MASTER'S THESIS

Classifying and Comparing Design Optimization Problems

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MS 2006-2

ABSTRACT

Title of Thesis: CLASSIFYING AND COMPARING DESIGN OPTIMIZATION PROBLEMS

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Research in product design optimization has developed and demonstrated a variety of modeling techniques and solution methods, including multidisciplinary design optimization. As new techniques migrate to the industrial world engineers are faced with much more complex problems often extending beyond their realm of knowledge. A novel classification scheme is proposed and demonstrated to offer engineers a method of organizing and searching for relevant example problems to assist in the production of their own optimization problem. To explore the tradeoff between information requirements and solution quality, computational experiments are conducted on two design problems, a bathroom scale and a universal electric motor. In particular, the results of these experiments identify the additional information required to solve a profit maximization problem, demonstrate the role of rules of thumb in formulating design optimization problems, show how decomposition affects solution quality and computational effort, and uncover the impact of using target matching in the objective function instead of as constraints. In addition, the results show how the values of targets

and objective function weights impact solution quality. In general, these results show the extent to which correct information is critical to finding a high quality solution, perhaps more critical than the optimization model selected. That is, the quality of the information used is more important than the amount of information used.

CLASSIFYING AND COMPARING DESIGN OPTIMIZATION PROBLEMS

by

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Thesis submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Master of Science
2006

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Acknowledgements

I would like to thank Dr. Jeffrey Herrmann for his continual support throughout this research. Without his patience, perseverance and guidance my understanding of the subject matter would not compare.

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List of Acronyms

AAO: All-at-Once
ATC: Analytical Target Cascading
BLISS: Bi-level Integrated System Synthesis
CO: Collaborative Optimization
CSSO: Collaborative Subspace Optimization
DBD: Decision-based Design
DCA: Discrete Choice Analysis
DV: Design Variable
EU: Expected Utility
EV: Expected Value
IDF: Individual-discipline Feasible
MDF: Multi-discipline Feasible
MDO: Multi-disciplinary Design Optimization
NaN: Not-a-Number
NVH: Noise-Vibration-Handling
PS: Probability of Satisfaction
WC: Worst Case

CHAPTER 1: INTRODUCTION

Product design optimization is an important step in the product development process. An optimal product may be defined as one that meets its performance requirements with the best economical outlook. In general product design optimization determines the best values for design variables based on some performance attributes that satisfy all constraints and maximize some overall objective. In its early stages design optimization was applied to detailed design problems with simple models that could be solved by hand. With the invention and advancement of computers, larger and more complex problems can be handled in a reasonable amount of time. Today design optimization is used in many stages of the product development process including conceptual design to evaluate possible concepts and multidisciplinary design to embrace interests from several departments within an engineering firm simultaneously.

Product design optimization, like other aspects of product development, continuously changes as new ideas and improved technology surface at an unprecedented rate. Much advancement in design optimization has focused on solution techniques and decomposition frameworks. A decomposition framework is a modeling architecture that allows a complex problem to be broken into more manageable subproblems. For example, multi-disciplinary design optimization techniques such as collaborative optimization, analytical target cascading, and concurrent subspace optimization are all decomposition frameworks. To demonstrate a new technique simple comparisons of the solution are made with previously accepted techniques to verify proper convergence and possibly improved performance. New decomposition frameworks attempt to link multiple disciplines to incorporate decisions and goals of each discipline without limiting the influence of each discipline's expertise.

With all of the advancements in optimization solution techniques, it seems that little importance has been placed on the information an engineer must have during the formulation of a design optimization problem. More information is needed to create more sophisticated optimization models. For example, maximizing profit requires information about how the design variables affect demand and cost. Studying the information requirements can help us explore the tradeoff between information requirements and solution quality. This thesis attempts to help design engineers formulate the optimization problem by providing a method of finding similar problems based on a classification scheme as well as providing insight into the tradeoffs involved in using different modeling approaches.

A classification scheme will be proposed and demonstrated in an attempt to standardize the taxonomy used by the optimization community in discussing product design optimization problems. The goals when developing this classification framework included both scientific and practical ones. The classification framework helps to organize and understand design optimization problems, an important step in any scientific discipline. A standardized taxonomy can assist design engineers when searching for similar examples previously handled by other engineers. The proposed classification scheme sorts design optimization problems based on the type of variables being considered and the objective functions being optimized. It does not focus on the algorithms used to solve the problems. This classification framework provides a new perspective that can help design engineers use optimization in the most appropriate way.

The second part of the thesis considers the tradeoff between information requirements and solution quality. Insight into the tradeoffs involved in using different modeling approaches are accrued by determining the information requirements needed to solve design optimization problems when formulated using different decomposition and solution techniques. The information available can greatly affect the effort needed to formulate the optimization model and the quality of results obtained during an optimization process. The majority of attention in available literature has been placed on finding new solution techniques rather than on the quantity of information needed to find an appropriate solution. Two example problems from previous literature will be optimized using different modeling approaches and disciplines to study information requirements and solution quality.

The questions we hope to answer from the computational experiments include the following: With the information that I have available right now, if I formulate problem P like example A and get solution X, how much effort will it take to get solution X and how good is solution X? On the other hand, if I formulate the problem like example B and get solution Y, what is the difference in effort required and quality compared to solution X? What other observations can be made from the analysis? What amount of information was needed to model P like A versus B relative to the quality of the solution?

Chapter 2 provides the relevant background information and literature review of the optimization field. Chapter 3 describes the methodology used in developing a classification scheme and analyzing relationships between a knowledge base and modeling. Chapter 4 details and demonstrates a classification framework for general product design optimization problems. Chapter 5 includes the analysis of a universal electric motor. Chapter 6 details the analysis of a bathroom scale. Chapter 7 offers a generalized discussion about the observed connections between heuristics and modeling. Finally, Chapter 8 summarizes and concludes the thesis.

CHAPTER 2: BACKGROUND

Product development has been a lively research area over the last few decades especially in system level design optimization. Much work has been done to develop new techniques and frameworks to aid in solving complex optimization problems. Adding disciplines and increasing problem complexity accented the major limitations of initial solution techniques thus driving researchers to find alternate approaches [1]. The growing complexity of optimization problems is forcing engineers to depend more on powerful software and computer technology, while improvements in computing technology allow engineers the opportunity to attempt complex problems.

Advancements in computer capabilities and software have helped bridge the gap between research and industrial applications. For example, software such as iSight, MAX, and Smart|Coupling can currently integrate several disciplines into one complete optimization [2, 3]. Third party analysis software such as computational fluid dynamics, finite element analysis, spreadsheet simulation, and in house code are currently being integrated through these programs to expand the scope of optimization capabilities. New research in optimization and improvements in software have generated two major shifts in the scope of optimization techniques.

First, a shift from single discipline optimization, e.g. structures, to multiple disciplines within the engineering domain, e.g. performance, structures, and aerodynamics, occurred. Multiple disciplines meant more objectives to satisfy and coupling considerations to handle during the optimization process. As a result, several multidisciplinary design optimization (MDO) techniques such as all-at-once (AAO), individual-discipline feasible (IDF), and multi-discipline feasible (MDF) approaches were developed [4, 5]. Since then, other MDO solution methods including collaborative optimization (CO), concurrent subspace optimization (CSSO), and bi-level integrated system synthesis (BLISS) have been created and demonstrated in example problems [6]. MDO techniques apply various decomposition and coordination methods to facilitate communication between several disciplines while utilizing common optimization solvers to find a solution. Sub-optimization functions can be contained within the subsystems, with appropriate coupling variables linking all the systems and subsystems together, to ensure a global objective is maintained.

Collaborative optimization (CO) and analytical target cascading (ATC) are two common frameworks used in multidisciplinary optimization that have been demonstrated frequently in the last decade. CO is a bi-level optimization method used for non-hierarchical systems [7-11]. The newer method of ATC, on the other hand, decomposes a hierarchical system into two or more levels [6, 14-17]. ATC is not directly a categorization of an MDO technique because it depends on how the problem is decomposed.

The second shift occurred as a result of the growing popularity of decision-based design (DBD). Since Hazelrigg [18] introduced a DBD framework for engineering design, applications have evolved to include decision-making and uncertainty [19-22]. MDO techniques are being applied to the DBD framework in an attempt to handle added variables from the marketing and manufacturing domains [6,11,16,17]. Many approaches have been developed and tested on example problems, however, the majority of available literature detailing these example problems is organized around modeling techniques and solution methods.

To the authors' knowledge there have been only two classification schemes related to single product design optimization. The first was a taxonomy for three MDO decomposition approaches developed by Cramer *et al*. [5] resulting in the AAO, IDF, and MDF approaches mentioned previously. The second classification scheme, developed by Balling and Sobieszczanski-Sobieski [9], was a more general and versatile taxonomy for the six fundamental approaches of MDO decomposition. The notation in this taxonomy distinguishes between single and multi-level optimization and whether the analysis is simultaneous or nested at the system and discipline levels. Both of these classification schemes focus on the details of the techniques used to solve multi-discipline problems.

In 1979, Graham et al. [12] created a classification scheme that helped generate ideas for the formulation proposed in this thesis. The three field $\alpha/\beta/\gamma$ notation classifies machine scheduling optimization problems based on various machine environments, job characteristics, and scheduling objectives, respectively [13].

To the author's knowledge there has not been any work conducted on evaluating optimization problems from the information flow perspective used in this research. Somewhat relevant literature related to the analyses performed in this thesis is found in Pan and Diaz [23]. Pan and Diaz discuss some inherent difficulties that arise when tightly coupled design problems are decomposed in a nonhierarchical fashion and solved sequentially. Sequential optimization is the process of obtaining a solution by first solving a subproblem, then using the outputs from that subproblem as inputs to a succeeding subproblem optimization. This process is repeated until all design variables have been determined.

CHAPTER 3: METHODOLOGY

This thesis consists of two activities: the development of a classification scheme for product design optimization and an investigation of relationships between information requirements and solution quality. This chapter will discuss the methodology used in each.

3.1 Classification Scheme

Developing a classification scheme that is general enough to span a large space of optimization problems requires knowledge of a large space of optimization problems. To gain this knowledge a literature review of single discipline as well as multi-discipline optimization frameworks, solution techniques, and examples was conducted. From this review comparisons were made based on common elements of optimization problems such as objective functions, constraints, variables, and disciplines involved. Important categories common to all optimization problems were selected with the intention of developing a classification scheme that can be used as means for comparing an unsolved optimization problem at hand with existing examples. These categories are a set of relevant metrics used in comparing various optimization problems to determine if existing examples can aid an engineer in solving their own optimization problem. Extensions to the three main categories were then analyzed for relevance and importance once the categories were decided on. These extensions included conceptual versus detailed design, single objective versus multi-objective optimization, and deterministic versus non-deterministic models.

3.2 Computational Experiments

Exploring the tradeoffs between information requirements and the solution quality was done using computational experiments on two example problems from the literature. The two example problems, including a bathroom scale and universal electric motor, were reformulated in several different ways to adjust the number of disciplines, constraints, objective functions, initial solutions, target values, and sequence used to solve them. Relation matrices were developed to determine alternate ways of decomposing the problems and to understand the degree of coupling involved in each example. A highly coupled system cannot be decomposed into a set of independent subsystems. The majority of subsystems will have direct dependence on one or more other subsystems. A weakly coupled system, on the other hand, can be decomposed into a set of subsystems with little dependence on other subsystems.

To study the tradeoff between information requirements and solution quality, the scope (i.e. number of disciplines) was increased by formulating a profit maximization problem that adds the marketing discipline to the engineering discipline. This requires more information but should yield a more profitable solution. Variations in constraints were generally related to the decomposition and scope of the problem. Adding the marketing discipline adds more constraints in an attempt to simultaneously satisfy more requirements. However, in some cases equality constraints were removed and replaced with target values in the objective function.

Many of the optimization setups were single discipline optimizations involving just engineering. For these cases the design variables were found using an engineering optimization model. The results were then evaluated by estimating the profitability, which required a price optimization.

When a problem is decomposed in different ways the constraints directly relevant to each subsection of the decomposition change depending on what variables are included in that respective subsection. Objective functions were changed in many different ways including single to multi-objective, weighting, and target settings. Finally, the sequence of solving the optimization was changed to determine the impact on solution quality, feasibility, and computational effort. Various sequences, if possible, were developed and solved.

For all the different variations described above, appropriate optimization models were created and solved using the optimization toolbox in MATLAB 7.0.4™. The MATLAB function *fmincon* was used in all cases to minimize the constrained optimization. In most cases default MATLAB settings were used for the constrained minimization problems, however, there were some cases where the number of allowable function calls was increased and the convergence criteria softened. The results were then compared and analyzed in an attempt to quantify the solution quality of different optimization models.

CHAPTER 4: A CLASSIFICATION FRAMEWORK

Research on design optimization has developed and demonstrated a variety of modeling techniques and solution methods, including techniques for multidisciplinary design optimization, and these approaches are beginning to migrate into product development practice. Software tools are appearing to assist with the optimization task. However, the complexity of the optimization problems being considered continues to increase because changing business strategies stress the importance of concurrent engineering and considering multiple disciplines simultaneously. This chapter presents a classification framework based on the examination of various design optimization problems from the perspective of information requirements and objectives. We are not directly concerned with decomposition or modeling techniques nor do we limit our classification to MDO problems. The generality of the proposed classification allows even the most basic optimization problems to be classified.

Our goals when developing this classification framework included both scientific and practical ones. First, this classification framework helps us to organize and understand design optimization problems, an important step in any scientific discipline. While this classification framework is not the only conceivable scheme, we believe that it concisely captures the most important attributes while remaining open to including other attributes in the future if so desired. Second, this classification framework provides practical help for design engineers considering design optimization. Using this scheme, a design engineer can locate similar design optimization problems, which can be useful guides for formulating a new problem. Moreover, the set of similar design optimization problems indicates the range of potential solution techniques. Of course, the design engineer must still choose a problem formulation and a solution technique. This classification framework does not replace modeling skill, but it does provide information that can help one develop it.

The remainder of the chapter proceeds as follows. After defining some key terms used in the chapter, we present the classification framework and then use available examples to demonstrate it.

4.1 Definitions

Many areas within a firm can influence the product development process. Engineering is obviously the basis of design while manufacturing and marketing are a major part of concurrent engineering. The engineering domain represents the perspective of design engineers and concerns about the product design and product performance. The manufacturing domain represents the perspective of manufacturing personnel and concerns about the manufacturing process and the corresponding metrics. The marketing domain represents the perspective of the product manager and concerns about finances, customer preferences, and demand.

Design optimization problems have three primary features: variables, constraints, and objective functions. Our classification framework will consider only variables and system level objectives. Secondary objectives, which may appear in constraints or subsystems, do not affect the classification scheme. Constraints are important because they can influence the choice of an optimization solver based on whether the constraints are linear, nonlinear, equality, or inequality constraints. However, constraints are generally created during the modeling process. Our classification framework is meant to describe the fundamental problem, not the model details.

Due to the nonconformity of terminology in design research, the following definitions are given along with possible synonyms to avoid any confusion.

4.1.1 Product Scope

The classification framework distinguishes between single product design optimization and product family optimization. Definitions for each of the two product types are given for clarity, however, only single-product design problems are treated in this thesis. Future work will extend this classification scheme to product families.

Single Product: This is a product that is designed with no regard to similar products. Component sharing and interconnection with other products do not influence the design decisions.

Product Family [24]:

1. A set of common elements, modules, or parts from which a stream of derivative products can be efficiently developed and launched.
2. A collection of common elements, especially the underlying core technology, implemented across a range of products.
3. A collection of assets (i.e., components, processes, knowledge, people and relationships) that are shared by a set of products.

4.1.2 Variables

Variables are sometimes referred to as parameters, design variables, and design parameters [25]. A designer must select the values for variables. Optimization is used to help find appropriate values of variables. The following three definitions refer to more specific types of variables.

Engineering Variables: These are variables specific to the product being designed. Typical engineering variables include product geometry, features, and material selection.

Manufacturing Variables: These are variables specific to the manufacturing domain. Every facility will have different manufacturing variables specific to the machine types and facility layout. Examples include number of machines, time allotment per machine, number of operations per part, force and energy requirements, feed rate, and depth of cut.

Price Variable: This variable is the price of the product or system being designed. Pricing is a critical but complex issue. For a new product, a successful pricing approach first determines the price that customers can be convinced to pay for the product concept, and then the firm designs a satisfactory product that can be manufactured profitably at the expected sales volume [26]. While the initial pricing strategy may be used to set a cost target for the product design, the product price will certainly change over time as the firm's pricing strategy influences their response to market forces. The product development team does not need to make pricing decisions that have not yet arrived. However, optimizing product profitability at the design stage requires understanding what the firm is likely to do. If alternative strategies are feasible (such as skim pricing or penetration pricing), the team may want to evaluate these strategies, since they control future prices.

4.1.3 Objective Functions

Design optimization (especially MDO) can include several subproblems depending on the system being designed. The classification framework considers only the system level objectives. The classification framework covers single objective as well as multi-objective optimization problems at the system level.

Attribute-based: These objective functions are related to product performance or product characteristics (i.e. attributes). For the purposes of this classification framework an attribute is a quantitative measure related to the object or system being designed. The objective is to maximize or minimize an attribute level, usually a performance measure, based on the product being designed. Although uncommon, it is possible to utilize demand information in the attribute-based objective function but it is not a requirement. Examples: minimize weight, minimize size, minimize stress, and maximize range. Alternatively, the objective may be to minimize the deviation from a target attribute value.

Cost-based: These objective functions are related to the engineering and manufacturing domains. The goal is to minimize the overall cost of the product based on one or more cost models. Generally this type of optimization will be more complex than the attribute-based objective because cost models will be necessary along with the design models. While one can consider a cost objective to be a performance measure equivalent to any attribute-based objective, we treat cost separately because product performance and product cost are fundamentally different and very important objectives, as discussed by Smith and Reinertsen [27]. Therefore it is useful to the designer if a distinction is made between the two types of objectives. Similar to the attribute-based objective function this can include situations where the objective is to minimize the deviation from a cost target. Demand can again be utilized as a weighting method in this objective but is not required.

Profit-based: These objective functions are directly related to the marketing domain. The goal of the optimization is to maximize the design value based on demand information. Although not stated explicitly in the classification it can be assumed that any profit-based objective will require some type of demand model. Another step in complexity is seen through the profit-based models, in comparison to the attribute-based and cost-based models, because more model evaluations are required for this type of optimization. Examples: maximize revenue, maximize profit, maximize expected utility of profit, maximize net present value, and maximize return on investment.

