ABSTRACT

Title of dissertation: FLAVOR SYMMETRY, LEPTOGENESIS AND GRAND UNIFICATION THEORIES

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Many neutrino experiments in the last few years have shown concrete evidence for neutrino mass and leptonic mixing; an indication of new physics beyond the standard model. In this thesis, we systematically study the flavor symmetry indicated by the low scale neutrino experiment data with the assumption that the seesaw mechanism is the reason for the light neutrino masses.

In the flavor basis, the testable exchange symmetry between muon neutrino and tau neutrino ($\mu - \tau$) is introduced to explain the near maximal atmospheric mixing angle and vanishing reactor mixing angle. This symmetry can reduce the seesaw parameters naturally and make it possible to connect the baryon asymmetry of our universe to the low scale neutrino data if leptogenesis causes the baryon asymmetry. We also show this leptonic symmetry can be extended to the quark sector and present a realistic supersymmetry $SU(5)$ grand unification model.

Motivated by solar mixing angle $\sin^2\theta_{\rm solar} \simeq 1/3$, we embed the $\mu - \tau$ symmetry in an $S3$ permutation symmetry and obtain a so-called tri-bimaximal mixing

pattern. We study the stability of the texture under radiative corrections. This $S3$ model is so constrained that the CP-violating phases of the low scale mixing are those generating the baryon asymmetry within leptongesis. Attempting to unify three families of fermions within the grand unification theories, we treat three families of fermions as the three dimensional irreducible representation of $S4$ and build a realistic model based on $SO(10)$ gauge group. This model predicts degenerate a right-handed neutrino mass spectrum.

In this thesis, we also address the issue of the natural realization of the seesaw mechanism in the supersymmetric minimal $SO(10)$ model. We realize the type II seesaw dominance by invoking a warped extra dimension, while keeping predictivity of the model.

FLAVOR SYMMETRY, LEPTOGENESIS AND GRAND UNIFICATION THEORIES

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Table of Contents

List of Tables

List of Figures

List of Abbreviations

SM	standard model
VEV	vacuum expectation value
PMNS	Pontecorvo-Maki-Nakagawa-Sakata
SUSY	supersymmetric
GUT	grand unification theory
MSSM	minimal supersymmetric standard model
RGE	renormalization group equation
TBM	tri-bimaximal mixing
RS	Randall-Sundrum model

Chapter 1

Introduction

1.1 Flavor Mixing in the Standard Model

In the past thirty years, the standard model (SM) of elementary particle physics has been tested in various experiments. This model provides a successful framework to describe fundamental interactions of matter (except gravity). The SM is based on the three gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$. The $SU(2)_L \times U(1)_Y$ unifies electromagnetism and the weak interaction as the electroweak interaction [1], and $SU(3)_c$ describes the strong interaction between quarks [2]. The SM is a chiral theory because only left-handed fields carry $SU(2)_L$ quantum number.

In the SM, $SU(2)_L \times U(1)_Y$ is spontaneously broken to $U(1)_{\rm em}$ by the Higgs mechanism [3], and $SU(3)_c$ remains unbroken. To break electroweak symmetry, we use a scalar doublet of $SU(2)_L$:

$$H = \begin{pmatrix} H_+ \\ H_0 \end{pmatrix} \tag{1.1}$$

with the Higgs potential given by

$$V(\phi) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2. \tag{1.2}$$

For $\mu^2 > 0$, the minimum of $V(H)$ occurs at

$$\langle H_0 \rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}. \tag{1.3}$$

In the Lagrangian, the kinematic term of H is given by

$$\mathcal{L} \supset |D_\mu H|^2. \tag{1.4}$$

The covariant derivative D_μ is defined as $D_\mu \equiv \partial_\mu - igA^a_\mu \tau^a - i\frac{1}{2}g' B_\mu$, where A^a_μ and B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge bosons respectively. $SU(2)_L$ and $U(1)_Y$ commute each other, so they can have different coupling constants g and g'; $\tau^a = \sigma^a/2$, where σ^a are the Pauli matrices.

When this spontaneous symmetry breaking occurs, three gauge bosons acquire mass due to the term in Eq. (1.4). They are two charged gauge bosons, $W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)$ with mass $m_W = \frac{1}{2}gv$, and one neutral gauge bosons $Z^0_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu)$ with mass $m_Z = \sqrt{g^2+g'^2}\frac{v}{2}$. The fourth vector field remains massless: $A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu)$ with mass $m_A = 0$.

One crucial question about this model is its renormalizability. In 1971, t'Hooft and Velteman proved that all spontaneously broken gauge theories only including interactions with mass dimension four or less are renormalizable [4].

As for the matter fields, there are known to be three generations (families) of quarks and leptons. There are three $SU(2)_L$ doublets of left-handed quarks:

$$Q^i_L = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L \tag{1.5}$$

where $u^i_L = (u_L, c_L, t_L)$ and $d^i_L = (d_L, s_L, b_L)$ with hypercharge $Y = +1/6$. There are six right-handed quarks, three with $Y = 2/3$ and three with $Y = -1/3$:

$$u^i_R = (u_R, c_R, t_R), d^i_R = d_R, s_R, b_R. \tag{1.6}$$

Each quark carries color quantum number and it transforms as the fundamental representation under $SU(3)_c$.

For leptons, we have three generations of $SU(2)_L$ doublets with $Y = -1/2$:

$$E_L^i = \begin{pmatrix} \nu_l^i \\ l^i \end{pmatrix}_L , \tag{1.7}$$

where $\nu^i = (\nu_e, \nu_\mu, \nu_\tau)$ and $l^i = (e_L, \mu_L, \tau_L)$. There are three right-handed charged leptons with $Y = -1$:

$$e_R^i = (e_R, \mu_R, \tau_R). \tag{1.8}$$

Leptons do not carry color quantum number and are therefore singlets under $SU(3)_c$. Note there is no right-handed neutrino in the SM.

The covariant derivative completely determines the coupling between fermions and gauge fields, once the quantum of fermion fields is specified.

$$\mathcal{L}_{\text{electroweak}} = g(W_\mu^+ J_W^\mu + W_\mu^- J_W^{mu-} + Z_\mu^0 J_Z^\mu) + eA_\mu J_{EM}^\mu \tag{1.9}$$

where the charged currents are

$$J_W^{\mu+} = \frac{1}{\sqrt{2}} \sum_i (\overline{\nu}_L^{0i} \gamma^\mu e_L^{0i} + \overline{u}_L^{0i} \gamma^\mu d_L^{0i}) \tag{1.10}$$

$$J_W^{\mu-} = \frac{1}{\sqrt{2}} \sum_i (\overline{e}_L^{0i} \gamma^\mu \mu_L^{0i} + \overline{d}_L^{0i} \gamma^\mu u_L^{0i}), \tag{1.11}$$

and the neutral currents are

$$\begin{aligned} J_Z^\mu = \frac{1}{\cos\theta_w} \sum_i \quad [\quad & \overline{\nu}_L^{0i} \gamma^\mu (\frac{1}{2}) \nu_L^{0i} + \overline{e}_L^{0i} \gamma^\mu (-\frac{1}{2} + \sin^2\theta_w) e_L^{0i} + \overline{e}_R^{0i} \gamma^\mu (\sin^2\theta_w) e_R^{0i} \\ + \quad & \overline{u}_L^{0i} \gamma^\mu (\frac{1}{2} - \frac{2}{3} \sin^2\theta_w) u_L^{0i} + \overline{u}_R^{0i} \gamma^\mu (-\frac{2}{3} \sin^2\theta_w) u_R^{0i} \\ + \quad & \overline{d}_L^{0i} \gamma^\mu (-\frac{1}{2} + \frac{1}{3} \sin^2\theta_w) u_L^{0i} + \overline{d}_R^{0i} \gamma^\mu (\frac{1}{3} \sin^2\theta_w) d_R^{0i} \ ]; \end{aligned} \tag{1.12}$$

$$J^{\mu}_{EM} = \sum_i \overline{e}^{0i}\gamma^{\mu}(-1)e^{0i} + \overline{u}^{0i}\gamma^{\mu}(\frac{2}{3}u^{0i}) + \overline{d}^{0i}\gamma^{\mu}(-\frac{1}{3})d^{0i}. \tag{1.13}$$

The superscript zero on fermions implies that they are not mass eigenstates. The mass eigenstates are determined only after fermions acquire masses in the process of spontaneous symmetry breaking.

In the SM, one cannot write the mass term like $m\overline{\Psi}_L\Psi_R$ in the Lagrangian because it violates gauge symmetry: the left-handed and right-handed components of one fermion field are two independent degree of freedom and carry different quantum numbers. To give masses to quarks and leptons, we must invoke the mechanism of spontaneous symmetry breaking. The gauge invariant terms involving left-handed and right-handed components of fermions and Higgs H are

$$\mathcal{L}_{\rm m} = Y_e^{ij}\overline{E}^{0i}He_R^{0j} + Y_d^{ij}\overline{Q}_L^{0i}Hd_R^{0j} + Y_u^{ij}\overline{Q}_L^{0i}\tilde{H}u_R^{0j} + h.c., \tag{1.14}$$

where $\tilde{H} = i\tau_2 H^*$. Again, the superscript zero implies that fermions are not in the mass eigenstates. When spontaneous symmetry breaking occurs, fermions acquire mass due to the Yukawa coupling given in Eq. (1.14):

$$\mathcal{L}_{\rm m} = \overline{e}_L^{0i}M_e^{ij}e_R^{0j} + \overline{d}_L^{0i}M_d^{ij}d_R^{0j} + u\overline{u}_L^{0i}M_u^{ij}u_R^{0j} + h.c., \tag{1.15}$$

where $M_e^{ij} = \frac{v}{\sqrt{2}}Y_e^{ij}$, $M_d^{ij} = \frac{v}{\sqrt{2}}Y_d^{ij}$, $M_u^{ij} = \frac{v}{\sqrt{2}}Y_u^{ij}$. In the SM, there are no right-handed neutrinos and one cannot write Yukawa couplings for neutrinos, so neutrinos are massless.

The mass matrices in Eq. (1.15) can be general 3×3 matrices and not necessarily to be real, symmetric or Hermitian. To find the mass eigenstates, one should make a chiral rotation on the fermion fields and diagnolize the mass matrices. To diagnolize an arbitrary matrix, one should use a bi-unitary transformation. Taking M_u as an example, we have

$$U_u^\dagger M_u V_u = \hat{M}_u \tag{1.16}$$

where $\hat{M}_u = \mathrm{diag}(m_u, m_c, m_t)$. The unitary matrices U_u and V_u are determined by $U_u^\dagger M_u M_u^\dagger U_u = \hat{M}_u^2$ and $V_u^\dagger M_u^\dagger M_u V_u = \hat{M}_u^2$. We then replace M_u in the Lagrangian $\mathcal{L}_\mathrm{m}$ by

$$M_u = U_u \hat{M}_u V_u^\dagger. \tag{1.17}$$

Now unitary matrices U_u and V_u appear in $\mathcal{L}_\mathrm{m}$, we can rotate them away by redefining the left-handed and right-handed up-type quark fields as

$$u_L^{0i} = U_u^{ij} u_L^j,\ u_R^{0i} = V_u^{ij} u_R^j. \tag{1.18}$$

With a similar process, we can eliminate other unitary matrices in $\mathcal{L}_\mathrm{m}$ by redefining down-type quark and charged lepton fields as

$$\begin{aligned} d_L^{0i} &= U_d^{ij} d_L^j,\ d_R^{0i} = V_d^{ij} d_R^j \\ e_L^{0i} &= U_e^{ij} u_L^j,\ e_R^{0i} = V_e^{ij} e_R^j. \end{aligned} \tag{1.19}$$

For neutrinos, we can have rotation

$$\nu^{0i} = U_\nu^{ij} \nu^j. \tag{1.20}$$

But since neutrinos are massless in the SM, the unitary matrix U_ν is not specified and can be arbitrary.

After the redefinition of fields, we get the fermion mass eigenstates. Recall that the charged and neutral currents given in Eq. (1.10), Eq. (1.12) and Eq. (1.13) are written in the the electroweak gauge interaction basis, so we need to check the change of form under the redefinition of fermion fields.

The neutral currents involve fermion fields of the form $\overline{\Psi}^i_L\gamma^\mu\Psi^i_L$ and $\overline{\Psi}^i_R\gamma^\mu\Psi^i_R$, so unitary rotations of fermion fields on the family space do not change the form of the neutral currents. Therefore, after we redefine fermion fields as in Eq. (1.18), Eq. (1.19) and Eq. (1.20), the form of J^μ_Z and J^μ_{EM} defined in Eq. (1.12) and Eq. (1.13) does not change and we just need to remove the superscript zero from the expressions (because now we are in the mass eigenstates of fermions).

However, for the charged weak currents which couple to $W^\pm_\mu$, the situation is different

$$\begin{aligned} J^{\mu+}_W &= \frac{1}{\sqrt{2}}\sum_i(\overline{\nu}^{0i}_L\gamma^\mu e^{0i}_L + \overline{u}^{0i}_L\gamma^\mu d^{0i}_L) \\ &= \frac{1}{\sqrt{2}}\sum_{i,j}(\overline{\nu}^{i}_L\gamma^\mu (U^\dagger_\nu U_e)^{ij} e^{j}_L + \overline{u}^{i}_L\gamma^\mu (U^\dagger_u U_d)^{ij} d^{j}_L) \end{aligned} \tag{1.21}$$

For the leptonic sector, because U_ν can be arbitrary unitary matrix, we can take $U_\nu = U_e$ such that $U^\dagger_\nu U_e = I_{3\times3}$, where $I_{3\times3}$ is 3×3 identity matrix. This is similar with the neutral currents case. However in quark sector, U_u and U_d are not arbitrary, so in general $U^\dagger_u U_d$ is not the identity matrix. The matrix

$$V_{ckm} = U^\dagger_u U_d \tag{1.22}$$

is known as Cabibbo-Kobayashi-Maskawa(CKM) mixing matrix [5].

V_{CKM} is a general, unitary 3×3 matrix which has 9 parameters including 3 rotation angles and 6 phases. However we can remove 5 phases by making phase rotations of the quark fields. The final V_{CKM} contains 3 angles and one phase. We parameterize V_{CKM} by using θ_{q12}, θ_{q23}, θ_{q13} and a phase θ_{q13}

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \qquad (1.23)$$

where $c_{ij} = \cos\theta_{qij}$ and $s_{ij} = \sin\theta_{qij}$. The current best fit value for the mixing angles are $\sin\theta_{q12} = 0.2272, \sin\theta_{13} = 0.00382, \sin\theta_{23} = 0.04178$ and $\delta_q = \pi/3$ [6].

In practice, it is convenient to use the Wolfenstein approximation [7]

$$V_{CKM} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^2(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \qquad (1.24)$$

where $\lambda = \sin\theta_{q12}$. A, ρ and η are order unity real numbers.

1.2 Neutrino Oscillations and Lepton Flavor Mixing

Neutrinos are color and electrically neutral, so they only can feel the weak interaction. The electron (ν_e), muon (ν_μ) and tau (ν_τ) neutrinos are produced in association with definite charged leptons i.e. e, μ and τ by weak interaction. ν_e, ν_μ and ν_τ are called flavor neutrinos.

Lepton flavor mixing means that the flavor neutrinos ν_α ,$\alpha = e, \mu, \tau$, are not coinciding with the neutrinos of definite mass ν_i $(i = 1, 2, 3)$ if neutrinos are massive. Flavor neutrinos are suppositions of mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (1.25)$$

This mixing matrix $U_{\alpha i}$ is known as Pontecorvo-Maki-Nakagawa-Sakata matrix [8].

U can parameterized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} .K \quad (1.26)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and K as defined as

$$K \equiv \begin{pmatrix} e^{-\varphi_1/2} & 0 & 0 \\ 0 & e^{-\varphi_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1.27)$$

If neutrinos are Majorana particles, one has the two Majorana phases φ_1 and φ_2.

As we discussed in the the previous section, the lepton part of the charged current in the mass basis should be written as

$$\sum_{i,j} \overline{e}^i_L \gamma^\mu (U_l^\dagger U_\nu) \nu^j_L. \quad (1.28)$$

Compare Eq. (1.25) and Eq. (1.28), we have

$$U = U_l^\dagger U_\nu. \quad (1.29)$$

If neutrinos are massive, U_ν is no longer arbitrary and U is not the unit matrix in general. Therefore, search for lepton flavor mixing can be used to study the mass of neutrinos. On the other hand, the lepton flavor mixing can cause neutrino flavor oscillations.

Neutrinos produced by weak interaction are flavor neutrinos which are the suppositions of the mass eigenstates. When neutrinos propagate in the vacuum, they are in mass states which are eigenstates of Hamiltonian in vacuum

$$|\nu_i(t)\rangle = e^{-i(E_i t - p_i L)}|\nu_i(0)\rangle. \tag{1.30}$$

In practice, neutrinos are extremely relativistic, so we can have the approximation $e^{-i(E_i t - p_i L)} \approx e^{-i(m_i^2/2E)L}$.

Imagine a neutrino ν_α with definite flavor α is produced at the beginning. After the neutrino propagates a distance L and reaches the detector, its state is

$$|\nu_\alpha(L)\rangle \approx \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L}|\nu_i\rangle = \sum_\beta \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L} U_{\beta i}^* |\nu_\beta\rangle. \tag{1.31}$$

It becomes a superposition of all the flavors, and the probability that it has flavor β is

$$P_{\nu_\alpha \to \nu_\beta} = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2 \tag{1.32}$$

$P_{\nu_\alpha \to \nu_\beta}$ can be easily calculated as

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} = \quad & \delta_{\alpha\beta} \quad -4\sum_{i>j} \mathrm{Re}(U_{\beta i}^* U_{\beta j} U_{\alpha i} U_{\alpha j}^*) \sin^2(\frac{L}{4E}\Delta m_{ij}^2) \\ + \quad & 2\sum_{i>j} \mathrm{Im}(U_{\beta i}^* U_{\beta j} U_{\alpha i} U_{\alpha j}^*) \sin(\frac{L}{2E}\Delta m_{ij}^2). \end{aligned} \tag{1.33}$$

To explicitly show the above results, let's take the two mass eigenstates and two flavor eigenstates case. The unitary matrix takes the form

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{1.34}$$

The Eq. (1.33) can be simplified as

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - \sin^2 2\theta \sin^2[\frac{L}{4E}\Delta m_{ij}^2]. \tag{1.35}$$

The concept of neutrino flavor oscillations was first introduced by Pontecorvo in the 1960s [9]. The observation of such neutrino oscillations was suggested to be an effective way to search for neutrino masses compared to the usual method of β decay. Experiments have been conducted over the years searching for neutrino mass by oscillation experiments. Since the 1960s, two kinds of neutrino oscillation have been carried out: one with atmospheric neutrinos and the other with solar neutrinos.

The atmospheric neutrinos are produced in the Earth's atmosphere by cosmic rays. The flux of cosmic rays that lead to neutrinos with energies above a few GeV is isotropic [10], so that these neutrinos are produced at the same rate all around the Earth. Therefore for the multi-GeV neutrinos with definite flavor, the detector on the Earth should also observe fluxes isotropically, which implies the downward and upward fluxes should be equal.

However, the underground Super-Kamiokande (Super-K) detector finds that for the multi-GeV atmospheric muon neutrinos [11],

$$\frac{\mathrm{Up} - \mathrm{Flux}(-1.0 < \cos\theta_Z < -0.2)}{\mathrm{Down} - \mathrm{Flux}(+2.0 < \cos\theta_Z < +0.1))} = 0.54 \pm 0.04, \tag{1.36}$$

where θ_Z is the zenith angle. This result strongly disagrees with the equality of upward and downward fluxes. Thus, some mechanism changes the ν_μ flux as the muon neutrinos travel to the detector. One candidate of such a mechanism is the neutrino oscillation i.e. the muon neutrinos from the atmosphere may oscillate to other flavor neutrinos when they travel. Compared to the downward muon neutrinos, the upward neutrinos have a longer distance to travel to reach the detector, and they have more chance to oscillate to other flavor neutrinos, therefore the downward neutrinos have more ν_μ than the upward neutrinos. The Super-K atmospheric neutrino data can be explained by the two-flavor $\nu_\mu - \nu_\tau$ oscillation with one mass splitting $\Delta m^2_{\rm atm}$ and one mixing angle $\theta_{\rm atm}$. At 90%C.L., the ranges are [12]

$$1.9 \times 10^{-3}{\rm eV}^2 \leq \Delta m^2_{\rm atm} \leq 3.0 \times 10^{-3}{\rm eV}^2 \text{ and } \sin^2 2\theta_{\rm atm} > 0.90. \qquad (1.37)$$

Solar neutrinos are produced in the fusion and decay of particles in core of sun. The sun neutrino fluxes observed at experiments are lower than the predication of the standard solar model calculations, and different detectors see different suppression ratios compared to the standard solar model. This is known as the solar neutrino puzzle. There are many solutions for this puzzle. After taking into account the matter effect, the most convincing solution is the so-called Large-Mixing-Angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) effect [13]. When neutrinos travel through matter, their forward scattering from particles they meet along the way can significantly change their propagation. In the sun, the number densities of electron, muon and tau are very different, therefore each flavor neutrino has different probability to be scattered by a corresponding charged lepton. As a result, the flavor change probability of neutrinos can be rather different than it is in the vacuum. The best fit values for solar neutrino oscillation parameters are [14]

$$\sin^2 \theta_{\rm solar} = 0.29, \text{ and } \Delta m^2_{\rm solar} = 6.0 \times 10^{-5}{\rm eV}. \qquad (1.38)$$

There are also oscillation experiments by using other neutrino sources such as reactor neutrinos (Kamland and CHOOZ), and accelerator neutrinos (K2K and MINOS). Remarkably, all of these experimental results can be explained by the oscillation hypothesis and three-flavor neutrino mixing. The current global fit values of the oscillation parameters (3σ) are [14]

$$\begin{aligned}
&7.1\times 10^{-5}\mathrm{eV}^2 \le \Delta m^2_{21} \le 8.9\times 10^{-5}\mathrm{eV}^2,\\
&2.0\times 10^{-3}\mathrm{eV}^2 \le \Delta m^2_{31} \le 3.2\times 10^{-3}\mathrm{eV}^2,\\
&0.24 \le \sin^2\theta_{12} \le 0.40,\\
&0.34 \le \sin^2\theta_{23} \le 0.68,\\
&\sin^2\theta_{13} \le 0.040,
\end{aligned} \tag{1.39}$$

with the best fit values $\Delta m^2_{21} = 7.9\times 10^{-5}\mathrm{eV}^2, \Delta m^2_{31} = 2.6\times 10^{-3}\mathrm{eV}^2, \sin^2\theta_{12} = 0.30, \sin^2\theta_{23} = 0.50$ and $\sin^2\theta_{13} = 0.00$. Currently, the CP-violating phases remain unknown.

Besides the data from oscillation experiments, there are bounds from non-oscillation neutrino experiments and observations from cosmology. These results are summarized as followed:

$$\begin{aligned}
&m_{ee} < 0.1 - 0.9\mathrm{eV}\ \text{(neutrinoless double beta decay) [15]},\\
&(\sum_i |U_{ei}|^2 m_i^2)^{1/2} < 2.3\ \text{(beta decay) [16]},\\
&\sum_i m_i < 0.62\mathrm{eV}\ \text{(WMAP) [17]}.
\end{aligned}$$

Evidence of $\overline{\nu}_\mu - \overline{\nu}_e$ oscillation was claimed by LSND collaboration in 1994 [18]. The LSND fit of data requires the mass-squared difference to be $\sim 1\mathrm{eV}^2$, which can not be simultaneously explained within the framework of only three neutrinos when combined with atmospheric and solar neutrino data. In March 2007, MiniBooNE experiment reported no evidence for ν_μ to ν_e oscillation in the LSND region [19].

We do not include the LSND result into our analysis.

1.3 Theory of Massive Neutrinos and Leptogenesis

The massive neutrinos and leptonic mixing provide concrete evidence for the new physics beyond the SM. To accommodate massive neutrinos in the SM, the simplest way is to introduce right-handed neutrinos and write the Yukawa coupling like other charged fermions

$$Y_{\nu ij}\overline{L_i}HN_j + h.c.. \tag{1.40}$$

After the electroweak symmetry breaking occurs, neutrinos get Dirac masses. Within this simple extension, to generate correct mass scale for neutrinos, one needs extremely small Yukawa coupling $Y_{\nu ij} \sim 10^{-13}$ if one takes the heaviest neutrino mass ~ 0.1eV.

On the other hand, if we treat the SM as an effective theory, using the light degree of freedom of the SM fields, we can write a dimension 5 operator as

$$\mathcal{L}_5 = \frac{y_{ij}L_iHL_jH}{\Lambda_N}. \tag{1.41}$$

Λ_N is the scale where new physics become important. When electroweak breaking occurs, this operator can generate neutrino mass as

$$m_{ij} = \frac{y_{ij}\langle H\rangle}{\Lambda_N}. \tag{1.42}$$

If we take the coupling constant $y_{ij} \sim \mathcal{O}(1)$, we need the new physics scale Λ_N to be order of $\sim 10^{14}$GeV if the heaviest neutrino mass is around 0.1eV.

Note the dimension 5 operator given in Eq. (1.41) violates lepton number by unit 2 i.e. $\Delta L = 2$. The lepton number and baryon number are accidental global symmetry of the SM, therefore the new physics associated with this operator should involve some process violating lepton number. Compared to the Yukawa coupling given by Eq. (1.40), the operator of Eq. (1.41) generates light neutrino mass more naturally in the sense that the coupling constant y_{ij} could be $\mathcal{O}(1)$.

There are two ways to realize this 5 dimensional operator in the more UV complete theory: add heavy Majorana mass terms to the right-handed neutrinos or add a heavy $SU(2)_L$ Higgs triplet (Δ).

Right-handed neutrinos are allowed to have Majorana mass because they are singlets of SM gauge group

$$M_{Rij} N_i^T C^{-1} N_j + h.c., \tag{1.43}$$

where C is the charge conjugate matrix $C = i\gamma^2\gamma^0$.

If $SU(2)_L$ triplet Δ exists, ν_L also can acquire mass from coupling

$$f_{\Delta ij} L_i^T L_j \Delta + h.c.. \tag{1.44}$$

The non-zero VEV of Δ gives $m_{Lij} = f_{\Delta ij}\langle\Delta\rangle$.

Therefore the most general mass structure for the neutrino is

$$\begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix}, \tag{1.45}$$

where m_L, m_D and M_R should be understood as 3×3 matrices.

If $M_R \gg m_D$ and m_L, the light Majorana masses are given by

$$m_\nu = m_L - m_D^T M_R^{-1} m_D. \tag{1.46}$$

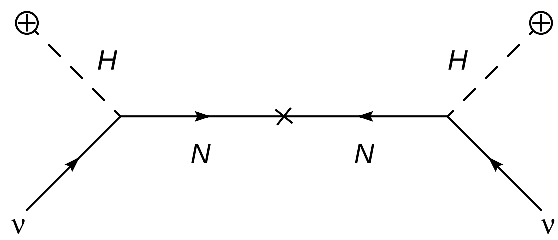

Figure 1.1: Feynman diagram for type I seesaw.

In the limit where $m_L = 0$, even eigenvalues of m_D have the same order of masses of charged leptons and quarks, if the mass scale of M_R is very high, the smallness of neutrino mass still can be explained naturally. This is the so-called type I seesaw mechanism [20].

In the case m_L dominates the contribution to the light neutrino mass, the smallness of neutrino mass is explain as follows: in general, the triplet VEV $\langle\Delta\rangle$ is an induced VEV, which is suppressed by the mass scale of the mass of Δ. For example, in non-supersymmetric case, the Lagrangian includes a term $\Lambda_T HH\Delta$, where Λ_T the coupling constant with mass dimension. When electroweak symmetry breaking occurs, Δ gets an induced VEV $\langle\Delta\rangle \sim \Lambda_T \langle H\rangle^2/M_\Delta^2$. In the supersymmetric case, the induced VEV is $\lambda_T\langle H\rangle^2/M_\Delta$, where λ_T is a dimensionless coupling and taken to be $\mathcal{O}(1)$. Therefore, if M_Δ is super heavy, we obtain the small neutrino mass as similar to the type I seesaw case. This scenario is called type II seesaw [21].

