TECHNICAL RESEARCH REPORT

Development of an Engine Idle Speed and Emission Controller

by Masaaki Nagashima and William S. Levine

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Abstract— This paper is focused on idling. An engine model is developed. The engine model includes an airflow dynamics model, a combustion model, a fuel injection model, and a catalytic converter model. These models will be used to compare controllers. The idle controller, air / fuel ratio controller, and the emission controller that Honda uses are installed. The measured data and simulated data are compared to evaluate the accuracy of the model. A linear model is developed by linearizing at nominal points. With this linear model, new controllers are developed and compared to the existing controllers.

Index Terms—idle control system, air / fuel control, emissions.

I. INTRODUCTION

In this paper, we focus on the idling condition for an internal combustion engine. When the vehicle is idling, it is difficult to stabilize engine speed because the engine speed is very low and is sensitive to the changing loads due to the Air Conditioner (A/C), Power Steering (P/S), and Alternator (ALT). Meanwhile, United States federal law requires better regulation of the exhaust emissions. Thus, it is also essential to reduce the emissions. Research upon these problems has been conducted. Most treat the engine speed control problem and the emission reduction problem separately. The combined objective of idle speed and A/F ratio control has been studied [3]. We are going to combine these problems, including the emission reduction and idle speed regulation problem, into one problem. We can thereby avoid the interference between two individual controllers.

There are two ways to develop an engine controller. One is by using an actual engine and another is by developing an engine model. The former has the advantage that we can test without any model. The latter has the advantage that we better understand engine dynamics and test controllers more easily. We choose the latter to comprehend dynamics of the engine.

After developing the engine model, we develop a new controller by using linearized model. To prove the advantage of the new controller, we installed the existing Honda controllers which contain separate idle, air and fuel ratio, and emissions controllers.

II. ENGINE MODEL

The goal of the engine model is to allow us to compare the new controller to the existing controllers. The engine model consists of four main components: airflow dynamics model, model of engine dynamics, fuel injection model, and catalytic converter model.

A. Airflow Dynamics Model

The only control of the air amount is the throttle angle. The airflow dynamics model is to describe the amount of air into the cylinders, which affects the combustion torque. We developed the airflow dynamics in

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the following steps. First, we take the throttle response delay into account, for the motor and controller of the throttle restrict the throttle action speed. Fig.1 is overall view of the airflow dynamics model.

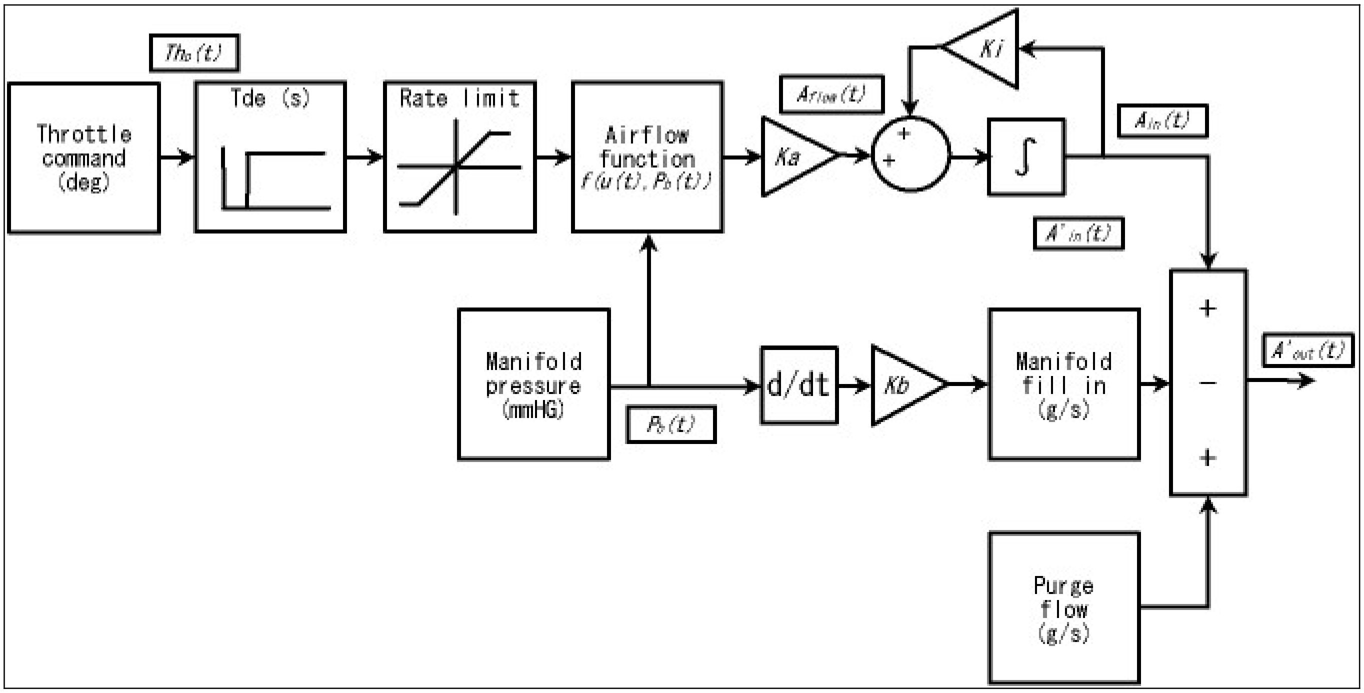

FIG.1 AIRFLOW DYNAMICS

$$Th_a(t) = Th_o(t - t_{de})\Big|_{\frac{dTh_a(t)}{dt} \le m}$$

$Th_o(t)$ Target throttle angle (deg)

$Th_a(t)$ Actual throttle angle (deg)

t_{de} Throttle delay from command to response (s)

m The motor response limitation (deg/s)

Second, the airflow rate is calculated. We assume that we can only use the negative pressure sensor. Basically, the airflow going though the throttle can be measured off-line, ignoring the variability of the throttle from vehicle to vehicle. Fig.2 shows the relationship between throttle angle and airflow rate on the supposition that the negative pressure is constant. Then we can determine the airflow rate by multiplying the effect of the gauge pressure. (Fig.3)

$$A_{flow}(t) = f_{air}(Th_a(t)) \cdot g_{air}(P_a - P_b(t))$$

$A_{flow}(t)$ Air mass flow rate through the throttle (l/min)

P_a Ambient pressure (mmHG)

$P_b(t)$ Negative pressure in the intake manifold (mmHG)

$f_{air}(\cdot)$ A function giving air mass flow from throttle angle (Fig.2)

$g_{air}(\cdot)$ A function compensating air flow for the gage pressure (Fig.3)

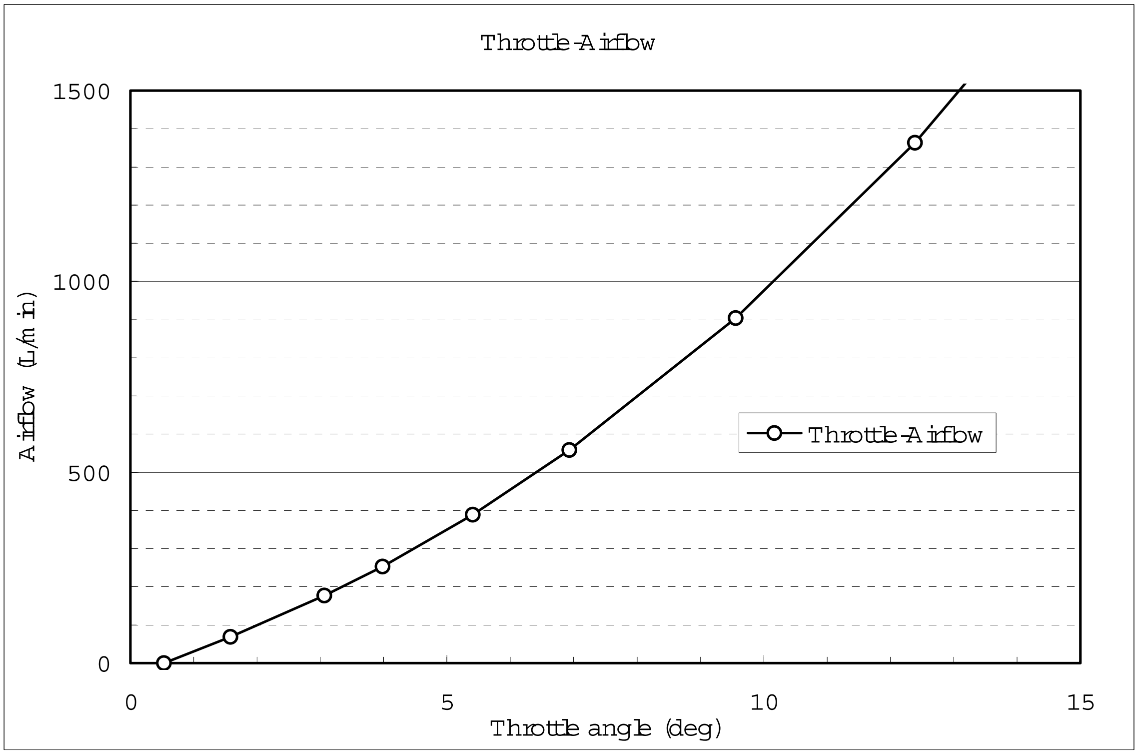

FIG.2 THROTTLE-AIRFLOW

Next, we change the units from (L/min) to (g/s).

$$A_{flow}(g/s) = \kappa_{air} \cdot \frac{A_{flow}(t)}{60}(l/min)$$

κ_{air} Air density at 0 degree centigrade (g/l)

From the experimental result, using the filter produces a better answer than only using delay. (Fig.4) By using the manifold temperature, the airflow amount is given by

$$(\frac{d}{dt}+3)A_{in}(t) = 3 \cdot \kappa_{air} \frac{T_0}{T_m(t)} \cdot \frac{A_{flow}(t)}{60}$$

$A_{in}(t)$ Air mass flow rate into the intake manifold (g/s)

$T_m(t)$ Manifold air absolute temperature (K)

T_0 Absolute temperature equivalent to 0 degree centigrade = 273.15 (K)

Here, s means the operator of the Laplace transform.

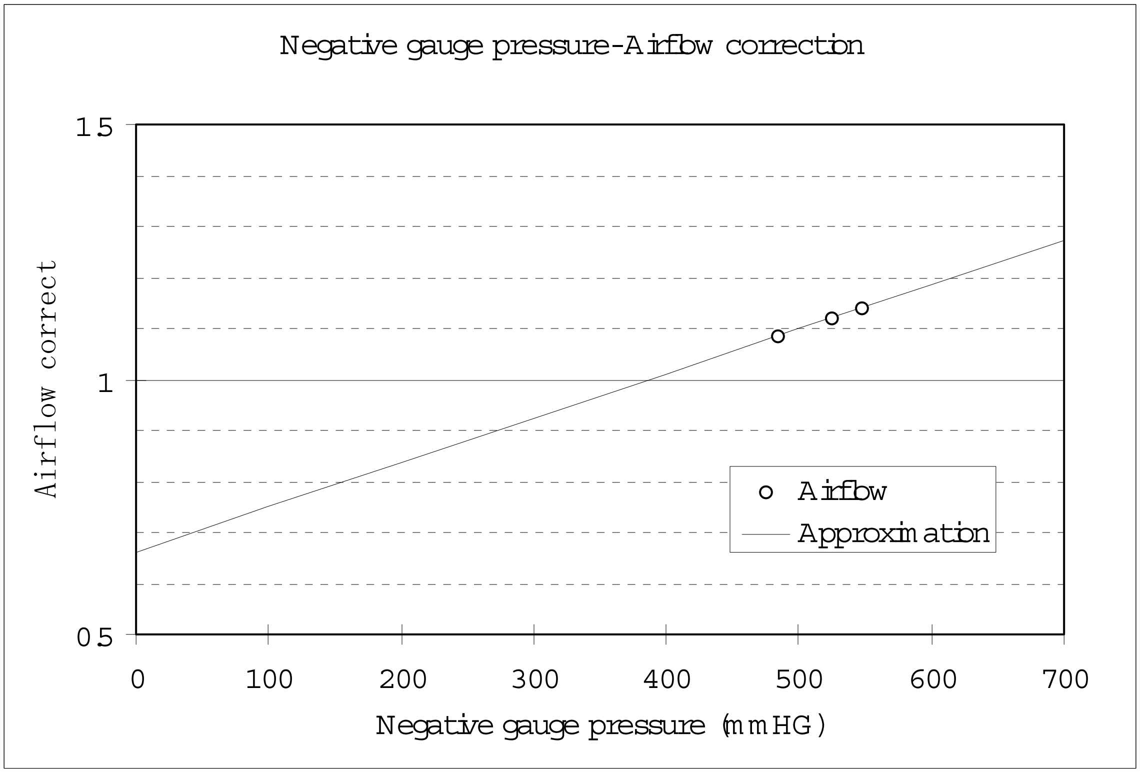

FIG.3 NEGATIVE GAUGE PRESSURE-AIRFLOW

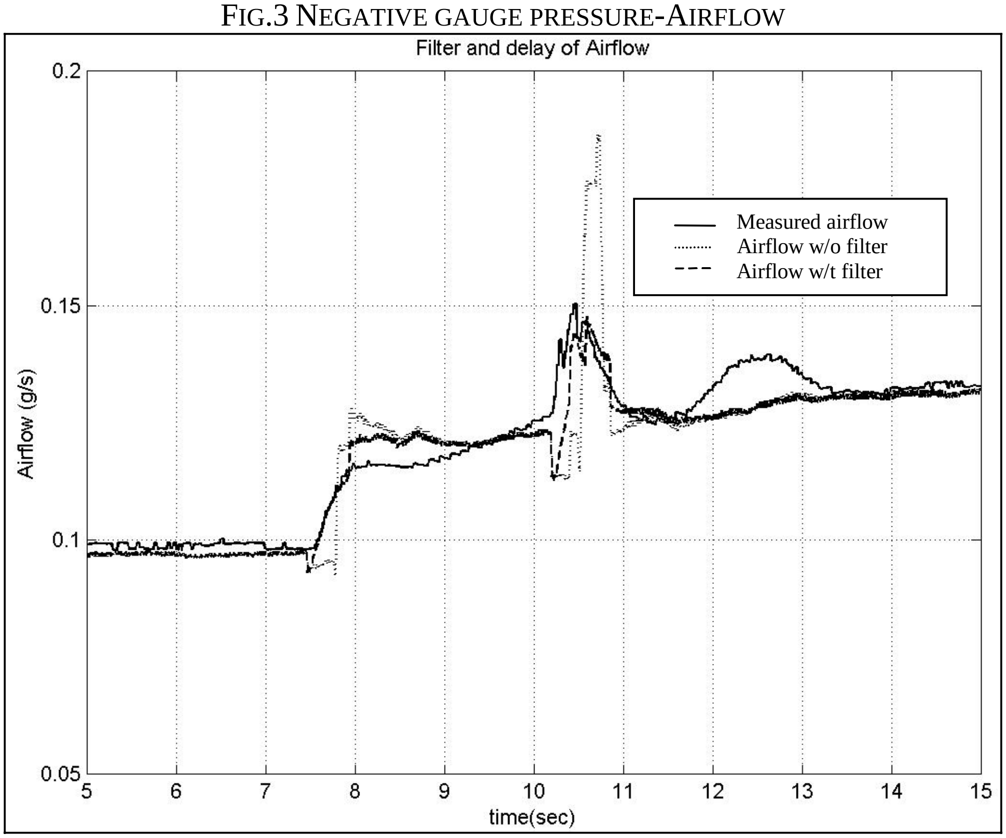

FIG.4 FILTER AND DELAY

Finally, we are able to calculate the air amount into the cylinders from the conservation law. In the actual engine, the purge flow plays an important role and makes the system more complicated. The purge is used to inhale the evaporated gas into the cylinder in order to prevent unburned gasoline from going outside. Suppose we can calculate the purge flow, the equation is given by

$$A'_{out}(t) = A_{in}(t) + A_{purge}(t - t_{purge}) - \frac{\frac{dP_b(t)}{dt}}{R_{air} \cdot T_m(t)} \cdot \frac{V_m}{760}$$

$A'_{out}(t)$	Air mass flow rate out from intake manifold (g/s)
$A_{purge}(t)$	Purge air mass flow rate (g/s)
t_{purge}	Purge flow delay (s)
R_{air}	Gas constant : 0.28703 (J/gK)
V_m	Intake manifold volume (m^3)

The last term represents the air amount that is used to change the intake manifold pressure.

B. Model of Engine Dynamics: airflow to torque to speed

In this model, we calculate the combustion torque of the engine; then we conduct the engine speed. Fig.5 is an overall view of the model of engine dynamics.

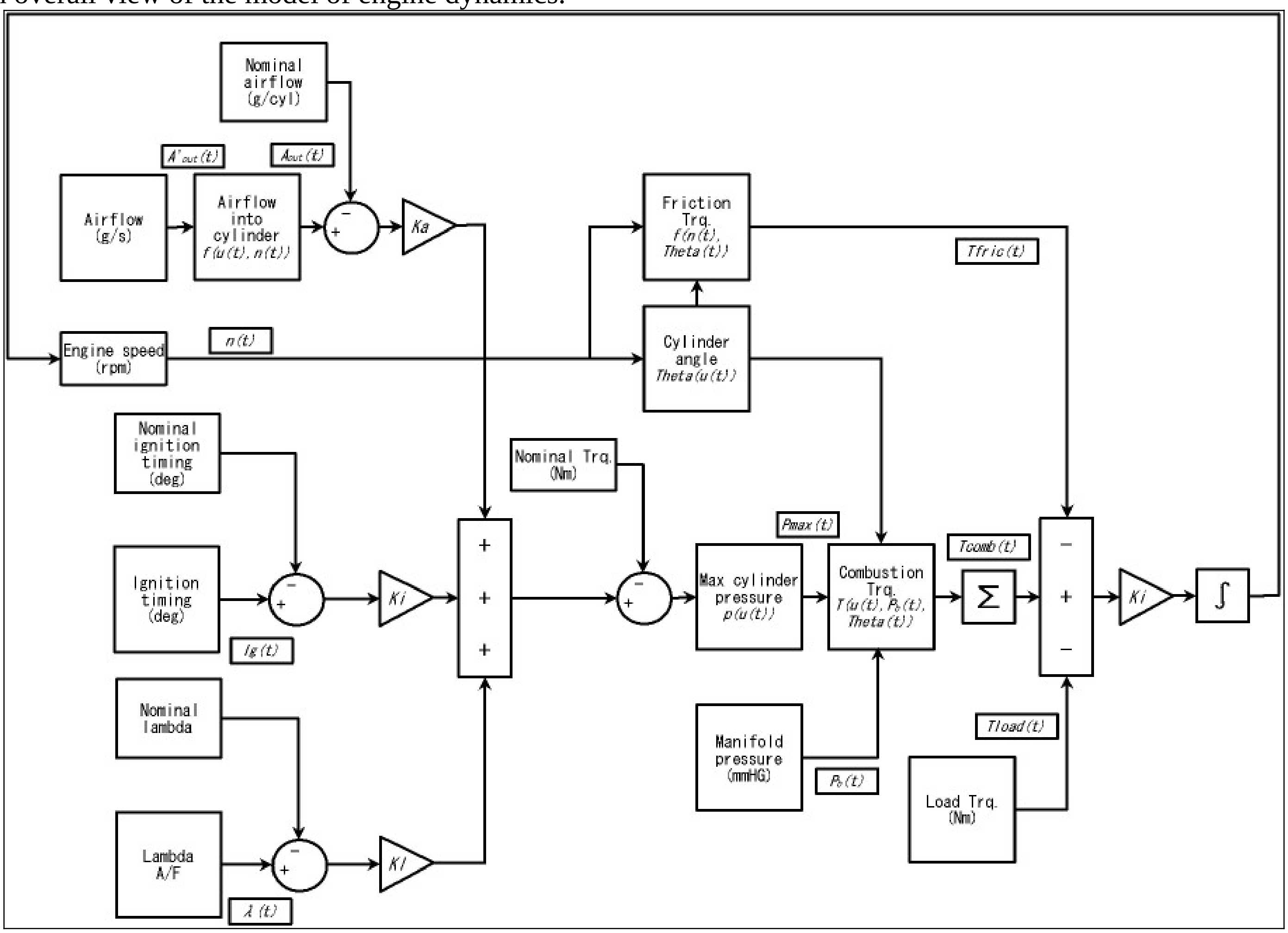

FIG.5 MODEL OF ENGINE DYNAMICS

The airflow amount, ignition timing, and the air/fuel ratio (lambda), affect the engine torque. To calculate the combustion torque, we need to assume that the combustion torque can be linearized at a nominal point. Temporarily ignoring the effect of the Internal Exhaust Gas Recirculation (Internal EGR), the relationship between the airflow amount and the engine combustion torque is almost linear. Also, when the engine is idling, we retard the ignition timing from Maximum Brake Torque (MBT), which allows us to make use of the torque curve in order to stabilize the engine speed. We can linearize near the target ignition timing. Lambda is an extremely important factor for the emissions; on the other hand, the effect of lambda on the engine speed has been neglected in most research. After the engine is warmed up, lambda stays close to stoichiometric, the ideal air/fuel ratio; therefore, we can linearize at the stoichiometric air/fuel ratio. In conclusion, we obtain the following equation,

$$\bar{T}_{comb}(t) = k_a(A_{out}(t) - A_{nom}) + K_{ig}(Ig(t) - Ig_{tgt}) + K_l(\lambda_{pre}(t) - \lambda_{stoich})$$

$\bar{T}_{comb}(t)$	Average combustion torque (Nm)
A_{nom}	Nominal airflow (g/cyl)
$Ig(t)$	The ignistion timing (deg)
Ig_{tgt}	Nominal ignition timing (deg)
$\lambda_{pre}(t)$	Pre-catalyst lambda (A/F)
λ_{stoich}	Nominal pre-catalyst lambda (A/F)

Where,

$$A_{out}(t) = A'_{out}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}}$$

$A_{out}(t)$	Air mass flow into each cylinder (g/cyl)
$n(t)$	Engine speed (revs)
N_{cyl}	The number of cylinders

Next, we take the internal EGR influence into account. The internal EGR is a reflux of burned gas during the overlap of the intake and exhaust valves. The amount of internal EGR is mostly dependent on the negative pressure. Increasing the internal EGR causes the torque to decrease. Fig.6 shows the effect of the internal EGR on the combustion torque. The combustion torque is given by:

$$\bar{T}_{comb,egr}(t) = f_{egr}(P_b(t)) \cdot \bar{T}_{comb}(t)$$

$f_{egr}(P_b(t))$ Torque coefficient (Fig.6)

The effect of the internal EGR should be multiplied by the airflow if internal EGR affects the negative pressure $P_b(t)$. However, the torque coefficient multiplied by the combustion torque instead of multiplying by the airflow. The reason is that this gives us better approximation of the airflow amount rather that multiplying to the airflow directly. Therefore, we assumed that internal EGR does not affect the negative pressure but only affect the combustion torque.

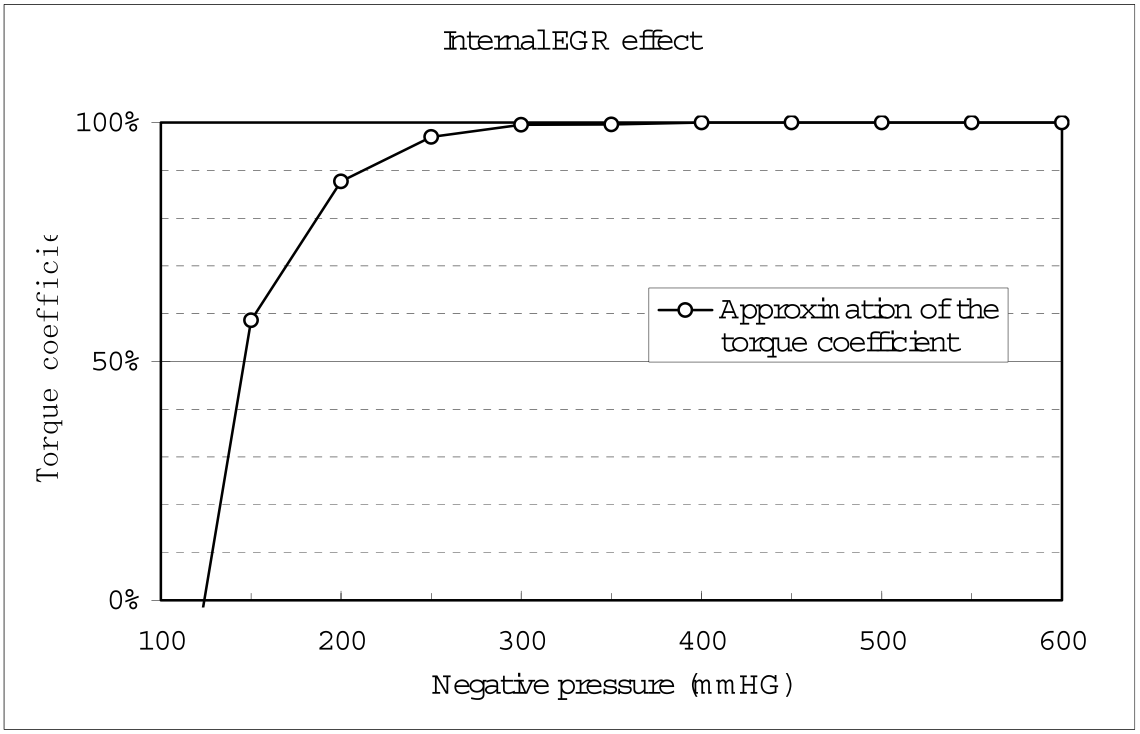

FIG.6 EFFECT OF INTERNAL EGR TO THE ENGINE TORQUE

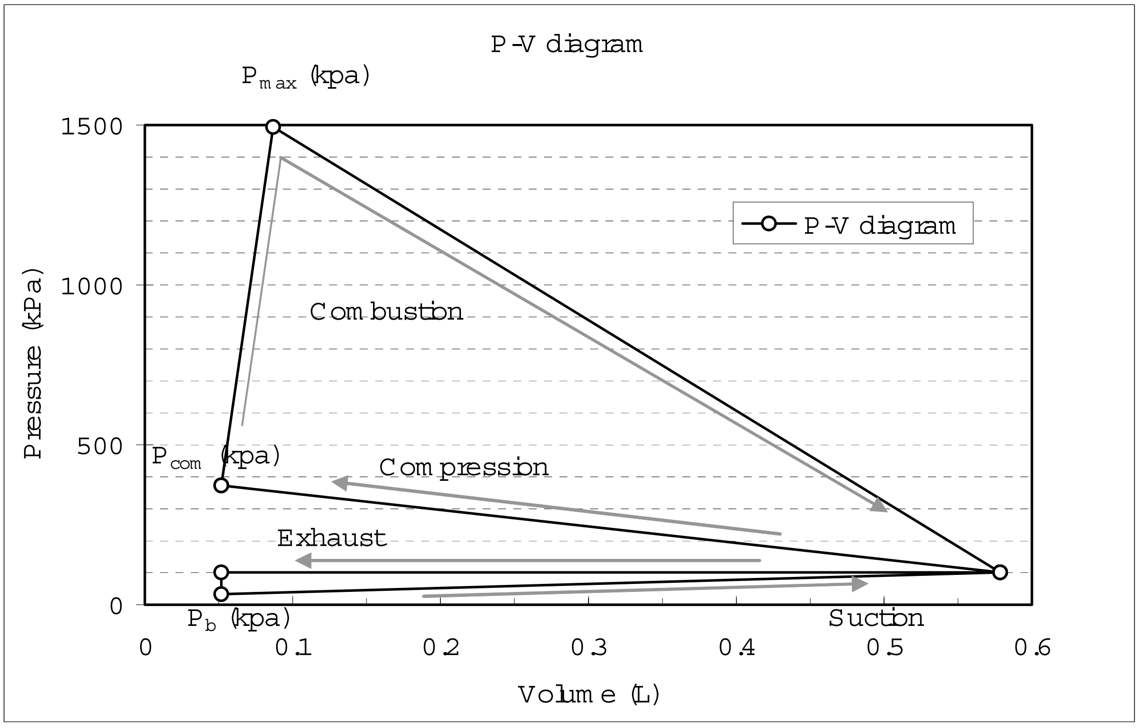

FIG.7 COMBUSTION P-V DIAGRAM

The torque we obtain above is a mean torque. The torque, however, is generated only during the explosion process. In addition, we need to include pumping loss, which is the loss during the induction process. Here we assume a simple model. (Fig.7)

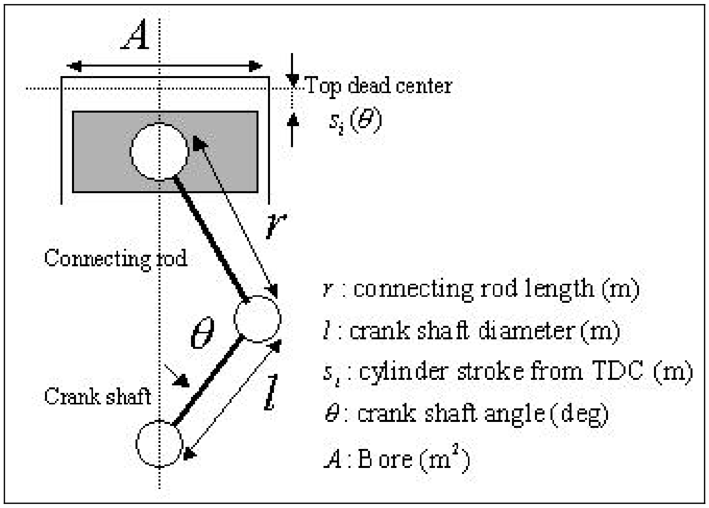

FIG.8 PISTON AND CRANKSHAFT MOTION

According to the Fig8, the rate of change of cylinder volume at θ is given by,

$$V_{o1}(\theta) = A \cdot s_i(\theta)$$

$$V_{o1}(\theta) \quad \text{The cylinder volume } (m^3)$$

Taking the derivative of the cylinder volume,

$$\frac{d}{d\theta} V_{o1}(\theta) = A \cdot \frac{d}{d\theta} s_i(\theta) = A \cdot r \left\{ 1 + \frac{r}{l} \frac{\cos\theta}{\sqrt{1 - \left(\frac{r}{l}\sin\theta\right)^2}} \right\} \sin\theta$$

$$\frac{d}{d\theta} V_{o1}(\theta) \quad \text{The derivative of the cylinder volume } (m^3/\text{deg})$$

A peak pressure can be derived from the mean torque. We suppose that the peak torque is gained at some angle After Top Dead Center (ATDC), while the cylinder pressure at TDC is constant. Now, we are to calculate the combustion torque generated to the crankshaft via connecting rods. First, an equation for the combustion cycle is given by

$$T'_{comb}(\theta) = \left\{ \frac{\theta}{Ang_{pmax}} (P_{max}(t) - P_{com}) + P_{com} \right\} \cdot \dot{V}_{o1}(\theta) \cdots 0 \le \theta \le Ang_{pmax}$$

$$Ang_{pmax} \quad \text{Cylinder angle that generate } P_{max}(t) \text{ (deg)}$$

$$T'_{comb}(\theta) = \left\{ \frac{\theta - Ang_{pmax}}{Ang_{pmax} - \pi} P_{max}(t) + P_{max} \right\} \cdot \dot{V}_{o1}(\theta) \cdots Ang_{pmax} \le \theta \le \pi$$

The angle 0 is the TDC of a cylinder. Next, we set the torque to zero during the exhaust process.

$$T_{comb}(\theta) = 0 \cdots \pi \le \theta \le 2\pi$$

Then, taking the pumping loss into consideration, the torque during the induction process is given by

$$T'_{comb}(\theta) = \left\{ \frac{\theta - 2\pi}{\pi} P'_b(t) \right\} \cdot \dot{V}_{o1}(\theta) \cdots 2\pi \le \theta \le 3\pi$$

Where $P'_b(t)$ is a negative pressure, but the units have changed from (mmHG) to (pa).

$$P'_b(t) = \left\{ \frac{P_b(t)}{760} - 1 \right\} \cdot \gamma_{pre}$$

γ_{pre} The constant to change units (mmHG $\rightarrow$ pa)

760 (mmHG) = 1 (atm)

Finally, an equation of the compression process is given by,

$$T'_{comb}(\theta) = \left\{ \frac{\theta - 3\pi}{\pi} P_{com} \right\} \cdot V_{ol}(\theta) \cdots 3\pi \le \theta \le 4\pi$$

The combustion torque for each cylinder is given by the integral of these equations above. To conclude, we can attain the total torque combustion equation such that,

$$T_{comb}(t) = \sum_{i=1}^{N_{cyl}} T_{comb}(\theta_i)$$

$$\theta_i = \left(\int_0^t 2\pi \cdot \frac{n(\sigma)}{60} d\sigma + \frac{4\pi i}{N_{cyl}} \right)\Big|_{Mod/4\pi}$$

Here, the subscript of θ represents the cylinder number.