4.2 Classification Framework

Three main categories become apparent when considering design optimization problems. Our classification framework sorts design optimization problems based on the following three characteristics: problem scope (i.e. single product versus product family), the variables that need to be decided (i.e. engineering, manufacturing, or price), and the system level objective function (or functions) of the optimization problem (i.e. attribute-based, cost-based, or profit-based).

To explain the classification framework, we will begin with the most basic types of deterministic optimization problems involving only a single objective function. Subsequent paragraphs will discuss problems with multiple objectives. After that, we will present a modifier to the objective function to describe typical methods of dealing with uncertainty.

The classification framework categorizes design problems using three fields corresponding to the three characteristics mentioned above. The first field notes the number of products. The second field notes the types of variables. The third field notes the type of objective function (or functions). Designing a single product with a single system level objective can include twelve possible optimization framework combinations. Six of the twelve combinations are more likely to be used due to the relationship between the objective function and the variables considered. For example, if the design process includes only engineering variables, then maximizing profit would not be a typical objective function since maximizing profit or the expected utility of profit would include the price variable. The twelve combinations for single objective optimization problems are shown in Table 1 with the six most logical in boldface.

The product type entry in field one can be either single product (S) or product family (F). Variables present in the optimization, shown in field two, may include engineering variables (E), manufacturing variables (M), or a price variable (P). Field three displays the objective functions for each combination of variables, which include attribute-based objectives (A), cost-based objectives (C), and profit-based objectives (Π).

Single Objective					
Field #			**Product Type**	**Variables Included**	**System Objective**
1	**2**	**3**			
S	**E**	**A**	**Single**	**Eng.**	**Attribute-based**
S	**E**	**C**	**Single**	**Eng.**	**Cost-based**
S	E	Π	Single	Eng.	Profit-based
S	**EM**	**A**	**Single**	**Eng. & Mfg.**	**Attribute-based**
S	**EM**	**C**	**Single**	**Eng. & Mfg.**	**Cost-based**
S	EM	Π	Single	Eng. & Mfg.	Profit-based
S	EP	A	Single	Eng. & Price	Attribute-based
S	EP	C	Single	Eng. & Price	Cost-based
S	**EP**	**Π**	**Single**	**Eng. & Price**	**Profit-based**
S	EMP	A	Single	Eng., Mfg. & Price	Attribute-based
S	EMP	C	Single	Eng., Mfg. & Price	Cost-based
S	**EMP**	**Π**	**Single**	**Eng., Mfg. & Price**	**Profit-based**

Table 1: Combinations of Single Product Optimization with a Single Objective.

The above classification framework is easy to use and self-explanatory. For instance, if a problem is classified as type S-E-A, one can immediately know that the optimization problem is for a single product, it has only engineering variables, and has an attribute-based objective.

The classification framework also includes multi-objective design optimization problems, resulting in eight more common combinations. The third field of the classification is further divided into two subfields (i.e. positions within the third field).

The first subfield may contain the entry “A” or “C.” The second field on the other hand can be either “A”, “C”, or “Π” to specify what other objectives are present.

The classification of an optimization problem with two or more attribute-based objectives would contain “AA” in the third field (e.g. S-E-AA or S-EM-AA). If “AC” appears in the third field of the classification then there are two or more attribute-based and cost-based objectives. Similarly, “AΠ” is used for the multi-objective case where attribute-based and profit-based objectives are present. The latter can be seen in multidisciplinary design optimization techniques such as ATC and CO when the multi-objective function is to minimize the deviation between attribute targets while maximizing profit [16, 17]. Note the specific number of objectives is not specified in the multi-objective case. The formulation of the objective function, as well as the choice of optimization program, may alter depending on the number of objectives (e.g., two versus four objectives) but from the perspective of the proposed classification scheme these differences are minor. Distinguishing between a single objective optimization and a multi-objective optimization plays a much larger role in selecting a solution technique than the difference between two objectives and four objectives.

The classification scheme also ignores the details of how a multi-objective problem is formulated. Multiple objectives are often formulated into a single objective function, for example minimizing the deviation between several targets and responses, but this distinction is too detailed when compared to the perspective used in developing the classification scheme.

The occurrence of a “CΠ” classification is unlikely because cost models are generally inputs to the profit model though this multi-objective problem is technically feasible. The sixteen possible combinations are shown in Table 2 with the eight most typical combinations in bold.

Multi-Objective					
Field #			Product Type	Variables Included	System Objective
1	2	3			
S	**E**	**AA**	**Single**	**Eng.**	**Attribute-based**
S	**E**	**AC**	**Single**	**Eng.**	**Att. & Cost-based**
S	**E**	**CC**	**Single**	**Eng.**	**Cost-based**
S	**EM**	**AA**	**Single**	**Eng. & Mfg.**	**Attribute-based**
S	**EM**	**AC**	**Single**	**Eng. & Mfg.**	**Att. & Cost-based**
S	**EM**	**CC**	**Single**	**Eng. & Mfg.**	**Cost-based**
S	EP	AA	Single	Eng. & Price	Attribute-based
S	EP	AC	Single	Eng. & Price	Att. & Cost-based
S	**EP**	**AΠ**	**Single**	**Eng. & Price**	**Att. & Profit-based**
S	EP	CC	Single	Eng. & Price	Cost-based
S	EP	CΠ	Single	Eng. & Price	Cost & Profit-based
S	EMP	AA	Single	Eng., Mfg. & Price	Attribute-based
S	EMP	AC	Single	Eng., Mfg. & Price	Att. & Cost-based
S	**EMP**	**AΠ**	**Single**	**Eng., Mfg. & Price**	**Att. & Profit-based**
S	EMP	CC	Single	Eng., Mfg. & Price	Cost-based
S	EMP	CΠ	Single	Eng., Mfg. & Price	Cost & Profit-based

Table 2: Combinations of Single Product Optimization with Multiple Objectives.

Deterministic models are preferred by engineers due to the simplicity of formulating and solving them. Unfortunately, it is a well known fact that the real world is not deterministic. Therefore, it is important to include uncertainty in the classification framework. An objective function subclass, including five methods of dealing with uncertainty, categorizes and clarifies optimization problems further. The first method of dealing with uncertainty is ignoring it, thus the problem is a deterministic optimization problem. Four other common methods include expected value (EV), expected utility (EU), worst-case (WC), and probability of satisfaction (PS). Although there are variations to the methods mentioned above (such as the Hurwicz criteria and maximum likelihood criteria), we believe the most common forms are accounted for.

The classification framework represents the uncertainty subclass using a subscript on the objective function terms in the third field. Deterministic objective functions would have no subscript in the third field while the four common methods for dealing with uncertainty described above would include a subscript of EV, EU, WC, or PS respectively. For example, the classification S-E-A_{WC} is used for problems that address a single product, have engineering variables, and optimize the worst-case performance.

The framework is deployed in the next section to classify available examples. Engineers will be able to use available examples to perform a case-based search and find design problems that are similar based on the three fields of the classification scheme and compare the different solution techniques previous designers used in solving the problem.

4.3 Examples

Available examples of various optimization problems, including MDO problems, will be classified using the proposed framework. The MDO problems used for demonstrating the framework were solved using either ATC or CO techniques.

An S-E-A type optimization is the most basic because it involves only the engineering domain. Therefore, the equations used to model this type of optimization rely only on principles of engineering science. First a general optimization problem is discussed followed by another example that employs one of the afore-mentioned MDO techniques.

A simple single discipline example of designing a fingernail clipper can be found in Otto and Wood [28]. In this example a model is formulated to represent finger force. The variables included in this model are finger force, cutting force at the blade, length of lever arm, distance to the blades, nail thickness, width of blade, and blade height. The deterministic attribute-based objective chosen in designing the fingernail clipper is to minimize the finger force required subject to stress and dimension constraints. It can easily be seen by looking at the variables involved that only engineering variables are included for the design of a single fingernail clipper. Thus, this problem can be classified as type S-E-A. Cost and manufacturing concerns are not present in the formulation although it is possible to extend this problem to include such domains.

Kroo *et al*. [8] present a system level aircraft design problem. The global objective function is to maximize range under the influence of an aerodynamics subsystem, a structures subsystem, and a performance subsystem. Range is an attribute of the system to be designed, which corresponds to the "A" in the classification. The variables in this problem are all related to the plane's design and include wing geometry, wing weight, twist angle, aspect ratio, gust loading, and lift-to-drag ratio, all of which affect range. This is a deterministic S-E-A problem because no distributions are applied to the input variables. Figure 1 shows the CO framework applied to solve this aircraft design problem.

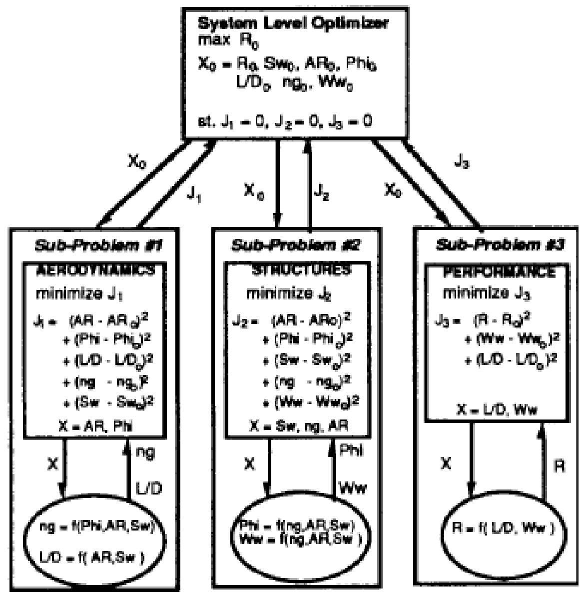


Figure 1: CO Framework for Aircraft Design [8].

The CO framework in this example clearly shows what variables are present during the optimization as well as the disciplines influencing the system level design. Notice the classification framework is not directly related to how the problem is divided or what disciplines within engineering are included. Sobieski and Kroo [10], Kim *et al.* [14, 15], Otto and Wood [28], and McAllister and Simpson [29] demonstrate other examples of S-E-A type optimization problems.

Although a fingernail clipper and aircraft design problem in the above examples intuitively seem very different, the difficulty in solving them is not all that different. The fingernail clipper is a detailed design problem while the aircraft wing is more conceptual. The aircraft design problem could have been formulated and solved as an AAO instead of using the collaborative optimization, in which case the two examples would appear to have a greater similarity. When a problem can be decomposed and modeled in different ways, a design engineer would probably want to see examples of several different methods to find the most appropriate one. Therefore, multiple examples of techniques based on similar problems seem more useful especially when dealing with a somewhat complex design problem.

Next, an example of an S-EP-Π type optimization problem is examined. Gu *et al.* [11] details an aircraft concept-sizing problem to maximize profit under the influence of engineering variables and a price variable. The authors chose to assume the utility of profit to be profit itself thus treating it as a deterministic problem for calculation purposes. If uncertainty were accounted for through a utility function this problem would be classified as S-EP-Π_{EU}. Figure 2 shows the general layout of the decision-based collaborative optimization approach.

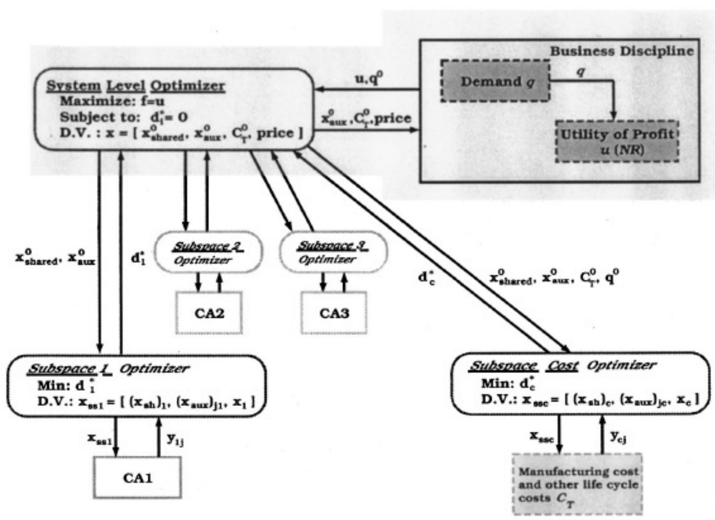


Figure 2: A General Decision-based CO Framework [11].

The engineering variables included in this single aircraft optimization example consist of aspect ratio, wing area, fuselage length, fuselage diameter, density of air at cruise altitude, cruise speed, and fuel weight. The price variable is also part of the optimization problem. In a profit-based optimization problem the cost models are present as inputs to a profit function but do not affect the classification framework because it is not a system level objective. The cost model term is present in the classification framework only for systems with a cost-based objective. Another example of a type S-EP-Π optimization can be found in Kumar *et al*. [6].

Next, an example is taken from Sues *et al*. [31] to demonstrate the uncertainty sub-class within the classification framework. This shape optimization of an airplane wing includes seven engineering variables related to the wing geometry. Values for aspect ratio, taper ratio, semi span wingtip incidence, structure skin thickness, structure span thickness, and wing sweep all need to be decided. The global objective of this single wing shape optimization is to maximize expected cruise range. Uncertainty appears through random distributions on all of the design variables to account for inconsistencies in the manufacturing processes. This example can be classified as type S-E-A_{EV}. Several other examples dealing with uncertainty can be found in Sues *et al*. [31]. An example of type S-EP-Π_{EU} can be found in [30].

Finally, a multi-objective optimization example will be classified using the framework. Azarm and Narayanan [32] discuss a multi-objective example regarding the design of a fleet of ships. The objectives of this example include minimizing construction and operating costs and maximizing the cargo capacity. The engineering variables present in the model of this optimization include: breadth, depth, deadweight, length, number of ships, draft, utilization factor, speed, and displacement. Due to the conceptual nature of this optimization problem, specific manufacturing construction variables were not considered. Manufacturing costs, however, were accounted for in the cost models. This problem can be classified as type S-E-AC. The "A" denotes the presence of an attribute-based objective (maximize cargo capacity) while the "C" denotes the presence of a cost-base objective (minimize construction and operating costs). Tappeta and Renaud [33] present an aircraft concept-sizing problem that can be classified as S-E-AA. The problem has two attribute-based objective functions: minimize mass and maximize range.

Classification	Reference #	Description
S-E-A	1	Launch Vehicle
S-E-A	2	Aircraft Engine
S-E-A	8	Aircraft Design
S-E-A	10	Aircraft Wing
S-E-A	14	Chassis Design
S-E-A	15	Chassis Design
S-E-A	28	Finger Nail
S-E-A_{EV}	31	Airplane Wing
S-E-C	1	Launch Vehicle
S-E-C	2	Aircraft Engine
S-EP-Π	11	Aircraft Concept
S-EP-Π_{EU}	6	Suspension
S-EP-Π_{EU}	30	Universal Motor
S-E-AA	33	Aircraft Concept
S-E-AC	32	Fleet of Ships
S-EP-AΠ	16	Weight Scale

Table 3: Classified Examples from Literature.

CHAPTER 5: ANALYSIS OF A BATHROOM SCALE

Modeling is a difficult task that generally requires experience, above and beyond academic knowledge, to truly perfect [34]. Decisions during the modeling process include things like objective function type, constraint type, decomposition method, optimization algorithms, and number of disciplines (i.e. scope). The next two chapters take a closer look at objective function formulation, demand modeling, decomposition, and information requirements.

This chapter begins the second part of the thesis, which addresses the questions raised in Chapter 1: With the information that I have available right now, if I formulate problem P like example A and get solution X, how much effort will it take to get solution X and how good is solution X? On the other hand, if I formulate the problem like example B and get solution Y, what is the difference in effort required and quality compared to solution X? What other observations can be made from the analysis? What amount of information was needed to model P like A versus B relative to the quality of the solution? Realizing that different models lead to different solutions is intuitive. We use computational experiments to get additional insight into the tradeoffs between information requirements and solution techniques.

A bathroom scale example, originally developed by Michalek *et al.* [16], will be used to help analyze and answer the questions mentioned in the introduction. The example in [16] was used to demonstrate the multi-disciplinary design optimization (MDO) technique known as analytical target cascading (ATC). A simple comparison of the ATC and a disjoint approach was given to show the effectiveness and correctness of the ATC approach. A disjoint approach is when an engineering optimization is solved first to determine all the engineering variables followed by a marketing optimization to determine price. No mention of information requirements or difficulty in programming the two approaches is given.

5.1 Original Model Formulation

The bathroom scale design problem includes fourteen design variables [x_1, …, x_{14}], six customer attributes [z_1, …, z_6], thirteen fixed model parameters [y_1, …, y_{13}], and eight constraints. The selection reasoning and derivation of the variables and equations for this model can be found in [16]. A relation matrix, shown in Table 4, shows the degree of coupling between design variables, constraints, and attributes (which are used as constraints in the all-at-once approach). The rows include all fourteen design variables while the columns include the eight geometric constraints along with the five customer attributes. A constraint or attribute with an "x" indicates it is a function of the variable x_i corresponding to row i where that respective "x" appears. The following matrix shows the degree of coupling in this example to be moderate, meaning it is not fully coupled nor is it completely uncoupled. The nomenclature and equations for the design variables, fixed model parameters, customer attributes, and constraints are also listed below.

Relation Matrix for Scale Optimization													
	Constraints												
Design Variables	1	2	3	4	5	6	7	8	z_1	z_2	z_3	z_4	z_5
x_1							x	x	x			x	x
x_2				x			x	x	x			x	x
x_3									x			x	x
x_4			x						x			x	x
x_5			x	x					x			x	x
x_6									x			x	x
x_7		x			x	x	x	x					
x_8					x	x							
x_9									x			x	x
x_{10}						x			x			x	x
x_{11}					x				x			x	x
x_{12}	x	x				x						x	x
x_{13}		x	x		x	x	x	x		x	x		
x_{14}	x						x			x	x		

Table 4: Relation Matrix for Bathroom Scale.

