ABSTRACT

Title of thesis: RANKING U.S. ARMY GENERALS OF THE
                    TWENTIETH CENTURY USING THE GROUP
                    ANALYTIC HIERARCHY PROCESS.

                    Todd Philip Retchless, Master of Science 2005

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The group analytic hierarchy process (GAHP) is a mathematically based decision making tool that allows groups of individuals to participate in the decision making process. In this thesis, we use the GAHP and the expert opinions of 10 professional and amateur military historians to rank seven U.S. Army generals of the 20th Century. We use two methods to determine the priority vectors: the traditional eigenvector method and the recently introduced interval linear programming method. We consider the effects of removing outlier data and compare the rankings obtained by each method.

RANKING U.S. ARMY GENERALS OF THE TWENTIETH CENTURY USING
THE GROUP ANALYTIC HIERARCHY PROCESS.

By

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Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Who is the greatest U.S. Army General of the 20th Century? This is an extremely difficult question with no unanimous answer. Presenting this question to 10 individuals may result in 10 different, yet justifiable, answers. The unique experiences and accomplishments of each general is one aspect that makes this decision difficult. How can we compare the achievements of General of the Army George Marshall to General of the Armies John Pershing? Each held entirely different positions in two different wars. Marshall was the World War 2 Army Chief of Staff while Pershing commanded the Army Expeditionary Force in World War 1. We face another complicating factor when we attempt to compare each general's strengths and weaknesses. Consider General of the Army Dwight Eisenhower and General George Patton. Eisenhower's greatest strength was his interpersonal skill, while Patton's tactical skill was his most well-known strength. Which of these skills is more important when ranking 20th Century U.S. Army Generals?

The obstacles faced in ranking 20th Century U.S. Army Generals are common in the world of decision making and can be overcome by using the Analytic Hierarchy Process (AHP). Complex decisions are composed of numerous components, or factors, which influence the overall goal (decision). Identifying these factors and determining the extent to which they influence our goal is the most critical aspect of the decision-making process. Comparing different factors two-at-a-time is an effective method of determining the relative importance of each factor. The AHP

provides a framework for translating these comparisons into relative weights for each factor. Only after the problem has been correctly structured and the weight of each factor assigned can the decision maker begin to consider which alternative best satisfies the overall goal.

Many decisions are best made after considering the opinions of multiple decision makers, yet demand an equitable means of synthesizing each member's input. Alternatives chosen without consideration of the preference of each group member fail to capitalize on the group's experience and knowledge. As decisions become more complex, the experience of each member of the group becomes more valuable in choosing the best alternative. The Group Analytic Hierarchy Process is a powerful decision-making tool that allows groups of decision makers to compare and select alternatives as part of the group decision-making process.

Our group of decision makers consists of 10 professional and amateur military historians. We used their expert opinions to identify and assign weights to the factors that best define great generals of the 20th Century. After structuring the problem, each decision maker rated seven U.S. Army Generals with respect to the factors determined in the previous step. The result is a ranking of the best U.S. Army Generals of the past century made by a panel of experts with diverse historical backgrounds. We devote the remainder of this chapter to introducing the decision makers who participated in our study.

Dr. Conrad Crane is the Director of the U.S. Army History Institute in Carlisle, Pennsylvania. His education includes a B.S from the U.S. Military Academy (USMA), and a M.A. and Ph.D. in History from Stanford. He is Trustee

for the Society of Military History and is a member of the World War 2 Studies Association. Dr. Crane's historical interests include 20th Century Military History, Airpower, and Generalship.

Major Andrew Dziengelski is a U.S. Army Operations Officer serving in Washington, D.C. He received a B.A. in History from Western Maryland College and is a member of the U.S. Army Armor Association. His historical interests include Operational Art and American Military History.

Robert Goldich is a Specialist in National Defense in the Foreign Affairs, Defense, and Trade Division of the Congressional Research Service at the Library of Congress. He holds a B.A. in History and Political Science from Claremont McKenna College, an M.A. in International Affairs from George Washington University, and a Diploma from the National War College. Mr. Goldich is a member of the American Historical Association, the Society for Military History, and the Centre for First World War Studies (UK). His historical interests include World War 1, U.S. Army Tactical Organization, and Ancient Military History.

COL Ken Hamburger (Ret) holds an M.A. and Ph.D. in History from Duke University. He has taught courses at West Point on the Korean and Vietnam Wars, Grand Strategy, and Leadership. COL Hamburger facilitates World War 2 Battlefield tours and his most recent book is a study of combat leadership in the Korean War.

Dr. Douglas Johnson II is a Research Professor in National Security Affairs at the U.S. Army War College in Carlisle, Pennsylvania. His education includes a B.S. from USMA, an M.A. in History from the University of Michigan, and a Ph.D. in History from Temple University. During his 30-year Army career, Dr. Johnson

served as an Associate Professor in History at USMA and was one of the original faculty of the School of Advanced Military Studies (SAMS). His primary historical interest is World War 1.

Dr. James Mennell was a Professor of History at Slippery Rock University for 32 years. He holds a Ph.D. in History from Iowa University and is primarily interested in World War 2. He is currently completing a research project on Lieutenant General Lesley McNair, Commanding General of Army Ground Forces during WW2.

Stephen H. Pound received a B.A in Psychology from Western Maryland College. His historical interests include European History (1871-1945) and U.S. Military History.

Dr. Edgar Raines is a Senior Historian at the U.S. Army Center for Military History in Washington, D.C. He received a Ph.D. in History from the University of Wisconsin, Madison and is a member of numerous historical organizations, including the Society for Military History and Military Classics Seminar. His primary historical interest is the History of the U.S. Army from 1880.

Dr. James Reseau is Director of the Molecular Diagnostic Laboratory at the Van Andel Institute in Grand Rapids, Michigan. He holds a B.A. in History and a Ph.D. in Biology. Dr. Reseau's historical interests include the American Civil War, World War 2, and the Roles of Leadership, Intelligence, and Personal Integrity in Military Operations.

David Reynolds is an Operations Research Analyst at the Center for Army Analysis at Fort Belvoir, Virginia. He holds a B.A. in Political Science from Mary

Washington University, a M.A. in Military Science from the American Military University, and an M.A. in Military Studies from the Marine Corps Command and Staff College. His historical interests include Strategic Leadership as well as Organizational Issues and War.

Chapter 2

Introduction to the Analytic Hierarchy Process

2.1   The Analytic Hierarchy Process

Our day-to-day lives are full of decisions such as what car to buy, what clothes to wear, and what food to eat. Most of these decisions loosely follow four key decision making steps: Identify the problem, develop alternatives, evaluate alternatives, and implement the best alternative. Of these four steps, evaluating the alternatives is arguably the most important and therefore demands most of the attention throughout the decision making process. Certainly, we must correctly identify the problem and develop good alternatives if we hope to be satisfied with our eventual decision. Yet, we rarely have control over the problem and often have limited alternatives to choose from. Thus, we tend to focus the majority of our energies on choosing the best alternative.

Important decisions receive more of our attention and are, by nature, more complex. Often, we find that none of our alternatives satisfy all aspects of the problem, but some fair well on a majority of aspects. When this occurs, we cannot choose an alternative until we identify the most important elements of the problem. Yet, this in itself can be very difficult as the problem may possess elements that are difficult to quantify and, therefore, hard to compare. The Analytic Hierarchy Process (AHP) is a popular and powerful tool used by decision makers to evaluate alternatives in problems that contain not only tangible and quantitative factors, but also intangible

and qualitative factors as well [14]. The rest of this chapter is devoted to providing an introduction to both the steps and underlying theory of the AHP while highlighting the adaptability of this effective decision making tool.

2.1.1 Hierarchy Development

When making decisions, it is important to correctly identify all relevant decision factors regardless of any inherent complexity. Simplifying assumptions can cause important decision factors to be misrepresented, or worse, completely overlooked and will almost certainly lead to unsatisfactory results. Therefore, decision makers must deal with problems as they exist by breaking down complex decisions into manageable elements that accurately portray a portion of the overall problem. When relationships become too numerous or complex for the human mind to intuitively grasp, it may become necessary to organize the relationships into a graphical representation [14]. For these reasons, construction of the hierarchy is the most critical aspect in the AHP.

At the top-level of the hierarchy we have the overall goal of the problem. We show a hierarchy in Figure 2.1. The goal is then broken down into the important decision criteria. These criteria can then be broken down further into subcriteria. It is important to identify those criteria that are absolutely necessary to adequately define all relevant and important aspects of the problem.

In Figure 2.1, we see that the overall goal is broken down into two criteria. In turn, each of these criteria is broken down into subcriteria; Criterion 1 has four subcriteria and Criterion 2 has two subcriteria. In this instance, no additional criteria
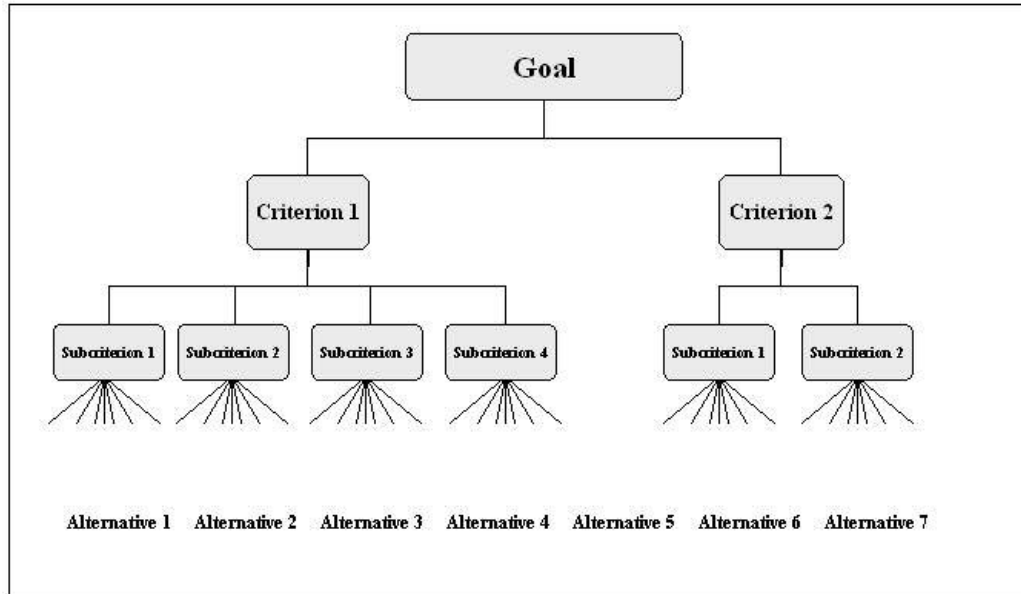
Figure 2.1: General Hierarchy

or subcriteria were needed to structure the problem. At the bottom level of the hierarchy, we see the seven alternatives. These seven alternatives will be considered with respect to each of the six subcriteria.

2.1.2 Pairwise Comparisons

The hierarchy in and of itself is not a powerful tool in the decision making process as it simply represents the relationships among the different criteria and subcriteria of a problem [14]. What makes the hierarchy a worthwhile aid is being able to assess the importance of the criteria and subcriteria. It is through our ability to assess the relative strengths of each criterion and subcriterion that we can accurately represent the decision-making problem.

The primary way AHP assesses the importance of criteria, subcriteria, and alternatives is through pairwise comparisons. The results of the pairwise comparisons

determine the weight, or priority, an element receives with respect to its siblings; that is, all elements at the same level under a parent. This method requires the decision maker to compare each element against each of its siblings, with respect to the impact each has on the parent. We will refer to this as a set of pairwise comparisons. The greatest strength of the pairwise comparison method is that it allows the decision maker to examine the unique relationship between any two factors.

The comparison process usually begins at the top of the hierarchy and moves down. For the general case depicted in Figure 2.1, Criterion 1 is compared against Criterion 2, with respect to their impact on the overall goal. The next two sets of pairwise comparisons would compare Subcriterion 1 through Subcriterion 4 with respect to Criterion 1 and Subcriterion 1 to Subcriterion 2 with respect to Criterion 2. At the bottom level of the hierarchy, we would compare the seven alternatives with respect to each of the six subcriteria; this would generate six more sets of pairwise comparisons. Each pairwise comparison generates a numerical value from 1 to 9, or a reciprocal thereof (see Saaty [14]). It is important to note that these values represent absolute magnitudes and are not mere ordinal numbers. For instance, if a decision maker believes criterion one is four times as important as criterion two, a value of 4 would be assigned to this comparison.

The values generated in a set of pairwise comparisons are stored in a pairwise comparison matrix, denoted by A. The comparison of n factors will require an $n \times n$ comparison matrix, where factor k is assigned to row k and column k. Each entry in A, denoted by a_{ij}, represents the comparison of factor i to factor j, and $a_{ii}=1$ for $i=1,2,...,n$. Correspondingly, the comparison of factor j to factor i is the

reciprocal of the entry for factor i compared to factor j. Thus $a_{ji} = 1/a_{ij}$ for all i,j and we observe that pairwise comparison matrices are positive reciprocal matrices. A general pairwise comparison matrix is of the form:

$$
A = \begin{bmatrix}
1 & a_{12} & a_{13} & \cdots & a_{1n} \\
\dfrac{1}{a_{12}} & 1 & a_{23} & \cdots & a_{2n} \\
\dfrac{1}{a_{13}} & \dfrac{1}{a_{23}} & 1 & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \ddots & \\
\dfrac{1}{a_{1n}} & \dfrac{1}{a_{2n}} & \dfrac{1}{a_{3n}} & & 1
\end{bmatrix}.
$$

2.1.3   Determining Priority Vectors with the Eigenvector Method

As stated earlier, the goal of the pairwise comparison process is to determine an overall weight for each element. The output of a pairwise comparison matrix is a set of numerical weights $(w_1, w_2,...,w_n)$ that reflects the information recorded in matrix A. If the decision maker has exact knowledge of the weights, then, $a_{ij} = w_i/w_j$ for all i,j and,

$$
a_{ij} \cdot \frac{w_j}{w_i} = 1 \qquad\qquad i, j = 1,2,...,n
$$

$$
\sum_{j=1}^{n} a_{ij} w_j \cdot \frac{1}{w_i} = n \qquad\qquad i = 1,2,...,n
$$

$$
\sum_{j=1}^{n} a_{ij} w_j = n w_i \qquad\qquad i = 1,2,...,n,
$$

which is equivalent to

$$Aw = nw, \tag{2.1}$$

where w is the principal right eigenvector of A with eigenvalue n. However, this holds true only in perfectly consistent cases. In the general case, the a_{ij} will deviate from the ideal w_i/w_j ratios and (2.1) will no longer hold. For cases such as this, Saaty [14] has shown that determining a pairwise comparison matrix's priority vector involves solving the modified eigenvalue problem for $\hat{w}$

$$\hat{A}\hat{w} = \lambda_{\max}\hat{w}, \tag{2.2}$$

where $\hat{A}$ is the pairwise comparison matrix generated by the decision maker, $\hat{w}$ estimates the true priority vector w, and $\lambda_{\max}$ is the largest eigenvalue of $\hat{A}$. The resultant priority vector, $\hat{w}$, is then normalized so the individual weights sum to one.

2.1.4   Consistency

The consistency of a set of pairwise comparisons must be considered before we accept the weights generated by this process. Consider the situation proposed earlier where the decision maker assessed factor one as four times as important as factor two. If the decision maker considered factor two twice as important as factor three, then factor one should be preferred eight times over factor three. This is an example of perfect consistency with respect to strength of preference, but perfect consistency is not guaranteed due to the human aspect of the process.

To measure consistency, Saaty [14] proposed the Consistency Index (*CI*) given by

$$CI = \frac{\lambda_{\text{max}} - n}{n - 1}.$$

(2.3)

This is a suitable equation for measuring the accuracy of λ_{max} for two reasons. First, small changes to non-diagonal elements in a positive reciprocal matrix will lead to only small changes in the eigenvalues. Second, the *n* eigenvalues of an $n \times n$ matrix with diagonal entries of one will always sum to *n*. Thus, the more consistent a matrix is, the less the a_{ij} entries will deviate from their actual values and the closer λ_{max} will be to *n* (recall 2.2). For different values of *n*, Saaty and others have computed the Consistency Index for a large number of matrices with random entries and averaged these results to produce the Random Index (*RI*). Saaty defines the consistency ratio for a matrix as

$$CR = \frac{CI}{RI}.$$

(2.4)

A matrix with a *CR* value less than 0.1 is considered by Saaty to have acceptable consistency.

2.1.5 Hierarchical Composition

After all weights for criteria, subcriteria, and alternatives have been generated, the decision maker is ready to determine the overall weight for each alternative. The decision maker uses hierarchical composition to generate the weight for each

12

alternative. This is illustrated in Figure 2.2. For each alternative, we multiply the alternative's weight for a specific subcriterion by the subcriterion's weight and then multiply the result by the parent's (criterion) weight. We then sum over all criteria to generate the alternative's final weight.
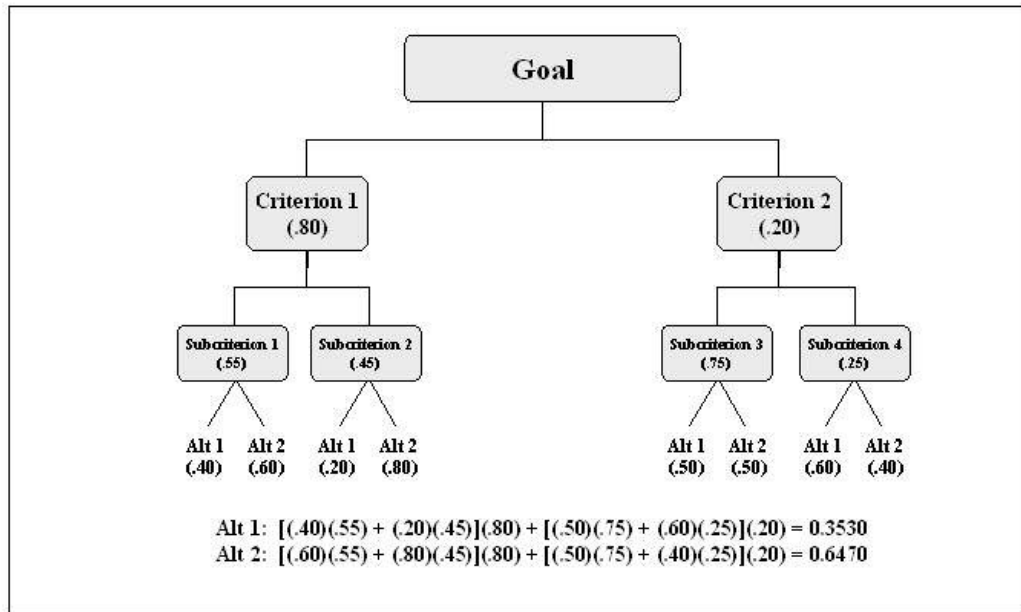
Figure 2.2: Determining Final Alternative Rankings

2.2 Variations of the Standard AHP

2.2.1 Ratings Hierarchy

The standard AHP process discussed above utilizes pairwise comparisons to generate the alternative weights under each subcriterion. However, this may be impractical when there are a large number of alternatives considered in a particular problem. The ratings hierarchy greatly facilitates the task of assigning weights to alternatives and is discussed in the next example.
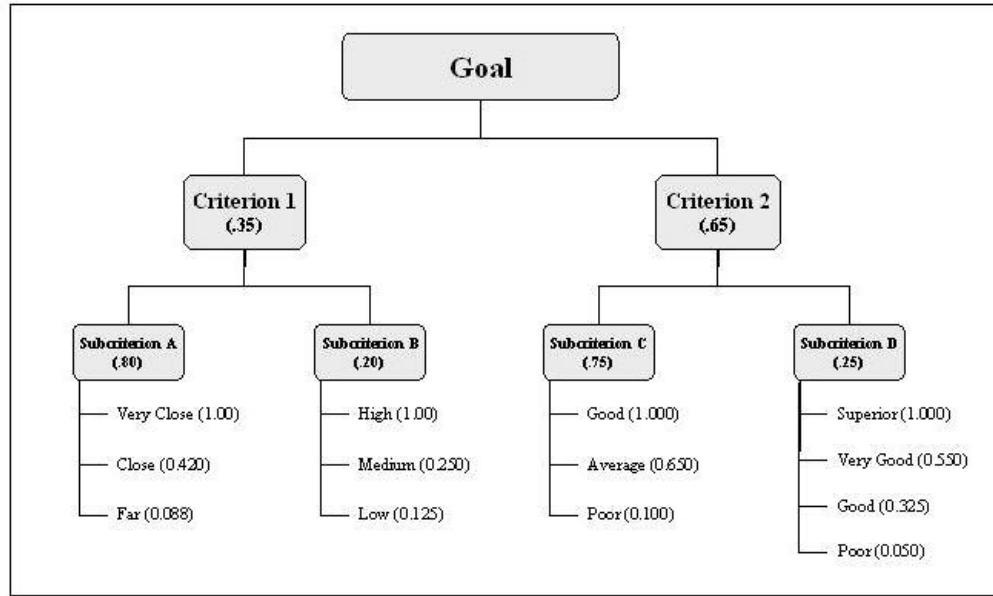
Figure 2.3: Ratings Hierarchy

Consider the hierarchy in Figure 2.3. Beneath each subcriterion is a list of ratings used to evaluate the alternatives. The weights of each rating were determined through pairwise comparison and the resultant eigenvector normalized so the largest weight is equal to one. Then, instead of pairwise comparing the alternatives under each subcriterion, the decision maker assigned a rating (and weight) to each alternative. Hierarchical composition is then used to determine the overall weights.

