ABSTRACT

Title of Dissertation: ESSAYS ON FIRM DYNAMICS AND MACROECONOMICS

Seula Kim
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Dissertation Directed by: Professor John C. Haltiwanger
Professor John Shea
Department of Economics

This dissertation describes a broad set of topics in firm dynamics and macroeconomics, including young firm dynamics, business dynamism, firm innovation, technological advance, and economic growth in the U.S. economy. In Chapter 1, I study how workers' uncertain job prospects affect young firms' pay and employment growth, and quantify macroeconomic implications. Building a heterogeneous-firm directed search model in which workers gradually learn about permanent firm productivity types, I find that the learning process creates endogenous wage differentials for young firms. In the model, a high performing young firm must pay a higher wage than that of high performing old firms, while a low performing young firm offers a lower wage than that of low performing old firms, to attract workers. This is because workers are unsure whether the young firm's performance reflects its fundamental type or a temporary shock given the lack of track records. I find that these wage differentials affect both hiring and retention margins of young firms and can dampen the growth of high-potential young firms. Furthermore, the

model indicates that higher uncertainty about young firms results in bigger wage differentials and thus hampers overall young firm activity and aggregate productivity. Using employee-employer linked data from the U.S. Census Bureau and regression specifications guided by the model, I provide empirical support for the novel predictions of the model.

Chapter 2 studies the effect of competition on firm innovation by developing a discrete-time endogenous growth model where multi-product firms do two types of innovation subject to friction in technology spillovers. Firms improve their existing products through internal innovation while entering others' product markets through external innovation. We introduce a novel friction, which we label as imperfect technology spillovers, which refer to frictions in learning others' technology in the process of external innovation. In contrast to existing models, this friction allows incumbent firms to defend themselves from competitors by building technological barriers through internal innovation. Using firm-level data from the U.S. Census Bureau integrated with firm-level patent data, we find regression results consistent with the model predictions. Our counterfactual analysis shows that rising competition by foreign firms leads domestic incumbent firms to undertake (i) more (less) internal innovation for the products in which they have (do not have) a technological advantage, and (ii) less external innovation. This compositional change in firm innovation affects overall innovation in the aggregate economy in different directions depending on the costs of external innovation. Specifically, the shift in innovation composition in response to rising competition decreases overall innovation in the U.S., but would increase overall innovation in an economy with high external innovation costs.

Lastly, Chapter 3 examines how increasing knowledge complexity and the accompanying rise in innovation cost affect firm innovation patterns and business dynamism in the U.S. economy. Using detailed firm-level data from S&P's Compustat and the U.S. Census Bureau, integrated with the U.S. patent database (USPTO PatentViews), we document the increasing trend in knowledge complexity in firm innovation activities. Specifically, the inventor team size, the number of technology types (technology subclasses), and the degree of interdependence across different technology subclasses associated with firms' patent portfolio have been increasing over time. Furthermore, we find the increasing trend of knowledge complexity is associated with the declining trend of business dynamism, such as firm entry, the share of young firms, and young firms' activity in job creation and reallocation. We offer a simple endogenous growth model in which different R&D inputs are interdependent (complementary) to each other and firms are required to use different types of inputs to generate a given amount of innovation. This increases more complexity in firm innovation process and makes small, young firms with less knowledge base more difficult to conduct innovation as before. This can impede firm entry and dampen the growth of small and young firms.

ESSAYS ON FIRM DYNAMICS AND MACROECONOMICS

by

Seula Kim

Dissertation submitted to the Faculty of the Graduate School of the
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Advisory Committee:
Professor John C. Haltiwanger, Co-Chair
Professor John Shea, Co-Chair
Professor Rajshree Agarwal
Professor S. Borağan Aruoba
Professor Thomas Drechsel

Dedication

To Karam and my parents (Nanhee and Youngin).

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I would also like to acknowledge the exceptional roles played by several individuals in making this thesis possible. In addition to my committee, I thank faculty members of the economics department for their helpful discussions that improved my research. I also thank Jason Faberman, Hyunseob Kim, Gadi Barlevy, and Joel David at the Federal Reserve Bank of Chicago for their very invaluable and constructive comments and feedback during my visit. I also thank Mike Ewens at Columbia Business School for providing me with valuable feedback and insights on my research, as well as opportunities to learn and grow through organizing a student workshop and interacting with other researchers. I also thank Joonkyu Choi at the Federal Reserve Board for his insightful advice and consistent support in completing my Ph.D. I also thank Annie Soyean Lee at Johns Hopkins University, SAIS, for her immense support and feedback on my research. I also thank the staffs at the department of economics, including Terry Davis, Vickie Fletcher, Jessica Gray, Wanda Hauser, Brian McLoughlin, Heather Nalley, Evan Schroeder, Amanda Statland, and Mark Wilkerson for helping me throughout the whole Ph.D. program.

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Disclaimer

Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

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Chapter 1: Workers' Job Prospects and Young Firm Dynamics

1.1 Introduction

Acquiring workers is essential for firms to grow, especially for young firms with high growth potential. High-growth young firms account for a disproportionate share of gross job creation and productivity growth in the U.S. economy, and have been at the center of recent research (Decker et al., 2016b, 2014a; Foster et al., 2018; Haltiwanger, 2012; Haltiwanger et al., 2013, 2016). However, young firms are nascent and have short track records. This increases workers' uncertainty about job prospects at the firms. The inherent uncertainty about the jobs offered by young firms could be important to understanding young firm dynamics, yet this mechanism has not been much studied. This paper proposes that uncertain job prospects affect worker pay and growth of young firms, and have macroeconomic implications for overall young firm activity, resource allocation, and aggregate productivity in the economy.

Workers evaluate the value of a job by considering its expected stream of wages, the possibility of being laid off, and potential future career development. To do so, workers consider the firm's current and historical performance, build beliefs about its growth potential, and decide whether or not to work for that firm. However, unlike mature firms, workers are less certain about young firms' performance as an indicator of their fundamentals, due to the firms' lack of history. Thus, workers may hesitate to work for young firms with high growth potential and may require compensating wage differentials relative to otherwise similar mature firms.

This paper formalizes and examines the proposed job prospects mechanism and its implications for young firm activity and aggregate outcomes, both theoretically and empirically. I construct a heterogeneous firm directed search model with symmetric Bayesian learning about firms' underlying productivity types, and test the model's predictions using two comprehensive databases hosted by the U.S. Census Bureau; the Longitudinal Business Database (LBD) and the Longitudinal Employer-Household Dynamics (LEHD).

I build on the directed search model of Schaal (2017) and introduce symmetric learning as in Jovanovic (1982). A novel feature of the model is that workers need to learn about firm types along the firm life cycle, and take jobs based on their beliefs about firm types. In the model, workers' learning and uncertain job prospects create endogenous wage differentials for young firms relative to otherwise similar mature firms. Specifically, I find that young firms with high demonstrated potential, defined as those with cumulative average performance above the cross-sectional prior mean, must offer wage premia to attract workers relative to otherwise similar mature firms. This is due to the relative lack of performance records for young firms, so that workers are not fully convinced by their average performance. Such wage differentials can create a barrier at the hiring or retention margin of those young firms, increasing their marginal costs and hampering their growth.

At the same time, young firms with low demonstrated potential, those with cumulative average performance below the cross-sectional prior mean, can pay wage discounts compared to their otherwise similar mature counterparts. This follows the same logic, where the low performing young firms benefit from the fact that their limited history gives them some upside risk.

The model further allows me to quantify the macroeconomic implications of this job prospects channel for overall young firm activity and aggregate productivity. A counterfactual analysis suggests that an increase in noise dispersion in the learning process (or an increase in the fundamental uncertainty regarding young firms' job prospects) can lead to declines in firm entry and the share of young firms. This is because higher uncertainty slows down the speed of learning about firm types, and increases gaps in workers' job prospects and the consequent wage differentials between young firms and otherwise similar mature firms.

In particular, more uncertain prospects amplify the wage premia paid by high performing young firms and hamper the growth of those young firms with high potential. Furthermore, more uncertain prospects allow low performing firms to pay less and linger in the economy. Thus, labor markets become tighter and overall hiring costs are raised for recruiting firms. This can in turn hamper overall allocative efficiency and decrease aggregate productivity. This establishes that workers' job prospects at young firms can have important macroeconomic impacts on overall business dynamism and allocative efficiency in the economy.

Next, I use the Census datasets and confirm these model predictions. In particular, I merge the LBD with LEHD, where the LBD tracks the universe of U.S. non-farm businesses and establishments annually, and the LEHD tracks the earnings, jobs, and demographics of workers reported in the Unemployment Insurance (UI) systems in most U.S. states. Using the linked data, I estimate an individual-level earnings regression informed by the model. I find that controlling for worker heterogeneity and observable firm characteristics, young firms with high demonstrated potential (or high average productivity) pay more than their mature counterparts with the same observable characteristics. At the same time, I find earnings discounts paid by young firms with low demonstrated potential (or low average productivity) relative to otherwise similar mature firms. This confirms the model's predictions about how learning and job prospects create wage differentials between young firms and their mature counterparts.

Next, I estimate the impact of the level of uncertainty on the earnings differentials of young firms by using industry-level variation in uncertainty (measured by the dispersion of firm-level productivity shocks) and interacting it with the earnings residuals. I find that the earnings differentials for young firms are more pronounced in industries with more dispersed firm-level productivity shocks. Lastly, I construct industry-level measures of business dynamism and examine their relationships with uncertainty to test the macroeconomic predictions of the model. I find that higher uncertainty with more dispersed noise has a negative impact on overall business dynamism at the industry level. These findings are consistent with the model's aggregate implications. To the best of my knowledge, these findings have not been documented in previous studies.

The remainder of this paper is structured as follows: Section 1.2 discusses the related literature along with the paper's main contribution; Section 1.3 develops a heterogeneous firm directed search model that extends Schaal (2017) by introducing a firm-type learning process; Section 1.4 lays out the model's main implications and mechanisms; Section 1.5 describes the model calibration and counterfactual exercises; Section 1.6 uses the U.S. Census Bureau data and tests the model implications for wage differentials of young firms and the effect of uncertainty on macroeconomic outcomes; and Section 1.7 concludes.

1.2 Literature Review

This paper is related to several strands of literature. First, it contributes to a broad line of work in firm dynamics and macroeconomics that studies potential factors or frictions affecting the post-entry dynamics and growth of young firms. Much previous research emphasizes the importance of financing constraints for entrepreneurship (Cooley and Quadrini, 2001; Evans and Jovanovic, 1989; Holtz-Eakin et al., 1994; Hurst and Lusardi, 2004; Kerr and Nanda, 2009; Robb and Robinson, 2014). Schmalz et al. (2017) and Davis and Haltiwanger (2019) specifically highlight the role of housing collateral values as a determinant of firm entry and post-entry growth of young firms. Foster et al. (2016) show that frictions in accumulating demand can also slow the growth of new businesses. Akcigit and Ates (forthcoming) and Jo and Kim (2021) suggest that barriers to knowledge spillovers can dampen firm entry and the growth of young firms. Meanwhile, Decker et al. (2020) attribute the recent decline in job reallocation, firm entry, and exit to the declining responsiveness of firms to idiosyncratic productivity shocks, possibly caused by an increase in adjustment costs or distortions positively correlated with fundamentals. This paper contributes to this literature by identifying workers' job prospects as a novel friction affecting firm entry and the growth of young firms.

Second, this paper is also relevant to a large set of literature that studies inter-firm wage differentials and compensation dynamics across different firms. Abowd et al. (1999), Abowd et al. (2002) and Abowd et al. (2004) develop an econometric framework (AKM, henceforth) and quantify earnings variation attributable to worker versus firm components using two-way fixed effects. This methodology has been applied in a broad range of subsequent studies analyzing firm wage differentials (Babina et al., 2019; Bloom et al., 2018; Card et al., 2018, 2013; Lopes de Melo, 2018; Song et al., 2019). Some studies mainly focus on wage differentials by firm age. Brown and Medoff (2003) find that older firms pay higher wages, but that relationship between firm age and wages becomes insignificant or negative controlling for worker characteristics. Haltiwanger et al. (2012) document an earnings gap between startups and established firms in the U.S., which exhibits an increasing trend in the last decade. Burton et al. (2018) and

Sorenson et al. (2021) find in Danish data that younger firms pay more while smaller firms pay less, controlling for employee characteristics, and that the size effect dominates the age effect. Kim (2018) uses data on MIT graduates and finds that startups pay wage premia relative to their counterparts at established firms. The closest study to this paper is Babina et al. (2019), who use U.S. Census data and show the existence of positive and significant wage premia for young firms, controlling for worker and firm selection. This paper contributes to this literature by providing a rich structural model that guides a concrete mechanism generating earnings differentials of young firms, which depend on the firms' performance history. Guided by the model, the paper develops and estimates an empirical specification that isolates the part of inter-firm earnings differentials attributed to workers' uncertain job prospects.

Lastly, this paper is grounded in the directed search literature. Menzio and Shi (2010) develop a general stochastic model of directed search on the job and find a block recursive equilibrium in which agents' value functions, policy functions and market tightness do not depend on the distribution of firms or workers. Menzio and Shi (2011) also develop a model of directed search on the job in which transitions of workers between unemployment and employment and across employers are driven by heterogeneity in the quality of firm-worker matches. My work is closely related to Kaas and Kircher (2015) and Schaal (2017), who link directed search to standard firm dynamics models. Kaas and Kircher (2015) investigate job reallocation and flow patterns across different-sized firms in a model in which firms actively compete for workers by offering long-term contracts in a frictional labor market. Schaal (2017) also builds a tractable directed search model with firm dynamics and time-varying idiosyncratic volatility and studies the impact of time-varying idiosyncratic risk at the establishment level on unemployment fluctuations. This paper contributes to this literature by adding a firm-type learning process to the directed search framework. The model generates endogenous wage differentials across different firm ages, even after controlling for firms' observable characteristics, and quantifies their macroeconomic implications.

1.3 Theoretical Model

In this section, I present a heterogeneous firm directed search model as a baseline framework to study how job prospects generate wage differentials for young firms seeking to hire or retain workers, compared to their otherwise similar mature counterparts. I analyze how such differentials affect young firms' hiring and retention margins as well as their employment growth and activity, and explore how job prospects affect overall young firm activity and aggregate productivity. The baseline model builds on Schaal (2017) by introducing a firm-type learning process as in Jovanovic (1982).

1.3.1 The Environment

The model is set in discrete time and consists of a continuum of heterogeneous firms with homogeneous workers within frictional labor markets. Both firms and workers are assumed to have symmetric information. The mass of workers is normalized to one, while the mass of firms is pinned down endogenously with free entry. Both firms and workers are risk neutral and have the same discount rate β. Firms all produce an identical homogeneous good which is the numeraire.

1.3.2 Firm-type Learning Process

Firms are born with different productivity types ν that are time invariant and unobserved to both firms and workers. Among entrants, ν is normally distributed with mean $\bar{\nu}_0$ and standard deviation σ_0. Entrants do not know their own ν, but know that their type ν has cross-sectional distribution $N(\bar{\nu}_0, \sigma_0^2)$. Given symmetric information, workers can also only observe the cross-sectional distribution of firm type among entrants. Thus, both entrants and workers start with a belief $\nu \sim N(\bar{\nu}_0, \sigma_0^2)$ at age 0. The dispersion of firm type σ_0 indicates the signal level in the economy. The more dispersed the type distribution is, the more signal agents can gain from observing firm productivity realizations.

Observed productivity for firm j at time t, P_{jt}, follows the following log-normal process, which depends on firm type ν_j:

$$P_{jt} = e^{\nu_j + \varepsilon_{jt}}, \tag{1.1}$$

where $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$ is a firm-specific shock that is independent over time and across firms. Here, the dispersion of firm-level shocks σ_ε indicates the degree of uncertainty in the economy, as higher shock dispersion generates more noise in the learning process.

Let a_{jt} denote the age of firm j at period t, which implies that the firm is born at $(t - a_{jt})$. Also, let $\bar{\nu}_{jt-1}$ and σ_{jt-1}^2 be the prior (or updated posterior) mean and variance about firm j's type at the beginning of period t, respectively. Note that $\nu_{jt-a_{jt}-1} = \bar{\nu}_0$ and $\sigma_{jt-a_{jt}-1}^2 = \sigma_0^2$ are the initial beliefs held at firm j's birth in period $(t - a_{jt})$. Upon observing the productivity level P_{jt}, both the firm and workers update their posterior beliefs about firm j's type using Bayes' rule.[1]

[1] See Appendix A.1 for more details on the Bayes' rule.

Consequently, the following posterior on ν_j is obtained:

$$\nu_j|_{P_{jt}} \sim N(\bar{\nu}_{jt}, \sigma^2_{jt}), \tag{1.2}$$

where

$$\bar{\nu}_{jt} = \frac{\sigma^2_\varepsilon \bar{\nu}_{jt-1} + \sigma^2_{jt-1} \ln P_{jt}}{\sigma^2_{jt-1} + \sigma^2_\varepsilon} = \frac{\frac{\bar{\nu}_{jt-1}}{\sigma^2_{jt-1}} + \frac{\ln P_{jt}}{\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_{jt-1}} + \frac{1}{\sigma^2_\varepsilon}}, \tag{1.3}$$

$$\sigma^2_{jt} = \frac{\sigma^2_{jt-1}\sigma^2_\varepsilon}{\sigma^2_{jt-1} + \sigma^2_\varepsilon} = \frac{1}{\frac{1}{\sigma^2_{jt-1}} + \frac{1}{\sigma^2_\varepsilon}}. \tag{1.4}$$

By iterating (1.3) and (1.4) backward and using $a_{jt} + 1 = a_{jt+1}$, I can rewrite (1.3) and (1.4) as follows:

$$\bar{\nu}_{jt} = \frac{\frac{\bar{\nu}_0}{\sigma^2_0} + \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_0} + (a_{jt}+1)\frac{1}{\sigma^2_\varepsilon}} = \frac{\frac{\bar{\nu}_0}{\sigma^2_0} + (a_{jt+1})\frac{\tilde{P}_{jt}}{\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_0} + (a_{jt+1})\frac{1}{\sigma^2_\varepsilon}} \tag{1.5}$$

$$\sigma^2_{jt} = \frac{1}{\frac{1}{\sigma^2_0} + (a_{jt+1})\frac{1}{\sigma^2_\varepsilon}} \tag{1.6}$$

where $\tilde{P}_{jt} \equiv \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{(a_{jt+1})} = \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{(a_{jt+1})}$ is the cumulative average of log productivity up to period t. $\bar{\nu}_{jt}$ and σ^2_{jt} are key state variables for firms and workers that summarize their posterior beliefs about firm j entering period $t+1$.

Equations (1.5) and (1.6) contain several noteworthy results. First, firm age and the average log productivity, $(a_{jt+1}, \tilde{P}_{jt})$, are sufficient statistics for the posterior about firm j's type at $t+1$. In particular, the posterior mean is a weighted sum of the initial prior mean and the average observed productivity, and the weights depend on firm age. Therefore, one can track job prospects for each firm using these two variables. This property comes from normal learning and makes the model tractable.

Second, the following relationships between the two sufficient statistics and the posterior mean at the beginning of each period t can be derived:

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}} > 0 \tag{1.7}$$

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}} = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_0^2 \sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} \begin{cases} \geqslant 0 & \text{if} \quad \tilde{P}_{jt-1} \geqslant \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}. \tag{1.8}$$

Equation (1.7) implies that the posterior mean increases in the average productivity level. As firms are observed to have higher average productivity, their prospects improve. Moreover, (1.8) shows that firm age affects job prospects differently depending on the firm's average productivity. Specifically, if firm j's cumulative average log productivity is above the initial cross-sectional mean, a higher age implies a better inferred type, while if a firm's average productivity is below the cross-sectional mean, a higher age implies a worse inferred type.

Definition 1. *Firms are "high performing" if their average productivity is above the cross-sectional prior mean, and "low performing" if their average productivity is below the cross-sectional prior mean.*

I will refer to firms as "high performing" and "low performing" throughout the paper, by the relationship between their average productivity and the initial prior mean. Note that in Bayesian learning, both firms and workers learn from observable performance to infer firms' fundamental types. Therefore, a firm's average observed productivity ($\tilde{P}_{jt-1}$) indicates their "potential" type in a given period t, which converges to the firm's time-invariant type ν_j in the long run.

Lastly, one can derive the following relationship between firm age and the posterior standard deviation:

$$\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}} = -\frac{1}{\sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} < 0, \tag{1.9}$$

which implies that as a firm ages, learning gets less noisy, and the posterior converges to a degenerate distribution centered at the true type ν_j.

Figure 1.1 summarizes the pattern of posterior beliefs across different firm ages, for a given level of average observed productivity (indicated by the red dashed line). The left panel shows the posteriors associated with low performing firms (having average productivity lower than the cross-sectional prior mean), and the right panel shows the counterpart for high performing firms (having average productivity higher than the cross-sectional prior mean). These panels clearly illustrate the properties in (1.7), (1.8), and (1.9): for low performing firms, the posterior mean is higher for younger firms than for older firms with the same average productivity, while the opposite is true for high performing firms; and the posterior variance declines in firm age.

1.3.3 Labor Market

The labor market is frictional. Following Schaal (2017), search is directed on both the worker and firm sides. Firms announce contracts to hire and retain workers each period. Following the convention in a standard directed search framework, a sufficient statistic to define labor markets in the economy is the level of promised utility that each contract delivers to workers upon matching. This is because firms that offer the same utility level to workers compete in the same labor market, and workers that require the same utility level search in the same market. Thus, the

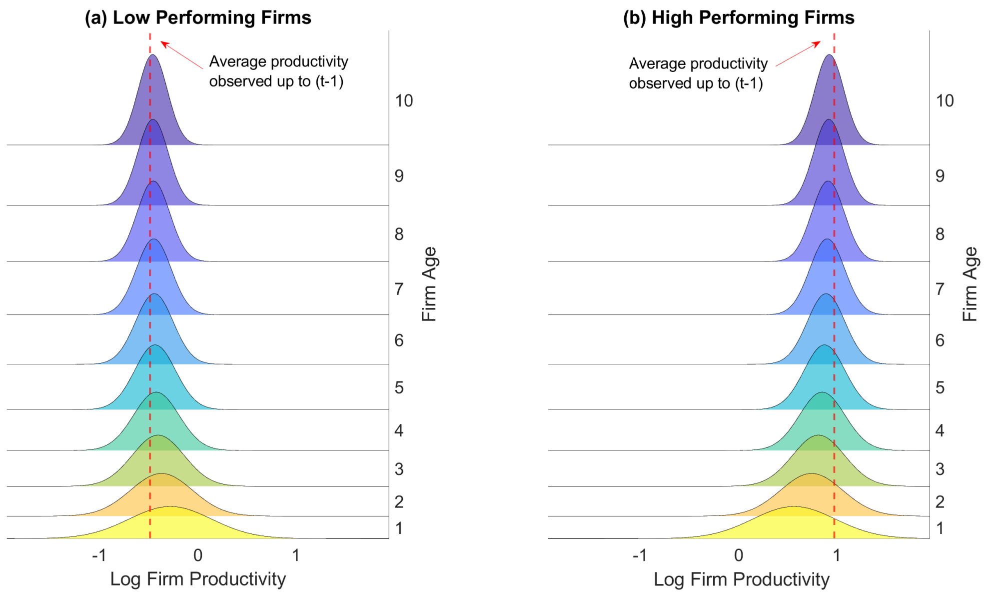


Figure 1.1: Posterior Distribution of Firm Type

labor market is a continuum of submarkets indexed by the total utility x_{jt} that firms (j) promise to workers.

Both firms and workers direct their search by choosing a submarket to search in and find their match. They both take into account a trade-off between the level of utility of a given contract and the corresponding probability of being matched. Matches are created within each market through a standard constant-returns-to-scale matching function. Firms post vacancies by paying a vacancy cost c to hire workers.

Let $\theta(x)$ denote the market tightness, defined as the vacancy-to-searchers ratio in each submarket x.[2] Also following the standard notation in search and matching models, let $f(\theta)$ and $q(\theta)$ be job finding and job filling rates for workers and firms, respectively. As is standard in the literature, I assume that $f'(\theta) > 0$, $f(0) = 0$, $q'(\theta) < 0$, and $q(0) = 1$. I also assume that firms and workers can only visit one submarket at a time. Lastly, there is both on-the-job and

[2]Note that searchers in a given market x are either unemployed workers or employed workers who are searching for a new job while on their current job. More details can be found in Section 1.3.8.

off-the-job search in the economy, so that both unemployed and employed workers are allowed to search. Here, λ denotes the relative search efficiency for employed workers compared to unemployed workers.

1.3.4 Dynamic Contracts

Contracts are written every period after matching occurs and before production takes place. As in Schaal (2017), contracts are recursive and are assumed to be state-contingent and fully committed for firms. A contract for workers employed at firm j at t, $\mathbf{\Omega_{jt}}$, specifies the current wage w_{jt}, the next period's utility level $\mathbf{\tilde{W}_{jt+1}}$, the firm's next-period exit probability $\mathbf{d_{jt+1}}$, and the worker's next-period separation probability $\mathbf{s_{jt+1}}$, where the last three terms are contingent on the firm's next period state variables $(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt})$, where l_{jt} is the number of workers employed at firm j at the end of period t. Thus, the contract can be written as

$$\mathbf{\Omega_{jt}} = \{w_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}, \tag{1.10}$$

where

$$\begin{aligned}
\mathbf{d_{jt+1}} &= \mathbf{d}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt}) \\
\mathbf{s_{jt+1}} &= \mathbf{s}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt}) \\
\mathbf{\tilde{W}_{jt+1}} &= \mathbf{\tilde{W}}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt}).
\end{aligned}$$

I assume firms offer common contracts across workers with the same ex-post heterogeneity (defined by the employment status of workers).[3] This is because there is neither worker ex-ante heterogeneity nor human capital accumulated within a firm. Since each firm j is committed to its contracts offered to workers each period, the firm writes new contracts at t taking as given the utility level $\mathbf{\tilde{W}_{jt}}$ promised in the previous period for the remaining incumbent workers at t, as well as the promised utility x_{jt} for the new hires at t.

1.3.5 Model Timeline

The model timeline is as follows. At the beginning of each period t, both incumbent firms and workers enter with priors (or updated posteriors) about firm type, characterized by firm age and the average log productivity, a_{jt} and $\tilde{P}_{jt-1}$.[4] The firms also enter with their employment level l_{jt-1} and the state-contingent contracts $\mathbf{\Omega_{jt-1}}$ that they offered in the previous period to their incumbent workers. Note that for new firms, both firms and workers start with initial priors about firm type based on the common initial firm type distribution firms are born with.

Next, an exogenous death shock hits incumbent firms, which drives a fraction δ of firms to exit. New firms enter afterwards by paying an entry cost c_e, where free entry is assumed. Firm productivity P_t is realized, after which firms decide whether to exit or stay, following the rule $\mathbf{d_{jt}}$ $(= \mathbf{d}(a_{jt}, \tilde{P}_{jt-1}, P_{jt}, l_{jt-1}))$, which is a function of the firm state variables at t, and is specified in their contract with workers at $t-1$. Also, they decide whether to lay off workers with probability $\mathbf{s_{jt}}$ $(= \mathbf{s}(a_{jt}, \tilde{P}_{jt-1}, P_{jt}, l_{jt-1}))$, which is also a function of the same set of the firm state variables

[3] This means firms offer the same state-contingent next-period variables to workers as workers obtain the same ex-post heterogeneity once they join the firm in the current period. However, the current wage can vary across workers depending on the workers' previous employment status before joining the firm or being retained by the firm in a given period.

[4] Note that the beginning-of-period priors for incumbent firms are the posteriors updated by the end of the previous period.

at t, following the rule announced in their contract at $t-1$.

Search and matching follows, with new and surviving incumbent firms on one side and unemployed and employed workers on the other side. Firms choose and search in market x_{jt}, post vacancies v_{jt} by paying the per-vacancy cost c, and hire new workers h_{jt} with a job filling rate determined by market tightness $q(\theta(x_{jt}))$. Here, the number of vacancies and new hires have the relationship $h_{jt} = q(\theta(x_{jt}))v_{jt}$, and the vacancy cost per hire is $\frac{c}{q(\theta(x_{jt}))}$. On the other hand, unemployed workers choose their market to search in, x_t^U, and employed workers at firm j choose search market x_{jt}^E. Unemployed and employed workers find a job with probability $f(\theta(x_t^U))$ and $f(\theta(x_{jt}^E))$, respectively.

At the end of this process, firms will end up with the following employment level l_{jt}:

$$l_{jt} = h_{jt} + (1 - \lambda f(\theta(x_{jt}^E)))(1 - s_{jt})l_{jt-1},$$

which is the sum of new hires and the remaining incumbent workers after the departure of those laid off and those moving to other jobs.

Finally, firms enter the last stage of each period, in which they write contracts to new and retained workers, and produce. They offer the workers the contract $\mathbf{\Omega_{jt}}$ as in (1.10). When writing this contract, firms are committed to providing utility $\tilde{\mathbf{W}}_{\mathbf{jt}}$ to surviving incumbent workers from $t-1$ and x_{jt} to new hires. Lastly, firms pay a fixed operating cost c_f, produce, and pay wages w_{jt} to workers as announced in the contract $\mathbf{\Omega_{jt}}$. Figure 1.2 shows the timeline.

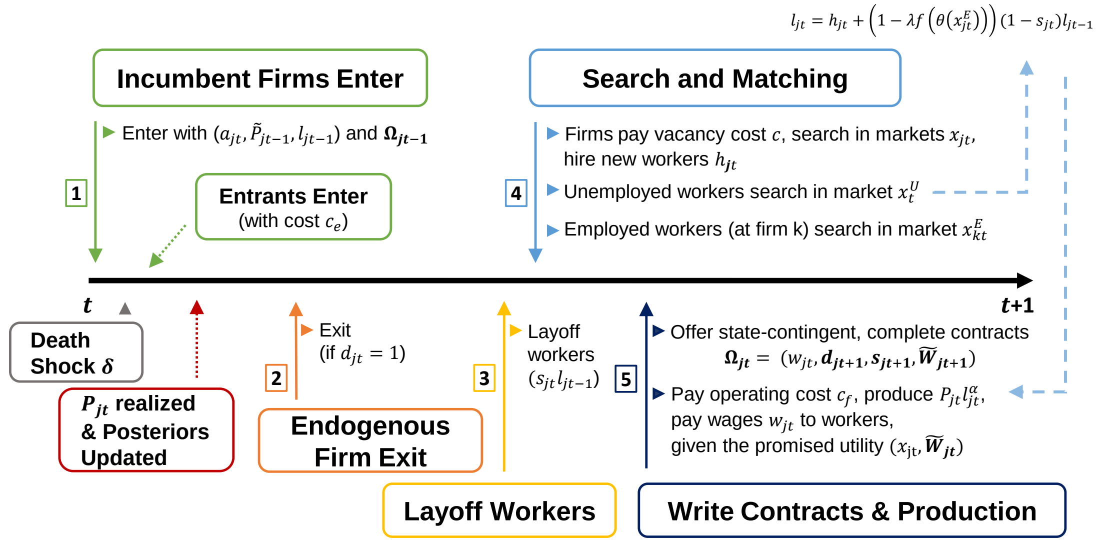


Figure 1.2: Timeline of the model

1.3.6 Workers' Problem

This subsection describes the workers' problem. First, unemployed workers have the following value function $\mathbf{U_t}$:

$$\mathbf{U_t} = b + \beta \mathbb{E}_t\Big[\max_{x_{t+1}^U}(1 - f(\theta(x_{t+1}^U)))\mathbf{U_{t+1}} + f(\theta(x_{t+1}^U))x_{t+1}^U\Big]. \tag{1.11}$$

Unemployed workers obtain unemployment insurance b and choose a market segment x_{t+1}^U for their job search, considering the job finding probability f as a function of labor market tightness $\theta(x_{t+1}^U)$. By doing so, they take into account a trade-off between the promised utility x_{t+1}^U and the labor market tightness $\theta(x_{t+1}^U)$. Here, workers do not save and are risk neutral.

Employed workers at firm j under the contingent contract $\mathbf{\Omega_{jt}} = \{w_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}$ have the following value function after the search and matching process is complete (at the stage of production):

$$\begin{aligned}\mathbf{W}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \mathbf{\Omega_{jt}}) =& w_{jt} + \beta \mathbb{E}_{jt}\Bigg[\bigg(\delta + \big(1-\delta\big)\big(\mathbf{d_{jt+1}} + (1-\mathbf{d_{jt+1}})\mathbf{s_{jt+1}}\big)\bigg)\mathbf{U_{t+1}} \\ &+ (1-\delta)(1-\mathbf{d_{jt+1}})(1-\mathbf{s_{jt+1}}) \max_{x^E_{jt+1}} \bigg(\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1} \\ &+ \big(1-\lambda f(\theta(x^E_{jt+1}))\big)\mathbf{\tilde{W}_{jt+1}}\bigg)\Bigg], \end{aligned} \tag{1.12}$$

where the firm's state variables are its age a_{jt}, average productivity $\tilde{P}_{jt-1}$ accumulated up to the beginning of t, productivity draw P_{jt} at t, and its employment level l_{jt-1} before search and matching at t, all of which determine the set of contracts $\mathbf{\Omega_{jt}} = \{w_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}$ for the workers employed at firm j, which the workers take as given.[5]

Equation (1.12) shows that workers employed at firm j first receive the wage w_{jt} as specified in their contracts. For the following period, they consider three possible cases: (i) they are dismissed, either because the firm exits (exogenously or endogenously) or because the firm lays off workers to cut back its employment level, (ii) they quit and move to other firms by successful search on the job, or (iii) they stay in the firm. In the case of firm exit or layoff, workers go to unemployment and get the value $\mathbf{U_{t+1}}$. Here, exogenous firm exit is caused by the exogenous death shock at rate $\delta \in [0, 1]$, while endogenous firm exit is characterized by the dummy variable $\mathbf{d_{jt+1}} \in \{0, 1\}$. $\mathbf{s_{jt+1}}$ is the firm's layoff probability at $t+1$, and layoffs are i.i.d. across incumbent workers. Note that both $\mathbf{d_{jt+1}}$ and $\mathbf{s_{jt+1}}$ are contingent on the firm's state at $t+1$, depending on

[5]Note that we need to list the average productivity $\tilde{P}_{jt-1}$ and the current productivity draw P_{jt} separately as a part of the firm's state variables. This is because the current productivity draw P_{jt} by itself directly affects the firm's production function, and the average productivity $\tilde{P}_{jt}$ through period t (the combination of the average productivity $\tilde{P}_{jt-1}$ up to $t-1$ and the current productivity draw P_{jt}) determines the firm's posterior belief about its own type and expected future value. Therefore, knowing $\tilde{P}_{jt}$ is not sufficient to understand the firm's optimal contract choice, and we need to consider both P_{jt} and $\tilde{P}_{jt-1}$ (or $\tilde{P}_{jt}$). This will become more clear from the firm's value function (1.13) in the following subsection.

its productivity draw P_{jt+1}.

Combining these possibilities, the first term inside the large bracket of the right-hand side of (1.12) shows the value when the worker becomes unemployed in the next period. Meanwhile, workers remain employed at $t+1$ with probability $(1-\delta)(1-\mathbf{d_{jt+1}})(1-\mathbf{s_{jt+1}})$ and are allowed to search on the job. With probability $\lambda f(\theta(x^E_{jt+1}))$ they are successful and quit, and with probability $1-\lambda f(\theta(x^E_{jt+1}))$ they remain in the firm and receive promised state-contingent utility $\mathbf{\tilde{W}_{jt+1}}$ from the firm. This is summarized by the remaining terms in the large bracket on the right-hand side of (1.12).

Note that $\mathbb{E}_{jt}(\cdot)$ refers to the workers' expectation of P_{jt+1} based on their updated beliefs on ν_j. The workers' posterior distribution of log productivity is

$$\ln P_{jt+1} \sim N(\bar{\nu}_{jt}, \sigma^2_{jt} + \sigma^2_{\varepsilon}),$$

following (1.5) and (1.6).

1.3.7 Firms' Problem

1.3.7.1 Incumbent Firms

Incumbent firm j ($a_{jt} \geqslant 1$) has the following problem at the search and matching stage in period t:

$$\mathbf{J}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \{\mathbf{\Omega^i_{jt-1}}\}_{i\in[0,l_{jt-1}]}) = \max_{\substack{\{\mathbf{\Omega^i_{jt}}\}_{i\in[0,l_{jt}]},\\ h_{jt}, x_{jt}}} P_{jt} l^{\alpha}_{jt} - \int_0^{l_{jt}} w^i_{jt} di - c_f - \frac{c}{q(\theta(x_{jt}))} h_{jt}$$

$$+\beta(1-\delta)\mathbb{E}_{jt}\Bigg[(1-d_{jt+1})\mathbf{J}(a_{jt+1},\tilde{P}_{jt},l_{jt},P_{jt+1},\{\mathbf{\Omega^i_{jt}}\}_{i\in[0,l_{jt}]})\Bigg], \tag{1.13}$$

where the firm produces with labor using the decreasing returns-to-scale technology $P_{jt}l_{jt}^{\alpha}$ ($\alpha < 1$), w_{jt}^i refers to the wage paid to worker $i \in [0, l_{jt}]$ as a component of the contract $\mathbf{\Omega^i_{jt}} \equiv \{w^i_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}$, h_{jt} is the new hires by firm j, x_{jt} is the market firm j searches in at t, and $q(\cdot)$ is the job filling probability, which is a function of labor market tightness $\theta(x_{jt})$ within the market.[6] The firm solves this problem subject to the following constraints:

$$l_{jt} = h_{jt} + (1-s_{jt})\big(1-\lambda f\big(\theta(x^E_{jt})\big)\big)l_{jt-1} \tag{1.14}$$

$$\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1} + (1-\lambda f(\theta(x^E_{jt+1})))\mathbf{\tilde{W}_{jt+1}} \geqslant U_{t+1} \tag{1.15}$$

$$x^E_{jt+1} = \mathbf{x^E}(\mathbf{\tilde{W}_{jt+1}}) \equiv \underset{x}{\operatorname{argmax}}\, f(\theta(x))\big(x - \mathbf{\tilde{W}_{jt+1}}\big) \tag{1.16}$$

$$\mathbf{W}(a_{jt},\tilde{P}_{jt-1},l_{jt-1},P_{jt},\mathbf{\Omega^i_{jt}}) \geqslant x_{jt} \quad \text{for new hires } i \in [0,h_{jt}] \tag{1.17}$$

$$\mathbf{W}(a_{jt},\tilde{P}_{jt-1},l_{jt-1},P_{jt},\mathbf{\Omega^i_{jt}}) \geqslant \mathbf{\tilde{W}_{jt}} \quad \text{for incumbent workers } i \in [h_{jt},l_{jt}]. \tag{1.18}$$

Note that (1.14) is the employment law of motion, which shows that total employment is the sum of new hires h_{jt} and incumbent workers remaining after firm layoffs and workers' successful on-the-job search.

(1.15) is a participation constraint, which prevents workers' return to unemployment unless separations take place, and (1.16) is an incentive constraint based on incumbent workers' optimal on-the-job search. The firm takes into account their workers' incentive to move to other firms

[6]Note that firms offer the common values for $\mathbf{d^i_{jt+1}} = \mathbf{d_{jt+1}}, \mathbf{s^i_{jt+1}} = \mathbf{s_{jt+1}}, \mathbf{\tilde{W}^i_{jt+1}} = \mathbf{\tilde{W}_{jt+1}}$ to workers as they become incumbents and no longer have ex-post heterogeneity in the next period.

and internalizes the impact of their utility promises on workers' on-the-job search behavior. In other words, firms' choice of promised utility to remaining incumbent workers $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ determines incumbent workers' choice of submarket for on-the-job search x^E_{jt+1} by the incentive condition. Therefore, the number of workers who quit upon successful on-the-job search, $\lambda f(\theta(x^E_{jt}))l_{jt-1}$, is predetermined by the state-contingent utility level $\tilde{\mathbf{W}}_{\mathbf{jt}}$ that the firm announced in the preceding period and is committed to in the current period. Furthermore, the firm optimally chooses the state-contingent utility level $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ to deliver in the next period as a component of the contract $\mathbf{\Omega}_{\mathbf{jt}}$, taking into account the workers' incentive constraint (1.16) in the next period.

In addition, (1.17) and (1.18) are promise-keeping constraints for new hires at t and surviving incumbent workers from the previous period, respectively, both of which the firm needs to satisfy. Because of the commitment assumption, the firm needs to announce contracts at t that deliver at least x_{jt} and $\tilde{\mathbf{W}}_{\mathbf{jt}}$ to their newly hired and incumbent workers, respectively.

After search and matching is complete, the firm enjoys an instantaneous profit equal to revenue $P_{jt}l^{\alpha}_{jt}$ minus the sum of the wage bill to its workers $\int_0^{l_{jt}} w^i_{jt}di$, the operating fixed cost c_f, and the vacancy cost $\frac{c}{q(\theta(x_{jt}))}h_{jt}$, as specified in the first line in (1.13). In the following period, conditional on surviving the exogenous death shock with probability $(1-\delta)$ and the state-contingent decision rule $\mathbf{d}_{\mathbf{jt+1}} = 0$, the firm enters the search and matching process again and obtains the next period value $\mathbf{J}_{\mathbf{jt+1}} = \mathbf{J}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \{\mathbf{\Omega}_{\mathbf{jt}}\}_i)$.

1.3.7.2 Entrants

New firms enter each period by paying entry cost c_e after the death shock hits incumbent firms, but before the entrants' initial productivity is realized. Entrants have initial beliefs about their types, characterized by the cross-sectional mean $\bar{\nu}_0$ and standard deviation σ_0. Based on their priors, they calculate the expected value from entry and keep entering until the expected value equals the entry cost, following the free-entry assumption. After entering and observing their initial productivity, new firms decide whether to exit or stay, and in the latter case they search and hire workers to produce as incumbents. They pay c for each vacancy they post and hire new workers with probability $q(\theta(x_t^e))$ as a function of the market tightness within the market x_t^e they choose to search in.

The entry mass is endogenously pinned down by the following free entry condition, which must hold when there is a positive entry mass M_t^e:

$$\int \max_{\substack{\mathbf{\Omega^e_{jt}}=\{w^e_{jt},\mathbf{d_{jt+1}},\mathbf{s_{jt+1}},\mathbf{\tilde{W}_{jt+1}}\},\\ d^e_{jt},l^e_{jt},x^e_{jt}}} (1-d^e_{jt})\Bigg(P_{jt}(l^e_{jt})^\alpha - w^e_{jt}l^e_{jt} - c_f - \frac{c}{q(\theta(x^e_{jt}))}l^e_{jt} + \beta(1-\delta)\mathbb{E}_{jt}\Big[(1-\mathbf{d_{jt+1}})\mathbf{J}(1,P_{jt},l^e_{jt},P_{jt+1},\mathbf{\Omega^e_{jt}})\Big]\Bigg)dF_e(P_{jt}) - c_e = 0, \tag{1.19}$$

where $\mathbf{\Omega^e_{jt}}$ is entrant firm j's contract decision, which consists of the four components in (1.10), where the wage w^e_{jt} only depends on the entrant's initial productivity P_{jt}, and the last three terms depend on the entrant's next-period state variables $(1, P_{jt}, l^e_{jt}, P_{jt+1})$ after drawing productivity P_{jt+1}. d^e_{jt}, l^e_{jt}, and x^e_{jt} stand for entrant firm j's exit, hiring, and search decisions, respectively, after the firm's initial productivity P_{jt} is observed at t. Note that these three terms are a function only of the initial productivity P_{jt} as the entrant does not have any previous history. Also, the distribution $F_e(P_{jt})$ of productivity is based on the entrant's initial prior about its own type ν_j, i.e.

$$\ln P_{jt} \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\varepsilon^2),$$

while $\mathbb{E}_{jt}(\cdot)$ stands for the firm's updated posterior after observing P_{jt}, which follows (1.5) and (1.6). Lastly, the firm is subject to the participation and incentive constraints (1.15) and (1.16) for retaining incumbent workers in the next period, and the following promise-keeping constraint for the newly hired workers in the current period:

$$\mathbf{W}(0, 0, 0, P_{jt}, \mathbf{\Omega^e_{jt}}) \geqslant x^e_{jt} \quad \text{for all workers } l^e_{jt}. \tag{1.20}$$

1.3.8 Labor Market Equilibrium

Equilibrium in each labor market is determined by workers' and firms' optimal search. First, workers trade off the utility level of a given contract and the corresponding probability of being matched. The trade-off depends on workers' current employment status, which determines their outside option of finding a job. In particular, unemployed workers choose a labor market x^U_t to search in by solving

$$x^U_t = \underset{x^U_t}{\text{argmax}}\, f(\theta(x^U_t))(x^U_t - \mathbf{U_t}), \tag{1.21}$$

where the outside option $\mathbf{U_t}$ is given by (1.11). In a similar fashion, employed incumbent workers at firm j solve

$$\mathbf{x^E}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) = \underset{x^E_{jt}}{\text{argmax}}\, f(\theta(x^E_{jt}))(x^E_{jt} - \tilde{\mathbf{W}}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})), \tag{1.22}$$

taking into account their outside option $\tilde{\mathbf{W}}_{\mathbf{jt}}$ provided by the current employer j. Equations (1.21) and (1.22) determine the optimal labor submarkets in which unemployed and employed workers choose to search.

Note that there exists ex-post heterogeneity among workers depending on their current employment status, although there is no ex-ante worker heterogeneity. This means that workers' choices and offers will be the same for all workers of a given employment status, being either unemployed or employed at a particular firm j with a given set of state variables $(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})$.

On firms' side, from (1.13) and (1.19) along with the promise-keeping constraints for new hires (1.17) and (1.20), imply that both entrants and incumbents face the same problem when choosing their optimal submarket x_{jt} to search in, which can be reduced to:

$$\min_{x_{jt}} \frac{c}{q(\theta(x_{jt}))} + x_{jt}, \tag{1.23}$$

which is common to every firm, independent of their state variables. This means that all firms are indifferent across the various submarkets x_{jt} that are solutions to (1.23). This result is due to the assumption of a constant vacancy cost that is common across firms.

Labor market equilibrium is pinned down by the (possibly multiple) intersection points between the workers' and firms' choices in (1.21), (1.22), and (1.23). These equilibria are computed as follows. Starting with the firms' problem, only submarkets that satisfy (1.23) are searched by firms. This implies that in equilibrium, the following complementary slackness condition should hold for any active labor submarket x_t:

$$\theta(x_t)\Big(\frac{c}{q(\theta(x_t))} + x_t - \kappa\Big) = 0, \tag{1.24}$$

where κ is the minimized cost value

$$\kappa \equiv \min\Big(\frac{c}{q(\theta(x_t))} + x_t\Big). \tag{1.25}$$

I assume a CES matching function

$$M(S(x_t), V(x_t)) = (S(x_t)^{-\gamma} + V(x_t)^{-\gamma})^{-\frac{1}{\gamma}},$$

which is common across labor submarkets x_t. $S(x_t)$ and $V(x_t)$ are the total number of searching workers and vacancies, respectively, in each labor submarket x_t.[7] Using the matching function, the job finding rate $f(\cdot)$ and filling rate $q(\cdot)$ for each submarket x_t are given by:

$$f(\theta(x_t)) = \theta(x_t)(1 + \theta(x_t)^{\gamma})^{-\frac{1}{\gamma}} \tag{1.26}$$

$$q(\theta(x_t)) = (1 + \theta(x_t)^{\gamma})^{-\frac{1}{\gamma}}, \tag{1.27}$$

where $\theta(x_t)$ is the ratio of total vacancies to searching workers, $\frac{V(x_t)}{S(x_t)}$, in each submarket x_t. Therefore, in equilibrium, labor market tightness in different submarkets must satisfy

$$\theta(x_t) = \begin{cases} \left(\left(\frac{\kappa - x_t}{c}\right)^{\gamma} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x_t < \kappa - c \\ 0 \quad \text{if} \quad x_t \geqslant \kappa - c. \end{cases} \tag{1.28}$$

Note that $\theta(\cdot)$ is decreasing in x_t, and if x_t is greater or equal to $\kappa - c$, there are no firms posting

[7]Note that the job searchers $S(x_t)$ are workers searching either from the unemployment pool (if x_t is the optimal market for unemployed workers to search in) or on the job (if x_t is the optimal market for workers employed at j to search in).

vacancies, so that the market becomes inactive, i.e. $\theta(x_t) = 0$.

We can derive the optimal choice for unemployed workers x_t^U by substituting out $\theta(x_t)$ in the unemployed workers' problem (1.21) with (1.28). In a similar fashion, one can derive a solution for x_{jt}^E using the employed workers' problem (1.22) and (1.28), which pins down the on-the-job search choice of workers employed at firm j. The solutions are provided in Appendix A.2.2.

1.3.9 Firm Distribution and Labor Market Clearing

In this subsection, I use the firms' decision rules and a labor market clearing condition to compute a stationary firm distribution and close the model. Since the model is solved at the steady state in a recursive form, I drop time subscripts. To be clear, I use x to denote state variables at the beginning of each period and use x' to express the next period value of x.[8]

Let $\mathbf{G}(a, \tilde{P}, l)$ be the steady state mass of firms aged a with average log-productivity $\tilde{P}$ and employment size l at the beginning of each period. This distribution satisfies the following law of motion for all $a \geqslant 1$, $\tilde{P}$, and l:

$$\mathbf{G}(a+1, \tilde{P}', l') = (1-\delta) \int_l \int_{\tilde{P}} \Big(1 - \mathbf{d}\big(a, \tilde{P}, l, P'\big)\Big) \mathbb{I}_{l'} \mathbf{G}(a, \tilde{P}, l) f_P(P') d\tilde{P} dl \tag{1.29}$$

subject to

$$P' = \mathrm{e}^{(a+1)\tilde{P}' - a\tilde{P}}, \tag{1.30}$$

[8]To avoid confusion, let me clarify that $\tilde{P}$ is the average productivity and l is the employment level that firms take as given when they enter the period, before observing their new productivity draw P. Thus, firm state variables are $(a, \tilde{P}, l, P)$. Also, $\tilde{\mathbf{W}}(a, \tilde{P}, l, P)$ is the utility level promised to incumbent workers by firms with $(a, \tilde{P}, l)$ at the beginning of each period and with P drawn subsequently.

where $\mathbb{I}_{l'}$ denotes an indicator function for firms choosing l', i.e. firms for whom $\mathbf{l}(a, \tilde{P}, l, P') = l'$, and P' is the next period productivity draw following (1.30). Note that $(\bar{\nu}, \sigma^2)$ are the mean and variance of the posterior distribution for a firm with age a and average log-productivity $\tilde{P}$ at the beginning of each period, given by:

$$\bar{\nu} \equiv \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{a\tilde{P}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + \frac{a}{\sigma_\varepsilon^2}}$$

$$\sigma^2 \equiv \frac{1}{\frac{1}{\sigma_0^2} + \frac{a}{\sigma_\varepsilon^2}},$$

and $f_P(\cdot)$ is the log-normal probability density function of productivity P, with the corresponding mean $\bar{\nu}$ and variance $\sigma^2 + \sigma_\varepsilon^2$.

(1.29) defines the next period mass of firms with age $(a+1)$, average log-productivity $\tilde{P}'$, and employment size l' as the sum of the surviving incumbents of age a that end up having the average log-productivity $\tilde{P}$, productivity draw P', and employment size $\mathbf{l}(a, \tilde{P}, l, P') = l'$.

We can track the stationary firm mass by iterating on the law of motion along with the following initial condition:

$$\mathbf{G}(1, \tilde{P}, l) = \begin{cases} M^e(1 - d^e(\mathrm{e}^{\tilde{P}}))f_e(\mathrm{e}^{\tilde{P}}) & \text{if} \quad l^e(\mathrm{e}^{\tilde{P}}) = l, d^e(\mathrm{e}^{\tilde{P}}) \neq 1 \\ 0 \quad \text{otherwise.} \end{cases}$$

Here, M^e is the firm entry mass, $f_e(\cdot)$ is the initial prior density of P, i.e. $\ln P \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\epsilon^2)$, and d^e and l^e are the entrants' exit and employment decision rules after observing their first realization of productivity P, following (1.19). This shows that the mass of firms with age 1 and average log-productivity $\tilde{P}$ is the mass of surviving entrants whose first productivity realization is $P = e^{\tilde{P}}$. Also, the mass of firms with age 1, average log-productivity $\tilde{P}$, and employment size l consists of surviving entrants whose initial productivity is $P = e^{\tilde{P}}$ and who choose initial employment size $l^e(e^{\tilde{P}}) = l$. Note that the entrant's log productivity $\ln P$ equals its average log productivity $\tilde{P}$ at the beginning of the next period when they become age 1.

To close the model, I impose the following labor market clearing condition:

$$\sum_{a\geqslant 1}\int_{\tilde{P}}\int_{l}\int_{P}\Bigg\{\Big(\delta+(1-\delta)\big(\mathbf{d}(a,\tilde{P},l,P) \\ +\big(1-\mathbf{d}(a,\tilde{P},l,P)\big)\mathbf{s}(a,\tilde{P},l,P)\big)\Big)lf_P(P)\mathbf{G}(a,\tilde{P},l)\Bigg\}dPdld\tilde{P} \\ = f\big(\theta(x^U)\big)\Big(N-\sum_{a\geqslant 1}\int_{\tilde{P}}\int_{l} l\mathbf{G}(a,\tilde{P},l)dld\tilde{P}\Big), \tag{1.31}$$

where $N = 1$ given the normalization of workers' total mass, and $f_P(\cdot)$ refers to the log-normal probability density function of productivity P as before, with mean $\bar{\nu}$ and variance $(\sigma^2 + \sigma_\epsilon^2)$.

Equation (1.31) says that in the steady state equilibrium, the inflow to the unemployment pool is equal to the outflow from the unemployment pool. To be specific, the left-hand side of (1.31) is the sum of the number of workers that lose their jobs because of firm exit (both exogenous δ and endogenous $\mathbf{d}$) or layoff $\mathbf{s}$ from their current employer with age a, average log-productivity $\tilde{P}$, employment size l and current productivity P, which characterizes the total inflow to the unemployment pool. Note that there is no loss of workers when entrant firms decide to exit, since entrants that immediately exit never hire workers. The right-hand side of (1.31) is the number of unemployed workers finding a job, which is the total outflow from the unemployment pool. The number of unemployed workers equals the total population of workers minus the number of employees before firm exit and layoffs. This is because of the timing assumption that workers laid off in period t cannot search until period $t+1$.

Furthermore, in a steady state equilibrium, total job creation by firms needs to be equal to total job finding by workers. For notational convenience, let $\tilde{\mathbf{G}}(a,\tilde{P},l,P)$ be the mass of firms who survive after observing the death shock and their productivity P, i.e. $\tilde{\mathbf{G}}(a,\tilde{P},l,P) \equiv (1-\delta)(1-\mathbf{d}(a,\tilde{P},l,P))f_P(P)\mathbf{G}(a,\tilde{P},l)$. Then, the following equation holds:

$$
\begin{aligned}
& M^e \int_P l^e(P)(1-d^e(P))f_e(P)dP + \sum_{a\geqslant 1}\int_{\tilde{P}}\int_l\int_P \Big\{\mathbf{h}(a,\tilde{P},l,P)\mathbb{I}_{\mathbf{h}>0}\tilde{\mathbf{G}}(a,\tilde{P},l,P)\Big\}dPdld\tilde{P} \\
= \;& f\big(\theta(x^U)\big)\Big(N - \sum_{a\geqslant 1}\int_{\tilde{P}}\int_l l\mathbf{G}(a,\tilde{P},l)dld\tilde{P}\Big) \\
& + \sum_{a\geqslant 1}\int_{\tilde{P}}\int_l\int_P \Big\{\lambda f(\theta(\mathbf{x}^{\mathbf{E}}(a,\tilde{P},l,P)))(1-\mathbf{s}(a,\tilde{P},l,P))l\tilde{\mathbf{G}}(a,\tilde{P},l,P)\Big\}dPdld\tilde{P}
\end{aligned}
$$

where the left-hand side is the sum of new jobs created by new entrants and recruiting incumbent firms, and the right-hand side is total job finding, which is the sum of newly hired unemployed and poached workers.

1.3.10 Stationary Recursive Competitive Equilibrium

Lastly, a stationary recursive competitive equilibrium can be defined as follows.[9]

Definition 2. *A stationary recursive competitive equilibrium for this economy consists of:*

1. *the posteriors on types $\{\bar{\nu},\sigma^2\}$, following (1.3) and (1.4);*
2. *a set of value functions $\mathbf{U}$, $\mathbf{W}(a,\tilde{P},l,P,\mathbf{\Omega})$, and $\mathbf{J}(a,\tilde{P},l,P,\mathbf{\Omega})$, characterized by (1.11), (1.12), and (1.13), respectively;*

[9]The derivation of the equilibrium is provided in Appendix A.2, and the computation algorithm is described in Appendix A.6.

3. *the associated unemployed workers' decision rule* x^U *solving (1.21), and employed workers' decision rule* $\{x^E\}$ *solving (1.22);*

4. *the associated incumbent firms' decision rule* $\{\mathbf{\Omega} = \{w, \{\mathbf{d}', \mathbf{s}', \tilde{\mathbf{W}}'\}\}, h, l, x\}$ *solving (1.13), and entrants' decision rule* $\{\mathbf{\Omega^e} = \{w^e, \{\mathbf{d}', \mathbf{s}', \tilde{\mathbf{W}}'\}\}, d^e, l^e, x^e\}$ *solving (1.19);*

5. *the mass of entrants* M^e*, following the free-entry condition (1.19);*

6. *the labor market tightness* $\{\theta(x)\}$ *for all active markets* x*, following (1.28);*

7. *the stationary firm distribution* $\mathbf{G}(a, \tilde{P}, l)$ *following (1.29);*

8. κ *characterizing the firms' indifference curve, pinned down by (1.31);*

given the exogenous process for P*, initial conditions* $(\bar{\nu}_0, \sigma_0^2)$ *and* $\mathbf{G}(1, \tilde{P}, l)$*, and the total number of workers, normalized as* $N = 1$.

1.4 Model Implications

In this section, I derive several implications of the model, which are the foundation of the quantitative analysis in Section 1.5.

1.4.1 Equilibrium Wages and Workers' Job Prospects

The propositions in this section discuss the determinants of equilibrium wages offered by firms to workers.

Lemma 1. *Firm promise-keeping constraints (1.17) and (1.18) bind.*

Proof. From (1.12), (1.13), (1.17), and (1.18), each firm j optimally chooses the lowest possible $\{w^i_{jt}\}_i$ that complies with the promise-keeping constraints. This does not change any incentive structure of the problem, and thus the promise-keeping constraints bind with equality. ■

Proposition 1. *Equilibrium current wages are determined by workers' outside options and their expected future value (job prospects) at a given firm.*

Proof. Using Lemma 1, the promise-keeping constraints (1.17) and (1.18) can be rephrased in terms of the current wage w. Then the following condition can be obtained for new hires:

$$\mathbf{w} = \mathbf{x} - \beta\mathbb{E}\Bigg[\Big(\delta + \big(1-\delta\big)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} + (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg], \tag{1.32}$$

where $\mathbf{x}$ is pinned down by the equilibrium submarket choices (A.1) and (A.2) for each unemployed and poached worker, as discussed in Appendix A.2.2. For incumbent workers, the corresponding condition holds:

$$\mathbf{w} = \tilde{\mathbf{W}} - \beta\mathbb{E}\Bigg[\Big(\delta + \big(1-\delta\big)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} + (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg], \tag{1.33}$$

where $\tilde{\mathbf{W}}$ is the total utility level firms promise to incumbent workers in equilibrium, as shown in Appendix A.2.4. The first term on the right hand side of (1.32) and (1.33) shows the promised utility level for each type of worker, which in equilibrium is determined by the worker's outside options and depends on the worker's previous employment status. The term in large brackets on the right hand side refers to workers' expected future value at a given firm, which depends on their posterior beliefs about firm type. Note that workers' expected future value is identical across workers for a given firm, regardless of whether they are new hires or incumbents, as they share the same information about the firm. ■

Proposition 1 shows that given the worker's previous employment status, as characterized by the promised utility level, the current wage solely depends on workers' expected future value at the firm. As workers' expectations are formed based on their posterior beliefs about the firm, the equilibrium wage varies by firm age, following (1.5) and (1.6).

Corollary 1. *Controlling for workers' previous employment status, the equilibrium current wage is determined by workers' expected future value at the firm, which is a function of firm age.*

Next, I discuss how workers' expected future value varies across firms depending on workers' job prospects. Lemma 2 establishes the existence of endogenous productivity cutoffs that determine firms' hiring status, and the following proposition 2 lays out the ranking of workers' value across their employers' hiring status.

Lemma 2. *There are four endogenous cutoffs for the current productivity draw P among operating firms: i) the upper cutoff $\mathcal{P}^h(a, \tilde{P}, l)$ between hiring versus inaction with no quits, ii) the middle cutoff $\mathcal{P}^q(a, \tilde{P}, l)$ between inaction with no quits versus inaction with quits, iii) the lower cutoff $\mathcal{P}^l(a, \tilde{P}, l)$ between quits only versus quits and layoffs, and iv) the exit cutoff $\mathcal{P}^x(a, \tilde{P}, l)$ below which firms endogenously exit. These cutoffs are endogenously determined by the beginning-of-period state variables $(a, \tilde{P}, l)$ before the current productivity draw P.*[10]

[10]Note that there does not exist any case in which firms find it optimal to both hire and lay off workers. More discussion can be found in Appendix A.2.4.1.

