

TECHNICAL RESEARCH REPORT

Evaluating Sheet Metal Nesting Decisions

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Evaluating Sheet Metal Nesting Decisions

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Abstract

This paper describes models that estimate the cost and time of sheet metal punching when nesting (batching) orders. These models help decision-makers plan production and evaluate the impact of changing the nesting policy. In addition, we use them to formulate a nesting optimization problem. Finally, we use the models to evaluate the sensitivity of the nesting policy to manufacturing parameters. We conclude that dynamic nesting can reduce the capacity requirements, material requirements, and cost of sheet metal punching.

KEYWORDS: algorithms, optimization, manufacturing operations, batching

1 Introduction & Problem Definition

Sheet metal is a popular material for many types of products. Forming a part from sheet metal includes preparatory and finishing operations such as blanking, deburring, and bending. A blank is an intermediate part that is cut from sheet metal and transformed by other operations to form the finished part. Many manufacturers use numerically controlled (NC) punch presses for creating blanks. Such a press is an extremely flexible machine and can form a wide variety of shapes from different types of sheet metal. It forms a part by using different tools to punch holes in the metal sheet. The metal sheet is clamped to a table, which the machine can move. By moving the sheet and cutting holes next to each other, the press forms the part's outline, leaving small pieces to hold the part in the sheet. During this process, the press follows the instructions listed in a NC program. These instructions tell the press how to punch one or more copies of the part from a sheet sheared to the correct size.

To complete a job (or order) that requires many parts, an operator first loads the NC program into the machine, arranges the clamps, and places the needed tools into the tool carousel, if they are not already in place. Then the operator loads the first sheared sheet and runs the NC program, which punches some parts from that sheet. Then the operator unloads the punched sheet, loads a new sheet, and repeats the process until the order is finished.

The press cannot punch parts from the whole sheet, since the clamps must cover some portion of the sheet. If the sheet is large, this clamping area is a small portion of the sheet. However, many sheet metal parts are very small and require small sheets that are sheared to the appropriate size. Then, the clamping area is a large portion of the sheet.

In the modern manufacturing environment, many companies are using smaller lot sizes and trying to decrease costs wherever possible. Nesting is a potential solution for sheet metal punching. Nesting combines multiple orders into one job to reduce the time for machine setup and sheet loading and to reduce material costs. If two or more orders require parts from the same type of sheet metal (the same material and the same thickness), one can nest those

orders by creating a new NC program that combines the existing NC programs and punches those parts from larger, standard sheets. Then the operator performs only one setup (instead of two or more) and loads fewer sheets. Clamping wastes less material, and standard sheets cost less per pound because they don't require a shearing operation. Section 2 describes nesting in more detail.

Nesting has great cost-saving potential, and we have worked with a manufacturer that is beginning to nest orders regularly. However, it raises some questions. If no orders are nested, it is straightforward to calculate the orders' capacity and material requirements [13]. However, nesting the orders requires new methods that model the requirements more accurately. This is especially important when using order release procedures that monitor work-in-process inventory and estimate the machine workloads. Overestimating the workload requirements will starve machines unnecessarily. If these machines are bottlenecks, then this decreases the shop's throughput. (This company is implementing such procedures [3, 5, 8].) In addition, the shop needs to monitor requirements for standard metal sheets so that sufficient stock is on-hand when needed. Finally, it is important to calculate accurate costs and to estimate the cost savings that nesting brings.

Although nesting can save time and money, nesting every order is not necessarily the best policy. There may be additional savings due to nesting a specific subset of the orders. Finding the optimal set of orders to nest is not a simple problem, however.

Various aspects of sheet metal manufacturing have attracted attention in the past. Some writers [11, 14] have described sequencing algorithms that reduce the total punching time due to table movement and tool changes. Sachs [9] introduces the fundamental issues in sheet metal fabrication and classifies sheet metal parts based on the part geometry. Other authors (see, for example, DeGarmo *et al.* [1]) also describe the associated manufacturing processes. Previous work on batching includes work on how to batch jobs to create good schedules [6, 7], cutting stock problems that seek to minimize waste [4], and multiperiod lot sizing problems [10] that are similar to the joint replenishment problem. Other authors have considered how setup time reductions affect average inventory costs (see, for example, DuPlaga *et al.* [2] and Trevino *et al.* [12]). However, we are not aware of any work that considers the problem of batching a set of orders to minimize setup and material costs.

Section 3 describes models that estimate the cost and time of sheet metal punching when nesting orders. These models help decision-makers plan production and evaluate the impact of nesting orders. The models are analytical expressions that approximate the capacity requirements, material requirements, and costs of a given set of orders and nests.

Section 4 presents an integer programming model for the optimal nesting problem. This problem is NP-complete, and we present a pseudo-polynomial time dynamic program to solve the problem.

Section 5 presents some examples of applying these models to the types of orders that we encountered in industry. We show that using the correct expressions lead to much more accurate requirements planning.

In Section 6, we use the models to evaluate the sensitivity of the nesting policy to manufacturing parameters. The material cost and setup time affect the benefits of optimization. In addition, nesting every order saves more money as the orders become smaller but more numerous.

Section 7 concludes the paper and presents some ideas for future work.

2 Dynamic Nesting

This section discusses the context for sheet metal nesting decisions. Although this discussion was motivated by our work with a specific manufacturer, the setting is typical and will apply to other manufacturers as well.

The manufacturer uses a manufacturing planning and control system that maintains a master production schedule. The planning system uses material requirements planning (MRP) to explode the end-item requirements into work orders for components and sub-assemblies. Also, the planning system has detailed routings (process plans) that specify the resources required for each product. Thus, production planners can identify, for next week and each week after that, the orders that will require the punch presses in the sheet metal area.

Because the product mix changes greatly each week, the sheet metal area will use *dynamic nesting* to nest the orders. That is, before each week, the production planners will identify the orders that will be released into the shop that week and nest them. Orders that require the same NC punch press, the same material type, and the same sheet thickness can form a nest. Then, the production planners will use existing software that arranges the required parts on the minimum number of unsheared sheets, leaving room for the clamping area and leaving room between the parts. This software also creates the required NC program by combining the NC programs for each different part. After the orders are released, the operator loads this NC program onto the punch press and starts punching the nest, loading unsheared sheets and removing punched sheets as required. Punched sheets are sent to a deburring operation, where an employee removes the individual parts from the sheet, removes any burrs from the part, and sorts the parts into the individual workorders for additional processing.

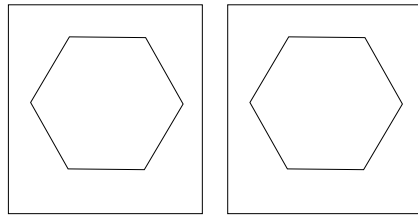
Dynamic nesting has great cost-saving potential since the nest will require only one setup and fewer, standard sheets. However, it complicates capacity and material planning, and it is difficult to estimate the cost savings. Finally, it may be possible to save more money by forming the nests carefully, but finding the optimal nest is not a simple problem. The remainder of this paper will discuss models that help decision-makers plan production and evaluate the nesting policy.

For a simple example, consider Figure 1. Order 1 contains two parts, one on each sheared sheet. Order 2 has four parts, all on one sheared sheet. Order 1 and Order 2 share the same material type and thickness. Thus, they can form a nest. Nesting both orders creates the nest in the bottom part of the figure. The nest requires two unsheared sheets. If the nest includes only Order 1, then the nest requires only one unsheared sheet, as shown in the middle part of the figure.

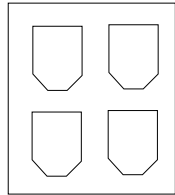
3 Approach

This section describes models that estimate the cost and time of sheet metal punching when nesting orders. The models are analytical expressions that approximate the capacity requirements, material requirements, and costs of a given set of orders and nests. First we will introduce the required notation and then we will present the models.