<u>Design Variables</u>
x_1 = length from base to force on long lever (inch)
x_2 = length from force to spring on long lever (inch)
x_3 = length from base to force on short lever (inch)
x_4 = length from force to joint on short lever (inch)
x_5 = length from force to joint on long lever (inch)
x_6 = spring constant (lb./in.)
x_7 = distance from base edge to spring (inch)
x_8 = length of rack (inch)
x_9 = pitch diameter of pinion (inch)
x_{10} = length of pivot's horizontal arm (inch)
x_{11} = length of pivot's vertical arm (inch)
x_{12} = dial diameter (inch)
x_{13} = cover length (inch)
x_{14} = cover width (inch)

<u>Fixed Model Parameters</u>
y_1 = gap between base and cover = 0.30 in.
y_2 = minimum distance between spring and base = 0.50 in.
y_3 = internal thickness of scale = 1.90 in.
y_4 = minimum pinion pitch diameter = 0.25 in.
y_5 = length of window = 3.0 in.
y_6 = width of window = 2.00 in.
y_7 = distance between top of cover and window = 1.13 in.
y_8 = number of pounds measured per tick mark = 1.0 lbs
y_9 = horizontal distance between spring and pivot = 1.10 in.

y_{10} = length of tick mark plus gap to number = 0.31 in.
y_{11} = number of pounds that number length spans = 16.00 lbs
y_{12} = aspect ratio of number = 1.29
y_{13} = minimum allowable distance of lever at base to centerline = 4.00 in.

Customer Attributes

z_1 = weight capacity (lbs) = $\dfrac{4\pi x_6 x_9 x_{10}\left(x_1+x_2\right)\left(x_3+x_4\right)}{x_{11}\left(x_1\left(x_3+x_4\right)+x_3\left(x_1+x_5\right)\right)}$

z_2 = aspect ratio = $\dfrac{x_{13}}{x_{14}}$

z_3 = platform area (in^2) = $x_{13}x_{14}$

z_4 = tick mark gap (in.) = $\pi\dfrac{x_{12}}{z_1}$

z_5 = number size (in.) = $\dfrac{\left(2\tan\left(\dfrac{\pi y_{11}}{z_1}\right)\right)\left(\dfrac{x_{12}}{2}-y_{10}\right)}{\left(1+\dfrac{2}{y_{12}}\tan\left(\dfrac{\pi y_{11}}{z_1}\right)\right)}$

z_6 = price ($)

Constraints and Bounds on Design Variables

1. $x_{12}-x_{14}+2y_1\le 0$
2. $x_{12}-x_{13}+2y_1+x_7+y_9\le 0$
3. $x_4+x_5-x_{13}+2y_1\le 0$
4. $x_5-x_2\le 0$
5. $x_7+y_9+x_{11}+x_8-x_{13}+2y_1\le 0$
6. $\left(x_{13}-2y_1\right)-\left(\dfrac{x_{12}}{2}+y_7\right)-x_7-y_9-x_{10}-x_8\le 0$
7. $\left(x_1+x_2\right)^2-\left(x_{13}-2y_1-x_7\right)^2-\left(\dfrac{x_{14}-2y_1}{2}\right)^2\le 0$
8. $\left(x_{13}-2y_1-x_7\right)^2+{y_{13}}^2-\left(x_1+x_2\right)^2\le 0$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
LB	0.125	0.125	0.125	0.125	0.125	1	0.5	1	0.25	0.5	0.5	1	1	1
UB	36	36	24	24	36	200	12	36	24	1.9	1.9	36	36	36
Units	in.	in.	in.	in.	in.	lb./in.	in.	in.	in.	in.	in.	in.	in.	in.

Table 5: Bounds on Engineering Design Variables.

	z_1	z_2	z_3	z_4	z_5	z_6
LB	200	0.75	100	0.063	0.75	10
UB	400	1.33	140	0.188	1.75	30
Units	lbs.	-	in^2	in.	in.	$

Table 5: Bounds on Product Attributes.

Marketing Related Models

The profit model is a basic model that incorporates demand (q), price (p), variable cost (c_v), and investment cost (c_i). Many marketing models superior to this can be found but for the sake of this analysis the model shown below will suffice.

$$\Pi = q\left(p - c_v\right) - c_i \tag{5.1}$$

The demand model was developed using discrete choice analysis (DCA) and a market survey. The total demand is population size multiplied by the probability that a consumer will select a particular design (i.e. estimated market share). Equation 5.2 shows the common DCA equation developed in [35, 36].

$$\begin{aligned} &q = sP = se^{v}\left[1+e^{v}\right]^{-1} \\ &s = population\ size \\ &v = \sum_{k=1}^{K-1} \Psi_k\left(\langle z \rangle_k\right) + \Psi_K(p) \end{aligned} \tag{5.2}$$

The attraction value "ν" is simply the summation of the beta values calculated from the spline functions for each attribute value and price. The spline functions are shown in Appendix A.

5.2 Optimization Setups

Seven different setups are created using the scale example in order to have a basis for comparing the information requirements and solution quality. The main goal of each setup is to create a product that will yield the most profit for a company. In general, Setups 1-6 do this using a disjoint two-step process. The first step is to optimize the engineering discipline with the assumption that marketing supplied appropriate target values. The second step is to take the result of step 1, along with customer demand and cost models, and determine a price to maximize profit. The problem is bounded by the eight geometric constraints mentioned above. Setup 7 is a joint optimization linking marketing and engineering. The objective is again to maximize profit but this time marketing decisions are made at the same time as the engineering decisions. This optimization problem is bounded by the eight geometric constraints as well as upper and lower limits on the six customer attributes. Figure 3 offers a clearer distinction between the seven setups used.

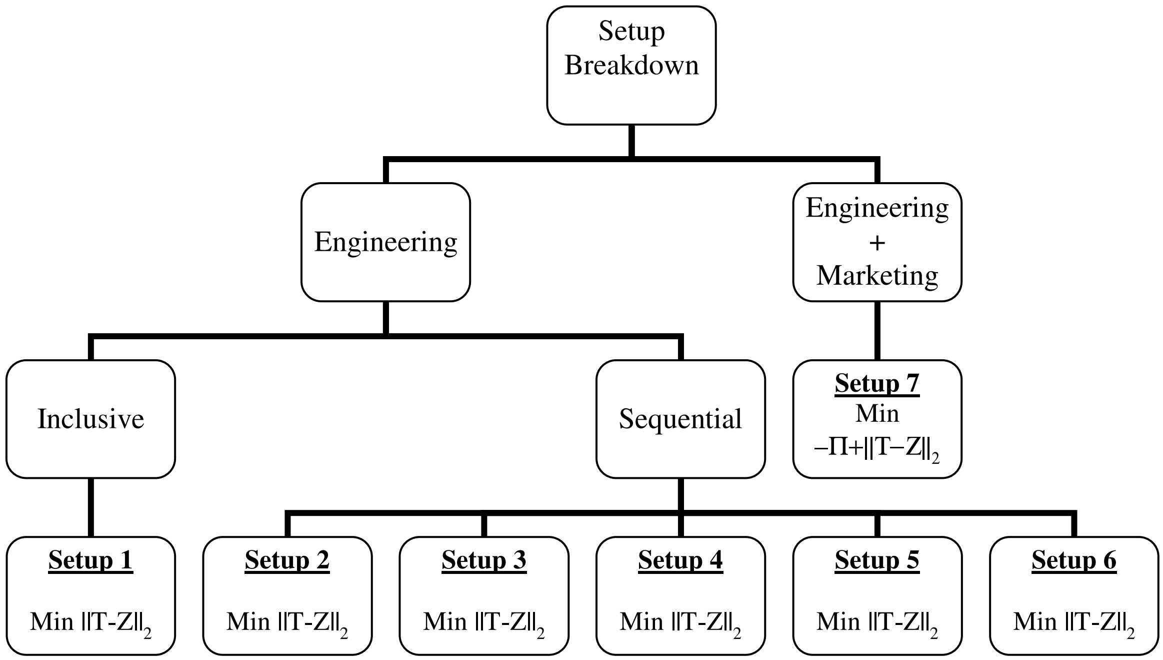


Figure 3: Breakdown of Setups for Scale Analyses.

Table 6 details the differences between the information requirements of the seven setups to further understand the differences between them. All the setups require information about the relationships between design variables and attributes. Setups 1-7a also require information about target settings. The major differences in information requirements can be seen in Setups 7a and 7b. Setup 7a requires the most information because of the multi-objective function. Information on cost modeling, demand, profit models, and pricing is needed. Setup 7b does not require target or weight information. The objective column in Table 6 details the sequence of objectives used. For example, Setup 2 requires two separate engineering optimizations. The first is a subproblem to match a target setting for capacity. The second is a subproblem optimization to meet the target settings for aspect ratio and platform area. The sequence of objectives will be discussed in greater detail later in section 5.2.

Setup #	Classification	Objective	Information Requirements	Outputs
1	S-E-AA	Meet all Targets	DV → Attributes, Targets	DV: $x_1 - x_{14}$
2	S-E-A S-E-AA	Capacity Ratio & Area	DV → Attributes, Targets	DV: $x_1 - x_{14}$
3	S-E-A S-E-AA S-E-AA	Capacity Ratio & Area Gap & Number	DV → Attributes, Targets	DV: $x_1 - x_{14}$
4	S-E-AA S-E-A	Ratio & Area Capacity	DV → Attributes, Targets	DV: $x_1 - x_{14}$
5	S-E-AA S-E-A S-E-AA	Ratio & Area Capacity Gap & Number	DV → Attributes, Targets	DV: $x_1 - x_{14}$
6	S-E-AA S-E-AA	Gap & Number Ratio & Area	DV → Attributes, Targets	DV: $x_1 - x_{14}$, Price
7a	S-EP-AΠ	Profit & Targets	DV → Attributes, Targets, Weights, Costs, Attr. → Demand Price → Demand Profit Model	DV: $x_1 - x_{14}$, Price
7b	S-EP-Π	Profit	DV → Attributes, Costs, Attr. → Demand	DV: $x_1 - x_{14}$, Price

Table 6: Breakdown of Seven Scale Setups.

The seven setups were solved using the *fmincon* function included in the optimization toolbox in MATLAB™. Within each setup various other parameters are changed as well such as weighting coefficients and initial solutions. The same seven initial solutions, shown in Table 7, were used for each of the seven setups. The feasibility of each initial solution was determined by entering the values into a spreadsheet model to check for constraint violation prior to running any optimizations. Initial solution 1 is the optimal result of the ATC approach used in [16]. The other six initial solutions were arbitrarily determined using trial and error in a spreadsheet model.

Initial Solutions for Engineering Optimization							
Design Variables	Initial Solution Number						
	1	2	3	4	5	6	7
x_1	2.30	18.00	1.00	5.00	1.00	3.39	3.00
x_2	8.87	18.00	1.00	10.00	7.00	7.79	8.50
x_3	1.34	12.00	1.00	12.00	1.00	1.40	1.34
x_4	1.75	12.00	1.00	5.00	3.00	1.49	1.75
x_5	0.41	18.00	1.00	5.00	5.00	0.88	0.41
x_6	95.70	95.50	1.00	9.00	60.00	95.10	95.70
x_7	0.50	6.00	1.00	2.00	2.00	0.50	0.50
x_8	7.44	18.00	1.00	9.00	6.00	6.91	7.44
x_9	0.25	12.00	1.00	1.00	1.00	0.30	0.25
x_{10}	0.50	1.00	1.00	1.00	3.00	0.55	0.50
x_{11}	1.90	1.00	1.00	1.00	1.00	1.84	1.90
x_{12}	9.34	18.00	1.00	10.00	7.00	9.33	9.34
x_{13}	11.54	18.00	1.00	15.00	11.00	11.53	11.54
x_{14}	11.57	18.00	1.00	18.00	10.00	11.08	11.57
Feasible?	NO	NO	NO	YES	NO	YES	NO

Table 7: Seven Initial Solutions Used in All Seven Setups.

5.2.1 Setup 1: Engineering Optimization

For the first setup, an optimization of the scale is performed using the original fourteen engineering variables with the attribute targets set as the most preferred level of each attribute based on a customer survey (see Appendix A). All of the geometric constraints (1-8) were applied as well as the upper and lower bounds on the design variables. The objective is to minimize the l_2 norm of the deviation between target (T) and response (Z) values. The procedure can be depicted as follows.

$$\begin{aligned} &\text{Minimize } f(x) = \left\| w \circ (T - Z) \right\|_2 \\ &\text{With respect to } [x_1, \ldots, x_{14}] \\ &\text{Subject to: Constraints 1-8; } x_{LB} \le x \le x_{UB} \end{aligned} \tag{5.3}$$

The optimization was then repeated with different target values and initial solutions. Target values were adjusted thrice. The first adjustment changed the targets to the actual attainable attribute levels found using the ATC approach detailed in Michalek *et al*. [16]. The second adjustment changed the attribute targets to the marketing optimization result mentioned in Michalek *et al*. [16]. Finally, the target attributes were set as the optimal values obtained through my own disjoint marketing optimization (see Appendix C). The four target setting values are shown in Table 8.

Target #	Attributes				
	z_1	z_2	z_3	z_4	z_5
1	300	1	120	0.125	1.75
2	254	0.997	133	0.116	1.33
3	283	0.946	124.2	0.136	1.75
4	288	0.9285	130.24	0.156	1.75

Table 8: Target Settings Used in Setup 1.

In practice the only target values that a designer will have knowledge of a priori will be from a marketing survey, which is the first target setting mentioned above. The other two settings were used to determine the sensitivity that target settings had, if any, to the solution.

5.2.2 Setup 2: Sequential Engineering Optimization

For this setup the engineering optimization described in Setup 1 is broken down into three sequential optimizations. The objective functions used in each sequential optimization again minimize the l_2 norm of the deviation between the target and response vectors as shown in equation 5.4. Target 1 from Table 8 is used for all the sequential engineering optimizations.

$$\min f = \|T - Z\|_2 \tag{5.4}$$

The first optimization determines design variables x_1-x_6 and x_9-x_{11} while meeting the target for capacity (z_1). The only constraint applied during this optimization is constraint

4. The next step is to take the results of the first optimization and simultaneously optimize for aspect ratio and platform area (z_2 and z_3) while satisfying constraints 3, 5, 7, and 8. This results in the determination of design variables x_{13} and x_{14}. The final step is to take the obtainable target for z_1, determined in the first step, and calculate x_{12} directly from equations z_4 or z_5. The sequence outline is as follows:

1. Optimize for z_1
    a. Given: a target for capacity z_1
    b. Find: x_1-x_6 and x_9-x_{11} and achievable capacity z_1
    c. Subject to: constraint 4
2. Optimize for z_2 and z_3
    a. Given: target values for z_2 and z_3; DV x_1-x_6 and x_9-x_{11}
    b. Find: x_{13}, x_{14}, and achievable z_2 and z_3
    c. Subject to: constraints 3, 5, 7, and 8
3. Calculate x_{12}
    a. Given: achievable capacity z_1
    b. Solve equations z_4 or z_5 for x_{12}
    c. Note: Steps 2 and 3 are interchangeable with no affect on the result

The relation matrix for this sequence is shown below with different color shading representing different steps of the sequence.

Relation Matrix for Scale Optimization: Setup 2														
		Constraints												
		4	3	5	7	8	z_4	z_5	1	2	6	z_1	z_2	z_3
Design Variables	x_1				x	x	x	x				x		
	x_2	x			x	x	x	x				x		
	x_3						x	x				x		
	x_4		x				x	x				x		
	x_5	x	x				x	x				x		
	x_6						x	x				x		
	x_9						x	x				x		
	x_{10}						x	x			x	x		
	x_{11}			x			x	x				x		
	x_7			x	x	x				x	x			
	x_8			x							x			
	x_{13}		x	x	x	x				x	x		x	x
	x_{14}				x				x				x	x
	x_{12}						x	x	x	x	x			

Table 9: Relation Matrix for Setup 2.

5.2.3 Setup 3: Sequential Engineering Optimization

Setup 3 is identical to Setup 2 except for the last step. Instead of calculating x_{12} directly from either z_4 or z_5 it is determined through a simultaneous optimization of z_4 and z_5. This was done because x_{12} was different depending on whether equation z_4 or z_5 was used to calculate it. Profitability as well as feasibility was checked for each result. The result of the optimization determined x_{12} to be 28.59 in. This is significantly different from the values of 11.94 in. and 13.68 in., which were calculated using equations for z_4 and z_5, respectively. Step three from Setup 2 is depicted below followed by the relation matrix for this setup. Notice the difference in the last row of Table 10 compared to Table 9.

3. Optimize z_4 and z_5
   a. Given: target values for z_4 and z_5
   b. Find: x_{12}
   c. Subject to: constraints 9, 10, and 14

Relation Matrix for Scale Optimization: Setup 3														
		Constraints												
		4	3	5	7	8	1	2	6	z_1	z_2	z_3	z_4	z_5
Design Variables	x_1				x	x				x			x	x
	x_2	x			x	x				x			x	x
	x_3									x			x	x
	x_4		x							x			x	x
	x_5	x	x							x			x	x
	x_6									x			x	x
	x_9									x			x	x
	x_{10}								x	x			x	x
	x_{11}			x						x			x	x
	x_7			x	x	x		x	x					
	x_8			x					x					
	x_{13}		x	x	x	x		x	x		x	x		
	x_{14}				x		x				x	x		
	x_{12}						x	x	x				x	x

Table 10: Relation Matrix for Setup 3.

5.2.4 Setup 4: Sequential Engineering Optimization

For this setup the engineering optimization described in Setup 1 is broken down into several sequential optimizations. The objective function and target settings used in each step of this set of sequential optimizations is the same as equation 5.4 shown in Setup 2.

The first optimization for this setup is to simultaneously determine the aspect ratio and platform area. Due to the nature of this problem the values for x_{13} and x_{14} can be calculated directly using the optimal targets for aspect ratio and area. In this case $x_{13} = x_{14} = 10.9545$ in. The next step is to optimize for capacity (z_1) utilizing x_{13}, x_{14}, and constraints 3, 4, 5, 7, and 8. This results in values for design variables x_1-x_{11}. Note here that by including x_7 and x_8 in the optimization for capacity, constraints 5, 7, and 8 could be applied. Variables x_7 and x_8 do not affect any of the attribute equations so adding them at this point simply allows more constraints to be used to help keep the design in a feasible region. The final step is identical to that of Setup 2: x_{12} is calculated directly using either z_4 or z_5. The sequence can be pictured as follows:

1. Calculate x_{13} and x_{14}
    a. Given: a target for aspect ratio z_2 and area z_3
    b. Solve equations z_2 and z_3 simultaneously
2. Optimize for z_1
    a. Given: a target value for z_1; DV x_{13} and x_{14}
    b. Find: x_1- x_{11}; achievable z_1
    c. Subject to: constraints 3, 4, 5, 7, and 8
3. Calculate x_{12}
    a. Given: achievable capacity z_1
    b. Solve equations z_4 or z_5 for x_{12}
    c. Note: Steps 2 and 3 are interchangeable with no affect on the result

The relation matrix for this setup is shown below with shading used to depict different steps in the sequence.

