ABSTRACT

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Most mathematics educators endorse the idea that important concepts and procedures should be taught by asking students to solve problems whose solutions can be derived by multiple solution methods. This vision for classroom activity involves the teacher routinely soliciting multiple ideas for solving a single problem; students communicating what they are thinking; students respectfully listening to what others say; and students discussing their solution methods and comparing the advantages or each.

This dissertation explores some of the practical challenges that teachers face when using multiple solutions in the mathematics classrooms, and considers how teachers might address these challenges. In addition, this dissertation puts forth a theoretical framework for analyzing how classrooms make use of students' multiple solutions. These issues were examined by utilizing a first-person research methodology in an

eighth grade classroom with students who had a history of behavioral concerns and low academic performance.

UNDERSTANDING THE CHALLENGES OF IMPLEMENTING A MULTIPLE SOLUTION NORM.

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Chapter 1: Introduction

Most mathematics educators endorse the idea that important concepts and procedures should be taught by asking students to solve problems whose solutions can be derived by multiple solution methods. The process of working on problems and assessing the relative advantages of alternate solution strategies can help students become critical thinkers of mathematics (Hiebert et al., 1997). However, despite calls for problem-solving approaches to teaching mathematics, most United States teachers have not incorporated the use of multiple solutions for problems as features of their classroom instruction (Silver et al., 2005).

This dissertation explores some of the practical challenges that teachers face when using multiple solutions in the mathematics classrooms, and considers how teachers might address these challenges. In addition, this dissertation puts forth a theoretical framework for analyzing how classrooms make use of students' multiple solutions. I examined these issues by studying my own teaching during the 2006/2007 school year in a sub-urban middle school situated about halfway between two densely populated urban areas in the mid-Atlantic region. My work takes place in an eighth grade pre-Algebra classroom consisting of eight African American students. Each of the students participated in the school's alternative education program[1] designed to support students with a history of behavior concerns and low academic performance.

In this study, I use the notion of social and sociomathematical norms (Yackel & Cobb, 1996) to focus on important features of social and mathematical activity within

[1] Note: Although it was not uncommon for students to receive services both through the district's alternative education program and through special education, none of the participants in the study received special education services.

a classroom environment where students are expected to generate and discuss multiple solutions to problems. I identified a collection of norms, called a multiple solution norm (Chapter 3), that I propose is important to instantiate in a classroom so that students routinely consider and discuss alternate ways to solve a challenging problem and use their solution strategies to build key mathematical understandings. My research goal in this study was to understand the challenges to implementing a multiple solution norm.

In the rest of this chapter, I will briefly describe how the origins of this work arose from working with a group of pre-service teachers who questioned the feasibility of constituting a multiple solution norm. Then, I will provide ideas for how this study can build from existing research and offer valuable contributions to both teachers and teacher educators.

Background

As a full-time doctoral student, I was fortunate to engage in a wide-array of experiences with the University of Maryland's secondary mathematics pre-service teacher education program. In particular, I had the opportunity to co-instruct a seminar course required for all teacher interns to take concurrently during their student teaching field experience. The origin of my research study took place as an instructor in this course.

A requirement for the seminar asked the teacher interns to show a video of their own instruction. In particular, one of the interns, Mike, shared a lesson in which he demonstrated how to set up and solve a proportion to find an unknown length given the lengths of the other five sides of two similar triangles. In the class discussion

regarding Mike's video, the comments from the teacher interns were generally positive. They commented Mike on the clarity, ease, and thoroughness of his explanations.

As a rule of thumb, I generally asked questions aimed at getting the teacher interns to think about the relative advantages of various instructional approaches. On this occasion, I asked Mike how he would find the length of the unknown side. His response was to find the unknown length by multiplying the known corresponding side by the scaling factor of the triangles. This was not the same way demonstrated in his video how to solve the problem. Next, I suggested to the group that it might be valuable to explore alternate ways of solving a problem with their students. To my surprise, almost each of the interns expressed an unwillingness to engage in such an activity. Many of the interns were fearful that showing their class more than one way to solve a problem would just confuse their students. Other reasons against the exploration of multiple solution methods that surfaced were: students only wanted to know one way to solve a problem; it was too difficult to manage classroom behavior when students are asked to explore different strategies; and this type of instruction takes too much time.

In the *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (NCTM, 2000) encourages teachers to solicit multiple solution techniques and explanations from their students. The *Standards* are laden with assumptions regarding the value of providing experiences for students where they solve problems in more than one way: "students gain insights into their thinking when they present their methods for solving problems" (NCTM, 2000, p. 60), "often,

a student who has one way of seeing a problem can profit from another student's view, which may reveal a different aspect of the problem" (NCTM, 2000, p. 62), and "by carefully listening to, and thinking about, the claims made by others, students learn to become critical thinkers about mathematics" (NCTM, 2000, p. 63).

More significantly, the capability to derive and analyze multiple solutions to problems relates to what it means to really know mathematics. The consensus among mathematicians and mathematics educators is that students should learn mathematics with understanding (NCTM, 2000; National Resource Council [NRC], 1989). Hiebert et al. (1997) defines understanding in terms of the ability to make connections with prior knowledge. Silver et al. (2005) suggest that analyzing different solution strategies can strengthen networks of related ideas by facilitating connections to different elements of knowledge with which a student may be familiar. In addition, Moschkovich (1998) suggests that when ideas are exchanged and subjected to thoughtful critiques, they are often refined and improved. As students express and defend their ideas and solutions with their classmates and question others' ideas, they are likely to recognize misconceptions, and reflect on and clarify their own thinking (Ball, 1993a; Lampert, 1990, 2001).

One vision for classroom activity that facilitates mathematical understanding involves the teacher routinely soliciting multiple ideas for solving a single problem; students communicating what they are thinking; students respectfully listening to what others say; and students discussing their solution methods and comparing the advantages or each.

Despite recommendations made by mathematics education reformers to get students involved in sharing their inventive strategies to problems, few classrooms in the United States resemble this model (Jacobs et al., 2006, Stigler & Hiebert, 1999). Clearly, for the teacher interns in the seminar course, a tension existed between this vision of teaching and learning and their concerns regarding the wisdom and practicality of using multiple solutions in the classroom. The interns used their experiences as teachers in the classroom to ground their argument against using multiple solutions. I did not know how to best challenge their viewpoints. Ultimately, I was endorsing a type of teaching activity that I supported and believed was feasible to do; however, in my prior work as a classroom teacher I had not experienced. I wanted to improve my understanding of the interns' position so that I would be better situated to deal with this in the future as a teacher educator.

Rationale for the Study

Creating a classroom environment predicated on the exploration and analysis of multiple solutions to problems entails a fundamental shift in the roles of teachers and students. One of the most deep-seated features of mathematics classrooms in the United States is that the teacher almost always demonstrates a procedure for solving problems before assigning them to students (Stigler & Hiebert, 1999). Jacobs et al. (2006) found that, despite recommendations to reform mathematics instruction, the predominant U.S. classroom culture still reflects more traditional mathematics pedagogy. Traditional forms of instruction in mathematics classrooms are typically characterized by direct instruction involving the transmission of factual information from the teacher to the students. Cuban (1993) argues that deeply rooted and

widespread cultural beliefs regarding how teaching should occur perpetuate the continuation of the transmission mode of communication. Although most teachers learn some things about problem-based teaching practices in their pre-service training, they learn mostly from their participation and observation of school mathematics classes (Lortie, 1975). The need exists to explore the complexities involved in this type of teaching and understand the practical obstacles of using students' multiple solutions as a central feature in mathematics instruction. Silver et al. (2005) state, "What has been left unclear is why the consideration of multiple solutions for a problem is an aspect of instructional practice that is rarely seen in U.S. classrooms and whether there are ways to support its more frequent use by mathematics teachers" (p. 289).

Research is needed to help teachers develop more complex and nuanced understandings of the instructional decisions a teacher makes when having students consider multiple solutions to problems (Silver et al., 2005). First-person research accounts by mathematics educators (i.e., Ball, 1993; Chazan, 2000; Heaton, 2000; Lampert, 1990, 2001; Romagnano, 1994; Schoenfeld, 1994) begin to develop a comprehensive representation of the work of reform teaching, yet the National Research Council (2001) notes that:

> too little of the extant research probes the work of teaching at a sufficiently fine grain to contribute to the development of a conceptual and practical language of practice. Much of the interactive work in instruction remains unexamined, which leaves to teachers the unnecessary challenge of reinventing their practice from scratch, armed with only general advice. Suggestions that a class "discuss the solutions to a problem" provides little specificity about what constitutes a productive discussion and runs the risk of a free-for-all session that resembles sharing more than instruction. (p. 359)

Research revealing how a teacher can overcome challenges, and successfully create a classroom environment where students consider multiple solutions to problems can potentially shape the beliefs of those who doubt that this type of instruction is feasible. Similar to Lampert's (1990) 'existence proof' that certain kinds of knowing and learning are possible in the school setting under ordinary conditions, this research can contribute to the vision of what is possible to do in the classroom. Successful adaptations in one class may not be successful in another, however what is learned about cultural assumptions underlying particular reformed practices in studies in one community can inform efforts to make practices more equitable in other settings (Lubienski, 2002).

The analysis of the data from this study can lead to the development of a theoretical framework that can build off of Hiebert et al.'s (1997) framework identifying five dimensions of classroom activity that can be used in examining whether a classroom is facilitating the development of mathematical understanding. In a similar way, I aim to identify a set of norms (multiple solution norm) to determine whether classrooms are making use of students' multiple solutions. This is connected to the work of Yackel and Cobb (1996), whose analysis of the emergence of norms contribute to developing a professional language of practice that describes the subtleties and intricacies of the work of teaching. In addition, my focus on a teacher's proactive role in the development of a classroom climate that regularly discusses alternate solution method could complement Simon's (1995) Mathematical Teaching Cycle modeling the interrelationships of a teacher's knowledge, thinking, and decision making process.

Having introduced my overarching research goals, briefly described some of the background that led to the study, and explained how results of this study might aid the work of both teachers and teacher educators, I now, in the following chapter, turn to a review of the literature to examine the challenges of instantiating a multiple solution norm. Then, in Chapter 3, I discuss the theoretical framework underlying my understanding of how social and sociomathematical norms are constituted in the classroom and explain how Cobb and Yackel's (1996) interpretive framework focusing on students' learning can be used to analyze the teacher's role in guiding and organizing the development of classroom activity. Also in Chapter 3, I define my construct of a multiple solution norm and present a series of classroom vignettes to illustrate some potential issues a practitioner could face when implementing a multiple solution norm. Finally, in Chapter 3, I use my definition of multiple solution norm and the conception of teaching explained in Simon's (1995) mathematical teaching cycle to introduce the research questions that guided this study.

In Chapter 4, I explain the methodology and the data collection and analysis methods that I used to examine the challenges of instantiating a multiple solution norm. In addition, I describe the context of working with a group of students with a history of academic failure and behavior concerns. Chapter 5 offers a presentation of the data in which I trace the development of classroom norms. In particular, I discuss the challenges of managing student behavior and describe how I fell short of my goal of instantiating a multiple solution norm. I examine the mathematical disposition of the students, and use the data to analyze six factors that influenced the students' disposition to expend effort on solving problems. In Chapter 6, I discuss how my

findings can contribute to a wider research base regarding student motivation and achievement goal theory. Also in Chapter 6, I use the analysis of my data to revise my original framework of a multiple solution norm. Lastly, in Chapter 7, I discuss some of the lessons I learned from this study that will inform my future work as a teacher educator.

Chapter 2: Review of Selected Literature

In this chapter, I review literature to investigate why it is rare to find secondary classrooms in the United States where teachers and students routinely work on finding more than one way to solve a mathematics problem. After presenting research regarding how seldom teachers in the United States afford students opportunities to explore multiple solutions, I examine three interdependent reasons why teachers and students do not typically engage in this type of activity: (1) Teaching is a cultural activity. Deeply engrained societal beliefs regarding the nature of mathematics and the role of mathematics teachers make it difficult to alter what happens in mathematics classrooms; (2) Teaching is student dependent. Instruction predicated on student ideas and discussion of those ideas requires the cooperation of students; and (3) Asking teachers to use multiple solutions as features of their instruction is difficult and uncertain work. In contrast, traditional teacher-centered instruction is comparatively well-defined with fewer demands on teachers.

United States Pattern of Mathematics Instruction

The Video Study of Teaching conducted as part of the Third International Mathematics and Science Study (TIMSS) (Stigler et al., 1999) offers one of the most extensive investigations of mathematics teaching in the United States. An examination of the video data reveals that recitation, where the teacher through telling or demonstrating presents new material, is the most common form of teaching. Results from TIMSS show that, in United States classrooms, 78% of the mathematical topics contain concepts that were stated by the teacher rather than

developed through examples or explanations. In contrast, that practice occurred for only 17% of the concepts in Japanese classrooms (Stigler & Hiebert, 1999). Further quantitative results from TIMSS found that only eight percent of lessons in eighth grade classrooms in the United States involved students presenting alternative solution methods. In Japanese middle grade classrooms, where students are more often expected to find and share their own solution methods to problems, 42% of lessons involved students presenting alternative solution strategies. Further, the study reveals that the average number of alternative solution methods presented by students per lesson in the United States is 0.2. In contrast, in Japanese classrooms, the average number of alternative solution methods presented by students per lesson is 1.7 (Stigler & Hiebert, 1999).

Consistent findings are reported from a longitudinal study examining fourth and fifth grade classrooms of successful mathematics teachers (Valli et al., 2005). In their study of teachers with higher than expected student performance, Valli et al. (2005) report that only seven percent of student activity in mathematics involved students responding with a conjecture, explanation, or alternate solution method. The most common student activity, occurring 33% of the time, involved students working on routine exercises and responding with a simple answer. Other student activity reported was: listening and watching (20%), engaging in non-instructional activities (17%); working on problems or extended writing (10%), asking questions, reading text, or writing on board (8%), and taking formal assessments (4%). Thus, even in classrooms where students are doing well, they are infrequently being asked to consider more than one way to solve a problem.

Emanating from the above study, Valli et al. (2008), in *Test Driven: High-Stakes Accountability in Elementary Schools,* note that the shift to high-stakes testing mandated by the No Child Left Behind (NCLB) Act has weakened the quality of mathematics teaching. During their study, the researchers found that pressure to "teach to the test" has undermined professional standards for teaching and learning. According to Valli et al., the era of NCLB has witnessed declines in teaching higher-order thinking, in the amount of time spent working on complex problems, and in the amount of high cognitive content in the curriculum.

In a similar vein, McKinney and Frazier (2008) report that the teacher-directed instruction continues to dominate many mathematics classrooms. In investigating the mathematics pedagogical and instructional skills of 64 in-service teachers who teach in high-poverty middle schools, McKinney and Frazier found that, although the teachers are implementing a variety of practices, they predominantly use traditional pedagogical practices such as lecture, drill and practice, and teacher-directed instruction. The researchers suggest that pressures to adhere to a mandated curriculum guide have restricted teachers' instructional freedom.

The American pattern of mathematics instruction that emanate from these studies is consistent with a general method of teaching that has persisted in the United States for over one hundred years (Cuban, 1993). In tracing three occurrences over the past century when reform-minded practitioners, administrators, and policymakers undertook an intense and widespread effort to alter what occurs in classrooms, Cuban (1993) concludes that the tradition of teacher-centered instruction continues to dominate elementary and secondary classrooms. In particular, secondary

mathematics classrooms are characterized by whole-class instruction, teachers talking most of the time while students listen, and reliance upon the textbook for authoritative knowledge (Cuban, 1993).

For more than two decades, mathematics education reformers have recommended that mathematics instruction shift away from traditional teacher led approaches; instead of following teachers' instructions, students should have the opportunity to invent, explain, and justify their own mathematical ideas and critique the ideas of other students (NCTM, 1989, 1991, 2000). Although many teachers report familiarity and adherence to these principles, their actual classroom teaching practices do not reflect a deep understanding of reform (Hiebert & Stigler, 2000). The majority of students in the United States appear to remain products of traditional teacher-centered instruction (Jacobs et al., 2006). The traditional role of students is to follow procedural instructions presented by the teacher and attempt to correctly obtain numerical answers to similar problems. Seldom are students given the opportunity or expected to arrive at a solution to a problem through their own inventive strategies.

The thinking and skills required for mathematical problem solving transfers to other areas of life. The consequence of using this same pattern of instruction is that young people in the United States are not being adequately prepared to meet the new demands in an ever changing workplace. Hiebert et al. (1997) note that, "In order to take advantage of new opportunities and to meet the challenges of tomorrow, today's students need flexible approaches for defining and solving problems" (p. 1). When teachers show students how to solve a problem and ask students to memorize rules for

moving symbols around on paper, students may be learning something, but they are not learning mathematics with deep understanding (Hiebert et al., 1997).

Creating a classroom environment that regularly makes use of students' multiple solutions requires changing the traditional and persistent roles of teachers and students. A significant obstacle prohibiting this shift is the notion that the routines of teaching have become so highly socialized that they are almost automatic and difficult to change.

Teaching is a Cultural Activity

In examining data from TIMSS, Stigler and Hiebert (1999) were "struck by the homogeneity of teaching methods within each culture" (p. x). Teaching, like other cultural activities, is largely influenced "through informal participation over long periods of time" (Stigler & Hieber, 1999, p. 86). Most Americans, through their schooling experience, have observed thousands of hours of classroom lessons. As a result of this *apprenticeship of observation* (Lortie, 1975), Stigler and Hiebert (1999) posit that nearly all Americans could enter a classroom at any moment and act like a teacher.

Teaching systems are composed of complex, reinforcing elements that interact with one another resulting in a system that is resistant to change (Stigler & Hiebert, 1999). Ernest (1989) notes, that the social context of the teaching situation is a key factor that influences mathematics instruction. Within the classroom, what teachers do can be attributed to the teacher's decisions, the nature of students, school and district policies, and the larger community in which the school is located (Cuban, 1993).

In attempting to posit why mathematics instruction in the United States has essentially resisted multiple reform efforts, one of Cuban's (1993) explanations is that deeply embedded and widespread cultural beliefs about the nature of knowledge, how teaching should occur, and how children should learn steer the thinking of policymakers, practitioners, and parents toward teacher-centered instruction. According to Cuban, these beliefs are rooted in Western religious educational traditions in which the role of the teacher was to impart basic factual knowledge to the uninformed. Cuban concludes that: "as long as the public schools' dominant social role in the culture . . . remains unchanged, and as long as schools remain organized as they currently are . . . teacher-centered instruction will remain pervasive" (p. 277).

Building on Cuban's (1993) view of the role of cultural beliefs, Chazan (2000) argues that a complex and deeply engrained web of beliefs about mathematical knowledge and mathematics instruction make if difficult to teach secondary school mathematics in a student-centered manner. According to Chazan, historical Western views about the certainty of mathematics and its importance mesh with views of school subject matter as unambiguous and unproblematically factual to shape teacher-centered mathematics instruction. In mathematics classrooms, these beliefs are exhibited as teachers traditionally attempt to clear up confusion as quickly as possible and adjudicate the correctness of ideas.

Similarly, Brousseau (1997) offers a theory explaining how cultural beliefs are sufficiently powerful to influence the decisions and actions of teachers. Brousseau's theory of *didactical situations* models the complex system of interactions between a

teacher and his/her students that is situated within the society that the teaching system is located. Brousseau suggests that teachers are influenced by a pair of systems: "the student and . . . a '*milieu*' that lacks any didactical intentions with regard to the student" (p. 40). According to Brousseau's theory, students have truly acquired knowledge only when they are able to apply it to an *adidactical situation* – a situation outside of any teaching context and in the absence of any intentional direction. Part of the role of the teacher is to *devolve* to the student an adidactical situation (i.e., convert non-contextualized mathematical ideas that are to be taught into situations or problems which provides the student with the most independent and fruitful interaction possible). In the devolution of objects of study, a teacher "has to formulate a method for making the answer explicit to the student: how to answer with the help of previous knowledge, how to understand and build new knowledge, how to 'apply' previous lessons, how to recognize questions, how to learn, guess, solve, etc." (p. 35). Brousseau asserts that this epistemology of the teacher "must also be the epistemology of the student and her parents. It must be present in the culture to allow justifications to function and be accepted. The teacher is not free to change it as she pleases" (p. 35). Thus, a teacher is constrained by the culture in which the teaching system is located to reorganize knowledge and modify its presentation (called the *didactical transposition*) so that it fits this epistemology.

Thus, in the United States, long-term cultural beliefs in an achievement based society drive teachers to satisfy organizational demands for children to obey authority, behave uniformly, and acquire a common body of knowledge (Cuban, 1993). Teachers' school experiences, as mathematics students, influence the

development of these beliefs (Raymond, 1997; Thompson, 1992). Research shows that most mathematics teachers, including pre-service teachers, have strongly-held beliefs about the nature of mathematics, student and teachers' roles, and how students learn best (Thompson, 1992). It is widely accepted premise that the type of mathematics instruction delivered by a teacher depends fundamentally on the teacher's belief system (Thompson, 1992). A cycle thus emerges that reinforces the nature of mathematics instruction in the United States. In the majority of American mathematics classrooms, the cultural and social context of schools pushes teaching toward a teacher-centered style of instruction; the systematic contact with mathematics significantly influences the beliefs of future teachers; and the belief system, in turn, affects the nature of instruction delivered by teachers.

Cohen (1988) refers to this cycle as the "ancient instructional inheritance" (p. 35). Three underlying beliefs of this inheritance are that knowledge is purely objective, teaching is equivalent to explaining, and learning is a passive process of accumulation. Cohen suggests that these beliefs result in a pervasive impact on teaching that makes it difficult to reform.

Recommendations to teachers to teach *through* problem solving (Schroeder & Lester, 1989) are filtered through teachers' existing beliefs, past experiences and current practices (Borko & Putnam, 1995). Even if teachers understand and seemingly embrace the potential value of creating an environment predicated on students' ideas and thinking, the obstacles created by the traditional nature of educational systems can limit teachers' ability and/or willingness to enact (Gregg,

1995; Haimes, 1996; Lloyd, 1999; Nelson, 1997; Lloyd & Wilson, 1998; Wilson & Lloyd, 2000).

A number of research findings illustrate cases where teachers, despite their intentions, are compelled to teach in certain and prescriptive ways. For example, Eisenhart et al. (1993) examined the tensions created to prospective teachers over teaching in a conceptual versus procedural way. In their university coursework, the teachers were encouraged to teach for conceptual understanding. Although administrators in their placement schools appeared to want teachers to teach for both conceptual understanding and procedural knowledge, in reality, administrators held teachers accountable only for the procedural knowledge. As a result, teachers in the schools taught for instrumental, rather than relational, understanding (i.e., see Skemp, 1976). Similarly, Lloyd (1999) reports on a teacher who admitted that his decision to teach in traditional ways was often influenced by departmental pressures.

In an ethnographic case study of a beginning high school mathematics teacher's acculturation into the school mathematics tradition, Gregg (1995) found evidence that the organization and structure of schools and the culture of teaching foster and perpetuate traditional teaching practices. Exploring why traditional practices have been so constant and durable, Gregg concludes that a successful movement to reform mathematics instruction must challenge the classroom, school, and societal obligations that characterize teachers' roles in the school mathematics tradition.

Additionally, Nelson (1997) presents a case study of an experienced teacher, who expressed intentions to create a classroom environment consistent with constructivist precepts, but found it difficult to accept student autonomy. Unable to resolve the

tension of providing opportunities for students to explore uncertain ideas on their own terms while ensuring direction and meaningfulness to their discussions, the teacher in Nelson's study followed fairly traditional patterns in which he maintained most of the authority for developing and maintaining discussion.

Similarly, in a case study of two experienced teachers, both of whom had committed a desire to integrate more cooperation and exploration into their instruction, Lloyd (1999) reports that the teachers struggled with whether students would learn appropriate mathematics without explicit teacher direction. Positing a potential loss of self-efficacy, Lloyd recommends that teachers need to find ways to feel efficacious as they adopt forms of instruction requiring them not to tell students what they need to know. Similarly, Smith (1996) argues that attempts to reform school mathematics create a sense of loss of efficacy on the part of the teacher by condemning the traditional expository model of teaching without replacing it with a clear alternative.

Like the teachers in Lloyd (1999) and Nelson's (1997) research, many practitioners are faced with pedagogical and emotional tensions regarding their role in the classroom (Frykholm, 2004). Before student-centered teaching practices are adopted, it is important that teachers overcome potential school pressures that expect the role of the teacher to be one who maintains authority for transmitting knowledge to students. The social context of schooling is such that teachers who are partisans of progressive pedagogy are overwhelmed by conflicting impulses to be simultaneously efficient, scientific, child-centered and authoritative (Cuban, 1993). Cuban (1993)

notes that teachers are often caught between demands from colleagues, community members, and parents to uphold conventions in the classroom.

In particular, parents, with a vital interest in the education of their children, can often exercise a strong influence on teachers to teach in traditional ways. Peressini (1998) notes that parents rely on their own mathematical experiences that were likely acquired under a regime of truth that in many ways stands in opposition to the regime of truth embodied in the mathematics reform literature. Cohen (1988) observes that the instructional practices that reformers wish to eliminate contain views of knowledge, teaching, and learning to which many parents, teachers, and students have deep loyalties. Parents, educators, and mathematicians, with concerns regarding the need to restore basic skills to mathematics education curricula, have led a public backlash against reform recommendations (Allen, 1997; Wu, 1999). According to Allen (1997), the primary role of a teacher should be an expositor and director of learning.

It is clear that teacher's actions in a classroom are influenced by their own beliefs, by district policies, by departmental and administrator pressures, and by the community in which they work. The complex interactions of these elements can help explain why traditional teaching practices are so robust and durable. Perhaps more importantly, students themselves, as active participants in a classroom, play a significant part in the production and reproduction of traditional teacher practices. Teaching practice is often a response to the students in the classroom.

Dependence on Students

Powell, Farrar, and Cohen (1985) argue that a tacit treaty exists between teachers and students in the majority of classrooms. Students agree to behave if, in return, their teachers do not make heavy intellectual demands. If teachers break the treaty by attempting to make students active agents in their learning, students will subsequently cause discipline problems in the classroom. Similarly, Brousseau (1997) theorizes that an implied *didactical contract* regulates the interactions between a teacher and students in a classroom. According to Brousseau, a relationship is formed which implicitly determines what the teacher and the student will have the responsibility for managing and be responsible to the other person for.

The teacher's responsibility in most U.S. classrooms is to present definitions of terms and demonstrate procedures for solving specific problems. Students are then asked to memorize the definitions and practice the procedures (Stigler & Hiebert, 1999). Thus, in most U.S. classrooms, the responsibility of students is not to understand mathematics. According to Cobb (1990), a student's goal in the classroom is not necessarily to learn mathematics, but to complete tasks in ways that are acceptable with respect to the classroom situation. Brousseau's (1997) theory suggests that when a student is unable to complete a task, a breach in the contract occurs and the student will rebel against what the teacher cannot give him/her the ability to do. According to Brousseau's theory, the subsequent renegotiation of the didactical contract often leads to instruction in which the teacher provides students with cues and hints on how to solve problems in order to spare the pain of revealing the holes in their students' knowledge.

Metz (1993) offers a plausible explanation for a phenomenon like this in terms of teacher intrinsic rewards. According to Metz, teachers receive little or no recognition of the effort they expend in the classroom; thus, teachers rely on intrinsic rewards for establishing job satisfaction. The most powerful way for teachers to obtain satisfaction is through the cooperation and success of their students. Therefore, teachers reduce the cognitive demand asked of students in exchange for student compliance and docility. Smith (1996) adds that many teachers are disposed to teach mathematics by *telling* (stating facts and demonstrating procedures) because it enables them to display their mastery of the content to their students, provides a clear model of what to do, and, in return, defines what students should do.

Through these classroom experiences, students form beliefs about their roles in a mathematics classroom, the nature of mathematics, and how mathematical knowledge is acquired. Schoenfeld (1992, p. 359) identifies some of the typical student beliefs regarding mathematical activity:

- Mathematics problems have one and only one right answer.
- There is only one correct way to solve any mathematics problem---usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically and without understanding.
- Mathematics is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.

A multiple solution norm (consisting of expectations that students persevere in solving challenging mathematical tasks, present solution strategies to their peers, try to make sense of and question other students' ideas, and rely on mathematical evidence and reasoning to determine the validity of a solution strategy) confronts the beliefs of most students. Constituting expectations among students that they should explore more than one way to solve a problem represents a significant departure from many students' prior experiences regarding their role in a mathematics classroom. A teacher intending to implement a multiple solution norm faces important obstacles from students who, in some cases, may resist activity that is inconsistent with their previous experiences in mathematics classrooms. Even if students do not overtly resist attempts at implementing a multiple solution norm, unresponsive, disengaged, and overly-quiet students present significant challenges for a teacher attempting to get students to share, evaluate, and modify their ideas.

For example, in *Inside Teaching: How Classroom Life Undermines Reform*, Kennedy (2005) portrays a case where the conduct of a single student was sufficiently powerful to undermine the teacher's effort to create an active and dynamic classroom community. To respond to a student who frequently disrupted lessons by inappropriately acting out in class, the teacher assumed a "calm, deliberate, and even boring" (p. 38) demeanor that was effective in supporting the student. However, the teacher's demeanor promoted a teacher-controlled classroom community that conflicted with her ideal of enthusiastic student participation and active intellectual engagement.

Similarly, Cooney (1985), in a study of a beginning mathematics teacher who was committed in belief and practice to problem solving instruction, found that classroom management problems arose from the conflict between the teacher's struggle to teach problem solving and students who preferred, and expected, more teacher-directed instruction. In a study of a high school Geometry teacher, Gregg (1995) focused on the teacher's instruction of doing proofs. Since the activity of doing proofs could not be reduced entirely to following a set of rules, the students appeared to dislike proofs and had considerable difficulty crafting them. To cope with this tension, the teacher, Ms. Weston, constituted her classes as procedurally as possible. Questions about thought processes were translated into questions about naming rules and procedures. Students were expected to name a rule or state a fact or theorem in response to the teacher's leading questions. Gregg contends that such proceduralization maximizes the appearance of student and teacher competence.

Lampert's (1990) teaching illustrates how instruction that makes use of student ideas is dependent on students' willingness and ability to effectively communicate their thinking. Lampert describes the challenges of teaching reticent students who are hesitant to publicly share their solution methods. Lampert posits that this reluctance is due to the fact that students do not have the words to tell anyone what mental processes led to a particular conclusion; they often lack the courage to expose the thinking behind an asserted answer; and students are often uncomfortable with having the class pay attention to their thinking. Conversely, in the same study, Lampert points out the challenge of working with students who, after coming up with different

answers to a problem, inappropriately attempted to shout down their opposition or intimidate someone who disagreed.

Clearly, instruction predicated on using students' multiple solution ideas to develop key mathematical understanding challenges the expectations for mathematical activity and behavior of students who have experienced years of traditional mathematics teaching. In my study, I purposefully intended to examine a specific context that research suggests provides unique obstacles to teachers. In particular, my study was designed to examine the challenges of instantiating a multiple solution norm to a group of low-tracked middle grade students.

Overall, there appears to be a general decline in school engagement of young adolescents. Middle school is a particularly sensitive time when some students have begun to purposefully withdraw effort, resist novel approaches to learning and avoid seeking academic help (Turner et al., 2002). The National Resource Council (NRC) and Institute of Medicine (IOM, 2004) found that, across all settings, student academic motivation decreases steadily from the early elementary grades into high school; they note that, "Even the best teachers, curricula, standards, and tests cannot be effective if the students to whom they are addressed are not engaged in learning" (p. ix).

Research suggests that tracking students into lower level classes can polarize students into developing negative academic attitudes and behaviors (Berends, 1995). For secondary students, an accumulation of past failure in mathematics classes likely leads to the expectation of future failure and the adoption of a range of miseducative behaviors. Schwartz (1981) contends that students in low-tracked trajectories often

attempt to sabotage a teacher's efforts for instruction. According to Schwartz, students in low-tracked classes often adopt an anti-academic subculture where status in this group is based on defiance of school and teacher norms.

In attempting to implement student-centered instruction to a lower-track Algebra high school class, Chazan (2000) found that sometimes students were actively not engaged and acted out, and at other times they simply disengaged passively by quietly retreating inside themselves. Chazan notes that his teaching was dependent on students' willingness to explore problems, share their ideas, and engage with the ideas of others. So, when students were not willing to participate in solving and discussing problems or attempt them in meaningful ways, Chazan described that his lessons would come to a "grinding halt." Chazan's teaching underscores the importance of selecting tasks that allow students to see value in the content and to create classroom norms that, among other goals, reconceptualizes the notion of right and wrong answers.

In addition to purposefully selecting a low-track classroom in a middle school setting, my study contained additional contextual variables that have been linked to students' academic resistant efforts. Seven of the eight participants in the study were enrolled in the school's alternative education program designed to assist at-risk students because they were identified as having serious academic and behavior issues. Although my school was not located in an urban area, six of the eight students had received significant portions of their prior education in nearby inner-city classrooms.

In designing a non-traditional science program aimed at getting at-risk urban students to construct their own explanations for scientific phenomena, Wong (1996)

describes how students' behaviors can undermine goals for instruction. Seventh and eighth grade students in Wong's class frequently offered explanations that were implicit and understated. Differences in student ideas did not lead to critical discussions. Wong found that students simply shrugged their shoulders and seemed quite comfortable with the fact that different people had different ideas. Wong's students were reluctant to evaluate other students' ideas and viewed this as an activity that was unfamiliar, uncomfortable, and frequently unproductive. Wong contends that the students' passiveness not only suggested that they were being asked to behave in an atypical manner but also indicated that students viewed evaluating one another's answers as aversive. Wong explains that his middle school students sought to conform and maintain social harmony as part of their unstable out-of-school world, and asking students to critically analyze other student ideas conflicted with peer social norms.

In examining teaching in urban contexts, Haberman (1991) continually observed traditional teacher-centered instruction characterized by a basic menu of teacher functions. Some of these core functions include dispensing information, monitoring seatwork, assigning and reviewing homework, and giving grades. Collectively, Haberman referred to these actions as the "pedagogy of poverty." Having learned to navigate in urban schools based on the pedagogy of poverty, Haberman argues that students will not readily abandon all their know-how to take on some new and uncertain system that they may not be able to control. According to Haberman, the pedagogy of poverty is so pervasive that even if a teacher seeks to involve students in a genuine learning activity, students who have been conditioned to accept the

pedagogy of poverty often respond with apathy or bedlam. Students reward teachers by complying and punish teachers by resisting. Thus, Haberman contends that the pedagogy of poverty is sufficiently powerful to undermine the implementation of any reform effort because it determines the way pupils spend their time, the nature of the behaviors they practice, and the bases of their self-concepts as learners.

Most urban educators are concerned about the academic performance of at-risk African American students. Within the past 25 years, a prominent body of research has recognized the negative affects that a mainstream American educational experience can have on African American and other minority students (Delpit, 1988, 1995; Heath, 1983; Lareau, 2003). For these theorists, the racial and cultural incompatibility between a teacher and his/her students is a significant component that could contribute to and limit the academic motivation and success for the students. Many urban African American students do not appear to share mainstream, middle-class perspectives or assumptions about learning and teaching and resist being forced to aspire to middle-class standards of success (McFarland, 1994). In my study, it is important to recognize that I was a White, middle-class male teacher teaching eight African-American students.

Steele (1992) refers to Black students' wish to disassociate themselves with mainstream school goals as *disidentification.* Similarly Ogbu's (1991) theory of cultural inversion describes the phenomenon that occurs when members of a minority group specifically reject behaviors, symbols, and meanings deemed characteristic of the majority culture. Cultural inversion eventually is manifested by the *low effort*

syndrome where Black students avoid, either tacitly or explicitly, acting White so as to remain culturally Black (Fordham & Ogbu, 1986).

Although I made no effort in my study to measure the students' socioeconomic status (SES), it was clear that each student in my class was a member of a working-class family. Lubienski (2000a) employed a sociocultural lens to study how students from socioeconomically diverse groups responded to a pedagogical approach in which students were expected to share, puzzle over, and make sense of mathematical ideas. Lubienski found that lower SES students preferred a teacher-directed style, and the lack of teacher directives seemed to create confusion for more of these students. The subsequent confusion and lack of confidence in their abilities kept many of them from wanting to participate in whole-class discussions. Lubienski raises the possibility that open discussions, in which a variety of methods and ideas are considered, may conflict with the beliefs and norms that lower-SES students bring into the classroom.

Instantiating norms where key mathematical understandings are developed from analyzing and comparing multiple solutions to a single problem requires a paradigm shift away from traditional models of teaching and learning. One clear message emanating from the preceding review of literature is that students can powerfully influence the nature of mathematical activity in a classroom. Students' expectations, interests, culture, and beliefs play a crucial role in shaping what is taught and learned. Changing the mathematics classroom to create different roles for teachers and students requires explicit negotiation on behalf of the teacher. Different contextual

variables influence the degree to which these negotiations are successful in creating a classroom environment predicated on student ideas and discussion of these ideas.

Even in an ideal context where teachers are fully supported and encouraged to teach in a student-centered way, and students willingly accept their role to generate and share solution strategies to problems, creating and maintaining a multiple solution norm is a challenging and difficult endeavor. Teachers must select quality tasks that provide students opportunities to learn the content by figuring out their own strategies and solutions. Teachers must skillfully orchestrate discussions about student ideas and find ways to engage students so that they critically evaluate each other's thinking. These challenges can be sufficiently powerful to undermine a practitioner's efforts to instantiate a multiple solution norm.

Reform Teaching is Difficult and Uncertain

To some, it may appear that a teacher's role is less demanding in a classroom where the discussion of students' ideas is used to develop key mathematical understandings. In reality, creating a multiple solution norm requires a substantial amount of work and effort on behalf of the teacher. A teacher must select tasks and present them in a way that has the capacity to engage all of the students in the class (Lampert, 1990). In orchestrating discussion around solution methods, a teacher must decide what ideas to pursue, when to provide information, and when to let students' struggle with a difficulty while continually assessing students' participation in the discussion (NCTM, 1991). A teacher must attend to the mathematics at hand, focus on the intellectual pace and liveliness of student discussion, and monitor the social and emotional tone of the class (Chazan & Ball, 1999).

Often, when engaging in this type of work, teachers are not allocated enough time to plan and organize rich experiences for their students. Shifter and Fosnot (1993) found that teaching in a student-centered manner required teachers to invest time outside of class to develop new materials and lesson plans. Simon's (1995) research describes data from a classroom teaching experiment designed to develop a model of teaching consistent with visions with constructivist views of learning. Simon's description of teaching makes clear the demanding and uncertain nature of this type of work. Simon notes that, teachers will need access to relevant research on children's mathematical thinking, innovative curriculum materials, and ongoing professional support in order to meet the demands of this role.

One aspect that makes this type of work particularly challenging is that it is non-prescriptive. In addressing concerns regarding the implementation of reform oriented strategies, Ball (1992) acknowledges that this kind of teaching is hard, and no one will be able to produce a system or a formula that can manufacture it. In creating a practice consistent with reform recommendations, Heaton (2000) realized that the vision of mathematics teaching offered by the reform documents is underdetermined. Teaching, predicated on student ideas, entails a continuous negotiation of moves dependent on context rather than prescribed in advance (Heaton, 2000). Stigler and Hiebert (1999) state:

> Unless one knows what to expect from students, it is a scary way to teach. Success depends on making many split-second decisions about which student suggestions to follow up on and which to ignore. What is learned by students during the lesson seems to depend on whether students hit upon the solution methods that make for good class discussions. Teachers can feel that they have lost control of the lesson, but they are told to "embrace the uncertainty" because this is what better teaching is like (pp. 155 – 156).

In contrast, traditional teaching styles are more prescriptive and certain. Traditional mathematics teaching involves telling: providing clear, step-by-step demonstrations of each procedure, and offering specific corrective support when necessary (Smith, 1996). Smith (1996) contends that telling enables teachers to develop a sense of efficacy by giving them a clear role and purpose in the classroom. Although telling cannot guarantee that students will learn, it narrows the scope of the content to manageable proportions, clearly defines what the central acts of teaching are, and provides structure for daily classroom life (Smith, 1996). Similarly, Brousseau (1997) suggests that demonstrating solution algorithms is a tool for teachers to solve didactical conflicts in the sense that it momentarily allows a clear division of responsibilities. According to Brousseau:

> The algorithm is practically the only "official" means of clearing a blockage in that teaching methods related to the algorithm are made explicit. It serves as a unique, or almost unique, model for any cultural approach to teaching (p. 38).

Cuban (1993) and Sizer (1984) suggest that traditional teaching methods have prevailed because they are a less intensive alternative than student-centered instruction. Cuban (1993) contends that the organizational structure of the district, school, and classroom have shaped teachers' dominant instructional practices, and the personal cost in time and energy likely deters many teachers from altering their roles in the mathematics classroom. According to Cuban, teachers ration their energy and time in order to cope with multiple and conflicting demands, and teacher-centered practices have emerged resilient, imaginative, and efficient compromises for dealing with large numbers of students in a small space for extended periods of time. In a five-year study of high schools, Sizer (1984) contends that classrooms are largely

teacher-centered because of the demands of the high school setting. Teachers make bargains with students in order to make their jobs manageable. Disengaged students and over-worked teachers make an unspoken agreement to demand the least amount of work possible from the other while still fulfilling their basic responsibilities.

Finding the time to cover a compulsory and, frequently, crowded mathematics curriculum is a major challenge facing teachers in today's high-stakes educational environment where student performance is made visible through the administration of mandated standardized tests. Smith (1996) argues that traditional teaching methods have remained resilient because they are more expeditious in covering a curriculum, and provide teachers more control over the content, pacing, and direction of the lesson. In contrast, Simon (1995) discovered that the experimental nature of student-centered mathematics teaching made it difficult to plan how long it takes to teach a particular concept. In one instance, Simon (1995) used eight periods to teach a concept that was planned for one or two.

Teachers are asked to trust that *Standards*-based instruction will benefit students and result in positive student achievement (Cuban, 1993). In an educational climate that makes demands for teachers to produce a set of ambitious student outcomes, teachers are likely to rely on previous experiential knowledge and avoid risks and experiments (Ball & Cohen, 1999). Cuban (2003) contends that the impact of standards-based performance and accountability has weakened progressive teaching practices while hardening traditional teaching patterns. Teachers are likely to avoid the looseness associated with instruction predicated on the discussion of student ideas. For example, Taylor (1990) attempted to assist a high school teacher to modify

his beliefs through a process of conceptual change. However, there were conflicting beliefs that he had to cover a mandated curriculum and teach for constant assessment. Given that he did not want to jeopardize students' learning with alternative strategies, change in the teacher's instructional behavior was restricted.

A commonly accepted supposition regarding the implementation of reform-based approaches is that it requires a deeper understanding of the mathematical content than do traditional teaching methods. Ball, Lubienski, and Mewborn (2001) note that interpreting reform ideas, using new curriculum materials, enacting new practices, and teaching new content all depend on teachers' knowledge of mathematics. Teachers are unlikely to be able to promote an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way (NRC, 2001).

Lampert's research on teaching (1990, 2001) reveals the depth of mathematical knowledge that is needed for a teacher in designing a problem, asking questions, and managing a complex discussion. In engaging her students in mathematical arguments, Lampert (1990) found it necessary to know more than the answer or the rule for how to find it. In changing her practice to incorporate reform recommendations, Heaton (2000) found it necessary to know a qualitatively different kind of mathematics. In a lesson involving composition of functions, Heaton (1995, 2000) realized she lacked a sense of purpose for her lesson, and did not really understand how composition of functions was relevant to the curriculum.

Consequently, Heaton was discouraged by her inability to manage a productive discourse and help students make sense of each other's ideas.

A teacher attempting to alter his/her traditional role in the mathematics classroom will likely feel a level of frustration when confronted with a situation in which it is evident that his/her knowledge is incomplete. Frykholm (2004) puts forward a framework that consists of four primary domains of discomfort that appear to be more prevalent in the teaching of reform-based mathematics. One of these domains, cognitive discomfort, entails uncertainty on behalf of the teacher over mathematical content knowledge, connections between mathematical concepts, and instrumental mathematical understandings versus procedural conventions (Frykholm, 2004). Frykholm suggests that a teacher's discomfort can be *debilitating* when, for example, a teacher simply does not have sufficient content knowledge to engage students in the mathematics of the curriculum. In such occurrences, a natural tendency is to resort to a more stable and comfortable teacher-centered practice. Being able to successfully manage these feelings of discomfort is a significant challenge faced by a practitioner attempting to implement a multiple solution norm.

A deep knowledge of and about mathematics is a necessary, but not sufficient condition to implementing a multiple solution norm. Teachers possessing a *profound understanding of fundamental mathematics* (Ma, 1999) are still faced with the task of managing an active learning environment (Frykholm, 2004). Orchestrating student discussion regarding their own solution methods presents teachers with unique challenges. For example, in a classroom discussion in which ideas are being exchanged spontaneously, Forman (2003) raises practical concerns regarding the

expectation that a teacher can fully appreciate several alternative solution paths without time to carefully examine each one. Silver et al. (2005) note that teachers are faced with decisions regarding selecting and sequencing student solution methods. Chazan (2000) describes the challenges of listening past his personal "enculturation" to appreciate the logic behind statements that were mathematically incorrect. And Ball (1993b) describes the frustrations involved in discussing ideas when students become confused and invent their own, nonstandard mathematics.

Classroom discussions are complex social events; "diverse students, the relationship among them, their emergent mathematical ideas, the curriculum, the clock – all of these and more interact as a class discussion evolves" (Chazan & Ball, 1999, p. 8). In whole-class discussion, students can inadvertently lead other students down mathematically unproductive paths (Ball, 1993b; Chazan & Ball, 1999; Lampert, 1990, 2001), they can become entrenched in their views as they defend their respective ideas (Chazan & Ball, 1999), students can reach a mathematically incorrect consensus (Chazan & Ball, 1999), and students may believe that a majority vote will resolve a conflict without exposing the incorrect assumptions or procedures that led to the divergence in the first place (Lampert, 1990). Chazan and Ball (1999) note that simply having students share their ideas will not necessarily generate learning. Decisions to intervene and provide an explanation or ask a pointed question are delicate ones a teacher needs to consider in shaping the direction of discourse (Ball, 1993b). According to Chazan and Ball, "Managing the differences among ideas in a discussion is one of the crucial challenges for teachers who seek to teach through student exploration and discussion" (p. 7).

Conclusion

Despite a general consensus among mathematics educators that students should have experiences in which they solve problems in more than one way, most classrooms in the United States appear to remain teacher-centered and deny students access to exploring multiple solutions. An important step to changing this tradition is to understand some of the obstacles against using students' multiple solutions as a key aspect in mathematics instruction. The literature presented here examined a subset of conditions affecting a teacher's instructional decisions. Reasons why teachers may avoid making students' multiple solutions a central focus in their instruction include that deeply rooted cultural beliefs make it difficult to reform mathematics teaching, students may offer active or passive resistance when required to become active participants in their own learning, and using student ideas and discussion of those ideas to develop key mathematical understandings is difficult and ambiguous work. While planning and conducting my study, I consistently drew upon this literature to anticipate obstacles and plan strategies for overcoming them.

Throughout this chapter, I continually referred to the notion of a multiple solution norm when referring to mathematical activity where students would solve a problem in more than one way and discuss and compare their various solutions strategies. In the next chapter, I will formally define the construct of a multiple solution norm and introduce the research questions that were used to guide the study. First, I begin the next chapter by discussing some theoretical issues related to classroom social and sociomathematical norms.

Chapter 3: Framework

The review of literature in the preceding chapter suggests that, by middle school, most students in the United States do not enter the mathematics classroom with an expectation that they can be able to solve one problem in many different ways. A goal for students to derive their own solution methods for a mathematical problem and talk about their ideas runs counter to most students' prior experiences. Thus, creating a classroom culture where students learn mathematics through analyzing each other's inventive solution strategies is dependent on getting students to change how they view their own role, their teacher's role, and other students' roles in the classroom.

Lampert's (1990) research illustrates a case of a teacher who aimed to create different kinds of roles and responsibilities for students. In challenging her own students' conventional assumptions about the nature of mathematics, Lampert explains, "I assumed that changing students' ideas about what it means to know and do mathematics was in part a matter of creating a social situation that worked according to rules different than those that ordinarily pertain in the classroom, and in part respectfully challenging their assumptions about what knowing mathematics entails" (p. 58). In *Teaching Problems and the Problems of Teaching*, Lampert (2001) mentions the need to establish and maintain norms of action and interaction to create a classroom culture in which students were publicly willing to reason their way from confusion to making mathematical sense and to talk about what they were thinking.