Within the seesaw frame, the lepton number violation can be understood as follows: in the type I case, because right-handed neutrinos carry lepton number, the Majorana mass term of the right-handed neutrinos break lepton number by unit 2; in the type II case, lepton number is broken by the VEV of the $SU(2)_L$ triplet Δ.

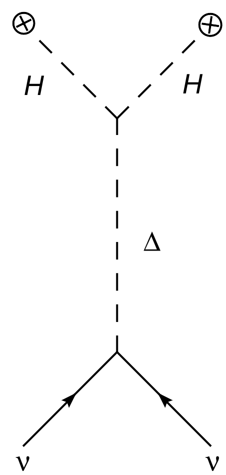

Figure 1.2: Feynman diagram for type II seesaw.

If the seesaw mechanism is the reason why neutrinos are light, the massive neutrino clearly implies the existence of a new physics scale Λ_N ($\sim 10^{14} - 10^{15}$GeV) beyond the electroweak scale $\sim$ 100GeV where N and Δ play important role on the physics above scale Λ_N. However, the above extension of the SM by introducing N and Δ is given by hand with the only purpose being to generate the 5 dimensional operator. N is a singlet under the SM, there is no gauge symmetry breaking associated with its Majorana mass scale, so its scale could be arbitrary-even as big as the Planck scale. For the Δ, although it is charged under the SM gauge group, its mass scale is also arbitrary and not necessarily to be the $\sim 10^{14}$GeV as required by seesaw mechanism to generate light neutrino masses. One hopes that the UV complete theory should include right-handed neutrinos or triplet as necessary ingredients and reproduce the SM at the low energy scale. On the other hand, in the minimal supersymmetric standard model (MSSM), the three coupling constants meet at 2×10^{16}GeV which is close to the Λ_N which is indicated by the seesaw mechanism. This suggests that the UV theory which gives rise to the seesaw mechanism will probably be some kind of Grand Unified Theory (GUT).

The SM gauge structure involves the products of three individual groups which have total rank 4. Thus the GUT group should at least have rank 4 and include the representations of the SM fields. It turns out the smallest rank 4 simple group which contains the SM group as a subgroup is $SU(5)$ [22]. The SM group also can be embedded into a larger group which contains $SU(5)$ as a subgroup for example $SO(10)$ [23] or E_6 [24]. $SO(10)$ has some advantages: (i) the **16** dimensional spinor representation of $SO(10)$ includes the SM singlet right-handed neutrino, which is a singlet under $SU(5)$; (ii) $U(1)_{B-L}$ is a gauged subgroup of $SO(10)$, thus the heavy right-handed neutrino Majorana mass directly corresponds to the breaking of local U_{B-L}; (iii) it contains the left-right symmetric model $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [25] which provide a natural way to explain parity breaking at the low energy scale.

Besides light neutrino masses, the seesaw scenario also provides a solution to one of the biggest mysteries of our universe i.e. why matter dominates our universe instead of anti-matter. To generate baryon asymmetry of our universe by dynamical process. In the early 1960s, Sakharov concluded that three conditions have to be satisfied [26]:

(i).Baryon number is violated;

(ii).Combined charge conjugate and parity symmetry (CP) is violated;

(iii).Non-thermal equilibrium of universe.

Among the many suggestions, leptogenesis [27] is particularly interesting given

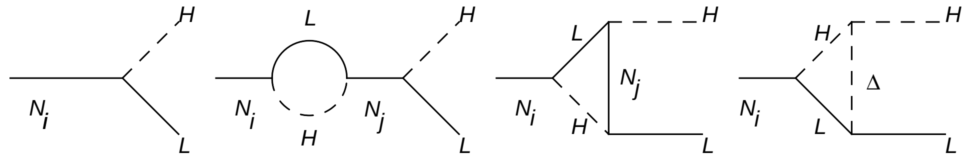

Figure 1.3: Leptogenesis.

the recent discovery of neutrino masses and mixing. We can see how the three Sakharov conditions can be satisfied in the seesaw scenario. First, the Majoana mass term of right-handed neutrinos violates lepton number conservation. The no-perturbative Spherelon process can convert lepton number asymmetry to baryon asymmetry while keeping $B - L$ conserved. Second, CP can be easily violated by the complex Yukawa coupling. Third, if the decay rate of the right-handed neutrinos is smaller than the Hubble expansion rate of universe, the non-equilibrium condition can be satisfied. The process is the following: The out-of-equilibrium decay of heavy right-handed Majorana neutrinos via Yukawa couplings can produce both leptons and anti-leptons due to their Majorana feature. The CP-violating phases in the Yukawa couplings can cause the asymmetry between lepton number and anti-lepton number. Once the primordial lepton asymmetry is produced, the Spherelon [28] converts it to the baryon asymmetry.

1.4 Flavor Symmetry and Grand Unification Theories

In general, the seesaw mechanism can explain the smallness of light neutrino masses, but can not explain the mixing pattern. Compared with quark mixing matrix V_{CKM} which is near unit matrix, the leptonic mixing pattern is quite special.

The θ_{23} is near maximal, θ_{12} is large but not maximal and θ_{13} almost vanishes. To understand this particular mixing pattern in the lepton sector, there are two main approaches in the literature: flavor (family) symmetry in the lepton sector; (ii) A class of $SO(10)$ grand unification models.

The low energy scale leptonic mixing pattern exhibits strong evidence for the possible flavor symmetry in the lepton sector. The $\mu - \tau$ symmetry [30] is one of the candidates for such a symmetry. This symmetry requires the light neutrino mass matrix to be invariant under the exchange of mu neutrino and tau neutrino in the basis where charged lepton mass matrix is diagonal. One can write the operator of the $\mu - \tau$ symmetry in the matrix form

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{1.47}$$

If the light neutrino mass matrix is invariant under the operation of P i.e. $M_\nu = PM_\nu P$, the M_ν has the form

$$M_\nu = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix} . \tag{1.48}$$

Diagonalization of this mass matrix leads to $\theta_{23} = \pi/2$ and $\theta_{13} = 0$ which agree with the best fit values of the oscillation data. Note $\mu - \tau$ does not give any bounds on the size of θ_{12} because it only acts on ν_μ and ν_τ. This symmetry is also testable by precise measurements of mixing angles and the correlation between the nonzero value of θ_{13} and the deviation of θ_{23} from $\pi/4$.

Flavor symmetry also makes it possible to connect low scale neutrino data to the seesaw parameters. Many attempts have been made on the derivation of seesaw parameters from the low scale data. This is particularly interesting because the baryon asymmetry of universe may be expressed as a function of neutrino oscillation data (if leptogenesis is the mechanism to cause the baryon asymmetry). One fascinating example is the possible connection between CP-violating phases generating the baryon asymmetry in the early universe and those which can be measured in the low energy experiments. But in general such derivation is impossible without additional assumptions, because the number of free parameters contained in seesaw framework is much more than the measurable quantities at low scale. Take the type I seesaw as an example: in the basis where right-handed neutrino mass matrix and charged lepton mass matrix are diagonal, all mixing and CP-violating phases are contained in the Yukawa coupling Y_ν which is a complex 3×3 matrix. Three of 9 phases in Y_ν can be rotated away by simultaneously rephasing charged lepton and neutrinos, so one is left with 6 CP-violating phases. Together with 3 right-handed neutrino masses and 15 parameters in neutrino Yukawa coupling, there are 18 total parameters. However there are only 9 parameters contained in the low scale data i.e. 3 light neutrino masses, 3 mixing angles, and three CP-violating phases. To establish such a connection, one has to reduce the seesaw parameters. Most of the work on this direction is based on the assumption of special texture of the Yukawa coupling with zero entries and two right-handed neutrinos seesaw scenario. However this approach is ad hoc. Actually the low scale mixing and masses of neutrinos correspond to an infinite set of textures. On the other hand, with the flavor symmetry, the texture of the Yukawa coupling and the mass matrix of right-handed neutrinos are constrained by the flavor symmetry, which provides a natural way to reduce the seesaw parameters.

Phenomenologically the flavor symmetry may work well to understand the neutrino oscillation data, however there is no guarantee such a symmetry can be extended to the more general family symmetry in the interaction basis where the forms of both charged lepton and neutrino mass matrices are determined by the family symmetry in the Lagrangian. Furthermore, this kind of extension faces more challenges when considering grand unification theories, because unlike in standard model where quarks and leptons have no intrinsic connection, in the grand unification theories, leptons and quarks are unified together. For example, in $SU(5)$ model, the anti-fundamental representation $\overline{\mathbf{5}}$ includes $SU(2)_L$ lepton doublet and $SU(2)_L$ singlet right-handed down-type quarks, and **10** includes $SU(2)_L$ quark doublets, $SU(2)_L$ singlet right-handed up-type quarks and right-handed charged leptons. In the $SO(10)$ model, the connection between quarks and leptons is even tighter: the spinor representation **16** includes one family of all fermions, and the large discrepancy between quark mixing and lepton mixing has to be overcome. In general, the family symmetry extracted from the lepton mixing may not be the symmetry in the quark sector.

Even if the flavor symmetry extracted from the low scale data can be extended to the more general family symmetry, quantum effects may break it if the light neutrino masses are generated by the high scale seesaw. This is because of the energy gap between the seesaw scale ($\sim 10^{14}$GeV), where we integrate out the heavy degree of freedom, and the low scale (a few GeV), where we make measurements: one should run the parameters of lepton mixing and neutrino masses from the seesaw scale to the electroweak scale by using renormalization group equations (RGEs). Although we start from the Lagrangian which is invariant under both gauge symmetry and family symmetry, and the light neutrino mixing matrix has the particular pattern right below the seesaw scale, the quantum corrections may distort the initial mixing pattern at the low scale. These RGE effects highly depend on the property of the family symmetry and mass spectrum of neutrinos.

Alternative to the family symmetry, a class of $SO(10)$ grand unification models provide a deep insight on the relation between lepton mixing and quark mixing, even though they look very different. In the supersymmetric minimal $SO(10)$ model, the almost maximal value of θ_{23} is due to the $b-\tau$ mass unification and the small θ_{13} is the result of the quark-lepton unification if the type II seesaw dominates the contribution to the light neutrino masses. The model is also very predictive, with 13 parameters as input without CP-violating phases (6 quark masses, 3 CKM mixing angles and three charged lepton masses), the neutrino sector is completely determined. Phenomenologically this model is very attractive; however, the type II seesaw dominance can not be realized when one minimizes the potential within the minimal Higgs sector because this model is constrained so that one can not generate the mass difference between the $SU(2)_L$ triplet and other submulitplets of $\overline{\mathbf{126}}$ multiplet of $SO(10)$ such that the perturbativity of the theory can be kept up to the GUT scale.

In this thesis, we address these issues systematically.

We extract the $\mu - \tau$ flavor symmetry from low scale neutrino oscillation data. With this $\mu - \tau$ symmetry, we derive a simple formula relating the lepton asymmetry and neutrino oscillation observables $\epsilon_l = (a\Delta m^2_{\text{solar}} + b\Delta m^2_{\text{atm}}\theta^2_{13})$ for three right-handed neutrinos and a relation of form $\epsilon_l \propto \theta^2_{13}$ for the case of two right-handed neutrinos. We extend $\mu - \tau$ symmetry successfully to the universal family symmetry which exchange the second and third generations of all fermions, and build a realistic supersymmetric $SU(5)$ grand unification model.

The $\mu - \tau$ symmetry does not act on the first generation which leaves the θ_{12} free. We study the possible embedding of this S2 symmetry to the more general $S3$ symmetry which is the permutation symmetry between three families of leptons and build a model which can generate the so-called tri-bimaximal (TBM) mixing pattern in the lepton sector if one has both type I and type II seesaws contribution to the light neutrino masses. We study the RGE effects on lepton mixing at low scale. The quasi-degenerate mass spectrum is strongly disfavored by the TBM mixing. In this model, the leptogensis includes contributions from both type I and type II, and the CP-violating phases in the neutrino mixing are directly responsible for the lepton asymmetry.

In this thesis, we also study the natural realization of the type II seesaw by extending the 4 dimensional minimal $SO(10)$ model to 5 dimensional theory with a warped extra dimension. With this setup, the ability of minimal supersymmetric $SO(10)$ model to explain the large θ_{23} and vanishing θ_{13} is still kept, and the type II dominance scenario can be realized naturally without adding new Higgs fields and tuning of the parameters. In the mini-warped minimal $SO(10)$ model, the GUT scale is introduced as a 4 dimensional theory cut-off, so the problem of non-perturbativity beyond GUT scale existing in 4 dimensional model can be avoided.

This thesis is organized as follows:

In Chapter 2, leptonic $\mu-\tau$ flavor symmetry is introduced to explain the lepton mixing pattern. We study the implications of this symmetry on the leptogenesis.

In Chapter 3, we extend the leptonic $\mu-\tau$ flavor symmetry into the universal family symmetry including the quark sector, and build a supersymmetric grand unification model based on $SU(5)$ group with type II seesaw mechanism.

In Chapter 4, we embed $\mu-\tau$ into larger permutation group $S3$ where $\mu-\tau$ symmetry is a $S2$ subgroup of $S3$ and obtain tri-bimaximal mixing pattern. We also study the radiative corrections to the exact TBM limit due to RGE effects.

In Chapter 5, we explore features of CP-violating phases our $S3$ model model and study the leptogenesis involving type II seesaw.

In Chapter 6, $S4$ permutation group is used as family symmetry to study unification of three families of fermions based on $SO(10)$ GUT. We build a realistic GUT model based on $S4 \times SO(10)$ symmetry.

In Chapter 7, we study the natural realization of the type II seesaw of minimal $SO(10)$ by extending 4 dimensional model to 5 dimension with a warped fifth extra dimension.

In Chapter 8, we present the summary and conclusion of this thesis.

Chapter 2

$\mu - \tau$ Symmetry, Leptogenesis and θ_{13}

2.1 Overview

There may be a deep connection between the origin of matter in the universe and the observed neutrino oscillations. This speculation is inspired by the idea that the heavy right-handed Majorana neutrinos that are added to the standard model for understanding small neutrino masses via the seesaw mechanism [20] can also explain the origin of matter via their decay. The mechanism goes as follows [27]: CP violation in the same Yukawa interaction of the right-handed neutrinos, which go into giving nonzero neutrino masses after electroweak symmetry breaking, lead to a primordial lepton asymmetry via the out of equilibrium decay $N \rightarrow L + H$ (where L are the known leptons and H is the standard model Higgs field). This asymmetry subsequently gets converted to baryon-anti-baryon asymmetry observed today via the the electroweak sphaleron interactions [28], above $T \geq v_{wk}$ (v_{wk} being the weak scale). Since this mechanism involves no new interactions beyond those needed in the discussion of neutrino masses, one would expect that better understanding of neutrino mass physics would clarify one of the deepest mysteries of cosmology both qualitatively as well as quantitatively.

This question has been the subject of many investigations in recent years [31, 32, 33, 34, 35, 36, 37, 38] in the context of different neutrino mass models and many interesting pieces of information about issues such as the spectrum of right-handed neutrinos, upper limit on the neutrino masses etc have been obtained. In a recent paper [39], the authors showed that if one assumes that the lepton sector of minimal seesaw models has a leptonic $\mu - \tau$ interchange symmetry [29, 30], then one can under certain plausible assumptions indeed predict the magnitude of the matter-anti-matter asymmetry in terms of low energy oscillation parameter, $\Delta m^2_{\rm solar}$ and a high scale CP phase. The choice of $\mu - \tau$ symmetry was dictated by the fact that it is the simplest symmetry of neutrino mass matrix that explains the maximal atmospheric mixing as indicated by data. Using present experimental value for $\Delta m^2_{\rm solar}$, one obtains the right magnitude for the baryon asymmetry of the universe.

The results of the paper [39] were derived in the limit that $\mu - \tau$ interchange symmetry is exact. If however a nonzero value for the neutrino mixing angle θ_{13} is detected in future experiments, this would imply that this symmetry is only approximate. Also, since in the standard model ν_μ and ν_τ are members of the $SU(2)_L$ doublets $L_\mu \equiv (\nu_\mu, \mu)$ and $L_\tau \equiv (\nu_\tau, \tau)$, any symmetry between ν_μ and ν_τ must be a symmetry between L_μ and L_τ at the fundamental Lagrangian level. The observed difference between the muon and tau masses would therefore also imply that the $\mu - \tau$ symmetry has to be an approximate symmetry. In view of this, it is important to examine to what extent the results of Ref. [39] carry over to the case when the symmetry is approximate. We find two interesting results under some very general assumptions:

(i) a simple formula relating the lepton asymmetry and neutrino oscillation observables for the case of three right-handed neutrinos, i.e. $\epsilon_l = (a\Delta m^2_{\rm solar} +$

$b\Delta m^2_{\rm atm}\theta^2_{13}$) and (ii) a relation of the form $\epsilon_l \propto \theta^2_{13}$ for the case of two right-handed neutrinos. Measurement of θ_{13} will have important implications for both the models; in particular we show that in a class of models with two right-handed neutrinos with approximate $\mu-\tau$ symmetry breaking, there is a lower limit on θ_{13}, which is between 0.1 to 0.15 depending on the values of the CP phase. These values are in the range which will be probed in experiments in near future [42].

The basic assumption under which the two results are derived are the following:

(A) type I seesaw formula is responsible for neutrino masses:

(B) $\mu-\tau$ symmetry for leptons is broken only at high scale in the mass matrix of the right-handed neutrinos.

We start with an extension of the minimal supersymmetric standard model (MSSM) for the generic the type I seesaw model for neutrino masses. The effective low energy superpotential for this model is given by

$$W \;=\; e^{cT}Y_\ell L H_d + N^{cT}Y_\nu L H_u + \frac{M_R}{2}N^{cT}N^c \qquad (2.1)$$

Here L, e^c, ν^c are leptonic superfields; $H_{u,d}$ are the Higgs fields of MSSM. Y_ν and M_R are general matrices where we choose a basis where Y_ℓ is diagonal. We do not display the quark part of the superpotential which is same as in the MSSM. After electroweak symmetry breaking, this leads to the type I seesaw formula for neutrino masses given by

$$M_\nu \;=\; -\,Y^T_\nu f^{-1} Y_\nu \frac{v^2_{wk}\tan^2\beta}{v_R} \qquad (2.2)$$

The constraints of $\mu-\tau$ symmetry will manifest themselves in the form of the Y_ν and M_R. It has been pointed out that if we go to a basis where the right-handed neutrino mass matrix is diagonal, we can solve for Y_ν in terms of the neutrino masses and mixing angles as follows [43]:

$$Y_\nu v \;=\; i\hat{M}_R^{1/2} R(z_{ij}) (\hat{M}_\nu)^{1/2} U^\dagger \tag{2.3}$$

where R is a complex matrix with the property that $RR^T = 1$. The unitary matrix U is the lepton mixing matrix defined by

$$M_\nu \;=\; U^* \hat{M}_\nu U^\dagger \tag{2.4}$$

The complex orthogonal matrices R can be parameterized as:

$$R(z_{12}, z_{23}, z_{13}) \;=\; R(z_{23}) R(z_{13}) R(z_{12}) \tag{2.5}$$

with

$$R(z_{12}) \;=\; \begin{pmatrix} \cos z_{12} & \sin z_{12} & 0 \\ -\sin z_{12} & \cos z_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.6}$$

and similarly for the other matrices. z_{ij} are complex angles.

Let us now turn to lepton asymmetry: the formula for primordial lepton asymmetry in this case, caused by right-handed neutrino decay is

$$\epsilon_l \;=\; \frac{1}{8\pi} \sum_j \frac{\mathrm{Im}[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{1j}^2}{(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{11}} F(\frac{M_1}{M_j}) \tag{2.7}$$

where $\tilde{Y}_\nu$ is defined in a basis where right-handed neutrinos are mass eigenstates and their masses are denoted by $M_{1,2,3}$ where $F(x) \;=\; -\frac{1}{x}\left[\frac{2x^2}{x^2-1} - \ln(1+x^2)\right]$ [44]. In the case where that the right-handed neutrinos have a hierarchical mass pattern i.e.

$M_1 \ll M_{2,3}$, we get $F(x) \simeq -3x$. In this approximation, we can write the lepton asymmetry in a simple form [45]

$$\epsilon_l \;=\; -\frac{3}{8\pi}\frac{M_1 \mathrm{Im}[Y_\nu M_\nu^\dagger Y_\nu^T]_{11}}{v^2(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{11}} \tag{2.8}$$

where using the expression for Y_ν given above, we can rewrite ϵ_l as:

$$\epsilon_l \;=\; -\frac{3}{8\pi}\frac{\mathrm{Im}[\hat{M}_R^{1/2}R(z_{ij})\hat{M}_\nu^2 R(z_{ij})\hat{M}_R^{1/2}]_{11}}{v^2|R^T(z_{ij})M_\nu R^\dagger(z_{ij})|_{11}^2} \tag{2.9}$$

We will now apply this discussion to calculate the lepton asymmetry in the general case without any symmetries. In the following sections, we follow it up with a discussion of two cases: (i) the cases of exact $\mu - \tau$ symmetry and (ii) the case where this symmetry is only approximate. Since the formula in Eq. (2.9) assumes that there are three right-handed neutrinos, we will focus on this case in the next two sections. In a subsequent section, we consider the case of two right-handed neutrinos (N_μ, N_τ), which transform into each other under the $\mu - \tau$ symmetry. Both cases are in agreement with the observed neutrino mass differences and mixings.

It follows from Eq. (2.9) that

$$\epsilon_l \;=\; -\frac{3M_1}{8\pi}\frac{\mathrm{Im}[m_1^2 R_{11}^2 + m_2^2 R_{12}^2 + m_3^2 R_{13}^2]}{v^2|R(z_{ij})M_\nu R^\dagger(z_{ij})|_{11}^2} \tag{2.10}$$

Since the matrix R is an orthogonal matrix, we have the relation

$$R_{11}^2 + R_{12}^2 + R_{13}^2 \;=\; 1 \tag{2.11}$$

Using this equation in Eq. 2.10, we get

$$\epsilon_l \;=\; -\frac{3M_1}{8\pi}\frac{\mathrm{Im}[\Delta m_{\mathrm{solar}}^2 R_{12}^2 + \Delta m_{\mathrm{atm}}^2 R_{13}^2]}{v^2\sum_j(|R_{1j}|^2 m_j)} \tag{2.12}$$

This relation connects the lepton asymmetry to both the solar and the atmospheric mass difference square [32]. To make a prediction for the lepton asymmetry, we need to the lengths of the complex quantities R_{1j}. The out of equilibrium condition does provide a constraint on $|R_{1j}|$ as follows:

$$\sum_{j=1,2,3} (|R_{1j}|^2 m_j) \leq 10^{-3}\ eV \tag{2.13}$$

It is clear from Eq. (2.13) that if neutrinos are quasi-degenerate i.e. $m_1 \simeq m_2 \simeq m_3 \equiv m_0$, then using Eq. (2.11), we find that the left hand side of Eq. (2.13) has a lower bound of m_0 which is clearly much bigger than the right hand side of the inequality. Defining $K \equiv \frac{\Gamma}{H}$, this means that $K \geq \frac{m_0}{2\times 10^{-3}\ eV} \gg 1$. This implies that the right-handed neutrinos decays are in equilibrium at $T \simeq M_1$. This will cause dilution of the lepton asymmetry generated with the dilution factor given by K. Using a parameterization for the dilution factor $\kappa_1 \simeq \frac{0.3}{K(\ln K)^{3/5}}$ [46], we get $\kappa_1 \simeq 10^{-3}$ which will make the baryon to photon ratio much too small. Based on this argument, we conclude that a degenerate mass spectrum with $m_0 \geq 0.1$ eV will most likely be in conflict with observations, if type I seesaw is responsible for neutrino masses. It must however be noted that a more appealing and natural scenario for degenerate neutrino masses is type II seesaw formula [21], in which case the above considerations do not apply. Therefore, it is not possible to conclude based on the leptogenesis argument alone that a quasi-degenerate neutrino spectrum is inconsistent.

In a hierarchical neutrino mass picture, Eq. (2.13) implies that $|R_{13}|^2 \leq 0.02$ and $|R_{12}|^2 \leq 0.1$. If we assume that the upper limit in the Eq. (2.13) is saturated, then we get the atmospheric neutrino mass difference square in Eq. (2.12) to give the dominant contribution. We will see below that if one assumes an exact $\mu - \tau$ symmetry for the neutrino mass matrix, the situation becomes different and it is the solar mass difference square that dominates.

2.2 Three Right-handed Neutrinos and Exact $\mu - \tau$ Symmetry

In this section, we consider the case of three right-handed neutrino with an exact $\mu - \tau$ symmetry in the Dirac mass matrix as well as the right-handed neutrino mass matrix. In this case, the right-handed neutrino mass matrix M_R and the Dirac Yukawa coupling Y_ν can be written respectively as:

$$M_R = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix} \quad (2.14)$$

$$Y_\nu = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{21} & h_{23} & h_{22} \end{pmatrix}$$

where M_{ij} and h_{ij} are all complex. An important property of these two matrices is that they can be cast into a block diagonal form by the same transformation matrix $U_{23}(\pi/4) \equiv \begin{pmatrix} 1 & 0 \\ 0 & U(\pi/4) \end{pmatrix}$ on the ν's and N's. Let us denote the block diagonal forms by a tilde i.e. $\tilde{Y}_\nu$ and $\tilde{M}_R$. We then go to a basis where the $\tilde{M}_R$ is subsequently diagonalized by the most general 2×2 unitary matrix as follows:

$$V^T(2 \times 2) U_{23}^T(\pi/4) M_R U_{23}(\pi/4) V(2 \times 2) = \hat{M}_R \quad (2.15)$$

where $V(2\times 2) = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}$ where V is the most general 2×2 unitary matrix given by $V = e^{i\alpha}P(\beta)R(\theta)P(\gamma)$. The 3×3 case therefore reduces to a 2×2 problem. The third mass eigenstate in both the light and the heavy sectors play no role in the leptogenesis as well as generation of solar mixing angle [39]. Note also that we have $\theta_{13} = 0$. The seesaw formula in the 1-2 subsector has exactly the same form except that all matrices in the left and right hand side of Eq. (2.9) are 2×2 matrices. The formula for the Dirac Yukawa coupling in this case can be inverted to the form:

$$\tilde{Y}_\nu(2\times 2) \;=\; i\hat{M}_R^{1/2}(2\times 2)R(z_{12})(\hat{M}_\nu)^{1/2}(2\times 2)\tilde{U}^\dagger \qquad (2.16)$$

where $U = U_{23}(\pi/4)\begin{pmatrix} \tilde{U} & 0 \\ 0 & 1 \end{pmatrix}$. Using this, we can cast ϵ_l in the form:

$$\epsilon_l \;=\; \frac{3}{8\pi}\frac{M_1}{v^2}\frac{\mathrm{Im}(\cos^2 z_{12})\Delta m^2_{\text{solar}}}{(|\cos z_{12}|^2 m_1 + |\sin z_{12}|^2 m_2)} \qquad (2.17)$$

This could also have been seen from Eq. (2.12) by realizing that for the case of exact $\mu-\tau$ symmetry, we have $z_{13} = 0$ and $z_{23} = \pi/4$.

The above result reproduces the direct proportionality between ϵ_l and solar mass difference square found in Ref. [39]. To simplify this expression further, let us note that out of equilibrium condition for the decay of the lightest right-handed neutrino leads to the condition:

$$\frac{M_1^2}{v_{wk}^2}[m_1|\cos z_{12}|^2 + m_2|\sin z_{12}|^2] \leq 14\frac{M_1^2}{M_{P\ell}} \qquad (2.18)$$

which implies that

$$|m_1|\cos z_{12}|^2 + m_2|\sin z_{12}|^2| \leq 2\times 10^{-3}\ eV \qquad (2.19)$$

Since solar neutrino data require that in a hierarchical neutrino mass picture $m_2 \simeq 0.9 \times 10^{-2}$ eV, in Eq. (2.19), we must have $|\sin z_{12}|^2 \sim 0.2$. If we parameterize $\cos^2 z_{12} = \rho e^{i\eta}$, we recover the conclusions of Ref. [39]. This provides a different way to arrive at the conclusions of Ref. [39].

