation is a woman-month.

Table 2.11 presents summary statistics for the HELP panel data and the NSFG panel data. Mean values for key variables are similar except for the fact that the fraction of observations in which the woman had a child is much higher in the NSFG panel data than in the HELP panel data. This difference is due to the fact that all individuals in the HELP panel data did not have children for months -12 up to (at least) month 7.

In Table 2.12, summary statistics for the HELP panel data and the NSFG panel data are presented, now by fertility status of the individual in the particular month. Note that participation rates for individuals in both samples are very similar once we condition by fertility status. This suggests that estimates of the impact of childbearing on employment are very similar across the two datasets.

Linear probability estimates of the impact of having at least a child (younger than a year old) on the probability of having a job are presented in columns 1 and 3 of Table 2.13. In the first column, results are presented for the model estimated using the HELP panel data. The main independent variable is *AnyChildren* (equal to 1 if the woman in that month had a child and 0 if not). The estimated impact (0.251) is very close to the estimates obtained in section 2.6. In the third column, results are presented for the same model estimated on the NSFG panel data. The

key result when comparing columns 1 and 3 is that the estimated impact using the NSFG panel data (0.259) is notably similar to the one obtained using the HELP panel data.<sup>18</sup>

In order to gauge the robustness of these results, I estimate fixed-effects models on both panel data sets. Results are presented in columns 2 and 4 of Table 2.13. For the HELP panel data the estimated impact slightly decreases in absolute value to 0.225. In the case of the NSFG panel data, the estimated impact decreases in absolute value to 0.219. This result provides some evidence that women that have children tend to have lower employment rates in months previous to get pregnant. Still, both estimates are very similar and the t-value of the test of equality of coefficients is just -0.02.

In the reminder of this subsection, I compare the estimated impact of having a child on work status (working full-time, part-time, maternity leave and no job) between these two panel data sets. Table 2.14 reports the distribution of women with respect to work status by fertility status for individuals in the HELP panel data and the NSFG panel data. While the fraction of women in the "no job" and "part-time" categories are very similar across data sets (conditional on fertility), the fraction of women that have a child and are on maternity leave is much larger for the HELP panel data than for the NSFG panel data (and the fraction working full-time is much lower).

In order to corroborate these results, I run a multinomial logit regression on each data set in order to estimate the impact of fertility on work status. Results are presented in Table 2.15. Again, the effect of fertility on the fraction of women in the "no job" and "part-time" categories are very similar across data sets while the impact on the fraction of maternity leave is much larger in the HELP panel data than in the NSFG panel data (and the impact on the fraction working full-time is smaller in absolute value).

These results can be explained by the interaction of two factors: 1) mothers in the HELP panel data tend to have younger babies; 2) mothers of young babies are more likely to be on maternity leave than working full-time. Regarding the first factor, Figure 2.3 presents the distribution.

<sup>&</sup>lt;sup>18</sup>the t-value of the test of equality of coefficients is 0.22.

ution of observations of mothers in each data set with respect to their babies' age in months. This distribution would be flat if for each mother in a panel data we had all observations while her baby was younger than a year old. While the distribution for the NSFG panel data is almost flat, its counterpart for the HELP panel data is sharply decreasing due to the way the latter panel data is constructed. Mothers in the HELP sample had a baby by month 21 and the HELP panel data includes monthly observations for months -12 to 21. Then, as women in this sample are having babies in different months (from around month 9 to month 21) all of them will be included in the HELP panel data when their baby is 0 months old but only a few of them (the ones that give babies very early) will have observations in the panel data in which their baby is close to one year old.

Figure 2.4 displays the distribution of women from the HELP sample that had a baby by month 21 with respect to work status by month (in this graph, month 0 corresponds to the month in which the baby was born). As expected, the fraction of women on maternity leave and working full-time is very dependant on the babies' age. Women that have just had babies tend to stop working full-time and rely heavily on maternity leave. This pattern, combined with the fact that women in the HELP panel data tend to have younger babies relative to the NSFG panel data, explains why the fraction of women on maternity leave is much larger in the HELP panel data than in the NSFG panel data. Note also that the fraction of women without a job is quite constant across women with babies of different age (in months) giving assurance that the similarity of the estimates of fertility on employment between both panel data sets is not dependant on the fact that the age in months distribution across samples is quite different.

Finally, to provide additional evidence in favor of the argument that we should get similar estimates if we condition on babies' age in months, I present Figures 2.5 and 2.6 which show the distribution of mothers in these two data sets with respect to work status by babies' age. Figure 2.5 presents the fraction of mothers working part-time and without a job by their baby's age. These distributions for both data sets line up very well except for the fraction of women working part-time with older babies. This reflects that as there are few women in the HELP panel data with older

babies, there is higher sampling variability when computing these fractions. Similar observations can be drawn from Figure 2.6 which presents the fraction of mothers working full-time and on maternity leave.

## 2.8.2 Comparison to estimates using Census 1980 and 1990 data

A first step in this exercise consists in choosing which Census years we should use as a comparison to the HELP sample. To make this decision, I check in which year fertility and other covariates are observed for each individual in the HELP sample. On average these variables are observed in 1986 and the 10th and 90th percentiles correspond to years 1978 and 1993, respectively. Taking this into account, it seems reasonable to compare the results obtained using the HELP sample to results obtained from two samples constructed using Census data for years 1980 and 1990.

Then, I construct two samples using the 5-percent Census Public Use Micro Samples for 1980 and 1990. These samples (from now on PUMS 1980 and PUMS 1990) include married women aged 21 to 40 years old, childless or with children younger than a year old. Only married women are kept in the sample in order to get women that are "at risk" of having a child. To make these samples comparable to the HELP sample, I keep only married women in the HELP sample for the analysis performed in this subsection.<sup>19</sup>

Table 2.16 presents descriptive statistics for the HELP, PUMS 1980 and PUMS 1990 samples. In the case of the HELP sample, the variable *Employed* is an indicator that equals to 1 if the woman had a job in month 21. For the PUMS samples, it equals to 1 if the woman had a job during the previous week to the survey. The variables *AnyChildren*, *Age*, *Education*, *Hispanic* and *Black* are similarly defined in the three samples and are all measured in month 21 (for the HELP sample) or at the time of the survey (for the PUMS samples). *AnyChildren* equals to 1 if the woman had at least a child. *Education* corresponds to the number of years of education. Finally, *Black* and *Hispanic* are dummy variables that equal to 1 if the woman belongs to each of these

<sup>&</sup>lt;sup>19</sup>Results obtained dropping the requirement of women in the HELP and PUMS samples to be married are very similar to those presented in this subsection (they are available from the author upon request).

groups.

Note in Table 2.16 that mean values of covariates in the HELP sample typically lie between the corresponding statistics for PUMS 1980 and 1990 suggesting that the latter two samples can be considered as sensible comparison data sets for the HELP sample. The only exception corresponds to the fraction of women that have a child which is significantly higher in the HELP sample. This should be expected, given that presumably all women in the HELP sample wanted to have children.

In Table 2.17 summary statistics by fertility status are presented for the three samples. Similar patterns are observed when comparing women who are childless and those with children in the three samples. The most important difference between the HELP sample and the PUMS sample is that women in the HELP sample, conditional on fertility status, present higher labor participation rate than women in the PUMS samples (this could stem from the fact that employment is not defined exactly in the same way in the NSFG compared to the Census). Finally, a remarkable finding from this table is that while participation rates for childless women increases slightly as we compare PUMS 1990 versus PUMS 1980, women with children have significantly higher average participation rates in 1990 compared to 1980 (0.598 versus 0.419) suggesting that the impact of having a child has drastically decreased over this time period.

Linear probability estimates of the impact of having a child (younger than a year old) on employment are presented in Table 2.18. Column 1 presents results for the HELP sample. The estimated impact is -0.277. Column 3 presents the estimated impact for the PUMS 1980 sample (0.365) and column 4 for the PUMS 1990 sample (0.228). Given the significant reduction in the estimated impact between 1980 and 1990, I proceed to augment the estimated model for the HELP sample in order to allow for a varying treatment effect over time. In order to do so, I rank observations in the HELP sample according to the month in which these women sought help to become pregnant and I divide these women in two groups: those who sought help before the median month at which women in the sample sought help (LateHelp = 0) and those who sought help later (LateHelp = 1). Women in the early help group were observed on average in 1981.4, while the corresponding statistic for woman in the late help group is 1991.0. This implies that

the estimated impact for women in the early help group could be compared to the results using the PUMS 1980 sample, while the estimated impact for women in the late help group could be compared to the results using the PUMS 1990 sample.

Linear probability estimates of the impact of having a child on employment allowing for differential effects for individuals in the early and later help group are presented in column 2 of Table 2.18. The estimated impact for women in the early help group is -0.377. This estimate is very close from the estimated impact using the PUMS 1980 sample (-0.365). Moreover, the estimated impact for women in the late help group is -0.192 (-0.377+0.185) which is also close to the estimated impact using the PUMS 1990 sample (-0.228).

From this set of results I can state two important conclusions. First, the estimated impacts obtained for the sample for which I can identify an exogenous change in the fertility variable (the HELP sample) are really close to the estimates obtained using OLS on comparable samples from Census data for which I do not control for the endogeneity of the fertility variable (and they are also very close to estimates obtained using a panel data from the NSFG as concluded in the previous subsection). Second, there is evidence of a significant reduction of about 40 to 50 percent in the short-term impact of childbearing on female labor supply in the 1980 to 1990 period.

## 2.9 Conclusions

This paper explores the question of the causal effect of childbearing on female labor supply. Answering this question is difficult for two reasons. First, some researchers believe that women that have children at a certain age may have different baseline labor supply from women with similar observed characteristics that do not have children at that age (Browning, 1992). This expected unobserved heterogeneity across groups suggests the existence of bias in simple cross-section comparisons. As noted by Nakamura and Nakamura (1992), we can try to deal with this problem adding to regressions of current labor supply on number of children the lagged values of labor supply.

However, there is a second problem that has complicated the estimation of the effect of

childbearing on female labor supply that cannot be solved by just using longitudinal data. This problem stems from the fact that the fertility decision may be endogenous to the woman and influenced by potential labor supply. Several studies starting with Rosenzweig and Wolpin (1980) have used the fact that having twins in the first birth changes (at least temporarily) family size. Angrist and Evans (1998) exploited the fact that parents typically prefer mixed-sex siblings in order to find exogenous variation to the fertility decision. Even though these papers have made a major contribution in answering the question posed, they are only able to estimate the effect of having a second or higher-order child.

In this paper, I estimate the short-term effects of having a first child on female labor supply. In order to deal with the problems of unobserved heterogeneity and endogeneity I restrict my attention to a group of women that sought help to get pregnant. In this sample, all the women wanted to have children and then the problem of endogeneity is minimized. Moreover, as a major fraction of the fertility variable is random, I can suspect that results will not be contaminated by unobserved heterogeneity across groups. In fact, the attractiveness of the strategy pursued is that, it mimics an ideal social experiment in which for a group of women that wanted to have a child, some women are assigned children while others are not. I provide evidence in favor of the empirical strategy pursued as I find that pre-treatment labor supply is uncorrelated with subsequent fertility.

Following this empirical strategy I estimate that having a first child younger than a year old reduces female labor supply by 26.3 percentage points. Interestingly, I obtain strong evidence that the estimates obtained using this strategy (which tackles the problem of the endogeneity of fertility) are very close to estimates derived from approaches that assume the exogeneity of fertility.

Given that studies which assume the exogeneity of fertility typically find larger impacts of fertility on female labor supply than those that treat as endogenous, a natural extension of this paper would be to attempt to understand why my empirical strategy reaches a different conclusion. One salient characteristic of this study is that all samples are defined consistently in order to make sure that the same treatment effect is being estimated across samples, i.e., the effect of having a child younger than a year old on employment. It could be the case that not holding constant

the age distribution of women's children could be an underlying reason for differences in estimates across studies.

Another interesting question that is left unanswered in this paper is why the empirical strategy seems to work. In other words, why do fertility and baseline employment seem to be uncorrelated? There are many potential hypotheses that it is possible to lay out in order to predict problems of the identification strategy. For example, using this strategy I restricted the sample to women that are homogeneous in that all wanted to have a child at certain point in time, but clearly they could differ in how much they wanted to have it and this could be correlated with baseline labor force attachment.

A potential explanation for the evidence that subsequent fertility is uncorrelated with pretreatment labor supply could be related to the fact that women in the HELP sample typically wait
a number of months until they seek help to get pregnant. This "waiting" could be reducing the
heterogeneity of individuals in the sample with respect to their baseline probability of being treated
(where treatment refers to having a child). Individuals with very high probability of being treated,
receive treatment early and then they are not included in the sample if we restrict it to individuals
that have not been treated after certain period of time. As individuals in the sample have more
similar probabilities of being treated, we tend to the ideal situation of random assignment which
is characterized as one in which all individuals have equal probability of being treated. If evidence
is found in favor to this hypothesis that "waiting" is a successful empirical strategy in the sense
that it increases the similarity between the treated and control groups, then this same strategy
could be applied to other evaluation problems where there is dynamic assignment of individuals
to treatment.

Table 2.1: Algorithm employed to construct the HELP sample

| Step  | Number of remaining observations |
|---|----------------------------------|
| 1- Start with the whole NSFG sample   | 10,847                           |
| 2- Drop women that did not seek help to get pregnant  | 895                              |
| 3- Drop women that sought help for the first time less than 21 months before the time of the interview                | 788                              |
| 4- Drop women that had adopted or step children when they first sought help to become pregnant                        | 766                              |
| 5- Drop women that were younger than 19 or older than 38 when sought help for the first time                          | 724                              |
| 6- Drop women that had already a child when sought help for the first time  | 536                              |
| 7- Drop women that were pregnant at some point of the month in which they sought help for the first time <sup>a</sup> | 500                              |
| 8- Drop women with missing values in some variable  | 499                              |

<sup>&</sup>lt;sup>a</sup> This group could include women that got pregnant right after seeking help for the first time (what occurred in the same month), or that were pregnant at the time when they sought help but did not know it. In fact 23 of the 36 reported as being pregnant the same month that first sought help to get pregnant got pregnant exactly that month or the previous one.

