

ABSTRACT

Title of Dissertation: THREE ESSAYS ON VERTICAL PRODUCT
 DIFFERENTIATION: EXCLUSIVITY,
 NON-EXCLUSIVITY AND ADVERTISING

Cesar Costantino, Doctor of Philosophy, 2004

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Since Hotelling's (1929) seminal work, economists have tried to understand how product differentiation affects price competition. I study the product location decisions, on a vertical characteristic space, of two sets of horizontal competitors when the inputs supplied by the "upstream" set (the manufacturers) and the input supplied by the "downstream" set (the retailers) are combined one-to-one to form a final good under the assumption that each manufacturer sells through one retailer exclusively. I find that the final product provided by each manufacturer-retailer pair shows maximum differentiation along one dimension and minimum differentiation along the other (MaxMin equilibrium). I conduct the same analysis under the assumption that each manufacturer sells to any retailer and each retailer buys from any manufacturer. I find a Nash Equilibrium in which each firm differentiates its product completely from its horizontal competitor. Finally, I estimate the effect of advertising on consumer brand choice and search behavior. Under imperfect information, advertising can affect consumer behavior by providing economically relevant information in a convenient way. I find that advertising has an increasing effect on consumers' search effort and on the probability of purchase associated with the featured brand.

THREE ESSAYS ON VERTICAL PRODUCT
DIFFERENTIATION: EXCLUSIVITY,
NON-EXCLUSIVITY AND ADVERTISING

by

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DEDICATION

To Mariana, Milena Lucia and Lucca.

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Chapter 1

Introduction

Since Hotelling's (1929) seminal work, economists have tried to understand how product differentiation affects price competition. In the following two chapters I study the product location decisions, on a vertical characteristic space, of two sets of horizontal competitors when the inputs supplied by the "upstream" set (the manufacturers) and the input supplied by the "downstream" set (the retailers) are combined one-to-one to form a final good.

The previous characterization captures many aspects of the relationship between manufacturers and retailers. Although it can be extended to other sectors, I discuss the results as applied to the new-car market and the groceries one-stop shopping market.

The economic function of any retail system is to provide consumers with a set of distribution services associated with the explicit items or services bought at retail. Typically, consumers cannot buy the distribution services provided by a retailer separately from the good or goods that retailer sells to form a bundle ("mix and match"). Thus, the location of any particular bundle on the product space will depend, among other things, on how do the firms providing the goods and the distribution services affect each other.

I start the analysis in Chapter 2, by studying the product location and pricing strategies of this set of retailers and manufacturers under the assumption that each manufacturer sells through one retailer exclusively. I find that when the affiliated manufacturers and retailers maximize profits independently from each other, the final product provided by each manufacturer-retailer pair does not show maximum differentiation along the product and distribution service vertical dimensions.

In Chapter 3, I conduct the same analysis under the assumption that each manufacturer sells to any retailer and each retailer buys from any manufacturer. I find that a Nash Equilibrium in which each firm differentiates its product completely from its horizontal competitor. The equilibrium mimics the unique equilibrium outcome in a game where each manufacturer and each retailer offer its good to the consumers directly, and then each consumer does the bundling (mix and match) at will.

Finally, in Chapter 4 I estimate the effect of advertising a set of brands and their prices on consumer brand choice and search behavior. Under imperfect information, advertising can affect consumer behavior by providing economically relevant information like price and other product characteristics in a convenient way. I find that advertising has a increasing effect on consumers' search effort and an increasing effect on the probability of purchase associated with the featured brand.

1.1 Vertical Differentiation and Oligopolistic Competition Theory

Two streams of the literature are merged in the models presented in Chapter 2 and Chapter 3: the product differentiation literature and the vertical control/integration literature.

The product differentiation literature distinguishes between vertical differentiation models (vector models in the marketing literature) and horizontal differentiation models (ideal point models in the marketing literature).

The seminal paper in this field is Hotelling (1929). He showed that given a price vector, firms have an incentive to locate in the same position. D'Aspremont et al., (1979) modified the Hotelling model to show that when price competition was considered, profit maximization led to maximal differentiation. They showed that when we let each firm choose its product location, two forces determine the location equilibrium. First, given prices, competition leads the two firms to locate as closely as possible. Second, a strategic force leads the two firms to differentiate from each other in order to diminish price competition.

Shaked and Sutton (1982) extended the analysis to consider vertical product differentiation along one dimension. They proved that both forces are present in the vertical product differentiation models and that they determine equilibria where firms are located at the extreme ends of the vertical dimension support.

While the models mentioned above are one-dimensional models, dePalma et al (1979), Neven and Thisse (1990), and Vandenbosch and Weinberg (1995), among others, have extended the analysis of product differentiation to more than one dimension. They all reach the conclusion that when two dimensions are con-

sidered, the typical result is an equilibrium where firms maximally differentiate themselves along one dimension and minimally differentiate along the other. So in equilibrium they forgo part of the potential rents that could come from further differentiation. This result is pervasive in the literature and has its own name, the "MaxMin" equilibrium.

In particular, Vandenbosch and Weinberg (1995) found that when the range of positioning options on each of the dimensions is not too different, $\frac{81}{128} \leq \frac{(S^{\max}-S^{\min})}{(D^{\max}-D^{\min})} \leq \frac{128}{81}$, one firm locates at the maximum quality in both dimension and the other at the maximum quality in one dimension and at the minimum quality in the other dimension. As expected, the firm located at the highest quality location in both dimensions earns the higher profits.

Again, there are two forces playing against each other in this model: a demand force drawing the firms together and a strategic force that pushes firms to differentiate. Specifically, if $\frac{(S^{\max}-S^{\min})}{(D^{\max}-D^{\min})} \geq \frac{128}{81}$, "...both firms want to have the highest quality, but because the strategic force, only one firm [firm 2] will locate there. The firm, which is unable to choose the highest quality position [firm 1], differentiates its product by choosing the minimum quality on only one dimension because of the demand force. This choice reduces price competition while at the same time maintains a sufficiently high quality level for the differentiating firm's product to appeal to a number of consumers." Firm 1 differentiates in both dimensions whenever the range of positioning options on each of the dimensions is such that $\frac{81}{128} \geq \frac{(S^{\max}-S^{\min})}{(D^{\max}-D^{\min})}$.

The vertical control literature studies the strategic relationship between "upstream" firms possessing monopoly power in an intermediate good market and the "downstream" firms demanding that good. As stated in Tirole (1988):

...[V]ertical relationships among firms are often much richer and more complex than those between a firm and its consumers. Ordinary consumers often just consume the good, but industrial consumers transform the good and/or market it. In other words, some further decisions are made after the intermediate good is sold by the upstream firm. Because these decisions affect its profit, the upstream firm has an incentive to control them. Beyond the pricing policy and the product specification for its good, it will exert further vertical control on downstream operations to the extent that such control is feasible.

The basic vertical framework is made of two firms. The "upstream" firm, a single monopolistic supplier producing an intermediate good, and the "downstream" firm, which is also a monopolist in the final good market. The "downstream" firm has a technology that transforms one unit of the intermediate good into one unit of the final good. These two firms sign a contract specifying the terms of their relationship. The literature on vertical control has focused extensively on studying the implications of different contractual arrangements under this simple environment.

In the next two chapters I study the interaction between a particular vertical arrangement and the location decisions of "upstream" and "downstream" oligopolistic firms.

Chapter 2

Vertical Product Differentiation under Exclusivity: The new-car market

2.1 Introduction

Many manufacturers offer their products through independent dealers with whom they maintain an exclusivity relationship. This is true in the new-car market as well as in those for sewing machines, agricultural machinery, and gasoline.

In these markets, the manufacturer's profit is affected by the dealer's pricing practices because the dealer's mark-up determine the volume of final sales. When the manufacturer and its dealer are monopolies in their own markets, there is a situation in which each side would prefer that the other did not have the power to set its price independently because of double-marginalization.

As in Bresnahan and Reiss (1985), this paper presents a market power explanation of rent distribution between manufacturers and their exclusive dealers. Unlike these authors, here the market power is not assumed but generated through vertical product differentiation along two dimensions, the quality of the car and the quality of the distribution services provided by the dealer, by two

pairs of duopolistic firms.

I show that manufacturers and their dealers can earn higher profits if each dealer is allowed to set its additive mark-up freely. Although this result is only valid within the confines of my assumptions, the intuition behind it can be extended easily to other settings. More generally, double-marginalization can drive profits up if the new-car market is duopolistic instead of monopolistic provided that the total quantity of both final goods offered in the market under total independence is not lower than the quantity offered under horizontal collusion when manufacturers have total control over the final price. McGuire and Staelin (1983) first showed that it was a Nash equilibrium for a set of oligopolistic manufacturers to sell through exclusive but independent dealers when the degree of price competition among them was high, and that in this equilibrium each firm earned a higher profit than under vertical integration. Coughlan (1985) finds a similar result under slightly different assumption and contributes with empirical evidence from the semiconductor industry that suggests that independent dealers help reduce price competition. However, they assumed the degree of price competition while in my model it is the result of firms' location decisions. My results show that going from a market structure where all manufacturer and dealers are vertically integrated like in Vandebosch and Weinberg (1995) to a market structure where the dealers are independent but exclusive does not erode the incentives for product differentiation.

I also show that not taking advantage of all the opportunities for vertical differentiation is the unique Sub-Game Perfect Equilibrium of this game. Firms chose to differentiate along the dimension that promises the highest profits, i.e. the dimension whose range is the widest as in the previous literature, and to

locate in the same point in the other dimension. For example, if the distribution service's quality range is larger than the product's quality range then the dealers differentiate from each other while the manufacturers locate themselves at the same point. The result is similar to the MaxMin equilibria found by dePalma et al., (1979), Neven and Thisse (1990) and Vandenbosch and Weinberg (1995) and it is an original contribution to the literature on vertical differentiation since the MaxMin result has only been found in two-dimensional models where each firm controls both dimensions and the final price.

This result can help explain why vertical differentiation among new-car dealers is not large. Typically, we see significant vertical differentiation among cars but much less vertical differentiation among dealerships: Hyundai and a Mercedes Benz are much more differentiated products than the distribution services provided by their respective dealerships. Data published by DealerRater.com supports this claim. DealerRater.com publishes a dealer quality rating based on spontaneous customer reviews. The rating scale is 0 to 5, being 5 the highest mark a dealer can get from a customer¹.

I regressed the dealers' average ratings on the car brand and state dummies in order to check if the perceived characteristics of each car brand were in any way related to the quality associated with each dealer. I included in the dataset the following car brands: Hyundai, Honda, Ford, Chevrolet, BMW, Mercedes Benz, Acura, Subaru and Toyota. In Table 2.1 I show the regression results for the brand dummies. The state dummies were included, but not shown in the table, in order to control for the effect of different regulatory legislation that could affect the level or quality of distribution services offered by the car dealers.

In Table 2.1, we can see that most of the dummies are not statistically signif-

Table 2.1: Car Brand Quality Vs. Dealer Distribution Service Quality

Variable	LSE		Tobit	
	Coef.	t	Coef.	t
Intercept	2.37	5.30	2.46	4.72
Toyota	-0.12	-0.25	-0.17	-0.31
Subaru	0.86	1.61	1.34	2.07
Mercedes	0.90	2.00	1.08	2.06
Honda	-0.02	-0.03	-0.02	-0.03
Acura	0.32	0.64	0.44	0.75
BMW	0.37	0.82	0.32	0.60
Ford	-0.19	-0.03	-0.03	-0.04
Chevrolet	0.04	0.08	0.11	0.19
Obs.	235		235	
F(43, 191)	1.05			
R2	0.19			
LR chi2(43)			57.05	
Pseudo R2			0.07	

icant at the usual significance levels, with the exception of Subaru and Mercedes Benz (Hyundai is the omitted brand and California is the omitted state). Although the coefficients are statistically significant and Subaru and Mercedes' signs partially support the hypothesis that brand quality might be positively related to dealer quality, their value suggest that the differentiation among dealers is at most weak and not very consistent. On the one hand, only 1.34 points on a 0 to 5 scale separate Hyundai dealers from Mercedes dealers. On the other hand, although our hypothesis imply that Mercedes should be of higher quality than Subaru's, the difference in coefficients is 0.26 points in favor of Subaru. Finally, we can see that BMW dealers are not statistically different from Hyundai dealers.

The model developed here also explains two additional facts: that in higher-quality cars the manufacturer and retailer margins are higher and that there appears to be a nearly proportional relation between manufacturer's margins and dealer's margins across products with different quality content (see Bresnahan and Reiss 1985 for a discussion about these stylized facts).

In order to derive my results, I use a two-stage vertical product differentiation model to study the location equilibria of a set of manufacturers' good quality and retailers' distribution services on a space defined by these two vertical characteristics, under the assumption that each manufacturer sells through one retailer exclusively and each firm sets its price independently.

New-car dealers typically enjoy extensive freedom in setting their mark-up

¹According to their website "Anyone who is not currently affiliated with any Dealership can write a review provided he/she has had first hand experience with the Dealership. Example of some one who can not write a review: Salesman A buys a car from Dealer B. Even though Salesman A does not work for Dealer B, he can not write a review since he is an employee of a Dealership." The data presented here was extracted from the website on July 29, 2004.

and in choosing the quality level associated with the distribution service provided. This is due, in part, to the fact that monitoring these two variables is very costly for car manufacturers.²

Evidence from past antitrust cases shows that manufacturers have tried to control the final price from time to time, with poor results. However, all cases found show that, contrary to the predictions from the successive monopolies literature, when they did it they tried to enforce minimum rather than maximum resale prices.³

In one case, for example, a car manufacturer tried to monitor its dealers' prices and made attempts to impose fines if the dealers sold below agreed prices or granted higher discounts than allowed.⁴

²It is not the aim of the paper to discuss the explicit legislation that regulates many aspects of the manufacturer-dealer relationship but only the regulation originated from antitrust cases dealing with vertical restrictions. For a discussion of the institutional restrictions on manufacturer-dealer relations see Smith (1982)

³For example, following an investigation carried out in 1994, New Zealand's Commerce Commission began a court action against Toyota New Zealand Limited because Toyota assisted its franchised dealers with discounts given by them on sales of new Toyotas to fleet owners. The aim of the scheme was to limit discounts Toyota dealers might offer when fleet owners were buying new vehicles, which was in contravention of the resale price maintenance provisions of the Commerce Act. The Commission acknowledged that Toyota had ceased the practice prior to the Commission's investigation and had co-operated with the Commission. Toyota agreed to having judgment entered against it, and a penalty of 250,000 dollars. Commission Chairman Dr Alan Bollard acknowledged that "There is no evidence in this case, that Toyota's guidelines worked, or that there was damage to buyers. However resale price maintenance is anti-competitive and can lead to considerably higher costs for consumers." New Zealand Commerce Commission Media Release. Source: <http://www.comcom.govt.nz>

⁴In March 2003 the Canadian Competition Bureau settled a price maintenance case involv-

The evidence shows that dealers often were able to circumvent such minimum resale price maintenance impositions by means of offering more generous trade-ins to buyers, or by putting some mileage on a new car and selling it for less as a "demo" unit⁵. As a result, car manufacturers usually have not put much of a fight when the antitrust authorities challenged their practices. Thus, we can conclude that dealers enjoy a *de facto*, due to informational asymmetries, and *de jure*, due to antitrust law, freedom to pursue their best benefit.

Thus, we can safely assume that dealers buy the cars at cost plus insurance plus freight (C.I.F.) and then attach a set of distribution services to them charging an additive mark-up over the purchase cost. The pricing game played by manufacturers and dealers can be best characterized as a non-cooperative game. I model the pricing game as a simultaneous game where the final price is made of two "mark-ups". The manufacturers charge a "mark-up" over the price of the distribution services charged by the dealers, and the dealers charge a "mark-up" over the wholesale price charged by the manufacturers (for expositional purposes we will continue to use the label "mark-up" for the prices charged by the retailers

ing Toyota Canada Inc. and its Access Toyota Program. The parties agreed to a Consent Prohibition Order issued by the Federal Court of Canada requiring Toyota to amend its sales, promotion, training and monitoring practices. As part of the settlement, Toyota made voluntary donations totalling 2.3 million dollars. The Bureau's inquiry addressed allegations that Toyota was prohibiting dealers under the Access Toyota Program from selling vehicles below "Access/Drive-Away Prices." Toyota fully cooperated with the Bureau's investigation and was willing to address its concerns regarding the challenged practice, without costly litigation.

⁵In the merger case between Unitrans Motors (Pty) Ltd and the motor division of Senwes Ltd, it emerged that manufacturers dictated the margins made on their products, the number of cars sold and even how the cars were sold but that dealers had many ways to circumvent these impositions. Source: <http://www.compcom.co.za>

and "wholesale price" for the price charged by the manufacturers).

The analysis is based on a two stage non-cooperative game. In the first stage each firm chooses the quality of its product/distribution service. In the second stage, and having observed its rivals' quality choices, each firm chooses its price. For the sake of tractability, I restrict the analysis to the case where there are two manufacturers and two retailers in the market and there is not entry or exit of firms. In the next section I present the formal model in detail.

2.2 The Model

Let S_j be the quality of the good produced by manufacturer j and D_r be the quality for the distribution service provided by retailer r . Then, the final product is the bundle (j, r) where $j = 1, 2$ and $r = 1, 2$. (See Figure 1.1).

Consumers are heterogeneous, their valuation for the pair (S_j, D_r) is idiosyncratic, and they have a unitary demand. A household i who buys good j from retailer r derives utility $U_{i,j,r}$, where:

$$U_{i,j,r} = A + \theta_i^S S_j + \theta_i^D D_r - P_{j,r}$$

The final price paid by the consumer is $P_{j,r}$, where:

$$P_{j,r} = w_j + x_r$$

w_j is the price charged by manufacturer r and x_r is the price charged by dealer r . For the chosen alternative, utility maximization requires that:

$$U_{i,j,r} = A + \theta_i^S S_j + \theta_i^D D_r - P_{j,r} \geq U_{i,-j,-r} = A + \theta_i^S S_{-j} + \theta_i^D D_{-r} - P_{-j,-r}$$

where A is high enough to allow all consumers to enjoy positive utility at the equilibrium prices. Under this assumption, price competition only affects market shares. This assumption makes tractable our model and allows us to concentrate on the effects of price competition among firms with respect to quality choice.

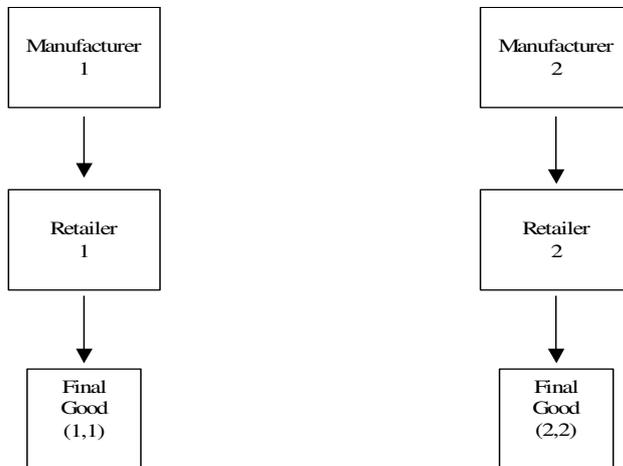


Figure 2.1: Exclusivity

We also assume that:

$$S_j \in [\underline{S}, \overline{S}] ; D_r \in [\underline{D}, \overline{D}]$$

$$f(\theta_i^S) = 1, \theta_i^S \in [0, 1]$$

$$f(\theta_i^D) = 1, \theta_i^D \in [0, 1]$$

The analysis presented in this section follows closely Vandenbosch and Weinberg (1995).

