ABSTRACT

Title of Dissertation: MODELS OF SUPPLY FUNCTION EQUILIBRIUM WITH APPLICATIONS TO THE ELECTRICITY INDUSTRY

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Electricity market design requires tools that result in a better understanding of incentives of generators and consumers. Chapter 1 and 2 provide tools and applications of these tools to analyze incentive problems in electricity markets.

In chapter 1, models of supply function equilibrium (SFE) with asymmetric bidders are studied. I prove the existence and uniqueness of equilibrium in an asymmetric SFE model. In addition, I propose a simple algorithm to calculate numerically the unique equilibrium. As an application, a model of investment decisions is considered that uses the asymmetric SFE as an input. In this model, firms can invest in different technologies, each characterized by distinct variable and fixed costs.

In chapter 2, option contracts are introduced to a supply function equilibrium (SFE) model. The uniqueness of the equilibrium in the spot market is established. Comparative statics results on the effect of option contracts on the equilibrium price are presented. A multi-stage game where option contracts are traded before the spot market stage is considered. When contracts are optimally procured by a central authority, the selected profile of option contracts is such that the

spot market price equals marginal cost for any load level resulting in a significant reduction in cost. If load serving entities (LSEs) are price takers, in equilibrium, there is no trade of option contracts. Even when LSEs have market power, the central authority's solution cannot be implemented in equilibrium.

In chapter 3, we consider a game in which a buyer must repeatedly procure an input from a set of firms. In our model, the buyer is able to sign long term contracts that establish the likelihood with which the next period contract is awarded to an entrant or the incumbent. We find that the buyer finds it optimal to favor the incumbent, this generates more intense competition between suppliers. In a two period model we are able to completely characterize the optimal mechanism.

MODELS OF SUPPLY FUNCTION EQUILIBRIUM WITH APPLICATIONS TO THE ELECTRICITY INDUSTRY

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Es para vos viejo,

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Chapter 1: Asymmetric Supply Function Equilibrium with Applications to Investment Decisions in the Electricity Industry

Abstract

The literature on supply function equilibrium (SFE) studies models of uniform-price auctions with complete information. Most results in this literature have been limited to symmetric environments, while asymmetric environments have proved to be very difficult to analyze. However, for almost any real-world application (e.g., electricity markets), an understanding is needed of SFE models in which different bidders exhibit different sizes and valuations. In this paper, significant progress is made toward filling this gap. I prove the existence and uniqueness of equilibrium in an asymmetric SFE model. In addition, I propose a simple algorithm to calculate numerically the unique equilibrium. As an application, a model of investment decisions is considered that uses the asymmetric SFE as an input. In this model, firms can invest in different technologies, each characterized by distinct variable and fixed costs. For this application, an asymmetric model is needed since the investment decisions endogenously generate differences in installed capacity. Simulations indicate that increasing price caps results in a higher capacity level, but this gain is outweighed be the negative impact on consumer welfare.

1 Introduction

The literature on supply function equilibrium (SFE) studies models of uniform-price auctions with demand uncertainty but otherwise complete information. SFE models have been used widely to study electricity markets. This is because the theoretical model matches closely the institution used in real day-ahead electricity markets, and the model is sufficiently simple and tractable to yield interesting conclusions.

Most results in this literature have been limited to symmetric environments, while asymmetric environments have proved to be very difficult to analyze. However, for almost any real-world application, it is essential to consider SFE models in which different bidders exhibit different sizes and valuations. For example, investment decisions will generate asymmetries endogenously even if the firms start out as ex-ante symmetric. To analyze the effects of mergers or divestitures, one also needs to understand how equilibrium supply functions and payoffs change in an asymmetric environment.

In this paper, significant progress is made toward filling this gap. I am able to prove the existence and uniqueness of equilibrium in an asymmetric SFE model. In addition, I propose a simple algorithm to calculate numerically the unique equilibrium. The algorithm reduces the problem of solving a system of differential equations to one of finding a point in a finite dimensional space. The existence of a convenient algorithm for the solution allows for empirical applications of the model.

My model can provide new insights in the discussion of electricity market design and how to secure adequate installed capacity in the least costly way. The interaction between market power and investment can be analyzed in a more realistic context. As an illustration of the power of these techniques, two key issues in electricity markets are analyzed. In the first application, I consider how incentives to invest and market power are affected by a change in the price cap. In the second analysis, I consider option contracts and the incentives of generators to sell these contracts in advance of the day-ahead market.

For the first application, a model of investment decisions is constructed that uses the asymmetric SFE model as the second period of a two-stage game. In the first period, firms can invest in different technologies, each characterized by distinct variable and fixed costs. For this application, an asymmetric model is needed since the investment decisions endogenously generate differences in installed capacity. This model is used to simulate investment decisions in the electricity industry. As expected, increasing the price cap results in greater incentives to invest. Nevertheless, the price cap is an extremely blunt instrument: if there is no accompanying market power mitigation measure, higher mark-ups result from the policy change. In simulations using reasonable parameter values, the price increase for electricity sometimes greatly outweighs the increment in the return on investment. I also find that raising the price cap may increase the distortion among alternative generating technologies. More specifically, in the examples considered, increasing the price cap intensifies the bias toward use of peaking, rather than baseload, technologies.

In the second exercise, option/forward contracts are introduced into the model of asymmetric SFE. These contracts allow generators and load-serving entities to hedge risks. I provide an example in which, despite the available gains from trade, generators might not sell these contracts due to market power considerations.

Before presenting the model, I will briefly review some related literature. The first SFE model was presented by Klemperer and Meyer [20]. The model was presented as a representation of an oligopolistic market in which firms can commit to a schedule of prices and quantities (i.e. a supply function). As indicated above, this model can also be interpreted as a representation of an auction in which bidders are sellers and the uniform pricing rule is used. Green and Newbery [14] noticed that the model captures key elements of wholesale electricity markets and applied this framework to study the British electricity market. Many setting result in multiplicity of equilibria, and in several other settings whether the equilibrium is unique was unknown. In a symmetric setting, Holmberg [15] establishes the uniqueness of the SFE in environments with capacity constraints. Holmberg [17] considers a model in which there is a single constant marginal cost technology and firms differ only in their installed capacity, and is then able to establish uniqueness of equilibrium. Holmberg [16] is able to calculate an equilibrium in a special model of asymmetric cost functions, but the asymmetries allowed are quite limited and uniqueness cannot be established. Baldick and Hogan [5] focus on stability analysis. Rudkevich [27] shows necessary conditions for optimality in asymmetric environments. Anderson and Xu [3] study the optimization problem of a bidder in a uniform-price auction.

Some authors have also tested the validity of these theoretical models using data from specific markets. Oren and Sioshansi [26] and Hortacsu and Puller [18] test the model of SFE in the Texas balancing market. They find that large bidders choose strategies that are close to the optimal strategies. Wolak [31] studies the National Electricity Market in Australia. He finds no evidence against the profit maximizing hypothesis.

My paper is organized as follows. In the next section, the model is presented. Uniqueness and existence of the equilibrium is demonstrated in section 3. Next, an algorithm for the numerical calculation of the equilibrium is developed. Section 5 considers extensions of the model. The two applications to electricity markets are presented in section 6. Section 7 concludes.

2 The model

There are n firms with respective cost functions $C_i(q_i)$ where $q_i \in [0, k_i)$ and $k_i > 0$ is the capacity of firm i. We assume each $C_i(.)$ is increasing, piecewise continuously differentiable and convex.

The game is a uniform price reverse auction with a price cap p^m. Proportional rationing is applied when needed. A strategy in this game is given by a piecewise continuously differentiable increasing supply function $q_i : [0, p^m] \rightarrow [0, k_i]$. Given a profile of supply functions $\mathbf{q} = \{q_i\}_{i=1}^n$, $q(.)$ denotes aggregate supply and $q_{-i}(.)$ indicates aggregate supply not including firm i's supply.

The demand function is inelastic, it is given by $d(p, x) = x$ where x is distributed according to a continuous and piecewise continuously differentiable strictly increasing cumulative distribution function $F : [0, M] \to [0, 1]$. Below we will consider an extension of the model in which we allow for a demand function that responds to price. We assume $M > \sum_{i=1}^{n} k_i$, that is, the quantity demanded is higher than installed capacity with positive probability. This is an important assumption that results in the uniqueness of equilibrium in Holmberg [15]. Also note that this is a reasonable assumption in electricity markets.

For a given profile of supply functions $\mathbf{q} = \{q_i\}_{i=1}^{n}$ and quantity demanded, x, the equilibrium price is given by:

$$p(x, \mathbf{q}) = \begin{cases} \inf\{p \in [0, p_m] : x \leq \sum_{i=1}^{n} q_i(p)\} & if \qquad x < q(p^m) \\ p^m & otherwise \end{cases} \tag{1}$$

The price cap, p^m, is the equilibrium price when there is excess demand at any price $p \in [0, p^m]$. When no rationing is required, the quantity supplied by each firm is given by: $\tilde{q}_i(x, q_i, q_j) = q_i(p(x, q_i, q_j))$. When rationing occurs, the supplied quantity is determined by the proportional rationing rule [1]. Let $\underline{q}_i(p) = \lim_{\epsilon \to 0} q_i(p - \epsilon)$ and $\overline{q}_i(p) = \lim_{\epsilon \to 0} q_i(p + \epsilon)$. Similar notation applies to the aggregate supply function and the demand function. Then, according to the proportional rationing rule:

$$\tilde{q}_i(x, \mathbf{q}) = \underline{q}_i(p) + \big(x - \underline{q}(p)\big) \frac{\overline{q}_i(p) - \underline{q}_i(p)}{\overline{q}(p) - \underline{q}(p)} \tag{2}$$

Where p is the equilibrium price $p(x, \mathbf{q})$. Given the definitions above, payoffs are given by:

$$\pi_i(\mathbf{q}) = \int_0^M [p(x, \mathbf{q})\tilde{q}_i(x, \mathbf{q}) - C_i(\tilde{q}_i(x, \mathbf{q}))]dF(x) \tag{3}$$

[1] As it will become clear in the proof the equilibrium analysis is robust to different specifications of the rationing rule. This is because in equilibrium the rationing rule is not used, it is only used in the proof when evaluating profitable deviations when the strategies are not an equilibrium.

The firms move simultaneously and independently in a static game where the cost functions and installed capacity of each firm are assumed to be common knowledge. Nash Equilibrium will be used as the solution concept. In the next section, the uniqueness and existence of the equilibrium will be verified.

3 Equilibrium analysis

In this section we establish conditions for the existence of a unique equilibrium in the model presented above. First, we present necessary conditions for an equilibrium. Next, we prove that there exist at most one profile of supply functions that satisfies these conditions. Finally, we verify that the profile of supply functions previously identified does, in fact, constitute an equilibrium.

3.1 Characterization of the equilibrium

In this section we present the equilibrium analysis for the case of two firms. (In section 5, we analyze the extension to the case of n firms.) It will be clear there that the n firm case is more complicated.

We will first prove that the equilibrium supply functions have no flat portions or discontinuities (except at the maximum price p^m or when the capacity constraint is binding). Next we establish results on the terminal conditions that the supply functions must satisfy. Using that, we can conclude in the next section that the equilibrium supply functions are uniquely determined by a system of differential equations.

Let $c_i(q_i)$ represent the marginal cost function. by assumption, this function is well defined almost everywhere. Also let $p_0 = max\,\{c_1(0), c_2(0)\}$.

Lemma 1

The equilibrium supply functions are continuous for every price $p \in (p_0, p^m)$.

Proof:

For $p < c_i(0)$ firm i's mark up is negative, so in equilibrium the supply must be zero, thus the function is continuous.

Suppose that at p^* with $c_j(0) < p^* < p^m$ we have $[\overline{q}_j - \underline{q}_j] > 0$. First, observe that for any subset $[p^*, p^* + \epsilon]$ there must be at least one other firm i that offers additional quantities in that range, otherwise j can deviate profitably by reducing supply at p^*. Let $p_i^\epsilon(p^*) = inf\,\{p : q_i(p) \geq q_i(p^*) + \epsilon\}$. Observe that $lim_{\epsilon \to 0} p_i^\epsilon(p^*) = p^*$.

We are going to prove that i can gain from offering more quantities at some price below p^*. Consider the following deviation:

$$\tilde{q}_i^\epsilon(p) = \begin{cases} \underline{q}_i(p^*) + \epsilon \quad\quad if \quad\quad p \in (p^* - \epsilon, p_i^\epsilon(p^*)) \\ \quad q_i(p) \quad otherwise \end{cases} \tag{4}$$

The new deviation results in a loss, L^ϵ, from lower prices and gains, G^ϵ, from more quantities sold. The loss from lower prices is bounded above by: $L^\epsilon < (p_i^\epsilon(p^*) - p^* + \epsilon))q_i(p_i^\epsilon(p^*))Pr^\epsilon(\Delta p)$. Where $Pr^\epsilon(\Delta p)$ is the probability that prices change with the new supply function, it converges to 0 as ϵ converges to 0. Since the first factor $(p_i^\epsilon(p^*) - p^* + \epsilon))$ divided by ϵ converges to a real number, we have that this upper bound has a derivative that equals 0 at $\epsilon = 0$.

The gain, G^ϵ, is bounded below by:

$$G^\epsilon > (p^* - \epsilon - c_i(\underline{q}_i(p^*) + \epsilon))\Delta E^\epsilon(q_i) \tag{5}$$

Now we prove that: $\Delta E^\epsilon(q_i)$, the variation in expected quantities, is strictly increasing as a function of ϵ at $\epsilon = 0$. Consider the case $\underline{q}_i(p^*) = \overline{q}_i(p^*)$, then, we have

$\Delta E^\epsilon(q_i) > \epsilon \Big[F(q_i(p^*) + \overline{q}_j(p^*)) - F(q_i(p^*) + \underline{q}_j(p^*))\Big]$.
Alternatively, suppose $\underline{q}_i(p^*) < \overline{q}_i(p^*)$, then we have:

$$\Delta E^\epsilon(q_i) = \int_{\underline{q}_i(p^*)+\underline{q}_j(p^*)}^{\underline{q}_i(p^*)+\underline{q}_j(p^*)+\epsilon} (x - \underline{q}_i(p^*) - \underline{q}_j(p^*))dF(x)$$
$$+ \int_{\underline{q}_i(p^*)+\underline{q}_j(p^*)+\epsilon}^{\overline{q}_i(p^*)+\overline{q}_j(p^*)} \left[\frac{\overline{q}_i(p^*) - \underline{q}_i(p^*) - \epsilon}{\overline{q}_i(p^*) + \overline{q}_j(p^*) - \underline{q}_i(p^*) - \underline{q}_j(p^*) - \epsilon}(x - \underline{q}_i(p^*) - \underline{q}_j(p^*) - \epsilon) + \epsilon\right] dF(x)$$
$$- \frac{\overline{q}_i(p^*) - \underline{q}_i(p^*)}{\overline{q}_i(p^*) + \overline{q}_j(p^*) - \underline{q}_i(p^*) - \underline{q}_j(p^*)} \int_{\underline{q}_i(p^*)+\underline{q}_j(p^*)}^{\overline{q}_i(p^*)+\overline{q}_j(p^*)} (x - \underline{q}_i(p^*) - \underline{q}_j(p^*))dF(x)$$

Then the derivative evaluated at $\epsilon = 0$ is positive:

$$\lim_{\epsilon\to 0} \frac{\Delta E^\epsilon(q_i)}{\epsilon}|_{\epsilon=0} = \int_{\underline{q}_j(p^*)+\underline{q}_i(p^*)}^{\overline{q}_j(p^*)+\overline{q}_i(p^*)} \frac{(\overline{q}_j(p^*) - \overline{q}_j(p^*))(\overline{q}_j(p^*) + \overline{q}_i(p^*) - x)}{(\overline{q}_j(p^*) + \overline{q}_i(p^*) - \underline{q}_j(p^*) - \underline{q}_i(p^*))^2} dF(x)) > 0$$

Since the other term in G^ϵ is strictly positive at $\epsilon = 0$, we have that the gain is strictly increasing at $\epsilon = 0$.

We conclude that for small enough ϵ the deviation is profitable. $\square$

Lemma 2

Let $\hat{p}$ be such that there exist $p^* < \hat{p}$ with $q_i(p^*) > 0$ for some i. Then, the equilibrium supply functions are strictly increasing at price $\hat{p}$.

Proof:

Suppose firm i offers the same quantity for $p \in [\underline{p}, \overline{p}]$ where $\hat{p} \in [\underline{p}, \overline{p}]$. Then observe that for that range no firm is offering additional units, otherwise there is a profitable deviation that consists in reducing supply for that range. This would increase prices, while keeping the quantities sold constant.

