ABSTRACT

Title of Document: FRIENDS AND PARTNERS: THE IMPACT OF NETWORK TIES AND COLLABORATION NETWORKS

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How does a high-tech entrepreneur find the most qualified engineer for her startup? How does a scientific inventor acquire funding or recruit the best partner for his project? In chapter 1 I develop a discrete matching model with heterogeneous values and an undirected social network to address these questions. My model offers a framework to study how relative network positions affect payoffs and incentives. While an entrepreneur's expected return increases with the size of her own network, the network externalities from competing entrepreneurs are more complex. There is a tradeoff between the size of an entrepreneur's network and the competitive externality she exerts. When an entrepreneur's network increases, her closest competitors are hurt, but her less similar competitors may actually have a better chance of finding a suitable partner. In a more connected network, fewer frictions interfere with compatible matches. Results are consistent with observable patterns in high-tech and biotechnology in Silicon Valley and Massachusetts, as well as the turn

of the 20th century German synthetic dye manufacturing. Initiatives to promote social networks within innovative sectors are critical and deserve future research.

In Chapter 2 I consider a two-period endogenous network search model in which entrepreneurs build relationships with specialists. The model includes a period of costly network search and applies results from my companion paper. In the presence of network externalities, entrepreneurs over-invest in networking. Networks in which is it not costly to build new relationships are the least efficient. While positive externalities reduce this problem some negative inefficiencies will likely prevail. Networks in which participation is cheap – such as online career networks LinkedIn or Monster.com – have limited information about individual specialists and are the most inefficient. A network that is costly to participate in, but is more effective at targeting entrepreneur's search for qualified candidates results in a more compatible and, likely, efficient partnership. These networks might include alumni groups, trade associations or head-hunters. This chapter provides one explanation for the varied successes of government programs in fostering effective business networks. Efficient networks foster fewer, more specific relationships.

FRIENDS AND PARTNERS:
THE IMPACT OF NETWORK TIES AND COLLABORATION NETWORKS

By

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1 Introduction

How does an entrepreneur's social network affect her profitability? How does her competitors network affect her chance of survival? Across scientific, technical and artistic fields, entrepreneurs seek out former colleagues, classmates and collaborators to gain access to necessary resources, including investment capital, technology and employees. (Gompers, Lerner and Scharfstein, 2005) Empirical evidence suggests that an entrepreneur's location within a social network is critical to her success and survival. Existing research, however, does not explicitly demonstrate the mechanism by which networks impact success. A central question is how does the individual network position and the interaction between agents in the network influence an entrepreneur's likelihood of success. (Goyal, 2005; Podolny, 2001) This paper explores how network structure and individual network connectedness affects the incentives and payoffs for agents. In particular, I focus on how competing entrepreneurs' social networks affect other entrepreneurs' outcomes and expectations.

I present a model in which each entrepreneur has a distinct innovative idea and must use her network connections to find a specialist partner. Once they match, an entrepreneur-specialist pair can together develop technology for startup. I show that an entrepreneur's ability to find a compatible match varies with both the size of her own network, as well as her competing entrepreneurs' networks. While her expected return increases with her own network, the externality from a competitor's network is more complex. There is a tradeoff between the size of an entrepreneur's network and the competition she exerts in the market. An entrepreneur's expected return decreases when her nearby competing entrepreneur's network increases. This effect reverses for less similar competitors. When the network of a more distant competitor increases, the original entrepreneur often has a better chance of finding a compatible match.

Think of each entrepreneur as a generalist with an idea for a new project and a set of preexisting relationships with industry specialists. The payoff to her project depends on partnering with a specialist capable of implementing her idea. I use a discrete matching model with heterogeneous values. A partnership between an entrepreneur and specialist constitutes a stable match. Each entrepreneur can only partner with a specialist with whom she has a preexisting relationship, whose type is thus known to her.[1] Stable matches follow pairwise stability and individual rationality so that every agent partners with its closest, most compatible known, available agent.

The equilibrium solution concept is a stable matching for each ex ante possible network structure. Ex ante, I characterize each entrepreneur's social network by the probability she has a connection with each specialist. The probability of any network graph implies a probability of a match between each unique entrepreneur-specialist pair. Three forces determine the distribution of match probabilities: (i) the probability the entrepreneur knows the specialist, (ii) the probability the specialist is the

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[1] According to Fleming and Frenken (2007), engineers and other inventors are often reluctant to ask for help or reveal their research. They rely mostly on trusted friends and select colleagues. Business disclosures to strangers are perilous and may result in stolen ideas. It is also difficult to transmit useful information about skills in nascent industries where these is often poor public information, immature markets, limited human capital and few standards. (Sorenson and Stuart, 2001)

entrepreneur’s best option and (iii) the probability the specialist does not partner with a competitor.

The model offers a general theory of how a change in network degree (i.e., the number of connections an agent has) impacts industry welfare and the individual payoffs. An entrepreneur’s expected profits are always higher with more connections. When an entrepreneur knows more people, she is more likely to obtain a compatible match and less likely to rely on a distant specialist. As an entrepreneur’s network increases, her nearby competitors are worse off, but her more distant competitors may actually receive a more compatible match. The network interaction between competing entrepreneurs depends on both agents’ network and their relative locations.

I compare the individual effects and social benefits across the most common network structures. In a complete network, the match is unique, assortative and socially optimal. In a random, one-degree network (i.e., where each agent has a single connection) expected returns are low. These two networks are synonymous with conventional labor markets of complete and incomplete information, respectively. Finally, I analyze a uniform, homogeneous network in which each agent has equal degree. A uniform network of very low degree has lower welfare than a random match, but as a network becomes more connected it approaches the socially optimal case. Each entrepreneur experiences relatively less competition for her most compatible specialists.

This approach to modeling network probabilities is in contrast to traditional discrete network graph analysis. It is difficult to develop results applicable to a broad class of networks by analyzing specific, discrete graphs. Galeotti, Goyal, Jackson, Vega-Redondo and Yariv (2008) use an approach similar to mine to model an agents' behavioral response to network externalities for a broad class of models. With this approach I develop general rules of how network structure relates to payoffs and incentives.

I integrate a discrete heterogeneous matching framework in the spirit of Becker (1973) with elements of the labor search literature and network theory. Recently authors apply this matching framework to a number of questions. Casella and Rauch (2002) model international trade through immigration networks. Hoppe, Moldevanu and Sela (2005) model two-sided matching with signaling. In the employer search literature, Lee and Schwarz (2007) model heterogeneous employee-employer matching with search.

The setup and results of this model are consistent with observed industry patterns, particularly in innovative sectors where the underlying technology is based on esoteric knowledge held by a small number of specialists and the risk of failure is high. Startups often lack resources, including customers, raw materials, technological expertise and employees. A well-connected entrepreneur uses her network of personal and professional relationships with former colleagues, fellow alumni and social contacts to access the resources she needs. (Castilla, Hwang, Granovetter and Granovetter, 2000; Gompers et al, 2005; Sorenson and Stuart, 2001)[2]

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[2] Castilla et al (2000) describe a densely interconnected, but diffuse, social network of engineers, entrepreneurs, financiers, professors and other business people in Silicon Valley, who are more loyal to the profession than a specific firm. This culture of mobility and innovation began when the first generation of entrepreneurs who left Fairchild Semiconductors to start their own firms. As more entrepreneurs left

Consider the history of one leading chemical and pharmaceutical company of the 20th century, Bayer Industries. In 1863 industrialist Freidrich Bayer partnered with colleague and dye chemist Friedrich Weskott to found Bayer Industries. The founders combined their respective business and scientific expertise to develop and market new synthetic dyes. It became one of the leading dye manufacturers. As Murmann (2003) describes, most of the successful early synthetic dye German entrepreneurs used an “informal network of ties that connected players in industry and academia” to partner with former colleagues with complementary expertise.

In this paper I focus on a static, exogenous network of existing ties of complete information. I assume it is prohibitively costly to endogenously create ties that contain such detail. In a companion paper I consider an endogenous network search model in which agents choose to build relationships with additional agents. Strong ties with extensive information are the critical foundation of efficient collaboration networks. For example, Van der Leij and Goyal (2006) use a coauthorship network of publishing economists to show that the random removal of strong ties increases the average network path length more than the random removal of weak ties.[3] Before exploring the impact of endogenous network search in this context, we must understand the impact of strong ties with complete information.

Fairchild and its spinoffs, this first generation of entrepreneurs lead to generations of startups. Years later the IBM post-doctoral program began employing new scientists for two years with the explicit purpose of creating ties and then sending them to other firms after two years. This created a diffuse network of scientists who were loyal to IBM. (Fleming and Frenken, 2007)

[3] Building even loose relationships with targeted individuals requires existing strong relationships to access the necessary pathways. Agents connect with *close* friends of *close* friends to build new ties through the pathways of their first-degree network links. Agents use their strong ties to make new connections and create partnerships with detailed information.

The paper proceeds as follows. Section 2 presents an overview of the model. Section 3 discusses the general definition of the equilibria, while section 4 describes the ex ante solution concept and its implications. Section 5 looks at the individual comparative static results. Section 6 discusses outcomes for three benchmark cases. Section 7 presents specific applications. Finally, Section 8 concludes by considering three canonical applications and the policy implications of the above results.

2 Model

The economy consists of two finite sets of *m* risk neutral agents: a set of generalist entrepreneurs $E = \{e_1, e_2, ..., e_m\}$ and a set of specialist $S = \{s_1, s_2, ..., s_m\}$. Agents in both sets are defined by specialized skills, positioned around a circle with circumference normalized to one.[4] The i^{th} entrepreneur is exogenously endowed with an idea for a project with specific skill requirements e_i, such that $e_i \in [0,1]$ for $i = 1, 2, \ldots, m$. The j^{th} specialist is exogenously endowed with skills s_j, such that $s_j \in [0,1]$ for $j = 1, 2, \ldots, m$. Each set of agents is ordered, equally spaced and equally located, $e_1 < e_2 < ... < e_m$ and $s_1 < s_2 < ... < s_m$. The incremental distance between neighboring agents is $\frac{1}{m}$. Skills represent areas of specialization within a specific field.[5] See Figure 1.

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[4] The model extends easily to a line in which agents are equally spaced between 0 and 1. Results for this case are parallel and similar to the circle.

[5] These results continue to hold if the two sets are skewed (i.e., not equally located). In fact, when two sets of agents are not equally located, an entrepreneur has a unique distance to each specialist. The only problematic case is when the sets are located

An entrepreneur must partner with a specialist to startup. Otherwise she does not implement her idea and pursues her outside option. Consistent with Lazear's (2005) interpretation, entrepreneurs are generalists who understand how to develop a good idea, recruit skilled employees, and oversee product development. Entrepreneurs, however, lack the specific human capital to build the underlying technology for their project and must partner with a specialist who can. Specialists, on the other hand, have highly developed technical expertise, but lack the general business skills to be an entrepreneur.

Agents are connected by a preexisting, exogenous network of ties between entrepreneurs and specialists. Network ties simply represent a preexisting social or professional relationship between two agents. I denote the entire network as a bipartite graph, $G_{m\times m}$, where g_{ij}=1 if agents e_i and s_j are connected; g_{ij}=0 otherwise. These ties are undirected so that if an entrepreneur knows a specialist, that specialist also knows the entrepreneur. An entrepreneur must have a network link to a specialist in order to partner with him. Agents have no way of revealing their types to a stranger.[6]

The vector $g_i = (g_{i1}, g_{i2}, ..., g_{im})$ represents entrepreneur e_i's set of connections to specialists. Each entrepreneur's set of network connections, or *network degree*, is observable and equal to $k_i = \sum_{j=1}^{m} g_{ij}$. Ex ante, the skills of the specific specialists with whom e_i is linked are not identifiable. Instead, the probability that e_i has a

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such that each specialist is located equidistant between two entrepreneurs. In this case, the complete network will yield more than one unique optimal matching.

[6] This is consistent with real world applications in which it is either prohibitively costly to verify skills or revealing types leave agents susceptible to theft of ideas. See footnote 2.

connection to any s_j is $\sigma_i(s_j) \in [0,1]$. Unless otherwise specified, assume entrepreneurs have equal probability of knowing any specialist around the circle so that $\sigma_i(s) = \frac{k_i}{m}$ for all s.

This paper characterizes the network by probability of a connection for a few reasons. An entrepreneur may know the number of connections she has, but must decide if to pursue a project before she knows her friends' exact types, or with whom her contacts are also connected to. Similarly, researchers often observe an agent's network degree, but not the specifics of each link. Using these probable expectations, the researcher can better understand the observed, real-world empirical patterns. Finally, it provides insight into the value of network connections, rather than a specific graph, more generally.

The pair-specific, heterogeneous return to a match between e_i and s_j is $\pi_{ij} = \pi(e_i, s_j) = \pi(d_{ij})$. Here d_{ij} is the minimum distance along the circle's circumference between agents e_i and s_j, and represents the dissimilarity or incompatibility of a pair of agents. The return π_{ij} of any pair-specific match is a decreasing one to one mapping from compatibility to payoff such that $\pi : \mathbb{R} \rightarrow \mathbb{R}$. For any entrepreneur e_i, the closest match is her *perfectly compatible* specialist at distance is $d_{ij} = 0$.

The entrepreneur and specialist split the return according to the Nash bargaining solution such that $\pi_{ij} = u_{ij} + v_{ij}$, entrepreneur e_i receives u_i and specialist s_j receives v_j. Each agent has respective outside option $\underline{u}$ and $\underline{v}$ where $u_i \geq \underline{u}$ and $v_j > \underline{v}$. The outside option for any partnership must be $\underline{\pi} = \underline{u} + \underline{v}$. Any partnership such that $\pi_{ij} < \underline{\pi}$ is not economically viable. I make one other restriction on the return.

Condition 1: For any e_i, e_k, s_j, s_l such that $d_{ij} < d_{kj} \leq d_{il}$ and $g_{ij} = g_{kj} = g_{il} = 1$ (i.e., e_i and s_j are closest to each other, but also connected to s_l and e_k, respectively) then $\pi_{ij} + \underline{\pi} \geq \pi_{il} + \pi_{kj}$.

Condition 1 implies a complementarity such that the expected marginal surplus of a match is increasing in compatibility. There is always a way to split the surplus from a closer match to benefit both agents. Condition 1 is sufficient, but not necessary, to ensure profit and compatibility maximizing matches in equilibrium.[7]

Payoff functions that fit the restrictions of condition 1 also capture the observed patterns for high-skill entrepreneurial partnerships. William J. Baumol (2004) argues that more innovative technologies tend to have higher risks of failure since the technologies are not fully understood. According to Sorenson and Stuart (2001) small, private entrepreneurial ventures are inherently risky, plagued by liabilities of newness and unproven business models. The reward to a good partnership is also great, but the risks to a poor partnership are also very high. Additionally, condition 1 is more likely to hold for relatively large outside options, or high values of $\underline{\pi}$. Both entrepreneurs and specialists in innovative, high skill industries have relatively high

[7] Heterogeneous matching models often rely on complementarity conditions. See Becker (1973) and Shimer and Smith (2000).

outside options, often working for large incumbent firms or scientific labs. Consider two possible functional forms:

Example 1: Suppose $\pi(d_{ij}) = R\left(1 - d_{ij}\right)^{\rho}$ where *R* represents optimal revenue, and ρ represents the importance of compatibility. For any $\rho > 1$, expected return is decreasing, convex in distance. The values of $\underline{\pi}$ and ρ that satisfy condition 1 vary with *m* and *R*.

Example 2: Suppose $\pi(d_{ij}) = (1 - r(d_{ij}))R$ where *R* represents optimal revenue and $r_{ij} = r(d_{ij})$ represents the pair's probability of failing. The risk of failure is increasing and convex in distance, $r'(d_{ij}) > 0$ and $r''(d_{ij}) < 0$, minimized for a perfectly compatible pair $r(0) = r_0$ and greatest for the most distant matches $r(\frac{1}{2}) \leq 1$. For instance, let $r(d_{ij};\phi) = d^{\phi}$ where $\phi > 1$ represents the project's failure rate.

3 Equilibrium

An entrepreneur partners with a specialist to maximize expected payoff. Following classic heterogeneous matching problems, equilibrium matching *M* is reduced form, one-to-one, individually rational and pairwise stable. Each pair consists of one specialist and one entrepreneur. Otherwise, an entrepreneur or specialist remains unmatched. Imposing condition 1 and pairwise stability ensures that agents have strict preferences for more compatible matches, regardless of the division of surplus (Lemma 1).[8]

Definition (pairwise stability). *Consider some matching M with individual matched pairs ij and i'j' such that* $M(e_i) = s_j$ *yields surplus* $\pi(e_i, s_j)$ *and* $M(e_{i'}) = s_{j'}$ *yields surplus* $\pi(e_{i'}, s_{j'})$. *Then M is pairwise stable if there is no pair of unmatched agents ij' or i'j whose surplus from matching,* $\pi(e_{i'}, s_j)$ *or* $\pi(e_i, s_{j'})$, *is greater than under M.*

Lemma 1. *Under condition 1, a closer match is always surplus maximizing for both the entrepreneur and specialist.*

Equilibrium matches follow two basic rationality (IR) and stability (PWS) conditions:

$$\text{(c1):} \quad \pi(e_i, s_j) \geq \underline{\pi} \qquad \text{(IR)}$$

$$\text{(c2):} \quad \begin{aligned} \pi(e_i, s_j) &\geq \max\nolimits_{s \in F_{e_i}} \pi(e_i, s) \\ \pi(e_i, s_j) &\geq \max\nolimits_{e \in F_{s_j}} \pi(e, s_j) \end{aligned} \qquad \text{(PWS)}$$

where F_x represents the set of available matches for agent x.

[8] Suppose matching proceeds as follows: The entrepreneur learns which specialists she has a connection to. She implicitly ranks her available specialists according to the pair-specific expected value or compatibility, and proceeds to make offers. She makes an offer to her most compatible match. If the specialist accepts, they partner. If the specialist rejects, she proceeds to her next best, and so on until she either gets a match or has no more available candidates. The specialist accepts his most compatible offer and rejects all others. This process continues until a stable matching exists for all agents.

Two agents e_i and s_j are *available* to partner if they have a preexisting relations and neither agent partners with a more compatible agent in her or his own respective subset of higher valued matches. When an agent has two available, equidistant candidates he or she has lexicographic preferences. The entrepreneur prefers the specialist who is clockwise to herself. A specialist accepts the entrepreneur who is counter clockwise to himself.[9]

Definition (availability). *The specialist s_j is available to entrepreneur e_i if (1) s_j and e_i are connected, $g_{ij} = 1$, and (2) s_j does not have a higher valued available match with some entrepreneur e_k. The entrepreneur e_i is available to s_j if (1) s_j and e_i are connected, $g_{ij} = 1$, and (2) e_i does not have a higher valued, available match with some specialist s_m.*

4 Ex Ante Solution Concept

For any discreet network graph $\boldsymbol{G}$ there is an explicit, pairwise stable match *M*. For each entrepreneur this stable match is a function of her set of available specialists. Prior to the realization of a specific graph, the ex ante probability of any match *M* is driven by the probability of any network graph and the associated set of connections. The ex ante equilibrium concept is thus a set of probabilities *P* over all possible individually rational and pairwise stable matches *M* as determined by the expected probability of graphs $\boldsymbol{G}$. Denote the ex ante probability of a match between

[9] This is a simplifying assumption. Results hold if entrepreneurs and specialists choose between equidistant agents with probabilities $\alpha \in [0,1]$ and $\gamma \in [0,1]$.

entrepreneur e_i and specialist s_j as $p_{ij} = p(e_i, s_j; \sigma)$ where $p_{ij} \in P$ and $\sigma_i \in \sigma$ for all i and j. With two sets of m agents, there are m^2 match probabilities.

Ex ante, the reduced form probability that $\{ij\}$ is a pairwise stable match, p_{ij}, has three components. For an entrepreneur and specialist to partner (i) the entrepreneur must know the specialist, (ii) the specialist must be the entrepreneur's closest available candidate, and, simultaneously, (iii) the entrepreneur must be most compatible for the specialist:

$$p_{ij} = \underbrace{prob\left\{e_i \text{ connected to } s_j\right\}}_{(i)} * \underbrace{prob\left\{s_j \text{ is } e_i\text{'s best available match}\right\}}_{(ii)} * \underbrace{prob\left\{s_j \text{ is available to } e_i\right\}}_{(iii)}$$

Terms (i) and (ii) represent the effects of an agent's own network and are driven by entrepreneur e_i's network connectedness, σ_i. Term (iii) reflects the probability that a competing entrepreneur wins the specialist s_j. It captures the interaction between entrepreneurs.

The probability over a match with any specialist is a function of her own network connectedness, σ_i, as well as the connectedness of nearby competitors, σ_k. The most compatible match that is simultaneously available for both agents is stable. For any pair $\{ij\}$ such that $\pi_{ij} \geq \underline{\pi}$, the probability of a pairwise stable match is:

$$p(e_i, s_j; \sigma) \equiv \sigma_i \cdot \left[1 - \sum\nolimits_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)\right] \cdot \left[1 - \sum\nolimits_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right] \quad (1.1)$$

Where subset $N(e_i, d_{ij})$ represents all specialists who are more compatible with, and thus preferred by, entrepreneur e_i than specialist s_j. This includes all specialists within distance d_{ij} of e_i and excludes s_j. Similarly, subset $N(s_j, d_{ij})$ contains all entrepreneurs who are more compatible with s_j than e_i. These entrepreneurs are located on the interval of distance d_{ij} from s_j, excluding e_i,. Each match probability $p(e_i, s_j)$ is decreasing and recursive in the probabilities over more compatible matches for e_i and s_j.[10] The probability $p(e_i, s_j)$ is a decreasing function of all match probabilities for more compatible matches for both e_i and s_j..

Example *m* = 4. *Consider the uniform network case when m = 4. Entrepreneur e_1's match probabilities are:*

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot (1 - p_{11}) \cdot (1 - p_{22}) \\
p_{13} &= \sigma_1 \cdot (1 - p_{11} - p_{12} - p_{14}) \cdot (1 - p_{23} - p_{33} - p_{43}) \\
p_{14} &= \sigma_1 \cdot (1 - p_{11} - p_{12}) \cdot (1 - p_{34} - p_{44})
\end{aligned}
$$

The probability that entrepreneur e_1 partners with specialist s_2 is a function of $p_{11} = \sigma_1$ her own probability of a perfectly compatible match, as well as the probability $p_{22} = \sigma_2$ that s_2 receives a perfectly compatible match. Since she prefers a match with s_2 over s_4, the probability p_{14} that she matches with specialist s_4 is conditional on her probability of matching with s_1 or s_2, $(1 - p_{11} - p_{12})$, as well as s_4's probability of not matching with e_3 and e_4, $(1 - p_{44} - p_{34})$. This continues on for all

[10] For ease of notation I suppress σ from the expression $p(e_i, s_j; \sigma)$ throughout the paper.

feasible entrepreneur, specialist probabilities. There is a similar set of probabilities for each entrepreneur.[11] *See figure 1.*

The example illustrates that competition between entrepreneurs in the network occurs over specific specialists. When two entrepreneurs are linked to the same specialist, the stable match is with the most compatible and preferred (in the case of equidistance) entrepreneur. For any entrepreneur, the probability that a linked specialist is unavailable due to competition is the aggregate probability that he partners with a more compatible entrepreneur. If e_i is available to specialist s_j , the probability that s_j partners with a more compatible entrepreneur $e_k \in N(s_j, d_{ij})$ is equal to one minus the aggregate probability over all of s_j's closer matches, $1-\sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)$. For any pair of agents, competition is driven by the network connectedness of nearby entrepreneurs σ_k and the match distance d_{ij}. Proposition 1 follows directly from these interactions.