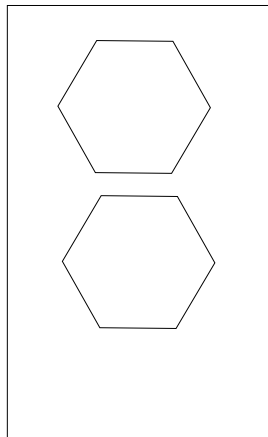
We limit our analysis to the most significant costs that nesting changes. These are the direct labor for performing machine setups and loading sheets, the time that the machine spends cutting the parts, and the cost of sheared and unsheared sheets. In this paper we are not explicitly considering the specific tool changes required for each order. The machine setup



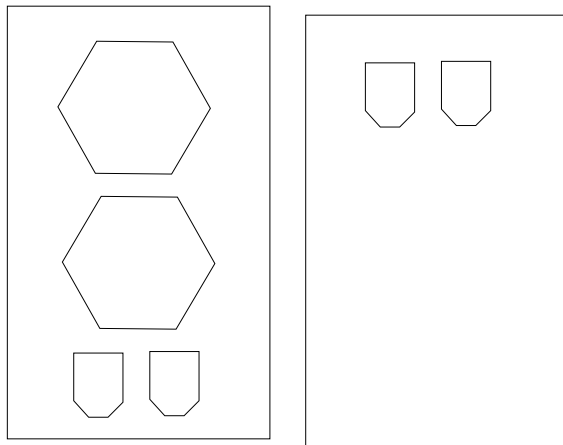
Order 1



Order 2



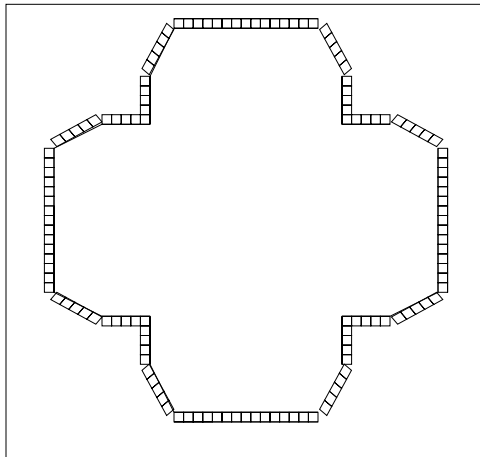
An incomplete nest



A complete nest

Figure 1: Two orders and two possible nests.

Figure 2: A blank on a sheared sheet



time includes time for tool changes. Also, we do not consider the part layout for a nest. Since parts are usually small compared to the sheet, we will assume that a part and the necessary space around it requires some specified area and that a sheet has a total usable area. Thus, the number of sheets required for a nest is at least the total area of the nested parts divided by the usable sheet area. We could use the same layout procedures that the nesting software uses. However, this would increase the computation effort for capacity planning.

We approximate the time needed to punch one part by a function of the part's perimeter, because the press punches many holes next to each other to form the part's outline. (See Figure 2.) The unit cutting time (in hours per inch) is the inverse of the machine's cutting speed. We do not include the time the punch spends moving from one part to another on the same sheet, since it is smaller than the time spent nibbling the part's outline, even when orders are nested.

Finally, we do not need to consider the sequencing of nests and orders on the punch press, since the setups are sequence-independent. Nesting an order does not change its priority (relative to the other orders waiting for processing), and so nesting will not delay its completion. (In fact, since nests require less time than orders, nested orders should complete sooner.)

The cost for setup or labor is determined by the cost coefficient (in dollars per hour) and by a performance index, which expresses operator time as a multiple of task time to model the average impact of fatigue and interruptions.

We do not include the time or cost of verifying NC programs. If a nest's NC program uses existing NC programs, it should need no testing.

3.1 Notation

For a given week w , there are n jobs (orders) to be processed on machine k . These n orders form the set $Y_{kw} = \{J_1, J_2, \dots, J_n\}$. These orders require p part types. These orders form g groups G_1, \dots, G_g . The jobs in a group require the same material and same sheet thickness. Thus, they can form a nest. The groups are disjoint subsets of Y_{kw} .

An order J_j requires q_j copies of part type i_j . Each part of type i_j has perimeter $P^P(i_j)$, area $P^A(i_j)$, thickness $t(i_j)$, and material $m(i_j)$. Each group has an associated thickness and

material so that $G_L = \{j \in Y_{kw} : t(i_j) = t(G_L) \text{ and } m(i_j) = m(G_L)\}$.

If not nested, an order J_j requires R_j^n sheared sheets. Each sheared sheet has a total area R_j^{TA} , which includes the clamping area and the effective area R_j^A . The number of parts per sheet is q_j^s ($q_j^s P^A(i_j) \leq R_j^A$). The time needed to load each sheared sheet is R_j^L . The cost of each sheet is R_j^C . The total setup time for order J_j is s_j if it is not nested. The total processing time for the parts in order J_j is a_j .

If some orders in group G_L are nested, the nest requires \overline{R}_L^n unsheared sheets, which depends upon the total area of the nested orders. Each unsheared sheet has a total area \overline{R}_L^{TA} , which includes the clamping area and the effective area \overline{R}_L^A . The time needed to load each unsheared sheet is \overline{R}_L^L . The cost of each sheet is \overline{R}_L^C . The total setup time for the nest is \overline{s}_L . The total processing time for the nest is \overline{a}_L . Both are a function of the nested orders.

The machine has a unit cutting time c_k (in hours per inch) and a performance index Π_k . The machine setup time for a non-nested order is s_k^m . The machine setup time for a nest is \overline{s}_k^m . Let C_s and C_a be the direct labor cost rates for setup time and run time (in dollars per hour).

3.2 Capacity Requirements without Nests

From this data, we can calculate the setup and run times for each order J_j , if it is not nested. The total setup time equals the machine setup time plus the total sheet loading time, which equals the number of sheets multiplied by the sheet loading time:

$$s_j = s_k^m + R_j^n R_j^L$$

The processing time equals the product of the part perimeter, unit cutting time, and order quantity:

$$a_j = P^P(i_j) c_k q_j$$

If no orders are nested, the total capacity requirements D'_{kw} for machine k in week w equals the sum of the setup and processing times, multiplied by the performance index:

$$D'_{kw} = \sum_{j \in Y_{kw}} (s_j + a_j) \Pi_k$$

3.3 Capacity Requirements with Nests

Because dynamic nesting batches orders, the equation presented above does not accurately describe the capacity requirements. Consider the orders J_j in one group G_L . Some of these orders may be nested. Let $X_j = 1$ if order J_j is in the nest, and $X_j = 0$ otherwise. Then, let the grand total setup time for the non-nested orders be S_L :

$$S_L = \sum_{j \in G_L} s_j (1 - X_j)$$

Let the grand total processing time for the non-nested orders be A_L :

$$A_L = \sum_{j \in G_L} a_j (1 - X_j)$$

If there are any orders in the nest, let $Y_L = 1$. Otherwise, let $Y_L = 0$. The number of sheets required depends upon the total area of the nested orders and the unshared sheet's effective area:

$$\bar{R}_L^n = \left\lceil \frac{\sum_{j \in G_L} X_j q_j P^A(i_j)}{\bar{R}_L^A} \right\rceil$$

where $\lceil x \rceil = \min\{i \in \mathbb{Z} : i \geq x\}$. The total setup time for the nest is the machine setup time and the total sheet loading time:

$$\bar{s}_L = Y_L \bar{s}_k^m + \bar{R}_L^n \bar{R}_L^L$$

Note that these times depend upon which orders, if any, are in the nest.

