Study of DS-CDMA, RAKE Receiver and Proposal for a Multicarrier DS-CDMA System over Multipath Fading Channel

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July 1999

Abstract

This paper gives a review of DS-CDMA, RAKE receiver and multicarrier DS-CDMA as well as a proposal and numerical result for a multicarrier DS-CDMA system. This paper is organized as follows: Section I introduces the development of methods for wireless communication over multipath fading channel. Section II gives Turin's evaluation of the effects of multipath and fading on the performance of DS-CDMA systems. Section III gives Pursley's evaluation of this issue. Section IV explains the structure and performance of conventional RAKE receiver. Section V shows Sourour's work on Multicarrier DS-CDMA. Section VI is a proposal for a multicarrier DS-CDMA system and some numerical results I have got. Section VII is devoted to the conclusions and future work.

1 Introduction

Code-divison multiple access (CDMA) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by modulating and spreading their information-bearing signals with preassigned signature sequences.

Recently, CDMA has been considered as a candidate to support multimedia services in mobile radio communications, because of its ability to cope with the asynchronous nature of multimedia data traffic, to add users without changing the setting of the system, and to combat the hostile channel frequency selectivity. Direct sequence (DS-) and frequency hopping (FH-) CDMA techniques have been subject to extensive research.

On the other hand, the multicarrier modulation scheme, often called orthogonal frequency-division multiplexing (OFDM), has drawn a lot of attention in the field of radio communications. This is mainly because of the lower signal rate and longer signal duration. To combat the problem, the OFDM seems to be a solution.

It was in 1993, an epoch of CDMA application, that three types of new multiple access schemes based on a combination of code division and OFDM techniques were proposed, such as 'multicarrier CDMA','multicarrier DS-CDMA' and 'multitone CDMA'. In this paper we discuss the multicarrier DS-CDMA in details. The basic idea is that the transmitter spreads the Serial to Parrellel converted data streams using a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation.

This paper is organized as following. In Section II gives Turin's evaluation of the effects of multipath and fading on the performance of DS-CDMA systems. Section III gives Pursley's evaluation of this issue. Section IV explains the structure and performance of conventional RAKE receiver. Section V shows Sourour's work on Multicarrier DS-CDMA. Section VI is a proposal for a multicarrier DS-CDMA system and some numerical results I have got. Section VII is devoted to the conclusions and future work.

2 DS-CDMA over Multipath and Fading Channel

In [4],[5], Turin gives two approximation results of CDMA performance. The first one is based on Single-path Equal-Power Nonfading Links, while the other is based on Multipath Fading Links.

2.1 Single-path Equal-Power Nonfading Links

The idea of Turin's work is to deduce the received signal and find out the signal term, noise term and other users' interference term. Then by using the Gaussian approximation, the variances of noise term and other users' interference term are calculated. Finally, the signal to noise ratio is calculated and the error probability is given.

In the single-path equal-power nonfading links, the received signal is as follows:

$$r(t) = n(t) + \sum_{k=1}^{K} \sqrt{2P} a_k(t - \tau_k) b_k(t - \tau_k) cos(\omega_c t + \theta_k) \tag{1}$$

Where n(t) is the Gaussian noise of double-sided power density $\frac{N_0}{2}$.K is number of users. P is the power of transmitted signal. ω_c is the carrier frequency. τ_k is the delay which is uniformly distributed over $[0, T)$. T is the symbol duration. θ_k is uniformly distributed over $[0, \pi)$. Assume that the Pseudo-Random sequence $a_k(t)$ and data sequence $b_k(t)$ are as following,where $T = NT_c$, Tc is the chip duration. P_{Tc} and P_T are the shaping impulses of Pseudo-Random sequence and data sequence.

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} P_{Tc}(t - jTc) \tag{2}$$

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l} P_T(t - lT) \tag{3}$$

Without loss of generosity for $\tau_1 = 0$ and $\theta_1 = 0$. The output of the first user's match filter is:

$$\begin{aligned} x_0 &= \int_0^T a_1(t) cos(\omega_c t) r(t) dt \\ &= \sqrt{\frac{P}{2}} (b_{1,0} + \sum_{k=2}^{K} \sum_{j=-\infty}^{\infty} cos\phi_{k,j} b_{k,j} \int_0^T a_1(t) a_k(t - jT - \tau_k)) \\ &\quad + \int_0^T n(t) a_1(t) cos\omega_c t dt \end{aligned}$$

The signal term is $\sqrt{\frac{P}{2}} b_{1,0}$.

The noise term has zero mean and variance $\frac{N_0 T}{4}$.

It can be shown that the interference of other users has zero mean and variance

$$Var = \frac{P}{2}(K-1)2[\frac{T^2}{3N} - \frac{T^2}{12N^2}]\frac{1}{2} = \frac{P(K-1)T^2}{6N} \tag{4}$$

where N is the Processing Gain.

The signal to noise ratio of the first user's match filter output is:

$$SNR = \frac{\frac{P}{2}T^2}{\frac{N_0}{4}T + \frac{P(K-1)T^2}{6N}} = [\frac{N_0}{2PT} + \frac{K-1}{3N}]^{-1} = [\frac{N_0}{2E_b} + \frac{K-1}{3N}]^{-1} \tag{5}$$

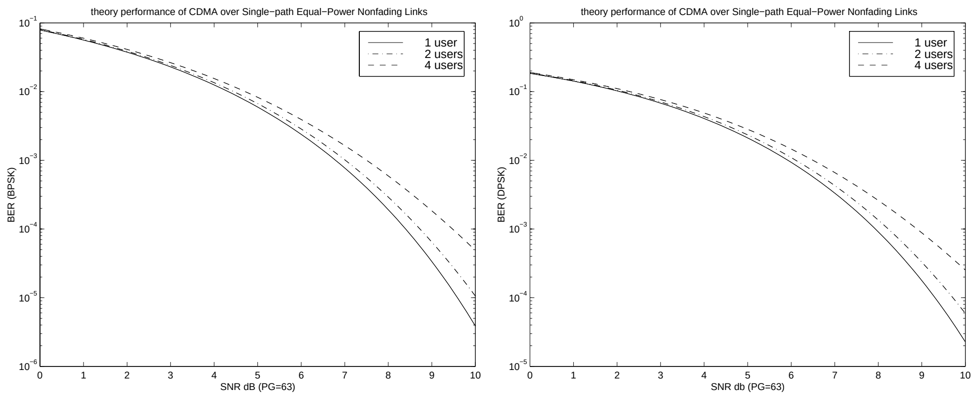

Figure 1: theory performance of CDMA over Single-path Equal-Power Nonfading Links (BPSK: left, DPSK: right)

