Magic Matter, the Computational Æther, and the Miner's Canary

Charles W. Misner
Albert Einstein Institute/Max Planck Institute
D-14476 Golm, Germany
and
Department of Physics, University of Maryland,
College Park MD 20742-4111 USA
e-mail: `misner@aei-potsdam.mpg.de`
`misner@physics.umd.edu`
(W O R K I N G   D R A F T, 26 January 2001; Revision 0.8)

Several viewpoints are proposed with the aim of promoting further approaches to the numerical integration of Einstein's equations, especially in support of attempts to detect astrophysically significant gravitational waves. Magic Matter suggests that one should instruct computer programs to ignore Einstein's equations inside the horizons of black holes, and to instead produce computationally convenient metrics there that will not interfere with the correct solution of the equations in the physically observable regions of the simulation. The Computational Æther is the spatial grid of coordinate vertices, conceived as an imaginary substance which spreads itself conveniently over the curved spacetime in ways that should simplify the computational effort. The dynamics of magic matter and the computational æther represent instances of Applied Science Fiction where physical laws inconsistent with our knowledge of nature are used in aspects of simulations that have no observable consequences, but are expected to improve computational efficiency. The Miner's Canary gives notice in such computations, not that the air in the mine is becoming poisoned, but that some regions of the computational grid probably lie inside apparent horizons so that one may take liberties with the Einstein equations to preserve the life of the computation. Satisfactory implementations of these three ideas are not provided here; rather some first steps toward such are proposed to stimulate further research.

I. INTRODUCTION

Words influence our creativity (Wheeler, Feynman). So here some ways of thinking about black hole calculations are proposed. Although I will use several different catch phrases to describe these approaches, they each attack one of two difficulties in numerical computation. One difficulty is the formation of spacetime singularities inside the horizon of any black hole that occur in the computation. The second is the likelihood that injudiciously chosen coordinates my require extreme computational efforts that halt the computation. These two difficulties are closely related since current experience has found coordinate distortions most prevalent near the horizons of black holes.

Applied Science Fiction (AFS) refers to the use of physically absurd dynamics where it has no observable consequences. On such place is inside black hole horizons where one may stuff a gravitating substance, Magic Matter, which allows singularities to be suppressed. The fluid version of Magic Matter may be called Aquavit (aqua vitæ, preserving the life of the computation) when it can be treated as a fluid. Another case of Applied Science Fiction is in assigning the dynamics of a possibly visco-elastic substance to the spatial coordinates (or grid vertices in a discrete computational approximation) — the Computational Æther. This dynamics is fictional because the motions of the coordinate grid should, from general covariance, have no influence on the geometry of the resulting spacetime. The stress-energy tensor of the computational æther may be a useful prop to our inventiveness in choosing a useful dynamics, but does not enter the Einstein equations or generate gravitational fields.

Before electrochemical technology supplied patentable devices, miners were said to carry a caged canary into the mines which, by showing sign of illness, would warn of poisoned air. A similar signal is needed by computational algorithms to warn that sectors of the grid lie within black holes, so that lifesaving science fiction should be invoked to preserve the computation. A scalar field playing the role that $\Phi = -M/r$ does in the Schwarzschild solution would be the ideal such Canary. Although a full scalar field dynamics might be invented to provide this service, our preliminary example of a dynamics for the computational æther suggests that such a scalar indicator may arise from some theories of the computational æther.

These ideas are partly an outgrowth of the Fat Particle idea (developed with Conrad Schiff, ref) which can be considered a first example of Applied Science Fiction. But some inspiration to use creative and (creativity producing) names came from the Lazarus Project [BBCL00] which showed that a black hole computation that was descending into computational hell could be resurrected by transforming to a fresh algorithm.