Besides the pumping loss, there exists friction between piston and sleeve. The friction torque increases in proportion to the piston speed, which is related to the engine speed. The friction torque is given by the Coulomb law such that,

$$T'_{fric}(t) = F_{fric}(t) \cdot \frac{ds_i(\theta)}{d\theta}$$

$T'_{fric}(t)$ A friction torque of each cylinder (Nm)

$F_{fric}(t)$ A friction force (N)

where,

$$F_{fric}(t) = c_{fric} \frac{ds_i(\theta)}{dt} = c_{fric} \frac{ds_i(\theta)}{d\theta} \frac{d\theta}{dt}$$

c_{fric} A friction coefficient

$\frac{d\theta}{dt}$ Angular velocity (rad/s)

$$\frac{ds_i(\theta)}{d\theta} = \frac{d}{d\theta} r \left\{ 1 - \cos\theta + \frac{l}{r}\left(1 - \sqrt{1 - \left(\frac{r}{l}\sin\theta\right)^2}\right) \right\}$$

$$\frac{d\theta}{dt} = 2\pi \frac{n(t)}{60}$$

Then the piston friction is given by

$$T'_{fric}(t) = c_{fric}\left(r\left(\sin\theta + \frac{r\sin\theta \cdot \cos\theta}{\sqrt{l^2 - (r\sin\theta)^2}}\right)\right)^2 \cdot 2\pi \frac{n(t)}{60}$$

In addition to the above friction, there exist other frictions such as transmission friction, camshaft friction, and oil viscosity. We assume these are constant. Now the total friction torque is given by

$$T_{fric}(t) = T'_{fric}(t) + T_{const}$$

$T_{fric}(t)$ A total friction torque (Nm)

The next step is calculating the load torque. A/C, P/S, and ALT loads are calculated. Besides these loads, we should have taken the automatic transmission engagement into account; however, we neglect it in this paper. Based on the above, the equation of the loads is given by,

$$T_{load}(t) = T_{A/C}(t) + T_{P/S}(t) + T_{ALT}(t)$$

$T_{load}(t)$ A total load torque (Nm)

Finally, we can obtain the crankshaft torque such that,

$$T_{crank}(t) = T_{comb}(t) - T_{fric}(t) - T_{load}(t)$$

$T_{crank}(t)$ A total crank torque (Nm)

Now that we have the crankshaft torque, we can derive the engine speed.

$$n(t) = \frac{60}{2\pi}\int_0^t \frac{T_{crank}(\sigma)}{J} d\sigma + n(0)$$

J Engine inertia moment (kgm^2)

We put results of the engine speed calculation model. Fig.9 shows the No Load (N/L) condition result. This gives a relatively good result.

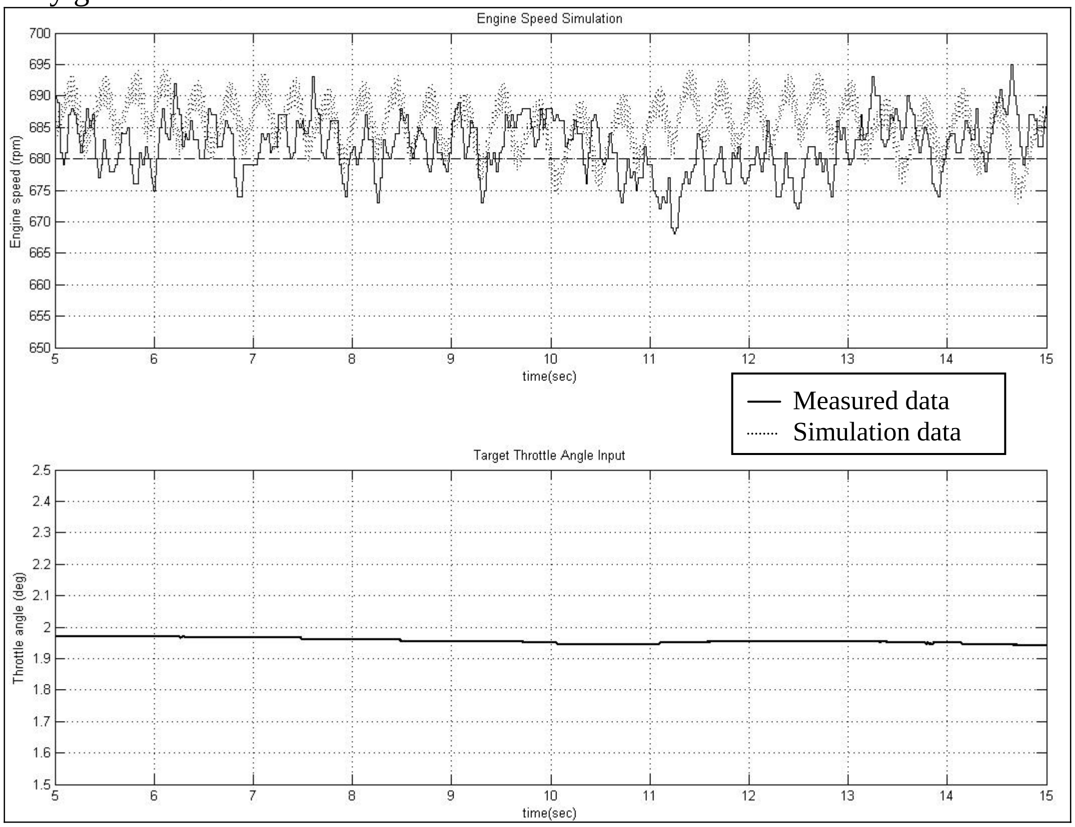

FIG.9 SIMULATION RESULT (N/L)

Secondly, fig.10 shows the result with ALT load. The engine speed dropped much more in the measured data than in the simulation. It is hard to know the true reason for this. We assume this comes from the specification of the ALT. Besides this specification, the ALT temperature perturbs the ALT load. (Fig.13)

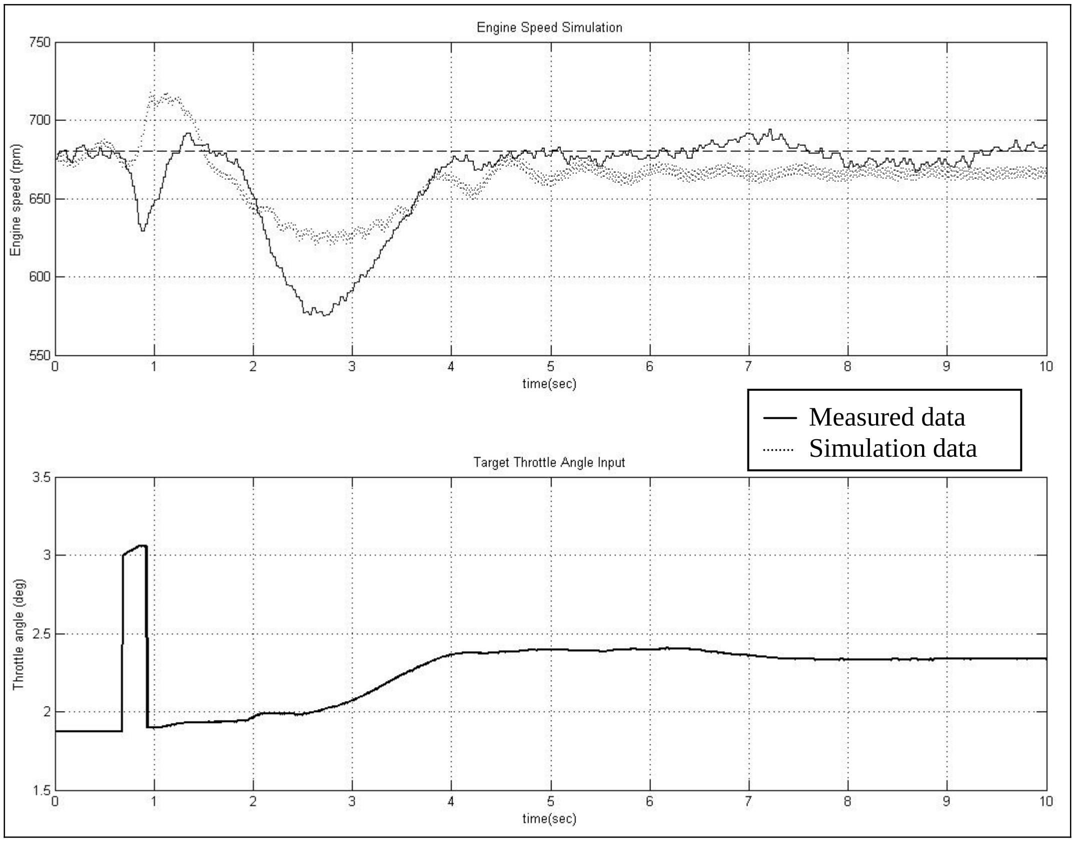

FIG.10 SIMULATION RESULT (ALT)

Next, the fig.11 shows the result with A/C load. The A/C load calculated here is already modified to compensate the delay from the throttle angle modification to the actual air amount change. Therefore, the A/C torque is overestimated during a few dozen ms from the addition of the A/C load. It causes the engine speed down of the simulation data.

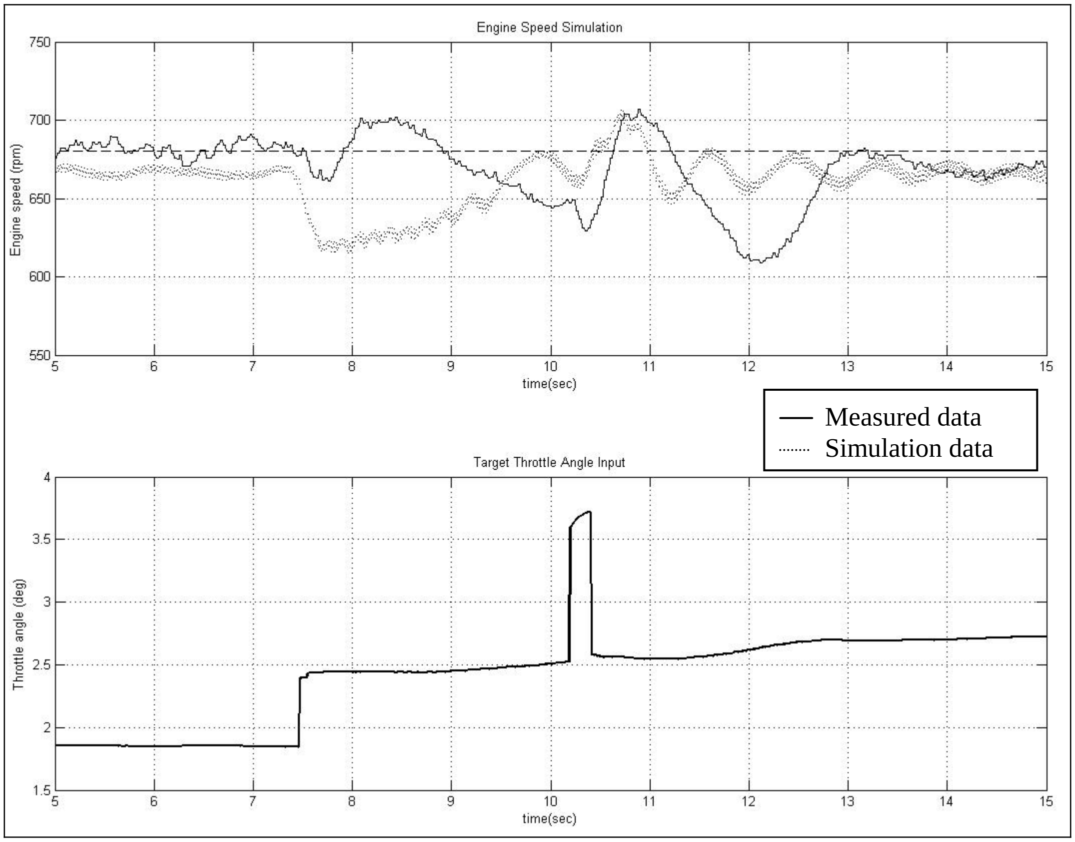

FIG.11 SIMULATION RESULT (A/C)

Finally, the fig.12 shows the result with P/S load. P/S load is very difficult to estimate because the load changes in accordance with the turning angle. When we turn full, P/S generates the maximum load, but when we turn half, P/S generates less. Since we have no way to estimate this amount, we supposed the P/S load from the measured data.

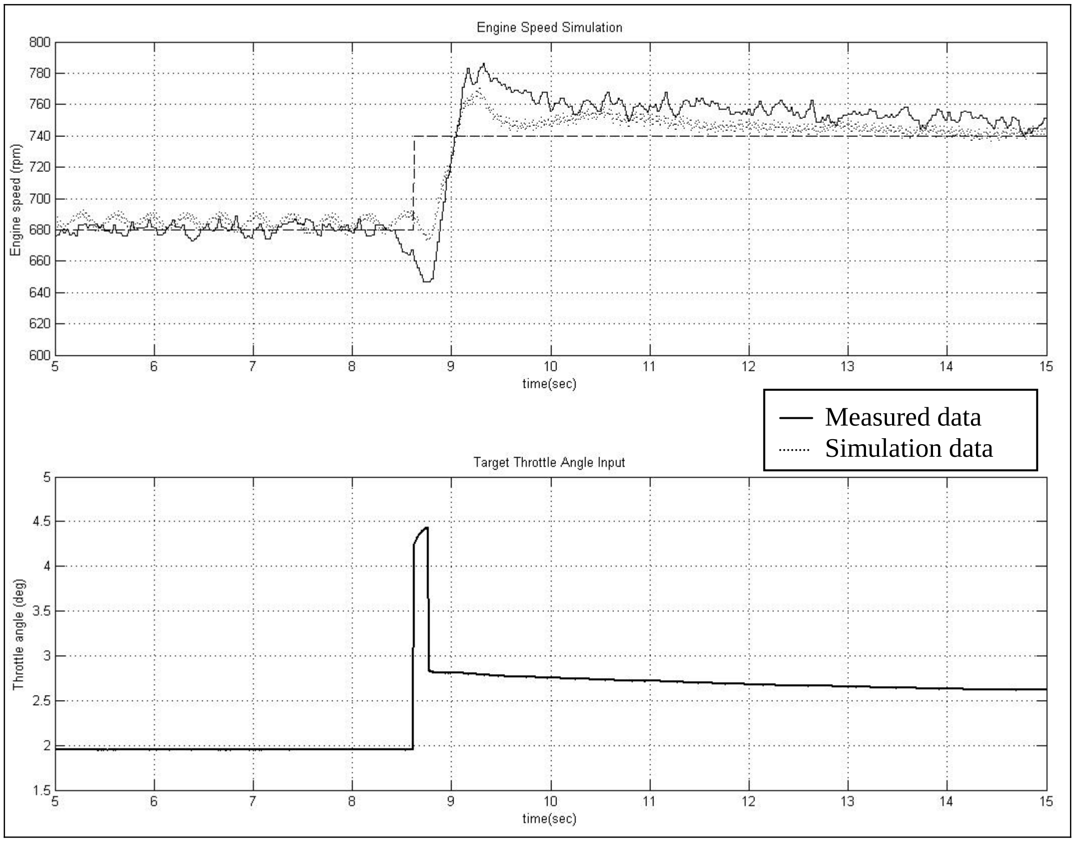

FIG.12 SIMULATION RESULT (P/S)

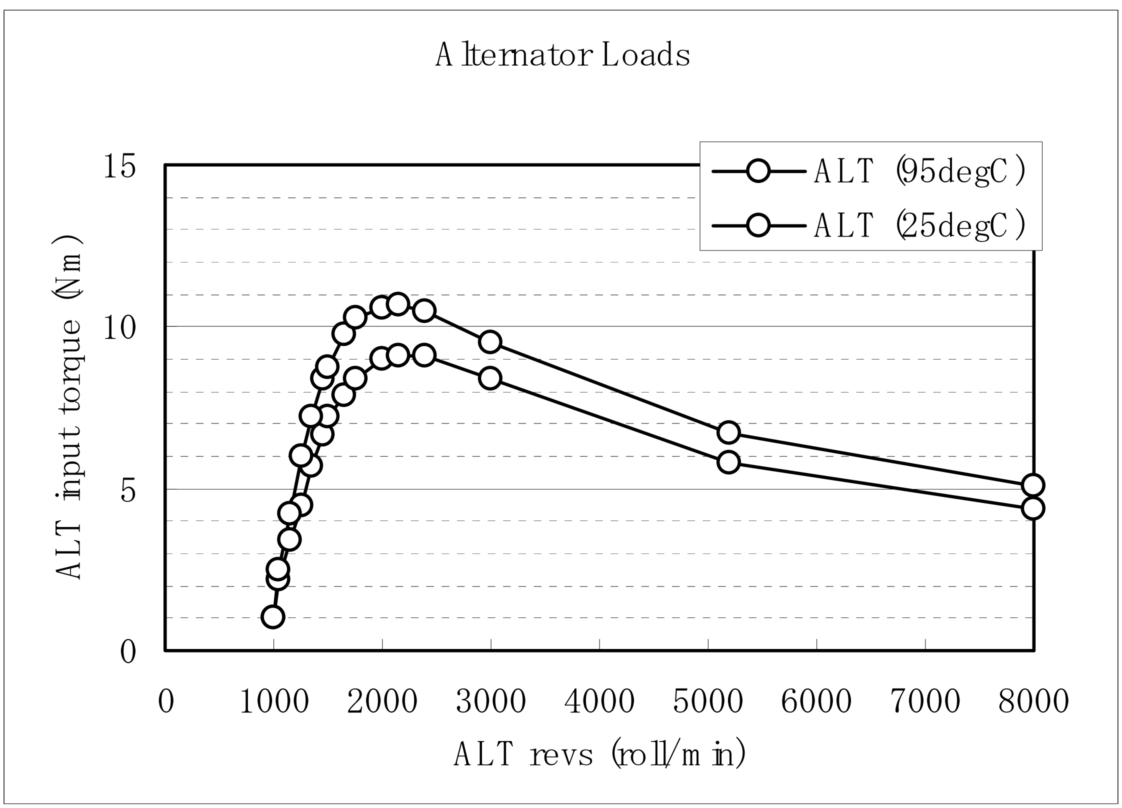

FIG.13 ALTERNATOR DISTURBANCE

We consider this model is enough to compare the superiority of the controller since the goal of the research is not to develop an engine plant model but to develop a better controller.

C. Fuel Injection Model

The fuel amount injected into the cylinder is calculated in this model. Fig.14 is the overall view of the model.

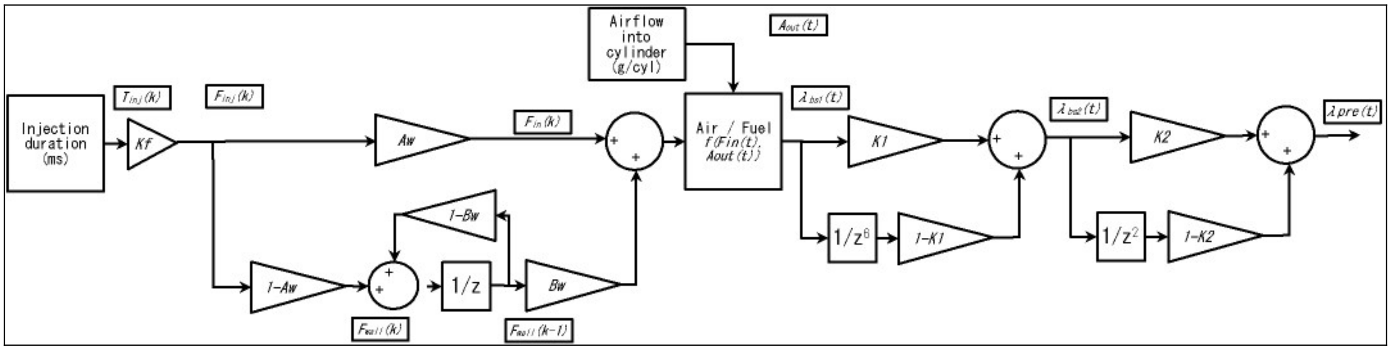

FIG.14 FUEL INJECTION MODEL

The input is the duration of the injection of each cylinder. We can compute the amount of the fuel injected from that duration.

$$F_{inj}(k) = f_{inj}(T_{inj}(k))$$

$T_{inj}(k)$ Injection duration (ms)

$F_{inj}(k)$ Injected fuel amount (g)

$f_{inj}(T_{inj}(k))$ A function calculating injected fuel amount

k means the TDC cycle of the cylinders. Then we introduce a wall-wetting effect. Some portion of the injected fuel doesn't enter the cylinder directly but adheres to the wall and valves around the injector. The fuel on the wall is absorbed into the cylinder within several steps. The equations below describe the phenomenon of wall wetting.

$$F_{in}(k) = a_{ww} \cdot F_{inj}(k) + b_{ww} \cdot F_{wall}(k - N_{cyl})$$

$F_{in}(k)$ Fuel amount sucked into the cylinder (g)

a_{ww} The rate of fuel mass directly absorbed into cylinder

b_{ww} The rate of fuel mass absorbed into cylinder from the cylinder wall

Similarly, the fuel on the wall can be obtained by

$$F_{wall}(k) = (1 - a_{ww}) \cdot F_{inj}(k) + (1 - b_{ww}) \cdot F_{wall}(k - N_{cyl})$$

$F_{wall}(k)$ Fuel amount on the wall (g)

We can calculate the amount of air into each cylinder from the airflow dynamics model; that is,

$$A_{out}(k) = A'_{out}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}}$$

Now we have both airflow and fuel amount, so we can calculate the lambda.

$$\lambda_{bs1}(k) = \frac{A_{out}(k)}{F_{in}(k) \cdot 14.5}$$

$\lambda_{bs1}(k)$ Lambda in the cylinder at each combustion cycle

14.5 Stoichiometric air/fuel ratio

The combustion gas change cycle effect, which is that the burned gas doesn't exhaust completely because there is a gap between the cylinder chamber ceiling and piston, is given by,

$$\lambda_{bs2}(k) = \frac{\alpha_{com} - 1}{\alpha_{com}} \lambda_{bs1}(k) + \frac{1}{\alpha_{com}} \lambda_{bs1}(k - N_{cyl})$$

α_{com} Combustion ratio

$\lambda_{bs2}(k)$ Lambda in the cylinder w/t in cylinder mixture effect

In this paper, we assume a V6 engine which has two bank and two air / fuel ratio sensors. Taking internal EGR effect into consideration, we get the calculated lambda:

$$\lambda_{pre,sim}(k) = \frac{f_{egr}(P_b(t)) - 1}{f_{egr}(P_b(t))}\lambda_{bs2}(k) + \frac{1}{f_{egr}(P_b(t))}\lambda_{bs2}(k-2)$$

$f_{egr}(P_b(t))$ Torque coefficient (Fig.6)

$\lambda_{pre,sim}(k)$ Lambda of the cylinder w/t internal EGR mixture

Fig.15 shows the result of the fuel injection model. The reaction of the lambda of the simulated data is faster than that of measured data.

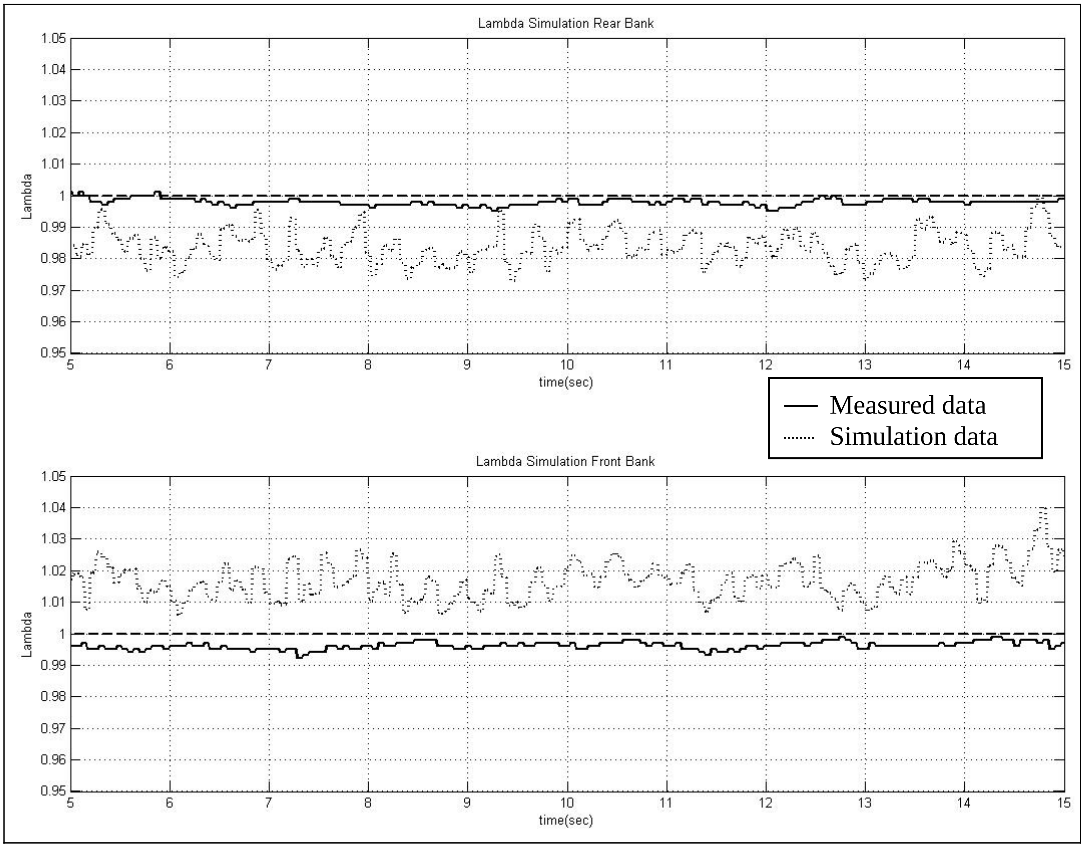

FIG.15 SIMULATION RESULT (A/F RATIO, N/L)

D. The Oxygen Storage Model: a catalytic converter model

It is essential to develop a catalytic converter model so that we can estimate the emission components. The dynamic model of a three-way catalyst has been conducted. [4] To begin with, we have calibrated the oxygen storage model of the catalyst and the emission estimation model. The catalytic converter has certain oxygen storage ability. Even if the pre-catalyst lambda is lean, the catalytic converter can absorb a certain amount of oxygen so that post-catalyst lambda will be at a stoichiometric. Conversely, if the pre-catalyst lambda is rich, yet the catalytic converter contains oxygen, the post-lambda will also stay at a stoichiometric because the catalytic converter releases oxygen. To estimate the post-catalyst lambda, we need to develop the oxygen storage model. According to [4], the oxygen storage model can be expressed as:

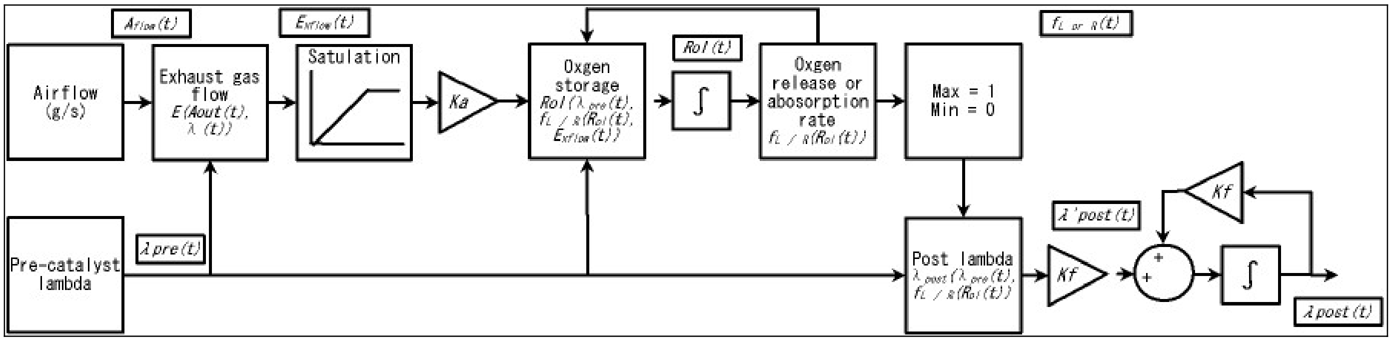

FIG.16 OXYGEN STORAGE MODEL

$$\frac{d}{dt}R_{ol}(t) = \frac{1}{C_{oxy}}0.21 \cdot sat(Ex_{flow}(t)) \cdot \left(\frac{\lambda_{pre}(t)-\lambda_{pre\_base}}{\lambda_{pre}(t)}\right) \cdot f_{LorR}(R_{ol}(t))$$

$$\lambda'_{pos}(t) = \lambda_{pre}(t) - f_{LorR}(R_{ol}(t)) \cdot (\lambda_{pre}(t) - 1)$$

$R_{ol}(t)$	Relative oxygen level (min=0,max=1)
C_{oxy}	Oxygen storage ability
$\lambda_{pre}(t)$	Pre-catalyst lambda
$\lambda_{pos}(t)$	Post-catalyst lambda
$Ex_{flow}(t)$	Exhaust gas flow (g/s)
$f_{LorR}(R_{ol}(t))$	A relative oxgen absorption or release function

Exhaust gas flow can be obtained from the airflow and the lambda.

$$Ex_{flow}(t) = A'_{out}(t) \cdot \left(1 + \frac{\lambda_{pre}(t)}{14.5}\right)$$

$A'_{out}(t)$ Air mass flow rate out from intake manifold (g/s)

We calibrated the parameters to get a better matching to the catalytic converter we use. Also, we added a low-pass filter to get better match to measured data.

$$10 \cdot \frac{d}{dt}\lambda_{pos}(t) = -\lambda_{pos}(t) + \lambda'_{pos}(t)$$

The following figure is the result of the oxygen storage model.