Consumers who are indifferent between the two final goods lie along the line:

$$\theta_i^D = \frac{(w_2 + x_2) - (w_1 + x_1)}{(D_2 - D_1)} + \theta_i^S \frac{(S_1 - S_2)}{(D_2 - D_1)}$$

Let us define “Asymmetric characteristics competition” as competition between firms when each firm has a relative advantage on one of the two characteristics only and “Dominated characteristics competition” as competition when one

of the firms has a relative advantage on both characteristics. Also, let us define “Characteristic S dominance” as the situation where $(S_1 - S_2) \geq (D_2 - D_1)$ or $(S_2 - S_1) \geq (D_2 - D_1)$ and “Characteristic D dominance” the situation where $(S_1 - S_2) \leq (D_2 - D_1)$ or $(S_2 - S_1) \leq (D_2 - D_1)$. Without loss of generality, it is assumed that under “Asymmetric characteristic competition” good 1 has the advantage on characteristic S and good 2 has the advantage on characteristic D , and that under “Dominated characteristics competition” good 2 has the advantage on both S and D .

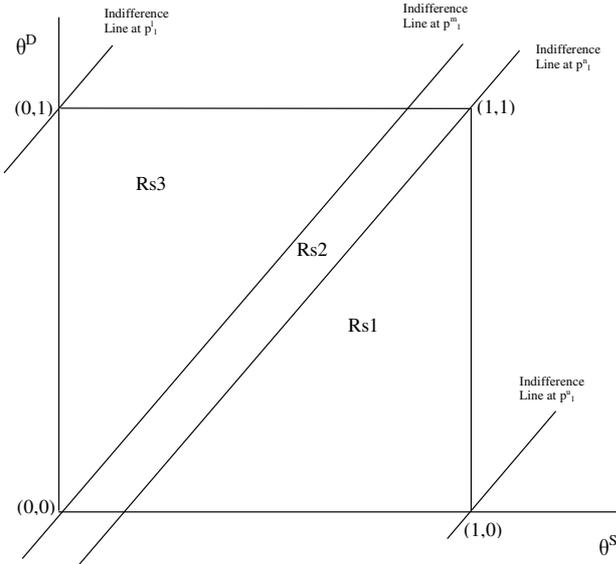


Figure 2.2: Asymmetric characteristics competition: characteristic S dominance

From the expression for the indifference line and given good 2's price, we can see that for good 1 there are four boundary price levels at which its demand function changes shape. These price levels are:

$(w_1 + x_1)^u$: at this price, the indifference line passes through $(\theta^S, \theta^D) = (1, 0)$.

$(w_1 + x_1)^m$: at this price, the indifference line passes through $(\theta^S, \theta^D) = (1, 1)$.

$(w_1 + x_1)^n$: at this price, the indifference line passes through $(\theta^S, \theta^D) = (0, 0)$.

$(w_1 + x_1)^l$: at this price, the indifference line passes through $(\theta^S, \theta^D) = (0, 1)$.

Thus:

$$(w_1 + x_1)^u = (w_2 + x_2) + (S_1 - S_2)$$

$$(w_1 + x_1)^m = (w_2 + x_2) + (S_1 - S_2) - (D_2 - D_1)$$

$$(w_1 + x_1)^n = (w_2 + x_2)$$

$$(w_1 + x_1)^l = (w_2 + x_2) - (D_2 - D_1)$$

For Retailer 2 and Manufacturer 2:

$$(w_2 + x_2)^u = (w_1 + x_1) + (D_2 - D_1)$$

$$(w_2 + x_2)^m = (w_1 + x_1)$$

$$(w_2 + x_2)^n = (w_1 + x_1) - (S_1 - S_2) + (D_2 - D_1)$$

$$(w_2 + x_2)^l = (w_1 + x_1) - (S_1 - S_2)$$

Thus, the following restrictions define each of the six relevant regions⁶.

$$Rs2 : S_1 - S_2 \geq D_2 - D_1 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^n, (w_1 + x_1)^m], (w_2 + x_2) \in [(w_2 + x_2)^n, (w_2 + x_2)^m]$$

⁶There are twelve regions in total. However, only these six regions are relevant for the analysis, as it is proved in Appendix A

$$\begin{aligned}
& Rd2 : D_2 - D_1 \geq S_1 - S_2 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^m, (w_1 + x_1)^n], (w_2 + x_2) \in \\
& [(w_2 + x_2)^m, (w_2 + x_2)^n] \\
& dRs1 : S_2 - S_1 \geq D_2 - D_1 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^l, (w_1 + x_1)^n], (w_2 + x_2) \in \\
& [(w_2 + x_2)^l, (w_2 + x_2)^n] \\
& dRs2 : S_2 - S_1 \geq D_2 - D_1 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^u, (w_1 + x_1)^l], (w_2 + x_2) \in \\
& [(w_2 + x_2)^u, (w_2 + x_2)^l] \\
& dRd1 : D_2 - D_1 \geq S_2 - S_1 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^u, (w_1 + x_1)^n], (w_2 + x_2) \in \\
& [(w_2 + x_2)^u, (w_2 + x_2)^n] \\
& dRd2 : D_2 - D_1 \geq S_2 - S_1 \geq 0, (w_1 + x_1) \in [(w_1 + x_1)^l, (w_1 + x_1)^u], (w_2 + x_2) \in \\
& [(w_2 + x_2)^l, (w_2 + x_2)^u]
\end{aligned}$$

In order to solve this game by backward induction, we must start deriving the price equilibrium in each one of these regions. For example, consider the region *Rs2*. From the FOCs of the profit functions we find that the price equilibrium is:

$$x_1 = \frac{1}{10} [6(S_1 - S_2) - (D_2 - D_1)]; w_1 = \frac{1}{10} [6(S_1 - S_2) - (D_2 - D_1)] \quad (2.1)$$

$$x_2 = \frac{1}{10} [4(S_1 - S_2) + (D_2 - D_1)]; w_2 = \frac{1}{10} [4(S_1 - S_2) + (D_2 - D_1)] \quad (2.2)$$

These prices are an equilibrium provided they lie in the intervals defining *Rs2*.

$$(x_1 + w_1) \in [(w_1 + x_1)^n, (w_1 + x_1)^m]; (x_2 + w_2) \in [(w_2 + x_2)^n, (w_2 + x_2)^m]$$

It is easy to prove that $(x_1 + w_1) \in [(w_1 + x_1)^n, (w_1 + x_1)^m]$ and $(x_2 + w_2) \in [(w_2 + x_2)^n, (w_2 + x_2)^m]$ if $S_1 - S_2 \geq D_2 - D_1$. Since this condition is always true

under “Asymmetric Characteristic S dominance”, there is no need to calculate the equilibrium prices in $Rs1$ and $Rs3$.

In Appendix B I derive the price equilibria for each relevant region. Equilibrium prices in all regions are higher than the respective equilibrium prices in Vandenbosch and Weinberg’s paper. This is due to the existence of double-marginalization between the manufacturer and its retailer.

After solving the second stage, we replace these prices into the respective profit functions and derive the location equilibrium. The location equilibrium is determined by comparing each firm’s most profitable product position, subject to the competitor’s position, in all relevant demand regions. First, the conditions determining the range of product positions allowable in each region are considered. Then, from the FOCs of the relevant profit functions for the product position we determine whether a firm’s profits are improved by increasing or decreasing the level associated with the vertical characteristic. Finally, each firm’s maximum profit in each relevant region, subject to the competitor’s location, are determined and compared. The product locations that yield the highest profit represent a best response to the competitor’s location.

Proposition 2.1 *Given our assumptions and if $(\bar{S} - \underline{S}) \leq (\bar{D} - \underline{D})$, there is a unique SPE in pure strategies such that $(S_1, D_1) = (\bar{S}, \underline{D})$ and $(S_2, D_2) = (\bar{S}, \bar{D})$*

(Proof in Appendix C).

Proposition 2.2 *Given our assumptions and if $(\bar{S} - \underline{S}) > (\bar{D} - \underline{D})$, there is a unique SPE in pure strategies such that $(S_1, D_1) = (\underline{S}, \bar{D})$ and $(S_2, D_2) = (\bar{S}, \bar{D})$*

(Proof in Appendix C).

The demand function will be different in each of the six relevant regions defined by the before mentioned cases. The first stage of the sequential game involves the firms' simultaneous choice of product location. These product decisions are dependent on the equilibrium prices. The procedure used to determine the product equilibrium involves, first, the determination of which demand regions need to be considered for the product equilibrium analysis. Second, the firms' profit functions in each of the relevant regions are calculated. Third, the FOCs of the profit functions, combined with the demand region restrictions, are used to determine the maximum profit equilibrium locations within each of the demand regions. Finally, the maximum profit levels in each of the relevant regions are compared to determine the equilibrium location representing the firm's optimal product location choice, given its competitor's choice.

In Appendix C I prove that in each case the equilibrium is unique. I show that if Retailer 2 and Manufacturer 2 provide the maximum quality possible in each dimension then Retailer 1 and Manufacturer 1 will always choose $(S_1, D_1) = (\bar{S}, \underline{D})$ whenever $\frac{4}{25}(\bar{D} - \underline{D}) \geq \frac{4}{25}(\bar{S} - \underline{S})$, and will always choose $(S_1, D_1) = (\underline{S}, \bar{D})$ whenever $\frac{4}{25}(\bar{D} - \underline{D}) \leq \frac{4}{25}(\bar{S} - \underline{S})$. For Retailer 2 and Manufacturer 2 it is always optimal to provide the maximum quality possible no matter in which of these two locations the Retailer 1 and Manufacturer 1 choose to be.

Table 2.2 shows each firm's profit. Under non-integration each and every firm earns higher profits than under complete vertical integration. Since the equilibrium location is the same under non-exclusivity and exclusivity, the higher profits are only due to the existence of double-marginalization under non-exclusivity. Unlike McGuire and Staelin (1983) I did not show whether non-exclusivity is an equilibrium of an extended three-stage game where in the first stage firms decide

Table 2.2: Firms' Profits

	$(\bar{D} - \underline{D}) \geq (\bar{S} - \underline{S})$	$(\bar{D} - \underline{D}) \leq (\bar{S} - \underline{S})$
Manufacturer 1's Profits	$\Pi_1^M = \frac{4}{25} (\bar{D} - \underline{D})$	$\Pi_1^M = \frac{4}{25} (\bar{S} - \underline{S})$
Retailer 1's Profits	$\Pi_1^R = \frac{4}{25} (\bar{D} - \underline{D})$	$\Pi_1^R = \frac{4}{25} (\bar{S} - \underline{S})$
Manufacturer 2's Profits	$\Pi_2^M = \frac{9}{25} (\bar{D} - \underline{D})$	$\Pi_2^M = \frac{9}{25} (\bar{D} - \underline{D})$
Retailer 2's Profits	$\Pi_2^R = \frac{9}{25} (\bar{D} - \underline{D})$	$\Pi_2^R = \frac{9}{25} (\bar{D} - \underline{D})$

whether they will be integrated or not. However, it would be interesting to study how the decision to vertically integrate interacts with the location decision.

2.3 Implications for Market Structure and Conclusion

In this model I have found that under separate profit maximization firms do not differentiate themselves to the maximum extent possible. This result is an original contribution to the literature on vertical differentiation since the MaxMin result has only been established for two-dimensional models where each horizontal competitor controls both dimensions and the final price.

It is interesting to note that despite the lack of explicit coordination between each manufacturer and its dealer, firms chose to differentiate along the dimension that promises the highest profits, i.e. the dimension whose range is the widest as in the previous literature.

Also, if we look at the price equilibria in the different regions we see that the firms which are not differentiated charge the same prices than the firms which are vertically differentiated. This is crucial in ensuring that no profitable devi-

ations towards vertical differentiation exist for the non-differentiated horizontal competitors. Thus, here, the differentiated firms transfer half of the profits from differentiation to the firms that are not vertically differentiated. Then, the model presented here is able to explain why, as pointed by Bresnahan and Reiss, "...there appears to be a nearly proportional relation between the manufacturer's margin and the dealer's margin across the product line [9, pp.253]."

In our model, the pricing game generates a double-marginalization situation. As a result of the double-marginalization, the equilibrium final prices under separate profit maximization are higher than under joint maximization. Double marginalization exists even when, under our assumptions, the product market demand is fixed because product differentiation generates a negatively sloped (with respect to price) market share function.

In this model, higher prices translate necessarily into higher profits. Thus, separate profit maximization is the Pareto optimal vertical structure, although we do not know whether it is an equilibrium. This result is driven by the assumption that the market is covered at the equilibrium prices, then double-marginalization does not have any cost in terms of reduction in the total quantity demanded by the market. In the real world, profits under double-marginalization will be higher if the equilibrium prices are higher than the prices under joint maximization but lower than the monopoly price. This is usually overlooked when studying the effects of double-marginalization on profits.

In addition, when we incorporate this possibility into the analysis we can understand why manufacturers have tried to enforce minimum resale prices, since prices in an oligopolistic market can be far from the monopoly price even when double-marginalization is present.

Finally, both results have important policy implications for predicting vertical structure in this market characterization. If separate profit maximization is the Pareto optimal vertical structure but not a Nash Equilibrium of the static game, firms might enforce it as the outcome of a repeated game.

Chapter 3

Vertical Product Differentiation under Non Exclusivity

3.1 Introduction

In this paper, I use a two-stage vertical product differentiation model to study the location equilibria of a set of retailers' distribution services quality and manufacturers' good quality on a space defined by these two vertical characteristics, under the assumption that each manufacturer sells to any retailer and each retailer buys from any manufacturer (non-exclusivity).

This characterization fits the market structure observed between manufacturers of groceries and their retailers. In particular, it is an accurate representation if the product market definition for retailer services is one-stop shopping distribution services as opposed to top-up, urgent or impulse shopping. In this market the retailers are supermarkets offering a sufficiently wide assortment of products and brands (for a discussion about why one-stop shopping is a separate product market from top-up, urgent or impulse shopping see UK Competition Commission's report on the retail market (2000)).

I found that there is a Nash Equilibrium to this game in which each firm differentiates its product completely from its horizontal competitor. The equilibrium mimics the Subgame Perfect Equilibrium outcome in a game where each manufacturer and each retailer offer its good to the consumers directly, and then each consumer does the bundling (mix and match) at will.

The analysis is based on a two stage non-cooperative game. In the first stage each firm chooses the quality of its product/distribution service. In the second stage, and having observed its rivals' quality choices, each firm chooses its price. I model the pricing game as a simultaneous game among all firms. In this game I assume that the final price is made of two "mark-ups". The manufacturers charge a "mark-up" over the price of the distribution services charged by the supermarkets, and the supermarkets charge a "mark-up" over the wholesale price charged by the manufacturers (for expositional purposes we will continue to use the label "mark-up" for the price charged by the retailers and "wholesale price" for the price charged by the manufacturers).

I solve this game by backwards induction by finding the Nash Equilibrium in pricing strategies and then the Nash Equilibrium in product locations.

For the sake of tractability, I restrict the analysis to the case where there are two manufacturers and two retailers in the market, there is not entry or exit of firms and the market is totally covered at the equilibrium prices.

The last assumption implies that firm decisions about quality and price do not affect the market demand but only each firm's market share. Although this simplifies the analysis a lot, simplicity comes at a cost. Suppose that for a given quality each firm decides to increase its price so that consumers' preferences among the differentiated products does not change. If we assume that the market

is covered then market shares do not change and actual quantities sold do not change either. However, this does not have to be the case. After the price increment, some consumers might find that it is optimal for them not to consume at all. We discuss this possibility further in the next section.

3.2 The Model

I model the pricing game by assuming that the final price is made of two “mark-ups”. The manufacturers charge a “mark-up” over the price of the distribution services charged by the supermarkets, and the supermarkets charge a “mark-up” over the wholesale price charged by the manufacturers. I look then for a Nash Equilibrium in pricing strategies.

Let S_j be the quality of the good produced by manufacturer j and D_r be the quality for the distribution service provided by retailer r . Then, the final product is the bundle (j, r) where $j = 1, 2$ and $r = 1, 2$.

Consumers are heterogeneous, their valuation for the pair (S_j, D_r) is idiosyncratic, and they have a unitary demand. A household i who buys good j from retailer r derives utility $U_{i,j,r}$, where:

$$U_{i,j,r} = A + \theta_i^S S_j + \theta_i^D D_r - P_{j,r}$$

For the chosen alternative, utility maximization requires that:

$$U_{i,j,r} = A + \theta_i^S S_j + \theta_i^D D_r - P_{j,r} \geq U_{i,-j,-r} = A + \theta_i^S S_{-j} + \theta_i^D D_{-r} - P_{-j,-r}$$

where A is high enough to allow all consumers to enjoy positive utility at the equilibrium prices. Under this assumption, price competition only affects market shares. This assumption makes tractable our model and allow us to concentrate on the strategic effects of price competition among firms with respect to quality choice.

We also assume that:

$$S_j \in [\underline{S}, \overline{S}] ; D_r \in [\underline{D}, \overline{D}]$$

$$f(\theta_i^S) = 1, \theta_i^S \in [0, 1]$$

$$f(\theta_i^D) = 1, \theta_i^D \in [0, 1]$$

I prove that this game has a SPE in which each manufacturer differentiates its good and each retailer differentiates its distribution service, from its respective horizontal competitors, as much as possible. I cannot prove that this is the unique SPE, except in the case where I restrict each manufacturer to charge the same wholesale price to each retailer, and each retailer to charge the same mark-up over each manufacturer's good.

My strategy is to find the SPE for the restricted game (Proposition 3.1) and then prove that this SPE is a SPE of the game where manufacturers can charge different wholesale prices to each retailer and retailers can charge a different mark-up on each manufacturer's good (Proposition 3.2).

Proposition 3.1 *If each manufacturer is restricted to charge the same wholesale price to each retailer and each retailer to charge the same mark-up over each manufacturer's good, the unique SPE in pure strategies involves quality differentiation to the maximum extent possible in both, the goods and the distribution services.*

Sketch of a Proof (Proof in Appendix C.):

Let

$$P_{jr} = w_j + x_r \quad j = 1, 2; r = 1, 2$$

Where w_j is the wholesale price charged by manufacturer j and x_r is the mark-up charged by retailer r . A consumer i who buys good 1 from retailer 1 must satisfy:

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_1 + \theta_i^D D_2 - P_{12} \quad (3.1)$$

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_2 + \theta_i^D D_1 - P_{21} \quad (3.2)$$

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_2 + \theta_i^D D_2 - P_{22} \quad (3.3)$$

Then, the taste parameter vector of consumers who buy good 1 from retailer 1 must satisfy:

$$\theta_i^D \leq \frac{x_2 - x_1}{D_2 - D_1}$$

$$\theta_i^S \leq \frac{w_2 - w_1}{S_2 - S_1}$$

$$\theta_i^D \leq \frac{(w_2 + x_2) - (w_1 + x_1) - \theta_i^S (S_2 - S_1)}{D_2 - D_1}$$

It is easy to show that when the two first inequalities hold, the third holds too. This is a result driven by the assumption stating that each retailer charges a uniform mark-up on each good. Then, we are allowed to derive the aggregate demand for final good (1, 1), AD_{11} , as:

$$AD_{11} = \left(\frac{x_2 - x_1}{D_2 - D_1} \right) \left(\frac{w_2 - w_1}{S_2 - S_1} \right) \quad (3.4)$$

The rest of the aggregate demand functions are derived in the same way. Since the demand that retailer 1 faces is made up by $(AD_{11} + AD_{21})$ and he charges the same mark-up on both, then profit for him is:

$$\Pi_1^R = x_1 \left(\frac{x_2 - x_1}{D_2 - D_1} \right) \quad (3.5)$$

From $(AD_{11} + AD_{21})$ it is clear that modeling the pricing game as a sequential game would produce the same profit function since wholesale prices do not enter in the profit function of the retailer (and mark-ups do not enter the profit function of the manufacturers). Moreover, retailer 1 will not care about what product is sold because they charge the same mark-up on both products. Also, manufacturer 1 will not care about the size of the mark-up charged by each retailer because the retailer charges the same mark-up on manufacturer 2 and the assumption that says that the market is always covered implies that what does not affect market shares does not affect demand for each firm.

