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Throughput Capacity of Random Ad Hoc Networks with Infrastructure Support

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***Abstract*— In this paper, we consider the transport capacity of ad hoc networks with a random flat topology under the present support of an infinite capacity infrastructure network. Such a network architecture allows ad hoc nodes to reach each other by purely using ad hoc nodes as relays. In addition, ad hoc nodes can also utilize the existing infrastructure fully or partially by reaching any access point (or gateway) of the infrastructure network in a single or multi-hop fashion. Using the same tools as in [1], we show that the per source node capacity of $\Theta(W/\log(N))$ can be achieved in a random network scenario with the assumptions that the number of ad hoc nodes per access points is bounded above and that N ad hoc nodes excluding the access points, each capable of transmitting at W bits/sec using a fixed transmission range, constitute a connected graph. This is a significant improvement over the capacity of random ad hoc networks with no infrastructure support which is found as $\Theta(W/\sqrt{N\log(N)})$ in [1]. Although better capacity figures are obtained by complex network coding or exploiting mobility in the network, infrastructure approach provides a simpler mechanism that has more practical aspects. We also show that even when less stringent requirements are imposed on topology connectivity, a per source node capacity figure that is arbitrarily close to $\Theta(1)$ can not be obtained. Nevertheless under these weak conditions, we can further improve per node throughput significantly.**

***Index Terms*— Transport capacity, random ad hoc networks, hybrid wireless networks.**

I. Introduction

Future network applications for commercial, scientific, or military use will necessitate utilization of different wireless technologies together for addressing the requirements of the specific scenarios [2]. Multi-hop wireless ad hoc networks with their paramount importance in establishing easily deployable, self-configurable, and highly flexible communication environment will probably be an indispensable component in these multiple technology and multiple layer network architectures. In a typical scenario, ad hoc networks can be visualized as an extension to the existing infrastructure networks such as cellular and wireless local area networks for further improvement in performance (e.g. higher system throughput/user capacity, reduced power consumption, etc.) [3], [4], [5], [6]. In another very likely and counter scenario, we can have the ad hoc network, which is fully capable of carrying out the communication tasks confined within the ad hoc domain by itself but at a limited level[1] and an infrastructure network with relatively abundant resources can help improving the networking performance of the ad hoc tier. This latter scenario will be the subject of our paper. More specifically, we will search for the theoretical gains of introducing an infrastructure overlay on top of an ad hoc network in terms of its transport capacity per source node, i.e. the maximum end-to-end data rate that can be uniformly obtained between pairs of the ad hoc nodes.[2]

We will define our problem on a disk domain as it is widely accepted in the literature [1], [7], [8]. Both the ad hoc nodes and the access points of the infrastructure network are assumed to be randomly distributed on this disk domain. The choice of random location for ad hoc nodes is a natural one, but it is legitimate to ask how proper it is to impose the same assumption on the access points. The answer depends on the specific scenario as usual. As a counter example, if we have the cellular networks as the infrastructure where access points are simply the base stations located at the center of hexagonal cells and they are connected to each other by a wireline network, then the locations of the access points are deterministic. On the other hand, if we have wireless local area networks (WLANs) as the infrastructure, then the shape of the serving area and hence the location of each access point is not well-determined [9]. Furthermore, when we consider the access points to be mobile/wireless routers with broadband connection to the infrastructure network, then our assumption becomes more sound. Although we do not have a control over the location of the access points, we will have control over their population: We require the number of ad hoc nodes per access point to be bounded from above.

The paper can be divided into two parts. In the first part, we obtain the throughput capacity under a notion of *strong connectivity* condition which mandates that the ad hoc nodes using the same fixed transmission range form a connected topology graph with high probability. In other words, we want to have a stand-alone ad hoc network which can provide connection between any pair of ad hoc nodes with probability close to

[1] Ad hoc nodes would probably have limited energy supplies while wireless channel impairments, multi-hop operations, and/or mobility would effectively reduce the available bandwidth significantly.

[2] As it will be clear in our network model, we assume that each source node will generate an equal amount of data for a random destination in a given time duration, hence we use the term *uniformly* here.

one and without the support of any existing infrastructure. This certainly is a very cautious constraint and does not take advantage of the existing infrastructure in its full extent. For instance, there can be partitions in the ad hoc tier, but when the overall topology construct is visualized, any pair of ad hoc nodes can still be connected. Therefore, at the expense of partitions, ad hoc nodes can reduce their transmission range below the value enforced by the strong connectivity. This eliminates excessive interference of ad hoc nodes on each other and increases the number of simultaneous transmissions in the ad hoc tier improving the upper bound of the transport capacity. Hence, in the second part of the paper, we introduce the second notion of connectivity, i.e. *weak connectivity*, that requires the overall network topology graph to be connected. We derive the necessary and sufficient conditions on the transmission range to satisfy the weak connectivity condition and show that any upper bound resulting from the weak connectivity condition can indeed be achieved. As a corollary, our results indicate that the transport capacity per node eventually converges to 0 as the ad hoc network size increases indefinitely. This is contrary to the recent studies that claim to achieve constant throughput rate per node under different networking constraints [10], [11], [8].

The rest of the paper is organized as follows. In section-II, we give a comprehensive overview of the most recent works in the literature. Section-III outlines the network model that will be considered in the rest of the paper. Section-IV presents the capacity result under the strong connectivity condition. Section-V derives the necessary and sufficient conditions on the transmission power to satisfy the weak connectivity condition and shows that $\Theta(1)$ bits/sec can not be achieved even under looser constraints. Section-VI shows that any upper bound based on the weak connectivity condition can indeed be achieved. Finally, in Section-VII, we conclude the paper.

II. Related Works

Transport capacity of wireless ad hoc networks have been a major research interest since the landmark paper of Gupta and Kumar [1]. In that paper, authors prove that per node throughput values of $\Omega(1/\sqrt{N})$ and $\Omega(1/\sqrt{N \log N})$ bits/sec are attainable for arbitrary and random networks respectively both on a planar disk domain and on the surface of a sphere. Achieving the throughput figure for arbitrary networks involves the freedom of placing the nodes and choosing the traffic patterns. On the other hand, random network scenarios encompass a uniform distribution of the nodes on the topology area as well as a random destination for each ad hoc node. Therefore, authors show the achievability results for random networks in the asymptotic sense by designing proper routing and transmission scheduling mechanisms. In [1], two different models are considered for determining the successful transmissions in the same channel: protocol and physical models. Protocol model ensures that given a transmitter-receiver pair, no other node in a disk centered at the intended receiver transmits. The radius of the disk depends both on the distance between the transmitter and the receiver as well as a protocol dependent constant. Whereas the physical model demands a certain signal to interference and noise (SINR) ratio threshold for successful transmission in the multiple access channel. The upper bounds that are derived for both transmission models in arbitrary network and for protocol model in random networks are found to be in the same order of the constructed lower bounds, hence capacity of ad hoc networks as modeled becomes $\Theta(1/\sqrt{N})$ and $\Theta(1/\sqrt{N \log N})$ correspondingly.

Although Gupta and Kumar consider stationary nodes with the rationale that mobility can only deteriorate the capacity, Grossglauer and Tse [10] demonstrate that mobility can achieve higher rates asymptotically as the number of nodes increase. They assume a stationary and ergodic distribution where the location of a node is uniformly distributed on a disk and SINR based physical model is assumed for determining successful transmissions. The key point in their analysis is that when each source or relay node transmits to the closest receiver, asymptotically, SINR requirement for each transmission pair is satisfied with a positive probability value. Hence, given θN nodes are randomly selected as transmitters (where $0 < \theta < 1$), transmitters always choose the closest receiver to send. Since all transmitter-receiver pairs are equally likely to be scheduled, each link is activated with the probability at the order of $\Theta(1/N)$. Authors define a two phase scheduling policy as follows. In the first phase source nodes transmit their packets to the closest receiver (which can be a relay or the destination node) and in the second phase transmitters (which can be source or relay node) forward the packets with the destination same as the closest receiver. Thus for any source-destination pair, $(N-2)$ relay nodes receive and transmit packets at rate $\Theta(1/N)$ while source nodes also transmits directly to destination with $\Theta(1/N)$. Summing over all paths, each flow identified by the source-destination pair acquires a fixed rate, i.e. $\Theta(1)$ which is a significant improvement over the results of Gupta and Kumar. But this result is achieved at the expense of possibly excessive delays.