Consider the following deviation by firm i:

$$\tilde{q}_i^{\epsilon}(p) = \begin{cases} q_i(\underline{p}-\epsilon) & if \qquad p \in \left(\underline{p}-\epsilon, \overline{p}\right) \\ \left[q_i\left(\underline{p}-\epsilon\right), q_i\left(\underline{p}\right)\right] & if \qquad p = \overline{p} \\ q_i(p) & otherwise \end{cases} \tag{6}$$

This deviation results in gains in terms of prices and losses in terms of quantities. Losses are bounded above by:

$$L^{\epsilon} < \underline{p}(q_i(\underline{p}) - q_i(\underline{p}-\epsilon))(F(q_i(\underline{p}) + q_j(\underline{p})) - F(q_i(\underline{p}-\epsilon) + q_j(\underline{p}-\epsilon))) \tag{7}$$

Where $(q_i(\underline{p}) - q_i(\underline{p}-\epsilon))$ converges to 0 as ϵ converges to 0 and $(F(q_i(\underline{p}) + q_j(\underline{p})) - F(q_i(\underline{p}-\epsilon) + q_j(\underline{p}-\epsilon)))$ divided by ϵ converges to a real number. This implies that the upper bound has a value of 0 and a derivative that equals 0 at $\epsilon = 0$.

Gains are bounded below by:

$$G^{\epsilon} > (\overline{p} - \underline{p})q_i(\underline{p}-\epsilon)\left(F(q_j(\underline{p}) + q_i(\underline{p})) - F(q_j(\underline{p}) + q_i(\underline{p}-\epsilon))\right) \tag{8}$$

Observe that the gain is strictly increasing in ϵ at $\epsilon = 0$.

We conclude that for sufficiently small ϵ the deviation is profitable. $\square$

The lemma below characterizes boundary conditions at the top.

Lemma 3

Both firms offer all of their capacity at the price cap. Additionally, the supply function is continuous at p^m for at least one firm.

Proof:

From lemma 2 we know that the equilibrium supply functions are strictly increasing up to p^m. Additionally we know that at p^m each firm offers all its capacity since there is no negative effect on price but more quantities are sold with positive probability. Suppose that for each firm we have $\lim_{p \to p^m} q_i(p) < k_i$ with $q_i^m < k_i$ then, the same reasoning presented in the proof of lemma 1 can be used to show that offering a larger quantity at prices below p^m constitutes a profitable deviation. $\square$

From lemmas 1 and 2 we conclude that equilibrium supply functions are continuous and their derivatives are strictly positive; this means that the monotonicity constraints do not bind. Thus, if we represent a firm's optimization problem as one in which firms select equilibrium prices for each demand level, we would observe the same outcome; additionally the monotonicity conditions would not be violated. If they did, we would contradict the property that the supply functions are best responses.

The associated unconstrained problem in which firm i selects the equilibrium price for each demand level x for a constant supply of firm j is given by:

$$max_{p(x)} \left(x - q_j(p(x))\right) p(x) - C_i \left(x - q_j(p(x)\right) \qquad (9)$$
$$s.t: \; q_j(x) \leq x$$

The first order condition for an interior solution is:

$$\frac{\partial \pi}{\partial p(x)} = \left(x - q_j(p(x)) - q_j'(p(x)) \left(p(x) - c_i \left(x - q_j(p(x))\right)\right)\right) = 0$$

In equilibrium the solution is interior since the supply functions are strictly increasing almost everywhere. Evaluating at equilibrium conditions $q_i(p(x)) = x - q_j(p(x))$, we have a system of differential equations that characterizes the equilibrium supply functions:

$$q_1(p) = q_2'(p)(p - c_1(q_1(p)))$$
$$q_2(p) = q_1'(p)(p - c_2(q_2(p)))$$

Additionally, we observe that there is an explicit expression for the slope of each supply function that depends on the state of the system at that price. This will be used later on in the proof on uniqueness and the algorithm for the numerical calculation.

The following lemma establishes the conditions that must be satisfied by the equilibrium supply functions for price below the marginal cost of the bidders. The logic behind the result is similar to what occurs in a model of Bertrand-like price competition.

Lemma 4

Let $j = argmax_i c_i(0)$ then, the equilibrium supply functions satisfy $q_1(p) = q_2(p) = 0$ $\forall p < c_j(0)$ and $q_j(c_j(0)) = 0$.

Proof:

First we observe that in equilibrium no positive quantities are offered below $c_j(0)$, j would not offer any since the mark up is negative in consequence i would never find it optimal to offer below that price.

Also observe that the minimum price at which firms offer positive quantities must coincide. Otherwise the firms that offers positive quantities at the low prices can profitably deviate by offering those quantities at higher prices.

Then, note that if the minimum price at which positive quantities are offered, p', is above $c_j(0)$ then the differential equations characterizing the equilibrium are not satisfied in the neighborhood of p'. Finally, note that if firm j offers positive quantities at $c_j(0)$ then firms i can deviate by offering positive quantities as long as the mark up is positive at that price. If the mark up is zero then no firm finds a best response to offer positive quantities at that price. $\square$

Figure 1 provides a representation of a pair of supply functions that satisfies the necessary conditions.

3.2 Uniqueness of SFE

From the results above, we learned that the supply functions that satisfy the equilibrium conditions are strictly increasing, continuous and there exist a system of differential equation that characterizes the equilibrium strategies for the price range $(c_j(0), p^m)$. Also, we found terminal conditions for the system of differential equation at p^m and $c_j(0)$.

We know that, in the equilibrium, the supply function of at most one of the firms might be discontinuous at p^m. Let $\bar{q}_i$ equal $\bar{q}_i = lim_{p \to p^m} q_i(p)$ and define $\mathbf{q}^{(\bar{q}_1, \bar{q}_2)}$ represent the set of supply functions that takes values $(\bar{q}_1, \bar{q}_2)$ at p^m and is constructed downwards using the equilibrium system of differential equations. These functions are reconstructed until the quantity of one firm is zero or the price equals $c_j(0)$, whatever happens first.

Lemma 5:

The function $q_i^{\bar{q}_1, \bar{q}_2}(p)$ is strictly increasing in $\bar{q}_i$ and strictly decreasing in $\bar{q}_j$ $j \neq i$ $\forall p < p^m$ for which $\mathbf{q}^{(\bar{q}_1, \bar{q}_2)}$ is defined.

Proof:

Consider the case in which $\bar{q}_1$ increases. For construction of the supply functions downward we use the following rate of change:

$$q_i'(p) = \frac{q_j(p)}{p - c_j(q_j(p))} \tag{10}$$

Observe that at p^m $q_1(p)$ is larger, then $q_2(p)$ decreases strictly faster at that price. Also observe that if at any price p, $q_1(.)$ is larger then $q_2(.)$ decreases at a faster pace and if $q_2(.)$ is lower, $q_1(.)$ decreases at a lower rate. Then the respective supply function will not intersect. $\square$

A graphical representation of this result can be found in Figure 2. The lemma above shows that the resulting supply functions react monotonically to a change in the boundary condition at p^m, this property is used in the theorem below to prove uniqueness of the equilibrium.

Theorem 1:

There is a unique set of strategies that satisfies the necessary conditions for an equilibrium.

Proof:

First, observe that Lemma 5 together with continuity of $q_i^{\bar{q}_1,\bar{q}_2}(p)$ in $\bar{q}_i$ imply that there exist a set of terminal conditions at the top such that the terminal conditions at the bottom are satisfied.

Note that the terminal conditions require the continuity of at least one of the supply functions at p^m. This means if $\mathbf{q}$ and $\mathbf{q}'$ satisfy the necessary conditions then we cannot have $(\bar{q}_1, \bar{q}_2) << (\bar{q}_1', \bar{q}_2')$ or $(\bar{q}_1, \bar{q}_2) >> (\bar{q}_1', \bar{q}_2')$, since any of these would imply that there is a firm for which not all the capacity is offered at the cap price.

Then we must have $\bar{q}_i \leq \bar{q}'_i$ and $\bar{q}_j \geq \bar{q}'_j$. From Lemma 4 we have that if one or both of the inequalities are strict inequalities then the necessary condition at $c_j(0)$ cannot be satisfied for both set of supply functions.

The only remaining possibility is $(\bar{q}_1, \bar{q}_2) = (\bar{q}'_1, \bar{q}'_2)$, which results in $\mathbf{q} = \mathbf{q}'$. $\square$

3.3 Existence of SFE

In this subsection we verify that the strategy profile identified in the previous subsection, $\mathbf{q}^*$, does constitute a pair of best responses.

With that purpose, we are going to consider an associated problem in which each firm i selects a clearing price for each demand level, taking the supply function of the other firm j as given. In other words, we will verify that the equilibrium supply functions are an ex-post equilibrium, that is, supply functions are a best response realization by realization.

Theorem 2:

The strategy profile q^* is a Nash Equilibrium.

Proof:

Consider the following associated problem in which firm i selects the equilibrium price for each demand level x given a constant supply of firm j:

$$\begin{aligned} max_{p(x)} \left(x - q_j(p(x))\right) p(x) - C_i \left(x - q_j(p(x)\right) \\ s.t: \;\; q_j(x) \leq x \end{aligned} \tag{11}$$

The derivative of the objective function is given by:

$$\frac{d\pi}{dp(x)} = (x - q_j(p(x)) - q_j'(p(x))\,(p(x) - c_i\,(x - q_j(p(x))))$$

Now, we will prove that for each x, the profit function is maximized at $p(x) = p(x, q_i^*, q_j^*)$, that is, the price corresponding to q^*. We know that $\mathbf{q}^*$ satisfies:

$$q_j^{*\prime}(p) = \frac{q_i^*(p)}{p - c_i(q_i^*(p))} \quad i = 1, 2 \tag{12}$$

Then derivative evaluated at $q_j = q_j^*$ equals:

$$\frac{\partial\pi}{\partial p(x)} = q_i - \frac{q_i^*(p)}{p - c_i(q_i^*(p))}\,(p(x) - c_i\,(q_i)))\,,$$

where q_i satisfies $q_i = x - q_j(p(x))$.

If $p(x) > p(x, q_i^*, q_j^*)$ then $q_i < q_i^*(p(x))$. Evaluating the derivative we find that the profit function is decreasing in price for that range of prices. Similarly, if $p(x) < p(x, q_i^*, q_j^*)$ then $q_i > q_i^*(p(x))$. After evaluating the derivative of the function we find that the profit function is increasing in price for that range. We conclude that the function is maximized at $p(x) = p(x, q_i^*, q_j^*)$.

Observe that for $x > k_1 + k_2$ no firm can affect the prices.

Finally for firm i and $x < q_i(c_j(0))$ we have that $\frac{\partial\pi}{\partial p} < 0$ for all $p > c_j(0)$ and $\frac{\partial\pi}{\partial p} > 0$ for prices below $c_j(0)$. This means that $c_j(0)$ is also a best response for these range of values for the demand function. Since $q_i = q_i^*$ is the solution to this more relaxed problem then we conclude that q_i^* is a best response to q_j^*, that is, it also solves the problem when the monotonicity constraints are considered. $\mathbf{q}^*$ is an equilibrium of the game $\square$

4 Numerical calculation

In this section we present a simple algorithm that solves numerically the system of equations that characterizes the equilibrium and finds the unique boundary condition consistent with an equilibrium. One difficulty presented by the system of equations is that the slope of the supply functions presents discontinuities.

Without loss of generality we assume $c_1(0) > c_2(0)$. The following is a description of the algorithm:

1. Start with $\{\bar{q}_1, \bar{q}_2\} = k_1, k_2$.
2. Calculate $q_i^{\bar{q}_1,\bar{q}_2}(p)$.
3. If the bottom terminal conditions are satisfied stop.
4. If $q_i^{\bar{q}_1,\bar{q}_2}(p) = 0$ for $p > c_1(0)$ then increase $\bar{q}_i$ or decrease $\bar{q}_j$.
5. If $q_1^{\bar{q}_1,\bar{q}_2}(c_1(0)) > 0$ then increase $\bar{q}_2$ or decrease $\bar{q}_1$.
6. Go back to step 2.

The updates of $\{\bar{q}_1, \bar{q}_2\}$ in 4 and 5 can be made using Newton steps. Observe that the solution problem is now simply a one dimensional search. We believe that this simple method will facilitate the use of the model in economic applications, both in simulations and empirical analysis.

Example 1:

In this and the subsequent examples we use a set of simple Matlab codes that solve the equilibrium for any given set of parameter values using the algorithm described above.

Consider the following parameter values:

$$c_b = 0 \quad c_p = 20 \quad p^m = 100 \quad M = 100$$
$$k_{1b} = 15 \quad k_{1p} = 15 \quad k_{2b} = 30 \quad k_{2p} = 30$$

Where k_{it} is the installed capacity of firm i in technology t and c_t is the marginal cost of technology t. Additionally we assume that quantities demanded are uniformly distributed on $[0, 100]$.

In Figure 3 we show the equilibrium supply functions (q_1^*, q_2^*). At $p = 64.5$, the baseload capacity for firm 1 is binding, at that price the slope of firm 1's supply function is larger than the slope of firm 2's supply function. At a price close to 92.6 firm 2 starts using peaker capacity. Starting at that price the gap in quantities offered decreases, since the slope of firm 1's supply function is smaller.

For comparison we include in Figure 3 an equilibrium in which firms are symmetric and the total capacity in the market is the same. Asymmetric capacities results in a significantly more collusive equilibrium.

5 Extensions

5.1 Electricity contracts

In most electricity markets, a large fraction of the capacity has been committed before the day-ahead market through electricity contracts. This means that for simulations it would be convenient to have this feature in a model of SFE. In this section we introduce bidders that have signed option contracts before participating in the spot electricity markets[2]. This affects their incentives to reduce supply.

We consider option contracts, in which the seller pays the difference between the

[2]Contract for differences are easier to introduce in the analysis since they do not result in discontinuities or flat sections in the equilibrium supply functions

equilibrium price and the strike price. Let p^s be the strike price and o_i be the amount of electricity contracts signed by firm i. If the equilibrium price is above the strike price, then firm i will pay $o_i(p(x) - p^s)$ to the holder of contract.

The objective function is:

$$\pi_i(\mathbf{q}) = \int_0^M [p(x,\mathbf{q})\tilde{q}_i(x,\mathbf{q}) - C_i(\tilde{q}_i(x,\mathbf{q})) - o_i[p(x,\mathbf{q}) - p^s]^+]dF(x) \qquad (13)$$

where $[a]^+ = max\,\{0, a\}$.

We will show that in this new setting, there can be discontinuities at the strike price and flat sections for the demand function above the strike price.

Proposition:

The equilibrium supply functions must satisfy:

$$\int_{\underline{q}(p^s)}^{\overline{q}(p^s)} \frac{\overline{q}_{-j}(p^s) - \underline{q}_{-j}(p^s)}{(\overline{q}(p^s) - \underline{q}(p^s))^2}(x - \underline{q}_{-j}(p^s))[p^s - c_i(\hat{q}_i(x,\mathbf{q})]dF(x) = (p^* - p^s)(\overline{q}_i(p^s) - o_i)f(\overline{q}(p^s))$$

For $i = 1, 2$ and $o_i \leq \overline{q_i}(p^s) - \underline{q_i}(p^s)$ for $i = 1, 2$ with equality for at least one of the firms and $[p^s, p^*]$ is a price range over which the supply functions q_i and q_j are constant.

Proof:

The conditions above imply that a firm does not have incentives to increase quantities offered at a price arbitrarily close but below p^s or decrease the quantities offered at p^s. The left side of the equation represents the gains in terms of more quantities sold at $p = p^s$ while the left side represent the losses in term of the fall in the price for $x = \overline{q}(p^s)$. Also observe that the critical price at which each supply functions is increasing p^* must be the same, otherwise the firm that it is offering incremental quantities at the lower price can profitably deviate by decreasing quantities offered at those prices.

Observe that if $o_i > \overline{q_i}(p^s) - \underline{q_i}(p^s)$ firm i could increase profits by offering additional quantities at p^s, this would not have any negative effect on prices while there is a positive effect on quantities.

Finally observe that if both firms have $o_i > \overline{q_i}(p^s) - \underline{q_i}(p^s)$ then just as we did in lemma 1, we can prove that there both firm find it profitable to deviate offering more quantities at prices close to but below p^s. $\square$

What we described above is the only new equilibrium feature introduced by option contracts. The rest of the analysis is not changed. In particular, it can easily be shown that the results on uniqueness and existence of the equilibrium still hold in this case.

5.2 Demand Response

We will consider the following stochastic demand function:

$$d(p, x) = max\,\{0, x - bp\}$$

Where $b > 0$ and x has cumulative distribution function $F : [0, 1] \to [0, M]$ strictly increasing and continuously differentiable.

