ABSTRACT

Title of Document: THE IMPACT OF *CURRICULUM FOCAL POINTS* ON STATE MATHEMATICS STANDARDS

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State mathematics standards from six states were analyzed to determine the impact of *Curriculum Focal Points* on recent revisions. The standards were analyzed to determine the alignment between the state documents and *Curriculum Focal Points*. In particular, a comparison of the framework used in each state standards document was compared to the framework used in *Curriculum Focal Points*, and the content within the state mathematics standards, as represented by the grade-level expectations for Grade 5, was compared to the content within *Curriculum Focal Points*. The results were used to compare state standards for consistency between one another and to determine what changes, if any, had occurred from standards developed prior to the release of *Curriculum Focal Points* to standards developed after the release of *Curriculum Focal Points*.

# THE IMPACT OF *CURRICULUM FOCAL POINTS* ON STATE MATHEMATICS STANDARDS

By

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# Chapter 1: Introduction and Background

The *Curriculum and Evaluation Standards for School Mathematics* — the landmark document released by the National Council of Teachers of Mathematics (NCTM) in 1989 — offered recommendations for evaluating the quality of mathematics curricula. The *Standards*, as this document came to be called, described the mathematical content and processes that should appear in each of three grade bands: K-4, 5-8, and 9-12. Eleven years later, NCTM released *Principles and Standards for School Mathematics* (*Principles and Standards*), which described the elements of the K-12 mathematics curriculum, as well as the principles upon which educational decisions should be made, according to four slightly narrower grade bands: preK-2, 3-5, 6-8, and 9-12.

A survey of state mathematics supervisors in 2001 attempted to identify the impact of both documents on state mathematics standards and state-level policy. Respondents from approximately half of the states claimed that *Principles and Standards* had a strong influence on the development of their state's standards (Martin, 2002). However, because the recommendations in both documents were presented by grade bands and were not offered grade-by-grade, one should not be surprised that Reys et al. (2007) found the placement of grade-level expectations (GLEs) within state mathematics standards to differ considerably from state to state.[1]

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[1] As a further example of the inconsistency of state standards, consider the Rosetta Stone Project undertaken by Educational Testing Service in 2003. The purpose of the project was to align state standards one with another, with the intent of creating a tool that would allow for the alignment of any resource to all states' standards. The conventional wisdom was that, if a question developed for the California state assessment aligned to a certain standard for California, and it was known that this standard aligned to particular standards from Indiana, Pennsylvania, New York, and Tennessee, then the question should align to those standards from the other states, too, and could therefore be used on

In contrast to the *Standards* and *Principles and Standards*, the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (*Curriculum Focal Points*), released by NCTM in 2006, attempted to identify the three most important topics for each grade, preK-8.[2] Consequently, if state departments of education subsequently accessed this document as a resource when developing or revising mathematics standards, then there ought to be greater consistency between state standards documents. In addition, state standards should reflect greater alignment to the focal points.

As the name implies, *Curriculum Focal Points* attempts to address the issue of curricular focus. As indicated, the document presents the three most important mathematical ideas at each grade level. This is in contrast to the de facto mathematics curriculum currently in place in the U.S., which includes a large number of topics but, according to critics, does not cover them with sufficient depth. As an example, Schmidt, Houang, and Cogan (2002) note, "eighth-grade mathematics textbooks in Japan have around 10 topics, but U.S. eighth-grade textbooks have over 30 topics" (p. 12).

Calls for the development of mathematics standards that are consistent between states and for the use of common curriculum materials have come from many sources. The American Federation of Teachers (2008) asserts that "states must develop common, coherent, grade-by-grade standards." (p. 1). They go on to say that

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the other states' assessments. Unfortunately, this was not the case. Though many hours were invested in the project, the inconsistency was so great that the project was abandoned after three months.

[2] Part of the reason that *Curriculum Focal Points* was organized by grade band was the testing mandate from No Child Left Behind for all students in Grades 3–8. Because of the mandate, NCTM believed that teachers would benefit from grade-by-grade guidance (F. Fennell, personal communication, November 5, 2009).

> Grade-by-grade content standards increase the likelihood that all students are exposed to a rigorous, sequenced curriculum that is consistent across grades, schools and school districts. Grade-specific standards also facilitate greater alignment of standards-based curriculum, assessments, textbooks, professional development and instruction. States that organize their standards grade by grade are best able to specify what students should learn and when they should learn it. (AFT, 2008, p. 2)

Likewise, Tinker (2009) suggests that the entire K-12 STEM curriculum should be revamped with the development of new curriculum materials. “The new curriculum would be tied to national standards, so students could move freely among schools during the school year” (p. 2).

Though not intended to serve as national standards, the *Curriculum Focal Points* do have the power to “drive discussion about what’s important in preK-8 mathematics” (Fennell, 2007, p. 315) and to “guide the thinking of the profession in the development of the next generation of curriculum standards, textbooks, and tests” (NCTM, 2006, p. 2). Grouws and McKnight (2008) argue that the current lack of consensus between states on where mathematical topics ought to be placed in the curriculum, as well as poor performance on international assessments, gave rise to discussions that ultimately led to the creation of *Curriculum Focal Points*. “The document provides a beginning point for states and districts to design more focused curricular expectations” (p. 345). When *Curriculum Focal Points* is used as a resource for developing state mathematics standards, the ideal result is consistency from state to state and an intended curriculum with the potential to provide all students with a common experience in mathematics.

States across the country have begun to develop mathematics standards using *Curriculum Focal Points* as the primary resource. In 2007, Fennell reported that at least 12 states were using *Curriculum Focal Points* to guide conversations about the mathematics curriculum, and a survey by the Center for the Study of Mathematics Curriculum [CSMC] (2007) found that 11 states used *Curriculum Focal Points* as a resource in their most recent revisions (Reys et al., 2005). In 2008, Rubillo reported that 22 states were using *Curriculum Focal Points* as the primary resource for revising their state mathematics standards (J. Rubillo, personal communication, November 12, 2008).

## *Background*

The first chapter within *Curriculum Focal Points* is titled, “Why Identify Curriculum Focal Points?” (NCTM, 2006, p. 3). The implied answers to this question were given by the headers used for the two sections in the chapter:

- Inconsistency in the Placement of Topics by Grade Level in U.S. Mathematics Curricula (p. 3)
- The Importance of Curricular Focus in Mathematics (p. 4).

The authors of the *Curriculum Focal Points* clearly hoped that the document would bring greater focus[3] to the U.S. mathematics curriculum as identified in state

[3] The authors of *Curriculum Focal Points* also hoped that the document would bring greater coherence to the U.S. mathematics curriculum, as implied by the full title. Yet, for a variety of reasons, including

mathematics standards. In addition, there was the suggestion that if states used *Curriculum Focal Points* as a guiding document, a greater level of consistency between state mathematics standards might be realized.

Schmidt (2008) claims that focus is an important aspect of mathematics standards. When standards exhibit focus, they address only a limited number of topics so that teachers can spend an adequate amount of time on each of them. A common criticism of the mathematics curriculum in the United States is that it contains too many topics and therefore lacks focus.

> …the curriculum that is enacted in the U.S. (compared to the rest of the world) is highly repetitive, unfocused, unchallenging, and incoherent… Our teachers work in a context that demands that they teach a lot of things, but nothing in-depth. We truly have standards, and thus enacted curricula, that are a "mile wide and an inch deep." (Schmidt, Houang, & Cogan, 2002, p. 1)

The content in the U.S. mathematics curriculum must "be sequenced in an understandable and logical way, a goal that is best accomplished when the focus is on important disciplinary concepts or topics" (Schmidt & Prawat, 2006, p. 642).

The *Principles and Standards* explain the need for focus within the curriculum. This document asserts, "a well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention

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time and the difficulties inherent in attempting to measure coherence in the curriculum, the analysis herein only attempts to analyze curricular focus in state standards documents. The decision to concentrate exclusively on curricular focus was made for logistical reasons and is not meant to diminish the importance of curricular coherence.

at different points in time. It also gives guidance about the depth of study warranted at particular times" (NCTM, 2000, p. 16). *Curriculum Focal Points* maintains that "a focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications" (NCTM, 2006, p. 4).

The authors of *Curriculum Focal Points* suggest that a curriculum organized around the focal points will comprise a connected body of mathematical knowledge based on "the most significant mathematical concepts and skills at each grade level" (NCTM, 2006, p. 1). In the introduction to *Curriculum Focal Points*, it is implied that focus is a highly sought outcome:

> Curriculum focal points are important mathematical topics for each grade level, pre-K–8. These areas of instructional emphasis can serve as organizing structures for curriculum design and instruction at and across grade levels. The topics are central to mathematics: they convey knowledge and skills that are essential to educated citizens, and they provide the foundations for further mathematical learning. Because the focal points are core structures that lay a conceptual foundation, they can serve to organize content… They are indispensable elements in developing problem solving, reasoning, and critical thinking skills, which are important to all mathematics learning. (NCTM, 2006, p. 5)

*Curriculum Focal Points* specifies three topics that should receive priority coverage at each grade level. In addition, it identifies several connections to the focal points in each grade that should also be integrated into the curriculum.[4] *Curriculum Focal Points* provides a limited number of important topics rather than an exhaustive list of all possible topics. Consequently, teachers can dedicate an adequate amount of classroom time to a few essential areas instead of providing insufficient coverage to a broad array of items.

Importantly, problem solving, reasoning, communication, making connections, and designing and analyzing representations, which are the five process standards previously documented in the *Principles and Standards*, are highlighted throughout *Curriculum Focal Points*. By making reference to *Principles and Standards*, the authors are showing that ideas previously presented in *Principles and Standards* provide the foundation upon which *Curriculum Focal Points* rests.

*Curriculum Focal Points* is meant to serve as a framework for creating focused mathematics curriculum in all states. The document expresses the importance of curricular focus and consistency from state to state by asserting

> Many factors have contributed to the need for a common mathematical focus for each grade level, pre-K–8. These include the increased emphasis on accountability testing, high levels of mobility of both students and teachers, and greater costs of curriculum development. A focused, coherent

[4] Within the document, the focal points for each grade are described on a separate page. The three focal points for the grade are listed in bold along the left side of the page, and under each bold heading is a description. The connections to the focal points for that grade are listed in a sidebar along the right side of the page.

> mathematics curriculum with a national scope has the potential to ease the impact of widely varying learning and assessment expectations on both students and teachers who relocate.
> (NCTM, 2006, p. 4)

## *Rationale*

Released on September 9, 2006, *Curriculum Focal Points* and its supporting documents were downloaded more than 1,000,000 times from the NCTM web site (http://nctm.org) — including 624,571 copies of the full document and another 296,952 copies of specific grade-level focal points — during its first 2 years (R. Aldridge, personal communication, October 23, 2008). These numbers suggest that the recommendations provided by *Curriculum Focal Points* are of great interest to mathematics educators and, likely, to the general public as well.

Many organizations and government entities are interested in the impact of *Curriculum Focal Points*. The Institute for Educational Sciences commissioned a study to determine the extent to which mathematics standards for Grades K-8 in Kentucky, Tennessee, Virginia and West Virginia align with *Curriculum Focal Points*. Although IES decided at the point of final revision that the methodology used "was not sufficiently scientific or replicable and therefore did not publish the study" (D. Walker, personal communication, January 5, 2009), the fact that the U.S. Department of Education (USDOE) desired this information in the first place speaks volumes about the respect given to *Curriculum Focal Points*.

In addition, publishers and school districts are quick to assert that their materials align with *Curriculum Focal Points*, and a cursory survey on the web found several references to *Curriculum Focal Points* that show its potential influence. A press release distributed by PLATO Learning contends that the Straight Curve™ Mathematics series is aligned with *Curriculum Focal Points* for difficult-to-master mathematics concepts (PLATO Learning, 2007). Similarly, the title of another press release asserts, "SRA Real Math Aligns With NCTM *Curriculum Focal Points*" (SRA McGraw-Hill, 2007). An information sheet released by the Frederick County Public Schools explains that, "*Investigations* [the elementary textbook series chosen by the district] more fully supports and corresponds with our curriculum and the NCTM focal points than the previous textbook series" (2008, p. 2). A web page about MathScore®, an online mathematics practice and assessment program, claims that, "Long before the *Curriculum Focal Points* were released, MathScore already supported nearly every focal point from grades 2 to 8, confirming that our approach is solid" (MathScore, 2008). Following the release of *Curriculum Focal Points*, Renaissance Learning distributed marketing materials that stated "New NCTM Focal Points Emphasize Fundamentals!" and a catalog of Innovative Learning Concepts, Inc., said, "NCTM Goes Back to Basics." NCTM sent letters to both organizations asking them to cease the use of "inaccurate, misleading statements" in marketing materials (K. Krehbiel, personal communication, November 5, 2009).

With the U.S. Department of Education investigating the alignment of selected state mathematics standards to *Curriculum Focal Points,* and with publishers and school districts so interested in giving the impression that their materials and curriculum adhere to the recommendations put forth in *Curriculum Focal Points,* a reasonable question is whether state mathematics standards do indeed reflect the recommendations of *Curriculum Focal Points*. More generally, how closely are publishers, policymakers, and others following the guidelines put forth by NCTM?

To determine if *Curriculum Focal Points* is having its intended impact, namely to "guide the thinking of the profession in the development of the next generation of curriculum standards" (NCTM, 2006, p. 2), the CSMC (2007) conducted a survey of 52 state supervisors of mathematics. The survey attempted to determine the degree to which *Curriculum Focal Points* has influenced the development of GLEs within the state-level K-8 mathematics standards. Of the 31 respondents who indicated that their states had not yet published, updated, revised or reviewed their state standards since the release of *Curriculum Focal Points*, 20 of them "indicated that *Curriculum Focal Points* will 'very likely' impact future revisions of state GLEs, and 8 states indicated that it is 'somewhat likely' that *Curriculum Focal Points* will have an influence" (p. 2). Eleven state supervisors reported that their states have published, updated, revised or reviewed their state mathematics GLEs since the fall of 2006 when *Curriculum Focal Points* was released, and all of them report that *Curriculum Focal Points* was used as a resource in the revision. "Eight states (Washington, Utah, Tennessee, New Mexico, Nevada, Minnesota, Florida, and Iowa) report a 'significant' impact" (p. 2).

Similarly, Fennell (2007) reported that "over one-fourth of the states and many local school districts have decided to use NCTM's *Curriculum Focal Points* to drive discussion about what's important in preK-8 mathematics curricula" (p. 315), and Rubillo reported that 22 states have indicated that *Curriculum Focal Points* served as the basis for the most recent revision of their state mathematics standards (J. Rubillo, personal communication, November 12, 2008).

The analysis undertaken for this thesis studied the impact of *Curriculum Focal Points* on state standards. In particular, it considered the standards of six states that used *Curriculum Focal Points* as a primary resource during their most recent revisions.[5] At the initiation of this study, no such research had been done, but Reys suggested that it was needed. Reys also indicated that CSMC was planning to conduct a similar study but wanted to "allow more time to pass" so that all states would have a chance to implement *Curriculum Focal Points* during the revision of their standards (B. Reys, personal communication, January 2, 2009). Other individuals, including former NCTM Executive Director Jim Rubillo, indicated that research about the extent to which *Curriculum Focal Points* has influenced state standards would be valuable to the Council (J. Rubillo, personal communication, October 22, 2008). Similar efforts have been undertaken previously to identify the impact of the *Curriculum and Evaluation Standards* and *Principles and Standards* on state mathematics standards (Martin, 2002).

In 1999, the Research Advisory Committee of NCTM proposed the development of the Standards Impact Research Group (SIRG) as a response to suggestions — both from within NCTM as well as from external groups — that research should be conducted on the impact of the *Standards* and to "insure a sustained, carefully conceptualized, and systematic assessment" of the *Principles and*

[5] Since the release of *Curriculum Focal Points* in September 2006, 21 states have revised their mathematics standards. Chapter 3 contains a description of how the six states used for this analysis were selected.

*Standards*. The report about the project stated, “just as teachers use information about what students are learning to inform their teaching, so too should NCTM learn about and consider the results of its practices in order to guide future decisions” (p. 484-485).

The SIRG was created to analyze the impact of *Principles and Standards*, but *Principles and Standards* describes the mathematical ideas on which the *Curriclum Focal Points* rest (NCTM, 2006, p. 8), and *Curriculum Focal Points* extends the ideas put forth in *Principles and Standards* “by describing an approach to curriculum development that focuses on areas of emphasis within each grade” (NCTM, 2006, p. 1). One can therefore conclude that the mathematics education research community would be interested in the impact of *Curriculum Focal Points*, given that *Curriculum Focal Points* is an extension of *Principles and Standards*.

The research agenda put forth by SIRG was meant to identify the influence of *Principles and Standards* in the areas of policy, assessment, instructional materials, teaching practices, and student achievement (Ferrini-Mundy, 2004). The analysis in this thesis will focus only on the impact of *Curriculum Focal Points* as it relates to state policy documents, specifically, the GLEs for Grade 5 in six selectively chosen states.

In 2001, the National Academy of Sciences released *Investigating the Influence of Standards: A Framework for Research in Mathematics, Science, and Technology Education*. This framework included several important questions, including, “How are nationally developed standards being received and interpreted? … What has changed as a result? … What components of the system have been affected and how?" (p. 36-37). If these questions "are used in the context of particular investigations, both producers and consumers of research can acquire important insights into possible benefits and limitations of nationally developed standards" (p. 38).

This analysis has addressed the question of what has changed as a result of the release of *Curriculum Focal Points*. Specifically, have state mathematics standards been affected and, if so, how? Do the state standards reflect the "look and feel" of *Curriculum Focal Points*? Are the big ideas within the state standards easily identified, and, for the most part, do these big ideas address the same topics as the focal points? Do the supporting ideas in the state standards cover the same topics identified by the connections in *Curriculum Focal Points*, or do the revised state standards contain other topics? Did revisions of state mathematics standards based on *Curriculum Focal Points* result in consistency from state to state? And finally, did *Curriculum Focal Points* have an influence on the state standards; that is, for states that used *Curriculum Focal Points* as a resource when revising their state mathematics standards, do the revised standards exhibit greater alignment to the focal points than the previous state standards?

## Purpose of the Study

The specific purpose of this study is to assess the impact of *Curriculum Focal Points* on state mathematics standards, but the more general purpose is to determine the impact of national standards on state level policy.

The Committee on Understanding the Influence of Standards of the National Research Council (2002) asserted that if standards affect the content taught in schools, then state mathematics standards should mirror national mathematics standards. “State content standards would be consistent with content specified by the nationally developed standards, providing comprehensive guidance on what should be taught at each grade level” (p. 44). Consequently, if *Curriculum Focal Points* is having any effect on standards, then the recommendations contained therein should be reflected in state policy documents.

This analysis looked at curriculum, one of the three “channels of influence” identified by the National Research Council (2002, p. 83) and referenced in the proceedings of the NCTM Research Catalyst Conference (NCTM, 2004). It attempted to address one small sliver of the research agenda put forth by the SIRG. This analysis did not attempt to identify the impact of *Curriculum Focal Points* on student achievement or even the enacted curriculum, but it has examined whether *Curriculum Focal Points* as a policy document had an effect on the intended curriculum.

By analyzing state mathematics standards and comparing them to *Curriculum Focal Points*, this investigation attempted to answer several questions:

- Do the state standards use a framework similar to that used in *Curriculum Focal Points*?
- Do the state standards address the same big ideas as *Curriculum Focal Points*?

- How well does the state document match the content of *Curriculum Focal Points*, as indicated by the percentage of GLEs that align to the focal points and connections?
- How do state documents compare to one another? That is, even if all state standards documents reflect a significant impact from *Curriculum Focal Points*, is there resultant consistency from state to state?
- Finally, how do the state mathematics standards developed using *Curriculum Focal Points* as a resource compare to the state mathematics standards developed prior to its release? Is greater alignment to the focal points and connections evident in the revised version?

These last two questions are particularly important. Much has been written about the lack of consistency between state standards documents. An analysis by Reys et al. (2007) showed that there is significant variation between state mathematics curriculum standards, with little consensus on the placement or emphasis of topics within specific grade levels. The analysis looked at the mathematics learning expectations for Grade 4 of the 10 most populous states and "found the intersection of learning expectations across the ten state documents quite small, while the union of these learning expectations was quite large and varied" (p. 9).

