ABSTRACT

Title of dissertation: VALUATION EFFECTS AND EXTERNAL ADJUSTMENT
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In the past two decades, cross country portfolio holdings of a large variety of assets have risen sharply. This has created an important role for changes in asset prices, or “valuation effects”. This dissertation examines the role of valuation effects in a country’s external adjustment. The dissertation is organized as follows: Chapter 1 is a brief introduction. Chapter 2 presents some facts about the U.S.’s valuation effects from stocks and bonds during 1994-2007. In particular, total valuation effects from stocks and bonds during this period were $1295 billions, offsetting about 22.8% the size of the U.S.’s total current account deficits. Much of the positive, stabilizing effects arose after 2002. Before 2002 the valuation effects were often negative and reinforcing the current account deficits.

Chapters 3, 4 and 5 present a two-country dynamic stochastic general equilibrium (DSGE) model to study valuation effects theoretically. Chapter 3 outlines the set up of the model, where output has a transitory and a trend component, both of which are subject to AR(1) shocks. Chapter 4 solves analytically a simplified version...
of the model that only considers transitory output shocks. It shows that valuation
effects are stabilizing in response to transitory shocks. That is, valuation effects move
in the opposite directions of the current account, and mitigate the impact of the cur-
rent account on the NFA position. Chapter 4 also shows analytically that the size of
valuation effects relative to the current account is positively related with the level of
financial integration, which in turn increases with risk aversion, with output volatility,
with output persistence, and decreases with the discount factor and with the cost of
investing abroad. For the benchmark calibration, when domestic investors hold about
40% of their financial wealth in foreign equity, valuation effects will completely offset
the current account.

Chapter 5 solves numerically for the full version of the model, where both tran-
sitory and trend output shocks are considered. It shows that valuation effects are not
always stabilizing. Following trend shocks on output, valuation effects are amplifying:
they move in the same direction as the current account and reinforce the impact of the
current account on net foreign assets. The results are illustrated by the external im-
balances between the U.S. and other industrialized countries since the 1990s. Chapter
6 concludes.
VALUATION EFFECTS AND EXTERNAL ADJUSTMENT

by

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DEDICATION

To my parents and my sister.
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Chapter 1

Introduction

Balance of payment (BOP) accounting has traditionally assumed that the evolution of a country’s net foreign asset (NFA) position is fully determined by a country’s current account balance. For example, countries that run a current account deficit experience a parallel reduction in their NFA position. This view was built on the implicit assumptions that the prices of foreign asset holdings were stable. This assumption was perhaps a reasonable approximation during the Bretton Woods period, when most international portfolio holdings consisted of bonds and exchange rates were by and large stable.

However, the past decades have witnessed a sharp rise in cross country portfolio holdings of a large variety of assets, most importantly equities. Furthermore, after the collapse of Bretton Woods, the world has experienced large exchange rate fluctuations, even among major industrialized nations. These developments highlight the role of changes in asset prices, recently referred to as “valuation effects”, in affecting a country’s BOP accounting. Valuation effects formally are changes in the value of a country’s gross external assets and liabilities due to asset price and exchange rate fluctuations. Positive valuation effects arise when the capital gains on foreign assets held by domestic agents are larger than those on domestic assets held by foreign agents. *Ceteris paribus*, positive valuation effects enhance a country’s external financial wealth.
and improve its NFA position. The “new” balance of payment accounting therefore should be changes in the NFA position consist of the current account plus valuation effects.

Figure 1.1: U.S. and other G7 countries’ gross external assets and liabilities

Following this argument, Gourinchas and Rey (2007) point out that large, persistent current account deficits of a country such as the U.S. do not necessarily lead to a sharp deterioration in the NFA position if the country experiences positive valuation effects. In such a situation, current account deficits can be much more sustainable than was previously thought and valuation effects exert a stabilizing role – they offset part of the current account deficit and mitigate the decline in the country’s NFA position.

Gourinchas and Rey (2007) impute net foreign asset returns from 1952 to 2004, and interpret these as the “valuation channel” of changes in NFAs. They find that the valuation channel was stabilizing and accounts for 27 percent of the U.S.’s cyclical external adjustments. However, Curcuru, Dvorak, and Warnock (2008), after correcting for measurement errors, find that the average return differential of U.S. claims over
U.S. liabilities (in stocks and bonds) was essentially zero during the period from 1994 to 2006.

Although net foreign asset returns and return differentials are the focus of the empirical literature, they are not precise measures of valuation effects. While total returns include asset price changes and dividend yields or interest payments, valuation effects are associated with asset price changes only (dividend payments and interest payments are captured in the current account). Large annual valuation effects, therefore, can exist even if Curcuru, Dvorak, and Warnock (2008) find a small average return differential. Chapter 2 of this dissertation shows that total valuation effects from stocks and bonds during 1994-2007 period were $1295 billion, offsetting about 22.8% of the U.S.’s total current account deficits. About 60% of the valuation effects came from the portfolio traded with developed countries, and 40% from the portfolio traded with emerging markets.

On the theoretical front, Devereux and Sutherland (2009) investigate valuation effects in a two-country dynamic model. However, similar to the empirical literature, they restrict valuation effects to return differentials, which, as discussed above, are not a precise measure of valuation effects. My dissertation considers asset prices and associates valuation effects with changes in asset prices only. Ghironi, Lee, and Rebuggi (2007) also explicitly consider asset prices. They illustrate the working of valuation effects, and show that the quantitative importance of valuation effects depends on features of the international transmission mechanism such as the size of financial frictions, substitutability across goods, and the persistence of the shocks. However, they do not focus on the role of valuation effects on NFAs.
Chapters 3, 4 and 5 of this dissertation present a two-country dynamic stochastic general equilibrium (DSGE) model to study the role of valuation effects in a country’s external adjustment. Chapter 3 outlines the set up of the model where output has a transitory and a trend components, both of which are subject to AR(1) shocks. Chapter 4 solves analytically a simplified version of the model that only considers transitory output shocks. It shows that valuation effects are stabilizing in response to transitory shocks. That is, they move in the opposite directions with the current account, and mitigate the impact of the current account on the NFA position. Chapter 4 also shows analytically that the size of valuation effects relative to the current account is positively related with financial integration, which in turn increases with risk aversion, with output volatility, with output persistence, and decreases with the discount factor and with the cost of investing abroad. For the benchmark calibration, when domestic investors hold about 40% or more of their financial wealth in foreign equity, valuation effects will completely offset the current account.

Chapter 5 solves numerically for the full version of the model, where both transitory and trend output shocks are considered. It shows that the impact of valuation effects depends critically on the nature of underlying output shocks. In response to transitory shocks, valuation effects are stabilizing; they counteract current account movements and help to soften the impact of the current account on a country’s NFA position. In response to trend shocks, valuation effects are amplifying; they move in the same direction as the current account, and reinforce, or “amplify” the impact of the current account on the NFA position.Unlike the conventional wisdom that valuation effects are stabilizing, as showcased in empirical find-
ings of Gourinchas and Rey (2007), and implicit in Ghironi, Lee, and Rebuffi (2007), Devereux and Sutherland (2009) and in Coeurdacier, Kollmann, and Martin (2008)’s theoretical results, this chapter shows that valuation effects can be amplifying too. This situation is clearly illustrated by the evolution of NFA position between the U.S. and other industrialized countries during the 1990s.

The theoretical results above critically depend on the cyclicality of the current account. For the U.S., the current account is slightly counter cyclical: the correlation between the current account and growth from 1960-2007 is -0.15. For other small developed countries, the average correlation is -0.17 (Aguiar and Gopinath (2007)).

The standard neo-classical framework with endowment economies cannot explain the counter-cyclicality of the current account. With production economies, the counter-cyclicality of the current account can be generated because investment increases following a positive productivity shock. However, when a reasonable adjustment cost is introduced, investment becomes more sluggish and the current account again becomes pro-cyclical (Backus, Kehoe, and Kydland (1992)).

My model uses trend shocks in the spirit of Aguiar and Gopinath (2007) to account for the counter-cyclicality of the current account. I calibrated the model to match the U.S. output from 1960-2007. In the simulation, the correlation between output growth and current account is -0.13 (negative and quite close the data).

\[ \text{The correlation between quarterly net export divided by output and output from Backus, Kehoe, and Kydland (1992) is -0.28.} \]
Chapter 2

The U.S.’s Valuation Effects from Stocks and Bonds

Utilizing the dataset constructed by Bertaut and Tryon (2007) and also used in Curcuru, Dvorak, and Warnock (2008), we find that the U.S.’s valuation effects from stocks and bonds were significant and in general stabilizing during 1994-2007: they partly offset the current account deficits and stabilized the net foreign asset position. In particular, total valuation effects from stocks and bonds during this period were $1295 billion, offsetting about 22.8% the size of the U.S.’s total current account deficits. Although the valuation effects were often negative from 1994-2002, they were always large and positive from 2002-2007. These facts imply that U.S.’s assets held overseas had a lower average return than U.S.’s liabilities before 2002, and a higher average return than U.S.’s liabilities after 2002. We also find that about 60% of the valuation effects came from the portfolio traded with developed countries, and 40% from the portfolio traded with emerging markets.

Unlike Gourinchas and Rey (2007) and Curcuru, Dvorak and Warnock (2008) which focus on asset returns, I focus on valuation effects based on asset price changes, as also in Ghironi, Lee, and Rebucci (2007).
2.1 Valuation effects in balance of payment accounting

As mentioned in the introduction chapter and will be discussed in depth in chapter 3, the “new” balance of payment accounting identity is that the changes in NFA position constitute of the current account balance and valuation effects.

\[ \Delta NFA_t = CA_t + VE_t \] (2.1)

The current account balance consists of trade balance (export \( EX_t \) minus import \( IM_t \)) and income balance (interest and dividend payments from abroad \( D^*_t \) minus interest and dividend payments to foreign investors \( D_t \)) and net transfer \( T_t \):

\[ CA_t = EX_t - IM_t + D^*_t - D_t + T_t \] (2.2)

Valuation effects comprise of the changes in the prices of foreign assets held by domestic investors minus the changes in the prices of domestic assets held by foreign investors.

\[ VE_t = W_{t-1} \frac{Q^*_t}{Q^*_{t-1}} - W^*_t \frac{Q_t}{Q_t} \] (2.3)

where \( W_{t-1} \) is the value of foreign assets held by domestic investors in the previous period; \( W^*_t \) is the value of domestic assets held by foreign investors in the previous period. \( Q_{t-1} \) and \( Q^*_{t-1} \) are domestic and foreign asset prices in the previous period, \( Q_t \) and \( Q^*_t \) are domestic and foreign asset prices this period. The current empirical literature on valuation effects associates valuation effects with returns. The return of domestic assets includes changes in asset prices and dividend/interest payments:

\[ R_t = \frac{Q_t}{Q_{t-1}} + \frac{D_t}{Q_{t-1}} \] (2.4)
It is important to note that while total returns include asset price changes, interest and dividend payments, valuation effects are associated with asset price changes only. Dividend and interest payments are captured in the current account. This chapter focuses on empirical evidence regarding the size of valuation effects based on asset price changes.

2.2 The U.S.’s valuation effects

2.2.1 The U.S.’s valuation effects with the rest of the world

Figure 2.1 presents the U.S.’s valuation effects in stocks and bonds with the rest of the world from 1994 to 2007. Data for valuation effects is from Bertaut and Tryon (2007). Data for the U.S.’s current account balances is from the U.S.’s Bureau of Economic Analysis (BEA). For 1994, valuation effect data is only from March 1994 to December 1994.