Proposition:

Let $\overline{p}_i = inf\,\{p : q_i(p) = k_i\}$ and $p_0 = c_1(0) > c_2(0)$. The following are the necessary and sufficient conditions for an equilibrium:

i - $lim_{p \to p_0} q_1(p) = 0$ and $q_2(p) = 0$ for all $p < p_0$.

ii - $q_i(p_m) = k_i \ \forall i.$

iii- Each supply function is continuous $(p_0, \overline{p}_i)$ and at most one of the supply functions is discontinuous at p^m.

iv - $q_i(p) = (b + q'_{-i}(p))(p - c_i(q_i(p))$ almost everywhere on $(p_0, \overline{p}_i)$

v - Suppose bidder i's supply function is constant on $(\underline{p}, \overline{p}) \subset (\underline{p}_i, \overline{p}_i)$ then

$$\int_{q(\underline{p})}^{q(p')} [\frac{q_i(p(x))}{q'_{-i}(p(x))+b} - (p(x) - c_i(q_i(p(x))))]dF(x) \leq 0 \text{ for all } p' \in (\underline{p}, \overline{p}) \text{ and}$$
$$\int_{q(\underline{p})}^{q(\overline{p})} [\frac{q_i(p(x))}{q'_{-i}(p(x))+b} - (p(x) - c_i(q_i(p(x))))]dF(x) = 0$$

vi - $\int_{q(\overline{p}_i)}^{q(p')} [\frac{k_i}{(b+q'_{-i}(p(x)))} - (p(x) - c_i(k_i))]dF(x) \leq 0 \ \forall p' \in (\overline{p}_i, p^m)$

Proof:

Observe that in this case, a firm might offer all its generating capacity, k_i, at a price below the price cap, $\overline{p}_i \leq p^m$. Condition vi checks that this is optimal. Suppose condition v is not satisfied, then there is a price p' such that $\int_{q(p_i)}^{q(p')} [\frac{k_i}{(b+q'_{-i}(p(x)))} - (p(x) - c_i(k_i))]dF(x) > 0$. This expression is equal to the rate of change on the profit function when the quantity offered at prices $(\overline{p}_i, p')$ decreases by the same quantity on that range. So this means that the deviation is profitable.

The equations in condition v are the derivative of the objective function when the supply function is moved horizontablly for a range of prices where the supply is constant. The second equation implies that increasing or decreasing quantities for a range in which the supply function is constant has zero marginal effect on the profit function. The first expresion on v implies that changing the quantity for a fraction of the range in the direction in which the monotonicity constraints is not violated does not result in higher profits. $\square$

Proposition:

There exists a unique equilibrium in the model with price response.

Proof:

The argument is very similar to the one presented in section 2 in which we considered changes in the terminal conditions at p^m. The major difference is that now there might be a range of prices where the supply function is constant. But the result on lemma 5 of section 3 is still valid and this guarantees uniqueness of the equilibrium. $\square$

5.3 n-firm case

In this section we show that some of the previous results for the 2-firm case can be extended to the case in which there are n firms. The analysis is more complicated because there can be price ranges in which the equilibrium supply functions are constant. This does not allow for a straightforward use of the monotonicity results used in the previous section. Nevertheless most of the logic used in the 2-firm case extends to the n-firm case.

We will first present the necessary conditions of an equilibrium. We also show how an such a profile of supply functions can be calculated numerically using an algorithm that is similar to the one used in the 2 firm case. Then we show that for the special case in which there are $n-1$ firms of one type and 1 for of another type, all the results of the 2-firm case still hold. Last we prove in the general case that the necessary conditions below are in fact sufficient.

The special case in which $n-1$ firms are of the same type is an important step for the study of incentives in electricity markets. It allows for the study of investment decisions in a context in which firms are ex-ante symmetric. Understanding this case allows for the characterization of main paths of subgame perfect equilibria since that way we can analyze the equilibrium of the spot market when one firm deviates from the equilibrium investment level and all the other bidders still select the equilibrium action in the investment stage.

For the n asymmetric firms' case, we will assume that the demand function satisfies: $d(p,x) = max\{0, x+\rho(p)\}$ where $\rho(p) < 0$ and $\rho'(p) < 0$. That is, we assume that there is a price responding demand for every price.

Proposition:[3]

Let $\underline{p}_i$ =max$\{c_i(0); min_{j\neq i}\{c_j(0)\}\}$, $1 = argmin_{i\in N} c_i(0)$ and $\overline{p}_i$ =inf$\{p : q_i(p) = k_i\}$ The equilibrium supply functions must satisfy:

i - $q_i(p_m) = k_i \ \forall i$.

ii - Supply functions are continuous except at p^m where at most one of the supply functions is discontinuous and at $\underline{p}_1$ for firm 1's supply function.

iii- $q_i(\underline{p}_i) = 0$ for all bidders except bidder 1 whose supply function equals 0 for any price below $\underline{p}_1$.

iv - Suppose bidder i's supply function is increasing at p, then

$$q_i(p) = (q'_{-i}(p) - \rho'(p))(p - c_i(q_i(p)) \text{ for all } p \in (\underline{p}_i, \overline{p}_i).$$

v - Suppose bidder i's supply function is constant on $(\underline{p}, \overline{p}) \subset (\underline{p}_i, \overline{p}_i)$ then

$$\int_{q(\underline{p})}^{q(p')} [\frac{q_i(p(x))}{(q'_{-i}(p(x)) - \rho'(p))} - (p(x) - c_i(q_i(p(x)))]dF(x) \leq 0 \text{ for all } p' \in (\underline{p}, \overline{p}) \text{ and}$$
$$\int_{q(\underline{p})}^{q(\overline{p})} [\frac{q_i(p(x))}{(q'_{-i}(p(x)) - \rho'(p))} - (p(x) - c_i(q_i(p(x)))]dF(x) = 0$$

vi - For prices above $\overline{p}_i$ we have:

$$\int_{q(\overline{p}_i)}^{q(p')} [(\frac{k_i}{q'_{-i}(p(x)) - \rho'(p))} - (p(x) - c_i(k_i)]dF(x) \leq 0 \text{ for all } p' \in (\overline{p}_i, p^m).$$

Proof:

The conditions are similar to the ones presented in the case of price responsive de-

[3] Similar conditions are presented in Rudkevich [27] and Anderson and Xu [3], the second paper focuses on the case of one bidder's best response.

mand. Conditions in 5 imply that it is not profitable to change quantities for a range of prices over which the present supply function is constant. The equations in v and vi are conditions on the derivatives of the supply function with respect to quantities offered on a specific price range. Conditions i, ii, iii and iv are proved in the same way as in section 3. □

We can describe the conditions at the top for each supply function on the real line, let:

$$\bar{t}_i = \begin{cases} \lim_{p \to p^m} q_i(p) - k_i & if \quad q_i(p) < k_i \forall p < p^m \\ p^m - \overline{p}_i & otherwise \end{cases}$$

This means that for a given vector of terminal conditions t we can use conditions iii through vi to produce a set of supply functions: $q^t(p)$. We start by using the equation in iv to find the derivatives of the supply functions. Let N_p be the subset of bidders whose supply functions are strictly increasing at p and $\hat{n}$ be the size of set N_p, then:

$$q_i'(p) = \frac{1}{(\hat{n}-1)} \sum_{j \in N_p} \left[\frac{q_j(p)}{p - c_j(q_j(p))} + \rho'(p) \right] - \frac{(\hat{n}-2)}{(\hat{n}-1)} \left[\frac{q_i(p)}{p - c_i(q_i(p))} + \rho'(p) \right] ,$$

If at any price the monotonicity constraint for a bidder is binding then, this bidder is excluded from that price up to the price in which the second equation in condition v is met. If at any point condition vi is violated then the process stops and we are left with the constructed supply functions up to that point. If at any point condition i is violated then we stop and we are left with the constructed supply functions up to that point. This means that the supply functions are defined up to that price.

We will select point t such that conditions i and ii are satisfied. That means that at most for one bidder we have $t_i < o$. Otherwise more than two supply functions would be discontinuous at p^m.

Observe that we obtain a unique supply function for each point t.

In figure 4 we present the numerical calculation of an equilibrium for the case of 4 asymmetric bidders. The dotted lines represent the cost functions of each firm. We have asymmetry both in terms of size (installed capacity) and technologies. The continuous lines represent the equilibrium supply functions. Observe that there is one firm for which the supply function is constant on a range of prices. Also note there are two firms the supply all their installed capacity at a price that is strictly below the price cap.

In this case, we have used an algorithm in which different bottom terminal conditions. These conditions are the slope of the supply function on the right for firms 2 through n at price $\underline{p}_i$ and the quantity offered at $\underline{p}_1$. We select different values until we find conditions such that when the corresponding supply function is constructed, all the necessary conditions are satisfied. For a given profile of bottom terminal conditions, the supply functions are constructed using the equation for the derivatives presented above. If at any price the monotonicity is violated, then that bidders' supply function is constant up to the point in which the second equation of condition v is satisfied.

Now we concentrate in the special case in which the problem is significantly simplified. There are $n-1$ bidders of one type and 1 bidder of another type. Also we assume that the demand function does not respond to price.

Proposition Consider the SFE model with $n-1$ firms of one type and 1 of another type, then there exists a unique equilibrium.

Proof:

Let $n-1$ firms be of type A and the other firm of type B. First we note that, by lemma

1, equilibrium supply functions are continuous (except at p^m). The supply functions take value 0 up to price $p_0 = max\{c_B(0); c_A(0)\}$. Also the supply function of firms of type A cannot have flat sections on (p_0, p^m) because if it did on a range $(\underline{p}, \overline{p})$ then in equilibrium firm B would not offer any quantity on that price range and then no firm find it profitable offering additional quantities at prices below $\underline{p}$. So only firm B can select an equilibrium supply function which is constant on a range of prices.

Finally we check that lemma 5 is still valid in this context, that is we still have that the supply functions respond monotonically to changes in the terminal conditions at the top. This guarantees existence and uniqueness using the same arguments as in section 3. $\Box$

Finally we provide a result regarding sufficient conditions.

Proposition:

The necessary conditions $i - vi$ are sufficient conditions for an equilibrium.

Proof:

Anderson and Xu [3] prove that there exists a solution to each firms' optimization problem. They prove it by representing the strategy space as a parametrized two dimensional curve. They show that the strategy space is compact and the function is continuous. This means that each firm's optimization problem has a solution.

For each firm we have a candidate solution, that satisfies the necessary conditions and there is no other supply function that satisfies the necessary condition. This means that the profile of supply functions is in fact an equilibrium. $\Box$

6 Applications

In this section we illustrate how a model of asymmetric SFE can be used to gain insight about incentives in electricity markets. We study the incentives, first, for firms to invest in generating capacity and, second, for firms to sell option contracts. We emphasize that an asymmetric model is needed to analyze these issues. Even if firms are ex-ante symmetric, the actions of investing in capacity and selling contracts result in ex-post asymmetries. We believe that the model presented in this paper will result in an improved understanding of incentives and conseguently in better electricity market design.

6.1 Investment in generating capacity

Incentives to invest in generating capacity are a critical issue in electricity markets. Incentives may be especially low for some peaker plants that might be dispatched only a few hours a year. If the electricity market design imposes a low price cap, then these plants would not recover their fixed costs and investment would not occur.

One possible policy measure that would result in stronger incentives to invest is to set higher price caps. This is one possible solution to the resource adequacy problem. In this section we evaluate how increasing the price cap impacts on the incentives to invest in peaking capacity. Also, we evaluate consumer welfare and firms' profits under each price cap.

In a perfectly competitive market, the price cap should be chosen to be equal to the cost of lost load: this would result in the efficient level of investment and each plant's profits in the spot market would equal the capital cost, that is, generators would break even[4]. If generators have market power the outcome can be significantly different.

We consider a model with four firms, each with installed capacity in two types of

[4]See Stoft [29] for more details on the perfectly competitive case.

technology: baseload and peaker. Baseload capacity (e.g. coal plants) have a lower marginal cost than peaker plants (e.g. gas turbines). However, the capital costs of baseload plants is higher than the capital costs associated with peaker plants. That results in an efficient technology portfolio with positive quantities of installed capacity of each technology. The parameters of the simulation are:

$k_{ib} = 10 \quad k_{ip} = 10 \quad i = 1, 2, 3, 4$

$c_b = 20 \quad c_p = 40 \quad M = 100 \qquad p^m = 100$

Where k_{it} is the installed capacity of firm i in technology t and c_t is the marginal cost of technology t. We also assume that the distribution is uniform on $[0, 100]$.

To add a more realistic element to the analysis, we assume all firms commit capacity under electricty contracts. There are two types of contracts, contracts of type b with a strike price of 0 and contracts of type p with a strike price of 40. At the original level of installed capacity, firms have contracts of type b for 75% of the baseload installed capacity and contracts of type p for 75% of their peaker capacity.

Generators' total profits, π_i^t, equal the sum of spot market profits plus the revenue that results from signing option contracts:

$$\pi_i^t(o, p_b, p_p) = \pi_i^s(o) + p_b o_{bi} + p_p o_{pi} \ ,$$

where o is the vector describing the profile of contracts signed by generators. The price of each type of contract is given by:

$$p_b = \int_0^M p(x) dF(x)$$
$$p_p = \int_0^M (p(x) - 40) dF(x)$$

That is, the price of each contract equals the marginal savings for a consumer, keeping the profile of firms' strategies constant, or in other words, is the price that we would expect when consumers are price takers and risk neutral. With risk averse consumers, the price would be higher but the qualitative results we show in this section still hold. Suppose that at the current market conditions, the regulator finds desirable the construction of a peaker plant that would increase by 2.5% the peaker capacity of the market. In our simulations we study the incentive that a firm has to make this investment.

In the simulations we analyze how an increment of 50% in the price cap affects the incentive to invest. We can think of the exercise as the study of a simple game in which each firm can only decide on one decision variable. In our model only one firm invests, is consistent with the property that investment decisions are strategic substitutes.

In Table 1 we summarize the results of the simulations. CC is the total cost of electricity to consumers. The last row indicates the total incentive to invest in an additional 2.5% of peaker capacity.

Table 1: Simulation results: Investment in peaking capacity

	$p_m = 100$	$p_m = 150$
$k_{1p} = 10$		
π_1	473.7	791.2
CC	3088.0	4352.7
$k_{1p} = 11$		
π_1	1692.3	812.6
$\Delta\pi_1^t$	11.6	21.6

The simulations indicate that there is an increase in the incentives to invest. With a price cap of 100 the change in the profits from generating electricity is 11.6 while with a price cap of 150 the change in profits for the firm that invested is 21.6. This is a significant increment in the return on investment. But we also observe that there is a much larger change in the total cost of electricity to consumers, there is an increment of more than 1100 in the total cost when the price cap jumps from 100 to 150. The change on the total cost to consumers is much greater than the change on the return on investment in new capacity.

From this numerical simulation we observe that, unless it is accompanied by some additional change in the market design, raising the price cap can be a very costly way to increase incentives to invest in new generating capacity. The change in the cost to consumer is generally much larger than the change in the returns on investment. The decision to raise the price cap should be made together with some other measures that would mitigate market power. If that is not available, then directly targeting investment through subsidies is a more appropriate solution[5].

In a second part of this exercise, we consider the incentives for firms to invest in different technologies. The efficient investment is the one that minimizes the sum of the cost of generating electricity and the capital costs. Let RI^b be the change in firm i's profits when the baseload capacity increases by Δ and RI^p be the change in i's profits when its peaker capacity increases by Δ. For investment to be efficient, the difference $RI^b - RI^p$ must equal the saving in generating cost that is attained when investment in baseload capacity is selected over investment in peaker capacity. This is because fixed costs that enter the calculations of firms' profits and variation in costs cancel each other out.

In table 2 we compare these values for the same setting as used in the previous analysis and $\Delta = 2$.

We observe the difference between the return on investment in each capacity is too small, thus there are values of the fixed costs for which equilibrium investment would be inefficient. For a price cap of 150 the difference between the change in profits is 18.2; this is 23% below the savings in the cost of generating electricity. This means

[5] In this simple exercise we are not evaluating capacities payments or more advanced market designs (e.g. forwards of installed capacity Cramton and Stoft [8]) that are used or have been proposed to solve the resource adequacy problem.

Table 2: Simulation results: Investment different technologies

	$p_m = 150$	$p_m = 225$
RI^b	60.0	87.1
RI^p	41.8	70.3
$RI^b - RI^p$	18.2	16.8
$\Delta Gen.Costs$	23.6	23.6

that, in this example, imperfectly competitive markets result in a bias toward investment in peaking capacity. Let f_t be the fixed cost associated with technology t. For fixed cost satisfying $\Delta Gen.Costs > f_b - f_p > RI^b - RI^p$ there will be investment in peaking capacity when the efficient choice is baseload capacity.

For a higher price cap, we observe that the bias is stronger, the savings in generation cost is 29% higher than the difference between the variation in profits. The bias is coming from market power, thus allowing for larger mark-ups results in a stronger bias.

6.2 Electricity Contracts

By signing electricity derivatives (e.g., option contracts) a generator is partially giving up its ability to exercise market power in the spot market. This implies that when studying pricing and trading of these contracts we need a model of imperfect competition in addition to an understanding of the traditional financial considerations.

As an illustration, in this section we show that a generator, despite gains from risk sharing, might choose optimally not to sell option contracts.