One stated goal of the Council in releasing *Curriculum Focal Points* is to achieve a level of consensus between state standards documents (NCTM, 2006). A major factor in the need for common curriculum at each grade level includes the percent of students and teachers who relocate. According to a report by the National

Center for Education Statistics, approximately 8.1% of teachers transfer to a new school each year (U.S. Department of Education, 2007). For school-aged children, the figures are even more dramatic. Data from the U.S. Census Bureau (2008) implies that more than 14.4% of children under the age of 17 relocate annually. Consistency between state standards would mitigate the negative consequences on students and teachers who change schools or districts.

### *Statement of the Problem*

The following question provides the primary motivation for this analysis:

- What is the impact of *Curriculum Focal Points* on state mathematics standards, especially in states for which *Curriculum Focal Points* served as a primary reference when the standards were revised?

### *Research Questions*

Four research questions are answered via this analysis:

1. Does the overall framework used in state mathematics standards match the framework employed in *Curriculum Focal Points*? In particular, do the state standards documents identify big ideas as well as supporting ideas at each grade level, and do they address the process standards of “problem solving, reasoning, communication, making connections, and designing and analyzing representations” (NCTM, 2006, p. 17)?

2. What percent of the GLEs within state documents address the focal points, what percent address the connections, and what percent address other ideas and topics?
3. For states that claim *Curriculum Focal Points* played a significant role during revisions (as indicated by state mathematics supervisors who responded to a survey conducted by CSMC), is there consistency between mathematics standards from state to state?
4. What major changes can be identified between the state mathematics standards developed prior to the release of *Curriculum Focal Points* and the revised state mathematics standards for which *Curriculum Focal Points* was used as a resource in the revision process?

Though not a research question per se, this thesis also attempted to determine what factors, if any, influenced the state standards revision processes.

# Chapter 2: Review of the Literature

Calls for focus and consistency in mathematics are nothing new. In 1894, the Committee of Ten called for greater focus in mathematics by suggesting that some topics ought to be omitted from the curriculum, while other topics ought to receive greater attention (NEA, 1894, p. 24). Similarly, in 1899, the Committee of the Chicago Section of the American Mathematical Society released a report which suggested that instruction in arithmetic should be confined to just 11 topics (Young et al., 1899, p. 138) and that those topics should comprise the entirety of the mathematics curriculum through Grade 6. In a sense, this list of 11 topics could be considered the first set of curriculum focal points in mathematics.

These same two reports made strong recommendations regarding consistency, as well. The report of the Committee of Ten (1894) claimed that secondary mathematics courses should have the same expectations for every student. Indeed, the CSMC (2004) claimed, “the primary purpose in convening the Committee of Ten was to provide a national force for standardizing the secondary school curricula” (p. 1).

Likewise, Young et al. (1899) recommended that there be consistency from school to school — and, by extension, from state to state — by adopting several resolutions. These resolutions stated that the same work in mathematics should be required for all students prior to secondary school, and “that in the secondary school the standard course in mathematics… should be required of all pupils, and that the instruction in this course should be the same for all pupils” (p. 136).

Although the students in the 1890's were no doubt different from the students of today, recent calls for focus and consistency have come from myriad groups. The following review of literature will examine various reports from several individuals and organizations, each suggesting that the mathematics curriculum needs greater focus, requires consistency from state to state, or both. Of necessity, this review will also give some attention to the recent push for national standards in mathematics.

*Focus in the Mathematics Curriculum*

The introduction to *Curriculum Focal Points* states that, "Many factors have contributed to the need for a common mathematical focus for each grade level, pre-K–8… a focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis" (NCTM, 2006, p. 4). The National Mathematics Advisory Panel echoed the need for more focus. "U.S. curricula typically include many topics at each grade level, with each receiving relatively limited development, while top-performing countries present fewer topics at each grade level but in greater depth" (USDOE, 2008, p. 20).

Schielack and Seeley (2007) stated that state curriculum documents often "evolved into lists of specific skills that are designed to be used more for assessment than instruction. A mathematics curriculum organized around focal points — that is, a focused curriculum — highlights the most important mathematical ideas for each grade" (p. 78). They recommended that states and districts who use *Curriculum Focal Points* as a guiding document ask:

- What key ideas are reflected in our existing mathematics curriculum? Are there any key ideas in NCTM's set of focal points that do not appear somewhere in our curriculum, or vice versa?
- Can we determine from our curriculum where to place the emphasis at each grade? Can we tell how a topic (like fraction operations) should be treated differently at grades 5, 6, 7, and 8 to help the student learn for long-term retention without repetition each year? (Schielack & Seeley, 2007, p. 79)

In 2005, the American Institute of Research compared the standards of several states with those of Singapore, the country with the highest performing students in mathematics on the Trends in International Mathematics and Science Study [TIMSS] 2003. For Grades 1-6, the state mathematics standards for every state included more topics per grade than the Singapore standards. At the low end, North Carolina included 20% more topics per grade than Singapore, and at the high end, Florida included 160% more topics in each grade (Ginsburg, Leinwand, Anstrom, & Pollock, 2005, p. 31).

Any discussion that concerns focus in the mathematics curriculum necessarily includes significant reference to the work of William H. Schmidt. A professor of education at Michigan State University, he was previously the Director of the U.S. National Research Center for TIMSS and the National Research Coordinator for TIMSS. He is the often connected with the phrase "a mile wide and an inch deep" (Schmidt, McKnight, & Raizen, 1996, p. 1) to describe the emphasis on breadth over depth in mathematics education in the U.S. He is the foremost expert regarding focus in the curriculum, and he served as a reviewer for the *Curriculum Focal Points* document.

In 1997, Valverde and Schmidt claimed that focus in the curriculum is vital, because teachers are likely to devote less time to each topic if the textbook contains an extensive list of topics. Research from TIMSS shows that U.S. textbooks cover more topics than the textbooks of other countries, and “the majority [of teachers] appear to be attempting the Herculean task of covering all the material in the textbook… the result is that U.S. teachers cover more topics per grade than is common in most TIMSS countries” (p. 62). This seems to be the reason that the authors of *Curriculum Focal Points* claim “a focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis” (p. 4).

Almost a decade later, Schmidt (2008) claimed, “The single most important result of the Third International Mathematics and Science Study (TIMSS) is that we now know that student performance is directly related to the nature of curricular expectations” (p. 22). Consequently, standards must include only a few important topics so that teachers are able to devote adequate time to teaching them. Whereas “top-achieving countries usually cover about four to six topics related to basic numeracy, measurement, and arithmetic operations” in the early grades, “state and district standards, as well as textbooks, often cram 20 topics into the first and second grades” (p. 22-23).

Many analyses of TIMSS data found that there was a lack of focus in the U.S. mathematics curriculum (NCES, 1997; Schmidt, Houang, & Cogan, 2002; Schmidt, McKnight, & Raizen, 1996). Schmidt (2006) claimed that the lack of focus in the

U.S. mathematics curriculum was due to “the large number of topics that needed to be and actually were covered” (p. 1); moreover, his analysis found that “covering too many topics does have a negative impact on student learning even when controlling for coherence” (Schmidt, 2006, p. 10).

To reach this conclusion, a 32 × 8 matrix was used to identify the grade-level coverage of each topic that occurred in at least four of the six top-scoring countries (Singapore, Korea, Japan, Hong Kong, Belgium, and the Czech Republic) on TIMSS. A dot was placed in the matrix to indicate the grade(s) at which each of 32 different topics was covered in Grades 1-8. For instance, a dot was placed in the columns for Grades 3-6 within the row for common fractions, because at least four of the six countries included common fractions in their curriculum for Grades 3, 4, 5, and 6.

Schmidt (2006) said that the “region of the matrix with ‘dots’ was taken as a model or ideal scenario” (p. 6). A similar matrix of grade-level coverage was then created using the curricula from various countries, and the matrices of those countries were compared to the ideal matrix. Yet Schmidt (2006) is quick to point out that this ideal matrix may not be the only satisfactory model. Consequently, when other countries were compared to this model, a low level of overlap only reflects “a deviation from the empirically derived ideal scenario” (p. 7), not necessarily an indicator of variation from a single, acceptable approach.

Schmidt (2006) summarizes the entire process by saying, “One way to think of this process is the model region was highlighted creating a ‘silhouette’… which was then superimposed on the country maps” (p. 6) and the degree of overlap was used to make a comparison.

Even though Schmidt's analysis attempted to identify curricular coherence, whereas this thesis attempts to analyze curricular focus, there are two implications from the work of Schmidt that are applicable to this thesis. First, because the inclusion of too many topics in the curriculum has a negative impact on student achievement, it seems vital that state standards contain only the several most important mathematical topics at each grade. It will therefore be important to examine the number of big ideas covered in the state mathematics standards.

Second, the "ideal or model scenario" used by Schmidt (2006) represents only one possible model, yet it serves as an acceptable model for evaluating the curriculum of other countries. Similarly, the list of topics presented in *Curriculum Focal Points* represents only "one possible response to the question of how to organize curriculum standards" (NCTM, 2006, p. 3), yet the focal points serve an acceptable model against which to compare state standards. Just as the model matrix in Schmidt's analysis could be superimposed on the matrix maps of other countries, so too can the focal points be superimposed on state standards.

## *Consistency from State to State*

In addition to calling for greater focus in the U.S. curriculum, the National Mathematics Advisory Panel also called for consistency between states. "States and districts should strive for greater agreement regarding the topics to be emphasized and covered at particular grades" (USDOE, 2008, p. 56).

Previously, there had been significant discrepancy between the breadth and depth of content contained in state mathematics standards. Webb (1997) reported that, "Arkansas prepared a mathematics framework based on 13 content standards applicable to all grades; South Carolina defined mathematics content standards by grade ranges for each of 6 core strands; New Jersey specified 18 mathematics standards, including content, process, and learning environment standards for K-12" (p. 7).

Schmidt, Houang, and Cogan (2002) argued that America's poor achievement on TIMSS is linked to the lack of a common curriculum and that the educational system would benefit greatly if all states included the same core content in their mathematics standards. Schmidt, Houang, and Cogan (2002) explained

> Teachers can work together with a shared language and shared goals; new teachers can receive clear guidance on what to teach; professional development may be anchored in the curriculum that teachers teach; textbooks may be more focused and go into greater depth with a smaller set of topics; and transient students (and teachers) may more easily adapt to new schools. All of this contributes to greater consistency and quality across schools. (p. 16)

More recently, Schmidt (2008) suggested that "states and districts should work together to establish national (if not federal) math standards" (p. 24).

A core curriculum and national standards, however, have not been developed. "Although documents published by NCTM in 1989 and 2000 influenced the content of state standards documents, states have generally worked independently of each other to create their own state mathematics curriculum framework" (Reys, 2006, p. 4).

Based on an analysis of state standards documents, the Council of Chief State School Officers (1997) found that GLEs differ considerably from state to state, and other groups have found similar results (Schmidt, Houang, & Cogan, 2002; Klein, Braams, Parker, Quirk, Schmid, & Wilson, 2005; Finn, Julian, & Petrilli, 2006). Reys and Lappan (2007) note that "mathematics learning expectations vary across the states along several dimensions, including level of specificity, language used to convey learning goals, and grade placement of specific learning expectations" (p. 677).

Perhaps the most important research that articulates the current lack of consistency in state standards document is *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion* (Reys, 2006), developed by the CSMC at the University of Missouri. CSMC is the foremost authority when it comes to analyzing state standards documents, and the principal investigator for their research is Barbara Reys, a reviewer for *Curriculum Focal Points* and a co-chair of the writing team for the most recent revisions to the Missouri mathematics standards.

The CSMC report draws attention to the discrepancy regarding the placement of GLEs in state standards documents. Specifically, GLEs related to number and operations, algebra, and reasoning were considered. One of the major findings of the study was the variability from state to state regarding the grade at which students are introduced to certain topics and the grade at which proficiency is expected. For instance, of the 41 state standards documents that include GLEs related to multi-digit addition, students in four states are expected to reach proficiency by Grade 2, whereas students in eight states are not expected to reach proficiency until Grade 5 or later. Further, students in some states are expected to have knowledge of variables as early as kindergarten, yet students in other states are not expected to have knowledge of variables until Grade 8.

In addition to variability in the grade-level placement of GLEs, the CSMC report also found that "the level of specificity or 'grain size' of learning expectations varies across state documents" (Reys, 2006, p. 8-9), the number of GLEs in the Grade 5 standards ranged from 26 in Minnesota to 77 in both Tennessee and Florida, and different terminology is used within GLEs from state to state.

The primary recommendation of the CSMC report was that state standards committees should "identify major goals or focal points at each grade level… These general goals may specify emphasis on a few strands of mathematics or a few topics within strands" (Reys, 2006, p. xxii). The report went on to provide a number of other recommendations:

- "Limit the number of grade-level learning goals to focus instruction and deepen learning" (p. xxii). Although the authors of this report did not specify the number of big ideas that the documents should include, they did recommend that state mathematics standards should contain 20-25 GLEs per grade.
- "Organize K-8 grade-level learning expectations by strand" (p. xxii). The authors elaborated to say that both the content standards and the process

standards ought to receive adequate attention within state standards documents.

- "Collaborate to promote consensus" (p. xxiii). With each state creating its own set of mathematics standards, a likely by-product will be textbooks that include a large number of topics, each of which is covered without depth. States should therefore work together to develop consistency between learning goals, in an attempt to ensure focused curriculum materials. The authors suggest that this level of consistency can be attained "if states build their curriculum standards from a 'core curriculum' offered by national groups" such as NCTM or Achieve, Inc. (p. xxiii). The authors recommend that a national core curriculum, focused on priority goals for Grades K-8, should be jointly developed through a partnership of national organizations.

It should be noted that this report analyzed the state mathematics standards as of May 2005, which is 17 months prior to the release of *Curriculum Focal Points*.

For the analysis in this thesis, the implication of the CSMC report is that state mathematics standards ought to be examined to determine if big ideas have been identified at each grade level; if the document contains a limited number of GLEs; if adequate attention has been given to both content and process standards; and if there is consistency between state standards documents. Relatedly, the post-*CFP* versions[6]

[6] The adjective pre-*CFP* will be used to describe state standards released before *Curriculum Focal Points* had been published, and post-*CFP* will be used to describe state standards released after *Curriculum Focal Points* was published. In particular, a state standards document will only be referred to as the "post-*CFP* version" if *Curriculum Focal Points* was used as a primary resource during the revision.

of state standards documents should be compared to pre-*CFP* versions to determine if there was a reduction in the number of GLEs and big ideas at each grade level.

*Methods of Comparison*

In 2009, based on an analysis of algebra GLEs in Grades 1-8, Chen found that there is marked variation in "the mathematics content, grade placement and cognitive level of learning expectations" (p. 1) when comparing the mathematics standards of Singapore, Taiwan, Japan, Minnesota, Missouri, and California. This finding corroborates the work of Reys (2006), yet the approach to this research was slightly different. In particular, Reys et al. (2006) used a method that counted the number of GLEs related to particular topics in each grade, whereas Chen (2009) used a method that looked not only at the number of GLEs but also at "the percent [of GLEs on a particular topic] with respect to the total number of LEs [learning expectations]" (p. 8). He also used percents to compare various strands within the standards documents. The rationale for using percent was to gauge the relative emphasis ("weight of topic") within the entire document (p. 8).

The implication of Chen's work for this thesis is two-fold. First, his research reinforces other findings regarding the inconsistent placement of GLEs within state standards documents. Second, and more importantly, his use of percents to indicate the relative weight of topics within standards documents might be useful in providing an accurate picture of the emphasis that state mathematics standards devote to each topic suggested in *Curriculum Focal Points*.

*The Movement toward National Mathematics Standards*

In 1991, a national education advisory panel under President George H. W. Bush suggested national standards and national tests, and President Bill Clinton proposed national tests for reading and math in 1997. Both proposals met strong resistance from Congress and were never actualized (McNeil, 2009).

More recently, an American Institutes for Research report suggested, “The United States should consider creating national mathematics standards that define a common core of mathematics skills and concepts at each grade” (Ginsburg, Leinwand, Anstrom, & Pollock, 2005, p. 138), and Schmidt (2008) said that “states and districts should work together to establish national (if not federal) math standards” (p. 24). Those responsible for developing learning expectations at the state level seem to concur. In a survey of state educational agencies, most respondents agreed that national organizations need to provide leadership to assist states in developing future mathematics standards (Reys, Dingman, Sutter, & Teuscher, 2005).

Myriad other groups, including the National Association of Secondary School Principals, the Thomas B. Fordham Foundation, the Council of Great City Schools, the Alliance for Excellent Education, and, perhaps most importantly, the U.S. Department of Education, have also called for the development of national standards in mathematics (Schmidt, Houang, & Cogan, 2002; Finn, Julian, & Petrilli, 2006; Reys, 2006; AFT, 2008; Sawchuk, 2008; Tinker, 2008; USDOE, 2008). Finn, Julian, and Petrilli (2006) claimed, “For the first time in almost a decade, people are seriously weighing the value of instituting national standards and tests in American K-12 education” (p. 1). The major difference between previous efforts to establish national standards and the current effort is that there is now political support for the idea.

At the 91st Annual Meeting of the American Council on Education, the U.S. Secretary of Education Arne Duncan said, “If we accomplish one thing in the coming years, it should be to eliminate the extreme variation in standards across America… the notion that we have fifty different goalposts is absolutely ridiculous” (USDOE, 2009). Further, Secretary Duncan has suggested that at least some of the money included in the recent economic stimulus package as incentive funds might be used to fund the development of national standards (McNeil, 2009).

Considering the support of the current administration as well as other recent occurrences, it seems that national standards may finally become a reality. In December 2008, the Council of Chief State School Officers [CCSSO], along with the National Governors Association released a report recommending that states pursue common standards. In a description of the Common Core State Standards Initiative, the CCSSO declared that “common standards will bring about real and meaningful transformation of our education system to benefit all students” (2009, ¶ 2). In June 2009, the state departments of education in 45 states and the District of Columbia agreed to “develop common national standards for what should be taught in classrooms from kindergarten through high school in reading and math” (Bowie, 2009).

# Chapter 3: Methodology

A number of decisions were required to conduct this analysis effectively, including deciding the process by which standards would be analyzed, selecting the states to include in the analysis, setting up a coding system to quantify the results, and determining a process for detecting any mitigating factors in the state standards writing processes that might have influenced the results. This chapter outlines those decisions and describes the methodology that was used to conduct the analysis.

## *Hermeneutics and the Interpretation of State Standards Documents*

Hermeneutics is the science of interpretation (Crotty, 1998). The term *hermeneutics* is derived from Greek mythology. Hermes, the messenger of the gods, was responsible for interpreting the Olympian gods' messages and communicating those ideas to mortals. Hermeneutics, similarly, is the process of interpreting texts to communicate their meaning. Although used originally to add rigor to the interpretation of religious texts, hermeneutics can be applied to the interpretation of any document or subject.

Heidegger (1962) explained that understanding comes from a circular process in which past knowledge and experience combine to facilitate deeper understanding. That awareness then enriches the understanding of future experiences, and the circle continues repeatedly to generate knowledge. This circular process has been termed the *hermeneutic spiral*. Interpretation within the hermeneutic spiral follows a back-and-forth process, in which current understanding of the whole is used to decipher a part, and then any new understanding of the part is used to enrich understanding of the whole.

In the research undertaken for this analysis, the interpretation of texts (in this case, the state mathematics standards documents) follows a hermeneutic spiral, with the four research questions identified in Chapter 1 providing four phases of investigation. This report presents an analysis of each question in turn, and a thorough examination of each question is completed before commencing the analysis of the next question.

Within hermeneutic analyses, qualitative and quantitative methods can be used in conjunction to measure different elements of a single phenomenon, yielding a greater understanding of the topics under investigation (Creswell, 2003). A blend of quantitative and qualitative analyses was used to determine the impact of *Curriculum Focal Points* when analyzing state standards. The first research question required a qualitative examination when determining the framework used for the arrangement of the state standards, while the second and fourth research questions necessitated a qualitative breakdown to ascertain the percentage of coverage of various topics. The third research question, which examines state-to-state consistency, required a combination of qualitative and quantitative analyses, primarily relying on the data and investigation completed for the first and second research questions.