2.3 Lepton Asymmetry and $\mu - \tau$ Symmetry breaking

In this section, we consider the effect of breaking of $\mu - \tau$ symmetry on lepton asymmetry. Within the seesaw framework, this breaking can arise either from the Dirac mass matrix for the neutrinos or from the right-handed neutrino sector or both. We focus on the case, when the symmetry is broken in the right-handed sector only. Such a situation is easy to realize in seesaw models where the theory obeys exact $\mu - \tau$ symmetry at high scale (above the seesaw scale) prior to B-L symmetry breaking as we show in a subsequent section. We will also show that in this case there is a simple generalization of the lepton asymmetry formula that we derived in the exact $\mu - \tau$ symmetric case [39][1].

In this case the neutrino Yukawa matrix is given in the mass eigenstates basis of the right-handed neutrinos by

$$\tilde{Y_\nu} = V^+_{1/3} V^+_{1/2} V^+_{2/3} Y_\nu \tag{2.20}$$

where Y_ν is the neutrino Dirac matrix in the flavor basis; The notation $V^+_{i/j}$ denotes a unitary 2×2 matrix in the (i, j) subspace. In the above equation, $V_{2/3} = V_{2/3}(\pi/4)$.

[1] Leptogenesis in a specific $\mu - \tau$ symmetric model where the Dirac Yukawa coupling has the form $Y_\nu = \mathrm{diag}(a, b, b)$ has been discussed in Ref. [40]. Our discussion applies more generally.

Now if we substitute for $\tilde{Y}_\nu$ the expression in Eq. (2.3) and use maximal mixing for the atmospheric neutrino we obtain

$$\begin{pmatrix} \tilde{Y}_{2\times 2} & 0 \\ 0 & \tilde{y_3} \end{pmatrix} = V_{1/3} M_R^{1/2} R_{1/2} R_{1/3} M_\nu^{1/2} U_{1/2}^+ U_{1/3}^+ \tag{2.21}$$

Since the $\mu - \tau$ symmetry breaking is assumed to be small and from reactor neutrino experiments $\theta_{13} << 1$ we will expand the mixing matrices in the $1 - 3$ subspace to first order in mixing parameter:

$$(V, R, U)_{1/3} \simeq 1 + (\epsilon, z, \theta)_{13} E \tag{2.22}$$

where

$$E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \tag{2.23}$$

To first order in ϵ_{13}, z_{13} and θ_{13} we have

$$z_{13} M_R^{1/2} R_{1/2} E M_\nu U_{1/2}^+ + \epsilon_{13} E M_R^{1/2} R_{1/2} M_\nu^{1/2} U_{1/2}^+ - \theta_{13} M_R^{1/2} R_{1/2} M_\nu^{1/2} U_{1/2}^+ E = 0 \tag{2.24}$$

It is straight forward to show that the perturbation parameters should satisfy the following equations

$$\begin{aligned} \epsilon_{13} M_{R_3} m_3 + z_{13} M_{R_1} m_3 R_{11} - \theta_{13} e^{-i\delta} M_{R_1} c_\theta (m_1 R_{11} - m_2 R_{12}) &\simeq 0, \\ \epsilon_{13} M_{R_2} (m_2 R_{12} s_\theta - m_1 R_{11} c_\theta) - z_{13} M_{R_3} m_1 c_\theta - \theta_{13} e^{-i\delta} M_{R_3} m_3 &\simeq 0, \\ \epsilon_{13} M_{R_2} (m_1 R_{11} s_\theta + m_2 R_{12} c_\theta) + z_{13} M_{R_3} m_1 s_\theta &\simeq 0, \\ z_{13} M_{R_2} m_3 R_{21} - \theta_{13} e^{-i\delta} M_{R_2} c_\theta (m_1 R_{21} - m_2 R_{22}) &\simeq 0 \end{aligned} \tag{2.25}$$

Where R_{ij} are the matrix elements of $R_{1/2}$ and c_θ and s_θ are the sine and cosine of the solar neutrino mixing angle. Hence one can see that the parameter z_{13} is proportional to the θ_{13} neutrino mixing angle and is given to first order by

$$z_{13} = [(\frac{m_1}{m_3})R_{21} - (\frac{m_2}{m_3})R_{22}]\theta_{13}e^{-i\delta}c_\theta \tag{2.26}$$

This proves that the matrix element R_{13} that goes into the leptogenesis formula is directly proportional to the physically observable parameter θ_{13}. This enables us to write $\epsilon_l = a\Delta m^2_{\text{solar}} + b\Delta m^2_{\text{atm}}\theta^2_{13}$. A consequence of this is that if the coefficient of proportionality is chosen to be of order one, then as experimental upper limit goes down, unlike the generic type I seesaw case in section II, the solar mass difference square starts to dominate for the LMA solution to the solar neutrino deficit.

2.4 Lepton Asymmetry for Two Right-handed Neutrinos

In this section, we consider the case of two right-handed neutrinos which transform into one another under $\mu - \tau$ symmetry. The leptogenesis in this model with exact $\mu - \tau$ symmetry was discussed in [39] and was shown that it vanishes. In this model therefore, a vanishing or very tiny θ_{13} would not provide a viable model for leptogenesis. Turning this argument around, enough leptogenesis should provide a lower limit on the value of θ_{13}.

To set the stage for our discussion, let us first review the argument for the exact $\mu - \tau$ symmetry case [39]. The symmetry under which ($N_\mu \leftrightarrow N_\tau$) and $L_\mu \leftrightarrow L_\tau$

whereas the $m_\mu \neq m_\tau$ constrains the general structure of Y_ν and M_R as follows:

$$M_R = \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22} \end{pmatrix} \tag{2.27}$$

$$Y_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{22} \end{pmatrix}$$

In order to calculate the lepton asymmetry using Eq. (2.7), we first diagonalize the righthanded neutrino mass matrix and change the Y_ν to $\tilde{Y}_\nu$. Since M_R is a symmetric complex 2×2 matrix, it can be diagonalized by a transformation matrix $U(\pi/4) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ i.e. $U(\pi/4)M_R U^T(\pi/4) = \mathrm{diag}(M_1, M_2)$ where $M_{1,2}$ are complex numbers. In this basis we have $\tilde{Y}_\nu = U(\pi/4)Y_\nu$. We can therefore rewrite the formula for n_ℓ as

$$\epsilon_l \propto \sum_j \mathrm{Im}[U(\pi/4)Y_\nu Y_\nu^\dagger U^T(\pi/4)]_{12}^2 F(\frac{M_1}{M_2}) \tag{2.28}$$

Now note that $Y_\nu Y_\nu^\dagger$ has the form $\begin{pmatrix} A & B \\ B & A \end{pmatrix}$ which can be diagonalized by the matrix $U(\pi/4)$. Therefore it follows that $\epsilon_\ell = 0$.

Let us now introduce $\mu-\tau$ symmetry breaking. If we introduce a small amount of $\mu - \tau$ breaking in the right-handed neutrino sector as follows: we keep the Y_ν symmetric but choose the right-handed neutrino mass matrix as:

$$M_R = \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22}(1+\beta) \end{pmatrix}. \tag{2.29}$$

After the right-handed neutrino mass matrix is diagonalized, the 3×2 Y'_ν takes the form (for $\theta_{13} \ll 1$ and in the basis where the light neutrino masses are diagonal):

$$\begin{pmatrix} A & B & w\theta_{13} \\ & & \\ x\theta_{13} & y\theta_{13} & D \end{pmatrix} \tag{2.30}$$

Here B, D, x, y, w are of order one and $\theta_{13} \propto \beta$.

To first order in the small mixing θ_{13}, the complex parameters A, B, D satisfy the constraint

$$A \sim \theta_{13}; \quad Bv^2 \simeq m_2 M_1; \quad Dv^2 \simeq m_3 M_2 \tag{2.31}$$

Using these order of magnitude values, we now find that

$$\epsilon_l \simeq \frac{3}{8\pi}\frac{M_1}{v^2}\frac{\sin\eta[m_3^2\theta_{13}^2\xi]}{m_2} \tag{2.32}$$

where ξ is a function of order one. It is clear that very small values for θ_{13} will lead to unacceptably small ϵ_l. In Fig. 2.1, we have plotted η_B against θ_{13} for values of the parameters in the model that fit the oscillation data and find a lower bound on $\theta_{13} \geq 0.1 - 0.15$ for two different values of the CP phases (Fig. 2.1). In this figure, we have chosen, $M_1 \simeq 7 \times 10^{11}$ GeV. For higher values of M_1 the allowed range θ_{13} moves to the lower range. Also we note that for values of $M_1 < 7 \times 10^{11}$ GeV, the baryon asymmetry becomes lower than the observed value.

2.5 A Model for $\mu - \tau$ Symmetry for Neutrinos

In this section, we present a simple extension of the minimal supersymmetric standard model (MSSM) by adding to it specific high scale physics that at low energies can exhibit $\mu - \tau$ symmetry in the neutrino sector as well as real Dirac masses for neutrinos.

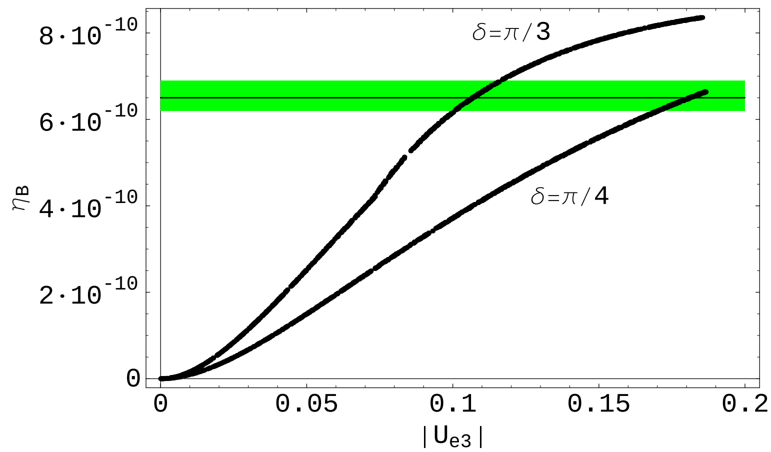

Figure 2.1: Plot of η_B vrs θ_{13} for the case of two right-handed neutrinos with approximate $\mu - \tau$ symmetry and CP phases $\delta = \pi/4$ and $\pi/3$. The values of θ_{13} are predicted to be 0.1 and 0.15 respectively. The horizontal line corresponds to $\eta_B^{obs} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}$ [47].

First we recall that MSSM needs to be extended by the addition of a set of right-handed neutrinos (either two or three) to implement the seesaw mechanism for neutrino masses [20]. We will accordingly add three right-handed neutrinos (N_e, N_μ, N_τ) to MSSM. We then assume that at high scale, the theory has $\mu - \tau$ $S2$ symmetry under which $N_\pm \equiv (N_\mu \pm N_\tau)$ are even and odd combinations; similarly, we have for leptonic doublet superfields $L_\pm \equiv (L_\mu \pm L_\tau)$ and leptonic singlet ones $\ell^c_\pm \equiv (\mu^c \pm \tau^c)$; two pairs of Higgs doublets ($\phi_{u,\pm}$ and $\phi_{d,\pm}$), and a singlet superfields $S_\pm$. Other superfields of MSSM such as N_e, L_e, e^c as well as quarks are even under the $\mu - \tau$ $S2$ symmetry. Now suppose that we write the superpotential involving the S fields as follows:

$$W_S = \lambda_1 \phi_{u,-} \phi_{d,+} S_- + \lambda_2 \phi_{u,-} \phi_{d_-} S_+ \tag{2.33}$$

then when we give high scale VEVs to $< S_\pm > \; = \; M_\pm$, then below the high scale there are only the usual MSSM Higgs pair $H_u \equiv \; \phi_{u,+}$ and $H_d \; \equiv \; (c\phi_{d,+} + s\phi_{d,-})$ that survive whereas the other pair becomes superheavy and decouple from the low energy Lagrangian. The effective coupling at the MSSM level is then given by:

$$\begin{aligned} W \;&=\; h_e L_e H_d e^c + h_1 L_e H_d \ell^c_+ + h_2 L_e H_d m^c_- + h_3 L_+ H_d e^c \\ &+ h_4 L_- H_d e^c + h_5 L_+ H_d \ell^c_+ + h_6 L_- H_d m^c_- + h_7 L_- H_d \ell^c_+ \\ &+ f_1 L_e H_{u,+} N_e + f_2 L_e H_{u,+} N_+ + f_3 L_+ H_{u,+} N_e + f_4 L_+ H_{u,+} N_+ \\ &+ f_5 L_- H_{u,+} N_- \end{aligned} \tag{2.34}$$

Note that the $\mu - \tau$ symmetry is present in the Dirac neutrino mass matrix whereas it is not in the charged lepton sector as would be required to .

We show below that it is possible to have a high scale supersymmetric theory which would lead to real Dirac Yukawa couplings (f_i) if we require the high scale theory to be left-right symmetric. To show how this comes about, consider the gauge group to be $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons assigned to left and right-handed doublets as usual [25] i.e. $Q(2, 1, 1/3)$, $Q^c(1, 2, -1/3)$; $L(2, 1, -1)$ and $L^c(1, 2, +1)$; Higgs fields $\Phi(2, 2, 0)$; $\chi(2, 1, +1)$; $\bar{\chi}(2, 1, -1)$; $\chi^c(1, 2, -1)$ and $\bar{\chi}_c(1, 2, -1)$. The new point specific to our model is that we have two sets of the Higgs fields with the above quantum numbers, one even and the other odd under the $\mu - \tau$ $S2$ permutation symmetry i.e. $\Phi_\pm$, $\chi_\pm$, $\bar{\chi}_\pm$, $\chi^c_\pm$ and $\bar{\chi}^c_\pm$ (plus for fields even under $S2$ and $-$ for fields odd under $S2$.) Furthermore, we will impose the parity symmetry under which $Q \leftrightarrow Q^{c*}$, $L \leftrightarrow L^{c*}$, $(\chi, \bar{\chi} \leftrightarrow \chi^{c*}, \bar{\chi^{c*}})$, $\Phi \leftrightarrow \Phi^\dagger$.

The Yukawa couplings of this theory invariant under the gauge group as well as parity are given by the superpotential:

$$W = h_{11}L_e^T\Phi_+L_e^c + h_{++}L_+^T\Phi_+L_+^c\ h_{--}L_-^T\Phi_+L_-^c\ h_{e+}L_e^T\Phi_+L_+^c + h_{e+}^*L_+^T\Phi_+L_e^c \tag{2.35}$$
$$+ h_{e-}L_e^T\Phi_-L_-^c + h_{e-}^*L_-^T\Phi_-L_e^c + h_{+-}L_+^T\Phi_-L_-^c + h_{+-}^*L_-^T\Phi_-L^c$$

where h_{11}, h_{++}, h_{--} are real.

The Higgs sector of the low energy superpotential is determined from this theory after left-right gauge group is broken down to the standard model gauge group by the VEV's of χ^c. The phenomenon of doublet-doublet spitting leaves only two Higgs doublets out of the four in $\Phi_\pm$ and is determined by a generic superpotential of type

$$W_{DD} = \sum_{i,j,k} \lambda_{ijk}\chi_i\Phi_j\chi_k^c + \lambda'_{ijk}\bar{\chi}_i\phi_j\bar{\chi}_k^c + M_1(\chi_\pm\bar{\chi}_\pm + \chi_\pm^c\bar{\chi^c}_\pm) \tag{2.36}$$

where i, j, k go over $+$ and $-$ for even and odd and only even terms are allowed by $\mu - \tau$ invariance e.g. $\lambda_{+++}, \lambda_{+--}, ...$ are nonzero. Now suppose that $<\chi_+^c>= 0$ but $<\chi_+^c>\neq 0$ and $<\bar{\chi^c}_\pm>\neq 0$. These VEVs break the left-right group to the standard model gauge group. It is then easy to see that below the $<\chi^c>$ scale, there are only one Higgs pair where $H_u = \phi_{u,+}$ and $H_d = \sum_{i=+,-,3,4} a_i\phi_{d,i}$. Here we have denoted the $\Phi \equiv (\phi_u, \phi_d)$ and $\phi_{d,3,4} = \chi_\pm$. The upshot of all these discussions is that the right-handed neutrino Yukawa couplings are $\mu - \tau$ even and therefore have the form:

$$Y_\nu = \begin{pmatrix} h_{11} & h_{e+} & 0 \\ h_{e+}^* & h_{++} & 0 \\ 0 & 0 & h_{--} \end{pmatrix} \tag{2.37}$$

It is easy to see that redefining the fields appropriately, we can make Y_ν real. So the only source of complex phase in this model is in the right-handed neutrino mass matrix, which in this model are generated by higher dimensional couplings of the form $L^c L^c \bar{\chi}^c \bar{\chi}^c$ as we discuss now.

The most general nonrenormalizable interactions that can give rise to right-handed neutrino masses are of the form:

$$\begin{aligned} W_{NR} &= \frac{1}{M}[(L^c_e \bar{\chi}^c{}_+)^2 + L^c_e \bar{\chi}^c{}_-)^2 + (L^c_+ \bar{\chi}^c{}_+)^2 \\ & (L^c_- \bar{\chi}^c{}_-)^2 + (L^c_- \bar{\chi}^c{}_+)^2 \ + (L^c_+ \bar{\chi}^c{}_-)^2 \\ & (L^c_+ \bar{\chi}^c{}_-)(L^c_- \bar{\chi}^c{}_+) \end{aligned} \tag{2.38}$$

Note that since both $\bar{\chi}^c{}_\pm$ acquire vevs, the last term in the above expression will give rise to $\mu - \tau$ breaking in the right-handed neutrino sector while preserving it in the Y_ν. The associated couplings in the above equations are in general complex. This leads to a realistic three generation model with approximate $\mu - \tau$ symmetry as analyzed in the previous sections.

In summary, we have studied the implications for leptogenesis in models where neutrino masses arise from the type I seesaw mechanism and where the near maximal atmospheric mixing angle owes its origin to an approximate $\mu - \tau$ symmetry. We derive a relation of the form $\epsilon_l = (a\Delta m^2_{\text{solar}} + b\Delta m^2_{\text{atm}}\theta^2_{13})$ for the case of three right-handed neutrinos, which directly connects the neutrino oscillation parameters with the origin of matter. We also show that if θ_{13} is very small or zero, only the LMA solution to the solar neutrino puzzle would provide an explanation of the origin of matter within this framework. Finally for the case of two right-handed neutrinos with approximate $\mu - \tau$ symmetry, we predict values for θ_{13} in the range $0.1 - 0.15$ for specific choices of the the high energy phase between $\pi/4$ and $\pi/3$.

Chapter 3

Grand Unification of $\mu - \tau$ Symmetry

3.1 Overview

Observation of nonzero neutrino masses and determination of two of their three mixing parameters by experiments have raised the hope that neutrinos may provide a clue to flavor structure among quarks. In order to make progress in this direction however, one needs knowledge of the detailed nature of the quark-lepton connection e.g. whether there is an energy scale where quarks and leptons are unified into one matter (or grand unification of matter). While there are similarities between quarks and leptons that make such an unification plausible, there are also many differences between them which may a priori point the other way: for instance, the mixing pattern among quarks is very different from that among leptons and the neutrino mass matrices in the flavor basis exhibit symmetries for which there apparently is no trace among quarks. Two examples of such apparent lepton-exclusive symmetries are : (a) discrete $\mu - \tau$ symmetry [29, 30] of the neutrino mass matrix in the flavor basis indicated by maximal atmospheric mixing angle and small θ_{13} and (b) continuous $L_e - L_\mu - L_\tau$ [48] symmetry, which will be indicated if the mass hierarchy among neutrinos is inverted.

If neutrinos are Majorana fermions, they are likely to acquire masses from very different mechanisms e.g. one of the various seesaw mechanisms which involve completely independent flavor structure (say for example from right handed neutrinos) than quarks. The apparent disparate pattern for quark and leptons mixings then need not argue against eventual quark-lepton unification. In fact there are now many grand unification models (where quarks and leptons are unified at short distances) where small quark mixings and large lepton mixings along with all their masses can be understood with very few assumptions in a seesaw framework [38].

In this chapter we address the question as to whether there could be an apparently pure leptonic symmetry such as $\mu - \tau$ symmetry in the neutrino mass matrix in the flavor basis (i.e. the basis where charged leptons are mass eigenstates), which is part of a general family symmetry within a quark-lepton unified framework such as a grand unified model. We particularly focus on this symmetry since there appears to be some hint in favor of this from the present mixing data. In the exact symmetry limit, the mixing parameter $\theta_{13} = 0$ [29] and breaking of the symmetry not only implies a small nonzero value for θ_{13} but also leads to a correlation between θ_{13} with $\theta_{23} - \pi/4$, which can be used to test for this idea [30]. This question has been discussed at a phenomenological level in several recent papers [49] but to the best of our knowledge no full-fledged gauge model has been constructed. Indeed most gauge models for $\nu_\mu - \nu_\tau$ symmetry discussed in the literature treat leptons separately from quarks [40].

One simple way to have quark flavor structure completely separated from that of leptons and yet have quark-lepton unification is to use the double seesaw [41] framework where neutrino flavor texture from "hidden sector" singlet fermions (e.g. SO(10) singlets) which are completely unrelated to quarks (for examples of such models, see [50, 51]). One can then have any pure "leptonic" symmetry on the hidden singlets without at the same time interfering with quark flavor texture. A necessary feature of such models is that one must introduce new fermions into the model. A question therefore remains as to whether one could do this without expanding the matter sector. In this paper, we propose such an approach without introducing new fermions within a realistic $SU(5)$ GUT framework that unifies quarks and leptons. We demand the full theory prior to symmetry breaking to obey a symmetry between the second and third generation (or a generalized version of $\mu - \tau$ symmetry). The neutrino masses are assumed to arise from a type II [21] mechanism, which disentangles the neutrino flavor structure from the quark flavor structure. The quark mass matrices are however constrained by the $\mu - \tau$ symmetry. The quark mixing angles then introduce departures from exact $\mu - \tau$ symmetry results and lead to nonzero θ_{13} as well as departures from maximal atmospheric mixing.

3.2 A SUSY $SU(5)$ Model with $\mu - \tau$ Symmetry

The model consists of a minimal set of Higgs bosons which are anyway required to reconcile the charged fermion masses in the minimal $SU(5)$ model. We find that the requirement of $\mu - \tau$ symmetry for neutrinos can be imposed on the model without contradicting observed charged fermion masses and mixings. As noted, the model predicts a nonzero value for θ_{13} correlated with the departure of θ_{23} from its maximal value.

As in the usual $SU(5)$ model, matter fields are assigned to $\bar{\mathbf{5}} \equiv F_\alpha$ and $\mathbf{10} \equiv T_\alpha$ (with $\alpha = 1, 2, 3$ denotes the generation index). We choose the Higgs fields to belong to the multiplets $\mathbf{24}$ (denoted by Φ and used to break the $SU(5)$ symmetry down to the standard model); $\mathbf{5} \oplus \bar{\mathbf{5}}$ (denoted by $h + \bar{h}$) and $\mathbf{45} \oplus \bar{\mathbf{45}}$ (denoted by $H + \bar{H}$) used to give masses to fermions) and $\mathbf{15} \oplus \bar{\mathbf{15}}$ (denoted by $S + \bar{S}$) to give masses to neutrinos via the type II seesaw mechanism [21].

The matter and Higgs fields transform under the $\mu - \tau$ discrete flavor symmetry as follows:

$$\begin{aligned} F_\mu &\leftrightarrow F_\tau \\ (h, \bar{h}) &\leftrightarrow (h, \bar{h}) \\ (H, \bar{H}) &\leftrightarrow (-H, -\bar{H}) \end{aligned} \tag{3.1}$$

and all other fields are singlets under this transformation. In this model, the matter part of the superpotential can be written as

$$W = Y_{15} F F S + Y_5 T T h + Y_{\bar{5}} T F \bar{h} + Y_{45} T F H. \tag{3.2}$$

After the electro-weak symmetry breaking, the mass matrices for the standard model fermions are given by

$$M_\nu = Y_{15}\langle S\rangle = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix} \tag{3.3}$$

$$M_u = Y_5\langle h\rangle = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix} \tag{3.4}$$

$$M_d = Y_{\bar{5}}\langle \bar{h}\rangle + Y_{45}\langle H\rangle = \begin{pmatrix} A_1 & B_1 & C_1 \\ E_1 & D_1 & F_1 \\ E_1 & D_1 & F_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ E_2 & D_2 & F_2 \\ -E_2 & -D_2 & -F_2 \end{pmatrix} \tag{3.5}$$

$$M_e = Y_{\bar{5}}^T\langle \bar{h}\rangle - 3Y_{45}^T\langle H\rangle = \begin{pmatrix} A_1 & E_1 & E_1 \\ B_1 & D_1 & D_1 \\ C_1 & F_1 & F_1 \end{pmatrix} - 3\begin{pmatrix} 0 & E_2 & -E_2 \\ 0 & D_2 & -D_2 \\ 0 & F_2 & -F_2 \end{pmatrix} \tag{3.6}$$

where the various parameters characterising the mass matrices are given in terms of the Yukawa couplings and vacuum expectation values of fields as follows: $\langle S\rangle, \langle h\rangle, \langle \bar{h}\rangle, \langle H\rangle$ are VEVs of $S, h, \bar{h}, H$ respectively.

The mass matrices depend on nineteen parameters if we ignore CP phases and there are seventeen experimental inputs (6 quark masses, 3 charged lepton masses, two neutrino mass difference squares plus five mixing angles values and an upper limit on θ_{13}). For the sake of comparison, we note that if we generated neutrino masses in the standard model using a Higgs triplet field, there would be 18 parameters in the absence of CP violation (9 from the quark sector, 3 from the charged lepton mass matrix and six from the neutrino sector). When one embeds the standard model into a GUT $SU(5)$, to be realistic, one needs to introduce **45** Higgs and its associated Yukawa couplings. In this case, the total number of parameters in the Yukawa sector is 30. In our model the requirement of $\mu-\tau$ symmetry has first led to a reduction in the total number by eleven and furthermore grand unification has strongly correlated the down quark and charged lepton mass matrix, as expected. It is therefore not obvious that the model will be consistent with known data on fermion masses.

To see if the model is phenomenologically acceptable, we first fit the masses of the charged leptons and down type quarks using the mass values of leptons and quarks at GUT scale given in the Table 3.1 [52]. These values are obtained by two-loop RGEs running from Z boson mass scale to the GUT scale. The initial values are from the experimental data.

The values of parameters in the model are found by scanning the whole parameter space under the constraint that we satisfy the current experiment requirements of θ_{13} and θ_{23}. Note that since in this model, neutrino mass matrix in the flavor basis is $\mu-\tau$ symmetric, it is diagonalized by the matrix:

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta_\nu & \sqrt{2}\sin\theta_\nu & 0 \\ -\sin\theta_\nu & \cos\theta_\nu & 1 \\ -\sin\theta_\nu & \cos\theta_\nu & -1 \end{pmatrix}, \tag{3.7}$$

where θ_ν is the solar mixing angle. The deviations of θ_{13} and θ_{23} from 0 and $\frac{\pi}{4}$ respectively should come from left-handed charged leptons mixing matrix. Since these deviations have upper bounds, this puts an nontrivial constraint on the charged

input observable	$\tan\beta = 10$
m_u (MeV)	$0.7238^{+0.1365}_{-0.1467}$
m_c (MeV)	$210.3273^{+19.0036}_{-21.2264}$
m_t (GeV)	$82.4333^{+30.2676}_{-14.7686}$
m_d (MeV)	$1.5036^{+0.4235}_{-0.2304}$
m_s (MeV)	$29.9454^{+4.3001}_{-4.5444}$
m_b (GeV)	$1.0636^{+0.1414}_{-0.0865}$
m_e (MeV)	$0.3585^{+0.0003}_{-0.0003}$
m_μ (MeV)	$75.6715^{+0.0578}_{-0.0501}$
m_τ (GeV)	$1.2922^{+0.0013}_{-0.0012}$

Table 3.1: The masses of charged fermions at the GUT scale.