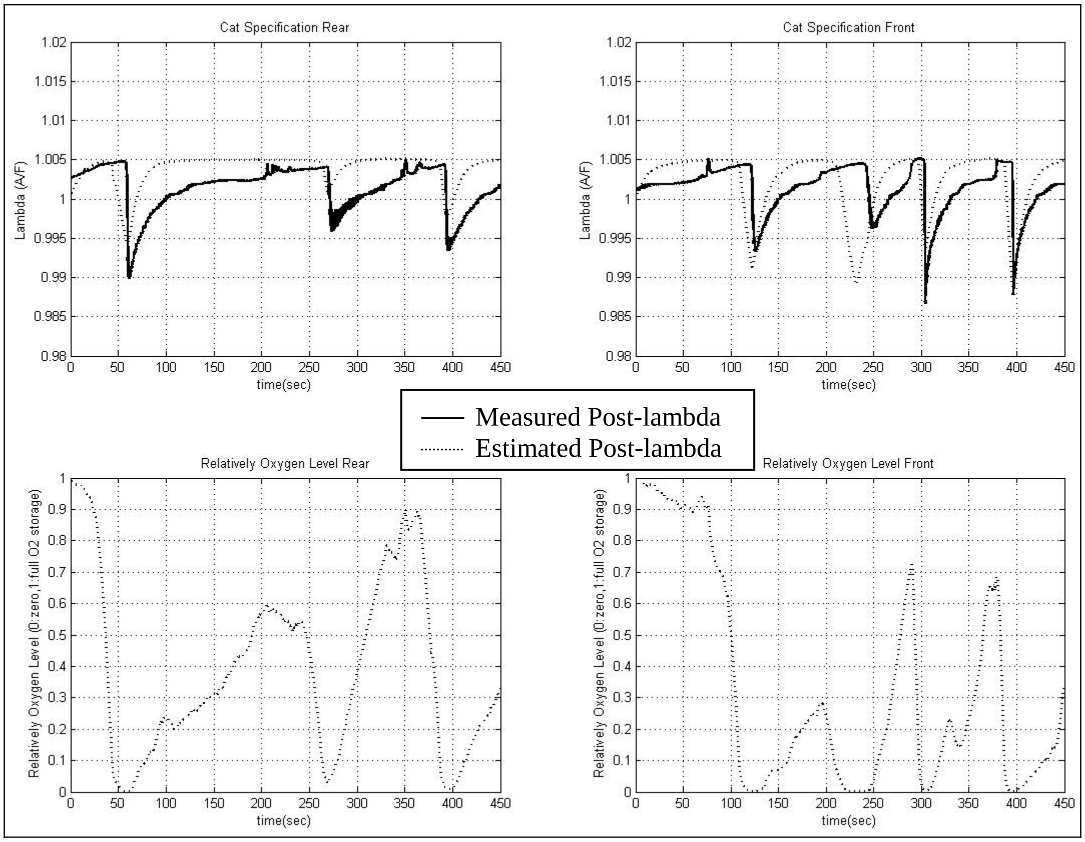

FIG.17 OXYGEN STORAGE MODEL

E. Feed-gas emissions and catalyst purification: a catalytic converter model

The pre-catalyst lambda decides the amount of feed-gas emissions, while the purification ratios are obtained from the post-catalyst lambda. Feed-gas emission is the emissions released from the cylinders.

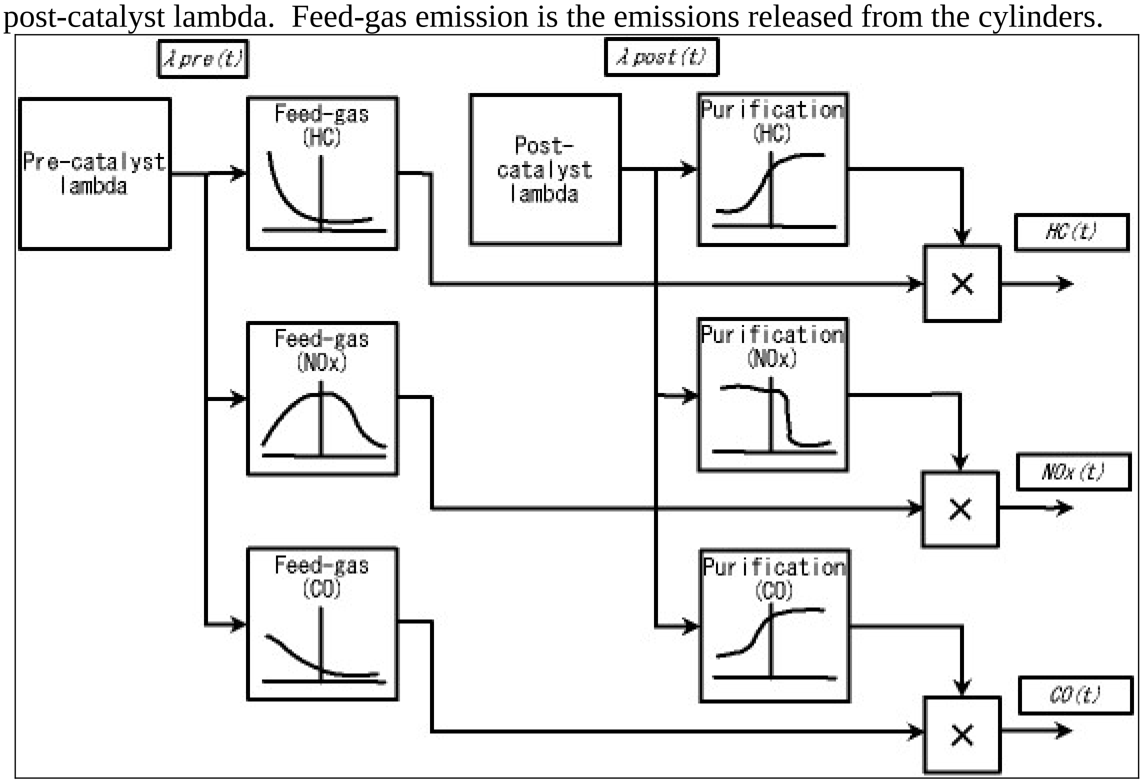

FIG.18 THE EMISSIONS ESTIMATION MODEL

Therefore, the emissions can be obtained by solving the equations:

$$HC(t) = f_{HC}(\lambda_{pre}(t)) \cdot p_{HC}(\lambda_{pos}(t)) \cdot Ex_{flow}(t)$$
$$NOx(t) = f_{NOx}(\lambda_{pre}(t)) \cdot p_{NOx}(\lambda_{pos}(t)) \cdot Ex_{flow}(t)$$
$$CO(t) = f_{CO}(\lambda_{pre}(t)) \cdot p_{CO}(\lambda_{pos}(t)) \cdot Ex_{flow}(t)$$

$HC(t)$ HC emission (g/s)
$NOx(t)$ NOx emission (g/s)
$CO(t)$ CO emission (g/s)
$f_{HC,NO,CO}$ Feedgas emissions (g/s)
$p_{HC,NO,CO}$ The purification ratios by the catalyst

By using measured results, we calibrate these functions. The result of each component is as follows.

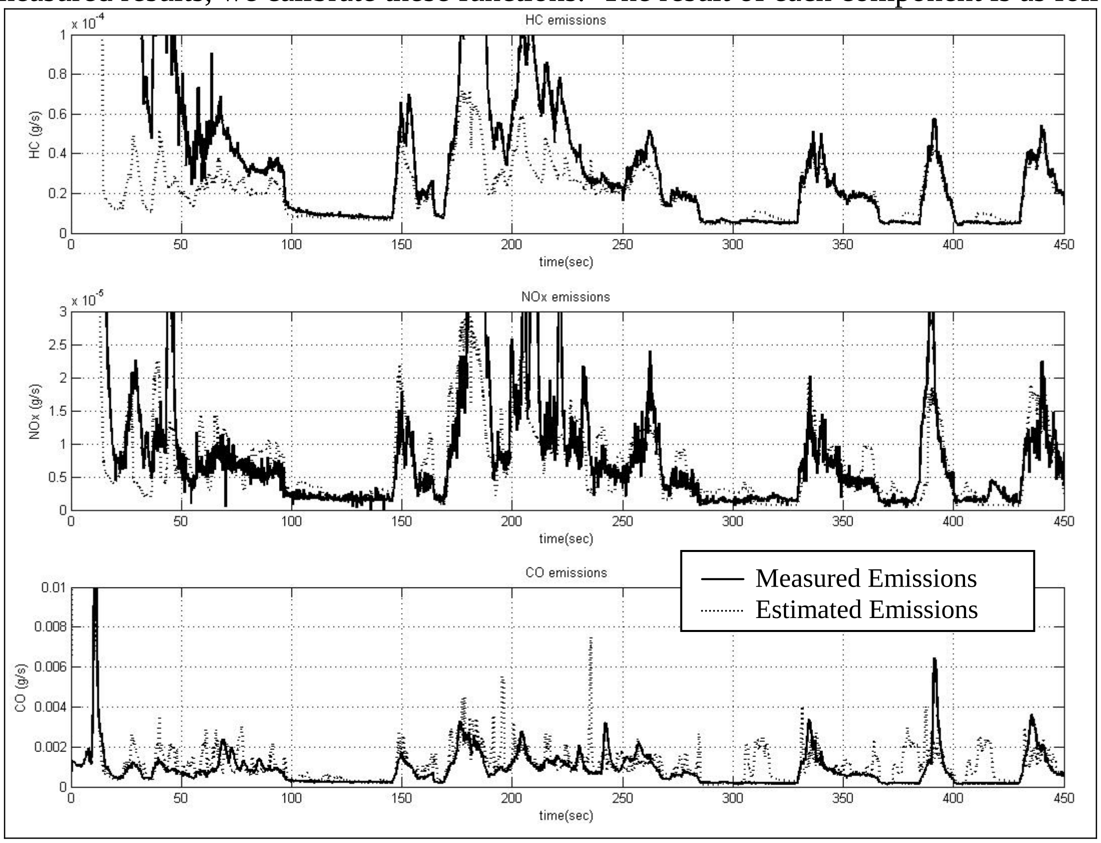

FIG.19-1 THE EMISSIONS ESTIMATION (0K MILE FRESH CATALYTIC CONVERTER)

This shows relatively good result. As the catalytic converter ages, the purification rate also changes. The Fig.19-2 is the result of 120K catalytic converter which is the most aged catalytic converter we should ensure the good emissions.

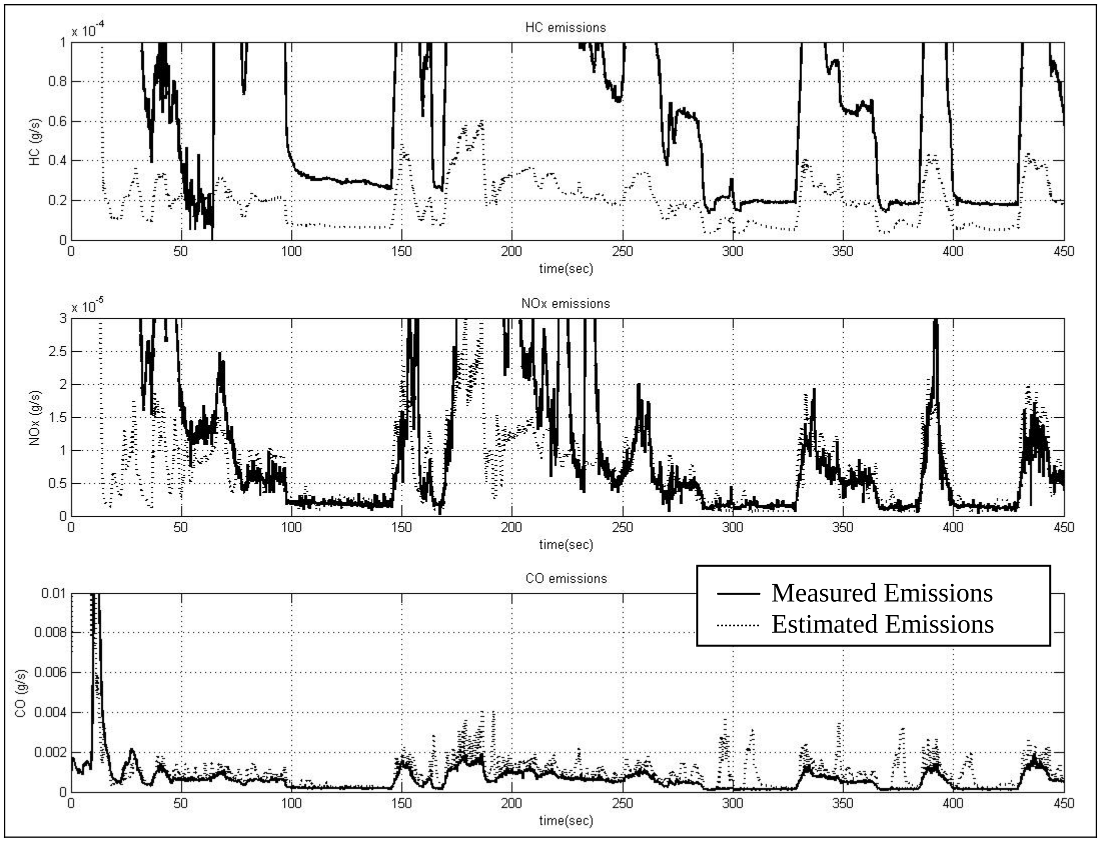

FIG.19-2 THE EMISSIONS ESTIMATION (120K MILE AGED CATALYTIC CONVERTER)

From this, we find that even if the purification ratio changes, the point that generates the emissions does not change. In other words, if we can reduce the emissions with certain catalytic converter, we can also reduce the emissions with the most aged catalyst. Also, the catalyst temperature affects the purification ratio: however, since we are assuming warmed-up condition, the catalytic converter temperature is above the light-off temperature, so we ignore the effects.

III. INTRODUCTION OF THE EXISTING HONDA CONTROLLERS

In this section, we introduce the existing Honda controllers of the engine speed and air and fuel ratio.

A. Engine Speed Controller

The engine speed controller consists of two separate controllers: the air amount PID controller and the ignition timing P controller.

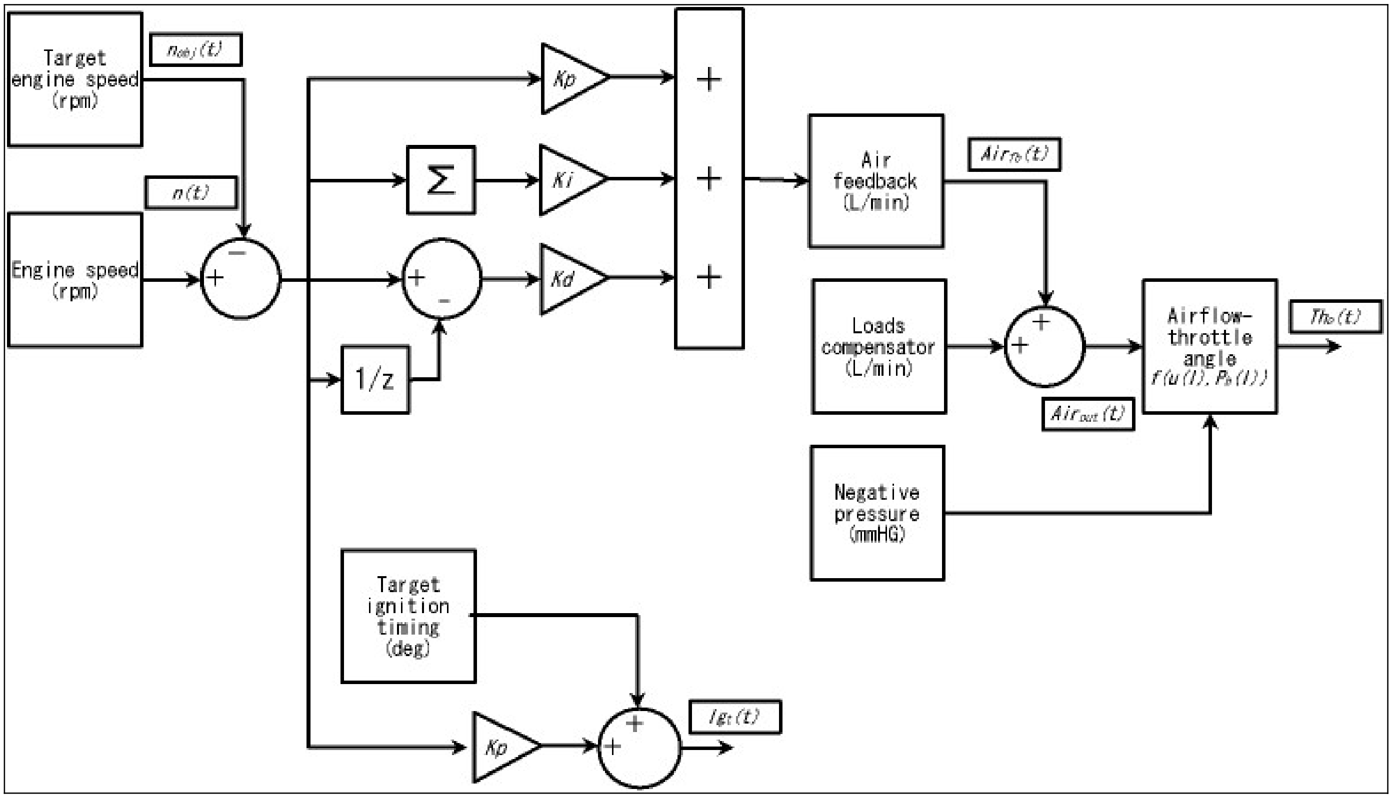

FIG.20 IDLE SPEED CONTROLLER

In addition, there is a feed forward compensator to minimize the disturbance caused by the ambient temperature and ambient pressure. The load variations are also compensated by adding a sufficient air amount. Since the controller is driven in fixed discrete time, we use l for each step.

$$\Delta n_{obj}(l) = n(l) - n_{tgt}(l)$$

$n(l)$ Engine speed (rpm)

$n_{tgt}(l)$ Target engine speed (rpm)

The total airflow feed back amount is calculated as

$$Air_{fb}(l) = Air_P(l) + Air_I(l) + Air_D(l)$$

$Air_{fb}(l)$ Air mass feedback flow rate (L/min)

Where, the airflow feed back controller is given by,

$$Air_P(l) = Kp_{air}\Delta n_{obj}(l)$$

$$Air_I(l) = Ki_{air}\Delta n_{obj}(l) + Air_I(l-1)$$

$$Air_D(l) = Kd_{air}(\Delta n_{obj}(l) - \Delta n_{obj}(l-1))$$

To compensate loads, a feed forward compensator is used. We omit the details here, but suppose we can calculate the compensated loads air amount, the total compensated airflow becomes

$$Air_{load}(l) = Air_{A/C}(l) + Air_{P/S}(l) + Air_{ALT}(l)$$

$Air_{load}(l)$ Air mass flow rate to compensate load disturbances (L/min)

Since the airflow varies slowly in comparison with throttle, we add an extra little pulse of air for short time. This air works to avoid dropping the engine speed. (See Fig.7-9) Now we can get the total air control output

$$Air_{total}(l) = Air_{fb}(l) + Air_{load}(l)$$

$Air_{total}(l)$ Total target air mass flow rate (L/min)

In conclusion, the target throttle angle can be attained.

$$Th_o(l) = f_{th}(Air_{total}(l))$$

$f_{th}(\cdot)$ A function to calculate target throttle angle from desired air flow

Meanwhile, we control the ignition timing at the same time. We use a P controller for the ignition-timing controller. The equation is given by

$$Ig_P(m) = Kp_{ig}\Delta n_{obj}(m) + Ig_{tgt}$$

Ig_{tgt} Ignition spark timing target during idle condition (deg)

In this equation, m means the TDC cycle. The effect of the ignition timing is faster than that of air feed back, yet restricted. We often use the ignition timing control instead of the D term of the airflow controller.

B. Air Fuel Ratio Controller

The air fuel ratio controller uses the self-tuning regulator algorithm. [2]

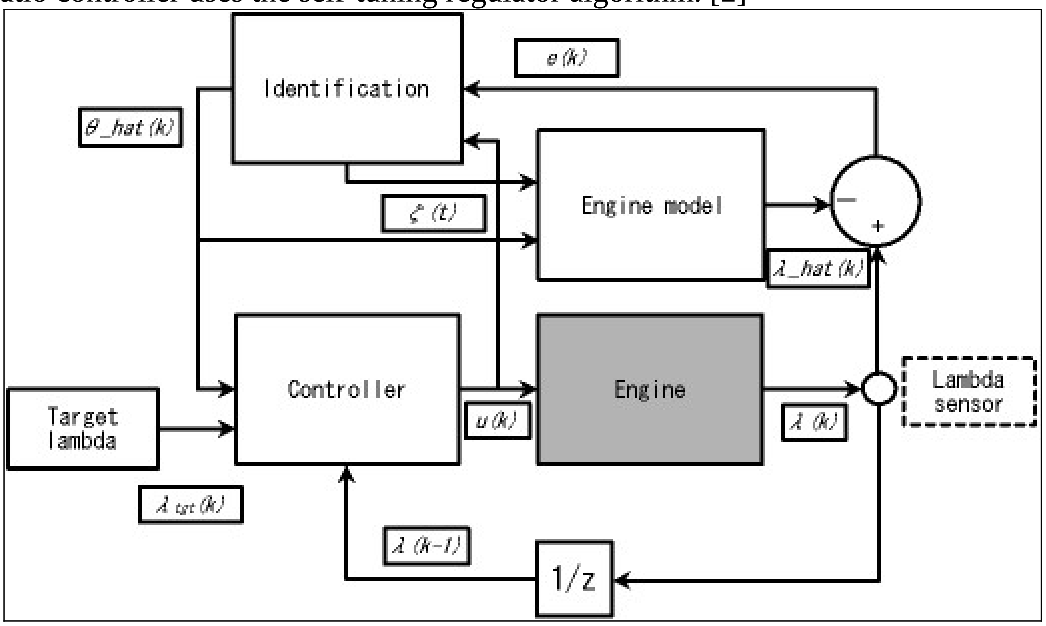

FIG.21 STR CONTROLLER

To make a controller, we create an approximate linear model, which can be written as

$$y(k) = b_0 u(k-d) + b_1 u(k-d-1) + a_1 y(k-1)$$

$u(k)$ The lambda correction input

$y(k)$ Measured lambda

Here k means each TDC step. Now we approximate d=4. Here we want to get the controller output, so by shifting 4 cycles ahead and 1 cycle back.

$$y(k+4) = b_0 u(k) + b_1 u(k-1) + a_1 y(k+3)$$
$$= b_0 u(k) + b_1 u(k-1) + a_1 (b_0 u(k-1) + b_1 u(k-2) + a_1 y(k+2)) \cdots$$

Finally, we get the following equation:

$$y(k+4) = b_0 u(k) + r_1 u(k-1) + r_2 u(k-2) + r_3 u(k-3) + r_4 u(k-4) + r_5 u(k-5) + s_0 y(k-1)$$

Suppose the controller works well; the output must be equal to the target lambda.

$$y(k+4) = \lambda_{tgt}(k)$$

$\lambda_{tgt}(k)$ Target A/F ratio

Therefore, we can obtain the controller equation with ignoring high order terms, k-4 and k-5

$$u(k) = \frac{1}{b_0}(\lambda_{tgt}(k) - r_1 u(k-1) - r_2 u(k-2) - r_3 u(k-3) - s_0 y(k-1))$$

Next, we identify the unknown parameters recursively. Ignoring the higher order terms, the model is described as

$$\hat{y}(k) = \hat{b}_0 u(k-4) + \hat{r}_1 u(k-5) + \hat{r}_2 u(k-6) + \hat{r}_3 u(k-7) + \hat{s}_0 y(k-4)$$

Changing into a matrix representation,

$$\hat{y}(k) = \hat{\theta}(k) \cdot \zeta(k)$$

Where,

$$\hat{\theta}(k) = \begin{bmatrix} \hat{b}_0(k) & \hat{r}_1(k) & \hat{r}_2(k) & \hat{r}_3(k) & \hat{s}_0(k) \end{bmatrix}$$

$$\zeta(k) = \begin{bmatrix} u(k) & u(k-1) & u(k-2) & u(k-3) & y(k) \end{bmatrix}$$

$\hat{\theta}(k)$ Unknow parameters vector

The error between the predicted output and measured output is defined as

$$e'(k) = y(k) - \hat{y}(k)$$

Introducing a forgetting factor, and applying to the recursive least square algorithms, the parameters are identified as

$$\theta(k) = \theta(k-1) + \gamma_{str} \zeta(k-4) e(k)$$

γ_{str} Forgetting factor

$$e(k) = \frac{e'(k)}{1 + \zeta(k-4)\gamma_{str}\zeta^T(k-4)}$$

To compensate for the higher order terms we neglected, we use the Sigma Correction method. Above all, we can get the fuel and air ratio controller such that,

$$u(k) = \frac{1}{\bar{b}_0}(\lambda(k) - \bar{r}_1 u(k-1) - \bar{r}_2 u(k-2) - \bar{r}_3 u(k-3) - \bar{s}_0 y(k-1))$$

$$\theta(k) = \sigma\theta(k-1) + \gamma_{str} \zeta(k-4) e(k)$$

$$\sigma = [1 \quad s \quad s \quad s \quad s] \cdots s = const. < 1$$

$$e(k) = \frac{e'(k)}{1 + \zeta(k-4)\gamma_{str}\zeta^T(k-4)}$$

In the controller, all parameters are averaged in order to avoid chattering.

C. The PRISM (PRediction and Identification type Sliding Mode) Controller

The PRISM controller has been developed to reduce emissions by stabilizing the post-catalyst lambda. This controller identifies the plant parameter recursively.

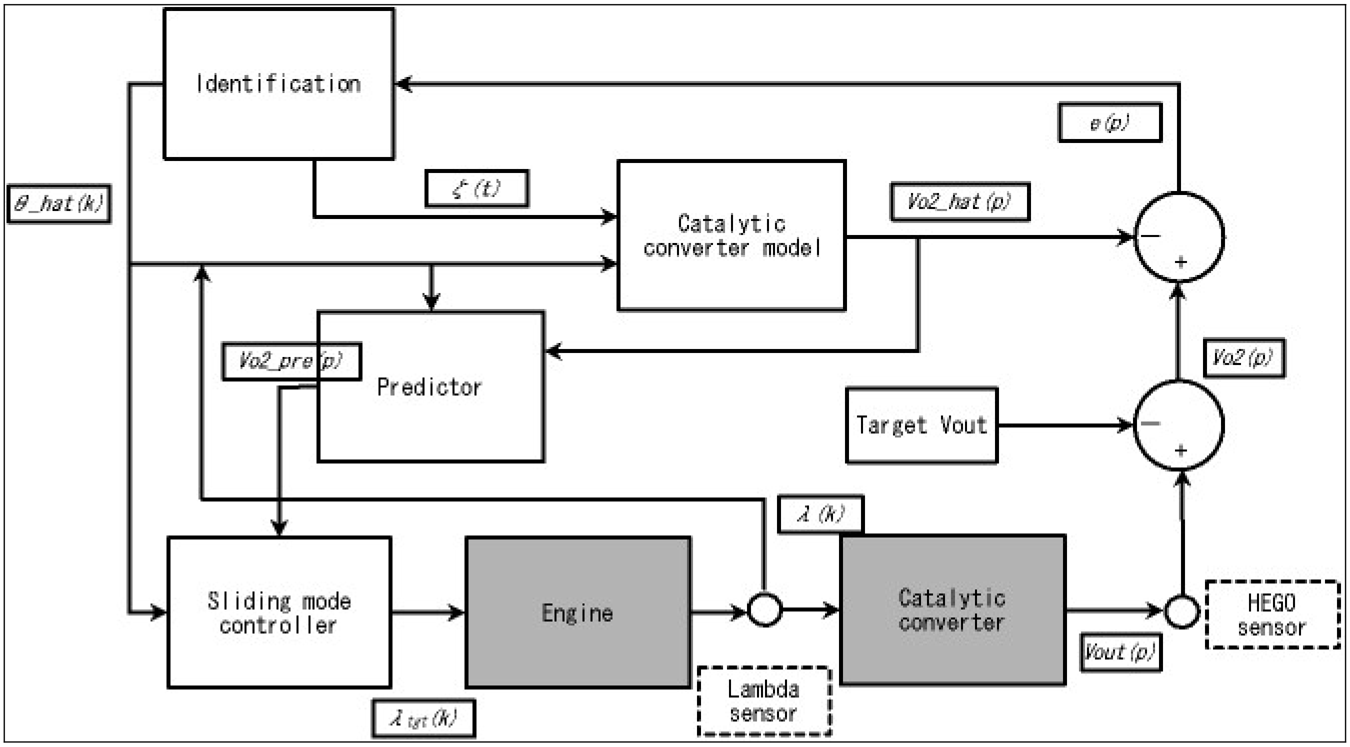

FIG.22 PRISM CONTROLLER

The catalytic converter model is as follows:

$$\delta VO_2(p+1) = a_1 \delta VO_2(p) + a_2 \delta VO_2(p-1) + b_1 \delta k_{act}(p-d_{cat})$$

Where,

$$\delta VO_2(p) = VO_2(p) - VO_2 target$$

$$\delta k_{act}(p) = \frac{1}{\lambda_{pre}(p)} - \frac{1}{\lambda\_base}$$

$VO_2(p)$	Secondary oxygen sensor voltage (V)
VO_2_target	Secondary oxygen sensor target voltage (V)
λ_base	The base lambda
d_{cat}	Catalyst plant delay
a_1, a_2, b_1	Catalytic converter model parameters

Applying for the recursive least square methods with a weighting factor,

$$\theta(p) = \begin{bmatrix} a_1(p) & a_2(p) & b_1(p) \end{bmatrix}$$

$$\zeta^T(p) = \begin{bmatrix} \delta VO_2(p) & \delta VO_2(p-1) & k_{act}(p-d_{cat}) \end{bmatrix}$$

$$\theta(p+1) = \theta(p) + KP(p) \cdot e_{id}(p)$$

$$e_{id}(p) = \delta VO_2(p) - \theta^T(p)\zeta(p-1)$$

$$KP(p) = \frac{P(p)\zeta(k-1)}{1+\zeta^T(k-1)P(p)\zeta(k-1)}$$

$$P(p+1) = \frac{1}{\lambda_{prism}}\left(I - \frac{P(p)\zeta(k-1)\zeta^T(k-1)}{\lambda_{prism} + \zeta^T(k-1)P(p)\zeta(k-1)}\right)P(p)$$

λ_{prism} Weighting factor

A predictor is used to compensate a delay from inputs to outputs. We can rewrite the catalytic converter model as,

$$\begin{bmatrix} VO_2(p+1) \\ VO_2(p) \end{bmatrix} = A \begin{bmatrix} VO_2(p) \\ VO_2(p-1) \end{bmatrix} + B \begin{bmatrix} k_{act}(p-d_{cat}) \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_1(p) & a_2(p) \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} b_1(p) \\ 0 \end{bmatrix}$$

Thus, the predicted secondary O2 sensors voltage is obtained as

$$\begin{bmatrix} VO_{2pre}(p) \\ VO_{2pre}(p-1) \end{bmatrix} = \begin{bmatrix} VO_2(p+d_{cat}) \\ VO_2(p+d_{cat}-1) \end{bmatrix}$$

$$\delta VO_{2\_pre}(p) = \alpha_1 \delta VO_2(p) + \alpha_2 \delta VO_2(p-1) + \sum_{i=1}^{d_{cat}} \beta_i \delta k_{act}(p-i)$$

$$\begin{bmatrix} \alpha_1(p) & \alpha_2(p) \\ * & * \end{bmatrix} = A^{d_{cat}}, \begin{bmatrix} \beta_i(p) \\ * \end{bmatrix} = A^{i-1}B$$

Honda applied the sliding mode controller in order to deal with the non-linearity they ignored. The basic idea of the sliding mode controller is to constrain the states on the ideal hyper plane (in this case, on the switching line) so as to restrict the states on the switching line. The switching function is given as:

$$\sigma_p(p) = \delta VO_{2\_pre}(p) + s_{pole} \delta VO_{2\_pre}(p-1)$$

Honda uses three inputs for the sliding mode controller, which are the equivalent input that controls the state to zero, the reaching input that constrains the state on the switching line, and the adaptive input that decay a chattering causes by delay or a model miss matching.

$$u_{sl}(p) = u_{eq}(p) + u_{rch}(p) + u_{adp}(p)$$

$$u_{eq}(p) = \frac{-1}{b_1}\{(a_1-1)+s_{pole}\}VO_{2\_pre}(p) + (a_2 - s_{pole})VO_{2\_pre}(p-1)$$

$$u_{rch}(p) = \frac{-F}{b_1}\sigma_p(p)$$

$$u_{adp}(p) = \frac{-G}{b_1}\sum_{j=1}^{p}\sigma_p(j)$$

$u_{sl}(p)$	Sliding mode controller input
$u_{eq}(p)$	Equivalent input
$u_{rch}(p)$	Reaching input
$u_{adp}(p)$	Adaptive input
F,G	Control gain

Finally the target lambda can be obtained as:

$$\lambda(p) = \frac{1}{u_{sl}(p)} + \lambda\_base$$

The following figures show the results of PRISM controller at idling condition.