### *Process for the Research Questions*

The first research question asks whether the frameworks used within the state mathematics standards match the framework used in *Curriculum Focal Points*. As such, an analysis of each state standards document was made by reviewing the document holistically. This examination of each document characterized:

- expectations enumerated by grade level (not by grade band)
- identification of big ideas[7]
- identification of supporting ideas[8]
- references to problem solving, reasoning, communication, making connections, and designing and analyzing representations (either explicitly stated in the front matter of the document, or implicitly contained within the GLEs)

This qualitative scrutiny attempted to confirm that observable manifestations of *Curriculum Focal Points* emerge within the state standards documents.

The second research question calls for an examination of the percent of alignment between state-level GLEs and the *Curriculum Focal Points*. To investigate this question, information was recorded in a spreadsheet to determine what percent of the GLEs within the state standards documents align to each focal point in

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[7] According to Charles (2005), a big idea is "an idea central to the learning of mathematics, one that coherently connects numerous mathematical understandings" (p. 10). By comparison, *Curriculum Focal Points* (NCTM, 2006) defines a focal point as a central topic in mathematics; the focal points "convey knowledge and skills that are essential to educated citizens, and they provide the foundations for further mathematical learning… they can serve to organize content, connecting and bringing coherence to multiple concepts and processes" (p. 5). The focal points can therefore be considered the big ideas within *Curriculum Focal Points*. For the purpose of this analysis, the term *focal point* will be used to refer to a major topic contained within the *Curriculum Focal Points* document, whereas the term *big idea* will be used to refer to a major topic contained within state standards documents.

[8] According to Charles (2005), a supporting idea is "an important idea that contributes to the understanding of a big idea" (p. 10). At each grade level, *Curriculum Focal Points* (NCTM, 2006) identifies a list of connections to the focal points that provides "introductory and continuing experiences related to focal points" and "brings in other important topics in meaningful ways" (p. 8). The connections to the focal points can therefore be considered the supporting ideas within *Curriculum Focal Points*. For the purpose of this analysis, the term *connection* will be used to refer to a secondary topic contained within the *Curriculum Focal Points* document, whereas the term *supporting idea* will be used to refer to a secondary topic contained within state standards documents.

*Curriculum Focal Points*. Each GLE was coded as relating to one of the focal points, to one of the connections, or to some other topic. Within the *Curriculum Focal Points* document, each focal point and connection is supported by a series of refined expectations. For example, the first focal point for Grade 5 is, “Developing an understanding of and fluency with addition and subtraction of fractions and decimals” (NCTM, 2006, p. 17). But additional documentation expounds on five expectations related to this focal point:

- Students apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators.
- They apply their understandings of decimal models, place value, and properties to add and subtract decimals.
- They develop fluency with standard procedures for adding and subtracting fractions and decimals.
- They make reasonable estimates of fraction and decimal sums and differences.
- Students add and subtract fractions and decimals to solve problems, including problems involving measurement.

State GLEs that pertain to *any* of these supporting expectations were codified as fulfilling the recommendation of this focal point. More generally, any GLE that pertains to a supporting expectation for any focal point or connection will be coded similarly.

Indubitably, a number of GLEs within the state standards documents do not apply to any of the focal points or connections. The presence of those GLEs was noted within the spreadsheet, and they were categorized as “other.” All of the GLEs that fall within this category were also analyzed to determine if they were associated with a big idea within the state mathematics standards that is not a focal point or connection, and such associations were noted. Such documentation was necessary when analyzing the standards for Missouri which, by design, included four big ideas at each grade level. It was anticipated that other states included additional big ideas in their state mathematics standards as well.

Analyses using methods similar to those just described were conducted by three research teams evaluating the placement of GLEs within state standards documents (Reys et al., 2006; Newton, Larnell, & Lappan, 2006; Kim & Lasner, 2006). As was done for this thesis, GLEs were codified and information recorded in a spreadsheet, so that an evaluation of the expectations within a particular topic could be tallied and analyzed. Reys advised that a similar spreadsheet should be prepared to record information about the GLEs that align to *Curriculum Focal Points* for this analysis (B. Reys, personal communication, January 2, 2009). The primary difference between the analyses conducted by those research teams and this analysis are that the research teams examined the *number* of GLEs related to a particular topic within state standards documents, whereas this thesis will rely on the *percentage* of GLEs related to the focal points. Despite this minor difference in output, the data needed was similar, and comparable data collection methods are appropriate.

The third research question addresses the degree to which there is consistency in state mathematics standards for those states whose state mathematics supervisors identified the *Curriculum Focal Points* as a substantive resource. The data collected and recorded in the spreadsheet for the second question was also used to compare the state standards documents one to another. Once the percentage of GLEs aligned to each focal point, connection, or other topic was identified, a comparison between states was conducted. The results were examined to determine the level of consistency between states as well as any outliers within the data.

The fourth research question addresses evidence that *Curriculum Focal Points* influenced revisions of state mathematics standards. Data from the spreadsheet was also used for this research question. In addition, the same data was collected and codified regarding the pre-*CFP* versions of the state standards documents. This entailed entering the same information described above. Once all data for both the pre-*CFP* and post-*CFP* versions of state mathematics standards was collected, an analysis was conducted to determine if the percent change is significant for each focal point and connection from state to state; that is, was there a conspicuous change in the number of big ideas or the number of GLEs between the pre-*CFP* and post-*CFP* versions? An analysis was also conducted for any big ideas or supporting ideas that occurred in multiple state documents but did not address the same topics as listed in the focal points or connections.

To assess the focus within the state mathematics standards, the percent of GLEs related to each big idea or supporting idea was calculated. The percentages were then compared to determine which topics receive the greatest emphasis. Of necessity, this comparison was done without statistical methods. Two statisticians recommended that analyses of variance should not be used for this investigation, because normality assumptions are violated when comparing percentages (D. Sundin, personal communication, February 27, 2009; G. Macready, personal communication, March 6, 2009).

*State Standards to be Analyzed*

For this thesis, the mathematics standards for Grade 5 from six states were examined: Florida, Minnesota, Utah, Washington, Missouri, and Kansas. The selected states were chosen for a variety of reasons.

Eight states (Florida, Iowa, Minnesota, Nevada, New Mexico, Tennessee, Utah, and Washington) recently revised their mathematics standards and claimed that *Curriculum Focal Points* had a significant impact on the revision (CSMC, 2007). However, Tennessee, New Mexico, Nevada, and Iowa had not yet completed their revisions when the research for this analysis was undertaken. Consequently, the mathematics standards of the four remaining states were analyzed to determine the impact, if any, of *Curriculum Focal Points*.[9] The revised standards for Florida and Utah were officially adopted in 2007, and the revised standards for Minnesota and Washington were officially adopted in 2008.

[9] Although Rubillo claimed that 22 states used *Curriculum Focal Points* as the primary resource when revising their standards, and Fennell indicated that one-fourth of the states have used *Curriculum Focal Points* to guide curriculum discussions, many fewer have received approval from their state departments of education for standards that were revised based on *Curriculum Focal Points*. The standards reviewed for this analysis were chosen from those states where the mathematics supervisor claimed that *Curriculum Focal Points* had a "significant impact" on the revision and where the state department of education has approved the revised version.

In addition, Missouri recently revised their standards using *Curriculum Focal Points* as the primary resource. The new mathematics standards in Missouri follow the structure of *Curriculum Focal Points* by identifying four big ideas per grade band — three of which address the same topics outlined in *Curriculum Focal Points*, as well as one additional idea chosen by the state standards revision committee (B. Reys, personal communication, 2009). Consequently, Missouri's state standards were also examined.

Finally, the state standards of Kansas were analyzed. The Kansas respondent to the CSMC study indicated that their standards, published in 2004, were reviewed and "members of the standards committee… compared the *Curriculum Focal Points* to the [state] standards item-by-item. It was determined that all the focal points were included in our standards" (CSMC, 2007, p. 3).

A number of factors contributed to the decision to analyze the state mathematics standards for Grade 5. First, due to the requirements of federal No Child Left Behind Act of 2001, all states are required to assess their students at Grade 5. Second, there appears to be a slight discrepancy between *Principles and Standards* and *Curriculum Focal Points* regarding the study of operations with fractions. *Principles and Standards* states that students in Grades 6-8 should "understand the meaning and effects of arithmetic operations with fractions, decimals, and integers" (NCTM, 2000, p. 214). By comparison, *Curriculum Focal Points* suggests that students in Grade 5 should develop "an understanding of and fluency with addition and subtraction of fractions and decimals" (NCTM, 2006, p. 17). Finally, though not least importantly, Barbara Reys agreed that analyzing the Grade 5 standards was appropriate, even though her analysis of standards had used mathematics standards from Grade 4 (B. Reys, personal communication, January 2, 2009).

For this analysis, state standards documents were obtained through the state standards database on the CSMC website at http://www.mathcurriculumcenter.org/states.php. Versions of the pre-*CFP* state mathematics standards were obtained from the state departments of education.

## *Structure of the Analysis*

The *Curriculum Focal Points* document lists all focal points and connections to the focal points in paragraph format. There is an implied order because of their arrangement on the page, but there is no hierarchy suggested by an explicit numbering system. The document presents all of these elements without implying a level of priority; that is, there is no indication that one focal point deserves more attention than any other focal point, nor is there any indication that one connection deserves more attention than any other connection. This format, in fact, was used deliberately, to prevent curriculum developers and other stakeholders from assuming a level of priority (F. Fennell, personal communication, May 6, 2009).

Despite this lack of priority, an organizational system was imposed for the purpose of this thesis. To facilitate uncomplicated communication, this thesis refers to the three focal points contained in Grade 5 as Focal Point 5.1, Focal Point 5.2, and Focal Point 5.3, respectively, and the four connections as Connections 5.A, Connections 5.2, Connections 5.3, and Connections 5.D, respectively.

This numbering system is only a matter of convenience and is not meant to suggest a hierarchy among the items. Further, it is not intended to imply that the topics listed within *Curriculum Focal Points* represent a checklist of topics to be included in a curriculum.

*Coding System for the Analysis*

Each GLE within each state standards document was reviewed to determine if there was any relation to a focal point, to a connection, or to some other topic. A tally of the number of GLEs related to each of the three focal points and four connections was recorded. In many cases, however, a particular GLE appeared to relate to multiple focal points or connections; and, in some cases, one portion of a GLE appeared to relate to a focal point or connection, but another portion may have no relation to any focal point or connection. Therefore, the method used for tallying these results deserves further explanation.

A point was added to the tally of a focal point or connection if there was a one-to-one correspondence between the content of a GLE and the content of a focal point or connection. For example, one GLE from the Washington Grade 5 mathematics standards states, "Fluently and accurately divide up to a four-digit number by one- or two-digit divisors using the standard long-division algorithm" (Office of the Superintendent of Public Instruction, 2008, p. 48). The entirety of the content included in this GLE is directly related to the first focal point for Grade 5, which pertains to "developing an understanding of and fluency with division of whole numbers" (NCTM, 2006, p. 17). Consequently, one point would be added to the tally for Focal Point 5.1 based on this GLE. The majority of GLEs exhibit this type of one-to-one correspondence, and determining how they affect the overall tally is rather straightforward.

That said, a number of GLEs include a large amount of content, and a fractional system was employed to cover these scenarios. As an example, one GLE from the Florida Grade 5 mathematics standards (2003) states, “Knows how to estimate the area and perimeter of regular and irregular polygons and how to estimate the volume of a rectangular prism” (p. 3). The latter portion of this GLE deals with volume and relates to the third focal point, which pertains to “describing three-dimensional shapes and analyzing their properties, including volume and surface area” (NCTM, 2006, p. 17), while the beginning portion of this GLE deals with area and perimeter, which does not relate to any of the focal points or connections for Grade 5. Because this GLE contains two roughly equal parts, a half-point was added to the tally for Focal Point 5.3, and a half-point was added to the tally for the category of *other*. (It is, of course, debatable as to whether these two parts are “roughly equal,” but attempting to divide GLEs into unequal fractions involved traversing a slippery slope. For instance, it could be argued that this GLE contains three mathematical concepts — area, perimeter, and volume — so perhaps this GLE should be divided into thirds, and since only one of these three concepts is directly referenced within Focal Point 5.3 (namely, volume), then only one-third point should be counted toward Focal Point 5.3. But slicing even finer, it could be said that this GLE actually references five mathematical topics — area of a regular polygon, area of an irregular polygon, perimeter of a regular polygon, perimeter of an irregular polygon, and volume of a rectangular prism — so perhaps this GLE should be divided into fifths, and therefore only one-fifth point should be counted toward Focal Point 5.3. To avoid absurd dissections of this nature, however, a reasonable attempt was made to determine the intent of each GLE, and fractional points were awarded accordingly.)

This fractional system was then extended for GLEs that contain multiple parts. Whereas a GLE that contains two separate but roughly equal parts donates a half-point to each of two categories, a GLE that contains *n* separate but roughly equal parts that relate to various focal points or connections will contribute 1/*n* points to each of the appropriate categories.

The following GLE shows the most extreme example. One of the GLEs within the Kansas state mathematics standards (2004) reads:

> Solves real-world problems using equivalent representations and concrete objects to:
>
> a. Compare and order –
>
> i. Whole numbers from 0 through 1,000,000; e.g., using base ten blocks, represent the attendance at the circus over a three day stay; then represent the numbers using digits and compare and order in different ways;
>
> ii. Fractions greater than or equal to zero (including mixed numbers), e.g., Frank ate 2 ½ pizzas, Tara ate 9/4 of the pizza. Franks says he ate more. Is he correct? Use a model to explain. With drawings and shadings, student shows

amount of pizza eaten by Frank and the amount eaten by Tara.

iii. Decimals greater than or equal to zero to hundredths place, e.g., uses decimal squares, money (dimes as tenths, pennies as hundredths), the correct amount of hundred chart filled in, or a number line to show that .42 is less than .59.

iv. Integers, e.g., plot winter temperature for a very cold region for a week (use Internet data); represent on a thermometer, number line, and with integers;

b. Add and subtract whole numbers from 0 through 100,000 and decimals when used as monetary amounts, e.g., use real money to show at least 2 ways to represent $846.00, then subtract the costs of a new computer setup;

c. Multiply through a two-digit whole number by a two-digit whole number, e.g., George charges $23 for mowing a lawn. How much will he make after he mows 3 lawns? Represent the $23 with money models – two $10 bills and three $1 bills and repeat that 3 times or represent the $23 using base ten blocks or 23 ✕ 3 or 23 + 23 + 23;

d. Divide through a four-digit whole number by a two-digit whole number, e.g., the Boy Scout troop collected cans and held bake sales for a year and earned $492.60. The money will be divided evenly among the 12 troop members to buy new uniforms.

> Represent each boy's share of the money at least 2 ways – traditional division; use 4 hundreds, 9 tens, 2 ones, and 6 dimes to act out the situation; or use base ten blocks to act it out. (p. 5-1 and 5-2)

This GLE contains four parts (a–d), and one of those parts (a) contains four sub-parts (i-iv). Each of the four parts contributes one-quarter point to each focal point or connection to which it relates, and each of the four sub-parts contributes one-sixteenth point to each focal point or connection to which is relates. And since half of one of the parts (b) relates to Connection 5.D, the connection regarding place value, and the other half relates to Focal Point 5.2, the focal point regarding adding and subtracting fractions and decimals, this part contributes one-eighth point to each category.

### *Determining Mitigating Factors*

Beyond the four research questions listed in Chapter 1, this analysis also considered the circumstances surrounding the state standards revision processes. By speaking with representatives from each state whose standards are to be analyzed, this thesis attempted to identify possible reasons why post-*CFP* mathematics standards do or do not exhibit a higher level of focus than the corresponding pre-*CFP* standards. In addition, it attempted to identify possible reasons why there is or is not a level of consistency between state mathematics standards.

For each state, the primary employees at the state departments of education were identified, and those representatives were contacted via email. Additional conversations occurred via phone, when necessary and possible. The initial contact with these people included a brief description of my thesis, an invitation to participate in this research, and a request for response to several questions:

- Briefly explain the process that was used to revise the mathematics standards within your state.
- Why did (or didn't) your state include the topics recommended in *Curriculum Focal Points*?
- Where differences occur between your state standards and the recommendations in *Curriculum Focal Points*, how were decisions made about which additional topics to include or exclude?
- Were there any external factors that helped or hindered the writing process?
- Did any outside groups influence the process in any way?
- Is there anything else you can tell me about the process that would inform my work?

This list of questions were tailored, as necessary, depending on prior information known about the process used in a particular state. For example, a phone conversation with Barbara Reys about the research to be conducted for this thesis yielded information to some of these questions previously, and asking them again would have been redundant.

In some cases, the information sought by these questions is confidential. In addition, discussions regarding state standards are often politically charged, and asking representatives to divulge such information may be putting them at political risk. Consequently, answers to these questions are reported to the extent possible, but pseudonyms are used to identify the source. The format “[state] representative, date” is used to reference information gleaned from individual conversations. This format is being used to maintain anonymity while permitting a reference to the knowledge base of the source.

# Chapter 4: Results

To determine the impact of *Curriculum Focal Points* on state mathematics standards, four research questions were considered. These questions investigate whether state standards documents exhibit the same structure as *Curriculum Focal Points*, examine the percentage of GLEs related to each focal point and connection, compare state documents one to another, and identify differences between the pre-*CFP* and post-*CFP* versions of the state mathematics standards.

## *Research Question 1*

State mathematics standards that reflect the intent of *Curriculum Focal Points* will likely have a structure similar to *Curriculum Focal Points*. Research Question 1 therefore compares the structure of state standards documents to the structure of *Curriculum Focal Points*.

1. **Does the overall framework used in state mathematics standards match the framework employed in *Curriculum Focal Points*? In particular, do the state standards documents identify big ideas as well as supporting ideas at each grade level, and do they address the process standards of "problem solving, reasoning, communication, making connections, and designing and analyzing representations" (NCTM, 2006, p. 17)?**

The *Curriculum Focal Points* document exhibits three defining characteristics.

First, “focal points are identified and described for each grade level, pre-K–8, along with connections” (NCTM, 2006, p. 10). This is a shift from previous recommendations put forth by NCTM. The *Curriculum and Evaluation Standards for School Mathematics*, published by the Council in 1989, divided the school experience into three grade bands: K-4, 5-8, and 9-12 (NCTM, 1989). The *Principles and Standards for School Mathematics*, released in 2000, used four grade bands: preK-2, 3-5, 6-8, and 9-12 (NCTM, 2000).

Second, the number of topics listed within *Curriculum Focal Points* is limited. Only three focal points are identified at each grade, because, “An approach that focuses on a small number of significant mathematical ‘targets’ for each grade level offers a way of thinking about what is important in school mathematics that is different from commonly accepted notions of goals, standards, objectives, or learning expectations” (NCTM, 2006, p. 1). In addition to the focal points, three connections are listed for Grades K, 1, 2, and 6; four connections are listed for Grades 3, 5, 7, and 8; and five connections are listed for Grade 4. Consequently, when the focal points and connections are considered collectively, the maximum number of topics covered in any grade is eight.

Third, significant attention is given to the process standards throughout the entire document. Communication, reasoning, representation, connections, and problem solving (NCTM, 2006) are mentioned on the first page of the document, and they are reiterated several pages later, when the authors recommend that each focal point should be addressed “in the context of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation” (NCTM, 2006, p. 5). At the beginning of the section where the focal points are described for each grade, the document describes the process standards more fully:

> To build students’ strength in the use of mathematical processes, instruction in these content areas should incorporate —
>
> - the use of the mathematics to solve problems;
> - an application of logical reasoning to justify procedures and solutions; and
> - an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics. (NCTM, 2006, p. 10)

Finally, the process standards are referenced in the paragraph preceding the focal points for every grade. The following sentence is repeated at the top of each of pages 12-20: “It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations” (NCTM, 2006, p. 12).

To reflect an impact from *Curriculum Focal Points*, the mathematics standards for a given state should exhibit these same three characteristics. They should (a) enumerate GLEs by grade level, not grade band, (b) explicitly define a limited number of big ideas and supporting ideas at each grade, and (c) display an awareness of the process standards.

The state mathematics standards were reviewed to evaluate the extent to which they demonstrate these characteristics. For this part of the analysis, only post-*CFP* versions of the mathematics standards from each state were considered.