We assume there are two firms, each with installed capacity of 40 in a constant marginal cost technology. Marginal cost, c, is a random variable, distributed uniformly on $[26, 62]$. This variation can be interpreted as the uncertainty over the price of fuel. Additionally we consider that there exist option contracts with a strike price equal to the realization of the marginal cost[6]. This way, consumers assume the fuel risk but hedge the additional risk associated with the mark-ups of the imperfectly competitive market. Let o_1 and o_2 be the respective quantities of contracts signed by each firm. Each firm is risk averse, with logarithmic Bernoulli utility function. The expected profit of each firm equals:

$$EP_i(o_i, o_j) = E_c(ln(\pi_i(o_i, o_j, c))) + p_o o_i,$$

where $\pi_i(.)$ is the profit in the spot market and p_o is the revenue that results from selling option contracts [7].

As in the previous examples, load is distributed uniformly on $[0, 100]$ and the price cap is 100. The spot market is a uniform auction and firms play the unique SFE.

Suppose firm 1 signed contracts for 75% of its capacity. We will focus on the incentives of firm 2 to sell contracts in an auction where firm 2 is the only supplier and the pricing is uniform. The demand side in this mechanism is a price taker. More specifically, the inverse demand function equals the expected savings from an additional unit of option contracts plus a risk premium, RP:

$$p_o = \int_{c}^{\overline{c}} \int_{0}^{M} max\left\{0, p(x, c, o_1, o_2) - c\right\} dF(x)dc + RP \tag{14}$$

In the numerical calculation we assume that the risk premium equals 20% of the savings that result from an additional unit of the contract. Given the inverse supply function, firm 2's optimization problem is reduced to choosing a quantity of contracts

[6] Given the parameter of this model, the option contracts we consider are equivalent to contract for differences with a reference price equal to the realization of the marginal cost

[7] Some gas fired plants sign take-or-pay contracts, under these arrangements the plant's owner pays for the gas even when it is not used. In this situation the opportunity cost of not providing electricity is higher and there are more incentives to sign option contracts. Our analysis reflects more closely the problem of a generator that does not incur the fuel cost when load is not served.

to be sold:

$$EP_i(o_i, o_j) = E_c(ln(\pi_i(o_i, o_j, c))) + p_o(o_i)o_i$$

We solve the optimization problem numerically. Our calculations show that this function is maximized at $o_i^* = 0$. The derivative of the function is provided in Figure 5. From this example we observe that generators might not sign option contracts due to market power considerations. Providing option contracts results in a lower prices for these contracts and additionally lower profits in the spot market.

In particular our example shows that selling contracts in a market is costly because an increase in the quantity provided results in lower prices for these contracts. This might explain why markets for contracts have not developed and most of this contracts are negotiated by two parts. In a two part negotiation the generator can extract more revenue from a consumer, in this way, a generator can find it optimal to sell some option contracts.

7 Concluding remarks

In this paper we make a significant contribution to the understanding of asymmetric SFE model. We prove the existence and uniqueness of the equilibrium and fully characterize the equilibrium strategies. Also, we provide a simple algorithm to solve numerically the system of differential equations that characterizes the equilibrium supply functions. We also present extensions of the basic model, we study cases with elastic demand and cases where bidders may sell option contracts before the auction.

The tools developed here can be used to gain insight into important issues in electricity market design, in particular, problems of resource adequacy and market power. As an example, we present a simple model of investment decisions. We evaluate how increasing the price cap affects incentives to invest and consumer welfare. We find that a price cap increment that is not accompanied by any market power mitigation measure may result in an increase in the cost to consumers that greatly outweighs the change in the returns on investment in new capacity. We observe that in our example there is a bias toward investment in peaking capacity. We also find that increasing the price cap results in a larger inefficient bias toward investment in peaking capacity.

As an additional illustration of how asymmetric SFE can provide insight into electricity market design, we consider generators' incentives to trade option contracts. We present a scenario in which, despite the existence of gains from risk sharing with LSEs, a generator does not sell electricity contracts due to market power considerations.

Figure 1: Necessary conditions of an equilibrium

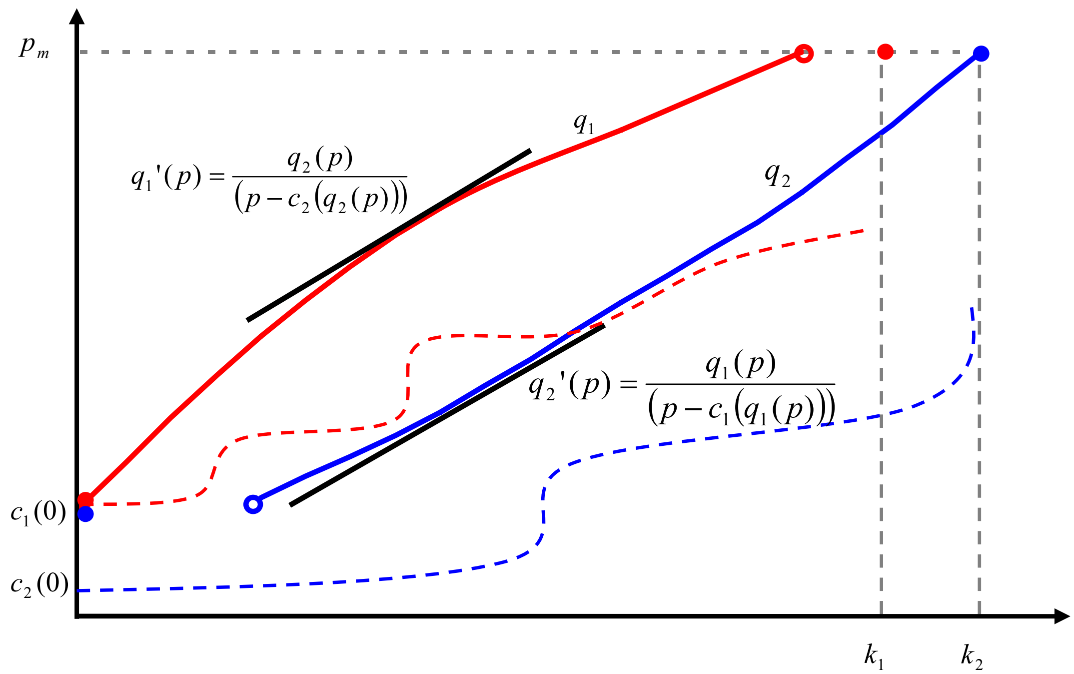


Figure 2: Example of the result on lemma 5

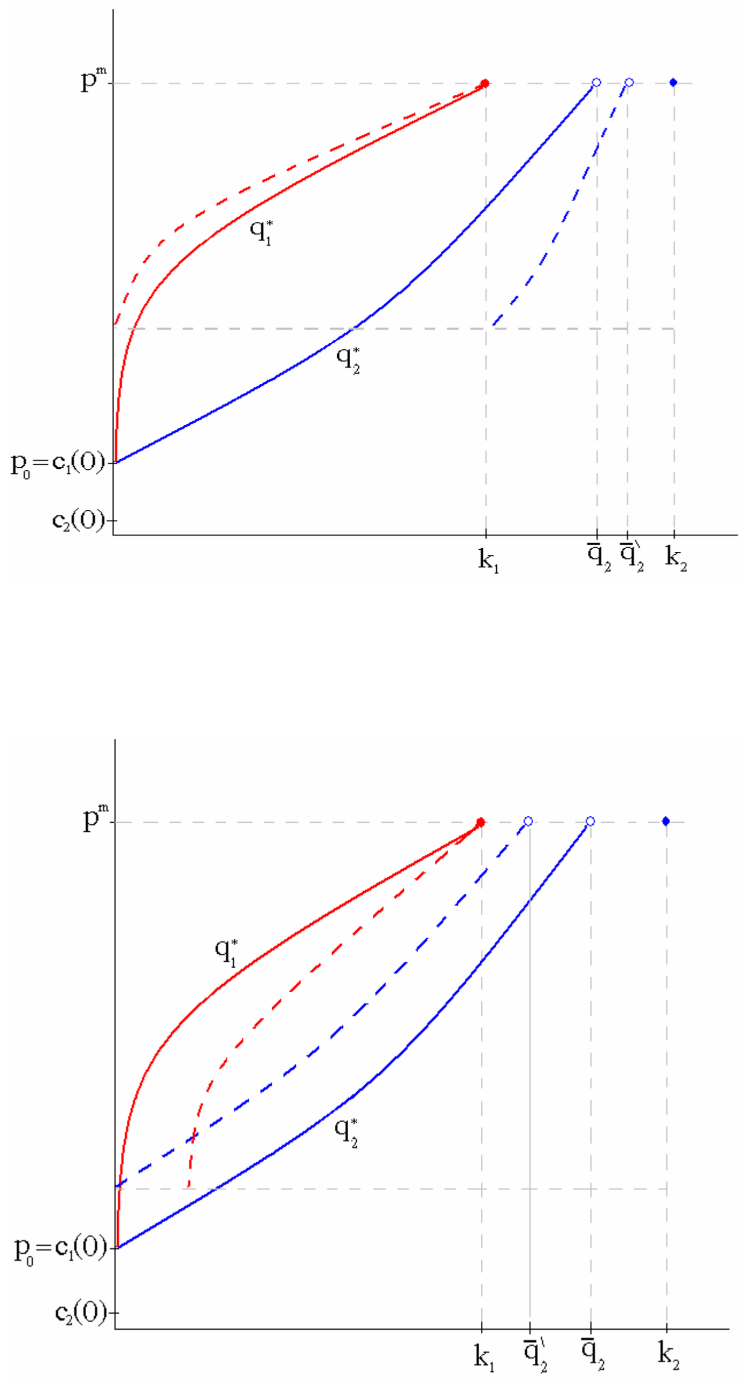


Figure 3: Numerical calculation of an asymmetric SFE

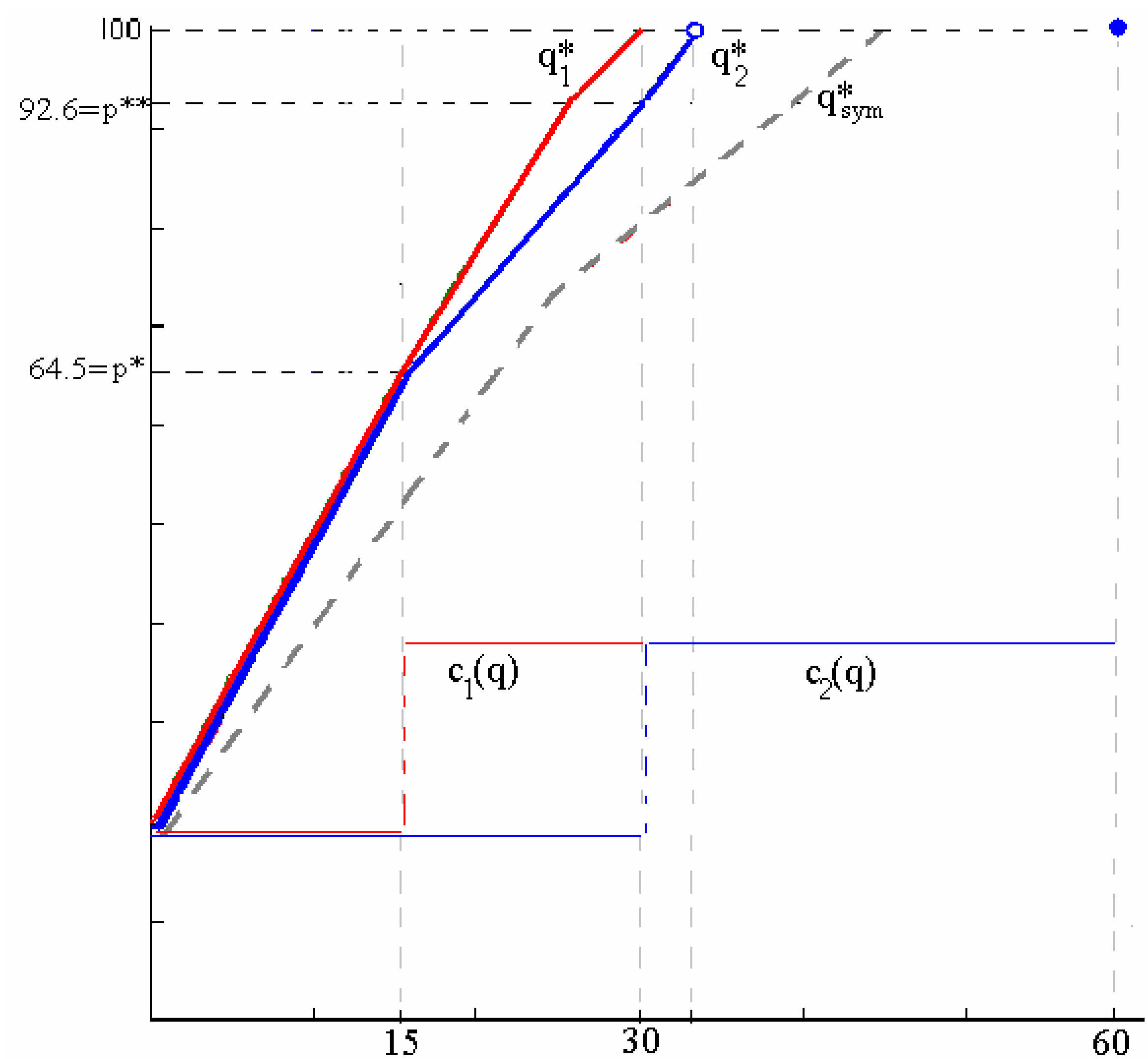


Figure 4: Numerical calculation of an asymmetric SFE with four asymmetric bidders

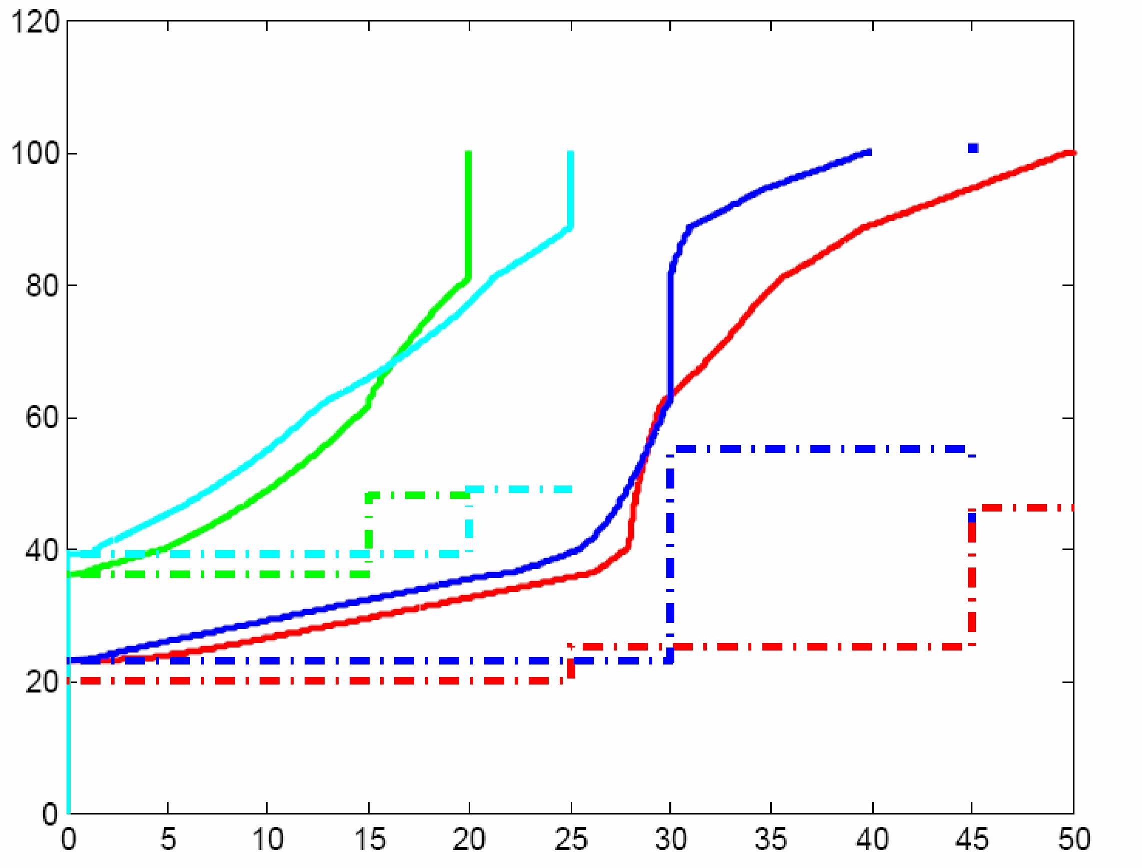
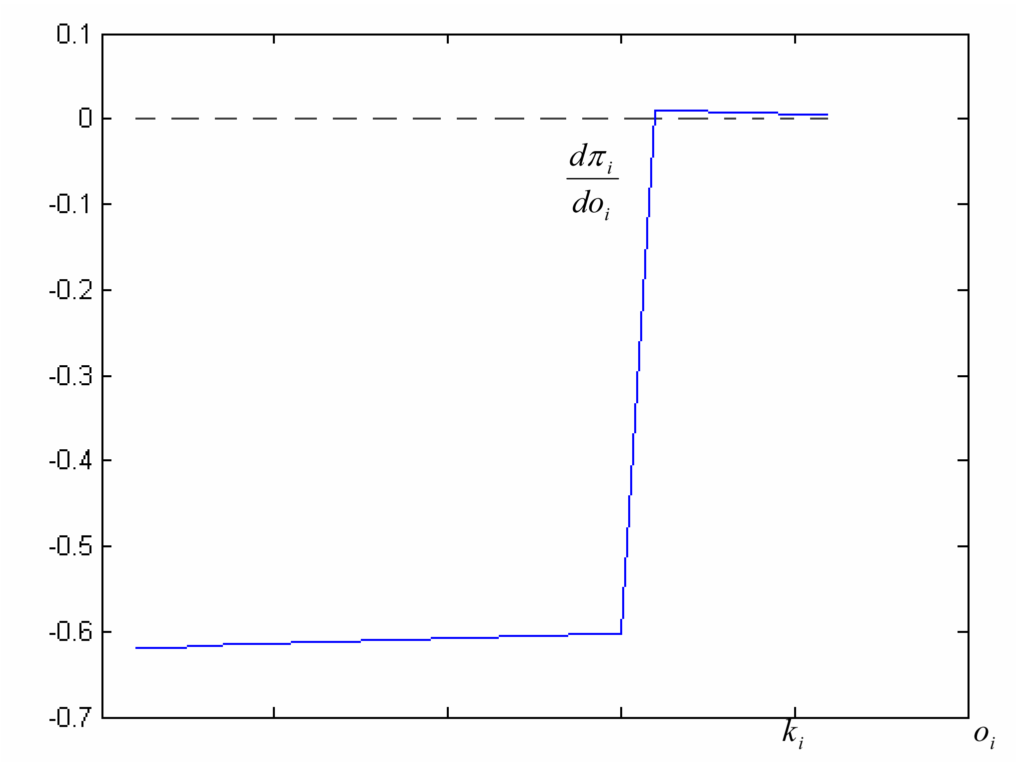


Figure 5: Derivative of firm 2's profit function with respect to number of contracts signed.