The manner in which Lampert (1990, 2001) examines mathematical activity in the classroom by accounting for the social interactions that take place is consistent with a wave of mathematics education research conducted over the past two decades. For Yackel and Cobb (1996), a fundamental feature of mathematics classrooms is that they are characterized by normative understandings regarding expectations and obligations for social interactions and for specifically mathematical interactions. Yackel and Cobb refers to the process in which a teacher and students cope with different expectations as the (re)negotiation of classroom norms.

In thinking through how best to study the challenges of using students' multiple solutions as a core feature of mathematics instruction, I decided to use the notion of norms as a lens to examine a teacher's attempts to create a specific kind of classroom environment. Several theoretical issues relating to the use of social and sociomathematical norms as an interpretive framework for analyzing a teacher's socially situated activity will be examined in this chapter. Also in this chapter, I define my construct of a multiple solution norm as a collection of specific social and sociomathematical norms. As a means of introducing and clarifying my construct of a multiple solution norm, I present a series of classroom vignettes to illustrate practical challenges to implementing a multiple solution norm. Similar to McGraw (2002), I use Simon's (1995) conceptual framework of the mathematical teaching cycle to situate my examination of the process of instantiating norms within the larger process of teaching described by this cycle. Finally, in this chapter, I introduce the research questions that were used to guide my data collection and analysis.

Social and Sociomathematical Norms

For Yackel and Cobb (1996), the use of social and sociomathematical norms arose as the result of finding a cognitive perspective limiting when attempting to develop accounts of students' mathematical learning. Unable to explain students' mathematical activity and learning in individualistic psychological terms, Yackel and Cobb explain that they needed to broaden their "interpretive stance by developing a sociological perspective on mathematical activity" (p. 459). Recognizing that "mathematical learning is both a process of active individual construction and a process of acculturation into the mathematical practices of a wider society" (Yackel & Cobb, 1996, p. 460), Cobb and Yackel (1996) developed an interpretive framework to analyze teachers' and student's activity in the classroom. Cobb and Yackel's interpretive framework is shown in Figure 1.

Social perspective	**Psychological perspective**
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions

Figure 1: Cobb and Yackel's (1996) Interpretive Framework

Although Cobb and Yackel's (1996) interpretive framework was developed with a focus on students' learning, they note that their framework can be adapted to analyze a teacher's instructional practices within the social context of a classroom. For example, Kazemi and Stipek (2001) used the construct of sociomathematical norms as a useful framework for understanding what teachers need to do to promote meaningful development of students' mathematical ideas. McClain and Cobb (2001)

used the framework to analyze a teacher's proactive role in the development of the classroom microculture in one first grade classroom. And Simon (1995) developed a model of mathematics teaching that was informed by a view that students' mathematical development occurs in the social context of the classroom.

As the column headings in Figure 1 indicate, the interpretive framework involves the explicit coordination of neo-Piagetian psychological constructivism with Vygotskian sociological perspectives (Cobb & Yackel, 1996). From a psychological perspective, mathematical knowledge development is fundamentally a cognitive process. Although social interaction can stimulate individual development, it is not integral to the cognizing individual's constructive activity (von Glasersfeld, 1990). From a sociological perspective, individuals have to interpret what the other is doing, and each person's actions are formed, in part, on the actions of others. Blumer (1969) refers to this process as social interactionism. According to Blumer, "[I]nteractionism sees meaning as social products, as creations that are formed in and through the defining activities of people as they interact" (p. 5). From a theoretical standpoint, Cobb and Yackel (1996) refer to the coordination of interactionism and psychologoical constructivism as the *emergent perspective.* Cobb et al. (2001) emphasizes that within the emergent perspective, neither interactionist nor psychological constructivist perspectives exist "without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective" (p. 122). From a sociological perspective a student's reasoning is located within an evolving microculture, and from a psychological perspective that microculture is treated as an emergent phenomenon

that is continually regenerated by the teacher and students in the course of their ongoing interactions (Cobb et al., 2001).

As indicated in Figure 1, the interpretive framework consists of three pairs of categories that are reflexively linked across social and psychological dimensions. The first pair of categories link social norms with students' beliefs about classroom roles and the general nature of mathematical activity. Social norms characterize regularities in collective classroom activity jointly established by the teacher and students as members of the classroom community and are themselves continually being (re)generated by and through interactions (Cobb & Yackel, 1996). At the same time, the teacher and students reorganize their beliefs about their own role, others' role, and the general nature of mathematical activity through these same interactions (Cobb, 2000). Consistent with the emergent perspective, Cobb (2000) posits that:

> [I]t is neither a case of a change in social norms causing a change in students' beliefs, nor a cause of students first reorganizing their beliefs and then contributing to the evolution of social norms. Instead, social norms and the beliefs of the participating students co-evolve in that neither is seen to exist independently of the other. (p. 69).

Lampert (2001) helps articulate the implications that this reflexive relationship has for teachers. According to Lampert, "Every teaching action, no matter how narrow its intent, has an impact on shaping the complex set of ongoing relationships aimed to enable every student in the class to learn mathematics over time, and conversely, those ongoing relationships are a constraint on every action" (p. 430).

Classroom social norms, such as expectations that students persist in solving challenging problems, listen to and attempt to make sense of other's solutions, and ask questions and raise challenges in situations of misunderstanding or disagreement,

are not specific to mathematics. Norms that are specific to the mathematical aspects of students' activity are referred to as sociomathematical norms (Yackel & Cobb, 1996). Normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant are examples of sociomathematical norms (Yackel & Cobb, 1996). According to Cobb and Yackel's (1996) framework, what becomes mathematically normative in a classroom is enabled and constrained by the students' changing mathematical beliefs and values. At the same time, these beliefs and values are themselves influenced by what is legitimized as acceptable mathematical activity.

The third aspect of the interpretive framework concerns the mathematical practices established by the classroom community and their psychological correlates, individual students' mathematical interpretations and actions. Cobb and Yackel (1996) explain,

> Students actively contribute to the evolution of classroom mathematical practices as they reorganize their individual mathematical activity, and conversely that these reorganizations are enabled and constrained by the students participation in the mathematical practice. (p. 180)

It should be noted that the interpretive framework in Figure 1 focuses on the classroom processes. The emergent perspective does not explicitly take into account that students are part of other communities that influence how they participate in the mathematics classroom. Although Cobb and Yackel (1996) recognize that they could often develop adequate explanations by referring solely to classroom processes, they found occasions when it was essential to take account of the broader institutional contexts in which such systems are embedded. Sociocultural theory proposes that teachers need to understand the mathematical knowledge that children bring with

them to school from the practices outside of school as well as the motives, beliefs, values, norms, and goals developed as a result of those practices (Forman, 2003). Cobb and Yackel (1996) point out that sociocultural perspectives are needed to account for disparate findings when different groups of students receive supposedly the same instructional treatment.

Identifying Social and Sociomathematical Norms

An observer can infer the existence of classroom norms by examining regularities in the interactions between a teacher and students, or by noting breaches that occur (Cobb, Yackel, & Wood, 1993). Yackel (2000) explains that understandings are normative if there is evidence from classroom activity that students' interpretations are compatible or taken-as-shared. Norms are not predetermined criteria set out in advance to govern classroom activity; instead "these normative understandings are continually regenerated and modified by the students and the teacher through their ongoing interactions" (Yackel & Cobb, 1996, p. 474). Although methodologically, both general social norms and sociomathematical norms are inferred by identifying regularities in patterns of social interaction (Yackel & Cobb, 1996), Cobb (2000) points out that normative taken-as-shared interpretations cannot be observed directly. Instead, conjectures about communal mathematical activity are developed and tested through the course of analyzing what the teacher and students say and do in the classroom (Cobb, 2000).

It is recognized that the differences between social and sociomathematical norms are not easily distinguished. While social norms refer to the general ways that students participate in classroom activities, sociomathematical norms concern the

normative aspects of classroom actions and interactions that are specifically mathematical (Yackel & Cobb, 1996). To clarify the subtle distinction between social norms and sociomathematical norms, Yackel and Cobb (1996) explain, "The understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm" (p. 461). Ultimately, Herbst (1997) recognizes that social and sociomathematical norms are social constructs and the distinction between the two is made by an observer studying classrooms, not the teacher or students.

Social and sociomathematical norms are frequently interdependent. A social norm that is described by the expectation that students regularly offer different solution strategies is likely related to the normative understanding of what counts as a mathematically different solution, a sociomathematical norm. Yet a classroom governed by such a social norm need not necessarily have constituted the related sociomathematical norm. It is conceivable that students can describe the steps they took to solve a problem without understanding how their solution compares and contrasts with others already offered. Many teachers find it easy to ask for different solution strategies, however it is a more challenging endeavor to engage students in genuine mathematical activity (Chazan & Ball, 1999; Kazemi & Stipek, 2001)

To identify and define general classroom social norms, several researchers have described the classroom participation structure (Lampert, 1990; Kazemi & Stipek, 2001; McClain & Cobb, 2001). Lampert (1990), drawing from the work of Florio (1978) and Erickson and Shultz (1981), explains that a participation structure

represents the "consensual expectations of the participants about what they are supposed to be doing together, their relative rights and duties in accomplishing tasks, and the range of behaviors appropriate within the event" (p. 34). When developing conjectures about social norms, Cobb et al. (2001) focuses on regularities in joint activity rather than an alternative approach that casts criteria for social norms in terms of the proportion of students who act in accord with a proposed norm. Cobb et al. (2001) explain that the latter criterion is "framed from a psychological perspective that is concerned with individual students' activity rather than from a social perspective that is concerned with how students' activity is constituted in the classroom" (p. 123).

In studies examining sociomathematical norms, researchers frequently examine the nature of classroom mathematical discourse and the teacher's role in those discussions (Kazemi & Stipek, 2001; McClain & Cobb, 2001; Pang, 2000; Yackel & Cobb, 1996). For example, Kazemi and Stipek (2001) used examples of classroom exchanges to suggest how sociomathematical norms governed classroom discussions. Lampert (2001) notes "each word and gesture the teacher uses has the potential to support the study of mathematics for all students" (p. 144). In analyzing the process by which sociomathematical norms emerge, McClain and Cobb (2001) point to the importance that the teacher's role in symbolizing students' offered solutions played in the development of sociomathematical norms.

A goal of this study is to examine my experiences attempting to negotiate an ambitious collection of norms governing students' social and mathematical activity. In the next section I detail the specific norms I sought to instantiate by formally

defining the construct of a multiple solution norm. To the extent that classroom norms constrain and enable learning, I believe it is possible for teachers to initiate and guide the constitution of norms in a purposeful manner. According to Cobb (2000), the teacher, as an institutionalized authority in the classroom, "expresses that authority in action by initiating, guiding, and organizing the renegotiation of classroom social norms" (p. 69). I recognize that, since norms are upheld through a process of social interactions within broader institutional settings, the specific norms that become constituted are unique to each classroom (Cobb & Yackel, 1996). Examining a teacher's role in guiding and organizing the development of the classroom microculture can offer valuable insights into the messiness and complexity of the classroom.

Defining a Multiple Solution Norm

It is common for many Americans to view that quality teaching is dependent on individual practitioners, and teaching can be improved by recruiting better teachers (Stigler & Hiebert, 1999). Star teachers are seen as individuals who possess a strong grasp of the subject matter, use questions to elicit student thinking, listens carefully to students, and injects enthusiasm and humor into exchanges with students (Boyer, 1983). Good teachers are seen to "pump up students' interests by increasing the pace of the activities, by praising students for their work and behavior, by the cuteness or real-lifeness of tasks, and by their own power of persuasion through their enthusiasm, humor, and 'coolness'" (Stigler & Hiebert, 1999, p. 93). Over time, these norms and expectations for quality teaching have become deeply entrenched.

In contrast to United States teachers, Japanese middle school teachers believe students learn best by first struggling to solve mathematics problems. Students then participate in discussions about how to solve them, and analyze the pros and cons of different methods and the relationship between them (Stigler & Hiebert, 1999). Students in middle grade Japanese classrooms are given time to explore different solution strategies, to make mistakes, to reflect, to construct connections between methods and problems, and to receive the needed information at an appropriate time (Stigler & Hiebert, 1999). Mathematics classrooms in Japan are guided by a different set of norms and expectations than American classrooms. Japanese teachers and students have constituted a set of norms in which students are expected to choose their own methods for solving a problem, share those methods with others, and analyze and evaluate the mathematics underlying those methods.

Any attempt to constitute a similar set of expectations and norms in American classrooms will require unlearning many deep-seated expectations about what classrooms should be like and what teachers and students should do. New roles and responsibilities must be negotiated, made explicit, and practiced by both the students and the teacher. Changing the fundamental nature of classroom interaction and learning is a difficult process. A teacher committed to creating a classroom environment in which student solutions are a key resource in teaching must explicitly aim to create social and sociomathematical norms different than what are found in most United States classrooms.

The collective set of norms in Figure 2 represented my original vision of a multiple solution norm. Consideration for the norms identified in this framework

came from a wide array of sources. The framework was influenced by the norms characterized by Yackel and Cobb (1996) and by Hiebert et al.'s (1997) features of social culture of the classroom that functions as communities of learners. The framework was influenced by the standards put forth by the National Council of Teachers of Mathematics (1991, 2000) and the National Resource Council (2001). The examination of Japanese lesson plans and video, together with research regarding teaching as a cultural activity (Stigler & Hiebert, 1999) factored significantly into the design of the framework. The framework was also influenced by educators who studied their own practice, and explicitly considered the norms of their classrooms in making a concerted effort to encourage students to generate, elaborate, share, evaluate, and modify their own ideas (i.e., Chazan 2000; Lampert, 1990, 2001; Wong, 1996). I used my own practice and reflected upon my efforts and challenges to implementing a multiple solution norm. In the study, my teaching was aimed at instantiating this set of norms. In the discussion of the results of the study, this framework will be updated to reflect the challenges and reality of teaching a group of low attaining students.

Social Norms	*Sociomathematical Norms*
Students possess a productive disposition	Students publicly present mathematical explanations for their solution strategies
Students listen to, respect, and comment on solution strategies	Students analyze/evaluate solution strategies
Teacher and students share responsibility for adjudicating correctness of solutions	Students use mathematical mistakes as learning opportunities
	Students compare, contrast, and make connections between solution strategies

Figure 2: Multiple Solution Norm

Social Norm: Students possess a productive disposition

Before students can share and discuss their solution strategies, it is essential that a classroom climate be created so that students accept the challenge to develop solutions to non-routine problems. Non-routine problems cannot be identified solely by reading them; they can look like traditional tasks if they are presented at the appropriate time, before a formal algorithm for its solution is well-developed (Lampert, 1990). Identifying it as one of the five strands of mathematical proficiency, the National Resource Council (2001) defines productive disposition as a habitual inclination to see mathematics as sensible, useful, and worthwhile. Productive disposition recognizes that hard work and one's own ability will lead to successful learning in mathematics. Students with a productive disposition are diligent workers, and are able to problematize a task and display a willingness to explore problems. The shared expectation of the teacher and students is that students will persevere their way from confusion to making mathematical sense (NCTM, 1991).

Traditionally, the tendency for students to rely on their problem-solving abilities is fragile (Hiebert et al., 1997). Too often, students have developed the idea that if they cannot answer a mathematical question almost immediately, then they might as well give up (NCTM, 1991). As mentioned in Chapter 2, Shoenfeld (1992) found that a prevailing belief among students is that any problem could be solved in five minutes or less and that there is only one correct way to solve any mathematics problem. Schoenfeld claims that students extract their beliefs in large measure from their

experiences in classrooms and these beliefs "shape their behavior in ways that have extraordinarily powerful (and often negative) consequences" (p. 359).

In most secondary mathematics classrooms, students come to expect that they should not have to struggle to solve a problem. Teachers in the United States frequently decompose a problem into tasks that are manageable for most students. Student confusion and frustration are signs that teachers have not done their job. When United States teachers notice confusion, they quickly assist students by providing whatever information it takes to get the students back on track (Stigler & Hiebert, 1999). Often the tasks are made routine in one of two ways: "the students may start pressing the teacher to reduce the challenge by specifying explicit procedures or steps for them to perform, or the teacher may take over the demanding aspects of the task when students encounter difficulty by either telling them or demonstrating what to do" (NRC, 2001, p. 325).

Sociomathematical Norm: Students publicly present mathematical explanations for their solution strategies

Once students problematize a task and arrive at various solution methods, an obvious next step for instantiating a multiple solution norm is for the teacher and students to jointly build a culture in which students are publicly willing to express their ideas and take intellectual risks. Hiebert et al. (1997) note that, "a student's responsibility does not end when she or he has used a method successfully. The student must then work out a way to present and explain the method" (p. 47). Many classrooms are governed by the social norm that students explain their thinking. However, Kazemi and Stipek (2001) point out that students can describe the steps

they took to solve a problem without explaining why the solution works mathematically.

Explanations must go beyond just a procedural description or summary. Students should be expected to display a sense of mathematical competence by justifying their solutions and validating their ideas with mathematical argument (NCTM, 1991). Students should not simply summarize the steps taken that led to a solution, but be able to provide a mathematical rationale for why they chose the steps they did. Additionally, a student should be able to show that the answer to a problem creates a reasonable and valid solution. In a classroom where a multiple solution norm is constituted through the actions of the teacher and students as they interact with one another in the course of classroom activity, students are able to differentiate between various types of mathematical reasons. In particular, students are able to distinguish between explanations that describe procedures and those that describe actions on experientially real objects (Yackel & Cobb, 1996).

The constitution of this sociomathematical norm is dependent on the students' willingness to engage in meaningful activity and to publicly share their ideas and the rationale behind them. Chazan (2000) found that student engagement was quite variable and fluctuated unpredictably. Adolescent students are often reluctant to stand out in any way and find it uncomfortable to publicly present their solution methods. Students need guidance and encouragement in order to willingly participate in the classroom (NCTM, 1991).

Students do not necessarily talk about mathematics naturally; teachers need to help them learn how to do so (NCTM, 2000). Wong (1996) recognizes that, when asked

to provide a rationale for their ideas, students may confuse level of explanation with detail of description. When pressed to present a mathematical argument or provide a deeper level of explanation, it is not uncommon for students to simply embellish their prior procedural explanations. For Wong, a critically important job for a teacher is to negotiate with students what constitutes an appropriate explanation.

Traditionally, the role of students in a mathematics classroom is one of passive acceptance of a large body of information provided by the teacher (Stigler & Hiebert, 1999). Students participate by responding to a teacher's request for information. Typically teacher questions can be answered with brief responses, often one word (Cazden, 1988). The aim of the student participating in this type of discourse is to display competence. In contrast, a classroom in which a multiple solution norm is constituted, students do the majority of the explaining. The function of these explanations is to promote learning as much as to demonstrate a student's aptitude (NCTM, 1991).

Social Norm: Students listen, respect, and comment on solution strategies

Hiebert et al. (1997) contends that, with the continual sharing of solution strategies, the potential exists for students to learn from each other by imitating a solution strategy to solve a problem they did not understand. A necessary condition to realize this potential is that students must listen to and respect the ideas of others. To create a norm in which student ideas are respected, it is important that students are given time to explain their reasoning without other students bursting in, frantically waving hands, or showing impatience (NCTM, 1991). Lampert (2001) worked to

create a classroom culture in which students respected what others had to say even if it did not seem to make sense.

It is important that students listen, not only out of politeness or respect, but also because of a genuine interest in what the speaker has to say (Paley, 1986). When students share their thinking, they are subjected to the judgment of the teacher and their peers. This creates the potential that students' differences will be placed in a spotlight.

Students should attempt to make sense of other's interpretations and solutions by asking questions and raising challenges in situations of misunderstanding or disagreement (Yackel, 2000). Students need to learn how to question another's conjecture or solution with respect for that person's thinking or knowledge. They also need to learn how to justify their own claims without becoming hostile or defensive.

For example, Ball (1993) describes a discussion in her third grade class where a student, named Sean, proposed that six is both even and odd because when six objects are grouped by twos, there is an odd number of groups. In response, students in the class rationally attempted to make sense of Sean's claim and express their disagreement by providing a mathematical argument. One of the students, Mei, restated Sean's conjecture and then disagreed by showing how, using Sean's argument, ten could be considered even and odd. Sean respectfully responded, by stating, "Thank you for bringing it up, and I agree. I say ten can be odd or even." Although Ball uses this episode to draw attention to the pedagogical dilemma of

validating a student's nonstandard idea, it illustrates a real example of students listening, respecting, and commenting on their classmates thinking.

Unfortunately, moments of classroom disagreements are not always handled in such a reasonable way. Lampert (1990) observed that it was not unusual for students to shout down the opposition or more indirectly intimidate someone who disagrees by addressing peers as dumb or stupid. Chazan and Ball (1999) and Wong (1996) found that, at times, individuals representing opposing explanations for a phenomenon actually became more entrenched in their views as they defended their respective ideas, and instead of facilitating progress, disagreement often led to greater polarization.

To prevent feelings of defensiveness or fear, students should come to understand that asking questions about their methods and their reasoning is a means of showing appreciation (Hiebert et al., 1997). The learning community must come to expect differences and appreciate the learning that comes from divergent ideas.

Sociomathematical Norm: Students analyze/evaluate solution strategies

A norm in which students respectfully listen to their classmates' explanations and ask questions in situations of misunderstanding are social norms that could be exhibited in any general classroom. For mathematical understanding to take place, students must also attend to the underlying mathematics of a student solution strategy. Students must be willing to publicly challenge mathematical claims and identify perceived mathematical errors. In a classroom where this norm is constituted, students understand what counts as an acceptable mathematical explanation (Yackel & Cobb, 1996), and incomplete explanations are not accepted and probed further by

students. Not every solution strategy is accepted equally. Students do not turn the idea that multiple solutions to a problem are possible into the relativistic notion that every solution should be accepted just because someone came up with it (Cooney, 1987 as cited in Lampert, 1990, p. 57).

When students are asked to critically evaluate each other's solution strategy, a teacher must carefully orchestrate around complex classroom interactions. Wong (1996) found, in trying to engage middle school students in the exploration and critique of their peers' explanations, some students became visibly upset and defensive, and their verbal and physical reactions revealed hurt and anger. Wong concluded that engaging students in particular scientific practices, such as critical analysis of each other's ideas, presented formidable instructional and ethical challenges because it seemed to work against some students' desire and need for conformity, harmony, and peace.

Sociomathematical Norm: Students use mathematical mistakes as learning opportunities

In traditional classrooms, where the emphasis is placed on getting the right answer and getting it quickly, students are often afraid to make mistakes or risk looking foolish. A point of emphasis in a classroom in which a multiple solution norm exists is that mistakes are seen as an important part of learning. Students are not ashamed or afraid to make mistakes.

Errors can be used to further students' mathematical understanding if mistakes are addressed in a way that allows student to learn from them (Hiebert et al., 1997). Inadequate solutions serve as "entry points for further mathematical discussion

involving justification and verification" (Kazemi & Stipek, 2001, p. 72). Creating a climate where mistakes are thought of in this way is challenging. Chazan (2000) suggests a teacher and his/her students need to reconceptualize the construct of right and wrong in the math classroom. If students think of solutions as only correct or incorrect, then Chazan expresses concerns that they will not be interested in examining ideas that are labeled wrong.

In a classroom with a multiple solution norm, students willingly put forth solution methods that seemingly do not work in order to identify errors in their strategy. Identifying a mathematical mistake or an anomaly in a solution strategy is a highly regarded skill in the classroom. Kazemi and Stipek (2001) found that teachers were better able to push students' conceptual thinking by "promoting the sociomathematical norm that mistakes are opportunities to reconceptualize a problem, explore contradictions to a solution approach, and try out alternative strategies" (p. 72)

Social Norm: Teacher and students share responsibility for adjudicating correctness of solutions

Creating a classroom environment in which students feel safe to share their solution strategies does not mean that students' errors go unchecked or that incorrect ideas are accepted as valid (NCTM, 1991). The whole class is responsible for making sense of mathematics. The students work with each other and the teacher to test and validate their ideas and methods rather than looking to the teacher as the sole voice of authority. The correctness of the solution comes from the logic of the mathematics

rather than from the word of the teacher (Hiebert et al., 1997). A teacher's questions are not a cue that a given answer is incorrect (Yackel & Cobb, 1996).

The correctness of solutions are held in suspension while the investigation of a given solution strategy is discussed (Hiebert et al., 1997). It is a key function of the teacher to develop and nurture students' abilities to learn with and from others, to clarify definitions and terms to one another, consider one another's ideas and solutions, and argue together about the validity of alternative approaches and answers (NCTM, 1991).

In traditional, teacher-centered classrooms, students often expect the teacher to act as the source of knowledge and validation. Students may become anxious when teachers probe their thinking rather than proclaiming their answer is correct or incorrect. Students will pressure the teacher for the correctness of their answers. In trying to establish this norm, Wong (1996) noted numerous occasions when students made comments at the end of a discussion such as, "Well, aren't you going to tell us the answer?" The more impatient ones, Wong noted, would remark during the middle of a discussion, "You're the teacher. You tell us." Some students, in an attempt to make sense of these unusual student and teacher roles, were prompted to conjecture aloud, "You don't really know the answer, do you?"

Sociomathematical Norm: Students compare, contrast, and make connections between solution strategies

Discourse should not be confined to the correctness of answers, but should include discussion of connections to other problems, alternate representations and solution methods (NRC, 2001). This sociomathematical norm is evident in the classroom

when students routinely juxtapose solution strategies to determine which methods are different and which are efficient and sophisticated.

The meanings of what constitutes different and sophisticated solution methods are jointly negotiated by the teacher and students (Yackel & Cobb, 1996). In a classroom with a multiple solution norm, students understand it is not appropriate to offer an explanation that essentially repeats a previous response. Connections between solution methods focus on mathematical aspects of students' strategies. Students routinely identify methods that have mathematically significant advantages over others.

Overall, to instantiate the collection of norms in Figure 2, a teacher will need to develop a shared vision of the classroom environment with their students. Through classroom interactions, this vision will be negotiated and adapted in ways so that the vision becomes compatible with students' ideas. As discussed in Chapter 2, students' can yield sufficient power to sway a teacher away from his/her goals for social and mathematical activity in the classroom. In addition, the pedagogical demands of attempting to instantiate a multiple solution norm can become overly burdensome. Next, in an attempt to clarify my definition of a multiple solution norm, I introduce several classroom vignettes designed to highlight practical challenges a practitioner might face when attempting to implement a multiple solution norm.

Vignettes: Challenges to Creating a Multiple Solution Norm

Creating a multiple solution norm is demanding and difficult work for a practitioner. This section will describe a series of annotated classroom vignettes in an attempt to describe a subset of the wide-array of challenges a teacher may confront

in his/her efforts to instantiate a multiple solution norm. Each vignette is meant to illustrate a potential challenge a practitioner may deal with in either constituting a given norm, dealing with the absence of a desired norm, or a challenge that arises as a consequence of a norm being constituted. The vignettes are not meant to infer what teacher action or process led to a challenge or how the challenge could be best addressed.

The vignettes, though fictional, are based on an actual lesson I taught to two sections of an eighth-grade Algebra class. The idea of using vignettes to highlight issues of creating a multiple solution norm came from vignettes in *Professional Standards for Teaching Mathematics* (NCTM, 1991) used to illustrate the roles of teachers and students in a student-centered classroom. In addition, the vignettes represent a collection of ideas from other research documents. Research by Kazemi and Stipek (2001), Pang (2000), and Yackel and Cobb (1996) examined issues regarding creating certain social and sociomathematical norms. Chazan and Ball (1999) and Lampert (2001) discuss the problems of teaching in ways predicated on student ideas.

Background Information:
The vignettes describe an episode from the classrooms of four teachers: Mr. Carl, Ms. Joyner, Ms. Robinson, and Mr. Lyttle. Each teacher reports a commitment to allow students the opportunity to be engaged in mathematical discourse in which they invent, explain, and justify, their own mathematical ideas, and critique the ideas of others.

The classroom participation structure in each of the four classrooms was similar: (a) students were seated in groups of four or five; (b) the teacher initiated an activity or gave students a mathematical problem; (c) students independently solved the given problems; (d) the teacher asked students to report their solution methods to the whole class; (e) students presented their solution methods; and (f) the teacher facilitated the classroom discussion.

Each teacher made a conscious effort not to adjudicate the correctness of ideas. All four teachers regularly asked students to explain their reasoning regardless of a correct or incorrect solution. Classes were dynamic and the students eagerly responded to requests made by the teacher. Each teacher had established a caring and permissive environment in which students' mistakes were welcomed and accepted without ridicule.

The issue of task selection was critical for each teacher. They took care to select problems and activities that would be accessible to each student. Each of the four teachers used tasks that would elicit alternative representations and solution strategies. The teachers purposefully posed problems before a formal algorithm had been reached.

Each teacher posed the following task for their students:

A person invests $1,000 into a savings account that earns 10% interest each year. How much money will the person have (a) After1 year? (b) After 6 years? (c) After 60 years?

Vignette #1:
Students in Mr. Carl's class predominantly worked quietly and independently on the problem. Although students were placed in groups of four, there was little collaboration between classmates. In two of the groups where students were working together, the distribution of work across a group was not equal; members of the groups often accepted another student's answer with little or no debate.

After several minutes, the majority of the class was no longer thinking of the problem and was engaged in conversations outside of mathematics. As Mr. Carl walked to different groups, he noticed most students had recorded a solution to part (a); however, Mr. Carl recognized that many students had made either procedural or conceptual errors in their solution. More than one student calculated the amount of money after one year as less than the original investment. Further, Mr. Carl observed many students made no visible attempt to solve the other two parts of the problem. One student who had not gone off task, asked Mr. Carl "I know how to get the first part, but how do I find how much there will be after six years?"

A challenge for Mr. Carl is to create an environment so that students reflect on mathematics and communicate ideas. In a classroom where a multiple solution norm is present, students will regularly ask questions of each other and evaluate solutions during student-student interactions.

In a classroom in which a multiple solution norm is evident, students will try to make sense of difficult problems and display a degree of persistence. Students may look to the teacher to clarify a task or point them in the right direction, but would not routinely ask the teacher how to solve a problem.

As Mr. Carl transitioned from student work to whole-class discussion, he managed to regain the attention of the class. Greg was the first student who spoke regarding part (a) of the task.

Greg: I think it's about one thousand and ten dollars.
Mr. C.: Why do you think that?
Greg: Because ten percent is like ten more dollars.
Mr. C.: Say more, why is that?
Greg: Because ten percent of something is about ten or eleven more dollars.

Mr. Carl asked the class what they thought of Greg's response. The students at first were largely unresponsive. Mr. Carl again urged students to comment on Greg's work. Erica spoke out.

Erica: Is he [Greg] right or wrong?
Mr. C.: What do you think?
Erica: I don't know.
Mr C. [To the class]: Does anybody have a different answer?
Erica: I think it is just one hundred.
Mr. C.: Why?
Erica: I remembered doing problems like this last year. Ms. Cooper taught us to write *n* over one thousand equals ten over one hundred. Then you criss-cross and get one hundred times *n* and one thousand times one hundred which is ten thousand, and then divide by one hundred and you get one hundred.
Mr. C.: Why do you write it out like that?
Erica: That's just how Ms. Cooper taught us to do them.

Although Mr. Carl twice attempted to get Greg to explain his answer, Greg did not offer a mathematical argument for his solution. Similarly, Erica's explanation involved a summary of the steps taken to solve the problem, but she was unable to provide a mathematical rationale for why those steps led to a correct solution. A challenge for a teacher when pressing for mathematical explanations is that students may not understand what is being asked, may not know what type of rationale is acceptable, or lack the language needed to explain.

A class predicated on student ideas requires a degree of participation on the students. A challenge for a teacher attempting to implement a multiple solution norm is to get students to respond to their classmates' solution strategies. Unresponsive students may lack sufficient knowledge of content or reasoning strategies to evaluate explanations; they may lack interest in the problem; and/or they may be unwilling to critically evaluate their classmates.

Erica's comment further suggests that she believes Mr. Carl is responsible for judging the correctness of student solutions. Erica may not be comfortable sharing her solution if she thinks she has the wrong answer. These are both challenges Mr. Carl needs to address to create a multiple solution norm.

Vignette #2:
In Ms. Joyner's class, students enthusiastically began working on the problem. At first, most of the students worked individually; then, as progress was made on the task, the students communicated their ideas with their group members. In the different groups, the distribution of labor was fairly equal and all students were engaged in solving the problem. Students shared their thoughts, asked questions of one another, and compared their solution strategies.

As groups calculated solutions, they were anxious to show Ms. Joyner what they had accomplished. Students waved their hands to get Ms. Joyner's attention and wanted Ms. Joyner to check their work. Some groups saw the task as a competition and were pleased when they solved the problem before other groups. Many of the groups suspended their work on parts (b) and (c) until they asked Ms. Joyner for confirmation for their answers on part (a).

A challenge for Ms. Joyner, in her attempts to constitute a multiple solution norm, is to create an environment in which her students regularly rely on mathematical logic and evidence to evaluate the validity of an answer. Students in a classroom in which a multiple solution norm is present would rarely seek the teacher's confirmation regarding the correctness of solutions. Additionally, the criteria for doing well would rest in making sense of mathematical ideas, not in the speed or pace of student work.

In the whole-class discussion for part (a), students expressed their ideas by freely speaking out. Ms. Joyner recorded the students' ideas on the board. Rob was the first to begin the conversation.

Rob:	I divided one thousand by ten and got one hundred, then I added that to one thousand and got one thousand one hundred dollars.
Megan:	I got the same answer, but I did it a different way. I multiplied one thousand by point ten because ten percent is point ten, and added what I got to one thousand.
Rob:	That's really the same thing.
Megan:	[to Rob] I don't understand why you divided by ten.
Kristen:	Megan, multiplying by point ten is the same as dividing by ten because it's a tenth
Ms. J.:	Any other questions or comments or different solutions or solution strategies?

John: I solved it using a proportion – is over of equals percent over one hundred; so I did *n* over one thousand equals ten over one hundred. I cross multiplied and divided to find *n* is one thousand one hundred.
Dylan: We just multiplied one thousand by one point one.
Students: Where is the one point one coming from?
Dylan: I'm not sure. Mac had explained it to me but I forgot why he did that. Mac, why did we use one point one?
Mac: Multiplying by one point one is the same as multiplying by point one and adding back what you started with because one point one is point one plus the one whole.

This episode demonstrates that Ms. Joyner's class has constituted norms in which students freely offer their solutions, and students listen and respond to one another.

In a class where a multiple solution norm exists, not all student solutions would be accepted as reasonable and valid. John's solution, which consisted of a procedural explanation, would be probed by students as they evaluate his method and compare and contrast in to others.

In monitoring whole-class student discourse, Ms. Joyner is faced with the challenge of making decisions that could potentially affect classroom norms. As students take charge in sharing solutions, Ms. Joyner needs to decide how to monitor participation so everyone has an equal chance to share their solutions or express their concerns. Ms. Joyner must decide how to attach notation and language to student ideas as she records them on the board. Ms. Joyner must use her knowledge of students, mathematics, and the curriculum to determine what ideas to pursue in depth among a potential wide-array of student strategies.

A bit later in the period, the class discussed part (b) of the task, finding how much money there would be after 6 years. The following conversation took place.

Olivia: After one year there was one hundred more dollars, so after six years there will be six hundred more dollars for a total of one thousand six hundred.
John: I multiplied one thousand by point ten and got one hundred, then multiplied one hundred six times to get one thousand six hundred.
Tressa: I agree with John and Olivia, but I did it more like Mac's way. Ten percent each year for six years is sixty percent, so one point six times one thousand is one thousand six hundred.
Ms. J.: Does anybody have a question? Or does anybody have a different solution or solution strategy?

After Tressa's comments, no other questions or solution strategies were put forth. Several students expressed they understood. Ms. Joyner had anticipated students

would not compound the interest each year and arrive at the solution obtained above. While students worked on the task earlier in the class, Ms. Joyner observed that Alison's group had compounded the interest and recursively found an answer for the amount of money at the end of six years. Knowing that a different solution was found, the following exchange took place.

Ms. J: Alison, I saw your group had found a different solution.
Alison: We had something different, but I see what we did wrong now.
Ms. J: You are satisfied that there is one thousand six hundred dollars after six years?
Alison: Yes.

A challenge for Ms. Joyner in creating a classroom environment predicated on student ideas is to have students explicitly appreciate similar and different solution strategies. Solution methods equivalent to each other are either not put forth by students, or identified by the class as being mathematically equivalent. Here, John offered a solution method isomorphic to Olivia's explanation. Whether or not John attempted to process Olivia's response is unknown; however the class raised no objection.

Ms. Joyner is faced with the dilemma of having the entire class reach a consensus on a mathematically objectionable solution. Although Alison and her group had calculated the desired answer, she seemed unwilling to share her perceived mistake with the class. In a class with a multiple solution norm, students would routinely share their (perceived) mistakes as an important component of learning.

Vignette #3:
The social and sociomathematical norms evident in Ms. Joyner's classroom also appeared in Ms. Robinson's class. Students collaboratively persevered in attempting to solve the task, students enthusiastically shared their different solution strategies, and there was evidence suggesting that students appropriated the responses of their classmates.

In the whole-class discussion for part (b), the following discussion occurred.

Kalie: In one year ten percent of one thousand is one hundred, so if you wait six years, you will get six hundred more dollars.
Alex: I agree with Kalie. If you do ten percent a year for six years you have sixty percent. Sixty percent of one thousand dollars is six hundred. So you will have one thousand six hundred dollars.
Cristal: I might be wrong, but I did something else and got a different answer. I started out like Kalie and found ten percent of one thousand is one hundred, so after one year there was one thousand one hundred dollars. Then I took ten percent of one thousand one

hundred which was one hundred ten and added it to get one thousand two hundred ten dollars. That is how much there was after two years. I kept going until I got to six.

Ms. R: You got a different answer than Kalie?

Cristal: I ended up getting one thousand seven hundred seventy one dollars.

Ms. R: Can anyone make an argument either for or against Cristal or Kalie?

At this request, a number of students expressed their allegiance to both sides. Students argued for their choice by essentially revoicing what Kalie and Cristal offered. One student suggested the class vote on which answer was right. Another student offered a compromise and suggested that "maybe they are both right."

Ms. Robinson, attempting to get her students to compare the discrepancies in the two solutions, asked: "Under what conditions would Kalie be correct, and what conditions or assumptions would Cristal's argument make more sense?" Again several students responded to Ms. Robinson's question by re-summarizing the procedures Kalie and Cristal used to get their respective answers. No new idea was put forth. At this point, students seemed frustrated regarding the stalemate and pressed Ms. Robinson to tell them which answer is correct. One student exclaimed: "I understand both ways, so if I know which one is correct, I can explain why it works."

Here, Ms. Robinson faces a challenge different than Ms. Joyner's. Whereas Ms. Joyner was challenged when the entire class arrived at an undesirable mathematical consensus, Ms. Robinson's dilemma centers on a class divided over the legitimacy of an answer. In a classroom with a multiple solution norm, students will routinely use mathematical arguments to support or refute a given solution strategy. The bases for student actions would be mathematical, not status-based. Students would unlikely suggest voting as a means for determining a correct answer. Students would not rely on the teacher's authority for adjudicating the correctness of a solution.

In the face of student frustration, Ms. Robinson needs to make difficult decisions regarding her next move. She must decide if and what information to give to her students versus letting her students struggle.

Vignette #4:

By all accounts, Mr. Lyttle and his students have jointly negotiated a multiple solution norm. During small group discussions, students described and defended their mathematical interpretations and solutions for the problem. When Mr. Lyttle approached a group, their mathematical work did not alter. Group discussions led to a consensus in which each member was accountable for understanding the accepted solution strategy.

During whole-class discussion, students offered detailed analysis of their solution methods. Students accepted explanations only if the explaining students included a mathematical justification for their answers. Students questioned and compared various solution methods. The basis on which each solution strategy was evaluated rested on the strength and logic of its mathematical argument.

Although multiple methods for part (a) were put forth, the class agreed that multiplying the original investment by one and one tenth was the most sophisticated and efficient way to arrive at solution. For part (b), the discussion, at first, resembled that of Ms. Robinson's class. However, instead of reaching a gridlock, the students' turned to the different ways of interpreting the problem. The students reached a consensus that the most logical interpretation was to compound the original investment each year. With that concord, the students agreed on the strategy of recursively calculating a solution by multiplying the previous year's amount by one and one tenth six times.

During the discussion to part (c), the students realized they could continue to recursively solve the problem to determine the amount of money after sixty years. This was not the preferred strategy of the class. Ruthie exclaimed: "there has to be a formula we can use." Jonah, referring to the table on the board from part (b), conjectured that the amount each year corresponded to a row of numbers in Pascal's triangle.

This episode illustrates how a teacher will inevitably be asked to deal with important challenges as a consequence of negotiating a multiple solution with his/her students. In this case, Mr. Lyttle is confronted with an unanticipated student response. Mr. Lyttle needs to be able to assess the mathematics in Jonah's idea, its level of sophistication, and student interest for it in order to decide if it is worthwhile to invest a substantial amount of class time discussing it.

Additionally, Mr. Lyttle must decide what type of questions to ask and what information to give to his class in facilitating a productive classroom discussion.

These decisions and the subsequent interaction with his students could positively or negatively impact the constitution of a multiple solution norm.

Having defined my vision for classroom activity, I next introduce a framework used to operationalize my overarching research goal of studying the challenges of negotiating a multiple solution norm. The framework, influenced by Simon's (1995)

Mathematical Teaching Cycle, views challenges in terms of confronting norms that are inconsistent with a teacher's goal for classroom activity.

Conceptual Framework

Concerned that a social constructivist view of knowledge development does not define a particular way of teaching, Simon (1995) utilized a constructivist perspective to develop a theoretical model of teacher decision-making called the Mathematics Teaching Cycle. The Mathematics Teaching Cycle models the "cyclical interrelationship of aspects of teacher knowledge, thinking, decision making, and activity" (Simon, 1995, p. 135). Central to the Mathematics Teaching Cycle is a hypothetical learning trajectory that the teacher constructs while planning a lesson. The hypothetical learning trajectory consists of a learning goal, activities that are to be used to achieve the goal, and a hypothesis of the learning process. The Mathematics Teaching Cycle highlights a process of ongoing modification in classroom activity by illustrating how a teacher's "assessment of student thinking . . . can bring about adaptations in the teacher's knowledge that, in turn, lead to a new or modified hypothetical learning trajectory" (Simon, 1995, p. 137).

Primarily, Simon (1995) portrays an image of a teacher whose decisions and actions are made with respect to the mathematical content. In Simon's (1995) Mathematics Teaching Cycle, the teacher's assessment of students' knowledge is the feedback mechanism that informs a teacher's decision making. From a social perspective, the "learning environment evolves as a result of interaction among the teacher and students as they engage in the mathematical content" (Simon, 1995, p. 133). For Simon (1995), norms for classroom mathematics activity, such as

determining what counts for mathematical justification, result from the purposeful selection of activities and discussion regarding those activities.

Based on my experience in attempting to initiate and guide the development of specific social and sociomathematical norms with middle grade students, I believe a model of mathematics teaching must make the development of norms more problematic. In describing work on how it might be possible to bring the practice of knowing mathematics in school closer too what it means to know mathematics within the discipline, Lampert (1990) writes,

> I needed to work on two teaching agendas simultaneously. One agenda was related to the goal of students acquiring technical skills and knowledge in the discipline, which could be called knowledge *of* mathematics or mathematical content. The other agenda was working toward the goal of students acquiring the skills and dispositions necessary to participate in disciplinary discourse, which could be knowledge *about* mathematics, or mathematical practice (p. 44 [Italics in the original]).

Consistent with Lampert's (1990) perspective, I believe that a practitioner will develop, in parallel, a hypothetical learning trajectory for how mathematical content might be learned, and a strategy for initiating and sustaining a learning environment that can characterized by specific social and sociomathematical norms. Building off the work of Simon (1995), I have designed a corollary to the Mathematics Teaching Cycle that highlights the problematic nature of developing these norms.

A Teaching Cycle for Constituting Classroom Norms

Secondary mathematics students often enter the classroom with a set of expectations and obligations regarding how the teaching and learning of mathematics should take place. The framework shown in Figure 3, presupposes that student expectations regarding the roles of the teacher and students are aligned with a

traditional model of teaching, and that the social and sociomathematical norms comprising a multiple solution norm would not occur spontaneously in a middle grades mathematics classroom with low-attaining students. In fact, a larger supposition is being made -- students pre-existing expectations and beliefs regarding the nature of mathematical activity in the classroom actually work against a practitioner's goal of constituting a multiple solution norm. Changing the nature of the classroom environment so that students and teacher share the responsibility for making sense of mathematic requires confronting existing norms for doing mathematics.

As a schematic model, Figure 3 represents the framework I used to study the problematic nature of initiating and guiding the constitution of classroom norms. A central feature of this framework is that a teacher's knowledge is continually being modified, and the development of classroom norms is dependent upon how a teacher draws from his/her evolving knowledge of students, pedagogy, mathematics, and curriculum. The framework begins by recognizing the significance of belonging to a school community. Through participating with other members in the community (administrators, other teachers, parents, etc . . .) a teacher gains knowledge of individual and collective students. This information contributes to a teacher's decision-making process regarding how to identify goals, choose activities, and manage classroom interactions affecting the development of classroom norms.

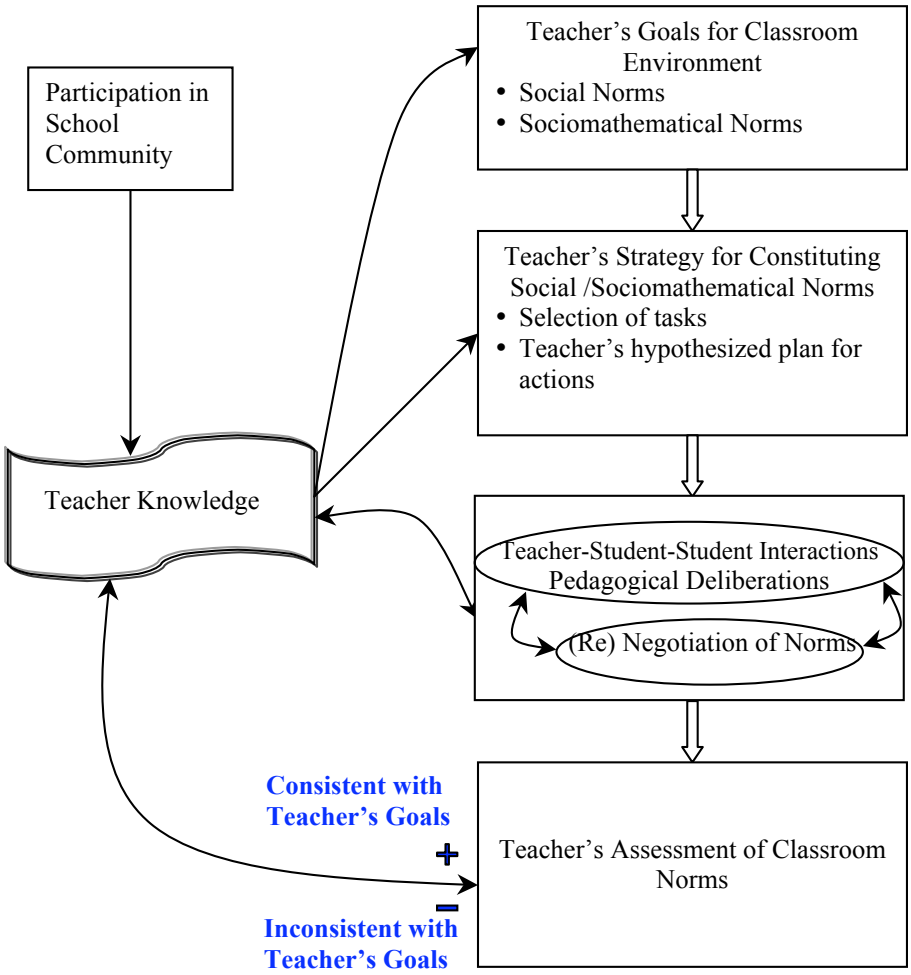


Figure 3: Conceptual framework for analyzing the development of classroom norms

A key component to changing the nature of mathematical activity requires that a teacher explicitly identify goals for the classroom environment. These include goals regarding the respective roles for the teacher and students, goals for the relationships between and among classroom participants, and goals for student behavior and mathematical activity. Similar to Simon's (1995) Mathematical Teaching Cycle, these goals and the teacher's knowledge contributes to the development of a strategy that a teacher intends to enact to facilitate the constitution of the desired norms. The strategy includes selection of tasks, purposeful actions, and guidelines for classroom activity that the teacher will communicate to his/her students. The teacher's plan for

action can be both thought out in advance during times of reflective planning or can occur in the moment of teaching a class.