Figure 2.1: U.S.’s valuation effects and the current account
Generally, the U.S.’s valuation effects have played a stabilizing role. They offset the current account deficits and helped stabilize the U.S.’s NFA position. Most of the significant stabilization came after 2002. For the entire period (1994-2007), the size of total valuation effects was $1295.32 billions, the size of total current account was -$5677.47 billions. Overall, valuation effects from stocks and bonds offset about 22.8% of the current account deficits.

A few additional comments are in order. Before 2002, except for 1999 and 1994, valuation effects were negative and quite significant. The negative valuation effects reflect a relatively good performance of the U.S. stock market (i.e. foreign investors benefited from the U.S. stocks they held). During that time, the U.S. experienced persistently higher productivity and economic growth than other industrialized countries. Negative valuation effects in these years, coupled with the current account deficits, imply a generally reinforcing role of valuation effects: they move in the same direction of the current account and “amplify” the impact of the current account on the NFA position. The year of 1999 is quite an exception. The year is marked by the peak of the dot com bubble, in which the bubble was even more severe in other countries. As a result, foreign stock markets rallied even more than the U.S.’s, causing a positive valuation effect for the U.S. in 1999. Since 2002 however, the U.S.’s valuation effects have been all positive and increasingly significant, accounting for as much as 4% of GDP and mitigating as much as two-thirds of the current account deficits in 2006 and 2007. The rising quantitative importance of the U.S.’s valuation effects is partly due to a sharp, continuous increase in international stock and bond holdings (Lane and Milesi-Ferretti (2007)).
The overall stabilization of valuation effects is consistent with the key result by Curcuru, Dvorak, and Warnock (2008) that the average return differential between U.S.’s foreign assets and liabilities (in stocks and bonds) was close to zero in the 1994-2006 period. The reason is that U.S.’s assets held overseas generally had a lower average return before 2002, and a higher average return after 2002 than that of U.S.’s liabilities. The average return differential over the entire period hence could be close to zero, but since the cross border portfolio holdings are larger after 2002, U.S. investors ended up making larger gains from foreign assets than foreign investors did from U.S.’s assets before 2002.

Most of the valuation effects come from changes in stock prices (figure 2.2).

![Figure 2.2: U.S.’s valuation effects by security types](image)

2.2.2 The U.S.’s valuation effects with developed countries

This part analyzes the U.S.’s valuation effects with developed countries. The list of developed countries follows that in Curcuru, Dvorak and Warnock (2008): Aus-
tria, Australia, Belgium, Luxembourg, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherland, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom.

Valuation effects coming from these developed countries totaled $840 billion in 1994-2007 period, constituting about 60% of the total U.S.’s valuation effects. They also were often negative and amplifying in 1990s, and always were positive, significant and stabilizing after 2002.

![U.S.'s valuation effects by country groups](image)

Figure 2.3: U.S.’s valuation effects by country groups

2.2.3 The U.S.’s valuation effects with emerging markets

Also following Curcuru, Dvorak, and Warnock (2008), we pick the list of emerging markets as follows: Argentina, Brazil, Chile, China, Colombia, Hungary, India, Korea, Malaysia, Mexico, Morocco, Peru, Philippine, Poland, Russia, South Africa, Thailand, Turkey, Venezuela. Total valuation effects from these countries during 1994-2007 were $504 billion, constituting about 40% of the U.S.’s total valuation effects.
Given that the size of stock and bond holdings between the U.S. and the emerging markets was smaller than that between the U.S. and developed countries (about one-third for bonds and one-fifth for stocks), the size of valuation effects for emerging markets was very sizable. This implies significant fluctuations of stock prices in these markets.

2.2.4 The U.S.’s valuation effects with China

As China plays a very large role in financing the U.S.’s current account deficits, this part investigates if valuation effects with China are significant. They appear modest (see Figure 2.4, note that data U.S.-China current account is from the U.S. BEA and only available after 1999). The reason for the insignificant valuation effects is because China has been holding mostly U.S.’s Treasury bonds, which do not fluctuate much in value.

Figure 2.4: U.S.’s valuation effects with China
Chapter 3

Model Setup

The next three chapters present a two-country dynamic stochastic general equilibrium (DSGE) model to study theoretically the role of valuation effects in a country’s external adjustment. This chapter presents the setup of the model.

The working framework is a simple stationary symmetric one-good two-country DSGE model. Output has a transitory and a trend component, both of which are subject to AR(1) shocks. There are two assets, each is a claim on a fraction of one country’s output, as in Lucas (1982). Agents observe output and choose their consumption, as well as the weight of two assets in their portfolios.

In the model, financial assets serve two purposes: for inter-temporal smoothing and for the purpose of risk sharing. Economic agents would like to insure themselves against the risk of undiversifiable labor income and domestic equity holdings. Ideally, in a frictionless asset market, agents would hold 50% of domestic endowment and 50% of foreign endowment to completely insure themselves against any country specific shocks (Lucas (1982)). In this case domestic and foreign agents would have exactly the same consumption and wealth in all states.

However in reality residents of most countries exhibit home bias in their portfolio holdings (French and Poterba (1991); Tesar and Werner (1995)). A number of explanations for the home bias puzzle have been presented; in this dissertation I as-
sume that there is a small cost of investing abroad (as in Heathcote and Perri (2004), Coeurdacier and Guibaud (2006), Tille and van Wincoop (2007)). These costs reflect a lack of market knowledge, market access and information, as well as cultural and language barriers. Such costs make investing abroad less attractive and create home bias in portfolio holdings. The portfolio home bias is also important to generate non-trivial current account. In the model, without the home bias, current account would be always zero because all agents are effectively insured (they would optimally hold 50% of home endowment and 50% of foreign endowment).

Note that there is only one good in the model, thus we cannot explicitly account for exchange rate movements. In practice, valuation effects consist of both movements in nominal asset prices and in foreign exchange rates. However, to the extent that exchange rate movements are equilibrium responses to fundamental shocks, the change in relative real asset prices in our model reflects both movements in nominal asset prices and exchange rates.

The detailed setup is as follows:

3.1 Production

Production of the home country takes the form of an endowment process:

$$Y_t = z_t \Gamma_t$$

(3.1)

We abstract from investment and labor for simplicity. However, a constant fraction $1 - \alpha$ of the endowment is considered as labor income. The rest can be considered as capital rent.
As in Aguiar and Gopinath (2007), $z_t$ and $\Gamma_t$ represent two productivity processes. The two processes are characterized by different stochastic properties. Specifically, $z_t$ follows an AR(1) process:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon^z_t$$

(3.2)

where $0 < \rho_z < 1$ and $\varepsilon^z_t$ represents iid draws from a normal distribution with zero mean and standard deviation $\sigma_z$.

The parameter $\Gamma_t$ represents a combination of a cumulative product of the growth shocks (as in Aguiar and Gopinath (2007)) and a convergence process. In particular:

$$\Gamma_t = g_t \Gamma_{t-1} \left( \frac{\Gamma^*_t}{\Gamma_{t-1}} \right)^\lambda$$

(3.3)

$$\log(g_t) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}) + \varepsilon^g_t$$

(3.4)

where $\Gamma^*_{t-1}$ is the permanent component of the foreign country, $\rho_g$ and $\lambda$ are between 0 and 1. $\varepsilon^g_t$ is iid normal with zero mean and standard deviation $\sigma_g$. $\bar{g} > 1$ is the long run mean growth rate.

A one time shock to $g$ changes the growth rate and has a permanent impact on the economy. The $\varepsilon^g_t$ is considered as trend shocks. Following a trend shock, agents will expect the economy to grow faster than its long run growth rate. This generates spending incentives in expectation of even higher output in the future. On the other hand, the $z_t$ shocks are temporary, and hence are called transitory shocks.

The permanent component $\Gamma_t$ is also affected by the output ratio of the two countries. All else equal, a lower home-foreign output ratio increases growth of the home country’s output, reflecting a convergence process. Eventually in the long run,
the output ratio goes to one, and the two countries grow at the same long run growth rate $\bar{g}$. This assumption is to generate long run output stationarity, which allows us to pin down a unique deterministic steady state and solve the model numerically.

The assumption is not unrealistic, particularly among countries and regions with similar institutional levels (for example, see Barro and Sala-i Martin (2003), chapter 1 for different states of the U.S., and Dowrick and Nguyen (1989) for OECD countries). Eaton and Kortum (1999) record that technology diffusions among G-7 countries are pervasive. Having said that, it is important to note that the main results of the paper do not depend on the assumption. In this paper, we firstly set $\lambda$ very close to zero (implying a very long convergence), and later redo the numerical exercise with a larger value for $\lambda$ (a faster convergence).

Similarly, production of the foreign country takes the form:

$$ Y_t^* = z_t^* \Gamma_t^* $$

(3.5)

The fraction $1 - \alpha$ of the endowment comes as labor income. The fraction $\alpha$ of the endowment is capital rent.

$z_t^*$ also follows an AR(1) process:

$$ \log(z_t^*) = \rho_z \log(z_{t-1}^*) + \varepsilon_t^{z*} $$

(3.6)

where $\varepsilon_t^{z*}$ is iid $\sim N(0,\sigma_z)$.

$\Gamma_t^*$ also contains an exogenous permanent component and a convergence component:

$$ \Gamma_t^* = g_t^* \Gamma_{t-1}^* \left( \frac{\Gamma_{t-1}^*}{\Gamma_{t-1}^*} \right)^\lambda $$

(3.7)

$$ \log(g_t^*) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}^*) + \varepsilon_t^{g*} $$

(3.8)
where $\varepsilon_t^g$ is iid $\sim \text{N}(0, \sigma_g)$.

For clarity, (3.3) and (3.7) can be rewritten as follow:

$$\frac{\Gamma_t^*}{\Gamma_t} = \frac{g_t^*}{g_t} \left( \frac{\Gamma_{t-1}^*}{\Gamma_{t-1}} \right)^{1-2\lambda} \quad (3.9)$$

If the system is in the long run equilibrium (i.e. $\frac{\Gamma_t^*}{\Gamma_t} = 1$ and $g_t = \bar{g}$), $\Gamma_t$ and $\Gamma_t^*$ will grow at the long run rate $\bar{g}$. In disequilibrium, the gap between $\log(\Gamma_t)$ and $\log(\Gamma_t^*)$ slowly narrows. The speed of convergence is dictated by $\lambda$. In the long run, $\Gamma_t$ and $\Gamma_t^*$ converge in ratio (i.e. $\frac{\Gamma_t^*}{\Gamma_t} \rightarrow 1$, or $\log(\Gamma_t) - \log(\Gamma_t^*) \rightarrow 0$).

Figure (3.1) shows two examples of the convergence process following a positive growth shock of 0.05% to the home country’s growth for $\lambda = 0.017$ (a fast convergence) and $\lambda = 0.001$ (a very slow convergence). Other parameters are $\bar{g} = 1.018; \rho_g = 0.930; \varepsilon_g = 0.0005$.

![Figure 3.1: Home-Foreign Output ratio after a 0.05 % trend shock](image)

(a) $\lambda = 0.017$  
(b) $\lambda = 0.001$

3.2 Assets

There are two assets: a claim on the Home capital stock and a claim on the Foreign capital stock, I refer to these as Home and Foreign equities (or assets). These two
terms will be used interchangeably below. The price at time $t$ of a unit of Home equity carried into the next period is denoted $Q_{t+1}$, measured in terms of the consumption good. The holder of this claim gets a dividend in period $t$ which is a share $\alpha$ of output, and can sell the claim for price $Q_{t+1}$. The overall return to the Home equity, in terms of the common good is:

$$R_t = \frac{Q_{t+1}}{Q_t} + \frac{\alpha Y_t}{Q_t}$$

(3.10)

Equation (3.10) states that the return to investment in domestic equity comprises of a dividend yield and an appreciation of the domestic equity.