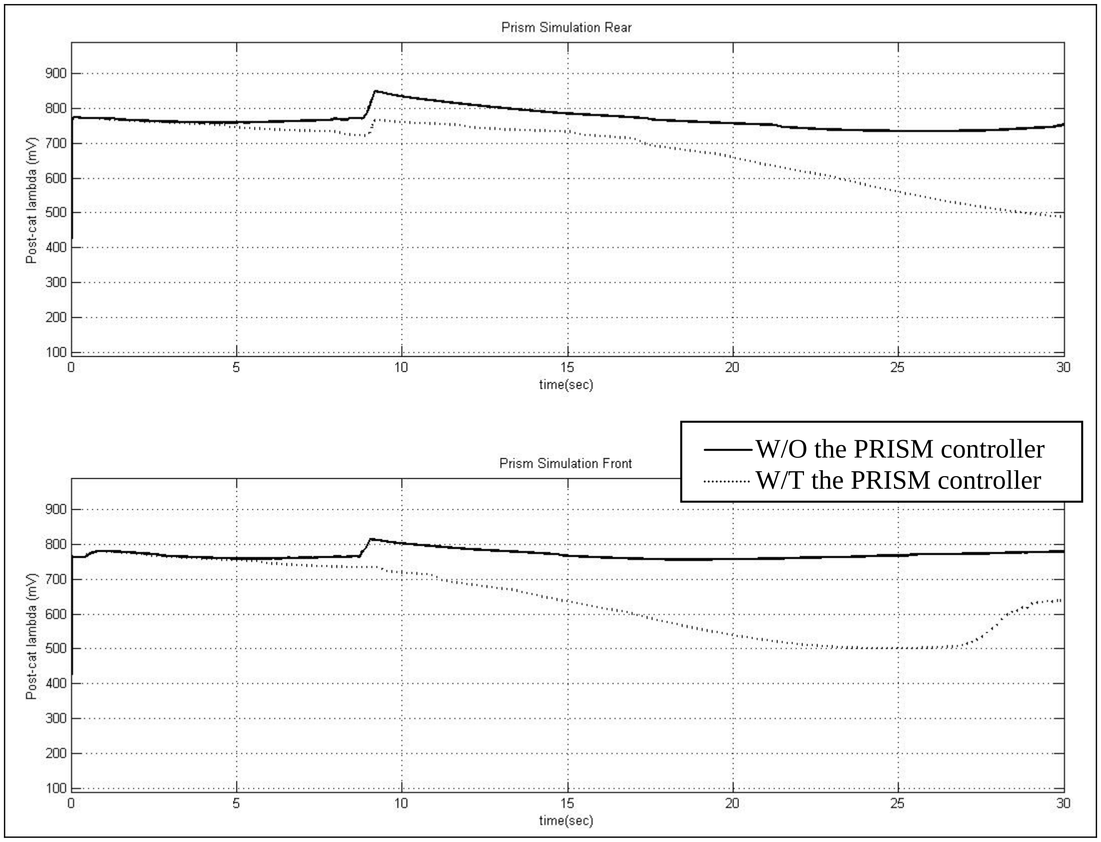

FIG.23-1 THE PRISM CONTROLLER (P/S, SECONDARY OXYGEN SENSOR VOLTAGE)

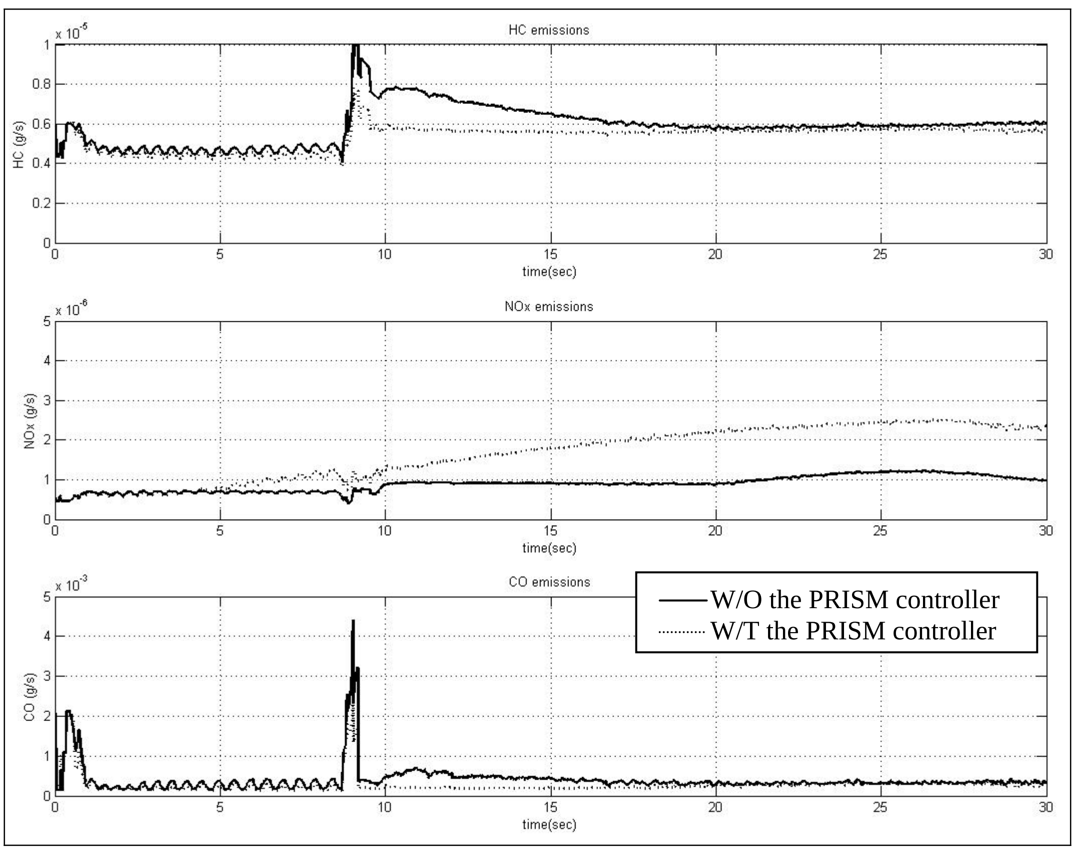

FIG.23-2 THE PRISM CONTROLLER (P/S, EMISSIONS)

There are little differences between with the PRISM controller and without. The PRISM controller is set to achieve low emissions at FTP-75. Basically, NOx and HC and CO emissions are trade-off relationship.

IV. ENGINE MODEL LINEARIZATION

By using the engine plant obtained previously, we create a linearized engine model. Since the engine model is a nonlinear model, we tried to linearize it at nominal points.

A. The Airflow Dynamics Linearization

Although the throttle actuator has a delay, the delay is small (0.03s) so that we ignore it. This is equal to set $t_{de} = 0$. When idling, we also don't have to care about a throttle response limitation.

$$Th_a(t) = Th_o(t - t_{de})\Big|_{\frac{dTh_a(t)}{dt} \le m} \approx Th_o(t)$$

$Th_o(t)$ Target throttle angle (deg)

$Th_a(t)$ Actual throttle angle (deg)

With assumption that we already obtained a nominal throttle angle, the throttle angle can be represented the nominal value and the error.

$$Th_o(t) = Th^*{}_o(t) + \delta Th_o(t)$$

$Th_o^*(t)$ Nominal target throttle angle (deg)

$\delta Th_o(t)$ Delta target throttle angle from the nominal point (deg)

Next, we approximate the specifications of throttle-airflow and negative gage pressure-airflow. Now we can estimate airflow rate as flowing equations.

$$A_{flow}(t) = f_{air}(Th_o(t)) \cdot g_{air}(P_a - P_b(t))$$

Where,

$$f_{air}(Th_o(t)) = 0.1548 Th_o^3(t) + 2.024 Th_o^2(t) + 64.087 Th_o(t) - 38.526 + Air_{leak}$$

$$g_{air}(P_a - P_b(t)) = -1.04552 \cdot 10^{-3} P_b(t) + 1.5856 = g'_{air}(P_b(t))$$

Air_{leak} Air leak through the throttle when closed (L/min)

Since $f_{air}(Th_o(t))$ can be linearized at the nominal throttle angle,

$$f_{air}(Th_o(t)) = f_{air}(Th_o^*(t) + \delta Th_o) = f_{air}(Th_o^*(t)) + \frac{\partial f_{air}(Th_o^*(t))}{\partial x} \delta Th_o$$

the nominal airflow and error are obtained as

$$A^*{}_{flow}(t) = f_{air}(Th_o^*(t)) \cdot g'_{air}(P_b(t))$$

$$\triangleq f_{aflow,nom}(Th_o^*(t), P_b(t))$$

$$\delta A_{flow}(t) = \frac{\partial f_{air}(Th_o^*(t))}{\partial x} \cdot g'_{air}(P_b(t)) \cdot \delta Th_o(t)$$

$$\triangleq f_{aflow}(Th_o^*(t), P_b(t)) \cdot \delta Th_o(t)$$

$A^*{}_{flow}(t)$ Nominal air mass flow rate through the throttle (l/min)

$\delta A_{flow}(t)$ Delta air mass flow rate (l/min)

Then we calculate the airflow amount into the manifold. On the nonlinear model, we added a Low Pass Filter to get better approximation of the actual airflow. Here, we also apply the LPF into the nominal model:

$$\left(\frac{d}{dt} + 3\right)(A^*{}_{in}(t) + \delta A_{in}(t)) = 3 \cdot \kappa_{air} \frac{273.15}{T_m(t)} \cdot \frac{A^*{}_{flow}(t) + \delta A_{flow}(t)}{60}$$

$A^*_{in}(t)$ Nominal air mass flow rate into the intake manifold (g/s)

$\delta A_{in}(t)$ Delta air mass flow rate (g/s)

Therefore, we attain the ordinal differential equations:

$$\frac{d}{dt}A^*_{in}(t) = -3A^*_{in}(t) + f_{air\_in,nom}(Th_o^*(t), P_b(t), T_m(t))$$

$$\frac{d}{dt}\delta A_{in}(t) = -3\delta A_{in}(t) + f_{air\_in}(Th_o^*(t), P_b(t), T_m(t)) \cdot \delta Th_o(t))$$

Where,

$$f_{air\_in,nom}(Th_o^*(t), P_b(t), T_m(t)) \square\ \kappa_{air}\frac{273.15 \cdot 3}{60} \cdot \frac{f_{aflow,nom}(Th_o^*(t), P_b(t))}{T_m(t)}$$

$$f_{air\_in}(Th_o^*(t), P_b(t), T_m(t)) \square\ \kappa_{air}\frac{273.15 \cdot 3}{60} \cdot \frac{f_{aflow}(Th_o^*(t), P_b(t))}{T_m(t)}$$

Finally we calculate the air mass flow rate into the cylinders. We ignore purge flow. Also, the conservation law term is quite small in comparison with the airflow through throttle, so we ignore this term as well.

$$A'_{out}(t) = A_{in}(t) + A_{purge}(t - t_{purge}) - \frac{\frac{dP_b(t)}{dt}}{R_{air} \cdot T_m} \cdot \frac{V_m}{760} \approx A_{in}(t)$$

$A'_{out}(t)$ Air mass flow rate with manifold pressure correction (g/s)

B. Nominal Model of Engine dynamics: airflow to torque to speed

In this model, we approximate the combustion torque of the engine. At first we change the units of airflow from {g/s) to {g/cyl).

$$A^*_{out}(t) = A'^*_{out}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}} = A^*_{in}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}}$$

$$\delta A_{out}(t) = \delta A'_{out}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}} = \delta A_{in}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}}$$

$A^*_{out}(t)$ Nominal air mass flow into the cylinder (g/cyl)

$\delta A_{out}(t)$ Delta air mass flow into the cylinder (g/cyl)

Similar to the model of engine dynamics, the combustion torque is obtained as

$$\bar{T}'_{comb}(t) = \bar{T}'^*_{comb}(t) + \delta\bar{T}'_{comb}(t) = \bar{T}'^*_{comb}(t) + k_a\delta A_{out}(t) + K_{ig}\delta Ig(t) + K_l\delta\lambda(t)$$

$$\delta A_{out}(t) = A_{out}(t) - A^*_{out}(t)$$

$$\delta Ig(t) = Ig(t) - Ig_{tgt}$$

$$\delta\lambda(t) = \lambda_{pre}(t) - \lambda_{stoich}$$

$\bar{T}'^*_{comb}(t)$ Nominal average combustion torque (Nm)

$\delta\bar{T}'_{comb}(t)$ Average combustion torque error (Nm)

$A_{out}(t)$ Air mass flow into the cylinder (g/cyl)

Ig_{tgt} Nominal ignition timing (deg)

λ_{stoich} Nominal air / fuel ratio

Separating the nominal value and delta term gives

$$\bar{T}'^{*}_{comb}(t) = k_a \cdot A^{*}_{in}(t) \cdot \frac{60}{n(t)} \cdot \frac{2}{N_{cyl}} + T_{comb,const}$$

$$\delta\bar{T}'_{comb}(t) \approx \frac{20 \cdot k_a}{n^{*}(t)} \delta A_{in}(t) + k_{ig} \cdot \delta Ig(t) + k_1 \cdot \delta\lambda(t)$$

Next, we take the internal EGR influence into account. From experimental result, $f_{EGR}(P_b(t))$ is given by,

$$f_{EGR}(P_b(t)) = -3.11742 \cdot 10^{-14} \cdot P_b^{\,6}(t) + 5.33881 \cdot 10^{-16} \cdot P_b^{\,5}(t) - 3.77043 \cdot 10^{-8} \cdot P_b^{\,4}(t) + $$
$$+1.4111 \cdot 10^{-5} \cdot P_b^{\,3}(t) - 2.97223 \cdot 10^{-3} \cdot P_b^{\,2}(t) + 3.37802 \cdot 10^{-1} \cdot P_b(t) - 15.445$$

$f_{EGR}(P_b(t))$ Torque coefficient

By using this, the linearized combustion torque is obtained as

$$\bar{T}^{*}_{comb}(t) = f_{EGR}(P_b(t)) \cdot \bar{T}'^{*}_{comb}(t)$$
$$= f_{comb}(P_b(t), n(t)^{*}) \cdot A^{*}_{in}(t) + f_{EGR}(P_b(t)) \cdot T_{comb,const}$$
$$\delta\bar{T}_{comb}(t) = f_{EGR}(P_b(t)) \cdot \frac{20 \cdot k_a \cdot \delta A_{in}(t)}{n(t)^{*}} + k_{ig} \cdot \delta Ig(t) + k_1 \cdot \delta\lambda(t)$$
$$= f_{comb}(P_b(t), n(t)^{*}) \cdot \delta A_{in}(t) + k_{ig} \cdot \delta Ig(t) + k_1 \cdot \delta\lambda(t)$$

Where,

$$f_{comb}(P_b(t), n(t)^{*}) \triangleq f_{EGR}(P_b(t)) \cdot \frac{20 \cdot k_a}{n(t)^{*}}$$

Next, we take the friction losses into account. The total of the friction is

$$\bar{T}_{fric}(t) = \bar{T}'_{fric}(t) + \bar{T}'_{pump}(t) + T_{fric,const}$$

$\bar{T}'_{fric}(t)$ Friction torque (Nm)

$\bar{T}'_{pump}(t)$ Pumping loss torque (Nm)

$T_{fric,const}$ The other friction loss torques (Nm)

These torques are approximated experimentally.

$$\bar{T}'_{fric}(t) = 2.49 \cdot 10^{-2} \cdot n(t) + T_{fric,const}$$
$$\bar{T}'_{pump}(t) = (-2.34779 \cdot 10^{-5} \cdot P_b(t) + 1.78432 \cdot 10^{-2}) \cdot n(t)$$

The same as the combustion torque, the friction torques can be separated into the nominal value and error, which are:

$$\bar{T}^{*}_{fric}(t) = (-2.34779 \cdot 10^{-5} \cdot P_b(t) + 4.27432 \cdot 10^{-2}) \cdot n^{*}(t) + T_{fric,const}$$
$$\delta\bar{T}_{fric}(t) = (-2.34779 \cdot 10^{-5} \cdot P_b(t) + 4.27432 \cdot 10^{-2}) \cdot \delta n(t) \triangleq f_{fric}(P_b(t)) \cdot \delta n(t)$$

$\bar{T}^{*}_{fric}(t)$ Nominal friction torque (Nm)

$\delta\bar{T}_{fric}(t)$ Delta friction torque (Nm)

The equation of the loads is the same as the engine plant model:

$$T_{load}(t) = T_{A/C}(t) + T_{P/S}(t) + T_{ALT}(t)$$

Finally, we obtain the crankshaft torque.

$$T^{*}_{crank}(t) = \bar{T}^{*}_{comb}(t) - \bar{T}^{*}_{fric}(t) - T^{*}_{load}(t)$$
$$\delta T_{crank}(t) = \delta\bar{T}_{comb}(t) - \delta\bar{T}_{fric}(t) - \delta T_{load}(t)$$
$$= f_{comb}(P_b(t), n(t)^{*}) \cdot \delta A_{in}(t) + k_{ig} \cdot \delta Ig(t) + k_1 \cdot \delta\lambda(t) - f_{fric}(P_b(t)) \cdot \delta n(t) - \delta T_{load}(t)$$

$$T^*_{crank}(t) \quad \text{Nominal crank torque (Nm)}$$

$$\delta T_{crank}(t) \quad \text{Delta crank torque error (Nm)}$$

Then we can calculate the engine speed from this crankshaft torque.

$$n(t) = \frac{60}{2\pi}\int_0^t \frac{T_{crank}(\sigma)}{J} d\sigma + n(0) = k_{inr} \cdot \int_0^t T_{crank}(\sigma) d\sigma + n(0)$$

$$k_{inr} = \frac{60}{2\pi J} \quad \text{Engine inertia (kgm}^2\text{)} \quad J = 3.65528$$

The nominal engine speed and error are describe as

$$n(t) = n^*(t) + \delta n(t)$$

$$n^*(t) \quad \text{Nominal engine speed (rpm)}$$

$$\delta n(t) \quad \text{Engine speed error (rpm)}$$

Taking derivative of both sides gives

$$\frac{d}{dt}\delta n(t) = \frac{d}{dt}n(t) - \frac{d}{dt}n^*(t) \approx \frac{d}{dt}n(t) = k_{inr} \cdot \delta T_{crank}(t)$$

$$= k_{inr}(-f_{fric}(P_b(t)) \cdot \delta n(t) + f_{comb}(P_b(t), n(t)^*) \cdot \delta A_{in}(t) + k_{ig} \cdot \delta Ig(t) + k_l \cdot \delta\lambda(t) - \delta T_{load}(t)$$

$$\frac{d}{dt}\delta A'_{in}(t) = -3\delta A_{in}(t) + f_{air\_in}(Th_o^*(t), P_b(t), T_m(t)) \cdot \delta Th_o(t)$$

Finally, we attain the linear state space equation such that:

$$\frac{d}{dt}\begin{bmatrix} \delta n(t) \\ \delta A_{in}(t) \end{bmatrix} = \begin{bmatrix} -k_{inr} \cdot f_{fric}(P_b(t)) & f_{comb}(P_b(t), n(t)^*) \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} \delta n(t) \\ \delta A_{in}(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & k_{inr} \cdot k_{ig} & k_{inr} \cdot k_l & -k_{inr} \\ f_{air\_in}(Th_o^*(t), P_b(t), T_m(t)) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta Th_o(t) \\ \delta Ig(t) \\ \delta\lambda(t) \\ \delta T_{load}(t) \end{bmatrix}$$

$$[\delta n(t)] = [1 \quad 0] \begin{bmatrix} \delta n(t) \\ \delta A_{in}(t) \end{bmatrix}$$

This is a linear state space equation for engine speed model. In the next chapter, We explain how to get these parameters.

C. Nominal Parameters Calculation Methods

It is essential to approximate the nominal throttle angle and torque. We introduce the way to attain these nominal points. At first, we show how we obtained the nominal torque. At the constant engine speed, the nominal torque must be zero because if it is not zero, the engine speed will change.

$$T^*_{crank}(t) = \bar{T}^*_{comb}(t) - \bar{T}^*_{fric}(t) - T^*_{load}(t) \to 0$$

If the load torque is added from outside, the combustion torque will increase to balance this, while the friction torque will not change unless the engine speed change.

$$\bar{T}^*_{comb}(t) = T^*_{load}(t) - \bar{T}^*_{fric}(t)$$

Thus, the combustion torque and load torque will change simultaneously. Now we try to get the nominal throttle angle. It is clear that if we can obtain the nominal airflow, we can estimate the nominal throttle angle. We calculate $f_{air}(Th_o^*(t))$ from nominal airflow rate:

$$f_{air}(Th_o^*(t)) = \frac{A^*_{flow}(t)}{g'_{air}(P_b(t))} = \frac{f_{aflow,nom}(Th_o^*(t), P_b(t))}{g'_{air}(P_b(t))}$$

Next, $f_{aflow,nom}(Th_o^*(t), P_b(t))$ also can be estimated as well:

$$f_{aflow,nom}(Th_o^*(t), P_b(t)) = \frac{f_{air\_in,nom}(Th_o^*(t), P_b(t), T_m(t))}{\kappa_{air}\dfrac{273.15\cdot 3}{60}} T_m(t)$$

$$= \frac{60\cdot T_m(t)}{\kappa_{air}\cdot 273.15\cdot 3}\left(\frac{d}{dt}A^*_{in}(t) + 3A^*_{in}(t)\right)$$

Where,

$$A^*_{in}(t) \approx A^*_{out}(t) = \frac{\bar{T}^*_{comb}(t)}{k_a}\frac{n(t)}{60}\cdot\frac{N_{cyl}}{2}$$

We can obtain the nominal combustion torque form the nominal friction torque and the nominal load torque:

$$\bar{T}^*_{comb}(t) = T^*_{load}(t) - \bar{T}^*_{fric}(t)$$

When the load changes, the nominal load torque also changes because there is a delay from a throttle movement to an increase of combustion torque. Then nominal throttle angle will change to balance to that load torque. After a while, the nominal throttle angle will increase or decrease, and finally, the nominal combustion torque will balance to the load torque. In this sense, this nominal torque and throttle angle estimator is like a feed-forward compensator of the nominal values.

D. Discrete Time Representation

In this section, we change the state-space equation we obtained previously into a discrete time state-space equation. The fuel control system must use the TDC cycles as one step because the fuel is injected at each TDC cycle. Therefore, we tried to get a discrete time model for idle speed system so as to control the engine speed and the lambda simultaneously. The TDC step is given by:

$$T_{TDC}(t) = \frac{60}{n(t)}\frac{2}{N_{CYL}}$$

$T_{TDC}(t)$ TDC step time (s)

TDC comes every two crank shaft rotations. By using Euler's method, the engine speed state space model can be rewritten as

$$\begin{bmatrix}\delta n(k+1)\\ \delta A_{in}(k+1)\end{bmatrix} = \begin{bmatrix}1 - T_{TDC}\cdot k_{inr}\cdot f_{fric}(P_b(t)) & T_{TDC}\cdot f_{comb}(P_b(t), n(t)^*)\\ 0 & 1-3\cdot T_{TDC}\end{bmatrix}\cdot\begin{bmatrix}\delta n(k)\\ \delta A_{in}(k)\end{bmatrix}$$

$$+\begin{bmatrix}0 & T_{TDC}\cdot k_{inr}\cdot k_{ig} & T_{TDC}\cdot k_{inr}\cdot k_l & -T_{TDC}\cdot k_{inr}\\ T_{TDC}\cdot f_{air\_in}(Th_o^*(t), P_b(t), T_m(t)) & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta Th_o(k)\\ \delta Ig(k)\\ \delta\lambda(k)\\ \delta T_{load}(k)\end{bmatrix}$$

$$[\delta n(k)] = [1 \quad 0]\begin{bmatrix}\delta n(k)\\ \delta A_{in}(k)\end{bmatrix}$$

E. Nominal Fuel Injection Model

Next we introduce a nominal fuel injection model. Fuel is injected every TDC cycle. Each cylinder has each combustion cycle so that we need to define the cylinder number. In this paper, we assume a V6 engine which has two bank and two air / fuel ratio sensors.

$$F_{inj}(k) = f_{inj}(T_{inj}(k))$$

$T_{inj}(k)$ Injection duration (ms)

$F_{inj}(k)$ Injected fuel amount (g)

$f_{inj}(T_{inj}(k))$ A function to calculate injected fuel amount

The effect of the wall wetting is taken into account. The fuel amount into the cylinder and on the wall is obtained as:

$$F_{in}(k) = a_{ww} \cdot F_{inj}(k) + b_{ww} \cdot F_{wall}(k-6)$$

$$F_{wall}(k) = (1-a_{ww}) \cdot F_{inj}(k) + (1-b_{ww}) \cdot F_{wall}(k-6)$$

$F_{inj}(k)$ Fuel amount into the cylinder (g/cyl)

$F_{wall}(k)$ Fuel amount on the wall (g/cyl)

By using z transform, the equation will become:

$$F_{in}(z) = a_{ww} \cdot k_{inj} \cdot T_{inj}(z) + b_{ww} \cdot F_{wall}(z) \cdot z^{-6}$$

$$F_{wall}(z) = (1-a_{ww}) \cdot F_{inj}(z) + (1-b_{ww}) \cdot F_{wall}(z) \cdot z^{-6}$$

By solving these, we can get transfer functions.

$$F_{in}(z) = \frac{a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}}{1-(1-b_{ww}) \cdot z^{-6}} \; f_{inj}(T_{inj}(z))$$

$$F_{wall}(z) = \frac{(1-a_{ww})}{1-(1-b_{ww}) \cdot z^{-6}} \; f_{inj}(T_{inj}(z))$$

We calculate the airflow in the nominal engine speed model, so the lambda in the cylinder at certain cycle is given by:

$$\lambda_{bs1}(k) = \frac{A_{in}(k)}{F_{in}(k) \cdot 14.5} \frac{60}{n(k)} \cdot \frac{2}{N_{cyl}}$$

Changing into z transform,

$$\lambda_{bs1}(z) = \frac{60 \cdot 2}{n(z) \cdot N_{cyl} \cdot 14.5} \frac{A_{in}(z)}{F_{in}(z)}$$

The gas mixture effect is taken into consideration.

$$\lambda_{bs2}(k) = \frac{\alpha_{com} - 1}{\alpha_{com}} \lambda_{bs1}(k) + \frac{1}{\alpha_{com}} \lambda_{bs1}(k-6)$$

α_{com} Combustion ratio

Similarly, the z transform of this equation is given as:

$$\lambda_{bs2}(z) = \frac{60 \cdot 2}{\alpha_{com} \cdot n(t) \cdot N_{cyl} \cdot 14.5} \frac{(\alpha_{com} - 1 + z^{-6}) A_{in}(z)}{F_{in}(z)}$$

Adding the effect of the internal EGR,

$$\lambda_{pre,sim}(k) = f_{EGR}(P_b(t)) \cdot \lambda_{bs2}(k) + (1 - f_{EGR}(P_b(t))) \cdot \lambda_{pre,sim}(k-2)$$

