Spectrum Auction Design

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Application: Spectrum auctions

- Many items, heterogeneous but similar
- Competing technologies
- Complex structure of substitutes and complements

- Government objective: Efficiency
  - Make best use of scarce spectrum

Simultaneous ascending auction

Auction rules

- Simultaneous
  - All lots at the same time
- Ascending
  - Can raise bid on any lot
- Stopping rule
  - All lots open until no bids on any lot
- Activity rule
  - Must be activity to maintain eligibility

Strategy in SAA

Auction as a Negotiation

- Learn what competitors need
- Learn how costly it is to ask for more
- Ask for more at the right time in the right place
- Manage eligibility

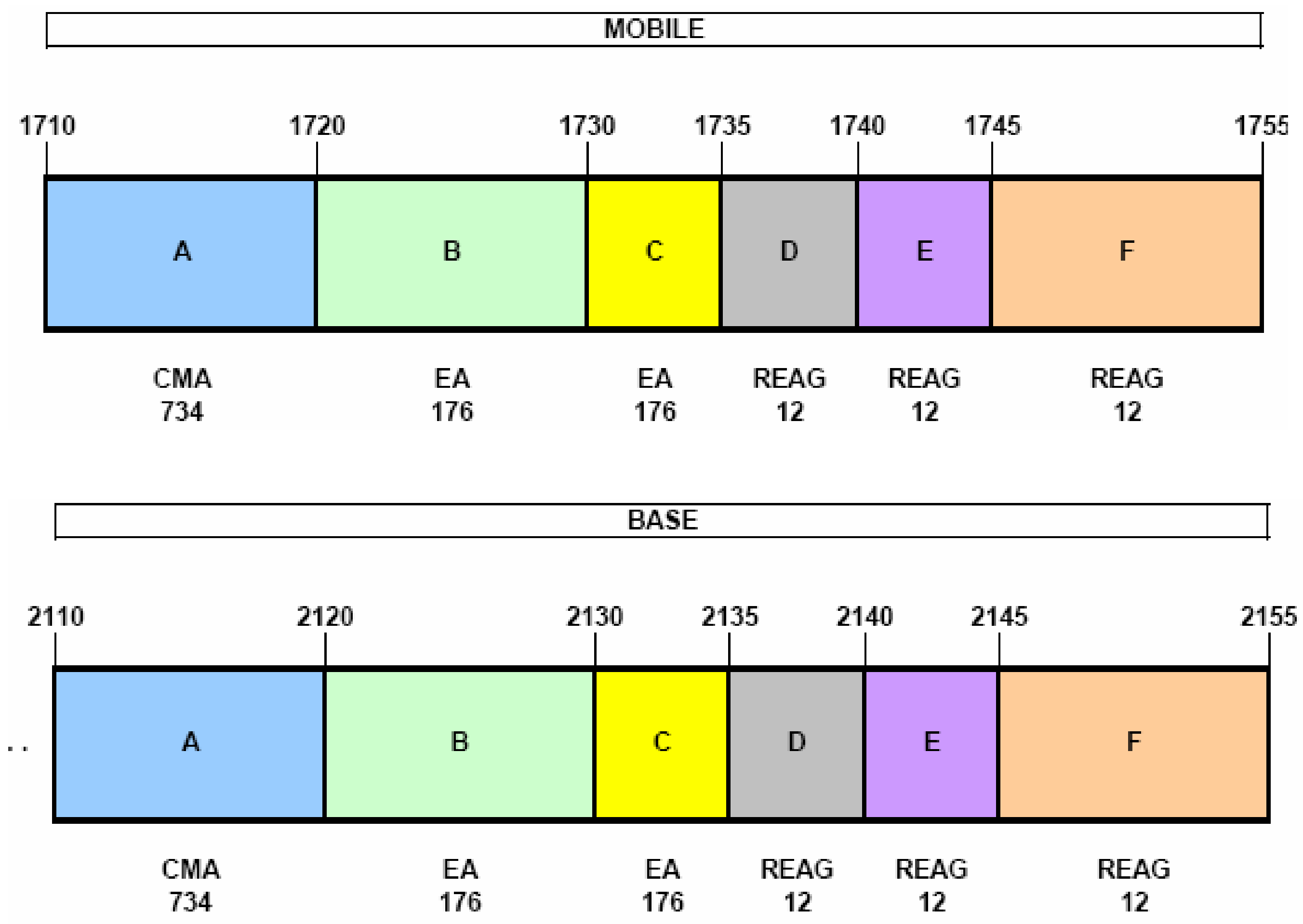
Retaliatory Bidding

- Stake a claim
