Optimal Preview Control of Markovian Jump Linear Systems

Kenneth D. Running, Nuno C. Martins

The
Institute for
Systems
Research

UNIVERSITY OF MARYLAND 18 56

A. JAMES CLARK
SCHOOL OF ENGINEERING

Optimal Preview Control of Markovian Jump Linear Systems

Kenneth D. Running and Nuno C. Martins

Abstract

In this paper, we investigate the design of controllers, for discrete-time Markovian jump linear systems, that achieve optimal reference tracking in the presence of preview (reference look-ahead). For a quadratic cost and given a reference sequence, we obtain the optimal solution for the full information case. The optimal control policy consists of the additive contribution of two terms: a feedforward term and a feedback term. We show that the feedback term is identical to the standard optimal linear quadratic regulator for Markovian jump linear systems. We provide explicit formulas for computing the feedforward term, including an analysis of convergence.

I. INTRODUCTION

This paper deals with the problem of designing control systems that achieve optimal reference tracking in discrete-time. More specifically, we consider the *servomechanism* problem, i.e., given a reference, the objective is to design feedback and feedforward strategies so that the state of the plant tracks the reference optimally, according to a quadratic cost. Here, we consider a plant that is linear but varies in time according to a Markovian process that takes values in a finite alphabet, such systems are called *Markovian jump linear systems*.

Basic notation: Throughout the paper, we adopt the following notation: (1) Boldface letters, such as $\mathbf{x}$, indicate (possibly vector valued) real random variables while the default font is used to represent particular realizations of a random variable or deterministic quantities. (2) If G is a matrix then $[G]_{i,j}$ is the entry located at the ith row and jth column. (3) if G is a matrix then G' indicates its transpose. Further notation will be introduced as needed.

Nuno C Martins is with the ECE Department and the ISR, University of Maryland, College Park. Address: Room 2259 AV Williams Bldg, University of Maryland, College Park, 20742 MD.(**E-mail:nmartins@isr.umd.edu**) Kenneth D. Running is with the Department of Aerospace Engineering, University of Maryland.

Definition 1.1: (**Markovian jump linear system: state space recursion**) Let $\bar{m}$, n and q be given positive integers along with a matrix of conditional probabilities $M \in [0, 1]^{\bar{m} \times \bar{m}}$ satisfying $\sum_{i=1}^{\bar{m}} [M]_{ij} = 1$, for each j in the set $\{1, \ldots, \bar{m}\}$. Consider also a given collection of matrices $\{A(i)\}_{i=1}^{\bar{m}}$ and $\{B(i)\}_{i=1}^{\bar{m}}$, where for each integer i in the set $\{1, \ldots, \bar{m}\}$ it holds that $A(i) \in \mathbb{R}^{n \times n}$ and $B(i) \in \mathbb{R}^{n \times q}$. In addition, consider two independent random variables $\mathbf{x}_0$ and $\mathbf{m}_0$ taking values in $\mathbb{R}^n$ and $\{1, \ldots, \bar{m}\}$, respectively. The following specifies the state recursion of a discrete-time Markovian jump linear system:

$$\mathbf{x}_{k+1} = A(\mathbf{m}_k)\mathbf{x}_k + B(\mathbf{m}_k)\mathbf{u}_k, \ \ k \geq 0 \tag{1}$$

where $\mathbf{m}_k$ is a Markovian process taking values in the set $\{1, \ldots, \bar{m}\}$ and whose statistical behavior is governed by $Pr(\mathbf{m}_{k+1} = i | \mathbf{m}_k = j) = [M]_{ij}$, $k \geq 0$. We also assume that $\mathbf{x}_0$ has a finite covariance matrix and that $\mathbf{m}_k$ has <u>no transient states</u>, i.e., $\liminf_{k \to \infty} Pr(\mathbf{m}_k = i) > 0$ for all $i \in \{1, \ldots, \bar{m}\}$. In this description, $\mathbf{u}_k$ takes values in $\mathbb{R}^q$ and it represents the system's input. In addition, we assume that $\{\mathbf{m}_l\}_{l=0}^{\infty}$ is independent of $\mathbf{x}_0$. The overall state of the system is denoted as follows:

$$\mathbf{s}_k \stackrel{def}{=} (\mathbf{x}_k, \mathbf{m}_k)$$

A. Survey of related results

The problem of designing controllers, for Markovian jump linear systems, that achieve optimal reference tracking has been discussed, for the continuous time case, in [15]. In this paper, we formulate the problem in discrete-time, and we show that the optimal solution results from the additive contribution of a feedforward and a feedback term. In addition, we give sufficient conditions for the convergence of the feedforward term, and when it converges we provide an explicit formula for its computation. In contrast to our work, where we assume that the reference is deterministic and known for the entire future, the authors of [35], [36], [37] have investigated tracking and the effects of perturbations, for the case where the reference or perturbation is a Markov process subject to causal processing. Subsequently, we give a short survey, of the state of the art in the design of optimal controllers of Markovian jump linear systems. This is followed by a brief discussion of existing results in optimal reference tracking for deterministic systems.

1) Results on Optimal Control of Markovian Jump Linear Systems: Motivated by a wide spectrum of applications, for the last thirty years, there has been active research in the analysis [3], [22] and in the design of controllers [10] for Markovian jump linear systems. More specifically,

in the last fifteen years, the classical paradigms of optimal control have been re-visited for Markovian jump linear systems, such as the ones defined by H_2 and mixed H_2/H_∞ measures of performance [13], [17], [15] (see [14] for a more detailed survey of existing work). The authors of [16] also propose an elegant method to analyze and design controllers for Markovian jump linear systems in continuous time. Other approaches aiming at the design of robust controllers can be found in [5], [4]. Not only optimal solutions were characterized, but also the optimal cost and its associated control law can be computed by means of solving *linear matrix inequalities* (LMIs) [12].

2) Brief Survey of Results on the Theory of Optimal Reference Tracking for Deterministic Systems (Optimal servomechanism design): Classical approaches guarantee asymptotic tracking of certain periodic references via the internal model principle, at the expense of *state augmentation techniques* [1]. In the late eighties, techniques based on operator theory were used to derive control laws for linear and time-invariant systems that guarantee optimal reference tracking, under the assumption of finite look-ahead and infinite look-ahead preview [26], [21]. The papers [18], [32], [31] are also relevant contributions for the particular case where the reference is available with no look-ahead (no preview). Examples of application can be found in [1], [28]. More recently, since the nineties, the theory of control leading to optimal reference tracking, for deterministic systems, has a wide portfolio of interesting results. In particular, more general performance metrics, such as H_∞, have been considered [11], [24]. There is also a substantial collection of results on fundamental limits of optimal reference tracking [25] for a variety of metrics [30], constraints [9], [8], [7] and plant classes [29], [6], [20], [34]. All of these results, in one way or another, conclude that reference look-ahead (preview) may lead to a substantial increase in the tracking performance.