Where $E_b = 2PT$ The error probability for BPSK and DPSK are

$$P_e = \begin{cases} \frac{1}{2}e^{-SNR} = \frac{1}{2}exp(-[\frac{N_0}{2E_b} + \frac{K-1}{3N}]^{-1}) & \text{for DPSK;} \\ Q(\sqrt{2SNR}) = Q([\frac{N_0}{E_b} + \frac{2(K-1)}{3N}]^{-1/2}) & \text{for BPSK.} \end{cases}$$

The numeric charts are shown in figure 1

2.2 Multipath Fading Links

For the multipath fading links, there is no analytical result to be found yet. However here we introduce Gaussain assumptions. The received signal is

$$r(t) = n(t) + \sqrt{2P} \sum_{k=1}^{K} \sum_{l=1}^{L_k} \beta_{k,l} a_k(t - \tau_{k,l}) b_k(t - \tau_{k,l}) cos(\omega_c t + \theta_{k,l}) \quad (6)$$

where it has the same definition of section 2.1. $\beta_{k,l}$ is the channel gain of kth user and lth path which is complex gaussian random variable with zero mean and unit variance. $\tau_{k,l}$ is the delay which is uniformly distributed over $[0, T)$. $\theta_{k,l}$ is uniformly distributed over $[0, \pi)$. $\beta_{k,l}, \tau_{k,l}, \theta_{k,l}$ are each assumed to be independent to each other with different k and l.

The front end of the receiver is still a filter matched to the first user. (This is followed by a demodulator and multipath combiner).The output of the first user's match filter has signal term, other users' interference term and channel noise terms.

The signal term is the same as the previous section

The channel noise has variance $\frac{N_0 T}{4}$

The interference of other users is

$$\sqrt{\frac{P}{2}}\sum_{k=2}^{K}\sum_{j=-\infty}^{\infty}\sum_{l=1}^{L_k}\cos(\phi_{k,j,l})b_{k,j}\int_0^T \beta_{k,l}a_1(t)a_k(t-jT-\tau_{k,l}) \tag{7}$$

Where $\phi_{k,j,l}$ is the phase term of kth user, jth chip and lth path. L_k is the multipath number of the kth user

Assume that $|\tau_{k,l}-\tau_{k,r}| \geq \frac{T}{M}$ for all $l \neq r$, the variance of the other users' interference term is shown in the paper as

$$Var = \frac{PT^2}{12N}\sum_{k=1}^{N}E[\sum_{l=1}^{L_k}|\bar{\beta}_{k,l}|^2] \tag{8}$$

where $|\bar{\beta}_{k,l}|^2$ is the mean square of $a_{k,l}$ and the expectation is over L_k.

So the factor by which interfering users increase the channel noise at the matched filter output is:

$$D = \frac{\frac{PT^2}{12N}\sum_{k=1}^{N}E[\sum_{l=1}^{L_k}|\bar{\beta}_{k,l}|^2]+\frac{N_0T}{4}}{\frac{N_0T}{4}} \tag{9}$$

Here we assume that the power control is optimal.

3 Analysis of an Asynchronous Phase-coded CDMA

In [6][7][8][9][10][11][12], Pursley introduces another mothed to analyse the Asynchronous Phase-coded CDMA system. He introduces a lot of definitions. By using these definitions, the conculation can be simplied. To be specific, the intergration of asynchronous user to the first user's match filter is changed to discrete time summation. In the first part of this section, the definition is introduced. Then the worst case is analysed. Finally the average performance is evaluated and compared to the Turin's work.

The system model is shown in figure 2. The definition is the same as the section above. The received signal is

$$r(t) = n(t) + \sum_{k=1}^{K}\sqrt{2P}a_k(t-\tau_k)b_k(t-\tau_k)cos(\omega_c t+\phi_k) \tag{10}$$

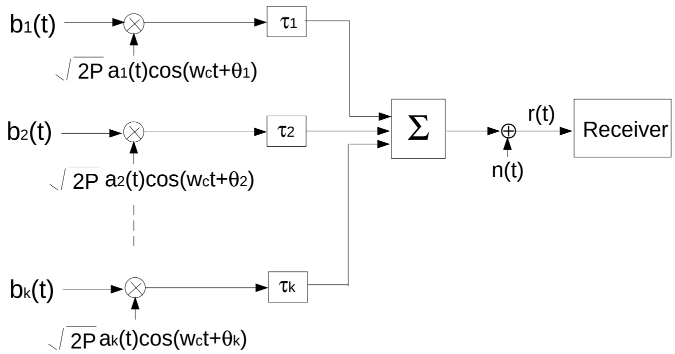

Figure 2: Phase-coded spread spectrum multipath-access system model

If the received signal r(t) is the input to a correlation receiver matched to the ith user, the output of matched filter is:

$$
\begin{aligned}
Z_i &= \int_0^T r(t) a_i(t) \cos \omega_c t dt \\
&= \int_0^T [n(t) + \sum_{k=1}^{K} \sqrt{2P} a_k(t-\tau_k) \sum_{j=-\infty}^{\infty} b_{k,j} P_t(t - jT - \tau_k) \cos(\omega_c t + \phi_k)] a_i(t) \cos \omega_c t dt \\
&= \int_0^T n(t) a_i(t) \cos \omega_c t dt \\
&+ \sqrt{\frac{P}{2}} [b_{i,0} T + \sum_{\substack{k=1 \\ \neq i}}^{K} \int_0^T a_k(t-\tau_k) a_i(t) \sum_{j=-\infty}^{\infty} b_{k,j} P_T(t - jT - \tau_k) \cos \phi_k dt] \\
&= \eta + \sqrt{\frac{P}{2}} b_{i,0} T \\
&+ \sqrt{\frac{P}{2}} \sum_{\substack{k=1 \\ \neq i}}^{K} [b_{k,-1} \int_0^{\tau_k} a_k(t-\tau_k) a_i(t) dt + b_{k,0} \int_{\tau_k}^{T} a_k(t-\tau_k) a_i(t) dt] \cos \phi_k \\
&= \eta + \sqrt{\frac{P}{2}} b_{i,0} T + \sqrt{\frac{P}{2}} \sum_{\substack{k=1 \\ \neq i}}^{K} [b_{k,-1} R_{k,i}(\tau_k) + b_{k,0} \hat{R}_{k,i}(\tau_k)] \cos \phi_k \\
&= \eta + \sqrt{\frac{P}{2}} [b_{i,0} T + \sum_{\substack{k=1 \\ \neq i}}^{K} \upsilon_{k,i}(\tau_k)]
\end{aligned}
$$

where we define the continous-time partial cross-correlation functions

$$R_{k,i}(\tau_k) = \int_0^{\tau_k} a_k(t-\tau_k) a_i(t) dt \tag{11}$$

$$\hat{R}_{k,i}(\tau_k) = \int_{\tau_k}^{T} a_k(t-\tau_k)a_i(t)dt \tag{12}$$

and

$$\upsilon_{k,i}(\tau_k) = [b_{k,-1}R_{k,i}(\tau_k) + b_{k,0}\hat{R}_{k,i}(\tau_k)]\cos\phi_k \tag{13}$$

The noise term has variance $\frac{N_0 T}{4}$

Since the discrete aperiodic cross-correlation function for the Pseudo-Random sequence of different users is

$$C_{k,i} = \begin{cases} \sum_{j=0}^{N-1-l} a_j^{(k)} a_{j+l}^{(i)} & 0 \le l \le N-1; \\ \sum_{j=0}^{N-1+l} a_{j-l}^{(k)} a_j^{(i)} & 1-N \le l < 0. \\ 0 & |l| \ge N \end{cases}$$

If we assume $0 \le l_k T_c \le \tau_k \le (l_k+1)T_c$, we could express the the continous-time partial cross-correlation functions as

$$\begin{aligned} R_{k,i}(\tau_k) &= \int_0^{\tau_k} \sum_{j=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_j^{(k)} a_m^{(i)} P_{T_c}(t-jT_c-\tau_k)P_{T_c}(t-mT_c)dt \\ &= C_{k,i}(l_k-N)T_c + [C_{k,i}(l_k+1-N) - C_{k,i}(l_k-N)](\tau_k - l_k T_c) \\ \hat{R}_{k,i}(\tau_k) &= C_{k,i}(l_k)T_c + [C_{k,i}(l_k+1) - C_{k,i}(l_k)](\tau_k - l_k T_c) \end{aligned}$$

Define the periodic cross-correlation function

$$\begin{aligned} \theta_{k,i}(l) &= \sum_{j=0}^{N-1} a_j^{(k)} a_{j+l}^{(i)} = C_{k,i}(l) + C_{k,i}(l-N) \\ \hat{\theta}_{k,i}(l) &= C_{k,i}(l) - C_{k,i}(l-N) \end{aligned}$$

then

$$\upsilon_{k,i}(\tau_k) = \begin{cases} b_{k,0}\{\theta_{k,i}(l_k)T_c + [\theta_{k,i}(l_k+1) - \theta_{k,i}(l_k)](\tau_k - l_k T_c)\}\cos\phi_k & b_{k,0} = b_{k,-1}; \\ b_{k,0}\{\hat{\theta}_{k,i}(l_k)T_c + [\hat{\theta}_{k,i}(l_k+1) - \hat{\theta}_{k,i}(l_k)](\tau_k - l_k T_c)\}\cos\phi_k & b_{k,0} \ne b_{k,-1}. \end{cases}$$

By using the above definations, we could get some convinience of approximation in the following analysis and following sections.

- Worst Case

  We want to choose the code that gives the smallest value for the ith user's maximum error probability $P_{max}^{(i)}$. We use minimax method. The maximum value of $\upsilon_{k,i}(\tau_k)$ is achieved when τ_k is an integer multiple of T_c and when $\phi_k = 0$. We define

$$u_{k,i}(\tau_k) = \frac{1}{T_c}\upsilon_{k,i}(\tau_k) = [b_{k,-1}C_{k,i}(l-N) + b_{k,0}C_{k,i}(l)] \tag{14}$$

which has value set of $\{\pm\theta_{k,i}(l), \pm\hat{\theta}_{k,i}(l)\}$ we define $\lambda_{k,i} = max(u_{k,i}(\tau_k))$, $\lambda = max\{\lambda_{k,i}\}$ and $\Lambda = max\Lambda_i$ the $P_{max}^{(i)}$ is minimized if the quantity $\Lambda_i = \sum_{k\neq i}\lambda_{k,i}$ is minimized, we have

$$P_{max}^{(i)} = 1 - \Phi([1-(\frac{\Lambda_i}{N})]\sqrt{\frac{2PT}{N_0}}) \tag{15}$$

The maximum error probability is

$$\begin{aligned} P_{max} &= \max P_{max}^{(i)} \\ &= 1 - \Phi([1-(\frac{\Lambda}{N})]\sqrt{\frac{2PT}{N_0}}) \\ &\leq 1 - \Phi([1-(K-1)(\frac{\lambda}{N})]\sqrt{\frac{2PT}{N_0}}) \end{aligned}$$

The maximum magnitude of the aperiodic cross-correlation is

$$C_c = max\{C_{k,i}(l)| : 1-N \leq l \leq N-1, 1 \leq i < k \leq K\} \tag{16}$$

Notice that $\lambda \leq 2C_c$ and hence

$$P_{max} \leq 1 - \Phi([1-(K-1)(\frac{2C_c}{N})]\sqrt{\frac{2PT}{N_0}}) \tag{17}$$

This analysis is only useful for the system which has large period N of the code sequence than the number of users K.