Nesting an order does not affect the part cutting times, so the nest processing time is the sum of the orders' processing times:

$$\bar{a}_L = \sum_{j \in G_L} X_j a_j$$

Note that the group's total processing time is constant regardless of the orders in the nest:

$$A_L + \bar{a}_L = \sum_{j \in G_L} a_j$$

The total capacity requirements D_{kw} for machine k in week w equals the sum of the setup and processing times, multiplied by the performance index:

$$D_{kw} = \sum_{G_L \subset Y_{kw}} (S_L + A_L + \bar{s}_L + \bar{a}_L) \Pi_k$$

3.4 Material Requirements

As with capacity requirements, dynamic nesting affects the material requirements and thus requires new models. Without nesting, the total material requirements F'_{kw} (in total sheet area) for machine k in week w equals the material requirements for each and every order:

$$F'_{kw} = \sum_{j \in Y_{kw}} R_j^n R_j^{TA}$$

Nesting orders changes the material requirements. For a given set of nests, the total material requirements F_{kw} (in total sheet area) for machine k in week w equals the material requirements for the non-nested orders plus the material requirements for the nests:

$$F_{kw} = \sum_{j \in Y_{kw}} R_j^n R_j^{TA} (1 - X_j) + \sum_{G_L \subset Y_{kw}} \bar{R}_L^n \bar{R}_L^{TA}$$

An important measure is material utilization, which evaluates whether the area is wasting material. Of course, material utilization cannot equal 100 percent because the required clamping area wastes some material. The total material consumed E_{kw} for machine k in week w equals the total area of the parts produced:

$$E_{kw} = \sum_{j \in Y_{kw}} q_j P^A(i_j)$$

Note that the material consumed does not depend upon the nesting decision. The material utilization U_{kw} for machine k in week w depends upon the nesting decision:

$$U_{kw} = \frac{E_{kw}}{F_{kw}}$$

3.5 Total Cost

The total cost includes the cost of material, the cost of setup time, and the cost of processing time. Without nesting, the total cost C'_{kw} for machine k in week w equals the cost for each and every order:

$$C'_{kw} = \sum_{j \in Y_{kw}} R_j^n R_j^C + \sum_{j \in Y_{kw}} s_j C_s \Pi_k + \sum_{j \in Y_{kw}} a_j C_a \Pi_k$$

If some orders are nested, then, for a given set of nests, the total cost C_{kw} for machine k in week w must include the total material cost H_{kw} and the total setup and processing time cost L_{kw} :

$$\begin{aligned} H_{kw} &= \sum_{j \in Y_{kw}} R_j^n R_j^C (1 - X_j) + \sum_{G_L \subset Y_{kw}} \bar{R}_L^n \bar{R}_L^C \\ L_{kw} &= \sum_{G_L \subset Y_{kw}} (S_L + \bar{s}_L) C_s \Pi_k + \sum_{G_L \subset Y_{kw}} (A_L + \bar{a}_L) C_a \Pi_k \\ C_{kw} &= H_{kw} + L_{kw} \end{aligned}$$

4 Cost Optimization

Since nesting reduces the number of machine setups and the number of sheets loaded, nesting orders should reduce the total setup time. However, minimizing the setup time is not the only objective. Nesting also significantly impacts the material requirements, and material is a significant cost. Since capacity is not the only concern, a more useful objective is to reduce the total cost of material, setup time, and processing time. Nesting orders should reduce the total cost, but nesting all orders is not necessarily an optimal solution.

4.1 Integer Programming

This section presents an integer programming model that describes the nesting decision for all orders that require a machine k in a week w . The objective function is the total cost, as defined in the previous section.

Minimize

$$C_{kw} = H_{kw} + L_{kw}$$

subject to

$$\begin{aligned} H_{kw} &= \sum_{j \in Y_{kw}} R_j^n R_j^C (1 - X_j) + \sum_{G_L \subset Y_{kw}} \bar{R}_L^n \bar{R}_L^C \\ L_{kw} &= \sum_{G_L \subset Y_{kw}} (S_L + \bar{s}_L) C_s \Pi_k + \sum_{G_L \subset Y_{kw}} (A_L + \bar{a}_L) C_a \Pi_k \\ A_L &= \sum_{j \in G_L} a_j (1 - X_j) \quad \forall G_L \subset Y_{kw} \\ S_L &= \sum_{j \in G_L} s_j (1 - X_j) \quad \forall G_L \subset Y_{kw} \\ \bar{a}_L &= \sum_{j \in G_L} X_j a_j \quad \forall G_L \subset Y_{kw} \\ \bar{s}_L &= Y_L \bar{s}_k^m + \bar{R}_L^n \bar{R}_L^L \quad \forall G_L \subset Y_{kw} \end{aligned}$$

$$\begin{aligned}
\bar{R}_L^n &= \left\lceil \frac{\sum_{j \in G_L} X_j q_j P^A(i_j)}{\bar{R}_L^A} \right\rceil \quad \forall G_L \subset Y_{kw} \\
X_j &\leq Y_L \quad \forall G_L \subset Y_{kw}, j \in G_L \\
X_j &\in \{0, 1\} \quad \forall j \in Y_{kw} \\
Y_L &\in \{0, 1\} \quad \forall G_L \subset Y_{kw}
\end{aligned}$$

4.2 Decomposition

Due to the independence of each group, we can decompose this problem by forming a subproblem for each group. Combining the solutions to the subproblems gives an optimal solution to the original problem. Also, we can use the fact that $A_L + \bar{a}_L = \sum_{j \in G_L} a_j$ to simplify the problem even more. Because the nesting decision does not change this constant, we will remove it. After this decomposition, we have the following subproblem for each group G_L :

Minimize

$$C_L = H_L + L_L$$

subject to

$$\begin{aligned}
H_L &= \sum_{j \in G_L} R_j^n R_j^C (1 - X_j) + \bar{R}_L^n \bar{R}_L^C \\
L_L &= (S_L + \bar{s}_L) C_s \Pi_k \\
S_L &= \sum_{j \in G_L} s_j (1 - X_j) \\
\bar{s}_L &= Y_L \bar{s}_k^m + \bar{R}_L^n \bar{R}_L^L \\
\bar{R}_L^n &= \left\lceil \frac{\sum_{j \in G_L} X_j q_j P^A(i_j)}{\bar{R}_L^A} \right\rceil \\
X_j &\leq Y_L \quad \forall j \in G_L \\
X_j &\in \{0, 1\} \quad \forall j \in G_L \\
Y_L &\in \{0, 1\}
\end{aligned}$$

The decision version of this problem is an NP-complete problem. For a proof, please see the Appendix. Moreover, this problem remains NP-complete if all of the setup costs are zero or if all of the material costs are zero.

We can solve the problem optimally using standard integer programming techniques. In addition, it is possible to solve the problem with a pseudo-polynomial dynamic programming algorithm.

4.3 Dynamic Programming

This section presents a pseudo-polynomial dynamic programming algorithm to solve the nesting problem for each group.

Given orders $1, 2, \dots, n$. Let $A = \bar{R}_L^A$ be the usable area of an unsheared sheet. Let $R = \bar{R}_L^C + \bar{R}_L^L C_s \Pi_k$ be the total material and setup cost of an unsheared sheet. Let $S = \bar{s}_k^m C_s \Pi_k$ be the machine setup cost for a nest. Let $F_j = R_j^n R_j^C + s_j C_s \Pi_k$ be the total material and setup cost of not nesting an order j . Let $O_j = q_j P^A(i_j)$ be the total area of an order j . Pick $e_j \geq 0$ and $c_j \geq 0$ such that $e_j < A$ and $O_j = e_j + c_j A$. c_j is the number of whole unsheared

sheets that order j requires, and e_j is the extra area that the order requires. Let $f(i, x)$ be the minimum cost of nesting the first i orders with area x on the last unsheared sheet.