*Enumerate GLEs by Grade Level*

Each of the six states considered in this analysis list their GLEs by grade level, not by grade band. It is worth noting that five of these six states also enumerated GLEs by grade level in the pre-*CFP* versions of their state standards, too. The only state whose pre-*CFP* standards were not enumerated by grade level was Florida; their standards had not been revised since 1996 and were based on the 1989 *Standards* (NCTM) document, which used the grade bands K-4, 5-8, and 9-12.

As shown in Table 1, the major differences from state to state were the inclusion of explicit descriptions of the big ideas and supporting ideas, the number of big ideas and supporting ideas that were identified, and the amount of attention given to the process standards. To provide a thorough analysis of the differences, the mathematics standards will be examined state by state.

Table 1

*Comparison of the Structure of post-CFP State Mathematics Standards*

| State | GLEs Are Enumerated by Grade Level | Big Ideas Are Explicitly Defined | Supporting Ideas Are Explicitly Defined | Process Standards Receive Adequate Attention |
|---|---|---|---|---|
| Florida | ✓ | ✓ | ✓ | ✓ |
| Kansas | ✓ | | | ✓ |
| Minnesota | ✓ | ✓ | | ✓ |
| Missouri | ✓ | ✓ | | ✓ |
| Utah | ✓ | ✓ | | |
| Washington | ✓ | ✓ | ✓ | ✓ |

*Florida*

The Florida mathematics standards identify big ideas for Grades K-8. In fact, the three big ideas defined by Florida at each grade level are identical to the focal points proposed in *Curriculum Focal Points*. Moreover, the GLEs contained in the Florida mathematics standards mimic the wording used in *Curriculum Focal Points*. For instance, the first focal point for Grade 5 in *Curriculum Focal Points* and the first big idea listed in the Florida mathematics standards both state, “Develop an understanding of and fluency with division of whole numbers” (Florida Department of Education, 2007, p. 49; NCTM, 2006, p. 17). Furthermore, the description of this focal point in *Curriculum Focal Points* asserts that students should “consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately” (NCTM, 2006, p. 17). Only slightly different is the third GLE listed under this big idea in the Florida mathematics standards, which states that students will “interpret solutions to division situations including those with remainders depending on the context of the problem" (Florida State Mathematics Standards, 2007, p. 49).

In addition to three big ideas, the Florida mathematics standards also identify several supporting ideas at each grade level, which are analogous to the connections listed in *Curriculum Focal Points*. For Grade 5, Florida identifies supporting ideas in algebra, geometry and measurement, number and operations, and data analysis, and these same areas are listed as the connections in *Curriculum Focal Points* for Grade 5. As it turns out, the supporting ideas defined at every grade level in the Florida mathematics standards are identical to the connections identified in *Curriculum Focal Points*.

The Florida mathematics standards identify three big ideas and no more than five supporting ideas at each grade. Consequently, they never propose more than eight topics per grade, similar to *Curriculum Focal Points*. The transition to a focused curriculum was a deliberate effort on the part of the authors of the Florida mathematics standards. The introduction to the *Sunshine State Standards: Mathematics* (Florida Department of Education, 2007) explains, "Responding to calls for clarity, coherence, and minimal redundancy, the numbers of K-8 grade level expectations were reduced from an average of more than 80 per grade to an average of less than 20 benchmarks per grade" (p. 4).

Within the Florida mathematics standards, a number of references are made to the process standards. The introduction states that the purpose of the supporting ideas is to establish connections between various mathematical strands and address mathematical ideas that are important for problem solving (Florida State Mathematics

Standards, 2007). In addition, elements of the process standards appear in several GLEs, with phrases like “describe the process” (p. 49) and “describe real-world situations” (p. 54) alluding to communication, while “determine ways to represent” (p. 55) insinuates the process standard of representation. In addition, seven of the 23 GLEs (30%) for Grade 5 refer to solving problems and to interpreting problem situations, alluding to the process standard of problem solving.

*Kansas*

In discussing the impact of *Curriculum Focal Points* on state mathematics standards, Kansas represents a unique situation. Although the current Kansas mathematics standards were approved January 31, 2004, they supposedly reflect the philosophy of *Curriculum Focal Points*. CSMC conducted a survey in which state mathematics supervisors were asked about the extent to which *Curriculum Focal Points* had influenced their most recent revisions. The respondent from Kansas indicated that the Kansas mathematics standards were compared to *Curriculum Focal Points*, and “it was determined that all the [Curriculum] Focal Points were included in our standards” (CSMC, 2007, p. 3).

The Kansas state mathematics standards contain 14 benchmarks for Grade 5. Because each benchmark has four to nine associated GLEs, the benchmarks appear to represent the big ideas within the Kansas mathematics standards. Although *Curriculum Focal Points* does not dictate the number of big ideas to include at each grade, it is probably safe to assume that the authors of *Curriculum Focal Points*, who included only three focal points at each grade,  would consider more than four times that number to be too many. In addition, because the objectives appear to be given equal weight, it is unclear if some of the benchmarks are meant to represent supporting ideas.

A reasonable amount of attention is paid to the process standards throughout the document. There is an explicit reference to the process standard of representation contained within a set of teacher notes following the list of GLEs for the first benchmark. It states, “Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures” (Kansas State Department of Education, 2004, p. 5-3). This sentence is then repeated in the teacher notes for each of the other 13 benchmarks, too. The process standards of problem solving, reasoning, communication, or making connections are also represented in the document, and at least 28 of the GLEs for Grade 5 reference the process standards by including phrases such as “describes mathematical relationships” (p. 5-16) and “solves real-world problems” (p. 5-21).

*Minnesota*

The Minnesota state mathematics standards do not explicitly refer to any topics as big ideas or supporting ideas, but they do identify “standards” for each grade. These standards read like big ideas, however. For instance, the first standard for Grade 5 is, “divide multi-digit numbers; solve real-world mathematical problems using arithmetic” (Minnesota Department of Education, 2007b, p. 15).

Despite the reluctance to explicitly define big ideas and supporting ideas, it is clear that *Curriculum Focal Points* played a significant role in the development of the post-*CFP* Minnesota mathematics standards. A considerable amount of wording within their standards is borrowed directly from *Curriculum Focal Points*; in fact, approximately half of the GLEs in the Minnesota mathematics standards use phrasing that is similar or identical to the wording used in *Curriculum Focal Points*. For instance, the following sentence is used in the description of Focal Point 5.2: "[Students] consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately." By comparison, one of the GLEs (benchmark 5.1.1.2) within the Minnesota mathematics standards states, "[Students] consider the context in which a problem is situated to select the most useful form of the quotient for the solution and use the context to interpret the quotient appropriately" (Minnesota Department of Education, 2007b, p. 15). Such similarities make it obvious that *Curriculum Focal Points* was used as a resource.

The introduction to the Minnesota mathematics standards mentions the process standards, saying, "The standards and benchmarks presented here describe a connected body of mathematical knowledge that is acquired through the processes of problem solving, reasoning and proof, communication, connections, and representation" (Minnesota Department of Education, 2007b, p. 2).

*Missouri*

Whereas *Curriculum Focal Points* contains three focal points at each grade level, the Missouri mathematics standards (2007) explicitly define four big ideas, which the document refers to as “core content areas” (p. 8) or “core concepts” (p. 35). Each of the three focal points from *Curriculum Focal Points* is included as a big idea in the Missouri standards, but the state standards committee also identified a fourth big idea for each grade. For instance, the mathematical content of the three focal points listed in *Curriculum Focal Points* for Grade 5 are division of whole numbers, addition and subtraction of fractions and decimals, and properties of three-dimensional shapes. In addition to these same three topics, the Missouri state mathematics standards also incorporate a fourth big idea, basic probability concepts. Although each grade contains one more topic than *Curriculum Focal Points*, the Missouri mathematics standards would still be said to identify a limited number of topics.

*Curriculum Focal Points* does not indicate exactly how much coverage each focal point should receive, but the Missouri mathematics standards suggest the percentage of instructional time that should be devoted to each big idea. Division of whole numbers as well as addition and subtraction of fractions and decimals are each allotted 30%, properties of three-dimensional shapes, volume and surface area is given 15%, and basic probability concepts are allocated 10%. Taken together, these percentages total less than 100%, an intentional decision so that districts have the ability to add content as needed (Missouri Department of Education, 2008).

The process standards receive significant coverage within the Missouri mathematics standards. Specific expectations are enumerated by grade band. For Grades 3-5, four process standards are explicitly stated in the introduction to the document:

- Apply and adapt a variety of strategies to solve problems.
- Make and investigate mathematical conjectures.
- Communicate mathematical thinking coherently and clearly to peers and teacher.
- Organize, record, communicate, and represent mathematical ideas.

Further, the following terms are used repeatedly within the GLEs: connections, representations, analyze, apply, solve, problems, describe, explain, justify. These terms are allusions to the same ideas as the process standards.

*Utah*

The introduction to the *Utah Elementary Mathematics Core Curriculum (K-6)* (Utah State Office of Education, 2007) declares, “The focal points within a grade are *not* the entire curriculum for that particular grade; however, Utah’s Core Curriculum was designed to include these areas of focus” (p. 4).

The Utah mathematics standards identify up to five big ideas for each grade K-6. The first standard for Grade 5 states, “Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals” (Utah Elementary Mathematics Core Curriculum, 2008, p. 33). Within this standard, there are six objectives comprised of 27 GLEs. For example, Objective 6 under this standard reads as follows:

> Demonstrate proficiency with multiplication and division of whole numbers and compute problems involving addition, subtraction, and multiplication of decimals and fractions.
>
> a. Multiply multi-digit whole numbers by a two-digit whole number with fluency, using efficient procedures.
> b. Divide multi-digit dividends by a one-digit divisor with fluency, using efficient procedures.
> c. Add and subtract decimals with fluency, using efficient procedures.
> d. Add and subtract fractions with fluency.
> e. Multiply fractions. (Utah State Office of Education, 2007, p. 34)

On the other hand, there are four other big ideas within the document, each of which contains only four to eight GLEs. The discrepancy between the number of GLEs associated with each big idea makes it difficult to discern if each of them deserve similar amounts of emphasis in the curriculum. Because the objectives associated with each big idea appear to be given equal weight, and because there are significantly more GLEs associated with the first big idea than the others, it appears that some of the big ideas might actually be meant to represent supporting ideas, but such a distinction is never explicitly stated.

There is a very clear relationship between *Curriculum Focal Points* and the Utah state mathematics standards in regards to terminology. In many cases, the wording is strikingly similar. One of the GLEs within the *Utah Elementary Mathematics Core Curriculum* (Utah Department of Education, 2008) states that students will be able to "recognize that a cube having a 1 unit edge is the standard unit for measuring volume expressed as a cubic unit" (p. 36). By comparison, *Curriculum Focal Points* (NCTM, 2006) states that students will be able to "understand that a cube that is 1 unit on an edge is the standard unit for measuring volume" (p. 17).

Within the Utah state mathematics standards, there is no explicit reference to the process standards. The document does make reference to the process standard of connections, saying, "Exploratory concepts and skills are included to establish connections with learning in subsequent grade levels" (Utah Elementary Mathematics Core Curriculum, 2008, p. 5), but this list often reads as a summary of the GLEs within the preceding section. For instance, the four exploratory concepts and skills listed with the number sense standard are (a) extend classification of whole numbers from 0-100 as prime, composite, or neither, (b) apply rules of divisibility, (c) explore adding and subtracting integers, and (d) divide multi-digit dividends by a two-digit divisor (Utah Department of Education, 2007, p. 34). Beyond that mention of connections within the exploratory concepts and skills, there are occasional words and phrases that allude to the process standards, such as "solve simple real-world problems" (p. 35) and "demonstrate multiple ways to represent" (p. 33), but only nine of the 54 GLEs for Grade 5 contain these references.

*Washington*

The *Washington State K-12 Mathematics Standards* (Office of the Superintendent of Public Instruction, 2008) explicitly lists big ideas, supporting ideas, and process standards at each grade level. However, the document refers to these elements as core content, additional key content, and core processes, respectively.

The strong alignment between Washington's state mathematics standards and *Curriculum Focal Points* can be illustrated with a specific example. For Grade 5, the Washington state mathematics standards identify four big ideas: (a) multi-digit division, (b) addition and subtraction of fraction and decimals, (c) triangles and quadrilaterals, and (d) representations of algebraic relationships. Three of these topics (a, b, and d) cover the same content as the focal points for Grade 5. The document also contains three supporting ideas, two of which reflect content contained within the connections for Grade 5. A minimum of four GLEs are included with each big idea, whereas each supporting idea is described by just one GLE, indicating which topics are to receive greater emphasis.

The document also includes ten GLEs specifically dedicated to the process standards. The introduction to the section with these GLEs contends that reasoning, problem solving and communication are the core processes for Grade 5. The GLEs related to process standards also includes reference to the process standard of representation, but there is no reference to the process standard of connections within the Grade 5 standards.

The structure and content of the other grade levels displays a similarly strong alignment to *Curriculum Focal Points*. In every grade, either three or four big ideas are identified, and a multitude of GLEs are included with each; up to eight supporting ideas are identified, each one described by just one GLE; and the process standards of reasoning, problem solving and communication are listed as core processes, while the process standards of representation and connections are referenced within the GLEs frequently.

*Research Question 2*

In addition to identifying big ideas, identifying supporting ideas, and referencing the process standards, state mathematics standards that reflect the intent of *Curriculum Focal Points* should display significant alignment with its content. Research Question 2 considers the GLEs within state standards documents to determine the amount of emphasis given to each focal point and connection.

2. **What percent of the GLEs within state documents address the focal points, what percent address the connections, and what percent address other ideas and topics?**

*Focal Points*

Three focal points are identified at each grade level within the *Curriculum Focal Points* document. The three focal points for Grade 5 are (a) Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers (Focal Point 5.1); (b) Number and Operations: Developing an understanding of and fluency with addition and subtraction of fractions and decimals

(Focal Point 5.2); and (c) Geometry and Measurement and Algebra: Describing three-dimensional shapes and analyzing their properties, including volume and surface area (Focal Point 5.3).

Complete descriptions of the focal points for Grade 5 are given in the appendix. Please note that the names ascribed in parentheses above are provided for discussion purposes only. They do not imply an official naming convention used by NCTM.

Within state mathematics standards, the percent of GLEs related to Focal Point 5.1 ranges from 2.13% (Kansas) to 18.92% (Missouri), with a mean of 12.94% and a median of 15.22%. Figure 1 shows the percent of GLEs related to Focal Point 5.1 for each of the six states.

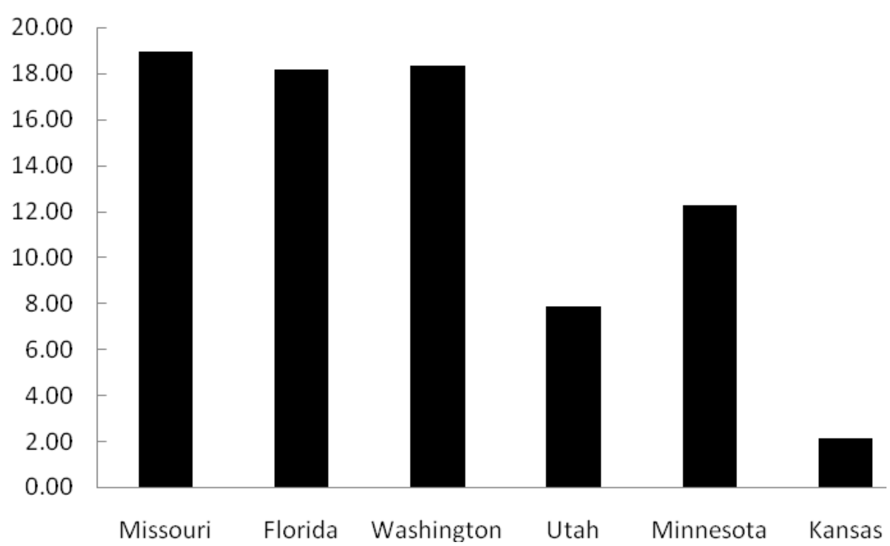


*Figure 1:* Percent of GLEs related to Focal Point 5.1

The percent of GLEs related to Focal Point 5.2 ranges from 3.65% (Kansas) to 35.14% (Missouri), with a mean of 16.57% and a median of 15.98%. Figure 2 shows the percent of GLEs related to Focal Point 5.2 for each of the six states.

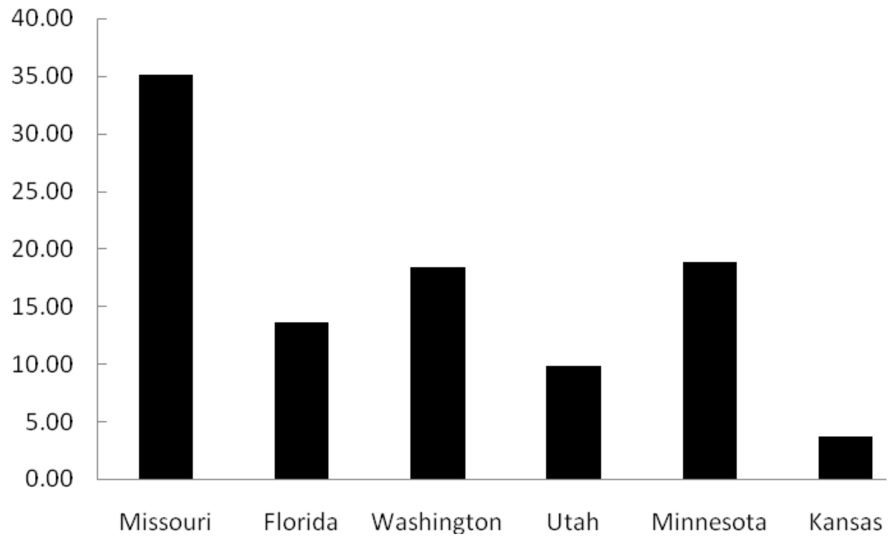


*Figure 2:* Percent of GLEs related to Focal Point 5.2

The percent of GLEs related to Focal Point 5.3 ranges from 0.00% (Washington) to 27.03% (Missouri), with a mean of 12.38% and a median of 11.41%. Figure 3 shows the percent of GLEs related to Focal Point 5.3 for each of the six states.

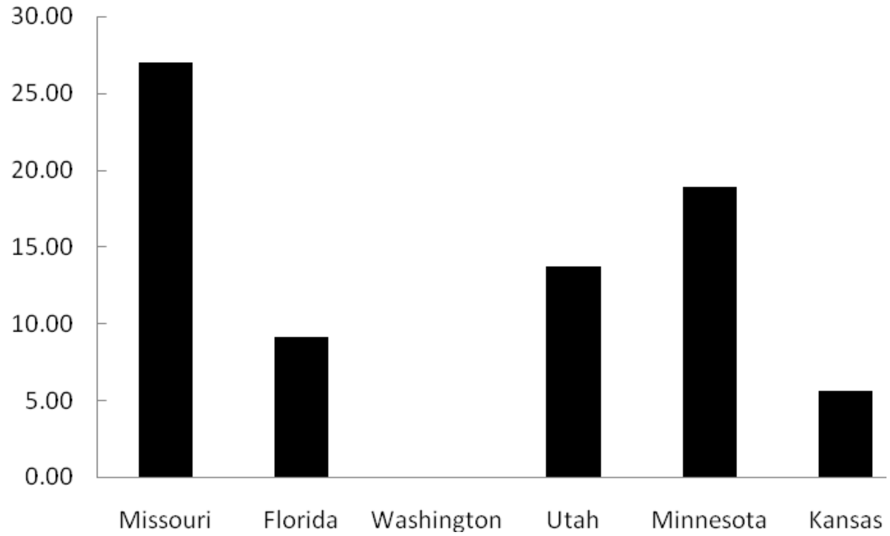


*Figure 3:* Percent of GLEs related to Focal Point 5.3

It is worth noting that Missouri has the highest percent of GLEs related to each of the three focal points in Grade 5. Interestingly, the Missouri mathematics standards suggest a percentage of instructional time that should be dedicated to each big idea. The document indicates that 30% of instructional time should be allocated to each of Focal Points 5.1 and 5.2 and that 15% of instructional time should be allocated to Focal Point 5.3. They also allocate 10% of instructional time to basic probability concepts, a fourth big idea that appears in the Missouri mathematics standards but not in *Curriculum Focal Points*. The sum of these percentages is only 85%. The Missouri standards document states that the "sum of these percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs" (Missouri Department of Education, 2007, p. 8).