Although this simplifies the analysis a lot, simplicity comes at a cost. Suppose that for a given quality each firm decides to increase its price so that consumers' preferences among the differentiated products does not change. If we assume that the market is covered then market shares do not change and actual quantities sold do not change either. However, this does not have to be the case. After the price increment, some consumers might find that it is optimal for them not to consume at all. Then, even when each retailer charges a uniform mark-up on both manufacturers' products and each manufacturer the same wholesale price to both retailers, we will find that the wholesale price charged by manufacturer 1 and 2 will enter the profit function of retailer 1 and that the mark-up charged by retailer 1 and 2 will enter the profit function of manufacturer 1.

However, from Proposition 3.1 we are allowed to conclude that this is the only channel through which price decisions in the "upstream" market will affect the "downstream" market, and the reverse, because when we assume that the market

is always covered what firms in the "downstream" market decide only affects that market.

In the second stage, each retailer and manufacturer reaction function is derived from its FOC. Then, we solve for the NE in prices given a quality level vector. We update each firm's profit function with the NE prices and let each manufacturer and retailer maximize its profit with respect to quality. The SPE outcome is quality differentiation to the maximum extent possible.

Proposition 2 *The SPE in pure strategies outcome from the restricted game is a NE equilibrium of a modified game (the unrestricted game) where each manufacturer can charge each retailer a different wholesale price, and each retailer can charge a different mark-up on each manufacturer's good.*

Sketch of a Proof (Proof in Appendix 3.2):

Let us assume, without loss of generality, that manufacturer 2 and retailer 2 are the high quality firms: $D_2 > D_1$, $S_2 > S_1$ and that $P_{22} - P_{21} > P_{12} - P_{11} > 0$ where P_{12} is the price charged on the good from manufacturer 1 and retailer 2.

Then, demand functions are:

$$AD_{11} = \left(\frac{(x_{12} + w_{12}) - (x_{11} + w_{11})}{D_2 - D_1} \right) \left(\frac{(x_{21} + w_{21}) - (x_{11} + w_{11})}{S_2 - S_1} \right) + \quad (3.6)$$

$$- \frac{(x_{22} + w_{22} + x_{11} + w_{11} - x_{12} - w_{12} - x_{21} - w_{21})^2}{2(D_2 - D_1)(S_2 - S_1)}$$

$$AD_{21} = \left(\frac{(x_{22} + w_{22}) - (x_{21} + w_{21})}{D_2 - D_1} \right) \left(1 - \frac{(x_{21} + w_{21}) - (x_{11} + w_{11})}{S_2 - S_1} \right) \quad (3.7)$$

$$AD_{12} = \left(1 - \frac{(x_{12} + w_{12}) - (x_{11} + w_{11})}{D_2 - D_1} \right) \left(\frac{(x_{22} + w_{22}) - (x_{12} + w_{12})}{S_2 - S_1} \right) \quad (3.8)$$

$$\begin{aligned}
AD_{22} = & \left(1 - \frac{(x_{22} + w_{22}) - (x_{21} + w_{21})}{D_2 - D_1}\right) \left(1 - \frac{(x_{22} + w_{22}) - (x_{12} + w_{12})}{S_2 - S_1}\right) + \\
& - \frac{(x_{22} + w_{22} + x_{11} + w_{11} - x_{12} - w_{12} - x_{21} - w_{21})^2}{2(D_2 - D_1)(S_2 - S_1)}
\end{aligned} \tag{3.9}$$

Let us replace these demand functions into the profit functions of retailers and manufacturers, and use the same equilibrium concept to solve this problem. Since, for each firm, the demand function is quadratic in its own price, profit functions are cubic in that price, and then FOCs are quadratic.

I prove that, at stage 2, each supplier does not have an incentive to deviate from charging the same wholesale price to both retailer and that each retailer does not have an incentive to deviate from charging the same mark-up on the goods from both manufacturers. For example, for retailer 1 it is a best response to charge $x_{11} = x_{12} = \frac{1}{3}(D_2 - D_1)$, given that retailer 2, manufacturer 1 and manufacturer 2 charges $x_{12} = x_{22} = \frac{2}{3}(D_2 - D_1)$, $w_{11} = w_{12} = \frac{1}{3}(S_2 - S_1)$, $w_{21} = w_{22} = \frac{1}{3}(S_2 - S_1)$ respectively. Because the pricing game played is symmetric for manufacturers and retailers we only need to derive the best responses for the manufacturers or the retailers. Profit functions for retailer 1 (retailer 2), given what retailer 2 (retailer 1), manufacturer 1 and manufacturer 2 charge, are:

$$\begin{aligned}
\Pi_1^R = & \frac{9(x_{11} - x_{21})^2(x_{11} + 2x_{21}) + 6(S_1 - S_2)(x_{11}^2 + 2x_{21}^2)}{18(D_1 - D_2)(S_1 - S_2)} + \\
& + \frac{4(D_1 - D_2)(3(x_{11} - x_{21})^2 + (S_1 - S_2)(x_{11} + 2x_{21}))}{18(D_1 - D_2)(S_1 - S_2)}
\end{aligned} \tag{3.10}$$

$$\begin{aligned}\Pi_2^R &= \frac{9(x_{12} - x_{22})^2(2x_{12} + x_{22}) + 6(S_1 - S_2)(x_{12}^2 + 2x_{22}^2)}{18(D_1 - D_2)(S_1 - S_2)} + \\ &+ \frac{8(D_1 - D_2)(3(x_{12} - x_{22})^2 + (S_1 - S_2)(x_{12} + 2x_{22}))}{18(D_1 - D_2)(S_1 - S_2)}\end{aligned}\quad (3.11)$$

Now, let us solve for the best response function from the respective FOCs. I get three critical point vectors for each retailer. For retailer 1, the critical point vectors are:

$$x_{11} = x_{21} = \frac{1}{3}(D_2 - D_1) \quad (3.12)$$

$$x_{11} = \frac{4}{27} \left(3(D_2 - D_1) + (S_2 - S_1) + \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right) \quad (3.13)$$

$$x_{21} = -\frac{2}{27} \left(6(D_1 - D_2) + 2(S_1 - S_2) + \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right)$$

$$x_{11} = \frac{4}{27} \left(3(D_2 - D_1) + (S_2 - S_1) - \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right) \quad (3.14)$$

$$x_{21} = -\frac{2}{27} \left(6(D_1 - D_2) + 2(S_1 - S_2) - \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right)$$

For retailer 2, the critical point vectors are:

$$x_{12} = x_{22} = \frac{2}{3}(D_2 - D_1) \quad (3.15)$$

$$x_{11} = -\frac{1}{54} \left(48(D_1 - D_2) + 5(S_1 - S_2) + \sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right) \quad (3.16)$$

$$x_{21} = -\frac{1}{54} \left(48(D_1 - D_2) + 23(S_1 - S_2) - 5\sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right)$$

$$x_{11} = -\frac{1}{54} \left(48(D_1 - D_2) + 5(S_1 - S_2) - \sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right) \quad (3.17)$$

$$x_{21} = -\frac{1}{54} \left(48(D_1 - D_2) + 23(S_1 - S_2) + 5\sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right)$$

The second order conditions for a local maximum only hold for (3.12) and (3.15). Then, for each retailer, charging a uniform additive mark-up on both manufacturers' goods is Sub-Game Perfect Equilibrium. By symmetry, the same happens to the manufacturers. Then, we can rely on Proposition 3.1 to derive the location equilibrium.

Table 3.1: Firms' Profits

Manufacturer 1's Profit	$\Pi_1^M = \frac{1}{9} (\bar{S} - \underline{S})$
Retailer 1's Profit	$\Pi_1^R = \frac{1}{9} (\bar{D} - \underline{D})$
Manufacturer 2's Profit	$\Pi_2^M = \frac{4}{9} (\bar{S} - \underline{S})$
Retailer 2's Profit	$\Pi_2^R = \frac{4}{9} (\bar{D} - \underline{D})$

If we compare the consolidated profits of all firms in Table 2.2 with the those in Table 3.1 we can see that whenever the range for product quality differentiation, $(\bar{S} - \underline{S})$, is not very different from the range for distribution service quality differentiation, $(\bar{D} - \underline{D})$, then the consolidated profits of all firms are higher under

non-exclusivity. Instead, if $(\bar{S} - \underline{S}) > \frac{125}{109}(\bar{D} - \underline{D})$ or if $(\bar{S} - \underline{S}) < \frac{109}{125}(\bar{D} - \underline{D})$ then the consolidated profits of all firms are higher under exclusivity. Since under exclusivity firms differentiate along the dimension whose range is larger, then they forgo less profits from differentiation whenever the ratio of the two vertical dimensions is closer to zero or infinity.

Thus, the trade-off is clear in these two models: having two sources of oligopolistic rent but no double-marginalization on each source versus double-marginalizing over the largest source of oligopolistic rent.

Moreover, manufacturer 1 and retailer 1 are always better off under exclusivity. This suggests that for them, double-marginalization under exclusivity more than compensates for the lower vertical differentiation. On the other hand, manufacturer 2 and retailer 2 are almost always better off under non-exclusivity.

Note that under non-exclusivity, the high quality firms get four times the profits of the low quality firms while under exclusivity, the high quality firms get approximately twice the profits of the low quality firms. These comparisons do not allow us to make any general statement about what would happen with profits under these two market structures in a more realistic setting. However, they highlight how double marginalization and vertical differentiation can interact with each other.

3.3 Implications for Market Structure and Conclusion

I have proved that if each manufacturer sells to any retailer and each retailer sells from any manufacturer and the market is totally covered, both types of

firms retain the incentives to fully differentiate their products as if they were selling them unbundled to the consumers.

This result is driven by the fact that when retailers carry both manufacturers' products, it is a Nash equilibrium for the retailers to charge the same additive mark-up on the manufacturers' product and for the manufacturers to charge the same wholesale price to each retailer. When this occurs, then wholesale prices do not enter in the profit function of the retailer and mark-ups do not enter the profit function of the manufacturers.

This result is important for its simplicity and implications. It says that, under our assumptions, whenever we analyze the strategic location of a set of vertically related firms offering a final product to the consumers we can abstract from considering the vertical relationship among the firms without affecting the results of our analysis.

It also says that whenever the location of the firms in the "upstream" market affect the profits and location of firms in the downstream market total demand, not only market shares, must be affected.

Chapter 4

Gone In Thirteen Seconds: Advertising and Search in the Supermarket

4.1 Introduction

In this paper, I estimate the effects associated with advertising a brand and its price on consumers' brand choice and search behavior inside the refrigerated orange juice category.

Optimal search models (Stigler 1961, Rothschild 1974, Weitzman 1979) suggest that consumers might restrict the number of brands they search (the consumer's choice set) if this is a costly activity, and that advertising can affect the search effort by providing economically relevant information in a convenient way

⁰Dickson and Sawyer [16] find that consumers spend an average of 13 seconds in selecting a brand out of the shelf. This is a very short time for a consumer to incorporate all the often-available marketing information associated with a given product category offered by a typical supermarket. Then, retailers and manufacturers' efforts to make available as much information as possible in a convenient way could lead consumers to make better decisions. Among the marketing tools that manufacturers and retailers often use we will analyze the effect of featuring a brand in newspapers and/or store leaflets.

(Betancourt and Gautschi 1988).

The evidence from the empirical literature suggests that advertising affects the economic returns from search and then the size of the choice set finally considered (Modjuszka et al. 2001, Akerberg 2001, Murthi and Srinivasan 1999, Allenby and Ginter 1995). For example, Akerberg (2001) and Murthi and Srinivasan (1999) find that advertising increases the size of the consumer's choice set. On the other hand, Allenby and Ginter (1995) find that advertising's main effect is to persuade the exposed consumers to reduce search outside the choice set made of the advertised brands.

Also, the economic literature recognizes that advertising can increase the probability of choosing the featured product or brand by "persuading" (Galbraith 1976) consumers to buy a particular brand, by generating utility in the same way goods do it (Stigler and Becker 1977), and by affecting positively the utility associated with the advertised good or brand (Becker and Murphy (1993) show the implications of treating advertisements and the advertised goods as complements).

I focus my attention on a particular type of advertising, i.e. brand and price ads that are featured in mail leaflets and local newspapers.

My results suggest that consumers make their choices after searching only a restricted set of alternatives even when the number of alternatives is not large and that advertising featuring increases the probability of purchase associated with the featured brand and the probability that consumers search larger choice sets.

I identify the effect of featuring on brand utility and search by proper parametrization of the probability of purchase associated with each brand and the probability

associated with each possible choice set. I used the GenL model, due to Swait (2001), in which choice probabilities associated to each brand and to each possible (latent) choice set are estimated from the sampled purchases. The model belongs to the Generalized Extreme Value family of discrete choice models and is a generalization of the Nested Multinomial Logit (NML) model. I implement a variation of this model where consumers are allowed to pick their choice sets using less information than available.

A discrete choice model seems appropriate for the product category I have information about since households typically buy only one brand in each purchase occasion. In the sample, households bought only one brand in 96% of the purchase occasions.

4.2 Effects of Advertising

Advertising can have different effects on consumer behavior. In the following paragraphs, I summarize its potential effects according to the literature.

Let us assume that goods have search and/or experience characteristics. Search characteristics are characteristics that are learned through inspection while experience characteristics are learned through consumption. Under different assumptions, advertising can be a good substitute for direct inspection (Stigler 1961, Grossman and Shapiro 1984) and consumption (Nelson 1974, Kihlstrom and Riordan 1984, and Milgrom and Roberts 1986). Advertising can also generate utility by itself or by giving favorable notice to other goods (Stigler and Becker 1977, Becker and Murphy 1993).

If advertising increases utility then, *ceteris paribus*, we should see an increase in the probability of purchase associated with the featured brand. Optimal search

models predict that if this effect is strong, consumers might choose to abort their search for additional alternatives. Finally, if featuring reduces search costs then it can generate an increase in the size of the choice set searched by the consumer.

The effect of featuring activity on search is due to the fact that the cost of search is an increasing function of the number of brands searched, for the chief cost is time. If being exposed to a feature reduces the time spent on searching the featured brand, then consumers' might be willing to increase the size of the choice set finally considered. The simplest example arises when consumers are subject to a time constraint that limits the number of brands that they can search. In the next paragraphs I provide a more elaborate example where featuring can have a positive effect on the size of the choice set finally considered even when there are no time or budget constraints involved.

Weitzman (1979) derived the optimal search strategy for a consumer facing n alternatives, who is uncertain about each alternative's potential reward. Let us assume that it costs c_i to learn the characteristics of item i . The learning is instantaneous once the consumer paid the cost. In the next paragraphs I use Weitzman's solution to show how featuring can increase the size of the choice set finally considered by a consumer.

Let us assume that there are a number of different brands of a good. Brand i gives a consumer potential utility u_i with probability distribution $F_i(x_i)$, independent of the other utilities. It costs c_i to learn all the characteristics of brand i . Every time the consumer decides to sample another brand, he must select the next one and spend the necessary time learning its characteristics. The consumer maximizes the expected utility's present value. Weitzman (1979) proves that the optimal strategy is a sequential decision rule that will tell the consumer at each

stage whether or not to continue searching, and if so, which brand to search next.

Let us assume that the disutility or opportunity cost of searching brand i is increasing in the amount of time already spent in the search process. With some loss of generality, let us assume that:

$$c_i = \left[\sum_{j=1}^i t_j - \sum_{j=1}^i f_j \right] \quad t_j \geq f_j \quad (4.1)$$

where t_j is the time the consumer must spend in searching brand j and f_j is the time saving brought by the brand j being featured. Brands j are the ones already searched before brand i .

In each stage, the consumer must choose if he searches another brand. In our example, suppose our consumer has searched one brand already which gave him utility equal to z_i . If he chooses to search an additional brand, he can expect a net utility equal to u_i :

$$u_i = -c_i + \beta_i \left[z_i \int_{-\infty}^{z_i} dF_i(x_i) + \int_{z_i}^{\infty} dF_i(x_i) \right] \quad (4.2)$$

The consumer will be indifferent between stopping or keeping the search if z_i equals the expression above, a condition that can be rewritten as

$$\left[\sum_{j=1}^i t_j - \sum_{j=1}^i f_j \right] = \beta_i \int_{z_i}^{\infty} (x_i - z_i) dF_i(x_i) - (1 - \beta_i) z_i \quad (4.3)$$

The value of z_i that satisfies the expression above is the reservation price of brand i . Weitzman (1979) proved that the optimal rule entails two parts:

Selection Rule: If a brand is to be searched, it should be the brand with the highest reservation utility.

Stopping Rule: Terminate search whenever the maximum sampled utility exceeds the reservation utility of every non-searched brand.

Given our assumptions, it is very easy to find examples where the choice set finally searched by the consumer increases in size if featuring reduces the time spent in searching enough. Suppose that our consumer is considering if he is going to search a third brand. Let us assume that when there is no feature the cost associated to searching the third brand is higher than its expected utility, inequality (4), while the opposite occurs if at least one brand is featured, inequality (5), then:

$$\sum_{j=1}^3 t_j > \beta_3 \int_{z_3}^{\infty} (x_3 - y_2) dF_3(x_3) - (1 - \beta_3) z_3 \quad (4.4)$$

$$\left[\sum_{j=1}^3 t_j - \sum_{j=1}^3 f_j \right] < \beta_3 \int_{z_3}^{\infty} (x_3 - y_2) dF_3(x_3) - (1 - \beta_3) z_3 \quad (4.5)$$

where y_2 is the utility reached if the consumer picks the highest value between the first two searched alternatives. Then if brand one is featured the consumer will prefer to search a third brand. More generally, if the opportunity cost or disutility of time of search associated to any brand increases with the number of brands already searched, the feature of any brand can have a positive effect on the size of the choice set finally considered.

Finally, if advertising is a perfect substitute for direct inspection or consumption then utility maximization implies that it will not affect the choice among brands in the sense that consumers that learned the information from the ad and consumers that learned it from other sources should show, *ceteris paribus*, the same choices. If learning through an ad is cheaper than searching, the advertised brand becomes more rewarding (see equation (2)), and so consumers might want to include that brand in their choice sets. This will have a positive effect on the probability of purchase of the advertised brand.