Initialization: $f(0, 0) = 0$. $f(0, A) = S$. $f(0, x) = \text{infinity}$ if $0 < x < A$.

Recursion: If $x = 0$, then

$$f(i, x) = f(i - 1, x) + F_i$$

In this case, no orders are nested.

If $0 < x \leq e_i$, then

$$f(i, x) = \min\{f(i - 1, x) + F_i, f(i - 1, A + x - e_i) + (c_i + 1)R\}$$

In this case, either order i was not nested, or the extra area of order i added another unsheared sheet to the nest. In this case, the portion of order i on the new unsheared sheet has area x , and the portion on the last unsheared sheet has area $e_i - x$. Thus, the area on the last unsheared sheet was $A - (e_i - x) = A + x - e_i$ before adding order i to the nest.

If $e_i < x \leq A$, then

$$f(i, x) = \min\{f(i - 1, x) + F_i, f(i - 1, x - e_i) + c_i R\}$$

In this case, either order i was not nested, or the extra area of order i fit onto the last unsheared sheet of the nest.

Answer:

$$\min_{0 \leq x \leq A} \{f(n, x)\}$$

The complexity of this algorithm is $O(nA)$.

This formulation allows us to identify a special case that can be solved immediately. Note that $(c_j + 1)R$ is an upper bound for the cost of adding order j to the nest. If $(c_j + 1)R \leq F_j$, then the cost of adding the order is not greater than the cost of excluding it. If $(c_j + 1)R \leq F_j \forall j$, then the least expensive nest has every order. The only solution that could cost less than nesting every order is nesting no orders. To decide, compare $\sum_j F_j$ (the cost of nesting no orders) to $S + \lceil \sum_j O_j / A \rceil R$ (the cost of nesting every order). If the first quantity is smaller, then nesting no orders is the optimal solution. Otherwise, nesting every order is an optimal solution.

5 Application

This section presents some examples of applying the above models to the types of orders that we encountered in industry. We show that using the correct expressions lead to much more accurate requirements planning. We will consider a small example that has orders from just two groups. The manufacturing firm provided the necessary part data. To protect proprietary information, we do not include in this report all of the data that we collected. In addition, all cost values are scaled.

First we identified 15 orders from two groups. Both groups used aluminum sheets. Group 1 used sheets that were 0.090" thick. Group 2 used sheets that were 0.125" thick. Table 1 presents some information about the orders and parts in Group 1. Table 2 presents information about the sheared sheets. Table 3 and Table 4 present the same information for Group 2. The load time R_j^L for all sheared sheets is 0.014 hours. The cost of sheared material is \$3 per pound.

Table 1: Group 1 Orders.

Order j	Part type i_j	Quantity q_j	Part Area $P^A(i_j)$ (inches ²)
1	11	15	9.59
2	12	50	75.35
3	13	5	410.27
4	14	2	128.80
5	14	10	128.80
6	15	5	26.42
7	16	5	34.08

Table 2: Group 1 Sheared Sheets.

Order j	Sheet total area R_j^{TA} (inches ²)	Sheets required R_j^n
1	159.21	3
2	1122.88	5
3	552.58	5
4	229.67	2
5	229.67	10
6	95.73	5
7	116.59	5

Table 3: Group 2 Orders.

Order j	Part type i_j	Quantity q_j	Part Area $P^A(i_j)$ (inches ²)
8	21	10	441.41
9	21	10	441.41
10	22	5	405.15
11	23	5	46.80
12	24	5	90.25
13	25	8	40.95
14	25	2	40.95
15	25	8	40.95

Table 4: Group 2 Sheared Sheets.

Order j	Sheet total area R_j^{TA} (inches ²)	Sheets required R_j^n
8	1099	5
9	1099	5
10	527	5
11	130	5
12	254	5
13	186	4
14	186	1
15	186	4

The machine setup time for a non-nested order is $s_k^m = 0.5$ hours. The part perimeters ranged from 12 to 85 inches.

For both groups, each (4' by 8') unsheared sheet has a total area $\overline{R}_L^{TA} = 4608$ square inches, which includes the clamping area and the effective area $\overline{R}_L^A = 4089$ square inches. The time needed to load each unsheared sheet is $\overline{R}_L^L = 0.021$ hours. The cost of unsheared material is \$1.7 per pound. The machine setup time for a nest is $\overline{s}_k^m = 1.25$ hours.

For different nests in these two groups, we compared the costs, the material requirements, and the capacity requirements. We considered the following four nests in Group 1: nesting all orders (Nest A), nesting the five smallest orders (Nest B), nesting the three largest orders (Nest C), and nesting no orders (Nest O). The results are shown in Table 5, Table 6, and Table 7. Nesting all orders is the least expensive solution. Using the notation of Section 4, the material cost is the term H_L . The setup cost is the quantity L_L . The run cost is $(A_L + \overline{a}_L)C_a\Pi_k$.

The number of unsheared sheets is the term \overline{R}_L^n . The quantity E_L is the total material consumed by the parts. For Group 1, this is 7808 square inches. For Group 2, this is 12,276 square inches. The material requirements F_L is the total area (in square inches) of the sheets required for the nest and the non-nested orders. The material utilization for the group is the quantity

$$U_L = \frac{E_L}{F_L}$$

where

$$E_L = \sum_{j \in G_L} q_j P^A(i_j)$$

and

$$F_L = \sum_{j \in G_L} R_j^n R_j^{TA} (1 - X_j) + \overline{R}_L^n \overline{R}_L^{TA}$$

Note that nesting reduces material requirements. Some nests can increase utilization by 20 per cent.

The capacity requirements (in hours) do not include the performance index. The setup time corresponds to the quantity $S_L + \overline{s}_L$. The run time corresponds to the quantity $A_L + \overline{a}_L$. Note that nesting every order reduces capacity requirements by approximately 50 per cent. This is an example of how important it can be to use accurate models for capacity planning.

Table 5: Group 1 Nests - Costs.

Nest	Orders in nest	Material cost	Setup cost	Run cost	Total cost
A	1-7	206	270	300	777
B	1,4-7	434	504	300	1238
C	2,3,5	285	733	300	1318
O	none	501	835	300	1635

Table 6: Group 1 Nests - Material Requirements.

Nest	Orders in nest	Unsheared sheets	Requirements (inches ²)	Material utilization
A	1-7	2	9,216	0.85
B	1,4-7	1	12,985	0.60
C	2,3,5	2	11,215	0.70
O	none	0	12,673	0.62

We also compared four different nests for Group 2: nesting all orders (Nest E), nesting the seven largest orders (Nest F), nesting the six smallest orders (Nest G), and nesting no orders (Nest H). The results are shown in Table 8, Table 9, and Table 10. For this group, nesting the seven largest orders is the least expensive solution, and it reduces the material and capacity requirements.

6 Sensitivity

Two obvious nesting policies are nesting everything and nesting nothing. Which is better? Each nest's cost and each nesting policy's desirability depend upon the cost parameters. Using the models from Section 3, we can gain some insight into how these cost parameters affect the nesting policies. This section describes that analysis and presents some calculations based on the examples presented above. In addition, we conducted some experiments to determine how different order sizes affected the nesting policies. This section presents some results of our experiments.

Consider the following solutions for a group: The first solution nests every order. We'll call this the complete nest. The second solution nests some of the orders. We'll call this an incomplete nest. The third solution nests no orders. We'll call this the empty nest. Increasing the cost parameters will increase each solution's cost. However, each solution's increase will be different. Thus, there may be scenarios (particular values of cost parameters) when the

Table 7: Group 1 Nests - Capacity Requirements.