Relation Matrix for Scale Optimization: Setup 4														
		Constraints												
		z_2	z_3	3	4	5	7	8	z_4	z_5	1	2	6	z_1
Design Variables	x_{13}	x	x	x		x	x	x				x	x	
	x_{14}	x	x				x				x			
	x_1						x	x	x	x				x
	x_2				x		x	x	x	x				x
	x_3								x	x				x
	x_4			x					x	x				x
	x_5			x	x				x	x				x
	x_6								x	x				x
	x_7					x	x	x				x	x	
	x_8					x							x	
	x_9								x	x				x
	x_{10}								x	x			x	x
	x_{11}					x			x	x				x
	x_{12}								x	x	x	x	x	

Table 11: Relation Matrix for Setup 4.

5.2.5 Setup 5: Sequential Engineering Optimization

The last step of Setup 4 was then modified slightly by simultaneously optimizing for z_4 and z_5 instead of directly calculating x_{12} from the equations for tick mark gap (z_4) and number size (z_5). This was done because x_{12} was 11.94 in. when calculated using the equation for z_4 and 13.68 in. when calculated using the equation for z_5. In order to determine the best value for x_{12} a tradeoff must be made between z_4 and z_5. The relation matrix for this setup is shown below.

3. Optimize for z_4 and z_5
   a. Given: target values for z_4 and z_5
   b. Find: x_{12}
   c. Subject to: constraints 1, 2, and 6

Relation Matrix for Scale Optimization: Setup 5														
		Constraints												
		z_2	z_3	3	4	5	7	8	1	2	6	z_1	z_4	z_5
Design Variables	x_{13}	x	x	x		x	x	x		x	x			
	x_{14}	x	x				x		x					
	x_1						x	x				x	x	x
	x_2				x		x	x				x	x	x
	x_3											x	x	x
	x_4			x								x	x	x
	x_5			x	x							x	x	x
	x_6											x	x	x
	x_7					x	x	x		x	x			
	x_8					x					x			
	x_9											x	x	x
	x_{10}										x	x	x	x
	x_{11}					x						x	x	x
	x_{12}								x	x	x		x	x

Table 12: Relation Matrix for Setup 5.

5.2.6 Setup 6: Sequential Engineering Optimization

For Setup 6 an optimization was performed on the tick mark gap (z_4) and the number size (z_5) first. Since these two attribute levels are functions of z_1, the equation for z_1 was input into z_4 and z_5 making them functions of variables x_1-x_{12}. The only constraint applied is the lower and upper bound on z_1, which are 200 lbs and 400 lbs, respectively. First a target value of 0.125 in. (z_4) and 1.75 in. (z_5) was set for six different initial solutions. Then the obtainable attribute values from the ATC optimization described in Michalek *et al.* 2005 were used as the target values. z_4 was set to 0.116 in. and z_5 to 1.33 in. Six starting values were again tried to determine the effects initial solutions have on the final solution.

The next step was to take the results from the previous step and use them to optimize z_2 and z_3. Variables x_1-x_{12} were taken from the first optimization and used as fixed values while trying to determine variables x_{13} and x_{14}. No feasible solutions could be found for x_{13} and x_{14} when the values for x_1-x_{12} and constraints 1 through 8 were utilized. The algorithm and relation matrix for this setup is shown below.

1. Optimize for z_4 and z_5
    a. Given: target values for z_4 and z_5
    b. Find: x_1-x_{12}
    c. Subject to: bounds on z_1
2. Optimize for z_2 and z_3
    a. Given: target values for z_2 and z_3; DV x_1- x_{12}
    b. Find: x_{13} and x_{14} and achievable z_2 and z_3
    c. Subject to: constraints 1-8

Relation Matrix for Scale Optimization: Setup 6														
		Constraints												
		z_1	1	2	3	4	5	6	7	8	z_2	z_3	z_4	z_5
Design Variables	x_1	x							x	x			x	x
	x_2	x				x			x	x			x	x
	x_3	x											x	x
	x_4	x			x								x	x
	x_5	x			x	x							x	x
	x_6	x											x	x
	x_7			x			x	x	x	x				
	x_8						x	x						
	x_9	x											x	x
	x_{10}	x						x					x	x
	x_{11}	x					x						x	x
	x_{12}		x	x				x					x	x
	x_{13}			x	x		x	x	x	x	x	x		
	x_{14}		x						x		x	x		

Table 13: Relation Matrix for Setup 6.

5.2.7 Setup 7: All-at-Once

Setup 7 combines the marketing information such as the spline functions, demand models, and profit model as well as the engineering variables into one optimization. The fourteen original engineering design variables remain the same, however, a price variable was added and ten more constraints were applied to assure that the selected design values kept the attribute levels within their bounds. These ten constraints were obtained by using the equations for z_1-z_5 along with their corresponding upper and lower bounds and setting them to be less than or equal to zero. Three feasible and four infeasible initial solutions were used to determine how the initial solution affects the result (same initial solutions as the disjoint engineering optimization except initial solution 5).

Two different objective functions were tried. The first method was a multi-objective function to minimize negative profit (i.e. maximize profit) and the l_2 norm of the deviation between target values and response values, see equation 5.5 (Setup 7a). Target Setting 1 is used in equation 5.5 for every optimization run. Initially no weighting coefficients were used to balance the magnitude of profit compared to the magnitude of the attribute levels (i.e. w = 1). The effect of weighting coefficient values of 10^3 and 10^5 is discussed later in this section.

$$\textit{Setup 7a}:\ \min f = -\Pi + w\left\|T - Z\right\|_2 \tag{5.5}$$

The second method (Setup 7b) used a single-objective function to minimize negative of profit (i.e. maximize profit) by itself.

$$\textit{Setup 7b}: \min f = -\Pi \tag{5.6}$$

During the iterations, design values may be temporarily selected that cause one or more attribute levels z_1-z_6 to extend beyond the spline function range. When this happens MATLAB does not update the Hessian matrix and the objective function evaluation returns not a number (NaN). To account for this a linear line is added on either side of the spline functions with a steep slope. An equation was determined such that the slope was 10 or -10 (arbitrarily chosen) with the line passing through the endpoint of the spline functions. This causes any value outside the range of the polynomial to have a large negative value, which decreases the preference level and forces the optimizer to stay within the bounds. For example, the left endpoint for the weight capacity polynomial is (200, -0.534) resulting in an equation $A = 10z_1 - 2000.534$, where A is the beta value for consumer preference and z_1 is the capacity calculated from the current design variables. Similarly, the right endpoint of the weight capacity polynomial is (400, 0.052) resulting in an equation $A = -10z_1 + 4000.052$. This procedure was done for all six spline functions shown in Appendix A.

5.3 Results of Bathroom Scale Analysis

Results of the seven setups along with several variations in weighting coefficients will be discussed in the following sections. First the best result of each target setting will be displayed for Setup 1. Conclusions can be drawn on how knowledge of target settings affects the result. In most situations there will be a large amount of uncertainty involved with target settings passed from marketing so this analysis will help us understand the sensitivity that an optimization may have to target information.

Next, the sequential optimization results, Setups 2-6, will be discussed. It is important to note that the sequential optimizations performed in this analysis each have a unique objective function. Pan and Diaz [23] have shown sequential optimization to be an inferior method when dealing with product design optimization. An assumption made in their work is that each decomposed problem has the same objective function while utilizing all the constraints. During the analysis of the bathroom scale a different objective function, namely a specific target setting, is used for each subsection of the overall problem. In addition only constraints that were directly linked to the variables being solved for were implemented since applying constraints that are not functions of the variables will do nothing.

Solutions from Setups 1-6 include only the engineering design variables. Evaluating the profitability of these solutions requires finding the optimal price for each one using the model shown in Appendix C.

The results of the more complex AAO approach will follow the sequential optimization results. AAO increases scope by incorporating marketing models and customer demand into the optimization in order to maximize profit.

5.3.1 Engineering Optimization Results

Seven initial solutions were tried for four different target settings. In addition weighting coefficients were varied to complete the analysis on Setup 1. The result for each target setting, with the objective function closest to zero, is shown in Table 14. More information on the characteristics of the optimization will be discussed in a later section. Since each of the five attributes is weighted equally an aggregated percent difference is calculated to determine which result matched its respective target setting most closely.

Best Results from Setup 1						
Attribute	z_1	z_2	z_3	z_4	z_5	Total % Difference
Target 1	299.998	1.157	120.003	0.100	1.201	66.7590
% Diff	6.67E-04	1.57E+01	2.58E-03	1.97E+01	3.14E+01	
Target 2	254.000	0.997	133.001	0.115	1.330	0.7378
% Diff	0.00E+00	4.01E-02	5.26E-04	6.90E-01	7.52E-03	
Target 3	282.999	1.154	124.203	0.109	1.285	68.7804
% Diff	4.95E-04	2.20E+01	2.33E-03	2.02E+01	2.65E+01	
Target 4	288.048	1.150	130.238	0.110	1.304	79.1934
% Diff	1.68E-02	2.39E+01	1.54E-03	2.98E+01	2.55E+01	

Table 14: Target Matching Results from Setup 1.

It can easily be seen that target setting 2 matched most closely with a total of just seven tenths of a percent difference. Recall that this target setting is the best result of the joint marketing and engineering optimization from [16]. It is important to note that just because a response matches a target setting closely there is no guarantee that the design is optimal unless all possible target settings are tried. If Target 1 could be matched it would result in the highest profit, however, the solution found when using Target 2 had the highest profit.

A feasible result was obtained for all 28 runs. Many equivalent solutions are apparent so selecting one is completely subjective. A comparison of the results is shown in Table 16 at the end of this chapter.

5.3.2 Sequential Engineering Optimization Results

Setups 2 to 6 decompose the original engineering analysis of a bathroom scale into five different sequential optimizations. Only Setup 5 resulted in a feasible solution. The best profit achieved using the result of Setup 5 was $33,371,000. For each infeasible case, by the time the last step of the sequence is performed, the design variable values from the previous steps were already out of range to obtain a feasible solution. This is most likely because a limited number of constraints were applied at each step. The design space is not constant for each step of the process thus causing problems as more variables become fixed and the design space shrinks. The subproblems of a decomposed design problem would need to have little to no dependence among them in order for a sequential optimization to yield a result consistently. This is consistent with [23], where issues common among strongly coupled, non-hierarchic problems are discussed.

5.3.3 All-at-Once Results

Seven initial solutions were tried for the AAO multi-objective optimization (Setup 7a), including four infeasible and three feasible starting points. Each of them converged to the same function value of -65,031,000 and all achieved the same attribute values of [251.6114, 0.9986, 134.1058, 0.1170, 1.3488, 26.0477] for z_1-z_6 respectively. The optimal price for all cases is $26.0477 resulting in a market share of 57.3%.

The same seven initial solutions were used to solve the profit maximization problem (Setup 7b). This time the feasibility of the initial solution played an important role. If the initial solution was infeasible the optimization terminated after two iterations claiming no feasible solution can be found. Results similar to the multi-objective case were observed in the case of the three feasible initial solutions. The objective function value was again -65,031,000 and all three initial solutions resulted in attribute levels of [251.6113, 0.9986, 134.1059, 0.1170, 1.3488, 26.0477] for z_1-z_6 respectively.

Now a discussion of how the weighting coefficients affect the result is mentioned. Table 15 displays results from three different weighting coefficients applied to the multi-objective function in Setup 7a. Weighting coefficients are used to balance the magnitude variation between the profit and l_2 norm terms within the objective function. Unlike decision-based design, weights are not used in this case to give preference to one objective over the other, rather they essentially remove any preference. The results intuitively make sense because a weighting coefficient of 10^5 increases the importance of the response deviation by bringing the attribute values to the same magnitude as profit. The tradeoff is that profit suffers due to the importance placed on reaching target attribute levels. This brings up an interesting question. Which objective is more important, matching target values as close as possible or maximizing profit? Intuitively one may assume that the closer a target is matched the higher the profit will be, however, this analysis shows differently. By reducing the weighting coefficients, profit increases by 7.67% and even more importantly the percent difference between response and target values actually decreases by 3%. In other words, equal weighting coefficients improved the design and the profit for this particular example. A solution was found for all seven initial solutions when w = 1 but initial solution five returned no solution for coefficients of $w = 10^3$ and $w = 10^5$.

Weighting Coefficient Effects on Setup 7a								
Coefficient Value	Initial Solution	z_1	z_2	z_3	z_4	z_5	Price	Profit
1	1	251.6114	0.9986	134.1058	0.1170	1.3488	26.0477	65,031,000
	2	251.6114	0.9986	134.1058	0.1170	1.3488	26.0477	65,031,000
	3	251.6114	0.9986	134.1058	0.1170	1.3488	26.0477	65,031,000
	4	251.6115	0.9986	134.1059	0.1170	1.3488	26.0477	65,031,000
	5	251.6113	0.9986	134.1058	0.1170	1.3488	26.0477	65,031,000
	6	251.6114	0.9986	134.1058	0.1170	1.3488	26.0477	65,031,000
	7	251.6111	0.9986	134.1057	0.1170	1.3488	26.0477	65,031,000
10^3	1	243.5258	1.0269	100.0000	0.1023	1.1561	24.0570	53,063,000
	2	253.5857	1.3300	140.0000	0.1197	1.3845	22.7450	44,785,000
	3	251.7499	0.9987	134.1053	0.1170	1.3482	26.0468	64,979,000
	4	251.7499	0.9987	134.1053	0.1170	1.3482	26.0468	64,979,000
	5	No Feasible Solution Found						
	6	251.7499	0.9987	134.1053	0.1170	1.3482	26.0468	64,979,000
	7	243.5258	1.0269	100.0000	0.1023	1.1561	24.0570	53,063,000
10^5	1	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000
	2	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000
	3	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000
	4	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000
	5	No Feasible Solution Found						
	6	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000
	7	265.9945	1.0056	133.9183	0.1111	1.2959	25.8201	60,387,000

Table 15: Weighting Coefficient Analysis Results.

5.4 Comparing Information Requirements

Setup 1 will be compared with Setup 7a and 7b in an attempt to understand how information requirements can affect the effort and solution quality of an optimization process. Alternatives will be discussed that can simplify the modeling process thus saving time and effort. Table 6 shows the difference in information requirements between the setups.

Setup 1 requires information on targets and relationships between design variables and attributes. There are three ways to obtain the necessary target information. One is directly from the marketing department, the second is through a marketing optimization (see Appendix C), and the third is through heuristics or experience.

Setup 7a requires information about the relationships between design variables and attributes, targets, weighting coefficients, cost models, demand models, profit models, and pricing. Setup 7b requires less information than 7a but more than Setup 1. Setup 7b requires information on the marketing models such as cost, profit, and pricing in addition to relationships between design variables and attributes.

For Setup 1, the target setting value changed the result significantly as shown in Table 16. The customer survey guided marketing to set infeasible target values for engineering to meet. This resulted in a profit of approximately $50.8 million. An improvement was made by creating a separate marketing program in MATLAB to optimize for the five attributes based on customer responses. Profit rose to $57.8 million when utilizing this new set of targets. An interesting observation is made from Table 14. The response attributes matched target setting 4 (the result of a separate marketing optimization) with a larger percent difference than the other three target settings yet still returned the second highest profit margin. This leads one to conclude that matching a target setting with a small deviation is only important if the target setting is "correct" in the first place. Information about the quality of a target setting is a good example of how information requirements can affect the solution quality.

The $7 million improvement requires the engineer to work closely with marketing in order to have the necessary models to perform a preliminary marketing optimization to determine target settings. Effectively communicating marketing and engineering disciplines has proven to cause problems [16]. In relatively simple models such as the scale example, collaboration with marketing would allow an engineer to model a single profit maximization problem instead of decomposing the problem into an engineering optimization, followed by a price optimization. A better result will generally be found as well. For example, Setup 7a returned a value of $60 million, topping Setup 1 by $3 million.

An AAO approach will result in a more profitable design simply because it integrates more objective and enterprise level goals simultaneously. This allows for multidisciplinary tradeoffs to be made which are otherwise unaccounted for. A downfall, however, is that the AAO approach requires more models, more input decisions have to be made, and the process is more computationally expensive.

Target setting and weighting coefficients have been an important topic in design optimization for several years. A disaggregate target setting technique has been developed and implemented by Cooper *et al.* [39]. Multidisciplinary design optimization techniques have been developed as well to integrate the target setting process (product planning) directly with design optimization (engineering analysis) [11,15,16]. The importance of target setting is obvious in the bathroom scale example. When comparing Setup 1 to Setup 7a it can be seen that with the right amount of information both setups could potentially result in the same profit level. The two key pieces of information to do this are the targets and the weighting coefficients. In Setup 1 (disjoint engineering optimization) target setting 2 resulted in a profit of $64,907,000. The problem with this solution is the simple fact that this particular target setting would likely not be known a priori. The AAO approach has drawbacks similar to the engineering optimization. For example, weighting coefficients play an important role in Setup 7a but have no affect on Setup 1. If information about what weighting coefficients should be used is unknown or difficult to determine, the AAO approach can result in a suboptimal solution. The AAO solution with weighting coefficients, based on mathematical equivalence between the magnitudes of profit and the target settings, resulted in $5 million less than the AAO solution with no weighting coefficients.

Solution time is extended in the AAO approach making weighting coefficient adjustments more manually intensive. Either case has the potential of yielding the same result. However, adjusting target settings is more of a random process compared to weighting coefficients. A reasonable weighting coefficient can usually be mathematically determined by simply comparing the magnitudes of the objectives.

It appears that an engineer essentially has two choices: 1) an engineering optimization could be performed with a significant amount of time spent finding a good target setting to try find the best solution or 2) an AAO optimization similar to Setup 7a could be developed with increased time spent developing and programming the models as well as significant time spent adjusting weighting coefficients (if present) to determine the optimal solution. These choices will be very dependent on the information available at the beginning of the optimization. A designer would likely choose Setup 1 with target setting 4 or Setup 7 if no prior knowledge of expected results or targets is known before an optimization is performed. Setup 7b would be preferred over 7a if appropriate initial solutions were known.

Knowledge of heuristics through experience and similar problems can fundamentally change the way a designer models a particular problem. If targets are known with a high degree of confidence the optimization can be simplified significantly. On the other hand, if targets are not known or assumed with a certain confidence level then an MDO technique may be needed to find a solution. Of course MDO techniques are generally more difficult to model, program, and analyze. An experienced designer will be able to assume some of these unknown values with a higher degree of accuracy than a novice.