Betancourt (2004) discusses the potentially positive prestige effect of advertising one product on non-advertised products. This can happen when consumers identify some characteristics as common among different products, then advertising of one product can spillover to competing products. The specification I use does not account for this possibility.

4.3 Effects of Other Covariates on Search and Evaluation Activity

I try to capture the effect of households' opportunity cost of time on their search effort by including as explanatory variables the consumer's income and the availability of a female head of household at home. Consumers with higher income should be less likely to engage in time-consuming search activities. Also, the set of households where the female head of household is employed full-time should be more time constrained than its complement. Finally, in some of the specifications tried we also controlled for the day of the week when the purchase was made (weekday vs. weekend).

4.4 Related Literature

When estimating discrete brand choice models, it is usually assumed that each consumer evaluates all the relevant information, such as price and other marketing variables, on every purchase occasion. The seminal work in this area is due to Guadagni and Little (1983). They estimate a multinomial logit model and include among the explanatory variables whether each brand was featured or

not, in a particular purchase occasion. They found that whenever each brand is featured, its associated representative utility was increased. The literature on this area is enormous and it is not my aim to do a review in this section but to include the closest references. Three works, by Allenby and Ginter (1995), Murthi and Srinivasan (1999) and Akerberg (2001), are closely related to my research questions. Also, Andrews and Srinivasan (1995), Siddarth et al., (1995) and Bronnenberg and Vanhonacker (1996) develop approaches to determine and analyze choice set formation where consumers first identify a subset of brands within the universal set of brands and then evaluate those brands and pick their most preferred one.

Murthi and Srinivasan (1999) uses a two stage multinomial choice model where in the first stage households decide if they are going to be in “extended evaluation” state or in “habitual evaluation” state. “Habitual evaluation” means that, in the second stage, the household makes his choice relying only on intrinsic brand preferences and loyalty. “Extended evaluation” means that the household incorporates information on price, feature and other marketing variables in addition to intrinsic brand preferences and brand loyalty when making a choice. The authors obtain estimates for the probability that, in a particular purchase occasion, a consumer will be in habitual or extended evaluation state and how that probability changes when marketing variables change. Among the explanatory variables they include whether there was featuring activity or not in that particular purchase occasion. The authors find that, on average, consumers are in an extended evaluation mode only on 59%-65% of the purchase occasions. They also find that feature activity increases the probability of being in “extended evaluation” mode.

The specification I use represents an improvement over Murthi and Srinivasan's specification since I estimate the probability associated to each possible choice set, not only the largest and the smallest. Also, unlike them, my key explanatory variable is the number of brands featured in a particular purchase occasion. Since the data set includes purchase occasions when more than one brand was featured, I can estimate the effect of multiple featuring on the probability that a given choice set is chosen.

Allenby and Ginter (1995) examine the influence of marketing variables, especially featuring, on household choice sets using a scanner data set of tuna purchases. They implement a single-stage heteroskedastic random utility model in which the random part of the utility is allowed to have alternative specific variances. Alternatives for which consumers are more responsive to changes in the deterministic part of the utility will have a smaller variance than the other alternatives. Hence, a large value in alternative i 's variance will reduce the relative effect of any alternative's price on the choice probability of alternative i . In the different formulations of the model, they allow display and feature to influence the deterministic component of utility and/or the alternatives' variances. These specifications allow them to investigate whether display and feature variables affect an alternative's deterministic utility or its price sensitivity. They find that feature and display have a positive influence on the utility associated to the alternative that is being displayed and/or featured, and have a negative effect on the alternative's price sensitivity by increasing the alternative's variance. As a result, displays and features serve to increase the utility of the alternative and to reduce the influence of price in the purchase decision. The authors define a household's choice set by the pair wise probabilities that the household simulta-

neously considers two brands. They find that when display and feature variables are included in the probability calculations, it is more likely that choice sets will include alternatives with the same brand but different characteristics (e.g., in oil and in water). In the absence of display and feature variables the opposite occurs. The authors suggest that the effect of featuring on reducing the price sensitivity of the featured alternative indicate that many households may identify their preferred brand before actually going to the store and observing the array of prices.

Finally, Akerberg (2001) estimates the effect of the information conveyed by the ads and the prestige effect. In order to disentangle the effects, he uses the following identification assumption: that advertisements that inform consumers should primarily affect consumers who have never tried the advertised brand, whereas advertisements that create prestige effects should affect both, inexperienced and experienced, users in the same way. He finds that the advertising of a newly introduced brand of yogurt primarily affected the inexperienced consumers.

Akerberg (2001) estimates a binomial discrete choice model where the discrete decision is whether or not to purchase the new brand, and the key explanatory variables are the interaction between a dummy that takes a value of 1 for each inexperienced consumer and the advertising exposure of that consumer, and the interaction between a dummy that takes value 1 for experienced consumers and the advertising exposure of that consumer. The advertising exposure of the consumer is proxied by the "... unweighted average of a household's past advertising intensities (i.e., the total Yoplait 150 [the new brand] advertisements seen up to t / the total hours of television watched up to t)."

 He finds a positive and statistically significant effect for the first interaction variable on the

probability of purchasing the new brand and a positive, but lower, statistically insignificant coefficient for the second interaction variable. Then, using his identification assumption, he concludes that an estimate for the informational effect on the population must be the difference between these two coefficients.

The approach used here to identify the effect of advertising on consumer behavior has been advocated by an increasing number of authors. Among them, Andrews and Srinivasan (1995), Siddarth et al., (1995) and Bronnenberg and Vanhonacker (1996) are previous examples of approaches to determine and analyze choice set formation based on a two-stage decision process. In Andrews and Srinivasan (1995) and Bronnenberg and Vanhonacker (1996) papers, in the first stage each brand is assigned a probability that represents the likelihood that the brand's utility exceeds some threshold level required for consideration. In the second stage, the consumer makes a probabilistic choice by selecting the considered brand with the highest overall utility. The main difference between these two models and Swait's is that the formers use exogenous covariates to explain choice set formation while the latter allows for the possibility these are simple functions of the underlying brand utilities rather than independent constructs. We will see in the next section how in the GenL model the probability that some set is the true choice set is a function of the expected maximum utility derived from the alternatives in the set.

4.5 The Model

Let us assume that consumers make decisions in two stages: in the first stage, alternatives are screened and evaluated using a subset of the information available and a restricted choice set is chosen, while in the second stage the selected

alternatives are evaluated using all available information.

I also assume that consumers know all the invariant characteristics associated with each alternative. Typically, each consumer will know if Tropicana's quality is higher than Minute Maid's but they may not know if in the current week Tropicana's price is higher, equal or lower than Minute Maid's.

The decision process can be characterized in the following way. In a given week, each orange juice brand is featured with positive probability. Each consumer is exposed to each brand's weekly feature with positive probability. If the consumer was exposed to a brand's feature we assume that he learns all the information contained in the feature at a negligible cost (compared to the cost associated to learning that information by going to the supermarket and retrieving it from the corresponding shelf). After learning the information, each consumer leaves to the supermarket. Once in the supermarket, each consumer starts searching the brands. Finally, he chooses the best alternative.

Thus, each consumer specify the number of brands to include in his choice set in advance of searching, very much like in Stigler's (1961) model. By far, the most interesting challenge is identifying the probability associated with an endogenously determined choice set. Given the available information, it is impossible to observe the set of alternatives actually searched by a consumer before making a choice. Therefore, choice sets are latent, as their probability of occurrence cannot be estimated from direct observational data. We follow the previous literature on discrete choice models with latent choice sets and use the GenL (Swait 2001) specification, which treat choice set generation as a probabilistic process.

From an heuristic perspective, we can characterize the identification of the ef-

fect of advertising on choice set probabilities through an example. Let us suppose that Tropicana is on sale and that due to the decrease in its price it becomes the most appealing brand for most of the consumers. If Tropicana is not advertised, then consumers with idiosyncratic high search costs may not include it in their choice set and then its probability of purchase would not increase because they will never know about the reduced price. On the other hand, if Tropicana is advertised, and advertising lower search costs, it will have more chances to be included in these consumers' choice sets. Thus, its probability of purchase would increase. The different probabilistic behavior of our dependent variable when conditioned on different values of the explanatory variables and parameters plus a sample showing enough variation is what will allow me to estimate the effect of advertising on consumer search behavior.

Let us denote the set of all possible subsets of M , the master set of alternatives, as $\Delta(M)$ and the number of possible subsets as K . Also, I denote choice set k as C_k , $k = 1, \dots, K$. Finally, each alternative belonging to M is indexed by $i = 1, \dots, M$.

Thus, the probability associated with brand i is the summation of the probabilities that brand i is chosen from choice set k , weighted by the probability that choice set k is chosen. If we interpret choice set generation probabilities as household population shares, then we will be able to infer which portions of the consumers evaluate only one, two or the complete set of available brands.

Consumer n derives utility from alternative i equal to:

$$U_{ni} = V_i + \varepsilon_{ni}$$

Where, V_i is the known deterministic part and ε_{ni} is the unknown and random part of utility. The error distribution is equivalent to the convolution of K

independent generalized extreme value variable vectors ε_k each with dimension $|C_k| \times 1$. In each of these conditional distributions, component errors are IID Type I extreme value with scale parameter μ_k (Swait 2001). Thus, the probability that brand i is chosen, can be expressed as:

$$P(i) = \sum_{k \in K_i} P(i, C_k) = \sum_{k \in K_i} P(i|C_k) Q(C_k) \quad (4.6)$$

where K_i , is the number of choice sets that contain alternative i .

$$P(i|C_k) = \frac{\exp(\mu_k V_{2,i})}{\sum_{j \in C_k} \exp(\mu_k V_{2,j})} \quad (4.7)$$

$$Q(C_k) = \frac{\exp(I_k)}{\sum_{r=1}^K \exp(I_r)} \quad (4.8)$$

$$I_k = \frac{1}{\mu_k} \ln \left(\sum_{j \in C_k} \exp(\mu_k V_{1,j}) \right) + S_k \quad (4.9)$$

Identification of choice set probabilities is achieved by proper parametrization of $Q(C_k)$. In the GenL model, the probability that some set C_k is chosen is a function of the expected maximum utility derived from the alternatives in the set. This value is a function of the utilities associated to the alternatives in the set and the degree of similarity of these alternatives, which is a function of μ_k , among other things. I_k is the expected utility that the consumer obtains from the choice situation when C_k is the choice set under consideration, sometimes referred to as the inclusive value of choice set k . Then, as alternative j becomes more attractive, all sets including j will have increased probability of being the true choice set. $V_{1,j}$ is the known deterministic utility associated to alternative j in the first stage (where consumers screen out alternatives from their choice sets),

while $V_{2,j}$ is the known deterministic utility associated to alternative j in the second stage (where consumers pick their most preferred alternative from their choice set). S_k is a set of choice set specific covariates, which enter directly into the probabilities $Q(C_k)$. I assume that $V_{1,j}$, $V_{2,j}$ and S_k are linear functions of the marketing variables, other environmental variables, demographic information and choice set characteristics.

Since, in the first stage, consumers may find it too costly to search all the characteristics of each alternative we can let V_{1j} be a function of only a subset of the covariates that appear in V_{2j} . If a covariate does not appear in V_{1j} that means the characteristic represented by this covariate is not being searched and not taken into account when the screening process is carried out. Since consumers can form expectations about non-searched covariates like the price, and as long as these expectations are constant throughout the sample period, we can allow the model to capture them by including alternative specific intercepts for all the alternatives.

As in the nested multinomial model, the vector $\mu = (\mu_1, \dots, \mu_k)$ characterizes the distribution of the random part of the utility $\varepsilon_n = (\varepsilon_{n1}, \dots, \varepsilon_{nJ})$. As Swait (2001) proves, it must be true that $\mu_k > 1$ for the GenL specification to be consistent with utility maximization.

In Table 4.1 I present a sketch of the three specifications tried for this paper.

In Model 1 I let brand intercepts to be the only brand-related covariates that explain choice set choice in the first stage. Brand intercepts will capture all the invariant attributes associated to the brands, which are the attributes all consumers are most probably going to be aware of. Therefore, we implicitly assume that consumers do not take into account current marketing information

Table 4.1: Model Specifications

	Model1	Model2	Model3
V_{1j}	Brand intercepts.	Brand intercepts and marketing variables.	Brand intercepts; marketing variables if the brand is featured.
V_{2j}	Brand intercepts and marketing variables.	Brand intercepts and marketing variables.	Brand intercepts and marketing variables.
S_k	Featuring activity and other variables.	Featuring activity and other variables.	Featuring activity and other variables.

when they first screen some of the brands out of their choice sets. In the second stage, consumers search all the brands that survive the screening.

Of the models presented here, this is the closest to Model E in Murthi and Srinivasan (Table 3 pp. 247). The results obtained are also the closest to Model E, in particular I get the same sign for the effect of feature activity, whether there is a female head of household employed full-time, and income, on the probability that a consumer chooses from larger choice set. In both models all estimates are statistically significant.

Model 2 is the GenL model applied to the fresh orange juice category. In estimating this model I am implicitly assuming that current marketing information is considered when determining the composition of choice sets, since all marketing variables are present in each of the two stages. This model will capture the behavior of consumers who could be inclined to search only a restricted subset of the available brands because, for any reason that is not related to search costs, he is captive to that subset. A hypothetical example: suppose that for some

medical reason a subset of the consumers cannot tolerate orange juice from concentrate. Then, no matter what value the marketing variables take, this subset of consumers will not search from-concentrate orange juice brands. In our case, this subset of consumers will not react to changes in utility associated to Minute Maid and will not search choice sets that contain this brand.

Finally, I begin the analysis of Model 3 with the first stage. I assume that consumers are ignorant about the price associated to each brand unless the brand is featured. If a particular brand is featured then a consumer will see the feature with positive probability. After being exposed to that information, the consumer must decide if he is going to keep it in his memory, and if it is worthwhile stopping his search for information about the other brands or not. For example, if a consumer decides, after learning the featured information on the first brand that it is not worthwhile to continue searching then his choice set will have only one element.

In this specification the consumer is characterized as someone who is exposed to the feature with positive probability. Conditional on the consumer being exposed, featuring introduces the featured brand in his choice set.

It is possible that the consumer is exposed to the feature, and that conditional on this he does search the featured brands but also he searches non featured brands. Since being exposed to a feature translates into a reduction in the time spent in searching the featured product, a feature will reduce the consumers' search costs and can have a positive effect on the size of the choice set finally considered. This effect will make more likely that a consumer searches a choice set larger than the number of brands featured. Again, the simplest example would be when consumers are subject to a time constraint that limits the number of

brands that they wish to search.

Finally, it is possible that the consumer is not exposed to the feature and that conditional on this he does not search the featured brands. This specification captures this event by estimating the probability associated to each choice set finally searched. Then, this specification allows for the possibility that the choice set made of all featured brands is searched with a probability lower than one.

The characterization described in the previous paragraphs is reflected in the specification of V_{1j} . As an example, let us assume that only Tropicana was featured in a given week and that the feature included price information. Then, consumers only know the price for Tropicana and must form expectations about the other brands price. This expectation can be captured by a brand specific dummy variable that takes value 1 when that brand is being evaluated and it was not featured, as long as that expectation is invariant across purchase occasions. This dummy variable not only captures the price expected value but other invariant attributes associated with the brand, like its quality or the fact that that brand was not featured.

$$V_{1,Florida} = \beta_1$$

$$V_{1,MMaid} = \beta_2$$

$$V_{1,PLabel} = \beta_3$$

$$V_{1,Tropic} = \beta_{11}Price_{Tropic} + \beta_8Price_{Tropic}Income + \beta_9Price_{Tropic}Size + \\ + \beta_{10}Price_{Tropic}Fful + \beta_{12}Feat_{Tropic}$$

$$S_k = \beta_{13}Set(3,4)Activity + \beta_{14}Set(3,4)Size + \beta_{15}Set(3,4)Fful + \\ + \beta_{16}Set(3,4)Income$$

Where, $Price_{Tropic}$ is Tropicana's price, $Income$ is the buyer's income, $Size$ is the size of the buyer's family, $Fful$ is a dummy that takes value 1 if the female

head of household is working full time, and $Feat_{Tropic}$ is a dummy that takes a value 1 if Tropicana was featured on that particular purchase occasion. Finally, $Set(3, 4)$ is a dummy that takes the value 1 when the choice set has more than two brands in it and $Activity$ is the number of brands that were advertised in that particular purchase occasion.

In the second stage I assume that consumers know all the available information associated to the brands included in the restricted choice set. I expect Model 3 to capture the “prestige” effect, through the coefficient β_{12} and the informational effect, through the coefficient β_{13} . Regarding the informational effect, β_{13} should be positive if learning the information through featuring implies a saving in time compared with learning it from direct inspection.

4.6 Data

I use a household-level panel data on beverages purchases to answer my research questions. The panel dataset was collected by Information Resources, Inc (IRI) and was made available by the Economic Research Service at the US Department of Agriculture (ERS-USDA). The data set spans over three years, 1997, 1998 and 1999. Each purchase made by each member of a cross-section of previously drafted households was recorded at the check. In order to estimate our model we had access to a sample containing 45975 purchase observations of 64 oz cartons of orange juice made by 1942 households at the retail chain Stop’n’Shop in the city of Pittsfield, Massachusetts . The brands analyzed are Minute Maid, Tropicana, Florida’s Natural and the supermarket private label. I observe which brand was purchased in a given day, the price paid as well as the price of the other three brands, which brands were featured in a store circular or advertisement on a

Table 4.2: Descriptive Statistics

Variable		Mean	Std. Dev.
Florida's Natural	market share	0.16	
Minute Maid	market share	0.11	
Private Label	market share	0.37	
Tropicana	market share	0.37	
Price Florida's Natural	dollar	2.61	0.27
Price Minute Maid	dollar	2.55	0.17
Price Private Label	dollar	1.77	0.20
Price Tropicana	dollar	2.85	0.23
Feature Florida's Natural	dummy	0.19	
Feature Minute Maid	dummy	0.15	
Feature Private Label	dummy	0.32	
Feature Tropicana	dummy	0.36	

particular week, and which brands had issued coupons on a particular week as well as who used those coupons. I also know if the purchase was made or not on a weekend, how much the consumer spent on each trip to the store and the income of the consumer, among other demographic variables.

The use of a discrete model of brand choice is supported by the fact that households typically buy only one brand in each purchase occasion. In the sample, households bought only one brand 96% of the purchase occasions.

Table 4.2 shows the descriptive statistics associated with a subset of the variables considered.

Our four brands accounted for more than 95% of the category brand choices.

Of the four brands analyzed, Tropicana and Minute Maid are well-established household brands while Florida's Natural has a more recent history though it has become a very strong player in the market of Not From Concentrate (NFC) orange juice behind Tropicana. Minute Maid is the only one made from concentrated (FC) juice. All three national brands use Florida oranges as opposed to other national brands like Sunkist, which use California oranges.

In Figure 4.1 we can see the behavior of each brand's weekly price.