Because the sum of percentages given above totals 85%, it would be expected that the percent of GLEs related to Focal Points 5.1 and 5.2, both of which are expected to receive 30% of instructional time, would be close to 30/85 = 35.29% of all GLEs; the percent of GLEs related to Focal Point 5.3, which is supposed to receive 15% of instructional time, would be close to 15/85 = 17.64%; and the percent of GLEs related to basic probability concepts would be close to 10/85 = 11.76%.

Based on this analysis of the Missouri mathematics standards, the percent of GLEs related to Focal Point 5.1 is 18.92%, the percent of GLEs related to Focal Point 5.2 is 35.14%, the percent of GLEs related to Focal Point 5.3 is 27.03%, and the percent of GLEs related to basic probability concepts is 10.81%.

Of the six states analyzed, Missouri is the only state to indicate a suggested percentage of instructional time for each big idea.

*Connections*

Anywhere from three to five connections are identified at each grade level within *Curriculum Focal Points* document. The four connections in Grade 5 are:

- Algebra (Connection 5.A) – includes patterns, equations, prime and composite numbers, and order of operations
- Measurement (Connection 5.B) – includes capacity, weight, mass, approximation, and precision
- Data Analysis (Connection 5.C) – includes double-bar graphs, line graphs, and ordered pairs on coordinate graphs

- Number and Operations (Connection 5.D) – includes place value, multiplication of large numbers, and negative numbers

Complete descriptions of the connections for Grade 5 are given in the appendix. Please note that the names ascribed in parentheses above are provided for discussion purposes only. They do not imply an official naming convention used by NCTM.

The percent of GLEs related to Connection 5.A ranges from 8.11% (Missouri) to 22.73% (Florida), with a mean of 14.07% and a median of 14.35%. Figure 4 shows the percent of GLEs related to Connection 5.A for each of the six states.

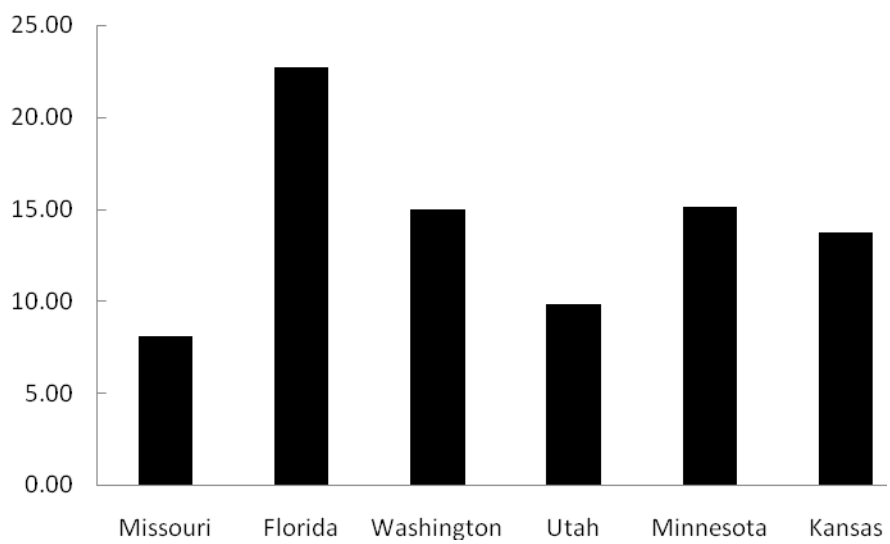


*Figure 4:* Percent of GLEs related to Connection 5.A

The percent of GLEs related to Connection 5.B ranges from 0.00% (Missouri, Washington, Utah, Minnesota) to 4.55% (Florida), with a mean of 1.04% and a median of 0.00%. Figure 4 shows the percent of GLEs related to Connection 5.B for each of the six states.

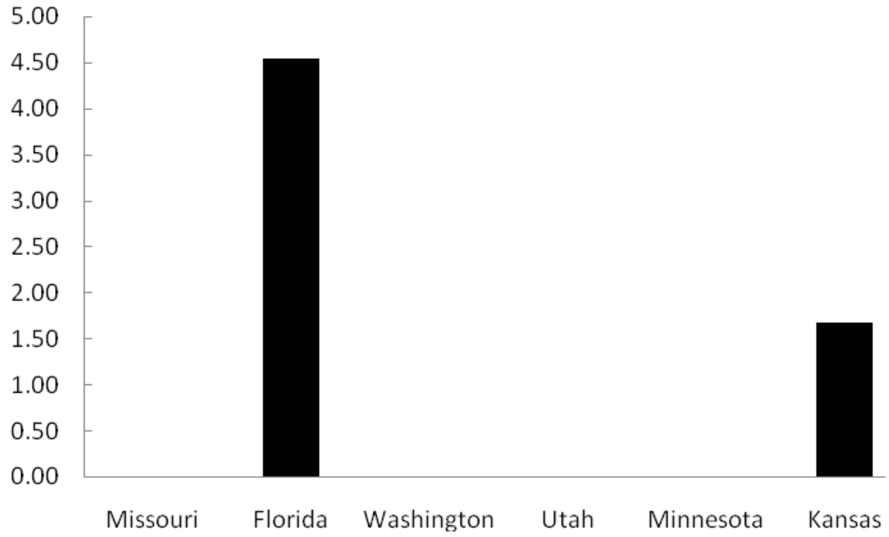


*Figure 5:* Percent of GLEs related to Connection 5.B

The percent of GLEs related to Connection 5.C ranges from 0.00% (Missouri) to 13.64% (Florida), with a mean of 6.30% and a median of 5.80%. Figure 6 shows the percent of GLEs related to Connection 5.C for each of the six states.

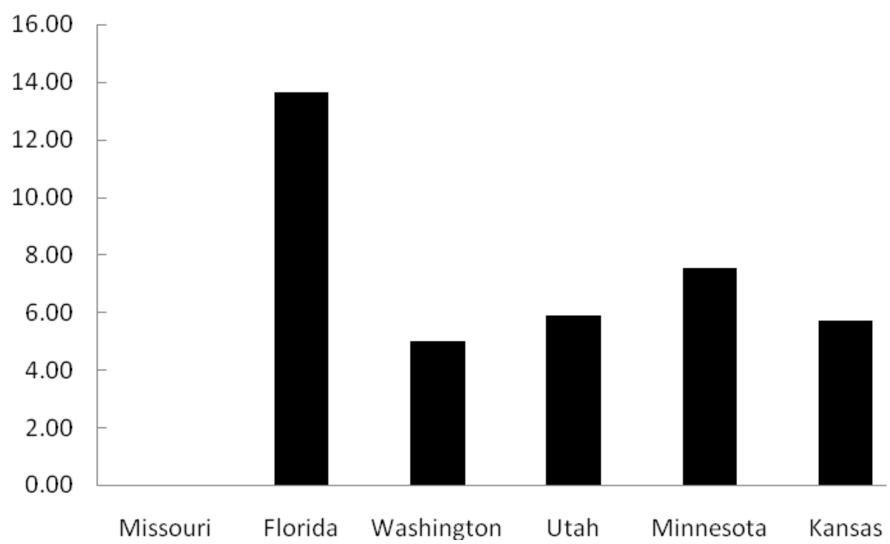


*Figure 6:* Percent of GLEs related to Connection 5.C

The percent of GLEs related to Connection 5.D ranges from 0.00% (Missouri) to 9.49% (Florida), with a mean of 4.01% and a median of 3.55%. Figure 7 shows the percent of GLEs related to Connection 5.D for each of the six states.

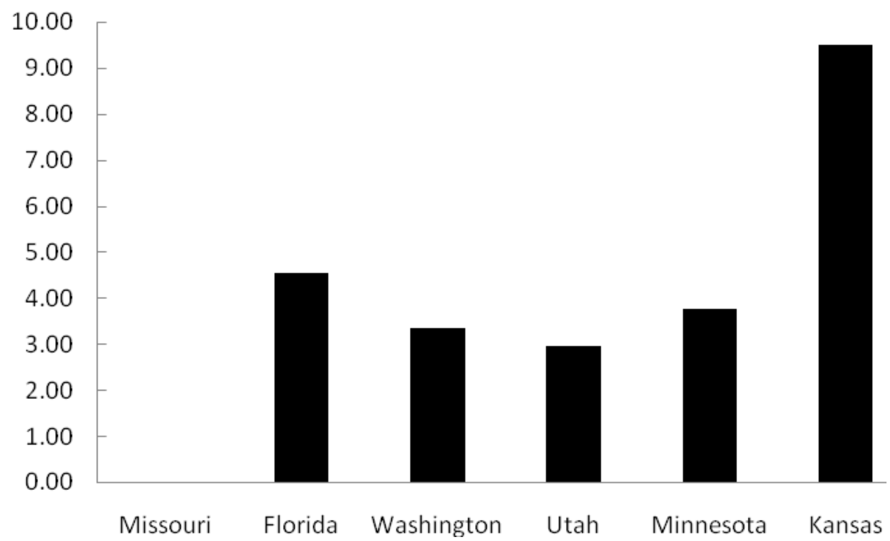


*Figure 7:* Percent of GLEs related to Connection 5.D

*Other Topics*

The percent of GLEs related to topics not covered within *Curriculum Focal Points* ranges from 10.81% (Missouri) to 58.10% (Kansas), with a mean of 32.69% and a median of 31.79%. Figure 8 shows the percent of GLEs related to topics not covered in *Curriculum Focal Points* for each of the six states.

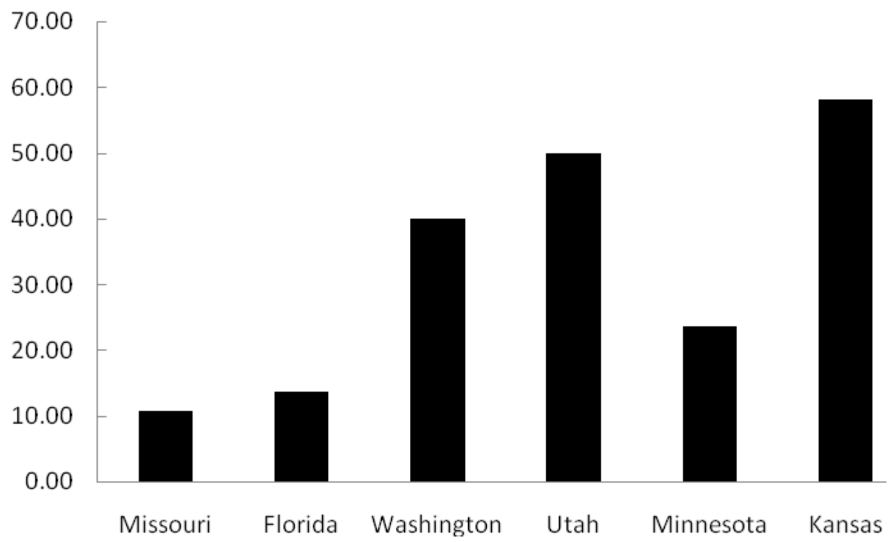


*Figure 8:* Percent of GLEs related to topics not covered in *Curriculum Focal Points*

Several of the states identified big ideas that are different from the focal points. Consequently, some of the GLEs covering topics not found in *Curriculum Focal Points* are related to these other big ideas.

Missouri explicitly identifies basic probability concepts as a fourth big idea in their mathematics standards for Grade 5. Approximately 10.81% of their GLEs are related to topics not covered in *Curriculum Focal Points*; likewise, 10.81% of their GLEs are related to basic probability concepts. That is to say, all of the GLEs in the Missouri mathematics standards not related to one of the focal points or connections are related to the additional big idea of basic probability concepts.

Like Missouri, Washington also explicitly defines four big ideas for Grade 5. The first two, multi-digit division and addition and subtraction of fractions and decimals, directly align with Focal Points 5.1 and 5.2. The fourth big idea is representations of algebraic relationships, which aligns with Connection 5.A and explains why a significant percentage of the GLEs in Grade 5 of the Washington mathematics standards relates to Connection 5.A. The third big idea, triangles and quadrilaterals, does not align with any of the focal points or connections from *Curriculum Focal Points*. However, 30.00% of the GLEs within the Washington mathematics standards relate to this big idea.

In Figure 8, the shaded regions of the bars for Missouri and Washington indicate the GLEs related to the fourth big idea that appears in the mathematics standards of each state, which shows that although there is a considerable amount of content not directly related to the focal points or connections, there is actually very little content not related to big ideas.

The Utah mathematics standards identify five big ideas, only one of which corresponds with any of the focal points or connections. Standard 2 in the Utah state mathematics standards (Utah Department of Education, 2007) states, “Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols” (p. 35). This corresponds to Connection 5.A, which states, “Students use patterns, models, and relationships as contexts for writing and solving simple equations” (NCTM, 2006, p. 17). This big idea contains four GLEs that are directly related to Connection 5.A.

The other four big ideas within the Utah state standards, however, do not correspond directly with any of the focal points or connections. The GLEs contained within these four big ideas may relate to a focal point, to a connection, or to another topic entirely. For instance, Standard 1 states, “Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals" (Utah Department of Education, 2007, p. 33), and it contains 4 GLEs related to Focal Point 5.1, 4 GLEs related to Focal Point 5.2, 2 GLEs related to Connection 5.A, 1.5 GLEs related to Connection 5.D, 12.5 GLEs related to other topics, and 3 GLEs related to process standards.

The Minnesota state mathematics standards explicitly identify nine big ideas for Grade 5. Three of these big ideas align with the focal points for multi-digit division, addition and subtraction of fractions and decimals, and understanding of surface area and volume, and each of these three big ideas accounts for 15.09% of the GLEs for Grade 5. None of the other big ideas are related to any of the focal points or connections for Grade 5, but the big idea for comparing fractions and decimals contains 18.87% of the GLEs. The remaining five big ideas collectively represent 35.86% of the GLEs, each accounting for 3.77% to 11.32% of the GLEs for Grade 5.

The Kansas mathematics standards identify 14 benchmarks for Grade 5, and each benchmark is associated with no more than 9.18% of the GLEs. Consequently, it is difficult to refer to any of these topics as big ideas.

## *Research Question 3*

The authors of *Curriculum Focal Points* state that "a focused, coherent mathematics curriculum with a national scope has the potential to ease the impact of widely varying learning and assessment expectations" (NCTM, 2006, p. 4). Research Question 3 therefore investigates the level of consistency between state mathematics standards.

### 3. For states that claim *Curriculum Focal Points* played a significant role during revisions (as indicated by state mathematics supervisors who responded to a survey conducted by CSMC), is there consistency between mathematics standards from state to state?

Five of the six states whose mathematics standards were considered in this analysis claimed that *Curriculum Focal Points* was a primary resource in their most recent revisions. The lone exception was Kansas, which claimed that a committee reviewed the standards and determined that no revisions were necessary because the standards were already in alignment with *Curriculum Focal Points*. Consequently, one would expect a level of consistency between the post-*CFP* versions of state standards documents.

Despite the fact that all six states used *Curriculum Focal Points* as a primary resource, the resulting sets of state mathematics standards are very different one from another. From state to state, there are five fundamental differences:

- Organizational structure of each document,
- Attention given to process standards,
- Percent of GLEs dedicated to each focal point,
- Total number of GLEs, and
- Number of big ideas identified at Grade 5.

*Organizational Structure*

The organizational structure of each state's post-*CFP* mathematics standards document was described in detail above. The discussion in this section will be limited to just the differences between the state documents.

All states explicitly list big ideas, but the state standards documents differ in the number of big ideas. Florida included three big ideas for Grade 5; Missouri and Washington included four big ideas; Utah included five big ideas; Minnesota included nine big ideas; and Kansas included 14 big ideas.

The state standards documents also differ in their treatment of supporting ideas. Florida and Washington explicitly list supporting ideas, but the other states did not.

The terminology used within state standards documents also differs from state to state. Table 2 shows the discrepancy in terminology between the documents. Note that Kansas used the term "benchmark" to describe a big idea, yet Florida and Minnesota use "benchmark" to describe a GLE.

Table 2

*Terminology Used to Describe Big Ideas and GLES in State Standards*

| State | Terminology Used to Describe Big Ideas | Terminology Used to Describe GLEs |
|---|---|---|
| Florida | Big Ideas | Benchmark |
| Kansas | Benchmarks | Knowledge Base Indicators and Application Indicators |
| Minnesota | Standards | Benchmarks |
| Missouri | Core Content | Learning Goals and Performance Indicators |
| Utah | Standards | Objectives |
| Washington | Core Content | Performance Expectations |

*Attention to Process Standards*

State mathematics standards differ in terms of the amount of attention given to the process standards. Minnesota, Missouri, and Washington include significant references to the process standards within the front matter as well as throughout the GLEs; Florida includes only a brief mention of the process standards within the front matter but then references the process standards significantly within the GLEs; Utah makes only minimal reference to the process standards within the front matter and GLEs; and, Kansas makes no reference to the process standards in the front matter but occasionally references the process standards within the GLEs.

Minnesota (2007b) references the process standards in the front matter of their document and within the body of the document. The introduction states, “The standards and benchmarks presented here describe a connected body of mathematical knowledge that is acquired through the processes of problem solving, reasoning and proof, communication, connections, and representation” (Minnesota Department of Education, 2007b, p. 2). One of their big ideas suggests that students should “understand and interpret equations and inequalities involving variables and whole numbers, and use them to represent and solve real-world and mathematical problems” (p. 17).

Missouri (2008) also includes references to the process standards within the front matter and through the GLEs. On the first page of the Missouri mathematics standards, the writing team asserts, “At every grade, students must be challenged to use mathematics to reason and solve problems… [and] to communicate about mathematics… mathematical processes are interwoven throughout the core content and learning goals” (Missouri Department of Education, 2008, p. 1-2). Likewise, the matrix on page 8 of the document identifies the most important process standards for each grade band. For Grades 3-5, it states that students should: (a) apply and adapt a variety of strategies to solve problems; (b) make and investigate mathematical conjectures; (c) communicate mathematical thinking coherently and clearly to peers and teacher: and (d) organize, record, communicate, and represent mathematical ideas.

Washington (2008) gives significant attention to problem solving, which is referenced in the introductory paragraph accompanying every big idea. The introductory paragraph often references other process standards, too, with statements such as, “Students apply these procedures… to solve a wide range of problems” (p. 50) and “Students make tables and graphs… to see the mathematical connections between algebra and geometry" (p. 54). In addition, the document contains five sections related to mathematical content and a sixth section dedicated to the process standards of reasoning, problem solving, and communications. There are nine performance expectations associated with the process standards, five of which explicitly mention problem solving, while the other four make reference to the process standards of communication, reasoning, and designing multiple representations.

Florida references connections and problem solving (Florida Department of Education, 2007) in the introduction to the document, but the attention to the process standards is more significant within the GLEs. Terms such as *describe* (p. 49), *verify* (p. 50), *solve* (p. 52), and *represent* (p. 55) allude to the process standards of communication, reasoning, problem solving, and representation and are included in many of the GLEs.

The front matter of the Utah state mathematics standards makes only cursory reference to the process standards. It states, "The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world" (p. 3), and, "This curriculum relates directly to student needs and interests" and will allow them "to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom" (Utah, 2007, p. 4). These statements do not refer to the process standards directly, and there are no other references to the process standards in the front matter.

Within the GLEs of the Utah mathematics standards, there are some references to the process standards. For instance, one GLE refers to multiple representations by asserting that students should “model multiplication of fraction and decimals… in a variety of ways” (p. 34). Another alludes to reasoning by stating that students should “interpret division-with-remainder problems as they apply to the environment” (Utah, 2007, p. 34). Unfortunately, these types of statements within the GLEs are rare; only 3 of the 54 GLEs for Grade 5 contain any reference to the process standards.

In the front matter of the Kansas state mathematics standards, there is no reference to the process standards. The process standards are referenced occasionally within the GLEs, appearing in 8 of the 98 GLEs for Grade 5. However, the table of contents for the document claims that an appendix will contain information on mathematical communication, reasoning, and problem solving (Kansas Department of Education, 2004). Unfortunately, no such appendix is included with the document. When asked about this appendix, a Kansas representative said, “the [state standards] document was written with the intent of having the committee continue to work developing additional resources, but their time ran out before they were all completed” (personal communication, June 1, 2009). Consequently, this appendix was never written.