Chapter 2:

Strategic Effect of Option Contracts

Abstract

Option contracts are introduced to a supply function equilibrium (SFE) model. The uniqueness of the equilibrium in the spot market is established. Comparative statics results on the effect of option contracts on the equilibrium price are presented. A multi-stage game where option contracts are traded before the spot market stage is considered. When contracts are optimally procured by a central authority, the selected profile of option contracts is such that the spot market price equals marginal cost for any load level resulting in a significant reduction in cost. If load serving entities (LSEs) are price takers, in equilibrium, there is no trade of option contracts. Even when LSEs have market power, the central authority's solution cannot be implemented in equilibrium.

1 Introduction

Wholesale electricity markets exhibit characteristics that can result in severe strategic supply reduction. Highly inelastic demand, transmission constraints and non-storability are among these characteristics. Supply reduction not only affects equilibrium prices but also network security and the cost of providing ancillary services. Option contracts and other electricity derivatives have been considered to mitigate market power while keeping an adequate level of investment (see [7] and [26]).

In this work, we study the potential benefits of option contracts in mitigating market power. For this, first, we evaluate the effects of option contracts on the spot market equilibrium. Next, we consider, for different market structures, the incentives that generators and LSEs have to trade these contracts.

One of the contributions of this work is to introduce option contracts to a Supply Function Equilibrium (SFE) model. Following common practice in the literature, we use the expression Supply Function Equilibrium (SFE) to refer to the equilibrium of a uniform price reverse auction, in which there is no asymmetric information. By issuing a "call" option, a supplier commits to provide a certain amount of units at a specific price in case these units are demanded. We analyze in detail how these contracts affect the incentives of generators to exercise market power.

Next, we use the spot market as an input to analyze multistage games in which option contracts are traded in rounds of negotiations that precede the spot market. We model these interactions explicitly. In this, we depart from common practice in the literature that simply imposes a no-arbitrage condition. Explicit modeling of bargaining is needed because in imperfectly competitive industries there are nontrivial interactions between spot and derivatives markets.

Two different settings will be considered. In a first scenario a central authority (or social planner) procures option contracts with the objective of minimizing the total cost of energy. Total cost of energy includes the cost of procuring option contracts and the net revenue in the spot market. The outcome from this setting will be used as a benchmark result reflecting the potential gains from option contracts in terms of market power mitigation. Alternatively, this setting can be viewed as a market in which there is a unique consumer of electricity, a monopsonistic market structure.

Next, we analyze the effect of option contracts in a decentralized environment. LSEs are modeled alternatively as price taking agents and agents with market power. LSEs are able to perform bilateral bargaining of option contracts with generators. We will focus on how much of the gains presented in the centralized setting can be attained in a setting in which LSEs act independently.

The analysis in this paper shows that when market power is present, some regulatory intervention on the option contracts markets can result in better market outcomes. If consumers are price takers, option contracts without regulatory intervention do not accomplish significant market power mitigation.

Our results can also be applied to other industries. Multiunit auctions are increasingly used in the private and public sector. For example, uniform price auctions are used by governments to sell bonds and lumber contracts.

In the following section we comment on the related literature. Section 3 presents the spot market model; this is followed by the analysis of multistage games in which financial positions are endogenously determined. The last section considers extensions and concludes.

2 Related Literature

The study of SFE was started by Klemperer and Meyer [20]. These models constitute a generalization of more traditional models used in the study of oligopolies. Instead of assuming that firms compete in price or quantities, these authors allowed firms to select a supply function, that is, a mapping that assigns to each price a quantity offered.

Most of the literature focuses on the case in which the position of the demand function is uncertain. Tractability and multiplicity of equilibrium are common problems when selecting this approach.

SFE has received renewed attention since it is considered an adequate representation of wholesale electricity markets. See for example Newbery [24], Green [13] and Rudkevich et al. [28].

Our model is close to the approach taken by Holmberg [15] in allowing for equilibrium excess of demand. That is, demand might be higher than firms installed capacity with strictly positive probability. That will result in a boundary condition that guarantees uniqueness of the spot market equilibrium.

To our knowledge, there is no piece of literature that considers option contracts in a SFE. There are some works considering contracts for differences (e.g. Green [13]).

Another related piece of literature is the one started by Allaz and Vilas [1]. They analyze the effect of futures contracts on market power in a model of Cournot competition. They conclude that future contracts result in significantly more competitive market outcomes. We extend their work in many dimensions. First we work with a model of supply function equilibrium, second, we are allowing for option contracts. Observe that futures contracts might be viewed as option contracts with a strike price of 0. We also model the financial contract trading stage explicitly, while in their work a simple no-arbitrage condition is imposed. Ferreira [10] and Hughes et alt. [19] extend this literature by analyzing the effect of observability in the model.

Finally, as mentioned in the introduction, there are some electricity markets policy papers that stress the role of financial derivatives in allowing for more competitive outcomes and an adequate level of investment(see Wilson et alt. [7] and Oren [25]).

3 The spot market

In this section we present a model of the spot wholesale electricity market in which generators have committed capacity under electricity option contracts. In the next section we will use the spot market as an input in a model in which option contracts are negotiated before the spot market.

There are n suppliers selecting supply functions in a uniform price reverse auction with a finite grid of prices $\{p_k\}_{k=0}^{M}$, with $p_0 = 0$. An strategy for firm i consist of a maximum quantity offered q_k^i for each price in the grid p_k, $\{q_k^i\}_{k=0}^{M}$. This sequence must be increasing in k. We use q_k to refer to the aggregate supply at p_k.

The demand function is inelastic, the quantity demanded equals x and is distributed according to $F(x)$ with support $[0, \infty)$. Suppliers face constant marginal cost normalized to zero up to *cap*, the total capacity of a firm. When the superindex i is omitted we refer to the aggregate variable.

There are also option contracts: o_k^i is the quantity committed by firm i at a price of k, $o_k = \sum_{i=1}^{n} o_j^i$ and $O_k^i = \sum_{j=0}^{k} o_j^i$. Firms always comply with options contracts which are of a physical nature. That is, generators must provide the energy commited under the contract. This implies that the maximum quantity that is offered by a firm in the spot market is $cap - O_M^i$, otherwise with positive probability a firm might not be able to supply quantities committed under option contracts. We are assumming that the sanctions to generators are high enough such they do not always comply with the obligation. Also, we are ruling out forced outages.

The equilibrium price is the minimum p_k such that $q_k + O_k \geq x$. We assumme proportional rationing over incremental quantities. There are two situations in which rationing might occur in equilibrium. If $q_{k-1} + O_{k-1} < x < q_{k-1} + O_k$, no spot market quantities are called at the clearing price. The quantity supplied by agent i equals the quantity offered below the clearing price (including exercised options) plus a fraction of the excess demand at p_{k-1} proportional to o_k^i / o_k. In the case $q_{k-1} + O_k < x < q_k + O_k$ similar proportional rationing is applied on the additional quantities offered at p_k.

In what follows, we will assume that the marginal cost of production is constant and we will normalize it to 0.

For a given demand level x the profit function equals:

$$\Pi(q_k^i, o_k^i, x) = p_k q_{k-1}^i + \sum_{j=0}^{k-1} p_j o_j^i + p_k o_k^i min\left[1, (x - q_{k-1} - O_{k-1})/o_k\right] + p_k max\left[0, (x - q_{k-1} - O_k)\frac{(q_k^i - q_{k-1}^i)}{(q_k - q_{k-1})}\right] \tag{1}$$

The first two terms refer to the units offered at a price below the clearing price. For the last two terms, the proportional rationing rule is applied, the quantities offered at the clearing price are used in the calculation. As x is not observed, the expected value must be calculated in order to find an expression for the objective function:

$$E_x\left(\Pi(q_k^i, o_k^i, x)\right) = \sum_{k=0}^{M} \int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} \left[p_k q_{k-1}^i + \sum_{j=0}^{k-1} p_j o_j^i + p_k(x - q_{k-1} - O_{k-1})o_k^i/o_k\right] dF(x) + \sum_{k=0}^{M} \int_{O_k+q_{k-1}}^{O_k+q_k} \left[p_k q_{k-1}^i + \sum_{j=0}^{k} p_j o_j^i + p_k(x - q_{k-1} - O_k))\frac{(q_k^i - q_{k-1}^i)}{(q_k - q_{k-1})}\right] dF(x) \tag{2}$$

The first group of integrals corresponds to the profit levels attained when only option contracts are demanded at the clearing price. The second group of integrals corresponds to the cases in which quantities offered at the clearing price in the spot market are also demanded. The constraints of the problem are:

$$q_k^i \geq q_{k-1}^i \forall k$$
$$q_M^i \leq cap - O_M \tag{3}$$

The first condition implies that the submitted supply function must be increasing. The second implies that the maximum quantity offered cannot be higher than the installed capacity minus the total amount committed according to the option contracts. This way, with a discrete grid, each supplier simply has to solve a programming problem in R^n with a set of linear constraints.

For future reference we provide here an expression for the partial derivative with respect to q_{k-1}^i:

$$\frac{\partial\Pi(.,.)}{\partial q_{k-1}^i} = -f(O_{k-1}+q_{k-1})q_{k-1}^i(p_k - p_{k-1})$$
$$+p_{k-1}\int_{O_{k-1}+q_{k-2}}^{O_{k-1}+q_{k-1}} \frac{(x - q_{k-2} - O_{k-1})(q_{k-1}^{-i} - q_{k-2}^{-i})}{(q_{k-1} - q_{k-2})^2} dF(x)$$
$$+p_k\int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} \frac{o_k^{-i}}{o_k} dF(x) + p_k\int_{O_k+q_{k-1}}^{O_k+q_k} \frac{(q_k^{-i} - q_{k-1}^{-i})(q_k + O_k - x)}{(q_k - q_{k-1})^2} dF(x) \tag{4}$$

The usual quantity-price trade off can be observed. The first term is negative and reflects the fall in the clearing price. The remaining three terms reflect the increase in quantities called as a consequence of the higher q_k^i.

The same expression for the case in which all suppliers are selecting the same strategy and have the same financial position(symmetry):

$$\frac{\partial\Pi(.,.)}{\partial q_{k-1}^i} = -f(O_{k-1}+q_{k-1})\frac{q_{k-1}}{n}(p_k - p_{k-1})$$
$$+p_{k-1}\frac{(n-1)}{n}\int_{O_{k-1}+q_{k-2}}^{O_{k-1}+q_{k-1}} \frac{(x - q_{k-2} - O_{k-1}))}{(q_{k-1} - q_{k-2})} dF(x)$$
$$+p_k\frac{(n-1)}{n}\int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} dF(x) + p_k\frac{(n-1)}{n}\int_{O_k+q_{k-1}}^{O_k+q_k} \frac{(q_k + O_k - x)}{(q_k - q_{k-1})} dF(x) \tag{5}$$

3.1 Equilibrium characterization

These propositions partially characterize the equilibrium:

Claim 3.1 In a symmetric equilibrium a positive quantity is offered at p_0.

Proof

Suppose not. Then it must be the case that the partial derivative of the objective function with respect to q_0^i is nonpositive. But observe that in that case the partial derivative with respect to q_0^i is positive as long as $q_1 > 0$ or $o_1 > 0$. If none of this condition is satisfied consider $q_{i+1} = 0$ where q_i is the variable just analyzed. We arrive to a contradiction. □

Claim 3.2 In a symmetric equilibrium all available capacity is offered at the cap price, that is $q_M^i + O_{M-1}^i = cap$.

Proof

Suppose the equation above is not satisfied. Then, observe that increasing the quantity offered at p_M results in a weakly larger number of quantities sold without any negative effect on price. This implies that the supply function is not a best response. □

According to our previous observation, in an equilibrium of the game, suppliers solve a programming problem in R^n. In particular, we have that if $q_{k-1} < q_k < q_{k+1}$ then the partial derivative of the profit function with respect to q_i must equal zero. Similarly if $q_{k-1} = q_k < q_{k+1}$, then the partial derivative with respect to q_k must be nonpositive. Finally if $q_{k-1} < q_k = q_{k+1}$, then, the optimality conditions imply that the partial derivative with respect to q_k must be nonnegative.

Observe that when all q's are different, the partial derivatives define a second order nonlinear difference equation.

3.2 Uniform Distribution

The simplest case to analyze is the case in which the quantity demanded (or load) is distributed uniformly. We will keep this assumption for the rest of the analysis. We claim that this results in a reasonable approximation since, as we will show below, for any continuous distribution, the partial derivative of the objective function converges to the uniform distribution case as the price grid becomes finer fine.

With a uniform distribution the objective function is given by:

$$\Pi(o,s) = \sum_{k=1}^{M} o_k[p_k[q^i_{k-1} - \frac{o^i_k}{2}] + \sum_{j=1}^{k-1} p_j o_j]$$
$$+(q_k - q_{k-1})[p_k[q^i_{k-1} + \frac{q^i_k - q^i_{k-1}}{2}] + \sum_{i=1}^{k} p_j o^i_j] \tag{6}$$

Note that the expression is, for each type of rationing (option contracts or spot market quantities), the sum of the probability of a certain price being the equilibrium price times the expected profit conditioned on a given equilibrium price. There are two terms since in some cases there is rationing in option contracts and in other cases the rationing is on the spot market quantities. With a uniform distribution we have a simpler expression for the partial derivative:

$$\frac{\partial \Pi(.,.)}{\partial q^i_k} = -q^i_k \Delta + p_k \frac{(q^{-i}_k - q^{-i}_{k-1})}{2} + p_{k+1} \frac{(q^{-i}_{k+1} - q^{-i}_k)}{2} + o^{-i}_{k+1} p_{k+1} \tag{7}$$

Where Δ is equal to the constant distance between prices in the grid.

Observe that the value of the partial derivative does not depend on each firm's financial position. That is to say, contracts affect incentives through the financial positions of the other suppliers. The effect is positive, that is, when the rivals increase the number of contracts at p_k there are more incentive to expand the supply at that price. Also observe that q^i_k is the only element of the supply function of generator i entering the partial derivative. Finally note that the partial derivative increases in q^{-i}_{k+1} and decreases in q_{k-1}^{-i}.

Note that this effect is the opposite of the one found in Allaz and Vilas.

With symmetry:

$$\frac{\partial\Pi(.,.)}{\partial q_k^i} = -q_k\Delta\frac{n+1}{2n} + \frac{n-1}{2n}\left[p_{k+1}q_{k+1} - p_k q_{k-1} + 2p_{k+1}o_{k+1}\right] \tag{8}$$

Theorem 3.1 In the symmetric spot market model there exists a unique symmetric equilibrium.