Proposition 1. *In the case of a strictly incomplete, conditional on finding a match, any entrepreneur e_i is most likely to partner with her most compatible specialist.*

The only obstacle to a perfectly compatible match is a network connection. If a perfectly compatible entrepreneur and specialist have a preexisting relationship, they will partner. The match probability is equal to the probability of a network

[11] See Appendix for the complete set of probabilities for $m = 3,4,5$.

connection, σ_i. If e_i and s_j are not a perfectly compatible match, $d_{ij} > 0$ and the chance of either having a more compatible, available specialist, or receiving competition for s_j from more compatible entrepreneurs are both higher. Both of these probabilities are increasing in d_{ij}. The probability of a pair of agents partnering diminishes with circumference distance for both the entrepreneur and specialist. Corollaries 1a and 1b capture this result.

Corollary 1a. *In the case of an incomplete network, for any entrepreneur e_i, the probability she partners with any specialist is decreasing in distance. If $d_{ij} < d_{il}$ then*

$$\left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right) > \left(1-\sum_{s\in N(e_i,d_{il})} p(e_i,s;\sigma)\right).$$

Corollary 1b. *In the case of an incomplete network, competition for any specialist s_j is increasing in distance.* If $d_{ij} < d_{kj}$ then

$$\left(1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)\right) > \left(1-\sum_{e\in N(s_j,d_{kj})} p(e,s_j;\sigma)\right).$$

Proposition 1 and its corollaries suggest one important consequences of a network. Conditional on finding a partner, an entrepreneur is most likely to find a perfectly compatible or very close match. A network partnership is often a good match, but this result varies with the density of an entrepreneur's network. Consider the cases of a well-connected entrepreneur with network probability $\bar{\sigma}$, and a poorly-connected entrepreneur with $\underline{\sigma}$. The well-connected entrepreneur has probability $\bar{\sigma}$

of knowing and partnering with her most compatible specialist. Her chance of partnering with a highly compatible specialist is less than $\bar{\sigma}$, but still relatively high. The poorly connected entrepreneur, on the other hand, has less chance $\underline{\sigma}$ of partnering with her most compatible specialists, and even lower chances of partnering with an alternative, highly compatible specialist. She will likely be competed out of matching with more distant competitors they do know.

Example *m* = 4 continued.

$$
\begin{aligned}
&p_{11} = \sigma_1 \\
&p_{12} = \sigma_1 \cdot (1 - p_{11}) \cdot (1 - p_{22}) \\
&p_{13} = \sigma_1 \cdot (1 - p_{11} - p_{12} - p_{14}) \cdot (1 - p_{23} - p_{33} - p_{43}) \\
&p_{14} = \sigma_1 \cdot (1 - p_{11} - p_{12}) \cdot (1 - p_{34} - p_{44})
\end{aligned}
$$

Entrepreneur e_i ranks her matches from most compatible to least as $\{s_1, s_4, s_2, s_3\}$*. The probability of matching with her perfectly compatible specialist is* $p_{11} = \sigma_1$*. The set of more compatible matches is null. For pair {12}, the probability that e_1 does not have a more compatible match is* $1 - \sigma_1$*; the probability s_2 does not have a more compatible match is* $1 - \sigma_2$*. Going one rank further, for matched pair {14} the probability that e_1 does not have a better match is* $1 - \sigma_1 - p_{12}$ *and the probability s_4 does not have a higher valued available match is* $1 - \sigma_4 - p_{34}$*. For matched pair {13} the probability that e_1 does not have a better match is* $1 - \sigma_1 - p_{12} - p_{14}$*; the probability s_4 does not have a better available match is* $1 - \sigma_3 - p_{23} - p_{34}$.

For each entrepreneur there is a critical tradeoff: the more specialists she knows the less likely she is to compete for these specialists. Each entrepreneur own network has two diverging effects on her likely match. First, the probability of having a network relationship is σ_i across the spectrum of specialists. Second, conditional on being available, the probability that s_j is e_i's best candidate, $1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)$, is decreasing in circumference distance. Herein is the tradeoff. When σ_i is high, entrepreneur e_i is more likely to know each specialist, but less likely to compete when she likely already has a more compatible candidate. I explore this tradeoff in the next section.

I use three measures to evaluate the network and understand its relationship to incentives and payoffs. First, the ex ante expected returns for every entrepreneur, $\pi_i \equiv \sum_{j=1}^{m} \pi_{ij} p_{ij}$, reflects each entrepreneur's value of the network. Next, ex ante expected welfare, $W = \sum_{i=1}^{m} \pi_i$, is the summation across all entrepreneurs. Depending on the compatibility and the number of matches, welfare may be markedly different from the individual returns. Finally, *entrepreneurship* is the proportion of entrepreneurs that form a partnership, $\varepsilon = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{m} p_{ij}$. This measure illustrates why rates of entrepreneurship vary across regions with similar resources but different network structures[12]

[12] For instance Silicon Valley and the Metro Boston Area have similar human capital, but Silicon Valley has both higher levels of entrepreneurship and a more connected social network of engineers, scientists and inventors. (Fleming and Frenken, 2007)

5 Comparative Statics

This section examines how the entrepreneur's expected return from the network changes as either she or her competitors become more connected. When entrepreneur e_i knows h more agents, her connectedness increases by $\frac{h}{m}$. Intuitively, a more connected entrepreneur is better off. An entrepreneur is more likely to receive a more compatible match and less likely to compete for a more distant specialist. Her own expected return increases while her nearby competitors' expectations decrease. Her less compatible competitors—those who require a different type of specialist—may experience less competition from the more connected entrepreneur e_i and have higher expectations.

The recursive nature of match probabilities implies that, for any pair $\{ij\}$, the chance of a partnership is decreasing in the probabilities of more compatible matches. Consider s_j's more compatible entrepreneur e_k. All else equal, if the probability that s_j partners with e_k decreases, the probability of partnership $\{ij\}$ increases. The opposite is also true. If the probability of s_j's more compatible match $p(e_k, s_j)$ increases, then the more distant $p(e_i, s_j)$ decreases. See Lemmas 2 and 3 in the Appendix for further discussion.

Less compatible agents exert no externality on match probabilities. For any entrepreneur and specialist pair, the probability of their match is only conditional on the networks and match probabilities of more each agent's more compatible counterparts. The more compatible a pair of networked agents are, the less competition either one will face. A more localized match faces fewer competitive frictions. Consider the m = 4 example again.

Example *m* = 4 continued. *Solving the entire system of probabilities:*

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot \left[1-\sigma_1\right] \cdot \left[1-\sigma_2\right] \\
p_{14} &= \sigma_1 \cdot \left[(1-\sigma_1)\cdot(1-\sigma_1\cdot(1-\sigma_2))\right] \cdot \left[(1-\sigma_4)\cdot(1-\sigma_3\cdot(1-\sigma_3))\right] \\
p_{13} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2))\cdot(1-\sigma_1(1-\sigma_4)\cdot(1-\sigma_3(1-\sigma_3)))\right] \\
&\quad \cdot \left[(1-\sigma_3)(1-\sigma_2(1-\sigma_2))\cdot(1-\sigma_4(1-\sigma_4)\cdot(1-\sigma_4(1-\sigma_1)))\right]
\end{aligned}
$$

The own network effect and competitive externality are demarcated in brackets. Each match probability is a function of the entrepreneur's own connectedness, σ_1, *and the connectedness of each competing entrepreneur who is closer to either the entrepreneur or the specialist. For each less compatible pair, there are more frictions to either agents' availability.*

Increasing her own chance of a more nearby network connection ensures that she is strictly better off. Ex ante, an entrepreneur's expected return is higher when she knows more specialists. Higher network connectedness has two countervailing effects. First, an entrepreneur who knows more specialists is more likely to know and thus partner with a more compatible specialist. By increasing her chance of a compatible network connection, she is less likely to seek a partnership with a less desirable candidate. This diminishes competition for more distant specialists.

Proposition 2. ***F****or any entrepreneur e_i, an increase in her own network degree strictly increases her ex ante expected return of a partnership:*

$$\text{if } \sigma_i^{'} > \sigma_i \text{ then } \pi_i(\sigma') > \pi_i(\sigma).$$

With a higher probability of knowing any agent, the probability of a match with a better specialist is increasing in σ_i. For a perfectly compatible pair, the marginal change in match probability is equal to one (i.e., the shift in p_{ij} is equal to the shift in σ_i). The likelihood of having a better match and having competition for the specialist erode this positive impact for less compatible. The impact is diminishing in dissimilarity.

In fact, there exists a compatibility threshold such that for any entrepreneur e_i the probabilities of a more compatible match is increasing in her network. The probability of a less compatible match decreases. Further, the more connected an entrepreneur is, the lower this threshold distance. In effect, as an entrepreneur e_i's network degree increases more compatible partnerships are more likely. The probability she will compete for less compatible matches diminishes. A higher σ_i hones the entrepreneur's search, increasing the probability of a close match and decreasing the chance that she has to search a wider skill set to find a specialist. This importance of this effect is far more remarkable for the entrepreneur's competitors.

Example *m* = 4 continued. *Suppose the network degree increases to $\sigma_1^{'} > \sigma_1$ for entrepreneur e_1. The probability of a match with specialist s_1 is now $p(e_1, s_1) = \sigma_1^{'}$.*

For her match with s_2, the probability of a network connection increases to $\sigma_1^{'}$*, but the probability she does not already have a better match with s_1 decreases to* $1-\sigma_1^{'}$*. For each less preferred match, even as the probability of a network link increases* $\sigma_1^{'}$*, but the probability of having a better match is also higher.*

As an entrepreneur's own match probabilities narrow around her more compatible agents, her effect on neighboring competitors may be positive or negative, depending on the distance between competitors and the connectedness of both the entrepreneur and competitor. From the perspective of a weakly connected competitor, a high network probability competitor may actually be very beneficial. Proposition 3 follows directly from this idea.

Proposition 3. *For any entrepreneur e_i and competing entrepreneur e_k, the effect of a shift in the competitor's network probability is negative for compatible competitors, but positive for less compatible competitors (i.e., high d_{ik}).*

Proposition 3 suggests that more connected agents make direct competition more difficult, but, by relieving friction in the matching process, actually alleviates competition in other regions of the same industry. When a competitor's network increases, the probability of a good match for a nearby competing entrepreneur actually decreases. An increase in σ_i strictly decreases the probability that any other entrepreneur will match with e_i's perfectly compatible specialist. The probability that e_i's compatible competitor at distance $1/m$ from herself matches with a good match ten decreases. In turn this strictly decreases her neighbor's expected return. As her σ_i increases she increases her probability of a compatible match. This negatively effects the match likelihood for entrepreneurs looking for similar specialists.

Example *m* = 4 continued. *Consider the effect of a change in nearby entrepreneur e_2's connectedness on entrepreneur e_1. Suppose σ_2 increases to σ_2'*

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot (1 - p_{11}) \cdot (1 - p_{22}) \\
p_{13} &= \sigma_1 \cdot (1 - p_{11} - p_{12} - p_{14}) \cdot (1 - p_{23} - p_{33} - p_{43}) \\
p_{14} &= \sigma_1 \cdot (1 - p_{11} - p_{12}) \cdot (1 - p_{34} - p_{44})
\end{aligned}
$$

The probability of e_1's perfectly compatible match, $p_{11} = \sigma_1$ is unaffected. The match probability p_{12} strictly decreases. Since e_2 is less compatible with s_4 than e_1, p_{14} is not affected by direct competition from e_2 for s_4. If p_{23} increases with σ_2' then p_{13} is also decreasing. Note from the solution below, the shift in p_{13} depends on the magnitude of $\sigma_2^{'}(1-\sigma_2^{'})$ relative to $\sigma_2(1-\sigma_2)$. The solution illustrates the complete pass through. For instance, as p_{12} decreases, however, σ_2' does indirectly change the likelihood that e_1 is available for s_4.

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot \left[1 - \sigma_1\right] \cdot \left[1 - \sigma_2^{'}\right] \\
p_{14} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2^{'}))\right] \cdot \left[(1-\sigma_4)\cdot(1-\sigma_3\cdot(1-\sigma_3))\right] \\
p_{13} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2^{'}))\cdot(1-\sigma_1(1-\sigma_4)\cdot(1-\sigma_3(1-\sigma_3)))\right] \\
&\quad \cdot \left[(1-\sigma_3)(1-\sigma_2^{'}(1-\sigma_2^{'}))\cdot(1-\sigma_4(1-\sigma_4)\cdot(1-\sigma_4(1-\sigma_1)))\right]
\end{aligned}
$$

As the distance between competitors increases, the effect of decreased competition becomes stronger, the likelihood of a positive externality increases. From Proposition 2, as entrepreneur e_i's network probability increases the match probability she competes for less compatible matches decreases. An increase in her network connectedness increases the aggregate probability of a better match and decreases the probability she will compete for her less compatible specialists. As she is less likely to compete for more distant agents, her more distant competitor's are more likely to obtain a match.

Example *m* = 4 continued. *Now consider the effect of a change in entrepreneur e_3's connectedness on entrepreneur e_1. Suppose σ_3 increases to σ_3':*

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot \left[1-\sigma_1\right] \cdot \left[1-\sigma_2\right] \\
p_{13} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2)) \cdot (1-\sigma_1(1-\sigma_4) \cdot (1-\sigma_3^{'}(1-\sigma_3^{'})))\right] \\
&\quad \cdot \left[(1-\sigma_3^{'})(1-\sigma_2(1-\sigma_2)) \cdot (1-\sigma_4(1-\sigma_4) \cdot (1-\sigma_4(1-\sigma_1)))\right] \\
p_{14} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2))\right] \cdot \left[(1-\sigma_4) \cdot (1-\sigma_3^{'}(1-\sigma_3^{'}))\right]
\end{aligned}
$$

Matches with both s_1 and s_2 are unaffected because they are more compatible. The probability of matching s_3 and s_4 are now dependent on the relative value of $\sigma_3^{'}(1-\sigma_3^{'})$ and $\sigma_3(1-\sigma_3)$. In fact, the probability of a match with s_3 is now strictly lower, but s_3 is a relatively low value match for e_1. Depending on the value of $\sigma_3^{'}(1-\sigma_3^{'})$, e_1 may have a higher chance of matching with the more compatible s_4.

This example provides insights on how these network effects change with the connectedness of both the impacted entrepreneur and the competitor, the size of the market and the distance between agents. First, if an entrepreneur's connectedness increases, she is more likely to exert a positive influence on her competitors, and entrepreneurship generally, if she has a high σ. The higher her probability of a very compatible match, the less competition she exerts on the market. Similarly, because this positive externality affects mid-distance matches, it is more likely to be beneficial for an entrepreneur who is not already likely to match with her perfectly compatible match.

Corollary 3. *A positive competitive externality is more likely to occur when the competing entrepreneur's network probability* σ_k *is high.*

Proposition 3 and its corollary highlight some critical tradeoffs of the network. First, they imply that networks have both an upside and a strong downside, depending on an agent's own location. If an entrepreneur's direct competitors are better connected, she is less likely to find a partner and startup. This result implies that in a market with segments of highly connected entrepreneurs, relatively unconnected entrepreneurs are less likely to be able to enter than they could if their competitors were not as highly connected. On the other hand, connected competitors alleviate competition for more distant market segments. Less connected entrepreneurs are more likely to startup if competitors in other parts of the market are more connected. As section 6 discusses there is some empirical evidence in support of this result.

6 Specific Network Structures

I consider three canonical network structures with a range of connectedness. In a *complete network* each entrepreneur knows every specialist, $g_{ij} = 1$ for all $\{ij\}$. This is equivalent to a market of complete information. Each agent partners with her perfectly compatible counterpart. In a *random, single degree* network, each entrepreneur is arbitrarily linked to a unique specialist. This is equivalent to a random matching with low expected returns. The *uniform, homogeneous network*, in which every entrepreneur has equal degree *k*, illustrates how the welfare benefit of a basic social network varies between these boundary cases. A network is *effective* for an individual entrepreneur if it increases the expected compatibility of her partnership over the market option. A *socially valuable* network raises total welfare above this random case. More specifically, I show that as the network becomes more connected, there are fewer competitive frictions and all entrepreneurs are more likely to find a compatible match.

a. Complete Network

In the complete network all agents are linked such that $g_{ij} = 1$ for all *ij* pairs and $\sigma_i = 1$ for all *i*. There is a unique equilibrium match, M^*, that is an assortative function between entrepreneurs and specialists such that $d_{ij} = 0$ for all *ij*. Since each agent partners with her perfectly compatible counterpart, M^* is socially optimal, because every entrepreneur receives her optimal expected return. Additionally, since every agent has complete information, this case is equivalent to a complete information market. Each startup receives $\pi^* = \pi(0)$ and total welfare is $W^* = M\pi^*$

Proposition 4. *In a complete network, there exists a unique stable equilibrium match M*. M* is positively assortative (i.e., strictly monotonic in agent types) and socially optimal.*

b. Random, One-Degree Network

In the next case, each entrepreneur's network consists of a single link to a unique specialist (i.e., no two entrepreneurs are linked to the same specialist). The exogenous network is generated by random assignment without replacement.[13] This case is equivalent to a market with no information and represents random pairing.

Remark. *Under a one-degree random network, for each entrepreneur e_i the ex ante expected return is equal to the average payoff,* $\sum_{j=1}^{m} \frac{\pi(e_i, s_j)}{m}$*, and is strictly less than the return in the complete network equilibrium,* π^*.

It follows from the definition that the return to a complete network is greater than the return to a random, one-degree network. Ex ante, the entrepreneur's expected return is at least as high as her outside option:

$$\pi^* \geq \sum_{j=1}^{m} \frac{\pi(e_i, s_j)}{m} \geq \underline{\pi}$$

[13] For instance, generate G by randomly drawing one specialist from the set S and assign him a link to e_1. Without replacement, select another specialist from S and assign him a link it to e_2. Continue this process until each specialist and entrepreneur has one link.

These two cases offer benchmarks to measure the relative value of any network. The complete network is equal to the optimal case represents the value that a network must exceed to be an improvement over a basic market. If the welfare of a network is below random, one degree network expected return, the actually network actually hinders matching, rather than alleviate an information problem.

c. Uniform, Homogeneous Network

Under the homogeneous network all entrepreneurs know an equal number of people, $\sigma_i = \sigma$ for all *i*. Ex ante each agent's expected impact on her surrounding entrepreneurs is symmetric and equal. This case illustrates how the effectiveness of the network changes with connectedness. I focus on the incomplete network cases in which $k < m$ and $\sigma < 1$. A homogeneous network with connectedness σ' is *more complete* than a homogeneous network with connectedness σ if $\sigma' \geq \sigma + \frac{1}{m}$.

Example *m* = 4 continued. *Replacing each* $\sigma_i = \sigma$ *implies the match probabilities*

$$p_{11} = \sigma$$
$$p_{12} = \sigma \cdot (1-\sigma)^2$$
$$p_{13} = \sigma \cdot \left((1-\sigma)\cdot(1-\sigma(1-\sigma))\cdot(1-\sigma(1-\sigma)(1-\sigma(1-\sigma)))\right)^2$$
$$p_{14} = \sigma \cdot \left((1-\sigma)(1-\sigma(1-\sigma))\right)^2$$

These probabilities are equal by compatibility rank for each entrepreneur.

Similar to the more general case, for every entrepreneur the probability of a perfectly compatible is highest, while the likelihoods of less compatible matches are diminishing with the compatibility rank. The probability of a match decreases with circumference distance. The match probabilities are equal for each entrepreneur and strictly decreasing around the agent's own type.

When each entrepreneur's network connectedness simultaneously increases there are two effects. First, the probability she knows and prefers a more compatible match increases, diminishing the likelihood she will try to rely on a less compatible candidate. Next, as each entrepreneur is more likely to know and match her most compatible specialist, it is less likely that less compatible specialist is available.

Proposition 5. *Under a homogeneous, incomplete network, ex ante welfare is less than the optimal matching. As the network becomes more complete, the expected welfare increases and approaches the socially optimal outcome. As* $\sigma \to \frac{1}{m}$ *the expected welfare is lower than the one-degree random network without replacement expected outcome.*

As the probability of the network degree increases, there are fewer competitive pressures between entrepreneurs and the network is increasingly efficient. Each entrepreneur is more likely to find a more compatible partner without searching broad skill ranges and being exposed to high levels of competition. A more complete network, with more links, has fewer competitive frictions. The likelihood of compatible partnerships increase and the probabilities of inefficiencies decrease. As I discuss in the next section this result is consistent with observed regional differences in entrepreneurship, even within the same industry.

7 Applications

The model has three important results consistent with empirical research and the observed experiences of entrepreneurs across a range of industries, regions and eras. First, consistent with Proposition 2, more connected entrepreneurs are more likely to succeed, specifically obtain funding, and successfully develop and market an idea. Second, as Proposition 5 suggests, even within the same industry with similar resources, regions with denser entrepreneurial networks have both higher levels of entrepreneurship and more successful entrepreneurs than regions with less connected networks. Thirdly, evidence indicates that there are positive externalities similar to results of Proposition 3.

The two critical challenges for empirical research on entrepreneurial network effects are measuring outcomes and network size. Since agents often form network connections with former colleagues, an entrepreneur's career history is a frequent estimate of network size and location. Common measures of entrepreneurial success include the level and number of capital investments; the number of patents and the breadth of their classifications; the number and variety of products the firm markets; and whether the startups survives to go public. In addition, researchers often measure innovativeness by the variation between patents or products between a founder's parent companies and her own entrepreneurial spin-offs.

Entrepreneurs with a greater preexisting network are more likely to receive early funding, survive through more rounds of funding and successfully produce and market more innovative products. Gompers, Lerner and Scharfstein (2005) find indirect evidence that access to a network of resources positively affects the success of venture-backed entrepreneurs who leave public companies to start a venture. Employees embedded in firms with a culture of entrepreneurship build strong relationships with a network coworkers, clients, and other partners who they rely on for their own startups. In the late 19th and early 20th centuries synthetic die industry, the most successful entrepreneurs in both Germany and England had central positions in the industry-university network. Murmann (2003).

Burton, Sorensen and Beckman (2002) show that entrepreneurs with more prominent networks are more likely to found innovative start-ups and successfully receive early funding than entrepreneurs in a less prominent network position. This is true regardless of previous education, entrepreneurial experience or whether ideas emerged while working for a previous employer. Entrepreneurs whose networks were linked with well-connected, entrepreneurially prominent firms such as IBM, Intel, Apple, HP and Stanford University are most likely to spawn successful start-ups.

Network effects exist, regardless of the entrepreneur's previous education and career experience, or if the entrepreneur's idea emerged from working for a previous employer. Agarwal et al (2002) use data on the Rigid Disk Industry between 1977 and 1997.[14] Even controlling for technological and marketing know-how, independent spin-outs are more likely to survive than de novo firms or those affiliated with their parent company. This is consistent with a network effect in which employees at cutting edge firms are more connected and, in turn, capable of connecting with the most compatible partners.

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[14] Spin-outs are entrepreneurial ventures, founded by incumbents of an existing firm. Spin-outs do not maintain a legal relationship with the original incumbent firm.

In the medical device industry, Chatterji (2007) finds that firms founded by entrepreneurs with preexisting relationships to the industry, who originally worked for a publicly traded medical device firm or have already created new startups, perform better and receive funding sooner. He finds little relationship between the patents of parents and the patents of entrepreneurs. This effect is not a function of knowledge pass through.

Across two very different contexts, one important factor for industry success is the capacity to link agents in private industry with university scientists. Both Darby et al's (1998) research on the biotechnology and gene sequencing industry, and Murmann's (2003) research on early German synthetic dye chemists point to a critical links between entrepreneurs, and star university scientists conducting bench-level research. In both cases the working relationships develop from, and foster further growth of, a network between industry leaders and university professors. Scientists find the business resources to market their research. Entrepreneurs gain access to cutting-edge technology.

According to Proposition 5 a more complete network is more likely to yield higher levels of entrepreneurship. A denser network is more efficient for its agents, more connected entrepreneurs are also better off. Specifically, Murmann (2003) attributes the informal 'academic-industrial knowledge network' with enabling higher rates of success and entrepreneurship in German synthetic dye industry over its British or U.S. counterparts to the dense university-industry network.