To illustrate this process, consider Alternative 1 for the problem depicted in Figure 2.3. Suppose that under Criterion 1 the decision maker rated Alternative 1 as "Close" for Subcriterion A, "High" for Subcriterion B, "Poor" for Subcriterion C, and "Superior" for Subcriterion D. Then the overall weight for Alternative 1 is

$$(.42)(.80)(.35) + (1.0)(.20)(.35) + (.10)(.75)(.65) + (1.0)(.25)(.65) = 0.3989.$$

The overall weights for the remaining alternatives are calculated in a similar manner.

2.2.2   Group AHP

Though we make the majority of our decisions individually, many times we find ourselves making decisions as a member of a group. As the group progresses through the four main decision-making steps, agreements must be reached in order to arrive at the final decision. Yet, it may be very difficult for even a small group to agree on the best alternative for a divisive and complex problem. Therefore, any differences of opinion must be taken into account in the decision-making process if all members of the group are to be satisfied with the result. The Group Analytic Hierarchy Process (GAHP) allows a group of individuals to participate in the decision-making process.

Consider a group of three individuals attempting to complete a pairwise comparison matrix using AHP. Though they may agree on many of the comparisons, it is unrealistic to expect them to agree on every entry in the matrix. In the GAHP, each member completes his or her own comparisons and records these in their individual pairwise comparison matrix. Each entry in the group pairwise comparison matrix is then determined as the geometric mean of the respective entries in the individual pairwise comparison matrices.

For an example of pairwise comparison aggregation, we examine the hierarchy presented in Figure 2.1 and consider a group of three individuals using GAHP. Let $A^k = (a_{ij}^k)$, $k=1,2,3$, represent the 4×4 pairwise comparison matrix generated by individual k when considering the four subcriteria of Criterion 1. Let $A = (a_{ij})$ be the group pairwise comparison matrix with entries given by

$$a_{ij} = \left(a_{ij}^1 \cdot a_{ij}^2 \cdot a_{ij}^3 \right)^{1/3} \qquad i, j = 1,2,3,4.$$

We are using the geometric mean to compute each entry of A. The geometric mean preserves the reciprocal nature that is required of pairwise comparison matrices, that is

$$a_{ij} = \left(a_{ij}^1 \cdot a_{ij}^2 \cdot a_{ij}^3 \right)^{1/3} = \left(\frac{1}{a_{ji}^1} \cdot \frac{1}{a_{ji}^2} \cdot \frac{1}{a_{ji}^3} \right)^{1/3} = \frac{1}{a_{ji}}$$

and $A = (a_{ij})$ is a positive reciprocal matrix. The group priority vectors are then determined using the Eigenvector method described in Section 2.1.3.

2.2.3 Interval Linear Programming

Interval linear programming has developed over the past decade as a method to capture the uncertainty a decision maker often faces when assigning judgments to the ratios w_i/w_j in the pairwise comparison matrix. In this variant of AHP, the decision maker assigns an interval judgment $[l_{ij}, u_{ij}]$ that defines the lower and upper bounds of w_i/w_j. General interval pairwise comparison matrices are of the form:

$$B = \begin{bmatrix} 1 & [l_{12}, u_{12}] & [l_{13}, u_{13}] & \cdots & [l_{1n}, u_{1n}] \\ [l_{21}, u_{21}] & 1 & [l_{23}, u_{23}] & \cdots & [l_{2n}, u_{2n}] \\ [l_{31}, u_{31}] & [l_{32}, u_{32}] & 1 & \cdots & [l_{3n}, u_{3n}] \\ \vdots & \vdots & \vdots & \ddots & \\ [l_{n1}, u_{n1}] & [l_{n2}, u_{n2}] & [l_{n3}, u_{n3}] & & 1 \end{bmatrix}.$$

16

Interval pairwise comparison matrices must preserve the reciprocal nature of the judgments. This requires $l_{ji} = 1/u_{ij}$ and $u_{ji} = 1/l_{ij}$ for all i, j with $i \neq j$.

There are currently several techniques for determining the priority vector w from an interval pairwise comparison matrix. We will use the three-stage interval linear programming (ILP) method recently described by Alford et al. [1]. This method is an extension of the two-stage method previously proposed by Chandran et al. [5]. For our purposes in this paper, we will refer to this method as ILP.

The ILP method is based on the premise that the best priority vector w will minimize the total error ε_{ij} between the decision maker's judgments $\sqrt{l_{ij}u_{ij}}$ and the weight ratios w_i/w_j as defined by the error relationship

$$\frac{w_i}{w_j} = \varepsilon_{ij}\sqrt{l_{ij}u_{ij}} \ . \tag{2.5}$$

This assumes the geometric mean $\sqrt{l_{ij}u_{ij}}$ is a good estimate of w_i/w_j. Additionally the priority vector w must satisfy the modified interval constraints

$$l_{ij}\gamma_{ij}^{-1} \leq \frac{w_i}{w_j} \quad \text{and} \quad \frac{w_i}{w_j} \leq u_{ij}\gamma_{ij} \tag{2.6}$$

where γ_{ij} is a "stretch factor" used to ensure the existence of a non-empty solution set. "Stretch factors" cannot shrink the interval and therefore must be greater than or equal to one. Since $l_{ji} = 1/u_{ij}$ and $u_{ji} = 1/l_{ij}$, it can be shown that the "stretch factors" and the errors terms are reciprocal in nature. That is $\gamma_{ji} = 1/\gamma_{ij}$ and $\varepsilon_{ji} = 1/\varepsilon_{ij}$.

The ILP method utilizes four transformed variables: $x_i = \ln(w_i)$, $y_{ij} = \ln(\varepsilon_{ij})$, $z_{ij} = |y_{ij}|$, and $g_{ij} = \ln(\gamma_{ij})$. The transformation to natural logarithm space serves two

purposes. First, it transforms the non-linear equations into linear equations. Second, if the decision maker's judgment is accurate in Equation 2.5, the transformed error term is equal to zero.

The goal of the first stage, Stage 0, is to determine the set of all "stretched" intervals that minimize the product of the "stretch factors" γ_{ij}. Remember, it may not be necessary to stretch any intervals, in which case $\gamma_{ij} = 1 \ \forall \, i, j$. The Stage 0 linear program for an $n \times n$ interval pairwise comparison matrix is given by:

$$\text{Min} \quad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} g_{ij}$$

s.t.

$$x_i - x_j - y_{ij} = \ln\left(\sqrt{l_{ij} u_{ij}}\right) \qquad \forall i, j \text{ with } i \neq j \qquad (2.7)$$

$$x_i - x_j + g_{ij} \geq \ln\left(l_{ij}\right) \qquad \forall i < j \qquad (2.8)$$

$$x_i - x_j - g_{ij} \leq \ln\left(u_{ij}\right) \qquad \forall i < j \qquad (2.9)$$

$$g_{ij} \geq 0 \qquad \forall i < j \qquad (2.10).$$

The Stage 0 objective function is the natural logarithm of the product of stretch factors γ_{ij}. Constraint 2.10 stems from the fact that $\gamma_{ij} \geq 1 \ \forall i < j$. Constraint 2.7 is the result of transforming Equation 2.5 into natural logarithm space and incorporating the necessary changes of variables. Finally, Constraints 2.8 and 2.9 are the logarithmic transformations of the inequalities presented in Equation 2.6. The Stage 0 solution set consists of all sets of intervals that minimize the "stretch" required to preserve feasible priorities that satisfy Equation 2.6. Let g^* be the Stage 0 optimal objective function value.

The Stage 1 goal is to determine the set of all priority vectors that minimize the product of the errors ε_{ij}. The Stage 1 linear program is given by:

$$\text{Min} \quad \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} z_{ij}$$

s.t.

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} g_{ij} = g^* \tag{2.11}$$

$$x_i - x_j - y_{ij} = \ln\left(\sqrt{l_{ij}u_{ij}}\right) \qquad \forall i, j \text{ with } i \neq j \tag{2.12}$$

$$x_i - x_j + g_{ij} \geq \ln\left(l_{ij}\right) \qquad \forall i < j \tag{2.13}$$

$$x_i - x_j - g_{ij} \leq \ln\left(u_{ij}\right) \qquad \forall i < j \tag{2.14}$$

$$z_{ij} - y_{ij} \geq 0 \qquad \forall i < j \tag{2.15}$$

$$z_{ij} - y_{ji} \geq 0 \qquad \forall i < j \tag{2.16}$$

$$g_{ij} \geq 0 \qquad \forall i < j \tag{2.17}$$

$$z_{ij} \geq 0 \qquad \forall i < j \tag{2.18}$$

The Stage 1 objective function minimizes the product of all positive error terms in natural logarithm space (recall the change in variables $y_{ij} = \ln\left(\varepsilon_{ij}\right)$ and $z_{ij} = \left|y_{ij}\right|$). Constraint 2.11 ensures that only optimal sets of intervals from Stage 0 are feasible sets in Stage 1. Constraints 2.15, 2.16, and 2.18 define $z_{ij} = \left|y_{ij}\right|$. The remaining constraints are identical to those in the Stage 0 linear program. The Stage 1 solution set consists of all priority vectors that minimize the product of the errors ε_{ij} while satisfying Equation and 2.6. Let z^* be the Stage 1 optimal objective function value.

The Stage 2 goal is to determine the priority vector that minimizes the maximum of the error terms ε_{ij}. The Stage 2 linear program is given by:

Min z_{max}

s.t.

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} z_{ij} = z^* \qquad\qquad (2.19)$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} g_{ij} = g^* \qquad\qquad (2.20)$$

$$x_i - x_j - y_{ij} = \ln\left(\sqrt{l_{ij}u_{ij}}\right) \qquad \forall i,j \text{ with } i \neq j \qquad (2.21)$$

$$x_i - x_j + g_{ij} \geq \ln(l_{ij}) \qquad \forall i < j \qquad (2.22)$$

$$x_i - x_j - g_{ij} \leq \ln(u_{ij}) \qquad \forall i < j \qquad (2.23)$$

$$z_{ij} - y_{ij} \geq 0 \qquad \forall i < j \qquad (2.24)$$

$$z_{ij} - y_{ji} \geq 0 \qquad \forall i < j \qquad (2.25)$$

$$g_{ij} \geq 0 \qquad \forall i < j \qquad (2.26)$$

$$z_{ij} \geq 0 \qquad \forall i < j \qquad (2.27)$$

$$z_{max} - z_{ij} \geq 0 \qquad \forall i < j \qquad (2.28).$$

Constraint 2.19 ensures that only optimal priority vectors from Stage 1 are feasible priority vectors in Stage 2. Constraint 2.28 ensures z_{max} is the maximum value of the positive transformed errors z_{ij}. The remaining constraints are identical to those in the Stage 1 linear program. The optimal priority vector w is determined by exponentiating the Stage 2 x_i values.

Interval pairwise comparison matrices and the ILP method are extremely adaptable to generating priority vectors for the GAHP. Instead of using the individual pairwise comparison matrices to create a group pairwise comparison matrix, we use them to compute a group interval pairwise comparison matrix. As earlier, let a_{ij}^k

represent the comparison of factor i to factor j for decision maker k. We then compute the interval bound $\left[l_{ij}, u_{ij} \right]$ through

$$l_{ij} = \min\{a_{ij}^1, a_{ij}^2, \ldots, a_{ij}^k\} \quad \text{and} \quad u_{ij} = \max\{a_{ij}^1, a_{ij}^2, \ldots, a_{ij}^k\}$$

If $l_{ij} = u_{ij}$ then we forego the interval and use a single number. For this, we refer the reader to Section 4.4 of Chandran et al. [5]. We can also eliminate the highest and lowest values (or two highest and two lowest values) and create interval bounds on the remaining $k - 2$ (or $k - 4$) values. On of the key goals of this thesis will be to compare the eigenvector GAHP approach and the interval linear programming GAHP approach on a data set involving the ranking of recent U.S. Army Generals. Comparable results from multiple methods will confirm the reliability of our study.

Chapter 3

Ranking U.S. Army Generals of the 20th Century

The United States Army has produced many great generals throughout the past 230 years, and the 20th Century was certainly no exception. Two global wars and many limited conflicts have provided historians, military scholars, and Army officers alike with a seemingly endless list of combat-tested general officers to study. Therefore, in our selection process we focused on finding the best-qualified generals for evaluation.

Naturally, the Second World War (WW2) produced the most of the great 20th Century commanders and it serves as the starting point for our selection process. Most notably among the WW2 alumni are Generals of the Army George Marshall, Douglas MacArthur, Dwight Eisenhower, and Omar Bradley, all of who are included in our study. However, WW2 also produced many lesser-known commanders who served as division, corps, army, and army group commanders. Among the best of this group are Joseph Collins, Jacob Devers, Robert Eichelberger, Leonard Gerow, Courtney Hodges, Walter Krueger, Alexander Patch, George Patton, Matthew Ridgway, and William Simpson, who collectively fought in the European, Pacific, and Mediterranean theaters of conflict. Despite the impressive combat records of these ten officers, we selected only Matthew Ridgway and George Patton for further consideration. Matthew Ridgway was selected for his success in directing early American airborne operations in WW2, coupled with his performance in Korea with Eighth Army, and later as Supreme Commander. George Patton, despite his

egregious slapping incident, was chosen for his leadership ability in North Africa and Europe. This led to him becoming the most feared Allied general in the eyes of the German high command. Though not selected for this study, Walter Krueger's performance with the Sixth Army throughout 21 successful amphibious operations in the Southwest Pacific was very notable.

As for the remaining conflicts of the 20th century, we considered seven generals. Six of these we did not select: William Walker, William Westmoreland, Creighton Abrams, Norman Schwarzkopf, Fred Franks, and Barry McCaffrey. Though William Walker performed well commanding XX Corps in Europe and later the Eighth Army in Korea, we did not feel we could distinguish him from the WW2 generals we did not select. As for the Vietnam War, we never felt William Westmoreland overcame the difficulties he faced in fighting a limited war or that the successful leadership of Creighton Abrams' during the later years of the war could compare with the accomplishments of the WW2 generals that had already been selected. Schwarzkopf's failure to complete the destruction of the Republican Guards Forces Command was too great a shortcoming to be dismissed, despite his accomplishments in both Grenada and the remainder of the Gulf War. Similarly, we felt that Fred Frank's responsibility in failing to destroy the RGFC in Desert Storm was too great to overcome. Though Barry McCaffrey was an excellent officer and a superb general, his lack of significant combat action in Iraq precluded him from being selected for further evaluation. Therefore, the only general we selected who did not serve in the Second World War was General of the Armies John Pershing, for his performance leading the American Expeditionary Force in World War 1 (WW1).

3.1 Discussion of Alternatives

The remainder of this section is devoted to examining the accomplishments of seven generals: Pershing, MacArthur, Marshall, Eisenhower, Patton, Bradley, and Ridgway. Our discussion of each general is brief and is intended to provide the reader with a basic understanding of each general. The progression is roughly chronological, in this case beginning with WW1 and ending with the Korean War.

The U.S. Army currently has four ranks of general that are distinguishable by the number of stars associated with each. A Brigadier General wears one star and is the lowest ranking general. Major Generals wear two stars, Lieutenant Generals wear three, and a General (also known as a full General) wears four. The rank of General of the Army is reserved for wartime only, is the highest ranking of all generals in the current system, and is signified by five stars. The rank of General of the *Armies* was assigned to John Pershing in 1919 to honor his wartime service. No other officer held that title until 1976, when George Washington was posthumously appointed General of the Armies of the United States to signify he ranks first among all Army officers, past and present [6].

3.1.1 General of the Armies John J. Pershing

John J. Pershing was born September 30, 1860 in Laclede, Missouri. Initially uninterested in a military career, Pershing attended West Point as a means of obtaining a top-notch education. He graduated in 1886 with an academic ranking in the middle of a class of 77 cadets. Though academically undistinguished, his natural

leadership ability earned him the top cadet rank for each of his four years at West Point [17].

Pershing spent the majority of his early years on field duty in the southwestern and northern plains serving with the 6th and the 10th Cavalry Regiments. He also served several years as professor of military science at the University of Nebraska and as assistant tactics instructor at West Point. During the Spanish American War, he again served with the 10th Cavalry and saw action in the Battle of San Juan Hill [6]. After a brief stint in Washington he was sent to the Philippines where he put down the Moro uprising, an accomplishment that earned Pershing positive headlines in many U.S newspapers. By the 20th anniversary of his Army career, Pershing was still only a captain and promotion seemed impossible in the seniority-based Army of the time. However, his exemplary service did not go unnoticed by Army or governmental officials. In September 1906, President Theodore Roosevelt promoted Pershing to Brigadier General over 862 senior ranking officers [17]. He spent his first nine years as a general officer serving in a variety of posts throughout the continental U.S and its possessions before being called to lead the Punitive Expedition in 1916.

In March of 1916, Pershing was selected to assemble and lead a sizable force into Mexico in response to Pancho Villa's murderous rampage in Columbus, New Mexico. He quickly assembled a force of nearly 7,000 infantry, cavalry, artillery, engineer, and support soldiers as well as eight airplanes and over 30 trucks (airplanes and trucks were new to the Army at the time). Pershing recognized the potential benefits of these technologies and readily employed them. The aviation elements

performed aerial reconnaissance and communications missions while the trucks tackled the daunting task of re-supplying an expeditionary force almost 500 miles from its logistical base [23].

Although the aviation elements experienced only limited success, the trucks quickly proved their worth on the modern battlefield as motorized transport hauled over 10,000 tons of supplies to Pershing's men from bases in the United States before the campaign ended on February 5, 1917 [23]. Despite never finding Villa, Pershing, who was promoted to Major General during the expedition, did project force quickly in support of American interests and displayed skills that would be called upon shortly as the United States entered the war in Europe [17].

Shortly after the United States declared war on Germany in April 1917, Pershing was selected to command the American Expeditionary Force (AEF). The AEF was created to fight under an American flag alongside the British and the French in order to assist in the defeat of the German armies on the Western Front. At its inception, the AEF consisted of only the newly formed 1st Division and had no historical precedent. The AEF numbered approximately 25,000 strong. Its soldiers were unfamiliar with the type of war being fought in Europe and were untrained in conducting large operations [7]. Before Pershing could command the AEF in combat, his first task was to create, equip, and train the largest army the U.S had ever fielded.

There were many difficulties in creating the AEF. Soldiers had to be trained to survive and fight in the trenches that came to define life on the Western Front. Staff officers needed to be trained to handle the complexities that were inherent with large-scale operations. Commanders needed to be identified and placed at all levels

from regiment to corps. Equipment and supplies needed to be procured and distributed throughout the AEF. Meanwhile, both the British and the French were demanding American soldiers to replenish their depleted ranks. Despite these obstacles, it took Pershing (promoted to the rank of General in October 1917) only one year to transform the AEF from a fledgling organization into a force prepared to fight and win on the fields on France.

In May of 1918, elements of the 1st Division successful attacked and seized Cantigny, France in the first American offensive action of the war. However, large operations would have to wait until the fall when Pershing activated and took command of the First Army. In early September 1918, Pershing and the First Army reduced the Saint Mihiel Salient (a German position) in the first large-scale American offensive of the war, thus certifying the AEF as a legitimate fighting force. Nearly two weeks later, Pershing directed the First Army in the successful allied Meuse-Argonne offensive while preparing the newly formed Second Army for action in Metz, France [7]. By November 11, 1918, the AEF consisted of over two million men and in a period of 18 months, Pershing had transformed it into a force comparable to those of America's allies.

3.1.2 General of the Army Douglas MacArthur

Unlike John Pershing, Douglas MacArthur always knew that the U.S. Army was his lifelong calling. MacArthur was born on January 26, 1880 in Little Rock, Arkansas. His father was a Civil War Congressional Medal of Honor Winner still on active service in the Army at that time. MacArthur enrolled at West Point in 1889

where he distinguished himself both academically and militarily. In 1903, he graduated first in his class academically and as Cadet First Captain, the top cadet position [9].

MacArthur spent the early years of his career serving in the Philippines before returning to Washington as an aide to President Theodore Roosevelt. After brief stints in Wisconsin and Kansas, he returned to Washington in 1913 to serve on the General Staff in the years leading up to the U.S involvement in the First World War. Shortly thereafter, in late 1917, MacArthur headed off to France as the Chief of Staff for the 42d Infantry Division [6].

As the fourth division to enter France, the 42d completed training and began to integrate into the defensive positions in eastern France in early 1918. MacArthur quickly earned fame and medals participating in several raids into enemy territory with elements of the 42d [9]. He was promoted to Brigadier General in June of 1918 and the following month, during the Second Battle of the Marne, he was made commander of the 84th Brigade, 42d Division. In September 1918 as 84th Brigade Commander, MacArthur successfully led his men in the reduction of the Saint Mihiel Salient and in the Meuse-Argonne Offensive. At the conclusion of the Meuse-Argonne operation, MacArthur temporarily commanded the 42d Division in the Sedan Offensive. At the end of the war, MacArthur found himself again commanding the 84th Brigade and the recipient of 12 U.S. and 19 Allied medals and decorations [9].