Similarly, the price at time $t$ of a unit of Foreign equity that is carried into the next period is denoted $Q_{t+1}^*$ expressed in terms of the good. The return to Foreign equity is:

$$R_t^* = \frac{Q_{t+1}^*}{Q_t^*} + \frac{\alpha Y_t^*}{Q_t^*}$$

(3.11)

3.3 Households

An infinitely-live representative household maximizes its expected discounted utility, with an endogenous discount factor, as in Schmitt-Grohe and Uribe (2003). This is a simplest technical device to induce uniqueness of the deterministic steady state and stationary responses to temporary shocks\(^1\). Specifically, the endogenous dis-

\(^1\)A well-known problem in open macroeconomics with incomplete markets is that transitory shocks to output have permanent effects on wealth. Without any mechanism to induce stationarity, long run wealth will be non-stationary (as in Evans and Hnatkovska (2007)). To obtain a stationary long run wealth distribution, Tille and van Wincoop (2007) assume agents die with a constant probability and consume all his wealth, and new agents are born at the same rate. Ghironi, Lee, and Rebucci (2007)
count factor decreases with the consumption-output ratio. Intuitively, this means that an agent whose consumption is growing relative to output has a larger discount rate for his future consumption. Note that with this specification, the endogenous discount factor is stationary, and consistent with long run growth.

\[
U = E_0 \sum_{t=0}^{\infty} e^{-\phi \sum_{\tau=1}^{t} \log(\frac{C_{\tau}}{Y_{\tau}})} \beta^t \frac{C_t^{1-\omega}}{1-\omega}
\] (3.12)

I assume a credit market friction. In particular, agents investing abroad receive the gross return times an “local expert” cost \(e^{-\tau}\), as in Tille and van Wincoop (2007). The cost captures expenses paid to local experts for local market access and information, as well as expenses spent to overcome cultural and language barriers. This friction generates a home-bias in portfolio holdings and market incompleteness. Follow Tille and van Wincoop (2007), \(\tau\) is second order (i.e. proportional to the variances of the shocks) so that the portfolio holding is well-behaved. This assumption implies that when the shock variances go to zero, the cost \(\tau\) will also go to zero. The “local expert” cost is paid in the host country; for instance, the cost could represent payments to experts in the local economy.

Denote \(\theta_t\) as the fraction of domestic wealth invested in domestic equity carried from the last period to the current period, and \(\theta_t^*\) the fraction of foreign wealth held in foreign equity. Domestic wealth in terms of the consumption good evolves according to the following law of motion:

\[
W_{t+1} = \theta_t W_t R_t + (1 - \theta_t) W_t R_t^* e^{-\tau} + (1 - \alpha) Y_t - C_t + (1 - \theta_t^*) W_t^* R_t (1 - e^{-\tau})
\] (3.13)

and Heathcote and Perri (2007) assume a convex cost of holding portfolios. DS avoid this problem altogether by assuming zero wealth.
where $\theta_t W_t R_t + (1 - \theta_t) W_t^* R_t e^{-\tau}$ is income from equities, $(1 - \alpha) Y_t$ is the labor income, and $(1 - \theta_t^*) W_t^* R_t (1 - e^{-\tau})$ is the local expert cost that foreign investors have to pay to domestic agents.

The timing of the agent’s problem is as follows: A representative agent enters the period knowing his wealth, his domestic and foreign equity holdings, and the domestic and foreign equity prices. Output is then observed. The agent then chooses consumption and portfolio holdings for the next period, taking the returns as given. However in equilibrium, the returns are affected by the agent’s portfolio choice.

Similarly, the budget constraint faced by foreign agents is:

$$W_{t+1}^* = (1 - \theta_t^*) W_t^* R_t e^{-\tau} + \theta_t^* W_t^* R_t + (1 - \alpha) Y_t^* - C_t^* + (1 - \theta_t) W_t R_t^* (1 - e^{-\tau}) \; (3.14)$$

Due to Walras law, only one budget constraint is relevant.

Without loss of generality, I only consider the dynamic programming problem of domestic agents. Denote $d_t \equiv \theta_t W_t$; $f_t \equiv (1 - \theta_t) W_t$ and $f_t^* \equiv (1 - \theta_t^*) W_t^*$ hence $d_{t+1} \equiv \theta_{t+1} W_{t+1}$ and $f_{t+1} \equiv (1 - \theta_{t+1}) W_{t+1}$.

The domestic agent’s Bellman equation is:

$$V(d_t, f_t) = \max_{C_t, d_{t+1}} \frac{C_t^{1-\omega}}{1 - \omega} + \beta \left( \frac{C_t}{Y_t} \right)^{-\phi} E_t V(d_{t+1}, d_t R_t + f_t R_t e^{-\tau} + (1-\alpha) Y_t - C_t + f_t^* R (1-e^{-\tau}) - d_{t+1}) \; (3.15)$$

where $\beta \left( \frac{C_t}{Y_t} \right)^{-\phi}$ is the discount factor. Following Schmitt-Grohe and Uribe (2003), I assume that agents do not internalize the discount factor. This can be rationalized by assuming that the discount factor depends not upon the agents own consumption and effort, but rather on the average per capita levels of these variables. If a small $\phi$ is imposed, the short run dynamics of the system will be very close to those of a standard
model with a fixed exogenous discount factor, except that now there exists a unique steady state and long run wealth distribution is stationary.

The Euler equations for domestic agents are:

\[
C_t^{-\omega} = \beta \left( \frac{C_t}{Y_t} \right)^{-\phi} E_t[C_{t+1}^{-\omega} R_{t+1}] \tag{3.16}
\]

\[
E_t[C_{t+1}^{-\omega} R_{t+1}] = E_t[C_{t+1}^{-\omega} R^*_{t+1}] e^{-\tau} \tag{3.17}
\]

Similarly, for foreign investors:

\[
C_t^{*,-\omega} = \beta \left( \frac{C_t^*}{Y_t^*} \right)^{-\phi} E_t[C_{t+1}^{*,-\omega} R^*_{t+1}] \tag{3.18}
\]

\[
E_t[C_{t+1}^{*,-\omega} R_{t+1}] e^{-\tau} = E_t[C_{t+1}^{*,-\omega} R^*_{t+1}] \tag{3.19}
\]

(3.17) and (3.19) describe optimal portfolio choice. Note that the portfolio shares do not enter these equations directly. They enter indirectly by affecting the portfolio returns, which affect wealth in the next period and hence the asset pricing kernels.

The intuition for (3.17) and (3.19) is standard. For example (15) states that domestic investors choose their portfolios such that the expected marginal utility gain from investing in domestic equity equals that from investing in foreign equity, after adjusting for the “local expert” cost \(\tau\).

3.4 Equilibrium conditions:

The goods market clearing condition is:

\[
Y_t + Y^*_t = C_t + C^*_t \tag{3.20}
\]
while asset market clearing conditions are:

\[ Q_{t+1} = \theta_{t+1} W_{t+1} + (1 - \theta_{t+1}^* W_{t+1}^* \] (3.21) 

\[ Q_{t+1}^* = (1 - \theta_{t+1}) W_{t+1} + \theta_{t+1}^* W_{t+1}^* \] (3.22)

(3.21) and (3.22) state that asset prices equate asset demand and asset supply (which are fixed at one unit). Adding up (3.21) and (3.22) yields:

\[ Q_{t+1} + Q_{t+1}^* = W_{t+1} + W_{t+1}^* \] (3.23)

### 3.5 Valuation effects

In standard inter-temporal models, the change in the net foreign asset position equals the current account. In this model, however, the change in NFAs needs not equal the current account, because the model explicitly considers capital gains/losses arising from changes in domestic and foreign asset prices. This is referred to as “valuation effects”. The valuation effects refer to changes in real value of international asset holdings due to changes in asset prices, or to changes in exchange rates. In the model, the valuation effects for the home country are:

\[ VE_t = (1 - \theta_t) W_t \left( \frac{Q_{t+1}^*}{Q_t} e^{-\tau} - 1 \right) - (1 - \theta_t^*) W_t^* \left( \frac{Q_{t+1}}{Q_t} e^{-\tau} - 1 \right) \] (3.24)

where \((1 - \theta_t) W_t \left( \frac{Q_{t+1}^*}{Q_t} e^{-\tau} - 1 \right)\) are home country’s capital gains from foreign asset holdings, after adjusting for the “local expert” costs, and \((1 - \theta_t^*) W_t^* \left( \frac{Q_{t+1}}{Q_t} e^{-\tau} - 1 \right)\) are the foreign investors’ capital gain from holding domestic equity.

The valuation effects can be split into “expected” components and “unexpected”
components. The expected component is:

\[ EV_E_t = (1 - \theta_t)W_t \left( \frac{E_t Q_{t+1}^* e^{-\tau}}{Q_{t}^*} - 1 \right) - (1 - \theta_t^*)W_t^* \left( \frac{E_t Q_{t+1}^* e^{-\tau}}{Q_{t}^*} - 1 \right) \]

while the unexpected components is:

\[ UV_E_t = (1 - \theta_t)W_t \frac{Q_{t+1}^* - E_t Q_{t+1} e^{-\tau}}{Q_{t}^*} - (1 - \theta_t^*)W_t^* \frac{Q_{t+1} - E_t Q_{t+1} e^{-\tau}}{Q_{t}^*} \]

The current account consists of the trade balance and net factor income:

\[ CA_t = Y_t - C_t + (1 - \theta_t)W_t \frac{\alpha Y_t^* e^{-\tau}}{Q_{t}^*} - (1 - \theta_t^*)W_t^* \frac{\alpha Y_t^* e^{-\tau}}{Q_{t}^*} \quad (3.25) \]

where \( \frac{\alpha Y_t^*}{Q_{t}^*} \) and \( \frac{\alpha Y_t^*}{Q_{t}^*} \) are foreign and home dividend yields respectively.

Net assets at time \( t \) equal gross assets minus gross liabilities: \( (1 - \theta_t)W_t - (1 - \theta_t^*)W_t^* \)

The change in NFAs hence equals:

\[ \Delta NFA_t = [(1 - \theta_{t+1})W_{t+1} - (1 - \theta_{t+1}^*)W_{t+1}^*] - [(1 - \theta_t)W_t - (1 - \theta_t^*)W_t^*] \quad (3.26) \]

The change in NFAs equals the current account plus the valuation effects:

\[ \Delta NFA_t = CA_t + VE_t \quad (3.27) \]

To see this, substituting (3.24),(3.25) and (3.26) into (3.27), and use (3.10) and (3.11), equation (3.27) can be expressed as:

\[ Y_t - C_t + (1 - \theta_t)W_t R_t e^{-\tau} - (1 - \theta_t^*)W_t^* R_t e^{-\tau} = [(1 - \theta_{t+1})W_{t+1} - (1 - \theta_{t+1}^*)W_{t+1}^*] \quad (3.28) \]

Subtracting (3.28) from the budget constraint (3.14) yields:

\[ - \alpha Y_t + \theta_t W_t R_t + (1 - \theta_t^*)W_t^* R_t = \theta_{t+1} W_{t+1} + (1 - \theta_{t+1}^*)W_{t+1}^* \quad (3.29) \]
Using the asset market clearing condition (3.21), (3.29) can be expressed as:

\[-\alpha Y_t + Q_t R_t = Q_{t+1}\]  

(3.30)

which is true because \( R_t = \frac{Q_{t+1}}{Q_t} + \frac{\alpha Y_t}{Q_t} \). Therefore, (3.27) holds.
Chapter 4

Financial Integration and Valuation Effects: An Analytical Investigation

This chapter solves analytically a simplified version of the model presented in chapter 3. In particular, it only considers transitory shocks.

\[ Y_t = z_t \]
\[ Y_t^* = z_t^* \]

and

\[ \log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t^z \]
\[ \log(z_t^*) = \rho \log(z_{t-1}^*) + \varepsilon_t^{z*} \]

where \( \varepsilon_t^z \) and \( \varepsilon_t^{z*} \) are iid \( \sim N(0, \sigma_z) \).