A handful of studies on more recent entrepreneurial industries and regions suggest that areas with more dense networks, such as Silicon Valley CA or Boston MA, are more successful. Gompers et al (2005) find that firms in Silicon Valley and the Boston area are more likely to spawn startups and these businesses are more likely to be unrelated to the original parent company. An entrepreneur who exploits technology less related to her previous employer is relying on her network to access resources, rather than the technical information from her former employers. Former employees benefit from a regional social network of easily accessible resources. Well-connected entrepreneurs are more capable of procuring critical resources, including compatible cofounders, for their startups.

Authors find that this effect is strongest for Silicon Valley and exists, but to a lesser extent, in Boston. This observation is consistent with Flemming and Frenken (2007) finding that the network of inventors and engineers are more complete in Silicon Valley than Boston. Fleming and Frenken show that the higher levels of entrepreneurship and invention in Silicon Valley over the Boston metropolitan area was driven by a denser co-inventor network in Silicon Valley.

Related research in these studies also suggest there may be positive externalities to some unconnected entrepreneurs within a strong network. Gompers et al (2005) show that, even for entrepreneurs who were likely less connected, were more likely to be successful than their counterparts in less connected regions. Even less connected entrepreneurs in Germany had a better chance of success than those in England. In addition, results from Burton et al (2002) suggest that entrepreneurs with less prominent positions in the entrepreneurial network are more successful in sectors, as measured by patent and product types, with fewer well-connected entrepreneurs.

8 Relevance to Policy

Both the model and empirical applications indicate the importance of dense social networks for innovation. My model motivates and provides a framework with which to think about additional questions. First, what is the most efficient network structure? For instance, there are likely different implications to a uniform network with equally connected agents than a few highly connected ‘stars’. The synthetic dye and gene sequencing networks were focused around well-connected stars. Silicon Valley, on the other hand, is a more uniform, widely diffuse network. Next, does the impact of network interactions shift with the level of risk, uncertainty or heterogeneity of payoffs? Burton et al (2002) suggest that network effects are particularly important for entrepreneurs pursuing an innovative, more risky idea.

It is also critical to consider the role of government and universities in fostering these networks. Evidence suggests that many of the most vibrant industries, characterized by successful, marketable, scientific innovations of the last century, had a strong inter-industry-university network. Cross university-industry networks are critical for high tech inventors particularly in Silicon Valley[15] and the Boston area (Fleming and Frenken, 2007), biotechnology (Zucker, Darby and Brewer, 1998) and turn of the 20th century synthetic dye manufacturing (Murmann, 2003).

[15] For instance, Stanford University’s initiatives have connected industry professionals with university researchers and have fostered a dense, interconnected network of experts who traversed between academia and industry. Three programs in particular fostered cross industry-university links. Two institutional programs started in ‘50s: University Honors Cooperative Program and the Stanford Industrial Park (Stanford Research Park) combined university research with nascent industry interests. Meanwhile, approx. 50 university research centers provided a forum for industry and university types to connect. (Castilla et al, 2002; Fleming and Frenken, 2007)

Policies and initiatives that foster social networks within a research field or industry affect the success of entrepreneurs, as well as the network-wide, often regional, development. Other examples of policies that promote industry-university networks include the California University system and Cornell University programs to promote research in wine-making, viticulture and enology, the U.S. government programs for Nanotechnology research, and the Stanford University programs to promote research in high tech and communication with Silicon Valley firms.

9 Conclusion

Through pathways of information, a strong social network enables its members to find compatible partners. I present a discrete matching model with heterogeneous values and an undirected social network to understand how an entrepreneur uses her network of preexisting contacts to find the best partner for her project. The model offers a framework to study how relative network positions affect payoffs and incentives within a network.

An entrepreneur's expected return to the network is a function of her own connectedness as well as the connectedness of her competitors. A more connected entrepreneur has a higher probability of finding a compatible partner. The externality of a competing entrepreneur's connectedness is more complex. When an entrepreneur's network increases, her closest competitors are hurt, but less similar competitors may be more likely to receive a suitable partner. I compare the individual effects and social benefits across the most common network structures. In a more connected network, every agent is more likely to find a ore compatible partner.

Results from the model are consistent with empirical evidence on entrepreneurial networks and "spawning," the process by which employees of a firm leave to become founders of a startup. First, more connected entrepreneurs are more likely to succeed. Second, while more networked entrepreneurs impede less connected, nearby competitors, they may make it easier for more distant competitors to find the best entrepreneurs. Third, regions with denser networks have both higher rates of entrepreneurship and higher rates of entrepreneurial success.

In a related paper I consider the consequences of extending this model to consider the impact of allowing for an endogenous network of weak ties. The present research focuses on a static, exogenous network. This paper uses results from above to explore the tradeoffs between an existing exogenous network and the possibility of creating a more efficient, but costly network.

Future research might also consider the impact of the above results on innovation and creativity. My results suggest that social networks promote connected insiders over newcomers. If creative, innovative ideas emerge from unconventional thinking, the above suggest that social networks that impede unconnected entrants might be harmful for innovation and creativity. These are concerns that are worth exploring through either a intertemporal dynamic model, or a model that considers the implications for learning, creativity or innovation.[16]

[16] For literature on the network implications for innovation and creativity see Baumol (2004), Burt (2003), Uzzi (1996), Uzzi (2005), Uzzi & Spiro (2004), Schilling and Phelps (2004).

10 Appendix

Lemma 1. *Under condition 1, each entrepreneur strictly prefers to match with a closer specialist, and each specialist prefers to match with a closer entrepreneur.*

Proof of Lemma 1. Let u_{ij} and v_{ij} represent the respective individual payoffs to entrepreneur e_i and specialist s_j for match $\{ij\}$, such that $\pi_{ij} = u_{ij} + v_{ij}$. Denote the respective outside options for any entrepreneur and any specialist as $\underline{u}$ and $\underline{v}$, where $\underline{\pi} = \underline{u} + \underline{v}$. Consider any e_i, $e_k \in E$ and s_j, $s_l \in S$ such that $g_{ij} = g_{kj} = g_{il} = 1$ and $d_{ij} < d_{kj} \leq d_{il}$ (i.e., both entrepreneurs are linked to specialist s_j and both specialists are linked to e_i, and e_i and s_j are closest to each other). Finally, suppose that e_k and s_l have no other feasible, intra-network options. For instance, e_k and s_l are only linked to s_j and e_i respectively. See figure.

Since e_k and s_l have no other partner options, e_k prefers to match with s_j if $u_{kj} \geq \underline{u}$ and s_l prefers to match with e_i if $v_{il} \geq \underline{v}$. In turn, if $u_{kj} \geq \underline{u}$ and $v_{il} \geq \underline{v}$, the match payoffs to e_i and s_j for matches $\{kj\}$ and $\{il\}$ are $v_{kj} \leq \pi_{kj} - \underline{u}$ and $u_{il} \leq \pi_{il} - \underline{v}$, respectively.

For $\{ij\}$ to be a pairwise stable match, the payoffs to e_i and s_j must be greater under $\{ij\}$ than under the alternative matches $\{il\}$ and $\{kj\}$. This implies that $u_{ij} \geq u_{il}$ and $v_{ij} \geq v_{kj}$. Substituting in $v_{kj} \leq \pi_{kj} - \underline{u}$ and $u_{il} \leq \pi_{il} - \underline{v}$, then $\{ij\}$ is stable if $u_{ij} > \pi_{il} - \underline{v}$ and $v_{ij} > \pi_{kj} - \underline{u}$. Combining these two conditions and recalling that $\pi_{ij} = u_{ij} + v_{ij}$:

$$\pi_{ij} = u_{ij} + v_{ij} \geq u_{il} + v_{kj} = \pi_{il} - \underline{v} + \pi_{kj} - \underline{u} = \pi_{il} + \pi_{kj} - \underline{\pi}.$$

Match $\{ij\}$ is stable as long as $\pi_{ij} - \pi_{kj} \geq \pi_{il} - \underline{\pi}$ or, rearranging terms, $\pi_{ij} + \underline{\pi} \geq \pi_{il} + \pi_{kj}$.

It is sufficient to prove the above strongest case. If e_k and/or s_l have an alternative, feasible, intra-network option, then the competing agent's rational payoff is strictly greater than the outside option: $u_{kj} > \underline{u}$ and $v_{il} > \underline{v}$ and this condition will continue to hold.

Q.E.D.

Proposition 1. *In the case of a strictly incomplete network of uniform network connectedness, conditional on finding a match, any entrepreneur e_i is most likely to partner with her closest specialist.*

Proof of proposition 1.

(1) For two perfectly compatible agents, the probability of a match is equal to the entrepreneur's local network probability σ_i.

Consider any entrepreneur e_i and her perfectly compatible specialist s_j. By definition $e_i = s_j$ and $d_{ij} = 0$. Further, the subsets of preferred agents $N(e_i, d_{ij})$ and $N(s_j, d_{ij})$ are empty. This implies that the probabilities of a better match are equal to zero for both e_i and s_j: $\sum_{s \in \varnothing} p(e_i, s; \sigma) = 0$ and $\sum_{e \in \varnothing} p(e, s_j; \sigma) = 0$. Following equation (1.1) the match probability is equal to the entrepreneur's network probability:

$$p(e_i, s_j; \sigma) = \sigma_i \cdot \left(1 - \sum_{s \in \varnothing} p(e_i, s; \sigma)\right) \cdot \left(1 - \sum_{e \in \varnothing} p(e, s_j; \sigma)\right) = \sigma_i .$$

(2) For not perfectly compatible agents, the probability of a match is strictly less than σ_i.

Consider any entrepreneur e_i and specialist s_l such that $d_{il} > 0$. Then the subsets of preferred agents $N(e_i, d_{il})$ and $N(s_l, d_{il})$ contain at least two agents (i.e., the perfectly compatible partner and the equidistant partner). This implies that the probabilities of a better match are positive and strictly greater than the network probability for both e_i and s_l: $\sum_{s \in N(e_i, d_{il})} p(e_i, s; \sigma) \geq p_{i,i} + p_{i,m}$ where $d_{im} = d_{il}$ and $s_l \neq s_m$, and

$\sum_{e \in N(s_l, d_{il})} p(e, s_j; \sigma) \geq p_{j,j} + p_{k,j}$ where $d_{kl} = d_{il}$ and $e_i \neq e_k$. Following equation (1.1) the match probability is less than the entrepreneur's network probability:

$$p(e_i, s_j; \sigma) = \sigma_i \cdot \left(1 - \sum_{s \in N(e_i, d_{il})} p(e_i, s; \sigma)\right)\left(1 - \sum_{e \in N(s_l, d_{il})} p(e, s_j; \sigma)\right) < \sigma_i .$$

From (1) and (2) the probability of a perfectly compatible match is higher than the probability for any non-perfectly compatible match: $p_{ij} = \sigma_i > p_{il}$ if $0 = d_{ij} < d_{il}$. Conditional on finding a match, an entrepreneur is most likely to match with her perfectly compatible specialist. Q.E.D

Corollary 1a. *In the case of an incomplete network with uniform probabilities, for any entrepreneur e_i, her preference over any match is decreasing in distance:*

$$\left(1 - \sum_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)\right) > \left(1 - \sum_{s \in N(e_i, d_{il})} p(e_i, s; \sigma)\right) \textit{if } d_{ij} < d_{il}$$

Proof of Corollary 1a. Consider any entrepreneur e_i and any two specialists s_j and s_l such that $d_{ij} < d_{il}$. From (1.1), the probabilities of matches *{ij}* and *{il}* are:

$$p_{ij} = p(e_i, s_j; \sigma) = \sigma_i \cdot \left[1 - \sum_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)\right] \cdot \left[1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right]$$

$$p_{il} = p(e_i, s_l; \sigma) = \sigma_i \cdot \left[1 - \sum_{s \in N(e_i, d_{il})} p(e_i, s; \sigma)\right] \cdot \left[1 - \sum_{e \in N(s_l, d_{ij})} p(e, s_l; \sigma)\right]$$

Recall that the ex ante probability that e_i prefers s_j to her other feasible options is $1 - \sum_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)$ where $N(e_j, d_{ij}) = \left[s \mid e_i - d_{ij} \leq s \leq e_i + d_{ij}\right] \setminus \{s_j\}$; the ex ante probability that e_i prefers s_l is $1 - \sum_{s \in N(e_i, d_{il})} p(e_i, s; \sigma)$ where $N(e_i, d_{il}) = \left[s \mid e_i - d_{il} \leq s \leq e_i + d_{il}\right] \setminus \{s_l\}$.

Assume that $\pi(d_{ij} + \frac{1}{m}) \geq \underline{\pi}$ so that e_i or s_j would prefer a match of distance $d_{ij} + \frac{1}{m}$ over the outside option. Since the $\sigma < 1$ for all entrepreneurs, the probability of a match for distance $d_{ij} + \frac{1}{m}$ is strictly positive: $p_{i, j\pm1} > 0$ and $p_{i\pm1, j} > 0$. This assumption ensures that matches beyond d_{ij} may occur with some probability.

If $d_{ij} < d_{il}$ then $N(e_i, d_{ij}) \subseteq N(e_i, d_{il})$ and

$$\sum_{N(e_i, d_{il})} p(e_i, s; \sigma) = \sum_{N(e_i, d_{ij})} p(e_i, s; \sigma) + \sum_{N(e_i, d_{il}) - N(e_i, d_{ij})} p(e_i, s; \sigma) > \sum_{N(e_i, d_{ij})} p(e_i, s; \sigma)$$

. Adding one to both sides and rearranging terms:

$$\left[1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s)\right] > \left[1-\sum_{s\in N(e_i,d_{il})} p(e_i,s)\right] \qquad (1.2)$$

By condition (1.2), for any e_i, s_j and s_l such that $d_{ij} < d_{il}$ the probability that e_i prefers s_j is greater than the probability that e_i prefers s_l. This implies that the probability that e_i prefers to match with any specialist is decreasing in distance between the entrepreneur and specialist. Q.E.D.

Corollary 1b. *In the case of an incomplete network of uniform network probabilities, competition for any specialist s_j is increasing in distance:*

$$\text{if } d_{ij} < d_{kj} \text{ then } \left(1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)\right) > \left(1-\sum_{e\in N(s_j,d_{kj})} p(e,s_j;\sigma)\right).$$

Proof of Corollary 1b. Consider any two entrepreneurs e_i and e_k and specialist s_j such that $d_{ij} < d_{kj}$. From (1.1), the probabilities of matches *{ij}* and *{kj}* are:

$$p_{ij} = p(e_i,s_j;\sigma) = \sigma_i \cdot \left[1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right] \cdot \left[1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)\right]$$

$$p_{kj} = p(e_k,s_j;\sigma) = \sigma_k \cdot \left[1-\sum_{s\in N(e_i,d_{kj})} p(e_i,s;\sigma)\right] \cdot \left[1-\sum_{e\in N(s_j,d_{kj})} p(e,s_j;\sigma)\right]$$

Recall that the ex ante probability that s_j is feasible for e_i is $1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)$ and the ex ante probability that s_j is feasible for e_k is $1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)$.

Assume that $\pi(d_{ij}+\frac{1}{m}) \geq \underline{\pi}$ so that e_i or s_j would prefer a match of distance $d_{ij}+\frac{1}{m}$ over the outside option. Since the $\sigma < 1$ for all entrepreneurs, the probability of a match for distance $d_{ij}+\frac{1}{m}$ is strictly positive: $p_{i,j\pm1} > 0$ and $p_{i\pm1,j} > 0$. This assumption ensures that matches beyond d_{ij} may occur with some probability.

If $d_{ij} < d_{kj}$ then $N(s_j,d_{ij}) \subset N(s_j,d_{kj})$ and $\sum_{e\in N(s_j,d_{kj})} p(e,s_j;\sigma) = \sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma) + \sum_{e\in N(s_j,d_{kj})-N(s_j,d_{ij})} p(e,s_j;\sigma)$. Similar to Step 2, it follows that $1-\sum_{e\in N(s_j,d_{kj})} p(e,s_j;\sigma) > 1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)$. This condition implies that, for any specialist s_j, and two entrepreneurs e_i and e_k such that $d_{ij} < d_{kj}$, e_k is less likely to be able to partner with specialist s_j (i.e., experiences more competition) than e_i. Q.E.D.

Lemma 2. *Consider any entrepreneur e_i, and specialists s_j and s_h such that e_i is more compatible with s_h than s_j, $d_{ij} > d_{ih}$. The probability that e_i is available to s_j, $\sigma_i\left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s)\right)$, is decreasing in e_i's own probability of matching with her more compatible specialists:*

$$\textit{If } s_h \in N(e_i,d_{ij}) \textit{ then } \frac{\partial}{\partial\sigma_k}\left[\sigma_i\left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s)\right)\right] < 0$$

$$\textit{If } s_h \notin N(e_i,d_{ij}) \textit{ then } \frac{\partial}{\partial\sigma_k}\left[\sigma_i\left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s)\right)\right] = 0$$

Proof of Lemma 2. First, recall the probability of any match $\{ij\}$

Consider any direct competitor to e_i for s_j, $e_k \in N(s_j, d_{ij})$, and the competitor's probability of matching with s_j, $p(e_k, s_j)$. For each $e_k \in N(s_j, d_{ij})$ the total effect of a shift in $p(e_k, s_j)$ on $p(e_i, s_j)$ is equal to the direct marginal effect, $-\sigma_i \cdot \left(1 - \sum_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)\right)$, plus the indirect effects of each shift in each of e_i's other competitors for s_j who are also effected by a shift in $p(e_k, s_j)$, $\sum_{e_m \in N(s_j, d_{ij})} \frac{\partial p(e_i, s_j)}{\partial p(e_m, s_j)} \cdot \frac{\partial p(e_m, s_j)}{\partial p(e_k, s_j)}$:

$$\frac{dp(e_i, s_j)}{dp(e_k, s_j)} = \frac{\partial p(e_i, s_j)}{\partial p(e_k, s_j)} + \sum_{e_m \in \{N(s_j, d_{ij}) - N(s_j, d_{kj})\}} \frac{\partial p(e_i, s_j)}{\partial p(e_m, s_j)} \cdot \frac{\partial p(e_m, s_j)}{\partial p(e_k, s_j)} \qquad (1.3)$$

A note on indirect effects (i.e., competitors of competitors) and the summation set $\{N(s_j, d_{ij}) - N(s_j, d_{kj})\}$. For any unique specialist s_j, if he prefers to a match with e_m over e_i then $e_m \in N(s_j, d_{ij})$. Further, if he prefers to match with e_k over e_m, $e_k \in N(s_j, d_{mj})$. It must also be that $d_{mj} < d_{ij}$ and $N(s_j, d_{mj}) \subset N(s_j, d_{ij})$. This implies that $e_k \in N(s_j, d_{ij})$. For entrepreneur e_i and any specialist s_j, if e_k is a competitor of e_i's competitor $e_{m\neq}$ she is also a direct competitor of e_i: if $e_k \in N(s_j, d_{kj})$ and $d_{kj} < d_{ij}$ then $e_k \in N(s_j, d_{ij})$. In addition to the direct effect of p_{kj}, the subset $\{N(s_j, d_{ij}) - N(s_j, d_{kj})\}$ include the indirect effects of p_{kj}, on e_i's other competitors, preferred over e_i but not preferred over e_k. Equation (1.3) is the complete, direct and indirect, effect of a shift in $p(e_k,s_j)$ on $p(e_i,s_j)$.

Since the direct effect of a shift in any competitor is equal,

$\frac{\partial p(e_i,s_j)}{\partial p(e,s_j)} = -\sigma_i \cdot \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right)$ for all $e \in N(s_j,d_{ij})$, rewrite (1.3) as

$$\frac{dp(e_i,s_j)}{dp(e_k,s_j)} = -\sigma_i \cdot \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right)\left(1+\sum_{e_m\in N(s_j,d_{ij})-N(s_j,d_{kj})} \frac{dp(e_m,s_j)}{dp(e_k,s_j)}\right) \quad (1.4)$$

where each

$$\frac{dp(e_m,s_j)}{dp(e_k,s_j)} = -\sigma_m \left(1-\sum_{s\in N(e_m,d_{mj})} p(e_i,s;\sigma)\right)\left(1+\sum_{e_n\in N(s_j,d_{kj})-N(s_j,d_{mj})} \frac{dp(e_n,s_j)}{dp(e_m,s_j)}\right).$$

To evaluate the value of (1.4), consider the term $\sigma_i \cdot \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right)$ for any e_i. For any $d_{ij} > 0$, $\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma) \geq \sigma_i$. This implies that $0 < \sigma_i \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right) < \sigma_i\left(1-\sigma_i\right)$. For any $\sigma_i \in \left(0,1\right)$, $\sigma_i\left(1-\sigma_i\right) \in \left(0,0.25\right]$. More specifically, $\sigma_i\left(1-\sigma_i\right) \to 0.25$ as $\sigma_i \to 0.5$ from above or below. Then, since $0 < \sigma_i \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right) < \sigma_i\left(1-\sigma_i\right) \in \left(0,0.25\right)$:

$$-\sigma_i \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right) \in -(0,0.25)$$

Further, as d_{ij} increases, the cumulative probability of a better match, $\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)$, increases and $\sigma_i \left(1-\sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)\right)$ decreases. A greater distance between agents implies a smaller competitive externality effect of a shift in competitor's probability.

With the recursive nature of the problem, this applies to both the direct effect as well as each of the indirect effects, but with off-setting signs. Referring back to (1.4), the second term, $\left(1+\sum_{e_m \in N(s_j,d_{ij})-N(s_j,d_{kj})} \frac{dp(e_m,s_j)}{dp(e_k,s_j)}\right)$, is strictly between zero and 1.

Each indirect effect, $\frac{dp(e_n,s_j)}{dp(e_m,s_j)}$, is also strictly between zero and -0.25, and diminishing in distance. Further, the sum of these indirect effects offset the direct effect, but never exceeds -1: $\sum_{e_m \in N(s_j,d_{ij})-N(s_j,d_{kj})} \frac{dp(e_m,s_j)}{dp(e_k,s_j)} \in -(0,1)$. Therefore, the direct effect of a shift in competitor's probability is always strictly negative, between zero and -0.25, but is diminishing in distance.

For any e_i and competitor e_k with match probabilities $p(e_i,s_j)$ and $p(e_k,s_l)$. If $s_j \neq s_l$ or $e_k \notin N(s_j,d_{ij})$ then a shift in the competitor's probability $p(e_k,s_l)$ has no effect on the competitive externality of match $p(e_i,s_j)$. *Q.E.D.*

Lemma 3. *The competitive externality of a match between any entrepreneur e_i matching with any specialist s_j is only affected by competing entrepreneurs, e_g, who are more compatible with s_j:*

$$\textit{If } e_g \in N(s_j,d_{ij}) \textit{ then } \frac{\partial}{\partial p(e_i,s_h)}\left(1-\sum_{e\in N(s_j,d_{ij})} p(e,s_j;\sigma)\right) < 0$$

$$If\ e_g \notin N(s_j, d_{ij})\ then\ \frac{\partial}{\partial p(e_i, s_h)} \left(1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right) = 0$$

Proof of Lemma 3. First, recall the probability of any match $\{ij\}$

$$p(e_i, s_j; \sigma) = \sigma_i \cdot \left[1 - \sum_{s \in N(e_i, d_{ij})} p(e_i, s; \sigma)\right] \cdot \left[1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right]$$

For any e_i consider the effect of a shift in match probabilities $p(e_i, s_l)$ on $p(e_i, s_j)$.