MacArthur returned to the United States in early 1919 and headed to West Point as Superintendent. His new posting allowed him to retain his wartime rank, a

rarity for officers during the interwar years. In 1922, MacArthur left West Point and spent the next 13 years jumping between postings in the Philippines and Washington, D.C. He was promoted to Major General in 1925 and was temporarily promoted to General while serving as Chief of Staff of the Army from 1930 to 1935 [6]. As Chief of Staff, MacArthur spent the majority of his time protecting the Army from further drawdowns and cutbacks during the early years of the depression. After a brief extension in Washington, MacArthur was reassigned to the Pacific as a Major General, this time as military advisor to the Philippines. Although he resigned from military service in 1937, he stayed on as military advisor until the war drums began to beat again.

In mid-1941, President Roosevelt brought MacArthur out of retirement and designated him the head of U.S. Army Forces, Far East with the rank of Lieutenant General and charged him with the defense of the Philippines. However, his command was unprepared for the Japanese air attack on December 8, 1941 and he remained on the defensive throughout the early months of 1942. In March 1942, as the fall of the Philippines seemed more certain, President Roosevelt ordered General MacArthur (promoted in December 1941) to Australia as Supreme Allied Commander, South West Pacific Area [9]. Two months later, the U.S. garrison in the Philippines surrendered.

In Australia, MacArthur built up his forces, defended New Guinea, and prepared to launch his offensive and make good on his promise to return to the Philippines. In mid 1943, he began a three-pronged attack that eventually developed into an extremely successful island-hopping campaign. In early 1944, MacArthur led

a highly successful foray against the Los Negros islands. Despite his staff's warnings, the attack succeeded brilliantly and the U.S. gained control of the Bismarck Sea. In the summer of 1944, MacArthur again defied conventional wisdom and seized Hollandia on New Guinea's northern coast, forcing the Japanese to leave their prepared defenses and attack his beachhead. In October, MacArthur's forces attacked the Philippines at Leyte, gaining a foothold in the country to which he had vowed to return. In January 1945, he continued his destruction of the Japanese garrison in the Battle of Luzon, leading a force of over 1000 ships, 3000 landing craft, and over 280,000 men. One month later, MacArthur fulfilled his promise as his forces completed the liberation of the Philippines. On September 2, 1945, General of the Army Douglas MacArthur (promoted in December 1944) accepted the formal Japanese surrender aboard the U.S.S. Missouri in Tokyo Bay. He then assumed duties as Supreme Allied Commander, Japan, supervising the reconstruction of the war-ravaged country [9]. MacArthur held this position until June, 1950, when as he stated, Mars provided one "last gift to an old warrior."

Shortly after the outbreak of the Korean War in 1950, MacArthur was designated Commander, United Nations Command, Far East. With UN Forces struggling to maintain the Pusan perimeter, MacArthur took drastic actions by ordering an extremely risky amphibious assault at Inchon in September. Once again, his intuition proved true and his flanking movement cut the North Korean supply lines and allowed him to regain the initiative. Throughout the fall of 1950, MacArthur directed UN Forces north from South Korea towards the Yalu River, until Chinese forces entered the war and completely shattered the UN offensive. As

fighting stabilized roughly at the prewar boundaries, MacArthur's demands for widening the war and his general distrust of Washington could no longer be tolerated. On April 11, 1951, MacArthur was relieved of his command [9]. He returned to the United States, and in his famous words to Congress, "faded away."

3.1.3 General of the Army George C. Marshall

George C. Marshall was born in Uniontown, Pennsylvania on December 31, 1880. He attended the Virginia Military Institute where he played left tackle on the varsity football squad and served as Cadet First Captain as a senior. After graduating in the spring of 1901, he pursued an appointment with the U.S. Army and was commissioned a Second Lieutenant of Infantry in February 1902 [11].

Marshall served in the Philippines and Oklahoma before heading to Ft. Leavenworth in 1906 to attend the Infantry and Cavalry School. After graduating first in his class, he remained at Leavenworth to attend the Army Staff College. Upon completion of the Staff College in 1908, Marshall was chosen to remain at Leavenworth as a Staff College instructor in the fields of engineering and military art [6]. After serving three years as an instructor, Marshall spent the next five years in various stateside duties as well as one more tour in the Philippines before being personally selected to serve as a staff officer in the 1st Division on the eve of the First World War.

In France, Marshall performed exceptionally, first as assistant Chief of Staff and later as Chief of Staff for Operations of the 1st Division. After planning the successful Cantigny assault, he was pulled by General Headquarters to serve in the

operations section of the AEF where his expertise in managing complex situations greatly aided in the success of the Saint Milhiel and Meuse-Argonne operations [7]. Finishing the war as a colonel, Marshall returned to the United States as Aide de Camp to the next Chief of Staff of the United States Army, John J. Pershing.

After serving at Pershing's side for five years, Marshall headed to China to join the 15th Infantry Regiment. In 1927, he returned to the States. He taught at the Army War College in Washington, D.C. and then at the Infantry School in Ft. Benning, Georgia. Marshall's five years in Georgia would prove invaluable as he began to lay the foundation of the World War II Army and developed close relationships with scores of future division and corps commanders, soon to be known as "Marshall's Men." As the clouds of war gathered over the next seven years, Marshall served in various commands, was promoted to Brigadier General, and in 1939, was selected to serve as Chief of Staff of the Army [11].

General Marshall was sworn in as the Chief of Staff of the world's seventeenth largest army and promoted immediately to full General just hours after Germany invaded Poland on September 1, 1939. Marshall knew that the Army was unprepared to enter the war. As the new Chief of Staff, he spent the majority of his time focusing on Army organization and spurring war production. His desire to build the Army around the nation's strategic objectives would eventually create a force custom tailored for the daunting task ahead [12].

As war loomed closer in the summer of 1941, Marshall directed efforts at reinforcing the Philippines as a result of the Japanese menace in Thailand. Throughout the summer, Marshall spent the majority of his time in pushing for the

passage of the draft extension, a motion that would prove critical in maintaining his fledgling fighting force of 1.5 million soldiers and airmen. At the end of the summer, he attended the Atlantic Charter Conference in Newfoundland, Canada with President Roosevelt, Prime Minister Churchill, as well as high-ranking Allied military officials [12]. After a brief discussion of Allied military strategy, Marshall returned to the U.S. to order extensive field maneuvers throughout the southeast, culminating in the Carolina Maneuvers just a month before Pearl Harbor.

In late December 1941, Marshall participated in the Arcadia conference and was the principal presenter of the American military plan to the military and political leaders of Great Britain. During this conference, Marshall pushed hard for a Combined Chiefs of Staff between the British and the Americans, a military run Munitions Assignment Board, and a unified command in the Pacific Theater. This proved to be the first of many conferences where President Roosevelt would rely heavily on General Marshall to serve as the primary U.S military representative to the British contingent [12]. Several months later, Marshall met with the British and discussed the creation of a second European front in France and potential operations in North Africa. In 1942, Marshall spent the majority of his time supplying the defense of Guadalcanal while preparing for the TORCH offensive in North Africa. Throughout the Casablanca and Tehran conferences in 1943, Marshall managed to keep the British focused on a cross-channel attack into France while delicately handling Stalin's demands of an immediate second front. Despite being the frontrunner for Supreme Command of the European Theater of Operations (ETO) and

the OVERLORD invasion, President Roosevelt believed Marshall was too valuable in Washington managing the global operations of a nearly eight million-man army.

At the beginning of 1944, Marshall was named *Time's* "Man of the Year." He was busy making the final coordination for the June landings in France. As well as recommending commanders for the upcoming offensive, he ensured sufficient supplies and naval fire-support would be available for the assault on Hitler's Atlantic Wall. Marshall also won what proved to be a long fight with Prime Minister Churchill over Operation DRAGOON. In August 1944, this operation successfully landed the Seventh Army in southern France.

As the war in Europe drew to a close, General of the Army Marshall (promoted in December 1944) became more concerned with the postwar borders and military government in Germany as well as the defeat of Japan [13]. Between V-E Day and the successful conclusion of the Manhattan Project, Marshall spent the majority of his time dealing with the Russians, as well as shifting forces and supplies in preparation for the presumed invasion of the Japanese home islands. By the time of the Japanese surrender, Marshall had transformed the world's seventeenth largest army into arguably the greatest fighting force ever. His expert handling of complex military and political discussions served the country well and prepared him for future postings as the Secretary of State and the Secretary of Defense.

3.1.4   General of the Army Dwight D. Eisenhower

Dwight D. Eisenhower was born October 14, 1890 in Denison, Texas. Though his original college plans involved playing football at the University of

Michigan, he accepted an appointment to U.S. Military Academy in 1911 and briefly played football before a knee injury ended his playing days. In 1915, Eisenhower graduated in the top third of his class (this class was later known as the "the class the stars would fall on," due to its high number of future WW2 generals) [2].

From 1915 to 1918, Eisenhower served with the Infantry in Texas and Georgia. Shortly after the American entrance into the Great War, Eisenhower was selected to command the Tank Corps at Gettysburg, Pennsylvania, a post he would hold throughout WW1. Over the next five years, Eisenhower served in various positions in both the Tank Corps and the Infantry before completing the Army Staff College, the Army War College, and the Army Industrial College. From 1933 until shortly after the beginning of WW2, Eisenhower served as chief military aide to General MacArthur, first in Washington and then in the Philippines [6]. Upon returning to the States in 1939, he served consecutively as Chief of Staff of the 3rd Division, 9th Corps, and 3rd Army where he received high praise for his successful performance in the Louisiana Maneuvers in the summer of 1941 [2]. Eisenhower was promoted to Brigadier General in September and was called to Washington, D.C. just days after Pearl Harbor to work in the War Department for George Marshall.

Eisenhower served in the War Plans Division and devoted much of the first months of America's involvement in the war to the defense of the Philippines. Shortly after the Arcadia conference in early 1942, Eisenhower became Chief of the War Plans Division, was promoted to Major General in March and then to Lieutenant General in July 1942. In his new position, he drafted a document outlining the Allied strategy for the first three years of the war. Though the contents of his document

were not new, his thorough understanding of its premise and the importance of Allied unity convinced both the American and British governments that he was best qualified to serve as the Commanding General of American forces in the European Theater of Operations [18]. Eisenhower's first responsibility was to oversee Operation BOLERO, the buildup of American forces in Britain in preparation of the eventual cross-channel invasion. However, due to President Roosevelt's desire to actively enter the war in 1942 and Prime Minister Churchill's wishes to increase Allied operations in the Mediterranean Theater, Eisenhower's first combat command came in November during TORCH, the Allied landings in North Africa.

Over the next year, General Eisenhower (promoted in February, 1943) directed operations in North Africa, Sicily, and Italy. In these operations, he dedicated significant effort to fighting the Germans, dealing with the Vichy French Government, and encouraging cooperation and teamwork among the Allied commanders [2]. Though his overall performance was far from superior, Eisenhower's ability to effectively manage the three armed forces of two different nations made him a logical consideration for Supreme Commander of the Allied Expeditionary Force. He was selected for this position in late 1943 and began planning for the invasion of France, which was scheduled for 1944.

As commander of the OVERLORD invasion of France, Eisenhower was responsible for the most important operation of the war [2]. He spent the months leading up to the invasion consumed in the difficulties of allocating scarce landing craft, directing strategic bombing operations, and determining the best utilization of airborne units in support of the amphibious assault. On June 6, 1944 and in the weeks

following, Eisenhower's success was evident as the Allies breached Hitler's vaunted Atlantic wall and landed over one million soldiers and one half million tons of supplies in France. Though the expansion of the beachhead was initially slow, the Allies did breakout after Operation COBRA and the collapse of the Falaise Pocket. By the end of the summer of 1944, Eisenhower was directing the coordinated march of over two million men through France on the way to Berlin [18]. His ability to handle the strong-willed personalities of both American and British subordinates was instrumental in the Allied success as they continued towards the German frontier.

Shortly after the beginning of the Battle of the Bulge on December 16, 1944, Eisenhower correctly identified the German thrust as a major attack and quickly seized the opportunity inherent in the overextension of German forces. As winter turned to spring, General of the Army Eisenhower (promoted on December 20, 1944) continued the offensive, thrusting into Germany and preparing for a double encirclement of the Ruhr Valley, Germany's industrial heartland. The unexpected seizure of a Rhine River crossing in March 1945 enabled the Allies to seriously consider racing the Russians to Berlin [18]. Despite intense British political pressure, as well as general American sentiment, Eisenhower opposed driving to Berlin as it could cost the lives of over 100,000 American and British soldiers. Furthermore, Berlin was within the occupation zone assigned to the Russians at the Yalta Conference [2]. On May 7, 1945, Eisenhower accepted the unconditional surrender of Germany and her armed forces – the success of the American Army in the ETO was absolute.

3.1.5 General George S. Patton, Jr.

George S. Patton, Jr. was born on November 11, 1885 at Lake Vineyard near San Gabriel, California. Despite his desire to attend the U.S. Military Academy in 1903, Patton was initially not accepted at West Point and instead enrolled at VMI after declining an offer to study at Princeton. In 1904, he was accepted at West Point, where he would shortly repeat his first year due to a failing math grade. Despite these initial setbacks, Patton graduated in the top half of the class of 1909, earning a varsity letter in track, and serving as Cadet Adjutant his senior year, second only to the Cadet First Captain [8].

Patton spent the early years after graduation with the Cavalry in Illinois, Washington, D.C., Kansas, and Texas. In 1912, he competed in the Modern Pentathlon in the Olympic Games in Stockholm, Sweden finishing a respectable fifth. The following year he became the Army's first master of the sword and taught swordsmanship at the Army's Mounted Service School. As tensions mounted with Mexico in early 1916, Patton pleaded directly with General Pershing for a role in the Punitive Expedition, earning a slot as Pershing's Aide. While leading a foraging expedition in Mexico, Patton killed three Villiastas in the first use of motorized vehicles in combat. This earned praise from Pershing and several northeastern newspapers. Just months after the conclusion of the Punitive Expedition, Patton set sail for France on the HMS *Baltic* as a member of Pershing's newly established AEF headquarters.

Patton served as the unofficial commander of the headquarters troop for the first several months of America's involvement in WW1. He was constantly searching

for ways to the front lines. In November 1917, Patton became the first soldier of the newly created U.S. Army Tank Corps where he organized and commanded the First Army Tank School. As commander of the 1st Tank Brigade (later re-designated the 304th), he participated in the reduction of the Saint Milhiel Salient and the Meuse Argonne Offensive [8]. After the war ended, Patton returned to the Cavalry, and spent the majority of the next two decades in various professional schools, as well as serving several tours each in Hawaii and in the Washington D.C. area. Shortly after war erupted in Europe in 1939, Patton left the Cavalry for good, assumed command of the 2d Armored Brigade, 2d Armored Division at Ft. Benning, Georgia, and was promoted to Brigadier General.

Despite the initial setbacks in preparing an under-manned and under-equipped force for combat, Patton excelled, was soon moved to division commander, and later promoted to Major General in April 1941. In 1941, the 2d Armored participated and performed very well in the Tennessee, Louisiana, and Carolina Maneuvers, where George Marshall noticed Patton's impressive performance. In January of 1942, Patton was promoted to command the I Armored Corps and directed to establish an armored training center in the California desert. Patton's continued success in armored operations and recognition as a fighter earned him a spot in TORCH, commanding the Western Task Force in the November 1942, invasion of Casablanca.

After Casablanca fell, Patton stayed in Morocco to establish the military government and begin planning for the upcoming invasion of Sicily, called Operation HUSKY. Several months later, after the Allied defeat at Kasserine Pass in February 1943, Eisenhower gave Patton command of II Corps. After leading II Corps to

victories at Gafsa and El Guettar during the drive through Tunisia, Eisenhower returned Patton to Morocco to continue planning for Operation HUSKY. In Sicily, Lieutenant General Patton (promoted in March 1943) led his Seventh Army to capture Palermo and Messina in July, before slapping two soldiers during separate visits to field hospitals in early August. Despite public outcry, Marshall and Eisenhower believed Patton could still contribute to the war effort [13]. In early 1944, he was brought to England to secretly command the Third Army while leading an elaborate deception as commander of the mythical First U.S. Army Group, a role critical to the later success of OVERLORD.

In early August, the Third Army became operational and Patton led them in the Normandy breakout and in the eventual closure of the Falaise gap. He pursued the Germans across France and finally halted on the Moselle River as the Allied logistical effort was directed to support Operation MARKET-GARDEN. Perhaps Patton's crowning achievement came in December 1944 during the Battle of the Bulge. As the German Army launched its last major counter-offensive of the war, Patton quickly redirected three divisions in terrible weather to relieve the defenders of Bastogne and reduce the German advance into the Ardennes. After breaching the West Wall, Patton's Third Army pushed through southern Germany and crossed the Rhine before halting in Czechoslovakia and Austria after the German surrender in May 1945. Promoted to four-star General just weeks before war's end, Patton was arguably the Allied commander the Germans feared most [8].

3.1.6 General of the Army Omar N. Bradley

Omar N. Bradley was born on February 12, 1893 in Clark, Missouri. In 1911, Bradley entered the United States Military Academy and joined Dwight Eisenhower in the "class the stars fell on." While at West Point, Bradley ranked in the academic top third of his class while lettering in both football and baseball [4].

After graduation in the spring of 1915, Bradley joined the infantry and first saw duty in the state of Washington. Despite his best attempts to be assigned overseas, Bradley spent the majority of America's involvement in the WW1 policing copper mines in Montana. He spent most of the 1920s and 1930s working in the Army educational system, serving as both instructor and student. He taught mathematics at West Point for four years before spending two years attending the Infantry School and the Command and General Staff School. Bradley then returned to the Infantry School to teach for four years before becoming a student once more at the Army War College. In 1938, he ended his academic career as an instructor in tactics at West Point [19].

In 1941, Bradley was promoted to Brigadier General and relocated to Georgia where, as head of the Infantry School, he developed a model Officer Candidate Program that would produce thousands of junior officers over the next four years [19]. In February 1942, shortly after America entered WW2, Bradley was promoted to Major General, assumed command of the 82d Infantry Division, and began preparing his untrained soldiers for combat. He performed so well in this task that after only four months he was asked to achieve the same results with another untrained division, the 28th Infantry. In February 1943, just after being designated

commander of X Corps, Eisenhower pulled Bradley to North Africa to serve as his personal representative in the field [4].

After Patton returned to Morocco in April 1943, Bradley assumed command of II Corps and was promoted to Lieutenant General. He led II Corps through the final fighting in North Africa, capturing Bizerte and over 40,000 German prisoners by the conclusion of hostilities in May. Bradley's corps spearheaded Seventh Army's assault on Sicily, eventually capturing Messina in mid-August 1943. As the forthcoming cross-channel invasion began to take shape, Marshall identified Bradley for two consecutive commands, first of an army and later of an army group [13].

In the months prior to the invasion, Bradley supervised the training of American soldiers. He continued to refine plans, most significantly pushing for the employment of airborne troops to secure the causeways leading inland from the invasion beaches. Bradley led the First Army in the D-Day assaults on Utah and Omaha beaches, and throughout the subsequent weeks as the Allies expanded the beachhead and their hold on Normandy. In late July, Bradley planned and led Operation COBRA that enabled a breakout of the Normandy beachhead and the beginning of a month-long pursuit of German forces across France. The success of this breakout allowed Eisenhower to activate the 12th Army Group, putting Bradley in charge of 21 divisions and over 900,000 soldiers. Bradley led the 12th Army Group in the elimination of the Falaise pocket and to the western edges of Germany before an ever-lengthening logistical line halted their pursuit of a routed enemy. Several months after the Battle of the Bulge, Bradley's forces seized a bridge over the Rhine and completed a double encirclement of a large enemy force in the Ruhr valley. By

the time they reached the Elbe River in mid-April, the 12th Army Group had taken over 315,000 German prisoners. By V-E Day, Bradley (promoted to General to General in March 1945) was in charge of the largest American force ever. It boasted 48 divisions and over 1.3 million men [19].

Bradley was selected as the country's first Chairman of the Joints Chiefs of Staff in August 1949, serving two terms before retiring in 1953. As advisor to President Truman during the Korean War, Bradley worked to contain the conflict in Asia while still maintaining a viable military presence in Europe [19].

3.1.7 General Matthew B. Ridgway

Matthew B. Ridgway was born on March 3, 1895 in Fort Monroe, Virginia, where his father was serving as a battalion commander. His earliest memories were of the parades and the Taps bugle call, so that a career in the military only seemed fitting. In 1912, Ridgway was temporarily set back when he failed to pass the West Point entrance exam due to a poor performance in geometry. He was accepted into West Point one year later and went on to serve as football team manager and cadet adjutant while maintaining an academic standing in the top half of his class of 139 cadets. Ridgway left West Point with the class of 1917. He graduated six weeks early due to the United States entrance into WW1 [10].