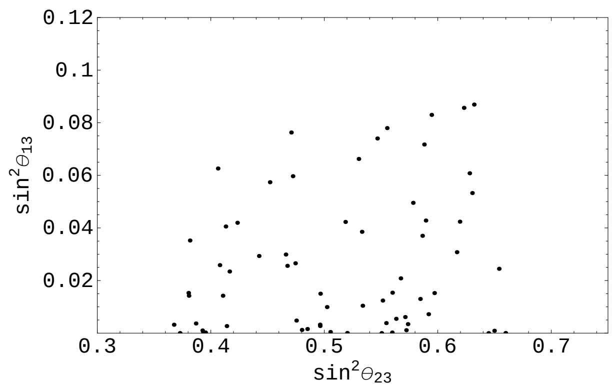

Figure 3.1: Scatter plot in the $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ plane.

lepton mass matrix of the model; but since the charged lepton mass matrix is already constrained by $\mu-\tau$ symmetry, it is nontrivial to get all masses and mixings to fit. It turns out that the fitting for the masses of leptons and quarks does not provide any bound on θ_{23}, however it gives quite stringent bound on θ_{13}. Using the relation $U_{MNS} = U_l^\dagger U_\nu$, one can write $\sin\theta_{13}$ and $\tan\theta_{23}$ as

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}|U_{l21} - U_{l31}| \tag{3.8}$$

$$\tan\theta_{23} = |\frac{U_{l22} - U_{l32}}{U_{l23} - U_{l33}}| \tag{3.9}$$

The 3σ experimental bounds of θ_{13} and θ_{23} are $0.34 \leq \sin^2\theta_{23} \leq 0.68$ and $\sin^2\theta_{13} \leq 0.051$ [14].

The scatter plot in Fig. 3.1 gives $\sin^2\theta_{13}$ as a function of $\sin^2\theta_{23}$ allowing for 3 σ uncertainty in all masses except m_e (chosen to be $0.3-0.4$ MeV), m_μ (chosen to be $73-76$ MeV) and m_d left free and θ_{23} within 3 σ.

Here, we give two typical fitting points for our model:

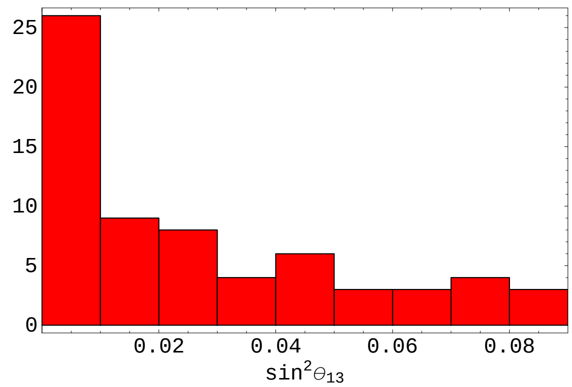

Figure 3.2: Value distribution of $\sin^2\theta_{13}$. 67 percent of fitting points have $\sin^2\theta_{13} \leq 0.03$ and 80 percent have $\sin^2\theta_{13} \leq 0.05$.

(i) Case 1:

$$m_d = 0.355117 \text{ MeV} \quad m_s = 34.0438 \text{ MeV} \quad m_b = 985.857 \text{ MeV} \tag{3.10}$$

$$m_e = 0.356047 \text{ MeV} \quad m_\mu = 75.1597 \text{ MeV} \quad m_\tau = 1336.14 \text{ MeV} \tag{3.11}$$

$$U_l = \begin{pmatrix} 0.999327 & 0.036688 & 0.0000316411 \\ 0.0366849 & -0.999231 & -0.0138381 \\ 0.000476075 & -0.01383 & 0.999904 \end{pmatrix} \tag{3.12}$$

For this case, we predict the following values for the neutrino mixing parameters θ_{13} and θ_{23}:

$$\theta_{13} \simeq 0.026 \tag{3.13}$$

$$\theta_{23} \simeq 44.3^\circ \tag{3.14}$$

(ii) Case 2:

$$m_d = 0.336552 \text{ MeV} \quad m_s = 38.4364 \text{ MeV} \quad m_b = 926.78 \text{ MeV} \tag{3.15}$$

$$m_e = 0.381779 \text{ MeV} \quad m_\mu = 73.112 \text{ MeV} \quad m_\tau = 1288.52 \text{ MeV} \tag{3.16}$$

$$U_l = \begin{pmatrix} 0.959961 & 0.280133 & 0.000326329 \\ 0.279872 & -0.959014 & -0.0443148 \\ 0.0121011 & -0.0426319 & 0.999018 \end{pmatrix} \tag{3.17}$$

giving us

$$\theta_{13} \simeq 0.19788 \tag{3.18}$$

$$\theta_{23} \simeq 41.2^\circ \tag{3.19}$$

We therefore note that the value of the most probable value for θ_{13} is in the range from $0.02 - 0.19$ with (as indicated in Fig. 3.2) values below 0.1 being much more probable.

Note that mass m_d in both cases has almost same magnitude as m_e and is smaller than the central value at the GUT scale by about $\sim$ 1MeV. The reason for this is that H is $\mu - \tau$ odd, leading to zero entries in the M_e, M_d. Note that in this model, we also have additional threshold correction from the exchange of the gauginos, which make larger contribution to quarks relative to the charged leptons of the corresponding generation due to strong coupling of the gluinos. In particular, the gluino contribution to the tree level masses of the quarks can be significant if the assumption of proportionality between the A-terms and the Yukawa couplings

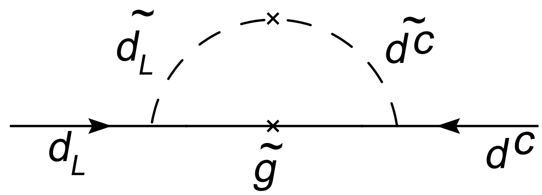

Figure 3.3: One-loop SUSY threshold correction to d quark mass due to gluino-squark exchange.

is abandoned. Fig. 3.3 gives a typical Feynman diagram contributing to the quark masses [54]. The generic contribution to the (i,j) element of the down quark mass matrix is given by:

$$\delta m_{d,ij} \simeq \frac{2\alpha_s}{3\pi} \frac{M_{\tilde{g}}}{m_{\tilde{q}}^2} (m_{d,ij}^0 \mu \tan\beta + A_{ij}^{(d)} m_0) \tag{3.20}$$

Including this radiative correction only in the 11 element of the down quark mass matrix, one can get the down quark mass to be in agreement with observations. We also note that the process of fitting the charged lepton and down quark masses gives a definite rotation matrix that diagonalizes the down quark mass matrix and contributes to the V_{CKM}. We then appropriately choose the parameters in the symmetric up-quark mass matrix so that we get the correct V_{CKM}.

3.3 Gauge Coupling Unification

This type-II seesaw requires that we have a medium scale for the mass of the SM triplet Higgs which is present in **15**-Higgs i.e. $M_T \sim 10^{14}$GeV; this is satisfied if we tune the coupling λ of $\lambda \Phi S \bar{S}$ to $\sim 10^{-2}$ or so since $M_T \sim \lambda v_U$. Once Φ get

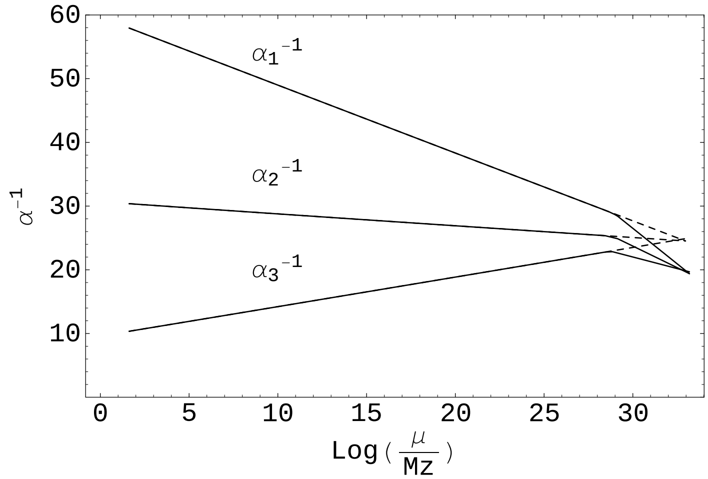

Figure 3.4: Unification of the gauge couplings at two-loop level for central values of low-energy oberservables. We find $M_{GUT} = 2.36 \times 10^{16}$GeV. The dashed lines in the figure show the pure MSSM running.

VEV and breaks $SU(5)$ to Standard Model, it also can induce the mass splitting of multiplets of $S, \bar{S}$. This will affect the unification of coupling. We display the effect of these mass splittings to the gauge coupling running as a threshold correction, in Fig. 3.4 and show that the unification of couplings is maintained and we get a slight increment in the value of $M_U \simeq 2.36 \times 10^{16}$ GeV.

3.4 **45** vrs its Higher Dimensional Equivalent

We also like to comment that a more economical possibility is to consider a model that uses a high dimension operator involving with Φ instead of the H. The matter part of the superpotenial in this case is given by:

$$W = Y_{15} FFS + Y_5 TTh + Y_{\bar{5}} FT\bar{h} + \frac{1}{M_P} Y_{24} FT\Phi\bar{h}, \tag{3.21}$$

where M_P is Planck scale and H_{24} is the $SU(5)$ adjoint representation used to break $SU(5)$ to $SU(3)\times SU(2)\times U(1)$. $M_P \sim 10^{19}$GeV,VEV of H is $\sim 10^{16}$GeV and VEV of $\bar{h}$ is $\sim 10^2$GeV ,thus the overall scale of the contribution of this higher dimensional operator to fermion mass matrices $\sim$ 100MeV. We have tried a fitting of data for this model and find it to be unacceptable, since it gives very large $\sin^2\theta_{23} \sim 0.76 - 0.8$ which is around 4-5 σ.

In summary, we have discussed the grand unification of apparently pure leptonic symmetries such as $\mu - \tau$ symmetry into the quark-lepton unifying supersymmetric SU(5) model for quarks and leptons and studied its implications for neutrino mixing angles. We find that it is possible to have a completely viable $SU(5)$ model of this type. In this model the neutrino masses arise from a triplet VEV induced type II seesaw mechanism. The presence of quark lepton unification leads to small deviations from maximal atmospheric mixing angle and vanishing θ_{13} implied in the exact symmetry limit.

Chapter 4

$S3$ Flavor Fymmetry and Tri-bimaximal Mixing

4.1 Overview

$\mu-\tau$ flavor symmetry has been proposed to understand lepton mixing pattern with near maximal atmospheric mixing angle and vanishing θ_{13}. Even though there is no such apparent "$\mu-\tau$" symmetry among quarks and charged leptons, in the previous chapter, we have shown that unified description of quarks and leptons is possible within $SU(5)$ GUT by extending neutrino $\mu-\tau$ symmetry to the permutation family symmetry between the second and third generations. A question raised by this is whether there are higher underlying symmetries of leptons.

A hint for a higher symmetry may be coming from the observation that the solar angle in the PMNS mixing matrix satisfies the relation $\sin^2\theta_{\text{solar}} \simeq \frac{1}{3}$. The resulting PMNS matrix has the simple form [53]:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{4.1}$$

and is called a tri-bimaximal mixing. The true nature of the symmetry responsible for this pattern is not clear, although there are many interesting suggestions [55, 56, 57].

In this chapter, we explore the possibility that the relevant symmetry may be the permutation symmetry $S3$ of three lepton generations. We show that a softly broken $S3$ symmetry for leptons can lead to tri-bimaximal mixing pattern if we use a combination of type I and type II seesaw mechanism to understand the smallness of neutrino mass. This approach appears to be different from previous attempts at building models for tri-bimaximal mixing [55, 56, 57].

We proceed in two steps: we first show how in a basis where charged leptons are diagonal, one can derive the mixing pattern in Eq. (4.1) using softly broken $S3$ symmetry under certain assumptions. We then show how this the $S3$ symmetry combined with $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$ symmetry can lead to a diagonal charged lepton mass matrix. We then extrapolate the neutrino mass matrix from the seesaw scale to the weak scale and obtain constraints on the mass ratios m_1/m_3 and m_2/m_3 so that the mixing angles match the observations. We obtain a prediction for θ_{13}, which turns out to be extremely small (~ 0.004). We further show that if the neutrino masses are quasi-degenerate and have the same CP property (i.e. are all positive), then the radiative corrections in the extrapolation to the weak scale are so large that the solar mixing angle is in disagreement with observations. This implies that in supersymmetric theories with large tanβ, seesaw scale tri-bimaximal mixing and degenerate neutrinos are mutually exclusive.

4.2 An $S3$ Model

We start with the Majorana neutrino mass matrix whose diagonalization at the seesaw scale leads to the tri-bimaximal mixing matrix:

$$M_\nu = \begin{pmatrix} a & b & b \\ b & a-c & b+c \\ b & b+c & a-c \end{pmatrix} \tag{4.2}$$

Diagonalizing this matrix leads to the U of Eq. (4.1) and the neutrino masses: $m_1 = a - b; m_2 = a + 2b$ and $m_3 = a - b - 2c$. Clearly if $|a| \simeq |b| \ll |c|$, we get a normal hierarchy for masses.

We now show that the mass matrix in Eq. (4.2) can be obtained from a softly broken $S3$ symmetry in the neutrino sector. For this purpose, we assign the three lepton doublets of the standard model (L_e, L_μ, L_τ) to transform into each other under permutation. The three right handed neutrinos $(N_{i=1,2,3})$ transform under three permutation and two cyclic operations of $S3$ as:

$$\begin{aligned} e \leftrightarrow \mu :&\ \ N_1 \leftrightarrow -N_1;\ N_2 \leftrightarrow -N_3 \\ \mu \leftrightarrow \tau :&\ \ N_2 \leftrightarrow -N_2;\ N_1 \leftrightarrow -N_3 \\ \tau \leftrightarrow e :&\ \ N_3 \leftrightarrow -N_3;\ N_1 \leftrightarrow -N_2 \\ e \to \mu \to \tau :&\ \ N_1 \to N_2;\ N_2 \to N_3;\ N_3 \to N_1 \\ e \to \tau \to \mu :&\ \ N_1 \to N_3;\ N_2 \to N_1;\ N_3 \to N_2 \end{aligned} \tag{4.3}$$

In order to obtain the neutrino mass matrix, we assume that there is a standard model triplet Higgs field Δ with $Y = 2$ which is $S3$ singlet that couples to the two lepton doublets and an $S3$ singlet Higgs doublet field H that gives the Dirac mass for the neutrinos. The triplet VEV can be made small and of the desired order if the mass of the triplet Higgs field is around 10^{14} GeV or so [38].

The first point to note is that the most general $S3$ invariant coupling of the triplet i.e. $f_{ab}L_aL_b\Delta$ is given by the coupling matrix:

$$f = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix} \tag{4.4}$$

For the Dirac neutrino coupling we choose to keep the following $S3$ invariant term:

$$L_D = h_\nu[\overline{N_1}H(L_e - L_\mu) + \overline{N_2}H(L_\mu - L_\tau) + \overline{N_3}H(L_\tau - L_e)] + h.c. \tag{4.5}$$

One other $S3$ invariant coupling is set to zero. This is natural in a supersymmetric theory due to the nonrenormalization theorem. We then get for the Dirac mass matrix for neutrinos

$$M_D = \begin{pmatrix} d & -d & 0 \\ 0 & d & -d \\ -d & 0 & d \end{pmatrix}. \tag{4.6}$$

where $d = h_\nu\langle H\rangle$. If we now assume the following hierarchy among the right handed neutrinos, i.e. $M_{N_{1,3}} \gg M_{N_2}$ so that a single right handed neutrino dominates the type I contribution to the seesaw formula [59], then in the strict decoupling limit, using the mixed type I+II seesaw formula:

$$M_\nu = M_0 - M_D^T M_R^{-1} M_D, \tag{4.7}$$

we get the desired form for the neutrino Majorana mass matrix (Eq. (4.2)) which leads to tri-bimaximal mixing. Note that the right handed neutrino masses being dimension three operators break the $S3$ softly.

In this discussion we have assumed that the charged lepton mass matrix is diagonal. A major challenge for any model for neutrino mixings is to have a consistent picture for both the charged lepton and neutrino sectors simultaneously so that the combination $U_\ell^\dagger U_\nu$ equals the observed PMNS matrix. Since in our case, the neutrino sector by itself gives the tri-bimaximal form for the PMNS matrix, the charged lepton sector should be diagonal or nearly so. We will now show that we can obtain a diagonal charged lepton mass matrix in a simple way using the $S3$ symmetry, provided we choose only one of two allowed $S3$ invariant Yukawa coupling terms.

In order to achieve this, we assume that there are three standard model Higgs doublets (H_e, H_μ, H_τ) transforming like the lepton doublets above under $S3$. We also assume that the right handed charged leptons (e_R, μ_R, τ_R) transform under $S3$ same way. We then assume a product of discrete symmetries $Z_{2e} \times Z_{2\mu} \times Z_{2\tau}$ under which all fields except the following are even: (e_R, H_e) odd under only Z_{2e} and similarly (μ_R, H_μ) are odd only under $Z_{2\mu}$ and (τ_R, H_τ) odd under $Z_{2\tau}$. The Yukawa couplings invariant under this are:

$$\begin{aligned} L'_Y \;=\; & h_e(\bar{L}_e H_e e_R + \bar{L}_\mu H_\mu \mu_R + \bar{L}_\tau H_\tau \tau_R) \;+\; h'_e(\bar{L}_e H_\mu \mu_R + \bar{L}_\mu H_e e_R \\ & + \bar{L}_\mu H_\tau \tau_R + \bar{L}_\tau H_\mu \mu_R + \bar{L}_e H_\tau \tau_R + \bar{L}_\tau H_e e_R) + h.c. \end{aligned} \tag{4.8}$$

By softly breaking the global $S3$ symmetry in the Higgs potential for the $H_{e,\mu,\tau}$, we can get $\langle H_e \rangle \ll \langle H_\mu \rangle \ll \langle H_\tau \rangle$ which allows us to obtain a realistic diagonal charged lepton mass matrix if we assume $h'_e = 0$. This model then gives us a tri-bimaximal neutrino mixing at the seesaw scale.

4.3 Radiative Stability of the Texture

In order to compare this model with observations, we need to extrapolate the seesaw scale neutrino mass matrix in Eq. (4.2) down to the weak scale [58] and then calculate the masses and mixing angles. This extrapolation depends on the mass hierarchy of the neutrinos. So comparing with observations, we can put limits on the mass hierarchy at low scale. From the expressions for the neutrino masses derived after Eq. (4.2), one might think that degenerate masses are compatible with tri-bimaximal pattern since there are three parameters and three masses to be fitted. However, in supersymmetric models, mixing angles can receive substantial contributions from RGE effects (specially for large tanβ) and will in general lead to distortion of the mixing angles away from the tri-bimaximal values. For the specific case of tan$\beta = 50$ we calculate the radiative corrections to the solar mixing angle θ_{12} in fig. 4.1. We plot $\sin^2\theta_{12}$ against m_2/m_3 with the input constraint being that $\Delta m^2_{\rm solar}/\Delta m^2_{\rm atm}$ is within 3 σ of its present value i.e. $0.024 \leq \Delta m^2_{\rm solar}/\Delta m^2_{\rm atm} \leq 0.060$ [14]. We see that for $m_2/m_3 > 0.3$ or so, the solar mixing angle goes outside the observed range and the agreement gets worse for larger values of this mass ratio which corresponds to quasi-degenerate neutrino spectrum. This leads us to conclude that tri-bimaximal mixing at the seesaw scale is incompatible with quasi-degenerate

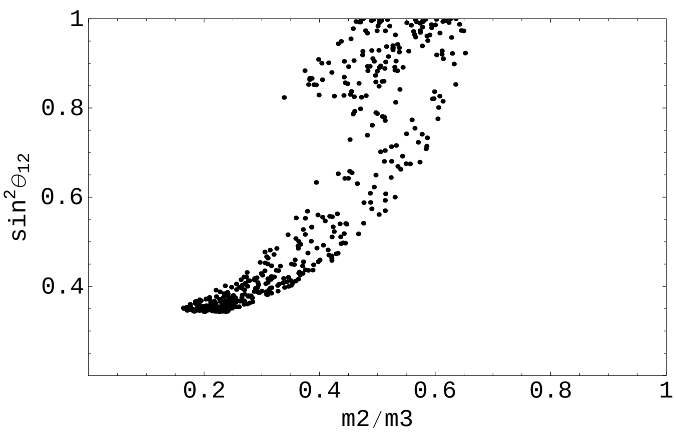

Figure 4.1: $\sin^2\theta_{12}$ at the weak scale for the case of quasi-degenerate neutrinos. Note that the higher the ratio m_2/m_3, the more degenerate the neutrinos are and further off the prediction for $\sin^2\theta_{12}$ is from the observed value.

neutrinos for large values of $\tan\beta$.

For the same value of $\tan\beta$, we show in fig. 4.2 the allowed ranges for the neutrino mass ratios for the case of normal hierarchy and in fig. 4.3, the prediction for θ_{13} for this model. In these figures, we have used the above 3σ experimental bounds for $\Delta m^2_{\rm solar}/\Delta m^2_{\rm atm}$ and also 3 σ bounds for $0.23 \leq \sin\theta^2_{12} \leq 0.38$ and $0.34 \leq \sin\theta^2_{23} \leq 0.68$ [14]. We find that the prediction for $\theta_{13} \sim 0.004$ which is much too small to be observable in near future. This is because the low energy theory in the absence of radiative corrections is $\mu - \tau$ symmetric. Clearly observation of θ_{13} higher than this value will rule out this model and indeed any simple model for tri-bimaximal mixing at the seesaw scale for the case of normal mass hierarchy.

In conclusion, in this chapter, we have presented a new way to obtain the tri-bimaximal mixing pattern for neutrinos by embedding $\mu - \tau$ symmetry of the

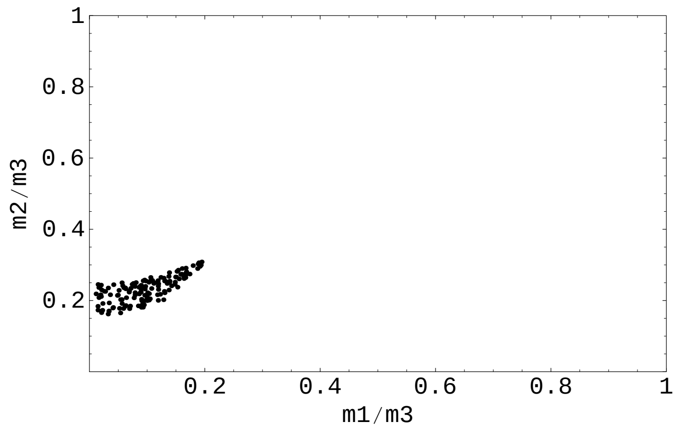

Figure 4.2: Allowed ranges of mass ratios at the weak scale for normal hierarchy case.

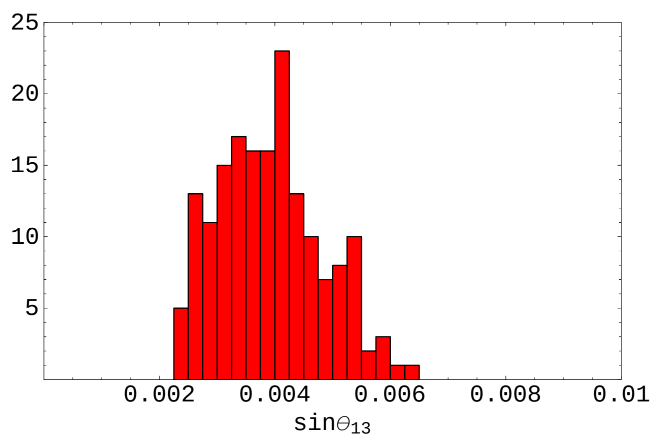

Figure 4.3: Distribution of $\sin\theta_{13}$ value.

neutrino mass matrix into a softly broken $S3$ permutation symmetry for leptons and using a simple combination of the type I and type II seesaw formulae along with the dominance of a single right handed neutrino [59]. We also find that tri-bimaximal mixing at the seesaw scale is incompatible with degenerate neutrino spectrum due to large radiative correction effects for large $\tan\beta$.

Chapter 5

Connecting Leptogenesis to CP Violation in Neutrino Mixings in a Tri-bimaximal Mixing model

5.1 Overview

In the Chapter 3, we derived two simple formulas connecting lepton number asymmetry and low scale neutrino oscillation data with the basic assumption that type I seesaw generates light neutrino mass and slightly broken $\mu - \tau$ symmetry at the high scale. The flavor symmetry reduces the seesaw parameters in a natural way and makes it possible to have such direct connection.

A further question of this connection is whether CP-violating phases in neutrino mixing that can be probed in long baseline as well as in neutrinoless double beta decay experiments are the ones that are responsible for the matter-anti-matter asymmetry. It turns out that in generic seesaw models there is no a priori connection between them and it is hoped that in a true theory of neutrino masses and mixing, such a connection may exist. By a direct connection, we mean the phase responsible for lepton asymmetry of the Universe is the same one that appears as either a Dirac or one of the two Majorana phases in neutrino mixings. The non-triviality of this problem stems from two facts: (i) in generic seesaw models, lepton asymmetry ϵ_ℓ depends only a subset of the phases of Dirac mass matrix M_D whereas low energy phases in the neutrino mass matrix involves all of them; and (ii) the seesaw formula "scrambles" up the phases due to multiplication of matrices so that any direct connection between low and high energy phases, if they exist at all becomes difficult to discern.

In this chapter, we show that the $S3$ model proposed in Chapter 4 for tri-bimaximal neutrino mixing, the structure of the neutrino mass matrix is so constrained by symmetry that a direct connection between the leptogenesis phase and neutrino mixing phases emerges. Thus within the context of this model, a measurement of the neutrino CP phases would provide a direct understanding of the origin of matter. In this model, the key flavor symmetry leading to tri-bimaximal mixing is the permutation symmetry of three lepton families. The resulting neutrino mass matrix is characterized by only three complex parameters, whose absolute values are constrained by already existing observations. We find that (i) in the exact tri-bimaximal limit, when there is no Dirac phase, one of the two Majorana phases is directly responsible for the lepton asymmetry of the Universe; (ii) even after we include small departures from the tri-bimaximal limit, the direct connection remains – there are then two contributions to ϵ_ℓ, one being proportional to the Dirac phase and the other to one of the two Majorana phases. This direct connection is possible due to the simple form of M_D dictated by the $S3$ symmetry of the model and the assumptions that in case (i) only one and in case (ii) only two right-handed neutrinos dominate the seesaw formula as well as the fact there is an $S3$ symmetric type II contribution to the neutrino masses in both cases.

This chapter is organized as follows: in section 2, we give a brief review the salient features of the $S3$ model for tri-bimaximal mixing and set up the notation; in section 3, we present a general discussion of leptogenesis in our model; in section 4, we calculate the baryon asymmetry in the exact tri-bimaximal mixing and establish the direct connection between one of the Majorana phases in the neutrino mixing and ϵ_ℓ; in section 5, we do the same for the case where we include deviations from tri-bimaximal limit and show the connection of ϵ_ℓ to the Dirac and the Majorana phases.

5.2 CP Violating Phases of the $S3$ Model

We start with the Majorana neutrino mass matrix whose diagonalization at the seesaw scale leads to the tri-bimaximal mixing matrix:

$$M_\nu = \begin{pmatrix} a' & b' & b' \\ b' & a'-c' & b'+c' \\ b' & b'+c' & a'-c' \end{pmatrix} \tag{5.1}$$

where the elements are chosen to be complex. Diagonalizing this matrix leads to the tri-bimaxial mixing pattern and the neutrino masses: $m_1 = a' - b'; m_2 = a' + 2b'$ and $m_3 = a' - b' - 2c'$. Clearly if $|a'| \simeq |b'| \ll |c'|$, we get a normal hierarchy for masses. It was pointed out in the Chapter 4 that the above Majorana neutrino mass matrix can be realized in a combined type I type II seesaw model with soft-broken $S3$ family symmetry for leptons. The type II contribution comes from an $S3$

invariant coupling $f_{\alpha\beta}L_\alpha L_\beta\Delta$,

$$f = \begin{pmatrix} f_a & f_b & f_b \\ f_b & f_a & f_b \\ f_b & f_b & f_a \end{pmatrix} \tag{5.2}$$

After the triplet Higgs field Δ gets VEV and decouples, its contribution to the light neutrino mass can written as

$$M_{II} = \begin{pmatrix} a' & b' & b' \\ b' & a' & b' \\ b' & b' & a' \end{pmatrix} \tag{5.3}$$

where $a' = \frac{v^2 \sin^2\beta\lambda}{M_T} f_a$ and $b' = \frac{v^2 \sin^2\beta\lambda}{M_T} f_b$. We denote M_T as the mass of the triplet Higgs and λ as the coupling constant between the triplet and doublets in the superpotential.