Paper Organization: This paper has five sections, besides the introduction: Section II gives preliminary definitions and a review of the linear quadratic optimal control of Markovian jump linear systems, while Section III focuses on the formulation of the optimal reference tracking problem, in the presence of preview. The optimal solution is derived in Sections IV and V. The paper ends with conclusions in Section VI.

---

[1]Notice that such state-augmentation techniques are impractical if the plant is a Markovian jump linear system with a large number of modes, because all of the A_i matrices will need augmentation leading to very high dimensional controllers.

II. PRELIMINARY DEFINITIONS AND REVIEW OF THE OPTIMAL LINEAR QUADRATIC REGULATOR (LQR) FOR MARKOVIAN JUMP LINEAR SYSTEMS

The results reviewed in this section will be used as a base for the extensions in Section III.

Definition 2.1: (State filtration) Let $\mathbf{s}_k = (\mathbf{x}_k, \mathbf{m}_k)$ be the state of a Markovian jump linear system (MJLS). The filtration that is generated by $\mathbf{s}_k$ is defined as:

$$\mathcal{S}_k \stackrel{def}{=} \sigma(\mathbf{s}_t; 0 \le t \le k) \tag{2}$$

where $\sigma(\mathbf{s}_t; 0 \le t \le k)$ is the smallest σ-field [23] with respect to which $\mathbf{s}_t$ is measurable for $t \in \{0, \dots, k\}$.

Definition 2.2: (**Regulator**) Let $\mathbf{s}_k = (\mathbf{x}_k, \mathbf{m}_k)$ be the state of an n dimensional Markovian jump linear system (MJLS) with input $\mathbf{u}_k$ taking values in $\mathbb{R}^q$. The class of regulators consists of all measurable feedback policies $\mathcal{R}_k$ with the following structure:

$$\mathbf{u}_k = \mathcal{R}_k(\{\mathbf{s}_l\}_{l=0}^k), \ \ k \ge 0 \tag{3}$$

Definition 2.3: (**Expectation and conditional expectation**) Let $\mathbf{s}_k = (\mathbf{x}_k, \mathbf{m}_k)$ be the state of an n dimensional Markovian jump linear system (MJLS) and assume that the input $\mathbf{u}_k$, taking values in $\mathbb{R}^q$, is adapted to $\{\mathcal{S}_k\}$ (such as in (3)). Given a non-negative integer k and a stochastic process $\mathbf{z}_k$ adapted to $\{\mathcal{S}_k\}$, throughout the paper we use $E_k[\mathbf{z}_k]$ to denote expectation of $\mathbf{z}_k$. Similarly, given two non-negative integers T and k satisfying $k < T$, we use the following notation for conditional expectation [23]:

$$E_{T|k}[\mathbf{z}_T] \stackrel{def}{=} E[\mathbf{z}_T | \mathcal{S}_k] \tag{4}$$

Problem 2.1: (**Optimal infinite horizon linear quadratic regulator (LQR): problem formulation**) Consider a Markovian jump linear system, as in Definition 1.1, and denote by n and q its order and dimension of the input, respectively. Given a regulator $\{\mathcal{R}_l\}_{l=0}^\infty$, and symmetric matrices $S \in \mathbb{R}^{n \times n}$ and $Q \in \mathbb{R}^{q \times q}$, which are symmetric and positive semi-definite and positive definite, respectively, we adopt the following cost function:

$$\mathcal{L}\left(\{\mathcal{R}_l\}_{l=0}^\infty\right) = \lim_{T \to \infty} E_T \left[\sum_{l=0}^T \mathbf{x}_l' S \mathbf{x}_l + \mathbf{u}_l' Q \mathbf{u}_l \right] \tag{5}$$

where $\mathbf{u}_k = \mathcal{R}_k(\{\mathbf{s}_l\}_{l=0}^k)$. The infinite horizon optimal *linear quadratic regulator* paradigm is defined by the following optimization problem:

$$\{\mathcal{R}_l^*\}_{l=0}^\infty \stackrel{def}{=} arg \min_{\{\mathcal{R}_l\}_{l=0}^\infty} \mathcal{L}\left(\{\mathcal{R}_l\}_{l=0}^\infty\right) \tag{6}$$

subject to $\mathcal{R}_l$ adapted to $\mathcal{S}_l$.

Definition 2.4: **Coupled Algebraic Riccati Equations (CARE)** Let all parameters needed in the definition of a MJLS be given. The CARE associated with Problem 2.1 is given by the following collection of matrix equations (see[2] [14, (Definition 4.4)]):

$$P(i) = S + A(i)'\bar{P}(i)A(i) - A(i)'\bar{P}(i)B(i)\left(Q + B(i)'\bar{P}(i)B(i)\right)^{-1}B(i)'\bar{P}(i)A(i),$$

$$\bar{P}(i) = \sum_{j \in \{1,\ldots,\bar{m}\}} [M]_{j,i}P(j), \ P(i) = P(i)' \geq 0, \ i \in \{1,\ldots,\bar{m}\} \quad (7)$$

Definition 2.5: (**Stabilizing solution**) Let $\{P(i)\}_{i=1}^{\bar{m}}$ be a solution of the CARE (7). In addition, consider the collection of matrices $\{K(i)\}_{i=1}^{\bar{m}}$ taking values in $\mathbb{R}^{q \times n}$ given by:

$$K(i) = -\left(Q + B(i)'\bar{P}(i)B(i)\right)^{-1}B(i)'\bar{P}(i)A(i), \ i \in \{1,\ldots,\bar{m}\} \quad (8)$$

A solution $\{P(i)\}_{i=1}^{\bar{m}}$ to the CARE (7) is qualified as stabilizing if the feedback law $\mathbf{u}_k = K(\mathbf{m}_k)\mathbf{x}_k$ stabilizes the MJLS, in the sense that $\lim_{k\to\infty} E_k[\mathbf{x}_k^T\mathbf{x}_k] = 0$.