5.5 Effects of Decomposition

The typical product design process involves decomposing a profit-based objective into smaller sections, usually by discipline. However, decomposing the moderately coupled problem beyond the engineering and marketing disciplines proved to be ineffective. Any models with moderate to high coupling will likely result in inferior solutions and require more programming and computational effort. As shown in Table 17 Setups 2-6 required multiple MATLAB programs, more iterations and often resulted in no solution. On the other hand Setup 1 only required 279 iterations with a profit of $50.8 million. Although there is a large difference in number of iterations, today's computing capabilities allow simple models like the ones in this experiment to be solved in less than five minutes. Even so, the results presented here give us a good idea of the extra effort required to formulate and solve decomposed engineering optimization problems using different techniques. Insight into how modeling may change depending on “what we know” can then be used to look for ways of simplifying the model and save time.

Scale Analysis Results																	
Result From:	Design Variables														Marketing Info		
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	Optimal Price ($)	Market Share	Profit ($)
Setup 1:Target 1	10.08	1.33	10.34	1.65	0.50	94.67	0.50	6.17	1.55	0.52	1.89	9.58	11.78	10.18	23.681	50.08%	50,786,000
Setup 1:Target 2	5.90	5.21	1.66	8.04	0.77	94.81	0.50	5.62	0.26	0.64	1.21	9.31	11.52	11.55	26.026	57.25%	64,907,000
Setup 1:Target 3	2.11	9.48	17.75	0.93	8.61	94.85	0.50	5.82	0.96	0.50	1.90	9.77	11.97	10.37	24.542	53.01%	56,092,000
Setup 1:Target 4	3.30	8.62	1.46	1.72	0.51	95.69	0.50	7.67	0.35	0.55	1.90	10.04	12.24	10.64	24.819	53.87%	57,764,000
Setup 2	36.00	0.13	24.00	0.13	0.13	94.50	0.50	7.44	0.50	1.00	1.00	11.94	36.00	19.26	Infeasible		
Setup 3	36.00	0.13	24.00	0.13	0.13	94.50	0.50	7.44	0.50	1.00	1.00	28.59	36.00	19.26	Infeasible		
Setup 4	8.67	0.34	23.00	0.13	0.17	94.50	2.73	4.53	1.85	0.50	1.89	11.94	10.95	10.95	Infeasible		
Setup 5	8.67	0.34	23.00	0.13	0.17	94.50	2.73	4.53	1.85	0.50	1.89	6.53	10.95	10.95	21.034	38.12%	33,371,000
Setup 6	17.18	23.32	3.53	23.39	10.56	94.50	0.50	7.44	0.25	1.25	1.84	9.34	No Feasible Solution		Infeasible		
Michalek [16]	2.30	8.87	1.34	1.75	0.41	95.70	0.50	7.44	0.25	0.50	1.90	9.34	11.54	11.57	26.041	57.28%	64,990,000
Setup 7a: $w = 10^5$	5.35	6.23	0.45	0.49	4.07	95.57	0.50	6.60	0.35	0.99	1.81	9.40	11.60	11.54	25.820	57.31%	60,387,000
Setup 7b: w =1	4.78	6.84	7.98	4.51	4.10	95.66	0.50	5.21	0.51	0.63	1.70	9.37	11.57	11.59	26.048	57.30%	65,031,000

Table 16: Results of Seven Scale Optimization Setups.

Computation and Effort Comparison								
MATLAB Programs		Setup Number						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Program 1	# of Variables	14	9	9	2	2	12	15
	# of Start Points	7	4	4	0	0	6	7
	Ave. # of Iterations/Start	39.8	18.75	18.75	0	0	20.167	461.7143
	Min # of Iterations	30	12	12	0	0	18	65
	Max # of Iterations	75	18	18	0	0	25	1058
	# of Equivalent Optimal Solutions	7	1	1	1	1	6	7
	Ave. Function Calls	677	287	287			271	1410
Program 2	# of Variables		4	4	11	11	2	
	# of Start Points		5	5	4	4	5	
	Ave. # of Iterations/Start		8	8	78	78	8	
	Min # of Iterations		8	8	19	19	8	
	Max # of Iterations		8	8	235	235	8	
	# of Equivalent Optimal Solutions		5	5	1	1	5	
	Ave. Function Calls		41	41	2750	2750	60	
Program 3	# of Variables			1		1		
	# of Start Points			7		7		
	Ave. # of Iterations/Start			173.5714		3.14		
	Min # of Iterations			3		3		
	Max # of Iterations			401		4		
	# of Equivalent Optimal Solutions			1		7		
	Ave. Function Calls			3174		6		
Totals/Results	# of Variables	14	13	14	13	14	14	15
	# of Start Points	7	9	16	4	11	11	7
	Total # of Iterations	278.60	115.00	1330.00	312.00	333.98	161.00	3232.00
	Total Function Calls	4188	1312	14000	11003	11027	1720	9875
	Price	$23.68				$21.03		$25.82
	Market Share	50.08%	Infeasible	Infeasible	Infeasible	38.12%	Infeasible	57.30%
	Profit	50,786,000				33,371,000		60,387,000

Table 17: Comparing Computational Effort of Scale Analysis.

CHAPTER 6: ANALYSIS OF AN ELECTRIC MOTOR

This chapter continues our investigation of the tradeoffs between information requirements and solution quality. In this chapter a common academic test problem, first appearing in the 1970's [37], will be used to help determine the necessary information requirements needed to perform an optimization resulting in a quality solution. The analysis of a universal electric motor problem originally developed by Simpson [38] will be detailed and modified to help answer the questions arisen in the introduction.

The motor design problem is naturally a single discipline non-hierarchic problem experiencing tight coupling among the customer attributes. Tight coupling causes problems when attempting to decompose a design problem into subsystems because there is no obvious way to divide the design space. Table 18 shows the degree of this coupling through a relation matrix. The rows of Table 18 are the eight design variables. The columns are the six constraints. An "x" indicates that the constraint is a function of the corresponding variable. Notice power and efficiency are functions of all eight design variables while mass and torque are functions of seven and six design variables, respectively. As a result of the high degree of coupling, unlike the scale problem in Chapter 5, this problem will be altered mainly through the objective function and constraint settings. The interested reader can refer to Pan and Diaz [23] to learn about proposed methods of dealing with decomposing tightly coupled problems into subsystems for sequential optimization.

A marketing discipline will be added to the analysis to increase the scope of the overall problem. This will help us understand the difference in information flow between multidisciplinary problems and single discipline problems. The cost models used in this experiment are taken from Wassenaar and Chen [30] and integrated into a simplified discrete choice analysis model to determine demand and profits as a function of design parameters.

Relation Matrix: Constraints vs. Design Variables						
	Constraints					
DV	H	$r_o > t_s$	Power (W)	η	Mass (kg)	Torque (Nm)
N_c	x		x	x	x	x
N_s			x	x	x	x
A_{rw}			x	x	x	
A_{sw}			x	x	x	
r_o	x	x	x	x	x	x
t_s	x	x	x	x	x	x
I	x		x	x		x
L			x	x	x	x

Table 18: Relation Matrix for the Universal Electric Motor.

6.1 Original Model Formulation

The optimization model for a universal electric motor problem includes nine design variables [x_1, …, x_9], five customer attributes [z_1, …, z_5], twenty-three intermediate engineering attribute calculations, five constraints, and five fixed engineering parameters. The equations used in the model of the universal electric motor problem will be mentioned here, however, the reader should reference Simpson [37] for the derivations of equations and other background information on universal electric motors. The nomenclature and equations for the design variables, fixed model parameters, customer attributes, and constraints are listed below.

Design Variables

N_c: Number of turns of wire on the armature
N_s: Number of turns of wire on the stator, per pole
A_{aw}: Cross sectional area of armature wire [mm^2]

A_{sw}: Cross sectional area of stator wire [mm^2]
r_o: Outer radius of the stator [m]
t_s: Thickness of the stator [m]
I: Electric current [Amperes]
L: Stack length [m]
P: Price [$]

Engineering Attributes

H: Magnetizing intensity [Ampere turns/m] => $H = N_c I/(l_c + l_r + 2l_g)$

l_c: Mean path length within the stator [m] => $l_c = \pi(2r_o + t_s)/2$

l_r: Diameter of armature [m] => $l_r = 2(r_o - t_s - l_g)$

P_{in}: Input power [W] => $P_{in} = V_t I$

P_{out}: Power losses due to copper and brushes [W] => $P_{out} = I^2(R_a + R_s) + 2I$

l_{aw}: Armature wire length [m] => $l_{aw} = 2L + 4(r_o - t_s - l_g)N_c$

l_{sw}: Stator wire length [m] => $l_{sw} = p_{st}(2L + 4(r_o - t_s))N_s$

R_a: Armature wire resistance [Ohm] => $R_a = \rho l_{aw}/A_{aw}$

R_s: Stator wire resistance [Ohm] => $R_s = \rho l_{sw}/A_{sw}$

M_w: Mass of windings [kg] => $M_w = (l_{aw}A_{aw} + l_{sw}A_{sw})\rho_{copper}$

M_s: Mass of stator [kg] => $M_s = \pi L(r_o^2 - (r_o - t_s)^2)\rho_{steel}$

M_a: Mass of armature [kg] => $M_a = \pi L(r_o - t_s - l_g)^2 \rho_{steel}$

K: Motor constant [dimensionless] => $K = N_c / \pi$

$\Im$: Magneto magnetic force [A turns] => $\Im = N_s I$

$\Re$: Total reluctance of the magnetic circuit [A turns/Wb] => $\Re = \Re_s + \Re_a + 2\Re_g$

$\Re_s$: Reluctance of stator [A turns/Wb] => $\Re_s = l_c/(2\mu_{steel}\mu_o A_s)$

$\Re_a$: Reluctance of armature [A turns/Wb] => $\Re_a = l_r/(\mu_{steel}\mu_o A_a)$

$\Re_g$: Reluctance of one air gap [A turns/Wb] => $\Re_g = l_g/(\mu_o A_g)$

A_s: Cross sectional area of stator [m^2] => $A_s = t_s L$

A_a: Cross sectional area of armature [m^2] => $A_a = l_r L$

A_g: Cross sectional area of air gap [m^2] => $A_g = l_g L$

μ_{steel}: Relative permeability of steel [dimensionless] =>

$$\mu_{steel} = -0.2279H^2 + 52.411H + 3115.8 \qquad H \le 220$$
$$\mu_{steel} = 11633.5 - 1486.33\ln(H) \qquad 220 < H \le 1000$$
$$\mu_{steel} = 1000 \qquad H > 1000$$

φ: Magnetic flux [Wb] => $\varphi = \Im / \Re$

Fixed Engineering Parameters

l_g: Length of air gap = 7.0×10^{-4} [m]
V_t: Terminal voltage = 115 [V]
ρ: Resistivity of copper = 1.69×10^{-8} [Ohms•m]
μ_o: Permeability of free space = $4\pi \times 10^{-7}$ [H/m]
p_{st}: Number of stator poles = 2

Customer Attributes

T: Torque [Nm] => $T=K\varphi I$
P: Power [W] => $P=P_{in}-P_{out}$
η: Efficiency [%] => $\eta = P/P_{in}$
M: Mass [kg] => $M=M_w+M_s+M_a$
B: Operating Time = η

Constraints and Bounds

$H \leq 5000$ [A turns/m]
$r_o > t_s$
$P = 300$ [W]
$T = \{0.05, 0.10, 0.125, 0.3, 0.5\}$ [Nm]
$\eta \geq 0.15$
$M \leq 2.0$ [kg]

Bounds on Design Variables								
	N_c	N_s	A_{aw}	A_{sw}	r_o	t_s	I	L
LB	100	1	0.01	0.01	0.01	0.0005	0.1	0.01
UB	1500	500	1.0	1.0	0.1	0.01	6	0.2
Unit	turns	turns	mm^2	mm^2	m	m	A	m

Table 19: Bounds on Design Variables.

Marketing Related Models

The profit model is a basic model that incorporates demand (q), price (p), and several design dependent costs. The cost model equations used for this analysis were originally derived in Wassenaar and Chen [30]. The following analysis simplifies the cost model slightly and creates a different discrete choice analysis model to predict demand. The profit model used in the analysis of the universal electric motor is shown in equation 6.1.

$$\Pi = q(p) - C \tag{6.1}$$

where C is a summation of cost functions as follows:

$$\begin{aligned}
C &= D_c + M_c + L_c + C_{cap} \\
D_c &= \$500,000 \\
M_c &= q\left(M_w P_{copper} + \left(M_s + M_r\right) P_{steel}\right) \\
L_c &= 30 M_c / 70 \\
C_{cap} &= 50\left(\left(q - 500,000\right)/1000\right)^2
\end{aligned}$$

The design cost D_c is assumed to be fixed at \$500,000 while the material cost M_c, labor cost L_c, and capacity cost C_{cap} varies with demand and engineering attributes.

The demand model was developed using discrete choice analysis (DCA) and synthetic spline functions for customer preference. The total demand is population size multiplied by the probability that a consumer will select a particular design (i.e. estimated market share). Equation 6.2 shows the common DCA equation developed in [34, 35].

$$\begin{aligned}
q &= sP = se^{v}\left[1 + e^{v}\right]^{-1} \\
s &= population\ size \\
v &= \sum_{k=1}^{K-1} \Psi_k\left(\langle z \rangle_k\right) + \Psi_K(p)
\end{aligned} \tag{6.2}$$

The attraction value "ν" is simply the summation of the beta values calculated from the spline functions for each attribute value and price. The spline functions are shown in Appendix B.

6.2 Optimization Setups

Five main setups are considered for the analysis on optimizing a universal electric motor. The first four setups include only the engineering discipline. An attribute-based objective function will be used to determine an optimal motor design. The results are then entered into a marketing optimization to determine price and profit so that a comparison can be made between the disjoint optimizations and the all-at-once (AAO) approach. The design space is bounded by the six constraints listed above. The fifth method adds the marketing domain and related models to create an AAO optimization. Figure 4 gives a clearer picture of the five setups. In this example no extra constraints are applied when marketing is added. Weighting coefficients will be studied for the multi-objective optimizations along with various other changing parameters. Two types of weighting coefficients are used in this example. The first is added to simply balance the difference in magnitudes between the objectives in the multi-objective case. The second set of weights adds preference to one or more attributes within the l_2 deviation segment of the objective function. Weighting coefficients will be discussed within each setup when relevant.

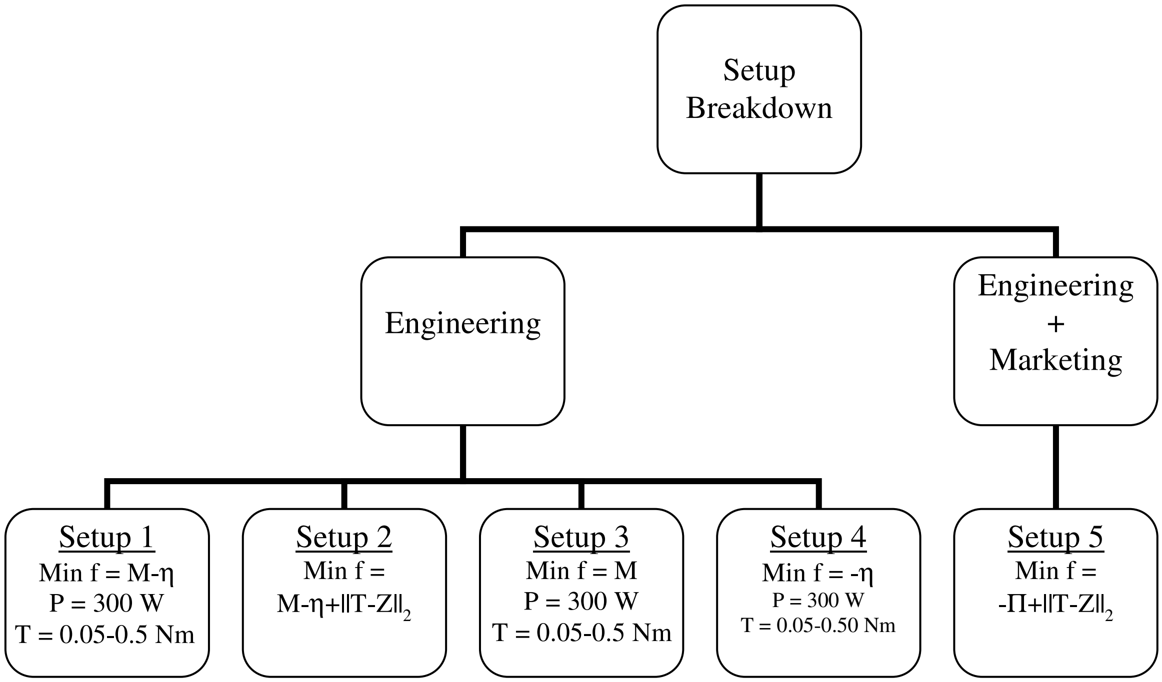


Figure 4: Breakdown of Setups for Motor Analyses

Setup	Class	Objective	Information Requirments	Output
1	S-E-AA	Min M - η	DV → Attributes	x_1-x_8
2	S-E-AA	Min M-η+ Dev.	DV → Attributes	x_1-x_8
3	S-E-A	Min M	DV → Attributes	x_1-x_8
4	S-E-A	Min -η	DV → Attributes	x_1-x_8
5a	S-EP-Π	Min –Π + Dev.	DV → Attributes, Attr. → Demand Targets, Weights, Price → Demand Cost & Profit Models	x_1-x_8, price
5b	S-EP-AΠ	Min -Π	DV → Attributes, Attr. → Demand Price → Demand, Cost & Profit Models	x_1-x_8, price

Table 20: Details of the Six Motor Setups.

The table above gives a breakdown of the six different setups used in the analysis of a universal electric motor. All six setups require information regarding the relationship between the design variables and attributes. Similar to the scale example, the major differences in information requirements can be found in Setups 5a and 5b. Setup 5a requires six more major pieces of information compared to Setups 1-4. This includes information on target settings, weights, pricing, cost models, profit models, and relationships between attributes and demand. Setup 5b is slightly less involved because information on targets and weights is not necessary. All six setups output values for the eight engineering design variables but Setups 5a and 5b also include a price output. The objective functions will be discussed in more detail later in this section.

