ABSTRACT

Title of dissertation: ESSAYS ON BUNDLING, COMPATIBILITY
AND COMPETITION

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Doctor of Philosophy, 2013

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The bundling literature has devoted much attention to the use of this pricing strategy as a deterrent to entry of a rival or the foreclosing of one. However, there are numerous industries where a two-product seller is a monopolist in one market and competes in the second, where neither entry was deterred, nor the rival was foreclosed. The first chapter makes progress in developing a framework that analyzes the profitability of bundling for such a two-product firm. Consumers are assumed to have negatively correlated valuations for the two types of products. We show that in equilibrium, the two-product seller will offer the products only in a bundle, thus preventing any consumer from forming his own bundle using the rival's product. Furthermore, he ensures himself highest profits by having all consumers interested in only the monopoly product purchase the bundle. The welfare results under this strategy show the consumers being harmed the most, as they would prefer variety in the bundle formation and/or being able to purchase only the monopoly product, should they choose to.

Compatibility of products has been addressed mostly in a mix and match scenario, where a consumer must purchase two different components to form the final system. The components themselves have no individual use. In the second chapter, we further extend this type of framework, by assuming that one of the two components (the platform) does have a stand alone use. The other type of product is offered by only one of the firms in the market, has no individual use, but could enhance the utility of one's platform. Therefore, the firm offering the complement must first decide whether to make it compatible with the rival platform and, furthermore, what pricing strategy is best. There are two types of consumers in the market: new and legacies. The new consumers are interested in platforms and, depending on compatibility, the complement. Legacy consumers are those who have already purchased a platform in a previous period and, currently, are only interested, if at all, in the complement.

We find that compatibility is optimal, as it reduces competition and maximizes profits. In such an outcome, the two-product seller offers the goods a la carte. If compatibility is not feasible due to exogenous factors, the legacy consumers in the market are of great importance. Our results indicate that the profit maximizing strategy depends on the mass of the two-product seller's legacy consumers in the market relative to the rival's.

ESSAYS ON BUNDLING, COMPATIBILITY AND COMPETITION

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2013

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Dedication

To all the loving, supportive people in my life, thank you for being by my side through thick and thin.

Acknowledgments

There are many people without whom this dissertation would not be possible. First and foremost I'd like to thank my advisor, Professor Daniel Vincent, for his invaluable guidance, endless hours of meetings and constant support. His insights and encouragement have been tremendously important.

I would also like to thank committee members Professor Erkut Ozbay, Professor Ginger Jin, Professor Peter Cramton and Professor Marvin Titus for their guidance, for teaching me everything I know, and for all their patience, advice and encouragements. A lot of faculty graduate students at the Department of Economics of the University of Maryland deserve a special mention. My research and graduate experience has greatly benefited from the interaction with them and all the constructive comments during seminar presentations.

I would like to acknowledge help and support from the staff at the Department of Economics, University of Maryland. Vickie Fletcher, Terry Davis, Kelly Fox and Heather Nalley were invaluable in handling all administrative matters and making them all seem so easy.

A big thank you goes to my wonderful undergraduate professors, who have believed in me, guided me on the right path and have taught me to always reach for the stars: Professor Fattahi, Professor Kenny, Professor Woirol, Professor Campbell and Professor Lamar.

Finally, I would like to thank my family and friends for all the support and encouragement they have provided me during the intricate life path that lead me to

the completion of my studies and the thesis in particular. My parents — Anuca and Mircea Vamosiu — supported me in everything I have ever attempted to do with my life. They have been an incredible example to me of people who work hard, enjoy life, and care deeply for others. My husband, John, has been the most supportive, understanding man and always believed in me, even when I didn't. I am so blessed to have him in my life forever.

Lastly, thank you all and thank God! It is impossible to remember all, and I apologize to those I have inadvertently left out.

Table of Contents

List of Figures

viii

List of Abbreviations

α	alpha
Π	Pi Profit
PU	Pure Unbundling
MB	Mixed Bundling
PB	Pure Bundling
Pl-Bdl	Platform and the Bundle
CG-Bdl	Complementary Good and the Bundle
$Pl1$	Platform 1
$Pl2$	Platform 2
$CG1$	Complementary Good

Chapter 1: Introduction

1.1 What is bundling and why is compatibility important ?

Bundling is the nonlinear pricing strategy of selling two or more separate goods and services as a specially priced package. The term "separate" implies that there already exists a demand for the stand alone product. Computers bundle screens, memory, modems, and are in turn bundled with operating systems and pre-loaded software. New cars come with bundle options such as Navigation Systems, heated seats and DVD players. Many bundles consist of complements (ie. burgers and fries), others of substitutes (ie. two-ticket combo to consecutive baseball games), while some are made up of independent products (ie. Hallmark type deals: buy 3 greeting cards and get a teddy bear at a fraction of its original price). A bundle can also consist of multiple units of the same product: detergent and cereal, for example, are sold in small and large packages.

The economics literature distinguishes between two different types of bundling. The first one, price bundling, refers to packages sold at a discount without any integration of the goods and services involved. A newspaper offers advertising both in its print and online version; the two products require no integration. The second, product bundling, adresses integrated products that give extra value to the

consumer, such as a new car with extra features. It is in this latter case that compatibility is an important issue, as many products either cannot be used without a compatible complement or, even if they do have a stand alone demand, they could have their value further enhanced by a compatible complement. A first well known set up is a mix and match type scenario, where two products are only useful if used together, thus forming a system (ie. computer and operating system). In another type of setting, the complements have no stand alone use, while the platform itself does: for example, the motion sensing input devices created for a specific game consoles (ie. Kinnect for XBox 360, Wii Remote Plus for Wii and PlayStation Move for PlayStation 3) cannot be used without the respective console and each is only compatible with its specific one. In the third possible scenario, the complements can each be used individually, but bring the consumer extra value when used together: cars have built in car technology that is compatible with a smart phone. While both of these products have a stand alone value and use, they create positive externalities when one is able to use them together.

The literature in this field considers three bundling strategies: (i) Pure Unbundling - offer the products individually, at component prices; a consumer interested in purchasing more than one type of good can do so and he will pay the sum of prices for each product, (ii) Mixed Bundling - the products are not only available individually, but also as part of a specially priced package. There are important subcases to mention and, for simplicity, we discuss the case of a two-product seller: he can always offer the bundle and only one of the products individually (this is a degenerate mixed bundling scenario, where the second product is available at a pro-

hibitively high/infinite price). The last potential pricing strategy is Pure Bundling - selling all products as part of a bundle only.

The bundling and compatibility literature carefully studies the impact of such pricing and design decisions on competition, entry and market outcomes. In the case of a two product monopolist, bundling can be used as a tool to discourage entry. In an imperfectly competitive market, both bundling and incompatibility could lead to market dominance or to pushing a competitor out of the market (foreclosure). Of great importance are the welfare implications of each of these strategies. There has been a growing interest in this topic due to the many recent court cases dealing with bundling, such as 3M v. LePage, HILTI v Eurofix (nails and nail guns), GE-Honeywell proposed merger, Kodak v ISO (Independent Service Organizations), US v Microsoft, etc.

The main goal of this dissertation is to study the role of bundling and the compatibility choice in an imperfectly competitive market. We first look at the case of price bundling only and assume a two-product seller facing competition in only one of its market serves consumers who have negatively correlated valuations for the two types products. We determine the best pricing strategy and market equilibrium and focus on the welfare implications. We then analyze the product bundling case, by assuming the two-product seller offers a monopoly complementary product that may or may not be compatible with the rival's competing platform. The consumers in this case have independently and uniformly distributed valuations for the two types of products. By synthesizing and extending ideas from both the price and product bundling literatures, this dissertation makes contributions to both by characterizing

3

equilibrium market outcomes under a given set of assumptions and by analyzing the respective welfare implications.

1.2 Literature Review

The questions of "why" and "when" bundling arises have been of great interest to both economists and entrepreneurs. Three distinct theories have emerged both from the economics and marketing fields to answer the first question. Firm side rationales argue that bundling arises due to lower sorting costs (Keeney and Klein, 1983), lower inventory holding costs (Eppen, Hanson and Martin, 1991) or greater economies of scope (Baumol, Panzer and Willig, 1980; Gilbert and Katz, 2001). Demand side rationales support the theory of price discrimination (Schmalensee, 1982; Venkatesh and Mahajan, 1993), the seek for variety and the need for balance within a portfolio (Farquhar and Rao, 1976; Bradlow and Rao, 2000) and complementarity (Tesler, 1979; Venkatesh and Kamakura, 2003). The competitor side rationales focus on the need for aggregation to reduce buyer heterogeneity (Bakos and Brynjolfsson, 1999), tie-in sales and entry deterrence (Whinston, 1990; Carbajo, de Meza and Seidmann, 1990) and the enabling of competition through unbundling to facilitate market growth (Wilson, Weiss and John, 1990).

In my dissertation, I show bundling is a form of price discrimination and a way to reduce the variation in consumer valuations. This is in line with the classic analysis of bundling by a multi product monopolist done by George Stigler (1963) in regards to block booking of feature films and the Adams and Yellen (1976) model of

consumers with negatively correlated valuations for the two types of products offered by a monopolist. Both theories show bundling is the best strategy as it reduces the variation in consumer valuations. Furthermore, Schmalensee (1982) extends Adams and Yellen's work by allowing for to the presence of a competitor in one of the two-product seller's markets. He argues that when the consumer demands for the two goods are independent, pure unbundling does at least as well as pure bundling. He further conjectures that mixed bundling may be the most profitable pricing strategy, particularly when there is a negative correlation between the consumer valuations for the two goods. His work is seminal in connecting the strategic foreclosure and the multiproduct monopolist literatures.

When a competitor is present in one of the two-product seller's markets, Greenlee et al. (2004) and Nalebuff (2004a) analyze how offering a bundling discount leads to foreclosure of the single product competitive firm without the monopolist having to price below the cost. Bundling has also been argued to be a good deterrent to entry in a two-product monopolist's market, thus avoiding the use of predatory pricing. Whinston (1990) reexamines the role of tying as an entry deterrent assuming the potential Entrant to offer a differentiated product from the monopolist's competing one. When the monopolist does not precommit to pure bundling prior to entry, bundling has zero strategic value, and the monopolist does just as well by selling the products a la carte. However, if the monopolist precommits to pure bundling, he will price less in order to make sure the monopoly good is being bought. Then the Entrant's profits are greatly diminished and entry is deterred. Nalebuff (2004b) shows that pure bundling is the dominant strategy for entry deterrence and most ef-

ficient when the consumers have positively correlated valuations for the Incumbent's products.

Another related literature, the mix-and-match compatibility one, analyzes the pricing and bundling incentives for a two-product seller offering the two components to a system and facing a competitor in each of those market. Much like the present dissertation, there is an assumption of no network effects (a consumer does not derive extra utility from building the same system as other consumers). Matutes and Regibeau (1988) develop a differentiated products model where two different producers offer each two components to a system. Consumers view each component type as horizonatally differentiated among the two firms. Their framework assumes the two types of products have no individual use and a consumer only values the system formed by putting a component of the first type and one of the second type together. A consumer is then faced with many choices of forming his system. Their main finding is that compatibility in components across firms is the market equilibrium, as it reduces competition and leads to higher profits. When it comes to best pricing strategies, in a follow up paper, they argue Mixed Bundling is the equilibrium for both firms, despite it not being profit maximizing. Einhorn (1999) extends these results to the case of vertical differentiation among firms. Boom (2001) allows each firm to choose its place in the product space only to find that under compatibility, firms choose maximum differentiation in both components' characteristics and under incompatibility, the firms prefer minimum differentiation in one and maximum differentiation in the other component.

More detailed literature review related to the specific research question of each

chapter can be found in the introduction sections 1.1 and 2.1. The overall structure of the thesis and a short preview of contributions are outlined next.

1.3 Summary of Contribution

The current thesis contributes to the growing body of literature on bundling in several important ways. First of all, our first model assumes an environment where two competitors, a two-product seller and a single product one, coexist and the two-product firm does not attempt to prevent the rival's entry or foreclose them, but simply to best respond to their presence via pricing. Second, consumers who have negatively correlated valuations for two different types of products had only previously been addressed in the case of a two-product monopolist offering both of those goods. Schmalensee (1982) is a notable exception to that, but his analysis did not extensively focus on these types of consumers, nor did it allow for product differentiation among the competitors. We develop a differentiated product model where consumers have negatively correlated valuations for the two-types of products available. A two-product seller starts out as a monopolist in both markets and he then faces competition in one of them from a firm offering a differentiated product.

Third, the compatibility literature with no network effects only focuses on systems that require two different types of products in order to form the final good. A unit of any of these two types of products has no utility without a unit of the other type. Our main contribution to this literature is to address a different scenario: two competing platforms who have stand alone values and whose utility can be enhanced

7

by a complementary product of no individual use, made available by only one of the firms. The two-product seller must first decide whether to make its monopoly product compatible to its rival's platform or not (product bundling). Afterwards, he will determine if price bundling is optimal.

The content of the thesis is organized in two separate chapters that do not rely on each other and can be read on their own in any order. The first chapter discusses bundling and competition and addresses the issue of price bundling in a market where consumers have negatively correlated valuations for the two types of products available. It examines the profitability of bundling for a two-market monopolist with horizontally differentiated products. We find that when consumers have negatively correlated valuations for the two types of goods, a bundle can extract the full consumer surplus. Then, consistent with the previous literature, the two-market monopolist's optimal pricing strategy is Pure Bundling.

When competition is present in one of the two-product seller's markets, Pure Bundling remains the optimal pricing strategy. Its main strength is derived from its ability to prevent any consumer interested in both types of products from forming his own bundle using the monopoly product from one firm and the competing product from the rival, as the monopoly good is only available in the bundle form. Furthermore, two out of the three types of goods available on the market (product type 1, product type 2 and the bundle) can only be obtained by purchasing the bundle from the two-product seller. Total welfare is maximized when the two-product supplier offers the monopoly good individually, thus showing that while Pure Bundling is certainly the profit maximizing strategy, it is not welfare maximizing.

The second chapter develops a framework of differentiated platforms with a stand alone use and analyzes the impact the entrance of a monopoly, no stand alone value complement has on the market outcomes. First, it addresses a product bundling problem, as the first decision being made is the level of compatibility between the complement and the rival's platform. Second, it examines the profitability of bundling for the two-product firm offering a differentiated stand alone use, durable platform and the monopoly durable complementary good. The complement brings a positive benefit to its owner only if one has a compatible platform for it. There are two types of consumers in the market: new and legacies. A legacy consumer has purchased a platform from either firm in a previous period and is only interested in the complement, if at all. Then, the two-product seller competes against a platform only provider for new consumers and he is a monopolist in the legacy market. In equilibrium, the two-product seller chooses to make the complement compatible with its rival's platform and offer its two products a la carte.

If compatibility cannot be achieved due to exogenous reasons, the two-product firm's strategy is dependent on the relative size of its own legacy consumers base. If these are a relatively high percentage of the total legacy consumers market, then the two-product seller maximizes profits by reaching out only to its own legacies and by offering the two goods a la carte. Otherwise, the rival's legacy customers become of greater importance and the two-product firm is best off offering a mixed bundle: its own legacy consumers will purchase, at best, the complement alone at a monopoly price, while the rival's legacies will buy, if at all, the bundle, since they do need the two-product seller's platform in order to derive any utility from the complement.

9

Chapter 2: Bundling and Competition

2.1 Introduction

2.1.1 Motivation

Bundling is the nonlinear pricing strategy of selling two or more separate goods and services as a specially priced package. The term "separate" implies that there already exists a demand for the stand alone product. Flights from Chicago to Miami are bundled with returns from Miami to Chicago; moreover, this round trip can be bundled with a hotel and/or a car rental. Pizza places bundle multiple toppings, fast food restaurants bundle burgers with fries and a drink. Computers bundle screens, memory, modems, etc. and are in turn bundled with operating systems and pre-loaded software. New cars come with bundle options such as Navigation Systems, heated seats, DVD players, etc. Many bundles consist of complements (pizza and toppings, burgers and fries, etc.), others of substitutes (two-ticket combo to consecutive baseball games), while some are made up of independent products (Hallmark type deals: buy 3 greeting cards and get a teddy bear at a fraction of its original price). A bundle can also consist of multiple units of the same product (toothpaste, detergent, cereal, for example, are sold in small and large packages).

It is important to make a distinction between product and price bundling. *Product bundling* defines integrated products that give extra value to the consumer (such as the new car with extra features), while *price bundling* refers to packages sold at a discount without any integration of the goods and services involved. This paper focuses on the second type.

The current work investigates a two-product supplier's incentives to bundle his products and the welfare implications of his actions under the scenario of a two-product monopolist then becoming a single product monopolist facing competition in his other market. The two products are assumed to have independent demands, the consumer reservation values are negatively correlated and there is full information about the products' quality. The motivation for this paper comes from both theoretical and empirical grounds. The current state of the single product monopolist with a competitive market good literature focuses on price bundling as a deterrent to entry or a way to foreclose an existing rival. Schmalensee (1982) is the exception to that, but he simply mentions the existence of a competitor and the effect of that on the two-product supplier's pricing menu.

From an empirical stand point, there are industries in which entry cannot be prevented by the monopolist. The news media industry is one such. Over fifty percent of the zip codes in the US are served by one local newspaper only[1]. Due to rapid developments in the world wide web and IT infrastructure, many readers obtain their news online. In response to that, newspapers created websites with an

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[1]Chandra (2009) finds that about half of the zip codes in the US are only served by one newspaper, consistent with the scenario presented here.

online version of their print one (both overlapping and extra content such as more pictures, videos, real time updates, etc.). Local TV stations have also responded to the internet boom by creating their own website with similar real time news updates. Research shows ninety-three percent of internet news readers check two or more online news sources each day[2]. The local news media (newspaper -print and online version- and local TV station website) are each platforms that offer news content to readers and sell ad space to advertisers. While advertisers value the number of readers that see their ads, for readers, advertisers come bundled with the news media of their choice; readers could derive positive or negative value from their ads, but these ads are not a factor in their media platform choice decision. This is the type of set up our paper models.

The current paper models competition explicitly. We assume spatial competition, à la Salop: the consumers' preferences are proxied by their location on the unit interval/circle, known to them ex ante and time invariant. Bundling is known to work best for a monopolist when the consumers have negatively correlated valuations for its two products. We further investigate if this result is robust to competition and find that it is.

We first look at the two-market monopolist's potential pricing strategies: (i) offer the independent demand products individually, where consumers may buy both, so to multihome between the two platforms, at component prices (Individual Pricing), (ii) add a bundle (Mixed Bundling) at a price that will extract the full surplus of the bundle buyers, (iii) only offer the bundle (Pure Bundling) or (iv) offer

12

the bundle and one of the individual goods separately. We analyze the case of all consumers having a positive valuation for each product and find the well known result that Pure Bundling is the optimal strategy. Furthermore, we consider the scenario of a strictly positive mass of consumers having a negative valuation for one of the two products, but a positive one for the other and learn that the two-market monopolist does strictly better when choosing the Mixed Bundling strategy because he can extract strictly more surplus from its customers. Moreover, consumers are better off when they have a negative valuation for one of the products, since Mixed Bundling leaves them with a positive amount of surplus from singlehoming.

In order to focus on the effect of competition on the market outcome, we assume entry to be exogenous. When a competitor enters one of the monopolist's markets, the latter must explore which pricing strategy (same choice set as in the two-market monopolist scenario) gives him highest profits. Pure Bundling emerges as the Incumbent's optimal pricing strategy, as (i) it prevents any consumer from forming their own bundle using the rival's product and (ii) it smooths over the variation in valuations consumers have for the Incumbent's products. The result that Pure Bundling is the optimal pricing strategy for a two-product supplier when consumers have negatively correlated valuations is indeed robust to competition. To further determine the ordering of optimality among the other strategies, we show that the Incumbent's profits are high when offering one product and the bundle only, followed by Mixed Bundling and, last, Individual Pricing.

The Entrant has highest profits under Mixed Bundling. In equilibrium, he is strictly worse off than under Mixed Bundling, worse off than under Product

"A"/Print and the Bundle when his competitor has a small exclusive mass of consumers, yet always better off than under Individual Pricing. Consumers are best off under Individual Pricing, followed by Product "A"/Print and the Bundle, Mixed Bundling and Pure Bundling. Total welfare is maximized under Product "A"/Print and the Bundle.

2.1.2   Literature Review

The questions of "why" and "when" bundling arises have been of great interest to both economists and entrepreneurs. Three distinct theories have emerged both from the economics and marketing fields to answer the first question. *Firm side rationales* argue that bundling arises due to lower sorting costs (Keeney and Klein, 1983), lower inventory holding costs (Eppen, Hanson and Martin, 1991) or greater economies of scope (Baumol, Panzer and Willig, 1980; Gilbert and Katz, 2001). *Demand side rationales* support the theory of price discrimination (Schmalensee, 1982; Venkatesh and Mahajan, 1993, etc.), the seek for variety and the need for balance within a portfolio (Farquhar and Rao, 1976; Bradlow and Rao, 2000) and complementarity (Tesler, 1979; Venkatesh and Kamakura, 2003). The *competitor side rationales* focus on the need for aggregation to reduce buyer heterogeneity (Bakos and Brynjolfsson, 1999), tie-in sales and entry deterrence (Whinston, 1990; Carbajo, de Meza and Seidmann, 1990) and the enabling of competition through unbundling to facilitate market growth (Wilson, Weiss and John, 1990).

The "when" question is addressed by focusing on the degree of market com-

petition in a multi good case (generally, the models are developed concerning two goods only[3]). At one end of the spectrum stands the classic analysis of bundling by a **multi product monopolist** done by George Stigler (1963) in regards to block booking of feature films. He debunked the leveraging theory and brought forth his price discrimination hypothesis that bundling the movies (called a pure bundle since one cannot buy the products separately) reduces the between variation in consumer valuations for each movie, thus increasing the seller's ability to extract more of the consumer surplus. Kenny and Klein (1983) disagreed with this theory and explain block booking as a way to minimize information costs both by increasing the number of transactions and the quantity of price information for the sales of a like product and by preventing the TV stations from investing in information of little social and private value.

Adams and Yellen (1976) take Stigler's theory a step further by looking at a two-product monopolist selling to consumers with negatively correlated valuations. They take into account the cost of producing the bundle, the possibility of mixed bundling, and the welfare implications arising from this. McAfee, MacMillan and Whinston (1989) follow up on Adams and Yellen's work by arguing that mixed bundling always dominates pure bundling and further coming up with sufficient conditions for mixed bundling to dominate individual pricing when the valuations for the two products are independently distributed. They generalize their result for any joint distribution of reservation values as long as the monopolist can monitor purchases.

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[3]Bakos and Brynjolfsson, (1999) are the exception to this.

Schmalensee (1984) reexamines the Adams and Yellen model under the assumption of Gaussian demand and focuses on non-negatively correlated consumer valuations. He shows that pure bundling is optimal since bundling of independent demands reduces the variation in the consumers' reservation values and allows the monopolist to extract more of the consumer surplus. Salinger (1995) goes back to the negative correlation assumption in Adams and Yellen, but introduces the concept of the monopolist benefiting from cost savings when offering a bundle. He then argues that low costs encourage bundling.

Bakos and Brynjolfsson (1999) further show that in a setting with uncorrelated demands and zero marginal costs, the more goods in a bundle, the more consumer surplus extracted. Their result extends to positive correlation in consumers' valuation only when the number of bundled products is large. Choi (2003) takes a completely different approach by looking at a two-product monopolist who has a well established product and a new one of uncertain quality. By bundling the two, the monopolist takes advantage of his reputation in the first market to correct for the information asymmetries in the other one; he incidentally also reduces the information costs and increases welfare and efficiency.

When the two-market monopolist faces competition in one of its markets, he becomes a **single product monopolist facing competition in its second good market**. Schmalensee (1982) adapts Adams and Yellen's theory to show that when the consumer demands for the two goods are independent, individual pricing does at least as well as pure bundling, yet mixed bundling may be the most profitable pricing strategy, particularly when there is a negative correlation between the con-

16

sumer valuations for the two goods. His work is seminal in connecting the strategic foreclosure and the multiproduct monopolist literatures. Greenlee et al. (2004) and Nalebuff (2004a) show that offering a bundling discount leads to foreclosure of the single product competitive firm without the monopolist having to price below the cost. Nalebuff (2004a) looks at a particular case where the demand for the competitive good is perfectly inelastic and proves that under exclusionary bundling, the firm makes public the threat of raising its unbundled prices if the bundle is not bought. In response to that, all consumers buy the bundle and so the threat never materializes. He further develops the Ortho test on whether or not an equally efficient competitor can be excluded by the bundling monopolist. Greenlee et al. (2004) relax the strictly inelastic demand assumption for the competitive good and focus on bundling rebates. The single product monopolist engages in mixed bundling. A consumer would receive a rebate if he purchased the competitive good from the single market monopolist; if the consumer is only interested in the monopoly good, he has the choice of purchasing it alone. They develop a test to expose bundling that would reduce the consumer welfare by checking whether the stand alone monopoly product bundling price is greater than in the case nonbundling.

Bundling has also been argued to be a good deterrent to entry in one of the two-market monopolist's markets, thus avoiding the use of predatory pricing. Whinston (1990) reexamined the role of tying as an entry deterrent assuming the potential Entrant to offer a differentiated product from the monopolist's competing one. When the monopolist does not precommit to pure bundling prior to entry, bundling has zero strategic value, and the monopolist does just as well by choosing individual

17

pricing. However, if the monopolist precommits to pure bundling, he will price less in order to make sure the monopoly good is being bought, thus keeping more customers. Then the Entrant's profits are greatly diminished and entry is deterred. Should entry still take place, the Incumbent would go back to his individual pricing strategy. It is important to note that had it not been for the threat of entry, the Incumbent would have not chosen pure bundling, as this lowers his profits both due to the lower prices in the competitive market and overproduction of the competitive good and the fewer sales of the monopoly product. Nalebuff (2004b) shows that pure bundling is the dominant strategy for entry deterrence and most efficient when the consumers have positively correlated valuations for the Incumbent's products. Profits double compared to individual pricing and, present entry, they are still 50% higher. The two main differences from Whinston's work are that competing market products are identical and that consumers have heterogeneous valuations for the monopoly product.

The paper is organized as follows: the two-market monopolist model is presented in section 2, while the single product monopolist one with competition in the other good market is discussed in section 3. The latter section also compares and contrasts the two pricing strategies and their respective equilibrium. Section 4 highlights the welfare implications and we conclude with section 5. All proofs are in the Appendix.

2.2 Two-Market Monopolist

2.2.1 Model

There is a two-market monopolist producing two different goods, each a monopoly in its respective market. Using Hotelling's framework of product differentiation, assume the two products are located at the opposite ends of the unit interval [0,1]. Firm 1 offers a type A good/service located at 0 and a distinct type $M - A$ located at 1 in Figure 1. Both products' quality is fully disclosed to the consumers without uncertainty. The monopolist is assumed to have zero costs when producing both goods. There is a group of consumers of mass normalized to one and uniformly distributed on the unit interval [0,1] according to Figure 1. The consumers' preferences are proxied by their location on the unit interval, known to them ex ante and time invariant. They do not value the interaction with other consumers from their group. Moreover, these consumers face a positive linear transportation cost of t per unit of distance, such that a consumer located at x will incur a transportation cost of tx should he purchase Product "A"/Print, a cost of $t(1 - x)$ if he chooses Product "M-A"/Online and a cost t (sum of the two transportation costs of reaching both products) if he decides to purchase both. The consumers derive a positive benefit of α per unit of goods and services obtained, so buying Product "A" gives them a benefit of αA, a benefit of $\alpha(M - A)$ if they purchase Product "M-A" and a larger benefit of αM if they buy both goods/multihome (simply the sum of the two individual benefits). Each consumer's marginal utility from a second unit of

19

Figure 2.1: Platform differentiaton a la Hotelling

a good he already purchased is zero. The demand for each good is independent, meaning that a consumer's demand for, say Product "M-A", is in no way affected by his ownership of Product "A".

We note that our uniformly distributed consumers have negatively correlated valuations for the two products. The closer a consumer is to Product "A", the higher his valuation for this product: $\alpha A - tx$ and the lower for Product "M-A": $\alpha(M - A) - t(1 - x)$. We make no assumption about the sign of the valuation for the second good, given one has positive valuation for the first one. Then, in this case, $\alpha(M - A) - t(1 - x)0$, when $\alpha A - tx \geq 0$. To better see the negative correlation in valuations, assume the consumer currently located at x moves ε units closer to Product "A", where $\varepsilon > 0$. Then the consumer's net valuation for this product has increased to $\alpha A - t(x - \varepsilon)$ and his valuation for the other product decreased to $\alpha(M - A) - t(1 - (x - \varepsilon))$, which, again, can have any sign. The change in the valuations is $t\varepsilon$ for Product "A" and, respectively, $-t\varepsilon$, for Product "M-A", a one to one correspondence. The perfectly negatively correlated valuations are a result of the Hotelling modeling framework chosen. Our goal is to fully model the competition once entry takes place by building upon the current set up and we argue this was the most realistic and tractable way to do this.

20
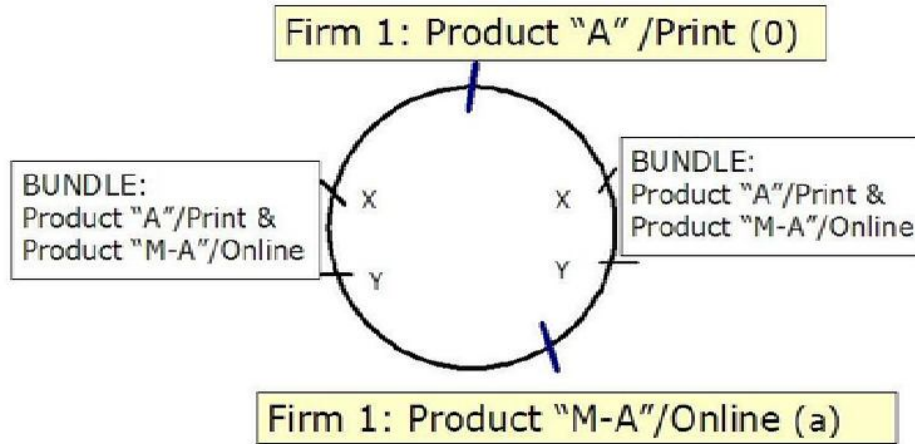
Figure 2.2: Salop Circle: differentiation a la Hotelling on the unit circle

Each consumer will have to pay a price for his purchase. First, the two-market monopolist will announce his pricing strategy and price menu and then the consumers will decide which good(s) to purchase. We have used the unit line to show the roots of our model and the basic set up. In order to prepare the reader for the competition model, we will move the current two-market monopolist from the unit line to the unit circle, as shows in Figure 1a. Product "A"'s location is normalized to zero and Product "M-A" is located at "a".

To make the arguments below more intuitive, we let $a = 1/2$ (same place the monopolist would chose if we solve the endogenous location model on the unit circle[4]), but the results hold for any value of "a".

This model applies to a myriad of markets where the monopolist sells two goods with negatively correlated customer valuations. Consider the cable industry, for example. A package of channels can contain all sports channel (ESPN, ESPN 2,

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[4]Proof available from the author upon request
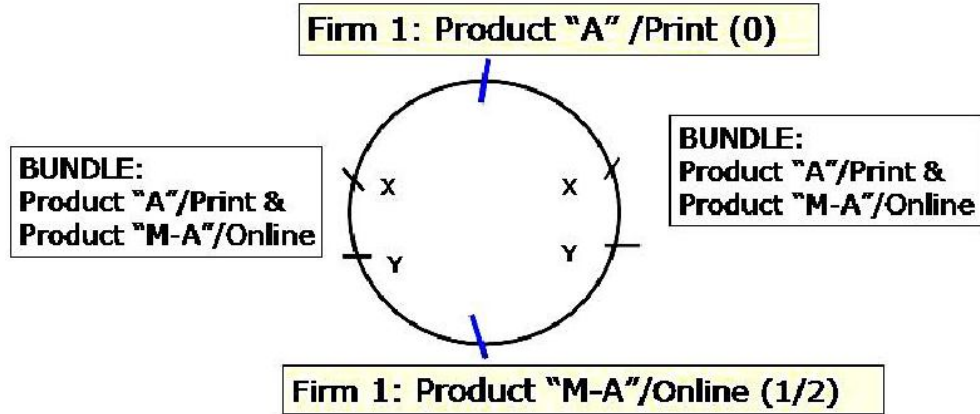
Figure 2.3: Salop Circle: differentiation a la Hotelling on the unit circle (special case a=1/2

Fox Sports, CSN, etc.), where the package itself is a bundle of channels viewers have positively correlated valuations for, as to attract customers with high valuations for each of these. Another such package would be made up of Lifetime type channels (Bravo, Style, TLC, Oxygen, Hallmark, E!, Lifetime, etc.). We argue that consumers who have high valuation for one of these packages, are likely to have a low one for the other. Some households may only purchase the first package (i.e. male college students dorm), others will only be interested in the second one (i.e. female college students apartment) and some consumers will be interested in both (mixed gender households)

Another such market deals with advertising choices in the media market. Without loss of generality, I would like to now present our model in the context of the news media market and use it as an application throughout the paper.

The Two-Market Monopolist

We can think of these two jointly owned monopolies as the two versions of a local newspaper: print and online. Readers see these as vertically differentiated platforms. Online news is inadvertently preferred due to real time updates, videos, more pictures, thus more informative content. Unless access is blocked, all readers would prefer internet media. We will refer to the two firms as platforms from now on whenever in the context of this industry. Advertisers see the two platforms as horizontally differentiated. For example, readers of the print version are considered to have restricted or inconsistent access to the internet and thus, advertisers associate them with a lower income bracket.

We assumed the two-market monopolist incurs zero costs. The monopolist will announce its pricing strategy: Individual Pricing (offering two separate prices only), Mixed Bundling (offering individual platform prices and a bundle price), Pure Bundling (one price only for the bundle, only the bundle is available for purchase) or selling one component and the bundle only. The advertisers will decide which platform(s) to sign with.

Readers

The type and amount A and type and amount of M-A goods and services each firm supplies map into a total mass of M readers divided among the two news platforms. An exogenous, fixed mass **A** of them only reads the Print version. These could be readers who do not have regular internet access or are not tech savvy. Furthermore, a distinct, exogenous, fixed mass **M-A** of readers choose the Online version only. To build better intuition for our results, we will assume they pay a null price for access to each of these platforms and none multihome between the two

23

media (assumption both supported empirically[5] and explained in our model by the vertical differentiation of the news platforms and lack of access/technological ability to the superior one by the group **A** of readers).

Advertisers

The *consumer* mass normalized to one *translates into a* unit mass of *advertisers* uniformly distributed on the unit circle. Advertisers only derive utility from reaching readers and have no benefit from the news content of the two platforms. Advertisers are aware there is a fixed mass A of Print only readers (most print readers have subscriptions; alternatively such information and demographics can be obtained from the Audit Bureau of Circulation) and a fixed mass $M - A$ of Online only readers (the newspaper is tracking e-readers via forced account creation, cookies, etc.). They derive a strictly positive utility α per unit of readers that will see their ads (i.e. advertising in Print only gives utility αA, Online only $\alpha(M - A)$ and both αM). Furthermore, since the platforms are differentiated à la Hotelling and located at diametrically opposite points on the unit circle, the advertisers have a strictly positive transportation cost t, such that $\alpha M - \frac{t}{2} > 0$. Possible interpretations of the transportation cost include the initial set up costs incurred when learning about the readers, the actual cost of signing up with the platform(s) or the actual physical cost of reaching the platform(s) or how close the demographics that read their ads are to their ideal customer base.

Advertisers' valuations for the two types of readers is negatively correlated: an advertiser more interested in Online readers targets a higher income group (i.e.

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[5]http://pewresearch.org/pubs/1508/internet-cell-phone-users-news-social-experience

luxury/new car dealership) and has little interest in low income readers (Print newspaper readers). A local community college will advertise in Print as it target low skill/low income workers and has little interest in educated, richer Online readers. A local dentist's office may value both types of customers similarly, so it will advertise in both. Advertisers endogenously decide whether to reach out to one platform's readers only (singlehome) and pay the individual price for advertising on that platform P_P or P_O or sign up with both news media (multihome) and pay the sum of the two individual prices or a multihoming price P_{OP}.

We refer to Figure 1a to see the net utilities for each of their potential choices (note that these are net utilities for the right arc, the left arc ones are symmetric):

$$U_P = \alpha A - P_P - t(x)$$ if an advertiser located at x chooses to place an ad in the Print newspaper version

$$U_O = \alpha(M - A) - P_O - t(\tfrac{1}{2} - y)$$ if an advertiser located at y chooses to place an ad in the Online newspaper version

$$U_{OP} = \alpha M - P_{OP} - \tfrac{1}{2}t$$ if an advertiser located anywhere on any arc chooses to place an ad in both the Print and Online newspaper versions

$$U_{Outside} = 0$$

2.2.2 Analysis

In what follows, we call consumers who purchase both products the monopolist offers multihomers, so the two-market monopolist will sell his bundle to the multihoming consumers only. The two independent demand products the monopo-

25

list offers will be referred to as goods and services or, in the context of our specific market application, readers. The bundling price is the price a multihomer would pay for obtaining both products or, when referring to our specific application, the price for reaching both Print and Online news readers (price of multihoming).

The Monopolist's Pricing Strategies

The monopolist can either practice (i) *Individual Pricing* by offering only two product (platform) specific prices $\{P_P, P_O\}$ to its consumers (advertisers) and having any bundle consumers (multihoming advertisers) pay the sum of the two $P_P + P_O$, (ii) *Mixed Bundling* by offering three separate prices $\{P_P, P_O, P_{OP}\}$, where the bundle price P_{OP} extracts the full consumer (multihoming advertiser) net surplus $\alpha M - \frac{t}{2}$, (iii) *Pure Bundling* $\{P_{OP}\}$ or (iv) offering the bundle and an individual good. The Mixed Bundling bundle price result is straightforward from (a) the full information the monopolist has about the consumers' (advertisers') valuation "α" for the good (readers), (b) the knowledge of a total transportation cost of $\frac{t}{2}$ (should one choose to multihome) and (c) the perfect negative correlation in consumers' (advertisers') valuations for the two products (types of readers).

In practice, often the producer may not know the exact valuations consumers have and how strong the negative correlation is, therefore, extracting the full surplus may not be feasible. However, offering a bundle price apart from two individual ones, does reduce the variation in perceived valuations and is guaranteed to extract more surplus from the nonextreme consumers (no strong preference for one good or the other).

In the current work, we always look for the equilibrium where the market is

fully covered.

Lemma 1 *Consumers (advertisers) have an elastic demand that leads to price independence among the two products (platforms).*

Proof. All proofs are contained in the Appendix. ■

Assume a consumer purchases one of the monopolist's products, therefore deriving positive utility from doing so. He will be interested in acquiring the other product too, as long as the additional utility from doing so is positive. The second purchase decision is independent from his initial one. For example, a consumer (advertiser) located at k on the unit circle and currently purchasing Product "M-A"/Online, derives a positive utility $\alpha(M-A) - P_O - t(\frac{1}{2} - k)$ by doing so. He would be interested in the other platform if and only the additional utility αA he could get from that would at least fully cover his transportation cost of getting to Product "A"/Print and the price on the second platform $P_P + tk$, therefore giving him an additional net utility of $\alpha A - P_P - tk$. If this extra utility is not positive, there is not incentive for the consumer to multihome. Thus, his decision to multihome is independent of the price of the platform he is already a customer of.

Proposition 2 *When all consumers have positive valuations for both of the monopolist's products, Pure Bundling is the optimal pricing strategy, with a full surplus extracting price of $P_{OP}^{PB} = \alpha M - \frac{t}{2}$ and profits $\pi^{PB} = \alpha M - \frac{t}{2}$.*

When there is a strictly positive mass of consumers with a negative valuation for one of the two products, a two-market monopolist's optimal strategy is to offer a Mixed Bundle $\{P_O^{MB}, P_P^{MB}, P_{OP}^{MB}\}$ where the bundle (multihoming) price P_{OP}^{MB}

27

extracts the full multihoming consumer's surplus. A unique equilibrium exists, where

the profit maximizing prices are all strictly positive:

$$P_O^{MB} = \frac{2\alpha(2M - A) - t}{4}, \quad P_P^{MB} = \frac{2\alpha(M + A) - t}{4}, \quad P_{OP}^{MB} = \alpha M - \frac{t}{2} \quad (2.1)$$

and the two-market monopolist will make a profit of

$$\pi^{MB} = \frac{2\alpha^2(2A^2 - 2AM + M^2) + 2\alpha Mt - t^2}{4t} \quad (2.2)$$

In regards to the first part of Proposition 1, Pure Bundling implies all consumers purchase the bundle at the full surplus extracting price, thus maximizing the two-product monopolist's profits. **Individual Pricing** leaves its multihoming consumers with a strictly positive surplus. To see that, we refer back to Figure 1a and notice than any consumer located between x and y, will have a positive surplus from signing up with Firm 1: Product "A"/Print (all consumers located [0,y) have a strictly positive net utility for doing so) and also a positive net utility from signing up with Firm 1: Product "M-A"/Online (all consumers located (x,1] derive a strictly positive surplus from doing so). Then $P_{OP}^{PBC} > \{ P_P^{IP}, P_O^{IP}, P_P^{IP} + P_O^{IP}\}$, so the seller's profits are strictly lower compared to Pure Bundling.

Mixed Bundling does extract the full surplus of the bundle consumers, but leaves its individual product customers with left over surplus. These buyers are charged a smaller unit price compared to the Pure Bundle $P_{OP}^{PBC} = \alpha M - \frac{t}{2}$. Therefore, the two-product monopolist's profits are lower compared to Pure Bundling.

If the producer offers **Firm 1: Product "A"/Print only and the bundle,** the latter being offered at the full surplus extracting price $P_{OP}^{P\&B} = \alpha M - \frac{t}{2}$, he still

leaves his Product "A" buyers with left over surplus and charges them a price below P_{OP}^{PBC}. Profits are below the Pure Bundle outcome. Same holds true for the **Firm 1: Product "M-A"/Online only and the bundle** marketing strategy. Therefore, Pure Bundling is the optimal pricing strategy for a two-market monopolist whose consumers have negatively correlated, but positive valued valuations for the two goods.

The following lemma will allow us to build intuition for the second part of Proposition 1.

Lemma 3 *Let consumers (advertisers) have a negative valuation for one of the two products. Under Mixed Bundling, the individual component prices $\{P_O^{MB}, P_P^{MB}\}$ are higher than the bundle one $P_{OP}^{MB} = \alpha M - \frac{t}{2}$.*

A consumer that has a negative valuation for the second product will actually reduce his net utility when buying the bundle (which leaves him with zero consumer surplus), compared to buying the higher valuation good only, even if the stand alone good is priced higher than the bundle. We argue the two-market monopolist can do no better by choosing Pure Bundling, as Pure Bundling is identical to Mixed Bundling when the consumers located at both extremes derive zero net benefit from the other good at a null price. If a strictly positive mass of consumers have strictly negative valuations for the second product, Mixed Bundling does better, as it manages to capitalize on this market segment via higher individual prices[6].

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[6]In our cable packaging example, a family with elementary/middle school children may consider certain channels/TV shows to go against the values and morals they want to instill in their

We have already shown that Individual Pricing leaves its multihoming consumers with a strictly positive surplus and its profits continue to be lowest. The other two potential strategies are offering (i) Firm 1: Product "A"/Print only and the bundle and (ii) Firm 1: Product "M-A"/Online only and the bundle, where the bundle is being offered at the full surplus extracting price $\alpha M - \frac{t}{2}$. Each of these two do worse than the Mixed Bundling strategy as we have shown that the component prices are higher than the bundle price under Mixed Bundling. Then, each of these two alternative pricing strategies eliminate one individual price that brought in more

offsprings (ie. Sixteen and Pregnant, The Housewives series, etc) and have a negative valuation for such a cable package. A cable customer may not support a certain TV station/set of station as they do not allign with his/her personal values (ie. the American Italian community movement against the TV show Jersey Shore, the Iranian American community's outrage over Shahs of Sunset TV show) or political views, so signing up for such a station/package would give him a negative utility even if it was for free. Other types of consumers are indifferent/ have a null valuation for a certain cable package. The avid sports watchers roommates have no value for the Lifetime and co. TV channels, but are also not bothered by their presence as long as they do not cost extra.

In our media market example, a high end jewelry store would only advertise to online readers, as these are considered higher income. Having their ad ran in the print daily newspaper could be of negative value even at a null price, as they would have their store frequented by low income window shoppers taking up staff time asking for assistance with jewelry that they would never be able to afford, thus taking away from staff assisting actual buyers. Moreover, this population would take away from the "exclusive" image of the jewelry store and negatively impact their reputation. A Cash Point type service (take in a car title or proof of future pay to get cash on the spot) has zero value from advertising in anything more than the print version, as online readers are most likely not in need of such a service.

profit per consumer than the bundle. They, however, increase profits compared to Pure Bundling.

Equilibrium prices and profit when consumers have negative valuations for one of the products are increasing in the valuation parameter α at different rates. Singlehoming prices only capture half of this benefit increase to the consumers, while the multihoming price will fully extract that. In the context of the media market, the higher the relative number of readers in each media (A vs $M - A$), the higher the relative monopolist singlehoming prices.

The current model assumes that a customer's purchase of a product is identical to him consuming it. However, in some real life instances[7], a consumer with negative valuation for a product may still buy the bundle if priced lower than the preferred individual components and discard the unwanted product. This way, he obtains the higher valued good at a smaller price. Other times[8], like in our model, the consumer may not discard the negative utility product.

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[7]In our scenario with the parents disagreeing with a certain set of TV shows/channels, a set of them would buy the bundle if priced lower, as long as they have a TV set that has parental controls features to block them.

[8]An advertiser buying a bundle of ad space in both the Print and Online newspaper cannot choose not to publish in only one of the two if he bought the bundle.