- Punish intruders
- Use code bids to clarify

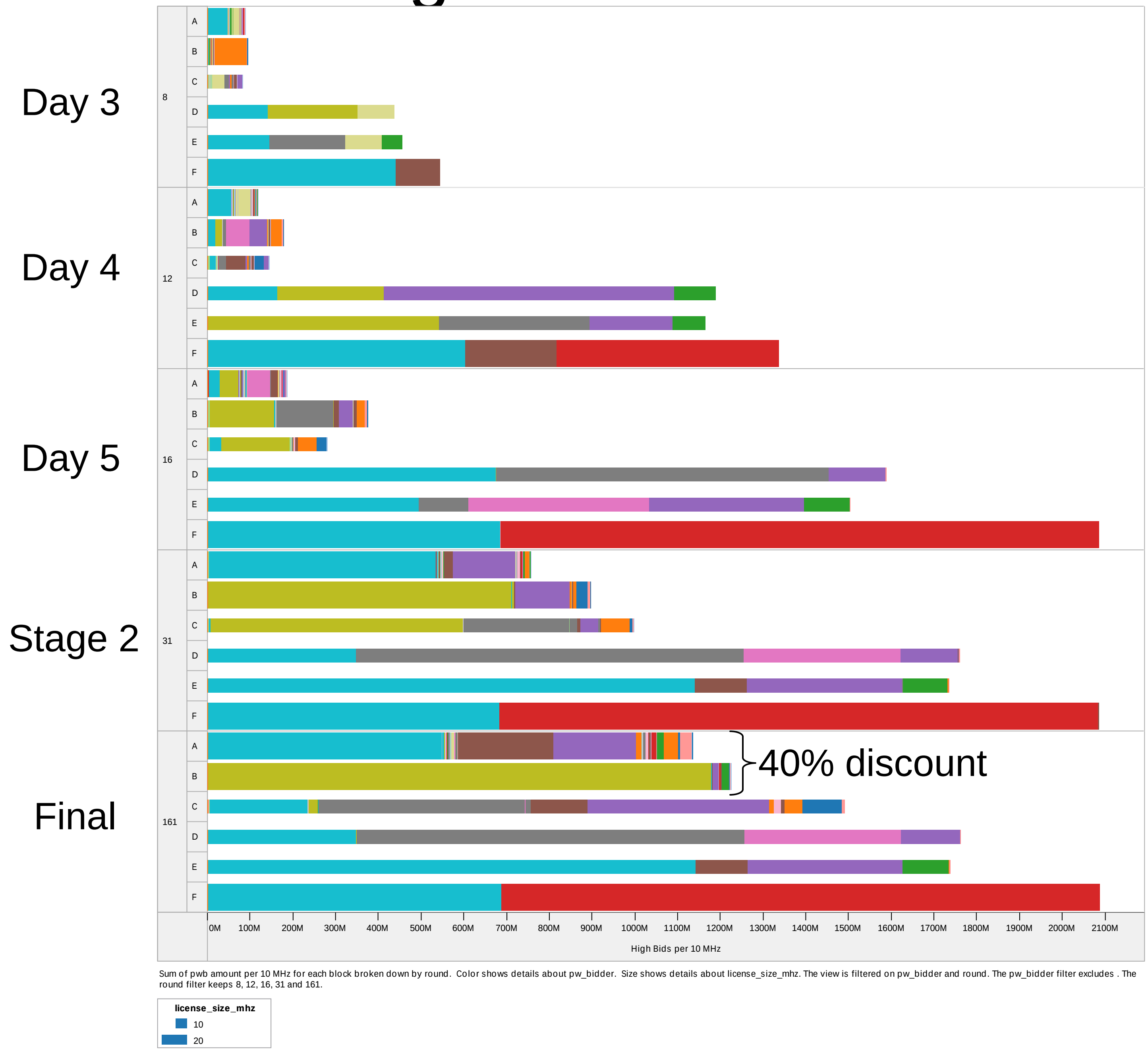
Simultaneous ascending auction

- Strengths
  - Simple price discovery process
  - Allows arbitrage across substitutes
  - Piece together desirable packages
  - Reduces winner's curse
- Weaknesses
  - Demand reduction
  - Tacit collusion
  - Parking
  - Exposure
  - Hold up
  - Limited substitution
  - Complex bidding strategies

Limited substitution: US AWS
90 MHz, 161 rounds, $14 billion



US AWS high bids in selected rounds



Sum of pwb amount per 10 MHz for each block broken down by round. Color shows details about pw_bidder. Size shows details about license_size_mhz. The view is filtered on pw_bidder and round. The pw_bidder filter excludes . The round filter keeps 8, 12, 16, 31 and 161.

Exposure problem