Coming to the type I contribution, the Dirac mass matrix for neutrinos comes from an $S3$ invariant Yukawa coupling of the form:

$$\mathcal{L}_D = h_\nu[\overline{N_1}H(L_e - L_\mu) + \overline{N_2}H(L_\mu - L_\tau) + \overline{N_3}H(L_\tau - L_e)] + h.c. \tag{5.4}$$

leading to

$$Y_\nu = \begin{pmatrix} h & -h & 0 \\ 0 & h & -h \\ -h & 0 & h \end{pmatrix}. \tag{5.5}$$

In the limit of $|M_{R1,R3}| \gg |M_{R2}|$, where a single right-handed neutrino dominates the type I contribution, the mixed type I+II seesaw formula

$$M_\nu = M_{II} - M_D^T M_R^{-1} M_D, \tag{5.6}$$

gives rise to the desired form for the neutrino Majorana mass matrix which leads to the tri-bimaximal mixing [62].

We can now do the phase counting in the model. When two of the above right-handed neutrinos decouple, there is only one Yukawa coupling. We can first redefine the phase of N_2 so that its mass is real and we then redefine all the lepton doublets by a common phase which now makes the Dirac Yukawa coupling h real. One cannot then do any more phase redefinitions and we are left with two phases in the neutrino mass matrix which in this basis reside in the entries a' and b' in Eq. (5.3). These two phases will appear as the Majorana phases in the low energy mass matrix as we show below.

As far as the charged lepton masses are concerned, the symmetry needs to be extended to $S3 \times (Z_2)^3$ to have a simple diagonal mass matrix and all their masses can be made real by separate independent phase redefinition of the right-handed charged leptons. No new phases enter the PMNS matrix.

Turning to the case where two of the right-handed neutrinos contribute to M_ν, there are three phases in the light neutrino mass matrix. This is because in this case there are two apriori complex right-handed neutrino masses and only one of them together with h can be made real by phase redefinition as in the first case. This leaves the phases of a' and b' and that of the second right-handed neutrino giving a total of three phases. This case represents a deviation from the tri-bimaximal mixing with the deviation being proportional to $|M_{R2}|/|M_{R3}|$. We will show in sec. 4 that the new phase in this case appears as the Dirac phase. Let us now proceed to discuss leptogenesis in both these cases. As noted, we choose f_a, f_b, M_{R3} to be complex and

h, M_{R2} to be real, and express them as $f_a = |f_a|e^{i\phi_a},\ f_b = |f_b|e^{i\phi_b}, M_{R3} = M_3 e^{-i\phi_3}$ and $M_{R2} = M_2$.

5.3 Leptogenesis in the Type II Seesaw Model

In this section, we present the calculation of lepton asymmetry in our model and show that for the parameter range of interest from neutrino mixing physics, one can explain the baryon asymmetry of the universe whose present value is given by the WMAP observations [63] to be

$$\frac{n_B}{n_\gamma} = 6.1 \pm 0.2 \times 10^{-10}. \tag{5.7}$$

Let us start by reminding ourselves of some well known facts about leptogenesis. In the type I seesaw scenario, lepton asymmetry is generated by the out-of-equilibrium decay of the right-handed neutrinos which participate in the seesaw mechanism to give neutrino masses and mixings. Most of the discussion of leptogenesis uses type I seesaw and there have been many papers [73] which have studied its connection to neutrino masses and mixings. In models with both type I [20] and type II seesaw [21], the presence of the triplet Higgs may also contribute to the lepton asymmetry in two ways: either the decay of one or more triplets [64] or the decay of right-handed neutrino with triplets running in the loop [65] [66]. Our model involves both type I and type II seesaw; however, it turns out that the first contribution (i.e. the one from triplet decay) is highly suppressed and only the lightest right-handed neutrino(sneutrino) decay is important, which we compute below.

The asymmetry from the decay of the right-handed neutrino N_i into a lepton(slepton) and a Higgs(Higgsino) is given by:

$$\varepsilon_i = \frac{\Gamma[N_i \to lH(\tilde{l}\tilde{H})] - \Gamma[N_i \to \bar{l}H^*(\tilde{\bar{l}}\tilde{H}^*)]}{\Gamma[N_i \to lH(\tilde{l}\tilde{H})] + \Gamma[N_i \to \bar{l}H^*(\tilde{\bar{l}}\tilde{H}^*)]}, \tag{5.8}$$

and we also have the sneutrino $\tilde{N}_i$ decay asymmetry, which we denote as $\tilde{\varepsilon}_i$. If one ignores the supersymmetry breaking effects, one has $\varepsilon_i = \tilde{\varepsilon}_i$.

In the basis where right-handed neutrinos mass matrix is diagonal, the decay asymmetry of right-handed neutrino from type I contribution is given by [44]

$$\varepsilon_i^I = -\frac{1}{8\pi}\frac{1}{[Y'_\nu Y'^\dagger_\nu]_{ii}}\sum_j \mathrm{Im}[\mathrm{Y}'_\nu \mathrm{Y}'^\dagger_\nu]^2_{\mathrm{ij}} F(\frac{M_j^2}{M_i^2}), \tag{5.9}$$

where $F(x) = \sqrt{x}(\frac{2}{x-1} + \ln[\frac{1+x}{x}])$ and for $x \gg 1$, $F(x) \simeq \frac{3}{\sqrt{x}}$.

The type II contribution has been calculated and is given in Ref. [65] [66] to be

$$\varepsilon_i^{II} = \frac{3}{8\pi}\frac{\mathrm{Im}[Y'_\nu f^* Y'^T_\nu \mu]_{ii}}{[Y'_\nu Y'^\dagger_\nu]_{ii} M_i}\ln(1 + \frac{M_i^2}{M_T^2}), \tag{5.10}$$

where $\mu \equiv \lambda M_T$ and λ is the coupling between triplet and two doublets in the superpotential. In general λ is complex, but its phase can be absorbed by rescaling phases of every elements of matrix f with same amount. We will treat it real in our discussion.

The total contribution to the lepton asymmetry then becomes

$$\varepsilon_i = \varepsilon_i^I + \varepsilon_i^{II}. \tag{5.11}$$

In our model, the lightest right-handed neutrino is N_2, and we will take $i = 2$.

The generated $B - L$ asymmetry can be written as

$$Y_{B-L} \equiv \frac{n_{B-L}}{s} = -\eta(\varepsilon_2 Y^{EQ}_{N_2} + \tilde{\varepsilon}_2 Y^{EQ}_{\tilde{N}_2}) \tag{5.12}$$

where

$$
\begin{aligned}
Y_{N_2}^{EQ} &= \frac{n_{N_2}^{EQ}}{s} = \frac{3}{4}\frac{45\zeta(3)}{\pi^4 g_{*s}} \\
Y_{\tilde{N}_2}^{EQ} &= \frac{n_{\tilde{N}_2}^{EQ}}{s} = \frac{45\zeta(3)}{\pi^4 g_{*s}},
\end{aligned}
\tag{5.13}
$$

g_{*s} is the effective degree of freedom contributing to entropy s with value 228.75 in MSSM, and η is the efficiency factor for leptogenesis. Ignoring the SUSY breaking effect, we have $\varepsilon_2 = \tilde{\varepsilon}_2$ and Y_{B-L} can be simplified as

$$
Y_{B-L} = -\frac{7}{4}\frac{45\zeta(3)}{\pi^4 g_{*s}}\eta\varepsilon_2. \tag{5.14}
$$

Lepton number asymmetry produced by decay of right-handed neutrino(sneutrino) can be converted to baryon number asymmetry by sphaleron effect. The baryon number is related to the $B-L$ asymmetry Y_{B-L} via

$$
Y_B = wY_{B-L}, \tag{5.15}
$$

where $w = \frac{8N_F+4N_H}{22N_F+13N_H}$ with N_F as generations of fermions and N_H as the number of the Higgs doublet. In MSSM, $N_F = 3$ and $N_H = 2$, one has $w = \frac{8}{23}$. Putting all this together, we get the baryon to photon ratio to be

$$
\frac{n_B}{n_\gamma} \simeq 7.04 Y_B = -1.04 \times 10^{-2}\varepsilon_2\eta. \tag{5.16}
$$

The efficiency factor η can be calculated by solving a set of coupled Boltzmann equations(See for example Refs. [67] [69]). We assume that to a good approximation the efficiency factor depends only on a mass parameter usually called the effective mass and the initial abundance of the right-handed neutrino(sneutrino). We also use the result for η in type I seesaw scenario. In our model, the effective mass for both the cases discussed below, is given by

$$\tilde{m}_2 = \frac{[Y_\nu Y_\nu^\dagger]_{22} v^2 \sin^2\beta}{M_2} = \frac{2h^2 v^2 \sin^2\beta}{M_2} \simeq \sqrt{\Delta m_A^2} \simeq 0.05\text{eV}, \tag{5.17}$$

which is larger than the equilibrium neutrino mass $m_* = \frac{16\pi^{5/2}\sqrt{g_*}}{3\sqrt{5}} \frac{v^2 \sin^2\beta}{M_{pl}} \simeq 1.50 \times 10^{-3}$eV, so it is in the strong washout region. In this region, the dependence of efficiency factor on the initial abundance of right-handed neutrino(senutrino) is small [68] [69]. We take the approximation formula from Ref. [69] to estimate the efficiency factor for our model

$$\frac{1}{\eta} \simeq \frac{3.3\times 10^{-3}\text{eV}}{\tilde{m}_2} + \left(\frac{\tilde{m}_2}{0.55\times 10^{-3}\text{eV}}\right)^{1.16}, \tag{5.18}$$

and find $\eta \simeq 5.3 \times 10^{-3}$, which we will use in the calculation of baryon to photon ratio for our model.

5.4 Exact Tri-bimaximal Limit

In this section, we establish the connection between ϵ_ℓ and the low energy phase in the neutrino mixing. In the limit of $|M_{R1,R3}| \to \infty$, light neutrino mass matrix has the form that leads to tri-bimaximal mixing pattern. In this limit, the contributions to lepton asymmetry from the exchange of N_1 and N_3 in the loops are negligible. As far as neutrino masses go, N_2 contribution dominates Δm_A^2 and triplet Higgs has the full contribution to $\Delta m^2_{\text{solar}}$. The observed values require that $M_T \sim (10^1 - 10^2)M_2$. This triplet can go into loop of the decay of N_2 and its interference with tree level diagram of N_2 decay can generate lepton asymmetry. In this case, Eq. (5.10) is simplified as

$$\varepsilon_2^{II} = \frac{3}{8\pi}\frac{\mathrm{Im}[Y_\nu f^* Y_\nu^T]_{22}\mu}{[Y_\nu Y_\nu^\dagger]_{22} M_2}\ln(1+\frac{M_2^2}{M_T^2}). \tag{5.19}$$

From Yukawa coupling matrices, one easily gets

$$\mathrm{Im}[Y_\nu f^* Y_\nu^T]_{22} = 2h^2(|f_b|\sin\phi_b - |f_a|\sin\phi_a) \tag{5.20}$$

$$[Y_\nu Y_\nu^\dagger]_{22} = 2h^2. \tag{5.21}$$

We also have

$$|f_a| = a\frac{M_T}{v^2\sin^2\beta\lambda}, |f_b| = b\frac{M_T}{v^2\sin^2\beta\lambda} \tag{5.22}$$

where $a \equiv |a'|$ and $b \equiv |b'|$, and ε_2^{II} can be written as

$$\varepsilon_2^{II} = \frac{3}{8\pi}\frac{(b\sin\phi_b - a\sin\phi_a)M_2}{v^2\sin^2\beta}\frac{M_T^2}{M_2^2}\ln(1+\frac{M_2^2}{M_T^2}). \tag{5.23}$$

Note that in the tri-bimaximal limit,

$$M_\nu = \begin{pmatrix} ae^{i\phi_a} & be^{i\phi_b} & be^{i\phi_b} \\ be^{i\phi_b} & ae^{i\phi_a} - c & be^{i\phi_b} + c \\ be^{i\phi_b} & be^{i\phi_b} + c & ae^{i\phi_a} - c \end{pmatrix}, \tag{5.24}$$

which can be diagnolized by U_{TB}

$$U_{TB}^T M_\nu U_{TB} = \begin{pmatrix} ae^{i\phi_a} - be^{i\phi_b} & 0 & 0 \\ 0 & ae^{i\phi_a} + 2be^{i\phi_b} & 0 \\ 0 & 0 & -2c + ae^{i\phi_a} - be^{i\phi_b} \end{pmatrix}. \tag{5.25}$$

Therefore one of the Majorana phases is given by

$$\varphi_1 \simeq \mathrm{Arc}\sin[\frac{a\sin\phi_a - b\sin\phi_b}{m_1}] \tag{5.26}$$

up to $O(\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}})$. And for $M_T \geq (10^1 - 10^2)M_2$, one has $\frac{M_T^2}{M_2^2}\ln(1+\frac{M_2^2}{M_T^2}) \simeq 1$. So the lepton asymmetry can be written as

$$\varepsilon_2^{II} \simeq -\frac{3}{8\pi}\frac{m_1 M_2 \sin\varphi_1}{v^2 \sin^2\beta}. \tag{5.27}$$

Thus we see that the Majorana phase φ_1 directly gives the lepton asymmetry, as noted in the introduction. This is the first main result.

To estimate the value of the baryon to photon ratio, we note that in this case $\varepsilon_2^I \simeq 0$ and $\varepsilon_2 = \varepsilon_2^{II}$, using Eq. (5.16) and Eq. (5.27), giving

$$\frac{n_B}{n_\gamma} \simeq 6.1 \times 10^{-10}(\frac{m_1}{2.8 \times 10^{-3}\text{eV}})(\frac{M_2}{10^{12}\text{GeV}})(\frac{\sin\varphi_1}{1})(\frac{\eta}{5 \times 10^{-3}}), \tag{5.28}$$

where we take $v = 170$Gev and $\tan\beta = 10$. To get the right range for baryon to photon ratio, the lightest right-handed neutrino mass should be larger than about 10^{12}GeV. Strict lower bound is on the product $m_1 M_2 \geq 2.8$ GeV^2. The thermal production of N_2 requires a reheat temperature of the Universe after inflation be $T_{reh} \geq 10^{12} - 10^{13}$GeV. It is to be noted that in most supersymmetry models, the reheat temperature is much below this scale- however, in more elaborate models, the reheat temperature can be different and will presumably include the higher values required in our mechanism [71]. An alternative mechanism is to use non-thermal leptogenesis. The interesting point however is that the high and low energy phase connections remain in both cases.

The right-handed neutrino mass M_2 is an input in our model. If we take as upper bound on M_2 to be 10^{14}GeV required to fit the atmospheric neutrino data, to get right baryon to photon ratio, we have to have a lower bound of $m_1 \sim 10^{-5}$eV.

On the other hand, if we take $M_2 \sim 10^{14}$GeV and $m_1 \sim 10^{-3}$eV, we get the lower bound of $\sin\varphi_1$ as $\sim 10^{-2}$.

Let us now address the question of whether the phase φ_1 can be measured in $\beta\beta_{0\nu}$ decay. The $\beta\beta_{0\nu}$ is proportional to $|m_1 \cos\theta_{12} e^{i\varphi_1} + m_2 \cos\theta_{12} e^{i\varphi_2}|$. First of all in our model $m_1 \ll m_2$ which leads to a suppression of the φ_1 effect and secondly there is the unknown φ_2. Therefore, without additional experimental input, it may not be possible to determine φ_1 from $\beta\beta_{0\nu}$ experiments.

5.5 Departure from Tri-bimaximal mMixing and New Contribution to Leptogenesis

In this section, we consider the case when we relax the mass constraint on the right-handed neutrinos and assume that $|M_{R2}| < |M_{R3}| \ll |M_{R1}|$. This will lead to departures from the exact tri-bimaximal mixing pattern [70]. In this case, there are three independent phases as noted above.

While the type II contribution to neutrino mass matrix in this case remains the same as in the exact tri-bimaximal case, the type I contribution changes and is given by

$$M_I = -M_D^T M_{\nu R}^{-1} M_D = -\begin{pmatrix} \sigma e^{i\phi_3} & 0 & -\sigma e^{i\phi_3} \\ 0 & c & -c \\ -\sigma e^{i\phi_3} & -c & c+\sigma e^{i\phi_3} \end{pmatrix}, \tag{5.29}$$

where $c \equiv \frac{h^2}{M_2} v^2 \sin^2\beta$ and $\sigma \equiv \frac{h^2}{M_3} v^2 \sin^2\beta$.

Combining the contributions from type I and type II, the light neutrino mass matrix is found to be

$$M_\nu = \begin{pmatrix} ae^{i\phi_a} - \sigma e^{i\phi_3} & be^{i\phi_b} & be^{i\phi_b} + \sigma e^{i\phi_3} \\ be^{i\phi_b} & ae^{i\phi_a} - c & be^{i\phi_b} + c \\ be^{i\phi_b} + \sigma e^{i\phi_3} & be^{i\phi_b} + c & ae^{i\phi_a} - c - \sigma e^{i\phi_3} \end{pmatrix}. \tag{5.30}$$

To diagnolize M_ν, we first consider $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$. The off-diagonal elements of $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$ are all zeros except $1-3$ and $3-1$ entries,

$$[U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}]_{13} = \sqrt{3}\sigma(ce^{-i\phi_3} + \sigma - a\cos(\phi_3 - \phi_a) + b\cos(\phi_3 - \phi_b)). \tag{5.31}$$

To further diagnolize $U_{TB}^\dagger M_\nu^\dagger M_\nu U_{TB}$, one needs another rotation in the $1-3$ plane. Because of the normal hierarchical mass spectrum of the light neutrinos, one has $c \gg a \simeq b$, and also $c \gg \sigma$ due to small upper bound of $\sin\theta_{13}$ value. In these approximation, the unitarity matrix in $1-3$ plane is

$$V = \begin{pmatrix} 1 & 0 & \xi \\ 0 & 1 & 0 \\ -\xi e^{i\phi_3} & 0 & e^{i\phi_3} \end{pmatrix} \tag{5.32}$$

where $\xi \simeq \frac{\sqrt{3}\sigma}{4c}$. Now the mixing matrix is given by $U = U_{TB}V$,

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\xi \\ -\frac{1}{\sqrt{6}} - \frac{e^{i\phi_3}\xi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{e^{i\phi_3}}{\sqrt{2}} - \frac{\xi}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} + \frac{e^{i\phi_3}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{e^{i\phi_3}}{\sqrt{2}} - \frac{\xi}{\sqrt{6}} \end{pmatrix}. \tag{5.33}$$

From this mixing matrix, we can read $\tan\theta_{12} = \frac{|U_{12}|}{|U_{11}|} = \frac{1}{\sqrt{2}}$, $\sin\theta_{13} = \sqrt{\frac{2}{3}}\xi$ and $\tan\theta_{23} = \frac{|U_{23}|}{|U_{33}|} \simeq 1 - \frac{2\xi}{\sqrt{3}}\cos\phi_3$. Note the correlation between θ_{13} and the departure of θ_{23} from its maximal value. For the Dirac phase, we use the Jarskog invariant [79] to extract it from above mixing matrix $J_{CP} = \mathrm{Im}[U_{11}U_{22}U_{12}^*U_{21}^*] = \frac{1}{8}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}\sin\delta$. From Eq. (5.33), one can easily get

$$\begin{aligned} \mathrm{Im}[U_{11}U_{22}U_{12}^*U_{21}^*] &= \frac{\xi}{3\sqrt{3}}\sin\phi_3 && (5.34) \\ \frac{1}{8}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}\sin\delta &= \frac{\xi}{3\sqrt{3}}\sin\delta. && (5.35) \end{aligned}$$

Therefore we have $\delta \simeq \phi_3$. Remarkably, although this model has three independent CP phase at the seesaw scale, the low energy scale Dirac phase is equal to one of the phases at the high energy scale up to $O(\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}})$. This is independent of the way to assign these three phases.

Coming to the calculation of lepton asymmetry in this case, with $|M_{R2}| < |M_{R3}| \ll |M_{R1}|$ limit, besides the contribution from type II to the lepton asymmetry, we should also consider the contribution from type I. From Eq. (5.9), we have

$$\varepsilon_2^I = -\frac{1}{8\pi}\frac{1}{[Y_\nu' Y_\nu'^\dagger]_{22}}\mathrm{Im}[Y_\nu' Y_\nu'^\dagger]_{23}^2 F(\frac{M_3^2}{M_2^2}), \tag{5.36}$$

and $Y_\nu' = U_R^\dagger Y_\nu$, where U_R is to diagnolize the right-handed neutrino mass matrix.

In the two light right-handed neutrinos limit, the phase of the mass of the heaviest right-handed neutrino is irrelevant to the lepton asymmetry and one can take $U_R = \mathrm{diag}(1, 1, e^{i\phi_3/2})$.

Therefore we have $[Y_\nu' Y_\nu'^\dagger]_{23} = -h^2 e^{i\phi_3/2}$, $[Y_\nu' Y_\nu'^\dagger]_{22} = 2h^2$ and $F(\frac{M_3^2}{M_2^2}) \simeq 3\frac{M_2}{M_3}$, and plugging them into Eq. (5.36), we get

$$\varepsilon_2^I = -\frac{3}{8\pi}\frac{h^2}{2}\sin\phi_3\frac{M_2}{M_3} \tag{5.37}$$

Notice that $\delta \simeq \phi_3$, $\sin\theta_{13} = \sqrt{\frac{2}{3}}\xi = \frac{\sqrt{2}}{4}\frac{M_2}{M_3}$, $\Delta m_A^2 \simeq 4c^2$ and $c = \frac{h^2}{M_2}v^2\sin^2\beta$, one can rewrite ε_2^I as function of the low energy scale observables,

$$\varepsilon_2^I \simeq -\frac{3}{8\pi}\frac{\sqrt{\Delta m_A^2}M_2}{\sqrt{2}v^2\sin^2\beta}\sin\delta\sin\theta_{13}. \tag{5.38}$$

Combining the contribution from ε_2^{II} given in Eq. (5.27), we have

$$\varepsilon_2 = \varepsilon_2^{II} + \varepsilon_2^I \simeq -\frac{3}{8\pi}\frac{M_2}{v^2\sin^2\beta}[\sqrt{\frac{\Delta m_A^2}{2}}\sin\delta\sin\theta_{13} + m_1\sin\varphi_1] \tag{5.39}$$

We again see that the phases in the leptogenesis formula are the same phases in the neutrino mixing matrix- one Dirac and one Majorana. This is the second main result. In this case also one can get the right value for the baryon to photon ratio by choosing the M_2 masses.

In conclusion, we have shown that in a model for tri-bimaximal neutrino mixing derived from an $S3$ permutation symmetry among lepton generations, the observable neutrino phases at low energies are directly responsible for the origin of matter (up to small corrections of order $\sqrt{\frac{\Delta m^2_{\text{solar}}}{\Delta m^2_{\text{atm}}}}$). Therefore, a measurement of the low energy neutrino phase in this model will provide a direct understanding of the high temperature early universe phenomenon of the origin of matter. This model is especially interesting in view of the fact that tri-bimaximal mixing pattern very closely resembles current experimental observations. Measurement of θ_{13} and θ_{23} can provide test of the tri-bimaximal mixing. If this pattern gets confirmed, experimental search for leptonic phases will become a matter of deep interest since it may hold the key to a fundamental mystery of cosmology.

Chapter 6

An SO(10) GUT Model with $S4$ Family Symmetry

6.1 Overview

The permutation symmetry have been used to understand lepton flavor mixing. In the Chapter 3, we studied the $S2$ ($\mu - \tau$) permutation symmetry within $SU(5)$ grand unification theory frame. In the Chapter 4, we studied the $S3$ family symmetry in the lepton sector and built a tri-bimaximal model with both type I and type II contributions. In this chapter, we focus on the group $S4 \times SO(10)$. $S4$ has certain good features to be a family symmetry. First, it has three dimensional irreducible representation to accommodate the three generations of fermions naturally. Note that this is different from $S3$ because the largest irreducible representation of $S3$ has dimension two and therefore we have to treat one family of fermions different from other two. Second, it can be embedded into continuous group $SU(3)$ or $SO(3)$ [72]. As we will show below, $S4$ symmetry also gives degenerate spectrum of the right-handed neutrinos naturally, which has some interesting consequences for the neutrino phenomenology. For example, in this case, one can use the resonant enhancement of leptogenesis for (quasi-)degenerate right-handed neutrinos to generate enough baryon asymmetry. With the degenerate heavy right-handed neutrinos, the low energy neutrino flavor structure is determined by Dirac mass matrix at the seesaw scale completely, which makes it easier to reconstruct high energy physics from low energy observables. Some work has been done in this direction. In Ref. [74], Lee and Mohapatra constructed a $S4 \times SO(10)$ model, which naturally gives quasi-degenerate spectrum of neutrinos masses with small solar angle, which already has been ruled out by large mixing angle MSW solution to the solar neutrino deficit. In principle radiative corrections may amplify the solar angle and keep the other two angles unchanged, but generally this needs extreme fine-tuning of parameters at the seesaw scale to realize it. On the other hand, in a recent paper [72] by Hagedorn, Linder, and Mohapatra, a low energy scale non-supersymmetric model is presented based on $S4$ flavor symmetry, which can accommodate current neutrino data. Our goal is to see if we can embed the model of Ref. [72] into a SUSY GUT framework without running into the small solar angle problem of Ref. [74]. Here, we address this question and find that we can build a realistic model based on $S4 \times SO(10)$ with the proper choice of the parameter space.

In this model, all the quarks and leptons of one generation are unified into a **16** spinor representation of $SO(10)$ and the Yukawa coupling structures of three generations are determined by $S4$. We use **10** and $\overline{\mathbf{126}}$ representations of $SO(10)$ for Yukawa couplings to account for all the fermions masses and mixing angles [75] [76]. Even though in the most general CP-violating case this model has 18 complex parameters, it is not obvious whether it can accommodate all observed masses and mixing angles because of constraints from $S4$ flavor symmetry and the correlations between quarks and leptons indicated by $SO(10)$ unification. For instance, with the particle assignment of $S4$ in this model, the heavy right-handed neutrino mass matrix is proportional to an identity matrix, and the Dirac mass matrix of neutrino determines the mixing among light neutrinos completely. The general mechanism to generate the lepton sector mixing independently from the quark sector by right-handed neutrinos does not work in this model. On the other hand, one may argue that since the total number of parameters is much larger than that of observables, this model may lose predicability even if it can fit all the observables. We find this not to be the case. It turns out that half of complex phases can be rotated away by choices of basis and redefinitions of the right-handed fields of charged leptons and down-type quarks. For the most general CP-violating case, this model gives wide range of $\sin\theta_{13}$ from zero to current bound with the most probable values $0.02-0.09$. The most probable values of leptonic CP phase are $2-4$ radians. With certain assumptions where the leptonic phases have same CP-violating source as the CKM phase, one gets narrower predicted range $0.03-0.09$ for $\sin\theta_{13}$ with the most probable values $0.04-0.08$.

Some issues about Higgs sector still need to be addressed. As we have six $\mathbf{10}$s and three $\overline{\mathbf{126}}$s, without analyzing the $S4 \times SO(10)$ invariant Higgs potential, whether or not we can get the desired vacuum configuration still remains an open question. We do not concern with doublet-doublet splitting and doublet-triplet splitting problems in this paper. With such rich Higgs fields, we assume they can be realized in some way. And another fact we should be careful is that generally the discrete flavor symmetry can enhance the accidental global symmetry of Higgs potential and lead to unwanted massless Nambu-Goldstone bosons. There are ways found in the literature to avoid it. One can introduce gauge singlet Higgs fields whose couplings are invariant under discrete symmetry but break the global symmetry [77],

Fields	Representation
Ψ_a,a=1,2,3	$\{\mathbf{3'}\} \times \{\mathbf{16}\}$
Φ	$\{\mathbf{1}\} \times \{\mathbf{210}\}$
$\overline{\Delta}_0$	$\{\mathbf{1}\} \times \{\overline{\mathbf{126}}\}$
H_0	$\{\mathbf{1}\} \times \{\mathbf{10}\}$
$H_{1,2}$	$\{\mathbf{2}\} \times \{\mathbf{10}\}$
$H_{3,4,5}$	$\{\mathbf{3}\} \times \{\mathbf{10}\}$

Table 6.1: Transformation property of fermions and Higgs multiplets under $S4 \times SO(10)$

or introduce soft terms which break discrete symmetry and global symmetry [78].