Proof. See Appendix A.2.4.5. ■

Proposition 2. *Define $\hat{W}$ as incumbent workers' value function after observing the firm's current productivity draw P, but before firms have completed endogenous exits and layoffs. Then, $\hat{W}$ is ranked from the highest to the lowest by the following order: workers at hiring or inactive firms ($\mathcal{P}^q < P$), quitting firms ($\mathcal{P}^l < P < \mathcal{P}^q$), firms laying off workers ($\mathcal{P}^x < P < \mathcal{P}^l$), and exiting firms ($P < \mathcal{P}^x$).*

Proof. See Appendix A.3.1. ■

The intuition behind this result is as follows. Following Lemma 1, the incumbent workers' value $\hat{W}$ right after observing firm productivity is determined by the state-contingent continuation utility level $\tilde{\mathbf{W}}$ promised by their employer and the workers' target utility in on-the-job search x^E. As these two terms are tightly linked to each other, through the workers' optimality condition (1.22), firms' choice of $\tilde{\mathbf{W}}$ depends on their desire to retain workers in the face of potential poaching by other firms.[11] Therefore, expanding firms with more willingness to retain workers offer higher value and deter poaching more successfully than contracting firms.[12] Lastly, workers' value in unemployment is lower than the value of being employed.

Next, let the workers' expected future value on the right hand side of (1.32) and (1.33) denoted as $\mathbb{E}[*']$ as follows:

$$\begin{aligned}\mathbb{E}[*'] \equiv & \mathbb{E}\Bigg[\Big(\delta + (1-\delta)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \\ & + (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x^{E}}'))\mathbf{x^{E}}' + \big(1-\lambda f(\theta(\mathbf{x^{E}}'))\big)\tilde{\mathbf{W}}'\Big)\Bigg],\end{aligned} \tag{1.34}$$

[11]In Appendix A.2.2, I show that workers' target utility in on-the-job search x^E is increasing in their promised utility $\tilde{\mathbf{W}}$ in the current employer. In other words, the higher utility $\tilde{\mathbf{W}}$ workers obtain from their current employer, the higher utility x^E an outsider firm needs to provide to poach them.

[12]This is due to the existence of vacancy costs as it is more costly to lose incumbent workers and hire new workers.

where $\mathbf{d}'$, $\mathbf{s}'$, $\tilde{\mathbf{W}}'$ are the firm's decision rules next period for exit, layoff, and the continuation utility promised to the workers, and $\mathbf{x}^{\mathbf{E}'}$ is the utility from successful on-the-job search in the next period. Thus, the terms in $[*']$ are a function of the firm's state variables after observing its productivity in the next period, $(a+1, \tilde{P}', l', P')$, and the worker's current expectation $\mathbb{E}[\cdot]$ is taken with respect to the firm's productivity P' in the next period, and is formed based on workers' current posterior beliefs about the firm.

Proposition 2 and (1.34) imply that workers expect higher future value at firms that are more likely to hire or stay inactive without allowing quits in the next period, which guarantees higher stability as well as better career options to workers. This is because these firms would not only offer higher continuation value to workers but also make workers more ambitious when targeting their on-the-job search options. On the other hand, if firms are expected to lose workers in the next period, either by poaching or layoffs, workers anticipate lower future value, as these are seen as less stable and less willing to retain workers with strong continuation utility. In other words, workers' future expected value $\mathbb{E}[*']$ in (1.34) is higher for firms with better posterior beliefs and less likelihood of laying off workers next period.

Given these results, one can establish the following result numerically for a broad range of parameters.[13]

Result 1 (**Worker's Expected Future Value across Firm Age**).

$$\mathbb{E}[*'](a_y, \tilde{P}, l, P) \leqslant \mathbb{E}[*'](a_o, \tilde{P}, l, P) \quad \text{if} \quad \tilde{P} > \bar{\nu}_0$$

$$\mathbb{E}[*'](a_y, \tilde{P}, l, P) \geqslant \mathbb{E}[*'](a_o, \tilde{P}, l, P) \quad \text{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geqslant 0.$$

[13]Note that the Gaussian integral does not allow for analytic solutions. An analytic proof of this result can be provided in a model assuming a simplified learning process based on a uniform distribution as in Pries (2004), where the main mechanism works the same.

Thus, when comparing two firms with the same observable characteristics ($\tilde{P}$, l, P) but different ages, workers have lower (higher) expected future values at younger firms if their cumulative average productivity is above (below) the cross-sectional mean.[14] In other words, for high performing firms with the same set of observable characteristics, workers' expected future value is lower at younger firms, while the opposite is true for low performing firms. This is due to the limited information available about younger firms, which makes workers pessimistic about job prospects at younger firms with high average performance, but optimistic at younger firms with low average performance.

Result 1 holds over a broad parameter space, and the main intuition is as follows. Note that the workers' expected future value in (1.34) is rooted in their posterior beliefs about firm type, defined by (ν_{jt}, σ_{jt}), and in particular their beliefs about the next-period productivity cutoffs and the workers' values (contingent on firms' hiring status) as defined in Lemma 2. The likelihood of drawing better productivity and expanding next period is higher for firms with better posterior beliefs, while the probability of laying off workers or exiting is higher for firms with worse posterior beliefs. Furthermore, as discussed in Appendix A.4.2, productivity cutoffs are (weakly) lower for firms with better prospects. This suggests that workers should generally perceive higher (lower) expected value at firms having better (worse) posterior beliefs.

Applying this insight to the firm age dimension, we know from (1.7) that younger firms have a lower (higher) posterior mean than their mature counterparts, if they are high (low) performing. This is because the posterior mean is a weighted sum of average performance and the initial prior mean, and a higher weight is put on average performance for older firms, given their

[14]Note that the equality holds when both firms are mature enough as the posterior converges to the firms' actual type.

longer track record. Thus, the posterior mean of older firms gets closer to the firms' observed performance. Therefore, if two firms have equally good performance, the posterior beliefs about the younger firm are relatively worse than for their mature counterpart. The opposite holds for two firms having the same low average performance.

Connecting this result with Proposition 1, firms can pay lower wages to workers all else equal if they are more likely to hire or stay inactive in the next period, whereas they need to pay higher wages if they have higher likelihood of losing workers by poaching or layoffs in the next period. These wage differentials are based on differences in expected future value due to differences in posterior beliefs. The following result shows how this insight applies to the wages paid by young firms:

Result 2 (**Wage Differentials across Firm Age**). *Given the firms' state variables* $(\tilde{P}, l, P)$*, equilibrium current wages offered to a given type of newly hired worker (unemployed or poached from a given firm) satisfy the following relationship across firm age:*

$$\begin{aligned} &\mathbf{w}^{\mathbf{type}}(a_y, \tilde{P}, l, P) \geqslant \mathbf{w}^{\mathbf{type}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0 \\ &\mathbf{w}^{\mathbf{type}}(a_y, \tilde{P}, l, P) \leqslant \mathbf{w}^{\mathbf{type}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geqslant 0, \end{aligned}$$

where $type \in \{U, E\}$ *for unemployed and poached workers, respectively. Also, given the firms' state variables* $(\tilde{P}, l, P)$ *and the number of incumbent workers the firm wants to retain (or equivalently, the promised utility* $\tilde{\mathbf{W}}$ *to incumbent workers), equilibrium current wages offered to incumbent workers satisfy:*

$$\mathbf{w}^{\mathbf{inc}}(a_y, \tilde{P}, l, P) \geqslant \mathbf{w}^{\mathbf{inc}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0$$

$$\mathbf{w}^{\mathbf{inc}}(a_y, \tilde{P}, l, P) \leqslant \mathbf{w}^{\mathbf{inc}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geqslant 0.$$

This result implies that high performing younger firms need to pay higher current wages than otherwise similar mature firms to hire or retain workers. On the other hand, low performing younger firms can pay lower current wages than otherwise similar mature firms.[15] These age gaps are due to different job prospects across firms with different ages and history of performance, conditional on the promised future utility $\mathbf{x}$ or $\tilde{\mathbf{W}}$.

Figure 1.1 displays workers' expected future value (the top left panel) and the equilibrium current wage to hire unemployed workers (in the top right panel), to poach workers from a median firm (in the bottom left panel), and to retain incumbent workers (in the bottom right panel). The figure shows the wage differentials across firms of different ages, controlling for the workers' previous employment status and the firms' observable characteristics (equally-sized firms that have equal above-average productivity). This confirms that wages decline with firm age for high performing firms. The counterparts for firms having low average productivity are displayed in Figure A.7.1 in Appendix A.7.[16,17]

[15] Note that since firms are indifferent across the various labor submarkets along their indifference curve characterized by (1.25), there can be multiple active labor submarkets in equilibrium, although there is no systematic linkage between firm characteristics and the specific submarkets they choose. In other words, there is no systematic pattern of sorting between firms (with heterogeneous characteristics) and workers (with different origins from the previous period) across submarkets. The labor market equilibrium is defined as a continuum of such submarkets, indexed by the promised utility level offered by firms. The wage relationships discussed above hold within each submarket, implying that on average, high performing young firms pay wage premia, while low performing young firms pay wage discounts.

[16] For this level of performance in Figure A.7.1, firms above age 4 no longer operate in the economy, while firms aged 4 and below operate and hire workers. Upon survival, younger firms pay lower wages to either newly hired or incumbent workers. Also, the dotted grey line indicates counterfactual wages that firms would have to pay if they continued operating, which shows that mature firms with the same observable characteristics would have to pay higher wages to hire or retain workers. Note that this pattern only applies to firms with low average performance.

[17] These figures are drawn for the baseline set of parameters calibrated in Tables 1.1 and 1.2 in the following Section 1.5. As discussed earlier, these patterns are robust across different sets of parameter values.

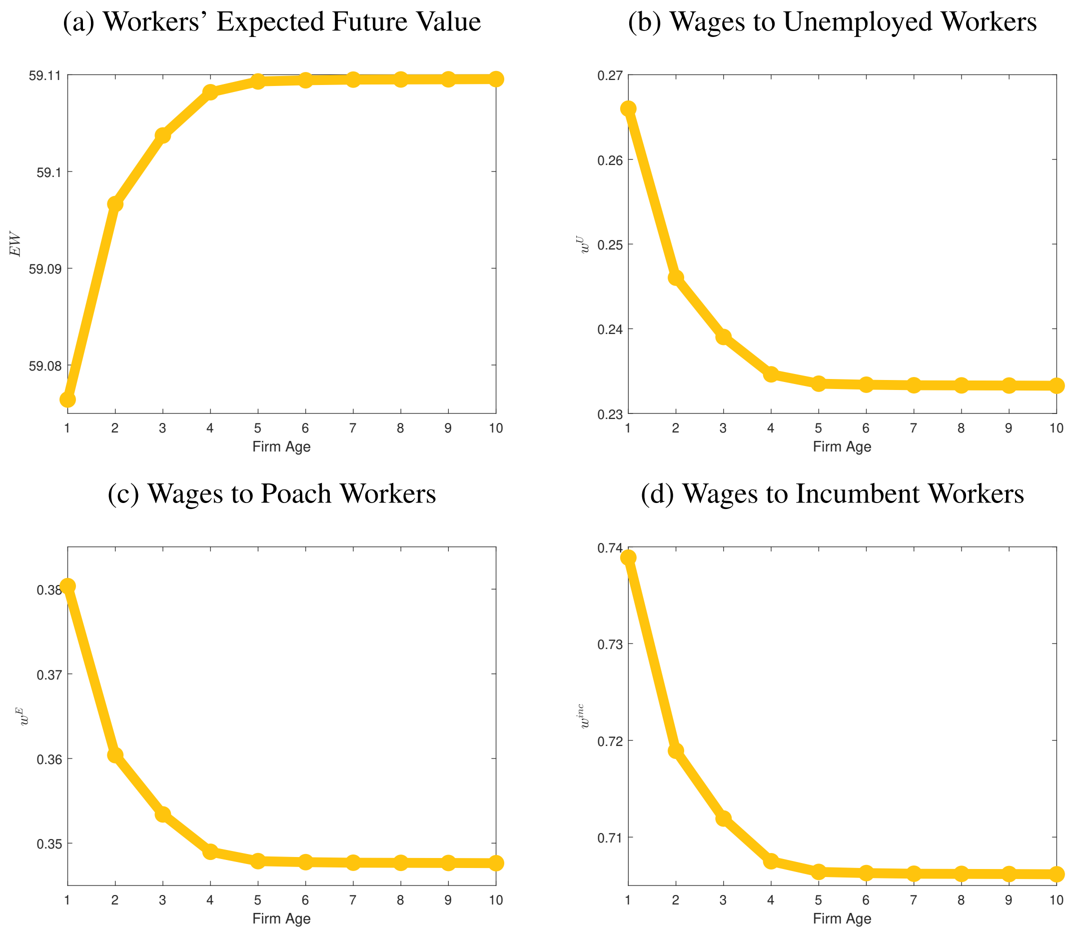


Figure 1.1: High Performing Firms (average size)

1.4.2 Equilibrium Employment Size

Given the wage differentials between young and mature firms, the following relationship between employment levels of firms at different ages can be derived:

Result 3 (**Employment Levels across Firm Age**). *Given the firms' state variables* $(\tilde{P}, l, P)$*,*

$$l(a_y, \tilde{P}, l, P) \leqslant l(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0$$

$$l(a_y, \tilde{P}, l, P) \geqslant l(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geqslant 0$$

This result shows that high performing younger firms have weakly lower employment levels (and growth) than older firms, all else equal, while low performing younger firms have weakly higher employment (and growth) than older firms. This result comes through the workers' learning and job prospects mechanism and the consequent wage differentials associated with firm age, which directly affect the hiring and retention margins of firms. Since high performing young firms pay higher wages relative to otherwise similar mature firms, they have a lower net marginal value of employment, as the wage premia increase the marginal cost of hiring or retaining a worker. High performing young firms also find it more beneficial to lay off workers relative to their mature counterparts, due to the increased wages needed to retain them. On the contrary, low performing young firms have a higher marginal value of hiring or retaining a worker as well as a lower marginal value of laying off a worker, compared to otherwise similar mature firms. I discuss details in Appendix A.4.3.

1.4.3 Uncertainty and Job Prospects

In this section, I discuss how the degree of uncertainty in the economy affects model outcomes. The following proposition shows how the learning process depends on the degree of productivity noise, σ_ε.[18]

Proposition 3. *If productivity noise σ_ε increases, high performing firms have a relatively lower posterior mean, while low performing firms have a relatively higher posterior mean, for any given age and average observed performance. Furthermore, higher noise increases the posterior variance for all firms.*

Proof. See Appendix A.4.4.1. ■

Proposition 3 implies that higher noise reduces the prospects at high performing firms, while improving the prospects of low performing firms, all else equal. This is because agents are less certain about firms' actual type.

Proposition 4. *As productivity noise σ_ε rises, firms' average observed productivity becomes less informative about firms' actual type.*

Proof. See Appendix A.4.4.2. ■

Proposition 4 shows that the positive relationship in (1.7) between the average productivity level and the posterior mean is dampened as productivity noise rises in the economy. Both Propositions 3 and 4 imply that slow learning harms the prospects of high performing firms.

[18] Recall that the dispersion of shock σ_ε refers to the degree of noise in the economy, while the dispersion of firm types σ_0 indicates the signal level. Thus, for a given level of signal σ_0, the dispersion σ_ε measures the degree of uncertainty in the economy. In the empirical section below, I directly estimate the noise-to-signal ratio $\frac{\sigma_\varepsilon}{\sigma_0}$ to proxy the level of uncertainty in different industries over time.

Proposition 5. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the effect of firm age on the speed of updating posteriors is more pronounced as noise increases.*[19]

Proof. See Appendix A.4.4.3. ■

Proposition 5 shows how the degree of noise affects the learning process at different firm ages. As in (1.8), firm age affects learning about firm type in a different way depending on firms' observed performance. Specifically, firms with high average performance have better prospects due to a higher posterior mean when they are older, while firms with low average performance have better prospects due to a higher posterior mean when they are younger. Furthermore, the posterior variance decreases monotonically in firm age as seen in (1.9). Proposition 5 shows that as the noise level rises, such age effects get more pronounced for $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

Corollary 2. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the difference in job prospects between otherwise similar firms of different ages increases in the degree of noise.*

Proof. See Appendix A.4.4.4. ■

Overall, higher noise particularly harms the job prospects of young firms with high performance. Although higher noise generally harms firms with high performance, as shown in Propositions 3 and 4, the damage is more pronounced to young firms, following Proposition 5 and Corollary 2. This is because the speed of updating over the firm life cycle is dragged out as noise increases, widening the gap in job prospects between young and mature firms.

[19] In Section 1.7, I externally calibrate both σ_ε and σ_0 using estimated values from the Census data. These estimates are consistent with the assumption that $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

1.4.4 Welfare Implications

Lastly, I discuss welfare implications of the model. I prove that the model's decentralized block-recursive allocation given the level of uncertainty is constrained efficient. However, the decentralized allocation is distorted relative to the social optimum if the planner could eliminate uncertainty about firm type. More discussion can be found in Appendix A.5.

1.5 Quantitative Analysis

1.5.1 Calibration

I calibrate the model to quarterly data for the U.S. economy from 1998Q1 to 2014Q4. There are thirteen model parameters, where the first six are externally calibrated and the remaining seven are internally calibrated. The model parameters are summarized in Tables 1.1 and 1.2. I discuss data sources more fully in the next section.

1.5.1.1 External Calibration

I externally calibrate the parameters $\{\beta, \alpha, N, \bar{\nu}_0, \sigma_0, \sigma_\varepsilon\}$. I set the discount rate β to 0.99 to match a quarterly interest rate of 1.2%. I set the curvature of the revenue function α to be 0.65, which is consistent with Cooper et al. (2007). I normalize the total number of workers, $N = 1$, and the initial prior mean of firm type, $\bar{\nu}_0 = 0$. Lastly, I estimate σ_0 and σ_ε using the LBD data described below. The results are presented in Table 1.1.[20]

[20]Note that $\sigma_\varepsilon < \sigma_0$ in the data, as assumed in Proposition 5 above.

Table 1.1: Externally Calibrated Parameters

Parameter	Definition	Value	From
β	Discount factor	0.99	Interest rate ($\beta = \frac{1}{(1+r)}$)
α	Revenue curvature	0.65	Cooper et. al. (2007)
N	Total number of workers	1	Normalization
ν_0	Initial prior on firm type mean	0	Normalization
σ_0	Initial prior on firm type dispersion	0.65	LBD
σ_ε	Idiosyncratic shock dispersion	0.47	LBD

1.5.1.2 Internal Calibration

I internally calibrate the remaining seven parameters $\{b, \lambda, c, \gamma, c_e, c_f, \delta\}$ to jointly match the following target data moments in the model's steady state: (i) the unemployment rate, (ii) the employment-employment (EE) job transition rate, (iii) the unemployment-employment (UE) rate, (iv) the elasticity of the UE rate with respect to the vacancy-employment ratio, (v) the firm entry rate, (vi) average firm size, and (vii) the young firm rate.[21]

I apply the simulated method of moments (SMM) which minimizes the following objective function over the parameter space Θ:

$$\min_{\Theta} \sum_{i=1}^{7} \left(\frac{M_i^{model}(\Theta) - M_i^{data}(\Theta)}{0.5(M_i^{model}(\Theta) + M_i^{data}(\Theta))} \right)^2,$$

which is the sum of squared percentage distances between the model-simulated moments $\{M_i^{model}(\Theta)\}_{i=1}^7$ and their counterpart moments in data $\{M_i^{data}(\Theta)\}_{i=1}^7$.

Although the parameters are jointly calibrated, in the following I discuss the most relevant moment for each parameter. The unemployment insurance b is set to match the average BLS

[21] The EE rate is defined as the share of employed workers who transition to a new job in the next period, the UE rate is defined as the share of unemployed workers who find a job in the next period, and the young firm rate is the share of firms aged five year or less in total firms.

Table 1.2: Internally Calibrated Parameters

Parameter	Definition	Value	Targets	Data	Model
b	Unemployment insurance	0.50	Unemployment rate	0.061	0.069
λ	Relative on-the-job search efficiency	0.90	EE rate	0.033	0.032
c	Vacancy cost	0.54	UE rate	0.244	0.296
γ	CES matching function parameter	0.78	Elasticity of UE rate w.r.t. θ	0.720	0.674
c_e	Entry cost	18.57	Firm entry rate	0.089	0.089
c_f	Fixed operating cost	0.78	Average employment size	23.04	22.40
δ	Exogenous death shock	0.01	Share of young firms	0.365	0.332

Notes: Target moments are based on literature and the author's calculation with the BLS, BDS, and J2J data.

quarterly unemployment rate. The relative on-the-job search efficiency λ is used to match average poaching separations as share of employment (the EE rate), as measured using the Census Job to Job flows database (J2J), which is a public version of the LEHD. To be consistent with the model, only hires with no observed interim nonemployment spell (so-called within-quarter job-to-job transitions) are used to define the EE rate.[22] The vacancy cost c is used to target the fraction of unemployed workers who are hired in a quarter (the UE rate), which is calculated from BLS data as the average ratio of unemployment-to-employment flows relative to total unemployment. The CES matching function parameter γ is set to target an elasticity of unemployed workers' job-finding rate with respect to labor market tightness of 0.72, following Shimer (2005). The firm entry rate, average employment size, and the young firm rate are calculated from the Business Dynamics Statistics (a public version of the LBD) and are targeted to calibrate the entry cost c_e, the operating fixed cost c_f, and the exogenous death shock δ, respectively.

Note that the target moments have mixed frequency in the data. The job flow moments and unemployment rate are measured using quarterly data, while the moments regarding firm dynamics are estimated using annual data. I calculate model moments using model data at the same frequency as the data counterparts.

[22]This variable is named "EEHire" in the J2J database. Note that the J2J data only begins in 2000Q2. I target the average of "EEHire" between 2000Q2 and 2014Q4.

Table 1.3: Implications of Uncertainty

Description	Baseline ($\sigma_\epsilon = 0.47$)	High Uncertainty ($\sigma_\epsilon = 0.58$)	% Changes
Firm entry rate (%)	8.93	8.19	-8.29%
Share of young firms (%)	33.22	32.54	-2.05%
Olley-Pakes covariance	0.50	0.46	-7.84%
Aggregate productivity	1.07	0.95	-11.21%
Low performing firm share (%)	13.89	23.44	+68.75%
Average job filling rate (%)	72.10	70.89	-1.68%
Welfare	72.33	69.93	-3.32%

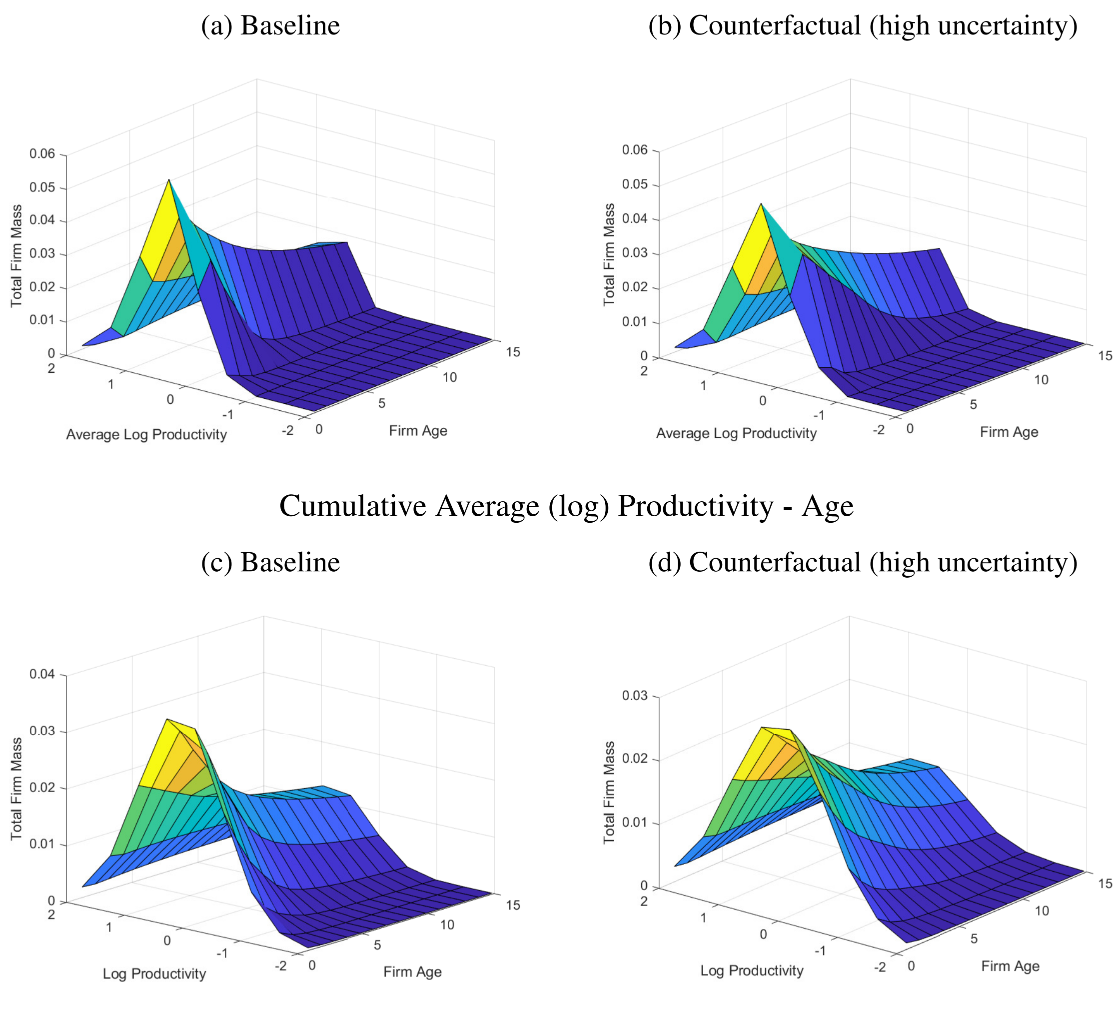
1.5.2 Aggregate Implications

In this section, I conduct a counterfactual analysis to draw out the aggregate implications of the job prospects mechanism, by changing the variance of productivity shocks σ_ε. From the baseline economy in which $\sigma_\varepsilon = 0.47$, I increase σ_ε to 0.58 (a one standard deviation increase) in the counterfactual economy.[23] Having a higher σ_ε implies slower learning due to higher noise in the economy. Therefore, there is higher uncertainty surrounding young firms.

The quantitative counterfactual analysis adds to the analytic results on the impact of uncertainty. First, as uncertainty rises, the wages offered by high performing firms to both unemployed and employed workers increase, while those offered by low performing firms decline. This reduces the exit of firms with low average performance. Second, as uncertainty increases, the compensating wage differentials that high performing young firms pay relative to their mature counterparts also increase, provided the mature firms are old enough. This result is closely related to the previous Proposition 5 and Corollary 2, where the age effects on job prospects are amplified with higher uncertainty.

Table 1.3 shows how changes in σ_ε affect macroeconomic variables at the steady state. As

[23]The standard deviation of σ_ε estimated in the LBD is approximately 0.11.



(log) Current Productivity - Age

Figure 1.2: Joint Distribution of Firm Productivity and Age

there is higher uncertainty about firm type in the economy, the firm entry rate and the startup share of employment decrease. Furthermore, resources are reallocated toward low performing firms and away from high performing firms, as indicated by the lowered covariance between firm size and productivity as in Olley and Pakes (1996). Therefore, aggregate productivity is decreased.

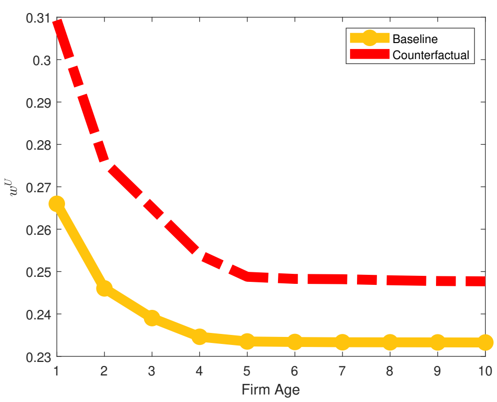
Figure 1.2 shows the equilibrium distribution of firms across productivity and age, comparing the baseline and counterfactual economies. In particular, the two panels on the top illustrate the joint distribution of firms' cumulative average log productivity and age, where the left panel shows the baseline economy and the right panel presents the counterfactual economy. These panels show that the mass of high performing firms (those having average productivity above zero, the cross-sectional prior mean) decreases in the counterfactual economy compared to the baseline, while the counterpart for low performing firms (with the average productivity below zero) increases. The same patterns are featured in the joint distribution of current log productivity and age. In the bottom panels, there is less mass of firms drawing high productivity and more firms drawing low productivity in the counterfactual economy relative to the baseline.

The intuition behind this result is simple. As the speed of learning about firm type slows down, the gap in job prospects between young and mature firms becomes larger. The compensating current wage differentials that high performing young firms need to pay workers at both the hiring and retention margins relative to otherwise similar established firms increase. Figure 1.3 compares wage differentials for high performing young firms between the baseline and counterfactual economies. Note that the wage differentials are more pronounced for high performing young firms in the counterfactual economy. The counterpart wages for low performing firms are shown in Figure A.7.2 in the Appendix.[24]

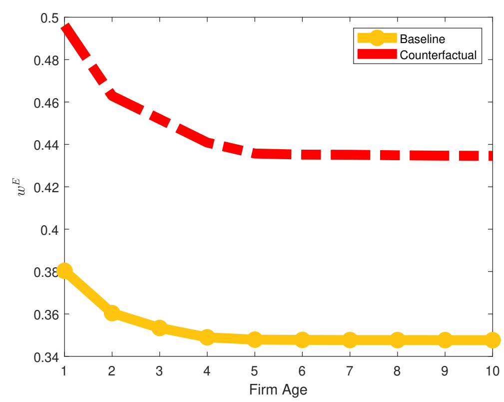
Thus, the growth of high performing young firms is dampened at both the hiring and retention margins, while low performing young firms absorb more workers. This increases the mass of surviving firms with low productivity. Total unemployment goes down, because more firms survive, including potentially bad types, and this induces higher labor market tightness and hiring costs. Consequently, the firm entry rate declines and the activity of young firms with high growth

[24]The wage discounts of young firms persist longer in the counterfactual economy. Furthermore, mature low performing firms no longer exit and continue operating in the counterfactual economy.

(a) Wages to Unemployed Workers



(b) Wages to Poached Workers



(c) Wages to Incumbent Workers

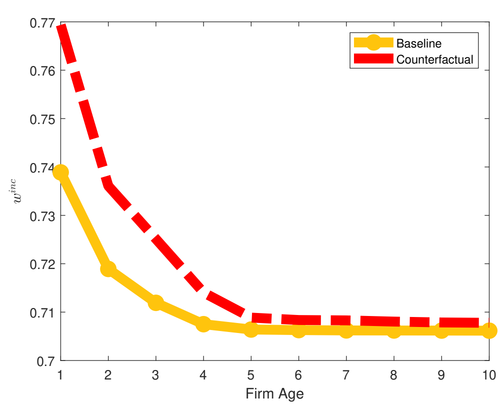


Figure 1.3: High Performing Firms: Baseline vs. Counterfactual (higher uncertainty)

potential is muted. These results suggest that magnified uncertainty about job prospects at young firms can be a source of declining business dynamism and lowered allocative efficiency in the economy.

1.6 Empirical Analysis

In this section, I use detailed firm-level and employee-employer linked data from the U.S. Census Bureau to test the model predictions developed in the previous sections. First, I estimate the firm productivity process in the data and empirically identify the part of productivity from which firms and workers learn about firms' fundamentals. Second, I define high (or low) performing firms as those whose cumulative average productivity is above (or below) the within-industry cross-sectional mean, and test whether young firms with high (low) average performance pay compensating wage premia (discounts) relative to their mature counterparts. Lastly, I measure the degree of uncertainty using the cross-sectional dispersion of firm productivity residuals at the industry level and test the effect of industry-level uncertainty on wage differentials as well as on overall business dynamism.

1.6.1 Data and Measures

I construct a comprehensive dataset containing firm-level measures, worker characteristics, employment records, and earnings using the Longitudinal Business Database (LBD), Revenue-enhanced Longitudinal Business Database (RELBD), and Longitudinal Employer Household Dynamics (LEHD) from 1998 through 2014 for the main analysis.

The LBD tracks the universe of U.S. business establishments and firms that have at least one paid employee. It covers all sectors and geographic areas of the economy annually from 1976 onward. Establishments that are owned by a parent firm are grouped under a common firm identifier, which allows me to aggregate establishment-level activities to the firm level. The LBD contains basic information such as employment, payroll, NAICS codes, employer identification numbers, business name, and location. The RELBD is a subset of the LBD that is merged with income tax filings that contain revenue data. The LBD and RELBD enable me to calculate measures of firm size, age, entry, exit, productivity, and employment growth.[25,26]

The LEHD data is constructed from quarterly Unemployment Insurance (UI) system wage reports of states participating in the program, which collect quarterly earnings and employment information. The data covers over 95 percent of private sector workers, and the length of time series varies across states covered by the LEHD. I have access to 29 states covering over 60 percent of U.S. private sector employment.[27]

The earnings data in the LEHD is reported on a quarterly basis, which include all forms of compensation that are taxable. The LEHD data also contains substantial information about individual characteristics such as gender, education, age, place of birth, race, ethnicity, and history of employment. The data enable me to identify worker heterogeneity, employment history, and job mobility. The UI data, the main source of the LEHD, assigns firms a state-level employer identification number (SEIN) that captures the activity of a firm within a state. The LEHD can be linked to the LBD through a crosswalk between employer identification numbers (EINs) and state-level employer identification numbers (SEINs), which allows me to track employer information for

[25] Jarmin and Miranda (2002) and Chow et al. (2021) contain more detailed information about the LBD, and Haltiwanger et al. (2016) provide more information on the RELBD.

[26] Fort and Klimek (2018) construct time-consistent NAICS codes for LBD establishments after the implementation of a change from the SIC to NAICS in 1997 in the U.S.

[27] The 29 states are AL, AZ, CA, CO, CT, DE, ID, IN, KS, MD, ME, ND, NE, NJ, NM, NV, NY, OH, OK, OR, PA, SD, TN, TX, UT, VA, WA, WI, and WY.

each job.

1.6.1.1 Firm Characteristics and Identifiers

Following Haltiwanger et al. (2013), I define firm age as the age of the oldest establishment that the firm owns when the firm is first observed in the data.[28] I label firms aged 5 years or below as young firms. Firm size is measured as total employment.[29] Lastly, I measure firm-level productivity as the log of real revenue per worker in the RELBD, which is normalized to 2009 U.S. dollars.[30]

One limitation of the LBD is the lack of longitudinally consistent firm identifiers. Although the redesigned LBD has a new firm identifier that links firms across time by correcting previous firm identifiers that are recycled in the old LBD, as described in Chow et al. (2021), it is still not yet a true longitudinal identifier.[31] However, longitudinal consistency of firm identifiers is necessary for my analysis to track firms' history of performance as well as to estimate noise components in firm type learning process. Therefore, I construct and use longitudinal firm identifiers from the LBD following Dent et al. (2018). Henceforth, I will use the term "firm identifier" to refer to the longitudinal firm identifiers constructed using this latter method.

1.6.1.2 Firm Type Learning Process

Using firm-level revenue productivity, I estimate a firm type learning process in my data. First, I take the deviation of firm-level log revenue productivity from its industry-year mean, and

[28] To be precise, I use firm age variables constructed by the Census using the method of Haltiwanger et al. (2013), which enables me to obtain firm age for the whole sample period and avoid any left-censoring issue.

[29] As a robustness check, I measure size using payroll.

[30] The revenue per worker is highly correlated with TFPQ within industries.

[31] The new firm identifiers have not yet resolved firm reorganization issues. See the discussion in Chow et al. (2021).

project the demeaned log productivity on its own lag. Thus, I estimate the following regression:

$$\ln P_{jt} \; = \rho \ln P_{jt-1} \; + \; \nu_j \; + \; \varepsilon_{jt}, \tag{1.35}$$

where $\ln P_{jt}$ refers to the log real revenue productivity for firm j demeaned at the industry-year level, and ν_j is a firm-level fixed effect. I include the lag term $\ln P_{jt-1}$ to factor out the productivity persistence observed in the data.[32] Removing industry-year means controls for the effects of fundamental industry-specific differences in technology or production processes as well as time trends or cyclical shocks.

The underlying assumption is that firms and workers can observe the industry-by-time means as well as the persistence in the firm-level productivity process, and filter these out when estimating the firm's fundamental. Therefore, they infer a firm's type using the remaining terms, which reflect the firm-level fixed effect ν_j and the residual ε_{jt}. This is the term that I map into the model productivity estimates, which I denote henceforth as $\ln \hat{P}_{jt}$, i.e. $\ln \hat{P}_{jt} \equiv \hat{\nu}_j \; + \; \hat{\varepsilon}_{jt}$. Then, I define noise in the learning process as the variance of the estimated residual $\hat{\varepsilon}_{jt}$ from (1.35).

Next, I construct average productivity ($\tilde{P}_{jt-1}$) over the firm life-cycle for each firm using the productivity estimates ($\hat{P}_{jt}$) and longitudinal firm identifiers. To do so, I restrict the sample to firms that have consecutively non-missing observations of $\ln \hat{P}_{jt}$ from their birth. I define $\tilde{P}_{jt-1}$ as follows:

$$\tilde{P}_{jt-1} \equiv \frac{\sum_{\tau=t-a_{jt}}^{t-1} \ln \hat{P}_{j\tau}}{a_{jt}}, \tag{1.36}$$

[32]This is a dynamic panel data model that includes the lagged dependent variable. To resolve the potential endogeneity bias, I adopt the Blundell and Bond (1998) Generalized method of moments (GMM) estimator.

where a_{jt} is the age of firm j in year t. I use $\ln \hat{P}_{jt}$ and $\tilde{P}_{jt-1}$ in my regression analysis below, where these measures represent the current productivity level and average productivity in the data as in the model.

Lastly, I indicate high performing firms as those having average productivity above the within-industry cross-sectional mean of firm-level estimated prior mean productivity:

$$\mathbb{I}^H_{jt} \equiv \begin{cases} 1 & \text{if} \quad \tilde{P}_{jt-1} > \frac{\sum_{j \in G(j,t)} \hat{\nu}_j}{N_{G(j,t)}} \\ 0 & \text{otherwise} \end{cases}, \tag{1.37}$$

where $N_{G(j,t)}$ is the number of firms in industry $G(j,t)$ in a given year t.[33]

1.6.1.3 Uncertainty Measure

I define the degree of uncertainty in the economy using the estimated parameters from (1.35). I estimate the within-industry cross-sectional dispersion of $\hat{\varepsilon}_{jt}$ and the fixed effect estimates $\hat{\nu}_j$, respectively, on a yearly basis. I denote these estimates by $\hat{\sigma}_{\varepsilon gt}$ and $\hat{\sigma}_{0gt}$, respectively, for each industry g, and I use the ratio of the former to the latter to measure industry-level uncertainty in period t, as follows:

$$Uncertainty_{gt} \equiv \frac{\hat{\sigma}_{\varepsilon gt}}{\hat{\sigma}_{0gt}}. \tag{1.38}$$

This measure is known as the "noise-to-signal" ratio in the literature. Note that the denominator can be translated into the initial dispersion of firm fundamentals, which captures the

[33] As a robustness check, I also use different thresholds to define high performing firms, such as the within-industry cross-sectional median or the within-industry-cohort mean of the estimated prior mean productivity.

informativeness of signals in each industry. The ratio (1.38) indicates the degree of uncertainty conditional on this fundamental dispersion, to take into account the differences of the informativeness of signals across industries observed in the data.

1.6.1.4 Jobs and Earnings

The LEHD is at the quarterly frequency and defines a job as the presence of an individual-employer match, with earnings defined as the amount earned from that job during the quarter. Following previous studies, a main job is defined for each worker-quarter pair as the job with the highest earnings. However, the LEHD does not record the start and end dates of a job, which makes the total number of weeks during that quarter unknown. Many researchers have used only full-quarter jobs in their analysis to avoid potential bias from this limitation. A full-quarter job is defined as an employer-employee match that has positive earnings consecutively in the previous, current, and following quarters.

I restrict my analysis to full-quarter main jobs that give the highest earnings in a given quarter and are present for the quarter prior to and the quarter after the focal quarter. Even after this restriction, there are some worker-quarter pairs that are associated with multiple jobs paying the same earnings. For these observations, I pick the job that shows up the most frequently in the worker's job history. This leaves one main job observation for each worker-quarter pair. All of my analysis is based on the log real quarterly earnings associated with these jobs, normalized to 2009 U.S. dollars.

1.6.2 Baseline Two-Stage Earnings Regression

The model predicts that equilibrium wages differ across firms depending on firm age and the firm's history of performance. Firm age and observable performance history are the two sufficient statistics characterizing job prospects at the firm, and these are the main sources generating wage differences across firms in the model. Based on the proposed model mechanism, we should expect to see wage premia for high performing young firms and wage discounts for low performing young firms, compared to their mature counterparts with similar current and average productivity, holding other characteristics fixed.

To test these model implications, I regress earnings on a young firm indicator, a high performing firm indicator, and their interaction, controlling for worker fixed effects along with time-varying worker characteristics, a measure of workers' previous employment status (to control for workers' outside options), firm-level observable characteristics, and fixed effects for time and industry.

Note that the theoretical model abstracts from ex-ante worker heterogeneity, although ex-post heterogeneity still exists in the model depending on workers' previous employment status, which affects wage offers provided by potential employers. In other words, whether the worker was hired from unemployment or poached from an existing job matters for their current wage, as does how much they were paid at the previous job. Prior job status matters for current wages regardless of the current firm's unobserved fundamentals or observed performance. Thus, I control for either the previous employer's firm fixed effect or the worker's previous earnings at the previous employer. I also control for an indicator for whether a worker was unemployed in the previous period.

Using these controls, I can estimate how firms pay workers across different ages and how the required compensation depends on workers' job prospects at the firm (as characterized by firm age and average productivity), all else equal.

I operationalize my empirical strategy using a two-stage regression. In the first stage, I use workers' full-quarter earnings in a given year and estimate worker and year fixed effects, controlling for time-varying worker characteristics. I subtract these fixed effects to get earnings residuals. I estimate this baseline regression at the worker-year level, using Q1 full-quarter earnings as the observation for each year.[34]

In the second stage, I regress the earnings residuals on the young firm indicator, the high performing firm indicator and their interaction, controlling for the worker's previous employment status and the current firm's time-varying characteristics, demeaned by industry and state. As a baseline, I control for the worker's previous employment status, using the AKM firm fixed effect estimate for the previous employer and a dummy indicating if the worker was not employed in the previous period.[35]

1.6.2.1 Stage 1: Estimating Earnings Residuals

In the first stage, I estimate earnings residuals controlling for worker and year fixed effects along with worker age, as follows:

$$y_{it} = \delta_i + \eta_t + X_{it}\gamma + \epsilon_{it}, \tag{1.39}$$

[34] Following the literature, I use the first quarter value of the full-quarter main jobs as a baseline. As robustness checks, I also use the second, third, and fourth quarter values.

[35] The AKM firm fixed effect is the firm fixed effect obtained from estimating the standard two-way fixed-effect framework in my data, following Abowd et al. (1999).

where y_{it} is the logarithm of the Q1 earnings of individual i in year t, δ_i is a time-invariant individual effect, η_t is a year effect, and X_{it} is a vector of controls for individual age, using quadratic and cubic polynomials centered around age 40, following Card et al. (2016), Crane et al. (2018), and Haltiwanger et al. (2021). In order to estimate the fixed effects, I implement the iterative algorithm proposed by Guimaraes and Portugal (2010). The algorithm helps to estimate a model with high-dimensional fixed effects without explicitly using dummy variables to account for the fixed effects.

1.6.2.2 Stage 2: Wage Differentials across Firm Age and Performance

In the second stage, I use the estimated earnings residuals from (1.39) as the main dependent variable. I regress the earnings residuals $\hat{\epsilon}_{it}$ on the young firm dummy and the high performing firm dummy in (1.37), along with their interaction. Recall that the high performing firm dummy in year t is based on the firm's historical productivity through period $t-1$.

Following the discussion above, I control for workers' previous employment status. I identify workers' previous jobs, which are also restricted to full-quarter main jobs. Following Haltiwanger et al. (2018), I can identify workers' previous job using a within/adjacent quarter approach, which allows for a brief nonemployment period between workers' last day on the previous job and their first day on the contemporaneous job. Therefore, workers are identified as previously employed if they had at least one full-quarter job within the most recent three quarters before t, while they are identified as non-employed if they had no full-quarter jobs within those three quarters.[36] For those workers previously employed before period t, their previous job is

[36]Note that restricting the sample to full-quarter main jobs makes use of the three-quarter duration to define previous jobs. For notational convenience, let $(t-q1)$ denote the quarter prior to t, and $(t-q2)$ denote two quarters prior to t, and so on. If a worker had any full-quarter jobs at either $(t-q1)$ or $(t-q2)$, this implies that the worker

identified as the most recent full-quarter main job within the three most recent quarters before t, and the employer of that job is denoted by $j(i, t-1)$.

Next, I run a standard two-way fixed-effect regression model as in Abowd et al. (1999) to estimate firm fixed effects on earnings.[37] I control for the fixed effect for the firm where worker i was employed in the previous period, i.e. $j(i, t-1)$ defined as above. For workers who were non-employed in the previous period, i.e. those who had no full-quarter earnings in any of the most recent three quarters before t, I set their previous employer fixed effect to zero and include a dummy variable for nonemployment.

Equation (1.40) presents the second stage regression, where the main coefficients of interest are β_1 and β_2, which capture the earnings differentials associated with young firms depending on their average performance.

$$\begin{aligned} \hat{\epsilon}_{it} \;&= \beta_1 Young_{j(i,t)t} \;+\; \beta_2 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} \;+\; \beta_3 \mathbb{I}^H_{j(i,t)t} \;+\; Z_{j(i,t)t}\gamma_1 \;+\; Z_{j(i,t-1)}\gamma_2 \\ &\quad + \; \mu_{G(j(i,t))} \;+\; \mu_{S(j(i,t))} \;+\; \alpha \;+\; \xi_{it} \end{aligned} \tag{1.40}$$

The regression is at the worker-year level, where $\hat{\epsilon}_{it}$ is the earnings residual of worker i in a given year t, $j(i, t)$ is the employer where worker i is employed at t, $Young_{j(i,t)t}$ is the young firm indicator for firm $j(i, t)$, $\mathbb{I}^H_{j(i,t)t}$ is the high performing firm indicator for firm $j(i, t)$, $Z_{j(i,t)t}$ is a

must have moved to the contemporaneous job within quarter $(t - q1)$. The latter could happen if the worker had some overlapping period between $(t - q1)$ and t in job transition. If a worker had any full-quarter jobs at $(t - q3)$, this means that the worker must have left the job at $(t - q2)$, had a brief nonemployment period between $(t - q2)$ and $(t - q1)$, and joined the contemporaneous job at $(t - q1)$. Alternatively, the within quarter approach identifies workers as previously employed if they had at least one full-quarter job within the latest two quarters before t, where the previous job is defined by the most recent main full-quarter job within the most recent two quarters before t.

[37] Note that the baseline fixed effect is estimated at the SEIN level. As a robustness check, I also use the fixed effects estimated at the (longitudinally consistent) firm identifier level.

vector of controls for time-varying properties of firm $j(i,t)$, and $Z_{j(i,t-1)}$ is a vector of controls for the worker's employer in the previous period. To be consistent with the model, I include average productivity, current productivity, and employment size of firm $j(i,t)$ in $Z_{j(i,t)t}$. For $Z_{j(i,t-1)}$, as a baseline, I use the AKM firm fixed effect associated with the worker's previous employer along with the non-employment indicator. Lastly, the regression includes industry fixed effects $\mu_{G(j(i,t))}$ and state fixed effects $\mu_{S(j(i,t))}$, where $G(j(i,t))$ is the industry that the firm belongs to and $S(j(i,t))$ is the state where the firm is located.

Note that the firm variables have the same values across all workers employed at that firm at t (i.e., workers employed at the SEINs associated with the same firm identifier). The novelty in (1.40) comes from the coefficients β_1 and β_2, which capture how firms with a given set of observable characteristics pay differently by firm age, and how the effect of age on wages depends on the firm's history of performance.

Table 1.4 presents the regression results with the full set of controls, which are consistent with the model predictions. For the sake of space, I only present the main coefficients; the full results can be found in Table H1 in Appendix. The first column controls for the current value of firm size and the second column uses the lagged value of firm size. Both columns include industry (NAICS6) and state fixed effects.

According to the model, there are wage differentials associated with young firms that depend nonlinearly on firms' historical performance: young firms pay more (less) than their mature counterparts only if these firms' average productivity is above (below) the industry cross-sectional mean. The underlying mechanism is that workers are not confident about young firms' fundamentals due to their limited information, so that their beliefs are less precise and more uncertain.

Table 1.4: Wage Differentials for Young Firms

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.002***	-0.003***
	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.016***
	(0.001)	(0.001)
High performing firm	0.002	0.002
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State
Controls	Full (current size)	Full (lagged size)

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

In the regression, the impact of being a young firm on earnings depends on β_1 and $\beta_2\mathbb{I}^H_{j(i,t)t}$, and the total impact depends on whether the observed average productivity $\tilde{P}_{j(i,t)}$ is below or above the industry mean. For low performing firms, the wage differential for young firms is given by β_1. For high performing firms, the wage differential for young firms is given by $\beta_1 + \beta_2$. Table 1.4 shows that $\hat{\beta}_1 < 0$, $\hat{\beta}_2 > 0$, and $\hat{\beta}_1 + \hat{\beta}_2 > 0$, where all of these point estimates are statistically significant. The results indicate that high performing young firms pay more than their otherwise similar mature counterparts, while low performing young firms pay less. This finding supports the model prediction about young firms' wage differentials attributed to workers' uncertain job prospects.

1.6.3 Robustness Checks

To further confirm the validity of the baseline results, I perform several robustness checks, with results reported in Appendix A.9.

1.6.3.1 Firm Size Effects

The baseline regression controls for firm size. However, firm size is highly correlated with firm age, and the firm size distribution varies by different firm age. For instance, most young firms are small in the U.S. economy. This may cause the size covariate to absorb firm age effects in the main regression. To check this possibility, I run regressions without controlling for firm size and using different sets of firm controls. Results are similar to the benchmark case, as shown in Appendix Table I3.

1.6.3.2 Correcting Sample Selection Bias

Another potential source of bias in the current analysis is sample selection. The current sample is drawn from the population of U.S. firms having consecutively non-missing observations of revenue data in the RELBD, and workers matched with these firms. Therefore, the sample drops firms with missing revenue data. To mitigate the possible resulting selection bias, I estimate a propensity score model using logistic regressions and weight the baseline regression sample with inverse propensity score weights. I use logistic regressions with a dependent variable equal to one if the firm belongs to the current sample and zero otherwise, along with firm characteristics such as firm size, age, employment growth rate, industry, and a multi-unit status indicator from the universe of the LBD. Using inverse probability weights calculated from the predicted values from the logistic regression, I weight the sample and rerun the regressions. The results are documented in Appendix Table I4, and are similar to the baseline results.

1.6.3.3 Standard Error Bootstrapping

The high performing firm indicator as well as firm control variables in the second-stage regression are constructed based on estimates from the regression in (1.35). This might cause the reported standard errors in Table 1.4 to be incorrect. To address this, I estimate the standard errors with bootstrapping and check the robustness of the results. To do so, I draw 5000 random samples with replacement repeatedly from the main dataset, estimate the main coefficients corresponding to these bootstrap samples, form the sampling distribution of the coefficients, and calculate the standard deviation of the sampling distribution for each coefficient. Appendix Table I5 lays out the results. The statistical significance of the main coefficient estimates stays robust across all columns.

1.6.3.4 Workers' Previous Employment Status

In the current specification, I control for workers' previous employment status using the AKM firm fixed effect for the previous job. Note that using the AKM fixed effect might be a conservative way to control for the worker's previous employment status. In the model, the equilibrium wages at both the hiring and retention margins (eventually) only depend on whether workers came from the unemployment pool, from an employer that wanted to expand or stay inactive, or from an employer that cut back on their size. Thus, another potential proxy to control for the worker's previous job and their place on the job ladder in the previous period would be the earnings associated with the previous employer (still also controlling for the non-employment status indicator).

Another reason for using earnings on the previous job is to control for workers' unobserved time-varying characteristics, which could be associated with the current employer's age or average performance regardless of the job prospects channel. For instance, high performing young firms might demand workers with more experience than their mature counterparts given the burden of training costs. Therefore, even controlling for where workers came from, high performing young firms might hire workers with longer tenure (which is highly correlated with earnings) at their previous employer. This scenario could provide an alternative interpretation of the main finding of earnings premia associated with high performing young firms, which has nothing to do with the uncertain prospects provided by the firms.

The results controlling for earnings on the previous job are shown in Appendix Table I6, where the first three columns replace the AKM firm fixed effect with the worker's previous earnings, and the next three columns use both variables to control for the worker's previous employment status properly. The baseline results stay robust in all cases.

1.6.4 The Impact of Uncertainty on Wages and Aggregate Outcomes

This section tests the model's implications about the effect of uncertainty on young firm earnings differentials and macroeconomic outcomes related to business dynamism.

1.6.4.1 Cross-sectional Implications on Wage Differentials

In the model, higher uncertainty drags out the speed of learning for young firms, as observable firm performance becomes less informative about fundamentals. Thus, higher noise implies that more time is required to obtain enough information to gain a given level of confidence. This can have potentially adverse effects on young firms with high potential, whose performance in each period has a high marginal effect on beliefs and wage differentials.

In order to test this implication, I add additional interaction terms involving the industry-level uncertainty measure (1.38) to the baseline earnings regression, as follows:

$$\begin{aligned}\hat{\epsilon}_{it} &= \beta_1 Young_{j(i,t)t} \ + \ \beta_2 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} \ + \ \beta_3 Young_{j(i,t)t} \times Uncertainty_{G(j,t)t-1} \\ &\quad + \ \beta_4 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} \times Uncertainty_{G(j,t)t-1} \ + \ \beta_5 Uncertainty_{G(j,t)t-1} \\ &\quad + \ \beta_6 \mathbb{I}^H_{j(i,t)t} \times Uncertainty_{G(j,t)t-1} \ + \ \beta_7 \mathbb{I}^H_{j(i,t)t} \ + \ Z_{j(i,t)t}\gamma_1 \ + \ Z_{j(i,t-1)}\gamma_2 \\ &\quad + \ \mu_{G(j(i,t))} \ + \ \mu_{S(j(i,t))} \ + \ \alpha \ + \ \xi_{it}, \end{aligned} \tag{1.41}$$

where $Uncertainty_{G(j,t)t-1}$ is the lagged value of the uncertainty measure in (1.38) for the main industry that firm $j(i,t)$ is associated with in year t. Note that I use lagged values to avoid any potential reverse causality. In this case, I define $\mu_{G(j(i,t))}$ as sectoral rather than industry fixed effects, where each sector is defined at the NAICS2 level. The regression can help capture how the wage differentials associated with high performing young firms vary across different industries with different levels of uncertainty.

As before, Table 1.5 displays the main regression results, using the current and lagged value of firm size in the first and second columns, respectively. The full results are provided in

Table 1.5: The Effect of Uncertainty on Young Firms' Wage Differentials

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.001	-0.002
	(0.002)	(0.002)
Young firm × High performing firm	0.003	0.005**
	(0.002)	(0.002)
Young firm × Uncertainty	-0.005**	-0.004*
	(0.002)	(0.002)
Young firm × High performing firm × Uncertainty (at $t-1$)	0.016***	0.015***
	(0.003)	(0.003)
Observations	50,170,000	50,170,000
Fixed effects	Sector, State	Sector, State
Controls	Full (current size)	Full (lagged size)

Notes: The table reports results for regression of earning residuals on young firm, high performing firm indicators, and the uncertainty measure. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect and a dummy for non-employed workers in the previous period, associated with the previous employer to capture time-varying components. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, sector and state fixed effects, and the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Appendix Table H2. The table shows that the coefficient estimate of the triple interaction term between the young firm indicator, the high performing firm indicator and the uncertainty measure is positive, i.e. $\beta_4 > 0$, which is consistent with the model prediction that higher uncertainty increases the wage premium that high performing young firms need to pay. Furthermore, the coefficient estimate for the interaction between the young firm indicator and the uncertainty measure is negative, i.e. $\beta_3 < 0$, which is consistent with the model result that the wage discount for low performing young firms gets larger as uncertainty rises.

1.6.4.2 Macroeconomic Implications

Next, I test the model's predictions regarding the effect of uncertain job prospects on overall business dynamism and aggregate productivity. The calibrated model predicts that more uncer-

Table 1.6: Aggregate Implications of Uncertainty

	(1) Entry rate	(2) Young firm share	(3) Young firm emp. share
Uncertainty (at $t-1$)	-0.011***	-0.050***	-0.021***
	(0.004)	(0.008)	(0.008)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year
	(4) HG young firm share	(5) HG young firm emp.share	(6) HG young firm avg. emp. growth
Uncertainty (at $t-1$)	-0.028***	-0.012***	-0.029***
	(0.005)	(0.003)	(0.008)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year

Notes: The table reports results for regression of young firm activities in each column on the lagged value of the uncertainty at the industry level. Observation counts are rounded to the nearest 100 to avoid potential disclosure risks. Estimates for constant, industry and year fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

tainty will reduce firm entry and young firm activity by slowing down learning and selection. To test these predictions, I estimate the following industry-level regression:

$$BusinessDynamism_{gt} = \beta Uncertainty_{gt-1} + \delta_g + \delta_t + \epsilon_{gt}, \tag{1.42}$$

where $BusinessDynamism_{gt}$ is either the firm entry rate, the share of young firms, the share of high-growth young firms, the employment share of young firms, the employment share of high-growth young firms, or the average employment growth rate of high-growth young firms at the industry level (industry g) in a given year t. Here, high-growth firms are those above the 90th percentile of the industry employment growth distribution, and high-growth young firms are high-growth firms aged five years or less. Industry is defined at the NAICS4 level. As before, I use lagged values of uncertainty. The regression takes out industry and year fixed effects, δ_g and δ_t, respectively.

Table 1.6 displays the results. Uncertainty has a negative and significant impact on the firm entry rate, as well as the share and employment share of young firms and high-growth young firms. The employment growth rate of high-growth young firms is also dampened when uncertainty rises. This shows that there is a negative association between uncertainty and business dynamism at the industry level, which holds consistently across different measures of young firm activity.

Next, I examine whether trends in industry-level uncertainty can account for the aggregate trend of declining business dynamism observed in recent decades. I extract year fixed effects for the measures of uncertainty and business dynamism using the following regressions:

$$BusinessDynamism_{gt} = \delta_g^{BD} + \delta_t^{BD} + \alpha^{BD} + \varepsilon_{gt}^{BD} \tag{1.43}$$

$$Uncertainty_{gt} = \delta_g^{U} + \delta_t^{U} + \alpha^{U} + \varepsilon_{gt}^{U}, \tag{1.44}$$

where δ_g^{BD} and δ_g^{U} are industry fixed effects, and δ_t^{BD} and δ_t^{U} are year fixed effects, for each measure of business dynamism and uncertainty, respectively.

Let $\hat{\delta}_t^{BD}$ and $\hat{\delta}_t^{U}$ denote the year fixed effects estimated from (1.43) and (1.44), respectively, which capture the aggregate time trends of business dynamism and uncertainty common to all industries. Next, I run the following regression to capture how the common year effect of uncertainty is associated with the common year effect of business dynamism:

$$\hat{\delta}_t^{BD} = \beta \hat{\delta}_{t-1}^{Uncertainty} + \alpha + \epsilon_t, \tag{1.45}$$

where I use the lagged value of the uncertainty year effect as before.

Table 1.7: Correlation between Year Fixed Effects of Uncertainty and Business Dynamism

	(1) Entry rate	(2) Young firm share	(3) Young firm employment share
Uncertainty (at $t-1$)	-0.083***	-0.215*	-0.144**
	(0.022)	(0.115)	(0.067)
Observations	20	20	20
	(4) High-growth young firm share	(5) High-growth young firm employment share	(6) High-growth young firm average employment growth
Uncertainty (at $t-1$)	-0.086***	-0.021*	-0.180***
	(0.026)	(0.010)	(0.051)
Observations	20	20	20

Notes: The table reports results for regression of year fixed effects associated with young firm activities in each column on lagged year fixed effects of the industry-level uncertainty. Observation counts are rounded to the nearest 10 to avoid potential disclosure risks. Estimates for constant are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

The results are shown in Table 1.7. In each column, I find a negative and statistically significant effect of the uncertainty year effects on the business dynamism year effects.

Overall, the evidence in this section suggests that higher uncertainty (relative to the initial informativeness of firm fundamentals) can hamper young firm activity, particularly for high-growth young firms, and can reduce business dynamism at the aggregate level, which is consistent with the model's macroeconomic implications.

1.7 Conclusion

In this paper, I study how workers' job prospects affect the potential growth of young firms, as well as overall young firm activity and aggregate productivity. I propose that workers' uncertain prospects at young firms can create hiring frictions for young firms with high potential by increasing wages and hiring costs. This paper develops a rich theoretical framework to understand wage differentials at young firms, and how such differentials create a barrier for high-potential young firms to grow.

Through this mechanism, the counterfactual analysis implies that rising uncertainty about young firms can dampen the growth of high performing young firms, redirect labor inputs to low performing young firms, and reduce business dynamism in the economy. This mechanism operates in part through higher market tightness as well as rising overall hiring costs caused by the increase in uncertain prospects about young firms. Overall, the model can provide a basis for understanding young firm wage dynamics and growth, generated by the inherent nascency and uncertainty surrounding young firms.