*Percent of GLEs Dedicated to Focal Points and Connections*

There is significant variation in the percent of GLEs dedicated to each focal point and connection in the state standards documents. In the most extreme instance, the percent of GLEs attributed to Focal Point 5.2 (number and operations) ranged from a low of 3.65% in Kansas to a high of 35.14% in Missouri, a difference of

31.49%. Figure 9 below visually depicts the variation for each of the focal points and connections.

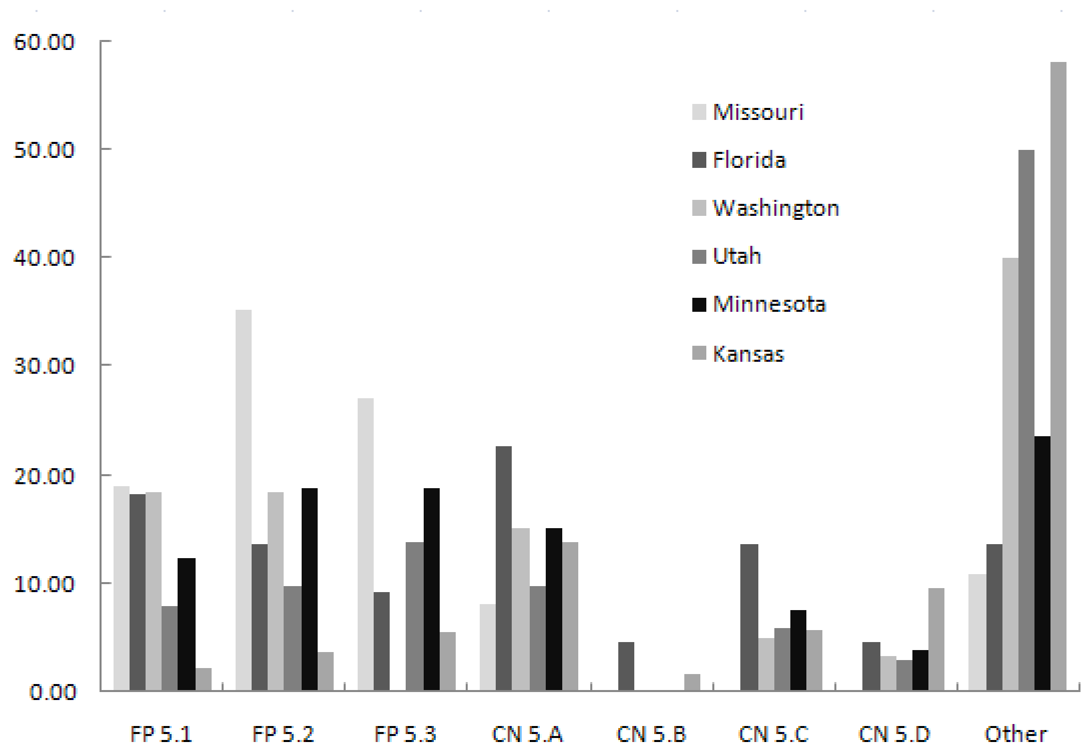


Figure 9: Percent of GLEs within state mathematics standards dedicated to each of the three focal points, to each of the four connections, and to other content not identified in *Curriculum Focal Points*

The greatest level of consistency regarding percent of GLEs occurs with Connection 5.B (measurement), which ranges from a high of 4.55% in Florida to a low of 0.00% in four of the six states. But rather than indicate agreement among the states, this merely suggests that most states did not seem to include measurement among their GLEs for Grade 5.

*Total Number of GLEs*

Although four of the six states realized a decrease in the number of GLEs in their standards documents from the pre-*CFP* to post-*CFP* versions (the number for Kansas did not change, and the number for Missouri increased slightly), there was still significant difference in the number of GLEs for Grade 5 between state standards documents.

The variation in GLEs ranged from a low of 23 (Florida) to 98 (Kansas), with a mean of 46 and a median of 39. By comparison, the pre-*CFP* versions ranged from a low of 33 (Minnesota) to a high of 98 (Kansas), with a mean of 61 and a median of 62. (In the post-*CFP* data set, Kansas is an outlier. But even with Kansas removed, the results are similar; the number of GLEs in the post-*CFP* versions ranged from 23 to 54, with a mean of 37 and a median of 39. Utah had 2.3 times as many GLEs as Florida.)

Table 3

*Number of GLEs Identified in State Standards for Grade 5*

| State | Number of GLEs in Post-*CFP* Standards | Number of GLEs in Pre-*CFP* Standards |
|---|---|---|
| Florida | 23 | 77 |
| Kansas | 98 | 98 |
| Minnesota | 27 | 33 |
| Missouri | 39 | 35 |
| Utah | 54 | 72 |
| Washington | 40 | 52 |

*Number of Big Ideas Identified at Grade 5*

Before describing the differences in the number of big ideas, it is necessary to explain the criteria used to determine what constitutes a big idea within state standards documents. Unfortunately, the term *big idea* is used often but typically presented without definition or description. Charles (2005) described a big idea as a concept central to the learning of mathematics that coherently connects numerous mathematical understandings. Though reasonable, this definition is not helpful in identifying the topics that constitute big ideas within state mathematics standards. Consequently, a very simple rule was implemented to identify big ideas: Big ideas were those topics or concepts that states explicitly identified as big ideas.

In some cases, identifying big ideas within state standards documents required little effort. For instance, Florida refers to their big ideas as “big ideas.” In other cases, states use terminology that is less obvious but still identifiable; for example, Washington distinguishes between big ideas and supporting ideas by referring to them as “core content” and “additional key content,” respectively, and Missouri used the term “core content” to describe their big ideas but explicitly defined the term as “important mathematical ideas… for curriculum design, instruction, and assessment for a grade level or course” (Missouri Department of Education, 2008, p. 1). Yet significant difficulty was encountered when analyzing the state standards for Utah and Kansas. In these two states, the presence of big ideas is not obvious, the terminology is not illuminating, there is no explicit description of big ideas, and similar structures are used in different ways.

Both Utah and Kansas employ a three-tier system in their state mathematics standards. In Utah, a standard consists of several objectives, and each objective contains several GLEs. In Kansas, an overarching statement consists of several benchmarks, and each benchmark consists of multiple indicators. Consequently, a decision had to be made for each state as to which level of the hierarchy constituted a big idea. For Utah, the standards seemed to reflect the intent of *Curriculum Focal Points* by stating a specific mathematical topic to be learned. In other words, the standards had the look and feel of focal points. For example, the first standard in the Utah state mathematics standards for Grade 5 reads, “Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals” (Utah State Office of Education, 2007, p. 33). On the other hand, the highest-level elements in the Kansas standards read more like headings than focal points. For example, the first overarching statement in the Kansas state mathematics standards for Grade 5 reads, “The student uses numerical and computational concepts and procedures in a variety of situations” (Kansas Department of Education, 2004, p. 5-1). It was determined that this statement was too vague; in fact, it reads more like a process standard than a big idea. On the other hand, the second-level elements (“benchmarks”) seemed to more satisfactorily reflect the look and feel of a focal point. As an example, the first benchmark for number sense reads, “The student demonstrates number sense for integers, fractions, decimals, and money in a variety of situations” (Kansas Department of Education, 2004, p. 5-1). Therefore, the highest-level elements in the Utah document but the second-level elements in the Kansas document were considered to be big ideas.

There is significant variation in the number of big ideas from state to state. At one end of the spectrum, Florida, Missouri, Utah, and Washington identify five or fewer big ideas in their mathematics standards for Grade 5. At the other end, Minnesota includes nine big ideas, and Kansas includes 14 big ideas. The distribution in the number of big ideas contained in state mathematics standards appears to be bimodal: four of the six states had a number of big ideas in Grade 5 very similar to the number of focal points in *Curriculum Focal Points*, while the other two had a number of big ideas triple or quadruple the number of focal points for the same grade.

Table 4

*Number of Big Ideas Identified in State Standards for Grade 5*

| State | Number of Big Ideas in Post-*CFP* Standards |
|---|---|
| Florida | 3 |
| Kansas | 14 |
| Minnesota | 9 |
| Missouri | 4 |
| Utah | 5 |
| Washington | 4 |

*<u>Research Question 4</u>*

To determine the extent to which *Curriculum Focal Points* influenced the revision of state mathematics standards, Research Question 4 compares pre-*CFP* standards to post-*CFP* standards.

4. **What major changes can be identified between the state mathematics standards developed prior to the release of *Curriculum Focal Points* and the revised state mathematics standards for which *Curriculum Focal Points* was used as a resource in the revision process?**

The terminology used to refer to big ideas within the post-*CFP* versions state mathematics standards changed for three of the six states (Florida, Missouri, and Washington). Interestingly, the same three states showed the greatest decrease in the number of big ideas from the pre-*CFP* to post-*CFP* versions. Table 5 shows the terminology used as well as the number of big ideas contained in both the pre-*CFP* and post-*CFP* versions of the standards.

Table 5

*Terminology and Number of Big Ideas in State Standards for Grade 5*

| State | Pre-*CFP* Terminology Used to Describe Big Ideas | Number of Big Ideas in Pre-*CFP* Standards | Post-*CFP* Terminology Used to Describe Big Ideas | Number of Big Ideas in Post-*CFP* Standards |
|---|---|---|---|---|
| Florida | Standards | 5 | Big Ideas | 3 |
| Kansas | Benchmarks | 14 | Benchmarks | 14 |
| Minnesota | Standards | 10 | Standards | 9 |
| Missouri | Big Ideas | 17 | Core Content | 4 |
| Utah | Standards | 5 | Standards | 5 |
| Washington | Essential Academic Learning Requirements | 12 | Core Content | 4 |

Four of the six states included a number of big ideas consistent with the inclusion of three focal points in the *Curriculum Focal Points* document. Florida included three big ideas within their state standards document, Missouri and Washington each included four big ideas, and Utah included five big ideas.

Four of the six states (Florida, Minnesota, Missouri, and Washington) realized a decrease in the number of big ideas covered by their standards. Respectively, the number of big ideas for these four states was reduced by 66%, 10%, 76%, and 66%. The number of big ideas in the Utah mathematics standards did not change, and the same or very similar big ideas were used in both the pre-*CFP* and post-*CFP* versions. For instance, the first standard in the two documents changed from, “Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals” (Utah Department of Education, 2003, p. 19) to, “Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals” (Utah Department of Education, 2007, p. 33). Likewise, the number of big ideas in the Kansas standards remained the same.

Four of the six states realized a significant decrease in the number of GLEs. Florida realized a 70% decrease (from 77 in the pre-*CFP* version to 23 in the post-*CFP* version); Utah, a 54% decrease (from 72 to 54); Washington, a 23% decrease (from 52 to 40); and Minnesota, an 18% decrease (from 33 to 27). Missouri actually realized an 11% increase (from 35 to 39), but given the significant reduction in the number of big ideas, the slight increase in GLEs is insignifcant. The number of GLEs in the Kansas standards remained the same, because their standards were not revised.

# Chapter 5: Mitigating Factors

The first chapter within *Curriculum Focal Points* is titled, "Why Identify Curriculum Focal Points?" (NCTM, 2006, p. 3). The implied answers to this question were given by the headers used for the two sections in the chapter:

- Inconsistency in the Placement of Topics by Grade Level in U.S. Mathematics Curricula (p. 3) and
- The Importance of Curricular Focus in Mathematics (p. 4).

The authors of *Curriculum Focal Points* clearly hoped that the document would bring greater focus to the U.S. mathematics curriculum as identified in state mathematics standards. In addition, there was the suggestion that if states used *Curriculum Focal Points* as a guiding document, a greater level of consistency between state mathematics standards might be realized.

Though greater focus within and consistency between state standards documents may have been an objective of *Curriculum Focal Points*, these were not necessarily the goals of the state standards writing committees. The development of state standards is subject to political and financial pressures. In an attempt to determine what factors, if any, influenced the state standards revision processes, an effort was made to contact a representative from each state. These representatives were members of the standards writing teams or employees of the state department of education. In all cases, the person contacted had intimate knowledge of the standards writing process.

Despite repeated efforts to contact a representative from Kansas, including multiple emails and several phone calls, no response was returned regarding the development of state standards. (Although an employee of the Kansas State Department of Education responded to a specific question regarding the lack of appendices and the apparent incompleteness of their state standards, he said that he had not been significantly involved in the standards writing process and would not be able to speak about it effectively.) Responses from the other five states, however, were returned very quickly, and the responses were generally more open and honest than could have been anticipated.

### *Florida*

In Florida, a committee of experts "recommended that the CFP be used as the foundation for the design of the new standards" (Dixon & Kersaint, 2008, p. 22). Two members of this committee of experts were Barbara Reys and Jane Schielack, both of whom have been mentioned previously. Barbara Reys served as a reviewer for *Curriculum Focal Points* (NCTM, 2006) and was a writing group co-chair for the most recent revision of the Missouri state mathematics standards (Missouri Department of Education, 2008). Jane Schielack served as the chair of the writing team for *Curriculum Focal Points* (NCTM, 2006). It is therefore not surprising that the final version of the Florida state mathematics standards reflect the vision of *Curriculum Focal Points* so strongly. Political factors contributed to this result.

Some members of the writing committee argued that the new state standards document ought to serve as a supplement to the 1996 Florida standards document, "in the same way that [*Curriculum Focal Points*] builds on and enhances the *Principles and Standards for School Mathematics*" (Dixon & Kersaint, 2008, p. 22). When the state department of education required that the new document must replace the old one, however, the writing committee decided to organize the standards around the same three focal points for each grade that are offered in *Curriculum Focal Points*. The only difference is that they would be called "big ideas" instead of "focal points."

A member of the Florida writing team (personal communication, May 13, 2009) explained that the revision process involved two phases. In the first phase, a group known as "the framers" reviewed curriculum and provided direction to the writing team. When the writers convened for the initial meeting, they were informed that the framers suggested *Curriculum Focal Points* ought to be used as the model for the K-8 standards. This information, however, had not been provided to the writers prior to their initial meeting.

In the second phase, the writers gathered for several face-to-face meetings. The first meeting of the writing group was "a most unusual meeting, because no one had resources, internet access was not available, and yet we were told to revise the curriculum" (Florida representative, personal communication, May 13, 2009).

The writing group informed the state department of education that an unrealistic timeline had been proposed and that additional time would be necessary. "Given the recent release [of *Curriculum Focal Points*], people needed time to make sense of the information in the document and to ensure that all members [of the writing team] had common views about the intent of the document. We did not [have enough time]." In the absence of an external authority to provide guidance, the writing team defaulted to statements contained within *Curriculum Focal Points*. “Because individuals had strong feeling and views, the language within the *CFP* became the compromise… why did we adhere to the [*Curriculum Focal Points*] so closely? Given the time provided to do this work, it would not have been possible to do anything else” (Florida representative, personal communication, May 13, 2009).

In addition to the circumstances described above, the Florida representative also included an unsolicited list of hindrances to the writing process:

- The Florida Department of Education had a strong desire to be the first to adopt new standards based on *Curriculum Focal Points*.
- It was difficult to disentangle teacher issues from curriculum issues. Writers were concerned with the ability of the current teaching force to teach the standards as intended, particularly because funds were not available for professional development.
- The structure and timeline provided did not allow time for the writers to really address and review the curriculum.
- Writers were not involved in the revision process after comments were received from the public. After public review of the document, modifications based on submitted comments were incorporated by Florida Department of Education staff. (personal communication, May 13, 2009)

*Minnesota*

Several members of the committee that wrote the pre-*CFP* version of the Minnesota state mathematics standards (Minnesota Department of Education, 2003) were also on the committee responsible for revising the standards. Many members of the committee believed that the previous version of the standards were unfocused, so "rather than revise the standards in 2007, the decision was made to start over" (Minnesota representative, May 28, 2009).

*Curriculum Focal Points* was used as the primary resource, and if disagreements arose, the committee always referred to *Curriculum Focal Points* for clarification.

The major issue encountered during the writing process concerned the inclusion of Algebra I by Grade 8. *Curriculum Focal Points* does not give specific advice in this regard, and "basically says that if you want to include Algebra I in eighth grade, you need to revise and figure out how to include it yourself" (Minnesota representative, personal communication, May 28, 2009).

Political influence played a role in the development of the Minnesota standards. The state assessment office exerted a considerable amount of influence on the first draft, "which caused a significant amount of repetition of content within the standards" (Minnesota representative, personal communication, May 28, 2009). The repetition of content was in direct conflict with the goals of the writing committee. In addition, comments solicited from national reviewers, all but one of whom were mathematicians, indicated that there should be less repetition of content.

Consequently, “big adjustments were made” after the reviews came back (Minnesota representative, personal communication, May 28, 2009).

*Missouri*

The standards revision process in Missouri was not initiated by the state department of education, because they did not want to change state assessments. Instead, a political push came from a STEM coalition within the state that pushed for more rigorous standards. Unlike previous standards, which were developed primarily to indicate the content of state assessments, the revised standards were meant to provide the outline for a full curriculum.

A Missouri representative (personal communication, May 14, 2009) indicated that political pressure was applied in a direction opposite to what was expected. After the writing team completed their first draft of the standards, the document was posted for public review. The overwhelming majority of comments, most of which were submitted by mathematicians, requested that the draft of the Missouri state standards be modified to reflect greater alignment to *Curriculum Focal Points*. Specifically, the writing team was “pressured to remove those places where [the writing team] deviated from [*Curriculum Focal Points*].”

The Missouri representative provided two specific examples of the types of changes that were suggested. *Curriculum Focal Points* includes the analysis, representation, and solution of systems of linear equations in Grade 8 (NCTM, 2006). The Missouri writing group had chosen not to include that topic in the draft of the standards, but reviewers of the draft document suggested that it ought to be included.

After much discussion, the writing team decided to disregard the reviewers' comments and held firm to their decision to exclude this topic.

In Grade 4, *Curriculum Focal Points* includes a focal point devoted to "developing an understanding of area and determining area of two-dimensional shapes" (NCTM, 2006, p. 16). In the draft version, the writing team included references to the area of rectangles but not to triangles, yet public comments suggested that the area of triangles ought to be included. Upon considering the comments, the writing team decided to align with NCTM and made the suggested modifications.

Because the writing team felt that *Curriculum Focal Points* was too narrow, a fourth and, in some cases, a fifth big idea was included in each grade. "This allowed for more probability and statistics to be included and allowed for an emphasis on measurement and geometry" in some grades, both of which are lacking in *Curriculum Focal Points* (Missouri representative, personal communication, May 14, 2009).

### *Utah*

In Utah, a number of political factors contributed to the state department of education deciding to revise the state mathematics standards. A Utah representative (personal communication) explained the process that was used in their most recent revisions:

> Our superintendent asked that the standards be revised in the winter of 2007 due to several factors. A committee of educators and mathematicians had recently reviewed the 2002 core and found it in

> need of some slight revision. The Fordham Report gave Utah a low score on curriculum. A petition of mathematicians was circulating in the state, focusing attention on mathematics education. The legislature was being encouraged by some to adopt the California Standards. The superintendent asked Utah State University to lead the revision process and Russell Thompson, Mathematics Department Chair at USU, led the committee. The committee consisted of mathematicians, university mathematics educators, and district-level mathematics supervisors and specialists, as well as the elementary and secondary mathematics specialists from the Utah State Office of Education (USOE).
>
> The committee examined several documents current at the time. At the elementary level, the *Curriculum Focal Points* were the primary resource. At the secondary level, the committee examined the Achieve Standards, the College Board Standards, and the [*Guidelines for Assessment and Instruction in Statistical Education*], along with [*Principles and Standards for School Mathematics*]. At both levels, other state standards were also referred to. (May 12, 2009)

The representative stated that the writing committee did not have a preconceived idea about the number of objectives to include, “but rather let our discussions determine the number of necessary objectives” (Utah representative, personal communication, May 12, 2009). However, the writing team did make a conscious effort to reduce the number of objectives and to avoid repetition at various grade levels. “The choice of objectives was based on the three focal points, but also on the connections and expectations in the *Focal Points* document” (Utah representative, personal communication, May 12, 2009).