Nest	Orders in nest	Setup time	Run time	Total time
A	1-7	1.29	1.43	2.72
B	1,4-7	2.41	1.43	3.84
C	2,3,5	3.50	1.43	4.93
O	none	3.99	1.43	5.42

Table 8: Group 2 Nests - Costs.

Nest	Orders in nest	Material cost	Setup cost	Run cost	Total cost
E	8-15	573	279	274	1126
F	8-13,15	440	382	274	1097
G	10-15	746	504	274	1525
H	none	945	937	274	2156

Table 9: Group 2 Nests - Material Requirements.

Nest	Orders in nest	Unsheared sheets	Requirements (inches ²)	Material utilization
E	8-15	4	18,432	0.67
F	8-13,15	3	14,101	0.88
G	10-15	1	15,598	0.79
H	none	0	17,216	0.71

Table 10: Group 2 Nests - Capacity Requirements.

Nest	Orders in nest	Setup time	Run time	Total time
E	8-15	1.33	1.31	2.64
F	8-13,15	1.83	1.31	3.14
G	10-15	2.41	1.31	3.72
H	none	4.48	1.31	5.79

complete nest is less expensive than the empty nest (as it was in the previous section). And there may be scenarios when the empty nest is less expensive. And there may be scenarios when an incomplete nest is best.

Of course, finding the optimal nest for a given group will always be a difficult problem. And no nesting policy will always be the best. However, it is possible to gain some insight into how the cost parameters affect the nesting policies.

6.1 Cost Parameters

Changing the cost parameters affects different nests differently. There are six primary cost parameters. Three affect only non-nested orders. Three affect only nested orders.

1. The unit material cost for sheared sheets.
2. The load time for sheared sheets.
3. The setup time for non-nested orders.
4. The unit material cost for unsheared sheets.
5. The load time for unsheared sheets.
6. The setup time for nests.

Suppose that any of the first three cost parameters increase. This does not change a complete nest's cost. It does increase an incomplete nest's cost some. However, it increases the empty nest's cost the most, since it has the most sheared sheets and setups. Conversely, suppose any of the first three cost parameters decrease. This does not change a complete nest's cost. It decreases an incomplete nest's cost some. However, it decreases the empty nest's cost the most. Thus, we might expect that there is a point where the incomplete nest or empty nest becomes less expensive than the complete nest. In fact, there might be a point where the empty nest is the least expensive solution. See Figures 3 and 4.

Consider the fourth and fifth cost parameters, and suppose either increases. This won't increase the empty nest's cost. This will increase an incomplete nest's cost some. This will increase the complete nest's cost the most, since it has the most unsheared sheets. So, there will be a point where the complete nest becomes the most expensive solution. See Figure 6.

Now suppose that the last cost parameter increases. This won't increase the empty nest's cost. It will increase an incomplete nest's cost and the complete nest's cost by the same amount, since these solutions have just one nest setup. Thus, this won't affect whether an incomplete solution is better than the complete solution, but there will be a point where the empty nest becomes the least expensive solution. See Figure 5.

6.2 Examples

Using the nests described in Section 5, we quantified how changing the cost parameters would affect the different solutions.

Specifically, we considered how the nest setup time and the unsheared material cost affected the total cost. We could do similar analysis on the other parameters. We chose these because they appeared to be the least certain, since nesting was just being implemented. (We excluded the sheet loading time because the total sheet loading time was much smaller than

Sensitivity to Machine Setup

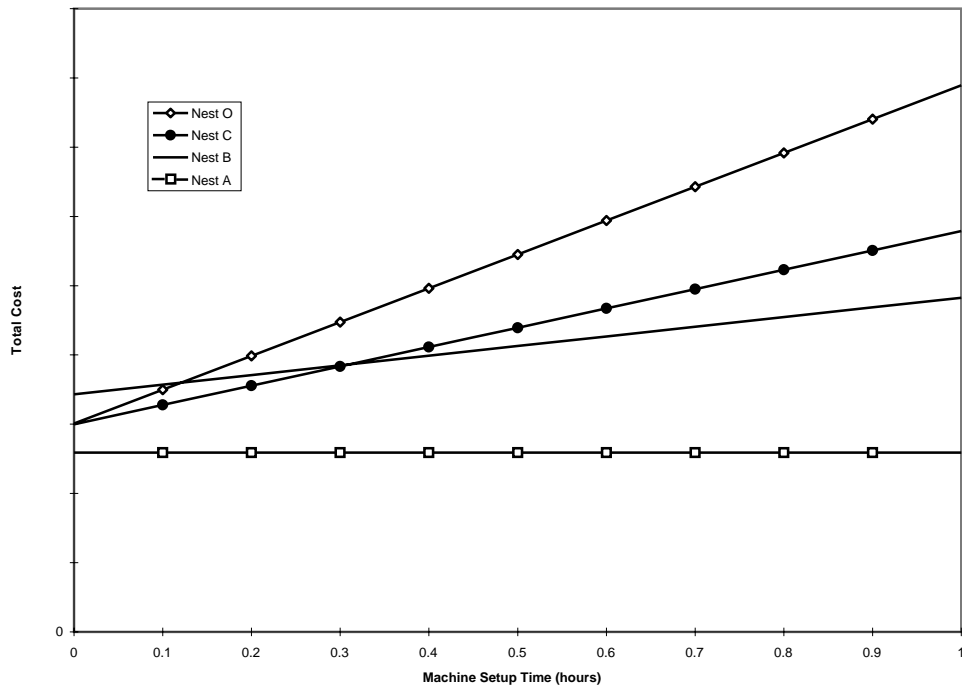


Figure 3: Total Cost as Machine Setup Time Changes - Group 1.

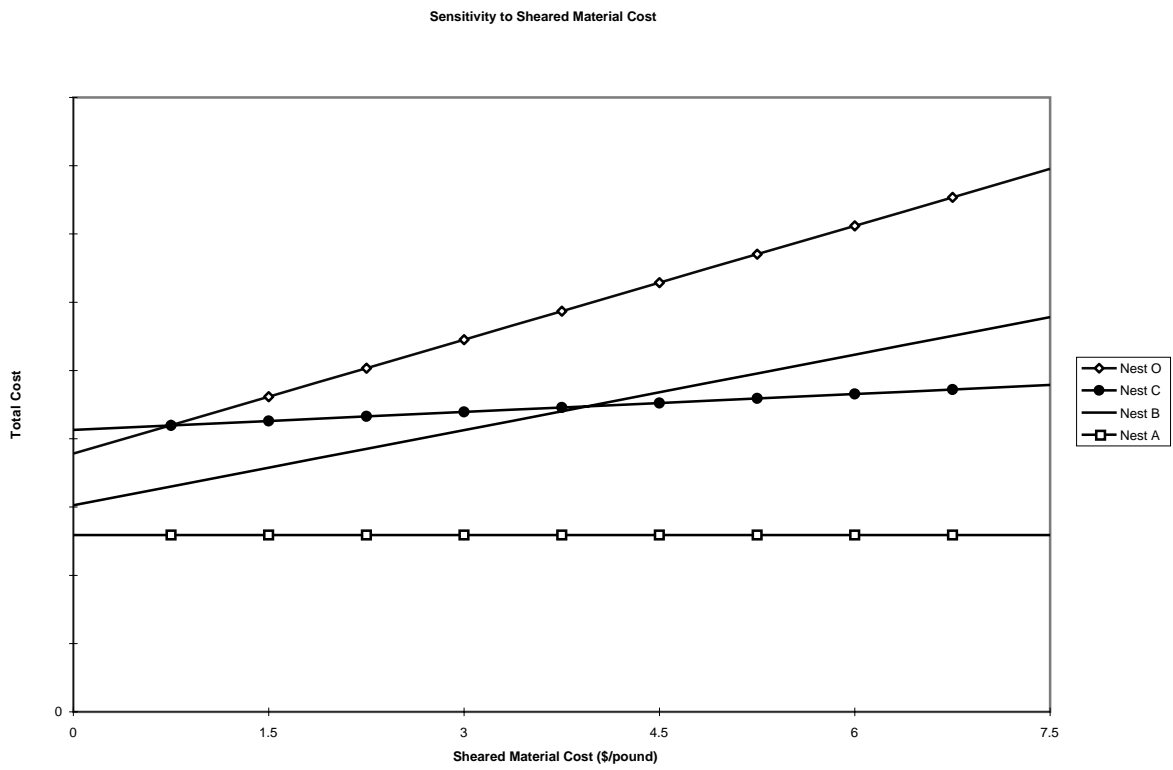


Figure 4: Total Cost as Sheared Material Cost Changes - Group 1.