II. APPLIED SCIENCE FICTION

Because the behavior of spacetime inside an event horizon (e.g., inside an apparent horizon) cannot influence the observable spacetime outside, some computational schemes (refs) have tried to avoid the treatment of singularities by excising a singularity prone region from the active computational grid. ASF similarly seeks to avoid solving Einstein's equations (with their singular consequences) inside apparent horizons. But computers don't accept the instruction "I don't care — do anything you like." So rather than directly attempting to avoid solving Einstein's equations inside a black hole, ASF asks that they be solved, but with some unphysical matter (e.g., Aquavit) replacing the vacuum there. Some suitable kinds of unphysical matter may be suggested by the science fiction studies of Thorne and Novikov (refs) on transversible wormholes.* I explore here some other avenues.

III. AQUAVIT

As suggested by the name (Aquavit, Akvavit, aqua vitæ), a simple example of ASF may be a fluid that has an unphysical equation of state designed to resist collapse to a singularity. Since the original example of collapse to form a black hole [OS39] can be reformulated [Bec62,Mis67,Mis69] as a sector of the pressure-free FRW closed universe matched to a Schwarzschild exterior solution, one first looks to avoid the *cosmological* singularity with a dose of Aquavit. For a homogeneous isotropic FRW cosmology the field equations can be taken, e.g. [MTW73, eqns. (27.39)], as

$$(dR/dt)^2 - (8\pi/3)R^2\rho(R) = -k \tag{3.1}$$

where R is the scale factor for the evolving universe (which we take as a stand-in for the interior of a collapsing star) and ρ is the density of mass-energy in the rest frame of the fluid. The properties of the matter are incorporated in the isentropic equation of state $\rho(n)$ where $n \propto 1/R^3$ is a conserved particle number density. Simple physical idealizations of matter include pressure-free dust with $\rho \propto n$ and massless radiation $\rho \propto n^{4/3}$. But inside an event horizon we are free to postulate any equation of state that might be useful for easing the computation. From this equation of state the pressure is computed as

$$P = n(d\rho/dn) - \rho \tag{3.2}$$

(cf. [MTW73, eqn. (22.7)]) which gives $P = (\gamma - 1)P$ for $\rho \propto n^\gamma$. The velocity of sound c, for small density perturbations from a homogeneous fluid in special relativity, is given by

$$c^2 = \frac{1}{\rho + P} n \left(\frac{\partial P}{\partial n}\right)_s \tag{3.3}$$

in units where the velocity of light is unity. This evaluates to $c^2 = \gamma - 1$ when $\rho \propto n^\gamma$.

For an example of a singularity avoiding equation of state (for a FRW universe) we suggest

$$\rho \propto n^\gamma (n_1^\gamma - n^\gamma) \tag{3.4}$$

which, for $n \ll n_1$ gives the familiar cases $\rho \propto n^\gamma$. With this equation of state, and with $n \propto 1/R^3$, the Friedmann equation (3.1) becomes

$$(dR/dt)^2 + V(R) = -K \quad , \tag{3.5}$$

a form corresponding to [MTW73, eqn. (27.74)]. Here the effective potential $V(R)$ which allows a rigorous qualitative description of the solutions of this equation is

$$V(R) = -V_N R^{-3\gamma+2}(1 - R^{-3\gamma}) \quad . \tag{3.6}$$

This function is plotted in Figure 1. [The scale factor $V_N = \frac{6\gamma-2}{3\gamma}\left(\frac{3\gamma-2}{6\gamma-2}\right)^{(2-3\gamma)/3\gamma}$ sets the minimum to -1.]