Furthermore, this paper finds empirical evidence consistent with the model implications using micro-level administrative data from the U.S. Census Bureau. Guided by the model, I identify the part of inter-firm earnings differentials for young firms that can be attributed to workers' learning and uncertain job prospects. Consistent with the model, I find that high (low) performing young firms need to pay higher (lower) earnings relative to their observationally identical mature counterparts. Moreover, I measure industry-level uncertainty using the cross-sectional dispersion of innovations to firm productivity, and find that the earnings differentials of young firms increase in industries having higher level of uncertainty. Lastly, I construct a set of industry-level business dynamism measures and regress them on industry-level uncertainty, and find a negative association between uncertainty and business dynamism at the industry-by-time level. In particular, I find that higher uncertainty hampers firm entry and (high-growth) young firm activity, which is consistent with the model's macroeconomic implications.

To the best of my knowledge, this paper is the first to offer a theory as well as evidence about how workers' uncertain job prospects generate wage differentials for young firms relative to otherwise similar mature firms. The paper develops a rich structural model that provides a novel channel that endogenously generates wage differentials for young firms that depend on firms' average productivity. Furthermore, the paper quantifies and highlights the importance of this channel by drawing out its implications for aggregate business dynamism and productivity.

There are several directions in which the current work can be extended. First, one could incorporate aggregate shocks to explore how the job prospects channel affects the cyclical dynamics of young firms. It is well documented that young firm activity is procyclical and that young firms are hit harder than mature firms during recessions. The main mechanism of my paper can potentially provide an explanation for the differential cyclicality of young firm activity. If the economy is in a downturn and there is higher uncertainty about finding a job, the costs of job loss should increase. Thus, workers may value stability more in recessions, which would reduce the value of working at young firms in general. In addition, if the degree of noise rises in recessions, learning would slow down and the uncertain prospects at young firms could be magnified. Thus, the wage differentials of young firms could be more pronounced in recessions. I find preliminary empirical evidence that supports this hypothesis.

Furthermore, one could introduce risk aversion and worker heterogeneity to the model. The current model abstracts from risk aversion components for tractability. Once risk aversion is added, there will be another source of wage differentials for young firms attributed to risk premia. Also, workers may have different outside options or opportunity costs that would affect their willingness to work at young firms. These differences could be correlated with worker age or skill.[38] Considering worker heterogeneity could enrich our understanding of the job prospects channel and its impact on potential young firm growth and business dynamism through the effects of sorting between firms and workers. This mechanism could potentially create a linkage between

[38]For instance, younger workers have more potential job opportunities in the event that their current job is destroyed and can be more risk tolerant of taking a job.

workforce composition and business dynamism in the economy. These extensions are left as interesting avenues for future research.

Chapter 2: Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers[1]

2.1 Introduction

Studies of the effect of competition on firm innovation have a long history, as economists broadly agree that innovation is a major source of economic growth. Researchers have studied the impact of competition on firm innovation within different product markets and across countries in different development stages, both empirically and theoretically. The results, however, are inconclusive. As documented in Gilbert (2006), different market structures, types of innovation and degrees of innovation protection can cause firms' incentive for innovation to move in different and potentially offsetting directions in response to increased competition.

In this paper, we first theoretically investigate the effect of competition on firm innovation by developing a discrete-time endogenous growth model where multi-product firms do two types of innovation—internal and external—subject to imperfect technology spillovers in the form of lagged learning of others' technology, extending the framework in Akcigit and Kerr (2018). Aided by this model, we decompose the overall changes in innovation in response to increasing competition into changes in the level and composition of the two types of innovation. We show that competition can either increase or decrease overall innovation, because i) competi-

[1]This chapter relies on a project co-authored with Karam Jo.

tion affects internal and external innovation differently, and ii) factors such as the innovation cost structure determine the relative changes in the two types of innovation in response to competition. We then provide firm-level regression results consistent with the model predictions.

In the real world, firms are multi-product firms, and they grow by both expanding their existing markets and entering other product markets. Thus, firms' growth paths depend on their product portfolio choices. In our model, we allow multi-product firms to choose their product portfolio through the two types of innovation. Firms use internal innovation to improve their existing product quality (or production processes), and use external innovation to enter new markets outside of their existing product scope and drive incumbent firms out.[2]

Also, in the real world, firms can defend their product markets from competitors by enhancing the quality of their existing products. We show that this channel is important to understanding firm entry and growth. If improving one's own products can be an effective tool to block competitors from either entering or expanding in one's existing product markets, internal innovation would affect not just an individual firm's own growth path but also firm entry within each product market. Nonetheless, existing models assume either that firms have a single product, or that they can't use innovation defensively.

In existing models that allow multi-product firms to grow through product scope expansion (e.g., Klette and Kortum (2004) and Akcigit and Kerr (2018)), firms cannot protect their markets because others can learn and copy the firms' frontier technology immediately without any friction. Thus, firms cannot escape competition by improving their own technology. Other previous frameworks with step-by-step innovation, such as Aghion et al. (2001) and Akcigit et al. (2018),

[2] A real-world example of external innovation is Apple developing the iPhone and getting into the cell phone industry back in 2007, when its major business at that time was computer manufacturing. A real-world example of internal innovation is Apple improving and producing iPhone 11 from iPhone 10.

incorporate certain forms of escaping competition but still assume single-product firms. This lack of realism in existing models limits their ability to account for the effect of competition on firm innovation and growth. To move forward, we allow multi-product firms to defend their product markets through internal innovation by introducing frictions in learning others' technology, which we label as imperfect technology spillovers.

When a firm attempts to enter other firms' markets and take them over through external innovation, it first needs to learn the technology of incumbent firms so that it can then improve on top. Realistically, however, there are barriers to learning others' technology. In our model economy, imperfect technology spillovers take the form of lagged learning, in which it takes one period for potential rival firms to learn incumbent firms' product-specific technology. Thus, internal innovation is built on the current frontier technology, while external innovation is built on the lagged frontier technology. Imperfect spillovers generate a technology gap between the current period frontier technology that incumbent firms have and the one-period lagged technology that potential rival firms can only learn through R&D.

Incumbent firms can use this gap to improve their technology further through internal innovation for defensive reasons, which makes it harder for competitors to catch up with their frontier technology and take over their markets. In other words, incumbent firms can build a technological advantage in their markets. In such an environment, individual firms use internal innovation not only to improve the profitability of their products but also to escape competition. In this sense, our framework brings together quality-ladder innovation models and step-by-step innovation models. The flip side is that defensive internal innovation by incumbents prevents rival firms from taking over other product markets through external innovation, as rivals need to overcome the technological advantage built by incumbent firms in those markets. Rising competition further increases this technological barrier, because more competition incentivizes incumbents to do more internal innovation.

The introduction of imperfect technology spillovers is our key theoretical contribution, and this allows us to distinguish the effect of competitive pressure on internal versus external innovation. In addition, we show that imperfect technology spillovers generate a novel technological barrier effect, in which firms' strategic choice to use defensive internal innovation influences the probability of successful external innovation and business takeover in the economy.

To our knowledge, this is the first paper that constructs a theoretical model of defensive innovation that allows multi-product firms to choose between two different types of innovation. Allowing for both internal and external innovation is important for understanding the effect of competition on firms' strategic innovation decisions, as well as firm-level and aggregate economic growth. Firms have different incentives to invest in two types of innovation, and they use innovation strategically to increase their profits and the probability of survival. Also, Akcigit and Kerr (2018) show that external innovation contributes more than internal innovation to both firm employment growth and aggregate economic growth. Thus, allowing for only one type of innovation, while overlooking potential compositional changes, may disguise the true effect of competition on overall firm innovation.

Our model shows how both types of innovation respond to increasing competition, by decomposing firms' innovation incentives into the following three terms: (i) the escape-competition effect, (ii) the Schumpeterian effect, and (iii) the technological barrier effect. We show that the technology gap, which measures the technological advantage incumbent firms have in their own market and determines their future profit gains from internal innovation, is the key to understanding firms' internal innovation decisions when competition increases. Internal innovation increases firms' expected future profits by improving their own product quality, thus widening the technology gap and lowering the probability of losing their product line to others. Thus, increasing competition induces firms to increase their internal innovation efforts, which is the escape-competition effect. On the other hand, increasing competition due to more firms doing external innovation raises the aggregate probability of losing a product line (the aggregate creative destruction arrival rate). This lowers the expected profits from each product line and discourages firms' internal and external innovation, which is the Schumpeterian effect. Lastly, the more effort incumbent firms put into internal innovation, the higher the average technology gap in the economy and the harder it becomes for competitors to take over incumbent firms' product markets. Thus, a higher average technology gap due to more internal innovation undertaken by incumbents dissuades firms from investing in external innovation. We define this as the technological barrier effect.

Whether firms increase or decrease their internal innovation depends on which of the first two effects is more dominant. We show that the escape competition effect dominates the Schumpeterian effect for firms that have innovated intensively in recent periods and have more existing technological advantages accumulated in their own product markets. Thus, increasing competition motivates innovation-intensive high-growth firms to increase their internal innovation for defensive reasons. These firms become better at protecting themselves from competitors by building technological barriers in their existing product markets.[3] Furthermore, firms' external innovation intensity will unambiguously decrease as a result of rising competition, through both the Schumpeterian and the technological barrier effects.

[3]For example, as of 2020, we hear that Apple is planning to introduce new iPhones more frequently, twice per year, because competition in the cellphone industry has become more intensified.

To test these model predictions empirically, we construct a unique dataset by combining firm-level data from the U.S. Census Bureau with patent data from the United States Patent and Trademark Office (USPTO) from 1976 to 2016. This comprehensive dataset has detailed information for the population of U.S. patenting firms, such as employment, international transactions, and the 6-digit NAICS industries in which each firm operates. We use China's WTO accession in 2001 as an exogenous change in competitive pressure from foreign firms and the patent self-citation ratio as a measure of the likelihood that each patent is used for internal innovation. Using this data, we provide regression results consistent with the model prediction for the escape-competition effect, as higher competition increases internal innovation among firms with existing technology advantages. We also show that the positive association between patenting and employment growth for innovation-intensive firms falls by one-third after competition increases, as more patents are used for internal innovation. Lastly, we find regression results consistent with the model prediction for the technological barrier effect by using changes in foreign patent growth (in other words, the recent innovation activity of other firms) as a measure of an exogenous variation in technological barriers.

To understand the effect of increasing competition on the composition of innovation and the aggregate economy, we calibrate our model to innovative firms in the U.S. manufacturing sector from 1987 to 1997 and perform the following three counterfactual exercises: i) increasing competitive pressure by foreign firms (so that the aggregate creative destruction arrival rate depends in part on foreign firms), ii) increasing competitive pressure by foreign firms in an economy where external innovation costs are much higher than in the U.S., and iii) lowering entry costs (specifically, lower external innovation costs for potential startups). We hold the change in the aggregate creative destruction arrival rate (equivalently, the change in competitive pressure) constant across the three counterfactual exercises, so the change in firm innovation decisions is also identical: incumbent firms undertake more (less) internal innovation for the existing products for which they have a (no) technological advantage, and less external innovation.

However, the counterfactual results differ in terms of aggregate implications. First, comparing the exercises i) and ii), we show that the average firm-level R&D to sales ratio decreases in response to rising foreign competition in the economy calibrated to the U.S., but increases in an economy with higher external innovation costs (with less creativity). In an economy with higher external innovation costs, firms invest fewer resources in external innovation even when competition is less intense. This implies that in such economy, there is very little room for external innovation to decline further with increasing competition. Thus, although the external innovation intensity falls after competition rises, the reduction is more than offset by increased internal innovation undertaken by incumbent firms for defensive reasons. On the other hand, in the economy calibrated to the U.S., firms are more active initially in external innovation. Thus, external innovation decreases more in response to increasing competition, which causes overall innovation to fall.

This result sheds light on the heterogeneous effect of increasing competition on overall innovation across different countries, as documented in Bloom et al. (2016) and Autor et al. (2019), along with our empirical results. This highlights that the change in innovation composition resulting from firms' strategic choices is an important margin to understand the effect of competition on firm innovation.

Exercise iii) shows that incumbent firms' response to increasing competitive pressure remains the same regardless of the source of competition. On the other hand, firm entry responds differently. The mass of domestic startups increases in the case of lowered domestic entry costs, while it decreases in response to a rise in competitive pressure induced by foreign firms. This finding may help researchers to identify the source of rising competitive pressure, in particular whether it results from foreign firms or domestic firm entry.

The rest of the paper proceeds as follows. Section 2.2 develops a baseline discrete-time infinite horizon general equilibrium model. Section 2.3 presents empirical results about the effect of international competition on the composition of firm innovation. Section 2.4 displays results from quantitative analysis of the baseline model. Section 2.5 concludes.

2.2 Baseline Model

In this section, we introduce a discrete time infinite horizon endogenous growth model with multi-product firms, two types of innovation, imperfect technological spillovers, and an exogenous source of competitive pressure. The exogenous competitive pressure can come from firms in foreign countries if we consider the aggregate economy, or from domestic incumbent firms in other sectors or states if we consider a certain sector or state. The baseline model extends Akcigit and Kerr (2018) in three dimensions: i) we impose imperfect technology spillovers by assuming that R&D expenditure on external innovation only allows rivals to learn the incumbent's technology lagged by one period, and by doing so, ii) we introduce the escape-competition effect, in which incumbent firms' internal innovation decision depends on the last period's innovation results, which are summarized by the technology gap—the ratio of the current-period technology $q_{j,t}$ to the last-period technology $q_{j,t-1}$, $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$. Lastly, iii) we allow for exogenous shifts of the aggregate creative destruction arrival rate to analyze the effect of increasing competitive pressure on firms' innovation and growth dynamics.

Hereafter, the time subscript is suppressed whenever there is no confusion. Superscript $'$ is used to denote next period variables at $(t+1)$, and subscript -1 is used for the previous period variables at $(t-1)$. The terms product quality and technology are used interchangeably.

2.2.1 Representative Household

The representative household has a logarithmic utility function and is populated by a measure one continuum of individuals. Each individual supplies one unit of labor each period inelastically and consumes a portion C_t of the economy's final good. Thus, the household's lifetime utility is

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t) \; .$$

Homogeneous workers are employed in the final goods sector (L). Thus in each period, the labor market satisfies

$$L = 1 \; . \tag{2.1}$$

2.2.2 Final Good Producer

The final good producer uses labor (L) and a continuum of differentiated intermediate products indexed by $j \in [0,1]$ to produce a final good. Denote $\mathcal{D}$ as the index set for differentiated products produced by domestic firms. Products with $j \notin \mathcal{D}$ are produced by foreign firms (or domestic incumbent firms in other sectors/states), as discussed later. The constant returns to scale production technology w.r.t. labor and differentiated products can be written as

$$Y = \frac{L^{\theta}}{1-\theta} \left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \in \mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \notin \mathcal{D}\}}\, dj \right],$$

where y_j is the quantity of differentiated product j, q_j is its quality, and $\mathcal{I}_{\{\cdot\}}$ are indicator functions. The final good price is normalized to be one in every period without loss of generality. The final good is produced competitively and input prices are taken as given.

2.2.3 Intermediate Producers

There is a set of measure $\mathcal{F}_d$ domestic firms and a set of measure $\mathcal{F}_o$ foreign firms with $\mathcal{F}_d + \mathcal{F}_o \in (0,1)$, which are determined endogenously in equilibrium, producing differentiated intermediate products each period and selling their products in monopolistically competitive domestic markets. Each differentiated product is produced in the producer's own region using domestic resources. Since each operating firm owns at least one product line, and each product line is owned by a single firm, a firm f can be characterized by the collection of its product lines $\mathcal{J}^f = \{j \,:\, j \text{ is owned by firm } f\}$. Only the owner of each product line can observe and use the product-line specific current period technology (product quality) $q_{j,t}$, where the technology gap between t and $t-1$ is defined as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$. Thus, each product line can be characterized by its quality and technology gap, (q_j, Δ_j). Each differentiated product $j \in [0,1]$ is produced at a unit marginal cost in terms of the final good.

2.2.4 Innovation by Intermediate Producers

Intermediate producers engage in two types of R&D—internal and external—to increase their profits from products they currently produce, to protect their product markets from competitors, to expand their businesses, and to enter new product markets, where the R&D output takes the form of improvements in product quality (equivalently, production technology). Innovation outcomes are realized at the beginning of the next period. To allow incumbent firms to protect their own product markets from competitors (the escape-competition effect) and to capture the fact that it is more difficult to take over other firms' product markets when incumbent firms are very innovative on average (the technological barrier effect), we introduce imperfect technological spillovers, which are captured by lagged learning: firms that don't own product line j can only learn the incumbent's last period technology, $q_{j,t-1}$. Thus, external innovation builds on the past-period technology. Also, we assume that a domestic firm can learn foreign firm's lagged technology if and only if that foreign firm sells its products in the domestic market.

In this setup, learning other firms' technology is costly in the sense that i) rivals can only learn incumbent firms' last period technology, and ii) learning involves R&D—only firms with strictly positive R&D expenditure can learn other firms' past technology through undirected learning.[4] For a particular product, the current period technology $q_{j,t}$ and the technology gap $\Delta_{j,t} \equiv \frac{q_{j,t}}{q_{j,t-1}}$ are observable only to the firm operating product line j in that period. However, aggregate variables and the technology gap distribution (the share of product lines with a certain level of technology gap) are publicly observable, and these are the objects individual firms need to know to make their optimal innovation decisions. Thus, an equilibrium with a station-

[4]Firms do not know which product line's lagged technology they will learn prior to their learning. This assumption helps keep the model tractable.

ary firm-product distribution is well defined. When two firms' technologies are neck and neck in a particular product line, a coin-toss tiebreaker rule applies as in Acemoglu et al. (2016) to make sure each product is produced by only one firm. An unused technology (idea) is assumed to depreciate by an amount sufficient to ensure that it becomes unprofitable to innovate on top of it next period.[5] Thus, only the winning firm from the coin toss keeps the product line until it is taken over by others through creative destruction (external innovation), while the losing firm never tries to enter the same market through internal innovation. Thus, the undirected nature of external innovation is ensured, and only the firm currently producing a product is allowed to do internal innovation on that product. Finally, to maintain tractability, we assume that each firm can do only one external innovation in each period regardless of the total number of product lines the firm owns.

2.2.4.1 Internal Innovation

Firms do internal innovation for each product they currently own and produce. Successful internal innovation improves the current quality $q_{j,t}$ of a firm's own product j by $\lambda > 1$. The probability of successful internal innovation, $z_{j,t}$, is determined by the level of R&D expenditure $R^{in}_{j,t}$ in units of the final good:

$$z_{j,t} = \left(\frac{R^{in}_{j,t}}{\widehat{\chi} q_{j,t}} \right)^{\frac{1}{\hat{\psi}}} ,$$

[5] This captures the idea that if you don't use your skill or idea frequently, you gradually forget about it. This is consistent with, for instance, the literature discussing displaced workers' human capital depreciation.

where $\widehat{\chi} > 0$ and $\widehat{\psi} > 1$. Thus, incumbent firm's good j quality realized at the beginning of $t+1$, assuming the firm is not displaced by creative destruction, is:[6]

$$\left\{q_{j,t+1}^{in}\right\} = \begin{cases} \left\{\lambda q_{j,t}\right\} & \text{with probability } z_{j,t} \\ \left\{q_{j,t}\right\} & \text{with probability } 1 - z_{j,t} \,. \end{cases}$$

2.2.4.2 External Innovation

Incumbents and potential startups attempt to take over other incumbents' markets through external innovation. Successful external innovation generates an improvement in product quality of $\eta > 1$ relative to the incumbent's lagged technology, where R&D results are realized at the beginning of next period. We assume $\lambda^2 > \eta > \lambda$. This assumption ensures that firms can protect their own product lines from potential rivals through internal innovation, while $\eta > \lambda$ reflects the idea that external innovation introduces a new way of producing existing products more efficiently. Thus, external innovation contributes more to both firm employment and aggregate growth than internal innovation, as found empirically in Akcigit and Kerr (2018). Both potential startups' and incumbent firms' external innovations are undirected, in the sense that they are realized in any other product line with equal probability.

Existing firms with at least one product line ($n_f > 0$) decide the probability of external innovation x_t by choosing R&D expenditures R_t^{ex} in units of the final good:

$$x_t = \left(\frac{R_t^{ex}}{\widetilde{\chi}\,\bar{q}_t}\right)^{\frac{1}{\widetilde{\psi}}} ,$$

[6]Hereafter, we write the quality of product j as a point set. This makes it easy to describe the case when external innovation fails and a firm does not acquire any product lines, in which case the product quality set is an empty set.

where $\widetilde{\chi} > 0$, and $\widetilde{\psi} > 1$, and $\bar{q}_t$ is the average quality in the country (or region) where the firm is located. Thus, for prospective external innovators whose takeover is not pre-empted by an incumbent's successful defensive innovation, the distribution of quality for product j at the start of the next period is:

$$\left\{q_{j,t+1}^{ex}\right\} = \begin{cases} \left\{\eta q_{j,t-1}\right\} & \text{with probability } x_t \\ \varnothing & \text{with probability } 1 - x_t \ . \end{cases}$$

With probability $1 - x_t$, the external innovation fails, which implies there is zero probability that the firm will take over product line j. In this case, product quality for potential entrants does not exist.

To better understand the firm's innovation decisions, external innovation and escape competition in detail, the following section graphically illustrates some specific cases.

2.2.4.3 Business Takeover and Escape Competition, an Illustration

Figure 2.1 illustrates how firms' product quality portfolio and technology gap portfolio evolve over time. Firm A owns the first three product lines and firm B owns the last four product lines in period t. Each bar represents a product and the height of the bar represents the log of product quality for each product, $\widehat{q}_{j,t} \equiv \log(q_{j,t})$. Product line 7 does not experience innovation by any firm in period t. Thus, its quality at $t + 1$ remains the same as at t, and it is still owned by firm B at $t + 1$. Example i) in the right panel illustrates the case in which firm B does external innovation in an attempt to take over firm A's product line 1. Firm A obtained this product line through successful external innovation at $t - 1$, but did not internally innovate at t. Thus, we have

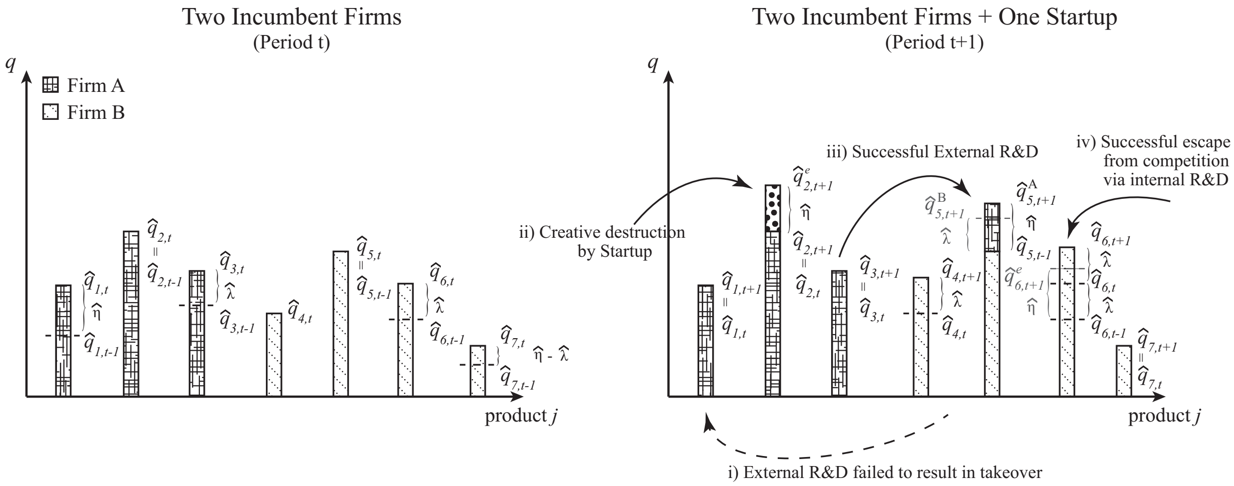


Figure 2.1: Firms' Innovation and Product Quality Evolution Example

$\Delta_{1,t} = \eta$, and $q^A_{1,t+1} = \eta q_{1,t-1}$ (implying in logs that $\widehat{q}^A_{1,t+1} = \widehat{\eta} + \widehat{q}_{1,t-1}$, where $\widehat{\eta} \equiv \log(\eta)$) for firm A. Meanwhile, firm B learns $q_{1,t-1}$ in period t and innovates itself, so that in period $t+1$, it realizes $q^B_{1,t+1} = \eta q_{1,t-1}$, which is the same as $q^A_{1,t+1}$. A coin is tossed, and firm A is the winner. Thus, firm A keeps product line 1. Case ii) illustrates how a firm can lose its existing product line through other firms' external innovation (creative destruction). Firm A failed to do internal innovation on product line 2 in periods $t-1$ and t. Thus, at the beginning of period $t+1$, the quality of product 2 for firm A is equal to $q^A_{2,t+1} = q_{2,t-1}$. A potential startup learns the product 2's last period technology (quality) by investing in R&D in period t and succeeds in external innovation. Thus, at the beginning of $t+1$, the quality of product 2 for the potential startup is equal to $q^e_{2,t+1} = \eta q_{2,t-1}$. Since $q^e_{2,t+1} > q^A_{2,t+1}$, the startup takes over product line 2. Case iii) illustrates how incumbent firm A can take over incumbent firm B's product line through external innovation, despite the internal innovation undertaken by incumbent firm B. Since there was no internal innovation between $t-1$ and t for product line 5, $q_{5,t} = q_{5,t-1}$. Thus, firm A's quality for product line 5 after its external innovation is $q^A_{5,t+1} = \eta q_{5,t}$. Firm B internally innovates product line 5 in period t, and its quality becomes $q^B_{5,t+1} = \lambda q_{5,t}$. Since $\eta > \lambda$, firm A takes over product line 5. Case iv) illustrates how firms can escape from competition (creative destruction) through successful internal innovation. Firm B succeeds in internally innovating product 6 for two consecutive periods. Thus, the quality of product line 6 for firm B in period $t + 1$ is equal to $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$. Because of the imperfect technology spillovers, rival firms can increase the quality for product line 6 only up to $q^e_{6,t+1} = \eta q_{6,t-1}$. Since $\lambda^2 > \eta$, firm B successfully protects product 6 from competitors. These examples illustrate an important feature that is unique to the economy with imperfect technology spillovers. Because incumbents can escape competition through internal innovation, not all firms that succeed in external innovation can successfully take over others' business. Thus, the probability of a successful business takeover is generally lower than the probability of external innovation, and depends on the existing technology gap in target markets (products).

2.2.4.4 Product Quality Evolution

As a rival firm can only learn the last period's technology, the technology gap, defined as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, is the most important factor determining an incumbent firm's ability to protect its product line through internal innovation. The technology gap summarizes technological advantages incumbent firms have in their own markets. In this model, there are four possible values for the technology gap:

Lemma 3. *There can be only four values for the technology gap in this economy,* $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, *and* $\Delta^4 = \frac{\eta}{\lambda}$, *and product lines with* Δ^3 *and* Δ^4 *can occur only through external innovation.*

Proof: See Appendix B.1.2.1.

To describe the evolution of product quality and the implied probabilities of retaining or losing a product from the perspective of an incumbent firm, consider a product line j with quality $q_{j,t}$ and technology gap $\Delta_{j,t}$ owned by a firm f. Denote z_j^ℓ as firm f's optimal choice of the probability of internal innovation for product line j when its technology gap is equal to $\frac{q_{j,t}}{q_{j,t-1}} = \Delta^\ell$, $\ell \in \{1, 2, 3, 4\}$. Suppose product line j has technology gap $\Delta_{j,t} = \Delta^1$. If the firm is successful at internal innovation with probability z_j^1, its product quality next period is $q_{j,t+1}^{in} = \lambda q_{j,t-1}$; otherwise, $q_{j,t+1}^{in} = q_{j,t-1}$.

If creative destruction arrives at rate $\overline{x}$—where $\overline{x}$ is the probability that an individual product market is faced with a rival that has made a successful external innovation—then the product quality of the rival will be $q_{j,t+1}^{en} = \eta q_{j,t-1}$. Since $q_{j,t+1}^{en} > \lambda q_{j,t-1} > q_{j,t-1}$, the rival takes over the product line j regardless of the firm's success at internal innovation. Thus, $\Delta_{j,t} = 1$, firm f loses its product line j next period with probability $\bar{x}$.

Based on the same arguments, the distributions of next period product quality of firm f in product line j conditional on $\Delta_{j,t} = \Delta^l$ can be defined as follows:

$$\left\{q_{j,t+1} \,\middle|\, \Delta_{j,t} = \Delta^1\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x} \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^1) \\ \{\lambda q_{j,t}\} & \text{, with prob. } (1-\overline{x})z_j^1 \end{cases} \tag{2.2}$$

$$\left\{q_{j,t+1} \,\middle|\, \Delta_{j,t} = \Delta^2\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x}(1-z_j^2) \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^2) \\ \{\lambda q_{j,t}\} & \text{, with prob. } z_j^2 \end{cases} \tag{2.3}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^3\right\} = \begin{cases} \varnothing & \text{, with prob. } \frac{1}{2}\overline{x}(1-z_j^3) \\ \left\{q_{j,t}\right\} & \text{, with prob. } \left(1-\frac{1}{2}\overline{x}\right)\left(1-z_j^3\right) \\ \left\{\lambda q_{j,t}\right\} & \text{, with prob. } z_j^3 \end{cases} \tag{2.4}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^4\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x}\left(1-\frac{1}{2}z_j^4\right) \\ \left\{q_{j,t}\right\} & \text{, with prob. } (1-\overline{x})(1-z_j^4) \\ \left\{\lambda q_{j,t}\right\} & \text{, with prob. } \left(1-\frac{1}{2}\overline{x}\right) z_j^4 \end{cases} \tag{2.5}$$

where product quality equal to $\varnothing$ means that firm f loses its product line j next period, and the terms $\frac{1}{2}$ in the probabilities reflect the coin-toss tiebreaker rule for neck and neck cases. Hence, for any Δ^ℓ except for Δ^1, firms can lower the probability of losing their product lines by investing more in internal innovation, where the magnitude of the decrease in the probability of losing the product depends on the technology gap. For this reason, firms have more incentive to increase their internal innovation intensity (through R&D investment that increases the probability of internal innovation) for products in which they have technological advantage ($\Delta^\ell > 1$) when they are faced with more competition, as represented by a higher creative destruction arrival rate $\overline{x}$.

The conditional takeover probability—the probability of product takeover, conditional on successful external innovation—can be computed as follows. If a rival firm succeeds in externally innovating a product line with technology gap Δ^1, then it takes over this product line with probability one. For a product line with technology gap Δ^2, this probability becomes $1 - z^2$; for technology gap Δ^3, it is $\frac{1}{2}(1 - z^3)$; and for technology gap Δ^4, it is $1 - \frac{1}{2}z^4$.[7] Thus, for a given technology gap distribution $\left\{\mu(\Delta^\ell)\right\}_{\ell=1}^4$, where $\mu(\Delta^\ell)$ represents the share of product lines with

[7]Here we assume that the internal innovation intensity z depends only on the technology gap Δ^ℓ. In the next section, we prove this is the case.

technology gap Δ^ℓ, the conditional takeover probability is equal to

$$\overline{x}_{takeover} = \mu(\Delta^1) + (1 - z^2)\mu(\Delta^2) + \frac{1}{2}(1 - z^3)\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right)\mu(\Delta^4)\,. \tag{2.6}$$

The higher the overall innovation intensity (both internal and external), the wider the average technology gap becomes in the economy. Thus, it becomes more difficult for rival firms to take over other firms' product markets. The conditional takeover probability defines the technological barrier channel, through which increases in either incumbent firms' internal innovation intensity or the aggregate creative destruction arrival rate $\bar{x}$ can lower domestic firms' incentives for external innovation, which will lower firm growth rates. This technological barrier effect is distinct from the well-known Schumpeterian effect, by which firms' innovation incentives decline following an increase in $\bar{x}$ due to lowered expected future profits conditional on successful innovation and business takeover. Higher overall innovation intensity in the economy will likely reduce $\overline{x}_{takeover}$, as the share of product lines with technology gap Δ^1 (where the probability of product takeover is the highest) will decrease, while at least some of the z^ℓ for $\ell = 2, 3, 4$ will increase. Since all firms, including potential startups, know the level of $\overline{x}_{takeover}$, firms will optimally choose to lower their external innovation intensity when $\overline{x}_{takeover}$ falls, unless expected profits from external innovation increase enough to offset the loss from a lowered conditional takeover probability.

Note that for a given technology gap distribution $\left\{\mu\left(\Delta^\ell\right)\right\}_{\ell=1}^4$, the unconditional probability of a firm failing in an attempted product takeover—the probability of not winning the product line, either due to the failure of external innovation (which occurs with probability of $1 - x$) or the successful escape-competition by incumbent firms—is

$$(1-x) + xz^2\mu(\Delta^2) + x\frac{1}{2}(1+z^3)\mu(\Delta^3) + x\frac{1}{2}z^4\mu(\Delta^4)$$
$$= 1 - x\left[1 - \left(z^2\mu(\Delta^2) + \frac{1}{2}(1+z^3)\mu(\Delta^3) + \frac{1}{2}z^4\mu(\Delta^4)\right)\right].$$

Given the above definition of the conditional takeover probability $\overline{x}_{takeover}$, the previous expression can be rewritten as $1 - x\,\overline{x}_{takeover}$. In what follows, we denote $x_{takeover} \equiv x\,\overline{x}_{takeover}$, which is the unconditional probability of successful product takeover. The probability distribution of the evolution of product quality from the perspective of a rival firm can also be defined in a similar way, which is described in Appendix B.1.2.2.

2.2.5 Potential Startups

The economy has a fixed mass of potential domestic startups $\mathcal{E}_d$, and an exogenously determined mass of foreign firms trying to start businesses in domestic markets.[8] To start a business, a potential startup invests in external R&D and, if successful, takes over a product line from an incumbent firm. Similar to incumbent firms, potential startups decide the probability of external innovation x_e by choosing their R&D expenditure R_e^{ex} in units of the final good:

$$x_e = \left(\frac{R_e^{ex}}{\widetilde{\chi}_e\,\bar{q}}\right)^{\frac{1}{\tilde{\psi}_e}},$$

[8]Strictly speaking, only the portion of the aggregate creative destruction arrival rate accounted for by outside firms is exogenously determined in this economy. However, this is effectively the same as having an exogenously determined mass of outside firms trying to start businesses in domestic markets, as will become clear in the following sections.

where $\widetilde{\chi}_e > 0$, and $\widetilde{\psi}_e > 1$, and $\bar{q}$ is the average product quality in the country where the potential startup is located.

Let $V(\{(q_j, \Delta_j)\})$ denote the value of a firm that has one product line with product quality q_j and technology gap Δ_j. Then a potential startup's expected profits from entering through R&D is

$$\Pi^e = \beta \mathbb{E}\Big[V(\{(q_j', \Delta_j')\}) \,\big|\, x_e\Big] - \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} \, ,$$

where the expectation conditioning on x_e is taken over the distribution of incumbents' product quality q_j and technology gap Δ_j due to the undirected nature of external innovation. Potential startups choose the probability of external innovation x_e that maximizes their expected profits from entry. Since there is no ex-ante heterogeneity among potential startups, they all choose the same optimal probability of external innovation x_e^*. Thus, the mass of potential domestic startups that succeed in external innovation and attempt to take over incumbent firms' product markets is $\mathcal{E}_d x_e^*$.

2.2.6 Exogenous Competitive Pressure and Creative Destruction

As explained in the previous section, the aggregate creative destruction arrival rate is the probability that an incumbent faces a rival (a domestic startup, a domestic incumbent or a foreign firm) that has succeeded in external innovation. Conditional on external innovation, whether the incumbent is replaced by the rival firm depends on the technology gap and internal innovation of the incumbent.

Each firm can externally innovate at most one product line each period, and there is a continuum of unit mass of product lines (markets). Thus, the total mass of firms that succeed in external innovation is equal to the total mass of product markets for which an incumbent faces a rival firm. Since external innovation is undirected, this implies that the probability an individual product market incumbent is faced with competition from other firms—the aggregate creative destruction arrival rate—is equal to the total mass of firms that succeed in external innovation. Denote $\overline{x}_d$ as the total mass of domestic firms that succeed in external innovation and $\overline{x}_o$ as the foreign firm counterpart. Then the aggregate creative destruction arrival rate $\overline{x}$ is

$$\overline{x} = \overline{x}_d + \overline{x}_o \, .$$

Note that increasing competitive pressure induced by foreign firms is defined as an exogenous increase in $\overline{x}_o$ in this model economy.

2.2.7 Equilibrium

We now turn to describing optimal decisions for each agent and the Markov Perfect Equilibrium of the economy, where optimal decisions depend only on individual characteristics, aggregate variables, and the technology gap distribution.

2.2.7.1 Optimal Production and Employment

The solution for the final good producer's profit maximization problem defines the optimal demands for labor and intermediate products. Denote p_j as the price for differentiated product j, and w as the wage rate in the domestic economy. Then the inverse demand for intermediate product j is

$$p_j = q_j^{\theta} L^{\theta} y_j^{-\theta} \; . \tag{2.7}$$

Here, we are assuming that each product is supplied by a single firm. However, previous incumbent firms in domestic markets, who have lost their technological leadership to the current leaders, could in principle try to produce and sell their products through limit pricing, as the marginal cost of production is equal for every firm. To avoid such a case and to simplify the model, we assume the following two-stage price-bidding game:

Assumption 1. *For each product line j in the economy, the current and any former incumbents enter a two-stage price-bidding game. In the first stage, each firm pays a fee of $\varepsilon > 0$. In the second stage, all firms that paid the fee announce their prices.*

This assumption ensures that only a technological leader enters the first stage and announces its price in equilibrium.

Intermediate producers (both domestic and foreign) take their product demand curves (2.7) as given and maximize operating profits (revenue net of production cost) for each individual product line $j \in \mathcal{J}^f$:

$$\pi(q_j) = \max_{y_j \geqslant 0} \left\{ L^{\theta} q_j^{\theta} y_j^{1-\theta} - y_j \right\} \; .$$

Since each intermediate product is produced at an unit marginal cost in terms of the final good, the intermediate producers' problem is the same for both domestic and foreign firms. The FOC of this problem yields the following optimal production level for each product j:

$$y_j = (1-\theta)^{\frac{1}{\theta}} L q_j \ , \tag{2.8}$$

and by plugging this into the demand curve (2.7), we get the monopoly price

$$p_j = \frac{1}{1-\theta} \ , \tag{2.9}$$

which is a markup $\frac{1}{1-\theta}$ over the unit marginal cost. Using (2.8), the profit from intermediate production is linear in its quality, holding all aggregate variables fixed:

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L}_{\equiv \pi} \ q_j \ .$$

From the final good producer's problem, the equilibrium wage rule follows

$$w = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q} \ , \tag{2.10}$$

which depends only on the average product quality in the economy. Since

$$L = 1 \tag{2.11}$$

in equilibrium, the optimal intermediate production becomes

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \tag{2.12}$$

and the scaling coefficient of the profit from intermediate production becomes

$$\pi = \theta(1-\theta)^{\frac{1-\theta}{\theta}} .$$

Finally, using (2.11) and (2.12), equilibrium final good production can be written as

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q} , \tag{2.13}$$

which grows at the same rate as average product quality.

2.2.7.2 Value Function for Incumbent Intermediate Firms

In this section, we solve for an intermediate firm's optimal R&D decisions. Define $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j\in\mathcal{J}^f}$ as a multi-set of product quality and technology gap pairs currently owned by intermediate producer f, where (q_j, Δ_j) defines product line j completely. Then firm f's value function can be written as

$$V\left(\Phi^f\right) = \max_{\substack{x\in[0,\bar{x}],\\ \{z_j\in[0,\bar{z}]\}_{j\in\mathcal{J}^f}}} \left\{ \sum_{j\in\mathcal{J}^f} \left[\pi q_j - \hat{\chi} z_j^{\hat{\psi}} q_j\right] - \bar{q}\widetilde{\chi} x^{\widetilde{\psi}} + \widetilde{\beta}\,\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right] \right\} ,$$

where πq_j is revenue net of production costs for product j. Thus, the first three terms define the current profits of a firm with the product quality and technology gap portfolio Φ^f, and the last term is the discounted expected future value, based on the conditional expectation taken over the success or failure of internal and external innovation, creative destruction, winning or losing coin tosses, the current period product quality distribution, and the current period technology gap distribution. $\widetilde{\beta}$ is the stochastic discount factor, which is constant over time as there is no uncertainty in this economy.

Proposition 6. *For a given technology gap distribution $\left\{\mu\left(\Delta^{\ell}\right)\right\}_{\ell=1}^{4}$, the value function of a firm with product quality and technology gap portfolio $\Phi^{f} \equiv \left\{(q_j, \Delta_j)\right\}_{j\in\mathcal{J}^f}$ is of the form:*

$$V(\Phi^f) = \sum_{\ell=1}^{4} A_\ell \left(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j \right) + \ B\,\overline{q}\,,$$

where

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big] \tag{2.14}$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big] \tag{2.15}$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right] \tag{2.16}$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right] \tag{2.17}$$

$$B = \frac{1}{1-\widetilde{\beta}(1+g)}\left[x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}\right]\,, \tag{2.18}$$

and optimal innovation probabilities are

$$z^1 = \left[\frac{\widetilde{\beta}\left[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.19}$$

$$z^2 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.20}$$

$$z^3 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.21}$$

$$z^4 = \left[\frac{\widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - \left(1-\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}} \right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.22}$$

$$x = \left[\frac{\widetilde{\beta}A_{takeover}}{\widetilde{\psi}\widetilde{\chi}} \right]^{\frac{1}{\widetilde{\psi}-1}}, \tag{2.23}$$

where g in (2.18) is the average product quality growth rate in the economy, and $A_{takeover}$ (2.18) and (2.23) is the ex-ante value of a product line obtained from successful takeover, which is defined as:

$$A_{takeover} \equiv \frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2). \tag{2.24}$$

Proof: See Appendix B.1.3.1

Comparing (2.24) and (2.6), we see that the determinants of $A_{takeover}$ include factors that determine the conditional takeover probability $\overline{x}_{takeover}$.

A_ℓ is the sum of discounted expected profits from owning a product line with a technology gap equal to Δ^ℓ, normalized by the current period product quality. The first two terms in (2.14) through (2.17) are the normalized instantaneous profits, net of optimal internal R&D spending, and the terms inside the brackets are the normalized future value from internal innovation. For instance, if a firm with gap Δ^1 succeeds in internally innovating its product and still owns that product next period, then the normalized value of that product is equal to A_2, as the next period technology gap is equal to Δ^2. If the firm fails to internally innovate its product but still owns that product next period, then the normalized value of that product is equal to A_1, as the next period technology gap is equal to Δ^1. B is the sum of the discounted expected profits from owning an additional product through external innovation, normalized by the average product quality. To understand this variable more clearly, we can re-express (2.18) as

$$B\overline{q} = x\widetilde{\beta}A_{takeover}\overline{q} - \widetilde{\chi}x^{\widetilde{\psi}}\overline{q} + \widetilde{\beta}(1+g)B\overline{q} \,.$$

After investing $\widetilde{\chi}x^{\widetilde{\psi}}\overline{q}$ in external innovation in the current period, the firm receives the discounted expected profit $\widetilde{\beta}A_{takeover}\overline{q}$ next period if external innovation succeeds, which happens with probability x. The firm owns at least one product line next period if current period external innovation is successful. Thus, it will invest in external innovation next period and receive an expected profit of $B\overline{q}'$ two periods later, where $\overline{q}' = (1+g)\overline{q}$. Thus, (2.18) shows that B is the annuity value of an infinite stream of constant payoffs $x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}$, evaluated at a constant discount rate $\widetilde{\beta}(1+g)$, the growth rate-adjusted discount factor.

For the optimal innovation intensities (2.19)-(2.22), the brackets in the numerator (after $\widetilde{\beta}$) is the future value from successful internal innovation, which increases quality by λ. The second term is the future value from no internal innovation, in which next period's technology gap is equal to one. Thus, the higher the future value for successful internal innovation, the higher is the optimal probability of internal innovation, holding $\overline{x}$ fixed. For this reason, the optimal probability of internal innovation for each product line depends on its technology gap. Intuitively, a wider technology gap should increase firms' internal innovation investment up to a point, as a wider gap implies that escape from competition via internal innovation is easier. However, past some point a wider technology gap should dissuade incumbent firms from investing in internal innovation, since it is much harder for other firms to take over a product line with a very high technology gap. Thus. there is less urgency to escape competition when a product line has a very high technology gap. Corollary 3 formalizes this argument.

Corollary 3. *In an equilibrium where $\{z^{\ell}\}_{\ell=1}^{4}$ are well defined, the probabilities of internal innovation satisfy $z^2 > z^3 > z^4 > z^1$.*

Proof: See Appendix B.1.3.2

Thus, for a product line with the widest technology gap $\Delta^3 = \eta$, firms invest less in internal innovation than they do for a product line with $\Delta^2 = \lambda$, as there is a lower probability that they will lose the product line even if they don't improve its quality—firms with technology gap Δ^3 lose a product line only when they are in a neck and neck case and lose the coin toss. Thus, $z^2 > z^3$, even though $\Delta^3 > \Delta^2$.

Since A_1 and A_2 depend on $\overline{x}$, it is difficult to sign the partial derivatives of $\{z^{\ell}\}_{\ell=1}^{4}$ w.r.t. $\overline{x}$. But holding the values for A_1 and A_2 fixed, we can determine the signs of these partial derivatives, which define the escape-competition effect:

Corollary 4. *With $\widetilde{\psi} \in (1, 2]$, the escape-competition effect is maximized and is positive for product lines with technology gap equal to Δ^2, whereas it is minimized and is negative for product lines with technology gap equal to Δ^1. The escape-competition effect is positive for the Δ^3 case, whereas its sign is ambiguous for the Δ^4 case. Thus,*

$$\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > 0, \quad \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} \lesseqgtr 0 \ , \quad \textit{and} \quad 0 > \left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} .$$

Proof: See Appendix B.1.3.3

As equation (2.2) shows, a firm cannot protect its product line from takeover through internal innovation if its technology gap is equal to Δ^1. This is why z^1 is a decreasing function of the creative destruction arrival rate $\overline{x}$, other things being equal. As equation (2.3) shows, the impact of internal innovation on the probability of losing the product is greatest in the Δ^2 case. Thus, the escape-competition incentive is the highest for this case. In the Δ^3 case, a marginal increase in z^3 decreases the probability of losing the product by 50% less than in the Δ^2 case. Thus the escape-competition effect is lower, but still positive. The sign of the escape-competition effect for the Δ^4 case is ambiguous, as the decrease in the probability of losing the product line resulting from higher z^4 is even smaller.

More innovation in the previous period increases the probability of having a high technology gap in the current period, and this helps firms to escape competition. Thus, Corollary 4 implies that firms who have innovated intensively in the previous period increase internal innovation more in response to higher competition (as measured as higher $\overline{x}$), compared to counterparts with less recent innovation. Corollary B.2.1 from the simple three-period model in Appendix B.2 formalizes this observation.

Meanwhile, the term A_2 in (2.19)-(2.22) reflects the Schumpeterian effect. The lower the expected future profits from keeping the product line through internal innovation, the lower is the incentive to invest in internal innovation.

The optimal probability of external innovation depends on internal innovation intensities, product values ($\{A_\ell\}_{\ell=1}^4$), and the technology gap distribution. The definition of $A_{takeover}$ and equation (2.23) indicate that higher overall innovation intensities (internal and external) in the economy lower the incentive for external innovation for an individual firm in partial equilibrium, holding product values fixed. This is the technological barrier effect, summarized in the conditional takeover probability $\overline{x}_{takeover}$. Corollary B.2.2 from the simple three-period model in

Appendix B.2 formalizes this observation.

Holding probabilities of internal innovation and the technology gap distribution fixed, a decrease in product values decreases an individual firm's incentive for external innovation. This is the Schumpeterian effect.

The direction of the changes in the probabilities of internal and external innovation in response to changes in the aggregate creative destruction arrival rate $\overline{x}$ are ambiguous in general equilibrium. They depend on the relative magnitudes and the directions of the escape-competition effect, the Schumpeterian effect, and the technological barrier effect. Nonetheless, results from the numerical exercise in Section 2.4.3.1 confirm that the partial equilibrium results for given $\{A_\ell\}_{\ell=1}^4$ and B still hold in general equilibrium for a plausible parameterization. Furthermore, $\{A_\ell\}_{\ell=1}^4$ and B also decrease as $\overline{x}$ increases exogenously.

2.2.7.3 Potential Startups

Recall that a potential startup's expected profits from entering through R&D are

$$\Pi^e = \widetilde{\beta}\mathbb{E}\Big[V(\{(q_j', \Delta_j')\}) \,\Big|\, x_e\Big] - \widetilde{\chi}_e(x_e)^{\widetilde{\psi}_e}\overline{q}\,.$$

By using the value function derived in Proposition 6, the optimal probability of external innovation for potential startups x_e can be computed as

$$x_e = \left(\widetilde{\beta}\frac{A_{takeover} + \overline{x}_{takeover}B(1+g)}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\right)^{\frac{1}{\widetilde{\psi}_e - 1}} . \tag{2.25}$$

The proof is in Appendix B.1.4.

As explained in the previous section, the total mass of domestic firms that succeed in external innovation defines the domestic portion of the aggregate creative destruction arrival rate. Since the optimal probabilities of external innovation for incumbent firms and potential domestic startups are equal to x and x_e respectively, and external innovation is undirected, the aggregate creative destruction arrival rate in this economy is defined as

$$\overline{x} = \underbrace{\mathcal{F}_d\, x + \mathcal{E}_d\, x_e}_{\equiv \overline{x}_d} + \overline{x}_o \,. \tag{2.26}$$

Since the mass of domestic incumbent firms $\mathcal{F}_d$ and the probabilities of external innovation x and x_e depend on $\overline{x}$, an exogenous increase in $\overline{x}_o$ may not increase $\overline{x}$ by the same amount in equilibrium. Thus, the level of $\overline{x}$ is endogenously determined even when $\overline{x}_o$ changes exogenously.

2.2.8 Growth rate

As equation (2.13) shows, the output growth rate in this model economy is equal to the product quality growth rate g. Proposition 7 presents an expression for this growth rate, and decomposes it according to the contributions made by different groups of firms and types of innovation.

Proposition 7. *The growth rate for aggregate variables in a Balanced Growth Path in this economy, g, satisfies the following:*

$$\begin{aligned} g = & \Big[(1-\overline{x})(1-z^1) + \Delta^2(1-\overline{x})z^1 + \Delta^3\overline{x}\Big]\mu(\Delta^1) \\ & + \Big[(1-\overline{x})(1-z^2) + \Delta^2 z^2 + \Delta^4\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \Delta^2 z^3\Big]\mu(\Delta^3) \\ & + \Big[(1-\overline{x})(1-z^4) + \Delta^2(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4) - 1 \,. \end{aligned} \tag{2.27}$$

Furthermore, this expression can be rearranged into four components:

$$
\begin{aligned}
1+g =& \Big[(1-\overline{x})(1-z^1)+\Delta^2(1-\overline{x})z^1\Big]\mu(\Delta^1)+\Big[(1-\overline{x})(1-z^2)+\Delta^2 z^2\Big]\mu(\Delta^2) \\
& \underbrace{+\left[\left(1-\frac{1}{2}\overline{x}\right)(1-z^3)+\Delta^2 z^3\right]\mu(\Delta^3)+\left[(1-\overline{x})(1-z^4)+\Delta^2\left(1-\frac{1}{2}\overline{x}\right)z^4\right]\mu(\Delta^4)}_{\text{internal innovation by both domestic incumbents and foreign firms}} \\
& +\underbrace{\Delta^3\mathcal{F}_d x\mu(\Delta^1)+\Delta^4\mathcal{F}_d x(1-z^2)\mu(\Delta^2)+\frac{1}{2}\mathcal{F}_d x(1-z^3)\mu(\Delta^3)+\Delta^2\mathcal{F}_d x\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\text{external innovation by domestic incumbent firms}} \\
& +\underbrace{\Delta^3\mathcal{E}_d x_e\mu(\Delta^1)+\Delta^4\mathcal{E}_d x_e(1-z^2)\mu(\Delta^2)+\frac{1}{2}\mathcal{E}_d x_e(1-z^3)\mu(\Delta^3)+\Delta^2\mathcal{E}_d x_e\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\text{external innovation by domestic startups}} \\
& +\underbrace{\Delta^3\overline{x}_o\mu(\Delta^1)+\Delta^4\overline{x}_o(1-z^2)\mu(\Delta^2)+\frac{1}{2}\overline{x}_o(1-z^3)\mu(\Delta^3)+\Delta^2\overline{x}_o\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\text{external innovation by foreign firms}}\ .
\end{aligned}
$$

Proof: See Appendix B.1.5.1

2.2.9 Firm Distribution

As the intermediate firm's decision rules show, the distribution of firms' technology gap portfolios completely describes the distribution of firms in this model economy.[9] In this section, we describe how we keep track of the evolution of this distribution. Let $\mathcal{N}=(n_f,n_f^1,n_f^2,n_f^3,n_f^4)$. Denote the technology gap composition for a firm with n_f product lines and with n_f^ℓ products with technology gap equal to Δ^ℓ, $\ell=1,2,3,4$, and denote the density of this object as $\mu(\mathcal{N})$.

2.2.9.1 Transition of the Technology Gap Portfolio Composition Distribution

Consider a firm with n_f-k products with $\Delta=\Delta^1$, k products with $\Delta=\Delta^2$, zero products with $\Delta=\Delta^3$ and zero products with $\Delta=\Delta^4$. Then for this firm, $\widetilde{\mathcal{N}}(n_f,k)\equiv(n_f,n_f-k,k,0,0)$, for $k\in[0,n_f]\cap\mathbb{Z}$, $n_f>0$. Ignoring external innovation, the probability of $\mathcal{N}=$

[9] The technology gap distribution can be computed from this distribution.

$\widetilde{\mathcal{N}}(n_f, k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \,|\, n_f, k\Big) = \begin{cases} \sum_{\widetilde{k}^1=\max\{0,\,\widetilde{k}-k\}}^{\min\{n_f-k,\,\widetilde{k}\}} \begin{pmatrix} n_f - k \\ \widetilde{k}^1 \end{pmatrix} \begin{pmatrix} k \\ \widetilde{k} - \widetilde{k}^1 \end{pmatrix} & \\ \quad \times \begin{bmatrix} (1-\overline{x})^{n_f-(\widetilde{k}-\widetilde{k}^1)} \, (1-z^1)^{n_f-k-\widetilde{k}^1} \, (z^1)^{\widetilde{k}^1} \\ \times (1-z^2)^{k-(\widetilde{k}-\widetilde{k}^1)} \, (z^2)^{\widetilde{k}-\widetilde{k}^1} \end{bmatrix} & \text{for } n_f \geqslant 1, \text{ and} \\ & 0 \leqslant \widetilde{k}, k \leqslant n_f \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}$$

is a combination of selecting k elements from n elements without repetition, where the order of selection does not matter. Thus, changes in the technology gap composition follow a binomial process, as in Ates and Saffie (2016). The range for $\widetilde{k}^1$ takes the form described above due to the fact that

i. For $0 \leqslant \widetilde{k} \leqslant \min\{n_f - k, k\}$, the two combinations preceding the term in brackets are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describe all the possible cases.

ii. If $n_f - k \geqslant k$, then $\widetilde{k} > k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\widetilde{k} - k \leqslant \widetilde{k}^1 \leqslant \widetilde{k}$.

iii. If $k \geqslant n_f - k$, then $\widetilde{k} > n_f - k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied.

Thus $\max\{0,\ \widetilde{k}-k\} \leqslant \widetilde{k}^1 \leqslant n_f - k$.

Since product lines can have technology gap equal to Δ^3 or Δ^4 only through external innovation, the probability of a technology gap composition $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ becoming $\mathcal{N}' = (n_f', n_f^{1\prime}, n_f^{2\prime}, n_f^{3\prime}, n_f^{4\prime})$ for any $n_f' \leqslant n_f + 1$ can be computed using $\widetilde{\mathbb{P}}(n_f, \widetilde{k} \mid n_f, k)$, and thus the change in the technology gap portfolio composition distribution can be tracked. The procedure is described in detail in Appendix B.1.6.

2.2.9.2 Technology Gap Distribution

By using the distribution of the firm-level technology gap composition for domestic firms $\mathcal{F}_d\, \mu(\mathcal{N})$, the aggregate distribution of the technology gap for the product lines owned by domestic firms $\left\{\widetilde{\mu}\left(\Delta^\ell\right)\right\}_{\ell=1}^4$ can be computed as

$$\widetilde{\mu}\left(\Delta^\ell\right) = \sum_{n_f=1}^{\overline{n}_f} \sum_{n_f^\ell=0}^{n_f} n_f^\ell\, \mathcal{F}_d\, \mu\left(n_f, n_f^1, n_f^2, n_f^3, n_f^4\right) . \tag{2.28}$$

Since this distribution describes the product lines owned by domestic firms, it should sum up to the total mass of product lines owned by domestic firms. Denote the total mass of product lines owned by domestic firms as s_d. Lemma 4 describes its relationship with the aggregate creative destruction arrival rate $\overline{x}$ in a stationary equilibrium:

Lemma 4. *In a stationary equilibrium, the total mass of product lines owned by domestic firms is equal to the share of the aggregate creative destruction arrival rate accounted for by domestic firms. That is,*

$$s_d = \frac{\overline{x}_d}{\overline{x}} .$$

Proof: See Appendix B.1.7.1

Thus,

$$\sum_{\ell=1}^{4} \widetilde{\mu}\big(\Delta^{\ell}\big) = \frac{\overline{x}_d}{\overline{x}} \,.$$

Since domestic incumbent firms and foreign firms operating in domestic markets are symmetric in terms of their R&D and production technology, their technology gap distribution should differ only by a constant multiple. Thus the aggregate technology gap distribution is equal to $\mu\big(\Delta^{\ell}\big) = \frac{\overline{x}}{\overline{x}_d}\widetilde{\mu}\big(\Delta^{\ell}\big)$ for $\ell = 1, \ldots 4$, and sums up to one:

$$\sum_{\ell=1}^{4} \mu\big(\Delta^{\ell}\big) = 1 \,.$$

2.2.9.3 Aggregate Variables and Balanced Growth Path Equilibrium

Given the optimal innovation decision rules, aggregate domestic R&D expenses can be computed as

$$R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j \, \mathcal{I}_{\{\Delta_j = \Delta^{\ell} \, , \, j \in \mathcal{D}\}} \, dj \right] (z^{\ell})^{\widehat{\psi}} + \mathcal{F}_d \widetilde{\chi} \overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} \,, \tag{2.29}$$

where $\mathcal{I}_{\{\Delta_j = \Delta^{\ell} \, , \, j \in \mathcal{D}\}}$ is an indicator function equal to one if product line j belongs to a domestic firm with technology gap equal to Δ^{ℓ}. Also, using the optimal intermediate production rule, the total final goods used as inputs by domestic product firms can be written as

$$Y_d = \int_0^1 y_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj$$

$$= (1-\theta)^{\frac{1}{\theta}} \int_0^1 q_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj \; .$$

Since R&D expenses and intermediate production costs are paid with final goods, aggregate consumption becomes

$$C = Y - R_d - Y_d \; . \tag{2.30}$$

The total intermediate goods produced by foreign firms in this economy are

$$\begin{aligned} Y_o &= \int_0^1 p_j y_j \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \\ &= (1-\theta)^{\frac{1-\theta}{\theta}} \int_0^1 q_j \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \; . \end{aligned}$$

Since there is no government expenditure, the Gross Domestic Product (GDP) in this economy is

$$GDP = Y - Y_o \; .$$

With these aggregate variables defined, we can define the balanced growth equilibrium of this economy:

Definition 3 (Balanced Growth Path Equilibrium)**.** *A balanced growth path equilibrium of this economy consists of* y_j^*, p_j^*, w^*, L^*, x^*, $\{z^{\ell *}\}_{\ell=1}^4$, $\overline{x}^*$, x_e^*, $\mathcal{F}_d^*$, R_d^*, Y^*, C^*, g^*, $\mu(\mathcal{N})$, $\{\widetilde{\mu}(\Delta^\ell)\}_{\ell=1}^4$ *for every* $j \in [0,1]$ *with* q_j *such that: (i)* y_j^* *and* p_j^* *satisfy* (2.12) *and* (2.9)*; (ii) the wage rate* w^* *satisfies* (2.10)*; (iii) total labor for final good production* L^* *satisfies* (2.11)*; (iv) the probabilities of internal innovation* $\{z^{\ell *}\}_{\ell=1}^4$ *satisfy* (2.19), (2.20), (2.21)*, and* (2.22)*, and the probability* *of external innovation by incumbents* x^* *satisfies* (2.23)*; (v) the aggregate creative destruction arrival rate* $\overline{x}^*$ *satisfies* (2.26)*; (vi) the probability of external innovation of potential startups* x_e^* *satisfies* (2.25)*; (vii) aggregate output* Y^* *satisfies* (2.13)*; (viii) aggregate domestic R&D expense* R_d^* *satisfies* (2.29)*; (ix) aggregate consumption* C^* *satisfies* (2.30)*; (x) the BGP growth rate* g^* *satisfies* (2.27)*; (xi) the invariant distribution of the technology gap portfolio composition* $\mu(\mathcal{N})$ *and the total mass of domestic firms* $\mathcal{F}_d^*$ *satisfy inflow(*$\mathcal{N}$*) = outflow(*$\mathcal{N}$*); and (xii) the invariant technology gap distribution* $\{\widetilde{\mu}(\Delta^\ell)\}_{\ell=1}^4$ *satisfies* (2.28).