Gupta and Kumar extend their work on capacity of large wireless networks and follow an information theoretical perspective to find the sufficient conditions for achieving a rate region by allowing arbitrarily complex network coding [11]. Authors group relay nodes in disjoint sets for each source-destination pair and order them such that lower order sets can only forward data to higher order sets, hence defining a forwarding graph. All possible forwarding graphs are considered to determine the achievable rates. Although it is not shown that their approach eventually yields a capacity result, nevertheless they demonstrate that a specific wireless network of N nodes located in a region of unit area can indeed achieve a network throughput of $\Theta(N)$ bit-meters/sec or $\Theta(1)$ bits/sec data rate per node which is a remarkable gain over their original capacity results that is limited by inherently assumed point-to-point communication.

Gastpar and Vetterli also consider the information theoretical capacity for a simple relay case [7]. The main difference in their problem setting as compared to the previous works is that they consider only one source-destination pair and remaining $(N-2)$ nodes act as pure relays helping the source node to convey as much information as possible to the destination by repeating the received signal. To make things analytically tractable, authors introduce a slotted scheme, where source node transmits in the even slots and relays repeat the received signal with proper amplification in the odd slots. Unlike [11], the total transmit power in the relays are constrained to be in the same order of the number of ad hoc nodes and no individual relay is allowed to transmit at an unbounded power level as N goes to infinity. Thus, the transmit powers of the relay nodes must be coordinated. The slotted scheme allows to use the separation principle for the source and channel coding although for multi-user communication this is not the case in general. It is proved that channel capacity behaves at best as $\log(N)$ imposing an additional constraint of an arbitrarily small but positive separation between the ad hoc nodes.

In a recent technical report, Duarte-Melo and Liu address a many-to-one communication paradigm in multi hop sensor networks [12]. They first consider a flat network architecture in which sensor nodes are assumed to be uniformly distributed on a planar disk domain with a single base station located at the center of the disk. All sensors generate data traffic at the same rate towards this single base station. They adopt the protocol model for packet transmissions and find out the conditions where the trivial upper bound $O(W/N)$ can not be achieved for a given channel bandwidth of W bits/sec. Under the same conditions, they demonstrate that $O(W/2N)$ is asymptotically feasible. Then authors introduce clustering where this time base stations are placed on equally separated grid points. Each sensor directs its traffic towards the closest base station. Base stations forward the sensory data again to a central node using a wireless channel non-interfering with the transmissions within the clusters. Also assuming that there is no interference between the clusters, authors show that trivial upper bound can be achieved asymptotically.

As it is clear from our overview, network capacity can be drastically improved when mobility, network coding, redundant relay nodes and/or clustering are effectively exploited. We instead work on a new perspective which searches for the achievable wireless network capacity when an infrastructure network support is available at random ingress and egress points to the ad hoc users. Such provisioning reduces the burden on the ad hoc tier in terms of coordination overhead when the alternatives such as complex network coding, adding redundant ad hoc nodes, and clustering are considered.

In a very recent work [8], which we discovered after the bulk of our work has been completed, authors investigate the throughput capacity of a similar network architecture. In that architecture, infrastructure network is depicted as a cellular network where the access points are located at the center of hexagonal cells and inter-connected via a broadband wireline network. Authors are mainly interested in how the number of access points (hence the hexagonal cells) should scale with the number of ad hoc nodes to gain substantial network capacity improvement over the pure ad hoc operations. They impose different routing strategies that segments the randomly distributed ad hoc nodes into two groups depending on whether they use the cellular network to reach the destination or not. The decision criteria in forming the groups rely on heuristic arguments and are not necessarily the optimum routing strategies. Under such circumstances, they

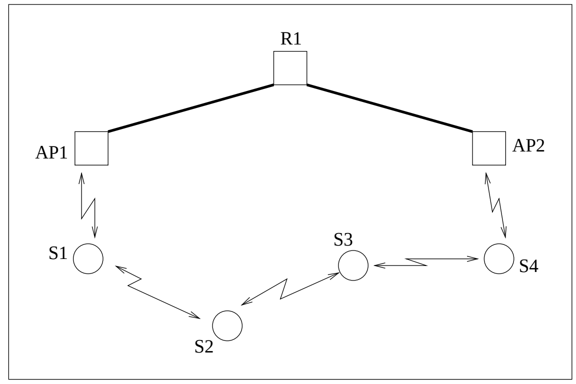

Fig. 1. Overlaid network architecture.

show that the number of access points should grow faster than $\sqrt{N}$ to have a noticeable gain. Their results also reveal that if all the bandwidth resources are allocated to the communication through the infrastructure network and number of access points are in the same order of ad hoc network size, then $\Theta(NW)$ bits/sec can be achieved as the total transport capacity. Note that such an allocation does not support all the source nodes and this capacity is mainly shared among the nodes which are routed through the infrastructure as determined by the routing layer.

Although there is a significant overlap between our network model and that of [8], there are also major differences as listed below that underlines the contribution of our work: (1) First of all, the type of the infrastructure network may not allow a hexagonal cell structure as we have already mentioned in the introduction. Assuming random locations for access points can give us a better capacity budget estimate of the scenarios where the access point locations are not on regular grid points. In fact, we will demonstrate in the latter sections that the network capacity of $\Theta(NW)$ bits/sec is not attainable in our random network model. (2) We specify the upper bound of throughput capacity over all routing and transmission strategies, then we design a specific routing and transmission scheme to achieve this upper bound. (3) Our constraints in terms of the connectivity requirements on the ad hoc network poses a different problem. (4) We show that the network throughput capacity can be achieved by a fair allocation of bandwidth among all users regardless of their destinations.

Having finished the overview of the related works and identified the distinguishing features of our problem, we are ready to proceed with the details of our network model in the next section.

III. Network Model

We consider a two-tier architecture where an ad hoc network is overlaid with an infrastructure network. Ad hoc nodes can communicate with each other along the paths that may reside entirely in the ad hoc tier, i.e. they cross only the ad hoc nodes. But, ad hoc nodes are also allowed to utilize the infrastructure network such that the flow paths can be partially overlapped with the infrastructure nodes and links. The infrastructure network is assumed to have relatively abundant bandwidth and the transmissions within each tier do not interfere with the other tier. The access between the two tiers is achieved through special nodes which will be referred as access points or gateway nodes. Without loss of generality and for clarity, access points are assumed only to relay the packets between each tier and they do not generate any data traffic themselves.

A typical scenario is depicted in Fig.1 which includes four ad hoc nodes (S1-S4), two access points (AP1-AP2), and a infrastructure router (R1). The infrastructure network can be a wireline network with exclusive links from R1 to AP1 and AP2, and vice versa. Suppose that we identify the amount of bandwidth resources used in the ad hoc network with the number of wireless hops involved for transmitting each packet from source to destination. Thus, when S2 has a packet for S4, it can be sent through S3 which uses *two hop resources* in the ad hoc network. The same packet could be sent through the path S2-S1-AP1-R1-AP2-S4 where three hop resources would be used instead, hence wasting an extra hop resources in the ad hoc tier compared to the previous path selection. Clearly, using the infrastructure did not really improve the efficient use of the ad hoc bandwidth resources. However, when all possible source-destination pairs are considered, we can save bandwidth resources of the ad hoc tier. Consider the case where S1 have packets to transmit for S4. Then choosing the path S1-AP1-R1-AP2-S4 spends two-hop resources whereas the alternative path S1-S2-S3-S4 wastes one extra hop of wireless bandwidth resources.