Table 2.2: Descriptive statistics -  $\operatorname{HELP}$  sample

|   | Means and (standard deviations) |                              |                              |  |  |
|---|---------------------------------|------------------------------|------------------------------|--|--|
| Variable                                      | All<br>women                    | AnyChildren21=0 <sup>a</sup> | AnyChildren21=1 <sup>a</sup> |  |  |
| Front and 121 (1 if amplement in month 21)    | 0.798                           | 0.877                        | 0.624                        |  |  |
| Employed21 (=1 if employed in month 21)       | (0.402)                         | (0.329)                      | (0.484)                      |  |  |
| Employed0 (=1 if employed in month 0)         | 0.862                           | 0.853                        | 0.881                        |  |  |
| Employedo (=1 ii employed iii montii 0)       | (0.345)                         | (0.354)                      | (0.324)                      |  |  |
| Employed_12 (=1 if employed in month -12)     | 0.855                           | 0.862                        | 0.841                        |  |  |
| Employed_12 (=1 if employed if filontifi -12) | (0.352)                         | (0.345)                      | (0.366)                      |  |  |
| OwnChildren21 (number of own children in      | 0.323                           | 0.000                        | 1.036                        |  |  |
| month 21) <sup>b</sup>                        | (0.491)                         | (0.000)                      | (0.185)                      |  |  |
| AnyOtherChildren21 (=1 if had adopted or      | 0.020                           | 0.027                        | 0.005                        |  |  |
| step children in month 21)                    | (0.141)                         | (0.162)                      | (0.073)                      |  |  |
| AgeO (age in month 0)                         | 26.3                            | 26.5                         | 25.9                         |  |  |
| Ageo (age in month o)                         | (4.3)                           | (4.1)                        | (4.7)                        |  |  |
| Year0 (year in month 0 and normalized as      | 14.7                            | 14.5                         | 15.0                         |  |  |
| 1970=0)                                       | (5.7)                           | (5.5)                        | (6.1)                        |  |  |
| Education0 (years of education in month 0)    | 13.6                            | 13.5                         | 13.8                         |  |  |
| Education (years of education in month o)     | (2.5)                           | (2.4)                        | (2.6)                        |  |  |
| Hispanic (=1 if Hispanic)                     | 0.069                           | 0.050                        | 0.113                        |  |  |
| mspane (–1 ii mspane)                         | (0.254)                         | (0.217)                      | (0.317)                      |  |  |
| Direct ( 1 :fbl-sl-)                          | 0.087                           | 0.091                        | 0.078                        |  |  |
| Black (=1 if black)                           | (0.281)                         | (0.287)                      | (0.267)                      |  |  |
|   | 0.884                           | 0.884                        | 0.884                        |  |  |
| Married0 (=1 if married in month 0)           | (0.320)                         | (0.321)                      | (0.320)                      |  |  |
| 6 1 0 / 1 / 6 1 1 1 2 2 2                     | 0.370                           | 0.408                        | 0.286                        |  |  |
| Smoke0 (=1 if smoke in month 0)               | (0.483)                         | (0.492)                      | (0.452)                      |  |  |
| InsuranceCovered (=1 if insurance covers      | 0.789                           | 0.787                        | 0.792                        |  |  |
| infertility treatments)                       | (0.408)                         | (0.409)                      | (0.406)                      |  |  |
| Number of observations                        | 499                             | 335                          | 164                          |  |  |

<sup>&</sup>lt;sup>a</sup> *AnyChildren21*=1 if the woman had at least an own child in month 21.
<sup>b</sup> Only 6 women had more than one child in month 21 and all them had exactly two children. From these women, 5 of them gave birth to twins and only one woman had twice one child.

Table 2.3: Comparison of HELP sample with women in the NSFG that had at least one child

## Means and (standard deviations)

|                          |                 |                      | NSFG –Mothers with       |
|--------------------------|-----------------|----------------------|--------------------------|
|                          |                 |                      | first birth when aged 19 |
| Variables a              | HELP sample     | NSFG – All Mothers b | to 38°                   |
| 4                        | 26.3            | 22.9 **              | 24.5 **                  |
| Age                      | (4.3)           | (4.9)                | (4.2)                    |
| Voqu                     | 14.7            | 14.0 *               | 15.0                     |
| Year                     | (5.7)           | (7.0)                | (6.4)                    |
| Employed 12d             | 0.855           | NT / A               | 0.787 **                 |
| Employed_12 <sup>d</sup> | (0.352)         | N/A                  | (0.409)                  |
| Education                | 13.6            | 12.3 **              | 12.8 **                  |
| Education                | (2.5)           | (2.6)                | (2.5)                    |
| Hignania                 | 0.069           | 0.125 **             | 0.112 **                 |
| Hispanic                 | (0.254)         | (0.331)              | (0.316)                  |
| Black                    | 0.087           | 0.150 **             | 0.110                    |
| Биск                     | (0.281)         | (0.357)              | (0.312)                  |
|                          | 0.884           | 0.702 **             | 0.782 **                 |
| Married                  | (0.320)         | (0.457)              | (0.413)                  |
| Const                    | 0.370           | 0.336                | 0.329                    |
| Smoke                    | (0.483)         | (0.472)              | (0.470)                  |
| Number of                | 499             | 6,911                | 5,150                    |
| observations             | <del>4</del> 77 | 0,911                | 5,150                    |

<sup>\*</sup> Significantly different from the mean of the HELP sample at the 5% significance level.

<sup>\*\*</sup> Significantly different from the mean of the HELP sample at the 1% significance level.

<sup>&</sup>lt;sup>a</sup> Variables for the two samples of mothers (second and third column) are measured at the month in which they gave birth to their first child (except from *Employed\_12*). Variables for women in the HELP sample (last column) are measured in the month in which they first sought help to get pregnant (except from *Employed\_12*).

<sup>&</sup>lt;sup>b</sup> This sample is constructed selecting in the NSFG sample all women that had at least one child.

<sup>&</sup>lt;sup>c</sup> Includes all women in the NSFG sample that gave birth their first child while being aged 19 to 38 years old

<sup>&</sup>lt;sup>d</sup> *Employed\_12* equals to 1 if the woman was employed 12 months before her first birth (third column) or 12 months before she first sought help to get pregnant (fourth column). In the case of the NSFG – All mothers sample (second column) I can not compute this variable as work status is asked in the survey only for months after the woman reaches 18 years old.

Table 2.4: Linear probability estimates. Impact of having a first child on employment. Dependent variable is Employed21 - HELP sample

| Independent variable   | (1)               | (2)               | (3)               | (4)               | (5)               | (6)               |
|------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| AnyChildren21          | -0.253<br>(0.045) | -0.246<br>(0.044) | -0.254<br>(0.044) | -0.263<br>(0.044) | -0.261<br>(0.043) | -0.263<br>(0.043) |
| Age0                   | -                 | 0.013<br>(0.005)  | 0.007<br>(0.005)  | 0.006<br>(0.005)  | 0.000<br>(0.005)  | 0.000<br>(0.005)  |
| Year0                  | -                 | -                 | 0.010<br>(0.004)  | 0.009<br>(0.004)  | 0.010<br>(0.004)  | 0.011<br>(0.004)  |
| Smoke0                 | -                 | -                 | -                 | -0.066<br>(0.040) | -0.045<br>(0.042) | -0.046<br>(0.041) |
| Education0             | -                 | -                 | -                 | -                 | 0.021<br>(0.007)  | 0.020<br>(0.007)  |
| Hispanic               | -                 | -                 | -                 | -                 | -0.131<br>(0.071) | -0.138<br>(0.069) |
| Black                  | -                 | -                 | -                 | -                 | 0.014<br>(0.050)  | -0.016<br>(0.051) |
| Married0               | -                 | -                 | -                 | -                 | -                 | -0.089<br>(0.038) |
| InsuranceCovered       | -                 | -                 | -                 | -                 | -                 | 0.109<br>(0.049)  |
| Constant               | 0.877<br>(0.019)  | 0.529<br>(0.128)  | 0.563<br>(0.126)  | 0.617<br>(0.137)  | 0.459<br>(0.156)  | 0.449<br>(0.149)  |
| Adj. R-squared         | 0.0854            | 0.1051            | 0.1190            | 0.1251            | 0.1467            | 0.1666            |
| Number of observations | 499               | 499               | 499               | 499               | 499               | 499               |

The mean of *Employed21* is 0.798. Standard errors in parenthesis.

Table 2.5: Fraction of women in each work status in month 21 - HELP sample

|                        | All women | Any<br>Children21=0 | Any<br>Children21=1 | Difference      |
|------------------------|-----------|---------------------|---------------------|-----------------|
|                        | (1)       | (2)                 | (3)                 | (4) = (3) - (2) |
| No Job                 | 20.20 %   | 12.31 %             | 37.64 %             | 25.33 %         |
| Maternity leave        | 4.31 %    | 0.39 %              | 12.94 %             | 12.55 %         |
| Part-time <sup>a</sup> | 7.18 %    | 5.98 %              | 9.83 %              | 3.85 %          |
| Full-time <sup>a</sup> | 68.31 %   | 81.32 %             | 39.59 %             | -41.73 %        |
| Total                  | 100 %     | 100 %               | 100 %               | 0 %             |

<sup>&</sup>lt;sup>a</sup> In the NSFG sample a woman is reported as working full-time if she was working 35 hours or more a week.

Table 2.6: Multinomial logit estimates of the impact of having a first child on work status - HELP sample

|                             | Marginal effects of changing <i>AnyChildren21</i> from 0 to 1 (and standard errors) |
|-----------------------------|---|
| No Job                      | 0.291<br>(0.047)  |
| Maternity leave             | 0.092<br>(0.027)  |
| Part-time <sup>a</sup>      | 0.048<br>(0.027)  |
| Full-time <sup>a</sup>      | -0.431<br>(0.050)   |
| Number of observations      | 499   |
| Log pseudo-likelihood value | -374.30   |
| Pseudo R-squared            | 0.1738  |

The dependent variable has four categories: no job, maternity leave, part-time and full-time. Covariates: *Age0*, *Year0*, *Smoke0*, *Education0*, *Hispanic*, *Black*, *Married0*, *InsuranceCovered*. Standard errors in parenthesis.

Table 2.7: Linear probability estimates. Predicting fertility using selected covariates. Dependent variable is AnyChildren21 - HELP sample

| Independent variable                    |                   |
|---|-------------------|
| Age0                                    | -0.016<br>(0.006) |
| Year0                                   | 0.007<br>(0.005)  |
| Smoke0                                  | -0.102<br>(0.046) |
| Education0                              | 0.013<br>(0.010)  |
| Hispanic                                | 0.198<br>(0.082)  |
| Black                                   | -0.052<br>(0.066) |
| Married0                                | -0.012<br>(0.051) |
| InsuranceCovered                        | 0.022<br>(0.054)  |
| Constant                                | 0.472<br>(0.184)  |
| Adj. R-squared                          | 0.0427            |
| P-value of F-test of joint significance | 0.0020            |
| Number of observations                  | 499               |

The mean of *AnyChildren21* is 0.312. Standard errors in parenthesis.

Table 2.8: Linear probability estimates. Explaining employment in month 0 using fertility status in month 21. Dependent variable is  $Employed\theta$  - HELP sample

| Independent variable   | (1)              | (2)              | (3)              | (4)               | (5)               | (6)               |
|------------------------|------------------|------------------|------------------|-------------------|-------------------|-------------------|
| AnyChildren21          | 0.028<br>(0.035) | 0.033<br>(0.035) | 0.027<br>(0.035) | 0.024<br>(0.035)  | 0.025<br>(0.034)  | 0.022<br>(0.034)  |
| Age0                   | -                | 0.009<br>(0.004) | 0.005<br>(0.004) | 0.004<br>(0.005)  | 0.002<br>(0.005)  | 0.002<br>(0.004)  |
| Year0                  | -                | -                | 0.007<br>(0.004) | 0.007<br>(0.004)  | 0.007<br>(0.004)  | 0.008<br>(0.004)  |
| Smoke0                 | -                | -                | -                | -0.029<br>(0.039) | -0.019<br>(0.041) | -0.020<br>(0.040) |
| Education0             | -                | -                | -                | -                 | 0.010<br>(0.006)  | 0.009<br>(0.006)  |
| Hispanic               | -                | -                | -                | -                 | -0.073<br>(0.067) | -0.075<br>(0.070) |
| Black                  | -                | -                | -                | -                 | 0.003<br>(0.047)  | -0.024<br>(0.048) |
| Married0               | -                | -                | -                | -                 | -                 | -0.071<br>(0.037) |
| InsuranceCovered       | -                | -                | -                | -                 | -                 | 0.119<br>(0.052)  |
| Constant               | 0.853<br>(0.022) | 0.605<br>(0.113) | 0.629<br>(0.112) | 0.653<br>(0.122)  | 0.580<br>(0.142)  | 0.553<br>(0.146)  |
| Adj. R-squared         | 0.0014           | 0.0149           | 0.0242           | 0.0258            | 0.0333            | 0.0592            |
| Number of observations | 499              | 499              | 499              | 499               | 499               | 499               |

The mean of *Employed0* is 0.862. Standard errors in parenthesis.

Table 2.9: Linear probability estimates. Explaining employment in month -12 using fertility status in month 21. Dependent variable is  $Employed\_12$  - HELP sample

| Independent variable   | (1)               | (2)               | (3)               | (4)               | (5)               | (6)               |
|------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| AnyChildren21          | -0.021<br>(0.038) | -0.014<br>(0.037) | -0.017<br>(0.038) | -0.017<br>(0.038) | -0.020<br>(0.037) | -0.022<br>(0.037) |
| Age0                   | -                 | 0.012<br>(0.004)  | 0.010<br>(0.005)  | 0.010<br>(0.005)  | 0.004<br>(0.005)  | 0.004<br>(0.005)  |
| Year0                  | -                 | -                 | 0.003<br>(0.004)  | 0.003<br>(0.004)  | 0.004<br>(0.003)  | 0.004<br>(0.003)  |
| Smoke0                 | -                 | -                 | -                 | -0.002<br>(0.040) | 0.020<br>(0.041)  | 0.018<br>(0.040)  |
| Education0             | -                 | -                 | -                 | -                 | 0.022<br>(0.007)  | 0.021<br>(0.007)  |
| Hispanic               | -                 | -                 | -                 | -                 | -0.069<br>(0.068) | -0.080<br>(0.070) |
| Black                  | -                 | -                 | -                 | -                 | -0.033<br>(0.050) | -0.064<br>(0.050) |
| Married0               | -                 | -                 | -                 | -                 | -                 | -0.096<br>(0.038) |
| InsuranceCovered       | -                 | -                 | -                 | -                 | -                 | 0.099<br>(0.053)  |
| Constant               | 0.862<br>(0.021)  | 0.551<br>(0.119)  | 0.562<br>(0.117)  | 0.564<br>(0.129)  | 0.399<br>(0.150)  | 0.400<br>(0.149)  |
| Adj. R-squared         | 0.0007            | 0.0212            | 0.0231            | 0.0232            | 0.0465            | 0.0717            |
| Number of observations | 499               | 499               | 499               | 499               | 499               | 499               |

The mean of *Employed\_12* is 0.855. Standard errors in parenthesis.

Table 2.10: Linear probability estimates. Impact of having a first child on employment. Dependent variable is Employed21 - HELP sample

| Independent variable   | (1)               | (2)               | (3)               | (4)               | (5)               | (6)               |
|------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| AnyChildren21          | -0.257<br>(0.045) | -0.249<br>(0.044) | -0.258<br>(0.044) | -0.268<br>(0.043) | -0.266<br>(0.043) | -0.270<br>(0.043) |
| Pregnant21             | -0.032<br>(0.054) | -0.031<br>(0.053) | -0.037<br>(0.052) | -0.045<br>(0.052) | -0.044<br>(0.052) | -0.052<br>(0.053) |
| Age0                   | -                 | 0.013<br>(0.005)  | 0.006<br>(0.005)  | 0.006<br>(0.005)  | 0.000<br>(0.005)  | 0.000<br>(0.005)  |
| Year0                  | -                 | -                 | 0.010<br>(0.004)  | 0.009<br>(0.004)  | 0.011<br>(0.004)  | 0.011<br>(0.004)  |
| Smoke0                 | -                 | -                 | -                 | -0.069<br>(0.040) | -0.048<br>(0.042) | -0.049<br>(0.041) |
| Educat0                | -                 | <del>-</del>      | -                 | -                 | 0.021<br>(0.007)  | 0.020<br>(0.007)  |
| Hispanic               | -                 | -                 | -                 | -                 | -0.130<br>(0.071) | -0.138<br>(0.070) |
| Black                  | -                 | -                 | -                 | -                 | 0.008<br>(0.051)  | -0.025<br>(0.051) |
| Married0               | -                 | -                 | -                 | -                 | -                 | -0.092<br>(0.038) |
| Inscoverage            | -                 | -                 | -                 | -                 | -                 | 0.109<br>(0.049)  |
| Constant               | 0.883<br>(0.021)  | 0.535<br>(0.128)  | 0.570<br>(0.125)  | 0.629<br>(0.136)  | 0.471<br>(0.155)  | 0.465<br>(0.148)  |
| Adj. R-squared         | 0.0862            | 0.1059            | 0.1201            | 0.1267            | 0.1482            | 0.1687            |
| Number of observations | 499               | 499               | 499               | 499               | 499               | 499               |

The mean of *Employed21* is 0.798. Standard errors in parenthesis.