2.3 Empirical Evidence on Model Predictions

Before we analyze the quantitative implications of our model, we empirically test the model predictions by identifying the causal effect of competition on the composition of firm innovation (internal vs. external). The rise of China in the U.S. markets after China's WTO accession in 2001 will be treated as a quasi-experimental increase in competition induced by foreign firms.

2.3.1 Data and Measurement

To construct a comprehensive firm-level dataset containing measures of innovation and foreign competition, we combine the following seven sources: the USPTO PatentsView database, the Longitudinal Business Database (LBD), the Longitudinal Firm Trade Transactions Database (LFTTD), the Census of Manufactures (CMF), the UN Comtrade Database, the NBER-CES database, and the tariff data compiled by Feenstra et al. (2002).

The LBD tracks the universe of establishments and firms in the U.S. non-farm private sector with at least one paid employee annually from 1976 onward.[10] An establishment corresponds to

[10] Details for the LBD and its construction can be found in Jarmin and Miranda (2002).

the physical location where business activity occurs. Establishments that are operated by the same entity, identified through the Economic Census and the Company Organization Survey, are grouped under a common firm identifier. We aggregate establishment-level information into firm-level observations using these firm identifiers. Firm size is measured by either total employment or total payroll. Firm age is based on the age of the oldest establishment of the firm when the firm is first observed in the data. The firm's main industry of operation is based on the six-digit North American Industry Classification System (NAICS) code associated with the establishment with the highest level of employment. Time-consistent NAICS codes for LBD establishments are constructed by Fort and Klimek (2018), and the 2012 NAICS codes are used throughout the entire analysis.

The LFTTD tracks all U.S. international trade transactions starting from 1992 onward at the firm level.[11] The LFTTD provides the U.S. dollar value of shipments, and the origin and destination country for each transaction, as well as a related-party flag, which indicates whether the U.S. importer and the foreign exporter are related by ownership of at least 6 percent.

The USPTO PatentsView database tracks all patents ultimately granted by the USPTO from 1976 onward.[12] This database contains detailed information for granted patents including application and grant dates, technology class, other patents cited, and the name and address of patent assignees. It also provides the list of inventors responsible for each patent with their locations. In the following analyses, we use the citation-adjusted number of utility patent applications as the main measure of firm innovation.[13] Using detailed information for each patent, we distinguish domestic innovation from foreign innovation, and measure the extent to which each patent repre-

[11] Bernard et al. (2009) describe the LFTTD in greater detail.

[12] See http://www.patentsview.org/download/.

[13] See Cohen (2010) for a comprehensive review of the literature on the determination of firms' and industries' innovative activity and performance and how patent-related measures are used.

sents internal innovation. The year in which a patent application is filed is used as a proxy for the innovation year. The citation-adjusted average of the internal innovation measure for the flow of patent applications in each firm-year is used as a proxy for the overall internal innovation at each firm in each year. We discuss the measure of internal innovation in detail shortly.

We match the USPTO patent database to the LBD to assign detailed firm-level information and firm-industry-level changes in trade flows to each patent. In the following analyses, we compare firms' patenting behavior across different years. Thus, match quality is important—failing to match a firm in the USPTO patent database in a particular year to its LBD counterpart will result in mismeasuring the changes in innovation. This problem arises because the USPTO doesn't track a consistent unique firm ID. The USPTO assigns patent applications to self-reported firm names. Thus, it is vulnerable to misspelling of firm names. To overcome this match quality issue, we adopt the Autor et al. (2019) methodology, which utilizes the machine-learning capacities of the internet search engine. We use all patents granted up to December 26, 2017 during the matching procedure, and use patent applications up to 2007 in the subsequent analyses. Thus, the following analyses are virtually free from the right censoring issue (mismeasuring firms' innovation activities due to patents applied for but not yet granted). Table C4 in the Appendix reports summary statistics for patenting firms in 1992.

The quinquennial CMF provides detailed information for activities by establishments in the manufacturing sector. It also provides detailed product codes and breaks down the value of shipments for all products each establishment sells. We use five-digit SIC codes for observations up to 1997, and seven-digit NAICS codes for observations from 2002 onward, to measure firms' product choices.

The UN Comtrade Database provides information for world trade flows at the six-digit

HS product-level from 1991 to 2016.[14] The six-digit HS codes are concorded to six-digit 2012 NAICS industries using the Pierce and Schott (2009, 2012) crosswalks. We construct industry-level imports and exports using the UN Comtrade Database. Also, we obtain U.S. tariff schedules from Feenstra et al. (2002) to measure industry-level Trade Policy Uncertainty (TPU), which is used to measure shocks to foreign competitive pressure. The construction of this measure is discussed in detail in the following section.

The NBER CES Manufacturing Industry Database, assembled by Becker et al. (2013), is used to obtain the industry-level deflator for the value of shipments for manufacturing industries from 1976 to 2011.[15] All nominal values are converted to 1997 U.S. dollars using this industry-level deflator for the value of shipments for manufacturing industries, and the BEA's Consumer Price Index for other industries. In the following analyses, we use subsets of a sample of USPTO patents matched to U.S. firms in the LBD and industry-level trade data from 1982 to 2007 for each regression specification.

2.3.1.1 Measure of Internal Innovation

In this paper, we use the self-citation ratio as a measure of whether a patent primarily reflects internal innovation. Each granted patent is required to cite all prior patents on which it builds itself. When a cited patent belongs to the owner of the citing patent, these citations are called self-citations. Akcigit and Kerr (2018) use the self-citation ratio—defined as the ratio of self-citations to total citations—as a measure of the likelihood each patent is used for internal innovation. The more an idea is based on the firm's internal knowledge stock (self-citation), the

[14] https://comtrade.un.org/db/default.aspx.
[15] http://www.nber.org/nberces/.

more likely the innovation is used for improving the firm's existing products (internal innovation). A higher self-citation ratio means that a patent is more likely to reflect internal innovation.[16]

2.3.1.2 Measures of Foreign Competition

As shown by Handley and Limão (2017), over one-third of the growth of imports from China to the U.S. in the first half of the 2000s can be explained by the U.S. granting permanent normal trade relations (PNTR) to China upon China's 2001 accession to the WTO. Nonmarket economies such as China are by default subject to relatively high tariff rates, originally set under the Smoot-Hawley Tariff Act of 1930, when they export to the U.S. These rates are known as non-Normal Trade Relations (non-NTR) or column 2 tariffs. On the other hand, the U.S. offers WTO member countries NTR or column 1 tariffs, which are substantially lower than non-NTR tariffs. The Trade Act of 1974 allows the President of the United States to grant temporary NTR status to nonmarket countries on an annually renewable basis after approval by Congress. Starting from 1980, U.S. Presidents granted such waivers to China.

While China never lost these waivers and the tariff rates applied to Chinese products were kept low, the process of annual approval by Congress created uncertainty about whether the low tariffs would revert to non-NTR rates. After the Tiananmen Square protests in 1989, Congress voted on a bill to revoke China's temporary NTR status every year from 1990 to 2001. Following the bilateral agreement on China's entry into the WTO between the U.S. and China in 1999, Congress passed a bill granting China PNTR status in October 2000. Upon China's accession to the WTO in December 2001, PNTR became effective and was implemented on January 1, 2002.

[16]Thus, 100% self-citation means the patent is used for internal innovation with a 100% probability, and 0% self-citation means the patent is used for external innovation with a 100% probability.

PNTR removed the uncertainty about U.S. trade policy toward China by permanently setting tariff rates on Chinese products at NTR levels. This lowered the expected U.S. import tariffs on Chinese products, and eliminated any option value of waiting for firms needing to incur large fixed costs in order to export products from China to the U.S. Thus, PNTR reduced trade policy uncertainty (TPU), the more so for industries with a large prior gap between tariff rates under NTR and non-NTR regimes.

We use the industry-level gap between NTR tariff rates reserved for WTO members and non-NTR tariff rates for non-market economies in the year 1999 as a proxy for the industry-level competitive pressure shock from China occurring in 2001.[17] Thus, for industry j,

$$NTRGap_j = Non\ NTR\ Rate_j \ - \ NTR\ Rate_j \ .$$

If a firm operates in multiple 6-digit NAICS industries, we use the employment-weighted average $NTRGap_j$. We use the unweighted average trade shock and the trade shock to firms' main industry as robustness checks. Table C1 and Table C2 in the Appendix report summary statistics for each trade shock measure.

2.3.2 Empirical Strategies and Main Results

The theory developed in the previous section provides the two following empirically testable predictions: i) the escape-competition effect, and ii) the technological barrier effect. We now test these two model predictions.

[17] We can consider the NTR gap as a first-order Taylor approximation of model-based TPU measures, such as Handley and Limão (2017), that is positively related to non-NTR rate and negatively related to NTR rate.

2.3.2.1 The Escape-Competition Effect

The first prediction of our model is that firms that have innovated intensively in recent periods increase internal innovation more when they are faced with higher competition, compared to their low innovation counterparts. This is because innovation-intensive firms can escape competition more easily through additional internal innovation, by leveraging the technological advantages (or technological barriers) that they built in their own markets through recent intensive innovation.

Following Pierce and Schott (2016), we use a Difference-in-Difference (DD) specification to identify the effect of the China competitive pressure shock on U.S. firm innovation for two periods, $p \in \{1992-1999\ ,\ \ 2000-2007\}$, for firm i in industry j:

$$\begin{aligned}\Delta y_{ijp} \ &= \beta_1 Post_p \times NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+\ \beta_2 Post_p \times NTRGap_{ijp0}\ +\ \beta_3 Post_p \times InnovIntens_{ijp0} \\ &+ \beta_4 NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+\ \beta_5 NTRGap_{ijp0}\ +\ \beta_6 InnovIntens_{ijp0} \\ &+\ \mathbf{X}_{ijp0}\,\gamma_1\ +\ \mathbf{X}_{jp0}\,\gamma_2\ +\ \delta_j\ +\ \delta_p\ +\ \alpha\ +\ \varepsilon_{ijp}\ ,\end{aligned} \tag{2.31}$$

where Δy_{ijp} is the DHS (Davis et al., 1996) growth rate of y between the start-year and end-year for each period $p \in \{1992-1999\,,\ 2000-2007\}$ for firm i in industry j, where y_{ijp} is either i) the total citation-adjusted number of patents, or ii) the citation-weighted average self-citation ratio. An increase in the self-citation ratio means that the firm's innovations became more internal. To maximize the sample size, we include firms that applied for at least one patent in the start-year and at least one patent in or before the end-year for each period, and compute the DHS growth rates for the longest span of years available. We also require firms to have at least one patent before the start-year of each period, or to have age > 0, to avoid the effect coming from firm entry. The sample includes all LBD firms matched to the USPTO patent database that meet these three criteria, except for firms in FIRE (finance, insurance, and real estate) industries.

$Post_p$ is a dummy variable equal to one for the period 2000-2007 and zero otherwise. This variable captures changes in firm innovation after China's WTO accession. $\mathbf{X}_{ijp0}$ is a vector of firm controls, and $\mathbf{X}_{jp0}$ is a vector of industry controls, both measured at the start-year for each period (1992 or 2000).[18] δ_j is an industry fixed effect (six-digit NAICS), and δ_p is a period fixed effect. All models are unweighted, and standard errors are clustered on the 6-digit NAICS industries.

$InnovIntens_{ijp0}$ is a continuous variable equal to the lagged five-year average of the ratio of the number of firm i's patent applications to total employment, measured in the start year for each period p. We control for industry-fixed effects for this measure by dividing it by its time-average at the 2-digit NAICS level. Thus, we are examining the impact of heterogeneity of innovation intensity within industries rather than differences across industries.

In these specifications, firms in low TPU industries are the control group, whereas firms in high TPU industries are the treatment group. We use the 1992 and 2000 cohorts of firms to measure firm innovation before and after the policy change in December 2001. In this way, the composition of firms in terms of their innovation is minimally affected by the policy change. The escape-competition hypothesis predicts β_1 to be positive when changes in the self-citation ratio

[18]Firm controls include: firm employment, firm age, past 5-year growth of U.S. patents in the CPC technology classes in which the firm operates, and dummy variables for publicly traded firms, exporters, importers, and offshoring firms. Industry control variables include NTR rates measured at the start of each period.

are used as the dependent variable.

Table 2.1 shows the estimates of β_1.[19] As indicated in column (4) of Table 2.1, the estimate for β_1 is positive and statistically significant when the growth rate of the self-citation ratio is the dependent variable, consistent with the model predictions. This estimated value for β_1 implies a 4.1 percentage points increase in the growth rate of the average self-citation ratio during the period 2000-2007 for a firm with average lagged innovation intensity (0.18) in an industry with an average NTR gap (0.291). The average value of the seven-year growth rate of the average self-citation ratio between 2000 and 2007 is 28.2 percentage points. Thus, this effect represents about a 14.6% increase in internal innovation.

The estimated effect is economically important as well. Table C11 in Appendix B.3.4 shows that for an average firm, creating 4 more patents is associated with a 3.4 percentage points increase in employment growth, but the association becomes smaller in magnitude if the average self-citation ratio of the new patents is high. The estimates in Table 2.1, combined with Table C11, suggest that the association between patenting and employment growth is decreased by 1.13 percentage points for firms with average innovation intensity following the competitive pressure shock from China.

Lastly, column (2) of Table 2.1 shows that the Chinese competitive pressure shock has no statistically significant effect on firms' overall innovation. Our model predicts that some firms increase their internal innovation while others decrease theirs in response to increased foreign competition, while all firms lower their external innovation. When these heterogeneous responses are combined, there need not be a significant overall effect. Thus, the regression results are con-

[19]To conserve space, Table 2.1 reports coefficients estimates for triple interaction terms only. Results including coefficients for all the interaction terms are reported in Table C8 in the Appendix.

Table 2.1: Escape-Competition Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post × Innov.-inten.	0.077	-0.017	0.732**	0.784***
	(0.231)	(0.233)	(0.299)	(0.268)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

sistent with the model prediction. And because firms do not change their overall innovation, the increasing self-citation ratio implies that innovative firms (firms with above-average innovation intensity) increase their internal innovation while decreasing their external innovation.

2.3.2.2 Discussion: PNTR as a Measure of Competitive Pressure

As discussed extensively in Pierce and Schott (2016) and Facchini et al. (2019), the main channel by which the removal of trade policy uncertainty affects trade between the U.S. and China is by persuading Chinese firms to export their products to the U.S. The two papers verify this channel by estimating the effect of the removal of TPU on changes in Chinese exports to the U.S. using the LFTTD at the product level, and Chinese Custom Data at the firm level. Table C9 in the Appendix shows OLS estimates of the effect of PNTR on changes in U.S. imports from China from 2000 to 2007 at the 8-digit HS level and the 6-digit NAICS level separately. As indicated in the table, the pre-2000 NTR gap is positively associated with post-2000 changes in U.S. imports from China regardless of the level of aggregation. However, statistical significance falls from the 1% to the 10% level as we move from the 8-digit HS level to the 6-digit NAICS level, where the latter is the level of aggregation used in this paper.

As is clear from the baseline model introduced in Section 2.2, firms consider competitive pressure when they decide how much to invest in internal innovation. In the real world, pressure can come from both realized competition (an increase in the number of competitors) and from anticipated competition (an increase in the number of potential entrants). Table C10 shows OLS results from regressing the two dependent variables of interest on interactions involving the realized changes in U.S. imports from China, to estimate the effect of realized competition on the composition of firm innovation. Here, we simply replace the NTR gap terms in equation (2.31) with the realized changes in U.S. imports from China and use the same two seven-year periods used in the previous analysis, 1992-1999 and 2000-2007. As the table indicates, changes in U.S. imports from China from 1992 to 2007 do not have any statistically significant effect on the composition of innovation after we control for firm characteristics.

This analysis, however, has two concerns: i) changes in U.S. imports from China (a measure of realized competition) are endogenous with respect to various factors affecting innovation, implying omitted variables bias, and ii) successful escape competition by U.S. firms can make realized competition low even if competitive pressure is substantial, implying reverse causality. The first concern can be addressed by using the imposition of PNTR as an instrument for changes in imports. However, as Table C9 shows, the NTR gap has low statistical power for predicting changes in U.S. imports from China at the 6-digit NAICS level. This indicates that the NTR gap is a weak instrument for realized competition.

Our model suggests that the second concern is important, and that measures of realized competition inherently cannot capture the amount of competition escaped. The removal of trade policy uncertainty, however, can be an excellent proxy for increased competitive pressure, as it is associated with an increase in Chinese firms' opportunity to enter the U.S. market. For example, Handley and Limão (2017) show that a reduction in TPU increases incentive for incumbents to incur irreversible investments to enter foreign markets. Erten and Leight (2019) further show that the imposition of PNTR induces Chinese manufacturing firms to increase their investment and their value-added per worker. These findings suggest a tight relationship between the imposition of PNTR and an increase in potential future competition. Finding direct evidence for this relationship, such as a link between PNTR and the number of Chinese startups or the number of Chinese firms with the ability to export their products to the U.S., is a priority for future research.

2.3.2.3 Validity of the Identification Strategy and Robustness Tests

Previous studies using PNTR with China as a competitive pressure shock, such as Pierce and Schott (2016) and Handley and Limão (2017), provide rich evidence for the exogeneity of PNTR for U.S. firms' decisions in the 1990s and 2000s. Thus, we focus on testing the parallel pre-trends assumption, the key identifying assumption for the DD model. To test the assumption for the dependent variables of interest, we estimate (2.31) for two seven-year periods before the policy change, 1984-1991 and 1992-1999. Table C12 in the Appendix shows the results, which support the validity of the parallel pre-trends assumption.

To further confirm the validity of our results, we perform several robustness checks, with results reported in the Appendix. First, we include upstream and downstream competitive pressure shocks as covariates in model (2.31). By using the 1992 BEA input-output table, we construct upstream and downstream competitive pressure shocks as weighted averages of industry-level trade shocks. The upstream measure shows the effect of trade is the effect of trade shocks propagating upstream from an industry's buyers, and the downstream measure shows the effect of trade shocks propagating downstream from its suppliers.[20] Table C13 in the Appendix shows that including controls for I-O linkages does not change the main results.

The second test uses different weights for constructing firm-level NTR gaps. Because patenting firms are typically multi-industry firms, in our baseline regressions we use employment in the start year of each period as weights and construct a weighted average of industry-level NTR gaps for all industries in which the firm operates. As a robustness check, we also use an unweighted average of this measure, as well as industry-level NTR gaps for firms' main industry (the industry with the most employment) as alternative measures for TPU in model (2.31). Table C15 in the Appendix shows that using these alternative measures does not change the main results.

The third test addresses possible selection bias resulting from including only firms with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis. This selection is inevitable as we need to compute the self-citation ratio for two years in each period. We correct for this bias by re-weighting the regression sample, using as weights the inverse of the propensity scores from a logit model with an indicator for being in the analysis sample as the dependent variable. Table C16 in the Appendix shows that this reweighting does not change the results. The fourth test adds the cumulative number of patents as a firm-level control variable in the model (2.31). The self-citation ratio can mechanically increase over time because the firm's patent stock increases as the firm becomes older. Adding the cumulative number of patents as a firm-level covariate addresses this issue, and Table C17 in

[20]Following Pierce and Schott (2016), for each 6-digit NAICS industry, we set the I-O weights to zero for both up and downstream industries belonging to the same 3-digit NAICS broad industries while computing the indirect effects to take into account the findings from Bernard et al. (2010) that U.S. manufacturing establishments often produce clusters of products within the same 3-digit NAICS sector.

the Appendix shows that this does not change the results.

Most variation in the firm-level NTR gap occurs at the industry-level. Thus, we cluster standard errors at the six-digit NAICS level in the main analysis. As a further robustness check, we cluster standard errors on firms, and Table C18 in the Appendix shows this does not change our inference on the main results. Finally, we test the robustness of our results by using the number of products added—an alternative measure for external innovation (the inverse of internal innovation)—as the dependent variable. Table C19 in the Appendix shows results that support the model prediction, that higher competitive pressure reduces the number of new products added for innovative firms, consistent with the model's predictions.

2.3.2.4 The Technological Barrier Effect

Another prediction from the model is that firms do less external innovation if other firms have performed more innovation in the past period. Intensive innovation by other firms raises the technology barrier in other markets on average, which implies that business take over through external innovation becomes more difficult. Thus, firms optimally reduce their R&D spending on external innovation. To test this theoretical prediction, we use the recent increase in the number of foreign patent applications as a proxy for increasing innovation intensity in other markets. Since we don't have product-market information for foreign firms, we use patent technology class (CPC) as a proxy for product in this exercise. Foreign patents are defined as patents filed by foreign firms whose first listed inventor is a foreigner. We use the pre-shock years from the period 1989 to 2000 and construct non-overlapping five-year first differences (DHS growth for

1989-1994 and 1995-2000) to estimate the following fixed-effects model:

$$\Delta Y_{ijt+5} = \beta_1 \overline{\Delta S}^{Own}_{ijt-5} + \beta_2 \overline{\Delta S}^{Outside}_{ijt-5} + \mathbf{X}_{ijt}\,\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}$$

ΔY_{ijt+5} is either the 5-year DHS growth rate of the citation-adjusted number of patents or the average self-citation ratio between t and $t+5$, and $\overline{\Delta S}^{tech}_{ijt-5}$ for $tech \in \{Own,\ Outside\}$ is the lagged average 5-year DHS growth rate of foreign patents inside firm i's own technology space (Own) or outside firm i's technology space ($Outside$).

To be more specific, for each technology class c in CPC, denote the total number of foreign patents filed in year t as $S_{c,t}$. Then the DHS growth rate of foreign patents belonging to c between year $t-5$ and t can be written as

$$\Delta S_{c,t-5} \equiv \frac{S_{c,t} - S_{c,t-5}}{0.5 \times (S_{c,t} + S_{c,t-5})} \; .$$

Let Q_t denote the set of all patent technology classes available through year t, and Q_{ijt} as the portfolio of patent technology classes firm i has accumulated through year t. This defines the technology space in which firm i operates. Furthermore, denote $\omega_{i,j,c,t}$ as the share of patent technology class c in firm i's technology portfolio accumulated through year t. Then the lagged growth in foreign innovation intensity in firm i's own technology space, $\overline{\Delta S}^{Own}_{ijt-5}$, is defined as

$$\overline{\Delta S}^{Own}_{ijt-5} \equiv \sum_{c \in Q_{ijt}} \omega_{i,j,c,t} \Delta S_{c,t-5} \; ,$$

Table 2.2: Technological Barrier Effect

	ΔPatents (1)	ΔSelf-cite (2)
Past 5 year Δforeign patent, outside of firm's own tech. fields	-5.984**	9.076***
	(2.756)	(2.711)
Observation	7,600	7,600
Fixed effects	jp	jp

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log payroll, firm age, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

while the counterpart measure outside of firm i's space, $\overline{\Delta S}_{ijt-5}^{Outside}$, is defined as

$$\overline{\Delta S}_{ijt-5}^{Outside} \equiv \frac{1}{\|Q_{ijt}^c\|} \sum_{c \in Q_{ijt}^c} \Delta S_{c,t-5} \ ,$$

where $Q_{ijt}^c \equiv Q_t \backslash Q_{ijt}$ is the complement of the set Q_{ijt}, and $\|Q_{ijt}^c\|$ is the number of technology classes in Q_{ijt}^c. Table C3 in the Appendix reports summary statistics for the technology shock measures.

Table 2.2 shows estimates of β_2.[21] The regression is unweighted and standard errors are clustered by firm. We include industry-period fixed effects to control for industry-level shocks. The theory predicts β_2 to be positive when the change in the self-citation ratio is the dependent variable, and insignificant or negative for changes in the total number of patents. As the table indicates, U.S. firms create fewer patent applications when recent outside innovation by foreign firms is high, and firms' innovation is more internal in nature. This suggests that U.S. firms perform less external innovation when the technological barrier in outside product markets is

[21]Table C20 in the Appendix shows the estimation results for own technology field shock, as well as the results including the interaction with firms' innovation intensities. We also run the same regression specification using concurrent technology shock, and Table C21 in the Appendix shows the results. The results are widely consistent with that of the lagged technology shock.

higher.

2.4 Quantitative Analysis

In this section, we calibrate the model to the average characteristics of the U.S. manufacturing sector from 1987 to 1997, and study how an increase in competitive pressure by foreign firms affects U.S. firms' innovation decisions. Then, we run the same exercise in a model economy in which external innovation is much more expensive than the U.S., and compare the results with those from the previous exercise. This comparison highlights how the same competitive pressure shock can lead to a decrease in overall innovation in an economy with high inherent creativity (an economy with less expensive external innovation), and an increase in overall innovation in an economy with low inherent creativity (an economy with more expensive external innovation). Lastly, we run an exercise in which we reduce the cost of external innovation by potential startups, which increases competitive pressure by domestic entrants.

2.4.1 Solution Algorithm

In the model, $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of $\mathcal{F}_d$, x, and x_e. Therefore, we solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

i) Guess a value for $\overline{x}$ and the technology gap portfolio composition distribution $\mu(\mathcal{N})$, which imply a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and total mass of domestic firms $\mathcal{F}_d$.

ii) Using the guess for $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$.

Table 2.3: Parameter Estimates

#	Parameter	Description	Value	Identification
1.	β	time discount rate	0.9615	annual interest rate of 4%
2.	$\widehat{\psi}$	curvature of internal R&D	2	Akcigit and Kerr (2018)
3.	$\widetilde{\psi}$	curvature of external R&D	2	Akcigit and Kerr (2018)
4.	$\widetilde{\psi}^e$	curvature of external R&D, startup	2	Akcigit and Kerr (2018)
5.	θ	quality share in final goods production	0.109	data
6.	$\widehat{\chi}$	scale of internal R&D	0.042	indirect inference
7.	$\widetilde{\chi}$	scale of external R&D	1.184	indirect inference
8.	$\widetilde{\chi}^e$	scale of external R&D, startup	7.696	indirect inference
9.	λ	quality multiplier of internal innovation	0.021	indirect inference
10.	η	quality multiplier of external innovation	0.038	indirect inference
11.	$\overline{x}_o$	exogenous foreign c.d. arrival rate	0.045	indirect inference

iii) Using the guesses for $\mu(\mathcal{N})$, $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, and $\mathcal{F}_d$,

a) Compute g, x, B, and x_e.

b) Compute stationary $\mu_\infty(\mathcal{N})$, thus $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, using the guesses for $\mu(\mathcal{N})$, innovation decision rules and the relationship

$$\mathcal{F}_{d,n+1}\,\mu_{n+1}(\mathcal{N}) = \mathcal{F}_{d,n}\,\mu_n(\mathcal{N}) + \text{inflow}_n(\mathcal{N}) - \text{outflow}_n(\mathcal{N})\,.$$

c) Compute g_∞, x_∞, B_∞, and x_{e_∞} using $\mu_\infty(\mathcal{N})$, and $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$.

iv) Compute $\overline{x}' = \mathcal{F}_{d,\infty}\,x_\infty + \mathcal{E}_d\,x_{e_\infty}$.

v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, and $\mu(\mathcal{N}) = \mu_\infty(\mathcal{N})$, use them as new guesses, and return to ii).

vi) iterate ii) to v) until $\overline{x}$ converges.

2.4.2 Calibration

The eleven structural parameters of the model, listed in Table 2.3, are calibrated in two ways. The first group of five parameters is externally calibrated according to the literature and the data. The second group of six parameters is internally calibrated to match target moments from firm level data and the import penetration ratio in the U.S. manufacturing sector from 1987 to 1997.[22] A sample of firms is drawn from the universe of innovative manufacturing firms in the 1987 through 1997 censuses.[23] The total mass of potential domestic startups ($\mathcal{E}_d$) is set equal to one.

2.4.2.1 Externally Calibrated Parameters

The time discount factor (β) is set equal to 0.9615, which corresponds to an annual interest rate of 4%. The curvatures of the three R&D cost functions ($\widehat{\psi}$, $\widetilde{\psi}$, $\widetilde{\psi}^e$) are taken from Akcigit and Kerr (2018), who base their parameterization on two lines of literature: one evaluating the empirical relationship between patents and R&D expenditure, and the other evaluating the impact of R&D tax credits on the R&D expenditure of firms. The average profit-to-sales ratio in the model is equal to $\int_f \frac{profit_f}{sales_f} df = \theta$, where profits include R&D expenditures. Thus the quality share in final goods production (θ) is set equal to the corresponding number from the data, which is 10.9% for the 1982-1997 period according to Akcigit and Kerr (2018).

2.4.2.2 Internally Calibrated Parameters

The remaining six parameters are estimated using an indirect inference approach. For a given set of six parameter values, we compute six model-generated moments, compare them

[22]The import penetration ratio in the manufacturing sector is defined as the ratio between manufacturing imports and manufacturing value added net of exports plus imports. The manufacturing imports and exports are taken from World Development Indicators, and manufacturing value added is taken from the Bureau of Economic Analysis.

[23]Innovative firms are defined as firms with positive R&D expenditure or a positive number of patent filings. The R&D to sales ratio, firm entry rate, and average sales growth rate are taken from Akcigit and Kerr (2018), whose sample period is from 1982 to 1997. The average number of products is taken from Bernard et al. (2010), and the high-growth firm growth rate is taken from Decker et al. (2016b).

Table 2.4: Target Moments

Moment	Data	Model	Moment	Data	Model
R&D to sales ratio (%)	4.1	4.1	avg. sales growth rate (%)	1.0	1.0
avg. number of products	3.5	1.5	high-growth firm growth rate (%)	22.8	22.8
firm entry rate (%)	5.8	5.8	import penetration in manuf. (%)	37.4	37.4

to the moments from the data, and find a set of parameter values that minimizes the objective function

$$\min \sum_{i=1}^{6} \frac{\left|\text{model moment}_i - \text{data moment}_i\right|}{\frac{1}{2}\left|\text{model moment}_i\right| + \frac{1}{2}\left|\text{data moment}_i\right|} .$$

The six moments are shown in Table 2.4 and are chosen in consideration of both their importance in answering the main question of this paper, and the relationships among the moments and the parameters coming from the choice of functional forms in the model. Although all the parameter values contribute substantially in determining the value for each model-generated moment, the tight relationship between certain sub-groups of parameters and moments can be noted.

Firms perform internal and external R&D to increase the number of product lines they operate. Since R&D cost is one of the important factors in determining the level of R&D intensity, and hence the number of product lines the firm owns, we discipline the scale of internal R&D ($\widehat{\chi}$) and the scale of external R&D ($\widetilde{\chi}$) through the R&D to sales ratio and the average number of products per firm.

Potential startups learn and improve existing technologies to enter the market, and the success probability of entry is tightly related to their level of R&D expenditure. Thus we discipline the scale of external R&D for startups ($\widetilde{\chi}^e$) using the firm entry rate.

Firms grow in terms of both sales and number of employees by improving the quality of their existing products and by adding new product lines to their product lines. How fast they can grow depends on how much improvement they can achieve in product quality. Thus we discipline the quality multiplier of internal innovation (λ) and the quality multiplier of external innovation (η) through the average sales growth rate and the employment growth rate of high-growth firms (the 90th percentile of the firm employment growth distribution). In the baseline model, intermediate producers use the final good for production. We compute the number of workers hired by the final good producer to produce the final goods used by intermediate producers to compute their (implied) employment growth rates.

Finally, we discipline the initial value for the exogenous foreign creative destruction arrival rate $\overline{x}_o$ using the import penetration ratio in the manufacturing sector, as the exogenous foreign creative destruction arrival rate is tightly related to the share of domestic differentiated product markets occupied by the foreign exporters.

Table 2.4 reports the model generated moments. The model matches the target moments very closely, except for the average number of products. This illustrates the drawback coming from the assumption that firms can make only one external innovation at a time. It becomes very hard for a model firm to add one more product line as its number of product lines increases. Roughly speaking, the probability of adding one more product line for a firm with n_f product lines is equal to $\overline{x}_{takeover}x(1-\overline{x})^{n_f}$, without considering internal innovation. The bar graphs in Figure 2.2 with solid lines show the distribution of the number of product lines and the technology gap distribution computed using the parameter values reported in Table 2.3. As we can see, the product line distribution resembles a Pareto distribution. Roughly 60% of the product lines have a technology gap equal to one under the calibrated parameter values. This might be another

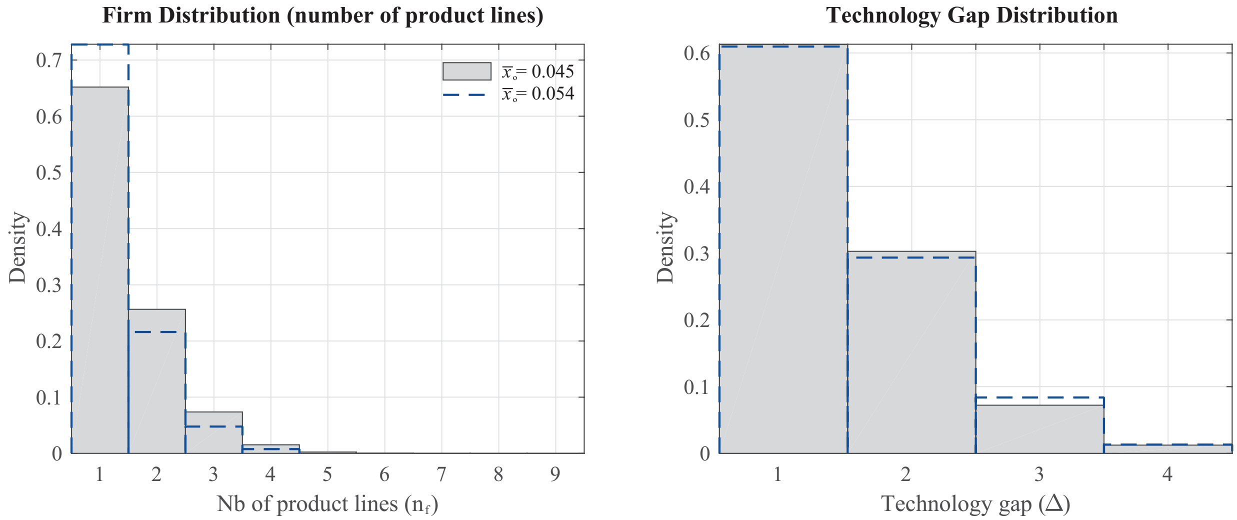


Figure 2.2: Firm Distribution and Technology Gap Distribution Changes

symptom of problems arising from the assumption of only one external innovation at a time, which influences the level of the technological barrier effect in the quantitative analysis.

2.4.3 Counterfactual Exercises

2.4.3.1 Increasing Competitive Pressure from Foreign Firms

In this section, we use the calibrated model to assess the impact of increasing competitive pressure from foreign firms on individual firms' behavior, particularly their overall innovation, composition of innovation, and employment growth rate. More specifically, we increase the value of $\overline{x}_o$ from 0.045 to 0.054 (20% increase). This is equivalent to an increase in the import penetration ratio in the U.S. manufacturing sector from 37.4% to 43.5% (a 6.1% increase).

Table 2.5 reports the resulting changes in variables related to firm-level innovation intensity. An exogenous increase in the foreign creative destruction arrival rate $\overline{x}_o$ increases the aggregate creative destruction arrival rate $\overline{x}$. As reported in Table 2.6, the expected profits from internal

Table 2.5: Changes in Innovation following Increases in Foreign Competition

description	variables	before	after	% change
foreign creative destruction arrival rate	$\overline{x}_o$	0.045	0.054	20.00%
creative destruction arrival rate	$\overline{x}$	0.120	0.123	2.98%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.224	0.224	-0.03%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.653	0.656	0.60%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.453	0.456	0.54%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.438	0.440	0.44%
prob. of external innovation, incumbents	x	0.097	0.095	-2.20%
prob. of external innovation, potential startups	x_e	0.033	0.032	-3.33%
conditional takeover probability	$\overline{x}_{takeover}$	0.747	0.746	-0.23%
unconditional takeover probability	$x_{takeover}$	0.073	0.071	-2.43%

innovation and production ($\{A_\ell\}_{\ell=1}^4$) and external innovation (B) decrease. These have negative Schumpeterian effects on firms' incentives for internal and external innovation. However, the escape-competition effect dominates for product lines with positive technology gaps. Thus, incumbent firms attempt to protect their existing product lines by increasing their internal innovation intensity for product lines with a technology gap higher than one, where the relative magnitudes of changes are in alignment with Corollary 4. Due to this increased internal innovation intensity and the heightened overall external innovation intensity—the higher value for the aggregate creative destruction arrival rate—the technology gap distribution changes, as reported in Table 2.7 and shown in Figure 2.2. Along with increased probabilities of internal innovation, this change in the technology gap distribution towards higher densities of Δ^3 and Δ^4 lowers the value of $\overline{x}_{takeover}$, the conditional takeover probability, which is what we call the technological barrier effect. Both the Schumpeterian effect and the technological barrier effect affect firms' incentive for external innovation negatively. Therefore, firms optimally lower their investment in external innovation. Recall that the optimal probability of external innovation x is an increasing

Table 2.6: Firm Value Changes following Increases in Foreign Competition

description	variables	before	after	% change
Firm Values	A_1	0.290	0.283	-2.18%
	A_2	0.305	0.299	-2.00%
	A_3	0.313	0.307	-1.95%
	A_4	0.295	0.289	-2.11%
	B	0.393	0.377	-4.03%

Table 2.7: Technology Gap Distribution Changes following Increases in Foreign Competition

description	variables	before	after	% change
Technology gap distribution (shares)	$\Delta^1 = 1$	0.613	0.612	-0.10%
	$\Delta^2 = \lambda$	0.303	0.301	-0.55%
	$\Delta^3 = \eta$	0.072	0.074	2.93%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.012	0.013	1.34%

function of $A_{takeover}$, where

$$A_{takeover} \equiv \frac{1}{2}(1 - z^3)A_1\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right) A_2\lambda\mu(\Delta^4)$$
$$+ A_3\eta\mu(\Delta^1) + (1 - z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2) .$$

Thus we can decompose the changes in x into two parts: one resulting from the Schumpeterian effect and the other from the technological barrier effect. Holding the expected future profits fixed at their initial levels, we find that 10.3% of the changes in x in our experiment are due to the technological barrier effect. Similarly, potential startups' external innovation intensity also drops, and this drives the decrease in the total mass of domestic startups.

The change in the technology gap distribution is affected by the assumption of only one external innovation at a time. Firms can have a product line with a technology gap equal to either Δ^3 or Δ^4 only through external innovation. Since incumbent firms are allowed to add only one product line per period, a large share of product lines with a technology gap equal to Δ^3

or Δ^4 belong to startups (both domestic and foreign). Thus, the share of product lines with the technology gap equal to Δ^3 or Δ^4 increases more than that of Δ^2 after an increase in competition from foreign startups. We conjecture this is the reason why we see a drop in the share of product lines with Δ^2 despite the overall increase in internal innovation intensity. This change in the technology gap distribution is one of the reasons for the mild decrease in the conditional takeover probability $\overline{x}_{takeover}$.

Table 2.8 reports changes in some of the model generated moments. Importantly, the overall domestic R&D to sales ratio drops as a result of increasing competitive pressure from foreign firms. This is because external innovation falls by more than the increase in internal innovation. Consequently, the external R&D intensity, measured as the ratio of total domestic R&D expenses for external innovation to total domestic R&D expenses for all innovation, also drops. The total masses of both domestic firms and domestic startups decrease. However, the total mass of domestic firms decreases by more, so that the domestic firm entry rate increases. The average number of products for each firm decreases after an increase in competitive pressure from foreign firms. This is in alignment with the empirical findings of Bernard et al. (2011). Using the U.S. Linked/Longitudinal Firm Trade Transaction Database and the U.S. Census of Manufactures, they find that firms experiencing higher tariff reductions after the Canada-U.S. Free Trade Agreement reduce the number of products they produce relative to firms experiencing smaller tariff reductions. The average firm sales growth rate, which is equal to the aggregate growth rate g in the model economy, increases after an increase in competitive pressure from foreign firms. However, this increase is completely driven by foreign exporters. Table 2.9 reports the decomposition of the change in the aggregate growth rate. After subtracting the contribution accounted for by foreign exporters, the aggregate growth accounted for by domestic firms falls by 0.69%

Table 2.8: Changes in Domestic Firm Entry, Exit, and Other Moments following Increases in Foreign Competition

description	before	after	% change
R&D to sales ratio (%)	4.124	4.064	-1.46%
external R&D intensity (%)	50.774	49.910	-1.70%
total mass of domestic firms	0.429	0.393	-8.23%
total mass of domestic startups	0.025	0.024	-3.56%
domestic firm entry rate (%)	5.833	6.130	5.09%
avg. number of products	1.461	1.435	-1.78%
avg. sales growth rate (%)	1.048	1.058	0.93%

Table 2.9: Aggregate Growth Decomposition

description	before	after	% change
aggregate growth ($1+g$)	1.0105	1.0106	0.01%
growth from internal innovation	0.9179	0.9154	-0.27%
growth from domestic external innovation	0.0322	0.0288	-10.45%
growth from domestic startups	0.0258	0.0249	-3.56%
growth from foreign external innovation	0.0346	0.0414	19.72%
growth from domestic firms	0.9759	0.9692	-0.69%

Table 2.10: Firm Employment Growth Rate Changes

description	before	after
p90 emp. growth rate (%)	22.843	20.997
p50 emp. growth rate (%)	0.254	0.246
p10 emp. growth rate (%)	-12.151	-12.082

after an increase in competitive pressure from foreign firms.

Lastly, Table 2.10 shows the 90th, 50th, and 10th percentiles of the employment-weighted distribution of firm employment growth rates before and after the increase in competitive pressure from foreign firms. The growth rate of high-growth firms, measured as the 90th percentile of the distribution, decreases from 22.8% to 21.0% after an increase in competitive pressure from foreign firms. The 50th percentile decreases after the increase in competitive pressure from foreign firms. The 10th percentile, however, increases, because firms are better at protecting their product markets with increased internal innovation.

2.4.3.2 Comparison I: Economy with High External Innovation Costs

To show how the effect of a given shock to competitive pressure changes if we consider an economy with low creativity—a low external innovation intensity due to increased friction—we run the same exercise of increasing the creative destruction arrival rate from outside firms, $\overline{x}_o$, by 20%, in an economy in which $\widetilde{\chi}$, the parameter governing the cost of external R&D, is 50 times higher than the baseline calibration of 1.184.

Columns 2 and 3 of Table 2.11 compare this low creativity economy with the economy calibrated to the U.S. (the baseline calibration with $\widetilde{\chi}$ = 1.184). As we can see, this economy is less dynamic compared to the U.S., with lower R&D, a lower number of startups, lower economic growth, and lower growth of high-growth firms than the baseline economy.

Columns 3 and 4 of Table 2.11 compare the moments of the low creativity economy before and after an increase in competitive pressure from foreign firms. Compared to the U.S. counterparts, all the moments except for the R&D to sales ratio move in the same direction, but the magnitudes are smaller. Importantly, the domestic R&D to sales ratio increases in this economy, whereas this ratio decreases in the baseline case. In this economy, firms put very little effort into external innovation. Thus, although external innovation decreases after an increase in foreign competitive pressure, the reduction is very small in absolute terms. Therefore, it is more than offset by the increased investment in internal innovation for defensive reasons. This result highlights the importance of examining changes in the composition of innovation along with the changes in overall innovation.

Table 2.12 shows the changes in innovation intensities in the low-creativity economy. Compared to the numbers reported in Table 2.5, we see that innovation intensities are smaller in mag-

Table 2.11: Moment Comparison: U.S. vs. Economy with High External Innov. Costs

Moment	Baseline	w/ high ext. innov. costs	after shock	% change
R&D to sales ratio (%)	4.124	1.451	1.480	2.02
avg. number of products	1.461	1.022	1.019	-0.31
total mass of domestic firms	0.429	0.355	0.300	-15.43
total mass of domestic startups	0.025	0.020	0.019	-7.39
avg. sales growth rate (%)	1.011	0.842	0.867	2.96
p90 emp. growth rate (%)	22.843	9.111	9.089	-0.24

Table 2.12: Changes in Innovation Intensities in Low-Creativity Economy

description	variables	before	after	% change
foreign creative destruction arrival rate	$\overline{x}_o$	0.045	0.054	20.00%
creative destruction arrival rate	$\overline{x}$	0.070	0.077	10.10%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.225	0.224	-0.14%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.581	0.594	2.24%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.411	0.418	1.76%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.403	0.409	1.57%
prob. of external innovation, incumbents	x	0.003	0.003	-6.43%
prob. of external innovation, potential startups	x_e	0.024	0.023	-6.67%
conditional takeover probability	$\overline{x}_{takeover}$	0.831	0.825	-0.78%
unconditional takeover probability	$x_{takeover}$	0.003	0.002	-7.16%

nitude in the economy with low creativity. However, the direction of changes in response to increasing competitive pressure from foreign firms are identical in both economies.

2.4.3.3 Comparison II: Increased Competitive Pressure from Domestic Startups

In this exercise, we lower $\widetilde{\chi}^e$, the parameter governing the cost of external R&D for potential startups, by 11.34%. This increases the aggregate creative destruction arrival rate $\overline{x}$ from 0.120 to 0.123 (a 2.98% increase), which is identical to the increase in the previous exercise in which we increased the foreign creative destruction arrival rate by 20%.

Table 2.13 shows the results. Since the aggregate creative destruction arrival rate is the same, all the moments related to individual incumbent firms are virtually identical to those reported in Tables 2.10, 2.8, and 2.5. However, the total mass of domestic firms, the total mass of

Table 2.13: Changes in Moments: Economy with Low Entry Costs

description	before	after	% change
total mass of domestic firms	0.429	0.444	3.54%
total mass of domestic startups	0.025	0.027	8.83%
R&D to sales ratio (%)	4.124	4.065	-1.44%
avg. number of products	1.461	1.435	-1.78%
avg. sales growth rate (%)	1.048	1.058	0.92%
p90 emp. growth rate (%)	22.843	20.997	-8.08%
prob. of external innovation, potential startups	0.033	0.037	9.08%

domestic startups, and the probability of external innovation by potential startups increase in this case. This is because the increasing competitive pressure is induced by an increase in the mass of domestic startups, rather than by foreign firms. This exercise shows that changes in moments related to the number of domestic firms and startups can be helpful in identifying whether an increase in competitive pressure is coming from the domestic entry margin or foreign firm entry.

2.5 Conclusion

In this paper, we investigate the effect of competition on overall firm innovation and its composition, by developing an endogenous growth model with heterogeneous innovation and imperfect technology spillovers, and by testing the model predictions empirically. Firms improve their own product quality through internal innovation and enter new product markets through external innovation by driving out incumbent firms. However, external innovation is subject to imperfect technology spillovers, in that it takes time to learn others' technology.

We show that having different types of innovation along with imperfect technology spillovers is crucial in analyzing the impact of increasing competition on firm innovation. Rising competition lowers firms' incentive to invest in external innovation, while it encourages firms' investment in internal innovation for existing product lines with a large technology gap accumulated through recent innovation.

We also show that the decomposition of innovation into two types is potentially crucial in understanding the differential effect of competition on firm innovation across different sectors or countries. The direction of incumbent firms' responses of internal and external innovation to competition are similar regardless of the costs of external innovation. However, overall innovation, which combines internal and external innovation, increases in an economy with high external innovation costs in response to increased competition, while it decreases in an economy with low external innovation costs, such as the U.S. This is because firms undertake very little external innovation in the first place in an economy with high external innovation costs even without any increase in competitive pressure. Thus, there is little room for external innovation to be further adjusted downward, and the decrease in external innovation is dominated by the increase in internal innovation.

To the best of our knowledge, this is the first attempt to develop an endogenous growth model incorporating the escape-competition effect with firm entry and exit, in which multi-product firms are allowed to grow through product scope expansion à la Klette and Kortum (2004), and the first attempt to identify the causal effect of competition on the composition of firm innovation empirically. Additionally, our model provides a rich framework that enables us to account for different responses of overall innovation to increasing competition across countries.

Chapter 3: Increasing Knowledge Complexity and Business Dynamism[1]

3.1 Introduction

As technology advances, innovation tends to require a larger knowledge base. Consequently, innovation is increasingly achieved by large teams composed of specialized experts (Jones, 2009), and the diversity and interdependence among specific technology fields increase. While the increase in the knowledge base and the accompanying rise in the diversity and interdependence of research inputs may have significant implications on firms' innovation costs, empirical evidence on trends in knowledge complexity is scarce and their implications on firm innovation, growth, and business dynamism are not well-understood. In this paper, we first document stylized facts about time trends in the complexity of knowledge used for innovation across different industries and analyze their implications for the patterns of firm innovation and business dynamism both empirically and theoretically.

To study knowledge complexity, we merge the U.S. patent database (USPTO PatentsView) with S&P's Compustat as well as administrative firm-level data (the Longitudinal Business Database, LBD henceforth) to trace innovation activities of firms in the U.S. Using these datasets, we construct several measures of the complexity of knowledge used for innovation and document two

[1] This chapter relies on a project co-authored with Serguey Braguinsky, Joonkyu Choi, Yuheng Ding, and Karam Jo.

broad sets of empirical facts. First, we find that there has been an upward trend in knowledge complexity in innovation activities. Specifically, the inventor team size, the number of technology subclasses (equivalent to CPC groups in the USPTO), and the degree of interdependence across technology subclasses in Compustat firms' patent portfolio have all been increasing over time. This trend also holds for all USPTO assignees and administrative firms. Second, the increasing trend of knowledge complexity is associated with the declining trend of business dynamism observed in the Business Dynamics Statistics (BDS) and documented in recent literature (Akcigit and Ates, forthcoming; Decker et al., 2016a, 2014b). We estimate industry and subperiod-specific time trends of knowledge complexity and business dynamism over a five-year horizon and show that the estimated trend of business dynamism is negatively correlated across industries and time periods with its counterpart for knowledge complexity. We measure trends in business dynamism using firm entry rates, the proportion of young firms, the employment share of young firms, the job creation share of young firms, and the labor reallocation share of young firms in a given industry. The results are robust across different outcome variables.

To seek potential mechanisms underlying this association between business dynamism and innovation, we offer a simple endogenous growth model with multi-product firms faced with a degree of interdependence (or complementarity) of diverse inputs. The model predicts that rising interdependence across different inputs dampens innovation activity by firms with a lower existing knowledge base and increases innovation in firms with a larger knowledge base. This is because increased knowledge complexity increases the cost of incorporating different inputs to produce a given amount of innovation, and only firms with a large enough knowledge base are willing to pay this higher cost.

These results show that rising knowledge complexity and interdependence, and the accompanying increase in research costs, can create burdens on small and young firms that lack a knowledge base. Thus, knowledge base can play an important role in declining business dynamism, in particular in innovation-intensive sectors in the U.S.

The rest of the paper proceeds as follows. Section 3.2 briefly discusses related literature. Section 3.3 describes data sources and the main measures used for our analysis. Section 3.4 documents empirical evidence on the secular trends of increasing knowledge complexity and declining business dynamism, and how they are associated. Section 3.5 lays out a theoretical model to understand the underlying mechanism behind this evidence. Section 3.6 concludes with the discussion of the remaining future work.

3.2 Literature Review

Existing literature has shown an increasing trend in the burden and complexity of knowledge needed for innovation (Jones, 2009), a substantial increase in the number of researchers required for a given rate of productivity growth (Bloom et al., 2020), as well as an increased use of teams composed of specialized researchers.

Furthermore, previous work has argued that complex innovations can be handled more efficiently by larger organizations that have better access to efficient knowledge hierarchies (Caliendo and Rossi-Hansberg, 2012; Garicano, 2000; Garicano and Rossi-Hansberg, 2004). Knowledge integration could also lead to superior performance by enhancing the firm's ability to appropriate rents in complex systems (Ethiraj et al., 2008; Rivkin, 2000).

The literature has also investigated the relationship between knowledge complexity and entrepreneurial decisions made by potential startup founders. As complex knowledge has been found to be difficult and costly to transfer, innovative employees possessing complex knowledge are less likely to move and become employee entrepreneurs (Ganco, 2013).

Turning to the macroeconomic and firm dynamics literatures, while the secular decline in business dynamism over the past few decades has been well documented (Decker et al., 2016b, 2014b), its causes and implications are not fully understood. Some of the underlying factors documented in recent literature include a slowdown in population growth (Hopenhayn et al., 2018; Karahan et al., 2019), an increase in lobbying activities by large incumbents (Gutiérrez and Philippon, 2019), an increase in competitive pressure driven by globalization (Jo, 2020; Jo and Kim, 2021), and the decline in knowledge diffusion from industry leaders to followers (Akcigit and Ates, 2021, forthcoming).

We contribute to these strands of literature by proposing increases in knowledge complexity and innovation costs as another source of the decline in business dynamism, and analyzing its implications on firm dynamics and innovation. In particular, we establish that the two patterns are associated across industries, for multiple measures of complexity and business dynamism. We build a novel model incorporating the notions of knowledge complexity and interdependence, to understand a potential mechanism linking knowledge complexity to business dynamism, and to show how rising complexity can hamper young firm activity.

3.3 Data and Measurement

The primary data sources for this paper are the U.S. patent database (the USPTO patent data) and S&P's Compustat. We merge the two databases to trace innovation activities of publicly listed firms in the U.S. between 1981 and 2010.[2] As an additional exercise, we also use the Longitudinal Business Database (LBD) from the U.S. Census Bureau and provide results in Appendix D.2.

3.3.1 USPTO PatentsView Database

The USPTO PatentsView tracks all patents ultimately granted by the USPTO from 1976 onward.[3] This database contains detailed information for granted patents including application and grant dates, technology class categories, patent inventors and citation information, and the name and address of patent assignees.

3.3.2 Compustat

For our main analyses, we use S&P's Compustat data to track publicly listed firms in the U.S. We create our own bridge between U.S. patenting firms in the USPTO patent database and Compustat firms through a standard name-matching and internet-based matching algorithm as in Autor et al. (2020). This linking method follows Ding et al. (2022).

First, we standardize firm names in both datasets using the algorithm provided by the NBER Patent Data Project (PDP) and use the standardized names in the matching process.[4] We define patenting firms as patent assignees that are located in the U.S. with assignee type equal to 2 (U.S. company or corporation) in the USPTO data.

The first match procedure involves identifying firms with precisely the same standardized

[2] Since the earliest year of the USPTO PatentsView data is 1976, we do not observe complete history of technological combinations (a key basis for our main complexity measures the following section) before then. Thus, we use the first few years, 1976-1980, as a buffer period to capture the history of technological components and we track combinations starting from 1981.

[3] See details from https://patentsview.org/download/data-download-tables.

[4] The PDP project can be found here: https://sites.google.com/site/patentdataproject/Home.

names in both datasets. Following previous studies, we do not use address information in Compustat during the match process, as Compustat address data only reports information for headquarters, which can be different from the exact address of establishments that file patent applications to the USPTO. For unmatched USPTO firms, we use the stem name (standardized firm names without suffixes) to find matches. For the unmatched U.S. patenting firms remaining after the standard name matching, we apply the internet-based matching algorithm to identify the same firms in Compustat.

Specifically, we put every patent assignee and Compustat firm name into the Google.com search engine, collect the URLs of the top five search results, and identify a patent assignee and Compustat firm as the same firm if they share at least two identical search results. For any patenting firms remaining unmatched, we identify a match if the top five search results of the unmatched patenting firm exactly matches the web-URL of a Compustat firm.

For all U.S. patenting firms reamining unmatched after the previous steps, we use the NBER PDP and find matches in Compustat. The NBER PDP did extensive manual matching to identify the same firms across the two datasets. Thus, this procedure helps us to reduce the burden of manually searching the unmatched USPTO firms. Lastly, we do our own manual matching to identify matches between the USPTO and Compustat firms. We manually inspect the match results at the end of each step to screen out false matches, especially for firms with many patent applications.

The above procedure matches 68.0% of utility patent applications filed by U.S. patenting firms, and 24.5% of U.S. patenting firms, to Compustat firms. We report more details on the match results in Appendix D.1.

3.3.3 Business Dynamics Statistics (BDS)

The Business Dynamics Statistics (BDS) provides annual measures of job creation and destruction, the entry and exit rate of firms and establishments, and other measures for the U.S. economy, aggregated by different firm characteristics, such as size, age, industry, and MSA. This data is aggregated from underlying administrative data collected by the Census Bureau.

3.3.4 Main Measures

We construct the following set of measures of knowledge complexity at the industry level (NAICS4): i) the average number of coinventors, ii) the average number of technology subclasses (equivalent to CPC groups in UPSTO), and iii) knowledge interdependence.[5,6]

We build each of the first two measures at the firm level using firms' patent portfolio, and take an unweighted average across firms within each industry. We take the average number of coinventors and technology subclasses associated with patents applied for by firms in each year.

The average number of coinventors is defined as follows:

$$\mathcal{AI}_{i,j,t} = \sum_{p \in \mathcal{P}_{i,j,t}} \frac{\mathcal{N}^{\mathcal{I}}_{p,t}}{\mathcal{N}^{\mathcal{P}}_{i,j,t}}, \tag{3.1}$$

where $\mathcal{P}_{i,j,t}$ is the set of patent applications for firm i (in industry j) in year t, $\mathcal{N}^{\mathcal{I}}_{p,t}$ is the count of inventors associated with patent application p for firm i, and $\mathcal{N}^{\mathcal{P}}_{i,j,t}$ is the count of patent applications for firm i in year t. (i.e. the count of patents in $\mathcal{P}_{i,j,t}$.)

[5] Henceforth, we will use technology subclasses and CPC groups interchangeably.

[6] Note that the CPC stands for Cooperative Patent Classification, which is a bilateral classification system jointly developed by the United States Patent and Trademark Office (USPTO) and the European Patent Office (EPO). The CPC group is a four-digit level of the categorization.

Note that we can also construct an alternative measure using backward citation information. Using the average number of coinventors associated with patents cited by the focal patent p instead of $\mathcal{N}^{\mathcal{I}}_{p,t}$, we have:

$$\mathcal{AI}^{cite}_{i,j,t} = \sum_{p \in \mathcal{P}_{i,j,t}} \frac{\mathcal{N}^{\mathcal{I},cite}_{p,t}}{\mathcal{N}^{\mathcal{P}}_{i,j,t}}, \tag{3.2}$$

where $\mathcal{N}^{\mathcal{I},cite}_{p,t}$ is the average number of coinventors for the cited patents. This captures the average size of coinventors that the focal patent's citations are based on.

In a similar fashion, the average number of technology subclasses is constructed as follows:

$$\mathcal{AC}_{i,j,t} = \sum_{p \in \mathcal{P}_{i,j,t}} \frac{\mathcal{N}^{\mathcal{C}}_{p,t}}{\mathcal{N}^{\mathcal{P}}_{i,j,t}}, \tag{3.3}$$

where $\mathcal{N}^{\mathcal{C}}_{p,t}$ is the count of technology subclasses (or CPC groups) associated with patent application p by firm i.

As an alternative, we can also use inventor information to identify CPC groups for patents.[7] We track inventors' patent portfolio and identify the modal CPC technology group in a given year, which can proxy for the specialty of inventors. Next, for each patent, we count the number of distinct CPC technology groups associated with its inventors and denote it by $\mathcal{N}^{\mathcal{C},inv}_{p,t}$. The resulting alternative measure is as follows:

$$\mathcal{AC}^{inv}_{i,j,t} = \sum_{p \in \mathcal{P}_{i,j,t}} \frac{\mathcal{N}^{\mathcal{C},inv}_{p,t}}{\mathcal{N}^{\mathcal{P}}_{i,j,t}}. \tag{3.4}$$

[7] We consider this method because CPC information is assigned to patents ex-post, and might not reflect the technology types "used" for patents in the application process. Alternatively, we can also use CPC information associated with cited patents, which is in progress.

Next, we take averages across firms within each industry j in year t:

$$\mathcal{AI}_{j,t} = \sum_{i \in \mathcal{F}_{j,t}} \frac{\mathcal{AI}_{i,j,t}}{\mathcal{N}^{\mathcal{F}}_{j,t}}, \quad \mathcal{AI}^{cite}_{j,t} = \sum_{i \in \mathcal{F}_{j,t}} \frac{\mathcal{AI}^{cite}_{i,j,t}}{\mathcal{N}^{\mathcal{F}}_{j,t}} \tag{3.5}$$

and

$$\mathcal{AC}_{j,t} = \sum_{i \in \mathcal{F}_{j,t}} \frac{\mathcal{AC}_{i,j,t}}{\mathcal{N}^{\mathcal{F}}_{j,t}}, \quad \mathcal{AC}^{inv}_{j,t} = \sum_{i \in \mathcal{F}_{j,t}} \frac{\mathcal{AC}^{inv}_{i,j,t}}{\mathcal{N}^{\mathcal{F}}_{j,t}} \tag{3.6}$$

where $\mathcal{F}_{j,t}$ is the set of firms and $\mathcal{N}^{\mathcal{F}}_{j,t}$ is the count of firms in industry j in year t.

The third measure of knowledge interdependence is directly constructed at the industry level. We base our measure of knowledge interdependence on the co-occurence of patent technology classifications. This is analogous to previous research that focuses on inventor knowledge complexity (Ganco, 2013) and on the complexity of patents (Fleming and Sorenson, 2001, 2004; Sorenson et al., 2010). Here, we exclude CPC Section Y when constructing the interdependence measure, since there rarely exists a patent assigned to Section Y as its main technology group.[8]

We construct this measure as follows. First, for each CPC technology group c in a given year t, we construct the following measure, called the standardized degree of interdependence $\mathcal{D}_{c,t}$ (or centrality around the technology group c):

$$\mathcal{D}_{c,t} = \frac{1}{\mathcal{N}_t - 1} \sum_{d \neq c} \mathcal{A}_{c,d}, \tag{3.7}$$

where $\mathcal{N}_t$ is the number of distinct CPC technology groups in year t, and $\mathcal{A}_{c,d}$ is a dummy variable indicating the existence of at least one patent assigned to both technology groups c and d in year

[8]According to CPC documentation, the primary purpose of tagging Section Y to a patent is "to monitor new technological development and to tag cross-sectional technologies that do not fit in a single other section of the IPC, so that the tagging codes of this section do not in any way replace the classification or indexing codes of the other sections." (https://www.uspto.gov/web/patents/classification/cpc/html/cpc-Y.html)

t ($\mathcal{A}_{c,d} = 1$ if there exists any patent assigned to both groups in year t, and $\mathcal{A}_{c,d} = 0$ otherwise.) Note that by definition, $\mathcal{D}_{c,t} \in [0,1], \forall c, t$, where $\mathcal{D}_{c,t}$ measures the fraction of distinct technology groups integrated with the focal technology.