We limit our attention on a random network scenario where ad hoc nodes and access points are randomly distributed on a disk of area $A_R = \pi \mathcal{R}^2$ where R is the disk radius[3]. Each ad hoc node generates data traffic of rate $\lambda(N,K)$ bits/sec for a random destination in the ad hoc tier. Here, N and K are the number of ad hoc nodes and access points respectively. We assume that the number of ad hoc nodes per access point is bounded and $\lim_{N\to\infty}(N/K) = \alpha$ where $\alpha \in (0,\infty)$. The transmission radius of ad hoc nodes is assumed to be fixed, but it can be arbitrarily small as N goes to infinity as long as the connectivity of the ad hoc network is guaranteed. At that point we can directly use the result from [13] which states that on a unit area disk the transmission radius r_T must at least satisfy the following bound for having a connected graph with probability one.

$$r_T \geq \sqrt{\frac{\log(N)}{\pi N}} \tag{1}$$

We assume a total available bandwidth of W bits/sec which can be carried over multiple orthogonal channels (i.e. frequency band and/or code). The contention over the same channel is resolved in time and space. As a simple interference scheme, we adopt the *protocol model*. Due to this model, transmission from node i to node j in a specific combination of ad hoc channel and time slot is called *interference-free* if the following two conditions are satisfied: (i) Euclidean distance between i and j is smaller than or equal to r_T, i.e. $|X_i - X_j| \leq r_T$, where X_l represents the position vector of node l. (ii) There are no other transmitters around j at a distance of $r_I = (1+\Delta) \times r_T$ on the same channel and time slot, where $\Delta \geq 0$. These two conditions along with the triangle inequality imply that disks of radius $\Delta r_T/2$ centered at the receivers must be disjoint to be able to schedule them simultaneously in the same channel and time slot [1].

The throughput capacity is computed over all possible time-space scheduling of transmissions and flow paths. A per node throughput of $\lambda(N,K)$ is called *feasible* if there exist satisfying time-space scheduling and routing paths with unlimited buffering capabilities in the intermediate nodes. We call the per node throughput capacity of the random network as described to be in the order of $\Theta(f(N,K))$ bits/sec if there are deterministic constants $0 < c < c' < \infty$, such that;

$$\lim_{N\to\infty} Prob(\lambda(N,K) = cf(N,K) \ is \ feasible) = 1 \ ,$$
$$\liminf_{N\to\infty} Prob(\lambda(N,K) = c'f(N,K) \ is \ feasible) < 1 \ .$$

In the next section, we provide some asymptotic results that will capture the benefits of using an infrastructure network even in random scenarios.

IV. Capacity Improvement with Infrastructure Layer

The tools to derive the capacity result for our network model will not be very different from the ones used in [1]. Using the interference-free transmission model, we can bound the number of simultaneous successful transmissions by the number of disks with radius $\Delta r_T/2$ that can be packed inside the disk of area A_R. But the boundary effects require modification in our argument. If a receiver is located close to the boundary of the disk domain such that the interference disk around the receiver is not completely inside the domain, then the region occupied by the transmissions to that receiver is smaller than the interference disk area. When the receiver is exactly on the boundary, then the occupied region has the smallest area. The occupied region is minimized as a ratio of interference disk area when the interference radius becomes $2R$ -i.e. diameter of the domain- and quarter of the interference disk area effectively occupies the domain. Hence, number of simultaneous transmissions must be smaller than $16A_R/(\pi\Delta^2 r_T^2)$. As a result, given the average number of hops $\bar{h}(N,K)$ within the ad hoc tier, total bandwidth W, and per node throughput $\lambda(N,K)$, following inequality holds.

$$N\lambda(N,K)\bar{h}(N,K) \leq \frac{16A_R W}{\pi\Delta^2 r_T^2} \tag{2}$$

The dependence of $\bar{h}$ on N is a natural consequence of letting transmission range to be smaller as N gets larger, while its dependence on K is the result of routing decisions which may be based on the location and number of the gateway nodes. Using the inequality (1) and the fact that $\bar{h}(N,K) \geq 1$, with probability of one (as N goes to ∞), the following upper bound holds under any routing and scheduling decision.

$$\lambda(N,K) \leq \frac{16A_R W}{\Delta^2 \log(N)} \tag{3}$$

[3]Although access points are also physically part of the ad hoc tier, we functionally treat them different. Unless otherwise is explicitly specified, when we call *ad hoc nodes*, we exclude the access points.

Next, we will show that $\Theta[W/\log(N)]$ is the actual per node throughput capacity by finding appropriate time-space scheduling and routing schemes which asymptotically achieve the upper bound in (3) with probability one. The following steps are involved in the construction of this *optimal* joint scheduling and routing scheme: (1) We create a Voronoi tessellation[4] on a disk with the area A_R where each Voronoi cell completely covers an area of $100A_R\log(N+K)/(N+K)$. We also set the transmission range such that any node can reach to other nodes in the same Voronoi cell. (2) We show that the number of Voronoi cells that interfere with the transmissions of a specific cell is bounded above by a constant C. (3) We prove that the number of ad hoc nodes including gateway nodes in each Voronoi cell is indeed less than $O(\log(N+K))$. (4) We demonstrate that each Voronoi cell includes at least one access point. (5) Finally we show that the number of destination nodes per access point within a Voronoi cell is $\Theta(1)$.

Before explaining each of these steps in detail, let's jump ahead and first examine their implications in our construction. Suppose that time is divided into slots with fine granularity and each node use the whole bandwidth W in the time slot it is transmitting. When steps 2 and 3 are considered together, we can schedule each node in a Voronoi cell including the access points without any conflict by assigning $W/[(C+1)\log(N+K)]$ amount of bandwidth to that node. On the other hand, steps 1 and 4 provide us the routing algorithm we are searching for: (i) If both the source and destination nodes are in the same Voronoi cell, then source node transmits to the destination node in single hop using its own share of bandwidth. (ii) Otherwise the source node can use its share of bandwidth to reach any access point in its own cell. Once, the data packets reach to the selected access point, they can be relayed up to one of the access points which share the same Voronoi cell as the destination node without any packet loss. Step 5 ensures that we can assign bounded number of destination nodes to each access point. Hence each access point divides its bandwidth share further by a constant value. Although access points before the destination nodes become the throughput bottleneck, nevertheless an end-to-end rate of $W/[C_1(\log(N)+\log(1+K/N))]$ per source node is supported. Since this result is asymptotic and $\lim_{N\to\infty} K/N = 1/\alpha$, we constructed the following lower bound which implies that per node throughput capacity for random network with infrastructure becomes $\Theta(W/\log(N))$.

$$\lambda(N,K) \geq \frac{W}{C_1\left[\log(N)+\log(1+\frac{1}{\alpha})\right]} \tag{4}$$

Now, we are ready to proceed with the individual steps to under-fill the result as found in (4).

***STEP 1*:**

We will repeat many procedures already established in [1] for the sake of completeness. Recall that the Voronoi tessellation of a closed region on $\mathcal{R}^2$ is defined by a set of points p on the region. Each Voronoi cell is identified by a point $p_i \in p$ and consists of the set of all nodes that are closer to p_i than any other point in p. Here on, the distance is measured simply in Euclidean distance. We provide a modified version of the lemma from [1] to make it directly applicable to disks in $\mathcal{R}^2$.

[4]Voronoi tessellation on a region is formed by a set of construction points on this region. Each construction point identifies a unique Voronoi cell and all the remaining points on the region are partitioned into disjoint Voronoi cells by assigning each point to the Voronoi cell represented by the closest construction point to that point [14].