This chapter is organized as follows: in Section 2, we present an $SO(10)$ model with $S4$ family symmetry and present the mass matrices of quarks and leptons; in Section 3, we present a detailed numerical analysis including CP-violating in quark and lepton sector.

6.2 SUSY SO(10) Model with $S4$ Family Symmetry

The group $S4$ is the permutation group of the four distinct objects, which has 24 distinct elements. It has five conjugate classes and contains five irreducible representations $\mathbf{1}$,$\mathbf{1'}$,$\mathbf{2}$,$\mathbf{3}$ and $\mathbf{3'}$. Our assignment of fermions and Higgs multiplets to $S4 \times SO(10)$ are shown in Table 6.1.

In this model, we assign three generations of **16** to $\mathbf{3'}$ irreducible representation of $S4$, because $\mathbf{3'}$ can be identified with the fundamental representation of continuous group SO(3) or SU(3) [72][1]. In Higgs sector, because of $\mathbf{3'} \times \mathbf{3'} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3'}$, to make Yukawa coupling $S4$ invariant, Higgs fields can not belong to $\mathbf{1'}$. **1** is necessary for phenomenological reason, otherwise all of the mass matrices would be traceless. To get symmetric mass matrices which is required by group structure of $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$ or $\mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}}$, Higgs should not belong to $\mathbf{3'}$. We include both **2** and **3** to get realistic mass and mixing of quark and lepton. One might think six **10** Higgs fields transforming as $\mathbf{1} + \mathbf{2} + \mathbf{3}$ under $S4$ are enough. But there are two reasons why we also need $\overline{\mathbf{126}}$, one is to give right-handed neutrinos heavy masses and the other is to fix the bad mass relation between quark sector and lepton sector indicated by $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$. In this sense, our choice of Higgs fields is minimal.

The breaking of $SO(10)$ to Standard Model(SM) can be realized in many ways. In this model, we choose **210** Higgs field, which is **1** under $S4$ transformation, to break $SO(10)$ to $SU(2)_L \times SU(2)_R \times SU(4)_C$ (G_{224}) while keep the $S4$ symmetry. We choose $(\mathbf{1}, \mathbf{3}, \mathbf{10})$ components of only $\overline{\Delta}_0$ (the numbers denote representation under the G_{224}) to get VEV v_R that breaks G_{224} down to the SM and gives heavy masses to right-handed neutrinos. With this breaking pattern, $S4$ symmetry is kept down to the electroweak scale.

To see what this model implies for fermion masses, let us first explain how the MSSM doublets emerge. Besides the $SU(2)_L$ Higgs doublets from submultim-

[1]If one gives up the possible embedding of $S4$ group to continuous group, one can choose **3** and the mass matrices for fermions do not change.

plets $(\mathbf{2},\mathbf{2},\mathbf{1})$ and $(\mathbf{2},\mathbf{2},\mathbf{15})$ contained in $\mathbf{10}$ and $\overline{\mathbf{126}}$ respectively, we also have Higgs doublets contained in $(\mathbf{2},\mathbf{2},\mathbf{10})\oplus(\mathbf{2},\mathbf{2},\overline{\mathbf{10}})$ from $\mathbf{210}$. Furthermore, to obtain anomaly-free theory, we need to introduce three $\mathbf{126}$, which we denote by Δ, that also contain Higgs doublets. Altogether, we have fourteen pairs of Higgs doublets: $\phi_u = (H_{iu}, \overline{\Delta}_{ju}, \Delta_{ju}, \Phi_{u1}, \Phi_{u2})$, $\phi_d = (H_{id}, \overline{\Delta}_{jd}, \Delta_{jd}, \Phi_{d1}, \Phi_{d2})$, where $i = 0, ..., 5$ and $j = 0, ...2$. As noted, six pairs from Hs, three pairs from $\overline{\Delta}$s, three pairs from Δs and two pairs from Φ. We can write Higgs doublet mass matrix as $\phi_u M_H \phi_d^T$. M_H can be diagonalized by XM_HY^T, which X and Y are unitarity matrices acting on ϕ_u and ϕ_d respectively. At the GUT scale, by some doublet-triplet and doublet-doublet splitting mechanisms, we assume only one pair of linear combinations of $X^*_{\alpha\beta}\phi_{u\beta}$ and $Y^*_{\alpha\beta}\phi_{d\beta}$, say $X^*_{1\beta}\phi_{u\beta}$ and $Y^*_{1\beta}\phi_{d\beta}$, has masses of order of the weak scale and all others are kept super heavy near GUT scale, which generally can be realized by one fine-tuning of the parameters in the Higgs mass matrix. The MSSM Higgs doublets are given by this lightest pair: $H_u^{\text{MSSM}} = X^*_{1\beta}\phi_{u\beta}$ and $H_d^{\text{MSSM}} = Y^*_{1\beta}\phi_{d\beta}$. Since we focus on the structures of Yukawa couplings, we do not discuss the details of the splitting mechanisms that lead to the above results.

With Higgs fields and fermions listed in Table 6.1, we can write down $S4 \times SO(10)$ invariant Yukawa coupling as [2]

[2]For the products and Clebsch-Gordan coefficients of $S4$ group, one can see Appendix B of this thesis.

$$
\begin{aligned}
W_{\text{Yukawa}} &= (\Psi_1\Psi_1+\Psi_2\Psi_2+\Psi_3\Psi_3)(h_0H_0+f_0\bar{\Delta}_0) \\
&+\frac{1}{\sqrt{2}}(\Psi_2\Psi_2-\Psi_3\Psi_3)(h_1H_1+f_2\bar{\Delta}_1) \\
&+\frac{1}{\sqrt{6}}(-2\Psi_1\Psi_1+\Psi_2\Psi_2+\Psi_3\Psi_3)(h_1H_2+f_2\bar{\Delta}_2) \\
&+h_3[(\Psi_2\Psi_3+\Psi_3\Psi_2)H_3+(\Psi_1\Psi_3+\Psi_3\Psi_1)H_4+(\Psi_1\Psi_2+\Psi_2\Psi_1)H_5].
\end{aligned}
\tag{6.1}
$$

After electroweak symmetry breaking, $(\mathbf{2},\mathbf{2},\mathbf{1})$ of $H_i(i=0,...,5)$ component acquires VEVs (denoted by $\langle H_i\rangle^u$ and $\langle H_i\rangle^d$). And $(\mathbf{2},\mathbf{2},\mathbf{15})$ sub-multiplet of $\overline{\Delta}_j(j=0,...,2)$ also get induced VEVs. Their VEVs are denoted by $\langle\overline{\Delta}_j\rangle^u$ and $\langle\overline{\Delta}_j\rangle^d(j=0,1,2)$.

The mass matrices for the quarks and the leptons have following sum rules:

$$M_u = M_u^{(10)} + M_u^{(126)}, \tag{6.2}$$

$$M_d = M_d^{(10)} + M_d^{(126)}, \tag{6.3}$$

$$M_\nu^D = M_u^{(10)} - 3M_u^{(126)}, \tag{6.4}$$

$$M_l = M_d^{(10)} - 3M_d^{(126)}, \tag{6.5}$$

$$M_\nu = -M_\nu^{D^T} M_\nu^D / f_0 v_R, \tag{6.6}$$

where

$$M_u^{(10)} = \begin{pmatrix} a_0 - 2a_2 & a_5 & a_4 \\ a_5 & a_0 + a_1 + a_2 & a_3 \\ a_4 & a_3 & a_0 - a_1 + a_2 \end{pmatrix}, \tag{6.7}$$

$$M_d^{(10)} = \begin{pmatrix} b_0 - 2b_2 & b_5 & b_4 \\ b_5 & b_0 + b_1 + b_2 & b_3 \\ b_4 & b_3 & b_0 - b_1 + b_2 \end{pmatrix}, \tag{6.8}$$

$$M_u^{(126)} = \begin{pmatrix} d_0 - 2d_2 & 0 & 0 \\ 0 & d_0 + d_1 + d_2 & 0 \\ 0 & 0 & d_0 - d_1 + d_2 \end{pmatrix}, \tag{6.9}$$

$$M_d^{(126)} = \begin{pmatrix} e_0 - 2e_2 & 0 & 0 \\ 0 & e_0 + e_1 + e_2 & 0 \\ 0 & 0 & e_0 - e_1 + e_2 \end{pmatrix}, \tag{6.10}$$

and where a_i and b_i are products of the type $h\langle H_i\rangle^u$ and $h\langle H_i\rangle^d$ respectively. Similarly, we use d_j and e_j to denote products of the type $f\langle\overline{\Delta}_j\rangle^u$ and $f\langle\overline{\Delta}_j\rangle^d$ respectively. The MSSM VEVs are given by $v_u = X^*_{1\beta}\langle\phi_{u\beta}\rangle$ and $v_d = Y^*_{1\beta}\langle\phi_{d\beta}\rangle$, where we use v_u and v_d to denote VEVs of H_u^{MSSM} and H_d^{MSSM} respectively. The Yukawa couplings and VEVs of Higgs fields in general are complex, and there are 18 complex parameters. We choose a basis in which the down-quark mass matrix is diagonalized and set $b_3 = 0, b_4 = 0$, and $b_5 = 0$. Note this is our main difference with Ref. [74], where they choose a basis in which up-quark mass matrix is diagonal and set off-diagonal entries of M_u to zeros, which leads to small solar mixing angle. In the basis we choose, the charged lepton mass matrix is also diagnolized. Therefore, the phases of b_0, b_1, b_2, e_0, e_1, and e_2 can be rotated away by redefining 3 right-handed down-type quarks fields and three right-handed charged leptons. We treat b_0, b_1, b_2, e_0, e_1, and e_2 as real parameters in later analysis, and they can be determined by the masses of down-quark and charged lepton completely.

Because the mass matrix of down-quark sector is diagnolized and M_u is symmetric, one can have

$$M_u = V_{CKM}^T \hat{M}_u V_{CKM}, \tag{6.11}$$

where $\hat{M}_u \equiv \mathrm{diag}(m_u, m_c, m_b)$. By fitting mass matrix of up-quark in Eq. (6.11), parameters a_3, a_4, a_5 can be determined. In addition, we get three conditions among the parameters a_0, a_1, a_2, d_0, d_1, and d_2. Therefore, there are three complex parameters left to be determined by masses and mixings of neutrino sector. Without loss of generality, we choose d_0, d_1, and d_2 to be determined by fitting of neutrino sector. And Dirac neutrino mass matrix can be written conveniently as

$$M_\nu^D = V_{CKM}^T \hat{M}_u V_{CKM} - 4m_t \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \tag{6.12}$$

with

$$x \equiv \frac{1}{m_t}(d_0 - 2d_2), y \equiv \frac{1}{m_t}(d_0 + d_1 + d_2), z \equiv \frac{1}{m_t}(d_0 - d_1 + d_2). \tag{6.13}$$

Because we know nothing about leptonic phases, in principle, there is no constraint on the phases of d_0, d_1, and d_2.

To see how this model can give a large atmospherical mixing angle, we give an approximate analysis first. Using first order Wolfenstein parameterization [7] for the quark mixing, $V_{CKM}^T \hat{M}_u V_{CKM}$ can be written as

$$m_t \begin{pmatrix} \lambda^6 + A^2\lambda^6(1-i\eta-\rho) & \cdot & \cdot\cdot \\ -\lambda^5 - A^2\lambda^5(1-i\eta-\rho) & \lambda^4 + A^2\lambda^4 & \cdots \\ A\lambda^3(1-i\eta-\rho) & -A\lambda^2 & 1 \end{pmatrix} \tag{6.14}$$

where we use $m_c/m_t \simeq \lambda^4$ and $m_u/m_t \simeq \lambda^8$. Therefore, to get near maximal mixing of θ_{23}, y and z should satisfy

$$\lambda^4(1+A) - 4y \simeq 1 - 4z. \tag{6.15}$$

6.3 Detailed Numerical Analysis

To see if the model is phenomenologically acceptable, we first fit the masses of the charged leptons and down-type quarks using the mass values of leptons and quarks at the GUT scale with $\tan\beta = 10$ given in the Table 3.1 [52].

We use standard parametrization form for the V_{CKM} and take the following values at the scale Mz [6]: $\sin\theta_{q12} = 0.2272, \sin\theta_{q13} = 0.00382, \sin\theta_{q23} = 0.04178$ and the CP phase $\delta_q = \frac{\pi}{3}$, where we use subscript q to distinguish them from the lepton section mixing angles. And we use RGE running factor $\eta = 0.8853$.

6.3.1 Quark and Charged Lepton Sector

Using the central values of charged lepton and down-quark masses at GUT scale, b_0, b_1, b_2, e_0, e_1 and e_2 are solved from Eq. (6.5) and Eq. (6.3) (in Mev)

$$b_0 = 387.756,\ b_1 = -539.649,\ b_2 = 193.27,$$

$$e_0 = -22.7734,\ e_1 = 22.8717,\ e_2 = -11.5298.$$

For up-quark sector, by solving Eq. (6.11) and Eq. (6.2), we get values of a_3, a_4, a_5 and three conditions for $a_0, a_1, a_2, d_0, d_1, d_2$ (in Mev):

$$\begin{aligned}
&a_3 = -2990.72 - i54.757,\ a_4 = 554.859 - i234.705,\ a_5 = -66.748 + i8.155,\\
&a_0 - 2a_2 + d_0 - 2d_2 = 14.628 - i3.162,\\
&a_0 + a_1 + a_2 + d_0 + d_1 + d_2 = 308.363 + i3.977,\\
&a_0 - a_1 + a_2 + d_0 - d_1 + d_2 = 82288.5 - i7.169 \times 10^{-6}.
\end{aligned} \tag{6.16}$$

We can see that accommodation of hierarchical structure of fermions masses is realized by adjusting the parameters, $S4$ flavor symmetry itself does not provide hints on it.

6.3.2 Neutrino Sector

In this model, the light neutrino mass matrix is given by type I seesaw [20]. The mass matrix of right-handed neutrinos is proportional to an identity matrix due to the $S4$ quantum number assignment, therefore the Dirac mass matrix M^D determines the lepton sector mixing because the charged lepton mass matrix is diagnolized.

$$M_\nu = -\frac{1}{f_0 v_R} M_\nu^{D^T} M_\nu^D. \tag{6.17}$$

This model gives hierarchical neutrino mass spectrum naturally. One can choose $f_0 \sim 1$ and $v_R \sim 10^{14}$GeV, so the mass of the heaviest light neutrino is around $10^{-2} - 10^{-1}$eV.

The fit of neutrino sector are found by scanning whole parameter space spanned by x, y and z under the constrain of the current experiment requirements.

We choose the standard parametrization for the lepton sector mixing, and take 3σ experiment values given in Eq. (1.39) [14].

As mentioned earlier x, y, and z generally are complex numbers. For the most general CP-violating case, we treat the phases of x, y, and z as random input numbers with range $0 - 2\pi$. The results are shown in fig. 6.1. In this case, $\sin\theta_{13}$ has wide range from zero to the current bound with the most probable values $0.02 - 0.09$ as shown in fig. 6.1 (a). fig. 6.1 (b) shows the correlation between $\sin\theta_{23}$ and $\sin\theta_{13}$. fig. 6.1 (c) is the value distribution of Dirac CP-violating phase in the lepton sector. The allowed range of δ is quite large from 0 to 2π radians with the most probable values $2 - 4$ radians. Two Majorana phases φ_1 and φ_2 have wide range from 0 to 2π as shown in fig. 6.1 (d), which is expected.

Now we consider an interesting special case where x, y, and z are all real. Note the complexity of $f_0 v_R$ only contributes an overall phase to the light neutrino mass matrix, which can be rotated away. Therefore, in this case leptonic CP-violating phases have same source as CKM phase.

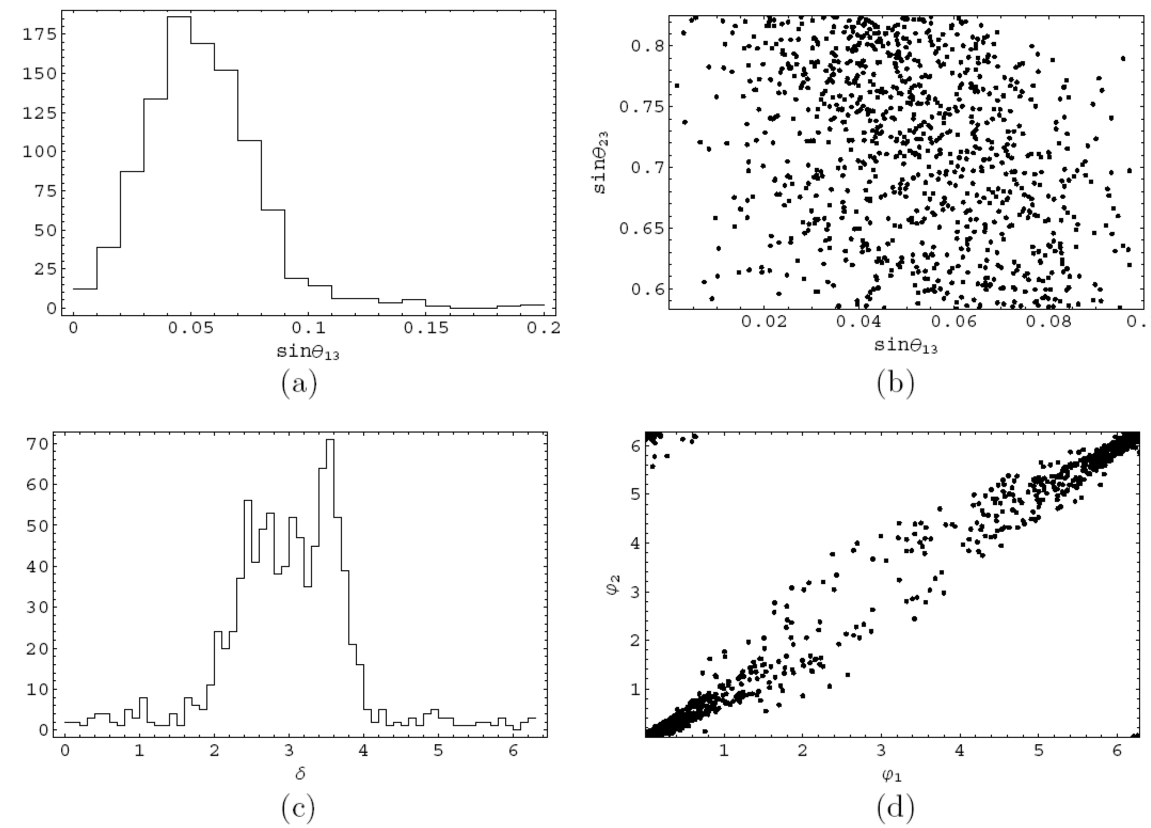

Figure 6.1: Numerical analysis for the most general case where x, y, and z are complex consistent with current experimental bound Eq. (1.39). (a) Value distribution of $\sin\theta_{13}$. (b) Correlation between $\sin\theta_{23}$ and $\sin\theta_{13}$. (c) Value distribution of leptonic Dirac CP-violating phase. (d) Scatter plot of two Majorana CP-violating phases φ_1 and φ_2.

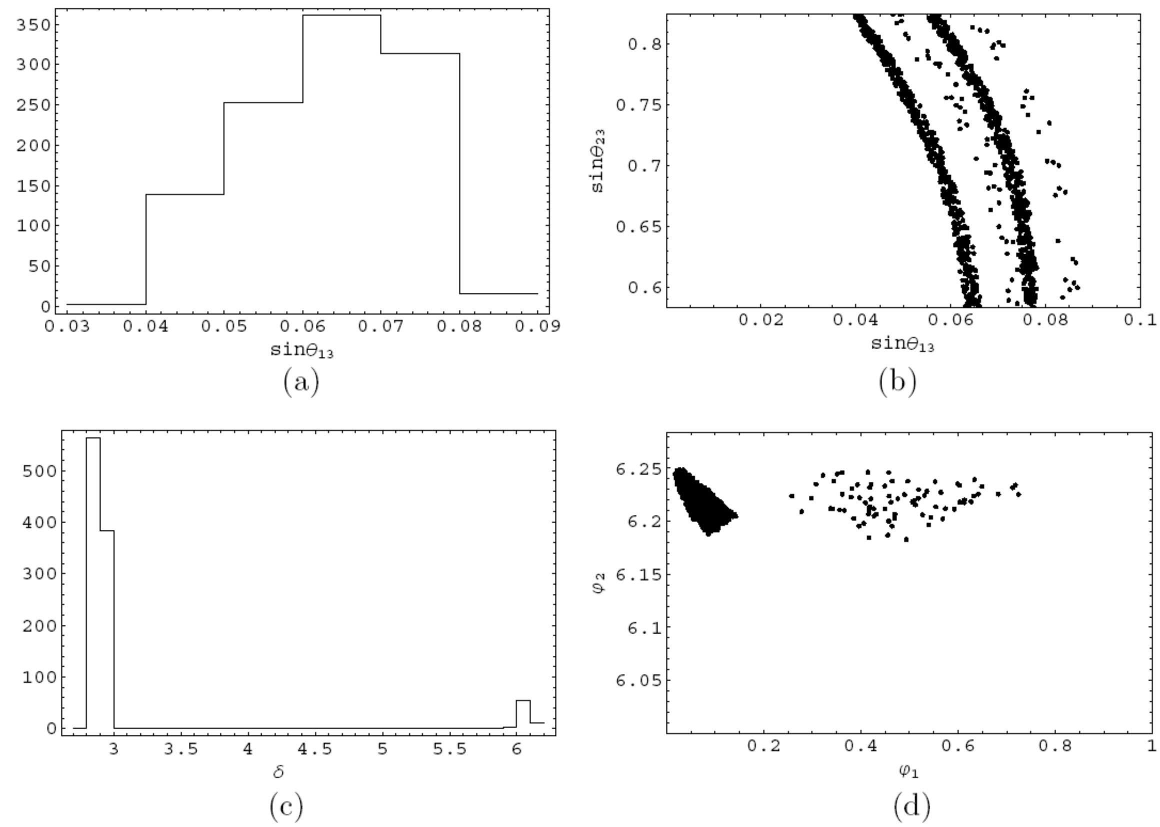

Figure 6.2: Numerical analysis for case where x, y, z are real consistent with current experimental bound Eq. (1.39). (a) Value distribution of $\sin\theta_{13}$. (b) Correlation between $\sin\theta_{23}$ and $\sin\theta_{13}$. (c) Value distribution of leptonic Dirac CP-violating phase. (d) Scatter plot of two Majorana CP-violating phases φ_1 and φ_2.

The allowed range $0.03 - 0.09$ for $\sin\theta_{13}$ is narrower compared to the general case, and the most probable range is $0.04 - 0.08$ as shown in fig. 6.2 (a). Unlike fig. 6.1 (b), fig. 6.2 (b) exhibits an interesting correlation between $\sin\theta_{23}$ and $\sin\theta_{13}$. If we take the central value of $\theta_{23} = \frac{\pi}{4}$, we can get two much narrower ranges for $\sin\theta_{13}$. One is $0.055 - 0.06$, and the other is $0.070 - 0.075$. The values of δ are $2.8 - 3$ radians, and $6.0 - 6.1$ with small possibility as shown in fig. 6.2 (c). fig. 6.2 (d) shows the allowed values of two Majorana phases. Note this parameter region is just left-up corner of fig. 6.1 (d) for the most general case. The most probable value ranges for φ_1 and φ_2 are $0.02 - 0.15$ radians and $6.19 - 6.25$ radians respectively.

For illustration, we give a typical example of fit for this case. We take

$$x = 0.0139726,\ y = 0.025914,\ z = 0.273173 \tag{6.18}$$

and solve $d_0, d_1, d_2, a_0, a_1, a_2$ from Eq. (6.13) and Eq. (6.16)(in Mev)

$$\begin{aligned} &d_0 = 8602.18,\ d_1 = -10191.2,\ d_2 = 3725.19,\ a_0 = 18935 + i0.271681, \\ &a_1 = -30798.9 + i1.9887,\ a_2 = 10036.1 + i1.71701. \end{aligned} \tag{6.19}$$

With these parameters values as input, one then obtains for the neutrino parameters

$$\begin{gathered} \sin\theta_{12} \simeq 0.53, \quad \sin\theta_{23} \simeq 0.73 \\ \sin\theta_{13} \simeq 0.054, \quad \Delta m^2_{\rm solar}/\Delta m^2_{\rm atm} \simeq 0.031. \end{gathered} \tag{6.20}$$

And light neutrino masses are $m_1 = 0.00774\text{eV},\ m_2 = 0.0118\text{eV},\ m_3 = 0.051\text{eV}$, which are normalized by $\Delta m^2_{31} = 2.6 \times 10^{-3}\text{eV}$. The Dirac phase appearing in MNS matrix is $\delta = 2.84$ radians. And two Majorona phases are (in radians): $\varphi_1 = 0.093,\ \varphi_2 = 6.21$. The Jarlskog invariant [79] has the value $J\text{cp} = 1.80 \times 10^{-3}$. One can evaluate the effective neutrino mass for the neutrinoless double beta decays process to be

$$|\sum U_{ei}^2 m_{\nu i}| \simeq 0.009 \text{ eV}.$$

In summary, we build a supersymmetric $SO(10)$ model with $S4$ flavor symmetry. The three dimensional irreducible representation of $S4$ group unify three generations of fermions horizontally. $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs fields have been used to give the Yukawa couplings and generate all the masses and mixings of quarks and leptons. This model accommodates all obervables including CKM CP-Violation phase. We studied the prediction of this model in the neutrino sector. For the most general CP-violating case, this model gives the most probable values $0.02 - 0.09$ for $\sin\theta_{13}$. In a special case where leptonic phases have same CP-violating source as CKM phase, one gets narrower range $0.03 - 0.09$ for $\sin\theta_{13}$ with the most probable values $0.04 - 0.08$.

Chapter 7

Natural Realization of Seesaw in Mini-Warped Minimal $SO(10)$ Model

7.1 Overview

In general, seesaw mechanism itself does not explain the lepton mixing pattern. As we have shown in the previous chapters, the lepton flavor mixing can be understood very well by adding flavor symmetry to the seesaw framework. And such flavor symmetry can be extended to include the quark mixing even in the grand unification models. But there is one class of $SO(10)$ grand unification models, the apparently different quark and lepton mixing patterns can be naturally accommodated without any flavor symmetry. And the near maximal atmospheric mixing angle and small rector mixing angle receive physical explanation.

In this chapter, we address an important aspect of embedding the seesaw mechanism in such grand unification model i.e. a minimal SUSY SO(10) model. We will discuss the class of models which we call minimal SO(10) models because of the Higgs content of **10**, $\mathbf{126} \oplus \overline{\mathbf{126}}$ and **210** and matter content in three **16** spinors [85]. In [80] and several subsequent papers [81], the neutrino mass discussion in this model was carried out using only the type I seesaw formula. But as is now well known, there are two contributions to the seesaw formula [21] in left-right symmetric as well as SO(10) models i.e.

$$\mathcal{M}_\nu \ = \ f v_L - M_D^T (f v)^{-1} M_D. \tag{7.1}$$

When the second term dominates, it is called type I seesaw whereas when the first one dominates, it is called type II seesaw. The advantage of the type II seesaw formula in understanding large atmospheric neutrino mixings in a two generations minimal SO(10) model was first observed in Ref. [82]. It was subsequently shown [83] that the same scenario can help to explain the large solar as well as small reactor mixing angle θ_{13} bringing these models to the mainstream of neutrino phenomenology. Other detailed questions in the model such as CP violation [87], proton decay [88] as well as symmetry breaking [89] have since been discussed. Because of predictivity in the neutrino sector while keeping the rest of fermion mass phenomenology in agreement with observations as well as general economy of the Higgs sector, these minimal models have become very attractive. One must therefore examine to what extent the model parameters needed for the neutrino predictions can be naturally obtained. It is this aspect of the models that we address in this chapter.