This chapter solves analytically for the first-order approximated solution of the current account, of valuation effects and of the changes in the NFA position and shows analytically that valuation effects are stabilizing in response to transitory shocks.

The mechanism of stabilizing valuation effects works as follows: in response to a positive transitory output shock, domestic asset prices appreciate relative to foreign asset prices. This is because asset prices are forward looking and agents incorporate expected domestic output growth into domestic asset prices. The appreciation of domestic asset prices creates a negative valuation effect, which reduces the NFAs.
the other hand, agents realize the shock is temporary and save a fraction of the additional output for future consumption. In other words, the domestic country runs a current account surplus which enhances the NFA position. Valuation effects, as a result, have a stabilizing property on NFAs as they counteract the fluctuations of the current account.

The chapter also shows that the size of valuation effects relative to the current account increases with financial integration, that is it increases with risk aversion, with output volatility, with output persistence, and decreases with the discount factor and with financial frictions. The main intuition is that with a higher degree of risk sharing, the size of the current account is smaller, whereas the size of valuation effects is larger. A higher level of financial integration implies that the changes in the foreign asset’s price will have larger impact on the domestic agents’ financial wealth. At the same time, since agents are holding less of their country’s assets, the impact of domestic output shocks on the agents’ income, and consequently, on the current account, is smaller.

In addition, output persistence also has a second channel to affect the relative size of the current account and valuation effects. A higher output persistence will reduce the size of the current account, as agents are confident that the output shocks will last longer and will consume more out of the additional output, and consequently save less. On the other hand, a higher output persistence leads to a more dramatic asset price changes, and therefore a larger valuation effects.

The individual impacts are summarized in the diagram below:
4.1 Solution of the model

About methodology, this chapter (as well as the next one) uses the approach of Tille and van Wincoop (2007) and Devereux and Sutherland (2007) to solve for portfolio choice. Tille and van Wincoop (2007) and Devereux and Sutherland (2007) develop an approximation method to characterize time-varying equilibrium portfolios in a two-country dynamic general equilibrium model, where financial markets are incomplete\(^1\). In my paper, market incompleteness, along with home bias in portfolio holdings, is assumed\(^2\), by the presence of an exogenous cost of investing in foreign equities.

The benchmark parameters are set as follows: the risk aversion, discount factor and capital share are set as standards. Persistence and standard deviation of output shocks are from Coeurdacier, Kollmann, and Martin (2008), which are the averages of industrialized countries’ statistics.

\(^1\)For different solution methods, see Evans and Hnatkovska (2007); Heathcote and Perri (2007); Pavlova and Rigobon (2008).

\(^2\)For papers that seek to explain home bias in portfolio holdings, see Kollman (2006); Engel and Matsumoto (2006); Heathcote and Perri (2007); Benigno (2007); Coeurdacier, Kollmann, and Martin (2008).
Table 4.1: Values for parameters

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>Output</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.97</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Output persistence</td>
<td>0.75</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Standard deviation of output shocks</td>
<td>0.015</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of the discount factor</td>
<td>0.001</td>
</tr>
</tbody>
</table>

It is well-known that up to a first order approximation, the values of the portfolio choice $\theta_t$ and $\theta_t^*$ are indeterminate, because at this level of approximation the two assets are perfect substitutes. Previous literature usually relies on perfect market structures that make portfolio choice irrelevant.

Following Tille and van Wincoop (2007) and Devereux and Sutherland (2007), I solve for the first order accurate solution. This involves taking a first order approximation of the system, and solving for first order approximations of the non-portfolio choice variables (i.e. conditional on the long run steady state portfolio choice). Subsequently, the conditional solution is substituted into the second order approximations of the portfolio choice equations to determine the values of the long run portfolio choice. It turns out that the current account, changes in NFAs and valuation effects can all be first order approximated. Note that the first order solution will be analytical.
The solution for the steady state equilibrium can be solved:

\[
Y = Y^* = 1
\]

\[
C = C^* = 1
\]

\[
R = R^* = \frac{1}{\beta}
\]

\[
W = W^* = Q = Q^* = \frac{\alpha Y}{R - 1}
\]

\[
\theta = \theta^*
\]  

(4.1)

The first step is take the first order approximations of the system. I derive a linear system of 11 equations and 11 variables: \( \hat{w}_{t+1}, \hat{w}^*_{t+1}, \hat{q}_{t+1}, \hat{y}_{t+1}, \hat{y}^*_{t+1}, \hat{\theta}_t, \hat{\theta}^*_t, \hat{\theta}_{t+1}, \hat{\theta}^*_{t+1} \) and \( \hat{\theta}_{t+1} - \hat{\theta}^*_{t+1} \), conditional on \( \hat{w}_t, \hat{w}^*_t, \hat{q}_t, \hat{q}^*_t, \hat{y}_t, \hat{y}^*_t, \hat{\theta}_t, \hat{\theta}^*_t \), \( \theta \) and \( \theta^* \) (note that in the steady state \( \theta = \theta^* \)). For all variables except for \( \hat{\theta}_t \) and \( \hat{\theta}^*_t \), \( \hat{x} \) indicates the log-deviation of \( x \) from the steady state (\( \hat{\theta}_t \) and \( \hat{\theta}^*_t \) are deviation in levels from the long run steady state portfolio choice). The system is in Appendix A.

Two aspects of portfolio decisions that enter the first order system is firstly, the steady state portfolio \( \theta \), and secondly, the term (\( \hat{\theta}_{t+1} - \hat{\theta}^*_{t+1} \)) . They enter the system only through the first order approximations of the budget constraint equation and the market clearing condition.

\[
W \hat{w}_{t+1} + C \hat{c}_t - (1 - \alpha)Y \hat{y}_t = \theta W R \hat{R}_t + (1 - \theta) W R \hat{R}^*_t + W R \hat{w}_t \quad (4.2)
\]

\[
Q \hat{q}_{t+1} - \theta W \hat{w}_{t+1} - (1 - \theta^*) W \hat{w}^*_{t+1} = W (\hat{\theta}_{t+1} - \hat{\theta}^*_{t+1}) \quad (4.3)
\]

Note that \( \hat{\theta}_{t+1} - \hat{\theta}^*_{t+1} \) enters the system as a choice variable and hence has no impact on other state variables. Denote \( \xi_t \equiv \hat{\theta}_{t+1} - \hat{\theta}^*_{t+1} \).

The system (conditional on long run values of portfolio choice \( \theta, \theta^* \)) can be solved
with \( \hat{w}_{t+1}, \hat{w}^*_{t+1}, \hat{q}_{t+1}, \hat{q}^*_{t+1}, \hat{y}_t, \hat{y}^*_t \) as the six endogenous state variables and \( \hat{c}_t, \hat{c}^*_t, \hat{R}_t, \hat{R}^*_t \) and \( \xi_t \) as the five choice variables. The solution is also in Appendix A.

Having the first order solution conditional on the long run portfolio choice, the next step is to derive the second order approximations of the two portfolio choice Euler equations:

\[
E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}^2_{t+1} - 2\omega \hat{c}_{t+1}\hat{R}_{t+1}] = E_t[\hat{R}^*_{t+1}] + \frac{1}{2} E_t[\hat{R}^*_{t+1} - 2\omega \hat{c}^*_{t+1}\hat{R}^*_{t+1} - \tau] \tag{4.4}
\]

\[
E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}^2_{t+1} - 2\omega \hat{c}^*_{t+1}\hat{R}_{t+1} - \tau] = E_t[\hat{R}^*_{t+1}] + \frac{1}{2} E_t[\hat{R}^*_{t+1} - 2\omega \hat{c}^*_{t+1}\hat{R}^*_{t+1}] \tag{4.5}
\]

Note that since the local expert cost \( \tau \) is of second order, it does not appear in the system of first order approximation, but does appear in (4.4) and (4.5). Subtracting (4.5) from (4.4), we obtain:

\[
E_t[(\hat{c}_{t+1} - \hat{c}^*_{t+1})(\hat{R}_{t+1} - \hat{R}^*_{t+1})] = \frac{\tau}{\omega} \tag{4.6}
\]

The above equation states that long run portfolio shares are chosen such that the covariance (approximated to second order) between the difference in consumption and the excess return is proportional to the local expert cost. Note that up to second order, the covariance is time-invariant. If the “local expert” cost \( \tau \) is zero, the covariance is zero because domestic and foreign agents will have the same level of consumption regardless of the interest rate differential. In other words, both domestic and foreign investors are completely insured against country-specific risk (i.e. the market is effectively complete). If \( \tau \) is positive, foreign investment becomes less attractive, which induces home biased portfolios and thus market incompleteness. As a result, the difference in consumption is positively correlated with the realized excess return because
a country whose assets yield a higher return can afford to consume more. Note also that when agents are more risk averse (i.e. larger $\omega$), the covariance is lower, implying more balanced portfolios and a higher degree of risk sharing.

I solve for the long run portfolio choice $\theta$ by substituting the conditional result of the system into (4.6). The value of the foreign agents’ long run portfolio choice of is simply $\theta^* = \theta$.

$$\theta = 1 - \frac{1}{2\alpha} + \frac{\tau}{4\alpha \sigma_y^2 \omega} \frac{(R - \rho)^2}{(R - 1)(R - 1 + \frac{\omega}{\phi})} \quad (4.7)$$

Additional comments are necessary. For $\tau=0$ and $\alpha = 1$, then $\theta = \frac{1}{2}$. What it means is that with no labor income and no investment cost, agents optimally choose to hold 50% of the domestic equity and 50% of the foreign equity to insure themselves against the idiosyncratic shocks.

If $\tau = 0$ and $\alpha = \frac{1}{3}$ (i.e. no investment cost and labor income constitutes two-third of total income), then $\theta = -\frac{1}{2}$, that is, agents go short on domestic equity and go long on the foreign equity to hedge against the labor risk (similar to Baxter and Jermann (1997)), so that at the end, they still hold 50% of the domestic endowment and 50% of the foreign endowment. In other words, the market is complete: agents perfectly insure against country-specific shocks.

Home-bias increases with the cost of investing abroad $\tau$ and with the discount rate, decreases with the output volatility, with risk aversion, and with the output persistence. Intuitively, when agents are more risk averse, or the economic environment is more risky, agents are willing to hold more foreign equity to get close to the complete-market scenario. Addition investment cost incurred can be considered as an insurance
premium to insure against the fluctuations of income.

To match the data ($\theta=0.85$), the last term $\frac{\tau}{4\alpha\sigma^2\omega}(R-\rho)^2(R-1)(R-1+\frac{\mu}{\omega})$ should equal 1.35. For that, the investment cost $\tau$ has to be very small (about 0.00001). We only need a very small investment cost to generate a level of home bias consistent with the data, even with the existence of labor income.

4.2 Current Account and Valuation Effects

Note that the first-order approximations of all the economic variables of interest can be expressed in first order terms as follows:

$$\hat{c}a_t = Y\hat{y}_t - C\hat{c}_t + \alpha Y(1 - \theta)(\hat{w}_t - \hat{w}_t^* + \hat{y}_t - \hat{y}_t^* + \hat{q}_t - \hat{q}_t^*) - \alpha Y\xi_t$$

$$\hat{v}e_t = (1 - \theta)W(\hat{q}_{t+1} - \hat{q}_{t+1} + \hat{q}_t - \hat{q}_t^*)$$

$$\hat{v}e_t = (1 - \theta)W(\hat{q}_{t+1} - \hat{q}_{t+1} + \hat{q}_t - \hat{q}_t^*)$$

The accounting identity also can be shown in first order approximations, that is $\Delta n\hat{f}a_t = \hat{c}a_t + \hat{v}e_t$.