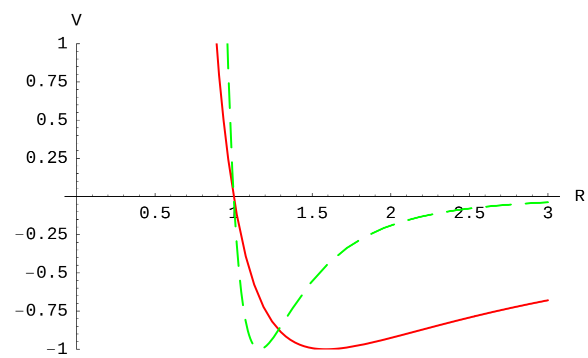

FIG. 1. The effective potential $V(R)$ plotted for two values of the parameter, $\gamma = 1$ and $\gamma = 2$. For all values of the constant energy-analog $-K \geq -1$ in equation (3.5) there exist nonsingular solutions with $R > 0$ for all time. The value $K = +1$ corresponds to a stable static Einstein universe populated with any $\gamma > 2/3$ brand of aquavit.

The use of unphysical fluids in inhomogeneous environments, such as the interior of black holes where one hopes they can be useful computationally, could bring in hydrodynamical problems as a trade-off for gravitational singularities. This can only be decided by computational tests. One problem that might be addressed is the thermodynamical instability of substances with negative compressibilities. But self-gravitating relativistic substances with stiff positive compressibilites are unstable (e.g., toward

*The NSF required careful accounting by Caltech to avoid the possibility that such ridiculous speculations could be linked to LIGO funding. Here I suggest that this science fiction can be applied to solve significant LIGO-related problems.

black hole formation) so Newtonian intuitions need to be upgraded by further investigation. The local properties of the $\gamma = 2$ aquavit are plotted in Figure 2.

A viscous aquavit might work better than the ideal fluids described above. The addition of bulk viscosity in the FRW prototype model for a black hole interior might result in an homogeneous collapse that was terminated, not by a bounce, but by a damped relaxation to the equilibrium Einstein universe represented by the bottom of the potential well in Figure 1.

As discussed in the next section, unphysical equations of state such as equation (3.4) need not be used only in regions of spacetime occupied by matter. When modified to allow $\rho = 0$ at low densities n, they can be considered either as modified equations of state for the vacuum, or as equations of state for the computational grid.

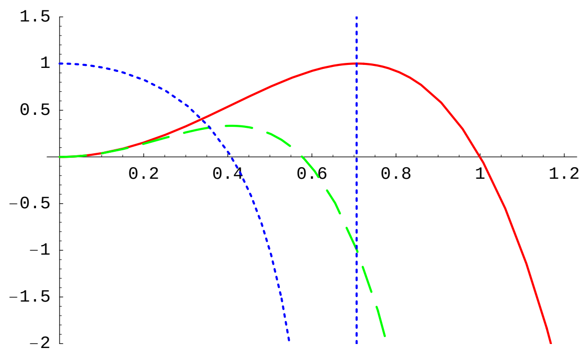

FIG. 2. The properties of the $\gamma = 2$ fluid defined by equation (3.4) against a normalized density n/n_1. The solid curve is the energy density ρ, while the dashed curve is the pressure P in the same units. The dotted curve is c^2, the dimensionless squared sound velocity. Note that the fluid becomes locally unstable ($c^2 < 0$) necessarily when $c^2 = 0$ where pressure gradient forces are negligible in the equation for density perturbations, so that the homogeneous model calculation for an FRW universe may there give meaningful suggestions of the behavior under self gravitation. Also the singularity in c^2, where $\partial\rho/\partial n = 0$ so $\rho + P = 0$, occurs at a state where the stress energy tensor reduces to a false vacuum condition (cosmological 'constant') which is not obviously troublesome.

IV. DYNAMIC GRIDS

It is commonly taught that the æther was eliminated from physics by Einstein with the introduction of special relativity. But a half century later it was not uncommon for physicists to talk of the new æther, the spacetime vacuum. Although the special relativitic æther had lost one familiar property of a substance — it's use as a reference to or from which a velocity could be defined — it retained many others. For instance even the special relativistic vacuum could be decorated with adornments such as the electromagnetic field. It also served as the medium for the propagation of electromagnetic waves. But with relativistic quantum field theory, and with classical general relativity, the vacuum became a quite substantial æther. It was visualized as a stew of virtual particles. It threatened to have a nonzero energy density. And it could bend and wave and deflect other types of matter. Now numerical relativity has brought us another æther beyond the discarded æther of the 19th century and the current æther of the 20th century. This is the computational æther — the coordinate system or computational grid — which is causing many of the problems facing scientists in the numerical solution of Einstein's gravitational equations.