Since in the minimal SO(10) model, GUT symmetry relates the Dirac masses of the neutrinos to the up quark masses, one can ask for a more quantitative understanding of the seesaw formula. For example, the atmospheric neutrino mass difference square $\Delta m^2_{\rm atm} \sim 0.0025$ eV^2 requires that at least one of the right handed neutrinos has a mass around 10^{14} GeV, if one uses the type I seesaw formula for neutrino masses. This is much less than the GUT scale which determines the B-L breaking and therefore implies a fine tuning of some Yukawa couplings. In the context of minimal SO(10) models, it in fact turns out that fitting charged fermion masses also requires a Yukawa coupling suppressed to that level [83]. Therefore they go together and clearly, it will be important to understand this mini-fine tuning from a more fundamental point of view.

In this chapter we concern ourselves with minimal SUSY SO(10) models that use type II seesaw where a different fine tuning becomes essential. The the magnitude of the type II seesaw contribution to neutrino masses is given by $f\frac{v_{wk}^2}{M_T}$ where M_T is the B-L=2, $SU(2)_L$ triplet mass and for $f \sim 1$, one needs $M_T \sim 10^{14}$ GeV whereas for $f \sim 0.01$ as may be required by charged fermion fitting, we need $M_T \sim 10^{12}$ GeV [1]. Since M_T is related to M_{GUT}, the discrepancy between them must be explained. An additional challenge for this class of models is that for type II term to dominate, one must not only have the first term dominate in Eq. (7.1) but the second term must also be simultaneously smaller. In the language of SU(5) submultiplets in the **126** field, M_T must be the mass of the **15** sub-multiplet.

The problem in understanding type II dominance was discussed in Ref. [90] where it was shown that the requirements given above for type II dominance cannot be satisfied in the minimal four dimensional SUSY SO(10) model with **10**$\oplus$**126**$\oplus$**210** Higgs fields. The reason is that at high scale there are only four parameters in the superpotential and constraints of supersymmetry imply that the triplet mass must be at the GUT scale, making then type II term subdominant. This calls into question

[1] Note that in non-SUSY SO(10) models, there is an additional enhancement factor in the type II seesaw of the form M_{GUT}/M_T making the fine tuning problem less severe. However such enhancement is absent in supersymmetric theories [86].

the viability of the minimal models. The solution to this suggested in [90] was that the model be extended to include a **54**-dim. Higgs field, in which case one can fine tune parameters to get a lower triplet mass while at the same time suppressing the type I term. Since **54** Higgs does not couple to matter fields, it does not affect the discussion of fermion masses and mixings.

In this chapter, we propose a different way to solve these fine tuning problems without adding extra Higgs fields but rather by embedding the minimal model into a warped 5-dimensional space time with warping between the Planck scale and the GUT scale and with all fields of the model in the bulk. We call this "mini-warping" since the warp factor required here is $\omega \equiv M_{GUT}/M_P \sim 10^{-2}$ rather than the usual m_W/M_P as in canonical Randall-Sundrum (RS) models. Two things happen in such models if the gauge group and other fields are in the bulk: (i) all mass parameters in the IR brane are suppressed by ω and (ii) depending on bulk mass and the gauge charge, there may be additional suppression factors [91]. A combination of these two factors provides a new way to resolve some of the fine tuning problems in these models.

An initial application of this idea to understand type I seesaw in minimal SO(10) has recently been discussed by Fukuyama, Kikuchi and Okada [92] where it was shown how the smallness of the right-handed neutrino mass can be understood as a consequence of mini-warping. In the present chapter, we show that mini-warping can also help to explain type II dominance of the seesaw formula. Unlike the case of type I seesaw dominance, type II case involves a lot of subtle issues such as the magnitude of the GUT scale, structure of the MSSM doublets in terms of the GUT Higgs multiplets etc. and is highly nontrivial due to interconnections between various terms in the superpotential. We have however succeeded in finding an example where this happens. This is the subject of this chapter. The significance of our result is that it restores the type II dominated minimal SUSY SO(10) into a viable model.

The chapter is organized as follows: in Section II we discuss the basic ingredients of the approach; in Section III, we discuss the minimal SO(10) and show how type II seesaw arises naturally without extra Higgs fields.

7.2 Basic Ingredients of a Mini-warped Model

Our basic approach consists of embedding the minimal SO(10) model in the warped five dimensional brane world scenario [93] with warping between the Planck scale to the GUT scale. The fifth dimension is compactified on the orbifold S^1/Z_2 with two branes, ultraviolet (UV) and infrared (IR), located on the two orbifold fixed points. As in the RS model, we use the warped metric [93],

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \ , \tag{7.2}$$

with $-\pi \leq y \leq \pi$ and $\eta_{\mu\nu} = (+,-,-,-)$. In the above expression, k is the AdS curvature, and r_c and y are the radius and the angle of S^1, respectively. As is well known, five dimensional $N = 1$ SUSY corresponds to $N = 2$ SUSY in four dimensions. We can therefore write the 5-D superfields in terms of $N = 2$ 4-D multiplets. The process of compactification leads to $N = 1$ SUSY on the brane as well as in 4-D.

The Lagrangian for a generic $U(1)$ gauge theory with matter and Higgs fields in the bulk can be written in terms of 4-D $N = 1$ superfields as [94]:

$$\begin{aligned}\mathcal{L} &= \int dy \left\{ \int d^4\theta \, r_c \, e^{-2kr_c|y|} \left(H_i^\dagger e^{-Q_i V} H_i + H_i^c e^{Q_i V} H_i^{c\dagger} \right) \right. \\ &+ \left. \int d^2\theta e^{-3kr_c|y|} H_i^c \left[\partial_y - (1 + C_i)\, kr_c \epsilon(y) - Q_i \frac{\chi}{\sqrt{2}} \right] H_i + h.c. \right\} , \quad (7.3)\end{aligned}$$

where C_i is a dimensionless (bulk mass) parameter, $\epsilon(y) = y/|y|$ is the step function, $H_i,\ H_i^c$ is the hypermultiplet with the charge Q_i under the gauge group, and

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\theta^2\bar{\theta}^2 D \, , \\ \chi &= \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F \, , \quad (7.4)\end{aligned}$$

are the vector multiplet and the adjoint chiral multiplets, which form an $N = 2$ SUSY gauge multiplet. Z_2 parity for H_i and V is assigned as even, while odd for H_i^c and χ. This technique is easily generalized to the case of SO(10) model. The point to emphasize is that in RS models, the mass scale of the IR brane is warped down by the warp factor [93], $\omega = e^{-kr_c\pi}$, in effective four dimensional theory. If we take the cutoff of the original five dimensional theory and the AdS curvature as $M_5 \simeq k \simeq M_P$, the four dimensional (reduced) Planck mass, the cutoff scale in the IR brane is $\Lambda_{IR} = \omega M_P$. In our case, we choose the warp factor to be such that $M_{GUT} = \Lambda_{IR} = \omega M_P$. In the IR brane, the theory becomes non-perturbative above this scale so that the question of large threshold corrections becomes moot.

Let us now assume that the gauge symmetry is broken down and the adjoint chiral multiplet χ develops a VEV. Since its Z_2 parity is odd, the VEV has to take the form,

$$\langle \Sigma \rangle = 2\alpha k r_c \epsilon(y). \tag{7.5}$$

In this case, the zero mode wave function of H_i satisfies the following equation of motion:

$$\left[\partial_y - (1 + C_i + Q_i\alpha)\, k r_c \epsilon(y)\right] H_i = 0 \tag{7.6}$$

which yields

$$H_i = \frac{1}{\sqrt{N_i}} e^{(1+C_i+Q_i\alpha)kr_c|y|}\; h_i(x^\mu)\ , \tag{7.7}$$

where $h_i(x^\mu)$ is the chiral multiplet in four dimensions. Here, N_i is a normalization constant which ensures that the kinetic term is canonically normalized. We have

$$\frac{1}{N_i} = \frac{2(C_i + Q_i\alpha)k}{e^{2(C_i+Q_i\alpha)kr_c\pi} - 1}\ . \tag{7.8}$$

There are now two typical cases to consider:

(i) if $e^{(C_i+Q_i\alpha)kr_c\pi} \gg 1$, the wave functions at $y = 0$ and $y = \pi$ are, respectively, given by

$$\begin{aligned} H_i(y=0) &\simeq \sqrt{2(C_i + Q_i\alpha)k}\; \omega^{C_i+Q_i\alpha}\; h(x^\mu). \\ H(y=\pi) &\simeq \sqrt{2(C_i + Q_i\alpha)k}\; \omega^{-1}\; h(x^\mu). \end{aligned} \tag{7.9}$$

(ii) whereas for $e^{(C_i+Q_i\alpha)kr_c\pi} \ll 1$, the wave functions are

$$\begin{aligned} H(y=0) &\simeq \sqrt{-2(C_i + Q_i\alpha)k}\; h(x^\mu), \\ H_i(y=\pi) &\simeq \sqrt{-2(C_i + Q_i\alpha)k}\; \omega^{-(C_i+Q_i\alpha)}\omega^{-1}\; h(x^\mu) \end{aligned} \tag{7.10}$$

In case (i), the wave function is localized around the IR brane while around the UV brane in case (ii). These non-trivial wave function profiles lead to important effects, namely suppression of couplings and masses, in effective four dimensional theory.

To see this, let us consider Yukawa couplings on the IR and UV branes for three bulk hypermultiplets:

$$\begin{aligned}\mathcal{L}_Y &= \int d^2\theta\omega^3 \frac{Y_1}{M_5^{3/2}} H_i(y=\pi)H_j(y=\pi)H_k(y=\pi) \\ &+ \int d^2\theta \frac{Y_2}{M_5^{3/2}} H_i(y=0)H_j(y=0)H_k(y=0) + h.c.,\end{aligned} \tag{7.11}$$

where $Q_i + Q_j + Q_k = 0$ has been assumed for the U(1) gauge invariance, and Y_1 and Y_2 are independent Yukawa coupling constants on the IR and UV branes, respectively. When all the bulk fields are localized around the IR brane ($C_{i,j,k} + Q_{i,j,k}\alpha > 0$), we obtain the Yukawa coupling constant in effective four dimensional theory as

$$Y_{4D} \sim Y_1 + Y_2 \omega^{C_i+Q_i\alpha}\omega^{C_j+Q_j\alpha}\omega^{C_k+Q_k\alpha} \sim Y_1. \tag{7.12}$$

There is no suppression for the Yukawa coupling constant on the IR brane while the Yukawa coupling constant on the UV brane is very much suppressed by the small wave function overlapping. A more non-trivial example is to assume H_i is localized around the UV brane ($C_i + Q_i\alpha < 0$) and the others are localized around the IR brane ($C_{j,k} + Q_{j,k}\alpha > 0$). This case leads to the effective Yukawa coupling constant as

$$Y_{4D} \sim Y_1 \omega^{-(C_i+Q_i\alpha)} + Y_2 \omega^{C_j+Q_j\alpha}\omega^{C_k+Q_k\alpha}. \tag{7.13}$$

Both of the coupling constants are suppressed according to the wave function overlapping between each field. Other cases are completely analogous and the effective Yukawa coupling constants are suppressed or not suppressed according to the wave function profiles.

Next let us consider mass terms on the IR and UV branes for two bulk hypermultiplet such as

$$\begin{aligned}\mathcal{L}_m &= \int d^2\theta\, \omega^3\, \frac{m_1}{M_5} H_a(y=\pi) H_b(y=\pi) \\ &+ \int d^2\theta \frac{m_2}{M_5} H_a(y=0) H_a(y=0) + h.c.\end{aligned} \tag{7.14}$$

Here two mass terms on the IR and UV branes have been generally introduced. If two bulk fields are localized around the IR brane ($C_{a,b} + Q_{a,b}\alpha > 0$), we obtain the mass term in effective four dimensional theory as

$$m_{4D} \sim m_1 + m_2\omega. \tag{7.15}$$

Although there is no suppression due to the wave function profiles in this case, the mass term on the IR brane is warped down. This is the characteristic feature of RS models mentioned above. More general cases are, again, analogous and we find that suppression factors (in addition to the warp factor) appear in the effective mass according to the wave function overlap.

In the next section, we apply these results to explain the naturalness of type I and type II seesaw in the minimal SO(10) model. We will see that this goal can more or less be achieved except we still need to do one fine tuning.

7.3 Relevant Aspects of the Minimal SUSY SO(10) Model

In order to apply the discussion of the previous section to the minimal SO(10) model, we provide a brief reminder of the salient aspects of these models. All the couplings and mass parameters in this model refer to four dimensions and we omit the superscript 4D for all of them for simplicity. As long as we allow only renormalizable couplings, the model has only two Yukawa coupling matrices: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. SO(10) has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we assume that CP violation arises from other sectors of the theory (e.g. squark masses) and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then there are only nine parameters describing the Yukawa couplings. Noting the fact that the **45** and $\bar{\mathbf{5}}$ SU(5)-submultiplets of $\overline{\mathbf{126}}$ has a pair of standard model doublets in addition to the **5** and $\bar{\mathbf{5}}$ multiplets of **10** that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows [80]:

$$\begin{aligned} M_u &= h\kappa_u + f v_u \\ M_d &= h\kappa_d + f v_d \\ M_\ell &= h\kappa_d - 3 f v_d \\ M_D &= h\kappa_u - 3 f v_u, \end{aligned} \tag{7.16}$$

where $\kappa_{u,d}$ are the VEVs of the up and down standard model type Higgs fields in the **10** multiplet and $v_{u,d}$ are the corresponding VEVs for the same doublets in **126**. This gives 13 parameters describing the fermion masses and mixings (for both leptons and quarks). If we input six quark masses, three lepton masses and three quark mixing angles and weak scale, these are a total of 13 parameters and all parameters are now determined. Thus all parameters of the model that go into fermion masses are determined. The neutrino sector therefore has no free parameters except an two overall scales (v_L and v_R) as we see below:

$$\mathcal{M}_\nu \; = \; 2fv_L - M_D^T(2fv_R)^{-1}M_D \tag{7.17}$$

If type I or type II seesaw dominates, except for an overall scale, all the rest of the parameters of the neutrino mass matrix are predicted. The problem addressed in this paper is to what extent one can understand the naturalness of parameters that make either type I or type II dominate. As noted earlier, a simple understanding of the large neutrino mixings [44, 83] as well as an explanation of the value of $\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}}}$ as being of order of the Cabibbo angle comes about in the case of type II dominance.

When one tries to understand CKM CP violation in these models, it is useful to extend it by the inclusion of a **120** Higgs field that couples to SM fermions [95]. We omit the **120** field from our considerations since our main point is not affected by this.

To see what fine tunings are needed to make type II seesaw dominate, let us write down the superpotential for the 4-D SUSY SO(10) model that we are discussing. Denoting the **126** fields by Σ, and **210** ones by Φ, we have

$$W \; = \; M_\Sigma^{4D}\Sigma\bar{\Sigma} \; + M_\Phi^{4D}\Phi^2 + \lambda_1^{4D}\Sigma\bar{\Sigma}\Phi + \lambda_2^{4D}\Phi^3 \tag{7.18}$$

where we have used the superscript 4-D to denote that this is a 4-D theory. It

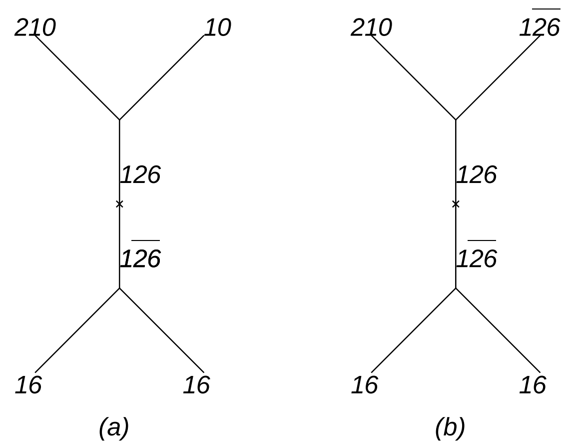

Figure 7.1: Supergraph for the type II seesaw in the minimal $SO(10)$ model.

is helpful to write down the SU(5)×U(1)$_X$ sub-multiplets of the various SO(10) multiplets used here:

$$\begin{aligned}
\mathbf{210} &= \mathbf{1}_0 \oplus \mathbf{5}_{-8} \oplus \overline{\mathbf{5}}_8 \oplus \mathbf{10}_4 \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \oplus \mathbf{75}_0 \oplus \mathbf{40}_{-4} \oplus \overline{\mathbf{40}}_4, \\
\mathbf{126} &= \mathbf{1}_{-10} \oplus \overline{\mathbf{5}}_{-2} \oplus \mathbf{10}_{-6} \oplus \overline{\mathbf{15}}_{+6} \oplus \mathbf{45}_2 \oplus \overline{\mathbf{50}}_{-2}, \\
\mathbf{10} &= \mathbf{5_2} \oplus \overline{\mathbf{5}}_{\mathbf{-2}}.
\end{aligned} \tag{7.19}$$

And the decomposition of matter field $\mathbf{16}$ is

$$\mathbf{16} = \mathbf{1_{-5}} \oplus \overline{\mathbf{5}}_{\mathbf{3}} \oplus \mathbf{10_{-1}}. \tag{7.20}$$

The supergraph responsible for type II seesaw term is given in fig. 7.1. An inspection of this graph reveals that the following conditions must be satisfied for the type II seesaw to be important for neutrino mass discussion:

(i) $M_{15} \sim f\ 10^{-2} M_{GUT}$;

(ii) coupling $\overline{\mathbf{15}} \cdot \mathbf{5} \cdot \mathbf{5} \subset \mathbf{210} \cdot \mathbf{126} \cdot \mathbf{10}$ or $\overline{\mathbf{15}} \cdot \mathbf{5} \cdot \mathbf{5} \subset \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}}$ must not be suppressed and be of order one.

We will show in the next section how we can have an understanding of these two conditions within a mini-warped model using the technique outlined in Sec. II.

7.4 Minimal SO(10) Theory in Five Dimensions

We take N=1 SUSY SO(10) model in five dimensions and put all the fields (matter as well as Higgs) in the bulk with different bulk mass terms for different fields. Note that all fields are paired with its complex conjugate field so that the bulk mass terms are allowed by gauge invariance and supersymmetry. Note that these mass terms play the role of a parameter describing the wave function profile of the field and are not the mass terms of 4-D theory.

We put the interaction terms on both IR and UV branes. Both **126** and **10** mass terms on the IR brane, and the mass term of **210** on the UV brane. The relevant part of the Lagrangian can be written as $\mathcal{L} = \int d^2\theta W_{IR} + \int d^2\theta W_{UV} + h.c.$, where

$$
\begin{aligned}
W_{IR} &= \omega^3 \Bigg[\frac{M_\Sigma}{M_5}\Sigma\overline{\Sigma} + \frac{M_H}{M_5}H^2 + \frac{\lambda_1}{M_5^{3/2}}\Phi^3 \\
&\qquad + \frac{\eta_1}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma} + \frac{1}{M_5^{3/2}}\Phi H(\alpha_1\Sigma + \overline{\alpha}_1\overline{\Sigma}) \Bigg]_{y=\pi} \\
W_{UV} &= \left[\frac{M_\Phi}{M_5}\Phi^2 + \frac{\lambda_2}{M_5^{3/2}}\Phi^3 + \frac{\eta_2}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma} + \frac{1}{M_5^{3/2}}\Phi H(\alpha_2\Sigma + \overline{\alpha}_2\overline{\Sigma}) \right]_{y=0} .
\end{aligned}
\tag{7.21}
$$

Suppose that the couplings on the UV and IR branes are of the same order.

Now we assume that the adjoint chiral multiplet of $U(1)_X$ has non-zero VEV as in Eq. (7.5) [2] and gives additional contributions to the bulk mass parameters for the

[2] Since Z_2 parity for this field is assigned as odd, the non-zero VEV leads to the Fayet-Iliopoulos

bulk fields. In the following, we denote each chiral field of SU(5)-submultiplets in H_i as $H_{im} = (\Phi_m, H_m, \Sigma_m, \overline{\Sigma}_m)$, where m specifies the dimension of the submultiplets. The zero mode solution of H_{im} is described as

$$H_{im}(x,y) = \kappa_{im}\sqrt{k}e^{kr_c|y|}e^{(C_i+\alpha Q_{im})kr_c|y|}h_{im}(x), \tag{7.22}$$

where $\kappa_{im} \equiv \sqrt{\frac{2(C_i+\alpha Q_{im})}{e^{2(C_i+\alpha Q_{im})kr_c\pi}-1}}$.

On the IR brane $H_{im}(x,\pi) = \kappa_{im}\sqrt{k}\omega^{-1}\omega^{-(C_i+\alpha Q_{im})}h_{im}(x)$ while $H_{im}(x,0) = \kappa_{im}\sqrt{k}h_{im}(x)$ on the UV brane.

We take M_Σ and M_H to be $\sim M_P$ and M_Φ to be $\sim M_{GUT}$. Because of the warp factor ω, the 4-D effective masses of the IR brane are warped down to $\omega M_P \simeq M_{GUT}$. Next note that

$$\begin{aligned} e^{(C_i+\alpha Q_{im})kr_c\pi} &\gg 1,\ \kappa_{im} \simeq \sqrt{2(C_i+\alpha Q_{im})}\,\omega^{C_i+\alpha Q_{im}} && (7.23)\\ e^{(C_i+\alpha Q_{im})kr_c\pi} &\ll 1,\ \kappa_{im} \simeq \sqrt{-2(C_i+\alpha Q_{im})} && (7.24)\\ e^{(C_i+\alpha Q_{im})kr_c\pi} &= 1,\ \kappa_{im} \simeq \sqrt{-\frac{1}{\ln\omega}}. && (7.25)\end{aligned}$$

The extent of suppression of couplings and masses in effective four dimensional theory are determined by parameters C_i and α. In this paper, we choose the parameters as listed in Tables.

D-terms localized on both the UV and IR branes [96], which should be canceled to preserve SUSY. For this purpose, we need to introduce new fields on both branes by which the D-terms are compensated. We can choose such fields to in representation **126** on of the branes and $\overline{\mathbf{126}}$ on the other, which have VEVs along $SU(5)$ singlet direction. The same fields can also generate the nonzero VEV for the Σ field. This does not affect the result of discussion.

7.4.1 Masses of Submultiplets of **126**

As noted in Sec. III, one main problem for the minimal 4-D SO(10) is that the SU(5)-submultiplets $\mathbf{15}, \mathbf{50}$ and $\mathbf{45}$ have the same mass M_Σ (up to the Clebsch-Gordan (CG) coefficients) [90]. When we lower the **15** Higgs mass so as to obtain type II dominance, other Higgs fields accordingly becomes light. As a result, gauge couplings blow up before they unite at the GUT scale. As we show now, the situation is very different in the mini-warped model.

Under the SU(5) decomposition, the mass term of the **126** pair on the IR brane can be written as

$$\int d^2\theta\omega^3 \left[\frac{M_\Sigma}{M_5}\Sigma\overline{\Sigma}\right]_{y=\pi} \sim \int d^2\theta\, m_\Sigma \left[\epsilon_{\sigma 0}\epsilon_{\overline{\sigma}0}\sigma_0\overline{\sigma}_0 + \epsilon_{\sigma 15}\epsilon_{\overline{\sigma}15}\sigma_{15}\overline{\sigma}_{15} + \epsilon_{\sigma 10}\epsilon_{\overline{\sigma}10}\sigma_{10}\overline{\sigma}_{10}\right.$$
$$\left. + \; \epsilon_{\sigma 50}\epsilon_{\overline{\sigma}50}\sigma_{50}\overline{\sigma}_{50} + \epsilon_{\sigma 45}\epsilon_{\overline{\sigma}45}\sigma_{45}\overline{\sigma}_{45} + \epsilon_{\sigma 5}\epsilon_{\overline{\sigma}5}\sigma_5\overline{\sigma}_5\right], \qquad (7.26)$$

where $m_\Sigma = \omega M_\Sigma \sim M_{GUT}$, and $\epsilon_{im} \equiv \kappa_{im}\omega^{-(C_i+\alpha Q_{im})}$. From Table 7.3 and Table 7.4, we have $\epsilon_{\sigma 15} \sim \omega^{3/2}$ and $\epsilon_{\overline{\sigma}15} \sim 1$, therefore the mass of **15** is suppressed by the factor $\omega^{3/2}$ and $M_{\mathbf{15}} \sim \omega^{3/2} M_{GUT} \sim 10^{13}$ GeV. On the other hand, we read $\epsilon_{\sigma 50} = \epsilon_{\sigma 50} \sim 1$, so the mass of **50** is $\sim M_{GUT}$. For **45**, $\epsilon_{\sigma 45} \sim \omega^{1/2}$ and $\epsilon_{\overline{\sigma}45} \sim 1$, and its mass is $\sim \omega^{1/2} M_{GUT} \sim 10^{15}$ GeV. In our mini-warped SO(10) model, there is no mass degeneracy between these submultiplets.

This mass splitting also leaves gauge coupling unification of MSSM unchanged, since the submultiplets are all full SU(5) multiplets. It is easy to check that the unified gauge coupling value at the GUT scale i.e. $\alpha_{GUT} \sim 0.2$ which is in the perturbative regime even though the $\mathbf{15} \oplus \overline{\mathbf{15}}$ multiplets with mass around 10^{13} GeV and the $\mathbf{45} \oplus \overline{\mathbf{45}}$ multiplets with mass around 10^{15} GeV are involved into the gauge coupling running.

7.4.2 Symmetry Breaking

Here we examine the realization of the SO(10) symmetry breaking. Let us first see the SO(10) gauge symmetry breaking down to SU(5). There are three SU(5) singlets: one in **210** and one in each of the **126** pair with non-zero B-L charge. Since supersymmetry must remain unbroken all the way down to the weak scale, F-flatness conditions determine vacuum expectation values. The relevant part in the superpotential in Eq. (7.21) is given by

$$\begin{aligned}
&\int d^2\theta\omega^3 \left[\frac{M_\Sigma}{M_5}\Sigma\overline{\Sigma} + \frac{\lambda_1}{M_5^{3/2}}\Phi^3 + \frac{\eta_1}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma}\right]_{y=\pi} \\
&+ \left[\frac{M_\Phi}{M_5}\Phi^2 + \frac{\lambda_2}{M_5^{3/2}}\Phi^3 + \frac{\eta_2}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma}\right]_{y=0} \\
\supset\ & m_\Sigma\epsilon_{\sigma0}\epsilon_{\overline{\sigma}0}\sigma_0\overline{\sigma}_0 + M_\Phi\kappa_{\phi0}^2\phi_0^2 + (\lambda_1\epsilon_{\phi0}^3 + \lambda_2\kappa_{\phi0}^3)\phi_0^3 \\
&+(\eta_1\epsilon_{\phi0}\epsilon_{\sigma0}\epsilon_{\overline{\sigma}0} + \eta_2\kappa_{\phi0}\kappa_{\sigma0}\kappa_{\overline{\sigma}0})\sigma_0\overline{\sigma}_0\phi_0 \\
\sim\ & m_\Sigma\omega^{3/2}\sigma_0\overline{\sigma}_0 + M_\Phi\phi_0^2 + (\lambda_1\omega^6 + \lambda_2)\phi_0^3 + (\eta_1\omega^{5/2} + \eta_2\omega^{7/2})\sigma_0\overline{\sigma}_0\phi_0. \quad (7.27)
\end{aligned}$$

F-flatness conditions for $\overline{\sigma}_0$ and ϕ_0 lead to

$$\begin{aligned}
&\sigma_0\left[m_\Sigma\omega^{3/2} + (\eta_1\omega^{5/2} + \eta_2\omega^{7/2})\phi_0\right] = 0, \\
&2M_\Phi\phi_0 + 3(\lambda_1\omega^6 + \lambda_2)\phi_0^2 + (\eta_1\omega^{5/2} + \eta_2\omega^{7/2})\sigma_0\overline{\sigma}_0 = 0, \quad (7.28)
\end{aligned}$$

and the solutions are

$$\langle\phi_0\rangle \simeq -\frac{m_\Sigma}{\eta_1\omega}, \ \ \langle\sigma_0\overline{\sigma}_0\rangle \simeq -\frac{2M_\Phi\langle\phi_0\rangle}{\eta_1\omega^{5/2}}\left(1 + \frac{3\lambda_2\langle\phi_0\rangle}{2M_\Phi}\right). \quad (7.29)$$

SO(10) gauge symmetry is broken down to SU(5)×U(1)$_X$ by $\langle \phi_0 \rangle$ at the scale $m_\Sigma/(\eta_1\omega)$. More correctly, when we carefully consider the CG coefficients and normalization of submultiplets of SO(10) under SU(5), we have an extra factor 10 accompanying with the coupling η_1 [90]. Thus, if we take, for example, $\eta_1 \sim 4\pi$ this symmetry breaking occurs around the GUT scale, $\langle \phi_0 \rangle \sim m_\Sigma/(10\eta_1\omega) \sim M_{GUT}$. On the other hand, in order to arrange the B-L breaking scale to be around the GUT scale, one needs to fine tune the coupling λ_2 to be $\lambda_2 \sim 1 - \omega^{3/2}$.