First, if e_i prefers s_j to s_l then there is no effect: if $s_j \notin N(e_i, d_{il})$ then $d_{ij} > d_{il}$ and $\frac{dp(e_i, s_l)}{dp(e_i, s_j)} = 0$. Next, $\frac{dp(e_i, s_j)}{dp(e_i, s_j)} = 1$. Finally, if $s_l \in N(e_i, d_{ij})$ then

$$\frac{dp(e_i, s_j)}{dp(e_i, s_l)} = \frac{\partial p(e_i, s_j)}{\partial p(e_i, s_l)} + \sum_{s_n \in \{N(e_i, d_{ij}) - N(e_i, d_{il})\}} \frac{\partial p(e_i, s_j)}{\partial p(e_i, s_n)} \cdot \frac{\partial p(e_i, s_n)}{\partial p(e_i, s_l)} \tag{1.4}$$

where the first term, $\frac{\partial p(e_i, s_j)}{\partial p(e_i, s_l)} = -\sigma_i \cdot \left(1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right)$, represents the direct effect of a shift in probabilities $p(e_i, s_l)$ on $p(e_i, s_j)$, and the second term represents the cumulative effect of all other shifts. Replacing the direct effect of a shift in any probability over a preferred, closer match is equal,

$\frac{\partial p(e_i, s_j)}{\partial p(e_i, s)} = -\sigma_i \cdot \left(1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right)$ for all $s \in N(e_i, d_{ij})$:

$$\frac{dp(e_i, s_j)}{dp(e_i, s_l)} = -\sigma_i \cdot \left(1 - \sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma)\right)\left(1 + \sum_{s_n \in \{N(e_i, d_{ij}) - N(e_i, d_{il})\}} \frac{dp(e_i, s_n)}{dp(e_i, s_l)}\right)$$

Following the logic from Lemma 2, the sum of the indirect effects,

$\sum_{s_n \in \{N(e_i,d_{ij})-N(e_i,d_{il})\}} \frac{dp(e_i,s_n)}{dp(e_i,s_l)}$, is never less than -1, so the total change must be negative. *Q.E.D.*

Proposition 2. ***F****or any entrepreneur e_i, an increase in her own network degree strictly increases her ex ante expected return of a partnership: if $\sigma_i' > \sigma_i$ then $\pi_i(\sigma') > \pi_i(\sigma)$.*

Proof of proposition 2. There are three steps to this proof. In step 1, I show that the aggregate probability of a match of any distance for e_i is increasing in σ_i. Step 2 analyzes shifts in marginal match probabilities with respect to a shift in σ_i. If a pair is perfectly compatible such that $d_{ij} = 0$ then $\frac{\partial p_{ij}}{\partial \sigma_i} = 1$. If a match is not perfectly compatible such that $d_{ij} > 0$ then $\frac{\partial p_{ij}}{\partial \sigma_i} < 1$. In fact, there exists a threshold distance $\bar{d}$ such that probabilities are increasing for closer matches, and probabilities of a match are decreasing for more distant matches: if $d_{ij} < \bar{d}$ then $\frac{\partial p_{ij}}{\partial \sigma_i} > 0$, and if $d_{ij} > \bar{d}$ then $\frac{\partial p_{ij}}{\partial \sigma_i} < 0$. In step 3 it must be that the expected return must be increasing in σ_I.

Step 1. *For any entrepreneur e_i, the aggregate probability of a match weakly preferred to a match of any distance d_{ij} is weakly increasing in σ_i over the subset $N(e_i, d_{ij})$.*

Proof. For any entrepreneur e_i, let the aggregate probability of a match with s_j or higher rank is:

$$P^{\sigma}_{N(e_i,d_{ij})} = \sum\nolimits_{s \in N(e_i,d_{ij})} p(e_i,s;\sigma) = \sum\nolimits_{s \in N(e_i,d_{ij})} \sigma_i \cdot \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right)\left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right)$$

The marginal change in aggregate probability with respect to σ_i is:

$$\frac{\partial P^{\sigma}_{N(e_i,d_{ij})}}{\partial \sigma_i} = \sum\nolimits_{s \in N(e_i,d_{ij})} \left\{ \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right)\left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right) - \sigma_i \cdot \frac{\partial P^{\sigma}_{N(e_i,d_{ij})}}{\partial \sigma_i} \cdot \left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right) - \sigma_i \cdot \frac{\partial P^{\sigma}_{N(s_j,d_{ij})}}{\partial \sigma_\iota} \cdot \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right) \right\}$$

where the first term is the added benefit of knowing more people. The second term represents the shift in aggregate probabilities over all matches weakly preferred to $\{ij\}$. The benefit of knowing more people is offset by the extent to which aggregate probabilities have increased for better matches. Similarly, the third term includes any indirect shift in competitive externality over s_j's preferred matches due to a shift in σ_i. This third term is outweighed by the entrepreneur's own shifts in preferences.

Proof by contradiction. Suppose the aggregate probability of a better match is, in fact, decreasing in σ_i for some subset of the most preferred matches, so that

$\frac{\partial P^{\sigma}_{N(e_i,d_{ij})}}{\partial \sigma_i} < 0$ for some e_i, d_{ij}. Then it must be true that the aggregate shift in probability over better matches by both entrepreneur e_i and specialist s_j exceeds the added benefit of knowing more agents (i.e., the aggregate conditional feasibility over agents), $\sum\nolimits_{s \in N(e_i,d_{ij})} \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right)\left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right)$:

$$\sum\nolimits_{N(e_i,d_{ij})} \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right)\left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right) < \sigma_i \cdot \sum\nolimits_{N(e_i,d_{ij})} \left(\frac{\partial P^{\sigma}_{N(e_i,d_{ij})}}{\partial \sigma_i} \cdot \left(1 - P^{\sigma}_{N(s_j,d_{ij})}\right) + \frac{\partial P^{\sigma}_{N(s_j,d_{ij})}}{\partial \sigma_\iota} \cdot \left(1 - P^{\sigma}_{N(e_i,d_{ij})}\right) \right)$$

For this to be true, however, the shift in aggregate probabilities over better matches increased by more than the added probability of knowing more agents. For this to be true, preference rankings over specialists must change with a shift in σ_i. That is, the aggregate probability that the match is feasible increases by more than the aggregate probability that entrepreneur match with an agent, but this is a contradiction.

Step 2. *The marginal probability of a perfectly compatible match is equal to one for* $d_{ij} = 0$. *The marginal probability for more distant matches is always less than one (and may be negative).*

Proof. Recall that the match probability for a perfectly compatible match $\{ij\}$ is $p(e_i, s_j; \sigma) = \sigma_i$. (Proposition 1) This implies that $\frac{\partial}{\partial \sigma_i} p(e_i, s_j; \sigma) = 1$ when $e_i = s_j$.

Next, consider any entrepreneur e_i and specialist s_j such that $d_{ij} > 0$. By definition $p(e_i, s_j; \sigma) = \sigma_i \cdot \left(1 - \sum_{s \in N_{e_i}^{d_{ij}}} p(e_i, s; \sigma)\right) \cdot \left(1 - \sum_{e \in N_{s_j}^{d_{ij}}} p(e, s_j; \sigma)\right)$. The marginal change in match probability for pair *{ij}* with respect to a change in σ_i is:

$$\frac{\partial p(e_i, s_j; \sigma)}{\partial \sigma_i} = \overbrace{\left(1 - \sum_{s \in N_{e_i}^{d_{ij}}} p(e_i, s; \sigma)\right) \cdot \left(1 - \sum_{e \in N_{s_j}^{d_{ij}}} p(e, s_j; \sigma)\right)}^{(i)} - \overbrace{\sigma_i \sum_{s \in N_{e_i}^{d_{ij}}} \frac{\partial p(e_i, s; \sigma)}{\partial \sigma_i} \cdot \left(1 - \sum_{e \in N_{s_j}^{d_{ij}}} p(e, s_j; \sigma)\right)}^{(ii)} - \underbrace{\sigma_i \left(1 - \sum_{s \in N_{e_i}^{d_{ij}}} p(e_i, s; \sigma)\right) \cdot \sum_{e \in N_{s_j}^{d_{ij}}} \frac{\partial p(e, s_j; \sigma)}{\partial \sigma_i}}_{(iii)} \quad (1.5)$$

Terms (*i*) and (*ii*) reflect the marginal change in entrepreneur e_i's local network effect. Term (*i*) is e_i's marginal benefit of being more likely to be linked to specialist s_j. It is positive, strictly less than one, and equal to the probability that both e_i prefers

s_j, and s_j prefers e_i. According to Proposition 1, $\left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)\right)$ is decreasing in distance. Term (*ii*) reflects e_i's marginal change in aggregate match probability over all preferred matches. Since the change in aggregate match probability is non-decreasing, the value of (*i*) and (*ii*) together is always less than one and less than the value of term (*i*):

$$
\begin{aligned}
1 &< \left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)\right)\cdot\left(1-\sum_{e\in N_{s_j}^{d_{ij}}} p(e,s_j;\sigma)\right) \\
&< \left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)\right)\cdot\left(1-\sum_{e\in N_{s_j}^{d_{ij}}} p(e,s_j;\sigma)\right)-\sigma_i\sum_{s\in N_{e_i}^{d_{ij}}}\frac{\partial p(e_i,s;\sigma)}{\partial \sigma_i}\cdot\left(1-\sum_{e\in N_{s_j}^{d_{ij}}} p(e,s_j;\sigma)\right) \\
&= \left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)-\sigma_i\sum_{s\in N_{e_i}^{d_{ij}}}\frac{\partial p(e_i,s;\sigma)}{\partial \sigma_i}\right)\cdot\left(1-\sum_{e\in N_{s_j}^{d_{ij}}} p(e,s_j;\sigma)\right)
\end{aligned}
$$

In fact, when $\left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)\right)<\left(\sigma_i\sum_{s\in N_{e_i}^{d_{ij}}}\frac{\partial p(e_i,s;\sigma)}{\partial \sigma_i}\right)$ an increase in local network probability decreases the probability of a match between e_i and s_j. Intuitively, when the aggregate change in probability of a preferred match exceeds the probability that entrepreneur e_i prefers a match with s_j a positive shift in σ_i actually decreases the probability that e_i prefers a match with s_j. The left hand side is strictly decreasing in distance (proposition 1) and the right hand side is increasing in distance as long as $\left(1-\sum_{s\in N_{e_i}^{d_{ij}}} p(e_i,s;\sigma)\right)>\left(\sigma_i\sum_{s\in N_{e_i}^{d_{ij}}}\frac{\partial p(e_i,s;\sigma)}{\partial \sigma_i}\right)$. This implies that, given the set of network probabilities σ, for each entrepreneur e_i and there exists a threshold distance, $\bar{\bar{d}}$, within which and entrepreneur's preference over matches is increasing and beyond which the probability is increasing. <Show that the threshold distance is greater for smaller values of σ_i.>0.

Term (*iii*) reflects the marginal change in entrepreneur e_i's competitive externality for specialist s_j. According to Lemma 2, term *(iii)* is positive and never outweighs terms (*i*) and (*ii*). Since (*ii*) is strictly positive, is suffices that (*iii*) is strictly opposite in sign and smaller in absolute magnitude to show that $\frac{\partial p(e_i,s_j;\sigma)}{\partial \sigma_i} < 1$. (Lemma 2)

Step 3. *The ex ante expected return is increasing in* σ_i *such that if* $\sigma_i^{'} > \sigma_i$ *then* $\pi_i(\sigma_i^{'},\sigma_{-i}) > \pi_i(\sigma_i,\sigma_{-i})$.

Proof. Recall the ex ante expected return is the average expected return, where probabilities are weighted according to the equilibrium probabilities: $\pi_i = \sum_{s\in S} \pi(e_i,s) p(e_i,s;\sigma)$. Consider any positive shift in σ_i such that if $\sigma_i^{'} > \sigma_i$. According to Step 1, $P(N(e_i,d_{ij});\sigma) > P(N(e_i,d_{ij});\sigma')$.

Next, suppose each individual match probability increased by a factor $\alpha > 1$ such that the sum of the aggregate match probabilities equals the updated aggregate match probability under $\sigma_i^{'}$:

$$P(N(e_i,d_{ij});\sigma) = \sum_{s\in N(e_i,d_{ij})} p(e_i,s;\sigma)$$
$$P(N(e_i,d_{ij});\sigma') = \sum_{s\in N(e_i,d_{ij})} \alpha p(e_i,s;\sigma)$$

Then the ex ante expected return also increases by a factor α:

$$\sum_{s\in N(e_i,d_{ij})} \pi(e_i,s;\sigma)\cdot p(e_i,s;\sigma) < \alpha \sum_{s\in N(e_i,d_{ij})} \pi(e_i,s;\sigma)\cdot p(e_i,s;\sigma).$$

Now consider the actual increase in individual match probabilities due to a shift in σ_i. For any match pair {*ij*} such that $d_{ij} < \bar{d}$ then $p(e_i,s_j;\sigma') > \alpha p(e_i,s_j;\sigma')$, and for any pair such that $d_{ij} > \bar{d}$ then $p(e_i,s_j;\sigma') < \alpha p(e_i,s_j;\sigma')$. Further, for perfectly compatible pair $p(e_i, s_j; \sigma') = \sigma_i' > \sigma_i = p(e_i, s_j; \sigma')$. Since the expected return to each match is decreasing in match distance this further implies that

$$\sum_{s \in N(e_i, d_{ij})} \pi(e_i, s) \cdot p(e_i, s; \sigma') > \alpha \sum_{s \in N(e_i, d_{ij})} \pi(e_i, s) \cdot p(e_i, s; \sigma) > \sum_{s \in N(e_i, d_{ij})} \pi(e_i, s) \cdot p(e_i, s; \sigma)$$

The ex ante expected return is increasing in σ_i.

Q.E.D.

Proposition 3. *For any entrepreneur e_i and competing entrepreneur e_k, the effect of a shift in the competitor's network probability is negative for compatible competitors, but positive for less compatible competitors (i.e., high d_{ik}).*

Proof of Proposition 3.

Preliminarily, consider the effect on competing entrepreneur e_k's match probabilities of a shift in own network probability from σ_k to σ_k'. From Proposition 2, her probability of a perfectly compatible match with specialist $s_k = e_k$, $p_{k,k} = \sigma_k$ shifts to $p_{k,k}' = \sigma_k'$. By Proposition 2 step 2, however, the change in $p_{k,k \pm l}$ (i.e., the probability of an $l+1$ ranked match) depends on the value of σ_k, her competitors' σ_{-k} and the value of *l*. There is some distance beyond which the probability of a match for e_k is decreasing in σ_k and the sign of $\frac{\partial p_{k,k \pm l}}{\partial \sigma_k}$ switches.

Next, consider the effect on entrepreneur e_i's match probabilities of a shift in competitor e_k's network probability from σ_k to σ_k'. Assume $e_i \neq e_k$. There are two types of effects: (1) The competitive externality of a shift in competitor's probability and (2) the subsequent local effect of any shift in $p(e_i,s)$ on e_i's more distant matches. From Lemma 2, the offsetting cumulateive effect on e_i's more distant matches is strictly less than the initial shift effect on $p(e_i,s_j)$. It follows that we must only consider the direct effect of $p(e_k,s_j)$ on $p(e_i,s_j)$. Two important implications emerge form Lemma 3. (i) A shift in $p(e_k,s_j)$ only effects $p(e_i,s_j)$ (i.e., $\frac{dp(e_i,s_j)}{dp(e_k,s_j)} \neq 0$) if $d_{ij} \geq d_{kj}$ and $e_k \in N(s_j,d_{ij})$. (ii) If $e_k \in N(s_j,d_{ij})$ then $p(e_k,s_j)$ has a negative effect on $p(e_i,s_j)$:

$$\frac{dp(e_i,s_j)}{dp(e_k,s_j)} = -\sigma_i \cdot \left(1 - \sum_{s \in N(e_i,d_{ij})} p(e_i,s;\sigma)\right)\left(1 + \sum_{e_m \in N(s_j,d_{ij}) - N(s_j,d_{kj})} \frac{dp(e_m,s_j)}{dp(e_k,s_j)}\right) < 0$$

For entrepreneur e_i and specialist s_j there is a competitive externality at effect if $e_k \in N(s_j,d_{ij})$, or if $d_{kj} \leq d_{ij}$.

Turning to the relationship between and $p(e_k,s_j)$ and a shift in σ_k. By Proposition 2, the value of $\frac{\partial p(e_k,s_j)}{\partial \sigma_k}$ is positive for close matches, diminishing in distance but turns, and negative for matches beyond some threshold distance $\bar{d}(e_k;\sigma)$.

Next, consider the effect of a shift in σ_k on two types of entrepreneurs. First, suppose $e_k = e_i + \frac{1}{m}$, so the entrepreneurs are neighboring competitors. When $e_k = e_i + \frac{1}{m}$ an increase in σ_k has no effect on $p(e_i,s \mid s = e_k)$, but always has a negative effect on e_i's next highest valued match $p(e_i, s \mid s = e_k)$. It follows that the subsequent indirect effects will not offset this initial shift in the second highest valued match. The expected surplus return is strictly lower for e_i.

Next suppose $e_k = e_i + \frac{l}{m}$ for $l > 1$. Now an increase in σ_k has no effect on $p(e_i, s \mid s = e_k)$ or e_i's next highest valued match $p(e_i, s \mid s = e_k)$. For entrepreneur e_i, the closest effect of a shift in σ_k is in competition over some specialist s_j such that $s_j \neq e_k$ but $d_{ij} \geq d_{kj}$. The impact of $p(e_k, s_j)$ on $p(e_i, s_j)$ is negative, but now the impact of σ_k on $p(e_k, s_j)$ is negative or positive depending on the value of σ_k and the distance d_{kj}. For every pair (e_k, s_j) such that $\frac{p(e_k, s_j)}{\sigma_k} < 0$ the cumulative impact on $p(e_i, s_j)$ is positive. It follows that the expected return for e_i may be increasing in σ_k. *Q.E.D*

Proposition 4. *In a complete network, there exists a unique stable equilibrium match M^* which is positively assortative (i.e., strictly monotonic in agent types) and socially optimal.*

Proof of proposition 4.

Any match such that at least one pair of agents closest to each other are not matched is unstable since there is at least one efficient deviation. Consider some match M' where at least one match {ij'} is not assortative, $d_{ij'} > 0$. Then e_i optimally deviates to match with $s_i = e_i$ instead of $s_{j'}$. Similarly, there is some entrepreneur $e_i' =$

s_j' such that s_j' prefers to match with e_i'. In this case, there is some mutually preferred, feasible matching. When she knows each specialist, the revenue-maximizing entrepreneur prefers the specialist whose skill is perfectly compatible.

In the complete network $g_{ij} = 1$ for all pairs $\{ij\}$. Every entrepreneur has a link to every specialist. By Lemma 1, each entrepreneur most prefers a match with her perfectly compatible specialist, $d_{ij} = 0$. Similarly, each specialist most prefers a match with his closest entrepreneur. $\pi_{ij} = \pi*$ for all *ij*. There is no agent who chooses to deviate from this distance minimizing matching. The assortative, strictly monotonic matching *M** is pairwise stable and individually rational. Further, since $\pi_{ij} > \max\left(\pi_{i'j}, \pi_{ij'}, \pi_0\right)$ it is socially optimal.

Next, consider any other match $M' \neq M$. There must be at least one match $\{ij'\}$ such that $d_{ij'} > 0$ and $\pi(d_{ij}) < \pi*$. Then e_i wishes to deviate by matching with $s_j = e_i$. Under matching *M'*, specialist s_j is either unmatched or matched with some entrepreneur e_i' such that $d_{i'j} > 0$ with associated payoff $\pi(d_{ij}) < \pi*$. Specialist *s* also prefers to deviate and match with e_i. Then any matching *M'* is unstable. There is no other pairwise stable match. *Q.E.D.*

Lemma A1. *Under a uniform, homogeneous network in equilibrium, the probabilities for every feasible match of a distance d are equal. Further, these match probabilities are decreasing in distance: for any {ij} such that* $d_{ij} = d$ *then* $p_{ij} = p(d)$ *where* $d = 0, \frac{1}{m}, \frac{2}{m}, \ldots,$ *and for any* $d' > d$ *then* $p(d') < p(d)$.

Proof of Lemma A1. There are three steps to this proof. Step 1 shows the existence of a symmetric equilibrium in which every match probability of a given distance is equal. This is the only stable, rational equilibrium. Under this equilibrium, the probabilities of a match are decreasing in match distance.

In a uniform, homogeneous network of *m* entrepreneurs and *m* specialists with uniform network degree $\sigma = \sigma_i$ for all *i*, there are m^2 possible equilibrium match probabilities, defined as follows:

- $p_{ij} = \sigma$ for all *{ij}* such that $e_i = s_j$ and $d_{ij} = 0$
- $p_{ij} = \sigma \cdot \left(1 - p_{i,i} - p_{i,i\mp1}\right) \cdot \left(1 - p_{j,j} - p_{j\pm1,j}\right)$ for all *{ij}* such that $s_j = e_i \pm \frac{1}{m}$ and $d_{ij} = \frac{1}{m}$
- $p_{ij} = \sigma \cdot \left(1 - p_{i,i} - p_{i,i\mp1} - p_{i,i\mp1} - p_{i,i\mp2}\right) \cdot \left(1 - p_{j,j} - p_{j\pm1,j} - p_{j\mp1,j} - p_{j\pm2,j}\right)$ for all *{ij}* $s_j = e_i \pm \frac{2}{m}$ and $d_{ij} = \frac{2}{m}$
- …

Suppose all match probabilities for each unique distance d_0 be equal and represented as $p(d_0)$ for $d_0 = 0, \frac{1}{m}, \frac{2}{m}, \ldots$. Replacing these values into the system of equations defined above:

- $p(0) = \sigma$

- $p(\frac{1}{m}) = \sigma \cdot \left(1 - \sigma - p(\frac{1}{m})\right) \cdot \left(1 - \sigma - p(\frac{1}{m})\right)$

- $p(\frac{2}{m}) = \sigma \cdot \left(1 - \sigma - 2p(\frac{1}{m}) - p(\frac{2}{m})\right) \cdot \left(1 - \sigma - 2p(\frac{1}{m}) - p(\frac{2}{m})\right)$

- …

- $p\left(\frac{1}{2}\right) = \sigma \cdot \left(1 - p(0) - 2p(\frac{1}{m}) - 2p(\frac{2}{m}) - ... - 2p\left(\frac{m-2}{2}\right)\right)^2$ if **value of *m* is even**, or

  $$p\left(\frac{(m-1)/2}{m}\right) = \sigma \cdot \left(1 - p(0) - 2p(\frac{1}{m}) - 2p(\frac{2}{m}) - ... - 2p\left(\frac{(m-3)/2}{m}\right) - p\left(\frac{(m-1)/2}{m}\right)\right)^2$$

  if **value of *m* is odd**.

Since $p(d) \leq 1$, starting with $p(0) = \sigma$, there is a unique solution to the system of simultaneous equations in which each probability is a function of σ.

Suppose there exists an equilibrium such that the match probabilities for some distance d_0 are not equal. For this to be true there must be some entrepreneur who strictly prefers one specialist at distance d_0 over her the other specialist at distance d_0. This is a contradiction.

Q.E.D.

Proposition A1. *In the case of homogeneous, uniform network probabilities, an entrepreneur's preference over a match is decreasing in distance, while her competition for any match is increasing.*

Proof of proposition A1. Propositions 1a and 1b show that the preferences over matches is decreasing in distance. We must further show that, in the case of uniform, homogeneous network probabilities, competition is weakly decreasing in distance for each entrepreneur.

For entrepreneur e_i, the probability of loosing a match with specialist s_j to a competitor is

$$\sum_{e \in N(s_j, d_{ij})} p(e, s_j; \sigma) \text{ where } N(s_j, d_{ij}) = \left[e \mid s_j - d_{ij} \le e \le s_j + d_{ij} \right] \setminus \{s_j\}$$

Similarly, the probability of loosing a match with specialist s_l to a competitor is

$$\sum_{e \in N(s_j, d_{il})} p(e, s_j; \sigma) \text{ where } N(s_l, d_{il}) = \left[e \mid s_l - d_{il} \le e \le s_l + d_{il} \right] \setminus \{s_l\}$$

From Proposition AI, the match probabilities of every feasible match at distance d are equal for any $d = \left\{0, \frac{1}{m}, \frac{2}{m}, \ldots\right\}$. Then, e_i's competition for specialist s_j is:

$$\sum_{e \in N_{s_j}^{d_{ij}}} p(e, s_j; \sigma) = p(d_{ij}) + p(d_{ij} - \frac{1}{m}) + \ldots + p(\frac{1}{m}) + p(0) + p(\frac{1}{m}) + \ldots + p(d_{ij} - \frac{1}{m}) \tag{1.6}$$

and her competition for specialist s_l is:

$$\sum_{e \in N_{s_l}^{d_{il}}} p(e, s_l; \sigma) = p(d_{il}) + p(d_{il} - \frac{1}{m}) + \ldots + p(\frac{1}{m}) + p(0) + p(\frac{1}{m}) + \ldots + p(d_{il} - \frac{1}{m}) \tag{1.7}$$

Then, for any e_i, s_j and s_l such that $d_{ij} < d_{il}$, $\sum_{e \in N_{s_l}^{d_{il}}} p(e, s_l; \sigma) > \sum_{e \in N_{s_j}^{d_{ij}}} p(e, s_j; \sigma)$

and the competition entrepreneur e_i faces at specialist s_l is greater than the competition she faces at s_j. Q.E.D.