Next, as the teacher and students interact, they are continually interpreting the actions of others. A teacher draws upon knowledge to interpret the actions of his/her students, and the interactions with students generate knowledge for the teacher. From the emergent perspective, joint activity between teacher and students and students and students simultaneously impact both the nature of these interactions and the (re)constitution of norms. In classroom interactions, the teacher is confronted with pedagogical deliberations such as how to facilitate mathematical discussions, how to record student ideas, and how to manage time. The teacher's response to these issues simultaneously impact classroom norms and will likely highlight or result in other pedagogical considerations.

The next stage in the framework is the teacher's identification of classroom norms. Social norms and sociomathematical norms are inferred by identifying regularities in patterns of social interaction or by identifying breaches that occur (Cobb, Yackel, & Wood, 1993). The norms a teacher perceives are either consistent or inconsistent with the teacher's goals for establishing a classroom environment and for the teacher's learning goals for his/her students. The teacher's assessment of classroom norms can bring about adaptation in the teacher's knowledge that can lead to new goals, strategies and (re)negotiation of norms.

The problematic nature in instantiating norms for social and mathematical activity is that norms are continually being (re)constituted in and through the actions of the participants as they interact (Yackel, 2000). Every teaching action can potentially

shape the complex set of classroom norms and expectations, and these evolving norms conversely influence how a teacher decides to act. One challenge in teaching is identifying and confronting norms that are inconsistent with the teacher's goals for classroom activity.

With regards to this framework, my overarching research problem of understanding the challenges of negotiating a multiple solution norm can be thought of as understanding how norms, inconsistent with a teacher's goals, are constituted. Pedagogical deliberations such as assigning seats, recording student work, managing time, and giving grades are of interest if they impact the constitution of norms that are inconsistent with the goals of a teacher. For example, consider the following scenario: When facilitating discussion of student work, a teacher is confronted with the pedagogical challenge of documenting student ideas on the board. Suppose, in the course of recording student work, a teacher routinely gives less space to students who solved a problem by using a guess and check strategy. Consequently, students come to expect that guessing and checking is not a favored way to solve a problem. Students who regularly rely on guessing and checking may then hesitate in the face of problem solving. This social norm is inconsistent with a teacher's goal of developing a productive disposition in his/her students. Thus, the pedagogical decision of how to record ideas is a significant factor to consider.

Research Questions

Emerging from the conceptual framework, the following research questions helped guide the study and address my research goal of understanding the challenges to implementing a multiple solution norm.

1) What norms are evident over time in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?
2) a) How are norms that are consistent with a practitioner's goals for classroom activity constituted in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?
   b) How are norms that are inconsistent with a practitioner's goals for classroom activity constituted in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?

Summary

As discussed in Chapter 1, my study evolved from conversations with a group of mathematics teacher interns who expressed a great deal of hesitation regarding the value of exploring multiple solution ideas in the classroom. In particular, the teacher interns expressed a view that many lower attaining students would be confused if they saw more than one way to solve a problem. To better understand their argument, I wanted to examine the challenges of using students' multiple solutions as a central feature of instruction to a group of low-attaining eighth grades students. My vision

for the use of multiple solutions extended well beyond the notion that teachers would simply ask their students, "Did anyone solve this problem a different way?"

In an attempt to articulate my vision for classroom activity, I used the notion that mathematics classrooms can be characterized by a set of shared expectations for social and mathematical behavior. The theoretical foundation of this study was built upon the work of Cobb and Yackel (1996) and Yackel and Cobb (1996) who used the language of social and sociomathematical norms to describe the social processes of a classroom. Employing the theory that individual constructive activities and classroom social processes are reflexive and mutually constraining, Cobb and Yackel's (1996) developed an interpretive framework (Figure 1) for analyzing students' mathematical development that reflexively linked the construct of social and sociomathematical norms to students' beliefs and values. Although my study does not consider students' mathematical development directly, I used this framework to analyze a teacher's efforts to engage students in the discovery and discussion of their own inventive problem solving strategies. As discussed in this chapter, my definition of a multiple solution norm (Figure 2) is a collection of specific social and sociomathematical norms.

The review of literature (Chapter 2) makes clear that instantiating a multiple solution norm is challenging and demanding work. A practitioner is faced with a range of challenges from managing potentially resistant student behavior to planning rich problem solving experiences to orchestrating discussion of student ideas to addressing curricular and time constraints to dealing with different levels of discomfort and potential loss of self-efficacy. Having defined a multiple solution

norm, a construct central to my dissertation, I used a conceptual framework (Figure 3) derived from Simon's (1995) Mathematical Teaching Cycle to operationalize my overarching research goal of studying the challenges of instantiating a multiple solution norm. Within this framework an object or action would be considered challenging if it could be inferred that it led to the constitution of a classroom norm inconsistent with a multiple solution norm. The conceptual framework helped to define the research question that guided the data collection and analysis methods for this study. These methods will be introduced in the next chapter. After briefly defining a first-person research methodology, the next chapter begins by providing a rationale for using this particular methodology to answer the above research questions. In addition, the next chapter describes the context of working with a group of students with a history of poor academic achievement and past behavior concerns.

Chapter 4: Research Design

The purpose of this chapter is to explain the methodology I employed and the data collection and analysis methods used to examine the challenges of instantiating a multiple solution norm. This chapter is divided into four sections. In the first section, I describe my rationale for utilizing a first-person researcher methodology. In the second and third sections, I describe my methods for data collection and data analysis, respectively. In the final section, I describe the context in which the study took place, including a description of the school and each of the participants.

Methodology

To examine the problem of initiating and maintaining a multiple solution norm in a class of low attaining students, I utilized a first-person research methodology (Ball, 2000) and assumed the role of teacher-researcher. Hubbard and Power (1999) define teacher-research as systemtatic and thoughtful analyses of teaching by teachers. Lyttle and Cochran-Smith (1990) define teacher-research as "systematic, intentional inquiry by teachers about their own school and classroom work" (p. 84). According to Lyttle and Cochran-Smith, teacher-research involves documenting information and experiences inside and outside of the classroom in ordered ways (systematic); teacher-research is a planned, rather than spontaneous, activity in which every lesson is purposefully designed (intentional); and teacher-research stems from questions and reflects teacher's desires to make sense of their experiences (inquiry). According to Ball (2000), two features differentiate first-person research from third-person case studies: (1) instead of merely studying what they find, researchers begin with an

issue and design a context in which to pursue it; and (2) the issue is at once theoretical and practical; rooted in everyday challenges of practice but also situated in a larger scholarly discourse.

Over the past 15 years, there exist a number of first-person research efforts examining pedagogical issues in mathematics education (i.e., Ball, 1993a, 1993b; Chazan, 2000; Heaton, 2000; Lampert, 1990, 2001; Lubineski, 2000a, 2000b; Shoenfeld, 1994; Simon, 1995). For example, Chazan (2000) explored the challenges and predicaments involved in teaching diverse learners in a high school Algebra class. Chazan (2000) used a function-based approach in teaching Algebra One, an identified problem of practice, to detail his experiences in motivating students and orchestrating whole-class discussions. Ultimately, one effect of Chazan's work is not to promote a function-based approach to Algebra, but to use the press of practice to portray teaching as a complicated and uncertain craft. Heaton (2000) attempted to understand the struggles that inhere while trying to change her teaching from a traditional approach to one grounded in reform ideas about good teaching. Heaton examined practical challenges such as the place of telling and responding to student ideas. More broadly, Heaton gets readers to understand that teaching entails a continuous negotiation of moves determined by the situation rather than defined and prescribed in advance. Lampert (1990, 2001) examined the problems of practice in creating a classroom culture designed to teach students what it meant to know and do mathematics. One consequence of Lampert's teaching is that it challenges traditional teachers' and students' conceptions of mathematics as a discipline and provides a model for what it might be possible to aim for pedagogically.

My study contributes to this genre of research. My goal of understanding how a multiple solution norm can be instantiated in a mathematics classroom is a problem embedded in practice. My findings as a teacher-researcher can provide thoughtful, vicarious accounts of practice that may potentially help practitioners reflect on and improve their craft (Anderson, 2002). At the same time, work centered on this problem can contribute to developing a professional language of practice by using the notion of norms as a descriptive framework to potentially capture the complexities of creating a classroom climate consistent with mathematics educators calls for reform.

There exist multiple reasons why I utilized a first-person research methodology to study challenges of instantiating a multiple solution norm. Foremost, I needed to look at a very specific kind of teaching. I wanted to examine a classroom in which a teacher purposefully attempted to initiate and guide the constitution of specific social and sociomathematical norms. A first-person research design permitted me the opportunity to create a context and a space in which to work. Simon (1995) notes that "researchers studying teachers' thinking, beliefs, and decision making have had little access to teachers who had well-developed constructivist perspectives and who understood and were implementing current reform ideas" (p. 118). Most of the research on teaching has focused, for the most part, on traditional instruction (Simon, 1995). Finding a non-inquiry based practice in which a multiple solution norm exists was not likely. Rather than having to find an instance of the type of teaching I was interested in, I was able to create it and study it myself.

Second, my analysis involved studying the fine-grain actions and decisions that are made in efforts to establish a multiple solution norm. Stigler and Hiebert (1999)

point out that researchers do not have access to the same information that teachers have as they confront real students in the context of real lessons with real learning goals. The knowledge a teacher-researcher possesses is deeper, more nuanced, and more visceral than an observer's knowledge (Anderson, 2002). Decisions and actions taken by a teacher are continuous; actions are not only based on what has happened immediately prior, but build off the entire history of the curriculum and from the relationships with all students in a class (Lampert, 2001). This is the quality of teachers' work that makes it difficult for outsiders to assess. Ball (2000) points out the participant-observers often miss nuances, make faulty connections, and inappropriately infer motives.

On a personal note, I was intently interested in studying my own practice. I had been pleased with my efforts to successfully guide the constitution of a multiple solution norm to classes of above grade level students in Algebra and Geometry. However, skeptical colleagues continually insisted that the same type of instruction would not *work* with below grade level students. As a teacher educator at the university level and as department chair at a public middle school, I stressed the importance of creating a multiple solution norm. It was important that I experience, first-hand, the challenges to the type of teaching I had espoused. Although my understanding of research set me uniquely apart from many teachers, I encountered the same struggles and obstacles that any teacher would encounter in teaching real students in a real classroom. My desire to be both a teacher and researcher is summed nicely by Cuban (1990):

> I wanted to maintain my credibility both as a teacher and as an academic who writes about teaching and public schools. I believe deeply in the idea of a scholar-

> practitioner – that is, someone who can bridge the two very different worlds of the university and the public school. Such switch hitters are uncommon, and I wanted to be one of that breed. (p. 480)

As a teacher who researched his own practice, I was aware that practitioner-research has a unique set of methodological and ethical dilemmas. For example, Wong (1995) pitted the concerns of the teacher in opposition to the researcher's agenda. For Wong, the goal of research is to learn through investigation and the goal for teaching is to bring others to understand. Wong expressed concerns that his actions as a teacher to be responsive to the needs of his students might impinge on goals of his research and its focus on understanding student thinking, and that his aims as a researcher might influence how he teaches. However, I do not believe this was the case for my study. My research did not require that I act any differently in the classroom than I always do. No split in attention was necessary for me to carry out my research. In fact, I could argue that the constant attention to detail and reflection I used as a researcher helped me teach in a more effective manner. Wilson (1995), in response to Wong (1995), offers a more appealing interpretation of the teacher-researcher role that I agree with. According to Wilson (1995):

> [I[t was in learning to be a researcher – learning to look, listen, respond, not assume, watch, entertain differences, and suspend belief (or disbelief) – that I developed greater capacity to act on my teacherly commitments to be moral, to hear and respect my students, to understand my own limitations. (p. 21)

An additional methodological concern for teacher-researchers is to treat their study as a matter of scrutiny and overcome a natural urge to defend against questions others raise (Ball, 2000). Anderson and Herr (1999) argue that a unique set of validity criteria are needed to evaluate the quality of practitioner research. In the next two sections, I will explain how data collection and analysis methods were purposefully

designed to address three of these criteria: process validity, democratic validity, and dialogic validity. According to Anderson and Herr, process validity deals with the problem of what counts as evidence to sustain assertions; process validity depends on the inclusion of multiple voices for triangulation. Democratic validity refers to the extent to which research is done in collaboration with all parties who have a stake in the research; democratic validity deals with ethical issues that guard against teacher-researchers finding solutions to problems at the expense of stakeholders. And dialogic validity deals the collaborative aspect of research; dialogic validity is a process of working with others who are familiar with the setting and can serve as devil's advocate for alternative explanations of research data.

Data Collection Methods

In conducting a first-person research study, a critical design issue is to collect data that allows the researcher to gain alternative perspectives and interpretations of his/her actions (Ball, 2000). To address the process validity criterion for practitioner-research, data from multiple perspectives were collected that guarded against viewing events in a simplistic or self-serving way (Anderson & Herr, 1999). As detailed in this section, data used for the analysis of this study came from three primary sources:

- Daily recordings from a reflective teaching journal
- Video taped class sessions
- Observation notes and conversations from observers

In addition, I collected classroom artifacts including classroom lesson plans and student notebooks, saved minutes from team meetings, and participated in informal conversations with faculty and staff. I also audiotaped two conversations (December,

2006 and June, 2007) with my advisor, Dr. Dan Chazan, where I offered detailed accounts and reflective thoughts from the class.

Reflective Teacher Journal

Following each class session, I documented my personal thoughts in a reflective journal maintained throughout the year. In daily journal entries, I addressed my research questions by making note of the overall classroom culture, summarizing events of the class, and reflecting on the perceived challenges to instantiating classroom norms. Data from a personal teaching journal captures the special kind of insider knowledge available to the first-person researcher that is difficult to gain from an outsider's perspective (Ball, 2000). During a teacher workday in January 2007, a more formal journal entry was recorded in the form on an essay where I purposefully noted my developing assertions and conjectures. Lyttle and Cochran-Smith (1990) note that, in the course of writing essays, teachers are able to connect practice to overarching concepts and show how broad theoretical frameworks apply to particular contexts. Maintaining a journal is an essential component for teacher-researchers as they provide a way to revisit, analyze, and evaluate experiences over time and in relation to broader frames of reference (Lyttle & Cochran-Smith, 1990).

Video and Audio Data

Video recordings of classes offer a promising method to study teaching as it preserves specific classroom interactions and discourse patterns (Stigler, Gallimore, & Hiebert, 2000). The video recordings allowed me to view my teaching from a different temporal perspective. Thus, in addition to recording thoughts and

reflections in a daily teacher journal throughout the 2006-2007 school year, I recorded my interpretations and assertions by adding marginal notes as each video was transcribed during the summer and fall of 2007. In this way, I was able to collect data on the same teaching episodes from different points of view. In a similar way, Heaton (1994) gained an alternate point of view and interpretation on her own teaching actions by separating her perspectives over time. Ball (2000) explains how Heaton (1994) invented a methodological device of distancing herself by using Ruth 1, Ruth 2, and Ruth 3 to separate her data and vantage point across her work. Ball (2000) explains:

> Ruth 1 is what she named the teacher teaching and struggling online to reconstruct her teaching. Ruth 2 is the teacher who was making sense of that teaching and learning and who offers perspectives gained through the reflective writing, recorded conversations and journal exchanges of the teacher during that same school year, but with a temporal distance from particular events. These entries capture the interpretive work of the teacher involved in the challenges of inventing and relearning how she teaches mathematics. Ruth 3 is the perspective of the teacher, 3 years later, who knows the experiences intimately but who has increased conceptual distance on them (p. 393).

To document the daily happenings within the classroom, I recorded 51 class sessions out of a total of 183 in the whole year. I audiotaped eight consecutive classes from August 28 to September 8, and video recorded 37 consecutive class sessions from September 8 to November 6 and six sessions in May (May 17, 21, 22, 23, 24, 25). Each class session lasted 45 minutes. In the original proposal for the study, I planned to limit my analysis to data collected during the first quarter of the year. Previous studies examining the emergence of social and sociomathematical norms found that norms appeared to be relatively stable within the first few weeks of the school year (Cobb et al., 2001; McClain & Cobb, 2001). However, in my

classroom, it was clear that the regular routines and expectations for academic behavior were qualitatively different in May than they were in the tenth week of the year. To capture this difference, the decision was made to collect additional video data towards the latter part of May.

An issue in video recording is deciding on the placement of the video camera. To protect student interests, the video camera was positioned in the back of the room and aimed to capture work recorded on the front board or overhead. McClain and Cobb (2001) found such a location valuable in serving to capture the teacher's written notation.

Outside Observations

I enlisted the participation from two individuals, Sandy Spitzer and Sue Pope, to assist in my study. According to Ball (2000), the quality of first-person research is dependent on how well the researcher can notice strange or discontinuous events or phenomena. Incorporating the unique perspectives of Sandy, a researcher from outside the school, and Sue, a teacher from inside the school, guarded against a naïve or biased interpretation of classroom events.

Sandy, from a nearby university, observed my teaching on four occasions (August 31, September 25, November 3, and May 17). Sandy provided copious field notes from each session. In addition to our informal conversations, I taped one 45-minute interview with Sandy.

Sue was the school's mathematical instructional support teacher. She regularly observed teachers and helped out in their classrooms. She often contributed in my classroom by working with students on an individual basis. On more than forty

occasions throughout the year, Sue was a participant-observer in either all or significant parts of class. My data includes eighteen (August 29, August 30, September 5, September 7, September 11, September 14, September 18, September 26, September 28, October 3, October 5, October 10, October 12, October 23, October 26, October 31, May 21, and May 22) recorded conversations (10 – 15 minutes in length) that took place immediately following a class. In addition, I taped two 30 minute interviews with Sue designed to elicit specific feedback regarding the constitution of the desired classroom norms. Incorporating the help of colleagues to view classroom events and offer assertions that challenge the teacher-researcher's ideas is vital to promoting democratic and dialogic validity in a first-person research study (Anderson & Herr, 1999).

Data Analysis Methods

Hubbard and Power (1999) suggest that teacher-researchers connect data collection and analysis throughout their study. For Ball (2000), an important component of first-person research is to take advantage of the insider position. According to Ball, it is important that teacher-researchers view their teaching apart from their efforts and desires, yet "to deny the personal is to undo the very project of first-person research, shutting out part of what is experienced on the inside" (p. 392). Throughout the data collection process, I regularly attempted to make sense of my data and interpret what was occurring in the classroom. Through my own reflections and conversations with Sandy and Sue, I used my journaling to record assertions regarding the normative behaviors in my classroom and what factors affected these.

Hubbard and Power (1999) strongly encourage teacher-researchers to draw on intuitive and past knowledge, and trust hunches.

During the school year, I attempted to transcribe as many class sessions as possible. The lengthy and time-consuming transcription process limited my efforts to only transcribing the first six class sessions. My original intention was to transcribe a subset of the classes; however, I could not come up with a suitable selection strategy for determining which classes would be transcribed and which would not. I decided it was important to include each recorded class into the data corpus.

During the summer of 2007, I resumed the extensive transcribing process. Working in chronological order, I reviewed each audio and video recording. For each recording, I chronicled the events of the class and kept notes of the inferences I made (Goetz & LeCompte, 1984). I recorded verbatim conversations of critical classroom episodes that occurred within and across class sessions. My research questions and framework for a multiple solution norm guided my determination of critical episodes. The unintentional postponement of the transcribing process during the school year resulted in a valuable methodological consequence. The viewing of successive class recordings helped to highlight critical classroom episodes that occurred across class sessions. Generally, an episode was deemed critical in two ways. One, if the activity in the episode appeared to provide clear evidence of normative behavior either consistent or inconsistent with my vision for the social and mathematical activity in the class; or, two, if the episode was unique from previously observed classes.

This methodological approach relates to Cobb and Whitenack's (1996) process for analyzing large sets of qualitative data generated from classroom video recordings.

Cobb and Whtenack's analysis involves a continual movement within and across episodes to find theoretically significant patterns and regularities in an empirically grounded way. Data are dealt with on an episode-by-episode basis in chronological order. Inferences made while analyzing one episode are viewed as initial conjectures that can be revised when analyzing subsequent episodes. These conjectures become data that are analyzed to create chronologies that are structured by general assertions and grounded in the data. Critical episodes are those that either refute a conjecture or substantiate an assertion. When viewed in isolation, these episodes may not appear important. Their significance only becomes "apparent when they are located within the chain of conjectures, refutations, and revisions that result from the first phase of analysis" (Cobb et al., 2001, p. 147).

The process of summarizing and transcribing portions of 51 class sessions provided an overwhelmingly large set of data. My next crucial methodological issue was to find a way to systematically organize and structure the analysis of the data. Coding is a key operation used in qualitative analysis to meaningfully reorganize data. Miles and Huberman (1994) define codes as "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (p. 56). Unfortunately, finding appropriate categories to place my data proved to be a challenging task. Initially, I used the social and sociomathematical norms from my definition of a multiple solution norm to create a provisional start list of codes (Miles & Huberman, 1994). Using a free public domain software tool (Weft QDA) for the analysis of textual data, I began to attach these codes to segments of my transcript data. While coding, I continually reflected on the purpose of my study and the degree

to which I was addressing my research questions. For each segment of data, I asked myself, "What does this tell us about the normative expectations in the classroom?" I became concerned that there were significant portions of my data that could not fit very well into my existing coding scheme. For example, the competitive nature in which my students approached their work was not adequately represented with the initial set of codes I was using. To address data like these, I created new categories. Ultimately, I was not satisfied with this structure. I felt I was creating too many categories, and I did not believe I was adequately capturing my experiences in the classroom.

Next, similar to the "grounded" approach originally advocated by Glaser and Strauss (1967), I focused on my specific research questions, and turned to an inductive system to build codes.

Looking at Question #1

> Question #1: *What norms are evident over time in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?*

Starting with the data from May, I read each of the transcripts from the six recorded class sessions line-by-line. Then, guided by my framework of a multiple solution norm, I brainstormed the following list of normative actions and behaviors that I believed were evident in the transcripts across each of these classes:

Students and teacher sat at one table
Students retrieved binders
I would announce it's time to begin work
Students began to work on problem
I would offer scaffolding help to individual students
Students used calculator
Students would blurt out ideas/observations

I would ask students to explain themselves
Students would get out of seat and do work at the board
Students would argue with each other
I would permit off-task chatter if they were making progress with problems
Students would re-focus when directed
Students would work several minutes on a problem
Students would have different ways to solve problems
Students would race against each other
Student were willing to offer explanations to other students
Students would ask me if they have the right answer
I would ask students to use the calculator to check work
I would ask students to check work with classmate
I would validate final answers
Students would celebrate being first to solve a problem

With the process validity criterion in mind, I then read my journal entries, and examined the observation feedback from Sue and Sandy over the same time frame to determine which (if any) of these assertions could be sustained across each of the multiple perspectives contained in my data. The following set of normative behaviors was evidenced across all data sources:

Students were willing to promptly engage in a mathematical task
Students engaged in meaningful math talk and sharing of ideas
Students supported their thinking with mostly procedural mathematical explanations Students persisted in solving non-routine tasks
Students looked for teacher to verify correctness of answers
Students used graphing calculator to self-monitor their own solution progress

I then used these six norms to develop a coding scheme that was used in categorizing all textural data. Simultaneous to the start of the coding process, I attempted to articulate and refine a clear-cut narrative description of the content of each category and subcategory. Determining clear operational definitions so they could be applied consistently throughout all textual data proved extremely valuable. Listed below are the categories and subcategories that I used in my coding scheme.

The definition of each category and subcategory and an exemplar of each is presented in Appendix A.

- Task Engagement
  - Willing
  - Not Willing

- Task Persistence
  - Persevere
  - Not Persevere

- Problem Solving - Sharing Ideas
  - Spontaneous
  - Induced
  - Resistant

- Explanation/Justification
  - Evidentiary
  - Non-evidentiary

- Sharing Solutions
  - Question
  - Alternate
  - Inattentive

- Adjudicate
  - Teacher
  - Student

- Nature of Mathematics Learning
  - Emphasis on Process
  - Emphasis on Answer

- Calculator Use / Concrete Reference
  - Meaningful
  - Not Meaningful

This coding scheme was used to examine how the set of six normative behaviors that were observed in May looked during the first ten weeks of the year.

In writing the surrounding narrative for this analysis, my concern was to present an accurate and meaningful representation of my experience and classroom events.

Ball (2000) notes that first-person researchers needs to guard against the tendency toward the personal on the basis of naïve ideas about what constitutes knowledge. My data collection and analysis methods were designed to guard against the presentation of data in a narrow and self-serving way.

Seale (1999), while establishing guidelines for high-quality qualitative research, states that the "trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability" (p. 266). The trustworthiness of my findings rests on the use of multiple data sources, data analysis methodology that only considered assertions sustained across each perspective, and member checks with Sue. I depended on Sue to assist me in determining whether I had fairly represented the data. Although I did not ask Sue to examine my coding scheme, I asked her to check the accuracy of the surrounding narrative and, specifically, if the presentation of data contained any bias or appeared narrow or self-serving. My account of the classroom experiences and presentation of the data resonated with Sue. Her endorsement of my writing, together with the validity criteria that were used in the design of the study help to ensure the trustworthiness of my work.

Looking at Question #2

> Question #2: *How are norms that are consistent or inconsistent with a practitioner's goals for classroom activity constituted in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?*

During the summer of 2008, I focused my attention on answering my second research question. My greatest challenge and concern throughout the study was managing student behavior to create an environment where the students where willing

to expend effort to solve challenging mathematical problems. Therefore, I narrowed my analysis to examining the factors influential in shaping the mathematical disposition of the students. To generate a list of possible factors, I read through my journal, and identified conjectures that I had made during the year regarding their social and mathematical behaviors. At different times over the year, I made an assertion that one of the following items in the list below affected my students' disposition:

My relationships with students
Relationships amongst students
Task selection
Assigning grades
Opportunities to experience success
Sue's presence in the room
Students' beliefs and attitudes
Parental influence
Student emotions
Ability grouping
Past performance history
Instructional style
Administrator pressure
Fear of retention
Team influence

Similar to my approach to the first research question, I attempted to utilize multiple perspectives to identify which of these factors appeared most significant. Following a review of the transcript data and the observation feedback and conversations from Sandy and Sue, I found the following list of factors were represented across data perspectives:

Creating a supportive environment with strong teacher-student relationships
Students' attitudes and beliefs
Selecting and/or designing appropriate tasks
External influences
Dense network of student relationships
Instructional decisions

I utilized varying approaches in the analysis of each of these items. To explore the issue of teacher-students' relationships and students' attitudes and beliefs, I created separate word documents consisting of cut and pasted clips from my data sources. I then read through these documents to identify important themes. On several occasions, I used my insider knowledge to recall classroom occurrences that I missed putting in these documents. On numerous occasions I found it necessary to attempt to go back to my data to record details of these episodes. To further investigate student beliefs, I parsed the data stored under the code of *Task Persistence, Not Persevering* and into four components: Ability, Problem, Teacher, and Student. A description of these codes are described in Appendix A.

To analyze how task design affected students' dispositions I used my journal to identify the five most productive and five least productive class sessions from the third to the tenth week of class. I decided to look at tasks after the third week because this was a point where meaningful relationships were being constructed and patterns of classroom structure were stable. Then I examined the tasks from these class sessions to gauge the cognitive demand (Stein, Grover, & Henningsen, 1996) of each day's work.

To address how external influences affected student disposition, I documented the mathematical activity of one student, Alan, who demonstrated marked swings in levels of interest and motivation. Similarly, I documented the activities of Jordan and Chris and the dynamic between Erika and Keisha to discuss how dense networks of student relationships affected the social and mathematical activity it the class.

It was clear that my decisions regarding task implementation and my actions with the students during instruction were important factors that contributed to the students' willingness to expend meaningful effort. To be as impartial as possible, I analyzed only the instructional strategies that were identified by Sandy and Sue. These strategies were: allowing students to collaborate, permitting students to use calculators to solve problems, using mistakes as learning opportunities, and judicious telling to assist students during problem solving.

Consistent with my analysis of the first research question, trustworthiness in the examination of this research questions was ensured by member checking the surrounding narrative with Sue. Sue's only challenge was that I potentially missed an additional factor that contributed to improving the disposition of some of the students. Sue argued that for, probably the first time in their lives, some of the students were now a top end student in a mathematics class and, potentially, this contributed in an improved disposition and attitudes towards mathematics. Although I recognize this as a plausible assertion, it was not sustained in the other data sources.

Context

The study of examining the challenges of implementing a multiple solution norm took place at Walker Middle School during the 2007 school year in a classroom consisting of eight African-American eighth grade students, each of whom had a history of behavioral issues and low academic performance. (All proper names are pseudonyms.)

I purposefully intended for the study to be conducted at the middle grade level. Middle school is a particularly important time when students' beliefs' towards

mathematics tend to crystallize. Many middle school students perceive that learning mathematics is attributable to innate ability and that putting forth effort has little or no influence on their ability to succeed (Kloosterman & Gorman, 1990). By early adolescence some students deflect attention away from their low performance by purposefully withdrawing effort, resisting novel approaches to learning, and avoiding seeking academic help when they need it (Turner et al., 2002).

The middle grade years are a time when students become increasingly aware of their appearance and are often reluctant to stand out in any way during group interactions. Although the *Standards* recognize this fact, an assertion made in this document is that teachers can succeed in creating communication-rich environments in middle-grades mathematics classrooms (NCTM, 2000). However, the *Standards* document lacks specifics regarding creating such an environment. Research is needed for this grade band that make teachers' decisions and considerations visible in attempting to implement what the *Standards* suggest is good to do in the classroom.

In addition, I desired a diverse setting for my research. There is seemingly contradictory evidence regarding the benefits of *Standards*-based teaching to students of low SES backgrounds. Boaler (2002) found that reform oriented instruction positively impacted student achievement and attitudes in classrooms with lower SES and minority student populations; however, Lubienski (2000a, 2000b) and Zvenbergen (1996) question the hidden assumptions with particular instructional approaches and suggest that underserved students may struggle when encountering these forms of instruction. Although my study did not examine the impact a specific

kind of instruction has on student achievement or beliefs, research in these setting is likely to challenge some assumptions that ordinarily pertain in classrooms.

School

Walker Middle School is part of a suburban school system situated about halfway between two densely populated urban areas in the mid-Atlantic region. In 2002, Walker Middle School was designated by the school district as an under-performing school. Served by the district's School Improvement Unit (SIU), Walker Middle School received targeted supervision and monitoring, and was assigned additional staff members and funding for supplies and materials, such as lap top computers for all staff members. The current principal, who took over after its designation as an SIU school, described Walker Middle School during this time as a "failing institution, with abysmal test scores, little PTA involvement, rampant disruptive behavior, poor attendance, and no business partners."

During the past six years, Walker Middle School has witnessed a remarkable turn-around. By 2008, the school had been recognized as a National PTA School of Excellence, received an award from the state for being a Character Education School of the Year, and was cited by a major news organization as one of the best 30 middle schools in the metropolitan area. Today, the school has nearly 500 PTA members and over 40 local business partners.

Of the 550 students enrolled at Walker Middle School during the 2007 school year, 43.1% were African American, 40.5% were White, 8.9% were Asian, and 6.9% were Hispanic. During SY2007, 21.7% of the students received Free/Reduced meals, and 6.7% of the students were supported through special education.

Walker Middle School has an *instructional intervention* and *kidtalk* teams which regularly meet with faculty and staff to address behavior and academic concerns regarding individual students. An important district policy is to make alternative education environments available for those students who, due to behavior concerns and low achievement, have demonstrated difficulty functioning in a regular instructional setting. Each student in the alternative education program has a plan to avoid future disciplinary action. Although these students follow a regular academic schedule, a contract room is available where the student, under the supervision of a staff member, can leave their instructional setting to work in another environment. Distinctly separate from special needs students who are provided with individualized education program (IEP) goals, these alternative education students share common academic and behavioral characteristics with students labeled learning disabled (LD) and emotionally disturbed (ED). Some of these common traits are performing approximately two or more grade levels behind their peers, difficulty attending to key dimensions of tasks, deficits employing metacognitive strategies, and exhibiting a general lack of persistence and concentration (Maccini & Gagnon, 2002). During the 2007 school year, twelve eighth grade students at Walker Middle School received support through the alternate education department. Although it is generally not uncommon for a student in the alternative education program to also receive services through special education, none of the participants in my study received accommodations through special education.

Participants

A component of Walker Middle School's *school improvement plan* for the 2007 school year included creating an additional mathematics course at each grade band designed to serve a small group of selected students who were identified as below grade level. The eighth grade course was titled *Accelerated pre-Algebra.* The participants for the study were all members in this course. At the beginning of the year, there were six male (Kyle, Cedric, Jordan, Jamaal, Alan, and Chris) and two female students (Erika and Keisha) enrolled in the class. Seven of the eight students in the pre-Algebra course received services from the school's alternative education program. In their seventh grade year, the eight students each scored at the lowest level on the state's mandated seventh grade mathematics test, each were labeled as below grade level on their end of year report cards, and each were enrolled in summer school prior to their eighth grade year. Six of the eight students received a significant portion of their primary education from one of the large urban school districts in the area prior to enrolling at Walker Middle School. Below is a brief description of each of the participants in the study.

Erica: Erika made no secret that she hated being in school and she seemingly did not respect anyone in position of authority in the school. At different times, while struggling to manage her frequent off-task behavior during the first quarter of the school year, I consulted with the school psychologist and assistant principal for ideas. Both noted that she was a difficult student, and neither one had been successful in building a relationship with her.

It was a struggle to engage Erika academically. She often displayed projective coping (Midgley, Kaplan, & Middleton, 2001) by referring to the work as dumb or stupid. Although she frequently received my attention for being a disruptive presence in the classroom, her behavior was not hostile or insubordinate. Often, she apologized for her actions and would pledge to do better. Although she seldom kept her promise, Erika maturely accepted consequences for her behavior. By spring, there was steady improvement. On returning from a suspension in March, she had been issued an ultimatum from the administration that any further misconduct would not be tolerated. On the last day of the year, she left me the following note in my faculty mailbox:

> Dear Mr. Hollenbeck, I enjoy having you as one of my 8th grade teachers. You were the 8th grade teacher [sic]. You done a lot of wonderful things for our class, like you bought us coupons and things like that. Thanks for having me in your class, Love your 8th grade student, Erika

Keisha: Keisha was a very strong willed individual. Prior to eighth grade, Keisha's reputation as being defiant and volatile made her known in advance to the eighth grade faculty. During a parent-teacher conference, her mother commented that Keisha thought the world revolved around her. Keisha was physically large in size and extremely out-spoken. She didn't shy away from verbal conflicts, and, on one occasion, she was suspended for a physical altercation with another student. In class, students seemed to treat her differently, perhaps aware of her rash nature.

Yet, she made a legitimate effort to start the year off well. She was exceedingly polite and wanted to please. She was particularly concerned about grades, and would spend an entire period trying to solve a single quiz question. Over time, and more than other students, Keisha increasingly resented my attempts to redirect her behavior

when it was inappropriate. As long as she perceived she was doing work, she seemed to believe she could do whatever else she wanted. On at least a half-dozen occasions, she overtly challenged my authority either verbally or by physically walking out of class without permission. Her damaging behavior was not unique to mathematics class. She was served with multiple suspensions including a suspension over the final two weeks of the year.

Kyle: Over six feet tall, and a talented basketball player, Kyle, was well respected by his classmates. In class, Kyle was extremely quiet and seldom joined in the conversations of others. Although Kyle struggled academically, his teachers glowingly referred to him when his name came up in kidtalk meetings. Teachers commented on how nice he was, and that he completed all assigned work. Mathematically, Kyle would answer any problem regardless of its difficulty. He often did so with error, and without a lot of reflection. However, if he did recognize a strategy for solving a problem, he was willing to invest time and energy in doing so, even if the strategy involved several tedious guess and check iterations.

Jordan: The time I spent with Jordan outside of class was more than with any other student. Jordan seemed genuinely interested in learning mathematics so he could be successful in life. He had specific career goals about owning his own business, and understood that his education was important. In class, Jordan wasn't always so focused. He had a witty sense of humor, and frequently used it to get laughter from his friends. Still, he seemed to understand when I felt his behavior was inappropriate, and his actions in class were a positive model for his classmates.

Jamaal: At 15 years of age, Jamaal was the oldest student in the class; however, he was physically the smallest in stature. He possessed a good sense of humor and an engaging personality. Although likable, he admitted to having a bad temper and a stubborn disposition, and he was suspended on three different occasions during the year. Academically, Jamaal struggled across all disciplines, and although he qualified for special services, his Mother denied support. Generally, at Walker Middle School, the protocol for dealing with students with severe academic concerns involved creating an individualized student action plan with specific interventions meant to address an identified academic or behavior goal. Typically the action plan would remain in effect for the school year. A last, and infrequent, resort for students showing no progress would be a referral to the school's instructional intervention team (IIT), which would often result in a recommendation to remove the student from the general education setting. Due to a lack of success with his sixth and seventh grade action plans, and a shared sense from his eighth grade teachers that Jamaal's aptitude was extraordinarily low, the usual protocol was bypassed, and he was immediately referred to the school's IIT in late December. Although, this referral was made, Jamaal remained in the school for the remainder year.

Cedric: Cedric was a sensitive student. When singled out by classmates he would either get angry, or, more frequently, quietly retreat inside himself and not respond to anyone. A concern expressed by all his teachers was the small amount of effort he seemed to put forward. In math class, it was common for Cedric to seemingly not to do any work unless he was afforded individual attention. His apparent disinterest and lack of motivation escalated over the first two quarters. By midyear, Cedric was

being mentioned as a candidate for retention. As part of an intervention on his action plan, Cedric's teachers met with him as a whole to discuss their concerns, and list specific steps for Cedric to do in and out of class. Cedric handled the meeting very well and his turn-around in class was immediate and lasting. One faculty member noted that the intervention was a success because Cedric was a student of high character.

Alan: Alan was the one student not part of Walker Middle School's alternate education program. Along with Chris (who will be introduced next), he was the strongest problem solver in the class; however he was very inconsistent in his willingness to engage in problems. One day, he would be very intent on solving a problem and take pride in the fact that he was one of the better students in the class; the next day he would sing rap songs and engage in off-task conversations and act like he didn't know how to do anything. After Chris was moved into an Algebra class, Alan openly expressed a desire to join him. For over a week in October, Alan came an hour before school to catch up on what he had missed in the Algebra class. His extra effort was admirable, and I let him know how proud I was with his commitment.

Undermining his improvement were some weighty issues outside of school. Alan's attendance had been a concern, and worsened as the first quarter progressed. Of all the students in the class, Alan seemed to have the closest ties to the metropolitan area, and would frequently talk of his visits to 'town.' Alan had a seemingly turbulent home life. It was unclear whom he lived with, and my only contact with his family was e-mail with an older sister. By the end of October, he

was no longer interested moving into Algebra, and stopped trying altogether in class. When I pressed him to do work he had previously demonstrated a strong understanding of, he would say he forgot how to do it. By November, he was no longer attending school and he was officially withdrawn before Thanksgiving.

Chris: Chris consistently finished his work before anyone else, and then used class time as an opportunity to engage in sophomoric behavior. He was the typical class clown. Chris was extremely likable, and his humor was innocent, but time and again he would be the source of disruptive behavior. It was almost as if he couldn't control his actions around his peers. After a conversation with his mother, on September 22, I moved Chris out of pre-Algebra and into an Algebra class. The change in his behavior was stark. He seldom talked, and when he did it was no more than a whisper. Throughout the year, as he gained confidence, he became more vocal in class. However, the nature of his talk was mathematical. Although he struggled with much of the content for most of the year, he successfully passed the state's high school algebra exam.

My Background

Shortly after identifying my research interest, I began the process to obtain a teaching license that would enable my employment as a full-time public school teacher. I was previously licensed in another state and had taught for five years in a public high school and one year as an adjunct instructor at a university. In addition to completing a secondary teacher certification program, I possess an undergraduate degree in electrical engineering and a graduate degree in applied mathematics. Reflecting on my teaching experience prior to enrolling in a doctoral program, it is

clear I taught from a relatively traditional perspective. Although I stressed to my students the importance of acquiring a conceptual understanding of mathematics, I assumed the responsibility of demonstrating how mathematical ideas were connected and why mathematical procedures worked. When students struggled with a problem, I often attempted to clear up any confusion as quickly as possible by attempting to provide clear and concise explanations. I was considered a very good teacher. Most of my students excelled on mandated assessments, and I received glowing evaluations and recommendations from administrators.

Prior to the 2006 school year, I accepted a position as both eighth grade teacher and instructional team leader at Walker Middle School. In this first year, I taught Algebra and Geometry to students identified as above grade level. My teaching practice was strongly influenced by my philosophy of instantiating a multiple solution norm. Over the course of the year, the normative behaviors of students in the class met my expectations for a multiple solution norm. In a visit to an Algebra class near the end of the year, my advisor, Dr. Dan Chazan concluded that a multiple solution norm had been constituted. My teaching during this year received some public notice. The Maryland Council of Teachers of Mathematics recognized me as outstanding middle school teacher of the year, and my instruction was videotaped and used for professional development purposes by the school district.

With these set of experiences behind me, I was eager to accept the challenge of teaching a group of students who had a history of low attainment. During the 2006-2007 school year, I taught two sections of Algebra, two sections of Geometry, and the Accelerated pre-Algebra class.

Summary

In this study, I used a first-person research methodology so that I could examine a specific kind teaching within a particular context. My study examines the challenges to instantiating a multiple solution norm to a group a low-attaining eighth grade students. Utilizing a first-person research design makes imperative the collection of multiple forms of data. As described in this chapter, my data consisted of video recorded classes, a reflective teaching journal, and observations by two individuals. My analysis depended on the triangulation of the data from these sources.

Having described how I collected and analyzed the data, I now turn to the Presentation of Data in then next chapter. I will answer the two research questions that guided the study by tracing the development of classroom norms and discussing my greatest challenge of managing student behavior so that students were willing to expend effort to solve challenging mathematical problems.

Chapter 5: Presentation of Data

My purpose in conducting this study was to understand the challenges a practitioner faces when attempting to incorporate students' multiple solutions as a regular feature of instruction, and to understand what a practitioner might do create a classroom environment where students are expected to solve a problem in more than one way. In designing this study, I used the notion of social and sociomathematical norms to define a construct of a multiple solution norm that outlined my goals for social and mathematical activity in a classroom that uses students' multiple solutions to develop key mathematical ideas (Chapter 3). I specifically aimed to examine my own practice of teaching a class of low attaining eighth grade students. My data collection and analysis was guided by the following two research questions:

3) What norms are evident over time in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?
4) How are norms that are consistent and/or inconsistent with a practitioner's goals for classroom activity constituted in an eighth grade pre-Algebra classroom where a practitioner intends to proactively initiate and guide the constitution of a multiple solution norm?

Throughout the study, it was clear that I had underestimated both the challenge of managing student behavior and the complexity of instilling in students the notion that persistence and effort were key components to problem solving. Much of my teaching effort centered on engaging students to do meaningful mathematical work. My original vision of constituting a multiple solution norm went unrealized, yet over

the course of the year, the students made much progress in their willingness to engage in a mathematical task and in their motivation to persist in solving non-routine problems.

To capture the challenges and successes I experienced throughout the year, I analyzed the data (Chapter 4) by narrowing the focus of the original wide-ranging research questions. To address the first research question, I identified the norms, relevant to the definition of a multiple solution norm (Chapter 3), that were evident near the end of the school year. Then, I examined the data to trace the development of these norms over the first quarter of the year. The second research question was narrowed to examine the issues related to managing student behavior and negotiating an obligation for students to expend effort to solve challenging mathematical problems. The revised research questions addressed in this narrative are:

1) What norms existed near the end of the year in a low attaining eighth grade pre-Algebra classroom where a practitioner attempted to initiate and guide the constitution of a multiple solution norms? How did these norms compare to the expectations and obligations for social and mathematical behavior that were evident in the first quarter of the school year?
2) What factors affected the students' behavior and disposition toward mathematics in a low attaining eighth grade pre-Algebra classroom where a practitioner attempted to proactively initiate and guide the constitution of a multiple solution norm?

I divide this chapter into two sections, each addressing one of the revised research questions. In the first section I present data that illustrates which norms were evident

near the end of the school year, and track these norms over the first quarter of the year. In the second section, I present data to highlight six factors that affected the students' mathematical disposition.

End-of-Year Classroom Norms

Like all teachers, I regularly encountered students who would come to class seemingly uninterested. On any given day, these students could have been hostile or confrontational, quietly noninvolved, or actively talking to others off-task. Over the years, I have developed a repertoire of actions and things to say and do to navigate around these difficult teaching moments. After the fifth class session, it was clear that I was facing classroom management challenges I had not expected nor experienced with my other classes. In my teaching journal, I wrote:

> [Creating a multiple solution norm] is going to be a very difficult task. I can see how it would be better behavior wise just to show them how we add integers and then ask them to practice. I cannot get their attention to even pose a question or a task. They are continually talking to one another, and barely acknowledge that I'm in the room . . . I can't seem to engage them in mathematics for any length of time. They either don't attempt the work or rush through it with no apparent reflection. They are so easily distracted by each other. They can hardly control themselves. I probably need a different approach. [Personal Teaching Journal, September 5, 2006]

Over time, my 'in the moment' impressions of the class continued to focus on how poorly the class was going, both in terms of managing their behavior and towards my goal of negotiating a multiple solution norm. In mid-October, I recorded the following journal entry:

> It was a struggle to get them to work on fractions. Cedric and Erika never seem willing to do anything. Cedric kept putting his head on the desk, I kept threatening to send him to Ms. Williams [alternative education teacher]. It is always difficult to get Kyle to try any problems, at least he is quiet. Jordan and Alan were awful; they kept talking about how hungry they were. Each

> time I tried to get their attention, Alan kept saying he already knew everything about fractions. Although Jamaal asked a question . . . , he did not seem very interested in talking about it. Everyone made fun at how he said the word 'other.' Keisha went to the board, and actually presented a good explanation for adding fractions, but I was the only one listening to her. [Personal Teaching Journal, October 17, 2006]

At mid-year, I took advantage of a professional development day to reflect back on the previous five months efforts at developing a multiple solution norm. By referring to my framework of the social and sociomathematical norms used to define my notion of a multiple solution norm, I noted some movement in improving the disposition of my students, but overall my sentiment was that a multiple solution norm was not being created, and their behaviors and actions in the classroom were a major concern.

> A significant challenge has been to establish a positive disposition toward mathematics for this group of students. Initially, I was struck by their apparent apathy and lack of motivation. Increasingly, they are showing a willingness to work on problems, but it is never very continuous. They seem to share the belief that as long as they are working on the task, they can talk or move around. . . . It is always a challenge to divert their off-task chatter. . . . I am concerned that I am doing too much telling, but at the same time, if I don't give significant help, they will just as soon shut down. . . . I am concerned by my seating arrangements for the group. I started off with all the students at one table, then I separated the whole group in two, then I switched the arrangement to four pairs of students, and now I have them arranged so they individually sit at different tables. I want them to share their ideas with each other, but when I put the group together or pair them up, they use that as an opportunity to socialize. . . . Whole-class discussion has been very difficult to facilitate. Although they are generally willing to give explanations and publicly present their solution strategies, I can't get them to listen well to one another. They frequently use discussions as a springboard to engage in off-task conversation, and there is never any rhythm or fluidity to what is discussed. [Personal Teaching Journal, January 17, 2007]

Near the end of the school year, my overall reaction to the class was that it had not gone well. I continued to be discouraged by the group's behavior. In a conversation with Dan Chazan, I expressed disappointment with my inability to negotiate desired

norms. Specifically, I believed the group was never able to effectively communicate or share their ideas with each other.

> On any given day, the class is just peppered with off-task chatter when I try to set up a task or go over students' solutions; there doesn't seem to have been a lot of movement from where I started to where I am now. . . . I have had very little success trying to orchestrate whole-class discussion, a central component to a multiple solution norm. . . . In whole-class discussions, they seem willing to share their solution methods, but I have no sense they are actively listening to what others are saying. . . . There are two girls in particular, so that in the absence of one of them the class seemed to go marginally well, but the two of them together made it very difficult to manage. [Conversation with Dan Chazan, June 4, 2007]

My classroom management issues were also clear to Sue and Sandy. However, neither one shared my disparaging perspective regarding the group's progress. In addressing the challenges the class presented, Sandy commented, "they had a hard time staying seated for any length of time" and "[they] would rather be talking about anything but math." However, Sandy perceived that the group made significant progress throughout the year. After an early November observation, she remarked that the students' level of engagement with the problems had improved, and after watching a lesson in May, she noted that there was definitely a "positive vibe" in the classroom and believed "the students were willing to talk to each other about math" and "they were doing some very serious math work."