Remark 2.1: (**Unicity and optimality properties of positive solutions to the CARE**) From [14, Lemma A.14] it follows that the CARE (7) has at most one stabilizing solution. General conditions for the existence of a stabilizing solution are given in [14, Theorem A.15]. In particular, if S is positive definite then, from [14, Proposition 3.42], it follows that the MJLS is stabilizable if and only if the CARE (7) has a solution. As such, we conclude from [14, Theorem A.15] that if S is positive definite then the CARE (7) has a solution if and only if such a solution is unique and stabilizing. If S is positive definite and the CARE (7) has a stabilizing solution $\{P(i)\}_{i=1}^{\bar{m}}$ then from [14, Theorem 4.5] it follows that the infinite horizon optimal regulator problem has a unique optimal solution given by $\mathbf{u}_k^* = K(\mathbf{m}_k)\mathbf{x}_k$, where $\{K(i)\}_{i=1}^{\bar{m}}$ is given by (8), and that the optimal cost is given by $\mathcal{L}^* = E_0[\mathbf{x}_0'P(\mathbf{m}_0)\mathbf{x}_0]$.

III. Optimal preview control: Problem formulation

In this section, we formulate the optimal preview control paradigm, under full-state feedback. We start by defining the following class of admissible preview controllers:

Definition 3.1: (**Preview controller**) Let $\mathbf{s}_k = (\mathbf{x}_k, \mathbf{m}_k)$ be the state of a Markovian jump linear system (MJLS) with input $\mathbf{u}_k$ and continuous state $\mathbf{x}_k$ taking values in $\mathbb{R}^q$ and $\mathbb{R}^n$, respectively. Given a time horizon $T \in \mathbb{N}\bigcup\{\infty\}$, a reference sequence $\{r_l\}_{l=0}^{T}$ taking values in

---

[2]In order to match our notation with [14], select $C_i^*C_i = R$, $D_i^*D_i = Q$, $X_i = P(i)$ and $\mathcal{E}_i(X) = \bar{P}(i)$.

$\mathbb{R}^n$, the class of preview (forecast) controllers consists of all feedback policies with the following structure:

$$\mathbf{u}_k = \mathcal{F}_{k,T}(\{\mathbf{s}_l\}_{l=0}^k, \{r_l\}_{l=k+1}^T) \tag{9}$$

Problem 3.1: (**Optimal preview control:problem statement**) Consider a Markovian jump linear system, as in Definition 1.1, and denote by n and q its order and dimension of the input, respectively. Given time horizon $T \in \mathbb{N}$, a sequence $\{r_l\}_{l=0}^T$ taking values in $\mathbb{R}^n$, a preview controller policy $\{\mathcal{F}_{l,T}\}_{l=0}^T$, and symmetric matrices $S \in \mathbb{R}^{n \times n}$ and $Q \in \mathbb{R}^{q \times q}$, which are positive semi-definite and positive definite, respectively, we adopt the following cost function:

$$\mathcal{J}_T\left(\{\mathcal{F}_{l,T}, r_l\}_{l=0}^T\right) \stackrel{def}{=} \frac{1}{T+1} E_T\left[\sum_{l=0}^T (\mathbf{x}_l - r_l)'S(\mathbf{x}_l - r_l) + \mathbf{u}_l'Q\mathbf{u}_l\right] \tag{10}$$

where $\mathbf{u}_k = \mathcal{F}_{k,T}(\{\mathbf{s}_l\}_{l=0}^k, \{r_l\}_{l=k+1}^T)$. The optimal *preview control* paradigm is defined by the following optimization problem:

$$\{\mathcal{F}_{l,T}^*\}_{l=k+1}^T \stackrel{def}{=} arg \min_{\{\mathcal{F}_{l,T}\}_{l=0}^T} \mathcal{J}_T\left(\{\mathcal{F}_{l,T}, r_l\}_{l=k+1}^T\right) \tag{11}$$

subject to $\mathcal{F}_{k,T}(\cdot, \{r_l\}_{l=k+1}^T)$ adapted to $\mathcal{S}_k$. The infinite horizon optimal preview control paradigm is defined via the minimization of the limit supremum of the cost (10), when T tends to infinity. Optimal solutions to the infinite horizon optimal preview control paradigm are written as $u_k = \mathcal{F}_{k,\infty}^*(\{s_l\}_{l=0}^k, \{r_l\}_{l=k+1}^\infty)$, where $\mathcal{F}_{k,\infty}^*$ denotes an optimal control policy.

IV. OPTIMAL SOLUTION TO THE INFINITE HORIZON ($T = \infty$) OPTIMAL PREVIEW CONTROL PARADIGM

In this Section, we solve the infinite horizon optimal preview control problem for Markovian jump linear systems. The main result is given in Theorem 4.1.

Theorem 4.1: Consider Problem 3.1 with S positive definite and the following two conditions: (**Condition 1**) The CARE (7) has a positive definite solution. (**Condition 2**) The reference sequence $\{r_l\}_{l=0}^\infty$ is such that the following limits exist for all $k \geq 0$ (Note: A proof of convergence, in the presence of bounded reference sequences, is provided in Section V):

$$B(i)'L_{k,\infty}(i, \{r_l\}_{l=k}^\infty) \stackrel{def}{=} \lim_{T \to \infty} B(i)'L_{k,T}(i, \{r_l\}_{l=k}^T), \ i \in \{1, \ldots, \bar{m}\} \tag{12}$$

where $\{L_{k,T}(1, \{r_l\}_{l=k}^T)\}_{k=0}^T$ through $\{L_{k,T}(\bar{m}, \{r_l\}_{l=k}^T)\}_{k=0}^T$ are sequences of vectors taking values in $\mathbb{R}^n$ defined via the following backward recursion:

$$L_{k,T}(i, \{r_l\}_{l=k}^T) \stackrel{def}{=} \begin{cases} (A(i) - B(i)K(i))' \left[\bar{P}(i)(A(i)r_k - r_{k+1}) + \bar{L}_{k,T}(i, \{r_l\}_{l=k+1}^T)\right] & k < T \\ 0 & k = T \end{cases}$$
(13)

$$\bar{L}_{k,T}(i, \{r_l\}_{l=k+1}^T) \stackrel{def}{=} \sum_{j \in \{1,\ldots,\bar{m}\}} [M]_{j,i} L_{k+1,T}(j, \{r_l\}_{l=k+1}^T) \tag{14}$$

If the two conditions above hold then the optimal solution to Problem 3.1, with S positive definite, exists and is given by:

$$\mathcal{F}_{k,\infty}^*(\{\mathbf{s}_l\}_{l=0}^k, \{r_l\}_{l=k+1}^\infty) = \underbrace{K(\mathbf{m}_k)\mathbf{x}_k}_{feedback}$$

$$+ \underbrace{\left(Q + B(\mathbf{m}_k)'\bar{P}(\mathbf{m}_k)B(\mathbf{m}_k)\right)^{-1} B(\mathbf{m}_k)' \left(\bar{P}(\mathbf{m}_k)r_{k+1} - \bar{L}_{k,\infty}(\mathbf{m}_k, \{r_l\}_{l=k+1}^T)\right)}_{feedforward}, \ k \geq 0 \quad (15)$$

where $K(1)$ through $K(\bar{m})$ are matrices in $\mathbb{R}^{q \times n}$ given by the optimal LQR solution (8).