2.3 Single Market Monopolist Facing Competition in his Second Product Market

2.3.1 Model

We now turn to a more complex case, where the Incumbent remains a monopolist in one of its markets, but faces competition in the other one. Bundling is known to work best when consumers have a negative correlation in valuations for the two products in the two-product monopoly case. We further investigate if this result is robust to competition. We assume Firm 1: Product "A"/Print is still the only one offering type A goods and services, but Firm 1: Product "M-A"/Online now faces an Entrant that also offers a differentiated type $M - A$ good. Following Salop's model of differentiation on the unit circle, we normalize Firm 1: Product "A"'s location at 0, assume the Incumbent Firm 1: Product "M-A"'s location exogenous and fixed at $a \in (0,1)$ (our goal is not to compare profits with the monopoly case, as that would be a trivial exercise, but rather to determine the new optimal pricing strategy. Therefore, we allow "a" to take more values than $a = 1/2$. As an exercise, our reader can always interpret our results with $a = 1/2$) and the Entrant's location exogenous and fixed at $b \in (0,1)^9$. Without loss of generality, we

[9]For the reader interested in the application to the media markets, we think of Firm 1 as being our Print newspaper, while Firm 2 being the Online version of this. The Entrant is the equivalent of a local news website or a local TV station website, a close substitute to the Incumbent's online newspaper that also offeres free access to readers. This market set up is consistent with Chandra (2009), whose data set shows that about half of the zip codes in the US are only served by one
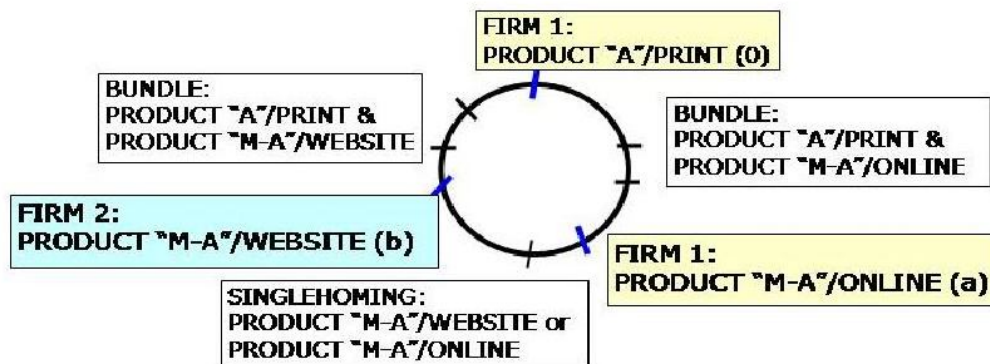
Figure 2.4: Platform differentiation a la Salop

assume $0 < a < b < 1$ and all consumer reservation values are positive.

The first direct result is that consumers have no reason to purchase from both competitors (advertisers have no incentive to multihome between Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website platforms, since they will reach the same mass $M - A$ of readers on each). We model the two competing platforms as differentiated à la Hotelling (advertisers may prefer Firm 1: Product "M-A"/Online,

newspaper. Moreover, empirical research shows that 93% of Americans check two or more online news media a day. We incorporate that in our model by assuming that every internet news reader (mass $M - A$) will multihome between the two web news media (Online Newspaper and Local Website).

When applying this model to the cable packaging example we had previously discussed, we can think of the Entrant as a website that offers access to all the shows and movies available on the Lifetime type channels for a fee (ie. Hulu). These two offers, though of the same content, are differentiated products to our viewers. Some customers will prefer the cable TV package, as they find viewing on the big screen more engaging, while younger, more tech savvy viewers value mobility, thus being able to watch a movie anytime, anywhere on various devices (laptop, IPad, phone, etc).

33

say, because they can keep on using the same graphics format, assuming they used to advertise Online before entry happened; others may prefer Firm 2: Product "M-A"/Website, because this media identifies with their own political views).

The second direct result is that post entry, consumers (advertisers) can form their own bundle by buying from different platforms: Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website do not offer a pre-made bundle, but any consumer located on arc $(b, 1)$ can purchase both goods (buy ad slots in both media) by just paying the individual prices.

The choice set for each consumer is dependent on their location on the Salop Circle. In Figure 2, consumers located on the $(0, a)$ arc have a strong preference for Firm 1: Product "M-A"/Online as opposed to Firm 2: Product "M-A"/Website, while the ones on $(b, 1)$ prefer Firm 2: Product "M-A"/Website to Firm 1: Product "M-A"/Online. Those who only want to target Internet readers are located on (a, b) and again their location is a proxy for their preference among the two products based on their characteristics (news media political views, ad formatting requirements, etc.). Note that the mass "a" of consumers (advertisers) distributed uniformly on the $(0, a)$ arc is exclusive to the Incumbent. We will refer to this arc as the Firm 1: Product "A"/Print - Firm 1: Product "M-A"/Online market. The mass "$b - a$" of consumers (advertisers) uniformly distributed on arc (a, b) is exclusive to Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website, so we will call this segment the Firm 1: Product "M-A"/Online - Firm 2: Product "M-A"/Website market. The mass "$1 - b$" of consumers (advertisers) uniformly distributed on arc $(b, 1)$ is exclusive to the Firm 1: Product "A"/Print - Firm 2: Product "M-

34

A"/Website market.

$$U_O = \alpha(M - A) - P_O - tx_{OW/OP}$$

$$U_W = \alpha(M - A) - P_W - tx_{OW/WP}$$

$$U_P = \alpha A - P_P - tx_{WP/OP}$$

$$U_{OP} = \alpha M - P_{OP} - ta$$

$$U_{PW} = \alpha M - P_P - P_W - t(1 - b) \qquad\qquad U_{Outside} = 0$$

The timing is as follows: first, firms announce their prices simultaneously and then consumers make their purchase(s).

2.3.2   Analysis

In order to solve for the market equilibrium, I analyze the interactions on each arc and incorporate them in the Incumbent and Entrant's maximization problems. Lemma 1 still holds when one consumer is faced with the choice of purchasing the monopoly good, so arcs $(0, a)$ and $(b, 1)$ in Figure 2. In the competitive market (a, b), the reaction curves depend directly on the competitor's price (see Appendix for proof).

The Incumbent's Pricing Strategies

The Incumbent can either practice:

(i) *Individual Pricing* by offering only two individual and product (platform) specific prices $\{P_P, P_O\}$ to its consumers (advertisers) and having any bundle consumers (multihoming advertisers) pay the sum of the two $P_P^{IP} + P_O^{IP}$ (or $P_P^{IP} + P_W^{IP}$)

(ii) *Mixed Bundling* by offering three separate prices $\{P_P^{MB}, P_O^{MB}, P_{OP}^{MB}\}$,

where the bundle (multihoming) price P_{OP} does not extracts the full consumer (advertiser) net multihoming surplus (different from the two-market monopolist case where $P_{OP}^{MB} = \alpha M - at$).

(iii) *Pure Bundling* by offering only one bundle price $\{P_{OP}^{PB}\}$ to its customers (advertisers)

(iv) The Bundle and One Individual Good: $\{P_{OP}^{OPB}, P_{O}^{OPB}\}$, $\{P_{OP}^{PPB}, P_{P}^{PPB}\}$.

The Entrant's Pricing Strategy

The Entrant chooses one price P_W. Any consumer forming his own bundle of (multihoming between) Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website will pay the sum of the two prices $P_{P}^{IP/MB} + P_W$.

In what follows, I will discuss each Incumbent strategy in the current market setting.

2.3.2.1 Incumbent Strategy 1: Individual Pricing

$\{P_{P}^{IP}, P_{O}^{IP}, P_{OP}^{IP} = P_{P}^{IP} + P_{O}^{IP}\}$

Under Individual Pricing, consumers on both the $(0, a)$ arc - interested in Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online - and $(b, 1)$ arc - interested in Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website - have a symmetric choice set: referring to Figure 2, if located on $(0, a)$, one can either buy Product "A"/Print, purchase Product "M-A"/Online or acquire both; a consumer on $(b, 1)$ can also either buy Product "A"/Print, purchase Product "M-A"/Website or both. Firm 1: Product "A"/Print is still a monopolist to both
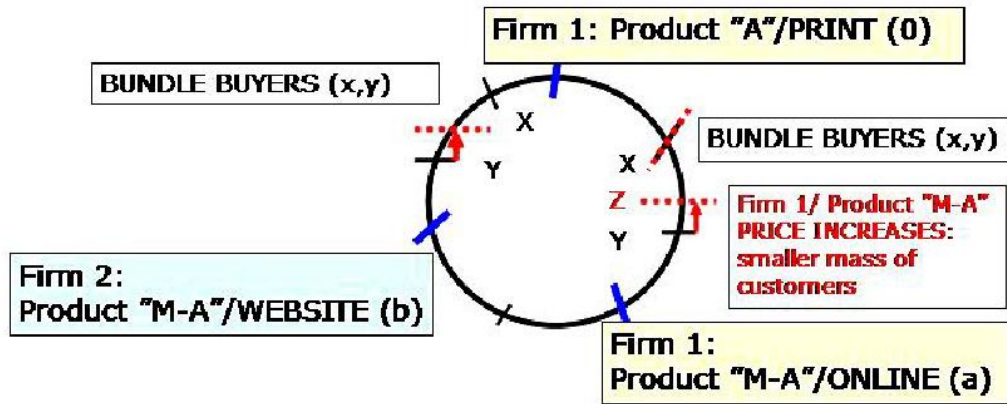
Figure 2.5: Figure 3: Individual Pricing with Competition: Increase in the Firm 1: Product "A"/Print price

the $(0, a)$ and $(b, 1)$ consumers, as he is the only provider of the good of type A (offers advertisers access to reader mass A) and prices as such. Its reaction curve is still constant. Firm 1: Product "M-A"/Online is still a monopolist when it comes to consumers on $(0, a)$ as the only supplier of good type $M - A$ (readers $M - A$) and Firm 2: Product "M-A"/Website is the same type of a monopolist to its consumers on $(b, 1)$. Better said, Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online still price their products independent of each other, just as Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website do.

Figure 3 analyzes the result of a Firm 1: Product "A"/Print price increase. Let $[x, y]$ represent the mass of consumers purchasing both Product "A"/Print and Product "M-A"/Online. If Firm 1: Product "A"/Print's price goes up, the consumer located at x is still indifferent between buying Product "A"/Print or both, as he has to pay the higher price under both circumstances. The consumer located at y

used to be indifferent between purchasing Product "M-A"/Online and the bundle. However, now he has a strict preference for the former, since acquiring both became more expensive due to the Product "A"/Print price increase. The new consumer indifferent between Product "M-A"/Online and the bundle is much closer to the Firm 1: Product "A"/Print. Firm 1: Product "A"/Print does have fewer paying consumers (advertisers) (mass z), while Firm 1: Product "M-A"/Online serves the same mass of $(1-x)$ consumers at the same price. The price increase for Product "A"/Print impacts not only the Firm 1: Product "A"/Print - Firm 1: Product "M-A"/Online market outcome, but also, symmetrically, the mass of consumers in the Firm 1: Product "A"/Print - Firm 2: Product "M-A"/Website (arc $(b,1)$). Moreover, we note that each firm makes no distinction between component and bundle consumers, since each type is charged the individual price for a product (advertising slot).

In a similar way, we can argue the case of the Product "M-A"/Online price decrease (see Figure 4): a decrease in the Product "M-A"/Online's price will gain the Incumbent more consumers (advertisers) in the Firm 1: Product "A"/Print - Firm 1: Product "M-A"/Online market. The difference appears in the Firm 1: Product "M-A"/Online - Firm 2: Product "M-A"/Website market, where there is a smaller gain in consumers (advertisers) compared to arc $(0,a)$, due to the direct price competition and identical products (same type of $M-A$ readers) the consumers (advertisers) have access to. The case of Firm 2: Product "M-A"/Website price change is similar.

Therefore, while the competitive firms' (internet news providers') reaction
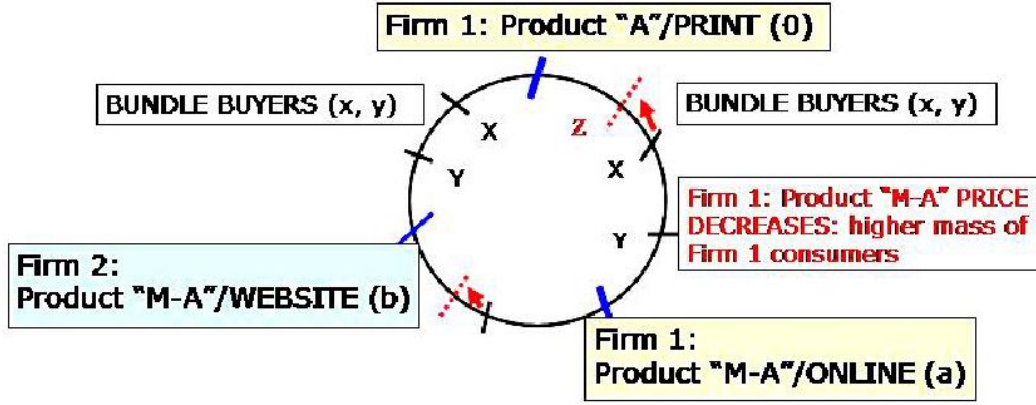
Figure 2.6: Individual Pricing with Competition: Decrease in the Firm 1: Product "M-A"/Online price

functions are independent of their monopolist counterpart, they are directly and symmetrically dependent on each other: $P_O = f(P_W)$ and $P_W = f(P_O)$.

Lemma 4 *When the Incumbent chooses a Individual Pricing pricing strategy, a unique equilibrium exists, where the two competing products have the same strictly positive price*

$$P_O^{IP} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{5}, \ P_W^{IP} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{5} \qquad (2.3)$$

and each is bought by half of the consumers (advertisers) in the competitive market. The monopoly product price is

$$P_P^{IP} = \frac{A\alpha}{2}. \qquad (2.4)$$

The Incumbent's profits are

$$\pi_{Incumbent}^{IP} = \frac{25A^2\alpha^2 + 3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t} \qquad (2.5)$$

39

and the Entrant will make a profit of

$$\pi_{Entrant}^{IP} = \frac{3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t} \qquad (2.6)$$

The higher the number of consumers on the (a, b) arc (having a distinct interest in Firm 1: Product "M-A"/Online or Firm 2: Product "M-A"/Website only), the higher the prices the competing firms (internet news providers) will charge. Their symmetric prices are directly related to the product type (mass $M - A$ of readers) they offer and the consumers' per unit benefit α. Firm 1: Product "A"/Print's monopoly price only depends on the product type and the consumers'(readers') per unit benefit α.

2.3.2.2   Incumbent Strategy 2: Mixed Bundling

$\{P_P^{MB}, P_O^{MB}, P_{OP}^{MB} < \alpha M - at\}$

Under Mixed Bundling, consumers on arc $(0, a)$ (Figure 2) can still purchase Product "A"/Print at a price P_P^{MB} or Product "M-A"/Online at a price P_O^{MB} or both at the bundle price P_{OP}^{MB}. Consumers interested in Firm 1: Product "A"/Print and/or Firm 2: Product "M-A"/Website (arc $(b, 1)$) have the option to buy one product at the respective product (platform) price or purchase both and pay the sum of the two individual prices $P_{WP}^{MB} = P_P^{MB} + P_W^{MB}$. Consumers interested in Product "M-A" only (arc (a, b)) will purchase it either from Firm 1 or Firm 2 at their respective price.

While in the two-product monopoly case, we showed that a fixed bundle price $P_{OP}^{MB} = \alpha M - at$, where $a = \frac{1}{2}$ that extracts all the $(0, a)$ exclusive consumers surplus

40

was optimal, this is no longer the case. While profits derived from the Incumbent's exclusive consumers found on the $(0, a)$ arc would always be maximized, in order for the Incumbent to push as many of these buyers as possible to purchase the bundle, he would have to price the individual products very high. But this harms the Incumbent's profits in the two adjacent markets: Product "A"/Print - Product "M-A"/Website and Product "M-A"/Online - Product "M-A"/Website. In the $(b, 1)$ market, Product "A"/Print is a monopolist, so any price increases will strictly decrease its profits there. In the (a, b) market, the two firms compete directly, thus an increase in the Product "M-A"/Online price leads to a smaller, but positive loss in profits due to the buffer of direct competition. Then, the Incumbent must weigh in the downward pressures on the individual Print and Online prices coming from the (a, b) and $(b, 1)$ market and the upward pressure to incentivize as many exclusive $(0, a)$ consumers as possible to purchase the bundle. Choosing a bundle price lower than the monopoly case one will generate higher profits for the Incumbent.

Since the bundle price is higher than the individual prices, the Incumbent has an incentive now to price higher $P_P^{MB} > P_P^{IP}$, $P_O^{MB} > P_O^{IP}$ and push most of its single product buyers on the $(0, a)$ arc to purchase the bundle at the higher price $P_{OP}^{MB} > P_P^{MB}$. Direct competition between Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website still results in interdependencies between the two firm (platform) prices, as seen in their reaction curves. However, the symmetry in their reaction functions found under Individual Pricing is lost, as consumers on the $(b, 1)$ arch can form their own bundle, pay the two individual prices, while the customers on the $(0, a)$ arch will have to purchase the bundle at the given price.

While we have argued that, strategically, Mixed Bundling creates an incentive for the Incumbent to price individual components higher than in the Individual Pricing case in order to push arc $(0, a)$ consumers to acquire the bundle instead and pay the higher bundle price $P_{OP}^{MB} > \max\{P_P^{MB}, P_O^{MB}\}$, this will cost the Incumbent consumers (advertisers), who will now choose the Entrant both on the (a, b) and $(b, 1)$ arc. In a later section we will discuss if and when this pricing strategy is best for the Incumbent. The Entrant also charges more compared to its Individual Pricing value, but it will be the least price increase $\Delta P_O^{MB-IP} > \Delta P_P^{MB-IP} > \Delta P_W^{MB-IP} > 0$.

Figure 5 works in detail through the differences and effects of the price increases compared to the Individual Pricing pricing strategy. Under Individual Pricing, all consumers on the right side arc $(0, y)$ purchased (subscribed to) Product "A"/Print, out of which, mass $(0, x)$ acquired the single product only. Therefore, the advertiser located at y was receiving zero utility from purchasing from Firm 1: Product "A"/Print. Under Mixed Bundling, Firm 1: Product "A"/Print's price increases, thus consumers much closer to Firm 1: Product "A"/Print (red line) is indifferent between Product "A" and the bundle. This same Firm 1: Product "A"/Print price increase also reduces the number of Firm 1: Product "A"/Print customers from x on the arc $(b, 1)$ to the new red line. Following a similar argument, the Firm 1: Product "M-A"/Online price increase moves the marginal Firm 1: Product "M-A"/Online buyer from x to the blue line. A loss in (a, b) consumers results under direct competition, due to the higher relative increase in prices for Firm 1: Product "M-A"/Online compared to its Individual Pricing value than for Firm 2: Product "M-A"/Website.

42
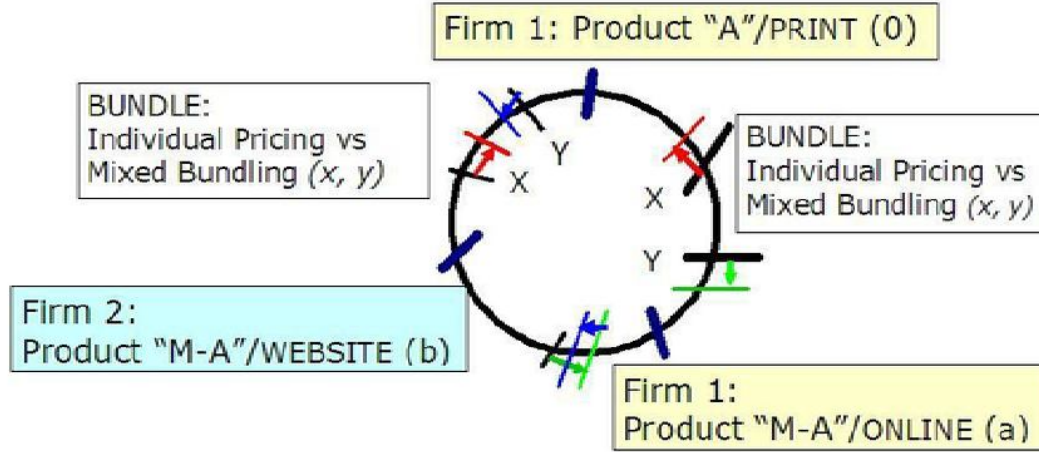
Figure 2.7: Mixed Bundling with Competition (comparing Mixed Bundling and Individual Pricing/Individual Pricing

The order of magnitude of the price changes compared to Individual Pricing pricing is straight forward given the discussion above $\Delta P_O^{MB-IP} > \Delta P_P^{MB-IP} > \Delta P_W^{MB-IP} > 0$. Firm 1 has the strongest incentive to raise the Product "M-A"/Online price as to push more consumers on arc $(0, a)$ to multihome at a higher price; the loss of consumers on the direct competition arc is softened by the existence of a competitor and, even further buffered by the price increase of Firm 2: Product "M-A"/Website. Firm 1 has the similar strategy with the price of Product "A"/Print when thinking of the mass "a" of consumers, but does take into consideration the full impact of its price increase on the adjacent arc $(b, 1)$ and choose to raise its price by a smaller amount. The Entrant has the smallest price increase compared to its Individual Pricing level since he does not have an exclusive mass of consumers to serve.

43

Lemma 5 *When the Incumbent chooses a Mixed Bundling pricing strategy, a unique equilibrium exists, where the Incumbent and Entrant offer the following prices:*

$$P_O^{MB} = \frac{-6A\alpha + 6\alpha M + (a + 7b)t}{19} > P_W^{MB} = \frac{22\alpha(-A + M) - 9at + 13bt}{57}, \quad (2.7)$$

$$P_P^{MB} = \frac{64A\alpha - 7\alpha M + 21at + 14bt}{114}, \quad (2.8)$$

$$P_{OP}^{MB} = \frac{14A\alpha + 43\alpha M - 15at + 28bt}{114} \quad (2.9)$$

The Incumbent's profits are

$$\pi_{Incumbent}^{MB} = \frac{\alpha^2(1624A^2 - 1082AM + 541M^2)}{2166t} \quad (2.10)$$

$$(2.11)$$

$$\frac{2(331a - 210b)\alpha(A - M)t}{2166t} \quad (2.12)$$

$$(2.13)$$

$$\frac{(366a^2 + 70ab + 245b^2)t^2}{2166t} \quad (2.14)$$

and the Entrant will make a profit of

$$\pi_{Entrant}^{MB} = \frac{(22A\alpha - 22\alpha M + 9at - 13bt)^2}{2166t} \quad (2.15)$$

The higher the mass of consumers interested only in Firm 1: Product "M-A"/Online or Firm 2: Product "M-A"/Website, the higher the prices the two firms (internet news providers) will charge, due to less intense competition. Their prices are also directly related to the good type (mass $M - A$ of readers) and the consumers' perceived benefit α. Firm 1: Product "A"/Print is the only one that offers good A (mass A of readers), yet under Mixed Bundling, its monopoly price no longer depends only on that quantity and the consumers' valuation for its good (readers),
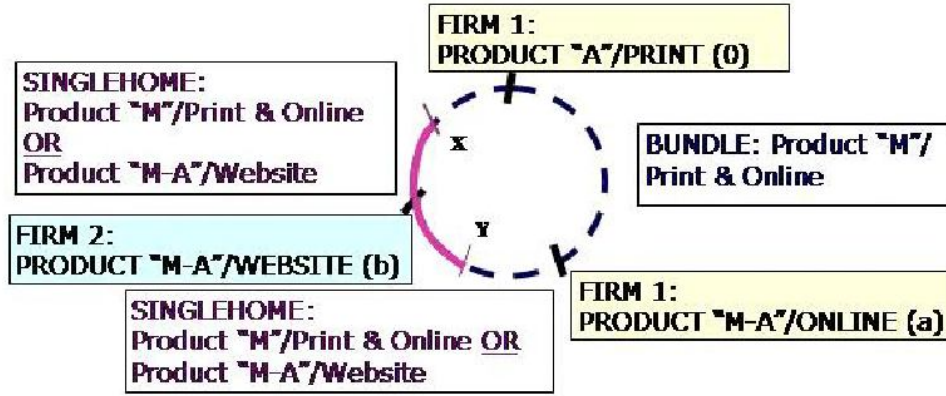
Figure 2.8: Pure Bundling with Competition

but also on the bundle (total number of readers). We explain this by the unique feature of the Mixed Bundling strategy of having an incentive to push as many consumers (advertisers) to bundling as possible by offering higher individual prices than under Individual Pricing. As expected, higher differentiation among the three products means each firm can price higher.

2.3.2.3 Incumbent Strategy 3: Pure Bundling

$\{P_{OP}^{MB}\}$

Pure Bundling is the best strategy in the two-product monopoly setting when consumers have negatively correlated and positive reservation values, as it smooths out the variation in valuations. We further inquire if this result is robust to competition being introduced. The Incumbent offers the bundle only, so the only way a consumer can obtain the Firm 1: Product "A"/Print or the Firm 2: Product "M-A"/Online is by purchasing the bundle.

45

When choosing Pure Bundling, the Incumbent names a price P_{OP}^{PB} for both the exclusive consumers (advertisers) on arc $(0, a)$ and the ones on the (a, b) and $(b, 1)$ arcs, as seen in Figure 6. Just as before, we still have direct competition on arc (a, b), but the goods are no longer similar, as consumers now choose between purchasing the Firm 2: Product "M-A"/Website or Firm 1: Product "M"/Print & Online. The argument for no multihoming on the consumer (advertiser) side still holds: no consumer would purchase from (sign on with) both Firm 2 and Firm 1, since consumers on (a, b) are interested in the similar products of Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website (or in online readers) and derive utility only from having one of them (readers already multihome between the two platforms).

While both Individual Pricing and Mixed Bundling led to no competition in the Firm 2: Product "M-A"/Website and Firm 1: Product "A"/Print market (arc $(b, 1)$), and, furthermore, the consumer was able to form its own bundle, now direct competition on the $(b, 1)$ determines that no consumer would acquire both Product "M-A"/Website and the Incumbent's bundle; those interested in Product "M-A"/Website product only can obtain it by purchasing it from that firm directly and those interested in the bundle or Product "A"/Print or Product "M-A"/Online only will have to buy the Firm 1/Print & Online bundle.

Lemma 6 *When the Incumbent chooses a Pure Bundling pricing strategy, a unique equilibrium exists where he prices*

$$P_{OP}^{PBC} = \frac{2\alpha A - (-3 + a)t}{6} \tag{2.16}$$

and makes a profit of

$$\pi^{PBC}_{Incumbent} = \frac{(-2\alpha A + (-3 + a)t)^2}{36t}.$$

(2.17)

The Entrant offers a price of

$$P^{PBC}_W = \frac{-2\alpha A + (3 + a)t}{6}$$

(2.18)

and makes a profit of

$$\pi^{PBC}_{Entrant} = \frac{(-2\alpha A + (3 + a)t)^2}{36t}.$$

(2.19)

The bundle price directly depends on the good the Incumbent is still a monopolist of: the more Print readers, the higher the price for the bundle, so the Incumbent leverages his advantage from that market. A higher exclusive market $(0, a)$ decreases the price. There are three forces acting here: first, for the Incumbent, a higher exclusive mass of consumers means (i) a lower price due to the higher transportation cost the bundle customers face, (ii) even stronger negative correlation in consumer valuations for the two bundled goods (a consumer located at $a_{new} > a_{initial}$ will value Product "A"/Print even less and viceversa for a consumer located closer to Product "A"/Print) and (iii) more intense competition in the (a, b) market, all exerting a downward pressure on price. While the Entrant's price faces the same downward pressure from the third factor, there is an upward force acting on it as well, since (i) Firm 2 is the only way a consumer on the (a, b) arch can obtain a Product "M-A" and (ii) there is a smaller transportation cost incurred. The higher the degree of differentiation among the products (higher t), the higher

47
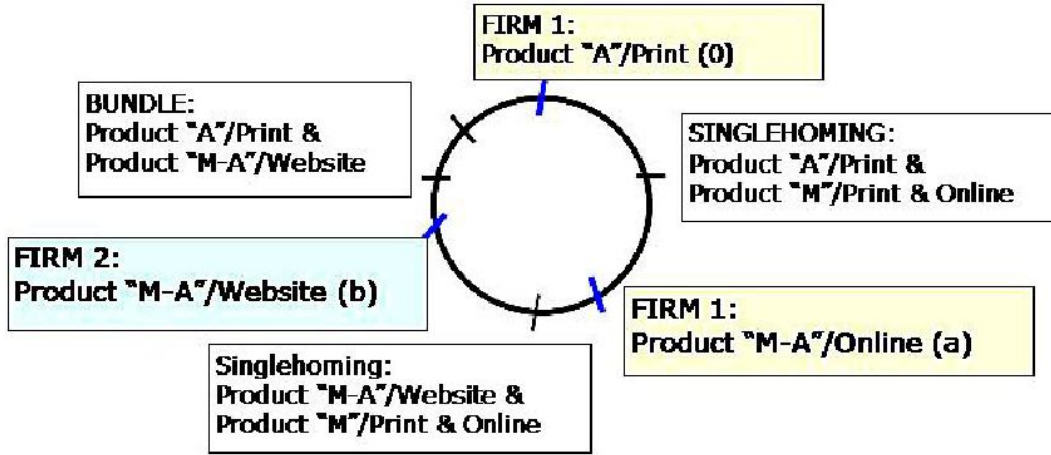
Figure 2.9: Product "A"/Print and the Bundle Pricing Strategy with Competition

the price both can charge. More differentiation means a more defined nonexclusive consumer taste for the good(s), thus a higher willingness to pay.

2.3.2.4 Incumbent Strategy 4: One Product and the Bundle

The Incumbent can also offer one product individually and the bundle only. In line with the literature on this topic, the current work only looks at the market covered outcomes. We find that the Incumbent would choose not offer Firm 1: Product "M-A"/Online and the bundle Firm 1: Product "M"/Print & Online under this assumption. However, it is feasible for him to offer Firm 1: Product "A"/Print alone and the bundle.

Under this strategy, all consumers on the $(b, 1)$ arc can form their own bundle by buying individually from the two rivals. The exclusive Incumbent consumers (arc $(0, a)$) must now choose between Firm 1: Product "A"/Print and the bundle, the bundle being their only way to access Firm 1: Product "M-A"/Online's product.

48

On the (a, b) arc, all consumers are interested in the same type of product (internet news readers) offered by Firm 2: Product "M-A"/Website and Firm 1: Product "M-A"/Online. Given that the latter is no longer available as a standalone product, they are faced with the choice of Firm 2: Product "M-A"/Website and the Incumbent's bundle only. Then, the bundle price is influenced by two distinct forces: (i) the downward pressure due to direct competition on the (a, b) arch and (ii) the incentive to reduce the price in order to ensure $(0, a)$ consumers buy the bundle instead of Product "A"/Print.

Product "A"/Print is a monopolist in the $(b, 1)$ market, therefore the Incumbent would feel the full impact of any price increases (no buffer from competition). Moreover, there is an upward pressure on the Print price in the $(0, a)$ market, where the Incumbent's goal is to push Product "A" consumers to purchase the bundle at the higher price.

Overall, the Incumbent's bundle price is below its Pure Bundling level. The Firm 1: Product "A"/Print price is lower than the Mixed Bundling one, yet higher than the Individual Pricing price.

We note that the price of the Incumbent's bundle is lower than the Mixed Bundling bundle price also, due to the competitive frontiers discussed above. However, the current bundle price is higher than the sum of the individual prices under Individual Pricing. Furthermore, competing against the bundle, Firm 2: Product "M-A"/Website's price choice is $P_W^{PPBC} < P_W^{PBC}$ due to its monopoly status in the $(b, 1)$ market.

Lemma 7 *When the Incumbent chooses to offer Product "A"/Print and the Bundle*

only, a unique equilibrium exists where he prices

$$P_P^{P\&Bdl} =$$

$$52A\alpha - 7\alpha M + 11at + 14bt \overline{}{92} \quad and \quad P_{OP}^{P\&Bdl} = \frac{3A\alpha + 8\alpha M - 6at + 7bt}{23} \quad (2.20)$$

and makes a profit of

$$\pi_{Incumbent}^{P\&Bdl} \quad = \quad \frac{\alpha^2(2188A^2 - 1732AM + 1041M^2)}{4232t} + \quad\quad (2.21)$$

$$(2.22)$$

$$+\frac{2\alpha(a(914A - 913M) + 56b(3A + 8M))t}{4232t} \quad\quad (2.23)$$

$$(2.24)$$

$$+\frac{(817a^2 - 672ab + 392b^2)}{4232t} \quad\quad$$

$$(2.25)$$

The Entrant offers a price of

$$P_W^{P\&Bdl} = \frac{-11A\alpha + 9\alpha M - at + 5bt}{23} \quad\quad (2.26)$$

and makes a profit of

$$\pi_{Entrant}^{P\&Bdl} = \frac{3(11A\alpha - 9\alpha M + (a - 5b)t)^2}{1058t}. \quad\quad (2.27)$$

The bundle price depends directly on the other product the Incumbent offers. We note that an increase in "b" means a larger consumer mass (a, b), softer competition and, thus, higher prices for both firms. The bundle price decreases with "a",

the partial distance any bundle buyer must travel to obtain the bundle. Similarly, the Incumbent's Product "A"'s price depends inversely on its other product "M-A" and increases with "a" as now the market Firm 1: Product "A" - Firm 1: Product "M-A" Print consumers have less of a valuation for the Online product. We note that again, Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website are both monopolists on the $(b, 1)$ arch. Then Product "M-A"/Website's price depends inversely on Product "A"/Print, via its participation in the Firm 1: Product "M" bundle and decreases with the distance between him and his direct competitor Product "M". For all prices, the higher the degree of differentiation "t", the higher the bundle price.

2.3.2.5 Comparative Statics and Profits

As the exclusive mass "a" of consumers in the Firm 1: Product "A"/Print - Firm 1: Product "M-A"/Online market increases and, holding everything else fixed (so "a" **moves closer to** "b", see Figure 2), there are fewer consumers (advertisers) Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website compete for. Referring to Figure 2, Firm 1: Product "M-A"/Online has the same mass of consumers (advertisers) to reach out to (mass "b"), but their distribution changes as a smaller amount of them are within the competitive interval (a, b). Firm 2: Product "M-A"/Website has a smaller mass of potential consumers to serve (mass "$1 - a$") and, while the noncompetitive market consumers are the same ones, the reduction takes place in the competitive market. Firm 1: Product "A"/Print can

reach a higher total mass of "$1 - b + a$" consumers (advertisers).

Under **Individual Pricing**, both Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website cut their singlehoming prices by the same amount. The two forces driving each are (i) the direct competition in the Firm 1: Product "M-A"/Online - Firm 2: Product "M-A"/Website market and (ii) the price change impact on the number of consumers (advertisers) they serve in their other respective market. Both Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website gain the same amount of advertisers on their noncompetitive arcs, and split the loss of consumers on the competitive arc (a, b).

Note that the lost mass Δa of consumers (advertisers) from the (a, b) arc is transferred to the $(0, a)$ arc, thus increasing the number of consumers that are interested in Firm 1: Product "A"/Print and/or Firm 1: Product "M-A"/Online. At a first glance, the reader may expect both P_P^{IP} and P_O^{IP} to further increase, since there are more consumers (advertisers) showing interest in them $(a_{initial} + \Delta a)$. However, we have showed that Firm 1: Product "A"/Print has a constant reaction function which does not depend on the other Firm's location. Firm 1: Product "M-A"/Online accounts for its direct competitor Firm 2: Product "M-A"/Website, but he is a monopolist to the consumers (advertisers) on arc $(0, a)$. Offering equal prices, both the profits of the Incumbent and the Entrant decrease by the same amount. The decrease rate in the Incumbent's profits is the second lowest (after Mixed Bundling), since he names two prices and is able to respond to market changes via both.

Under **Mixed Bundling**, two forces act on Firm 1: Product "M-A"/Online:

52

Profits

$\Pi^{IP}_{Incumbent}$

a
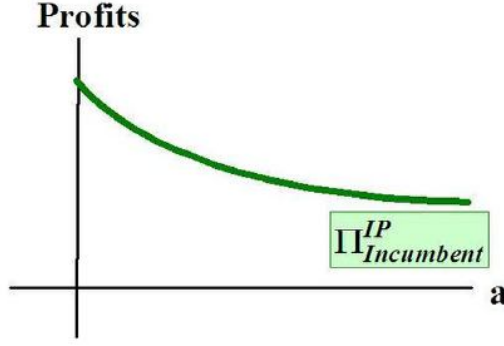
Figure 2.10: Incumbent's Profits under Individual Pricing (Pure Unbundling) and Competition

(i) the incentive to reduce its price since the direct competition against Firm 2: Product "M-A"/Website just became stronger and (ii) the need increase its price in order to push as many arc $(0, a)$ consumers (advertisers) to buy the bundle (multi-home) in order to charge them the higher price P^{MB}_{OP}. As the mass "a" of consumers, P^{MB}_{OP} is *reduced most by the increase in "a"*. In other words, the transportation cost of multihoming increases one to one with the distance between Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online, therefore leaving less surplus to be extracted by the Incumbent from its bundle consumers. The Incumbent is looking to make up part of that loss by putting more emphasis on the second factor above and P^{MB}_{O} increases by a small amount.

Similarly, Firm 1: Product "A"/Print's need to push as many arc $(0, a)$ consumers (advertisers) to multihome means an increased price compared to the Individual Pricing outcome. Then, this higher-than-monopoly price hurts the Incumbent in the Firm 1: Product "A"/Print - Firm 2: Product "M-A"/Website market. As

53

a result, P_P^{MB} will increase its value the most as the size of the exclusive market increases. As a quick note, we mention that the Entrant reduces its price, as he simply reacts to the more intense competition in the market (a, b), given he is a monopolist in the $(b, 1)$ one. This points further helps understand why the increase in P_O^{MB} is less compared to P_P^{MB}, given the two rival firms' differentiated products are strategic complements.

The Incumbent's profits are decreasing in "a". Intuitively, for small "a"s, the market Product "A"/Print-Product "M-A"/Online consumers have high positive valuations for both of the Incumbent's products and the P_{OP}^{MB} is highest due to the small arc one has to travel to reach both Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online. Moreover, the individual prices are lowest in order to make up for the small exclusive market and gain profits in the adjacent ones. However, as "a" gets larger, the bundle price decrease to account for the higher transportation cost of the bundle buyers (less valuation for both products) and the individual prices increase to make up the loss in profits. Overall, the profits on the adjacent arcs (a, b) and $(b, 1)$ are decreasing because of the higher Incumbent component prices Incumbent profits start decreasing (similar to Figure 8 or see Figure 9). We note that the decrease in profits is lowest under Mixed Bundling, as the Incumbent has the ability to name three prices and more flexibility in responding to any changes.

The **Pure Bundling (PBC)** price decreases at a much slower rate as the size of the exclusive market increases. The Incumbent faces stronger competition on the (a, b) arc and less surplus to extract from its exclusive and market $(b, 1)$ consumers due to the higher distance they have to travel. The fact that the Incumbent has

customers outside of his exclusive market and that he starts with a price less than the full surplus extracting one P_{OP}^{MB} allows him to cut price by a lesser amount as "a" increases. The shape of the profit function is much like the one for Individual Pricing (see Figure 8), but a bit steeper (see Figure 9).

Under **Product "A"/Print and the Bundle**, as "a" increases, competition becomes more intense in the Firm 1: Product "M"/Print & Online - Firm 2: Product "M-A"/Website market and the bundle price decreases at a faster rate than under Pure Bundling. The Incumbent will be raising its standalone Firm 1: Product "A"/Print price as the mass of exclusive consumers increases since now multihoming between Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online is getting more expensive due to the larger distance travelled. Then, Firm 1: Product "A"/Print is suddenly more attractive to the consumers particularly interested in this product (closer to 0). Firm 1: Product "A"/Print can now extract higher rents from the exclusive consumers by increasing its price by less than under Mixed Bundling.

The consumers' ability to form their own bundle of Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website is hurting the Incumbent compared to Pure Bundling. Profits are decreasing at an increasing rate. The decrease in profits rate as "a" increases is highest under Product "A"/Print and the Bundle when "a" is small since the Incumbent must respond to the intensifying competition in the (a, b) market and highest under Pure Bundling for large "a"s, as the two-product seller must take into account the large transportation cost all of its consumers must incur in order to buy his bundle.

2.3.3 Equilibrium

The key point about Pure Bundling is its ability to restrict the consumer's ability to form its own bundle on the $(b, 1)$ arc, as the only available goods are the Incumbent's bundle or the Entrant's product. The Pure Bundling price is higher than both the Incumbent's individual product prices under Individual Pricing and the bundle one too. The Entrant's price is higher too, as he is the only option now for an individual good for arc (a, b) and $(b, 1)$ consumers. In the context of our newspaper market, an advertiser only interested in reaching internet readers is willing to pay the Entrant much more as he is the only one offering him that opportunity.

The Pure Bundling price is also higher than the Incumbent's individual good prices under Mixed Bundling. The Entrant's price is higher under Pure Bundling than its Mixed Bundling equivalent. Since under Product "A" and the Bundle, the consumers can still form their own bundle by purchasing Product "M-A" from the rival, the bundle price is below its Pure Bundling value, which also reduces the Entrant's price.

Having talked about the forces and incentives present with each of the four pricing strategies, we are now ready to compare the Incumbent's profits under the four regimes.

We argue that the Pure Bundling pricing strategy will always bring the Incumbent strictly higher profits than Individual Pricing. Under the former, Firm 1/Print & Online sells one product only and competes head on with the Entrant

both in the (a, b) and $(b, 1)$ market. Moreover, no consumer can form his own bundle using the rival's good. In the Individual Pricing scenario, the direct competition only happens on arc (a, b), between the providers of similar goods (Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website) and any consumer on the $(b, 1)$ arc can form his own bundle by purchasing each component separately. The difference between the two profits is decreasing at an increasing rate.

We further find that the Pure Bundling price strategy will always yield the Incumbent strictly higher profits than Mixed Bundling. For large values of "a", the Mixed Bundling bundle price is higher than the Pure Bundling one, since under the former any Incumbent's bundle buyer will travel exactly the distance "a" to obtain the bundle, while under Pure Bundling, bundle buyers from the (a, b) and $(b, 1)$ arch have to add to that travel distance. Mixed Bundling loses profits on its singlehomer consumers who could be purchasing the bundle if there was no other way to obtain the Incumbent's products. That is how Pure Bundling extracts more surplus and it further restricts any consumer from using the rival's product to form his own bundle. As "a" increases, the difference between the Pure Bundling (PBC) and Mixed Bundling profits is decreasing at an increasing rate, yet Mixed Bundling never quite catches up with Pure Bundling, making the latter the optimal strategy.

Product "A" and the Bundle offers a bundle price lower than its Pure Bundling equivalent, as now consumers on the $(b, 1)$ arch can form their own bundle using the Entrant's product. Currently, the Incumbent is selling the bundle to some of its exclusive consumers and those with a high preference for the rival goods (arc (a, b)). As "a" increases, the difference between the Pure Bundling (PBC) and Product
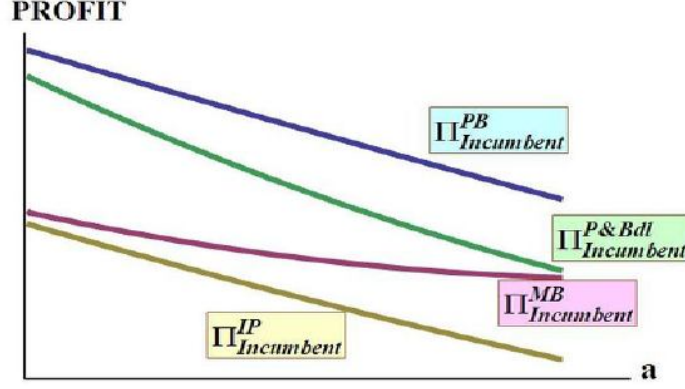
Figure 2.11: Individual Pricing vs Mixed Bundling vs Pure Bundling vs Product "A"/Print and the Bundle

"A" and the Bundle profits is decreasing at an increasing rate, yet Product "A" and the Bundle profits never quite reach the Pure Bundling ones, making the latter the optimal strategy.

Proposition 8 *Once entry occurred in one of the Incumbent's markets, the Incumbent's profit maximizing strategy is* **Pure Bundling** *with* $P_{OP}^{PBC} = \frac{2\alpha A - (-3+a)t}{6}$ *yielding a profit of* $\pi_{Incumbent}^{PBC} = \frac{(-2\alpha A + (-3+a)t)^2}{36t}$.

Therefore, the result that Pure Bundling is the best strategy in the two-product monopoly setting when the consumers have positive and negatively correlated valuations is robust to the presence of competition.

Deviations from the Market Equilibrium

In this section, we want to see if the Incumbent would ever deviate from the Pure Bundling strategy. In order to search for profitable deviations, we take the Entrant's Pure Bundling reaction function as given and allow the Incumbent to

deviate to:

(i) offering Product "A" and the Bundle;

(ii) offering Product "M-A" and the Bundle;

(iii) offering the products both individually and in a bundle (Mixed Bundling)

(iv) offering the products individually only (Individual Pricing).

I particularly focus on cases (i) and (iii) in the discussion below, though all four cases are covered in detail in the Appendix.

If the Incumbent is to *deviate to the Firm 1: Product "A"/Print and the Bundle pricing strategy*, all consumers on the $(b, 1)$ arc can now form their own bundle by buying individually from the two competitors. Since there is no competition between Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website, there is also no buffer for any price of Product "A" changes (the two-product seller will face the full effect of a price change). Moreover, the price of Product "A"/Print faces an upward pressure on the $(0, a)$ arch in order to push as many exclusive consumers as possible to purchase the bundle.

The exclusive Incumbent consumers (arc $(0, a)$) now choose between Firm 1: Product "A"/Print and the bundle, the bundle being their only way to access Firm 1: Product "M-A"/Online's product. The bundle price must respond both (i) to the direct competition against the Entrant's product in the Firm 1: Product "M"/Print & Online - Firm 2: Product "M-A"/Online market (downward pressure) and (ii) to the incentive to offer a lower price in order to determine more exclusive consumers in the $(0, a)$ market to purchase the bundle instead of Product "A"/Print. Therefore,

$P_{OP}^{PPBC} < P_{OP}^{PBC}$ and while $P_P^{PPBC} > P_P^{IP}$, its value is still below the Mixed Bundling one.