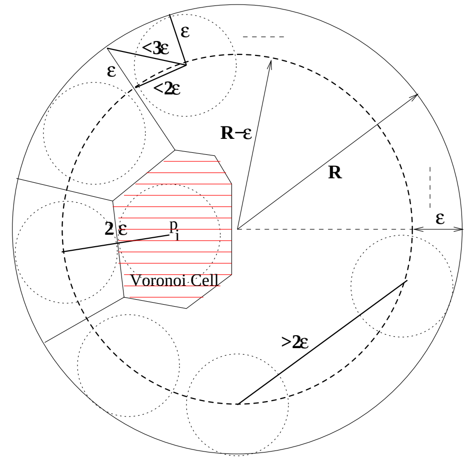

Fig. 2. Formation of a Voronoi tessellation on the disk domain with radius R. Each Voronoi cell can be sandwiched between disks of radius ϵ and 3ϵ

Lemma 1: For every $R \geq \epsilon > 0$, there is a Voronoi tessellation of a disk of radius R in $\mathcal{R}^2$ with the property that each Voronoi cell contains a disk of radius ϵ and is contained in a disk of radius 3ϵ.

Proof: Let $D(x,\epsilon)$ denotes the disk centered at point x with radius ϵ. We form the tessellation in two rounds. We start the first round with a construction point p_1 which is exactly at a distance of ϵ from the disk domain boundary (see fig.2). Given the first $(i-1)$ points, the next construction point p_i is selected such that the distance between p_i and the disk domain boundary is exactly ϵ while the distances between p_i and previously selected points are at least 2ϵ. Since these points lie on the finite perimeter of a circle which is concentric with domain disk and has a radius $(R-\epsilon)$, the first round terminates eventually. In the second round, we add a new construction point p_j only on the inner disk of radius $(R-\epsilon)$ and only if $D(p_j,\epsilon)$ does not intersect $D(p_i,\epsilon)$ for already selected p_i's. Since we have a bounded area and each addition of a point removes a finite portion of the available area to select a point from, second round eventually halts. The Voronoi tessellation generated by points p_i satisfy the properties of the lemma. To be precise, suppose point x is closer to construction point p_i than to any other construction point. If x lies on the inner disk of radius $(R-\epsilon)$, it is at most 2ϵ away from p_i otherwise it would be at a distance larger than 2ϵ from all construction points and the disk $D(x,\epsilon)$ would not intersect with the disks $D(p_j,\epsilon)$ contradicting to our construction. On the other hand if x lies outside the disk of radius $(R-\epsilon)$, using triangular inequality it must be at most 3ϵ away from p_i. It is also clear from our construction that each Voronoi cell covers a disk of radius ϵ, otherwise at least two disks $D(p_i,\epsilon)$ and $D(p_j,\epsilon)$ for $i \neq j$ would intersect by again violating our construction. ■

Thus, when we choose ϵ and the transmission range r_T such that $\pi\epsilon^2 = 100 A_R \log(N+K)/(N+K)$ and $r_T = 6\epsilon$, lemma-1 guarantees us that we have a tessellation of which each Voronoi cell covers at least an area of $100 A_R \log(N+K)/(N+K)$ while each node can reach to other nodes in the same cell in single hop. The following steps will provide the basis of designing a joint routing and scheduling scheme built upon this particular tessellation.

STEP 2:

Any Voronoi cell V' interferes with another Voronoi cell V if V' and V include points that are at most $(r_T + r_I) = (2+\Delta)r_T$ apart. Assuming the worst case condition, these points could be just on the boundaries of each cell and since the Voronoi cells have a diameter less than or equal to 6ϵ, any interfering cell for V must be located in a region with a radius of $9\epsilon + (2+\Delta)r_T$. Since each cell area is lower bounded by $\pi\epsilon^2$ and we already have set $r_T = 6\epsilon$, there can be at most,

$$C = \left\lfloor \frac{\pi(9\epsilon + (2+\Delta)r_T)^2}{\pi\epsilon^2} \right\rfloor - 1 = \left\lfloor (21+6\Delta)^2 \right\rfloor - 1 \ ,$$

interfering cells in the neighborhood of V. Notice that C is a constant which depends only on the medium access protocol specific parameter Δ. Now, it is a straight-forward application of the graph theory to demonstrate that $(C+1)$ slots are enough to schedule one transmission for each cell in a conflict-free manner. When each Voronoi cell is represented by a vertex and we draw an edge between any two vertices if the corresponding cells are interfering with each other, we have a graph coloring problem in our hands where colors correspond to different time slots. Since this graph has a maximum degree of C, we can color it using at most $(C+1)$ colors. The corollary of this result is that we have a scheduling of length $(C+1)$ slots to allocate for each Voronoi cell in a round robin fashion. In each slot, the corresponding cell utilizes the entire bandwidth. We can then further introduce sub-slots within each slot to allocate equal amount of bandwidth among the ad hoc nodes and the access points over the ad hoc channels in the same cell. The order of the number of these sub-slots will be same as the order of the number of users in the same cell which is obtained in the next step.

STEP 3:

We use the *Vapnik-Chervonenkis Theorem* and a lemma from [1] to prove that each Voronoi cell include less than $O(\log(N+K))$ nodes.

Theorem 1 (The Vapnik-Chervonenkis Theorem): If $\mathcal{F}$ is a set of finite VC-dimension VC-d($\mathcal{F}$), and $\{X_i\}$ is a sequence of i.i.d. random variables with common probability distribution P, then for every $\epsilon, \delta > 0$,

$$Prob\left(\sup_{F\in\mathcal{F}} |\frac{1}{L}\sum_{i=1}^{L} I(X_i \in F) - P(F)| \le \epsilon\right) > 1-\delta \ ,$$

whenever

$$L > \max\left(\frac{8VC - d(\mathcal{F})}{\epsilon}\log\frac{16e}{\epsilon}, \frac{4}{\epsilon}\log\frac{2}{\delta}\right).$$

Lemma 2: The Vapnik-Chervonenkis dimension (VC-d) of the set of disks in $\mathcal{R}^2$ is 3.

Then, by letting the sequence $\{X_i\}$ be the random positions of ad hoc nodes and access points, L equal to $N+K$, and $\mathcal{F}$ be the set of disks in $\mathcal{R}^2$ with area $900 A_R \log(N+K)/(N+K)$ so that the disk area entirely covers a Voronoi cell, we obtain:

$$Prob\left(\sup_{D\in\mathcal{F}} |\frac{Number\ of\ nodes\ in\ D}{N+K} - P(D)| \le \epsilon\right) > 1-\delta \ , \quad (5)$$

whenever

$$N+K > \max\left(\frac{24}{\epsilon}\log\frac{16e}{\epsilon}, \frac{4}{\epsilon}\log\frac{2}{\delta}\right) \ . \quad (6)$$

Equation (5) implies that;

$$Prob\left(\frac{Number\ of\ nodes\ in\ D}{N+K} \le \sup_{D\in\mathcal{F}}[P(D)] + \epsilon\right) > 1-\delta \ . \quad (7)$$

But, $\sup_{D\in\mathcal{F}}[P(D)] = 900\log(N+K)]/(N+K)$ and setting $\epsilon = \delta = 100\log(N+K)/(N+K)$ satisfies (6) at least for large $N+K$. Hence, we have

$$Prob\{Number\ of\ nodes\ in\ any\ Voronoi\ cell \le 1000\log(N+K)\} > 1 - \delta(N+K) \ .$$

We basically proved that with probability one, total number of access points and ad hoc nodes within each Voronoi cell in the constructed tessellation is $O(\log(N+K))$ as $(N+K) \to \infty$. Now, what remains to prove is that there are enough number of access points in each Voronoi cell to be able to route the packets from source nodes to infrastructure[5] and from access points to the destination nodes without effecting the order of bandwidth allocated to each transmitter.