Proof

First, for each possible value of q_{M-1} find there is a unique value for q_M such that the first order condition is satisfied. This is because the partial derivative is always decreasing in q_k for any k. Also observe that the value for q_M that satisfies the first order condition is decreasing in q_{M-1}.

Now we are going to use an induction argument. Consider k such that for all j with $k < j \leq M$ there exist a unique decreasing function $q_j(q_{j-1})$ that gives the quantity for which the first order conditions are satisfied for all $z > j-1$. Then we can prove that there exist a function $q_{j-1}(q_{j-2})$ with the same properties. Replacing $q_j(q_{j-1})$ on the partial derivative with respect to q_{j-1} and the applying the implicit function theorem, we can check that in fact the inductive hypothesis is satisfied. Finally we have that for $k = 0$, the value below is equal to 0 and this results in the unique equilibrium that can be recovered by using the functions mentioned above.□

As we mentioned above, we consider that the assumption of a uniform distribution is a reasonable one. We have that, for any continuous distribution, as the price grid becomes finer, the conditions characterizing the first order conditions converge to the conditions of the continuous grid case which is independent of the distribution of the quantities demanded (see Holmberg [15]).

We already observed that option contracts have a procompetitive effect on the incentives of generators. The partial derivative for a quantity at any price level, increases with the number of contracts. Below we extend that result by prensenting comparative statics of the best response function.

Claim 3.3 For given supply functions and financial position of other generators, an increase in the level of financial contracts for generator g results in lower equilibrium prices for all the quantities in the support. Also, the spot market supply function for firm g remains the same except the maximum quantity falls one to one with the change in the sum of option contracts.

Proof

Simply observe that the partial derivative is not affected, so the optimum quantities offered in the spot market of each price do not change but given that more contracts have been signed the equilibrium price must decrease for all quantities.□

The following lemma is a comparative statics results of the equilibrium. It will be useful in the following section, when we characterize the optimal procurement policy of a central planner whose objective is to minimize the total cost of electricity.

Lemma 3.1 Consider the spot market model with option contracts such that p_k is the maximum price in equilibrium then, an increment in o_k^i for all i results in a new symmetric equilibrium with $q_k^{i\prime} = q_k^i - \Delta o_k^i$ and higher quantities offered in the spot market for all prices below p_k.

Proof

First we note that at the new $q_k^{i\prime}$ keeping k_{k-1}^i constant, the partial derivative with respect to quantity is still nonnegative. For q_{k-1}^i by inspecting the partial derivative we check that the positive effect of a higher o_k is higher than the negative effect of the lower q_k. This implies that for any q_{k-2} the new q_{k-1} that satisfies the equilibrium is higher. Now applying induction we can check that this is true for any q_j^i with $j \leq k-1$. This concludes the proof.□

Also we can consider the partial derivative of the objective function with respect to the financial position, this will be useful below when using the envelope theorem.

$$\frac{\partial \Pi(.,.)}{\partial o_k^i} = p_k(cap - q_k - O_k) + p_k q_k^i + \sum^{k-1} p_j o_j^i + \frac{p_k o_k^i}{2} + \frac{p_k o_k}{2}$$
$$- \left[p_M q_{M-1}^i + \sum^{M-1} o_j^i p_j + p_M \frac{q_M^i - q_{M-1}^i}{2} \right] - p_M \frac{q_M - q_{M-1}}{2} - o_M p_M \tag{9}$$

3.2.1 Special Case

For sufficiently low levels of option contracts the unique equilibrium supply function is strictly increasing and is characterized by the linear difference equation below:

$$q_{k-1} = \frac{2(n-1)p_k(o_k)}{(n+1)(p_k - p_{k-1})} + \frac{(n-1)(p_k q_k - p_{k-1} q_{k-2})}{(n+1)(p_k - p_{k-1})} \tag{10}$$

This is a second degree nonhomogeneous difference equation. There are two boundary conditions: zero quantities are offered below p_0 and all the available capacity is offered at p_K.

Note that this difference equation converges to a differential equation:

$$q_{k-1} = \frac{2(n-1)p_k(o_k)}{(n+1)\Delta} + \frac{(n-1)(p_k q_k - p_{k-1}q_{k-2})}{(n+1)\Delta} \tag{11}$$

$$\rightarrow \lim_{\Delta \to 0} \frac{2(n-1)p_k(O_k - O_{k-\Delta})}{(n+1)\Delta} +$$

$$\lim_{\Delta \to 0} \frac{(n-1)(p_k(q_k - q_{k-1}) + p_{k-1}(q_{k-1} - q_{k-2})}{(n+1)\Delta} + \frac{(n-1)q_{k-1}}{(n+1)} \tag{12}$$

$$\tag{13}$$

$$q(p) = \frac{(n-1)p}{n}\frac{\partial q(p)}{\partial p} + \frac{(n-1)po(p)}{n} \tag{14}$$

This equation can be compared to similar expression obtained in continuous price models. This seems to indicate that as long as the cost of submitting an additional step in the bidding function is not too costly, the continuous case and the discrete case with a sufficiently dense grid should predict similar results.

The system of linear difference (or recurrence equations) can also be solved explicitly. We can rewrite the equation as:

$$q_k = a_k q_{k-1} + b_k q_{k+1} + c_k \tag{15}$$

Where:

$$\begin{aligned} a_k &= -\frac{(n-1)p_k}{(n+1)(p_{k+1} - p_k)} \\ b_k &= \frac{(n-1)p_{k+1}}{(n+1)(p_{k+1} - p_k)} \\ c_k &= \frac{2(n-1)p_{k+1}(o_{k+1})}{(n+1)(p_{k+1} - p_k)} \end{aligned} \tag{16}$$

Since the lowest and the highest value for q_i are known, this second order difference equation defines a system of linear equations. The solution is unique if the matrix of coefficient has a determinant different from zero. We find the unique solution by solving the system through substitution and applying induction. The computed solution equals:

$$q_k = \sum_{t=k}^{M-1} \left[\prod_{i=t}^{M-1} b_i \frac{\sum_{i=1}^{t} c_i d_i \prod_{j=i+1}^{t} a_j}{\prod_{i=t}^{M} d_i} + \prod_{i=t}^{M-1} b_i \frac{d_t}{d_M} [cap - O_M] \right] \tag{17}$$

Where:

$$d_1 = d_2 = 1 \tag{18}$$

$$d_k = d_{k-1} - d_{k-2} b_{k-2} a_{k-1}$$

For sufficiently high level of option contracts, there might be some k's such that $q_k = q_{k+1}$.

4 Option markets

Two different settings will be studied, in both we introduce a previous stage in which option contracts are traded. In the first one we establish a benchmark result. A central planner procures option contracts to minimize the total cost of electricity provision. This can also be interpreted as a monopsonistic market structure. In the second setting we allow LSEs and generators bargain bilateral contracts. First we assumme that there are LSEs that are price taking in the spot market, that is, each LSE's financial position does not affect the equilibrium spot market price. Later we allow for LSEs that are large and affect the equilibrium spot market price through their financial positions.

4.1 Centralized problem

Now suppose that there is a central authority that procures option contracts with the objective of minimizing the total cost of electricity. The central planner makes take it or leave it offers for a profile of financial contracts, $o(p)$. This function indicates at each price the quantity of call options being provided. A strategy for a generator consists of accepting or rejecting the offer for the contracts (a_i) plus a mapping ,$s_i(p)$, that constitutes the supply function in the spot market. Then if a generator accepts the offer, the objective function is given by:

$$\Pi^T(a, s) = T + \Pi(o, s) \tag{19}$$

Where T is the payment to the generator for signing the contracts. If a generator rejects the offer the objective function is given by:

$$\Pi(a, s) = \Pi(0, s) \tag{20}$$

To implement a profile $o(.)$ as an equilibrium it must be the case that accepting the offer is a best response when all the other generators also accept the offer, that is:

$$T + \Pi((o_i, o_{-i}), (s_i*, s_{-i}*)) \geq \Pi((0, o_{-i}), (s_0, s_{-i}*)) \ \forall s_0 \tag{21}$$

where $s*(.)$ is the unique symmetric equilibrium of the spot market when every generator accepts the offer for o_i. Note that the condition can be simplified by considering s_0 the best response to s_i* for the corresponding financial position.

The central planner's objective is to minimize the cost of electricity, this implies that T will be selected such that the inequality above is satisfied as an equality. So the objective function of the planner depends only on the profile of contracts o_i:

$$min_{o_i}\Pi(o_i, s*) + \Pi((0, o_{-i}), (s_0, s_{-i}*)) - \Pi((o_i, o_{-i}), (s_i*, s_{-i}*)$$
$$= \Pi((0, o_{-i}), (s_0, s_{-i}*)) \qquad (22)$$

The following result characterizes the solution to the central planner's problem:

Theorem 4.1 The solution to the planner's problem is a profile of option contracts such that price equals marginal cost for any load level.

Proof

Suppose that o_k is such that $p_k > 0$ is the equilibrium highest price. We are going to prove that increasing the quantity of contracts at that price, o_k, results in a lower total cost of electricity. There is a direct gain in the spot market that is given by the lower price for a given supply functions. This gain more than offsets the increase in the cost of procuring contracts. We can establish that by noting, as shown above, that the planner's objective function can be expressed simply as: $\Pi((0, o_{-i}), (s_0, s_{-i}*))$, the maximum profit of a generator that rejects the offer for the option contracts when all other generators accept. Note that this is always decreasing since, first, an increase in the number of option contracts signed by the other generators o_{-i} has a direct negative effect on $\Pi_i(.)$. Additionally there is a negative effect due to the associated increase in the equilibrium supply function as proved in the lemma presented in the previous section.$\square$

The central authority makes offers to the generators, this implies that there is an additional stage, the contracting stage, in which they compete. This double round of competition allows the central authority to reduce the cost of electricity.

In a model with asymmetric producers (e.g. peakers and baseload generators) options could also allow for a more fair compensation for peakers. Note that without option contracts, baseload producers would be unnecessarily compensated during peak hours. Observe that we assumed risk neutrality for generators and the central authority, nevertheless the optimal allocation is such that the price is flat.

Under the central authority offer, there might exist other equilibria in which generators do not accept the offer for option contracts. Requiring unique implementation of the allocation would make the procurement of contracts more costly and might change the results.

To quantify the relevance of the result above we present a numerical example in the next subsection.

4.1.1 Numerical example

Consider a market with 3 generators, demand is distributed uniformly between 0 and 100. As before marginal cost equals zero up to a per generator capacity of 100/3. We assume that the grid price has 101 points between 0 and 100. In that case the equilibrium supply function of an equilibrium with no option contracts is represented in figure 1.

We can compute the cost of electricity associated with that equilibrium by using the expression of the generators profit function that in this case coincides with the cost of electricity. The cost is equals to 254,170.

To compute the cost under optimal procurement, note that if the remaining generators are submitting a flat supply curve at marginal cost, the best response is to offer all the capacity at a price equal to the cap price. To note that this is the best response, observe that the only increment in quantities can be attained by offering at a price equal to the marginal cost, but this results in a mark-up of zero. This way we can easily compute the maximum profit level of a firm that rejects the offer for contracts. Remember that this is the cost associated with procuring the contracts. In this case, the sum of the profits of the generators (and total cost of electricity) equals 166,830. This implies that the reduction in the cost of electricity is larger than 33 percent of the original level. We conclude this section emphasizing that option contracts can result in a significant reduction in the cost of electricity. Next we analyze different environments to study if and to what extent decentralized markets attain those potential gains.

4.2 Decentralized market

Now we are going to compare the benchmark result above with what can result in specific market environments. First we consider a case in which LSEs are small enough so that their respective financial position does not affect prices in the spot market. This is an extreme assumption but it will be used to illustrate that the potential gains presented in the previous section might not be realized in certain market environments. Next, we will present some results for the case in which LSE are large, that is, their individual actions and financial positions have an effect on the distribution of prices in the spot market.

4.2.1 Price taking LSEs

As indicated we first analyze a model in which LSEs are price takers in the spot market. The financial position of an individual LSE has no effect on the equilibrium distribution of the spot price. This can be considered as an approximation to the case in which LSEs are small. Another interpretation of the environment is that we are considering LSEs that are naive and do not anticipate the effect of their financial positions on the average price. They only consider the effect on net revenue for a given equilibrium distribution of the spot price. Independently of the interpretation, the analysis of this environment results in some useful insights for the analysis of the gains that can result from the existence of option contracts.

Additionally, this exercise is an extension and robustness check of the literature started by Allaz and Vilas (av) which was mentioned in the Related Literature section. In all these papers, the demand side agents are modeled as price takers.

For the following proof we will make a small change to the model presented above. We will assume that when rationing a quantity at a given price, first the quantities offered in the spot market are called and then option contracts are exercised. The assumption regarding proportional rationing described above still stands. The modification should not be viewed as conceptual but as one made for expositional convenience.

The following is an illustration of the difficulties that can obstruct the realization of the potential benefits of option contracts:

Theorem 4.2 Suppose LSEs are price taking in the spot market, then, there are no gains from trading option contracts.

Proof

For each load level x, the gains for an LSE from procuring option contracts are simply given by the difference between the equilibrium price p and the corresponding strike price of the contract in the case the option is exercised. Next we claim that the loss to a generator is larger than the gain to a LSE. The gain to a LSE is a lower bound for the loss to a generator since by not selling the contract the generator can still mimic the prices of the equilibrium with those contracts. Under this strategy, a generator commits the same number of units at each price, but, for each price there are more units offered in the spot market and less units committed under option contracts. Now observe that this results in delivering a strictly higher number of quantities since, for a given price, quantities offered at the spot market, q_k^i, are called before the quantities committed under contracts o_k^i. We conclude that the cost to generators is higher than the benefit to a price taking LSE. $\square$

Corollary 4.1 Suppose LSEs are price taking in the spot market, then, in no symmetric equilibrium there is trade of option contracts.

Proof

This is a trivial consequence of the previous theorem.$\square$

Observe that the result above does not depend on the bargaining position of each agent. That is, even if we allow LSEs to make take it or leave it offers to generators there will be no trade of option contracts in a symmetric equilibrium. This is a consequence of LSEs not internalizing the externalities of option contracts through the spot market price. Small LSEs do not consider that effect, while generators with market power do consider that effect.

Note that this result is in sharp contrast with the result by Allaz and Vilas [1]. In

their work the agents on the demand side do not have market power, nevertheless there exist trade of futures contracts and this result in a more competitive outcome. In their work, there are no capacity contraints, and competitors reduce the quantity supplied when they observe that a rival has increased the number of contracts signed. This implies that signing contracts is convenient for a generator/supplier. Our result does not depend on observability, since of other generators observe a higher number of contracts signed, then they have less incentives to reduce the supply level.
It would be desirable to consider intermediate cases in which LSEs do not act as a single agent but are large enough so that a fraction of the effect on the equilibrium spot price is taken into account. This is what we consider in the following subsection.

4.2.2 LSEs with market power

Now we turn to large LSEs that do not act as a single agent. In line with the result in the previous setting, we show that, even when we allow for large LSEs with market power, a significant fraction of the potential benefits of option contracts might not be attained in a decentralized market environment.

Consider the following market protocol:

- LSEs submit offers for bilateral option contracts simultaneously.
- Generators accept or reject those contracts simultaneously.
- Generators observe their respective financial position.
- The spot market is run as a uniform price auction.

The above procedure should be considered as an example. The result presented below holds for wide-ranging market protocols. For example we could have generators making the offers or a centralized market for option contracts. Since the proof is in terms of gains from trade, the bargaining position of agent does not modify the result.

Theorem 4.3 The central authority solution cannot be implemented in a symmetric decentralized market equilibrium.

Proof

The proof consists in showing that there are no gains from trade. Suppose that the planner's solution is implemented in an equilibrium. Then, the cost to a generator of accepting the offers made for quantities o_0^i by LSEs equals the product of p_M and the expected quantity called in the spot market when the price at which capacity is offered is the highest price. This is because, if contracts are not traded, a generator would select the maximum price for those units.

Observe that since that value is also an upper bound for the benefits to an LSE, then that must be the equilibrium price for the contracts.

The benefit of a contract of o_0^i units is smaller since alternatively a LSE can offer contracts for ko_0^i units to two generators at the price of k times the price for the original contract. For k close to 1, these contracts will be accepted and the difference in net revenue in the spot market will be smaller since both generators are competing and will submit a supply function which is strictly below p_M. Since the cost to generators is higher that the benefit to a LSE, we conclude there are no gains from trade and, thus, such a contract is not traded in equilibrium. □

This result is another example of the limitations of decentralized markets in capturing the potential benefits of option contracts. The intuition behind the result is that LSEs are not able to internalize the effect of their respective financial positions of the net revenue of other LSEs in the subsequent spot market. As a consequence demand is too low to implement the solution of the central planner.

It would be interesting to simulate numerically an equilibrium of the market presented above. We would like to asses which is the fraction of the gains that are captured when LSEs are big, that is, individual LSEs financial positions affect the spot market equilibrium price distribution. Considering a market with risk averse agents would also help to asses the impact of option contracts on market power mitigation.