Since the Entrant's best response function is the same as under Pure Bundling, he only takes the rival's bundle price into consideration. The Incumbent's bundle best response takes the Entrant's reaction function as given and directly depends on its other product. Then, under this deviation, Firm 2: Product "M-A"/Website's price choice is $P_W^{PPBC} < P_W^{PBC}$, due both to its monopoly status in the $(b, 1)$ market (the Entrant faces the full effect of a price change) and the direct competition on the (a, b) arch.

We find that the Incumbent's profits from deviating are below the Pure Bundling level. The consumers' ability to form their own bundle of Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website is hurting the Incumbent compared to Pure Bundling.

If the Incumbent is to *deviate to the Mixed Bundling pricing strategy*, all consumers on the $(b, 1)$ arc can now form their own bundle by buying individually from the two competitors. Since there is no competition between Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website, there is also no buffer for any price of Product "A" changes (the two-product seller will face the full effect of a price change). Moreover, the price of Product "A"/Print faces an upward pressure on the $(0, a)$ arch in order to push as many exclusive consumers as possible to purchase the bundle.

In the (a, b) market, all consumers now choose between the two rival goods:

Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website. There are two forces determining the price of the Incumbent's product: (i) the direct competition on the (a, b) arch that pushes the price down and (ii) the incentive to price higher individually as to determine as many arch $(0, a)$ buyers as possible to purchase the bundle instead of the individual Incumbent goods. We note that the Entrant's best response function is the same as under Pure Bundling, he only takes the rival's bundle price into consideration. Then both the Incumbent's bundle price and the Entrant's are higher than under Pure Bundling. Moreover, all individual product prices are higher than under Mixed Bundling, but since the Entrant's best response is that of Pure Bundling, its price increase is least, as it only depends on the deviation bundle price. We find that the Incumbent's profits from deviating are below the Pure Bundling level. The consumers' ability to form their own bundle of Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website is hurting the Incumbent compared to Pure Bundling.

We have argued that there are no profitable deviations for the Incumbent, thus Pure Bundling is his optimal pricing strategy.

2.3.4 Entrant's Profits

While in the current paper we do not model the entry decision and take firms' locations as exogenous, we note that Whinston (1990) and Nalebuff (2004a) show that Pure Bundling does deter entry (see the Literature Review section for more details on this result and their model set up). While our model may resem-

61

ble Whinston's more, since we too assume the Entrant's good to be differentiated from the Incumbent's (Hotelling/Salop differentiation). We also explicitly model the competition and assume (indirectly, via the nature of the type of differentiation chosen) negative correlation in consumers' valuations for the two Incumbent products (two types of readers: Print and Online). In our model, since we assume no costs, the Entrant's profits are always positive.

Proposition 9 *Under Pure Bundling (PBC), the Entrant is strictly worse off than under Mixed Bundling, strictly worse off than under Product "A"/Print and the Bundle for*

$$a < a^{W:PB-P\&Bdl} =$$

$\frac{1652A\alpha t - 69\sqrt{6}\sqrt{t^2(-13A\alpha + 9\alpha M + 3t + 5bt)^2}] - 3t(162\alpha M + (529 + 90b)t)}{475t^2}$ *and always better off than*

Individual Pricing.

Following our discussion for the Incumbent's dominant strategy, under Mixed Bundling, the Entrant faces competition only on the (a, b) arch from the product similar to his. Moreover, under Mixed Bundling, all prices are higher compared to Individual Pricing and Product "A"/Print and the Bundle, thus yielding the Entrant highest profits. If the Incumbent were to choose the Product "A"/Print and the Bundle strategy, the Entrant would only face competition in its (a, b) market and it would be against the Incumbent's bundle. Then, the price for Product "M-A"/Website is lower than under Mixed Bundling and Pure Bundling, thus yielding Firm 2 lower profits than under Mixed Bundling. Moreover, when "a" is small, the (a, b) market competition is very relaxed under Product "A" and the Bundle and

since there is no competition in the $(b, 1)$ market, Firm 2 accrues higher profits than in the Pure Bundling case.

Pure Bundling introduces direct competition in both the (a, b) and $(b, 1)$ market. There are two competitive forces the Entrant must take into account: (i) now the Entrant faces competition on the (a, b) arc against an even more differentiated product "M"/Print & Online, and (ii) he is no longer a monopolist in the $(b, 1)$ market and must respond to direct competition against the same product as in its first market. Then the Entrant's profits are lower under Pure Bundling compared to Mixed Bundling. However, Pure Bundling profits are still higher than the Individual Pricing ones, since under the latter, all prices are lowest.

2.4   Welfare

2.4.1   Two-Market Monopolist

We quickly note that when all consumers have positive valuations for both products, Pure Bundling is the optimal pricing strategy. Consumers (Advertisers) have their full surplus extracted. At the opposite end of the spectrum, they have most surplus left under Individual Pricing. The monopolist has lowest profits under Individual Pricing and highest under Pure Bundling. Readers are best off under Pure Bundling as all see all ads. Overall, total welfare is highest under Individual Pricing for extreme values of "A" (readers in Print) and low differentiation among the products.

Proposition 1 states that when a strictly positive mass of consumers have

a negative valuation for one of the products, a **two-market monopolist** prefers a Mixed Bundling pricing strategy, where the bundle price extracts the full surplus of its bundle buyers, while the individual prices are able to extract even more net surplus for the consumers with a negative valuation for one of the products. While the monopolist benefits from choosing Mixed Bundling, consumers (advertisers) are worse off than under Individual Pricing and better off than under Pure Bundling. Under Individual Pricing, we have shown prices are lower, so all consumers (advertisers) are left with positive surplus after making their purchase(s). Under Pure Bundling, all are forced to buy the bundle, even if it brings them a negative marginal utility. The total **consumers'/advertisers'** surplus is substantially lower under Mixed Bundling, compared to Individual Pricing and higher compared to Pure Bundling. It is important to note that consumers are better off if they have a negative valuation for one of the products, as they are left with positive surplus.

Overall, Mixed Bundling generates lower total welfare than Individual Pricing and higher total welfare than Pure Bundling and any product and the bundle only strategy, since some of the consumers with negative valuations will acquire only the product they have a positive valuation for and have some surplus left over after their purchase. If we go back to our application of the model to the media market, with the two markets being the print version of a newspaper and the online one, **readers** of each of these versions (note they are a disjoint group, no readers multihome among the two platforms) benefit from Mixed Bundling compared to Individual Pricing, as they are exposed to more advertising due to the more multihoming. However, they are always better off under Pure Bundling, as they are able to see all ads.

2.4.2 One Market Monopolist and Competition

The first thing to note under Pure Bundling is that consumers are denied the choice of making their own bundle with a product from Firm 1: Product "A"/Print and another from Firm 2: Product "M-A"/Website or buying only one of the Incumbent's bundle components when that would maximize their net utility. The welfare comparison analysis can no longer be done based on price only, so we take each type of consumer, analyze his net utility under one strategy, then find his type under Pure Bundling (i.e., a Firm 1: Product "A"/Print single product consumer on the $(b, 1)$ arc can now become a bundle only buyer or a Firm 2: Product "M-A"/Website consumer) and do the same. Comparing the net utilities will give us a detailed break down of the welfare implications.

Pure Bundling (PBC) makes the consumers (advertisers) worse off than *Individual Pricing*. As we have shown before, both the Incumbent and the Entrant's Pure Bundling prices are higher than the Individual Pricing corresponding ones. All **consumers** who were purchasing both Product "A"/Print and Product "M-A"/Online now derive a lower net utility as we have proved that $P_{OP}^{PB} > P_O^{IP}$, $P_{OP}^{PB} > P_P^{IP}$ and $P_{OP}^{PB} > P_{OP}^{IP}$. On the $(b, 1)$ arc, previous Firm 1: Product "A"/Print single product consumers all buy the bundle now, which makes them worse off. Those acquiring both Product "A"/Print and Product "M-A"/Website under Individual Pricing will either buy the bundle or Firm 2: Product "M-A"/Website's product and, in both cases, do worse. On the (a, b) arc, some Product "M-A"/Online only consumers buy the bundle, while others purchase from Firm 2: Product "M-A"/Website

and, in both cases, derive less net surplus. Firm 2: Product "M-A"/Website customers are strictly worse off due to a higher individual price $P_W^{PBC} > P_W^{IP}$. Therefore, advertisers are worse off than under Individual Pricing. We have shown both the Incumbent and the Entrant makes higher profits under Pure Bundling compared to Individual Pricing. Overall, **total surplus** is higher under Individual Pricing for larger values of "a".

A Print reader will see all but the Website ads, while an internet news (Online and Website, since all internet news readers multihome) reader will see all ads. Print readers are strictly better off compared to Individual Pricing, Internet news readers are strictly better off with access to all ads, and, overall, **readers** are better off.

Mixed Bundling means lower Incumbent and Entrant individual prices than Pure Bundling. We note that the Mixed Bundling Product "M"/Print & Online price is higher than under Pure Bundling for large values of "a", since under the former every bundle exclusive consumer incurs a travel cost of ta, while under the latter, bundle consumers from the $(b, 1)$ and (a, b) market have higher transportation costs.

The previous Product "A"/Print Mixed Bundling consumers in the $(b, 1)$ market are worse off under Pure Bundling, as they (i) can no longer form their own bundle using the rival's good and (ii) must purchase the Incumbent's bundle in order to gain access to Product "A"/Print. The Mixed Bundling multihomers in this market separate into Entrant and Incumbent customers under Pure Bundling. Each are worse off: the former since now they must consume a bundle that contains the least preferred differentiated good Product "M-A"/Online instead of their first choice

Product "M-A"/Website; the latter since they will now only consume the Entrant's good and derive less utility that from the self formed bundle under Mixed Bundling. All other Firm 2: Product "M-A"/Website previous and current singlehomers are worse off since they pay higher prices under Pure Bundling $P_W^{PBC} > P_W^{MB.}$.

The same is true for the Firm 2's consumers in the (a, b) market. The Entrant consumers under Mixed Bundling that are still its buyers under Pure Bundling are worse off because of the Product "M-A"/Website price increase. We find that some of Entrant's previous customers under Mixed Bundling will purchase the Incumbent's bundle under Pure Bundling, but that still brings them lower surplus. All previous Firm 1 consumers in this market have an ambiguous change in welfare under Pure Bundling, depending on the size of "a" and the degree of differentiation "t" among the products. All exclusive consumers have an ambiguous change in welfare as well under Pure Bundling, depending on the same two parameters.

We have found that overall, **consumers (advertisers)** are better off under Mixed Bundling**.** The Incumbent is better off under Pure Bundling, but the Entrant is worse off. Overall, **total surplus** is higher under Pure Bundling for lower values of "a". If we go back to our example of consumers being advertisers and the products being reader masses of the Print newspaper or Internet news, we argue that both Print and Internet news **readers** see more ads under Pure Bundling compared to Mixed Bundling, thus readers are better off.

Under the *Product "A"/Print and the Bundle* pricing strategy, all **consumers (advertisers)** are better off due to lower prices compared to Pure Bundling, since (i) buyers can make their own bundle on the $(b, 1)$ arch using the rival's product,

(ii) customers interested in Product "A"/Print only can buy it individually and (iii) all prices are lower. The Incumbent is worse off under Product "A"/Print and the Bundle, while the Entrant is better off for small values of "a" only. Overall, **total welfare** is higher under the Product "A"/Print and the Bundle strategy. Again, if we think of our consumers are advertisers purchasing slots in a news source with a mass "A" or "M-A" readers or both, Print readers see the most ads compared to all other marketing strategies.

All in all, consumers (advertisers) are best off under Individual Pricing, followed by Product "A"/Print and the Bundle, Mixed Bundling and Pure Bundling (the dominant strategy brings least surplus to the buyers). Print readers are best off under Product "A"/Print and the Bundle, followed by Pure Bundling and, respectively, Individual Pricing and Mixed Bundling. Internet readers are best off under Pure Bundling, then Product "A"/Print and the Bundle, then Individual Pricing and, at last, Pure Bundling. Total welfare is maximized under Product "A"/Print and the Bundle, thus showing that while Pure Bundling is certainly the profit maximizing strategy, it is not welfare maximizing.

2.5   Conclusion

The current work investigates a two-product supplier's incentives to bundle his products and the welfare implications of his actions under the scenario of a two-product monopolist then becoming a single product monopolist facing competition in his other market. The two products are assumed to have independent

demands, with consumers having negatively correlated valuations.

The two-market monopolist can (i) offer the independent demand products individually and allow consumers to buy both (multihome between the two platforms) at separate prices (Individual Pricing), (ii) add a bundle to the mix (Mixed Bundling) at a price that will extract the full surplus of the bundle buyers, (iii) sell the bundle only (Pure Bundling) or (iv) offer the Bundle and one individual product. Under the assumption of Hotelling differentiation of the two products, when all consumers have a positive valuation for both products, we obtain the standard result in the literature: Pure Bundling is the optimal pricing strategy. If a strictly positive mass of consumers have negative valuations for one of the two products, the monopolist can do strictly better by choosing the Mixed Bundling strategy because he can extract strictly more surplus from its single-product customers. This result does not contradict the previous literature arguing that bundling is best when consumers have positive valued, but negatively correlated valuations, but further extends it by showing that a negative valuation for one of the products can be exploited by the monopolist via Mixed Bundling.

We show that when a competitor enters one of the monopolist's markets, the latter must explore which pricing strategy gives him higher profits: Individual Pricing (offering the two goods separately only, allowing the consumer to make his own bundle, should he be interested), Mixed Bundling (offering the two goods individually and as a bundle), Pure Bundling or offering the bundle and one individual product. Pure Bundling manages to improve the Incumbent's outcome compared to all other pricing strategies, as it smooths out the variation in valuations and pre-

69

vents any buyer from forming his own bundle using the rival's product. Therefore, the result that Pure Bundling is the best strategy in the two-product seller setting when the consumers have negatively correlated valuations is robust to competition being introduced.

The Entrant has highest profits under Mixed Bundling. In equilibrium, he is strictly worse off than under Mixed Bundling and, when "a" is below a cut off value, also worse off than under Product "A"/Print and the Bundle, yet always better off than under Individual Pricing. Consumers (advertisers) are best off under Individual Pricing, followed by Product "A"/Print and the Bundle, Mixed Bundling and Individual Pricing. Total welfare is maximized under Product "A"/Print and the Bundle. We note that in the context of our newspaper advertisers, Print readers are best off under Product "A"/Print and the Bundle, followed by Pure Bundling and, respectively, Individual Pricing and Mixed Bundling. Internet readers are best off under Pure Bundling, Product "A"/Print and the Bundle, then Individual Pricing and, at last, Pure Bundling.

Chapter 3: Compatibility and Bundling in a Market with a Monopoly Complementary Product

3.1 Introduction

3.1.1 Motivation

Many products that have a stand alone demand can have their value further enhanced when used with a compatible complement. In some cases, the complements have no value on their own: for example, the motion sensing input devices created for a specific game consoles (i.e. Kinnect for XBox 360, Wii Remote Plus for Wii and PlayStation Move for PlayStation 3) cannot be used without the console and each is only compatible with its specific one. Phone applications are only useful if one has a smart phone that the application is compatible with (most IPhone developed applications cannot be used on another platform due to the different operating systems). In other cases, the complements each have their own stand alone demand, but bring the consumer extra value when used together: cars have built in car technology that is compatible with a smart phone. While both of these products have a stand alone value and use, they create positive synergies when one is able to use, for example, the phone and the car together.

Each of these complementary products was the first of its kind in the market at a point in time or is still the only complement available. Initially, their producer was faced with the decision of making them compatible with their competitor's platform or not. The game consoles producers made their motion sensing input devices incompatible with their competitors', as did the IPhone software developers with many of their applications. However, Apple made its ITunes available for the competing MP3 player, but not its videos due to its DRM (Digital Rights Management- technology that inhibits uses of digital content that are not desired or intended by the content provider).

Network effects are one important focus of the compatibility literature[1] . Network effects are a special type of externality in which consumers' utility are directly affected by the number of consumers or firms' profits are directly affected by the number of producers using either a compatible or the same technology. Consumption network effects can be (i) positive: consumers benefit from an increase in the number of consumers using the same or a compatible brand or (ii) negative: customers are worse off when more buyers use the same or a compatible brand. Some authors distinguish between direct and indirect network effects. Direct network effects involve the presence of an extra user having a direct (positive or negative) effect on the other members of the network: They get positive or negative value from being able to interact directly with that extra (new) member. Indirect network effects do not have that direct component but instead involve economies of scale. For example, in a credit card network, a user does not directly gain if one more person has

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[1]See e.g. Katz and Shapiro [1985], Farrell and Saloner [1992] for this line of inquiry.

a credit card; but the additional person will encourage more merchants to accept a credit card, so from a cardholder's perspective there is more choice and variety of merchants accepting this card.

The compatibility literature in the absence of network externalities to date focuses on two producers each offering both components needed for a system. Both the complement and platform produced by each do not have an independent use and each producer is faced with the choice of making its complementary products compatible with its competitor's. Compatibility would mean a consumer can buy the complement for his platform from any of the duopolists, thus being able to create a larger number of different systems and build one closer to his ideal.

The current work develops a differentiated products model where two rivals offer each a durable, stand alone use platform with an individual demand. Furthermore, one of them also sells a unique, durable complementary product with no stand alone use. A consumer needs a compatible platform in order to be able to use the complement. There are two types of consumers in the market: new consumers and legacy consumers belonging to each of the two platforms. We think of the latter as customers who purchased a stand alone use platform from either of the two sellers in a time period prior to the existence of the complementary product. In the first stage, the two-product seller decides whether to make the complementary product compatible to its rival's platform. In the second stage, he decides what pricing strategy would maximize its profits. Prices are chosen by each firm in the third stage. We further explore the impact targeting own versus rival legacy consumers with the new complementary good has on both the compatibility and pricing decisions. We

show that making the complementary product always compatible with the rival's platform is the optimal strategy and the two-product seller is always best off selling the products individually. Compatibility reduces competition, thus leading to higher prices and profits.

It is important to acknowledge that in many industries, due to copyright laws or high costs, compatibility is not a realistic option. Therefore, we further explore the two marketing strategies the two-product seller has in the case where compatibility is not feasible: (i) targeting only its own legacy consumers and the new ones with its new complementary product or (ii) reaching to all legacy consumers (including those owning a rival platform) and the new ones. The latter case implies that a legacy consumer owning an incompatible platform will have to purchase both the complementary good and the compatible platform. Moreover, previous period preferences are still relevant, as a rival's legacy consumer must have had a lower valuation for the two-product seller's compatible platform relative to the one he owns (otherwise, he would have purchased the two-product seller's platform in a previous time period instead), so such a consumer has a relative dislike for the compatible platform compared to the one he previously bought. A rival's legacy consumer purchases the two-product seller's platform and complement only if that increases his utility compared to the status quo of using just the platform he already owns. Our results show that when a relatively small mass of legacy consumers belong to the two-product seller, reaching out to all legacy consumers (including his rival's) and offering a Mixed Bundle (the platform and the complement sold both individually and as a specially priced package) is profit maximizing. The two-product firm will

74

choose to only reach out to its own legacy consumers when they are a relatively large part of the legacy market. It is then best to sell the two products individually.

3.1.2 Literature Review

The differentiated products compatibility model presented in this paper draws upon two different literatures: the *mix-and-match compatibility* one and the works analyzing the pricing and bundling incentives for *a two-product seller that is a single product monopolist with a competitive market for its second good.* In the former, Matutes and Regibeau (1988) develop a differentiated products model where two different producers offer two components to a system, each component type being horizontally differentiated among the two firms. Their framework assumes the two types of products have no individual use and a consumer only values the system formed by putting a component of the first type and one of the second type together. A consumer is then faced with many choices of forming his system. The authors show compatibility is the Nash equilibrium of such a game when producers price a la carte; producers realize that the competition between them becomes less aggressive when the two components are compatible across manufacturers. Consumer surplus is also higher under full compatibility. Economides (1989) proves that the above conclusion still holds for n firms and a more general specification of demand. In a follow up paper, Matutes and Regibeau (1992) use the same set up from their previous paper and investigate the possibility of price bundling. They find that the results of the literature on monopoly bundling do not directly extend. Duopolists

selling compatible components choose to offer a discount on their full system even though they would be better off if they could agree not to bundle.

Einhorn (1992) expands on Matutes and Regibeau's work and further shows that producers of components still earn higher profits under compatibility even in industries with quality (vertical) differentiation. The finding is robust to both components produced by one firm being of higher quality or to each firm having a quality advantage in only one component. Economides and Salop (1992) generalize Cournot's model of complementary duopoly to the case of multiple brands of compatible components. They describe the conditions on the degree of component substitutability under which the equilibrium system price is lower under joint ownership than with independent ownership of component producing firms. Boom (2001) uses the same framework to focus on the compatibility decision when the two firms can choose the location of their two components in the product space. She finds that under compatibility, firms choose maximum differentiation in both components' characteristics and under incompatibility, the firms prefer minimum differentiation in one and maximum differentiation in the other component. While compatibility is the equilibrium outcome, contrary to the literature with exogenous horizontal product differentiation, consumer surplus is decreased.

Matutes and Regibeau (1989) extend their initial model by assuming each of the components needed for a system to function is produced by a separate manufacturer. In the first stage the consumers buy the first component (computer) they need from one of the two possible sellers and in the second stage they buy the second components (the software). Two competing software sellers enter the market

sequentially and must decide whether to sell software that works with both types of computers or create one for each. In equilibrium, the first entrant chooses to offer a software that works with both computers while the second one will offer a complementary good that works only with one of the computers.

Economides and Salop (1992) generalize Cournot's model of complementary duopoly to the case of multiple brands of compatible components. Their main focus is on defining the conditions on the degree of component substitutability for the equilibrium system price to be lower under joint ownership than with independent ownership of component-producing firms.

Farrell, Monroe and Saloner (1998) compare and contrast two forms of organizing production: firms producing and competing in fully assembled systems (closed organization) versus components (open organization). They account for costs and argue that open interaction is always the socially efficient choice because it minimizes costs. However, that doesn't make it the most profitable, as low costs are only one of the determinants of profits. Viecens (2009) develops a model where firms are horizontally differentiated à la Hotelling and are asymmetric in the value of their fully compatible application. Applications are being produced by a third party and the platform creates a two sided market environment facilitating access of developers to consumers and viceversa. The author quantifies firm dominance by a premium in consumer valuations and studies its impact on firms' attitudes towards compatibility and, furthermore, how the compatibility decision influences incentives on investment in product improvement. She finds the dominant firm is against compatibility and that compatibility reduces the incentive to invest in

product enhancement significantly more for the dominant firm than the small one.

Since the two-market seller discussed in the current model is a single product monopolist (the complement market) facing competition in his other market (the platform market), it is important to mention some results from the literature on a *single product monopolist with a competitive market for its second good.* An important difference between our model and the mentioned literature is that our two-product seller does not start out as a two-product monopolist that faces entry in one of its product markets. He initially produces one good only and is a direct competitor in a two sellers platform market. He later introduces a monopoly product with no stand alone use that is a complement to the platforms.

The single product monopolist facing competition in his other market literature mostly focuses on preventing entry or foreclosing a rival. Schmalensee (1982) is the exception to that. He adapts Adams and Yellen's theory about consumers with negatively correlated valuations for the two goods a two-product monopolist sells to show that when the consumer demands for the two goods are independent, individual pricing does at least as well as pure bundling, yet mixed bundling may be the most profitable pricing strategy, particularly when there is a negative correlation between the consumer valuations for the two goods. His work is seminal in connecting the strategic foreclosure and the multiproduct monopolist literatures. Vamosiu (2013) looks at a two-product seller who offers differentiated goods for which the consumers have negatively correlated valuations. Once a competitor enters one of the two markets, the two-product seller is best off offering a Pure Bundle only, thus being the only one to provide a desirable product for two out of the three types of

consumers (the monopoly product buyers and the bundle buyers).

Greenlee et al. (2004) and Nalebuff's (2004a) line of research argues that offering a bundling discount leads to foreclosure of the single product competitive firm without the monopolist having to price below the cost. Bundling has also been argued to be a good deterrent to entry in one of the two-market monopolist's markets, thus avoiding the use of predatory pricing. Whinston (1990) reexamines the role of tying as an entry deterrent assuming the potential Entrant to offer a differentiated product from the monopolist's competing one. When the monopolist does not precommit to pure bundling prior to entry, bundling has zero strategic value, and the monopolist does just as well by choosing individual pricing. However, if the monopolist precommits to pure bundling, he will price less in order to make sure the monopoly good is being bought, thus keeping more customers. Then the Entrant's profits are greatly diminished and entry is deterred. Should entry still take place, the Incumbent would go back to his individual pricing strategy. It is important to note that had it not been for the threat of entry, the Incumbent would have not chosen pure bundling, as this lowers his profits both due to the lower prices in the competitive market and overproduction of the competitive good and the fewer sales of the monopoly product. Nalebuff (2004b) shows that pure bundling is the dominant strategy for entry deterrence and most efficient when the consumers have positively correlated valuations for the Incumbent's products. Profits double compared to individual pricing and, present entry, they are still 50% higher. The two main differences from Whinston's work are that competing market products are identical and that consumers have heterogeneous valuations for the monopoly

product.

The paper is organized as follows: the two-product firm model with a complement of no individual use is presented in Section 2. Section 3 discusses the potential pricing strategies and the equilibrium. Welfare implications are presented in Section 4. Conclusions are in Section 5 and all proofs are in the Appendix.

3.2   Model

There are two firms in a market, each offering a durable, differentiated good (platform) that has a stand alone use to the consumers. Using Hotelling's framework of product differentiation, assume the two platforms are located at the opposite ends of the unit segment [0,1]. One of the firms (without loss of generality, we will assume Firm 0) also offers a complementary good, also durable, but with no independent use (unless a consumer already owns or will buy a compatible Firm 0 or Firm 1 platform, the complement brings no benefit to them). The complement is the only one on the market of its kind. This product is located at point 0 on the vertical unit segment (see Figure 1). We assume there are no economies of scope and that all three products are produced at a constant marginal cost which we normalize to zero. All products' quality is fully disclosed to the consumers without uncertainty.

There is a group of new consumers of mass normalized to one and uniformly distributed on the unit square according to Figure 1. These consumers decide whether to purchase a platform from Firm 0 or Firm 1 and, simultaneously, if to also buy the complement. It is important to note that the valuations the consumers

have for the two types of products (platform and complementary good) are independently distributed. The consumers' preferences are proxied by their location in the unit square, known to them ex ante and time invariant. They do not value the interaction with other consumers from their group. The consumers derive a gross positive benefit of h per platform and c per complementary product (as long as the consumer has a compatible platform to use it with). Each consumer's marginal utility from a second unit of a certain type of good he has already purchased is zero. Moreover, these consumers face a positive linear transportation cost of t per unit of distance, such that a consumer located at (x, y) will incur a transportation cost of tx should he purchase the platform from Firm 0, a cost of $t(1 - x)$ if he chooses Firm 1's platform and a cost ty if he decides to purchase the complement. Note that the demand for the complementary good is not independent, as it is affected by one's ownership or lack of of the product from Firm 0 or Firm 1.

Then, a consumer located at (x, y) would have the following utility functions, depending on his purchase:

$U_0 = h - t * (x) - P_0$

$U_1 = h - t * (1 - x) - P_1$

$U_{0\&CG} = h + c - t * (x + y) - P_0 - P_C$; utility function when purchasing Platform 0 and the Complementary Good (CG)

$U_{1\&CG} = h + c - t * (1 - x + y) - P_1 - P_C$; utility function when purchasing Platform 1 and the Complementary Good (CG). Note that his last equation is only valid when there is full compatibility between Firm 0's Complement and Firm 1's Platform. We will discuss next the utility function when this assumption does not
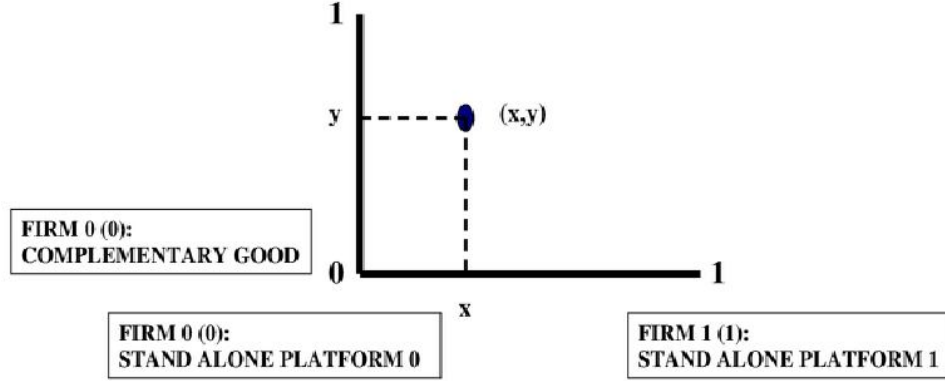
Figure 3.1: New consumer valuations for the two types of products (the platform and the complement)

hold true.

We further assume each firm has been in the market for at least one prior time period, so each is expected to have an installed base of preexistent consumers. If we think of the two firms as having been in the market for the same n time periods, each would have a legacy customer base of $\frac{n-1}{2}$. However, to make our results applicable to more general cases, we do not impose the restriction that the two firms have participated in the market for the same number of periods, but simply that, in the current time period, there is a total mass $A > 0$ of previous consumers who already own a platform from either of the two firms (but not both) and that Firm 0 has a mass M of legacy consumers, while Firm 1 has the other mass $A - M$ legacy customers. A legacy consumer is interested in the complement only if it is compatible with the durable product he already has. Otherwise, the preexistent consumer would have to buy both a new, compatible platform and the complementary good, should that bring him a net positive utility. Each legacy

82
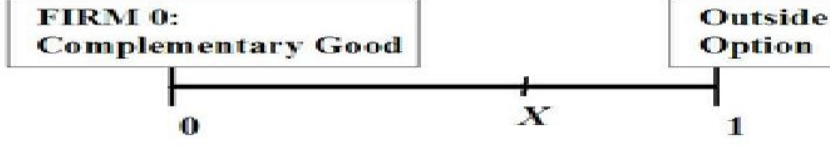
Figure 3.2: All legacy consumers' valuations for the complement under full compatibility / Two-product seller's consumers' valuations for the complement under incompatibility

consumer has a gross valuation c for the complement and faces a positive linear transportation cost of t per unit of distance.

If the platform the legacy consumer currently holds is compatible with the complement, then the legacy consumer located at z (see Figure 2) will incur a transportation cost of tx should he purchase the complementary good.

$$U_{CG}^{Legacy} = c - t * x - P_C$$

$$U_{OutsideOption} = 0$$

If the platform the legacy consumer currently owns is not compatible with the complement, then he must acquire the rival's platform should he want to purchase and be able to use the complement. Then, rival legacy consumers who are interested in the complementary good, must also acquire Firm 0's platform in order to be able to use the complement. We assume that once this purchase is made, the new Firm 0 platform becomes their primary use platform. This conjecture reflects a large

subset of real life choices consumers make: a buyer switching from an Android to an IPhone because of applications that are only compatible with the IPhone and that he values, will now start using the IPhone for all purposes (more likely than him carrying around two phones, the old one for every day use and the new one simply to use when he needs the complementary applications he wanted). A photographer purchasing a new camera and its compatible detachable lenses he values (and only work with the new camera he also purchased) will most likely start using this one as his primary camera. For simplicity, we do not account for the phone user or the photographer in our examples selling their previous platform in a secondary market and subsidizing their current period purchase with those funds.

A rival's legacy dislikes Firm 0's platform relative to Firm 1's, preference revealed in their previous period choice. The reasoning is as follows: in a previous period, when the complement was not yet on the market and consumers did not foresee its existence in a future period, a then new consumer had to choose between Firm 0 and Firm 1's platform. Assuming the same type of Hotelling differentiation, the classical result of each firm serving half of the market and offering a price equal to the transportation cost follows. Therefore, it is safe to assume that a present time rival platform owner was a consumer that purchased Firm 1's platform in a previous period, so his location on the unit interval is to the right of $\frac{1}{2}$, as shown in Figure 3a. Then, in order for him to purchase Firm 0's bundle, he must travel a longer distance to Platform 0, therefore the relative dislike for the compatible platform compared to the one he has previously bought. Moreover, given the independent distribution of valuations consumers have between the platform and the complement, a rival legacy

84
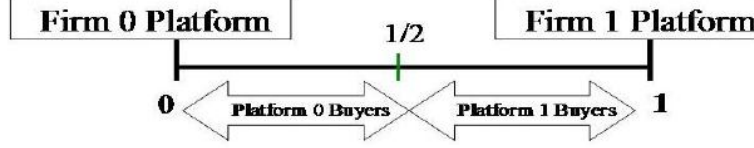
Figure 3.3: Legacy consumers' previous period choices

located at (x, y) in Figure 3b has the following utility function:

$U_{0\&CG}^{Rival's Legacy} = h - tx + c - ty - P_B$, where $x \geq \frac{1}{2}$ and P_B is the cost of

purchasing both the platform and the complement from Firm 0

or, written more intuitively based on Figure 3b,

$U_{0\&CG}^{Rival's Legacy} = h - t(\frac{1}{2} + X_R) + c - ty - P_B$

A rival legacy consumer will find Firm 0's bundle worth his time only if it

bring him a positive marginal utility over his current one from just using Firm 1's

platform

$U_1^{Rival's Legacy} = h - t(1 - x)$, where $x \geq \frac{1}{2}$,

or we can rewrite it as

$U_1^{Rival's Legacy} = h - t(\frac{1}{2} - X_R)$.

An important point to make is related to the two-product seller's ability to

identify and separate consumers into groups. If we are thinking of the complement

as something that must be installed on the platform during the purchase process, it

is reasonable to think of the complement buyer having to bring the platform at the

store. Then, the two-product seller will be able to tell if this consumer is his own

legacy or a rival platform owner. Note that if the consumer brings a rival platform,

there is no way for the two product seller to tell if he is dealing with a new consumer

85
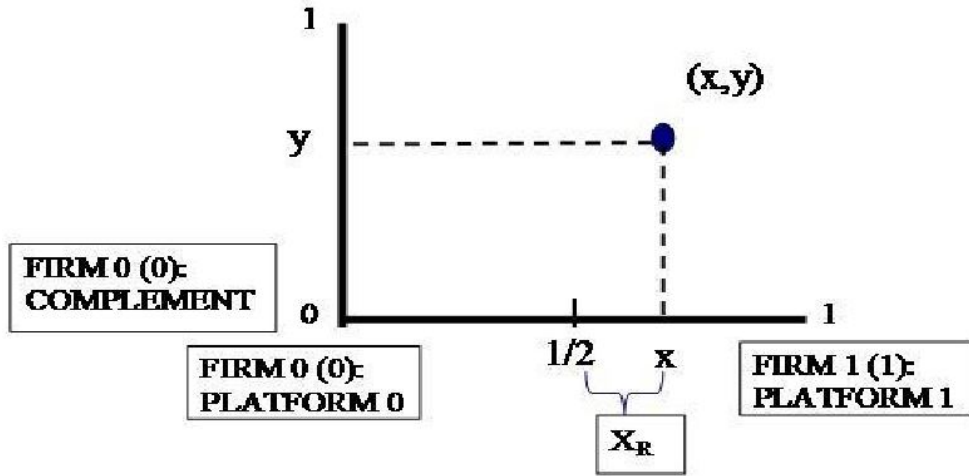
Figure 3.4: Rival's legacy consumers valuations for the bundle under incompatibility

who purchased the rival's platform in the current time period or a rival legacy. In the current paper I will work out both cases: the two-product seller being able to separate out his own legacies and not being able to and address any differences in results in each section.

The timing of the game is as follows: in Stage 1, Firm 0 (who puts the new, unique complementary good on the market) has to decide whether to make it compatible with its competitor's platform or not. We further assume this decision is irreversible. In Stage 2, Firm 0 chooses a pricing strategy: Individual Pricing (selling the Firm 0 product and the Complementary Good separately), Mixed Bundling (selling the two products together at a special price, but also offering them separately), Pure Bundling (offering the two products only as a package), offering Firm 0's platform and the Bundle only and offering the Complementary Good and the Bundle only. Some of these strategies are not rational depending on compatibility,

86

as we will show. In what follows, we will work backwards to solve for the market equilibrium and only look at the outcomes where the market is fully covered.

3.3 Analysis

3.3.1 Case I: Full Compatibility between the Complement and the Rival's Platform

Let us assume Firm 0 decides to produce a Complementary Good that is also compatible with its rival's product. In choosing a pricing strategy, Firm 0 must also consider the mass of legacy consumers, who, due to the full compatibility of the complementary product with both platforms, are only interested, if at all, in the Complementary Good (see Figure 2). We will show that the two-product firm is always better off offering the complement a la carte.

The two-product seller has the following pricing strategies to consider: offering the two product individually (Individual Pricing), selling them both individually and as a specially priced package/the platform only and the bundle/the complementary good only and the bundle (Mixed Bundling) and offering the two products as a bundle only (Pure Bundling). We note that whenever the complementary good is not available for individual sale, the legacy consumers would have to buy the bundle in order to obtain in, which includes Firm 0's platform (a product type they already own, either from Firm 0 or Firm 1 from a previous time period purchase).

Under **INDIVIDUAL PRICING,** Firm 0 offers the competing platform and

the complementary good individually at component prices. All legacy consumers are

be interested in the complement only and a positive mass of them will purchase it.

New consumers have the option to purchase a platform only, or a platform and the

complement, depending on what maximizes their utility. If the complement does not

have to be installed on the platform at the time of the purchase, the two-product

seller will only be able to separate consumers the following way: any consumer

purchasing the platform must be a new Firm 0 one, any customer purchasing the

bundle must be a new Firm 0 one and any customer purchasing the complement

could be either a new rival buyer, an own legacy or a rival legacy. If the platform

was needed upon purchase of the complement, the Firm 0 legacies could be teased

out. However, this makes no difference, since the two-product firm already offers

its complementary good at the monopoly price, thus highest possible profits on this

market segment are already attained. The platforms, strategic complements, are

sold at the identical Hotelling prices and share the market equally.

Remark 10 *When the two-product seller's complementary good and the rival's platform are compatible, an Individual Pricing strategy implies profit maximizing prices*

$$P_0^{IP\ Comp} = P_1^{IP\ Comp} = t \tag{3.1}$$

$$P_C^{IP\ Comp} = \frac{c}{2} \tag{3.2}$$

the new platform consumers market is shared equally

$$Pl_0^{IP\ Comp} = Pl_1^{IP\ Comp} = \frac{1}{2} \tag{3.3}$$

88

and there are

$$CG_{New}^{IP\ Comp} = \frac{c}{2t} \tag{3.4}$$

complement buyers and

$$CG_{Legacy}^{IP\ Comp} = A\frac{c}{2t} \tag{3.5}$$

legacy purchases. Each firm will make the following profits:

$$\Pi_0^{IP\ Comp} = \frac{(1+A)c^2}{4t} + \frac{t}{2} and \tag{3.6}$$

$$\Pi_1^{IP\ Comp} = \frac{t}{2} \tag{3.7}$$

Firm 0's profits are higher by the revenue made on the complement. Their profit is increasing in the mass A of legacy consumers and the valuation c for the complement. We note this case is identical to having three distinct firms each selling one of the products; there is no internalization of complementarity for Firm 0 under this pricing strategy.

We now turn to the **MIXED BUNDLING** pricing strategy and discuss the possible cases.

Case 1: Offering Each Product Individually and as a Specially Priced Package

When the typical mixed bundle is offered, again, all new consumers will choose between purchasing (i) an individual platform, (ii) the two-product seller's bundle or (iii) the rival's platform and the two-product seller's complement. Assuming the two-product seller chooses an optimal strategy, the bundle price would be higher than the complement's individual one; therefore, no consumer interested in the

89

complement alone would purchase the bundle and just discard of the platform. Then legacy consumers will buy, if anything, in the complement. The best strategy for Firm 0 is to capitalize on the legacy and rival platform consumers by selling the complement at its monopoly price. Again, following the logic above, there is no benefit to being able to sort out the Firm 0 legacy consumers. Moreover, the rival platforms are again sold at equal Hotelling prices and the new buyers market is shared equally between the two firms. Then, the bundle price is simply the sum of the component prices.

We note that it is not profitable for the two-product seller to offer a higher platform price in order to push the consumers to purchase the bundle instead (this would be a behavior specific to a two-market monopolist). A consumer currently indifferent between Firm 0's platform and its bundle would choose the bundle if such a platform price increase were to take place. A new buyer currently indifferent between Firm 0's platform and the rival's would then be prompted to purchase Firm 1's platform if Firm 0's price went up. Therefore, some current platform 0 consumers would purchase Firm 0's bundle, others would stick to Firm 0's platform, others would purchase the rival's platform and some would form their own bundle using the competing platform. Then, when the two types of products are compatible and consumers have independently distributed valuations for the two types of products, it is in the two-market seller's best interest to offer the same platform price as its rival.

Remark 11 *When the two-product seller's complementary good and the rival's plat-*

form are compatible, a Mixed Bundling strategy implies profit maximizing prices

$$P_0^{MB\ Comp} = P_1^{MB\ Comp} = t \tag{3.8}$$

$$P_C^{MB\ Comp} = \frac{c}{2} \tag{3.9}$$

The new platform consumers market is shared equally

$$Pl_0^{MB\ Comp} = Pl_1^{MB\ Comp} = \frac{1}{2} \tag{3.10}$$

and there are

$$CG_{New}^{MBComp} = \frac{c}{2t} \tag{3.11}$$

complement buyers and

$$CG_{Legacy}^{MB\ Comp} = A\frac{c}{2t} \tag{3.12}$$

legacy purchases. Each firm will make the following profits:

$$\Pi_0^{MB\ Comp} = \frac{(1+A)c^2}{4t} + \frac{t}{2}and \tag{3.13}$$

$$\Pi_1^{MB\ Comp} = \frac{t}{2} \tag{3.14}$$

Lemma 12 *When there is full compatibility between the two-product seller's complementary good and the rival's platform, Individual Pricing and Mixed Bundling (offering each product individually and as a specially priced package) are equivalent pricing strategies.*

The intuition for this result comes from the monopoly position the complementary product has and the separability of demand. Since the types of consumers

interested only in the complementary good are some of the legacy ones and some of the new ones purchasing the rival's platform, the two-product seller is best off offering the complement individually at its monopoly price. Moreover, by selling the platform at the Hotelling price, just as his competitor does, pricing the bundle as the sum of the individual prices makes the marginal consumer indifferent between the two platforms, but sure to purchase at least the complement from Firm 0.

Case 2: Offering the Platform Individually and the Complement Only as Part of the Bundle

With the complement not being available for individual purchase, the rival platform new consumers are no longer able to form the bundle themselves using Firm 1's platform and Firm 0's complement. This case is identical to one of incompatibility between the rival's platform and the complement. We note that any legacy consumer interested in the complementary good can only obtain it by purchasing the bundle, which includes Firm 0's platform. Since own legacy consumers already have a Firm 0 platform from a previous time period, they value a second unit at zero. The rival legacies have Firm 1's platform from a previous time period. They choose to purchase Firm 0's bundle as long as the utility they receive from using Firm 0's platform and the complement is higher than from simply using their current Firm 1 platform.

We first focus on the legacy consumers; they are of two types: owners of platform 0 and owners of platform 1. They have the following utility functions:

Firm 0 Legacy Consumers buying the bundle:

Figure 3.5: Compatibility: The two-product seller's legacy consumers' valuations for the bundle when the complement is not available individually

$$U_{1\&CG}^{F1Legacy} = c - t * z - P_B$$

Firm 1 Legacy Consumers buying Firm 0's bundle have the following utility function from it:

$$U_{0\&CG}^{Rival'sLegacy} = h - t(\tfrac{1}{2} + X_R) + c - ty - P_B$$

which we compare against their net utility from just using Platform 1, which they have purchased in a previous period

$$U_1^{Rival'sLegacy} = h - t(\tfrac{1}{2} - X_R)$$

The two-product seller cannot differentiate among the legacy consumers, since both make the exact same purchase, if any: the bundle. Then, there is no need to bring in their previous period purchased platform to install the complement, which is the only way a two-product seller could tell if a legacy is his own or the rival's. Moreover, the two-product firm cannot differentiate its own new consumer bundle buyers and the two types of legacies, since they all make the same purchase: the bundle.

In order for Firm 0 to attract *legacy consumers*, he would have to price its bundle below the classical mixed bundling price. The first thing to consider is
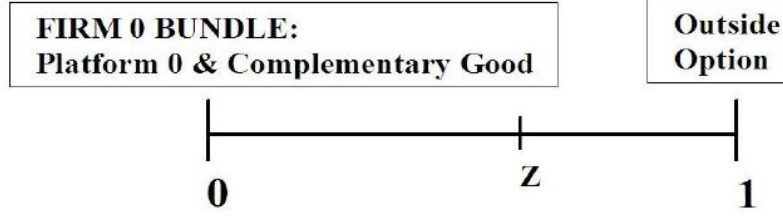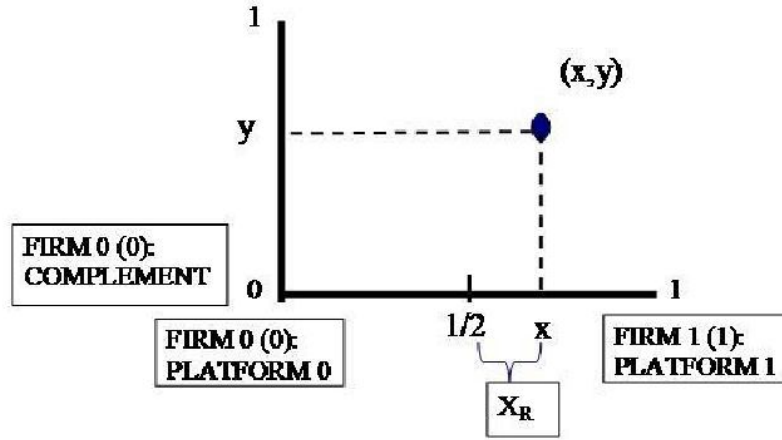
Figure 3.6: Compatibility: The rival's legacy consumers' valuations for the bundle when the complement is not available individually

whether to price based on its own legacy consumers only or target all legacies. When choosing the profit maximizing prices taking into consideration only the new consumer market and own legacies, the bundle price is influenced by the size of the its own legacy market. If the rival's legacies are also being targeted as potential buyers, the bundle price is dependent on the relative mass of its own legacy consumers to its rival's and lower due to the rival legacies' relative dislike for Firm 0's platform. To reiterate, the bundle price when reaching out to all legacies is the lowest compared to the first type of Mixed Bundling, followed by the bundle price when targeting only its own legacies (still below the Mixed Bundling value). Moreover, due to strategic complementarity, both platform prices decrease compared to their classical mixed bundle levels. Firm 0's platform and the bundle are also strategic complements, and the bundle views the rival's platform as a strategic complement too. Then, Firm 0's platform price is smaller than its competitor's and both below their classical mixed

94

bundling values. These lower prices are profit maximizing for the two-product seller (due to the complexity of the closed form solutions, they are not written out here).