- Average SNR
  Here the phase shifts, time delays and data symbols are treated as mutually independent random variables. Without loss of generosity, assume $\phi_i = 0$ and $\tau_i = 0$. So the variance of the ith user's match filter output is

$$\begin{aligned} Var(Z_i) &= (\frac{P}{4T}) \sum_{\substack{k=1 \\ \neq i}}^{K} \int_0^T (R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau))d\tau + \frac{N_0T}{4} \\ &= (\frac{P}{4T}) \sum_{\substack{k=1 \\ \neq i}}^{K} \sum_{l=0}^{N-1} \int_{lT_c}^{(l+1)T_c} (R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau))d\tau + \frac{N_0T}{4} \\ &= (\frac{PT^2}{12N^3})(\sum_{\substack{k=1 \\ \neq i}}^{K} r_{k,i}) + \frac{N_0T}{4} \end{aligned}$$

where

$$\begin{aligned} r_{k,i} = & \sum_{l=0}^{N-1}[C_{k,i}^2(l-N) + C_{k,i}(l-N)C_{k,i}(l-N+1) \\ & + C_{k,i}^2(l-N+1) + C_{k,i}^2(l) + C_{k,i}(l)C_{k,i}(l+1) \\ & + C_{k,i}^2(l+1)] \end{aligned}$$

Define the cross-correlation parameters $\mu_{k,i}(n)$

$$\mu_{k,i}(n) = \sum_{l=1-N}^{N-1} C_{k,i}(l)C_{k,i}(l+N) \tag{18}$$

Notice that

$$\begin{aligned}\mu_{k,i}(0) &= \sum_{l=1-N}^{N-1} C_{k,i}^2(l) = \sum_{l=0}^{N-1} C_{k,i}^2(l-N) + C_{k,i}^2(l) \\ &= \sum_{l=0}^{N-1} C_{k,i}^2(l-N+1) + C_{k,i}^2(l+1)\end{aligned}$$

and

$$\begin{aligned}\mu_{k,i}(1) &= \sum_{l=1-N}^{N-1} C_{k,i}(l)C_{k,i}(l+1) \\ &= \sum_{l=0}^{N-1} C_{k,i}(l-N)C_{k,i}(l-N+1) \\ &\quad +C_{k,i}(l)C_{k,i}(l+1)\end{aligned}$$

therefore

$$r_{k,i} = 2\mu_{k,i}(0) + \mu_{k,i}(1) \tag{19}$$

So the final signal to noise ratio is

$$SNR_i = \{(6N^3)^{-1} \sum_{\substack{k=1 \\ \neq i}}^{K} [2\mu_{k,i}(0) + \mu_{k,i}(1)] + \frac{N_0}{2PT}\}^{-1/2} \tag{20}$$

If we have the appoximation

$$(6N^3)^{-1} \sum_{\substack{k=1 \\ \neq i}}^{K} r_{k,i} \approx (K-1)/3N \tag{21}$$

which yields

$$SNR_i \approx \{\frac{K-1}{3N} + \frac{N_0}{2PT}\}^{-1/2} \tag{22}$$

It is exactly the same as Turin's result.

4 RAKE receiver

In [1], Proakis introduce the structure of coherent RAKE receiver shown in figure 3.

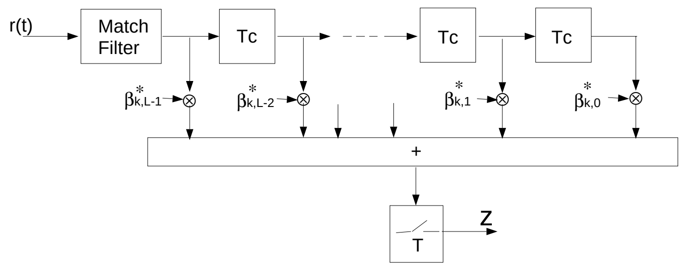

Figure 3: Coherent RAKE receiver model

Here we assume that we have perfect timing and channel estimation. The error rate for the corresponding modulation method in a pure Gaussian Channel is

$$P_e(\gamma_b) = \begin{cases} \frac{1}{2}e^{-\frac{E_b}{N_0}} & \text{for DPSK;} \\ Q(\sqrt{2E_b/N_0}) & \text{for BPSK.} \end{cases}$$

Where γ_b is the overall signal to noise ratio. We try to find out the distribution of $\gamma_b : p(\gamma_b)$, and then use the following equation to find overall probability of error.

$$P_e = \int_0^\infty P_e(\gamma_b)p(\gamma_b)d\gamma_b \tag{23}$$

We suppose that there are L paths, the SNR per bit is

$$\gamma_b = \frac{E_b}{N_0}\sum_{l=0}^{L-1}|\beta_l|^2 = \sum_{l=0}^{L-1}\gamma_l \tag{24}$$

Where $\gamma_l = \frac{E_b|\beta_k|^2}{N_0}$ is the instantaneous SNR on the lth path. If there is only one path. The characteristic function of γ_1 is easily shown to be

$$\psi_{\gamma_1}(ju) = E(e^{ju\gamma_1}) = \frac{1}{1 - ju\bar{\gamma}_c} \tag{25}$$

where $\bar{\gamma}_c$ is the average SNR per path, which is assumed to be indentical for all paths.

$$\bar{\gamma}_c = \frac{E_b}{N_0}E(|\beta_k|^2) \tag{26}$$

Since we assume that the fading on the L paths is mutually statistically independent. Hence the characteristic function for the γ_b is simply result in raised to the Lth power, i.e.