Another way to measure interdependence is to replace the dummy indicator $\mathcal{A}_{c,d}$ with the number of patents that are co-assigned to technology groups c and d. This measure captures the intensity of interdependence between technology classes c and d:

$$\mathcal{D}'_{c,t} = \frac{1}{\mathcal{N}_t - 1} \sum_{d \neq c} \mathcal{P}_{c,d}, \tag{3.8}$$

where $\mathcal{P}_{c,d}$ is the number of patents assigned to both technology groups c and d in year t. This further captures how active firm patenting is in each pair of interdependent technology groups.

Next, after applying (3.7) and (3.8) to each CPC technology group, we construct the following industry-level interdependence measure:

$$\mathcal{K}_{j,t} = \frac{1}{\mathcal{N}^{\mathcal{C}}_{j,t}} \sum_{c \in \mathcal{C}_{j,t}} \mathcal{D}_{c,t} \tag{3.9}$$

and

$$\mathcal{K}'_{j,t} = \frac{1}{\mathcal{N}^{\mathcal{C}}_{j,t}} \sum_{c \in \mathcal{C}_{j,t}} \mathcal{D}'_{c,t}, \tag{3.10}$$

where $\mathcal{C}_{j,t}$ is the set of all technology groups assigned to at least one patent applied for by industry j in year t, and $\mathcal{N}^{\mathcal{C}}_{j,t}$ is the count of technology groups in industry j in year t.

Note that both (3.9) and (3.10) can alternatively be constructed using inventors' CPC information as before. This simply replaces $\mathcal{C}_{j,t}$ and $\mathcal{N}^{\mathcal{C}}_{j,t}$ with the set and count of inventors' modal CPC technology groups, $\mathcal{C}^{inv}_{j,t}$ and $\mathcal{N}^{\mathcal{C},inv}_{j,t}$, respectively.

To measure business dynamism, we use the BDS and LBD and measure the firm entry rate (the share of startups), the share of young firms, the employment share of young firms, the job creation share of young firms, and the job reallocation (the sum of job creation and destruction) share of young firms in each industry. Let $\mathcal{BD}_{j,t}$ denote industry-level business dynamism in industry j and year t.

3.4 Empirical Evidence

3.4.1 The Trend in Knowledge Complexity

In this section, we document the recent trends of knowledge complexity, as defined by the above set of measures. Figure 3.1 presents the time trends of knowledge complexity for the universe of USPTO assigness across different measures. We find that there has been an upward trend in knowledge complexity in firm innovation. Specifically, the inventor team size, the number of technology groups, and the degree of interdependence across technology groups in firms' patent portfolio have all been increasing over time.

Figure 3.2 presents these trends for Compustat firms. This shows that the trend of increasing knowledge complexity is also observed for Compustat firms across different measures. Furthermore, as described in Appendix D.2.2, this pattern also holds for administrative firms, and is more pronounced for large firms in the Census.

In Figure 3.3 and 3.4, we further investigate the trend of knowledge interdependence of CPC technology groups within each CPC section.[9] Figure 3.3 plots the standardized measure of knowledge interdependence as in (3.7), and Figure 3.4 presents the intensity measure as in (3.8).

[9] A CPC section is a one-digit level of the CPC categorization, which is the highest layer. More details can be found in https://www.uspto.gov/web/offices/pac/mpep/s905.html.

Figure 3.1: Knowledge Complexity (USPTO assignees)

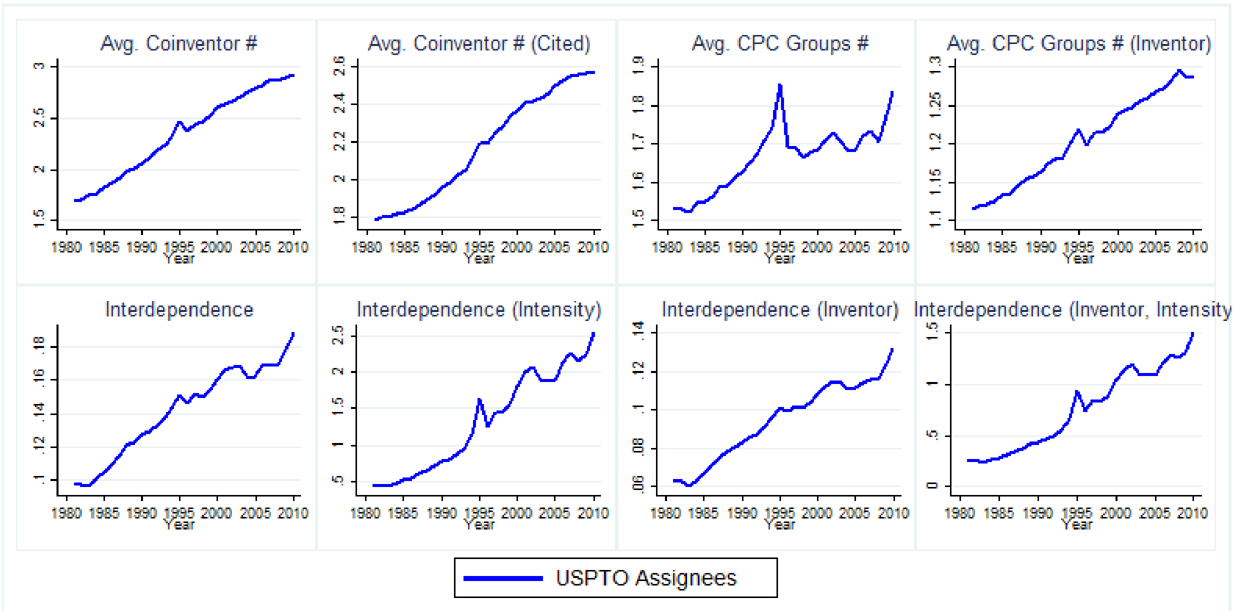


Figure 3.2: Knowledge Complexity (Compustat firms)

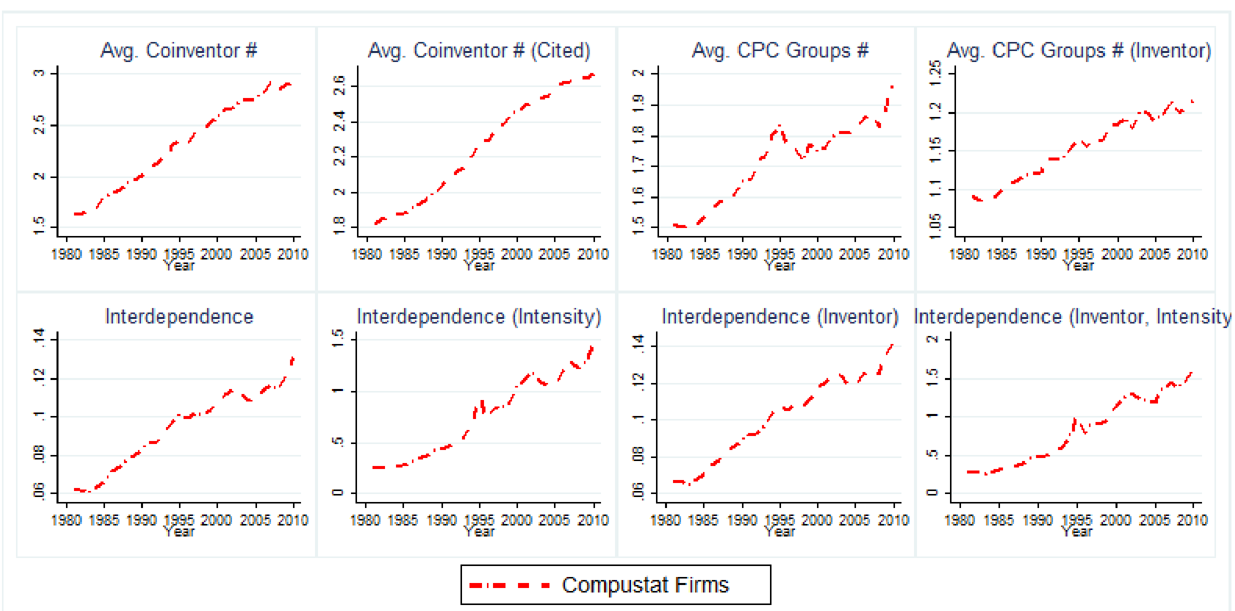
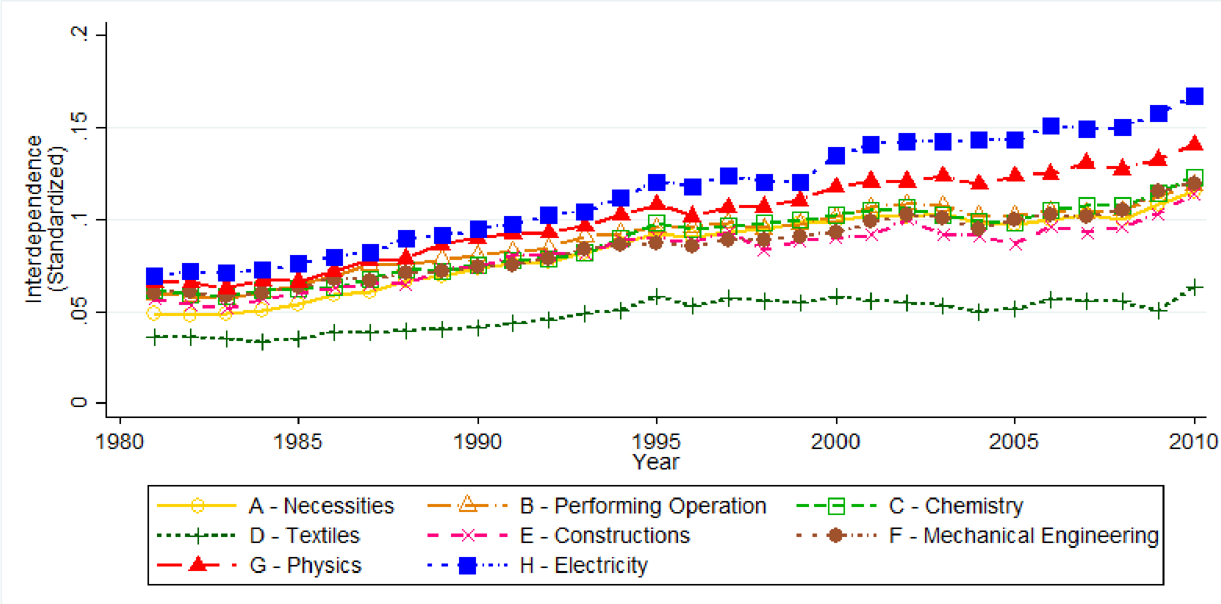


Figure 3.3: Standardized Degree of Interdependence



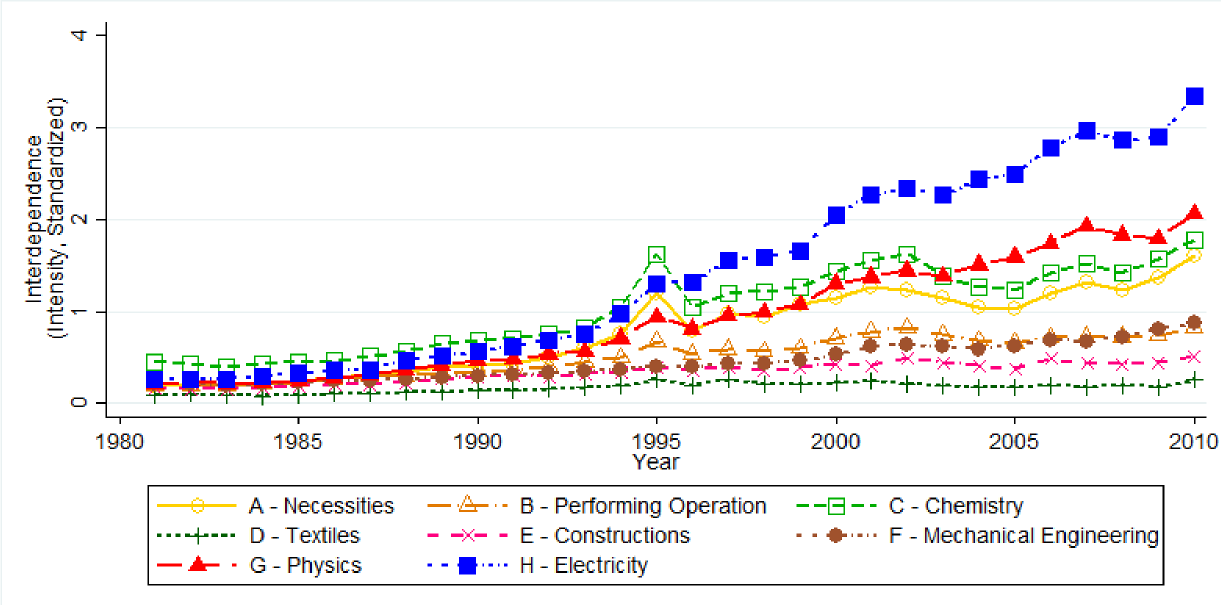
Both figures indicate that the degree of interdependence across different technologies has been increasing over time, with different growth rates across technology classes. In particular, Physics and Electricity have shown the highest growth rates in interdependence time, while Textiles has a flatter growth path.

This heterogeneity in the rise of interdependence across different technology groups is more pronounced if we take into account the intensity of co-occurrence of patent classes, as in Figure 3.4. The growth of interdependence for Physics and Electricity has especially been more pronounced since the late 2000s. This shows that more patents co-assigned to other technology classes have been integrated with Physics and Electricity in recent years.[10]

Next, we further investigate the origin of the rise in knowledge interdependence. To do so, we re-construct the measures (3.7) and (3.8) by only counting the number of integrated tech-

[10]Note that this is not just driven by the rise of patenting. As shown in Appendix Figure D.1.6, the number of patents has been increasing over time since the beginning of the sample period, but it is not reflected on the patenting intensity in the technology interdependence until the late 2000s. This shows that the degree of interdependence does not capture the rise of patenting by itself, but the rise of patents co-assigned to different technology groups.

Figure 3.4: Intensity-Based Degree of Interdependence



nology groups within each section and subsection, respectively. For instance, within-section knowledge interdependence only counts the number of integrated technology groups that lie in the same technology section.

Figure 3.5 and 3.6 present the decomposition of each measure. Both figures suggest that most of the increasing degree of interdependence originates from cross-section (or subsection) knowledge integration. This pattern is robust across all technology sections. Also, it is noteworthy to see that within-subsection technology integration has been relatively more stable over time, while within-section technology integration shows more time variation. Still, the latter only explains a small fraction of the overall trend of rising knowledge interdependence. This suggests that the increasing interdependence is largely driven by the integration between different technology groups that are distant rather than close to each other.

Figure 3.5: Decomposition of Standardized Interdependence

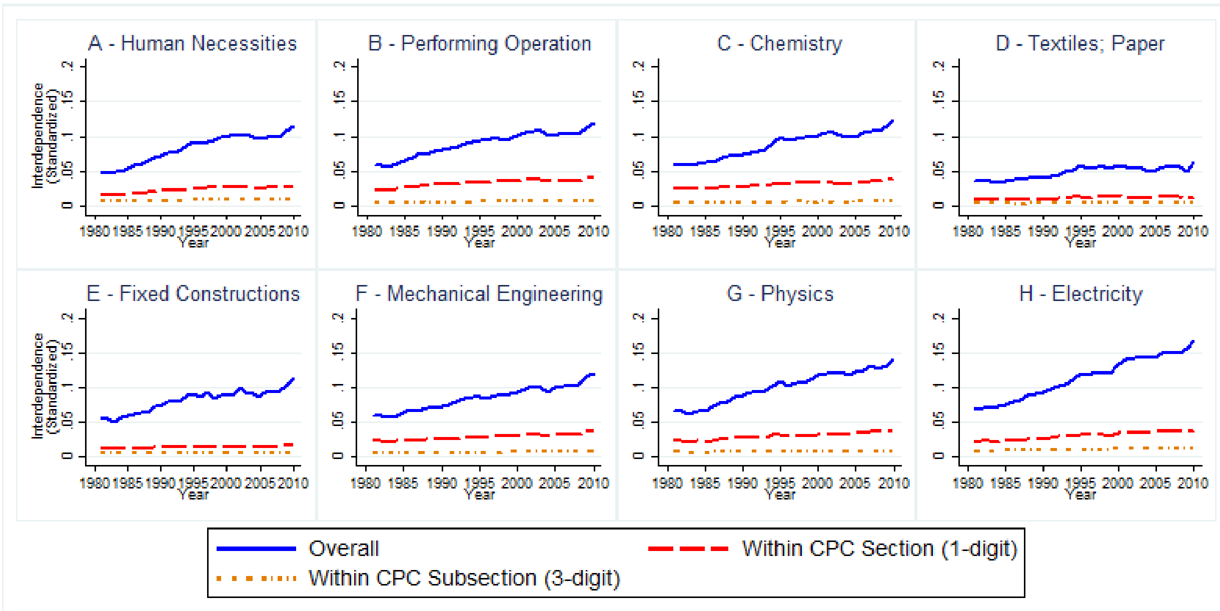


Figure 3.6: Decomposition of Intensity-Based Interdependence

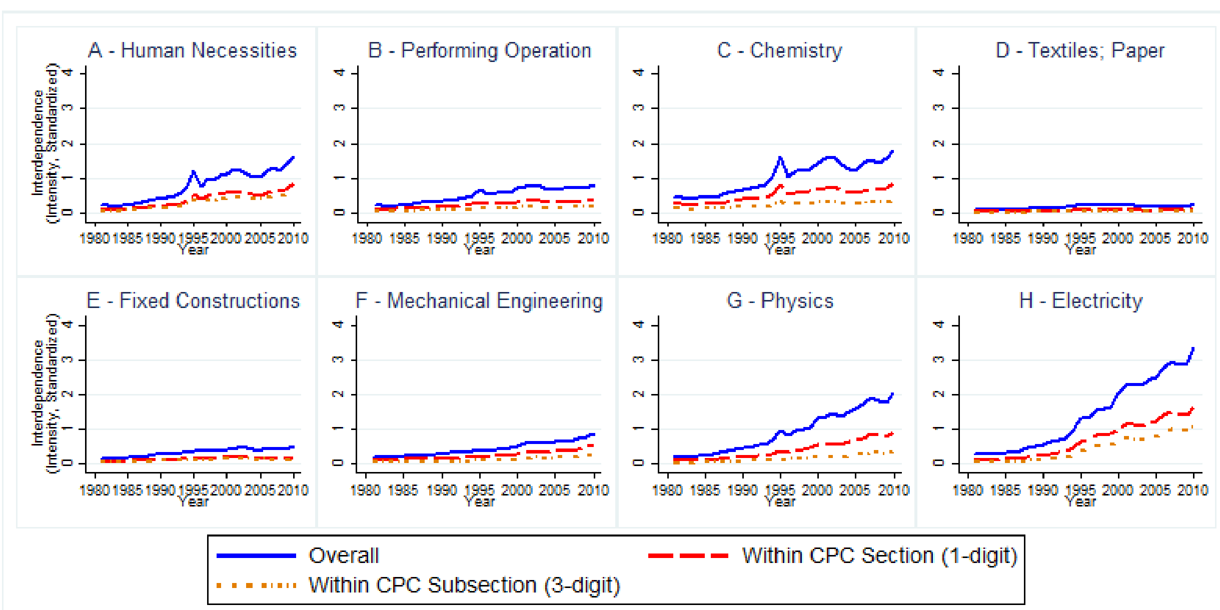
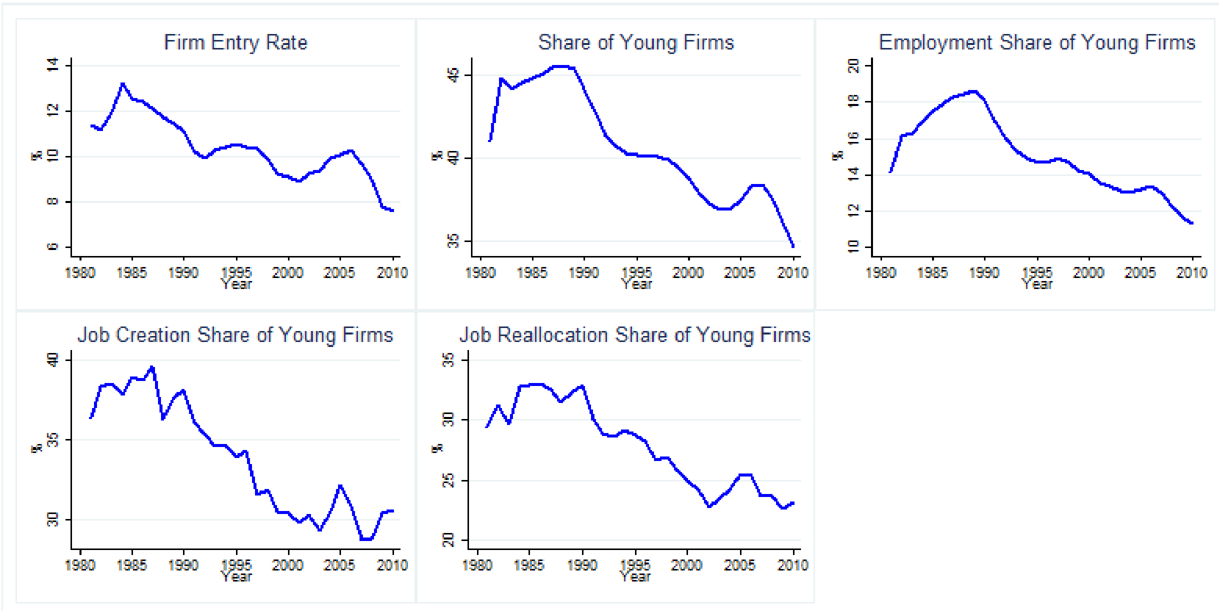


Figure 3.7: Business Dynamism



3.4.2 The Trend in Business Dynamism

Next, Figure 3.7 presents the trends of the measures of business dynamism constructed from the BDS database. Across all measures, there has been a secular decline in business dynamism in the U.S., especially since the mid 1980s. Both the startup rate and the share of young firms (firms less than five years of age) in the economy have declined. In particular, the firm entry rate was around 13% in the mid 1980s, but decreased to 9% before the start of the Great Recession, and has declined further since then. The share of young firms also declined from around 46% to 39% over this period. Furthermore, the economic activity of young firms has also been dampened over time. The share of employment and jobs created at young firms declined from about 18% to 13%, and from 39% to 30%, respectively. These trends are consistent with previous literature (Akcigit and Ates, forthcoming; Decker et al., 2016b, 2014a,b; Hopenhayn et al., 2018; Karahan et al., 2019).

3.4.3 The Relationship between Knowledge Complexity and Business Dynamism

To investigate whether the two trends of increasing knowledge complexity and declining business dynamism are associated with each other, we use the following two-stage regressions.

First, we fit linear trends that can vary by industry and across the non-overlapping five year intervals, 1981-1985, 1986-1990, 1991-1995, 1996-2000, 2001-2005, 2006-2010, and use their slopes to capture time-varying industry-level time trends of each measure of knowledge complexity and business dynamism. Here, we use a balanced panel, restricting the sample to industries having full observations over the entire sample period, to avoid potential bias attributed to missing observations within the subperiods.

Specifically, we run the following regression to estimate time trends for each industry j over each of the five-year subperiods $s \in \{1981-1985, 1986-1990, 1991-1995, 1996-2000, 2001-2005, 2006-2010\}$:

$$\mathcal{Y}_{j,t} = \beta^{\mathcal{Y}}_{j,s} Year_t + \alpha_{j,s} + \varepsilon_{j,t} \tag{3.11}$$

where $Year_t$ indicates year t, and where $\mathcal{Y}_{j,t}$ is either the average number of coinventors as in (3.5), the average number of technology groups as in (3.6), the average level of knowledge interdependence as in (3.9), or a measure of business dynamism, for each industry j in year t. Here, $\beta^{\mathcal{Y}}_{j,s}$ captures the time trend of $\mathcal{Y}$ in industry j over subperiod s.

Next, we take the coefficient estimates from (3.11) and run the following regression to see how the time trends of knowledge complexity and business dynamism are associated across industry and time:

$$\hat{\beta}^{\mathcal{BD}}_{j,s} = \gamma \hat{\beta}^{\mathcal{KC}}_{j,s} + \delta_j + \delta_s + \varepsilon_{j,s}, \tag{3.12}$$

where $\hat{\beta}^{\mathcal{KC}}_{j,s}$ stands for the coefficient estimate for the one of the knowledge complexity measures, $\hat{\beta}^{\mathcal{BD}}_{j,s}$ is the counterpart for one of the business dynamism measures, δ_j is an industry fixed effect, and δ_s is a fixed effect for the five-year horizon subperiod. Here, γ captures how the trends of knowledge complexity and business dynamism are correlated across industries and subperiods.

Table 3.1 shows results focusing on the trend of young firm entry, and Table 3.2 presents the counterpart results for the intensive margin of young firm. Both tables indicate that there exists a negative association between the trends of knowledge complexity and business dynamism, and that this pattern is robust across different measures.

In Table 3.1, the trends of knowledge complexity are in general negatively correlated with the trends of firm entry and the share of young firms. In particular, the trends of the number of inventors' technology groups and knowledge interdependence are strongly correlated with the trends of firm entry and the mass of young firms. On the other hand, the correlation of firm entry and the number of young firms with the average number of technology classes is negative but not statistically significant.

Furthermore, the trend of knowledge complexity is also associated with the trend of young firm activity at the intensive margin. Table 3.2 shows that the trend of increasing knowledge complexity is strongly associated with the trend of declining young firms' employment, job creation, as well as job reallocation share. This suggests that even for surviving young firms, the trend of their growth is negatively associated across industries and time periods with the trend of rising breadth and integration of different technology groups.

Table 3.1: Relationships between the Trends in Knowledge Complexity and Firm Entry

	Firm Entry	Share of Young Firms
Avg. Number of Coinventors	-0.011***	-0.009*
	(0.004)	(0.005)
Avg. Number of Coinventors (citing patents)	-0.040***	-0.035*
	(0.014)	(0.018)
Avg. Number of Tech. Groups	-0.010	-0.011
	(0.007)	(0.009)
Avg. Number of Tech. Groups (inventor CPC)	-0.054***	-0.083***
	(0.018)	(0.023)
Knowledge Interdependence	-0.137***	-0.183***
	(0.041)	(0.053)
Knowledge Interdependence (inventor CPC)	-0.325***	-0.411***
	(0.108)	(0.138)
Obs	426	426
FE	Subperiod & Industry	Subperiod & Industry

Notes: Knowledge Interdependence refers to (3.9). Observations are weighted by the number of patents per employee (averaged value between 1981 and 2010). * p<0.1, ** p<0.05, *** p<0.01.

Table 3.2: Relationships between the Trends in Knowledge Complexity and Young Firm Activity

	Employment Share	Job Creation Share	Job Reallocation Share
Avg. Number of Coinventors	-0.019***	-0.033***	-0.020***
	(0.004)	(0.011)	(0.004)
Avg. Number of Coinventors (citing patents)	-0.045***	-0.050	-0.054***
	(0.017)	(0.041)	(0.016)
Avg. Number of Tech. Groups	-0.025***	-0.056***	-0.020**
	(0.009)	(0.021)	(0.008)
Avg. Number of Tech. Groups (inventor CPC)	-0.087***	-0.037	-0.081***
	(0.022)	(0.053)	(0.021)
Knowledge Interdependence	-0.142***	-0.200*	-0.158***
	(0.049)	(0.120)	(0.047)
Knowledge Interdependence (inventor CPC)	-0.252*	-0.244	-0.371***
	(0.129)	(0.313)	(0.124)
Obs	426	426	426
FE	Subperiod & Industry	Subperiod & Industry	Subperiod & Industry

Notes: The dependent variable is defined by the employment share of young firms, the job creation share of young firms, and the job reallocation share of young firms, respectively. Knowledge Interdependence refers to (3.9). Observations are weighted by the number of patents per employee (averaged value between 1981 and 2010). * p<0.1, ** p<0.05, *** p<0.01.

3.5 Theoretical Framework

In this section, we lay out a theoretical framework to propose a plausible mechanism explaining the impact of increasing knowledge complexity on business dynamism. We present a simple model that shows how knowledge interdependence and the accompanying rise of innovation costs affect firm innovation and the firm entry rate. We first consider an innovation cost that is constant across all firms, and demonstrate that the impact of complexity on business dynamism is even more pronounced if the cost becomes a function of firms' knowledge base (the set of product quality). For simplicity, we assume a two-period partial equilibrium model in which firms take prices and aggregate variables as given.

3.5.1 Baseline Setup

Intermediate firms are monopolistic producers of differentiated goods, and sell those to final good producers. We assume that intermediate firms are multi-product producers and can innovate to improve product quality, where the outcome of the process depends on the amount of inputs used. We build on Klette and Kortum (2004) and treat firm innovation as adding a product line to its product portfolio.

In period t, firm j is endowed with quality q_{jt}^{i} for product i in its portfolio. In what follows, we assume that this endowment of quality is determined by the firm knowledge base, and we use the terms "quality" and "knowledge base" interchangeably. Firms decide how much input to invest to innovate their knowledge base in period $(t + 1)$.

3.5.1.1 Production Process

The production processes are as follows. The final good production function is

$$Y_t = \frac{L^\theta}{1-\theta}\Big[\int_0^1 (q_t^i)^\theta (y_t^i)^{1-\theta} di\Big], \tag{3.13}$$

where L is production workers used by the final good sector (taken as given), y_t^i denotes the quantity of intermediate good i, and q_t^i is its quality (or knowledge base). This gives the standard inverse demand function for each good i as follows:

$$p_t^i = L^\theta q_t^i (y_t^i)^{-\theta}, \tag{3.14}$$

where p_t^i is the price of good i.

Firm j takes into account the demand function (3.14) and produces the intermediate good i to maximize the following profit function:

$$\pi_{jt}^i \equiv \max_{y_{jt}^i} p_{jt}^i y_{jt}^i - y_{jt}^i \tag{3.15}$$

subject to (3.14), where we assume a unit marginal cost of intermediate production. Solving (3.15), we obtain:

$$y_{jt}^i = (1-\theta)^{\frac{1}{\theta}} L q_{jt}^i \tag{3.16}$$

$$p_{jt}^i = \frac{1}{1-\theta} \tag{3.17}$$

$$\pi^i_{jt} = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L}_{\equiv \pi} q^i_{jt}. \tag{3.18}$$

3.5.1.2 Innovation Process

Firms invest in innovation to improve their knowledge base. A firm is defined by its product portfolio, which is summarized by a set of product qualities (or technologies) $\{q^i_{jt}\}_{i=1}^{n_{jt}}$ for each product i owned by the firm. Firms add new product lines through successful innovation by having the highest technology (quality) and becoming a leader in the target product market. The outcome of innovation—success or failure—is realized at the beginning of the subsequent period (period $t+1$). As explained in Klette and Kortum (2004), the number of products n_{jt} measures the knowledge stock of firms, accumulated through past innovation. n_{jt} is a sufficient statistic for the knowledge base of firms, and characterizes firm size.

Successful innovation increases the quality of the target product q^{target}_{jt} by $\eta > 1$ as follows:

$$q^{target}_{jt+1} = \begin{cases} \eta q^{target}_{jt} & \text{with probability} \quad \phi_{jt} \\ q^{target}_{jt} & \text{with probability} \quad 1-\phi_{jt} \end{cases}. \tag{3.19}$$

ϕ_{jt} is the success probability of the innovation, which is determined by the level of R&D investment along with the current knowledge base as follows:

$$\phi_{jt} = \chi n_{jt}^{\frac{1-\sigma}{\psi}} R_{jt}(x^1_{jt}, x^2_{jt})^{\frac{1}{\psi}}, \tag{3.20}$$

where n_{jt} is the number of product lines the firm j currently operates, and $R_{jt}(x^1_{jt}, x^2_{jt})$ is the amount of R&D input firms invest. We assume that there are two R&D inputs available in the economy, denoted by x^1 and x^2. Also, following literature, $\chi > 0$ and $\psi > 1$ are assumed as scale parameters, and $0 < \sigma < 1$ is assumed. This indicates that existing knowledge base (with higher n_{jt}) helps firms to create new ideas more easily.

3.5.2 Knowledge Complexity (Interdependence)

We embed the notion of knowledge complexity and interdependence as the degree of complementarity across R&D inputs that are required for firm innovation. As different inputs become more complementary to each other in the R&D process, firms are required to use them in more equal amounts to create the same probability of innovation success as before. In other words, the complementarity generates interdependence across different types of inputs and makes the innovation process more complex.

To illustrate the idea of interdependence in the simplest way, consider the following two extreme cases for the R&D function:

$$R_{jt} = (x_{jt}^1 + x_{jt}^2) \tag{3.21}$$

$$R_{jt} = \min(x_{jt}^1, x_{jt}^2), \tag{3.22}$$

where (3.21) assumes perfect substitutability, and (3.22) assumes perfect complementarity between the two inputs. Note that the former is the standard problem with a unit input type, while the latter forces firms to obtain equal amounts of both input types to attain a given likelihood of successful innovation. The latter function implies a more complex innovation process, requiring a more diverse set of inputs.

3.5.3 Firms' Problem

We assume that per-unit input costs are the same across these two types, denoted by w (i.e., wages for the two input types are the same). We further assume that firms need to pay a fixed cost c (e.g., search or learning costs associated with knowledge) to enter into each type of input market. Thus, the fixed cost incurred is c if firms hire either x^1 or x^2 alone, and $2c$ if firms hire both x^1 and x^2.

Successful firm innovation translates into firms taking over other firms' product lines. In other words, an incumbent firm in a given product line is faced with a positive probability of losing the product if an entrant firm successfully innovates and takes over this market. We let τ denote the arrival rate of creative destruction, which firms take as given.

Given this, firms have the following maximization problem:

$$\begin{aligned}\Pi_{jt}\left(\{q_{jt}^i\}_{i=1}^{n_{jt}}\right) \equiv \max_{x_{jt}^1, x_{jt}^2} \; & \pi \sum_{i=1}^{n} q_{jt}^i - c\mathbb{I}_{(x_{jt}^1>0)} - c\mathbb{I}_{(x_{jt}^2>0)} - w(x_{jt}^1 + x_{jt}^2) \\ & + \beta \left\{ (1-\tau)\left[\phi_{jt}\pi\left(\sum_{i=1}^{n_{jt}} q_{jt}^i + \overline{q}'\right) + (1-\phi_{jt})\pi \sum_{i=1}^{n_{jt}} q_{jt}^i\right]\right. \\ & \left. + \tau \sum_{i'=1}^{n_{jt}} \frac{1}{n_{jt}}\left[\phi_{jt}\pi\left(\sum_{i=1}^{n_{jt}} q_{jt}^i \mathbb{I}_{i\neq i'} + \overline{q}'\right) + (1-\phi_{jt})\pi \sum_{i=1}^{n_{jt}} q_{jt}^i \mathbb{I}_{i\neq i'}\right]\right\},\end{aligned} \tag{3.23}$$

subject to the R&D production function (3.20). The first line sums the firm's instantaneous profit net of R&D input costs across all product lines. The second line is the expected future value of innovation if the firm experiences no creative destruction. The third line is the expected future value of innovation if the firm is faced with creative destruction and loses one of its product lines. Product losses are randomly assigned across the firm's portfolio with equal probability $\frac{1}{n_{jt}}$. Also, innovation is undirected, so that the new product is randomly chosen upon successful innovation, where $\overline{q}'$ is the economy's average product quality level in the next period, which is the expected quality for an added product.

By collecting common terms, the above problem can be rephrased in the following way:

$$\Pi_{jt}\left(\{q_{jt}^i\}_{i=1}^{n_{jt}}\right) \equiv \max_{x_{jt}^1, x_{jt}^2} \; \pi \sum_{i=1}^{n_{jt}} q_{jt}^i - c\mathbb{I}_{(x_{jt}^1>0)} - c\mathbb{I}_{(x_{jt}^2>0)} - w(x_{jt}^1 + x_{jt}^2) + \beta\left[\phi_{jt}\pi\overline{q}' + \frac{n_{jt}-\tau}{n_{jt}}\pi\sum_{i=1}^{n_{jt}} q_{jt}^i\right], \tag{3.24}$$

subject to the R&D function (3.20).

Conditional on paying the fixed cost and hiring each input type $k \in \{1, 2\}$, the first-order condition with respect to x_{jt}^k ($k \in \{1, 2\}$) gives us an expression for the optimal value for x_{jt}^k as

$$x_{jt}^{k*} = \left(\frac{\beta\pi\chi}{\psi w}\right)^{\frac{\psi}{\psi-1}} (\overline{q}')^{\frac{\psi}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}}. \tag{3.25}$$

Note that for both cases in (3.21) and (3.22), the optimal value for ϕ_{jt} is derived as follows:

$$\phi_{jt}^* = \chi^{\frac{\psi}{\psi-1}} \left(\frac{\beta\pi}{\psi w}\right)^{\frac{1}{\psi-1}} (\overline{q}')^{\frac{1}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}}\;. \tag{3.26}$$

With the optimal choice for x_{jt}^k and ϕ_{jt}, we can rewrite the firm's value function as follows assuming that it conducts innovation:

$$\Pi_{jt}\left(\{q_{jt}^i\}_{i=1}^{n_{jt}}\right) = \pi\sum_{i=1}^{n_{jt}} q_{jt}^i + (\psi - \mathbb{L})\left(\frac{\beta\pi\chi\overline{q}'}{\psi}\right)^{\frac{\psi}{\psi-1}} w^{-\frac{1}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}} + \beta\pi\frac{n_{jt}-\tau}{n_{jt}}\sum_{i=1}^{n_{jt}} q_{jt}^i - \mathbb{L}c, \tag{3.27}$$

where $\mathbb{L}$ is an indicator function for the degree of complementarity between the two R&D inputs as follows:

$$\mathbb{L} = \begin{cases} 1 & \text{if perfect substitutes as in (3.21)} \\ 2 & \text{if perfect complements as in (3.22)} \end{cases} . \tag{3.28}$$

Otherwise, if the firm does not do any innovation, it obtains the following value:

$$\Pi^0_{jt}\left(\{q^i_{jt}\}^{n_{jt}}_{i=1}\right) = \pi \sum_{i=1}^{n_{jt}} q^i_{jt} + \beta\pi \frac{n_{jt} - \tau}{n_{jt}} \sum_{i=1}^{n_{jt}} q^i_{jt}. \tag{3.29}$$

3.5.4 Firm Entry

Potential startups take into account the expected value of entry, which is the continuation value of operating in the market multiplied by the success probability of entry. Conditional on successful entry, they are randomly assigned a product line. We assume that they enter the market with a single product given the fact that a majority of startups are single-product firms.[11]

This gives the following value function for startups conditional on entering the economy successfully and conducting innovation after entry:

$$\Pi_{jt}(q^e_{jt}) = \pi q^e_{jt} + (\psi - \mathbb{L}) \left(\frac{\beta\pi\chi\overline{q}'}{\psi}\right)^{\frac{\psi}{\psi-1}} w^{-\frac{1}{\psi-1}} + \beta\pi(1-\tau) q^e_{jt} - \mathbb{L}c\,, \tag{3.30}$$

which has the same form as (3.27), substituting out $n_{jt} = 1$ and replacing the existing set of product qualities with q^e_{jt}, which refers to the quality startup j attains from a randomly assigned product market e. $\mathbb{L}$ is the indicator as in (3.28).

[11]This setup is consistent with Klette and Kortum (2004).

Otherwise, if they choose not to innovate after entry, they obtain the following:

$$\Pi^0_{jt}(q^e_{jt}) = \pi q^e_{jt} + \beta\pi(1-\tau)q^e_{jt}\,. \tag{3.31}$$

3.5.5 Main Implications of Knowledge Complexity

Proposition 8. *If the required inputs for successful innovation become more complex (interdependent), the total amount of R&D inputs needed to produce a given probability of innovation increases.*

Proof. If inputs are perfectly substitutable as in (3.21), then firms would only use one of the input types and are indifferent between using either of them. Thus, the optimal solution is either x^{1*}_{jt} satisfying (3.25) and $x^{2*}_{jt} = 0$, or $x^{1*}_{jt} = 0$ and x^{2*}_{jt} satisfying (3.25). Therefore, the total optimal input level is equal to

$$x^{1*}_{jt} + x^{2*}_{jt} = \left(\frac{\beta\pi\chi}{\psi w}\right)^{\frac{\psi}{\psi-1}} (\overline{q}')^{\frac{\psi}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}}. \tag{3.32}$$

On the other hand, if the R&D inputs are perfectly complementary (or interdependent) to each other, as in (3.22), (3.25) holds for both inputs $k \in \{1, 2\}$. Thus, the amount of inputs needed increases as follows:

$$x^{1*}_{jt} + x^{2*}_{jt} = 2\left(\frac{\beta\pi\chi}{\psi w}\right)^{\frac{\psi}{\psi-1}} (\overline{q}')^{\frac{\psi}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}}. \tag{3.33}$$

Note that the optimal innovation intensity does not vary across the two cases, as seen in (3.26). In other words, the amount of R&D input needed to produce the same amount of innovation increases. This suggests that higher interdependence across innovation inputs implies a higher cost for firms to produce the same amount of innovation. ■

Lemma 5. *There exists a cutoff for firm knowledge base (size) above which firms conduct innovation.*

Proof. Recall that if firms choose to conduct innovation, their value is given by (3.27). Otherwise, firms' value is given by (3.29). Comparing (3.27) and (3.29), we can derive firms' marginal value of innovation as follows:

$$\underbrace{\psi\left(\frac{\beta\pi\chi\overline{q}'}{\psi}\right)^{\frac{\psi}{\psi-1}} w^{-\frac{1}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}}}_{\text{marginal gain}} \underbrace{- \mathbb{L}\left(\frac{\beta\pi\chi\overline{q}'}{\psi}\right)^{\frac{\psi}{\psi-1}} w^{-\frac{1}{\psi-1}} n_{jt}^{\frac{1-\sigma}{\psi-1}} - \mathbb{L}c}_{\text{marginal cost (wage + search/learning cost)}}, \tag{3.34}$$

which is equal to $\Pi_{jt}\left(\{q_{jt}^i\}_{i=1}^{n_{jt}}\right) - \Pi_{jt}^0\left(\{q_{jt}^i\}_{i=1}^{n_{jt}}\right)$. Here, $\mathbb{L}$ is the indicator as in (3.28).

Note that the first term refers to the marginal gain of innovation, which depends positively on firms' existing knowledge base and innovation intensity, and the last two terms stand for the marginal cost of innovation, which is the sum of R&D input costs. If the total is positive, firms select into doing innovation, and otherwise, do not innovate.

Thus, the n_{jt} for which (3.34) is zero is the threshold for firm innovation. The following shows the threshold for firms' knowledge base (size) above which firms do innovation:

$$\bar{n} = \left[\mathbb{L}c\left(\psi - \mathbb{L}\right)^{-1}\left(\frac{\beta\pi\chi\overline{q}'}{\psi}\right)^{-\frac{\psi}{\psi-1}} w^{\frac{1}{\psi-1}}\right]^{\frac{\psi-1}{1-\sigma}}. \tag{3.35}$$

■

Proposition 9. *If the knowledge required for successful innovation becomes more complex (in-*

terdependent), the threshold size for conducting innovation increases.

Proof. Recall the cutoff in (3.35). Note here that $\mathbb{L}c\left(\psi-\mathbb{L}\right)^{-1}$ is an increasing function of $\mathbb{L}$, and thus, the threshold $\bar{n}$ increases in the degree of knowledge complexity (interdependence). This implies that only firms with a large enough knowledge base (size) will conduct innovation, and that the minimum size increases in knowledge complexity and interdependence. ■

This implies that if knowledge complexity (interdependence) in the innovation process rises, only large firms with a broad knowledge base would optimally continue to innovate. On the other hand, small and young firms with less knowledge stock will find it even less profitable to undertake innovation if they have to invest in diverse R&D inputs. This slows down the innovation activities and growth of small and young firms. With the same logic, the following proposition shows how knowledge complexity would affect firm entry.

Proposition 10. *Increasing knowledge complexity (interdependence) decreases the potential firm entry rate.*

Proof. Increasing knowledge complexity, proxied by moving from $\mathbb{L}=1$ to $\mathbb{L}=2$ as before, decreases the post-entry value of a successful business conducting innovation (3.30). This dampens potential startups' incentives to open a new business, which reduces the entry of firms. ■

These results imply that as R&D becomes more complex by requiring different types of inputs with higher interdependence, the overall R&D costs as well as the minimum threshold knowledge base for innovation increase, even if the probability of innovation is held fixed. This creates a force to hamper firm entry and the innovation activity of small and young firms with less of a knowledge base.

The current model can be extended to have multiple inputs ($\mathbb{L} > 2$), and the model predictions should remain the same. Furthermore, we can endogenize the search/learning cost structure c as a function of firms' knowledge base. For instance, firms with larger existing knowledge base may have more of a network of information across different markets, and this can reduce their search/learning cost. Therefore, the fixed cost can further be assumed to be a decreasing function of firms' knowledge base, as follows:

$$c = c(n_{jt}), \quad c'(n_{jt}) < 0.$$

Incorporating this structure can amplify the marginal value of innovation for firms with a large knowledge base in the case of increasing knowledge interdependence.

3.6 Concluding Remarks

In this paper, we study the trend of increasing knowledge complexity and how it affects the patterns of firm innovation, and small and young firm activity in the U.S. Using Compustat and the Census firm-level data integrated with the UPSTO patent database, we document several stylized facts on knowledge complexity. We find that knowledge complexity has been rising over time, a pattern that is robust across different measures, including the number of coinventors, technology groups per patent, and the degree of knowledge interdependence across patent classes. Moreover, these patterns are observed for USPTO assignees, Compustat firms, and administrative firms from the Census.

We show that rising knowledge complexity is associated with declining business dynamism at the industry level, for measures including firm entry, the share of young firms, and young firms' share of employment, job creation, and job reallocation.

To investigate a potential mechanism to explain the linkage between increasing knowledge complexity and declining business dynamism, we build a simple endogenous growth model with complementary innovation inputs. The model predicts that rising knowledge complexity in the innovation process, proxied by higher complementarity or interdependence between inputs, requires firms to use multiple sets of inputs together and incurs rising innovation costs. This can negatively affect the innovation of young and small firms that do not have enough of a knowledge base, and thus can hamper firm entry.

There are several issues remaining to be addressed in this paper. First, the current theoretical framework is based on partial equilibrium. We need to solve the general equilibrium of the model and quantify the mechanism, which is in progress. Second, it would be worthwhile to further explore different measures of knowledge complexity in the data, to better understand the fundamental sources behind the rising knowledge complexity. There could be specific types of technology that drive the observed trend, such as information technology, semiconductor, AI, and etc. Lastly, we intend to develop an identification strategy to test the causal effects of knowledge complexity on young firm activity. All of these remain as our to-do list.

Appendix A: Chapter 1 Appendix

A.1 Bayesian Learning

Suppose that initial prior is $\nu_j \sim N(\bar{\nu}_0, \sigma_0^2)$, and there is an observation of $\ln P_{jt} = \nu_j + \varepsilon_{jt}$ such that $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$, $\ln P_{jt}|_{\nu_j} \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\varepsilon^2)$. Following the Bayes' rule,

$$f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt} | \nu_j),$$

we have:

$$\begin{aligned} f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt} | \nu_j) &= \left(\frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\nu_j - \bar{\nu}_0)^2}{2\sigma_0^2} \right) \right) \left(\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{(\ln P_{jt} - \nu_j)^2}{2\sigma_\varepsilon^2} \right) \right) \\ &\propto \left(\frac{1}{\sqrt{2\pi\sigma_0^2\sigma_\varepsilon^2}} \exp\left(-\frac{\left(\nu_j - \left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2} \right) \right)^2}{2 \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}} \right) \right), \end{aligned}$$

which implies that

$$f(\nu_j | \ln P_{jt}) \sim N\left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2}, \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2} \right).$$

A.2 Derivation of the Stationary Recursive Competitive Equilibrium

A.2.1 Matching Function and Labor Market Tightness

Based on the CES matching function and the job finding and filling rates (1.26), (1.27) in the main text, the firm's complementary slackness condition (1.24) can be rewritten as follows:

$$\theta(x)\left(\frac{c}{(1+\theta(x)^{\gamma})^{-\frac{1}{\gamma}}}+x-\kappa\right)=0.$$

Thus, for active market x, such that $\theta(x)>0$,

$$\theta(x)=\left((\frac{\kappa-x}{c})^{\gamma}-1\right)^{\frac{1}{\gamma}},$$

and this holds for x in the range of $x<\kappa-c$. On the other hand, if $x\geqslant\kappa-c$, this market is inactive, and we have $\theta(x)=0$. Therefore, (1.28) is proved.

A.2.2 Workers' Problem

A.2.2.1 Unemployed Workers

Using the job finding rate (1.26), the unemployed workers' problem can be simplified as follows:

$$\max_{x^U}\theta(x^U)(1+\theta(x^U)^{\gamma})^{-\frac{1}{\gamma}}\big(x^U-\mathbf{U}\big).$$

The first-order condition with respect to x^U gives

$$\theta(x^U)+\frac{1}{(1+\theta(x^U)^{\gamma})}\theta'(x^U)\big(x^U-\mathbf{U}\big)=0.$$

Using (1.28), if $x^U < \kappa - c$, the following holds:

$$\theta(x^U) = \Big((\frac{\kappa - x^{\mathbf{U}}}{c})^{\gamma} - 1 \Big)^{\frac{1}{\gamma}}$$

Plugging it back to the unemployed workers' first-order condition, the following can be derived:

$$x^U = \kappa - (c^{\gamma}(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}}. \tag{A.1}$$

The result shows that x^U is constant with respect to firms' state variables. This is because unemployed workers have no heterogeneity (both ex-ante and ex-post) and thus all choose the same market to search.

Thus,

$$\theta(x^U) = \begin{cases} \Big(c^{-\frac{\gamma}{1+\gamma}} (\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1 \Big)^{\frac{1}{\gamma}} & \text{if} \quad x^U < \kappa - c \\ 0 \quad \text{if} \quad x^U \geqslant \kappa - c, \end{cases}$$

and

$$f(\theta(x^U)) = \begin{cases} \Big(c^{-\frac{\gamma}{1+\gamma}} (\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1 \Big)^{\frac{1}{\gamma}} \Big(c^{-\frac{1}{1+\gamma}} (\kappa - \mathbf{U})^{\frac{1}{1+\gamma}} \Big) & \text{if} \quad x^U < \kappa - c \\ 0 \quad \text{if} \quad x^U \geqslant \kappa - c, \end{cases}$$

which implies that if $x^U \geqslant \kappa - c$, the market x^U is inactive and workers remain unemployed.

Furthermore, $\mathbf{U}$ is a fixed point of the following equation:

$$\mathbf{U} = b + \beta \Bigg(\mathbf{U} + \max \Bigg[0, \Big(c^{-\frac{\gamma}{1+\gamma}} (\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1 \Big)^{\frac{1}{\gamma}} \Big(c^{-\frac{1}{1+\gamma}} (\kappa - \mathbf{U})^{\frac{1}{1+\gamma}} \Big) \Big(\kappa - (c^{\gamma}(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} - \mathbf{U} \Big) \Bigg] \Bigg).$$

A.2.2.2 Employed Workers

In a similar fashion, the employed workers' problem can be solved, and a similar solution for $x^E(a, \tilde{P}, l, P)$ can be obtained for workers employed at a firm having $(a, \tilde{P}, l, P)$. That is, given the promised utility $\tilde{\mathbf{W}}(a, \tilde{P}, l, P)$ offered by the firm, the workers will direct their on-the-job search to:

$$x^E(a, \tilde{P}, l, P) = \kappa - (c^\gamma(\kappa - \tilde{\mathbf{W}}(a, \tilde{P}, l, P)))^{\frac{1}{1+\gamma}}, \tag{A.2}$$

as far as the market is active, i.e. $\mathbf{x^E}(a, \tilde{P}_{a-1}, l, P) < \kappa - c$. This depends on workers' opportunity cost of moving to other firms, which is a function of the current employer's state variables. x^E is increasing in the workers' opportunity cost $\tilde{\mathbf{W}}$, which means that the higher utility $\tilde{\mathbf{W}}$ workers receive from their current employer, the higher utility x^E another firm needs to deliver to poach them successfully. In other words, workers only climb up to a labor market that provides higher utility than what they currently have, which captures the standard job ladder property in a directed search framework.

Notably from the solutions (A.1) and (A.2), firms' promised utility to both unemployed and employed workers in the search market does not depend on recruiting firms' characteristics, but rather only on workers' employment status. In other words, workers are not indifferent across active submarkets, and search in a specific submarket that provides a certain promised utility (at least equal to or above their outside options) upon successful job match, while firms are indifferent across active submarkets in equilibrium.

Also, the equilibrium market tightness and job finding rate for the market x^E are derived as follows:

$$\theta(x^E(a, \tilde{P}_{a-1}, l, P)) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a, \tilde{P}, l, P))^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x^E < \kappa - c \\ 0 \quad \text{otherwise,} & \end{cases}$$

and

$$f(\theta(x^E)) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a, \tilde{P}, l, P))^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}} \left(c^{-\frac{1}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a, \tilde{P}, l, P))^{\frac{1}{1+\gamma}}\right) & \text{if} \quad x^E < \kappa - c \\ 0 \quad \text{if} \quad x^E \geqslant \kappa - c. & \end{cases}$$

A.2.3 Joint Surplus Maximization

Using Lemma 1 and substituting out $\{w^i_{jt}\}_i$ in (1.13), I have:

$$
\begin{aligned}
&\mathbf{J}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \Omega^{-w}_{jt-1}) \\
&= \max_{\substack{\Omega^{-w}_{jt}=\{d_{jt+1}, s_{jt+1}, \tilde{\mathbf{W}}_{\mathbf{jt+1}}\},\\ x_{jt}, h_{jt}}} P_{jt} l^{\alpha}_{jt} - x_{jt} h_{jt} - \frac{c}{q(\theta(x_{jt}))} h_{jt} - \tilde{\mathbf{W}}_{\mathbf{jt}} (1 - s_{jt})(1 - \lambda f(\theta(x^E_{jt})))l_{jt-1} - c_f \\
&\quad + \beta \mathbb{E}_{jt} \Bigg[(1-\delta)(1-d_{jt+1}) \Big(\mathbf{J}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \Omega^{-w}_{jt}) + (1 - s_{jt+1}) \lambda f(\theta(x^E_{jt+1})) x^E_{jt+1} l_{jt} \\
&\quad + \tilde{\mathbf{W}}_{\mathbf{jt+1}} (1 - s_{jt+1})(1 - \lambda f(\theta(x^E_{jt+1})))l_{jt} \Big) + \Big(\delta + (1-\delta)(d_{jt+1} + (1 - d_{jt+1}) s_{jt+1}) \Big) \mathbf{U_{t+1}} l_{jt} \Bigg],
\end{aligned}
\tag{A.3}
$$

subject to (1.14), (1.15), and (1.16). For notation, I use Ω^{-w}_{jt} to denote the contract abstracting from the wage $\{w^i_{jt}\}_i$. Note here that the problem can be solved without the participation constraint first, and one can prove that the solution satisfies the participation constraint. Also, using the incentive constraint, the problem can be rephrased as the firm choosing x^E_{jt+1} and pinning down $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ indirectly. In other words, the firm indirectly controls the job-hopping rate $\lambda f(\theta(x^E_{jt+1}))$ by taking into account the workers' optimal job search behavior and offers $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ backed out from the worker's incentive constraint. Following this, once the solution is obtained, I prove in section A.3.1 that the participation constraint holds.

Reformatting (A.3) to be at the production stage after search and matching, the firm value function can be rewritten as:

$$
\begin{aligned}
&\mathbf{J}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}, \Omega^{-w}_{jt-1}) \\
&\quad = \max_{\substack{\Omega^{-w}_{jt}=\{d_{jt+1}, s_{jt+1}, \tilde{\mathbf{W}}_{\mathbf{jt+1}}\},\\ x_{jt+1}, h_{jt+1}}} P_{jt} l^{\alpha}_{jt} - x_{jt} h_{jt} - \tilde{\mathbf{W}}_{\mathbf{jt}} (1 - s_{jt})(1 - \lambda f(\theta(x^E_{jt})))l_{jt-1} - c_f \\
&\quad + \beta \mathbb{E}_{jt} \Bigg[(1-\delta)(1-d_{jt+1}) \Big(\mathbf{J}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \Omega^{-w}_{jt}) - \Big(x_{jt+1} + \frac{c}{q(\theta(x_{jt+1}))} \Big) h_{jt+1} \\
&\quad + (1 - s_{jt+1}) \lambda f(\theta(x^E_{jt+1})) x^E_{jt+1} l_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt+1}} (1 - s_{jt+1})(1 - \lambda f(\theta(x^E_{jt+1})))l_{jt} \Big) \\
&\quad + \Big(\delta + (1-\delta)(d_{jt+1} + (1 - d_{jt+1}) s_{jt+1}) \Big) \mathbf{U_{t+1}} l_{jt} \Bigg],
\end{aligned}
\tag{A.4}
$$

Let $\mathbf{V}^{prod}_{jt} \equiv \mathbf{J}^{prod}_{jt} + x_{jt}h_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f\big(\theta(x^E_{jt})\big))l_{jt-1}$ be the joint surplus of the firm and its workers at the production stage. Using this and rewriting (A.3), I obtain:

$$\begin{aligned}
\mathbf{V}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}) = & \max_{d_{jt+1}, s_{jt+1}, x_{jt+1}, x^E_{jt+1}, h_{jt+1}} P_{jt}l^{\alpha}_{jt} - c_f \\
& + \beta\mathbb{E}_{jt}\Bigg[(1-\delta)(1-d_{jt+1})\Big(\mathbf{V}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt+1}, P_{jt+1}) - \Big(x_{jt+1} + \frac{c}{q(\theta(x_{jt+1}))}\Big)h_{jt+1} \\
& + (1-s_{jt+1})\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1}l_{jt}\Big) + \Big(\delta + (1-\delta)\big(d_{jt+1} + (1-d_{jt+1})s_{jt+1}\big)\Big)\mathbf{U_{t+1}}l_{jt}\Bigg],
\end{aligned} \tag{A.5}$$

subject to (1.14), (1.15) and (1.16). The firm's original profit maximization can be fully replicated by the joint surplus maximization in (A.5), given that the last two terms defining $\mathbf{V}^{prod}$, $x_{jt}h_{jt}$ and $\tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))l_{jt-1}$, are predetermined so that maximizing $\mathbf{V}^{prod}$ gives the same results as maximizing $\mathbf{J}^{prod}_{jt}$ and thus $\mathbf{J}_{jt}$. Furthermore, using (A.5) simplifies the set of state variables in $\mathbf{J}_{jt}$ and increases tractability. Lastly, (1.17) and (1.18) (assumed to hold with equality) characterize the equilibrium wages that the firm needs to pay $\{w^i_{jt}\}_i$.

In a similar fashion, the free-entry condition (1.19) can be rephrased as follows:

$$\int \max_{d^e_{jt}, l^e_{jt}, x^e_{jt}} \Bigg[(1-d^e_{jt})\Big(\mathbf{V}^{prod}(0, 0, l^e_{jt}, P_{jt}) - x^e_{jt}l^e_{jt} - \frac{c}{q(\theta(x^e_{jt}))}l^e_{jt}\Big)\Bigg]dF_e(P_{jt}) - c_e = 0. \tag{A.6}$$

A.2.4 Firms' Decision Rules

As discussed in the previous section, the firm profit maximization can be replicated by the following joint surplus maximization problem:

$$\begin{aligned}
& \mathbf{V}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}) \\
& = \max_{d_{jt+1}, s_{jt+1}, h_{jt+1}, x^E_{jt+1}} P_{jt}l^{\alpha}_{jt} - c^f + \beta\mathbb{E}_{jt}\Big[\delta\mathbf{U_{t+1}}l_{jt} + (1-\delta)\big(d_{jt+1} + (1-d_{jt+1})s_{jt+1}\big)\mathbf{U_{t+1}}l_{jt} \\
& \quad + (1-\delta)(1-d_{jt+1})\Big(\mathbf{V}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt+1}, P_{jt+1}) - \kappa h_{jt+1} + (1-s_{jt+1})\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1}l_{jt}\Big)\Big].
\end{aligned}$$

Given that choice variables are contingent on future productivity, it can be transformed with the following value function defined at the beginning of each period, $\mathbf{V}_t^{init}$:

$$\mathbf{V}^{init}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) = \max_{d_{jt}, s_{jt} h_{jt}, x^E_{jt}} \delta \mathbf{U_t} l_{jt-1} + (1-\delta)(d_{jt} + (1-d_{jt})s_{jt})\mathbf{U_t} l_{jt-1}$$
$$+ (1-\delta)(1-d_{jt})\Big(P_{jt} l_{jt}^{\alpha} - c^f - \kappa h_{jt} + (1-s_{jt})\lambda f(\theta(x^E_{jt}))x^E_{jt} l_{jt-1} + \beta \mathbb{E}_{jt} \mathbf{V}^{init}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1})\Big)$$

subject to $l_{jt} = h_{jt} + (1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))l_{jt-1}$. This can also rephrase the free-entry condition as follows:

$$\int \max_{d^e_{jt}, l^e_{jt}} (1-d^e_{jt})\Big(P_{jt}(l^e_{jt})^{\alpha} - c^f - \kappa l^e_{jt} + \beta \mathbb{E}_{jt} \mathbf{V}^{init}(1, \ln P_{jt}, l^e_{jt}, P_{jt+1})\Big) dF_e(P_{jt}) - c^e = 0. \tag{A.7}$$

Note that this value function has the following relationship with the firm's original value function in the main text (A.3):

$$\mathbf{V}^{init}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) = \Big(\delta + (1-\delta)\big(d_{jt} + (1-d_{jt})s_{jt}\big)\Big)\mathbf{U_t} l_{jt-1}$$
$$+ (1-\delta)(1-d_{jt})(1-s_{jt})\Big(\lambda f(\theta(x^E_{jt}))x^E_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))\Big) l_{jt-1}$$
$$+ (1-\delta)(1-d_{jt})\mathbf{J}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \Omega_{jt-1}), \tag{A.8}$$

where the first two lines are the workers' future expected value as of the previous period, and the last line is the firm's value (1.13) in the search and matching stage. Note that d_{jt}, s_{jt}, l_{jt}, h_{jt}, x_{jt}, and $\tilde{\mathbf{W}}_{\mathbf{jt}}$ are the firm's policy functions, each of which is a function of the following set of state variables: $(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})$. This relationship will be useful to draw out interpretations of equilibrium equations in the following section.