Proof: This proof has three parts. (**Part 1: Definitions**) We start by defining the following cost-to-go for any two non-negative integers k and T satisfying $k \leq T$:

$$\mathcal{G}_{k,T}\left(\{\mathcal{F}_{l,T}, r_l\}_{l=k}^T, \{\mathbf{s}_l\}_{l=0}^k\right) \stackrel{def}{=} \begin{cases} \tilde{\mathbf{x}}_k'S\tilde{\mathbf{x}}_k + \mathbf{u}_k'Q\mathbf{u}_k + E_{T|k}\left[\sum_{l=k+1}^T \tilde{\mathbf{x}}_l'S\tilde{\mathbf{x}}_l + \mathbf{u}_l'Q\mathbf{u}_l\right] & k < T \\ \tilde{\mathbf{x}}_T'S\tilde{\mathbf{x}}_T + \mathbf{u}_T'Q\mathbf{u}_T & k = T \end{cases}$$
(16)

where u_k must be of the form $u_k = \mathcal{F}_{k,T}(\{s_l\}_{l=0}^k, \{r_l\}_{l=k+1}^T)$ and $\tilde{x}_k$ is defined as:

$$\tilde{x}_k \stackrel{def}{=} x_k - r_k$$

Notice that since $\mathbf{s}_k$ is Markov, the conditional expectation in (16) does not change if we condition with respect to $\{\mathbf{s}_l\}_{l=0}^k$ or just $\mathbf{s}_k$. In addition, $\mathcal{G}_{k,T}$ as defined above satisfies:

$$\mathcal{J}_T\left(\{\mathcal{F}_{l,T}, r_l\}_{l=0}^T\right) = \frac{1}{T+1}E_0\left[E_{T|0}\left[\sum_{l=0}^T (\mathbf{x}_l - r_l)'S(\mathbf{x}_l - r_l) + \mathbf{u}_l'Q\mathbf{u}_l\right]\right] =$$

$$\frac{1}{T+1}E_0\left[\mathcal{G}_{0,T}\left(\{\mathcal{F}_{l,T}, r_l\}_{l=0}^T, \mathbf{s}_0\right)\right] \quad (17)$$

At a later stage, we proceed to determine the control law $u_k = \mathcal{F}_{k,T}(\{s_l\}_{l=0}^k, \{r_l\}_{l=0}^T)$, with $k \in \{0, \ldots, T\}$, that minimizes $\mathcal{G}_{0,T}\left(\{\mathcal{F}_{l,T}, r_l\}_{l=0}^T, s_0\right)$ for any given s_0 in $\mathbb{R}^n$. The optimal

cost-to-go is represented as:

$$\mathcal{G}_{k,T}^* \left(\{r_l\}_{l=k}^T, \{s_l\}_{l=0}^k \right) \overset{def}{=} \min_{\{\mathcal{F}_{l,T}\}_{l=k}^T} \mathcal{G}_{k,T} \left(\{\mathcal{F}_{l,T}, r_l\}_{l=k}^T, \{s_l\}_{l=0}^k \right) \tag{18}$$

We also define symmetric and positive-definite matrices $P_{k,T}(1)$ through $P_{k,T}(\bar{m})$, with $k \in \{0, \ldots, T\}$, via the following backward recursion:

$$P_{k,T}(i) \overset{def}{=} \begin{cases} S & \text{if } k = T \\ S + A(i)' \bar{P}_{k,T}(i) A(i) - K_{k,T}(i)' \left(Q + B(i)' \bar{P}_{k,T}(i) B(i) \right) K_{k,T}(i) & \text{if } k < T \end{cases} \tag{19}$$

$$\bar{P}_{k,T}(i) \overset{def}{=} \sum_{j=1}^{\bar{m}} [M]_{j,i} P_{k+1,T}(j), \ k < T \tag{20}$$

where

$$K_{k,T}(i) \overset{def}{=} - \left(Q + B(i)' \bar{P}_{k,T}(i) B(i) \right)^{-1} B(i)' \bar{P}_{k,T}(i) A(i), \ k \leq T, i \in \{1, \ldots, \bar{m}\} \tag{21}$$

Similarly, we need to define functions $\wp_{k,T}(i, \{r_l\}_{l=k}^T) \mapsto \mathbb{R}$ with $i \in \{1, \ldots, \bar{m}\}$ and $k \in \{0, \ldots, T\}$, according to the following backward recursion:

$$\wp_{T,T}(i, r_T) \overset{\Delta}{=} 0 \tag{22}$$

$$\wp_{k,T}(i, \{r_l\}_{l=k}^T) \overset{def}{=} \bar{\wp}_{k,T}(i, \{r_l\}_{l=k+1}^T) + z_k' \bar{P}_{k,T}(i) z_k + 2 z_k' \bar{L}_{k,T}(i, \{r_l\}_{l=k+1}^T)$$
$$- \left(\bar{P}_{k,T}(i) z_k + \bar{L}_{k,T}(i, \{r_l\}_{l=k+1}^T) \right)' \left(Q + B(i)' \bar{P}_{k,T}(i) B(i) \right)^{-1} \left(\bar{P}_{k,T}(i) z_k + \bar{L}_{k,T}(i, \{r_l\}_{l=k+1}^T) \right),$$
$$k < T \quad (23)$$

$$\bar{\wp}_{k,T}(i, \{r_l\}_{l=k+1}^T) \overset{def}{=} \sum_{j=1}^{\bar{m}} [M]_{j,i} \wp_{k+1,T}(j, \{r_l\}_{l=k+1}^T), \ k < T \tag{24}$$

where the n dimensional real vectors $L_{k,T}(1, \{r_l\}_{l=k}^T)$ through $L_{k,T}(\bar{m}, \{r_l\}_{l=k}^T)$ are computed from (13)-(14), while z_k is given by:

$$z_k \overset{def}{=} A_{m_k} r_k - r_{k+1} \tag{25}$$

(**Part 2: optimal solution for finite** T) The following fact is key in our proof of Theorem 4.1:

Fact 1: Given integers k and T satisfying $k \leq T$ and $k \geq 1$, and a reference sequence $\{r_l\}_{l=0}^T$, if (26) holds then (27) is true.