Expecting to see combat duty in France shortly after graduation, Ridgway was disappointed to spend his first assignment after West Point serving along the Mexican border. In 1918, he returned to West Point to teach Spanish and head the athletic program. After spending six more years at West Point as an instructor, Ridgway

attended the Infantry School and served in infantry units in China and the U.S. In late 1927, he served on the American Electoral Commission in Nicaragua (forgoing a possible berth on the 1928 Olympic pentathlon team). He spent five years in South America before heading to the Pacific as military advisor to the Governor General of the Philippines. Upon returning from the Philippines in 1933, Ridgway spent the next four years in stateside staff assignments and as a student at the Army Command and General Staff School and the Army War College. Shortly after the outbreak of war in Europe in September 1939, General George Marshall reassigned Ridgway to the War Department in Washington, D.C [6]. This was a sure sign of increased future responsibilities for Ridgway.

In early 1942, Ridgway was promoted to Brigadier General and left the War Department to work for Omar Bradley as deputy commander of the newly reactivated 82d Infantry Division. Over the next four months, Ridgway helped turn a group of draftees into a cohesive unit ready for combat. He assumed command of the division in June 1942 and was promoted to Major General two months later [4]. After transforming the division from light to airborne infantry, Ridgway planned the Army's first major airborne operations as fighting progressed in the Mediterranean Theater. After parachuting on Gela during the summer invasion of Sicily in 1943, Ridgway and the 82d prepared for a September assault of Salerno, Italy. Several months later, the 82d Airborne Division set sail for England in order to begin preparations for OVERLORD.

After the division participated in another airborne drop on D-Day, Ridgway assumed command of the XVIII Airborne Corps that consisted of both the 82d and 101st Airborne Divisions.

In early fall, 1944, Ridgway led his men in MARKET-GARDEN. The XVIII Airborne Corps participated in numerous airborne drops and nearly 60 days of hard fighting in Holland before being sent to refit in France. Just one month later, Ridgway and the XVIII Airborne Corps were called forward to Belgium as Eisenhower's strategic reserve during the early stages of the Battle of the Bulge. After defeating "Hitler's last gamble" in early 1945, the XVIII Airborne pushed into Germany, and participated in the fighting in the Ruhr Valley before it linked up with Soviet troops along the Baltic in the spring of 1944 [3].

After the war ended in Europe, Lieutenant General Ridgway (promoted in June 1945) briefly served as Commander of the Mediterranean Theater of Operations. In 1946 he assumed another diplomatic post as the U.S. representative to the United Nations Military Staff Committee. In 1948, he served as Commander of the Caribbean Command before returning to Washington to work in the Pentagon for the Chief of Staff of the Army. Shortly after Korea erupted in June, 1950, Douglas MacArthur identified Ridgway as the future commander of the Eighth Army, which was then actively engaged in combat on the Korean Peninsula. Several months later, the Eighth Army Commander was killed in a Jeep accident [3]. Ridgway assumed command of a demoralized and defeated army that had just finished the longest retreat in U.S. military history. He quickly regrouped his forces and slowed the Communist advance.

Launching a series of offensives in early 1951, Ridgway began to push the invaders back. He eventually retook Seoul and stabilized the front around the 38th Parallel. As Washington became more dissatisfied with MacArthur's behavior, Ridgway began to see increased responsibility. In 1951, he replaced MacArthur as the American and Supreme Allied Commander in the Far East. Several months later, Ridgway was promoted to full General and began what would become two years of peace negotiations that eventually culminated in an armistice on July 27, 1953 [16].

Ridgway completed his 38-year military career as Army Chief of Staff during the presidency of Dwight Eisenhower.

3.2   Hierarchy Development

3.2.1   Introduction

The generals selected for this study reached their wartime positions based on a career's worth of formal and informal performance evaluations. These evaluations rated each general's competencies and past performances in an attempt to assess their capacity for increased responsibility in future positions. Similarly, we will use competencies and past performances as the two criteria most influential in determining the best U.S. Army wartime general of the 20th Century. For ease of discussion, we will use the analogous terms of Skills and Actions to define competence and performance, respectively.

The Skills criterion is further defined by the Conceptual, Interpersonal, Tactical, and Technical subcriteria. The Actions criterion permits evaluation of the
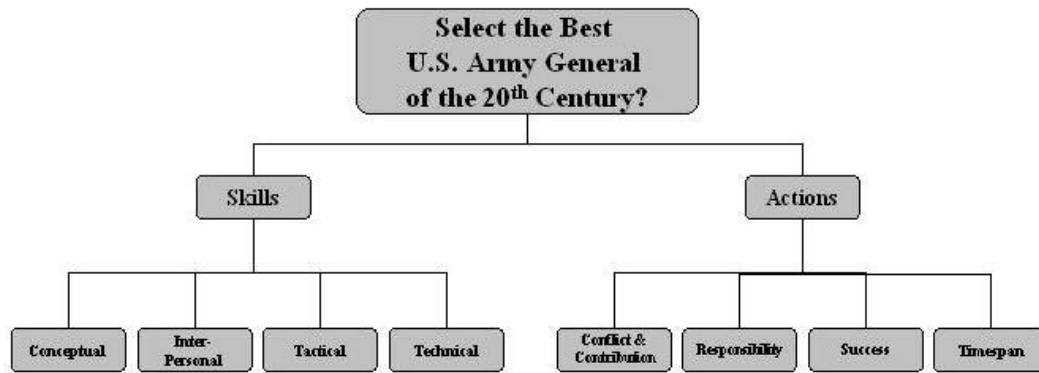
Figure 3.1: Best U.S. Army General Hierarchy

wartime performance of each general through the subcriteria Contribution to Conflict, Responsibility, Success, and Timespan. The complete hierarchy is displayed in Figure 3.1

Under each of the eight subcriteria, we use ratings to assign weights to each general. All eight categories used a scale of Superior, Very Good, Good, and Poor ratings. A Superior rating is assigned when a general's wartime performance with respect to that subcriterion is judged as the best of all generals throughout the century. A Very Good rating is described as performance comparable to a small number of generals throughout the century. A Good rating is assigned when the performance is as good as the majority of generals throughout the century. A Poor rating is given when the performance rates as among the worst of U.S. Army generals in the 20th century. Weights were assigned to each of the ratings through pairwise comparisons and the results are given in Table 3.1. The CR of this matrix is 0.0045. We devote the remainder of this section to further discussion of the hierarchy's criteria and subcriteria.

	Superior	Very Good	Good	Poor	Adjusted Ratings
Superior	1	2	3	4	1.0000
Very Good	-	1	2	3	0.5932
Good	-	-	1	2	0.3426
Poor	-	-	-	1	0.2042

Table 3.1: Pairwise Comparison matrix for ratings

3.2.2 Skills Criterion

The structure of the Skills criterion was directly influenced by the current Army Leadership Manual, which cites conceptual, interpersonal, tactical, and technical as the four types of skills needed by successful Army leaders. We selected these traits as those most important in determining the best general in our study.

Conceptual skills are those that enable us to handle ideas. For the purposes of this study, they are essential in establishing intent, filtering information, and understanding how organizations function. They are also necessary in envisioning and developing proper frames of reference, as well as in dealing with ambiguity and uncertainty.

Interpersonal skills influence the way we communicate and work with others. They affect each general's ability to successfully communicate, conduct dialogue, and negotiate with superiors, peers, and subordinates. In addition, they are critical in achieving consensus and building teams within large and often diverse groups. In this study, we also consider the negative aspects of each general's personality to belong to the Interpersonal subcriterion.

Tactical skills describe a general's proficiency in required professional knowledge and judgment as it is applied to warfighting. They reflect the capacity to synchronize activities and make correct tactical decisions on the battlefield. For the purposes of this study, tactical skills are extended to knowing when not to interfere with the tactical decisions and operations of subordinates.

Technical skills encompass a general's capacity to operate at the highest levels of the military command structure. These skills allow a general to adequately resource equipment, personnel, time, budgets and facilities in support of mission accomplishment. These skills provide for a clear understanding of second- and third-order effects as well as permit the translation of political goals into military objectives [20].

3.2.3 Actions Criterion

The purpose of the Actions criterion is to appraise the wartime performance of each general from an historical viewpoint. We decomposed the Actions criterion into four subcriteria.

The first subcriterion used to define Actions is Timespan, which considers the number of wars each general served in as well as the total time of his wartime service.

The second subcriterion developed to define the Actions category was Success. Each general was extremely successful during wartime, some as battlefield commanders and some as coalition builders. The Success subcriterion evaluates a general's overall success throughout all responsibilities of his position, from tactical victories to strategic-level planning.

The Contribution to Conflict category was created as a way of measuring a general's impact on the wartime conflicts he participated in. This subcriterion considers the specific contributions made by each general to the eventual outcome of the conflict. It also includes the importance of respective theaters as well as development of new tactics, techniques, and procedures. For instance, General Pershing's contribution towards ending WWI was significantly different than General Eisenhower's contribution to resolving WWII. Pershing was the senior American commander while Eisenhower was a theater commander.

The Responsibility subcriterion was created to consider the overall scope of the general's position during war. It includes the general's geographical area of responsibility, the size of his forces, and the importance of decisions made throughout the conflict.

3.3   Pairwise Comparison Matrices

Each of the 10 decision makers participating in this study completed three sets of pairwise comparisons in order to assign their preferences to criteria and subcriteria. The first set of comparisons determined which of the two top-level criteria, Skills and Actions, were judged most important in determining the best overall general. The second set of comparisons considered the four Skills subcriteria while the third set of comparisons considered the four Actions subcriteria.

This section is devoted to summarizing the results of the 10 decision makers, as well as synthesizing the individual pairwise comparison matrices into group pairwise comparison matrices. The first three subsections discuss the three sets of

pairwise comparisons: Top-Level, Skills, and Actions. The tables in these sections provide the priorities of the criteria and subcriteria based on the judgments in the individual pairwise comparison matrices. The final subsection provides the group pairwise comparison matrices and priorities determined by the methods presented in Sections 2.1 and 2.2.

3.3.1 Top-level Pairwise Comparisons Matrices

The 10 individual results comparing Skills (SK) and Actions (AC) were almost evenly split between the two top-level criteria. The matrices are given in Table 3.2 (2×2 matrices are completely consistent). Four decision makers felt Actions were more important than Skills, citing that results are more important than potential. The three decision makers who favored Skills believed that action without the necessary skills often results in failure. Three decision makers felt both criteria were equally important as greatness is determined by action, which in turn is predicated by the necessary skills. Table 3.3 summarizes the rankings by decision maker.

3.3.2 Skills Pairwise Comparison Matrices

The results comparing Conceptual (CO), Interpersonal (IN), Tactical (TA), and Technical (TE) with respect to the Skills criterion for the 10 decision makers are given in Table 3.4 (the CR of each matrix is provided in parenthesis). Four of the decision makers gave Tactical the largest priority. Three decision makers gave

	Decision Maker 1		Priority
Skills	1	7	0.8750
Actions	-	1	0.1250

	Decision Maker 2		Priority
Skills	1	1/9	0.1000
Actions	-	1	0.9000

	Decision Maker 3		Priority
Skills	1	1/5	0.1667
Actions	-	1	0.8333

	Decision Maker 4		Priority
Skills	1	1	0.5000
Actions	-	1	0.5000

	Decision Maker 5		Priority
Skills	1	1	0.5000
Actions	-	1	0.5000

	Decision Maker 6		Priority
Skills	1	1/3	0.2500
Actions	-	1	0.7500

	Decision Maker 7		Priority
Skills	1	5	0.8333
Actions	-	1	0.1667

	Decision Maker 8		Priority
Skills	1	6	0.8571
Actions	-	1	0.1429

	Decision Maker 9		Priority
Skills	1	1	.5000
Actions	-	1	.5000

	Decision Maker 10		Priority
Skills	1	1/7	0.1250
Actions	-	1	0.8750

Table 3.2: Top-level pairwise comparison matrices and priorities

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Skills	1	2	2	1	1	2	1	1	1	2
Actions	2	1	1	1	1	1	2	2	1	1

Table 3.3: Top-level criteria rankings

Conceptual the highest weight and three gave Interpersonal the highest weight. Technical was not rated highest by any decision maker.

The decision makers who cited Tactical as the most important subcriterion remarked that every general is involved in tactics on some level, either directly when on the battlefield or indirectly when in a theater headquarters. They also cited the importance of a general knowing when they should not interfere with the tactical decisions of their subordinates. Several decision makers believed that a sound understanding of tactics best supported a general's job, which is to win wars while caring for those under his command. We note that four decision makers gave Tactical a weight larger than 0.60 – for them, Tactical was more important than the remaining three subcriteria combined.

The three decision makers who scored the Conceptual subcriterion as most important believed the best generals provide a clear vision for their organization, whether in peace or in war. They felt the best generals were those who could handle the uncertainty and risk often hidden in the fog of war. In addition, the best generals must be able to apply their mental abilities to successfully overcome adversity faced during war.

For the Interpersonal subcriterion, four decision makers felt it was the least important. These four decision makers proposed that history is abound with generals who had serious personality flaws yet managed to be incredibly successful on the battlefield and that only certain postings truly demand great interpersonal ability. The three decision makers who believed Interpersonal was the most important subcriteria

Decision Maker 1 (0.0969)

					Priority
CO	1	9	7	1	0.4859
IN	-	1	1/6	1/7	0.0383
TA	-	-	1	1/3	0.1274
TE	-	-	-	1	0.3484

Decision Maker 2 (0.0545)

					Priority
CO	1	8	5	3	0.5656
IN	-	1	1/4	1/6	0.0463
TA	-	-	1	1/3	0.1229
TE	-	-	-	1	0.2652

Decision Maker 3 (0.0225)

					Priority
CO	1	1/4	1/2	1	0.1357
IN	-	1	1	1	0.3858
TA	-	-	1	2	0.3190
TE	-	-	-	1	0.1595

Decision Maker 4 (0.0327)

					Priority
CO	1	4	1/2	1/2	0.2017
IN	-	1	1/5	1/5	0.0639
TA	-	-	1	2	0.4307
TE	-	-	-	1	0.3037

Decision Maker 5 (0.0190)

					Priority
CO	1	1/2	1/8	1	0.0808
IN	-	1	1/5	3	0.1723
TA	-	-	1	7	0.6702
TE	-	-	-	1	0.0767

Decision Maker 6 (0.0083)

					Priority
CO	1	1/3	5	2	0.2352
IN	-	1	9	5	0.5928
TA	-	-	1	1/2	0.0565
TE	-	-	-	1	0.1155

Decision Maker 7 (0.0994)

					Priority
CO	1	1/5	1/7	1	0.0586
IN	-	1	1/6	6	0.2191
TA	-	-	1	9	0.6705
TE	-	-	-	1	0.0519

Decision Maker 8 (0.0634)

					Priority
CO	1	1/5	6	2	0.1997
IN	-	1	8	6	0.6428
TA	-	-	1	1/3	0.0474
TE	-	-	-	1	0.1101

Decision Maker 9 (0.0718)

					Priority
CO	1	5	1/5	2	0.1909
IN	-	1	1/9	1/5	0.0424
TA	-	-	1	5	0.6309
TE	-	-	-	1	0.1358

Decision Maker 10 (0.0878)

					Priority
CO	1	5	9	7	0.6550
IN	-	1	6	2	0.1889
TA	-	-	1	1/5	0.0394
TE	-	-	-	1	0.1187

Table 3.4: Skills pairwise comparison matrices and priorities

	Decision Maker									
	1	2	3	4	5	6	7	8	9	10
Conceptual	1	1	4	3	3	2	3	2	2	1
Interpersonal	4	4	1	4	2	1	2	1	4	2
Tactical	3	3	2	1	1	4	1	4	1	4
Technical	2	2	3	2	4	3	4	3	3	3

Table 3.5: Skills subcriteria rankings

felt the ability to work well with other armed services and other nations is what distinguished the best generals of the 20th century.

No decision makers considered Technical as the most important subcriterion. Those who rated it somewhat high believed that a general's mastery of strategic art is the culmination of a career's worth of training and experience, thus proving his proficiency across the entire spectrum of leadership. Those who rated it somewhat low believed it was critical for only a small number of positions and therefore not necessarily for a great general. Table 3.5 summarizes each decision maker's rankings of the four Skills subcriteria.

3.3.3   Actions Pairwise Comparison Matrices

The pairwise comparison matrices for the four subcriteria of Actions, Contribution to Conflict (CC), Responsibility (RE), Success (SU), and Timespan (TS) are given in Table 3.6 (the CR of each matrix is provided in parenthesis).

Seven decision makers considered Success as the most important subcriterion. All seven decision makers stated that a general's ultimate responsibility is to win our

Decision Maker 1 (0.0952)

					Priority
CC	1	1/4	1/7	3	0.0924
RE	-	1	1/5	3	0.2019
SU	-	-	1	9	0.6533
TS	-	-	-	1	0.0524

Decision Maker 2 (0.0872)

					Priority
CC	1	5	1/4	7	0.2624
RE	-	1	1/7	3	0.0815
SU	-	-	1	9	0.6150
TS	-	-	-	1	0.0411

Decision Maker 3 (0.0632)

					Priority
CC	1	4	1/6	1	0.1604
RE	-	1	1/6	1/3	0.0618
SU	-	-	1	4	0.6198
TS	-	-	-	1	0.1580

Decision Maker 4 (0.0571)

					Priority
CC	1	2	4	6	0.4932
RE	-	1	3	4	0.2971
SU	-	-	1	4	0.1483
TS	-	-	-	1	0.0614

Decision Maker 5 (0.0229)

					Priority
CC	1	1/5	1	3	0.1446
RE	-	1	5	9	0.6460
SU	-	-	1	4	0.1574
TS	-	-	-	1	0.0520

Decision Maker 6 (0.0506)

					Priority
CC	1	3	5	7	0.5975
RE	-	1	1/2	2	0.1389
SU	-	-	1	3	0.1936
TS	-	-	-	1	0.0699

Decision Maker 7 (0.0609)

					Priority
CC	1	1/3	1/7	3	0.0965
RE	-	1	1/5	4	0.1948
SU	-	-	1	9	0.6599
TS	-	-	-	1	0.0487

Decision Maker 8 (0.0634)

					Priority
CC	1	2	1/4	7	0.2206
RE	-	1	1/5	5	0.1354
SU	-	-	1	9	0.6043
TS	-	-	-	1	0.0397

Decision Maker 9 (0.0225)

					Priority
CC	1	1/2	1/2	5	0.2179
RE	-	1	1	5	0.3604
SU	-	-	1	5	0.3604
TS	-	-	-	1	0.0613

Decision Maker 10 (0.0953)

					Priority
CC	1	1/3	1/7	5	0.1093
RE	-	1	1/4	7	0.2302
SU	-	-	1	9	0.6225
TS	-	-	-	1	0.0380

Table 3.6: Actions pairwise comparison matrices and priorities

	1	2	3	4	5	6	7	8	9	10
Contribution	3	2	2	1	3	1	3	2	3	3
Responsibility	2	3	4	2	1	3	2	3	1	2
Success	1	1	1	3	2	2	1	1	1	1
Timespan	4	4	3	4	4	4	4	4	4	4

Table 3.7: Actions subcriteria rankings

nation's wars. Six of the seven decision makers felt Success was more important that all three other subcriteria combined and gave it a priority greater than 0.60.

The decision makers who gave Contribution to Conflict a high priority argued that contribution is what truly distinguishes generals. They believed that true greatness is measured through overall contribution and can be achieved in one instant, through one key decision, regardless of the level of success achieved throughout the remainder of a career.

Two decision makers believed that Responsibility was the most significant subcriterion. The best generals are continually entrusted to larger commands and more sizable areas of operation. They believed generals must be decisive leaders and make critical decisions without being affected by the enormous moral and emotional pressures of their positions.

Timespan was ranked the least important by nine of the ten decision makers. Although length of service and number of wars fought can be impressive, it is the accomplishments of a career that determine greatness. Table 3.7 contains each decision maker's rankings of the Actions subcriteria.