If the two-product seller chooses to *only focus on the new consumers* and not worry about offering a bundle price to attract the legacy ones, then the two-product firm offers the bundle at a price equal to the sum of its platform price and the complement's monopoly one. The compatible platform's price is lower than its rival. Clearly, when Firm 0 lowers the price of one of its components (the platform, in this case), it attracts additional consumers who buy one or both components from the firm. The intuition is as follows: since the platform's price is a part of the bundle price, a lower platform price relative to its classical mixed bundling equivalent and to its rival's ensures a lower bundle price, both leading to more sales and higher profits. To better see that, let's start with the scenario of the two platform prices being equal, just like under the classical mixed bundling scenario and see the changes in choices when Firm 0 reduces its price further than Firm 1. The first thing we note is that since the complementary product still has a monopoly price (though not offered individually, the bundle price is the sum of Firm 0's platform and the complement), all the consumers who previously did not purchase the complement did not because it brought them a negative marginal utility. Given the implicit complement price has not changed, they are still not interested in the complement, since it still brings them negative utility. So a consumer that chose a platform only under the classical mixed bundling scenario, will choose a platform only during the current scenario.

A consumer who, under Mixed Bundling, is indifferent between purchasing

the platform from Firm 0 and Firm 1 will now go with Firm 0 when its platform price goes down more than Firm 1's. A customer otherwise indifferent between Firm 0's bundle and Firm 1's self-formed one will make a purchase from Firm 0 now. A customer indifferent between the bundle and the rival's platform opts for the former once it's price becomes lower (that can only happen if Firm 0's platform price becomes lower, since the monopoly complement price is fixed). However, a buyer indifferent between Firm 0's platform and its bundle sees not change when the platform price is reduced. Firm 0 affords to offer a lower platform price since the platform is not its only source of profits.

Remark 13 *When the two-product seller's complementary good and the rival's platform are compatible, a strategy where only the platform is available individually and only the new consumers are being targeted yields profit maximizing prices*

$$P_0^{Pl\&Bdl} = -\frac{c^2}{6t} + t \quad < \quad P_1^{Pl\&Bdl} = -\frac{c^2}{12t} + t \tag{3.15}$$

$$P_B^{Pl\&Bdl} = \frac{c}{2} - \frac{c^2}{6t} + t \tag{3.16}$$

More new consumers purchase Firm 0's platform

$$Pl_0^{Pl\&Bdl} = \frac{1}{24}(12 + \frac{c^2}{t^2}) > Pl_1^{Pl\&Bdl} = \frac{1}{24}(12 - \frac{c^2}{t^2}) \tag{3.17}$$

Firm 0 will sell its complement to

$$CG_{new}^{Pl_1} = \frac{1}{24}(12 + \frac{c^2}{t^2})\frac{c}{2t} \tag{3.18}$$

new complement consumers Each firm will make the following profits:

$$\Pi_0^{Pl\&BdlComp} = \frac{(c^2 + 12t^2)^2}{288t^3} and \tag{3.19}$$

$$\Pi_1^{Pl\&BdlComp} = \frac{(c^2 - 12t^2)^2}{288t^3} \tag{3.20}$$

If there is a relatively high mass of legacy consumers, offering the platform and the complement bundle at a lower price that would attract consumers from this market segment is optimal.

This optimal price choice is quite different from a more common one in the bundling literature: a two-product monopolist will increase the price of its components to push more consumers to purchase the bundle. We argue that the departure from such a strategy can be explained by (i) the independent valuations a consumer has for the two types of products and (ii) the presence of a competing platform. By increasing its platform's price, the two-product seller will see some of its platform only consumers shifting to purchasing the bundle, some continuing to purchase the platform only and some switching to the rival's platform. However, sticking to the current pricing scheme brings Firm 0 more than half of the new consumer market and highest profits. While prices are decreasing in the complement valuation c, profits increase in it at a higher rate for Firm 0.

We note that in this case it is worth reaching out to the legacy consumers with the low priced bundle as long as there is a relatively high mass of them or they have a high valuation for the complementary product. Moreover, Firm 0 should reach out to all if most legacies are rival platform owners and only to its own otherwise. However, by not offering the complement alone, Firm 0 forgoes monopoly profits from both its own and rival legacies and the rival's new consumers. Thus, such a strategy can never bring the two-product seller higher profits than a strategy where the monopoly product is sold individually.

Case 3: Offering the Complementary Good Individually and the Firm 0 Platform as Part of the Bundle

All legacy consumers now have the option to purchase the only product on the market they have an interest in: the complement. All new consumers choose between (i) Firm 0's bundle, (ii) Firm 1's platform and (iii) Firm 1's platform and Firm 0's complementary good. Firm 0's bundle is a strategic complement to both the rival's platform and the monopoly product. Firm 1's platform is a strategic complement to its rival's bundle, but views the complement as a strategic substitute. The consumers have independent valuations between the platform and the complementary good. When only offering the bundle, Firm 0 must be able to attract consumers who may have a high valuation for its platform but a very low one for the complement (those who were previous platform 0 only buyers under mixed bundling). Before, those type of consumers would have been Firm 0 platform buyers, but since they no longer have that option, Firm 0 attempts to keep them as customers by offering a cheaper priced bundle compared to the mixed bundling bundle price. Due to the strategic complementarity, Firm 1 lowers its platform price too. Then, Firm 0's optimal strategy is to offer a complement price such that all new rival platform consumers would purchase it. Then, between the Firm 0 bundle buyers and the Firm 1 consumers who form their own bundle, all new consumers buy the complement. The bundle price is simply the sum of the rival's platform price (Hotelling price) and the complement, thus having the two firms share the new buyers's market equally. Therefore, the price a new consumer pays for the

bundle is the same whether he obtains it from the Firm 0 or forms it himself using Firm 0's complement and Firm 1' platform. Both the bundle price and the price of the complementary product are lower than in the case of the Pure Unbundle and the Mixed Bundle.

Remark 14 *When the two-product seller's complementary good and the rival's platform are compatible, a strategy where only the complement is available individually implies profit maximizing prices*

$$P_B^{Bdl\&CG} = c > \quad P_1^{Bdl\&CG} = t \tag{3.21}$$

$$P_C^{Bdl\&CG} = c - t \tag{3.22}$$

More new consumers purchase Firm 0's platform

$$Pl_0^{Bdl\&CG} = Pl_1^{Bdl\&CG} = \frac{1}{2} \tag{3.23}$$

Firm 0 will sell its complement to all new consumers

$$CG_{new}^{Bdl\&CG} = 1 : \quad CG_{new}^{Pl_1} = CG_{new}^{Pl_2} = \frac{1}{2} \tag{3.24}$$

and all legacies

$$CG_{Legacy}^{Bdl\&CG} = A \tag{3.25}$$

Each firm will make the following profits:

$$\Pi_0^{Bdl\&CGComp} = c + A(c - t) - \frac{t}{2} and \tag{3.26}$$

$$\Pi_1^{Bdl\&CGComp} = \frac{t}{2} \tag{3.27}$$

The two-product firm would benefit from being able to separate out its own legacies and offer them the complement at the monopoly price. These consumers would have no incentive to cheat and just purchase a new bundle, since the price of a new bundle is higher than the monopoly price of the complement. Firm 0 would be better off when selling a complement that required installation on the platform at the time of purchase.

An immediate observation for **PURE BUNDLING** is that the two-product seller will not be able to make a sell to the new consumers interested in the rival platform, since the complementary product is not for individual sale. When Firm 0 offers its platform and complementary product together only and targets only the new consumer market, the outcome is indenticaly with the one in Case 3 under Mixed Bundling, when the two-product seller was offering the bundle only to its new consumers, he is effectively requiring the legacy consumers to purchase its platform too in order to have access to the complement or just not make a purchase at all. The argument is similar to the Case 2 under Mixed Bundling. If the two-product seller is planning to target the legacy consumers, he must take into account there are two types: its own and its rivals', the latter having a relative dislike for Firm 0's platform. He will offer a very low bundle price, depending on the relative size of the legacy market, in order to entice some of the legacy consumers to buy the bundle (note that the new platform that is part of the bundle is of no individual value to the legacy consumers, as they already have the one of their choice from a previous time period). Again, the legacy consumers' valuations are depicted in

Figures 4 and 5, together with their corresponding utility functions. We find that Firm 0 considers targeting all legacies when its own legacy consumer mass is a small share of the legacy market.

Alternatively, when Firm 0 is only focusing on the new consumers, the bundle price is higher. The market outcome is identical to the one when Firm 0 offers the complement individually and cannot differentiate its own legacies from the rival's new and legacy consumers.

Remark 15 *When the two-product seller's complementary good and the rival's platform are compatible, a strategy where the two types of products are available individually (***Pure Bundling***) but **only** the **new consumers** are **targeted** implies profit maximizing prices*

$$P_B^{PBOwn} = \frac{c+2t}{3} > P_1^{PB} = \frac{-c+4t}{3} \tag{3.28}$$

the new platform consumers market is shared

$$Pl_0^{PB} = \frac{1}{6}(2 + \frac{c}{t}) \ , \ Pl_1^{PB} = \frac{2}{3} - \frac{c}{6t} \tag{3.29}$$

and there are

$$CG_{new}^{Pl_1} = \frac{1}{6}(2 + \frac{c}{t}) \tag{3.30}$$

complement buyers and Each firm will make the following profits:

$$\Pi_0^{PB} = \frac{(c+2t)^2}{18t} and \tag{3.31}$$

$$\Pi_1^{PB} = \frac{(c-4t)^2}{18t} \tag{3.32}$$

If there is a relatively high mass of legacy consumers, offering the platform and

the complement bundle at a lower price that would attract some consumers from this market segment is optimal.

Then, it is worth reaching out to the legacy consumers with the low priced bundle as long as there is a relatively high mass of them or they have a high valuation for the complement. However, not offering the complement alone can never bring the two-product seller highest profits: not making monopoly profits off the complement in the legacy consumer market and on the competitor's new consumers is not a profit maximizing strategy.

Firm 0's profits are only higher due to being able to sell the monopoly complement to its rival's consumers and its own legacies. The more legacy consumers (A), the higher the profits to be made. Higher consumer valuations for the complement c also leads to a higher monopoly price and higher profits for Firm 0. We have previously argued that it is essential for the two-product seller to offer the complement alone and sell it at the monopoly price. Any pricing strategy when this is not done, either completely ignores legacy consumers and the monopoly profits these bring or offers very low prices in order to determine these consumers to purchase the complement as part of a bundle, where the other bundle component is a platform (which legacy consumers already own from a previous purchase and have no value for a second unit of it). Moreover, such a pricing strategy also prevents the rival's new consumers from additionally purchasing the complement.

Proposition 16 *When the two-product seller's complementary good and the rival's platform are compatible, the two-product seller's optimal strategy is to offer the plat-*

form and the complement individually.

3.3.2 Case II: Incompatibility between the Complement and the Rival's Platform

We look at two scenarios: (i) the two-product seller reaches out only to its own legacy consumers and (ii) the two-product seller makes its complement attractive to the rival's legacy consumers too. It is important to note that in the second case, the rival platform's legacy consumers must also purchase and use Firm 0's platform in order to be able to enjoy the complementary product. These legacy consumers do not own Firm 0's platform because in a previous time period they chose Firm 1's over Firm 0's, due to their relative distaste for Firm 0's. As previously shown in Figure 3b, we incorporate this feature into their utility function, by assuming that their valuation for Firm 0's platform is lower to begin with ($U_{0\&CG}^{Rival'sLegacy} = h - t(\frac{1}{2} + X_R) + c - ty - P_B$).

3.3.2.1 A: Targeting only its Own Legacy Consumers

Let us start with the first case, where **the two-product seller only targets its own legacy consumers** as potential buyers of the complement. Due to incompatibility between the complementary product and the rival's platform, any new consumer that makes a purchase from Firm 1 will not be able to use the complement. The only potential buyers of the complement are Firm 0's legacy consumers and its new consumers. We argue it is always best to offer the complementary

product for individual sale.

The two-product seller considers the same potential pricing strategies as before: Individual Pricing, Mixed Bundling and Pure Bundling. In all but the individual pricing case, the two-product seller is now able to separate out the new consumers and own legacies. Firm 0 knows that any consumer purchasing the complement alone is a legacy, so charging the monopoly price for this product is profit maximizing. Therefore, the two-product seller finds it optimal to always offer the complement alone at the monopoly price.

When the two-product seller offers the goods separately (**INDIVIDUAL PRICING**), legacy consumers are interested in the complement only, if at all; its new consumers may purchase the platform or both the platform and the complement, while the rival's new consumers make no purchase from the two-product seller due to incompatibility between the complement and Firm 1's platform. Compared to the full compatibility case, both Firm 0 and Firm 1 reduce the price of their platforms, with Firm 0 doing it the most, while still offering the complement at its monopoly price. As expected, by lowering the price of one component, Firm 0 attracts more platform or platform and complement consumers. We will now compare and contract the full compatibility individual pricing case with the current one by focusing on the marginal consumers the two firms compete for. A consumer that used to be indifferent between Firm 0's bundle and Firm 1's self-formed bundle will now become a Firm 0 bundle buyer. A new customer previously indifferent between Firm 0's bundle and Firm 1's platform will now purchase Firm 0's bundle. A buyer

who, under compatibility, is indifferent between Firm 0's platform and Firm 1's, will now purchase Firm 0s. The new consumers who would otherwise be indifferent between Firm 0's platform and the Firm 1's self-formed bundle, will go with Firm 0's platform since (i) it is priced lower and (ii) a self-formed bundle using Firm 1's platform and Firm 0's complement is no longer possible due to incompatibility. Any consumer previously indifferent between Firm 0's bundle and its platform does not change his preferences. Lastly, any new buyer that used to be indifferent between Firm 1's self-formed bundle and Firm 1's platform, no longer has the option of the former and may purchase either platform. Since Firm 0 can no longer sell the complement to the rival's new consumers, it is optimal for them to determine as many new buyers as possible to purchase its platform, thus also increasing the number of sales of the complement. Firm 1 cannot afford to decrease its price by the same amount, given that the platform is its only source of profits.

Under compatibility, any consumer who purchased a platform only had a negative marginal valuation from buying the complement. Since the complement is still priced as a monopoly product, its price does not change under incompatibility. Then, all previous platform only consumers will remain platform only consumers. However, since Firm 0's platform price is lower, more will choose its product over the rival's. Those consumers who, under compatibility, formed their own bundle using Firm 0's complement and Firm 1's platform, now no longer have this option. By reducing the price of its bundle under incompatibility via a reduction in the price of the platform, Firm 0 attracts most of the rival's previous bundle formers. The remainder become Firm 1 platform consumers.

105

Remark 17 *When the two-product seller's complementary good and the rival's platform are incompatible, an Individual Pricing strategy yields profit maximizing prices*

$$P_0^{IP \ IncompOwn} = -\frac{c^2}{6t} + t \quad < \quad P_1^{IP \ IncompOwn} = -\frac{c^2}{12t} + t \tag{3.33}$$

$$P_C^{IP \ IncompOwn} = \frac{c}{2} \tag{3.34}$$

the new platform consumers market is shared equally

$$Pl_0^{IP \ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2}) > Pl_1^{IP \ IncompOwn} = \frac{1}{24}(12 - \frac{c^2}{t^2}) \tag{3.35}$$

and there are

$$CG_{New}^{IP \ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2})\frac{c}{2t} \tag{3.36}$$

new complement buyers and

$$CG_{OWNLegacy}^{IP \ IncompOwn} = M\frac{c}{2t} \tag{3.37}$$

own legacy purchases. Each firm will make the following profits:

$$\Pi_0^{IP \ IncompOwn} = \frac{c^4 + 24c^2(1 + 3M)t^2 + 144t^4}{288t^3} + max\{0, \Pi_0^{RivalLegacy}\} \ and \tag{3.38}$$

$$\Pi_1^{IP \ IncompOwn} = \frac{(c^2 - 12t^2)^2}{288t^3} \tag{3.39}$$

While the two-product seller does not target rival legacies directly, some may still find the bundle worth their time, as shows in the profit function extra term $+ max\{0, \Pi_0^{RivalLegacy}\}$. Prices are both decreasing in the complement valuation c, with Firm 0 having the most intense negative reaction to any positive changes. Both prices are increasing in the degree of differentiation t. The competing platforms no longer share the market equally, but Firm 0 becomes the leader platform. Profits are

106

substantially lower for both firms compared to the compatibility case, but increasing in the valuation c for the complement. Note that there would be no benefit to Firm 0 to be able to separate out the new and legacy consumers; they are viewed as a homogenous group of consumers when it comes to the complementary product target market.

Under **MIXED BUNDLING,** the two-product seller is able to discriminate against the new consumers and own legacies. Firm 0 knows that any consumer purchasing the complement alone is a legacy, so charging the monopoly price for this product is profit maximizing. We note that it makes no difference to the two-product seller if the legacy consumer can be identified as own or not (we had previously assessed that it could be the case that a legacy must bring his platform at time of purchase for the complement to be professionally installed), since we assume in this section that the firm only focuses on targeting its own legacy consumers.

Case 1: Offering both product individually and as a package

Legacy consumers interested in the complementary product will be able to purchase it individually. All new consumers must choose between Firm 1's platform and Firm 0's platform or bundle. As we have argued above, due to incompatibility, platform prices are lower, with Firm 0 reducing its price the most. Due to the two-product seller's ability to separate the new and legacy consumers and discriminate between the two, the complement is sold at its monopoly price. The two-product seller's bundle price is the sum of the monopoly price for the complement and platform price, the latter being lower than the rival's price for the platform. Therefore,

we can rule out a case where the bundle price would be below the complement's, thus determining the legacies to purchase the bundle instead and discard of the extra platform, as this is not profit maximizing for the two-product seller.

Remark 18 *When the two-product seller's complementary good and the rival's platform are incompatible, a Mixed Bundling strategy implies profit maximizing prices*

$$P_0^{MB\ IncompOwn} = -\frac{c^2}{6t} + t \quad < \quad P_1^{MB\ IncompOwn} = -\frac{c^2}{12t} + t \tag{3.40}$$

$$P_C^{MB\ IncompOwn} = \frac{c}{2} \tag{3.41}$$

the new platform consumers market is shared equally

$$Pl_0^{MB\ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2}) > \ Pl_1^{MB\ IncompOwn} = \frac{1}{24}(12 - \frac{c^2}{t^2}) \tag{3.42}$$

and there are

$$CG_{New}^{MB\ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2})\frac{c}{2t} \tag{3.43}$$

complement buyers and

$$CG_{OWNLegacy}^{MB\ IncompOwn} = M\frac{c}{2t} \tag{3.44}$$

own legacy purchases. Each firm will make the following profits:

$$\Pi_0^{MB\ IncompOwn} = \frac{c^4 + 24c^2(1 + 3M)t^2 + 144t^4}{288t^3} + max\{0, \Pi_0^{RivalLegacy}\} and \tag{3.45}$$

$$\Pi_1^{MB\ IncompOwn} = \frac{(c^2 - 12t^2)^2}{288t^3} \tag{3.46}$$

While the two-product seller does not target rival legacies directly, some may still find the bundle worth their time, as shows in the profit function extra term $+ max\{0, \Pi_0^{RivalLegacy}\}$. We note that the two-product monopolist strategy of offering

108

the components at higher prices to determine more consumers to purchase the bundle does not apply here. Since consumers have independently distributed valuations for the two types of products, offering the Firm 0 platform at a higher prices will determine some of the current platform 0 only consumers to either purchase the bundle or continue purchasing the platform from Firm 0 or buy Firm 1's platform instead. Thinking at the margin, a consumer who is currently indifferent between Firm 0's bundle and their platform would purchase the bundle is the platform price increased. A new customer indifferent between Firm 0's platform and Firm 1's will go with the latter after the former's price increase. Then, the two-product seller is better off offering the current pricing scheme, where its platform's price is lower than its rival's and the bundle price is the sum of the components (with the complement being offered at the monopoly price).

The market outcome we saw under compatibility in Lemma 1 exists here too: platform prices are identical to their Individual Pricing equivalents and the bundle price is the sum of the individual prices. The outcome is identical to the Individual Pricing case.

Lemma 19 *When there is incompatibility between the two-product seller's complementary good and the rival's platform, Individual Pricing and Mixed Bundling (offering each product individually and as a specially priced package) are equivalent pricing strategies.*

Case 2: Offering the complement individually and the bundle

The new consumers choose between Firm 0's bundle and the rival's platform.

109

The legacy consumers can purchase the complementary product separately, should they be interested in it. Therefore, the complement is sold at its monopoly price. The bundle price is much lower than its counterpart under compatibility, since Firm 0's only source of profit from the new consumer is the bundle, as the complementary good can no longer be sold to the rival platform's new users, due to incompatibility. The consumers have independent valuations between the platform and the complementary good. When only offering the bundle, Firm 0 must be able to attract consumers who may have a high valuation for its platform but a very low one for the complement. Before, those type of consumers would have been Firm 0 platform buyers, but since they no longer have that option, Firm 0 attempts to keep them as customers by selling them a cheaper priced bundle compared to the mixed bundling bundle price.

We find it is optimal to charge a bundle price higher than the complement's, therefore expecting no legacy would purchase the bundle and discard the platform in order to get a better deal on the complement. Firm 1's platform is also cheaper than under classical mixed bundling and under the same scenario assuming compatibility, as the two products are strategic complements. More new consumers purchase the bundle, but profits for both firms are lower than under compatibility. It is important to note that the bundle price is less than the sum of the individual prices under individual pricing and the first type of mixed bundling: since Firm 0 is no longer offering the platform, offering a lower bundle price is the best strategy to attempt to keep most of its previous platform consumers from going with the rival's bundle and to attract new consumers that used to purchase the rival's bundle.

Remark 20 *When the two-product seller's complementary good and the rival's platform are incompatible, offering the platform only as part of the bundle yields profit maximizing prices*

$$P_B^{Bdl\&CG\ IncompOwn} = \frac{c+2t}{3} > P_1^{Bdl\&CG\ IncompOwn} = \frac{-c+4t}{3} \tag{3.47}$$

$$P_C^{Bdl\&CG\ IncompOwn} = \frac{c}{2} \tag{3.48}$$

the new platform consumers market is shared

$$Pl_0^{Bdl\&CG\ IncompOwn} = \frac{1}{6}(2 + \frac{c}{t}) \ , \ Pl_1^{Bdl\&CG\ IncompOwn} = \frac{2}{3} - \frac{c}{6t} \tag{3.49}$$

and there are

$$CG_{new}^{Pl_1} = \frac{1}{6}(2 + \frac{c}{t}) \tag{3.50}$$

complement buyers and

$$CG_{OWNlegacy}^{Bdl\&CG\ IncompOwn} = M\frac{c}{2t} \tag{3.51}$$

own legacy purchases. Each firm will make the following profits:

$$\Pi_0^{Bdl\&CG\ IncompOwn} = \frac{c^2(2+9M)+8ct+8t^2}{36t} + max\{0, \Pi_0^{RivalLegacy}\} \ and \tag{3.52}$$

$$\Pi_1^{Bdl\&CG\ CompOwn} = \frac{(c-4t)^2}{18t} \tag{3.53}$$

While the two-product seller does not target rival legacies directly, some may still find the bundle worth their time, as shows in the profit function extra term $+ max\{0, \Pi_0^{RivalLegacy}\}$. Moreover, Firm 1 charges its highest price when its competitor offers the platform as part of the bundle, since it is now the only firm to offer the platform individually and, thus, is more differentiated from Firm 0's product. Then,

111

a consumer that was indifferent under mixed bundling between the bundle and Firm 1's platform will still purchase the bundle since the bundle price decreased while the platform 1 one increased. A consumer previously indifferent between the competing platforms will now choose Firm 1's. Both firm's profits are increasing in the valuation of the complement c. Firm 1's profits directly change with the size of its own legacy consumers mass.

Proposition 21 *When (i) the two-product seller's complementary good and the rival's platform are incompatible and (ii) the two-product seller targets its own legacy consumers only, he will maximize profits by offering the platform and the complement individually (Individual Pricing). All profits are lower compared to the full compatibility case.*

While the two-product seller does not target rival legacies directly, some may still find the bundle worth their time, as shows in the profit function extra term $+ \, max\{0, \Pi_0^{RivalLegacy}\}$. Looking at Remark 6, the higher the valuation c for the complement, the lower the price Firm 0 will charge for its platform (and Firm 1 will decrease its price too due to the strategic complementarity, but by a smaller amount, as discussed before). Since any new consumer can only use the complement if he also buys the compatible platform, Firm 0's strategy is to attract as many new consumers as possible to purchase its platform, thus indirectly increasing the number of complement sales. Since a high complement valuation means more consumers interested in the complement and a higher complement price, the two-product seller has a strong incentive to reduce its platform price in order to attract those new

customers to purchase both Firm 0 products. Profits are also increasing in c. A higher mass of own legacies M will also increase the number of sales made to them and Firm 0's profit level. Finally, as we have previously discussed, offering the complement alone will always increase profits since the two-product seller will be able to offer the product at a monopoly price and make the highest attainable profits on his own legacy consumers.

3.3.2.2   B: Targeting All Legacy Consumers

We now turn to the second case, where **the two-product seller not only targets its own legacy consumers** as potential buyers of the complementary product, **but also its rival's**. The rival's legacy consumers who are interested in the complementary good, must also acquire Firm 0's platform in order to be able to use the complement. However, they dislike Firm 0's platform relative to Firm 1's, feature that is reflected in their previous period choice (they did purchase a platform from Firm 1 previously and not Firm 0) and in their current period utility function (see Figure 3b):

$$U_{0\&CG}^{Rival's Legacy} = h - t(\tfrac{1}{2} + X_R) + c - ty - P_B$$

A two product seller can never separate out a new consumer purchasing both its platform and the complement from a rival's legacy, as the latter must make the same purchase, but has a lower utility for it. While in the Individual Pricing scenario, we cannot identify Firm 0's legacies, under all other pricing strategies, the two-product seller can price discriminate against this group (being the only type of

113

consumers buying the complement only) by always offering the complement at the monopoly price. We do find that, in certain situations, the bundle price is lower than the monopoly complement's, which determines the Firm 0 legacies to simply purchase the bundle and discard of the extra platform. We note that having to have the platform present at the time of purchase of the complement in order to install the complement (mechanism we had previously argued could help identify a legacy consumer purchase the complement only) does not help the two-product seller separate out its own legacies, as these are better off showing up empty handed (as if they did not own a platform already) and just purchasing the bundle, like any other new or rival legacy consumer.

Under **INDIVIDUAL PRICING,** the new consumers choose between Firm 0's platform, Firm 0's platform and complementary product and Firm 1's platform. Firm 0's legacy consumers are the only ones interested in the complement alone, if anything. The rival's legacy consumers either purchase both Firm 0's platform and the complement or nothing. Since the rival's legacy consumers must purchase both Firm 0's products, both prices depend on the relative magnitude of the legacy consumer masses of Firm 0 and Firm 1. We note that the two-product seller is not able to tell if a bundle buyer is a new one or a rival's legacy.

Being able to identify its own legacies does make a difference in this scenario. Assuming a legacy must bring his platform when purchasing the complement for it to be installed, the two-product seller could price discriminate against this group and charge them the monopoly price for the complement. This result holds if the sum

of the components is higher than the monopoly price of the components, which is always true when we look at the profit maximizing outcome in this case. However, if identifying its own legacies is not an option for Firm 0, these become a homogenous group with the new Firm 0 consumers of both products and the rival legacies. We make a note that in the previous section, we showed that by targeting only its own legacy consumers, Firm 0 offered the complement at its monopoly price. Reaching out to its rival's legacy consumers changes its incentives. The complement is a strategic substitute of the compatible platform and a strategic complement of the rival's product. Its price directly depends on the ratio of legacy consumers by firm: the more legacy consumers that belong to the two-product seller, the closer the complement price is to its monopoly value (fewer rival legacy consumers to target and lower the price for).

Again, due to the attention given to the rival's legacy consumers, Firm 0's platform price also decreases the higher the ratio of the rival's legacy consumers in the market. While Firm 1 makes no sale to any legacy consumers, due to strategic complementarity, its platform price indirectly depends on their ratio. Profits are the lowest for both firms compared to all other Individual Pricing scenarios and (in)compatibility cases due to them attempting to attract the rival's legacy consumers. Moreover, profits are higher when the two-product seller can separate out its own legacies. He would then charge them the monopoly price for the complement (due to the complexity of the closed form solutions, they are not written out here).

When offering a **MIXED BUNDLE**, the only type of consumers interested

in the complement only are Firm 0's own legacy consumers. Then, the two-product seller can yet again offer the complement at its monopoly price and make the highest possible profits on their own legacy consumers. This outcome will take place if there is a substantial mass of own legacy consumers in the market. If the legacy market is dominated by the rival's legacies, the bundle price is often lower than the complement monopoly price. Then, any own legacy has an incentive to simply purchase the bundle instead and discard of the platform. The rival's legacy consumers will purchase, it anything, the bundle.

Case 1: Offering both product individually and as a package

The only type of consumers interested in individual platforms only are the new ones. Some will prefer Firm 0's platform, some Firm 1's. Other new consumers will go for Firm 0's bundle. Then the platforms, strategic complements, are sold at higher prices than under individual pricing, since they do not have to be affordable to the rival's legacy consumers. Firm 1's platform is priced higher compared to its rival's since it is its only source of profit. However, the bundle price is lower than under individual pricing as to capture that previously marginal new consumers who was undecided between buying Platform 0 or the Firm 0 bundle, as well as to attract more of the rival's legacy consumers. Compared to individual pricing, any new consumer indifferent between Firm 0's platform and the bundle will now purchase the bundle (higher platform 0 price, lower bundle price). A new customer previously indifferent between Firm 0's platform and Firm 1's will go with Firm 0's due to the lower price. Any new buyer who, under individual pricing, used to be indifferent

116

between the bundle and Firm 1's platform will now choose the bundle, priced lower than before and relatively cheaper than Firm 1's platform.

Since the rival's legacy consumers are an important part of the market, the bundle price is a function of the distribution of the legacy consumers among the two firms and it increases the fewer legacy consumers belong to the rival. Due to strategic complementarity, higher percentage of own legacies also increases the platform prices. The intuition for this result is as follows: the more of the legacy market is composed of the two-product seller's legacies M, the more important these ones become, the less the two-product seller has to worry about attracting the rival's legacies and accounting for their dislike for the platform 0 part of the bundle and the more weight Firm 0 puts on its own preexistent consumers when determining its optimal pricing. Then, the prices converge towards the same classical mixed bundling ones from the previous section (when targeting only its own Firm 0 legacies). All prices then increase as the share of the the rival's legacies in the market decreases, but never quite reach the levels they did when only focusing on its own legacies.

We have previously shown that when Firm 0's legacies are the only ones in the market interested in the complement only, by offering the complement alone, the two-product seller may discriminate against them by selling it at the monopoly price. We now discuss the circumstances under which Firm 0 legacies have an incentive to make a different purchase than the complement and/or hide their identify. We have argued the two-product seller is interested in selling its bundle both to its new customers and rival legacies; when there is a relatively large mass of rival

legacy consumers, it is optimal for him to choose a bundle price that is below the complement's monopoly price. Then, its own legacy consumers choose to purchase the bundle themselves rather than just the complement and just discard of the extra platform. There is no way for the two-product firm to identify its own legacies in this scenario; we had previously argued that if the complement must be installed on the platform at the time of purchase, a legacy consumer buying the complement only would bring his platform with him and the two-product seller would know this is his own legacy customer. But in this scenario, a Firm 0 legacy could do better by going to the store without their platform and pretending to be either a new or a rival legacy in order to get the better bundle price and the two-product seller would not be able to identify him as its own legacy. When the legacy market is small to begin with and with most of it being dominated by its own legacies, the rival legacies become of little importance. The complement is sold and bought at the monopoly price, the bundle price is higher than the complement's and all prices are at their highest. Higher valuations c for the complement also leads to higher prices: it directly increases the bundle prices and complement price and indirectly, via the strategic complementarity, the platform prices increase.

All prices and Firm 1 profits are lower (due to the complexity of the closed form solutions, they are not written out here) compared to the scenario of Firm 0 reaching out only to its own preexistent customers. Firm 0 does makes higher profits than when reaching out to its own legacy consumers only, as long as its own preexistent customer base is relatively small.

Case 2: Offering the complement individually and the bundle

Firm 0's bundle buyers are both the rival's legacies and the new consumers. The bundle price is lower than in all other cases and under all other marketing strategies, as it must account for (i) the rival's legacy consumer who have a relative dislike for Firm 0's platform that they must also buy in order to be able to use the complement and (ii) the new consumers who previously only purchased Firm 0's platform and not the bundle and whom the two-product seller would like to incentivize to now purchase the bundle instead. Firm 1 offers its highest platform price from all other pricing strategies when assuming Firm 0 targets all legacies. Then, the consumer who, under mixed bundling, was indifferent between Firm 0's bundle and Firm 1's platform will purchase the former. A customer previously indifferent between Firm 0's platform and bundle may now either go with the bundle or the rival's platform. The bundle price increases as fewer rival legacy consumers exist, since the two-product seller now focuses on the relatively higher segment of the market, the new consumers.

In the beginning of the section we mentioned that Firm 0's legacies would only be interest in the complement under the current marketing strategy. However, when Firm 0's own legacies are a small share of the legacy market, the focus is on the rival's legacies. Then the bundle price will be at its lowest and below the complement's monopoly price. Any Firm 0 legacy will cheat by purchasing the bundle at the relatively lower prices. Their strategy would be to enter the store without their platform and pretend to be either a new or a rival legacy in order to get the better

bundle price. The two-product seller would not be able to identify them as their own legacy. But once Firm 0's legacies become the lion' share of the legacy market, the bundle price increases as little attention is given to the rival legacies and the focus is to extract most surplus from the new and own legacy consumers. The Firm 0 legacies are then better off simply purchasing the complement.

Remark 22 *When the two-product seller's complementary good and the rival's platform are incompatible, a strategy targeting all legacy consumers with the platform only available as part of the bundle yields profit maximizing prices*

$$P_B^{Bdl\&CG\ IncompAll} = \frac{c(-1-A+M)+4t+3(A-M)t}{3+4A-4M} > \tag{3.54}$$

$$P_1^{Bdl\&CG\ IncompAll} = \frac{c(1+2A-2M)+2t-2t(A-M)}{3+4A-4M} \tag{3.55}$$

$$P_C^{Bdl\&CG\ IncompAll} = \frac{c}{2} \tag{3.56}$$

the new platform consumers market is shared

$$Pl_0^{Bdl\&CG\ IncompAll} = \frac{c(1+A-M)+2t+5(A-M)t}{2(3+4A-4M)} > \tag{3.57}$$

$$Pl_1^{Bdl\&CG\ IncompAll} = \frac{c(-1-A+M)+4t+3(A-M)t}{2(3+4A-4M)} \tag{3.58}$$

and there are

$$CG_{new}^{Pl_1} = \frac{c(1+A-M)+2t+5(A-M)t}{2(3+4A-4M)} \tag{3.59}$$

new complement buyers and

$$CG_{OWNlegacy}^{Bdl\&CG\ IncompAll} = M\frac{c}{2t} \tag{3.60}$$

own legacy purchases and

$$RL_{0\&CG} = \frac{(A-M)(2c(1+A-M)-(5+2A-2M)t)}{2(3+4A-4M)} \tag{3.61}$$

rival legacy purchases Each firm will make the following profits:

$$\Pi_0^{Bdl\&CG\ IncompAll} = \frac{c^2(2(1+A)(1+2A)^2 - (1+8A(1+A))M - 8(1+A)M^2 + 8M^3) -}{4(3+4A-4M)^2t} \tag{3.62}$$

$$-\frac{8c(1+2A-2M)(1+A-M)(-1+A-M)t+}{4(3+4A-4M)^2t} \tag{3.63}$$

$$+\frac{+8(1+A-M)(1-A+M)^2t^2)}{4(3+4A-4M)^2t} \tag{3.64}$$

$$and \tag{3.65}$$

$$\Pi_1^{Bdl\&CG\ IncompAll} = \frac{(c(1+A-M) - 4t - 3(A-M)t)^2}{2(3+4A-4M)^2t} \tag{3.66}$$

Remark 23 *When* $P_B^{Bdl\&CG\ IncompAll} < P_C^{Bdl\&CG\ IncompAll}$, *no own legacy purchases*

the complement alone, but rather the bundle. Then

$$P_B^{Bdl\&CG\ IncompAll} = \frac{c(1+2A+2M) + 2t - 2t(A-M)}{3+4A+4M} > \tag{3.67}$$

$$P_1^{Bdl\&CG\ IncompAll} = \frac{-c(1+A+M) + (4 - A(-3) + 5M)t}{3+4A+4M} \tag{3.68}$$

the new platform consumers market is shared

$$Pl_0^{Bdl\&CG\ IncompAll} = \frac{c(1+A+M) + (2+5A+3M)t}{2(3+4A+4M)t} \tag{3.69}$$

$$Pl_1^{Bdl\&CG\ IncompAll} = \frac{-c(1+A+M) + (4 - A(-3) + 5M)t}{2(3+4A+4M)t} \tag{3.70}$$

and there are

$$CG_{new}^{Pl_1} = \frac{c(1+A+M) + (2+A(5)+3M)t}{2(3+4A+4M)t} \ newcomplementbuyersand \tag{3.71}$$

$$CG_{OWNlegacy}^{Bdl\&CG\ IncompAll} = \frac{2M(c(1+A+M) + (-1+A-M)t)}{(3+4A+4M)t} ownlegacypurchasesand \tag{3.72}$$

$$RL_{0\&CG} = \frac{(A-M)(2c(1+A+M) - (5+2A+6M)t)}{2(3+4A+4M)t} rivallegacypurchases \tag{3.73}$$

121

Each firm will make the following profits:

$$\Pi_0^{Bdl\&CG\ IncompAll} = \frac{2M(c(1+A+M)+(-1+A-M)t)}{(3+4A+4M)t} and \quad (3.74)$$

$$\Pi_1^{Bdl\&CG\ IncompAll} = \frac{(c(1+A+M)+(-4+A(-3)-5M)t)^2}{2(3+4A+4M)^2t} \quad (3.75)$$

We have established Firm 1's platform price is highest under pure bundling, as it is the only provider of a sole platform and more differentiated from the rival's only product: the bundle. When bundle price is higher than the complement's monopoly price, the Firm 0 legacies will purchase the complement only. Prices and profits are lowest than under any other "complement and the bundle" scenario. All are increasing in the complement valuation c and the share of Firm 0's legacies M.

Proposition 24 *When (i) there is incompatibility among the two-product seller's complementary good and the rival's platform and (ii) the two-product seller targets both types of legacy consumers, the two-product seller will maximize profits by offering a Mixed Bundle (both products sold individually and as a specially priced package).*

We note that while the Mixed Bundle charges the highest Firm 0 platform price, it offers the second highest bundle price after Individual Pricing and the second highest Firm 1 price, after Pure Bundling. This result holds among all strategies and compatibility levels.

Having discussed all these potential marketing strategies, we are ready to determine the profit maximizing outcome. We are first going to focus on the case where incompatibility is not feasible due to really high costs or copyrights issues.

122

Proposition 25 *If compatibility is **not** feasible due to exogenous market conditions and the two-product seller's legacy consumer mass is relatively high compared to its rival, the two-product seller focuses on his own legacy consumers only and maximizes profits by offering the platform and the complement individually. The profit maximizing prices are*

$$P_0^{IP\ IncompOwn} = -\frac{c^2}{6t} + t < P_1^{IP\ IncompOwn} = -\frac{c^2}{12t} + t \qquad (3.76)$$

$$P_C^{IP\ IncompOwn} = \frac{c}{2} \qquad (3.77)$$

and the new platform consumers market is shared equally

$$Pl_0^{IP\ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2}) > Pl_1^{IP\ IncompOwn} = \frac{1}{24}(12 - \frac{c^2}{t^2}) \qquad (3.78)$$

and there are

$$CG_{New}^{IP\ IncompOwn} = \frac{1}{24}(12 + \frac{c^2}{t^2})\frac{c}{2t} \qquad (3.79)$$

complement buyers and

$$CG_{OWNLegacy}^{IP\ IncompOwn} = M\frac{c}{2t} \qquad (3.80)$$

own legacy purchases. Each firm will make the following profits:

$$\Pi_0^{IP\ IncompOwn} = \frac{c^4 + 24c^2(1 + 3M)t^2 + 144t^4}{288t^3} and \qquad (3.81)$$

$$\Pi_1^{IP\ IncompOwn} = \frac{(c^2 - 12t^2)^2}{288t^3} \qquad (3.82)$$

If the rival's legacy consumers make up most of the preexistent customer market segment, a Mixed Bundle, where the two goods are sold both individually and as a bundle, is the optimal strategy.

The Proposition above is the case we often see in various industries, as copyrights and patents play an important role in a firm's strategy. It is important to note that the higher the complementary good valuation c, the lower the M mass that determines the two-product seller to reach out to all rival's legacy consumers.

3.3.2.3 Equilibrium

We further analyze if, in the absence of exogenous factors preventing it, compatibility is desired and, if Firm 1 would ever have an incentive to forego the copyrights for its platform and allow the two-product seller to make a complement compatible to Firm 1's platform.

We first investigate when Firm 1 enjoys highest profits. We find it maximizes profits when the rival's complement is compatible with its platform and when the rival's platform and complement are available for individual purchase (Individual Pricing). This marketing strategy allows Firm 1 to compete directly with Firm 0 in the same product market and compete against comparable products (both offer a platform consumers see differentiated, both offer the possibility of using the complement with the platform, despite the complement only being sold by Firm 0). We have previously argued in Proposition 1 that Firm 0 would always choose to sell its products individually under compatibility. Therefore, this is our subgame perfect equilibrium.

Proposition 26 *In equilibrium, the two-product seller will maximize profits by always making his complement **compatible** with its rival's platform and by offering*

124

*his platform and his complement individually (**Individual Pricing**). The equilibrium market prices are*

$$P_0^{IP\ Comp} = P_1^{IP\ Comp} = t \tag{3.83}$$

$$P_C^{IP\ Comp} = \frac{c}{2} \tag{3.84}$$

the new platform consumers market is shared equally

$$Pl_0^{IP\ Comp} = Pl_1^{IP\ Comp} = \frac{1}{2} \tag{3.85}$$

and there are $CG_{New}^{IP\ Comp} = \frac{c}{2t}$ complement buyers and

$$CG_{Legacy}^{IP\ Comp} = A\frac{c}{2t} \tag{3.86}$$

legacy purchases. Each firm will make the following profits:

$$\Pi_0^{IP\ Comp} = \frac{(1+A)c^2}{4t} + \frac{t}{2} and \Pi_1^{IP\ Comp} = \frac{t}{2} \tag{3.87}$$

Firm 0's profit is increasing in the mass A of legacy consumers and the valuation c for the complement. We note this case is identical to having three distinct firms each selling one of the products; there is no internalization of complementarity for Firm 0 under this pricing strategy.

In the current paper, we chose to model the product bundling and pricing choices as taking place in consecutive stages of the game. We considered products where the design is dependent on the compatibility decision and, then, so is the bundling option (i.e. a camera and professional lens, camera and tripods, Mac laptops and their specially shaped batteries). In such scenarios, the product bundling decision is not as easy one to reverse. There are also cases where a platform and

a complement are sold at a discount without actual integration (i.e. new laptop and accessories: USB, noise cancellation headphones, wireless headphones). Then, we can safely assume that there are only two stages to the game: the compatibility decision and the prices choice stage. Our results do not change, in equilibrium the two-product seller chooses compatibility and offers the two products individually. There are no profitable deviations.

3.4 Welfare

Both Firm 0 and Firm 1 derive highest profits under compatibility. Therefore, producer surplus is highest under compatibility, when the two-product seller offers its goods a la carte.

Consumer surplus ordering is dependent on the share of the own legacy consumers mass in the legacy market and the valuation c for the complement. *Incompatibility with a focus on all legacy consumers* assumes new consumers interested in Firm 1's platform do not have access to the complement due to incompatibility. Moreover, Firm 1's legacies are required to purchase Firm 0's platform they have a relative dislike for (otherwise they would have purchased it in a previous period instead of choosing Firm 1's over it) in order to be able to derive any utility from the complementary good. Under this scenario, the two-product firm's legacies are, at times, strictly better off under this marketing strategy compared both to compatibility and incompatibility with own legacies as the only target legacy market, when the bundle price is often below the complement's monopoly one and then own legacies

can keep more of their consumer surplus by just purchasing the bundle instead and discarding the extra platform. If the bundle price is higher than the complement's, then they are just as well off as under compatibility and incompatibility with own legacies as the only targeted legacy market segment. Firm 0's new consumers are relatively better than under compatibility and incompatibility with a focus on its own legacies only, because they pay lower prices for any platform and the bundle.