$$\mathcal{G}_{k,T}^* \left(\{r_l\}_{l=k}^T, \{s_l\}_{l=0}^k \right) = \tilde{x}_k' P_{k,T}(m_k) \tilde{x}_k + 2 \tilde{x}_k' L_{k,T}(m_k, \{r_l\}_{l=k}^T) + \wp_{k,T}(m_k, \{r_l\}_{l=k}^T) \tag{26}$$

$$\mathcal{G}^*_{k-1,T}\left(\{r_l\}^T_{l=k-1}, \{s_l\}^{k-1}_{l=0}\right) =$$

$$\tilde{x}'_{k-1}P_{k-1,T}(m_{k-1})\tilde{x}_{k-1} + 2\tilde{x}'_{k-1}L_{k-1,T}(m_{k-1}, \{r_l\}^T_{l=k-1}) + \wp_{k-1,T}(m_{k-1}, \{r_l\}^T_{l=k-1}) \quad (27)$$

(**Proof of Fact 1**) Notice that for any integers k and T satisfying $k \leq T$ and $k \geq 1$, we can express the optimal cost-to-go as:

$$\mathcal{G}^*_{k-1,T}\left(\{r_l\}^T_{l=k-1}, \{\mathbf{s}_l\}^{k-1}_{l=0}\right) =$$

$$\min_{\mathcal{F}_{k-1,T}}\left[\tilde{\mathbf{x}}'_{k-1}S\tilde{\mathbf{x}}_{k-1} + \mathbf{u}'_{k-1}Q\mathbf{u}_{k-1} + \min_{\{\mathcal{F}_{l,T}\}^T_{l=k}} E_{T|k-1}\left[\sum^T_{l=k}\tilde{\mathbf{x}}'_l R\tilde{\mathbf{x}}_l + \mathbf{u}'_l Q\mathbf{u}_l\right]\right] \quad (28)$$

where each realization u_k must be of the form $u_k = \mathcal{F}_{k,T}(\{s_l\}^k_{l=0}, \{r_l\}^T_{l=k+1})$. Now notice that the following holds:

$$\min_{\{\mathcal{F}_{l,T}\}^T_{l=k}} E_{T|k-1}\left[\sum^T_{l=k}\tilde{\mathbf{x}}'_l S\tilde{\mathbf{x}}_l + \mathbf{u}'_l Q\mathbf{u}_l\right] = \min_{\{\mathcal{F}_{l,T}\}^T_{l=k}} E_{k|k-1}\left[E_{T|k}\left[\sum^T_{l=k}\tilde{\mathbf{x}}'_l S\tilde{\mathbf{x}}_l + \mathbf{u}'_l Q\mathbf{u}_l\right]\right] =$$

$$E_{k|k-1}\left[\min_{\{\mathcal{F}_{l,T}\}^T_{l=k}} E_{T|k}\left[\sum^T_{l=k}\tilde{\mathbf{x}}'_l S\tilde{\mathbf{x}}_l + \mathbf{u}'_l Q\mathbf{u}_l\right]\right] = E_{k|k-1}\left[\mathcal{G}^*_{k,T}\left(\{r_l\}^T_{l=k}, \{\mathbf{s}_l\}^k_{l=0}\right)\right] \quad (29)$$

From (28)-(29), we can write the following backward (Bellman's) equation [33] for the optimal cost-to-go as:

$$\mathcal{G}^*_{k-1,T}\left(\{r_l\}^T_{l=k-1}, \{\mathbf{s}_l\}^{k-1}_{l=0}\right) =$$

$$\min_{\mathbf{u}_{k-1}} \tilde{\mathbf{x}}'_{k-1}S\tilde{\mathbf{x}}_{k-1} + \mathbf{u}'_{k-1}Q\mathbf{u}_{k-1} + E_{k|k-1}\left[\mathcal{G}^*_{k,T}\left(\{r_l\}^T_{l=k}, \{\mathbf{s}_l\}^k_{l=0}\right)\right] \quad (30)$$

Now notice that under (26), we have:

$$E_{k|k-1}\left[\mathcal{G}^*_{k,T}\left(\{r_l\}^T_{l=k}, \{\mathbf{s}_l\}^k_{l=0}\right)\right] =$$

$$\tilde{\mathbf{x}}'_k\bar{P}_{k-1,T}(\mathbf{m}_{k-1})\tilde{\mathbf{x}}_k + 2\tilde{\mathbf{x}}'_{k-1}\bar{L}_{k-1,T}(\mathbf{m}_{k-1}, \{r_l\}^T_{l=k}) + \bar{\wp}_{k-1,T}(\mathbf{m}_{k-1}, \{r_l\}^T_{l=k}) \quad (31)$$

where we also used the fact that $\tilde{x}_k = A(m_{k-1})\tilde{x}_k + B(m_{k-1})u_{k-1} + z_{k-1}$. The **proof of Fact 1** follows by substituting (31) in (30) and completing the squares to arrive at (27) and to conclude that the optimal control $\mathbf{u}^*_k$ is given by the following:

$$\mathbf{u}^*_k = \mathcal{F}^*_{k,T}(\{\mathbf{s}_l\}^k_{l=0}, \{r_l\}^T_{l=k+1}) = \underbrace{K_{k,T}(\mathbf{m}_k)\mathbf{x}_k}_{feedback}$$

$$+ \underbrace{\left(Q + B(\mathbf{m}_k)'\bar{P}_{k,T}(\mathbf{m}_k)B(\mathbf{m}_k)\right)^{-1}B(\mathbf{m}_k)'\left(\bar{P}_{k,T}(\mathbf{m}_k)r_{k+1} - \bar{L}_{k,T}(\mathbf{m}_k, \{r_l\}^T_{l=k+1})\right)}_{feedforward} \quad (32)$$

Now notice that since (26) holds for $k = T$, by repeated application of Fact 1, we can conclude that (32) is the optimal solution to the finite horizon problem of minimizing $\mathcal{J}_T\left(\{\mathcal{F}_{l,T}, r_l\}_{l=0}^T\right)$. (**Part 3: optimal solution for the infinite horizon case:**) The infinite horizon solution (15) will be well defined provided that the feedback and the feedforward terms in (32) converge, for every k, as T tends to infinity. As such, we proceed to proving the convergence of the feedback and feedforward terms. (**Convergence of the feedback term**): We start by noticing, from (19)-(20), that $\{P_{0,T}(i)\}_{i=1}^{\bar{m}}$ converges to some $\{P_\infty(i)\}_{i=1}^{\bar{m}}$ as T tends to infinity if and only if, for all k, $\{P_{k,T}(i)\}_{i=1}^{\bar{m}}$ also converges $\{P_\infty(i)\}_{i=1}^{\bar{m}}$ as T tends to infinity. Hence it suffices to study the convergence of $\{P_{0,T}(i)\}_{i=1}^{\bar{m}}$ and $\{K_{0,T}(i)\}_{i=1}^{\bar{m}}$. Recall that the CARE (7) characterize the solution of the infinite horizon optimal LQR for MJLS (see Remark 2.1). In particular, if condition 1 holds, i.e., (7) has a positive definite solution $\{P_{0,T}(i)\}_{i=1}^{\bar{m}}$, then the infinite horizon optimal LQR cost is finite (see Remark 2.1) and the following holds for any choice of independent random variables $\mathbf{m}_0$ and $\mathbf{x}_0$ (with finite second moment):