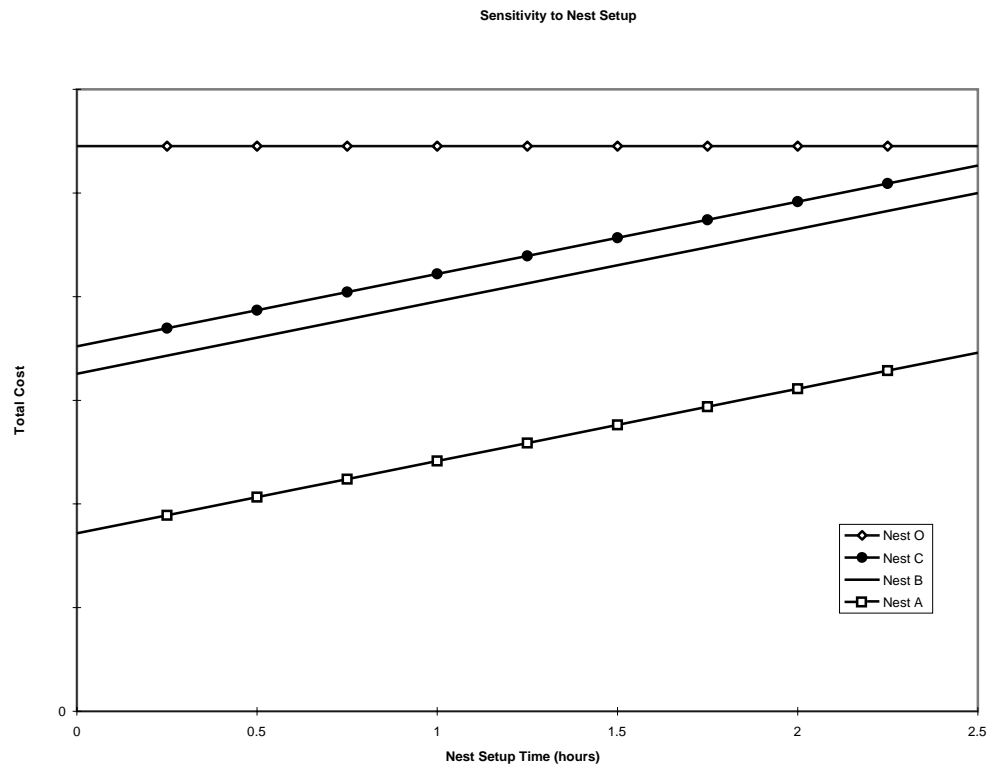


Figure 5: Total Cost as Nest Setup Time Changes - Group 1.

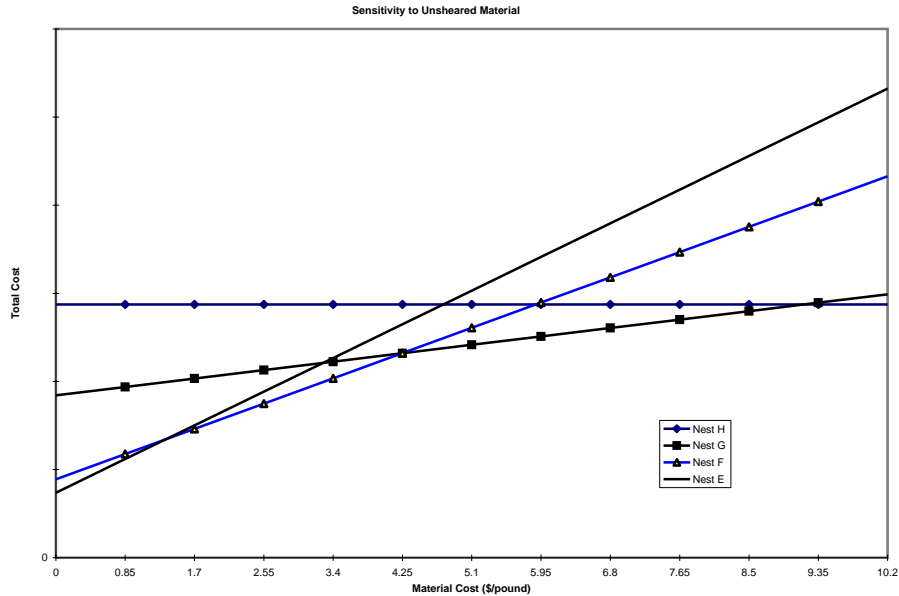


Figure 6: Total Cost as Unsheared Material Cost Changes - Group 2.

the machine setup times.) Figure 6 shows the four nests for Group 2 and each nest's total cost as the unsheared material cost changes. The predicted value is \$1.70 per pound. The empty nest's total cost does not change. The smaller nest's total cost increases less quickly because it uses less unsheared material. The complete nest's total cost increases most quickly. As the cost changes, different nests are optimal. The empty nest is optimal only when unsheared material costs are much higher than predicted. This chart shows that there can be some benefit to finding the optimal nest, but the most benefit occurs when unsheared material costs are in a middle region. For instance, when the unsheared material cost is 5.1 dollars per pound, Nest G is 16 per cent less expensive than the empty nest and 20 per cent less expensive than the complete nest.

Of course, the nest setup time also affects each nest's optimality. Specifically, increasing the nest setup time increases the complete nest's cost and any incomplete nest's cost. Thus, this makes the empty nest more desirable.

Figure 7 shows regions where each nest has the least total cost (of the four nests considered). The regions correspond to different values of the two most significant nest parameters: the nest setup time and the unsheared material cost. When the unsheared material cost is very low, the complete nest (Nest E) is the least expensive nest. As the unsheared material cost increases, Nest F is least expensive. Then Nest G is the least expensive. Finally, the empty nest (Nest H) is the least expensive. As the nest setup time increases, the threshold between Nest G and Nest H decreases. Near the bottom of the chart is a small circle that corresponds to the predicted values of the two parameters.

Figure 8 shows a similar analysis for Group 1. For Group 1, Nest C is never the least expensive nest.