Dropping the time subscripts, it becomes:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \max_{d,s,h,x^E} \delta U l + (1-\delta)(d + (1-d)s)U l + (1-\delta)(1-d)\Big(P l'^{\alpha} - c^f - \kappa h$$
$$+ (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E} \mathbf{V}^{init}(a+1, \tilde{P}', l', P')\Big) \tag{A.9}$$

subject to $l' = h+(1-s)(1-\lambda f(\theta(x^E)))l$. The solution of x^E pins down $\tilde{\mathbf{W}}$ following (A.2). The expectation of P' is formed based on the posterior updated after observing P, which is $\ln P' \sim (\frac{\frac{\bar{\nu}_0}{\sigma_0^2}+\frac{a\tilde{P}+\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2}+\frac{(a+1)}{\sigma_\epsilon^2}})$.

Note that the first term δUl is independent of the variables to maximize and $(1-\delta)$ in the remaining two terms just scales the objective function. Thus, the maximization problem is simplified to maximize the following terms:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = \max_{d,s,h,x^E}(d+(1-d)s)Ul + (1-d)\Big(Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l + \beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Big),$$

subject to $l' = h+(1-s)(1-\lambda f(\theta(x^E)))l$ and $\tilde{P}' = \frac{a\tilde{P}+\ln P}{a+1}$.

I first solve the problem for s, h, x^E, and then for d, which rephrases the above maximization problem as:

$$\max\Bigg[Ul, \max_{s,h,x^E} sUl + Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l + \beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Bigg]. \tag{A.10}$$

Let's first focus on the maximization in the large bracket, which solves for optimal s, h, and x^E. Note that there is no case in which firms hire and separate workers at the same time. In other words, if $s > 0$, then $h = 0$ should hold, and if $h > 0$, then $s = 0$. This is discussed in detail in the following Section A.2.4.1.

A.2.4.1 Nonexistence of the case $h > 0$ and $s > 0$

Proposition A.2.1. *Hiring firms with productivity P drawn above the hiring cutoff $\mathcal{P}^h(a,\tilde{P},l)$ do not layoff workers. Similarly, firms that layoff workers with productivity P drawn between the middle and lower productivity cutoffs $\mathcal{P}^q(a,\tilde{P},l)$ and $\mathcal{P}^l(a,\tilde{P},l)$ do not hire new workers.*

Proof. Suppose that firms both hire and separate workers, e.g. $h > 0$ and $s > 0$, so that they solve the following maximization problem:

$$\max_{h,s,x^E} sUl + Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{A.11}$$

The first-order conditions with respect to h, s, and x^E are as follows (in the same order):

$$\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] = \kappa, \tag{A.12}$$

$$Ul - \lambda f(\theta(x^E))x^E l - (1 - \lambda f(\theta(x^E)))l\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] = 0, \tag{A.13}$$

$$\lambda f'(\theta(x^E))\theta'(x^E)x^E l + \lambda f(\theta(x^E))l - \lambda f'(\theta(x^E))\theta'(x^E)l\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] = 0. \tag{A.14}$$

Using (A.12) to substitute out the term $\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big]$ in (A.14), and using (1.28), I can rewrite the left-hand side of (A.14) as follows:

$$\frac{(\kappa - x^E)^{\gamma} c^{-\gamma}\Big((\frac{\kappa - x^E}{c})^{\gamma} - 1\Big)^{\frac{1}{\gamma}}(\frac{\kappa - x^E}{c})^{\gamma}}{(\frac{\kappa - x^E}{c})^{\gamma} - 1} = \frac{\Big((\kappa - x^E)^{\gamma} c^{-\gamma}\Big)^2}{\Big((\frac{\kappa - x^E}{c})^{\gamma} - 1\Big)^{1-\frac{1}{\gamma}}} > 0.$$

This term can be proved to be strictly positive given that $x^E < \kappa - c$ for any active markets x^E. This means that the marginal value of x^E is strictly positive, and thus optimal x^E reaches the upper bound:

$$x^E = \kappa - c. \tag{A.15}$$

Thus, for hiring firms it follows that $f(\theta(\kappa - c)) = 0$, which makes the marginal value of s from (A.13) negative as follows:

$$U - \Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] < 0. \tag{A.16}$$

This is due to (A.12) and $\kappa > U$, and shows that hiring firms can never have any marginal value of separating workers and would never separate workers.

In a similar fashion, contracting firms would never hire workers, given that their marginal value of a new hire from (A.12) is always negative:

$$\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\right] - \kappa < 0, \tag{A.17}$$

given (A.24) and $\kappa > U$. Therefore, this completes the proof that if $h > 0$, $s = 0$ needs to hold, and vice versa.

The proof enables me to split the firm's problem into the following three cases from A.2.4.2 through A.2.4.4. ■

A.2.4.2 Hiring Firms: $s = 0$ and $h > 0$

$$\max_{h, x^E} P l'^{\alpha} - c^f - \kappa h + \lambda f(\theta(x^E)) x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{A.18}$$

subject to $l' = h + (1 - \lambda f(\theta(x^E)))l$ and $\tilde{P}' = \frac{a\tilde{P} + \ln P}{a+1}$.

As before, the first-order conditions with respect to h and x^E are (A.12) and (A.14), respectively. We know the optimal x^E is pinned at the upper bound as in (A.15).

Lastly, using (A.2), the utility level $\tilde{\mathbf{W}}$ that firms will offer to their incumbent workers under this case is determined by:

$$\tilde{\mathbf{W}} = \kappa - c. \tag{A.19}$$

A.2.4.3 Inactive Firms: $s = 0$ and $h = 0$

Note that this case holds only when

$$\left[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{l'=l}\right] < \kappa,$$

where the marginal value of h is strictly less than zero and $h = 0$ is optimal. Under this case, firms need to solve the following problem:

$$\max_{x^E} P l'^{\alpha} - c^f + \lambda f(\theta(x^E)) x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{A.20}$$

subject to $l' = (1 - \lambda f(\theta(x^E)))l$.

Using the first-order condition with respect to x^E in (A.14), and evaluating l' at $(1 - \lambda f(\theta(x^E)))l$, we have the following equation to determine x^E:

$$x^E + \frac{f(\theta(x^E))}{f'(\theta(x^E))\theta'(x^E)} - \left[\alpha P \Big((1 - \lambda f(\theta(x^E)))l\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{l'=(1-\lambda f(\theta(x^E)))l}\right] = 0. \tag{A.21}$$

Using (A.2), the equilibrium utility level $\tilde{\mathbf{W}}$ firms offer to their incumbent workers is pinned down by:

$$\tilde{\mathbf{W}} = \kappa - (\kappa - x^E)^{1+\gamma} c^{-\gamma}. \tag{A.22}$$

Note that this only holds when the optimal x^E is in the range of $\kappa \leqslant c$. If P is high enough so that the left-hand side of (A.21) becomes strictly greater than 0, then as before for the hiring firms, the optimal solution is bound by the upper bound, i.e. $x^E = \kappa - c$. This holds when

$$\kappa - c < \left[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{l'=l}\right],$$

so that the marginal value of x^E is strictly positive, and hence, the optimal x^E is bound by the upper bound

$\kappa - c$. In this case, firms would not just stay inactive but also not allow workers quitting. In other words, they stay inactive not allowing quitting, i.e. $l' = l$. More details about the productivity cutoff will be supplemented in A.2.4.5.

A.2.4.4 Separating Firms with Layoffs: $s > 0$ and $h = 0$

$$\max_{s,x^E} sUl + Pl'^{\alpha} - c^f + (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{A.23}$$

subject to $l' = (1-s)(1-\lambda f(\theta(x^E)))l$.

Note that the first-order conditions with respect to s and x^E hold the same as in (A.13) and (A.14), respectively. Rewriting (A.13) by canceling out l and using (1.26) as before, the following is obtained:

$$\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\right] = \frac{U - \lambda x^E \Big(\theta(x^E)(1+\theta(x^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}{1 - \lambda\Big(\theta(x^E)(1+\theta(x^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}. \tag{A.24}$$

Substituting out the term $\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\right]$ in (A.14) using (A.24), x^E is determined by the following equation:

$$\kappa - U = c\Big[(1+\theta(x^E)^{\gamma})^{1+\frac{1}{\gamma}} - \lambda\theta(x^E)^{1+\gamma}\Big]. \tag{A.25}$$

Again, the equilibrium utility level $\tilde{\mathbf{W}}$ is determined as (A.22).

A.2.4.5 Productivity Cutoffs

Now, next step is to determine the following three productivity cutoffs among operating firms: i) a hiring cutoff above which firms decide to hire workers and below which firms decide to stay inactive without quits; ii) a quitting cutoff above which firms stay inactive without quits and below which firms stay inactive but allow quits; and iii) a separation cutoff above which firms stay inactive with quits and below which firms decide to layoff workers. And lastly, a productivity cutoff for firm exit can be determined, below which firms endogenously exit.

Note that all these cutoffs should depend on the other firm state variables, which are a, $\tilde{P}$, and l. Let the hiring cutoff denoted by $\mathcal{P}^h(a, \tilde{P}, l)$, the quitting cutoff denoted by $\mathcal{P}^q(a, \tilde{P}, l)$, the layoff cutoff denoted by $\mathcal{P}^l(a, \tilde{P}, l)$, and the exit cutoff denoted by $\mathcal{P}^x(a, \tilde{P}, l)$.

First, to determine the hiring cutoff, it is determined by (A.12) evaluated at $l' = l$. The reason behind this is that given $(a, \tilde{P}, l)$, if P lies in a range in which the marginal value of hiring (the right-hand side of (A.12) becomes less than κ, then firms no longer hire any workers. The threshold of P is determined at a point where it is optimal to choose $h = 0$ from the hiring firms' problem, below which firms would never hire workers due to the reason marginal value of hiring a new worker is not high enough.

Therefore, the following equation determines the hiring productivity cutoff $\mathcal{P}^h(a, \tilde{P}, l)$:

$$\Big[\alpha \mathcal{P}^h l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^h}{a+1}, l'=l}\Big] = \kappa, \tag{A.26}$$

where the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with the firm age $a+1$ and the average productivity $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^h}{a+1}$ at the beginning of the next period.

Next, the quitting cutoff can be obtained as follows. Note that firms would not hire workers when

$$\Big[\alpha P\Big((1-\lambda f(\theta(x^E)))l\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{l'=(1-\lambda f(\theta(x^E)))l}\Big] < \kappa, \tag{A.27}$$

as before. At the same time, if the marginal value of x^E is still high enough, then firms should also set x^E to the upper bound. This happens when:

$$\lambda f'(\theta(x^E))\theta'(x^E)x^E l + \lambda f(\theta(x^E))l - \lambda f'(\theta(x^E))\theta'(x^E)l\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] > 0,$$

which can be rephrased as

$$\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] > x^E + \frac{f(\theta(x^E))}{f'(\theta(x^E))\theta'(x^E)},$$

given $\theta'(x^E) < 0$ and $f'(\theta(x^E)) < 0$. Also, given $x^E = \kappa - c$, this can further rephrased as

$$\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}\Big] > \kappa - c. \tag{A.28}$$

Combining (A.27) and (A.28), firms would stay inactive without allowing quits in the following range

$$\kappa - c < \Big[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{l'=l}\Big] < \kappa, \tag{A.29}$$

in other words, the quitting cutoff $\mathcal{P}^q(a, \tilde{P}, l)$ is determined by the following:

$$\Big[\alpha \mathcal{P}^q l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^q}{a+1}, l'=l}\Big] = \kappa - c, \tag{A.30}$$

below which firms start allowing quits. Again, the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with $a+1$ and $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^q}{a+1}$ as before.

Lastly, in regards to the layoff cutoff, it is determined by (A.24) evaluated at $l' = (1 - \lambda f(\theta(x^E)))l$ where x^E is the root of (A.25). Similar to the hiring cutoff, given $(a, \tilde{P}, l)$, if P lies in a range in which the marginal value of layoff (the left-hand side of (A.24) becomes less than its cost (the right-hand side of (A.24), then firms no longer lay off any workers. Therefore, the cutoff is determined at where it is optimal to choose $s = 0$ from the separating firms' problem, above which firms would never lay off workers.

Therefore, the following equation determines the layoff productivity cutoff $\mathcal{P}^l(a, \tilde{P}, l)$:

$$\begin{aligned} &\Big[\alpha \mathcal{P}^l((1 - \lambda f(\theta(x^E)))l)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^l}{a+1}, l'=(1-\lambda f(\theta(x^E)))l}\Big] \\ &= \frac{U - \lambda x^E\Big(\theta(x^E)(1+\theta(x^E)^\gamma)^{-\frac{1}{\gamma}}\Big)}{1 - \lambda\Big(\theta(x^E)(1+\theta(x^E)^\gamma)^{-\frac{1}{\gamma}}\Big)}, \end{aligned} \tag{A.31}$$

where $x^E = \mathbf{x^E}(a, \tilde{P}, l, \mathcal{P}^l)$ is the root of (A.25) with the set of state variables $(a, \tilde{P}, l, \mathcal{P}^l)$. Here also, the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with $a + 1$ and $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^l}{a+1}$ as before.

Once the three cutoffs are determined, I refer to a firm value in each case – hiring, inaction, quitting, and layoffs – as $\mathbf{V}^{init,h}$, $\mathbf{V}^{init,i}$, $\mathbf{V}^{init,q}$, and $\mathbf{V}^{init,l}$, respectively. Using these terms, the value function (A.10) can be rewritten as follows:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \delta Ul + (1-\delta)\bigg(dUl + (1-d)\max\Big[\mathbf{V}^{init,h}, \mathbf{V}^{init,i}, \mathbf{V}^{init,q}, \mathbf{V}^{init,l}\Big]\bigg).$$

A.2.4.6 Exiting Firms

Lastly, firms' optimal exit decision is chosen by:

$$\mathbf{d}(a, \tilde{P}, l, P) = \begin{cases} 1 & \text{if} \quad \mathbf{U}l > \max\Big[\mathbf{V}^{init,h}, \mathbf{V}^{init,i}, \mathbf{V}^{init,q}, \mathbf{V}^{init,l}\Big] \\ 0 & \text{otherwise.} \end{cases}$$

Letting the productivity cutoff denoted by $\mathcal{P}^x(a, \tilde{P}, l)$, it is determined by the following equation:

$$\mathbf{U}l = \max\Big[\mathbf{V}^{init,h}(a, \tilde{P}, l, \mathcal{P}^x(a, \tilde{P}, l)), \mathbf{V}^{init,q}(a, \tilde{P}, l, \mathcal{P}^x(a, \tilde{P}, l)), \mathbf{V}^{init,l}(a, \tilde{P}, l, \mathcal{P}^x(a, \tilde{P}, l))\Big]. \tag{A.32}$$

A.3 Workers' Future Expected Value (Job Prospects)

Recall that the employed worker's value in (1.12). Incorporating the decision rules of firms obtained in the previous section, the worker's value function can be rephrased as the following:

$$\begin{aligned}\mathbf{W}(a, \tilde{P}, l, P) =& \mathbf{w} + \beta\mathbb{E}\Bigg[\bigg(\delta + \big(1-\delta\big)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\bigg)\mathbf{U} \\ &+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\bigg(\lambda f(\theta(\mathbf{x^{E}}'))\mathbf{x^{E}}' + \big(1-\lambda f(\theta(\mathbf{x^{E}}'))\big)\tilde{\mathbf{W}}'\bigg)\Bigg],\end{aligned} \tag{A.33}$$

where $\mathbf{w} = \mathbf{w}(a, \tilde{P}, l, P)$ is the equilibrium wage offered by the firm, $\mathbf{d}' = \mathbf{d}(a+1, \tilde{P}', l', P')$, $\mathbf{s}' = \mathbf{s}(a+1, \tilde{P}', l', P')$, $\mathbf{x}^{\mathbf{E}\prime} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$, and $\tilde{\mathbf{W}}' = \tilde{\mathbf{W}}(a+1, \tilde{P}', l', P')$ are the firm's exit, layoff, retention decision rules in the next period, contingent on the realization of P'. Note that $l' = \mathbf{h}(a, \tilde{P}, l, P) + (1 - \lambda f(\mathbf{x}^{\mathbf{E}}(a, \tilde{P}, l, P)))l$ is the next period initial employment size of the firm as a result of its hiring and retention activity in the current period. Hence, the worker value function ends up being a function of the employer's current state variable, $(a, \tilde{P}, l, P)$.

As seen in Lemma 1, the promise keeping constraints (1.17) and (1.18) hold with equality at equilibrium for new hires and incumbent workers, respectively. Thus, following Proposition 1, given workers' outside option, equilibrium wages depend on the workers' expected value based on their beliefs about firms.

Now, I would like to delve into the large bracket in (A.33), which is associated with workers' expected future value. Incorporating the firm's decision rules and the productivity cutoffs, these terms be rephrased as the following:

$$\begin{aligned}
&\delta \mathbf{U} + (1-\delta)\Bigg(\int_{\mathcal{P}^q}^{\infty} (\kappa - c)\, dF(P') + \int_{\mathcal{P}^l}^{\mathcal{P}^q} \Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}\prime}))\mathbf{x}^{\mathbf{E}\prime} + \big(1 - \lambda f(\theta(\mathbf{x}^{\mathbf{E}\prime}))\big)\tilde{\mathbf{W}}' \Big) dF(P') \\
&+ \int_{\mathcal{P}^x}^{\mathcal{P}^l} \Big(\mathbf{s}'\mathbf{U} + (1-\mathbf{s}')\big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}\prime}))\mathbf{x}^{\mathbf{E}\prime} + \big(1 - \lambda f(\theta(\mathbf{x}^{\mathbf{E}\prime}))\big)\tilde{\mathbf{W}}'\big) \Big) dF(P') + \int_{\infty}^{\mathcal{P}^x} \mathbf{U}\, dF(P') \Bigg),
\end{aligned} \tag{A.34}$$

where $F(\cdot)$ is the log-normal cumulative density function of productivity P', based on the worker's posterior about the firm with the corresponding mean $\bar{\nu} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + (a+1)\frac{\tilde{P}'}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + (a+1)\frac{1}{\sigma_\varepsilon^2}}$ and variance $\sigma^2 + \sigma_\varepsilon^2$ where $\sigma^2 = \frac{1}{\frac{1}{\sigma_0^2} + (a+1)\frac{1}{\sigma_\varepsilon^2}}$. Also, the productivity cutoffs $\mathcal{P}^q$, $\mathcal{P}^l$, $\mathcal{P}^x$ are from (A.30), (A.31), and (A.32), respectively, which are a function of the firm's state variables $(a+1, \tilde{P}', l')$ at the beginning of the next period.

The first term is the worker's value when the employer is hit by the exogenous death shock. And conditional on surviving from the shock, workers further consider the following cases expressed in the large bracket following the first term.

First, the first term in the bracket in (A.34) shows that workers will get $\kappa - c$ conditional on the case in which their employer hires or stay inactive without losing any workers in the next period, i.e. P' is drawn above $\mathcal{P}^q(a+1, \tilde{P}', l')$. As seen in the previous sections A.2.4.2 and A.2.4.3, this firm would not allow any quits by setting the promised utility to incumbent workers to the maximum value, i.e. $\kappa - c$. Thus, in either case, workers end up obtaining the value $\kappa - c$ and staying at the firm.

Next, the second term in the bracket presents the worker's expected value when the firm stays inactive but allows quits, i.e. P' is realized in between $\mathcal{P}^l(a+1, \tilde{P}', l')$ and $\mathcal{P}^q(a+1, \tilde{P}', l')$. In this case, with probability $\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$, the worker can make his on-the-job search successful and gain $\mathbf{x}^{\mathbf{E}'}$. Otherwise, the worker stays at the current employer and obtains $\tilde{\mathbf{W}}'$. Note that $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$ and $\tilde{\mathbf{W}} = \tilde{\mathbf{W}}(a+1, \tilde{P}', l', P')$ are the employer's equilibrium retention choice (taking into account the worker's choice for x^E) following (A.2), (A.21), and (A.22).

The third term in the bracket is the worker's expected value when the firm has a possibility to lay off workers in the next period, i.e. (with P' realized between $\mathcal{P}^x$ and $\mathcal{P}^l$). Then, in the case of firm layoffs, the worker goes to the unemployment pool and consumes the value $\mathbf{U}$, which is the first term of the integral in this bracket. Otherwise, the worker needs to consider the possibility of being poached (with the probability $\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$) or staying at the current firm (with the probability $1 - \lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$) as before, which is expressed by the remaining terms in the integral. Here, $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$ is the employer's layoff decision rule following (A.24), and $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$, $\tilde{\mathbf{W}} = \tilde{\mathbf{W}}(a+1, \tilde{P}', l', P')$ are the employer's retention decision rules as before following (A.2), (A.22), and (A.25).

Lastly, conditional on the firm observing P' below the exit cutoff $\mathcal{P}^x(a+1, \tilde{P}', l')$, the firm will endogenously stop operating and exit, and the worker becomes unemployed. This is reflected on the last term.

A.3.1 The Ranking of Workers' Value (Proof of Proposition 2)

Now, comparing the worker's value after observing P, the followings can be established:

i) Workers at hiring or inactive employers (with $P \geqslant \mathcal{P}^q$) obtain the highest value, $(\kappa - c)$;

ii) Workers at quitting employers (with $P \in [\mathcal{P}^l, \mathcal{P}^q]$) have a value lower than those at hiring or inactive firms (without quits) and higher than those at firms laying off workers, $\Big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\Big)$;

iii) Workers at employers that lay off workers (with $P \leqslant \mathcal{P}^l$) have a value lower than those at quitting or inactive or expanding firms but higher than unemployed workers, $\Big(\mathbf{sU} + (1-\mathbf{s})\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big)\Big)$;

iv) Unemployed workers have the lowest value, U.

The proof is as follows. First, it is already known that any inactive markets x^E need to be ranged below $\kappa - c$. And following (A.22), $\tilde{\mathbf{W}}$ has to be bound by $\kappa - c$. In other words,

$$x^E \leqslant \kappa - c \quad \text{and} \quad \tilde{\mathbf{W}} \leqslant \kappa - c \quad \text{for any active } x^E,$$

which confirms that

$$\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big) \leqslant \kappa - c, \quad \forall \mathbf{x^E}, \tilde{\mathbf{W}}. \tag{A.35}$$

Next, consider a firm in the inaction region, $P \in [\mathcal{P}^l, \mathcal{P}^q]$, but allowing quits. Using (A.22), the worker's value at this firm can be rephrased as follows:

$$\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big) = \mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^\gamma + c\theta(\mathbf{x^E})^{1+\gamma}, \tag{A.36}$$

which is the weighted average of the promised utility in the current firm and the target utility in the worker's on-the-job search. Here, $\mathbf{x^E}$ is the solution of the equation (A.21). Furthermore, this firm finds $s = 0$ to be optimal and stays inactive with quits allowed. Therefore, the marginal value of s, the left-hand side of (A.13), has to be strictly negative with any $s > 0$ and equals to zero with $s = 0$.

Combining this with (A.21), it can be proved that

$$\mathbf{U} \leqslant \left(\mathbf{x^E} + \frac{(1-\lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f(\theta)\theta(\mathbf{x^E})}\right),$$

which can further be rewritten with (1.26) and (1.28) as follows:

$$\mathbf{U} \leqslant \mathbf{x^E} - \theta(\mathbf{x^E})^{\gamma}(\kappa - \mathbf{x^E}). \tag{A.37}$$

With (A.36) and (A.37), it is proved that

$$\mathbf{U} \leqslant \left(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \left(1 - \lambda f(\theta(\mathbf{x^E}))\right)\tilde{\mathbf{W}}\right), \tag{A.38}$$

for any firms staying inactive with quits and choosing $\mathbf{x^E}$ following (A.21).

Similarly, let's consider a firm laying off workers after observing $P \in [\mathcal{P}^x, \mathcal{P}^l]$ in a given period. Based on (A.36), the worker's value at this firm is

$$\mathbf{sU} + (1-\mathbf{s})\left(\mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + c\theta(\mathbf{x^E})^{1+\gamma}\right), \tag{A.39}$$

where $\mathbf{x^E}$ is the solution of the equation (A.25). Furthermore, (A.25) implies that

$$\mathbf{U} = \mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + \lambda c\theta(\mathbf{x^E})^{1+\gamma}. \tag{A.40}$$

Hence, (A.39) and (A.40) confirm that

$$\mathbf{U} \leqslant \mathbf{sU} + (1-\mathbf{s})\left(\mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + c\theta(\mathbf{x^E})^{1+\gamma}\right). \tag{A.41}$$

for any firms laying off workers with $\mathbf{s}$ and $\mathbf{x^E}$ following (A.13) and (A.21).

Combining (A.35), (A.38), and (A.41) proves i) and iv), meaning that workers obtain the highest value at a hiring or inactive firm and get the lowest value in the unemployment pool.

Lastly, the rank order of workers' value between quitting firms and those laying off workers needs to be confirmed to verify ii) and iii). This can be established with the following two proofs. First, it can be proved that (A.36) is weakly increasing in $\mathbf{x^E}$, implying that workers get weakly higher values at a firm with higher $\mathbf{x^E}$. Second, the other proof to be confirmed is the equilibrium $\mathbf{x^E}$ is higher for quitting firms than contracting firms with layoffs. In other words, $\mathbf{x^E}$ satisfying (A.21) is higher than $\mathbf{x^E}$ satisfying (A.25). Then, the two proofs along with (A.38) can confirm that workers obtain higher values at quitting firms than those laying off workers.

Let's start with the first one by getting the derivative of (A.36) with respect to x^E.

$$\begin{aligned}\frac{\partial\big(\lambda f(\theta(x^E))x^E + \big(1-\lambda f(\theta(x^E))\big)\tilde{\mathbf{W}}\big)}{\partial x^E} &= \frac{\partial\Big(x^E - (\kappa - x^E)\theta(x^E)^\gamma + c\theta(x^E)^{1+\gamma}\Big)}{\partial x^E} \\ &= 1 - \frac{\theta^\gamma(\frac{\kappa - x^E}{c})^{-\gamma}}{1-(\frac{\kappa-x^E}{c})^{-\gamma}} = 0. \end{aligned} \tag{A.42}$$

This implies that for any non-binding optimal solutions for x^E in (A.21), worker values conditional on not being separated are the same.

Second, it is already seen in the previous discussion from the equations (A.21) and (A.21) that the optimal choice x^E of quitting firms follows:

$$\mathbf{U} \leqslant x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)},$$

while the choice of firms laying off workers is pinned down by the following:

$$\mathbf{U} = x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}.$$

Thus, in order to confirm the former is higher than the latter, it is sufficient to prove the following terms are increasing in x^E:

$$x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}.$$

Using (1.26), the above terms can be rephrased by

$$x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)} = x^E - \theta^\gamma(\kappa - x^E) + \lambda c\theta^{\gamma+1}.$$

These terms satisfy the following property:

$$\frac{\partial\Big(x^E - \theta^\gamma(\kappa - x^E) + \lambda c\theta^{\gamma+1}\Big)}{\partial x^E} = \frac{\partial\Big(x^E - \theta^\gamma(\kappa - x^E) + c\theta^{\gamma+1}\Big)}{\partial x^E} - (1-\lambda)c(\gamma+1)\theta^\gamma\frac{\partial\theta(x^E)}{\partial x^E} > 0,$$

given that (A.42) makes the first term on the right-hand side being zero and $\frac{\partial\theta(x^E)}{\partial x^E} < 0$. Thus, the following is proved:

$$\frac{\partial\Big(x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}\Big)}{\partial x^E} > 0,$$

implying that the optimal x^E is higher for quitting firms than those laying off workers. Lastly, this fact along with (A.38) and (A.42) finalizes the proof for ii) and iii).

Linking the findings i)-iv) to the equation (A.34), it can be shown that workers would expect higher future values at a hiring or inactive firm than a contracting firms with poaching or layoffs.

A.4 Implications of Workers' Job Prospects

A.4.1 The Ranking of $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$

In this section, I analyze how workers' job prospects matter for firms' decision making at the hiring or retention margin. Recalling (A.9), it can be rephrased as follows:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = [*]l + (1-\delta)(1-d)\Bigg(J(a,\tilde{P},l',P,x,\tilde{\mathbf{W}}) - \frac{c}{q(\theta(x))}h\Bigg), \tag{A.43}$$

where

$$[*] \equiv \delta\mathbf{U} + (1-\delta)(d+(1-d)s)\mathbf{U} + (1-\delta)(1-d)(1-s)(\lambda f(\theta(x^E))x^E + (1-\lambda f(\theta(x^E)))\tilde{\mathbf{W}}).$$

Then, iterating it one period forward and taking expectation, the following holds:

$$\frac{\partial \mathbb{E}\mathbf{V}^{init'}}{\partial l'} = \mathbb{E}[*'] + \frac{\partial \mathbb{E}[*']}{\partial l'}l' + (1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big], \tag{A.44}$$

which shows the expected future marginal value of a labor input. Note that this is the sum of the following three components associated with workers' job prospects and firms' own prospects about their type: i) the first term on the right-hand side is workers' future expected value, ii) the second term is the indirect effect of firm size on workers' future expected value, and iii) the last term is the firms' expected future value. These terms play a key role in firms' decision making.

Now, I prove that $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$ varies across firms depending on their employment status, and the ranking holds the same as the workers' future expected value as seen in the previous section.

A.4.1.1 Hiring Firms: $s = 0$ and $h > 0$

For hiring firms, their value function becomes:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \delta \mathbf{U} l + (1-\delta)\Big[P l'^{\alpha} - c^f - \kappa \mathbf{h} + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')\Big],$$

where $\mathbf{h} \equiv \mathbf{h}(a, \tilde{P}, l, P)$ is the firm's hiring decision rule and $l' \equiv \mathbf{h}(a, \tilde{P}, l, P) + l$. Then, we have the following derivative with respect to l:

$$\begin{aligned}\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} =& \delta \mathbf{U} + (1-\delta)\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'}\Big] \\ &+ (1-\delta)\frac{\partial \mathbf{h}}{\partial l}\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'} - \kappa\Big],\end{aligned}$$

where the first line is a direct effect of l, and the second line is an indirect effect of l through its optimal hiring on the value function. With (A.12), the indirect effect becomes zero, which is consistent with the Envelope theorem. Therefore, it gets simplified as follows:

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \delta \mathbf{U} + (1-\delta)\kappa. \tag{A.45}$$

A.4.1.2 Inactive Firms: $s = 0$ and $h = 0$

Next, consider inactive firms who do not allow quits. Their hiring, layoff, and retention decisions are $\mathbf{h} = 0$, $\mathbf{s} = 0$, and $\mathbf{x}^{\mathbf{E}} = 0$, all of which are the function of $(a, \tilde{P}, l, P)$, and this makes $l' = l$. Thus, their value function is

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \delta \mathbf{U} l + (1-\delta)\Big[P l^{\alpha} - c^f + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l, P')\Big],$$

and the first derivative of it with respect to l is

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \delta \mathbf{U} + (1-\delta)\Big[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l, P')}{\partial l}\Big].$$

Note that this case can only happen with the range (A.29), and thus this term should be in between $[\kappa - c, \kappa]$. In other words, the following holds for this type of firms:

$$\delta\mathbf{U} + (1-\delta)(\kappa - c) \leqslant \frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} \leqslant \delta\mathbf{U} + (1-\delta)\kappa. \tag{A.46}$$

Now, consider the other case of inactive firms who allow quits. Their value function is as follows:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \delta\mathbf{U}l + (1-\delta)\Big[Pl'^{\alpha} - c^f + \lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l + \beta\mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')\Big],$$

where $\mathbf{x^E} \equiv \mathbf{x^E}(a, \tilde{P}, l, P)$ is their optimal retention choice, which is a root of (A.21), and $l' \equiv (1 - \lambda f(\theta(\mathbf{x^E}(a, \tilde{P}, l, P))))l$.

Getting the derivative as before, the following can be obtained:

$$\begin{aligned}\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} &= \delta\mathbf{U} + (1-\delta)\Big[(1-\lambda f(\theta(\mathbf{x^E})))\Big(\alpha P l'^{\alpha-1} + \beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'}\Big) + \lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}\Big] \\ &+ (1-\delta)\frac{\partial \mathbf{x^E}}{\partial l}\Big[-\lambda f'(\theta)\theta'(\mathbf{x^E})l\Big(\alpha P l'^{\alpha-1} + \beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'}\Big) + \lambda f(\theta(\mathbf{x^E}))l + \lambda f'(\theta)\theta'(\mathbf{x^E})\mathbf{x^E}l\Big],\end{aligned}$$

where the first line is a direct effect of l, and the second line is an indirect effect of l through its optimal retention on the value function. As before, using (A.21), the indirect effect becomes zero. Thus, the terms can be rephrased as follows:

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \delta\mathbf{U} + (1-\delta)\Big[\mathbf{x^E} + \frac{(1-\lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f'(\theta)\theta'(\mathbf{x^E})}\Big].$$

Note that this term has to be in the following range:

$$\mathbf{U} \leqslant \frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} \leqslant \delta\mathbf{U} + (1-\delta)(\kappa - c). \tag{A.47}$$

The upper bound comes from $f'(\theta)\theta'(\mathbf{x^E}) < 0$ and $\mathbf{x^E} \leqslant \kappa - c$. The lower bound is from the fact that this firm never finds $s > 0$ to be optimal, which is consistent to say the left-hand side of (A.13) is strictly negative with any $s > 0$ or zero with $s = 0$. Combining this with (A.21), it can be proved that

$$\mathbf{U} \leqslant \Big[\mathbf{x^E} + \frac{(1-\lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f'(\theta)\theta'(\mathbf{x^E})}\Big]$$

which gives the lower bound of (A.47).

A.4.1.3 Separating Firms with Layoffs: $s > 0$ and $h = 0$

For firms separating workers with explicit layoffs, their value function is:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = \delta\mathbf{U}l + (1-\delta)\Big[\mathbf{sU}l + Pl'^{\alpha} - c^f + (1-\mathbf{s})\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l + \beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Big],$$

where $\mathbf{s} \equiv \mathbf{s}(a,\tilde{P},l,P)$ is their layoff decision, $\mathbf{x^E} \equiv \mathbf{x^E}(a,\tilde{P},l,P)$ is their retention decision, and $l' \equiv (1-\mathbf{s}(a,\tilde{P},l,P))(1-\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P))))l$.

Making the first derivative of it with respect to l, it can be obtained that

$$\begin{aligned}
&\frac{\partial\mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}\\
&= \delta\mathbf{U} + (1-\delta)\Big[\mathbf{sU} + (1-\mathbf{s})(1-\lambda f(\theta(\mathbf{x^E})))\Big(\alpha Pl'^{\alpha-1} + \beta\frac{\partial\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big)(1-\mathbf{s})\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}\Big]\\
&+ (1-\delta)\frac{\partial\mathbf{s}}{\partial l}\Big[\mathbf{U}l - (1-\lambda f(\theta(\mathbf{x^E})))l\Big(\alpha Pl'^{\alpha-1} + \beta\frac{\partial\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big) - \lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l\Big]\\
&+ (1-\delta)(1-\mathbf{s})\frac{\partial\mathbf{x^E}}{\partial l}\Big[-\lambda f'(\theta)\theta'(\mathbf{x^E})l\Big(\alpha Pl'^{\alpha-1} + \beta\frac{\partial\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big) + \lambda f(\theta(\mathbf{x^E}))l\\
&+ \lambda f'(\theta)\theta'(\mathbf{x^E})\mathbf{x^E}l\Big],
\end{aligned}$$

where the first line is a direct effect of l, the second line is an indirect effect of l through its optimal layoffs, the last two lines are an indirect effect of l through its optimal retention on the value function. Note that, consistent with the Envelope theorem again, (A.24) and (A.14) make the indirect effects zero. Also, using (A.24), the first line gets even more simplified. Ultimately, the derivative becomes:

$$\frac{\partial\mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l} = \mathbf{U}. \tag{A.48}$$

A.4.1.4 Exiting firms: $d = 1$

Lastly, for exiting firms, their value function is:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \mathbf{U}l,$$

and the derivative with respect to l is

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \mathbf{U}. \tag{A.49}$$

Combining (A.45), (A.46), (A.47), (A.48), and (A.49), it can be proved that for $\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}$, hiring firms have the highest value, inactive firms without quits have the second highest value, quitting firms have the third highest value, and firms laying off workers or exiting have the lowest value. Therefore, this implies that firms that are more expected to draw higher P' and expand in the next period will obtain a higher expected future marginal value of a labor input, $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$.

This indicates that the expected future marginal value of a labor input goes in the same direction as the workers' expected value in the previous section. In other words, even after considering the indirect effect of firm size on the workers' expected value as well as the firms' own prospects, the direct effect through the workers' job prospects remains dominant to the expected future marginal value of a labor input. In the following sections, I discuss how workers' job prospects can matter for firms' choice for hiring and retention by showing that the expected future marginal value of a labor input directly affects their decision.

A.4.2 Implications on Productivity Cutoffs

Note from the previous section A.2.4 that the term $\frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}$ matters to determine variations of the productivity cutoffs across firms with different job prospects. Recall that there are four endogenous productivity cutoffs among operating firms, $\mathcal{P}^h$, $\mathcal{P}^q$, $\mathcal{P}^l$, and $\mathcal{P}^x$, which are determined by (A.26), (A.30), (A.31), and (A.32), respectively.

In order to see how the productivity cutoffs vary across firms with different posteriors, let's consider the following case. Suppose a firm with $(a, \tilde{P}, l)$ has the equilibrium productivity cutoffs denoted by $\mathcal{P}^h(a, \tilde{P}, l)$, $\mathcal{P}^q(a, \tilde{P}, l)$, and $\mathcal{P}^l(a, \tilde{P}, l)$, following the equations (A.26), (A.30), and (A.31), respectively. Let's consider another firm having the same age a and size l, but higher average productivity $\tilde{P}$ than the focal firm. Thus, this firm has a better posterior mean, with the same posterior variance.

Now suppose that the three productivity cutoffs remain the same for this firm at the equilibrium. Since this firm has a better posterior, it is more likely to expand and less likely to contract in the next period, and thus has a higher level of $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'}$ following the previous discussion. However, this contradicts to the equilibrium conditions for the productivity cutoffs, as the left-hand sides of (A.26), (A.30), and (A.31) become greater than the right-hand sides of the equations that remain constant. This confirms that firms having different posteriors cannot have the same productivity cutoffs.

Note that the exact level of the productivity cutoffs can only be solved numerically. However, it can still be inferred that the productivity cutoffs would be lower for the firm having a better posterior from the following. The firm with a better posterior is expected to draw higher productivity in the next period, and this increases the expected future marginal value of a labor input following the discussion in the previous section. Thus, from the equations (A.26), (A.30), and (A.31), the expected marginal future values on the left-hand side are higher for this firm, and this will require the productivity cutoffs to go down to equate with the right-hand side by lowering the spontaneous marginal product of the firm.

A.4.3 Implications on Hiring and Retention Margins (Result 3)

In this section, I show how workers' job prospects affect firms' decision to hire or retain workers.

A.4.3.1 Hiring Margin

The equation (A.12) associated with firms' optimal hiring choice can be rephrased as follows by using (A.43) and the firms' indifference curve (1.25):

$$\alpha P l'^{\alpha-1} + \beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big] = x - \beta\mathbb{E}[*'] + \frac{c}{q(\theta(x))} - \beta\frac{\partial\mathbb{E}[*']}{\partial l'}l'. \tag{A.50}$$

The left-hand side of the equation shows the marginal benefit of hiring and the right-hand side of the equation is the marginal cost of hiring.

Here, workers' job prospects are associated with two terms as follows: i) workers' expected future value ($\mathbb{E}[*']$) and ii) the derivative of workers' expected value with respect to firm size ($\frac{\partial\mathbb{E}[*']}{\partial l'}l'$). First, workers' job prospects affect firms' hiring costs. Different job prospects from the perspective of workers create wage differentials conditional on the promised utility x, which are reflected in the first two terms in the second bracket. Second, workers' expected future value is further indirectly affected by new hires, which also affect hiring costs and the net marginal value of hiring. This is the last term in the second bracket.

Furthermore, firms' own uncertain prospects about themselves affect their expected future value and hiring decision under the symmetric information structure. This is reflected in the second term in the first bracket: iii) $\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big]$.

As seen in the previous section, workers' expected future value is higher (lower) for high (low) performing young firms with better (worse) posterior beliefs. Thus, high performing young firms end up facing higher wage differentials and hiring costs compared to otherwise similar mature firms, while low performing young firms can pay a discount. This causes high performing young firms to have a lower net marginal value of hiring, while low performing young firms have a higher marginal value of hiring, compared to otherwise similar mature firms. This effect is through the term i) due to workers' uncertain job prospects at young firms.

Furthermore, as seen in the previous section A.4.1, the sum of the above three terms i)-iii) is higher (lower) for firms having better (worse) posterior beliefs and thus increases (decreases) their marginal value of hiring.[1] In other words, even after considering the indirect effect of new hires in ii) and the firms' own uncertain prospects in iii), the direct effect through workers' uncertain job prospects and the consequent wage differentials remains influential. Thus, workers' uncertain job prospects can create a hiring friction for young firms with high potential, leading to employment gaps between young and mature firms.

A.4.3.2 Retention Margin

The retention margin is another channel through which young firms' uncertain job prospects affect firm size and growth. It is characterized by (A.14). Using the relationship (A.43) again and putting the equation differently, the following can be obtained:

$$\begin{aligned}&\alpha P l'^{\alpha-1} + \beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big] \\ &\quad = \tilde{\mathbf{W}} - \mathbb{E}[*'] - \beta\frac{\partial \mathbb{E}[*']}{\partial l'}l' + \frac{f(\theta(x^E)) + f'(\theta)\theta'(x^E)(x^E - \tilde{\mathbf{W}})}{f'(\theta)\theta'(x^E)}. \end{aligned} \tag{A.51}$$

As before, the left-hand side of the equation indicates the marginal benefit and the right-hand side shows the marginal cost of retaining workers (by increasing a unit of promised utility to incumbent workers). Note that unlike (A.50), the marginal cost here does not include vacancy cost, but does include the marginal effect of adjusting contract offers on the previously employed workers by affecting their wages in the previous period (the last term on the right-hand side). The marginal benefit and cost are equalized at the equilibrium.

Here also, the aforementioned three terms are related to beliefs about firm type: i) workers' expected future value ($\mathbb{E}[*']$), ii) the derivative of workers' expected value with respect to firm size ($\frac{\partial \mathbb{E}[*']}{\partial l'}$), and iii)

[1] The separate signs of ii) and iii) cannot be analytically derived, but only the total sign can be shown.

the firm's expected future marginal value $\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J'}-\frac{c}{q(\theta(x'))}h')\Big]$, where the first two terms are related to workers' uncertain job prospects, and the last term is associated with firms own prospects. As discussed before, given the utility level $\tilde{\mathbf{W}}$ promised to incumbent workers, the wage differentials through the the term i) reduce (increase) the marginal value of high (low) performing young firms relative to heir mature counterparts. And this effect is dominant to the other effects through the terms ii) and iii) as shown in Appendix A.4.1. This indicates that firms' retention decision is also importantly affected by workers' uncertain job prospects and the consequent wage differentials.

Related to the retention margin, the workers' job prospects also affect the layoff decision of firms. Firms' layoff decision rule is determined by (A.13), which can be rephrased as follows:

$$\begin{aligned}&-(1-\lambda f(\theta(x^E)))l\Bigg(\alpha P l'^{\alpha-1}+\beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J'}-\frac{c}{q(\theta(x'))}h')\Big]\Bigg)\\&=-(1-\lambda f(\theta(x^E)))l\Big(\tilde{\mathbf{W}}-\mathbb{E}[*']-\beta\frac{\partial\mathbb{E}[*']}{\partial l'}l'\Big)-\mathbf{U}l+\lambda f(\theta(x^E))lx^E+(1-\lambda f(\theta(x^E)))l\tilde{\mathbf{W}},\end{aligned}\tag{A.52}$$

where the left-hand side is the marginal benefit and the right-hand side is the marginal cost of laying off a worker. To be specific, the first term on the left-hand side shows the marginal decrease of firm size with respect to an increase in separation probability, i.e. $-(1-\lambda f(\theta(x^E)))<0$. Thus, the left-hand side characterizes a marginal effect of laying off a worker on the firm's marginal product and expected future value through employment size change.

Similarly, the first three terms on the right-hand side present the reduction of wages for dismissed workers. The remaining terms on the right-hand side show the marginal effect of higher layoff possibility on the wages paid to workers in the previous period. This is another source of the marginal cost associated with firm layoff.

As before, the aforementioned three terms are engaged in the same direction, and the firms' layoff decision is mainly affected by workers' job prospects. More importantly, firms with better (worse) workers' job prospects now have a higher (lower) marginal cost of laying off workers and are less (more) likely lay off workers due to the negative sign of $-(1-\lambda f(\theta(x^E)))$.

A.4.4 Uncertainty and Job Prospects

A.4.4.1 Proof of Proposition 3

Proof.

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \sigma_\varepsilon^2} = \Big(\frac{a_{jt}}{\sigma_\varepsilon^2\sigma_0^2}\Big)\frac{(\bar{\nu}_0 - \tilde{P}_{jt-1})}{\Big(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\Big)^2}\begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \end{cases}$$

$$\frac{\partial \sigma_{jt-1}}{\partial \sigma_\varepsilon^2} = \Big(\frac{a_{jt}}{\sigma_\varepsilon^2}\Big)\frac{1}{\Big(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\Big)^2} > 0$$

■

A.4.4.2 Proof of Proposition 4

Proof.

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\Big(\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}}\Big) = -\Big(\frac{a_{jt}}{\sigma_\varepsilon^4\sigma_0^2}\Big)\frac{1}{\Big(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\Big)^2} < 0$$

■

A.4.4.3 Proof of Proposition 5

Proof. With $\frac{\sigma_\varepsilon}{\sigma_0} < 1, \forall\, a_{jt} \geqslant 1$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\Big(\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}}\Big) = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_\varepsilon^4\sigma_0^2\Big(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\Big)^3}\Big(\frac{a_{jt}}{\sigma_\varepsilon^2} - \frac{1}{\sigma_0^2}\Big)\begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}$$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}}\right) = -\frac{\left(\frac{a_{jt}}{\sigma_\varepsilon^2} - \frac{1}{\sigma_0^2}\right)}{\sigma_\varepsilon^4\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^3} < 0.$$

■

A.4.4.4 Proof of Corollary 2

Proof. Suppose there are two firms, firm 1 and firm 2, having the same average productivity $\tilde{P}$. Let a_1 and a_2 be the ages of firms 1 and 2, respectively, where $a_1 > a_2 \geqslant 1$. Also, let $\bar{\nu}_1$ and $\bar{\nu}_2$ be the posterior means for firms 1 and 2, respectively. From previous results, we have

$$\bar{\nu}_1 > \bar{\nu}_2 \quad \text{if} \quad \tilde{P} > \bar{\nu}_0$$

$$\bar{\nu}_1 < \bar{\nu}_2 \quad \text{if} \quad \tilde{P} < \bar{\nu}_0.$$

Then the following relationship holds:

$$\frac{\partial(\bar{\nu}_1 - \bar{\nu}_2)}{\partial \sigma_\varepsilon^2} = \frac{\frac{(a_1-a_2)(\tilde{P}-\bar{\nu}_0)}{\sigma_0^2\sigma_\varepsilon^4}\left(\frac{a_1 a_2}{\sigma_\varepsilon^4} - \frac{1}{\sigma_0^4}\right)}{\left(\frac{1}{\sigma_0^2} + \frac{a_1}{\sigma_\varepsilon^2}\right)^2\left(\frac{1}{\sigma_0^2} + \frac{a_2}{\sigma_\varepsilon^2}\right)^2} \begin{cases} > 0 & \text{if} \quad \tilde{P} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P} < \bar{\nu}_0, \end{cases}$$

so that the gap between $\bar{\nu}_1$ and $\bar{\nu}_2$ increases in σ_ε^2. ■

A.5 Welfare Implications

In this subsection, I derive welfare implications of the model as follows.

Proposition A.5.1. *Given the level of uncertainty about firms' productivity type (given σ_ε and σ_0), the model's block-recursive equilibrium can be replicated by a constrained social planner's problem and thus is efficient.*

Proof. Suppose that a social planner is constrained by both of the search and information frictions as in the market economy. The social planner aims to maximize the following welfare function:

$$
\begin{aligned}
\max_{\substack{u_t, v_t, M_t^e, \mathbf{G}(a_{t+1}, \tilde{P}_t, l_t),\\ \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{s}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{h}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta^{\mathbf{E}}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{l}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta_t^U, d_t^e(P_t), l_t^e(P_t)}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ u_t b - c v_t \\
& + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), a_t \geqslant 1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \\
& \qquad * (1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)) \left(P_t l_t^{\alpha} - c_f\right) \\
& + M_t^e \left(\sum_{P_t} f^e(P_t)(1 - d_t^e(P_t)) \left(P_t (l_t^e(P_t))^{\alpha} - c_f\right) - c_e \right) \Big\}, \qquad \text{(A.53)}
\end{aligned}
$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E)) l_{t-1} + h_t \qquad \text{(A.54)}$$

$$v_t = \theta_t^U u_t + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} \lambda \theta_t^E(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \qquad \text{(A.55)}$$

$$
\begin{aligned}
u_t = & (1 - f(\theta_t^U)) u_{t-1} \\
& + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} (d_t + (1 - d_t) s_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \qquad \text{(A.56)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{G}(a_{t+1}, \tilde{P}_t, l_t) = & \sum_{\tilde{P}_{t-1}, l_{t-1}} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_{t+1}, \tilde{P}_t)} \Big((a_t + 1)\tilde{P}_t - a_t \tilde{P}_{t-1} \Big) \\
& * (1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)) \mathbb{I}_{l(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = l_t} \text{ for } a_t \geqslant 1 \qquad \text{(A.57)}
\end{aligned}
$$

$$
\mathbf{G}(1, \tilde{P}_{t-1}, l_{t-1}) = \begin{cases} M_t^e f^e(\tilde{P}_{t-1})(1 - d^e(\tilde{P}_{t-1})), \text{ if } l_{t-1} = l_t^e(\tilde{P}) \\ \\ 0, \text{ otherwise} \end{cases}
$$

$$h_t(1-d_t) = f(\theta_t^U)u_t \text{ for firms searching in market } \theta^U \text{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^U) \tag{A.58}$$

$$h_t(1-d_t) = \lambda f(\theta_t^E)(1-s_t)l_{t-1} \text{ for firms poaching from market } \theta^E \text{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^E) \tag{A.59}$$

The first line in the objective function shows the utility for unemployed workers and search cost that the social planer takes into account. The second line presents the value of operating incumbent firms, and the last line indicates the value of successful entrant firms.

Equation (A.60) can be rephrased as the following problem with an identifier j for each firm j and their birth year t_0^j:

$$\begin{aligned}\max_{\substack{u_t, v_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}, l_t^j\right\}_{j, a_t^j \geqslant 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ \int_j \bigg(\Big(\prod_{\tau=t_0^j}^{t} (1-d_\tau^j) \left(P_t^j (l_t^j)^\alpha - c_f \right) \Big) \mathbb{I}_{t_0^j < t} \\ & + \left((1-d_t^j) \left(P_t^j (l_t^j)^\alpha - c_f \right) M_t^e - M_t^e c_e \right) \mathbb{I}_{t_0^j = t} \bigg) dj \\ & + u_t b - c v_t \Big\},\end{aligned} \tag{A.60}$$

subject to

$$l_t^j = (1-s_t^j)(1-\lambda f(\theta_t^{Ej}))l_{t-1}^j + h_t^j \tag{A.61}$$

$$v_t = \theta_t^U u_t + \int_j \left(\prod_{\tau=t_0^j}^{t} (1-d_\tau^j)(1-s_t^j)\lambda \theta_t^{Ej} l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{A.62}$$

$$u_t = (1-f(\theta_t^U))u_{t-1} + \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1-d_\tau^j)(d_t^j + (1-d_t^j)s_t^j)l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{A.63}$$

$$h_t^j(1-d_t^j) = f(\theta_t^U)u_t \text{ for firm } j \text{ searching in market } \theta^U \tag{A.64}$$

$$h_t^j(1-d_t^j) = \lambda f(\theta_t^{Ek})(1-s_t^k)l_{t-1}^k \text{ for firm } j \text{ poaching workers in market } \theta^{Ek} \tag{A.65}$$

$$M_t^e \int_j (1-d_t^j)\mathbb{I}_{t_0^j=t} dj = \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1-d_\tau^j) d_t^j \right) \mathbb{I}_{t_0^j<t} dj \tag{A.66}$$

Combining (A.62), (A.64), and (A.65), along with the relationship $\theta_t = \frac{f(\theta_t)}{q(\theta_t)}$ gives the following equation:

$$v_t = \int_j \left(\prod_{\tau=t_0^j}^{t} (1-d_\tau^j) \frac{h_t^j}{q(\theta_t^j)} \right) dj, \tag{A.67}$$

where θ_t^j is the market that firm j search in, i.e. $\theta_t^j \in \{\theta_t^U, \{\theta_t^{Ek}\}_k\}$.

Then, rephrasing (A.60) by replacing l_t^j with (A.61), v_t with (A.67), and using Langrangian multipliers μ_t for (A.63) and $\eta(\theta_t^j)$ for (A.64) and (A.65), the following is obtained:

$$\begin{aligned}\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geqslant 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau=t_0^j}^{t-1} (1-d_\tau^j)(1-d_t^j) \Bigg(P_t^j \left((1-s_t^j)(1-\lambda f(\theta_t^{Ej})) l_{t-1}^j + h_t^j \right)^\alpha \\ & - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j) h_t^j + \eta(\theta_t^{Ej}) \lambda f(\theta_t^{Ej})(1-s_t^j) l_{t-1}^j \\ & + \mu_t (d_t^j + (1-d_t^j) s_t^j) l_{t-1}^j \Bigg) \mathbb{I}_{t_0^j<t} \\ & + (1-d_t^j) \left(P_t^j h_t^j - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j) h_t^j - c_e \right) M_t^e \mathbb{I}_{t_0^j=t} \Bigg) dj \\ & + u_t b - \mu_t (u_t - u_{t-1}(1-f(\theta_t^U))) + \eta(\theta_t^U) u_{t-1} f(\theta_t^U) \Bigg\},\end{aligned} \tag{A.68}$$

Here, pick a competitive equilibrium U_t and $x(\theta_t^j)$ and replace $\mu_t = U_t$, $\eta_t(\theta_t^j) = x_{jt}$ s.t. $\theta_t^j = \theta(x_{jt})$, $\eta_t(\theta_t^{Ej}) = x_{jt}^E$ s.t. $\theta_t^{Ej} = \theta(x_{jt}^E)$, and $\eta_t(\theta_t^U) = x_t^U$ s.t. $\theta_t^U = \theta(x_t^U)$.

Rewriting (A.68), I have:

$$\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geqslant 1}}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau=t_0^j}^{t-1} (1-d_\tau^j)(1-d_t^j) \Bigg(P_t^j \left((1-s_t^j)(1-\lambda f(\theta(x_{jt}^E)) l_{t-1}^j + h_t^j \right)^\alpha$$

$$
\begin{aligned}
&- c_f - (\frac{c}{q(\theta(x_{jt}))} + x_{jt})h_t^j + x_{jt}^E(\lambda f(\theta(x_{jt}^E))(1-s_t^j)l_{t-1}^j \\
&+ U_t(d_t^j + (1-d_t^j)s_t^j)l_{t-1}^j\Big)\mathbb{I}_{t_0^j<t} \\
&+ \left((1-d_t^j)\left(P_t^j(h_t^j)^\alpha - c_f - (\frac{c}{q\theta(x_{jt})} + x_t^j)h_t^j - c_e\right)M_t^e\right)\mathbb{I}_{t_0^j=t}\Big)dj \\
&+ u_t b - U_t(u_t - u_{t-1}(1-f(\theta_t^U))) + \eta(\theta_t^U)u_{t-1}f(\theta_t^U)\Big\}. \qquad \text{(A.69)}
\end{aligned}
$$

Note that the first three lines are equivalent to the incumbent firms' and entrants' problems in the market equilibrium. Solving the last line with respect to u_t and θ_t^U gives the following two first-order conditions:

$$b - U_t + \beta\left(U_t(1-f(\theta_{t+1}^U)) + f(\theta_{t+1}^U)x_{t+1}(\theta_{t+1}^U)\right) = 0 \qquad \text{(A.70)}$$

$$-f'(\theta_t^U)U_t + f'(\theta_t^U)x_t(\theta_t^U) + x_t'(\theta_t^U)f(\theta_t^U) = 0, \qquad \text{(A.71)}$$

where (A.70) is equivalent to the unemployed workers' value function, and (A.71) is identical to their optimal choice in the competitive equilibrium. ■

Therefore, this shows that we can find a solution for the constrained social planner's problem to be competitive equilibrium. In other words, under both search and information frictions, the competitive equilibrium is the first best allocation. This is consistent with standard directed search literature.

The following corollary holds under no uncertainty.

Corollary A.5.1. *If there is no uncertainty about the firm's productivity type ($\sigma_\varepsilon = 0$ and given σ_0), the model's decentralized block-recursive equilibrium can be replicated by a social planner's problem with a search friction only, and thus is efficient.*

Proof. Now we assume that the social planner can see exact firm type. Thus, the information friction is no longer existent. In that case, the social planner's problem can be written as:

$$\max_{\substack{u_t, v_t, M_t^e, g(l_t), \\ \mathbf{d}(\nu, l_{t-1}), \\ \mathbf{s}(\nu, l_{t-1}), \\ \mathbf{h}(\nu, l_{t-1}), \\ \theta(\nu, l_{t-1}), \\ \theta^{\mathbf{E}}(\nu, l_{t-1}), \\ \mathbf{l}(\nu, l_{t-1}), \\ \theta_t^U, d_t^e(\nu), l_t^e(\nu)}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ u_t b - c v_t + \sum_{(\nu, l_{t-1})} g(l_{t-1}) f(\nu)(1 - \mathbf{d}(\nu, l_{t-1})) \left(e^{\nu} l_t^{\alpha} - c_f\right)$$

$$+ M_t^e \left(\sum_{\nu} f(\nu)(1 - d_t^e(\nu)) \left(e^{\nu} l_t^e(\nu)^{\alpha} - c_f\right) - c_e \right) \Bigg\}, \tag{A.72}$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E)) l_{t-1} + h_t \tag{A.73}$$

$$v_t = \theta_t^U u_t + \sum_{(\nu, l_{t-1})} \lambda \theta_t^E(\nu, l_{t-1}) l_{t-1} g(l_{t-1}) f(\nu) \tag{A.74}$$

$$u_t = (1 - f(\theta_t^U)) u_{t-1} + \sum_{\nu, l_{t-1}} (d_t + (1 - d_t) s_t) l_{t-1} g(l_{t-1}) f(\nu) \tag{A.75}$$

$$g(l_t) = \sum_{\nu, l_{t-1}} f(\nu) g(l_{t-1})(1 - \mathbf{d}(\nu, l_{t-1}) \mathbb{I}_{l(\nu, l_{t-1}) = l_t} \tag{A.76}$$

$$+ \sum_{\nu} M_t^e f(\nu)(1 - d_t^e(\nu)) \mathbb{I}_{l_t^e(\nu) = l_t} \tag{A.77}$$

$$h_t(1 - d_t) = f(\theta_t^U) u_t \text{ for firms searching in market } \theta^U \text{ (i.e. } \theta(\nu, l_{t-1}) = \theta^U) \tag{A.78}$$

$$h_t(1 - d_t) = \lambda f(\theta_t^E)(1 - s_t) l_{t-1} \text{ for firms poaching from market } \theta^E \text{ (i.e. } \theta(\nu, l_{t-1}) = \theta^E) \tag{A.79}$$

Following the same trick, it is obvious to prove that the competitive equilibrium under the full information is also socially optimal as it can be replicated by the social planner's problem (A.72). ■

These results verify that the model's decentralized block-recursive allocation given the level of uncertainty is socially optimal. If the planner could resolve uncertainty, the decentralized allocation would be distorted due to the uncertainty.