STEP 4 & 5:

Steps 4 and 5 are again straightforward applications of the Vapnik-Chervonenkis Theorem and lemma 2. But, this time, we let the sequence $\{X_i\}$ be the random positions of *access points*, $\mathcal{F}$ be the set of disks in $\mathcal{R}^2$ with area $100 A_R \log(N+K)/(N+K)$, and we set $\epsilon = \delta = 50\log(N+K)/(N+K)$ to obtain the following result as $N \to \infty$.

$$Prob\left\{Number\ of\ access\ points\ in\ any\ Voronoi\ cell \ge \frac{50\log(N+K)}{(1+\alpha)}\right\} > 1-\delta \ .$$

Asymptotic lower bound given in equation 8 is also valid for number of ad hoc nodes if we substitute α with $1/\alpha$. These lower bounds and step 2 together imply that both number of ad hoc nodes and access points belonging to same Voronoi cell are asymptotically in the same order, i.e. $\Theta(\log(N+K))$, hence number of distinct destination points per access point is bounded by $\Theta(1)$ for large (N+K). But, since the source-destination pairs are selected randomly, different source nodes can generate packets for the same destination with a finite probability. In fact, this turns out to be a small technicality in

[5] Actually one access point per cell is enough for this purpose

the asymptotic results. Suppose Y_i denote the position vector of the destination node corresponding to the source node i in our disk domain. Then, $\{Y_i\}$ is a sequence of uniformly distributed i.i.d. random variables. This allows us to use the same $\mathcal{F}$ and $\epsilon = \delta = 100\log(N+K)/(N+K)$ as in step 3, except for now we have upper-bounded the number of destination points with $O(\log(N+K))$.

Hence, we completed all the steps required for deriving the lower bound as given in inequality (4). Upper and lower bounds in (3) and (4) states that the throughput capacity for each ad hoc node is $\Theta(W/\log(N))$. This also becomes the first main result of our paper. In the next section, we will modify our connectivity assumption to search for the full benefits of having the infrastructure network in terms of the throughput capacity.

V. Looser Connectivity Constraints and Achievability of $\Theta(1)$

So far, we have assumed a strong connectivity condition in our network model, i.e. the network graph consisting of only the ad hoc nodes excluding the access points is required to be connected. The underlying logic is simple; it is often desirable to have an ad hoc network which can function without any infrastructure. However, this constraint does not fully capture the benefits of exploiting the infrastructure either. Accordingly we should relax our connectivity condition as follows: each ad hoc node should be connected to at least one access point with high probability and this probability must be approaching to one as number of nodes increases. This is equivalent to considering the ad hoc network and the infrastructure as a single topology graph and define the connectivity according to this broaden topology. We will refer to this specific definition of connectivity as *connectivity in the weak sense* or *weak connectivity*. This section is dedicated towards obtaining first the necessary and sufficient conditions on the transmission range to achieve the weak connectivity, then to show that even under weaker connectivity condition, we can not have a per node transport capacity of $\Theta(1)$ as it can be widely seen under different network scenarios [11], [10], [8].

In the simplest form of weak connectivity, there exists at least one access point within the transmission range of the ad hoc node. Hence given that there are K gateway nodes, X_i denote the location of node i uniformly distributed on disk domain, each node i has a capture area $A_c^i(X_i)$ where its neighbors can be located, and A_ϵ denote the disk area with radius $\epsilon = r_T$, the following relations hold:

$$\begin{aligned}
&Prob[Node\ i\ connected\ to\ any\ access\ point \mid X_i = x] \\
&\geq Prob[Node\ i\ has\ an\ adjacent\ access\ point \mid X_i = x] \\
&\overset{(a)}{=} 1 - \left(1 - \frac{A_c^i(x)}{A_R}\right)^K \\
&\overset{(b)}{\geq} 1 - \left(1 - \frac{A_\epsilon}{4A_R}\right)^K \quad . \qquad (8)
\end{aligned}$$

Here, step (a) follows directly considering the case where no access point is located in the capture area of node i and step (b) follows from the boundary effect of the disk domain, i.e. at least quarter of a disk centered at the ad hoc node with radius equal to transmission range must be totally covered by the capture area. Integrating both sides of (8) over the disk domain and taking the limit, we find the asymptotic lower bound as:

$$\begin{aligned}
&Prob[Node\ i\ connected\ to\ any\ access\ point] \\
&\geq 1 - \lim_{K\to\infty}\left(1 - \frac{A_\epsilon}{4A_R}\right)^K \quad . \qquad (9)
\end{aligned}$$

We can also obtain an upper bound similar to the right hand side of the expression in (9) for the probability of connectivity. Let N denote the number of ad hoc nodes. The event that node i is not connected to an access point includes the event that i is isolated. Hence, the upper bound can be derived as follows.

$$\begin{aligned}
&Prob[Node\ i\ disconnected\ from\ access\ points \mid X_i = x] \\
&\geq Prob[Node\ i\ is\ isolated \mid X_i = x] \\
&\overset{(a)}{=} \left(1 - \frac{A_c^i(x)}{A_R}\right)^{N+K-1} \\
&\overset{(b)}{\geq} \left(1 - \frac{A_\epsilon}{A_R}\right)^{c_2 K} \quad . \qquad (10)
\end{aligned}$$

Step (a) again is a result of having no other nodes including access points within the capture area uniquely defined by the position of node i on the disk domain and transmission radius. And step (b) comes from the observation that $A_\epsilon/4 \leq A_c^i(x) \leq A_\epsilon$ in addition to the initial assumption $K = \Theta(N)$. Again integrating both sides of inequality (10) over the disk domain, we get rid of the conditional probability;

$$\begin{aligned}
&Prob[Node\ i\ disconnected\ from\ access\ points] \\
&\geq \left(1 - \frac{A_\epsilon}{A_R}\right)^{c_2 K} \quad . \qquad (11)
\end{aligned}$$

By simple manipulations and taking the limit, we obtain an asymptotic upper bound for weak connectivity.

$$\begin{aligned}
&Prob[Node\ i\ connected\ to\ any\ access\ point] \\
&\leq 1 - \lim_{K\to\infty}\left(1 - \frac{A_\epsilon}{A_R}\right)^{c_2 K} \quad . \qquad (12)
\end{aligned}$$

Next, we introduce a lemma that will help us to compute the limits in the lower and upper bound expressions given in (9) and (12) respectively.

Lemma 3: Let $a(x)$ and $b(x)$ be differentiable functions such that following properties are satisfied: (i) There exists x_1 such that $1/b(x) \neq 0$ for all $x \in (x_1, \infty)$, (ii) $\lim_{x\to\infty} a(x) = \pm\infty$ and $lim_{x\to\infty} b(x) = \pm\infty$. Then

$$\lim_{x\to\infty}\left(1 + \frac{1}{a(x)}\right)^{b(x)} = \exp\left[\lim_{x\to\infty}\left(\frac{b(x)^2}{a(x)^2}\frac{\dot{a}(x)}{\dot{b}(x)}\right)\right] ,$$

provided that the limit on the right hand side exists in $\mathcal{R}^+ = \mathcal{R}\cup\{\infty, -\infty\}$. Above, $\dot{a}(x)$ and $\dot{b}(x)$ represent the derivatives of $a(x)$ and $b(x)$ with respect to x.

Proof: See appendix. ■

To apply lemma-3, we need to overcome an obvious technicality. Our upper and lower bound expressions are sequences with non-negative integer indices, but the lemma considers continuous functions. For that reason, we will consider the sequence $A_\epsilon(K)$ as a sampling from a continuous function that will capture the desired features of transmission range r_T as a function of number of nodes in the network.

Definition 1: $A_\epsilon(K) = \int_0^\infty A_\epsilon(x)\delta(x-K)dx$ where $\delta(x-K)$ is the Dirac-Delta function, $A_\epsilon(x)$ is a non-increasing differentiable function of x and has a lower bound 0. Hence $\lim_{x\to\infty} A_\epsilon = 0$.

From the definition, it is clear that $A_\epsilon(K)$ and $r_T(K)$ are assumed to be monotonically non-increasing sequences with limits 0. The underlying rationale is simple: we are looking for necessary and sufficient conditions for $A_\epsilon(K)$ that will make the asymptotic probability of connectivity arbitrarily small yet we want to minimize $A_\epsilon(K)$ so that we can pack as many transmission as we can in the same channel maximizing the upper bound. Putting more access points while keeping the r_T same would increase the probability of connectivity as seen from (9). Then, we can reduce r_T at a smaller pace than the increase in number of access points, and yet improve the probability of connectivity. Next lemma will introduce the sufficiency condition for the existence of limits in upper and lower bounds.