For comparing the nest everything policy to the nest nothing policy, these charts show that the nest everything policy will be better unless unsheared material costs and nest setup times are much larger than predicted. In addition, there are regions where finding the optimal

Sensitivity to Nest Parameters - Group 2

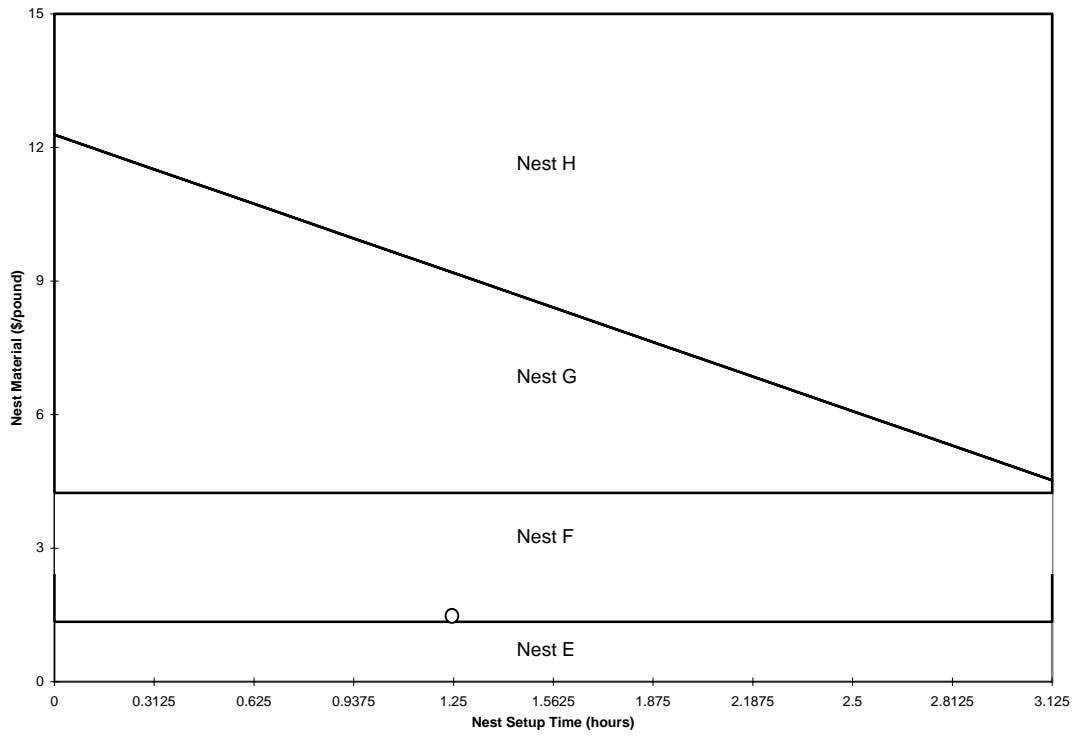


Figure 7: Optimal Group 2 Nests.

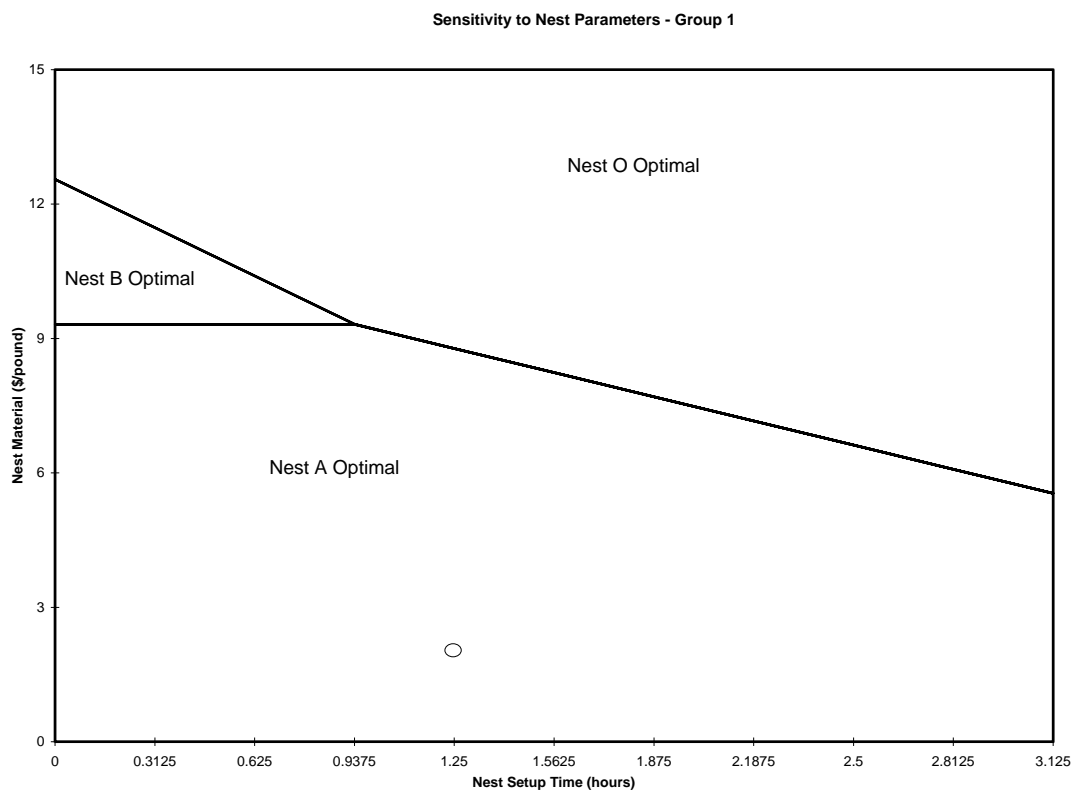


Figure 8: Optimal Group 1 Nests.

Table 11: Problem Set Parameters

Group	Problem Set	Number of orders	Minimum number of parts	Maximum number of parts
1	1	4	30	40
	2	8	10	25
	3	14	5	15
2	1	2	20	60
	2	4	10	30
	3	8	5	15

Table 12: Problem Set Results

Group	Problem Set	Nesting no orders	Nesting every order
1	1	2.42	1.00
	2	2.83	1.00
	3	3.72	1.00
2	1	1.84	1.00
	2	2.07	1.00
	3	2.69	1.00

nest can save additional money.

6.3 Order Size

The two instances that Section 5 describes represent a typical set of orders at the time we conducted the research. For these orders, nesting everything was much less expensive than nesting nothing. However, in the future, the shop might see a change in the order size: there might be more, smaller orders, or there might be fewer, larger orders. Thus we wanted to determine if the order sizes affected the savings due to nesting. To evaluate this, we generated 60 instances of the nesting problem, 30 instances for each group. Note that there are six part types in Group 1 and five part types in Group 2.

For each group, we defined three problem sets and generated 10 instances for each problem set. The parameters for each set are given in Table 11. This lists, for each group and each problem set, the number of orders in each instance, the minimum number of parts in an order, and the maximum number of parts in an order. To generate an order, we randomly chose one of that group’s part types, which were all equally likely. The order size was uniformly distributed between the minimum and maximum number of parts. All of the cost parameters remained the same.

After forming these 60 instances, we determined the setup and material cost of nesting no orders, the cost of nesting every order, and the cost of the optimal nest. We found the optimal nest by solving the corresponding integer program with a commercial solver. For every problem set, the cost of nesting every order dominated the cost of nesting no orders. The benefit of nesting every order increased as the number of orders increased (and the order size was decreasing). In addition, for almost every instance, nesting every order was an optimal solution. Table 12 summarizes these results. Each policy’s performance is the average performance relative to the optimal solutions.

7 Conclusions

Dynamic nesting has great cost-saving potential for NC punch presses that create sheet metal blanks. It reduces setups and material requirements. However, it complicates capacity and material planning, and it is difficult to estimate the cost savings. This paper presents models that help decision-makers plan production and evaluate the nesting policy. The models are analytical expressions that approximate the capacity requirements, material requirements, and costs of a given set of orders and nests.

With these expressions we can construct an integer program that models the nesting decisions for a given punch press and a given week. The objective is to minimize the total cost of setup, processing, and material. We can decompose this problem into subproblems for each group of orders that require the same material type and thickness. Finding an optimal solution is an NP-complete problem, however. This paper describes a pseudo-polynomial time dynamic program to solve the problem.