$\lambda_{pre,sim}(k)$ Lambda of the cylinder w/t internal EGR mixture

We obtain the discrete time transfer function:

$$\lambda_{pre,sim}(z) = (1 - f_{EGR}(P_b(t))) \cdot \lambda_{pre,sim}(z) \cdot z^{-2} + \frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n(t) \cdot N_{cyl} \cdot 14.5} \frac{(\alpha_{com} - 1 + z^{-6}) A_{in}(z)}{F_{in}(z)}$$

The next step is linearizing this equation. We suppose the nominal injection duration:

$$T_{inj}(k) = T_{inj}^{*}(k) + \delta T_{inj}(k)$$

$T_{inj}^*(k)$ Nominal injection duration (ms)

$\delta T_{inj}(k)$ Delta injection duration from the nominal point (ms)

This gives a nominal fuel amount into the cylinder.

$$F_{in}(z) = F_{in}^* + \delta F_{in}(z)$$
$$= \frac{a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}}{1-(1-b_{ww}) \cdot z^{-6}} (f_{inj}(T_{inj}^*(z)) + f_{inj}(\delta T_{inj}(z)))$$

$F^*{}_{in}(z)$ Nomianl fuel amount into the cylinder (g/cyl)

$\delta F_{in}(z)$ Delta fuel amount into the cylinder (g/cyl)

The nominal airflow is already calculated in the engine speed model.

$$A_{in}(z) = A_{in}^*(z) + \delta A_{in}(z)$$

We can linearize the equation as follows:

$$\frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n(t) \cdot N_{cyl} \cdot 14.5} \frac{(\alpha_{com} - 1 + z^{-6}) A_{in}(z)}{F_{in}(z)} =$$
$$= \frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n^* \cdot N_{cyl} \cdot 14.5} \{ \frac{(\alpha_{com} - 1 + z^{-6})(1-(1-b_{ww}) \cdot z^{-6})(A_{in}^*(z) + \delta A_{in}(z))}{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}) \cdot f_{inj}(T_{inj}^*(z))} -$$
$$- \frac{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6})(1-(1-b_{ww}) \cdot z^{-6}) f_{inj}(\delta T_{inj}(z)))}{((a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}) \cdot f_{inj}(T_{inj}^*(z)))^2} \}$$

By substituting this into $\lambda_{pre,sim}(z)$, the nominal lambda is obtained as

$$\lambda^*{}_{pre,sim}(z) = (1 - f_{EGR}(P_b(t))) \cdot \lambda^*{}_{pre,sim}(z) \cdot z^{-2} +$$
$$+ \frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n^* \cdot N_{cyl} \cdot 14.5} \frac{(\alpha_{com} - 1 + z^{-6})(1-(1-b_{ww}) \cdot z^{-6}) A_{in}^*(z)}{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}) \cdot f_{inj}(T_{inj}^*(z))}$$

$\lambda^*{}_{pre,sim}(k)$ Nominal pre-catalyst lambda

Since we know the target lambda, we can calculate the nominal injection duration.

$$f_{inj}(T_{inj}^*(z)) = \frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n^* \cdot N_{cyl} \cdot 14.5} \frac{(\alpha_{com} - 1 + z^{-6})(1-(1-b_{ww}) \cdot z^{-6}) A_{in}^*(z)}{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6})(1-(1- f_{EGR}(P_b(t))) \cdot z^{-2}) \cdot \lambda_{tgt}(z)}$$

We assume the target lambda is 1, the stoichiometric, here, so the equation becomes simpler.

$$f_{inj}(T_{inj}^*(z)) = \frac{60 \cdot 2}{\alpha_{com} \cdot n^* \cdot N_{cyl} \cdot 14.5 \cdot b_{ww} \cdot \lambda_{tgt}} (\alpha_{com} - 1 + z^{-6})(1-(1-b_{ww}) \cdot z^{-6}) A_{in}^*(z)$$

The delta from the nominal lambda is given as:

$$\delta\lambda_{pre,sim}(z) = (1 - f_{EGR}(P_b(t))) \cdot \delta\lambda_{pre,sim}(z) \cdot z^{-2} +$$
$$+ \frac{f_{EGR}(P_b(t)) \cdot 60 \cdot 2}{\alpha_{com} \cdot n^* \cdot N_{cyl} \cdot 14.5} (\frac{(\alpha_{com} - 1 + z^{-6})(1-(1-b_{ww}) \cdot z^{-6})}{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}) \cdot f_{inj}(T_{inj}^*(z))} \cdot \delta A_{in}(z) -$$
$$- \frac{(a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6})(1-(1-b_{ww}) \cdot z^{-6})}{((a_{ww} - (a_{ww} - b_{ww}) \cdot z^{-6}) \cdot f_{inj}(T_{inj}^*(z)))^2} \cdot f_{inj}(\delta T_{inj}(z)))$$

$\delta\lambda_{pre,sim}(z)$ Delta nominal pre-catalyst lambda

We define time-variant parameters.

$$k_{th}(P_b(t),T_{inj}^*(z)) = \frac{f_{EGR}(P_b(t))\cdot 60\cdot 2}{\alpha_{com}\cdot n^*\cdot N_{cyl}\cdot 14.5\cdot(a_{ww}-(a_{ww}-b_{ww})\cdot z^{-6})\cdot f_{inj}(T_{inj}^*(z))}$$

$$k_{inj}(P_b(t),T_{inj}^*(z)) = \frac{f_{EGR}(P_b(t))\cdot 60\cdot 2}{\alpha_{com}\cdot n^*\cdot N_{cyl}\cdot 14.5\cdot((a_{ww}-(a_{ww}-b_{ww})\cdot z^{-6})\cdot f_{inj}(T_{inj}^*(z)))^2}$$

Substituting these, the delta pre-catalyst lambda are attained.

$$\delta\lambda_{sim}(z) = k_{th}(P_b(t),T_{inj}^*(z))\frac{(\alpha_{com}-1+z^{-6})(1-(1-b_{ww})\cdot z^{-6})}{1-(1-f_{EGR}(P_b(t)))\cdot z^{-2}}\delta A_{in}(z)-$$

$$-k_{inj}(P_b(t),T_{inj}^*(z))\frac{1-(1-b_{ww})\cdot z^{-6}}{1-(1-f_{EGR}(P_b(t)))\cdot z^{-2}} f_{inj}(\delta T_{inj}(z))$$

Here, we introduce a lambda state.

$$\delta\lambda_{sim,state}(z) =$$

$$= k_{th}(P_b(t),T_{inj}^*(z))\frac{(\alpha_{com}-1)\cdot(1-f_{EGR}(P_b(t)))\cdot z^{-2}+(\alpha_{com}b_{ww}-\alpha_{com}-b_{ww}+2)\cdot z^{-6}-(1-b_{ww})\cdot z^{-12}}{1-(1-f_{EGR}(P_b(t)))\cdot z^{-2}}\delta A_{in}(z)-$$

$$-k_{inj}(P_b(t),T_{inj}^*(z))(\frac{(1-f_{EGR}(P_b(t)))\cdot z^{-2}-(1-b_{ww})\cdot z^{-6}}{1-(1-f_{EGR}(P_b(t)))\cdot z^{-2}} f_{inj}(\delta T_{inj}(z))$$

$\delta\lambda_{sim,state}(z)$ The lambda state

This state can be expressed in a different way such that:

$$\begin{aligned}
\delta\lambda_{sim,state}(k+1) &= (1-f_{EGR}(P_b(t)))\cdot\delta\lambda_{sim,state}(k-1)+\\
&+k_{th}(P_b(t),T_{inj}^*(k+1))\cdot(\alpha_{com}-1)\cdot(1-f_{EGR}(P_b(t)))\cdot\delta A_{in}(k-1)+\\
&+k_{th}(P_b(t),T_{inj}^*(k+1))\cdot(\alpha_{com}b_{ww}-\alpha_{com}-b_{ww}+2)\cdot\delta A_{in}(k-5)-\\
&-k_{th}(P_b(t),T_{inj}^*(k+1))\cdot(1-b_{ww})\cdot\delta A_{in}(k-11)-\\
&-k_{inj}(P_b(t),T_{inj}^*(k+1))\cdot(1-f_{EGR}(P_b(t)))\cdot f_{inj}(\delta T_{inj}(k-1))+\\
&+k_{inj}(P_b(t),T_{inj}^*(k+1))\cdot(1-b_{ww})\cdot f_{inj}(\delta T_{inj}(k-5))
\end{aligned}$$

Then the lambda will become,

$$\delta\lambda_{sim}(k) = \delta\lambda_{sim,state}(k)-k_{th}(P_b(t),T_{inj}^*(k))(\alpha_{com}-1)\cdot\delta A_{in}(k)+k_{inj}(P_b(t),T_{inj}^*(k))\cdot f_{inj}(\delta T_{inj}(k))$$

F. Nominal Catalytic Converter Model

In this chapter, we linearize the catalytic converter model. At first, we simplify the oxygen storage model.

$$\frac{d}{dt}R_{ol}(t) = \frac{1}{C_{oxy}}0.21\cdot sat(Ex_{flow}(t))\cdot(\frac{\lambda_{pre}(t)-\lambda_{pre\_base}}{\lambda_{pre}(t)})\cdot f_{LorR}(R_{ol}(t))$$

$R_{ol}(t)$ Relative oxygen level (min=0,max=1)

C_{oxy} Oxygen storage ability

$\lambda_{pre}(t)$ Pre-catalyst lambda

$\lambda_{pos}(t)$ Post-catalyst lambda

$Ex_{flow}(t)$ Exhaust gas flow (g/s)

$f_{LorR}(R_{ol}(t))$ A relative oxygen absorption or release function

Where,

$$Ex_{flow}(t) = A'_{out}(t)\cdot(1+\frac{\lambda_{pre}(t)}{14.5})$$

We don't have to think about the saturation since the airflow is very low when idling.

$$\frac{d}{dt}R_{ol}(t)=\frac{1}{C_{oxy}}0.21\cdot A'_{out}(t)\cdot(1+\frac{\lambda_{pre}(t)}{14.5})\cdot(\frac{\lambda_{pre}(t)-\lambda_{pre\_base}}{\lambda_{pre}(t)})\cdot f_{LorR}(R_{ol}(t))$$

$$=\frac{1}{C_{oxy}}0.21\cdot A'_{out}(t)\cdot(\frac{14.5-\lambda_{pre\_base}}{14.5}-\frac{1}{\lambda_{pre}(t)}+\frac{\lambda_{pre}(t)}{14.5})\cdot f_{LorR}(R_{ol}(t))$$

The airflow and pre-catalyst lambda are normalized and given by:

$$A'_{out}(t)\approx A_{in}(t)=A_{in}^{*}(t)+\delta A_{in}(t)$$

$$\lambda_{pre}(t)=\lambda_{pre\_base}+\delta\lambda_{pre}(t)$$

The relative oxygen absorption or release function is approximated as:

$$f_{LorR}(R_{ol}(t))\approx\begin{cases}\begin{cases}-\frac{1}{6}R_{ol}(t)+1 & \lambda_{pre}(t)>\lambda_{pre\_base}\ \&\ R_{ol}(t)<0.9\\ -8.5R_{ol}(t)+0.85 & \lambda_{pre}(t)>\lambda_{pre\_base}\ \&\ R_{ol}(t)\geq 0.9\end{cases}\\ \begin{cases}6.37R_{ol}(t) & \lambda_{pre}(t)<\lambda_{pre\_base}\ \&\ R_{ol}(t)<0.15\\ 0.706R_{ol}(t)+0.85 & \lambda_{pre}(t)<\lambda_{pre\_base}\ \&\ R_{ol}(t)<0.15\end{cases}\end{cases}$$

Therefore, the oxygen storage parameter is obtained as:

$$\frac{d}{dt}R_{ol}(t)=\frac{1}{C_{oxy}}0.21\cdot(A_{in}^{*}(t)+\delta A_{in}(t))\cdot(1-\frac{1}{\lambda_{pre\_base}}+(\frac{1}{\lambda_{pre\_base}}+\frac{1}{14.5})\delta\lambda_{pre}(t))\cdot f_{LorR}(R_{ol}(t))$$

$$=\frac{1}{C_{oxy}}0.21\cdot\{A_{in}^{*}(t)\cdot(1-\frac{1}{\lambda_{pre\_base}})\cdot f_{LorR}(R_{ol}(t))+$$

$$+(1-\frac{1}{\lambda_{pre\_base}})\cdot f_{LorR}(R_{ol}(t))\cdot\delta A_{in}(t)+$$

$$+A_{in}^{*}(t)\cdot(\frac{1}{\lambda_{pre\_base}}+\frac{1}{14.5})\cdot f_{LorR}(R_{ol}(t))\cdot\delta\lambda_{pre}(t))\}$$

Then the post-catalyst lambda is given by:

$$\frac{d}{dt}\lambda_{pos}(t)=-\frac{1}{10}\lambda_{pos}(t)+\frac{1}{10}\lambda'_{pos}(t)$$

$$=-\frac{1}{10}\lambda_{pos}(t)+\frac{1}{10}(\lambda_{pre}(t)-f_{LorR}(R_{ol}(t))\cdot(\lambda_{pre}(t)-1))$$

The normalized valuables are obtained by:

$$\frac{d}{dt}\delta\lambda_{pos}(t)=-\frac{1}{10}\delta\lambda_{pos}(t)+\frac{1}{10}(1-f_{LorR}(R_{ol}(t)))\cdot\delta\lambda_{pre}(t)$$

The feed-gas emissions are approximated as the following equations.

$$f_{HC}(\lambda_{pre}(t))\approx\begin{cases}-4.7125\cdot10^{-4}\cdot(\lambda_{pre}(t)-1.0345)+1.65\cdot10^{-4} & \lambda_{pre}(t)\leq 1.0345\\ 1.65\cdot10^{-4} & \lambda_{pre}(t)\geq 1.0345\end{cases}$$

$$f_{NOx}(\lambda_{pre}(t))\approx\begin{cases}7.5521\cdot10^{-5}\cdot(\lambda_{pre}(t)-0.98276)+1.776\cdot10^{-5} & \lambda_{pre}(t)\leq 0.98276\\ -7.5521\cdot10^{-5}\cdot(\lambda_{pre}(t)-0.98276)+1.776\cdot10^{-5} & \lambda_{pre}(t)\geq 0.98276\end{cases}$$

$$f_{CO}(\lambda_{pre}(t))\approx\begin{cases}-1.3292\cdot10^{-1}\cdot(\lambda_{pre}(t)-1)+1.25\cdot10^{-3} & \lambda_{pre}(t)\leq 1\\ -3.625\cdot10^{-3}\cdot(\lambda_{pre}(t)-1)+1.25\cdot10^{-3} & \lambda_{pre}(t)\geq 1\end{cases}$$

Also, the purification ratios are approximated as follows:

$$p_{HC}(\lambda_{pos}(t)) \approx \begin{cases} -6.9601 \cdot (\lambda_{post}(t) - 0.96552) + 0.02 & \lambda_{post}(t) \le 0.96552 \\ -1.45 \cdot (\lambda_{post}(t) - 0.96552) + 0.02 & \lambda_{post}(t) \ge 0.96552 \end{cases}$$

$$p_{NOx}(\lambda_{pos}(t)) \approx \begin{cases} 6.7383 \cdot 10^{-1} \cdot (\lambda_{post}(t) - 1.0138) + 0.08 & \lambda_{post}(t) \le 1.0138 \\ 1.1237 \cdot 10^{1} \cdot (\lambda_{post}(t) - 1.0138) + 0.08 & \lambda_{post}(t) \ge 1.0138 \end{cases}$$

$$p_{CO}(\lambda_{pos}(t)) \approx \begin{cases} -5.51 \cdot (\lambda_{post}(t) - 1) + 0.03 & \lambda_{post}(t) \le 1 \\ -3.77 \cdot 10^{-1} \cdot (\lambda_{post}(t) - 1) + 0.03 & \lambda_{post}(t) \ge 1 \end{cases}$$

The linearized emissions are given by the following equation (we show the HC emission, for example)

$$HC(t) = f_{HC}(\lambda_{pre}(t)) \cdot p_{HC}(\lambda_{pos}(t)) \cdot Ex_{flow}(t)$$

$$HC(t) = HC^* + \delta HC(t)$$

HC^*	Nominal HC emission (g/s)
$\delta HC(t)$	Delta HC emission (g/s)

$$\begin{aligned} HC(t) &= HC^* + \delta HC(t) \\ &= (a_{HC} \cdot \delta\lambda_{pre}(t) + cf_{HC}) \cdot (b_{HC} \cdot \delta\lambda_{pos}(t) + cp_{HC}) \cdot (A_{in}^*(t) + \delta A_{in}(t)) \cdot (1 + \frac{\lambda_{pre\_base} + \delta\lambda_{pre}(t)}{14.5}) \\ &= cf_{HC} \cdot cp_{HC} \cdot A_{in}^*(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) + \\ &+ cp_{HC} \cdot A_{in}^*(t) \cdot (a_{HC}(1 + \frac{\lambda_{pre\_base}}{14.5}) + \frac{cf_{HC}}{14.5}) \cdot \delta\lambda_{pre}(t) + \\ &+ cf_{HC} \cdot b_{HC} \cdot A_{in}^*(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta\lambda_{pos}(t) + \\ &+ cf_{HC} \cdot cp_{HC} \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta A_{in}(t) \end{aligned}$$

$$HC^* = cf_{HC} \cdot cp_{HC} \cdot A_{in}^*(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5})$$

$$\begin{aligned} \delta HC(t) &= cp_{HC} \cdot A_{in}^*(t) \cdot (a_{HC}(1 + \frac{\lambda_{pre\_base}}{14.5}) + \frac{cf_{HC}}{14.5}) \cdot \delta\lambda_{pre}(t) + \\ &+ cf_{HC} \cdot b_{HC} \cdot A_{in}^*(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta\lambda_{pos}(t) + \\ &+ cf_{HC} \cdot cp_{HC} \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta A_{in}(t) \end{aligned}$$

Similarly, we can estimate all emission components. Finally, we get the state-space equations for the emissions such that:

$$\delta\lambda_{pos}(k+1) \approx (1 + T_{TDC} \cdot \frac{1}{10}) \cdot \delta\lambda_{pos}(k-1) + T_{TDC} \cdot \frac{1}{10}(1 - f_{LorR}(R_{ol}(t))) \cdot \delta\lambda_{pre}(k)$$

$$\delta HC(k) = cp_{HC} \cdot A_{in}^{*}(t) \cdot (a_{HC}(1 + \frac{\lambda_{pre\_base}}{14.5}) + \frac{cf_{HC}}{14.5}) \cdot \delta\lambda_{pre}(t) +$$
$$+ cf_{HC} \cdot b_{HC} \cdot A_{in}^{*}(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta\lambda_{pos}(t) +$$
$$+ cf_{HC} \cdot cp_{HC} \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta A_{in}(t)$$

$$\delta NOx(k) = cp_{NOx} \cdot A_{in}^{*}(t) \cdot (a_{NOx}(1 + \frac{\lambda_{pre\_base}}{14.5}) + \frac{cf_{NOx}}{14.5}) \cdot \delta\lambda_{pre}(t) +$$
$$+ cf_{NOx} \cdot b_{NOx} \cdot A_{in}^{*}(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta\lambda_{pos}(t) +$$
$$+ cf_{NOx} \cdot cp_{NOx} \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta A_{in}(t)$$

$$\delta CO(k) = cp_{CO} \cdot A_{in}^{*}(t) \cdot (a_{CO}(1 + \frac{\lambda_{pre\_base}}{14.5}) + \frac{cf_{CO}}{14.5}) \cdot \delta\lambda_{pre}(t) +$$
$$+ cf_{CO} \cdot b_{CO} \cdot A_{in}^{*}(t) \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta\lambda_{pos}(t) +$$
$$+ cf_{CO} \cdot cp_{CO} \cdot (1 + \frac{\lambda_{pre\_base}}{14.5}) \cdot \delta A_{in}(t)$$

Combined with the state-space equations we obtained before, the total state-space equation will be:

$$x(k+1) = A(k)x(k) + B(k)u(k)$$
$$y(k) = C(k)x(k)$$

$$x(k) = \begin{bmatrix} \delta n(k) \\ \delta A_{in}(k) \\ \delta A_{in}(k-1) \\ \delta A_{in}(k-2) \\ \delta A_{in}(k-3) \\ \delta A_{in}(k-4) \\ \delta A_{in}(k-5) \\ \delta A_{in}(k-6) \\ \delta A_{in}(k-7) \\ \delta A_{in}(k-8) \\ \delta A_{in}(k-9) \\ \delta A_{in}(k-10) \\ \delta A_{in}(k-11) \\ \delta\lambda_{sim,state}(k) \\ \delta\lambda_{sim,state}(k-1) \\ f_{inj}(\delta T_{inj}(k)) \\ f_{inj}(\delta T_{inj}(k-1)) \\ f_{inj}(\delta T_{inj}(k-2)) \\ f_{inj}(\delta T_{inj}(k-3)) \\ f_{inj}(\delta T_{inj}(k-4)) \\ f_{inj}(\delta T_{inj}(k-5)) \\ \delta\lambda_{pos}(k) \\ \delta\lambda_{pos}(k-1) \end{bmatrix} \quad u(k) = \begin{bmatrix} \delta Th_0(k) \\ \delta Ig(k) \\ \delta T_{load}(k) \\ f_{inj}(\delta T_{inj}(k)) \end{bmatrix} \quad y(k) = \begin{bmatrix} \delta n(k) \\ \delta\lambda_{pre}(k) \\ \delta\lambda_{post}(k) \\ f_{inj}(\delta T_{inj}(k)) \\ \delta HC(k) \\ \delta NOx(k) \\ \delta CO(k) \end{bmatrix}^{T}$$

$$
A(k)=\begin{bmatrix}
a_{1,1} & a_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1,14} & 0 & a_{1,16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{14,3} & 0 & 0 & 0 & a_{14,7} & 0 & 0 & 0 & 0 & 0 & a_{14,13} & 0 & a_{14,15} & 0 & a_{14,17} & 0 & 0 & 0 & a_{14,21} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & a_{22,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22,15} & 0 & a_{22,17} & 0 & 0 & 0 & 0 & 0 & a_{22,23} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

$$a_{1,1}=1-T_{TDC}\cdot k_{inr}\cdot f_{fric}(P_b(t))$$

$$a_{1,2}=T_{TDC}\cdot f_{comb}(P_b(t),n(t)^*)+k_{th}(P_b(t),T_{inj}^*(k))\cdot(\alpha_{com}-1)\cdot T_{TDC}\cdot k_{inr}\cdot k_1$$

$$a_{1,14}=T_{TDC}\cdot k_{inr}\cdot k_1$$

$$a_{1,16}=-T_{TDC}\cdot k_{inr}\cdot k_1\cdot k_{th}(P_b(t),T_{inj}^*(k))$$

$$a_{2,2}=1-3\cdot T_{TDC}$$

$$a_{14,3}=k_{th}(P_b(t),T_{inj}^*(k))\cdot(\alpha_{com}-1)\cdot(1-f_{EGR}(P_b(t)))$$

$$a_{14,7}=k_{th}(P_b(t),T_{inj}^*(k))\cdot(\alpha_{com}b_{ww}-\alpha_{com}-b_{ww}+2)$$

$$a_{14,13}=-k_{th}(P_b(t),T_{inj}^*(k))\cdot(1-b_{ww})$$

$$a_{14,15}=(1-f_{EGR}(P_b(t)))$$

$$a_{14,17}=-k_{inj}(P_b(t),T_{inj}^*(k))\cdot(1-f_{EGR}(P_b(t)))$$

$$a_{14,21}=k_{inj}(P_b(t),T_{inj}^*(k))\cdot(1-b_{ww})$$

$$a_{22,3}=\frac{T_{TDC}}{10}(1-f_{LorR}(R_{ol}(t)))\cdot k_{th}(P_b(t),T_{inj}^*(k))(\alpha_{com}-1)$$

$$a_{22,15}=\frac{T_{TDC}}{10}(1-f_{LorR}(R_{ol}(t)))$$

$$a_{22,17}=-\frac{T_{TDC}}{10}(1-f_{LorR}(R_{ol}(t)))\cdot k_{inj}(P_b(t),T_{inj}^*(k))$$

$$a_{22,23}=1-T_{TDC}\cdot\frac{1}{10}$$

$$
B(k) = \begin{bmatrix}
0 & b_{1,2} & b_{1,3} & 0 \\
b_{2,1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

$$b_{1,2} = T_{TDC} \cdot k_{inr} \cdot k_{ig}$$

$$b_{1,3} = -T_{TDC} \cdot k_{inr}$$

$$b_{2,1} = T_{TDC} \cdot f_{air\_in}(Th_o^*(t), P_b(t), T_m(t))$$

$$C(k)=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&c_{2,2}&0&0&0&0&0&0&0&0&0&0&0&1&0&c_{2,16}&0&0&0&0&0&0&0\\0&0&c_{3,3}&0&0&0&0&0&0&0&0&0&0&0&1&0&c_{3,17}&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\0&c_{6,2}&c_{6,3}&0&0&0&0&0&0&0&0&0&0&c_{6,14}&c_{6,15}&c_{6,16}&c_{6,17}&0&0&0&0&c_{6,22}&c_{6,23}\\0&c_{7,2}&c_{7,3}&0&0&0&0&0&0&0&0&0&0&c_{7,14}&c_{7,15}&c_{7,16}&c_{7,17}&0&0&0&0&c_{7,22}&c_{7,23}\\0&c_{8,2}&c_{8,3}&0&0&0&0&0&0&0&0&0&0&c_{8,14}&c_{8,15}&c_{8,16}&c_{8,17}&0&0&0&0&c_{8,22}&c_{8,23}\end{bmatrix}$$

$$c_{2,2}(k)=k_{th}(P_b(t),T_{inj}^{*}(k+1))(\alpha_{com}-1)$$
$$c_{2,16}(k)=-k_{inj}(P_b(t),T_{inj}^{*}(k+1))$$
$$c_{3,3}(k)=c_{2,2}(k-1)$$
$$c_{3,17}(k)=c_{2,16}(k-1)$$
$$c_{6,2}(k)=c_{6,3}(k-1)$$
$$c_{7,2}(k)=c_{7,3}(k-1)$$
$$c_{8,2}(k)=c_{8,3}(k-1)$$
$$c_{6,14}(k)=c_{6,15}(k-1)$$
$$c_{7,14}(k)=c_{7,15}(k-1)$$
$$c_{8,14}(k)=c_{8,15}(k-1)$$
$$c_{6,16}(k)=c_{6,17}(k-1)$$
$$c_{7,16}(k)=c_{7,17}(k-1)$$
$$c_{8,16}(k)=c_{8,17}(k-1)$$
$$c_{6,22}(k)=c_{6,23}(k-1)$$
$$c_{7,22}(k)=c_{7,23}(k-1)$$
$$c_{8,22}(k)=c_{8,23}(k-1)$$

$$c_{6,3}=cp_{HC}\cdot A_{in}^{*}(k)\cdot(a_{HC}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{HC}\cdot\frac{1}{14.5})\cdot k_{th}(P_b(t),T_{inj}^{*}(k))(\alpha_{com}-1)+cf_{HC}\cdot cp_{HC}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$
$$c_{6,15}=cp_{HC}\cdot A_{in}^{*}(k)\cdot(a_{HC}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{HC}\cdot\frac{1}{14.5})$$
$$c_{6,17}=-cp_{HC}\cdot A_{in}^{*}(k)\cdot(a_{HC}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{HC}\cdot\frac{1}{14.5})\cdot k_{inj}(P_b(t),T_{inj}^{*}(k+1))$$
$$c_{6,23}=cf_{HC}\cdot b_{HC}\cdot A_{in}^{*}(k)\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$
$$c_{7,3}=cp_{NOx}\cdot A_{in}^{*}(k)\cdot(a_{NOx}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{NOx}\cdot\frac{1}{14.5})\cdot k_{th}(P_b(t),T_{inj}^{*}(k+1))(\alpha_{com}-1)+cf_{NOx}\cdot cp_{NOx}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$
$$c_{7,15}=cp_{NOx}\cdot A_{in}^{*}(k)\cdot(a_{HC}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{NOx}\cdot\frac{1}{14.5})$$
$$c_{7,17}=-cp_{NOx}\cdot A_{in}^{*}(k)\cdot(a_{NOx}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{NOx}\cdot\frac{1}{14.5})\cdot k_{inj}(P_b(t),T_{inj}^{*}(k+1))$$
$$c_{7,23}=cf_{NOx}\cdot b_{NOx}\cdot A_{in}^{*}(k)\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$
$$c_{8,3}=cp_{CO}\cdot A_{in}^{*}(k)\cdot(a_{CO}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{CO}\cdot\frac{1}{14.5})\cdot k_{th}(P_b(t),T_{inj}^{*}(k+1))(\alpha_{com}-1)+cf_{CO}\cdot cp_{CO}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$
$$c_{8,15}=cp_{CO}\cdot A_{in}^{*}(k)\cdot(a_{CO}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{CO}\cdot\frac{1}{14.5})$$
$$c_{8,17}=-cp_{CO}\cdot A_{in}^{*}(k)\cdot(a_{CO}\cdot(1+\frac{\lambda_{pre\_base}}{14.5})+cf_{CO}\cdot\frac{1}{14.5})\cdot k_{inj}(P_b(t),T_{inj}^{*}(k+1))$$
$$c_{8,23}=cf_{CO}\cdot b_{CO}\cdot A_{in}^{*}(k)\cdot(1+\frac{\lambda_{pre\_base}}{14.5})$$

G. Nominal Model Results

The Fig.24 shows the result of the nominal model. Fig.24-1 is the result of the engine speed nominalization, Fig.24-2 is the pre-catalyst lambda, and Fig.24-3 is the post-catalyst lambda.