Proof of Proposition 5.

Under Proposition 4, the optimal social welfare is $W = m\pi(0)$, where each entrepreneur partners with her perfectly compatible specialist with probability one, $p(e_i,s_j) = \left(1 \,|\, e_i = s_j\right)$. In the case of an incomplete, homogeneous network of network connectedness $\sigma < 1$ each entrepreneur partners with her perfectly compatible specialist with probability σ, $p(e_i,s_j) = \left(\sigma \,|\, e_i = s_j\right)$. Since the expected return to a perfectly compatible partnership is strictly higher than any other partnership, it follows that the expected return to an incomplete network is less than optimal for each entrepreneur and the social welfare is strictly less than the optimal.

Suppose the network connectedness increases from σ to σ'. Then the probability of a perfectly compatible match increases from $p(e_i,s_j) = \left(\sigma \,|\, e_i = s_j\right)$ to $p(e_i,s_j) = \left(\sigma' \,|\, e_i = s_j\right)$, and the probabilities over less compatible matches shift accordingly. It follows that the expected social welfare also increases.

When $\sigma = \frac{1}{m}$, there always exists some probability that two agents are connected to the same entrepreneur, so the probability of a match is actually less than $\frac{1}{m}$. It follows that expected social welfare is less than the random network without replacement. *Q.E.D.*

Example (*m* = 3). When $m = 3$, the match probability matrix P is:

$$P = \begin{bmatrix} \sigma_1 & \sigma_2(1-p_{22}-p_{23})(1-p_{11}-p_{31}) & \sigma_3(1-p_{33})(1-p_{11}) \\ \sigma_1(1-p_{11})(1-p_{22}) & \sigma_2 & \sigma_3(1-p_{33}-p_{13})(1-p_{22}-p_{12}) \\ \sigma_1(1-p_{11}-p_{12})(1-p_{33}-p_{23}) & \sigma_2(1-p_{22})(1-p_{33}) & \sigma_3 \end{bmatrix}$$

The probability matrix implies a system of 9 equations and 9 unknowns:

$$p_{11} = \sigma_1$$
$$p_{12} = \sigma_1 \cdot (1-p_{11}) \cdot (1-p_{22})$$
$$p_{13} = \sigma_1 \cdot (1-p_{11}-p_{12}) \cdot (1-p_{23}-p_{33})$$

$$p_{21} = \sigma_2 \cdot (1-p_{22}-p_{23}) \cdot (1-p_{11}-p_{31})$$
$$p_{22} = \sigma_2$$
$$p_{23} = \sigma_2 \cdot (1-p_{22}) \cdot (1-p_{33})$$

$$p_{31} = \sigma_3 \cdot (1-p_{33}) \cdot (1-p_{11})$$
$$p_{32} = \sigma_3 \cdot (1-p_{33}-p_{31}) \cdot (1-p_{12}-p_{22})$$
$$p_{33} = \sigma_3$$

Example (*m* = 4). When $m = 4$, the match probability matrix P is:

$$\begin{bmatrix} \sigma_1 & \sigma_2 \cdot (1-\sum_{j=2,3} p_{2j}) \cdot (1-\sum_{i=1,4} p_{i1}) & \sigma_3 \cdot (1-\sum_{j=2,3,4} p_{3j}) \cdot (1-\sum_{i=1,2,4} p_{i1}) & \sigma_4 \cdot (1-p_{44}) \cdot (1-p_{11}) \\ \sigma_1 \cdot (1-p_{11}) \cdot (1-p_{22}) & \sigma_2 & \sigma_3 \cdot (1-\sum_{j=3,4} p_{3j}) \cdot (1-\sum_{i=1,2} p_{i2}) & \sigma_4 \cdot (1-\sum_{j=1,3,4} p_{4j}) \cdot (1-\sum_{i=1,2,3} p_{i2}) \\ \sigma_1 \cdot (1-\sum_{j=1,2,4} p_{1j}) \cdot (1-\sum_{i=2,3,4} p_{i3}) & \sigma_2 \cdot (1-p_{22}) \cdot (1-p_{33}) & \sigma_3 & \sigma_4 \cdot (1-\sum_{j=1,4} p_{4j}) \cdot (1-\sum_{i=2,3} p_{i3}) \\ \sigma_1 \cdot (1-\sum_{j=1,2} p_{1j}) \cdot (1-\sum_{i=3,4} p_{i4}) & \sigma_2 \cdot (1-\sum_{j=1,2,3} p_{2j}) \cdot (1-\sum_{i=1,3,4} p_{i4}) & \sigma_3 \cdot (1-p_{33}) \cdot (1-p_{44}) & \sigma_4 \end{bmatrix}$$

The probability matrix implies a system of 16 equations and 16 unknowns:

$$p_{11} = \sigma_1$$
$$p_{12} = \sigma_1 \cdot (1-p_{11}) \cdot (1-p_{22})$$
$$p_{13} = \sigma_1 \cdot (1-p_{11}-p_{12}-p_{14}) \cdot (1-p_{23}-p_{33}-p_{43})$$
$$p_{14} = \sigma_1 \cdot (1-p_{11}-p_{12}) \cdot (1-p_{34}-p_{44})$$

$$p_{21} = \sigma_2 \cdot (1-p_{22}-p_{23}) \cdot (1-p_{11}-p_{41})$$
$$p_{22} = \sigma_2$$
$$p_{23} = \sigma_2 \cdot (1-p_{22}) \cdot (1-p_{33})$$
$$p_{24} = \sigma_2 \cdot (1-p_{21}-p_{22}-p_{23}) \cdot (1-p_{34}-p_{44}-p_{14})$$

$$p_{31} = \sigma_3 \cdot (1-p_{32}-p_{33}-p_{34}) \cdot (1-p_{11}-p_{21}-p_{41})$$
$$p_{32} = \sigma_3 \cdot (1-p_{33}-p_{34}) \cdot (1-p_{12}-p_{22})$$
$$p_{33} = \sigma_3$$
$$p_{34} = \sigma_3 \cdot (1-p_{33}) \cdot (1-p_{44})$$

$$p_{41} = \sigma_4 \cdot (1-p_{44}) \cdot (1-p_{11})$$
$$p_{42} = \sigma_4 \cdot (1-p_{41}-p_{43}-p_{44}) \cdot (1-p_{12}-p_{22}-p_{32})$$
$$p_{43} = \sigma_4 \cdot (1-p_{41}-p_{44}) \cdot (1-p_{23}-p_{33})$$
$$p_{44} = \sigma_4$$

Example ($m = 5$). When $m = 5$, the match probability matrix P is:

$$\begin{bmatrix} \sigma_1 & \sigma_2(1-\sum_{j=2,3} p_{2j})\cdot(1-\sum_{i=1,5} p_{i2}) & \sigma_3(1-\sum_{j=2,3,4,5} p_{3j})\cdot(1-\sum_{i=1,2,4,5} p_{i1}) & \sigma_4(1-\sum_{j=3,4,5} p_{4j})\cdot(1-\sum_{i=1,2,5} p_{i1}) & \sigma_5(1-p_{55})\cdot(1-p_{11}) \\ \sigma_1(1-p_{11})\cdot(1-p_{22}) & \sigma_2 & \sigma_3(1-\sum_{j=3,4} p_{3j})\cdot(1-\sum_{i=1,2} p_{i2}) & \sigma_4(1-\sum_{j=1,3,4,5} p_{4j})\cdot(1-\sum_{i=1,2,3,5} p_{i2}) & \sigma_5(1-\sum_{j=1,4,5} p_{5j})\cdot(1-\sum_{i=1,2,3} p_{i2}) \\ \sigma_1(1-\sum_{j=1,2,5} p_{1j})\cdot(1-\sum_{i=2,3,4} p_{i3}) & \sigma_2(1-p_{22})\cdot(1-p_{33}) & \sigma_3 & \sigma_4(1-\sum_{j=4,5} p_{4j})\cdot(1-\sum_{i=2,3} p_{i3}) & \sigma_5(1-\sum_{j=1,2,3,4} p_{5j})\cdot(1-\sum_{i=1,2,4,5} p_{i5}) \\ \sigma_1(1-\sum_{j=1,2,3,5} p_{1j})\cdot(1-\sum_{i=2,3,4,5} p_{i4}) & \sigma_2(1-\sum_{j=1,2,3} p_{2j})\cdot(1-\sum_{i=3,4,5} p_{i4}) & \sigma_3(1-p_{33})\cdot(1-p_{44}) & \sigma_4 & \sigma_5(1-\sum_{j=1,5} p_{5j})\cdot(1-\sum_{i=3,4} p_{i4}) \\ \sigma_1(1-\sum_{j=1,2} p_{1j})\cdot(1-\sum_{i=4,5} p_{i5}) & \sigma_2(1-\sum_{j=1,2,3,4} p_{2j})\cdot(1-\sum_{i=1,3,4,5} p_{i5}) & \sigma_3(1-\sum_{j=2,3,4} p_{3j})\cdot(1-\sum_{i=2,4,5} p_{i5}) & \sigma_4(1-p_{44})\cdot(1-\sum_{i=3,4} p_{i5}) & \sigma_5 \end{bmatrix}$$

The probability matrix implies a system of 25 equations and 25 unknowns:

$$p_{11} = \sigma_1$$
$$p_{12} = \sigma_1(1-p_{11})\cdot(1-p_{22})$$
$$p_{13} = \sigma_1(1-p_{11}-p_{12}-p_{15})\cdot(1-p_{23}-p_{33}-p_{43})$$
$$p_{14} = \sigma_1(1-p_{11}-p_{12}-p_{15}-p_{13})\cdot(1-p_{24}-p_{34}-p_{44}-p_{54})$$
$$p_{15} = \sigma_1(1-p_{11}-p_{12})\cdot(1-p_{45}-p_{55})$$

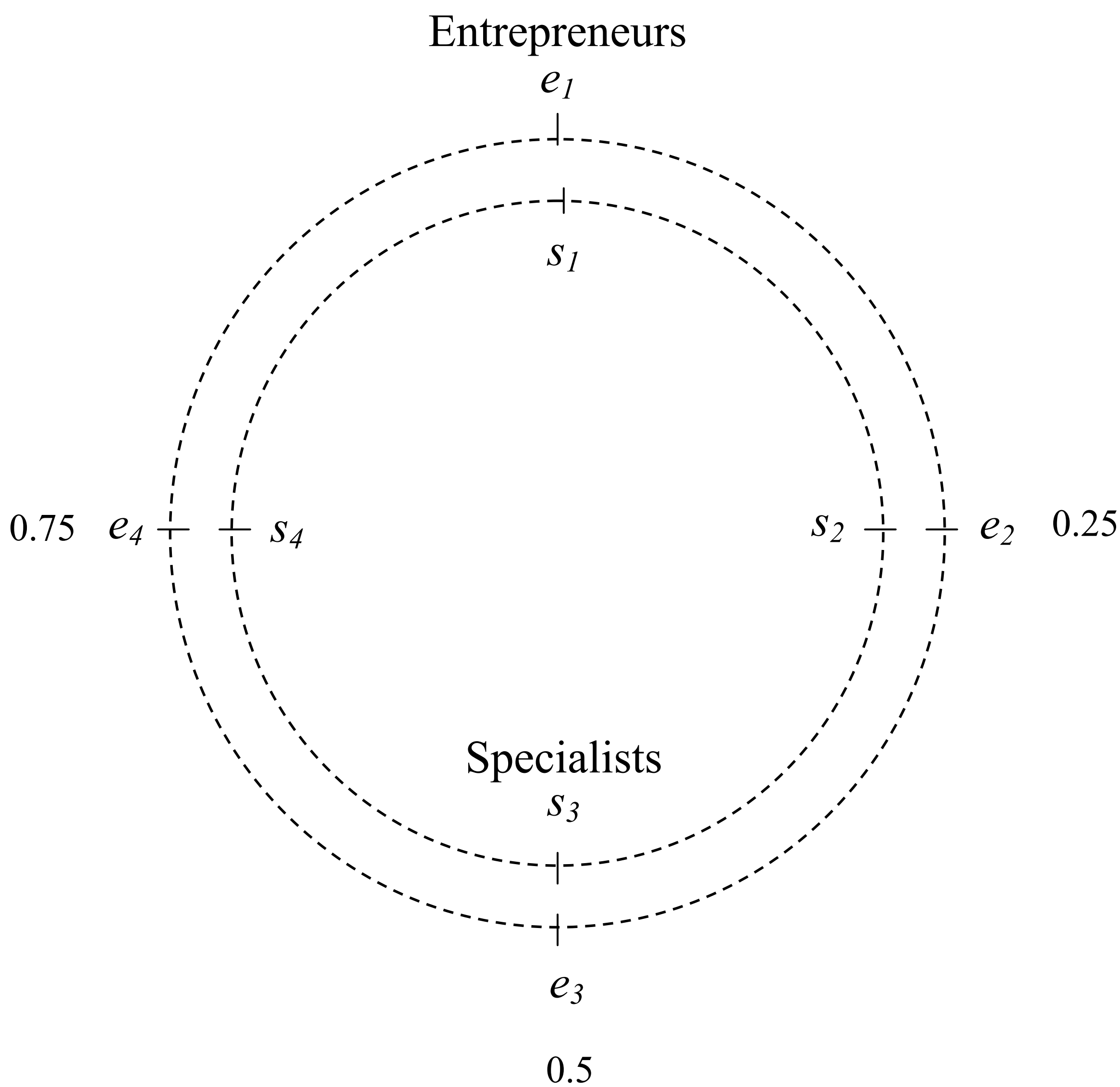
$$p_{21} = \sigma_2(1-p_{22}-p_{23})\cdot(1-p_{11}-p_{31})$$
$$p_{22} = \sigma_2$$
$$p_{23} = \sigma_2(1-p_{22})\cdot(1-p_{33})$$
$$p_{24} = \sigma_2(1-p_{21}-p_{22}-p_{23})\cdot(1-p_{34}-p_{44}-p_{54})$$
$$p_{25} = \sigma_2(1-p_{21}-p_{22}-p_{23}-p_{24})\cdot(1-p_{35}-p_{45}-p_{55}-p_{15})$$

$$p_{31} = \sigma_3(1-p_{32}-p_{33}-p_{34}-p_{35})\cdot(1-p_{11}-p_{21}-p_{41}-p_{51})$$
$$p_{32} = \sigma_3(1-p_{33}-p_{34})\cdot(1-p_{12}-p_{22})$$
$$p_{33} = \sigma_3$$
$$p_{34} = \sigma_3(1-p_{33})\cdot(1-p_{44})$$
$$p_{35} = \sigma_3(1-p_{32}-p_{33}-p_{34})\cdot(1-p_{55}-p_{45}-p_{15})$$

$$p_{41} = \sigma_4(1-p_{43}-p_{44}-p_{45})\cdot(1-p_{11}-p_{21}-p_{51})$$
$$p_{42} = \sigma_4(1-p_{43}-p_{44}-p_{45}-p_{15})\cdot(1-p_{12}-p_{22}-p_{32}-p_{52})$$
$$p_{43} = \sigma_4(1-p_{44}-p_{45})\cdot(1-p_{23}-p_{33})$$
$$p_{44} = \sigma_4$$
$$p_{45} = \sigma_4(1-p_{44})\cdot(1-p_{55})$$

$$p_{51} = \sigma_5(1-p_{55})\cdot(1-p_{11})$$
$$p_{52} = \sigma_5(1-p_{54}-p_{55}-p_{51})\cdot(1-p_{12}-p_{22}-p_{32})$$
$$p_{53} = \sigma_5(1-p_{54}-p_{55}-p_{51}-p_{52})\cdot(1-p_{13}-p_{23}-p_{33}-p_{43})$$
$$p_{54} = \sigma_5(1-p_{55}-p_{51})\cdot(1-p_{34}-p_{44})$$
$$p_{55} = \sigma_5$$

Figure 1. Example with $m = 4$



Chapter 2
The Value of Networking:
Searching Endogenous Social Networks

1 Introduction

When is networking efficient? More specifically, when should an entrepreneur invest in building new relationships rather than rely on a qualified individual she already knows? With the rise of online social networks and low-cost communication tools, there is increasing focus on the process of networking to obtain useful, career related information and foster relationships with potential business partners. Networking is especially critical for entrepreneurs who use their network connections to raise capital, gain access to existing resources, and connect with cofounders or employees with complimentary business or technical skills. This paper focuses on three fundamental questions about an entrepreneur's networking incentives. First, given that networks have positive effects for the individual entrepreneur, as well as significant externalities on her competitors, when should an entrepreneur invest in networking to expand her contacts? Second, given the negative externalities competitors exert on each other through this network, is the level of networking chosen by an entrepreneur socially optimal? Finally, given these incentives, what types of policies improve the efficiencies of networks?

In Chapter 1, I develop a discrete matching model with heterogeneous values to analyze how an entrepreneur's and her competitors' network positions effect stable partnerships in a network. The model, however, is intractable for understanding the incentives for building network connections. In this paper, I integrate a reduced form version of this original model into a dynamic model to analyze networking incentives. Results from this paper explain why efforts to create general, regional business networks for entrepreneurs have been less successful than highly specialized networks, such as university-industry business networks, in which participation is costly. Targeted networks that are more costly to search and build connections within minimize inefficiencies and increase payoffs.

Results from the discreetly distributed model of stable entrepreneurial partnerships in a network setting suggest an entrepreneur is better off with a denser network. An entrepreneur, therefore, should have strong incentives to expand the number of people she knows, particularly those with whom she is most compatible. By fostering new relationships with friends of friends, attending alumni events of former employers or schools, or even becoming a member of social or business group or association, the entrepreneur increases her chances of finding a business partner or capital investor. But these activities are also costly and exert externalities on other entrepreneurs in the market. The critical question is when is social networking privately efficient or socially optimal?

Using a setup similar to the discreet model, I introduce a two-period game of incomplete information. Both sets of agents are distributed continuously around a circle. Entrepreneurs and specialists are initially linked through a set of undirected, first-degree network connections.[17] In the first period, each entrepreneur may choose to expand her network through costly investment. Examples of networking include

[17] Undirected ties simply represent some preexisting social or professional relationship between agents. Since such relationships emerge over time and often by chance, they are exogenous for the purpose of this model.

meeting new individuals through existing contacts, or attending social events for alumni groups and trade associations. She cannot choose which specific specialists she will meet. In the second period, the market of entrepreneurs and specialists undergo a stable matching process. Each entrepreneur partners with an available, complementary specialist in her updated network. In the presence of a negative network externality, I show there is both a unique symmetric Nash equilibrium and social planner's solution.

In order to estimate the ex ante probability of stable matches in the first period, I use the results from the discreet, pairwise stable match probabilities in Chapter 1. Based on these previous results, the probability of a stable match between any entrepreneur and specialist is dependent on the entrepreneur's own network densities, the network density of similar, competing entrepreneurs and the compatibility between the entrepreneur and specialist. While the pairwise stable match probabilities from the discreetly distributed model are more accurate, they are also intractable for investigating the incentives for networking. In the present paper, to determine the optimal network investment in the first period, agents estimate the likely outcomes of the second period stable matching.

The functional form for these pairwise stable match probabilities reflect the results from the pairwise stable matching in the discreetly distributed model. According to Chapter 1, the entrepreneur's likelihood of a compatible match is increasing in her own network density, while the externalities from her nearby competitors decreases her probability of a match. The externality for less compatible competitors is more complex. In the Appendix of this paper, I discuss two functional forms for the second period pairwise stable matching that capture the most interesting components of the matching process in detail. For the purposes of tractability, I focus on a functional form with an entirely negative externality.

Under strictly negative externalities, in comparison to the optimal network investment, the non-cooperative entrepreneurs over-invest in networking. Entrepreneurs build more network connections than under the cooperative solution. Intuitively, if each entrepreneur networks less, there are fewer connections and less competition for each specialist. Each partnership is likely to be less compatible. The added compatibility of the noncooperative outcome, however, does not make up for the marginal investment cost or its negative impact on other entrepreneurs. The effect is akin to a networking 'rat race.' Faced with a denser network and more connected competitors, entrepreneurs invest more in their own network just to find any compatible partner. Unless a partnership is highly compatible, the specialist will likely partner with a more well-suited entrepreneur.

Seeing that the most efficient networks are costly and less connected, there are two circumstances that may improve the optimality of noncooperative networks. First, the presence of positive externalities between less compatible agents diminishes the negative effects of networking. This more indirect positive impact likely will not outweigh the entire negative effect of more direct competition. Alternatively, a network search that is targeted to find the most compatible candidates is more efficient and may result in more compatible matches. This is consistent with observations that programs to encourage more general business networks in a region are not particularly successful, whereas policies to developed specialized networks for a specific industry appear very effective.

The context of this paper is an industry with significant entrepreneurial entry and a preexisting network of ties among entrepreneurs and specialist experts. For instance, Chatterji (2007) discusses the biomedical device industry in which physician entrepreneurs have an idea for a new biomedical device, but need a biomedical engineer with the expertise to develop the product. The physician networks to find an expert with complimentary skills.[18] Similarly, a university scientist with a technology for the private market must decide whether to partner with a business colleague he already knows or search for someone whose expertise is closer to his needs.

The model also applies to entrepreneurs more broadly. For instance, a manager of an existing business with a new project must consider whether to rely on existing employees or conduct an outside search for a specialist with additional skills. If he chooses to search outside, he must either rely on those he knows or search more broadly. Other applications extend well beyond science and technology into artistic skilled fields, including Broadway production teams or designers starting a new fashion house.[19]

This paper draws from elements of strains of literature. The modeling and topic are close in spirit to Lee and Schwartz (2009). These authors analyze interviewing choices in two-sided markets, but are more concerned with the impact of interview

[18] For more information, Chatterji and Fabrizio (2008) explore the role of physician-entreprenurs in the medical device industry.
[19] For more information on networks of Broadway musical producers, see Uzzi and Spiro (2005)

overlap than efficiency of search choices. In addition, the present paper addresses questions to the employer search literature, for instance Atakan (2006). Both chapters consider questions similar to those found in the endogenous network search literature surveyed extensively in Jackson (2005). The present modeling borrows elements from literature related to matching with costly signaling, particularly Hoppe, Molduvanu and Sela (2005). Results are related to previous literature on over entry and investment including inefficient signaling (Akerlof, 1976), over-entry in a market setting (Mankiw and Whinston, 1986) and auctions with entry (Levin and Smith, 1994).

The paper proceeds as follows. Section 2 presents an overview of the model. Section 3 compares the equilibrium and social planner's solutions. Section 4 considers the efficiency of extensions such as positive externalities and targeted search. Section 5 concludes by considering future research. The appendix has an extensive discussion of the functional forms and equilibrium under more general forms and circumstances.

2 Model

a. Agents

Consider two equal sets of risk neutral entrepreneurs and specialists. Agents in each set are located around a circle with circumference normalized to one and distributed uniformly. The set of entrepreneurs are denoted $E = \{e_1, e_2, .e_3, ...\}$. Each entrepreneur has an exogenous idea for a new project.[20] The value of any individual entrepreneur $e_i \in [0,1]$ denotes the skill requirements of her project and represents her location around the circle. In order to enter the market and create a startup, the entrepreneur must find a complementary specialist partner to develop the technology.[21]

The set of specialist partners is $S = \{s_1, s_2, s_3, ...\}$ where each element $s_j \in [0,1]$ represents the skills of the j^{th} specialist for j = 1, 2, 3, …, m. Both sets of agents are ordered numerically around the circle such that $e_1 < e_2 < e_3 < ... < e_m$ and $s_1 < s_2 < s_3 < ... < s_m$. Since both sets of agents are located around a circle, any interval represents a continuum of agents whose skills are most similar to each other.