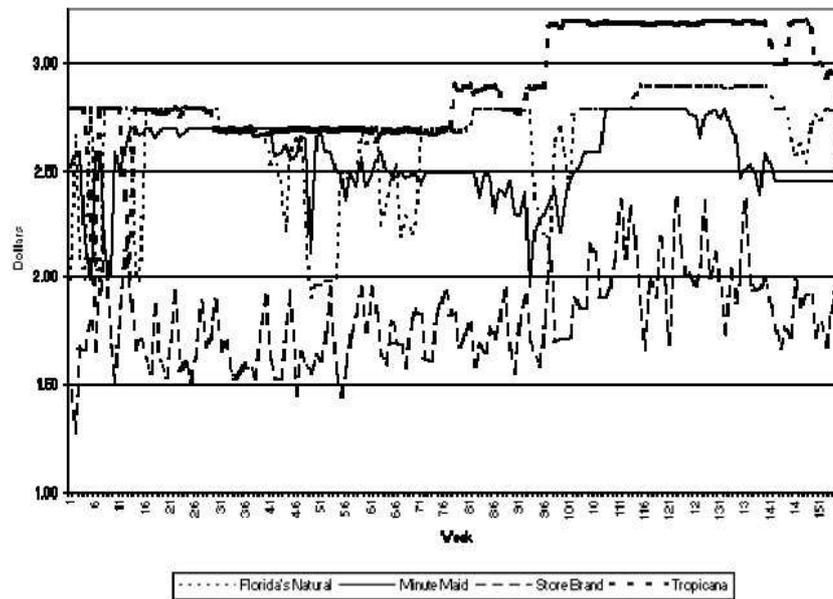


Figure 4.1: Weekly Prices

Using a discrete-choice framework simplifies the analysis although as a result there is no way to account for household inventory behavior, which means that we cannot perfectly assimilate brand preference share among households with market share. In the Table 4.3 I show the frequency of multiunit buys.

Table 4.3: Multiunit Buys Frequency

Variable	Frequency	Relative Frequency
1 unit	30631	66.6%
2 units	12991	28.2%
3 units	1036	2.3%
4 units	934	2.0%
5 units or more	506	0.9%

Two-thirds of the purchases, consumers bought only one unit. Also, 66.2% of the households bought one and two units on at least once occasion. Although we cannot perfectly assimilate brand preference share behavior with market share behavior, the data shows that consumers do not respond stockpiling large quantities when a brand is on sale. Behind this behavior there is the fact that, as with all products that need refrigeration and have a relatively large size relative to its price, refrigerated orange juice probably has a much higher storage opportunity cost than products that can be stored in a closet or basement.

During the period covered by this sample, featuring of at least one brand occurred 98.4% of the purchase occasions. Simultaneous featuring is a rare event in this sample: simultaneous featuring occurred only 2.7% of the purchase occasions. Since we want to identify the effect of single and simultaneous featuring on choice set size choice, we used a large sample in order to avoid the event of a sample with no multiple featuring in it. Table 4.4 shows the frequency of multiple featuring in my sample.

Table 4.4: Simultaneous Featuring Frequency

Variable	Frequency	Relative Frequency
0 Brands Featured	795	1.7%
1 Brand Featured	43936	95.6%
2 Brands Featured	926	2.0%
3 Brands Featured	318	0.7%
4 Brands Featured	0	0.0%

4.7 History and Stylized Facts in the Fresh Orange Juice Market

The modern history of orange juice starts in 1954 when Tropicana introduced flash pasteurization of juice, which extended the shelf life while retaining the fresh taste. This allowed the company to expand its geographic market beyond Florida (e.g., the market of New York upscale hotels). Until then, orange juice drinkers outside the growing areas had only two options; they could mix frozen concentrate with water or squeeze the oranges on their own.

Effectively, concentrated orange juice dates back to the early forties. The U.S. Army was one of the first customers of this new product (in the form of powder and concentrated syrup). The powder was first produced in 1945, following the same process that the National Research Corporation of Boston, Massachusetts (they would change its name to Florida Foods Corporation first and then to The Minute Maid Company) had utilized for dehydrating penicillin among other things.

After the WWII, and seeing its sales plunge, the company started exploring

the potential of the product for the household market, trying different product specifications. Finally, the chosen one was in the form of frozen orange juice liquid concentrate, which was sold under the name Minute Maid.

In the late 40's the brand had been nationally established, with Bing Crosby advertising it in his radio broadcasts. In 1973, the company (already bought by Coca Cola in 1960) introduced the ready-to-drink orange juice from concentrate. Finally, in 1996 the product was further improved and started being sold as Minute Maid Premium.

Florida's Natural is a relatively recent player in the market for NFC orange juice. The company history dates back to 1934, though its first inroad in the orange juice market came when the company started producing concentrated orange juice for the U.S. Army in 1943. The company did not launch its own consumer brand until 1987, when they entered the household market with Florida's Natural brand (a NFC orange juice). National recognition as a household brand was secured in 1991, after the firm launched its first television ad campaign.

Finally, Stop'N'Shop store brand is called Sunrise Valley. It is made from concentrate from Florida and Brazil oranges.

The main stylized facts in this market are:

- Tropicana is the leading national brand in the NFC segment of the market.
- Tropicana is leader in product development in the NFC segment of the market.
- Minute Maid is the leading national brand in the FC segment of the market.
- Minute Maid is the leader in product development in the FC segment of the market.

- Ceteris paribus, the consumers perceive NFC juices as the highest quality segment in this market.
- Florida's Natural is the second national brand in the NFC segment of the market, with a product line that replicates Tropicana's.
- Florida oranges are the most praised among consumers.
- Ceteris paribus, Stop'n'Shop store brand should be regarded as the lowest quality brand due to the fact that it is from concentrate and does not use Florida oranges exclusively.

4.8 Estimation and Results

I estimate the models using Maximum Likelihood. Table 4.5 shows the results for the three models compared against a standard Multinomial Logit specification. Only the coefficients of the representative utility function and S_k are shown. Coefficients with a double asterisk means that they are statistically significant at the one percent level while a single asterisk means significance at the five percent level.

A coefficient without asterisks means that the coefficient was not estimated but is the result of a restriction introduced in the model's specification. For example, in models 1 and 2 each brand intercept was restricted to be equal in both stages. In both cases we show the coefficients twice, in stage 1 and in stage 2, but we show the asterisks only once.

Table 4.5: Estimation Results

Variables	MNL	Model 1	Model 2	Model 3
<i>First Stage Brand Intercepts</i>				
F Natural	-	-1.27**	-0.59**	-2.79**
M Maid	-	-1.53**	-0.71**	-1.48**
Private Label	-	-0.76**	-0.44**	-0.90**
Tropicana	-	-	-	0.12
<i>Second Stage Brand Intercepts</i>				
F Natural	-0.80**	-1.27	-0.59	-0.16**
M Maid	-1.13**	-1.53	-0.76	-0.34**
Private Label	-0.42**	-0.76	-0.46	-0.06*
Tropicana	-	-	-	-
<i>Marketing and Other Variables</i>				
Price	-0.54**	-0.96**	-0.32**	-0.22**
Feat	0.39**	8.87**	0.59**	0.85**
Price*Feat	0.37**	-1.85**	0.18**	-0.22**
Price*Size	-0.18**	-0.30**	-0.11**	-0.05**
Price*Fful	-0.18**	-0.56**	-0.15**	-0.12**
Price*Income	0.07**	0.13**	0.05**	0.01**
<i>Choice Set Variables</i>				
Set(3,4)*Activity	-	0.53**	0.76**	0.37**
Set(3,4)*Size	-	0.20**	-0.04	0.44**
Set(3,4)*Fful	-	-0.51**	-0.32**	0.03
Set(3,4)*Income	-	-0.10**	-0.89**	-0.32**
- Log Likelihood	11030.7	10925.3	10918.1	10877.2

4.8.1 Nested Models

The MNL model is nested in Model 1 (four restrictions) and Model 2 (seven restrictions). Model 1 is nested in Model 2. Model 1 and 2 are not nested in Model 3. Based on the likelihood ratio test (LR) we reject the MNL model in favor of Model 1 and Model 2 (LR statistic for Model 1 vs. MNL is 210.6; LR statistic for Model 2 vs. MNL is 225.2), and Model 1 in favor of Model 2 (LR statistics for Model 1 vs. Model 2 is 14.4).

Rejection of the MNL model allows us to conclude that in any purchase occasion a consumer will pick his preferred brand from a restricted set of alternatives, instead of considering all the available options, with positive probability.

4.8.2 Non-Nested Models: Quality of Fit

As discussed in Guadagni and Little, probabilistic models pose special difficulties in overall evaluation especially when the models are non-nested. I use here McFadden's likelihood ratio index, ρ , and Betancourt and Clague's [7] measure of predictive performance, I , as a quality of fit measure (for discussions on measures of quality of fit see Greene pp. 831-834 [18], Guadagni and Little pp. 210-211 [20] and Betancourt and Clague pp. 86).

The likelihood ratio index is:

$$\rho^2(m) = 1 - \frac{L(m)}{L(0)}$$

where $L(m)$ is the mean log likelihood of the model the researcher proposes,

Table 4.6: Quality of Fit

Measure of Fit	MNL	Model 1	Model 2	Model 3
ρ	0.244	0.251	0.252	0.255
I	0.096	0.093	0.096	0.108

and $L(0)$ is the mean log likelihood of the null model. When the null model is the one that defines maximum entropy for the particular case, $\rho^2(m)$ describes the fraction of uncertainty empirically explained by the model proposed by the researcher relative to the null model. Here, the null is a Multinomial Logit Model with only brand intercepts as explanatory variables.

Betancourt and Clague's is a summary measure that scores each prediction by giving it points not only in accordance with whether the prediction is right or wrong but also in a way that reflects the degree of certainty of the prediction. This measure penalizes incorrect predictions in the same way. For example, in the dichotomous case, a higher score is given to a correct prediction that is close to 1 than to a correct prediction that is close to 0.5 and a lighter penalty is associated to an incorrect prediction that is close to 0.5 than to an incorrect prediction that is close to 1. The measure also includes a degrees-of-freedom correction that penalizes specifications with more explanatory variables.

The fact that Model 3 best fits the data suggests that in deciding what alternatives to include in his choice set, a consumer sometimes does not use current available marketing information about each available alternative. When it was

assumed that, in the first stage, consumers used a strict subset of the available information (Model 3) the model fitted better the sample information than when it was assumed that consumers used all the available information (Model 2), even after penalizing for the use of three more degrees of freedom.

4.8.3 Coefficients of Model 3

The coefficient for the dummy that takes value 1 when the brand is featured, β_8 , takes a positive value. This suggests that consumers gain utility from a particular brand being advertised. This is consistent with the works by Stigler and Becker and Becker and Murphy and with all empirical works cited in this paper.

The value of the coefficient associated with the variable that captures the effect of featuring on choice set choice is positive and statistically significant, which suggests that featuring activity has a positive effect on search and evaluation activity. This result is consistent with findings by Murthi and Srinivasan and Akerberg.

The estimated price coefficient is negative and statistically significant, as expected. The sign of the coefficients associated with the interaction between income and price and income and choice set size indicate that consumers with higher income are less sensitive to price changes and choose from smaller choice sets. This result is consistent with the hypothesis that more affluent consumers have higher opportunity costs associated to search.

The size of the household has two effects: larger households are more sensitive to price differentials than smaller ones, for any given choice set size, and are also more prone to pick from equal or larger choice sets.

Finally, the set of households where the female head of household is working full time is more price sensitive than its complement, for any choice set. The effect on choice set size is statistically insignificant.

4.8.4 Robustness of the Results

The principal hypotheses consider the effect of featuring on the size of the set of alternatives finally evaluated in all their attributes. I estimated a number of different specifications in order to check the robustness of our results, three of them are shown in Table 4.5. In addition, I show a variation of Model 3 in Appendix F, where we interacted the variable with dummy variables that account for the number of brands that were being featured at the moment of the purchase. We refer to this specification as Model 4.

Model 4 fits the data as well as Model 3. Of all the interaction terms, the only one that is positive and significant is the one associated with two brands being simultaneously featured. The rest of the coefficients are positive but not significant.

In the case of the interaction between the dummies that identify large choice sets and purchase occasions when only one brand was featured, the coefficient

is positive but relatively small and not significant. In this case I am inclined to conclude that in the population there is not a positive effect from one-brand featuring. In the case of the interaction between the dummies that identify large choice sets and purchase occasions when two brands were featured, the coefficient is positive, relatively large and significant. Finally, in the case of the interaction between the dummies that identify large choice sets and purchase occasions when three brands were featured, the coefficient is positive, the largest of all coefficients but statistically insignificant. This leaves open the possibility that the effect is positive in the population, but that we were unable to identify it due to the small number of purchase occasions where three brands were simultaneously featured.

4.9 Marginal Effects and Elasticities: The Two Faces of Featuring

In the next table I show the effects of simultaneous featuring on brand probability and choice set probability. It is possible to see that when a brand is featured its choice probability increases dramatically. Regarding the choice set probabilities we see how larger choice sets see their probabilities increased when simultaneous featuring takes place. The final effect on choice set probability is a function of :

- the effect of featuring on brand's utility,
- the effect of featuring activity on choice set size, and

- the weight associated to choice set that captures the level of correlation among the alternatives inside that choice set.

Regarding the change in choice set probability when there is no featuring versus when there is featuring it is interesting to note that, despite the positive effect of featuring on choice set size, the choice set with Florida's Natural as the only element sees its probability increased after the simultaneous featuring. The explanation comes from the fact that Florida's Natural brand intercept when this brand is not featured is negative and relatively large, meaning that this brand suffers the most when it is not featured. As a consequence, the positive effect of being featured on its utility and then on the utility of all choice sets that include it is larger than the negative effect that featuring has on the probability associated to choice sets of size one and two. The probabilities associated to small choice sets for the remaining brands change in an expected way when simultaneous featuring takes place.

When all brands are simultaneously featured, Model 3 boils down to the model implemented by Swait. In this case and for a particular brand, an increment in the probability associated with large choice sets that contain this brand will have a positive impact in the marginal effect of price:

- the larger the weight of the large choice set, i.e. the larger the correlation among the random terms

Table 4.7: Effect of Featuring

	No Featuring	FN MM Featured	All Featured
<i>Brand Probability</i>			
P[FN]	0.132	0.473	0.173
P[MM]	0.094	0.313	0.105
P[PL]	0.420	0.130	0.496
P[Tr]	0.334	0.084	0.225
<i>Choice Set Probability</i>			
ChS1: Tropic	0.100	0.045	0.026
ChS2: PL	0.051	0.023	0.034
ChS3: PL Tr	0.100	0.045	0.035
ChS4: MM	0.029	0.044	0.021
ChS5: MM Tr	0.100	0.050	0.027
ChS6: MM PLabel	0.051	0.044	0.035
ChS7: MM PLabel Tr	0.087	0.100	0.151
ChS8: FN	0.008	0.052	0.024
ChS9: FN Tr	0.100	0.055	0.028
ChS10: FN PLabel	0.051	0.052	0.035
ChS11: FN PLabel Tr	0.086	0.109	0.155
ChS12: FN MM	0.029	0.055	0.025
ChS13: FN MM Tr	0.083	0.120	0.128
ChS14: FN MM PL	0.045	0.109	0.147
ChS15: FN MM PL Tr	0.082	0.097	0.130

- if the probability associated to large choice sets that contain this brand is large,
- if the probability that this brand is chosen from the large choice set is close to one half.

As the weight increases, the positive correlation among the random terms increase. When this happens, the observed part of the utility becomes more important as a determinant of brand choice. The second bullet says that the effect of an increment in the probability associated to large choice sets that contain the brand is increasing. Finally, all specifications derived from the standard multinomial logit model share the property stated in the third bullet.

In the next table we show direct and cross price marginal effects and elasticities, for a four-member household with an income of sixty-thousand dollars a year and where the female head of household is not employed full-time, evaluated at the average prices.

We can see that, when comparing No Feature against All Featured the direct price elasticity of the probability for Florida's Natural increases, in absolute value, from -2.12 to -2.47 , while the cross price elasticity of the probability for Florida's Natural with respect to Tropicana's price decreases from 1.34 to 0.84 .

In Model 3, the effect of price on brand utility is almost the same whether the brand was featured or not. Thus, the changes seen in Table 4.8 in the direct and cross marginal effects of the price on brand probability are explained exclusively

Table 4.8: Direct and Cross Price Marginal Effects and Elasticities of Share (Model 3)

	No Feature Mg. Effect	No Feature Elasticity	All Featured Mg. Effect	All Featured Elasticity*
FNatural	-0.107	-2.12	-0.125	-2.47
vs. MMaid	0.016	0.31	0.022	0.43
vs. PLabel	0.030	0.40	0.064	0.86
vs. Tropicana	0.062	1.34	0.039	0.84
MMaid	-0.069	-1.87	-0.081	-2.19
vs. FNatural	0.016	0.44	0.022	0.61
vs. PLabel	0.015	0.28	0.036	0.68
vs. Tropicana	0.039	1.18	0.024	0.74
PLabel	-0.113	-0.48	-0.189	-0.80
vs. FNatural	0.030	0.19	0.064	0.40
vs. MMaid	0.015	0.09	0.036	0.22
vs. Tropicana	0.068	0.46	0.089	0.60
Tropicana	-0.168	-1.43	-0.152	-1.30
vs. FNatural	0.062	0.49	0.039	0.31
vs. MMaid	0.039	0.30	0.024	0.18
vs. PLabel	0.068	0.36	0.089	0.47

by the increase in search activity, i.e. the increment in the size of the choice set considered, on the part of the consumers.

The most obvious regularity we found is the increase in the cross price marginal effects between any brand and the Private Label. In this particular case this regularity is consistent with the fact that featuring made more likely that consumers consider choice sets including the Private Label and any of its competitors relative to one element choice sets.

At the aggregate level and after simultaneous featuring, the sum of the marginal effects and the elasticities (in absolute value) is close to twenty percent larger in the case of the former and close to twelve percent larger in the case of the latter. Then, we can conclude that featuring has a competition enhancing effect on this category.

4.10 Summary of Findings and Economic Implications

Regarding the questions discussed in the introduction, my findings suggest that in any purchase occasion a consumer will pick his preferred brand from a restricted set of alternatives, instead of searching all the available options, with positive probability. Also, that in deciding what alternatives to include in the choice set, sometimes consumers will not use current available marketing information about

each available alternative. When it was assumed that in the first stage consumers used a strict subset of the available information (Model 3) the model fitted better the sample information than when it was assumed that consumers used all the available information (Model 2) or nothing (Model 1). Finally, unlike the previous literature, I was able to identify the effect of simultaneous featuring on the size of the choice set searched. I found that, most probably, the information provided through featuring affects positively the size of the choice set considered.

The fact that the informational effect increases with the number of brands featured has important economic implications for each firm, especially the manufacturers of the branded products. If we assume that in order for a retailer to feature a brand he needs to obtain authorization from the manufacturer, the informational and “prestige” effect could explain why simultaneous featuring is such a rare event in this sample.

The prestige effect acts as a vertical characteristic, since it increases the utility associated to the featured brand. If consumers’ valuation for the feature differs, it could be profitable for the competing firms to differentiate from each other by avoiding simultaneous featuring, in order to reduce competition among brands.

Appendix A

Definition of Relevant Regions

A.1 Asymmetric Characteristics Competition and Characteristic S Dominance

This case is characterized by the condition:

$$S_1 - S_2 \geq D_2 - D_1 \geq 0 \tag{A.1}$$

A.1.1 R_{s2}

In order to be in region R_{s2} it must be true that:

$$(w_1 + x_1) \in [(w_1 + x_1)^n, (w_1 + x_1)^m] \tag{A.2}$$

$$(w_2 + x_2) \in [(w_2 + x_2)^n, (w_2 + x_2)^m] \quad (\text{A.3})$$

It is easy to show that A.2 and A.3 hold if and only if $S_1 - S_2 \geq D_2 - D_1 \geq 0$. Thus, *Rs2* is the only region in which Assymmetric Characteristic Competition and Characteristic S Dominance is satisfied in equilibrium. Then we do not need to calculate the equilibrium prices for regions *Rs1* and *Rs3* because the equilibrium prices will violate the condition $S_1 - S_2 \geq D_2 - D_1 \geq 0$.