Table 2.11: Descriptive statistics - HELP panel data and NSFG panel data

|                        | Means (and standard deviations)   |  |  |  |  |  |
|------------------------|---|--|--|--|--|--|
| Data                   | NSFG – Cycle 5 (1995)   | NSFG – Cycle 5 (1995)  |  |  |  |  |
| Sample                 | HELP panel data. It includes<br>women in the HELP sample. For<br>each individual includes<br>observations of months -12 up to<br>21 | NSFG panel data. It includes cohabitating/married women aged 21 to 40 childless or with children younger than 1 year old |  |  |  |  |
| Unit of observation    | Woman-month   | Woman-month  |  |  |  |  |
| Employed               | 0.844<br>(0.363)  | 0.808<br>(0.394)   |  |  |  |  |
| AnyChildren            | 0.066<br>(0.249)  | 0.171<br>(0.376)   |  |  |  |  |
| Age                    | 26.7<br>(4.4)   | 27.0<br>(4.4)  |  |  |  |  |
| Year (1970=0)          | 14.7<br>(5.7)   | 15.6<br>(5.8)  |  |  |  |  |
| Smoke                  | 0.370<br>(0.483)  | 0.327<br>(0.469)   |  |  |  |  |
| Education              | 14.0<br>(2.5)   | 14.0<br>(2.6)  |  |  |  |  |
| Hispanic               | 0.069<br>(0.254)  | 0.066<br>(0.248)   |  |  |  |  |
| Black                  | 0.087<br>(0.281)  | 0.056<br>(0.230)   |  |  |  |  |
| Married                | 0.863<br>(0.344)  | 0.891<br>(0.311)   |  |  |  |  |
| Number of observations | 16,966  | 237,751  |  |  |  |  |
| Number of women        | 499   | 4,786  |  |  |  |  |

Table 2.12: Descriptive statistics by fertility status - HELP panel data and NSFG panel data

| Data                         | NSFG – Cyo                              | cle 5 (1995)   | NSFG – Cy     | cle 5 (1995)  |
|------------------------------|---|--|---------------|---|
| Sample                       | the HELP sample. I includes observation | HELP panel data. It includes women in<br>the HELP sample. For each individual<br>includes observations of months -12 up<br>to 21 |               | ata. It includes<br>ed women aged 21<br>or with children<br>in 1 year old |
| Unit of observation          | Woman                                   | -month   | Womar         | n-month   |
| Variables                    | AnyChildren=0                           | AnyChildren=1  | AnyChildren=0 | AnyChildren=1   |
| Employed                     | 0.860                                   | 0.610  | 0.853         | 0.588   |
|                              | (0.347)                                 | (0.488)  | (0.354)       | (0.492)   |
| Age                          | 26.7                                    | 27.3   | 27.2          | 26.3  |
|                              | (4.4)                                   | (4.7)  | (4.4)         | (3.9)   |
| Year (1970=0)                | 14.6                                    | 15.0   | 15.7          | 15.2  |
|                              | (5.7)                                   | (6.0)  | (5.7)         | (5.8)   |
| Smoke                        | 0.376                                   | 0.292  | 0.333         | 0.296   |
|                              | (0.484)                                 | (0.454)  | (0.471)       | (0.457)   |
| Education                    | 14.0                                    | 14.2   | 14.1          | 13.6  |
|                              | (2.6)                                   | (2.4)  | (2.6)         | (2.6)   |
| Hispanic                     | 0.066                                   | 0.116  | 0.060         | 0.097   |
|                              | (0.249)                                 | (0.320)  | (0.237)       | (0.296)   |
| Black                        | 0.087                                   | 0.087  | 0.056         | 0.056   |
|                              | (0.281)                                 | (0.282)  | (0.230)       | (0.230)   |
| Married                      | 0.860                                   | 0.913  | 0.875         | 0.969   |
|                              | (0.347)                                 | (0.282)  | (0.330)       | (0.173)   |
| Number of observations       | 15,727                                  | 1,239  | 195,020       | 42,731  |
| Number of women <sup>a</sup> | 499                                     | 164  | 4,291         | 3,635   |

<sup>&</sup>lt;sup>a</sup> Numbers of women in each cell correspond to the number of women that at least in some month of each panel data where childless or had children.

Table 2.13: Linear probability estimates. Impact of having a first child on employment. Dependent variable is Employed - HELP panel data and NSFG panel data

| Data                   | NSFG – C  | ycle 5 (1995)     | NSFG – Cycle 5 (1995)  |                   |  |
|------------------------|---|-------------------|--|-------------------|--|
| Sample                 | HELP panel data. It includes women in<br>the HELP sample. For each individual<br>includes observations of months -12 up<br>to 21  Woman-month |                   | NSFG panel data. It includes<br>cohabitating/married women aged<br>21 to 40 childless or with children<br>younger than 1 year old<br>Woman-month |                   |  |
| Unit of observation    |   |                   |  |                   |  |
| Regression model       | OLS   | Fixed effects     | OLS  | Fixed effects     |  |
| AnyChildren            | -0.251<br>(0.036)   | -0.225<br>(0.032) | -0.259<br>(0.010)  | -0.219<br>(0.010) |  |
| Pregnant               | -0.055<br>(0.020)   | -0.039<br>(0.016) | -0.074<br>(0.008)  | -0.052<br>(0.007) |  |
| Age                    | 0.004<br>(0.004)  | -                 | 0.000<br>(0.002)   | -                 |  |
| Year (1970=0)          | 0.007<br>(0.003)  | -                 | 0.005<br>(0.001)   | -                 |  |
| Smoke                  | -0.028<br>(0.035)   | -                 | 0.033<br>(0.011)   | -                 |  |
| Education              | 0.013<br>(0.005)  | -                 | 0.016<br>(0.003)   | -                 |  |
| Hispanic               | -0.118<br>(0.063)   | -                 | -0.073<br>(0.019)  | -                 |  |
| Black                  | -0.023<br>(0.044)   | -                 | 0.024<br>(0.018)   | -                 |  |
| Married                | -0.023<br>(0.039)   | -                 | -0.020<br>(0.010)  | -                 |  |
| Constant               | 0.511<br>(0.133)  | 0.584<br>(0.012)  | 0.576<br>(0.045)   | 0.931<br>(0.073)  |  |
| Adj. R-squared         | 0.0762  | 0.7008            | 0.0880   | 0.5753            |  |
| Number of observations | 16,966  | 16,966            | 237,751  | 237,751           |  |

Fixed effects models include dummies for individuals and year. Observations clustered by individual. Standard errors in parenthesis.

Table 2.14: Distribution of women with respect to work status by fertility status - HELP panel data and NSFG panel data

| Data                   | NSFG – Cycle 5 (1995)   |                   | NSFG – Cycle 5 (1995)  |                   |
|------------------------|---|-------------------|--|-------------------|
| Sample                 | HELP panel data. It includes<br>women in the HELP sample. For<br>each individual includes<br>observations of months -12 up to<br>21 |                   | NSFG panel data. It includes cohabitating/married women aged 21 to 40 childless or with children younger than 1 year old |                   |
| Unit of observation    | Woman-month   |                   | Woman-month  |                   |
| Variables              | Any<br>Children=0   | Any<br>Children=1 | Any<br>Children=0  | Any<br>Children=1 |
| No Job                 | 13.98   | 38.97             | 14.73  | 41.16             |
| Maternity leave        | 0.35  | 23.02             | 0.41   | 12.38             |
| Part-time <sup>a</sup> | 7.44  | 9.17              | 10.19  | 10.2              |
| Full-time <sup>a</sup> | 78.23   | 28.84             | 74.67  | 36.26             |

<sup>&</sup>lt;sup>a</sup> In the NSFG sample, a woman is reported as working full-time (opposed to be part-time) if she was working 35 hours or more a week.

Table 2.15: Multinomial logit estimates of the impact of having a first child on work status - HELP panel data and NSFG panel data

| Data                        | NSFG – Cycle 5 (1995)   | NSFG – Cycle 5 (1995)  |  |
|-----------------------------|---|--|--|
| Sample                      | HELP panel data. It includes<br>women in the HELP sample.<br>For each individual includes<br>observations of months -12 up<br>to 21 | NSFG panel data. It includes<br>cohabitating/married women<br>aged 21 to 40 childless or with<br>children younger than 1 year<br>old |  |
| Unit of observation         | Woman-month   | Woman-month  |  |
|                             | Marginal effects of changing <i>AnyChildren</i> from 0 to 1 (and standard errors)   |  |  |
| No Job                      | 0.273<br>(0.040)  | 0.246<br>(0.009)   |  |
| Maternity leave             | 0.190<br>(0.022)  | 0.116<br>(0.004)   |  |
| Part-time <sup>a</sup>      | 0.021<br>(0.022)  | 0.005<br>(0.006)   |  |
| Full-time <sup>a</sup>      | -0.484<br>(0.034)   | -0.368<br>(0.009)  |  |
| Number of observations      | 16,966  | 237,751  |  |
| Log pseudo-likelihood value | -11,584.29  | -195,128.98  |  |
| Pseudo R-squared            | 0.1194  | 0.0904   |  |

<sup>&</sup>lt;sup>a</sup> The dependent variable has four categories: no job, maternity leave, part-time and full-time. Covariates: *Age*, *Year*, *Smoke*, *Education*, *Hispanic*, *Black*, *Married*. Observations clustered by individual. Standard errors in parenthesis.

Table 2.16: Descriptive statistics - HELP, 1980 PUMS and 1990 PUMS samples

|                        | Means (and standard deviations)                       |  |  |  |  |
|------------------------|---|--|--|--|--|
| Data                   | NSFG – Cycle 5<br>(1995)                              | 5-percent 1980 PUMS  | 5-percent 1990 PUMS  |  |  |
| Sample                 | Married women in HELP sample                          | Married women aged 21 to 40 childless or with children younger than 1 year old | Married women aged 2<br>to 40 childless or with<br>children younger than 1<br>year old |  |  |
| Variables measured     | 21 months after<br>seeking help for the<br>first time | in 1980  | in 1990  |  |  |
| Employed               | 0.793   | 0.726  | 0.796  |  |  |
|                        | (0.405)   | (0.446)  | (0.403)  |  |  |
| AnyChildren            | 0.324   | 0.158  | 0.128  |  |  |
|                        | (0.468)   | (0.364)  | (0.334)  |  |  |
| Age                    | 28.2  | 27.2   | 29.3   |  |  |
|                        | (4.2)   | (4.9)  | (5.3)  |  |  |
| Education              | 13.8  | 13.4   | 13.9   |  |  |
|                        | (2.5)   | (2.6)  | (2.5)  |  |  |
| Hispanic               | 0.066   | 0.053  | 0.077  |  |  |
|                        | (0.247)   | (0.223)  | (0.266)  |  |  |
| Black                  | 0.067   | 0.061  | 0.061  |  |  |
|                        | (0.249)   | (0.239)  | (0.240)  |  |  |
| Number of observations | 440   | 287,292  | 301,371  |  |  |

Table 2.17: Descriptive statistics by fertility status - HELP, 1980 PUMS and 1990 PUMS samples

|                        | Means (and standard devi                        |            |  |            | ns)  |            |  |
|------------------------|---|------------|--|------------|--|------------|--|
| Data                   | NSFG – Cycle 5 (1995)                           |            | 5-percent 1980 PUMS  |            | 5-percent 1990 PUMS  |            |  |
| Sample                 | Married women in HELP sample                    |            | Married women aged 21 to 40 childless or with children younger than 1 year old |            | Married women aged 21 to 40 childless or with children younger than 1 year old |            |  |
| Variables measured     | 21 months after seeking help for the first time |            | in 1   | in 1980    |  | in 1990    |  |
| Variables              | Any   | Any        | Any  | Any        | Any  | Any        |  |
|                        | Children=0                                      | Children=1 | Children=0   | Children=1 | Children=0   | Children=1 |  |
| Employed               | 0.881   | 0.612      | 0.783  | 0.419      | 0.825  | 0.598      |  |
|                        | (0.324)   | (0.487)    | (0.412)  | (0.493)    | (0.380)  | (0.490)    |  |
| Age                    | 28.4  | 27.6       | 27.5   | 25.8       | 29.5   | 27.7       |  |
|                        | (4.0)   | (4.4)      | (5.0)  | (3.7)      | (5.3)  | (4.3)      |  |
| Education              | 13.7  | 13.9       | 13.5   | 13.3       | 13.9   | 14.0       |  |
|                        | (2.5)   | (2.6)      | (2.6)  | (2.5)      | (2.5)  | (2.4)      |  |
| Hispanic               | 0.052   | 0.094      | 0.050  | 0.066      | 0.076  | 0.085      |  |
|                        | (0.222)   | (0.292)    | (0.218)  | (0.249)    | (0.264)  | (0.279)    |  |
| Black                  | 0.073   | 0.053      | 0.062  | 0.056      | 0.062  | 0.055      |  |
|                        | (0.261)   | (0.224)    | (0.240)  | (0.229)    | (0.241)  | (0.228)    |  |
| Number of observations | 293   | 147        | 242,003  | 45,289     | 262,052  | 39,319     |  |

Table 2.18: Linear probability estimates. Impact of having a first child on employment - HELP, 1980~PUMS and 1990~PUMS samples

| Data   | NSFG – Cycle 5 (1995)                           |                   | 5-percent 1980 PUMS   | 5-percent 1990 PUMS   |
|--|---|-------------------|---|---|
| Sample                                       | Married women in HELP sample                    |                   | Married women aged<br>21 to 40 childless or<br>with children younger<br>than 1 year old | Married women aged<br>21 to 40 childless or<br>with children younger<br>than 1 year old |
| Variables measured                           | 21 months after seeking help for the first time |                   | in 1980   | in 1990   |
| Mean of dependent variable – <i>Employed</i> | 0.793   | 0.793             | 0.726   | 0.796   |
| Independent variables                        | (1)   | (2)               | (3)   | (4)   |
| AnyChildren                                  | -0.277<br>(0.045)                               | -0.377<br>(0.071) | -0.365<br>(0.002)   | -0.228<br>(0.003)   |
| AnyChildren *LateHelp                        | -   | 0.185<br>(0.090)  | -   | -   |
| Age  | -0.005<br>(0.006)                               | -0.005<br>(0.006) | -0.004<br>(0.000)   | -0.001<br>(0.000)   |
| Education                                    | 0.024<br>(0.007)                                | 0.023<br>(0.007)  | 0.030<br>(0.000)  | 0.031<br>(0.000)  |
| Hispanic                                     | -0.154<br>(0.077)                               | -0.155<br>(0.078) | -0.047<br>(0.004)   | -0.087<br>(0.004)   |
| Black  | -0.012<br>(0.061)                               | -0.010<br>(0.062) | -0.017<br>(0.003)   | -0.033<br>(0.004)   |
| Year0  | 0.015<br>(0.004)                                | 0.010<br>(0.004)  | -   | -   |
| Constant                                     | 0.484<br>(0.159)                                | 0.568<br>(0.163)  | 0.489<br>(0.006)  | 0.417<br>(0.007)  |
| Adj. R-squared                               | 0.1646  | 0.1767            | 0.1222  | 0.0814  |
| Number of observations                       | 440   | 440               | 287,292   | 301,371   |

Standard errors in parenthesis.