Lemma 4: Let $\Gamma_K = [1 - a_1 A_\epsilon(K)]^{a_2 K}$ and $\Gamma(x) = [1-a_1 A_\epsilon(x)]^{a_2 x}$. If $\lim_{x\to\infty}\Gamma(x)$ exists, then $\lim_{K\to\infty}\Gamma_K = \lim_{x\to\infty}\Gamma(x)$.

Proof: From the definition of limit, $\forall\epsilon$, $\exists K_0$ such that if $x > K_0$ then $|\Gamma(x) - \Gamma^*| < \epsilon$. Substituting K_0 with $\lfloor K_0 \rfloor$ and x with K completes the proof. ■

Since we have established a relation between Γ_K and $\Gamma(x)$, we are ready to apply lemma-3 to compute the limit of $\Gamma(x)$. To do this, we set $a(x) = -1/a_1 A_\epsilon(x)$ and $b(x) = a_2 x$. Since conditions of lemma-3 are satisfied, we have the following relations given that the limit on right hand side of the equation exists in set of extended real numbers.

$$\begin{aligned}
&\lim_{x\to\infty} [\Gamma(x) = (1 - a_1 A_\epsilon(x))^{a_2 x}] \\
&\quad = \exp\left[\lim_{x\to\infty}\left(\frac{a_2^2 x^2}{1/a_1^2 A_\epsilon^2(x)}\frac{(-1/a_1)\dot{A_\epsilon^{-1}}(x)}{a_2\dot{x}}\right)\right] , \\
&\quad = \exp\left[\lim_{x\to\infty}\left(a_1 a_2 x^2 \dot{A}_\epsilon(x)\right)\right] . \qquad (13)
\end{aligned}$$

Equation (13) provides us valuable insights about the necessary and sufficient conditions for connectivity in the weak sense as stated below in theorem-2. But, first we provide some useful properties of $\dot{A}_\epsilon(x)$.

Property 1: $\dot{A}_\epsilon(x) \le 0$ for all x.

Proof: Follows from non-increasing property of $A_\epsilon(x)$. ■

Property 2: If there exists a X_0 such that $\dot{A}_\epsilon$ is continuous for all $x \ge X_0$, then $\lim_{x\to\infty}\dot{A}_\epsilon(x) = 0$.

Proof: Suppose limit does not exist or it is not zero. Then there exists $\epsilon_i > 0$ for all $X_i \ge X_0$ such that $|\dot{A}_\epsilon(x)| \ge \epsilon_i$ in a non-zero length interval (x_i, x_{i+1}) where $x_{i+1} > x_i \ge X_i$. Here, non-zero length interval is a consequence of continuity. Since this statement is true for all $X_i = x_{i+1}$, there are infinitely many finite intervals where $\dot{A}_\epsilon(x) \le -\min_i \epsilon_i$ and in other intervals $\dot{A}_\epsilon(x)$ is at most 0 (using property-1), the integral (hence $A_\epsilon(x)$) diverges to -∞. This contradicts to the definition of $A_\epsilon(x)$. ■

Theorem 2: Given that $\dot{A}_\epsilon(x)$ is continuous, the network is asymptotically connected in the weak sense with probability approaching one if and only if

$$\lim_{x\to\infty}\left(x^2\dot{A}_\epsilon(x)\right) = -\infty \ .$$

Proof: If we show that $\lim_{x\to\infty}(x^2\dot{A}_\epsilon(x))$ exists in $\mathcal{R}^+ = \mathcal{R}\cup\{\infty,-\infty\}$, then using relation (13) and lemma-4, we prove the existence of limits for upper and lower bounds. Clearly these limits are equal to 1 if and only if $\lim_{x\to\infty}(x^2\dot{A}_\epsilon(x)) = -\infty$. Thus for completing the proof of the lemma, we are bound to demonstrate existence of $\lim_{x\to\infty}(x^2\dot{A}_\epsilon(x))$ in set of extended real numbers. We will use the way of contradiction to do it.

Suppose there is no limit, then for every real number x^*, there exists $x_0 > X_0$ and $\epsilon_0 > 0$ for all X_0 such that,

$$|x_0^2\dot{A}_\epsilon(x_0) - x^*| \ge \epsilon_0 \ .$$

Otherwise the limit would exist and be equal to x^*. Using our freedom of choosing any x^*, let's set $x^* = 0$. Then,

$$|x_0^2\dot{A}_\epsilon(x_0)| \ge \epsilon_0 \iff |\dot{A}_\epsilon(x_0)| \ge \epsilon_0/x_0^2 \ ,$$

for some $x_0 > X_0$, $\epsilon_0 > 0$ and any X_0. But we know by property-2, $\lim_{x\to\infty}\dot{A}_\epsilon(x)$ is 0, then for all $\epsilon_1 > 0$ there exists X_1 such that $|\dot{A}_\epsilon(x)| < \epsilon_1$ when $x > X_1$. If we set $\epsilon_1 = \epsilon_0/x_0^2$ and $X_0 = X_1$, we have our contradiction. ■

Corollary 1: Given that $\dot{A}_\epsilon(x)$ is continuous, the network is asymptotically disconnected in the weak sense with probability approaching one if and only if

$$\lim_{x\to\infty}\left(x^2\dot{A}_\epsilon(x)\right) = 0 \ .$$

Corollary 2: The network is **not** asymptotically connected in the weak sense with probability approaching one if $A_\epsilon(K) \le c_3/K$ for any positive finite number c_3.

Proof: First, observe that if the network is disconnected in the weak sense for the disk area $A_\epsilon(K)$, it is also disconnected for any other disk area $A_{\epsilon'}(K) \le A_\epsilon(K)$. Suppose $A_\epsilon(K) = c_3/K$, then clearly $A_\epsilon(x) = c_3/x$ satisfies the definition-1 as well as the hypothesis of theorem-2. Since $x^2\dot{A}_\epsilon(x) = -c_3 > -\infty$, theorem-2 states that we do not have weak connectivity with arbitrarily high probability. Thus, it is also true for all $A_{\epsilon'}(K) \le c_3/K$. ■

We can actually prove a more stringent requirement by conditioning connectivity on all nodes, i.e. instead of any node i, all the ad hoc nodes in the network must be asymptotically connected to the infrastructure access points with probability one.

Theorem 3: Let $\mathcal{Y}$ denote the number of nodes that are connected to at least one access point. Then expected value of $\mathcal{Y}$, i.e. $E[\mathcal{Y}]$, becomes $\Theta(N)$ for large N if $\lim_{x\to\infty}(x^2\dot{A}_\epsilon(x)) < 0$. In addition, if any node i is connected to at least one access point with arbitrarily high probability as increasing N (or K), it is also true that all nodes are asymptotically connected to at least one access point with arbitrarily high probability.

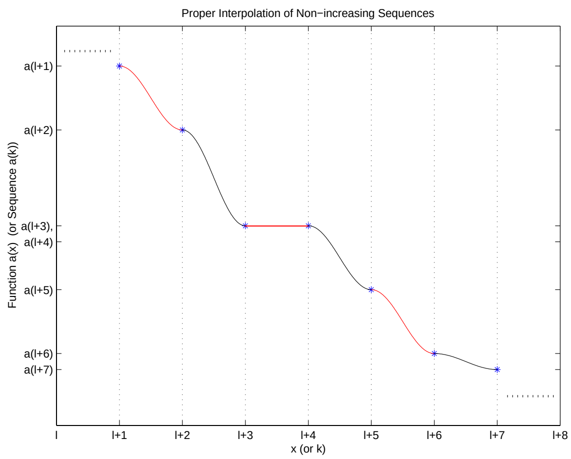

Fig. 3. Representing non-increasing sequences by differentiable functions with continuous first order derivatives.