The first order approximated solution of the current account, valuation effects and changes in net foreign assets is:

$$\begin{pmatrix}
\Delta n\hat{f}a_t \\
\hat{v}e_t \\
\hat{c}a_t
\end{pmatrix} = A
\begin{pmatrix}
w_t \\
w_t^* \\
q_t \\
q_t^* \\
yt \\
y_t^*
\end{pmatrix}$$

(4.9)
where
\[
A = \begin{pmatrix}
nfa_1 & 0 & nfa_3 & nfa_4 & nfa_5 & nfa_6 \\
0 & 0 & ve_3 & ve_4 & ve_5 & ve_6 \\
ca_1 & 0 & ca_3 & ca_4 & ca_5 & ca_6
\end{pmatrix}
\] (4.10)

I will focus on analyzing the response of valuation effects and of the current account when there is a positive output shock. That is, I will study the following coefficients \(ve_5\) and \(ca_5\) in depth.

4.2.1 Valuation Effects

First we consider the absolute size of valuation effects:

Consider a positive shock to the domestic output, the (unexpected) valuation effect at time \(t\) is:
\[
\hat{ve}_t = ve_5 \hat{y}_t.
\]

Consider \(ve_5\). We solve for
\[
ve_5 = -W(1 - \theta)(q_5 - q_6) = -W(1 - \theta) \frac{\rho(R - 1)}{R - \rho}.
\]

A few comments are in order. First, unexpected valuation effects are negative for positive output shocks. Second, unexpected valuation effects are zero for i.i.d. shocks (i.e. \(\rho = 0\)) since in the next period, dividend payments for the two assets are expected to be the same, hence the relative asset prices do not change. Third, for a given \(\theta\), unexpected valuation effects are larger when output shocks are more persistent, because asset price changes are more dramatic in this case. Fourth, the less home-biased and more risk sharing (smaller \(\theta\)), the more significant valuation effects are.

It is intuitive, as a higher level of risk sharing implies that changes in foreign asset prices have a larger impact on the domestic agents’ wealth. As a result from the fourth comment, we can infer that any factors that encourage risk sharing also increase the
size of valuation effects. Hence the absolute size of valuation effects increases with output volatility, with risk aversion and with the persistence of output. Note that a higher output persistence raises the size of valuation effect via two channels: the first is via more dramatic responses of asset prices; and the second is via increased financial integration.

Substitute (4.7) into $\nu e_5$

$$\nu e_5 = -\frac{1}{2} \frac{\rho}{R - \rho} + \frac{\tau\rho}{4\omega\sigma^2_y} \frac{R - \rho}{(R - 1)(R - 1 + \phi)}$$

(4.11)

For the benchmark parameters, the size of the surprise negative valuation effect equals 13.06% of the additional output.

We also consider the size of expected valuation effects. Consider $\nu e_3 = W(1 - \theta)$ and $\nu e_4 = -W(1 - \theta)$: since $\nu e_3 > 0$ and $\nu e_4 < 0$, expected valuation effects positively depend on expected changes in domestic asset prices and negatively depend on expected changes in foreign asset prices. In response to a positive shock to the domestic output, domestic prices are expected to increase more than foreign prices. As a result, expected valuation effects are positive in response to a positive shock to the domestic output.

Consider a positive shock to domestic output at time $t$, expected valuation effects at time $t + 1$ is:

$$\hat{\nu} e_{t+1} = W(1 - \theta)\hat{q}_t - W(1 - \theta)\hat{q}^*_t + W(1 - \theta)(q_6 - q_5)\hat{y}_{t+1}$$
$$= W(1 - \theta)q_5\hat{y}_t - W(1 - \theta)q_6\hat{y}_t - W(1 - \theta)(q_5 - q_6)\rho\hat{y}_t$$
$$= W(1 - \theta)(q_5 - q_6)(1 - \rho)\hat{y}_t$$

(4.12)

Following a positive transitory output shocks, the expected valuation effect in the following period is positive, and is smaller in size than the surprise valuation effect.
(i.e., \(\frac{\text{eve}_{t+1}}{\text{eve}_t} = 1 - \rho\)). When the output shock is more persistent, expected valuation effects play a smaller role relative to unexpected ones, because the surprise changes in asset price are significant.

Similarly, expected valuation effects at time \(t+i\) is:

\[
e v \hat{e}_{t+i} = W(1 - \theta)(q_5 - q_6)(1 - \rho)^i \hat{y}_t \tag{4.13}
\]

Total (non-discounted) expected valuation effects are:

\[
\sum_{i=1}^{\infty} e v \hat{e}_{t+i} = W(1 - \theta)(q_5 - q_6) \frac{1 - \rho}{\rho} \hat{y}_t \tag{4.14}
\]

### 4.2.2 Current Account

In this section we consider the size of the current account in response to a domestic output shock. The coefficient for the response of the current account is \(ca_5\):

\[
ca_5 = \alpha(\theta - 1 + \frac{1}{2\alpha}) \left( \frac{1 - \rho - \phi}{R - \rho} \right)
= \tau \frac{R - \rho}{4\omega\sigma_y^2} \frac{R - 1}{R - 1 + \frac{\phi}{\omega} - 1} \tag{4.15}
\]

We consider some special cases. The first case is when \(\tau = 0\), which is the case of complete markets, current account is zero. In this situation, in response to a positive domestic output shock, the domestic country runs a trade surplus, and at the same time, repatriate pays dividends to the foreign investors. In other words, the income payment exactly offsets the trade surplus. The second special case is the case of a random walk \((\rho = 1)\), and \(\phi\) is very small, the current account is very close to zero, since agents consume all the additional income.
For the realistic values of the parameters, $ca_5$ is always positive, implying that a current account surplus will follow a positive transitory shock. Intuitively, agents realize that the shock is temporary and desire to save a fraction of additional output for future consumption, hence they smooth consumption and run a current account surplus accordingly. For the benchmark parameters, $ca_5 = 0.4$, which implies the current account is 40\% of the additional output.

From (4.15), we can see that the size of the current account increases with $\theta$. In particular, it increases with the investment cost $\tau$, decreases with output volatility, with risk aversion, and with output persistence. It is intuitively, since when agents are holding more of their country’s assets, the impact of a domestic output shock on the agents’ income, and hence, on the current account, is larger.

It is worthwhile to note that an increase in output persistence could reduce the size of the current account via another channel, that is, agents could raise consumption as high as their income as they know income shocks are persistent.

4.2.3 Relative size of Valuation Effects

This is the most interesting part, which investigates analytically when valuation effects can most significant offset the movement of the current account. Intuitively it is straightforward. In the last two sections, we have established that higher financial integration implies smaller current account and larger valuation effects. Therefore, the relative size of the (surprise) valuation effects relative to the current account should increase with the level of financial integration.
To verify, now consider the two coefficients $ve_5$ and $ca_5$:

$$
ve_5 = -\frac{1}{2} \frac{\rho}{R - \rho} + \frac{\tau \rho}{4\omega\sigma_y^2} \frac{R - \rho}{(R - 1)(R - 1 + \frac{\phi}{\omega})}
$$

$$
ca_5 = \frac{\tau}{4\omega\sigma_y^2} \frac{R - \rho}{R - 1} \left[ \frac{R - \rho}{R - 1 + \frac{\phi}{\omega}} - 1 \right]
$$

Note that $ve_5 < 0$ with $1 > \theta > 0$. Assuming this is the case, the relative size of surprise valuation effects to the current account therefore varies with this ratio:

$$
\Phi \equiv \frac{|ve_5|}{ca_5} = \frac{-ve_5}{ca_5} = \frac{\rho}{1 - \rho - \frac{\phi}{\omega}} \left[ \frac{2\omega\sigma_y^2 (R - 1)(R - 1 + \frac{\phi}{\omega})}{\tau (R - \rho)^2} - 1 \right]
$$

$$
= \frac{\rho}{1 - \rho - \frac{\phi}{\mu}} \left[ \frac{1}{2\alpha (\theta + \frac{1}{2\alpha} - 1)} - 1 \right] \quad (4.16)
$$

For the benchmark parameters, $\Phi = 0.326$, implying that the surprise negative valuation effects offsets 32.6% the current account surplus. In this sense, valuation effects are significant and plays a stabilizing role, it offsets the movement of the current account and stabilizes the NFA position.

With $ve_5 < 0$ and $\Phi > 0$, the relative size of valuation effects increases with financial integration, in particular it increases with risk aversion, with output volatility, and with output persistence. It decreases with financial friction.

### 4.2.4 Sensitivity analysis

In this section I present the sensitivity analysis regarding the relative size of valuation effects compared to the current account.

Rewrite the ratio:

$$
\Phi = \frac{\rho}{1 - \rho - \frac{\phi}{\mu}} \left[ \frac{1}{2\alpha (\theta + \frac{1}{2\alpha} - 1)} - 1 \right] \quad (4.17)
$$
In the first analysis, I examine the impact of risk sharing to the relative size of valuation effects. In particular, I change any of the following parameters: risk aversion $\omega$, output volatility $\sigma_y$ or investment cost $\tau$, and keep $\rho$ unchanged. The changes in $\omega$, $\sigma_y$ or $\tau$ will affect $\theta$, leading to changes in $\Phi$. In other words, this exercise shows the direct impact of financial integration to the relative size of valuation effects. The figure below graphs $\Phi$ against $\theta$:

![Figure 4.1: Impact of financial integration on the relative size of valuation effects](image)

The figure shows a powerful offsetting effect of valuation effects. If agents hold about 62% of their wealth in domestic equity, valuation effects will more than offset the current account, implying a deterioration of the net foreign asset position in response to a positive output shock. Note that with a long run portfolio of 62% being held in domestic equity, the economies are still far from having complete markets. With the existence of labor income, complete markets imply heavy shorting of the domestic equity to hedge against the labor income risk.

In the second analysis, I examine the impact of the output persistence to in-
ternational risk sharing and the size of valuation effects. Note that a higher output persistence increases the relative size of valuation effects via two channels. The first one is via an increase in financial integration (a smaller \( \theta \)), the second one is via the appreciation of asset prices. The two channels are reinforcing each other. In this exercise I change the value of \( \rho \) and keep other parameters unchanged. The first observation is that \( \theta \) and \( \Phi \) are very sensitive to \( \rho \). The second observation is that the change in \( \rho \) magnifies the impact of financial integration. In the second diagram, the dotted line represents the sensitivity of the relative size of valuation effects with respect to \( \theta \) when \( \rho \) changes. The solid line is the one from figure 4.1, representing the sensitivity of the relative size of valuation effects with respect to \( \theta \) when \( \rho \) is kept constant. The additional impact we observe is due to the second channel we discussed, namely, more significant changes of asset prices because of a higher output persistence.

Figure 4.2: Impact of output persistence on the relative size of valuation effects
Chapter 5

Valuation Effects with Transitory and Trend Output Shocks

This chapter solves numerically for the full version of the model, which is presented in chapter 3. Both transitory and trend output shocks are considered.

It shows that the impact of valuation effects depends critically on the nature of underlying output shocks. In response to transitory shocks, valuation effects are stabilizing; they counteract current account movements and help to soften the impact of the current account on a country’s NFA position. In response to trend shocks, valuation effects are amplifying; they move in the same direction as the current account, and reinforce, or “amplify” the impact of the current account on the NFA position. The theoretical predictions are illustrated by the evolution of the NFA position between the U.S. and other industrialized countries since the 1990s.