A.2 Assymmetric Characteristics Competition and Characteristic D Dominance

This case is characterized by the condition:

$$D_2 - D_1 \geq S_1 - S_2 \geq 0 \quad (\text{A.4})$$

A.2.1 Rd2

In order to be in region *Rd2* it must be true that:

$$(w_1 + x_1) \in [(w_1 + x_1)^m, (w_1 + x_1)^n] \quad (\text{A.5})$$

$$(w_2 + x_2) \in [(w_2 + x_2)^m, (w_2 + x_2)^n] \quad (\text{A.6})$$

It is easy to show that A.5 and A.6 hold if and only if $0 \leq S_1 - S_2 \leq D_2 - D_1$. Thus, *Rd2* is the only region in which Assymmetric Characteristic Competition and Charactersitic D Dominance is satisfied in equilibrium.

A.3 Dominated Characteristics Competition and Characteristic S Dominance

This case is characterized by the condition:

$$S_2 - S_1 \geq D_2 - D_1 \geq 0 \tag{A.7}$$

A.3.1 dRs1

In order to be in region *dRs1* it must be true that:

$$(w_1 + x_1) \in \left[(w_1 + x_1)^l, (w_1 + x_1)^n \right] \tag{A.8}$$

$$(w_2 + x_2) \in \left[(w_2 + x_2)^l, (w_2 + x_2)^n \right] \tag{A.9}$$

It is easy to show that A.8 and A.9 hold if and only if $0 \leq \frac{2}{3}(S_2 - S_1) \leq D_2 - D_1$.

A.3.2 dRs2

In order to be in region $dRs2$ it must be true that:

$$(w_1 + x_1) \in \left[(w_1 + x_1)^u, (w_1 + x_1)^l \right] \quad (\text{A.10})$$

$$(w_2 + x_2) \in \left[(w_2 + x_2)^u, (w_2 + x_2)^l \right] \quad (\text{A.11})$$

It is easy to show that A.10 and A.11 hold if and only if $\frac{2}{3}(S_2 - S_1) \geq D_2 - D_1 \geq 0$. From this and the previous condition, there cannot be a Dominated S Characteristic price equilibrium in region $dRs3$.

A.4 Dominated Characteristics Competition and Characteristic D Dominance

This case is characterized by the condition:

$$D_2 - D_1 \geq S_2 - S_1 \geq 0 \quad (\text{A.12})$$

A.4.1 dRd1

In order to be in region $dRd1$ it must be true that:

$$(w_1 + x_1) \in [(w_1 + x_1)^u, (w_1 + x_1)^n] \quad (\text{A.13})$$

$$(w_2 + x_2) \in [(w_2 + x_2)^u, (w_2 + x_2)^n] \quad (\text{A.14})$$

It is easy to show that A.13 and A.14 hold if and only if $\frac{3}{2}(S_2 - S_1) \geq D_2 - D_1 \geq 0$.

A.4.2 dRd2

In order to be in region *dRd2* it must be true that:

$$(w_1 + x_1) \in [(w_1 + x_1)^l, (w_1 + x_1)^u] \quad (\text{A.15})$$

$$(w_2 + x_2) \in [(w_2 + x_2)^l, (w_2 + x_2)^u] \quad (\text{A.16})$$

It is easy to show that A.15 and A.16 hold if and only if $0 \leq \frac{3}{2}(S_2 - S_1) \leq D_2 - D_1$. From this and the previous condition, there cannot be a Characteristic D Dominance price equilibrium in region *dRd3*.

Appendix B

Demands, Profit Functions and Price Equilibria.

In this appendix I present the demand , profit functions and price equilibria for each one of the relevant regions as defined in the Appendix A.

Asymmetric Characteristic and Characteristic S Dominance ($0 < D_2 - D_1 < S_1 - S_2$)

$$D_{Rs1}^1 = \frac{(S_1 - S_2 - w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rs2}^1 = \frac{-D_1 + D_2}{2(S_1 - S_2)} + \frac{D_1 - D_2 + S_1 - S_2 - w_1 + w_2 - x_1 + x_2}{S_1 - S_2}$$

$$D_{Rs3}^1 = 1 - \frac{(-D_1 + D_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rs1}^2 = 1 - \frac{(S_1 - S_2 - w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rs2}^2 = 1 - \frac{-D_1 + D_2}{2(S_1 - S_2)} - \frac{D_1 - D_2 + S_1 - S_2 - w_1 + w_2 - x_1 + x_2}{S_1 - S_2}$$

$$D_{Rs3}^2 = \frac{(-D_1 + D_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

Price Equilibrium in Region Rs2

Retailer 1

$$\Pi_{Rs2}^{R1} = \frac{x_1(D_1 - D_2 + 2(S_1 - S_2 - w_1 + w_2 - x_1 + x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{Rs2}^{R1}}{\partial x_1} = \frac{D_1 - D_2 + 2(S_1 - S_2 - w_1 + w_2 - 2x_1 + x_2)}{2(S_1 - S_2)} = 0$$

Retailer 2

$$\Pi_{Rs2}^{R2} = -\frac{(D_1 - D_2 - 2(w_1 - w_2 + x_1 - x_2))x_2}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{Rs2}^{R2}}{\partial x_2} = \frac{D_1 - D_2 - 2w_1 + 2w_2 - 2x_1 + 4x_2}{-2S_1 + 2S_2} = 0$$

Manufacturer 1

$$\Pi_{Rs2}^{M1} = \frac{w_1(D_1 - D_2 + 2(S_1 - S_2 - w_1 + w_2 - x_1 + x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{Rs2}^{M1}}{\partial w_1} = \frac{D_1 - D_2 + 2(S_1 - S_2 - 2w_1 + w_2 - x_1 + x_2)}{2(S_1 - S_2)} = 0$$

Manufacturer 2

$$\Pi_{Rs2}^{M2} = -\frac{w_2(D_1 - D_2 - 2(w_1 - w_2 + x_1 - x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{Rs2}^{M2}}{\partial w_2} = \frac{D_1 - D_2 - 2w_1 + 4w_2 - 2x_1 + 2x_2}{-2S_1 + 2S_2} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium

$$x_1 = \frac{1}{10}(D_1 - D_2 + 6S_1 - 6S_2), x_2 = \frac{1}{10}(-D_1 + D_2 + 4S_1 - 4S_2)$$

$$w_1 = \frac{1}{10}(D_1 - D_2 + 6S_1 - 6S_2), w_2 = \frac{1}{10}(-D_1 + D_2 + 4S_1 - 4S_2)$$

Asymmetric Characteristic and Characteristic D Dominance ($0 < S_1 - S_2 < D_2 - D_1$)

$$D_{Rd1}^1 = \frac{(S_1 - S_2 - w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rd2}^1 = \frac{S_1 - S_2}{2(-D_1 + D_2)} + \frac{-w_1 + w_2 - x_1 + x_2}{-D_1 + D_2}$$

$$D_{Rd3}^1 = 1 - \frac{(-D_1 + D_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rd1}^2 = 1 - \frac{(S_1 - S_2 - w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

$$D_{Rd2}^2 = 1 - \frac{S_1 - S_2}{2(-D_1 + D_2)} - \frac{-w_1 + w_2 - x_1 + x_2}{-D_1 + D_2}$$

$$D_{Rd3}^2 = \frac{(-D_1 + D_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(S_1 - S_2)}$$

Price Equilibrium in Region Rd2

Retailer 1

$$\Pi_{Rd2}^{R1} = \frac{x_1(-S_1 + S_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{R1}}{\partial x_1} = \frac{S_1 - S_2 + 2(-w_1 + w_2 - 2x_1 + x_2)}{-2D_1 + 2D_2} = 0$$

Retailer 2

$$\Pi_{Rd2}^{R2} = \frac{x_2(2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 2x_2)}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{R2}}{\partial x_2} = \frac{2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 4x_2}{2D_1 - 2D_2} = 0$$

Manufacturer 1

$$\Pi_{Rd2}^{M1} = \frac{w_1(-S_1 + S_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{M1}}{\partial w_1} = \frac{S_1 - S_2 + 2(-2w_1 + w_2 - x_1 + x_2)}{-2D_1 + 2D_2} = 0$$

Manufacturer 2

$$\Pi_{Rd2}^{M2} = \frac{w_2(2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 2x_2)}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{M2}}{\partial w_2} = \frac{2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 4w_2 - 2x_1 + 2x_2}{2D_1 - 2D_2} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium

$$x_1 = \frac{1}{10}(-4D_1 + 4D_2 + S_1 - S_2), x_2 = \frac{1}{10}(-6D_1 + 6D_2 - S_1 + S_2)$$

$$w_1 = \frac{1}{10}(-4D_1 + 4D_2 + S_1 - S_2), w_2 = \frac{1}{10}(-6D_1 + 6D_2 - S_1 + S_2)$$

Dominated Characteristic and Characteristic S Dominance ($0 < D_2 - D_1 < S_2 - S_1$)

$$D_{dRs1}^1 = \frac{(-w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRs2}^1 = \frac{-D_1 + D_2}{2(-S_1 + S_2)} + \frac{D_1 - D_2 - w_1 + w_2 - x_1 + x_2}{-S_1 + S_2}$$

$$D_{dRs3}^1 = 1 - \frac{(-D_1 + D_2 - S_1 + S_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRs1}^2 = 1 - \frac{(-w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRs2}^2 = 1 - \frac{-D_1 + D_2}{2(-S_1 + S_2)} - \frac{D_1 - D_2 - w_1 + w_2 - x_1 + x_2}{-S_1 + S_2}$$

$$D_{dRs3}^2 = \frac{(-D_1 + D_2 - S_1 + S_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

Price Equilibrium in Region dRs2

Retailer 1

$$\Pi_{dRs2}^{R1} = \frac{x_1(-D_1 + D_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{R1}}{\partial x_1} = \frac{D_1 - D_2 + 2(-w_1 + w_2 - 2x_1 + x_2)}{-2S_1 + 2S_2} = 0$$

Retailer 2

$$\Pi_{dRs2}^{R2} = \frac{x_2(D_1 - D_2 + 2(S_1 - S_2 - w_1 + w_2 - x_1 + x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{R2}}{\partial x_2} = \frac{D_1 - D_2 + 2S_1 - 2S_2 - 2w_1 + 2w_2 - 2x_1 + 4x_2}{2S_1 - 2S_2} = 0$$

Manufacturer 1

$$\Pi_{dRs2}^{M1} = \frac{w_1(-D_1 + D_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{M1}}{\partial w_1} = \frac{D_1 - D_2 + 2(-2w_1 + w_2 - x_1 + x_2)}{-2S_1 + 2S_2} = 0$$

Manufacturer 2

$$\Pi_{dRs2}^{M2} = \frac{w_2(D_1 - D_2 + 2(S_1 - S_2 - w_1 + w_2 - x_1 + x_2))}{2(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{M2}}{\partial w_2} = \frac{D_1 - D_2 + 2S_1 - 2S_2 - 2w_1 + 4w_2 - 2x_1 + 2x_2}{2S_1 - 2S_2} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium in Region dRs2

$$x_1 = \frac{1}{10}(D_1 - D_2 - 4S_1 + 4S_2), x_2 = \frac{1}{10}(-D_1 + D_2 - 6S_1 + 6S_2)$$

$$w_1 = \frac{1}{10}(D_1 - D_2 - 4S_1 + 4S_2), w_2 = \frac{1}{10}(-D_1 + D_2 - 6S_1 + 6S_2)$$

Price Equilibrium in Region dRs1

Retailer 1

$$\Pi_{dRs1}^{R1} = \frac{x_1(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs1}^{R1}}{\partial x_1} = \frac{(w_1 - w_2 + x_1 - x_2)(w_1 - w_2 + 3x_1 - x_2)}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Retailer 2

$$\Pi_{dRs1}^{R2} = \left(1 - \frac{(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}\right) x_2$$

$$\frac{\partial \Pi_{dRs1}^{R2}}{\partial x_2} = \frac{2(D_1 - D_2)(S_1 - S_2) - (w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Manufacturer 1

$$\Pi_{dRs1}^{M1} = \frac{w_1(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs1}^{M1}}{\partial w_1} = \frac{(w_1 - w_2 + x_1 - x_2)(3w_1 - w_2 + x_1 - x_2)}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Manufacturer 2

$$\Pi_{dRs1}^{M2} = w_2 \left(1 - \frac{(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}\right)$$

$$\frac{\partial \Pi_{dRs1}^{M2}}{\partial w_2} = \frac{2(D_1 - D_2)(S_1 - S_2) - (w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium in Region dRs1

$$x_1 = -\frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}, w_1 = -\frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}$$

$$x_2 = -\sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}, w_2 = -\sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}$$

$$x_1 = \frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}, w_1 = \frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}$$

$$x_2 = \sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}, w_2 = \sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}$$

Dominated Characteristic and Characteristic D Dominance ($0 < S_2 - S_1 < D_2 - D_1$)

$$D_{dRd1}^1 = \frac{(-w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRd2}^1 = \frac{-S_1 + S_2}{2(-D_1 + D_2)} + \frac{S_1 - S_2 - w_1 + w_2 - x_1 + x_2}{-D_1 + D_2}$$

$$D_{dRd3}^1 = 1 - \frac{(-D_1 + D_2 - S_1 + S_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRd1}^2 = 1 - \frac{(-w_1 + w_2 - x_1 + x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

$$D_{dRd2}^2 = 1 - \frac{-S_1 + S_2}{2(-D_1 + D_2)} - \frac{S_1 - S_2 - w_1 + w_2 - x_1 + x_2}{-D_1 + D_2}$$

$$D_{dRd3}^2 = \frac{(-D_1 + D_2 - S_1 + S_2 + w_1 - w_2 + x_1 - x_2)^2}{2(-D_1 + D_2)(-S_1 + S_2)}$$

Price Equilibrium in Region dRd2

Retailer 1

$$\Pi_{dRd2}^{R1} = \frac{x_1(-S_1 + S_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{R1}}{\partial x_1} = \frac{S_1 - S_2 + 2(-w_1 + w_2 - 2x_1 + x_2)}{-2D_1 + 2D_2} = 0$$

Retailer 2

$$\Pi_{dRd2}^{R2} = \frac{x_2(2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 2x_2)}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{R2}}{\partial x_2} = \frac{2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 4x_2}{2D_1 - 2D_2} = 0$$

Manufacturer 1

$$\Pi_{dRd2}^{M1} = \frac{w_1(-S_1 + S_2 + 2(w_1 - w_2 + x_1 - x_2))}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{M1}}{\partial w_1} = \frac{S_1 - S_2 + 2(-2w_1 + w_2 - x_1 + x_2)}{-2D_1 + 2D_2} = 0$$

Manufacturer 2

$$\Pi_{dRd2}^{M2} = \frac{w_2(2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 2w_2 - 2x_1 + 2x_2)}{2(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{M2}}{\partial w_2} = \frac{2D_1 - 2D_2 + S_1 - S_2 - 2w_1 + 4w_2 - 2x_1 + 2x_2}{2D_1 - 2D_2} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium in Region dRd2

$$x_1 = \frac{1}{10}(-4D_1 + 4D_2 + S_1 - S_2), x_2 = \frac{1}{10}(-6D_1 + 6D_2 - S_1 + S_2)$$

$$w_1 = \frac{1}{10}(-4D_1 + 4D_2 + S_1 - S_2), w_2 = \frac{1}{10}(-6D_1 + 6D_2 - S_1 + S_2)$$

Price Equilibrium in Region dRd1

Retailer 1

$$\Pi_{dRd1}^{R1} = \frac{x_1(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRd1}^{R1}}{\partial x_1} = \frac{(w_1 - w_2 + x_1 - x_2)(w_1 - w_2 + 3x_1 - x_2)}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Retailer 2

$$\Pi_{dRd1}^{R2} = \left(1 - \frac{(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}\right) x_2$$

$$\frac{\partial \Pi_{dRd1}^{R2}}{\partial x_2} = \frac{2(D_1 - D_2)(S_1 - S_2) - (w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Manufacturer 1

$$\Pi_{dRd1}^{M1} = \frac{w_1(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRd1}^{M1}}{\partial w_1} = \frac{(w_1 - w_2 + x_1 - x_2)(3w_1 - w_2 + x_1 - x_2)}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Manufacturer 2

$$\Pi_{dRd1}^{M2} = w_2 \left(1 - \frac{(w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)}\right)$$

$$\frac{\partial \Pi_{dRd1}^{M2}}{\partial w_2} = \frac{2(D_1 - D_2)(S_1 - S_2) - (w_1 - w_2 + x_1 - x_2)^2}{2(D_1 - D_2)(S_1 - S_2)} = 0$$

Retailer 1 and 2 and Manufacturer 1 and 2 Price Equilibrium in Region dRd1

$$x_1 = -\frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}, w_1 = -\frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}$$

$$x_2 = -\sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}, w_2 = -\sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}$$

$$x_1 = \frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}, w_1 = \frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{\sqrt{6}}$$

$$x_2 = \sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}, w_2 = \sqrt{\frac{2}{3}}\sqrt{(D_1 - D_2)(S_1 - S_2)}$$

Quality Equilibria

Updated Profit Functions in Each of the Relevant Regions

Retailer 1 Profits in Rs2

$$\Pi_{Rs2}^{R1} = \frac{(D_1 - D_2 + 6S_1 - 6S_2)^2}{100(S_1 - S_2)}$$
$$\frac{\partial \Pi_{Rs2}^{R1}}{\partial D_1} = \frac{1}{50} \left(6 + \frac{D_1 - D_2}{S_1 - S_2} \right) > 0$$

Retailer 2 Profits in Rs2

$$\Pi_{Rs2}^{R2} = \frac{(D_1 - D_2 - 4S_1 + 4S_2)^2}{100(S_1 - S_2)}$$
$$\frac{\partial \Pi_{Rs2}^{R2}}{\partial D_2} = \frac{1}{50} \left(4 + \frac{D_1 - D_2}{-S_1 + S_2} \right) > 0$$

Manufacturer 1 Profits in Rs2

$$\Pi_{Rs2}^{M1} = \frac{(D_1 - D_2 + 6S_1 - 6S_2)^2}{100(S_1 - S_2)}$$
$$\frac{\partial \Pi_{Rs2}^{M1}}{\partial S_1} = \frac{1}{100} \left(36 - \frac{(D_1 - D_2)^2}{(S_1 - S_2)^2} \right) > 0$$

Manufacturer 2 Profits in Rs2

$$\Pi_{Rs2}^{M2} = \frac{(D_1 - D_2 - 4S_1 + 4S_2)^2}{100(S_1 - S_2)}$$
$$\frac{\partial \Pi_{Rs2}^{M2}}{\partial S_2} = \frac{1}{100} \left(-16 + \frac{(D_1 - D_2)^2}{(S_1 - S_2)^2} \right) < 0$$