$$E_0\left[\mathcal{G}_{0,T}^*(0, \mathbf{s}_0)\right] = E_0\left[\mathbf{x}_0' P_{0,T}(\mathbf{m}_0)\mathbf{x}_0\right] \leq \mathcal{L}^* < \infty, \; T \geq 0 \tag{33}$$

where $\mathcal{L}^*$ denotes the optimal infinite horizon LQR cost. In (33) we adopt an abuse of notation, where $\mathcal{G}_{0,T}^*(0, \mathbf{s}_0)$ represents the cost to go when $r_k = 0$, for $k \in \{0, \ldots, T\}$. The equality in (33) follows from Fact 1, i.e., $\mathcal{G}_{0,T}^*(0, s_0) = x_0' P_{0,T}(m_0)x_0$ results from (27), where we also use the fact that $\{L_{0,T}(i, 0)\}_{i=1}^{\bar{m}} = 0$ and $\{\wp_{0,T}(i, 0)\}_{i=1}^{\bar{m}} = 0$. The equality $\mathcal{G}_{0,T}^*(0, s_0) = x_0' P_{0,T}(m_0)x_0$ also implies that the sequence $E_0\left[\mathbf{x}_0' P_{0,T}(\mathbf{m}_0)\mathbf{x}_0\right]$ is non-decreasing in T, and since it is bounded it must converge. In addition, $E_0\left[\mathbf{x}_0' P_{0,T}(\mathbf{m}_0)\mathbf{x}_0\right]$ must converge to $\mathcal{L}^*$, otherwise this would contradict the optimality of $\mathcal{L}^*$. Moreover, from [14, Theorem 4.5] we know that $\mathcal{L}^* = E_0\left[\mathbf{x}_0' P(\mathbf{m}_0)\mathbf{x}_0\right]$, where $\{P(i)\}_{i=1}^{\bar{m}}$ is the positive definite solution to the CARE (see also Remark 2.1). Hence, we conclude that $E_0\left[\mathbf{x}_0'\left(P(\mathbf{m}_0) - P_{0,T}(\mathbf{m}_0)\right)\mathbf{x}_0\right]$ converges to zero as T goes to infinity, and since the choice of $\mathbf{m}_0$ and $\mathbf{x}_0$ was arbitrary, we conclude that $\{P_{0,T}(i)\}_{i=1}^{\bar{m}}$ must converge to $\{P(i)\}_{i=1}^{\bar{m}}$. Hence $\{K_{k,T}(i)\}_{i=1}^{\bar{m}}$ also converges to $\{K(i)\}_{i=1}^{\bar{m}}$. The **convergence of the feedforward term** follows from condition 2. $\square$ (**End of Proof of Theorem 4.1**) Notice that if r_l is such that $A(i)r_l = r_{l+1}$ then (15) reduces to $\mathcal{F}_{k,\infty}^*(\{\mathbf{s}_l\}_{l=0}^k, \{r_l\}_{l=k}^\infty) = K(\mathbf{m}_k)(r_k - \mathbf{x}_k)$, which is the solution we would obtain from the well known internal model principle [19].

Remark 4.1: **Dealing with positive semi-definite** S**:** Notice that our proof of Theorem 4.1 requires that the matrix S is positive definite, which guarantees that the CARE (7) has a unique positive definite solution (see Remark 2.1). If S is only positive semi-definite then the CARE

(7) may have multiple solutions. In particular, if S is positive semi-definite then the iteration (19) converges to a minimal solution, which might not be stabilizing (see [15, Chapter V] for a discussion of this issue for the continuous time case). Our proof remains valid, for S positive semi-definite, provided that we impose that the MJLS is stochastically detectable with respect to $S^{1/2}$ (see [14, Remark A.22]). Indeed, in the presence of stochastic detectability, the CARE (7) has only one solution, which is positive definite, and the recursion (19) will converge to it.

V. COMPUTATION OF THE FEEDFORWARD TERM

The main result of this Section is given in Theorem 5.1, where we give an explicit formula for computing $\{L_{k,\infty}(i, \{r_l\}_{l=k}^{\infty})\}_{i=1}^{\bar{m}}$, which is a central quantity in (15), in the presence of bounded reference sequences.

Notation: Given a collection of matrices (or vectors) $W(1)$ through $W(\bar{m})$, we denote the corresponding block diagonal matrix as:

$$\mathfrak{D}\left(\{W(i)\}_{i=1}^{\bar{m}}\right) \overset{def}{=} \begin{pmatrix} W(1) & 0 & \cdots & 0 \\ 0 & W(2) & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & W(\bar{m}) \end{pmatrix} \tag{34}$$

Given vectors $v(1)$ through $v(\bar{m})$, we use the following notation to denote vectorization:

$$\mathfrak{V}(\{v(i)\}_{i=1}^{\bar{m}}) \overset{def}{=} \begin{pmatrix} v(1) \\ \vdots \\ v(\bar{m}) \end{pmatrix} \tag{35}$$

The Kronecker product between two matrices X and $Y \in \mathbb{R}^{n_1 \times n_2}$ is denoted as:

$$X \otimes Y \overset{def}{=} \begin{pmatrix} X\,[Y]_{1,1} & X\,[Y]_{1,2} & \cdots & X\,[Y]_{1,n_2} \\ X\,[Y]_{2,1} & \cdots & \cdots & X\,[Y]_{2,n_2} \\ \vdots & \vdots & \vdots & \vdots \\ X\,[Y]_{n_1,1} & X\,[Y]_{n_1,2} & \cdots & X\,[Y]_{n_1,n_2} \end{pmatrix} \tag{36}$$

Theorem 5.1: Given a Markovian jump linear system, as in Definition 1.1, select a reference sequence $\{r_l\}_{l=0}^{\infty}$ and consider the optimal preview control paradigm of Problem 3.1 with S positive definite. If the CARE (7) have a stabilizing solution $\{P(i)\}_{i=1}^{\bar{m}}$ and the reference sequence is bounded, then $L_{k,\infty}(1, \{r_l\}_{l=k}^{\infty})$ through $L_{k,\infty}(\bar{m}, \{r_l\}_{l=k}^{\infty})$, defined in (12)-(14), can be computed via the following general formula:

$$\mathfrak{V}(\{L_{k,\infty}(i, \{r_l\}_{l=k}^{\infty})\}_{i=1}^{\bar{m}}) = \sum_{j=k}^{\infty} F^{j-k} \mathfrak{V}\left(\{(A(i) - B(i)K(i))'\bar{P}(i)(A(i)r_j - r_{j+1})\}_{i=1}^{\bar{m}}\right) \quad (37)$$

where $K(1)$ through $K(\bar{m})$ are the gains (8) and $F \in \mathbb{R}^{n\bar{m} \times n\bar{m}}$ is a stable matrix (spectral radius less than one) given by:

$$F \overset{def}{=} \mathfrak{D}\left(\{(A(i) - B(i)K(i))'\}_{i=1}^{\bar{m}}\right)(I_{n\times n} \otimes M') \quad (38)$$