$$\psi_{\gamma_b}(ju) = \frac{1}{(1 - ju\bar{\gamma}_c)^L} \tag{27}$$

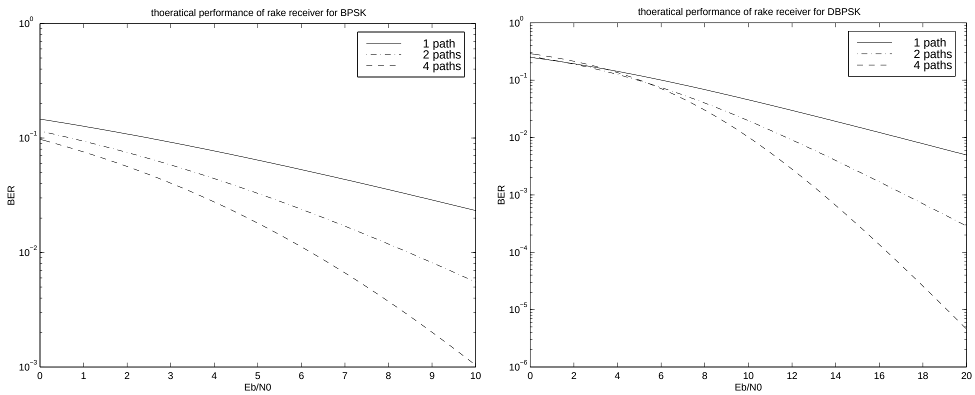

Figure 4: theory performance of RAKE receiver over Multipath Fading Channel (BPSK: left, DPSK: right)

So that the probabilty density function is

$$p(\gamma_b) = \frac{1}{(L-1)!\bar{\gamma}_c^L}\gamma_b^{L-1}e^{-\gamma_b/\bar{\gamma}_c} \tag{28}$$

We obtain a common expression for both coherent BPSK and DPSK

$$P_e = [\frac{1}{2(1-\mu)}]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} [\frac{1+\mu}{2}]^l \tag{29}$$

where

$$\mu = \begin{cases} \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}} & \text{for BPSK;} \\ \frac{\bar{\gamma}_c}{1+\bar{\gamma}_c} & \text{for DPSK.} \end{cases}$$

The simulation results are shown in figure 4

5 Multicarrier CDMA in a Multipath Fading Channel

In [13][15][16][17][18], many combinations of CDMA and OFDM are introduced. In [14], Sourour proposes a new multicarrier DS-CDMA system. Transmitted data bits are serial to parallel converted to a number of parallel branches. On each branch, each bit is direct-sequence spread-spectrum modulated and transmitted using orthogonal carriers. This procedure provides the following advantages over traditonal SS-CDMA: the transmission bandwidth is more efficiently utilized, the effect of frequency selective multipath interference can be mitigated, and frequency/time diversity is achieved. The system is analyzed with both

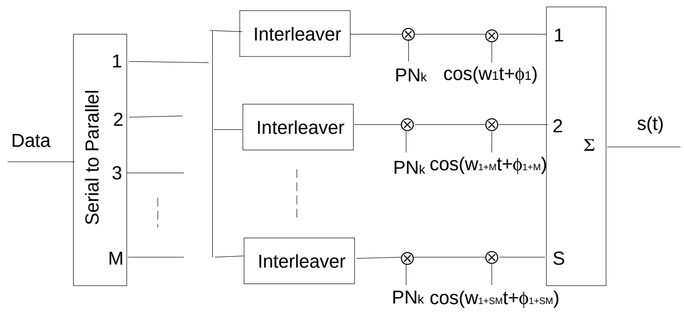

Figure 5: Multicarrier CDMA Transmitter

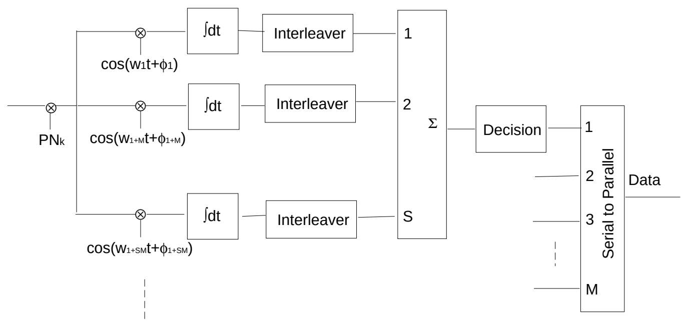

Figure 6: Multicarrier CDMA Receiver

a conventional matched-filter receiver and a RAKE receiver for each carrier. The performance is compared to that of the conventional single carrier system with RAKE receiver. It is shown that the multicarrier system is able to out-perform the RAKE receiver when the system parameters are selected properly.

The transmitter and receiver structure are shown in figure 5 and figure 6. The different carrier frequencies have relation

$$\omega_m = \omega_1 + (m-1)\frac{2\pi}{T_c}, \ where\ m = 1, 2, \ldots, MS \tag{30}$$

The adjacent frequency bands are 50% overlapped to each other. In order to have the same bandwidth with the single carrier CDMA, the new chip duration T_c and the single carrier CDMA chip duration T_{c1} must follow

$$T_c = \frac{MS+1}{2} T_{c1} \tag{31}$$

Consequently, with $T_c = MT_b/N$ and $T_{c1} = T_b/N_1$, the period N of the PN sequence must follow:

$$N = \frac{2M}{MS+1} N_1 \tag{32}$$

Define the spectral gain(SG) of the MC system as the ratio between the BW required by the MS carriers if no spectral overlapping is untilized, and the actual BW used for transmission when 50% overlapping is allowed

$$SG = \frac{MS(2/T_c)}{(MS+1)(1/T_c)} = \frac{2MS}{MS+1} \tag{33}$$

Utilizing power control among the CDMA users, the channel is assumed to be statistically independent but indentical for all users. The Complex low-pass impulse response of the channel for carrier m of user k is given by