The mechanism of valuation effects works as follows: in response to a positive output shock, either trend or transitory, domestic asset prices appreciate relative to foreign asset prices. This is because asset prices are forward looking and agents incorporate expected domestic output growth into domestic asset prices. In both cases, the appreciation of domestic asset prices creates a negative valuation effect.

However, the role of the valuation effect in the two scenarios is very different. Following a positive transitory shock on home output, agents smooth consumption and save; the domestic country runs a current account surplus. The valuation effect, which
is negative, then partly offsets the current account surplus. As a result, the increase in
the NFAs is smaller than the current account surplus. In other words, valuation effects
have a stabilizing property on NFAs as they counteract the fluctuations of the current
account.

On the other hand, after a positive trend output shock, the role of the valuation
effect is amplifying. A positive trend output shock implies that growth is sustained,
i.e. higher output today will be followed by even higher output tomorrow. Put differ-
ently, the increase in current income is lower than the increase in permanent income.
Consumption smoothing incentive implies that consumption rises more than output,
and the domestic country runs a current account deficit. The negative valuation effect
then moves in the same direction with the current account, and reinforces the current
account deficit. As a result, the decrease in the NFAs is now more than the current
account deficit, which means valuation effects are amplifying. Simulation results indi-
cate sizable valuation effects, especially in response to trend shocks because asset price
appreciations are more dramatic in this case.

As discussed in the introduction, the theoretical results above critically depend
on the cyclicality of the current account. For the U.S., the current account is slightly
counter cyclical: the correlation between the current account and growth from 1960-
2007 is \(-0.15\)^1. For other small developed countries, the average correlation is \(-0.17\)
(Aguirar and Gopinath (2007)).

The standard neo-classical framework with endowment economies cannot explain

---

1. The correlation between quarterly net export divided by output and output from
Backus, Kehoe, and Kydland (1992) is \(-0.28\).
the counter-cyclicality of the current account. Following a positive shock, agents save a fraction of the additional output for future consumption, resulting in a current account surplus. With production economies, the counter-cyclicality of the current account can be generated because investment increases following a positive productivity shock. However, when a reasonable adjustment cost is introduced, investment becomes more sluggish and the current account again becomes pro-cyclical (Backus, Kehoe, and Kydland (1992)).

My model uses trend shocks in the spirit of Aguiar and Gopinath (2007) to account for the counter-cyclicality of the current account. I calibrated the model to match the U.S. output from 1960-2007. In the simulation, the correlation between output growth and current account is -0.13 (negative and quite close the data).

5.1 Solution of the model

The solution method is also borrowed from Tille and van Wincoop (2007) and Devereux and Sutherland (2007), as in chapter 4. However, we can not realistically solve for the analytical solution of this full version, and therefore resort to the numerical solution.

5.1.1 De-trending the system

First of all, with the trend shocks, output is non-stationary with a stochastic trend, i.e. a realization of \( g \) permanently affects \( \Gamma \). Therefore, for any home variable \( X \), following Aguiar and Gopinath (2007), I introduce a lower-case \( x \) to denote its
detrended counterpart.

\[ x_t = \frac{X_t}{\Gamma_{t-1}} \]

For any foreign variable \( X^* \), I also use introduce \( x^* \):

\[ x^*_t = \frac{X^*_t}{\Gamma_{t-1}} \]

This insures that if \( X_t \) and \( X^*_t \) are in the information set of time \( t \), so are \( x_t \) and \( x^*_t \).

The system in terms of detrended variables is presented in Appendix B. Note that there is now a new variable, \( \pi_t = \frac{\Gamma^*_t}{\Gamma_{t-1}} \), which is the ratio of the two trend processes. \( \pi_t \) is a state variable and converges to one in the steady state.

5.1.2 Solving the de-trended system

Solving the detrended system involves taking a first order approximation of all the detrended equations, and solving for first order approximations of the non-portfolio choice variables (conditional on the long run steady state portfolio choice). Subsequently, this conditional solution is substituted into the second order approximations of the portfolio choice equations to determine the value of the long run portfolio choice. The current account, changes in NFAs and valuation effects can also be first order approximated.

After solving for the detrended variables, level variables are recovered. Note that interest rate and portfolio choice decisions are invariant to this conversion.
The solution for the steady state equilibrium can be solved:

\[ y = y^* = \bar{y} \]
\[ c = c^* = \bar{y} \]
\[ R = R^* = \frac{g^*}{\beta} \]
\[ w = w^* = q = q^* = \frac{\alpha y}{R - \bar{y}} \]
\[ \theta = \theta^* \]  \tag{5.1}

The first step is to pin down the values of the long run portfolio choice. First, I take first order approximations of the model’s Euler equations and equilibrium conditions. The 16 equations are numbered (B1) to (B17), except (B10). Note that (B10) is redundant as the two portfolio choice Euler equations (B8) and (B10) yield the same first order approximations. I derive a linear system of 16 equations and 16 variables:

\[ \hat{w}_{t+1}, \hat{\xi}_{t+1}, \hat{q}_{t+1}, \hat{g}_{t+1}, \hat{\pi}_{t+1}, \hat{\zeta}_t, \hat{\xi}_t, \hat{g}_t, \hat{c}_t, \hat{\xi}_t, \hat{g}_t, \hat{R}_t, \hat{R}_t^* \text{ and } \hat{\theta}_{t+1} - \hat{\theta}_{t+1}^* \], conditional on \[ \hat{w}_t, \hat{\xi}_t, \hat{q}_t, \hat{\xi}_t, \hat{g}_t, \hat{\pi}_t, \hat{\zeta}_t, \hat{g}_t, \hat{c}_t, \hat{\xi}_t, \hat{g}_t, \hat{R}_t, \hat{R}_t^* \text{ and } \hat{\theta}_t, \hat{\theta}_t^* \] (note that in the steady state \( \theta = \theta^* \)). For all variables except for \( \hat{\theta}_t \) and \( \hat{\theta}_t^* \), \( \hat{x} \) indicates the log-deviation of \( x \) from the steady state (\( \hat{\theta}_t \) and \( \hat{\theta}_t^* \) are deviation in levels from the long run steady state portfolio choice).

Two aspects of portfolio decisions that enter the first order system is firstly, the steady state portfolio \( \theta \), and secondly, the term \( (\hat{\theta}_{t+1} - \hat{\theta}_{t+1}^*) \). They enter the system only through the first order approximations of the budget constraint equation (B12) and the market clearing condition (B16):

\[ \bar{g}w(\hat{w}_{t+1} + \hat{g}_t + \lambda \hat{\pi}_t) + c\hat{c}_t - (1 - \alpha)y\hat{\gamma}_t = \theta wR\hat{R}_t + (1 - \theta)wR\hat{R}_t^* + wR\hat{w}_t \]  \tag{5.2}
\[ q\hat{q}_{t+1} - \theta w\hat{w}_{t+1} - (1 - \theta^*)w\hat{w}_{t+1}^* = w(\hat{\theta}_{t+1} - \hat{\theta}_{t+1}^*) \]  \tag{5.3}

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Equation (5.3) shows that $\hat{\theta}_{t+1} - \hat{\theta}^*_{t+1}$ enters the system as a choice variable and hence has no impact on other state variables.

As discussed above, the two portfolio choice Euler equations (B8) and (B10) have the same first order approximations, written below:

$$E_t[\hat{R}_{t+1} - \hat{R}^*_{t+1}] = 0 \quad (5.4)$$

Equation (5.4) indicates that to a first order approximation, the expected excess return is zero.

The system (conditional on long run values of portfolio choice $\theta, \theta^*$) can be solved by any standard solution method for linear rational expectations models with $\hat{w}_{t+1}, \hat{w}^*_{t+1}, \hat{q}_{t+1}, \hat{q}^*_{t+1}$ and $\hat{R}_{t+1}$ as the five endogenous state variables and $\hat{y}_t, \hat{y}^*_t, \hat{c}_t, \hat{c}^*_t, \hat{R}_t, \hat{R}^*_t$ and $(\hat{\theta}_t - \hat{\theta}^*_t)$ as the seven choice variables.

Next, the second order approximations of the two portfolio choice Euler equations are derived:

$$E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}^2_{t+1} - 2\omega \hat{c}_{t+1} \hat{R}_{t+1}] = E_t[\hat{R}^*_{t+1}] + \frac{1}{2} E_t[\hat{R}^*_{t+1} - 2\omega \hat{c}^*_{t+1} \hat{R}^*_{t+1} - \tau] \quad (5.5)$$

$$E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}^2_{t+1} - 2\omega \hat{c}^*_{t+1} \hat{R}_{t+1} - \tau] = E_t[\hat{R}^*_{t+1}] + \frac{1}{2} E_t[\hat{R}^*_{t+1} - 2\omega \hat{c}^*_{t+1} \hat{R}^*_{t+1}] \quad (5.6)$$

Note that since the local expert cost $\tau$ is of second order, it does not appear in the system of first order approximation, but does appear in (5.5) and (5.6). Subtracting (5.6) from (5.5), we obtain:

$$E_t[(\hat{c}_{t+1} - \hat{c}^*_{t+1})(\hat{R}_{t+1} - \hat{R}^*_{t+1})] = \frac{\tau}{\omega} \quad (5.7)$$

Equation (5.7) is the detrended version of equation (4.6).
I solve for $\theta$ by substituting the conditional result of the system into (5.7). The value of the foreign agents’ long run portfolio choice of is simply $\theta^* = \theta$.

Denote the normalized current account, NFAs and Valuation Effects as $ca_t \equiv \frac{CA_t}{\Gamma_{t-1}}$; $nfa_t \equiv \frac{NFA_t}{\Gamma_{t-1}}$; and $ve_t \equiv \frac{VE_t}{\Gamma_{t-1}}$. Furthermore, we can define normalized Expected Valuation Effects and Unexpected Valuation Effects as $eve_t \equiv \frac{EVE_t}{\Gamma_{t-1}}$ and $uve_t \equiv \frac{UV E_t}{\Gamma_{t-1}}$.

Note that first-order approximations of all the economic variables of interest can be expressed in first order terms.

\[
\hat{ca}_t = y\hat{y}_t - c\hat{c}_t + \alpha y(1-\theta)(\hat{w}_t - \hat{w}_t^* + \hat{y}_t - \hat{q}_t - \hat{q}_t^*) - \alpha y(\hat{\theta}_t - \hat{\theta}_t^*)
\]

\[
nfa_t = (1-\theta)w(\hat{w}_t - \hat{w}_t^*) - w(\hat{\theta}_t - \hat{\theta}_t^*)
\]

\[
ve_t = (1-\theta)w\bar{g}(\hat{q}_{t+1} - \hat{q}_{t+1} + \hat{w}_t - \hat{w}_t^* + \hat{q}_t - \hat{q}_t^*) + (1-\theta)w(\hat{w}_t^* - \hat{w}_t)
\]

\[
+ w(1-\bar{g})(\hat{\theta}_t - \hat{\theta}_t^*)
\]

\[
eve_t = (1-\theta)w\bar{g}(E_t[\hat{q}_{t+1}] - E_t[\hat{q}_{t+1}] + \hat{w}_t - \hat{w}_t^* + \hat{q}_t - \hat{q}_t^*) + (1-\theta)w(\hat{w}_t^* - \hat{w}_t)
\]

\[
+ w(1-\bar{g})(\hat{\theta}_t - \hat{\theta}_t^*)
\]

\[
uve_t = ve_t - eve_t
\]

Note that $(\hat{\theta}_t - \hat{\theta}_t^*)$, which can be interpreted as the relative portfolio choice, is a first order term and solved in the first order system. The dynamics of the current account, changes in NFAs and valuation effects can be recovered and analyzed with the first order system.
5.2 A numerical exercise

5.2.1 Parameters

The coefficient of risk aversion, discount factor and labor share are set as standards. To estimate parameters pertaining to the shocks, I filter annual log of real GDP per capita of the U.S. and from 1960-2007 by a Hodrick Prescott (HP) filter with a smoothing parameter of 100 to recover the trend and transitory components of the series. The estimated parameters of the transitory component are $\rho_z = 0.539$; $\sigma_z = 0.016$. The estimated parameters of the permanent component are $\rho_g = 0.930$; $\sigma_g = 0.0005$ and $\bar{g} = 0.018$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of transitory shocks</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence of growth shocks</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of transitory shocks</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>Standard deviation of growth shocks</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>Long run growth rate</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Foreign investment cost</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Convergence rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of the discount factor</td>
</tr>
</tbody>
</table>

Table 5.1: Values for parameters

$\phi$ is set at an arbitrarily small value of 0.001. Recall the role of $\phi$ is to induce stationarity of long run wealth distribution. Statistics of simulated series are robust to changes in $\phi$, as long as $\phi$ remains small.