The minimum circumference distance $d_{ij} = \min\left[\left|e_i - s_j\right|, 1 - \left|e_i - s_j\right|\right]$ between any entrepreneur e_i and specialist s_j represents the pair's skill *compatibility* or dissimilarity. With a circle or circumference equal to one, the maximum dissimilarity is one half and $d_{ij} = d(e_i, s_j) \leq \frac{1}{2}$ for all ij pairs. The narrower the circumference between complementary agents, the more the specialist's skills meet the entrepreneur's requirement. For any pair of agents e_i and s_j, the subset $E(s_j, d_{ij})$ denotes the interval of entrepreneurs who are more compatible with specialist s_j than

[20] The model applies to cases in which the entrepreneur has an existing firm and is looking to implement a new project.

[21] This interpretation of entrepreneurs and specialists is consistent with Lazear (2005). A successful entrepreneur is a generalist with some understanding of the industry, who is capable of identifying a good idea, hiring the experts and managing project development. For the purposes of this paper, each entrepreneur defines her project by the type of specialist who is most compatible to complete the task and develop the technology.

is e_i. This subset includes all entrepreneurs located on the interval of distance d_{ij} in either direction of s_j. Throughout this paper, I refer to these agents who are similarly located as compatible. Two entrepreneurs who are trying to partner with the same specialist are competitors.

Definition. For entrepreneur e_i, matching with specialist s_j, the entrepreneur e_k is a *nearby* competitor of e_i for s_j if and only if $d_{ij} \quad d_{kj}$.

b. Network and Networking Costs

Entrepreneurs most often use their network of preexisting relationships to obtain information about available specialists. A network of undirected ties represents the preexisting social or professional relationships between entrepreneurs and specialists. Since these links typically emerge over time, they are exogenous at $t = 0$. At $t = 1$, any entrepreneur may expand her network through a costly search for weak ties. In either period, when $t = 0,1$, the element $g_{ij}^t = 1$ if agents e_i and s_j are *connected* and $g_{ij}^t = 0$ if they are otherwise *unconnected*. The vector $g_i^t = (g_{i1}^t, g_{i2}^t, g_{i3}^t, ...)$ represents entrepreneur e_i's set of *local network* connections. The matrix G^t represents the entire graph of all g_{ij} values.

Similar to Chapter 1, this paper also focuses on the *network density*, or proportion of specialists with which each entrepreneur is connected. Entrepreneur e_i's local network vector g_i^t implies an exogenous *local network density.* This paper will focus primarily on the impact of uniform networks in which all entrepreneurs are equally connected. As in the previous chapter, σ_i represents original, exogenous network density in period zero. This initial σ_i only effects the total and marginal costs of search. In the first period each entrepreneur e_i chooses whether to expand her network density to δ_i.[22]

The total cost of networking is $c(\delta_i;\sigma_i) = (\delta_i - \sigma_i)c(\sigma_i)$ where $c(\sigma_i)$ is the constant marginal cost of networking and $\delta_i > \sigma_i$. Total search cost is dependent on the density of the original exogenous network and the endogenous updated network. Each entrepreneur enters the networking phase with some connections with which to build new contacts. She may talk to people in her initial network, attend alumni events, and consult trade associations. Since forming a relationship with any specialist requires one period, each entrepreneur can only use her preexisting ties to make additional connections. The marginal cost of networking is dependent on her initial network, rather than the target density. Marginal cost is decreasing in density $c'(\sigma_i) < 0$ since the fewer people she knows, the harder it is to meet new people. There is no cost to not networking.

c. Timing and Information

Information is revealed through costly networking over two periods. Initially (i.e., $t = 0$), agents realize their own types and local network densities. Each entrepreneur learns her project requirement $e_i \in [0,1]$ and her exogenously endowed initial

[22] If an entrepreneur's network density varies by location, assume the probability of a connection is dependent on the specialist type, $\sigma_i(s_j)$. Networking expands the local network proportionally to $\delta_i(s_j)$.

network degree $\sigma_i \in [0,1]$. Each specialist knows his skill type $s_j \in \left[0,1\right]$. The distribution of types and the vector of network densities $\sigma = \left(\sigma_1, \sigma_2, \sigma_3, ...\right)$ are common knowledge.

At $t = 1$, each entrepreneur e_i chooses if she will invest in costly networking to extend her network density to $\delta_i > \sigma_i$. Each entrepreneur e_i may increase her network density to $\delta_i \in \left(\sigma_i, 1\right]$. By the end of the first period, networking results in network graph G^1 with a vector of densities $\delta = \left(\delta_1, \delta_2, \delta_3, ...\right)$. This vector of densities is common knowledge. Based on the entrepreneur's own network density, and the network densities of all other entrepreneurs, in $t = 1$ each entrepreneur e_i expects a pairwise stable match with specialist s_j with probability $p(e_i, s_j; \delta_i, \delta_{-i})$.

At $t = 2$, entrepreneurs and specialists match according to network G^1 and the individually rational and pairwise stable matching mechanism, $\mu : E \to S$. Each partnership $\{ij\}$ receives $\pi(e_i, \mu(e_i); G^1)$ where $s_j = \mu(e_i)$. Partners split the payoff according to Nash bargaining, subject to the outside options.

d. Payoffs and Assumptions

Let $\pi_{ij} = \pi(e_i, s_j) = \pi(d_{ij})$ denote the pair-specific expected return to a partnership between any e_i and s_j. In addition to increasing in compatibility, payoffs follow three basic assumptions.

Assumption 1 (Supermodular Expected Returns). *Given any two entrepreneurs e_i and e_k, and any two specialists s_h and s_j such that* $d_{ij} = \min_d \{d_{ij}, d_{ih}, d_{kj}, d_{kh}\}$ *and* $d_{kh} = \max_d \{d_{ij}, d_{ih}, d_{kj}, d_{kh}\}$ *then it must be that:*

$\pi(e_i, s_j) + \pi(e_k, s_h) \geq \pi(e_i, s_h) + \pi(e_k, s_j)$.

This supermodularity condition is sufficient, but not necessary, to ensure the existence of a pairwise stable match in which each agent partners with the closest available complementary counterpart. Under supermodular payoffs, the expected surplus to a more compatible partnership increases in compatibility. Supermodularity is a fairly standard assumption. Matching models with heterogeneous match values, including Becker (1973), often rely on supermodularity or complementarity to ensure assortative matching.[23] Supermodularity applies nicely to entrepreneurial applications. For startups in high skill industries, the risks to a poor partnership are high, but the payoffs to a highly compatible match increase dramatically.[24]

The following three assumptions restrict analysis to relevant cases. A positive outside option for the entrepreneur ensures that she can make a costly investment in her network, and that each agent matches with the most compatible available counterpart. The third assumption requires partners to have a network connection. The final cost assumption ensures that any entrepreneur can always afford to invest in

[23] For a complete discussion of supermodularity and complementarity assumptions, see Amir (2003).

[24] Consider the payoff function $\pi(e_i, s_j) = R\left(1 - d_{ij}\right)^{\rho}$, where $R > 0$ is the optimal revenue under perfect compatibility and $\rho > 1$ reflects the uncertainty of a poor match.

networking. Total investment cost must be less than the entrepreneur's outside option: $C(\delta_i, \sigma_i) \leq \underline{u}$. Since the marginal cost is highest for an entrepreneur with no preexisting network $\overline{c} = c(0)$, the maximum total cost is $C(1,0) = \overline{c}$.

Assumption 2 (Positive Outside Option). *The outside option for any partnership is* $\underline{\pi} = \underline{u} + \underline{w}$, *where* $\underline{u} > 0$ *and* $\underline{w} > 0$ *are the outside options for entrepreneur's and specialist's respectively.*

Assumption 3 (Network Connection). *If* $g_{ij} = 1$ *then* $\pi_{ij} = \pi(e_i, s_j) = \pi(d_{ij})$. *Without this assumption, there is an infinitely negative return to an unconnected set of agents such that if* $g_{ij} = 0$ *then* $\pi(e_i, s_j) = -\infty$.

Assumption 4 (Affordable Investment). The maximum marginal cost must be less than the entrepreneur's outside option: $\overline{c} \leq \underline{u} \leq \underline{\pi}$.

3 Solution Concept

Prior to the second period stable partnering, each entrepreneur $e_i \in E$ chooses whether to expand her network density to $\delta_i \in (\sigma_i, 1]$ in order optimize the ex ante expected value function in the next period. The solution choice is, therefore, the vector of optimal network densities $\delta = \{\delta_1, \delta_2, \delta_3, ...\}$ for each entrepreneur.

a. Second Period Pairwise Stable Matching

I model the second period stable partnerships as a reduced form, one-to-one, pairwise stable and individually rational matching mechanism $\mu : E \to S$ between entrepreneurs and specialists. Each partnership must be mutual. If e_i partners with s_j then $\mu(e_i) = s_j$ and the inverse $\mu^{-1}(s_j) = e_i$ must also be a function. If e_i remains unmatched then $\mu(e_i) = e_i$. According to Lemma 1, under supermodularity, these stability and rationality constraints imply every agent partners with his or her most compatible and available counterpart. This match is by definition efficient.

Definition (Pairwise Stability). *A matching μ is pairwise stable if there is no pair of unmatched agents ij' or i'j who simultaneously benefit from blocking μ for some alternate matching μ'. For any pairwise stable match μ with matched pairs ij and i'j' such that $\mu(e_i) = s_j$ and $\mu(e_{i'}) = s_{j'}$, then $\mu^{-1}(s_j) = e_i$ and $\mu^{-1}(s_{j'}) = e_{i'}$.*

Definition (Efficiency). *Given a specific network G^1 the matching μ is efficient in $t = 2$ if there exists no alternate, feasible matching μ' such that:*

$$\sum_{e \in E} \pi(e, \mu'(e); G^1) \geq \sum_{e \in E} \pi(e, \mu(e); G^1)$$

Lemma 1 (Pairwise Stability). *Under pairwise stability and supermodular payoffs, any entrepreneur e_i and specialist s_j matches with the closest available agent. This is a pairwise stable, t = 2 efficient match.*

b. Ex Ante Expected Matching Payoffs at *t* = 1

By the second period matching, each entrepreneur's first period expected return is

$$v_i(\delta_i, \delta_{-i}; \sigma_i) = \pi_i(\delta_i, \delta_{-i}) - c(\delta_i; \sigma_i) \tag{1}$$

where $\pi_i(\delta_i, \delta_{-i})$ represents the ex ante expected payoff to a project

$$\pi_i(\delta_i, \delta_{-i}) = \int_{s \in S} \pi_i(s) p(e_i, s; \delta_i, \delta_{-i}) f(s) ds \tag{2}$$

The first period probability of a match between entrepreneur e_i and any specialist s_j is

$$p(e_i, s_j; \delta_i, \delta_{-i}) = \delta_i \cdot m(e_i, s_j; \delta_i) \cdot q(e_i, s_j; \delta_{-i}) \tag{3}$$

Before networking results in a specific network graph, this ex ante match probability depends on the entrepreneur's own network density δ_i and her competitors' network densities δ_{-i}. Function (3) corresponds to the specific match probabilities in Chapter 1.

Using a discreetly distributed model, Chapter 1 calculates the specific match probability for each entrepreneur-specialist pair. The ex ante probability of a partnership between entrepreneur e_i and specialist s_j is a function of the vector of network densities for the entrepreneur and her nearby competitors, $\left(\delta_i, \delta_{e \in E(s_j, d_{ij})}\right)$. In other words, this probability is a function of three factors:

$$
\begin{aligned}
p_{ij} = {} & \underbrace{prob\left\{e_i \text{ connected to } s_j\right\}}_{(i)} * \underbrace{prob\left\{s_j \text{ is } e_i\text{'s best available match}\right\}}_{(ii)} \\
& * \underbrace{prob\left\{s_j \text{ is available to } e_i\right\}}_{(iii)}
\end{aligned}
$$

The terms of equation (3) correspond to these three factors. The network density δ_i is the probability that the pair has a connection. Then $m_i(\cdot)$ represents e_i's probability of being available to specialist s_j. Similarly, $q_j(\cdot)$ represents specialist s_j's network externality and probability of not partnering with a more compatible entrepreneur.

For example, when m = 4 in Chapter 1, the match probabilities for entrepreneur e_1 are

$$
\begin{aligned}
p_{11} &= \sigma_1 \\
p_{12} &= \sigma_1 \cdot \left[1-\sigma_1\right] \cdot \left[1-\sigma_2\right] \\
p_{13} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2)) \cdot (1-\sigma_1(1-\sigma_4) \cdot (1-\sigma_3'(1-\sigma_3')))\right] \\
&\quad \cdot \left[(1-\sigma_3')(1-\sigma_2(1-\sigma_2)) \cdot (1-\sigma_4(1-\sigma_4) \cdot (1-\sigma_4(1-\sigma_1)))\right] \\
p_{14} &= \sigma_1 \cdot \left[(1-\sigma_1)(1-\sigma_1(1-\sigma_2))\right] \cdot \left[(1-\sigma_4) \cdot (1-\sigma_3'(1-\sigma_3'))\right]
\end{aligned}
$$

where σ_i is the exogenous network density for each entrepreneur e_i. The first term of each expression represents e_1's network connectedness. The following first bracket is the probability that e_1 is available. The second bracket is the probability the specialist is available. There is a complex interdependence between entrepreneurs that depends on their relative compatibility.

For analyzing networking incentives, however, these interdependencies and externalities between entrepreneurs become computationally intractable and must be simplified. The current paper resolves this problem by incorporating the key properties of the true match function into a simplified function in the general form of expression (3). Consider the following functional form for the match probability between e_i and s_j:

$$p(e_i, s_j; \delta_i, \delta_{-i}) = \delta_i \cdot \left(1 - \delta_i\right)^{2d_{ij}} \cdot \left(1 - \bar{\delta}_{\mathrm{E}(s_j, d_{ij})}\right)^{2d_{ij}} \tag{4}$$

where $\bar{\delta}_{E(s_j, d_{ij})}$ is the mean network density for the subset of all nearby competing entrepreneurs $e_{-i} \in E(s_j, d_{ij})$. Equation (4) behaves similar to the actual match probabilities in Chapter 1.

The expression $\delta_i(1-\delta_i)^{2d_{ij}}$ represents the total impact of an entrepreneur's own network density. According to the original discretely distributed model, for entrepreneur e_i the probability of not matching with a more compatible agent is decreasing and concave in network density. Further, the probability she is available is strictly decreasing in dissimilarity (i.e., increasing in compatibility) with any specialist. Accordingly, expression (4) is decreasing and concave in network density d_i Œ$\left(0, 1\right)$, as well as decreasing in dissimilarity d_{ij} or increasing in compatibility. Since the circumference between the entrepreneur and specialist equals the proportion of specialists in the market who are more compatible with e_i. than s_j., $2d_{ij}$ represents the proportion of agents who are more compatible with e_i-and s_j than they are to each other.

Also similar to Chapter 1, there is a tradeoff between the likelihood of a network connection and the probability of a partnership. There is a threshold compatibility $\bar{d}$ such that as an entrepreneur becomes more connected, the probability of a match that is more compatible than $\bar{d}$ increases, while the probability of less compatible matches decreases. Further, this threshold distance $\bar{d}$ is decreasing in the entrepreneur's connectedness.

The reduced expression for network externality is $q(e_i, s_j; \delta_{-i}) = \left(1 - \bar{\delta}_{E(s_j, d_{ij})}\right)^{2d_{ij}}$.[25] Similar to the original function, this expression is a function of all nearby competitors $E(s_j, d_{ij})$, and it is also a decreasing, concave function of all competing entrepreneurs' local networks. It implies the probability any specialist is available is also decreasing in compatibility. This expression, however, includes a strictly negative network externalities and does not incorporate the non-monotonic nature of original network externalities. See the appendix for an extensive discussion of expression (4).

4 Results

A solution to the model is a vector of network investment choices in the first period that optimize the second period value function. Under a uniform network there exists both a non-cooperative symmetric Nash equilibrium (SNE) and socially optimal

---

[25] Where $\bar{\delta}_{E(s_j, d_{ij})} = \sum_{e \in E(s_j, d_{ij})} \delta(e)\lambda(e, s_j)$ such that $1 = \sum_{e \in E(s_j, d_{ij})} \lambda(e, s_j)$. The weights for $\bar{\delta}_{E(s_j, d_{ij})}$, $\lambda(e_i, s_j)$ are nondecreasing in compatibility so that the most compatible competitors have the greatest impact.

social planner's solution. By definition, a uniform network implies each entrepreneur's exogenous network density is equal, $\sigma_i = \sigma$ for all e_i. In turn each entrepreneur has equal marginal cost to networking. Comparing these two solutions reveals that non-cooperative entrepreneurs overinvestment in networking.

a. Nash Equilibrium

In the first period, each entrepreneur e_i simultaneously chooses her own network connectedness δ_i to optimize her second period expected net return:

$$v_i^*(\delta_i,\delta_{-i}) = \max_{\delta_i}\left[\int_{s\in S}\pi_i(s)p_i(s;\delta_i,\delta_{-i})f(s)ds - c_i(\delta_i;\sigma_i)\right] \tag{5}$$

where the probability of a match between entrepreneur e_i and any specialist s_j is (4).

Proposition (Entrepreneur's Solution). *Under a uniform network with supermodular payoffs and match probability* $p(e_i,s_j;\delta_i,\delta_{-i}) = \delta_i\cdot\left(1-\delta_i\right)^{2d_{ij}}\cdot\left(1-\bar{\delta}_{\mathrm{E}(s_j,d_{ij})}\right)^{2d_{ij}}$*, for any marginal cost c there exists a symmetric Nash equilibrium in which each entrepreneur invests to achieve network density* $\bar{\delta}$ *if* $\bar{\delta} > \sigma$*. She receives expected net return*

$$\bar{v} = \pi - c\bar{\delta} = 2\frac{\bar{\delta}^2}{1-\bar{\delta}}\int_{s\in S}\pi(s)\cdot d(e,s)(1-\bar{\delta})^{4d(e,s)}f(s)ds$$

Intuitively, each entrepreneur expands her network until her marginal benefit of networking is equal to the marginal cost, c. She does not account for the impact she has on her competitors because she is only concerned with her own expected payoff.

Since every entrepreneur's original network density is equal, each entrepreneur's symmetric best response is also equal and $\bar{\delta} = \delta_i(\bar{\delta})$ for all *i*. The first order equilibrium condition

$$c = \int_{s \in S} \pi_i(s) \cdot (1-\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s)\frac{\bar{\delta}}{1-\bar{\delta}}\right) f(s)ds \tag{6}$$

holds for each entrepreneur e_i. Three key observations prove for any marginal cost *c* there is a unique $\bar{\delta} = \delta(\bar{\delta})$ that solves the equilibrium condition. The marginal payoff is positive and greater than *c* as δ approaches zero and the network is unconnected. This marginal payoff is also strictly decreasing in δ until the marginal payoff is zero as δ approaches one and the network is fully connected. For any marginal cost, therefore, there is a unique equilibrium value of $\bar{\delta}$. If the initial network density is already greater than the equilibrium investment, it is not beneficial to invest in networking.

The network investment $\bar{\delta}$ is decreasing in marginal cost *c*. If marginal cost is low, entrepreneurs invest in a highly-connected network in which $\bar{d} > \frac{1}{2}$. In this case, networking narrows entrepreneurs' search. There exists some threshold distance $2\bar{d}_{ij} = \frac{1 - d_i}{d_i}$ such that the probability of more a compatible match is increasing and the probability of a less compatible match is decreasing. When the marginal cost of networking is high, the network investment results in an unconnected network in which $\bar{\delta} < \frac{1}{2}$. Each entrepreneur networks to increase her probability of any match.

b. Social Planner's Solution

In the first period, the Social Planner's symmetric problem is to choose the network investment for each entrepreneur, $\delta^* = \{\delta_1^*, \delta_2^*, \delta_3^*, ...\}$. In the following period, the socially optimal match optimizes the sum of all net expected returns. The social planner's choice problem is

$$V(\delta^*) = \max_{\delta=[\delta_1,\delta_2,\delta_3,...]} \sum_{i=1,2,3,...} \left(\pi_i(\delta_i,\delta_{-i}) - c_i(\delta_i;\sigma_i)\right) \tag{7}$$

The social planner simultaneously chooses the optimal vector of network densities for all entrepreneurs $d^* = \{d_1^*, d_2^*, ..., d_i^*, ...\}$. In contrast to the self-interested entrepreneur, the social planner's solution accounts for the externalities each entrepreneur exerts.

Proposition (Social Planner's Solution). *Under a uniform network with supermodular payoffs and match probability*

$p(e_i,s_j;\delta_i,\delta_{-i}) = \delta_i \cdot (1-\delta_i)^{2d_{ij}} \cdot \left(1-\bar{\delta}_{\mathrm{E}(s_j,d_{ij})}\right)^{2d_{ij}}$ *, for any marginal cost c there is a unique, symmetric Social Planner solution in which each entrepreneur invests to reach $\delta^*$and receives expected value:*

$$v^* = \pi - c\delta^* = 4\frac{\delta^{*2}}{1-\delta^*}\int_{d=0}^{\frac{1}{2}} d\pi(d)(1-\delta^*)^{4d} g(d)dd$$

Each entrepreneur networks up to the density δ^* at which marginal cost is equal to the marginal welfare impact of each entrepreneur's network. The proof is similar to the symmetric Nash equilibrium. The first order condition requires that δ^* satisfy:

$$c = \int_{s \in S} \pi_i(s)(1-\delta^*)^{4d(e_i,s)} \left(1 - 4d(e_i,s)\frac{\delta^*}{1-\delta^*}\right) f(s)ds \qquad (8)$$

This condition is similar to (6), but the marginal value of networking now includes the entrepreneur's negative externality. Similar to the Nash equilibrium condition, the right hand side of this expression is greater than *c* for values of δ^* close to zero. When the right hand side of condition (8) is positive, it is also strictly decreasing in δ^* and approaching zero as the value of δ^* approaches one.

c. Comparing Results

When a social planner chooses all networks simultaneously, he takes into account the externality each entrepreneur's network has on her nearby competitors. The social planner's marginal social return to each entrepreneur's investment includes the entrepreneur's marginal payoff, as well as the externality she exerts over competitors. The entrepreneur, on the other hand, invests until only her marginal ex ante expected return is equal to cost. The individual entrepreneur invests more because she fails to account for her effect on other entrepreneurs and in turn experiences a higher marginal return to her network.

Proposition (Compare First Best and Entrepreneur's Reaction). *Under a uniform network with supermodular payoffs and negative externalities each entrepreneur over-invests in her network so that* $\overline{\delta} > \delta^*$.

For both the social planner and the entrepreneur, the marginal return from the network investment equals marginal cost. By condition (6) the individual entrepreneur networks until the marginal expected payoff to her startup is equal to the marginal cost:

$$c = \int_{s \in S} \pi_i(s) \cdot (1-\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s) \frac{\bar{\delta}}{1-\bar{\delta}} \right) f(s) ds$$

A social planner also accounts for the entrepreneur's externality in condition (8):

$$c = \int_{s \in S} \pi_i(s)(1-\delta^*)^{4d(e_i,s)} \left(1 - 4d(e_i,s) \frac{\delta^*}{1-\delta^*} \right) f(s) ds$$

Since it includes the negative externality, the right hand side of (8) decreases in δ faster than the right hand side of (6). For the two expressions to both be equal to *c*, it must be that $\bar{\delta} > \delta^*$, or when the competitive networking is higher than the socially optimal networking. When this occurs, there is overinvestment.