My biggest challenge for the year, according to Sue, was "getting them to focus, just to sit still long enough for them to realize that there was something going on in class." By the end of the first marking period, Sue had noticed a qualitative change in how the students were behaving. After observing a class on October 26, Sue stated, "I am just amazed that they sat for 45 minutes and talked about math. And used math terms to talk about it." In an interview at the end of the school year, I asked Sue if

anything surprised her about the year; she answered, "about two months into the year, what surprised me, was you were able to spend 30 to 35 minutes getting them to do math work. . . . Out of respect for you, they were willing to do mathematics and talk about what they were doing."

Sue did not agree with my assessment that the students did not work well together. She acknowledged, "in the beginning no one listened to anybody whenever they had something to say," and they went through a time when, "if they did listen, and they didn't agree, they were very antagonistic toward each other." However, Sue believed that, two-thirds into the year, things were different. She stated, "around spring break time they were listening to each other and processing what each other had to say. , . . They communicated with each other and if someone figured out how to solve a problem, they would share their ideas with everyone else, and they were excited to share their ideas."

The following summer, after some temporal distance from my in the moment journaling, I began to view the class differently. In particular, the process of transcribing the tapes afforded the opportunity to detect some of the same things noted by Sandy and Sue. Viewing the class sessions in chronological order provided a kind of time-lapse effect illustrating that student behavior became more adaptive and appropriate, and their actions more consistent with many of my goals for the class.

There were numerous incongruities between my daily journals and what I observed from the tapes. My journaling frequently reflected my disappointment regarding the amount of academic rigor the students offered. Too often, my journal

entries were biased towards particular student behaviors or conflicts. However, the videotape helped me understand that the behaviors were often not as severe as I had initially thought. In addition, there were moments of productive student work and sophistication that was captured by the video camera in which I was unaware.

Inasmuch as the existence of norms are social constructs interpreted through an observer's lens, an examination of any classroom near the end of a school year is likely to reveal a wide array of normative behaviors and expectations. Observations of my lessons late in the year suggest that a number of constituted norms, relevant to my construct of a multiple solution norm, were in place. In particular, a visitor to my classroom would notice the following mutually dependent expectations governing the behavior and actions of me and my students: (1) students were willing to promptly engage in a mathematical task; (2) students engaged in meaningful math talk and sharing of ideas; (3) students supported their thinking with mostly procedural mathematical explanations; (4) students persisted in solving non-routine tasks; (5) students looked for teacher to verify correctness of answers; and (6) students used graphing calculator to self-monitor their own solution progress.

To highlight the existence of these norms, a partial transcript from a lesson in May is provided below. The transcript is broken into three sections. After the first two parts of the transcript, I analyze the existence and evolution of the above norms. Following the third piece, I discuss to what degree a multiple solution norm was constituted.

Six lessons in May were included in the data set (May 17, 21, 22, 23, 24, 25). Each of these lessons was part of the same unit on representing linear relationships.

The normative understandings regarding the expectations and obligations for social and mathematical interactions were stable across the six days. The lesson from Wednesday, May 23 is highlighted below. Wednesdays were typically a day, especially in late spring, when the class was most productive. This example is not presented as an exemplar of how to teach a lesson on linear relationships. It is likely that the task and how I managed student work could be criticized on several different levels.

Lesson on Wednesday, May 23, 2007

Six[2] desks are arranged together in a two by three rectangular configuration. Before anyone has entered the room, graphing calculators have been placed on each student desk, and student binders are piled in the usual location in one corner of the room. I am sitting in the middle desk along the side facing the doorway. Cedric is the first to enter, and I greet him in my usual way by saying, "How's Cedric?" Cedric says "good," finds his binder, and sits down at a desk adjacent to mine. Jamaal, Jordan, and Kyle are the next to come in. Jamaal and Jordan are loud and pushing each other as they get their binders, and the three of them sit at the row of desks opposite Cedric and myself. I greet them by saying, "How you guys doing today?" Jamaal says, as he usually does, "I'm hungry" and Jordan asks, "What is to eat today?" Prior to the bell, Erika enters and takes the seat next to mine, across from Kyle. I greet Erika by saying, "Alright, Erika's here." As the bell rings, they are either opening or have already opened their binders, as I announce "O.K., let's see

[2] Keisha is in the midst of serving a ten-day suspension. The class this day consists of Kyle, Erika, Cedric, Jordan, Jamaal, and myself.

how we can do today, I've given you a problem like we have been doing, only I have made the numbers different to try to make a little bit harder."

The task I assigned is shown below. Students were familiar with the problem of finding a linear relationship given a numerical representation. The uniqueness of this task was that the slope was between zero and one:

A box containing 10 paperclips has a mass of 9 grams. The same box containing 15 paperclips has a mass of 11 grams. The table below shows the relationship between the number of paperclips and the mass of the box and paperclips.

Number of Paperclips	Mass of Box and Paperclips
10	9 grams
15	11 grams

Find an equation showing the relationship between the number of paperclips in the box and the mass of the box and paperclips.

Part I: Students attempt to solve initial task.

Jamaal:	It's going up five right? [Jordan: Yeah]. So the first one got to be five. It'll be five plus another five is ten, plus another five is 15. Because if you have five right now and add another five, that's ten; and add another five, that's 15; and another five, that's 20. On this side, it's going up two.
Jordan:	I don't see how you got to find the equation.
Jamaal:	It's going by five, and starting by ten.
Erika:	No its not.
Jamaal:	Starting by five.
Jordan:	No it's not, it's going by five.
Jamaal:	If you start off at five, and go up by five it's ten, and go up by another five it's 15.
Jordan:	No it's not yo.
Jamaal:	Listen to what I'm saying, yo!
Jordan:	[Loudly] I'm listening to what you're saying, yo!
RH:	Stop yelling, yo [Laughter]. Your goal is to find an equation that will give you this table. Across from the ten you'll have a nine, and across from the 15, you'll have an 11.
Jamaal:	[Leaning over to Jordan] You always got to go up by right there, what he put up there. So it got to be five right? [Jordan: yeah]. And another five is ten, and another five is 15, and another five is 20.

Jordan:	That's going by five. [Jamaal: Exactly]. You said it starts by five.
Jamaal:	It do start at five.
Cedric:	Like this [*Cedric hands me calculator; Cedric has entered two equations* $y = 5 + 5x$ *and* $y = 7 + 2x$*, and he is showing me a table*]
RH:	O.k., tell me how you got this.
Cedric:	For the first one, I started at five and it goes by five so it's five plus five x, and the other one starts at seven and goes by two.
Jordan:	Where did seven come from?
Cedric:	I'm not sure, I first tried nine but that was too big, so then I tried eight and then seven.
Jamaal:	I don't get this Mr. Hollenbeck. I can't get this.
RH:	[To Cedric] O.K., that's really good, what if I asked you to make this table using only one equation, like make a table where these are the x's and these are the y's.
Jordan:	Mr. Hollenbeck, what am I forgetting?
Jamaal:	I went to town yesterday, yo, We got lost coming home.
RH:	Jamaal, even if you've given up, lets give everybody else a chance to try it.
Jamaal:	I didn't give up.
RH:	Does this problem seem too hard?
Jordan:	It's not hard, I just don't know how to make the other side.
Jamaal:	I'm fresh out. I need to see somebody do it first so I can understand where they're coming from.

Norm: Students were willing to promptly engage in a mathematical task

All teachers strive to create a positive classroom learning environment. A class that begins with disruptive behavior and student apathy toward work is clearly not a conducive setting for teaching and learning. The most critical component in establishing a multiple solution norm is developing a students' productive disposition toward learning mathematics. Students with a productive disposition believe mathematics is useful and valuable, and that steady effort is an essential component to learning. Being genuinely interested in solving an assigned mathematical task is a necessary condition for a student to possess a productive disposition. A multiple solution norm cannot be developed in an environment where students either avoid or overtly resist doing mathematical work.

During the first quarter, I frequently encountered problem behavior and high levels of resistant conduct. In particular, the first ten minutes of class was often unruly and loud. In her field notes following an observation on September 25, Sandy wrote, "five minutes into the period, students are rowdy, sharpening pencils, and fooling around." Sue attributed the hectic behavior at the start of class to other contexts. She believed "a lot of the students [had] a hard time leaving what was going on with them personally or carry over from other classes or home life at the door, and [found it difficult to] focus on what they needed to do for the class."

My disappointment in managing students' behavior and their lack of productive work was a frequent topic in my journaling. In an October 9 journal entry, I described what the start of class was like:

> When the bell rang, no one was in his or her seat. They were gathered around Erika's table. Erika had her cell phone out, I took it away. I asked each of them to take their seats and get started. They slowly complied with my request, and took circuitous routes back to their seats. It was at least 12:10 [class started at Noon] before anyone even looked at the problems. Jamaal immediately said he didn't know how to do any of them. He refused to read number one out loud. Jordan and Alan were talking and singing about rap songs. Keisha asked if she could do number one on the board, and they all began arguing about who should go to the board. Soon, half of them were out of their seats, racing to put their names on the board. When I went to check with Cedric, he was just beginning to read the problems. This is all too common – they don't start working until I am near them. Not a good norm to have. [Personal Journal Entry, October 9]

In the majority of class sessions during the first ten weeks I perceived that the class did not behave well. As shown in Table 1, in journaling from August to November, I explicitly referenced the pervasiveness of my students' off-task behavior in 28 (62%) out of 45 entries. In only 11 out of 45 (24%) journal entries did I praise their task engagement. In the remaining six (13%) journal entries, no reference was

made regarding their conduct. I expended a great amount of effort managing student behavior and trying to get the students involved in doing mathematics throughout the class period. Sandy commented in her field notes that, "Rick seems to be spending most of his time getting students to attend."

Number of Journal Entries Focused on Students Off-Task Behavior	28	62%
Number of Journal Entries Praising Student Behavior	11	24%
Number of Journal Entries with No Mention of Behavior	6	13%

Table 1: August to November journal entries (45 classes) relating to student behaviors

A journal entry on September 26 is typical of the issues I confronted in my attempts to get students to focus on mathematics:

> A lot of non-math talk took place. When we tried to get to the math objective, it didn't go very smoothly. Alan is very non-responsive to requests to do anything, and Erika is very similar. They talk a lot and don't seem interested in anything I try to do. Keisha also doesn't seem too responsive sometimes, especially when she is talking with Erika. Jamaal is very reluctant to do anything he is not sure off. Kyle is very quiet, and always does the work but he is a very low-achiever. Jordan did seem interested in doing the work. Erika, Keisha, and Jamaal were out of their seats and felt free to sit on their desks. I didn't stop it today, but asked them sit in chairs tomorrow. [Personal Teaching Journal, September 26, 2007]

I perceived that the dense friendship relationship between Erika and Keisha contributed to many moments of disruptive conduct. I lamented to Sue after an early September class that "they constantly talk, joke, and snicker without any regard to my requests to stop." Sandy noted that, even though Erika and Keisha were seated on opposite sides of the room, they still managed, due to their "strength in will," to "yell back and forth across from each other." In particular, I singled out Erika as a source of unruly, non-academic behavior in the first quarter. I went as far as approaching my assistant principal with my concerns regarding Erika and suggested that the class

could be much more effective without her. This conjecture was based on my experience with the class during Erika's absences. Six times during the first 45 days of school Erika missed class. Out of the 11 journal entries where I praised the groups' on-task behavior, Erika was not present in five of them. Following an absence on October 23, I wrote:

> The problem solving part of class went very well. Cedric and Kyle were particularly strong. Jordan continues to improve. I think working with him after school is having a great effect. Jamaal seemed discouraged and complained that he couldn't do any of the problems. Keisha did well working with him. I commended Jamaal on his effort, and told him how proud I was that he didn't give up. Alan had little difficulty with the task. I wish he would work with others better than he does. Erika was absent, so the trend continues where minus Erika, the class is really strong. Keisha is much more helpful and engaged when she isn't here. I wonder how I can get the rest of the group to realize that, and put pressure on Erika to go along with us. [Personal Teaching Journal, October 23, 2006]

In examining the videotapes from May, it is clear that dramatic improvement in both student behavior and their willingness to engage in mathematical work occurred. By year's end, there were norms in place guiding students' actions to engage in a mathematical problem as soon as class started. In each of the six videotaped class sessions from May, students promptly begin their work. In the May 23 class, Jamaal and Jordan take immediate interest in the problem. It is evident that Cedric is thinking about the task; Erika seems initially involved but it is difficult to ascertain her and Kyle's level of engagement. In a conversation with Sue in mid-May, she commented that, unless there was some issue from a prior context, in which case a student needed to be "calmed down and refocused," the group would start their work right away. A breach in late May of this expectation illustrates the presence of this norm. Instead of having a mathematical task already placed in their binder, I had

planned an activity that required collecting some data before we could begin our work. Invariably, as each student retrieved and opened their binders, they individually announced that their worksheet was missing or asked why they didn't have one. They appeared genuinely bewildered that there was no problem to start class with. When I surveyed the group, and asked them to describe to a student next year what this class is like, Kyle succinctly referred to the structure of the class in his description, "don't be late, get your binder, do your work, stay on-task."

By May, the amount and nature of the off-task behavior was undeniably influenced by the attendance of Keisha. This particular class was characteristic of the group's behavior in her absence. By year's end, without Keisha's presence, I was able to easily, and without conflict, manage moments of disruptive or off-task behaviors, and refocus students attention on the task at hand. In watching a Keisha-less class in May, Sandy commented that "they still find it difficult to behave, but you know that they want to." The following journal entry suggests that the students even understood that the class functioned better when Keisha was absent:

> It's amazing how good the class is since Keisha's suspension. I think even the class recognizes that. Cedric said something about how, in the beginning, I thought she was all nice and all, but now I see how she really is. Jamaal mentioned how lost Keisha is going to be when she comes back. I said that they would have to help her catch up, but they all said not me. [Personal Teaching Journal, May 30, 2007]

In the May 23 class, until Jamaal's comment about going to town and getting lost coming home, there is no action overtly off-task at this point in the lesson. Only a couple of weeks earlier, Keisha's presence would have likely created a qualitatively different classroom environment. In a journal entry, I wrote the following:

> I had to send Keisha to the office. She was a constant distraction today – talking, singing, laughing, moving around the room – I gave repeated warnings for her to stop, and when I sat down next to her she told me to 'get out of my face.' My relationship with Keisha is now non-existent. Her behavior is increasingly unmanageable. She seems to believe that as long as she is 'doing her work', she can do anything else. [Personal Teaching May 10, 2007]

The data presented here suggests that norms governing student expectations and obligations to engage in assigned mathematical work can be completely transformed throughout a school year. An examination of the first ten minutes of class from August to May reveals two distinctly different patterns for student on-task behavior. In addition, an examination of the data reveals the powerful influence individual students can potentially have on the constitution of classroom norms. In the fall, the attendance of Erika had a discernable affect on the class, while by May, Keisha's attendance was key. Particularly interesting was the dichotomous nature of class in May with or without Keisha.

Norm: Students engaged in meaningful math talk and sharing of ideas

In addition to their prompt willingness to work on the task, the May 23 lesson illustrates that students would naturally share their ideas with one another in a purposeful way. This spontaneous interaction supplies evidence that, by May, the students had appropriated social norms regarding the public sharing of ideas, which I valued in my attempt to instantiate a multiple solution norm.

A productive discussion, where students purposefully thought about a mathematics problem, did not preclude them from yelling or insulting one another. A consistent challenge throughout the year was trying to have the students share their ideas in a polite and respectful way. Sue noted their antagonistic talk, and commented that the

nature of their conversations in class was the same way they communicated with each other outside of school. In her field notes following an August observation, Sandy recorded, "Students discuss problem between themselves, but it easily turns to some yelling at each other." Students appeared just as likely to try to shout down each other's ideas or quarrel with one another in May, as they were earlier in the year. In the May 23 lesson above, Jordan and Jamaal quickly grew frustrated and start yelling at one another. In a May 21 episode, Jordan dismissed a solution posed by Cedric. Consequently, they engaged in several iterations of an increasingly louder shouting match with Jordan yelling to Cedric 'you're wrong,' and Cedric countering 'no I'm not.' After I break in and ask for one of them to justify their opinion, Jamaal correctly assesses that Cedric's answer cannot be right. Jordan's boastful response is, "What are you talking about, got what? You stupid, you stupid, ahhhhh. Your wrong, your wrong. You ain't doing nothing, you just hollering, your're making all this noise for nothing."

In the August to November classes, students infrequently shared their ideas in a spontaneous manner. It should be noted that the lack of this kind of interaction was likely influenced by the fact that, unlike the physical arrangement of desks in May, the students were not in close proximity to one another. The isolated seating arrangement in the beginning of the year was a result of an explicit negotiation with the students regarding their persistent lack of productivity and off-task chatter. In a class on September 6, after making a plea for their cooperation, the group started blaming each other for their misbehaviors. Keisha suggested I should change their seating because, as she stated, "We are all too close; that's how everybody gets in

trouble when we turn around and talk to each other." In my journal for that day, I wrote, "After I changed their seats, the group did well for the last five minutes. . . . Ideally, they wouldn't be separated, I will see if I can get them back together after some time. If they will stay on-task I will keep things this way for awhile."

Although the physical layout of the room was changed early in the year so that students were not seated next to each other, I often asked them to pair or team together to discuss a problem. As summarized in Table 2, the successes of this induced collaboration was hit and miss. From August to November, I coded 102 instances where students were given the opportunity to cooperatively share their ideas; approximately half the time, the students appeared to use this as an opportunity to goof around or engage in off-task conversations. The students engaged in the productive sharing of ideas approximately 40% of the time. A discussion was considered productive if the majority of the discourse focused on the assigned mathematical task. A discussion was considered non-productive if the students generally used the opportunity to collaborate as a time for socializing. It should be noted that when multiple groups of students were working together, the placement of the video camera generally captured the discourse from only one group.

On at least nine occasions, students refused to work in the group I had selected for them. In one of these instances, on September 11, Jamaal declined to work with another student and faithfully asserted that, "I can't learn from other students. I can only learn from myself or a teacher." These became critical moments of (re)negotiation. In the aforementioned episode with Jamaal, I steadfastly held onto my conviction that he needed to discuss the problem with his group, and admonished

Jamaal by sternly responding to his belief that he couldn't learn from others by over-emotionally exclaiming, "That's not true, it's dumb thing to say. You're acting like a stubborn two-year old. I'm not going to help you do this, until you try to work with Keisha and Cedric." Jamaal's subsequent action was to suspend all effort for the rest of the class period.

Number of Productive Group Discussions	42	41%
Number of Non-Productive Group Discussions	51	50%
Number of Times a Student Resisted Working in a Group	9	9%

Table 2: August to November summary of productive and non-productive cooperative learning opportunities

It is important to note that there were several instances during the first quarter where I observed a productive and interesting dialogue between students. As early as the first week of school in August, there were moments when students constructively worked together to find a solution to a problem. Below is a brief exchange between Chris and Alan on August 31 regarding the solution of $15+(-22)$. This episode is not meant to document whether any understanding was shared, in fact it is not clear how Alan thought about the problem. It does illustrate that, early in the year, my students arrived to class with willingness and a capacity to share their ideas in a meaningful way.

RH:	You don't have the same answer as Alan.
Alan:	I got this being positive seven.
Chris:	It's negative because, if you add
Alan:	No, look, look, no, look, let me explain why boy. If you subtract 22 from 15, you're going all the way up to the positives and its going to be positive seven, see positive seven
Chris:	But if you think about it then, that's left over, so you going to have to add the 15.
Alan:	So we subtract the 22 and not 15?

Chris:	You can do it both ways, but you did it wrong because there are more negatives than positives, which means there are going to be more negatives left over.

In the sharing of ideas, the students consistently appeared to struggle to effectively communicate them. Erika, in a lesson on September 28, eagerly wanted to publicly explain how she thought of a task; however, she grew frustrated with her inability to articulate herself and ultimately avoided attempts at an explanation by claiming she did not understand the problem: "The x is equal to one, and so two of them is equal to one. I don't know how to explain it . . . one of them is equal to two, so, I don't know. I don't get this." In the May 23 class, Jamaal had difficulty in clarifying his idea to Jordan, and essentially made his point by reiterating the same argument three times. Sandy observed that "they had a hard time telling each other what their ideas were" and conjectured that "their knowledge of their own solutions wasn't explicit enough [to effectively communicate their ideas], they only understood what they did tacitly." Sue posited that, "they lacked basic mathematical skill and knowledge, and did not really have the vocabulary necessary to explain themselves."

Each of the eleven class periods during the first quarter of the year where I praised my students' behavior had multiple moments of purposeful math talk. Below is a transcript from an episode on October 26 when Kyle, Jordan, and Alan, reviewing for the school district's quarterly assessment, was working on the following task:

> *At 8:00 am, a thermometer reads a temperature of* 8° *Celsius. A cold front causes the temperature to drop* 3° *Celsius per hour. What will the thermometer read at 11:00 am?*

Jordan:	The regular temperature was eight and then it drops three degrees per hour.
Alan:	I know how to do these. [*Reading problem*] At eight o'clock the thermometer reads a temperature of eight Celsius, a cold front

	causes the temperature to drop three Celsius per hour.
Jordan:	What's the temperature at 11 am? So it goes up.
Kyle:	It goes down, it drops.
Jordan:	I know that, the temperature goes down, but the hours go up, so it's eight right now, it will be five, that's one hour.
Alan:	Alright, one hour it's five degrees.
Jordan:	Huhh?
Alan:	It drops three degrees so in one hour it goes from eight to five.
Jordan:	Yeah, yeah, yeah.
Alan:	It will be two.
Jordan:	At ten it will be two.
Alan:	At 11, it will be one, you count zero? You don't count zero.
Jordan:	You don't?
Kyle:	Yes
Jordan:	Mr. Hollenbeck, you don't count zero?
RH:	Try to figure out what makes the most sense.
Alan:	It's negative two.
Kyle:	I think, it's negative three.
Jordan:	Hold up, hold up.
Kyle:	Look you all.
Alan:	Everybody, look, look,
Kyle:	Two, one, zero.
	[*Alan and Kyle refer to a number line* – 8,7,6,5,4,3,2,1,0,–1,–2,–3]
Alan:	It going by two, look that will be positive and that will be negative. That's three, that's three, and that's three.
Jordan:	Wait, wait, wait, it's eight right now, and goes down three per hour, the next hour it will be five, that's nine o'clock, the next hour it'll be two, and the next hour, it's [negative] two right there.
Alan:	No, it's negative one.
Jordan:	It's negative one?
Kyle:	It's negative one.
Alan:	You got to count that one [zero] there.
Jordan:	Alright, alright, it's negative one.
Alan:	Alright, we got it.

In addition to the cooperative nature of their problem solving and their productive sharing of ideas, it is important to note that the group did not look at me to reduce the complexity of the task or press me to give them ideas on how to solve it. Although, Alan asked a question that would have made the task more straight forward, neither he nor his group were the least bit discouraged or frustrated when I deflected away

his question. During moments of productive collaborative effort, this normative expectation was relatively stable throughout the year.

According to Sandy, "A major expectation I observed across the four classes was that you expected students should make a serious attempt to decide if their answer is correct without recourse to you, and that talking about their math ideas is something that is really valued." In the fifth class session of the year, Erika, Chris, and Cedric were grouped together when Erika tried to solicit my help. Before I had the opportunity to respond, Chris said, "He's not going tell us." Although I believe Chris was mimicking how he expected me to react, it highlights the fact, that from day one, I explicitly and consistently held off giving students assistance *if* they were working well together.

Conversely, when students were working individually, or not functioning productively as a group, they would routinely expect me to help. I frequently complied by providing suggestions and assessing their solutions and solution strategies. A journal entry from October 9 illustrates how my role in giving information was vastly different from when students were working well together:

> To get them focused on the problems I had to do too much telling and adjudicating of answers. Erika asked me if she was correct about one of the problems and I said she was; I pointed out to Keisha that her work for number one was not correct because she was missing the negatives; I had to explain to Alan that 8 + 3n meant eight plus three times a number; and I virtually solved several of the problems for Cedric. I asked for Jamaal and Jordan to compare their answers but I ended up telling them which ones were right. [Personal Journal Entry, October, 9, 2006]

In attempt to break this dependency, as well as improve the groups' on-task behavior, I changed the structure of the class in late October and created an extrinsic reward system. For about a three-week period, I offered the group a weekly reward

if, for each class period, they followed a set of explicit directions. With no teacher assistance permitted, the students were directed to begin class with five minutes of individual work, ten minutes of small-group time, and then ten minutes of whole-group collaboration on the problem. Only after this twenty-five minute window would students be permitted to ask for help or seek my evaluation of a solution. In the first day of the new routine, students continued to press me for assistance, and increasingly, I obliged them. I sensed if I didn't address their calls for help and they became too frustrated, then they would easily give up. After a November observation, Sandy remarked, "I noticed that you gave some hints, but it seemed like if it weren't for the hints, students didn't have faith in their own efficacy to continue working on a problem."

Thus, the shared expectation that I would not reduce the difficulty of an assigned task depended on the nature of classroom work. If I asked the students to work independently or if they were in a group that was not focused on the task, it was common that I would immediately offer varying degrees of assistance. If students were collaboratively working on the assigned task, then it was more likely for me to hold off lending support.

By the end of the year, a visitor to my class would have observed the students seated at a single table engaged in spontaneous, meaningful dialogue regarding an assigned mathematical task. Sue believed that this was the best arrangement for the group because they were in close proximity to me and was able to imitate my behavior. When I asked why the same arrangement failed in August, Sue posited that, by May, the students liked me and I had earned their trust and respect. The non-

induced mathematical conversations that a visitor would hear would likely contain moments when the tenor of discourse would seem inappropriate. In an instant, a student might become visibly frustrated, and yell and sling insults at others.

In May, and in the absence of Keisha, I was able to effectively manage these outbursts and keep the lesson moving forward. If students reached an impasse in solving a problem, the students and I appeared to share the same expectation that I would not reduce the complexity of the task or immediately give help. Norms governing these expectations evolved throughout the year. Through the first ten weeks of school, students did not always work well when they were given the opportunity to sit together. In addition, it was frequently expected that I assist students by giving them an idea, working out a partial solution, or evaluating the correctness of a solution. The evolution of specific normative actions in the classroom is intertwined with other norms. Again, in Keisha's absence in May, students were seated together because they were willing to promptly engage in a task, and not use the setting as an opportunity to socialize. A combination of the physical arrangement, and a commitment toward problem solving, then made the important sharing of ideas a natural occurrence.

Norm: Students supported their thinking with mostly procedural mathematical explanations

The two norms discussed above (students were willing to attempt a problem and publicly share their ideas) are expectations for any subject classroom. A critical goal in my teaching was to instantiate sociomathematical norms guiding students' actions to specifically provide a mathematical rationale for their ideas and solutions. When

students asked me if a solution was correct, I frequently asserted that the correctness of solutions should come from the logic of a mathematical argument rather than from my evaluation. In a disagreement between Alan and Chris on August 29, I stated, "Here's what I want to happen. We're not going to decide if something is right or wrong by who can say it the loudest. I want you both to explain what you did and see whose explanation makes the most sense." At least 12 times during the first week of school, I harangued the group regarding the importance of providing an explanation for their work. As I did with Cedric in the May 23 class, regardless of it's correctness, I regularly asked students to explain how they arrived at a solution.

Shortly in the year, students, at least rhetorically, accepted an obligation to support their mathematical thinking. After checking Alan's solution to a problem in a class during the second week of school, he asked, "Don't you want to know how I got my answer?" Mid-way through the first quarter, I was impressed by the students' tendency to give explanations.

> There is definitely a shared expectation that, when talking to me, they will explain what they think they should do, or explain what they did to get an answer. I like that they even seem to be trying to explain why they don't understand something. Today, Keisha wasn't quite sure what to do, but instead of just saying 'I don't get it,' she basically described how she didn't know what the word profit meant. All year long I've been trying to emphasize how unhelpful it is to when someone simply states, "I don't know how to do this." [Personal Teaching Journal, October 3, 2006]

When a solution to a non-routine task was proposed, students generally would not accept answers from classmates without some evidentiary support. Early in the year, on September 8, Alan volunteered to come to the front of the room to show his solution to a problem. As he started to explain his reasoning, he apparently lost his train of thought, and only wrote a final numeric answer on the overhead. Jordan

laughed and responded, “He ain’t going to try to explain it.” Erika stated, “You got to explain how you got it.” A month later, on October 12, Sue found it interesting that Jamaal mocked Kyle when Kyle explained he found his answer by guessing. Suggesting that Jamaal was learning to value the notion of justifying answers, Sue stated, “You wouldn’t have seen [Jamaal] tease Kyle that way in the beginning of the year.” By spring, it was common for students to share the responsibility of asking their classmates to justify aspects of their thinking. In the May 23 class, both Jordan and myself asked Cedric questions about how he obtained his solution.

Although students were willing to supply a rationale for their mathematical ideas, they often provided a procedural description explaining how they arrived at an answer rather than a conceptual explanation explaining why that process was selected. For example, consider the problem below assigned to students on May 21.

> *Kyle works at Electronic City selling high-definition televisions. He makes a fixed amount of money each week plus commission on each television sold. The table below shows the relationship between the number of televisions sold in a week and Kyle’s weekly salary. Determine an equation showing this relationship.*

Number of Televisions Sold in a Week	**Kyle’s Weekly Salary**
5	$300
8	$420
11	$540
14	$660

By entering equations into the graphing calculator, students were able to monitor their own solution process. Jamaal was the first to show me a table on his calculator proving he had the correct equation, $y = 100 + x \cdot 40$. When asked how he found the numbers 100 and 40, he explained, “I saw that Kyle’s pay went up $140 [sic]. I broke that into three parts, and I found 40, 40, and 40 was 120, and then did 300 minus 40,

minus 40, minus 40, five times to find what it would be for zero." When I pressed Jamaal to explain why he performed the arithmetic calculations in that way, it was clear that Jamaal possessed a relational understanding of slope and *y*-intercept for this problem. When Erika became frustrated because she could not obtain a correct solution, I said to Jamaal, "Go see if you can help Erika, but don't just tell her the answer, explain where your answer came from." Although Jamaal possessed a conceptual understanding of the problem, his help to Erika consisted of providing a step-by-step procedural description.

My expectation was for students to justify their solutions with a mathematically rigorous explanation. In large part, I wanted students to evaluate the legitimacy of a solution strategy by analyzing the strength of their explanation. Such a norm never materialized. The students did not appear to frame the intent of mathematical explanations the way I had envisioned. Although they would balk at a student if no support were provided for a solution, very rarely did students raise concerns regarding someone's explanations, or even ask clarifying questions. This suggests that, instead of explanations being used to provide significant learning opportunities, the students viewed them as a ritualized part of discourse.

Early in the year, I tried to make explicit what constituted an appropriate mathematical argument. For example, on August 30, I attempted to clarify that a mathematical explanation must include a rationale for why certain decisions are made, and not just a procedural summary. During a whole-class discussion, the episode below, illustrates my attempts to negotiate this expectation.

From a second floor flight of stairs, Cedric walks up 10 stairs and Keisha down 4 stairs, how many steps separate Cedric and Keisha?

Chris:	I got six.
RH:	Because?
Chris:	Because I subtracted ten and four.
RH:	Why did you subtract?
Chris:	I minused them.
RH:	I know this is the hard part. You might think this is a lot, but I want you to try to explain everything you can about the problem. Like Chris gave me his answer, six, that's good; and then he told me what he did to get the answer, he said he subtracted ten and four, and that's good; but what I still want him, or anyone, to explain to me is, like, why did you subtract ten and four, and not multiply, add, or divide them. I'm interested in why you did something, and not in your final answer. In fact, a lot of times, I won't even care what your final answer is, as long as your explanation is good. So, who can tell me why Chris subtracted ten and four?
Keisha:	For what one?
	[*Erika and Cedric do not seem to be paying attention*]
RH:	This one, for your and Cedric's problem with the stairs?
Alan:	Oh, I got 14.
Jordan:	Yeah, yeah, yeah.
RH:	Why did you get 14?
Alan:	I added ten and four.
RH:	Great, this is my point, why did you add and not subtract? Jamaal what do you think?
Jamaal:	I already got my answer.
Keisha:	Can I do the next one?
	[*Several students start waving their hands, and shouting out that they want to do the next problem*]
Rick:	What did I say about patience? We have lots of time, and we don't have to rush through these. We need patience, patience, patience. I don't care about the final answer. I care about what you think about to get the answer. For this one, I still don't understand if we should subtract or add.
Jamaal:	Which one's right?
RH:	That's exactly why I want to hear your explanation. I want to take whatever one has the better explanation.
Jamaal:	You got to tell me if it's right or wrong. We don't know which one [is right] unless you tell us.
RH:	That's not true. If I asked you what two plus two is, and you say four, do I need to tell you it's right?
Jamaal:	No, but that's not the same, we already know that.
	[*Instead of pressing for an explanation, I polled each student to seek a consensus for which answer was correct. The group agrees that 14 is the correct answer.*]

RH:	But, Chris do you still think six is the right answer.
Chris:	I see what I did wrong; It should be 14.
RH:	What did you do wrong?
Chris:	I subtracted instead of added.

Clearly, I was not able to elicit a suitable mathematical explanation. The students noticeably seem to place a greater emphasis on the final solution rather than the reasoning supporting it. Referring across all her observations, Sandy stated, "I could tell that you wanted them to feel that the process was more important than the final answer, but that was something that they were resistant to. . . . [The] students held onto the idea that a right answer was something that was highly valued."

It often appeared that students would not attend to their classmate's explanations. For example, consider an episode from September 13, where students were discussing their solution for the following task:

> *In a recent basketball game against the Wizards, Lebron James made eight free throws, nine two-point baskets, and two three-point baskets. How many points did Lebron score?*

Chris:	I added his nine two-pointers and got 18, and then two three-pointers and got six, and added eight to that and got 28.
	[*As Chris is talking, I try to, verbatim, write out his explanation on the board*]
Erika:	That's wrong.
RH:	Can you look at what he did? Do you have different number than he did?
Erika:	I got 32.
RH:	How did you get 32?
Erika:	I added 8, 18, and 6.
	[*Verbatim, I record Erika's words on the board.*]
RH:	O.k., so we have Chris's work here and Erika's here and they have different answers. Can you either find a mistake in one or both of these, or can you tell me which one you agree with?
Jamaal:	[*To Erika*] What did you get?
Erika:	32.
Jamaal:	Alright then.

Typical of many whole-group discussions, students did not compare or contrast Chris and Erika's work. Sandy observed that comparing ideas was challenging for the students: "They had a hard time resolving if one person had something and somebody had something different, . . . and even if two people solved a problem a different way, you couldn't really be sure because they had a hard time expressing themselves." Sue posited that the students did not have interest in listening to each other's solutions. She stated, "They were not necessarily selfish, but they were self-centered and when it got so it didn't apply to them in the scheme of things, they weren't too interested in listening to someone else." My perception is that the students and I did not share the same expectations regarding their roles as active participants. I made numerous attempts to make my goals explicit to the class during the first several weeks. On September 20, I took advantage of a positive moment of classroom interaction to state, "This was great; Erika gave an answer, Keisha asked her to explain it, Cedric thought he had a question, but realized he didn't. If we can do that every time, we will have a great class. But the problem is we start breaking down, lose our focus, lose our patience, and don't listen to each other."

Mid-way through the first quarter, due to a perceived lack of progress in comparing and contrasting solution strategies, I made a conscious decision to move away from formal whole-group discussions. After a class on October 18, I made the following journal entry:

> Whole-class discussions have not been productive uses of our time. The students are too enthusiastic about giving their answer. Alan, Jordan, Erika, and Keisha constantly want to get up to show their answer. They only want to give their way, and not listen to others. There is no reflection on different solutions. When a wrong answer is given . . . ideally we would try to find the flaw in the reasoning and not be interested in showing how "I did it." I'm

> very frustrated in how they behave during whole-group discussions. I would like them to be more focused. I would like them to listen to each other more. Instead of trying to facilitate whole-group discussions, I think I'm going to selectively ask them to explain their solutions to different individuals in the class. [Personal Teaching Journal, October, 18, 2006]

The notion of whole-group discussions shifted during the year. Instead of eliciting and recording student ideas at the board, I took advantage of the class size and would have moments in the class where we convened at a single table to discuss key ideas. By May, we gathered at a single table for an entire period. Once students arrived at solutions, I would direct a discussion of student work by asking students to describe their solution strategies to another student who either needed help or solved it a different way.

In May, the students regularly asked classmates how they found a solution. It was normative that students would be accountable for explaining and justifying their thinking. It seemed accepted that as long as students gave some support for a solution, then that was the limit of their obligation. Explanations were largely procedural summaries of student work, and there was rarely any follow-up regarding an explanation.

Part II: Temporary suspension from task

As I resume with the May 23 class session, it is evident that the students were thinking of the two variables as separate sequences, rather than the functional correspondence between them. I did not anticipate students thinking in this way. In prior class sessions, students had solved similar tasks by finding a single equation showing the relationship between the two variables.

I realized that the task was not at the correct level of difficulty. The students were genuinely willing to solve the problem, but their efforts were not resulting in significant progress. It is essential that tasks be structured so they present an appropriate level of challenge. A task must allow for a degree of success given appropriate effort by the students. Students should be encouraged to attribute their successes to a combination of ability and effort, and not be given cause to believe that their failures are due to lack of ability. With that philosophy in mind, I decided to temporarily suspend work on the task, and spontaneously think of new problems.

RH:	Let's start off with an easier problem first.
Jordan:	No, we can do this, we just need a little push.
Jamaal:	Jordan can't get it, he couldn't even get the one from yesterday. How Jordan going to get this, if he couldn't even get the question from yesterday.
RH:	Lets start off with one we can do. And we will get to the class work later. Can you give me [a linear equation] for this one?
	[*I write a table on the board with the following points: (0,2), (1,5), (2,8), (3, 11)*]
Erika:	I know how to do that
RH:	Let's do that then.
Jamaal:	It start out at two, and going up by three.
Erika:	It going up three.
Jamaal:	That's what I said.
Erika:	Shut up, I said it before you. [*Jamaal stands up and makes motion towards Erika*]
RH:	Sit down Jamaal. I can't have you start something completely irrelevant when we are in the middle of something. How are you going to make an equation?
Cedric:	Jamaal got it right.
RH:	So what's the equation?
Cedric:	Two plus three *x*.
Jordan:	I did do that. I swear to god on my life, I did that by myself.
RH:	How about this one?
	[*I write a table on the board with the following points: (2, 7), (4, 9)*]
Erika:	I want to do it. Two. It going up by two. The first one is, ain't eight going to be zero?
RH:	Try it.
	[*Nearly two minutes of individual, silent working*]

Erika:	It's going up one.
Cedric:	No it's not.
Erika:	Yes it is, if you put those numbers in between them, it's going up by one. So its, 7, 8, 9, 10 and 4, 5, 6. It's going up one right?
RH:	What do you think? Why is Erika saying it's going up by one?
Erika:	Because you put the numbers in between in.
Jordan:	What's the equation?
RH:	We still don't have the equation. We have the number that it goes up by, what have we called that?
Jordan:	The rate of change.
RH:	We sometimes call it the slope too.
Cedric:	Is it going to be one plus one *x*?
Kyle:	It starts at zero. Starts at one.
RH:	Does that work?
Cedric:	Look how close I am.
Erika:	I know it. Look.
Kyle:	Got it.
Jordan:	What is it?
Cedric:	Don't tell me, I don't want to know.
RH:	Have you figured out where it starts?
Cedric:	I got it. Five.
RH:	Does that work? [Students: 'yeah'] Does anybody know why is starts at five?
Jordan:	If you subtract nine minus seven, that's two; seven minus two is five.
Kyle:	It's a pattern.
Jordan:	Put another up.
RH:	Hold up for a minute, how did Cedric or Erika figure it starts at five.
Cedric:	I tried seven plus one *x*, and then kept going down.
RH:	And Erika; [Erika: 'what']; How did you figure out it starts at five?
Erika:	I don't know, it went five, six, seven, eight; hold up, I don't know, like it has to start at five and then go six, seven, eight.

Norm: Students persisted in solving non-routine tasks

A multiple solution norm cannot exist in an environment where students surrender their pursuit of a challenging problem. The construct of a multiple solution norm is predicated on the assumption that, when faced with a non-routine mathematical task, students will apply their prior knowledge in ways that result in unique solution

strategies. A task may be considered non-routine if an algorithmic process for obtaining a solution has not been well developed. In many mathematics classes, the task for finding a linear equation is reduced to following a formulaic procedure for finding the slope and *y*-intercept for the line. Analysis of the transcript from the class on May 23 illustrates my students had not constructed such a prescribed routine. Any success of the lesson from that day was dependent on my students' motivation to, not only to show an initial interest in the task, but also their willingness to put forth effort in the problem, and develop strategies for obtaining its solution.

A comparison of classes in May with those from the first quarter reveals that students' displayed a greater commitment towards problem solving by the end of the year. In contrast to the first 45 classes, the group, in May, was willing to work longer on problems, be more resilient when challenged, and displayed a greater sense in their own efficacy. In the May 23 class, the students clearly struggled with the initial task of finding a linear equation showing the relationship between the number of paper clips and the mass of the paper clips in the box, yet they invested over five minutes trying to figure out a solution. Although Jamaal publicly announced that he couldn't get it, he also resisted the notion that he had given up. Jordan did not want to suspend working on the task and refused to acknowledge that the problem was too hard. Later, Cedric did not want an answer revealed until he had solved it on his own. There was a general enthusiasm and willingness to attempt additional related problems.

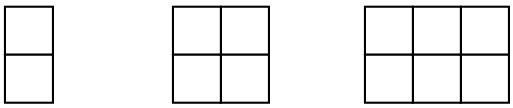
The nature of my students' effort towards solving a non-routine task shifted significantly over the course of the year. The level of intrinsic persistence toward

problem solving illustrated in the May 23 class was not evident during the first quarter. Even in one of my most productive lessons during the first ten weeks, students seemed to need a great amount of scaffolding and individual attention to prevent them from quitting work on a challenging task. An example of such a case occurred on September 18. This was one of the 11 coded classes from the first quarter in which I perceived that the students were predominantly on-task. In my journal for that day, I wrote:

> Overall, one of my better classes so far. I'm definitely taking advantage of the small size. It resembled an after school help session where Sue and I went around helping students individually. Once they understood what the problem was asking for, they appeared somewhat interested in finding an answer. It wasn't as hectic as usual, and they were working towards solving the problem! . . . I was pleased by how they were thinking of the problem. Kyle and Cedric drew out to the tenth pattern; Jordan and Alan counted up by two's; Chris multiplied two and ten; Sue sat beside Erika for most of the class, and was helping Jamaal; Sue commended Erika for her work. Keisha had the right answer, but didn't want to tell me how she did the problem. She said she doesn't like it when teachers ask her questions until she understands. I don't think she understood what the problem was asking, and just wrote down the number she heard Jordan say. [Personal Teaching Journal, September 18, 2006]

The objective for the lesson was for students to recognize, describe, and extend patterns and functional relationships. Unlike the May 23 class where everyone is seated around a single table, the physical arrangement of the students has them isolated from one another. Although the seating arrangement was made in an attempt to reduce off-task chatter, the students would frequently, and without hesitation, speak out. As class begins, Sue, who is observing for the day, and I circulate throughout the room. The videotape captures several instances in which we attempt to redirect off-task behavior, ask students to sit down, and make repeated requests for students to begin their work. The task presented to the students was as follows:

Square blocks are arranged in the pattern shown below:



1st Pattern 2nd Pattern 3rd Pattern

If the pattern continues, how many squares will be in the 10th pattern? Explain how you arrived at your solution. Use words, symbols, or both in your explanation.

As students start working on the problem, it is apparent that the task is not readily accessible to most of them. The transcript below captures the first few moments of class after students have taken their seats and begun to read the task:

Jordan:	I don't get what it's asking.
Alan:	I don't get this.
RH:	Tell me what you don't get.
Alan:	Like I don't get it. I don't get what they're saying.
RH:	Read the problem for me.
	[*Alan reads problem*]
Erika:	This is dumb, I'm not doing it.
Jamaal:	Stupid right, how you suppose [inaudible]?
RH:	[To the whole class] One of the key words in this problem is pattern. What does a pattern mean to you?
	[*The group is highly inattentive and most are engaging in off-task chatter*]
RH:	Can everyone try to focus on the problem please? I know it seems hard but if we think about it, I think we can get it. Basically there is a pattern here. Here is the first one, the second one, the third one; we're missing the fourth one but we want to find out what the tenth one would be.
Keisha:	My whole thing is I don't know how to write out the problem.
Cedric:	This is hard.
RH:	We have to keep in mind we are looking for the number of squares.
Jamaal:	Two, four, six, eight, ten, twelve. It will be 12? I'm not sure, but I think it would.
RH:	Part of this says to explain your answer.
Jamaal:	I can't explain it. Is it right?
RH:	Let me hear your explanation to see if it makes sense.
Jamaal:	I don't know how I got it.
Alan:	I don't get the pattern.
Jordan:	Two, four, six, eight, ten, twelve, fourteen,

Jamaal:	Why you still going?
Jordan:	It says go up to ten.
Jamaal:	Exactly ten.
Jordan:	Ten, ten, ten, ten!
Keisha:	I don't know how to do this.

As the class ensues, Sue and I continue to help students make sense of the task, and provide them with ideas about how to proceed. Below is a transcript of my individual conversation with Keisha:

RH:	What ideas have you thought of?
Keisha:	That's the thing. I don't know how to even start to write out the problem.
RH:	Like I said, an important word here is pattern. What do you think that means?
Keisha:	Like, I don't know, it's something, I don't know. I don't know how to do this.
RH:	O.K., here are three figures that make a pattern, that means we can use these three to predict what the next one will look like. Ignore the question for a minute, can you tell me or draw out what the fourth pattern will look like?
	[*Keisha correctly draws a 2 by 4 grid*]
RH:	This looks good, how did you know to make it like this?
Keisha:	I just added these two onto that one.
RH:	Now, the question we're trying to answer is what will the tenth one look like, or at least how many squares is in the tenth one. Try this for a few minutes.
	[*After checking with several other students, I come back to Keisha several minutes later. Keisha still has the 2 by 4 grid she drew a few moments earlier and below it she has written the number 20*]
RH:	Keisha, can I see what you've done?
Keisha:	I don't know if it's right or not.
RH:	Well tell me how you counted the squares?
Keisha:	I don't like when teachers read my work unless we go over it and I understand.

In reference to other classes during the first quarter, the quality of this lesson was good. It was generally devoid of the maladaptive behaviors I frequently managed, the students were eventually willing to put forth effort in the problem, and they arrived at a range of solution strategies. However, if not for the explicit attention Sue and I

offered, I am confident that the students would have quickly given up. It was clear from the tenor of student discourse in the beginning of the lesson that they were frustrated with the task. Almost immediately, the students were coming to a consensus that the task was confusing, dumb, and unsolvable. Keisha's disposition toward the problem was disappointing. If a multiple solution norm had been constituted, she would have likely viewed my interest in her work as an opportunity to understand the material rather than an evaluation of her solution.

Generally, the degree in which students persisted in solving a non-routine problem in the beginning of the year was qualitatively different from how they persisted in solving non-routine problems at the end of the year. In the overall data corpus, there were 68 occurrences in which student behavior was coded as persevering and 243 occurrences in which student behavior was coded as non-persevering. In the August to November data set, there were there were 32 occurrences (12%) when student action was coded as persevering and 225 occurrences (88%) when student action was coded as non-persevering. In the six May classes, 36 times (67%) student action was coded as persevering and 18 times (33%) student action was coded as non-persevering. Table 3 summarizes the frequency of these student actions.

	Number of Persevering Occurrences	Number of Non-Persevering Occurrences	Total
August - November	32	225	257
May	36	18	54
Total	68	243	311

Table 3: Frequency of persevering and non-persevering student action in August - November and May

Table 4 illustrates the nature of student talk representative of the persevering and non-persevering behavior observed through the year.

Student Talk Representative of Persevering Behavior	Student Talk Representative of Non-Persevering Behavior
Can somebody help me?	This is dumb/stupid.
I can do this.	I don't get this.
What am I forgetting?	I don't understand.
It's not hard.	This is hard.
Let's do another one.	I'm not doing this.
I remember doing this before.	You never taught us this.

Table 4: Student comments illustrative of persevering and non-persevering behavior

The manner of student talk clearly changed, as did the students observable persistence with problems. Thus, the overwhelming shift in the data strongly supports the hypothesis that classroom norms evolved during the year in which students' became more persistent in attempting to solve non-routine mathematical tasks.