Prices are highest under *compatibility*, but any new consumer interested in both types of products has the option of creating the bundle that best suits his preferences, as the complement is compatible with all platforms and the platform prices are identical. Firm 0's new consumers interested in the platform only are always worse off under compatibility due to the highest platform prices. When the valuation for the complement c is high, a rival new consumer does have a higher benefit from being able to form his own bundle using Firm 0's complement and Firm 1's platform. But when the valuation c for the complement is low, consumers overall are better off under incompatibility with a focus on own legacies only. Under compatibility, the rival's legacies do not have to purchase a second platform that brings them a relative disutility in order to use the complement, so they are better off too. An own legacy under compatibility pays the monopoly price for any product, which makes it just as well off as under incompatibility with own legacies as the target market and at most as well off as when all legacy consumers are targeted under incompatibility.

When own legacies are not the vast majority of the legacy market, compatibility leaves consumers with most surplus. Overall, incompatibility with a marketing

strategy focusing on *own* legacies brings consumers highest welfare only when the ratio $\frac{M}{A}$ is close to one (high value). Very few rival legacy consumers are then hurt by incompatibility and all the new consumers benefit from the lower prices. The actual own legacies derive the same net surplus as under compatibility, as they still pay the monopoly price for the complement.

Total surplus is always highest under compatibility, as the profits are substantially higher under this strategy than any other (see the appendix for further details). If there were no legacy consumers in the market, the lower prices under incompatibility would make this the consumer surplus maximizing strategy. Total surplus would still be highest under compatibility due to the high profits the two-product seller derives under this pricing strategy.

3.5   Conclusion

We developed a differentiated products model where two rivals offer each a durable, stand alone use platform with an individual demand. Furthermore, one of them also sells a unique, durable complementary product with no stand alone use. There is a unit mass of consumers whose valuations for the two types of products is uniformly and independently distributed on the unit square. We also assume the existence of a pre-established customer base of the durable platforms, clients who purchased a platform from either of the two sellers in a prior time period. In the first stage, the two-product seller must decide whether to make the complementary product compatible to its rival's platform. In the second stage, he chooses a pricing

strategy to maximize its profits.

We find that making the complementary product always compatible with the rival's platform is the optimal strategy and the two-product seller is always best off selling the products individually (Individual Pricing). Compatibility brings highest profits to the competitor too, thus being the market equilibrium.

We do acknowledge that in many industries, due to copyright laws or high costs, compatibility is not a realistic option. Therefore, we further explore the two marketing strategies the two-product seller has in the case where compatibility is not feasible: (i) reaching out with its new complementary product both to its own legacy consumers and the new ones or (ii) targeting its new complementary product both legacy consumers (including those owning a rival platform) and the new ones. The latter case implies that a rival's legacy consumer, who owns an incompatible platform, will have to purchase both the complementary good and the compatible platform, for which he has a relative dislike. Previous period preferences are still relevant, as a rival's legacy consumer must have had a lower valuation for the compatible platform relative to the Firm 1 one he owns (otherwise, he would have purchased it in a previous time period instead).

Our results show that when the two-product seller has a relatively small mass of its own legacy consumers compared to the total legacy market, reaching out to all legacy consumers (including his rival's) and offering a Mixed Bundle (the platform and the complement sold both individually and as a specially priced package) is profit maximizing. The two-product firm will choose to only reach out to its own legacy consumers when they are a substantial part of the legacy market. He will

then sell the two products individually.

The welfare implications of the model for our consumers depend heavily on the proportion of the two types of legacy consumers and the valuation for the complementary product. Compatibility benefits the consumers most when the legacy market is not dominated by own legacies. Otherwise, incompatibility allows consumers to keep more of their surplus due to lower prices for the new buyers. Profits are significantly higher under compatibility, and, therefore, total surplus is always maximized when the complement is compatible to both platforms.

Chapter A: Appendix: Bundling and Competition

A.1 Two-Market Monopolist

Proof. Lemma 1, Lemma 2 and Proposition 1:

CASE I: Individual Pricing $P_{OP}^{IP} = P_P^{IP} + P_O^{IP}$

Figure 1a shows the Unit Circle where Firm 1: Product "A"/Print is located at 0 and Firm 1: Product "M-A"/Online is located at 1/2. We first focus on the right arc $(0, \frac{1}{2})$.

A consumer/advertiser will choose platform O only iff

$$\alpha(M - A) - t(\tfrac{1}{2} - y) - P_O^{IP} \geq \alpha M - (P_P^{IP} + P_O^{IP}) - \tfrac{t}{2}$$

$$\tfrac{1}{2} - y \leq \frac{-\alpha A + P_P^{IP} + \frac{t}{2}}{t} = Ad_{OR} \quad \text{consumers will singlehome on O}$$

A consumer/advertiser will choose platform P only iff

$$\alpha A - P_P^{IP} - t(x) \geq \alpha M - (P_P^{IP} + P_O^{IP}) - \tfrac{t}{2}$$

$$x \leq \frac{\alpha A - \alpha M + P_O^{IP} + \frac{t}{2}}{t} = Ad_{PR} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will multihome iff

$$\alpha M - (P_P^{IP} + P_O^{IP}) - \tfrac{t}{2} > \max\{\alpha(M - A) - P_O^{IP} - t(\tfrac{1}{2} - y), \alpha A - P_P^{IP} - tx\}$$

$$\frac{\alpha M - P_O^{IP} - P_P^{IP} - \frac{t}{2}}{t} = Ad_{OPR} \quad \text{consumers will multihome on OP}$$

Due to symmetry, the left arc $(\frac{1}{2}, 1)$ will yield the same results

Therefore, the monopolist must solve the following profit maximization problem:

$$P_O^{IP}, P_P^{IP} \max P_O^{IP}(Ad_O + Ad_{OP}) + P_P^{IP}(Ad_P + Ad_{OP}) =$$

$$P_O^{IP}, P_P^{IP} \max P_O^{IP}(2\frac{-\alpha A + P_P^{IP} + \frac{t}{2}}{t} + 2\frac{\alpha M - P_O^{IP} - P_P^{IP} - \frac{t}{2}}{t}) + P_P^{IP}(2\frac{\alpha A - \alpha M + P_O^{IP} + \frac{t}{2}}{t} + 2\frac{\alpha M - P_O^{IP} - P_P^{IP} - \frac{t}{2}}{t})$$

$$= P_O^{IP}, P_P^{IP} \max P_O^{IP}(2\frac{\alpha(M-A) - P_O^{IP}}{t}) + P_P^{IP}(2\frac{\alpha A - P_P^{IP}}{t})$$

Notice above the price independence between P_O^{IP} and P_P^{IP}. We can further see this in the First Order Conditions (FOC)

FOC(P_O^{IP}) : $\alpha(M-A) - 2P_O^{IP} = 0$ $\boxed{P_O^{IP} = \frac{\alpha(M-A)}{2}}$ P_O^{IP}'s reaction

function is independent of P_P^{IP}

FOC(P_P^{IP}) : $\alpha A - 2P_P^{IP} = 0$ $\boxed{P_P^{IP} = \frac{\alpha A}{2}}$ P_P^{IP}'s reaction function

is independent of P_O^{IP}

$$Ad_{OR}^{IP} = \frac{-\alpha A + t}{2t} \qquad Ad_{PR}^{IP} = \frac{-\alpha(M-A) + t}{2t} \qquad Ad_{OPR}^{IP} = \frac{\alpha M - t}{2t}$$

$$Ad_{OL}^{IP} = \frac{-\alpha A + t}{2t} \qquad Ad_{PL}^{IP} = \frac{-\alpha(M-A) + t}{2t} \qquad Ad_{OPL}^{IP} = \frac{\alpha M - t}{2t}$$

and the total mass of each type

$$\boxed{Ad_O^{IP} = \frac{-\alpha A + t}{t} \qquad Ad_P^{IP} = \frac{-\alpha(M-A) + t}{t} \qquad Ad_{OP}^{IP} = \frac{\alpha M - t}{t}}$$

$$\pi^{IP} = P_O^{IP} * (Ad_O^{IP} + Ad_{OP}^{IP}) + P_P^{IP} * (Ad_P^{IP} + Ad_{OP}^{IP}) = \frac{\alpha^2(2A^2 - 2AM + M^2)}{2t}$$

CASE II: Mixed Bundling $P_{OP}^{MB} = \alpha M - \frac{t}{2}$

Figure 1a shows the Unit Circle where Firm 1: Product "A"/Print is located at 0 and Firm 1: Product "M-A"/Online is located at 1/2. We first focus on the right arc $(0, \frac{1}{2})$. A consumer/advertiser will choose platform O only iff

$$\alpha(M-A) - P_O^{MB} - t(\frac{1}{2} - y) \geq \alpha M - (\alpha M - \frac{t}{2}) - \frac{t}{2} = 0$$

$$\frac{1}{2} - y \leq \frac{\alpha(M-A) - P_O^{MB}}{t} = Ad_{OR} \quad \text{consumers will singlehome on O}$$

A consumer/advertiser will choose platform P only iff

$$\alpha A - P_P^{MB} - tx \geq 0$$

$$x \leq \frac{\alpha A - P_P^{MB}}{t} = Ad_{PR} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will multihome iff

$$\alpha M - (\alpha M - \tfrac{t}{2}) - \tfrac{t}{2} > \max\{\alpha(M-A) - P_O^{MB} - t(\tfrac{1}{2} - y), \alpha A - P_P^{MB} - tx, 0\}$$

$$\frac{-\alpha M + P_O^{MB} + P_P^{MB} + \tfrac{t}{2}}{t} = Ad_{OP} \quad \text{consumers will multihome on OP}$$

Similarly, we solve the left arc problem too.

Therefore, the monopolist must solve the following profit maximization problem:

$$P_O^{MB}, P_P^{MB} \max P_O^{MB}(Ad_O) + P_P^{MB}(Ad_P) + (\alpha M - \tfrac{t}{2})(Ad_{OP}) =$$

$$= P_O^{MB}, P_P^{MB} \max P_O^{MB}(2\tfrac{\alpha(M-A) - P_O^{MB}}{t}) + P_P^{MB}(2\tfrac{\alpha A - P_P^{MB}}{t}) + (\alpha M - \tfrac{t}{2})(2\tfrac{-\alpha M + P_O^{MB} + P_P^{MB} + \tfrac{t}{2}}{t})$$

We take First Order Conditions (FOC)

$$\text{FOC}(P_O^{MB}): \quad \alpha(M-A) - 2P_O^{MB} + \alpha M - \tfrac{t}{2} = 0 \quad \boxed{P_O^{MB} = \frac{\alpha(2M-A) - \tfrac{t}{2}}{2}} \quad P_O^{MB}\text{'s}$$

reaction function is independent of P_P^{MB}

$$\text{FOC}(P_P^{MB}): \quad \alpha A - 2P_P^{MB} + \alpha M - \tfrac{t}{2} = 0 \quad \boxed{P_P^{MB} = \frac{\alpha(M+A) - \tfrac{t}{2}}{2}} \quad P_P^{MB}\text{'s}$$

reaction function is independent of P_O^{MB}

On each arc: $Ad_{OR}^{MB} = \frac{-2\alpha A + t}{4t} \quad Ad_{PR}^{MB} = \frac{-2\alpha(M-A) + t}{4t} \quad Ad_{OPR}^{MB} = \frac{\alpha M}{2t}$ and

$$Ad_{OL}^{MB} = \frac{-2\alpha A + t}{4t} \quad Ad_{PL}^{MB} = \frac{-2\alpha(M-A) + t}{4t} \quad Ad_{OPL}^{MB} = \frac{\alpha M}{2t}$$

$$\boxed{Ad_O^{MB} = \frac{-2\alpha A + t}{2t} \quad Ad_P^{MB} = \frac{-2\alpha(M-A) + t}{2t} \quad Ad_{OP}^{MB} = \frac{\alpha M}{t}}$$

Then,

$$\pi^{MB} = P_O^{MB} * Ad_O^{MB} + P_P^{MB} * Ad_P^{MB}$$

$$+(\alpha M - \tfrac{t}{2}) * Ad_{OP}^{MB} = \frac{2\alpha^2(2A^2 - 2AM + M^2) + 2\alpha M t - t^2}{4t} \quad (A.1)$$

133

We further show that for a consumer to singlehome under Mixed Bundling, he must have a negative valuation for the second product:

The marginal consumer of good A only under MB has the following utility from good $M - A$: $\alpha(M - A) - t(\frac{1}{2} - Ad_P^{MB}) = \frac{-2A\alpha + 2\alpha M - t}{4} \leq 0$

The marginal consumer of good $M - A$ only under MB has the following utility from good A: $\alpha A - t(\frac{1}{2} - Ad_O^{MB}) = \frac{2A\alpha - t}{4} \leq 0$

CASE III: Pure Bundling $\boxed{P_{OP}^{PB} = \alpha M - \frac{t}{2}}$

Under Pure Bundling, all advertisers multihome at price $P_{OP}^{PB} = \alpha M - \frac{t}{2}$, making a profit of the same value.

CASE IV: Product "A"/Print and the Bundle $\boxed{P_{OP}^{P\&B} = \alpha M - \frac{t}{2}}$

This is a special case of Mixed Bundling, where the bundle consumers have all their surplus extracted and Product "M-A" is not available for individual sale. We focus first on arch $(0, \frac{1}{2})$

A consumer/advertiser will choose platform P only iff

$$\alpha A - P_P^{P\&B} - tx \geq 0$$

$$x \leq \frac{\alpha A - P_P^{P\&B}}{t} = Ad_{PR} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will multihome iff

$$\alpha M - (\alpha M - \frac{t}{2}) - \frac{t}{2} > \max\{\alpha(M - A) - P_O^{P\&B} - t(\frac{1}{2} - y), \alpha A - P_P^{P\&B} - tx, 0\}$$

$$\frac{-\alpha M + P_O^{P\&B} + P_P^{P\&B} + \frac{t}{2}}{t} = Ad_{OPR} \quad \text{consumers will multihome on OP right arch}$$

(0,1/2)

Similarly, we solve the left arc problem too. We then solve the profit maxi-

mization problem and find that

$$\boxed{P_P^{P\&B} = \frac{\alpha(M+A)-\frac{t}{2}}{2}} \text{ and } \boxed{Ad_{OP}^{P\&B} = \frac{2\alpha(M-A)+t}{2t} \qquad Ad_P^{P\&B} = \frac{-2\alpha(M-A)+t}{2t}}$$

$$\pi^{P\&B} = P_P^{P\&B} * Ad_P^{P\&B} + (\alpha M - \tfrac{t}{2}) * Ad_{OP}^{P\&B} = \tfrac{1}{8}(4\alpha(A+M) + \tfrac{4\alpha^2(A-M)^2}{t} - 3t)$$

CASE V: Product "M-A"/Online and the Bundle $\boxed{P_{OP}^{O\&B} = \alpha M - \tfrac{t}{2}}$

This is a special case of Mixed Bundling, where the bundle consumers have all their surplus extracted and Product "A" is not available for individual sale. We follow the method in Case IV and find that

$$\boxed{P_O^{O\&B} = \frac{\alpha(2M-A)-\frac{t}{2}}{2}} \text{ and } \boxed{Ad_{OP}^{O\&B} = \frac{2A\alpha+t}{2t} \qquad Ad_O^{O\&B} = \frac{-2A\alpha+t}{2t}}$$

$$\pi^{O\&B} = P_O^{O\&B} * Ad_O^{O\&B} + (\alpha M - \tfrac{t}{2}) * Ad_{OP}^{O\&B} = \tfrac{1}{8}(-4\alpha(A-2M) + \tfrac{4\alpha^2 A^2}{t} - 3t)$$

COMPARISON:

We first note that Pure Bundling and Individual Pricing are the only strategies available to the two-market monopolist when **ALL consumers** are assumed to have **strictly positive valuations for both products**[1]. **Pure Bundling is the optimal strategy**, since

$$\pi^{IP} - \pi^{PB} = \frac{\alpha^2(2A^2-2AM+M^2)}{2t} - \left(\alpha M - \tfrac{t}{2}\right) = \frac{2A^2\alpha^2-2A\alpha^2 M+(-\alpha M+t)^2}{2t} < 0 \text{ using}$$

our set of parameter values.

If **a strictly positive mass of consumers have negative valuation for one of the products**, the two-market monopolist can choose between Individual

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[1]We first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We impose that the mass of customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \le A_j^i \le a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies all constraints above.

Pricing, Pure Bundling, Mixed Bundling, Product "A"/Print and the Bundle and Product "M-A"/Online and the Bundle. We compare all these profits.

$\pi^{MB} = \pi^{IP} + \frac{2t(\alpha M - t)}{4t}$. We look for the set of parameters both Individual Pricing and Mixed Bundling exist for (all advertiser types are positive and all derive a positive utility from their choice); we find that $\alpha M - t > 0$ on this subset, so

$$\boxed{\pi^{MB} > \pi^{IP}}$$

Then

$$\pi^{PB} - \pi^{MB} = (\alpha M - \frac{t}{2}) - \frac{2\alpha^2(2A^2 - 2AM + M^2) + 2\alpha Mt - t^2}{4t} =$$

$$= -\left(\frac{4A^2\alpha^2 - 4AM\alpha^2 + 2M^2\alpha^2 - 2Mt\alpha + t^2}{4t} \right) \leq 0, \text{ so}$$

We now compare Pure Bundling with Individual Pricing:

$$\pi^{PB} - \pi^{IP} = (\alpha M - \frac{t}{2}) - \frac{\alpha^2(2A^2 - 2AM + M^2)}{2t} = -\frac{2A^2\alpha^2 - 2A\alpha^2 M + (-\alpha M + t)^2}{2t} > 0, \text{ so}$$

$$\boxed{\pi^{PB} > \pi^{IP}}$$

We follow the same method for the pricing strategies including the bundle and one of the individual products only and find that:

If **a strictly positive mass of consumers have (strictly) negative valuation for one of the products**, the two-market monopolist's profits are ranked as follows:

$$\boxed{\pi^{MB} \geq (>)\pi^{P\&B} \& \pi^{O\&B} \geq (>)\pi^{PB} > \pi^{IP}}$$

To build further intuition for the impact of the bundle introduction on the market outcome, we compare prices under the Individual Pricing and Mixed Pricing strategies when a positive mass of consumers have (strictly) negative valuation for one of the products:

$$P_O^{IP} - P_O^{MB} = \frac{\alpha(M-A)}{2} - \frac{\alpha(2M-A)-\frac{t}{2}}{2} = -\frac{\alpha M+\frac{t}{2}}{2} < 0 \text{ since } \alpha M - \frac{t}{2} > 0$$

$$P_P^{IP} - P_P^{MB} = \frac{\alpha A}{2} - \frac{\alpha(M+A)-\frac{t}{2}}{2} = -\frac{\alpha M+\frac{t}{2}}{2} < 0 \text{ since } \alpha M - \frac{t}{2} > 0$$

$$P_{OP}^{IP} - P_{OP}^{MB} = \frac{\alpha(M-A)}{2} + \frac{\alpha A}{2} - (\alpha M - \frac{t}{2}) = \frac{-\alpha M+t}{2} < 0 \text{ since } Ad_{OP}^{IP} = \frac{\alpha M-t}{2t} > 0.$$

Compare consumers/advertisers mass under the Individual Pricing and Mixed Pricing strategies when a positive mass of consumers have (strictly) negative valuation for one of the products:

Recall $Ad_O^{IP} = \frac{-\alpha A+t}{t}$ $\qquad Ad_P^{IP} = \frac{-\alpha(M-A)+t}{t}$ $\qquad Ad_{OP}^{IP} = \frac{\alpha M-t}{t}$ and

$$Ad_O^{MB} = \frac{-\alpha A+t}{2t} \qquad Ad_P^{MB} = \frac{-\alpha(M-A)+t}{2t} \qquad Ad_{OP}^{MB} = \frac{\alpha M}{t}$$

$$Ad_O^{IP} - Ad_O^{MB} = \frac{-\alpha A+t}{t} - \frac{-\alpha A+t}{2t} = \frac{1}{2} > 0$$

$$Ad_P^{IP} - Ad_P^{MB} = \frac{-\alpha(M-A)+2t}{2t} - \frac{-\alpha(M-A)+t}{2t} = \frac{1}{2} > 0$$

$$Ad_{OP}^{IP} - Ad_{OP}^{MB} = \frac{\alpha M-2t}{2t} - \frac{\alpha M}{2t} = -1 < 0 \quad \blacksquare$$

Proof. <u>Lemma 3</u>:

<u>ARC $(\mathbf{0}, \mathbf{a})$</u>: Referring to Figure A1 below, any consumer located right of x and left of y has to travel both to Firm 1: Product "A"/Print located at 0 and Firm 1: Product "M-A"/Online located at a to get the bundle. We make the assumption that this travelling will always happen on the arc not including a third firm (in this case $(0, a)$ and not $(0, b, a)$ arc, as can be seen in Figure 2) and impose conditions on the model's parameters such that bundling is incentive compatible.

- Figure A1: **consumers must choose between singlehoming on Firm 1: Product "A"/Print at price P_P^{IP}, singlehoming on Firm 1: Product "M-A"/Online at price P_O^{IP} and multihoming at $P_{OP}^{IP} = P_P^{IP} + P_O^{IP}$**
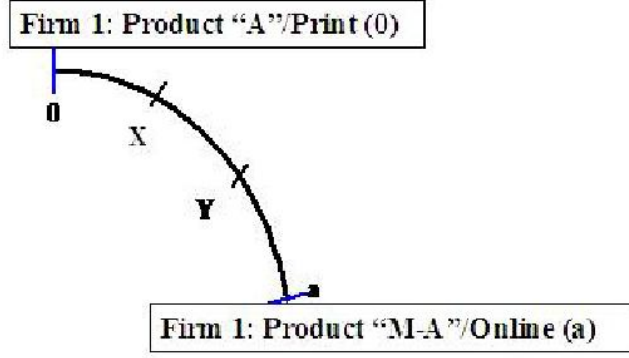
Figure A.1: Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online differentiation a la Hotelling

A consumer/advertiser will choose Firm 1: Product "M-A"/Online only iff

$$\alpha(M - A) - P_O^{IP} - t(a - y) \geq \alpha M - (P_P^{IP} + P_O^{IP}) - at$$

$$a - y \leq \frac{-\alpha A + P_P^{IP} + at}{t} = Ad_{OR}^{IP} \quad \text{consumers will singlehome on O}$$

A consumer/advertiser will choose Firm 1: Product "A"/Print only iff

$$\alpha A - P_P^{IP} - tx \geq \alpha M - (P_P^{IP} + P_O^{IP}) - at$$

$$x \leq \frac{\alpha A - \alpha M + P_O^{IP} + at}{t} = Ad_{PR}^{IP} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will multihome iff

$$\alpha M - (P_P^{IP} + P_O^{IP}) - at > \max\{\alpha(M - A) - P_O^{IP} - t(a - y), \alpha A - P_P^{IP} - tx, 0\}$$

$$\frac{\alpha M - P_O^{IP} - P_P^{IP} - at}{t} = Ad_{OP}^{IP} \quad \text{consumers will multihome on OP}$$

Similarly, on **ARC** $(b, 1)$:

Referring to Figure A2, any consumer located left of x when looking at the arc or right of x when using the line set up, has to travel both to Firm 1: Product
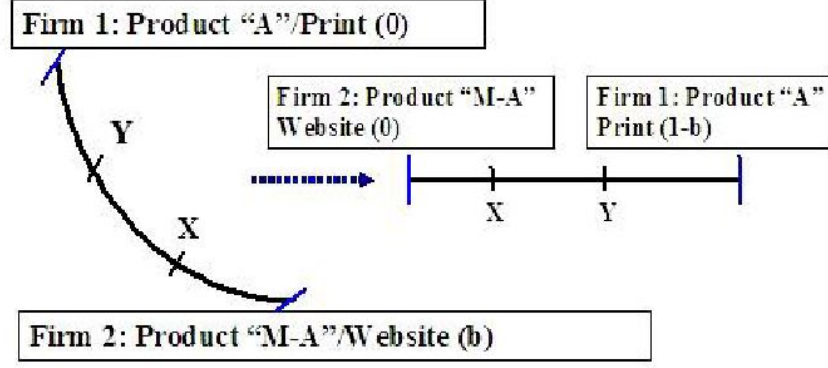
Figure A.2: Firm 1: Product "A"/Print and Firm 2: Product "M-A"/Website differentiation a la Hotelling

"A"/Print located at 0 and Firm 2: Product "M-A"/Website located at b to get the bundle. We make the assumption that this travelling will always happen on the arc not including a third firm (in this case $(b, 1)$ and not $(b, a, 0)$ arc, as Figure 2 shows) and impose conditions on the model's parameters such that bundling is incentive compatible.

A consumer/advertiser will choose Firm 1: Product "A"/Print only iff

$$\alpha A - P_P^{IP} - t(1 - b - y) \geq \alpha M - (P_P^{IP} + P_W^{IP}) - (1 - b)t$$

$$1 - b - y \leq \frac{\alpha A - \alpha M + P_W^{IP} + (1-b)t}{t} = Ad_{PL}^{IP} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will choose Firm 2: Product "M-A"/Website only iff

$$\alpha(M - A) - P_W^{IP} - tx \geq \alpha M - (P_P^{IP} + P_W^{IP}) - (1 - b)t$$

$$x \leq \frac{-\alpha A + P_P^{IP} + (1-b)t}{t} = Ad_{WL}^{IP} \quad \text{consumers will singlehome on W}$$

A consumer/advertiser will multihome iff

139
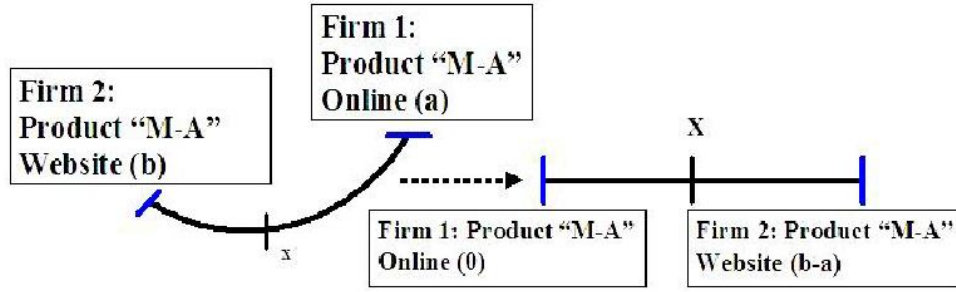
Figure A.3: Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website differentiation a la Hotelling

$$\alpha M - (P_P^{IP} + P_W^{IP}) - t(1-b) > \max\{\alpha(M-A) - P_W^{IP} - tx, \alpha A - P_P^{IP} - t(1-b-x),$$
$$0\}$$

$\frac{\alpha M - P_W^{IP} - P_P^{IP} - (1-b)t}{t} = Ad_{WP}^{IP}$ consumers will multihome on WP

ARC (a, b): consumers/advertisers will always singlehome with Firm 1: Product "M-A"/Online for P_O^{IP} or Firm 2: Product "M-A"/Website for P_W^{IP}

A consumer/advertiser will choose Firm 1: Product "M-A"/Online only iff

$$\alpha(M - A) - P_O^{IP} - tx \geq \alpha(M - A) - P_W^{IP} - (b - a - x)t$$

$x \leq \frac{P_W^{IP} - P_O^{IP} + (b-a)t}{2t} = Ad_{OL}^{IP}$ consumers will singlehome on P

$\frac{-P_W^{IP} + P_O^{IP}}{2t} = Ad_{WR}^{IP}$ consumers will singlehome on W

PROFIT MAXIMIZATION

Incumbent: $P_O^{IP}, P_P^{IP} max P_O^{IP}(Ad_{OL}^{IP} + Ad_{OR}^{IP} + Ad_{OP}^{IP}) + P_P^{IP}(Ad_{PL}^{IP} + Ad_{PR}^{IP} + Ad_{OP}^{IP} + Ad_{WP}^{IP})$

Plug in the functions derives above and take FOCs:

$\text{FOC}(P_P^{IP})$: $\boxed{P_P^{IP} = \frac{A\alpha}{2}}$ Firm 1: Product "A"/Print's reaction curve-constant

$\text{FOC}(P_O^{IP})$: $\boxed{P_O^{IP} = \frac{-2A\alpha + 2\alpha M + P_W^{IP} + bt - at}{6}}$ Firm 1: Product "M-A"/Online's

reaction curve - only depends on the direct competitor's price P_W^{IP}

Entrant: $P_W^{IP} max P_W^{IP} (Ad_{WL}^{IP} + Ad_{WR}^{IP} + Ad_{WP}^{IP})$

$\text{FOC}(P_O^{IP})$: $\boxed{P_W^{IP} = \frac{-2A\alpha + 2\alpha M + P_O^{IP} + bt - at}{6}}$ Firm 2: Product "M-A"/Website's

reaction curve - only depends on the direct competitor's price P_O^{IP}

Equilibrium:

$\boxed{P_P^{IP} = \frac{A\alpha}{2}}$

$\boxed{P_O^{IP} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{6}}$

$\boxed{P_W^{IP} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{6}}$

$Ad_{OR}^{IP} = a - \frac{A\alpha}{2t}$

$Ad_{PR}^{IP} = \frac{3A\alpha - 3\alpha M + bt + 4at}{5t}$

$Ad_{OP}^{IP} = -\frac{A\alpha - 6\alpha M + 2bt + 8at}{10t}$

$Ad_{PL}^{IP} = -\frac{-3A\alpha + 3\alpha M + (-5 + 4b + a)t}{5t}$

$Ad_{WP}^{IP} = \frac{-A\alpha + 6\alpha M + 2(-5 + 4b + a)t}{10t}$

$Ad_{WL}^{IP} = 1 - b - \frac{A\alpha}{2t}$

$Ad_{OL}^{IP} = \frac{b-a}{2}$

$Ad_{WR}^{IP} = \frac{b-a}{2}$

$\boxed{\pi_{Incumbent}^{IP} = \frac{25A^2\alpha^2 + 3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t}}$

$\boxed{\pi_{Entrant}^{IP} = \frac{3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t}}$ ∎

141

A.2 Single Market Monopolist

Proof. <u>Lemma 4</u>:

<u>**ARC** $(0, a)$</u>, see Figure A1: **consumers must choose between single-homing on Firm 1: Product "A"/Print at a price** P_P^{MB} **, singlehoming on Firm 1: Product "M-A"/Online at a price** P_O^{MB} **and multihoming at price** P_{OP}^{MB}. Same travelling rules apply (only via the $(0, a)$ arc if multihoming and not the $(1, b, a)$ arc).

A consumer/advertiser will choose Firm 1: Product "M-A"/Online only iff

$$\alpha(M - A) - P_O^{MB} - t(a - y) \geq \alpha M - P_{OP}^{MB} - at$$

$$a - y \leq a - \frac{Aa + P_O^{MB} - P_{OP}^{MB}}{t} = Ad_{OR}^{MB} \quad \text{consumers will singlehome on O}$$

A consumer/advertiser will choose Firm 1: Product "A"/Print only iff

$$\alpha A - P_P^{MB} - tx \geq \alpha M - P_{OP}^{MB} - at$$

$$x \leq a + \frac{Aa - aM + P_{OP}^{MB} - P_P^{MB}}{t} = Ad_{PR}^{MB} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will multihome iff

$$\alpha M - P_{OP}^{MB} - at > \max\{\alpha(M - A) - P_O^{MB} - t(a - y), \alpha A - P_P^{MB} - tx, 0\}$$

$$\frac{aM + P_O^{MB} - 2P_{OP}^{MB} + P_P^{MB} - at}{t} = Ad_{OP}^{MB} \quad \text{consumers will multihome on OP}$$

Similarly, on <u>**ARC** $(b, 1)$</u>, consumers/advertisers can choose to singlehome with Firm 1: Product "A"/Print for P_P^{MB} or Firm 2: Product "M-A"/Website for P_W^{MB} or multihome at $P_{WP}^{MB} = P_P^{MB} + P_W^{MB}$. Same travelling rules apply (only via

the $(b, 1)$ arc to multihome and not the $(b, a, 0)$ one).

A consumer/advertiser will choose Firm 1: Product "A"/Print only iff

$$\alpha A - P_P^{MB} - t(1 - b - y) \geq \alpha M - (P_P^{MB} + P_W^{MB}) - (1 - b)t$$

$$1 - b - y \leq \frac{\alpha A - \alpha M + P_W^{MB} + (1-b)t}{t} = Ad_{PL}^{MB} \quad \text{consumers will singlehome on P}$$

A consumer/advertiser will choose Firm 2: Product "M-A"/Website only iff

$$\alpha(M - A) - P_W^{MB} - tx \geq \alpha M - (P_P^{MB} + P_W^{MB}) - (1 - b)t$$

$$x \leq \frac{-\alpha A + P_P^{MB} + (1-b)t}{t} = Ad_{WL}^{MB} \quad \text{consumers will singlehome on W}$$

A consumer/advertiser will multihome iff

$$\alpha M - (P_P^{MB} + P_W^{MB}) - t(1 - b) > \max\{\alpha(M - A) - P_W^{MB} - tx, \ \alpha A - P_P^{MB} - t(1 - b - x), 0\}$$

$$\frac{\alpha M - P_W^{MB} - P_P^{MB} - (1-b)t}{t} = Ad_{WP}^{MB} \quad \text{consumers will multihome on WP}$$

<u>**ARC**</u> (a, b), consumers/advertisers will always singlehome with Firm 1: Product "M-A"/Online for P_O^{MB} or Firm 2: Product "M-A"/Website for P_W^{MB}

A consumer/advertiser will choose Firm 1: Product "A"/Online only iff

$$\alpha(M - A) - P_O^{MB} - tx \geq \alpha(M - A) - P_W^{MB} - (b - a - x)t$$

$$x \leq \frac{P_W^{MB} - P_O^{MB} + (b-a)t}{2t} = Ad_{OL}^{MB} \quad \text{consumers will singlehome on P}$$

$$\frac{-P_W^{MB} + P_O^{MB}}{2t} = Ad_{WR}^{MB} \quad \text{consumers will singlehome on W}$$

<u>PROFIT MAXIMIZATION</u>

Incumbent: $\quad P_O^{MB}, P_P^{MB}, P_{OP}^{MB} \max P_O^{MB}(Ad_{OL}^{MB} + Ad_{OR}^{MB}) + P_P^{MB}(Ad_{PL}^{MB} + Ad_{PR}^{MB} + Ad_{WP}^{MB}) + P_{OP}^{MB}(Ad_{OP}^{MB})$

Plug in the functions derives above and take FOCs:

FOC(P_P^{MB}): $\boxed{P_P^{MB} = \frac{64Aa - 7aM + 21at + 14bt}{114}}$ Firm 1: Product "A"/Print's reaction curve depends on the bundle price P_{OP}^{MB}

FOC(P_O^{MB}): $\boxed{P_O^{MB} = \frac{-6Aa + 6aM + (a+7b)t)}{19}}$ Firm 1: Product "M-A"/Online's reaction curve - depends on the direct competitor's price P_W^{MB} and the bundle price P_{OP}^{MB}

FOC (P_{OP}^{MB}): $\boxed{P_{OP}^{MB} = \frac{14Aa + 43aM - 15at + 28bt}{114}}$ Firm 1: Product "M"/Print & Online's reaction curve - depends on the two individual Incumbent prices

Entrant: $P_W^{MB} max P_W^{MB}(Ad_{WL}^{MB} + Ad_{WR}^{MB} + Ad_{WP}^{MB})$

FOC(P_W^{MB}): $\boxed{P_W^{MB} = \frac{22a(-A+M) - 9at + 13bt}{57}}$ Firm 2: Product "M-A"/Website's reaction curve - only depends on the direct competitor's price P_O^{MB}

Equilibrium:

$\boxed{P_P^{MB} = \frac{64Aa - 7aM + 21at + 14bt}{114}}$

$\boxed{P_O^{MB} = \frac{-6Aa + 6aM + (a+7b)t}{19}}$

$\boxed{P_{OP}^{MB} = \frac{14Aa + 43aM - 15at + 28bt}{114}}$

$\boxed{P_W^{MB} = \frac{22a(-A+M) - 9at + 13bt}{57}}$

$Ad_{OR}^{MB} = \frac{-64Aa + 7aM + 93at - 14bt}{114t}$

$Ad_{PR}^{MB} = \frac{32Aa - 32aM + 39at + 7bt}{57t}$

$\underline{Ad_{OP}^{MB} = \frac{-a + (aM)/t}{2}}$

$Ad_{PL}^{MB} = \frac{35a(A-M) + (57 - 9a - 44b)t}{57t}$

$Ad_{WP}^{MB} = \frac{-20Aa + 77aM + (-3(38+a) + 74b)t}{114t}$

$$Ad_{WL}^{MB} = \frac{-50Aa - 7aM + (114 + 21a - 100b)t}{114t}$$

$$Ad_{OL}^{MB} = \frac{-4Aa + 4aM - 69at + 49bt}{114t}$$

$$Ad_{WR}^{MB} = \frac{4Aa - 4aM - 45at + 65bt}{114t}$$

$$\boxed{\pi_{Incumbent}^{MB} = \frac{\alpha^2(1624A^2 - 1082AM + 541M^2) + 2(331a - 210b)\alpha(A-M)t + (366a^2 + 70ab + 245b^2)t^2}{2166t}}$$

$$\boxed{\pi_{Entrant}^{MB} = \frac{(22Aa - 22aM + 9at - 13bt)^2}{2166t}}$$

We compare Mixed Bundling singlehoming vs multihoming prices:

$$P_{OP}^{MB} - P_P^{MB} = \frac{14Aa + 43aM - 15at + 28bt}{114} - \frac{64Aa - 7aM + 21at + 14bt}{114} > 0, \text{ so } \boxed{P_P^{MB} \leq P_{OP}^{MB}}$$

$$P_{OP}^{MB} - P_O^{MB} = \frac{14Aa + 43aM - 15at + 28bt}{114} - \frac{-6Aa + 6aM + (a + 7b)t}{19} > 0, \text{ so } \boxed{P_O^{MB} \leq P_{OP}^{MB}} \quad \blacksquare$$

Proof. <u>Lemma 5</u>:

The Incumbent and the Entrant compete head on both the (a, b) and $(b, 1)$ arcs. The latter is a result of the Incumbent selling the bundle only, therefore no consumer on $(b, 1)$ would buy from both Firm 1: Product "M"/Print&Online and Firm 2: Product "M-A"/Website, since access to internet readers is already provided by the bundle.

ARC $(0, a)$: all a consumers buy the bundle, multihome with the Incumbent at P_{OP}^{PBC}

ARC $(b, 1)$: consumers/advertisers can choose to singlehome with Firm 1/Print&Online at a price P_{OP}^{PBC} or Firm 2: Product "M-A"/Website for P_W^{PBC}. No consumer would multihome, since the bundle already ensures access to the good of Firm 1: Product "M-A"/Online (or the internet readers that multihome among Online and Website, so any of the internet news media will ensure the advertiser reaches all $M - A$ of

145

Figure A.4: Pure Bundling: Competition on the $(b, 1)$ arch

them), which is a close substitute to Firm 2: Product "M-A"/Website's product (same readers). Referring to Figure A4 below, any consumer located left of x when looking at the arc or right of x when using the line set up, has to travel both to Firm 1: Product "A"/Print located at 0 and Firm 1: Product "M-A"/Online located at a to get the bundle. We make the assumption that this travelling will always happen via the $(0, a)$ arc and not $(0, b, a)$ and impose conditions on the model's parameters such that is always incentive compatible.

A consumer/advertiser will choose Firm 1: Product "M"/Print&Online iff

$$\alpha M - P_{OP}^{PBC} - t(1 - b - x + a) \geq \alpha(M - A) - P_W^{PBC} - xt$$

$$x \leq \frac{-\alpha A + P_{OP}^{PBC} - P_W^{PBC} + t(1 - b + a)}{2t} = Ad_{WL}^{PBC} \text{ consumers will singlehome/purchase}$$

from Firm 2: Product "M-A"/Website only and

$$Ad_{OPL}^{PBC} = 1 - b - x = 1 - b - \frac{-\alpha A + P_{OP}^{PBC} - P_W^{PBC} + t(1 - b + a)}{2t}$$

ARC (a, b): consumers/advertisers can choose to singlehome with Firm 1: Product "M"/Print&Online for P_{OP}^{PBC} or Firm 2: Product "M-A"/Website for

146
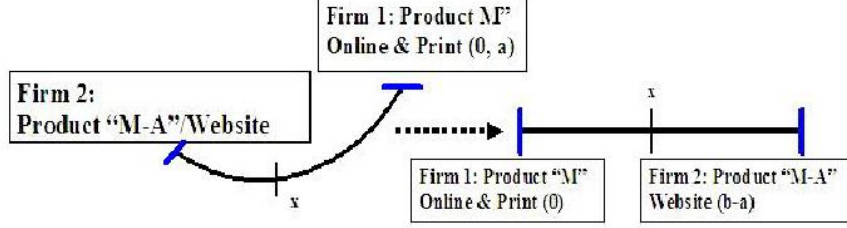
Figure A.5: Pure Bundling: Competition on the (a, b) arch

P_W^{PBC}. No consumer would multihome, since the bundle already ensures access to the good of Firm 1: Product "M-A"/Online (or the internet readers that multihome among Online and Website, so any of the internet news media wil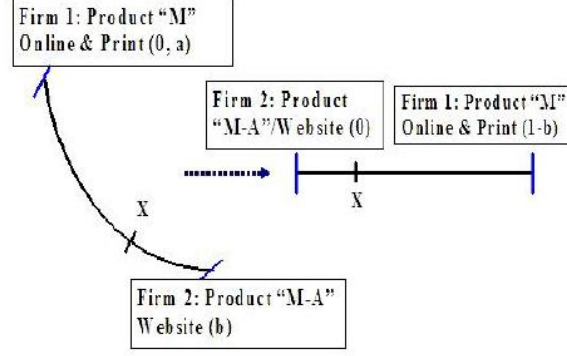l ensure the advertiser reaches all $M - A$ of them), which is a close substitute to Firm 2: Product "M-A"/Website's product (same readers). Referring to Figure A5 below, any consumer located right of x when looking at the arc or left of x when using the line set up, has to travel both to Firm 1: Product "A"/Print located at 0 and Firm 1: Product "M-A"/Online located at a to get the bundle. We make the assumption that this travelling will always happen via the simple $(0, a)$ arc and impose conditions on the model's parameters such that this is always incentive compatible.

A consumer/advertiser will choose Firm 1: Product "M"/Print&Online iff

$$\alpha M - P_{OP}^{PBC} - t(x + a) \geq \alpha(M - A) - P_W^{PBC} - (b - a - x)t$$

$$x \leq \frac{\alpha A - P_{OP}^{PBC} + P_W^{PBC} + t(b - 2a)}{2t} = Ad_{WR}^{PBC} \text{ consumers will singlehome/purchase}$$

from Firm 1: Product "M"/Print&Online and

$$Ad_{WR}^{PBC} = b - a - x = b - a - \frac{\alpha A - P_{OP}^{PBC} + P_W^{PBC} + t(b - 2a)}{2t} \text{ buy from Firm 2: Product}$$

"M-A"/Website

147

PROFIT MAXIMIZATION

Incumbent: $\quad P_{OP}^{PBC} max P_{OP}^{PBC}(Ad_{OPL}^{PBC} + Ad_{OPR}^{PBC} + a)$

Plug in the functions derives above and take FOCs:

FOC(P_{OP}^{PBC}): $\quad \boxed{P_{OP}^{PBC} = \frac{2Aa + 2P_W^{PBC} + t - at}{4}}$ Firm 1: Product "M"/Print&Online's

reaction curve depends on the direct competitor's price P_W^{PBC}

Entrant: $\quad P_W^{PBC} max P_W^{PBC}(Ad_{WL}^{PBC} + Ad_{WR}^{PBC})$

FOC(P_W^{PBC}): $\quad \boxed{P_W^{PBC} = \frac{-2Aa + 2P_{OP}^{PBC} + t + at}{4}}$ Firm 2: Product "M-A"/Website's

reaction curve depends on the direct competitor's price P_{OP}^{PBC}

Equilibrium:

$$\boxed{P_{OP}^{PBC} = \frac{2\alpha A - (-3+a)t}{6}}$$

$$\boxed{P_W^{PBC} = \frac{-2Aa + (3+a)t}{6}}$$

$$Ad_{OPL}^{PBC} = \frac{3 - 3b - 2a + (Aa)/t}{6}$$

$$Ad_{OPR}^{PBC} = \frac{Aa + 3bt - 5at}{6t}$$

$$Ad_{WR}^{PBC} = -\frac{Aa - 3bt + at}{6t}$$

$$Ad_{WL}^{PBC} = \frac{3 - 3b + 2a - (Aa)/t}{6}$$

$$\boxed{\pi_{Incumbent}^{PBC} = \frac{(-2\alpha A + (-3+a)t)^2}{36t}}$$ Under Pure Bundling with the market covered,

the Incumbent offers only one product, so the competition between the two can be

simplified to the Hotelling model of product differentiation, with the two firms being

located at each at one end point of the unit segment. Therefore, the profit does not

depend on b, the location of the Entrant.

$$\boxed{\pi_{Entrant}^{PBC} = \frac{(-2Aa + (3+a)t)^2}{36t}} \quad \blacksquare$$
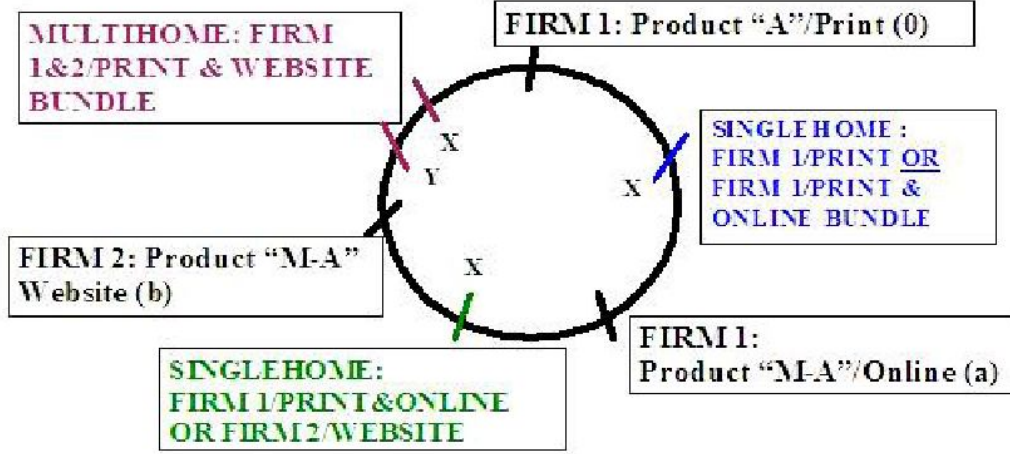
Figure A.6: Incumbent offers Firm 1: Product "A"/Print or the Bundle only

Proof. <u>Lemma 6:</u>

We follow the same methodology used to determine the market outcome under Individual Pricing, Mixed Bundling and Pure Bundling.