Overinvestment is greatest when low costs motivate extensive networking. When δ is low, the difference between $\left(4d(e_i,s) \frac{\delta^*}{1-\delta^*} \right)$ and $\left(2d(e_i,s) \frac{\bar{\delta}}{1-\bar{\delta}} \right)$ is relatively minimal. As δ increases, both the externality and the disparity between the optimal networking choice increases. This suggest networks in which the cost of building new relationships is low, such as Facebook or Linkedin, are the most inefficient for high-skill labor searches. By limiting the incentives for intensive search, networks in which it is costly to build new relationships encourage more efficient network investment levels.

A second implication of this overinvestment is that it is more efficient for entrepreneurs to rely on less compatible partnerships with a lower expected return than search for a perfect partner. Under a more intensive search the expected gross return to is higher, but the added cost of networking exceeds this benefit. While greater network density increases an entrepreneur's chance of a more compatible match, this benefit does not outweigh the costs.

5 Extensions

The previous results prompt a critical question. If networks are more efficient when they are most costly to use, how can policies improve either the efficiency or expected return to using a business network? Two possibilities come to mind. First, if a network actually exhibits positive externalities to increasing network densities, then there is less disparity between the socially optimal and competitive solutions. Second, networks in which it is more costly to build relationships may be more informative. An effective network may limit inefficiencies and motivate more compatible partnerships by targeting searches.

a. Targeted Networking

One possibility is the objective of social policies and valuable networking institutions is to help target search so that an entrepreneur only networks with her most compatible candidates. Such targeted networking may occur through exclusive association memberships, alumni clubs for business schools or other graduate programs, or headhunters. There is a tradeoff. Participation in a network that allows for more a more informative search is also generally more costly. Participating in an alumni network requires both attending the school and often annual donations. Exclusive associations often require significant annual dues. Headhunters charge fees.

Suppose each entrepreneur only meets specialists within her $\frac{1}{r}$ most compatible specialists. In other words, the entrepreneur only networks with $\frac{1}{r}$ proportion of the circle. Each entrepreneur chooses to increase her network density to $r\delta_i$ of the subset of specialists, $s \in S\left[e_i, \frac{1}{r}\right]$, within circumference distance $d = \frac{1}{2r}$ of herself. The probability that she partners with any of these most compatible specialists is

$p(e_i, s_j; \delta_i, \delta_{-i}) = r\delta_i \cdot \left(1 - r\delta_i\right)^{2d_{ij}} \cdot \left(1 - r\bar{\delta}_{\mathrm{E}(s_j, d_{ij})}\right)^{2d_{ij}}$.

The entrepreneur's network is now denser for the $\frac{1}{r}$ most compatible specialists who also yield the highest ex ante expected return and are least susceptible to being competed away. The updated condition for the symmetric Nash equilibrium is

$$c = r \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s)\frac{r\bar{\delta}}{1 - r\bar{\delta}}\right) f(s)ds \tag{6a}$$

The cooperative social planner's solution is now

$$c = r \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\delta^*)^{4d(e,s)} \left(1 - 4d(e,s)\frac{r\delta^*}{1 - r\delta^*}\right) f(s)ds \tag{6b}$$

Limiting the maximum distance to $\frac{1}{2r}$ eliminates the least compatible matches for which the discrepancy between $2d(e,s)\frac{r\bar{\delta}}{1 - r\bar{\delta}}$ and $4d(e,s)\frac{r\delta^*}{1 - r\delta^*}$ is greatest.

Targeted networks reduce frictions between less compatible competitors because any two entrepreneurs no longer compete for the same subset of specialists. It also increases the expected compatibility of a match and expected return.

Proposition. *Under a uniform network with targeting, supermodular payoffs and negative externalities, as network targeting becomes more specific ex ante expected partnerships are more compatible.*

For the same marginal cost, entrepreneurs search at least as intensively in the most compatible regions under targeted search. The total network-wide density and total cost, however, may be lower. This implies the probability of a most compatible match is greater, but the total cost may be lower. Participation in more targeted networks are often costly and, there are fewer negative externalities from excessive networking. This will also increase the likely efficiency of a match.

Proposition (Compare General to Targeted Cases). *Under a uniform network with supermodular, negative network externalities and constant marginal cost, the network density in the search region is greater under a targeted search when* $r > 1$*, than under a general search when* $r = 1$*:* $\bar{\delta} < r\delta$ *and* $\delta^{*} < r\delta$.

b. Positive Externalities

What is particularly interesting about the discreetly distributed case in Chapter 1 is, while there is a coordination problem at close range, such density actually alleviates competition for less compatible entrepreneurs. By introducing a nonlinear externality that is negative for compatible competitors and positive at a longer range, the present optimization problem becomes considerably less tractable. Nevertheless, this positive externality may counteract some of the inefficiencies of the competitive equilibrium. It is, however, likely these positive effects will not make up for the entire discrepancy.

Referring to the discreet solutions from Chapter 1, the likely positive network densities occur between the least compatible agents and, therefore, have less impact. Consistent with this intuition, the functional form

$$p_{ij}(\delta_i, \delta_{-i}) = \delta_i \cdot \left(1-\delta_i\right)^{2d_{ij}} \cdot \prod_{k \in E(s_j, d_{ij})} \left(1-\delta_k(1-\delta_k)^{2d_{ij}}\right)$$

incorporates the negative externality that is similar to Chapter 1. Conducting a similar analysis to the above using this form does not appear to have a significant positive externality. See Appendix for the extended discussion of this form.

In the symmetric Nash equilibrium, the short-range negative impact of competition is paramount. Suppose e_i's increased network density has a positive effect on her less compatible competitor, e_k. In the symmetric case, e_k will also be impacted by the stronger negative effects of her closer competitors increasing their network density. The negative externality is still stronger. In addition, since the less compatible partnerships become irrelevant in the targeted network case, the positive effects are also diminished.

6 Discussion and Conclusion

Generalized networking creates a "rat race" in which entrepreneurs must create a dense network to find a highly compatible specialist, even if a specialist whom they are already know is a sufficient partner. Despite the total welfare loss to the network and the individual entrepreneurs, specialists gain under the network equilibrium when the cost of search is born entirely by entrepreneurs. The gross return from the non-cooperative solution is higher, so both partners receive a higher payoff. For the entrepreneur, however, this added payoff is diminished by the cost of investment. Fiercer competition creates better opportunities for specialists in successful startups with no investment.

The consequences of excessive networking are similar to those of business stealing. Additional networking decreases the expected return to close competitors by diminishing their likelihood of a compatible partnership. In response, the competitors must also increase their network to ensure they can also find a compatible specialist who will be available. While there is a welfare gain to a more compatible specialist, the gain is more than offset by the investment cost. This is a coordination problem. Each entrepreneur may know an acceptable specialist, but this specialist will probably not be available. In turn, the entrepreneur must network to meet the most compatible specialists.

In real world applications, this manifests itself as a dense social network in which individual agents have many connections, but few successfully find a partner. This is consistent with a conversation with Auren Hoffman, an entrepreneur and the highly connected president of Rapleaf in San Francisco, CA. Despite his dense network of contacts throughout the industry and politics, it is nearly impossible to find a skilled computer engineer. If one of his friends knows of a good engineer, he would likely already hire him. Even when a network is dense with links, it may still be difficult to compete when competitors have the same network connections.

The most efficient networks are those in which participation is costly, but that disseminate more specialized information. Entrepreneurs are forced to compete at high costs to get even better partners. This is consistent with the real-world observations about successful university-industry research networks I discuss in more detail in Chapter 1. To foster more effective, efficient business networks policies should create networks in which participation is costly, but that are also informative and successful in building relationships between the most compatible individuals.

There are a number of interesting research extensions to consider. Given the implications for these coordination problems, future extensions include analyzing the impact of cooperation between entrepreneurs who agree to share contacts and not compete. How does the equilibrium change when the distribution of agents is not equal? Is this problem dissipated when there is an overabundance of specialists? What if entrepreneurs choose more than one specialist?

Future research should also focus on how this overinvestment impacts agents in a heterogeneous network where individuals are not initially equally connected. More focus on how individuals respond depending on their own connectedness is illuminating. Such research may also explain how an existing network may hinder or help newcomers in an industry. These questions are, in turn, particularly pertinent to innovation and new ideas within an industry.

7 Appendix

a. Match Probabilities and Functional Form

From Chapter 1, the probability of a match between any entrepreneur e_i and specialist s_j is the product of three factors:

$$p(e_i, s_j) = prob\left\{\text{network connection}\right\} \cdot prob\left\{e_i \text{ most prefers } s_j \text{ from available partners}\right\}$$
$$\cdot prob\left\{s_j \text{ most prefers } e_i \text{ from available partners}\right\}$$

First, the agents must have a network connection. Otherwise the cost of a partnership is infinitely high and the match does not satisfy individual rationality. Next, the agents must be available to each other. Any entrepreneur or specialist is available if she or he does not already partner with a more compatible agent. From her subset of available specialists, the entrepreneur partners with specialist s_j if he is her most compatible option. Similarly, specialist s_j partners with his most compatible entrepreneur in his available subset.

A functional form that approximates this probability must account for these local network effects and the network externalities from nearby entrepreneurs as follows:

$$p_{ij}(\delta_i, \delta_{-i}) = \delta_i \cdot m_{ij}\left(\delta_i\right) \cdot q_{ij}\left(\delta_{E(s_j, d_{ij})}\right) \tag{9}$$

where $\delta_{E(s_j, d_{ij})}$ is the subset of network densities for all nearby, competing entrepreneurs $e_{-i} \in E(s_j, d_{ij})$ who are more compatible with specialist s_j than entrepreneur e_i. Here $m_i(\square)$ represents e_i's probability of not partnering with a more compatible specialist. The probability specialist s_j partners with a more compatible entrepreneur, $q_j(\square)$, is a function of the entrepreneur's nearby competitors' network densities. Two possible forms are:

Example 1. $p_{ij}(\delta_i,\delta_{-i}) = \delta_i \cdot \left(1-\delta_i\right)^{2d_{ij}} \cdot \left(1-\bar{\delta}_{\mathrm{E}(s_j,d_{ij})}\right)^{2d_{ij}}$

Example 2. $p_{ij}(\delta_i,\delta_{-i}) = \delta_i \cdot \left(1-\delta_i\right)^{2d_{ij}} \cdot \prod_{k\in E(s_j,d_{ij})} \left(1-\delta_k(1-\delta_k)^{2d_{ij}}\right)$

i. Impact of local network

Consider the function

$$m(e_i,s_j;\delta_i) = (1-\delta_i)^{2d(e_i,s_j)} \tag{7}$$

for the conditional probability that e_i is available to match with s_j if the pair have a network connection. From Chapter 1, the probability of not matching with a more compatible agent is decreasing and concave in network density. Further, e_i is strictly less likely to be available for a less compatible match. The probability that e_i is available is decreasing in the circumference distance to a partner. Accordingly, function (7) is indeed decreasing in $d \in \left[0,\frac{1}{2}\right]$, and decreasing concave for any values of d_i Œ$\left(0,1\right)$.[26]

[26] The first and second derivatives of (7) are:
$\frac{\partial m(e_i,s_j;\delta_i)}{\partial \delta_i} = -2d(e_i,s_j)(1-\delta_i)^{2d(e_i,s_j)-1} < 0$ and
$\frac{\partial^2 m(e_i,s_j;\delta_i)}{\partial \delta_i^2} = -2d(e_i,s_j)\left(1-2d(e_i,s_j)\right)(1-\delta_i)^{2d(e_i,s_j)-2} < 0$ for all values of
$d \in \left[0,\frac{1}{2}\right]$.

The form of (7) follows the spirit of the entrepreneur's own local network effect in Chapter 1. Consider the special case of the discreet model when entrepreneur e_i has network density $\delta_i \in (0,1)$ and all other entrepreneurs are unconnected, $\delta_{-i} = 0$. In this case, e_i experiences no competition or network externality. The entrepreneur's outcome is dependent only on her own network and the probabilities over network connections are

$$\begin{aligned}
&p_{i(1)} = \delta_i \\
&p_{i(2)} = \delta_i \cdot (1 - p_{i,i}) = \delta_i \cdot (1 - \delta_i) \\
&p_{i(3)} = \delta_i \cdot (1 - p_{i,i} - p_{i,i+1}) = \delta_i \cdot (1 - \delta_i - \delta_i \cdot (1 - \delta_i)) = \delta_i \cdot (1 - \delta_i)^2 \\
&p_{i(4)} = \delta_i \cdot (1 - p_{i,i} - p_{i,i+1} - p_{i,i-1}) = \delta_i \cdot (1 - \delta_i - \delta_i \cdot (1 - \delta_i) - \delta_i \cdot (1 - \delta_i)^2) = \delta_i \cdot (1 - \delta_i)^3 \\
&\qquad \ldots \\
&p_{i(n)} = \delta_i \cdot (1 - \delta_i)^n
\end{aligned}$$

Here n represents e_i's incompatibility ranking of specialists (i.e., 1 is the most compatible), which s a function of circumference distance.

Along a continuous distribution, the circumference distance between the entrepreneur and specialist equals the proportion of specialists in the market who are more compatible with e_i. For any entrepreneur-specialist pair, e_i and s_j, $2d_{ij}$ of the agents are more compatible. Independent of competition, the probability that e_i is linked and available for partnership $\{ij\}$ is $p_{ij}(\delta_i, 0) = \delta_i (1 - \delta_i)^{2d_{ij}}$. In turn,

$m_{ij}(\delta_i) = \dfrac{p_{ij}}{\delta_\iota} = (1 - \delta_i)^{2d_{ij}}$ is the conditional probability e_i is available to a specialist d_{ij}.

The expression $\delta_i (1 - \delta_i)^{2d_{ij}}$ behaves similarly to the discreet model. In Chapter 1 there is a tradeoff between the likelihood of a network connection and the probability of a partnership. There is an incompatibility or distance threshold, $\bar{d}$. As an entrepreneur becomes more connected, the probability of a match that is more compatible than $\bar{d}$ increases, while the probability of less compatible matches decreases. Further, this threshold distance is dependent on the entrepreneur's connectedness.

This same threshold exists here. The probability of being connected and available is increasing for any match $\{ij\}$ such that $2d_{ij} < \frac{1-\delta_i}{\delta_i}$ and decreasing otherwise.[27] When an entrepreneur increases her network, the probability she knows any specialist, δ_i, is higher, but the probability she is available $m_{ij}(\delta_i)$ might, in turn be lower.

There is a key difference in the experience of connected and unconnected agents. For any unconnected entrepreneur whose d_i £ $\frac{1}{2}$, increasing the network increases the probability of a match at every distance. An unconnected entrepreneur broadens her search possibilities and becomes more likely to achieve any partnership as her network increases. A well-connected entrepreneur, whose network is $d_i > \frac{1}{2}$, effectively narrows her search by building new relationships. She is more likely to partner with a more compatible specialist, while her probability of a less compatible specialist falls.

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[27] The derivative $\frac{d}{d\delta_i}\left(\delta_i(1-\delta_i)^{2d_{ij}}\right) = (1-\delta_i)^{d_{ij}}\left(1-2d_{ij}\frac{\delta_i}{1-\delta_i}\right)$ is positive whenever $2d_{ij} < \frac{1-\delta_i}{\delta_i}$. Further, if $\delta_i \le \frac{1}{2}$ then $\frac{1-\delta_i}{\delta_i} < 1$ and $2d_{ij} < \frac{1-\delta_i}{\delta_i}$ for all d_{ij}.

ii. Network externality

The function $q_i(\cdot)$ is the conditional probability that s_j is available to match with e_i. It represents the network externality that entrepreneur e_i's competitors exert on her likelihood of a match. I consider two possible functional forms for the externality that incorporate the following properties found in Chapter 1. First, the probability that a match is available is a function of all nearby, more compatible entrepreneurs $e_k \in E(s_j, d(e_i, s_j))$. Second, the externality that any entrepreneur exerts is increasing in her compatibility with s_j. Consider two equally connected entrepreneurs, e_k and e_m. If $d_{kj} < d_{mj}$ entrepreneur e_k exerts a greater impact on the probability that s_j is available. Third, the probability any specialist is available to some entrepreneur is decreasing and concave in incompatibility. Finally, Example 2 also incorporates the nonmonotonicity of the externality.

When e_k extends her network it only affects e_i's chances over the half of agents who are closer to e_k than e_i. In other words, $\frac{\partial q_{ij}(\delta_{-i \in E[s, d(e_k, s)})}{\partial \delta_k} \neq 0$ if and only if $e_k \in E(s_j, d_{ij})$. When e_i and e_k are searching for similar specialist types (i.e., d_{ik} is low), e_k exerts an externality on e_i's probability over her partnering with some of her most compatible specialists. As the compatibility between competitors approaches ½, the change effects the probabilities over e_i's least compatible specialists. Lemma 2 summarizes this result.

Lemma 2. *For any e_i and e_k there exists some $\left[\underline{s},\overline{s}\right]$ around e_i such that if*

$s_j \in \left[\underline{s},\overline{s}\right]$ then $d_{ij} < d_{kj}$ and $\dfrac{\partial q_{ij}(\delta_{-i})}{\partial \delta_k} = 0$. If $s_j \notin \left[\underline{s},\overline{s}\right]$ then $\dfrac{\partial q_{ij}(\delta_{-i})}{\partial \delta_k} \neq 0$. The circumference of this interval is ½ the interval of the circle.

Proof. Consider any two entrepreneurs e_i and e_k located on a circle. Let the minimum circumference distance between entrepreneurs e_i and e_k be $d_{ij}^{\min} = \min\left[\left|e_i - e_k\right|, 1 - \left|e_i - e_k\right|\right]$, let the counter maximum distance be $d_{ij}^{\max} = \max\left[\left|e_i - e_k\right|, 1 - \left|e_i - e_k\right|\right]$ and note that $d_{ij}^{\max} = 1 - d_{ij}^{\min}$. For the purposes of this proof, suppose e_i is located $d_{ij}^{\min}$ clockwise to e_k. The same logic holds for all cases.

Along the narrow interval between e_i and e_k (i.e., the interval with circumference $d_{ij}^{\min}$), a specialist s_j is more compatible with e_i if $d_{ij} < d_{kj}$ *or* $s_j \in \left[e_i, e_i + \frac{1}{2}d_{ij}^{\min}\right]$. Similarly, along the wide interval a specialist s is more compatible with e_i if $s \in \left[e_i - \frac{1}{2}d_{ij}^{\max}, e_i\right]$. Then the interval of all specialists more compatible with e_i is $s \in \left[e_i - \frac{1}{2}d_{ik}^{\max}, e_i + \frac{1}{2}d_{ij}^{\min}\right]$. Define $\underline{s} = e_i - \frac{1}{2}d_{ik}^{\max}$ and $\overline{s} = e_i + \frac{1}{2}d_{ij}^{\min}$ such that $s \in \left[\underline{s},\overline{s}\right]$. The circumference of this subset of specialists is $\frac{1}{2}d_{ik}^{\max} + \frac{1}{2}d_{ij}^{\min} = \frac{1}{2}\left(1 - d_{ik}^{\min} + d_{ij}^{\min}\right) = \frac{1}{2}$. *Q.E.D.*

Competitive Externality Example 1. Consider $q(e_i, s_j; \delta_i) = \left(1-\bar{\delta}\right)^{2d_{ij}}$ where $\bar{d}_{E(s_j,d_{ij})}$ is the weighted average connectedness of the nearby entrepreneurs who are more compatible with s_j.[28] Similar to $m(e_i, s_j; \delta_i)$, the exponent $2d_{ij}$ represents the proportion of entrepreneurs who are more compatible with s_j than e_i. This network externality function is a decreasing, concave function of the average network density of all competing entrepreneurs. The first and second order effects are decreasing for all $k, m \in E(s_j, d_{ij})$.[29] Otherwise, by Lemma 2 there is no competitive externality from the other half of specialists.

Similar to the discreet model of Chapter 1, the ex ante expected return to entrepreneur e_i is decreasing in the network density of any other entrepreneur e_k where e_k š e_i. Further, the value of this impact is a function of the supermodularity of payoffs, entrepreneur e_i's own local network density and the network density of nearby competing entrepreneurs. **T**he marginal impact on the entrepreneur's ex ante expected return is strictly negative:

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[28] For instance, let the vector $\boldsymbol{\lambda}$ reflects the weights for each less compatible entrepreneur with respect to s_j: $\bar{\delta}_{E(s_j,d_{ij})} = \sum_{e\in E(s_j,d_{ij})} \delta(e)\lambda(e,s_j)$ such that $1 = \sum_{e\in E(s_j,d_{ij})} \lambda(e,s_j)$. More compatible entrepreneurs have a greater impact relative to their density when the $\lambda(e_i, s_j)$ is increasing in compatibility.

[29]
$$\frac{\partial q(e_i,s_j;\delta_{-i})}{\partial \delta_k} = -\lambda_k 2d(e_i,s_j)\left(1-\bar{\delta}_{E(s_j,d_{ij})}\right)^{2d(e_i,s_j)-1} < 0$$
$$\frac{\partial q^2(e_i,s_j;\delta_{-i})}{\partial \delta_k^2} = -\lambda_k^2 2d_{ij}(1-2d_{ij})\left(1-\bar{\delta}_{E(s_j,d_{ij})}\right)^{2d_{ij}-2} < 0$$
$$\frac{\partial q^2(e_i,s_j;\delta_{-i})}{\partial \delta_k \partial \delta_m} = -\lambda_k\lambda_m 2d_{ij}(1-2d_{ij})\left(1-\bar{\delta}_{E(s_j,d_{ij})}\right)^{2d_{ij}-2} < 0$$

$$\frac{\partial \pi_i}{\partial \delta_k} = -\lambda_k \int_{s \in [S | d(e_k, s) < d(e_i, s)]} 2d(e_i, s_j)(1-\delta_i)^{2d(e_i, s_j)} \left(1 - \bar{\delta}_{E(s_j, d_{ij})}\right)^{2d(e_i, s_j)-1} \pi_i(s) f(s) ds \leq 0$$

where the subset of specialists only includes the half that are closer to e_k than e_i.

Competitive Externality Example 2. The functional form $q(e_i, s_j; \delta_{-i}) = \prod_{e_k \in E(s_j, d_{ij})} \left(1 - \delta_k (1-\delta_k)^{2d_{kj}}\right)$ is more complex, but integrates the nonmonotonicity property in network externalities. The probability that any entrepreneur e_k is available to s_j is $\delta_k (1-\delta_k)^{2d_{kj}}$. The counter probability that e_k is not linked or available to s_j is $1 - \delta_k (1-\delta_k)^{2d_{kj}}$. Finally, the probability that s_j is available to e_i because every more compatible entrepreneur is not connected or available to s_j is $\prod_{e_k \in E(s_j, d_{ij})} \left(1 - \delta_k (1-\delta_k)^{2d_{kj}}\right)$.

When a competitor increases her network, she exerts a negative impact on her most compatible competitors' match probability, but may increase the match probability for others. If $2d_{kj} < \frac{1-\delta_k}{\delta_k}$ then e_k has a negative impact on less compatible entrepreneurs' ability to partner with s_j. Otherwise, e_k extends her network, she has a positive effect on other competitors' probabilities of partnering with s_j. Taking the partial derivative:

$$\frac{\partial q(e_i, s_j; \delta_{-i})}{\partial \delta_k} = -(1-\delta_k)^{2d_{kj}} \left[1 - 2d_{kj} \frac{\delta_k}{(1-\delta_k)}\right] \prod_{e_m \in E\left(s_j, d_{ij}\right) \setminus \{e_k\}} \left(1 - \delta_m (1-\delta_m)^{2d_{mj}}\right)$$

The externality is negative if $2d_{kj} \leq \frac{1-\delta_k}{\delta_k}$. An unconnected entrepreneur always exerts a negative externality. The externality is positive if $2d_{kj} > \frac{1-\delta_k}{\delta_k}$. Well connected entrepreneurs have a positive effect on some competitors. This effect is increasing in her network density.

b. Proofs

Lemma 1. *Under pairwise stability and supermodular payoffs, any entrepreneur e_i or specialist s_j match with the closet available agent.*

Proof of Lemma 1. Consider any two entrepreneurs $\left[e_i, e_k\right] \in E$ and any two specialists $\left[s_h, s_j\right] \in S$ such that $d_{ij} = \min_d \left\{d_{ij}, d_{ih}, d_{kj}, d_{kh}\right\}$ and $d_{kh} = \max_d \left\{d_{ij}, d_{ih}, d_{kj}, d_{kh}\right\}$, and each agent has a network link to both opposing agents, $g_{ij} = g_{ih} = g_{kj} = g_{kh} = 1$. These assumptions imply that agents e_i and s_j are closest, or most compatible, to each other. Similarly, agents e_k and specialists s_h are least compatible. Let the entrepreneur's outside option be $\underline{u} \geq c$. Note that the network investment is always affordable. Let $\underline{w} > 0$ be the specialists outside option such that $\underline{\pi} = \underline{u} + \underline{w}$. Let $u(e_i, s_j) = u_{ij}$ represent e_i's payoff to $\{ij\}$, and $w_{ij} = w(e_i, s_j)$ represent specialist s_j's payoff to $\{ij\}$, such that $\pi_{ij} = u_{ij} + w_{ij}$.