Figure 2.1: Distribution of women by age at: a) first birth (mothers with first birth when aged 19 to 38), b) which first sought help to get pregnant (women in the HELP sample)

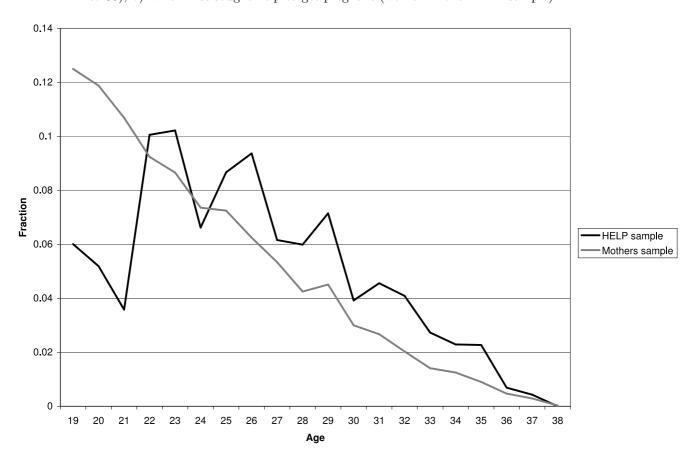


Figure 2.2: Employment rates by month for women that: a) had a baby by month 21 b) did not have a baby by month 21 - HELP sample

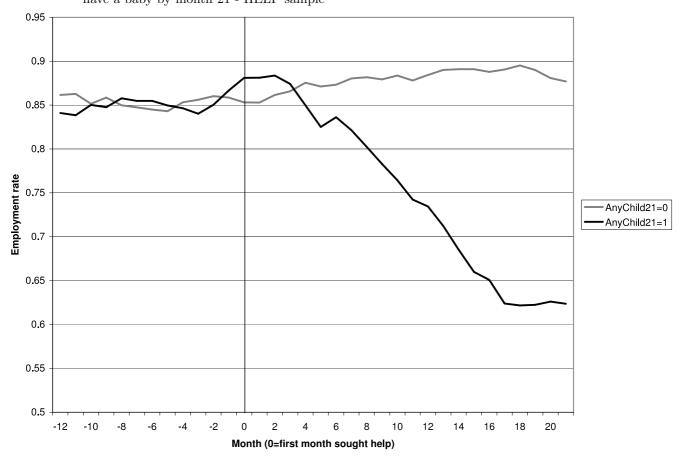


Figure 2.3: Distribution of mothes by babies' age in months - NSFG panel data and HELP panel data

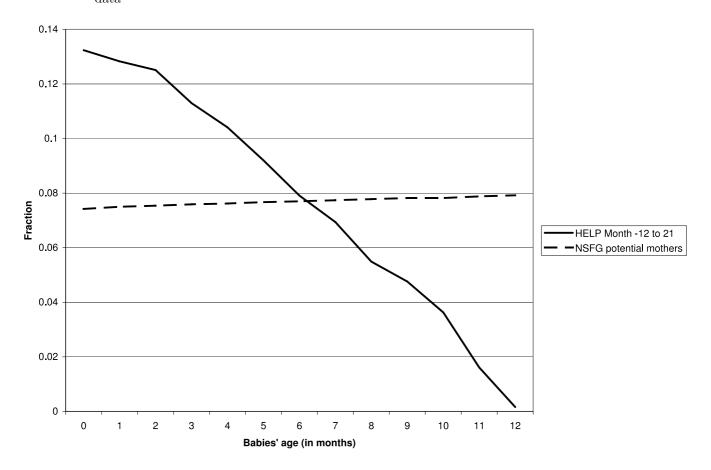


Figure 2.4: Distribution of work status around birth month - women that gave birth to a child by

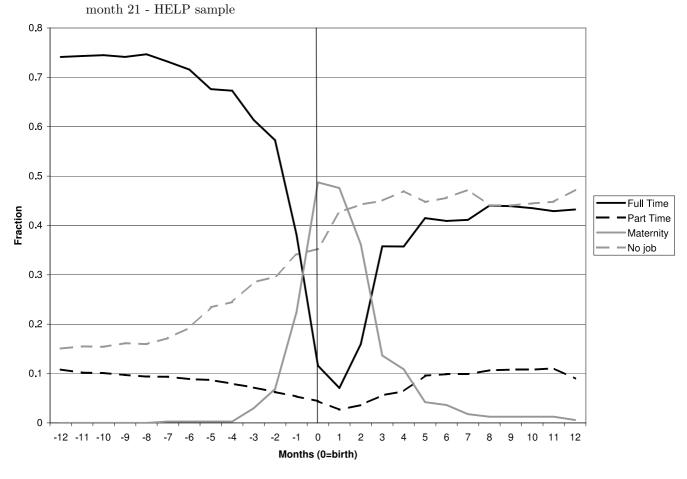


Figure 2.5: Fraction of mothers working part-time and without a job by babies' age - NSFG panel data and HELP panel data

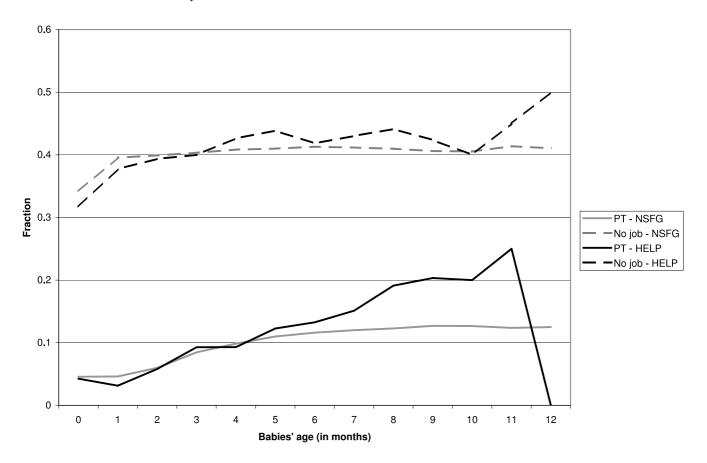
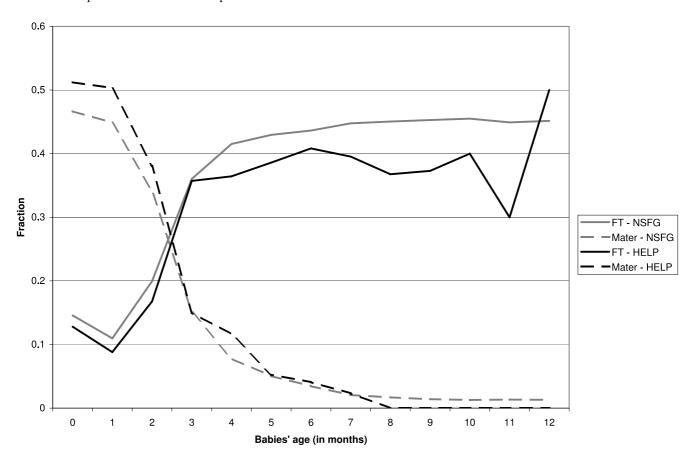


Figure 2.6: Fraction of mothers working full-time and on maternity leave by babies' age - NSFG panel data and HELP panel data



# Chapter 3

On the optimal revelation policy for educational achievement

### 3.1 Introduction

Over the last thirty years real expenditures per student in the U.S. have doubled. In the same period, test results reported in the National Assessment of Educational Progress (NAEP) have remained almost constant. Almost simultaneously, an empirical literature has arisen that tried to estimate an education production function where the main input was considered to be expenditures per student and the output some measure of cognitive skills as test scores. However, this view of the educational process has faced some challenges that can be summarized in the following question. How is it possible that educational inputs increase dramatically and output nearly does not increase at all?

More recently, some researchers have tried to address the previous paradox by aiming at a neglected education input: students' effort (Costrell, 1994; Betts, 1998). This new strand of the literature analyzes the educational process in a principal-agent setting where educators try to maximize educational output setting up an optimal scheme that takes into account students' preferences and incentives.

The central message of this paper is that the way schools and universities reveal information about students is a very useful tool to influence students decisions about effort. In particular, the optimal revelation policy should be dependent on the way students are distributed with respect to ability or other personal characteristics that influence their effort decisions.

To show this point, I present a simple model in which I analytically demonstrate that the optimal revelation policy depends on the distribution of students with respect to ability. In this model, if this density is decreasing, the educational authority should reveal all information about students' productivity. If this density is increasing, he should classify individuals in two groups

and only reveal this information. Finally, if the density is single-peaked, he should set a critical standard, group together all individuals below this level and provide full information for students that surpass it.

This model share features of other models in this new literature. I assume that firms do not observe individuals' productivity but rather to which group the student was classified into by the educational authority. Due to perfect competition in labor markets, wages for individuals in each group are set at the expected productivity level for the group. Therefore, the educational authority can influence students' behavior by deciding his classification policy. In short, the educational authority has better information than firms about students' productivity and decides an optimal revelation policy for this information.

Typically in previous models, the educational authority was constrained to classify individuals into two groups and then he could only decide where to set the only one standard. The only exception is Costrell (1994) who explores whether the educational authority should classify individuals in two groups and reveal this information or whether he should reveal all the information. Based on simulations, he found that this decision will be affected by the degree of students' heterogeneity and also by how egalitarian the educational authority is. Nonetheless, he asserted that under most plausible cases, perfect information increases GDP as well as social welfare.

This paper contributes to this literature analyzing the more real and interesting situation where the educational authority is not constrained to choose whether to classify students into two groups or to reveal perfect information, but rather he can choose into how many groups he wants to classify students (potentially infinite) and the standards for each group. An interesting finding of this paper is that for the plausible case where the density of students is single-peaked, it is shown that the optimal schedule combines the two "extremes": the educational authority should set a critical standard and provide no information below it (just say that the student failed to attain the standard) and all the information above it.

Costrell (1994) also analyzed how the optimal standard, in a setting where the educational authority can only classify students into two groups, varies under different environments. One of

the main conclusions of his work was that an egalitarian educator will set a lower standard than an income maximizing one. However, Betts (1998) presents a model where individuals' ability affects productivity even when students provide the lowest amount of effort and shows that, in this context, an egalitarian standard setter may choose a higher standard than an income maximizing one.

These theoretical developments were accompanied by many empirical studies that focused on the effects of the creation of high stakes exams in different states over students wages, number of drop-outs and educational achievements.<sup>1</sup> This research effort was fueled by an important public debate about the educational system in the U.S.

In particular, there has been a major and growing concern about the causes and consequences of the poor achievement levels of American students when compared to their counterparts in other developed countries. One of the areas of debate is about the way that schools and universities should convey information to the marketplace and in particular whether they should provide only information about graduation (a discrete variable) or they should provide all the available information. John L. Bishop (1997, 2000, 2001) has been a major supporter of the full information option and has provided a number of policy recommendations about the way to attain this goal.

The model presented in this paper contributes to this discussion as it provides some guidance about how this revelation policy should be decided. Taking the model at face value, this implies that policy recommendations are going to vary depending on the school environment. In particular, we can think that in the case of high schools, which a big fraction of the population attends, the distribution of students with respect to ability will be single-peaked. Consequently, in this case the educational authority should set a critical standard, provide full information above that standard and no information below it. However, in the case of graduate studies, we may suspect that this distribution is characterized by a high number of individuals with high ability and a low number of individuals with low ability. Following the results of this paper, in this case the educational 1000 100

authority should classify students just in two groups and reveal whether a student graduated or not.

The remainder of the paper is organized as follows. Section 3.2 presents the model and discuss its features and assumptions. Section 3.3 describes the solution of the model. Finally, section 3.4 concludes.

# 3.2 The model

In this paper, I analyze the problem that an educational authority faces when he tries to maximize mean education level of a heterogeneous population of students. In this problem there are three types of agents: the educational authority, students-workers and firms. These actors pursue different objectives and interact in a setting where information (and the lack of it) plays a central role.

I assume that students' preferences are defined over lifetime wages and effort provided during the education process. Their productivity in the marketplace is a function of innate ability and the effort provided while they were at school (their decision variable). In particular, I assume that:

$$U = w - \frac{e^2}{2} (3.1)$$

$$p = e\sqrt{a} \tag{3.2}$$

where U represents the utility of the student, w his lifetime wages, e his effort level, p his productivity and finally a his ability level.

Solving equation 3.2 for effort and replacing it in equation 3.1:

$$U = w - \frac{p^2}{2a} \tag{3.3}$$

That is, students are going to have higher levels of utility the higher lifetime wages, but lower levels of utility the more productive they are (keeping wages constant) as it is costly for them to be more productive.

In this model students are different from each other and educators cannot specify different standards for different students. If all students are identical with respect to preferences and ability, then the professor's problem is straightforward (maximize his utility function subject to the participation constraint of the representative student). Similarly, if the professor can set one standard for each student, again he is going to solve a simple maximization problem but now he solves one for each student. The interesting and real case is the one where students are heterogeneous and the professor has to set the same policy for all students.

Throughout this paper, I model student heterogeneity by assuming that students only differ in the ability parameter a. That is, I will assume that  $F(\cdot)$  represents the cumulative distribution of students with respect to ability and  $f(\cdot)$  its density, with supports from 0 to  $\bar{a}$ . All the results of this paper will hold if I alternatively assume that students differ in their disutility of effort or in the relative weight that they give to current disutility of effort and future consumption provided I keep the same distributional assumptions over these parameters.<sup>2</sup>

I assume that firms operate under perfect competition in the labor market. In a world of perfect information and perfect competition, the relationship between a worker's productivity and his wage should be proportional, i.e. a worker who can produce double the amount of goods than another worker should receive double pay. However, a key fact in labor markets is that firms do not perfectly observe individuals' productivity. As it is customary in this type of models, I assume that firms observe educational attainments and pay each individual wages equal to the expected productivity of their educational group. Then, wages are going to depend on the group that the individual belongs to rather than his actual productivity. But how are individuals assigned to groups, and how are these groups defined? To answer these questions I present the third actor:

$$U = w - \beta p^2$$

and then we can see that changes in the ability parameter a just affect the parameter  $\beta$ . That is, students ultimately differ in their parameter  $\beta$ . But this heterogeneity clearly could have arisen if they value differently current effort versus future consumption or if they differ in their disutility of effort.

<sup>&</sup>lt;sup>2</sup>this claim can be supported by observing that the assumptions over heterogeneity in ability ultimately affect the way that students weight lifetimes wages and productivity. That is, the students utility function can be represented as:

the educational authority.

The educational authority is concerned about the educational level (or productivity) that students attain after the educational process. In particular, I assume that the educational authority minimizes the following objective function:

$$\int_0^{\bar{a}} |z(a) - p(a)| f(a) da$$

where z(a) represents the school preferred productivity level for a student of ability a. That is, if the school were able to force a student of ability a to attain certain productivity level, the school would make the student to attain a productivity level equal to z(a). As p(a) represents the productivity level chosen by student of type a, then |z(a) - p(a)| represents how different is the actual productivity level of the student of ability a compared with what the school would like him to attain. Finally, the objective function is just the average of this deviation computed over all students.

I assume that:

$$z(a) \ge 2a$$

Later I will show that a student of type a attains his first best when he chooses a productivity level equal to a. Then, the assumption just presented is used to model the fact that there exists a conflict of interest between students and educators with respect to what the optimal educational level is for these students (educators would like to make students attain higher levels of education than what students would like).