The competition on the $(b, 1)$ arc is the same as under Mixed Bundling and Individual Pricing. The differences appear in the Incumbent's exclusive market (arc $(0, a)$), where each consumer is faced with the choice of Firm 1: Product "A"/Print only and the bundle, and in the Firm 2-Firm 3 market.

<u>ARC $(0, a)$</u>: A consumer will choose Print iff $\alpha A - P_P^{P\&Bdl} - tx \geq \alpha M - at - P_{OP}^{P\&Bdl}$

<u>ARC (a, b)</u>: A consumer will choose the Bundle iff $\alpha M - (a + x)t - P_{OP}^{P\&Bdl} \geq \alpha(M - A) - t(b - a - x) - P_W^{P\&Bdl}$

Therefore, following the proofs for each of the three strategies before (Individual Pricing, Mixed Bundling and Pure Bundling), we find:

149

$$P_P^{P\&Bdl} = \frac{52Aa - 7aM + 11at + 14bt}{92}$$

$$P_{OP}^{P\&Bdl} = \frac{3Aa + 8aM - 6at + 7bt}{23}$$

$$P_W^{P\&Bdl} = \frac{-11Aa + 9aM - at + 5bt}{23}$$

$$Ad_{PL}^{P\&Bdl} = -\frac{-12Aa + 14aM + (-23 + a + 18b)t}{23t},$$

$$Ad_{PR}^{P\&Bdl} = \frac{52Aa - 53aM + 57at + 14bt}{92t}$$

$$Ad_{OPR}^{P\&Bdl} = \frac{-52Aa + 53aM + 7(5a - 2b)t}{92t}$$

$$Ad_{OPL}^{P\&Bdl} = \frac{9Aa + aM - 41at + 21bt}{46t}$$

$$Ad_{WR}^{P\&Bdl} = -\frac{9Aa + aM + 5(a - 5b)t}{46t}$$

$$Ad_{WL}^{P\&Bdl} = \frac{-40Aa - 7aM + (92 + 11a - 78b)t}{92t}$$

$$Ad_{PWL}^{P\&Bdl} = \frac{-8Aa + 63aM + (-92 - 7a + 58b)t}{92t}$$

$$\pi_{OP}^{P\&Bdl} = \frac{a^2(2188A^2 - 1732AM + 1041M^2) + 2a(a(914A - 913M) + 56b(3A + 8M))t + (817a^2 - 672ab + 392b^2)t^2}{4232t}$$

$$\pi_W^{P\&Bdl} = \frac{3(11Aa - 9aM + (a - 5b)t)^2}{1058t}$$

The Incumbent NEVER offers the Firm 1: Product "M-A"/Online product and the Bundle (Firm 1/Print & Online) only.

In determining the set of parameters over which our model is defined and keeping the market covered assumption used in the literature, wefocus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. As before, we first impose that customers that sign up with each firm on each arc are a nonnegative mass and less than the size of the customers in that market (i.e. $0 \leq Ad_{OP} \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters on which we will compare the profit under the deviation with the Pure Bundling one by creating the union of the current set

and the preexistent one created to compare Individual Pricing, Mixed Bundling, Pure Bundling and Product "A" and the Bundle. We find they have no common values, therefore the Incumbent would not find it feasible to offer Firm 1: Product "M-A"/Online product and the Bundle only. We also construct the set of parameters on which we could compare the this current strategy to Pure Bundling and still find no common parameters. Therefore, when choosing a pricing strategy on the set of parameters including the other four strategies, the Incumbent would not take Firm 1: Product "M-A"/Online product and the Bundle into consideration. Firm 1: Product "M-A"/Online product and the Bundle is an optimal strategy on its own set of disjoint parameters.

COMPARING PRICES ACROSS THE MODELS

We want to look at some basic comparisons across the four models.

INCUMBENT

$P_P^{IP} = \frac{A\alpha}{2}$

$P_P^{MB} = \frac{64Aa - 7aM + 21at + 14bt}{114}$

$P_P^{P\&Bdl} = \frac{52Aa - 7aM + 11at + 14bt}{92}$

$P_{OP}^{PBC} = \frac{2\alpha A - (-3+a)t}{6}$

$P_P^{IP} - P_P^{MB} = \frac{A\alpha}{2} - \frac{64Aa - 7aM + 21at + 14bt}{114} < 0$

$P_P^{MB} - P_P^{P\&Bdl} = \frac{64Aa - 7aM + 21at + 14bt}{114} - \frac{52Aa - 7aM + 11at + 14bt}{92} > 0$

$P_P^{IP} - P_P^{P\&Bdl} = \frac{A\alpha}{2} - \frac{52Aa - 7aM + 11at + 14bt}{92} < 0$ so $\boxed{\mathbf{P}_P^{IP} < \mathbf{P}_P^{P\&Bdl} < \mathbf{P}_P^{MB}}$

$P_{OP}^{PBC} - P_P^{IP} = \frac{2\alpha A - (-3+a)t}{6} - \frac{A\alpha}{2} = -\frac{1}{6}A\alpha - \frac{1}{6}t(a-3) > 0$

$P_{OP}^{PBC} - P_P^{MB} = \frac{2\alpha A - (-3+a)t}{6} - \frac{64Aa - 7aM + 21at + 14bt}{114} > 0$

151

$$\boxed{P_{OP}^{PBC} > P_P^{MB} > P_P^{IP}}$$

Similarly, we show that $\boxed{P_{OP}^{PBC} > \mathbf{P}_O^{MB} > \mathbf{P}_O^{IP}}$ and $\boxed{P_{OP}^{PBC} > P_P^{IP} + P_O^{IP}}$,

but $\boxed{P_{OP}^{PBC} > P_{OP}^{MB} \text{ if "}a\text{" is small and } P_{OP}^{PBC} \leq P_{OP}^{MB} \text{ if "}a\text{" is large}}$

<u>ENTRANT</u>

$P_W^{IP} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{6}$

$P_W^{MB} = \frac{22a(-A+M) - 9at + 13bt}{57}$

$P_W^{PBC} = \frac{-2A\alpha + (3+a)t}{6}$

$P_W^{P\&Bdl} = \frac{-11Aa + 9aM - at + 5bt}{23}$

$P_W^{IP} - P_W^{MB} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{6} - \frac{22a(-A+M) - 9at + 13bt}{57} < 0$

$\boxed{P_W^{MB} > P_W^{IP}}$

$P_W^{PBC} - P_W^{IP} = \frac{-2A\alpha + (3+a)t}{6} - \frac{-2A\alpha + 2\alpha M + (b-a)t}{6} = -\frac{1}{6}\left(-3t + 2M\alpha - 2at + bt\right) >$

0, so $\boxed{P_W^{PBC} > P_W^{IP}}$

$P_W^{PBC} - P_W^{MB} = \frac{-2A\alpha + (3+a)t}{6} - \frac{22a(-A+M) - 9at + 13bt}{57} > 0$, so $\boxed{P_W^{PBC} > P_W^{MB}}$

$P_W^{MB} - P_W^{P\&Bdl} = \frac{22a(-A+M) - 9at + 13bt}{57} - \frac{-11Aa + 9aM - at + 5bt}{23} > 0$

$P_W^{IP} - P_W^{P\&Bdl} = \frac{-2A\alpha + 2\alpha M + (b-a)t}{6} - \frac{-11Aa + 9aM - at + 5bt}{23} < 0$ so $\boxed{P_W^{IP} < P_W^{P\&Bdl} < P_W^{MB}}$

Then $\boxed{\mathbf{P}_W^{IP} < \mathbf{P}_W^{P\&Bdl} < \mathbf{P}_W^{MB} < \mathbf{P}_W^{PBC}}$ ∎

Proof. COMPARATIVE STATICS

1. CHANGES IN THE LOCATION OF FIRM 1: PRODUCT "M-A"/ONLINE: "a"

$\boxed{\mathbf{a} \Uparrow, \ (b-a) \Downarrow, \ (1-b) \Longleftrightarrow}$

Potential Market: Firm 1/Print: $(1-b+a)\Uparrow$ Firm 1/Online: $b\Longleftrightarrow$ Firm 2/Website: $(1-a)\Downarrow$

1A. INDIVIDUAL PRICING: $a \Uparrow$

Holding all prices constant, fewer advertisers from the WO arc joint the two firms, since $(b - a) \Downarrow$

Competition becomes stronger on the WO arc because of the lower mass of advertisers available and unchanged on the WP & OP arcs (since we have no competition there to begin with).

$$\mid \mathbf{P_W^{IP}} \Downarrow (-\mathbf{t/5})\mid \, = \, \mid \mathbf{P_O^{IP}} \Downarrow (-\mathbf{t/5})\mid \, > \, \mid \mathbf{P_P^{IP}} \Longleftrightarrow \mid$$

W has a smaller number of advertisers to reach $(1-a)$ and since it faces fiercer competition on WO it must cut its price. O has the same mass of advertisers b to compete for, his mass of advertisers he is a monopolist for on the OP arc increases, but the one he competes for decreases. O will also decrease its price, as the reaction curves (RC) show. P is a monopolist on both OP and WP, so he does not make any changes to his current strategy.

$P|\_\_\_\_\_MH \; at initial prices|_____|_____O$

"a" increases, so when preexistent advertisers choose between P and OP, more choose P only since it's more expensive to travel all the way to O due to the higher distance "a". This does not change the number of advertisers signed with P.

$P|\_\_\_\_\_a|_____MH|\_\_\_|_____a_____O$

On the O side, again holding all prices constant, same number of advertisers sign with O. The only change is that as "a" increases and advertisers are choosing

153

between O and OP, choose O more.

P|_____MH|___|_____a_____O

As $\mathbf{P_O^{IP}}$ decreases, O will attract more advertisers on the OP arc.

P|_____MH|_____|_____a_____O

$$Ad_{WL}^{IP}(0) + Ad_{WR}^{IP}(-\mathbf{a/2}) + Ad_{PW}^{IP}(a/5) = Ad_W^{IP}(-\mathbf{3a/10}) \Downarrow$$

$$Ad_{PL}^{IP}(-a/5) + Ad_{PR}^{IP}(4a/5) + Ad_{OP}^{IP}(-4a/5) + Ad_{PW}^{IP}(a/5) = Ad_P^{IP}(\mathbf{0}) \Longleftrightarrow$$

$$Ad_{OL}^{IP}(-\mathbf{a/2}) + Ad_{OR}^{IP}(a) + Ad_{OP}^{IP}(-4a/5) = Ad_O^{IP}(-\mathbf{3a/10}) \Downarrow$$

O's loss is smaller than W's because O loss some "$b-a$" advertisers to itself on another arc (OP), while W lost them for good.

OUTCOME: An increase in a hurts Website and Online equally and does not affect Print.

CHANGES IN PROFIT:

-Incumbent:$\frac{\partial \pi_{Incumbent}^{IP}}{\partial a} = \frac{\partial \frac{25A^2\alpha^2 + 3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t}}{\partial a} = \frac{3}{25}\left(-2\alpha(M-A) - t(b-a) < 0$ since $M > 0$, $A > 0$, $b > a > 0$.

-Entrant:$\frac{\partial \pi_{Entrant}^{IP}}{\partial a} = \frac{\partial \frac{3(2A\alpha - 2\alpha M + (-b+a)t)^2}{50t}}{\partial a} = \frac{3}{25}\left(-2\alpha(M-A) - t(b-a) < 0$

The decrease in the two profits is equal, since Firm 1: Product "A"/Print profit is unaffected by changed in "a".

1B. **MIXED BUNDLING: a $\Uparrow$**

$$\boxed{\mid \mathbf{P_{OP}^{MB}} \Downarrow (\mathbf{15t/114})\mid \; > \mid\mathbf{P_O^{MB}} \Uparrow (\mathbf{t/19})\mid \; = \mid \mathbf{P_W^{MB}} \Downarrow (\mathbf{9t/57})\mid \; > \mid\mathbf{P_P^{MB}} \Uparrow (\mathbf{21t/114})\mid}$$

The Print product is a monopolist in both the (0,a) and (b,1) market. In the former, it faces an upward pressure in order to push as many of the exclusive con-

154

sumers as possible to purchase the bundle, since it's priced higher than any of the individual Incumbent goods. In the latter, it must account for the full effect of any price changes, due to the lack of competition and, thus, no buffer. Similarly, the Online product faces the same upward pressure in the (0,a) market, an a downward one in the (a,b) market due to direct competition with the rival's differentiated product. Then, the Incumbent must weigh in the downward pressures on the individual Print and Online prices coming from the (a, b) and $(b, 1)$ market and the upward pressure to incentivize as many exclusive $(0, a)$ consumers as possible to purchase the bundle. Choosing a bundle price lower than the monopoly case one $\alpha M - ta$ will generate higher profits for the Incumbent.

Since the bundle price is higher than the individual prices, the Incumbent has an incentive now to price higher $P_P^{MB} > P_P^{IP}$, $P_O^{MB} > P_O^{IP}$ and push most of its single product buyers on the $(0, a)$ arc to purchase the bundle at the higher price $P_{OP}^{MB} > P_P^{MB}$. Direct competition between Firm 1: Product "M-A"/Online and Firm 2: Product "M-A"/Website still results in interdependencies between the two firm (platform) prices, as seen in their reaction curves. However, the symmetry in their reaction functions found under Individual Pricing is lost, as consumers on the $(b, 1)$ arch can form their own bundle, pay the two individual prices and have surplus left over, while the customers on the $(0, a)$ arch will have their full surplus extracted when purchasing the bundle.

P|_____MH $at initial prices$|_____|_____O

"a" increases

P|_____MH|_____|----------------a_____O

prices are held constant

P|_____$M\alpha\ + a$|_____|----------a_____O

$\mathbf{P_O^{MB}}$ and $\mathbf{P_P^{MB}}$ decrease

P|_____$M\alpha\ + a$|_____|-----------a_____O

There is a *net gain in MH* advertisers on OP compared to the initial "a" case.

On the PW arc, $\mathbf{P_W^{MB}}$ decreases and $\mathbf{P_P^{MB}}$ increases. The same is true on the OW arch, thus the Website ends up with more advertisers from both markets.

CHANGES IN PROFIT:

$$\pi_{Incumbent}^{MB} = \frac{\alpha^2(1624A^2 - 1082AM + 541M^2) + 2(331a - 210b)\alpha(A-M)t + (366a^2 + 70ab + 245b^2)t^2}{2166t}$$

$$\pi_{Entrant}^{MB} = \frac{(22Aa - 22aM + 9at - 13bt)^2}{2166t}$$

-Incumbent: $\dfrac{\partial \pi_{Incumbent}^{MB}}{\partial a} = \dfrac{\partial \frac{\alpha^2(1624A^2 - 1082AM + 541M^2) + 2(331a - 210b)\alpha(A-M)t + (366a^2 + 70ab + 245b^2)t^2}{2166t}}{\partial a} <$

0

-Entrant: $\dfrac{\partial \pi_{Entrant}^{MB}}{\partial a} = \dfrac{\partial \frac{(22Aa - 22aM + 9at - 13bt)^2}{2166t}}{\partial a} < 0$

1C. PURE BUNDLING: a $\Uparrow$

Holding all prices constant, fewer advertisers from the WO arc joint the two firms, since $(b-a) \Downarrow$

Competition becomes stronger on the WO arc because of the lower mass of advertisers available and unchanged on the WP & OP arcs.

$$P_{OP}^{PBC} = \frac{2\alpha A - (-3+a)t}{6}$$

$$\boxed{\mid \mathbf{P_{OP}^{PBC}} \Downarrow (\mathbf{-t/6}) \mid = \mid \mathbf{P_W^{IP}} \Uparrow (\mathbf{t/6}) \mid}$$

Since now the competition is down to two products essentially (the bundle

156

or the Entrant's good), and it happens on both arcs, a higher a means a higher transportation cost for anyone on the arc $(b,1)$ interested in purchasing the bundle, as Firm 1: Product "A"/Print and Firm 1: Product "M-A"/Online are further apart. This allows the Entrant to charge higher prices, as the loss in consumers on the (a,b) arc is less than the gain on $(b,1)$ arc.

CHANGES IN PROFIT:

-Incumbent:$\frac{\partial \pi^{PB}_{Incumbent}}{\partial a} = \frac{\partial \frac{(-2\alpha A + (-3+a)t)^2}{36t}}{\partial a} = \frac{-2A\alpha + (-3+a)t}{18} < 0$ since $P^{PBC}_W = $

$\frac{-2Aa + (3+a)t}{6} > 0$

-Entrant:$\frac{\partial \pi^{MB}_{Entrant}}{\partial a} = \frac{\partial \frac{(-2Aa + (3+a)t)^2}{36t}}{\partial a} == \frac{-2A\alpha + (3+a)t}{18} < 0$ since $P^{PBC}_{OP} = \frac{2\alpha A - (-3+a)t}{6} > $

0

1D. PRODUCT "A"/PRINT and the BUNDLE: a $\Uparrow$

Holding all prices constant, fewer advertisers from the WO arc joint the two firms, since $(b-a) \Downarrow$

Competition becomes stronger on the WO arc because of the lower mass of advertisers available and unchanged on the WP & OP arcs.

$$\boxed{\mid \mathbf{P^{PBC}_{OP}} \Downarrow (\mathbf{6t/23}) \mid \, > \, \mid \mathbf{P^{IP}_P} \Uparrow (\mathbf{11t/92}) \mid \, > \, \mid \mathbf{P^{IP}_W} \Downarrow (\mathbf{t/23}) \mid}$$

Since any bundle consumer has the to travel the extra distance, the bundle price goes down. The Product "A"/Print price increases in order to push as many consumers on the $(0,a)$ arch as possible to multihome and to extract more surplus from those who are not willing to incur the higher transportation cost to get the bundle. Since the Entrant only competes directly on arch (a,b) against the bundle, they are strategic complements, thus his price also decreases.

CHANGES IN PROFIT:

-Incumbent:

$$\frac{\partial \pi_{Incumbent}^{P\&Bdl}}{\partial a} =$$

$$\frac{\partial \frac{\alpha^2(2188A^2-1732AM+1041M^2)+2a(a(914A-913M)+56b(3A+8M))t+(817a^2-672ab+392b^2)t^2}{4232t}}{\partial a} < 0 (A.3)^2$$

-Entrant: $\frac{\partial \pi_{Entrant}^{P\&Bdl}}{\partial a} = \frac{\partial \frac{3(11Aa-9aM+(a-5b)t)^2}{1058t}}{\partial a} == \frac{-2A\alpha+(3+a)t}{18} < 0$ since $P_{OP}^{PBC} =$

$\frac{2\alpha A-(-3+a)t}{6} > 0$ ∎

Proof. <u>Proposition 2</u>

Case 27 *We compare the Pure Bundling (PBC) and Individual Pricing (IP) profits.*

$$\boxed{\Delta\pi_{Incumbent}^{PBC-IP} = \pi_{Incumbent}^{PBC} - \pi_{Incumbent}^{IP}} = \frac{-450A^2a^2+25(-2Aa+(-3+a)t)^2-54(2Aa-2aM+(-b+a)t)^2}{900t}$$

Since our difference is quadratic in a, we take First and Second Order Conditions to see its behavior:

FOC: $\frac{\partial \Delta\pi_{Incumbent}^{PBC-IP}}{\partial a} = \frac{\partial((-450A^2a^2+25(-2Aa+(-3+a)t)^2-54(2Aa-2aM+(-b+a)t)^2)/(900t))}{\partial a} =$

$\frac{-158Aa+108aM+(-75+54b-29a)t}{450} 0$

SOC: $\frac{\partial^2 \Delta\pi_{Incumbent}^{PBC-IP}}{\partial a^2} = \frac{\partial \frac{\partial((-450A^2a^2+25(-2Aa+(-3+a)t)^2-54(2Aa-2aM+(-b+a)t)^2)/(900t))}{\partial a}}{\partial a} = -\frac{29t}{450} <$

0 the difference is concave, and we can be both on the increasing and decreasing portion of it.

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[2]In determining the sign of the Mixed Bundling profits FOC in a, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that customers that sign up with each firm on each arch are a non negative mass and less than the size of the customer mass in that market (i.e. $0 \le A_j^i \le a$), then assume each consumer is deriving positive utility from its purchaset. All the prices are positive. We have now constructed the set of parameters on which we will determine the sign.

Let's find the roots of the difference quadratic equation:

$$\Delta \pi_{Incumbent}^{PBC-IP} = 0,$$

then

$$\Delta \pi_{Incumbent}^{PBC-IP} = 0,$$

then $a_1 = \frac{t(-158Aa + 108aM - 75t + 54bt)}{29t^2} < 0$

$a_2 = \frac{t(-158Aa + 108aM - 75t + 54bt)}{29t^2} +$

$+ \frac{15\sqrt{2(t^2(19A^2a^2 - 24Aa(2aM + (-3+b)t) + 3(2aM + (-3+b)t)^2))}}{29t^2} > 1$

Since $0 < a < b < 1$, $\Delta \pi_{Incumbent}^{PBC-IP} > 0$, so $\boxed{\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{IP}}$

Case 28 *We compare the Pure Bundling (PBC) and Mixed Bundling (MB) profits.*

$$\boxed{\Delta \pi_{Incumbent}^{MB-PBC} = \pi_{Incumbent}^{MB} - \pi_{Incumbent}^{PBC}} =$$

$$= \frac{-361(-2Aa + (-3+a)t)^2 + 6(a^2(1624A^2 - 1082AM + 541M^2) + 2(331a - 210b)\alpha(A - M)t + (366a^2 + 70ab + 245b^2)t^2)}{12996t}$$

Since our difference is quadratic in a, we take First and Second Order Conditions to see its behavior:

FOC: $\frac{\partial \Delta \pi_{Incumbent}^{PBC-MB}}{\partial a} = \frac{2708Aa - 1986aM + (1083 + 1835a + 210b)t}{6498} < 0$

SOC: $\frac{\partial^2 \Delta \pi_{Incumbent}^{PBC-MB}}{\partial a^2} = \frac{1835t}{6498} > 0$ the difference is convex, and we are on the decreasing portion of it.

Let's find the roots of the difference quadratic equation:

$\Delta \pi_{Incumbent}^{PBC-MB} = 0$, then $a_1^{MB-PBC} =$

$-\frac{2708Aat + 3t(-662aM + (361 + 70b)t)}{} -$

$-\frac{19\sqrt{6}\sqrt{t^2(\alpha^2(-3646A^2 + 534AM - 929M^2) + 2a((3189 + 1330b)A - 3(331 + 420b)M)t + (3294 + 35(6 - 35b)b)t^2)}}{1835t^2}$

$a_2^{MB-PBC} = -\frac{2708Aat + 3t(-662aM + (361 + 70b)t)}{} +$

$+\frac{19\sqrt{6}\sqrt{t^2(\alpha^2(-3646A^2 + 534AM - 929M^2) + 2a((3189 + 1330b)A - 3(331 + 420b)M)t + (3294 + 35(6 - 35b)b)t^2)}}{1835t^2}$

159

Using our set of parameter restrictions, we find that Since $0 < a_1^{MB-PBC} <$ $a < a_2^{MB-PBC} < b < 1 : \pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{MB}$. Then $\boxed{\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{MB}}$

Case 29 *We compare the Pure Bundling (PBC) and Product "A" and the Bundle (P&Bdl) profits.*

We follow exactly the same methodology as above and find that $\boxed{\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{P\&Bdl}}$

Therefore, $\boxed{\pi_{Incumbent}^{PBC} > \max\{\pi_{Incumbent}^{MB}, \pi_{Incumbent}^{IP}, \pi_{Incumbent}^{P\&Bdl}\}}$

Case 30 *I further determine the ordering of the profits under the nonoptimal pricing strategies.*

I use the same method as above and compare the three strategies (Mixed Bundling, Individual Pricing and Product "A" and the Bundle)

- $\pi_{Incumbent}^{MB}$ and $\pi_{Incumbent}^{IP}$ to find $\pi_{Incumbent}^{MB} > \pi_{Incumbent}^{IP}$

- $\pi_{Incumbent}^{MB}$ and $\pi_{Incumbent}^{P\&Bdl}$ to find $\pi_{Incumbent}^{P\&Bdl} > \pi_{Incumbent}^{MB}$

- $\pi_{Incumbent}^{IP}$ and $\pi_{Incumbent}^{P\&Bdl}$ to find $\pi_{Incumbent}^{P\&Bdl} > \pi_{Incumbent}^{IP}$

to find that

$\boxed{\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{P\&Bdl} > \pi_{Incumbent}^{MB} > \pi_{Incumbent}^{IP}}$ ∎

Proof. <u>NO PROFITABLE DEVIATION</u>

<u>FROM THE PURE BUNDLING STRATEGY</u>
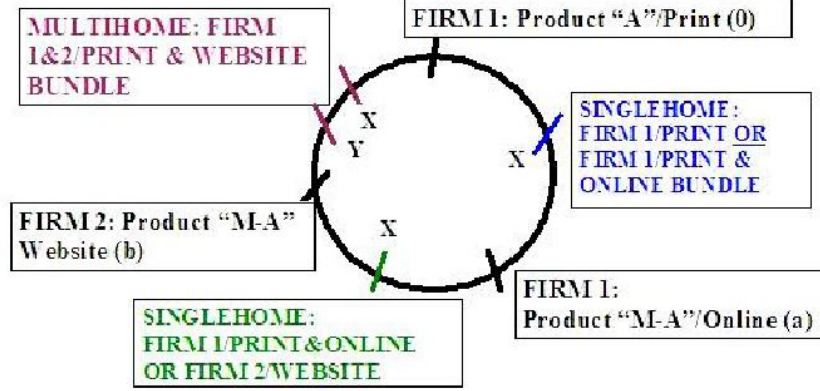
Figure A.7: Incumbent offers Firm 1: Product "A"/Print or the Bundle only (repost)

CASE 1: **Incumbent offers the Firm 1: Product "A"/Print product and the Bundle (Firm 1/Print & Online) only.**

We keep the Entrant's reaction function constant, so that is depends only on the bundle price, as we show in Lemma 6. The Incumbent deviates to offering Firm 1: Product "A"/Print product and the Bundle, while the Entrant's best response curve is kept fixed. The competition on the $(b, 1)$ arc is the same as under Mixed Bundling and Individual Pricing. The differences appear in the Incumbent's exclusive market (arc $(0, a)$), where each consumer is faced with the choice of Firm 1: Product "A"/Print only and the bundle, and in the Firm 2-Firm 3 market (see Figure A6, repasted here for the reader's convenience).

<u>**ARC $(0, a)$**</u>: A consumer will choose Print iff $\alpha A - P^{P\&Bdl}_{P-DEV} - tx \geq \alpha M - at - P^{P\&Bdl}_{OP-DEV}$

<u>**ARC (a, b)**</u>: A consumer will choose the Bundle iff $\alpha M - (a+x)t - P^{P\&Bdl}_{OP-DEV} \geq \alpha(M - A) - t(b - a - x) - P^{PB}_{W-DEV}$

161

There is no need to find the Entrant's best response as we are taking the Pure Bundling one as given.

We solve the Incumbent's profit maximization problem and find that

$$P^{P\&Bdl}_{P-DEV} = \frac{14Aa-2aM+t+3at+4bt}{24}$$

$$P^{P\&Bdl}_{OP-DEV} = \frac{2Aa+4aM+t-3at+4bt}{12}$$

$$P^{P\&Bdl}_{W-DEV} = \frac{-10Aa+4aM+(7+3a+4b)t}{24}$$

$$\pi^{P\&Bdl}_{OP-DEV} = \frac{20a^2(5A^2-4AM+2M^2)+4a((1+21a+4b)A+2(1-9a+4b)M)t+(33a^2-6a(1+4b)+(1+4b)^2)t^2}{192t}$$

$$\pi^{P\&Bdl}_{W-DEV} = -\frac{(38Aa-44aM+(19+15a-20b)t)(-10Aa+4aM+(7+3a+4b)t)}{1152t}$$

We compare prices with the Pure Bundling ones and find that

$$P^{P\&Bdl}_{OP-DEV} - P^{PB}_{OP} = \frac{2Aa+4aM+t-3at+4bt}{12} - \frac{2\alpha A-(-3+a)t}{6} > 0 \text{ so } P^{P\&Bdl}_{OP-DEV} > P^{PB}_{OP}$$

$$P^{P\&Bdl}_{P-DEV} - P^{MB}_{P} > 0 \text{ thus } P^{P\&Bdl}_{P-DEV} > P^{MB}_{P}(> P^{IP}_{P})$$

$$P^{P\&Bdl}_{W-DEV} - P^{PB}_{W} > 0 \text{ therefore } P^{P\&Bdl}_{W-DEV} > P^{PB}_{W}$$

The difference $P^{P\&Bdl}_{OP-DEV} - P^{PB}_{OP}$ is much higher than $P^{P\&Bdl}_{W-DEV} - P^{PB}_{W}$, the Incumbent makes lower profits on the (a,b) arch. Moreover, $P^{P\&Bdl}_{P-DEV} - P^{MB}_{P}$, so the Incumbent makes even lower profits on the $(b,1)$ arch. He does make higher profits on its exclusive consumers though, but these do not make up for the lower profits in the nonexclusive markets..

Overall, we find that $\pi^{P\&Bdl}_{OP-DEV} < \pi^{PB}_{OP}$, so Firm 1: Product "A"/Print only and the bundle is not a profitable deviation.

CASE 2: Incumbent deviates to the Mixed Bundling strategy.

We keep the Entrant's reaction function constant, so that is depends only on the bundle price, as we show in Lemma 6. The Incumbent deviates to Mixed

Bundling, while the Entrant's best response curve is kept fixed. There is no competition on the $(b, 1)$ arc, since each firm is a monopolist. In the (a, b) market, the rivals compete directly, each offering Product "M-A". The Entrant's best response, however, only depends on the Mixed Bundling bundle price.

We repeat the analysis above and find that

$$P^{MB}_{OP-DEV} > P^{PB}_{OP}$$

$$P^{MB}_{W-DEV} > P^{PB}_{W}.$$

$$P^{MB}_{P-DEV} > P^{MB}_{P}$$

$$P^{MB}_{O-DEV} > P^{MB}_{O}$$

Again, we compare the deviation profit to the Pure Bunding one and find that $\pi^{MB}_{OP-DEV} < \pi^{PB}_{OP}$, so Mixed Bundling is not a profitable deviation.

CASE 3: Incumbent deviates to the Firm 1: Product "M-A"/Online product and the Bundle (Firm 1/Print & Online) strategy.

In determining the set of parameters over which our model is defined and keeping the market covered assumption used in the literature, we focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. As before, we first impose that customers that sign up with each firm on each arc are a nonnegative mass and less than the size of the customers in that market (i.e. $0 \leq Ad_{OP} \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters on which we will compare the profit under the deviation with the Pure Bundling one by creating the union of the current set and the preexistent one created to compare Individual Pricing, Mixed Bundling,

Pure Bundling and Product "A" and the Bundle. We find they have no common values, therefore the Incumbent would not find it feasible to deviate to offer Firm 1: Product "M-A"/Online product and the Bundle only.

CASE 4: Incumbent deviates to the Pure Bundling strategy.

The same result as in case 3 applies.

We have shown that if the Incumbent chose Pure Bundling, he would have no incentive to deviate from this strategy. ∎

Proof. <u>Proposition 3</u>:

We compare Firm 2: Product "M-A"/Website's profits under the Incumbent's Individual Pricing (IP) and Mixed Bundling (MB) strategies.

$$\boxed{\Delta\pi_{Entrant}^{MB-IP} = \pi_{Entrant}^{MB} - \pi_{Entrant}^{IP}} = -\frac{4(112a(A-M)+(51a-61b)t)(Aa-aM+3at+2bt)}{27075t} > 0$$

Since our difference is quadratic in a, we take First and Second Order Conditions to see its behavior:

$$\text{FOC:} \frac{\partial \Delta\pi_{Entrant}^{MB-IP}}{\partial a} = -\frac{12(43Aa-43aM+34at-9bt)}{9025} > 0$$

$$\text{SOC:} \frac{\partial \Delta\pi_{Entrant}^{MB-IP}}{\partial a^2} = -\frac{408t}{9025} < 0 \text{ the difference is concave, and we are on the}$$

increasing portion of it.

Let's find the roots the difference:

$\Delta\pi_{Entrant}^{MB-IP} = 0$, and we show that given our parameters, $a_1^{MB-IP} < a < \min$

$\{b, a_1^{MB-IP}\}$

Then $\boxed{\pi_{Entrant}^{IP} < \pi_{Entrant}^{MB}}$

Following the same method and steps, we compare

- $\pi_{Entrant}^{PB}$ and $\pi_{Entrant}^{MB}$ to find $\pi_{Entrant}^{MB} > \pi_{Entrant}^{PB}$

- $\pi_{Entrant}^{PB}$ and $\pi_{Entrant}^{MB}$ to find $\pi_{Entrant}^{PB} > \pi_{Entrant}^{IP}$

- $\pi_{Entrant}^{P\&Bdl}$ and $\pi_{Entrant}^{MB}$ to find $\pi_{Entrant}^{MB} > \pi_{Entrant}^{P\&Bdl}$

- $\pi_{Entrant}^{P\&Bdl}$ and $\pi_{Entrant}^{IP}$ to find $\pi_{Entrant}^{P\&Bdl} > \pi_{Entrant}^{IP}$

- $\pi_{Entrant}^{P\&Bdl}$ and $\pi_{Entrant}^{PB}$ to find $\pi_{Entrant}^{P\&Bdl} > \pi_{Entrant}^{PB}$ when "a" is small and $\pi_{Entrant}^{P\&Bdl} < \pi_{Entrant}^{PB}$ for larger values of "a"

Overall $\boxed{\pi_{Entrant}^{MB}(> \pi_{Entrant}^{P\&Bdl}) > \pi_{Entrant}^{PB}(> \pi_{Entrant}^{P\&Bdl}) > \pi_{Entrant}^{IP}}$ ■

Proof. <u>WELFARE</u>

1. <u>TWO-MARKET MONOPOLIST</u>

Positive Valuation: Pure Bundling is the optimal pricing strategy. Consumers/Advertisers have their full surplus extracted. At the opposite end of the spectrum, they have most surplus left under Individual Pricing.

Consumers'/Advertisers' Surplus (IP):

$P_O^{IP} = \frac{\alpha(M-A)}{2}$ $P_P^{IP} = \frac{\alpha A}{2}$ $Ad_{OR}^{IP} = \frac{-\alpha A + t}{2t}$ $Ad_{PR}^{IP} = \frac{-\alpha(M-A)+t}{2t}$ $Ad_{OPR}^{IP} = \frac{\alpha M - t}{2t}$

$Ad_{OL}^{IP} = \frac{-\alpha A + t}{2t}$ $Ad_{PL}^{IP} = \frac{-\alpha(M-A)+t}{2t}$ $Ad_{OPL}^{IP} = \frac{\alpha M - t}{2t}$

- Purchasing Product "A"/Print consumers/advertisers have a total surplus of:

2*Total Surplus for all Product "A"/Print consumers arc $(0, 1/2)$ and $(1/2, 1$

$\int_0^{\frac{-\alpha(M-A)+t}{2t} + \frac{\alpha M - t}{2t}} [\alpha * (A) - t * (x) - \frac{\alpha A}{2}] dx$

165

- Purchasing Product "M-A"/Online consumers/advertisers have a total surplus of:

Total Surplus for all Product "M-A"/Online consumers arc

$$(0, 1/2$$

) and $(1/2, 1) + 2 * \int_0^{\frac{-\alpha A + t}{2t} + \frac{\alpha M - t}{2t}}$

$$[\alpha * (M - A) - t * (x) - \frac{\alpha(M-A)}{2}]dx (A.4)$$

Producer Surplus (IP): $\frac{\alpha^2(2A^2 - 2AM + M^2)}{2t}$

Consumers'/Advertisers' Surplus (PB):

- All purchase the bundle and are left with zero surplus

Producer Surplus (IP): $\alpha M - t/2$

Then $\boxed{\mathbf{CS}^{Ad}(\mathbf{IP}) > \mathbf{CS}^{Ad}(\mathbf{PB})}$

and $\boxed{\mathbf{PS}(\mathbf{IP}) < \mathbf{PS}(\mathbf{PB})}$

Overall

$\boxed{\mathbf{TS}(\mathbf{IP}) < \mathbf{TS}(\mathbf{PB})}$ when A takes on extreme values or t is small

$\boxed{\mathbf{TS}(\mathbf{IP}) \geq \mathbf{TS}(\mathbf{PB})}$ otherwise

Negative Valuation: We have already shown that the two-market monopolist strictly prefers Mixed Bundling to Individual Pricing (see proof of Lemma 1 and Proposition 1). Consumers/Advertisers have no surplus left under Pure Bundling and some net surplus under Mixed Bundling. Following the proof for the positive valuation case, we show that Mixed Bundling always generates higher Total Surplus.

166

Then $\boxed{\mathbf{CS}^{Ad}(\mathbf{MB}) > \mathbf{CS}^{Ad}(\mathbf{PB})}$

and $\boxed{\mathbf{PS(MB)} > \mathbf{PS(PB)}}$

Based again on proof of Lemma 1 and Proposition 1, we show that

$$\boxed{\mathbf{CS}^{Ad}(\mathbf{IP}) > \mathbf{CS}^{Ad}(\mathbf{MB}) > \mathbf{CS}^{Ad}(\mathbf{P\&Bdl})/\mathbf{CS}^{Ad}(\mathbf{O\&Bdl}) > \mathbf{CS}^{Ad}(\mathbf{PB})}$$

and $\boxed{\mathbf{PS(MB)} > \mathbf{PS(P\&Bdl)}/\mathbf{PS(O\&Bdl)}}$

Overall

$$\boxed{\mathbf{TS(MB)} \geq \mathbf{TS(P\&Bdl)}/\mathbf{TS(O\&Bdl)} > \mathbf{TS(PB)} > \mathbf{TS(IP)}}$$

Print Readers view:

- $Ad_P^{IP} + Ad_{OP}^{IP} = \frac{-\alpha(M-A)+t}{t} + \frac{\alpha M - t}{t} = \frac{\alpha A}{t}$ ads under Individual Pricing and

- $Ad_P^{MB} + Ad_{OP}^{MB} = \frac{-2\alpha(M-A)+t}{2t} + \frac{\alpha M}{t} = \frac{2\alpha A + t}{2t}$ ads under Mixed Bundling

  Then $Ad_P^{IP} + Ad_{OP}^{IP} < Ad_P^{MB} + Ad_{OP}^{MB}$, print readers benefit from more ads under Mixed Bundling.

  Total Ads^{ReadersPrint}(IP) <**Total Ads^{ReadersPrint}(MB)**¡

¡**Total Ads^{ReadersPrint}(O\&Bdl)** <**Total Ads^{ReadersPrint}(P\&Bdl)** <

Total Ads^{ReadersPrint}(PB)

Online Readers view:

- $Ad_O^{IP} + Ad_{OP}^{IP} = \frac{-\alpha A + t}{t} + \frac{\alpha M - t}{t} = \frac{\alpha(M-A)}{t}$ ads under Individual Pricing and

- $Ad_O^{MB} + Ad_{OP}^{MB} = \frac{-2\alpha A + t}{2t} + \frac{\alpha M}{t} = \frac{2\alpha(M-A)+t}{2t}$ ads under Mixed Bundling

  Then $Ad_O^{IP} + Ad_{OP}^{IP} < Ad_O^{MB} + Ad_{OP}^{MB}$, online readers benefit from more ads under Mixed Bundling.

167

$$\boxed{\textbf{Total Ads}^{ReadersInternet}(\textbf{IP}) < \textbf{Total Ads}^{ReadersInternet}(\textbf{MB})}$$

¡**Total Ads**^{ReadersPrint}(**P&Bdl**) <**Total Ads**^{ReadersPrint}(**O&Bdl**) <

Total Ads$^{ReadersPrint}(PB)$

2. ONE MARKET MONOPOLIST and COMPETITION

PURE BUNDLING vs. INDIVIDUAL PRICING

Consumers'/Advertisers' Surplus (PB):

- Singlehoming/buying Firm 2: Product "M-A"/Website only consumers/advertisers have a total surplus of:

Total Surplus for all Entrant consumers arc $(b,1)$ $\int_0^{Ad_{WL}=1/6(3-3b+2a-(A\alpha)/t)}[\alpha *$
$(M-A)-t*(x)-1/6(-2A\alpha+(3+a)t)]dx$

Total Surplus for all Entrant consumers arc $(a,b)+\int_0^{Ad_{WR}=-((A\alpha-3bt+at)/(6t))}[\alpha *$
$(M-A)-t*(x)-1/6(-2A\alpha+(3+a)t)]dx$

- Multihoming/buying both from on Firm 1: Product "M-A"/Online and Firm 1: Product "A"/Print consumers/advertisers have a total surplus of:

Total Surplus for all Incumbent exclusive consumers arc $(0,a)$a$*(\alpha * M - t *$
$a-1/6(2A\alpha-(-3+a)t))$

Total Surplus for all Incumbent consumers on the arc $(b,1)$

$+\int_0^{Ad_{OPL}=1/6(3-3b-2a+(A\alpha)/t}[\alpha * M - t*(x+a)-1/6(2A\alpha-(-3+a)t)]dx$

+Total Surplus for all Incumbent consumers on the arc (a, b)

$$\int_0^{Ado_{PR}=(A\alpha+3bt-5at)/(6t)}[\alpha*M-t*(x+a)-1/6(2A\alpha-(-3+a)t)]dx$$

Consumers'/Advertisers' Surplus (IP):

- Multihoming/buying both from on Firm 2: Product "M-A"/Website and Firm 1: Product "A"/Print consumers/advertisers have a total surplus of:

$$(a*M-t*(1-b)-\frac{Aa}{2}-\frac{-2Aa+2aM+(b-a)t}{5})\frac{-Aa+6aM+2(-5+4b+a)t}{10t}$$

- Singlehoming/buying Firm 1: Product "A"/Print only consumers/advertisers have a total surplus of:

$$\int_0^{Ad_{PR}=(3Aa-3aM+bt+4at)/(5t)}[a*A-t*x-(Aa)/2]dx+$$

$$+\int_0^{Ad_{PL}=(-((-3Aa+3aM+(-5+4b+a)t)/(5t)))}[a*A-t*x-(Aa)/2]dx+$$

- Singlehoming/buying Firm 1: Product "M-A"/Online only consumers/advertisers have a total surplus of:

$$\int_0^{Ad_{OR}=a-(Aa)/(2t)}[a*(M-A)-t*(x)-1/5(-2Aa+2aM+(b-a)t)]dx+$$

$$\int_0^{Ad_{OL}=(b-a)/2}[a*(M-A)-t*(x)-1/5(-2Aa+2aM+(b-a)t)]dx+$$

- Singlehoming/buying Firm 2: Product "M-A"/Website only consumers/advertisers have a total surplus of:

$$\int_0^{Ad_{WL}=1-b-(Aa)/(2t)}[a*(M-A)-t*(x)-1/5(-2Aa+2aM+(b-a)t)]dx+$$

$$\int_0^{Ad_{WR}=(b-a)/2}[a*(M-A)-t*(x)-1/5(-2Aa+2aM+(b-a)t)]dx+$$

169

- Multihoming/buying both Firm 1: Product "M-A"/Online and Firm 1: Product "A"/Print consumers/advertisers have a total surplus of:

$$(a * M - t * a - \frac{Aa}{2} - \frac{-2Aa + 2aM + (b-a)t}{5}) * (-\frac{Aa - 6aM + 2bt + 8at}{10t})$$

Then $CS^{Ad}(PBC) - CS^{Ad}(IP) < 0$

$$\boxed{\mathbf{CS}^{Ad}(\mathbf{PBC}) < \mathbf{CS}^{Ad}(\mathbf{IP})}[3]$$

When we look at welfare by arcs, we start with $(0, a)$. Here, all the multihomers are better off under IP since $P^{PBC} > P_P^{IP} + P_O^{IP}$. Consumers who singlehome in this interval are worse off:

Remember that $Ad_{OR}^{IP} = a - \frac{A\alpha}{2t}$ and $Ad_{PR}^{IP} = \frac{3A\alpha - 3\alpha M + bt + 4at}{5t}$

$Ad_{PR}^{IP} : U_{Ad_{PR}^{IP}}^{IP} = aA - t\frac{3A\alpha - 3\alpha M + bt + 4at}{5t} - (Aa)/2$

$Ad_{PR}^{PB} : U_{Ad_{PR}^{IP}}^{PB} = aM - ta - 1/6(-2A\alpha + (3+a)t)$

$U_{Ad_{PR}^{IP}}^{IP} - U_{Ad_{PR}^{IP}}^{PB} > 0$, so the same last advertiser on the $(0, a)$ arc interested in Firm 1: Product "A"/Print only receives more net utility under IP from singlehoming than under PB when purchasing the bundle. How about the first singlehomer on this arc for each firm:

Consumer located at "0": $U_0^{IP} = a * A - (Aa)/2$

$U_0^{PB} = a * M - t * a - 1/6(-2A\alpha + (3+a)t)$, then $U_0^{IP} > U_0^{PB}$

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[3]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

Consumer located at "a": $U_a^{IP} = a * (M - A) - 1/6(-2A\alpha + (3 + a)t)$

$U_a^{PB} = a * M - t * a - 1/6(-2A\alpha + (3 + a)t)$, then $U_0^{IP} > U_0^{PB}$

each are better off under IP.

Same way, we show this result holds for the last singlehomer on the $(0, a)$ arc interested in Firm 1: Product "M-A"/Online under IP, the last singlehomer on the (a, b) arc interested in Firm 1: Product "M-A"/Online under IP, the last singlehomer on $(b, 1)$ interested in Firm 1: Product "A"/Print under IP, all the Firm 2: Product "M-A"/Website consumers under IP and all the bundle buyers on the $(b, 1)$ arc.