3.3.4 Group Pairwise Comparison Matrices

The group pairwise comparison matrices were compiled using the geometric mean method as described in Section 2.2.2 (Group AHP). To illustrate this process, consider determining the Top-level group pairwise comparison judgment a_{12}, the comparison of Skills to Actions. Let a_{12}^k denote the comparison of Skills to Actions for decision maker k ($k = 1,2,\ldots,10$). Then,

$$
\begin{aligned}
a_{12} &= \left(a_{12}^1 \cdot a_{12}^2 \cdot a_{12}^3 \cdot a_{12}^4 \cdot a_{12}^5 \cdot a_{12}^6 \cdot a_{12}^7 \cdot a_{12}^8 \cdot a_{12}^9 \cdot a_{12}^{10} \right)^{\frac{1}{10}} \\
&= \left(7 \cdot 1/9 \cdot 1/5 \cdot 1 \cdot 1 \cdot 1/3 \cdot 5 \cdot 6 \cdot 1 \cdot 1/7 \right)^{\frac{1}{10}} \\
&= \left(0.2222 \right)^{\frac{1}{10}} \\
&= 0.8604.
\end{aligned}
$$

The three group pairwise comparison matrices are given in Tables 3.8 – 3.10. The consistency ratios are provided at the top left corner of each. All consistency rations are less than 0.10. The associated priorities were calculated using the eigenvector method introduced in Section 2.1.3

	Skills	Actions	Priority
Skills	1	0.8604	0.4625
Actions	-	1	0.5375

Table 3.8: Top-level group pairwise comparison matrix and priorities

(0.0045)	CO	IN	TA	TE	Priority
CO	1	1.2821	1.2377	1.5575	0.3087
IN	-	1	0.6494	1.0748	0.2121
TA	-	-	1	1.1665	0.2712
TC	-	-	-	1	0.2080

Table 3.9: Skills group pairwise comparison matrix and priorities

(0.0070)	CC	RE	SU	TS	Priority
CC	1	0.9603	0.4449	4.1141	0.2338
RE	-	1	0.4522	3.2958	0.2255
SU	-	-	1	5.9614	0.4730
TS	-	-	-	1	0.0678

Table 3.10: Actions group pairwise comparison matrix and priorities

3.4   Alternative Ratings

Each general's group rating with respect to each subcriterion was determined by averaging the individual ratings assessed with respect to that same subcriterion. To illustrate this process, consider determining Bradley's group rating with respect to the Conceptual subcriterion. Each of the 10 decision makers rated Bradley as either Superior (S), Very Good (VG), Good (G), or Poor (P) with regards to Conceptual as indicated below.

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	G	G	VG	G	G	G	VG	G	G	VG

These ratings were assigned numerical values in Table 3.1. Substituting the numerical values for the verbal ratings and calculating the average yields,

$$\frac{(.3426)+(.3426)+(.5932)+(.3426)+(.3426)+(.3426)+(.5932)+(.3426)+(.3426)+(.5932)}{10} = 0.41778.$$

Thus, Bradley's group Conceptual rating is 0.41778. This value is recorded in Table 3.11.

In this section, we briefly highlight the reasoning behind each general's group priority through comments provided by the decision makers. A table is included at the conclusion of each subcriterion's discussion that contains each general's group priority as well as the highest and lowest ratings received from the 10 decision makers. Appendix A contains the complete ratings by subcriterion.

3.4.1 Conceptual Ratings

Marshall rated highest with respect to the Conceptual subcriterion primarily due to his ability to quickly grasp and master new concepts. He had a superior ability to navigate the unique politico-military issues associated with his position as wartime Chief of Staff. Eisenhower ranked second in this category as he displayed excellent judgment in launching the D-Day invasion. However, several decision makers faulted him for not intervening in closing the Falaise Gap. MacArthur was judged by

	Group Rating	High Rating	Low Rating
Marshall	0.8780	Superior	Very Good
Eisenhower	0.7715	Superior	Good
MacArthur	0.7076	Superior	Poor
Ridgway	0.6495	Superior	Good
Pershing	0.5994	Superior	Good
Patton	0.4930	Very Good	Good
Bradley	0.4178	Very Good	Good

Table 3.11: Group Ratings for Conceptual

some to be the most brilliant Army leader ever. Yet, he received the only Poor rating

given in this category due to his poor judgment in permitting the destruction of his

entire Army Air Force at Clark Field in the opening days of WW2 and in disregarding

the Chinese threat in the Korean War. Ridgway's ability to quickly grasp new

concepts and his ability to understand the political and military situation in Korea

earned him the fourth spot. Pershing received high marks for his work in building the

AEF, yet some decision makers felt he should have stepped aside earlier as First

Army Commander. Patton was a creative thinker but displayed extremely poor

judgment when he slapped two soldiers suffering from combat fatigue and accused

them of cowardice. Bradley was never able to overcome his failure to realize the

opportunity to close the Falaise Gap, and this placed him last in Conceptual.

3.4.2   Interpersonal Ratings

Eisenhower ranked first with respect to this subcriterion. He received a

Superior rating from nine decision makers. He was cited as the most successful

	Group Rating	High Rating	Low Rating
Eisenhower	0.9593	Superior	Very Good
Marshall	0.8122	Superior	Good
Pershing	0.6244	Superior	Good
Ridgway	0.5086	Superior	Good
Bradley	0.4930	Very Good	Good
MacArthur	0.4039	Very Good	Poor
Patton	0.3235	Very Good	Poor

Table 3.12: Group Ratings for Interpersonal

commander of the largest and most complex military coalition in history. Marshall was excellent in assessing and empowering subordinates and was primarily responsible for placing nearly all of the top Army leaders in WW2. Ironically, he was also extremely distant in his professional relationships; this was a trait common to Pershing, his mentor. Pershing built efficient and well-trained staffs, yet would relieve subordinate commanders extremely quickly when they failed to perform to his expectations. Ridgway inspired confidence and impressed his aggressiveness on every unit he commanded in WW2 and the Korean War. Bradley was an excellent coach and trainer as commander of both the 82d and 28th Divisions. MacArthur and Patton were two of the most dynamic U.S. Army generals of the 20th century. However, both displayed the inability to work well with their military and civilian superiors. Unfortunately, MacArthur probably displayed his best interpersonal skills during peacetime in the occupation of Japan after WW2.

	Group Rating	High Rating	Low Rating
Patton	0.8936	Superior	Good
Ridgway	0.8780	Superior	Very Good
MacArthur	0.6401	Superior	Good
Marshall	0.4930	Very Good	Good
Bradley	0.4585	Superior	Good
Pershing	0.4290	Very Good	Poor
Eisenhower	0.3400	Very Good	Poor

Table 3.13: Group Ratings for Tactical

3.4.3 Tactical Ratings

Patton ranked first in the Tactical subcriterion. He understood tactical details and performed well at both exploitation and positional warfare. His mastery of exploitation warfare often overshadowed his success in positional warfare. Patton's best performances were in France in late summer 1944 and during the Battle of the Bulge. Ridgway also performed well in the Battle of the Bulge. He was at his best in Korea after he assumed command of Eighth Army. MacArthur performed ably as a WW1 division commander and throughout the majority of WW2. His best performance was at Inchon in Korea. Marshall, Bradley, Pershing, and Eisenhower were all assessed as better planners and organizers than combat leaders.

3.4.4 Technical Ratings

Marshall's greatest strength was his ability to understand the broad political, economic, and strategic matters involved in creating the Army for WW2. He was

	Group Rating	High Rating	Low Rating
Marshall	0.8122	Superior	Good
Eisenhower	0.7715	Superior	Good
Ridgway	0.7058	Superior	Good
MacArthur	0.6401	Superior	Good
Patton	0.6401	Superior	Good
Pershing	0.5587	Superior	Good
Bradley	0.5086	Superior	Good

Table 3.14: Group Ratings for Technical

proficient in translating political goals into military objectives; this was a strength he shared with Eisenhower. Many decision makers felt that Eisenhower was the only WW2 general who possessed the military, political, and diplomatic skills to serve as Supreme Commander of the Allied Expeditionary Force. Ridgway certainly understood the political situation in Korea better than MacArthur and fought extremely well despite being constrained in a limited war. MacArthur was a great strategist though he generally failed to understand political goals that were set by others. Patton clearly understood weapons effects and was cited as the Army's best tank man by Marshall. [8]. Pershing understood the political importance of the AEF remaining independent in WW1 and managed to realize this goal despite intense allied pressure.

3.4.5 Contribution to Conflict Ratings

Many decision makers viewed Marshall's role in the WW2 victory as indispensable. His simultaneous management of the Army's 4000% growth and the global war were arguably the greatest military achievements in U.S. history. Eisenhower scored slightly lower than Marshall, as his contributions were restricted to the Mediterranean and European theaters by virtue of his positions. Pershing was required to develop emerging AEF doctrine while simultaneously ensuring his forces were prepared to fight. MacArthur made impressive contributions in three wars. However, his role in WW2 was restricted to the Pacific, which was a secondary theater of war. Despite an impressive WW2 record, Ridgway's most significant contributions were made during the Korean War when he rebuilt a defeated Eighth Army and forced the enemy to come to the bargaining table in just over six months. Bradley's most significant contributions were Operation COBRA, the Normandy breakout, and the seizure and exploitation of the Remagan bridgehead. Patton made many contributions as an army commander. However, the influence he had as army commander could never come close to the influence exhibited by the other generals in the study. This fact certainly affected him negatively in this rating.

3.4.6 Responsibility Ratings

As Supreme Commander in Europe, Eisenhower was primarily responsible for coalition building and making critical decisions. His decision to launch the D-Day invasion was cited as perhaps the most significant in the war. This decision was only equaled by Truman's decision to use atomic weapons on Japan. Marshall was the

	Group Rating	High Rating	Low Rating
Marshall	0.9593	Superior	Very Good
Eisenhower	0.8373	Superior	Very Good
Pershing	0.7465	Superior	Good
MacArthur	0.6513	Superior	Poor
Ridgway	0.5587	Superior	Good
Bradley	0.5242	Superior	Good
Patton	0.4290	Very Good	Poor

Table 3.15: Group Ratings for Contribution to Conflict

only general in this study who managed operations on a global level. In the process of building the largest army ever mobilized, Marshall made critical decisions on force structure and doctrinal emphasis. He was ultimately responsibility for raising, equipping, and training 89 American divisions. Pershing had similar responsibilities in WW1 where he was responsible for 30 American divisions (one WW1 division was roughly equal in size to two WW2 divisions). MacArthur held high responsibility theater commander positions in WW2 and Korea. The importance of his decisions could never match those of Eisenhower. Ridgway assumed the least responsibility of all generals in WW2. He was later responsible for the entire theater during the Korean War. Bradley's 12th Army was the largest American force ever and consisted of nearly 48 American divisions. Bradley never managed high-level political matters. He never faced decisions as important as those generals rated above him. Patton's responsibilities were almost exclusively military due, in large part, to

	Group Rating	High Rating	Low Rating
Eisenhower	0.9593	Superior	Very Good
Marshall	0.9593	Superior	Very Good
Pershing	0.9186	Superior	Very Good
MacArthur	0.8529	Superior	Good
Ridgway	0.6807	Superior	Good
Bradley	0.5994	Superior	Good
Patton	0.4446	Superior	Poor

Table 3.16: Group Ratings for Responsibility

his position as army commander where he never commanded a force larger than 16 divisions.

3.4.7 Success Ratings

Marshall was unanimously viewed as the most successful general. He received a Superior rating from all 10 decision makers. His effort in developing the WW2 Army was cited as the foundation of all American success throughout the war. Eisenhower's greatest achievement was building the Allied coalition and then keeping it together as the war progressed across Europe. Many decision makers believed this was a feat only Eisenhower could have achieved. Patton placed third based on the strength of his battlefield success. His exploits in the French frontier in late summer 1944 and his response in the opening days of the Battle of the Bulge later that year were cited as his biggest accomplishments. Ridgway commanded in WW2

	Group Rating	High Rating	Low Rating
Marshall	1.0000	Superior	Superior
Eisenhower	0.8122	Superior	Good
Patton	0.6902	Superior	Good
Ridgway	0.6746	Superior	Very Good
Pershing	0.6495	Superior	Good
MacArthur	0.6012	Superior	Poor
Bradley	0.5838	Superior	Good

Table 3.17: Group Ratings for Success

with great success. He was most successful in the Korean War after he revitalized

Eighth Army and forced the North Koreans and Chinese to agree to ceasefire talks.

Pershing's greatest successes were building the AEF and keeping it independent from

the British and French Armies. MacArthur was a poor coalition builder who failed in

the defense of the Philippines and later in Korea. His remarkable victories at

Hollandia and Leyte in WW2 and at Inchon in Korea kept him out of the last position.

Bradley was assessed as an officer who never experienced great success or great

defeat. His best moments were the Normandy Breakout, the Rhine River crossing,

and the pursuit to the Elbe in the closing days of the war. He also performed well in

Tunisia and Sicily.

3.4.8   Timespan Ratings

MacArthur's nearly 62 months of service across three wars earned him 10

"Superior" ratings in this category. Marshall served as Chief of Staff during the

	Group Rating	High Rating	Low Rating
MacArthur	1.0000	Superior	Superior
Marshall	0.7559	Superior	Very Good
Pershing	0.6150	Very Good	Good
Eisenhower	0.5180	Very Good	Good
Ridgway	0.5180	Very Good	Good
Bradley	0.4930	Very Good	Good
Patton	0.4679	Very Good	Good

Table 3.18: Group Ratings for Timespan

United States involvement in WW2 for nearly 45 months, from December 1941 until September 1945. Pershing served in two wars, with nearly 32 combined months in both the Punitive Expedition and WW1. Eisenhower spent 41 months as a general in WW2. Ridgway was a wartime general for 41 months, with 33 months in WW2 and the eight months in Korea. Bradley spent 41 months as a general in WW2 and 14 months as a general during the Korean War (which was generally discounted as his involvement in the Korean War was negligible). Patton spent 41 months fighting in WW2.

3.5 Eigenvector GAHP Results

We used hierarchical composition to determine the overall weights of the seven generals in our study. Figure 3.2 contains the eigenvector-generated group priorities and the group alternative ratings, as determined in Sections 3.3 and 3.4, respectively.

|	Conceptual (0.3087)	Inter-Personal (0.2121)	Tactical (0.2712)	Technical (0.2080)	Conflict & Contribution (0.2338)	Responsibility (0.2255)	Success (0.4730)	Timespan (0.0678)
Bradley	0.4178	0.4930	0.4585	0.5086	0.5242	0.5994	0.5838	0.4930
Eisenhower	0.7715	0.9593	0.3400	0.7715	0.8373	0.9593	0.8122	0.5180
MacArthur	0.7076	0.4039	0.6401	0.6401	0.6513	0.8529	0.6012	1.0000
Marshall	0.8780	0.8122	0.4930	0.8122	0.9593	0.9593	1.0000	0.7559
Patton	0.4930	0.3235	0.8936	0.6401	0.4290	0.4446	0.6902	0.4679
Pershing	0.5994	0.6244	0.4290	0.5587	0.7465	0.9186	0.6495	0.6150
Ridgway	0.6495	0.5086	0.8780	0.7058	0.5587	0.6807	0.6746	0.5180

Figure 3.2: Best U.S. Army General hierarchy (Completed)

To illustrate the hierarchical composition process, consider determining Bradley's overall weight. The process given in Section 2.1.5 yields

$$((.3086)(.4178)+(.2121)(.4930)+(.2712)(.4585)+(.2080)(.5086))(.4625)+$$
$$((.2338)(.5242)+(.2255)(.5994)+(.4730)(.5838)+(.0678)(.4930))(.5375)=.5193$$

The overall weights for the remaining six generals are calculated in an identical manner.

The overall weight of each general is listed in Table 3.19 and groups the generals into three distinct tiers. The top tier consists of Marshall and Eisenhower. The middle tier consists of MacArthur, Ridgway and Pershing. The lower tier consists of Patton and Bradley. The middle tier is clearly the most competitive.

70

	Overall Weight
Marshall	0.8636
Eisenhower	0.7679
Ridgway	0.6637
MacArthur	0.6570
Pershing	0.6470
Patton	0.5761
Bradley	0.5193

Table 3.19: GAHP Overall Weights

It is noteworthy that each of the top four finishes directed operations at the theater or global level at some point in their careers. None of the bottom three finishers commanded at the theater level of higher. Pershing, ranked fifth, commanded the AEF, which contained three armies and nearly two million men at the end of WW1. Though not technically a theater commander, his command was more similar to the generals who finished ahead of him than to Patton's or Bradley's.

3.6   Interval Linear Programming GAHP Results

3.6.1   Interval Pairwise Comparison Matrices

We used the 10 decision makers' individual pairwise comparison matrices to create three sets of interval pairwise comparison matrices representing the preferences of the group. The first set of interval pairwise comparison matrices was determined by considering the pairwise judgments of all 10 decision makers $\left(a_{ij}^{k}, k = 1,2,\dots,10\right)$. To illustrate this process, consider determining the Top-level interval pairwise comparison matrix. This matrix is of the form

$$\begin{bmatrix} 1 & [l_{12},u_{12}] \\ [l_{21},u_{21}] & 1 \end{bmatrix}.$$

Then, $[l_{12},u_{12}]$ is the interval judgment representing the comparison of Skills to Actions. It is calculated by

$$l_{12} = \min\{a_{12}^1,a_{12}^2,\ldots,a_{12}^{10}\} = \min\{7,1/9,1/5,1,1,1/3,5,6,1,1/7\} = 1/9$$

$$u_{12} = \max\{a_{12}^1,a_{12}^2,\ldots,a_{12}^{10}\} = \max\{7,1/9,1/5,1,1,1/3,5,6,1,1/7\} = 7.$$

Similarly, $[l_{21},u_{21}]$ represents the comparison of Actions to Skills and is calculated by

$$l_{21} = \min\{a_{21}^1,a_{21}^2,\ldots,a_{21}^{10}\} = \min\{1/7,9,5,1,1,3,1/5,1/6,1,7\} = 1/7$$

$$u_{21} = \max\{a_{21}^1,a_{21}^2,\ldots,a_{21}^{10}\} = \max\{1/7,9,5,1,1,3,1/5,1/6,1,7\} = 9.$$

Continuing this process for the Skills and Actions interval pairwise comparison matrices provides the matrices contained in Tables 3.22 – 3.24. The priorities for each criterion and subcriterion were determined by applying the interval linear programming (ILP) method to each matrix. For our purposes, we designated this set ILP-10. The linear programming formulations for all sets of matrices presented in this section are provided in Appendix B.

Determining the interval judgments in this manner reduces the opinion of the group to that of the highest and lowest individual judgments. This will not skew the group's preference when the 10 individual judgments are fairly consistent. However, the opinion of the group will not be accurately presented if the highest and/or lowest individual judgments are drastically different than the remainder of the group.

Eliminating the outlying judgments before determining the group interval judgment is a method to ensure the group's opinion is accurately recorded.

We created the second set of interval pairwise comparison matrices by determining the interval judgments after removing the lowest and highest individual judgments. To illustrate, consider determining the top-level interval judgment comparing Skills to Actions. After removing the lowest judgment $\left(a_{12}^1 = 1/7\right)$ and the highest judgment $\left(a_{12}^2 = 9\right)$, the interval $\left[l_{12}, u_{12}\right]$ is determined by

$$l_{12} = \min\{5,1,1,3,1/5,1,1/6,1,7\} = 1/5$$

$$u_{12} = \max\{5,1,1,3,1/5,1,1/6,1,7\} = 7 .$$

This set of matrices and the ILP determined priorities are given in Tables 3.25 – 3.27. For our purposes we designated this set ILP-8.

We created the third set of interval pairwise comparison matrices by determining the interval judgments after removing the two highest and two lowest individual judgments. This set of matrices and the ILP determined priorities are given in Tables 3.28 – 3.30. For our purposes, we designated this set ILP-6.

			Priority
SK	1	[1/9,7]	0.4686
AC	[1/7,9]	1	0.5314

Table 3.22: Top-level group interval pairwise comparison matrix (ILP-10)

					Priority
CO	1	[1/5,9]	[1/8,9]	[1/2,7]	0.3368
IN	[1/9,5]	1	[1/9,9]	[1/7,6]	0.2416
TA	[1/9,8]	[1/9,9]	1	[1/5,9]	0.2416
TE	[1/7,2]	[1/6,7]	[1/9,5]	1	0.1800

Table 3.23: Skills group interval pairwise comparison matrix (ILP-10)

					Priority
CC	1	[1/5,5]	[1/7,5]	[1,7]	0.2932
RE	[1/5 ,5]	1	[1/7,5]	[1/3,9]	0.2932
SU	[1/5,7]	[1/5,7]	1	[3,9]	0.3468
TS	[1/7,1]	[1/9,3]	[1/9,1/3]	1	0.0668

Table 3.24: Actions group interval pairwise comparison matrix (ILP-10)

			Priority
SK	1	[1/7,6]	0.4807
AC	[1/6,7]	1	0.5193

Table 3.25: Top-level group interval pairwise comparison matrix (ILP-8)

					Priority
CO	1	[1/5,8]	[1/7,7]	[1,3]	0.3421
IN	[1/8,5]	1	[1/6,8]	[1/6,6]	0.2704
TA	[1/7,7]	[1/8,6]	1	[1/3,7]	0.2342
TE	[1/3,1]	[1/6,6]	[1/7,3]	1	0.1533

Table 3.26: Skills group interval pairwise comparison matrix (ILP-8)

					Priority
CC	1	[1/5,4]	[1/7,4]	[1,7]	0.2874
RE	[1/4,5]	1	[1/6,3]	[2,7]	0.2689
SU	[1/4,7]	[1/3,6]	1	[4,9]	0.3803
TS	[1/7,1]	[1/7,1/2]	[1/9,1/4]	1	0.0634

Table 3.27: Actions group interval pairwise comparison matrix (ILP-8)

					Priority
SK	1	[1/5,5]			0.5000
AC	[1/5,5]	1			0.5000

Table 3.28: Top-level group interval pairwise comparison matrix (ILP-6)

					Priority
CO	1	[1/4,5]	[1/5,6]	[1,2]	0.2872
IN	[1/5,4]	1	[1/6,6]	[1/5,5]	0.2569
TA	[1/6,5]	[1/6,6]	1	[1/3,5]	0.2569
TE	[1/2,1]	[1/5,5]	[1/5,3]	1	0.1990

Table 3.29: Skills group interval pairwise comparison matrix (ILP-6)

					Priority
CC	1	[1/4,3]	[1/7,1]	[3,7]	0.2295
RE	[1/3,4]	1	[1/6,1]	[3,6]	0.2295
SU	[1,7]	[1,6]	1	[4,9]	0.4869
TS	[1/7,1/3]	[1/6,1/3]	[1/9,1/4]	1	0.0541

Table 3.30: Actions group interval pairwise comparison matrix (ILP-6)

3.6.2 Adjusted Alternative Ratings

Just as outlying decision maker input can adversely affect the group criteria and subcriteria priorities, the highest and lowest ratings can adversely affect the group ratings. Consider the example of Patton with respect to the Tactical subcriterion. The 10 decision makers rated Patton as indicated below.