Many of the difficulties which Einstein and others had in understanding his theory of general relativity were founded in a tendency to attribute physical significance to coordinates. Fifty years after 1915 it became possible to conceive of a curved spacetime without giving prominence to coordinates, and this is now commonplace among relativists. But numerical relativists are finding that the coordinate system is a scaffolding which is essential in building a spacetime. I suggest that it be thought a second æther interacting with the primary æther, spacetime. The principle of general covariance says that this second æther cannot influence the structure of the physical spacetime. But it does not say that the physical spacetime cannot influence the scaffolding. And, when sheltered by an event horizon, this principle can be violated; the computational æther can be allowed to influence the physical æther.

A. Fluid Æther

For an example of a computational æther which can actively change the spacetime geometry consider the equation of state

$$\rho \propto \begin{cases} 0, & n \le n_0 \\ (n-n_0)^\gamma[(n_1-n_0)^\gamma - (n-n_0)^\gamma], & n_0 \le n \end{cases} \tag{4.1}$$

in which $0 \le n_0 < n_1$ is assumed. This is just equation (3.4) offset to give $\rho = 0$ for $n \le n_0$. This is an example of an (unphysical) equation of state for the vacuum, or an equation of state for a computational æther. At low densities $n < n_0$ it has no energy density ρ and no pressure P and thus is a conventional classical vacuum or a computational æther which does not enter the Einstein equations. At higher densities, expected to be found only inside black holes, it might provide the protection against singularities suggested for the simpler aquavit of the previous section. But if no other matter is introduced, as in a binary black hole system, what is n?

To interpret equation (4.1) as the equation of state of a computational æther or of a coordinate system one must interpret n as the density of spatial coordinates. In a discrete form this would make $1/n$ the volume of a grid cell in its own rest frame. In the continuum description n is defined by its conservation law $(nu^\mu)_{;\mu} = 0$ where

u^μ is the 4-velocity of the spatial coordinates, thus in a coordinate system comoving with this computational æther one has $u^i = 0$ for $i = 1, 2, 3$. The conservation law for n then reads

$$\partial_0[n\sqrt{-g}/\sqrt{-g_{00}}] = 0 \tag{4.2}$$

and defines n directly in terms of the metric and some (arbitrary) initial density assignment. The covariant spatial components of the æther's 4-velocity $u_i = u^0 g_{0i}$ are essentially the shift vector. In regions where $\rho + P > 0$ the u_i will be determined algebraically by the momentum constraints $G^0_i \propto T^0_i = (\rho+P)u^0 u_i = (\rho+P)g_{0i}/(-g_{00})$.

This simple picture of the grid as a fluid computational æther is likely inadequate. The properties of the fluid of equation (4.1) are too violent at the transition $n = n_0$ from test matter to self-gravitating matter. And the shift vector control enters too abruptly as $\rho + P$ increases or decreases from zero. But further attempts can be made to define shift conditions by thinking of the grid (spatial coordinates) as a kind of æther or test (nongravitating) matter. From this viewpoint the 4-velocity of the grid, u^μ with $u^i = 0$ for $i = 1, 2, 3$, would play an increased role in our thinking.