In order to maximize his objective function the educational authority (from now on the professor) chooses a revelation policy. This revelation policy consists in partitioning the productivity space in subsets, where each subset corresponds to an educational group. Then, all students that choose productivity levels which belong to the same subset are going to be assigned to the same and a subset of the papers in this literature assume that educators want to maximize mean productivity level. I relax somewhat this assumption by assuming that the school preferred level of productivity for each student is finite but all the same, significantly higher than what students want.

educational group and then they will receive the same wage (I assume that the professor perfectly observes students' productivity). In this way, the professor chooses how to split students in groups indirectly when choosing how to partition the productivity space in subsets. For example, a simple revelation policy is to partition the productivity space in just two subsets: [0,p) and  $[p,\infty)$ . Then students that choose productivity levels below p are going to be assigned to a group (drop-outs) and the ones that attain it or surpass it are going to be classified into another group (graduates). This way of modeling the revelation policy allows an enormous flexibility but at the same time makes the professor's problem complex given that he can choose any possible partition of the productivity space when maximizing his objective function. Fortunately, the set of all potential revelation policies can be significantly reduced as I show next.

Let's define a standard as the minimum productivity level for each subset defined by the professor from the productivity space. As I will show later, individuals only choose productivity levels that are standards. Then the revelation policy of the professor simplifies to just choosing the set of productivity levels that are standards. That is, the only real decision that the professor takes is determining the standards.

This implies that the professor's revelation policy can be represented by the set S that contains all the standards.<sup>4</sup> As productivity cannot be a negative number then  $S \subset \Re^+$ . Moreover, students can always supply zero effort and attain zero productivity. This participation constraint is introduced by requiring that  $0 \in S$ . As an example, if the set S is just  $S = \{0, \hat{s}\}$ , then the students are going to be assigned to only two groups. Students with productivity  $p \geq \hat{s}$  are assigned to the high productivity group while students with productivity  $p < \hat{s}$  are assigned to the low productivity group. It is important to note that the set S can include intervals which are greater than a point. For example,  $S = \{[0, \hat{s}]\}$  with  $\hat{s} \neq 0$ . In this case, for the whole interval  $[0, \hat{s}]$  productivity is perfectly revealed. The reason for this is that as there is a continuum of standards then each group is going to be formed by agents with only one level of productivity. Finally note that the professor can set  $S = \Re^+$  in which case firms can perfectly observe students' productivity.

 $<sup>^4</sup>$ Strictly speaking, S is allowed to be a finite union of disjoint closed and convex intervals, where an interval can also be a point.

# 3.3 Solution

This section presents the solution of this dynamic game. As it is customary in this type of game, the problem is solved by backward induction. For that, it is useful to state the timing of this game:

- 1) The professor announces his revelation policy, i.e. he announces the set S containing all the standards. This set S defines a classification function  $s(\cdot)$  where s represents the group that the student is assigned to (a group is defined by its lower standard) and its argument is the student productivity. Then, a student of ability a is going to be assigned to group  $s_t$  if this is the highest standard equal or lower than his productivity level.
- 2) Given the professor's strategy, each type of student chooses his optimal strategy and these strategies are represented with the function p(a, S). That is, the productivity level for each student is going to depend on his type (a) and on the revelation policy chosen by the professor (S).
- 3) The professor implements his policy by assigning each student to his corresponding group. This is done by applying the classification function to the students' strategy: s(p(a, S)).
- 4) Firms compute expected productivity for each group and due to perfect competition wages are set at that level. That is:

$$w_i = E\left[p(a, S)|s(p(a, S)) = s_i\right] \quad \forall i \tag{3.4}$$

where  $w_i$  is the wage of group i and  $s_i$  is the standard which defines that group (i.e. the lower standard of that group).

I now apply backwards induction to solve this game. I start by analyzing the way that wages are going to be set by firms.

## 3.3.1 Firms

As said before, due to perfect competition in the labor market, wages are going to be set at mean productivity for each group. The following result simplifies the method to get the equilibria of the game.

#### Lemma 1.

$$p^*(a,S) \in S \quad \forall \ a, \ S \tag{3.5}$$

This lemma says that students optimal strategies involve choosing a productivity level that is equal to a standard set by the professor. This statement can be shown by contradiction. Assume that it is not true. Then there exists at least one agent  $\hat{a}$  for which his optimal productivity level is  $\hat{p} = p(\hat{a})$  and  $\hat{p} \notin S$ . Then,  $s(\hat{p}) < \hat{p}$  (i.e. his productivity level is higher than the standard which defines his group). This implies that this agent is optimally choosing to surpass a standard but not enough to reach a higher standard. But then,  $\exists \ \tilde{p}$  where  $\tilde{p} < \hat{p}$  and  $|\hat{p} - \tilde{p}|$  is so small that  $s(\tilde{p}) = s(\hat{p})$ . This implies that the agent could be receiving the same wage (as he belongs to the same group) but providing less effort by choosing  $\tilde{p}$  instead of  $\hat{p}$ . Hence,  $\hat{p}$  was not optimal and the result is shown by contradiction.

## Corollary 1.

$$w_i = s_i \quad \forall \ i \tag{3.6}$$

That is, wages for each group are equal to the lower standard of that group. The reason is that if all agents choose a productivity level equal to a standard, then the expected productivity for each group is exactly the standard that defines that group.

We now turn to the students' problem.

# 3.3.2 Students

The original student problem is:

$$Max w - \frac{p^2}{2a} st (3.7)$$

$$s(p) = \underset{x}{Max} \quad x \quad st \quad x \in S \quad \land \quad x \le p \tag{3.8}$$

$$w_i = E[p(a, S)|s(p(a, S)) = s_i]$$
 (3.9)

That is, students choose productivity in order to maximize utility subject to the assignment of the student to the group of the highest standard chosen by the professor that the student is able to attain and the fact that wages are set equal to average productivity for each educational group.

The nice feature of equation 1 is that it shows us that the computation of an equilibrium in this case of certainty is much simpler than in other settings. The key is that, as students only choose productivity levels equal to standards, then wages for each group are independent of students' strategies. This implies that there is no connection between the problems of different types of agents and this greatly simplifies the solution of the problem.<sup>5</sup>

Using lemma 1 and corollary 1 the problem simplifies to:

$$Max_{p \in S} \quad w - \frac{p^2}{2a} \qquad s.t. \quad s(p) = p$$

$$w = s(p) \tag{3.10}$$

Eliminating both constraints and variables w and s:

$$p^*(a, S) = Arg \underset{p \in S}{Max} \quad p - \frac{p^2}{2a}$$
 (3.11)

That is, the student is going to maximize his utility function by choosing a productivity level equal to some standard set by the professor. This expression shows the real power that the 5In the absence of this result (as it happens in a situation where the professor observes a signal of productivity rather than its true level) we cannot solve each student problem independently as their mutual decisions are going to affect the expected productivity for each group (and wages) and consequently, we need to solve the problem of all agents simultaneously.

professor enjoys. He can decide the productivity levels that can be chosen by the student and conveyed to the labor market. If the student chooses another productivity level that is different from the set of standards determined by the professor, this information cannot be conveyed to the firms as they do not observe actual productivity but rather which diploma the student has obtained.

Lemma 2. 
$$p^*(a, S) = ArgMin_{p \in S} |a - p|$$

That is, the optimal strategy for each student involves choosing the standard closest to his ability level. Its proof involves just a number of transformations of the utility function (See the Appendix for the proof).

The intuition of this result comes from the fact that, if a student were in a situation of perfect information  $(S = \Re^+)$ , he would choose his first best productivity level. Given his utility function, his first best occurs at  $p^*(a) = a$ . In a situation where there is imperfect information in the labor market, the agent is constrained to choose another productivity level if there is no standard at his first best. It is intuitive to think that he is going to tend to choose the standard which is closest to his first best. In this model, the utility is symmetric with respect to p = a, then the students will choose the standard which is closest to his first best a.

In particular, given that  $0 \in S$  (a student can always drop out and attain productivity equal to zero), under no circumstance p(a) > 2a, as in that case the student should just chooses the zero standard.<sup>6</sup> Then:

$$p(a) \le 2a$$

Throughout this paper, I assume that if an individual is indifferent between choosing two standards, he is going to choose the higher one. This assumption is going to be implicit each time that I present the constraint for the professor's problem about the decision of each student.

<sup>&</sup>lt;sup>6</sup>It is more intuitive to think that productivity and wages for this group are not going to be zero but rather the wage set for raw labor.

## 3.3.3 The professor

The professor solves the following problem,

$$\underset{S \subset \Re^+}{Max} \quad \Omega = \int_0^{\bar{a}} |z(a) - p(a)| f(a) da \qquad s.t. \quad p(a) = \underset{p \in S}{ArgMin} |a - p|$$
 (3.12)

I have assumed that  $z(a) \geq 2a$  and it was shown above that  $p(a) \leq 2a$ . Combining these two inequalities:

$$z(a) \ge p(a)$$

Then,

$$|z(a) - p(a)| = z(a) - p(a)$$

Replacing this in the professor's objective function:

$$\underset{S \subset \mathbb{R}^+}{Min} \ \Omega = \int_0^{\bar{a}} \left[ z(a) - p(a) \right] f(a) da$$

$$\min_{S \subset \Re^+} \Omega = \int_0^{\bar{a}} z(a) f(a) da - \int_0^{\bar{a}} p(a) f(a) da$$

$$\underset{S \subset \Re^+}{Max} \Omega = \int_0^{\bar{a}} p(a)f(a)da - Z$$

Then, the professor's problem simplifies to:

$$\underset{S \subset \mathbb{R}^+}{Max} \Omega = \int_0^{\bar{a}} p(a)f(a)da \qquad s.t. \ p(a) = \underset{p \in S}{ArgMin} |a - p|$$
 (3.13)

That is, he maximizes mean productivity (or educational output) subject to incentive compatibility and participation constraints. He has to decide the set S which contains all the standards.

Assuming that a solution exists, we can clearly see that there are going to be multiplicity of equilibria in a naive manner. As an example, observe that given certain optimal set  $S^*$  where

 $\hat{s} \notin S^*$  and  $\hat{s} > 2\bar{a}$  (recall that  $\bar{a}$  corresponds to the highest ability level),  $\hat{S} = S^* \cup \{\bar{s}\}$  is also going to be optimal. The reason is that the addition of this new standard does not change the value of the objective function as no agent is going to choose it (as it is so high) and at the same time it will not break any constraint for the same reason. Then, it is useful to trim any set S that is solution eliminating all the elements of S which are not going to be chosen by any agent in equilibrium.

It is possible to simplify the problem for the case that the set S has a finite number of elements. In this case, we can order, from the lowest to the highest, all elements in set S starting from  $s_0 = 0 < s_1 < s_2 < \ldots < s_N$ . Then, the lowest ability agents will prefer the standard  $s_0$  and as the ability level increases, agents start choosing higher standards  $s_1, s_2, \ldots, s_N$ . Moreover, defining the agent who is going to be indifferent between two neighboring standards as  $\bar{s}_{ij} = \frac{s_i + s_j}{2}$ , the objective function can be expressed as the following:

$$\Omega = \int_0^{\frac{s_1}{2}} p(x) f(x) dx + \int_{\frac{s_1}{2}}^{\frac{s_1 + s_2}{2}} p(x) f(x) dx + \int_{\frac{s_1 + s_2}{2}}^{\frac{s_2 + s_3}{2}} p(x) f(x) dx + \dots$$

$$+ \int_{\frac{s_{N-1} + s_N}{2}}^{\bar{a}} p(x) f(x) dx$$
(3.14)

As all agents choose the standard which is closest to his ability level:

$$p(x) = s_i \ \forall \frac{s_{i-1} + s_i}{2} \le x < \frac{s_i + s_{i+1}}{2} \ \forall i$$
 (3.15)

This means that given a standard  $s_i$  all agents who have ability levels close enough to that standard are going to choose it. In particular, agents who have ability level lower than the standard, are going to choose that standard provided it is closest to their first best (a), than the immediately lower standard  $s_{i-1}$ . Then, they are going to choose it provided their ability level is higher than the ability level of the agent who is indifferent between standards  $s_{i-1}$  and  $s_i$  (this  $\overline{\phantom{a}}$  This is exactly the same situation as when we are analyzing a problem of price discrimination with 2 types of consumers. Typically a solution is presented that consists of two pairs of prices and quantities. All the same, other solutions could be presented that contain the two pairs of prices and quantities mentioned and other pairs no chosen in equilibrium by any agent, but there is no point in presenting these other solutions.

agent is called  $\bar{s}_{i-1,i}$  and his ability level is equal to  $\bar{s}_{i-1,i} = \frac{s_{i-1}+s_i}{2}$ ). In the same way, all agents who have an ability level higher than  $s_i$  but lower than  $\bar{s}_{i,i+1}$  are going to choose the standard  $s_i$ . Combining the last three equations, it is possible to eliminate all the p(x) functions and the constraints and then the problem simplifies to an unconstrained maximization of the following objective function:<sup>8</sup>

$$\Omega = \int_0^{\frac{s_1}{2}} 0f(x)dx + \int_{\frac{s_1}{2}}^{\frac{s_1+s_2}{2}} s_1f(x)dx + \int_{\frac{s_1+s_2}{2}}^{\frac{s_2+s_3}{2}} s_2f(x)dx + \dots + \int_{\frac{s_{N-1}+s_N}{2}}^{\bar{a}} s_Nf(x)dx$$
(3.16)

$$\Omega = s_1 \left[ F\left(\frac{s_1 + s_2}{2}\right) - F\left(\frac{s_1}{2}\right) \right] + s_2 \left[ F\left(\frac{s_2 + s_3}{2}\right) - F\left(\frac{s_1 + s_2}{2}\right) \right] + \ldots +$$

$$s_N \left[ 1 - F\left(\frac{s_{N-1} + s_N}{2}\right) \right] \tag{3.17}$$

$$\Omega = \sum_{i=0}^{N} s_i \left[ F\left(\frac{s_i + s_{i+1}}{2}\right) - F\left(\frac{s_{i-1} + s_i}{2}\right) \right]$$
(3.18)

where I define  $s_{N+1} = 2\bar{a} - s_N$ .

Presenting the problem in this way gives us a useful insight. The problem of deciding an intermediate standard  $s_i$  is, in its structure, exactly like the problem of deciding any other standard  $s_j$ . The only difference between both problems are the values of the neighboring standards  $(s_{i+1}, s_{i-1} \text{ and } s_{j+1}, s_{j-1}, \text{ respectively})$  and the values that the  $F(\cdot)$  function takes between these neighboring standards. This implies that, if we can find certain results which are independent of these elements (the value of the neighboring standards and the  $F(\cdot)$  function), then these results are going to be applied for all intermediate standards.<sup>10</sup>

Also note that the decision of taking out an intermediate standard from S only affects the value of the objective function, by changing the mass of agents who were choosing that standard

 $<sup>^8 {\</sup>rm Still}$  we have to satisfy that  $s_0 < s_1 \ldots < s_N < 2 \bar{a}$ 

 $<sup>{}^9</sup>s_{N+1}$  is not an element of the set S but rather just a variable that we define in order for  $F(\frac{s_N+s_{N+1}}{2})$  to be equal to one.

 $<sup>^{10}</sup>$ An intermediate standard is one that it is not the lowest element of S (i.e. it is not zero) and it is not the highest one.

(after eliminating it, it is going to be zero) and the mass of agents that choose the neighboring standards. But the change does not affect the mass of agents who were choosing other standards.

These insights are used to prove the following set of theorems.

**Theorem 1.** If 
$$F(x) = x \ \forall \ x \in [0, \bar{a}] \Rightarrow \forall \ S \ / \ \bar{a} \in S, \ S \ is \ a \ solution \ to \ 3.13$$

See the Appendix for the proof.