A.6 Computation Algorithm

A.6.1 Guess $\mathbf{V}^{init}$

We start with our guess $\mathbf{V}^{init0}(a, \tilde{P}, l, P)$ for $\mathbf{V}^{init}(a, \tilde{P}, l, P)$.[2]

A.6.2 Use Free-entry Condition

1. Get $\mathbb{E}_{P'}\mathbf{V}^{init}(1, \ln P, l^e, P')$

   For each possible grid points for P, use $\ln P' \sim N(\frac{\frac{\bar{\nu}_0}{\sigma_0^2}+\frac{\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma_\epsilon}}, \frac{1}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma_\epsilon}} + \sigma_\epsilon^2)$.

2. Guess κ

3. Find l^e and d^e that solves:

$$\max_{d^e, l^e}\Big[(1-d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta\mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big], \tag{A.80}$$

   for each possible P, and adjust κ with a bisection method until it satisfies

$$\int \max_{d^e, l^e}\Big[(1-d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta\mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big]dF_e(P) = c^e,$$

   where $\ln P \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\epsilon^2)$.

[2] Here, for notational convenience, I will use $\tilde{P}$ and l to refer to the average log productivity and employment size in the previous period, respectively. Note that P is the current period productivity. Variables with $'$ refer to their value in the next period, i.e. $\tilde{P}'$ is the average log productivity up to the current period, l' is the current period employment size after all decisions made (for hiring, retention, and layoffs, etc.), and P' is the next period productivity.

A.6.3 Unemployed Workers' Problem

Use the solution for x^U,

$$x^U = \kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} \tag{A.81}$$

and solve a fixed-point problem for $\mathbf{U}$ from the following:

$$\mathbf{U} = b + \beta\Big((1 - f(\theta(x^U))\mathbf{U} + f(\theta(x^U))x^U\Big), \tag{A.82}$$

using (1.28).

A.6.4 Value Function Iteration

1. Generate $\mathbb{E}\mathbf{V}^{init0}(a+1, \tilde{P}', l', P') = \mathbb{E}\mathbf{V}^{init0}(a+1, \frac{a\tilde{P}+\ln P}{(a+1)}, l', P')$.

   Given state variables $(a, \tilde{P}, l, P)$ and $\ln P' \sim (\frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{a\tilde{P}+\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2} + \frac{a+1}{\sigma_\epsilon^2}}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{a+1}{\sigma_\epsilon^2}} + \sigma_\epsilon^2)$, I use the interpolation of $\mathbf{V}^{init0}$ evaluated at each $(a+1, \frac{a\tilde{P}+\ln P}{a+1}, l', P')$ and take expectation across $\ln P'$.

2. Use grid search to max $\mathcal{V}$ and obtain the argmax gridpoint l'.

   For each possible combination of l and l', given $(a, \tilde{P}, l, P)$:

   (a) Step 1: for hiring/inaction case ($l' \geqslant l$)

$$x^E = \kappa - c \tag{A.83}$$

$$s = 0 \tag{A.84}$$

$$h = l' - (1 - \lambda f(\theta(x^E)))l = l' - l \tag{A.85}$$

(b) Step 2: for separation case $(l' < l)$

$$x^E = \max\Big(x_1^E, x_2^E\Big) \tag{A.86}$$

$$s = 1 - \frac{l'}{(1 - \lambda f(\theta(x^E)))l} \tag{A.87}$$

$$h = 0 \tag{A.88}$$

where x_1^E refers to the promised utility level to incumbent workers in a firm facing both layoffs and quits, and is pinned down by the root of the following:

$$\kappa - \mathbf{U} = c\Big((1 + \theta(x^E)^\gamma)^{1+\frac{1}{\gamma}} - \lambda\theta(x^E)^{1+\gamma}\Big), \tag{A.89}$$

and x_2^E refers to that in a firm having quits only, and is the root of the following:

$$\begin{aligned} \frac{l - l'}{\lambda l} &= f(\theta(x^E)) = \Big(1 - (\frac{\kappa - x^E}{c})^{-\gamma}\Big)^{\frac{1}{\gamma}} \\ x^E &= \kappa - c\Big(1 - (\frac{l - l'}{\lambda l})^\gamma\Big)^{-\frac{1}{\gamma}} \end{aligned} \tag{A.90}$$

Thus, from the above steps, we have

$$\mathbf{x^E}(a, \tilde{P}, l, P, l'), \mathbf{s}(a, \tilde{P}, l, P, l'), \mathbf{h}(a, \tilde{P}, l, P, l') \tag{A.91}$$

and

$$\tilde{\mathbf{W}}(a, \tilde{P}, l, P, l') = \kappa - (\kappa - x^E(a, \tilde{P}, l, P, l'))^{1+\gamma}c^{-\gamma} \tag{A.92}$$

for each possible set of (l, l') and the state variables.

Using it, we find a gridpoint l' that solves the following maximization:

$$\mathcal{V}(a,\tilde{P},l,P) \equiv \max_{l'} \mathbf{s}(a,\tilde{P},l,P,l')\mathbf{U}l + Pl'^{\alpha} - c^f - \kappa\mathbf{h}(a,\tilde{P},l,P,l')$$
$$+ (1-\mathbf{s}(a,\tilde{P},l,P,l'))\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P,l')))x^E(a,\tilde{P},l,P,l')l + \beta\mathbb{E}\mathbf{V}^{init0}(a+1,\frac{a\tilde{P}+\ln P}{a+1},l',P'). \tag{A.93}$$

3. Spline approximation for l'

   Let I be the optimal index for l' that maximizes $\mathcal{V}$, given $(a,\tilde{P},l,P)$. Now, we would like to spline approximate $\mathcal{V}$ across the points l_{I-1}, l_I, and l_{I+1} to get a proper policy function.

   (a) Step 1: use the spline approximated form of $\mathcal{V}$

   $$\mathcal{V} = \mathcal{V}_i(l) \quad \text{if} \quad l_i \leqslant l \leqslant l_{i+1}$$

   where

   $$\begin{aligned}\mathcal{V}_i(l) &= a_i(l-l_i)^3 + b_i(l-l_i)^2 + c_i(l-l_i) + \mathcal{V}_i(l_i)\\ \mathcal{V}_i'(l) &= 3a_i(l-l_i)^2 + 2b_i(l-l_i) + c_i\\ \mathcal{V}_i''(l) &= 6a_i(l-l_i) + 2b_i.\end{aligned}$$

   (b) Conditions to use

   $$\begin{aligned}\mathcal{V}_i(l_i) &= \mathcal{V}_{i-1}(l_i)\\ \mathcal{V}_i'(l_i) &= \mathcal{V}_{i-1}'(l_i)\\ \mathcal{V}_i''(l_i) &= \mathcal{V}_{i-1}''(l_i)\end{aligned}$$

→ Using the functional form for $\mathcal{V}_i$ above, these conditions are rephrased as follows:

$$\Delta\mathcal{V}_i(l_i) = a_{i-1}(l_i - l_{i-1})^3 + b_{i-1}(l_i - l_{i-1})^2 + c_{i-1}(l_i - l_{i-1}) \tag{A.94}$$

$$c_i = 3a_{i-1}(l_i - l_{i-1})^2 + 2b_{i-1}(l_i - l_{i-1})^2 + c_{i-1} \tag{A.95}$$

$$2b_i = 6a_{i-1}(l_i - l_{i-1}) + 2b_{i-1} \tag{A.96}$$

(c) Generate coefficient matrix

We can convert (A.94), (A.95), and (A.96), for $i = 2, 3, ..., N$ (N is the number of l grid points), into a matrix form. Let

$$Coeff = \begin{pmatrix} a_1 & b_1 & c_1 & ... & a_{N-1} & b_{N-1} & c_{N-1} \\ ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... \end{pmatrix}. \tag{A.97}$$

Then, we could get this by

$$Coeff = DV * inv(H), \tag{A.98}$$

where

$$H = \begin{pmatrix}
(l_2-l_1)^3 & 0 & 0 & ... & 0 & 3(l_2-l_1)^2 & 0 & ... & 0 & 6(l_2-l_1) & 0 & ... & 0 & 0 & 0 \\
(l_2-l_1)^2 & 0 & 0 & ... & 0 & 2(l_2-l_1) & 0 & ... & 0 & 2 & 0 & ... & 0 & 0 & 0 \\
(l_2-l_1) & 0 & 0 & ... & 0 & 1 & 0 & ... & 0 & 0 & 0 & ... & 0 & 1 & 0 \\
0 & (l_3-l_2)^3 & 0 & ... & 0 & 0 & 3(l_3-l_2)^2 & ... & 0 & 0 & 6(l_3-l_2) & ... & 0 & 0 & 0 \\
0 & (l_3-l_2)^2 & 0 & ... & 0 & 0 & 2(l_3-l_2) & ... & 0 & -2 & 2 & ... & 0 & 0 & 0 \\
0 & (l_3-l_2) & 0 & ... & 0 & -1 & 1 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & 0 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & 0 & ... & 0 & 0 & -2 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & -1 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & 0 & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & 1 & 0 & 0 & ... & 0 & 0 & 0 \\
0 & 0 & 0 & ... & (l_N-l_{N-1})^3 & 0 & ... & ... & 0 & 0 & 0 & ... & 0 & 0 & 3(l_N-l_{N-1})^2 \\
0 & 0 & 0 & ... & (l_N-l_{N-1})^2 & 0 & ... & ... & 0 & 0 & 0 & ... & -2 & 0 & 2(l_N-l_{N-1}) \\
0 & 0 & 0 & ... & (l_N-l_{N-1}) & 0 & ... & ... & -1 & 0 & 0 & ... & 0 & 0 & 1
\end{pmatrix}$$

and

$$DV = \begin{pmatrix} \Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\ \Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\ \Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \end{pmatrix}$$

where the number of each matrix is the same as $3*(N-1)$, and the number of rows in $Coeff$ and DV is $(na*n\tilde{P}*N*nP)$, and each row is for each pair of state variables $(a, \tilde{P}, l, P')$.

(d) Get the root of l'

Once we have $Coeff$, we derive the root of l' from each $\mathcal{V}_{I-1}$ and $\mathcal{V}_I$. This means to find l', such that

$$\mathcal{V}'_{I-1}(l) = a_{I-1}(l-l_{I-1})^2 + b_{I-1}(l-l_{I-1}) + c_{I-1} = 0$$

and

$$\mathcal{V}'_{I}(l) = a_{I}(l-l_{I})^2 + b_{I}(l-l_{I}) + c_{I} = 0$$

Thus, we have four possible roots of l' from the spline approximation:

$$l' = \Big[\frac{-B_{I-1} \pm \sqrt{B_{I-1}^2 - 4A_{I-1}C_{I-1}}}{2A_{I-1}}, \frac{-B_I \pm \sqrt{B_I^2 - 4A_IC_I}}{2A_I}\Big] \tag{A.99}$$

where

$$A_i = 3a_i$$

$$B_i = 2b_i - 6a_il_i$$

$$C_i = 3a_il_i^2 + 2b_il_i + c_i, \quad for \quad i \in \{I-1, I\}$$

(e) Evaluate $\mathcal{V}$ and the corresponding policy function l'

We evaluate

$$\max[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}],$$

and obtain

$$l'(a, \tilde{P}, l, P) = argmax[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}]. \tag{A.100}$$

Note that $l_1' \sim l_4'$ are the roots based on (A.99), and the first $\mathcal{V}(l_1') \sim \mathcal{V}(l_4')$ are spline approximated $\mathcal{V}$ evaluated at each root, and the last $\mathcal{V}$ is the maximized value from the grid search.

(f) Managing inaction ranges

For the inaction range, such that $l_I(a, \tilde{P}, l, P) = l$, we don't use spline approximation for $\mathcal{V}(a, \tilde{P}, l, P)$.

4. Policy functions

We use (A.91) and (A.100) to back out policy functions for

$$\begin{aligned}
\mathbf{x}^{\mathbf{E}}(a, \tilde{P}, l, P) &\equiv \mathbf{x}^{\mathbf{E}}(a, \tilde{P}, l, P, l') \\
\mathbf{s}(a, \tilde{P}, l, P) &\equiv \mathbf{s}(a, \tilde{P}, l, P, l') \\
\mathbf{h}(a, \tilde{P}, l, P) &\equiv \mathbf{h}(a, \tilde{P}, l, P, l'),
\end{aligned}$$

and

$$\mathbf{d}(a, \tilde{P}, l, P) = \begin{cases} 1 & \text{if} \quad \mathbf{U}l > \mathcal{V}(a, \tilde{P}, l, P) \\ 0 & \text{otherwise.} \end{cases} \tag{A.101}$$

5. Update the Guess

$$\mathbf{V}^{init1}(a,\tilde{P},l,P) = \bigg(\delta + (1-\delta)\mathbf{d}(a,\tilde{P},l,P)\bigg)\mathbf{U}l + (1-\delta)(1-\mathbf{d}(a,\tilde{P},l,P))\mathcal{V}(a,\tilde{P},l,P) \quad \text{(A.102)}$$

If $|\mathbf{V}^{init0} - \mathbf{V}^{init1}| < \epsilon$, with sufficiently small ϵ, then it's done! Otherwise, replace $\mathbf{V}^{init0}$ with a new guess $\mathbf{V}^{init1}$ and reiterate from the part B.2.

A.7 Figures for Low performing Firms

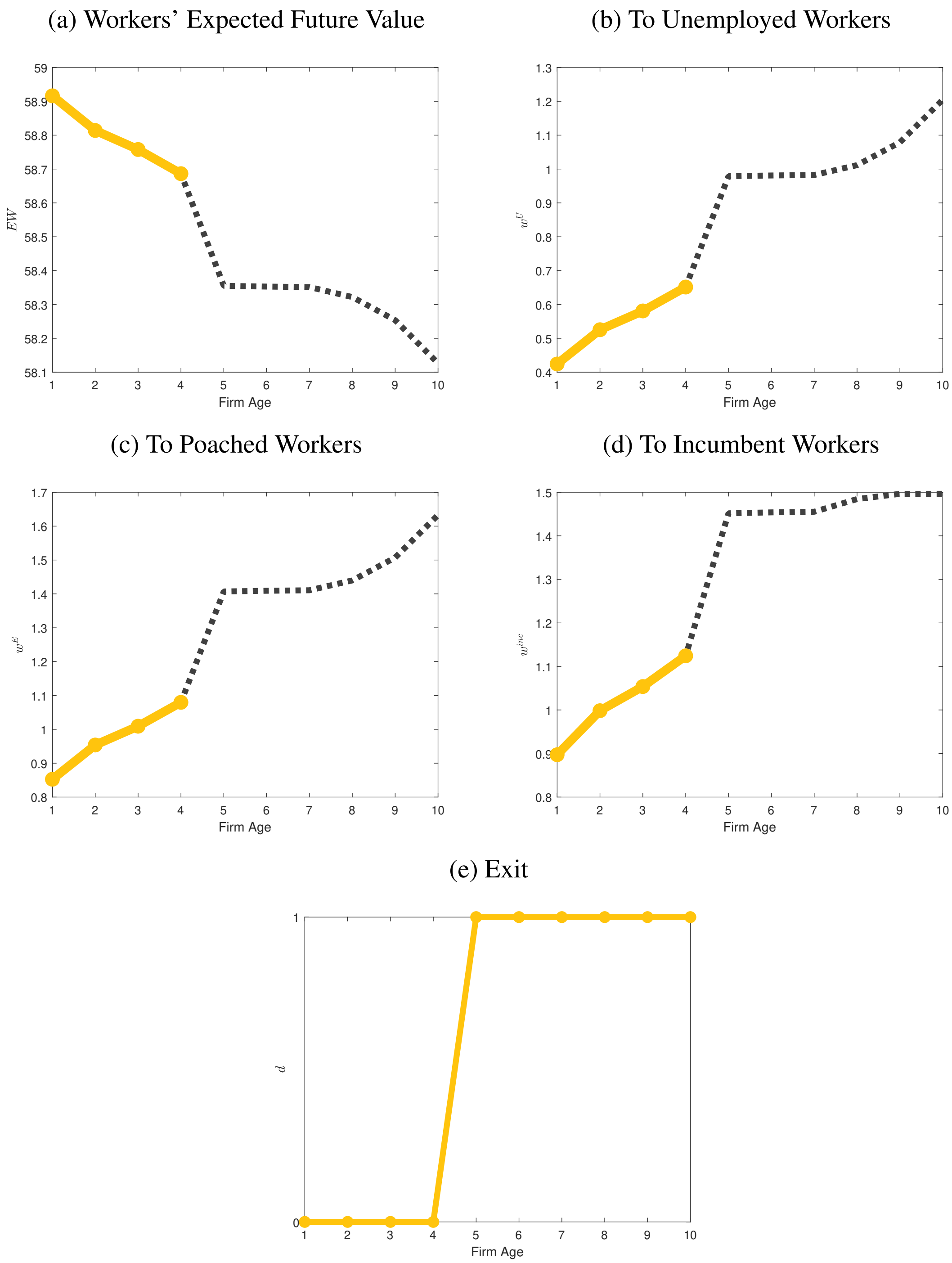


Figure A.7.1: Low Performing Firms (average size)

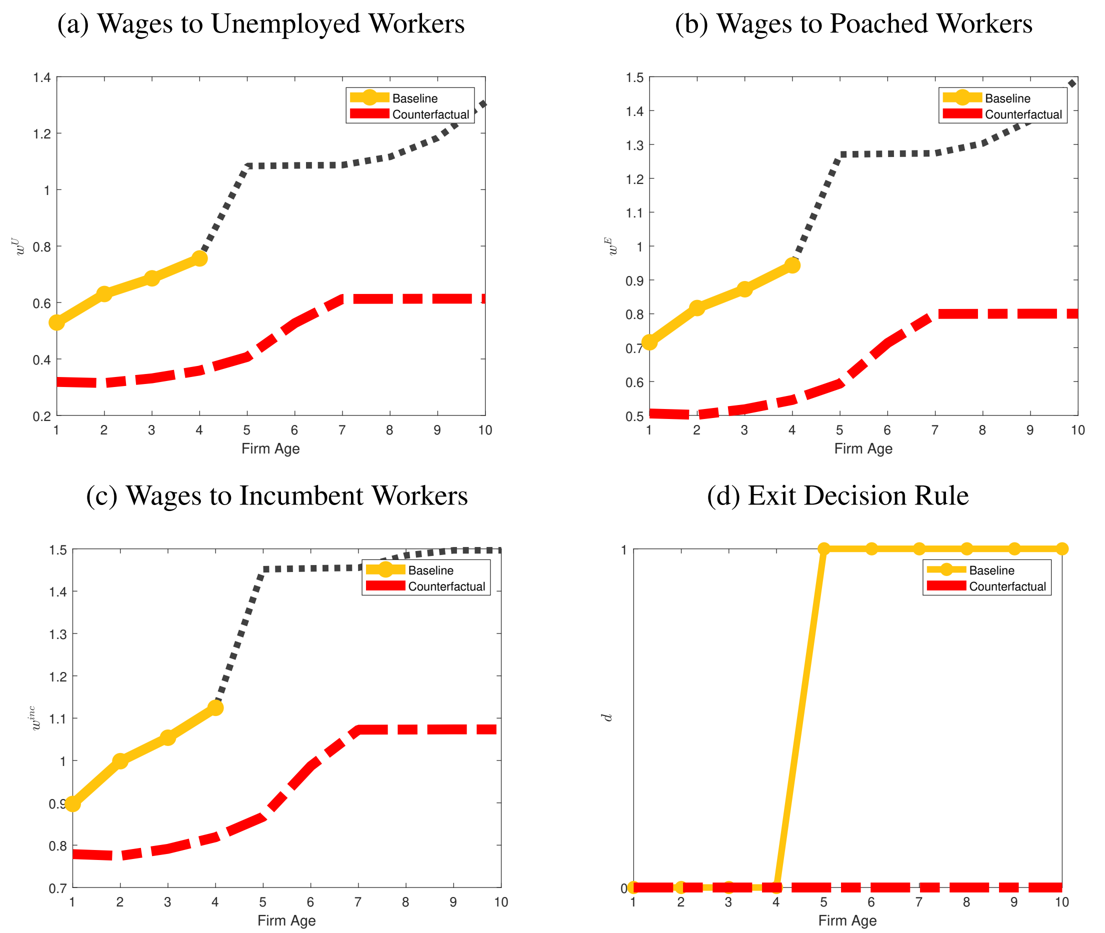


Figure A.7.2: Low Performing Firms: Baseline vs. Counterfactual (higher uncertainty)

A.8 Full Tables

Table H1: Wage Differentials for Young Firms

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.002***	-0.003***
	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.016***
	(0.001)	(0.001)
High performing firm	0.002	0.002
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.009***	0.012***
	(0.001)	(0.001)
Current Productivity (at t)	0.020***	0.015***
	(0.001)	(0.001)
Firm Size (at t)	0.017***	
	(0.001)	
Firm Size (at $t-1$)		0.013***
		(0.001)
Previous Employer (AKM)	0.267***	0.270***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table H2: The Effect of Uncertainty on Young Firms' Wage Differentials

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.001	-0.002
	(0.002)	(0.002)
Young firm × High performing firm	0.003	0.005**
	(0.002)	(0.002)
Young firm × Uncertainty	-0.005**	-0.004*
	(0.002)	(0.002)
Young firm × High performing firm × Uncertainty (at $t-1$)	0.016***	0.015***
	(0.003)	(0.003)
Uncertainty (at $t-1$)	-0.067***	-0.071***
	(0.002)	(0.002)
High performing firm	0.004**	0.003*
	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.009***	0.011***
	(0.000)	(0.000)
Current Productivity (at t)	0.020***	0.016***
	(0.000)	(0.000)
Firm Size (at t)	0.012***	
	(0.000)	
Firm Size (at $t-1$)		0.010***
		(0.000)
Previous Employer (AKM)	0.269***	0.272***
	(0.000)	(0.000)
Observations	50,170,000	50,170,000
Fixed effects	Sector, State	Sector, State

Notes: The table reports results for regression of earning residuals on young firm, high performing firm indicators, and the uncertainty measure. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect and a dummy for non-employed workers in the previous period, associated with the previous employer to capture time-varying components. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, sector and state fixed effects, and the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

A.9 Robustness Checks for Regressions

Table I3: Wage Differentials for Young Firms (without firm size)

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.006***	-0.007***
	(0.001)	(0.001)
Young firm × High performing firm	0.013***	0.015***
	(0.001)	(0.001)
High performing firm	0.005***	0.004***
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***
	(0.001)	(0.001)
Current Productivity (at t)		0.015***
		(0.001)
Previous Employer (AKM)	0.283***	0.281***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level and current productivity level. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I4: Compensating Differentials for Young Firms (propensity score weighted)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals
Young firm	-0.007***	-0.008***	-0.003***	-0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.004***	0.002*	-0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Average Firm Productivity	0.017***	0.003***	0.005***	0.009***
(up to $t-1$)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.021***	0.027***	0.021***
		(0.001)	(0.001)	(0.001)
Firm Size			0.020***	
			(0.000)	
Firm Size (at $t-1$)				0.015***
				(0.000)
Previous Employer (AKM)	0.281***	0.278***	0.266***	0.269***
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I5: Wage Differentials for Young Firms (bootstrapped standard errors)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.006***	-0.007***	-0.002***
	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.013***	0.015***	0.015***
	(0.002)	(0.002)	(0.002)
High performing firm	0.005***	0.004*	0.002
	(0.002)	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***	0.009***
	(0.000)	(0.001)	(0.001)
Current Productivity (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators by using bootstrapped standard errors. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the standard errors. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I6: Wage Differentials for Young Firms (with previous earnings)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals	(5) Earnings Residuals	(6)	(7)
Young firm	-0.003***	-0.003***	-0.004***	-0.005***	-0.005***	-0.000	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.014***	0.014***	0.016***	0.018***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.001***	0.001***	-0.004	-0.010***	-0.010***	-0.012***	-0.012***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.006***	0.003***	0.006***	-0.009***	-0.007***	-0.005***	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.005***	0.012***		-0.003***	0.003***	0.000
		(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Firm Size (at t)			0.028***			0.018***	
			(0.001)			(0.000)	
Firm Size (at $t-1$)							0.014***
							(0.000)
Previous Employer (AKM)				0.155***	0.155***	0.141***	0.160***
				(0.001)	(0.001)	(0.001)	(0.001)
Previous Earnings	0.194***	0.194***	0.190***	0.167***	0.167***	0.167***	0.165***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are previous earning level (in all columns) along with AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period (in the last three columns). Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I7: Aggregate Implications of Uncertainty (current value)

	(1) Entry rate	(2) Young firm share	(3) Young firm emp. share
Uncertainty (at t)	-0.009***	-0.013***	-0.013***
	(0.002)	(0.005)	(0.005)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year
	(4) HG young firm share	(5) HG young firm emp.share	(6) HG young firm avg. emp. growth
Uncertainty (at t)	-0.010***	-0.008***	-0.020***
	(0.003)	(0.002)	(0.005)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year

Notes: The table reports results for regression of young firm activities in each column on the contemporary value of the uncertainty at the industry level. Observation counts are rounded to the nearest 100 to avoid potential disclosure risks. Estimates for constant, industry and year fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Appendix B: Chapter 2 Appendix

B.1 Baseline Model

B.1.1 Optimal Production and Employment

Final goods producer's production function is of the form:

$$Y = \frac{L^{\theta}}{1-\theta} \left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \in \mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \notin \mathcal{D}\}}\, dj \right] ,$$

where $\mathcal{D}$ is the index set for differentiated products produced by domestic firms, and final good price is normalized to one $P = 1$. Thus profits are

$$\Pi^{\text{FG}} = Y = \frac{L^{\theta}}{1-\theta} \left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \in \mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \notin \mathcal{D}\}}\, dj \right] - wL - \int_0^1 p_j\, y_j\, dj \; .$$

FONCs of final good producer's profit maximization problem w.r.t. k_j and L are

$$\frac{\partial}{\partial y_j} : \quad p_j = q_j^{\theta}\, L^{\theta}\, y_j^{-\theta} \tag{B.1}$$

$$\frac{\partial}{\partial L} : \quad w = \frac{\theta}{1-\theta}\, L^{\theta-1} \left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \in \mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j \notin \mathcal{D}\}}\, dj \right] . \tag{B.2}$$

Intermediate good producers, both domestic firms and foreign exporters, take differentiated product demand (B.1) as given and solve for the profit maximization problem:

$$\pi(q_j) = \max_{y_j \geqslant 0} \left\{ L^\theta \, q_j^\theta \, y_j^{1-\theta} - y_j \right\} \ .$$

The FOC of this problem gives us:

$$\frac{\partial}{\partial y_j} : \quad (1-\theta) L^\theta \, q_j^\theta \, y_j^{-\theta} = 1 \quad \Rightarrow \quad y_j = (1-\theta)^{\frac{1}{\theta}} L \, q_j \text{ , and } p_j = \frac{1}{1-\theta} \ .$$

By plugging in the two optimal choices, differentiated product producer's profits from a product line j become

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} \, L}_{\equiv \pi} \, q_j \ .$$

By plugging in optimal differentiated product production rule to (B.2), we get the wage rule that depends only on average product qualities

$$\begin{aligned} w &= \frac{\theta}{1-\theta} \, L^{\theta-1} \left[\int_0^1 q_j^\theta \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \, q_j^{1-\theta} \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj + \int_0^1 q_j^\theta \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \, q_j^{1-\theta} \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \right] \\ &= \frac{\theta}{1-\theta} \, L^{\theta-1} \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \int_0^1 q_j \, dj \\ &\Rightarrow w = \theta(1-\theta)^{1-2\theta} \overline{q} \qquad \text{(B.3)} \end{aligned}$$

Finally, using the labor market clearing condition

$$L = 1 \, , \tag{B.4}$$

we get the equilibrium conditions:

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}}\overline{q} \tag{B.5}$$

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \tag{B.6}$$

$$p_j = \frac{1}{1-\theta} \tag{B.7}$$

$$\pi = \theta(1-\theta)^{\frac{1-\theta}{\theta}} \, . \tag{B.8}$$

B.1.2 Product Quality Determination

In this section, we will consider all possible cases where firm keeps or loses its product lines next period and compute the probabilities as functions of internal innovation intensities and creative destruction arrival rate. Clearly, past period technology gap $\Delta_t = \frac{q_t}{q_{t-1}}$ is the only information needed to compute these probabilities, as incumbent firm and outside firm trying to take over incumbent firm's product line compete with the level of next period product qualities they come up with, where product quality in period $t+1$ the incumbent firm will have after internal innovation improves or fail to improve the product quality by $\Delta_{j,t+1}$ is $q^{in}_{j,t+1} = \Delta_{j,t+1}\Delta_{j,t}q_{j,t-1}$, and product quality the outside firm will have after successful external innovation is $q^{en}_{j,t+1} = \eta q_{j,t-1}$. We will first show Δ_t can assume only four values, $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$.

B.1.2.1 Proof of Lemma 3

Proof. To make argument clearer, let's consider the cases where 1) there is no ownership change between $t-1$ and t, and 2) there is ownership change between $t-1$ and t.

1) No ownership change between $t-1$ **and** t**:** In this case, $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ should hold, where only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ are possible due to the fact that $\Delta_{j,t}$ is an outcome of internal innovation.

2) Ownership change between $t-1$ **and** t**:** In this case, $q_{j,t} = \eta q_{j,t-2}$ should hold. Let's consider all potentially possible cases where i. $\Delta_{j,t} = 1$, ii. $\Delta_{j,t} = \lambda$, iii. $\Delta_{j,t} = \eta$, iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$, v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$, and vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only potentially possible values Δ can assume, as there are only three step sizes (1, λ, and η) product quality can change between two periods and there cannot be a technology regression ($q_t < q_{t-1}$). In the end, we will see that only the first four cases are possible.

case 2)-i. $\Delta_{j,t} = 1$

For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, this implies $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was external innovation between $t-2$ and $t-1$, and no internal innovation between $t-3$ and $t-1$, thus $q_{j,t-2} = q_{j,t-3}$. Thus $\Delta_{j,t} = 1$ is possible with ownership change between $t-1$ and t.

case 2)-ii. $\Delta_{j,t} = \lambda$

For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there is internal innovation between $t-3$ and $t-2$, and external innovation between $t-2$ and $t-1$, but no internal innovation between

$t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$, and $q_{j,t-1} = \eta q_{j,t-3}$. Thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$. So we proved both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through external innovation between $t-1$ and t.

case 2)-iii. $\Delta_{j,t} = \eta$

For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there is no ownership change and no internal innovation between $t-1$ and $t-2$. Thus $\Delta_{j,t} = \eta$ is possible.

case 2)-iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$

The possibility of this case is shown in case 2)-ii.

case 2)-v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$

Let's suppose this is the case. Since $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$ there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ should hold, and this implies $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. $m \leqslant n-1$ is not possible as this implies technology regression. Let's suppose $m > n-1$. Since $n \geqslant m > 0$, this implies $m = n$ should hold. Suppose this is the case, thus $g_{j,t-2} = \frac{\lambda^m}{\eta^{m-1}} q_{j.t-1}$. If the values for λ, η, and m are such that $\frac{\lambda^m}{\eta^{m-1}} < 1$, then this means technology regression, which is not possible. Let's suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$ is true. If $m = 1$, we are back in the case 2)-ii and case 2)-iv. Let's suppose $m > 1$. Since $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus $q_{j,t-1} = \eta q_{j,t-3}$, and this implies $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$.

Thus if $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible, then

$$q_{j,t-s} = \begin{cases} \frac{\eta^m}{\lambda^m} q_{j,t-s-1} & , s\text{: even number} \\ \\ \frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1} & , s\text{: odd number} . \end{cases}$$

Thus in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ should hold, which is not possible (or we assume this case out). Thus $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$ is not possible.

case 2)-vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$

With a similar argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only four values, $\left\{1, \lambda, \eta, \frac{\eta}{\lambda}\right\}$. ■

B.1.2.2 Product Quality Evolution for Outsider Firms

Let's denote z_j^ℓ as an internal innovation intensity for product line j when it's technology gap is $\frac{q_{j,t}}{q_{j,t-1}} = \Delta^\ell$, such that $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$. Then product quality in period $t+1$ evolves probabilistically as:

$$q_{j,t+1}(\Delta_t = 1) = \begin{cases} \lambda q_{j,t-1} \, , & \text{with prob. } (1 - \overline{x})\, z_j^1 \\ q_{j,t-1} \, , & \text{with prob. of } (1 - \overline{x})\,(1 - z_j^1) \\ \eta q_{j,t-1} \, , & \text{with prob. } \overline{x} \, , \end{cases}$$

where $q_{j,t-1} = q_{j,t}$,

$$q_{j,t+1}(\Delta_t = \lambda) = \begin{cases} \lambda^2 q_{j,t-1} \ , & \text{with prob. } \ z_j^2 \\ \lambda q_{j,t-1} \ , & \text{with prob. } \ (1-\overline{x})\,(1-z_j^2) \\ \eta\, q_{j,t-1} \ , & \text{with prob. } \ \overline{x}(1-z_j^2) \ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\lambda} q_{j,t}$,

$$q_{j,t+1}(\Delta_t = 1+\eta) = \begin{cases} \lambda\eta\, q_{j,t-1} \ , & \text{with prob. } \ z_j^3 \\ \eta\, q_{j,t-1} \ , & \text{with prob. } \ (1-\overline{x})(1-z_j^3) + \frac{1}{2}\,\overline{x}\,(1-z_j^3) \\ \eta\, q_{j,t-1} \ , & \text{with prob. } \ \frac{1}{2}\,\overline{x}\,(1-z_j^3) \ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\eta} q_{j.t}$, and

$$q_{j,t+1}\left(\Delta_t = \frac{\eta}{\lambda}\right) = \begin{cases} \lambda\,\frac{\eta}{\lambda}\, q_{j,t-1} \ , & \text{with prob. } \ (1-\overline{x})\, z_j^4 + \frac{1}{2}\,\overline{x}\, z_j^4 \\ \frac{\eta}{\lambda}\, q_{j,t-1} \ , & \text{with prob. } \ (1-\overline{x})\,(1-z_j^4) \\ \eta q_{j,t-1} \ , & \text{with prob. of } \ \overline{x}\,(1-z_j^4) + \frac{1}{2}\,\overline{x}\, z_j^4 \ , \end{cases}$$

where $q_{j,t-1} = \frac{\lambda}{1+\eta} q_{j,t}$.

B.1.2.3 Product Quality Evolution for an Incumbent Firm

For each Δ^ℓ, transition dynamics for product quality and technology gap for product line j_i can be represented using two indicator functions I_i^z and $I_i^{\overline{x}}$, where $\Delta'_{j_i} = 0$ (or equivalently $\{q'_{j_i}\} = \phi$) implies firm loses product line j_i in the next period. Here, we write down the expressions as if incumbent firm is doing coin-tossing at all times.

B.1.2.3.1 i) $\Delta_{j_i} = \Delta^1 = 1$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \Big\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\Big\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \Big\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\Big\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

B.1.2.3.2 ii) $\Delta_{j_i} = \Delta^2 = \lambda$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

B.1.2.3.3 iii) $\Delta_{j_i} = \Delta^3 = \eta$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \lambda I_i^z & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{(\lambda I_i^z)\, q_{j_i}\right\}\backslash\{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\}\backslash\{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

B.1.2.3.4 iv) $\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda}$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1-I_i^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\}\backslash\{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{(1-I_i^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\}\backslash\{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

B.1.3 Value Function and Optimal Innovation Decisions

Conditional expectation inside of the expression for the value function is over the success/failure of internal and external innovation, creative destruction shock arrival, winning/losing from coin-tosses (c-t), the current period product quality q distribution, and the current period technology gap Δ^{ℓ} distribution. Thus $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^{f}\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ is equal to

$$
\begin{aligned}
&\sum_{I_1^{\overline{x}}, I_2^{\overline{x}}, \ldots, I_{n_f}^{\overline{x}}=0}^{1} \; \sum_{I_1^{z}, \ldots, I_{n_f}^{z}=0}^{1} \; \sum_{\text{c-t}_1, \ldots, \text{c-t}_{n_f} = \text{win}}^{\text{lose}} \; \sum_{I^x=0}^{1} \left[\prod_{i=1}^{n_f} \overline{x}^{I_i^{\overline{x}}} (1-\overline{x})^{1-I_i^{\overline{x}}} z_i^{I_i^z} (1-z_i)^{1-I_i^z}\right] \\
&\times \left[x^{I^x}(1-x)^{1-I^x}\right] \left(\frac{1}{2}\right)^{n_f} \\
&\times \mathbb{E}_{q,\Delta}\, V\left(\left[\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid (q_{j_i}, \Delta_{j_i}), I_i^{\overline{x}}, I_i^z, \text{c-t}_i\right\} \backslash \{\mathbf{0}\}\right]\right] \right. \\
&\qquad\qquad \left. \bigcup \left[\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\ \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \backslash \{\mathbf{0}\}\right]\right).
\end{aligned}
$$

The first term inside of the value function, $\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid (q_{j_i}, \Delta_{j_i}), I_i^{\overline{x}}, I_i^z, \text{c-t}_i\right\} \backslash \{\mathbf{0}\}\right]$, depicts subsets of possible realizations for $\Phi^{f\prime}$ from internal innovation, creative destruction, and coin-toss, and the second term, $\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\ \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \backslash \{\mathbf{0}\}$, depicts subsets of possible realizations for $\Phi^{f\prime}$ from external innovation, where $\{q'_{j_i}\} = \{\Delta'_{j_i} q_{j_i}\} \backslash \{0\}$, and $\{q'_{-j}\} = \{\frac{\eta}{\Delta_{-j}} I^x q_{-j}\} \backslash \{0\}$. If $\Delta'_{j_i} = 0$, then firm f loses product line j_i and $\{(q'_{j_i}, \Delta'_{j_i})\} \backslash \{\mathbf{0}\} = \{\mathbf{0}\} \backslash \{\mathbf{0}\} = \varnothing$.

B.1.3.1 Proof of Proposition 6

Proof. Due to the linearity of expectation, $\sum_{\ell=1}^{4} A_\ell \sum_{j \in \mathcal{J}^f | \Delta_j = \Delta^\ell} q_j$ portion of conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be written as

$$\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \sum_{j\in\mathcal{J}^{f\prime}|\Delta_j'=\Delta^\ell} q_j'\right] = \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^{f}|(\Delta_j'|\Delta_j)=\Delta^\ell} \Delta^\ell q_j\right] + \mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j} q_j\right],$$

where the first term is expected value from existing product lines and the second term is expected value from a new product line added through external innovation.

Since realization of internal innovation success/failure and creative destruction shock are independent from realization of external innovation success/failure, expected value from a new product line is

$$\begin{aligned}
\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j} q_j\right] &= \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x}\mathbb{E}_{q_j,\Delta_j}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} I^x \frac{\eta}{\Delta_j} q_j\right] \\
&= x\,\mathbb{E}_{q_j}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) \\
&\qquad + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg] q_j \\
&= x\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) \\
&\qquad + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\overline{q}\,.
\end{aligned}$$

The second equality follows from the fact that randomly chosen product line with a quality q_j can have technology gap Δ^ℓ with the probability $\mu(\Delta^\ell)$ and probability of taking over this product line depends on its technology gap. The third equality follows by integrating product quality over all product line indices.[1]

First expectation can further divided into four cases, depending on current period technology gap Δ:

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^\ell)} \Delta^\ell q_j\right] = \sum_{\tilde{\ell}=1}^{4} \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^{\tilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right].$$

To make formulas easy to write, let's re-order the product quality portfolio q_j according to technology gap Δ^ℓ and renumber them according to:

$$q^f = \Big\{ \underbrace{q_{j_1},\ q_{j_2},\ \ldots,\ q_{j_{n_f^1}}}_{\Delta^1},\ \underbrace{q_{j_{n_f^1+1}},\ \ldots,\ q_{j_{n_f^1+n_f^2}}}_{\Delta^2},\ \underbrace{q_{j_{n_f^1+n_f^2+1}},\ \ldots,\ q_{j_{n_f^1+n_f^2+n_f^3}}}_{\Delta^3},$$
$$\underbrace{q_{j_{n_f^1+n_f^2+n_f^3+1}},\ \ldots,\ q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}}_{\Delta^4}\Big\}.$$

Then for $i = 1, 2, \ldots, n_f^1$ ($\Delta_{j_i} = \Delta^1 = 1$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^1)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=1}^{n_f^1}\Big[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\Big]q_{j_i}\,,$$

for $i = n_f^1+1, \ldots, n_f^1+n_f^2$ ($\Delta_{j_i} = \Delta^2 = \lambda$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^2)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+1}^{n_f^1+n_f^2}\Big[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\Big]q_{j_i}\,,$$

[1] Only the share of technology gap $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and average quality $\overline{q}$ are contained in individual firm's information set in terms of firm distribution. That is, for an individual firm, technology gap and product quality are independent.

for $i = n_f^1 + n_f^2 + 1, \dots, n_f - n_f^4$ ($\Delta_{j_i} = \Delta^3 = \eta$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i \in \mathcal{J}^f | (\Delta'_{j_i} | \Delta_{j_i} = \Delta^3) = \Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} \left[A_1 \left(1 - \frac{1}{2}\overline{x}\right)(1 - z_i^3) + \lambda A_2 z_i^3\right] q_{j_i} \, ,$$

and for $i = n_f - n_f^4 + 1, \dots, n_f$ ($\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda}$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i \in \mathcal{J}^f | (\Delta'_{j_i} | \Delta_{j_i} = \Delta^4) = \Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f-n_f^4}^{n_f} \left[A_1(1 - \overline{x})(1 - z_i^4) + \lambda A_2 \left(1 - \frac{1}{2}\overline{x}\right) z_i^4\right] q_{j_i} \, .$$

$B\overline{q}$ portion of conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \middle| \{z_j\}_{j \in \mathcal{J}^f}, x\right]$ can be written as

$$\mathbb{E}B\overline{q}' = B(1+g)\overline{q} \, ,$$

where g is a growth rate of product qualities in balanced growth path (BGP). Thus by plugging in the conjectured value function, the original value function can be written as

$$\sum_{i=1}^{n_f^1} A_1 q_{j_i} + \sum_{i=n_f^1+1}^{n_f^1+n_f^2} A_2 q_{j_i} + \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} A_3 q_{j_i} + \sum_{i=n_f-n_f^4+1}^{n_f} A_4 q_{j_i} + B\overline{q} =$$

$$
\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}}
\left\{
\begin{array}{l}
\sum_{i=1}^{n_f}\left[\pi q_{j_i}-\hat{\chi} z_i^{\hat{\psi}} q_{j_i}\right]-\bar{q}\widetilde{\chi} x^{\tilde{\psi}} \\[2ex]
+\widetilde{\beta}\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1)+\lambda A_2(1-\overline{x})z_i^1\right]q_{j_i} \\[2ex]
+\widetilde{\beta}\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2)+\lambda A_2 z_i^2\right]q_{j_i} \\[2ex]
+\widetilde{\beta}\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3)+\lambda A_2 z_i^3\right]q_{j_i} \\[2ex]
+\widetilde{\beta}\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4)+\lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z_i^4\right]q_{j_i} \\[2ex]
+\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)\right. \\[2ex]
\qquad\qquad \left. +A_3\eta\mu(\Delta^1)+(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q} \\[2ex]
+\widetilde{\beta}B(1+g)\overline{q}
\end{array}
\right\}
$$

Optimal innovation intensities from FONCs are

$$
\frac{\partial}{\partial z_i^1}: -\widehat{\psi}\widehat{\chi}(z_i^1)^{\hat{\psi}-1}q_{j_i}+\widetilde{\beta}(1-\overline{x})\left[\lambda A_2-A_1\right]q_{j_i}=0
$$

$$
\Rightarrow z^1=\left[\frac{\widetilde{\beta}(1-\overline{x})\left[\lambda A_2-A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}
$$

$$
\frac{\partial}{\partial z_i^2}: -\widehat{\psi}\widehat{\chi}(z_i^2)^{\hat{\psi}-1}q_{j_i}+\widetilde{\beta}\left[\lambda A_2-(1-\overline{x})A_1\right]q_{j_i}=0
$$

$$
\Rightarrow z^2=\left[\frac{\widetilde{\beta}\left[\lambda A_2-(1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}
$$

$$
\frac{\partial}{\partial z_i^3}: -\widehat{\psi}\widehat{\chi}(z_i^3)^{\hat{\psi}-1}q_{j_i}+\widetilde{\beta}\left[\lambda A_2-\left(1-\frac{1}{2}\overline{x}\right)A_1\right]q_{j_i}=0
$$

$$
\Rightarrow z^3=\left[\frac{\widetilde{\beta}\left[\lambda A_2-\left(1-\frac{1}{2}\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}
$$

$$
\begin{aligned}
\frac{\partial}{\partial z_i^4} :& -\widehat{\psi}\widehat{\chi}(z_i^4)^{\widehat{\psi}-1}q_{j_i} + \widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right]q_{j_i} = 0 \\
&\Rightarrow z^4 = \left[\frac{\widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x} :& -\widetilde{\psi}\widetilde{\chi}\overline{q}x^{\widetilde{\psi}-1} \\
&+ \widetilde{\beta}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\overline{q} \\
&= 0 \\
&\Rightarrow x = \left[\frac{\widetilde{\beta}\left[\frac{(1-z^3)A_1\mu(\Delta^3)}{2} + \left(1-\frac{z^4}{2}\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]}{\widetilde{\psi}\widetilde{\chi}}\right]^{\frac{1}{\widetilde{\psi}-1}}
\end{aligned}
$$

By plugging in optimal innovation intensities and equating the LHS to the RHS, we get the five coefficients of the conjectured value function of the form

$$
\begin{aligned}
A_1 &= \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big] \\
A_2 &= \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big] \\
A_3 &= \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right] \\
A_4 &= \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right] \\
B &= \frac{1}{1-\widetilde{\beta}(1+g)}\Bigg[\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1)\right. \\
&\qquad\qquad \left. + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right] - \widetilde{\chi}(x)^{\widetilde{\psi}}\Bigg] \\
&= \frac{1}{1-\widetilde{\beta}(1+g)}\left(\widetilde{\psi}\widetilde{\chi}\right)^{-\frac{1}{\widetilde{\psi}-1}}\left(1-\frac{1}{\widetilde{\psi}}\right)\Bigg[\widetilde{\beta}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) \\
&\qquad\qquad + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\Bigg]^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}.
\end{aligned}
$$

■

B.1.3.2 Proof of Corollary 3

Proof. Define $\widetilde{z^{\ell}} = \frac{\widehat{\psi}\widehat{\chi}}{\widetilde{\beta}} \left(z^{\ell}\right)^{(\widehat{\psi}-1)}$. Then $z^{\ell} > z^{\ell'} \Leftrightarrow \widetilde{z^{\ell}} > \widetilde{z^{\ell'}}$ for $\ell, \ell' \in [1,4] \cap \mathbb{Z}$ with $\widehat{\psi} > 1$. Since $\widetilde{z^2} - \widetilde{z^3} = \frac{1}{2}\overline{x}A_1 > 0$, $\widetilde{z^2} - \widetilde{z^1} = \overline{x}\lambda A_2 > 0$, $\widetilde{z^2} - \widetilde{z^4} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, and $\widetilde{z^4} - \widetilde{z^1} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, we have $z^2 > z^3$, $z^2 > z^1$, $z^2 > z^4$, and $z^4 > z^1$. Now, if we know the sign for $\widetilde{z^3} - \widetilde{z^4} = \frac{1}{2}\overline{x}\left[\lambda A_2 - A_1\right]$ then we know the entire relationships among $\{z^{\ell}\}_{\ell=1}^{4}$. But in an equilibrium, $\widetilde{z^1} = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ should hold, which implies $\lambda A_2 - A_1 > 0$. Thus $\widetilde{z^3} > \widetilde{z^4} \Leftrightarrow z^3 > z^4$. Therefore, $z^2 > z^3 > z^4 > z^1$. ■

B.1.3.3 Proof of Corollary 4

Proof. The partial derivatives of $\{z^{\ell}\}_{\ell=1}^{4}$ w.r.t. $\overline{x}$, holding A_1 and A_2 fixed are

$$\begin{aligned}
\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\left(z^1\right)^{2-\widehat{\psi}}\left[\,\lambda A_2 - A_1\,\right] < 0 \\
\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\left(z^2\right)^{2-\widehat{\psi}} A_1 > 0 \\
\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\left(z^3\right)^{2-\widehat{\psi}} \frac{1}{2}A_1 > 0 \\
\left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\left(z^4\right)^{2-\widehat{\psi}}\left[\,\frac{1}{2}\lambda A_2 - A_1\,\right] \gtrless 0\,.
\end{aligned}$$

Since we know $\lambda A_2 - A_1 > 0$, $\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2}$ should be negative. Also, since $z^2 > z^3$, $\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2}$. Since $z^3 > z^4$ and $A_1 > A_1 - \frac{1}{2}\lambda A_2$, $\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2}$ but the sign for $\frac{1}{2}\lambda A_2 - A_1$ is ambiguous. ■

B.1.4 Potential Startups

By plugging in the value function defined in the previous section, the expected term becomes

$$
\begin{aligned}
\mathbb{E}V(\{(q_j', \Delta_j')\}) &= \mathbb{E}_{q_j}\Bigg[\frac{1}{2}x_e(1-z^3)\Big[A_1 q_j + B\overline{q}'\Big]\mu(\Delta^3) + x_e\left(1-\frac{1}{2}z^4\right)\Big[A_2\lambda g_j + B\overline{q}'\Big]\mu(\Delta^4) \\
&\qquad + x_e\Big[A_3\eta q_j + B\overline{q}'\Big]\mu(\Delta^1) + x_e(1-z^2)\Big[A_4\frac{\eta}{\lambda}q_j + B\overline{q}'\Big]\mu(\Delta^2)\Bigg] \\
&= x_e\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) \\
&\qquad + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\overline{q} + x_e\Bigg[\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) \\
&\qquad + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\Bigg]B(1+g)\overline{q}\,.
\end{aligned}
$$

Thus from FOSC, optimal external innovation intensity for potential startups x_e is

$$
\begin{aligned}
x_e = \Bigg[\Bigg[&\left(\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right) \\
&+ \left(\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\right)B(1+g)\Bigg] \\
&\times \frac{\widetilde{\beta}}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\Bigg]^{\frac{1}{\widetilde{\psi}_e - 1}}\,.
\end{aligned}
$$

B.1.5 Growth rate

B.1.5.1 Proof of Proposition 7

Proof. In this model economy, output growth rate is equal to product quality growth rate. Pick any q_j. Then it's technology gap is equal to $\Delta_j = \Delta^\ell$ with the probability $\mu(\Delta^\ell)$ and the probability of Δ'_j becoming a certain technology gap depends on this.

$$
\begin{aligned}
\text{If } \Delta_j = \Delta^1, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^1) \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad (1-\overline{x})z^1 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad \overline{x} \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad 0 \\
\text{If } \Delta_j = \Delta^2, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^2) \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad z^2 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad \overline{x}(1-z^2) \\
\text{If } \Delta_j = \Delta^3, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad 1-z^3 \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad z^3 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad 0
\end{aligned}
$$

$$
\begin{aligned}
\text{If } \Delta_j = \Delta^4, \quad q_j' &= \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^4) \\
q_j' &= \Delta^2 q_j \quad \text{w/ prob.} \quad z^4 + \overline{x}(1-z^4) \\
q_j' &= \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
q_j' &= \Delta^4 q_j \quad \text{w/ prob.} \quad 0
\end{aligned}
$$

Thus

$$
\begin{aligned}
\mathbb{E}\big[q_j' \mid q_j\big] = &\Bigg[\Big[(1-\overline{x})(1-z^1) + \lambda(1-\overline{x})z^1 + \eta\overline{x}\Big]\mu(\Delta^1) \\
&+ \Big[(1-\overline{x})(1-z^2) + \lambda z^2 + \frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \lambda z^3\Big]\mu(\Delta^3) \\
&+ \Big[(1-\overline{x})(1-z^4) + \lambda(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg] q_j \ ,
\end{aligned}
$$

and

$$
\begin{aligned}
g = &\Bigg[\Big[(1-\overline{x})(1-z^1) + \lambda(1-\overline{x})z^1 + \eta\overline{x}\Big]\mu(\Delta^1) \\
&+ \Big[(1-\overline{x})(1-z^2) + \lambda z^2 + \frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \lambda z^3\Big]\mu(\Delta^3) \\
&+ \Big[(1-\overline{x})(1-z^4) + \lambda(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg] - 1 \ .
\end{aligned}
$$

The decomposition follows from the straightforward application of the definition of $\overline{x}$ and product quality evolution. ■

B.1.6 Technology Gap Portfolio Composition Distribution Transition

Let's define technology gap portfolio composition with $n_f - k$ number of $\Delta = \Delta^1$, k number of $\Delta = \Delta^2$, zero number of $\Delta = \Delta^3$ and zero number of $\Delta = \Delta^4$ as $\widetilde{\mathcal{N}}(n_f, k) \equiv$

$(n_f, n_f - k, k, 0, 0)$, for $k \in [0, n_f] \cap \mathbb{Z}$, $n_f > 0$. Then without considering external innovation, probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \mid n_f, k\Big) =$$

$$\begin{cases} \sum_{\widetilde{k}^1=\max\{0,\,\widetilde{k}-k\}}^{\min\{n_f-k,\,\widetilde{k}\}} \begin{pmatrix} n_f - k \\ \widetilde{k}^1 \end{pmatrix} \begin{pmatrix} k \\ \widetilde{k} - \widetilde{k}^1 \end{pmatrix} & \\ \qquad \times \begin{bmatrix} (1-\overline{x})^{n_f - (\widetilde{k}-\widetilde{k}^1)} \, (1-z^1)^{n_f - k - \widetilde{k}^1} \, (z^1)^{\widetilde{k}^1} \\ \qquad \times (1-z^2)^{k-(\widetilde{k}-\widetilde{k}^1)} \, (z^2)^{\widetilde{k}-\widetilde{k}^1} \end{bmatrix} & \text{for } n_f \geqslant 1, \text{ and } 0 \leqslant \widetilde{k}, k \leqslant n_f \\ 0 & \text{otherwise} \end{cases}$$

where

$$\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}$$

is a combination of selecting k elements from n elements without repetition, where the order of selection does not matter. Range for $\widetilde{k}^1$ is of the form described as above due to the fact that

i. For $0 \leqslant \widetilde{k} \leqslant \min\{n_f - k, k\}$ case, the two combinations are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describes all the possible cases.

ii. For $n_f - k \geqslant k$ case, $\tilde{k} > k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\widetilde{k} - k \leqslant \widetilde{k}^1 \leqslant \widetilde{k}$.

iii. For $k \geqslant n_f - k$ case, $\widetilde{k} > n_f - k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\max\{0,\ \widetilde{k} - k\} \leqslant \widetilde{k}^1 \leqslant n_f - k$.

By using $\widetilde{\mathbb{P}}(n_f, \widetilde{k} \mid n_f, k)$, probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f,\ k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ for any $h \geqslant 0$ without considering external innovation can be defined as follows. Take out h^1 number of product lines with $\Delta = \Delta^1$, and $h - h^1$ number of product lines with $\Delta = \Delta^2$ from $\widetilde{\mathcal{N}}(n_f,\ k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f - h,\ k - (h - h^1))$ becoming $\widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ by using $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f - h,\ k - (h - h^1)\Big)$ for all feasible h^1:

$$\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f,\ k\Big) =$$

$$\begin{cases} \sum_{h^1=\max\{0,\,h-k\}}^{\min\{h,\,n_f-k\}} \left[\begin{pmatrix} n_f - k \\ h^1 \end{pmatrix} \begin{pmatrix} k \\ h - h^1 \end{pmatrix} \overline{x}^h (1 - z^2)^{h-h^1} \right. & \\ \qquad \left. \times \widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f - h,\ k - (h - h^1)\Big) \right] & \text{for } 0 \leqslant h < n_f, \\ & n_f \geqslant 1, \\ & 0 \leqslant \widetilde{k} \leqslant n_f - h, \\ & \text{and } 0 \leqslant k \leqslant n_f \\ \overline{x}^{n_f}\, (1 - z^2)^k & \text{for } h = n_f \geqslant 1, \\ & \widetilde{k} = 0, \\ & \text{and } 0 \leqslant k \leqslant n_f \\ 0 & \text{otherwise.} \end{cases}$$

Range for h^1 is defined as above, due to the fact that for any h^1, $0 \leqslant h - h^1 \leqslant k$ and $0 \leqslant h^1 \leqslant n_f - k$ should be satisfied.

By using $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f,\ k\Big)$, other possible technology gap portfolio composition transition probabilities can be described conveniently.

1-i. Probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \;\Big|\, n_f, n_f - k, k, 0, 0\Big) =$$

$$\begin{aligned} &\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f, k\Big)\,(1 - x\overline{x}_{takeover}) \\ &+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\,\mu(\Delta^3)\,\frac{1}{2}\,x\,(1 - z^3) \\ &+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f, k\Big)\,\mu(\Delta^4)\,x\,\left(1 - \frac{1}{2}z^4\right) . \end{aligned}$$

The first term is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ directly via firm's existing technology gap portfolio composition change, while external innovation fails. The second term is the probability of $\mathcal{N}$ becoming $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, then successful external innovation adds one product line with $\Delta' = \Delta^1$. Since next period technology gap of product line j from successful external innovation is equal to $\Delta'_j = \frac{q'_j}{q_j} = \frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}} = \frac{\eta}{\Delta_j}$, firm needs to take over product line with technology gap $\Delta = \Delta^3 = 1 + \eta$ to have a product line with technology gap Δ^1 next period. The third term is the probability of $\mathcal{N}$ becoming $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k} - 1)$, then successful external innovation adds one product line with $\Delta' = \Delta^2$ by taking over a product line with technology gap $\Delta = \Delta^4$. For $h = -1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\,\cdot \mid \cdot\,)$. Thus this probability is well defined for any $h \geqslant -1$.

1-ii. Probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - k, k, 0, 0\Big) = \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\,\mu(\Delta^1)\,x\,.$$

Firm's existing technology gap changes from $\widetilde{\mathcal{N}}(n_f,k)$ to $\widetilde{\mathcal{N}}(n_f-h-1,\widetilde{k})$, then successful external innovation adds $\Delta' = \Delta^3 = 1+\eta$.