However, it should be cautioned that looking at the comments students made might not always be a reliable indicator of these positive classroom norms. The possibility exists that the students made or avoided the use of specific comments because they became increasingly knowledgeable of the type of talk I valued, not because of a change in how they viewed mathematics teaching and learning. For example, there were 22 occurrences in the data during August and September where a student referred to a task as stupid or dumb. In October, the number dropped to 4 times, and by May there were no such instances. It is reasonable to posit that this pattern of talk shifted because the students responded to my cues regarding what type of comments were acceptable, not because of a qualitative change in their beliefs or attitudes. Both Sandy and Sue commented, on more than one occasion, that they believed the students were interested in pleasing me and seemed to mimic how I talked about math.

Additionally, just because a student publicly expressed frustration and avoidance with a problem, doesn't mean the student was not interested or was not willing to invest energy to finding a solution. In a lesson early in the year on representing equivalent fractions, Erika twice made comments that she didn't understand how to solve the problem and wasn't going to do it. However, when pressed to think about the task, she offered valuable contributions to the class. In my journal for that day, I noted:

> This seems pretty interesting -- Erika said twice she didn't know how to do it, yet she was a pivotal figure in the discussion. How can you determine what a student really means when he/she says I don't know how to do it? When does that have literal meaning versus when is a student trying to avoid doing some work? I think Erika took confidence in the fact that no one else was certain of the problem, and so she wasn't afraid of sharing her ideas. [Personal Teaching Journal, September 7, 2006].

Further, students expressing a determination to find a solution could have superficially been exhibiting a positive demeanor toward problem solving. In a class on May 21, Jordan stubbornly held on to a desire to solve a problem regarding finding the equation of a liner relationship. After making an assessment that he was not on a trajectory that would result in a successful solution, I asked Cedric and Jamaal, who had successfully solved the problem, to offer a strategy for Jordan to consider. Jordan refused to listen to any advice. He comically replied, "No, no, no. I say mind your business and stay in school. I can get this." Although Jordan's effort was initially commendable, his dogged insistence that he could solve the problem detracted from his opportunity to learn.

Ultimately, the way in which the class, as a whole, responded to a challenging problem undeniably changed. In the September 18 lesson, the students mutually

supported and influenced each other in their denouncement of the task. For approximately the first four weeks, this type of herd mentality appeared to be the norm. The students bonded together in either their resistance or acceptance of the mathematical work I asked them to do. The renegotiation of this norm was explicit in an episode just a few class periods later. Below is a moment on September 28 when, for the first time, students expressed a strong interest in solving a challenging problem void of any teacher assistance:

Keisha:	Could you just tell us?
Jordan:	No, let us figure it out.
Keisha:	Could you just tell us so we could know?
Cedric:	No, don't say anything.
Keisha:	Shut up, how we going to do this if he don't help us.
RH:	I could tell you, but let's take another minute, like Jordan said, and try to figure it out, because you're real close.
Keisha:	I'm not doing it, I don't get it.

I greatly underestimated the challenge of instilling in my students the notion that effort and persistence was an essential component to problem solving. Of the many changes that occurred throughout the year, the progress this group made in accepting the challenge of solving a novel task was extremely satisfying. Sue observed that "more than teaching them math, which you did, you gave them the confidence to try and not be afraid to fail." In talking about Jamaal, Sue stated:

> Jamaal has come a long way. I observed him a lot in seventh grade class. Jamaal didn't want to try in seventh grade. Being in the accelerated class with you, he tried, if it didn't work out well he tried again, and he had more than one way to solve a problem. He was very good with a calculator, and I think that helped him. [Conversation with Sue Pope, June, 10, 2007]

Norm: Students looked for teacher to verify correctness of answers

An analysis of the transcript data provides convincing evidence that my students expected that I should be the primary source of knowledge and valuator of solutions.

On the third day of school, the class was confronted with the challenge of determining the legitimacy of two explanations that resulted in different solutions. In an ensuing discussion, I asked Jamaal his opinion on one of the explanations. Jamaal stated, "You got to tell me if it's right or wrong. We don't know which one [is right] unless you tell us." Such beliefs posed a challenge to my vision of classroom activity. My goal to create a multiple solution norm included a negotiation of expectations so that students would share the responsibility for adjudicating the correctness of solutions.

On numerous occasions during the first two weeks, I attempted to deflect student inquiries regarding the correctness of an answer by placing emphasis on the process that was used to obtain the solution. For example, on August 31, I responded to Erika's request to evaluate the correctness of her solution by stating, "I don't want to tell you if you're right or wrong right away. We will find out if it's right or not, but I want to hear how you found your answer and see if your explanation makes sense." I knew it was important to downplay the value the students attached to correct solutions, and to reduce the stigmatization of incorrect answers. Also on August 31, I sermonized, "I don't necessarily care about right or wrong answers. In fact, I sometimes would rather someone make a mistake, because that is how I believe we can learn best, by correcting the mistakes we make."

By mid-September, there were moments when students appeared to value the notion that the process was as important as the final answer. For example, on September 14, Jordan found the correct final answer for the problem below.

> *Cedric shared 28 baseball cards with his friends. Six of his friends received exactly one card each, five of his friends received exactly two cards, and the*

rest of his friends received exactly three cards. How many friends received exactly three cards? Explain how you determined your answer.

After listening to some of my conversation with Keisha where we used index cards as a concrete manipulative to model the task, Jordan said, "Mr. Hollenbeck, I got the same answer, but I didn't do nothing like Keisha and you." Jordan based his correct answer of four friends because "these are going down and the cards keep going up." I needed Jordan to repeat his rationale multiple times before I realized he was continuing an apparent pattern of six friends get one card, five friends get two cards, and so four friends get three cards. After pointing out to Jordan that he never used the 28 cards in the problem and that his rationale would apply even if Cedric started of with any number of cards, Jordan asked, "So I wasn't really doing it right?"

Overwhelmingly, most students placed a high emphasis on final results and gave less attention to solution strategies. This belief remained resilient from August to May. In an occurrence with Alan similar to above, Alan held on to the notion that, since the net result of his answer was correct, then it did not matter he had applied a faulty strategy. Sandy observed that the students could not get beyond their belief that obtaining a correct answer was a highly valued commodity, regardless of what process led to that answer. In addition, the students prized obtaining solutions quickly. They often competed with one another over who would be the first to solve a problem.

Throughout the year, it was important for students to obtain my evaluation of their final answer. In the May 23 class, even when students had used a graphing calculator to check their own solution, they frequently insisted on showing me their calculator and would still ask, "Is this right?" When I asked Sue at the end of the year if she

thought that students were willing to share in the adjudication of solutions, she stated "I saw some evidence of it, but not as much as I'm sure you hoped it would be." Sandy observed, "The students clearly valued getting a correct final answer and it was important for them for you to validate that correctness."

I frequently pleaded with the students not to look at me as an arbiter of correct solutions. However, in an examination of the data from the first quarter, my actions were extremely inconsistent when confronted with a student inquiry regarding the correctness of a solution. On average, I was asked to evaluate a solution four times per class. As shown in Table 5, over a quarter of those times, I gave an absolute assessment of the student answer.

Number of times an absolute assessment of student answer was given.	52	28%
Number of times student was asked for an explanation.	74	40%
Number of times student was asked to compare solution with a classmate.	61	33%
Total	187	

Table 5: August to November responses to student inquiries regarding the correctness of a solution

Although I verbalized the importance for students to check the validity of an answer by examining their solution strategy and not to rely on me as the sole authority for adjudicating the correctness of solutions, I still offered students a definite assessment of their answers on a number of occasions.

Overall, obtaining an accurate final answer was the coin of the realm in problem solving. In May, an observer to my class would witness unsolicited student competition regarding who could be the first to solve a problem. After solving a mathematical task, students frequently demonstrated a desire to show me their

solution and ask if it was right. Even if it was clear that a correct answer had been obtained, the students sought out my assessment as a necessary step in the solution process.

Norm: Students used graphing calculators to self-monitor their own solution progress

In a classroom with a multiple solution norm, students should be able to respond to detailed questions about their solution strategy as well as juxtapose their solution method with others. For the most part, these were activities that my students were not skilled at. Sandy conjectured that the students in my class may not have had an explicit enough knowledge of their own solutions to talk about them with any type of sophistication, and "the only thing that could clue them in that something was different was if they got two different answers."

It did appear that my students' ability to effectively apply their knowledge was very situated and context dependent. In a May 17 class, Jordan presented a suitable explanation for the following task:

> *Jonathan weighed 90 pounds on September 1st. Eight months later, on May 1st, Jonathan weighed 106 pounds. How much weight did Jonathan gain each month. For each month, assume that Jonathan gained the same amount of weight.*

A few moments after reading the problem, the following discourse took place

Jordan:	[*Using his fingers to count up from 90 to 106*] He gained 16 pounds for eight months, and so eight times two is 16.
RH:	So how many pounds did Jonathan gain each time?
Jordan:	Uh, eight pounds.
RH:	He gained eight pounds each month? He started at 90 and then went to
Jordan:	92. No two pounds.
RH:	Jordan thinks it might be two pounds.
Jamaal:	I got 16, then I divided by eight and got two.
RH:	So, Jordan, you realized eight times two got you 16, and so you

	knew he had to gain two pounds every month; and Jamaal, you did something a little different; you did 16 divided by eight to get two pounds?
Jordan:	So I could have just divided? [RH: yes] For Real?

The very next class, on May 21, Jordan struggled with the following isomorphic problem:

Use a pattern to fill in the missing numbers in the table below:

0	1	2	3	4	5	6	7	8
90								106

After Jordan asked for "a push", I suggested, "What if I told you this was 90 pounds and eight months later, this was 106 pounds." Jordan replied, "Uhh Hahhh, This is like yesterday. I gotch ya. . . . Hold up. Hold up. I know we had to do something with the eight months. What do I got to do? Divide 106 by eight?"

When working on a non-routine task, students need the ability to monitor and evaluate their own thinking. Creating a classroom climate predicated on student ideas and discussion of those ideas presupposes that students have an intimate knowledge of their individual solution strategies. Students appeared to use the calculator to consciously reflect on and revise their own solution methods.

Incorporating graphing calculators as a normative component of problem solving allowed the students a greater opportunity to monitor their own solution progress. In the May 23 lesson, students were checking and revising their solutions by creating and displaying a table of values for a given function. Several times, I responded to student questions regarding the validity of their solutions by directing them to check

their own answer. By May, Sue saw the prevalent[3] use of graphing calculators as "the primary means of solving problems." Sue believed that the graphing calculator allowed students to gain access to mathematics beyond their level of computational skill. Sandy observed that the students' lack of basic skills knowledge often limited their ability to successfully find solutions to problems, and the use calculators to perform tedious arithmetic calculations positively impacted their motivation to work on problems.

Obviously, there were times in the curriculum where the use of a graphing calculator was more beneficial than others. The graphing calculator was particularly useful in developing student understanding of the relationships between the graphical, numerical, and symbolic representations of linear function. Students were capable of entering equations into a graphing calculator to produce graphs and table. The capability to change the parameters of an equation and quickly observe the corresponding effects on a graph or a table of values gave the students opportunities to reflect on and revise their strategies for finding desired linear equations.

Part III: Scaffolding to build knowledge of slope and y-intercept

Reducing the cognitive demand of a task by adjusting the quantities or changing the context of a problem so to make it more accessible were typical strategies I used when students struggled with an assigned task. Once a problem was modified in such a way that allowed students to use their prior knowledge and problem solving skill to obtain a reasonable solution, I typically adjusted the quantities or context again in

[3] The county's local assessment for the first quarter prohibited calculator use. Thus, in preparation for the county's assessment, graphing calculators were not regularly used from August to November.

order to make it more challenging. This way my typical strategy for scaffolding. In the last part, I asked the students to find the equation of a line containing the points (0, 2), (1, 5), (2, 8), and (3, 11) – a relationship where it was clear where the relationship "started at" and "what it goes by." Then, I asked them to find the equation of the line containing the points (2, 7) and (4, 9), and assessed that Erika, at least, found the slope by filling in "the numbers in between."

Continuing with the May 23 transcript, I'm concerned that the students have still not constructed a reliable method for finding the *y*-intercept, and it's not entirely clear they understand how to find the slope if the independent variable is not in unit increments. To further assess my students' knowledge and provide them additional opportunities to develop understanding of these concepts, I present another example that is intended to push them to think of a reliable way to find the rate of change and *y*-intercept.

RH:	O.k., let's try another one.
	[*I write a table on the board with the following points: (3, 20), (5, 24)*]
Kyle:	Starts at three, and going up four. Hold up, what did we say why [the previous problem] it start at five?
	[*Erika and Jordan shout over each other trying to answer Kyle's question*]
Erika:	It goes up by four, ain't that right, if you fill in three, four, five, six, seven.
RH:	O.k., let me add three, four, five, six, and seven [to the table] what will the other [*y*-values] be.
Britney:	Go by one.
RH:	So 20, 21, 22?
Erika:	No, it won't be that. Sixteen. Two would be sixteen.
Jordan:	Why?
Erika:	No it wouldn't. Ain't that right? Hold up. Yes it is, two will be 16.
Jordan:	What was I doing the other day? When I said subtract the number.
RH:	Is two 16?
Erika:	Yes
Jordan:	No it's not.

Erika:	Yes it is.
Jordan:	Two plus 16 *x*?
Erika:	No, on the table.
Jamaal:	One would be 12?
Erika:	Yeah, one would be 12.
RH:	More details, why?
Jamaal:	We need zero.
Erika:	Nine.
Jamaal:	No, it will start at eight. That's what it is.
Kyle:	[*Showing me calculator*] Like this.
RH:	Close, but we want this to be three, five, seven, nine.
Jordan:	Close, close.
Cedric:	Almost got it [*shows Jamaal calculator*]; Jamaal, that's what I got.
Jamaal:	Hold up, let me do something real quick.
Kyle:	I got it, got it, got it.
RH:	No you want to see (5, 24).
Jordan:	Hah!
RH:	Get back to Erika's idea. If you filled in every number, what would it have to go by?
Jamaal:	Two, two you idiots.
RH:	Erika, you're not trying this, are you?
Erika:	Cause I don't know.
Cedric:	I got it! Starts, at two, no it starts at 14 and goes by two.
Jamaal:	Where did you get that from?
	[*Cedric, Jordan, Kyle are out of seats and quickly off-task*]
RH:	Unless we can figure out where these numbers are coming from, we're going to have the same problem doing these equations every time. We might get them, but we have to figure out a better way to figure out where it starts, and what it goes by.
Jordan:	I thought my way was getting it. Like I do 24 minus 20, get four, and that's, what's that?
RH:	So Jordan, it went from 20 to 24, up four, in how many steps?
Jordan:	Two.
RH:	So what do we have to do with that four?
Jamaal:	Divide it, divide it; [RH: by?]; by two.
Jordan:	For real? Put another one up there, I want to use that.
Erika:	Why did you divide by two?
RH:	Because it went up four but it skipped that one. Just like what you did in yours, you had to fill in that number in between.
Erika:	Where did you get the 14 from? Oh, that's where it starts at.
RH:	How do you know that?
Erika:	If you go back, you get 18.
Cedric:	You have to start at zero, Erika.
Jamaal:	It's going down by two's.
Cedric:	Look, Erika.

Erika:	Shut up.
RH:	No, what were you going to say Cedric?
Erika:	No, I get it.
RH:	No, Erika, I want to listen, Cedric, what were you going to say?
Cedric:	I started off with 20, and then went back to the negative numbers and kept trying those, until I got something that worked.

Multiple Solution Norm

My vision of a mathematics classroom governed by a multiple solution norm would see students struggling to solve a given mathematical task, participating in discussions about how to solve them, and analyzing the relative advantages of different methods and the connections between them. When given time to work on a novel task, students would routinely accept the challenge of trying to find an appropriate solution. Through perseverance and cooperative work, students would arrive at a range of tested solution strategies, which they could support by articulating a mathematical argument. In a public presentation of solution methods, students would intently listen to one another and strive to understand the rationale of a mathematical explanation. During this process, a teacher would attend to the mathematics being brought forth and continually monitor student actions and behavior to assess how well students were attending to and appropriating the explanations of others. Students would evaluate the strengths and limitations of solution methods and construct connections and understand differences of various strategies.

In reflecting on the course, Sandy stated, "It seems like your expectations for the class were quite ambitious." The data presented here makes clear that this vision of a multiple solution norm was not realized in my classroom. A significant concern for

the first part of the year regarded the nature of the students' behavior and their apparent unwillingness to put forth effort. Additionally, the students and I struggled when attempting to facilitate the sharing of ideas in meaningful ways. My immense difficulty early in the year to manage student behavior during whole-group discourse guided the eventual constitution of classroom norms that eliminated whole-group discussions altogether. Students were obligated to explain their reasoning to individual classmates; however, they had difficulty communicating and appropriating each other's ideas. Too frequently, students resorted to shouting or yelling and made inappropriate comments to one another. The students highly valued final answers and commonly viewed problem solving as a competition.

Yet, the data also indicates that, by the end of the year, several critical components of a multiple solution norm were realized. Students appeared to hold the expectation that there was not a single way for a solution to be found. They were willing to share their ideas, and at least procedurally, explain the steps they took to solve a problem. During the unit on linear functions, the students displayed skill at using a graphing calculator to help monitor their own solution progress and try alternate strategies if they were not able to obtain a desired solution. Sue noted, "They were not afraid to make mistakes," and they took great satisfaction in correctly solving a difficult task. Perhaps most importantly, students were willing to work on a non-routine task that was not readily solvable. Unlike the beginning of the year, by May, students were generally on-task and non-academic behaviors were easily managed. In the next section, I discuss factors that influenced the constitution of these norms.

Instantiating a Productive Disposition

In my experiences instantiating a multiple solution norm, I have found that the beginning of the school year is a critical period for establishing management related routines and norms for social and mathematical behavior. My prior experience with eighth grade Algebra and Geometry classes is that the first few days of school are a time when students are particularly attentive and compliant as they enter their new environment with a mix of excitement and apprehension. The first day with the students is an opportune time to clearly communicate expectations and goals for the year. During this important class, I attempt to identify a non-routine task whose solution is accessible to the students in multiple ways. As students work on the problem, I explicitly explain what I am expecting from them, and offer guidance regarding how they should be working with other students they are grouped with. Similarly, in whole class discussion of solution methods, I clearly point out my expectations for student behavior. All the while, I make a conscious effort to have a strong affective focus and be sensitive to students who are uncomfortable with their role in the class. I carefully and continually monitor the classroom environment and promptly recognize occurrences that are consistent with my goals for social and mathematical behavior, and purposefully address any incidents that are undesirable.

Unfortunately, on the first day of the pre-Algebra class, I was not afforded the luxury of working with quiet, attentive, and compliant students. Part of my journal entry from this first day addressed the uniqueness of the group:

> As the students started to enter the room, it was obvious that they were not like my other classes. They clearly were not intimidated by my presence at all. They were talkative and loud, they complained about my seating arrangements, and I had to plea for their attention. As I tried to explain my expectations and goals for

> the year, I was interrupted multiple times by their off-task chatter. [Personal Journal Entry, August 28, 2006]

The task I had planned for this first day was:

> *Find a four digit number so that:*
> 1. *All four digits are different.*
> 2. *The number is even.*
> 3. *The sum of the digits is 20.*
> 4. *The hundreds digit is twice the ones digit.*

My frustration in trying to engage the students in meaningful mathematical activity is highlighted in my journal entry that day:

> When I asked them to come up with their four-digit number, Cedric and Erika were completely disengaged. Erika tried to withdraw from the group by sliding her chair several feet away from the table. When I asked her to join the group, she only moved an inch or two. The other students answered the problem in a matter of seconds [only Chris's number met all four criteria] and quickly started talking and joking around. Chris in particular, kept making jokes. When I asked if they could come up with a different four-digit number, they again answered the problem too quickly, with almost no reflection. Most of their numbers did not meet all the criteria. When I pointed this out, students pressed me to tell them if their number was correct. . . . When I tried to publicly compare their solutions, I couldn't keep the focus on one of their numbers long enough to have any meaningful discussion. Jamaal, Chris, Alan, and Jordan seemed interested in the problem, but they only wanted to discuss their number and kept pressing me to tell them if they were right. [Personal Journal Entry, August 28, 2006]

My construct of a multiple solution norm was predicated on having students demonstrate high levels of task engagement, effort, and persistence when challenged with a non-routine problem. Sadly, my students did not approach a mathematical task with a desire to significantly engage in the problem. Moreover, it was not uncommon for my students to actively avoid an academic task and engage in a range of maladaptive behaviors. The following quote from Holt (1982) captures my interpretation of how the pre-Algebra students thought of a mathematical task during the beginning of the school year.

> For children, the central business of school is not learning, whatever this vague word means; it is getting these daily tasks done, or at least out of the way, with a minimum of effort and unpleasantness. Each task is an end in itself. The children don't care how they dispose of it. If they can get it out of the way by doing it, they will do it; if experience has taught them that this does not work very well, they will turn to other means, illegitimate means that wholly defeat whatever purpose the task given may have had in mind (pp. 37 – 39).

Perhaps the most significant obstacle in establishing a classroom culture predicated on the cooperation of students to engage in meaningful academic work was overcoming the tendency of my students to abruptly lose focus on a mathematical task and become occupied in mathematically pointless chatter. After an observation in September, I asked Sandy what she viewed as some of my biggest challenges. Her response was, "The management of the students. You can tell they have a hard time staying seated for any length of time and they cannot avoid talking to each other." More than the quantity of off-task behavior, Sandy noted the challenge of creating an environment where the focus would remain on mathematics. After a November observation, Sandy stated, "Students had periods of working with periods of not working, and it is hard to maintain continuity across that."

As discussed in the previous section, I had significantly underestimated the challenge of simply maintaining on-task behavior and creating a culture where students would be willing to do mathematical work. By May, much progress had been made; students were willing to engage in a mathematical task and persist in solving non-routine problems; moments of unproductive behavior, although still present, were more easily managed and redirected. An examination of the data suggests that the following six interrelated factors were influential in shaping student behavior and my students' willingness to do mathematics: (1) creating a supportive

environment with strong teacher-student relationships; (2) selecting and/or designing an appropriate task; (3) instructional decisions such as encouraging student collaboration, promoting the use of calculators, valuing mistakes as learning opportunities, and use of scaffolding; (4) students' attitudes and beliefs; (5) external events both within and outside of school; and (6) dense network of student relationships.

Teacher-student relationships

Teacher-student relationships

In addressing classroom situations characterized by a lack of student discipline and an apparent apathy toward academics, a common refrain from my school's principal was, "Many students don't care to learn, unless they learn you care." Right away, I recognized the importance of building positive teacher-student relationships as an important component of classroom management to counteract unproductive patterns of student behavior. On August 30, after the third class meeting, I wrote in my journal, "I think by building strong relationships, their willingness to do some of the things I'm asking will improve." In some ways, it seems trivial suggesting that a constructive teacher-student relationship can positively impact a student's willingness to engage in a mathematics problem and persist when challenged. Nonetheless, unlike my other classes, the successful building of relationships with my pre-Algebra students was a necessary condition in my efforts to constitute an environment where, minimally, students would not partake in disruptive, off-task behaviors. At the end of the first week of school, I believed I needed to approach the pre-Algebra students differently than students in my other classes:

> I know building relationships is key, but Carol [School's Psychologist] only echoed that today. I want them to see me more than just a teacher. I suspended some of the mathematical work for the day in order to answer their questions about my age, background, and family. I shared pictures of my wife and daughter, and Keisha and Erika seemed real interested in these. I also asked them questions about their families and what some of their interests are. This was a good investment in time, as they seemed to focus a bit better today. These are things that I haven't done with my other classes. In my Algebra and Geometry classes, it seems like the students, as a whole, have intrinsic respect for me. As long as I prevent any feelings of negativity toward the class, or myself then these students should continue to respond in positive ways. But for this class, it is more difficult than that. Not only do they show less respect for me, there seems to be some distrust. I need to earn their respect and confidence. I think it will be important for them to like me. [Personal Journal Entry, August 31, 2006]

Because I felt it was important for students to like me, I was reluctant to discipline students; however, due to the pervasiveness of their off-task conduct, I ultimately felt compelled to follow some of the traditional consequences for disruptive behavior. After growing frustrated by my students' lack of attention during a class on the second day of school, I warned them about behaviors that would receive infractions:

> I know you are all aware that we have these things called infractions, and that there are consequences for getting these. You will be assigned lunch detention, we will have a conference with your parents, and if you get, more than six combined, that means six total from all your teachers, I think, you will either get a Friday school or a suspension, and you will not be able to take part in our end-of-quarter activity. You probably won't be allowed to go on any field trips. These are not things that you want to get, you don't like getting them, and personally, I don't like to give infractions. I don't think there should be a need. I know that each of you know how to behave, and more importantly I know that each of you are good kids. But if you keep interrupting with things that have nothing to do with math, then I will have no choice than to give you an infraction. So try to keep us both happy, and when I ask you to stop talking, or singing, or anything like that, that you respect me enough to listen and do the right thing and stop. [Classroom Transcript, August 29, 2006]

Despite this expressed reluctance, I, nonetheless, by the end of the second week of school, had written numerous infractions, and assigned lunch detentions to all students except for Keisha. Each infraction was for continued off-task behavior. For

Erika, Chris, and Alan, who received more than one during the first two weeks, I made calls to their parents or guardians. Admittedly, it was difficult to differentiate student conduct that would be singled out to receive an infraction. There was no well-defined action in which an infraction was issued. The students did not act out violently, or speak to me in an insubordinate way. When I explicitly addressed a student to stop talking, the student would frequently apologize, and temporarily suspend the behavior I singled out. However, in a given moment, even with repeated individual warnings, any member of the class might be engaged in an off-task conversation. The threat of issuing infractions seemed to offer little deterrence or prevent the continuance of the disruptive behaviors in the classroom. Further, if a student received an infraction, they seemed to perceive, correctly, that I would be hesitant to write a second one during the same period.

With the exception of Chris's mother and Alan's older sister, I did not sense that efforts to contact a parent or guardian had positive effects regarding the conduct of the students. Although the level of parental backing improved throughout the year, I did not feel well supported when I made contact to a parent or guardian during the first several weeks of school. In a September 5 journal entry, I recorded some of my perceptions after calling Erika's mother:

> At lunch, I called Erika's mom at work to discuss my concerns with how she'd been behaving, and to let her know that Erika had been given two infractions for being off-task. Unfortunately, I couldn't articulate very well to her why I had given Erika infractions. Her mom kept questioning me about what was wrong with Erika's actions and seemed very suspicious of my decision to issue these infractions. I felt like I was being put on the defensive to justify why I had disciplined Erika. [Personal Teaching Journal, September 5, 2006]

As I noted in the following journal entry at the start of the fourth week of school, my role as a teacher, a supposed position of authority, provided minimal influence in my attempts to persuade students to behave in desired ways. I believed creating a positive rapport with students was instrumental in any success I had experienced in changing the culture in the classroom:

> It is still humbling to realize that my authority over them is limited. Today, Kyle and Chris lingered at the start of class, and Chris blatantly carried on a conversation, and was laughing while I was right there with him, trying to get his attention to start class. I asked him if he had any respect for me at all. He said that he did respect me, and finally moved to his seat. Relationships and respect are how I am most effective getting them to put forth effort. [Personal Teaching Journal, September 18, 2006]

Sue also shared this perspective. After observing a class on October 26, Sue was particularly impressed by the nature of student work and behavior. When I asked her why she thought the students did so well, Sue said, "You bring a personableness to the room, and they want to please you, they want you to like them. When you talk to them, you kneel down so that they can look you in the eye, and that really means something to them."

An examination of the transcript data reveals I consistently made sincere attempts to acknowledge my disappointments with the class. For example, after much frustration during a class on September 6, I passionately stated:

> It makes me sad, I want to try to teach you so much, and the opportunity's here, we're a small class, we meet everyday, and each of you has shown me you can do this. I'm not saying I'm the greatest teacher in the world, but I have had a lot of success for a lot of students, classes that start of a lot like this, but I've always had some degree of success. For some reason, things are not going positively here at all. I need your cooperation, not just some of the time, or from some of you, but all of you, all of the time. You've got to make a commitment to come in here and do 40 minutes of math each day, then you're going to be all that you can be as far as achieving and moving on and being successful. Have you been successful in math class? Some of you yes and some of you no, but everybody can, and I'm

here to help you. I don't like to give speeches, but I want you to know how much I care about every single one of you, your welfare, and it hurts me when I see us wasting time like this because it ultimately takes away time from what we need to do in order to prepare you for high school and beyond. [Classroom Transcript, September 6, 2006]

The ensuing discourse, although chaotic, captures when, for the first time, the students expressed concern regarding their own behavior. Additionally their actions were not aligned, as they expressed explicit dissatisfaction with the conduct of other class members.

Jordan:	I'm fully down with what you said, but I think there are some kids back here who [inaudible]
	[*Jordan is interrupted by a chorus of students; Chris accuses Jordan of "snitching;" Keisha simultaneously shouts at Jordan telling him that, "You're not perfect."*]
RH:	I just want to get to some math today.
Alan:	Everybody, shut down y'all.
Cedric:	Stop talking.
Erika:	I'm going to stop talking for the rest of class.
	[*Chris yells something at Erika*]
Erika:	[*To Chris*] Shut up!
RH:	You all are still worrying me that we won't ever get anything done.
Keisha:	You all just being rude.
Jordan:	Like he said, you can only control yourself.
Alan:	I'm just going to be quiet. I'm not saying nothing else.
Keisha:	I think you need to send somebody to the office.

Also on this day, upon the suggestion of the students, I rearranged the seating so they were separated from one another. Overall, this was a pivotal moment in improving the climate of the classroom. On at least three occasions in the ensuing three weeks, I referred back to the September 6 class in negotiating how I expected them to behave. For example, on September 20, after being visibly discouraged that the students repeatedly disregarded my efforts asking them to quit their off-task conversations, I, yelled out:

> Everybody stop. Can I have your attention? Excuse me. Can everyone focus for a minute? You all are being rude. Look at how you're acting. This is too much like the first two weeks. I know you can do better. You proved that to me when you asked me to change your seats. [Classroom Transcript, September 20, 2006]

As discussed in the previous section, the disposition of my students to engage in mathematical work was a frequent theme in my journaling. Out of 45 first quarter journal entries, 28 times I addressed concerns that the behavior of the group was negatively impacting my goals for their mathematical activity. Eleven times, I expressed satisfaction with the nature of their mathematical work; and six times I did not make a clear judgment regarding their conduct. After reviewing the transcripts from these six classes, it is apparent that the behavior and the disposition of the students during these days more closely resembled the 11 class periods in which I favorably commented on the students' behavior than the 28 times I did not. Figure 4 illustrates the distribution of these class periods. For the first three weeks of school (August 28 - September 15), I had favorable impressions for only two out of 12 class sessions (16.7%). In the final seven weeks (September 18 – November 3), I had favorable impressions for 15 out of 33 class session (45.5%). A reasonable interpretation of this data is that as my relationships with students evolved, they became more willing to attend to mathematical activity, and this was reflected in my daily journaling.

August 28 -	**August 29** -	**August 30** -	**August 31** +	**September 1:** No School
September 4: No School	**September 5** -	**September 6** -	**September 7** -	**September 8** -
September 11 -	**September 12;** No School	**September 13** -	**September 14** +	**September 15** -
September 18 +	**September 19** -	**September 20** -	**September 21** +	**September 22** No Mention
September 25 -	**September 26** -	**September 27** No Mention	**September 28** No Mention	**September 29** -
October 2 No Mention	**October 3** +	**October 4** -	**October 5** -	**October 6** -
October 9 -	**October 10** -	**October 11** +	**October 12** +	**October 13** No Mention
October 16 -	**October 17** -	**October 18** -	**October 19:** Class Field Trip	**October 20:** No School
October 23 +	**October 24** No Mention	**October 25** -	**October 26** +	**October 27** -
October 30 -	**October 31** +	**November 1** -	**November 2** +	**November 3** -

Figure 4: Favorable (+) and unfavorable (-) impressions of student behavior from August-November

Relationships were not built in a uniform way and were greatly influenced by the amount of time spent with students outside of class. The building of relationships is not a prescriptive act and is undoubtedly dependent on context and personalities of

students and teachers. After the first month, my rapport was strongest with Jordan, Chris, Jamaal, and Alan. Starting in mid-September, each of these students took part in the county's supplemental education services program and remained after school for two hours every day Monday through Thursday. On most days, I met with them for nearly the first hour. We spent a mix of doing mathematics and playing around. They seemed to enjoy the freedom of hanging out in my room rather than taking part in the structured after-school program. Being able to draw upon our shared time outside of class appeared to be particularly helpful in managing classroom behavior and getting the students willing to do the assigned work during the class period. In an October journal entry, I recorded how this opportunity to work with the students was helping:

> Working with Jordan, Chris, Alan, and Jamaal after school is making a big difference. It's hard to understand, but they are more willing to do the same math work after school than they are during the class period. They don't like going to their room, so they are willing to do the work, and really try to do it, instead of going back. I also let them have a lot more freedom after school to do things like sit on the desks, throw a ball around, and write on the board. Not only are they starting to understand what we doing in class, their behaviors in class are not as bad, and they will listen to me more when I ask them to stop doing things. On a number of occasions, I have gotten them to work in class by threatening not to allow them to work with me after school unless they tried the work in class first. [Personal Journal Entry, October 23, 2006]

In addition, as documented in my teaching journal, I found that the after-school time was a good opportunity to address concerns I had about their classroom behaviors:

> In Cougar Time today, [Jordan, Cedric, and Jamaal] were talking about the need to fight someone if they felt it was necessary. Jamaal mentioned he had a bad temper. In a non-threatening, light way, I brought up the fact that Jamaal's temper will likely get him in trouble. I used that as an invitation to address some of his behavior in class. I brought up the other day when he refused to do any work, and how frustrated that made me. I told him that when he gets mad, he

> tends to shut down. I tried to convey to him, that that is not a desirable way to deal with a conflict. [Personal Teaching Journal, November 2, 2006]

Over time, as my relationships strengthened with students, I started to gain confidence in my ability to more sternly single out and address maladaptive behaviors as they occurred in class. In the previous section, I referred to an exchange with Jamaal on September 11 in which I harshly responded to his belief that he couldn't learn from others by stating, "That's not true; it's a dumb thing to say. You're acting like a stubborn two-year old." Strong relationships with students helped me earn the collateral I felt was needed in order to confront a student without fear of public recourse. For the first few weeks, when faced with inappropriate classroom behaviors, this was not the case. I tended to make respectful requests for them to improve their behavior; and I would defeatedly issue an infraction if I felt too many requests were made. In conversing with Sue about Erika's behavior during the second week of school, I stated, "I feel if I confront her, she will just assume fight than do what I ask."

For individual students, the building of mutually respectful relationships helped to avert any form of maladaptive behaviors and aided in motivating these students to do the assigned mathematical work. In particular, by the end of the first quarter, I had built a strong allegiance with Jordan. Most of the time, if he was off-task, I could say something like, "That's enough, let's focus here," and I could redirect his attention. Frequently when his classmates would veer off-task, he seemed to sense my frustration and would yell to everyone to "hush." Often, he also accepted my challenge to persevere with a mathematical task. He presented a positive model in the classroom for what could be done. For example, in solving the following task

posed to the class on October 31, not only did Jordan respond positively to the challenge, he offered assistance to Jamaal.

1) *Keisha bought 4 candy bars and 3 deck of cards. If the candy bars each cost $1.35 each and the deck of cards cost $2.50 each, much money did Keisha spend? Explain how you determined your answer.*

2) *At a different store, Jamaal also bought 4 candy bars and 3 deck of cards. The candy bars cost $1.35 each. All you know is that Jamaal spent no more than $15.00. How much could each deck of cards cost? Explain how you determined your answer*

Jordan:	I can't do this one [*number 2*].
RH:	First, how did you do this one?
Jordan:	[*Uses calculator*] Like this [*Shows me calculator*]
RH:	O.k., so you did
Jordan:	I did four times a dollar thirty five and three times two fifty.
RH;	And you got?
Jordan:	Twelve point nine.
RH:	Which is twelve dollars and ninety cents. [Jordan: yeah]. So why is this one harder?
Jordan:	I don't know that two fifty.
RH:	But we do know that Jamaal spends no more than fifteen dollars.
Jordan:	Oh, so do I like, like need to put something here [*points to Calculator*] to get fifteen.
RH:	Yeah, see if you can find that.
Jordan:	Alright, Alright, I can do this.
	[*Keisha and Jamaal both announce they don't get it*]
Jordan:	It's easy yo, you just got to try something more than two fifty.
Jamaal:	What you get?
Jordan:	I don't got nothing yet gee, but I'm getting closer. It's more than three dollars.
Jamaal:	What's more than three dollars?
Jordan:	Your cards yo, try more than three.

The consequences of a damaging relationship had an opposite effect on a student's willingness to behave appropriately and engage in meaningful mathematical work. For over half the school year, Keisha and I had a very productive relationship, although, to an outsider, her conduct in class would seem far from ideal. She was loud, and easily distracted. She seemed to want to take part in any conversation. In

particular, her exchanges with Erika were often very disruptive. However, for the most part, when I approached her one-on-one, she was rather compliant and willing to focus on the mathematics work in a meaningful way. I sensed she liked my style of instruction, and was proud that she was doing well in a mathematics class.

An incident in February changed the dynamics of our relationship for the remainder of the year. Keisha brought to class a project she needed to complete for social studies. She explained she had the right to work on the project, because, as she stated, "I already did the math." After several requests to put the social studies work away, I issued the ultimatum, "If you don't put it away, I'm going to give you a lunch detention." Keisha's refusal to stop working on the project, led to an issuance of an infraction with an assigned lunch detention. When I placed the infraction on her desk, Keisha madly crumbled up the slip and tossed it on the floor. This overt act of defiance led me to seek the referral of an administrator. Upon an escalation of her behavior in the presence of an administrator, Keisha was consequently suspended. Keisha had a history of volatile behavior in school. In hindsight, I should not have confronted Keisha in this public way. I should have, in private, given her the option of reporting to the alternative education classroom. Sadly, I chose a different set of actions for this day.

Academically, for the remainder of the year, Keisha retreated. Her interest and desire to perform declined. In my opinion, she reasoned that her sliding grades were a consequence of our damaged relationship, and not due to the diminished quality of her work. At times she behaved quietly and simply finished the work quickly and without a meaningful effort. On many occasions, as pointed out in the previous

section, she was a disruptive presence in the room. I believe my positive relationships with the other students prevented a mutinous outcome or any setbacks to the classroom culture I had worked hard to create.

In reflecting on the year, Sandy opined, "I think people in schools of education underestimate the issue of classroom management and behavior specifically." The data clearly points to the building of constructive relationships as a way to address classroom management concerns. Ultimately, how teachers build relationships is not as important as how a teacher taps into these relationships. Establishing caring and supportive relationships provided me a right to speak to them in firm and demanding ways. A consistent press for appropriate behavior and meaningful mathematical work contributed to the creation of an environment where students displayed a readiness to engage in a mathematical task and a willingness to persevere when confronted with uncertain work.

Task selection

It is generally recognized by mathematics educators, that the selection of meaningful and interesting tasks is one of the most important pedagogical decisions a teacher can make (NCTM, 1991). Not only does a task impact a students' opportunity to learn, it conveys implicit messages about the nature of mathematics and what is worth knowing and doing in mathematics. Many educators tacitly argue that students can be sufficiently motivated to do mathematics through the selection of appropriate problems or activities. They further suggest that many discipline issues in a classroom are the result of students becoming bored, not understanding the teacher directions, or simply finding little relevance in the task. The data suggests

that by mid-September, buoyed by burgeoning relationships, the selection of a task did influence the behavior of the students and their motivation to do work. However, during the first few weeks of school, the nature of the task did not seem to significantly affect the actions and dispositions of my students.

In a journal entry at the conclusion of first week, I expressed concern that I was not posing worthwhile mathematical tasks. I posited that the tasks I had selected lacked a level of cognitive demand that was needed to motivate students. My goal for August 31 was for students to develop strategies for computing integer sums. I presented a task that asked students to pair together forks and knifes out of a basket of silverware, and determine what is left over after the pairings. Then, I replaced the context of pairing forks and knifes, to the pairing of plus signs and minus signs. Finally, I asked the same questions using the traditional integer representation. Although this was a day in which the students were relatively well behaved, I didn't feel like I made any significant progress in extending the students' knowledge of mathematics:

> The problems I'm giving are being answered too quickly without reflection. As soon as I take away the context of the forks and knifes or the pluses and minuses, they're going right back to their previously learned procedural knowledge regarding the addition of integers, and this procedural knowledge is too disconnected and fragile. For (-5) + 4, they all answered either positive or negative nine, even though they all successfully paired up five minus signs with four positive signs to find one minus left over. They're not making the connection with the concrete representation, and it's hard to engage them in a discussion about it. I need tasks that will require some more thought and time. Maybe I'll try to ask the converse question, like after pairing up the forks and knifes, we have 8 forks left over, what could have been in the basket of silverware. [Personal Teaching Journal, August 31, 2006]

I believed that challenging students to find an answer to a question with countless solutions would raise the cognitive demand for the task and result in increased student

thinking. Unfortunately, on this occasion, that was not the case. The next class, students were seemingly uninterested in the task. The modest improvement in their willingness to focus on mathematics the class before did not continue:

> They are continually talking to one another, and barely acknowledge that I'm in the room. I cannot get all of their attention to even pose a task or ask anything about it. It's virtually impossible to get them to listen to me. When I asked them to think of how I might have 8 forks left over, the overwhelming response was, "Why are we doing this again." They seemed to have no interest in the problem. I had to take advantage of the small class size and pose the question individually to each student and point out how it was different than Thursday's questions. Even then, as soon as my attention left a student, they continued with their conversations, and put no effort with the task [Personal Teaching Journal, September 5, 2006]

During the initial ten to twelve class sessions, spanning the first three weeks of school, I was consistently confronted with these types of challenges. Regardless of the nature of mathematical activity, the students seemed to remain largely disengaged with the mathematics. As my relationships with the students began to strengthen, issue of task design became more prominent in determining the ways in which students engaged in the mathematics. Juxtaposed against the backdrop of establishing caring and supportive relationships, it is likely that the tasks I designed, the manner in which I attempted to implement them, and my interactions with the students around their mathematical work contributed to changing student expectations and beliefs in significant ways.

Commonly, I attempted to construct tasks that would extend the previous day's work. Thus, the task I selected each day was not completely designed until the day before. Although I regularly examined exercises from commercial textbooks and internet resources, the majority of the tasks I used were teacher developed. The first quarter tasks I posed to my students were a mixture of problem-based activities and

drill and practice work. A consistent tension I felt during the first quarter was trying to fit my vision of teaching and learning into a crowded, mandated curriculum where students were required to take a district wide assessment. Appendix *B* contains a list of the first quarter objectives the school district expected students to understand. The district wide assessment primarily required students to be proficient with symbolic manipulation and fluent in evaluating number sentences. For a large percentage of the test, students were not permitted access to calculators.

As described in the data analysis section in Chapter 4, to investigate the importance of task selection on student disposition, I referred to my teaching journal to identify ten pivotal class sessions from September 18 to November 3. I singled out five class sessions where I made a record that student behavior was most appropriate and their engagement was relatively high, and five class sessions where my impression was that students were not engaged and their behavior was most disappointing. September 18 was a point far enough in the quarter where meaningful relationships were being constructed and patterns of classroom structure were stable. Appendices *C* and *D* contain, in chronological order, the key mathematical task(s) from the five most productive class sessions and five least productive class sessions, respectively.

An examination of the tasks from the five most productive class sessions reveals that, in four of the five days, students were presented with a problem-based activity whose solution was not accessible through the application of a prescribed or memorized routine. Students could use their current understandings, and did not need explicit instructions, to find solutions. These tasks were embedded in contexts that

helped provide students access to the mathematics. They were worded using simple sentence structures in a straightforward way. Each had an accompanying figure, picture, or table. The problems could be modified in a natural way so that a student who did not know how to proceed could make progress with the task. For example, in spite of the fact that the class had explored the notion of profit in an earlier class, they generally seemed confused on how to proceed on the task from October 3:

Eisenhower Middle School Raffle Ticket *Price: $10.00*

First Prize: Panasonic 9" LCD Portable DVD Player 

Second Prize: Apple Ipod MP3 Player 

Third Prize: Motorola Cordless Phone with Digital Answering System 

The eighth grade class at Eisenhower Middle School decided to hold a raffle to raise money for their school. They purchased three prizes to raffle off. They purchased a portable DVD player for $400.00, a MP3 player for $250, and cordless phone system for $120.

a) If each raffle ticket will be sold for $10, how many tickets will need to be sold before they start making a profit for their school? Explain how you determined your answer.

b) If the eighth grade class goal is make a profit of $500, how many tickets will need to be sold? Explain how you determined your answer.

As an aid to students, I reworded the first part of the problem, and asked them to, "Find how many tickets we need to sell so we can pay for all of these prizes." Although this prompt seemed to help several of the students, Erika remained puzzled.

RH:	How you doing Erika?
Erika:	I don't get it.
RH:	Tell me, what don't you get?
Erika:	None of it.
RH:	Do you know what a raffle is?
Erika:	Yeah.
RH:	For this raffle we have to first buy all of these. But let's not

	worry about all of that. Let's pretend we are only going to raffle away the DVD player. O.k. [Erika: o.k.]. How much money do we need to raise just so we can pay off the DVD player?
Erika:	Four hundred.
RH:	Good. Now how many tickets to we need to sell to make four hundred?
Erika:	I don't know, like four hundred.
RH:	O.k., how much do we get for each raffle ticket?
Erika:	Ten dollars.
RH:	O.k., so if you sell 400 hundred tickets, how much money will you make?
Erika:	Four hundred?
RH:	You're right, we want to make four hundred dollars, but what if you just sell one raffle ticket, one to Keisha. How much do you raise?
Erika:	Ten dollars.
RH:	O.k., so we need to make a lot more to pay off the DVD player. What if you sold two raffle tickets, one to Keisha and one to Jordan. How much would you have raised?
Erika:	Twenty.
RH:	O.k., we need more than twenty, we need to make, how much? [Erika: four hundred]. What if I sold three raffle tickets, how much would I make [Erika: thirty], and four? [Erika: forty], and five? [Erika: fifty] O.k. so do you think you can work on this for a few minutes to see if you can find how many tickets we need to sell to get up to four hundred? [Erika: yeah]

In addition to modifications, each of the tasks provided the opportunity to add extensions. For the above raffle problem, I asked Alan to answer the same questions assuming that raffle tickets were only sold for fifty cents each. Having an opportunity to increase or decrease the difficulty level of a problem, yet use the same context and not reduce the complexity of the challenge appeared to be an important component in the design of a task. Evolving knowledge of my students' prior understanding and problem solving skills factored significantly in the creation of tasks and how I decided to modify and extend them.

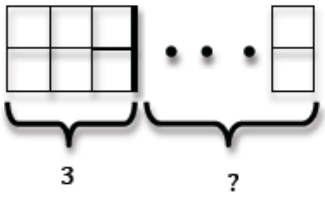
Addressing my goals of constituting a multiple solution norm, it was important that tasks have multiple entry points with multiple solution paths. As a general rule

of thumb, I attempted to design problem-based activities whose solution was accessible using a guess and check strategy. Encouraging students to use a guess and check strategy helped to emphasize the significance of putting forth effort as a critical component in the problem solving process.

An examination of the activities from the least productive class sessions reveals that four of the five classes involved low level cognitive demanding tasks. The task from October 20, although set in a context, provided no alternative pathways for a solution. Class sessions from September 20, September 26, and October 16 were designed only to give students opportunities to practice solving traditional, non-problem-based, procedural exercises. The unproductive nature of mathematical activity during these class periods is captured in an October 16 journal entry:

> They wanted to use calculators, so I had to explain that they will have to do problems like this on the county's assessment without the aid of a calculator. The problem is, they can't do these. They lack the necessary basic skills. I don't know if anyone got any of them right the first time. Jamaal refused to attempt any of them, saying he couldn't do them. Most everyone else, quickly wrote down answers and it was a waste of time trying to get them to compare their answers. Most of the conversations were way off-task and when they did compare, there was no meaningful evaluation of each other's work. At one point they were all out of their seats and everything was very hectic. Sue came by about mid-way, and I was embarrassed that there was no productive work going on. I asked for students to go to the overhead and explain how they got an answer, but they would just write their answer and would have no real explanation. To quiet everyone down, I finally explained how I would do each one, but I know this wasn't helpful. [Personal Teaching Journal, October 16, 2006].