B. Elastic Æther

An æther conceived as a material substance with timelike 4-velocity u^μ has as an important descriptor the strain of its rest frame. This strain is described by the projection tensor $h_{\mu\nu}$ and by the strain rate $L_{\alpha\beta}$ where[†]

$$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \quad \text{and} \quad L_{\alpha\beta} = -\tfrac{1}{2}\mathcal{L}_u h_{\alpha\beta} \quad . \tag{4.3}$$

These clearly depend only on the choice of spatial coordinates and their world lines to which u^μ is tangent. In coordinates tied to this æther so that $u^i = 0$ these tensors satisfy $h_{0\mu} = 0 = L_{0\mu}$ and have spatial components h_{jk} and $L_{jk} = -\frac{1}{2}\Gamma\partial_0 h_{jk}$. Here $\Gamma \equiv u^0 = 1/\sqrt{-g_{00}}$. An alternative definition of h_{jk} in these coordinates is that it is the 3×3 matrix inverse of the matrix g^{jk}, the spatial components of the contravariant 4-metric. Thus these components, like $h_{\mu\nu}$ and $L_{\mu\nu}$, are independent of changes in the time slicing (choice of the x^0 coordinate) and depend only on the choice of spatial grid. Using these desciptors as tools, one could imagine controlling the grid (and thus the shift vector) by requiring this æther to act as a viscoelastic solid or even a kind of Silly Putty (i.e., fluid on large scales, but solid for some higher frequencies). However, some considerations of even a simple Schwarzschild black hole with its singularity tamed by unphysical matter shows that a practical computational æther needs to be made of grid points that can move above the speed of light, i.e., with spacelike worldlines.

[†]In coordinates with zero shift so $g_{0k} = 0$ the strain parameters h_{jk} and L_{jk} reduce to the components of the first and second fundamental forms of a time slice as used by Dedonder and ADM.

C. Magic Matter

To get some idea of the kind of unphysical matter needed to tame a computed black hole model we modify the Schwarschild metric and learn what it costs. In a long known but little used form of the Schwarzschild metric one has

$$ds^2 = -dt^2 + \left(dx^i + x^i\sqrt{\frac{2U}{r^2}}dt\right)^2 \tag{4.4}$$

where a Euclidean sum of squares occurs on the space indices. This form is closely related to the ingoing Eddington-Finkelstein coordinates (e.g., [MTW73, Box 31.2]) and satisfies the vacuum Einstein equations when $U = M/r$ with $r^2 = x^i x^i$. It allows one to describe infalling matter or light and the formation of the singularity and has the convenience of a unit lapse, Euclidean geometry on the $t =$ constant slices and $\sqrt{-g} = 1$. An important Kerr generalization of this presentation of a black hole has been given recently by Doran [Dor00]. We will modify this metric in the region $r \le M$, well inside the horizon, to tip the light cones back to a nonsingular configuration there. The inward and outward edges of the light cones in this metric are given by the radial null vectors

$$\begin{aligned} \ell &= \partial_t + (x^i/r)(1-\sqrt{2U})\partial_i \\ &\text{and} \\ n &= \partial_t - (x^i/r)(1+\sqrt{2U})\partial_i \end{aligned} \tag{4.5}$$

so one sees that the entire light cone tips inward (toward smaller r) whenever $2U > 1$. For this metric to be differentiable at the origin $r = 0$ one needs that $\sqrt{U(r)/r^2}$ be differentiable there. But $\sqrt{f(u)}$ is differentiable provided $f(u)$ is and while also $f(u) > 0$. So with $r^2 = x^i x^i$ differentiable (which r is not at the origin $r = 0$), we easily achieve a differentiable metric by choosing $U(r)/r^2 = f(r^2) > 0$ for any differentiable function f(u). This requirement includes $U(0) = 0$ so the light cones at the origin are oriented just as in Minkowski spacetime.

The Einstein tensor computed for this metric (in spherical coordinates) we write as $G^\mu{}_\nu = \mathrm{diag}\lceil -\rho, P_r, P_\perp, P_\perp)\rfloor$ suggesting its interpretation, when nonzero, as generated by a stress-energy tensor for some exotic matter. The computed values are

$$\rho = -P_r = 2(rU)'/r^2 \quad , \quad P_\perp = -(r^2U')'/r^2 \tag{4.6}$$

and are all zero when $U = M/r$. It is, however, not difficult to "draw by hand" values of U for $r \le M$ that smoothly join to the Schwarzschild values at larger r but leave the metric nonsingular. An example with M=1 is

$$U = \begin{cases} 1/r & \text{if } r \geq 1 \\ r^2(35 - 42r^2 + 15r^4)/8 & \text{if } r \leq 1 \end{cases} \tag{4.7}$$

This choice makes all metric components in equation (4.4) C^∞ functions everywhere except at $r = 1$ where they are only C^2. The nonzero components of $G^\mu{}_\nu$ for this example are plotted in Figure 3.