The examples based on industry data show that nesting orders can reduce material and capacity requirements and reduce total cost. Moreover, it shows that ignoring the nesting decision leads to highly inaccurate capacity and material plans. By using the correct models in their planning procedures, the shop can make better decisions about material and capacity requirements. Accurate capacity requirements are an essential tool for order release. In addition, these more accurate models help quantify the savings due to dynamic nesting.

The cost of any nest and the benefit of nesting in general depends upon various cost parameters. The material, capacity, and cost models presented here allow one to measure that sensitivity. Different cost parameters affect different nests differently. For our application, nesting every order is significantly better than nesting no orders, and this will remain true unless the cost parameters change drastically. Moreover, this remains true even if the number of orders and order size should change. Nesting every order saves more money as the orders become smaller but more numerous.

Future work should consider how nesting will affect the workload at subsequent operations, since many orders will complete in a short range of time. Even though punched sheets can begin deburring while other the punch press finishes other sheets, nesting may lead to waves of parts moving through the sheet metal area. The area will need to allocate space to store the work-in-process.

8 Appendix: Complexity

This section considers the problem of finding the least expensive nest for a group. Let us formally state the decision version of the nesting optimization problem:

NESTING. Instance: A set G_L of orders, with costs as given in Section 3, and a cost constraint \mathcal{C} .

Question: Is there a nest such that the total cost C_L is less than or equal to \mathcal{C} ?

Total cost for group:

$$\begin{aligned}
 C_L &= \sum_{j \in G_L} R_j^n R_j^C (1 - X_j) + \overline{R}_L^n \overline{R}_L^C \\
 &+ \sum_{j \in G_L} (s_k^m + R_j^n R_j^L) (1 - X_j) C_s \Pi_k \\
 &+ (Y_L \overline{s}_k^m + \overline{R}_L^n \overline{R}_L^L) C_s \Pi_k
 \end{aligned}$$

To show that this problem is NP-complete, we define a transformation from the NP-complete problem PARTITION.

PARTITION. Instance: A finite set $A = \{d_1, d_2, \dots, d_n\}$, where each d_i is a positive integer.

Question: Is there a subset $S \subset \{1, 2, \dots, n\}$ that partitions the set into two equal subsets?

$$\sum_{i \in S} d_i = \sum_{i \notin S} d_i$$

Given any instance of PARTITION, we can construct (in polynomial time) an instance of NESTING and show that there is a solution to PARTITION if and only if there is a solution to NESTING. This will prove that NESTING is NP-complete.

For an instance of PARTITION, let $B = \frac{1}{2} \sum_{i \in A} d_i$. Thus, there is a solution to PARTITION if and only if there is a subset S such that $\sum_{i \in S} d_i = B$. Now, construct the following instance of NESTING:

Let $G_L = \{0, 1, 2, \dots, n\}$. For each order $j > 0$, let $P^A(i_j) = 1$, $q_j = d_j$, $R_j^n = d_j$, and $R_j^C = 1$. These orders have many small parts, and a sheared sheet holds one part. For order 0, let $P^A(i_0) = 2B$, $q_0 = 1$, $R_0^n = 1$, and $R_0^C = 2B$. This order has one large part and requires one expensive sheared sheet. For all orders, let $R_j^L = 0$. Let $\overline{R}_L^C = 2B$ and $\overline{R}_L^A = 3B$. Let $s_k^m = 0$ and $\overline{s}_k^m = 0$ and $\overline{R}_L^L = 0$. Let $C_s \Pi_k = 0$. Let $C = 3B$. Substitute these values into the equations for \overline{R}_L^n and C_L :

$$\overline{R}_L^n = \left\lceil \frac{X_0 2B + \sum_{j>0} X_j d_j}{3B} \right\rceil$$

$$C_L = \overline{R}_L^n 2B + 2B(1 - X_0) + \sum_{j>0} d_j(1 - X_j)$$

If there is a solution to PARTITION, then there exists a partition S such that $\sum_{i \in S} d_i = B$. Form a solution to NESTING as follows: nest order 0 and those orders $j \in S$. Don't nest the other orders. That is, $Y_L = 1$ and $X_0 = 1$; for $j > 0$, $X_j = 1$ if and only if $j \in S$.

$$\overline{R}_L^n = \left\lceil \frac{2B + \sum_{j \in S} d_j}{3B} \right\rceil = 1$$

$$\begin{aligned} C_L &= 2B + \sum_{j \notin S} d_j = 3B \\ &= C \end{aligned}$$

If there is no solution to PARTITION, then, for any subset S , $\sum_{j \in S} d_j \neq B$. Now, we need to show that the cost of any nest is greater than C , so there is no solution to NESTING. For any nest, let $S = \{j > 0 : X_j = 1\}$. There are four cases that we must consider. First, suppose $Y_L = 1$, $X_0 = 1$, and $B < \sum_{j \in S} d_j \leq 2B$. This nest will require two unshared sheets and the total cost will exceed C :

$$\overline{R}_L^n = \left\lceil \frac{2B + \sum_{j \in S} d_j}{3B} \right\rceil = 2$$

$$C_L = 2 \times 2B + \sum_{j \notin S} d_j \geq 4B > \mathcal{C}$$

Second, suppose $Y_L = 1$, $X_0 = 1$, and $\sum_{j \in S} d_j < B$. Thus, $\sum_{j \notin S} d_j > B$. This nest will require one unsheared sheet, but the total cost still exceeds \mathcal{C} :

$$C_L = 2B + \sum_{j \notin S} d_j > 3B = \mathcal{C}$$

Third, suppose $Y_L = 1$, $X_0 = 0$, and $0 < \sum_{j \in S} d_j \leq 2B$. This nest will require one unsheared sheet, and the total cost will exceed \mathcal{C} due to the cost of order 0 ($R_0^C = 2B$):

$$\overline{R}_L^n = \left\lceil \frac{\sum_{j \in S} d_j}{3B} \right\rceil = 1$$

$$C_L = 2B + 2B + \sum_{j \notin S} d_j \geq 4B > \mathcal{C}$$

Finally, suppose no orders are nested: $Y_L = 0$, $X_0 = 0$, and $\sum_{j \in S} d_j = 0$. Thus, $\sum_{j \notin S} d_j = 2B$. This solution will require no unsheared sheets, but the total cost will exceed \mathcal{C} due to the cost of order 0 ($R_0^C = 2B$):

$$C_L = 2B + \sum_{j \notin S} d_j = 4B > \mathcal{C}$$

This completes the proof, for we have shown that there is no solution to NESTING.

Note that this proof shows that NESTING is NP-complete even if all setup costs are zero. This problem remains NP-complete if the setup costs are positive but the material costs are zero. To prove this, construct the following transformation: Let $G_L = \{0, 1, 2, \dots, n\}$. For each order $j > 0$, let $P^A(i_j) = 1$, $q_j = d_j$, $R_j^n = d_j$, $R_j^C = 0$, and $R_j^L = 1$. For order 0, let $P^A(i_0) = 2B$, $q_0 = 1$, $R_0^n = 1$, $R_0^C = 2B$, and $R_0^L = 2B$. Let $\overline{R}_L^C = 0$, $\overline{R}_L^L = 2B$, and $\overline{R}_L^A = 3B$. Let $s_k^m = 0$ and $\overline{s}_k^m = 0$. Let $C_s \Pi_k = 1$. Let $\mathcal{C} = 3B$.

This yields the same equations for \overline{R}_L^n and C_L :

$$\begin{aligned} \overline{R}_L^n &= \left\lceil \frac{X_0 2B + \sum_{j>0} X_j d_j}{3B} \right\rceil \\ C_L &= \overline{R}_L^n 2B + 2B(1 - X_0) + \sum_{j>0} d_j (1 - X_j) \end{aligned}$$

Thus, this problem has a solution ($C_L \leq \mathcal{C}$) if and only if PARTITION has a solution.

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