The five initial solutions used for all five of the setups are shown in Table 21. The first initial solution is the lower bound on all eight design variables. The second initial solution is the upper bound on all eight design variables. Initial solution three is the median of the range for each of the eight design variables. Initial solutions four and five were arbitrarily chosen with values for r_o and t_s extending beyond the actual range for each respective design variable. Finally, the sixth and seventh initial solutions are the 75% and 25% values among the range for each design variable, respectively.

DV	Initial Solutions						
	1	2	3	4	5	6	7
N_c	100	1500	800	1062	730	1150	450
N_s	1	500	250.5	54	45	375.25	125.75
A_{aw}	0.01	1	0.505	0.241	0.203	0.7525	0.2575
A_{sw}	0.01	1	0.505	0.376	0.205	0.7525	0.2575
r_o	0.01	0.1	0.055	2.59	3.62	0.0775	0.0325
t_s	0.0005	0.015	0.00775	6.66	9.69	0.011375	0.004125
I	0.1	6	3.05	4.29	3.65	4.525	1.575
L	0.01	0.2	0.105	2.6	0.998	0.1525	0.0575

Table 21: Initial solutions for Universal Electric Motor Optimizations.

Five different torque requirements will be used to optimize five different motor designs. Each motor design can be thought of as an individual optimization with no coupling between the other motors. If there were a link between the five torque settings then the optimization could become a product family, which is performed in Simpson *et* *al.* [37]. Benchmark motor designs shown in Table 22 will be used to compare the correctness of the optimization results.

Benchmark Motor Designs												
Motor	N_c (turns)	N_s (turns)	A_{aw} (mm^2)	A_{sw} (mm^2)	r_o (cm)	t_s (mm)	I (A)	L (cm)	T (Nm)	P (W)	η (%)	M (kg)
1	730	45	0.203	0.205	3.62	9.69	3.65	0.998	0.05	300	71.4	0.5
2	750	76	0.186	0.203	3.31	11.77	3.73	1.28	0.1	300	70.6	0.5
3	760	89	0.19	0.203	3.12	11.2	3.73	1.41	0.125	300	70	0.5
4	1030	73	0.253	0.23	2.44	6.35	4.19	2.74	0.3	300	62.2	0.712
5	1087	72	0.284	0.247	2.71	7.15	4.71	3.16	0.5	300	55.3	0.985

Table 22: Benchmark Motor Designs [37].

6.2.1 Setup 1: Multi-objective Tradeoff between Mass and Efficiency.

This setup is based off examples found in [30, 36, 37]. The overall objective is to minimize mass while maximizing efficiency. Power and torque are set as equality constraints to match the company decided necessary requirements. The equality constraints make all the initial solutions infeasible initially since none of the seven initial solutions meet the exact torque and power requirements. The objective function used for all five motors designs in Setup 1 is shown in equation 6.3. No weighting coefficients are used during this optimization because the magnitude of mass and efficiency are comparable. Mass ranges from 0 to 2.0 kilograms while efficiency ranges from 0 to 100 percent (0 – 1.0).

$$\min f = M - \eta \qquad (6.3)$$

Once the engineering optimization is complete the solutions are evaluated in the following way: They are entered as fixed values into a secondary price optimization to determine the best price and maximum achievable profit (see Appendix C).

6.2.2 Setup 2: Multi-objective Tradeoff between Mass and Efficiency

Setup 2 is a modification to Setup 1 that will provide information on the effect that target settings have on an optimization solver. For this setup the equality constraints for torque and power are replaced with target settings in the objective function. Using the same initial solutions, an attempt to meet the torque and power requirements through target settings will be made to study how the optimizer reacts and see if a quality solution can be found. This can be useful in a situation where the equality constraints cannot be met but a satisfactory solution within a specified tolerance may be found. Removing the equality constraints allows some of the initial solutions to be feasible because the stiff requirements are initially removed. Equation 6.4 shows the objective function used in Setup 2. The l_2 norm of the difference between target and response values for torque and power are added to the original multi-objective function in Setup 1.

$$\min f = M - \eta + \left\| w \circ (T - Z) \right\|_2 \tag{6.4}$$

Once the engineering optimization is complete the solutions are evaluated in the following way: They are entered as fixed values into a secondary price optimization to determine the best price and maximum achievable profit (see Appendix C).

It is important to have weights on the deviations of torque and power since torque ranges from 0.05 – 0.5 Nm and power is fixed at 300. For example, with a torque requirement of 0.05 Nm the vector w is set to [600 1]. Element by element matrix multiplication takes the difference in torque and multiplies it by 600 while the difference in power is multiplied by one. This balances the difference in magnitude between torque and power. These weights are adjusted to study how different values change the result.

6.2.3 Setup 3: Minimizing Mass

Setup 3 simplifies the objective function be removing the efficiency objective. It is expected that minimizing mass using equality constraints for torque and power would return a solution quicker than the previous two setups simply because it is single objective. Equation 6.5 shows the simplified objective function.

$$\min f = M \tag{6.5}$$

Once the engineering optimization is complete the solutions are evaluated in the following way: They are entered as fixed values into a secondary price optimization to determine the best price and maximum achievable profit (see Appendix C).

6.2.4 Setup 4: Maximizing Efficiency

Setup 4 is again a single objective optimization to maximize efficiency. Torque and power are set as equality constraints as in Setup 1. Setups 3 and 4 will allow a comparison to be made between the difference in solution quality when a problem that logically should be solved using a multi-objective function is solved using each objective individually. Equation 6.6 shows the objective function used for this setup.

$$\min f = -\eta \tag{6.6}$$

Once the engineering optimization is complete the solutions are evaluated in the following way: They are entered as fixed values into a secondary price optimization to determine the best price and maximum achievable profit (see Appendix C).

6.2.5 Setup 5: All-at-Once

This setup is important for several reasons. Advantages or disadvantages of a joint optimization can be studied using an AAO approach and comparing it to the previous disjoint optimization setups. Also, although the all-at-once approach is limited to small multi-disciplinary problems, important information about the knowledge required to perform such an optimization can be studied through simple examples. AAO increases scope and complexity by considering customer preference directly in the optimization.

The AAO optimization of a universal electric motor is tried using both equality constraints and target settings to meet torque and power requirements. Target settings in the AAO approach are different from other multi-disciplinary optimization techniques because no coordination among disciplines is required. It is common for various decomposition techniques in multi-disciplinary optimization to use target settings to coordinate disciplines and guide them toward the same solution. The objective functions for these setups are shown in equation 6.7. The first equation of 6.7 has been used for the MDO technique known as analytical target cascading [16, 17]. The second equation of 6.7 is a common objective function for decision-based design (DBD). Typically DBD optimizations are profit-based single objective optimizations.

$$\begin{aligned} &Setup\ \ 5a: \min f = -\Pi + w_1 \left\| w_2 \circ (T - Z) \right\|_2 \\ &Setup\ \ 5b: \min f = -\Pi \end{aligned} \qquad (6.7)$$

6.3 Results of Universal Electric Motor Analysis

The results will be displayed and compared in several different tables. First the result of each setup with the lowest objective function value (i.e. "best") for each torque requirement will be displayed. This will allow easy comparison with the benchmark motor designs shown in Table 22. Then Section 6.4 will compare the information requirements between several setups, Section 6.5 will discuss how "rules of thumb" can affect the selection of an objective function, and finally Section 6.6 will analyze how the use of target settings in the objective function affects the solution.

6.3.1 Results of each Setup

	Best Result: Setup 1				
	0.05 Nm	0.10 Nm	0.125 Nm	0.3 Nm	0.5 Nm
N_c (turns)	698.3939	873.8862	931.4766	1062.5550	1161.1770
N_s (turns)	57.5732	58.0332	56.8460	40.2552	46.3483
A_{aw} (mm^2)	0.2087	0.2087	0.2087	0.2155	0.2737
A_{sw} (mm^2)	0.2087	0.2087	0.2087	0.2156	0.2737
r_o (m)	0.0161	0.0184	0.0191	0.0214	0.0249
t_s (m)	0.0026	0.0031	0.0033	0.0037	0.0047
I (A)	3.1967	3.5952	3.8081	6.0000	6.0000
L (m)	0.0196	0.0219	0.0226	0.0246	0.0287
Efficiency	0.8161	0.7256	0.6850	0.4348	0.4348
Mass (kg)	0.2553	0.3614	0.3992	0.5215	0.8352
Power (W)	300.0000	300.0000	300.0000	299.9999	299.9999
Profit ($)	2,691,980	2,316,635	2,169,453	1,492,425	894,234

Table 23: Results of Multi-Objective Optimization.

It can be seen from Table 23 that the torque and power requirements have been met for each of the five motor designs. As expected the mass increases and efficiency decreases as the torque increases from 0.05 through 0.5 Nm. The efficiency is dramatically less for the larger motors because a large current is necessary to meet the power constraint.

	Best Result: Setup 2				
	0.05 Nm	0.10 Nm	0.125 Nm	0.3 Nm	0.5 Nm
N_c (turns)	867.2270	732.7830	1023.9600	1059.6100	1061.7800
N_s (turns)	38.6611	58.4106	34.2572	104.0420	101.5650
A_{aw} (mm^2)	0.3072	0.5523	0.2752	0.3404	0.3815
A_{sw} (mm^2)	0.6949	0.3528	0.4971	0.3339	0.6086
r_o (m)	0.0198	0.0256	0.0304	0.0393	0.0336
t_s (m)	0.0051	0.0045	0.0037	0.0039	0.0056
I (A)	3.0708	2.9592	4.0591	4.5190	3.7542
L (m)	0.0224	0.0290	0.0217	0.0121	0.0290
Efficiency	0.8495	0.8815	0.6427	0.5773	0.6949
Mass (kg)	0.4935	1.0075	0.8936	1.0748	1.5737
Power (W)	300.0000	300.0000	300.0000	300.0000	300.0000
Profit ($)	2,249,784	1,298,245	1,095,600	610,753	-36,241

Table 24: Results of MO Optimization with Target Settings.

Similar to Setup 1 the mass increases more dramatically in Setup 2 than efficiency. Torque and power requirements have been met for all five motor designs with similar results to Setup 1. Apparently mass was not given as much importance as efficiency because both are higher in Setup 2. No weights were given to either because their magnitudes are similar so there is no clear explanation for why there is a large difference in mass from Setup 1. Efficiency seems to improve greatly as torque increases, however, mass also increases greatly.

	Best Result: Setup 3				
	0.05 Nm	0.10 Nm	0.125 Nm	0.3 Nm	0.5 Nm
N_c (turns)	732.3850	880.4442	908.7270	1056.4000	1159.3206
N_s (turns)	24.8018	29.7585	31.7158	40.3948	46.3208
A_{aw} (mm^2)	0.0907	0.1294	0.1429	0.2150	0.2734
A_{sw} (mm^2)	0.0908	0.1291	0.1429	0.2148	0.2751
r_o (m)	0.0127	0.0155	0.0166	0.0214	0.0249
t_s (m)	0.0017	0.0023	0.0026	0.0037	0.0047
I (A)	6.0000	6.0000	6.0000	6.0000	6.0000
L (m)	0.0152	0.0177	0.0189	0.0247	0.0288
Efficiency	0.4348	0.4348	0.4348	0.4348	0.4348
Mass (kg)	0.1012	0.1902	0.2334	0.5215	0.8352
Power (W)	299.9998	299.9999	299.9999	299.9999	299.9999
Profit ($)	2,314,632	2,136,571	2,052,662	1,492,400	899,324

Table 25: Results of Single Objective Optimization to Minimize Mass.

Minimizing mass alone had dramatic effects on efficiency. As expected, mass decreased substantially from the multi-objective cases with the adverse affect of also decreasing efficiency. Current reached its upper bound of 6.0 amps for every motor design while the remaining variables were relatively constant compared to Setups 1 and 2. This makes sense because mass is not a function of current, therefore, increased current helps meet the power and torque constraints but is detrimental to efficiency.

	Best Result: Setup 4				
	0.05 Nm	0.10 Nm	0.125 Nm	0.3 Nm	0.5 Nm
N_c (turns)	443.8787	275.5630	781.1160	1056.4000	1062.3700
N_s (turns)	92.4786	111.2370	101.1030	40.3948	103.5470
A_{aw} (mm^2)	1.0000	1.0000	1.0000	0.2150	0.5592
A_{sw} (mm^2)	1.0000	1.0000	1.0000	0.2148	0.5592
r_o (m)	0.0288	0.0347	0.0322	0.0214	0.0312
t_s (m)	0.0133	0.0147	0.0148	0.0037	0.0068
I (A)	2.7260	2.7478	2.7847	6.0000	3.2902
L (m)	0.0208	0.0434	0.0233	0.0247	0.0388
Efficiency	0.9570	0.9494	0.9368	0.8897	0.7929
Mass (kg)	0.9892	2.0000	1.5843	2.0000	2.0000
Power (W)	300.0000	300.0000	300.0000	300.0000	300.0000
Profit ($)	1,445,608	-158,592	179,748	-654,731	-714,666

Table 26: Results of Single Objective Optimization to Maximize Efficiency.

Setup 4 is just the opposite of Setup 3. Priority is placed on efficiency thus increasing mass proportionately. Mass reached its constraint of 2.0 kg in three of the five motor designs. Efficiency increases substantially as well. For example, when T = 0.05 Nm efficiency increased from 82% to 96% for Setups 1 and 4, respectively.

	Best Result: Setup 5				
	0.05 Nm	0.1 Nm	0.125 Nm	0.3 Nm	0.5 Nm
N_c (turns)	770.9107	638.4883	749.7149	928.3510	1013.6319
N_s (turns)	53.0176	81.3443	65.7928	79.5408	79.0881
A_{aw} (mm^2)	0.2573	0.3184	0.3167	0.2637	0.2993
A_{sw} (mm^2)	0.2362	0.2927	0.3046	0.2712	0.4038
r_o (m)	0.0145	0.0224	0.0188	0.0289	0.0311
t_s (m)	0.0025	0.0041	0.0033	0.0042	0.0042
I (A)	3.1068	3.0952	3.2112	4.2013	4.5787
L (m)	0.0225	0.0244	0.0348	0.0231	0.0287
Price ($)	7.1279	6.7501	6.5867	5.6217	4.9559
Efficiency	0.8397	0.8428	0.8124	0.6209	0.5697
Mass (kg)	0.2898	0.5565	0.6039	0.8253	1.1926
Power (W)	300.0	300.0	300.0	300.0	300.0
Profit ($)	2,641,216	2,159,373	1,986,651	1,274,171	507,865

Table 27: Results of AAO Optimization

The results shown in Table 27 are from Setup 5a. Again the targets for power and torque were met for all five motor designs. Notice that efficiency does not decrease as quickly as in Setup 1, however, mass increases faster in Setup 5 than Setup 1.

Notice the large deviation in profit between the 0.05 Nm motor and the 0.5 Nm motor. This is believed to have occurred because of the simplicity in the spline functions used to estimate customer demand. Mass plays a large role in determining customer preference and a larger motor will obviously have a larger mass automatically making it less appealing, even if the torque is higher. Realistically there should be separate demand models for each motor size to more accurately represent the market segments.

6.3.2 Comparing Five Setups for T = {0.05, 0.10, 0.125, 0.30, 0.50} Nm

Table 28 displays the results of all five setups for motor design 1 (i.e. T = 0.05 Nm). Information from the optimization such as number of iterations, number of function calls, number of infeasible solutions, active constraints (ineqlin and ineqnonlin), and which initial solution returned the optimal design are displayed as well.

Each motor design from the first four setups was entered into a marketing optimization to determine price and maximum profit. This gives a basis for comparing the engineering optimizations with the AAO approach. It can be seen that the best result is from Setup 1. Setup 5b returned the next best solution based on profit margin alone.

Table 29 displays the results of motor design 2 (T = 0.10 Nm). Similar trends can be seen in these results as in Table 28. The main difference in this motor design is seen in Setup 4. The efficiency optimization results in a negative profit even though it has a value of almost 95%. Because mass is sacrificed at the expense of efficiency and the demand and cost models are both functions of mass, a small increase in mass will have a large role in profit margin. Efficiency is only included in the demand function and therefore has less importance.

Table 30 displays the results of motor design 3 (T = 0.125 Nm). Similar trends to the first two motors are seen here except in this case the single objective optimization to minimize mass results in a higher profit than three of the other setups.

The results of motor designs 4 and 5 are displayed in Tables 31 and 32, respectively. Again similar trends can be seen in all the optimizations. As the torque requirement increases the single objective mass optimization becomes the more preferred method. Tables 28 – 32 are at the end of Chapter 6.

6.4 Comparing Information Requirements

The universal electric motor example is a better example at demonstrating how information requirements affect the solution effort and quality when using different modeling approaches. Setup 1 will be compared to Setup 5b and Setup 2 will be compared to Setup 5a to understand the importance of information requirements on the solution quality. Table 20 provides a detailed breakdown of the information requirements involved in the different setups. Setup 1 is most similar to Setup 5b because both use equality constraints to meet torque and power requirements. Setup 2 is most similar to Setup 5a because both utilize a target deviation term in the objective function to meet torque and power requirements.

Both Setups 1 and 5b require information about how attributes are related to design variables, however, Setup 5b also requires information about the relationships between attributes and demand, price and demand, as well as information on cost models, and profit models. This is a significant amount of information to gather prior to creating an optimization model and there is no clear way to quantify this amount or the effort required to gather it. The most interesting result is that Setup 5b does not find a solution for any starting point. All the effort that went into developing the extra models would have been a waste of time. Setup 1 finds the highest profit of all the setups, so in this example developing marketing models is a waste of time anyway.

The information requirements between Setups 2 and 5a are more involved than for Setups 1 and 5b. Setup 2 requires information about the relationships between design variables and attributes as well as target settings. Setup 5b, on the other hand, requires information about the relationships between the design variables, attributes, price and demand models. In addition cost models, profit models, weighting coefficients, and targets have to be considered. The advantage of performing the necessary work to develop all of these models is that Setup 5a returns \$391,433 more than Setup 2. Determining if the extra effort and time it took to develop such models is worth close to

$400,000 is a difficult task. Setup 5b required user adjustments within MATLAB to find a solution.

Similar issues to those of the scale example also arise with the AAO approaches in this example problem. For example, adding spline functions from discrete choice analysis for customer preference required additional statements in the computer program to help keep the optimizer within the customer attribute bounds. The additional statements were arbitrarily chosen and it is not known if the optimization algorithm is sensitive to it.