We have showed that $CS^{Ad}(PBC) < CS^{Ad}(IP)$ and $\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{IP}$ and $\pi_{Entrant}^{PBC} > \pi_{Incumbent}^{IP}$. We further combine these to find compare Total Surplus and find that

$\boxed{\textbf{TS}(\textbf{PBC}) < \textbf{TS}(\textbf{IP}) \text{ for high values of "}a\text{"}}$

READERS' WELFARE

Total Ads Readers Can View:

- *Print Readers* can view the following number of ads:

  Total Ads (PBC): $Ad_{OP}^{PBC} = 1/6(3 - 3b - 2a + (A\alpha)/t) + (A\alpha + 3bt - 5at)/(6t) + a$

  Total Ads (IP): $Ad_{OP}^{IP} + Ad_P^{IP} = ((3Aa - 3aM + bt + 4at)/(5t) - (-3Aa + 3aM + (-5 + 4b + a)t)/(5t) -$

  $-(Aa - 6aM + 2bt + 8at)/(10t) + (-Aa + 6aM + 2(-5 + 4b + a)t)/(10t)$

  Then

$\boxed{\textbf{Total Ads}^{ReadersPrint}(\textbf{PBC}) > \textbf{Total Ads}^{\textbf{ReadersPrint}}(\textbf{IP})}$

- *Internet Readers* can view the following number of ads:

Total Ads (PBC):1 (all ads)

Total Ads (IP): $Ad_{OP}^{IP} + Ad_{O}^{IP} = [(-Aa + 6aM + 2(-5 + 4b + a)t)/(10t) + 1 -$

$b - (Aa)/(2t) + (b - a)/2 +$

$+a - (Aa)/(2t) - (Aa - 6aM + 2bt + 8at)/(10t) + (b - a)/2$

Overall, $\boxed{\textbf{Total Ads}^{Readers}(\textbf{PBC}) > \textbf{Total Ads}^{\textbf{Readers}}(\textbf{IP})}$

PURE BUNDLING vs MIXED BUNDLING

Total Consumers'/Advertisers' Surplus (PBC):

- Singlehoming/buying Firm 2: Product "M-A"/Website only consumers/advertisers have a total surplus of:

Total Surplus for all Entrant consumers arc $(b, 1)$

$\int_0^{Ad_{WL}^{PBC} = 1/6(3 - 3b + 2a - (A\alpha)/t)} [\alpha * (M - A) - t*(x) - 1/6(-2A\alpha + (3 + a)t)]dx$

Total Surplus for all Entrant consumers arc (a, b)

$+\int_0^{Ad_{WR}^{PBC} = -((A\alpha - 3bt + at)/(6t))} [\alpha * (M - A) - t*(x) - 1/6(-2A\alpha + (3 + a)t)]dx$

- Multihoming/buying both Firm 1: Product "M-A"/Online and Firm 1: Product "A"/Print consumers/advertisers have a total surplus of:

Total Surplus for all Incumbent exclusive consumers arc

$(0, a)$a$*(\alpha * M - t * a - 1/6(2A\alpha - (-3 + a)t))$

Total Surplus for all Incumbent consumers on the arc $(b, 1)$

172

$$+\int_0^{Ad_{WL}^{PBC}=1/6(3-3b-2a+(A\alpha)/t[\alpha*M-t*(x+a)-1/6(2A\alpha-(-3+a)t)]dx}$$

Total Surplus for all Incumbent consumers on the arc (a,b)

$$\int_0^{Ad_{WR}^{PBC}=(A\alpha+3bt-5at)/(6t)[\alpha*M-t*(x+a)-1/6(2A\alpha-(-3+a)t)]dx}$$

- Multihoming/buying both from on Firm 2: Product "M-A"/Website and Firm 1:

  Product "A"/Print consumers/advertisers have a total surplus of:

  $$\left(a*M-t*(1-b)-\frac{22a(-A+M)-9at+13bt}{57}-\frac{64Aa-7aM+21at+14bt}{114}\right)*\frac{-20Aa+77aM+(-3(38+a)+74b)t}{114t}$$

  - Singlehoming/buying Firm 1: Product "A"/Print only consumers/advertisers

    have a total surplus of:

    $$\int_0^{Ad_{PR}^{MB}=\frac{32Aa-32aM+39at+7bt}{57t}}[a*A-t*x-\frac{64Aa-7aM+21at+14bt}{114}]dx+$$

    $$+\int_0^{Ad_{PL}^{MB}=\frac{35a(A-M)+(57-9a-44b)t}{57t}}[a*A-t*x-\frac{64Aa-7aM+21at+14bt}{114}]dx+$$

  - Singlehoming/buying Firm 1: Product "M-A"/Online only consumers/advertisers

    have a total surplus of:

    $$\int_0^{Ad_{OR}^{MB}=\frac{-64Aa+7aM+93at-14bt}{114t}}[a*(M-A)-t*x-\frac{-6Aa+6aM+(a+7b)t}{19}]dx+$$

    $$\int_0^{Ad_{OL}^{MB}=\frac{-4Aa+4aM-69at+49bt}{114t}}[a*(M-A)-t*x-\frac{-6Aa+6aM+(a+7b)t}{19}]dx+$$

  - Singlehoming/buying Firm 2: Product "M-A"/Website only consumers/advertisers

    have a total surplus of:

    $$\int_0^{Ad_{WR}^{MB}=\frac{4Aa-4aM-45at+65bt}{114t}}[a*(M-A)-t*x-\frac{22a(-A+M)-9at+13bt}{57}]dx+$$

    $$\int_0^{Ad_{WL}^{MB}=\frac{-50Aa-7aM+(114+21a-100b)t}{114t}}[a*(M-A)-t*x-\frac{22a(-A+M)-9at+13bt}{57}]dx+$$

  - Multihoming/buying both from on Firm 1: Product "M-A"/Online and Firm

    1: Product "A"/Print consumers/advertisers have a total surplus of: 0

Then $\boxed{\mathbf{CS}^{Ad}(\mathbf{PBC}) < \mathbf{CS}^{Ad}(\mathbf{MB})}$[4]

We do the same type of analysis as before for MB. We look on each arc at the previous type of each consumer, his current type and how his utility changed. On arc $(0, a)$ all previous multihomers are better off under PB as the bundle price is lower and each singlehomer gets more profits from buying the bundle. Take the first Firm 1: Product "A"/Print singlehomer and compare his utility under MB and PB

$$(a * A - 1/4(2Aa + aM - at)) - (aM - ta - 1/6(2Aa - (-3 + a)t) =$$

$$= 1/12(10Aa - 15aM + (6 + 13a)t) < 0$$

and the first Firm 1: Product "M-A"/Online only singlehomer

$$(a(M - A) - 1/35(-14Aa + 26aM - 19at + 7bt)) - (aM - ta - 1/6(2Aa - (-3 + a)t) =$$

$$= 1/210(-56Aa - 156aM + (105 - 42b + 289a)t) < 0.$$ Thus, all previous arc $(0,a)$ singlehomers are better off under PB.

On arc $(b, 1)$, we find that all Firm 1: Product "A"/Print singlehomers now buy the bundle and have a higher net utility, which is also true for the multihomers on those arc and the Firm 2: Product "M-A"/Website only consumers.

On the (a, b) arc, all previous Firm 1: Product "M-A"/Online singlehomers

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[4]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

174

now purchase the bundle and derive more net utility. Some of the previous Firm 2: Product "M-A"/Website consumers buy the bundle and some stick to purchasing from Firm 2: Product "M-A"/Website only. The former derive a higher utility the closer they are to "a" and smaller as they are further away from it. The latter derive strictly more utility than under MB.

Since $CS^{Ad}(PBC) < CS^{Ad}(MB)$ and

$\pi_{Incumbent}^{PBC} > \pi_{Incumbent}^{MB}$ and $\pi_{Entrant}^{PBC} < \pi_{Entrant}^{MB}$,

$\boxed{\mathbf{TS(PBC) > TS(MB)} \text{ for low values of "}a\text{"}}$

PURE BUNDLING vs PRODUCT "A"/PRINT and the BUNDLE

We do the same detailed analysis as above and find that $\boxed{\mathbf{CS}^{Ad}(\mathbf{PBC}) < \mathbf{CS}^{Ad}(\mathbf{P\&BDL})}$ and $\boxed{\mathbf{TS(PBC) < TS(P\&BDL)}}$[5]

PURE UNBUNDLING vs PRODUCT "A"/PRINT and the BUNDLE

We do the same detailed analysis as above and find that $\boxed{\mathbf{CS}^{Ad}(\mathbf{IP}) > \mathbf{CS}^{Ad}(\mathbf{P\&BDL})}$ and $\boxed{\mathbf{TS(IP) < TS(P\&BDL)}}$[6]

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[5]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[6]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e.

MIXED BUNDLING vs PRODUCT "A"/PRINT and the BUNDLE

We do the same detailed analysis as above and find that $\boxed{\mathbf{CS}^{Ad}(\mathbf{MB}) < \mathbf{CS}^{Ad}(\mathbf{P\&BDL})}$ and $\boxed{\mathbf{TS(MB)} < \mathbf{TS(P\&BDL)}}$[7]

MIXED BUNDLING vs PURE UNBUNDLING

We do the same detailed analysis as above and find that $\boxed{\mathbf{CS}^{Ad}(\mathbf{MB}) < \mathbf{CS}^{Ad}(\mathbf{IP})}$ and $\boxed{\mathbf{TS(MB)} < \mathbf{TS(IP)}}$[8]

Overall,

$$\boxed{\mathbf{CS(IP)} > \mathbf{CS(P\&Bdl)} > \mathbf{CS(MB)} > \mathbf{CS(PB)}}$$

$$\boxed{\mathbf{TS(P\&Bdl)} > \max\{\mathbf{TS(IP)},\ \mathbf{TS(MB)}, \mathbf{TS(PB)}\}}$$

$0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[7]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[8]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

- **READERS' WELFARE**

PURE BUNDLING vs MIXED BUNDLING

We follow the same method as above to show that the following total surplus

inequalities hold:

$$\text{Total Ads}^{ReadersPrint}(\text{PBC}) > \text{Total Ads}^{\text{ReadersPrint}}(\text{MB})$$

$$\text{Total Ads}^{ReadersInternet}(\text{PBC}) > \text{Total Ads}^{\text{ReadersInternet}}(\text{MB})\ [9]$$

INDIVIDUAL PRICING vs MIXED BUNDLING

Following the steps above, we find that:

$$\text{Total Ads}^{ReadersPrint}(\text{IP}) = \text{Total Ads}^{\text{ReadersPrint}}(\text{MB})$$

$$\text{Total Ads}^{ReadersInternet}(\text{IP}) > \text{Total Ads}^{\text{ReadersInternet}}(\text{MB})\ [10]$$

INDIVIDUAL PRICING vs PRODUCT "A" and the BUNDLE

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[9]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[10]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

Similarly, we show that:

$$\text{Total Ads}^{ReadersPrint}(\textbf{IP}) < \text{Total Ads}^{\textbf{ReadersPrint}}(\textbf{P\&Bdl})$$

$$\text{Total Ads}^{ReadersInternet}(\textbf{IP}) < \text{Total Ads}^{\textbf{ReadersInternet}}(\textbf{P\&Bdl})^{[11]}$$

MIXED BUNDLING vs PRODUCT "A" and the BUNDLE

Similarly, we show that:

$$\text{Total Ads}^{ReadersPrint}(\textbf{MB}) < \text{Total Ads}^{\textbf{ReadersPrint}}(\textbf{P\&Bdl})$$

$$\text{Total Ads}^{ReadersInternet}(\textbf{MB}) < \text{Total Ads}^{\textbf{ReadersInternet}}(\textbf{P\&Bdl})^{[12]}$$

PURE BUNDLING vs PRODUCT "A" and the BUNDLE

Similarly, we show that:

$$\text{Total Ads}^{ReadersPrint}(\textbf{PB}) < \text{Total Ads}^{\textbf{ReadersPrint}}(\textbf{P\&Bdl})$$

$$\text{Total Ads}^{ReadersInternet}(\textbf{PB}) > \text{Total Ads}^{\textbf{ReadersInternet}}(\textbf{P\&Bdl})^{[13]}$$

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[11]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[12]In comparing the two results, we first focus on the restrictions on our parameter set $\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A_j^i \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

[13]In comparing the two results, we first focus on the restrictions on our parameter set

Overall,

$$\overline{\textbf{Total Ads}^{ReadersPrint}(P\&Bdl) > \textbf{Total Ads}^{ReadersPrint}(PB) >}$$

$$\overline{>\textbf{Total Ads}^{ReadersPrint}(IP) = \textbf{Total Ads}^{Readers}(MB)}$$

$$\overline{\textbf{Total Ads}^{\textbf{ReadersInternet}}(\textbf{PB}) > \textbf{Total Ads}^{\textbf{ReadersInternet}}(\textbf{P\&Bdl}) >}$$

$$\overline{>\textbf{Total Ads}^{ReadersInternet}(\textbf{IP}) > \textbf{Total Ads}^{\textbf{ReadersInternet}}(\textbf{MB})}$$

and, when looking at all readers, $\textbf{Total Ads}^{\textbf{Readers}}(\textbf{P\&Bdl}) > \textbf{Total Ads}^{\textbf{Readers}}(\textbf{PB})$

for large "a" values. ∎

$\{M, A, t, a, b, \alpha\}$. We first impose that under each model, customers that sign up with each firm on each arch are a non negative mass and less than the size of the customers in that market (i.e. $0 \leq A^i_j \leq a$), then assume each consumer is deriving positive utility from its purchase. All the prices are positive. We then construct the set of parameters that satisfies the restrictions above and are able to sign the difference.

Chapter B: Appendix: Compatibility and Bundling in a Market with a Monopoly Complementary Product

B.1   Compatibility

<u>Unit Mass of New Consumers</u>

We refer to Figure 1 in the paper and use the notation found there.

$U_0 = h - t * (x) - P_0$

$U_1 = h - t * (1 - x) - P_1$

$U_{0\&CG} = h + c - t * (x + y) - P_0 - P_C$; utility function when purchasing Platform 0 and the Complementary Good (CG)

$U_{1\&CG} = h + c - t * (1 - x + y) - P_1 - P_C$; utility function when purchasing Platform 0 and the Complementary Good (CG)

Referring to Figure 2, a **legacy consumer**'s utility is $U_{CGlegacy} = c - t * (x) - P_C$

<u>INDIVIDUAL PRICING:</u>

To determine how many <u>new consumers</u> will purchase Firm 0's platform only, Firm 1's platform only and the complement, we solve the following system of simultaneous equations, where Pl_0 stands for Firm 0's platform, Pl_1 stands for Firm 1's platform and CG refers to the complementary product:

$$\left\{ \begin{array}{c} h - t*(Pl_0) - P_0 = h - t*(Pl_1) - P_1 \\[2mm] h - t*(Pl_0) - P_0 = h + c - t*(Pl_0 + CG_{new}) - P_0 - P_C \\[2mm] h - t*(Pl_0) - P_0 = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[2mm] h - t*(Pl_1) - P_1 = h + c - t*(Pl_0 + CG_{new}) - P_0 - P_C \\[2mm] h - t*(Pl_1) - P_1 = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[2mm] h + c - t*(Pl_0 + CG_{new}) - P_0 - P_C = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[2mm] c - t*CG_{legacy}/A - P_C = 0 \end{array} \right\}$$

where $Pl_0 + Pl_1 = 1$, and $0 \le CG_{legacy} \le A$

We find $Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

$Pl_1 = \frac{P_0 - P_1 + t}{2t}$

$CG_{new} = \frac{c - P_C}{t}$

$CG_{legacy} = A\frac{c - P_C}{t}$

The profit maximization problem is

$P_0, P_c Max P_0 * Pl_0 + P_C * (CG_{new} + CG_{legacy}) = P_0, P_1, P_c Max P_0 * \frac{-P_0 + P_1 + t}{2t} +$

$P_C * (\frac{c - P_C}{t} + A\frac{c - P_C}{t})$

and

$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$

We take First Order Conditions (FOCs) with respect to each price and set them equal to zero:

$P_0 = \frac{P_1 + t}{2}$ $\qquad P_1 = \frac{P_0 + t}{2}$ $\qquad P_C = \frac{c}{2}$. We then solve the system of these three simultaneous equations to find:

$\overline{P_0 = t}$

$P_1 = t$

$$\underline{P_C = \frac{c}{2}}$$

Plugging these back into our consumer equations, we get

$$\overline{Pl_0 = \frac{1}{2}}$$

$$Pl_1 = \frac{1}{2}$$

$CG_{new} = \frac{c}{2t}$, where $CG_{new}^{Pl_0} = \frac{c}{4t}$ purchase Platform 0 to use the complement

with and $CG_{new}^{Pl_1} = \frac{c}{4t}$ purchase Platform 1.

$$\underline{CG_{legacy} = A\frac{c}{2t}}$$

and each firm will earn profits:

$$\Pi_1^{IPComp} = ProfitsfromCG\underbrace{\frac{(1+A)c^2}{4t}} + \frac{t}{2}$$

$$\Pi_2^{IPComp} = \frac{t}{2}$$

MIXED BUNDLING:

A. Each product is sold individually and as part of a bundle:

(SAME outcome as INDIVIDUAL PRICING)

New consumers purchase either Firm 0's platform, Firm 1's platform, Firm

0's bundle or Firm 1's platform and Firm 0's complementary product.

Legacy consumers will purchase, at most, the complementary product. Our

system of equations reflecting their new utility functions:

182

$$\left\{ \begin{array}{c} h - t*(Pl_0) - P_0 = h - t*(Pl_1) - P_1 \\[4pt] h - t*(Pl_0) - P_0 = h + c - t*(Pl_0 + CG_{new}) - P_B \\[4pt] h - t*(Pl_0) - P_0 = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[4pt] h - t*(Pl_1) - P_1 = h + c - t*(Pl_0 + CG_{new}) - P_B \\[4pt] h - t*(Pl_1) - P_1 = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[4pt] h + c - t*(Pl_0 + CG_{new}) - P_B = h + c - t*(Pl_1 + CG_{new}) - P_1 - P_C \\[4pt] c - t*CG_{legacy}/A - P_C = 0 \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \le CG_{new} \le 1$ and $0 \le CG_{legacy} \le A$

We find that

$P_B = P_0 + P_C$, same as the Individual Pricing case.

$Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

$Pl_1 = \frac{P_0 - P_1 + t}{2t}$

$CG_{new} = \frac{c - P_C}{t}$

$CG_{legacy} = A\frac{c - P_C}{t}$

The profit maximization problem is

$$P_0, P_c Max P_0 * Pl_0 + P_C * (CG_{new} + CG_{legacy}) = P_0, P_c Max P_0 * \frac{-P_0 + P_1 + t}{2t} + P_C *$$

$(\frac{c - P_C}{t} + A\frac{c - P_C}{t})$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$$

First, we derive the reaction function:

$P_0 = \frac{P_1 + t}{2}$

$P_1 = \frac{P_0 + t}{2}$, so the two platforms are strategy complements.

$P_C = \frac{c}{2}$

Solving for all prices and consumer mass:

$$\overline{P_0 = t}$$

$$P_1 = t$$

$$P_C = \frac{c}{2}$$

$$P_B = t + \frac{c}{2}$$

$$\overline{Pl_0 = \frac{1}{2}}$$

$$Pl_1 = \frac{1}{2}$$

$CG_{new} = \frac{c}{2t}$, where $CG_{new}^{Pl_0} = \frac{c}{4t}$ purchase Firm 0's bundle and $CG_{new}^{Pl_1} = \frac{c}{4t}$ form their own bundle using Firm 1's platform and Firm 0's complement. Our results are identical to the Individual Pricing case:

$$\underline{CG_{legacy} = A\frac{c}{2t}}$$

and each firm will earn profits:

$$\Pi_1^{IP=MBComp} = ProfitsfromCG\underbrace{\frac{(1+A)c^2}{4t}} + \frac{t}{2}$$

$$\Pi_2^{IP=MBComp} = \frac{t}{2}$$

B. The platform is sold individually and as part of a bundle

(the complement is only sold in the bundle)

New consumers purchase either Firm 0's platform, Firm 1's platform or Firm 0's bundle. This case looks exactly like a scenario where compatibility was not possible, therefore Firm 1's new consumers cannot use the complement. The difference here is that it is not a compatibility issue that prevents them from enjoying the complement, but the marketing choice of not offering the complement individually.

When considering the legacy consumers, we have two plausible scenarios:

184

Then, only the new consumers are of importance. The rival's platform buyers

no longer have access to the complement.

$$\left\{ \begin{array}{c} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B \\ h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1, 0 \leq CG_{new} \leq 1$

Then

$Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

$Pl_1 = \frac{P_0 - P_1 + t}{2t}$

$CG_{new} = \frac{c + P_0 - P_B}{t}$

We solve profit maximization problem

$P_0, P_B Max P_0 * Pl_0 * (1 - CG_{new}) + P_B * (Pl_0 * CG_{new}) =$

$P_0, P_B Max P_0 * \frac{-P_0 + P_1 + t}{2t} * (1 - \frac{c + P_0 - P_B}{t}) + P_B * (\frac{-P_0 + P_1 + t}{2t} * \frac{c + P_0 - P_B}{t})$

and

$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$

To build some intuition, we first look at the best response function:

$P_0 = \frac{1}{3} \left(-c + P_1 + 2(P_B + t) - \sqrt{c^2 + P_1 + c(P_1 - P_B - t) + P_1(-2P_B + t) + (P_B + t)^2} \right)$

$P_B = \frac{c}{2} + P_0$

$P_1 = \frac{P_0 + t}{2}$

After taking derivatives:

$\frac{\partial P_0}{\partial P_B} > 0$ and $\frac{\partial P_B}{\partial P_0} > 0$, the bundle and platform 0 are strategic complements

$\frac{\partial P_0}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_0} > 0$, platform 0 and its rival are strategic complements

$\frac{\partial P_B}{\partial P_1} = \frac{\partial P_1}{\partial P_B} = 0$

Solving the system of equations:

$$\overline{P_0 = -\frac{c^2}{6t} + t}$$

$$P_1 = -\frac{c^2}{12t} + t > P_0$$

$$P_B = \frac{c}{2} - \frac{c^2}{6t} + t$$

$$\overline{Pl_0 = \frac{1}{24}\left(12 + \frac{c^2}{t^2}\right)}$$

$$Pl_1 = 1 - \frac{1}{24}\left(12 + \frac{c^2}{t^2}\right)$$

$CG_{new} = \frac{c}{2t}$, where $CG_{new}^{Pl_0} = \frac{c}{2t} * \frac{1}{24}\left(12 + \frac{c^2}{t^2}\right)$ purchase Firm 0's bundle and

the rest make no purchase.

$$\underline{CG_{legacy} = 0}$$

and each firm will earn profits:

$$\Pi_0^{Pl\&BdlComp} = \frac{(c^2 + 12t^2)^2}{288t^3}$$

$$\Pi_1^{Pl\&BdlComp} = \frac{(c^2 - 12t^2)^2}{288t^3}$$

Case 2: The Two-Product Seller Reaches Out to the Legacy Consumers

to Make a Purchase

Legacy consumers are solely interested, at most, the complementary product. However, the differentiation among the legacy consumers comes into play:

OWN legacy consumers: must purchase the whole bundle to obtain the complement. However, they already own Firm 0's platform from a previous period, so they have zero valuation for a second unit of it. Their utility function associated with Figure 4 is

$U_{0\&CGOWN\ legacy} = c - t * z - P_B$

Rival's legacy consumers: must purchase the whole bundle to obtain the complement. They already own Firm 1's platform from a previous period. By choosing Firm 1's platform in a previous period over Firm 0's, they show a relative dislike for the latter, thus we incorporate their previous period preference in their utility function. Their utility function associated with Figure 3 is

$U_{0\&CG}^{Rival'sLegacy} = h - t(\frac{1}{2} + X_R) + c - ty - P_B$ and we must compare it against

their utility from just using their Firm 1 platform purchased in a past time period without the complement $U_1^{Rival'sLegacy} = h - t(\frac{1}{2} - X_R)$

Subcase 1: Target ALL legacy consumers

Our system of equations reflecting their new utility functions:

$$\begin{cases} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B \\ h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \\ c - t * CG_{OWN\ legacy}/M - P_B = 0 \\ h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R) \end{cases},$$

where $Pl_0 + Pl_1 = 1$, $0 \leq CG_{new} \leq 1$ and $0 \leq CG_{OWN\ legacy} \leq M$ and

$0 \leq RL_{0\&CG} \leq A - M$.

We first focus on our rival legacies, where the marginal rival legacy consumer indifferent between purchasing Firm 0's bundle and just continuing to use his Platform 1 bought in a previous time period is $h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R)$

$h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R)$

$X_R = \frac{c - P_B - yt}{2t}$ or, to make it more intuitive, a consumer's total travel distance

187

for both the bundle components (Platform 0 and the complement)

$$\left(\tfrac{1}{2}\right) + 2X_R + y$$

should be at least as high as the cost

$$\frac{c - P_B}{t} + \left(\tfrac{1}{2}\right).$$

Since all consumers have independently distributed valuations between the two types of products: the platform and the complement, then the number of sales made to the rival legacies is

$$RL_{0\&CG} = X_R * y * (A - M) = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We find that

$$\overline{Pl_0} = \frac{-P_0 + P_1 + t}{2t}$$

$$Pl_1 = \frac{P_0 - P_1 + t}{2t}$$

$$CG_{new} = \frac{c + P_0 - P_B}{t}$$

$$CG_{OWN \; legacy} = M * \frac{c - P_B}{t}$$

$$RL_{0\&CG} = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We then solve profit maximization problem

$$P_0, P_B \, Max \, P_0 * Pl_0 * (1 - CG_{new}) + P_B * (Pl_0 * CG_{new} + CG_{OWN \; legacy} + RL_{0\&CG})$$

and

$$P_1 \, Max \, P_1 * Pl_1$$

To build some intuition, we first look at the best response functions.

$$P_0 = \tfrac{1}{3} \left(-c + P_1 + 2(P_B + t) - \sqrt{c^2 + P_1 + c(P_1 - P_B - t) + P_1(-2P_B + t) + (P_B + t)^2} \right)$$

$$P_1 = \frac{P_0 + t}{2}$$

$$P_B = \frac{-2P_0^2 + y(-A + M)t^2 + 2P_0(P_1 + t) + c(-P_0 + P_1 + (1 + A + M)t)}{2(-P_0 + P_1 + (1 + A + M)t)}$$

The first thing to notice is that we have three equations with four unknowns:

188

each of the prices P_0, P_1, P_B and y. We remember y is the distance a consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement).

In order to be able to solve our model, we must make some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B - t}{2t} \text{ and } RL_{0\&CG} = X_R * y * (A - M) = \frac{c - P_B - t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B}{2t} \text{ and } RL_{0\&CG} = \frac{c - P_B}{2t} * (A - M)$$

After taking derivatives:

$\frac{\partial P_0}{\partial P_B} > 0$ and $\frac{\partial P_B}{\partial P_0} > 0$, the bundle and platform 0 are strategic complements

$\frac{\partial P_0}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_0} > 0$, platform 0 and its rival are strategic complements

$\frac{\partial P_B}{\partial P_1} > 0$, but $\frac{\partial P_1}{\partial P_B} = 0$ the bundle sees platform 1 as a strategic complements

Our closed form solutions are lengthy and available upon request[1]. We note

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[1]Mathematica was used to solve the system of all these equations given the constraints of having all net utilities positive and the consumers masses within appropriate ranges by type. The code is available upon request.

189

that all prices are lower than before (regardless of the extreme value choice for y, but always lower when $y = 1$ due to the higher distance a consumer has to travel to reach the complement), in order to determine the legacy consumers to make a purchase. The profits are below the classical Mixed Bundle ones for both firms.

Subcase 2: Target OWN legacy consumers only

Our system of equations reflecting their new utility functions:

$$\left\{ \begin{array}{c} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B \\ h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \\ c - t * CG_{OWN\ legacy}/M - P_B = 0 \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \leq CG_{new} \leq 1$ and $0 \leq CG_{OWN\ legacy} \leq M$

We find that

$$\overline{Pl_0} = \frac{-P_0 + P_1 + t}{2t}$$

$$Pl_1 = \frac{P_0 - P_1 + t}{2t}$$

$$CG_{new} = \frac{c + P_0 - P_B}{t}$$

$$CG_{OWN\ legacy} = M * \frac{c - P_B}{t}$$

We then solve profit maximization problem

$$P_0, P_B Max P_0 * Pl_0 * (1 - CG_{new}) + P_B * (Pl_0 * CG_{new} + CG_{OWN\ legacy})$$

and

$$P_1 Max P_1 * Pl_1$$

To build some intuition, we first look at the best response function:

$$P_0 = \tfrac{1}{3} \left(-c + P_1 + 2(P_B + t) - \sqrt{c^2 + P_1 + c(P_1 - P_B - t) + P_1(-2P_B + t) + (P_B + t)^2} \right)$$

$$P_1 = \frac{P_0 + t}{2}$$

$$P_B = \frac{2P_0(-P_0+P_1+t)+c(-P_0+P_1+t+2Mt)}{2(-P_0+P_1+t+2Mt)}$$

After taking derivatives:

$\frac{\partial P_0}{\partial P_B} > 0$ and $\frac{\partial P_B}{\partial P_0} > 0$, the bundle and platform 0 are strategic complements

$\frac{\partial P_0}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_0} > 0$, platform 0 and its rival are strategic complements

$\frac{\partial P_B}{\partial P_1} > 0$, but $\frac{\partial P_1}{\partial P_B} = 0$ the bundle sees platform 1 as a strategic complements

Our closed form solutions are lengthy and available upon request[2]. We note that all prices are lower than before, in order to determine the legacy consumers to make a purchase. The profits are below the classical Mixed Bundle ones for both firms.

$$\Pi_{0AllLegacy}^{Pl\&BdlComp} > \Pi_{0OwnLegacy}^{Pl\&BdlComp} \text{ if } \frac{M}{A} <<< 1.$$

C. The complement is sold individually and as part of a bundle (the platform is only sold in the bundle)

New consumers purchase either Firm 0's bundle, Firm 1's platform or Firm 1's platform and Firm 0's complement.

Legacy consumers are only interested in the complement, if at all.

Then, we conjecture the two-product seller will offer the complement at the monopoly price and we show below that is true. First, let's look at our system of equations reflecting their new utility functions:

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[2]Mathematica was used to solve the system of all these equations given the constraints of having all net utilities positive and the consumers masses within appropriate ranges by type. The code is available upon request.

$$\left\{ \begin{array}{c} h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h + c - t * (Pl_1 + CG_{new}) - P_1 - P_C \\ h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \\ h - t * (Pl_1) - P_1 = h + c - t * (Pl_1 + CG_{new}) - P_1 - P_C \\ c - t * CG_{legacy}/A - P_C = 0 \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \le CG_{new} \le 1$ and $0 \le CG_{legacy} \le A$

We find that

$$\overline{Pl_0} = \frac{P_1 - P_B + P_C + t}{2t}$$

$$Pl_1 = \frac{-P_1 + P_B - P_C + t}{2t}$$

$$CG_{new} = 1$$

$$CG_{legacy} = A\frac{c - P_C}{t}$$

We then solve profit maximization problem

$$P_C, P_B Max P_B * (Pl_0 * CG_{new}) + P_C * (CG_{legacy} + Pl_1 * CG_{new}) = P_C, P_B Max P_B *$$

$$(\frac{P_1 - P_B + P_C + t}{2t} * 1) + P_C * (A * \frac{c - P_C}{t} + \frac{-P_1 + P_B - P_C + t}{2t})$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{-P_1 + P_B - P_C + t}{2t}$$

To build some intuition, we first look at the best response function:

$$P_B = \frac{P_1 + 2P_C + t}{2} = \frac{P_1 + 2c - t}{2}$$

$$P_1 = \frac{P_B - P_C + t}{2} = \frac{P_B - c + 2t}{2}$$

$$P_C = c - t$$

After taking derivatives:

$\frac{\partial P_B}{\partial P_C} = 1$, the bundle and complement are strategic complements

$\frac{\partial P_1}{\partial P_C} = \frac{-1}{2}$, platform 1 and the complement are strategic substitutes

$\frac{\partial P_B}{\partial P_1} = 0$, but $\frac{\partial P_1}{\partial P_B} = \frac{1}{2}$ platform 1 considers the bundle as a strategic comple-

ments

Solving the system of equations:

$$\overline{P_C = c - t}$$

$$P_1 = t$$

$$P_B = c$$

$$\overline{Pl_0 = \tfrac{1}{2}}$$

$$Pl_1 = \tfrac{1}{2}$$

$CG_{new} = 1$, where $CG_{new}^{Pl_0} = \tfrac{1}{2}$ purchase Firm 0's bundle and the rest purchase

Firm 1's platform and Firm 0's complement.

$$\underline{CG_{legacy} = A}$$

and each firm will earn profits:

$$\Pi_0^{Bdl\&CGComp} = c + A(c - t) - \tfrac{t}{2}$$

$$\Pi_1^{Bdl\&CGComp} = \tfrac{t}{2}$$

PURE BUNDLING:

The two-product supplier offers platform 0 and the complement only as part of a bundle. **New consumers** purchase either Firm 0's bundle or Firm 1's platform. This case looks exactly like a scenario where compatibility was not possible, therefore Firm 1's new consumers cannot use the complement. The difference here is that it is not a compatibility issue that prevents them from enjoying the complement, but the marketing choice of not offering the complement individually.

When considering the legacy consumers, we have two plausible scenarios:

Case 1: The Two-Product Seller Does NOT Reach Out to the Legacy Consumers

Then, only the new consumers are of importance. The rival's platform buyers no longer have access to the complement.

$$\left\{ h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \right\},$$

where $Pl_0 * CG_{new} + Pl_1 = 1$ and $0 \leq CG_{new} \leq 1$

Then

$$\overline{Pl_0} = \frac{c + P_1 - P_B}{2t}$$

$$Pl_1 = -\frac{c + P_1 - P_B - 2t}{2t}$$

$$CG_{new} = 1$$

We then solve profit maximization problem

$$P_B Max P_B * (Pl_0 * CG_{new}) = P_B Max P_B * (\frac{c + P_1 - P_B}{2t} * 1)$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * (-\frac{c + P_1 - P_B - 2t}{2t})$$

To build some intuition, we first look at the best response function:

$$P_B = \frac{c + P_1}{2}$$

$$P_1 = \frac{-c + P_B + 2t}{2}$$

After taking derivatives:

$\frac{\partial P_B}{\partial P_1} = \frac{1}{2}$ and $\frac{\partial P_1}{\partial P_B} = \frac{1}{2}$, the bundle and rival's platform are strategic complements

Solving the system of equations:

$$\overline{P_C} = c - t$$

$$P_1 = t$$

$$P_B = c$$

194

$$\overline{Pl_0 = \tfrac{1}{2}}$$

$$Pl_1 = \tfrac{1}{2}$$

$CG_{new} = 1$, where $CG_{new}^{Pl_0} = \tfrac{1}{2}$ purchase Firm 0's bundle and the rest make no purchase.

$$\underline{CG_{legacy} = A}$$

and each firm will earn profits:

$$\Pi_0^{PBComp} = c + A(c - t) - \tfrac{t}{2}$$

$$\Pi_1^{PBComp} = \tfrac{t}{2}$$

Case 2: The Two-Product Seller Reaches Out to

the Legacy Consumers to Make a Purchase

Legacy consumers are solely interested, at most, in the complementary product. However, this is not available individually, so the two-product seller must reach out either to its own or to both types of legacy consumers and sell them the bundle, which includes both the platform and the complement. We know the legacy consumers have no use for another platform.

OWN legacy consumers: must purchase the whole bundle to obtain the complement. However, they already own Firm 0's platform from a previous period, so they have zero valuation for a second unit of it. Their utility function associated with Figure 4 is

$$U_{0\&CGOWN\ legacy} = c - t * (z) - P_B$$

Rival's legacy consumers: must purchase the whole bundle to obtain the complement. However, they already own Firm 1's platform from a previous period, so

195

they have no stand alone use for a second platform. Even more so, by choosing Firm 1's platform in a previous period over Firm 0's, they show a relative dislike, which we must incorporate in their utility function. Their utility function associated with Figure 3 is

$$U_{0\&CG}^{Rival'sLegacy} = h - t(\tfrac{1}{2} + X_R) + c - ty - P_B$$

Subcase 1: Targeting ALL legacy consumers

Our system of equations reflecting their new utility functions:

$$\left\{\begin{array}{c} h + c - t*(Pl_0 + CG_{new})_{Bundle} - P_B = h - t*(Pl_1) - P_1 \\[2mm] c - t*CG_{OWN\ legacy}/M - P_B = 0 \\[2mm] h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R) \end{array}\right\},$$

where $Pl_0 * CG_{new} + Pl_1 = 1$, $0 \le CG_{new} \le 1$ and $0 \le CG_{OWN\ legacy} \le M$ and $0 \le RL_{0\&CG} \le A - M$

We first focus on the rival legacies, where the marginal rival legacy consumer indifferent between purchasing Firm 0's bundle and just continuing to use his Platform 1 bought in a previous time period: $h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R)$

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - P_B - yt}{2t}$ or, to make it more intuitive, a rival legacy consumer's total travel distance for both bundle components (Platform 0 and the complement):

$$(\tfrac{1}{2}) + 2X_R + y$$

should cost the rival legacy at most:

$$\frac{c - P_B}{t} + (\tfrac{1}{2}).$$

Since all consumers have independently distributed valuations between the two types of products: the platform and the complement, and a rival legacy is only

interested in both types (Platform 0 and the Complement or none), the number of sales made to the rival legacies is

$$RL_{0\&CG} = X_R * y * (A - M) = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We find that

$$\overline{Pl_0} = \frac{c + P_1 - P_B}{2t}$$

$$Pl_1 = -\frac{c + P_1 - P_B - 2t}{2t}$$

$$CG_{new} = 1$$

$$CG_{OWN\ legacy} = M * \frac{c - P_B}{t}$$

$$RL_{0\&CG} = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We then solve profit maximization problem

$$P_B Max P_B * (Pl_0 * CG_{new} + CG_{OWN\ legacy} + RL_{0\&CG})$$

and

$$P_1 Max P_1 * Pl_1$$

and find the following best response functions

$$P_1 = \frac{-c + P_B + 2t}{2}$$

$$P_B = \frac{c(1 + A + M) + P_1 - yt(A - M)}{2(1 + A + M)}$$

The first thing to notice is that we have two equations with three unknowns: each of the prices P_1, P_B and y. We remember y is the distance a consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement).

After taking derivatives:

$\frac{\partial P_B}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_B} > 0$, the bundle and rival's platform are strategic comple-

197

ments

Solving the system of equations:

$$P_1 = \frac{-c(1+A+M)+(4-A(-4+y)+(4+y)M)t}{3+4A+4M}$$

$$P_B = \frac{c(1+2A+2M)+2t-2yt(A-M)}{3+4A+4M}$$

$$\overline{Pl_0 = \frac{c(1+A+M)+(2+A(4+y)+4M-yM)t}{2t(3+4A+4M)}}$$

$$Pl_1 = \frac{-c(1+A+M)+(4-A(-4+y)+(4+y)M)t}{2t(3+4A+4M)}$$

$$CG_{new} = \frac{c(1+A+M)+(2+A(4+y)+4M-yM)t}{2t(3+4A+4M)}, \text{ where } CG_{new}^{Pl_0} = CG_{new}* \ Pl_0 \text{purchase}$$

Firm 0's bundle and the rest make no purchase.

$$CG_{OWN \ legacy} = \frac{2M(c(1+A+M)+(-1+Ay-yM)t)}{t(3+4A+4M)}$$

$$\underline{RL_{0\&CG} = \frac{(A-M)(2c(1+A+M)-(2+y(3+2A+6M))t)}{2t(3+4A+4M)}}$$

and each firm will earn profits:

$$\Pi_0^{PBComp} = \frac{(1+A+M)(c(1+2A+2M)+2(1-Ay+yM)t)^2}{2(3+4A+4M)^2 t}$$

$$\Pi_1^{PBComp} = \frac{(c(1+A+M)+(-4+A(-4+y)-(4+y)M)t)^2}{2(3+4A+4M)^2 t}$$

We notice the distance a rival legacy would have to travel to reach the complement appears in all our results, as previously explained. In order to be able to fully solve our model, we must make some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t*1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c-P_B-t}{2t} \text{ and } RL_{0\&CG} = X_R * y = \frac{c-P_B-t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same

high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \tfrac{c - P_B}{2t} \text{ and } RL_{0\&CG} = \tfrac{c - P_B}{2t} * (A - M)$$

We note that all prices and profits are lower when $y = 1$ compared to $y = 0$ due to the higher distance a consumer has to travel to reach the complement. The profits are below the classical Mixed Bundle ones for both firms.

Subcase 2: Targeting OWN legacy consumers

Using the same method above in Subcase 1 and comparing the two levels of profits for Firm 0, we find it is optimal to only target own legacies when they are the predominant share of the market.

COMPARING PROFITS UNDER COMPATIBILITY

We first put together the set of inequalities that guarantee that the mass of new consumer is always in the unit interval, the mass of legacy consumers is in its respective interval: $[0, A]$ for all, $[0, M]$ for own legacy and $[0, A - M]$ for rival's legacy and net utilities are always positive.

We compare Firm 0's profits from all of the pricing strategies above and find that:

$$\Pi_0^{IP=MBComp} > \Pi_0^{Bdl\&CGComp} >$$

$$> (>if\ Aislarge \underbrace{\Pi_{0All/OwnLegacy}^{PBComp} \Pi_{0NoLegacy}^{PBComp}}) >$$

$$> (>if\ Aislarge (> if \tfrac{M}{A} small \underbrace{\Pi_{0AllLegacy}^{Pl\&BdlComp} \underbrace{\Pi_{0OwnLegacy}^{Pl\&BdlComp}) \Pi_{0NoLegacy}^{Pl\&BdlComp}})}$$

Therefore, Firm 0 is best off offering the two products individually at $P_0 = t$, $P_1 = t$ and $P_C = \frac{c}{2}$. They share the new consumers market equally. and each serve $\frac{c}{4t}$ of the legacy consumers.

In a similar manner we compare prices and find that

$$P_B^{IP=MBComp}(= P_0 + P_C^{IP=MBComp})$$

$$> P_B^{Bdl\&CGComp} > P_{BNoLegacy}^{Pl\&BdlComp} >$$

$$> P_{BOwnLegacy}^{Pl\&BdlComp} > P_{BAllLegacy}^{Pl\&BdlComp} >$$

$$> P_{BNoLegacy}^{PBComp} > P_{BAll/OwnLegacy}^{PBComp}$$

and

$$P_1^{IP=MBComp} = P_1^{Bdl\&CGComp} > P_{1NoLegacy}^{Pl\&BdlComp} >$$

$$> P_{1OwnLegacy}^{Pl\&BdlComp} > P_{1AllLegacy}^{Pl\&BdlComp} > P_{1NoLegacy}^{PBComp} > P_{1AllLegacy}^{PBComp}$$

and

$$P_0^{IP=MBComp} > P_{0NoLegacy}^{Pl\&BdlComp} > P_{0AllLegacy}^{Pl\&BdlComp}$$

and

$$P_C^{IP=MBComp} > P_C^{Bdl\&CGComp}$$

B.2   Incompatibility

B.2.1   A: Targeting only its Own Legacy Consumers

Unit Mass of New Consumers

We refer to Figure 1 in the paper and use the notation found there.

$U_0 = h - t * (x) - P_0$

$U_1 = h - t * (1 - x) - P_1$

$U_{0\&CG} = h + c - t*(x+y) - P_0 - P_C$; utility function when purchasing Platform

0 and the Complementary Good (CG). There is no $U_{1\&CG}$ since no rival platform

consumer can use the complement along with his platform, due to incompatibility.

Referring to Figures 2 and 3, a **legacy consumer**'s utility is

$U_{CGOWN\ legacy} = c - t * (x) - P_C$

$U_{CGRIVAL's\ legacy} = h - t(\frac{1}{2} + X_R) + c - ty - P_B$ (which we do not focus on in

this section)

INDIVIDUAL PRICING:

To determine how many <u>new consumers</u> will purchase Firm 0's platform only,

Firm 1's platform only or Firm 0's platform and the complement, we solve the

following system of simultaneous equations, where Pl_0 stands for Firm 0's platform,

Pl_1 stands for Firm 1's platform and CG refers to the complementary product:

$$\begin{cases} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new}) - P_0 - P_C \\ h - t * (Pl_1) - P_1 = h + c - t * (Pl_0 + CG_{new}) - P_0 - P_C \\ c - t * CG_{OWN\ legacy}/M - P_C = 0 \end{cases},$$

where $Pl_0 + Pl_1 = 1$, $0 \le CG_{new} \le 1$ and $0 \le CG_{OWN\ legacy} \le M$

We find $Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

$Pl_1 = \frac{P_0 - P_1 + t}{2t}$

$CG_{new} = \frac{c - P_C}{t}$

$$CG_{legacy} = M * \frac{c - P_C}{t}$$

The profit maximization problem is

$$P_0, P_c Max P_0 * Pl_0 * (1 - CG_{new}) + P_C * CG_{OWN\ legacy} + (P_0 + P_C) * Pl_0 *$$

$$CG_{new}\ (+ \max\{0, (P_0 + P_C) * RL_{0\&CG}),$$

where the last term we assume it is not taken into consideration in the maximization problem, but it is part of the final profits. It states that a rival's legacy may find it worth its time to form a bundle and purchase it, or he may not. However, when choosing prices, the two-product seller does not focus on this market segment.

$$= P_0, P_c Max P_0 * \frac{-P_0 + P_1 + t}{2t} * (1 - \frac{c - P_C}{t}) + P_C * (M * \frac{c - P_C}{t}) + (P_0 + P_C) * \frac{-P_0 + P_1 + t}{2t} *$$

$$\frac{c - P_C}{t}$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$$

We take First Order Conditions (FOCs) with respect to each price and set them equal to zero. We derive the following reaction functions:

$$P_0 = \frac{P_1 + t}{2} \qquad P_1 = \frac{P_0 + t}{2} \qquad P_C = \frac{c}{2}.$$ The two platforms are strategic complements and the complement is a monopoly product priced accordingly. We then solve the system of these three simultaneous equations to find:

$$\overline{P_0 = -\frac{c^2}{6t} + t}$$

$$P_1 = -\frac{c^2}{12t} + t > P_0$$

$$P_C = \frac{c}{2}$$

$$\overline{Pl_0 = \frac{1}{24}(12 + \frac{c^2}{t^2})}$$

$$Pl_1 = 1 - \frac{1}{24}(12 + \frac{c^2}{t^2})$$

$$CG_{new} = \frac{c}{2t}, \text{ where } CG_{new}^{Pl_0} = \frac{c}{2t} * \frac{1}{24}(12 + \frac{c^2}{t^2}) \text{ purchase Firm 0's bundle and}$$

the rest make no purchase.

$$CG_{legacy} = \frac{cM}{2t}$$

To determine how many rival legacies would purchase both products from Firm 0, we solve

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_0 - P_C = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_0 - P_C - yt}{2t}, \text{ where } RL_{0\&CG} = \max\{0, X_R * y * (A - M) = \frac{c - P_0 - P_C - yt}{2t} * y * (A - M)\}$$

We remember y is the distance a rival legacy consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement). We notice the distance a rival legacy would have to travel to reach the complement appears in all our results, as previously explained. In order to be able to fully solve our model, we must make some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_0 - P_c - t}{2t} \text{ and } RL_{0\&CG} = X_R * y = \frac{c - P_0 - P_C - t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_0 - P_C}{2t} \text{ and } RL_{0\&CG} = \frac{c - P_0 - P_C}{2t} * (A - M)$$

203

Each firm will earn profits:

$$\Pi_0^{IPIncomp} = \frac{c^4 + 24c^2(1+3M)t^2 + 144t^4}{288t^3} + \max\left\{0, \quad \left(-\frac{c^2}{6t} + t + \frac{c}{2}\right) * RL_{0\&CG}\right\}$$

$$\Pi_1^{IPIncomp} = \frac{(c^2 - 12t^2)^2}{288t^3}$$

MIXED BUNDLING:

A. Each product is sold individually and as part of a bundle:

(SAME outcome as INDIVIDUAL PRICING)

New consumers purchase either Firm 0's platform, Firm 1's platform, Firm 0's bundle or Firm 1's platform and Firm 0's complementary product.