It is not possible to state that this magic matter is moving at velocities greater than light. This for the reason that, as with false vacuum, the rest frame of the matter is not defined. One would normally define the 4-velocity u^μ of matter as the timelike eigenvector of the matter stress-energy tensor, $T^\mu{}_\nu u^\nu = -\rho u^\mu$, as for an ideal fluid. But *every* vector v^μ in the rt plane satisfies $G^\mu{}_\nu v^\nu = -\rho v^\mu$ for the metric of equation (4.4), and such vectors can be spacelike or null as well as timelike.

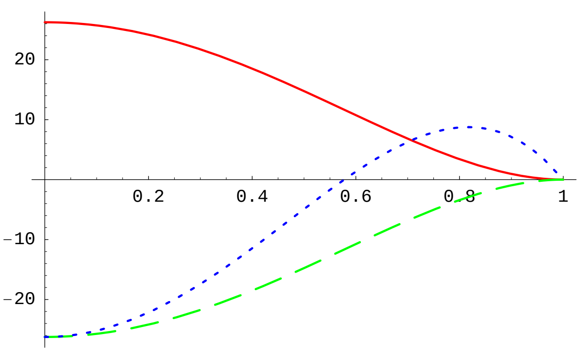

FIG. 3. The properties of some magic matter which removes the singularity in a Schwarzschild black hole are plotted here. These are the nonzero (diagonal) components of the Einstein tensor for (the spherical coordinate version of) the metric defined by equations (4.4) and (4.7), plotted against r/M. The solid curve is the implied energy density ρ, while the dashed curve is the radial stress (pressure) P_r in the same units. The dotted curve is $P_\perp$, the required stress in the transverse (θ, ϕ) directions.

D. A Computational Æther

Although the above example of a nonsingular black hole stuffed with magic matter did not require the matter to move faster than light, its presentation as a stationary metric did require — most importantly in the vacuum regions just inside the horizon — that the vertices $x^i =$ const of the spatial grid move faster than light. Thus the 4-velocity of the spatial grid, the computational æther, cannot be assumed to be a unit vector. We therefor propose to describe the motion of this computational æther by a vector v^μ whose direction in spacetime is tangent to the world lines of this æther, so $v^i = 0$, but whose magnitude will be normalized by the chosen time coordinate so that we set $v^0 = 1$. Then $v^\mu v_\mu = g_{00} \equiv -\sigma$ has an unspecified sign even when we restrict the choice of time coordinate $x^0 \equiv t$ to ones that make constant t hypersurfaces spacelike so that $t_{;\mu} t^{;\mu} = g^{00}$ is always negative. [In the preceding example one had $\sigma = -g_{00} = 1 - 2U$ but $t_{;\mu} t^{;\mu} = g^{00} = -1$.] Associated with this measure of the grid velocity v^μ are two tensors which we hope will be useful in defining a dynamics for the grid that will help computer programs lay out convenient coordinates on the spacetimes they are being asked to generate. One such tensor tries to measure the strains in the æther. It is

$$p^{\mu\nu} = v^\mu v^\nu - (v_\alpha v^\alpha) g^{\mu\nu} \quad . \tag{4.8}$$

The other measures the strain rate

$$\kappa_{\mu\nu} = -\tfrac{1}{2} \mathcal{L}_v p_{\mu\nu} \quad . \tag{4.9}$$

These two tensors live essentially on the spacelike hypersurfaces $t =$ const since one finds that

$$v^\mu p_{\mu\nu} = 0 = v^\mu \kappa_{\mu\nu} \tag{4.10}$$

The first of these equations is just simple algebra; the second follows by taking the Lie derivative $\mathcal{L}_v$ of the first and using $\mathcal{L}_v v^\mu = 0$. In the associated coordinates where $v^\mu = \delta^\mu_0$ these read $p_{0\mu} = 0 = \kappa_{0\mu}$.