Weighting coefficients for Setups 2 and 5a played an important role as well. If the norm of the deviation (w_1 in equation 6.7) was not multiplied by a value of 100,000 or higher no solution was found. However, a lower profit resulted for a value higher than 100,000. Similarly, the weights placed on the target attributes altered the solution significantly. If a coefficient less than 125 was placed on the torque deviation the optimizer found no solution. I do not see any significance in this value since, for example, one might expect the weight on torque for T = 0.05 Nm to be 6000 in order to balance with power. An optimization was run using Setup 2 with w from equation 6.4 as a variable because of the large range of possible weights. When started at 6000 the solution returned an optimal value of 5839. A solution was found in the same number of iterations as the case when w was fixed and also improved mass by 13.07% while decreasing efficiency by only 1.53%. It is determined through trial and error that the initial guess for the weight must be at least 160 or no feasible solution is found. When the weight is started at 160 the optimal result is 242 but the solution is suboptimal, based on profit alone, compared to the result when w is 5839.

6.5 Using Rules of Thumb in Choosing an Objective Function

Setup 1 will be compared to Setups 3 and 4 to see how rules of thumb, usually obtained through experience and sometimes called heuristics, could affect how an objective function is chosen along with the possible solutions that can be obtained. All three setups use equality constraints. The only difference between them is seen in the objective function formulation.

The universal electric motor design is a tightly coupled model with many intermediate functions. The objective functions in this example were fundamentally different from those in the scale example because target settings were not required. The original goals were to minimize mass and maximize efficiency with an enterprise goal to maximize profit. The objective function type again played an important role in maximizing profit. There appears to be a sensitive relation between the objective function used and the demand model. Comparing Tables 28, 30, and 31 shows how this relation affects the result. As the motor torque requirement increases from 0.05 Nm to 0.30 Nm, the single objective mass optimization (Setup 3) continuously returns higher profits relative to Setups 1 and 4. This probably occurs because of the way the cost modeling and customer demand modeling was carried out. The only customer attribute that the cost depended on was mass so it makes sense that a lighter motor is going to yield higher profits. Marketing information is not always readily available or accurate so target setting is still a difficult task.

Comparing results from Setups 1, 3, and 4 used in the motor design analyses brings to light another example of how a designer's experience and knowledge base can affect the way an optimization problem is formulated. The cost models used in the analysis are functions of only one out of four attributes. Mass appears in two of the four equations used to model cost. As a result cost is very sensitive to mass. For larger motors such as when T = 0.50 Nm the profit will automatically be less simply because the motor is heavier.

This shows the importance of cost and demand modeling but it also gives a solid relationship relative to the qualitative nature of the problem. If an engineer is aware of this relationship through previous examples or past experience the model may be simplified. Minimizing mass solely results in a higher profit for the larger motors than any of the other four objective functions. Maximizing efficiency, on the other hand, returns very poor solutions based on profit. Although customers desire high efficiency, choosing to maximize efficiency with no consideration for mass would be a bad rule of thumb to follow. Since single objective functions are generally simpler to formulate and solve, especially in single discipline optimizations, an appropriate rule of thumb could reduce the effort needed to find an optimal result. Essentially an optimization to maximize profit could be performed without actually solving a profit maximization problem.

6.6 Target Matching in the Objective

Target matching in the objective function played a large role in reducing the affects of initial solutions. For example, take motor designs 2-5 (T = 0.1, 0.125, 0.3, and 0.5 Nm). The multi-objective optimization of Setup 1 experienced at least one infeasible solution for the set of initial solutions mentioned in Table 21. Setup 1 uses equality constraints to meet torque and power requirements. Setup 2, which adds a target matching term to the objective function, finds a solution for every starting point of every motor design.

The tradeoff is seen when the results of Setups 1 and 2 are entered into a price optimization to determine profit. Setup 2 returns a profit margin that is $442,196 less than Setup 1. This seems like a huge tradeoff, but there may be design problems that cannot find a feasible solution when equality constraints are used. Target matching may be an alternative method to help get a solution. Of course one can assume the result is not the global optima, but it's better than not finding any solutions.

This exact situation can be seen when Setups 5a and 5b are compared. Setup 5b is a profit maximization problem utilizing equality constraints on power and torque. When this model is solved no solution is found for any of the starting points. The optimization terminates after just a few iterations claiming no feasible solution can be found. In an attempt to find a solution, Setup 5a was used. This setup adds a target matching objective to the profit objective. When this model is solved a solution is found for every starting point.

Since it would not be known if the optimizer will find a solution before a model is built and solved, a comparison cannot be made between the advantages of solving Setup 1 versus Setup 5a. However, it is useful to know that a simple modification to the objective function and constraints could improve the possibility of finding a solution.

Comparison of Results for Motor Design 1 (T = 0.05 Nm)						
		Formulations				
Design Variables	**Nomenclature**	**Multi-objective**	**MO with Targets**	**Mass**	**Efficiency**	**AAO**
# wire turns on armature	**N_c (turns)**	698.3916	867.2270	732.3850	443.8787	770.9107
# wire turns on stator	**N_s (turns)**	57.5732	38.6611	24.8018	92.4786	53.0176
x-sect. area of arm. Wire	**A_{aw} (mm^2)**	0.2087	0.3072	0.0907	1.0000	0.2573
x-sect. area of stator wire	**A_{sw} (mm^2)**	0.2087	0.6949	0.0908	1.0000	0.2362
stator radius	**r_o (m)**	0.0161	0.0198	0.0127	0.0288	0.0145
stator thickness	**t_s (m)**	0.0026	0.0051	0.0017	0.0133	0.0025
current	**I (A)**	3.1967	3.0708	6.0000	2.7260	3.1068
stack length	**L (m)**	0.0196	0.0224	0.0152	0.0208	0.0225
Customer Attributes	**Power (W)**	300.0000	300.0000	299.9998	300.0000	300.0000
	Torque (Nm)	0.0500	0.0500	0.0500	0.0500	0.0500
	Efficiency	0.8161	0.8495	0.4348	0.9570	0.8397
	Mass (kg)	0.2553	0.4935	0.1012	0.9892	0.2898
	Price ($)					7.1279
	Profit ($)					**2,641,217**
Properties of Optimization	**Obj. Funct.**	-0.5607	-0.3560	0.1012	-0.9570	-2641195.333
	Lower					
	Upper			7	3,4	
	ineqlin					
	ineqnonlin	1		1	1	
	# iterations	99	56	227	21	111
	# function calls	921	824	2215	201	1500
	# infeasible sln.	0	0	0	0	0
	Initial solution #	4,5	2	5	7	1
Price Optimization Properties	**Price ($)**	7.0860	6.8699	6.0349	6.5400	
	Profit ($)	**2,691,980**	**2,249,784**	**2,314,632**	**1,445,608**	

Table 28: Comparison of Five Setups on Motor Design 1

Comparison of Results for Motor Design 2 (T = 0.10 Nm)						
		Formulations				
Design Variables	**Nomenclature**	**Multi-objective**	**MO with Targets**	**Mass**	**Efficiency**	**AAO**
# wire turns on armature	**N_c (turns)**	873.9287	732.7830	880.4442	275.5630	638.4883
# wire turns on stator	**N_s (turns)**	58.0334	58.4106	29.7585	111.2370	81.3443
x-sect. area of arm. Wire	**A_{aw} (mm^2)**	0.2087	0.5523	0.1294	1.0000	0.3184
x-sect. area of stator wire	**A_{sw} (mm^2)**	0.2087	0.3528	0.1291	1.0000	0.2927
stator radius	**r_o (m)**	0.0184	0.0256	0.0155	0.0347	0.0224
stator thickness	**t_s (m)**	0.0031	0.0045	0.0023	0.0147	0.0041
current	**I (A)**	3.5952	2.9592	6.0000	2.7478	3.0952
stack length	**L (m)**	0.0219	0.0290	0.0177	0.0434	0.0244
Attributes	**Power (W)**	300.0000	300.0000	299.9999	300.0000	300.0000
	Torque (Nm)	0.1000	0.1000	0.1000	0.1000	0.1000
	Efficiency	0.7256	0.8815	0.4348	0.9494	0.8428
	Mass (kg)	0.3614	1.0075	0.1902	2.0000	0.5565
	Price ($)					6.7501
	Profit ($)					**2,159,373**
Properties of Optimization	**Obj. Funct.**	-0.3642	0.1260	0.1902	-0.9494	-2159371.4532
	lower					
	upper			7	3,4	
	ineqlin					
	ineqnonlin			1		
	# iterations	109	121	337	54	131
	# function calls	1034	1635	3455	514	1592
	# infeasible sln.	1	0	1	0	0
	Initial solution #	5,7	3	7	2	7
Price Optimization Properties	**Price ($)**	6.6369	6.2594	5.9108	5.0309	
	Profit ($)	**2,316,635**	**1,298,245**	**2,136,571**	**-158,592**	

Table 29: Comparison of Five Setups on Motor Design 2

Comparison of Results for Motor Design 3 (T = 0.125 Nm)						
		Formulations				
Design Variables	**Nomencl.**	**Multi-objective**	**MO with Targets**	**Mass**	**Efficiency**	**AAO**
# wire turns on armature	**N_c (turns)**	931.4766	736.8520	908.7270	781.1160	749.7149
# wire turns on stator	**N_s (turns)**	56.8460	61.9368	31.7158	101.1030	65.7928
x-sect. area of arm. Wire	**A_{aw} (mm^2)**	0.2087	0.4707	0.1429	1.0000	0.3167
x-sect. area of stator wire	**A_{sw} (mm^2)**	0.2087	0.5277	0.1429	1.0000	0.3046
stator radius	**r_o (m)**	0.0191	0.0285	0.0166	0.0322	0.0188
stator thickness	**t_s (m)**	0.0033	0.0053	0.0026	0.0148	0.0033
current	**I (A)**	3.8081	3.0242	6.0000	2.7847	3.2112
stack length	**L (m)**	0.0226	0.0298	0.0189	0.0233	0.0348
Attributes	**Power (W)**	300.0000	300.0000	299.9999	300.0000	300.0000
	Torque (Nm)	0.1250	0.1250	0.1250	0.1250	0.1250
	Efficiency	0.6850	0.8626	0.4348	0.9368	0.8124
	Mass (kg)	0.3992	1.1261	0.2334	1.5843	0.6039
	Price ($)					6.5867
	Profit ($)					**1,986,652**
Properties of Optimization	**Obj. Funct.**	-0.2858	0.2634	0.2334	-0.9368	-1986645.0986
	lower					
	upper			7	3,4	
	ineqlin					
	ineqnonlin	1		1	1	
	# iterations	50	76	489	29	1586
	# function calls	465	1076	5104	304	35498
	# infeasible sln.	1	0	1	0	0
	Initial Guess #	2	3	7	3	6
Price Optimization Properties	**Price ($)**	6.4492	6.0171	5.8501	5.6728	
	Profit ($)	**2,169,453**	**1,095,600**	**2,052,662**	**179,748**	

Table 30: Comparison of Five Setups on Motor Design 3

Comparison of Results for Motor Design 4 (T = 0.3 Nm)

Design Variables	Nomenclature	Formulations				
		Multi-objective	**MO with Targets**	**Mass**	**Efficiency**	**AAO**
# wire turns on armature	**N_c (turns)**	1062.5534	1059.6100	1062.6300	827.3550	928.3510
# wire turns on stator	**N_s (turns)**	40.2552	104.0420	40.2505	109.6810	79.5408
x-sect. area of arm. Wire	**A_{aw} (mm^2)**	0.2155	0.3404	0.2156	0.7945	0.2637
x-sect. area of stator wire	**A_{sw} (mm^2)**	0.2156	0.3339	0.2155	0.7946	0.2712
stator radius	**r_o (m)**	0.0214	0.0393	0.0214	0.0298	0.0289
stator thickness	**t_s (m)**	0.0037	0.0039	0.0037	0.0068	0.0042
current	**I (A)**	6.0000	4.5190	6.0000	2.9320	4.2013
stack length	**L (m)**	0.0246	0.0121	0.0246	0.0371	0.0231
Attributes	**Power (W)**	299.9999	300.0000	299.9999	300.0000	300.0000
	Torque (Nm)	0.3000	0.3000	0.3000	0.3000	0.3000
	Efficiency	0.4348	0.5773	0.4348	0.8897	0.6209
	Mass (kg)	0.5215	1.0748	0.5215	2.0000	0.8253
	Price ($)					5.6217
	Profit ($)					**1,274,172**
Properties of Optimization	**Obj. Funct.**	0.0868	0.4976		-0.8897	-1274151.8457
	lower					
	upper	7		7		
	ineqlin					
	ineqnonlin	1		1	1,4	
	# iterations	42	146	275	34	169
	# function calls	400	2417	2807	348	2653
	# infeasible sln.	3	0	1	2	0
	Initial Guess #	2,5,7	6	2	3	1
Price Optimization Properties	**Price ($)**	5.4472	5.1674	5.4472	4.9422	
	Profit ($)	**1,492,425**	**610,753**	**1,492,400**	**-654,731**	

Table 31: Comparison of Five Setups on Motor Design 4

Comparison of Results for Motor Design 5 (T = 0.5 Nm)						
		Formulations				
Design Variables	Nomencl.	Multi-objective	MO with Targets	Mass	Efficiency	AAO
# wire turns on armature	**N_c (turns)**	1161.1825	1061.7800	1144.3277	1062.3700	1013.6319
# wire turns on stator	**N_s (turns)**	46.3483	101.5650	46.4379	103.5470	79.0881
x-sect. area of arm. Wire	**A_{aw} (mm^2)**	0.2737	0.3815	0.2715	0.5592	0.2993
x-sect. area of stator wire	**A_{sw} (mm^2)**	0.2737	0.6086	0.2741	0.5592	0.4038
stator radius	**r_o (m)**	0.0249	0.0336	0.0249	0.0312	0.0311
stator thickness	**t_s (m)**	0.0047	0.0056	0.0047	0.0068	0.0042
current	**I (A)**	6.0000	3.7542	6.0000	3.2902	4.5787
stack length	**L (m)**	0.0287	0.0290	0.0290	0.0388	0.0287
Attributes	**Power (W)**	299.9999	300.0000	299.9999	300.0000	300.0000
	Torque (Nm)	0.5000	0.5000	0.5000	0.5000	0.5000
	Efficiency	0.4348	0.6949	0.4348	0.7929	0.5697
	Mass (kg)	0.8352	1.5737	0.8353	2.0000	1.1926
	Price ($)					4.9559
	Profit ($)					**507,865**
Properties of Optimization	**Obj. Funct.**	0.4004	0.8789	0.8353	-0.7929	-507862.6302
	lower					
	upper	7		7		
	ineqlin					
	ineqnonlin	1		1	1,4	
	# iterations	68	110	164	35	154
	# function calls	643	1461	1830	320	21225
	# infeasible sln.	2	0	2	0	0
	Initial Guess #	3,5	4	6	4	2
Price Optimization Properties	**Price ($)**	5.0095	4.8430	5.0081	4.5958	
	Profit ($)	**894,234**	**-36,241**	**899,324**	**-714,666**	

Table 32: Comparison of Five Setups on Motor Design 5

CHAPTER 7: DISCUSSION

Product development is a decision making process that selects values for a set of variables in order to create a product that will be profitable. Design optimization is a tool to help decision makers overcome cognitive limitations during the selection of variable values in order to create the best, most profitable design possible. Design optimization, however, is by no means a trivial process. There are many pieces of information that are not known to a design engineer during this decision making process. The computational experiments conducted in Chapters 5 and 6 were used to study the information flow surrounding the design optimization stage of product development in an attempt to understand the importance of information requirements in selecting an optimization model. A more general view of how information such as rules of thumb affects model selection and input decisions was discussed as well.

Information flow surrounding the design optimization process can be broken down into three main tasks, i.e. groups of decisions, two pre-optimization tasks and one post-optimization task, as shown in Figure 5. The first task is model selection, where relevant knowledge and past experience, along with new modeling techniques, are used to build a quantitative model of the product. Engineering principles, experience, and background help design engineers make decisions related to modeling such as objective function formulation, decomposition techniques, algorithm selection, cost modeling, and customer demand modeling. A good example of this can be seen from the results of the motor analyses. The choice of Setup 1 or Setup 3 may be subjective, but obviously the results are going to strongly depend on the decision.

The second task deals with input decisions. This assignment is where design engineers make decisions about what values should be used as model inputs. Experience and background are a few pieces of knowledge used to make decisions such as selecting design variables, weighting coefficients, initial solutions, and determining values for fixed parameters during an optimization. Many of the decisions in the input model are based on experience, personal preference, and intuition rather than objective analysis. Examples include selecting values for weights and initial solutions and determining how many initial solutions to use.

There is some degree of coupling between the first two tasks because model type and solution technique will play a role in what types of inputs are needed. Issues involved with inputs, such as initial solutions and weighting coefficients, are inherent to most design optimization problems. An example of the coupling between model selection and input decisions can be seen in both example problems. If it is decided that marketing models will be developed, the design engineer must then choose between a larger set of objective function options. The problem could be solved using an engineering optimization or a joint engineering and marketing optimization, in which case targets and weights could also be affected.

The third task deals with evaluating the optimization results. The output evaluation task involves validating and examining the results for correctness. A designer needs to determine if the results make sense and are robust to change. Understanding the design space is useful in determining this but when the problem consists of more than three variables it becomes difficult to picture the design space. For example, if the initial goal was to perform a profit maximization problem, as in Setup 5b of the motor analysis, it would be determined that no solution can be found using the current model. In this case it would be necessary to loop back to the Model Selection section in Figure 5 to make necessary adjustments. In the case of the motor, it would be to add target settings in the objective function or possibly switch to separate marketing and engineering optimizations.

The analyses performed in Chapters 5 and 6 examine particular elements of the information flow relevant to Figure 5. The focus of this analysis is placed on key portions of the optimization process typically involving engineering judgment such as initial solution selection, weighting coefficient determination, cost modeling, objective function formulation, and customer demand modeling. Individuals have attempted to improve each and every one of these important aspects of the product design optimization process individually.

To my knowledge no one has ever compared the significance of each to determine what information affects various problem types and how modeling changes because of it. For instance, if adjusting weighting coefficients has a larger variance in profit than initial solutions, more time and effort should probably be placed on altering weighting coefficients to determine the highest profit. If time is not an issue all key portions of an optimization should be thoroughly studied, however, time is usually one of the largest factors in the product design process. If a designer is under time constraints, insight into where attention should be focused could prove to be useful. After performing the scale analysis, it became apparent that a significant amount of time could be saved in performing the optimization if the correct target settings were known a priori.