$$h_{k,m}(t) = \sum_{l=1}^{L} \beta_{k,m,l} \delta(t - t_{k,l}) \tag{34}$$

where L is the number of resolvable paths, $\beta_{k,m,l}$ is a complex Gaussian r.v. with zero mean and variance σ_l^2, and $t_{k,l} = (l-1)T_c + \Delta_{k,l}$ is the delay of lth path of the kth user, assumed equal for all carriers of the same user. $\{\Delta_{k,l}\}$ are i.i.d. r.v.'s for all k an l, uniformly distributed in $[0, T_c)$, the path gains $\{\beta_{k,m,l}\}$ are independent for different k, independent for different l, i,i,d, for different k and same l due to power control, and identical but correlated for different carriers transmitted by the same user undergo the same path. We consider a time nonselective channel. The unit energy constraint on the fading process covariance function implies

$$\sum_{l=1}^{L} \sigma_l^2 = 1 \tag{35}$$

We consider uniform and exponential multipath power profile, that have, respectively

$$\sigma_l^2 = \frac{1}{L} \quad \sigma_l^2 = \sigma_1^2 e^{-\frac{l-1}{L}} \quad \sigma_1^2 = \frac{1 - e^{-1/L}}{1 - e^{-1}} \tag{36}$$

When the maximum delay spread of the channel is T_m, the number of resolvable paths L is give by $L = \lfloor T_m/T_c \rfloor + 1$, L_1 is the number of resolvable paths for the single carrier case. we have relation

$$L = \lfloor \frac{2(L_1 - 1)}{MS+1} \rfloor + 1 \tag{37}$$

If $MS \geq 2L_1 - 2$ then L=1. the number of resolvable paths decreases as the number of carriers increases. The received signal is

$$r(t) \;\; = \;\; n(t) + \sqrt{2P} \sum_{k=1}^{K} \sum_{m=1}^{MS} \sum_{l=1}^{L}$$

$$\beta_{k,m,l} b_{k,p}(t - t_{k,l} - \tau_k) a_k(t - t_{k,l} - \tau_k) \cos(\omega_m t + \phi_{k,m,l})$$

The output of user one's MF of carrier q and nth path is

$$Z_{q,n} = \int_{t_{1,n}}^{t_{1,n}+T} r(t) a_1(t - t_{1,n}) \cos(\omega_q t + \phi_{1,q,n}) dt \tag{38}$$

Each RAKE receiver consists of λ MF's that lock to the first λ paths. For a bit transmitted group p, p=1,2...M and $q = p + M(\nu - 1)$,The decision statistics of the $S\lambda$ MF's are added to form the final decision statistics.

$$\begin{aligned} Z|p &= \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} Z_{q,n} \\ &= \eta + D + I_1 + I_2 + I_3 + I_4 \end{aligned}$$

where $\eta, D, I_1, I_2, I_3, I_4$ are defined as following

- Noise term η
  η is a Gaussian r.v. with zero mean and variance $N_0 T S \lambda / 4$
- Desired signal D
  D is the desired signal by setting k=1, l=n and q=m

$$D = \sqrt{\frac{P}{2}} T b_{1,p}^0 \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \beta_{1,q,n} \tag{39}$$

- Self, Same Carrier Interference I_1, with $k = 1, q = m\ and\ l \neq n$

$$\begin{aligned} I_1 &= \sqrt{\frac{P}{2}} \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \sum_{\substack{l=1 \\ \neq n}}^{L} \beta_{1,q,l} \cos(\phi_{1,q,l} - \phi_{1,q,n}) \\ & \int_{t_{1,n}}^{t_{1,n}+T} b_{1,p}(t - t_{1,l}) a_1(t - t_{1,l}) a_1(t - t_{1,n}) dt \end{aligned}$$

- Self, Other Carrier Interference I_2, with $k = 1, m \neq q\ and\ l \neq n$

$$\begin{aligned} I_2 &= \sqrt{\frac{P}{2}} \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \sum_{\substack{m=1 \\ \neq q}}^{MS} \sum_{\substack{l=1 \\ \neq n}}^{L} \beta_{1,m,l} \\ & \int_{t_{1,n}}^{t_{1,n}+T} b_{1,g}(t - t_{1,l}) a_1(t - t_{1,l}) a_1(t - t_{1,n}) \\ & cos[(\omega_m - \omega_q)t + \phi_{1,m,l} - \phi_{1,q,n}] dt \\ g &= 1 + [(m-1)\ mod\ M] \end{aligned}$$

- Other Users, Same Carrier Interference I_3, with $k > 1$ *and* $m = q$

$$\begin{aligned} I_3 &= \sqrt{\frac{P}{2}} \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \sum_{k=2}^{K} \sum_{l=1}^{L} \beta_{k,q,l} \cos(\phi_{k,q,l} - \phi_{1,q,n}) \\ &\quad \int_{t_{1,n}}^{t_{1,n}+T} b_{k,p}(t - \tau_k - t_{k,l}) a_k(t - \tau_k - t_{k,l}) a_1(t - t_{1,n}) dt \end{aligned}$$

- Other Users, Other Carrier Interference I_4, with $k > 1$ *and* $m \neq q$

$$\begin{aligned} I_4 &= \sqrt{\frac{P}{2}} \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \sum_{k=2}^{K} \sum_{\substack{m=1 \\ \neq q}}^{MS} \sum_{l=1}^{L} \beta_{k,m,l} \int_{t_{1,n}}^{t_{1,n}+T} b_{k,p}(t - \tau_k - t_{k,l}) \\ &\quad a_k(t - \tau_k - t_{k,l}) a_1(t - t_{1,n}) \cos[(\omega_m - \omega_q)t + \phi_{k,m,l} - \phi_{1,q,n}] dt \end{aligned}$$