The problem from September 19, was more cognitively demanding, however, it was not an accessible task:



- **If, above the question mark, there were four columns of squares, how many total squares would you have? Explain how you determined your answer.**
- **If, above the question mark, there were five columns of squares, how many total squares would you have? Explain how you determined your answer.**
- **If, above the question mark, there were 20 columns of squares, how many total squares would you have? Explain how you determined your answer.**
- **If, above the question mark, there were *x* columns of squares, how many total squares would you have? Explain how you determined your answer.**

The diagram was not a helpful aid to the students. Keisha asked, "How am I supposed to count these dots?" My attempts to clarify the problem by drawing out the figures represented by the first two bulleted items diminished the demand for the task. They were able to quickly count the number of squares, without reflecting on the concept I was attempting to build. Kyle commented, "That's all we have to do." As illustrated by my conversation with Alan, the students did not seem to possess a deep enough understanding of variable and the task design did not provide the students an opportunity to generalize their thinking in a meaningful way.

Alan:	What's *x*?
RH:	It might be anything. Basically, I want you to try to figure out if I told you some number, that would be *x*, then you could tell me how many squares there would be.
Alan:	Could it be, like 10?
RH:	It might, but let's try this. For this one, when it was four, how many squares were there [Alan: eight]. But total, if you count these? [Alan: six]. So there would be eight and six which would be? [Alan: sixteen] Close, eight and six would be fourteen. How did you know it was eight and six?
Alan	I counted those.
RH:	Then for this one, how many did you get? [Alan: ten]. Again, there would be ten over the question mark and these six more, so

	there would be [Alan: sixteen]. O.k., how did you do the one with twenty? Did you draw out twenty and count?
Alan:	No. [RH: How did you do it?] I just did two, four, six, eight and I did that all the way up to twenty
RH:	You got?
Alan:	Forty.

While I worked with individual students, the rest of the class became very loud and restless. At one point, while working with Keisha on the first part of the problem, Jordan, Kyle, and Cedric were throwing pieces of tangram puzzles, which were stored on a shelf in the back of the room, at each other. My frustration was evident in my journal entry for that day:

> The class did not go well. They continually carried on conversations and ignored my repeated efforts to get them on-task. To get anyone working on the problem, I practically had to do it for them. I don't think anyone took anything away from today's class. I am really concerned that they expect that they don't have to do any work. [Personal Teaching Journal, September 19, 2006]

It should be noted that the classroom activity on October 23 illustrates a case of constructive student work in the setting of assigned tasks that were cognitively low demanding, and non-problem based. Sue believed a point was reached during the year when, regardless of task selection, the students' social and mathematical behavior was appropriate. She believed that the behavior and actions of the students were most sensitive to my building friendly and supportive relationships with the class. When I asked Sue if she thought that the nature of tasks influenced student work, Sue stated, "I don't think for those students, any task in and of itself would be engaging, without you possessing a likable personality, and them wanting to please you. They were interested in doing a task to please you, and then the task became engaging because they had become successful in some part."

However, the data suggests that task selection was an important component. For the October 23 class session, other factors likely contributed to the productive outcome for this class. This was the first day when, as described in the previous section, I attempted to orchestrate a system of extrinsic rewards for appropriate student behavior and work. In addition, Erika, a consistent behavior challenge, was not present. Although vitally important, task selection is only one feature of classroom work that can affect students' disposition towards mathematics.

During the first part of the year, my task selection oscillated between problem-based tasks to ones that required rote practicing of procedures. I realized early on that my students would not possess the procedural skill and fluency necessary to be successful on the district wide assessment, yet I felt pressure to continue a trajectory that would result in covering each topic. Three weeks before the district wide assessment, the journal entry below captures the way in which Sue, who as an instructional support teacher had a vested interest to increase student assessment scores, pushed me to consider alternate ways to teach:

> I also don't think they will be prepared to take the county assessment. I still have to cover exponents, square roots, scientific notation, and the Pythagorean Theorem. I don't think they will be able to solve the type of procedural questions the county exam will assess. Sue thinks I should consider more of an expository teaching model in order to prepare the students for the exam. She suggested that, as a professional, I need to consider every mode of teaching and be open to the idea that the manner in which I'm teaching now is doing a disservice to the students. [Personal Teaching Journal, October 13, 2006]

By May, I generally did not consider the mandated curriculum in my task selection decisions. In a mid-year reflection I wrote:

> I do not believe I'm serving my students well by trying to adhere to the pacing and content objectives set forth by the county. Ultimately, my students are not at a level where they can pass the assessment regardless of whether I cover all the

> objectives or not. I have decided not to cover every objective and try to concentrate on the big ideas [Personal Teaching Journal, January 17, 2007]

At the end of the year, I had exclusively adopted a problem-based approach towards teaching. I aimed to design tasks that were problematic, and permitted multiple solution strategies. I believe that the use of accessible and challenging tasks contributed to the constitution of norms governing students' willingness to engage in a mathematical task and persist in the face of mathematical uncertainty. It became important to know students' well in order to select contexts with motivational appeal and at an appropriate level of difficulty.

Instructional decisions

In addition to building personal relationships and designing tasks that were cognitively demanding and engaging to students, the instructional decisions I made during task implementation inevitably influenced how students' thought about mathematics and their role in mathematics classrooms. Sandy and Sue identified the following strategies that positively affected the students' mathematical disposition:

- Encouraging student collaboration
- Encouraging students to use calculators to solve problems
- Using mistakes as learning opportunities
- Providing scaffolding to assist students during problem solving

Sandy noted that a point of emphasis across all of her observations was the press that I made for students to work together to determine the validity of a final answer. Although she noted that the students often had difficulty resolving differences, it was a significant instructional decision because of the autonomy it provided students.

According to Sandy, "You made it clear that multiple solution methods were a valued outcome" and affording students the opportunity to collaborate allowed them to "recognize that there was more than one way to solve a problem." Sue saw the strategy of asking students to work together as a way to help build the confidence of some of the students. According to Sue, many of students did not feel comfortable with how they solved a problem, and letting students work together helped them gain assurances that they were capable learners.

A second instructional decision contributing to improving the students' willingness to engage and persist in solving problems regarded student use of calculators. Sue saw that the calculator "was the primary means of solving problems" as it "allowed [students] to compensate for weaknesses with basic skills." Similarly, Sandy commented that without the aid of graphing calculators, the students would likely become frustrated by their inability to perform basic computations.

Both Sandy and Sue observed that is was important that I valued mistakes as a critical component in the learning process. Sandy noted, "Mistakes were viewed as opportunities for improvement." At different times throughout the year, Sue made observations that "no response was too trivial," "every one felt valued," and "they were not afraid to make wrong responses."

A fourth strategy identified by Sandy and Sue was that I provided students with useful hints to help them make progress. Sandy observed that I provided students with ideas and suggestions or helped students understand what they did wrong by pointing out a counterexample. According to Sandy, "I noticed when students had a wrong answer, you several times asked them a series of questions until they realized

where they made an error." Sue noted that my questioning did not signal to students that they necessarily made a mistake since, according to Sue, I responded to correct and incorrect solutions in the same way. Sue recognized that I often helped students with a problem by referring to previous class work. Sue stated, "When students struggled with a problem, you wouldn't necessarily show them what to do, but you instead would go back to a problem they may have solved the day before and ask them about that."

Attitudes and beliefs

The fundamental tenet of the emergent perspective (Chapter 3) is that the constitution of social and sociomathematical norms coevolves with students' beliefs and values. An examination of classroom norms governing students' social and mathematical behavior must be made with the awareness of their beliefs. By middle school, students enter the classroom with experiential knowledge and deeply held beliefs about the nature of mathematics, about their ability to learn mathematics, and about the roles of teachers and students in a mathematics classroom.

Each of the students in my class entered the year with an accumulation of past failures in mathematics. They had taken summer school classes prior to the start of the year, they were identified as below grade level on their report cards, and they scored in the lowest level of the state's assessment of mathematical knowledge. It is reasonable to assume that these experiences had an affect on the students' self-image and influenced how they chose to participate in learning opportunities. Early in the year, when I asked Sue why it was such a challenge to engage the group in an academic way, she posited, "It's a combination of their past performances, past

expectations, and their own expectations. They have low expectations of themselves, and are looking anywhere and everywhere for other people to like them, and trying to fit in."

It is recognized that beliefs are not attributes that are directly observable; however, the day-to-day interactions and conversations with students made an interpretation of their beliefs accessible. In particular, an examination of classroom transcript data suggests that students' beliefs regarding their ability to learn mathematics and their beliefs regarding their own role and others' roles were important factors that likely affected the disposition of students.

For some students, as illustrated in the following two episodes, the small size of the group sent a clear message regarding their ability to do mathematics. The first exchange took place during the opening minutes of the first day of school, and the second was recorded near the end of the year on May 21:

Alan:	This all of us?
RH:	I think there might be one or two more, but this will be good. I think having a small class will be great.
Cedric:	No it won't. [RH: Why?]. It mean's we're stupid.
RH:	No, that's not true. It might mean that you didn't do so well last year, but it doesn't mean you won't do good this year.,
Jamaal:	Cedric right. This is the same as summer school, except for Josh.
Jordan:	Summer school was easy, yo.
Cedric:	It was easy, because we're dumb.

RH:	How is this math class different than other math classes?
Cedric:	Little.
Jamaal:	We're dumb.
Jordan:	No we're not.
Jamaal:	I don't care what you all say we're retarded. Why do you think we're in this class?
RH:	Because the class is small?
Jamaal:	That, and we're dumb.
RH:	Come on, you guys have done a lot of difficult mathematics this

	year. Ms. Pope [Sue] is always saying how impressed she is with you guys.
Jamaal:	I don't care. We still dumb.
	[several moments later]
RH:	Describe to someone who will be in this class next year, what they can expect.
Jamaal:	That they will be retarded.

It is important to note that these individual statements regarding the generalized ability of the class went, for the most part, unchallenged by group. Although Jordan objected to Jamaal's initial comment in May, he did not respond when Jamaal challenged him to consider why they were placed in this particular class. Blatant commentaries such as these were rare; however it is conceivable that the above episodes illustrate the students' true sense of self-efficacy or lack thereof.

It is plausible to conjecture that my students' evaluation of their own ability and their desire to avoid further mathematical failures were factors that influenced their decisions to resist engaging in meaningful mathematical work. According to Sandy, a likely reason why students did not put forth effort in solving mathematical tasks was that "students didn't have faith in their own efficacy to continue working on a problem." Of the 243 coded occurrences in the transcript data where student behavior was coded as non-persevering, 36 times students provided a cue indicating a belief that they were not capable of finding a solution. For example, the following exchange took place on October 9:

RH:	Cedric, how's it going? Can you show me how you're thinking about the problem?
Cedric:	I don't get this.
RH:	What don't you get?
Cedric:	I don't know how to do it.
RH:	What do you understand?
Cedric:	I don't get any of it.

It was more common for a student to find fault with the problem itself by calling it dumb (26 times) or too hard (48 times); or cast blame on me for not providing adequate instruction (58 times); or simply shrug off the task and appear indifferent about the work (75 times). Table 6 breaks down the reasons students gave for suspending work on a task. In light of Cedric and Jamaal's comments about what it meant to be in a small class, it could be posited that student beliefs in their (in)ability to do mathematics is not adequately represented in the table below. Student claims that I did not teach them enough, or that a problem was stupid may have been defensive strategies for coping with an underlying belief that they were not capable of doing the work.

Not capable	36	15%
Problem is too hard or not relevant	74	30%
Not provided adequate instruction	58	24%
Indifference	75	31%

Table 6: Student reasons for suspending work on a task

Logically equivalent to the conditional statement that certain problems cannot be answered because students lack an ability to solve them, is the claim that students who possess mathematical ability can solve problems. Although there are not many occurrences in the data where students made comments about why they were capable of solving a problem, the few instances that do exist provide evidence that the students believed the above claim to be true. The episode below is from a September 21 discussion with Alan where, after much tribulation, he successfully solved the problem of finding how many chairs could be seated around 50 rectangular tables joined together (see Appendix *C*):

Alan:	I'm right?
RH:	What do you think?
Alan:	Yeah.
RH:	Yeah, great job. Now what does that show you?
Alan:	That I'm smart.
RH:	That, but it really shows you that if you don't give up, and keep trying different things then you can be successful.
Alan:	Has anyone else got this right?
RH:	Not yet.
Alan:	Why am I always the first one to get these?
RH:	I don't know. I don't care. It's not a race.

Students with a productive disposition would recognize that persistence is a key element to problem solving. Clearly, Alan did not acknowledge this aspect, and it was important to him that he was able to finish the problem before anyone else.

My vision of teaching and learning was inconsistent with the students' previous experiences in mathematics classrooms. In the third week of class, Sue pointed out, "The way you're teaching is new to them and it will take time for them to get used to it. They have very few math skills and you need to build their confidence so that they can be successful." It is likely they previously experienced traditional instruction filled with low-level recall of facts and procedures, with few opportunities to engage in solving problems that required independent and original thought. Consequently, my teaching style conflicted with students who preferred and expected more teacher-directed instruction.

On a number of occasions, these moments of conflict resulted in resistance on behalf of the students to engage in the assigned mathematical activity. Jamaal frequently challenged my motives for instruction and appeared to stubbornly hold to his convictions. In particular, there were at least three occasions during the first quarter when Jamaal expected that I should evaluate the correctness of student

solutions and believed it was important to know if an answer was correct before explaining or justifying how it was obtained. Below is an episode from September 27 in which Jamaal struggled with the second part of the following task:

Find the missing numbers:
1) Eight plus this number is 12.
2) Eight plus this number is 2.

Jamaal:	How we supposed to do this one?
RH:	How did you do the other one?
Jamaal:	But this is different, this is a negative.
Kyle:	It's easy.
Jordan:	I know, I just read it that's why, but it's easy though. I understand it.
Jamaal:	Eight plus negative 12, 13 or 14. Something like that?
RH:	O.k., write down what you think then explain why you have that.
Jamaal:	But I don't know.
RH:	If you don't know what to do, what are our options?
Jamaal:	Not to do it.
RH:	Your're just going to wait? That's one option, but you just had an idea that it was negative 12, 13 or 14, and we have all this time, so is there something you can do to try and figure it out?
Jamaal:	Am I right?
RH:	Can you explain why you think it is negative 12, 13, or 14?
Jamaal:	Am I right? That's what I want to know.
RH:	Do you think your answers make sense?
Jamaal:	I won't be able to tell you until you tell me if I'm right or wrong.

Over the course of the year, Jamaal appeared to reorganize this belief so that it became more aligned with my goals for mathematical activity. He even admitted to liking my insistence on explanation. At the same time, as described in the following episode from May 21, Jamaal provided a valid rationale supporting his original beliefs.

RH:	How would you describe how I teach?
Jordan:	Confusing at first.
RH:	What made it confusing?
Jordan:	How you explain things.
Jamaal:	I wouldn't say that, because he makes you explain what you do first.

RH:	So you think because I have you explain it first, it's confusing?
Jamaal:	I don't think that. I like that.
RH:	Didn't we argue about that one time? When I asked you to explain something, you kept saying, "But I'm asking you."
Jamaal:	Yeah, cause sometimes it's frustrating when you can't do it, and you know your wrong, but you keep saying, "How you do it," but you know your wrong.
RH:	So is it still as frustrating as when we started?
Jordan:	Not for me.
Jamaal:	Me either.

Thus, students' actual mathematical knowledge is intertwined with students' beliefs about their ability to do mathematics, and together these are powerful influences that guide students' decisions to engage in mathematical activity. This underscores the importance of choosing appropriate tasks. Certainly, most students would suspend effort on a mathematical problem if it required knowledge that students did not possess. Both Sandy and Sue made comments regarding the challenge of working with students whose knowledge of mathematics was so fragile. Sandy used the analogy of asking how a math teacher might respond if given a question from organic chemistry.

An additional belief conveyed by some students that seemed to negatively affect their disposition to do mathematics was the idea that they would be willing to work on problems only if they were given an appropriate incentive. The transcript data from September 29 highlights the students' expectation that Fridays should be days when they are not pressed to do significant mathematics:

Jamaal:	Man, you making us do work today?
RH:	Yeah, I want to try and do this problem today.
Kyle:	It's Friday.
Keisha:	We should have a party.
Erika:	Ms. Smith would let us watch the Price is Right.
	[*Several students begin conversations about Ms. Smith's class*]
RH:	I'm sorry, but we have a lot we got to do.

Jamaal:	Why don't you play any games?
RH:	I never liked playing games, I never found them a good way to do math.
Keisha:	That's not true. We learn better by playing games. Jeopardy we can play jeopardy.
Jamaal:	I'm not doing this.
RH:	What do I have to do so you will try?
Jamaal:	Pay us.
RH:	Pay you? How much money do you think I make [*Laughter*]?

It is apparent that the students' previous experiences created an expectation that there should be moments of relaxed academic rigor. Additionally, as previously mentioned, some students believed that their obligation to attend to a mathematics problem ended when they finished a task, regardless of their understanding of the pertinent mathematics. Keisha, for instance, would frequently justify her inappropriate behavior by explaining that she had done the assigned work.

Overall, my students entered the classroom with a range of prior experiences and deeply seated beliefs regarding the nature of mathematics and the roles of teachers and students. My goals for mathematical activity, which depended on the students' willingness to solve and discuss challenging problems, were new to my students. The conflict between my and the students' vision of mathematics teaching and learning likely contributed to the students' decisions to actively avoid an academic task and engage in a range of maladaptive behaviors.

External influences

Although adolescence can be a turbulent time, most middle grade students I worked with at Walker Middle School appeared capable of managing their emotions and coping with the inevitable distractions and stress that accompany adolescent growth. However, several of the students in the pre-Algebra class seemed to lack a

self-regulatory capacity to control their emotions. Sue observed that, "A lot of students had a hard time leaving what was going on with them personally to focus on what they needed to do." According to Sue, before they would be willing to do work, "Some kids would need to be calmed down at the door." Broader school and societal contexts appeared to affect the disposition of the students to engage in mathematics during class.

An illustration of this is contained in the data from the October 4 class. At the start of the period, Kyle and Cedric were in the office being questioned regarding their role in a physical altercation that occurred between two other students in their preceding class. Rumors of the fight had quickly spread in halls during the movement between classes. Details of this incident and its potential repercussions were the sole focus of the students throughout the period. Despite several pleas for the students to "wait until lunch" to discuss the fight, I could not deter their attention away from talking about it. Ultimately, as highlighted in the following transcription, I did not attempt to press on with my lesson, and instead tried to offer advice for dealing with conflict:

	[*Alan and Jamaal are talking about why sometimes you have to fight*]
RH:	That's where you're wrong. I don't understand why that is always your first reaction. You guys are too big and too strong and at a point when someone could get seriously hurt, or more likely get themselves arrested and in big trouble. I've had these conversations with Mr. Fisher, you guys are hanging out with the wrong people. Alan, you even said yourself you have known people from town [a nearby metropolitan area] who have been killed. Do you want that to happen to you?
Alan:	No.
RH:	Then, why are you so willing to fight just because you heard somebody say something about someone who disrespected somebody who knew you [*Laughter*]

Jamaal:	No, but still sometimes you got to handle things.
RH:	Maybe so, but I don't want none of you handling things.

As noted previously, Keisha, Erika, Jamaal, Cedric, Jordan, Kyle, and Chris were all part of the alternative education program for students at academic and behavioral risk. According to Sue, "They did not have very effective filters on what they had to say." Several times, what appeared to be innocent teasing and joking escalated into larger conflicts. Alan, the one student in the class not part of the alternative education program, frequently struggled keeping his emotions in check. In Sandy's second observation on September 25, Alan and Keisha entered class arguing with each other. Despite several requests for them to stop, they continued their verbal insults at each other as if I was not present. After Alan angrily threw his books off his desk, I asked both students to step out into the hall where a nearby administrator, who was fortunately walking by, was able to quickly help diffuse the situation. Although both students returned to class in a civil manner, neither one participated in the work for the day. In her field notes, Sandy recorded, "The meltdown at the beginning seriously undermined your academic efforts."

Throughout the first several months, Alan was a particularly intriguing student. Along with Chris, he was the best problem solver in the class. Many times he seemed to embrace this role. He frequently boasted about being the first one to solve a problem. After successfully solving a task on September 28, he commented, "Why am I so good at math?" Just two days prior, Alan seemed completely disinterested. Although the low cognitive demand of the task on September 26 may not have provided sufficient motivation to engage in the problem, my journal entry from that day noted Alan's inconsistent behavior:

> I wish Alan would be more consistent in his effort. There are days when he does great. He is interested, asks questions, participates; and then there are too many days like this when he is consistently off-task, mostly with Jordan and Erika, sings rap-songs, and doesn't try. When I asked to check his work today, he said, "I forgot how to do this." He says this a lot when he hasn't done any work. It appears to be his way of deflecting responsibility for not doing the work. When I ask him to tell me what he forgot or doesn't understand, he then, suddenly, seems capable of figuring out the problem. [Personal Teaching Journal, September 26, 2006]

Alan appeared to assume a more positive attitude towards mathematics during the second week in October. During this week, he displayed a temporary, but strong commitment to move out of the pre-Algebra class. Weeks after Chris was placed into Algebra, Alan asked me why he wasn't moved. I told him that I was concerned that he didn't always put forth effort in his work, but I would be willing to move him if he could show that he was able to understand some of the Algebra he had missed. For one week Alan came in up to an hour before school to receive extra help. At the end of that week, I recorded the following journal entry:

> Alan came in at 7:30 again this morning to work on Algebra stuff. He clearly doesn't like that Chris has moved up to Algebra and he didn't, but there seems to be something more to it. I'm excited that he is motivated to move to Algebra. He is doing well. . . . He is able to grasp the concept of rates of change and starting value. Maybe I should have moved him with Chris, but he has shown too much of a tendency to shut down. I think I can use this as motivation to keep him more involved in class. I told him that he has to be focused in class, before I can consider moving him into Algebra, but I will. Today, he talked too much, but I was able to more quickly redirect his focus. He will rise to the challenge and solve problems. He genuinely seems pleased when he figures something out. He often wants me to look at his answers, and is willing to explain how he thought about it. [Personal Teaching Journal, October 13, 2006]

Unfortunately, the next week Alan stopped seeking extra help and his behavior in class seemed to worsen. When I approached him about moving into Algebra, he simply shrugged it off, and, suddenly, no longer seemed interested. I know that Alan did not have a traditional home life. He lived with an older sister and a younger

cousin. He had several other family members who lived in a nearby city. I believe his situation was unstable. In talking about Alan, Sue described, "A bright kid, but so many personal problems that I don't think he can get beyond them." By the beginning of November, he was out of school more than he was present. When present, he was not engaged. In an October 30 journal entry, I made the following note:

> Alan was awful. At the same table, he would blatantly continue talking. I wrote an infraction and e-mailed his sister. I should have sent him out. He refused to try any problems saying he can't do them. I rather sternly said that these are the same problems you have done all year, and how good a problem solver he is, but he told me that they are too hard. [Personal Teaching Journal, October 30, 2006]

My attempts to contact his sister were unsuccessful. Jordan indicated that Alan was living in the city. By the middle of November, Alan had officially withdrawn from school. Throughout the first quarter Alan's behavior and effort greatly oscillated. By the end of the quarter, it was clear that there was something going on in his personal life that likely contributed to his miseducative efforts.

Dense network of student relationships

An examination of the data suggests that students' behavior and disposition towards mathematics can more profitably be understood by considering the informal organization of the class. The well-defined and close friendships amongst the students seemed to promote some of their non-intellectual behavior. In particular, Chris's actions and conduct appeared closely tied to the dense social network that existed between himself and his classmates.

Although his standardized test results were similar to his classmates' scores, I signaled Chris out within the first couple of class periods because of his problem

solving ability and sophisticated ways he tended to approach mathematics. After the second day of school I referred to Chris in the following journal entry:

> I think Chris will do real well. He is very likeable and polite. More than the others, he seems to have a good grasp of basic facts and can reason things out. My one concern is how social he is. It seems like every time I finally get the attention of the whole group, he is the first one to speak out and cause everyone to laugh and lose their attention. [Personal Teaching Journal, August 29, 2006]

Chris's tendency to socialize worsened over the first two weeks. It was clear that he and Jordan were close friends. Together, they seemed to circumvent the demands of a task by quickly answering a problem and then engage in good-natured, but off-task conversations. On September 8, I issued Chris a second infraction for off-task behavior and made my first call to his mother. His mother was extremely appreciative, and pleaded with me to help Chris measure up to the potential she saw in him. By September 13, I began to consider if Chris could make the move to an above grade level Algebra class:

> Chris is having no difficulty with this work. The problem is, when he finishes, he uses that as a cue to begin his typically silly behavior. He will not, or only momentarily, listen to my requests to stop talking/laughing. Today, I told him that his actions were demonstrating a complete lack of respect for me. He apologized but did not stop being a distraction. When he is around his peers, he doesn't seem capable of controlling his actions. I think a different environment could make a big difference. I wonder how he might do in an Algebra class, where students more motivated to do work surround him. [Personal Teaching Journal, September 13, 2006]

During the next few days, I attempted to assess how well Chris was able to think proportionately and reason with linear patterns to gauge if he might be able to succeed in my Algebra class. I made the case to the eighth grade team that I was considering moving Chris to Algebra. Although some were mildly skeptical about moving Chris a year ahead in the curriculum, I received the teams' and

administrations' endorsement. Chris's mom enthusiastically supported the pending decision. When I approached Chris with my recommendation, he was hesitant and did not want to be moved. Ultimately, his mom pressed forward and Chris was placed into one of my Algebra sections on September 22.

The change in Chris's demeanor was immediate. He appeared completely overwhelmed by both the content and setting. I questioned if I had made a responsible decision:

> I feel sorry for Chris. He is having a hard time understanding how to find a linear relationship, and he was nearly in tears today. He is so reluctant to work with anyone at his table, and he only asks me for help. He asks me in this meek, quiet voice. I think he is really trying hard. I wonder if it was the right idea moving him. He is such a different student behavior wise; he never speaks out and goofs off. I only hope that getting a portion of Algebra is better than being in a class where he is not challenged. [Personal Teaching Journal, September 28, 2006]

I made it a priority to work with Chris outside of class. Eventually, he became more comfortable and experienced a degree of success. He ultimately passed the state's assessment of Algebra. By mid-year he seemed proud of the fact that he was passing my course. Late in the year, reflecting back on the change of Chris's placement, Sue commented, "I think it has been a good move. I was skeptical at first. I think that Chris is a child that didn't know how to handle being successful, and putting him in a class with more focused kids with better behavior has helped."

Besides the immediate reaction of the students on the first day after Chris switched classes, I didn't notice a direct affect that Chris's absence had on the social and mathematical activity in the class. On the first day without Chris, the students seemed envious that he had been placed in Algebra. It was as if Chris had obtained some kind of desired status. However, their interest in Algebra did not appear to

translate into an improved classroom dynamic. One week after moving Chris, I recorded the following in a September 29 journal entry:

> I'm not real sure how the class is different without Chris. It is nice not having to constantly address his off-task chatter, but they still seem just as disinterested in what we're doing, and at any moment either Erika, Keisha, Alan, or Jamaal can be just as much off-task and distracting as Chris ever was. [Personal Teaching Journal, September 29, 2006].

Over time, however, the data indicates that Chris's departure from the class had some influence on students. According to Sue, "Getting Chris out made a big difference in that class." As discussed above, Alan temporarily attempted to take on increased academic responsibility so that he could also be moved into an Algebra class. Interestingly, near the end of the year, Jordan made an unsolicited reference to Chris's leaving as pivotal to his mathematical disposition. In a class on May 21, Jordan stated, "You know I think it was a good thing you moved Chris. I mean, at first, I was like why does he get to go to Algebra, but then it really made me want to do better. So it was good that you moved him."

Like Chris, Jordan also displayed a different pattern of behaviors when surrounded by his peers in class. I first noticed this difference about a week after I started working with Jordan and other students after school as part of the county's supplemental education services program. In a September 19 journal entry, I made the following note:

> Jordan seems to want to do well, but he can easily be distracted and will join in the conversations of others. After school, he is more motivated to do and understand the work. He asks questions and always wants me to give him another problem to try. Unfortunately, he gets too distracted in class. [Personal Teaching Journal, September 19, 2006]

Although, generally Jordan's behavior in class was an infrequent concern, he did not demonstrate the same focus that he did outside of class, a fact that Jordan seemed to realize himself. For example, in the following episode from October 24, I, partly tongue-in-cheek addressed an off-task Jordan:

RH:	I don't understand how you can be so interested and so on-task and think so well when we work after school, but in class its like you
Jordan:	Don't' try?
RH:	I wouldn't say you don't try
Jordan:	Off-task?
RH:	It's not that your always off-task
Jordan:	Loud?
RH:	Thanks for trying to help me out, but it's like I'm always giving you more of a push than I have to do after school
Jordan:	Well you see, it's just my associates over here, sometimes don't [inaudible]
RH:	Oh, so I see, it's out of your hands. So when you're at lunch are you unable to eat because of your associates? [*Laughter*]
Jamaal:	No, Jordan always eating everyone else's food.

Whereas Chris and Jordan displayed a different disposition towards mathematics during class time as compared to outside of it, Erika and Keisha, seemed capable of influencing the routines of the class itself. As described in the previous section, during the first ten weeks of school, Erika's attendance appeared to be an important variable affecting the willingness of the students to try and persist in solving mathematical problems. When she was absent, the output of the class was noticeably more productive than when she was present. When present, Erika's behavior often fueled the miseducative behaviors of the group. Other students, most frequently Keisha, Alan, and Jamaal, would accompany and build off of Erika's disruptions. For example, on September 11, the students seemed to be working well, and at low-volume. On the video, I can be heard working with Cedric and Kyle, while Jordan's,

Alan's, and Chris's voices can be heard talking about the problem. Keisha, Erika, and Jamaal are initially quiet. Suddenly, Erika started to loudly sing. In effort to get her to stop, I quickly yelled out Erika's name. Without delay, Erika apologized by stating, "I'm sorry," but the focus on the problem was already lost. Keisha started to sing the same song, and Erika joined in again. Alan, Jordan, Chris, and Jamaal yelled at them to "shut up," not because Erika and Keisha were disrupting the class, but because they did not like the song. These types of disruptions were common throughout the first ten weeks of school. Although Erika was not always such a clear catalyst for these disruptions, the class clearly contained fewer interruptions, and more constructive student work, during Erika's six absences during the first quarter.

By the end of the year, as described in the previous section, Keisha was often a source of resistive and non-academic behavior. Although Keisha's actions would not result in the same kind of disruptive and chain reactive talk that Erika's did, the magnitude of her behavior adversely affected the nature of mathematical activity in the room. Many times, Erika would respond to Keisha in a manner that condoned her behavior, however, Cedric, Kyle, Jordan, and Jamaal typically did not. For the most part they overlooked Keisha's outbursts. At the same time, perhaps due to Keisha's prominence in the classroom, they never appeared to reproach her. Thus, it is likely that Keisha believed they tacitly endorsed her behavior and she felt empowered by her classmates to act out.

Many times in my reflections I made the assertion that my issues of classroom management would have been less severe if the students were in a social context where they were not surrounded by a dense network of friends. Particularly for the

first part of the year, the students' close and personal relationships with one another appeared to support and reinforce the group's resistive behavior towards mathematics. Thus, the informal organization of the classroom contributed students' decisions to engage and persist in working on challenging mathematical tasks.

Summary

Despite my expressed and explicit aim to create a classroom governed by a multiple solution norm, I was largely unsuccessful in accomplishing this goal. Managing student behavior and motivating students so that they would engage in a mathematical problem were challenges that I vastly underestimated. Attempts to facilitate whole-class discussion of ideas were not productive, and within the first two months I was no longer trying to orchestrate whole-class discussions. Students struggled communicating mathematical explanations for their thinking and they did not appear to listen to each other's ideas. Consistently, the students looked to me to validate a final answer and viewed problem solving as a competition, rather than a means to learn mathematics. Overall, the students did not enter the classroom with an interest in solving mathematics problems or a natural tendency to talk about mathematics.

Over the course of the year, negotiating expectations that students would promptly engage in a mathematical task and persist in solving non-routine problems were major accomplishments in the activity of the group. By year's end, the students were willing to publicly share their ideas and they regularly used graphing calculators to regulate their own solution methods. The strength of teacher-student relationships was a significant factor affecting the students' willingness to expend effort to solve a

mathematical problem. Designing cognitively demanding tasks at an appropriate level of difficulty contributed to students' efforts to do mathematics. Implementation decisions, such as valuing student mistakes and encouraging student collaboration, were factors that affected students' disposition toward mathematics.

My attempts to challenge students to use their own inventive strategies for solving problems came up against many deep-seated students' beliefs and attitudes about the nature of mathematics and teacher's and students' roles in the classroom. Broader school and societal contexts seemed to work against my efforts to constitute a productive disposition within the class.

Having presented the data in this chapter, I turn now to a discussion regarding how the results of my study can contribute to a wider research base regarding student motivation and achievement goal theory. In addition, in the next chapter I use the preceding analysis to present a revised definition of a multiple solution norm.

Chapter 6: Discussion

This chapter is divided into two sections. In the first section, I aim to frame my findings regarding factors that promote a student's productive disposition with an existing research base on student motivation. In particular, I look at the findings in relation to literature that examines students' activity through the lens of achievement goal theory. In the second section, I revise the original framework I used to define a multiple solution norm to account for the accomplishments I experienced and the context dependent obstacles I encountered in my teaching.

Productive Disposition and Achievement Goal Theory

Unfortunately, the students in my study did not approach mathematics with a desire to engage in and persist at academic tasks. In the beginning of the year, they avoided academic work by using a range of strategies. Cedric frequently seemed to purposefully withhold effort; Erika's public off-task comments would often be a disruptive influence in the classroom; Alan would sometimes avoid asking for help even though he recognized he needed assistance; and Jamaal would openly challenge my instructional decisions.

Certainly these behaviors and avoidance strategies are not unique to my context. The National Mathematics Advisory Panel (2008) recognizes that a challenge for most mathematics teachers is to instill in students the notion that effort and persistence are necessary components to mathematical problem solving. When faced with a non-routine mathematical task, it is common for students to quickly yield to a problem or not attempt it at all (Schoenfeld, 1987). Teachers

who desire to implement what the *Standards* suggest is good to do in the classroom depend on students' cooperation and willingness to engage in challenging mathematical activity. I greatly underestimated the challenge of instilling in my students the idea that effort and persistence were an important part of problem solving. Results of my study suggest some factors that were influential in shaping my students' willingness to do mathematics. These findings can build on a wider research base regarding the question of how do students develop positive work habits in school.

Dweck (1986) notes that it "has long been known that factors other than ability influence whether children seek or avoid challenges, whether they persist or withdraw in the face of difficulty, and whether they use and develop skills effectively" (p. 1040). In educational research, "motivation theories are most often used to explain students' activity choice, engagement, persistence, help seeking, and performance in school" (Meece, Anderman, & Anderman, 2006, p. 489). Contemporary theories view motivation as a social-cognitive construct (e.g., Ames, 1992; Bandura, 1986; Dweck & Leggett, 1988; Weiner, 1972). In a given context, motivations are mediated through how a student construes a situation, interprets events in the situation, and processes information about the situation (Dweck, 1986). Middleton and Spanias (1999) refer to motivations as simply reasons individuals have for behaving in a given manner in a given situation; they guide student's decisions, and help determine whether or not students will engage in mathematical activity.

Over the past 25 years, achievement goal theory has emerged as one of the most prominent frameworks used by educational psychologists for understanding academic motivation. Ames (1992) explains that this theory describes "how different goals elicit qualitatively different motivational patterns and how these goals are reflected in the broader context of classroom learning environments" (p. 261). Goal theory assumes that students' motivational behavior can be influenced by the unique interaction between an individual's personal dispositions and beliefs and their classroom environment.

Although different terminologies are employed, goal theorists believe students adopt either a *mastery goal orientation* or a *performance goal orientation.* Students possessing a mastery goal orientation seek to increase their competence, and are focused on learning as something valuable and meaningful in itself (Dweck, 1986). Students with a focus on mastery goals are more willing to take risks, and consistently demonstrate high levels of task engagement, effort and persistence (Grant & Dweck, 2003; Wolters, 2004). Mastery learning goals are associated with positive perceptions of academic ability and self-efficacy (Midgley et al. 1998; Wolters, 2004), and have been related to lower avoidance behaviors (Ryan, Pintrich, & Midgley, 2001). Students with a performance goal orientation seek favorable judgments of their academic ability from teachers, parents, and peers, or aim to avoid negative judgments of their competence (Dweck, 1986). In recent years, researchers have begun to parse performance goals into approach and avoidance components (Covington, 2000). Middleton and Midgley (1997) linked performance avoidance goals with maladaptive

student behavior. Students with these goals often avoid asking questions if they feel that doing so would demonstrate a lack of knowledge or ability, are more likely to engage in projective coping like blaming their teachers for their low performance, and are often disruptive in class in order to deflect attention from their difficulty.

Middle school is a particularly sensitive time for analyzing students' goal orientations. In early elementary grades, students tend to be highly motivated to learn mathematics and believe that working hard will enable them to be successful (Kloosterman, 1991). By middle school, however, many students perceive that learning mathematics is attributable to innate ability and that putting forth effort has little or no influence on their ability to succeed (Kloosterman & Gorman, 1990). In a review of research on motivation in the middle grades, Anderman and Maehr (1994) cite convincing evidence indicating that students often exhibit a disturbing downturn in motivation; they find an overall pattern that "supports the view of decreased investment in academic activities and increased investment in nonacademic activities during the middle grades" (p. 288). Tuner et al. (2002) note that, by adolescence, low-attaining students often deflect attention from their low ability by withdrawing effort and resisting novel approaches to learning.

Applying a goal theory lens to view my classroom shows how students consistently held onto a performance goal orientation. A normative expectation in the class was that I would verify the correctness of student solutions. Even when it was clear what a final answer would be, it was still important for students to seek my favorable judgment for their work. Students with a mastery goal

orientation are intrinsically rewarded by improving their level of competence or acquiring some new understanding and would not always need a teacher's endorsement for their work. The manner in which my students expressed a low self-worth of their own ability to do mathematics is central to a performance orientation (Dweck, 1986). The group's unsolicited competition regarding who could be the first to solve a problem and the way they constantly valued a correct answer are clear indicators of performance goals. Meece, Blumenfeld, and Hoyle (1988) found that public recognition that one has done better than others is especially important to a performance orientation. The students in my class frequently were inspired to do work, or claimed they would be more willing to do work, if they were offered an award. Students with a mastery goal orientation would not need to seek out an external prize in exchange for their effort.

How students form goal orientations

Through a goal theory perspective, a multiple solution norm can most easily be instantiated in classrooms where students exhibit a mastery goal orientation. Before exploring how achievement goal theory suggests that goal orientations can shift, it is important to consider how goal orientations are formed. Although it is reasonable to assume that the particular goal a student adopts may be influenced by past academic failures and achievement history (Wentzel, 1991), research by Dweck and colleagues (Dweck, 1975; Dweck & Leggett, 1988; Dweck & Reppucci, 1973) has demonstrated that students who avoid challenges and show impairment in the face of difficulty are initially equal in ability to students willing

to seek challenges and show persistence. A critical question is why do individuals of equal ability adopt such marked goal orientations.

Dweck and Leggett (1988) present a model in which goals are mediated by individuals' beliefs and values. Likely influenced by parents' goals and beliefs (Ames & Archer, 1987), Dweck and Leggett argue that a child's implicit beliefs about ability are a consistent predictor of that child's goal orientation. Children who believe intelligence is incremental tend to adopt a mastery goal orientation, whereas those who believe intelligence is a fixed entity are more likely to possess a performance goal orientation. Dweck and Leggett's framework integrates cognitive and affective components of goal-directed behavior. Their framework suggests a cycle where students' self-conceptions foster their adoption of achievement goals; students' goal orientations set up a pattern of responding to academic challenges; the outcome of students' academic behavior, in turn, shapes their beliefs and values.

The impact of students' beliefs about mathematics and school is well documented (e.g., Cooney, 1985; Schoenfeld, 1987; Thompson, 1984, 1985). Middle school appears to be an important time to account for student beliefs. Anderman and Maehr (1994) point to findings suggesting students' beliefs, definitions, and attributions concerning ability change substantially during late childhood and early adolescence.

Success, or lack thereof, in mathematics is a powerful influence on the motivation to achieve (Middleton & Spanias, 1999). Because of repeated lack of success and the attribution of failure to lack of ability, students can develop a

sense of learned helplessness and view success as unattainable (Dweck, 1986). Helpless[4] individuals are more likely to adopt a range of maladaptive academic and social behaviors.

By the eighth grade, students in my class had received consistence evidence of their perceived incompetence. Each of the students had been enrolled in summer school prior to their eighth grade year, they performed at the lowest level on the state's standardized mathematics tests, and their report card data commented that they were below grade level. My students exhibited a range of helpless behaviors. For example, many times during the first quarter students would not even attempt to work on a problem until I approached them. They appeared to lack the confidence in their own ability to even read the problem. They displayed defensive strategies for coping with failure like avoiding schoolwork, blaming me for not adequately preparing them, and acting disruptively in class.

Seven of the eight students were participants in the school's alternative education program, indicating that they had a history of significant behavior and academic concerns. For many students with classroom behavioral issues, there exists an underlying academic cause. Finn (1989) argues that when a student becomes more and more embarrassed and frustrated by school failure, he or she may exhibit increasingly inappropriate behavior. Insubordinate behavior becomes the focus of a teacher's attention, further reducing learning opportunities, and in extreme cases, according to Finn, the "problem behavior is exacerbated until the student withdraws or is removed entirely from participating in the school

[4] Dweck and Leggett (1988) use the notion of *helpless* to characterize children who tend to avoid challenges and whose performance deteriorates in the face of obstacles.

environment" (p. 122). To disrupt the cycle, Finn argues that schools are faced with the difficult challenge of "increasing students' performance, not to mention self-esteem, perhaps against high resistance on the student's part and a host of external influences" (p. 122). Although challenging, it appears that disengaged and academically withdrawn middle school students can develop more positive work habits.

Malleability of goal orientations

Middle school should be a time of urgency when addressing issues of student motivation and achievement. Middleton and Spanias (1999) argue that middle grade students' motivations toward mathematics tend to crystallize into their adult forms. Students' beliefs and values in the middle grades predict the courses taken and mathematics achievement in high school and college (Meyer & Fennema, 1985). Pintrich, Conley, and Kempler (2003) observes that, although achievement goals were once seen along a single continuum, current research findings suggest that students can endorse multiple goals simultaneously, and may even actively select which type of goal to adopt depending on the affordances of the circumstance.

Dweck (1986) describes a situation where an overconcern with ability may lead students to avoid difficult tasks. Concerned that even a mere exertion of effort might threaten a student's demonstration of ability, a student with a strong performance-approach goal orientation can slip into an avoidance orientation. Through the use of longitudinal survey data, Middlton, Kaplan, and Midgley (2004) found that students who expressed high self-efficacy and performance-

approach goals early in middle school shifted toward performance-avoidance goals later in middle school. Their findings suggest that there are cases when performance-approach and performance-avoidance orientations may be the same achievement goal, and the adoption of approach and avoidance orientations is merely a matter of the situation.

Utilizing a social-cognitive perspective to view goals, it should be expected that, as contexts change, students reevaluate their goals and actions. In fact, a change in school environment often fosters a change in students' goal orientation (Anderman & Midgley, 1997). Although the majority of adolescents make the transition from elementary to middle school without excessive trauma, the changes in environments can be profound to many students. In their *stage-environment fit* theory, Eccles and Midgley (1989) provides a plausible explanation for the declines in behavior and academic motivation by pointing out how the learning environment of typical middle school classrooms do not fit the developmental needs of young adolescents. For example, the shift to middle school is associated with an increase in practices such as whole-class task organization, between-classroom ability grouping, and public evaluation of the correctness of work at a time when young adolescents have a heightened concern about their status in relation to their peers. Adolescents' desires for increased autonomy and participation in classroom decision making arise when many middle grade classrooms are characterized by a greater emphasis on teacher control and discipline. Using the lens of goal theory, Anderman and Anderman (1999) attests to the plausibility of Eccles and Midgley's results by finding that

mastery goals decreased and performance goals increased as students make the transition from elementary to middle school. Thus, it is plausible that mismatches between the psychological needs of students and the middle school environment contributes to a decline in the adolescents' motivation and interest towards school (Eccles et al., 1993).

In theory, changes in context can influence students' goal orientations. There is limited research on students' motivation in reform-oriented mathematics settings (Middleton & Spanias, 1999). Most of the research in this area has seemed to focus on shifts from mastery to performance goals or changes in students' avoidance orientations. I was unable to find research documenting shifts in middle school students from performance to mastery goals. Overall, mastery and performance goals appear to be relatively stable during middle school (Middleton, Kaplan, & Midgley, 2004). Although my study did not include an a priori plan to examine students' goal orientations, a post-hoc analysis of the data reveals that the students clearly shifted from a performance-avoidance orientation to a performance-approach one. Over the course of the year, they demonstrated an increased willingness to engage and persist in solving cognitively demanding tasks. No evidence suggests my students had acquired a mastery goal orientation.

Teacher influences on goal orientations

My findings indicate that teacher dependent contextual factors can influence shifts in a student's goal orientation. The decisions and actions I made regarding task design, my style of instruction, and the quality of teacher-student interpersonal

relationships appeared to be significant factors associated with the personal goals of students. Findings from my research provide valuable information regarding classroom level decisions that influence students' motivations. Although research evidence suggests that students' achievement goals in mathematics are related to a combination of both student factors and features of the classroom context (e.g. Ames, 1992; Patrick, Turner, Meyer, & Midgley, 2003; Turner et al., 2002; Turner & Patrick, 2004), the need exists to examine specific teacher behaviors and classroom structures that are important in influencing students' perceptions of the classroom goal structure (Pintrich, Conley, & Kepler, 2003).

The need to provide students with a caring and supportive environment was a chief finding from my study. The students' motivation to meaningfully engage in a mathematics problem was linked with earning their trust and respect. Similar findings exist among goal theorists who have examined issues of affect in classrooms. Following 248 students from sixth to eighth grade, Wentzel (1997) explored whether adolescent students' motivation to participate changed in response to feelings of being supported and cared for by teachers. After controlling for previous motivation and beliefs, Wentzel's research provides strong evidence that students are more likely to engage in classroom activities if they feel supported and valued. Caring teachers in Wentzel's study were described as "demonstrating democratic interaction styles, developing expectations for student behavior in light of individual differences, modeling a 'caring' attitude toward their own work, and providing constructive feedback" (pp. 415-416). Similarly, Roeser, Midgley, and Urdan (1996) found that middle

school students' goals toward mastery were positively correlated with their perception of caring, respectful teachers and positive student-teacher relationships. For Middleton and Spanias (1999), the most important conclusion drawn from the findings of several studies on motivation, is that a "supportive, authoritative teacher serving as a model and as a friend gives children the confidence and feelings of self-worth necessary to be comfortable in mathematics" (p. 82).

Another key finding from my study was that the tasks and learning activities a teacher selects conveys messages to students about their ability, and affects students' willingness to apply effortful strategies. I found that ideal tasks should be at an appropriate level of difficulty based on knowledge of students' prior understanding and problem solving skills. Tasks should allow students control of choosing problem solving strategies by having multiple entry points. Tasks should be interesting, relevant and offer students a personal challenge. This notion is supported by a review of evidence that found that mastery orientations are promoted in classrooms affording students' autonomy and decision-making in both the organization of the class and in developing strategies for task solutions (Ames, 1992). Ames (1992) suggests that variety, diversity, challenge, and control are task dimensions shown to affect student perceptions of classroom goals and personal goal orientations. Meece (1991) reported on teachers who created highly mastery-oriented classrooms by adapting instruction to students' level of understanding, supported students' autonomy, and provided opportunities for collaboration.

In contrast, I found that tasks with a low cognitive demand were associated with less effort and more problematic student behaviors. This notion is supported by research suggesting that students more frequently display motivational patterns of avoiding challenging tasks if they are routinely presented with tasks that can be completed successfully with little effort (Jagacinski & Nicholls, 1984) or asked to solve problems which require the use of short term learning strategies, such as memorizing and rehearsing (Meece, Blumenfeld, & Hoyle, 1988).

Tasks are embedded in the social organization of the classroom. Cognitive engagement patterns are shaped not only by the structure of the task, but how the task is implemented and how the class interacts with the task. In classrooms, "teachers employ an array of instructional practices that are, in high probability, a mixture of different messages and cues that can influence the endorsement of both mastery and performance goals" (Pintrich, Conley, & Kempler, 2003, p. 327).