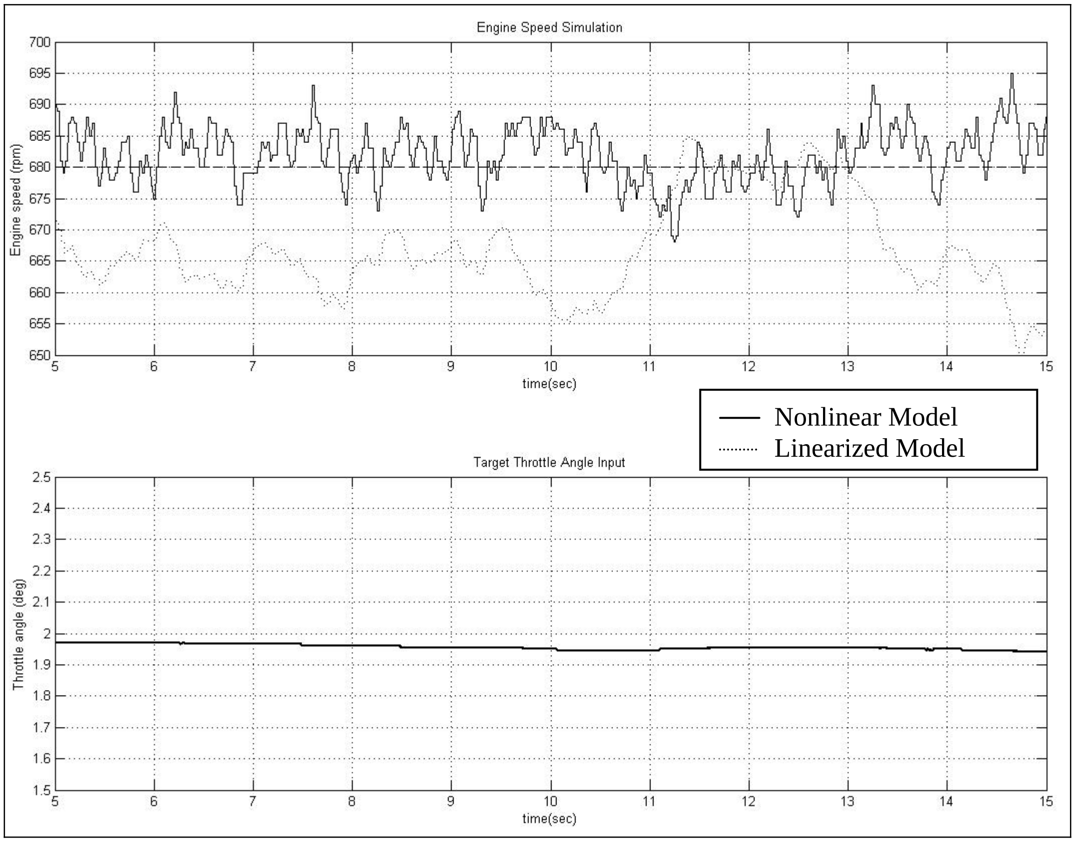

FIG.24-1 NOMINAL MODEL SIMULATION RESULT (N/L)

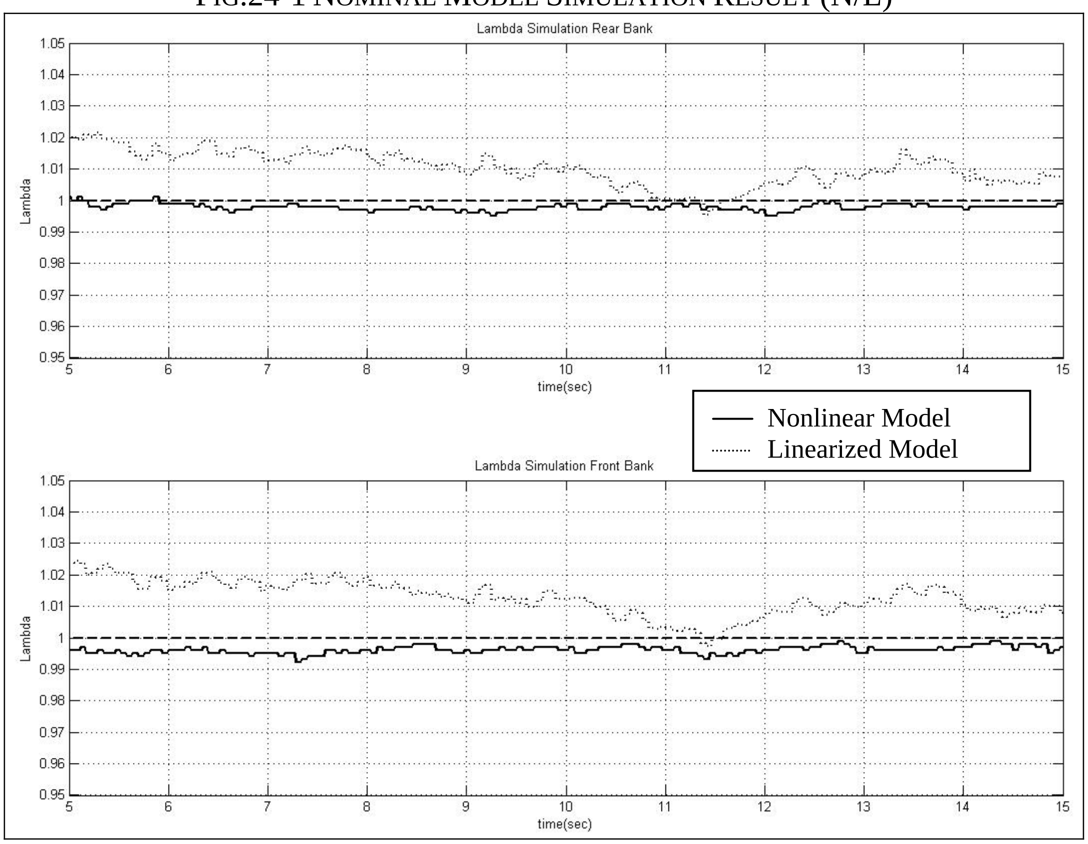

FIG.24-2 NOMINAL MODEL SIMULATION RESULT (PRE-CAT LAMBDA)

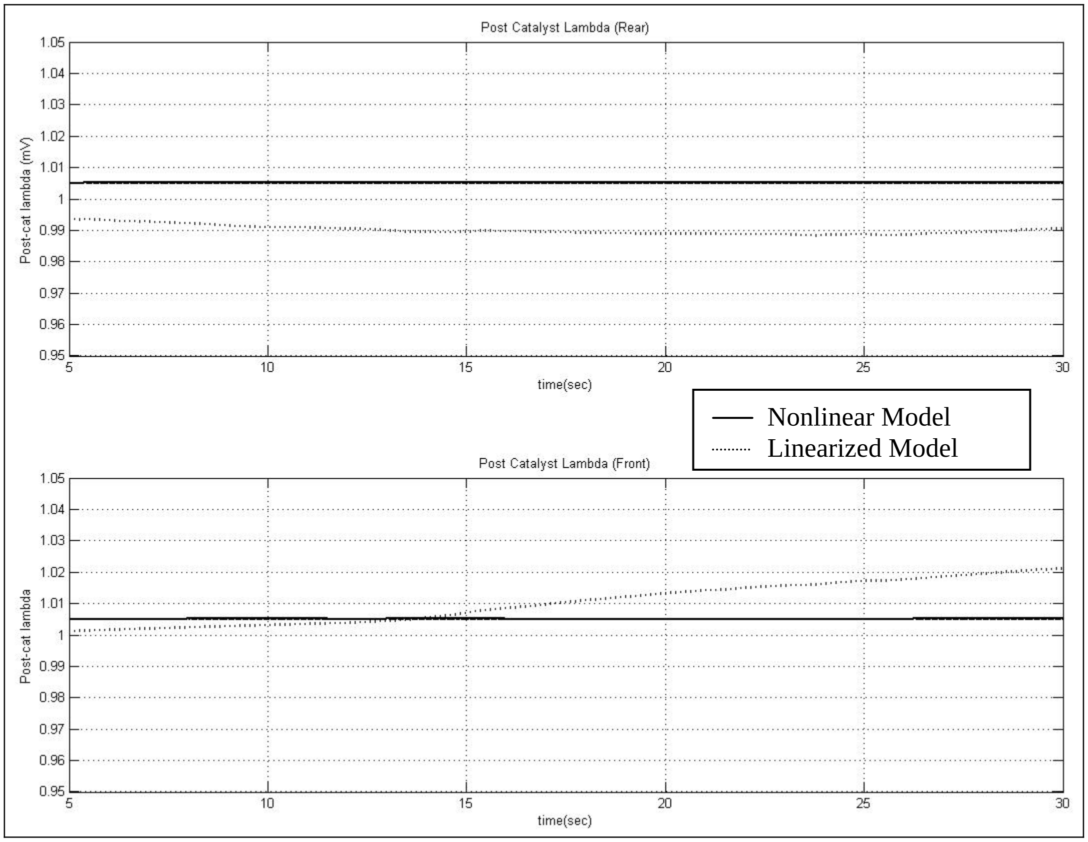

FIG.24-3 NOMINAL MODEL SIMULATION RESULT (POST-CAT LAMBDA)

V. VARIOUS NEW CONTROLLERS USING NOMINAL MODEL

The goal of this research is to develop a new controller which gives a better performance than the existing controller. We introduced the linearized state-space equations previously. In this chapter, we develop various controllers based on the state-space equations.

A. Pole Assignment Controller

Pole assignment technique is fundamental of other controllers. We start with this algorithm. It is well-known that if the system is controllable, we can assign eigenvalues to wherever we want. However, we are restricted by the inputs gain. Also, there are other restrictions in these controllers. In the engine speed control, we cannot control the lambda and load torque. Similarly, we cannot control the throttle angle as well. In addition, we need observers for the controller so that we can define the states. Therefore, we can get state-space equations with pole assignment algorithm such that:

$$\hat{x}(k+1) = (A(t) + B(t)K_{pole}) \cdot \hat{x}_n(k) + H_{pole}(y(k) - \hat{y}(k))$$

$$\hat{y}(k) = C(t) \cdot \hat{x}(k)$$

$$\delta u(k) = K_{pole} \cdot \hat{x}(k)$$

$$e(k+1) = (A(t) - H_{pole} \cdot C(t))e(k)$$

We assigned poles by using the parameters K_{pole} and H_{pole} below.

$$K_{pole}=\begin{bmatrix} -0.05 & -0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1 & -0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0 \end{bmatrix}$$

$$H_{pole}=\begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This assigns the poles to the following positions.

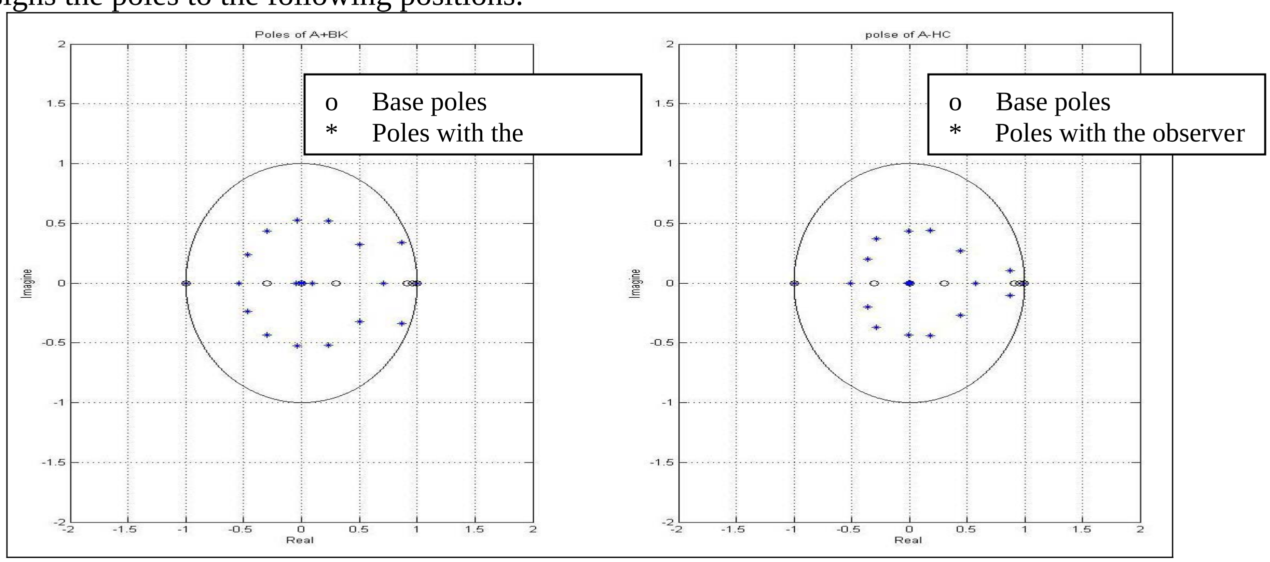

FIG.25 POLES OF THE CONTROLLER AND OBSERVER

We put two results of this controller with the results of Honda existing controller that we explained above.

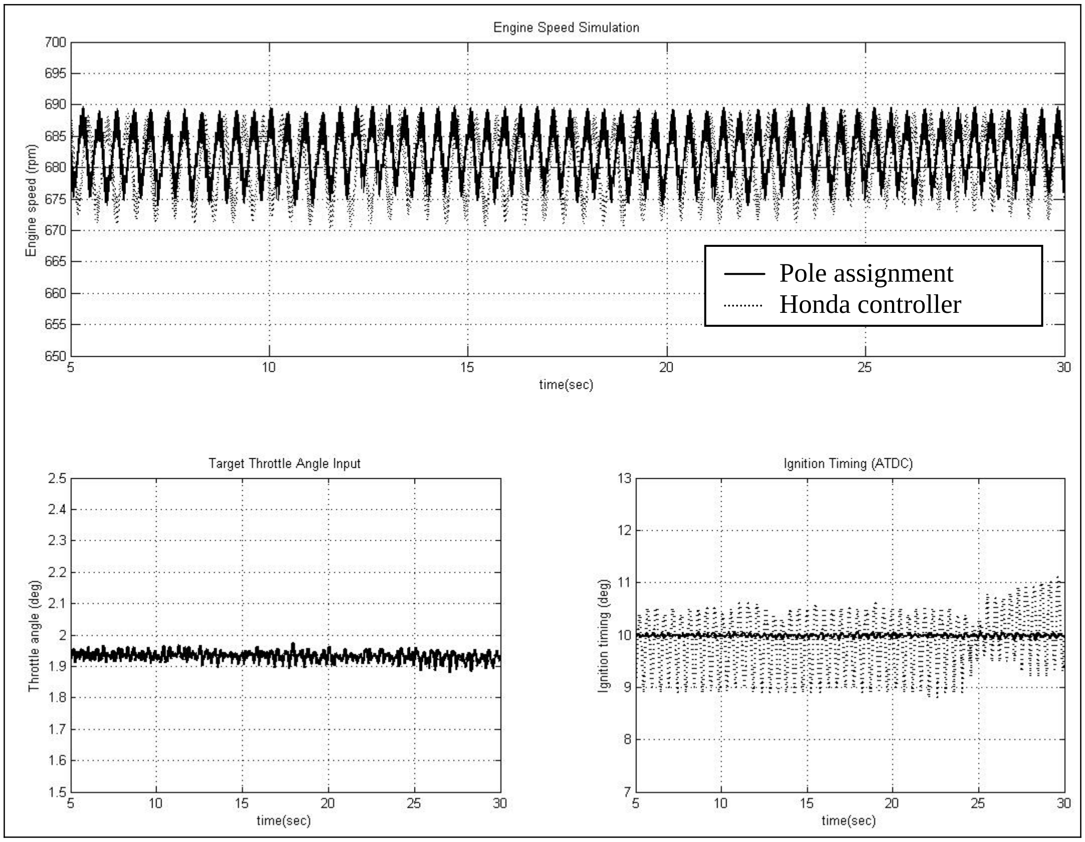

FIG.26-1 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, REVS)

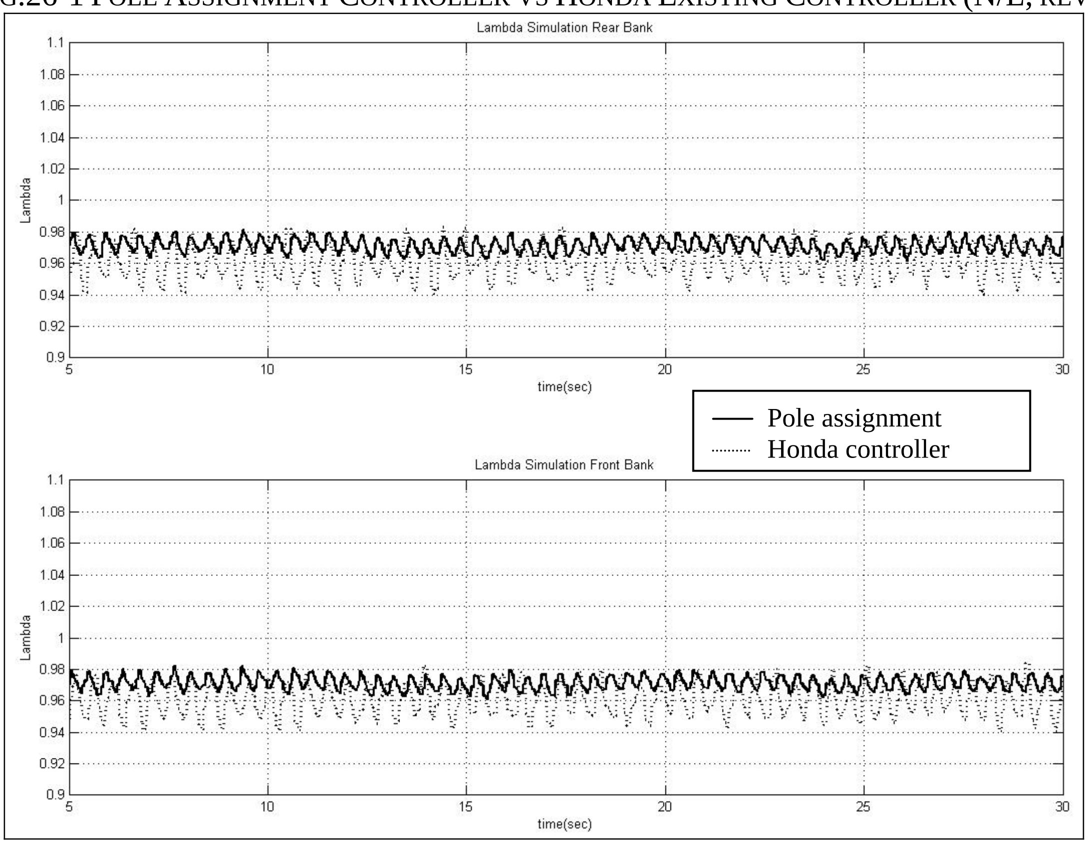

FIG.26-2 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, PRE-CAT LAMBDA)

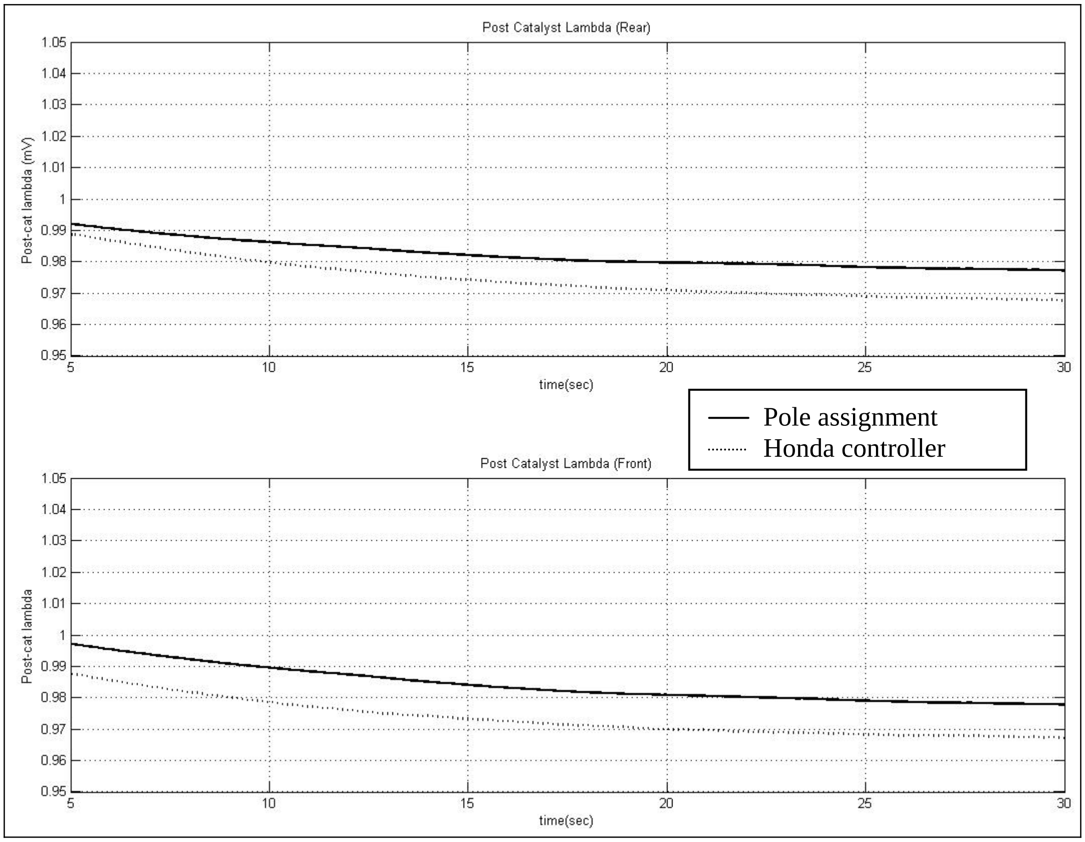

Fig.26-3 Pole Assignment Controller vs Honda Existing Controller (N/L, Post-cat lambda)

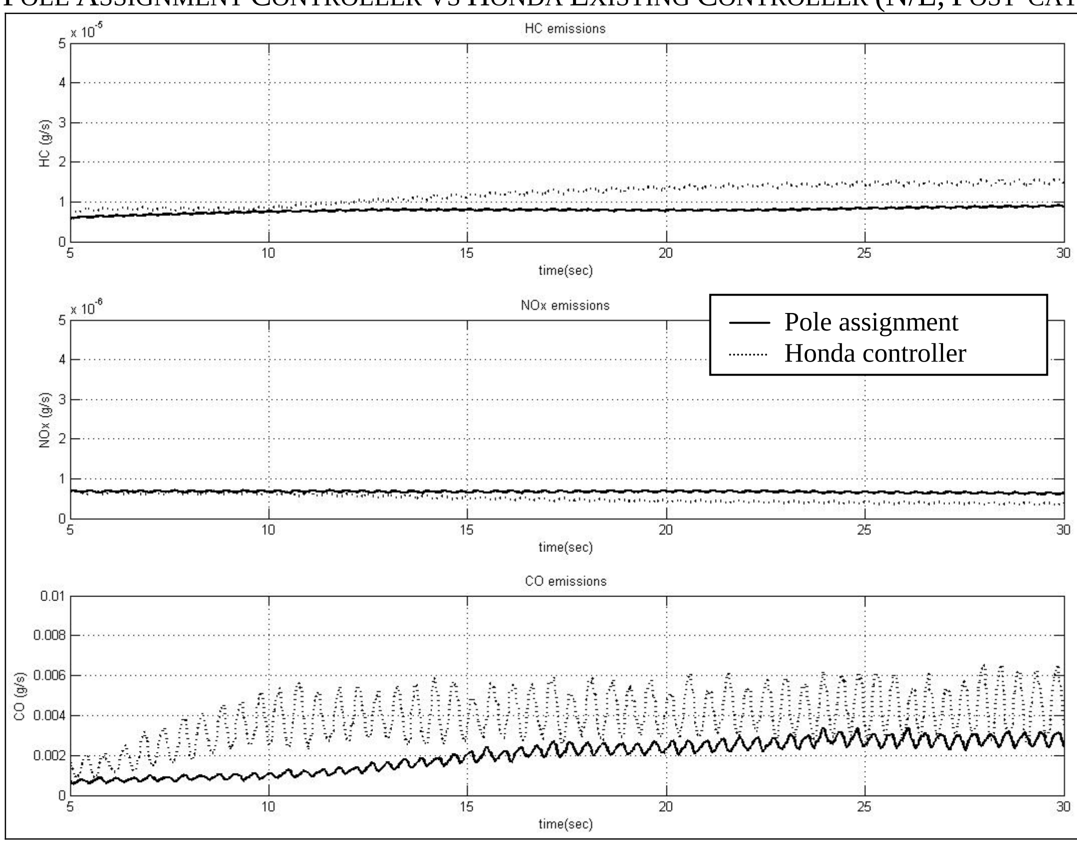

Fig.26-4 Pole Assignment Controller vs Honda Existing Controller (N/L, Emissions)

The results with the P/S load are followings.

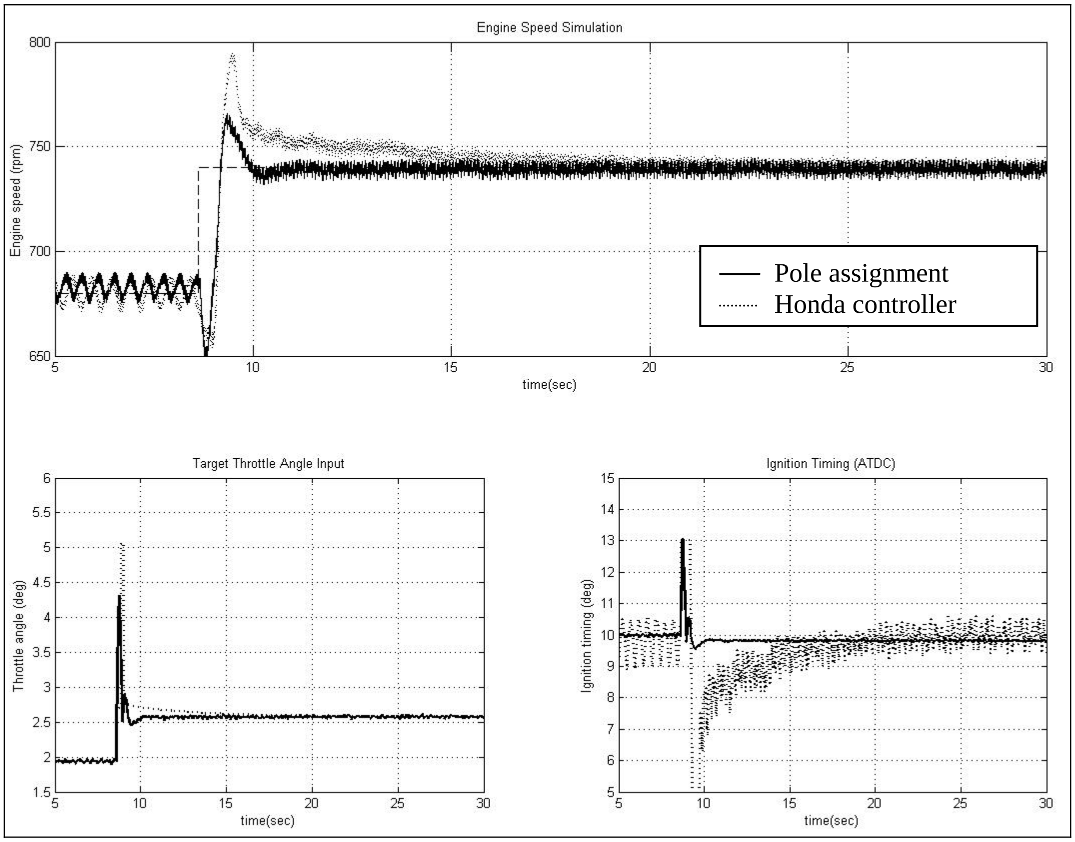

FIG.27-1 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, REVS)

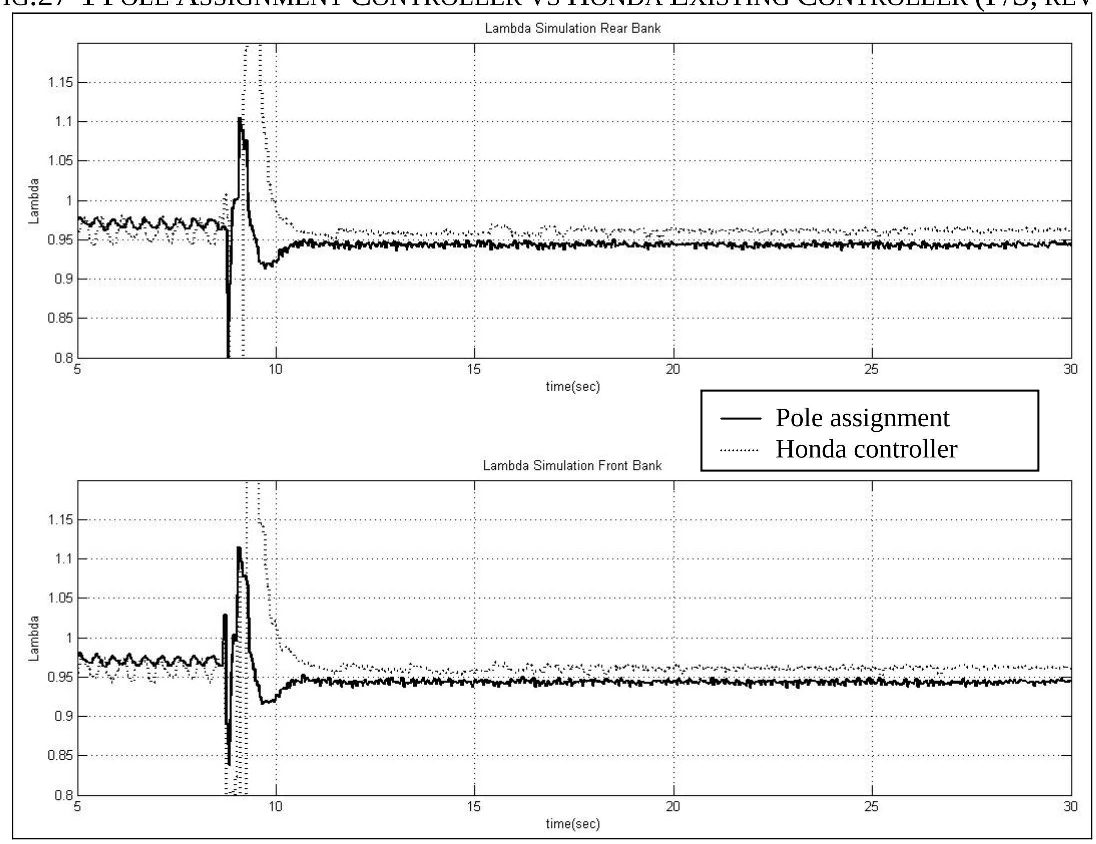

FIG.27-2 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, PRE-CAT LAMBDA)