Use the mathematical method introduced in the previous two sections, we could get the following result

$$E[Z_1|p] = \sqrt{\frac{PT^2}{2}} B \tag{40}$$

and

$$Var[Z_1|p] = \frac{N_0 T S \lambda}{4} + \frac{PST_c^2}{6}\chi_1 + \frac{PT_c^2}{3\pi^2}\chi_2 + \frac{PS\lambda T_c^2}{6N} r + \frac{P\lambda T_c^2}{2N\pi^2}\mu Q \tag{41}$$

where

$$B = \sum_{\nu=1}^{S} \sum_{n=1}^{\lambda} \beta_{1,q,n} \tag{42}$$

$$Q = \sum_{\nu=1}^{S} \sum_{\substack{m=1 \\ \neq p+(\nu-1)M}}^{MS} \frac{1}{[m - (p + (\nu - 1)M]^2} \tag{43}$$

$$\chi_1 = \sum_{n=1}^{\lambda} \sum_{l=1 \neq n}^{L} \sigma_l^2 A_1(|l - n|) + \frac{\pi}{2} \sum_{n=1}^{\lambda-1} \sum_{l=n+1}^{\lambda} \sigma_l \sigma_n A_2(l - n) \tag{44}$$

$$\chi_2 = Q \sum_{n=1}^{\lambda} \sum_{\substack{l=1 \\ \neq n}}^{L} \sigma_l^2 A_3(|l - n|) - \frac{\pi}{2} \tilde{Q} \sum_{n=1}^{\lambda-1} \sum_{l=n+1}^{\lambda} \sigma_l \sigma_n A_4(l - n) \tag{45}$$

$$\tilde{Q} = \frac{1}{M^2} \sum_{\nu=1}^{S-1} \sum_{d=\nu+1}^{S} \frac{1}{(d - \nu)^2} \tag{46}$$

$$r = \sum_{k=2}^{K} r_{k,1} \tag{47}$$

$$\mu = \sum_{k=2}^{K} \mu_{k,1}(0) - \mu_{k,1}(1) \tag{48}$$

$$A_1(f) = A_2(f) + A_2(f - N) \tag{49}$$

$$A_2(f) = C_1^2(f+1) + C_1^2(f) + C_1(f+1)C_1(f) \tag{50}$$

$$A_3(f) = A_4(f) + A_4(f - N) \tag{51}$$

$$A_4(f) = [C_1(f+1) - C_1(f)]^2 \tag{52}$$

Considering $Z_1|p$ as a Gaussian r.v. the probability of error conditioned on B and p is given by

$$P[e|p, B] = \frac{1}{2} erfc(B\sqrt{Y}) \tag{53}$$

where Y is given by

$$Y^{-1} = \frac{N_0 S\lambda}{PT} + \frac{2S}{3N^2}\chi_1 + \frac{1}{\pi^2 N^2}\chi_2 + \frac{2S\lambda}{3N^3} r + \frac{2\lambda}{N^3 \pi^2}\mu Q \tag{54}$$

The r.v. B is the sum of λS Rayleigh r.v.'s that constitute λ sets. Each set represents one of the multipaths. All the r.v.'s in one set have the same parameter σ_l^2 $l = 1, 2..., \lambda$. An r.v. in any set is independent of any r.v. in any other set. Each set contains S identically distributed, correlated, Rayleigh r.v.'s representing the envelops of the S identical-bit carriers. Note that although the probability of error is conditioned on the Rayleigh r.v.'s included in B, Y is not conditioned on these r.v.'s that are involved in I_1 and I_2. This is due to the Gaussian assumption for the interference. p(B) is the probability of B. The final probability of error is given by

$$P[e] = \frac{1}{M} \sum_{p=1}^{M} \int_0^\infty \frac{1}{2} erfc(B\sqrt{Y}) p(B) dB \tag{55}$$

6 Proposal and Numerical Results for a Multicarrier CDMA system

The difference between the proposed system and the system above is that there is no overlap. We also want to apply joint source channel coding onto the system. In the system in the previous section, we assume that the channel estimation and timing are perfect. We want to do some research on the channel estimation and timing. But here we just give out some results of very basic simulation.

$$\begin{aligned} r(t) &= n(t)+\sqrt{2P}\sum_{k=1}^{K}\sum_{m=1}^{MS}\sum_{l=1}^{L} \\ & \beta_{k,m,l}b_{k,p}(t-t_{k,l}-\tau_k)a_k(t-t_{k,l}-\tau_k)\cos(\omega_m t+\phi_{k,m,l}) \end{aligned}$$

The definition is the same as the previous section, except that $\omega_m = (m-1)\frac{4\pi}{T_c}$. The output of user one's MF of carrier q and nth path is

$$Z_{q,n} = \int_{t_{1,n}}^{t_{1,n}+T} r(t)a_1(t-t_{1,n})\cos(\omega_q t+\phi_{1,q,n})dt \quad (56)$$

The decision statistics of the group p and the first λ paths is as follows, where $q = p+M(\nu-1)$

$$\begin{aligned} Z|p &= \sum_{\nu=1}^{S}\sum_{n=1}^{\lambda} Z_{q,n} \\ &= \sum_{\nu=1}^{S}\sum_{n=1}^{\lambda}\int_{t_{1,n}}^{t_{1,n}+T} r(t)a_1(t-t_{1,n})\cos(\omega_q t+\phi_{1,q,n})dt \\ &= \sum_{\nu=1}^{S}\sum_{n=1}^{\lambda}\int_{t_{1,n}}^{t_{1,n}+T}(n(t)+\sqrt{2P}\sum_{k=1}^{K}\sum_{m=1}^{MS}\sum_{l=1}^{L}\beta_{k,m,l}b_{k,p}(t-t_{k,l}-\tau_k) \\ & a_k(t-t_{k,l}-\tau_k)\cos(\omega_m t+\phi_{k,m,l}))a_1(t-t_{1,n})\cos(\omega_q t+\nu_{1,q,n})dt \\ &= \eta+\sqrt{\frac{P}{2}}\sum_{\nu=1}^{S}\sum_{n=1}^{\lambda}\sum_{k=1}^{K}\sum_{m=1}^{MS}\sum_{l=1}^{L}\beta_{k,m,l}\int_{t_{1,n}}^{t_{1,n}+T} b_{k,p}(t+t_{1,n}-t_{k,l}-\tau_k) \\ & a_k(t+t_{1,n}-t_{k,l}-\tau_k)a_1(t)\cos((\omega_m-\omega_q)t+\phi'_{k,m,l}-\phi'_{1,q,n})dt \end{aligned}$$

where $Var(\eta) = \frac{N_0 TS\lambda}{4}$, The simulation results are shown and analyzed as following