This theorem says that when the distribution of agents with respect to ability is uniform, there are multiple solutions and the only requirements that the solution set has to satisfy is that  $\bar{a}$  belongs to that set. This implies that, any particular solution, to separate students in just two groups, N groups or to perfectly disclose information about their productivity, is going to be optimal (provided  $\bar{a} \in S$ ).

To understand the intuition of this result it is convenient to use graphs. Let's think in a situation where we have only 2 standards besides 0, i.e.  $S = \{0, s_1, s_3\}$ . The value of the objective function can be graphed as  $\Omega$  is just:

$$\Omega = s_1 \left[ F(s_{13}) - F(s_{01}) \right] + s_3 \left[ F(\bar{a}) - F(s_{13}) \right]$$
(3.19)

where

$$s_{ij} = \frac{s_i + s_j}{2}$$

The value of  $\Omega$  is represented in Figure 3.1 as the grey area delimited by black dash lines. It can be graphed due to the fact that it is just the sum of the productivity of each group of agents choosing the same productivity level multiplied by the mass of agents that choose that level. In the figure, we can see that  $\Omega$  is composed by two rectangles. The upper represents the total productivity of agents that are choosing standard  $s_3$ . This total productivity is just equal to the productivity that each agent attains  $(s_3)$  multiplied by the mass of agents choosing this productivity level  $(F(\bar{a}) - F(s_{13}))$ . Then, the area of the upper rectangle is just  $s_3 [F(\bar{a}) - F(s_{13})]$ . Similarly, the lower rectangle represents the total productivity added by agents that choose to attain the  $s_1$ 

standard. This is just the productivity that each of them attain  $(s_1)$  multiplied by the mass of agents that choose that level  $(F(s_{13}) - F(s_{01}))$ . Finally, adding up the area of both rectangles we get the total productivity of the population of students and this is just  $\Omega$ .

Suppose we add an intermediate standard exactly in the midpoint between  $s_1$  and  $s_3$ . That is, we add  $s_2 = s_{13}$ . The new value of the objective function is represented in Figure 3.2 as the area composed of three rectangles (delimited by solid grey lines) due to the fact that there are three different productivity levels that are chosen by agents. The lower rectangle  $(s_1 [F(s_{12}) - F(s_{01})])$  represents the total productivity of agents that choose standard  $s_1$ . The middle rectangle  $(s_2 [F(s_{23}) - F(s_{12})])$  represents total productivity added by agents choosing  $s_2$  standard. Finally, the upper rectangle represents total productivity added by agents choosing the standard  $s_3$ .

Figure 3.3 presents  $\Omega$  for the case of two standards and also when we add an intermediate standard. In this figure, we can see that some agents maintain their decisions with respect to productivity while others change it and decide to choose the new standard  $s_2$ . To start with, agents with ability level lower than  $s_{01}$  were previously choosing the zero standard and they continue doing so. Secondly, agents with ability level between  $s_{01}$  and  $s_{12}$  were previously choosing the  $s_1$  standard and they continue doing so, as this standard remains the closest to their preferred first best (a). Similarly, agents with ability level between  $s_{23}$  and  $\bar{a}$  also do not change their decisions as  $s_3$  continues being the closest standard. However, there are agents that do change their productivity levels.

Agents with ability level close to  $s_2$  do change their decision. In particular, agents with ability level between  $s_{12}$  and  $s_2$  were previously choosing standard  $s_1$  but now they start choosing  $s_2$ . This is the positive effect of adding an intermediate standard. There were some agents that wanted to attain higher levels of productivity than  $s_1$  but they did not do so because the standard  $s_3$  was "too far". Now, as they have an intermediate option, they increase effort and attain standard  $s_2$ . The mass of agents that increase productivity is equal to  $F(s_2) - F(s_{12})$  and their increase in productivity is equal to  $s_2 - s_1$ .

On the other hand, agents with ability levels between  $s_2$  and  $s_{23}$  decrease their productivity level from  $s_3$  to  $s_2$ . The reason for this change is that previously they reluctantly chose standard level  $s_3$  because they wanted to choose a lower productivity level, but  $s_1$  was "too low". Now, as they have an intermediate choice,  $s_2$ , they decrease their productivity from  $s_3$  to  $s_2$ . This negative effect is equal to the mass of agents that decrease their productivity level  $(F(s_{23}) - F(s_2))$  times how much they decrease productivity  $(s_3 - s_2)$ .

Figure 3.4 presents the impact of adding an intermediate standard on the value of the objective function. In this figure, the positive effect is represented by rectangle I which is delimitated by solid grey lines. The negative effect is represented by rectangle II delimitated by dash black lines. If area I is bigger than the area II we should add the intermediate standard. As we can see, both areas are equal and therefore the net effect is zero. This means that the value of the objective function is completely independent of whether we add an intermediate standard or not.

The reason why we are able to compare both areas so easily is because we considered the case of adding a standard that lies exactly in the midpoint between  $s_1$  and  $s_3$ . This makes the increase that some agents make  $(s_2 - s_1 = s_{13} - s_1 = \frac{s_{3+}s_1}{2} - s_1 = \frac{s_3-s_1}{2})$  exactly equal to the decrease made by those agents that lower their productivity level  $(s_3 - s_2 = s_3 - s_{13} = s_3 - \frac{s_3+s_1}{2} = \frac{s_3-s_1}{2})$ . However, the key to this result that the net effect is zero lies in the fact that  $F(\cdot)$  is linear. This implies that the number of agents that decrease their productivity level is exactly the same as the number of agents that increase their productivity level. Moreover, this highlights the crucial role that  $F(\cdot)$  plays in this problem. It is clear to see that under different shapes for  $F(\cdot)$  we should get completely different results.

**Theorem 2.** If 
$$F''(x) > 0 \ \forall \ x \in [0, \bar{a}] \Rightarrow S^* = \{0, s_1^*\}$$
 is the unique solution to 3.13

See the Appendix for the proof.

The intuition for this result derives from the analysis made above. We just need to think of the same situation as before (starting with two standards and adding a third one at the midpoint) but now we acknowledge that, as  $f(\cdot)$  is increasing, the mass of individuals that decrease their productivity level is higher than the mass of those that increase it. This implies that the net effect of adding an intermediate standard is negative and then it provides the intuition of why a solution in this setting will not have any intermediate standard. Hence, in this case, the professor's optimal revelation policy consists on choosing one standard and then classifying individuals in two groups and just conveying this information to the marketplace.

A way to grasp the intuition for this result is by thinking about a situation where there are many high ability students and a low number of low ability students. Given this situation, the optimal revelation policy consists on dividing all students in just two groups: high productivity and low productivity students. As the lower standard for the low productivity group is zero, applying our previous result, we know that all students who are in this group are going to have zero productivity and then wages for this group are going to be zero. As a consequence, we can set a standard really high aiming at the high ability students as the penalty of not attaining it is really big as they have to choose a zero productivity level (where their first best productivity is high). Setting this high standard forces students with low and intermediate ability to aim for the zero productivity standard, but this is a cost which we are willing to pay, as we started assuming that there was a low number of low ability students and a high number of high ability ones. In short, by giving them only two choices we are able to extract a high degree of effort from high ability students that is not possible to extract in a situation where there are intermediate standards which they can choose from.

**Theorem 3.** If 
$$F''(x) < 0 \ \forall \ x \in (0,\bar{a}) \Rightarrow \ S^* = \{[0,\bar{a}]\}$$
 is the unique solution to 3.13

See the Appendix for the proof.

This theorem states that when f(x) is decreasing, the professor chooses to perfectly reveal the information about students' productivity to the labor market. The intuition for this result comes from the fact that adding an intermediate midpoint standard has a positive net effect as the mass of students that increase their effort is higher than the mass of students reducing it.

Another way to understand this result is by thinking about a situation where there are many individuals with low ability and just a few with high ability. In this case, if we think about setting up a standard and dividing students into two groups, we can clearly see that the standard should be set up very low in order not to force the high number of agents with low ability to drop out. If we just set that standard, individuals with ability higher than that standard will opt to attain it, but this means that we are inducing them to choose a productivity level lower than their first best. Clearly, we could improve the solution adding new standards in order to allow them to choose their first best. This gives the intuition for why perfect information is optimal in this situation.

**Theorem 4.** If 
$$f(x)$$
 is single-peaked  $\Rightarrow S^* = \{0, [\hat{s}, \bar{a}]\}$  or  $S^* = \{0, \hat{s}\}$  with  $\hat{s} > \bar{a}$ 

See the Appendix for the proof.

This theorem analyzes the most interesting case where the density of students with respect to ability is single-peaked. In this case, the optimal set S is going to be equal to  $\{0, [\hat{s}, \bar{a}]\}$  or  $S^* = \{0, \hat{s}\}\ \hat{s} > \bar{a}$ . Note that these two possibilities in fact correspond to only one characterization of S. That is, the professor just needs to set a critical standard  $\hat{s}$  and disclose all information above this standard. This means that  $S = \{0, [s, \infty]\}$ . However, recall that we trim the set S in order to include in this set only productivity levels that are chosen in equilibrium, then depending on whether  $\hat{s}$  is smaller or greater than  $\bar{a}$ , the solution will be  $S^* = \{0, [\hat{s}, \bar{a}]\}$  or  $S^* = \{0, \hat{s}\}$  with  $\hat{s} > \bar{a}$ , respectively.

There is a strong intuition for this solution. Given the shape of f(x), the mode of this density  $(x_m)$  is at an intermediate value between 0 and  $\bar{a}$ . This can be interpreted as that the density is concentrated around the value  $x_m$ . Then, it makes sense to establish a standard  $\hat{s}_2$  above this level  $x_m$  and no intermediate standard below as in this way we are going to give an incentive to this bulk of students to attain this standard. If we introduced an intermediate standard below  $\hat{s}_2$ , a big fraction of these students would lower their productivity aiming to this new intermediate standard. However, above  $\hat{s}_2$  the situation is exactly the same as in the case of a decreasing density and then it makes sense to provide perfect information above that level.

## 3.4 Conclusions

This paper explores the problem of an educational authority who has to decide his revelation policy about students educational attainments. I find that this optimal policy completely depends on the way in which students are distributed with respect to their optimal productivity level.<sup>11</sup>

In the model presented, when the density of students with respect to their optimal productivity level is decreasing, I find that schools should disclose all the information they have about students' productivity. In the opposite case, when this density is increasing, the educational authority should classify students only in two groups and just disclose to which group every student belongs.

Finally, in the most plausible case where this density is single-peaked I show that schools should define a critical standard and group all individuals below this standard together and perfectly disclose information for all students that attain or surpass this standard. This result suggests that policies that involve full disclosure or just classifying students in two groups can be improved by a combination of both: full information above a critical standard, no information below it.

<sup>&</sup>lt;sup>11</sup>In the model it is assumed that students differ only in ability, but recall that this was just an assumption for simplicity and clarity of exposition.

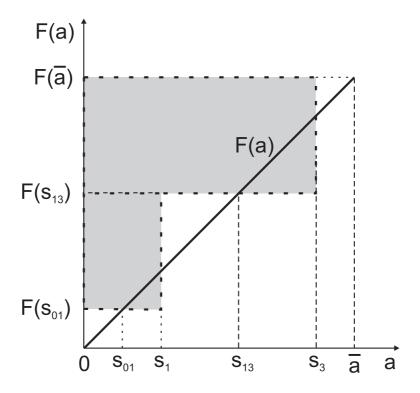


Figure 3.1: Value of the objective function with two standards

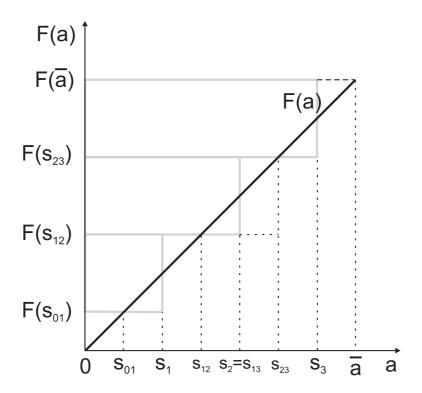


Figure 3.2: Value of the objective function with three standards

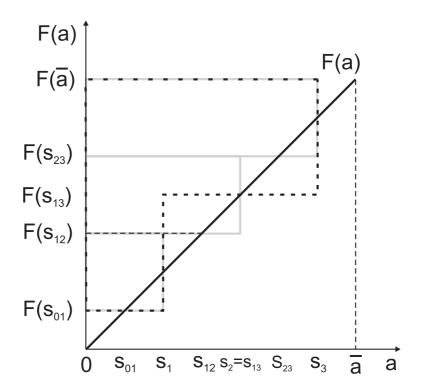


Figure 3.3: Value of the objective function with two and three standards

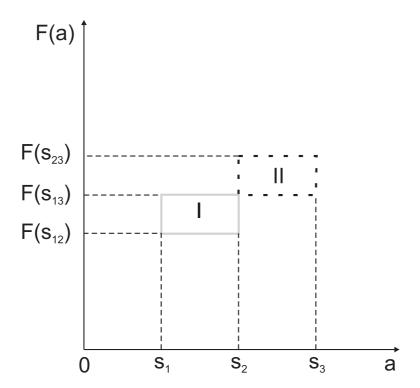


Figure 3.4: Net effect of adding an intermediate standard

# Appendix A

Proofs of lemmas and theorems

$$Lemma \ 2. \ p^*(a,S) = \underset{p \in S}{ArgMin} \ |a-p|$$

## Proof

From equation 3.11,

$$p^*(a,S) = \underset{p \in S}{ArgMax} \quad p - \frac{p^2}{2a}$$
 (A.1)

$$= Arg \underset{p \in S}{Max} - \frac{1}{2a} (p^2 - 2ap)$$
 (A.2)

$$= Arg Max_{p \in S} - \frac{1}{2a} (p^2 - 2ap + a^2 - a^2)$$
 (A.3)

$$= Arg \underset{p \in S}{Max} - \frac{1}{2a}(p-a)^2 + \frac{a}{2}$$
 (A.4)

$$= Arg \underset{p \in S}{Max} - \frac{1}{2a}(p-a)^2 \tag{A.5}$$

$$= Arg \underset{p \in S}{Min} \ (p-a)^2 \tag{A.6}$$

$$= Arg \underset{p \in S}{Min} |p - a| \tag{A.7}$$

**Lemma 3.** If S contains an interval C with positive mass then taking out the interior elements of C from S will increase (decrease)  $\Omega$  if f(x) is increasing (decreasing) over  $[0, \bar{a}]$ . If f(x) is constant then removing the interior elements of C from S will not affect  $\Omega$ .