5 Concluding remarks

We considered a supply function equilibrium (SFE) model with option contracts. Uniqueness of the symmetric equilibrium was verified. We also checked that option contracts result in less supply reduction and lower prices in the spot market.

In the benchmark results, the centralized setting, we show that option contracts can result in significant reduction in the total cost of electricity. Additionally the volatility of the cost is greatly diminished since in the solution to the planner's problem the spot market price always equals marginal cost.

The realization of these gains depends on market characteristics. More specifically, the social planner's solution cannot be implemented when LSEs act in a decentralized market. LSEs do not internalize the benefits of procuring their respective option contracts on the spot price paid by other LSEs. Additionally, we find that with price taking LSEs there are no gains from trading option contracts. Thus, in equilibrium, there is no trade of option contracts and there is no benefit resulting from option contracts. In this case, the result does not depend on the bargaining power of the different agents. This results are a consequence of LSE not internalizing the social benefits of procuring option contacts.

Market structure and regulatory measures shape the effect of option contracts on imperfectly competitive markets. Centralized procurement of call options and requiring generators to sell call options are among the regulatory measures that could improve the performance of wholesale electricity markets. Ignoring these possibilities might result in not implementing an important fraction of the potential gains of option contracts.

The model can be extended to include more realistic features of wholesale electricity markets. For example, allowing for asymmetric generators, LSEs with market power, and introducing risk aversion seem to be desirable features. It is not clear if numerical methods will be required to conduct those exercises. In particular with asymmetric producers, we could study how option contracts might compensate peakers that are called only during extremely high load period, that is, with low probability.

A more general cost structure is also going to be considered in future research. We speculate that our observations will stand. That is, an important fraction of the market power mitigation gains from option contracts might not be attained unless the adequate regulation is put in place.

Transmission constraints is another salient characteristic of wholesale electricity markets that has important consequences when considering market power related issues. Network congestion generates local market power stochastically. Naturally, the modeling of such a market might result in considerable technical difficulties.

Market power reduction and investment in capacity have generally been conflicting objectives in electricity markets. It would be interesting to establish how much of this conflict goes away once when option contracts are considered. Not only the volume of investment but also the resulting technology mix should be considered.

Chapter 3:
Repeated Procurement and Long Term Contracts

This chapter has been coauthored with Wedad Elmaghraby.

Abstract

We consider a game in which a buyer must repeatedly procure an input from a set of firms. In our model, the buyer is able to sign long term contracts that establish the likelihood with which the next period contract is awarded to an entrant or the incumbent. We find that the buyer finds it optimal to favor the incumbent, this generates more intense competition between suppliers. In a two period model we are able to completely characterize the optimal mechanism.

1 Introduction

We consider a game in which a buyer must repeatedly procure an input from a set of firms. In our model, the buyer is able to sign long term contracts that establish the likelihood with which the next period contract is awarded to an entrant or the incumbent. This is our key departure from previous literature.

Our main contribution is to show that the buyer minimizes procurement costs by signing long term contracts that favor next period's incumbent. The intuition is that, by favoring an incumbent in the following period, the competition is more fierce in the current period. The gains from this second effect are, for a relevant range, larger than the losses from the resulting inefficient allocation of the following period. These results go in the opposite direction of what would most commonly be observed in single period problems. In this case the weak seller (in most of the cases the entrant) would be favored by the contract allocation rule.

Most of the literature on procurement, mechanism design and auctions consider environments with a single transaction. Additionally, when considering dynamic analysis, the focus is on short term(i.e. single period) optimization by the auctioneer.

The next section of the paper discusses related literature. Next the model is presented. Section 4 studies the solution of the buyers problem. The two period problem is analyzed in section 5. A simple auction thorugh which a buyer could implement these ideas is presented in section 6. The last section concludes.

2 Related Literature

This paper results in new insights that can be contrasted to results in the study of both dynamic and static procurement problems.

Lewis and Yildirim [21] consider a model of repeated procurement with learning by doing in which the buyer is not able to sign long term contracts. They study how a buyer should optimally take into account the impact of current procurement policies on the future cost for sellers.

The literature on ratchet effect, see for example Freixas, Guesnerie and Tirole [11], study dynamic adverse selection problems in which the revealing private information in early period is harmful for players that typically participate in a principal-agent game. In our paper we assume that private information is indepently distributed on different periods, thues the ratchet effect is not present in this dynamic setting.

Our contribution is also related to the literature on optimal mechanism design. A well known result in static mechanism design with asymmetric players is that favoring players that are more likely to be of a low type is an optimal strategy for an auctioneer; see for example the seminal work by Myerson [23]. In our environment, incumbents, even when they are stronger than entrants, are the favored players.

Our work is also related to Luton and McAfee [22]. They study repeated a repeated procurement problem in which the buyer is able to charge fee to bid in the second project at the time of the first project. In their paper, the optimal mechanism favours the entrant when allocating the second period contract. In our model entry fees are not considered, and the result will be the opposite.

Characterizing revenue maximizing/minimizing mechanism in dynamic environment has proved a difficult task. This paper is one step toward the understanding of this problem.

3 The model

A firm must buy one unit of an input in each of periods 1 through T. There is a set of sellers $1, ..., n$ that produce the input. The cost for producer i in period t is x_{it}. This parameter, x_{it}, is drawn in period t from the distribution $F_t^j(x_{it})$ with support X_t^j, where $j = I$ if i is an incumbent in period t, $j = E$ otherwise. This means that all entrants are symmetric but we allow for incumbents whose cost level is drawn from a different distribution. We assume that x_{it} is private information of player i.

To allow for a more general analysis we will use the mechanism design approach. In each period, producers report their cost by selecting one element in the support of the distribution function: $\hat{x}_{it}$. The contract is awarded to a supplier according to a function of the messages sent $c_t(\hat{x}_t)$ where $\hat{x}_t$ is the vector including all the messages for period t and $c_t(\hat{x}_t)$ is a vector whose ith entry is equal to the probability that i is awarded the procurement contract for period t. Similarly the monetary transfers are a function of the messages $p_t(\hat{x}_t)$.

For simplicity we will restrict attention to functions $c_t(\hat{x}_t)$ where the producer with the lowest message is awarded the contract with the exception that the incumbent's message is corrected by a parameter a_t when making the ranking of messages. As a result, suppose that $\tilde{i}$ is the incumbent, then $c_{it}(\hat{x}_t)$ for $i \neq \tilde{i}$ equals:

$$c_{it}(\hat{x}_t) = \begin{cases} 1 \quad if & \hat{x}_{it} \leq min\left\{min_{j \neq \tilde{i}} \hat{x}_{jt}, \hat{x}_{\tilde{i}t} + a_t\right\} \\ 0 \quad otherwise & \end{cases} \tag{1}$$

and for $i = \tilde{i}$, the incumbent:

$$c_{\tilde{i}t}(\hat{x}_t) = \begin{cases} 1 \quad if & \hat{x}_{\tilde{i}t} + a_t \leq min\left\{min_{j \neq \tilde{i}} \hat{x}_{jt}, \hat{x}_{\tilde{i}t} + a_t\right\} \\ 0 \quad otherwise & \end{cases} \tag{2}$$

If $a < 0$ then, the incumbent is more likely to be awarded the contract than what would be observed if every submitted message is treated equally. On the contrary, if $a > 0$ then entrants are favored against the incumbent.

The payoff of a bidder that reports $\hat{x}_{it}$ and has cost x_{it} equals:

$$\hat{U}_{it}(x_{it}, \hat{x}_{it}) = P_{it}(\hat{x}_{it}) - x_{it} C_{it}(\hat{x}_{it}) + \beta E[\hat{U}_{it+1}(x_{it+1}, \hat{x}_{it+1}) / \hat{x}_{it}] \tag{3}$$

where:

$$C_{it}(x_{it}) = E_{x_{-it}}[c_{it}(x_t)] \tag{4}$$
$$P_{it}(x_{it}) = E_{x_{-it}}[p_{it}(x_t)]$$

And β is the discount factor. Note that period $t+1$ expected utility function depends on the report, $\hat{x}_{it}$, but does not depend on the type of the supplier on period t, x_{it}. This is a result of the assumptions on independence we made on the distribution of cost level.

3.1 Mechanism Design Results

In this subsection we will present results from the mechanism design theory. For more on the mechanism design theory see Fudenberg and Tirole [11].

Let $U_{1t+1}(x_{it+1})$ be the indirect utility function of player i.

The revelation principle will allow us to have an expression for the payoffs of suppliers that only depends on the allocation rule and a constant. Suppose that a mechanism $\{c(.), p(.)\}$ is incentive compatible. Then, through an application of the envelope theorem, we get:

$$\frac{dU_{it}(x_{it})}{dx_{it}} = C_{it}(x_{it}) \tag{5}$$

Integrating the equation above we have:

$$U_{it}(x_{it}) = \int_{x_{it}}^{\overline{x}} C_{it}(s)ds + U_{it}(\overline{x}) \tag{6}$$

Note that the expression above indicates that the only way through which the future enters the payoff function of a supplier is through the expected payment of the least efficient type. This property will prove critical when considering the design of optimal procurement policies. Observe that in this setting changes in the allocation rule for tomorrow affects the current payoffs of the buyer in a way that is significantly different from what occurs in a static mechanism design problem. In particular, the rents captured by the worst possible type of each player enters the objective function. But any rents in the second period that is not captured by the worst type does not affect the cost of procurement. This will prove important when calculating the optimal mechanism.

Note that we can now use the definition of the utility function to find an expression for the interim payment function $P_{it}(.)$:

$$P_{it}(x_{it}) = \int_{x_{it}}^{\overline{x}_{it}} C_{it}(s)ds + C_{it}(x_{it})x_{it} + U_{it}(\overline{x}) - \beta E[U_{it+1}(x_{it+1})] \tag{7}$$

We will now use the expression above to show that the mechanism is incentive compatible. Incentive compatibility means that all participants in the mechanism have incentives to report their true type.

From the optimality principle, the mechanism is incentive compatible if the following objective function is maximized with respect to $\hat{x}_{it}$ at $\hat{x}_{it+1} = x_{it+1}$:

$$\hat{U}_{it}(x_{it}, \hat{x}_{it}) \geq P_{it}(\hat{x}_{it}) - x_{it}C_{it}(\hat{x}_{it}) + \beta E[\hat{U}_{it+1}(x_{it+1}, \hat{x}_{it+1})/\hat{x}_{it}] \tag{8}$$

Lemma 3.1 *The mechanism described above is implementable.*

Proof:

From the principle of optimality we know that it is suficient to check that truthtelling is a best response in period t given that reports are going to be truthful thereafter. That is we require that:

$$U_{it}(x_{it}) = P_{it}(\hat{x}_{it}) - x_{it}C_{it}(\hat{x}_{it}) + \beta E[\hat{U}_{it+1}(x_{it+1}, \hat{x}_{it+1})/\hat{x}_{it}]$$

For all $\hat{x} \in X_{it}$. Replacing by the expressions for the utility function $U_{it}(x_{it})$, this can be rearranged as follows:

$$P_{it}(x_{it}) - P_{it}(\hat{x}_{it}) + \beta E[\hat{U}_{it+1}(x_{it+1}, \hat{x}_{it+1})/x_{it}] - \beta E[\hat{U}_{it+1}(x_{it+1}, \hat{x}_{it+1})/\hat{x}_{it}] \geq x(C_{it}(x_{it}) - C_{it}(\hat{x}_{it}))$$

Now we can use the payment function of equation (9) to obtain:

$$\int_{x_{it}}^{\hat{x}_{it}} C_{it}(s)ds \geq C_{it}(\hat{x}_{it})(\hat{x}_{it} - x_{it}) \tag{9}$$

This expression means that as long as the function $C_{it}(.)$ is decreasing, truthful reporting is a best response. Since the allocation function is a decreasing function according to our description, we conclude that incentinve compatibility is satisfied.□

In what follows, we will assume that the distribution for entrants and the incumbent have a common support $[\underline{x}_t, \overline{x}_t] = X_t^I = X_t^E$. We will relax this assumption later when considering the two period case.

Using the expression for the utility function of the suppliers, we can find an expression for the procurement cost. Before presenting the complete expression we present the discounted sum of the expected payments to an individual supplier i starting on period $\hat{t}$:

$$\sum_{t=\hat{t}}^{T} E[\beta^t P_{it}(x_{it})] = \int_{\underline{x}_{\hat{t}}}^{\overline{x}_{\hat{t}}} U_{it}(x_{it}) dF^j(x_{it}) + \sum_{t=\hat{t}}^{T} E[\beta^t x_{it} C_{it}(x_{it})]$$

$$\sum_{t=\hat{t}}^{T} E[\beta^t P_{it}(x_{it})] = \int_{\underline{x}_{\hat{t}}}^{\overline{x}_{\hat{t}}} \left[\int_{x_{it}}^{\overline{x}_{\hat{t}}} C_{it}(s) ds \right] dF^j(x_{it}) + U_{i\hat{t}}(\overline{x}_{\hat{t}}) + \sum_{t=\hat{t}}^{T} E[\beta^t x_{it} C_{it}(x_{it})]$$

The last term in the equation is equal to the expected physical cost of producing the input while the first two terms are the expected rents of a specific supplier. Where we use the same notation as above that is, $j = E$ if i is an entrant, $j = I$ otherwise.

We will consider the problem of a buyer that takes as given the allocation rule used in the current period but selects the allocation rule, that is, the value of a_{t+1} for period $t + 1$.

Additionally, we consider that the mechanism is such that if an agent is of the most inefficient type, $x_{it} = \overline{x}_t$ then this agent gets no rents in period t, that is:

$$U_{jt}(\overline{x}_t) = \beta P_{jt}(\overline{x}_t) \int_{\underline{x}_{t+1}}^{\overline{x}_{t+1}} U_{It+1}(x) dF_I(x) + \beta(1 - P_{jt}(\overline{x}_t)) \int_{\underline{x}_{t+1}}^{\overline{x}_{t+1}} U_{Et+1}(x) dF_E(x)$$

for $j = I, E$. In particular the expression above implies that the buyer is not able to charge an entry fee for the period to procurement process at the time the period one contract is awarded. If that was the case, the buyer would charge each supplier an entry fee which equals the ex-ante gains from participating.

Then, we can find an expression for the objective function of the buyer on period t. For simplicity we assume that $i = 1$ is the incumbent on period t and since all entrants are symmetric we will simply label them 2. When we sum up expression (9) for all the players we obtain the total procurement costs starting on period t:

$$\begin{aligned} PC_t(a_{t+1}) &= \sum_{\hat{t}=t}^{T} E[\beta^{\hat{t}-t} P_{i\hat{t}}(x_{i\hat{t}})] \\ &= U_{1t}(\overline{x}_t) + (n-1)U_{2t}(\overline{x}_t) \\ +\beta E[x_{1t+1}C_{1t+1}(x_{1t+1}) &+ (n-1)x_{2t+1}C_{2t+1}(x_{2t+1})] + K \end{aligned} \tag{10}$$

The parameter K includes all the terms that do not depend on a_{t+1}.