## Washington

The writing team in Washington reviewed many documents during the revision process. A Washington representative (personal communication, May 9, 2009) claimed that among the documents reviewed were a number of reports by Linda Plattner, the draft version of the Florida state mathematics standards, the Texas state mathematics standards, the Georgia state mathematics standards, the National Assessment of Education Progress (NAEP) results, *Curriculum Focal Points*, the *Guidelines for Assessment and Instruction in Statistics Education* from the American Statistical Association, *Adding It Up* from the National Research Council, and an article by Lynn Arthur Steen.

The Washington state mathematics standards are remarkably similar to *Curriculum Focal Points* in structure and format, except that more than three big ideas are included in each grade. In most cases, the additional content is pulled from probability and statistics. The Washington representative explained that the inclusion of probability concepts within the Washington state mathematics standards was “an outgrowth of attention to the NAEP results” (Washington representative, personal communication, May 9, 2009).

Although the representative believes that “the resulting standards are remarkably sound” mainly because of “the wonderful Washington folks on the writing team," the process was fraught with political tension. An external group, which had no interest in group processes, continually bullied policymakers into requiring additional reviews of the writing team's work by an external consultant. Because of this group, the development of the standards "was one of the most intense and absolutely the most negative professional environments" in which the representative had worked (Washington representative, personal communication, May 18, 2009).

### *Commonalities in the Circumstances*

Political pressure influenced all states' revision process. In the case of Utah, pressure from internal and external sources pushed the state department of education to begin the standards revision process, and in other states, politics directly affected the work of the writing teams.

It is also interesting to note that many of the state writing teams used other states' standards for reference. It would therefore seem likely that greater consistency might be realized between state standards documents.

# Chapter 6: Discussion

*Curriculum Focal Points* has clearly had an impact on the most recent revisions of state mathematics standards, but the level of impact differs significantly from state to state. The post-*CFP* state documents vary in several characteristics, specifically (a) the framework and organizational structure, (b) the number of big ideas, (c) the number of GLEs, and (d) the overlap between the content in *Curriculum Focal Points* and the GLEs. While states that used *Curriculum Focal Points* as a reference during their most recent revisions were able to narrow the focus of their mathematics standards, consistency between state documents was not a corollary outcome.

In the discussion that follows, it may appear that the six states have been divided into three groups. The division was not intentional, but the reason for the apparent division has to do with the amount of influence *Curriculum Focal Points* had on the standards revision process.

The first group into which the six states can be divided includes those that revised their standards with *Curriculum Focal Points* as a primary resource and received additional assistance from a key player in the development of *Curriculum Focal Points*. The three states in this group are Florida, Missouri, and Washington.

Jane F. Schielack served as the chair of the writing team for *Curriculum Focal Points*, and R. James Milgram and Barbara Reys served as formal reviewers of the document. Each of them is also included on a list of national experts who presented research to the framers of the Florida Sunshine State Standards in mathematics

(Florida Department of Education, 2008). Barbara Reys also served as one of the co-chairs of the writing team for the Missouri mathematics standards.

Former NCTM President Cathy Seeley served as one of the lead consultants for the revisions of the Washington state mathematics standards. Although not directly involved with the writing or reviewing of *Curriculum Focal Points*, Seeley was the president of the Council when the project was initiated, and she generally supports the philosophy of the Council.

It should therefore come as no surprise that the mathematics standards in Florida, Missouri, and Washington display characteristics similar to those of *Curriculum Focal Points*.

The second group into which the states can be divided is those that used *Curriculum Focal Points* as a primary resource but did not receive any additional assistance in interpreting the document. The two states in this group were Minnesota and Utah. Although *Curriculum Focal Points* is referenced in the introduction to the Utah state standards document and is described as a resource that "provided the basis for the selection of content in the [Minnesota] standards and benchmarks" (Minnesota Department of Education, 2007a, question 4), there is no indication that any of the key players who helped to develop *Curriculum Focal Points* offered assistance to either of these states.

The third group is those states for whom *Curriculum Focal Points* played an insignificant role in the standards revision process. Of the six states analyzed, the lone state in this category is Kansas. The state standards committee reviewed the pre-*CFP* mathematics standards and determined that no revisions were necessary because the

Kansas document sufficiently covered all topics within *Curriculum Focal Points* (Reys, 2007). Consequently, no revisions were undertaken, the pre-*CFP* and post-*CFP* standards are identical, and by definition *Curriculum Focal Points* could not have had much influence.

Though purely speculative, it is possible that the reluctance to revise the state standards was due to political pressures. Revising standards is costly and time-consuming. A Kansas representative indicated that the pre-*CFP* standards "document was written with the intent of having the committee continue to work developing additional resources, but their time ran out before they were all completed" (personal communication, June 1, 2009). Given that the pre-*CFP* document was never completed, it is understandable that the Kansas State Department of Education would not want to begin the process anew. Further, assessments based on the pre-*CFP* version of the Kansas standards were administered for the first time in 2006. Revising the standards based on *Curriculum Focal Points*, which was released in September 2006, would likely necessitate modifications to the state assessments, which is also costly and time-consuming. Therefore, no judgment is meant to be implied when claiming that *Curriculum Focal Points* did not play a significant role in the development of the post-*CFP* Kansas state standards, and it is completely reasonable that the Kansas State Department of Education opted not to revise the standards when *Curriculum Focal Points* was released. Still, this reality makes it difficult to ascribe any impact to *Curriculum Focal Points* when there was no resulting change to the Kansas state mathematics standards.

*Framework of the State Standards*

The state standards reflected varying levels of alignment to the structure of *Curriculum Focal Points*.

The framework employed in the Florida, Missouri, and Washington mathematics standards is strongly aligned with *Curriculum Focal Points*, identifying big ideas and supporting ideas at each grade level and referring to the process standards in the introduction as well as referencing them within the GLEs.

The framework employed in the Minnesota and Utah mathematics standards is partially aligned with *Curriculum Focal Points*. The Minnesota document uses some of the same terminology employed in *Curriculum Focal Points*, but it does not explicitly identify big ideas, nor does it distinguish between big ideas and supporting ideas. On the other hand, the process standards are referenced in both the introduction and within the GLEs of the Minnesota state standards. Conversely, the Utah mathematics standards identify big ideas at each grade level, but the process standards are given minimal treatment within the document, referenced in only a small portion of the GLEs.

The framework employed in the Kansas mathematics standards is minimally aligned with the framework of *Curriculum Focal Points*, identifying a large number of big ideas but no supporting ideas at each grade level. And while the terminology employed within the GLEs makes some reference to the process standards, the document does not include a summary statement about the importance of the process standards.

*Big Ideas in the State Standards*

State mathematics standards exhibit mathematical focus by identifying a limited number of big ideas. Although *Curriculum Focal Points* does not recommend a particular number of big ideas that should be identified at each grade, the document identifies three focal points for Grade 5, and the number of big ideas identified within a state standards document should be similar.

The graph in Figure 10 shows the reduction in the number of big ideas between the pre-*CFP* and post-*CFP* versions of each state's standards. While no state has more big ideas than before, it's clear that not every state put forth effort to greatly reduce the number of topics.

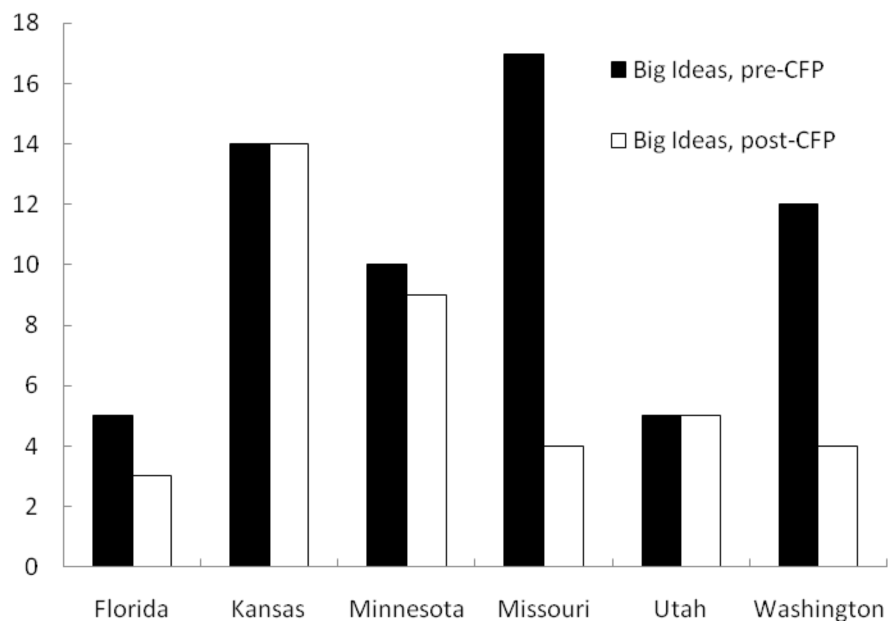


Figure 10: Change in number of big ideas from pre-*CFP* to post-*CFP* versions of state standards

The number of big ideas identified within the post-*CFP* versions of the Florida, Missouri, Utah, and Washington mathematics standards is similar to the number of focal points identified in *Curriculum Focal Points*. For Grade 5, Florida identified three big ideas, Minnesota and Washington identified four big ideas, and Utah identified five big ideas. It seems reasonable that states with five or fewer big ideas in their state standards documents have reasonably interpreted the intent of *Curriculum Focal Points*. Indeed, *Curriculum Focal Points* implies that students benefit from mathematical instruction that "focuses on a small number of key areas of emphasis" (NCTM, 2006, p. 5).

It may be illustrative to consider a specific example. The pre-*CFP Florida Sunshine State Standards* explicitly identified five big ideas, although these were nothing more than headers indicating five content standards: Number Sense, Concepts, and Operations; Measurement; Geometry and Spatial Sense; Algebraic Thinking; and Data Analysis and Probability. A cursory analysis of this document makes it clear that there are far more than five major topics to be addressed. The GLEs within the document represent a laundry list of topics and do not indicate curricular focus in any way. On the other hand, the post-*CFP* Florida mathematics standards developed after the release of *Curriculum Focal Points* contain only three big ideas, and these big ideas are explicitly stated.

In contrast, Minnesota identified nine big ideas in its post-*CFP* state standards documents, and Kansas identified 14 big ideas. Given that the numbers of big ideas in these documents are, respectively, three and almost five times greater than the number of focal points, it seems that they do not focus on a small number of key mathematical topics. It seems that these states did not reasonably interpret the intent of *Curriculum Focal Points* with respect to mathematical focus.

Further, it appears that *Curriculum Focal Points* had an impact in terms of encouraging states to reduce the number of big ideas. Four of the six post-*CFP* standards documents contained fewer big ideas than the pre-*CFP* versions. The two states that did not realize a decrease were Utah, which had only five big ideas in both versions, and Kansas, which did not revise their standards.

In addition to the number of big ideas, it is interesting to compare the content of the big ideas and supporting ideas with the content of the focal points and connections. Table 6 shows that five of the six states include Focal Points 5.1 and 5.2 as two of their big ideas. In addition, five states give significant attention to Connection 5.A, and five states give little attention to Connection 5.B.

Yet despite the areas of overlap, this table highlights the inconsistency between state standards documents. Focal Point 5.3, which pertains to three-dimensional shapes, is not included as a big idea in the *Washington State K-12 Mathematics Learning Standards* (OSPI, 2008); instead, Washington includes a big idea about the geometric properties of triangles and quadrilaterals in Grade 5, while big ideas about three-dimensional shapes are included in Grades 6 and 7.

There is also discrepancy regarding Connection 5.A, which pertains to algebraic equations. One state (WA) includes the topic as both a big idea and supporting idea; two states (MN and UT) include the topic as a big idea only; one state (FL) includes the topic as a supporting idea; one state (KS) does not include it as either a big idea or supporting idea but dedicates a large percentage of GLEs to the topic; and one state (MO) does not include the topic at all.

Table 6

*Alignment of Big Ideas and Supporting Ideas within State Standards Documents to the Focal Points and Connections in* Curriculum Focal Points

| | FL | KS | MN | MO | UT | WA |
|---|---|---|---|---|---|---|
| Focal Point 5.1 | ● | | ● | ● | ● | ● |
| Focal Point 5.2 | ● | | ● | ● | ● | ● |
| Focal Point 5.3 | ● | | ● | ● | ● | |
| Connection 5.A | ○ | ⊗ | ● | | ● | ●○ |
| Connection 5.B | ○ | | | | | |
| Connection 5.C | ○ | | ● | | | ○ |
| Connection 5.D | ○ | ⊗ | | | | |

A ● indicates that a state lists the focal point or connection as a big idea in their document.
A ○ indicates that a state lists the focal point or connection as a supporting idea.
A ⊗ indicates that a state does not explicitly list the focal point or connection as either a big idea or supporting idea, but a large percentage of GLEs are dedicated to the topics contained within that focal point or connection.

*GLEs in the State Standards*

Including a limited number of big ideas and including fewer big ideas in the post-*CFP* standards than in the pre-*CFP* version are two indicators that states attempted to attain greater focus by concentrating on fewer topics. Similarly, limiting the number of GLEs and reducing the number of GLEs from the pre-*CFP* to post-*CFP* versions are two additional indicators.

*Curriculum Focal Points* does not provide recommendations regarding the number of GLEs that should be included in state mathematics standards. However, Reys (2006) suggested that state standards documents should contain 20-25 GLEs per grade. Florida, with just 23 GLEs for Grade 5, was the only state of the six to fall within that recommended range; Minnesota, with 27 GLEs for Grade 5, came very close. Interestingly, Missouri exceeded the recommendation by including 39 GLEs for Grade 5, even though Barbara Reys was the lead author on the study suggesting that the number of GLEs be limited as well as a co-chair of the writing team for the Missouri state mathematics standards. Washington, Utah, and Kansas also exceeded the recommendation by including 40, 54, and 98 GLEs, respectively. Although five of the six states were above the recommended range, four of them showed a significant decrease in the number of GLEs from the pre-*CFP* to post-*CFP* versions of their state standards, with the percent decrease ranging from 18% (Minnesota) to 70% (Florida). The number of GLEs within the Missouri state standards for Grade 5 increased from 35 to 39, yet this increase could be considered unimportant when viewed in light of the drastic decrease in the number of big ideas (from 17 to 5). Kansas showed no change in the number of GLEs, because their standards did not change as a result of *Curriculum Focal Points*.

*Content Alignment to Curriculum Focal Points*

*Curriculum Focal Points* (NCTM, 2006) was meant to provide “one possible response to the question of how to organize curriculum standards” (p. 3), yet it was not meant to imply that there is only acceptable organization of content within a focused curriculum. Consequently, it would be unwise to claim that any set of state mathematics standards does not align with *Curriculum Focal Points* simply because there is not a one-to-one correspondence between the content in the two documents.

That said, just as Schmidt (2006) used a matrix of dots as an overlay to compare the standards of various states with a model scenario, it seems reasonable to use the content within *Curriculum Focal Points* as an overlay to examine the content within state mathematics standards. While this method may not identify all possible acceptable arrangements of mathematics curricula, it will provide a reasonable estimate of the consistency between state standards documents.

The Missouri state mathematics standards document dedicates 76.92% of its GLEs to the content described by Focal Points 5.1, 5.2, and 5.3. Of the 9 GLEs that are not dedicated to one of these three focal points, 3 GLEs (7.69%) are dedicated to Connection 5.A, 4 GLEs (10.26%) are dedicated to other topics (in particular, they are dedicated to the additional big idea of probability identified in the document), and 2 GLEs (5.13%) are dedicated to process standards. Although there are no GLEs related to the content of Connections 5.B, 5.C, or 5.D, there is still significant correlation between the Missouri state mathematics standards and *Curriculum Focal Points*.

In the Florida mathematics standards, only 13.64% of the GLEs were related to content not contained in the focal points or connections. The remainder of the GLEs was divided roughly equally between the focal points and connections, with 40.91% dedicated to content identified in the focal points and 45.45% dedicated to content identified in the connections. More of the GLEs within the Florida state mathematics standards were dedicated to Connections 5.A and 5.C than were dedicated to either Focal Point 5.2 or 5.3. Yet despite these numbers, the Florida state mathematics standards still bear a strong relation to *Curriculum Focal Points*. The big ideas within the document match the focal points directly, and much of the wording used within the GLEs is borrowed directly from the text of *Curriculum Focal Points*.

Half of the GLEs within the Minnesota state mathematics standards are dedicated to the content identified in the focal points, and 26.41% of the GLEs are dedicated to the content identified in the connections. However, 23.59% of the GLEs are related to content not identified in either the focal points or connections, and very little of the GLEs are related to Connections 5.B, 5.C, or 5.D.

For all six states analyzed, more attention is given to the focal points than to the connections, which is in agreement with the recommendations provided in *Curriculum Focal Points*. Yet for three of the states — Washington, Kansas, an Utah — a significant number of GLEs are related to content not addressed within *Curriculum Focal Points*.

In the Washington state mathematics standards, approximately 40% of the GLEs are not related to either the focal points or the connections. However, the fourth big idea in the Washington state mathematics standards, triangles and quadrilaterals, accounts for 30.00% of the GLEs for Grade 5. That is, the majority of the GLEs not related to the focal points or connections is related to an additional big idea explicitly identified within the state standards document. Therefore, the Washington state mathematics standards bear a strong relationship to and reflect the intent of *Curriculum Focal Points*.

The content covered by the GLEs in the state mathematics standards for Utah and Kansas, on the other hand, bear little resemblance to the content identified in *Curriculum Focal Points*. Half of the GLEs within the Utah state mathematics standards, and almost 60% of the GLEs within the Kansas state mathematics standards, are related to content that does not appear in the focal points or connections for Grade 5.

That said, there is an important point that should not be missed regarding the Utah state mathematics standards. Approximately half of the content within the Utah state mathematics standards does not relate directly to *Curriculum Focal Points*, and the five big ideas within the Utah state standards do not correspond directly with the Grade 5 focal points. However, the big ideas within the Utah state standards are reasonable attempts at identifying the most important topics for Grade 5. Moreover, the first standard within the Utah document contains 27 GLEs, which seems to indicate a lack of focus, but the other big ideas contain 4 to 8 GLEs each, which seems reasonable.

As indicated by the number of GLEs related to the focal points and connections, it is evident that there is little consistency between state standards documents. This lack of consistency is visually evident in Figure 9 (see page 68). There is uneven coverage in the percent of GLEs dedicated to each focal point and connection. Moreover, the relationship between the state standards documents and *Curriculum Focal Points* is highly variable, with some states dedicating nearly four-fifths of their GLEs to the content identified in *Curriculum Focal Points*, while other states dedicate less than two-fifths of their GLEs to the same content.

*Changes from pre-CFP to post-CFP Standards*

A comparison of the pre-*CFP* and post-*CFP* versions of the state mathematics standards indicate that five of the six states attempted to incorporate changes based on the recommendations contained in *Curriculum Focal Points*. Obviously, there was no difference between the pre-*CFP* and post-*CFP* versions of the Kansas mathematics standards. Among the other states, however, four of the five reduced the number of big ideas; only Utah, which had the same number of big ideas in the pre-*CFP* and post-*CFP* versions, did not. Similarly, four of the five states reduced the number of GLEs within their state standards document; only Missouri, which realized an 11% increase, did not. In addition, several of the states used wording within their state standards documents similar to the wording within *Curriculum Focal Points*.

# Chapter 7: Conclusions and Implications

The framework for this thesis consisted of four research questions, and the preceding chapters attempted to answer those research questions as thoroughly as possible. But at the heart of it, there were essentially three questions that I hoped to answer by conducting an analysis of state standards documents:

- Do post-*CFP* state mathematics standards exhibit greater focus than pre-*CFP* standards?
- Do post-*CFP* mathematics standards exhibit consistency from state to state?
- And perhaps most importantly, does the development of national-level documents like *Curriculum Focal Points* have an impact on state-level policy?

The reason for these primary questions comes directly from *Curriculum Focal Points*. On pages 8-9 of the document, two main sections dedicated to "Inconsistency in the Placement of Topics by Grade Level in U.S. Mathematics Curricula" and "The Importance of Curricular Focus in Mathematics" serve as answers to the question "Why Identify Curriculum Focal Points?"