- With complements, bidding on individual lots is risky
  - Bidder must “go for it” or drop out early
  - Outcome is often inefficient
  - Experiments sometimes get high revenues
- Exposure problem eliminated with package bids

Variations of SAA

Variations of SAA

- Anonymous bidding
- Generic lots
- Package bidding

Package clock auction

- Porter-Rassenti-Roopnarine-Smith (2003)
- Ausubel-Cramton-Milgrom (2006)

- Clock auction for packages, followed by
- Best-and-final (proxy) round

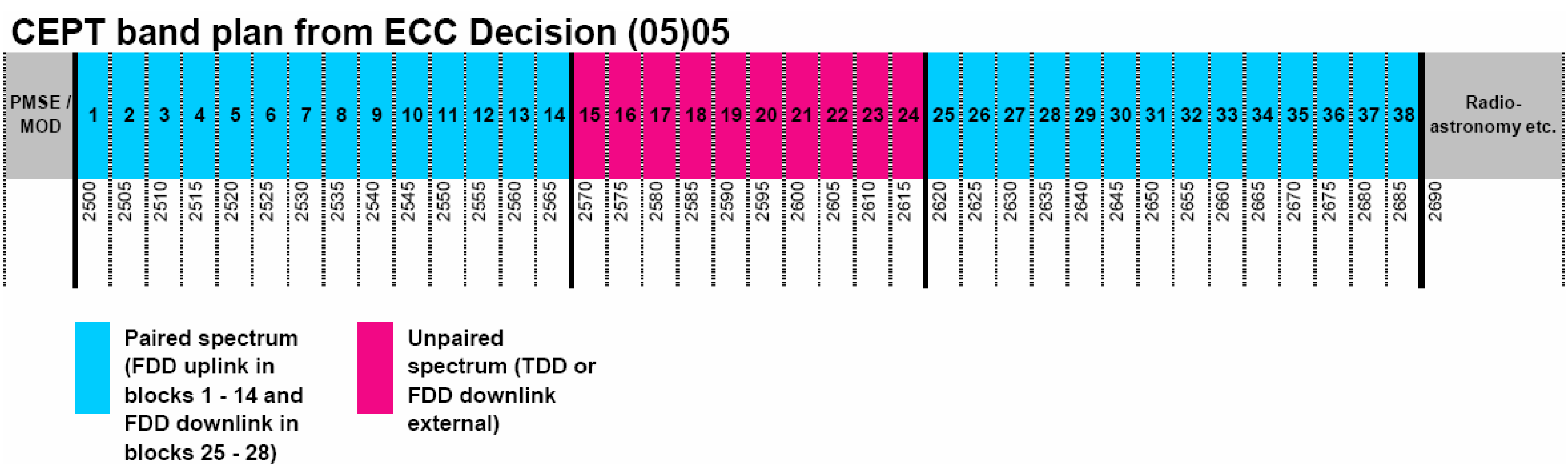
Package clock auction

- Auctioneer names prices; bidders name only quantities
  - Price adjusted according to excess demand
  - Process repeated until market clears
- No exposure problem (package auction)
- Activity rule to promote price discovery
- Final round to fine-tune assignment