Suppose μ represents the pairwise stable match. If the stable match is always distance minimizing, then $\mu(e_i) = s_j$ and $\mu(e_k) = s_h$. For this matching to hold, entrepreneur e_i must prefer a match with s_j, to a match with s_h. In addition, $u_{ij} > u_{ih}$. Given the outside option, it must be true that $w_{ij} \geq \underline{w}$ for any network G^1. Replacing u with the definition, $u_{ij} = \pi_{ij} - w_{ij}$ and $u_{ih} = \pi_{ih} - w_{ih}$:

$$\pi_{ij} - w_{ij} \geq \pi_{ij} - \underline{w} \geq \pi_{ih} - w_{ih}$$

Following the same logic, if s_j prefers a match with e_i to a match with e_k, $w_{ij} > w_{jk}$ then:

$$\pi_{ij} - u_{ij} \geq \pi_{ij} - \underline{u} \geq \pi_{kj} - w_{kj}$$

Combining the above two conditions and solving,

$$\pi_{ij} - w_{ij} + \pi_{ij} - u_{ij} \geq \pi_{ih} - \underline{w} + \pi_{kj} - \underline{u}$$

$$\pi_{ij} \geq \pi_{ih} + \pi_{kj} - \underline{\pi}$$

$$\pi_{ij} + \underline{\pi} \geq \pi_{ih} + \pi_{kj}$$

Finally, given $\underline{\pi} \leq \pi_{kj}$ then $\pi_{ij} + \pi_{kh} > \pi_{ih} + \pi_{kj}$. Supermodularity holds when the pairwise stable match is distance minimizing.

Now suppose μ is not distance minimizing. Then it must be true that $\{ij\} \succ_{e_i} \{ih\}$ and $\{kh\} \prec_{s_h} \{ih\}$, or $\{kj\} \succ_{s_j} \{ij\}$ and $\{kj\} \succ_{e_k} \{kh\}$. For the first case to hold, then $u_{ij} < u_{ih}$ and $w_{kh} < w_{ih}$. By definition and substitutions, this implies $\pi_{ij} - w_{ij} \leq \pi_{ij} - \overline{w} \leq \pi_{ih} - w_{ih}$ and $\pi_{kh} - u_{kh} \leq \pi_{kh} - \overline{u} \leq \pi_{ih} - u_{ih}$. Combining these conditions and solving, $\pi_{ih} + \pi_{kj} \geq \pi_{ih} + \overline{\pi} \geq \pi_{kh} + \pi_{ij}$. This contradicts the supermodular payoff assumption! Similar logic holds for the second case. *Q.E.D.*

Proposition (Entrepreneur's Solution). *Under a uniform network with supermodular payoffs and match probability* $p(e_i, s_j; \delta_i, \delta_{-i}) = \delta_i \cdot (1 - \delta_i)^{2d_{ij}} \cdot \left(1 - \overline{\delta}_{\mathrm{E}(s_j, d_{ij})}\right)^{2d_{ij}}$*, for any marginal cost c there exists a symmetric Nash equilibrium in which each entrepreneur invests to achieve network density* $\overline{\delta}$ *if* $\overline{\delta} > \sigma$*. She receives expected net return*

$$\bar{v} = \pi - c\bar{\delta} = 2\frac{\bar{\delta}^2}{1-\bar{\delta}} \int_{s \in S} \pi(s) \cdot d(e,s)(1-\bar{\delta})^{4d(e,s)} f(s) ds$$

Proof. Under a uniform, symmetric network, the entrepreneur's problem (5) reduces to choosing a best response $\delta(\bar{\delta})$ to all other entrepreneurs' symmetric choice, $\bar{\delta}$:

$$\delta_i(\bar{\delta}) = \arg\max_{\delta} \int_{s \in S} \pi_i(s) \cdot \delta(1-\delta)^{2d(e,s)}(1-\bar{\delta})^{2d(e,s)} f(s) ds - c(\delta)$$

If a best response $\delta_i(\bar{\delta})$ exists then it must satisfy the first order condition:

$$\frac{\partial}{\partial \delta} v = \int_{s \in S} \pi_i(s) \cdot (1-\delta(\bar{\delta}))^{2d(e,s)} \left(1 - 2d(e,s)\frac{\delta(\bar{\delta})}{1-\delta(\bar{\delta})}\right)(1-\bar{\delta})^{2d(e,s)} f(s) ds - c = 0 \quad (10)$$

where by the affordable investment assumption, marginal cost is $0 < c < \underline{u}$.

A Nash equilibrium is a vector of best responses $\delta_i(\bar{\delta})$ for all *i*. Given that second order condition strictly less than zero, a unique best response function does exist:

$$\frac{\partial^2}{\partial \delta^2} v = -\left(2 + \frac{\delta(\bar{\delta})}{1-\delta(\bar{\delta})}\right) \int_{s \in S} \pi_i(s) \cdot 2d(e,s)(1-\delta(\bar{\delta}))^{2d(e,s)}(1-\bar{\delta})^{2d(e,s)} f(s) ds < 0 \quad (11)$$

Recall that in the uniform network case, each entrepreneur has the same exogenous network density at $t = 0$ and therefore faces a symmetric problem. If there exists a symmetric Nash equilibrium, each entrepreneur's best response is $\bar{\delta} = \delta_i(\bar{\delta})$ for all *i*. Substituting $\bar{\delta} = \delta_i(\bar{\delta})$ into (10) the condition for a symmetric Nash equilibrium is:

$$c = \int_{s \in S} \pi_i(s) \cdot (1-\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s)\frac{\bar{\delta}}{1-\bar{\delta}}\right) f(s) ds \quad (12)$$

The SNE exists for marginal cost $0 < c < \underline{u}$ (by assumption) and if there is a unique $\bar{\delta} \in (0,1)$ such that (12) holds. Three key points proves this existence:

0. Preliminarily, the right hand side of (12) is equal to the marginal expected payoff

$$\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} = \int_{s\in S} \pi_i(s)\cdot(1-\bar{\delta})^{4d(e,s)}\left(1-2d(e,s)\frac{\bar{\delta}}{1-\bar{\delta}}\right)f(s)ds \tag{13}$$

1. There exist some values for $\delta \in (0,1)$ such that (13) is positive.

   If $\left(1-2d(e,s)\frac{\bar{\delta}}{1-\bar{\delta}}\right) > 0$ for all $2d(e,s) \le 1$ then (13) must be positive. If $\delta \le \frac{1}{2}$ then $\frac{1-\delta}{\delta} \ge 1 \ge 2d$ for all possible $d \in \left[0,\frac{1}{2}\right]$, and if $\frac{1-\delta}{\delta} \ge 2d$ then (rearranging) $\left(1-2d\frac{\delta}{1-\delta}\right) > 0$. Therefore for any $\delta \le \frac{1}{2}$, (13) must be positive.

2. For any unique value of $\bar{\delta} \in (0,1)$, (13) decreasing in $\bar{\delta}$ and, therefore, there is a unique value of (13). The derivative of expression (13) with respect to $\bar{\delta}$ is

   $$\frac{\partial}{\partial \bar{\delta}}\left(\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i}\right) = -\int_{s\in S} \pi_i(s)\cdot 2d(e,s)(1-\bar{\delta})^{4d(e,s)-1}\left(2d(e,s)+\frac{1-\bar{\delta}4d^2(e,s)}{1-\bar{\delta}}\right)f(s)ds$$

   (14)For all $d \in \left[0,\frac{1}{2}\right]$ then $4d^2(e,s) \le 1$ and $\frac{1-\bar{\delta}4d^2(e,s)}{1-\bar{\delta}} \ge 1$. Therefore, (14) is strictly less than 0 for all $0 < \bar{\delta} < 1$

3. There exists some interval of values $\bar{\delta} \in (0,1)$ such that for any $\bar{\delta}$ there is a unique value of $\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} \in \left[0,\frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}}\right]$. By definition, the average payoff is $\frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}} = \int_{s\in S} \pi_i(s)\cdot(1-\bar{\delta})^{4d(e,s)} f(s)ds$. Substituting into (13), the marginal payoff is equal to the average payoff, less the marginal change in availability:

$\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} = \frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}} - \frac{\bar{\delta}}{1-\bar{\delta}} \int_{s \in S} \pi_i(s) \cdot 2d(e,s)(1-\bar{\delta})^{4d(e,s)} f(s)ds$. Given

$\frac{\bar{\delta}}{1-\bar{\delta}} \int_{s \in S} \pi_i(s) \cdot 2d(e,s)(1-\bar{\delta})^{4d(e,s)} > 0$ then $\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} < \frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}}$. Further, if

$\delta \to 0$ then $\frac{\bar{\delta}}{1-\bar{\delta}} \int_{s \in S} \pi_i(s) \cdot 2d(e,s)(1-\bar{\delta})^{4d(e,s)} \to 0$ and $\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} \to \frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}}$.

From points 1 through 3, for any marginal cost $c \in \left(0, \underline{u}\right)$ there is a unique value $\bar{\delta}$ that satisfies expression (12). Given that $\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i}$ is decreasing in $\bar{\delta}$ and $0 < \underline{u} \leq \frac{\pi_i(\bar{\delta},\bar{\delta})}{\bar{\delta}}$ then there exists some interval of values $\bar{\delta}$ such that for any $c \in \left(0, \underline{u}\right)$ $\frac{\partial \pi_i(\bar{\delta},\bar{\delta})}{\partial \delta_i} = c$.

Rearranging (10) the value to each entrepreneur is:

$$v_i = \pi_i - c\bar{\delta} = \frac{\bar{\delta}^2}{1-\bar{\delta}} \int_{s \in S} \pi_i(s) \cdot 2d(e,s)(1-\bar{\delta})^{4d(e,s)} f(s)ds$$

This is, by definition, the ex ante expected value to the network investment $\bar{\delta}$.

QED

Proposition (Social Planner's Solution). *Under a uniform network with supermodular payoffs and match probability* $p(e_i,s_j;\delta_i,\delta_{-i}) = \delta_i \cdot \left(1-\delta_i\right)^{2d_{ij}} \cdot \left(1-\bar{\delta}_{\mathrm{E}(s_j,d_{ij})}\right)^{2d_{ij}}$*, for any marginal cost c there is a* *unique, symmetric Social Planner solution in which each entrepreneur invests to reach δ*and receives expected value:*

$$v^* = \pi - c\delta^* = 4\frac{\delta^{*2}}{1-\delta^*}\int_{d=0}^{\frac{1}{2}} d\pi(d)(1-\delta^*)^{4d} g(d)dd$$

Proof. Under a uniform, symmetric network, the social planner's problem in (7) reduces to choosing a target network density δ* for all entrepreneurs.

$$\delta^* = \arg\max_\delta M\left[\int_{s\in S} \pi(s)\delta(1-\delta)^{4d(e,s)} f(s)ds - c(\delta)\right]$$

When all agents are uniform, each entrepreneur's ex ante payoff is symmetric and can be rewritten as a function of distance, where $g_i(d)$ is the uniform distribution of distances to each specialist from e_i and $G_i(\frac{1}{2}) = 1$ for all *i*.

$$\delta^* = \arg\max_\delta M\left[\int_{d=0}^{\frac{1}{2}} \pi(d)\delta(1-\delta)^{4d} g(d)dd - c(\delta)\right]$$

A symmetric Social Planner's solution for δ^* must satisfy the first order condition:

$$\frac{\partial}{\partial\delta}V = \int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta^*)^{4d}\left[1-4d\frac{\delta^*}{1-\delta^*}\right]g(d)dd - c = 0$$

Similar to the SNE, the Social Planner's symmetric solution implies that at the optimal solution, marginal cost must equal the marginal ex ante payoff to the network:

$$\frac{\partial\pi(\delta^*)}{\partial\delta} = \int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta^*)^{4d}\left[1-4d\frac{\delta^*}{1-\delta^*}\right]g(d)dd = c \qquad (15)$$

Three points prove that for any $c \in \left(0,\underline{u}\right)$ there exists a unique $\delta^* \in \left(0,1\right)$ satisfying (15):

1. There exist values of $0 < \delta^* < 1$ such that (15) is positive. If $\left[1 - 4d\frac{\delta}{1-\delta}\right] > 0$, or $4d < \frac{1-\delta}{\delta}$, then $\int_{d=0}^{\frac{1}{2}} \pi(d)\delta(1-\delta)^{4d}\left[\frac{1}{\delta} - \frac{4d}{(1-\delta)}\right]g(d)dd > 0$. Given $d \le \frac{1}{2}$, if $\frac{1-\delta}{\delta} \ge 4d$ or $\delta \le \frac{1}{3}$ then (15) is positive.

2. For any *c*, the value of δ^* is unique. For any δ^* such that (15) is positive then expression (15) is strictly decreasing and the second order condition is strictly negative:

$$\frac{\partial^2}{\partial\delta^2}V = -\int_{d=0}^{\frac{1}{2}} \pi(d)4d(1-\delta^*)^{4d-1}\left[\frac{2-\delta^*(1+4d)}{1-\delta^*}\right]g(d)dd < 0$$

The above expression is less than 0 if $\frac{1-\delta}{\delta} \ge 4d$. For any feasible value of δ^*, $\int_{d=0}^{\frac{1}{2}} \pi(d)\delta(1-\delta)^{4d}\left[\frac{1}{\delta} - \frac{4d}{(1-\delta)}\right]g(d)dd$ is decreasing in δ^*.

3. For any value of $0 < \delta^* < 1$, $\frac{\partial\pi(\delta^*)}{\partial\delta} < \frac{\pi(\delta^*)}{\delta^*}$. First, on the left hand side of (11), the marginal expected welfare return is equal to the average payoff, $\int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta^*)^{4d} g(d)dd = \frac{\pi(\delta^*)}{\delta^*}$. Substituting into (15), $\frac{\partial\pi(\delta^*)}{\partial\delta} = \frac{\pi(\delta^*)}{\delta^*} - \frac{2\delta^*}{1-\delta^*}\int_{d=0}^{\frac{1}{2}} \pi(d)2d(1-\delta^*)^{4d} g(d)dd$. Given

$\frac{2\delta^*}{1-\delta^*}\int_{d=0}^{\frac{1}{2}}\pi(d)2d(1-\delta^*)^{4d}g(d)dd > 0$ then $\frac{\partial\pi(\delta^*)}{\partial\delta} < \frac{\pi(\delta^*)}{\delta^*}$. Further, as $\delta \to 0$

then $\frac{2\delta^*}{1-\delta^*}\int_{d=0}^{\frac{1}{2}}\pi(d)2d(1-\delta^*)^{4d}g(d)dd \to 0$ and $\frac{\partial\pi(\delta^*)}{\partial\delta} \to \frac{\pi(\delta^*)}{\delta^*}$.

From points 1 through 3, there is some interval of values for δ^* such that $\frac{\partial\pi(\delta^*)}{\partial\delta} \in \left(0, \frac{\pi(\delta^*)}{\delta^*}\right)$. Further, since $\frac{\partial\pi(\delta^*)}{\partial\delta}$ is decreasing in δ^* there is some more narrow interval of δ^* values such that $\frac{\partial\pi(\delta^*)}{\partial\delta} \in \left(0, \underline{\pi}\right)$ where $\underline{\pi} < \frac{\underline{\pi}}{\delta^*} < \frac{\pi(\delta^*)}{\delta^*}$.

Finally, since $\frac{\partial\pi(\delta^*)}{\partial\delta}$ is strictly decreasing in δ^*, the second order condition holds and this solution is unique.

Rearranging (15) the socially optimal payoff is equal to:

$$v^* = \pi - c\delta = 4\delta^2\int_{d=0}^{\frac{1}{2}}d\pi(d)(1-\delta)^{4d-1}g(d)dd > 0.$$

Q.E.D.

Proposition (Compare First Best and Entrepreneur's Reaction). *Under a uniform network with supermodular payoffs and negative externalities each entrepreneur over-invests in her network so that* $\bar{\delta} > \delta^*$.

Proof. For any fixed marginal cost *c*, the two first order conditions from the previous two proofs are equal:

$$\int_{d=0}^{\frac{1}{2}}\pi(d)(1-\delta)^{4d}\left[1-\frac{4d\delta}{(1-\delta)}\right]g(d)dd = \int_{d=0}^{\frac{1}{2}}\pi_i(d)\cdot(1-\bar{\delta})^{4d}\left(1-\frac{2d\bar{\delta}}{1-\bar{\delta}}\right)g(d)dd$$

By contradiction, suppose that investment is equal so that $\bar{\delta} = \delta^*$. Then

$$\int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta)^{4d}\left[1-\frac{4d\delta}{(1-\delta)}\right]g(d)dd = \int_{d=0}^{\frac{1}{2}} \pi_i(d)\cdot(1-\bar{\delta})^{4d}\left(1-\frac{2d\bar{\delta}}{1-\bar{\delta}}\right)g(d)dd$$

$$4\frac{\delta}{1-\delta}\int_{d=0}^{\frac{1}{2}} d\pi(d)g(d)dd = 2\frac{\delta}{1-\delta}\int_{d=0}^{\frac{1}{2}} d\pi_i(d)g(d)dd$$

but this is clearly a contradiction, so $\bar{\delta} \neq \delta^*$.

Next suppose there is underinvestment so that $\bar{\delta} < \delta^*$. Then $(1-\delta^*)^{4d} < (1-\bar{\delta})^{4d}$ and

$$\begin{aligned} \frac{\delta^*}{1-\delta^*} &> \frac{\bar{\delta}}{1-\bar{\delta}} \Rightarrow \\ \frac{4d\delta^*}{1-\delta^*} &> \frac{2d\bar{\delta}}{1-\bar{\delta}} \Rightarrow \\ 1-\frac{4d\delta^*}{1-\delta^*} &< 1-\frac{2d\bar{\delta}}{1-\bar{\delta}} \end{aligned}$$

Combining these expressions $(1-\delta^*)^{4d}\left(1-\frac{4d\delta^*}{1-\delta^*}\right) < (1-\bar{\delta})^{4d}\left(1-\frac{2d\bar{\delta}}{1-\bar{\delta}}\right)$ and

$$\int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta)^{4d}\left[1-\frac{4d\delta}{(1-\delta)}\right]g(d)dd < \int_{d=0}^{\frac{1}{2}} \pi_i(d)\cdot(1-\bar{\delta})^{4d}\left(1-\frac{2d\bar{\delta}}{1-\bar{\delta}}\right)g(d)dd$$

but this is a contradiction to the initial condition.

This leaves the case of entrepreneurial overinvestment, $\bar{\delta} > \delta^*$, in which $(1-\delta^*)^{4d} > (1-\bar{\delta})^{4d}$ and $\frac{\delta^*}{1-\delta^*} < \frac{\bar{\delta}}{1-\bar{\delta}}$. This implies that the relationship between $\frac{4d\delta^*}{1-\delta^*}$ and $\frac{2d\bar{\delta}}{1-\bar{\delta}}$ is ambiguous. Then it is feasible that $\bar{\delta} > \delta^*$ and

$$\int_{d=0}^{\frac{1}{2}} \pi(d)(1-\delta)^{4d}\left[1-\frac{4d\delta}{(1-\delta)}\right]g(d)dd = \int_{d=0}^{\frac{1}{2}} \pi_i(d)\cdot(1-\bar{\delta})^{4d}\left(1-\frac{2d\bar{\delta}}{1-\bar{\delta}}\right)g(d)dd$$

Since this is the only feasible case it follows that $\bar{\delta} > \delta^*$ and there is entrepreneurial overinvestment in the network. *Q.E.D.*

Proposition. *Under a uniform network with targeting, supermodular payoffs and negative externalities, as network targeting becomes more specific ex ante expected partnerships are more compatible.*

Proof. Following the same logic as the original entrepreneur's SNE and social planner's problem, the updated respective first order conditions are:

$$c = r \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s) \frac{r\bar{\delta}}{1 - r\bar{\delta}}\right) f(s) ds \qquad \text{(SNE)}$$

$$c = r \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\delta^*)^{4d(e,s)} \left(1 - 4d(e,s) \frac{r\delta^*}{1 - r\delta^*}\right) f(s) ds \qquad \text{(SPS)}$$

Following the immediately preceding proof, the relevant expression to compare the entrepreneur's choice $\bar{\delta}$ with the social planner's efficient solution δ^* is:

$$\int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\bar{\delta})^{4d(e,s)} \left(1 - 2d(e,s) \frac{r\bar{\delta}}{1 - r\bar{\delta}}\right) f(s) ds$$
$$= \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\delta^*)^{4d(e,s)} \left(1 - 4d(e,s) \frac{r\delta^*}{1 - r\delta^*}\right) f(s) ds$$

In the case of targeted search, the maximum skill incompatibility distance is $\bar{d} = \frac{1}{2r}$. As *r* increases, the maximum distance decreases. As the maximum distance decreases, the externality to the entrepreneur, $2d(e,s)\frac{r\bar{\delta}}{1 - r\bar{\delta}}$, and social planner,

$4d(e,s)\frac{r\delta^*}{1-r\delta^*}$, diminish. The difference between the $\bar{\delta}$ and δ^* that solve these expression must be decrease as *r* increases and $\bar{d} = \frac{1}{2r}$ decreases. In turn, the efficiency of the entrepreneur's solution increases.

Given $\bar{\delta} < r\delta$ *and* $\delta^* < r\delta$, the probability of a more compatible partnership is at least at factor *r* higher. The probability of any partnership is at least as great and the probability of a more compatible partnership is increasing in *r*. The expected return is, therefore increasing.

Q.E.D.

Proposition (Compare General to Targeted Cases). *Under a uniform network with supermodular, negative network externalities and constant marginal cost, the network density in the search region is greater under a targeted search when* $r > 1$*, than under a general search when* $r = 1$*:* $\bar{\delta} < r\delta$ *and* $\delta^* < r\delta$.

Proof of Proposition (Compare General to Targeted Cases). The proof follows the same logic as the previous comparison. For the SNE entrepreneur's equilibrium:

$$r\int_{s\in S(e_i,\frac{1}{2r})} \pi_i(s)\cdot(1-r\bar{\delta})^{4d(e,s)}\left(1-2d(e,s)\frac{r\bar{\delta}}{1-r\bar{\delta}}\right)f(s)ds$$
$$=\int_{s\in S(e_i,\frac{1}{2r})} \pi_i(s)\cdot(1-\bar{\delta})^{4d(e,s)}\left(1-2d(e,s)\frac{\bar{\delta}}{1-\bar{\delta}}\right)f(s)ds$$

For the social planner's solution:

$$r \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - r\delta^*)^{4d(e,s)} \left(1 - 4d(e,s) \frac{r\delta^*}{1 - r\delta^*} \right) f(s) ds$$

$$= \int_{s \in S(e_i, \frac{1}{2r})} \pi_i(s) \cdot (1 - \delta^*)^{4d(e,s)} \left(1 - 4d(e,s) \frac{\delta^*}{1 - \delta^*} \right) f(s) ds$$

Following a proof by contradiction in either case, it follows that $\bar{\delta} < r\bar{\bar{\delta}}$ and $\delta^* > r\delta^{*\tau}$. Q.E.D.

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