Next we consider the SU(5) symmetry breaking by **24** VEV. The relevant superpotential is given by

$$\begin{aligned} &\int d^2\theta \omega^3 \left[\frac{\lambda_1}{M_5^{3/2}}\Phi^3\right]_{y=\pi} + \left[\frac{M_\Phi}{M_5}\Phi^2 + \frac{\lambda_2}{M_5^{3/2}}\Phi^3\right]_{y=0} \\ \supset\ & M_\Phi \kappa_{\phi 24}^2 \phi_{24}^2 + (\lambda_1 \epsilon_{\phi 0}\epsilon_{\phi 24}^2 + \lambda_2 \kappa_{\phi 0}\kappa_{\phi 24}^2)\phi_0\phi_{24}^2 + (\lambda_1\epsilon_{\phi 24}^3 + \lambda_2\kappa_{\phi 24}^3)\phi_{24}^3 \\ \sim\ & M_\Phi \phi_{24}^2 + (\lambda_1\omega^6 + \lambda_2)\phi_0\phi_{24}^2 + (\lambda_1\omega^6 + \lambda_2)\phi_{24}^3. \end{aligned} \tag{7.30}$$

Through the F-flatness condition for ϕ_{24}, we obtain

$$\langle \phi_{24} \rangle \sim -\frac{M_\phi + \lambda_2 \langle \phi_0 \rangle}{\lambda_2} \sim M_{GUT}. \tag{7.31}$$

Once ϕ_0 gets the VEV, a new contribution appears to the mass of **15** through the superpotential,

$$\begin{aligned} &\int d^2\theta\, \omega^3 \left[\frac{\eta_1}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma}\right]_{y=\pi} + \left[\frac{\eta_2}{M_5^{3/2}}\Phi\Sigma\overline{\Sigma}\right]_{y=0} \\ \supset\ & \left[\eta_1\epsilon_{\phi 0}\epsilon_{\sigma 15}\epsilon_{\overline{\sigma}15} + \eta_2\kappa_{\phi 0}\kappa_{\sigma 15}\kappa_{\overline{\sigma}15}\right]\langle \phi_0 \rangle\, \sigma_{15}\overline{\sigma}_{15} \\ \sim\ & \left[\eta_1\omega^{7/2} + \eta_2\omega^{5/2}\right]\langle \phi_0 \rangle\, \sigma_{15}\overline{\sigma}_{15}. \end{aligned} \tag{7.32}$$

Substituting the above $\langle \phi_0 \rangle$ into this formula, we find the additional contribution of order $\omega^{3/2}M_{GUT}$, that is the same order as the one from the tree level mass term in Eq. (7.26).

7.5 Neutrino Mass and Type II Dominance

In this section we show how type II dominance emerges in our model. Yukawa couplings on both the IR and UV branes are given by

$$\int d^2\theta\omega^3 \left[\frac{f_{1ab}}{M_5^{3/2}}\Psi_a\Psi_b\overline{\Sigma} + \frac{h_{1ab}}{M_5^{3/2}}\Psi_a\Psi_b H\right]_{y=\pi} + \left[\frac{f_{2ab}}{M_5^{3/2}}\Psi_a\Psi_b\overline{\Sigma} + \frac{h_{2ab}}{M_5^{3/2}}\Psi_a\Psi_b H\right]_{y=0}, \tag{7.33}$$

where Ψ_a is the **16** matter field of the a-th generation ($a = 1, 2, 3$).

We first consider the Yukawa coupling for $\overline{\mathbf{5}} \cdot \overline{\mathbf{5}} \cdot \mathbf{15}$, which is extracted as

$$\left[f_{1ab}\epsilon^2_{\psi\bar{5}}\epsilon_{\bar{\sigma}15} + f_{2ab}\kappa^2_{\psi\bar{5}}\kappa_{\bar{\sigma}15}\right]\psi_{\bar{5}}\psi_{\bar{5}}\overline{\sigma}_{15} \sim \left[f_{1ab}\omega^{1/2} + f_{2ab}\omega^{5/2}\right]\psi_{\bar{5}}\psi_{\bar{5}}\overline{\sigma}_{15}. \tag{7.34}$$

Now the effective Yukawa coupling in 4-D is found to be $\sim f_{1ab}\omega^{1/2}$.

In fig. 7.1, there are two vertexes between Higgs fields involved in type II seesaw formulas, $\mathbf{210} \cdot \mathbf{126} \cdot \mathbf{10}$ or $\mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}}$. From the superpotential in Eq. (7.21) the vertex in fig. 7.1 (a) can be read off as

$$\left[\alpha_1\epsilon_{\phi 5}\epsilon_{h5}\epsilon_{\sigma\overline{15}} + \alpha_2\kappa_{\phi 5}\kappa_{h5}\kappa_{\sigma\overline{15}}\right]\phi_5 h_5\sigma_{\overline{15}}. \tag{7.35}$$

From Tables, $\epsilon_{\phi 5} \sim \epsilon_{h5} \sim 1$, $\epsilon_{\sigma\overline{15}} \sim \omega^{3/2}$, and $\kappa_{\phi 5} \sim \kappa_{h5} \sim \kappa_{\sigma\overline{15}} \sim 1$, so that we have the coupling $\sim \alpha_2\phi_5 h_5\sigma_{\overline{15}}$ un-suppressed. On the other hand, for the vertex in fig. 7.1(b), we have

$$\left[\eta_1\epsilon_{\phi 5}\epsilon_{\bar{\sigma}5}\epsilon_{\sigma\overline{15}} + \eta_2\kappa_{\phi 5}\kappa_{\bar{\sigma}5}\kappa_{\sigma\overline{15}}\right]\phi_5\overline{\sigma}_5\sigma_{\overline{15}}. \tag{7.36}$$

This contribution is negligible compared to the previous one, since $\epsilon_{\overline{\sigma}5} \sim 1$ and $\kappa_{\overline{\sigma}5} \sim \omega^{1/2}$.

We are now ready to estimate the relative magnitudes of the two different seesaw contributions to neutrino mass in our model. For this purpose, we note that in terms of the original SO(10) Yukawa couplings the $f_1 \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}}$, we can rewrite the seesaw formula as

$$\mathcal{M}_\nu \; = \; 2 f_1 v_L - M_D^T (2 f_1 v_R)^{-1} M_D \tag{7.37}$$

The magnitude of the neutrino mass from the Type II seesaw contribution is estimated as

$$M_\nu^{II} \simeq \frac{2(f_1)_{33}\omega^{1/2} v_{10} v_{210} \alpha_2}{M_{GUT}\omega^{3/2}}, \tag{7.38}$$

where $v_{10,210}$ is the VEV of up-type Higgs doublets in $\mathbf{10}$ and $\mathbf{210}$. If we take $(f_1)_{33} \sim 1$, $\alpha_2 \sim 0.5$ and assume $v_{10} \simeq v_{210} \sim 100$ GeV, we arrive at the reasonable value for the atmospheric neutrino oscillation data, $M_\nu^{II} \simeq 0.05$ eV. Note however that $b - \tau$ unification as well as charge fermion fitting implies that $(f_1)_{33} \sim 0.037$ [84]. In this case also one can get type II term to be 0.046 eV if $\alpha_2 \sim 4\pi$ and is perturbative.

Next let us examine type I seesaw contribution. The right-handed neutrino mass can be read as

$$\left[f_{1ab} \epsilon_{\psi 1}^2 \epsilon_{\overline{\sigma}0} + f_{2ab} \kappa_{\psi 1}^2 \kappa_{\overline{\sigma}0} \right] \langle \overline{\sigma}_1 \rangle \sim \omega^{3/2} f_{1ab} M_{GUT}. \tag{7.39}$$

Thus, the type I seesaw contribution is found to be

$$M_\nu^I = M_D^T M_R^{-1} M_D \simeq \frac{m_t^2 \omega^{1/2}}{2(f_1)_{33} M_{GUT} \omega^{3/2}}, \tag{7.40}$$

where m_t is top quark mass, and we have used the natural relation $M_D \sim m_t$ in GUT models. Using $m_t \sim 100$ GeV at the GUT scale, the type I seesaw gives the contribution to the neutrino mass as $M_\nu^I \simeq 0.025$ eV for $(f_1)_{33} \sim 1$, which is already smaller than the type II seesaw contribution. Again for the case of $(f_1)_{33} \sim 0.037$ obtained from charged fermion fitting in Ref. [84], even though the naive order of magnitude estimate for m_ν from type I seesaw may appear to be large, full matrix effects from M_D and M_R indeed gives the desired neutrino masses. For example, if we use the explicit forms for the coupling matrices given in Ref. [84], with $(f_1)_{33} \simeq 0.035$ using Eq. (V.7), we get the right order for m_3 even though naive estimates would have suggested $m_\nu \simeq 0.5$ eV.

In conclusion, we have shown that unlike the 4-dimensional minimal SUSY SO(10) models where it is not possible to achieve type II dominance of the seesaw formula, embedding into a mini-warped 5-D space-time cures this problem and leads to an effective 4-D theory where either type II or mixed seesaw can dominate the neutrino mass. Thus the simple understanding of the large neutrino mixings as well as the right solar mass difference square obtained in minimal SUSY SO(10) models is based on sound theoretical footing and no new Higgs fields need be added. We have also analyzed the symmetry breaking of SO(10) down to the standard model in this framework and we found that to maintain the SU(5) and SO(10) scales at 10^{16} GeV in this model, we need to fine tune only one parameters by a factor of 10^{-3}. Note that in the minimal 4-D SO(10) model, we could not even do any fine tuning to get the desired feature of type II dominance. We have also checked that the SU(5) multiplets below the GUT scale not only do not affect unification as expected but they also keep the GUT couplings $\alpha_{GUT} \sim 0.2$ meaning that one can use perturbation theory up to the GUT scale without any problem.

$\mathbf{16}$ components	$C_{\mathbf{16}} + \alpha Q_i$
$\mathbf{1_{-5}}$	$7/4$
$\mathbf{10_{-1}}$	$3/4$
$\mathbf{\overline{5}_3}$	$-1/4$

Table 7.1: Effective mass parameters of submultiplets **16** with $C_{\mathbf{16}} = 1/2$ and $\alpha = -1/4$.

$\mathbf{10}$ components	$C_{\mathbf{10}} + \alpha Q_i$
$\mathbf{5_2}$	0
$\mathbf{\overline{5}_{-2}}$	1

Table 7.2: Effective mass parameters of submultiplets of **10** with $C_{\mathbf{10}} = 1/2$ and $\alpha = -1/4$.

$\mathbf{126}$ components	$C_{\mathbf{126}} + \alpha Q_i$
$\mathbf{1_{-10}}$	$5/2$
$\overline{\mathbf{5}}_{\mathbf{-2}}$	$1/2$
$\mathbf{10_{-6}}$	$3/2$
$\overline{\mathbf{15}}_{\mathbf{6}}$	$-3/2$
$\mathbf{45_{2}}$	$-1/2$
$\overline{\mathbf{50}}_{\mathbf{-2}}$	$1/2$

Table 7.3: Effective mass parameters of submultiplets of $\mathbf{126}$ with $C_{\mathbf{126}} = 0$ and $\alpha = -1/4$.

$\overline{\mathbf{126}}$ components	$C_{\overline{\mathbf{126}}} + \alpha Q_i$
$\mathbf{1_{10}}$	$-3/2$
$\mathbf{5_{2}}$	$1/2$
$\overline{\mathbf{10}}_{\mathbf{6}}$	$-1/2$
$\mathbf{15_{-6}}$	$5/2$
$\overline{\mathbf{45}}_{\mathbf{-2}}$	$3/2$
$\mathbf{50_{2}}$	$1/2$

Table 7.4: Effective mass parameters of submultiplets of $\overline{\mathbf{126}}$ with $C_{\overline{\mathbf{126}}} = 1$ and $\alpha = -1/4$.

$\mathbf{210}$ components	$C_{\mathbf{210}} + \alpha Q_i$
$\mathbf{1_0}$	-2
$\mathbf{5_{-8}}$	0
$\overline{\mathbf{5}}_{\mathbf{8}}$	-4
$\mathbf{10_4}$	-3
$\overline{\mathbf{10}}_{\mathbf{-4}}$	-1
$\mathbf{24_0}$	-2
$\overline{\mathbf{40}}_{\mathbf{4}}$	-3
$\mathbf{40_{-4}}$	-1
$\mathbf{75_0}$	-2

Table 7.5: Effective mass parameters of submultiplets of **210** with $C_{\mathbf{210}} = -2$ and $\alpha = -1/4$.

Chapter 8

Summary and Conclusion

In this thesis, we assume neutrinos are Majorana particles and use the seesaw mechanism to generate light neutrino masses. We studied the possible permutation flavor symmetry indicated by the low scale neutrino data, its implications on leptogenesis and grand unification theories.

Leptonic $\mu-\tau$ symmetry is introduced to explain the lepton mixing data. This symmetry has interesting consequences for the leptogenesis. It provides a natural way to reduce the seesaw parameters and make it possible to connect the baryon asymmetry of our universe to the low scale neutrino experiment data if leptogenesis is the mechanism to generate baryon asymmetry.

We have shown that it is possible to apply a $\mu - \tau$ symmetry extracted from the lepton sector to the quark sector by extending this symmetry to a permutation symmetry between the second and third generations. A supersymmetric $SU(5)$ GUT model with this extended $\mu - \tau$ symmetry has been proposed to describe the mixing and mass of all fermions consistently.

Motivated by the agreement between the mixing angle data and so-called tri-bimaximal mixing pattern, the $\mu - \tau$ symmetry has been extended to the higher permutation symmetry $S3$. A simple leptonic model based on permutation symmetry between three families of leptons has been built to realize the tri-bimaximal mixing. The stability of texture under quantum correction has been studied. The quasi-degenerate spectrum of neutrino spectrum is excluded. This model is so constrained that the CP-violating phases are directly related to the phases that can be measured at low energy experiments.

A supersymmetric $SO(10)$ GUT model with $S4$ family symmetry has been proposed. The three families of fermions is described by three irreducible representation of $S4$. This model predicts degenerate right-handed neutrino spectrum due to $S4$.

We also discussed the issue of a natural realization of the seesaw mechanism in the supersymmetric minimal $SO(10)$. We embedded the 4 dimensional model in 5 dimension model with a warped fifth dimension. This setup provides a way to reduce the tuning of parameters needed in the 4 dimensional case and realizes the type II seesaw dominance without adding Higgs fields. The good features and predictivity of the model remains.

In this thesis, we have focused on the flavor mixing indicated by the results of neutrino oscillation experiments. To fully understand new physics indicated by massive neutrinos and lepton mixing, we need more information from experiments. The Majorana nature of neutrinos has not been confirmed yet, which is essential for the seesaw mechanism. More precise measurements of θ_{12}, θ_{23} and the determination of θ_{13} value as well as the order of the mass eigenstate order are important to test the leptonic flavor symmetry and judge different models. The future neutrino experiments will help us uncover this new physics beyond the standard model. The work presented in this thesis is based on a series of published papers of the author and his collaborators [97].

Chapter A

Branching Rules for $SO(10)$

For convenience, we list the branching rules for $SO(10)$ under its sub-group $SU(5) \times U(1)$ and $SU(2) \times SU(2) \times SU(4)$ [99].

A.1 $SO(10) \supset SU(5) \times U(1)$

$$
\begin{aligned}
\mathbf{10} &= \mathbf{5_2} + \overline{\mathbf{5}}_{\mathbf{-2}} \\
\mathbf{16} &= \mathbf{1_{-5}} + \overline{\mathbf{5}}_{\mathbf{3}} + \mathbf{10_{-1}} \\
\mathbf{45} &= \mathbf{1_0} + \mathbf{10_4} + \overline{\mathbf{10}}_{\mathbf{-4}} + \mathbf{24_0} \\
\mathbf{54} &= \mathbf{15_4} + \overline{\mathbf{15}}_{\mathbf{-4}} + \mathbf{24_0} \\
\mathbf{120} &= \mathbf{5_2} + \overline{\mathbf{5}}_{\mathbf{-2}} + \mathbf{10_{-6}} + \overline{\mathbf{10}}_{\mathbf{6}} + \mathbf{45_2} + \overline{\mathbf{45}}_{\mathbf{-2}} \\
\mathbf{126} &= \mathbf{1_{-10}} + \overline{\mathbf{5}}_{\mathbf{-2}} + \mathbf{10_{-6}} + \overline{\mathbf{15}}_{\mathbf{6}} + \mathbf{45_2} + \overline{\mathbf{50}}_{\mathbf{-2}} \\
\mathbf{144} &= \overline{\mathbf{5}}_{\mathbf{3}} + \mathbf{5_7} + \mathbf{10_{-1}} + \mathbf{15_{-1}} + \mathbf{24_{-5}} + \mathbf{40_{-1}} + \overline{\mathbf{45}}_{\mathbf{3}} \\
\mathbf{210} &= \mathbf{1_0} + \mathbf{5_{-8}} + \overline{\mathbf{5}}_{\mathbf{8}} + \mathbf{10_4} + \overline{\mathbf{10}}_{\mathbf{-4}} + \mathbf{24_0} + \mathbf{40_{-4}}\overline{\mathbf{40}}_{\mathbf{4}} + \mathbf{75_0}
\end{aligned}
$$

A.2 $SO(10) \supset SU(2) \times SU(2) \times SU(4)$

$$
\begin{aligned}
\mathbf{10} &= (\mathbf{2},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{6}) \\
\mathbf{16} &= (\mathbf{2},\mathbf{1},\mathbf{4}) + (\mathbf{1},\mathbf{2},\overline{\mathbf{4}}) \\
\mathbf{45} &= (\mathbf{3},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{15}) + (\mathbf{2},\mathbf{2},\mathbf{6}) \\
\mathbf{54} &= (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{20}) + (\mathbf{2},\mathbf{2},\mathbf{6}) \\
\mathbf{120} &= (\mathbf{2},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{1},\overline{\mathbf{10}}) + (\mathbf{3},\mathbf{1},\mathbf{6}) + (\mathbf{1},\mathbf{3},\mathbf{6}) + (\mathbf{2},\mathbf{2},\mathbf{15}) \\
\mathbf{126} &= (\mathbf{1},\mathbf{1},\mathbf{6}) + (\mathbf{3},\mathbf{1},\overline{\mathbf{10}}) + (\mathbf{1},\mathbf{3},\mathbf{10}) + (\mathbf{2},\mathbf{2},\mathbf{15}) \\
\mathbf{144} &= (\mathbf{2},\mathbf{1},\mathbf{4}) + (\mathbf{1},\mathbf{2},\overline{\mathbf{4}}) + (\mathbf{3},\mathbf{2},\overline{\mathbf{4}}) + (\mathbf{2},\mathbf{3},\mathbf{4}) + (\mathbf{2},\mathbf{1},\mathbf{20}) + (\mathbf{1},\mathbf{2},\overline{\mathbf{20}}) \\
\mathbf{210} &= (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{15}) + (\mathbf{2},\mathbf{2},\mathbf{6}) + (\mathbf{3},\mathbf{1},\mathbf{5}) + (\mathbf{1},\mathbf{3},\mathbf{15}) + (\mathbf{2},\mathbf{2},\mathbf{10}) + (\mathbf{2},\mathbf{2},\overline{\mathbf{10}})
\end{aligned}
$$

Chapter B

$S4$ Permutation Symmetry

In this appendix, we show some features of the $S4$ permutation group [72].

B.1 Representations

$S4$ is the permutation group of four distinct objects and it contains five irreducible representations.

$$\begin{aligned} \mathbf{1} &: \quad \text{one dimensional symmetricrepresentation;} \\ \mathbf{1'} &: \quad \text{one dimensional anti} - \text{symmetric representation;} \\ \mathbf{2} &: \quad \text{two dimensional representation;} \\ \mathbf{3} &: \quad \text{three dimensional symmetric representation;} \\ \mathbf{3'} &: \quad \text{three dimensional anti} - \text{symmetric representation.} \end{aligned}$$

B.2 Products

$$\begin{aligned} &\mathbf{1} \times \mathbf{1} = \mathbf{1}, \mathbf{1} \times \mathbf{1'} = \mathbf{1'}, \mathbf{1'} \times \mathbf{1'} = \mathbf{1} \\ &\mathbf{2} \times \mathbf{1} = \mathbf{1}, \mathbf{2} \times \mathbf{1'} = \mathbf{2}, \mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1'} + \mathbf{2}, \mathbf{2} \times \mathbf{3} = \mathbf{3} + \mathbf{3'}, \mathbf{2} \times \mathbf{3'} = \mathbf{3} + \mathbf{3'} \\ &\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3'}, \mathbf{3} \times \mathbf{3'} = \mathbf{1'} + \mathbf{2} + \mathbf{3} + \mathbf{3'}, \mathbf{3'} \times \mathbf{3'} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3'} \end{aligned} \tag{B.1}$$

Note for any representations A and B, one has $A \times B = B \times A$.

B.3 Clebasch Gordan Coefficients

$$A_0, B_0 \sim \mathbf{1}, \quad A_0', B_0' \sim \mathbf{1}', \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \sim \mathbf{2},$$
$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}, \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} \sim \mathbf{3}, \begin{pmatrix} C_1' \\ C_2' \\ C_3' \end{pmatrix}, \begin{pmatrix} D_1' \\ D_2' \\ D_3' \end{pmatrix} \sim 3'.$$

$$\mathbf{1} \times \mathbf{1} : A_0 B_0 \sim \mathbf{1}, \quad \mathbf{1} \times \mathbf{1}' : A_0 B_0' \sim \mathbf{1}, \quad \mathbf{1}' \times \mathbf{1} : B_0' A_0' \sim \mathbf{1} \quad \mathbf{1}' \times \mathbf{1}' : B_0' B_0' \sim \mathbf{1}. \tag{B.2}$$

$$\mathbf{1} \times \mathbf{2} : \begin{pmatrix} A_0 A_1 \\ A_0 A_2 \end{pmatrix} \sim \mathbf{2}, \mathbf{1} \times \mathbf{3} : \begin{pmatrix} A_0 C_1 \\ A_0 C_2 \\ A_0 C_3 \end{pmatrix} \sim \mathbf{3}, \mathbf{1} \times \mathbf{3}' : \begin{pmatrix} A_0 C_1' \\ A_0 C_2' \\ A_0 C_3' \end{pmatrix} \sim \mathbf{3}. \tag{B.3}$$

$$\mathbf{1}' \times \mathbf{2} : \begin{pmatrix} -A_0' A_2 \\ -A_0' A_1 \end{pmatrix} \sim \mathbf{2}, \mathbf{1}' \times \mathbf{3} : \begin{pmatrix} A_0' C_1 \\ A_0' C_2 \\ A_0' C_3 \end{pmatrix} \sim \mathbf{3}', \mathbf{1}' \times \mathbf{3}' : \begin{pmatrix} A_0' C_1' \\ A_0' C_2' \\ A_0' C_3' \end{pmatrix} \sim \mathbf{3}. \tag{B.4}$$

$$\mathbf{2} \times \mathbf{2} : A_1 B_1 + A_2 B_2 \sim \mathbf{1}, -A_1 B_2 + B_2 A_2 \sim \mathbf{1}', \begin{pmatrix} A_1 B_2 + A_2 B_1 \\ A_1 B_1 - A_2 B_2 \end{pmatrix} \sim \mathbf{2}. \tag{B.5}$$

$$\mathbf{3}\times\mathbf{3}: C_1D_1+C_2D_2+C_3D_3\sim\mathbf{1},\ \begin{pmatrix}\frac{1}{\sqrt{2}}(C_2D_2-C_3D_3)\\ \frac{1}{\sqrt{6}}(-2C_1D_1+C_2D_2+C_3D_3)\end{pmatrix}\sim\mathbf{2},$$
$$\begin{pmatrix}C_2D_3+C_3D_2\\ C_1D_3+C_3D_1\\ C_1D_2+C_2D_1\end{pmatrix}\sim\mathbf{3},\ \begin{pmatrix}C_3D_2-C_2D_3\\ C_1D_3-C_3D_1\\ C_2D_1-C_1D_2\end{pmatrix}\sim\mathbf{3'}\qquad\text{(B.6)}$$

$$\mathbf{3'}\times\mathbf{3'}: C'_1D'_1+C'_2D'_2+C'_3D'_3\sim\mathbf{1},\ \begin{pmatrix}\frac{1}{\sqrt{2}}(C'_2D'_2-C'_3D'_3)\\ \frac{1}{\sqrt{6}}(-2C'_1D'_1+C'_2D'_2+C'_3D'_3)\end{pmatrix}\sim\mathbf{2},$$
$$\begin{pmatrix}C'_2D'_3+C'_3D'_2\\ C'_1D'_3+C'_3D'_1\\ C'_1D'_2+C'_2D'_1\end{pmatrix}\sim\mathbf{3},\ \begin{pmatrix}C'_3D'_2-C'_2D'_3\\ C'_1D'_3-C'_3D'_1\\ C'_2D'_1-C'_1D'_2\end{pmatrix}\sim\mathbf{3'}.$$

$$\mathbf{2}\times\mathbf{3}: \begin{pmatrix}A_2C_1\\ -\frac{1}{2}(\sqrt{3}A_1C_2+A_2C_2)\\ \frac{1}{2}(\sqrt{3}A_1C_3-A_2C_3)\end{pmatrix}\sim\mathbf{3},\ \begin{pmatrix}A_1C_1\\ \frac{1}{2}(\sqrt{3}A_2C_2-A_1C_2)\\ -\frac{1}{2}(\sqrt{3}A_2C_3+A_1C_3)\end{pmatrix}\sim\mathbf{3'}.\qquad\text{(B.7)}$$

$$\mathbf{2}\times\mathbf{3'}: \begin{pmatrix}A_1C'_1\\ \frac{1}{2}(\sqrt{3}A_2C'_2-A_1C_2)\\ -\frac{1}{2}(\sqrt{3}A_2C'_3+A_1C'_3)\end{pmatrix}\sim\mathbf{3},\ \begin{pmatrix}A_2C_1\\ -\frac{1}{2}(\sqrt{3}A_1C_2+A_2C_2)\\ \frac{1}{2}(\sqrt{3}A_1C_3-A_2C_3)\end{pmatrix}\sim\mathbf{3'}.\qquad\text{(B.8)}$$

$$
\mathbf{3}\times\mathbf{3}' : C_1C_1' + C_2C_2' + C_3C_3' \sim \mathbf{1}', \begin{pmatrix} \frac{1}{\sqrt{6}}(2C_1C_1' - C_2C_2' - C_3C_3') \\ \frac{1}{\sqrt{2}}(C_2C_2' - C_3C_3') \end{pmatrix} \sim \mathbf{2},
$$

$$
\begin{pmatrix} C_3C_2' - C_2C_3' \\ C_1C_3' - C_3C_1 \\ C_2C_1' - C_1C_2' \end{pmatrix} \sim \mathbf{3}, \begin{pmatrix} C_2C_3' + C_3C_2' \\ C_1C_3' + C_3C_1' \\ C_1C_2' + C_2C_1' \end{pmatrix} \sim \mathbf{3}'. \quad \text{(B.9)}
$$

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