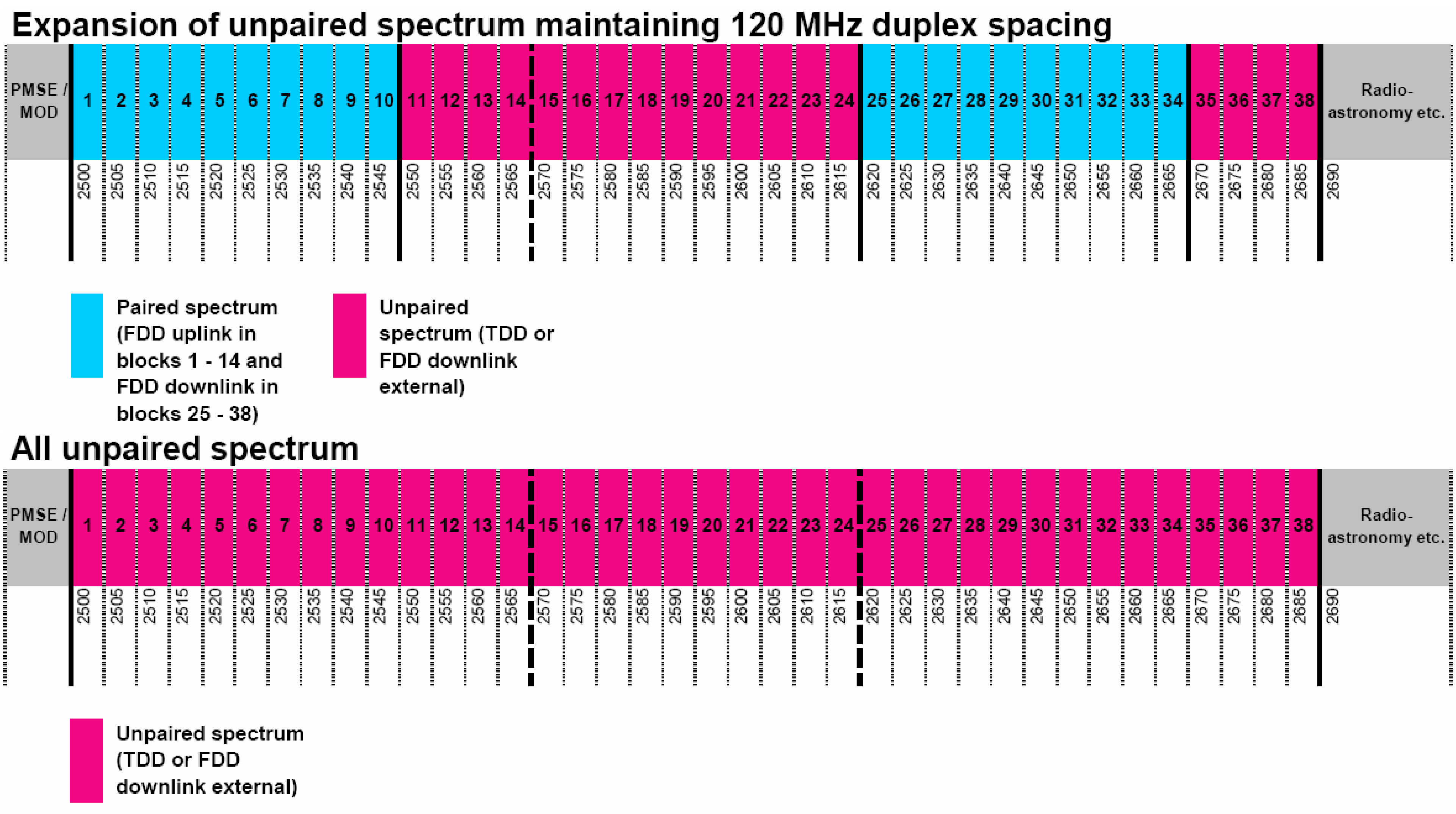
UK
spectrum auctions

UK 2.6 GHz auction proposal

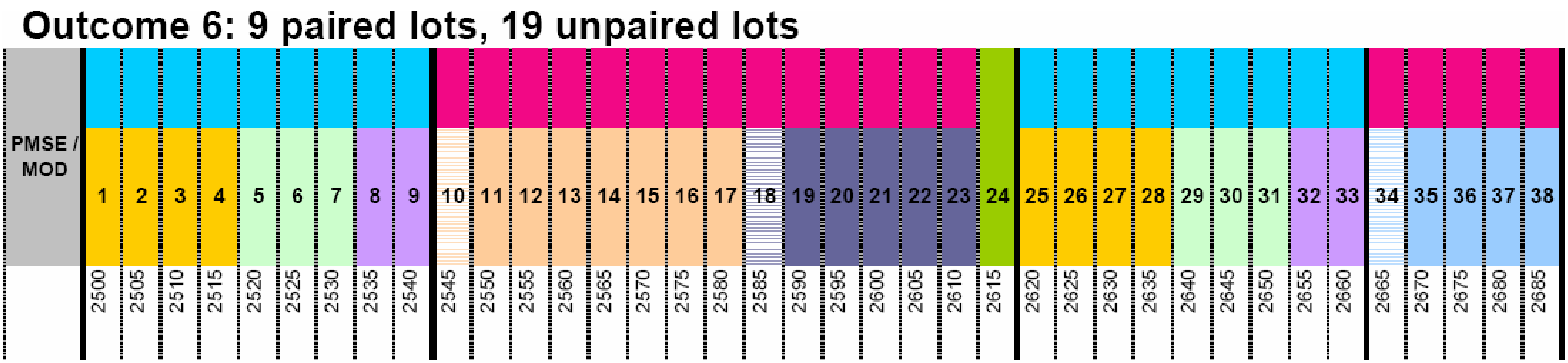
- 190 MHz (38 lots of 5 MHz)
  - – Compare US AWS, 90 MHz, $14 billion
- How much paired vs. unpaired?



Let auction determine band plan



3 paired winners; 3 unpaired



Key design choices

- Generic 5 MHz lots
  - Lots are perfect substitutes
- Package bids
  - No exposure problem
- Clock stage
  - How many paired? How many unpaired? Supply = 38
  - Continue until no excess demand
- Activity rule
  - Demands for paired and unpaired cannot increase
- Best-and-final bids
  - Demand (p,u) = (p paired lots, u unpaired lots)
  - When drop below (p,u), give best bid for (p,u) lots
- Best-and-final round
  - Find value maximizing generic assignment
- Assignment stage
  - Require contiguous spectrum if possible
  - Top-up bid to determine specific assignment

Information policy in clock stage

- Anonymous: Aggregate demands reported
- Transparent: All demands reported
- Anonymous is simplest for bidders
  - Focus on what is most important
- Transparent is best if bidders have “need to know”
  - For example, need to know likely winners to estimate value

Pricing rule

- In clock stage? In assignment stage?
- Pay-as-bid pricing
  - Incentives for demand reduction, bid shading
- Bidder-optimal core pricing
  - Stronger incentives for truthful bidding

Bidder-optimal core pricing

- Minimize payments subject to core constraints
- Core = assignment and payments such that
  - Efficient: Value maximizing assignment
  - Unblocked: No subset of bidders prefers to offer seller a better deal

Optimization

- Core point that minimizes payments readily calculated
- Use constraint generation method: find most violated core constraint and add it (Day and Raghavan 2005)
- Tie-breaking rule for prices is important
  - Minimize square deviation from Vickrey prices

Why core pricing?

- Truthful bidding nearly optimal
  - Simplifies bidding
  - Improves efficiency
- Same as Vickrey if Vickrey in core (substitutes)
- Avoids Vickrey problems with compliments
  - Prices that are too low
- Revenue is monotonic in bids and bidders
- Minimizes incentive to distort bids

Where do we see core pricing?