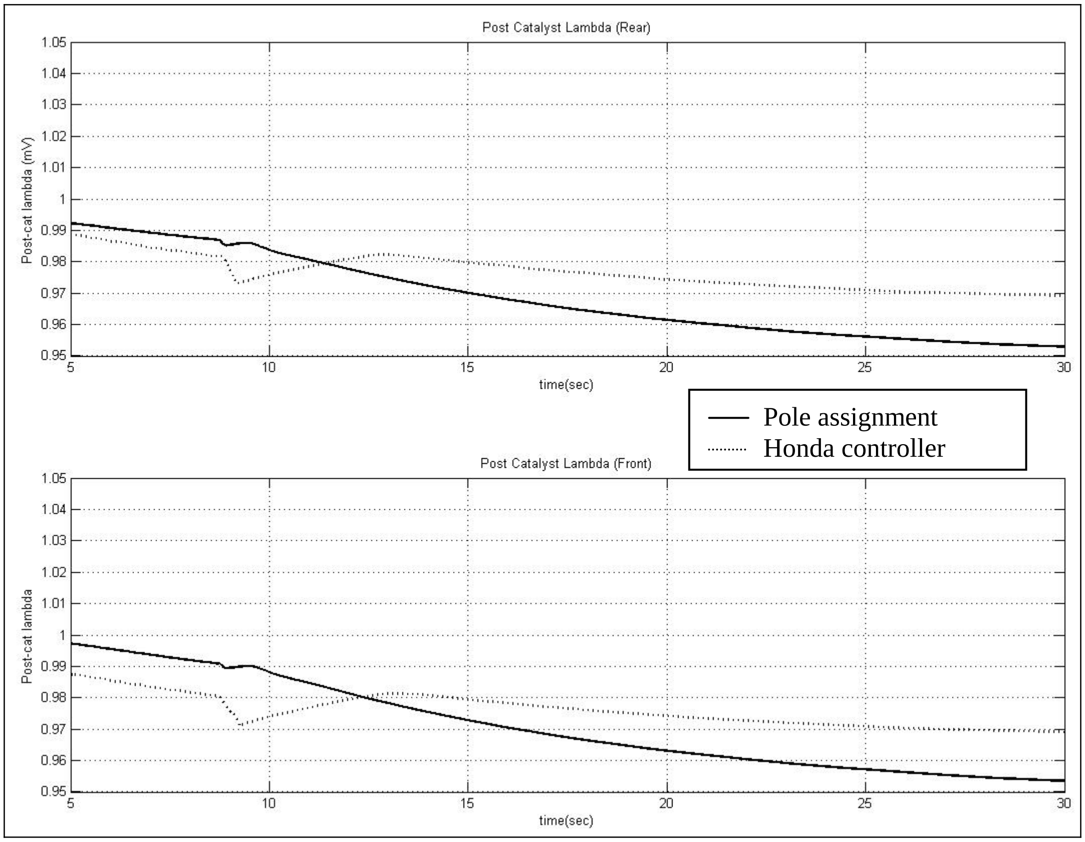

FIG.27-3 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, POST-CAT LAMBDA)

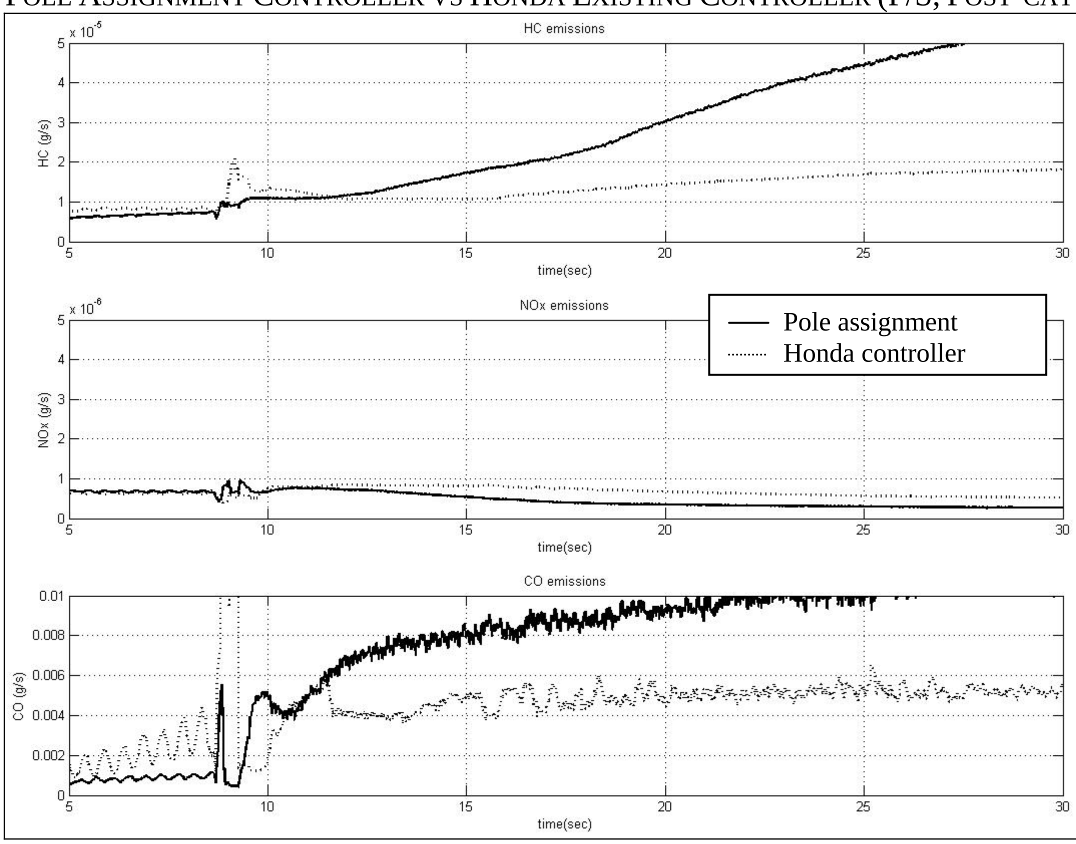

FIG.27-4 POLE ASSIGNMENT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, EMISSIONS)

From these, we know that the pole assignment controller performs almost the same. Note that with P/S load, the air / fuel moves very quickly, and this does not occur with the actual vehicle or cannot measure by the air / fuel sensor.

B. The Dead Beat Controller

Decoupling is the useful method with multi-inputs and multi-output case. Here, we begin with the simplest decoupling method, the dead beat, which is letting $\hat{x}(k+1)$ equal to 0.

$$0 = A(t)\cdot \hat{x}_h(k) + B(t)\cdot \delta u(k) + H_{pole}(y(k) - \hat{y}(k))$$

$$\hat{y}(k) = C(t)\cdot \hat{x}(k)$$

$$\delta u(k) = -B^{\dagger}(t)\cdot (A(t)\cdot \hat{x}(k) + H_{pole}(y(k) - \hat{y}(k)))$$

$$e(k+1) = (A(t) - H_{pole}\cdot C(t))e(k)$$

Where $B^{\dagger}(t)$ is pseudo-inverse of $B(t)$.

$$B^{\dagger}(t) = (B^T(t)\cdot B(t))^{-1}\cdot B^T(t)$$

The results of this are given as:

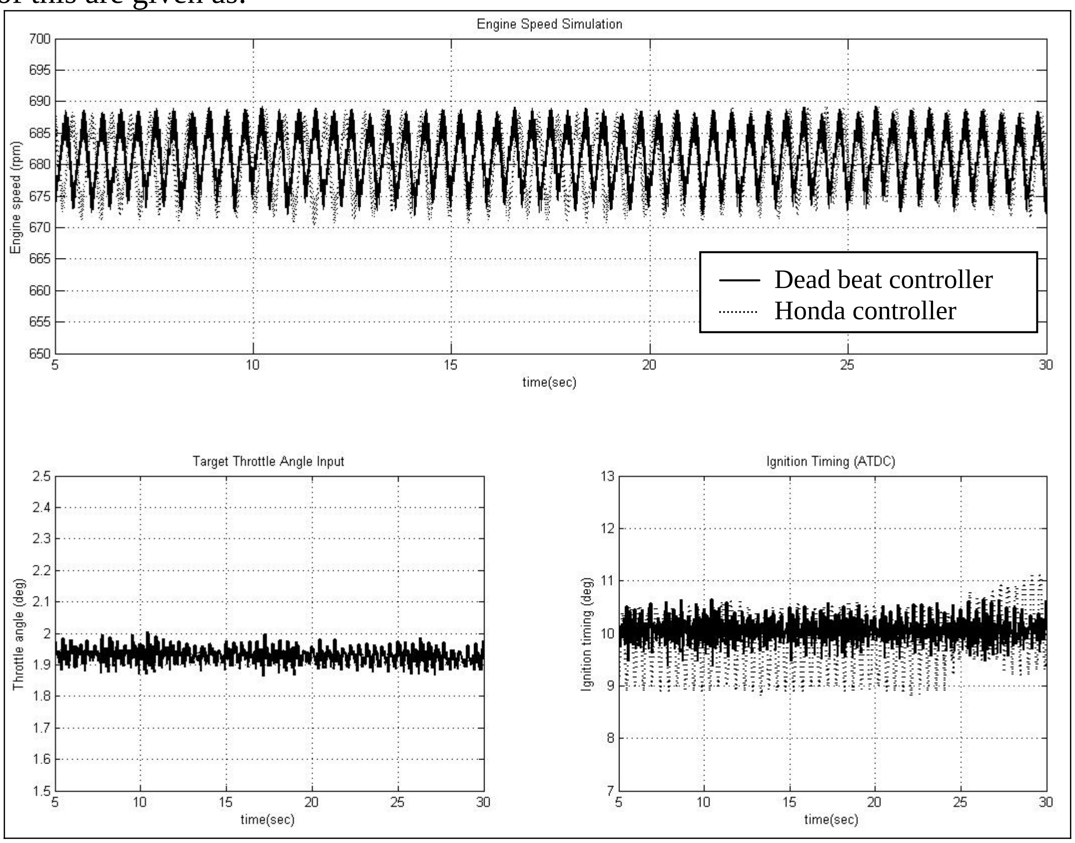

FIG.28-1 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, REVS)

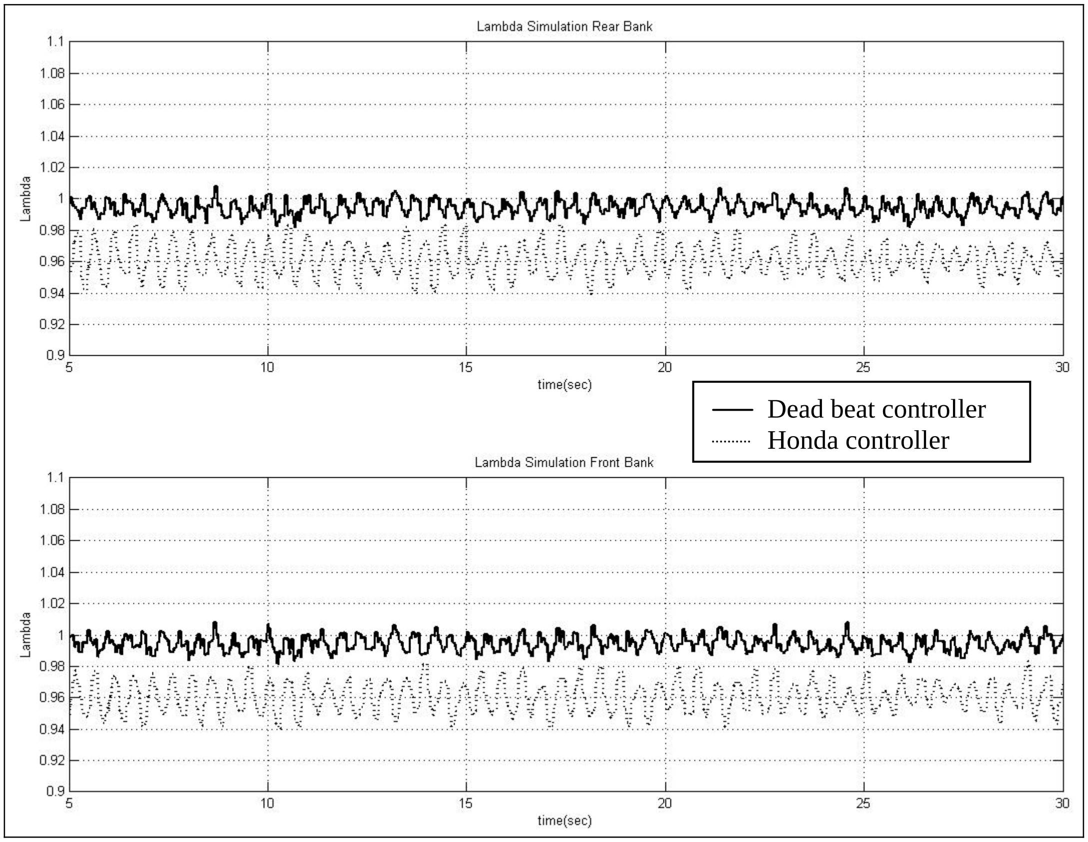

FIG.28-2 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, PRE-CAT LAMBDA)

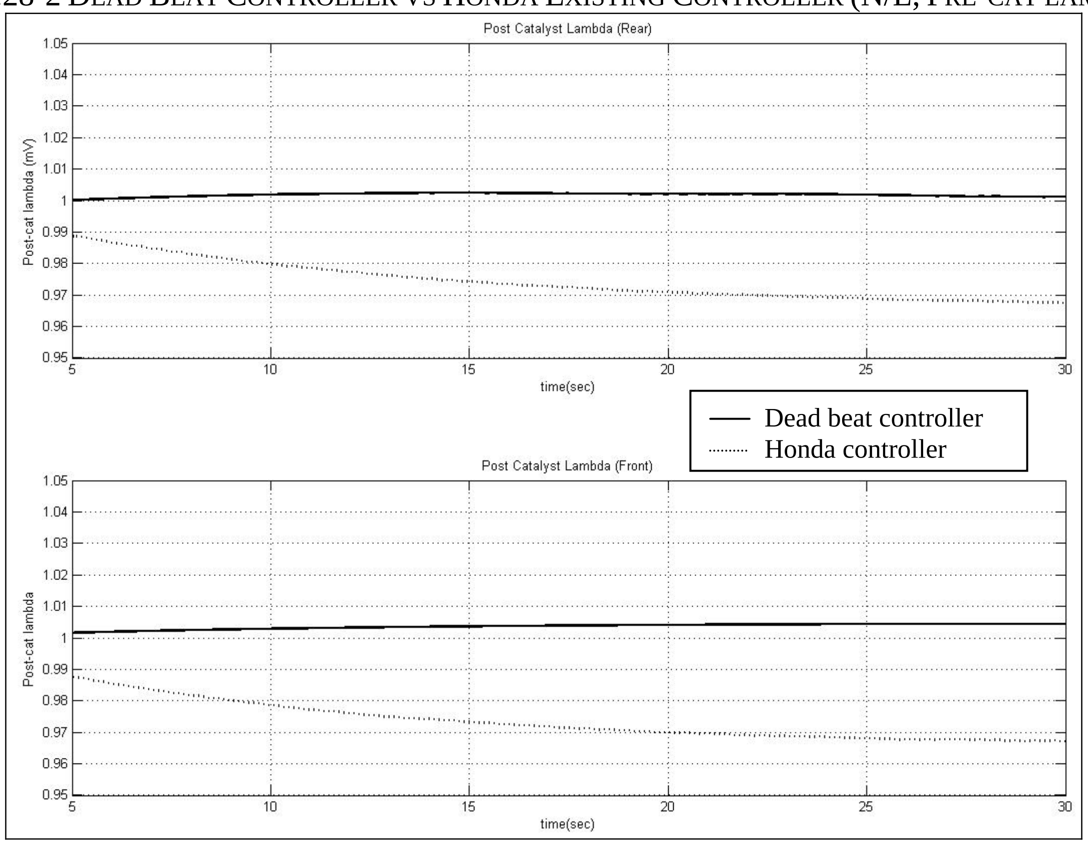

FIG.28-3 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, POST-CAT LAMBDA)

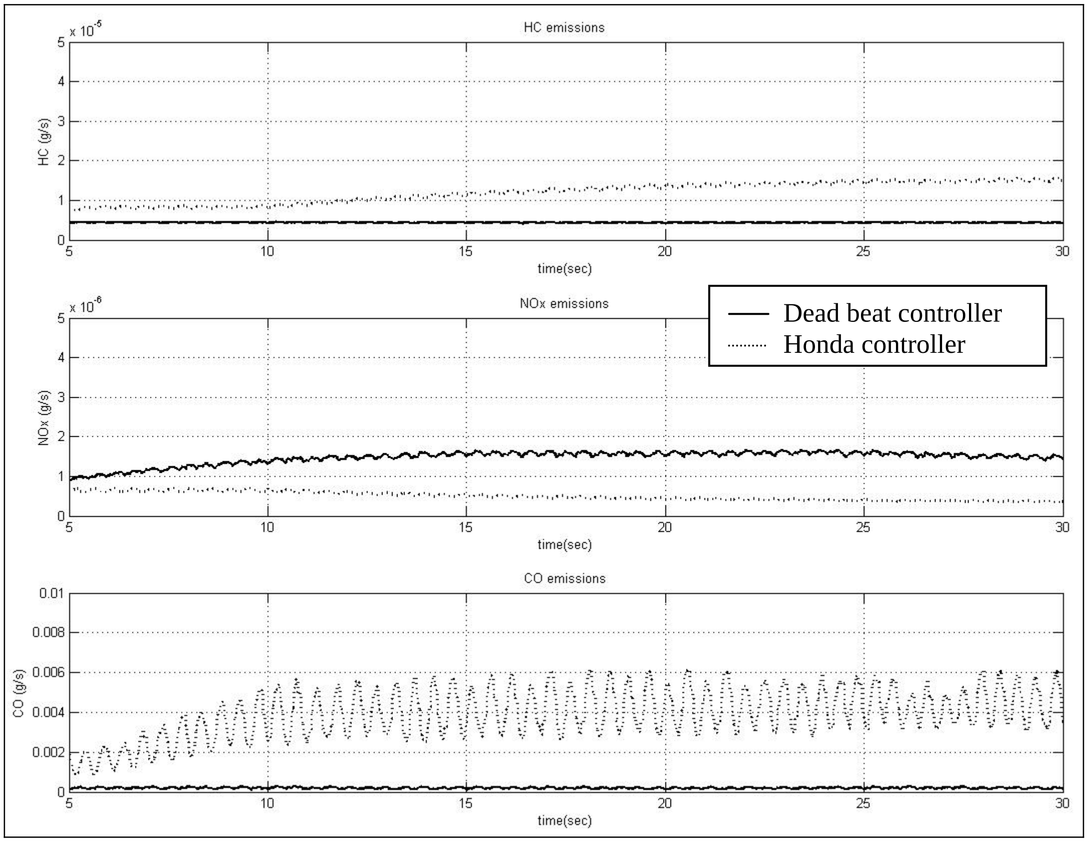

FIG.28-4 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, EMISSIONS)

The results with the P/S load are followings.

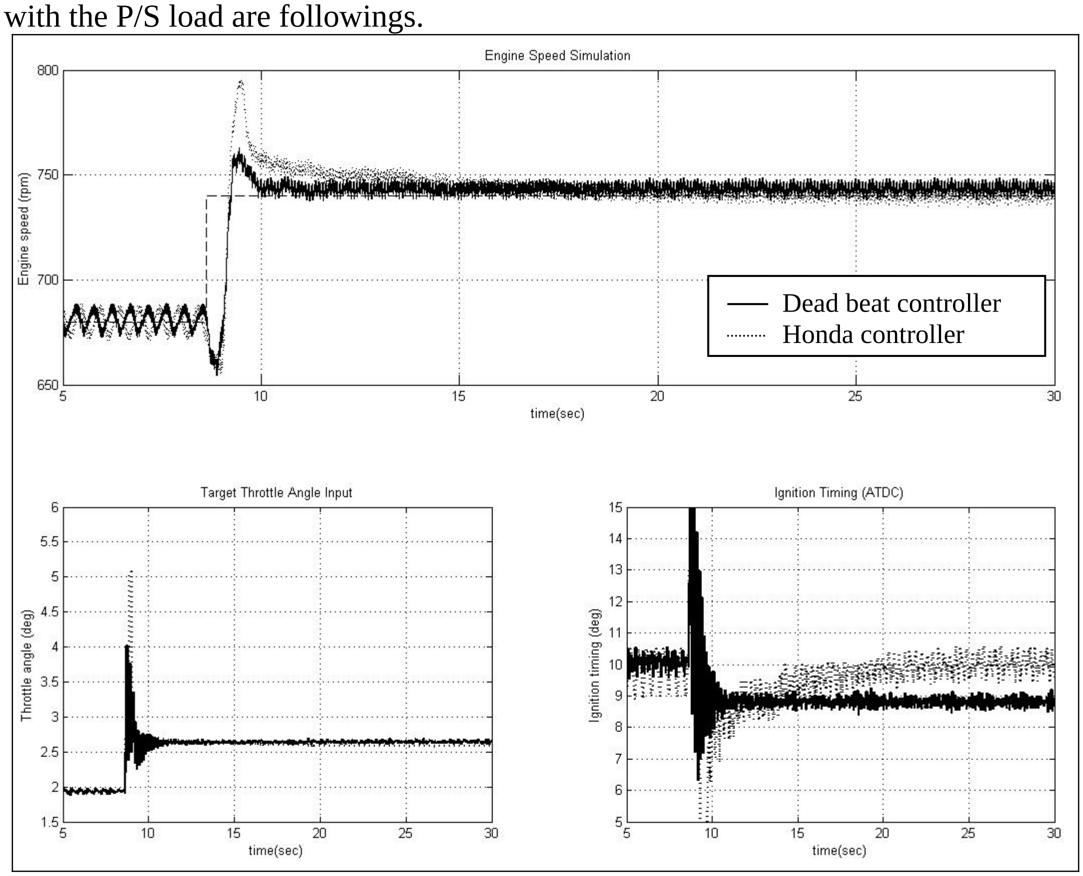

FIG.29-1 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, REVS)

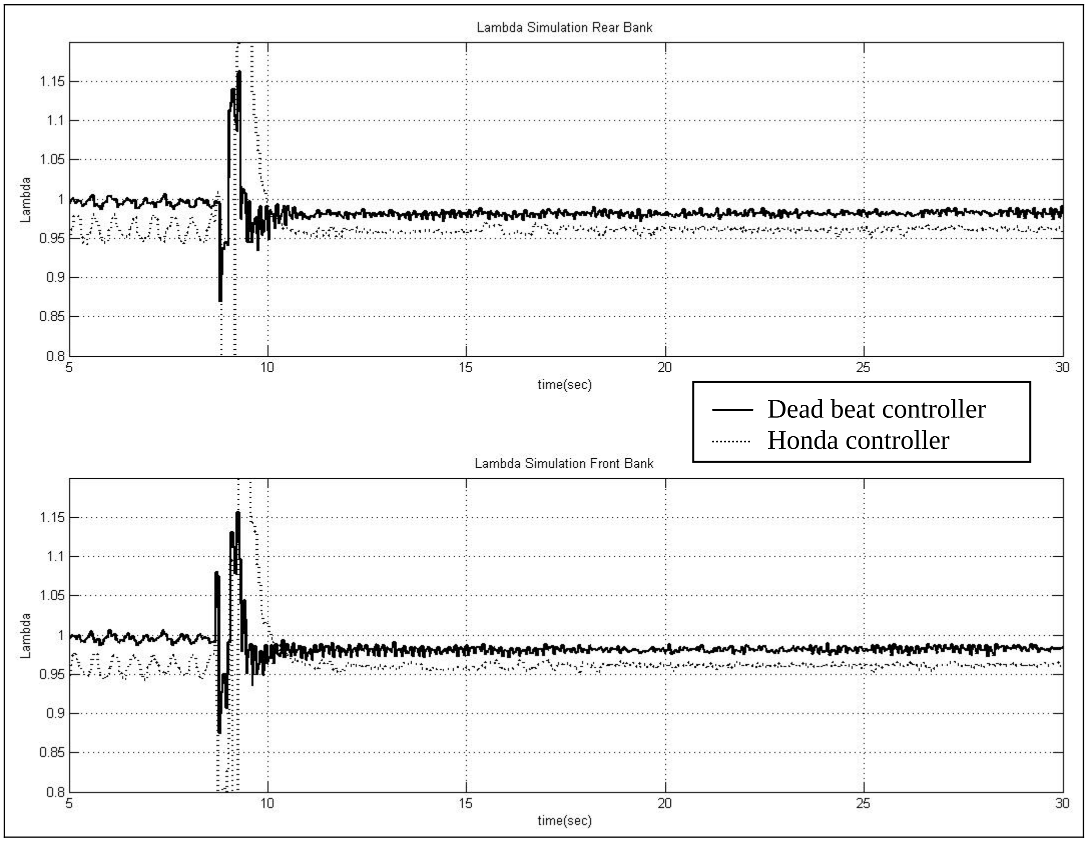

FIG.29-2 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, PRE-CAT LAMBDA)

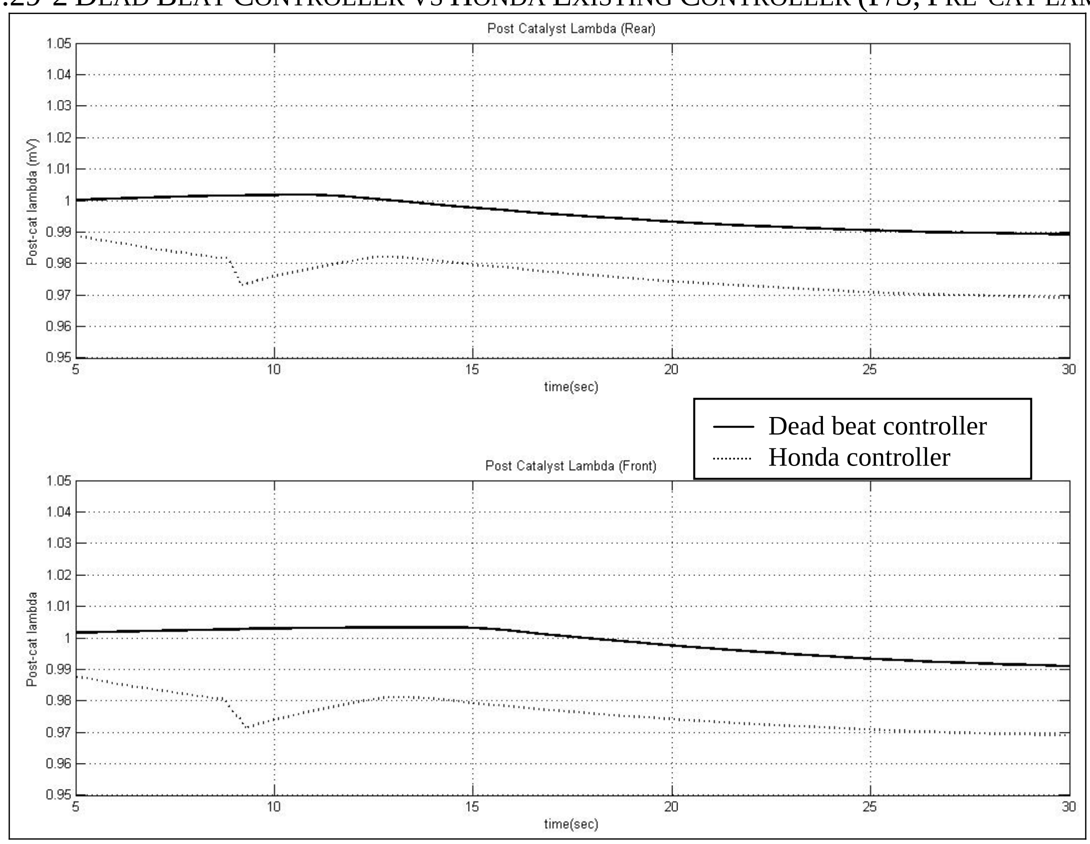

FIG.29-3 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, POST-CAT LAMBDA)

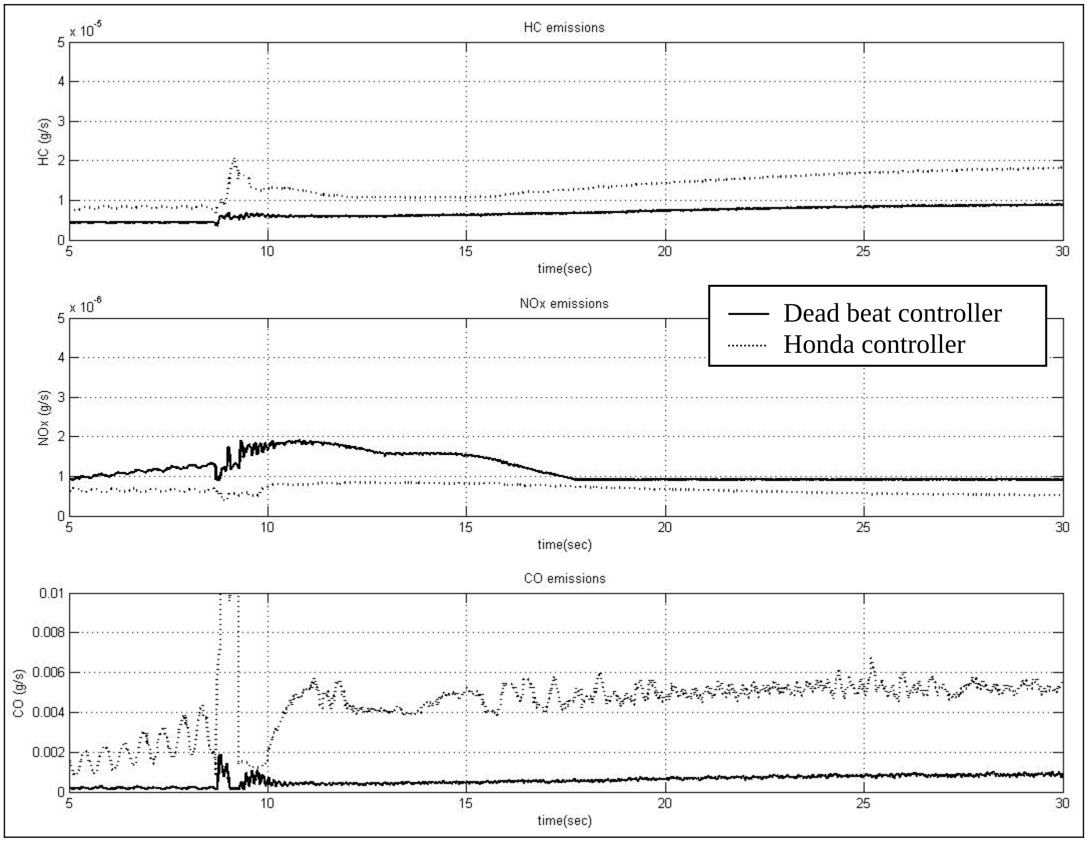

FIG.29-4 DEAD BEAT CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, EMISSIONS)

As the results show, when the condition is steady, the controller is working well; however, at the transient condition, the controller works poorly. Also, this controller always tries to set the states to 0 in one step, inputs gains becomes higher so that the oscillation width becomes bigger.

C. The Linear Quadratic Regulator

We applied the discrete LQR (Linear Quadratic Controller) for the above state-space model. The basic idea of the LQR is to get the optimal controller for the following cost function:

$$J = \sum \{x^T Q x + u^T R u + 2x^T N u\}$$

$$R > 0, Q - NR^{-1}N^T \geq 0$$

Here, we ignore the cross-term cost by setting N=0,

$$J = \sum \{x^T Q x + u^T R u\}$$

$$R > 0, Q \geq 0$$

Suppose that the state matrices are time-invariant, the control inputs that minimize the cost function J is given by:

$$u = Kx(k)$$

$$K = (B^T SB + R)^{-1} B^T SA$$

Where S is the solution of the algebraic Ricatti equation:

$$0 = A^T SA - S - A^T SB(B^T SB + R)^{-1} B^T SA + Q$$

The results of the LQR controller are as follows.