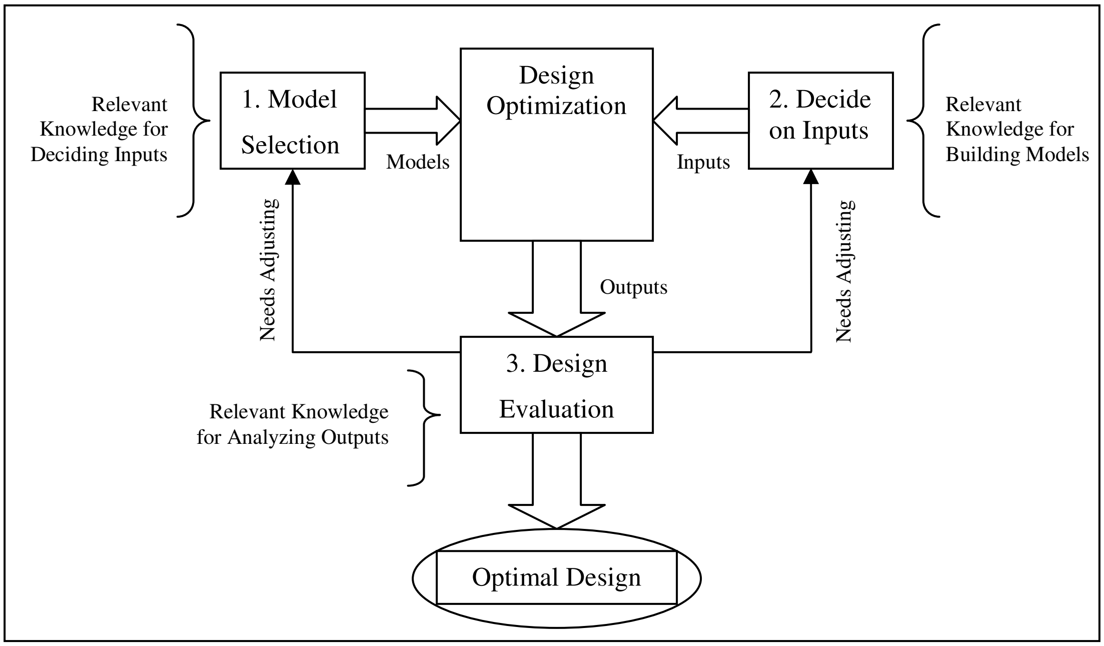


Figure 5: Design Optimization Information Flow.

7.2.3 Other Examples from Literature

While physical relationships, expressed using mathematics, generally determine the details of the optimization model, qualitative factors influence the model structure as discussed above. In particular, a design engineer's knowledge base and experience influence the information available, which determines the engineer's formulation of the design problem, unless time and resources are sufficient to gather more information. The computational experiments in Chapters 5 and 6 clearly demonstrate how different sets of information lead to different optimization models. The following examples from the literature provide additional illustrations.

An S-E-AA optimization problem in Kim *et al.* [15] demonstrates target cascading on a suspension design of a half vehicle model. The objectives are to minimize deviations from target settings for NVH (noise-vibration-harshness) and packaging (relative displacement of sprung and unsprung masses). Both of these objectives are related to ride quality and handling of a passenger vehicle. No market survey was conducted to determine customer preference but the design engineer chose these two objectives probably based on years of data from previous designs and maybe even personal experience of what makes a car appealing. The target values were probably a common threshold learned through experience to make a car quiet and smooth.

A second example of type S-E-A is taken from Sobieski and Kroo [10] where a wing design problem is used to demonstrate collaborative optimization. The design problem objective is to maximize range. Choosing this objective function when designing an aircraft wing and not an entire aircraft seems a little random. Why not minimize mass or maximize lift-to-drag ratio? The design engineer likely has knowledge through experience or senior staff that makes range the more important objective. A more fuel efficient aircraft will fly farther yet the Breuget range equation used in [10] is not a function of fuel efficiency. The engineer knows that reshaping the wing can improve aerodynamics and decrease weight, which in turn improves fuel efficiency, which in turn increases range. Range is an important performance measure to a customer so the engineer models all of the related equations into one optimization rather than optimizing each element individually.

A third and final example is an S-EP-Π optimization of an aircraft concept-sizing problem developed by Gu *et al.* [11]. The aircraft concept-sizing problem is developed to demonstrate decision-based design using collaborative optimization. Building customer demand models is beyond the realm of many design engineers, as in this case, so industry trends were used to develop reasonable models. Heuristics based on gross take-off weight, aircraft range, stall speed, fuselage volume, cruise speed, and price are used to develop a demand function. For example, it is known through experience that lower take-off weight generates higher demand but a very light aircraft is undesired. Also, the longer the range the higher the demand but no significant increase in demand is seen after range reaches 600 miles. This example includes engineering models, cost models, demand models, and profit models to maximize net revenue. How the customer attributes were chosen and the fact that there are six of them likely helped guide the engineer to decide on the scope of the optimization. A large number of objectives make formulating and solving a multi-objective problem quite difficult. The problem could be simplified and solved with greater ease if a design group knew if one or two of the six customer attributes affected the profit more significantly than the others. In this case it is assumed that all six attributes are important so the appropriate models had to be developed to account for all necessary goals.

CHAPTER 8: SUMMARY AND CONCLUSIONS

This thesis attempts to aid design engineers in formulating optimization problems by providing a method of finding similar problems based on a classification scheme as well as providing insight into the tradeoffs involved in using different modeling approaches. Conformation and organization of design optimization terminology and existing example problems can improve this important part of the product design process.

8.1 Design Optimization Classification

A novel classification framework for design optimization problems has been presented. Several example problems (including multidisciplinary design optimization problems) were considered to show the versatility and usefulness of the classification framework. Designers can use this classification framework and the reference examples as an initial solution for considering the scope of the design optimization problem and reviewing relevant examples before working on the details of the problem formulation and programming the optimization software.

The classification framework does not cover every characteristic of design optimization problems. For instance, the classification framework does not cover qualitative but important measures such as safety and environmental impact unless a specific objective function can be found. It does not consider important issues such as the linear (or nonlinear) nature of the constraints and objective functions. It does not distinguish between optimization problems used in different phases of product design (e.g., conceptual design or detailed design).

The first contribution of the classification framework is to begin organizing the ever-increasing variety of design optimization problems using characteristics that are relevant to design engineers. Its second, related, contribution is to provide guidance to design engineers and product development teams who want to use design optimization.

A set of design optimization problems that receive the same classification may cover a range of formulations, solved using a variety of techniques. This diversity is useful since it provides a range of relevant examples so that the designer (or design team) can find one that is most appropriate for their situation and their abilities.

In addition, the classification framework provokes the designer (or design team) to consider a broader perspective of the entire process. Abstraction early in the design phase allows a designer to focus on the high level understanding of the problem at hand before getting immersed in the details. The design classification indicates in a rough way the type and amount of information required to solve the problem. Second, a methodical review of what the major goals and decisions for the project are can clarify and guide the process.

8.2 Design Optimization Comparison

Effectively incorporating design optimization into the product development process requires not only understanding the problem objectives and design attributes but also addressing the tradeoff between the information required to formulate the optimization model and the quality of the solution that is found. It may be intuitive that a more comprehensive optimization problem will require more information and yield a better solution (for instance, by maximizing profit directly instead of optimizing a customer attribute). This thesis attempts to explore that concept systematically through a set of computational experiments on two design problems.

In particular, the results of computational experiments on the bathroom scale design problem and the universal electric motor design problem identify the additional information required to solve a profit maximization problem (as discussed in Sections 5.4 and 6.4), demonstrate the role of rules of thumb in formulating design optimization problems (Section 6.5), show how decomposition affects solution quality and computational effort (Section 5.5), and uncover the impact of using target matching in the objective function instead of as constraints (Section 6.6). In addition, the results show how the values of targets and objective function weights impact solution quality (Sections 5.4 and 6.4).

Compared to optimization models that include only engineering variables and optimize design attributes, formulating profit maximization problems requires additional information about the relationships between attributes and demand, price and demand, cost models, and profit models. While such models should yield the most profitable designs, this result is not guaranteed. If high-quality attribute targets are available, optimizing the design to meet those targets can be just as profitable, as demonstrated in the bathroom scale experiments.

Similarly the universal electric motor demonstrated how different objective functions could yield the same or better results depending on cost and demand models. More specifically, as torque requirements increase the single objective minimization of mass became more appealing and eventually exceeded the profit returned by the otherwise superior multi-objective optimization that used equality constraints. We conclude that the use of a good rule of thumb (such as "minimize mass") can, without the information required to directly maximize profit, lead to solutions that are just as profitable. Of course, a bad rule of thumb (e.g., "maximize efficiency") leads to unprofitable designs. A review of the literature identified other rules of thumb used when formulating design optimization problems.

When matching critical attributes, targets must be included in the objective function (instead of as constraints) in order to find feasible solutions. The profit maximization may find a solution that is worse than that found from an engineering-only design optimization that includes the targets as constraints, as demonstrated in the electric motor experiments. More feasible solutions are found when equality constraints are replaced with target settings in the objective function, but target settings often result in a lower profit margin. Using the wrong weights to combine multiple objectives (such as profit and matching target attributes) can yield poor solutions, as demonstrated in both sets of experiments.

In general, these results show the extent to which correct information is critical to finding a high quality solution, perhaps more critical than the optimization model selected. That is, the quality of the information used is more important than the amount of information used.

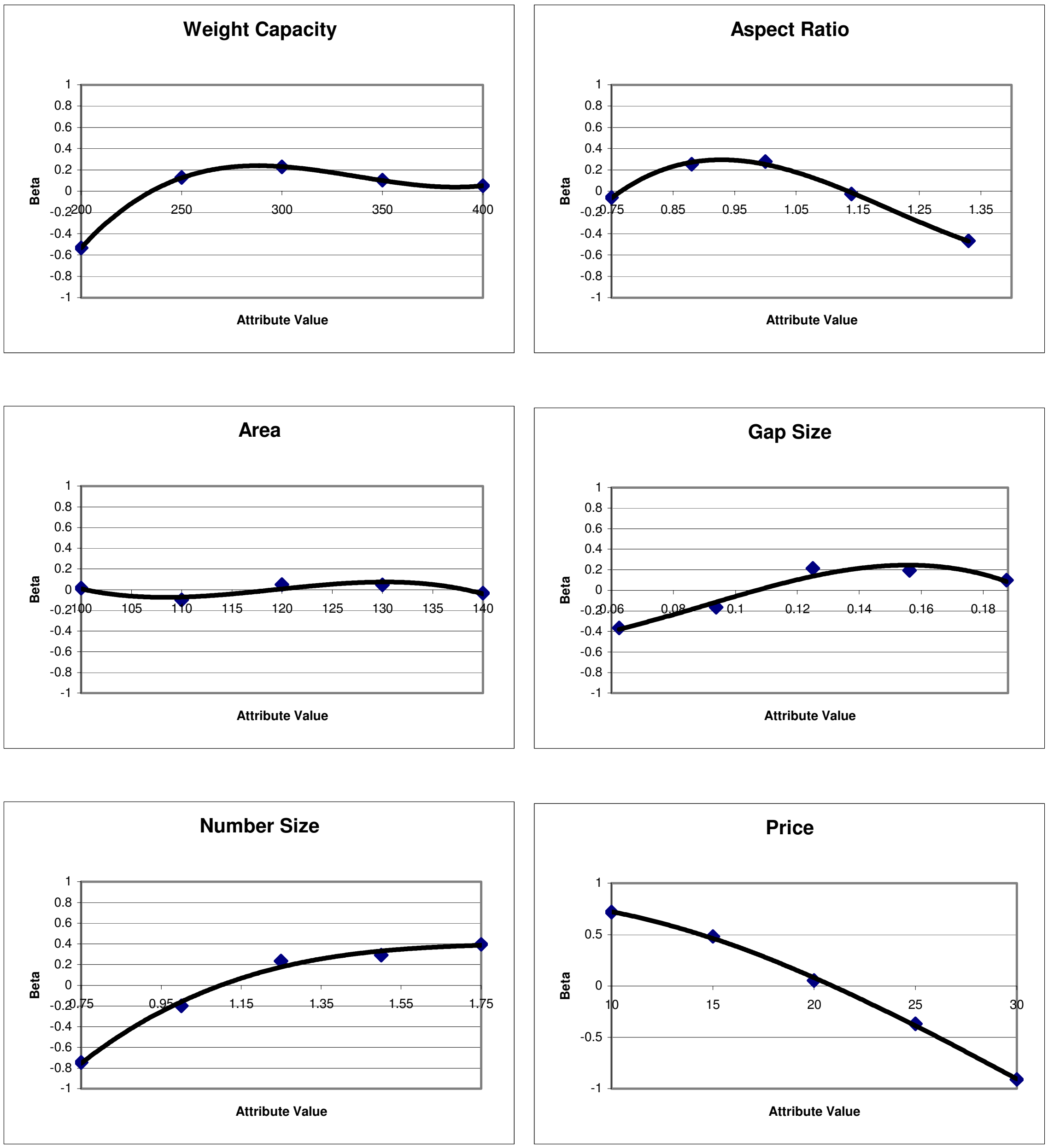
8.3 Future Work

The ideas and propositions made in this thesis are the beginning of a long journey. Currently there are few optimization problems documented, especially MDO problems. To enhance the classification scheme the optimization community needs to work together to classify and compile a large set of example problems with easy access. The classification scheme will continuously become more effective as the number of example problems grow and improve. A larger set of example problems, whether academic or industrial, may also bring to light more heuristics relating to knowledge and modeling.

An interesting extension to this study of information flow surrounding the design optimization process would be to collaborate with industry to search for and identify important heuristics that affect modeling and solution techniques. From there it may be possible to develop more quantitative ways of imposing such heuristics into the optimization process.

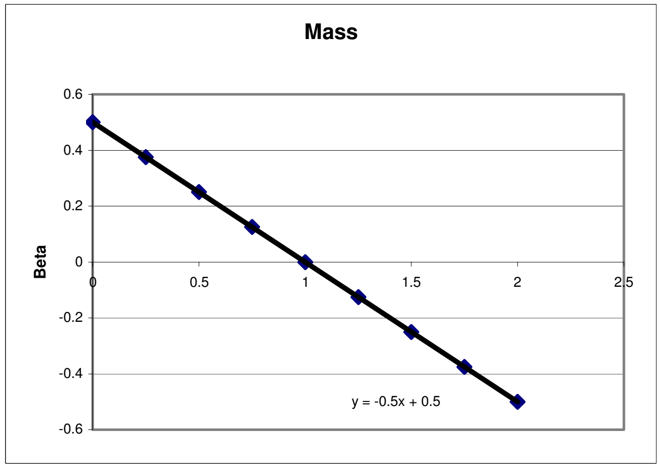
The computational experiments performed in this thesis did not cover all aspects of current optimization techniques. For example, no comparison was made between the performance of different optimization algorithms such as sequential quadratic programming and genetic algorithms. Different algorithms may show improved performance for specific types of models. Also no comparison was made between multidisciplinary optimization frameworks such as collaborative optimization and analytical target cascading. Each of these new frameworks has been individually compared to standard single level optimization approaches such as all-at-once and sequential optimization, but no direct comparison between the two methods has ever been done because CO is a framework for nonhierarchical problems while ATC has typically been used for hierarchical problems. This difference limits the set of possible problems that can be used for comparison.

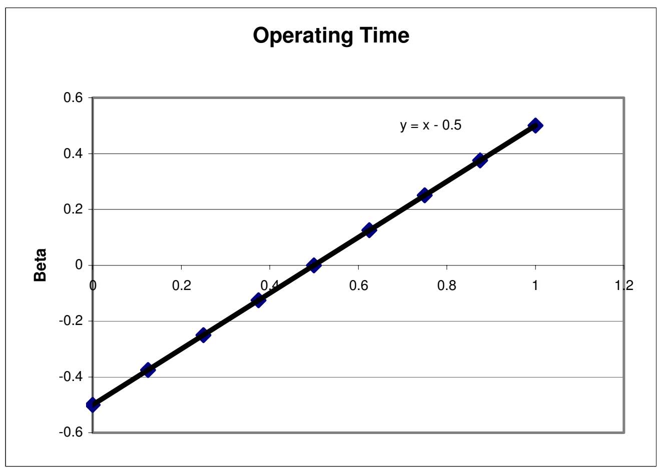
APPENDIX A

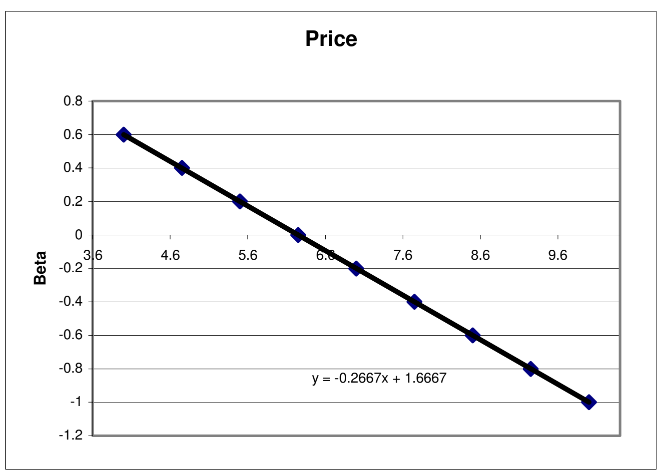


Appendix A: Spline Functions for Scale Analysis.

APPENDIX B







Appendix B: Spline Functions for Universal Electric Motor Analysis.

APPENDIX C

Marketing Optimization Model (Scale Example)

Max $\Pi = q\left(p - c_v\right) - c_i$
With respect to z_1-z_6
Subject to: $z_{LB} \leq z \leq z_{UB}$
Where:

$$q = sP = se^{v}\left[1 + e^{v}\right]^{-1}$$

$$s = 5{,}000{,}000$$

$$v = \sum_{k=1}^{K-1} \Psi_k\left(\langle z \rangle_k\right) + \Psi_K(p)$$

c_v = \$3/piece
c_i = \$1,000,000

Price Optimization Model (Scale and Motor Examples)

Max Π
With respect to price
Subject to: $\text{price}_{LB} \leq \text{price} \leq \text{price}_{UB}$
Where:

$$q = sP = se^{v}\left[1 + e^{v}\right]^{-1}$$

$$v = \sum_{k=1}^{K-1} \Psi_k\left(\langle z \rangle_k\right) + \Psi_K(p)$$

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