Retailer 1 Profits in Rd2

$$\Pi_{Rd2}^{R1} = -\frac{(4D_1 - 4D_2 - S_1 + S_2)^2}{100(D_1 - D_2)}$$
$$\frac{\partial \Pi_{Rd2}^{R1}}{\partial D_1} = \frac{-16(D_1 - D_2)^2 + (S_1 - S_2)^2}{100(D_1 - D_2)^2} < 0$$

Retailer 2 Profits in Rd2

$$\Pi_{Rd2}^{R2} = -\frac{(6D_1 - 6D_2 + S_1 - S_2)^2}{100(D_1 - D_2)}$$
$$\frac{\partial \Pi_{Rd2}^{R2}}{\partial D_2} = -\frac{-36(D_1 - D_2)^2 + (S_1 - S_2)^2}{100(D_1 - D_2)^2} > 0$$

Manufacturer 1 Profits in Rd2

$$\Pi_{Rd2}^{M1} = -\frac{(4D_1-4D_2-S_1+S_2)^2}{100(D_1-D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{M1}}{\partial S_1} = \frac{4D_1-4D_2-S_1+S_2}{50D_1-50D_2} > 0$$

Manufacturer 2 Profits in Rd2

$$\Pi_{Rd2}^{M2} = -\frac{(6D_1-6D_2+S_1-S_2)^2}{100(D_1-D_2)}$$

$$\frac{\partial \Pi_{Rd2}^{M2}}{\partial D_2} = \frac{6D_1-6D_2+S_1-S_2}{50D_1-50D_2} > 0$$

Retailer 1 Profits in dRs2

$$\Pi_{dRs2}^{R1} = -\frac{(D_1-D_2-4S_1+4S_2)^2}{100(S_1-S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{R1}}{\partial D_1} = \frac{1}{50} \left(4 + \frac{D_1-D_2}{-S_1+S_2} \right) > 0$$

Retailer 2 Profits in dRs2

$$\Pi_{dRs2}^{R2} = -\frac{(D_1-D_2+6S_1-6S_2)^2}{100(S_1-S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{R2}}{\partial D_2} = \frac{1}{50} \left(6 + \frac{D_1-D_2}{S_1-S_2} \right) > 0$$

Manufacturer 1 Profits in dRs2

$$\Pi_{dRs2}^{M1} = -\frac{(D_1-D_2-4S_1+4S_2)^2}{100(S_1-S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{M1}}{\partial S_1} = \frac{1}{100} \left(-16 + \frac{(D_1-D_2)^2}{(S_1-S_2)^2} \right) < 0$$

Manufacturer 2 Profits in dRs2

$$\Pi_{dRs2}^{M2} = -\frac{(D_1-D_2+6S_1-6S_2)^2}{100(S_1-S_2)}$$

$$\frac{\partial \Pi_{dRs2}^{M2}}{\partial S_2} = \frac{1}{100} \left(36 - \frac{(D_1-D_2)^2}{(S_1-S_2)^2} \right) > 0$$

Retailer 1 Profits in dRs1 or dRd1

$$\Pi_{dRs1}^{R1} = \frac{\sqrt{(D_1-D_2)(S_1-S_2)}}{3\sqrt{6}}$$

$$\frac{\partial \Pi_{dRs1}^{R1}}{\partial D_1} = \frac{S_1-S_2}{6\sqrt{6}\sqrt{(D_1-D_2)(S_1-S_2)}} < 0$$

Retailer 2 Profits in dRs1 or dRd1

$$\Pi_{dRs1}^{R2} = \frac{2}{3} \sqrt{\frac{2}{3}} \sqrt{(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs1}^{R2}}{\partial D_2} = \frac{\sqrt{\frac{2}{3}}(-S_1 + S_2)}{3\sqrt{(D_1 - D_2)(S_1 - S_2)}} > 0$$

Manufacturer 1 Profits in dRs1 or dRd1

$$\Pi_{dRs1}^{M1} = \frac{\sqrt{(D_1 - D_2)(S_1 - S_2)}}{3\sqrt{6}}$$

$$\frac{\partial \Pi_{dRs1}^{M1}}{\partial S_1} = \frac{D_1 - D_2}{6\sqrt{6}\sqrt{(D_1 - D_2)(S_1 - S_2)}} < 0$$

Manufacturer 2 Profits in dRs1 or dRd1

$$\Pi_{dRs1}^{M2} = \frac{2}{3} \sqrt{\frac{2}{3}} \sqrt{(D_1 - D_2)(S_1 - S_2)}$$

$$\frac{\partial \Pi_{dRs1}^{M2}}{\partial S_2} = \frac{\sqrt{\frac{2}{3}}(-D_1 + D_2)}{3\sqrt{(D_1 - D_2)(S_1 - S_2)}} > 0$$

Retailer 1 Profits in dRd2

$$\Pi_{dRd2}^{R1} = -\frac{(4D_1 - 4D_2 - S_1 + S_2)^2}{100(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{R1}}{\partial D_1} = \frac{-16(D_1 - D_2)^2 + (S_1 - S_2)^2}{100(D_1 - D_2)^2} < 0$$

Retailer 2 Profits in dRd2

$$\Pi_{dRd2}^{R2} = -\frac{(6D_1 - 6D_2 + S_1 - S_2)^2}{100(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{R2}}{\partial D_2} = -\frac{-36(D_1 - D_2)^2 + (S_1 - S_2)^2}{100(D_1 - D_2)^2} > 0$$

Manufacturer 1 Profits in dRd2

$$\Pi_{dRd2}^{M1} = -\frac{(4D_1 - 4D_2 - S_1 + S_2)^2}{100(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{M1}}{\partial S_1} = \frac{4D_1 - 4D_2 - S_1 + S_2}{50D_1 - 50D_2} > 0$$

Manufacturer 2 Profits in dRd2

$$\Pi_{dRd2}^{M2} = -\frac{(6D_1 - 6D_2 + S_1 - S_2)^2}{100(D_1 - D_2)}$$

$$\frac{\partial \Pi_{dRd2}^{M2}}{\partial S_2} = \frac{6D_1 - 6D_2 + S_1 - S_2}{50D_1 - 50D_2} > 0$$

Appendix C

Proof of Proposition 2.1 and 2.2

Proposition 1: *If $\bar{S} - \underline{S} \leq \bar{D} - \underline{D}$, then $(S_1, D_1) = (\bar{S}, \underline{D})$ and $(S_2, D_2) = (\bar{S}, \bar{D})$ is a SPE of the game.*

Proposition 2: *If $\bar{S} - \underline{S} \geq \bar{D} - \underline{D}$, then $(S_1, D_1) = (\underline{S}, \bar{D})$ and $(S_2, D_2) = (\bar{S}, \bar{D})$ is a SPE of the game.*

Proof:

Let us assume that $(S_2, D_2) = (\bar{S}, \bar{D})$.

In *Rs2*, it must be true that $S_1 - S_2 \geq D_2 - D_1$. Thus, the only response for both firms is (\bar{S}, \bar{D}) . This results in zero profits.

In *Rd2*, since $D_2 - D_1 \geq S_1 - S_2 \geq 0$ then S_1 is constrained to take the value $S_1 = \bar{S}$. The best response for the retailer is to choose the lowest quality level. Then, their best responses are $(S_1, D_1) = (\bar{S}, \underline{D})$. This results in a profit of $\Pi_1^M = \frac{4}{25} (\bar{D} - \underline{D})$ and $\Pi_1^R = \frac{4}{25} (\bar{D} - \underline{D})$.

In $dRs1$, $S_2 - S_1 \geq D_2 - D_1$. From both firms' FOCs, their best responses are $(S_1, D_1) = (\underline{S}, \underline{D})$. However, for equilibrium prices to remain within this region, it must be true that $(S_2 - S_1) \leq \frac{3}{2}(D_2 - D_1)$. Under this condition, profits will be: $\Pi_1^M = \frac{\sqrt{(\overline{D}-\underline{D})(\overline{S}-\underline{S})}}{3\sqrt{6}}$ and $\Pi_1^R = \frac{\sqrt{(\overline{D}-\underline{D})(\overline{S}-\underline{S})}}{3\sqrt{6}}$. When $(S_2 - S_1) > \frac{3}{2}(D_2 - D_1)$, to stay in this region the manufacturer 1 and the retailer 1 must choose $D_1 = \overline{D} - \frac{2}{3}(\overline{S} - \underline{S})$ and $S_1 = \underline{S}$. This results in a profit of: $\Pi_1^M = \frac{(\overline{S}-\underline{S})}{9}$ and $\Pi_1^R = \frac{(\overline{S}-\underline{S})}{9}$.

In $dRs2$, $S_2 - S_1 \geq D_2 - D_1$. For the price equilibrium to be in $dRs2$ it must be true that $(S_2 - S_1) \geq \frac{3}{2}(D_2 - D_1) \geq 0$. From the location's FOCs we know that profits increases for the manufacturer if S_1 decreases and increases for the retailer if D_1 increases. Then, their best responses are $(S_1, D_1) = (\underline{S}, \overline{D})$ because the second constraint does not bind. This results in profits of $\Pi_1^M = \frac{4}{25}(\overline{S} - \underline{S})$ and $\Pi_1^R = \frac{4}{25}(\overline{S} - \underline{S})$.

In $dRd1$, from the FOCs we know that profit increases for each firm if they decrease the quality level to the maximum extent possible. However, to remain within this region, it must be true that $\frac{2}{3}(D_2 - D_1) \leq (S_2 - S_1)$. Complete differentiation will take place if $\frac{2}{3}(D_2 - D_1) \leq (S_2 - S_1)$. This results in a profit of $\Pi_1^M = \frac{\sqrt{(\overline{D}-\underline{D})(\overline{S}-\underline{S})}}{3\sqrt{6}}$ and $\Pi_1^R = \frac{\sqrt{(\overline{D}-\underline{D})(\overline{S}-\underline{S})}}{3\sqrt{6}}$. When $(\overline{S} - \underline{S}) < \frac{2}{3}(\overline{D} - \underline{D})$, to stay in this region the manufacturer 1 and the retailer 1 must choose $D_1 = \underline{D}$ and $S_1 = \overline{S} - \frac{2}{3}(\overline{D} - \underline{D})$. This results in a profit of: $\Pi_1^M = \frac{(\overline{D}-\underline{D})}{9}$ and $\Pi_1^R = \frac{(\overline{D}-\underline{D})}{9}$.

In $dRd2$, the constraint for staying in the region does not bind. Then each

firm's best response is $(S_1, D_1) = (\bar{S}, \underline{D})$ respectively. This results in profits of $\Pi_1^M = \frac{4}{25} (\bar{D} - \underline{D})$ and $\Pi_1^R = \frac{4}{25} (\bar{D} - \underline{D})$.

Retailer 1 and Manufacturer 1 will choose $(S_1, D_1) = (\bar{S}, \underline{D})$ whenever $\frac{4}{25} (\bar{D} - \underline{D}) \geq \frac{\sqrt{(\bar{D}-\underline{D})(\bar{S}-\underline{S})}}{3\sqrt{6}}$ and $\frac{4}{25} (\bar{D} - \underline{D}) \geq \frac{4}{25} (\bar{S} - \underline{S})$, and will choose $(S_1, D_1) = (\underline{S}, \bar{D})$ whenever $\frac{4}{25} (\bar{S} - \underline{S}) \geq \frac{\sqrt{(\bar{D}-\underline{D})(\bar{S}-\underline{S})}}{3\sqrt{6}}$ and $\frac{4}{25} (\bar{D} - \underline{D}) \leq \frac{4}{25} (\bar{S} - \underline{S})$. Since the second inequality implies the first in both cases, then Retailer 1 and Manufacturer 1 choose to differentiate incompletely always.

Let us assume that $(S_1, D_1) = (\bar{S}, \underline{D})$.

In *Rs2*, it must be true that $S_1 - S_2 \geq D_2 - D_1$. From the FOCs, profits increase when S_2 decreases and D_2 increases. Thus, the best response is $(S_2, D_2) = (\underline{S}, \bar{D})$ as long as $\bar{S} - \underline{S} \geq \bar{D} - \underline{D}$. If this condition is not true, the best response for Retailer 2, when manufacturer 2 chooses $S_2 = \underline{S}$, is to choose $D_2 = \underline{D} - (\underline{S} - \bar{S})$. The profits associated with these responses are maximized when the above condition holds with equality. This results in a profit of $\Pi_2^M = \frac{4}{25} (\bar{D} - \underline{D})$ and $\Pi_2^R = \frac{4}{25} (\bar{D} - \underline{D})$.

In *Rd2*, it must be true that $D_2 - D_1 \geq S_1 - S_2$. From the FOCs, profits increase when S_2 and D_2 increase. Thus, the best responses are $(S_2, D_2) = (\bar{S}, \bar{D})$. This results in profits of $\Pi_2^M = \frac{9}{25} (\bar{D} - \underline{D})$ and $\Pi_2^R = \frac{9}{25} (\bar{D} - \underline{D})$.

In *dRs1*, since it must be true that $S_2 - S_1 \geq D_2 - D_1$ and $S_2 - S_1 \leq \frac{3}{2} (D_2 - D_1)$ then $(S_2, D_2) = (\bar{S}, \underline{D})$ and profits are zero for all firms.

In *dRs2*, since it must be true that $S_2 - S_1 \geq D_2 - D_1$ and $S_2 - S_1 >$

$\frac{3}{2}(D_2 - D_1)$ then $(S_2, D_2) = (\bar{S}, \underline{D})$ and profits are zero for all firms.

In $dRd1$, it must be true that $D_2 - D_1 \geq S_2 - S_1$ and $\frac{2}{3}(D_2 - D_1) \leq (S_2 - S_1)$.

From the FOCs, profits increase when S_2 and D_2 increase; however to remain in this region the best response must be $(S_2, D_2) = (\underline{S}, \bar{D})$. This results in profits of zero for all firms (due to profit function form).

In $dRd2$, it must be true that $D_2 - D_1 \geq S_2 - S_1$ and $\frac{2}{3}(D_2 - D_1) > (S_2 - S_1)$.

From FOCs, profits increase whenever D_2 and S_2 increase. Thus, the best responses are $(S_2, D_2) = (\bar{S}, \bar{D})$. This results in profits of $\Pi_2^M = \frac{9}{25}(\bar{D} - \underline{D})$ and $\Pi_2^R = \frac{9}{25}(\bar{D} - \underline{D})$

Profits associated with $(S_2, D_2) = (\bar{S}, \bar{D})$ are always greater than the other options.

Let us assume that $(S_1, D_1) = (\underline{S}, \bar{D})$.

In $Rs2$, it must be true that $S_1 - S_2 \geq D_2 - D_1$. From the FOCs, profits increase when S_2 decreases and D_2 increases; however to stay in this region $(S_2, D_2) = (\underline{S}, \bar{D})$, which results in zero profits.

In $Rd2$, it must be true that $D_2 - D_1 \geq S_1 - S_2$. From the FOCs, profits increase when S_2 and D_2 increase; however to remain in the region the firms must locate at $(S_2, D_2) = (\underline{S}, \bar{D})$, which results in zero profits.

In $dRs1$, since it must be true that $S_2 - S_1 \geq D_2 - D_1$ and $S_2 - S_1 \leq \frac{3}{2}(D_2 - D_1)$ then $(S_2, D_2) = (\underline{S}, \bar{D})$ and profits are zero for all firms.

In $dRs2$, since it must be true that $S_2 - S_1 \geq D_2 - D_1$ and $S_2 - S_1 >$

$\frac{3}{2}(D_2 - D_1)$. From the FOCs, profits increase when S_2 and D_2 increase, then $(S_2, D_2) = (\bar{S}, \bar{D})$ and profits $\Pi_2^M = \frac{9}{25}(\bar{S} - \underline{S})$ and $\Pi_2^R = \frac{9}{25}(\bar{S} - \underline{S})$.

In $dRd1$, it must be true that $D_2 - D_1 \geq S_2 - S_1$ and $\frac{2}{3}(D_2 - D_1) \leq (S_2 - S_1)$. From the FOCs, profits increase when S_2 and D_2 increase; however to remain in this region the best response must be $(S_2, D_2) = (\bar{S}, \underline{D})$. This results in profits of zero for all firms (due to profit function form).

In $dRd2$, it must be true that $D_2 - D_1 \geq S_2 - S_1$ and $\frac{2}{3}(D_2 - D_1) > (S_2 - S_1)$. From FOCs, profits increase whenever D_2 and S_2 increase. However to remain in this region the best response must be $(S_2, D_2) = (\bar{S}, \underline{D})$. This results in zero profits for all firms (due to profit function form). Thus, the best responses are $(S_2, D_2) = (\bar{S}, \bar{D})$. This results in profits of $\Pi_2^M = \frac{9}{25}(\bar{D} - \underline{D})$ and $\Pi_2^R = \frac{9}{25}(\bar{D} - \underline{D})$.

Profits associated with $(S_2, D_2) = (\bar{S}, \bar{D})$ are always greater than the other options.

Appendix D

Proof of Proposition 3.1

Let

$$P_{jr} = w_j + x_r \quad j = 1, 2; r = 1, 2$$

Where w_j is the wholesale price charged by manufacturer j and x_r is the mark-up charged by retailer r . A consumer i who buys good 1 from retailer 1 must satisfy:

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_1 + \theta_i^D D_2 - P_{12} \quad (\text{D.1})$$

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_2 + \theta_i^D D_1 - P_{21} \quad (\text{D.2})$$

$$\theta_i^S S_1 + \theta_i^D D_1 - P_{11} \geq \theta_i^S S_2 + \theta_i^D D_2 - P_{22} \quad (\text{D.3})$$

Then, the taste parameter vector of consumers who buy good 1 from retailer 1 must satisfy:

$$\theta_i^D \leq \frac{x_2 - x_1}{D_2 - D_1}$$

$$\theta_i^S \leq \frac{w_2 - w_1}{S_2 - S_1}$$

$$\theta_i^D \leq \frac{(w_2 + x_2) - (w_1 + x_1) - \theta_i^D (D_2 - D_1)}{D_2 - D_1}$$

It is easy to show that when the two first inequalities hold, the third holds too. This is a result driven by the assumption stating that each retailer charges a uniform mark-up on each good. Then, we are allowed to derive the aggregate demand for final good (1, 1), AD_{11} , as:

$$AD_{11} = \left(\frac{x_2 - x_1}{D_2 - D_1} \right) \left(\frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.4})$$

$$AD_{21} = \left(\frac{x_2 - x_1}{D_2 - D_1} \right) \left(1 - \frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.5})$$

$$AD_{12} = \left(1 - \frac{x_2 - x_1}{D_2 - D_1} \right) \left(\frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.6})$$

$$AD_{11} = \left(1 - \frac{x_2 - x_1}{D_2 - D_1} \right) \left(1 - \frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.7})$$

Since the demand that retailer 1 faces is $(AD_{11} + AD_{21})$ and he charges the same mark-up on each manufacturer's good, his profit is;

$$\Pi_1^R = x_1 \left(\frac{x_2 - x_1}{D_2 - D_1} \right) \quad (\text{D.8})$$

Profit functions for the rest of the firms are derived in the same way:

$$\Pi_2^R = x_2 \left(1 - \frac{x_2 - x_1}{D_2 - D_1} \right) \quad (\text{D.9})$$

$$\Pi_1^M = w_1 \left(\frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.10})$$

$$\Pi_2^M = w_2 \left(1 - \frac{w_2 - w_1}{S_2 - S_1} \right) \quad (\text{D.11})$$

By inspection, we can see that retailer 1's profit do not depend on wholesale prices or good quality. In the second stage, each retailer and manufacturer reaction function is derived from its FOC. Then, we solve for the NE in prices given a quality level vector. The unique SPE outcome of this game is:

$$x_1^* = \frac{1}{3} (D_2 - D_1) \quad (\text{D.12})$$

$$x_2^* = \frac{2}{3} (D_2 - D_1) \quad (\text{D.13})$$

$$w_1^* = \frac{1}{3} (S_2 - S_1) \quad (\text{D.14})$$

$$w_2^* = \frac{2}{3} (S_2 - S_1) \quad (\text{D.15})$$

We update each firm's profit function and let each manufacturer and retailer maximize its profit by choosing the most profitable location. The FOCs are:

$$\frac{\partial \Pi_1^R}{\partial D_1} = -\frac{1}{9} < 0 \quad (\text{D.16})$$

$$\frac{\partial \Pi_2^R}{\partial D_2} = \frac{4}{9} < 0 \quad (\text{D.17})$$

$$\frac{\partial \Pi_1^M}{\partial S_1} = -\frac{1}{9} < 0 \quad (\text{D.18})$$

$$\frac{\partial \Pi_2^M}{\partial S_2} = \frac{4}{9} < 0 \quad (\text{D.19})$$

Then the SPE outcome is quality differentiation to the maximum extent possible.