Legacy consumers will purchase, at most, the complementary product. Our system of equations reflecting their new utility functions:

To determine how many <u>new consumers</u> will purchase Firm 0's platform only, Firm 1's platform only or Firm 0's platform and the complement, we solve the following system of simultaneous equations, where Pl_0 stands for Firm 0's platform, Pl_1 stands for Firm 1's platform and CG refers to the complementary product:

$$\left\{\begin{array}{c} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new})_{bundle} - P_B \\ h - t * (Pl_1) - P_1 = h + c - t * (Pl_0 + CG_{new})_{bundle} - P_B \\ c - t * CG_{OWN\ legacy}/M - P_C = 0 \end{array}\right\},$$

where $Pl_0 + Pl_1 = 1, 0 \leq CG_{new} \leq 1$ and $0 \leq CG_{OWN\ legacy} \leq M$

We find $Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

$Pl_1 = \frac{P_0 - P_1 + t}{2t}$

$CG_{new} = \frac{c + P_0 - P_B}{t}$

$$CG_{legacy} = M * \frac{c - P_C}{t}$$

The profit maximization problem is

$$P_0, P_C, P_B Max P_0 * Pl_0 * (1 - CG_{new}) + P_C * CG_{OWN\ legacy} + P_B * Pl_0 * CG_{new} \ (+ \max\{0, (P_B)*$$

$$CG_{RIVAL's\ legacy}),$$

where the last term we assume it is not taken into consideration in the maximization problem, but it is part of the final profits. It states that a rival's legacy may find it worth its time to purchase Firm 0's bundle, or he may not. However, when choosing prices, the two-product seller does not focus on this market segment.

$$= P_0, P_C, P_B Max P_0 * \frac{-P_0 + P_1 + t}{2t} * (1 - \frac{c + P_0 - P_B}{t}) + P_C * (M * \frac{c - P_C}{t}) + P_B * \frac{-P_0 + P_1 + t}{2t} *$$

$$\frac{c + P_0 - P_B}{t}$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$$

We take First Order Conditions (FOCs) with respect to each price and set them equal to zero. We derive the following reaction functions:

$$P_0 = \frac{-2c + P_1 + 4P_B + t}{6} \qquad P_1 = \frac{P_0 + t}{2} \qquad P_B = \frac{c}{2} + P_0 \qquad P_C = \frac{c}{2}.$$ The two platforms are strategic complements and the complement is a monopoly product priced accordingly. We then solve the system of these four simultaneous equations to find:

$\overline{P_0 = -\frac{c^2}{6t} + t}$ same as under Individual Pricing (see case above)

$P_1 = -\frac{c^2}{12t} + t > P_0$ same as under Individual Pricing (see case above)

$P_B = P_0 + P_C = \frac{c}{2} - \frac{c^2}{12t} + t$ sum of the component prices

$P_C = \frac{c}{2}$ monopoly price

$\overline{Pl_0 = \frac{1}{24}(12 + \frac{c^2}{t^2})}$

$Pl_1 = 1 - \frac{1}{24}(12 + \frac{c^2}{t^2})$

$CG_{new} = \frac{c}{2t}$, where $CG_{new}^{Pl_0} = \frac{c}{2t} * \frac{1}{24}(12 + \frac{c^2}{t^2})$ purchase Firm 0's bundle and

the rest make no purchase.

$\underline{CG_{legacy} = \frac{cM}{2t}}$

To determine how many rival legacies would purchase both products from

Firm 0, we solve

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_0 - P_C = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - P_B - yt}{2t}$, where $RL_{0\&CG} = \max\{0, X_R * y * (A - M) = \frac{c - P_0 - P_C - yt}{2t} * y *$

$(A - M)\}$

We remember y is the distance a rival legacy consumer would travel in order

to reach the complement. Moreover, we know that any rival legacy consumer is only

interested in a bundle purchase (and has no use for only one of the items involved:

Platform 0 or the Complement). We notice the distance a rival legacy would have to

travel to reach the complement appears in all our results, as previously explained.

In order to be able to fully solve our model, we must make some approximations for

y or offer upper and lower bounds. We then solve two extreme cases: the first one

refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have

to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - P_0 - P_c - t}{2t}$ and $RL_{0\&CG} = X_R * y = \frac{c - P_0 - P_C - t}{2t} * (A - M)$

The second extreme is assuming $y = 0$, so all rival legacies have the same

high valuation for the complement and do not have to travel any extra distance to

achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \tfrac{c - P_0 - P_C}{2t} \text{ and } RL_{0\&CG} = \tfrac{c - P_0 - P_C}{2t} * (A - M)$$

Each firm will earn profits:

$$\Pi_1^{IP=MBIncompOWN\ LegacyOnly} = \tfrac{c^4 + 24c^2(1+3M)t^2 + 144t^4}{288t^3} + \max\{0,\ (P_B)*RL_{0\&CG}\}$$

$$\Pi_2^{IP=MBIncompOWN\ LegacyOnly} = \tfrac{(c^2 - 12t^2)^2}{288t^3}$$

since $\Pi_1^{IPIncompOWN\ LegacyOnly} = \Pi_1^{MBIncompOWN\ LegacyOnly}$ and

$$\Pi_2^{IPIncompOWN\ LegacyOnly} = \Pi_2^{MBIncompOWN\ LegacyOnly}.$$

B. The complement is sold individually and as part of a bundle (the platform is only sold in the bundle)

New consumers purchase either Firm 0's bundle or Firm 1's platform

Legacy consumers are only interested in the complement, if at all.

Then, we conjecture the two-product seller will offer the complement at the monopoly price and we show below that is true. First, let's look at our system of equations reflecting their new utility functions:

$$\left\{ \begin{array}{c} h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \\ c - t * CG_{OWNlegacy}/M - P_C = 0 \end{array} \right\},$$

where $Pl_0 * CG_{new} + Pl_1 = 1$, $0 \le CG_{new} \le 1$ and $0 \le CG_{OWNlegacy} \le M$ and

$0 \le RL_{0\&CG} \le A - M$

We find that

$$\overline{Pl_0} = \tfrac{c + P_1 - P_B}{2t}$$

$$Pl_1 = 1 - \tfrac{c + P_1 - P_B}{2t}$$

$$CG_{new} = Pl_0 * 1 = \tfrac{c + P_1 - P_B}{2t}$$

$$CG_{OWNlegacy} = M * \tfrac{c - P_C}{t}$$

We then solve profit maximization problem

$$P_C, P_B Max P_B * (Pl_0 * CG_{new}) + P_C * CG_{legacy} \quad (+\max\{0, (P_B) * CG_{RIVAL's\ legacy}),$$

where the last term we assume it is not taken into consideration in the maximization problem, but it is part of the final profits. It states that a rival's legacy may find it worth its time to purchase Firm 0's bundle, or he may not. However, when choosing prices, the two-product seller does not focus on this market segment. $= P_C, P_B Max P_B * (\frac{c+P_1-P_B}{2t} * 1) + P_C * M * \frac{c-P_C}{t}$

and

$$P_1 Max P_1 * Pl_1 = P_1 * (1 - \frac{c+P_1-P_B}{2t})$$

To build some intuition, we first look at the best response function:

$P_B = \frac{c+P_1}{2}$

$P_1 = \frac{-c+P_B}{2} + t$

$P_C = \frac{c}{2}$

After taking derivatives:

$\frac{\partial P_B}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_B} > 0$, the bundle and rival's platform are strategic complements

Solving the system of equations:

$\overline{P_C = \frac{c}{2}}$

$P_1 = \frac{-c+4t}{3} < P_1$

$P_B = \frac{c+2t}{3} < P_B^{COMP}$

$\overline{Pl_0 = \frac{1}{6}(2 + \frac{c}{t})}$

$Pl_1 = \frac{2}{3} - \frac{c}{6t}$

$CG_{new} = 1$, where $CG_{new}^{Pl_0} = \frac{1}{6}(2 + \frac{c}{t})$ purchase Firm 0's bundle and the rest

208

make no purchase.

$$CG_{OWNlegacy} = \frac{cM}{2t}$$

To determine how many rival legacies would purchase both products from Firm 0, we solve

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B - yt}{2t}, \text{ where } RL_{0\&CG} = \max\{0, X_R * y * (A - M) = \frac{c - P_B - yt}{2t} * y * (A - M)\}$$

We remember y is the distance a rival legacy consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement). We notice the distance a rival legacy would have to travel to reach the complement appears in all our results, as previously explained. In order to be able to fully solve our model, we must make some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B - t}{2t} \text{ and } RL_{0\&CG} = X_R * y = \frac{c - P_B - t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B}{2t} \text{ and } RL_{0\&CG} = \frac{c - P_B}{2t} * (A - M)$$

Each firm will earn profits:

$$\Pi_1^{Bdl\&CGIncompOWN\ LegacyOnly} = \frac{c^2(2+9M)+8ct+8t^2}{36t} + \max\{0, \quad RL_{0\&CG}\}$$

$$\Pi_2^{Bdl\&CGIncompOWN\ LegacyOnly} = \frac{(c-4t)^2}{18t}$$

COMPARING PROFITS UNDER INCOMPATIBILITY when FIRM 1 ONLY REACHES OUT TO ITS OWN LEGACY CONSUMERS

We first put together the set of inequalities that guarantee that the mass of new consumer is always in the unit interval, the mass of legacy consumers is in its respective interval: $[0, A]$ for all, $[0, M]$ for own legacy and net utilities are always positive.

We compare Firm 0's profits from all of the pricing strategies above and find that:

$$\Pi_1^{IP=MBIncompOWN\ LegacyOnly} > \Pi_1^{Bdl\&CGIncompOWN\ LegacyOnly}$$

Therefore, Firm 0 maximizes profits when offering the two products individually at $P_0 = -\frac{c^2}{6t}+t < P_1 = -\frac{c^2}{12t}+t$ and $P_C = \frac{c}{2}$. Firm 0 serves more new consumers. Fewer new customers purchase the complement due to incompatibility.

In a similar manner we compare prices and find that

$$P_B^{IP=MBIncompOWN\ LegacyOnly}(= P_0 + P_C^{IP=MBIncompOWN\ LegacyOnly})$$

$$> P_B^{Bdl\&CGIncompOWN\ LegacyOnly}$$

and

$$P_C^{IP=MBIncompOWN\ LegacyOnly} = P_C^{Bdl\&CGIncompOWN\ LegacyOnly}$$

and

$$P_1^{IP=MBIncompOWN\ LegacyOnly} > P_{1NoLegacy}^{Pl\&BdlIncompOWN\ LegacyOnly} > P_{1AllLegacy}^{Pl\&BdlIncompOWN\ LegacyOnly}$$

B.2.2 B: Targeting All Legacy Consumers

Unit Mass of New Consumers

We refer to Figure 1 in the paper and use the notation found there.

$$U_0 = h - t * (x) - P_0$$

$$U_1 = h - t * (1 - x) - P_1$$

$U_{0\&CG} = h + c - t*(x+y) - P_0 - P_C$; utility function when purchasing Platform

0 and the Complementary Good (CG). There is no $U_{1\&CG}$ since no rival platform

consumer can use the complement along with his platform, due to incompatibility.

Referring to Figures 2 and 3, a **legacy consumer**'s utility is

$$U_{CG}^{OWN\ legacy} = c - t * (z) - P_C$$

$U_{0\&CG}^{Rival's Legacy} = h - t(\frac{1}{2} + X_R) + c - ty - P_B$ which we compare with the utility

a rival legacy consumer derives from simply sticking to his previously purchased

Platform 1 $U_1^{Rival's Legacy} = h - t(\frac{1}{2} - X_R)$

INDIVIDUAL PRICING:

To determine how many <u>new consumers</u> will purchase Firm 0's platform only,

Firm 1's platform only or Firm 0's platform and the complement, we solve the

following system of simultaneous equations, where Pl_0 stands for Firm 0's platform,

Pl_1 stands for Firm 1's platform and CG refers to the complementary product:

$$\left\{ \begin{array}{c} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new}) - P_0 - P_C \\ h - t * (Pl_1) - P_1 = h + c - t * (Pl_0 + CG_{new}) - P_0 - P_C \\ c - t * CG_{OWN\ legacy}/M - P_C = 0 \\ h - t(\frac{1}{2} + X_R) + c - ty - (P_0 + P_C) = h - t(\frac{1}{2} - X_R) \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \le CG_{new} \le 1$, $0 \le CG_{OWN\ legacy} \le M$ and $0 \le$

$RL_{0\&CG} \le A - M$

We first focus on the rival legacies, where the marginal rival legacy consumer indifferent between purchasing Firm 0's bundle and just continuing to use his Platform 1 bought in a previous time period: $h - t(\frac{1}{2} + X_R) + c - ty - (P_0 + P_C) = h - t(\frac{1}{2} - X_R)$

$$h - t(\tfrac{1}{2} + X_R) + c - ty - (P_0 + P_C) = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - (P_0 + P_C) - yt}{2t}$ or, to make it more intuitive, a rival legacy consumer's total travel distance for both bundle components (Platform 0 and the complement):

$$(\tfrac{1}{2}) + 2X_R + y$$

should cost the rival legacy at most:

$\frac{c - (P_0 + P_C)}{t} + (\tfrac{1}{2})$.

Since all consumers have independently distributed valuations between the two types of products: the platform and the complement, and a rival legacy is only interested in both types (Platform 0 and the Complement or none), the number of sales made to the rival legacies is

$$RL_{0\&CG} = X_R * y * (A - M) = \frac{c - (P_0 + P_C) - yt}{2t} * y * (A - M)$$

We find $Pl_0 = \frac{-P_0 + P_1 + t}{2t}$

212

$$Pl_1 = \frac{P_0 - P_1 + t}{2t}$$

$$CG_{new} = \frac{c - P_C}{t}$$

$$CG_{OWN \; legacy} = M * \left(-\frac{-c + P_0 + P_C}{t}\right)$$

$$RL_{0\&CG} = \frac{c - (P_0 + P_C) - yt}{2t} * y * (A - M)$$

The profit maximization problem is

$$P_0, P_c Max P_0 * Pl_0 * (1 - CG_{new}) + P_C * (CG_{OWN \; legacy}) + (P_0 + P_C) * (Pl_0 *$$

$$CG_{new} + RL_{0\&CG})$$

$$= P_0, P_c Max P_0 * \frac{-P_0 + P_1 + t}{2t} * \left(1 - \frac{c - P_C}{t}\right) + P_C * \left(M * \frac{c - P_C}{t}\right) + (P_0 + P_C) * \left(\frac{-P_0 + P_1 + t}{2t} * \right.$$

$$\left. \frac{c - P_C}{t} + \frac{c - (P_0 + P_C) - yt}{2t} * y * (A - M)\right)$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$$

We take First Order Conditions (FOCs) with respect to each price and set them equal to zero. We derive the following reaction functions:

$$P_0 = \frac{P_C(-c + P_C) + (P_1 + (A - M)(c - 2P_C))t + (1 - y(A - M))t^2}{2(1 + A - M)t}$$

$$P_1 = \frac{P_0 + t}{2}$$

$$P_C = \frac{-(A - M)t(2P_0 + yt) + c(-P_0 + P_1 + (1 + A + M)t)}{2(-P_0 + P_1 + (1 + A + M)t)}$$

The first thing to notice is that we have three equations with four unknowns: each of the prices P_0, P_C, P_1 and y. We remember y is the distance a consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement). Then, the distance a rival legacy would have to travel to reach the complement would appear in all our final results, as previously explained. In order to be able to fully solve our model, we must make

213

some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - (P_1 + P_C) = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \tfrac{c - (P_1 + P_C) - t}{2t} \text{ and } RL_{0\&CG} = X_R * y = \tfrac{c - (P_1 + P_C) - t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - (P_1 + P_C) = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \tfrac{c - (P_1 + P_C)}{2t} \text{ and } RL_{0\&CG} = \tfrac{c - (P_1 + P_C)}{2t} * (A - M)$$

Building upon some relevant comparative statics:

$\frac{\partial P_0}{\partial P_1} = \frac{\partial P_1}{\partial P_0} > 0$, the platforms are strategic complements

$\frac{\partial P_0}{\partial P_C}, \frac{\partial P_C}{\partial P_0} < 0$ the complement and its compatible platform are strategic substitutes

$\frac{\partial P_C}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_C} = 0$, the complement considers the rival platform a strategic complement

$\frac{\partial P_0}{\partial M} > 0$ the fewer rival legacy consumers, the more emphasis on its own who only need the complement, the higher the platform price Firm 0 can charge

$\frac{\partial P_C}{\partial M} => 0$ the fewer rival legacy consumers, the more emphasis on its own who only need the complement, the higher the complement price Firm 0 can charge

We then solve the system of these four simultaneous equations. We find that

our closed form solutions are lengthy (available upon request[3]). We note that all prices and profits are lower when $y = 1$ compared to $y = 0$ due to the higher distance a consumer has to travel to reach the complement. We note that all prices are lower than under any other Individual Pricing scenario, in order to determine the rival's legacy consumers to make a purchase. The profits are below any other Individual Pricing scenario too for both firms.

MIXED BUNDLING:

A. Each product is sold individually and as part of a bundle:

New consumers purchase either Firm 0's platform, Firm 1's platform or Firm 0's bundle.

Firm 0's own **legacy consumers** will purchase, at most, the complementary product. Firm 1's legacy consumers are interested in the bundle only.

$$\left\{ \begin{array}{c} h - t * (Pl_0) - P_0 = h - t * (Pl_1) - P_1 \\ h - t * (Pl_0) - P_0 = h + c - t * (Pl_0 + CG_{new})_{bundle} - P_B \\ h - t * (Pl_1) - P_1 = h + c - t * (Pl_0 + CG_{new})_{bundle} - P_B \\ c - t * CG_{OWN\ legacy}/M - P_C = 0 \\ h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R) \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \leq CG_{new} \leq 1$, $0 \leq CG_{OWN\ legacy} \leq M$ and $0 \leq RL_{0\&CG} \leq A - M$

We first focus on the rival legacies, where the marginal rival legacy consumer

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[3]Mathematica was used to solve the system of all these equations given the constraints of having all net utilities positive and the consumers masses within appropriate ranges by type. The code is available upon request.

indifferent between purchasing Firm 0's bundle and just continuing to use his Platform 1 bought in a previous time period: $h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R)$

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - P_B - yt}{2t}$ or, to make it more intuitive, a rival legacy consumer's total travel distance for both bundle components (Platform 0 and the complement):

$$(\tfrac{1}{2}) + 2X_R + y$$

should cost the rival legacy at most:

$$\frac{c - P_B}{t} + (\tfrac{1}{2}).$$

Since all consumers have independently distributed valuations between the two types of products: the platform and the complement, and a rival legacy is only interested in both types (Platform 0 and the Complement or none), the number of sales made to the rival legacies is

$$RL_{0\&CG} = X_R * y * (A - M) = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We find that

$$\overline{Pl_0} = \frac{c + P_1 - P_B}{2t}$$

$$Pl_1 = -\frac{c + P_1 - P_B - 2t}{2t}$$

$$CG_{new} = 1$$

$$CG_{OWN\ legacy} = M * \frac{c - P_B}{t}$$

$$RL_{0\&CG} = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We then find

$$Pl_0 = \frac{-P_0 + P_1 + t}{2t}$$

$$Pl_1 = \frac{P_0 - P_1 + t}{2t}$$

$$CG_{new} = \frac{c + P_0 - P_B}{t}$$

$$CG_{OWN\ legacy} = M * \frac{c - P_C}{t}$$

$$RL_{0\&CG} = \frac{c - P_B - yt}{2t} * y * (A - M)$$

The profit maximization problem is

$$P_0, P_C, P_B Max P_0 * Pl_0 * (1 - CG_{new}) + P_C * CG_{OWN\ legacy} + P_B * (Pl_0 * CG_{new} + RL_{0\&CG})$$

$$= P_0, P_C, P_B Max P_0 * \frac{-P_0 + P_1 + t}{2t} * (1 - \frac{c + P_0 - P_B}{t}) + P_C * (M * \frac{c - P_C}{t}) + P_B * (\frac{-P_0 + P_1 + t}{2t} * \frac{c + P_0 - P_B}{t} + \frac{c - P_B - yt}{2t} * y * (A - M))$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * \frac{P_0 - P_1 + t}{2t}$$

We take First Order Conditions (FOCs) with respect to each price and set them equal to zero. We derive the following reaction functions:

$$P_0 = \tfrac{1}{3}(-c + P_1 + 2(P_B + t) - \sqrt{c^2 + P_1 + c(P_1 - P_B - t) + P_1(-2P_B + t) + (P_B + t)^2}$$

$$P_1 = \frac{P_0 + t}{2}$$

$$P_B = \frac{-2P_0^2 + y(-A + M)t^2 + 2P_0(P_1 + t) + c(-P_0 + P_1 + (1 + A - M)t)}{2(-P_0 + P_1 + (1 + A - M)t)}$$

$$P_C = \frac{c}{2}$$

The first thing to notice is that we have four equations with five unknowns: each of the prices P_0, P_C, P_1 and P_B and y. We remember y is the distance a consumer would travel in order to reach the complement. Moreover, we know that any rival legacy consumer is only interested in a bundle purchase (and has no use for only one of the items involved: Platform 0 or the Complement). Then, the distance a rival legacy would have to travel to reach the complement would appear in all our final results, as previously explained. In order to be able to fully solve our model, we must make some approximations for y or offer upper and lower bounds. We then

217

solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B - t}{2t} \text{ and } RL_{0\&CG} = X_R * y = \frac{c - P_B - t}{2t} * (A - M)$$

The second extreme is assuming $y = 0$, so all rival legacies have the same high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c - P_B}{2t} \text{ and } RL_{0\&CG} = \frac{c - P_B}{2t} * (A - M)$$

For certain model parameters, $P_C < P_B$ (when $A - M$ or c are high), so then own legacy consumers are better off purchasing the bundle instead and discarding of the new platform. Profits are then recalculated and always lower.

Then

$\frac{\partial P_0}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_0} > 0$ the two platforms are strategic complements

$\frac{\partial P_0}{\partial P_C} = \frac{\partial P_B}{\partial P_C} = \frac{\partial P_1}{\partial P_C} = 0$

$\frac{\partial P_0}{\partial P_B} > 0$ and $\frac{\partial P_B}{\partial P_0} > 0$ the bundle and the Firm 0 platform are strategic complements

$\frac{\partial P_B}{\partial P_1} > 0$, but $\frac{\partial P_1}{\partial P_B} = 0$ the bundle considers the rival's platform a strategic complement

We then solve the system of these simultaneous equations to find closed form solutions. These are quite complicated and available upon request from the author[4].

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[4]Mathematica was used to solve the system of all these equations given the constraints of having

We note that all prices and profits are lower when $y = 1$ compared to $y = 0$ due to the higher distance a consumer has to travel to reach the complement.

The market outcome is different than under Individual Pricing as all prices and profits are higher $\Pi_0^{IPIncompALL} < \Pi_0^{MBIncompALL}$.

We had previously argued that for certain values of the parameters, $P_C > P_B$. Then, any own legacy will simply purchase the bundle and discard of the platform. In that case, a two-product seller should always resolve his profit maximization problem assuming all legacies will purchase the bundle and continue doing that for as long as $P_C > P_B$.

B. The complement is sold individually and as part of a bundle (the platform is only sold in the bundle)

New consumers purchase either Firm 0's bundle or Firm 1's platform.

Firm 0's own **legacy consumers** are only interested in the complement, if at all. Therefore, being they are the only buyers of the component, the two-product seller is best off offering it at the monopoly price. The rival's legacy consumers will purchase the bundle, at most.

First, let's look at our system of equations reflecting their new utility functions:

$$\left\{ \begin{array}{c} h + c - t * (Pl_0 + CG_{new})_{Bundle} - P_B = h - t * (Pl_1) - P_1 \\ c - t * CG_{legacy}/M - P_C = 0 \\ h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R) \end{array} \right\},$$

where $Pl_0 + Pl_1 = 1$, $0 \leq CG_{new} \leq 1$, $0 \leq CG_{OWN\ legacy} \leq M$ and $0 \leq$

all net utilities positive and the consumers masses within appropriate ranges by type. The code is available upon request.

$RL_{0\&CG} \leq A - M$

We first focus on the rival legacies, where the marginal rival legacy consumer indifferent between purchasing Firm 0's bundle and just continuing to use his Platform 1 bought in a previous time period: $h - t(\frac{1}{2} + X_R) + c - ty - P_B = h - t(\frac{1}{2} - X_R)$

$$h - t(\tfrac{1}{2} + X_R) + c - ty - P_B = h - t(\tfrac{1}{2} - X_R)$$

$X_R = \frac{c - P_B - yt}{2t}$ or, to make it more intuitive, a rival legacy consumer's total travel distance for both bundle components (Platform 0 and the complement):

$$(\tfrac{1}{2}) + 2X_R + y$$

should cost the rival legacy at most:

$$\frac{c - P_B}{t} + (\tfrac{1}{2}).$$

Since all consumers have independently distributed valuations between the two types of products: the platform and the complement, and a rival legacy is only interested in both types (Platform 0 and the Complement or none), the number of sales made to the rival legacies is

$$RL_{0\&CG} = X_R * y * (A - M) = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We find that

$$\overline{Pl_0} = \frac{c + P_1 - P_B}{2t}$$

$$Pl_1 = 1 - \frac{c + P_1 - P_B}{2t}$$

$$CG_{New} = Pl_0 * 1 = \frac{c + P_1 - P_B}{2t}$$

$$CG_{OWNlegacy} = M * \frac{c - P_C}{t}$$

$$RL_{0\&CG} = \frac{c - P_B - yt}{2t} * y * (A - M)$$

We then solve profit maximization problem

$$P_C, P_B Max P_B * (CG_{new} + RL_{0\&CG}) + P_C * CG_{OWNlegacy} =$$

$$= P_0, P_1, P_B Max P_B * (\frac{c+P_1-P_B}{2t} * 1 + \frac{c-P_B-yt}{2t} * y * (A-M)) + P_C * M * \frac{c-P_C}{t}$$

and

$$P_1 Max P_1 * Pl_1 = P_1 * (1 - \frac{c+P_1-P_B}{2t})$$

To build some intuition, we first look at the best response function:

$$P_B = \frac{c+2Ac-2cM+P_1-ty(A-M)}{2+4A-4M}$$

$$P_1 = \frac{-c+P_B+2t}{2}$$

$$P_C = \frac{c}{2}$$

After taking derivatives:

$\frac{\partial P_B}{\partial P_1} > 0$ and $\frac{\partial P_1}{\partial P_B} > 0$, the bundle and rival's platform are strategic complements·

$\frac{\partial P_B}{\partial M} > 0$, the more of its own legacy consumer, the less emphasis on the rival's

bundle buyers and the more on its new consumer bundle buyers.

Solving the system of equations:

$$\overline{P_C = \frac{c}{2}}$$

$$P_1 = \frac{c(-1-A+M)+4t-(-4+y)(A-M)t}{3+4A-4M}$$

$$P_B = \frac{c(1+2A-2M)+2t-2ty(A-M)}{3+4A-4M}$$

The first thing to notice is that we have three equations with four unknowns:

each of the prices P_C, P_B, P_1 and y. We remember y is the distance a consumer

would travel in order to reach the complement. Moreover, we know that any rival

legacy consumer is only interested in a bundle purchase (and has no use for only

one of the items involved: Platform 0 or the Complement). Then, the distance a

rival legacy would have to travel to reach the complement would appear in all our

final results, as previously explained.

We note that for certain model parameters, $P_C < P_B$ (when $A - M$ or c are

221

high), so then own legacy consumers are better off purchasing the bundle instead and discarding of the new platform. Profits are then recalculated and always lower.

$$\overline{Pl_0} = \frac{c(1+A-M)+2t+(4+y)(A-M)t}{2(3+4A-4M)}$$

$$Pl_1 = \frac{c(-1-A+M)+4t-(-4+y)(A-M)t}{2(3+4A-4M)}$$

$$CG_{new}^{Pl_0} = \frac{c(1+A-M)+2t+(4+y)(A-M)t}{2(3+4A-4M)}$$ purchase Firm 0's bundle and the rest make

no purchase.

$$RL_{0\&CG} = \frac{(A-M)(2c(1+A-M)-(2+y(3+2A-2M))t)}{2(3+4A-4M)}$$

$$\underline{CG_{OWNlegacy}} = \frac{cM}{2t}$$

and each firm will earn profits:

$$\Pi_0^{Bdl\&CGIncompALL} = \frac{1}{4(3+4A-4M)^2t}(c^2(2(1+A)(1+2A)^2 - (1+8A(1+A))M -$$

$$8(1+A)M^2 + 8M^3)$$

$$-8c(1+2A-2M)(1+A-M)(-1+Ay-yM)t+$$

$$+8(1+A-M)(1-Ay+yM)^2t^2)$$

calculated assuming $P_C < P_B$, We obtain lower profits if the parameters of the model yield the opposite inequality.

$$\Pi_1^{Bdl\&CGIncompALL} = \frac{(c(1+A-M)-4t+(-4+y)(A-M)t)^2}{2(3+4A-4M)^2t}.$$

In order to be able to fully solve our model, we must make some approximations for y or offer upper and lower bounds. We then solve two extreme cases: the first one refers to $y = 1$ and ensures any rival legacy purchasing Firm 0's platform will have to travel the maximum distance in order purchase the complement:

$$h - t(\tfrac{1}{2} + X_R) + c - t * 1 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \frac{c-P_B-t}{2t}$$ and $RL_{0\&CG} = X_R * y = \frac{c-P_B-t}{2t} * (A - M)$

The second extreme is assuming $y = 0$, so all rival legacies have the same

222

high valuation for the complement and do not have to travel any extra distance to achieve it

$$h - t(\tfrac{1}{2} + X_R) + c - t * 0 - P_B = h - t(\tfrac{1}{2} - X_R)$$

$$X_R = \tfrac{c - P_B}{2t} \text{ and } RL_{0\&CG} = \tfrac{c - P_B}{2t} * (A - M)$$

We note that all prices and profits are lower when $y = 1$ compared to $y = 0$ due to the higher distance a consumer has to travel to reach the complement.

For certain values of the parameters, $P_C > P_B$. Then, any own legacy will simply purchase the bundle and discard of the platform. In that case, a two-product monopolist should always resolve his profit maximization problem assuming all legacies will purchase the bundle and continue doing that for as long as $P_C > P_B$.

$$\overline{Pl_0 = \tfrac{c(1+A+M)+(2+A(4+y)+(4-y)M)t}{2(3+4A+4M)t}}$$

$$Pl_1 = \tfrac{-c(1+A+M)+(4-A(-4+y)+(4+y)M)t}{2(3+4A+4M)t}$$

$$CG_{new}^{Pl_0} = \tfrac{c(1+A+M)+(2+A(4+y)+(4-y)M)t}{2(3+4A+4M)t} \text{ purchase Firm 0's bundle and the rest}$$

make no purchase.

$$RL_{0\&CG} = \tfrac{(A-M)(2c(1+A+M)-(2+y(3+2A+6M))t)}{2(3+4A+4M)t}$$

$$CG_{OWNlegacy} = \tfrac{2M(c(1+A+M)+(-1+Ay-yM)t)}{(3+4A+4M)t}$$

$$P_B^{Bdl\&CG\ IncompAll} = \tfrac{c(1+2A+2M)+2t-2yt(A-M)}{3+4A+4M}$$

$$P_1 = \tfrac{-c(1+A+M)+(4-A(-4+y)+(4+y)M)t}{3+4A+4M}$$

$$\Pi_0^{Bdl\&CGIncompALL} = \tfrac{(1+A+M)(c(1+2A+2M)+2(1-Ay+yM)t)^2}{2(3+4A+4M)^2t}$$

$$\Pi_1^{Bdl\&CGIncompALL} = \tfrac{(c(1+A+M)+(-4+A(-4+y)-(4+y)M)t)^2}{2(3+4A+4M)^2t}$$

COMPARING PROFITS UNDER INCOMPATIBILITY when

FIRM 1 ONLY REACHES OUT TO ALL LEGACY CONSUMERS

We first put together the set of inequalities that guarantee that the mass of new consumer is always in the unit interval, the mass of legacy consumers is in its respective interval: $[0, A]$ for all, $[0, M]$ for own legacy and net utilities are always positive.

We compare Firm 0's profits from all of the pricing strategies above and find that:

$$\Pi_0^{MBIncompALL\ LegacyOnly} > \Pi_0^{Bdl\&CGIncompALL\ LegacyOnly} > \Pi_0^{IPIncompALL\ LegacyOnly}$$

In a similar manner we compare prices and find that

$$P_B^{IPIncompALL\ LegacyOnly}(= P_0 + P_C^{IPIncompALLLegacyOnly}) >$$
$$> P_B^{MBIncompALLLegacyOnly} > P_{BNoLegacy}^{Bdl\&CGIncompALL\ LegacyOnly}$$

and

$$P_C^{MBIncompALL\ LegacyOnly} = P_C^{Bdl\&CGIncompALL\ LegacyOnly} > P_C^{IPIncompALL\ LegacyOnly}$$

$$P_1^{Bdl\&CGIncompALL\ LegacyOnly} > P_1^{MBIncompALL\ LegacyOnly} > P_1^{IPIncompALL\ LegacyOnly}$$

and

$$P_0^{MBIncompOWN\ LegacyOnly} > P_0^{IPIncompOWN\ LegacyOnly}$$

EQUILIBRIUM:

We have shown Individual Pricing is the optimal strategy when the two-product seller chooses incompatibility and targets only its own legacy consumers. If he reaches out to the rival's legacy consumers as well, he is best off offering a

224

Mixed Bundle, where the two products are available both for individual sale and in a package. Comparing the two profits under the conditions of the model, we find that

$$\Pi_0^{MBIncompALL\ LegacyOnly} > \Pi_0^{IPIncompALL\ LegacyOnly}, \text{ when } M \text{ is small, so the}$$

rival's legacy consumers make most of the legacy market.

$$\Pi_0^{MBIncompALL\ LegacyOnly} \leq \Pi_0^{IPIncompALL\ LegacyOnly}[5], \text{ otherwise.}$$

Once we take one step back and look at the compatibility decision, we find that

$$\Pi_0^{IPComp} > \{\Pi_0^{MBIncompALL\ LegacyOnly}, \Pi_0^{IPIncompOWN\ LegacyOnly}\} \text{ always. There-}$$

fore, compatibility is the profit maximizing strategy.

We further look at Firm 1's profits and, following the analysis above, we find that:

$$\Pi_1^{IPComp} > \{\Pi_1^{iComp}, \Pi_1^{jIncompALL\ LegacyOnly}, \Pi_1^{jIncompOWN\ LegacyOnly}\}, \text{ where}$$

$i = MB, Pl\ \&\ Bdl$ (with or without legacy), $Bdl\ \&\ CG$, PB (with or without legacy) and $j = IP, MB, Pl\ \&\ Bdl$ (with or without legacy), $Bdl\ \&\ CG$, PB (with or without legacy)

Therefore, both firms prefer compatibility and Individual Pricing in the subgame perfect equilibrium.

$$\overline{P_0 = t}$$

$$P_1 = t$$

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[5]Mathematica was used to sign this inequality given the constraints of having all net utilities positive and the consumers masses within appropriate ranges by type. The code is available upon request.

$$\overline{P_C = \frac{c}{2}}$$

$$\overline{Pl_0 = \frac{1}{2}}$$

$$Pl_1 = \frac{1}{2}$$

$CG_{new} = \frac{c}{2t}$, where $CG_{new}^{Pl_0} = \frac{c}{4t}$ purchase Platform 0 to use the complement

with and $CG_{new}^{Pl_1} = \frac{c}{4t}$ purchase Platform 1.

$$\overline{CG_{legacy} = A\frac{c}{2t}}$$

and each firm will earn profits:

$$\Pi_1^{IPComp} = ProfitsfromCG \underbrace{\frac{(1+A)c^2}{4t}}_{} + \frac{t}{2}$$

$$\Pi_2^{IPComp} = \frac{t}{2}$$

What if there are only two stages in the game: compatibility decision in time period 0 and the firms choosing prices in period 1 ?

We argue Individual Pricing under Compatibility is the market equilibrium. We check for any Firm 0 profitable deviations by computing all the profits where Firm 1's reaction function is held constant and Firm 0 deviates to another pricing strategy.

Firm 1⇓ Firm 0⟹	IP=MB	Pl&Bdl1	Pl&Bdl2	Bdl&CG	PB1	PB2
IP=MB, Pl&Bdl $P_1 = \frac{P_0+t}{2}$	**A**,B	C,D	E,F	X	X	X

where

$A = \frac{(1+A)c^2}{4t} + \frac{t}{2}$

$B = \frac{t}{2}$

$C = \frac{(c^2+12t^2)^2}{288t^3}$

226

$D = \frac{(c^2 - 12t^2)^2}{288t^3}$

and E, F (long closed form solutions)

Using Mathematica, we find that $A > C$ and $A > E$, so it's not profitable for the two-product seller to deviate from his Individual Pricing strategy under compatibility.

WELFARE

COMPATIBILITY-INDIVIDUAL PRICING

A. Consumer Surplus

Firm 0: Individual Pricing Welfare: we use the market outcome from the Compatibility case when the two-product seller offers the two goods a la carte. We first look at the consumer surplus:

-new consumer purchasing platform 0 $\int_0^{1/2} (h - t * (Pl_0) - t)dx$

-new consumer purchasing platform 1 $\int_0^{1/2} (h - t * (Pl_1) - t)dx$

-new consumer purchasing Firm 0's complement $\int_0^{\frac{c}{2t}} (c - t * (CG_{new}) - \frac{c}{2})dx$

-legacy consumer purchasing the complement $A * \int_0^{\frac{c}{2t}} (c - t * (CG_{new}) - \frac{c}{2})dx$

Then Consumer Surplus $CS^{COMP\ IP} = h + \frac{(1+A)c^2}{8t} - \frac{5t}{4}$

B. Producer Surplus

$\Pi_1^{IPComp} = \frac{(1+A)c^2}{4t} + \frac{t}{2}$

$\Pi_2^{IPComp} = \frac{t}{2}$

C. Total Surplus

$$TS^{COMP\ IP} = CS^{COMP\ IP} + PS^{COMP\ IP} = h + \frac{(1+A)c^2}{8t} - \frac{5t}{4} + \frac{(1+A)c^2}{4t} + \frac{t}{2} + \frac{t}{2} =$$

$$\frac{1}{4t} - t^2 + 4ht + c + Ac)$$

INCOMPATIBILITY with OWN LEGACIES - INDIVIDUAL PRICING

A. Consumer Surplus

Firm 0: Individual Pricing Welfare: we use the market outcome from the Compatibility case when the two-product seller offers the two goods a la carte. We first look at the consumer surplus:

-new consumer purchasing platform 0 $\int_0^{\frac{1}{24}(12+\frac{c^2}{t^2})} (h - t*(Pl_0) - (-\frac{c^2}{6t} + t))dx$

-new consumer purchasing platform 1 $\int_0^{\frac{1}{24t^2}(12t^2 - c^2)} (h - t*(Pl_1) - (-\frac{c^2}{12t} + t))dx$

-new consumer purchasing Firm 0's complement $\int_0^{\frac{c}{2t}} (c - t*(CG_{new}) - \frac{c}{2})dx$

-own legacy consumer purchasing the complement $M * \int_0^{\frac{c}{2t}*\frac{1}{24}(12+\frac{c^2}{t^2})} (c - t * (CG_{new}) - \frac{c}{2})dx$

Then Consumer Surplus $CS^{INCOMPOWN\ IP} = h - \frac{c^6 - 32c^4t^2 - 144c^2(7+4M)t^4 + 5760t^6)}{4608t^5}$

B. Producer Surplus

$\Pi_1^{IPIncompOwnLegacy} = \frac{c^4 + 24c^2(1+3M)t^2 + 144t^4}{288t^3} + \max\{0, \Pi_{0\&CG_{Rival'sLegacy}}^{IPIncompOwnLegacy}\}$

$\Pi_2^{IPIncompOwnLegacy} = \frac{(c^2 - 12t^2)^2}{288t^3}$

C. Total Surplus

$$TS^{INCOMPOWN\ IP} = CS^{INCOMPOWN\ IP} + PS^{INCOMPOWN\ IP} =$$

h-$c^6 - 32c^4t^2 - 144c^2(7+4M)t^4 + 5760t^6) \frac{}{4608t^5 + \frac{c^4 + 24c^2(1+3M)t^2 + 144t^4}{288t^3} + \frac{(c^2 - 12t^2)^2}{288t^3}} =$

$$\frac{1}{11\,520t^3}\left(4160ct^3 - 3ct^2 + 11\,520ht^3 + 80c^4 - 17\,280t^3 + 11\,520t^4 + 2304Mct^3 + 2880Mc^2t^2\right)$$

$$+ \max\{0, \Pi_{0\&CG_{Rival's Legacy}}^{IPIncompOwnLegacy}\}$$

INCOMPATIBILITY with ALL LEGACIES - MIXED BUNDLING

The closed form solutions are quite lengthy and complicated and are available upon request from the author.

WELFARE ANALYSIS

Using Mathematica, we compile the final set on which all our parameters have appropriate values and consumer masses are positive and within the appropriate bounds. We then analyze consumer welfare by marketing strategy and by type of consumers (new vs legacy, own vs rival's legacy, new own vs new legacy) and find that:

<u>Case 1: $\frac{M}{A}$ is very small (closer to zero)</u>

$CS^{COMP\ IP} > CS^{INCOMPALL\ MB} > CS^{INCOMPOWN\ IP}$, if c is large

$CS^{COMP\ IP} > CS^{INCOMPOWN\ IP} > CS^{INCOMPALL\ MB}$, otherwise

<u>Case 2: $\frac{M}{A}$ is very large (closer to one)</u>

$CS^{INCOMPOWN\ IP} > CS^{COMP\ IP} > CS^{INCOMPALL\ MB}$,

where $CS^{INCOMPOWN\ IP} > CS^{COMP\ IP}$ iff $c > 0$ & $M > 0$ & $A > 0$ &

$A + \frac{c^4}{576t^4} < \frac{3}{4} + M + \frac{c^2}{18t^2}$ & $((h > c\ \&\ 2c \geq 3t\ \&\ c \leq 2\ t)||(3t > 2c\ \&\ 2h \geq 3t))$

<u>Case 3: $\frac{M}{A}$ takes all other values</u>

$CS^{COMP\ IP} > CS^{INCOMPALL\ MB} > CS^{INCOMPOWN\ IP}$ any c values

Note that $\frac{\partial(CS^{COMP\ IP}-CS^{INCOMPOWN\ IP})}{\partial M} < 0$ and $\frac{\partial(CS^{COMP\ IP}-CS^{INCOMPOWN\ IP})}{\partial c} > 0$

Moving to total welfare (we already know the producers' surplus from the profits derivation and comparison parts

of the appendix)

Case 1: $\frac{M}{A}$ is very small (closer to zero)

$TS^{COMP\ IP}_{NEWFirm1} > TS^{INCOMPALL\ MB}_{NEWFirm1} > TS^{INCOMPOWN\ IP}_{NEWFirm0}$, if c is large

$TS^{COMP\ IP}_{NEWFirm1} > TS^{INCOMPALL\ MB}_{NEWFirm1} > TS^{INCOMPOWN\ IP}_{NEWFirm0}$, otherwise

Case 2: $\frac{M}{A}$ is very large (closer to one)

$TS^{COMP\ IP}_{NEWFirm1} > TS^{INCOMPOWN\ IP}_{NEWFirm0} > TS^{INCOMPALL\ MB}_{NEWFirm1}$

Case 3: $\frac{M}{A}$ takes all other values

$TS^{COMP\ IP}_{NEWFirm1} > TS^{INCOMPOWN\ IP}_{NEWFirm0} > TS^{INCOMPALL\ MB}_{NEWFirm1}$, if c is small

$TS^{COMP\ IP}_{NEWFirm1} > TS^{INCOMPALL\ MB}_{NEWFirm1} > TS^{INCOMPOWN\ IP}_{NEWFirm0}$, otherwise

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