- Core methods have a long history in matching
  - Match interns to hospitals
  - Match students to schools
- Core methods in auctions are much more recent
  - Auctioning many items with complements
  - All standard auctions find point in core

10-40 GHz Auction

Category and band	Number of lots	Spectrum endowment	Eligibility pts per lot	Reserve price per lot
A: 10 GHz national	10	2 x 10 MHz	1	£10,000
B: 28 GHz national	2	2 x 112 MHz	6	£60,000
C: 28 GHz sub-region 1	1	2 x 112 MHz	2	£20,000
D: 28 GHz sub-region 2	1	2 x 112 MHz	1	£10,000
E: 28 GHz sub-region 3	1	2 x 112 MHz	3	£30,000
F: 32 GHz national	6	2 x 126 MHz	6	£60,000
G: 40GHz national	6	2 x 250 MHz	3	£30,000

Package clock auction

- Principal stage
  - Primary rounds (clock auction)
  - Supplementary bids
  - Optimization: Winners and base prices
- Assignment stage
  - Assignment bids
  - Optimization: Specific assignments and additional payments

Activity rule

- Whenever reduce package size, value on all larger packages limited by prices at the time of reduction
  - Example
    - Bidder drops from package of size 40 to 36 at prices p
    - For all packages x of size 37 to 40,
      bid $\leq p \cdot x$
- Implication
  - Profit maximization is poor strategy
  - Bid to maximize package size subject to profit ≥ 0

Full-scale test of design (Maryland PhD students)

- Experienced subjects
  - Course in advanced game theory
  - Prior participation in package clock auction
- Motivated subjects
  - Average subject payment = $400
- Realistic scenarios
- 4 mock auctions with moderate competition (6 bidders)
- 8 mock auctions with weak competition (4 bidders)

Six bidder auctions

Mock	Truth	Value	Efficiency	Profit	Share
6a.1	93%	5,782	93%	2,706	43%
6a.2	97%	5,540	89%	3,755	60%
6a.all	100%	6,248	100%	2,646	42%
6b.1	96%	4,176	93%	1,751	39%
6b.2	100%	4,303	96%	1,147	26%
6b.all	100%	4,494	100%	1,101	25%

Truth = average of bid/value; Share = profit/value.
Mock *.all includes all bids (about 50,000) at value.

- Deviation from truth is caused by activity rule, not strategic behavior
- Activity rule
  - Destroys price discovery
  - Undermines efficiency

Four bidder auctions

Mock	Truth	Value	Efficiency	Profit	Share
4a.1	100%	2,858	96%	1,446	49%
4a.2	100%	2,859	96%	1,698	57%
4a.3	100%	2,880	97%	1,296	44%
4a.4	100%	2,970	100%	1,370	46%
4a.all	100%	2,978	100%	976	33%
4b.1	100%	1,865	99%	1,033	55%
4b.2	100%	1,590	84%	410	22%
4b.3*	100%	1,891	100%	1,054	56%
4b.4	100%	1,819	97%	1,016	54%
4b.all	100%	1,882	100%	879	47%

Truth = average of bid/value; Share = profit/value.

Mock *.all includes all bids (about 34,000) at value.

* White's package violates minimum size constraint.

- Activity rule destroys price discovery

Activity rule is readily fixed

- Revealed-preference activity rule
  - Compare times s and t (s < t),
    Prices: p_s, p_t Demands: x_s, x_t
  - At time s, x_s is better than x_t: $v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t$
  - At time t, x_t is better than x_s : $v(x^t) - p^t \cdot x^t \geq v(x^s) - p^t \cdot x^s$
- Adding inequalities yields the RP activity rule:

$$(RP) \qquad\qquad (p^t - p^s) \cdot (x^t - x^s) \leq 0 .$$

Revealed preference for supplementary bids

- For each package T in clock stage, provide a value v(T), consistent with revealed preference
- For each supplementary package S, provide a value V(S) consistent with revealed preference:
  $v(S) - P^t(S) \leq v(T) - P^t(T)$
  (T was more profitable than S at time t)

Evaluation from test

- *Activity rule destroys price discovery*
  - *Readily fixed by switching to revealed preference*
- Efficient
- Robust
- Low risk
- Simple for bidders
- Competitive revenues
- Simple for the auctioneer

Conclusion

- Package clock auction
  - Eliminates exposure
  - Eliminates gaming
  - Enhances substitution
  - Allows auction to determine band plan
  - Readily customized to a variety of settings