When implementing tasks, a consistent observation from Sandy and Sue was the press I made to students to focus more on the process of solving a problem, and less on a final answer. Sue noted that I did not differentiate my responses to correct and incorrect solutions, and that student mistakes were lauded as learning opportunities. Turner et al. (2002) highlighted the importance of this instructional move. In an analysis of 65 sixth grade classrooms exploring the relationship between the classroom learning environment and students' use of avoidance strategies, they found that lower incidences of avoidance strategies were evident in classrooms where teachers conveyed mastery messages to their students. Achievement mastery messages were conveyed, in part, through explicit

admonitions to students not to feel inadequate or ashamed when they did not understand something and by emphasizing that a necessary part of learning involved being unsure and learning from mistakes.

Conversely, discourse patterns that emphasized cognitive aspects, such as final answers or sharing reasoning without adequate understanding, or did not explicitly address the motivation concerns of students were typical of high-avoidance classrooms (Turner et al., 2002). Additionally, telling students what to think or do limits their opportunity for autonomy. Turner et al. (2003) found that teacher use of nonsupportive instructional discourse patterns, such as telling, were typically characteristic of high avoidance settings. When teachers engaged students in supportive instructional discourse, or scaffolding, students demonstrated increased competence and ownership over their own learning.

In addition to accepting student solution ideas without adjudicating the correctness of a response, Sue remarked how unusual it seemed not to praise students when they answered a question correctly. According to Sue, "It really means something to them when they get the right answer. But, you never really say to the students 'good job' or anything like that." This subtle move may have had significant consequences. Dweck (1999) points out that every time teachers give feedback to students, they convey messages that affect students' motivation. A common fallacy among educators, contends Dweck (1999), is to think that giving students many opportunities to experience success and then praising them for their intelligence will increase a student's confidence and motivation to succeed. However, in a study with more than 400 fifth-grade students, Mueller

and Dweck (1998) found that when children are praised for their intelligence, they become overly concerned about making mistakes and either choose simple tasks or avoid challenges altogether. Alternatively, children praised for their effort, their concentration, the effectiveness of their strategies, or their interesting ideas, demonstrated a desire to work on challenging tasks and held a higher sense of self-esteem. To that end, Middleton and Spainas (1999) recommends the practice of allowing children to struggle and to value their mistakes when solving challenging problems so that their confidence is not shattered when they encounter a problem that cannot be solved in a routine fashion.

I consciously made an attempt to select tasks so that a guess and check strategy could lead to progress in solving a problem. I emphasized that guessing and checking was a legitimate means to solve a problem, and pointed out that patience and effort were key attributes to students using a guess and check strategy. Research exists suggesting that attribution training can be effective in helping students develop positive motivational patterns; students who received attribution training displayed superior self-efficacy gains and fewer avoidance characteristics compared with students receiving no attribution training (Middlton & Spanias, 1999). According to Dweck (1986), "retraining children's attributions for failure (teaching them to attribute their failures to effort or strategy instead of ability) has been shown to produce sizable changes in persistence in the face of failure, changes that persist over time and generalize across task" (p. 1046).

Another decision I made that appeared to positively affect my students' disposition was the routine use of graphing calculators and concrete reference.

Sandy noted that over time students became more skilled at using manipulatives to monitor their own thinking strategies. Sue saw that students used a graphing calculator as a primary tool to help in solving problems. Research supplies ample evidence of the positive benefits graphing calculators has on both students' affect and their mathematical understandings (e.g. Dunham, 1996). In a meta-analysis of the effects of calculators on students' achievements and attitudes, Ellington (2003) found that students' operational skills and problem-solving skills improved when calculators were an integral part of testing and instruction, and that students calculator counterparts. No existing research was found relating goal orientations to calculator use.

Socio-cultural and school influences on goal orientations

Even if a teacher adopts a range of actions that research says is related with student mastery behavior, the larger context of school and society may moderate students' selection of adaptive goal orientations. Students' goal orientations are mediated through the climate of the school and their relationships with parents and peers. Ethnographic research findings demonstrate that cultural and social-class differences can significantly affect student behavior and their educational outcomes (Murrell, 1994).

The existing structure of many middle schools can create a climate that undermines both teacher and student motivation (Eccles et al., 1993). The shift to middle school typically involves an increased emphasis on testing and the assignment of grades. In the current educational climate of *No Child Left Behind*, middle schools are mandated to assess the yearly mathematical progress of every

student. To prepare for these high stakes tests, many middle schools, like the one I worked in, opt to give local assessments intermittingly throughout the year. These assessments are used to make a judgment on student progress and teacher effectiveness. The focus on test scores is likely to increase social comparison, concerns about evaluation, and competitiveness, which, for young adolescents, are negatively related to intrinsic motivation and adaptive forms of behavior. The public scrutiny on test scores and evaluation, which make aptitude differences more salient to both teachers and students, places emphasis on performance rather than mastery (Eccles et al., 1993). Meece, Anderman, and Anderman (2006) note a "careful examination of the effects of NCLB on student achievement, motivation, and emotional well-being is needed" (p. 498).

In my study, I found that the social goals for my students seemed to interfere with their concerns for academic work. Summers, Schallert, and Ritter (2003) found that middle school mathematics students who expressed a low level of mastery goals were more influenced by comparisons to close friends than to other students in the class. The students in my class were from the same social clique. The tight cohesiveness and friendships in the group likely contributed to their miseducative behaviors. In particular, Erika and Keisha seemed to support and accentuate one another's behavior. Chris presented a unique case study of a student whose actions and behavior starkly changed when I moved him into an Algebra group.

In a case study examining patterns of defiant student behavior in two different schools, McFarland (2001) found that the density of one's friendship network and

access to public discourse are associated with high levels of resistant efforts and disruptive behavior on the part of students. McFarland explains:

> Students with dense friendship networks, rebellious friends, and prominence in the classroom friendship network are more likely to disrupt class tasks and enter disputes with the teacher. Dense networks buffer young people in conflicts and provide them with social support. Rebellious friends act as a reference group, to whose behavior the student is pressured to conform. Social prominence in classroom friendship networks affords the student support beyond his or her own clique and attributes status value to their actions. [p. 663]

For McFarland, when looking at disruptive acts of behavior, the unit of analysis is a student within a particular classroom. Changing either the classroom or student would change the decision to resist academic work. In addition to the characteristics of social networks, McFarland argues that the type of instruction a teacher delivers can contribute to student decisions to rebel. When students with dense friendship networks become disenchanted with the subject or alienated from the teacher, student-centered tasks affords students the opportunity to express and spread their discontent. Although not recommending teacher-centered tasks as a preferred form of instruction, McFarland suggests that "[t]eachers can use teacher-centered tasks to minimize student opportunities at voicing resistance. . . . it prevents dense cliques from expressing their discontent in collective fashion" (p. 666).

Students' beliefs about their academic situations can factor into how they behave in class. Like Jamaal and Cedric, students who associate their classrooms with academic stigmatization are likely to react against and avoid academic activities (Oakes, 1985; Schwartz, 1981). While not denying the powerful impacts of teachers, Schwartz (1981) ultimately concludes that "classroom behavior of student and teachers alike is organized by a powerful system of

institutional expectations in which rank predominates" (p. 118). In an ethnographic study of four inner-city schools, Schwartz found that low-track social ties hindered and subverted participation in class work, and low-track students related to peers in an overt and disruptive manner. Other characteristics of low-track classrooms identified by Schwartz were that low-track students often tease one another, accuse each other of cheating and being stupid, move around the classroom, and use academic resources inappropriately.

In addition to the influence of schools, peer networks, and parents, cultural and social class differences can play a significant part in determining the actions and motivations of traditionally underserved students. In my study, I am aware that cultural and racial differences existed between the students and me. With a different, more critical lens, I recognize that a different interpretation can be posited regarding my students' disposition toward mathematics.

Mainstream, middle-class teachers are increasingly being called upon to teach mathematics in urban school contexts to schoolchildren of color who do not share their same assumptions about learning and teaching (e.g., Delpit, 1995; Heath, 1983). Ogbu (1991) theorizes that, as involuntary minorities, African American students tend to view education as a system controlled by the group that subjugated and oppressed them and their ancestors. Many African American students have developed a belief system that discounts formal education as a tool for social mobility. Suffering from "low effort syndrome" (p. 437), Ogbu argues that many African American students do not see any point in working hard or maintaining their efforts long enough to achieve academically.

In a critical ethnography of middle school mathematics classrooms, Murrell (1994) sought to account for reasons why *Standards*-based teaching practices, meant to promote deeper understanding of mathematics, actually diminish African American students opportunities to learn. Focusing on one of NCTM's (2000) five process standards, communication, Murrell looked at patterns of classroom discourse. The NCTM standards are based on the expectation that key mathematics will be developed through eliciting thoughtful student explanations and justifications of their solutions to problems. In analyzing the discourse patterns and speech events in mathematics classrooms, Murrell showed that the African American male students in the study framed the instructional intent of discourse differently than their teachers. While the teachers' goal for discourse was to use the rationales of student solutions to build conceptual understanding of mathematics, the African American male students tended to engage in superficial aspects of math talk. These students placed a high emphasis on their verbal adroitness as a criterion for doing well in mathematics class. Applying suggestions from Delpit (1988), Murrell argues that "making explicit the rules, codes, and expected performances of classroom discourse is essential to helping non-mainstream students develop reasoning competence in discourse" (p. 567).

Conclusion

Teachers do not choose whom they teach. On any given day, a teacher is certain to come across students who are resistant to participate in any kind of mathematical activity. Even worse, the behavior of these students can divert the teacher's and class attention away from their important work. Motivating

students so that they are willing to engage and persist in solving challenging mathematical tasks can be a daunting challenge, especially for teachers of students who have consistently had negative experiences and persistent failures in mathematics classes. There are a myriad of complex, intertwining reasons why students fail to put forth effort. Ames (1992) hypothesizes that structures of task selection, evaluation, and authority are mutually dependent on each other and interact in a multiplicative manner. Mathematics education research, from an achievement goal theory lens, may provide important insights regarding how specific teaching behaviors and structure can influence students' motivation to engage in mathematics.

Revised Multiple Solution Norm

In reflecting on the results of my study, I am concerned that someone who doubted the feasibility of instantiating a multiple solution norm to a group of low attaining students would walk away thinking that my research provides firm evidence against doing this type of teaching in certain contexts. I'm worried that these individuals will use my study to draw the conclusion that students do not benefit from exploring alternate solution strategies and that using problems to teach low attaining students takes up too much time. Mostly, it troubles me to think that some teachers will maintain that the most effective, if not only, way to teach low attaining students is to draw from a core menu of teaching acts consistent with traditional teacher-centered instruction that Haberman (1991) referred to as the *pedagogy of poverty*. These teachers will, for the good of their students, attempt to simplify the mathematics by reducing it to a sequence of small smooth steps that can be easily traversed by leading

students through a chain of reasoning and asking them to fill in gaps with arithmetic answers, or low-level recall of facts (Watson, 2002). Watson (2002) observes that this style of pedagogy does not indicate that anything new has been learnt. According to Watson, although low attaining students have likely "experienced a melee of concrete images, money calculations, helpful hints, special cases, partially-remembered rules and so on," they have "no access to any secure conceptual framework on which future number work can be developed, or which can be called into play when necessary in other mathematical contexts." (p. 462).

I suppose skeptics of the work I attempted to do could evaluate the effectiveness of my teaching efforts by judging it against my definition of a multiple solution norm and conclude that my teaching was by and large unsuccessful. Unfortunately, throughout my study, student explanations seldom went beyond a step-by-step procedural description, and by year's end, the students still were not good at listening to their classmates' ideas. However, if one looked at the progress the students made regarding their willingness to engage in mathematical activity and persist in solving challenging mathematical tasks, then an argument could be made that I enjoyed a successful year. The improvement in the students' mathematical disposition is particularly noteworthy when accounting for the context of working with students with a history of low academic performance and a range of miseducative behaviors.

In light of these accomplishments and the lessons learned from the study, I am making a number of revisions to my original framework for a multiple solution norm (Chapter 3).

Within the context of using students' inventive solution strategies as a central feature of instruction in teaching middle grade students with a history of academic and behavior concerns, my revised multiple solution norm parses the original social norms into more well-defined expectations of observable student actions and proposes significant changes to the desired sociomathematical norms. In this section, I will explain that it is important to consider the context of a given class when analyzing how classrooms utilize students' multiple solutions. Figure 5 shows the revised multiple solution norm framework, and Figures 6 and 7 compare the original and revised social norms and sociomathematical norms respectively.

Social Norm	Sociomathematical Norm
Students display appropriate academic behavior and willingness to attempt assigned mathematical work	Students and Teacher publicly present mathematical explanations for solution strategies
Students willing to put forth effort on non-routine problem-solving task	Students and Teacher differentiate solution strategies
Students willing to share ideas in cooperative group settings	Students and Teacher analyze/evaluate soundness of solution strategies
Students publicly present solution strategies	Students and Teacher use mathematical mistakes as learning opportunities
Students listen to and respect the ideas of others	Students and Teacher make connections between solution strategies and understand relative advantages of different approaches
Students comment on solution strategies	
Students share responsibility with teacher for adjudicating correctness of solution	

Figure 5: Revised Multiple Solution Norm

Social Norms	
Original Framework	**Revised Framework**
Students possess a productive disposition	Students display appropriate academic behavior and willingness to attempt assigned mathematical work
	Students willing to put forth effort on non-routine problem-solving task
Students listen to, respect, and comment on solution strategies	Students willing to share ideas in cooperative group settings
	Students publicly present solution strategies
	Students listen to and respect the ideas of others
	Students comment on solution strategies
Teachers and students share responsibility for adjudicating correctness of solutions	Students share responsibility with teacher for adjudicating correctness of solution

Figure 6: Original vs. Revised Social Norms

Sociomathematical Norms	
Original Framework	**Revised Framework**
Students publicly present mathematical explanations for their solution strategies	Students and Teacher publicly present mathematical explanations for solution strategies
Students analyze/evaluate solution strategies	Students and Teacher differentiate solution strategies
	Students and Teacher analyze/evaluate soundness of solution strategies
Students use mathematical mistakes as learning opportunities	Students and Teacher use mathematical mistakes as learning opportunities
Students compare, contrast, and make connections between solution strategies	Students and Teacher make connections between solution strategies and understand relative advantages of different approaches

Figure 7: Original vs. Revised Sociomathematical Norms

Original versus revised social norms

In my original framework, the two social norms that students possess a productive disposition and that students listen to, respect, and comment on solution strategies encompassed too large a range of social activity to adequately capture the challenges I encountered and the progress the students made in response to academic work. From the onset of the study, I was aware that convincing students to persist and expend effort on an uncertain task might be a difficult thing to do, however, I did not anticipate the challenges I faced in attempting to create an engaging classroom. My students apparent lack of motivation to engage in mathematical work and their related miseducative behaviors were significant obstacles I needed to overcome in order to negotiate the expectation that students should give serious attempt to solve a non-routine task. Sue recognized that student behavior was a chief concern in the class. After an observation early in the year, Sue stated, "You've got a hard group. . . . What I see as one of your biggest obstacles is just getting the students in their seats and quiet long enough for you to explain what you want them to do." As discussed in the previous chapter, significant progress was made when students were willing to engage in a problem within the first ten minutes of the period. In the proposal for this study, I did not foresee the struggles dealing with classroom management issues. The revised framework reflects the challenges of addressing student behavior and student effort as distinct components of productive disposition.

In the original framework for a multiple solution norm, I used one norm to group together activity where students would listen to, respect, and comment on other students' solution strategies (Figure 6). Within this norm, I tacitly assumed that

students would be willing to share their ideas, both during cooperative work and when speaking to the entire group. Unfortunately, in my context, students did not arrive to class with a natural tendency to talk about mathematics. As noted in the previous chapter, a significant development in the class was the constitution of the normative expectation that students would engage in meaningful math talk and sharing of ideas. By May, together around a single table, students would shout out various ideas and solution strategies, and some students would privately collaborate together. My revised multiple solution norm makes it an explicit goal to negotiate an expectation for students to share their thinking with their classmates while working in cooperative settings and be willing to publicly present their ideas.

Although the sharing of ideas marked a positive development in student activity, the nature of their sharing lacked focus, and often escalated into students shouting at, and speaking over one another. In examining discourse in a second grade classroom in which the teaching practices were fundamentally different from conventional mathematics instruction found in most elementary schools, Wood (1999) found that it was important to establish expectations for students to explain their solutions to others, however it was an even more significant to establish expectations for students as listeners. According to Wood, "listeners were expected to follow the thinking and reasoning of others to determine whether what was presented was logical and made sense" (p. 189). Thus, consistent with my original framework, the revised multiple solution norm contains expectations that students respectfully listen to and appropriate the ideas of their classmates. I propose that it makes sense to think of these as two separate norms. One where students respectfully listen to other students,

and two, where students are expected to comment on solution strategies by asking clarifying questions or summarizing another student's ideas.

Original versus revised sociomathematical norms

In a classroom governed by a multiple solution norm, students assist one another's learning of mathematics by engaging in productive mathematical discourse. The successful implementation of a multiple solution norm requires that students explain, defend, and justify their own ideas, and critique the ideas of others. As described in the review of literature in Chapter 2, establishing a discourse community is challenging and complex work that involves the (re)negotiation of classroom norms and expectations regarding both the purpose of discussion and the roles of the teacher and students during discussions.

The examination of classrooms of students engaged in genuine mathematical dialogue has provided important insights regarding ways that teachers might establish norms and expectations conducive to promoting productive discourse. For example, Manouchehri and Enderson (1999) recommend that tasks be designed with multiple entry points so that all students have the opportunity to engage in the activity, regardless of their level of mathematical knowledge. To establish norms for discourse, Manouchehri and Enderson recommend that teachers insist their students "explain personal solutions to their peers, listen to and try to make sense of one another's explanations, attempt to achieve consensus about an answer, and resolve conflicting interpretations and solutions" (p. 221). They stress the importance of making explicit these norms as rules or principles to be followed from the first day of school, and to capitalize on specific incidents in which students' activity either

instantiated or transgressed a norm by using them as opportunities to discuss desired expectations and behavior. Similarly, Sherin, Mendez, and Louis (2000) concluded that significant effort is required at the beginning of the school year to create a classroom community in which discourse is used to support the mathematical learning of all students.

Prior to conducting this study, I abided by suggestions such as these to successfully establish and manage a discourse community in above-grade level Algebra and Geometry classes. In these classes, students routinely discovered important mathematical concepts by explaining, questioning, and agreeing or disagreeing on solutions to problems. In moments of disagreement, the pattern of discourse closely resembled the following argumentative interaction pattern described by Wood (1999, p.179):

1. A child (*or group*) provided an explanation of her or his solution to the problem (italics added).
2. A challenge was issued from a listener who disagreed with the solution presented. The challenger might or might not tell why he or she disagreed.
3. The explainer offered a justification for her or his explanation.
4. At this point, the challenger might accept the explanation or might continue to disagree by offering a further explanation or rationale for his or her position.
5. The explainer continued to offer further justification for her or his solution.
6. This process continued and other listeners sometimes contributed in an attempt to resolve the contradiction.

7. The exchange continued until the members of the class (including the teacher) were satisfied that the disagreement was resolved.

I contend that a number of intertwined instructional features helped create this particular context with my students. Each day I started class with a challenging problem and provided students with the time and space to cooperatively work together to derive a solution. Within this cooperative group setting, I pressed students to share their ideas and explain their personal solutions to each other. I pushed each group to agree upon one solution strategy that would be presented to the class, and made clear that I valued the strength of their reasoning rather than their final answer. I expected each group member be able to articulate the group's consensus strategy. I used time to listen to each group's strategy, and, without judging the correctness of their ideas, often asked questions to indicate where their explanation needed more support.

In whole-group discussions, I explicitly stressed the importance of carefully attending to the ideas being presented. To constitute the expectation that students listen to and appropriate other's ideas, I held students responsible for either using their own words to revoice a group's explanation, or expected them to ask a clarifying question. After an explanation was offered, I typically would say something like, "At this point, you should be able to summarize what the strategy is, or you should have a question that you can ask."

Student work was generally recorded on a front board. At times, I recorded what was being said by using group names to label the strategies and conjectures that were being put forth, and other times, group members would use board space to present

their solutions. Like a teacher in Wilson and Lloyd's (2000) study, I typically facilitated discussion from the back and sides of the classroom in an attempt to give students more opportunities to share control over the flow and, to some extent, the content of the discussions. To emphasize the importance of multiple solutions, I consistently asked groups to offer a different way to solve a problem, or asked if anyone had something to add to a solution. Often I used my assessment of student work during problem solving to solicit specific responses from groups. In asking students to compare the relative advantages of different solution strategies, I often sent them back to their small groups for further investigation.

Creating this type of classroom culture did not proceed linearly. In fact, I found that during the first few days of school, students readily complied with my requests for classroom structure and it was typical to facilitate an interesting discussion of student ideas during these first classes. After a few productive discussions, however, it was not uncommon for some students to grow frustrated with my goals for classroom activity. There were times when students whose previous experience suggested that the teacher should provide more direction, who were not accustomed to finding mathematics problematic, or who were not used to spending so much time and effort with a single problem would either tacitly or overtly resist my efforts to shift their roles as learners. I found it important to manage these moments by displaying empathy for the students' concerns, offering to give additional support outside of class, and to provide students with the rationale behind my approach. It was these kinds of teacher moves I employed in constituting a multiple solution norm that I intended to systematically study and make record of in the current study.

Unfortunately, a similar set of teaching moves that resulted in the constitution of a multiple solution norm in my Algebra and Geometry classes did not result in a successful instantiation in my pre-Algebra class. The sociomathematical norms I sought to create were never evident in my classroom, and I rarely felt that whole-class discussion was a productive use of class time. My efforts to engage students in discourse were informed and constrained by my perceptions of their understanding. I made decisions about the appropriateness of discussions based on my hypothesis of the process of learning and with my interactions with students. Although I was working towards creating and maintaining a desired set of norms, I, as their teacher, was concerned that the unproductive use of instructional time during whole-class discussion, and the manner in which students appeared to misbehave and lose focus, was detracting from the students' learning.

The following excerpt from a whole-class discussion on September 14 is presented as an illustrative example of my concerns regarding the use of valuable class time to discuss students' solutions:

	Cedric shared 28 baseball cards with his friends. Six of his friends received exactly one card each, five of his friends received exactly two cards, and the rest of his friends received exactly three cards. How many friends received exactly three cards? Explain how you determined your answer.
RH:	Let's have someone discuss how you did it, and then if you have something to add to it you can. Alan, do want to start?
Alan:	I did 5 times 2 and got 10, then I added the 6 and got 16, then I . . . [*quietly*] what did I do?
	[*Long pause as Alan tries to look back over his work. Conversations in the class become louder and off-task*]
Alan:	It's 4.
RH:	Can the conversations end for a minute please? What's 4?
Alan:	The answer. Four friends received exactly 3 cards each.
Keisha:	Can I show how I got it?
RH:	Wait a minute, does anyone know how Alan got the 4?

Keisha:	I can tell you how I got 4?
RH:	But, I'm still not sure how Alan got it.
Keisha:	Can I just show you what I did?
RH:	You think you did it a different way?
Keisha:	Yeah, this is what I did
	[*Keisha feels free to go to the board. She draws 28 lines representing a card and then crosses off sixteen of them and makes 4 circles each containing 3 lines. The class is very chatty and off-task during this*.]
RH:	What does this show us?
Keisha:	That it's four because that's the circles.
RH:	O.k., you also have 4. Can anyone compare the difference between Alan's and Keisha's?
Jordan:	They're different?
RH:	Alan, you got the same answer, did the way you get 4, was it the similar or different than Keisha's?
Alan:	I didn't draw out all those sticks [*laughter*].
Jamaal:	Alan's way is way easier.
RH:	And why's that?
Jamaal:	It's shorter, why's she always doing it the long way and making it harder and all?

It is likely I could have been more skillful in facilitating this discussion and perhaps a different set of actions existed so that the conversation would have created more meaning to the students. However, the set of actions and behaviors in this episode is representative of the challenges I encountered when asking students to publicly share and compare their solution strategies. In whole-class discussions it was common for students, like Alan, to have difficulty articulating their solution strategies. What made these moments particularly problematic was how quickly other students would use these hesitations to lose focus and get off-task. Additionally, as discussed in Chapter 5, important mathematical conceptions were rarely put forth as students typically explained how an answer was obtained with mostly procedural explanations. Also illustrated in the above episode, was my concern that students infrequently attempted to listen to and understand other

students' strategies. Keisha, although eager to demonstrate her solution process, short-circuited my attempts for a more detailed discussion of Alan's solution.

Although, throughout the study, I became increasingly pleased at my students willingness to engage in mathematical work and their ability to produce legitimate solutions and think in mathematically sophisticated ways, I was not able to resolve the above dilemmas in getting the students to share their ideas in a productive manner. My current thinking regarding the lack of success I experienced in establishing a discourse community is that I did not recognize my students' struggles to use appropriate language and vocabulary for describing their ideas.

As discussed in Chapter 5, both Sandy and Sue noted that the students' fragile mathematical knowledge and limited vocabulary were factors preventing them from communicating their ideas in ways that other students would find helpful. As highlighted in the previous chapter, Sandy posited that the students had a hard time talking about math because, "Their knowledge of their own solutions wasn't explicit enough, they only understood what they were doing tacitly." Sue noted that it was difficult for the students to express their ideas because they did not possess a sufficient vocabulary to communicate their thinking. From this perspective, the sociomathematical norms in the original framework were not instantiated because the students found it difficult, if not impossible, to make an adequate interpretation of other students' work and to juxtapose their ideas with their classmates.

The revised sociomathematical norms in Figure 7 recognize that, when facilitating whole-class discussion, the teacher may need to do greater amounts of telling than I initially imagined. The revised sociomathematical norms suggest that a teacher

explain, analyze, and make connections between students' solution strategies. I conjecture that if I had assumed a greater responsibility regarding explaining and comparing students' ideas, then more progress towards the constitution of a multiple solution norm would have been made. Unfortunately, I did not consider assuming such a proactive role during my teaching. My philosophy was in part guided by Reinhart's (2000) article, "Never say anything a kid can say." I strongly believe that "by encouraging students to share and discus methods of solution, they have a chance to clarify their ideas for themselves and others. When students' intuitive strategies are made public, they can be analyzed more deeply and everyone can learn from them" (Hiebert et al., 1997, p. 45). At the time of the study, I held tightly onto the notion that it was critically important for students to express and compare their own ideas. I purposefully resisted the move to speak for a student or make a connection between different solution methods. This resistance was reinforced by my concerns that excessive telling would result in creating norms that would be inconsistent with my visions for classroom activity. For example, I was wary that assuming such an active role in explaining ideas may bolster expectations that students would not have to listen to one another (O'Connor & Michaels, 1993).

My concerns regarding the potential negative consequences of too much telling is consistent with the telling/not-telling dilemma expressed by a number of mathematics educators (i.e., Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005; Romagnano, 1994; Smith, 1996). Lobato, Clarke, and Ellis (2005) state, "Whether or not one should tell is a central tension for teachers and teacher educators concerned with developing a practice connected to constructivist tenets" (p. 102). Chazan and Ball

(1999) note that the term telling is insufficiently precise and there exist a number of opportunities when it is appropriate to tell students something. It should be emphasized that my issue regarding telling is not centered on new kinds of information teachers might tell students when they struggle solving a mathematical problem. My chief concern, here, is what to do after students have arrived at solution strategies, but appear unable to communicate their ideas in a useful way. It was not until analyzing my data, when I recognized the need to address the question, "What should you do when a kid doesn't know what to say?"

Huffard-Ackles, Fuson, and Sherin (2004) created a useful framework for thinking about such a question. Their framework provides a way to describe and evaluate the process a class goes through in the building of a *math-talk* learning community – "a community in which individuals assist one another's learning of mathematics by engaging in meaningful mathematical discourse" (p. 81). The framework, which examines four categories related to establishing productive mathematical discourse (questioning, explaining math thinking, source of mathematical ideas, and responsibility for learning), provides teachers with steps to develop their classrooms into a rich math-talk learning community. In the framework, a scale of 0 to 3 is used, where Level 0 represents a traditional, teacher-directed classroom and Level 3 represents a classroom with meaningful collaborative math-talk where students are able to explain, defend, and justify their mathematics thinking with confidence. Stein's (2007) condensed version of Huffard-Ackles, Fuson, and Sherin's (2004) framework is shown in Figure 8.

Levels of Discourse in a Mathematics Classroom	
Levels	**Characteristic of Discourse**
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's question.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

Figure 8: Stein's (2007) adaption of Huffard-Ackles, Fuson, and Sherin's (2004) framework for mathematical discourse.

Huffard-Ackles, Fuson, and Sherin (2004) recognize that students "must have a grasp of the language of the domain of mathematics in order to carry on math talk both to describe one's own thinking question or extend the work of others" (p. 111) and that teachers throughout the year might need to "fall back to Level 1 or Level 2 to assist students in building vocabulary and concepts in new content areas" (p. 112). In positioning my framework for a multiple solution norm against the framework of Huffard-Ackles, Fuson, and Sherin, it is evident that my revised set of sociomathematical norms is consistent with Level 1, whereas my original set of sociomathematical norms is consistent with Level 3. Thus, my revised sociomathematical norms suggest that teachers in classrooms of learners who find it challenging to communicate their ideas should first aim to create a classroom climate

consistent with Level 1 of Huffard-Ackles, Fuson, and Sherin's framework and assume a proactive role in explaining and analyzing student strategies.

Based on the assertion that the students' fragile mathematical knowledge restricted their ability to share their ideas in ways other students would find helpful, I do not believe it was appropriate to expect my low attaining students to approach whole-class discussion in the same way that my higher attaining students did. Students in my Algebra and Geometry classes appeared to have more ways to speak about their mathematical knowledge and ideas than the students in my pre-Algebra class. Next, I will explore how, given another opportunity to teach the same group of pre-Algebra students, I would attempt to instantiate the revised multiple solution norm.

Suggestions for instantiating the revised multiple solution norm

Since my construct of a multiple solution norm is dependent on negotiating normative expectations for students to be willing to engage and persist in solving non-routine problems, my first goal with the group would be to attend to the six factors identified in Chapter 5 (creating a supportive environment with strong teacher-student relationships; selecting and/or designing an appropriate task; instructional decisions; students' attitudes and beliefs; external influences; and dense network of student relationships) as being influential in shaping students' behavior and their willingness to do mathematics. I recommend beginning class with a problem designed to build on the students' existing understandings (Fennema, Franke, Carpenter, & Carey, 1993). Because the strength of my students' social relationships were so dense, I suggest physically organizing the classroom space so that students would be separated from one another.

A necessary condition to constituting a multiple solution norm is that students arrive at solutions using a variety of strategies. Thus, in planning a lesson, it would be important to consider a range of alternatives so that the task could either be extended or modified to make it more accessible or familiar to students. I maintain that, once expectations were negotiated so that my students readily attended to a mathematical task, they were capable of using their knowledge and problem-solving skills to derive mathematically sophisticated solutions. For example, in the above episode from September 14, Alan used an addition strategy to calculate the number of cards that were given away and then reasoned that four more three's were needed to account for all the cards. Keisha, using a semi-concrete representation, drew 28 line segments to represent the number of cards, crossed out the line segments representing the cards that were given away, and grouped the remaining line segments by three's and observed she had four groupings. Not included in the above transcript, Chris's solution was a more abstract version of Keisha's. Chris symbolically subtracted the known quantity of cards given away from 28 and then divided the remaining amount by three. As discussed in Chapter 5, Jordan found the correct answer by using a pattern that did not make mathematical sense in the context of the problem. The strategies that Jordan and the other students used are summarized in Figure 9. It should be noted that other researchers have similarly found that low attaining students are able to think in ways that enable others to succeed (i.e., Boaler, 2000; Chazan, 2000; Watson, 2002). For example, in a classroom research project designed to examine the mathematical thinking of low attaining 14-year old students, Watson (2002) found that the students were able to think in ways normally attributable to

successful mathematicians.

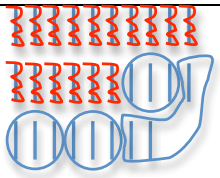
Summary of Jordan's Solution	Summary of Keisha's Solution	Summary of Alan's Solution	Summary of Chris's Solution
Six friends were given 1 card, five friends were given 2 cards, so four friends would get 3 cards		First $5 \cdot 2 = 10$, Then $10 + 6 = 16$, Then $16 + 3 + 3 + 3 + 3 = 28$ So the answer is 4.	First $28 - 6 = 22$, Then $22 - 10 = 12$, Then $12 \div 3 = 4$

Figure 9: Four students' solutions for the problem: *Cedric shared 28 baseball cards with his friends. Six of his friends received exactly one card each, five of his friends received exactly two cards, and the rest of his friends received exactly three cards. How many friends received exactly three cards?*

Unlike the students in my Algebra and Geometry classes, I believe that my pre-Algebra students needed support to help them make sense of their mathematical knowledge and ideas. I recommend that a teacher carefully attend to the explanations of as many students as possible. It is imperative that a teacher carefully listens to students and concentrates on their reasoning. While working with an individual student, I suggest that it is very important that the teacher suspend judgment on the quality and correctness of the student work, and instead aim to reformulate the student's ideas and communicate them back to the student. When reformulating student ideas, a teacher must be consistent with a student's true understandings and not recast a student's strategy so that it aligns with the teacher's ideas (O'Connor & Michaels, 1993). At the same time, a teacher should help students understand the mathematics behind their own strategies. In addition, I recommend that the teacher and student work together to record the student's ideas on paper. This written record will be used in the discussion of the problem and can be archived to serve as a

resource in future problem solving. For solution methods that will connect in important ways with upcoming content, it is recommended that the teacher and students display solution strategies on posters that can be displayed prominently around the room.

In whole class discussion, I propose that teachers follow a predictable routine for classroom discourse. Specifically, I recommend that the teacher start by explaining strategies that led to different answers. The students in my class highly valued obtaining a correct answer despite my attempts to emphasize the process over the product of ideas. Often, my students appeared to be unable to focus on how a problem was solved when the correctness of a final answer was in doubt. Thus, my first goal in a discussion of solution strategies would be to, when necessary, compare and contrast two different solution strategies: a strategy that led to an incorrect final answer and the least robust strategy that was used to obtain a correct final answer. An obvious concern with creating this structure is that students would be hesitant to share solutions they fear might not be correct. Therefore, it would be critical to value both strategies, praise the efforts of both students' work, and embrace the learning opportunities created through mathematical mistakes.

To make the structure and purpose of discourse transparent to students, I suggest a teacher should make an announcement, such as, "The first thing to do is see what the correct answer is. I saw two different answers, and I want to explain how these answers were obtained to see which one seems most reasonable." Because the students in my class often lost focus when a classmate stumbled over an idea or mispronounced a word, I suggest the teacher take the lead in explaining the two

different strategies. After explaining both strategies, it would be natural to ask students which strategy makes the most sense mathematically. Depending on student responses, I suggest that a teacher should point out the relative advantages and limitations of both solution methods and clearly communicate which method produced a correct answer.

Following an evaluation of the correctness of an answer, I recommend that a teacher continue the discussion of student ideas in similar ways by presenting solutions in order of increasing robustness. I argue that it might be important to start with solution strategies that are less efficient so as to preserve the opportunity to discuss important mathematical ideas and connections contained by these ideas. Typically, I found that once a succinct and efficient strategy was put forth, my students would view less robust solution methods as overly burdensome, and not be interested in considering these as viable solution strategies.

Throughout my study, I was concerned that the students seemed too eager to have their method explained without listening to other student's solutions. Unlike discussions in my Algebra and Geometry classes, students' comments in my pre-Algebra class were random and chaotic and did not build off one another. The purposeful presentation of least to most robust solution strategies is intended to create a structure that would set a useful pace and course for discussion and encourage students to listen and think about other strategies. Teachers should continually give students the opportunity to make comments and share their thinking by asking students what they think of a new strategy or idea. Yet, it is the teacher's responsibility to make sure key mathematical connections are pointed out. A

schematic representation summarizing these recommendations for organizing classroom discourse to a group of low attaining students is provided in Figure 10. These suggestions are not intended to be a prescription to cure classrooms where discourse is not a productive use of time, but to help a teacher think of ways to structure whole class discussion so that its purposes and roles for students are more transparent and mathematically valuable.

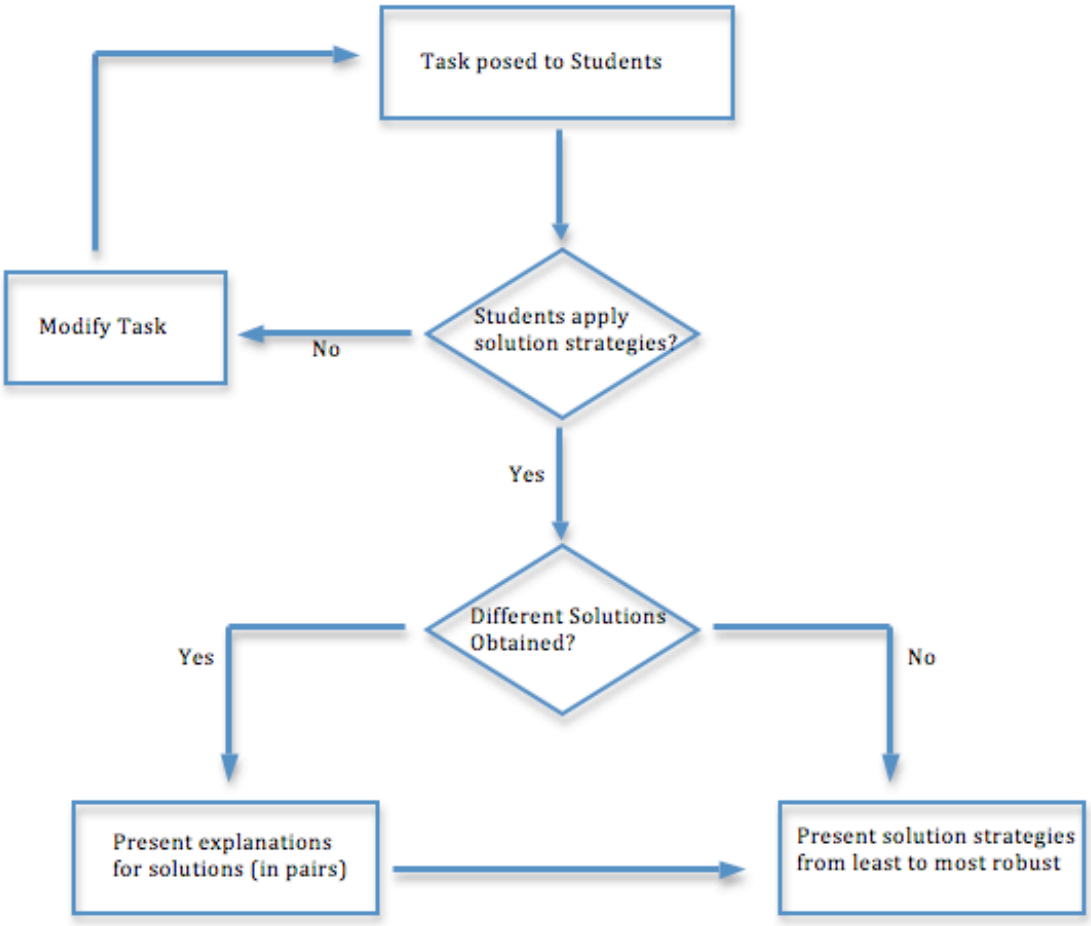


Figure 10: A schematic diagram for organizing classroom discourse to low attaining students.

My intention for recommending that teachers share a large responsibility regarding explaining, evaluating, and comparing different solution strategies is that this will

open up a space for students to think about alternate solutions that might otherwise not exist. Although my students were capable of finding various solution strategies to solve a problem, there was rarely any discussion of the relative advantages of these methods. By assuming a more proactive role in explaining and comparing solution strategies, the potential exist that a teacher could stimulate students' mathematical thinking beyond the problem solving process. Throughout, it is important for teachers to monitor students' interest level and participation, and assess if normative expectations are being created that are inconsistent with the teacher's goal for classroom activity. For example, by doing too much explaining and comparing the relative advantages of solution strategies, it is easy to imagine an environment where students sit passively, unengaged, and listen to their teacher as he/she talks past the students. Lobato, Clarke, and Ellis (2005) point out, because the purpose of a teacher's telling is "to stimulate novel mathematical thoughts for students, one must consider students' responses to the teacher's initiating action." (p. 111).

Ultimately, students should understand the importance of having multiple ways of thinking about solving problems. They should become increasingly proficient at explaining how they solved problems and be able to appropriate the ideas of their classmates. Following the whole-class presentation of different solution methods it is recommended that a teacher adjust the quantities and/or the context of the task so as to create dissonance with the least robust solution strategies, and to encourage students to consider using a more robust method. For example the following problems are three possible extensions to the September 14 task. Each of these problems presents challenges using Keisha or Alan's strategy. Chris's solution

method would be a more efficient one for students to use.

> *Cedric shared 202 baseball cards with his friends. Six of his friends received exactly one card each, five of his friends received exactly two cards, and the rest of his friends received exactly three cards. How many friends received exactly three cards?*
>
> *Cedric shared $54 with his friends. Six of his friends received $1.50 each, five of his friends received $2.25 each, and the rest of his friends received exactly $3.75. How many friends received exactly $3.75?*
>
> *Cedric spent exactly $30 buying fruit. He bought six pounds of apples for exactly $1 per pound, five pounds of oranges for exactly $2 per pound, and the rest he spent on grapes that cost exactly $3 per pound. How many pounds of grapes did Cedric buy?*

In posing any of these questions to the class, it is recommended a teacher use the archived work from the original task to help students recall the different solution strategies that were used. A teacher should give the students the option of using any of the previously discussed solution methods or a different one altogether. Students that choose an inefficient method should be pressed to explain the limitations of that method. While working with individual students, a teacher can assess whether students appear to possess a useful way to communicate their ideas, and if so, provide students a greater opportunity to explain their thinking during a whole group discussion. In this way, a teacher can move towards the goal of creating a set of sociomathematical norms consistent with my original definition.

Summary

All teachers are concerned by students who arrive to class unmotivated to learn mathematics. Students' possession of a productive disposition is a necessary condition for instantiating a multiple solution norm. The findings from my study, together with extensive research on students' goal orientations indicate that there are

a number of variables that a teacher can influence to affect a student's disposition to engage in mathematical work. A hope is that my study can assist those interested in understanding the complexities involved in having students explore and analyze multiple solutions to a problem.

In addition, I contend that identifying a set of normative expectations and obligations for social and mathematical activity appropriate for a specific classroom can be used to assess how well classrooms are making use of students' multiple solutions. Chazan and Ball (1999) recognize that a professional language of practice that describes what occurs in classrooms will "enhance opportunities for sustained, critical and insightful discourse about teaching among researchers, teachers and teacher educators" (p. 9). The development of a multiple solution norm can provide a shared basis for which to discuss details of practice.

As described in Chapter 1, my study originated from working with a group of pre-service teachers who suggested that having students explore multiple ways to solve problems was not a feasible thing to do in their teaching. In the next, concluding, chapter, I report on some main lessons I learned from conducting this study that I will use in my future work as a teacher educator.

Chapter 7: Conclusion

> Despite general agreement that students should consider more than one way to solve a complex mathematical problem, this practice is rarely observed in U.S. classrooms (Silver et. al, 2005, p. 287).

As described in Chapter 1, the idea for exploring the challenges of using students' multiple solutions as a regular feature of mathematics instruction sprung from conversations with pre-service teachers who raised a number of pedagogical and cognitive concerns about the wisdom and practicality of using alternate solution strategies with their students. The interns expressed concerns that there was not enough classroom time to teach in this way. They generally seemed to believe that this type of instruction might only work with above-grade level students. One of the interns, Mike, expressed the view that lower attaining students would be confused if they saw more than one solution to a problem. In his teaching, Mike attempted to make mathematics as non-problematic as possible by leading the students in a step-by-step manner through the lesson material; students were shown one standard method of solving a task, and were then expected to practice the demonstrated method on a set of similar problems.

My suggestion to develop important mathematical understandings through analyzing and comparing multiple solution strategies to a single problem was clearly a departure from the type of teaching that Mike and some of the other teacher interns were using in their classrooms. They indicated that my suggestions for classroom activity were simply not a feasible thing to do. Thus, one of the goals for this study was to *prove* that a multiple solution norm governing students' social and mathematical behavior could be instantiated in a classroom of low attaining middle

grade students amid the demands of a compulsory curriculum with rigorous time demands and the pressures of testing.

Although, as documented in the previous two chapters, I did not accomplish all of my goals for the social and mathematical activity in the classroom I firmly believe that my students greatly benefited from my efforts to instantiate a multiple solution norm. At the beginning of the year, the students displayed a lack of motivation and disdain toward mathematics. They lacked confidence in their ability to do mathematics. They demonstrated a weak and fragile understanding of important mathematical ideas, and of a mixed-up sense of basic math facts, procedures, and rules. Analysis of the data makes clear that student expectations and actions significantly shifted during the year. The disposition of my students qualitatively changed. They demonstrated an increased willingness to engage and persist in solving cognitively demanding mathematical tasks. Most importantly, they displayed a capacity to do some serious mathematical work; they were able to apply their previous knowledge and use their developing problem solving skills to discover inventive ways to solve new problems. Allowing students the freedom to explore more than one way to solve a problem promoted richer learning opportunities as they made connections to different domains of knowledge.

The results of this study provide a number of lessons learned that can be used to help teachers guide the constitution of a multiple solution norm. I learned that, in my context, the building of supportive relationships was a necessary condition in negotiating the roles and expectations for social and mathematical activity I desired. Task selection, and the ability to quickly modify or extend a task, was an important

factor that contributed to student effort and their ability to derive inventive solutions to problems. Although I found that my students could derive solution strategies, I learned that the mathematics underlying a specific strategy often remained tacit to the students. Thus, I recommend that teachers need to carefully monitor their students' ability to effectively communicate their own ideas and be prepared and willing to explain important mathematical ideas embedded in students' solutions.

I learned that the context of teaching a prescribed curriculum with mandated assessments adds to the challenge of teaching in ways that promote the development of a multiple solution norm. In my context, I was asked to prepare students for a year-end standardized test by meeting a set of benchmark objectives set in advance by the school district. The district assessed these objectives at the end of each quarter. Teachers were expected to have covered each objective by the specified date in the curriculum guide and were held accountable for their students' achievement. I believe that placing such a rigid constraint on the pace and content of the curriculum is a significant hindrance to a teacher attempting to instantiate a multiple solution norm. To constitute a multiple solution norm, teachers, particularly early in the year, need to have space and time to negotiate desired norms and to meet the learning needs of their students. Once a multiple solution norm becomes instantiated, I argue that a curriculum can be covered in an expeditious way because students will be continually discovering, discussing, and making personal connections with important mathematical ideas. Thus, school districts that do not provide teachers the autonomy to pace out the teaching of objectives based on the needs of their students tacitly dictate a style of instruction that more closely resembles Haberman's (1991)

pedagogy of poverty rather than instruction that is consistent of a multiple solution norm.

I learned that the context of teaching a dense network of low-tracked students adds a unique challenge toward the constitution of a multiple solution norm. In a classroom with a multiple solution norm, students are expected to work well with one another and share and listen to each other's ideas. In my context, students often took advantage of opportunities to compare their answers and share their work as a time to engage in off-task conversations. Chris's story highlights the value of placing students in classrooms where both the students and teacher hold high expectations. As described in Chapter 5, Chris constantly exhibited a range of miseducative behaviors while placed within the pre-Algebra class; however, after assigning him to an above-grade level Algebra class, Chris's inappropriate behavior immediately ceased. More importantly, Chris became an active participant in discussions of solution strategies, and his success in the class was culminated with a passing grade on the state's Algebra exam.

In my future work as a teacher educator, I will continue to endorse the notion that students need to experience more than one way to solve problems. I conclude this dissertation by expanding on four lessons learned:

1. *Demands of compulsory curricula and mandated assessments can powerfully influence teachers' decisions and actions.*

   Many teachers are faced with the challenge of covering an ambitious set of curricular objectives to prepare students for mandated standardized tests. In addition, low attaining students are likely to enter the classroom without an

adequate understanding of prerequisite material. These curricular and testing constraints are significant impediments to teachers intending to implement a multiple solution norm in the classroom. The constitution of a multiple solution norm requires ample room for teachers to adjust their instruction to meet the learning needs of their students. It takes a lot of time for students to construct their own mathematical understandings, share their ideas with their classmates, and discuss the relative advantages of different solution methods.

In contrast, for teachers who feel pressure to address the demands of a crowded curriculum, it is easy to abandon the notion of using multiple solutions simply because it will take too much time. I recognize the enticement of adopting a traditional, teacher-centered approach to instruction. Direct instruction provides teachers with more control over the pace and direction of a lesson. Teachers can plan out a day-to-day schedule for covering a set of objectives in a prescribed amount of time regardless of students' academic ability or motivation. Smith (1996) notes that teaching by telling is an expeditious way to cover the curriculum because it simplifies issues of planning and classroom management, narrows the scope of the content to manageable pieces, and provides a prescriptive structure for teaching.