Appendix E

Proof of Proposition 3.2

Let us assume, without loss of generality, that manufacturer 2 and retailer 2 are the high quality firms: $D_2 > D_1$, $S_2 > S_1$ and that $P_{22} - P_{21} > P_{12} - P_{11} > 0$ where P_{12} is the price charged on the good from manufacturer 1 and retailer 2.

Then, demand functions are:

$$AD_{11} = \left(\frac{(x_{12} + w_{12}) - (x_{11} + w_{11})}{D_2 - D_1} \right) \left(\frac{(x_{21} + w_{21}) - (x_{11} + w_{11})}{S_2 - S_1} \right) + \quad (\text{E.1})$$
$$- \frac{(x_{22} + w_{22} + x_{11} + w_{11} - x_{12} - w_{12} - x_{21} - w_{21})^2}{2(D_2 - D_1)(S_2 - S_1)}$$

$$AD_{21} = \left(\frac{(x_{22} + w_{22}) - (x_{21} + w_{21})}{D_2 - D_1} \right) \left(1 - \frac{(x_{21} + w_{21}) - (x_{11} + w_{11})}{S_2 - S_1} \right) \quad (\text{E.2})$$

$$AD_{12} = \left(1 - \frac{(x_{12} + w_{12}) - (x_{11} + w_{11})}{D_2 - D_1} \right) \left(\frac{(x_{22} + w_{22}) - (x_{12} + w_{12})}{S_2 - S_1} \right) \quad (\text{E.3})$$

$$\begin{aligned}
AD_{22} = & \left(1 - \frac{(x_{22} + w_{22}) - (x_{21} + w_{21})}{D_2 - D_1}\right) \left(1 - \frac{(x_{22} + w_{22}) - (x_{12} + w_{12})}{S_2 - S_1}\right) + \\
& - \frac{(x_{22} + w_{22} + x_{11} + w_{11} - x_{12} - w_{12} - x_{21} - w_{21})^2}{2(D_2 - D_1)(S_2 - S_1)}
\end{aligned} \tag{E.4}$$

Let us replace this demand functions into the profit functions of retailers and manufacturers. I prove that, at stage 2, each supplier does not have an incentive to deviate from charging the same wholesale price to both retailer and that each retailer does not have an incentive to deviate from charging the same mark-up on the goods from both manufacturers. Because the pricing game played is symmetric for manufacturers and retailers we only need to derive the best responses for the manufacturers or the retailers.

Profit functions for retailer 1 (retailer 2), given what retailer 2 (retailer 1), manufacturer 1 and manufacturer 2 charge, are:

$$\begin{aligned}
\Pi_1^R = & \frac{9(x_{11} - x_{21})^2(x_{11} + 2x_{21}) + 6(S_1 - S_2)(x_{11}^2 + 2x_{21}^2)}{18(D_1 - D_2)(S_1 - S_2)} + \\
& + \frac{4(D_1 - D_2)(3(x_{11} - x_{21})^2 + (S_1 - S_2)(x_{11} + 2x_{21}))}{18(D_1 - D_2)(S_1 - S_2)}
\end{aligned} \tag{E.5}$$

$$\begin{aligned}
\Pi_2^R = & \frac{9(x_{12} - x_{22})^2(2x_{12} + x_{22}) + 6(S_1 - S_2)(x_{12}^2 + 2x_{22}^2)}{18(D_1 - D_2)(S_1 - S_2)} + \\
& + \frac{8(D_1 - D_2)(3(x_{12} - x_{22})^2 + (S_1 - S_2)(x_{12} + 2x_{22}))}{18(D_1 - D_2)(S_1 - S_2)}
\end{aligned} \tag{E.6}$$

From the FOCs with respect to its own mark-ups, we solve for the best response function under the assumption that the strategy of the other players is to charge uniform mark-ups and wholesale prices. For retailer 1, the critical point vectors are:

$$x_{11} = x_{21} = \frac{1}{3}(D_2 - D_1) \quad (\text{E.7})$$

$$x_{11} = \frac{4}{27} \left(3(D_2 - D_1) + (S_2 - S_1) + \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right) \quad (\text{E.8})$$

$$x_{21} = -\frac{2}{27} \left(6(D_1 - D_2) + 2(S_1 - S_2) + \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right)$$

$$x_{11} = \frac{4}{27} \left(3(D_2 - D_1) + (S_2 - S_1) - \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right) \quad (\text{E.9})$$

$$x_{21} = -\frac{2}{27} \left(6(D_1 - D_2) + 2(S_1 - S_2) - \sqrt{S_1 - S_2} \sqrt{-3(D_2 - D_1) - 4(S_2 - S_1)} \right)$$

For retailer 2, the critical point vectors are:

$$x_{12} = x_{22} = \frac{2}{3}(D_2 - D_1) \quad (\text{E.10})$$

$$x_{12} = -\frac{1}{54} \left(48(D_1 - D_2) + 5(S_1 - S_2) + \sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right) \quad (\text{E.11})$$

$$x_{22} = -\frac{1}{54} \left(48(D_1 - D_2) + 23(S_1 - S_2) - 5\sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right)$$

$$x_{12} = -\frac{1}{54} \left(48(D_1 - D_2) + 5(S_1 - S_2) - \sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right) \quad (\text{E.12})$$

$$x_{22} = -\frac{1}{54} \left(48(D_1 - D_2) + 23(S_1 - S_2) + 5\sqrt{S_1 - S_2} \sqrt{24(D_1 - D_2) + 25(S_1 - S_2)} \right)$$

The second order conditions for a local maximum only hold for (36) and (39).

Then, for each retailer, charging a uniform additive mark-up on both manufacturers' goods is Sub-Game Perfect Equilibrium. By symmetry, the same happens to the manufacturers.

Retailer 1 second order derivatives for the first critical vector are:

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} = \frac{2}{3(D_1 - D_2)} + \frac{1}{3(S_1 - S_2)} < 0 \quad (\text{E.13})$$

$$\frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} = \frac{4}{3(D_1 - D_2)} + \frac{1}{3(S_1 - S_2)} < 0 \quad (\text{E.14})$$

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} \frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} - \left(\frac{\partial^2 \Pi_1^R}{\partial x_{11} \partial x_{21}} \right) = \frac{2 \left(4 + \frac{3(D_1 - D_2)}{S_1 - S_2} \right)}{9(D_1 - D_2)^2} > 0 \quad (\text{E.15})$$

Retailer 1 second order derivatives for the second critical vector are:

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} = \frac{2 + \frac{4\sqrt{3(D_1-D_2)+4(S_1-S_2)}}{\sqrt{S_1-S_2}}}{9(D_1-D_2)} < 0 \quad (\text{E.16})$$

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} \frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} - \left(\frac{\partial^2 \Pi_1^R}{\partial x_{11} \partial x_{21}} \right) = \frac{4 \left(-4 + \frac{3(D_1-D_2)}{S_2-S_1} \right)}{9(D_1-D_2)^2} < 0 \quad (\text{E.17})$$

Retailer 1 second order derivatives for the third critical vector are:

$$\frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} = \frac{8 + \frac{8\sqrt{3(D_1-D_2)+4(S_1-S_2)}}{\sqrt{S_1-S_2}}}{9(D_1-D_2)} < 0 \quad (\text{E.18})$$

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} \frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} - \left(\frac{\partial^2 \Pi_1^R}{\partial x_{11} \partial x_{21}} \right) = \frac{4 \left(-4 + \frac{3(D_1-D_2)}{S_2-S_1} \right)}{9(D_1-D_2)^2} < 0 \quad (\text{E.19})$$

Retailer 2 second order derivatives for the first critical vector are:

$$\frac{\partial^2 \Pi_2^R}{\partial x_{12}^2} = \frac{2}{3} \left(\frac{1}{(D_1-D_2)} + \frac{1}{(S_1-S_2)} \right) < 0 \quad (\text{E.20})$$

$$\frac{\partial^2 \Pi_2^R}{\partial x_{22}^2} = \frac{4}{3(D_1-D_2)} + \frac{2}{3(S_1-S_2)} < 0 \quad (\text{E.21})$$

$$\frac{\partial^2 \Pi_2^R}{\partial x_{12}^2} \frac{\partial^2 \Pi_2^R}{\partial x_{22}^2} - \left(\frac{\partial^2 \Pi_2^R}{\partial x_{11} \partial x_{21}} \right) = \frac{4 \left(2 + \frac{3(D_1-D_2)}{S_1-S_2} \right)}{9(D_1-D_2)^2} > 0 \quad (\text{E.22})$$

Retailer 2 second order derivatives for the second critical vector are:

$$\frac{\partial^2 \Pi_1^R}{\partial x_{22}^2} = \frac{1 + \frac{5\sqrt{24(D_1-D_2)+25(S_1-S_2)}}{\sqrt{S_1-S_2}}}{18(D_1-D_2)} < 0 \quad (\text{E.23})$$

$$\frac{\partial^2 \Pi_2^R}{\partial x_{12}^2} \frac{\partial^2 \Pi_2^R}{\partial x_{22}^2} - \left(\frac{\partial^2 \Pi_1^R}{\partial x_{11} \partial x_{21}} \right) = \frac{\sqrt{S_1 - S_2} (24(D_1 - D_2) + 25(S_1 - S_2))}{9(D_1 - D_2)^2 (S_1 - S_2)^{\frac{3}{2}}} +$$

(E.24)

$$+ \frac{3\sqrt{24(D_1 - D_2) + 25(S_1 - S_2)}}{9(D_1 - D_2)^2 (S_1 - S_2)^{\frac{3}{2}}} < 0$$

Retailer 2 second order derivatives for the third critical vector are:

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} = \frac{25 + \frac{7\sqrt{24(D_1 - D_2) + 25(S_1 - S_2)}}{\sqrt{S_1 - S_2}}}{18(D_1 - D_2)} < 0$$

(E.25)

$$\frac{\partial^2 \Pi_1^R}{\partial x_{11}^2} \frac{\partial^2 \Pi_1^R}{\partial x_{21}^2} - \left(\frac{\partial^2 \Pi_1^R}{\partial x_{11} \partial x_{21}} \right) = \frac{-3\sqrt{24(D_1 - D_2) + 25(S_1 - S_2)}\sqrt{S_1 - S_2}}{9(D_1 - D_2)^2 (S_1 - S_2)^{\frac{3}{2}}} +$$

(E.26)

$$+ \frac{-24(D_1 - D_2) - 25(S_1 - S_2)}{9(D_1 - D_2)^2 (S_1 - S_2)} < 0$$

Appendix F

Model 4

Table F.1: Estimation Results

Variables	Model 4
<i>First Stage Brand Intercepts</i>	
F Natural	-2.94**
M Maid	-1.69**
Private Label	-0.95**
Tropicana	0.09
<i>Second Stage Brand Intercepts</i>	
F Natural	-0.16**
M Maid	-0.35**
Private Label	-0.04
Tropicana	-
<i>Marketing and Other Variables</i>	
Price	-0.24**
Feat	0.86**
Price*Feat	-0.20**
Price*Size	-0.05**
Price*Fful	-0.14**
Price*Income	0.01**
<i>Choice Set Variables</i>	
Set(3,4)*Feat1	0.14
Set(3,4)*Feat2	2.26 *
Set(3,4)*Feat3	5.39
Set(3,4)*Size	0.49**
Set(3,4)*Fful	0.17
Set(3,4)*Income	-0.35**
- Log Likelihood	10876.4

BIBLIOGRAPHY

- [1] D.A. Akerberg. Empirically distinguishing informative and prestige effects of advertising. *RAND Journal of Economics*, 32(2):216–333, 2001.
- [2] G.M. Allenby and J.L. Ginter. The effects of in-store displays and feature advertising on consideration sets. *Journal of Research in Marketing*, 12:67–80, 1995.
- [3] R.L. Andrews and T.C. Srinivasan. Studying consideration effects in empirical choice models using scanner panel data. *Journal of Marketing Research*, 32:30–41, 1995.
- [4] A. Ansari, N. Economides, and A. Ghosh. Positioning in markets with nonuniform preferences. *Marketing Science*, 13(3):248–273, 1994.
- [5] G.S. Becker and K.M. Murphy. A simple theory of advertising as a good or bad. *Quarterly Journal of Economics*, 108(4):942–964, 1993.
- [6] R.R. Betancourt. *The Economics of Retailing and Distribution*. Edward Kegan Publishing Co., 2004.

- [7] R.R. Betancourt and C.K. Clague. *Capital Utilization: A Theoretical and Empirical Analysis*. Cambridge University Press, 1981.
- [8] R.R. Betancourt and D. Gautschi. The economics of retail firms. *Managerial and Decision Economics*, 9(2):133–144, 1988.
- [9] T.F. Bresnahan and P.C. Reiss. Dealer and manufacturer margins. *RAND Journal of Economics*, 16(2):253–268, 1985.
- [10] G.J. Bronnenberg and W.R. Vanhonacker. Limited choice sets, local price response and implied measures of price competition. *Journal of Marketing Research*, 33:163–173, 1996.
- [11] J.A. Carlson and R.J. Gieseke. Price search in a product market. *Journal of Consumer Research*, 9(4):357–365, 1983.
- [12] UK Competition Commission. Supermarkets: A report on the supply of groceries from multiple stores in the united kingdom. Technical report, UK Competition Commission, 2000.
- [13] A. Coughlan. Competition and cooperation in marketing channel choice: Theory and application. *Marketing Science*, 4(2):110–129, 1985.
- [14] C. D’aspremont, J. Gabszewicz, and J.F. Thisse. On hotelling’s ‘stability in competition’. *Econometrica*, 47(5):1145–1150, 1979.

- [15] A. dePalma, V. Guinsburgh, Y.Y. Papageorgiou, and J.F. Thisse. The principle of minimum differentiation holds under sufficient heterogeneity. *Econometrica*, 53(4):767–781, 1985.
- [16] P.R. Dickson and A.G. Sawyer. The price knowledge and search of supermarket shoppers. *Journal of Marketing*, 54:42–53, 1990.
- [17] J.K. Galbraith. *The Affluent Society*. Houghton-Mifflin, 4th edition, 1976.
- [18] W.H. Greene. *Econometric Analysis*. Prentice Hall, 4th edition, 2000.
- [19] G. Grossman and C. Shapiro. Informative advertising with differentiated products. *Review of Economic Studies*, 51(1):63–81, 1984.
- [20] P.M. Guadagni and J.D.C. Little. A logit model of brand choice. *Marketing Science*, 2:203–238, 1983.
- [21] J.R. Hauser. Competitive price and positioning strategies. *Marketing Science*, 7(1):76–91, 1988.
- [22] H. Hotelling. Stability in competition. *The Economic Journal*, 39:41–57, 1929.
- [23] D.C. Jain, N.J. Vilcassim, and P.K. Chintagunta. A random-coefficients logit brand-choice model applied to panel data. *Journal of Business and Economic Statistics*, 12(3):317–328, 1994.

- [24] K. Kalyanam and D.S. Putler. Incorporating demographic variables in brand choice models: An indivisible alternatives framework. *Marketing Science*, 16(2):166–181, 1997.
- [25] R. Kihlstrom and M. Riordan. Advertising as a signal. *Journal of Political Economy*, 92(3):427–450, 1984.
- [26] R. Manning and P.B. Morgan. Search and consumer theory. *The Review of Economic Studies*, 49(2):203–216, 1982.
- [27] C.F. Manski and S.R. Lerman. The estimation of choice probabilities from choice based samples. *Econometrica*, 45(8):1977–1988, 1977.
- [28] T.W. McGuire and R. Staelin. An industry equilibrium analysis of downstream vertical integration. *Marketing Science*, 2(2):161–191, 1983.
- [29] P. Milgrom and J. Roberts. Price and advertising signals of product quality. *Journal of Political Economy*, 94(4):796–821, 1986.
- [30] E.M. Modjuszka, J.A. Caswell, and J.M. Harris. Consumer choice of food products and the implications for price competition and government policy. *Agribusiness*, 17(1):81–104, 2001.
- [31] B.P.S. Murthi and K. Srinivasan. Consumers’ extent of evaluation in brand choice. *Journal of Business*, 72(2):229–256, 1999.

- [32] P. Nelson. Advertising as information. *Journal of Political Economy*, 82(4):729–753, 1974.
- [33] D. Neven and J.F. Thisse. *Economic Decision Making: Games, Econometrics and Optimization*, chapter On Quality and Variety Competition, pages 175–199. North Holland, 1990.
- [34] B. Ratchford. Cost benefit models for explaining consumer choice and information seeking behavior. *Management Science*, 28:197–212, 1982.
- [35] M. Rotschild. Searching for the lowest price when the distribution of prices is unknown. *Journal of Political Economy*, 82:689–711, 1974.
- [36] A. Shaked and J. Sutton. Relaxing price competition through product differentiation. *Review of Economic Studies*, 49(1):3–13, 1982.
- [37] S. Shugan. The cost of thinking. *Journal of Consumer Research*, 7:99–111, 1980.
- [38] S. Siddarth, R.E. Bucklin, and D.G. Morrison. Making the cut: Modeling and analyzing choice set restriction in scanne panel data. *Journal of Marketing Research*, 32:255–266, 1995.
- [39] R.L. Smith. Franchise resgualtion: An economic analysis of state restrictions on automobile distribution. *Journal of Law and Economics*, 25:125–157, 1982.

- [40] G.J. Stigler. The economics of information. *Journal of Political Economy*, 69:213–225, 1961.
- [41] G.J. Stigler and G.S. Becker. De gustibus non est disputandum. *American Economic Review*, 67:76–90, 1977.
- [42] J. Swait. Choice set generation within the generalized extreme value family of discrete choice models. *Transportation Research Part B*, 35:643–666, 2001.
- [43] J. Tirole. *The Theory of Industrial Organization*. MIT Press, 1988.
- [44] M.B. Vandenbosch and C.B. Weinberg. Product and price competition in a two-dimensional vertical differentiation model. *Marketing Science*, 14(2):224–249, 1995.
- [45] M.L. Weitzman. Optimal search for the best alternative. *Econometrica*, 47(3):641–654, 1979.