				Decision Maker						
	1	2	3	4	5	6	7	8	9	10
Patton	S	S	S	G	S	S	S	S	S	VG

Despite earning Superior ratings from eight decision makers, Patton's group rating was 0.8936. Eliminating the highest and lowest ratings, in this case eliminating one Superior rating and the Good rating, and calculating the average of the remaining eight ratings yields an adjusted group rating of 0.9492. This rating is much closer to the overall group trend. We can repeat this process on the remaining eight ratings. Eliminating one Superior rating and the Very Good rating, and calculating the average of the remaining six ratings provides an adjusted group rating of 1. In this case, removing the three highest and three lowest individual ratings would eliminate more than half of the groups' contribution, and is therefore not beneficial.

We used three sets of alternative ratings in our study. The standard set of alternative ratings was based on the input of all 10 decision makers. This set was discussed in Sections 3.4.1-3.4.8 and was used in hierarchical composition with the group priorities determined through the Eigenvector method (Section 3.5). This set of alternative ratings will also be used in hierarchical composition with the ILP-10 priorities in the following section.

We created the second alternative set by removing the highest and lowest ratings, and calculating the adjusted group alternative ratings with the remaining eight individual ratings. Similarly, we created the third alternative set by removing the two highest and two lowest ratings, and calculating the adjusted group alternative ratings with the remaining six individual ratings. These two sets of alternative ratings will be used in hierarchical composition with the ILP-8 and ILP-6 priorities, respectively, in the following section. The three sets of alternative ratings are provided in Appendix A.

3.6.3   Hierarchical Composition

Completing hierarchical composition with the ILP-10 priorities and the standard set of alternative ratings yielded the results in tables 3.31. The hierarchical composition results for the ILP-8 priorities and associated alternative ratings are contained in Tables 3.32. The hierarchical composition results for the ILP-6 priorities and associated alternative ratings are contained in Tables 3.33.

These results support the three tiered ranking structure presented in Section 3.5. In addition, these results define MacArthur as the top ranked general of the highly competitive middle tier.

	Overall Weight
Marshall	0.8649
Eisenhower	0.7820
MacArthur	0.6646
Pershing	0.6620
Ridgway	0.6549
Patton	0.5495
Bradley	0.5167

Table 3.31: ILP-10 Overall Weights

	Overall Weight
Marshall	0.8874
Eisenhower	0.8072
MacArthur	0.6672
Pershing	0.6572
Ridgway	0.6397
Patton	0.5495
Bradley	0.4930

Table 3.32: ILP-8 Overall Weights

	Overall Weight
Marshall	0.8892
Eisenhower	0.8053
MacArthur	0.6490
Ridgway	0.6385
Pershing	0.6318
Patton	0.5640
Bradley	0.4859

Table 3.33: ILP-6 Overall Weights

Chapter 4

Conclusions and Ideas for Future Work

This thesis ranked seven U.S. Army Generals of the 20th Century using the Group Analytic Hierarchy Process (GAHP). We used the expert opinions of 10 military historians to determine four sets of group priorities for the criteria and subcriteria in our study. The first set was created through the eigenvector method (EM) and the remaining three were created through the interval linear programming method (ILP-10, ILP-8, ILP-6). For comparison purposes, the four sets of priorities are contained in Table 5.1. The overall weights determined through the four methods are provided in Table 5.2. These results unanimously support the three-tiered ranking structure:

<u>Top Tier</u>
General of the Army George Marshall
General of the Army Dwight Eisenhower

<u>Middle Tier</u>
General of the Army Douglas MacArthur
General of the Armies John Pershing
General Matthew Ridgway

<u>Bottom Tier</u>
General George Patton, Jr.
General of the Army Omar Bradley.

	EM	ILP-10	ILP-8	ILP-6
Skills	0.4625	0.4686	0.4807	0.5000
Actions	0.5375	0.5314	0.5193	0.5000
Conceptual	0.3086	0.3368	0.3421	0.2872
Interpersonal	0.2121	0.2416	0.2704	0.2569
Tactical	0.2712	0.2416	0.2342	0.2569
Technical	0.2080	0.1800	0.1533	0.1990
Contribution to Conflict	0.2338	0.2932	0.2874	0.2434
Responsibility	0.2255	0.2932	0.2689	0.2254
Success	0.4730	0.3468	0.3803	0.4781
Timespan	0.0678	0.0668	0.0634	0.0531

Table 5.1: Priority comparison

EM		ILP-10		ILP-8		ILP-6	
Marshall	0.8641	Marshall	0.8649	Marshall	0.8874	Marshall	0.8892
Eisenhower	0.7683	Eisenhower	0.7820	Eisenhower	0.8072	Eisenhower	0.8053
Ridgway	0.6637	MacArthur	0.6646	MacArthur	0.6672	MacArthur	0.6490
MacArthur	0.6573	Pershing	0.6620	Pershing	0.6572	Ridgway	0.6385
Pershing	0.6472	Ridgway	0.6549	Ridgway	0.6397	Pershing	0.6318
Patton	0.5759	Patton	0.5495	Patton	0.5495	Patton	0.5640
Bradley	0.5193	Bradley	0.5167	Bradley	0.4930	Bradley	0.4859

Table 5.2: Overall weight comparison

The highly competitive middle tier is an excellent starting point for future research. We could better define the relative rankings of MacArthur, Pershing, and Ridgway by increasing the number of decision makers in our group. This could easily be completed by combining the input from our original 10 decision makers with that from a new group of 10 decision makers.

Another area of future research would be to consider additional generals. The use of the ratings hierarchy allows us to do this without having to discount the decision makers' original alternative ratings. Collecting ratings on additional generals from the original group of decision makers does not require any changes to the criteria of subcriteria priorities. The overall weights for the newly considered generals could easily be determined and incorporated into the final rankings.

Appendix A

GAHP Appendix

A.1 Sammon Maps

Sammon's non-linear mapping is a visualization tool which provides a two dimensional representation of the relationships in higher dimensional data sets [15]. In our study, we used Sammon maps to visualize the relationship between the decision maker's individual priorities and the eigenvector generated group priorities. Closely spaced Sammon map points represent a similarity between two sets of priorities while widely spaced points represent a great difference between two sets of priorities. This permits us to identify clusters of similarly thinking decision makers as well as identify a decision maker whose priorities may be vastly different from the remaining members of the group.

A.1.1 Skills Sammon Map

The Skills Sammon map provides a two dimensional representation of 11 four-dimensional data points. The four dimensions represent the Conceptual, Interpersonal, Tactical, and Technical subcriteria. The individual priorities provided by the 10 decision makers and the eigenvector generated group priorities comprise the 11 data points. Individual decision makers are represented by their respective numbers and the group is indicated by "+."

With regards to Skills, the decision makers can be grouped into three clusters. The first cluster consists of Decision Makers 4,5, 7, and 9. Each of these decision makers gave Tactical the highest priority. The second cluster consists of Decision Makers 1, 2, and 10. These decision makers gave Conceptual the highest priority. Decision Makers 3, 6, and 8 all gave the highest priority to Interpersonal. They comprise the third cluster. No decision maker displayed preferences in great contrast to the other nine members of the group.
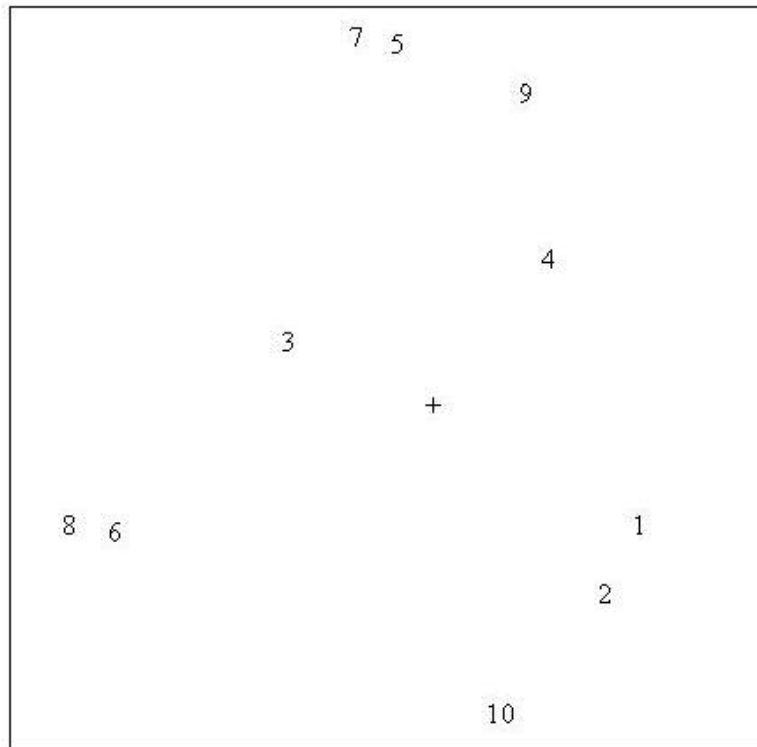
Figure A.1: Skills Sammon Map

A.1.2 Actions Sammon Map

The Actions Sammon map provides a two dimensional representation of 11 four-dimensional data points. The four dimensions represent the Contribution to Conflict, Responsibility, Success, and Timespan subcriteria. The individual priorities

provided by the 10 decision makers and the eigenvector generated group priorities comprise the 11 data points. Individual decision makers are represented by their respective numbers and the group is indicated by "+."

With regards to Actions, the decision makers can be grouped into one large cluster and two smaller clusters. The largest cluster consists of Decision Makers 1, 2, 3, 7, and 8; all of who gave Success the highest priority. Decision Maker 9 felt that Success and Responsibility were equally important. His location in the Sammon Map is nearly midway between the first cluster and Decision Maker 5, the only other decision maker who gave Responsibility the highest priority. Decision Makers 4 and 6 form the last cluster and both gave Conflict to Contribution the highest priority.
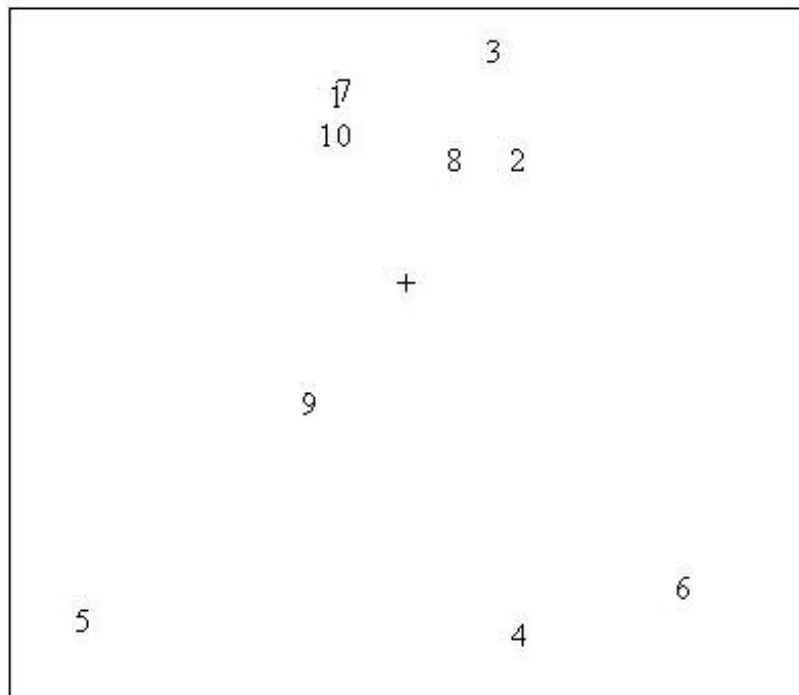
Figure A.2: Actions Sammon Map

A.2 Individual Alternative Ratings

The tables in this section contain the decision makers' complete individual alternative ratings, by subcriterion. The possible ratings were Superior (S), Very Good (VG), Good (G), and Poor (P).

A.2.1 Conceptual Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	G	G	VG	G	G	G	VG	G	G	VG
Eisenhower	S	G	VG	VG	VG	S	S	VG	S	S
MacArthur	S	S	G	VG	P	S	VG	G	S	S
Marshall	S	VG	VG	S	S	S	VG	S	S	S
Patton	VG	VG	G	VG	VG	VG	VG	G	G	G
Pershing	VG	G	VG	VG	VG	G	S	G	S	VG
Ridgway	VG	VG	VG	G	S	VG	VG	VG	VG	S

A.2.2 Interpersonal Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	VG	G	VG	G	G	VG	VG	G	VG	VG
Eisenhower	S	S	S	S	S	S	S	VG	S	S
MacArthur	G	G	VG	VG	P	VG	G	G	G	G
Marshall	S	S	VG	S	VG	VG	G	S	S	S
Patton	P	VG	P	G	G	VG	G	P	P	P
Pershing	S	VG	VG	VG	VG	VG	G	VG	G	S
Ridgway	G	G	G	G	S	VG	VG	G	VG	VG

A.2.3   Tactical Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	G	G	VG	VG	G	G	G	G	G	S
Eisenhower	G	P	VG	G	G	G	P	G	G	G
MacArthur	VG	VG	S	VG	G	G	G	VG	S	S
Marshall	G	VG	G	VG	G	VG	VG	G	VG	VG
Patton	S	S	S	G	S	S	S	S	S	VG
Pershing	VG	P	VG	G	G	VG	G	G	G	VG
Ridgway	S	S	S	VG	S	S	S	VG	VG	S

A.2.4   Technical Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	G	G	VG	G	G	S	G	VG	VG	VG
Eisenhower	VG	S	S	VG	VG	S	VG	S	S	G
MacArthur	S	S	G	VG	G	VG	G	VG	S	VG
Marshall	VG	S	S	S	S	VG	G	VG	S	S
Patton	VG	VG	G	VG	G	S	S	G	S	VG
Pershing	VG	VG	VG	G	VG	G	G	VG	S	VG
Ridgway	VG	S	G	G	S	S	VG	VG	VG	S

A.2.5 Contribution to Conflict Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	G	G	VG	G	G	G	VG	G	S	S
Eisenhower	S	S	S	VG	VG	S	VG	VG	S	S
MacArthur	VG	S	S	VG	VG	S	G	VG	P	VG
Marshall	S	S	VG	S	S	S	S	S	S	S
Patton	VG	G	VG	G	VG	VG	G	G	P	G
Pershing	S	VG	S	S	S	S	VG	VG	G	G
Ridgway	VG	VG	G	G	VG	VG	S	G	VG	VG

A.2.6 Responsibility Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	VG	G	VG	VG	VG	G	VG	G	S	S
Eisenhower	S	S	S	S	S	S	S	VG	S	S
MacArthur	S	S	S	S	S	S	G	VG	S	VG
Marshall	S	S	S	S	S	S	VG	S	S	S
Patton	VG	G	G	G	G	G	VG	G	S	4
Pershing	S	VG	S	S	S	S	S	VG	S	S
Ridgway	G	S	G	VG	VG	VG	S	G	S	S

A.2.7 Success Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	VG	VG	VG	G	VG	G	VG	VG	VG	S
Eisenhower	S	S	S	VG	VG	S	G	VG	S	S
MacArthur	G	VG	G	VG	P	S	G	VG	S	S
Marshall	S	S	S	S	S	S	S	S	S	S
Patton	VG	S	VG	VG	S	VG	VG	VG	S	G
Pershing	VG	G	S	VG	VG	VG	S	VG	VG	VG
Ridgway	VG	VG	VG	VG	S	VG	S	VG	VG	VG

A.2.8 Timespan Ratings

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	VG	G	VG	G	G	VG	VG	G	VG	VG
Eisenhower	VG	G	VG	G	G	VG	VG	VG	VG	VG
MacArthur	S	S	S	S	S	S	S	S	S	S
Marshall	VG	VG	VG	VG	S	S	S	VG	VG	S
Patton	VG	G	VG	G	VG	G	VG	G	VG	G
Pershing	VG	G	G	S	S	G	S	G	VG	VG
Ridgway	VG	VG	VG	G	VG	G	VG	G	VG	VG

A.3 Group Alternative Ratings

The tables in this section contain the three sets of group alternative ratings

A.3.1 Skills Alt-10 Alternative Ratings

	Conceptual	Interpersonal	Tactical	Technical
Bradley	0.4178	0.4930	0.4585	0.5086
Eisenhower	0.7715	0.9593	0.3400	0.7715
MacArthur	0.7076	0.4039	0.6401	0.6401
Marshall	0.8780	0.8122	0.4930	0.8122
Patton	0.4930	0.3235	0.8936	0.6401
Pershing	0.5994	0.6244	0.4290	0.5587
Ridgway	0.6495	0.5086	0.8780	0.7058

A.3.2 Actions Alt-10 Alternative Ratings

	Contribution	Responsibility	Success	Timespan
Bradley	0.5242	0.5994	0.5838	0.4930
Eisenhower	0.8373	0.9593	0.8122	0.5180
MacArthur	0.6513	0.8529	0.6012	1.0000
Marshall	0.9593	0.9593	1.0000	0.7559
Patton	0.4290	0.4446	0.6902	0.4679
Pershing	0.7465	0.9186	0.6495	0.6150
Ridgway	0.5587	0.6807	0.6746	0.5180

A.3.3 Skills Alt-8 Alternative Ratings

	Conceptual	Interpersonal	Tactical	Technical
Bradley	0.4052	0.4992	0.4052	0.4679
Eisenhower	0.7966	1.0000	0.3253	0.7966
MacArthur	0.7340	0.4052	0.6322	0.6322
Marshall	0.8983	0.8474	0.4992	0.8474
Patton	0.4992	0.3047	0.9492	0.6322
Pershing	0.5814	0.6127	0.4366	0.5306
Ridgway	0.6440	0.4679	0.8983	0.7144

A.3.4 Actions Alt-8 Alternative Ratings

	Contribution	Responsibility	Success	Timespan
Bradley	0.4874	0.5814	0.5619	0.4992
Eisenhower	0.8474	1.0000	0.8474	0.5306
MacArthur	0.6636	0.8983	0.6009	1.0000
Marshall	1.0000	1.0000	1.0000	0.7457
Patton	0.4366	0.4052	0.6949	0.4679
Pershing	0.7653	0.9492	0.6440	0.6009
Ridgway	0.5306	0.6831	0.6440	0.5306

A.3.5 Skills Alt-6 Alternative Ratings

	Conceptual	Interpersonal	Tactical	Technical
Bradley	0.3844	0.5097	0.3844	0.4679
Eisenhower	0.7966	1.0000	0.3426	0.7966
MacArthur	0.7548	0.3844	0.6192	0.6192
Marshall	0.9322	0.8644	0.5097	0.8644
Patton	0.5097	0.2734	1.0000	0.6192
Pershing	0.5514	0.5932	0.4261	0.5514
Ridgway	0.5932	0.4679	0.9322	0.7288

A.3.6 Actions Alt-6 Alternative Ratings

	Contribution	Responsibility	Success	Timespan
Bradley	0.4261	0.5512	0.5932	0.5097
Eisenhower	0.8644	1.0000	0.8644	0.5514
MacArthur	0.6610	0.9322	0.5775	1.0000
Marshall	1.0000	1.0000	1.0000	0.7288
Patton	0.4261	0.3844	0.6610	0.4679
Pershing	0.7966	1.0000	0.5932	0.5775
Ridgway	0.5514	0.3870	0.5932	0.5514

A.4 Individual AHP Results

A.4.1 Individual AHP Priorities

- Decision Maker 1

	Priority
MacArthur	0.8742
Eisenhower	0.8001
Marshall	0.8001
Patton	0.6255
Ridgway	0.6238
Pershing	0.6218
Bradley	0.3794

- Decision Maker 2

	Priority
Marshall	0.9569
Eisenhower	0.9287
MacArthur	0.7668
Patton	0.7365
Ridgway	0.6377
Bradley	0.4813
Pershing	0.4263

- Decision Maker 3

	Priority
Eisenhower	0.9156
Pershing	0.8457
Marshall	0.8218
MacArthur	0.6019
Bradley	0.5932
Patton	0.5646
Ridgway	0.5456

- Decision Maker 4

	Priority
Marshall	0.8999
Pershing	0.6744
MacArthur	0.6661
Eisenhower	0.6050
Ridgway	0.4524
Patton	0.4245
Bradley	0.4338

- Decision Maker 5

	Priority
Ridgway	0.8286
Marshall	0.7447
Pershing	0.6806
Eisenhower	0.6692
Patton	0.6494
MacArthur	0.5618
Bradley	0.4433

- Decision Maker 6

	Priority
Eisenhower	0.9694
Marshall	0.9222
MacArthur	0.9187
Pershing	0.7828
Ridgway	0.5975
Patton	0.5714
Bradley	0.4119

- Decision Maker 7

	Priority
Ridgway	0.8850
Patton	0.7883
Marshall	0.5912
Pershing	0.4777
Eisenhower	0.4556
Bradley	0.4423
MacArthur	0.3602

- Decision Maker 8

	Priority
Marshall	0.9326
Eisenhower	0.6214
Pershing	0.5387
Ridgway	0.4410
MacArthur	0.4145
Bradley	0.3879
Patton	0.3147

- Decision Maker 9

	Priority
MacArthur	0.8993
Marshall	0.8592
Patton	0.8212
Eisenhower	0.7801
Ridgway	0.6665
Pershing	0.6213
Bradley	0.6079

- Decision Maker 10

	Priority
Marshall	0.9980
Eisenhower	0.9735
Bradley	0.9376
MacArthur	0.8578
Ridgway	0.7165
Pershing	0.6607
Patton	0.3164

A.4.2 Individual AHP Rankings

The table below is a summary of the results contained in A.4.1. It displays each general's ranking with respect to each decision maker.