1-iii. Probability of $\mathcal{N} = (n_f, n_f-k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f-h, n_f-h-1-\widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f-h, n_f-h-1-\widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f-k, k, 0, 0\Big) =$$

$$\widetilde{\mathbb{P}}\Big(n_f-h-1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^2)\, x\, (1-z^2)\,.$$

2-i. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f-1-k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f-h, n_f-h-\widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f-h, n_f-h-\widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f-1-k, k, 1, 0\Big) =$$

$$\begin{aligned}
&\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f-h, \widetilde{k} \,\Big|\, n_f-1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-1, \widetilde{k} \,\Big|\, n_f-1, k\Big)\, \left(1-\tfrac{1}{2}\overline{x}\right)\, (1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-1, \widetilde{k}-1 \,\Big|\, n_f-1, k\Big)\, z^3
\end{array}\right] \times (1 - x\,\overline{x}_{takeover}) \\
+&\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f-h-1, \widetilde{k} \,\Big|\, n_f-1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2, \widetilde{k} \,\Big|\, n_f-1, k\Big)\, \left(1-\tfrac{1}{2}\overline{x}\right)\, (1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2, \widetilde{k}-1 \,\Big|\, n_f-1, k\Big)\, z^3
\end{array}\right] \times \mu(\Delta^3)\, \frac{1}{2}\, x\, (1-z^3) \\
+&\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f-h-1, \widetilde{k}-1 \,\Big|\, n_f-1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2, \widetilde{k}-1 \,\Big|\, n_f-1, k\Big)\, \left(1-\tfrac{1}{2}\overline{x}\right)\, (1-z^3) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2, \widetilde{k}-2 \,\Big|\, n_f-1, k\Big)\, z^3
\end{array}\right] \times \mu(\Delta^4)\, x\, \left(1-\frac{1}{2}z^4\right)\,.
\end{aligned}$$

Three probabilities in the brackets are the probabilities when the existing product line with $\Delta = \Delta^3$ is taken over by other firm, internal innovation fails but firm keeps it, and internal innovation succeeds and firm keeps it. The first bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when external innovation fails, the second bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when successful external innovation adds a product line with technology gap $\Delta' = \Delta^1$, and the third bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when successful external innovation adds a product line with $\Delta' = \Delta^2$. Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,1,0,0,0 \,\Big|\, 1,0,0,1,0\Big) = \left(1 - \frac{1}{2}\overline{x}\right)(1-z^3)(1 - x\,\overline{x}_{takeover}) \\ + \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^3)\,\frac{1}{2}\,x\,(1-z^3)\,,$$

and

$$\mathbb{P}\Big(1,0,1,0,0 \,\Big|\, 1,0,0,1,0\Big) = z^3\,(1 - x\,\overline{x}_{takeover}) + \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^4)\,x\,\left(1 - \frac{1}{2}z^4\right)\,.$$

2-ii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) = \\ \left[\begin{array}{c} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\tfrac{1}{2}\,\overline{x}\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\left(1 - \tfrac{1}{2}\overline{x}\right)\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\,z^3 \end{array}\right] \times \mu(\Delta^1)\,x$$

$\mathcal{N}$ becomes $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$ through internal innovations, then successful external innovation adds a product line with $\Delta' = \Delta^3$ by taking over a product line with $\Delta = \Delta^1$. Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,1,0 \,\Big|\, 1,0,0,1,0\Big) = \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^1)\,x\,.$$

2-iii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) =$$
$$\begin{bmatrix} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \frac{1}{2}\,\overline{x}\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \left(1 - \frac{1}{2}\overline{x}\right)\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, z^3 \end{bmatrix} \times \mu(\Delta^2)\,x\,(1-z^2)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,0,1 \,\Big|\, 1,0,0,1,0\Big) = \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^2)\,x\,(1-z^2)\,.$$

3-i. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f - 1 - k, k, 0, 1\Big) =$$

$$
\begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \overline{x}\ \left(1 - \tfrac{1}{2}z^4\right) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, (1 - \overline{x})\, (1 - z^4) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\ \left(1 - \tfrac{1}{2}\overline{x}\right)\ z^4
\end{bmatrix} \times (1 - x\, \overline{x}_{takeover})
$$

$$
+ \begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \overline{x}\ \left(1 - \tfrac{1}{2}z^4\right) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, (1 - \overline{x})\, (1 - z^4) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\ \left(1 - \tfrac{1}{2}\overline{x}\right)\ z^4
\end{bmatrix} \times \mu(\Delta^3)\, \frac{1}{2}\, x\, (1 - z^3)
$$

$$
+ \begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, \overline{x}\ \left(1 - \tfrac{1}{2}z^4\right) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, (1 - \overline{x})\, (1 - z^4) \\
+ \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 2 \,\big|\, n_f - 1, k\Big)\ \left(1 - \tfrac{1}{2}\overline{x}\right)\ z^4
\end{bmatrix} \times \mu(\Delta^4)\, x\ \left(1 - \frac{1}{2}z^4\right) .
$$

Similarly, for $n_f = 1$,

$$
\begin{aligned}
\mathbb{P}\Big(1, 1, 0, 0, 0 \,\big|\, 1, 0, 0, 0, 1\Big) =& (1 - \overline{x})(1 - z^4)(1 - x\, \overline{x}_{takeover}) \\
& + \overline{x}\, \left(1 - \frac{1}{2}z^4\right) \mu(\Delta^3) \frac{1}{2}\, x\, (1 - z^3)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{P}\Big(1, 0, 1, 0, 0 \,\big|\, 1, 0, 0, 0, 1\Big) =& \left(1 - \frac{1}{2}\overline{x}\right)\ z^4 (1 - x\, \overline{x}_{takeover}) \\
& + \overline{x}\ \left(1 - \frac{1}{2}z^4\right)\ \mu(\Delta^4)\, x\ \left(1 - \frac{1}{2}z^4\right) .
\end{aligned}
$$

3-ii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ becoming $\mathcal{N}' = (n_f - h, n_f -$

$h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - 1 - k, k, 0, 1\Big) = \left[\begin{array}{c} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \overline{x}\, \left(1 - \frac{1}{2}z^4\right) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, (1 - \overline{x})\, (1 - z^4) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, \left(1 - \frac{1}{2}\overline{x}\right)\, z^4 \end{array}\right] \times \mu(\Delta^1)\, x\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1, 0, 0, 1, 0 \,\Big|\, 1, 0, 0, 0, 1\Big) = \overline{x}\left(1 - \frac{1}{2}z^4\right)\, \mu(\Delta^1)\, x\,.$$

3-iii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - 1 - k, k, 0, 1\Big) = \left[\begin{array}{c} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \overline{x}\, \left(1 - \frac{1}{2}z^4\right) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, (1 - \overline{x})\, (1 - z^4) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, \left(1 - \frac{1}{2}\overline{x}\right)\, z^4 \end{array}\right] \times \mu(\Delta^2)\, x\, (1 - z^2)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1, 0, 0, 0, 1 \,\Big|\, 1, 0, 0, 0, 1\Big) = \overline{x}\left(1 - \frac{1}{2}z^4\right)\, \mu(\Delta^2)\, x\, (1 - z^2)\,.$$

Now that the probabilities of any particular technology gap portfolio composition becoming other particular technology gap portfolio composition is computed, we can specify the inflows and outflows of a particular technology gap portfolio. Let $\mathcal{F}$ be a total mass of firms in the economy and let $\mu(\mathcal{N})$ be a share of firms with technology gap portfolio $\mathcal{N}$.

i) For $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ with $n_f \geqslant 2$, any firms with technology gap portfolio next period not equal to $\mathcal{N}$ accounts for outflows. Thus

$$\begin{aligned}\text{outflow}\big(n_f, n_f - k, k, 0, 0\big) =& \Big[\, 1 - \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n_f, n_f - k, k, 0, 0\big) \,\Big] \\ & \times \mathcal{F}\, \mu\big(n_f, n_f - k, k, 0, 0\big)\ .\end{aligned}$$

Any firms with total number of product line $n \geqslant n_f - 1$ can have technology gap portfolio composition equal to $\mathcal{N}$ through combinations of internal and external innovations. Thus for the maximum number of product lines $\bar{n}_f$,

$$\begin{aligned}&\text{inflow}\big(n_f, n_f - k, k, 0, 0\big) = \\ &\mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\ &\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\ &\qquad\quad \times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\ &\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\ &\qquad\quad \times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \,\Bigg] \\ &\qquad - \mathcal{F}\, \mu\big(n_f, n_f - k, k, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n_f, n_f - k, k, 0, 0\big)\ .\end{aligned}$$

ii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ with $n_f \geqslant 2$

$$
\begin{aligned}
&\text{outflow}\big(n_f, n_f - 1 - k, k, 1, 0\big) \\
&\quad = \Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big) \,\Big] \\
&\quad\quad \times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\ .
\end{aligned}
$$

$$
\begin{aligned}
&\text{inflow}\big(n_f, n_f - 1 - k, k, 1, 0\big) = \\
&\mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n - \widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n - \widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\quad + \mu\big(n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\quad\quad \times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\quad + \mu\big(n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\quad\quad \times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \,\Bigg] \\
&\quad - \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big)\ .
\end{aligned}
$$

iii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ with $n_f \geqslant 2$

$$
\begin{aligned}
&\text{outflow}\big(n_f, n_f - 1 - k, k, 0, 1\big) \\
&\quad = \Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n_f, n_f - 1 - k, k, 0, 1\big) \,\Big] \\
&\quad\quad \times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 0, 1\big)\ .
\end{aligned}
$$

$$
\begin{aligned}
&\text{inflow}\big(n_f, n_f-1-k, k, 0, 1\big) = \\
&\mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad\quad \times \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\qquad\quad \times \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
&\qquad - \mathcal{F}\, \mu\big(n_f, n_f-1-k, k, 0, 1\big)\, \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n_f, n_f-1-k, k, 0, 1\big)\,.
\end{aligned}
$$

iv) $\mathcal{N} = (1, 1, 0, 0, 0)$

$$
\text{outflow}\big(1, 1, 0, 0, 0\big) = \Big[\, 1 - \mathbb{P}\big(1, 1, 0, 0, 0 \,\big|\, 1, 1, 0, 0, 0\big) \Big]\, \mathcal{F}\, \mu\big(1, 1, 0, 0, 0\big)\,.
$$

$$
\begin{aligned}
\text{inflow}(1, 1, 0, 0, 0) =\; & \mathcal{E}\, x_e\, \mu(\Delta^3) \frac{1}{2}(1-z^3) \\
& + \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1, 1, 0, 0, 0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad\quad \times \mathbb{P}\big(1, 1, 0, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\qquad\quad \times \mathbb{P}\big(1, 1, 0, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
&\qquad - \mathcal{F}\, \mu\big(1, 1, 0, 0, 0\big)\, \mathbb{P}\big(1, 1, 0, 0, 0 \,\big|\, 1, 1, 0, 0, 0\big)\,.
\end{aligned}
$$

v) $\mathcal{N} = (1,0,1,0,0)$

$$\text{outflow}\big(1,0,1,0,0\big) = \Big[\, 1 - \mathbb{P}\big(1,0,1,0,0 \,\big|\, 1,0,1,0,0\big) \,\Big]\, \mathcal{F}\, \mu\big(1,0,1,0,0\big)\,.$$

$$\begin{aligned}
\text{inflow}(1,0,1,0,0) =\ & \mathcal{E}\, x_e\, \mu(\Delta^4)\, \left(1 - \frac{1}{2} z^4\right) \\
& + \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
& \quad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
& \qquad \times \mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
& \quad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
& \qquad \times \mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \,\Bigg] \\
& \quad - \mathcal{F}\, \mu\big(1,0,1,0,0\big)\, \mathbb{P}\big(1,0,1,0,0 \,\big|\, 1,0,1,0,0\big)\,.
\end{aligned}$$

vi) $\mathcal{N} = (1,0,0,1,0)$

$$\text{outflow}\big(1,0,0,1,0\big) = \Big[\, 1 - \mathbb{P}\big(1,0,0,1,0 \,\big|\, 1,0,0,1,0\big) \,\Big]\, \mathcal{F}\, \mu\big(1,0,0,1,0\big)\,.$$

$$\begin{aligned}
\text{inflow}(1,0,0,1,0) =\ & \mathcal{E}\, x_e\, \mu(\Delta^1) \\
& + \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
& \quad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)
\end{aligned}$$

$$\times\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\, \mathcal{F}\, \mu\big(1,0,0,1,0\big)\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, 1,0,0,1,0\big)\,.$$

vii) $\mathcal{N} = (1,0,0,0,1)$

$$\text{outflow}\big(1,0,0,0,1\big) = \Big[\, 1 - \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big)\, \Big]\, \mathcal{F}\, \mu\big(1,0,0,0,1\big)\,.$$

$$\text{inflow}(1,0,0,0,1) = \,\mathcal{E}\, x_e\, \mu(\Delta^2)\, (1-z^2)$$

$$+\, \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$\times\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\, \mathcal{F}\, \mu\big(1,0,0,0,1\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big)\,.$$

B.1.6.1 Number of Points in Technology Gap Portfolio Composition Distribution

Let's denote $N(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines, $\left(n_f, n_f^1, n_f^2, n_f^3, n_f^4\right)$, where $n_f = \sum_{\ell=1}^{4} n_f^\ell$, $n_f^3, n_f^4 \in \{0, 1\}$, and $n_f^3 = n_f^4 = 1$ is not possible.

Let's denote $\widetilde{N}(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines with no product line that has Δ^3 or Δ^4, $\left(n_f, n_f^1, n_f^2, 0, 0\right)$. Then

$$N(n_f) = \widetilde{N}(n_f) + 2\widetilde{N}(n_f - 1) \ ,$$

as

$$\left(n_f, n_f^1, n_f^2, 1, 0\right) = \left(n_f - 1, n_f^1, n_f^2, 0, 0\right) + (1, 0, 0, 1, 0) \ ,$$

and

$$\left(n_f, n_f^1, n_f^2, 0, 1\right) = \left(n_f - 1, n_f^1, n_f^2, 0, 0\right) + (1, 0, 0, 0, 1) \ .$$

Since $\widetilde{N}(n_f) = n_f + 1$, $N(n_f) = 3n_f + 1$. Thus for a maximum number of product line individual firm can have, $\overline{n}_f$, total number of points in technology gap portfolio composition distribution is

$$N_{\text{total}} = \sum_{n_f=1}^{\overline{n}_f} \left(3n_f + 1\right) = \frac{\left(3\overline{n}_f + 5\right) \overline{n}_f}{2} \ .$$

B.1.7 Total Mass of Product Lines Owned by the Domestic Firms

B.1.7.1 Proof of Lemma 4

Proof. Since the optimal probability of external innovation for both domestic firms and foreign exporters are the same, the aggregate creative destruction arrival rate can be decomposed into:

$$\overline{x} = \underbrace{\mathcal{F}_d x + \mathcal{E}_d x_e}_{\overline{x}_d} + \underbrace{\mathcal{F}_{fx} x + \mathcal{E}_{fx} x_e}_{\overline{x}_{fx}} \ .$$

In any stationary equilibrium, the share of domestic incumbent firms should be equal to the share of potential domestic startups. Thus,

$$\frac{\mathcal{F}_d}{\mathcal{F}_d + \mathcal{F}_{fx}} = \frac{\mathcal{E}_d}{\mathcal{E}_d + \mathcal{E}_{fx}} \ .$$

Since all the incumbent firms are homogeneous in terms of their optimal R&D decisions, and external innovation is undirected, the share of domestic incumbent firms should be equal to s_d in an equilibrium. Then by rearranging $\overline{x}$ and multiplying it by s_d, we get

$$\begin{aligned}
s_d \overline{x} &= s_d \big(\mathcal{F}_d x + \mathcal{F}_{fx} x + \mathcal{E}_{fx} x_e + \mathcal{E}_d x_e\big) \\
&= s_d \big(\mathcal{F}_d + \mathcal{F}_{fx}\big) x + s_d \big(\mathcal{E}_d + \mathcal{E}_{fx}\big) x_e \\
&= \frac{\mathcal{F}_d}{\mathcal{F}_d + \mathcal{F}_{fx}} \big(\mathcal{F}_d + \mathcal{F}_{fx}\big) x + \frac{\mathcal{E}_d}{\mathcal{E}_d + \mathcal{E}_{fx}} \big(\mathcal{E}_d + \mathcal{E}_{fx}\big) x_e \\
&= \mathcal{F}_d x + \mathcal{E}_d x_e \\
&= \overline{x}_d \ ,
\end{aligned}$$

and $(1 - s_d)\overline{x} = \overline{x}_{fx}$. Therefore,

$$s_d = \frac{\overline{x}_d}{\overline{x}} \, .$$

■

B.2 Simple Three-Period Heterogeneous Innovation Model

To understand firms' incentives for internal and external innovation, and to derive empirically testable model predictions, we will consider a three-period economy with two product markets and three firms. In period 0, the economy starts with two product markets, market 1 and 2, with initial market-specific technologies $q_{1,0}$, and $q_{2,0}$, and two firms, firm A and B. Product market 1 is given to firm A and is ready for production. Firm A is also given an initial probability of internally innovating product 1, $z_{1,0}$. Firm B, on the other hand, is given only a probability of externally innovating product 2 $x_{2,0}$. Thus, firm B can start operating and producing in period 1 but not in period 0. If external innovation fails, then firm B still keeps market 2 but produces with initial product quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are equal to:

$$q_{1,1} = \begin{cases} \lambda \, q_{1,0} & \text{with probability } z_{1,0} \\ q_{1,0} & \text{with probability } 1 - z_{1,0} \, , \end{cases}$$

and

$$q_{2,1} = \begin{cases} \eta\, q_{2,0} & \text{with probability } x_{2,0} \\ q_{2,0} & \text{with probability } 1 - x_{2,0} \,. \end{cases}$$

where $\lambda^2 > \eta > \lambda > 1$ are innovation step sizes.

In period 1, the main period of interest, there is an outside firm (potentially from a foreign country) that does external innovation hoping to take over the two product markets in period 2. The outside firm succeeds in doing external innovation with probability x_1^e in each product market. Also, there is a news shock about period 2 profit (potentially including an increase in foreign demand) announced in period 1. Afterwards, the two incumbent firms produce using the given technologies, invest in internal innovation to improve the quality of their own products, and invest in external innovation to take over the other firm's product market. At the beginning of period 2, all innovation outcomes are realized. Then, technological competition in each product market takes place, and only the firm with the highest technology in each product market produces. The economy ends after period 2.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R_{j,1}^{in}$ on internal innovation, $j \in \{1, 2\}$ (e.g., for $i = A$, $j = 1$), implying a success probability $z_{j,1}$ using the R&D production function

$$z_{j,1} = \left(\frac{R_{j,1}^{in}}{\widehat{\chi} q_{j,1}} \right)^{\frac{1}{2}} .$$

Successful internal innovation increases the next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes

$$q_{j,2}^{i} = \begin{cases} \lambda\, q_{j,1} & \text{with probability } z_{j,1} \\ q_{j,1} & \text{with probability } 1 - z_{j,1}\ . \end{cases}$$

Similarly, firm i invests $R^{ex}_{-j,1}$ to learn the period 0 technology used by firm $-i \neq i$, implying a success probability of external innovation $x_{-j,1}$ using the R&D production function

$$x_{-j,1} = \left(\frac{R^{ex}_{-j,1}}{\widetilde{\chi} q_{-j,0}} \right)^{\frac{1}{2}} \ ,$$

where $-j$ is owned by $-i$. Successful external innovation increases product quality relative to the past-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i becomes

$$q_{-j,2}^{i} = \begin{cases} \eta\, q_{-j,0} & \text{with probability } x_{-j,1} \\ \varnothing & \text{with probability } 1 - x_{-j,1}\ , \end{cases}$$

where $\varnothing$ means firm i failed to acquire a production technology for product $-j$.

B.2.1 Optimal Innovation Decisions and Theoretical Predictions

Assume that in each product market j in each period t, firms receive instantaneous profit of $\pi_{j,t}\, q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the outside firm can perform external innovation on the same product. To keep the model simple, further assume that the outside firm can do external innovation only if an incumbent fails to do external innovation, following Garcia-Macia et al. (2019). Then the profit maximization problem of firm i that has product market j with quality $q_{j,1}$ in period 1 can be written as

$$V(q_{j,1}) = \max_{\{z_{j,1},\, x_{-j,1}\}} \left\{ \begin{array}{l} \pi q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} \\ \\ +(1-x_{j,1})(1-x_1^e)\Big[(1-z_{j,1})\pi_{j,2} q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\Big] \\ \\ +\big[x_{j,1} + (1-x_{j,1})\, x_1^e\big]\Big[z_{j,1}\pi_{j,2}\, \lambda\, q_{j,1}\, \mathcal{I}_{\{\lambda q_{j,1} > \eta q_{j,0}\}} \\ \qquad\qquad\qquad\qquad\quad + \frac{1}{2}(1-z_{j,1})\pi_{j,2} q_{j,1}\, \mathcal{I}_{\{q_{j,1} = \eta q_{j,0}\}}\Big] \\ \\ +x_{-j,1}\Big[(1-z_{-j,1})\, \pi_{-j,2}\, \eta\, q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} > q_{-j,1}\}} \\ \qquad\quad + z_{-j,1}\, \pi_{-j,2}\, \eta\, q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} > \lambda q_{-j,1}\}} \\ \qquad\quad + \frac{1}{2}(1-z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} = q_{-j,1}\}} \\ \qquad\quad + \frac{1}{2} z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} = \lambda q_{-j,1}\}}\Big] \end{array} \right\},$$

where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the two technologies among the three firms in period 2 in a given market. The first line shows the period 1 profit net of the total R&D cost. The second line represents the incumbent's period 2 expected profit from market j when the other incumbent and the outside firm fail to externally innovate the market j technology. The third and the fourth line represent the period 2 expected profit from market j when one of the two other firms succeeds in externally innovating the market j technology. The fifth to eighth lines represent the period 2 expected profit from market $-j$ when firm i succeeds in externally innovating the market $-j$ technology. The terms following $\frac{1}{2}$ are for the cases in which two firms can produce the same quality product, so that a coin-toss tiebreaker rule applies.

The interior solutions to this problem are

$$
z^*_{j,1} = \begin{cases} \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\,(\lambda-1)(1-x^*_{j,1})(1-x^e_1) & \text{, when } q_{j,1} = q_{j,0} \\[2ex] \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\left[\lambda-(1-x^*_{j,1})(1-x^e_1)\right] & \text{, when } q_{j,1} = \lambda\, q_{j,0} \\[2ex] \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\left[\lambda-\dfrac{1}{2}-\dfrac{1}{2}(1-x^*_{j,1})(1-x^e_1)\right] & \text{, when } q_{j,1} = \eta\, q_{j,0} \end{cases}
$$

and

$$
x^*_{-j,1} = \begin{cases} \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}} & \text{, when } q_{-j,1} = q_{-j,0} \\[2ex] \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,(1-z^*_{-j,1}) & \text{, when } q_{-j,1} = \lambda\, q_{-j,0} \\[2ex] \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,\dfrac{1}{2}(1-z^*_{-j,1}) & \text{, when } q_{-j,1} = \eta\, q_{-j,0}\,. \end{cases}
$$

The above results show that the firm's optimal innovation decisions depend on the (expected) future profit, the technology gap in both its own market and the other firm's market, and other firms' internal and external innovation decisions. From these interior solutions, I draw the following results.

Proposition B.2.1. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order internal innovation intensities as*

$$
z^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad z^*_{1,1}\big|_{q_{j,1}=\eta q_{j,0}} \quad > \quad z^*_{j,1}\big|_{q_{j,1}=q_{j,0}}\,.
$$

Furthermore,

$$\left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\eta q_{j,0}} \quad > \quad 0 \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=q_{j,0}} .$$

Proof: See Appendix B.2.2.1

The second part of proposition B.2.1 implies that firms with no local technology gap lower their internal innovation investment when they are faced with a higher probability of creative destruction in their own markets, as they cannot increase the probability of escaping competition by improving their products through internal innovation. On the other hand, if a firm has very high technological advantage, then the firm doesn't increase its internal innovation investment much in response to outsiders' investment in external innovation, because the probability of losing its own product market is small. In the intermediate case, firms increase their internal innovation investment more strongly in response to outsiders' external innovation, as they can lower the probability of losing their market by doing so.

Higher innovation in period 0 increases the probability of having a high local technology gap in period 1 and this helps firms to escape competition. To understand how past innovation intensity affects the firm's current internal innovation decision when the firm is faced with a higher probability of encountering a competitor, x^e_1, define the expected value of internal innovation intensity in period 1 as

$$\overline{z}^*_1 = z^*_{1,1}\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1-z_{1,0}) \;+\; z^*_{2,1}\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1-x_{2,0}) \;+\; z^*_{1,1}\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2} z_{1,0} \;+\; z^*_{2,1}\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2} x_{2,0} \,,$$

where $\frac{1}{2}$ comes from the fact that there are two products. Then, proposition B.2.1 gives us:

Corollary B.2.1 (Escape Competition Effect). *The impact of period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$ on expected internal innovation in period 1 satisfies:*

$$\frac{\partial \overline{z}_1^*}{\partial x_1^e \partial z_{1,0}} > 0 \,, \text{ and } \frac{\partial \overline{z}_1^*}{\partial x_1^e \partial x_{2,0}} > 0 \,.$$

Proof: See Appendix B.2.2.2

Corollary B.2.1 implies that intensive innovation in the previous period induces firms to increase the response of their internal innovation to higher product market competition. As the optimal decision rule shows, firms' external innovation decision also depends on past innovation decisions of other firms:

Proposition B.2.2. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order external innovation intensities as*

$$x_{j,1}^*\big|_{q_{j,1}=q_{j,0}} \quad > \quad x_{j,1}^*\big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad x_{j,1}^*\big|_{q_{j,1}=\eta q_{j,0}}$$

Furthermore,

$$\left.\frac{\partial x_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=q_{j,0}} = 0 \,, \quad \left.\frac{\partial x_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\lambda q_{j,0}} < 0 \,, \text{ and } \left.\frac{\partial x_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\eta q_{j,0}} < 0 \,.$$

Proof: See Appendix B.2.2.1

Proposition B.2.2 implies that firms do less external innovation if other firms have a higher technology advantage, as it becomes more difficult to take over their markets through external innovation. For product markets with a technological barrier (local technology gap > 1), firms also lower their external innovation if the outside firm does more external innovation, as incumbents in these markets will respond by doing more internal innovation with defensive motive (proposition B.2.1). To understand how the past innovation intensity of other firms affects a firm's current external innovation decision, define the expected value of external innovation intensity in period 1 as

$$\overline{x}_1^* = x_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1-z_{1,0}) \; + \; x_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1-x_{2,0}) \; + \; x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2} z_{1,0} \; + \; x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2} x_{2,0} \, .$$

Then, the first part of proposition B.2.2 implies the following:

Corollary B.2.2 (Technological Barrier Effect). *For a given technology $q_{j,1}$ and period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$, we have*

$$\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} < 0 \text{ , and } \frac{\partial \overline{x}_1^*}{\partial x_{2,0}} < 0 \, .$$

Proof: See Appendix B.2.2.3

Corollary B.2.2 implies that higher technology levels in other markets, which are due to previous innovation, serve as an effective technological barrier that makes it difficult for outside firms to take over another firm's product market. This reduces firms' incentive for external innovation. Because innovation is forward looking, changes in future profit π' are an important factor affecting current period innovation intensity. Proposition B.2.3 summarizes this:

Proposition B.2.3 (Ex-post Schumpeterian Effect). *Given expected period 2 profit $\pi_{j,2}$, we have*

$$\frac{\partial z_{j,1}^*}{\partial \pi_{j,2}} > 0 \, , \quad \forall \, q_{j,1} \text{ , and } \quad \frac{\partial x_{j,1}^*}{\partial \pi_{j,2}} > 0 \, , \quad \text{for } q_{j,1} = q_{j,0} \, .$$

Signs for $\frac{\partial x^*_{j,1}}{\partial \pi_{j,2}}$ for other local technology gaps are ambiguous.

Proof: See Appendix B.2.2.4

Proposition B.2.3 implies that any factor that affects future profits may affect firms' internal and external innovation. These include market size changes (such as an opportunity to access foreign markets), changes in input costs, and the future survival probability. More specifically, an increase in the expected profit from one's own market induces firms to increase their internal innovation. However, the effect of increasing expected profit in other markets on firms' external innovation is ambiguous for cases with local technology gap > 1. This is because incumbents in these markets increase their internal innovation in response to increasing expected profit, and this helps them escape competition. For the case with local technology gap $= 1$, incumbents cannot escape competition through internal innovation. Thus, an increase in expected future profit unambiguously increases external innovation for this case. The above results outline various factors affecting internal, external, and total innovation.

B.2.2 Proofs for the Simple Model

B.2.2.1 Proof for Proposition B.2.1

Proof. The first part of proposition B.2.1 follows from simple algebra. I prove the second part here. For $q_{j,1} = q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{x_1^e} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)\left[(1 - x_{j,1}) + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\right],$$

and

$$\frac{\partial x_{j,1}}{\partial x_1^e} = 0\,.$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)(1 - x_{j,1}) < 0\,.$$

For $q_{j,1} = \lambda q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\left[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\right]\,,$$

and

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e}\,.$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = (1 - x_{j,1})\left[\frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e)\right]^{-1} > 0\,,$$

hence

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0\,.$$

For $q_{j,1} = \eta q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\left[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\right] ,$$

and

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial x_1^e} .$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = (1 - x_{j,1})\left[\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1 - x_1^e)\right]^{-1} > 0 ,$$

hence

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0 .$$

From $x_{j,1}^*$, we see that $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}} \in (0,1)$. Then, under a parameter restriction $4\widehat{\chi} > \pi_{j,2}$,

$$\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1 - x_1^e) > \frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e) .$$

Thus, $\left.\frac{\partial z_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\lambda q_{j,0}} > \left.\frac{\partial z_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\eta q_{j,0}}$ ■

B.2.2.2 Proof of Corollary B.2.1

Proof. From $\overline{z}_1^*$, we know that

$$\frac{\partial \overline{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \; - \; z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) > 0 \, ,$$

and

$$\frac{\partial \overline{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) > 0 \, ,$$

where the signs of the two derivatives follow from proposition B.2.1. Then, the results follow from proposition B.2.1 ■

B.2.2.3 Proof of Corollary B.2.2

Proof. From $\overline{x}_1^*$, we have

$$\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - x_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) < 0 \, ,$$

and

$$\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - x_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) < 0 \, ,$$

where the signs for the two derivatives follow from proposition B.2.2 ■

B.2.2.4 Proof of Proposition B.2.3

Proof. For $q_{j,1} = q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda - 1)(1 - x_{j,1})(1 - x_1^e) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)(1 - x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} \, ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda - 1)(1 - 2x_{j,1})(1 - x_1^e) \, ,$$

and this is positive iff $x_{j,1} < \frac{1}{2}$. $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} > 0$ unambiguously.

For $q_{j,1} = \lambda q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\big[\lambda - (1 - x_{j,1})(1 - x_1^e)\big] + \frac{\pi_{j,2}}{2\widehat{\chi}}(1 - x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} \, ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{x_{j,1}}{\pi_{j,2}} - \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \pi_{j.2}} \, .$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - (1-2x_{j,1})(1-x_1^e)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}(1-x_1^e)\right]^{-1} ,$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous.

For $q_{j,1} = \eta q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda - \frac{1}{2} - \frac{1}{2}(1-x_{j,1})(1-x_1^e)\right] + \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1-z_{j,1}) - \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}} .$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - \frac{1}{2} - \frac{1}{2}(1-2x_{j,1})(1-x_1^e)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1-x_1^e)\right]^{-1} ,$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous. ■

B.3 Data Appendix

B.3.1 Summary Statistics

Table C1: Foreign Competition Shocks

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.
Mean	0.291	0.138	0.203	0.027	0.303
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)
cov(, NTR gap)		0.485	0.434	0.412	0.969
cov(, Up. NTR g.)		0.204			

Table C2: Firm-level NTR Gaps with Different Weights

	NTR gap, unweighted	NTR gap, main industry
Mean	0.333	0.336
(Std. dev.)	(0.107)	(0.116)
cov(, NTR gap)	0.78	0.86
cov(, NTR gap, main industry)	0.906	

Table C3: Technology Shocks

	Past 5 years			5 years onward	
	own US shock	own foreign shock	outside f. shock	own f. shock	outside f. shock
Mean	0.388	0.342	0.188	0.344	0.257
(Std. dev.)	(0.306)	(0.299)	(0.064)	(0.304)	(0.161)
cov(, past own f.)	0.593			-0.059	
cov(, past out f.)	-0.191	0.151			-0.991
cov(, onward out f.)				0.541	

Table C4: All Patenting Firms vs. Regression Sample in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Table C5: Export Share of Total Value of Shipments (CMF exporters)

	1992	2002	2007
Avg. of firm-level exp/vship	4.99%	5.27%	6.41%
Avg. of firm-level CN exp/vship	0.70%	0.89%	1.17%
Aggregate-level exp/vship	7.76%	9.29%	10.46%
Aggregate-level CN exp/vship	0.19%	0.38%	0.64%

Table C6: Share of Exporters (LBD firms)

Year	1992	2002	2007
Share of exporters	15.90%	22.10%	24.00%
Share of firms exporting to CN	0.60%	2.30%	4.00%

B.3.2 Overall and Escape-Competition Effect

Table C7: Overall Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.226	0.049	0.025	0.052
	(0.230)	(0.279)	(0.260)	(0.291)
NTR gap	-2.222***	0.569	1.104***	-0.117
	(0.372)	(0.405)	(0.317)	(0.393)
Post	-0.276***	-0.198**	-0.092	-0.021
	(0.077)	(0.082)	(0.080)	(0.084)
Past 5yr Δpat in own tech.		0.170*		0.282***
		(0.087)		(0.091)
Log employment		0.134***		0.014
		(0.013)		(0.014)
Firm age		-0.005**		-0.009***
		(0.002)		(0.002)
NTR rate		-2.273		1.222
		(1.690)		(2.267)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

Table C8: Escape-Competition Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.238	0.054	-0.075	-0.051
	(0.237)	(0.287)	(0.257)	(0.295)
× Innovation-intensity	0.077	-0.017	0.732**	0.784***
	(0.231)	(0.233)	(0.299)	(0.268)
NTR gap	-2.206***	0.593	1.101***	-0.067
	(0.375)	(0.409)	(0.315)	(0.397)
× Innovation intensity	-0.226	-0.213	-0.198	-0.379
	(0.158)	(0.175)	(0.231)	(0.231)
Post	-0.277***	-0.202**	-0.071	-0.002
	(0.078)	(0.083)	(0.080)	(0.083)
× Innovation-intensity	-0.053	0.017	-0.179*	-0.198**
	(0.070)	(0.075)	(0.095)	(0.085)
Innovation-intensity	0.080*	0.057	0.059	0.086
	(0.048)	(0.046)	(0.070)	(0.066)
NTR rate		-2.403		1.021
		(1.703)		(2.272)
× Innovation-intensity		0.593		0.539
		(0.507)		(0.484)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.3.3 Import Competition

Table C9: Effect of PNTR on US Imports from China

	Δlog(CN imp) HS8-level (1)	Δlog(CN imp) NAICS6-level (2)
NTR gap	0.631***	0.846*
	(0.216)	(0.509)
Δlog(NTR rate)	-6.497**	-7.696*
	(3.210)	(4.206)
Δlog(Transport cost)	-2.638**	-2.509
	(1.119)	(1.613)
Obsevations	6862	490

Notes: Table reports results of OLS regressions of changes in US imports from China from 2000 to 2007 on NTR gap at the 8-digit HS level, and 6-digit NAICS level. NTR rates at the 8-digit HS level are from the United States International Trade Commission (https://dataweb.usitc.gov/tariff/annual). Data for 8-digit HS level US imports from China and transport cost is from Schott (2008) (https://sompks4.github.io/sub_data.html). NTR rates and transport costs are in their iceberg form (e.g. from 10% to log(1.1)). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C10: Regression using 7-year Changes in the U.S. Imports from China

(a) 7-year changes in the US imports from China

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
7yr ΔUS imports from CN	-0.273***	-0.041	-0.277***	-0.043	0.082	-0.030	0.081	-0.030
	(0.047)	(0.041)	(0.047)	(0.041)	(0.061)	(0.058)	(0.061)	(0.058)
× Innovation intensity			0.037**	0.017			0.001	-0.001
			(0.017)	(0.015)			(0.020)	(0.015)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p
Controls	no	full	no	full	no	full	no	full

Notes: Table reports results of OLS regression results estimating the relationship between the U.S. firms' innovation and realized changes in the U.S. imports from China. Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.3.4 Firm Growth and Two Types of Innovation

Akcigit and Kerr (2018) show that internal innovation contributes less to firm employment growth by using the LBD. Here, we replicate their result while including firm controls for the Census years: 1982, and 1992 and construct non-overlapping five-year first differences (DHS growth) by using the LBD matched USPTO patent database. We estimate the following fixed-effect regression model:

$$\Delta Y_{ijt+5} = \beta_1 Pat_{ijt} + \beta_2 Internal_{ijt} + \mathbf{X}_{ijt}\,\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}$$

For firm i in industry j, ΔY_{ijt+5} is a 5-year DHS growth rate of i) firm employment growth from year t to $t+5$, and ii) number of six-digit NAICS industries added. Pat_{ijt} is a log of citation adjusted number of patents in year t, and $Internal_{ijt}$ is an citation-adjusted average self-citation ratio in year t. Firm and industry controls include firm age, and log of payroll. The regression is unweighted and standard errors are clustered on firm. Based on Akcigit and Kerr (2018) we expect β_1 to be positive while β_2 to be negative, as internal innovation contributes less to firm employment growth. We run the same regression model with the number of products (seven-digit NAICS product codes) added by using the CMF firms.

Table C11: Real Effect of Innovation: Employment Growth, Industry add, and Product add

	LBD firms		CMF firms
	ΔEmployment	Log nb. of industries added	Log nb. of products added
	(1)	(2)	(3)
Log nb. of patents	0.031***	0.098***	0.078***
	(0.010)	(0.011)	(0.013)
Avg. self-citation	-0.269**	-0.154**	-0.343***
	(0.106)	(0.078)	(0.102)
Log payroll	-0.025***	0.083***	0.154***
	(0.009)	(0.006)	(0.008)
Firm age	-0.004**	-0.004**	-0.007***
	(0.002)	(0.002)	(0.002)
Innovation intensity	0.032	0.009	0.076'***
	(0.029)	(0.015)	(0.017)
Observations	5,400	5,400	5,700
Fixed effects	jp	jp	jp

Notes: Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.3.5 Pre-trend and Robustness

Table C12: Parallel Pre-Trend Test

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap	-0.393	-0.379	-0.559	-0.551
	(0.487)	(0.488)	(0.403)	(0.403)
$\times$ Innovation intensity		-0.193		-0.0057
		(0.162)		(0.394)
NTR gap $\times$ $\mathcal{I}_{\{1992\}}$	0.520	0.498	0.254	0.261
	(0.355)	(0.361)	(0.294)	(0.290)
$\times$ Innovation intensity		0.092		-0.114
		(0.243)		(0.490)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, and dummy for publicly traded firms. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table C13: Foreign Competition Shock with I-O

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.111	-0.111	-0.290	-0.415
	(0.332)	(0.343)	(0.355)	(0.354)
× Innovation intensity		0.054		0.825***
		(0.319)		(0.282)
NTR gap	0.580	0.613	-0.096	-0.038
	(0.406)	(0.411)	(0.382)	(0.387)
× Innovation intensity		-0.275		-0.407
		(0.203)		(0.262)
Post	-0.254**	-0.264**	-0.145	-0.137
	(0.110)	(0.111)	(0.122)	(0.123)
× Innovation intensity		0.158		-0.098
		(0.142)		(0.139)
Innovation intensity		0.057		0.089
		(0.047)		(0.068)
NTR rate	-2.314	-2.512	1.129	0.900
	(1.670)	(1.704)	(2.237)	(2.240)
× Innovation intensity		1.027		0.666
		(0.874)		(0.765)
Downstream X Post	0.501	0.492	0.965	0.979
	(0.597)	(0.602)	(0.707)	(0.715)
× Innovation intensity		-0.241		-0.019
		(0.525)		(0.348)
Upstream X Post	0.161	0.196	0.430	0.491
	(0.443)	(0.447)	(0.480)	(0.482)
× Innovation intensity		-0.497		-0.382
		(0.381)		(0.418)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

Table C14: Overall Response: Different Weights for Firm-Level Tariffs

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.139	-0.017	0.133	0.091
	(0.331)	(0.247)	(0.311)	(0.260)
NTR gap	0.943**	omitted	-0.240	omited
	(0.374)		(0.349)	
Post	-0.146	-0.194***	-0.024	-0.036
	(0.107)	(0.074)	(0.106)	(0.076)
NTR rate	-1.763	-2.360	1.614	0.368
	(1.533)	(1.871)	(1.792)	(2.373)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Weights for tariffs	unweighted	major indust.	unweighted	major indust.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level tariff measures are constructed with different weights. Controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties $>$ 0 (full controls). Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C15: Escape-Competition effect: Different Weights for Firm-Level Tariffs

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.131	-0.015	0.017	0.021
	(0.339)	(0.251)	(0.310)	(0.260)
× Innovation intensity	0.038	0.017	0.754***	0.745***
	(0.218)	(0.218)	(0.261)	(0.263)
NTR gap	0.962**	omitted	-0.189	omitted
	(0.376)		(0.350)	
× Innovation intensity	-0.268	-0.235	-0.380*	-0.395*
	(0.168)	(0.173)	(0.228)	(0.229)
Post	-0.150	-0.197***	0.004	-0.024
	(0.109)	(0.074)	(0.105)	(0.075)
× Innovation intensity	0.002	0.008	-0.191**	-0.185**
	(0.071)	(0.071)	(0.082)	(0.083)
Innovation intensity	0.065	0.056	0.085	0.085
	(0.045)	(0.046)	(0.066)	(0.066)
NTR rate	-1.839	-2.482	1.468	0.256
	(1.541)	(1.874)	(1.795)	(2.372)
× Innovation intensity	0.583	0.584	0.576	0.666
	(0.517)	(0.525)	(0.489)	(0.477)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Weights for tariffs	unweighted	major indust.	unweighted	major indust.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level tariff measures are constructed with different weights. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C16: Inverse Propensity Scores

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.085	-0.058	-0.065	-0.294
	(0.417)	(0.420)	(0.362)	(0.351)
× Innovation intensity		-0.033		0.794***
		(0.271)		(0.269)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which observations are weighted by the inverse of the propensity scores from logit model (y = indicator for analysis sample). Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C17: Cumulative Number of Patents Included in Firm-Level Controls

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.000	0.004	0.088	-0.015
	(0.279)	(0.287)	(0.290)	(0.289)
× Innovation intensity		-0.011		0.786***
		(0.231)		(0.268)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level cumulative number of patents are included as a control. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C18: Cluster Standard Errors on Firms

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.004	0.010	0.103	-0.000
	(0.287)	(0.290)	(0.308)	(0.311)
× Innovation intensity		-0.012		0.784***
		(0.235)		(0.274)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
se. cluster	firmid	firmid	firmid	firmid

Notes: Table reports results of OLS generalized difference-in-differences regressions in which robust standard errors are adjusted for clustering at the firm-level. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C19: Effect of Foreign Competition on Product Add

	Log number of products added	Log number of products added
	(1)	(2)
NTR gap × Post	-0.209***	-0.208***
	(0.067)	(0.068)
× Innovation intensity		-0.554***
		(0.196)
Post × Innovation intensity		0.024
		(0.088)
Innovation intensity		0.227***
		(0.042)
Observations	497,000	497,000
Fixed effects	j, p	j, p

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log payroll, firm age, NTR rate and its interaction with innovation intensity, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.3.6 Technological Barrier Effect

Table C20: Technological Barrier Effect

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
Past 5yr Δforeign patent, outside of own technology field	-5.984**	-5.209*	9.076***	8.712***
	(2.756)	(2.733)	(2.711)	(2.740)
$\times$ Innovation intensity		0.161		-0.365
		(0.240)		(0.264)
Past 5yr Δforeign patent, inside of own technology field	0.005	-0.006	0.033	0.021
	(0.079)	(0.081)	(0.081)	(0.082)
$\times$ Innovation intensity		0.048		0.047
		(0.055)		(0.059)
Observation	7,600	7,600	7,600	7,600
Fixed effects	jp	jp	jp	jp

Notes: Full controls except for the NTR rate are included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C21: Effect of Concurrent Technological Shocks

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
5yr Δforeign patent, outside of own technology field	-8.680**	-7.637**	14.15***	13.56***
	(3.546)	(3.521)	(3.540)	(3.565)
$\times$ Innovation intensity		-0.063		0.081
		(0.114)		(0.122)
5yr Δforeign patent, inside of own technology field	0.212***	0.228***	0.133*	0.118
	(0.075)	(0.077)	(0.075)	(0.076)
$\times$ Innovation intensity		-0.069		0.067
		(0.062)		(0.074)
Observation	7,600	7,600	7,600	7,600
Fixed effects	jp	jp	jp	jp

Notes: Description the same as Table C20.

Appendix D: Chapter 3 Appendix

D.1 Appendix Figures

Figure D.1.1: Share of Patent Application

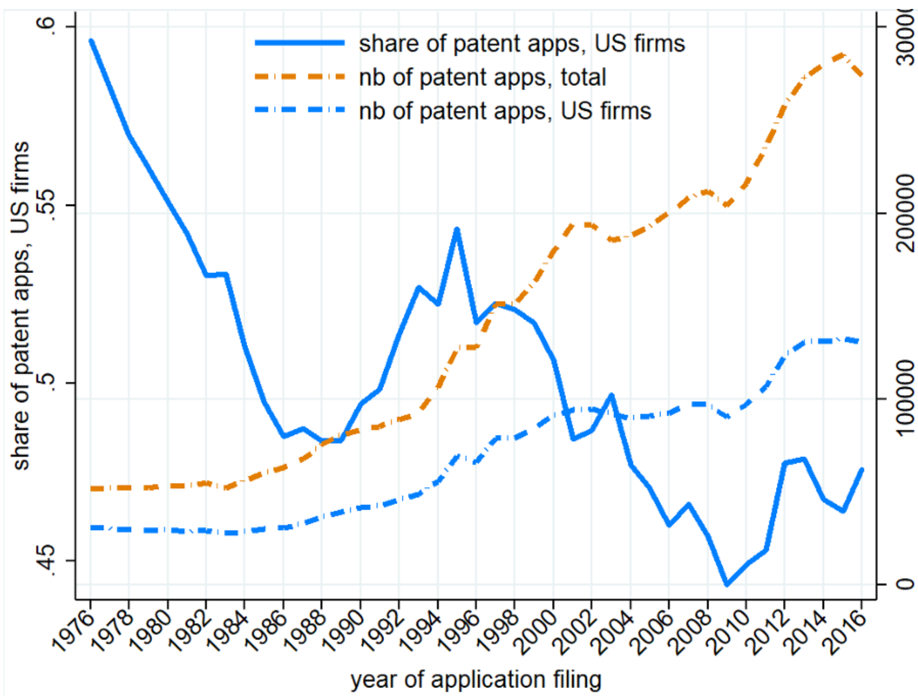


Figure D.1.2: Share of Matched Patents

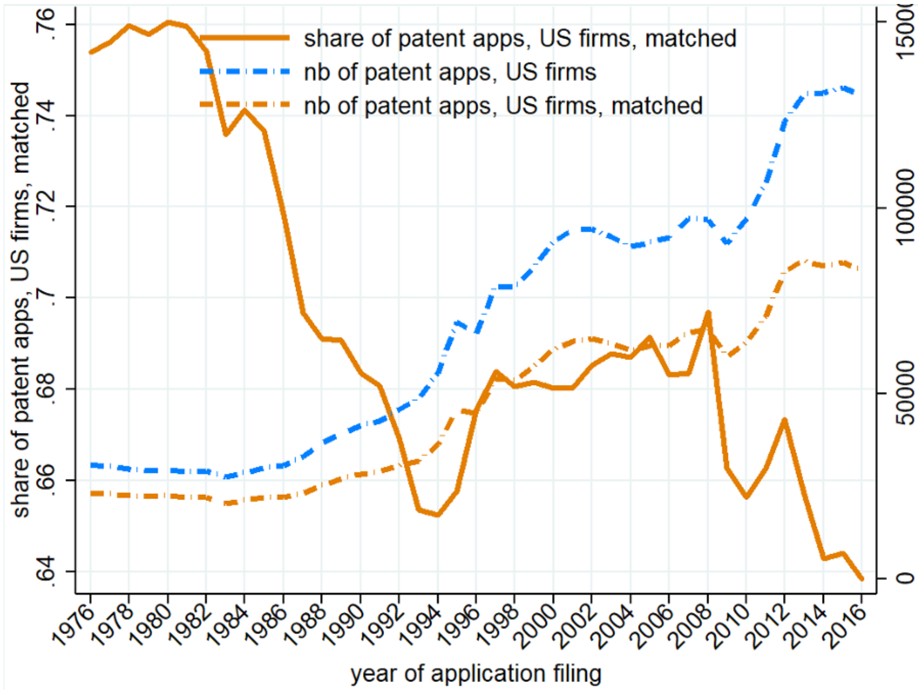


Figure D.1.3: Share of Matched Patenting Firms

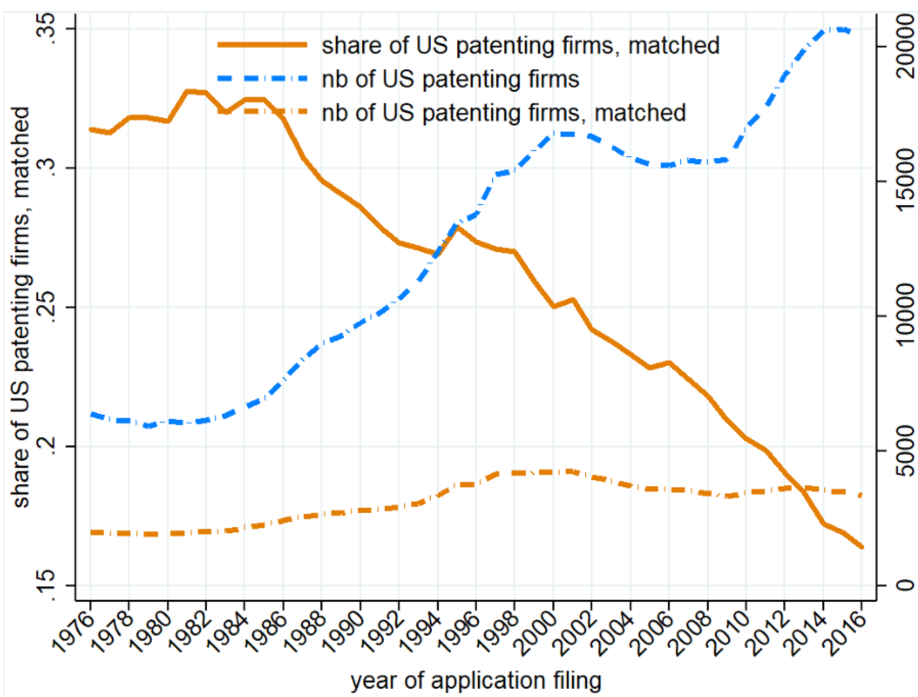


Figure D.1.4: Cumulative USPTO-Compustat Match (by method, patent level)

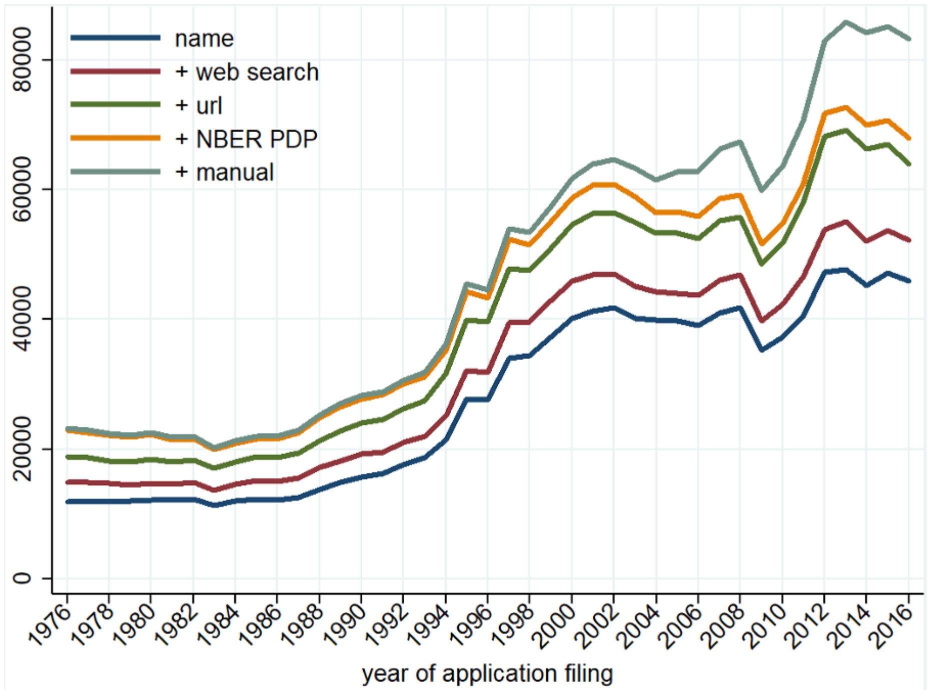


Figure D.1.5: Cumulative USPTO-Compustat Match (by method, firm level)

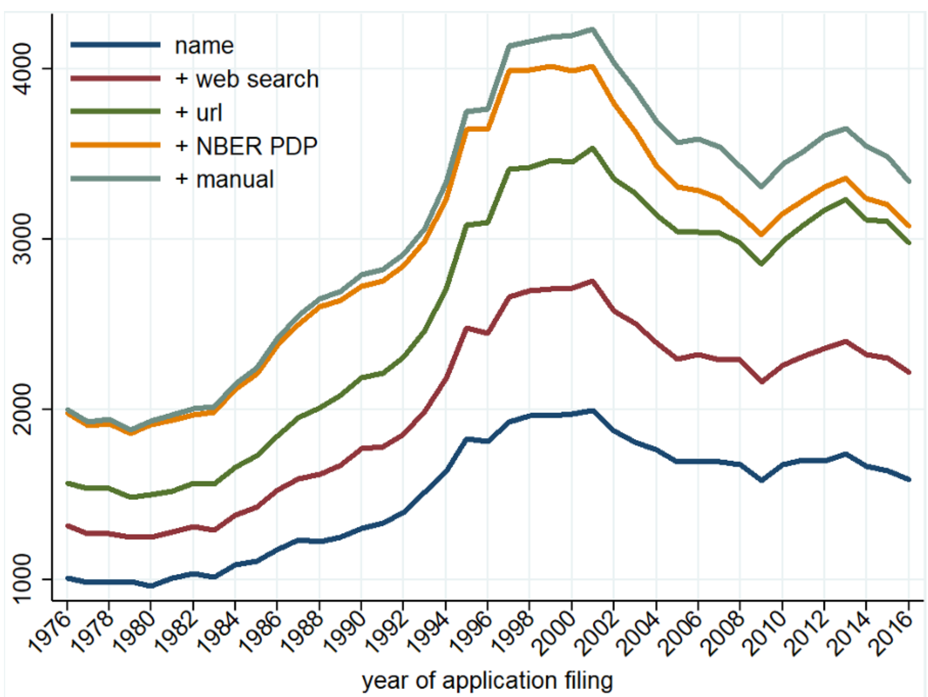
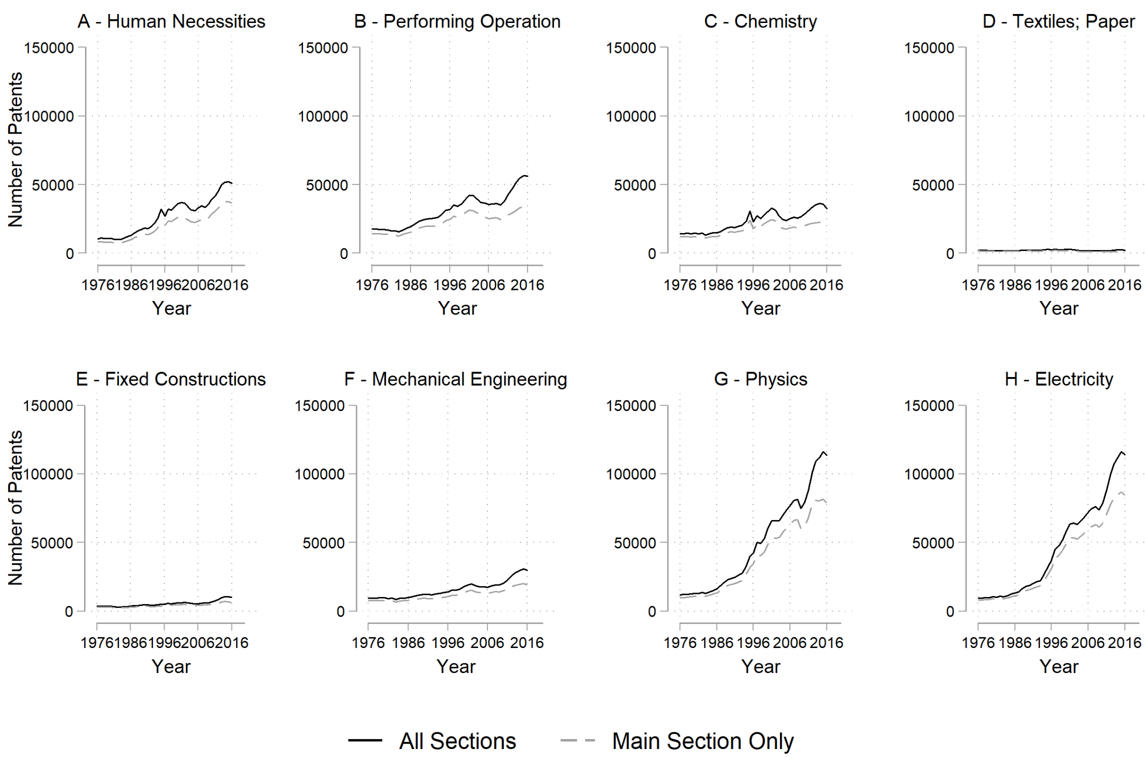


Figure D.1.6: Number of Patents by CPC Sections



D.2 Additional Analyses using the Census Data

D.2.1 The Longitudinal Business Database (LBD)

The Longitudinal Business Database (LBD, henceforth) is administrative firm-level data from the U.S. Census Bureau, which tracks the universe of non-farm establishments and firms with at least one paid employee in the U.S. The data contains a comprehensive set of firm-level information such as firm size, age, growth, industry diversification, and etc.[1] As additional analyses, we link the LBD to the USPTO following Ding et al. (2022), and find consistent results.

D.2.2 Trend in Knowledge Complexity

First, we regress each of the following measures on years as follows to indicate overall firm innovation activities and the complexity of knowledge used for them:

$$Y_{i,t} = \beta Year_t + \alpha + \varepsilon_{i,t}, \tag{D.1}$$

where $Y_{i,t}$ is either the number of patents applied by firm i in year t, the average number of coinventors as in (3.1), the average number of technology groups as in (3.3), or the knowledge interdependence measure (3.9) constructed at the firm level (replacing industry j with firm i), and $Year_t$ indicates years.

Table B1 presents the estimation results. We find that there has been an upward trend in knowledge complexity in innovation activities. Specifically, the inventor team size, the number of technology subclasses and the degree of interdependence across technology subclasses in firms'

[1] More details can be found in Jarmin and Miranda (2002) and Chow et al. (2021).

patent portfolio have all been increasing over time. The results are robust to limiting the sample to innovation-intensive industries defined by either patent intensity or patent share as in the following tables.

Table B1: Overall Trend of Knowledge Complexity

	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(***)	(***)	(***)	(***)
FE	No	No	No	No

Notes: Estimates for the constant are suppressed. The dependent variable is defined by the number of patents, the average number of coinventors, the average number of technology groups, and the knowledge interdependence in columns 1, 2, 3, and 4, respectively. Due to Census Bureau qualitative disclosure procedures, only signs and significance of the coefficients are allowed to disclose at this moment. Thus, observation counts, exact magnitude of the coefficients and standard errors associated with them are not yet disclosed. Observations are unweighted. * p<0.1, ** p<0.05, *** p<0.01.

Table B2: Overall Trend of Knowledge Complexity in Innovative Industries

(by patent intensity)	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(***)	(***)	(***)	(***)
FE	No	No	No	No
(by patent share)	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(***)	(***)	(***)	(***)
FE	No	No	No	No

Notes: Estimates for the constant are suppressed. The dependent variable is defined by the number of patents, the average number of coinventors, the average number of technology groups, and the knowledge interdependence in columns 1, 2, 3, and 4, respectively. The top panel defines innovative industries by patenting intensities, and the bottom panel defines those by patent share. Due to Census Bureau qualitative disclosure procedures, only signs and significance of the coefficients are allowed to disclose at this moment. Thus, observation counts, exact magnitude of the coefficients and standard errors associated with them are not yet disclosed. Observations are unweighted. * p<0.1, ** p<0.05, *** p<0.01.

Next, we interact the time trend with a dummy equal to one for large firms (those with the number of employees over 500) and zero, otherwise, to examine if such a trend is even more pronounced for large firms. The baseline estimating equation is:

$$Y_{i,t} = \beta_1 Year_t + \beta_2 \mathbb{I}_{i,t}^{Large} + \beta_3 Year_t \times \mathbb{I}_{i,t}^{Large} + \alpha + \varepsilon_{i,t}, \tag{D.2}$$

where $\mathbb{I}_{i,t}^{Large}$ is the large firm dummy.

Furthermore, we estimate the same equation while controlling for industry-year specific effects. That is, we include fixed effects for each pair of industry and year as follows:

$$Y_{i,t} = \beta_1 \mathbb{I}_{i,t}^{Large} + \beta_2 Year_t \times \mathbb{I}_{i,t}^{Large} + \delta_{j(i,t),t} + \alpha + \varepsilon_{i,t}, \tag{D.3}$$

where $\delta_{j(i,t),t}$ is the fixed effect for industry j (that firm i belongs to in year t) and year.

Table B3 lays out the results of these two estimations, showing that the increasing trends in inventor team size and knowledge interdependence are significantly more pronounced among larger firms. This implies a growing role of large firms in achieving complex innovation. The results for the number of patents and knowledge interdependence are once again robust to limiting the sample to innovation-intensive industries.

Table B3: Overall Trend of Knowledge Complexity for Large Firms

	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(***)	(***)	(***)	(***)
Large Firms	-	-	+	-
	(***)	(***)	(**)	
Year × Large Firms	+	+	-	+
	(***)	(***)	(**)	
FE	No	No	No	No

	(1)	(2)	(3)	(4)
Large Firms	-	-	-	-
	(***)	(***)		(***)
Year × Large Firms	+	+	+	+
	(***)	(***)		(***)
FE	Year & Industry	Year & Industry	Year & Industry	Year & Industry

Notes: Estimates for the year-industry fixed effect and the constant are suppressed. The dependent variable is defined by the number of patents, the average number of coinventors, the average number of technology groups, and the knowledge interdependence in columns 1, 2, 3, and 4, respectively. The top panel shows the results for (D.2), and the bottom panel displays the results for (D.3). Due to Census Bureau qualitative disclosure procedures, only signs and significance of the coefficients are allowed to disclose at this moment. Thus, observation counts, exact magnitude of the coefficients and standard errors associated with them are not yet disclosed. Observations are unweighted. * $p<0.1$, ** $p<0.05$, *** $p<0.01$.

Table B4: Overall Trend of Knowledge Complexity for Large Firms in Innovative Industries

(by patenting intensity)	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(*)	(***)	(***)	(***)
Large Firms	-	-	+	-
	(***)		(***)	(*)
Year × Large Firms	+	+	-	+
	(***)		(***)	(*)
FE	No	No	No	No

(by patent share)	(1)	(2)	(3)	(4)
Year	+	+	+	+
	(**)	(***)	(***)	(***)
Large Firms	-	-	+	-
	(***)		(*)	(**)
Year × Large Firms	+	+	-	+
	(***)		(*)	(**)
FE	No	No	No	No

(by patenting intensity)	(1)	(2)	(3)	(4)
Large Firms	-	-	-	-
	(***)			(***)
Year × Large Firms	+	+	+	+
	(***)			(***)
FE	Year & Industry	Year & Industry	Year & Industry	Year & Industry

(by patent share)	(1)	(2)	(3)	(4)
Large Firms	-	-	+	-
	(***)			(***)
Year × Large Firms	+	+	-	+
	(***)	(*)		(***)
FE	Year & Industry	Year & Industry	Year & Industry	Year & Industry

Notes: Estimates for the year-industry fixed effect and the constant are suppressed. The dependent variable is defined by the number of patents, the average number of coinventors, the average number of technology groups, and the knowledge interdependence in columns 1, 2, 3, and 4, respectively. The top two panels show the results for (D.2), where the first defines innovative industries by patenting intensity and the second panel defines those by patent share. The bottom two panels display the results for (D.3), with the same order of the definitions of innovative industries. Due to Census Bureau qualitative disclosure procedures, only signs and significance of the coefficients are allowed to disclose at this moment. Thus, observation counts, exact magnitude of the coefficients and standard errors associated with them are not yet disclosed. Observations are unweighted. * p<0.1, ** p<0.05, *** p<0.01.

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