We can replace the expression for the payoff functions of suppliers at $\overline{x}$, equation (10) above, this way we get the following expression:

$$\begin{aligned} PC_t(a_{t+1}) = \beta(P_{1t}(\overline{x}_t) + (n-1)P_{2t}(\overline{x}_t)) \int_{\underline{x}_{t+1}}^{\overline{x}_{t+1}} U_{It+1}(x)dF_I(x) \\ +\beta(n - P_{1t}(\overline{x_t}) - (n-1)P_{2t}(\overline{x}_t)) \int_{\underline{x}_{t+1}}^{\overline{x}_{t+1}} U_{Et+1}(x)dF_E(x) \\ +\beta E[x_{1t+1}C_{1t+1}(x_{1t+1}) + (n-1)x_{2t+1}C_{2t+1}(x_{2t+1})] + K \end{aligned} \tag{11}$$

4 Optimal contracts

In this section, we establish that a buyer would always wish to sign a contract in which the incumbent in the next period has a higher probability of being allocated the contract. There is a positive probability that the incumbent is awarded the contract even when there is an entrant with a lower cost. For simplicity we will assume from now on that the support of the distribution functions do not depend on time, that is:

$X_t^I = X_t^E = X.$

Our first result deals with the best response of a seller:

Theorem 4.1 *Suppose $n > 2$ then, it is always the case that the buyer favors the incumbent, that is, it is never the case that $a \geq 0$ in a solution to the buyer's problem.*

Proof:

The derivative of the objective function is:

$$\begin{aligned}\frac{dPC(a_{t+1})}{da_{t+1}} &= \beta(P_{1t}(\overline{x}) + (n-1)P_{2t}(\overline{x}))\int_{\underline{x}}^{\overline{x}} \frac{dC_{It+1}(x)}{da_{t+1}} F_I(x)dx \\ &+\beta(n - P_{1t}(\overline{x}) - (n-1)P_{2t}(\overline{x}))\int_{\underline{x}}^{\overline{x}} \frac{dC_{Et+1}(x)}{da_{t+1}} F_E(x)dx \\ &+\beta^2(P_{1t}(\overline{x}) + (n-1)P_{2t}(\overline{x}))\frac{dC_{It+1}(\overline{x})}{da_{t+1}} E(U_{It+2}(x) - U_{Et+2}(x)) \\ &+\beta^2(n - P_{1t}(\overline{x}) - (n-1)P_{2t}(\overline{x}))\frac{dC_{Et+1}(\overline{x})}{da_{t+1}} E(U_{It+2}(x) - U_{Et+2}(x)) \\ &+\beta\frac{dFC_{t+1}(a_{t+1})}{da_{t+1}} \end{aligned} \tag{12}$$

where: $FC_{t+1}(a_{t+1}) = E[\sum_{i=1}^{n}[x_{it+1}c_{it+1}(x_{t+1})]]$, that is $FC_{t+1}(.)$ is the expected technological cost associated with a rule to award contracts. Now if $n > 2$ then there are at least two entrants, which means that $C_{2t}(\overline{x})) = 0$ also we have $\frac{dC_{Et+1}(\overline{x})}{da_{t+1}} = 0$. Additionally suppose $a_{t+1} > 0$ then $P_{1t}(\overline{x}) = 0$. Then the derivative equals:

$$\begin{aligned}\frac{dPC(a_{t+1})}{da_{t+1}} &= \beta n \int_{\underline{x}}^{\overline{x}} \frac{dC_{Et+1}(x)}{da_{t+1}} F_E(x)dx \\ &+\beta\frac{dFC_{t+1}(a_{t+1})}{da_{t+1}} \end{aligned} \tag{13}$$

Observe that $\frac{dC_{Et+1}(x)}{da_{t+1}} \leq 0$ and $\frac{dFC_{t+1}(a_{t+1})}{da_{t+1}} > 0$ since by increasing a_{t+1} the inefficiency in the allocation of contracts is augmented. The derivative is positive at for any $a \in [0, +\infty)$ which means that the solution, if any, is in $(\infty, 0)$. Finally note that the derivative is continuous and that for sufficiently low a_{t+1} it takes negative values, the intermediate value theorem guarantees the existence of a solution at $a < 0$.$\square$

Observe that this is a very general result, it does not depend on the number of periods, or shape of the distributions of values for entrants and incumbent. One assumption we are making is that the supremum of the support of entrants' distribution coincides with the supremum of the support of the distribution of values for incumbents. In the next section we will provide examples that show that similar results also hold when this assumption is relaxed.

5 Optimal mechanism with 2 periods

In this section we will completely characterize the optimal mechanism in the case in which there are only two periods. We will show that the insights of the previous analysis still hold in a scenario in which the buyer is able to use a richer set of tools in the procurement process. More specifically, the buyer would be able to commit in period 1 to any allocation rule for period 2 that results in an incentive compatible mechanism.

We will consider that in period 1 all suppliers are symmetric entrants and the allocation is efficient. Let $m = \left\{ \{c^i(.)p^i(.)\}_{i=1}^n \right\}$ be the mechanism selected for period 2, where $c^i(.)$ is the allocation rule for the second period contract when the incumbent is supplier i, we define the other function, $p^i(.)$ in a similar way. Additionally, without loss of generality we consider mechanisms for which the identity of sellers is irrelevant. In other words, the allocation rule for entrants and incumbents is independent of the identity of the incumbent. Following the same framework as in the previous section, the revelation principle results in the following objective function for the buyer:

$$PC(m) = \sum_{i=1}^{n} \int_{\underline{x}}^{\overline{x}} \left[\frac{F_1(x_{1i})}{f_1(x_{1i})} + x_{i1} \right] (1 - F_1(x_{1i}))^{n-1} dF_1(x_{1i})$$
$$+\frac{\beta}{n} \sum_{i=1}^{n} \int \left[x_{i2} c_i^i(x_2) + \sum_{j \neq i} \left[\frac{F_2^E(x_{j2})}{f_2^E(x_{j2})} + (x_{j2}) \right] c_j^i(x_2) \right] dF_2^i(x_2) \tag{14}$$

Where $F_2^j(x_2)$ is the joint probability of $x_2 = (x_{12}, x_{22}, ..., x_{n2})$ when j is the incumbent.

The first line of the expression above does not depend on the mechanism selected on the second period. On the second line we observe that we have an expression that looks similar to what is observed in the classical optimal mechanism design approach (see for example Myerson [23]). The difference now is that the incumbent's rents in the second period do not enter the expression. This means that allocating the contract to the incumbent would result in higher procurement costs only through an increase in the physical cost of production.

Theorem 5.1 *Assume $F_2^E(x_{j2})/f_2^E(x_{j2})$ is increasing, then when i is the incumbent, the optimal mechanism assigns to:*
i *if* $x_{i2} \leq min_{j \neq i} \left[\frac{F_2^E(x_{j2})}{f_2^E(x_{j2})} + x_{2j} \right]$
$j \neq i$ *if* $x_{i2} \geq min_{k \neq i} \left[\frac{F_2^E(x_{k2})}{f_2^E(x_{k2})} + x_{2k} \right] = \frac{F_2^E(x_{j2})}{f_2^E(x_{j2})} + x_{2j}$

Proof:

Consider the optimal allocation $c_2(x_2)$ for each x_2. Note that it is optimal to allocate to the incumbent i, if and only if the coefficient of $c_i^i(x_2)$: x_{i2} is lower than any of the coefficients for $c_j^i(x_2)$ for $j \neq i$: $x_{j2} + \frac{F_2^E(x_{j2})}{f_2^E(x_{j2})}$. Then, we find that the rule above is optimal when doing case by case unconstrained optimization. Since interim probabilities, $C^i(x_{i2})$, that result from the allocation rule $\{c^i(x_2)\}_{i=1}^n$ are monotonic, we conclude that the IC constraints are satisfied. $\square$

Observe that under this result the incumbent is allocated the second period contract more often than what would occur in the efficient solution. An entrant of type x_{E2} is allocated the contract only if there is a cost saving of at least $\frac{F_2^E(x_{E2})}{f_2^E(x_{E2})}$ when compared to the cost of the incumbent, x_{I2}. The nature of the inefficiency coincides with what we have shown in the previous section. And the reason is the same as in the previous section: the buyer introduces inefficiencies because these reduce the rents of the most inefficient types in period 1 which results in reduction of the suppliers rents that outweigh the losses from inefficient allocation.

The solution doest not necessarily coincide point by point with the types of mechanisms studied in the previous section. This is because in the optimal mechanism the maximum cost differential, $x_I - x_E$, under which the incumbent is awarded the contract changes for different ranges of costs. In the previous section we restricted this differential cost to be a constant, a.

5.1 Static optimal mechanism design

The solution of the optimal mechanism with long term contracts is significantly different from the static solution. As an example, we will consider here the static problem of optimal mechanism design.

Suppose that $\frac{F_2^E(t_{j2})}{f_2^E(t_{j2})} \leq \frac{F_2^I(x)}{f_2^I(x)}$ this means that incumbent have a distribution that compared to the entrants' distribution is more likely to draw low levels for the cost parameter. From well known optimal mechanism design results, in the second period solution without commitment the entrants would be the ones receiving the allocation more often than what is prescribed by efficiency considerations. This is sometimes described in terms of the virtual valuation of each player. More specifically, in that case the rule would be:

- Allocate to the incumbent if $max_{j\neq i}\left[\frac{F_2^E(x_{j2})}{f_2^E(x_{j2})} + x_{2j}\right] \geq \frac{F_2^I(x_{i2})}{f_2^I(x_{i2})} + x_{2j}$.

This is the typical solution in which gains from trade plus informational rents of each agent form the marginal revenue (in our case cost of procurement), see for example Myerson [23]. In our example, incumbents have a distribution with lower costs, which in turn implies that for a given cost level, the associated informational rents are higher.

In contrast, in the two-period mechanism, only entrants' second period informational rents enter the calculation. This is because second period rents enter the objective function through the indirect utility of the most inefficient type in period 1.

5.2 Uniform distribution

Consider x_{i1} distributed uniformly on the segment $[0, 1]$ for all i. In the second period entrants' and incumbent's cost distribution are the equal to the distribution for the first period values. There are three suppliers ($n = 3$). We are going to compare the optimal mechanism allocation to the allocations that would result from eroding price contracts (a mechanism that is described below) and sequential efficient auctions.

With uniform distribution and three suppliers, under the optimal mechanism with long term contracts, an incumbent i is allocated the contract in period 2 if and only if $x_{i2}^I < min_{j \neq i} 2x_{2j}$. This means that the cost of the incumbent must be twice the cost of the most efficient entrant for the contract to be awarded to the entrant.

Comments:

- The allocation of the optimal contract cannot be implemented through the type of mechanisms studied in section 3. The optimal mechanism requires more flexibility in terms of how entrants' and incumbent's costs determine the allocation process, for low draws of cost levels the inefficiency is low while for higher draws the gap between a incumbent being awarded the contract and an entrant can be as high as 1/2.

- If we assume the buyer is able to charge agents to be able to participate in the second period auction, then, all the second period rents can be extracted in the first period, and the second period allocation would be efficient. This is because as of period one, there is no asymmetric information regarding period 2 values. This is the approach followed by Luton and McAfee [22], they allow for entry fees, then the allocation of the second period contracts in the optimal mechanism takes the form of what would be observed in a static problem.

- We can also propose examples with a different distribution in which the solution of the problem is such that the incumbent is always allocated the contract in the second period. In this case the second period's payment would be equal to the highest cost in the support of the second period distribution.

5.3 Other mechanisms

We would like to compare the optimal mechanism described above to what would result in a single period mechanism design problem. Also in this subsection we compare the mechanism with long term contracts to two institutions that are commonly used in repeated procurement environments: sequential auctions and eroding price contracts.

Consider the static mechanism design problem at period 2, the optimal allocation would assign a contract to the incumbent i if and only if $x^I_{i2} < min_{j \neq i} x_{2j}$. This means that the incumbent is allocated the contract only when the marginal cost is lower than the marginal cost of the most efficient entrant. This is the case for symmetric problems.

We can also compare the allocation in this example to the allocations that would result under two alternative procurement mechanism that have been studied in the literature, see for example Elmaghraby and Oh [9]. These are second price auctions and eroding price contracts. An eroding price contract is a mechanism in which the incumbent offered the contract for a prearranged payment equal to R; if the incumbent does not accept then the contract is awarded to the entrants thorugh an auction.

In a second price auction the allocation would be efficient, this would result in the entrants being awarded contracts with a probability that is higher than the probability that would occur in an optimal mechanism (see figure 1).

The comparison with the eroding price contract requires a more detailed explanation. The resulting allocation is presented in figure 1-B. As can be seen, an eroding price contract differs from an optimal mechanism in two areas. First, there are low values of the entrants' cost for which the contract is awarded to the incumbent in an eroding price contract while an optimal mechanism would allocate to an entrant. Second, when both the incumbent and the most efficient entrant have high costs, an eroding price contract would allocate the contract to an entrant while the optimal mechanism would award the contract to the incumbent.

We should note however that for certain parameter values, the optimal contract and the eroding price contract coincide. For example, this is the case when random costs are uniformly distributed on $[1, 2]$. In these case an eroding price contract of $R = 1$ would result in the allocation of the optimal mechanism.

6 Implementation through a simple mechanism

In this section we provide an example of how a buyer could favour the incumbent, and thus reduce procurement costs, through a simple auction format.

Suppose that there are two suppliers per period. Consider in period 2 a modified second price auction in which the entrant is allocated the contract if its bid, b_E is below $b_I + a$ where a is a constant and b_I is the bid of the incumbent. If the entrant wins the contract, then the payment is equal to $b_I + a$. Similarly, the incumbent wins the contract if b_I is below $b_E - a$ and, in that case, the payment equals $b_E - a$. If $a = 0$ then this auction is a second price auction. With $a > 0$ this auction favors the entrant, with $a < 0$ the auction favors incumbents.

It is straightforward to show that truthtelling is an equilibrium in weakly dominant strategies.

The equilibrium of this auction would then implement the allocation of contracts described in section 4. Now suppose that cost are distributed uniformly on $[0, 1]$ on periods 1 and 2 and that $a < 0$. In this case we can calculate the ex-ante payoff for period 2:

$$\begin{aligned}\int_0^1 U_2^E(x_{i2})dx_{i2} &= -\frac{(1+a)^3}{2}\\ \int_0^1 U_2^I(x_{i2})dx_{i2} &= \frac{1}{6} - \frac{a}{2} + \frac{a^3}{6}\end{aligned} \tag{15}$$

Additionally suppose that in the first period a regular second price auction is used to allocate the first period contract. Then, in period 1 all players would bid truthfully, but taking into account that the continuation payoffs are different for entrants and the incumbent. That is, there is an added value to being awarded the contract. Then the equilibrium bids are:

$$\begin{aligned}b_1(x_{i1}) = x_{i1} + \beta \int_0^1 &[U_2^I(x_{i2}) - U_2^E(x_{i2})]dx_2\\ &= x_{i1} - \beta a + \beta\frac{a^2}{2}\end{aligned} \tag{16}$$

From the expression above we can see how suppliers bid below their marginal cost when $a < 0$, there is a positive probability that negative payoffs are realized on period 1, but in expectation this is compensated by the gains of being the incumbent on period 2.

7 Concluding Remarks

We have studied the problem of a buyer that is able to commit to a long term procurement policy. Our contribution is to show that the buyer has incentives to sign a contract that would favor the incumbent. This is because by doing that, the buyer is able to generate more competition between the sellers in the current period since being awarded a contract today is more valuable.

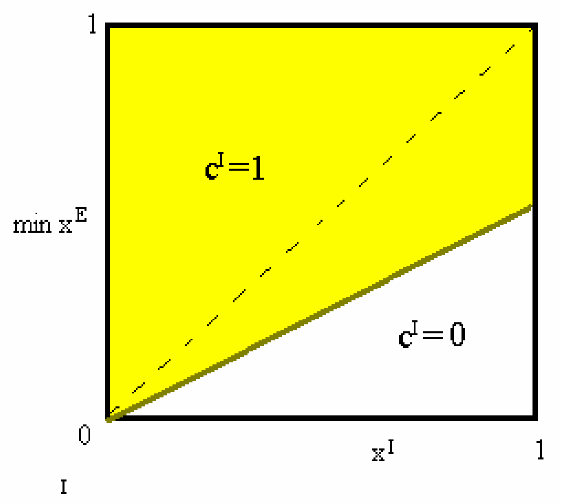
There is a sharp contrast between our result and what can be expected from a static mechanism design approach. In the latter an optimal mechanism would favor the weak player (typically the entrant). Our results also indicate that two mechanism that are commonly used in repeated procurement environments (eroding price contracts and sequential auctions) are not optimal.

Our results present a new perspective on the repeated procurement problem. It is also a contribution to the study of dynamic mechanism design. In particular, it is a step forward in the study of revenue optimization problems in dynamic environments.

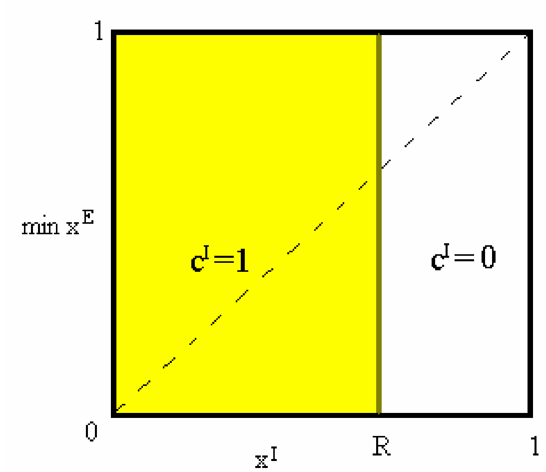
The study of richer dynamic environments in which the current cost level affects the distribution of future costs is one of our future research projects. We would also like to study how our results are matched by current practice in the field.

Figure 1

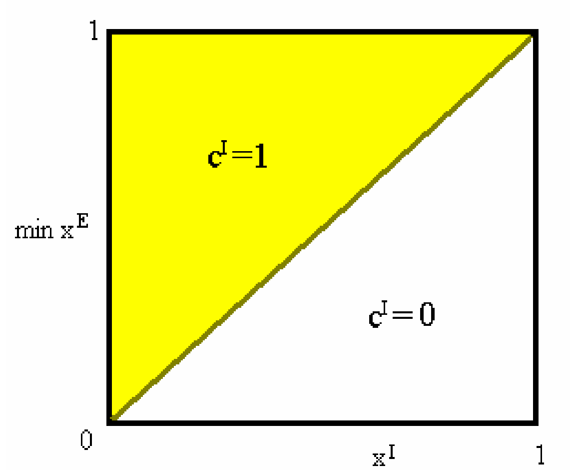
A- Optimal mechanism



B- Eroding price contract:



C- Second price auction:



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