Proof: See appendix. ∎

We are ready to state the main result of this section by revisiting the upper bound expression as given in (2). The corollary-2 necessitates that $r_T > c_4/\sqrt{\pi N}$, therefore;

$$\lambda(N,K) < \frac{16 A_R W}{c_4^2 \Delta^2} \ , \tag{14}$$

for any positive finite number c_4. In other words, per node throughput can **not** reach to $\Theta(1)$ as $N \rightarrow \infty$. Hence, we have the following theorem.

Theorem 4: Even under weak connectivity condition, per node transport capacity of $\Theta(1)$ can **not** be achieved with probability one.

Now, there remains one subtle point to make the arguments that we made so far rigorous. We started from non-increasing sequences as an index of number of nodes, then we have found necessary and sufficient conditions in terms of any non-increasing differentiable function $A_\epsilon(x)$ with following conditions: $A_\epsilon(x)$ (i) has samples at non-negative integer points equal to the sequence of interest, (ii) has a limit 0, and (iii) has a continuous derivative function.[6] Let's define the set of all such functions as $S_\epsilon = \{A_\epsilon(x)\}$. Our results are general in the sense that we can pick any function from S_ϵ and yet use the result given in theorem-2, corollary-1, corollary-2 and theorem-3. The question is if we can find at least one such function for every sequence of interest. We pictorially demonstrate below that it is indeed the case. Thus, the set of $\{S_\epsilon\}$ represents all possible sequences in which we are interested.

In fig.3, we interpolate any two different valued consecutive sequence points with a cosine function with period 2 in interval $[0, pi]$ where it is a monotonically decreasing function. The amplitude of cosine is shifted in time and amplitude such that it exactly fits into the corresponding interval. If two consecutive points are same, then we interpolate between these points with a straight horizontal line. Clearly this piecewise defined function is differentiable. Moreover, the derivative is equal to zero at integer points and behaves as a sine function in between preserving continuity.

[6]Note that, since we are mainly interested in the asymptotic behavior, we can modify the statements of definitions, lemmas, and theorems in this section by requiring continuity and monotonicity features only for large K or x values.

The necessary and sufficient conditions as stated in theorem-2 provide us the mechanisms to check the order of transmission radius and consequently the upper bound above which per node transport capacity can not be achieved with probability one. The question of whether one can find a minimal function on the order of transmission radius (equivalently the maximum upper bound) that conforms with these conditions is not addressed in this paper. Rather, we show that any upper bound conforming with the necessary and sufficient conditions can indeed be achieved with probability one as N goes to ∞.

VI. Achievability of the Upper Bounds for Connectivity in the Weak Sense

The design steps to show the achievability of any upper bound derived from a transmission area $A_\epsilon(N)$ satisfying the requirements of weak sense connectivity with probability one are exactly same as the steps in section-IV. Though, there are two nuances; first the disk areas covered by Voronoi cells in the new tessellation are different and secondly we can not apply Vapnik-Chervonenkis Theorem for any disk area of interest.

Without loss of generality, let us define $A_\epsilon(N)$ as $g(N)/N$ and suppose $A_\epsilon(N)$ satisfies the hypothesis of theorem-2. Thus, using equation (2) and assuming $r_T \geq \sqrt{A_\epsilon(N)/\pi}$, the upper bound for per node throughput capacity becomes

$$\lambda(N,K) \leq \frac{16 A_R W}{\Delta^2 g(N)} \ . \tag{15}$$

To show that the upper bound given above is achievable with probability one, we form the tessellation such that $\pi\epsilon^2 = A_R g(N)/N$ and $r_T = 6\epsilon$ (see step one in section-IV). As usual, each Voronoi cell is confined between two disks of radii ϵ and 2ϵ respectively. Hence, we need to prove that each Voronoi includes $\Theta(g(N))$ ad hoc nodes, access points, and destination points with arbitrarily high probability as $N \rightarrow \infty$.

As usual let X_i denote the position of node i in the disk domain. Note that, we do not differentiate between node i being a source node, an access point, or a destination node, since clearly $\{X_i\}$ are i.i.d. random variables with uniform distribution across the disk domain in all cases. Define $Y_L \triangleq \sum_{i=1}^{L} I(X_i \in \mathcal{V})$, where $\mathcal{V}$ is a particular Voronoi cell. Since L will be either N or K, we have $\lim_{N\rightarrow\infty}(L/N) = \Theta(1)$. Thus, we have $\overline{Y} = E[Y] = LP(X_i \in \mathcal{V})$ and $\sigma_Y^2 = Var[Y] = LP(X_i \in \mathcal{V})(1 - P(X_i \in \mathcal{V}))$. Since $P(X_i \in \mathcal{V}) = \Theta(g(N)/N)$, we can use the well-known Chebyshev's inequality [15] as follows:

$$P\left[|Y - L\Theta(\frac{g(N)}{N})| < \gamma\right] \geq 1 - \frac{L\Theta(g(N)/N)(1 - \Theta(g(N)/N))}{\gamma^2} \ .$$

But, here γ can assume any positive value and setting $\gamma = \Theta(g(N))$ simplifies the inequality above further as;

$$P\left[Y = \Theta(g(N))\right] \geq 1 + \frac{1}{\Theta(N)} - \frac{1}{\Theta(g(N))} \ .$$

The results from the previous section require that $g(N)$ can not be bounded above with a finite value and $g(N)/N$ must be defined for all positive integers N. Therefore $\lim_{N\to\infty} g(N) = \infty$. In other words, number of regular ad hoc nodes, access points, and destination nodes in any Voronoi cell is asymptotically in the order of $\Theta(g(N))$ with probability one. Hence, we can actually achieve any upper bound that conforms to the condition given in theorem-2.

This section completes our results on per node throughput capacity of random ad hoc networks with infrastructure support. Illustrative examples that signify the strength of the results presented in last two section are given below before we conclude our paper.

Example 1: Let $g(N)$ be $N^{1/p}$ where $p > 1$ is a constant number. Then, $A_\epsilon(N)$ becomes $N^{1/p}/N = N^{1/p-1}$. Trivially choosing $A_\epsilon(x) = x^{1/p-1}$ provides us a continuously differentiable and monotonically decreasing function for $x > 0$. Since

$$\lim_{x\to\infty} x^2 \dot{A}_\epsilon(x) = \lim_{x\to\infty} \left(\frac{1}{p} - 1\right) x^{1/p} = -\infty \ ,$$

$A_\epsilon(x) = x^{1/p-1}$ satisfies weak connectivity condition with probability one. Thus, the corresponding upper bound $\Theta(1/N^{1/p})$ by selecting $A_\epsilon(N) = \Theta(N^{1/p-1})$ is achievable. ■

Example 2: Let $g(N)$ behave as a recursive logarithm function [16] for large N, i.e. $g(N) = \ln^{(m)}(N)$ for $N \geq N_0$ where m, N_0 are positive finite numbers and $\ln^{(m)}(\cdot)$ denotes taking natural logarithm of the argument m times. Then, $A_\epsilon(N)$ becomes $\ln^{(m)}(N)/N$. Simply substituting discrete variable N with continuous variable x gives us a continuously differentiable function $A_\epsilon(x)$ which is monotonically decreasing for sufficiently large x. Since

$$\lim_{x\to\infty} x^2 \dot{A}_\epsilon(x) = \lim_{x\to\infty} \left[\frac{1}{\prod_{i=1}^{m-1} \ln^{(i)}(x)} - \ln^{(m)}(x)\right] = -\infty,$$

$A_\epsilon(N) = \ln^{(m)}(N)/N$ satisfies weak connectivity constraint with probability one. Moreover $\lim_{N\to\infty} \ln^{(m)}(N) = \infty$, thus per node throughput $\Theta[1/\ln^{(m)}(N)]$ is feasible with probability one for any constant $m > 0$. ■

VII. Conclusion

In this paper, we addressed the benefits of using a hybrid network architecture over pure ad hoc wireless networks with no infrastructure support in terms of per node throughput capacity. We showed that adding an infrastructure which provides access to the ad hoc users at random locations improve the per node throughput significantly over the infrastructure-less operation. Such a hybrid network model is adequate especially when the access points of the infrastructure network are not placed on regular grid points. Supporting examples can be given from a wide span of scenarios, e.g. sensor networks formed by scattering the sensors, some of which have long-range radio trancievers, over a terrain, cellular/WLAN networks with wireless/mobile relays, ad hoc networks with airborne communication node (ACN) support, etc.