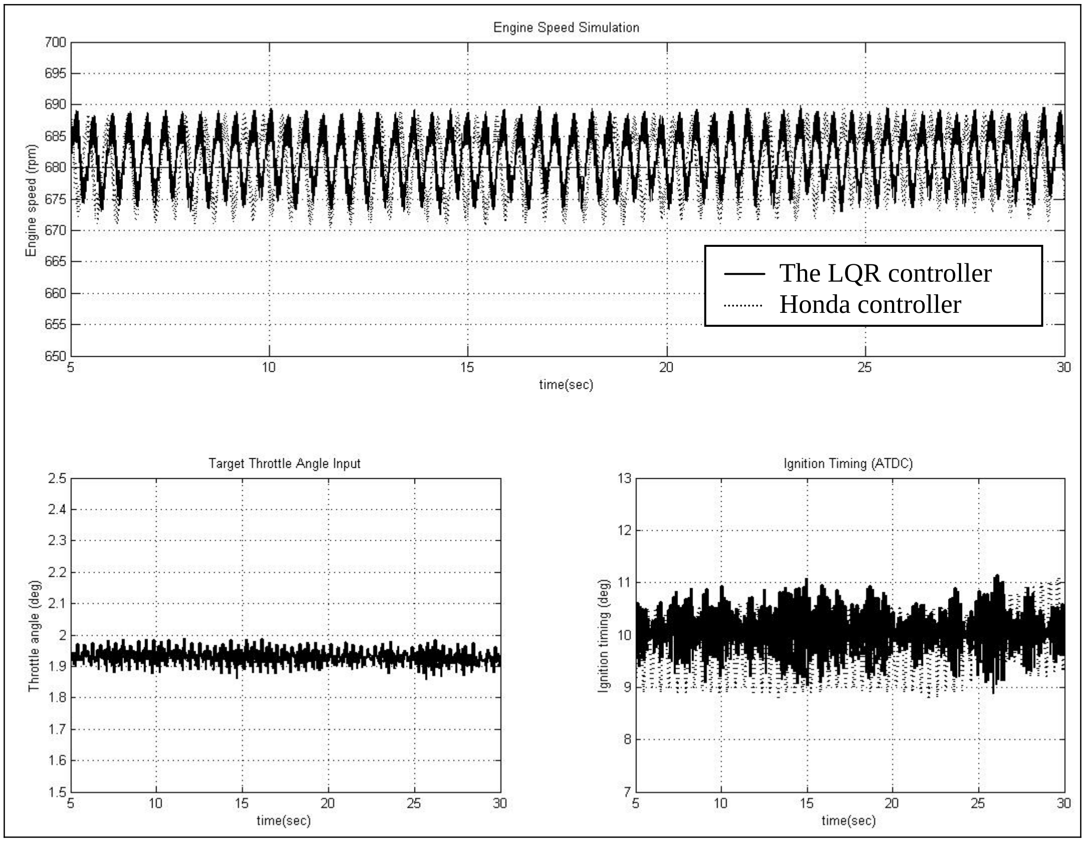

FIG.30-1 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, REVS)

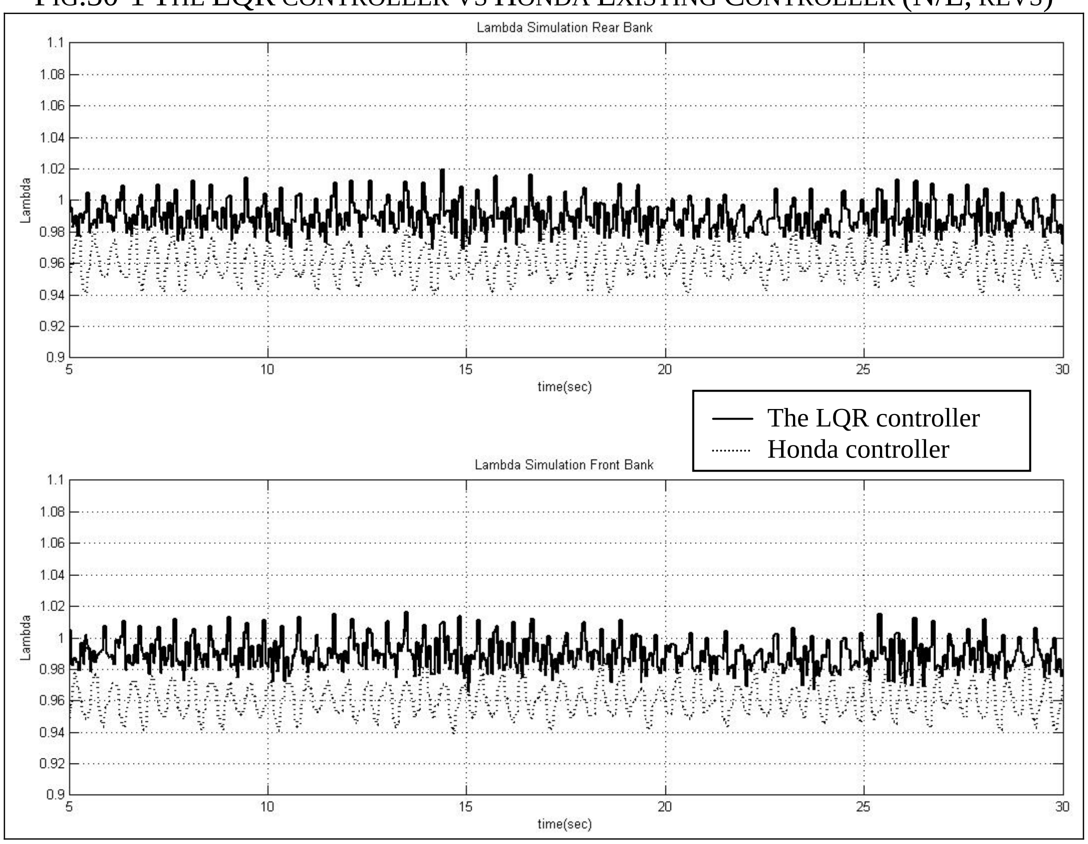

FIG.30-2 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, PRE-CAT LAMBDA)

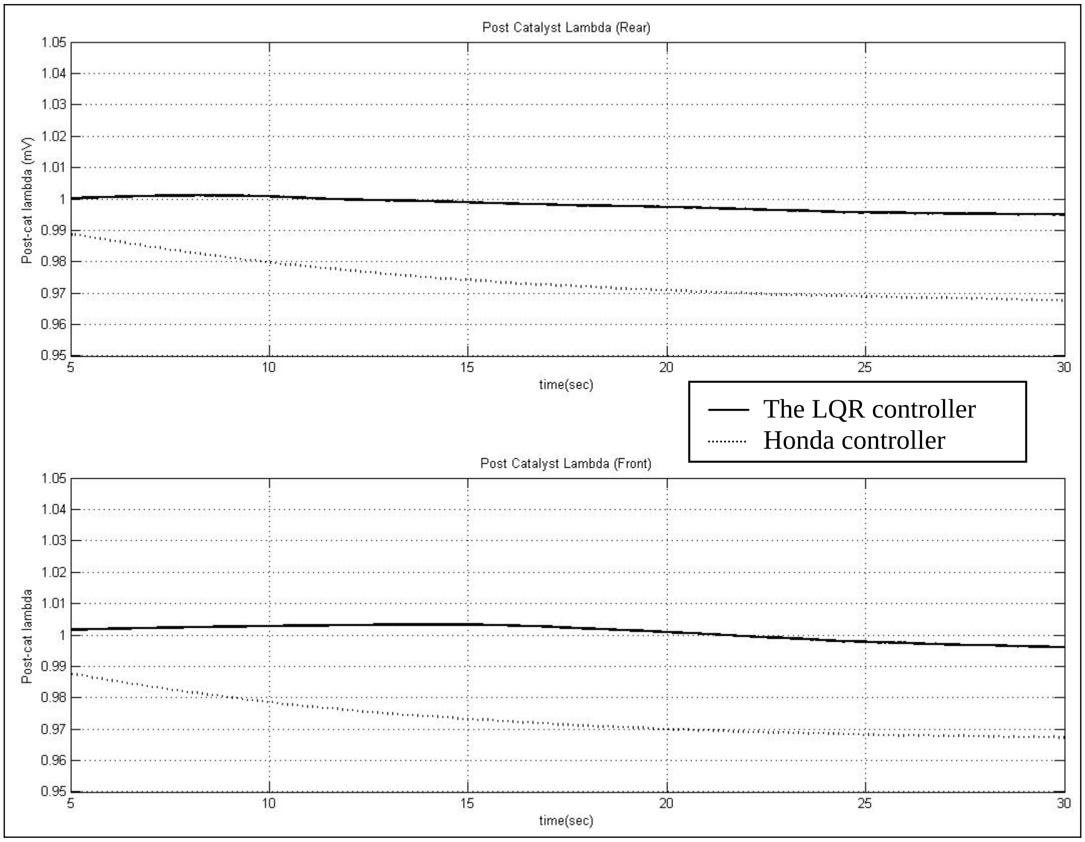

FIG.30-3 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, POST-CAT LAMBDA)

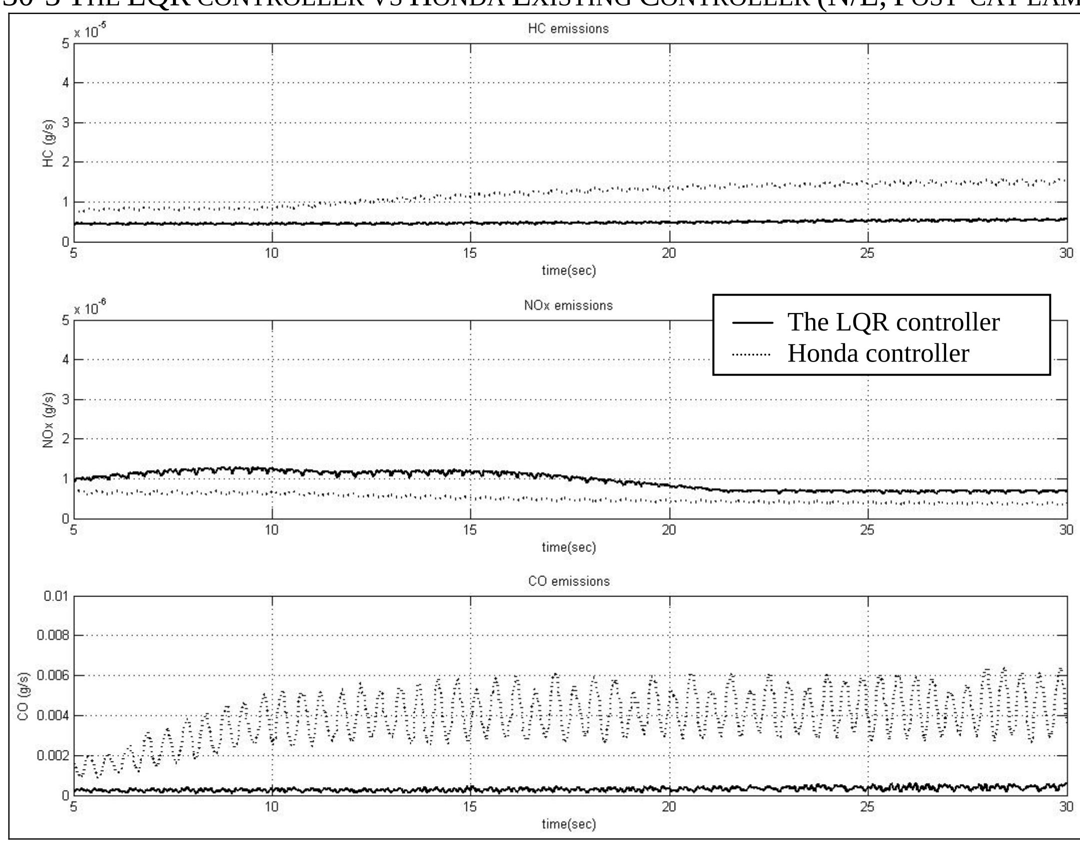

FIG.30-4 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (N/L, EMISSIONS)

The results with the P/S load are followings.

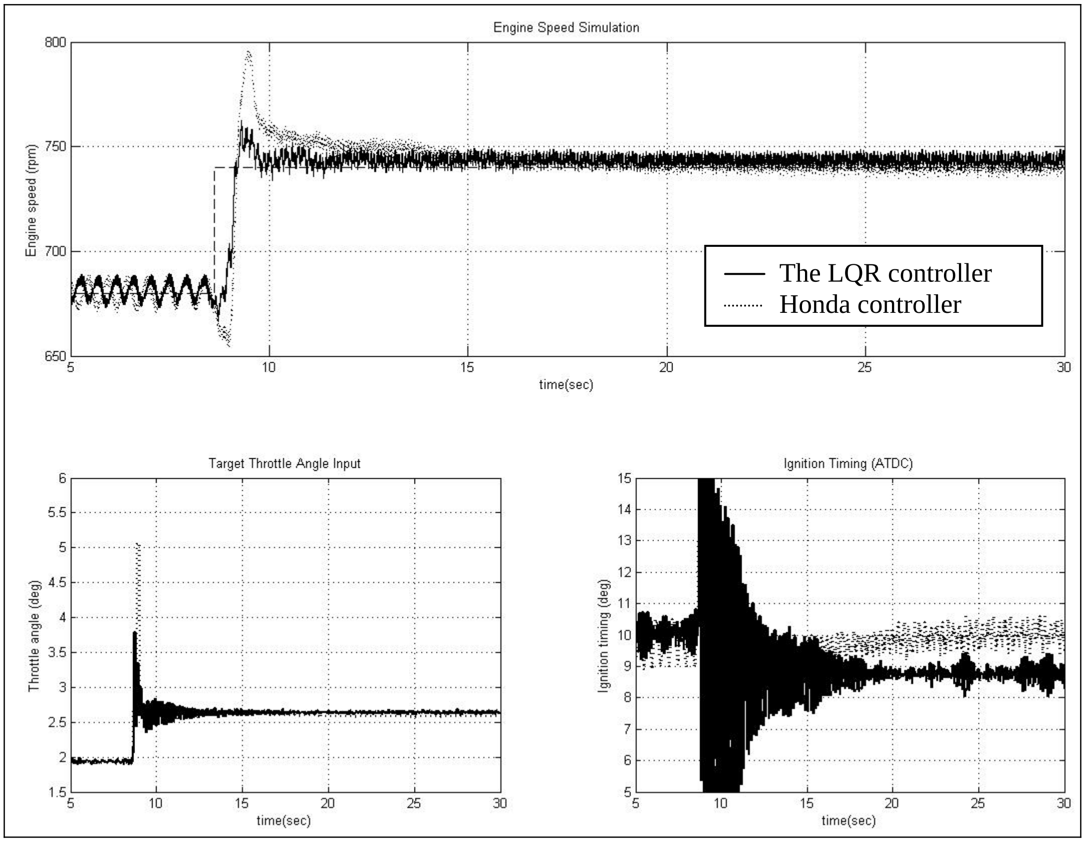

FIG.31-1 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, REVS)

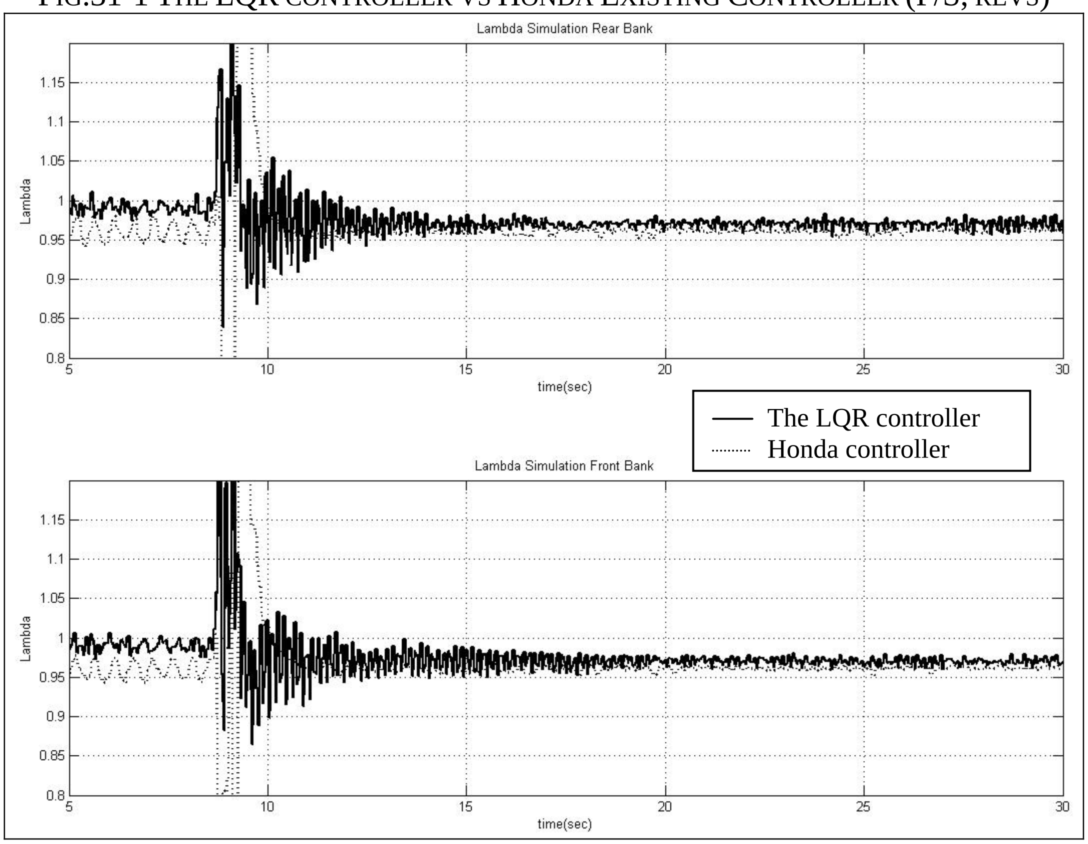

FIG.31-2 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, PRE-CAT LAMBDA)

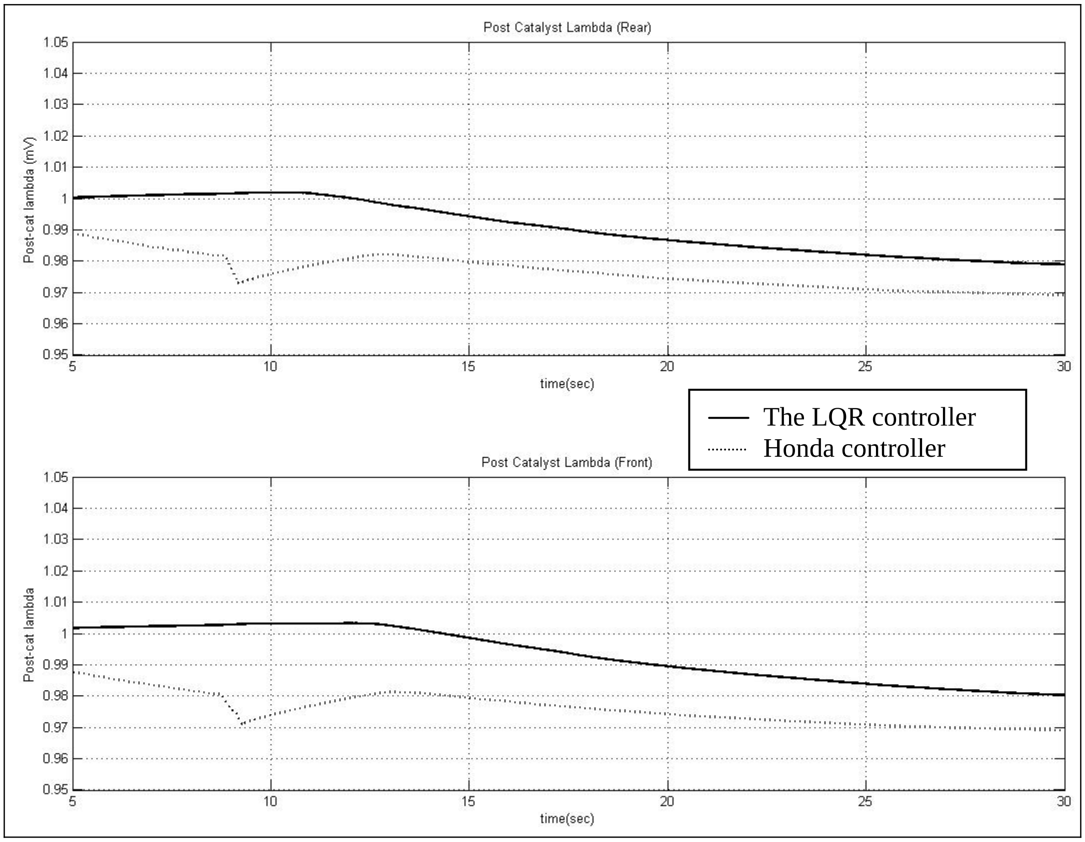

FIG.31-3 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, POST-CAT LAMBDA)

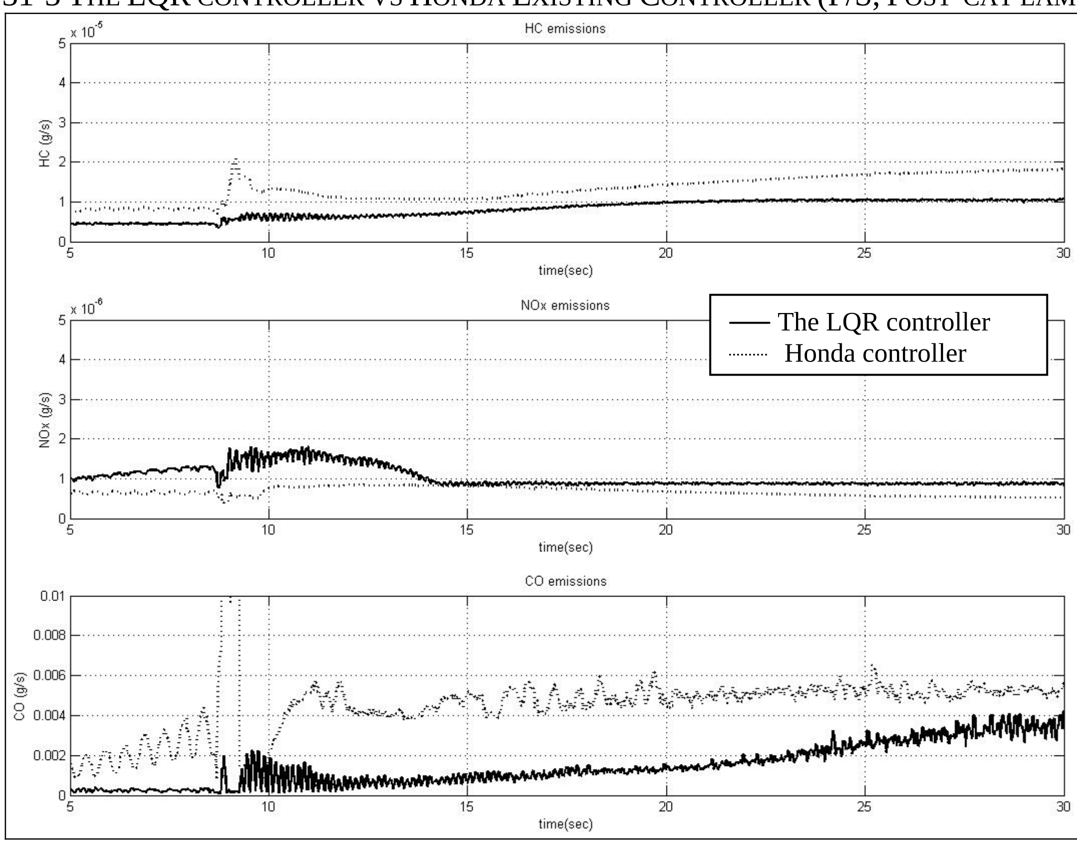

FIG.31-4 THE LQR CONTROLLER VS HONDA EXISTING CONTROLLER (P/S, EMISSIONS)

D. The H-2 Controller

We try to apply for the H-2 controller. This controller can divide the system into two inputs which are controllable inputs and disturbances and two outputs which are measurable outputs and immeasurable outputs. This controller recognizes the loads torque as a disturbance and gives less sensitivity to the engine speed and the immeasurable outputs which are emissions. The preview of our system of the controller is given in Fig.32.

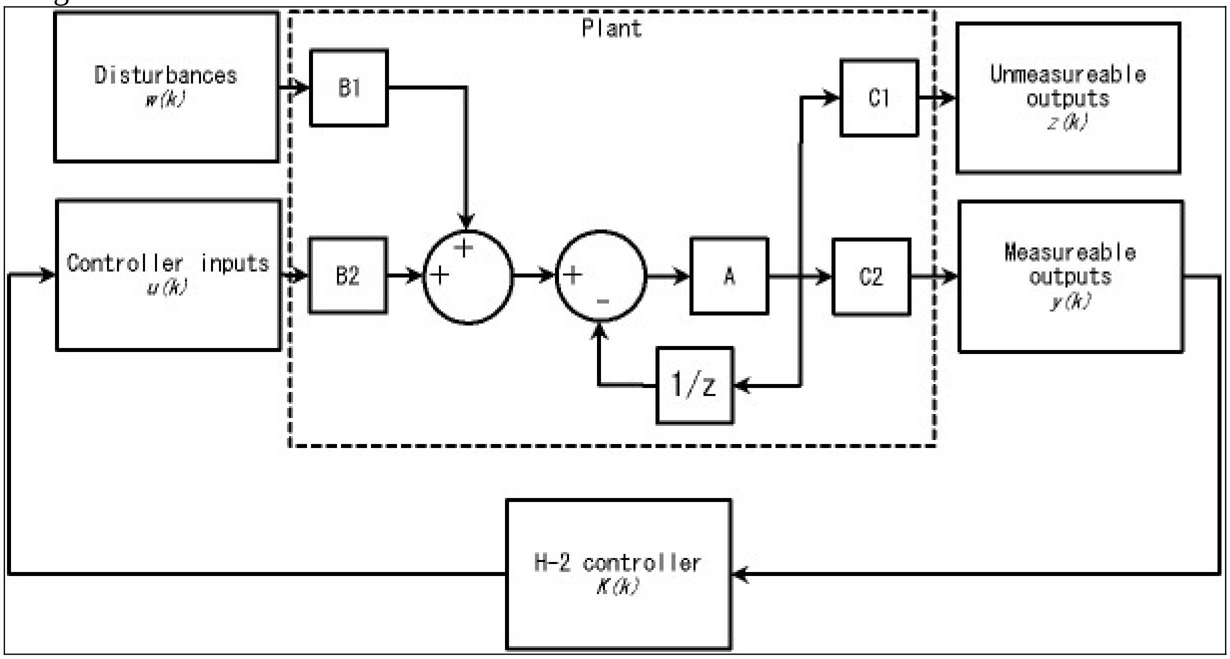

FIG.32 H-INFINITY CONTROLLER

At first, we would like briefly introduce about the discrete H-2 controller. The system matrix can be written as:

$$x(k+1) = Ax(k) + B_1 w(k) + B_2 u(k)$$
$$z(k) = C_1 x(k) + D_{11} w(k) + D_{12} u(k)$$
$$y(k) = C_2 x(k) + D_{21} w(k) + D_{22} u(k)$$

The H-2 control problem is to find a proper and real-rational controller K_{H2} that stabilizes the plant and minimizes the H-2 norm of the transfer matrix T_{zw}. There are some assumptions to solve this problem easier.

(A1) (A, B_2) is stabilizable and (C_2, A) is detectable;

(A2) D_{12} is full column rank with $\begin{bmatrix} D_{12} & D_{\perp} \end{bmatrix}$ unitary and

D_{21} is full row rank with $\begin{bmatrix} D_{21} \\ D_{\perp} \end{bmatrix}$ unitary;

(A3) $\begin{bmatrix} A - e^{j\theta} I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all $\theta \in [0, 2\pi]$;

(A4) $\begin{bmatrix} A - e^{j\theta} I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all $\theta \in [0, 2\pi]$.

The unique optimal controller is

$$x_{H2}(k+1) = A_{H2} x_{H2}(k) + B_{H2} y(k)$$
$$u(k) = A_{H2} x_{H2}(k) + C_{H2} y(k)$$

Where,

$$A_{H2} = \hat{A}_2 - B_2 L_0 C_2,\ B_{H2=} - (L_2 - B_2 L_0)$$
$$C_{H2} = F_2 - L_0 C_2,\ D_{H2} = L0$$

There parameters are defined as:

$$R_b := I + B_2^T X_2 B_2$$
$$F_2 := -(R_b)^{-1}(B_2^T X_2 A + D_{12}^T C_1)$$
$$F_0 := -(R_b)^{-1}(B_2^T X_2 B_1 + D_{12}^T D_{11})$$
$$L_2 := -(A Y_2 C_2^T + B_1 D_{21}^T)(I + C_2 Y_2 C_2^T)^{-1}$$
$$L_0 := -(F_2 Y_2 C_2^T + B_1 D_{21}^T)(I + C_2 Y_2 C_2^T)^{-1}$$

X2 and Y2 are the solution of the algebraic Ricatti equation:

$$0 = A_x^T X_2 A_x - X_2 - A_x^T X_2 B_2 (B_2^T X_2 B_2 + I)^{-1} B_2^T X_2 A_x + C_1^T D_\perp D_\perp^T C_1$$
$$0 = A_y^T Y_2 A_x - Y_2 - A_y^T Y_2 C_2^T (C_2 Y_2 C_2^T + I)^{-1} C_2 Y_2 A_y + B_1 \tilde{D}_\perp^T \tilde{D}_\perp B_1^T$$

Where

$$A_x = A - B_2 D_{12}^T C_1$$
$$A_y = A - B_1 D_{21}^T C_2$$

VI. CONCLUSION

We developed an engine plant that can be used to compare the existing and new controller. The engine plant model can calculate the engine speed, the pre-catalytic converter lambda, and the post-catalytic converter lambda from the throttle angle, the ignition timing, and the injection duration.

We linearized the engine model in order to apply for the various controller algorithms.

The Honda current controller is included with the engine model. We can compare to the Honda controller by the simulation.

We tried three fundamental linear control algorithms. With the dead beat controller and the LQR controller, the results were a little better than the Honda controllers especially for the engine speed control problem. Also, the calibration labor will be reduced since we don't need to calibrate the feed-forward air compensation.

As a further research, we would like to try the development of the H-inf robust controller which considers the load torque as a disturbance and try to reduce the sensitivity from the load torque to the engine speed and emissions.

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