Decision Maker

	1	2	3	4	5	6	7	8	9	10
Bradley	7	6	5	7	7	7	6	6	7	3
Eisenhower	2	2	1	4	4	1	5	2	4	2
MacArthur	1	3	4	3	6	3	7	5	1	4
Marshall	2	1	3	1	2	2	3	1	2	1
Patton	4	4	6	6	5	6	2	7	3	7
Pershing	6	7	2	2	3	4	4	3	6	6
Ridgway	5	5	7	5	1	5	1	4	5	5

Appendix B

Interval Linear Programming Appendix

This section contains the linear programming formulations used to determine the ILP-10, ILP-8,and ILP-6 priorities. Stage 0 is not required in calculating the top-level priorities as 2×2 interval pairwise comparison matrices are completely consistent. This ensures a non-empty solution set for Stages 1 and 2.

B.1   Interval Linear Programming -10 (ILP-10)

B.1.1   Top-level ILP-10 Formulations and Output

- Top-level ILP-10 Pairwise Comparison Matrix

	SK	AC
SK	1	[1/9,7]
AC	[1/7,9]	1

- Top-level ILP-10 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$$x_1 - x_2 - y_{12} = -0.1257 \qquad\qquad x_1 - x_2 + g_{12} \geq -2.1972$$

$$x_2 - x_1 - y_{21} = 0.1257 \qquad\qquad x_1 - x_2 - g_{12} \leq 1.9459$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{12} - y_{21} \geq 0 \qquad\qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

- Top-level ILP-10 Stage 2 Formulation

Min $\quad z_{max}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0$$

$$x_1 - x_2 - y_{12} = -0.1257 \qquad x_1 - x_2 + g_{12} \geq -2.1972$$

$$x_2 - x_1 - y_{21} = 0.1257 \qquad x_1 - x_2 - g_{12} \leq 1.9459$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{12} - y_{21} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

$$z_{max} \geq z_{ij} \quad \forall i,j$$

- Top-level ILP-10 Output

	Obj.	x_1	x_2
Stage 1	0	0	0.1257
Stage 2	0	0	0.1257

- Top-level ILP-10 Priorities

	Priority
Skills	0.4686
Actions	0.5314

B.1.2   Skills ILP-10 Formulations and Output

- Skills ILP-10 Pairwise Comparison Matrix

	CO	IN	TA	TE
CO	1	[1/5,9]	[1/8,9]	[1/2,7]
IN	[1/9,5]	1	[1/9,9]	[1/7,6]
TA	[1/9,8]	[1/9,9]	1	[1/5,9]
TE	[1/7,2]	[1/6,7]	[1/9,5]	1

- Skills ILP-10 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = 0.2939 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$
$$x_1 - x_3 - y_{13} = 0.0589 \qquad x_1 - x_3 + g_{13} \geq -2.0794$$
$$x_1 - x_4 - y_{14} = 0.6264 \qquad x_1 - x_4 + g_{14} \geq -0.6931$$
$$x_2 - x_3 - y_{23} = 0 \qquad x_2 - x_3 + g_{23} \geq -2.1972$$
$$x_2 - x_4 - y_{24} = -0.0771 \qquad x_2 - x_4 + g_{24} \geq -1.9459$$
$$x_3 - x_4 - y_{34} = 0.2939 \qquad x_3 - x_4 + g_{34} \geq -1.6094$$
$$x_2 - x_1 - y_{21} = -0.2939 \qquad x_1 - x_2 - g_{12} \leq 2.1972$$
$$x_3 - x_1 - y_{31} = -0.0589 \qquad x_1 - x_3 - g_{13} \leq 2.1972$$
$$x_4 - x_1 - y_{41} = -0.6264 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$
$$x_3 - x_2 - y_{32} = 0 \qquad x_2 - x_3 - g_{23} \leq 2.1972$$
$$x_4 - x_2 - y_{42} = 0.0771 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$
$$x_4 - x_3 - y_{43} = -0.2939 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Skills ILP-10 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$x_1 - x_2 - y_{12} = 0.2939$ $x_1 - x_2 + g_{12} \geq -1.6094$

$x_1 - x_3 - y_{13} = 0.0589$ $x_1 - x_3 + g_{13} \geq -2.0794$

$x_1 - x_4 - y_{14} = 0.6264$ $x_1 - x_4 + g_{14} \geq -0.6931$

$x_2 - x_3 - y_{23} = 0$ $x_2 - x_3 + g_{23} \geq -2.1972$

$x_2 - x_4 - y_{24} = -0.0771$ $x_2 - x_4 + g_{24} \geq -1.9459$

$x_3 - x_4 - y_{34} = 0.2939$ $x_3 - x_4 + g_{34} \geq -1.6094$

$x_2 - x_1 - y_{21} = -0.2939$ $x_1 - x_2 - g_{12} \leq 2.1972$

$x_3 - x_1 - y_{31} = -0.0589$ $x_1 - x_3 - g_{13} \leq 2.1972$

$x_4 - x_1 - y_{41} = -0.6264$ $x_1 - x_4 - g_{14} \leq 1.9459$

$x_3 - x_2 - y_{32} = 0$ $x_2 - x_3 - g_{23} \leq 2.1972$

$x_4 - x_2 - y_{42} = 0.0771$ $x_2 - x_4 - g_{24} \leq 1.7918$

$x_4 - x_3 - y_{43} = -0.2939$ $x_3 - x_4 - g_{34} \leq 2.1972$

$z_{12} - y_{12} \geq 0$ $z_{ij} \geq 0 \ \forall i, j$

$z_{13} - y_{13} \geq 0$ $g_{ij} \geq 0 \ \forall i, j$

$z_{14} - y_{14} \geq 0$

$z_{23} - y_{23} \geq 0$

$z_{24} - y_{24} \geq 0$

$z_{34} - y_{34} \geq 0$

$z_{12} - y_{21} \geq 0$

$z_{13} - y_{31} \geq 0$

$z_{14} - y_{41} \geq 0$

$z_{23} - y_{32} \geq 0$

$z_{24} - y_{42} \geq 0$

$z_{34} - y_{43} \geq 0$

- Skills ILP-10 Stage 2 Formulation

Min $\quad z_{\max}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0.683199$$

$$x_1 - x_2 - y_{12} = 0.2939 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_1 - x_3 - y_{13} = 0.0589 \qquad x_1 - x_3 + g_{13} \geq -2.0794$$

$$x_1 - x_4 - y_{14} = 0.6264 \qquad x_1 - x_4 + g_{14} \geq -0.6931$$

$$x_2 - x_3 - y_{23} = 0 \qquad\qquad x_2 - x_3 + g_{23} \geq -2.1972$$

$$x_2 - x_4 - y_{24} = -0.0771 \qquad x_2 - x_4 + g_{24} \geq -1.9459$$

$$x_3 - x_4 - y_{34} = 0.2939 \qquad x_3 - x_4 + g_{34} \geq -1.6094$$

$$x_2 - x_1 - y_{21} = -0.2939 \qquad x_1 - x_2 - g_{12} \leq 2.1972$$

$$x_3 - x_1 - y_{31} = -0.0589 \qquad x_1 - x_3 - g_{13} \leq 2.1972$$

$$x_4 - x_1 - y_{41} = -0.6264 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$

$$x_3 - x_2 - y_{32} = 0 \qquad\qquad x_2 - x_3 - g_{23} \leq 2.1972$$

$$x_4 - x_2 - y_{42} = 0.0771 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$

$$x_4 - x_3 - y_{43} = -0.2939 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{\max} \geq z_{ij} \quad \forall i,j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Skills ILP-10 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	0.6264	0.3325	0.3325	0
Stage 1	0.6832	0.6264	0.3325	0.3325	0
Stage 2	0.3710	0.6264	0.2939	0.2939	0

- Skills ILP-10 Priorities

	Priority
Conceptual	0.3368
Interpersonal	0.2416
Tactical	0.2416
Technical	0.1800

B.1.3 Actions ILP-10 Formulations and Output

• Actions ILP-10 Pairwise Comparison Matrix

	CC	RE	SU	TS
CC	1	[1/5,5]	[1/7,5]	[1,7]
RE	[1/5 ,5]	1	[1/7,5]	[1/3,9]
SU	[1/5,7]	[1/5,7]	1	[3,9]
TS	[1/7,1]	[1/9,3]	[1/9,1/3]	1

• Actions ILP-10 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = 0 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$
$$x_1 - x_3 - y_{13} = -0.1682 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$
$$x_1 - x_4 - y_{14} = 0.9730 \qquad x_1 - x_4 + g_{14} \geq 0$$
$$x_2 - x_3 - y_{23} = -0.1682 \qquad x_2 - x_3 + g_{23} \geq -1.9459$$
$$x_2 - x_4 - y_{24} = 0.5493 \qquad x_2 - x_4 + g_{24} \geq -1.0986$$
$$x_3 - x_4 - y_{34} = 1.6479 \qquad x_3 - x_4 + g_{34} \geq 1.0986$$
$$x_2 - x_1 - y_{21} = 0 \qquad x_1 - x_2 - g_{12} \leq 1.6094$$
$$x_3 - x_1 - y_{31} = 0.1682 \qquad x_1 - x_3 - g_{13} \leq 1.6094$$
$$x_4 - x_1 - y_{41} = -0.9730 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$
$$x_3 - x_2 - y_{32} = 0.1682 \qquad x_2 - x_3 - g_{23} \leq 1.6094$$
$$x_4 - x_2 - y_{42} = -0.5493 \qquad x_2 - x_4 - g_{24} \leq 2.1972$$
$$x_4 - x_3 - y_{43} = -1.6479 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Actions ILP-10 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$x_1 - x_2 - y_{12} = 0$ $x_1 - x_2 + g_{12} \geq -1.6094$

$x_1 - x_3 - y_{13} = -0.1682$ $x_1 - x_3 + g_{13} \geq -1.9459$

$x_1 - x_4 - y_{14} = 0.9730$ $x_1 - x_4 + g_{14} \geq 0$

$x_2 - x_3 - y_{23} = -0.1682$ $x_2 - x_3 + g_{23} \geq -1.9459$

$x_2 - x_4 - y_{24} = 0.5493$ $x_2 - x_4 + g_{24} \geq -1.0986$

$x_3 - x_4 - y_{34} = 1.6479$ $x_3 - x_4 + g_{34} \geq 1.0986$

$x_2 - x_1 - y_{21} = 0$ $x_1 - x_2 - g_{12} \leq 1.6094$

$x_3 - x_1 - y_{31} = 0.1682$ $x_1 - x_3 - g_{13} \leq 1.6094$

$x_4 - x_1 - y_{41} = -0.9730$ $x_1 - x_4 - g_{14} \leq 1.9459$

$x_3 - x_2 - y_{32} = 0.1682$ $x_2 - x_3 - g_{23} \leq 1.6094$

$x_4 - x_2 - y_{42} = -0.5493$ $x_2 - x_4 - g_{24} \leq 2.1972$

$x_4 - x_3 - y_{43} = -1.6479$ $x_3 - x_4 - g_{34} \leq 2.1972$

$z_{12} - y_{12} \geq 0$ $z_{ij} \geq 0 \quad \forall i,j$

$z_{13} - y_{13} \geq 0$ $g_{ij} \geq 0 \quad \forall i,j$

$z_{14} - y_{14} \geq 0$

$z_{23} - y_{23} \geq 0$

$z_{24} - y_{24} \geq 0$

$z_{34} - y_{34} \geq 0$

$z_{12} - y_{21} \geq 0$

$z_{13} - y_{31} \geq 0$

$z_{14} - y_{41} \geq 0$

$z_{23} - y_{32} \geq 0$

$z_{24} - y_{42} \geq 0$

$z_{34} - y_{43} \geq 0$

- Actions ILP-10 Stage 2 Formulation

Min z_{max}

s.t.

$$\sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3} \sum_{j=i+1}^{4} z_{ij} = 1.4371$$

$$x_1 - x_2 - y_{12} = 0 \qquad\qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_1 - x_3 - y_{13} = -0.1682 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$

$$x_1 - x_4 - y_{14} = 0.9730 \qquad x_1 - x_4 + g_{14} \geq 0$$

$$x_2 - x_3 - y_{23} = -0.1682 \qquad x_2 - x_3 + g_{23} \geq -1.9459$$

$$x_2 - x_4 - y_{24} = 0.5493 \qquad x_2 - x_4 + g_{24} \geq -1.0986$$

$$x_3 - x_4 - y_{34} = 1.6479 \qquad x_3 - x_4 + g_{34} \geq 1.0986$$

$$x_2 - x_1 - y_{21} = 0 \qquad\qquad x_1 - x_2 - g_{12} \leq 1.6094$$

$$x_3 - x_1 - y_{31} = 0.1682 \qquad x_1 - x_3 - g_{13} \leq 1.6094$$

$$x_4 - x_1 - y_{41} = -0.9730 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$

$$x_3 - x_2 - y_{32} = 0.1682 \qquad x_2 - x_3 - g_{23} \leq 1.6094$$

$$x_4 - x_2 - y_{42} = -0.5493 \qquad x_2 - x_4 - g_{24} \leq 2.1972$$

$$x_4 - x_3 - y_{43} = -1.6479 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i, j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i, j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{max} \geq z_{ij} \quad \forall i, j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Actions ILP-10 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	1.4797	1.4797	1.6479	0
Stage 1	1.4371	1.4797	1.4797	1.6479	0
Stage 2	0.9304	1.4797	1.4797	1.6479	0

- Actions ILP-10 Priorities

	Priority
Contribution to Conflict	0.2932
Responsibility	0.2932
Success	0.3469
Timespan	0.0668

B.2 Interval Linear Programming -8 (ILP-8)

ILP-8 removed from consideration the highest and lowest a_{ij} values (outliers) when creating the intervals $[l_{ij}, u_{ij}]$. ILP-8 therefore determined the optimal priority vector on the interval bounds created by the remaining eight a_{ij} values. Determining interval bounds after removing outlying data provides a more accurate depiction of the group's preferences.

B.2.1 Top-level ILP-8 Formulations and Outputs

- Top-level ILP-8 Pairwise Comparison Matrix

	SK	AC
SK	1	[1/7,6]
AC	[1/6,7]	1

- Top-level ILP-8 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$$x_1 - x_2 - y_{12} = -0.0771 \qquad x_1 - x_2 + g_{12} \geq -1.9459$$

$$x_2 - x_1 - y_{21} = 0.0771 \qquad x_1 - x_2 - g_{12} \leq 1.7918$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{12} - y_{21} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

- Top-level ILP-8 Stage 2 Formulation

Min z_{max}

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0$$

$$x_1 - x_2 - y_{12} = -0.0771 \qquad x_1 - x_2 + g_{12} \geq -1.9459$$

$$x_2 - x_1 - y_{21} = 0.0771 \qquad x_1 - x_2 - g_{12} \leq 1.7918$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i, j$$

$$z_{12} - y_{21} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i, j$$

$$z_{max} \geq z_{ij} \quad \forall i, j$$

- Top-level ILP-8 Output

	Obj.	x_1	x_2
Stage 1	0	0	0.0771
Stage 2	0	0	0.0771

- Top-level ILP-8 Priorities

	Priority
Skills	0.4807
Actions	0.5193

B.2.2 Skills ILP-8 Formulations and Outputs

- Skills ILP-8 Pairwise Comparison Matrix

	CO	IN	TA	TE
CO	1	[1/5,8]	[1/7,7]	[1,3]
IN	[1/8,5]	1	[1/6,8]	[1/6,6]
TA	[1/7,7]	[1/8,6]	1	[1/3,7]
TE	[1/3,1]	[1/6,6]	[1/7,3]	1

- Skills ILP-8 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = 0.2350 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$
$$x_1 - x_3 - y_{13} = 0 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$
$$x_1 - x_4 - y_{14} = 0.5493 \qquad x_1 - x_4 + g_{14} \geq 0$$
$$x_2 - x_3 - y_{23} = 0.1438 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$
$$x_2 - x_4 - y_{24} = 0 \qquad x_2 - x_4 + g_{24} \geq -1.7918$$
$$x_3 - x_4 - y_{34} = 0.4236 \qquad x_3 - x_4 + g_{34} \geq -1.0986$$
$$x_2 - x_1 - y_{21} = -0.2350 \qquad x_1 - x_2 - g_{12} \leq 2.0794$$
$$x_3 - x_1 - y_{31} = 0 \qquad x_1 - x_3 - g_{13} \leq 1.9459$$
$$x_4 - x_1 - y_{41} = -0.5493 \qquad x_1 - x_4 - g_{14} \leq 1.0986$$
$$x_3 - x_2 - y_{32} = -0.1438 \qquad x_2 - x_3 - g_{23} \leq 2.0794$$
$$x_4 - x_2 - y_{42} = 0 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$
$$x_4 - x_3 - y_{43} = -0.4236 \qquad x_3 - x_4 - g_{34} \leq 1.9459$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Skills ILP-8 Stage 1 Formulation

Min $\quad \displaystyle\sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$x_1 - x_2 - y_{12} = 0.2350$ $\qquad x_1 - x_2 + g_{12} \geq -1.6094$

$x_1 - x_3 - y_{13} = 0$ $\qquad x_1 - x_3 + g_{13} \geq -1.9459$

$x_1 - x_4 - y_{14} = 0.5493$ $\qquad x_1 - x_4 + g_{14} \geq 0$

$x_2 - x_3 - y_{23} = 0.1438$ $\qquad x_2 - x_3 + g_{23} \geq -1.7918$

$x_2 - x_4 - y_{24} = 0$ $\qquad x_2 - x_4 + g_{24} \geq -1.7918$

$x_3 - x_4 - y_{34} = 0.4236$ $\qquad x_3 - x_4 + g_{34} \geq -1.0986$

$x_2 - x_1 - y_{21} = -0.2350$ $\qquad x_1 - x_2 - g_{12} \leq 2.0794$

$x_3 - x_1 - y_{31} = 0$ $\qquad x_1 - x_3 - g_{13} \leq 1.9459$

$x_4 - x_1 - y_{41} = -0.5493$ $\qquad x_1 - x_4 - g_{14} \leq 1.0986$

$x_3 - x_2 - y_{32} = -0.1438$ $\qquad x_2 - x_3 - g_{23} \leq 2.0794$

$x_4 - x_2 - y_{42} = 0$ $\qquad x_2 - x_4 - g_{24} \leq 1.7918$

$x_4 - x_3 - y_{43} = -0.4236$ $\qquad x_3 - x_4 - g_{34} \leq 1.9459$

$z_{12} - y_{12} \geq 0$ $\qquad z_{ij} \geq 0 \quad \forall i, j$

$z_{13} - y_{13} \geq 0$ $\qquad g_{ij} \geq 0 \quad \forall i, j$

$z_{14} - y_{14} \geq 0$

$z_{23} - y_{23} \geq 0$

$z_{24} - y_{24} \geq 0$

$z_{34} - y_{34} \geq 0$

$z_{12} - y_{21} \geq 0$

$z_{13} - y_{31} \geq 0$

$z_{14} - y_{41} \geq 0$

$z_{23} - y_{32} \geq 0$

$z_{24} - y_{42} \geq 0$

$z_{34} - y_{43} \geq 0$

- Skills ILP-8 Stage 2 Formulation

Min z_{max}

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 1.1993$$

$$x_1 - x_2 - y_{12} = 0.2350 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_1 - x_3 - y_{13} = 0 \qquad\qquad x_1 - x_3 + g_{13} \geq -1.9459$$

$$x_1 - x_4 - y_{14} = 0.5493 \qquad x_1 - x_4 + g_{14} \geq 0$$

$$x_2 - x_3 - y_{23} = 0.1438 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$

$$x_2 - x_4 - y_{24} = 0 \qquad\qquad x_2 - x_4 + g_{24} \geq -1.7918$$

$$x_3 - x_4 - y_{34} = 0.4236 \qquad x_3 - x_4 + g_{34} \geq -1.0986$$

$$x_2 - x_1 - y_{21} = -0.2350 \qquad x_1 - x_2 - g_{12} \leq 2.0794$$

$$x_3 - x_1 - y_{31} = 0 \qquad\qquad x_1 - x_3 - g_{13} \leq 1.9459$$

$$x_4 - x_1 - y_{41} = -0.5493 \qquad x_1 - x_4 - g_{14} \leq 1.0986$$

$$x_3 - x_2 - y_{32} = -0.1438 \qquad x_2 - x_3 - g_{23} \leq 2.0794$$

$$x_4 - x_2 - y_{42} = 0 \qquad\qquad x_2 - x_4 - g_{24} \leq 1.7918$$

$$x_4 - x_3 - y_{43} = -0.4236 \qquad x_3 - x_4 - g_{34} \leq 1.9459$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{\text{max}} \geq z_{ij} \quad \forall i,j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Skills ILP-8 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	0.8024	0.5674	0.4236	0
Stage 1	1.1993	0.8024	0.5674	0.4236	0
Stage 2	0.5674	0.8024	0.5674	0.4236	0