### Proof

I define  $s_c^L$  and  $s_c^H$  as the infimum and supremum of C, respectively. The value of  $\Omega$  when S contains this interval C is:

$$\Omega(S) = \int_0^{s_c^L} p(x)f(x)dx + \int_{s_c^L}^{s_c^H} p(x)f(x)dx + \int_{s_c^H}^{\bar{a}} p(x)f(x)dx$$

$$\text{As } p(x) = x \ \forall \ x \in \left[s_c^L, s_c^H\right] = C \text{ as } C \subset S:$$

$$(A.8)$$

$$\Omega(S) = \int_{0}^{s_{c}^{L}} p(x)f(x)dx + \int_{s^{L}}^{s_{c}^{H}} xf(x)dx + \int_{s^{H}}^{\bar{a}} p(x)f(x)dx$$
 (A.9)

The value of  $\Omega$  taking out all the interior values of C is:

$$\Omega(S - (s_c^L, s_c^H)) = \int_0^{s_c^L} p(x)f(x)dx + \int_{s_c^L} \frac{s_c^L + s_c^H}{2} p(x)f(x)dx$$

$$+ \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} p(x)f(x)dx + \int_{s_c^H}^{\bar{a}} p(x)f(x)dx$$

$$\text{as } p(x) = s_c^L \quad \forall \quad x \in \left[s_c^L, \frac{s_c^H + s_c^L}{2}\right] \text{ and } p(x) = s_c^H \quad \forall \quad x \in \left[\frac{s_c^H + s_c^L}{2}, s_c^H\right]$$

$$\Omega(S - (s_c^L, s_c^H)) = \int_0^{s_c^L} p(x)f(x)dx + \int_{s_c^L}^{\frac{s_c^L + s_c^H}{2}} s_c^L f(x)dx$$

$$+ \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} s_c^H f(x) dx + \int_{s_c^H}^{\bar{a}} p(x) f(x) dx$$
 (A.11)

Define  $\Delta\Omega$  as the increase in the objective function if I eliminate all the interior elements of

C:

$$\Delta\Omega = \Omega(S - (s_c^L, s_c^H)) - \Omega(S) = \tag{A.12}$$

$$= \int_{s_c^L}^{\frac{s_c^L + s_c^H}{2}} s_c^L f(x) dx + \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} s_c^H f(x) dx - \int_{s_c^L}^{s_c^H} x f(x) dx$$
 (A.13)

$$= \int_{s_c^L}^{\frac{s_c^L + s_c^H}{2}} s_c^L f(x) dx + \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} s_c^H f(x) dx - \int_{s_c^L}^{\frac{s_c^L + s_c^H}{2}} x f(x) dx - \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} x f(x) dx$$
(A.14)

$$= \int_{\frac{s_c^L + s_c^H}{2}}^{s_c^H} \left( s_c^H - x \right) f(x) dx - \int_{s_c^L}^{\frac{s_c^L + s_c^H}{2}} \left( x - s_c^L \right) f(x) dx \tag{A.15}$$

Next, I change the variable of integration for both integrals in the previous equation. For the first integral I change it for  $y=s_c^H-x$  and for the second integral I replace the variable of integration for  $y=x-s_c^L$ . Then,

$$\Delta\Omega = \int_{\frac{s_c^H - s_c^L}{2}}^0 y f(s_c^H - y)(-1) dy - \int_0^{\frac{s_c^H - s_c^L}{2}} y f(y + s_c^L) dy$$
(A.16)

$$= \int_{0}^{\frac{s^{H}-s^{L}}{c}} y f(s^{H}_{c}-y) dy - \int_{0}^{\frac{s^{H}-s^{L}}{c}} y f(y+s^{L}_{c}) dy$$
 (A.17)

$$= \int_{0}^{\frac{s_{c}^{H} - s_{c}^{L}}{2}} y \left[ f(s_{c}^{H} - y) - f(y + s_{c}^{L}) \right] dy \tag{A.18}$$

As  $s_c^H - y > y + s_c^L \iff \frac{s_c^H - s_c^L}{2} > y > 0$  and I am integrating from 0 to  $\frac{s_c^H - s_c^L}{2}$ , then  $s_c^H - y > y + s_c^L$  over the integration range. This implies that if  $f'(x) \ngeq 0 \ \forall \ x \in [0, \bar{a}]$  then  $f(s_c^H - y) \gtrless f(y + s_c^L)$  for all values of y between the integration limits. Finally, if  $f'(x) \gtrless 0$  then  $\Delta \Omega \gtrless 0$  and this completes the proof of this Lemma.

Theorem 1. If 
$$F(x) = x \ \forall \ x \in (0, \bar{a}) \Rightarrow \ \forall \ S \ / \ \bar{a} \in S, \ S$$
 is a solution to 3.13

## Proof

Using lemma 3, I can remove all intervals with positive mass from S and replace them for its extremes and the value of  $\Omega$  is not going to change. Doing these replacements and defining  $\bar{s}$  as the highest element of  $\bar{S}$  and  $\hat{s}$  as the second highest, the value of the objective function can be expressed as:

$$\Omega\left(S\right) = \sum_{i=1}^{N-1} s_i \left[ F\left(\frac{s_i + s_{i+1}}{2}\right) - F\left(\frac{s_{i-1} + s_i}{2}\right) \right] + \bar{s} \left[ F\left(\bar{a}\right) - F\left(\frac{\hat{s} + \bar{s}}{2}\right) \right]$$
(A.19)

$$= \sum_{i=1}^{N-1} s_i \left[ \frac{s_i + s_{i+1}}{2} - \frac{s_{i-1} + s_i}{2} \right] + \bar{s} \left[ \bar{a} - \frac{\hat{s} + \bar{s}}{2} \right]$$
 (A.20)

$$= \sum_{i=1}^{N-1} s_i \left[ \frac{s_{i+1} - s_{i-1}}{2} \right] + \bar{s} \left[ \frac{2\bar{a} - \hat{s} - \bar{s}}{2} \right]$$
 (A.21)

$$= \frac{1}{2} \left[ \sum_{i=1}^{N-1} \left( s_i s_{i+1} - s_i s_{i-1} \right) + \bar{s} \left( 2\bar{a} - \hat{s} - \bar{s} \right) \right]$$
(A.22)

Define  $h_{i+1} = s_{i+1}s_i$  and replace in the first term of the previous sum:

$$\sum_{i=1}^{N-1} (s_i s_{i+1} - s_i s_{i-1}) = \sum_{i=1}^{N-1} (h_{i+1} - h_i) = \sum_{i=1}^{N-1} h_{i+1} - \sum_{i=1}^{N-1} h_i =$$
(A.23)

$$= h_N + \sum_{i=2}^{N-1} h_i - h_1 - \sum_{i=2}^{N-1} h_i = h_N - h_1 = \bar{s}\hat{s} - s_1 s_0 = \bar{s}\hat{s} \quad as \quad s_0 = 0$$
 (A.24)

Replacing this result in  $\Omega(S)$ :

$$\Omega(S) = \frac{1}{2} \left[ \bar{s}\hat{s} + \bar{s} \left( 2\bar{a} - \hat{s} - \bar{s} \right) \right] = \frac{1}{2} \left[ \bar{s}\hat{s} + 2\bar{a}\bar{s} - \bar{s}\hat{s} - \bar{s}^2 \right] = \frac{1}{2} \left[ 2\bar{a}\bar{s} - \bar{s}^2 \right]$$
(A.25)

$$\Omega(S) = \bar{s} \left[ \bar{a} - \frac{\bar{s}}{2} \right] = \bar{s} \left[ F(\bar{a}) - F\left(\frac{\bar{s}}{2}\right) \right] \tag{A.26}$$

Then, for any arbitrary set S, if the distribution of agents with respect to ability is uniform, the value of the objective function is going to only depend on the value of the highest element of S.<sup>1</sup> This implies that the only requirement for a set S to be optimal is that its higher element has to be chosen optimally. But the problem of choosing that standard optimally is just equivalent to the problem of choosing the standard optimally in a situation where the professor can only set one standard (besides  $s_o = 0$ ) and then divide students into just two groups. That is,

$$\Omega(\bar{s}) = \underset{\bar{s} > 0}{Max} \ \bar{s} \left[ F(\bar{a}) - F\left(\frac{\bar{s}}{2}\right) \right] \tag{A.27}$$

$$\Omega(\bar{s}) = \underset{2\bar{a} > \bar{s} > 0}{Max} \ \bar{s} \left[ \bar{a} - \frac{\bar{s}}{2} \right] \tag{A.28}$$

$$\Omega'(\bar{s}) = \bar{a} - \frac{\bar{s}}{2} - \frac{\bar{s}}{2} \tag{A.29}$$

Remember that we trim all elements of S that are not going to be chosen by any agent.

Setting  $\Omega'(\bar{s}) = 0$  and solving for  $\bar{s}$ :

$$\bar{s}^* = \bar{a} \tag{A.30}$$

Moreover,  $\Omega''(\bar{s}) = -1 \ \forall \ \bar{s}$  and then  $\bar{s}^* = \bar{a}$  is its unique maximum.

Theorem 2. If 
$$F''(x) > 0 \ \forall \ x \in (0, \bar{a}) \Rightarrow$$
 Exists a unique solution and it is  $S^* = \{0, s_1^*\}$ 

### Proof

Using lemma 3 I know that no solution set S includes an interval with positive mass as the value of the objective function will increase if we remove the interior elements of these intervals.

Next, I will show that in any solution I cannot have more than two standards. For that, suppose that there exists more than two standards in a solution. Then, there should exists an intermediate standard that I call  $s_2$ . I call the neighbouring standards as  $s_1$  and  $s_3$ , where  $s_1 < s_2 < s_3$ . In a solution, the standard  $s_2$  has to be set optimally given all the other standards and in particular,  $s_1$  and  $s_3$ . In particular, the following condition has to hold:

$$s_2 = Arg \underset{s_1 \le s_2 \le s_3}{Max} \Omega(S) = \int_0^{\bar{a}} p(x)f(x)dx \text{ where } p(x) = Arg \underset{p \in S}{Min} |p - x|$$
(A.31)

$$s_2 = Arg \max_{s_1 \le s_2 \le s_3} \int_0^{s_1} p(x)f(x)dx + \int_{s_1}^{\frac{s_1 + s_2}{2}} p(x)f(x)dx +$$
(A.32)

$$\int_{\frac{s_2+s_3}{2}}^{\frac{s_2+s_3}{2}} p(x)f(x)dx + \int_{\frac{s_2+s_3}{2}}^{s_3} p(x)f(x)dx + \int_{s_3}^{\bar{a}} p(x)dx$$
 (A.33)

As in this problem  $s_2$  is constrained to lie between  $s_1$  and  $s_3$ , then the first and last term of the RHS of the previous expression are independent of the value of  $s_2$ . Then,

$$s_2 = Arg \underset{s_1 \le s_2 \le s_3}{Max} \int_{s_1}^{\frac{s_1 + s_2}{2}} p(x)f(x)dx + \int_{\frac{s_1 + s_2}{2}}^{\frac{s_2 + s_3}{2}} p(x)f(x)dx + \int_{\frac{s_2 + s_3}{2}}^{s_3} p(x)f(x)dx$$
(A.34)

Moreover, using the constraint about p(x):

$$s_2 = Arg \max_{s_1 \le s_2 \le s_3} \int_{s_1}^{\frac{s_1 + s_2}{2}} s_1 f(x) dx + \int_{\frac{s_1 + s_2}{2}}^{\frac{s_2 + s_3}{2}} s_2 f(x) dx + \int_{\frac{s_2 + s_3}{2}}^{s_3} s_3 f(x) dx$$
(A.35)

$$s_2 = Arg \max_{s_1 \le s_2 \le s_3} s_1 \left[ F\left(\frac{s_1 + s_2}{2}\right) - F\left(s_1\right) \right] +$$
 (A.36)

$$s_{2}\left[F\left(\frac{s_{2}+s_{3}}{2}\right)-F\left(\frac{s_{1}+s_{2}}{2}\right)\right]+s_{3}\left[F\left(s_{3}\right)-F\left(\frac{s_{2}+s_{3}}{2}\right)\right] \tag{A.37}$$

$$s_2 = Arg \max_{s_1 \le s_2 \le s_3} s_1 F\left(\frac{s_1 + s_2}{2}\right) + s_2 \left[F\left(\frac{s_2 + s_3}{2}\right) - F\left(\frac{s_1 + s_2}{2}\right)\right] - s_3 F\left(\frac{s_2 + s_3}{2}\right) \tag{A.38}$$

Rearranging,

$$s_2 = Arg \max_{s_1 \le s_2 \le s_3} -(s_3 - s_2) F\left(\frac{s_2 + s_3}{2}\right) - (s_2 - s_1) F\left(\frac{s_1 + s_2}{2}\right)$$
(A.39)

If  $s_2$  maximizes this function, it has to minimize the function multiplied by  $-\frac{1}{s_3-s_1}$ :

$$s_2 = Arg \min_{s_1 \le s_2 \le s_3} \frac{(s_3 - s_2)}{(s_3 - s_1)} F\left(\frac{s_2 + s_3}{2}\right) + \frac{(s_2 - s_1)}{(s_3 - s_1)} F\left(\frac{s_1 + s_2}{2}\right)$$
(A.40)

I define  $g(s_2)$  as the function

$$g(s_2) = \frac{(s_3 - s_2)}{(s_3 - s_1)} F\left(\frac{s_2 + s_3}{2}\right) + \frac{(s_2 - s_1)}{(s_3 - s_1)} F\left(\frac{s_1 + s_2}{2}\right)$$
(A.41)

I evaluate  $g(\cdot)$  at  $s_1$  and  $s_3$ :

$$g(s_2 = s_1) = \frac{(s_3 - s_1)}{(s_3 - s_1)} F\left(\frac{s_1 + s_3}{2}\right) + \frac{(s_1 - s_1)}{(s_3 - s_1)} F\left(\frac{s_1 + s_1}{2}\right) = F\left(\frac{s_1 + s_3}{2}\right) \tag{A.42}$$

$$g(s_2 = s_3) = \frac{(s_3 - s_3)}{(s_3 - s_1)} F\left(\frac{s_3 + s_3}{2}\right) + \frac{(s_3 - s_1)}{(s_3 - s_1)} F\left(\frac{s_1 + s_3}{2}\right) = F\left(\frac{s_1 + s_3}{2}\right) \tag{A.43}$$

Defining  $\alpha = \frac{(s_3 - s_2)}{(s_3 - s_1)}$ , I get:

$$g(s_2) = \alpha F\left(\frac{s_2 + s_3}{2}\right) + (1 - \alpha)F\left(\frac{s_1 + s_2}{2}\right)$$
 (A.44)

As the function F(.) is strictly convex, then:

$$\alpha F\left(\frac{s_2 + s_3}{2}\right) + (1 - \alpha) F\left(\frac{s_1 + s_2}{2}\right) > F\left(\alpha \frac{s_2 + s_3}{2} + (1 - \alpha) \frac{s_1 + s_2}{2}\right) \ \forall \ \alpha \in (0, 1) \quad (A.45)$$

Eliminating  $\alpha$  and  $(1 - \alpha)$  from the RHS:

$$\alpha F\left(\frac{s_2+s_3}{2}\right) + (1-\alpha)F\left(\frac{s_1+s_2}{2}\right) > F\left(\frac{(s_3-s_2)}{(s_3-s_1)}\frac{(s_2+s_3)}{2} + \frac{(s_2-s_1)}{(s_3-s_1)}\frac{(s_1+s_2)}{2}\right) \quad (A.46)$$

$$\alpha F\left(\frac{s_2 + s_3}{2}\right) + (1 - \alpha) F\left(\frac{s_1 + s_2}{2}\right) > F\left(\frac{\left(s_3^2 - s_2^2\right)}{2\left(s_3 - s_1\right)} + \frac{\left(s_2^2 - s_1^2\right)}{2\left(s_3 - s_1\right)}\right) \tag{A.47}$$

$$\alpha F\left(\frac{s_2 + s_3}{2}\right) + (1 - \alpha) F\left(\frac{s_1 + s_2}{2}\right) > F\left(\frac{s_3^2 - s_1^2}{2(s_3 - s_1)}\right) \tag{A.48}$$

$$\alpha F\left(\frac{s_2+s_3}{2}\right) + (1-\alpha)F\left(\frac{s_1+s_2}{2}\right) > F\left(\frac{s_1+s_3}{2}\right) \ \forall \ \alpha \in (0,1)$$
(A.49)

$$g(s_2 \in (s_1, s_3)) > g(s_2 = s_1) = g(s_2 = s_3) \text{ as } s_1 < s_2 < s_3 \implies \alpha \in (0, 1)$$

This implies that this function is minimized at  $s_1$  or  $s_3$ . Given equation A.40, this implies that  $s_2 = s_1$  or  $s_2 = s_3$ . But I started assuming that  $s_1 < s_2 < s_3$ . Then, by contradiction, I have showed that there is no solution that has an intermediate standard and then any solution can have at most two standards.