If the reference sequence is constant, i.e., $r_k = r$ for some n dimensional real vector r, then the sequences $L_{k,\infty}(1, \{r_l\}_{l=k}^{\infty})$ through $L_{k,\infty}(\bar{m}, \{r_l\}_{l=k}^{\infty})$ are constant with respect to k and they can be computed as:

$$\mathfrak{V}(\{L_{k,\infty}(i, \{r_l\}_{l=k}^{\infty})\}_{i=1}^{\bar{m}}) = \left(I - F\right)^{-1} \mathfrak{V}\left(\{(A(i) - B(i)K(i))'\bar{P}(i)(A(i) - I)r\}_{i=1}^{\bar{m}}\right), \; k \geq 0 \quad (39)$$

Proof: We start by representing the backward equations (13)-(14) in the following equivalent form, which will immediately lead to (37):

$$\mathfrak{V}(\{L_{k,T}(i, \{r_l\}_{l=k}^{T})\}_{i=1}^{\bar{m}}) = F \; \mathfrak{V}(\{L_{k+1,T}(i, \{r_l\}_{l=k+1}^{T})\}_{i=1}^{\bar{m}}) +$$

$$\mathfrak{V}\left(\{(A(i) - B(i)K(i))'\bar{P}(i)(A(i)r_k - r_{k+1})\}_{i=1}^{\bar{m}}\right), \; k < T \quad (40)$$

This type of vectorization technique has been used in [27] to obtain a forward recursion for the state covariance of a linear and time-invariant plant, under a networked control formulation. If r_l is constant and equal to r then we use (37) to arrive at (39). In order to finish this proof, below we show that $\varrho(F)$, the spectral radius of F, is strictly less than one. Let $\bar{w}(1)$ through $\bar{w}(\bar{m})$ be arbitrarily selected vectors taking values in $\mathbb{R}^n$. Further, define $\mathbf{w}_0 = \bar{w}(\mathbf{m}_0)$. Now consider a stochastic process $\{\mathbf{w}_l\}_{l=0}^{\infty}$ specified by the following forward recursion:

$$\mathbf{w}_{k+1} = (A(\mathbf{m}_k) - B(\mathbf{m}_k)K(\mathbf{m}_k))\mathbf{w}_k, \; k \geq 0 \quad (41)$$

Since $K(1)$ through $K(\bar{m})$ are the optimal LQR gains with S positive definite then the second moment of $\mathbf{w}_k$ will converge to zero (see Remark 2.1), in particular, the following also holds

for all $i \in \{1, \ldots, \bar{m}\}$:

$$\underbrace{\lim_{k \to \infty} \sum_{j=1}^{\bar{m}} E_k[(\mathbf{w}_k)^2 | \mathbf{m}_k = j] Pr(\mathbf{m}_k = j)}_{E_k[(\mathbf{w}_k)^2]} = 0 \implies \lim_{k \to \infty} E_k[\mathbf{w}_k | \mathbf{m}_k = i] = 0 \qquad (42)$$

We continue by defining sequences $\{v_l(1)\}_{l=0}^{\infty}$ through $\{v_l(\bar{m})\}_{l=0}^{\infty}$ as:

$$v_k(i) \stackrel{def}{=} E_k[\mathbf{w}_k | \mathbf{m}_k = i] Pr(\mathbf{m}_k = i), \ i \in \{1, \ldots, \bar{m}\} \qquad (43)$$

In addition, select $\mathbf{m}_0$ uniformly distributed, i.e., $Pr(\mathbf{m}_0 = i) = \frac{1}{\bar{m}}$, with $i \in \{1, \ldots, \bar{m}\}$. Now notice that the following holds:

$$v_k(i) = \begin{cases} \sum_{j=1}^{\bar{m}} [M]_{i,j} (A(j) - B(j)K(j)) v_{k-1}(j) & \text{if } k \geq 1 \\ \bar{w}(i) \frac{1}{\bar{m}} & \text{if } k = 0 \end{cases} \qquad (44)$$

which can be equivalently expressed as:

$$\mathfrak{V}(\{v_k(i)\}_{i=1}^{\bar{m}}) = (F')^k \, \mathfrak{V}(\{v_0(i)\}_{i=1}^{\bar{m}}), \ k \geq 0 \qquad (45)$$

From (42) we know that $v_k(1)$ through $v_k(\bar{m})$ will also converge to zero, as k goes to infinity. Since $\bar{w}(1)$ through $\bar{w}(\bar{m})$ can be selected arbitrarily, convergence of $\mathfrak{V}(\{v_k(i)\}_{i=1}^{\bar{m}})$ to zero and (45) imply that $\varrho(F) < 1$. $\square$

VI. CONCLUSIONS

We obtain the solution to the infinite horizon optimal preview control problem, for Markovian jump linear systems, in the presence of infinite look-ahead. The optimal control policy comprises a feedback term and a feedforward term. The feedback term can be obtained via an immediate adaptation of the standard linear quadratic regulator for Markovian jump linear systems. However, this paper is the first to provide an explicit characterization of the feedforward term via efficiently computable quantities. Notice that, in the infinite horizon optimal preview control of linear and time-invariant plants, the convergence of the feedforward term follows immediately from the stability of the closed loop state-space representation [33, (pp. 68, Corollary 4.1)]. However, this convergence result cannot be extended to Markovian jump linear systems. In Theorem 5.1, we solve such a problem by providing a convergence analysis for the Markovian jump linear systems case.