$\lambda$ is also set at 0.001. I later repeat the exercise for a larger value $\lambda = 0.017$. At
this value of $\lambda$, the gap of log output between the two countries in the model is reduced by half after 20 years. This is to correspond to an estimate of Eaton and Kortum (1999) that among G7 countries, about half of new domestic patents will be adopted overseas after 20 years or less.

$\tau$ is calibrated to be 0.000075, that is the cost of overseas investment is 0.0075 percent of the total return. $\tau$ is set so that the long run domestic equity holding $\theta$ is about 87 percent of an agent’s total portfolio. In other words, people holds 87 percent of their wealth in domestic equity (which implies that long run gross external assets are about 140 percent percent of output, which is about the level of U.S. in 2005). Overseas investing, although more costly, serves as an insurance mechanism against domestic income shocks (including labor income and equity income).

5.2.2 Impulse Responses

5.2.2.1 A transitory shock of one standard deviation

The left columns of figures 5.1 to 5.4 present impulse responses to a 1.6 percent transitory shock to home output. The shock decays quickly due to a small $\rho_z = 0.539$. The shock, together with the long run growth of $\Gamma_t$ implies that the home economy will grow at a rate of $1.016 \times 1.018 = 1.034$ or 3.4 percent right after the shock, but quickly return to its long run growth path of 1.8 percent after about ten years.

Since the shock is entirely transitory, home agents smooth consumption, and save for future consumption when output falls. As a result, trade and current account surpluses follow the shock in After that, trade and current account turn into deficits.
Trade balance will eventually get balance thanks to the endogenous discount factor ($\phi > 0$). Without this mechanism, the transitory shock would have a permanent impact on wealth distribution. The home country would be forever richer than the foreign country and could afford to run trade deficit forever.

Home equity prices increase more than foreign asset prices. The asset price ratio jumps close to 1.001, about 6% the magnitude of the shock, since the shock is transitory. This has an implication on the magnitude of the valuation effects. Valuation effects after transitory shocks are modest, since domestic asset price appreciations are small. Following the initial jump, asset prices quickly converge and go back to the steady state equilibrium ratio. This implies an expected relative decline in the prices of domestic assets, beginning in the period following the shock.

The increase in domestic asset prices coupled with the current account surplus raises the home country’s wealth, only modestly however, as the domestic price appreciation is small. Wealth ratio goes back to 1 in the long run. This is because of the stationary inducing mechanism.

In terms of valuation effects, the transitory shock causes an immediate negative effect of about about 0.08 percent of the GDP, or about a quarter of the size of the current account surplus. The “unexpected valuation effect” partly offsets the current account surplus. However beginning period 2, the expected valuation effects become positive, since relative domestic asset prices are expected to fall.

In the case of a transitory shock, most of the portfolio movements are due to the foreign investment costs, because expected marginal utilities decrease. As a result, I see substantially more home biased portfolios following a positive transitory shock.
Domestic agents decide to hold more domestic equity and less foreign equity; whereas foreign agents hold more foreign equity and less domestic equity. Therefore, gross assets and gross liabilities of the home country decrease after the shock.

After the initial shock, gross assets bounce back faster than gross liabilities, making net assets quickly rise to close to 1.5% of home output. This is mostly due to the savings of home investor and partly to the relative increase in the prices of foreign assets.

5.2.2.2 A growth shock of one standard deviation

The impulse responses to a 0.05% trend (growth) shock are shown in the right column of figures 5.1 through 5.4. Following the shock, the output ratio grows for 40 years and peaks at 1.0065 before converging to unity. Note that this convergence is assumed (as $\lambda > 0$). Without the assumption, output ratio would not go back to 1. In that sense, the shock is truly permanent.

Since the trend shock implies that the relative growth is sustained for a long time, domestic agents smooth consumption and runs both trade and current account deficits. The trade deficit lasts for 12 years, while the current account deficit lasts 40 years. This causes net assets to decrease. After that, agents anticipate a potential catch-up and start to save. In the long run, as the output ratio slowly decreases, the current account goes to surplus and gets balanced. Net assets therefore become positive and slowly converge to zero in the long run.

The asset price ratio jumps even larger to 1.0045, ten times larger than the magni-
tude of the shock. The trend shock produces a hump-shaped response in relative asset prices, reflecting sustained relative output growth. The reason is because asset prices are forward looking, they incorporate the future relative output growth. Domestic asset prices keep increase for more than 40 years after the shock.

Thanks to a huge domestic asset price appreciation, despite the current deficit, the domestic country is still a lot richer. The wealth ratio jumps close to 1.003, six times larger than the shock. After that wealth ratio goes down when the home run current account deficits. When domestic agents start to save again, the wealth ratio would picks up and converges to one in the long run, due to the stationarity inducing mechanism.

In term of the “valuation effects”, there is a huge unexpected negative valuation effect after the shock, amounting to about 0.4% of GDP, larger than the size of the current account deficit itself. This greatly exacerbates the NFAs. Furthermore, the expected valuation effects remain negative for a long period, since domestic asset prices are expected to rise for a long time.

If we increases the value of \( \lambda \), the convergence process will be faster and consequently, trade and current account deficits will more short-lived, and domestic asset price appreciation would not be as dramatic. Having said that, every qualitative results of model would hold for a larger value of \( \lambda \).

Most of the portfolio adjustment is due to risk sharing, since expected marginal utilities do not change significantly. After the growth shock, both foreign and home agents increase their holding of home equity, which correspond to the appreciation of the domestic asset prices. However foreign investors move to home equity more
aggressively. Net assets fall, reflecting the spending motive of domestic agents after the growth shock. However in the long run, net assets increase and become positive, and slowly converge to zero in the long run.

1.6% positive transitory shock 0.05% positive trend shock

Figure 5.1: Home-Foreign output and wealth ratios

Figure 5.2: Trade balance and current account as percentages of GDP
1.6 % positive transitory shock  

0.05 % positive trend shock

\[ \begin{align*}
&\text{Home–Foreign Output ratio} \\
&\text{Home–Foreign Price ratio}
\end{align*} \]

\[ \begin{align*}
&\text{Year} \\
&\text{Home–Foreign Output ratio} \\
&\text{Home–Foreign Price ratio}
\end{align*} \]

Figure 5.3: Asset price dynamics

\[ \begin{align*}
&\text{Current Account} \\
&\text{Valuation Effects} \\
&\text{Changes in NFAs}
\end{align*} \]

\[ \begin{align*}
&\text{Year} \\
&\text{Current Account} \\
&\text{Valuation Effects} \\
&\text{Changes in NFAs}
\end{align*} \]

Figure 5.4: Valuation effects as percentages of GDP

\[ \begin{align*}
&\text{5.2.3 Simulations} \\
&\text{I run a simulation exercise to investigate the quantitative importance of valuation effects. Parameters will be set to match U.S.’s business cycles. Limited data on the U.S., however, prevents meaningful matching of valuation effects’ moments.}
\end{align*} \]
### Table 5.2: Volatility of current account, valuation effects and changes in NFAs

In the simulation I generate 100 histories, each of 100 periods. Each period corresponds to one year. I run three separate simulations. First I have both shocks, then I shut off the trend shocks, and finally I shut off the temporary shocks.

Table 5.2 reports averaged simulated standard deviations of output growth, consumption growth, the trade balance, the current account, valuation effects, including the expected and unexpected components and changes in NFAs. Numbers in brackets are the standard deviations of the statistics.  


<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Both shocks</th>
<th>Only Transitory</th>
<th>Only Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{std} \left( \frac{Y_{t+1}}{Y_t} \right)$</td>
<td>Output growth</td>
<td>0.019</td>
<td>0.0188</td>
<td>0.0187</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0014)</td>
<td>(0.0014)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{C_{t+1}}{C_t} \right)$</td>
<td>Consumption growth</td>
<td>0.0138</td>
<td>0.0135</td>
<td>0.0132</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0009)</td>
<td>(0.0010)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{T B_t}{Y_t} \right)$</td>
<td>Trade balance</td>
<td>0.0185</td>
<td>0.0150</td>
<td>0.0130</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0023)</td>
<td>(0.0014)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{C A_t}{Y_t} \right)$</td>
<td>Current account</td>
<td>0.019</td>
<td><strong>0.0129</strong></td>
<td><strong>0.0109</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0015)</td>
<td>(0.0011)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{\Delta N F A_t}{Y_t} \right)$</td>
<td>Changes in NFAs</td>
<td>0.022</td>
<td><strong>0.0151</strong></td>
<td><strong>0.0104</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0019)</td>
<td>(0.0012)</td>
<td>(0.0015)</td>
</tr>
<tr>
<td>$\text{corr} \left( \frac{V E_t}{Y_t}, \frac{C A_t}{Y_t} \right)$</td>
<td>Corr(Val. Eff, CA)</td>
<td>-0.64</td>
<td>0.1412</td>
<td>-0.3884</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1158)</td>
<td>(0.1012)</td>
<td>(0.1216)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{U V E_t}{Y_t} \right)$</td>
<td>Unexpected Val. Eff.</td>
<td>n/a</td>
<td>0.0057</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0004)</td>
<td>(0.0001)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{E V E_t}{Y_t} \right)$</td>
<td>Expected Val. Eff.</td>
<td>n/a</td>
<td>0.0028</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0012)</td>
<td>(0.0004)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td>$\text{std} \left( \frac{V E_t}{Y_t} \right)$</td>
<td>Valuation Effects</td>
<td>0.018</td>
<td>0.0061</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0008)</td>
<td>(0.0004)</td>
<td>(0.0007)</td>
</tr>
</tbody>
</table>
If only transitory shocks are present, the consumption growth is less volatile than output growth, since domestic agents only consume a fraction of additional output. The valuation effects are mitigating, they move in opposite directions of the current account and help soften the impact of the current account’s volatility on the country’s NFAs. As a result, changes in NFAs are slightly less volatile than the current account. Valuation effects are said to be small because their average standard deviation is about one seventh of the output shocks’. In this exercise, valuation effects are on average .79% of output.

We can see an entirely different picture with trend shocks. If only trend shocks are present, consumption growth is more volatile than output growth. Valuation effects are positively correlated with the current account; they amplify the impact of the current account on the country’s NFA position. As a result, the changes in net foreign assets are much more volatile than the current account, their average deviation is one and a half time as large as the current account’s. Finally, valuation effects are larger in magnitude with trend shocks, their standard deviation is almost twice as large as the shocks’. Valuation effects in this exercise are on average about 3.4% of output.

When both shocks are present the changes in NFAs are more volatile than the current account, indicating a dominance of growth shocks over transitory shocks. The correlation between valuation effects and the current account is positive, but not statistically significant.