In general, it appears that *Curriculum Focal Points* had an impact on recent state standards revisions. Post-*CFP* state mathematics standards seem to exhibit somewhat more focus than their pre-*CFP* counterparts, though there is still a significant lack of consistency from state to state.

The data reviewed for this study provide an unequivocal response to the third research question. Clearly, the release of *Curriculum Focal Points* had an impact on state-level policy, as reflected in the changes to state mathematics standards. Five of the six states reduced the number of big ideas, reduced the number of GLEs, or borrowed wording directly from *Curriculum Focal Points*. In addition, the representatives with whom I spoke indicated that *Curriculum Focal Points* served as a primary reference for their writing teams. Further, former NCTM Executive Director Jim Rubillo and NCTM Past President Skip Fennell both indicated that NCTM worked with a number of state policymakers to help them understand the intent of *Curriculum Focal Points* for developing the next generation of mathematics standards.

Unfortunately, the degree to which *Curriculum Focal Points* influenced state mathematics standards varies greatly from state to state. *Curriculum Focal Points* presents a suggested "framework on which the next generation of state and district-level mathematics curricula *might* be built" (NCTM, 2006, p. 7, emphasis added), yet the authors freely admit that the framework is not meant to represent the only possible solution to curriculum design. Instead, the intent of the document is to "launch an ongoing, far-reaching, significant discussion with the potential to guide the thinking of the profession in the development of the next generation of curriculum standards," and that the focal points would be used to "guide discussions as [states] review, refine, and revise mathematics curricula" (NCTM, 2006, p. 2).

The results of this analysis seem to indicate that some states, such as Washington and Missouri, are engaging in the type of discussions that *Curriculum*

*Focal Points* recommends. The resulting state mathematics standards borrow a portion of their content from *Curriculum Focal Points* but also include other content deemed important by the standards writing committee. On the other hand, Florida appears to be taking the majority of their content directly from *Curriculum Focal Points* without these types of discussions. As the Florida representative indicated, writing team members were often not able to reach consensus when working on a compressed timeline, and the content of *Curriculum Focal Points* served as a compromise. As a result, the majority of the Florida mathematics standards contain much of the same content as *Curriculum Focal Points*.

The answer to the second research question is also evident. Despite exerting some influence on state-level policy, the release of *Curriculum Focal Points* seems not to have resulted in consistency of standards from state to state. The state standards documents exhibit an inconsistent relationship to *Curriculum Focal Points*. Some states limit the number of big ideas, yet others still include a laundry list of topics. The six state standards documents analyzed for this thesis contain anywhere from 3 to 14 big ideas in Grade 5 and anywhere from 23 to 98 GLEs. Moreover, the amount of content within state mathematics standards that overlaps with the content of *Curriculum Focal Points* ranges from just over 40% to almost 90%.

Two possible reasons for the lack of consistency are (1) insufficient time for policymakers and writing teams to thoroughly understand the intent of *Curriculum Focal Points* and (2) lack of collaboration between states, consortiums, and national organizations. The Florida Department of Education had a strong desire to be the first to adopt new standards based on *Curriculum Focal Points*, yet the Florida representative indicated that members of the writing team did not have adequate time to make sense of the information in the document, nor did they have common views regarding its intent. From personal experience, I suspect that members of the Florida writing team were not alone. During a presentation at the 1998 NCTM Annual Meeting, a conference participant asked, "Why are the [NCTM] *Standards* being revised? Most of us have just figured out what they mean!" The *Standards* documents were released in 1989, and this educator implied that it took almost 9 years to understand their intent. Though said tongue-in-cheek, the message was clear — significant time is needed to develop a comprehensive understanding of complicated documents. *Curriculum Focal Points* was released in September 2006, and the states analyzed for this study began work on revisions in 2006 or 2007. That may have been too soon for states to adequately reflect its intent when revising state mathematics standards.

To promote consensus among states in the development of consistent mathematics standards, Reys (2006) suggested that states work together under national leadership, with a core curriculum developed jointly through a partnership of national organizations. Despite that recommendation, *Curriculum Focal Points* were developed entirely by NCTM without the cooperation of other organizations. In defense of the Council, the lack of collaboration may have been motivated by the urgency of the topic. Releasing the document prior to the work of the National Mathematics Advisory Panel gave the Council a tremendous political advantage. Moreover, while many states were quick to adopt the recommendations of

*Curriculum Focal Points*, states that have used the document to guide recent revisions have done so without collaboration.

Data collected to address the first research question revealed inconsistent patterns across the six states whose standards were reviewed. The majority of state standards documents exhibited a reduction in the number of big ideas and GLEs. Based on this statistic alone, it may appear that greater focus has been achieved. However, there may be more to the story.

Table 7 below shows the ratio of GLEs to big ideas within state standards documents for Grade 5. With only three GLEs per big idea, the Minnesota state mathematics standards may not provide enough coverage for any of the big ideas to receive adequate attention. In this regard, however, Minnesota is a clearly an outlier. The other five states have ratios between 7.00 and 10.80, suggesting that each big idea in these states receives double or triple the level of attention as the big ideas in Minnesota.

Table 7

*Ratio of GLEs to Big Ideas within State Standards for Grade 5*

| State | Number of GLEs in Post-*CFP* Standards | Number of Big Ideas in Post-*CFP* Standards | Ratio of GLEs to Big Ideas |
|---|---|---|---|
| Florida | 23 | 3 | 7.67 |
| Kansas | 98 | 14 | 7.00 |
| Minnesota | 27 | 9 | 3.00 |
| Missouri | 39 | 4 | 9.75 |
| Utah | 54 | 5 | 10.80 |
| Washington | 40 | 4 | 10.00 |

When Minnesota is removed from this list, a line of best fit for the other data has a correlation coefficient of $r = 0.97$ with equation $y = 6.13x + 14.00$, where $y$ is the number of GLEs and $x$ is the number of big ideas. Though this equation is not terribly useful for the purpose of developing state standards, it is interesting to note that there is such a strong correlation, and the most important part of this equation may be the $y$-intercept. With a value of 14, the $y$-intercept may suggest that many state standards documents contain a large number of GLEs not related to the big ideas; at a minimum, it indicates that the amount of attention per big idea varies considerably from state to state.

I believe this ratio is important to consider when discussing curricular focus, because the two components of focus within state standards are a limited number of big ideas and GLEs and an adequate amount of attention given to each big idea. While the states analyzed for this thesis certainly seem to have made progress toward achieving the first component, it is difficult to say if improvement has been attained regarding the second.

Another reason that greater focus may not have been realized is that the Grade 5 mathematics standards of at least two states, Minnesota and Kansas, contain a large number of big ideas with no indication of which are most important. Each big idea contains roughly the same number of GLEs, and it is therefore indeterminable which of the big ideas deserves greatest emphasis. On the other hand, the Utah state mathematics standards contain a limited number of big ideas, yet half of the GLEs within the document are associated with just one big idea. Ostensibly, this big idea seems to warrant more attention than the others, yet the document draws no distinction between them.

*Implications for National Organizations, States and Districts*

Many research studies attempt to develop international benchmarks by comparing each state's standards to those of the highest performing countries (Schmidt, 2006). Policymakers and curriculum developers in the U.S. have to analyze and deal with 50 state standards, whereas other countries have just one set of standards. This means that considerable research time is devoted to running an analysis multiple times to get a complete picture of mathematics education in the United States.

Consistency between state standards is necessary to eliminate this inefficient use of resources and to ensure equitable opportunities for all students. National organizations like NCTM, with political power and the support of 100,000 members, can influence the use of common state standards.

Both Jim Rubillo and Francis (Skip) Fennell indicated that they had met with key policymakers, Congressional aides, testified before Congress, and met with representatives in several states after the release of *Curriculum Focal Points*. The purpose of these meetings was to assist states in understanding the intent of the document and to discuss how the document could provide assistance as they revised their standards. I believe that these meetings, while helpful, may indicate a missed opportunity to improve consistency from state to state.

Rather than meetings with individual states, I believe that NCTM could have capitalized on the opportunity by working in collaboration with a consortium of other national organizations as well as representatives from all 50 states. Admittedly, special materials containing information about *Curriculum Focal Points* were created and distributed through the Association of State Supervisors of Mathematics, and a number of presentations were made for the National Council of Supervisors of Mathematics. However, a consistent and unified message regarding the potential impact of *Curriculum Focal Points* was not presented by a variety of organizations. Instead, NCTM delivered the message in the hopes that other organizations would follow their lead. Recent comments by U.S. Secretary of Education Arne Duncan suggest that the time is right, politically, for a move to national standards (USDOE, 2009), and the recent success of the National Governors Association, the Council of Chief State School Officers, and Achieve, Inc., to collaborate on the development of common standards suggest that the Council may have had much to gain by forming an alliance with other groups.

The representative from Washington explained that a local group exerted significant influence on the writing process. Because of this local group's insistence, policymakers continually required additional reviews of the writing team's work. I can't help but wonder if the Washington writing team would have had less difficulty justifying their decisions had *Curriculum Focal Points* been developed jointly by several national organizations rather than being the work of just one group.

While one goal of *Curriculum Focal Points* was to influence consistency between state standards, that was not necessarily the goal of the states. The states' purpose was to create sound mathematics standards for their students; matching their standards to those of other states was likely irrelevant to them. Consequently, while national organizations such as NCTM continue to declare that national consistency is a goal, without a concerted effort to make that happen by being directly involved with the states, or without a policy in place to enforce state-to-state consistency, I do not believe that it will become a reality.

For national documents such as *Curriculum Focal Points* to promote consistency from state-to-state, professional development should be made available to curriculum developers from all states. NCTM developed “Questions and Answers” regarding the *Curriculum Focal Points,* and a series of books offering further explanations of *Curriculum Focal Points* are in production, yet the delay between the release of the *Curriculum Focal Points* and these supplementary materials may be diminishing their effectiveness.  NCTM could have offered personalized training to state departments of education to ensure that the intent, not just the word, was understood by all who are responsible for developing standards and curriculum. By offering professional development sessions in central locations, curriculum developers from various states could have met to discuss the document, thus increasing the likelihood of collaboration between states and thereby encouraging consistency.

Similarly, when new standards are developed based on *Curriculum Focal Points*, professional development should also be provided to teachers so that they, too, understand the intent of any changes. The need for this is highlighted by the following anecdote from Dixon and Kersaint (2007):

> A member of the Florida Department of Education shared a reaction by a teacher during an open forum regarding the new Florida standards. The teacher looked at the short list of curricular topics in a grade and said, ‘I can teach this in 20 days; what do I do the rest of the year?’ Although this comment may cause a jarring reaction, when we consider the list of topics from the perspective of a teacher who has taught a new topic every two days in the past, this teacher’s misperception is not far-fetched. (Florida has had as many as 93 grade-level expectations to be taught in a given year).” (Dixon & Kersaint, 2008, p. 24)

*Implications for Further Research*

It may be that the research conducted for this thesis occurred too soon. Only 22 of 50 states have revised their state standards since 2006, and the 2007 Reys study indicates that only a subset of them used *Curriculum Focal Points* as a primary resource. In personal communication with Barb Reys (January 2, 2009), she indicated that she hoped to do research similar to the analysis in this thesis. However, she said that she would “wait for more time to pass” so that states would have an opportunity to incorporate the recommendations of *Curriculum Focal Points* into their standards.

The limited sample used for this thesis identified a level of inconsistency between state standards that may have only appeared because of the sample size or because of the specific states that were considered. If other states’ standards were to be analyzed, the level of inconsistency may be less than noted here. In addition, the research for this thesis was based on state standards developed soon after the release of *Curriculum Focal Points*; state standards developed more recently may better reflect the suggestions in the document, because more time has allowed for a greater understanding of its intent.

### *Possible Modifications to the Methodology*

The primary motivation for conducting the investigation described in this thesis was to answer a fundamental question regarding the impact of *Curriculum Focal Points* and of national policy documents in general: Did the *Curriculum Focal Points* alleviate any of the inconsistency in the placement of topics within state curricula? Although the process used to analyze state mathematics standards for this thesis provided a clear answer to that question, perhaps stronger results could have been obtained with some modifications to the methodology.

All of the data for this study was collected from Grade 5 mathematics standards for six states that claimed to use *Curriculum Focal Points* as a primary resource. At a very basic level, the results may have been improved by looking at the standards from additional states. The six states selected for this analysis were primarily selected because of the responses of state mathematics supervisors in the Reys et al. (2005) study. At the initiation of this research, only a handful of states had approved standards that were revised after the release of *Curriculum Focal Points*. However, the number of states who have recently approved revised standards has increased significantly. Since the release of *Curriculum Focal Points* in

September 2006, 21 states have revised their standards. It is likely that more than six of these states used *Curriculum Focal Points* as a primary resource, and additional information could be obtained by analyzing the standards of these states.

It is also possible that some of these additional states did not use *Curriculum Focal Points* as a primary resource. At least one state mathematics supervisors who responded to the Reys et al. (2005) survey indicated that *Curriculum Focal Points* might not be used as a primary resource during the revision process. The supervisor stated that their "revisions were based on alignment with NAEP. Due to the NAEP alignment, there are several places we were not permitted to follow CFP" (Reys, 2007, p. 3). Consequently, an additional modification to the methodology that may have yielded useful information would be to compare those states that used *Curriculum Focal Points* with those that did not and identify any differences.

The Reys (2007) analysis of state standards considered the GLEs for Grade 4. The analysis of state standards in this thesis considered GLEs for Grade 5. In both cases, useful information was gathered even though standards from only one grade level were explored. That said, additional insight may have been gleaned had multiple grade levels been examined. In particular, by looking at the standards for Grades 4 and 6 of the six selected states, it might have been possible to identify the inclusion of topics identified as focal points or connections that were placed one grade level above or below the recommended placement in *Curriculum Focal Points*.

To investigate the inconsistent placement of GLEs within state standards documents, it may not have been necessary to use *Curriculum Focal Points* as a framework. That is to say, a list of all GLEs from all analyzed states could have been compiled, and an analysis of the intersection between states could have been conducted. Such a process would have identified areas of inconsistency as well as the method employed in this analysis. However, it would have also nullified the ability to determine the impact of *Curriculum Focal Points* on state standards.

Though likely a separate thesis question entirely, a quantitative analysis to determine the specific level of usage of *Curriculum Focal Points* in the revision process could have been conducted. Such an analysis would have been extremely time-intensive, however, and given the issues that often accompany the state standards writing processes, it is difficult to estimate how effective such a study would have been. It is certainly possible that identifying individuals to provide adequate and accurate information would have been a daunting task.

Finally, the creation of state standards is a process fraught with political issues. Many of the pre-*CFP* standards were developed using *Principles and Standards* as a primary resource, and in many, if not most, cases, the post-CFP standards are revisions to the pre-CFP standards. Consequently, the post-*CFP* standards may show significant alignment to the *Principles and Standards*. It would therefore be interesting to conduct the same analysis using *Principles and Standards* as the comparison document. As noted by one of the supervisors who responded to the Reys (2005) survey, "…it is important for NCTM to produce a standards document that is in the spirit of the focal points. …the focal points will [not] have the impact that they are intended to have without revision to [*Principles and Standards*]" (Reys, 2007, p. 3).

Despite numerous suggestions for how the methodology could have been modified to provide additional information or different results, the process used in this thesis allowed for an adequate analysis of the research questions under investigation.

*In Conclusion*

I believe that each of the states analyzed in this study made a reasonable attempt to embody the intent of *Curriculum Focal Points*. At the same time, I am disheartened that states who used *Curriculum Focal Points* as a guiding document produced state mathematics standards that are somewhat different from one another. State mathematics standards are less unfocused than they used to be, but I'm not certain that they have reached the level of curricular focus for which the authors of *Curriculum Focal Points* were striving. Moreover, the state standards appear to be as inconsistent today as they were prior to the release of *Curriculum Focal Points*.

Despite the efforts of NCTM to provide guidance, it seems that we still have not attained the vision of "a focused, coherent mathematics curriculum with a national scope" sought by *Curriculum Focal Points*.

# Appendix

Curriculum Focal Points for Grade 5

**Number and Operations and Algebra:** Developing an understanding of and fluency with division of whole numbers. Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.

**Number and Operations:** Developing an understanding of and fluency with addition and subtraction of fractions and decimals. Students apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They apply their understandings of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standard procedures for adding and subtracting fractions and decimals. They make reasonable estimates of fraction and decimal sums and differences. Students add and subtract fractions and decimals to solve problems, including problems involving measurement.

**Geometry and Measurement and Algebra:** Describing three-dimensional shapes and analyzing their properties, including volume and surface area. Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.

Connections to the Focal Points for Grade 5

**Algebra:** Students use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities. They create graphs of simple equations. They explore prime and composite numbers and discover concepts related to the addition and subtraction of fractions as they use factors and multiples, including applications of common factors and common multiples. They develop an understanding of the order of operations and use it for all operations.

**Measurement:** Students' experiences connect their work with solids and volume to their earlier work with capacity and weight or mass. They solve problems that require attention to both approximation and precision of measurement.

**Data Analysis:** Students apply their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on coordinate grids.

**Number and Operations:** Building on their work in grade 4, students extend their understanding of place value to numbers through millions and millionths in various contexts. They apply what they know about multiplication of whole numbers to larger numbers. Students also explore contexts that they can describe with negative numbers (e.g., situations of owing money or measuring elevations above and below sea level.)

# Glossary

**Big Idea –** a concept central to the learning of mathematics, one that coherently connects numerous mathematical understandings (Charles, 2005). For the purpose of this analysis, the term *big idea* will be used to refer to a major topic contained within state standards documents. Compare with *focal point*.

**Connection (to a Focal Point) –** an introductory or continuing experience related to a focal point (NCTM, 2006). For the purpose of this analysis, the term *connection* will be used to refer to a secondary topic contained within the *Curriculum Focal Points* document, whereas the term *supporting idea* will be used to refer to a secondary topic contained within state standards documents.

**Curriculum Focal Points –** *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (aka, "*Curriculum Focal Points,*" "*Curriculum Focal Points*"); a document released by NCTM in September 2006 that identifies important mathematical topics for each grade level, K-8 (NCTM, 2006). According to information on the NCTM web site, the focal points are "the most important mathematical topics for each grade level. They comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and lasting learning" (NCTM, http://www.nctm.org/standards/content.aspx?id=270).

**Focal Point –** one of the three major topics at each grade band in the *Curriculum Focal Points* document. For the purpose of this analysis, the term *focal point* will be used to refer to a major topic contained within the *Curriculum Focal Points* document, whereas the term *big idea* will be used to refer to a major topic contained within state standards documents.

**Grade Level Expection (GLE) –** a statement that defines the content that all students should know and be able to do by the end of a grade level; the specific learning objectives included in state mathematics standards.

**NCTM –** National Council of Teachers of Mathematics; the trade association for mathematics teachers in the United States, and the world's largest organization dedicated to mathematics education, with almost 100,000 members. According to their mission statement, the Council is "a public voice of mathematics education, providing vision, leadership and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students" (NCTM.org, http://nctm.org/about).

**Pre-*CFP* –** before the release of *Curriculum Focal Points*. Specifically, this adjective will be used to describe state mathematics standards that were in use prior to the release of *Curriculum Focal Points*. For example, the *Grade Level Expectations for the Florida State Sunshine Standards*, which was released in 1996, would be referred to as the pre-CFP version of the Florida mathematics standards.

**Principles and Standards –** *Principles and Standards for School Mathematics* (aka, "*Principles and Standards*"); a document released by NCTM in April 2000 that identifies the mathematical knowledge and skills that students are expected to acquire during their education from pre-kindergarten through grade 12. This document serves as the primary model for standards-based mathematics and is a revision of the 1989

*Curriculum and Evaluation Standards for School Mathematics*, also published by NCTM.

**Post-*CFP* –** after the release of *Curriculum Focal Points*. Specifically, this adjective will be used to describe the state mathematics standards that were developed with *Curriculum Focal Points* as a primary resource. For instance, the *Sunshine State Standards: Mathematics*, which was released in 2007, would be referred to as the post-CFP version of the Florida mathematics standards.

**Supporting Idea –** an important concept that contributes to the understanding of a big idea (Charles, 2005). For the purpose of this analysis, the term *supporting idea* will be used to refer to a secondary topic contained within state standards documents. Compare with *Connection (to a Focal Point)*.

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