An important feature of $\kappa_{\mu\nu}$ is that when v^μ is a Killing vector, as in the stuffed black hole example, then $\kappa_{\mu\nu} = 0$. A Killing vector satisfies $\mathcal{L}_v g_{\mu\nu} = 0$ and, with $\mathcal{L}_v v^\mu = 0$ plus derivative product rules, the formula $p_{\mu\nu} = (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\nu} g_{\alpha\beta}) v^\alpha v^\beta$ immediately gives $\kappa_{\mu\nu} = 0$. Consequently, with a viscous stress proportional to this strain rate, one could hope to write a dynamics for the grid motion v^μ that would damp toward $\kappa_{\mu\nu} = 0$ when the geometry of the spacetime permitted a stationary solution. A possible dynamics for study could be

$$(v_{\mu;\nu} + v_{\nu;\mu}) v^\nu + 2\xi \kappa_{\mu\nu}{}^{;\nu} = 0 \tag{4.11}$$

where $\xi > 0$ is a coefficient of viscosity. Another prospective dynamics would be

$$-(v_{\mu;\nu} + v_{\nu;\mu})^{;\nu} + 2\xi \kappa_{\mu\nu}{}^{;\nu} = 0 \quad . \tag{4.12}$$

Each of these equations is satisfied when v^μ is a Killing vector. The first is probably a parabolic equation, the second probably hyperbolic. They could be combined to give a damped wave equation. Their relations to the shift conditions introduced by Alcubierre (refs) have not yet been established.

Although the grid velocity v^μ and its associated strain and strain rate may be the appropriate fields for formulating a dynamics for the computational æther, it will still be important to be able to describe independently the choices being made for the time slicing $x^0 = t$ and for the space coordinates or computational grid. Thus it is important to recognize that the g^{jk} components of the 4-metric are independent of the time slicing condition. Consider the inner product $\Phi_{;\mu} \Psi^{;\mu} = \Phi_{,\mu} \Psi_{,\nu} g^{\mu\nu}$ in the case $\Phi = x^j$ and $\Psi = x^k$. The result is $\Phi_{;\mu}\Psi^{;\mu} = g^{jk}$ which is clearly independent of one's choice of slicing funcion $t = x^0$. Still another way to interpret the g^{jk} is to consider the distance from one grid point to a neighbor. To be dynamically important in the reaction of the æther to distortions in the grid, this distance should be measured in the rest frame of the grid, i.e., along a small vector s^μ orthogonal to the grid 4-velocity v^μ. But since $v^i = 0$, the orthogonality condition $v^\mu s_\mu = 0$ gives $s_0 = 0$. Consequently, the invariant norm of this vector is just $s^2 = s_\mu s_\nu g^{\mu\nu} = s_j s_k g^{jk}$ which reduces to sums only over the spatial indices. Inner products among such vectors to worldlines of nearest neighbor grid vertices similarly involve only g^{jk}, again establishing these metric components as the descriptors of grid cell shapes. When grid vertices move at greater than the velocity of light, this 3×3 matrix g^{jk} could have a non-Euclidean signature or be degenerate with vanishing determinant.

The simplest property of the time coordinate is the norm of its gradient $t_{;\mu}t^{;\mu} = g^{00} \equiv -1/N^2$ which is usually reported via the lapse function N. As long as this gradient is timelike (N real), the induced metric ${}^3g_{jk} = {}^4g_{jk}$ will have a Euclidean signature and the $t =$ const surfaces will be spacelike.