REFERENCES

[1] E. K. Bender, *"Optimum linear preview control with application to vehicle suspension,"* ASME J. Basic Engineering, Volume 90, no. 2, 1968, Page(s) 213-221

[2] D. P. Bertsekas, *"Dynamic Programming and Optimal Control,"* vol 1, Athena Scientific, 1995

[3] W. P. Blair, and D. D. Sworder, *"Feedback control of a class of linear discrete systems with jump parameters and quadratic cost criteria,"* International Journal of Control, 21:833-844, 1975

[4] E.K. Boukas, A. Swierniak, H. Yang, *" On the robustness of jump linear quadratic control"*, International Journal of Robust and Nonlinear Control, Volume 7, Issue 10 , 1998, Pages 899 - 910

[5] E.K. Boukas and H. Yang, *"Robust LQ Regulator for Jump Linear Systems with Uncertain Parameters"*, Dynamics and Control, Springer, Issue Volume 9, Number 2 / April, 1999, Pages 125-134

[6] G. Chen; J. Chen; R. Middleton, *"Optimal tracking performance for SIMO systems,"* IEEE Transactions on Automatic Control, Volume 47, Issue 10, Oct 2002 Page(s): 1770 - 1775

[7] J. Chen; L. Qiu; O. Toker, *"Limitations on maximal tracking accuracy,"* IEEE Transactions on Automatic Control, Volume 45, Issue 2, Feb 2000 Page(s):326 - 331

[8] J. Chen; Z. Ren; S. Hara; L. Qin, *"Optimal tracking performance: preview control and exponential signals,"* IEEE Transactions on Automatic Control, Volume 46, Issue 10, Oct 2001 Page(s):1647 - 1653

[9] J. Chen; S. Hara; G. Chen, *"Best tracking and regulation performance under control energy constraint,"* IEEE Transactions on Automatic Control, Volume 48, Issue 8, Aug. 2003 Page(s): 1320 - 1336

[10] H. J. Chizeck, A. S. Willsky, and D. Castanon, *"Discrete-time Markovian-jump linear quadratic optimal control,"* Int. J. Control, vol. 43, no. 1, 1986, Page(s): 213-231.

[11] A. Cohen, U. Shaked, *"Linear discrete-time Hinf-optimal tracking with preview,"* IEEE Transactions on Automatic Control, Volume 42, Issue 2, Feb 1997 Page(s):270 - 276

[12] O. L. V. Costa, *"Mean square stabilizing solutions for discrete-time coupled algebraic Riccati equations,"* IEEE Transactions on Automatic Control, 41:593-598, 1996

[13] O. L. V. Costa and R. P. Marques, *" Mixed H_2/H_∞ control of discrete time Markovian jump linear systems,"* Automatic Control, IEEE Transactions on, Volume 43, Issue. 1, Jan 1998, Page(s): 95-100.

[14] O.L.V. Costa, M. D. Fragoso, R. P. Marques, *"Discrete-Time Markovian Jump Linear Systems,"* Springer, 2005

[15] V. Dragan, T. Morozan, A.-M. Stoica, *" Mathematical Methods in Robust Control of Linear Stochastic Systems "*, Springer, 2006

[16] R. J. Elliot and D. Sworder, *"Control of a hybrid conditionally linear Gaussian process,"* JOTA, 74 (1992), pages 75-85

[17] D. P. de Farias, J. C. Geromel, J. B. R. do Val, and O. L.V. Costa, *" Output feedback control of Markov jump linear systems in continuous-time,"* Automatic Control, IEEE Transactions on, Volume 45, Issue 5, May 2000, Page(s): 944-949.

[18] B. Francis, *"The optimal linear-quadratic time-invariant regulator with cheap control,"* IEEE Transactions on Automatic Control, vol AC-24, no 4, Aug. 1979, pages 616-621

[19] B. Francis and W. Wonham, *"The internal model principle of control theory,"* Automatica, vol 12, no 5, 1976, pages 457-465

[20] M. J. Grimble, *"Model reference predictive LQG optimal control law for SIMO systems,"* IEEE Transactions on Automatic Control, Volume 37, Issue 3, Mar 1992 Page(s):365 - 371

[21] M. E. Halpern, *"Preview tracking for discrete-time SISO systems,"* IEEE Transactions on Automatic Control, Volume 39, Issue 3, Mar 1994 Page(s):589 - 592

[22] Y. Ji, H. J. Chizeck, X. Feng, and K. A. Loparo, *"Stability and control of discrete-time jump linear systems,"* Control Theory Adv. Technol., vol. 7, no. 2, 1991, Page(s): 247-270.

[23] I. Karatzas and S. E. Shreve, *"Brownian Motion and Stochastic Calculus,"* Second Edition, Springer, 1991

[24] H. Katoh, *"H_∞-optimal preview controller and its performance limit,"* IEEE Transactions on Automatic Control, vol 48, Issue 11, Nov 2004, Pages 2011-2017

[25] R. H. Middleton, *"Trade-offs in linear control systems design,"* Automatica, vol 27, no 2, March 91, pages 281-292

[26] E. Mosca, G. Zappa, *"Matrix fraction solution to the discrete-time LQ stochastic tracking and servo problems,"* IEEE Transactions on Automatic Control, Volume 34, Issue 2, Feb 1989 Page(s):240 - 242

[27] J. Nilsson, *"Real-time control systems with delays,"* Ph.D. dissertation, Department of Automatic Control, Lund Institute of Technology, 1998.

[28] M. Peng, M. Tomizuka, M., *"Preview control for vehicle lateral guidance in highway automation,"* ASME Journal of Dynamical Systems, Measurement and Control, 115, Page(s): 679- 686.

[29] L. Qui and E. J. Davidson, *"Performance limitations of nonminimum phase systems in the servomechanism problem,"* Automatica, vol 29, no 2, pages 337-349, 1993

[30] M. M. Seron, J. H. Braslavsky and G. C. Goodwin, *"Fundamental limitations in Filtering and Control,"* London, UK: Springer-Verlag, 1997

[31] M. Sternad and T. Soderstrom, *"LQG-Optimal feedforward regulators,"* Automatica, vol. 24, no. 4, pages 557-561, 1988.

[32] M. Tomizuka, *"Optimal continuous finite preview problem,"* Automatic Control, IEEE Transactions on, Volume 20, Issue 3, March 1975, Page(s): 362-265

[33] P. Whittle, *"Optimization over time, Volume I"* Wiley, 1982

[34] A. Woodyatt, M. Seron, J. Freudenberg, and R. Middleton, *" Cheap control tracking performance for nonright-invertible systems,"* Int. Journal of Robust Nonlinear Control, vol 12, no 15, Dec 2002, pages 1253-1273

[35] T. Morozan, *"Optimal stationary control for dynamic systems with Markovian perturbations,"* Stoch. Anal. Appl., 3, pp. 299-325, 1983

[36] T. Morozan, *"Stability and control for linear discrete-time systems with Markov perturbations,"* Rev. Roum. Pures et Applique, 40, pp. 471-494, 1995

[37] V. Dragan, T. Morozan, *" Discrete-time Riccati type equations and tracking problems,"* ICIC Express Letters, vol. 2, no. 2, pp. 109-116, June 2008