Unlike Devereux and Sutherland (2009), we consider asset prices explicitly and hence can study the quantitative importance of expected and unexpected valuation
effects with first order approximation\textsuperscript{3}. From the exercise, in all three cases, expected and unexpected components of valuation effects have more or less equal quantitative importance.

5.3 The U.S.’s valuation effects with other G7 countries

The paper’s theoretical results have some important implications for the U.S.’s external imbalances. The U.S. has experienced persistently higher economic and productivity growth than other industrialized countries in the 1990s. The average annual growth rate of U.S. PPP GDP during 1990-2000 was 1.94\%, compared to 1.47\% for other G7 countries (henceforth referred to as G6). At the same time, U.S.’s relative stock prices have been in an upward trend, while the U.S.-G6 current account balance has continued to worsen (Figures 5.5 and 5.6).

![Figure 5.5: U.S.-G6 Normalized Total Factor Productivity ratios](image)

The theoretical results imply that if the U.S indeed had a positive trend output

\textsuperscript{3}Ghironi, Lee, and Rebooki (2007) also follow the same approach.
shock relative to other industrialized countries, the valuation effects between the U.S. and these countries were negative and they worsened the impact of the current account deficit on the U.S.’s NFA position.

Figure 5.6: Log of normalized stock price index ratios, 1990-2007

Figure 5.7 confirms that from 1994 to 2001, U.S.-G6 valuation effects were negative (except in 1999), and they exacerbated the impact of the current account deficit
on the NFAs\textsuperscript{4}. After 2002, valuation effects became positive, reflecting a slowdown of the U.S. economy and the decline of the U.S. stock market. The large sizes of the valuation effects after 2002 are partly due to the increase in cross country portfolio holdings.

\textsuperscript{4}Unfortunately data on valuation effects are only available after May 1994 (the 1994 position in the graph only covers the last seven months of the year).
Chapter 6

Conclusions

This dissertation investigates empirically and theoretically the role of valuation effects in a country’s external adjustment.

It first shows that the U.S.’s valuation effects from stocks and bonds were significant and in general stabilizing during 1994-2007: they partly offset the current account deficits and stabilized the net foreign asset position. In particular, total valuation effects from stocks and bonds during this period were $1295 billion, offsetting about 22.8% of the U.S.’s total current account deficits. Although the effects were often negative from 1994-2002, they were always large and positive from 2002-2007. These facts imply that U.S.’s assets held overseas had a lower average return than that of U.S.’s liabilities before 2002, and a higher average return than that of U.S.’s liabilities after 2002. We also find that about 60% of the U.S.’s valuation effects (from stocks and bonds) came from developed countries; and 40% came from emerging markets.

This subsequent chapters investigate analytically the role of valuation effects in a two country DSGE model with both transitory output shocks and trend shocks. They show that whether valuation effects are indeed stabilizing depends on the nature of underlying output shocks. In response to transitory shocks, valuation effects are stabilizing; they counteract current account movements and partly offset the current account. In response to trend shocks, valuation effects are amplifying, they move in
the same direction with the current account and reinforce the impact of the current account on changes in net foreign asset position. Unlike conventional wisdom that valuation effects tend to be stabilizing, the paper shows that valuation effects can be amplifying too. This is clearly illustrated by the evolution of NFA position between the U.S. and other industrialized countries in the 1990s, when the U.S. experienced persistently higher economic growth. During the period, the U.S. had current account deficits and negative valuation effects with other G7 countries.

In a simplified version of the model, the dissertation analyzes analytically that the size of valuation effects relative to the current account is positively related with financial integration, which in turn increases with risk aversion, with output volatility, with output persistence, and decreases with the discount factor and with the cost of investing abroad. For the benchmark calibration, when domestic investors hold about 40% or more of their financial wealth in foreign equity, valuation effects will completely offset the current account.
Appendix A

The log-linearized system of the simplified model

The first order system is:

\[ \hat{y}_{t+1} = \rho \hat{y}_t + \varepsilon_t^y \] (A.1)

\[ \hat{y}^*_{t+1} = \rho \hat{y}^*_t + \varepsilon_t^{y^*} \] (A.2)

\[ R\hat{R}_t = \hat{q}_{t+1} - \hat{q}_t + \frac{\alpha Y}{Q}(\hat{y}_t - \hat{q}_t) \] (A.3)

\[ R\hat{R}^*_t = \hat{q}^*_{t+1} - \hat{q}^*_t + \frac{\alpha Y}{Q}(\hat{y}^*_t - \hat{q}^*_t) \] (A.4)

\[ -\omega \hat{c}_t = -\phi \hat{c}_t + E_t(-\omega \hat{c}_{t+1} + \hat{R}_{t+1}) \] (A.5)

\[ -\omega \hat{c}^*_t = -\phi \hat{c}^*_t + E_t(-\omega \hat{c}^*_{t+1} + \hat{R}^*_{t+1}) \] (A.6)

\[ E_t\hat{R}_{t+1} = E_t\hat{R}^*_{t+1} \] (A.7)

\[ \hat{y}_t + \hat{y}^*_t = \hat{c}_t + \hat{c}^*_t \] (A.8)

\[ \hat{w}_{t+1} + \hat{w}^*_{t+1} = \hat{q}_{t+1} + \hat{q}^*_t \] (A.9)

\[ W\hat{w}_{t+1} + C\hat{c}_t - (1 - \alpha)Y\hat{y}_t - WR\hat{w}_t = \theta WRR\hat{R}_t + (1 - \theta)WR\hat{R}^*_t \] (A.10)

\[ Q\hat{q}_{t+1} - \theta W\hat{w}_{t+1} - (1 - \theta^*)W\hat{w}^*_{t+1} = W(\hat{\theta}_{t+1} - \hat{\theta}^*_{t+1}) \] (A.11)
The first order solution of the system is:

\[
\begin{pmatrix}
\dot{c}_t \\
\dot{c}_t^* \\
\dot{\xi}_t \\
\dot{R}_t \\
\dot{R}_t^* \\
\dot{w}_t \\
\dot{w}_t^* \\
\dot{q}_t \\
\dot{q}_t^* \\
\end{pmatrix} =
\begin{pmatrix}
c_1 & 0 & c_3 & c_4 & c_5 & 1 - c_5 \\
-c_1 & 0 & -c_3 & -c_4 & 1 - c_5 & c_5 \\
\xi_1 & 0 & \xi_3 & \xi_4 & \xi_5 & \xi_6 \\
0 & 0 & -1 & 0 & r_5 & r_6 \\
0 & 0 & 0 & -1 & r_6 & r_5 \\
w_1 & 0 & w_3 & w_4 & w_5 & q_5 + q_6 - w_5 \\
-w_1 & 0 & -w_3 & -w_4 & q_5 + q_6 - w_5 & w_5 \\
0 & 0 & 0 & 0 & q_5 & q_6 \\
0 & 0 & 0 & 0 & q_6 & q_5 \\
\end{pmatrix}
\begin{pmatrix}
w_t \\
w_t^* \\
q_t \\
q_t^* \\
y_t \\
y_t^* \\
\end{pmatrix}
\]

(A.12)

where:

\[
c_1 = \frac{W}{C} (R - 1 + \frac{\phi}{\omega}) \quad c_3 = -\theta \frac{W}{C} (R - 1 + \frac{\phi}{\omega}) \quad c_4 = -(1 - \theta) \frac{W}{C} (R - 1 + \frac{\phi}{\omega})
\]

\[
c_5 = \frac{1}{2} \frac{1 - \rho - \frac{\omega}{R}}{R - \rho} + \frac{(R-1+\frac{\phi}{C})(1-\alpha+\alpha\theta)}{R-\rho}
\]

\[
r_5 = \frac{1}{2} \frac{\omega(1-\rho)-\phi}{R-\rho} + \frac{R-1}{R-\rho} \quad r_6 = \frac{1}{2} \frac{\omega(1-\rho)-\phi}{R-\rho}
\]

\[
w_1 = 1 - \frac{\phi}{\omega} \quad w_3 = -\theta \frac{1 - \frac{\phi}{\omega}}{\omega} \quad w_4 = -(1 - \theta) \frac{1 - \frac{\phi}{\omega}}{\omega}
\]

\[
w_5 = -\frac{C}{W} c_5 + (1 - \alpha)y + \frac{1}{2} \frac{R\omega(1-\rho)-\phi}{R-\rho} + \theta \frac{R(R-1)}{R-\rho}
\]

\[
q_5 = \frac{R\omega(1-\rho)-\phi}{2} + \frac{R-1}{R-\rho} \quad q_6 = \frac{R\omega(1-\rho)-\phi}{2} + \frac{R-1}{R-\rho}
\]
Appendix B

The de-trended system of the full model

Exogenous processes:

\[
\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_t^z \quad (B.1)
\]

\[
\log(z_t^*) = \rho_z \log(z_{t-1}^*) + \varepsilon_t^{z*} \quad (B.2)
\]

\[
\log(g_t) = (1 - \rho_g) \log(g_{t-1}) + \varepsilon_t^g \quad (B.3)
\]

\[
\log(g_t^*) = (1 - \rho_g) \log(g_{t-1}^*) + \varepsilon_t^{g*} \quad (B.4)
\]

Output processes:

\[
y_t = z_t g_t (\pi_t)^\lambda \quad (B.5)
\]

\[
y_t^* = z_t^* g_t^* (\pi_t)^{1-\lambda} \quad (B.6)
\]

\[
\pi_t = \frac{g_t}{g_t-1} \pi_{t-1}^{1-2\lambda} \quad (B.7)
\]

where \( \pi_t = \frac{g_t^*}{g_t} \pi_{t-1}^{1-2\lambda} \) is the ratio of the two trend processes. \( \pi_t \) is a state variable at time \( t \) and converges to one in the steady state.

Euler equations in detrended form become:

\[
c_t^{-\omega} = \beta \left( \frac{c_t}{y_t} \right)^{-\phi} E_t[c_{t+1}^{-\omega}(g_t \pi_t^\lambda)^{-\omega} R_{t+1}] \quad (B.8)
\]

\[
E_t[c_{t+1}^{-\omega} R_{t+1}] = E_t[c_{t+1}^{-\omega} R_{t+1}^*] e^{-\tau} \quad (B.9)
\]

\[
c_t^*^{-\omega} = \beta \left( \frac{c_t^*}{y_t} \right)^{-\phi} E_t[c_{t+1}^*^{-\omega}(g_t \pi_t^\lambda)^{-\omega} R_{t+1}^*] \quad (B.10)
\]

\[
E_t[c_{t+1}^*^{-\omega} R_{t+1}] e^{-\tau} = E_t[c_{t+1}^*^{-\omega} R_{t+1}] \quad (B.11)
\]
while the domestic agent’s budget constraint and market clearing conditions are now:

\[
g_t \pi_t^\lambda w_{t+1} + c_t = \theta_t w_t R_t + (1 - \theta_t) w_t R_t^* e^{-\tau} + (1 - \theta^*) w_t^* R_t (1 - e^{-\tau}) + (1 - \alpha) y_t
\]  
\[(B.12)\]

\[
R_t = g_t \pi_t^\lambda \frac{q_{t+1}}{q_t} + \frac{\alpha y_t}{q_t}
\]  
\[(B.13)\]

\[
R_t^* = g_t \pi_t^\lambda \frac{q_{t+1}^*}{q_t^*} + \frac{\alpha y_t^*}{q_t^*}
\]  
\[(B.14)\]

\[
y_t + y_t^* = c_t + c_t^*
\]  
\[(B.15)\]

\[
q_{t+1} = \theta_{t+1} w_{t+1} + (1 - \theta_{t+1}^*) w_{t+1}^*
\]  
\[(B.16)\]

\[
q_{t+1} + q_{t+1} = w_{t+1} + w_{t+1}^*
\]  
\[(B.17)\]
Bibliography


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