E. The Miner's Canary

Notice of the formation and location of black holes in current numerical computations is obtained primarily through the use of apparent horizon finders. It is possible that less accurate but more economical identification of the grid sectors inside black holes might be obtained from the behavior of test fields propagating on the evolving background. One possible direction for study would be the construction of scalar field theories where the scalar field for a suitable self-interaction potential might want to fall into any black hole and reveal the black hole's presence by some characteristic values or behaviours. A simple useful scalar field in the Kerr metric is any function $\Psi(r)$ where r is the usual Kerr and Boyer-Lindquist coordinate as in [MTW73, Box 33.2]. Since g^{rr} vanishes at the horizon in these metrics, one has $\Psi_{;\mu}\Psi^{;\mu} = 0$ at the horizon, and this might serve as an horizon alarm even in metrics that were only approximately Kerr black holes. But for this thought to be fruitful, one would need a way to characterize such fields Ψ. A simple wave equation $\Psi^{;\mu}_{;\mu} = 0$ has, in the Kerr metric, solutions $\Psi(r)$ which are unfortunately singular at the horizon: $\Psi = \Psi_0 + \ln[(r-r_+)/(r-r_-)]$ where $r_\pm$ are the horizon locations. A function $\Psi = (r-r_+)/(r-r_-)$ which is well behaved from spatial infinity down through the outer horizon satisfies an equation which, while meaningful in other metrics, appears difficult to handle numerically, namely $(\Psi^2)^{;\mu}_{;\mu} - 4\Psi_{;\mu}\Psi^{;\mu} = 0$. Were it possible to find a wave equation for Ψ which (with suitable boundary conditions) stably evolves to a function of r alone in the Kerr geometry, and which was meaningful in other geometries, then the condition $\Psi_{;\mu}\Psi^{;\mu} = 0$ might warn that one was in the neighborhood of an apparent horizon.

Another possibility is that the 4-velocity v^μ of the computational æther itself, for a suitable dynamics, could detect the presence of black holes. As a first attempt in this direction I suggest the study of æther dynamics based on equations similar to (4.11) and (4.12). If the viscosity drives $\kappa_{\mu\nu}$ toward zero it should also be driving v^μ toward an approximate Killing vector. But some Killing vectors in the Schwarzschild and Kerr metrics become null vectors with $-\sigma \equiv v^\mu v_\mu = 0$ at the horizon. It would suffice for many purposes if regions with, say $\sigma < -1$ (or some other constant) were normally within black hole horizons. At the least such a signal could provide a starting point for apparent horizon searches to verify the presence of a black hole.

[BBCL00] John Baker, Bernd Brügmann, Manuella Campanelli, and Carlos O Lousto. Gravitational waves from black hole collisions via an eclectic approach. *Class. Quantum Grav.*, 17:L149–L156, 2000. 'The Lazarus project'.

[Bec62] D. L. Beckedorff. senior thesis, Princeton University, 1962. Summarized in [MTW73, Sec. 32.4] and in [Mis67,Mis69].

[Dor00] Chris Doran. A new form of the Kerr solution. *Physical Review*, D61:067503, 2000.

[Mis67] Charles W. Misner. Infinite red-shifts in general relativity. In T. Gold, editor, *The Nature of Time*, pages 75–89, Ithaca NY, 1967. Cornell University Press.

[Mis69] Charles W. Misner. Gravitational collapse. In M. Chretein, S. Deser, and J. Goldstein, editors, *Astrophysics and General Relativity, 1968 Brandeis Summer Institute*, volume 1, pages 113–215, New York, 1969. Gordon and Breach.

[MTW73] C. W. Misner, K. S. Thorne, and J. A. Wheeler. *Gravitation.* W. H. Freeman and Company, New York, 1973.

[OS39] J. R. Oppenheimer and H. Snyder. On continued gravitational contraction. *Physical Review*, 56:455–459, 1939.