- different number of paths
  Because the signal experiences uncorrelated fading and noise by different paths, it is obvious that the more the paths, the better the performance. When there is only one path, the performance of CDMA system is the same as the BPSK over AWGN channel. Our simulation shows the point exactly on the line of theoratical curve shown in figure 7. With the increasing of the number of paths, the performace becomes better and better. We have theoratical curves shown in the section of RAKE receiver. But because some assumptions there are not hold in the real system, the simulation result is a little bit worse than the theoratical one.

- different M
  From figure 8, we find that there is no change with the increasing of groups. As matter

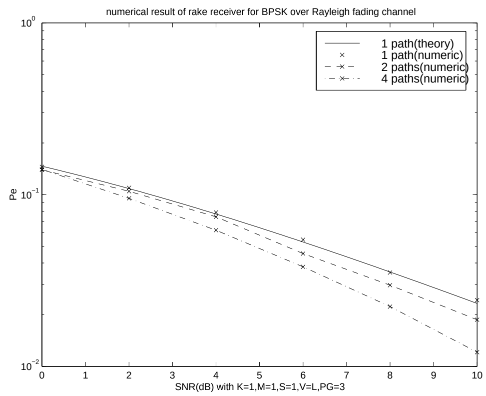

Figure 7: effect of different path numbers

of fact, it is not true for the real system. First, when different carriers transmitted by the same user undergo the same path, they are subject to a correlated fading. In the simulation, we assume they are independent. Second, with the increasing of M, the resolvable path number will decrease, but here we assume that there is only one path, so the performance will not be affected by this fact.

- different S
  If we increase the number of carriers for each bits, the perform will definitely become better. Here we assume that the resolvable path is one. If it is not one, the performace would be gained by increasing of S and lost by decreasing of resolvable paths. The performance curve is shown in figure 9

- different Processing Gain
  It is shown that with increasing of Processing Gain, there is no gain at all when there is only one resolvable path, figure 10. In fact, with higher PG, the system will have larger bandwidth and resolve more paths. It is a kind of tradeoff, and we will discuss it in the future.

- different user numbers
  The performance will drop when there are more users. The simulation result is shown in figure 11. We will apply channel coding and some other techniques to the system and discuss about the maximum of number of users in the near future.

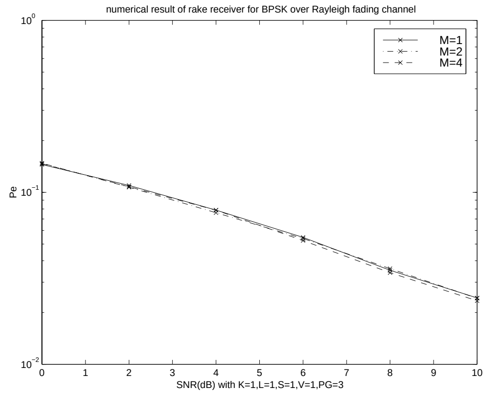

Figure 8: effect of different M

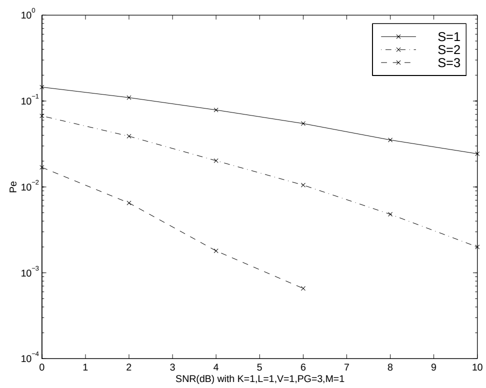

Figure 9: effect of different S

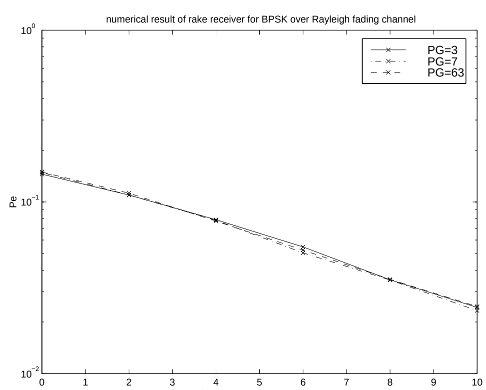

Figure 10: effect of different Processing Gain

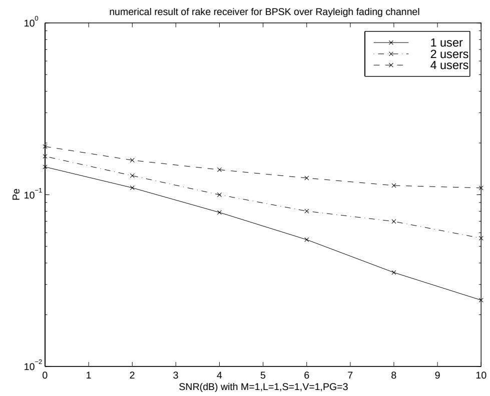

Figure 11: effect of different user numbers

7 Conclusion and future work

By reading the books and papers listed in the reference, I get to understand the basic structure of Multicarrier CDMA over multipath fading channel. By developing some simulations, I get to know the effect of different parameters to the overall performance. By deducing the equations in Turin and Pursley's papers, I know how to get theoratical evaluation of the system.

The future work is listed as following:

- Develop a more thourough system under C.
- Apply joint source channel coding onto the system.
- Evaluate the theoratical performance of system.
- Study the effect of timing and channel estimation on the performance of the system.
- Explore the network and security issue.

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