We started with a strict connectivity constraint under which ad hoc tier must preserve the connectivity with arbitrarily high probability for stand-alone functionality. Asymptotic capacity figures are derived under this regimen. Results reveal that $\Theta(\sqrt{N/\log(N)})$ folds better performance than the pure ad hoc operations can be obtained despite of the randomness imposed on the locations of the access points. The gain in performance is mainly due to the fact that the mean number of hops from source to destination in the ad hoc tier is effectively reduced to a constant factor as opposed to the case of pure ad hoc network where the mean number of hops increases as a function of N.

In the second part of the paper, we relaxed the connectivity constraint to fully utilize the infrastructure network. Under this weaker connectivity constraint, the combined network topology graph is required to be connected. We devised an analytical tool to find the necessary and sufficient conditions on the radio transmission range which effectively determines the upper bound on the per node throughput capacity. The consequence of the necessary condition indicates that even under weaker connectivity assumptions, per node throughput asymptotically goes to zero in contrast to the constant rates obtained under different problem constructions as reported in the literature. But the rate of convergence to zero can be made remarkably small at the expense of increased confidence interval for weak connectivity. Although, we could not provide a minimal function on the transmission radius which leads us to the maximum upper bound on capacity without compromising the weak connectivity condition, we proved that this maximum upper bound can in fact be achieved with probability one.

Appendix

Proof of Lemma-3 - The proof follows from the L'Hospital Rule and properties of the $\log$ function. We can express $\lim_{x\to\infty}(1 + 1/a(x))^{b(x)}$ as

$$exp\left(\lim_{x\to\infty} \frac{\ln(1 + 1/a(x))}{1/b(x)}\right) \ .$$

We have an indeterminate form of $\frac{0}{0}$ and conditions (i)-(ii) in the lemma allow us to apply L'Hospital Rule which states that the above limit exists in set of extended real numbers if

$$\begin{aligned} &exp\left(\lim_{x\to\infty} \frac{d(\ln(1 + 1/a(x)))/dx}{d(1/b(x))/dx}\right) \\ &\quad = exp\left(\lim_{x\to\infty} \frac{-\dot{a}(x)/a(x)^2}{1 + 1/a(x)} (-b(x)^2/\dot{b}(x))\right) \\ &\quad = exp\left(\lim_{x\to\infty} \frac{b(x)^2}{a(x)^2} \frac{\dot{a}(x)}{\dot{b}(x)}\right) \ . \end{aligned} \tag{16}$$

exists and it is equal to the limit in (16). Hence, we proved the lemma. ■

Proof of Theorem-3 - We can express $\mathcal{Y}$ as,

$$\mathcal{Y} = \sum_{i=1}^{N} I(i \ is \ connected \ to \ an \ access \ point) \ , \tag{17}$$

where I is the indicator function. Clearly, $\mathcal{Y} \leq N$, hence;

$$\begin{aligned}
N &\geq E[\mathcal{Y}] \\
&= E\left[\sum_{i=1}^{N} I(i \ is \ connected \ to \ an \ access \ point)\right] \ , \\
&= \sum_{i=1}^{N} E[I(i \ is \ connected \ to \ an \ access \ point)] \ , \\
&\stackrel{(a)}{=} N \times Prob[i \ is \ connected \ to \ an \ access \ point] \\
&\stackrel{(b)}{\geq} N \times \left[1 - \left(1 - \frac{A_\epsilon(K)}{4A_R}\right)^K\right] \ . \qquad (18)
\end{aligned}$$

Here, step (a) follows from the fact that each node has the same marginal distribution of being connected to an access point although they are not independent. And step (b) is a direct application of the lower bound as given by relation (8). Define $\beta(K) = [1 - (1 - A_\epsilon(K)/4A_R)^K]$ and suppose $\beta(K)$ has a limit $\beta^* > 0$. Then for all $\epsilon > 0$, there exists a real number K_0 such that $|\beta(K) - \beta^*| < \epsilon$ for all $N > K \geq K_0$. Choose $\epsilon = 1/N^2$, thus we have $N\beta > N(\beta^* - \epsilon) = N\beta^* - 1/N$. Or equivalently

$$N \geq E[\mathcal{Y}] > \beta^* N - \gamma \ , \ \forall N \geq K_0 \ , \ \gamma > 0 \ ,$$

where γ is arbitrarily small. Corollary-1 implies the existence of $\beta^* > 0$ completing the first part of the theorem.

Proving the second statement of the theorem is again a brute-force application of theorem-2. Weak connectivity of node i with arbitrarily high probability forces β^* to be 1 and $E[\mathcal{Y}]$ becomes arbitrarily close to N. Considering this result along with the observation $E[\mathcal{Y}] = N$ if and only if Prob[all nodes are connected to an access point] = 1 suffices to prove the second part of the theorem. ■

REFERENCES

[1] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[2] Joseph P. Macker, Vincent D. Park, and M. Scott Corson, "Mobile and Wireless Internet Services: Putting the Pieces Together," *IEEE Communications Magazine*, vol. 39, no. 6, pp. 148–155, June 2001.

[3] Ying-Dar Lin and Yu-Ching Hsu, "Multihop Cellular: A New Architecture for Wireless Communications," in *IEEE INFOCOM 2000*, March 2000, pp. 1273–1282.

[4] John W. Noerenberg II, "Bridging Wireless Protocols," *IEEE Communications Magazine*, vol. 39, no. 11, pp. 90–97, Nov. 2001.

[5] George Neonakis Aggélou and Rahim Tafazolli, "On the Relaying Capability of Next-Generation GSM Cellular Networks," *IEEE Personal Communications*, pp. 40–47, Feb. 2001.

[6] A.N. Zadeh, B. Jabbari, R. Pickholtz, B. Vojcic, "Self-organizing packet radio ad hoc networks with overlay (SOPRANO)," *IEEE Communications Magazine*, vol. 40, pp. 149–157, June 2002.

[7] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: The relay case," in *Proceedings of the IEEE Infocom*, 2002.

[8] Benyuan Liu, Zhen Liu, and Don Towsley, "On the capacity of hybrid wireless networks," in *Proceedings of the IEEE Infocom*, 2003.

[9] Jiang Xie and Ian F. Akyildiz, "A Distributed Dynamic Regional Location Management Scheme for Mobile IP," in *Proceedings of IEEE Infocom*, June 2002.

[10] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," in *Proceedings of the IEEE Infocom*, 2001.

[11] P. Gupta and P.R. Kumar, "Towards an information theory of large networks: An achievable rate region," in *IEEE Int. Symp. Info. Theory, Washington DC*, June 2001.

[12] Enrique J. Duarte-Melo and Mingyan Liu, "Data-Gathering Wireless Sensor Networks: Organization and Capacity," Uni. of Michigan, Ann Arbor, Tech. Rep., 2002.

[13] P. Gupta and P.R. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Application: A Volume in Honor of W.H. Fleming*, March 1998.

[14] Thomas M. Cover and Joy A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 1991.

[15] Athanasios Papoulis, *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill Inc., 1991.

[16] Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest, *Introduction to Algorithms*. MIT Press, 1990.

[17] P.T. Olivier Dousse and M. Hasler, "Connectivity in ad-hoc and hybrid networks," in *Proceedings of the IEEE Infocom*, 2002.

[18] Vladimir Vapnik, *Estimation of Dependences Based on Empirical Data*. Springer-Verlag, 1982.

[19] J. Li, C. Blake, D.S.J. DeCouto, H.I. Lee, and R. Morris, "Capacity of ad hoc wireless networks," in *Proceedings of Mobicom*, 2001.