As  $s_0 = 0 \in S$ , then the solution set S is going to have another standard  $s_1$  besides  $s_0$ , as a solution set S containing only  $s_0$  can easily be improved choosing any  $s_1 \in (0, 2\bar{a})$ . Finally,  $s_1$  is going to be set solving the same problem that a professor that can only classify two students is going to face. That is,

$$Max \ \Omega(s_1) = s_1 \left[ F(\bar{a}) - F\left(\frac{s_1}{2}\right) \right]$$
 (A.50)

The solution to this problem exists and it is unique. This can be shown from the following properties of  $\Omega(s_1)$ :

$$\Omega'(s_1) = F(\bar{a}) - F\left(\frac{s_1}{2}\right) - \frac{s_1}{2}f\left(\frac{s_1}{2}\right) \tag{A.51}$$

$$\Omega'(s_1 = 0) = F(\bar{a}) > 0 \tag{A.52}$$

$$\Omega'\left(s_{1}=2\bar{a}\right)=F\left(\bar{a}\right)-F\left(\frac{2\bar{a}}{2}\right)-\frac{2\bar{a}}{2}f\left(\frac{2\bar{a}}{2}\right)=-\bar{a}f\left(\bar{a}\right)<0\tag{A.53}$$

$$\Omega''(s_1) = -\left[f\left(\frac{s_1}{2}\right) + \frac{s_1}{4}f'\left(\frac{s_1}{2}\right)\right] < 0 \ \forall \ s_1 \in [0, 2\bar{a})$$
 (A.54)

As  $\Omega'(s_1)$  is positive at  $s_1 = 0$ , negative at  $s_1 = 2\bar{a}$  and decreasing over this range, then there is one and only one value of  $s_1$  for which this derivative is equal to zero.

Theorem 3. If 
$$F''(x) < 0 \quad \forall \ x \in (0, \bar{a}) \Rightarrow \ S^* = [0, \bar{a}]$$
 is the unique solution to 3.13

# Proof

Suppose that there exists a solution set S for which there exists an interval with positive mass C such that  $C = (s_c^L, s_c^H)$  where  $s_c^L, s_c^H \in S$  and  $C \cap S = \emptyset$ . Using lemma 3 we know that including all the interior elements of this interval to S will increase the value of the objective function. Then, by contradiction the result is shown.

Theorem 4. If 
$$f(x)$$
 is single-peaked  $\Rightarrow S^* = \{0, [\hat{s}, \bar{a}]\}$  or  $S^* = \{0, \hat{s}\}$  with  $\hat{s} > \bar{a}$ 

### Proof

To show this theorem, I use other results stablished in previous theorems. In order to do so, I proceed sequentially showing the following results for this case where f(x) is single-peaked.

1) S cannot include neither an interval with positive mass in the increasing region of f(x) nor more than one standard (besides zero) in this region.

- 2) S cannot contain the extrems of an interval with positive mass of agents but not all the interior elements, in the decreasing region of f(x).
  - 3) S cannot includes an standard at  $x_m$ .
  - 4) S has to include an interval above  $x_m$ .
  - 5) S cannot contain an standard in the increasing region of f(x).

Combining these five results, I conclude that S is composed only by zero and a closed and convex interval above  $x_m$ . If  $\hat{s}$  is higher than  $\bar{a}$  then, S is just  $\{0, \hat{s}\}$ . Now, I show these five results.

1) S cannot include neither an interval with positive mass in the increasing region of f(x) nor more than one standard (besides zero) in this region.

#### Proof:

Given lemma 3, we know that the set S will not include an interval in the increasing region of f(x) as replacing it by its extremes, the solution will be improved. Secondly, a solution to this problem cannot contain two standards besides zero in the increasing region of f(x) as the proof of theorem 2 shows that the solution can be improved by eliminating any intermediate standards.

2) S cannot contain the extrems of an interval with positive mass of agents but not all the interior elements, in the decreasing region of f(x).

## Proof:

Using lemma 3, we know that a solution cannot include the extremes of an interval but not all the interior elements as the solution can be improved by including these interior elements.

3) S cannot includes an standard at  $x_m$ .

## Proof:

Combining results 1) and 2), I know that any solution can have (at most) an standard in the decreasing region of f(x) (besides zero), an standard at  $x_m$  and an interval above  $x_m$ . Now, I will show that there is no solution with an standard at  $x_m$ . To do that, I assume that there exists a solution where the set S has an standard at  $x_m$ . Then, I cannot have an standard in the increasing region of f(x) (as it would be an intermediate standard) and moreover the set S will

 $<sup>^{2}</sup>x_{m}$  corresponds to the mode of the ability distribution.

include the whole decreasing region of f(x). This implies that the solution will be  $S = [\hat{s}, \bar{a}]$  where  $\hat{s} = x_m$ . As this is an optimal solution, there is no change that that can be made over the set S that will increase the value of the objective function  $\Omega$ . In particular, the solution cannot be improved by increasing the lower extreme of the interval  $[\hat{s}, \bar{a}]$ . Now, I compute the first derivative of the objective function with respect to its interval lower extreme and I evaluate it at  $\hat{s} = x_m$ 

$$\Omega\left(\hat{s}\right) = \int_{\frac{\hat{s}}{2}}^{\hat{s}} \hat{s}f\left(x\right)dx + \int_{\hat{s}}^{\bar{a}} xf\left(x\right)dx = \hat{s}\left[F\left(\hat{s}\right) - F\left(\frac{\hat{s}}{2}\right)\right] + \int_{\hat{s}}^{\bar{a}} xf\left(x\right)dx \tag{A.55}$$

$$\Omega'(\hat{s}) = F(\hat{s}) - F\left(\frac{\hat{s}}{2}\right) + \hat{s}\left[f(\hat{s}) - \frac{1}{2}f\left(\frac{\hat{s}}{2}\right)\right] - \hat{s}f(\hat{s})$$
(A.56)

$$\Omega'(\hat{s} = x_m) = F(x_m) - F\left(\frac{x_m}{2}\right) - x_m \frac{1}{2} f\left(\frac{x_m}{2}\right) = \int_{\frac{x_m}{2}}^{x_m} f(x) dx - \int_{\frac{x_m}{2}}^{x_m} f\left(\frac{x_m}{2}\right) dx \quad (A.57)$$

$$\Omega'(\hat{s}) = \int_{\frac{x_m}{2}}^{x_m} \left[ f(x) - f\left(\frac{x_m}{2}\right) \right] dx > 0 \quad as \quad f(x) > f\left(\frac{x_m}{2}\right) \quad \forall \quad \frac{x_m}{2} < x < x_m$$
 (A.58)

This last result implies that I can improve the solution by increasing  $\hat{s}$  and then the result is shown by contradiction. Hence, I conclude that a the solution set S cannot include an standard at  $x_m$ .

4) S has to include an interval above  $x_m$ .

Proof:

Suppose that is not the case. Then, given the results obtained above, I will have a solution set with only one standard, that I call  $s_1$ , in the increasing region of f(x) besides the standard that lies at zero. However, this cannot be a solution, as I can just add another standard  $s_2$  in the decreasing region of f(x) and its only effect is going to be to make some agents that were choosing  $s_1$  to choose  $s_2$ , and then the solution will be improved. Then, this was not a solution and by contradiction the result is shown.

5) S cannot contain an standard in the increasing region of f(x).

Proof:

Combining the four results presented above, I know that S is going to include only one interval above  $x_m$  and it may also include only one standard below  $x_m$ . Then, the problem just becomes:

$$\underset{s_{1}, s_{2}}{Max} \ s_{1} \left[ F\left(\frac{s_{1} + s_{2}}{2}\right) - F\left(\frac{s_{1}}{2}\right) \right] + s_{2} \left[ F\left(s_{2}\right) - F\left(\frac{s_{1} + s_{2}}{2}\right) \right] + \int_{s_{2}}^{\bar{a}} x f\left(x\right) dx \tag{A.59}$$

$$st \quad s_1 < x_m < s_2$$

I want to show that  $s_1^* = 0$  (i.e. there is no standard in the increasing region of f(x) besides zero). To do so, let's start assuming that  $s_1^* > 0$  (remember that  $s_2^*$  has to be greater than  $x_m$ ). Then, there can be two different situations: a)  $\frac{s_1^* + s_2^*}{2} \ge x_m$  or b)  $\frac{s_1^* + s_2^*}{2} < x_m$ .

a) 
$$\frac{s_1^* + s_2^*}{2} \ge x_m$$

The derivative of  $\Omega$  with respect to  $s_2$  is:

$$\frac{\partial\Omega}{\partial s_2} = \frac{s_1}{2} f(\frac{s_1 + s_2}{2}) + F(s_2) - F\left(\frac{s_1 + s_2}{2}\right) + s_2 f(s_2) - \frac{s_2}{2} f\left(\frac{s_1 + s_2}{2}\right) - s_2 f(s_2) \quad (A.60)$$

$$\frac{\partial\Omega}{\partial s_2} = F\left(s_2\right) - F\left(\frac{s_1 + s_2}{2}\right) - \frac{\left(s_2 - s_1\right)}{2} f\left(\frac{s_1 + s_2}{2}\right) \tag{A.61}$$

$$\frac{\partial\Omega}{\partial s_2} = \int_{\frac{s_1 + s_2}{2}}^{s_2} f(x)dx - \int_{\frac{s_1 + s_2}{2}}^{s_2} f(\frac{s_1 + s_2}{2})dx \tag{A.62}$$

$$\frac{\partial \Omega}{\partial s_2} = \int_{\frac{s_1 + s_2}{2}}^{s_2} \left[ f(x) - f(\frac{s_1 + s_2}{2}) \right] dx \tag{A.63}$$

As  $f'(x) < 0 \quad \forall \ x > x_m$  and  $\frac{s_1 + s_2}{2} > x_m$  then  $f'(x) < 0 \ \forall \ x \in \left[\frac{s_1 + s_2}{2}, s_2\right]$  and then  $f(x) < f\left(\frac{s_1 + s_2}{2}\right) \ \forall \left(\frac{s_1 + s_2}{2}, s_2\right)$ . This implies that I am integrating a function that is negative over the whole range of integration. Then:

$$\frac{\partial \Omega}{\partial s_2} < 0$$

But I initially assumed that  $(s_1, s_2)$  was a solution. Clearly it cannot be the case that  $\frac{\partial \Omega}{\partial s_2}$  evaluated at the solution is different from zero. Hence, by contradiction, I conclude that in no solution  $\frac{s_1^* + s_2^*}{2} \geq x_m$ .

b) 
$$\frac{s_1^* + s_2^*}{2} < x_m$$
.

As this is a solution, I know that:

$$s_{1}^{*} = Arg Max \left\{ s_{1} \left[ F\left(\frac{s_{1} + s_{2}}{2}\right) - F\left(\frac{s_{1}}{2}\right) \right] + s_{2} \left[ F\left(s_{2}\right) - F\left(\frac{s_{1} + s_{2}}{2}\right) \right] + \int_{s_{2}}^{\bar{a}} x f\left(x\right) dx \right\}$$

$$(A.64)$$

Removing the terms that are independent of  $s_1$ :

$$s_1^* = ArgMax \left\{ s_1 \left[ F\left(\frac{s_1 + s_2}{2}\right) - F\left(\frac{s_1}{2}\right) \right] - s_2 F\left(\frac{s_1 + s_2}{2}\right) \right\}$$
 (A.65)

Multiplying the objective function by  $-\frac{1}{s_2}$ :

$$s_1^* = ArgMin_{s_1} \left\{ \left( 1 - \frac{s_1}{s_2} \right) F\left( \frac{s_1 + s_2}{2} \right) + \frac{s_1}{s_2} F\left( \frac{s_1}{2} \right) \right\}$$
 (A.66)

I call this objective function  $W(s_1)$ . Using the previous equation:

$$W(s_1) \le W(0) = F\left(\frac{s_2}{2}\right) \tag{A.67}$$

Then,

$$\left(1 - \frac{s_1^*}{s_2^*}\right) F\left(\frac{s_1^* + s_2^*}{2}\right) + \frac{s_1^*}{s_2^*} F\left(\frac{s_1^*}{2}\right) \le F\left(\frac{s_2^*}{2}\right) \tag{A.68}$$

On the other side, I know that  $f'(x) > 0 \ \forall \ x < x_m$  and that  $\frac{s_1 + s_2}{2} < x_m$  then  $f'(x) > 0 \ \forall \ s < \frac{s_1 + s_2}{2}$ . Similarly,  $F''(x) > 0 \ \forall \ x < \frac{s_1 + s_2}{2}$ . This implies that:

$$\alpha F(a) + (1 - \alpha) F(b) > F(\alpha a + (1 - \alpha) b) \quad \forall \quad \alpha \in (0, 1)$$
(A.69)

Now, setting:  $\alpha = \frac{s_1^*}{s_2^*}$ ,  $a = \frac{s_1^*}{2}$  and  $b = \frac{s_1^* + s_2^*}{2}$  the right hand side of this inequality becomes:

$$F(\alpha a + (1 - \alpha)b) = F\left(\frac{s_1^*}{s_2^*} \frac{s_1^*}{2} + \left(1 - \frac{s_1^*}{s_2^*}\right) \frac{s_1^* + s_2^*}{2}\right) = F\left(\frac{\left(s_1^*\right)^2}{2s_2^*} + \frac{\left(s_2^* - s_1^*\right)\left(s_2^* + s_1^*\right)}{2s_2^*}\right)$$
(A.70)

$$= F\left(\frac{\left(s_1^*\right)^2}{2s_2^*} + \frac{\left(s_2^*\right)^2 - \left(s_1^*\right)^2}{2s_2^*}\right) = F\left(\frac{\left(s_2^*\right)^2}{2s_2^*}\right) = F\left(\frac{s_2^*}{2}\right)$$
(A.71)

This implies that the inequality states that:

$$\left(1 - \frac{s_1^*}{s_2^*}\right) F\left(\frac{s_1^* + s_2^*}{2}\right) + \frac{s_1^*}{s_2} F\left(\frac{s_1^*}{2}\right) > F\left(\frac{s_2^*}{2}\right)$$
(A.72)

Clearly, this inequality contradicts inequality A.68. Then, there cannot be a solution where  $\frac{s_1^* + s_2^*}{2} < x_m$  and  $s_1^* > 0$ .

Combining the two results obtained above, I conclude that there cannot be a solution with  $s_1^* > 0$ . Then, by contradiction it is shown that  $s_1^* = 0$  and then that there is no standard below  $x_m$  besides zero.

Additionally, I can assert that the solution exists as I am maximiziming a continuous function on a closed and convex set. $^3$ 

Moreover, it can be shown that this solution is unique. To do that, I need to get equate the first derivative of  $\Omega$  with respect to  $s_2$  equal to zero (I call this value  $s_2^*$ ) and then I can check that for all values of  $s_2$  lower than  $s_2^*$  I know that this first derivative is positive and that for all values of  $s_2$  that are higher than  $s_2^*$  the first derivative first is negative and once that they are higher than  $2\bar{a}$  the derivative become zero (as the value of the function remains at zero).

<sup>&</sup>lt;sup>3</sup>To see that, just think that instead of maximizing the function subject that  $x_m < s_2$ , assume that  $x_m < s_2 \le 2\bar{a}$ . The solution of both problems is equivalent as  $s_2$  is going to be always smaller or equal than  $2\bar{a}$  as if not, all agents are going to choose the zero standard.

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