- Skills ILP-8 Priorities

	Priority
Conceptual	0.3421
Interpersonal	0.2704
Tactical	0.2342
Technical	0.1533

B.2.3 Actions ILP-8 Formulations and Outputs

- Actions ILP-8 Pairwise Comparison Matrix

	CC	RE	SU	TS
CC	1	[1/5,4]	[1/7,4]	[1,7]
RE	[1/4,5]	1	[1/6,3]	[2,7]
SU	[1/4,7]	[1/3,6]	1	[4,9]
TS	[1/7,1]	[1/7,1/2]	[1/9,1/4]	1

- Actions ILP-8 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = -0.1116 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$
$$x_1 - x_3 - y_{13} = -0.2798 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$
$$x_1 - x_4 - y_{14} = 0.9730 \qquad x_1 - x_4 + g_{14} \geq 0$$
$$x_2 - x_3 - y_{23} = -0.3466 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$
$$x_2 - x_4 - y_{24} = 1.3195 \qquad x_2 - x_4 + g_{24} \geq 0.6931$$
$$x_3 - x_4 - y_{34} = 1.7918 \qquad x_3 - x_4 + g_{34} \geq 1.3863$$
$$x_2 - x_1 - y_{21} = 0.1116 \qquad x_1 - x_2 - g_{12} \leq 1.3863$$
$$x_3 - x_1 - y_{31} = 0.2798 \qquad x_1 - x_3 - g_{13} \leq 1.3863$$
$$x_4 - x_1 - y_{41} = -0.9730 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$
$$x_3 - x_2 - y_{32} = 0.3466 \qquad x_2 - x_3 - g_{23} \leq 1.0986$$
$$x_4 - x_2 - y_{42} = -1.3195 \qquad x_2 - x_4 - g_{24} \leq 1.9459$$
$$x_4 - x_3 - y_{43} = -1.7918 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Actions ILP-8 Stage 1 Formulation

Min $\displaystyle\sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$x_1 - x_2 - y_{12} = -0.1116$ $x_1 - x_2 + g_{12} \geq -1.6094$

$x_1 - x_3 - y_{13} = -0.2798$ $x_1 - x_3 + g_{13} \geq -1.9459$

$x_1 - x_4 - y_{14} = 0.9730$ $x_1 - x_4 + g_{14} \geq 0$

$x_2 - x_3 - y_{23} = -0.3466$ $x_2 - x_3 + g_{23} \geq -1.7918$

$x_2 - x_4 - y_{24} = 1.3195$ $x_2 - x_4 + g_{24} \geq 0.6931$

$x_3 - x_4 - y_{34} = 1.7918$ $x_3 - x_4 + g_{34} \geq 1.3863$

$x_2 - x_1 - y_{21} = 0.1116$ $x_1 - x_2 - g_{12} \leq 1.3863$

$x_3 - x_1 - y_{31} = 0.2798$ $x_1 - x_3 - g_{13} \leq 1.3863$

$x_4 - x_1 - y_{41} = -0.9730$ $x_1 - x_4 - g_{14} \leq 1.9459$

$x_3 - x_2 - y_{32} = 0.3466$ $x_2 - x_3 - g_{23} \leq 1.0986$

$x_4 - x_2 - y_{42} = -1.3195$ $x_2 - x_4 - g_{24} \leq 1.9459$

$x_4 - x_3 - y_{43} = -1.7918$ $x_3 - x_4 - g_{34} \leq 2.1972$

$z_{12} - y_{12} \geq 0$ $z_{ij} \geq 0 \quad \forall i, j$

$z_{13} - y_{13} \geq 0$ $g_{ij} \geq 0 \quad \forall i, j$

$z_{14} - y_{14} \geq 0$

$z_{23} - y_{23} \geq 0$

$z_{24} - y_{24} \geq 0$

$z_{34} - y_{34} \geq 0$

$z_{12} - y_{21} \geq 0$

$z_{13} - y_{31} \geq 0$

$z_{14} - y_{41} \geq 0$

$z_{23} - y_{32} \geq 0$

$z_{24} - y_{42} \geq 0$

$z_{34} - y_{43} \geq 0$

- Actions ILP-8 Stage 2 Formulation

Min z_{max}

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0.8431001$$

$$x_1 - x_2 - y_{12} = -0.1116 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_1 - x_3 - y_{13} = -0.2798 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$

$$x_1 - x_4 - y_{14} = 0.9730 \qquad x_1 - x_4 + g_{14} \geq 0$$

$$x_2 - x_3 - y_{23} = -0.3466 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$

$$x_2 - x_4 - y_{24} = 1.3195 \qquad x_2 - x_4 + g_{24} \geq 0.6931$$

$$x_3 - x_4 - y_{34} = 1.7918 \qquad x_3 - x_4 + g_{34} \geq 1.3863$$

$$x_2 - x_1 - y_{21} = 0.1116 \qquad x_1 - x_2 - g_{12} \leq 1.3863$$

$$x_3 - x_1 - y_{31} = 0.2798 \qquad x_1 - x_3 - g_{13} \leq 1.3863$$

$$x_4 - x_1 - y_{41} = -0.9730 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$

$$x_3 - x_2 - y_{32} = 0.3466 \qquad x_2 - x_3 - g_{23} \leq 1.0986$$

$$x_4 - x_2 - y_{42} = -1.3195 \qquad x_2 - x_4 - g_{24} \leq 1.9459$$

$$x_4 - x_3 - y_{43} = -1.7918 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{max} \geq z_{ij} \quad \forall i,j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Actions ILP-8 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	1.5120	1.4452	1.7918	0
Stage 1	0.8431	1.5120	1.4452	1.7918	0
Stage 2	0.5390	1.5120	1.4452	1.7918	0

- Actions ILP-8 Priorities

	Priority
Contribution to Conflict	0.2874
Responsibility	0.2689
Success	0.3803
Timespan	0.0634

B.3　Interval Linear Programming -6 (ILP-6)

ILP-6 removed from consideration the two highest and two lowest a_{ij}^{k} values

(outliers) when creating the intervals $[l_{ij}, u_{ij}]$. ILP-6 therefore determined the optimal

priority vector on the interval bounds created by the remaining six a_{ij} values.

Determining interval bounds after removing outlying data provides a more accurate

representation of the group's preferences.

B.3.1　Top-level ILP-6 Formulations and Outputs

- Top-level ILP-6 Pairwise Comparison Matrix

	SK	AC
SK	1	[1/5,5]
AC	[1/5,5]	1

- Top-level ILP-6 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0$$

$$x_1 - x_2 - y_{12} = 0 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_2 - x_1 - y_{21} = 0 \qquad x_1 - x_2 - g_{12} \leq 1.6094$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{12} - y_{21} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i,j$$

- Top-level ILP-6 Stage 2 Formulation

Min z_{max}

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0$$

$$x_1 - x_2 - y_{12} = 0 \qquad x_1 - x_2 + g_{12} \geq -1.6094$$

$$x_2 - x_1 - y_{21} = 0 \qquad x_1 - x_2 - g_{12} \leq 1.6094$$

$$z_{12} - y_{12} \geq 0 \qquad z_{ij} \geq 0 \quad \forall i,j$$

$$z_{12} - y_{21} \geq 0 \qquad g_{ij} \geq 0 \quad \forall i,j$$

$$z_{max} \geq z_{ij} \quad \forall i,j$$

- Top-level ILP-6 Output

	Obj.	x_1	x_2
Stage 1	0	0	0
Stage 2	0	0	0

- Top-level ILP-6 Priorities

	Priority
Skills	0.5000
Actions	0.5000

B.3.2 Skills ILP-6 Formulations and Outputs

- Skills ILP-6 Pairwise Comparison Matrix

	CO	IN	TA	TE
CO	1	[1/4,5]	[1/5,6]	[1,2]
IN	[1/5,4]	1	[1/6,6]	[1/5,5]
TA	[1/6,5]	[1/6,6]	1	[1/3,5]
TE	[1/2,1]	[1/5,5]	[1/5,3]	1

- Skills ILP-6 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = 0.1116 \qquad x_1 - x_2 + g_{12} \geq -1.3863$$
$$x_1 - x_3 - y_{13} = 0.0912 \qquad x_1 - x_3 + g_{13} \geq -1.6094$$
$$x_1 - x_4 - y_{14} = 0.3466 \qquad x_1 - x_4 + g_{14} \geq 0$$
$$x_2 - x_3 - y_{23} = 0 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$
$$x_2 - x_4 - y_{24} = 0 \qquad x_2 - x_4 + g_{24} \geq -1.6094$$
$$x_3 - x_4 - y_{34} = 0.2554 \qquad x_3 - x_4 + g_{34} \geq -1.0986$$
$$x_2 - x_1 - y_{21} = -0.1116 \qquad x_1 - x_2 - g_{12} \leq 1.6094$$
$$x_3 - x_1 - y_{31} = -0.0912 \qquad x_1 - x_3 - g_{13} \leq 1.7918$$
$$x_4 - x_1 - y_{41} = -0.3466 \qquad x_1 - x_4 - g_{14} \leq 0.6931$$
$$x_3 - x_2 - y_{32} = 0 \qquad x_2 - x_3 - g_{23} \leq 1.7918$$
$$x_4 - x_2 - y_{42} = 0 \qquad x_2 - x_4 - g_{24} \leq 1.6094$$
$$x_4 - x_3 - y_{43} = -0.2554 \qquad x_3 - x_4 - g_{34} \leq 1.6094$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Skills ILP-6 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij} = 0$$

$x_1 - x_2 - y_{12} = 0.1116$	$x_1 - x_2 + g_{12} \geq -1.3863$
$x_1 - x_3 - y_{13} = 0.0912$	$x_1 - x_3 + g_{13} \geq -1.6094$
$x_1 - x_4 - y_{14} = 0.3466$	$x_1 - x_4 + g_{14} \geq 0$
$x_2 - x_3 - y_{23} = 0$	$x_2 - x_3 + g_{23} \geq -1.7918$
$x_2 - x_4 - y_{24} = 0$	$x_2 - x_4 + g_{24} \geq -1.6094$
$x_3 - x_4 - y_{34} = 0.2554$	$x_3 - x_4 + g_{34} \geq -1.0986$
$x_2 - x_1 - y_{21} = -0.1116$	$x_1 - x_2 - g_{12} \leq 1.6094$
$x_3 - x_1 - y_{31} = -0.0912$	$x_1 - x_3 - g_{13} \leq 1.7918$
$x_4 - x_1 - y_{41} = -0.3466$	$x_1 - x_4 - g_{14} \leq 0.6931$
$x_3 - x_2 - y_{32} = 0$	$x_2 - x_3 - g_{23} \leq 1.7918$
$x_4 - x_2 - y_{42} = 0$	$x_2 - x_4 - g_{24} \leq 1.6094$
$x_4 - x_3 - y_{43} = -0.2554$	$x_3 - x_4 - g_{34} \leq 1.6094$
$z_{12} - y_{12} \geq 0$	$z_{ij} \geq 0 \quad \forall i, j$
$z_{13} - y_{13} \geq 0$	$g_{ij} \geq 0 \quad \forall i, j$
$z_{14} - y_{14} \geq 0$	
$z_{23} - y_{23} \geq 0$	
$z_{24} - y_{24} \geq 0$	
$z_{34} - y_{34} \geq 0$	
$z_{12} - y_{21} \geq 0$	
$z_{13} - y_{31} \geq 0$	
$z_{14} - y_{41} \geq 0$	
$z_{23} - y_{32} \geq 0$	
$z_{24} - y_{42} \geq 0$	
$z_{34} - y_{43} \geq 0$	

- Skills ILP-6 Stage 2 Formulation

Min $z_{\max}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 0.2962$$

$$x_1 - x_2 - y_{12} = 0.1116 \qquad x_1 - x_2 + g_{12} \geq -1.3863$$

$$x_1 - x_3 - y_{13} = 0.0912 \qquad x_1 - x_3 + g_{13} \geq -1.6094$$

$$x_1 - x_4 - y_{14} = 0.3466 \qquad x_1 - x_4 + g_{14} \geq 0$$

$$x_2 - x_3 - y_{23} = 0 \qquad\qquad x_2 - x_3 + g_{23} \geq -1.7918$$

$$x_2 - x_4 - y_{24} = 0 \qquad\qquad x_2 - x_4 + g_{24} \geq -1.6094$$

$$x_3 - x_4 - y_{34} = 0.2554 \qquad x_3 - x_4 + g_{34} \geq -1.0986$$

$$x_2 - x_1 - y_{21} = -0.1116 \qquad x_1 - x_2 - g_{12} \leq 1.6094$$

$$x_3 - x_1 - y_{31} = -0.0912 \qquad x_1 - x_3 - g_{13} \leq 1.7918$$

$$x_4 - x_1 - y_{41} = -0.3466 \qquad x_1 - x_4 - g_{14} \leq 0.6931$$

$$x_3 - x_2 - y_{32} = 0 \qquad\qquad x_2 - x_3 - g_{23} \leq 1.7918$$

$$x_4 - x_2 - y_{42} = 0 \qquad\qquad x_2 - x_4 - g_{24} \leq 1.6094$$

$$x_4 - x_3 - y_{43} = -0.2554 \qquad x_3 - x_4 - g_{34} \leq 1.6094$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i, j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i, j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{\max} \geq z_{ij} \quad \forall i, j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Skills ILP-6 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	0.3670	0.2554	0.2554	0
Stage 1	0.2962	0.3670	0.2554	0.2554	0
Stage 2	0.2554	0.3670	0.2554	0.2554	0

- Skills ILP-6 Priorities

	Priority
Conceptual	0.2872
Interpersonal	0.2569
Tactical	0.2569
Technical	0.1990

B.3.3 Actions ILP-6 Formulations and Outputs

- Actions ILP-6 Pairwise Comparison Matrix

	CC	RE	SU	TS
CC	1	[1/4,3]	[1/7,1]	[3,7]
RE	[1/3,4]	1	[1/6,1]	[3,6]
SU	[1,7]	[1,6]	1	[4,9]
TS	[1/7,1/3]	[1/6,1/3]	[1/9,1/4]	1

- Actions ILP-6 Stage 0 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij}$$

s.t.

$$x_1 - x_2 - y_{12} = -0.1438 \qquad x_1 - x_2 + g_{12} \geq -1.3863$$
$$x_1 - x_3 - y_{13} = -0.9730 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$
$$x_1 - x_4 - y_{14} = 1.5223 \qquad x_1 - x_4 + g_{14} \geq 1.0986$$
$$x_2 - x_3 - y_{23} = -0.8959 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$
$$x_2 - x_4 - y_{24} = 1.4452 \qquad x_2 - x_4 + g_{24} \geq 1.0986$$
$$x_3 - x_4 - y_{34} = 1.7918 \qquad x_3 - x_4 + g_{34} \geq 1.3863$$
$$x_2 - x_1 - y_{21} = 0.1438 \qquad x_1 - x_2 - g_{12} \leq 1.0986$$
$$x_3 - x_1 - y_{31} = 0.9730 \qquad x_1 - x_3 - g_{13} \leq 0$$
$$x_4 - x_1 - y_{41} = -1.5223 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$
$$x_3 - x_2 - y_{32} = 0.8959 \qquad x_2 - x_3 - g_{23} \leq 0$$
$$x_4 - x_2 - y_{42} = -1.4452 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$
$$x_4 - x_3 - y_{43} = -1.7918 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$
$$g_{ij} \geq 0 \quad \forall i, j$$

- Actions ILP-6 Stage 1 Formulation

$$\text{Min} \quad \sum_{i=1}^{3} \sum_{j=i+1}^{4} z_{ij}$$

s.t.

$$\sum_{i=1}^{3} \sum_{j=i+1}^{4} g_{ij} = 0$$

$$x_1 - x_2 - y_{12} = -0.1438 \qquad x_1 - x_2 + g_{12} \geq -1.3863$$

$$x_1 - x_3 - y_{13} = -0.9730 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$

$$x_1 - x_4 - y_{14} = 1.5223 \qquad x_1 - x_4 + g_{14} \geq 1.0986$$

$$x_2 - x_3 - y_{23} = -0.8959 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$

$$x_2 - x_4 - y_{24} = 1.4452 \qquad x_2 - x_4 + g_{24} \geq 1.0986$$

$$x_3 - x_4 - y_{34} = 1.7918 \qquad x_3 - x_4 + g_{34} \geq 1.3863$$

$$x_2 - x_1 - y_{21} = 0.1438 \qquad x_1 - x_2 - g_{12} \leq 1.0986$$

$$x_3 - x_1 - y_{31} = 0.9730 \qquad x_1 - x_3 - g_{13} \leq 0$$

$$x_4 - x_1 - y_{41} = -1.5223 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$

$$x_3 - x_2 - y_{32} = 0.8959 \qquad x_2 - x_3 - g_{23} \leq 0$$

$$x_4 - x_2 - y_{42} = -1.4452 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$

$$x_4 - x_3 - y_{43} = -1.7918 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad\qquad z_{ij} \geq 0 \quad \forall i, j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad\qquad g_{ij} \geq 0 \quad \forall i, j$$

$$z_{14} - y_{14} \geq 0$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Actions ILP-6 Stage 2 Formulation

Min $z_{\max}$

s.t.

$$\sum_{i=1}^{3}\sum_{j=i+1}^{4} g_{ij} = 0 \qquad\qquad \sum_{i=1}^{3}\sum_{j=i+1}^{4} z_{ij} = 1.0683$$

$$x_1 - x_2 - y_{12} = -0.1438 \qquad x_1 - x_2 + g_{12} \geq -1.3863$$

$$x_1 - x_3 - y_{13} = -0.9730 \qquad x_1 - x_3 + g_{13} \geq -1.9459$$

$$x_1 - x_4 - y_{14} = 1.5223 \qquad x_1 - x_4 + g_{14} \geq 1.0986$$

$$x_2 - x_3 - y_{23} = -0.8959 \qquad x_2 - x_3 + g_{23} \geq -1.7918$$

$$x_2 - x_4 - y_{24} = 1.4452 \qquad x_2 - x_4 + g_{24} \geq 1.0986$$

$$x_3 - x_4 - y_{34} = 1.7918 \qquad x_3 - x_4 + g_{34} \geq 1.3863$$

$$x_2 - x_1 - y_{21} = 0.1438 \qquad x_1 - x_2 - g_{12} \leq 1.0986$$

$$x_3 - x_1 - y_{31} = 0.9730 \qquad x_1 - x_3 - g_{13} \leq 0$$

$$x_4 - x_1 - y_{41} = -1.5223 \qquad x_1 - x_4 - g_{14} \leq 1.9459$$

$$x_3 - x_2 - y_{32} = 0.8959 \qquad x_2 - x_3 - g_{23} \leq 0$$

$$x_4 - x_2 - y_{42} = -1.4452 \qquad x_2 - x_4 - g_{24} \leq 1.7918$$

$$x_4 - x_3 - y_{43} = -1.7918 \qquad x_3 - x_4 - g_{34} \leq 2.1972$$

$$z_{12} - y_{12} \geq 0 \qquad\qquad z_{ij} \geq 0 \quad \forall i, j$$

$$z_{13} - y_{13} \geq 0 \qquad\qquad g_{ij} \geq 0 \quad \forall i, j$$

$$z_{14} - y_{14} \geq 0 \qquad\qquad z_{\max} \geq z_{ij} \quad \forall i, j$$

$$z_{23} - y_{23} \geq 0$$

$$z_{24} - y_{24} \geq 0$$

$$z_{34} - y_{34} \geq 0$$

$$z_{12} - y_{21} \geq 0$$

$$z_{13} - y_{31} \geq 0$$

$$z_{14} - y_{41} \geq 0$$

$$z_{23} - y_{32} \geq 0$$

$$z_{24} - y_{42} \geq 0$$

$$z_{34} - y_{43} \geq 0$$

- Actions ILP-6 Output

	Obj.	x_1	x_2	x_3	x_4
Stage 0	0	1.5223	1.4452	2.1436	0
Stage 1	1.0683	1.5223	1.4452	2.1972	0
Stage 2	0.4054	1.5223	1.4452	2.1972	0

- Actions ILP-6 Priorities

	Priority
Contribution to Conflict	0.2434
Responsibility	0.2254
Success	0.4781
Timespan	0.0531

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