Early in my study, I struggled with the tension of how I could cover a long list of curricular objectives in a fixed amount of time. Initially, I was compelled to attempt to squeeze in as many objectives as I could. I wanted my students to be successful on the district's assessment. However, it became convincingly clear that, if I were to be consistent in my goals for mathematical activity, then I would

have to make decisions regarding what content to address and which to leave out. Ultimately, I was able to resolve this tension by relying on my beliefs and knowledge regarding how students learn mathematics. I strongly believe that students' do not develop a relational understanding of mathematics by demonstration and practice, and that problems of motivation are related to students' understanding. Hiebert et al. (1997) suggest that students are more likely to withdraw from mathematics if they lack understanding and believe that success in mathematics depends on their ability to memorize definitions, rules, and procedures.

The loose and fragile collection of basic skills and procedures that my students time and again demonstrated proved to me that past teaching efforts led to a weak and misconception filled understanding of mathematical concepts and procedures. Thus, I was confident that covering a list of objectives for the sake of an assessment would have no lasting benefit to my students, and my decision to leave things out would not do harm.

Still, even with these beliefs and knowledge, the lack of success my students had on their assessments created an element of doubt regarding how best to teach these children. Conversations with colleagues from the university helped to reinforce my belief that I was teaching in a responsible and acceptable way. Sue and Sandy provided vital support. In particular, Sue's frequent comments regarding how impressed she was by the progress of the group was a source of continued inspiration.

To support teachers in their efforts to adopt instructional practices consistent with recommendations for mathematics reform, Smith (1996) recognizes the need to provide teachers "new moorings for efficacy." For me, a crucial mooring was the support I received from colleagues both within the school and from the university. I also received the backing from my principal and the district's mathematics leader. In the future, I am hopeful that NCTM's *Curriculum Focal Points* (2006) will help create a movement among school districts to target fewer and more focused mathematical objectives. Until that time, to manage the dilemmas of a mandated curriculum and the demands of constant assessment, it is important for teachers to seek out others who can help them consider mathematical and pedagogical alternatives.

2. *Low attaining students can generate productive and unique mathematical ideas when challenged to solve problems using their own inventive strategies.*

Silver et al. (2005) found that a prevailing sentiment among teachers is that the exploration of multiple solutions for a single problem was an activity that they viewed as feasible with high attaining students but not one that should be used in the heterogeneous classes they taught. Similarly, in anecdotal discussions with other teachers, I have found a common belief is that low attaining students need teachers to demonstrate one way to solve a problem and then be provided frequent opportunities for guided and independent practice. Yet, research shows that even students with relatively poor skills can engage in instruction that allows deep analysis of problems (RAND Mathematics Study Panel, 2002). Karsenty, Arcavi, and Hadas (2007) found that low attaining students were capable of generating

sophisticated and useful mathematical ideas; and even though the students' thinking contained inaccuracies and were often expressed in non-conventional ways, their ideas contained valuable information and needed to be nurtured. Similarly, I found that, given an appropriate non-routine task, each of my students were capable of using their prior knowledge to approach a problem from different mathematical perspectives and were able to generate reasonable solution strategies.

Task selection was paramount. Cognitively demanding tasks with multiple entry points that presented students with a moderate challenge were most likely to evoke students' effort and result in a variety of solution techniques. Intimate knowledge and reflection on each student's understandings was needed to determine when a task would be moderately challenging. In many cases, tasks were developed that could be individualized to meet the relative strengths of students. Through the process of extended problem analysis (Usiskin et al., 2003), the quantities, context, and/or parameters of a task could be adjusted to make a problem more or less accessible. I did not find it necessary to teach specific problem-solving skills or heuristic processes. Tasks that permitted the use of concrete representations, or included multiple representations that allowed students to use a graphing calculator as a problem-solving tool were particularly effective.

The process of designing tasks adds an additional layer in the work of teaching. Because an appropriate task may be below grade level expectations, it is important to resolve the dilemma of teaching content that is not an explicitly

part of a curriculum. I found the situated and context dependent nature of task design precludes the exclusive use of published curricular materials. Designing rich experiences for students requires a substantial amount of work and investment of time outside of class. In my study, task selection appeared to be a significant factor that influenced students' willingness to engage in problems and their capacity to find inventive ways to solve problems.

3. *Provide students with a caring and supportive classroom.*

Consistent with the students in my class, NCTM (1989) recognizes that students' "beliefs about mathematics exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition" (p. 233). By middle school, students with a history of low attainment are likely to exhibit a range of academically unproductive conduct in the mathematics classroom. Students may passively resist demands for academic work by engaging in private socializing, ignoring teacher directives, and choosing not to complete assignments or participate in class activities. In other cases, students may more actively seek to disrupt classroom activity by publicly challenging a teacher's authority or the legitimacy of a task.

Although some forms of resistant behavior are inevitable in any class, contexts exist where classroom disruptions and defiance of teacher authority can bring the instructional process to a grinding halt (McFarland, 2001). These situations challenge a tacit assumption among many educators that oppositional behavior by students will disappear, or at least be moderated in classrooms, if students are

presented with interesting and appropriate curricular experiences. It is important to recognize that many non-instructional aspects of classroom activity are related to student effort and engagement.

In the beginning of the year, regardless of the lesson I had planned, the students in my class did not seem interested in mathematics and engaged in a number of resistive efforts. Before my students were willing to offer focused academic effort, I found it necessary to establish caring and supportive, but firm relationships with students.

Bondy and Ross (2008) refer to teachers who communicate warmth along with a nonnegotiable demand for student effort and mutual respect as *warm demanders*. According to Bondy and Ross, warm demanders take time to know their students; they approach students with unconditional positive regard; and insist that students perform to a high standard.

Although there are countless and immeasurable factors affecting how teachers and students build relationships, Bondy and Ross (2008) recommends teachers consistently hold student behavior to high standards by respectfully and insistently reminding students of their expectations, and calmly and matter-of-factly delivering consequences if students do not comply. The building of supportive relationships is not a prescriptive act and is undoubtedly dependent on context and personalities of students and teachers. Additionally, Winograd (2002) found that teacher-student relations depended on the teacher's understanding of students; the teacher's ability to articulate his/her vision of classroom culture; the

willingness and ability of students to align with the teacher's goals; and the teacher's willingness and ability to see the students' point of view.

4. *Teachers may need to help low attaining students communicate and compare their ideas.*

Key components of a multiple solution norm are the establishment of social norms that empower students to discuss mathematics and the constitution of sociomathematical norms that guide how students compare the relative advantages of different solution strategies. Attempting to engage adolescents in a whole-class discussion of their ideas presents unique challenges. While some middle grade students are reluctant to publicly share their thinking, others can dominate discussions. Teachers can become overly concerned by the uncertainty and direction of classroom discourse and limit student opportunities to share their thinking. Teachers can struggle with pedagogical issues such as how to sequence solution strategies, how to respond to errors, and when and how to intervene. Facilitating discussion of student ideas requires a sophisticated knowledge of mathematics. Although each of these can be a significant issue in creating a classroom culture characterized by meaningful discussion of student ideas, these were challenges that I had successfully navigated in constituting a multiple solution norm in my Algebra and Geometry classes.

In my pre-Algebra class, I never felt that I was able to orchestrate a productive discussion of student ideas. Whole-class discussions were typically linked with frequent off-task behavior. When a student was speaking, other students would be visibly inattentive and often make inappropriate comments. Students struggled

to explain their reasoning in a way that other students found helpful. The sharing of ideas appeared to become a *ritualization of discourse* (Williams and Baxter, 1996) where students seemingly shared their solution strategies to appease me rather than a way for them to make sense of each other's understandings. Ultimately, by the third month of school, I abandoned attempts at whole-class discourse.

The process of reflection and analysis has led to the conjecture that my attempts to orchestrate whole-class discussion failed, in part, because I underestimated how difficult in was for my students to make sense of and articulate their own solution methods. Sandy observed that, when comparing solutions, the only cue that provided an indication that two students solved a problem in a different way was if they obtained different answers. I currently assert that it was an unreasonable expectation to ask my students to meaningfully compare their solution strategies with other students.

If given another opportunity with the group, I recommend assuming a greater role in helping students communicate their ideas to the class. Although my students were capable of solving problems, they frequently seemed unaware of the mathematics they used in their own solution methods. As reflected in my revised framework for a multiple solution norm (Chapter 6), I suggest that my students needed assistance to explain their solution methods and to compare the relative advantages of different solution methods.

To conclude, my attempts to instantiate a multiple solution norm to a group of low attaining students clearly proved more difficult than imagined. A number of

factors, such as motivational behavior, the students' fragile mathematical knowledge and vocabulary, and the curricular and testing demands of the school district impacted my goals for social and mathematical activity more than anticipated. Using students' inventive solution strategies as a regular feature of instruction is demanding work, but work that must be done if the goal is for students to learn mathematics with understanding. My study raises a number of critical issues where future research is needed to help practitioners balance the vision of instantiating a multiple solution norm in all classrooms with the realities of the classroom life:

- *How do educational accountability, prescriptive curriculum, and high-stakes comparative assessments affect the beliefs and instructional practices of teachers?*
- *How is the performance of low attaining students on high-stakes assessments affected by instruction that is guided by the goals of instantiating a multiple solution norm?*
- *What content, pedagogical, and curriculum support do teachers need to instantiate a multiple solution norm?*
- *How can teacher education courses and in-service professional development opportunities help teachers in classrooms where students avoid and/or actively resist engaging in academic work?*
- *How can teachers promote rich mathematical discourse in contexts where students are unable to effectively communicate their ideas?*

- *How can teachers manage normative expectations that simultaneously promote and conflict with their learning goals for students, their expectations for appropriate behavior, and/or their goals for instantiating a multiple solution norm?*

Appendix A: Description of Codes

Code	Description	Example
Task Engagement		
Willing	Students immediately begin working on an assigned task or comply with teacher's request to begin work.	Students work very well on the problem. Cedric asks what the $75 means. Jamaal replies to Cedric, "Didn't you even read this?"
Not Willing	Students do not work on an assigned task or only begin work after several explicit teacher requests.	The class is difficult to get started with the problem. I ask several of the students to give me their best effort and to focus. I ask what I can do so they will do the problem. Several of them joke to pay them. Alan, Erika, and Jamaal continue their off-task chatter.

Task Persistence		
Persevere	Students make perceived effort to understand problem. When challenged, students will continue to work on problem or elicits/accepts a suggestion on how to proceed.	Jordan: Can't get this. RH: Someone give Jordan a hint. Cedric: You want a hint? Jordan: I don't need nothing. I'm going to get this. I don't care if its next year, I'm going to come back and show Mr. Hollenbeck I got it.
Not Persevere	No perceived effort is given to understand a problem. When challenged, students will suspend work on problem, and often engage in non-academic behavior.	Erika: I don't get it. RH: Tell me what you don't get. Erika: The whole thing. RH: When you don't understand something what are our options? Erika: To not do it.

Task Persistence – Not Persevere		
Ability	Students indicate that they are not capable of doing or understanding how a problem should be solved.	Erika: I don't get it. RH: Tell me what you don't get. Erika: The whole thing. RH: When you don't understand something what are our options? Erika: To not do it.
Problem	Students indicate that the problem is too hard or that it is somehow not relevant – often referring to a problem as dumb or stupid.	RH: Read the problem for me [Alan reads problem] RH: Tell me what you don't get. Erika: This is dumb. I'm not doing it. Jamaal: Stupid right, how you suppose [inaudible]?
Teacher	Students indicate that a problem cannot be solved because they were not given adequate instruction.	Alan: How we suppose to do this, you never taught us this? RH: You might figure it out if you try. Alan: Not if you never taught us.
Student	Students are indifferent about solving a problem. Students indicate that they have no desire or interest in solving a problem.	RH: Why haven't you started? Jamaal: I don't feel like it. RH: Is it too hard. Jamaal: It's not hard. I just don't want to do it.

Problem Solving -- Sharing Ideas			
Spontaneous	Students spontaneously engage in purposeful discussion regarding a mathematical task.	Jordan: Alan: Jordan: Alan: Jordan: Alan:	Let me go over to my associate over here. [moves to Alan's desk] Do you get how to do this? No, I don't get this; like how do we know how much a ticket is. Right here it says each ticket is $10. Where? Oh, so 50 tickets is $500. How do you get that Because 50 ten times is 500.
Induced	Students are asked to share their ideas with each other. After asking students to work together, students either engage in a productive discussion regarding a mathematical task (perhaps facilitated by teacher) or student discussion is off-task.	RH: Alan: RH: Kyle: RH: Cedric:	What makes this one harder? Because when you add you get five; how you going to add to nine to get five? Yeah, that's a good question. Cedric, the question for this one, think about it for a second, how are we going to add a number to get five? Subtract. Kyle says subtract. Does that help us? Because the question is add. Oh, you would add to a negative number.
Resistant	Students consciously avoid working with or sharing ideas with classmates.	RH: Jamaal: RH: Jamaal:	Go over and see how Kyle and Cedric are doing the problem. I don't learn that way. What do you mean? I don't learn from other students, I either learn it myself or a teacher explains it.

Explanation/Justification		
Evidentiary	Students support and provide rationale for their solutions and/or solution strategies. Rationale is provided either spontaneously or after prompting from teacher or classmate.	Keisha: If she plays 10 games it equals $32, so three more games is going to be nine more dollars, so $41 altogether, and then one more game is going to be three more after that so $44. Alan: So, what she play, 13 games? Keisha: No, she play 14 because she paid $44.
Non-evidentiary	Students share solutions or procedures without providing a rationale for their thinking. In moments of disagreements, these exchanges commonly become quarrelsome.	Alan: That's wrong. Jordan: How's it wrong? Alan: The answer's 12. Jordan: No it's not. Alan: Yo, it is. You wrong. Jordan: No one's saying you ain't wrong G, but you can't be saying I'm wrong neither.

Sharing Solutions		
Question	Student asks a question about a classmate's solution or solution strategy.	Cedric: I added seven plus three plus 16 and got 26. Jordan: That's what I did. Erika: Hold up, where did you get the 16? Jordan: I got the 16 from the eight two pointers; eight times two is 16.
Alternate	Student offers a different solution, offers to correct a classmate's solution, or offers a different strategy for obtaining a solution.	Cedric: I counted up from 12 to 20, got eight, and then I counted by two's to get eight. Chris: Couldn't you also subtract, 12 minus 20, and then divide it by two?
Inattentive	Students seemingly do not listen nor appropriates the solution strategies of their classmates.	Jamaal: If you put the two on the six then they would be equal, so you need two positives. RH: Chris, is it clear what Jamaal's doing? Chris: No, but I know what he meant though RH: O.K., explain it in your own words. Chris: Which one is he doing? Jamaal: You weren't listening. Chris: I was listening, but I wasn't listening a lot.

Adjudicate		
Teacher	Students rely on the teacher to verify solutions or solution strategies.	Alan: Eleven because I subtracted 90 and 81. Chris: I did it the other way. Alan: But he got nine; could you just tell me what answer is right?
Student	Students rely on classmates to verify solutions or solution strategies.	Alan: Ain't a negative plus a positive always a positive? Cedric: I don't think so Alan. Erika: What you saying? RH: Alan asked is a negative plus a positive always a positive. Erika: Yes it is. No it ain't. Yes it is. I don't know. Cedric: You can have more negatives.
Self	Students verify their own solutions or solution strategies; mainly by utilizing a graphing calculator or manipulatives.	Kyle: Got it. Look. [Shows Cedric calculator] Cedric: I'm close. I don't have them lined up yet.

Nature of Mathematics Learning		
Emphasis on Process	Students seek to understand how/why an answer is obtained. Students value an incorrect answer if an appropriate strategy was employed.	Alan: No you wrong. Keisha: No I'm not. I'm about to show you right now. It's these negative; that five right there is negative. You group these together. Alan: I thought this was negative. Keisha: No, [inaudible] Alan: Well, if that was negative that would be right [Keisha laughs]. So I'm still right, I'm right so I don't care, I got the answer right, I thought of it the wrong way. I thought it was the other way.
Emphasis on Answer	Students value final answer with minimal regard to understanding how or why the answer is obtained. Often students compete to find the answer first.	RH: Jamaal, could you please listen to Kyle's explanation. Jamaal: I already know what the answer is. RH: But maybe he did something different. Jamaal: What did you get? Kyle: Fourteen. Jamaal: Alright then.

Calculator Use / Concrete Reference		
Meaningful	Students appear to use the calculator and/or manipulatives as a problem solving tool and to monitor their own solution strategies	Being in the accelerated class with you, he tried, if it didn't work out well he tried again, and he had more than one way to solve a problem. He was very good with a calculator, and I think that helped him.
Not Meaningful	Although provided, students do not appear to use the calculator and/or manipulatives to help them solve problems.	As soon as I take away the context of the forks and knifes or the pluses and minuses, they're going right back to their previously learned procedural knowledge regarding the addition of integers, and this procedural knowledge is too disconnected and fragile.

Appendix B: First Quarter Objectives

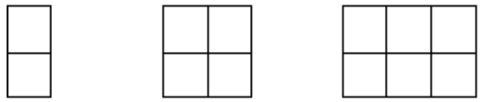
Unit/Obj	Objective / Assessment Limit
1.1	Compare, order, and describe rational numbers, with and without relational symbols (<, <, =) *Assessment Limit: Use no more the 4 integers (-100 to 100) or positive rational numbers (0 – 100) using equivalent forms or absolute value*
1.2	Add, subtract, multiply, and divide integers. *Assessment Limit: Use one operation (-1000 to 1000)*
1.3	Evaluate numeric expressions using the order of operations *Assessment Limit: Use no more than 5 operations including exponents of no more than 3 and 2 sets of parentheses, brackets, a division bar, or absolute value with rational numbers (-100 to 100)*
1.4	Evaluate algebraic expressions *Assessment Limit: Use one or two unknowns and up to three operations and rational numbers (-100 to 100)*
1.5	Write algebraic expressions to represent unknown quantities *Assessment Limit: Use one unknown and no more than 3 operations and rational numbers (-1000 to 1000)*
1.6	Identify and use the laws of exponents to simplify expressions *Assessment Limit: Use the rules of power times power or power divide by power with the same integer as a base (-20 to 20) and exponents (0 – 10)*
1.7	Read, write, and represent rational numbers *Assessment Limit: Use exponential notation or scientific notation (-10,000 to 1,000,000,000)*
1.8	Use properties of addition and multiplication to simplify expressions *Assessment Limit: Use the commutative property of addition or multiplication, associative property of addition or multiplication, additive inverse property, the distributive property, or the identity property for one or zero with integers (-100 to 100)*
1.9	Simplify algebraic expressions by combining like terms *Assessment Limit: Use no more than 3 variables with integers (-50 to 50) or proper fractions with denominators as factors of 20 (-20 to 20)*
1.10	Write equations to represent relationships *Assessment Limit: Use a variable, the appropriate relational symbols and no more than 3 operational symbols (+,-,x,/) on either side and rational numbers (-1000 to 1000)*
1.11	Solve for the unknown in a linear equation *Assessment Limit: Use one unknown no more than 3 times on one side and up to 3 operations (same or different but only one division) and rational numbers (-2000 to 2000)*

1.12	Identify equivalent equations *Assessment Limit: Use one unknown no more then 3 times times on one side and up to 3 operations (same or different but only one division) and rational numbers (-2000 to 2000)*
1.13	Apply given formulas to a problem-solving situation *Assessment Limit: Use no more than four variables and up to three operations with rational numbers*
1.14	Calculate powers of integers and square roots of perfect square whole numbers *Assessment Limit: Use powers with bases no more than 12 and exponents no more than 3, or square roots of perfect squares no more than 144*
1.15	Estimate the square roots *Assessment Limit: Use whole numbers (0 – 100)*

Appendix C: Tasks from Productive Classes

Monday, September 18:

Square blocks are arranged in the pattern shown below.

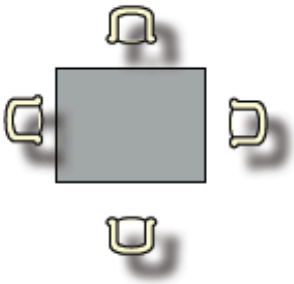


1st Pattern 2nd Pattern 3rd Pattern

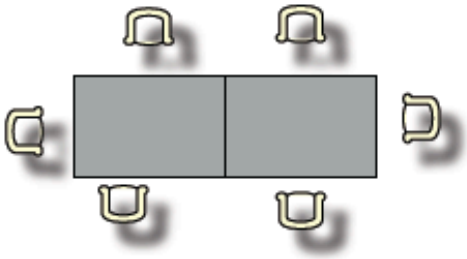
If the pattern continues, how many squares will be in the 10th pattern? Explain how you arrived at your solution. Use words, symbols, or both in your explanation.

Thursday, September 21

In a restaurant, 4 chairs can fit around a rectangular table as shown below



To accommodate more than 4 people, the tables are pushed together:



1. How many chairs can fit around 3 tables pushed together? Explain!!
2. How many chairs can fit around 5 tables pushed together? Explain!!
3. How many chairs can fit around 10 tables pushed together? Explain!!
4. How many chairs can fit around 50 tables pushed together? Explain!!

Tuesday, October 3

Eisenhower Middle School Raffle Ticket *Price: $10.00*

First Prize: Panasonic 9" LCD Portable DVD Player 

Second Prize: Apple Ipod MP3 Player 

Third Prize: Motorola Cordless Phone with Digital Answering System 

The eighth grade class at Eisenhower Middle School decided to hold a raffle to raise money for their school. They purchased three prizes to raffle off. They purchased a portable DVD player for $400.00, a MP3 player for $250, and cordless phone system for $120.

a) If each raffle ticket will be sold for $10, how many tickets will need to be sold before they start making a profit for their school? Explain how you determined your answer.

b) If the eighth grade class goal is make a profit of $500, how many tickets will need to be sold? Explain how you determined your answer.

Wednesday, October 11

Brittany has $200 in her bank account at the beginning of the year. Each month she deposits the same amount into her account. She does not withdraw any money from her account, and the account pays no interest. After 12 months, Brittany has $656 in her account.

a) Complete the table below showing the relationship between Brittany's money and the number of months she has saved.

Number of Months	0	1	2	3	4	5	6	7	8	9	10	11	12
Money Saved	$200												$656

b) How much money will Brittany have after 20 months? Explain how you determined your answer.

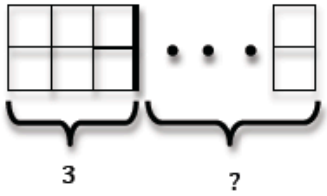
c) When will Brittany have $1,530? Explain how you determined your answer.

Monday, October 23

1. $\frac{8 \cdot -4}{2} =$
2. $4 - 17 =$
3. $(4 - 17) + \frac{8 \cdot -4}{2} =$
4. $(10 - 8)^3 + (-5) =$
5. $9x + y - 5y + 8x =$
6. $4(5x - 6) =$

Appendix D: Tasks from Unproductive Classes

Tuesday, September 19



- If, above the question mark, there were four columns of squares, how many total squares would you have? Explain how you determined your answer.
- If, above the question mark, there were five columns of squares, how many total squares would you have? Explain how you determined your answer.
- If, above the question mark, there were 20 columns of squares, how many total squares would you have? Explain how you determined your answer.
- If, above the question mark, there were *x* columns of squares, how many total squares would you have? Explain how you determined your answer.

Wednesday, September 20 1) $(3 + 5) + (3 + 5) =$ 2) $2(3 + 5) =$ 3) $3 + x + x =$ 4) $(3 + x) + (3 + x) =$ 5) $2(3 + x) =$
Tuesday, September 26 1) $-8 + 9 =$ 2) $2(3x + 4) =$ 3) $2x + 8 + x + (-5) =$ 4) **What number plus nine equals 21?** 5) **What number plus nine equals 2?**

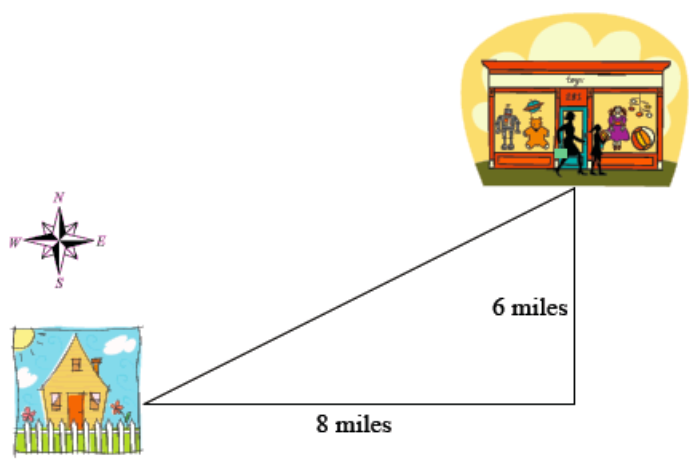
Monday, October 16

1) $\frac{4 \cdot -5}{-2} =$
2) $6 - 11 =$
3) $2^3 =$
4) $\frac{4 \cdot -5}{-2} + (6 - 11) =$
5) $5 + 2(x + 3) =$
6) Solve for n: $14 = 8 + 3n$

Wednesday, October 20

To find the longest side (called the hypotenuse) of a right triangle, the equation $c^2 = a^2 + b^2$ can be used. In the equation, c represents the length of the hypotenuse and a and b represent the lengths of the legs of the right triangle.

To travel to a store, Keisha leaves her house and drives 8 miles east, then 6 miles north. Use the equation $c^2 = a^2 + b^2$ to find how far Keisha's house is from the store.



References

Allen, F. (1997). A plan for improving the quality of exposition in high school mathematics. Retrieved February 16, 2009, from Mathematically Correct Web site: http://www.mathematicallycorrect.com/plan.htm

Ames, C. (1992). Classroom goals, structures, and student motivation. Journal of *Educational Psychology*, 84(3), 261-271.

Ames, C., & Archer, J. (1987). Mothers' beliefs about the role of ability and effort in school learning. *Journal of Educational Psychology*, 79, 409-414.

Anderman, E. M., & Maehr, M. L. (1994). Motivation and schooling in the middle grades. *Review of Educational Research*, 64, 287-309.

Anderman, E. M. & Midgley, C. (1997). Changes in achievement goals orientations, perceived academic competence, and grades across the transition to middle-level schools. *Contemporary Educational Psychology*, 22, 269–298.

Anderman L. H., & Anderman, E. M. (1999). Social predictors of changes in students' achievement goal orientations. *Contemporary Educational Psychology,* 25, 21-37.

Anderson, G.L. (2002). Reflecting on research for doctoral studies in education. *Educational Researcher,* 31*,* 22-25.

Anderson, G. L. & Herr, K. (1999). The new paradigm wars: Is there room for rigorous practitioner knowledge in schools and universities? *Educational Researcher,* 28*,* 12-21.

Ball, D. L. (1992). *Implementing NCTM standards: Hopes and hurdles*. Issue paper 92-2. East Lansing, MI: National Center for Research on Teacher Learning. ED 352 264.

Ball, D. L. (1993a). Halves, pieces, and twoths: Constructing and using representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema, T. A. Ronberg (Eds.), *Rational numbers: An integration of research* (pp. 157-195). Hillsdale, NJ: Lawrence Erlbaum.

Ball, D. L. (1993b). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal,* 93(4), 373-397.

Ball, D. L. (2000). Working on the inside: Using one's own practice as a site for studying mathematics teaching and learning. In Kelly, A. & Lesh, R. (Eds.), *Handbook of research design in mathematics and science education* (pp. 365-402). Mahwah, NJ: Lawrence Erlbaum Associates.

Ball, D.L. & Cohen, D. K. (1999). Developing practice, developing practitioners: toward a practice-based theory of professional development. In L. Darling-Hammond & G. Skyes (Eds.), *Teaching as the learning professional: Handbook of policy and practice*. (pp. 3-32). San Francisco: Jossey-Bass.

Ball, D. L., Lubienski, S., and Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching, 4th ed*., (pp. 433-456). New York: Macmillan.

Bandura, A. (1986). *Social foundations of thought and action: A social-cognitive theory*. Upper Saddle River, NJ: Prentice-Hall.

Berends, M. (1995). Educational stratification and students' social bonding to school. *British Journal of Sociology of Education,* 16(3), 327-351.

Blumer, H. (1969). *Symbolic interactionism.* Englewood Cliffs, NJ: Prentice-Hall.

Boaler, J. (2000). Exploring situated insights into research and learning. *Journal for Research in Mathematics Education,* 31(1), 113–119.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education,* 33*,* 239-258.

Bondy, E., & Ross, D. D. (2008). The teacher as warm demander. *Educational Leadership,* 66(1), 54-58.

Borko, H., & Putnam, R. T. (1995). Expanding a teachers' knowledge base: A cognitive psychological perspective on professional development. In T. R. Guskey & M. Huberman(Eds.), *Professional development in education: New paradigms and practices* (pp. 35–65). New York: Teachers College Press.

Brousseau G. (1997) *Theory of didactical situations in mathematics*. Dordrecht: Kluwer Academic Publishers

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom.* New York: Teachers College Press.

Chazan, D. & Ball, D. L. (1999). Beyond being told not to tell. *For the Learning of Mathematics,* 19(2), 2-10.

Cobb, P. (1990). Multiple perspectives. In L. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 200 - 215). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Cobb, P. (2000). The Importance of a Situated View of Learning to the Design of Research and Instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematical teaching and learning* (pp. 45-82). Stamford, CT: Ablex.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences,* 10(1&2), 113-163.

Cobb, P., & Whitenack, J. (1996). A method for conducting longitudinal analyses of classroom videorecordings and transcripts. *Educational Studies in Mathematics,* 30(3), 213-228.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist,* 31*,* 175-190

Cobb, P., Yackel, E., & Wood, T. (1993). Theoretical Orientation. In T. Wood, P. Cobb, E. Yackel & D. Dillon (Eds.), *Rethinking Elementary School Mathematics: Insights and Issues, Monograph #6*. Reston, VA: NCTM

Cohen, D. K. (1988). Teaching practice: Plus que ca change. In P. W. Jackson (Ed.), *Contributing to educational change* (pp. 27-84). Berkeley: McCutchan.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education,* 16*,* 324-336.

Covington, M. V. (2000). Goal theory, motivation, and school achievement: An integrative review. *Annual Review of Psychology*, 51, 171-200.

Cuban, L. (1990). What I learned from what I had forgotten about teaching: Notes from a professor. *Phi Delta Kappan,* 71, 479-482.

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms, 1890-1990* (2nd ed.). New York: Teachers College Press.

Cuban, L. (2003). *Why is it so hard to get good schools?* New York: Teachers College Press.

Delpit, L. D. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. *Harvard Educational Review,* 58(3), 280-298.

Delpit, L. D. (1995). *Other people's children: Cultural conflict in the classroom*. New York: New Press.

Dunham, P. H. (1996). Looking for trends: What's new in graphing calculator research? In P. Bogack (Managing Ed.), E. D. Fife, & L. Husch (Eds.), *Proceedings of the Eighth Annual International Conference on Technology in Collegiate Mathematics* (pp. 120-124). Reading, MA: Addison Wesley Publishing Company.

Dweck, C. S. (1975). The role of expectations and attributions in the alleviation of learned helplessness. *Journal of Personality and Social Psychology,* 31, 674-685.

Dweck, C. S. (1986). Motivational processes affecting learning. *American Psychologist*, 41(10), 1040-1048.

Dweck, C. S. (1999). Caution: Praise can be dangerous. *American Educator,* 23(1), 4-9.

Dweck, C.S., & Leggett E.L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review*, 95, 256-273.

Dweck, C. S., & Reppucci, N. D. (1973). Learned helplessness and reinforcement responsibility in children. *Journal of Personality and Social Psychology,* 25, 109-116.

Eccles, J. S., & Midgley. C. (1989). Stage/environment fit: Developmentally appropriate classrooms for early adolescents. In R. E. Ames & C. Ames (Eds.), *Research on motivation in education* (Vol. 3, pp. 139-186). San Diego, CA: Academic Press.

Eccles, J. S., Midgley, C., Wigfield, A., Buchanan, C. M., Reuman, D., Flanagan, C., & MacIver, D. (1993). Development during adolescence: The impact of stage-environment fit on young adolescents' experiences in schools and families. *American Psychologist*, 48(2), 90-101.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education,* 24(1), 8-40.

Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education,* 34(5), 433-463.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the Mathematics Teacher: A model. *Journal of Education for Teaching.* 15(1), 13-34.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's knowledge in instruction. *American Educational Research Journal*, 30(3), 555-583.

Finn, J. D. (1989). Withdrawing from school. *Review of Educational Research,* 59(2), 117-142.

Fordham, S. & Ogbu, J. (1986). Black students' school success: Coping with the burden of 'acting White.' *Urban Review,* 18(3), 1-31.

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G. Marin & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 333-353). Reston, VA: National Council of Teachers of Mathematics.

Frykholm, J. A. (2004). Teachers' tolerance for discomfort: Implications for curriculum reform in mathematics education. *Journal of Curriculum and Supervision*, 19(2), 125-149.

Glaser, B. G. & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* Aldine, New York.

Goetz J., & LeCompte, M. (1984). *Ethnography and qualitative design in educational research*. Orlando, Fl: Academic Press.

Grant, H. & Dweck, C.S. (2003). Clarifying achievement goals and their impact. *Journal of Personality and Social Psychology*, 85, 541-553.

Gregg, J. (1995). The tensions and contradictions of the school mathematics tradition. *Journal for Research in Mathematics Education*, 26(5), 442-466.

Haberman, M. (1991). Pedagogy of poverty versus good teaching. *Phi Delta Kappan, 73,* 290-294.

Haimes, D. H. (1996). The implementation of a "function" approach to introductory algebra: A case study of teacher cognitions, teacher actions, and the intended curriculum. *Journal for Research in Mathematics Education,* 27(5), 582-602.

Heath, S. B. (1983). *Ways with words: Language, life, and work in communities and classrooms.* Cambridge: Cambridge University Press.

Heaton, R. M. (1994). *Creating and studying a practice of teaching elementary mathematics for understanding.* Unpublished dissertation. E. Lansing, MI: Michigan State University.

Heaton, R. M. (1995). *What is a pattern?: An elementary teacher's early efforts to teach mathematics for understanding.* Issue paper, 95-1. E. Lansing, MI: Michigan State University, National Center for Research on Teacher Learning.

Heaton, R. M. (2000). *Teaching mathematics to the new Standards: Relearning the dance.* New York: Teachers College Press.

Hiebert, J., Carpenter, T.P, Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from the TIMSS Video Study. *Elementary School Journal*, 101(1), 3-20.

Holt, J. (1982). *How children fail*. Rev. ed. New York: Perseus Books.

Hubbard, R. S., & Power, B. M. (1999). *Living the questions: A guide for teacher-researchers*. Portland, Maine: Stenhouse.

Huffard-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education,* 35(2), 81-116.

Jacobs, J. K., Hiebert, J., Givvin, K., Hollingsworth, H., Garnier, H., & Wearne, D. (2006). Does eighth-grade teaching in the United States align with the NCTM Standards? Results from the TIMSS 1995 and 1999 video studies. *Journal for Research in Mathematics Education*, 37, 5-32.

Jagacinski, C. M., & Nicholls, J. G. (1984). Conceptions of ability and related affects in task involvement and ego involvement. *Journal of Educational Psychology,* 76, 909-919.

Karsenty, R., Arcavi, A., & Hadas, N. (2007). Exploring informal mathematical products of low achievers at the secondary school level. *Journal of Mathematical Behavior*, 26(2), 156-177.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal,* 102(1), 59-80.

Kennedy, M. M. (2005). *Inside teaching: How classroom life undermines reform.* Cambridge, MA: Harvard University Press.

Kloosterman, P. (1991). Beliefs and achievement in seventh-grade mathematics. *Focus on Learning Problems in Mathematics*, 13(3), 3-15.

Kloosterman, P., & Gorman, J. (1990). Building motivation in the elementary mathematics classroom. *School Science and Mathematics*, 90, 375-382.

Lampert, M. L (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-64.

Lampert, M. L. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

Lareau, A. (2003). *Unequal childhoods: Class, race, and family life.* Berkeley: University of California Press.

Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education,* 2, 227-252.

Lloyd, G. M., & Wilson, M. R. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education.* 29(3), 248-274.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformation of telling. *Journal for Research in Mathematics Education, 36*(2), 101-136.

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Lubienski, S. T. (2000a). A clash of class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics classroom. *Elementary School Journal,* 100*,* 377-403.

Lubienski, S. T. (2000b). Problem solving as a means toward "mathematics for all": An exploratory look through a classroom lens. *Journal for Research in Mathematics Education,* 31*,* 454-482.

Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning,* 4*,* 103-125.

Lyttle, S. L. & Cochran-Smith, M. (1990). Learning from teacher research: A working typology. *Teachers College Record,* 92*,* 83-103.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. New York: Erlbaum.

Maccini, P., & Gagnon, J. C. (2002). Perceptions and application of NCTM's standards by special and general education teachers: Implications for practice for secondary students with emotional and learning disabilities. *Exceptional Children*, 68, 325-344.

Manouchehri A., & Enderson, M. C. (1999). Promoting mathematical discourse: Learning from classroom examples. *Mathematics Teaching in the Middle School*, 4(4), 216-222.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education,* 32(3)*,* 236-266.

McFarland, D. A. (2001). Student resistance: How the formal and informal organization of classrooms facilitate everyday forms of student defiance. *The American Journal of Sociology,* 107(3), 612-678.

McGraw, R. (2002). *Facilitating whole-class discussion in secondary mathematics classrooms*. Unpublished doctoral dissertation, Indiana University, Indiana.

McKinney, S. & Frazier, W. (2008). Embracing the principles and standards for school mathematics: An inquiry into the pedagogical and instructional practices of mathematics teachers in high-poverty middle school. *The Clearing House,* 81(5), 201-210.

Meece, J. L. (1991). The classroom context and students' motivational goals. In M. L. Maehr & P. R. Pintrich (Eds.), *Advances in motivation and achievement* (Vol. 7, pp. 261-285). Greenwich, CT: JAI Press.

Meece, J. L., Anderman, E. M., & Anderman L. H. (2006). Classroom goal structure, student motivation, and academic achievement. *Annual Review of Psychology,* 57(1), 487-503.

Meece, J. L., Blumenfeld, P. C., & Hoyle, R. H. (1988). Students' goal orientations and cognitive engagement in classroom activities. *Journal of Educational Psychology*, 4, 514-523.

Metz, M. H. (1993). Teachers' ultimate dependence on their students. In J. W. Little & M. W. McLaughlin (Eds.) *Teachers' work. Individuals, colleagues and contexts* (pp.104-136). New York: Teachers College Press.

Meyer, M. R., & Fennema, E. (1985). Predicting mathematics achievement for females and males from causal attributions. In S. K. Damarin & M. Shelton (Eds.), *Proceedings of the seventh annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 201–206). Columbus, OH: Authors.

Middleton, M. J., Kaplan, A., & Midgley, C. (2004). The change in middle school students' achievement goals in mathematics over time. *Social Psychology of Education,* 7, 289-311.

Middleton, M. J., & Midgley, C. (1997). Avoiding the demonstration of lack of ability: An under-explored aspect of goal theory. *Journal of Educational Psychology*, 89, 710-718.

Middleton, M. J., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, 30, 65-88.

Midgley, C., Kaplan, A., & Middleton, M. (2001). Performance-approach goals: Good for what, for whom, under what circumstances, and at what cost? *Journal of Educational Psychology*, 93(1), 77-86.

Midgley, C., Kaplan, A., Middleton, M., Maehr, M. L., Urdan, T., Anderman, L. H., Anderman, E., & Roeser, R. (1998). The development and validation of scales assessing students' achievement goal orientations. *Contemporary Educational Psychology*, 23, 113-131.

Miles, M. B. & Huberman, A. M. (1994). *Analyzing qualitative data: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: Sage.

Moschkovich, J. N. (1998). Resources for refining mathematical conceptions: Case studies in learning about linear functions. *Journal of the Learning Sciences,* 7(2), 209-237.

Mueller, C. M., & Dweck, C. S. (1998). Intelligence praise can undermine motivation and performance. *Journal of Personality and Social Psychology,* 75, 33-52.

Murrell, P. C. (1994). In search of responsive teaching for African American males: An investigation of students' experiences of middles school mathematics curriculum. *The Journal of Negro Education,* 63(4), 556-569.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. Reston, VA: NCTM.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Pane*l. Retrieved March 26, 2009, from http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council and the Institute of Medicine. (2004). *Engaging schools: Fostering high school students motivation to learn.* Committee on Increasing High School Students' Engagement and Motivation to Learn. Board on Children, Youth, and Families, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Nelson, R. S. (1997). *Developing mathematical knowledge through class discussion: One teacher's struggles in implementing reform.* Unpublished dissertation. Virginia Polytechnic Institute and State University.

Oakes, J. (1985). *Keeping on track*. New Haven, CT: Yale University Press

O'Conner, M., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly,* 24, 318-335.

Ogbu, J. (1991). Minority coping responses and school experience. *Journal of Psychohistory,* 18, 433-456.

Patrick, H., Turner, J. C., Meyer, D. K., & Midgley, C. (2003). How teachers establish psychological environments during the first days of school: Associations with avoidance in mathematcis. *Teachers College Record,* 105(8), 1521-1558.

Peressini, D. (1998). The portrayal of parents in the reform of mathematics education: Locating the context for parental involvement. *Journal for Research in Mathematics Education*, 29(5), 555-582.

Pintrich, P. R., Conley, A. M., & Kempler, T. M. (2003). Current issues in achievement goal theory and research. *International Journal of Educational Research,* 39, 319-337.

Powell, A., Farrar, E. & Cohen, D. (1985). *The shopping mall high school*. Boston: Houghton Mifflin.

RAND Mathematics Study Panel. (2002). Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education. Retrieved February 16, 2009, from http://www.rand.org/multi/achievementforall/math/finalreport.html

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal of Research and Development Education,* 28*,* 550-576*.*

Roeser, R. W., Midgley, C., & Urdan, T. C. (1996). Perceptions of the school psychological environment and early adolescents' psychological and behavioral functioning in school: The mediating role of goals and belonging. *Journal of Educational Psychology,* 88, 408-422.

Romagnano, L. (1994). *Wrestling with change: The dilemmas of teaching real mathematics*. Portsmouth, NH: Heinemann.

Ryan, A. M., Pintrich, P. R., & Midgley, C. (2001). Avoiding help-seeking in the classroom: Who and why? *Educational Psychology Review*, 13, 93-114.

Schoenfeld, Alan H. (1987). What's all the fuss about metacognition? In Alan H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992), Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grousw (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53-70). New York: MacMillan.

Schroeder, T. L., & Lester, F. K., Jr. (1989). Developing understanding in mathematics via problem solving. In P. R. Trafton (Ed.), *New directions for elementary school mathematics* (pp. 31-42). Reston, VA: NCTM.

Schwartz, F. (1981). Supporting or subverting learning: Peer group patterns in four tracked schools. *Anthropology & Education Quarterly,* 12(2), 99-121.

Seale, C. (1999). Quality in qualitative research. *Qualitative Inquiry,* 5(4), 465-478.

Sherin, M., Mendez, E. & Louis, D. (2000). Talking about math talk. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 188-196). Reston, VA: National Council of Teachers of Mathematics.

Shifter, D. & Fosnot, C. T. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York: Teachers College Press.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., Font Strawhum, B. T. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior*, 24, 287-301.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education,* 26*,* 114-145.

Sizer, T. (1984). *Horace's compromise: The dilemma of the American high school*. Boston: Houghton Mifflin.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Smith, J. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education,* 27*,* 387-402.

Steele, C. (1992). Race and the schooling of African-Americans. *Atlantic Monthly,* 269(4), 68-78.

Stein, C. S. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher,* 101(4), 285-289.

Stein M. Grover B. & Henningsen M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488.

Stigler, J. W., Gallimore, R., & Hiebert, J. (2000). Using video surveys to compare classrooms and teaching across cultures: Examples and lessons from the TIMSS video studies. *Educational Psychologist*, 35, 87-100.

Stigler, J.W., Gonzales, P., Kawanaka, T., Knoll, S., and Serrano, A. (1999). The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States. (NCES 1999-074). U.S. Department of Education. Washington, DC: National Center for Education Statistics.

Stigler, J. W. & Hiebert, J. (1999). *The teaching gap.* New York: Free Press.

Summers, J. J., Schallert, D. L., & Ritter, P. M. (2003). The role of social comparison in students' perceptions of ability: An enriched view of academic motivation in middle school students. *Contemporary Educational Psychology*, 28(4), 510-523.

Taylor, T. (1990). Mathematical attitude development from a Vygotskian perspective. *Mathematical Education Research Journal,* (4)3, 8-23.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A. G. (1985). Teachers' conceptions of mathematics and the teaching of problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 281-294). Hillsdale, NJ: Lawrence Erlbaum.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research in mathematics teaching and learning.* (p. 127 - 146) New York: MacMillan.

Turner, J. C., Meyer, D. K., Anderman, E. M., Midgley, C., Gheen, M., & Kang, Y. (2002). The classroom environment and students' reports of avoidance strategies in mathematics: A multimethod study, *Journal of Educational Psychology,* 94(1), 88-106.

Turner, J. C., Meyer, D. K., Midgley, C., & Patrick, H. (2003). Teacher discourse and sixth graders' reported affect and achievement behaviors in two mastery/high performance mathematics classrooms. *Elementary School Journal,* 103, 357-382.

Turner, J. C., & Patrick, H. (2004). Motivational influences on student participation in classroom learning activities. *Teachers College Record,* 106(9), 1759-1785.

Usiskin, Z., Peressini, A. L., Marchisotto, E., & Stanley, D. (2003). *Mathematics for high school teachers: An advanced perspective.* New Jersey: Prentice Hall.

Valli, L., Croninger, R. G., Chambliss, M. J., Graeber, A. O., Alexander, P., Buese, D., & Eick, C (2005). *High quality teaching of foundational skills in mathematics and reading: Interim report , July 1, 2005.* Retrieved, February 16, 2009, from: www.education.umd.edu/EDCI/hqtstudy/HQTReport%2007-01-05.pdf

Valli, L., Croninger, R. G., Chambliss, M. J., Graeber, A. O., & Buese, D. (2008). *Test driven: high-stakes accountability in elementary Schools*. NY: Teachers College Press.

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 19-29). Reston, VA: National Council of Teachers of Mathematics.

Watson, A. (2002). Instances of mathematical thinking among low attaining students in an ordinary secondary classroom. *Journal of Mathematical Behavior,* 20, 461-475.

Weiner, B. (1972). *Theories of motivation: From mechanism to cognition.* Morristown, NJ: General Learning Press.

Wentzel, K. R. (1991). Social competence at school: Relation between social responsibility and academic achievement. *Review of Educational Research,* 61(1), 1-24.

Wentzel, K. R. (1997). Student motivation in middle school: The role of perceived pedagogical caring. *Journal of Educational Psychology,* 89(3), 411-419.

Williams, S., & Baxter, A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal,* 97, 21-38.

Wilson, M. R., & Lloyd, G. M. (2000). The challenge to share mathematical authority with students: High school teachers reforming classroom roles and activities through curriculum implementation. *Journal of Curriculum and Supervision,* 15, 146-169.

Wilson, S. M. (1995). Not tension but intention: A response to Wong's analysis of the researcher/teacher. *Educational Researcher,* 24(8), 19-22.

Wolters, C. A. (2004). Advancing achievement goal theory: using goal structures and goal orientations to predict students' motivation, cognition and achievement. *Journal of Educational Psychology*, 96(2), 236–250.

Wong, E. D. (1995). Challenges confronting the researcher/teacher: Conflicts of purpose and conduct. *Educational Researcher,* 24(3), 22-28.

Wong, E. D. (1996). Students' scientific explanations and the contexts in which they occur. *Elementary School Journal,* 96(5), 495-511.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education,* 30(2), 171-191.

Wu, H. (1999). Basic skills versus conceptual understanding: A bogus dichotomy in mathematics education. *American Educator,* 23(3), 14-19, 50-52.

Yackel, E. (2000). *Creating a mathematics classroom environment that fosters the development of mathematical argumentation.* Proceeding at the Ninth International Congress of Mathematical Education, Tokyo/Makuhari, Japan.

Yackel, E. & Cobb, P. (1996). Sociomathemaical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education,* 27(4), 458-477.

Zvenbergen, R. (1996). Constructivism as a liberal Bourgeois discourse. *Educational Studies in Mathematics,* 31*,* 95-113.